

Class-XII

MHT-CET

Study Material

MATHEMATICS

Booklet-1

(Solutions)

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Mathematical Logic

Exercise 1 :

WARM-UP Topic-wise MCQs

- (d) Mumbai is the capital of India is a statement.
- (c) "The sun is a star" is a statement.
- (d) "Alas! I have failed" is not a statement.
- (a) "New Delhi is in India" is true. So, it is a statement.
- (d) "Everyone in the room is bold" is not a statement because it is not clear which room is referred here and the word bold is not clearly defined.
- (d) "Mathematics is fun" is not always true. So it is not a statement.
- (c) This sentence is not a statement because it is a question.
- (a) "x is a natural number" is an open statement.
- (d) $x + 3 = 10, x \in \mathbb{R}$ is not a statement.
- (a) "Please do me a favour" is not a statement.
- (d) 'Two plus two is four', is a statement.
- (d) Only declarative sentences are statements.
- (a) We know that a month has 30 or 31 days. It is false to say that a month has 35 days. Hence, it is a statement.
- (d) (a) Everyone in this room is bold. This is not a statement because from the context, it is not clear which room is referred here and the term 'bold' is not clearly defined.
(b) She is an engineering student. This is also not a statement because it is not clear, who she is.
(c) $\sin^2\theta$ is greater than $1/2$. This is not a statement because we cannot say whether the sentence is true or not.
(d) We know that, $3 + 3 = 6$. It is true. Hence, the sentence is a statement.
- (c) "Where are you going?" is not a statement.
- (a) "Paris is in England" is a statement.
- (c) Mathematics is interesting is not a logical sentence. It may be interesting for some persons are may not be interesting for others.
 \therefore This is not a propositions.
- (d) x is variable upon which the truth value depends.
- (c) Roots will be ω and ω^2 , which are imaginary.
- (a) (a) It is false. (b) It is true.
(c) It is true.
- (b) Let p and q the statements such that $\sqrt{5}$ an integer $q = 5$ is an irrational number.

Then, negation of the given statement, $\sqrt{5}$

is not an integer and 5 is not an irrational number

$$\sim (p \vee q) = \sim p \wedge \sim q$$

22. (b) p : It rains, q : I shall go to school

Thus, we have $p \Rightarrow q$

Its negation is $\sim (p \Rightarrow q)$ i.e. $p \wedge \sim q$

i.e. It rains and I shall not go to school.

23. (a) S: "If you are born in India, then you are a citizen of India."

Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

So contrapositive of statement S will be :

"If you are not a citizen of India, then you are not born in India."

24. (b) p : A number is a prime

Q : It is odd.

We have $p \Rightarrow q$

The inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$

i.e., if a number is not a prime then it is not odd.

25. (b) It is obvious.

26. (c) Required conditional has positive antecedent and negative consequent.

27. (c) It is obvious.

28. (a) Correct result is $(\sim p \vee \sim q) \Rightarrow (r \wedge s)$

So, $\sim (p \wedge q) \Rightarrow (r \wedge s)$

29. (d) $\sim s \vee (\sim r \wedge s) \equiv (\sim s \vee \sim r) \wedge (\sim s \vee s)$

$$\equiv (\sim s \vee \sim r) \quad (\because \sim s \vee s \text{ is tautology})$$

$$\equiv \sim (s \wedge r)$$

30. (c) $p \Rightarrow q \equiv \sim p \vee q \therefore \sim (p \Rightarrow q) \equiv p \wedge \sim q$

31. (b) Inverse $\equiv \sim (p \wedge \sim q) \rightarrow \sim r \equiv (\sim p \vee q) \rightarrow \sim r$

32. (c) $\sim [(p \vee q) \wedge (q \vee \sim r)] \equiv \sim (p \vee q) \vee \sim (q \vee \sim r)$
 $\equiv (\sim p \wedge \sim q) \vee (\sim q \wedge r)$

33. (d)
- | p | q | $\sim p$ | $p \wedge q$ | $q \vee \sim p$ | $(p \wedge q) \rightarrow (q \vee \sim p)$ |
|---|---|----------|--------------|-----------------|--|
| T | T | F | T | T | T |
| T | F | F | F | F | T |
| F | T | T | F | T | T |
| F | F | T | F | T | T |

34. (b) Required proposition is a negation of conjunction.

35. (d) It is obvious.

36. (d) (a) $p \vee (q \vee r) \equiv T \vee (T \vee F) \equiv T \vee T \equiv T$

$$(b) (p \wedge (q \wedge r)) \vee \sim ((p \vee q) \wedge (r \vee s))$$

$$\equiv (T \wedge (T \wedge F)) \vee \sim (T \wedge F) \equiv (T \wedge F) \vee T \equiv T$$

$$(c) (\sim (p \wedge q) \vee \sim r) \equiv \sim (T \wedge F) \vee \sim F$$

$$\equiv T \vee T \equiv T. \text{ Hence its dejunction too is T.}$$

37. (a) $\sim p \vee q$ means $F \vee F = F$, $\sim r$ means $F(\sim p \vee q) \wedge \sim r$ means F
 $\therefore [(\sim p \vee q) \wedge \sim r] \Rightarrow p$ means **T**
 [\therefore in $p \Rightarrow q$ we have FTT]
38. (b) Since $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$ and $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$
 So option (a) and (d) are not true.
 $(p \rightarrow q) \equiv p \wedge \sim q$, so option (c) is not true.
 Now $p \rightarrow q \sim p \vee q$
 $\sim q \rightarrow \sim p \equiv [\sim(\sim q) \vee \sim p] \equiv q \vee \sim p \equiv \sim p \vee q$
 $p \rightarrow q \equiv \sim q \rightarrow \sim p$

39. (c)

p	q	$\sim q$	$p \wedge \sim q$	$p \rightarrow (p \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

40. (b) $(\sim p \rightarrow q) \vee r \equiv (T \rightarrow F) \vee F \equiv F \vee F \equiv F$

41. (a)

p	q	$\sim p$	$\sim q$	$\sim(\sim p \vee \sim q)$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	F

42. (b) $(p \wedge q) \wedge (q \wedge r)$ is true means $p \wedge q \wedge r$ are both true. $\Rightarrow p, q, r$ are all true.

43. (c) $\sim(p \Rightarrow q) \equiv p \wedge \sim q$
 $\therefore \sim(\sim p \Rightarrow \sim q) \equiv \sim p \wedge \sim(\sim q) \equiv \sim p \wedge q$

Thus $\sim(\sim p \Rightarrow \sim q) \equiv \sim p \wedge q$

44. (c) "q only if p" is not equivalent " $p \rightarrow q$ ".

45. (c)

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$	$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

46. (a) $\sim[(\sim p \wedge q) \vee (p \wedge \sim q)] \equiv (p \vee \sim q) \wedge (\sim p \vee q)$

47. (d) I and II are converse of each other.

48. (a) Let $p : x, y \in Z$ such that x and y are odd.
 $q : xy$ is odd.

To check the validity of the given statement, assume that if p is true, then q is true.

p is true means that x and y are odd integers. Then, $x = 2m + 1$, for some integer m.

$y = 2n + 1$, for some integer n.

Thus, $xy = (2m + 1)(2n + 1) = 2(2mn + m + n) + 1$

This shows that xy is odd. Therefore, the given statement is true.

Also, if we assume that q is not true. This implies that we need to consider the negation of the statement q. This gives the statement.

$\sim q : \text{product } xy \text{ is even.}$

This is possible only, if either x or y is even. This shows that p is not true. Thus, we have shown that

$\sim q \Rightarrow \sim p$

Note: The above problem illustrates that to prove $p \Rightarrow q$. It is enough to show $\sim q \Rightarrow \sim p$ which is the contrapositive of the statement $p \Rightarrow q$.

49. (d) It is obvious.
50. (a) It is obvious.
51. (b) Since, the statement $\sim q \rightarrow \sim p$ is contrapositive of the statement $p \rightarrow q$.
52. (c) Let $p : \text{Ram secures 100 marks in maths}$
 $q : \text{Ram will get a mobile}$
 Converse of $p \rightarrow q$ is $q \rightarrow p$
 i.e. If Ram will get a mobile, then he secures 100 marks in maths.
53. (c) $\sim[(p \wedge q) \rightarrow (\sim p \vee r)] \equiv (p \wedge q) \vee [\sim(\sim p \vee r)] \equiv (p \wedge q) \wedge (p \wedge \sim r)$
54. (a) Given result means $p \wedge \sim r$ is true, $q \vee r$ is false.
55. (b) $\sim(p \vee (\sim q)) \equiv \sim p \wedge \sim(\sim q) \equiv (\sim p) \wedge q$

56. (c)

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$	Required
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

57. (a) $\sim[p \vee (\sim p \wedge q)] \equiv \sim p \wedge \sim(\sim p \vee q)$
 $\equiv \sim p \wedge (\sim(\sim p) \wedge \sim q)$
 $\equiv \sim p \wedge (p \wedge \sim q)$

58. (c) It is a property

59. (c) **Statement:** $\Rightarrow 2 + 3 = 5$ and $8 < 10$

Negation: $2 + 3 \neq 5$ and $8 \not< 10$

60. (a) The contrapositive statement is "If Chandigarh is not in India, then Chandigarh is not the capital of Punjab".

61. (a) $p \rightarrow p \vee q$ is a tautology.

62. (a)

p	q	$p \wedge q$	$p \vee q$	$\sim(p \vee q)$	$(p \wedge q) \wedge \sim(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

$\therefore (p \wedge q) \wedge (\sim(p \vee q))$ is a contradiction.

63. (d) $(q \rightarrow p) \rightarrow p$ is not a tautology.

64. (a) $(p \wedge \sim q) \wedge (\sim p \wedge q) = f$

65. (a) Only (i) correct statement.

66. (a)

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Clearly, $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction.

67. (c)

p	$\sim p$	$p \Rightarrow \sim p$	$\sim p \Rightarrow p$	$(p \Rightarrow \sim p) \wedge (\sim p \Rightarrow p)$
T	F	F	T	F
F	T	T	F	F

Clearly, $(p \Rightarrow \sim p) \wedge (\sim p \Rightarrow p)$ is a contradiction.

68. (b) $p \Rightarrow q$ is logically equivalent to $\sim q \Rightarrow \sim p$
 $\therefore (p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a tautology but not a contradiction.

69. (d) $(\sim p \wedge q) \rightarrow (p \vee q)$
 $\Rightarrow \sim \{(\sim p \wedge q) \wedge (\sim p \wedge \sim q)\} \equiv (p \vee \sim q) \vee (p \vee q)$

70. (b) $(p \vee q) \vee (p \vee \sim q) = p \vee (q \vee \sim q) \vee \sim q$
 $= (p \vee p) \vee (q \vee \sim q) = p \vee T = T$
 Hence, second statement is tautology.

71. (a)

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$	$q \rightarrow p \wedge (p \rightarrow q)$	$p \wedge q$	$p \vee (p \wedge q)$	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	T	T	T
F	T	T	F	T	F	F	F	T	F
F	F	T	F	T	T	F	F	F	F

72. (b) q is false and $[(p \wedge q) \leftrightarrow r]$ is true
 As $(p \wedge q)$ is false
 $[\text{False} \leftrightarrow r]$ is true
 Hence r is false
 Option (a): says $p \vee r$,
 Since r is false
 Hence $(p \vee r)$ can either be true or false
 Option (b): says $(p \wedge r) \rightarrow (p \vee r)$
 $(p \wedge r)$ is false
 Since, $F \rightarrow T$ is true and
 $F \rightarrow F$ is also true

$$\sim [(\sim(p \rightarrow (\sim q))) \vee (\sim((\sim q) \rightarrow p))]$$

Here, $\sim(p \rightarrow q) \equiv (p \wedge \sim q)$

So, $[(p \wedge q) \vee (\sim q \wedge \sim p)] \wedge q$
 $\Rightarrow p \wedge q$

76. (c) (a) $p \vee (q \rightarrow p) \equiv p \vee (\sim q \vee p) \equiv p \vee \sim q$ (Not T)
 (b) $\sim q \rightarrow \sim p \equiv p \rightarrow q$ (Not T)
 (c) $(\sim q \vee p) \vee (\sim p \wedge q) \vee (p \wedge q) = T$
 (d) F

73. (a) By truth table :

p	q	$\sim q$	$p \vee \sim q$	$\sim p$	$p \wedge \sim q$	$p \vee q$	$p \rightarrow p \vee q$	$p \wedge q$	$(p \wedge q) \rightarrow p$	$\sim p \vee q$
T	T	F	T	F	F	T	T	T	T	T
T	F	T	T	F	T	T	T	F	T	F
F	T	F	F	T	F	T	T	F	T	T
F	F	T	T	T	F	F	T	F	T	T

$(p \wedge q) \rightarrow (\sim p) \vee q$	$(p \vee q) \rightarrow (p \vee (\sim q))$
T	T
T	T
T	F
T	T

74. (a) $\therefore p \vee (\sim p) = t$

75. (d) Given statement $(\sim(p \Leftrightarrow q)) \wedge q$

$$\sim((p \rightarrow (\sim q)) \wedge ((\sim q) \rightarrow p)) \wedge q$$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee q$	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	F	T

$p \Rightarrow (p \vee q)$ is also a tautology.

77. (c) We consider following truth table.

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \vee q$	$(\sim(p \vee q))$	$(p \wedge q) \wedge (\sim(p \vee q))$
T	T	F	F	T	T	F	F
T	F	F	T	F	T	F	F
F	T	T	F	F	T	F	F
F	F	T	T	F	F	T	F

Clearly last column of the above truth table contains only F. Hence, $(p \wedge q) \wedge (\sim(p \vee q))$ is a contradiction.

78. (d) The truth value of $\sim(\sim p) \leftrightarrow p$ as follow

p	$\sim p$	$\sim(\sim p)$	$\sim(\sim p) \rightarrow p$	$p \rightarrow \sim(\sim p)$	$\sim(\sim p) \leftrightarrow p$
T	F	T	T	T	T
F	T	F	T	T	T

Since last column of above truth table contains only T.
Hence $\sim(\sim p) \leftrightarrow p$ is a tautology.

79. (a) $\sim(p \rightarrow q) \rightarrow [(\sim p) \vee (\sim q)] \equiv (p \wedge \sim q) \rightarrow \sim(p \wedge q)$

80. (a) $(p \rightarrow q) \vee ((\sim p) \wedge q)$

p	q	$\sim p$	$p \rightarrow q$	$\sim p \wedge q$	$(p \rightarrow q) \vee (\sim p) \wedge q$
T	T	F	T	F	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	F	T

Not a tautology

p	q	$\sim p$	$q \rightarrow p$	$(\sim p) \wedge q$	$(q \rightarrow p) \rightarrow (\sim p) \wedge q$
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	F	F

Not a contradiction

81. (b) Given (i) $p \rightarrow (p \wedge (p \rightarrow q))$

$$(\sim p) \vee (p \wedge (\sim p \vee q))$$

$$(\sim p) \vee (p \wedge q)$$

$$\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q) = \sim p \vee q$$

(ii) $(p \wedge q) \rightarrow (\sim p \rightarrow q)$

$$\sim(p \wedge q) \vee (p \vee q) = t$$

$$\{a, b, d\} \vee \{a, b, c\} = \vee$$

Tautology

(iii) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$

$$\sim(p \wedge (\sim p \vee q)) \vee \sim q = \sim(p \wedge q) \vee \sim q = \sim p \vee \sim q$$

Not tautology

(iv) $p \vee (p \wedge q) = p$

Not tautology.

So, option (b) is correct.

82. (d)

p	q	$p \Rightarrow q$	$\sim q$	$p \wedge \sim q$	$(p \Rightarrow q) \vee (p \wedge \sim q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

$\sim p$	$\sim q$	$\sim p \Rightarrow \sim q$	$\sim p \vee q$	$((\sim p) \Rightarrow (\sim q)) \wedge (\sim p \vee q)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

83. (b) Given expression is

$$((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$$

We know, $p \Rightarrow q$ is equivalent to $\sim p \vee q$

$$(\sim(p \wedge q) \vee (r \vee q)) \wedge (\sim(p \wedge r) \vee q)$$

$$\equiv (\sim(p \vee \sim q \vee r \vee q)) \wedge (\sim p \vee \sim r \vee q)$$

$$\equiv (\sim p \vee r \vee t) \wedge (\sim p \vee \sim r \vee q)$$

$$\equiv (t) \wedge (\sim p \vee \sim r \vee q)$$

$$\equiv \sim p \vee \sim r \vee q$$

For this to be tautology, $(\sim p \vee \sim r \vee q)$ must be always true which follow for $r = \sim p$ or $r = q$.

84. (b) $S_1 \equiv ((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

p	q	r	$p \vee q$	$(p \vee q) \Rightarrow r$	$p \Rightarrow r$	$((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	T

$$S_2 \equiv (p \vee q) \Rightarrow r \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow e))$$

p	q	r	$(p \vee q) \Rightarrow r$	$p \Rightarrow r$	$q \Rightarrow r$	$(p \Rightarrow r) \vee (q \Rightarrow r)$	S_2
T	T	T	T	T	T	T	T
T	T	F	F	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	F	F	T	T	F
F	T	T	T	T	T	T	T
F	T	F	F	F	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$S_2 \rightarrow$ not a tautology

85. (b) Given, $(p \wedge \sim q) \rightarrow (p \rightarrow \sim q)$

$$\equiv (\sim(p \wedge \sim q)) \vee (\sim p \vee \sim q)$$

$$\equiv (\sim p \vee q) \vee (\sim p \vee \sim q)$$

$$\equiv \sim p \vee t = t$$

86. (c) By definition

87. (a) By definition

88. (d) By definition.

89. (c) By definition.

90. (b) By definition.

91. (b) By definition.
 92. (a) **Case-I** : If $\Delta \equiv \nabla \equiv \vee$
 $(p \vee q) \rightarrow ((p \vee q) \vee r) \equiv$ tautology

then $(p \vee q) \vee r \equiv (p \Delta r) \vee q$

Case-II : If $\Delta \equiv \nabla \equiv \wedge$

$(p \wedge q) \rightarrow ((p \wedge q) \wedge r)$

It will be false if r is false.

So not a tautology

Case-III : If $\Delta \equiv \vee, \nabla \equiv \wedge$

then $(p \wedge q) \rightarrow \{(p \vee q) \wedge r\}$

Not a tautology

(Check $p \rightarrow T, q \rightarrow T, r \rightarrow F$)

Case-IV : If $\Delta \equiv \wedge, \nabla \equiv \vee$

$(p \wedge q) \rightarrow \{(p \wedge q) \vee r\}$

Not a tautology

93. (c) By options
 (a)

$p=r$	q	$\sim p$	$r \vee (\sim p)$	$(p \wedge q)$	$(p \wedge q) \vee r$	$r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$
T	F	F	T	F	T	T
F	T	T	T	F	F	F
T	T	F	T	T	T	T
F	F	T	T	F	F	T

(b)

p	$\sim p$	$r \vee (\sim p)$	$q=r$	$(p \wedge q)$	$(p \wedge q) \vee r$	$r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$
T	F	T	T	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	T	T	F	F	F	F

(c)

p	q	$r = \sim p$	$r \vee (\sim p)$	$(p \wedge q)$	$(p \wedge q) \vee r$	$r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$
T	T	F	F	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	F	T	T	F	T	T

(d)

$\sim p$	p	q	$r \vee (\sim p)$	$r = \sim q$	$(p \wedge q)$	$(p \wedge q) \vee r$	$r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$
F	T	T	F	F	T	T	T
F	T	F	T	T	F	T	T
T	F	T	T	F	F	F	F
T	F	F	T	T	F	T	T

Now final answer is option no. 3.

94. (b) Given tautology $((p \Delta \sim q) \vee ((\sim p) \Delta q))$ will be always true when the choices of Δ would be ' \vee ' and ' \Rightarrow '. Therefore, number of the total choices are 2.

95. (c) $(p \wedge \sim q) \rightarrow (p \vee q)$
 $= \sim (p \wedge \sim q) \vee (p \vee q) = (\sim p \vee q) \vee (p \vee q)$
 $= (\sim p \vee p) \vee (q \vee q) = t \vee q = t$

96. (a) Explanation : All should be on in-order to complete the circuit

97. (c) Explanation : Anyone should be on in-order to complete the circuit.

98. (b) Switches in series use ' \wedge ' and switches in parallel use ' \vee '.

99. (a) Switches in series use ' \wedge ' and switches in parallel use ' \vee '.

100. (a) Explanation : The circuit will be complete if (P) is true, $Q \vee \sim Q$ will always be true

Exercise 2 :

ACCELERATOR

Topic-wise MCQs

- (c) $x^2 - 3x - 4 = 0, x \in \mathbb{R}$ is not statement.
- (c) Unity and number itself are the only two factors.
- (b) For $x \in \{1, 2\}, 3^x < 3x + 5$
- (d) For $x \in \mathbb{N}, x + 5 > 4$
- (a) Every element of option (a) satisfies the given inequality.
- (c) We note that p and q both are false statements, so compound statement $p \vee q$ is false.
- (c) Let us consider one by one:
The connecting word is and. Both the statements are false. So, the given statement is false.
- (a) Both these statements are true. Therefore, the compound statement is true.
- (a) We know that both these statements are true. The connecting word is and. So, the given statement is true.
- (d) It should be $19 \not> 13$ and $12 \not< 15$.
- (a) The true quantified statement is 'the square of every real number is positive'.
- (a) Inclusive "or". 17 is a real number or a positive integer or both.
- (b) $p \rightarrow (q \vee r)$ If 2 is even number then 2 is a prime number or $2 + 2 = 2^2$.
- (c) $p \wedge q$ means Mathematics is interesting and Mathematics is difficult.
- (b) P : there is a rational number $x \in S$ such that $x > 0$
 $\sim P$: Every rational number $x \in S$ satisfies $x \leq 0$
- (b) It is obvious.
- (c) p : we control population, q : we prosper
 \therefore we have $p \Rightarrow q$
 Its negation is $\sim (p \Rightarrow q)$ i.e. $p \wedge \sim q$
 i.e., we control population but we do not prosper.
- (b) Since the statement $q \rightarrow p$ is the converse of the statement $p \rightarrow q$.

19. (b) $\sim p$: Ashok does not work hard
Use ' \rightarrow ' symbol for then
($\sim p \rightarrow q$) mean = If Ashok does not work hard then he gets good grade.
20. (b) It is correct.
 $\therefore \sqrt{3}$ is not rational but it is real.
21. (b) It is obvious.
22. (b) It is obvious.
23. (a) From given statement
 $P \rightarrow (\sim Q \wedge R)$
 $\equiv \sim P \vee (\sim Q \wedge R) \equiv (\sim P \vee \sim Q) \wedge (\sim P \vee R)$
24. (c) According to option take truth value of p, q and r
- | p | q | r | $\sim p$ | $\sim q$ | A:(p∨q) | B:(~p∨r) | A∧B | C=~q∨r | (A∧B)→C |
|---|---|---|----------|----------|---------|----------|-----|--------|---------|
| T | F | T | F | T | T | T | T | T | T |
| T | T | F | F | F | T | F | F | F | T |
| F | T | T | T | F | T | T | T | F | F |
| T | F | F | T | T | T | F | F | T | T |
- \therefore option c is correct
25. (a) Given, statement p is true (T) and statement q is false (F).
So, $p \rightarrow q \equiv T \rightarrow F = F$ and $p \leftrightarrow q \equiv T \leftrightarrow F = F$
26. (d) Given statement is $(P \wedge (R)) \rightarrow ((\sim R) \wedge Q)$
 $X \Rightarrow Y$ is a false
when X is true and Y is false
So, $P \rightarrow T, Q \rightarrow F, R \rightarrow F$
(a) $P \vee Q \rightarrow \sim R$ is T
(b) $P \vee Q \rightarrow \sim P$ is T
(c) $\sim(P \vee Q) \rightarrow \sim R$ is T
(d) $\sim(P \vee Q) \rightarrow \sim R$ is F
27. (d) $(p \wedge q) \rightarrow (\sim q \vee r)$
 $= \sim(p \wedge q) \vee (\sim q \vee r) = (\sim p \vee \sim q) \vee (\sim q \vee r)$
 $= (\sim p \vee \sim q \vee r)$
 $\therefore (\sim p \vee \sim q \vee r)$ is false, then $\sim p, \sim q$ and r all these must be false.
 $\Rightarrow p$ is true, q is true and r is false.
28. (a) Given statement $p \rightarrow (\sim q \vee r)$ is False.
 $\Rightarrow p$ is True and $\sim q \vee r$ is False
 $\Rightarrow p$ is True and $\sim q$ is False and r is False
 \therefore truth values of p, q, r are T, T, F respectively.
29. (d) $P \equiv$ Ramesh listens to music
 $\sim q \equiv$ He is in village.
 $r \vee s \equiv$ Saturday or Sunday
 \therefore Given statement is satisfy.
 $P \Rightarrow ((\sim q) \wedge (r \vee s))$
30. (d) According to given statements 'Ramu is intelligent and honest if and only if Ramu is not rich'.
So, $(P \wedge \sim R) \Leftrightarrow \sim Q$
We need to find negation of above statement.

Then, $\sim[(P \wedge \sim R) \Leftrightarrow \sim Q]$
 $\Rightarrow P \Leftrightarrow Q \equiv [P \wedge Q \vee (\sim P \wedge \sim Q)]$
 $\sim[\sim(P \wedge \sim R) \vee \sim Q] \wedge [Q \vee (P \wedge \sim R)]$
Therefore, $((P \wedge \sim R) \wedge Q) \vee (\sim Q \wedge (\sim P \vee R))$.

31. (c) Given statement $S_1 : (\sim p \vee q) \vee (\sim p \vee r)$
 $\equiv \sim p \vee (q \vee r)$
 $S_2 : p \rightarrow (q \vee r)$
 $\equiv \sim p \vee (q \vee r) \rightarrow$ By conditional law

$$S_1 \equiv S_2$$

32. (c) If we take

p	q	r	s
F	F	T	F

The truth value of all the propositions will be true.

33. (a) Statement : for all $M > 0$, there exists $x \in S$ such that $x \geq M$.
Negation of statement : there exists $M > 0$ such that $x < m$ for all $x \in S$
(\therefore Negation of 'there exists' is 'for all'.)
34. (c) Let p : weather is good and q : ground is not wet
 $\therefore \sim(p \wedge \sim q) = \sim p \vee q$
So, negative statement : The match will played only is the weather is not good or ground is wet
35. (c) We know that, Contrapositive of $A \rightarrow B$ is $\sim B \rightarrow \sim A$
 \therefore Contrapositive of the given statement will be
 $\sim(\text{you will earn money}) \rightarrow \sim(\text{you will work})$
i.e., if you will not earn money, you will not work.
36. (b) Negation of given statement is $\sim((A \wedge C) \rightarrow B)$
 $\sim(\sim(A \wedge C) \vee B)$. Using De-Morgan's law,
So, $(A \wedge C) \wedge (\sim B)$
37. (b) $(\sim(\sim p)) \wedge q = p \wedge q$
38. (d) $p \Rightarrow (\sim p \vee q)$ is false means p is true and $\sim p \vee q$ is false.
 $\Rightarrow p$ is true and both $\sim p$ and q are false
 $\Rightarrow p$ is true and q is false.
39. (b) $\sim[(p \vee q) \wedge r] \equiv \sim(p \vee q) \vee \sim r \equiv (\sim p \wedge \sim q) \vee \sim r$
40. (c) The inverse of the proposition $(p \wedge \sim q) \rightarrow r$ is
 $\sim(p \wedge \sim q) \rightarrow \sim r$
 $\equiv \sim p \vee \sim(\sim q) \rightarrow \sim r$
 $\equiv \sim p \vee q \rightarrow \sim r$
41. (d) We know that $\sim(p \rightarrow q) \equiv p \wedge \sim q$
 $\therefore \sim((p \wedge r) \rightarrow (r \vee q)) \equiv (p \wedge r) \wedge [\sim(r \vee q)]$
 $\equiv (p \wedge r) \wedge (\sim r \wedge \sim q)$
42. (d) Converse of $((\sim p) \wedge q) \Rightarrow r$
 $\equiv r \Rightarrow (\sim p \wedge q)$ [As $p \rightarrow q \equiv \sim p \vee q$]
 $= \sim r \vee (\sim p \wedge q) = (\sim r \vee \sim p) \wedge (\sim r \vee q) = (\sim(r \vee p)) \wedge (\sim(r \vee \sim q))$

43. (a) $\sim[(p \vee q) \wedge (q \vee (\sim r))]$
 $\Rightarrow \sim(p \vee q) \vee \sim(q \vee (\sim r))$

$\Rightarrow (\sim p \wedge \sim q) \vee (\sim q \wedge r)$

Apply distribution law

$\Rightarrow \sim q \wedge (\sim p \vee r) \Rightarrow (\sim p \vee r) \wedge (\sim q)$

44. (c) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

$\Rightarrow \sim[\sim(p \rightarrow q) \vee q \rightarrow p]$ [As $p \rightarrow q \equiv \sim p \vee q$]

$\Rightarrow p \rightarrow q \wedge \sim(q \rightarrow p)$

$\Rightarrow (\sim p \vee q) \wedge (q \wedge \sim p) \Rightarrow q \wedge \sim p.$

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$	$\sim[(p \rightarrow q) \rightarrow (q \rightarrow p)]$
T	T	T	T	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	F

45. (a)

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$(p \wedge \sim q) \vee (\sim p)$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

Now, $(\sim p \vee q) \wedge p = (\sim p \wedge p) \vee (q \wedge p) = p \wedge q$

46. (d) $\sim(q \vee ((\sim q) \wedge p))$

$\equiv \sim q \wedge \sim((\sim q) \wedge p) \equiv \sim q \wedge (q \vee \sim p)$

$\equiv (\sim q \wedge q) \vee (\sim q \wedge \sim p)$

$\equiv C \vee (\sim q \wedge \sim p) \equiv (\sim q \wedge \sim p) = (\sim p) \wedge (\sim q)$

47. (c) Given boolean expression are

$\sim p \vee q \equiv p \rightarrow q$ and $\sim q \wedge p \equiv \sim(p \rightarrow q)$

Negation of $\sim(p \rightarrow q) \rightarrow (p \rightarrow q)$

is $\sim(p \rightarrow q) \wedge (\sim(p \rightarrow q))$ i.e., $\sim(p \rightarrow q)$

48. (a) $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p) = \sim(p \wedge q) \vee ((r \wedge q) \wedge p)$

$= \sim(p \wedge q) \vee ((r \wedge q) \wedge (p \wedge q))$

$= [\sim(p \wedge q) \vee (p \wedge q)] \wedge (\sim(p \wedge q) \vee (r \wedge p))$

$= t \wedge [\sim(p \wedge q) \vee (r \wedge p)] = \sim(p \wedge q) \vee (r \wedge p)$

$= (p \wedge q) \Rightarrow (r \wedge p)$

49. (d) Given: $(p \rightarrow q) \wedge (q \rightarrow \sim p)$

$= (\sim p \vee q) \wedge (\sim q \vee \sim p)$ $\{\because p \rightarrow q = \sim p \vee q\}$

$= (\sim p \vee q) \wedge (\sim p \vee \sim q)$ $\{\text{commutative property}\}$

$= \sim p \vee (q \wedge \sim q)$ $\{\text{distributive property}\}$

$= \sim p \vee F = \sim p$

50. (b) $\because (p \wedge \sim q) \Rightarrow (q \vee \sim p)$

$= \sim(p \wedge \sim q) \vee (q \vee \sim p) = (\sim p \vee q) \vee (\sim p \vee q)$

$= \sim p \vee q \equiv p \Rightarrow q$

51. (b)

p	q	$\sim q$	$p \wedge \sim q$	$\sim p$	$p \rightarrow \sim(p \wedge \sim q)$	$\sim p \vee q$
T	T	F	F	F	T	T
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	T	F	T	T	T

$\therefore p \rightarrow \sim(p \wedge \sim q)$ is equivalent to $\sim p \vee q$

52. (a) $\sim((p \vee r) \rightarrow (q \vee r))$

$= (p \vee r) \wedge (\sim q \vee \sim r)$

$= (p \vee r) \wedge (\sim(q \wedge r))$

$= ((p \vee r) \wedge \sim r) \wedge (\sim q)$

$= ((p \vee \sim r) \vee (r \wedge \sim r)) \wedge (\sim q)$

$= ((p \wedge \sim r) \vee f) \wedge (\sim q)$

$= (p \wedge \sim r) \wedge (\sim q)$

$= p \wedge \sim q \wedge \sim r$

53. (d) $\because (P \vee Q) \wedge (\sim P) \rightarrow Q$

$= \sim(P \vee Q) \vee P \vee Q \Rightarrow$ It is a tautology

only option (d) is a tautology because

$\sim(P \rightarrow Q) = \sim(\sim P \vee Q) = P \wedge \sim Q$

54. (c) Given propositions P1: $\sim(p \rightarrow \sim q)$

P2: $(p \wedge \sim q) \wedge ((\sim p) \vee q)$

Required table is shown below.

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \rightarrow (\sim p \vee q)$	$p \rightarrow \sim q$	$\sim(p \rightarrow \sim q)$	$p \wedge \sim q$	$(p \wedge \sim q) \wedge ((\sim p) \vee q)$
T	T	F	F	T	T	F	T	F	F
T	F	F	T	F	F	T	F	T	F
F	T	T	F	T	T	T	F	F	F
F	F	T	T	T	T	T	F	F	F

If p is true, q is false see from table P1 and P2 both are false.

55. (a) Contrapositive statement will be
 "For an integer n , if n is not odd then $n^3 - 1$ is not even".
 or
 "For an integer n , if n is even then $n^3 - 1$ is odd".
56. (b) Contrapositive statement will be "If a function is not continuous at 'a', then it is not differentiable at 'a'".
57. (d) Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$
 i.e. contrapositive of 'if p then q ' is 'if not q then not p '.
58. (a) Since, $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$
 $\equiv (p \wedge (\sim q)) \vee ((\sim p) \wedge (q \vee (\sim q))) \equiv (p \wedge (\sim q)) \vee ((\sim p) \wedge t)$
 $\equiv (p \wedge (\sim q)) \vee (\sim p) \equiv (\sim p \vee p) \wedge (\sim p \vee \sim q)$
 $\equiv t \wedge (\sim p \vee \sim q) \equiv \sim p \vee \sim q$

59. (d)

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$q \vee r$	$(p \vee q) \wedge (p \vee r) \rightarrow q \vee r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	F	F	F	T

Hence total no of ordered triplets are 7

60. (d) $\sim [p \vee (\sim (p \wedge q))]$
 $\equiv \sim p \wedge (p \wedge q)$ [$\therefore \sim(\sim p) = p$]
 $\equiv (p \wedge q) \wedge (\sim p)$ [\therefore commutative]
61. (c) $\sim (p \wedge (p \rightarrow \sim q))$
 $\equiv \sim (p \wedge (\sim p \vee \sim q)) \equiv \sim \{ (p \wedge \sim p) \vee (p \wedge \sim q) \}$
 $\equiv \sim \{ c \vee (p \wedge \sim q) \} \equiv \sim \{ (p \wedge \sim q) \} \equiv \sim p \vee q$
62. (a) Let $r = (\sim (P \wedge Q)) \vee ((\sim P) \wedge Q) : s = ((\sim P) \wedge (\sim Q))$

P	Q	$\sim (P \wedge Q)$	$(\sim P) \wedge Q$	r	s	$r \rightarrow s$
T	T	F	F	F	F	T
T	F	T	F	T	F	F
F	T	T	T	T	F	F
F	F	T	F	T	T	T

By options

A: $\sim P \vee Q$	B: $(\sim Q \vee P)$	$A \wedge B$
T	T	T
F	T	F
T	F	F
T	T	T

63. (b) Given $B \rightarrow ((\sim A) \vee B)$
 As $p \rightarrow q = \sim p \vee q$
 So $\sim B \vee (\sim A \vee B) \equiv (\sim B \vee B) \vee \sim A$ [\therefore Commutative]
 $\equiv T \vee \sim A \equiv T$

Now checking options

- (a) $B \rightarrow (A \rightarrow B) \equiv \sim B \vee (A \rightarrow B) \equiv \sim B \vee \sim A \vee B = T$
 (b) $A \rightarrow (A \leftrightarrow B) \equiv \sim A \vee (A \leftrightarrow B) \equiv \sim A \vee [(A \rightarrow B) \wedge (B \rightarrow A)]$
 $\equiv \sim A \vee [(\sim (A \vee B)) \wedge (\sim B \vee A)]$
 $\equiv \sim A \vee [\sim (A \vee B) \equiv \sim A \vee \sim B \wedge \sim B]$
 $\equiv \sim A \wedge \sim B \equiv \sim (A \vee B)$
 (c) $A \rightarrow (\sim A \rightarrow B) \equiv \sim A \vee (\sim A \rightarrow B) \equiv \sim A \vee A \vee B \equiv T$
 (d) $B \rightarrow (\sim A \rightarrow B) \equiv \sim B \vee (\sim A \rightarrow B) \equiv \sim B \vee A \vee B \equiv T$
 Here, T is used for tautology.
64. (a) We have $P \Rightarrow Q \equiv \sim P \vee Q$
 $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$
 $\Rightarrow (\sim P \vee Q) \wedge (\sim R \vee Q) \Rightarrow (\sim P \wedge \sim R) \vee Q$
 $\Rightarrow \sim (P \vee R) \vee Q \Rightarrow (P \vee R) \Rightarrow Q$
65. (b) We know that $p \rightarrow q$ is false only when p is true and q is false.
 So $p \rightarrow (\sim p \vee q)$ is false only when p is true and $(\sim p \vee q)$ is false.
 But $(\sim p \vee q)$ is false if q is false because $\sim p$ is false.
 Hence $p \rightarrow (\sim p \vee q)$ is false when truth value of p and q are T and F respectively.
66. (d) $(p \wedge q) \Rightarrow (p \wedge r)$
 $\equiv \sim (p \wedge q) \vee (p \wedge r)$
 $\equiv (\sim p \vee \sim q) \vee (p \wedge r)$
 $\equiv (\sim p \vee (P \wedge r)) \vee \sim q$
 $\equiv (\sim p \vee p) \wedge (\sim p \vee r) \vee \sim q$
 $\equiv (\sim p \vee r) \vee \sim q$
 $\equiv (\sim p \vee \sim q) \vee r$
 $\equiv \sim (p \wedge q) \vee r$
 $\equiv (p \wedge q) \Rightarrow r$
67. (b) Given statement is
 $(p \rightarrow q) \vee (p \rightarrow r) = (\sim p \vee q) \vee (\sim p \vee r) = \sim p \vee (q \vee r)$
 $= p \rightarrow (q \vee r) \equiv (c)$ is true.
 Now, (a) $(p \wedge \sim r) \rightarrow q$
 $\sim (p \wedge \sim r) \vee q = (\sim p \vee r) \vee q$
 $= \sim p \vee (r \vee q) = p \rightarrow (q \vee r)$
 From option (d).
 $(p \wedge \sim q) \rightarrow r = \sim (p \wedge \sim q) \vee r \equiv (\sim p \vee q) \vee r$
68. (c) Given $(p \wedge r) \Leftrightarrow (p \wedge (\sim q)) \equiv (\sim p)$
 taking $r = q$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$(p \wedge q) \Leftrightarrow (p \wedge (\sim q))$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

69. (b) Let us make the truth table for the given statements, as follows :

p	q	$p \vee q$	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$	$p \rightarrow (p \vee q)$
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	T	T

From table we observe
 $p \rightarrow (q \rightarrow p)$ is equivalent to $p \rightarrow (p \vee q)$

70. (a) $\sim (p \vee q) \vee (\sim p \vee q)$
 $\equiv (\sim p \wedge \sim q) \vee (\sim p \vee q)$
 $\equiv \sim p \wedge (\sim q \vee q) \equiv \sim p$
71. (a) $\sim (\sim p \rightarrow q) \equiv \sim (p \vee q) \equiv \sim p \wedge \sim q$
72. (c) Consider the expression

$$\begin{aligned} & ((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q) \\ & \equiv (p \wedge \sim q) \wedge (\sim p \wedge \sim q) \\ & \equiv ((p \vee \sim q) \wedge \sim p) \wedge ((p \vee \sim q) \wedge \sim q) \\ & \equiv ((p \wedge \sim p) \vee (\sim q \wedge \sim p)) \wedge \sim q \\ & \equiv (\sim p \wedge \sim q) \wedge \sim q \equiv (\sim p \wedge \sim q) \end{aligned}$$

77. (c)

p	q	r	$p \vee q$	$q \rightarrow r$	$\sim r$	$(p \vee q) \wedge (q \rightarrow r) \wedge \sim r$	$p \wedge q$	$((p \vee q) \wedge (q \rightarrow r) \wedge \sim r) \rightarrow p \wedge q$
T	T	T	T	T	F	F	T	T
T	T	F	T	F	T	F	T	T
T	F	T	T	T	F	F	F	T
T	F	F	T	T	T	T	F	F
F	T	T	T	T	F	F	F	T
F	T	F	T	F	T	F	F	T
F	F	T	F	T	F	F	F	T
F	F	F	F	T	T	F	F	T

78. (b) Negation of given statement $= \sim (p \vee (\sim p \wedge q))$

$$\begin{aligned} & = \sim p \wedge \sim (\sim p \wedge q) = \sim p \wedge (p \vee \sim q) \\ & = (\sim p \wedge q) \vee (\sim p \wedge \sim q) \\ & = F \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q \end{aligned}$$

79. (a) $p : x \leftrightarrow \sim y = (x \rightarrow \sim y) \wedge (\sim y \rightarrow x)$

$$\begin{aligned} & = (\sim x \vee \sim y) \wedge (y \vee x) \\ & = \sim (x \wedge y) \wedge (x \vee y) \quad (\because \sim (x \wedge y) = \sim x \vee \sim y) \end{aligned}$$

Negation of p is

$$\sim p = (x \wedge y) \vee \sim (x \vee y) = (x \wedge y) \vee (\sim x \wedge \sim y)$$

80. (d)

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

73. (c) Logical statement,

$$\begin{aligned} & = [\sim (\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r) \\ & = [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r) \\ & = [(p \wedge \sim q) \wedge (\sim q \wedge r)] \vee [(p \wedge r) \wedge (\sim q \wedge r)] \\ & = [p \wedge \sim q \wedge r] \vee [p \wedge r \wedge \sim q] \\ & = (p \wedge \sim q) \wedge r \\ & = (p \wedge r) \wedge \sim q \end{aligned}$$

74. (d) $\sim p \wedge q = \sim (q \rightarrow p)$

75. (a) $p : ((A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A$

$$\begin{aligned} & [\sim (A \wedge (B \vee C)) \vee (A \vee B)] \Rightarrow A \\ & [(A \wedge (B \vee C)) \wedge \sim (A \vee B)] \vee A \\ & (f \vee A) = A \sim p \equiv A \end{aligned}$$

76. (d) $\sim [p \wedge (q \wedge \sim (p \wedge q))]$

$$\begin{aligned} & \equiv \sim p \vee (\sim q \vee (p \wedge q)) \\ & \equiv \sim p \vee ((\sim q \vee p) \wedge (\sim q \vee q)) \\ & \equiv \sim p \vee (\sim q \vee p) \\ & \equiv (p \wedge q) \vee p \end{aligned}$$

options	1	2	3	4
	T	T	T	T
	T	F	T	T
	F	T	T	T
	F	T	F	T

81. (a) Given conditional statement is

$$((p \wedge q) \rightarrow ((\sim p) \vee r)) \vee (((\sim p) \vee r) \rightarrow (p \wedge q))$$

\Rightarrow Here, $(A \rightarrow B)$ is equal to $(\sim A \vee B)$.

From given statement,

$$\begin{aligned} & \Rightarrow (\sim p \vee \sim q) \vee (\sim p \vee r) \vee (p \wedge q) \\ & \Rightarrow \sim p \vee (r \vee \sim q) \vee p \wedge (\sim r \vee q) \end{aligned}$$

If negation of p and only p is present with union, then it represents tautology.

82. (c) $p \vee q \Rightarrow q$

$$\begin{aligned} & \Rightarrow \sim (p \vee q) \vee q \\ & \Rightarrow (\sim p \wedge \sim q) \vee q \end{aligned}$$

$$\Rightarrow (\sim p \vee q) \wedge (\sim q \vee p)$$

$$\Rightarrow (\sim p \vee q) \wedge t = \sim p \vee q$$

from options we can say option (c) is correct

$$(p \wedge q) \Rightarrow \sim p \vee q \Rightarrow \sim p \vee \sim q \vee \sim p \vee q \Rightarrow t$$

83. (b) For (b) : * = \vee , • = \wedge

$$(p \vee q) \vee (p \vee \sim q) \equiv t$$

84. (d) $(A \wedge (A \rightarrow B)) \rightarrow B = (A \wedge (\sim A \vee B)) \rightarrow B$

$$= ((A \wedge \sim A) \vee (A \wedge B)) \rightarrow B = (A \wedge B) \rightarrow B$$

$$= \sim (A \wedge B) \vee B = (\sim A \vee \sim B) \vee B = T$$

85. (a) $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$

$$\equiv (p \wedge (\sim p \vee q) \vee (\sim q \vee r)) \rightarrow r$$

$$\equiv ((p \wedge q) \wedge (\sim p \vee r)) \rightarrow r \equiv (p \wedge q \wedge r) \rightarrow r$$

$$\equiv \sim (p \wedge q \wedge r) \vee r = (\sim p) \vee (\sim q) \vee (\sim r) \vee r$$

\Rightarrow tautology

86. (d) (a) $(p \rightarrow q) \vee (\sim q \rightarrow p) = (\sim p \vee q) \vee (q \vee p)$

$$= (\sim p \vee p) \vee q = t \vee q = t$$

It is tautology.

$$(b) (q \rightarrow p) \vee (\sim q \rightarrow p) = (\sim q \vee p) \vee (q \vee p)$$

$$= (\sim q \vee q) \vee p = t \vee t = t$$

It is tautology.

$$(c) (p \rightarrow \sim q) \vee (\sim q \rightarrow p) = (\sim p \vee \sim q) \vee (q \vee p)$$

$$= (\sim p \vee q) \vee (\sim q \vee p) = t \vee t = t$$

It is tautology.

$$(d) (\sim q \rightarrow q) \vee (\sim q \rightarrow p)$$

$$= (p \vee q) \vee (q \vee p) = (p \vee p) \vee (q \vee p) = p \vee q$$

Which is not a tautology.

87. (b) $p \Rightarrow q \Leftrightarrow q * (\sim p)$ is a tautology.

$\therefore p \Rightarrow q$ and $q * (\sim p)$ have same truth value for all logical possibility.

$$\therefore q * (\sim p) \equiv p \Rightarrow q$$

$$\text{So, } p * \sim q \equiv q \Rightarrow p$$

88. (a) We have $p \rightarrow q = \sim p \vee q$

$$\therefore (p \wedge q) \rightarrow (p \rightarrow q) = \sim (p \wedge q) \vee (p \rightarrow q)$$

$$= (\sim p \vee \sim q) \vee (\sim p \vee q)$$

$$= \sim p \vee (q \vee \sim q) = \sim p \vee t = t \text{ is a tautology.}$$

89. (a) We know that $p \rightarrow q = \sim p \vee q$

$$\text{So, } (p \wedge q) \rightarrow (p \rightarrow q) = \sim (p \wedge q) \vee (p \rightarrow q)$$

$$= (\sim p \vee \sim q) \vee (\sim p \vee q)$$

$$= \sim p \vee q \vee \sim q = \sim p \vee t = t$$

is a tautology.

So, option (a) is correct.

90. (d) $F_1 = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$

$$= [(A \wedge \sim B) \vee \sim A] \vee [\sim C \wedge (A \vee B)]$$

$$= [(A \vee \sim A) \wedge (\sim B \vee \sim A)] \vee [\sim C \wedge (A \vee B)]$$

$$= t \wedge (\sim B \vee \sim A) \vee [\sim C \wedge (A \vee B)]$$

$$= (\sim B \vee \sim A) \vee [\sim C \wedge (A \vee B)]$$

$$= [(\sim B \vee \sim A) \vee \sim C] \wedge [(\sim B \vee \sim A) \vee (A \vee B)]$$

$$= [(\sim B \vee \sim A) \vee \sim C] \wedge t = \sim A \vee \sim B \vee \sim C \neq t$$

$$F_2 = (A \vee B) \vee (B \rightarrow \sim A) = (A \vee B) \vee (\sim B \vee \sim A) = t$$

91. (d) The truth table of both the statements is

p	q	$\sim p$	$\sim q$	$q \vee p$	$p \leftrightarrow \sim q$	(S ₁)	$\sim p \leftrightarrow q$	(S ₂)
T	T	F	F	T	F	F	F	F
T	F	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	F
F	F	T	T	F	F	T	F	F

$\therefore S_1$ is not tautology and S_2 is not fallacy.

Hence, both the statements (S_1) and (S_2) are not correct.

92. (c) Take the options one by one.

$$(a) (\sim q \wedge p) \wedge q = (\sim q \wedge q) \wedge p = f$$

$$(b) (\sim q \wedge p) \wedge (p \wedge \sim p) = \sim q \wedge (p \wedge \sim p) = f$$

$$(c) (\sim q \wedge p) \vee (p \vee \sim p) = (\sim q \wedge p) \vee (t) = t$$

$$(d) (p \wedge q) \wedge (\sim p \wedge q) = f$$

93. (b)

(A)

p	q	$\sim q$	$p \rightarrow q$	$\sim p$	$(\sim q \wedge (p \rightarrow q))$	(A)
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T

(B)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	(B)
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Both are tautologies.

94. (d) The truth table of $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$ is

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$	$(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	T	T

Hence, the statement is tautology.

95. (d) $S_1 : (P \Rightarrow q) \wedge (P \wedge \sim q)$
 $\equiv (\sim P \vee q) \wedge (P \wedge \sim q)$
 $\equiv (\sim P \wedge P \wedge \sim q) \vee (q \wedge P \wedge \sim q)$
 $\equiv (f \wedge \sim q) \vee (f \wedge P) \equiv f \vee f \equiv f$
 $S_2 : (P \wedge q) (\sim P \wedge q) \vee (P \wedge \sim q) \vee (\sim P \wedge \sim q)$
 $\equiv ((P \vee \sim P) \wedge q) \vee ((P \vee \sim P) \wedge \sim q)$
 $\equiv q \vee \sim q \equiv t$
96. (c) Explanation : The circuit will be complete if R is true and Q is false or P is false.
97. (a) Explanation : The circuit will be complete if R is true and Q is false or P is false.
98. (a) Explanation : The circuit will be complete if ($\sim P$) is true and S is true.
99. (a) Explanation : This can be achieved by turning one of switches N or M on.
100. (a) The circuit will be complete if P is true and Q is false.

Exercise 3 :

PREVIOUS YEARS MCQs

1. (a) $\sim((\sim p) \wedge q) \equiv \sim(\sim p) \vee \sim q \equiv p \vee (\sim q)$
 2. (c) $\sim q \rightarrow \sim p$
 3. (c) $\sim P \wedge \sim q$
 4. (d) Given expression is $(\sim(p \wedge q)) \vee q$
 $= (\sim p \vee \sim q) \vee q = \sim p \vee \sim q \vee q = \sim p \vee T$
 make truth table

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee q$	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	F	T

$p \rightarrow (p \vee q)$ is also a tautology.

12. (b)

p	q	r	$\sim r$	$\sim p$	$p \wedge q$	$\sim p \wedge q$	$\sim r \vee (p \wedge q)$	$(p \wedge q) \wedge [\sim r \vee (p \wedge q)]$	$(p \wedge q) \wedge [\sim r \vee (p \wedge q)] \vee (\sim p \wedge q)$
T	T	T	F	F	T	F	T	T	T
T	T	F	T	F	T	F	T	T	T
T	F	T	F	F	F	F	F	F	F
T	F	F	T	F	F	F	T	F	F
F	T	T	F	T	F	T	F	F	T
F	T	F	T	T	F	T	T	F	T
F	F	T	F	T	F	F	F	F	F
F	F	F	T	T	F	F	T	F	F

$\therefore (p \wedge q) \wedge [\sim r \vee (p \wedge q)] \vee (\sim p \wedge q) \equiv q$

5. (a) Suman is brilliant and dishonest if and only if Suman is rich is expressed as: $Q \leftrightarrow (P \wedge \sim R)$
 Negation of it will be $\sim(Q \leftrightarrow (P \wedge \sim R)) \equiv \sim Q \leftrightarrow \sim(P \wedge \sim R)$
6. (d) P is True, Q is False and R is True
 (a) $(\sim P) \vee (Q \wedge R) \equiv F \vee (F \wedge T) \equiv F \vee F = F$
 (b) $(P \wedge Q) \vee (\sim R) \equiv (T \wedge F) \vee (F) \equiv F \vee F = F$
 (c) $(\sim P) \wedge (\sim Q \wedge R) \equiv F \wedge (T \wedge T) \equiv F \wedge T = F$
 (d) $P \vee (\sim Q \wedge R) \equiv T \vee (T \wedge T) \equiv T \vee T = T$
7. (d) Given expression is $p \leftrightarrow (q \Rightarrow p)$
 $\sim(p \leftrightarrow (q \Rightarrow p))$
 $[\because \sim(p \leftrightarrow q) = (p \wedge \sim q) \vee (q \wedge \sim p)]$
 $\sim(p \leftrightarrow (q \Rightarrow p)) = (p \wedge \sim(q \Rightarrow p)) \vee ((q \Rightarrow p) \wedge \sim p)$
 $[\because (p \wedge \sim(q \Rightarrow p)) = p \wedge (q \wedge \sim p) = (p \wedge \sim p) \wedge q = c]$
 $(q \Rightarrow p) \wedge \sim p = (\sim q \vee p) \wedge \sim p = \sim p \wedge (\sim q \vee p)$
 $= (\sim p \wedge \sim q) \vee (\sim p \wedge p) = \sim p \wedge \sim q$
 $\therefore \sim(p \leftrightarrow (q \Rightarrow p)) = c \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q$
8. (b) $\sim[(T \wedge T) \vee (F \vee F)] \equiv \sim(T \vee F) \equiv \sim T \equiv F$
9. (c) Statement given in option (c) is correct.
 $\sim[p \vee (\sim q)] = (\sim p) \wedge \sim(\sim q) = (\sim p) \wedge q$
10. (c) Contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$
 \therefore contrapositive of $(p \vee q) \Rightarrow r$ is
 $\sim r \Rightarrow \sim(p \vee q)$ i.e. $\sim r \Rightarrow (\sim p \wedge \sim q)$
11. (b) $(p \wedge \sim q) \wedge (\sim p \wedge q) = (p \wedge \sim p) \wedge (\sim q \wedge q)$
 $= c \wedge c = c$
 (By using associative laws and commutative laws)
 $\therefore (p \wedge \sim q) \wedge (\sim p \wedge q)$ is a contradiction.

13. (a) $\sim(p \rightarrow \sim q) = p \wedge \sim(\sim q) = p \wedge q$
14. (b) p = The weather is fine
 q = My friends will come and we go for a picnic.
 \therefore given statement can be written as : $p \rightarrow q$
 \therefore its contrapositive is : $\sim q \rightarrow \sim p$
 i.e. If my friends do not come or we do not go for picnic,
 then weather will not be fine.
15. (b) $p \wedge (\sim p \wedge q)$
 $= (p \wedge \sim p) \wedge q$ (Associative law)
 $= c \wedge q$ (Compliment law)
 $= c$ (Identity law)
16. (b) The truth table is given below

p	q	$\sim p$	$p \vee q$	$\sim p \wedge q$	$(p \vee q) \wedge \sim q$
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	F	F

$$\therefore (\sim p \wedge q) \equiv (p \vee q) \wedge \sim p$$

17. (c) Option (a) : $(p \rightarrow q) \wedge (p \vee q) \equiv (\sim p \vee q) \wedge (p \vee q)$
 $\equiv (\sim p \wedge p) \vee q \equiv C \vee q = q$
 It is not tautology.
 Option (b) : $(p \rightarrow q) \vee (p \wedge q) \equiv (\sim p \vee q) \vee (p \wedge q)$
 $\equiv \sim p \vee [q \vee (p \wedge q)] \equiv \sim p \vee q$
 It is not tautology.
 Option (c) : $(p \rightarrow q) \vee (p \vee q) \equiv (\sim p \vee q) \vee (p \vee q)$
 $\equiv (\sim p \vee p) \vee (q \vee q) \equiv T \vee q = T$
 It is tautology

$$\text{Option (d) : } (p \rightarrow q) \wedge (p \wedge q) \equiv (\sim p \vee q) \wedge (p \wedge q)$$

$$\equiv [(\sim p \vee q) \wedge q] \wedge p \equiv q \wedge p$$

It is not tautology.

18. (a) After replacing ' \vee ' by ' \wedge ' and ' \wedge ' by ' \vee ', we get dual of the statement $\sim p \wedge (q \vee c)$ is $\sim p \vee (q \wedge t)$
19. (d) Since $\sim(p \Rightarrow q) \equiv p \wedge \sim q$
 $\sim(\sim p \Rightarrow q) \equiv \sim p \wedge \sim q$
20. (c) Check each option
 (a) $(p \vee q) \wedge (\sim p \wedge q) = (\sim p \wedge q)$
 (b) $(p \vee q) \wedge (\sim p \vee q) = q$
 (c) $(p \wedge q) \wedge (\sim p \vee q) = p \wedge q$
 (d) $(p \wedge q) \wedge (\sim p \wedge q) = F$
21. (b) When p is true and q is false, then $p \vee q$ is true,
 $q \Rightarrow p$ is true and $p \wedge (\sim q)$ is true. (\therefore both p and $\sim q$ are true)
 Here, $p \Rightarrow q$ is not true as a true statement cannot imply a false statement.
22. (d) Here S_1 represent by p and S_2 represent by q .
 So, given circuit is represented as
 $(p \wedge q) \vee (p \wedge q)$.
 This is not equivalent to any of the given options.
23. (a) We know that the contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$. So contrapositive of $p \rightarrow (\sim q \rightarrow \sim r)$ is
 $\sim(\sim q \rightarrow \sim r) \rightarrow \sim p$
 $\equiv \sim q \wedge [\sim(\sim r)] \rightarrow \sim p$
 $\therefore \sim(p \rightarrow q) \equiv p \wedge \sim q$
 $\equiv \sim q \wedge r \rightarrow \sim p$

Exercise 1 :

WARM-UP
Topic-wise MCQs

1. (c) There are a total of 6 elementary operations that are possible on matrices, three on rows and three on columns.
2. (d) The elementary operation $R_i \rightarrow 1 + kR_j$ is incorrect, the valid elementary operations on matrices are as follows.
- Interchanging any two rows and columns
 - The multiplication of the elements of any row or column by a non-zero number.
 - The addition to the elements of any row or column, the corresponding elements of any other row and column multiplied by any non-zero number.

3. (b) Given that, $A = \begin{bmatrix} 2 & 3 \\ 6 & 4 \end{bmatrix}$

Applying the elementary operation, $R_1 \rightarrow R_1 + R_2$ we get

$$B = \begin{bmatrix} 2+6 & 3+4 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 4 \end{bmatrix}$$

4. (b) Consider matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, applying the

elementary operation $R_1 \rightarrow 2R_1 + 3R_2$

$$\begin{bmatrix} 2(0)+3(0) & 2(0)+3(0) & 2(0)+3(0) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, the matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, remains same after

applying the elementary operation.

5. (d) The given matrix is $A = \begin{bmatrix} 8 & 5 \\ 2 & 8 \end{bmatrix}$

Applying the elementary operation $R_1 \rightarrow R_1 + 2R_2$, we get

$$\begin{bmatrix} 8+2(2) & 5+2(8) \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 12 & 21 \\ 2 & 8 \end{bmatrix}$$

6. (b) Given that, $A = \begin{bmatrix} 4 & 5 & 2 \\ 6 & 7 & 1 \\ 3 & 9 & 5 \end{bmatrix}$

Applying row operation, $R_1 \rightarrow 2R_1 + 3R_2$

$$\begin{bmatrix} 2(4)+3(6) & 2(5)+3(7) & 2(2)+3(1) \\ 6 & 7 & 1 \\ 3 & 9 & 5 \end{bmatrix} = \begin{bmatrix} 24 & 31 & 7 \\ 6 & 7 & 1 \\ 3 & 9 & 5 \end{bmatrix}$$

Applying the row operation, $R_2 \rightarrow 3R_2 - 2R_3$

$$\begin{bmatrix} 24 & 31 & 7 \\ 3(6)-2(3) & 3(7)-2(9) & 3(1)-2(5) \\ 3 & 9 & 5 \end{bmatrix} = \begin{bmatrix} 24 & 31 & 7 \\ 12 & 3 & -7 \\ 3 & 9 & 5 \end{bmatrix}$$

7. (a) Consider $A = \begin{bmatrix} 2 & 5 & 4 \\ 5 & 2 & 6 \\ 7 & 2 & 1 \end{bmatrix}$, after applying

$R_2 \rightarrow 2R_2 + 3R_1$

$$\begin{bmatrix} 2 & 5 & 4 \\ 2(5)+3(2) & 2(2)+3(5) & 2(6)+3(4) \\ 7 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 4 \\ 16 & 19 & 24 \\ 7 & 2 & 1 \end{bmatrix}$$

8. (b) Given matrix $A = \begin{bmatrix} 5 & 8 \\ -1 & 2 \\ 3 & -4 \end{bmatrix}$ Applying the column

operation, $C_1 \rightarrow 4C_1$ we get $\begin{bmatrix} 4(5) & 8 \\ 4(-1) & 2 \\ 4(3) & -1 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ -4 & 2 \\ 12 & -4 \end{bmatrix}$

9. (d) Given that, $A = \begin{bmatrix} -7 & 2 & 6 \\ -2 & 3 & -5 \\ 2 & 1 & 3 \end{bmatrix}$

Applying the column operation, $C_2 \rightarrow 2C_1 + C_2$ we get

$$\begin{bmatrix} -7 & 2(-7)+2 & 6 \\ -2 & 2(-2)+3 & -5 \\ 2 & 2(2)+1 & 3 \end{bmatrix} = \begin{bmatrix} -7 & -12 & 6 \\ -2 & -1 & -5 \\ 2 & 5 & 3 \end{bmatrix}$$

Applying the column operation, $C_3 \rightarrow 3C_1 + 2C_3$ we get

$$\begin{bmatrix} -7 & -12 & 2(6)+3(-7) \\ -2 & -1 & 2(-5)+3(-2) \\ 2 & 5 & 2(3)+3(2) \end{bmatrix} = \begin{bmatrix} -7 & -12 & -9 \\ -2 & -1 & -16 \\ 2 & 5 & 12 \end{bmatrix}$$

Therefore, the resulting new matrix is $\begin{bmatrix} -7 & -12 & -9 \\ -2 & -1 & -16 \\ 2 & 5 & 12 \end{bmatrix}$

10. (c) The column operation $C_2 \rightarrow 2 + 2C_2$ is incorrect. A non-zero number cannot be directly added to any column or row in a matrix.

11. (a) We have $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$
 $= \cos 75^\circ \cos 15^\circ - \sin 75^\circ \sin 15^\circ$
 $= \cos (75^\circ + 15^\circ) = \cos 90^\circ = 0$

12. (b) Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

We have

If A is a square matrix of order n then

$$A(\text{adj } A) = |A| \cdot I_n$$

Here, $n = 2$

$$\therefore A(\text{adj } A) = I_2 |A|$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (12 - 2) = 10 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

13. (b) The element $a_{11} = 2$. Its minor is given by determinant of the matrix obtained by deleting the rows and column which contain element $a_{11} = 2$

$$\text{i.e., minor of } a_{11} = \begin{vmatrix} 7 & 8 \\ 4 & 5 \end{vmatrix} = 35 - 32 = 3$$

14. (c) $\det(B^{-1}AB) = \det(B^{-1}) \det A \det B$
 $= \det(B^{-1}) \cdot \det B \cdot \det A = \det(B^{-1}B) \cdot \det A$
 $= \det(I) \cdot \det A = 1 \cdot \det A = \det A$

15. (b) Let $A = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & 51-50 \\ 2+0 & 34-0 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 34 \end{bmatrix}$$

$$\Rightarrow |AB| = \begin{vmatrix} 3 & 1 \\ 2 & 34 \end{vmatrix} = 102 - 2 = 100$$

16. (b) $|A| = 0$ as the matrix A is singular

$$\therefore |A| = \begin{vmatrix} 1 & 3 & \lambda + 2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix} = 0$$

Apply $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$ and expand.

$$-2(4 - 3\lambda) + 4(4 - 2\lambda) = 0 \Rightarrow 8 - 2\lambda = 0 \Rightarrow \lambda = 4$$

17. (d) We have

$$\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} = (a+ib)(a-ib) - (c+id)(-c+id)$$

$$= (a^2 + b^2) - (c^2 - d^2) = a^2 + b^2 + c^2 + d^2$$

18. (c) Since A is singular matrix

$$\Rightarrow |A| = 0$$

$$\Rightarrow 2k^2 - 32 = 0 \Rightarrow k = \pm 4$$

19. (b) Let $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$

$$C_{11} = \delta, C_{12} = -\gamma, C_{21} = -\beta, C_{22} = \alpha$$

$$\therefore \text{adj } A = \begin{bmatrix} \delta & -\gamma \\ -\beta & \alpha \end{bmatrix} = \begin{bmatrix} \delta & -\beta \\ -\gamma & \alpha \end{bmatrix}$$

20. (d) Let A and B be square matrices and A^{-1} , B^{-1} be of same order then $(AB)^{-1} = B^{-1} \cdot A^{-1}$.

21. (a) $M_{12} = \begin{vmatrix} -\sin \theta & \sin \theta \\ 1 & 1 \end{vmatrix} = -\sin \theta + \sin \theta = 0$
 $\Rightarrow c_{12} = -M_{12} = 0$

$$M_{22} = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 1 + 1 = 2 \Rightarrow c_{22} = M_{22} = 2$$

$$M_{32} = \begin{vmatrix} 1 & 1 \\ -\sin \theta & \sin \theta \end{vmatrix} = \sin \theta + \sin \theta = 2 \sin \theta$$

$$\Rightarrow c_{32} = -M_{32} = -2 \sin \theta$$

22. (a) Here, $a_{32} = 5$, then,

$$c_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix}$$

$$= (-1)^5 (8 - 30) = -(-22) = 22$$

$$\therefore a_{32} \cdot c_{32} = 5 \times 22 = 110$$

23. (a) We have,

$$\Delta = \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \Rightarrow \Delta = \begin{vmatrix} \log_3 2^9 & \log_2 2^3 \\ \log_3 2^3 & \log_2 2^3 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 9\log_3 2 & \frac{1}{2}\log_2 3 \\ 3\log_3 2 & \frac{2}{2}\log_2 3 \end{vmatrix} \left[\because \log_{a^p} m^n = \frac{n}{p} \log_a m \right]$$

$$\Rightarrow \Delta = (9\log_3 2) \times (\log_2 3) - \left(\frac{1}{2}\log_2 3\right)(3\log_3 2)$$

$$\Rightarrow \Delta = 9(\log_3 2 \times \log_2 3) - \frac{3}{2}(\log_2 3 \times \log_3 2)$$

$$\Rightarrow \Delta = 9 - \frac{3}{2}$$

$$\Rightarrow \Delta = \frac{15}{2} \quad [\because \log_b a \times \log_a b = 1]$$

24. (b) $M_{32} = \begin{vmatrix} 1 & y+z \\ 1 & z+x \end{vmatrix} = z+x-y-z = x-y$

$$\Rightarrow c_{32} = -M_{32} = y-x$$

25. (a) $M_{11} = \begin{vmatrix} ca & b \\ ab & c \end{vmatrix} = c^2a - ab^2 = a(c^2 - b^2)$

$$\Rightarrow C_{11} = M_{11} = a(c^2 - b^2)$$

$$M_{21} = \begin{vmatrix} bc & a \\ ab & c \end{vmatrix} = bc^2 - a^2b = b(c^2 - a^2)$$

$$\Rightarrow C_{21} = -M_{21} = b(a^2 - c^2)$$

$$M_{31} = \begin{vmatrix} bc & a \\ ca & b \end{vmatrix} = b^2c - a^2c = c(b^2 - a^2)$$

$$\Rightarrow C_{31} = M_{31} = c(b^2 - a^2)$$

26. (a) Let M_{ij} and c_{ij} respectively denote the minor and cofactor of element a_{ij} in A. Then,

$$M_{21} = \begin{vmatrix} 3 & -2 \\ 5 & 2 \end{vmatrix} = 6 + 10 = 16 \Rightarrow c_{21} = -M_{21} = -16$$

$$M_{31} = \begin{vmatrix} 3 & -2 \\ -5 & 6 \end{vmatrix} = 18 - 10 = 8 \Rightarrow c_{31} = M_{31} = 8$$

27. (d) Minor of 6 = $M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = 8 - 14 = -6$

$$\text{Cofactor of 4} = c_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} = -(18 - 24) = 6$$

28. (a) Cofactors can be written as,

$$A = ei - hf, \quad B = gf - di, \quad C = dh - ge,$$

$$D = ch - bi \quad \text{and} \quad G = bf - ec$$

$$\text{Now, } bB + cC - dD - gG$$

$$= bgf - bdi + cdh - cge - cdh + bdi - gbf + gec = 0$$

29. (a) $M_{23} = \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = 10 + 3 = 13$

30. (a) $(A^{-1} B^{-1})^T = (B^{-1})^T (A^{-1})^T$
 $= (B^T)^{-1} (A^T)^{-1} = B^{-1} A^{-1}$
 $= (AB)^{-1} = (BA)^{-1}$
 $= A^{-1} B^{-1}$

Hence, $A^{-1} B^{-1}$ is symmetric.

31. (b) We know, matrix A is singular, when $|A| = 0$

$$\therefore \begin{vmatrix} 6-x & 4 \\ 3-x & 1 \end{vmatrix} = 0$$

$$\Rightarrow 6-x-12+4x=0 \Rightarrow 3x=6 \Rightarrow x=2$$

32. (b) $A^{-1} = \frac{1}{|A|} (\text{adj } A)$

33. (a) $A (\text{adj } A) = |A| I$

34. (b) $\begin{vmatrix} 2 & 4 & 0 \\ 0 & 5 & 16 \\ 0 & 0 & 1+p \end{vmatrix} = 20$

On expanding along C_1 ,

$$2\{5(1+p) - 0\} = 20$$

$$\Rightarrow 1+p=2 \Rightarrow p=1$$

35. (b) We know that $A^{-1} = \frac{(\text{adj } A)}{|A|}$

$$\text{or, } A A^{-1} = \frac{A \cdot \text{adj } A}{|A|}$$

$$\text{or, } I_n = \frac{A \cdot \text{adj } A}{|A|}$$

$$A (\text{adj } A) = |A| I_n = |A| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

36. (a) Let $A = \begin{bmatrix} 1 & p & q \\ 0 & x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -p & -q \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A A^{-1} = I$$

$$\Rightarrow \begin{bmatrix} 1 & p & q \\ 0 & x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -p & -q \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow x=1$$

$$37. \text{ (c) Cofactor matrix} = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$$

$$\therefore \text{adj } A = (\text{Cofactor matrix})^T = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$$

$$\therefore |\text{adj } A| = \begin{vmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{vmatrix} = a^6.$$

$$38. \text{ (c) } A \cdot \text{adj } A = |A| I$$

$$|A| = xyz - 8x - 3(z - 8) + 2(2 - 2y)$$

$$|A| = xyz - (8x + 3z + 4y) + 28 \Rightarrow 60 - 20 + 28 = 68$$

$$\Rightarrow (\text{adj } A)^{-1} \text{ always exists whenever, } (A)^{-1} \text{ exists.}$$

$$\therefore A \cdot \text{adj } A = |A| I$$

$$= 68 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$$

$$39. \text{ (a) Given, } \Delta = \begin{vmatrix} k^2 + 2k & 2k + 1 & 1 \\ 2k + 1 & k + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= (k^2 + 2k)(k + 2 - 3) - (2k + 1)(2k + 1 - 3) +$$

$$[6k + 3 - 3k - 6]$$

$$= k^3 - k^2 + 2k^2 - 2k - 4k^2 + 4k - 2k + 2 + 3k - 3$$

$$= k^3 - 3k^2 + 3k - 1$$

$$= (k - 1)^3$$

$$\text{(I) } (k - 1)^3 > 0 \Rightarrow k > 1$$

$$\text{(II) } (k - 1)^3 < 0 \Rightarrow k < 1$$

$$\text{(III) } (k - 1)^3 = 0 \Rightarrow k = 1$$

$$40. \text{ (c) Given relation is } AB = I.$$

$$\text{Take, } |A| = \frac{1}{8} \text{ and } |AB| = 1 \Rightarrow |A| |B| = 1$$

$$\Rightarrow \frac{1}{8} |B| = 1 \Rightarrow |B| = 8$$

$$\text{Now } |\text{adj } (B \text{ adj } (2A))| = |B \text{ adj } (2A)|^2$$

$$= |B|^2 |\text{adj } (2A)|^2$$

$$= |B|^2 (|2A|^2)^2 = |B|^2 (2^6 |A|^2)^2$$

$$\text{Put the value of } |B| \text{ \& } |A|.$$

$$\text{required value} = 64$$

$$41. \text{ (c) } A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{1} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$A^{-2} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \quad A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$$

$$42. \text{ (b) } |A| = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{vmatrix} = 2$$

$$\therefore |\text{adj } (\text{adj } A)| = |A|^{(n-1)^2} = |A|^{2^2} \quad [\therefore \text{Here } n = 3]$$

$$= 2^4 = 16$$

$$43. \text{ (d) } [M(\alpha) M(\beta)]^{-1} = M(\beta)^{-1} M(\alpha)^{-1}$$

$$\text{Now } M(\alpha)^{-1} = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} = M(-\alpha)$$

$$M(\beta)^{-1} = \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix} = \begin{bmatrix} \cos(-\beta) & 0 & \sin(-\beta) \\ 0 & 1 & 0 \\ -\sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix}$$

$$= M(-\beta) [M(\alpha) M(\beta)]^{-1} = M(-\beta) M(-\alpha)$$

$$44. \text{ (b) } M = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$$

$$|M| = 6, \text{ adj } M = \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix}$$

$$M^{-1} = \frac{1}{6} \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 1/6 \end{bmatrix}$$

$$45. \text{ (b) } A = \begin{bmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{bmatrix}$$

$$C_{11} = (-1)^2 \begin{vmatrix} -4 & 2 \\ 2 & 8 \end{vmatrix} = -32 - 4 = -36$$

$$C_{12} = (-1)^3 \begin{vmatrix} 5 & 2 \\ 2 & 8 \end{vmatrix} = -(40 - 4) = -36$$

$$C_{13} = (-1)^4 \begin{vmatrix} 5 & -4 \\ 2 & 2 \end{vmatrix} = 10 + 8 = 18$$

$$C_{21} = (-1)^3 \begin{vmatrix} -5 & -2 \\ 2 & 8 \end{vmatrix} = -(-40 + 4) = 36$$

$$C_{22} = (-1)^4 \begin{vmatrix} 4 & -2 \\ 2 & 8 \end{vmatrix} = (32 + 4) = 36$$

$$C_{23} = (-1)^5 \begin{vmatrix} 4 & -5 \\ 2 & 2 \end{vmatrix} = -(8 + 10) = -18$$

$$C_{31} = (-1)^4 \begin{vmatrix} -5 & -2 \\ -4 & 2 \end{vmatrix} = -10 - 8 = -18$$

$$C_{32} = (-1)^5 \begin{vmatrix} 4 & -2 \\ 5 & 2 \end{vmatrix} = -(8 + 10) = -18$$

$$C_{33} = (-1)^6 \begin{vmatrix} 4 & -5 \\ 5 & -4 \end{vmatrix} = -16 + 25 = 9$$

$$\therefore \text{adj}(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

$$= \begin{bmatrix} -36 & -36 & 18 \\ 36 & 36 & -18 \\ -18 & -18 & 9 \end{bmatrix}' = \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}$$

46. (d) We have $|\text{adj} A| = |A|^{n-1}$, where n is order of matrix A
 $= (-4)^2 = 16$

47. (c) $(\text{adj} A) = \begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix} \Rightarrow \text{adj} A = \begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$

$$\Rightarrow (\text{adj} A)' = \begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$$

48. (d) $AB = 6I \Rightarrow B^{-1} = \frac{1}{6}A$

49. (b) Given, $A + iB = \begin{bmatrix} 2 & 3+i & -1 \\ 3-i & 0 & i-1 \\ -1 & -1-i & 1 \end{bmatrix}$

$$= 2(i^2 - 1) - (3+i)(3-i+i-1) + 1(3-i)(i+1)$$

$$= 2(-1-1) - (3+i)2 + 3i - i^2 + 3 - i$$

$$= -4 - 6 - 2i + 3i + 1 + 3 - i$$

$$\Rightarrow A + iB = -6 = -6 + 0i$$

$$\therefore A + B = -6 + 0 = -6$$

50. (b) $|A| = -7$

$$\therefore \sum_{i=1}^3 a_{i2}A_{i2} = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} = |A| = -7$$

51. (c) $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125 \Rightarrow |A|^3 = 125$

$$\text{Now, } |A| = \alpha^2 - 4 \Rightarrow (\alpha^2 - 4)^3 = 125 = 5^3$$

$$\Rightarrow \alpha^2 - 4 = 5 \Rightarrow \alpha = \pm 3$$

52. (c) Inverse of an identity matrix is the matrix itself.

53. (a) For invertible matrix, $\begin{vmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{vmatrix} \neq 0$

$$\Rightarrow \lambda(0-1) + 1(-6+1) + 4(-3-0) \neq 0$$

$$\Rightarrow -\lambda - 5 - 12 \neq 0 \Rightarrow \lambda \neq -17$$

54. (a) $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

clearly $A \neq 0$. Also $|A| = -1 \neq 0$

$$\therefore A^{-1} \text{ exists, further } (-1)I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$$

$$\text{Also } A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

55. (a) $[A(\text{adj} A)A^{-1}]A$
 $= [|A|I A^{-1}]A = |A|I = 6I$

56. (d) Let $A = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

Then the matrix equation is $AX = B$.

$$\therefore |A| = \begin{vmatrix} 1 & -4 \\ 3 & -2 \end{vmatrix} = -2 + 12 \neq 0.$$

So A is an invertible matrix

$$\text{Also } \text{adj} A = \begin{bmatrix} -2 & -3 \\ 4 & 1 \end{bmatrix}' = \begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}$$

$$\text{So, } A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{10} \begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}$$

Now, $AX = B$

$$\Rightarrow A^{-1}(AX) = A^{-1}B \Rightarrow (A^{-1}A)X = A^{-1}B$$

$$\Rightarrow IX = A^{-1}B \Rightarrow X = A^{-1}B$$

$$\Rightarrow X = \frac{1}{10} \begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 11/2 & 2 \end{bmatrix}$$

57. (c) $\text{Adj}(4A) = 4^2 \text{Adj}(A) = 16 \text{Adj}(A)$

$$\Rightarrow |\text{Adj} 4A| = 16^3 |\text{Adj} A| = 16^3 \cdot 5^2$$

58. (a) We know that for any non-singular matrix A

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

Now, put $A = P^{-1}$ then we have

$$(P^{-1})^{-1} = \frac{1}{|P^{-1}|} \text{adj}(P^{-1})$$

$$\Rightarrow P = |P| \text{adj}(P^{-1}) \Rightarrow \text{adj}(P^{-1}) = \frac{P}{|P|}$$

59. (c) $|\text{adj } A^T| + |(\text{adj } A^{-1})^{-1}|$
 $= |\text{adj } A^T| + \frac{1}{|\text{adj } A^{-1}|} = |A^T|^2 + \frac{1}{|A^{-1}|^2}$
 $= |A|^2 + |A|^2 = 2^2 + 2^2 = 8$
60. (a) $A(A+B)^{-1}B^{-1}(AB)$
 $= B^{-1}((A+B)^{-1})^{-1}A^{-1}(AB) = B^{-1}(A+B)A^{-1}(AB)$
 $= (B^{-1}A + I)A^{-1}(AB) = (B^{-1}AA^{-1} + A^{-1})(AB)$
 $= (B^{-1} + A^{-1})(AB) = B^{-1}AB + A^{-1}AB$
 $= B^{-1}BA + A^{-1}AB = A + B$
61. (c) $B = (I - A)(I + A)^{-1}$
 $B^T = (I + A^T)^{-1}(I - A^T)$
 $= (I - A)^{-1}(I + A)$
 $BB^T = (I - A)(I + A)^{-1}(I - A)^{-1}(I + A)$
 $= (I - A)(I - A)^{-1}(I + A)^{-1}(I + A) = I$
 $[\because (I - A)(I + A) = (I + A)(I - A)]$
 So, B is orthogonal matrix.

62. (d) $B = A^{2^n} = A^{2 \cdot 2^{n-1}} = (A^2)^{2^{n-1}} = (A^{-1})^{2^{n-1}}$
 $= (A^{2^{n-1}})^{-1} = (A^{2 \cdot 2^{n-2}})^{-1} = ((A^2)^{2^{n-2}})^{-1}$
 $= ((A^{-1})^{-1})^{2^{n-2}} = A^{2^{n-2}}$
 $B = C \Rightarrow (B - C) = O \Rightarrow \det. (B - C) = 0$

63. (c) We have, $A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$

$$\therefore \text{adj } A = \begin{bmatrix} 4 & 0 & 0 \\ -4 & 4 & 0 \\ 0 & -4 & 4 \end{bmatrix}$$

$$\therefore \text{adj}(\text{adj } A) = \begin{bmatrix} 16 & 0 & 0 \\ 16 & 16 & 0 \\ 16 & 16 & 16 \end{bmatrix} = 16 \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

64. (c) Let M and N two real matrices of order 2 where

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ and } N = \begin{bmatrix} n_1 & 0 \\ 0 & n_2 \end{bmatrix}$$

M and N are invertible matrices, so

$$M^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Therefore,

$$MNM^{-1} = \frac{1}{ad - bc} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} n_1 & 0 \\ 0 & n_2 \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{ad - bc} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} n_1 d & -n_1 b \\ -n_2 c & n_2 a \end{bmatrix}$$

$$= \frac{1}{ad - bc} \begin{bmatrix} an_1 d - bn_2 c & -an_1 b + bn_2 a \\ cn_1 d - dn_2 c & -cn_1 d + dn_2 a \end{bmatrix}$$

It is given that MNM^{-1} is a diagonal matrix, then

$$ab(n_2 - n_1) = 0 = cd(n_1 - n_2)$$

and if M is a diagonal matrix, means $b = c = 0$, then the above requirement getting satisfy.

65. (b) Let $A = \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix} \Rightarrow \text{adj}(A) = \begin{bmatrix} 5 & 0 \\ a & 5 \end{bmatrix}$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \begin{bmatrix} 5 & 0 \\ a & 5 \end{bmatrix} = \frac{1}{25} \begin{bmatrix} 5 & 0 \\ a & 5 \end{bmatrix}$$

$$\Rightarrow A^{-2} = (A^{-1})^2 = \frac{1}{25} \begin{bmatrix} 5 & 0 \\ a & 5 \end{bmatrix} \frac{1}{25} \begin{bmatrix} 5 & 0 \\ a & 5 \end{bmatrix}$$

$$= \frac{1}{625} \begin{bmatrix} 25 & 0 \\ 10a & 25 \end{bmatrix} = \begin{bmatrix} \frac{1}{25} & 0 \\ \frac{2a}{125} & \frac{1}{25} \end{bmatrix}$$

$$\text{Now, } \begin{bmatrix} \frac{1}{25} & 0 \\ x & \frac{1}{25} \end{bmatrix} = \begin{bmatrix} \frac{1}{25} & 0 \\ \frac{2a}{125} & \frac{1}{25} \end{bmatrix} \Rightarrow x = \frac{2a}{125}$$

66. (d) $\because |A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix} = 3(-3+4) - 2(-3+4) + 0 = 1$

$\therefore \text{adj}(\text{adj } A) = |A|^{3-2} A = A$ and $|\text{adj}(\text{adj } A)| = |A| = 1$
 Also, $|\text{adj } A| = |A|^{3-1} = |A|^2 = 1^2 = 1$

67. (a) Here $|A| = 1$
 $\Rightarrow \text{Adj}(\text{Adj } A) = |A|^{3-2} \cdot A = A$
 $\therefore \text{Trace}(A) = 3 - 3 + 1 = 1$

68. (d) We have

$$\begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}^{-1} = \frac{1}{1 + \tan^2 \theta} \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \cos^2 \theta \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix}$$

$$= \cos^2 \theta \begin{bmatrix} 1 - \tan^2 \theta & -2 \tan \theta \\ 2 \tan \theta & 1 - \tan^2 \theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$\therefore a = \cos 2\theta, b = \sin 2\theta$

69. (a) $|Q| = 6$

$$\text{Now, } |\text{adj } Q| = |Q|^2 = 36 = |P| = \begin{vmatrix} 1 & k & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & k & 3 \\ 1 & 3 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 36$$

$$\Rightarrow -k(1-3) + 3(1-3) = 36 \Rightarrow 2k = 42 \Rightarrow k = 21$$

70. (a) $|P||Q|=1 \Rightarrow |Q|=\frac{1}{9}$
 $\Rightarrow |\text{adj}(P\text{adj}(3Q))|=|\text{adj adj}(3Q)||\text{adj } P|$
 $=|3Q|^4|P|^2=(3^3)^4 \times |Q|^4|P|^2=9^4$

71. (b) $|A| \neq 0$

72. (c) The equation of given lines are
 $3y+4x=1$... (i)
 $y=x+5$... (ii)
 and $5y+bx=3$... (iii)

On solving Eqs. (i) and (ii), we get
 $x=-2$ and $y=3$
 If these lines are concurrent, then these values must satisfy the third equation
 $15-2b=3 \Rightarrow 2b=12 \Rightarrow b=6$

ALTERNATE SOLUTION:

Given equation of lines are
 $3y+4x-1=0$
 $x+5-y=0$ and
 $bx+5y-3=0$
 Since, the given lines are concurrent
 \therefore The value of determinant made by coeff of equations is 0.

$$\text{ie, } \begin{vmatrix} 4 & 3 & -1 \\ 1 & -1 & 5 \\ b & 5 & -3 \end{vmatrix} = 0$$

$$\Rightarrow 4(3-25)-3(-3-5b)-1(5+b)=0$$

$$\Rightarrow -88+4+14b=0 \Rightarrow -84=-14b \Rightarrow b=6.$$

73. (d) $\Delta=0$

74. (c) We know,
 System of a pair of linear equations in two variables are given as

$$a_1x+b_1y+c_1=0 \quad \dots(i)$$

$$\text{and } a_2x+b_2y+c_2=0 \quad \dots(ii)$$

This system has no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Now, on comparing given equations with (i) and (ii), we get
 $a_1=k, a_2=3, b_1=2, b_2=1, c_1=-5, c_2=-1$
 For no solution,

$$\frac{k}{3} = \frac{2}{1} \Rightarrow k=6$$

75. (a) $\therefore \Delta=0$

76. (a) Consider first two equations:
 $2x+3y=-4$ and $3x+4y=-6$

We have $\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1 \neq 0$

$$\Delta_x = \begin{vmatrix} -4 & 3 \\ -6 & 4 \end{vmatrix} = 2 \quad \text{and} \quad \Delta_y = \begin{vmatrix} 2 & -4 \\ 3 & -6 \end{vmatrix} = 0$$

$\therefore x=-2$ and $y=0$

Now this solution satisfies the third, so the equations are consistent with unique solution.

77. (a) Since the system is homogeneous and has a non-trivial solution

$$0 = \begin{vmatrix} p+a & b & c \\ a & q+b & c \\ a & b & r+c \end{vmatrix} = \begin{vmatrix} p & -q & 0 \\ 0 & q & -r \\ a & b & r+c \end{vmatrix}$$

using $R_1 \rightarrow R_1 - R_2$ and $R_2 \rightarrow R_2 - R_3$.

Expanding along C_1 we get

$$\Rightarrow p[q(r+c)+br]+aqr=0 \Rightarrow pqr+pqc+prb+qra=0$$

$$\Rightarrow \frac{a}{p} + \frac{b}{q} + \frac{c}{r} = -1.$$

78. (c) $\Delta = \Delta_1 = 0$

79. (c) $D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0; D_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

\Rightarrow Given system, does not have any solution.

\Rightarrow No solution

80. (c) The given system of equations are

$$kx+y+z=k-1$$

$$x+ky+z=k-1$$

$$x+y+kz=k-1$$

$$A = \begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}, B = \begin{bmatrix} k-1 \\ k-1 \\ k-1 \end{bmatrix} \text{ and } x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix}$$

$$= k(k^2-1) - 1(k-1) + 1(1-k)$$

$$= k^3 - k - k + 1 + 1 - k = k^3 - 3k + 2$$

The given system of equations has no solution, if

$$|A|=0$$

$$\Rightarrow k^3 - 3k + 2 = 0 \Rightarrow (k-1)^2(k+2) = 0$$

$$\Rightarrow k=1 \text{ or } k=-2$$

81. (c) The system is homogenous system.

\therefore it has either unique solution or infinite many solution depend on $|A|$

$$\therefore |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \end{vmatrix} = 2 \times 1 + 1(-3) + (4-3)$$

$$= 2 - 3 + 1 = 0$$

Hence, system has infinitely many solution.

82. (b) $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

83. (b) A system of linear equations like

$$a_1x + b_1y = c_1.$$

$$a_2x + b_2y = c_2.$$

can be represented in matrix form as

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

84. (c) Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & a & 3 \end{vmatrix}$

$$= 6 - 2a - 1 + a - 2 = 3 - a$$

For unique solution

$$|A| \neq 0 \Rightarrow 3 - a \neq 0 \Rightarrow a \neq 3, b \in \mathbb{R}$$

85. (c) Given that system of equation has no solution

$$\therefore \begin{vmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \lambda^2 - |\lambda| \end{vmatrix} = 0$$

$$\Rightarrow 2(3\lambda^2 - 3|\lambda| - 1) + 3(\lambda^2 - |\lambda| + 3) + 5(-1 - 9) = 0$$

$$\Rightarrow 6\lambda^2 - 6|\lambda| - 2 + 3\lambda^2 - 3|\lambda| + 9 - 50 = 0$$

$$\Rightarrow 9|\lambda|^2 - 9|\lambda| - 43 = 0$$

$$D = 81 + 4 \times 9 \times 43 > 0$$

So λ has two distinct real roots.

86. (a) unique solution.

87. (a) The system of equations will be consistent if

$$\Delta = \begin{vmatrix} 1 & \lambda & 2 \\ \lambda & 1 & -2 \\ \lambda & \lambda & 3 \end{vmatrix} = 0$$

To evaluate Δ we use $R_1 \rightarrow R_1 + R_2$ followed by

$C_2 \rightarrow C_2 - C_1$ to obtain

$$\Delta = \begin{vmatrix} \lambda+1 & \lambda+1 & 0 \\ \lambda & 1 & -2 \\ \lambda & \lambda & 3 \end{vmatrix} = \begin{vmatrix} \lambda+1 & 0 & 0 \\ \lambda & 1-\lambda & -2 \\ \lambda & 0 & 3 \end{vmatrix}$$

$$= 3(\lambda+1)(1-\lambda) = 3(1-\lambda^2)$$

For the system to be consistent, we must have

$$1 - \lambda^2 = 0 \text{ or } \lambda = \pm 1.$$

88. (b) $\therefore A^2 + B^2 + C^2 = 0$

$$\therefore A = B = C = 0$$

$$\therefore \begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

89. (a) $\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 0 & 0 & p+2 \\ 0 & 2p+1 & 1 \end{vmatrix} = -(p+2)(2p+1)$

$$\Delta_1 = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 0 & p+2 \\ 0 & 2p+1 & 1 \end{vmatrix}$$

$$= -(2p+1)(p+2) - 2(3-2p-4) - 9(2p+1)$$

$$= (2p+1)(-p-2+2-9) = -(2p+1)(p+9)$$

Clearly for $p = -2, \Delta = 0, \Delta_1 \neq 0$.

Hence, the given system of equation is inconsistent.

90. (d) Since the system has no solution

$$\begin{vmatrix} 2 & -1 & -4 \\ 1 & -2 & -1 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \Rightarrow 2(-2\lambda+1) + 1(\lambda+1) - 4(3) = 0$$

$$\Rightarrow -4\lambda + 2 + \lambda + 1 - 12 = 0 \Rightarrow -3\lambda = 9 \Rightarrow \lambda = -3$$

91. (b) $\begin{vmatrix} 1 & a & -1 \\ 2 & -1 & a \\ a & 1 & 2 \end{vmatrix} = 0$

Applying $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - aR_1$, we get

$$\begin{vmatrix} 1 & a & -1 \\ 0 & -1-2a & a+2 \\ 0 & 1-a^2 & 2+a \end{vmatrix} = 0$$

Expanding along C_1 , we get

$$(a+2)(a^2-2a-2) = 0 \Rightarrow a = -2, 1 \pm \sqrt{3}$$

92. (c) Since the system has a non-trivial solution,

$$\text{therefore } \begin{vmatrix} \lambda & \sin \alpha & \cos \alpha \\ 1 & \cos \alpha & \sin \alpha \\ -1 & \sin \alpha & -\cos \alpha \end{vmatrix} = 0$$

$$\Rightarrow \lambda(-\cos^2 \alpha - \sin^2 \alpha) - (-\sin \alpha \cos \alpha - \sin \alpha \cos \alpha)$$

$$- (\sin^2 \alpha - \cos^2 \alpha) = 0$$

$$\Rightarrow -\lambda + \sin 2\alpha + \cos 2\alpha = 0 \Rightarrow \lambda = \sin 2\alpha + \cos 2\alpha$$

$$\Rightarrow \lambda = \sqrt{2} \cos \left(2\alpha - \frac{\pi}{4} \right)$$

$$\text{Since } -1 < \cos \left(2\alpha - \frac{\pi}{4} \right) < 1 \forall \alpha \in \mathbb{R}$$

$$\therefore -\sqrt{2} < \lambda < \sqrt{2} \text{ i.e. } \lambda \in [-\sqrt{2}, \sqrt{2}]$$

93. (d) Given, $x + y + z = 6$... (i)

$$\alpha x + \beta y + 7z = 3 \text{ ... (ii)}$$

$$x + 2y + 3z = 14 \text{ ... (iii)}$$

By equations (i) and (iii)

$$y + 2z = 8; y = 8 - 2z \text{ and } x = -2 + z$$

Now putting in equation (ii)

$$\alpha(z-2) + \beta(-2z+8) + 7z = 3$$

$$\Rightarrow (\alpha - 2\beta + 7)z = 2\alpha - 8\beta + 3$$

For unique solutions, $\alpha - 2\beta + 7 \neq 0$

and for no solutions,

$$\alpha - 2\beta + 7 = 0 \text{ and } 2\alpha - 8\beta + 3 \neq 0$$

for infinite solutions,

$$\alpha - 2\beta + 7 = 0 \text{ and } 2\alpha - 8\beta + 3 = 0$$

94. (c) Given system of equations are $x + y + z = 6$

$$2x + 5y + \alpha z = \beta$$

$$x + 2y + 3z = 14$$

From the given equations.

$$x + y = 6 - z$$

$$x + 2y = 14 - 3z$$

Subtract (i) from (ii),

$$\Rightarrow y = 8 - 2z, \text{ then } x = z - 2.$$

Now, put the values x & y

in eq. $2x + 5y + \alpha z = \beta$.

$$2(z - 2) + 5(8 - 2z) + \alpha z = \beta$$

$$(\alpha - 8)z = \beta - 36$$

For having infinite solutions

$$\alpha - 8 = 0 \text{ \& } \beta - 36 = 0$$

$$\alpha = 8, \beta = 36$$

$$\text{Required sum} = \alpha + \beta = 44$$

95. (d) $kx + y + z = 1$

$$x + ky + z = k$$

$$x + y + kz = k^2$$

These equations will have no solution of $\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0$.

$$\Rightarrow k(k^2 - 1) - 1(k - 1) + (1 - k) = 0$$

$$\Rightarrow k(k + 1)(k - 1) - 1(k - 1) - (k - 1) = 0$$

$$\Rightarrow (k - 1)[k(k + 1) - 1 - 1] = 0$$

$$\Rightarrow (k - 1)(k^2 + k - 2) = 0$$

$$\Rightarrow k = 1 \text{ or } -2.$$

For $k = 1$, all equations are same and have infinite solution. So, for $k = -2$, equations have no solution.

96. (b) System has no solution when determinant is equal to 0.

$$\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0$$

Expand w.r.t. R_1

$$\Rightarrow |\lambda| = 7 \Rightarrow \lambda = \pm 7$$

System:

$$2x + 3y - z = -2$$

$$x + y + z = 4$$

$$x - y + |\lambda|z = 4\lambda - 4$$

Eliminating y from equal (ii) & (iii) we get

$$x + 4z = 14$$

Add equation (iii) & (iv),

$$\Rightarrow x + \left(\frac{|\lambda| + 1}{2}\right)z = 2\lambda$$

For $\lambda = 7$, Δ gives equal to 0 & for $\lambda = -7$, Δ would be not equal to 0.

97. (a) For infinite many solutions

$$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow 1(2 + \alpha) - 1(1 - 2\alpha) - 1(-1 - 4) = 0$$

$$\Rightarrow 3\alpha + 6 = 0 \Rightarrow \alpha = -2$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & -1 & \beta \end{vmatrix} = 0$$

$$\Rightarrow 1(2\beta + 1) - 1(\beta - 2) + 2(-1 - 4) = 0 \Rightarrow \beta - 7 = 0 \Rightarrow \beta = 7$$

$$\text{So, } \alpha + \beta = 5$$

98. (c) For non-trivial solution

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \mu\lambda + 2\mu - 6\lambda - 12 = 0 \Rightarrow (\lambda + 2)(\mu - 6) = 0$$

$$\Rightarrow \lambda = -2 \text{ or } \mu = 6$$

99. (a) $\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix} = (20) - 2(25) - 3(-10) = 0$

$$\Delta_1 = \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix} = 4(5a - 2b - c)$$

$$\Delta_2 = \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix} = -5(5a - 2b - c)$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix} = -2(5a - 2b - c)$$

For infinite solution $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Rightarrow 5a = 2b + c.$$

100. (a) For non-zero solution, $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ \lambda - 1 & 4\lambda - 2 & \lambda + 3 \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$$\Rightarrow 6\lambda^3 - 36\lambda^2 + 54\lambda = 0 \Rightarrow 6\lambda[\lambda^2 - 6\lambda + 9] = 0$$

$$\Rightarrow \lambda = 0, \lambda = 3 \quad [\text{Distinct values}]$$

Then, the sum of distinct values of $\lambda = 0 + 3 = 3$.

Exercise 2 :

ACCELERATOR
Topic-wise MCQs

1. (c) Given matrix is $A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$
Now apply rules to check the options then,

(i) $R_1 \rightarrow R_1 + R_2 : \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ possible

(ii) $R_1 \leftrightarrow R_2 : \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ possible

(iii) Option is not possible

(iv) $R_1 \rightarrow R_2 + 2R_1 : \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$ possible

2. (b) After applying $R_2 \rightarrow R_2 - 2R_1$ to $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, we get

$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}$ (first multiply all elements of R_1 by 2 and then subtract these elements from R_2).

3. (a) If A is any square matrix, then $AA^{-1} = I$ and $A^{-1}I = A^{-1}$

Since, $A^2 - A + I = 0$

$\Rightarrow A^{-1}A^2 - A^{-1}A + A^{-1}I = 0$

$\Rightarrow (A^{-1}A)A - (A^{-1}A) + A^{-1} = 0$

$\Rightarrow IA - I + A^{-1} = 0 \Rightarrow A - I + A^{-1} = 0$

$\Rightarrow A^{-1} = I - A$

4. (d) Since, B is the inverse of matrix A .

So, $10A^{-1} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$\Rightarrow 10A^{-1} \cdot A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

$\Rightarrow 10I = \begin{bmatrix} 4+4+2 & -4+2+2 & 4-6+2 \\ -5+0+\alpha & 5+0+\alpha & -5+0+\alpha \\ 1-4+3 & -1-2+3 & 1+6+3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -5+\alpha & 5+\alpha & -5+\alpha \\ 0 & 0 & 10 \end{bmatrix}$

$\Rightarrow -5 + \alpha = 0 \Rightarrow \alpha = 5$

5. (d) $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}, A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

We know that, $AA^{-1} = I$

$\Rightarrow \begin{bmatrix} 2x & 0 \\ \pi & x \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On comparing, we get $2x = 1 \Rightarrow x = \frac{1}{2}$

6. (b) Minor of element 9 = $\begin{vmatrix} 19 & 2 \\ 13 & 1 \end{vmatrix} = 19 - 26 = -7$

7. (b) We have $|A| = (a + ib)(a - ib) - (-c + id)(c + id)$
 $= a^2 + b^2 + c^2 + d^2 = 1.$

and $\text{adj}(A) = \begin{bmatrix} a - ib & -c - id \\ c - id & a + ib \end{bmatrix},$

then $A^{-1} = \begin{bmatrix} a - ib & -c - id \\ c - id & a + ib \end{bmatrix}$

8. (a) $\text{Adj}(A^{-1}) = (\text{Adj } A)^{-1}$ (Property).

9. (c) $A = \begin{bmatrix} -1 & x & 3 \\ -4 & -5 & -6 \\ -7 & y & 9 \end{bmatrix}$

Here, $A^{23} = (-1)[-y + 7x] = -7x + y$

$\Rightarrow -7x + y = 22$

$\Rightarrow A^{31} = (-1)^{3+1}[-6x + 15] = 27 \Rightarrow x = -2$... (i)

From eqn (i), $14 + y = 22 \Rightarrow y = 8$

Now $5x + y = 5x(-2) + 8 = -2$

10. (c) $|A| = \begin{vmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{vmatrix} = 0$

$\Rightarrow (x-3)^2 x = 0 \Rightarrow x = 0, 3.$

11. (d) $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$

$= 6i[3i^2 + 3] + 3i[4i + 20] + 1[12 - 60i]$

$= 6i[-3 + 3] + 12i^2 + 60i + 12 - 60i$

$= -12 + 12 = 0 = x + iy$

$\therefore x = 0$

12. (a) We have, $A = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \text{adj}(A) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

13. (d) $\text{adj } A = \begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}^T = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} = A$

$\therefore \text{adj } A = A.$

and $|A| = -4(0-4) + 3(3-4) - 3(4-0) = 1$

$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \text{adj } A = A$

14. (a) $|A| = 1(1-4) - 2(2-4) + 2(4-2)$
 (Expanding along R_1)

$= -3 + 4 + 4 = 5 \neq 0$

$\therefore A^{-1}$ exists.

So, $\text{adj } (A) = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}^T = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|} \cdot \text{adj } (A) = \frac{1}{5} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

15. (a) We have, $A^2 = I$

Since, A is non-singular matrix

$\therefore |A| \neq 0$, So, A^{-1} exists.

Multiplying (i) by A^{-1} on both sides, we get

$A^{-1}(A^2) = A^{-1}(I)$

$\Rightarrow (A^{-1}A)A = A^{-1}I \Rightarrow IA = A^{-1}$

$\therefore A^{-1} = A$

16. (c) Let $D = \begin{vmatrix} 1 & z & -y \\ -z & 1 & x \\ y & -x & 1 \end{vmatrix}$

Expanding along R_1

$\Rightarrow D = 1 \begin{vmatrix} 1 & x \\ -x & 1 \end{vmatrix} - z \begin{vmatrix} -z & x \\ y & 1 \end{vmatrix} - y \begin{vmatrix} -z & 1 \\ y & -x \end{vmatrix}$

$= (1+x^2) - z(-z-xy) - y(xz-y)$

$= 1+x^2+z^2+xyz-xyz+y^2$

$= 1+x^2+y^2+z^2 = 1+1=2$

17. (b) Given matrices are :

$A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

$AA^{-1} = I$

$\Rightarrow \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow 2x=1$

18. (c) $|\text{adj } A^{-1}| = |A^{-1}|^2 = \frac{1}{|A|^2}$

$\left| (\text{adj } A^{-1})^{-1} \right| = \frac{1}{|\text{adj } A^{-1}|} = |A|^2 = 2^2 = 4$

19. (a) $A^2 + A \cdot \text{adj } A = AA^{-1}$

$\Rightarrow A^2 = (1-|A|)I \Rightarrow |A|^2 = (1-|A|)^2 \Rightarrow |A| = 1/2$

$\Rightarrow |3A^{-1}| = 9|A^{-1}| = 9 \times 2 = 18$

20. (d) $P = \text{adj } (A) \Rightarrow |P| = |\text{adj } (A)| = |A|^{n-1} = |A|^3$

$|P| = \left| \frac{A}{2} \right| = \frac{1}{2^4} |A|$

$|A|^3 = \frac{1}{2^4} |A| \Rightarrow |A|^2 = \frac{1}{16} \Rightarrow |A| = \pm \frac{1}{4}$

$|A^{-1}| = \frac{1}{|A|} = \pm 4$

21. (a) $\therefore |A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix} = 3(-3+4) - 2(-3+4) + 0 = 1$

$\therefore \text{adj}(\text{adj } A) = |A|^{3-2} A = A$ and $|\text{adj}(\text{adj } A)| = |A| = 1$

Also, $|\text{adj } A| = |A|^2 = 1^2 = 1$

$\therefore |\text{adj}(\text{adj } A)| + |\text{adj } A| = 1 + 1 = 2$

22. (c) $B = A^{-1} \Rightarrow AB = I$

$\Rightarrow \frac{1}{10} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 \\ -5 & 0 & \alpha \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \frac{1}{10} \begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & 4+\alpha-9 \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow 4 + \alpha - 9 = 0 \Rightarrow \alpha = 5$

23. (b) $\|A^{-1}\|(AB)^T = \|A^{-1}\| \cdot (A \cdot \text{adj } A)^T$

$= \left| \frac{1}{|A|} \cdot (|A| \cdot I)^T \right| = \left| \frac{1}{|A|} \cdot |A| \cdot I^T \right| = |I^T| = 1$

24. (c) We have,

$\begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix}^{-1} \begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 14 & S \\ E & F \end{bmatrix}$

$\Rightarrow \frac{1}{(2A+1)A-20} \begin{bmatrix} A & 5 \\ 4 & 2A+1 \end{bmatrix} \begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 14 & S \\ E & F \end{bmatrix}$

On comparing corresponding elements, we get

$\frac{(A-5)A+10A-10}{2A^2+A-20} = 14$

$\Rightarrow A^2+5A-10 = 28A^2+14A-280$

$\Rightarrow 27A^2+9A-270=0 \Rightarrow A_1+A_2 = \frac{-9}{27}$

$\Rightarrow -27(A_1+A_2) = 9$

25. (a) We have $A(\alpha, \beta)^{-1}$

$$= \frac{1}{e^\beta} \begin{bmatrix} e^\beta \cos \alpha & -e^\beta \sin \alpha & 0 \\ e^\beta \sin \alpha & e^\beta \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= A(-\alpha, -\beta)$$

26. (c) Since, the matrix $\begin{bmatrix} 1 & -3 & 2 \\ 2 & -8 & 5 \\ 4 & 2 & \lambda \end{bmatrix}$ is not an invertible

matrix.

therefore it's determinant is zero.

$$\Rightarrow \begin{vmatrix} 1 & -3 & 2 \\ 2 & -8 & 5 \\ 4 & 2 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 1(-8\lambda - 10) + 3(2\lambda - 20) + 2(4 + 32) = 0$$

$$\Rightarrow -8\lambda - 10 + 6\lambda - 60 + 72 = 0$$

$$\Rightarrow -2\lambda + 2 = 0 \Rightarrow \lambda = 1$$

27. (d) $\therefore \Delta = (1 - 3 \sin \theta \cos \theta) - 3 \cos \theta (\sin \theta - 3 \cos \theta) + 1(\sin^2 \theta - 1)$

$$= 1 - 6 \sin \theta \cos \theta + 9 \cos^2 \theta + \sin^2 \theta - 1$$

$$= (\sin \theta - 3 \cos \theta)^2$$

$$\therefore -\sqrt{10} \leq \sin \theta - 3 \cos \theta \leq \sqrt{10}$$

$$\therefore 0 \leq (\sin \theta - 3 \cos \theta)^2 \leq 10$$

$$\therefore |\text{Maximum value of } \Delta - \text{Minimum value of } \Delta|^3$$

$$= |10 - 0|^3 = 1000$$

28. (a) Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ be a determinant of order 3.

$$\text{Then, } \Delta = (a_1 b_2 c_3 + a_3 b_1 c_2 + a_2 b_3 c_1) - (a_1 b_3 c_2 + a_2 b_1 c_3 + a_3 b_2 c_1)$$

Since each element of Δ is either 1 or 0, therefore the value of the determinant cannot exceed 3.

Clearly, the value of Δ is maximum when the value of each term in first bracket is 1 and the value of each term in the second bracket is zero. But $a_1 b_2 c_3 = a_3 b_1 c_2 = 1$ implies that every element of the determinant Δ is 1 and in that case $\Delta = 0$.

$$\text{Thus, we may have } \Delta = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 2$$

29. (c) $\therefore B = A^{-1} \Rightarrow |B| = \frac{1}{|A|}$

$$\text{Now, } |B| = \begin{vmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{vmatrix} = 2\alpha^2 - 2\alpha - 25$$

Given, $\det. (A) + 1 = 0$

$$\Rightarrow \frac{1}{2\alpha^2 - 2\alpha - 25} + 1 = 0$$

$$\Rightarrow \frac{2\alpha^2 - 2\alpha - 24}{2\alpha^2 - 2\alpha - 25} = 0$$

$$\Rightarrow \alpha = 4, -3 \Rightarrow \text{Sum of values} = 1$$

30. (c) Given $|\text{adj}(24A)| = |\text{adj } 3(\text{adj } 2A)|$

$$\Rightarrow |24a|^2 = |3 \text{adj}(2A)|^2$$

$$\Rightarrow (24^3 |A|)^2 = (3^3 |\text{adj}(2A)|)^2 \Rightarrow 3^6 (|2A|^2)^2$$

$$\Rightarrow 24^6 |A|^2 = (24^3 |A|)^2 = 3^6 \times 2^{12} |A|^4$$

$$\Rightarrow |A|^2 = \frac{24^6}{3^6 \times 2^{12}} = 64$$

31. (c) $\begin{vmatrix} 2 & 3 & 4 \\ 1 & k & 2 \\ 4 & 1 & 5 \end{vmatrix} = 0$

$$\Rightarrow 2(5k - 2) - 3 \times (-3) + 4(1 - 4k) = 0 \Rightarrow k = \frac{3}{2}$$

Required quadratic equation

$$\left(x - \frac{3}{2}\right)\left(x - \frac{2}{3}\right) = 0 \Rightarrow 6x^2 - 13x + 6 = 0$$

32. (d) If matrix A has no inverse, then $\det(A) = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & x \\ 1 & x & 1 \\ x & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow x^3 - 3x + 2 = 0$$

$$\Rightarrow x^2(x - 1) + x(x - 1) - 2(x - 1) = 0$$

$$\Rightarrow (x - 1)(x + 2)(x - 1) = 0$$

$$\Rightarrow (x - 1)^2(x + 2) = 0 \Rightarrow x = -2, 1, 1$$

Sum of distinct values of $x = -2 + 1 = -1$.

33. (c) We have $A + \begin{bmatrix} 1 & -5 & 7 \\ -9 & 2 & 6 \\ -10 & 5 & 2 \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\therefore A = \begin{bmatrix} 2 & 5 & -7 \\ 9 & 1 & -6 \\ 10 & -5 & 1 \end{bmatrix} \Rightarrow |A| = -18$$

$$\therefore \det\{(\text{adj } A) \cdot P^{-1}\} = \det(\text{adj } A) \det(P^{-1})$$

$$= (|A|)^2 \cdot \frac{1}{|P|} = \frac{(-18)^2}{6} = 54$$

$$34. (a) A = \begin{vmatrix} a & 1 & 2 \\ 1 & 2 & b \\ c & 1 & 3 \end{vmatrix}, \text{Adj}A = \begin{vmatrix} 7 & -1 & -5 \\ -3 & 9 & 5 \\ 1 & -3 & 5 \end{vmatrix}$$

$$\text{We know, } \text{Adj}A = \begin{vmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{vmatrix}$$

Where, c_{ij} is co-factor of a_{ij}

Now, $c_{11} = 7$

$$\Rightarrow \begin{vmatrix} 2 & b \\ 1 & 3 \end{vmatrix} = 7 \Rightarrow 6 - b = 7 \Rightarrow b = -1$$

Also, $c_{13} = 1$

$$\Rightarrow \begin{vmatrix} 1 & 2 \\ c & 1 \end{vmatrix} = 1 \Rightarrow 1 - 2c = 1 \Rightarrow c = 0 \text{ and } c_{33} = 5$$

$$\Rightarrow \begin{vmatrix} a & 1 \\ 1 & 2 \end{vmatrix} = 5 \Rightarrow 2a - 1 = 5 \Rightarrow a = 3$$

So, $a^2 + b^2 + c^2 = 9 + 1 + 0 = 10$

$$35. (a) |A| = \begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix} = 1(8 - 6) - 0 + 3(-3 - 4)$$

$$= -19 \neq 0$$

$\therefore A^{-1}$ exists.

(Expanding along C_1)

$$\text{Now } \text{adj}(A) = \begin{bmatrix} 2 & -9 & -6 \\ -8 & -2 & 5 \\ -7 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -8 & -7 \\ -9 & -2 & 3 \\ -6 & 5 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj}(A) = \frac{-1}{19} \begin{bmatrix} 2 & -8 & -7 \\ -9 & -2 & 3 \\ -6 & 5 & 2 \end{bmatrix}$$

$$\text{So, } (AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix} \frac{-1}{19} \begin{bmatrix} 2 & -8 & -7 \\ -9 & -2 & 3 \\ -6 & 5 & 2 \end{bmatrix}$$

$$= \frac{-1}{19} \begin{bmatrix} -16 & -12 & -1 \\ -21 & -11 & 7 \\ -10 & 2 & -3 \end{bmatrix} = \frac{1}{19} \begin{bmatrix} 16 & 12 & 1 \\ 21 & 11 & -7 \\ 10 & -2 & 3 \end{bmatrix}$$

$$36. (a) \text{ Let } A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} \text{ and } C = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$\text{Then, } |A| = \begin{vmatrix} 1 & -2 \\ 1 & 4 \end{vmatrix} = 6 \neq 0$$

So, A is invertible.

The given matrix equation is

$$B \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \Rightarrow BA = C \Rightarrow (BA)A^{-1} = CA^{-1}$$

$$\Rightarrow BA = CA^{-1} \Rightarrow B = CA^{-1}$$

$$\therefore \text{adj}A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj}A = \frac{1}{6} \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Now, $B = CA^{-1}$

$$\Rightarrow B = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \cdot \frac{1}{6} \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow B = \frac{1}{6} \begin{bmatrix} 24 + 0 & 12 + 0 \\ 0 - 6 & 0 + 6 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

37. (b) Given : A and B is a 3×3 matrix, So, $n = 3$

$$\frac{|\text{adj}B|}{|C|} = \frac{|\text{adj}(\text{adj}A)|}{|5A|} = \frac{|A|^{(3-1)^2}}{5^3 |A|}$$

$$[|\text{adj}(\text{adj}A)| = |A|^{(n-1)^2}]$$

$$= \frac{|A|^3}{5^3} = \frac{|A|^p}{5^q} \therefore p + q = 3 + 3 = 6$$

38. (b) Since, the matrix A is,

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \Rightarrow |A| = 4$$

$$\therefore \det [3\text{adj}(2A^{-1})] = |3 \cdot 2^2 \text{adj}(A^{-1})|$$

$$= 12^3 |\text{adj}(A^{-1})| = 12^3 |A^{-1}|^2 = \frac{12^3}{|A|^2} = \frac{12^3}{16} = 108$$

39. (a) Given $|\text{adj}(\text{adj}(\text{adj}A))| = 12^4$

$$\Rightarrow |A|^{(n-1)^3} = 12^4, \text{ and } n = 3$$

$$\Rightarrow |A|^8 = 12^4 \Rightarrow |A|^2 = 12 \Rightarrow |A| = 2\sqrt{3}$$

$$\text{Now, } |A^{-1} \cdot \text{adj}A| = |A^{-1}| \cdot |\text{adj}A| = \frac{1}{|A|} \cdot |A|^{3-1} = |A| = 2\sqrt{3}$$

40. (d) Equating co-factor of A_{21} with B_{21}

$$(2\alpha^2 - 3\alpha) = \alpha$$

$$\alpha = 0, 2 \text{ (accept)}$$

Now, Equating cofactor of A_{11} with B_{11}

$$2\alpha^2 - \alpha\beta = 3\alpha$$

$$\alpha = 2 \quad \beta = 1$$

$$|AB| = |A| \cdot |B| = |A| |\text{Adj}(A)^T| = |A| |A|^2 = |A|^3$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{bmatrix} = 6 - 2(9) + 3(6) = 6$$

$$\therefore |AB| = 216$$

41. (c) $Adj(A) = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}; (AdjA)^2 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$

$(AdjA)^n = \begin{bmatrix} 1 & -2n \\ 0 & 1 \end{bmatrix}; (AdjA)^{10} = \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} + \dots + \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 11 & -110 \\ 0 & 11 \end{bmatrix} \Rightarrow$ sum of elements of B

= -88

42. (b) Let $A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$

$\begin{bmatrix} a & b \\ b & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}, |A| = 1 \Rightarrow ad - b^2 = 1$

$a + b = 3 \Rightarrow a = 3 - b, b + d = 7 \Rightarrow d = 7 - b,$

$(3 - b)(7 - b) - b^2 = 1$

$21 - 10b = 1 \Rightarrow b = 2, a = 1, d = 5$

$A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}, A^{-1} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$

$A^{-1} = \alpha A + \beta I$

$\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \alpha + \beta & 2\alpha \\ 2\alpha & 5\alpha + \beta \end{bmatrix}$

$\alpha = -1, \beta = 6 \therefore \alpha + \beta = 5$

43. (a) Since, given $A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}, |A - d(adjA)| = 0$

$\Rightarrow |A - d(adjA)| = \begin{vmatrix} m & n \\ p & q \end{vmatrix} - d \begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$

$\left(\text{Since, } adj(A) = \begin{bmatrix} q & -n \\ -p & m \end{bmatrix} \right)$

$\Rightarrow \begin{vmatrix} m - qd & n(1 + d) \\ p(1 + d) & q - md \end{vmatrix} = 0$

$\Rightarrow (m - qd)(q - md) - np(1 + d)^2 = 0$

$\Rightarrow mq - m^2d - q^2d + mqd^2 - np(1 + d)^2 = 0$

$\Rightarrow (mq - np) + d^2(mq - np) - d(m^2 + q^2 + 2np) = 0$

$\Rightarrow d + d^3 - d((m + q)^2 - 2d) = 0$

$\Rightarrow 1 + d^2 = (m + q)^2 - 2d \Rightarrow (1 + d)^2 = (m + q)^2$

44. (c) Given, $A = \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} \frac{2}{5x} & 0 & \frac{7}{5x} \\ 0 & 1 & 0 \\ \frac{1}{5x} & -2 & \frac{1}{5x} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5x} & 0 & \frac{7}{5x} \\ 0 & 1 & 0 \\ \frac{1}{5x} & -2 & \frac{1}{5x} \end{bmatrix}$

Equating the matrices, we get

$\frac{2}{5x} = 2 \Rightarrow x = \frac{1}{5}$

So, $\begin{vmatrix} x & x+1 & x+2 \\ x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$\begin{vmatrix} x & x+1 & x+2 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{vmatrix} = 0 = 5 \times \frac{1}{5} - 1$

= $5x - 1$.

45. (c) $\therefore A = \begin{bmatrix} \alpha^2 & 5 \\ 5 & -\alpha \end{bmatrix}, \det(A) = -\alpha^3 - 25$

also we know that $\det(A^n) = (\det A)^n (n \in I)$

$\therefore \det(A^{10}) = (\det A)^{10} = 1024$

$\Rightarrow (-\alpha^3 - 25)^{10} = 1024 = (2)^{10} \Rightarrow -\alpha^3 - 25 = 2$

$\Rightarrow \alpha = -3$

46. (a) Expanding along R_1 ,

$\det(A) = 5(-25 - 1) - \sin^2\theta(-5\sin^2\theta - \cos^2\theta) + \cos^2\theta$

$(-\sin^2\theta + 5\cos^2\theta)$

$= -130 + 5\sin^4\theta + 5\cos^4\theta = -130 + 5[\sin^4\theta + \cos^4\theta]$

$= 5 - 130 - \frac{1}{2}\sin^2 2\theta$

for maximum value of $\det(A)$, $\sin 2\theta$ must be minimum i.e.,

$\sin 2\theta = 0$

$\therefore (\det A)_{\max} = 5 - 130 = -125$

47. (d) Since if we put $x = -1$

We can see R_1 and R_3 became identical so $A = 0$

Hence $x = -1$ is a root of the given equation

\Rightarrow from option, $x^2 - x - 2 = 0$

48. (d) Given that $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & \sin \theta & -\cos \theta \end{bmatrix}$

$|A| = -\cos^2\theta - \sin^2\theta = -1$

$adj(A) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \theta & -\sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix}^T$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore A + \text{adj}(A) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{Null matrix.}$$

$$A - A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{bmatrix} - (-1) \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{Null matrix}$$

$$A^{-1} + \text{adj}(A) = (-1) \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$+ \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{Null matrix}$$

$$49. \text{ (a)} \quad \begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$$

(Applying $C_1 \rightarrow C_1 + C_2 + C_3$)

$$\begin{vmatrix} x+1+\omega+\omega^2 & \omega & \omega^2 \\ x+1+\omega+\omega^2 & x+\omega^2 & 1 \\ x+1+\omega+\omega^2 & 1 & x+\omega \end{vmatrix} = 0$$

$$\begin{vmatrix} x & \omega & \omega^2 \\ x & x+\omega^2 & 1 \\ x & 1 & x+\omega \end{vmatrix} = 0$$

$$= x \begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & x+\omega^2 & 1 \\ 1 & 1 & x+\omega \end{vmatrix} = 0$$

$$\therefore x = 0$$

$$50. \text{ (d)} \quad \text{Given, } [x \ 1 \ 1] \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = [45]$$

$$\Rightarrow [x+11 \ 2x+13 \ 3x+15] \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = [45]$$

$$\Rightarrow [x+11+2x+13+3x^2+15x] = [45]$$

$$\Rightarrow 3x^2+18x+24=45$$

$$\Rightarrow x^2+6x-7=0$$

On solving we get, $x = -7$ and 1

51. (d) For singular matrix,

$$\begin{bmatrix} 0 & k & 4 \\ -k & 0 & -5 \\ -k & k & -1 \end{bmatrix} = 0$$

$$k(5k-k)+4(-k^2)=0$$

$$4k^2-4k^2=0$$

Hence, for all values of k , the given matrix is singular matrix.

52. (d) Putting $x = \pi$ in given function

$$f(\pi) = \begin{bmatrix} \cos\pi & \sin\pi \\ -\sin\pi & \cos\pi \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$[f(\pi)]^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

53. (d) Given $|A| = \frac{1}{2\sqrt{2}}$, $|B| = \frac{1}{729}$

$$|2B(\text{adj}(3A))| = |2B| |\text{adj}(3A)|$$

$$\text{We know that } |\text{adj} A| = |A|^{n-1} \text{ and } |kA| = k^n |A|$$

$$= 2^3 |B| |3A|^{3-1} = 8 |B| (3^3 |A|)^2$$

$$= 8 |B| 3^6 |A|^2 = 8 \cdot \frac{1}{729} \times 3^6 \left(\frac{1}{2\sqrt{2}}\right)^2 = 8 \times \frac{1}{8} = 1$$

54. (c)

$$55. \text{ (d)} \quad A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\text{So, } A^2 = A \cdot A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 8 & 6 \\ 5 & 7 & 6 \\ 6 & 6 & 5 \end{bmatrix}$$

$$\therefore |A^2| = 9$$

$$\text{Hence, } |\text{adj}(A^2)| = 9^2 = 81$$

$$56. \text{ (b) } A = \begin{bmatrix} 1 & 5 & 2 \\ 4 & 1 & 3 \\ 2 & 6 & 3 \end{bmatrix}$$

$$|A| = 1(3-18) - 5(12-6) + 2(24-2) = -1$$

$$\text{As, } |\text{Adj } A| = |A|^{n-1}$$

$$= |A|^{3-1} = (-1)^2 = 1$$

$$\text{Also, } |X^{-1}| = \frac{1}{|X|}$$

$$\Rightarrow |(\text{Adj } A)^{-1}| = \frac{1}{|\text{Adj } A|} = \frac{1}{1} = 1$$

$$57. \text{ (a) } |A| = 1(52-66) + 3(-26+42) - 5(22-28) = 64$$

$$\text{We know } |\text{Adj } A| = |A|^{n-1} \text{ [Here } n=3]$$

$$= |A|^2 = (64)^2$$

$$\therefore \sqrt{|\text{Adj } A|} = \sqrt{(64)^2} = 64$$

$$58. \text{ (a) We have given, } A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \text{ and } |A| = -1(-1) + 1 \cdot 1 = 2$$

$$\text{Hence, } A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \quad \dots(i)$$

$$\text{and } A^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\text{Now, } \frac{A^2 - 3I}{2} = \frac{1}{2} \left\{ \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right\}$$

$$= \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = A^{-1} \quad \text{[Using (i)]}$$

$$59. \text{ (a) } A^2 = A \cdot A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$$\text{Now, } |A^2| = 3(3-0) + 4(0) + 4(0-2) = 1$$

$$\text{Now, cofactors of } A^2 \text{ to obtain } \text{adj}(A^2)$$

$$C_{11} = 3, C_{12} = 0, C_{13} = -2, C_{21} = -4, C_{22} = -1,$$

$$C_{23} = 2, C_{31} = -4, C_{32} = 0, C_{33} = -3$$

$$\therefore \text{adj}(A^2) = \begin{bmatrix} 3 & 0 & -2 \\ -4 & -1 & 2 \\ -4 & 0 & -3 \end{bmatrix}^T = \begin{bmatrix} 3 & -4 & -4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$$\therefore (A^2)^{-1} = \frac{1}{|A^2|} \text{adj}(A^2) = \frac{1}{1} \begin{bmatrix} 3 & -4 & -4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} = A^2$$

$$60. \text{ (c) } |\text{adj } A| = |A|^{n-1} = |A|^2$$

$$|\text{adj } A^2| = |\text{adj } A|^2 = (|A|^2)^2 = |A|^4$$

$$|\text{adj}(\text{adj } A^2)| = (|A|^4)^{3-1} = |A|^8$$

$$61. \text{ (d) For unique solution } \Delta \neq 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow 1(k+2) - 1(2k+3) + 1(4-3) \neq 0$$

$$\Rightarrow -k + 2 - 3 + 1 \neq 0 \Rightarrow k \neq 0 \therefore S = R - \{0\}$$

$$62. \text{ (d) We have,}$$

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 3 & -1 & 5 \\ 1 & -3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\text{Here, } |A| = 0$$

Since, the system of linear equations has more than one solution, so $(\text{adj } A)(B) = 0$

$$\Rightarrow \begin{bmatrix} 13 & -13 & 13 \\ -1 & 1 & -1 \\ -8 & 8 & -8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow b = a + c \Rightarrow b - c - a = 0$$

$$63. \text{ (a) The given system of linear equations has a unique solution if}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 5 & \lambda \end{vmatrix} \neq 0$$

Expanding along R_1 , we get
 $\lambda - 8 \neq 0$ or $\lambda \neq 8$.

$$64. \text{ (c) } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 0$$

Since $\Delta = 0$, solution is not unique.

The system will have infinite solutions

$$\text{if } \Delta_1 = 0, \Delta_2 = 0, \Delta_3 = 0.$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ p & 2 & 4 \\ p^2 & 4 & 10 \end{vmatrix} = 0 \Rightarrow p^2 - 3p + 2 = 0 \Rightarrow p = 1 \text{ or } 2$$

$$\text{Required sum} = 1 + 2 = 3$$

$$65. \text{ (a) Given system has non-trivial solution then}$$

$$\begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} = 0 \Rightarrow \alpha_1 \beta_2 = \alpha_2 \beta_1$$

$$\Rightarrow \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = \frac{\alpha_1 + \alpha_2}{\beta_1 + \beta_2} = \sqrt{\frac{\alpha_1 \alpha_2}{\beta_1 \beta_2}}$$

$$\Rightarrow \frac{-b}{-a} = \sqrt{\frac{c}{r}} \Rightarrow \frac{bp}{aq} = \sqrt{\frac{cp}{ar}}$$

$$\Rightarrow b^2 p^2 ar = a^2 q^2 cp \Rightarrow b^2 pr = q^2 ac$$

66. (b) $A = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 3 & -2 \\ 2 & 7 & 7 \end{vmatrix} = 1(21+14) - 2(21+4) + 1(21-6) = 0$

So, system has no solution.

67. (b) We have $\Delta = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 3 & -3 \end{vmatrix} = 0$

Hence for at least one solution

If $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Rightarrow \Delta_1 = \begin{vmatrix} a & -2 & 1 \\ b & 1 & -2 \\ c & 3 & -3 \end{vmatrix} = 0 \Rightarrow a - b + c = 0$$

From $\Delta_2 = 0$ and $\Delta_3 = 0$, we get the same condition.

68. (c) For unique solution $\begin{vmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{vmatrix} \neq 0$

$$1(1-3\mu) - 1(-5-2\mu) + 1(15+2) \neq 0$$

$$\mu \neq 23 \text{ for all } \lambda \in \mathbb{R}$$

69. (a) For no real solution, $\begin{vmatrix} 1 & 1 & k \\ 2 & 2 & 0 \\ 1 & 2 & 2k \end{vmatrix} = 0$

$$1(4k-0) - 1(4k-0) + k(4-2) = 0 \Rightarrow k = 0$$

70. (a) We have, $\Delta = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix} = 0$

$$\Delta_x = \begin{vmatrix} a & 1 & 1 \\ b & -2 & 1 \\ c & 1 & -2 \end{vmatrix} = 3a + 3b + 3c = 3(a + b + c)$$

$$\Delta_y = \begin{vmatrix} -2 & a & 1 \\ 1 & b & 1 \\ 1 & c & -2 \end{vmatrix} = 3a + 3b + 3c = 3(a + b + c)$$

$$\Delta_z = \begin{vmatrix} -2 & 1 & a \\ 1 & -2 & b \\ 1 & 1 & c \end{vmatrix} = 3a + 3b + 3c = 3(a + b + c)$$

For no solution, $a + b + c \neq 0$

71. (b) For the system to have nontrivial solution $\Delta = 0$

$$\begin{vmatrix} \sin \theta & -2 \cos \theta & -a \\ -1 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

i.e., $\sin \theta + 4 \cos \theta = 3a$

$$\Rightarrow -\frac{\sqrt{17}}{3} \leq a \leq \frac{\sqrt{17}}{3}$$

So, a has three integer values.

72. (b) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 3 \\ 3 & 4 & 3 \end{vmatrix} = 3 - 2(12-9) + 3 \times (16-15) = 0$

For equation to be consistent

$$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 4 & 5 & 5 \\ 3 & 4 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 5\lambda - 20 - 2(4\lambda - 15) + 4 = 0 \Rightarrow \lambda = \frac{14}{3}$$

$$\Rightarrow 3\lambda = -14 = n + 100 \Rightarrow 14 = n + 100 \Rightarrow n = -86$$

73. (c) System of equation is given as :

$(k+1)x + 8y = 4k$... (i)

and $kx + (k+3)y = 3k-1$... (ii)

Here, $a_1 = k+1, b_1 = 8, c_1 = 4k, a_2 = k, b_2 = k+3$

and $c_2 = 3k-1$

Such a system of equations will have infinite number of solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \text{ i.e., } \frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1}$$

Taking last two we get

$$8(3k-1) = 4k(k+3)$$

$$\Rightarrow 24k - 8 = 4k^2 + 12k$$

$$\Rightarrow 4k^2 - 12k + 8 = 0$$

$$\Rightarrow k^2 - 3k + 2 = 0$$

$$\Rightarrow (k-1)(k-2) = 0$$

$$\Rightarrow k = 1, 2$$

Taking first two $(k+1)(k+3) = 8k$

$$\Rightarrow k^2 + 4k + 3 - 8k = 0 \Rightarrow k^2 - 4k + 3 = 0$$

$$\Rightarrow (k-1)(k-3) = 0$$

So, $k = 1, 3$.

Combining both, $k = 1, 2, 3$.

Thus, this system have 3 values of k .

74. (c) The given system of equations are :

$p^3x + (p+1)^3y = (p+2)^3$... (i)

$px + (p+1)y = (p+2)$... (ii)

$x + y = 1$... (iii)

This system is consistent, if values of x and y from first two equation satisfy the third equation.

$$\Rightarrow \begin{vmatrix} p^3 & (p+1)^3 & (p+2)^3 \\ p & (p+1) & (p+2) \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Applying $C_2 \rightarrow C_2 - C_1$

$$\Rightarrow \begin{vmatrix} p^3 & (p+1)^3 - p^3 & (p+2)^3 - p^3 \\ p & 1 & 2 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$\begin{aligned} \Rightarrow 2(p+1)^3 - 2p^3 - (p+2)^3 + p^3 &= 0 \\ \Rightarrow 2(p^3 + 1 + 3p^2 + 3p) - 2p^3 - (p^3 + 8 + 12p + 6p^2) + p^3 &= 0 \\ \Rightarrow 2p^3 + 2 + 6p^2 + 6p - 2p^3 - p^3 - 8 - 12p - 6p^2 + p^3 &= 0 \\ \Rightarrow -6 - 6p &= 0 \\ \Rightarrow p &= -1 \end{aligned}$$

75. (b) For infinitely many solutions,

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & b \\ 2 & 3 & -1 \end{bmatrix} = 0$$

$$1 - 3b + 1 + 2b + 5 = 0 \Rightarrow b = 7$$

From given 1st and 2nd equations, we get

$$2x + 8z = a + 2 \Rightarrow x + 4z = \frac{a+2}{2} \quad \dots(i)$$

From given 1st and 3rd equations

$$x + 4z = 3a - 1 \quad \dots(ii)$$

from equations (i) and (ii), we get

$$\frac{a+2}{2} = 3a - 1 \Rightarrow 5a = 4 \Rightarrow b - 5a = 7 - 4 = 3$$

76. (d) The given system of equations is

$$x + y + z = 2 \quad \dots(i)$$

$$2x + y - z = 3 \quad \dots(ii)$$

$$\text{and } 3x + 2y + kz = 4 \quad \dots(iii)$$

This system has a unique solution if

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$

$$\text{We get } \begin{vmatrix} 1 & 0 & 0 \\ 3 & -1 & -3 \\ 3 & -1 & k-3 \end{vmatrix} \neq 0$$

$$\Rightarrow -1(k-3) - 3 \neq 0 \text{ or } -k + 3 - 3 \neq 0 \Rightarrow k \neq 0$$

77. (d) For no solution

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{vmatrix} = 0$$

$$\Rightarrow 1(3a - 25) - 2(a - 10) + 3(5 - 6) = 0$$

$$\Rightarrow a = 8.$$

78. (b) The given system of equations is

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$\text{and } a_3x + b_3y + c_3z = d_3$$

$$\text{Let } \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

This system has a unique solution x_0, y_0, z_0 if $\Delta \neq 0$ and

$$x_0 = \frac{\Delta_x}{\Delta} = 0 \Rightarrow \Delta_x = 0$$

$$\Rightarrow \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = 0$$

79. (a) $x + 2y + 3z = 1$

$$2x + y + 3z = 2$$

$$5x + 5y + 9z = 4$$

Writing in matrix form, $Ax = B$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{bmatrix}$$

$$|A| = 1(9 - 15) - 2(18 - 15) + 3(10 - 5) = -6 - 2(3) + 3(5) = -12 + 15 = 3 \neq 0$$

So, these equations have unique solution

80. (b) For getting infinite solutions $\Delta = 0, \Delta_1 = \Delta_2 = \Delta = 0$ then check all the three equations

$$\text{Let } \Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = 0$$

$$\Rightarrow \delta = -3$$

$$\text{And } \Delta_1 = \begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ K & 4 & -3 \end{vmatrix} = 0 \Rightarrow K = 6$$

$$\Rightarrow \delta + K = 3$$

81. (b) We are given that

$$2x - 3y = \gamma + 5 \text{ and } \alpha x + 5y = \beta + 1$$

\therefore Here system of linear eqns has infinite many solution

$$\therefore \frac{\alpha}{2} = \frac{5}{-3} = \frac{\beta + 1}{\gamma + 5}$$

$$\Rightarrow \alpha = \frac{-10}{3}, 5\gamma + 25 = -3\beta - 3$$

$$\Rightarrow 9\alpha = -30, 3\beta + 5\gamma = -28 \Rightarrow 9\alpha + 3\beta + 5\gamma = -58$$

$$\therefore |9\alpha + 3\beta + 5\gamma| = 58$$

82. (b) For unique solution $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{vmatrix} \neq 0$

$$\Rightarrow 3a - 25 - 2(a - 10) + 3(5 - 6) \neq 0$$

$$\Rightarrow a - 8 \neq 0 \Rightarrow a \neq 8 \text{ and } b \in \mathbf{R}.$$

83. (b) Let $\Delta = \begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0$

$$3(-8a - 9) + 2(5a - 18) + 1(21) = 0$$

$$\Rightarrow a = -3$$

Now $\Delta_1 = \begin{vmatrix} b & -2 & 1 \\ 3 & -8 & 9 \\ -1 & 1 & -3 \end{vmatrix}$

$$\text{If } b = \frac{1}{3}$$

$$\Delta_1 = 0$$

For no solution $\Delta = 0$, and at least one of Δ_1 , Δ_2 or Δ_3 should be not equal to zero.

84. (c) For infinitely many solutions,

$$\Delta = 0 = \Delta_1 = \Delta_2 = \Delta_3$$

$$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \alpha(10 - 9) - 1(5 - 3) + 1(3 - 2) = 0$$

$$\Rightarrow \alpha - 2 + 1 = 0$$

$$\Rightarrow \alpha = 1$$

$$\Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 4 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow 5(10 - 9) - 1(20 - 3\beta) + 1(12 - 2\beta)$$

$$\Rightarrow 5 - 20 + 3\beta + 12 - 2\beta \Rightarrow -3 + \beta = 0$$

$$\Rightarrow \beta = 3$$

85. (d) Given system of linear equations is consistent when determinant is not equal to zero.

$$\Delta = \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} = 121 - k^2$$

Take $\Delta \neq 0$ $121 - k^2 \neq 0$, $k \neq \pm 11$

For $k = -11, 11$, the system has no solution.

Therefore, the system has unique solution with $k \in \mathbf{R} - \{11, -11\}$.

86. (b) Given system of linear equations are,

$$x + y + z = \alpha; \alpha x + 2\alpha y + 3z = -1; x + 3\alpha y + 5z = 4$$

System is inconsistent when $|\mathbf{A}| = 0$.

$$\mathbf{A} = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix} = 0$$

expand w.r.t. R_1 ,

$$\Rightarrow (\alpha - 1)^2 = 0 \Rightarrow \alpha = 1$$

Now, we have

$$x + y + z = 1 \quad \dots(i)$$

$$x + 2y + 3z = -1 \quad \dots(ii)$$

$$x + 3y + 5z = 4 \quad \dots(iii)$$

Multiply (i) by 2 and subtract (iii), then

$$x + y + z = -6 \quad \dots(iv)$$

So, equation (iv) does not satisfy all points of eq. (i).

Therefore, for $\alpha = 1$ the system of equation has inconsistent solution.

87. (b) Given that $\alpha + \beta + \gamma = 2\pi$

$$\begin{vmatrix} 1 & \cos \gamma & \cos \beta \\ \cos \gamma & 1 & \cos \alpha \\ \cos \beta & \cos \alpha & 1 \end{vmatrix}$$

$$\begin{aligned} &= 1 + 2\cos\alpha \cdot \cos\beta \cdot \cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma \\ &= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + (\cos(\alpha + \beta) + \cos(\alpha - \beta)) \cos\gamma \\ &= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + \cos^2\gamma + \cos(\alpha - \beta) \cos\gamma \\ &= \sin^2\alpha - \cos^2\beta + \cos(\alpha - \beta) \cos(\alpha + \beta) \end{aligned}$$

$$\therefore \cos(\alpha + \beta) \cdot \cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta$$

$$= \sin^2\alpha - \cos^2\beta + \cos^2\alpha - \sin^2\beta = 0$$

88. (a) $3x + 5y + \lambda z = 3$

$$7x + 11y - 9z = 2$$

$$97x + 155y - 189z = \mu$$

$$\begin{bmatrix} 3 & 5 & \lambda \\ 7 & 11 & -9 \\ 97 & 155 & -189 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ \mu \end{bmatrix}$$

$$\mathbf{AX} = \mathbf{B}$$

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$$

$$X = \frac{\text{adj}A}{|A|}B$$

For Infinitely many solution

$$|A| = 0 \text{ and } (\text{adj}A)B = 0$$

$$\begin{vmatrix} 3 & 5 & \lambda \\ 7 & 11 & -9 \\ 97 & 155 & -189 \end{vmatrix} = 0$$

$$\begin{aligned} -2052 + 2250 + 18\lambda &= 0 \\ \Rightarrow \lambda &= -11 \end{aligned}$$

$$\text{adj} A = \begin{bmatrix} -684 & -760 & 76 \\ 450 & 500 & -50 \\ 18 & 20 & -2 \end{bmatrix}$$

$$(\text{adj}A)B = \begin{bmatrix} 684 & -760 & 76 \\ 450 & 500 & -50 \\ 18 & 20 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ \mu \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \Rightarrow 54 + 40 - 2\mu &= 0 \\ \Rightarrow 2\mu &= 94 \\ \Rightarrow \mu &= 47 \\ \Rightarrow \mu + 2\lambda &= 25 \end{aligned}$$

89. (c) Since the system of equation

$$\begin{aligned} x + 2y + 3z &= 3 && \dots(i) \\ 4x + 3y - 4z &= 4 && \dots(ii) \\ 8x + 4y - \lambda z &= 9 + \mu && \dots(iii) \end{aligned}$$

Now

$$(i) \times 4 - (ii) \Rightarrow 5y + 16z = 8 \quad \dots(iv)$$

$$(ii) \times 2 - (iii) \Rightarrow 2y + (\lambda - 8)z = -1 - \mu \quad \dots(v)$$

$$(iv) \times 2 - (iii) \times 5 \Rightarrow (32 - 5(\lambda - 8))z = 16 - 5(-1 - \mu)$$

$$\text{For infinite solutions} \Rightarrow 72 - 5\lambda = 0 \Rightarrow \lambda = \frac{72}{5}$$

$$21 + 5\mu = 0 \Rightarrow \mu = \frac{-21}{5}$$

$$\text{thus, } (\lambda, \mu) = \left(\frac{72}{5}, \frac{-21}{5} \right)$$

90. (d) System of linear equation is inconsistent when $\Delta = 0$.

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 0$$

$$= (6 + 1) - 2(2\alpha - \alpha) + 1(\alpha + 3\alpha) = 0$$

$$= 7 - 2\alpha + 4\alpha = 0 = 7 + 2\alpha = 0 \text{ So, } \alpha = -\frac{7}{2}$$

For inconsistent solution, atleast one of $\Delta_1, \Delta_2, \Delta_3$ not equal to 0.

$$\text{Then } \Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 14 + 2\alpha$$

Now, put $\alpha = -\frac{7}{2}$ in above equation,

$$\Delta_1 = 14 + 2 \times \left(-\frac{7}{2} \right) = 7; \Delta_1 \neq 0$$

91. (b) Augmented matrix

$$[A : B] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 3 & 2 & -1 & 5 \\ 2 & -1 & -2 & -3 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 3R_1, R_3 \rightarrow R_3 - 2R_1$$

$$[A : B] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -1 & 2 & -13 \\ 0 & -3 & 0 & -15 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 3R_2$$

$$[A : B] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -1 & 2 & -13 \\ 0 & 0 & -6 & 24 \end{array} \right]$$

$$\Rightarrow -6z = 24 \Rightarrow z = -4 \Rightarrow -y + 2z = -13 \Rightarrow y = 5 = \beta$$

$$x + y - z = 6 \Rightarrow x = -3 = \alpha$$

$$\therefore \alpha + \beta = 5 - 3 = 2$$

92. (d) Equations are

$$x - y + 2z = 0 \quad \dots(i)$$

$$kx - y + z = 0 \quad \dots(ii)$$

$$3x + y - 3z = 0 \quad \dots(iii)$$

System of equations posses a unique solution, if

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ k & -1 & 1 \\ 3 & 1 & -3 \end{vmatrix} \neq 0$$

$$\text{Applying } C_1 \rightarrow C_1 + C_2 \text{ and } C_2 \rightarrow C_2 + \frac{1}{2}C_3$$

$$|A| = \begin{vmatrix} 0 & 0 & 2 \\ k-1 & -\frac{1}{2} & 1 \\ 4 & -\frac{1}{2} & -3 \end{vmatrix} = -\frac{1}{2} \begin{vmatrix} 0 & 0 & 2 \\ k-1 & -1 & 1 \\ 4 & -1 & -3 \end{vmatrix}$$

$$= \frac{1}{2} \times 2[(k-1)(-1) - (4)(-1)], 0$$

$$\Rightarrow -(-k + 1 + 4) \neq 0 \Rightarrow k - 5 \neq 0 \Rightarrow k \neq 5$$

Thus, the system does not possess unique solution, if $k = 5$

93. (d) Here, $A = \begin{bmatrix} 3 & -4 & 1 \\ 2 & 3 & -1 \\ 1 & -2 & -3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} -7 \\ 10 \\ 3 \end{bmatrix}$

$$\therefore |A| = 3(-9 - 2) + 4(-6 + 1) + 1(-4 - 3) = -60$$

$$A^{-1} = \frac{1}{-60} \begin{bmatrix} -11 & -14 & 1 \\ 5 & -10 & 5 \\ -7 & 2 & 17 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{-60} \begin{bmatrix} -11 & -14 & 1 \\ 5 & -10 & 5 \\ -7 & 2 & 17 \end{bmatrix} \begin{bmatrix} -7 \\ 10 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

$$\therefore x = \alpha = 1.$$

94. (a) We have, $\Delta = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{vmatrix} = 7\lambda + 35$

If $7\lambda + 35 \neq 0$ i.e., $\lambda \neq -5$, system has a unique solution. [$\Delta \neq 0 \Rightarrow$ unique solution]

But if $\lambda = -5$, we have $\Delta = 0$. Solution exists in this case if

$$\Delta_x = \Delta_y = \Delta_z = 0.$$

Now for $\lambda = -5$,

$$\text{and } \Delta_x = \begin{vmatrix} 3 & -1 & 4 \\ -2 & 2 & -3 \\ -3 & 5 & -5 \end{vmatrix} = 0, \Delta_y = \begin{vmatrix} 3 & 3 & 4 \\ 1 & -2 & -3 \\ 6 & -3 & -5 \end{vmatrix} = 0,$$

$$\Delta_z = \begin{vmatrix} 3 & -1 & 3 \\ 1 & 2 & -2 \\ 6 & 5 & -3 \end{vmatrix} = 0$$

Thus, $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ so there exists infinite number of solutions.

95. (b) $\Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$

$$\Rightarrow -1(1 - \cos^2 A) + \cos C(\cos A \cos B + \cos C + \cos B(\cos A \cos C + \cos B))$$

$$= -\sin^2 A + \cos^2 B + \cos^2 C + 2 \cos A \cos B \cos C$$

Clearly, we can observe that $\Delta > 0$.

Hence, system of equations has a non-zero solution.

96. (a) $\therefore \Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 1 & 5 & 10 \end{vmatrix} = 0$

\therefore Given system of equation is consistent.

Therefore, $\Delta_1 = 0$

$$\Rightarrow \Delta_1 = \begin{vmatrix} 1 & 2 & 1 \\ k & 3 & 4 \\ k^2 & 5 & 10 \end{vmatrix} = 0$$

$$\Rightarrow 1(30 - 20) - 2(10k - 4k^2) + (5k - 3k^2) = 0$$

$$\Rightarrow (k - 2)(k - 1) = 0 \Rightarrow k = 2, 1$$

Hence, the real values of k i.e.

$$A = 2 \text{ and } B = 1 \Rightarrow A + B = 2 + 1 = 3$$

97. (b) Let $\Delta = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix}$

$$= 1(1 - 4) + 1(3 - 4) + 2(3 - 1) = 0$$

$$\text{and } \Delta_1 = \begin{vmatrix} 4 & -1 & 2 \\ 6 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix} = 4(1 - 4) + 1(6 - 4) + 2(6 - 1)$$

$$= -12 + 2 + 10 = 0$$

\therefore These equations have infinitely many solutions.

98. (d) The given system can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 5 \\ 1 & 4 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix} \Rightarrow |A| = 0$$

As $|A| = 0$, this system can have no solution (or) infinitely many solutions. So we check $(\text{adj } A)B$.

$$\text{adj } A = \begin{bmatrix} -22 & 10 & 14 \\ 11 & -5 & -7 \\ 11 & -5 & -7 \end{bmatrix}$$

$$(\text{adj } A)B = \begin{bmatrix} -22 & 10 & 14 \\ 11 & -5 & -7 \\ 11 & -5 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \neq 0$$

Hence, there exist no solution

99. (d) $D = \begin{vmatrix} 1 & -2 & 3 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 42$

$D \neq 0$ therefore unique solution

$$D_2 = \begin{vmatrix} 1 & -2 & 4 \\ 3 & 1 & 7 \\ 2 & 3 & 6 \end{vmatrix} = 21; z = \frac{D_2}{D} = \frac{21}{42} = \frac{1}{2}.$$

100. (d) Since, system of equation have non trivial solution, then $\Delta = 0$

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ \lambda - 1 & 4\lambda - 2 & \lambda + 3 \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0 \Rightarrow \lambda = 0 \text{ or } 3$$

If $\lambda = 0$, the equations become

$$\begin{aligned} -x + y &= 0, \\ -x - 2y + 3z &= 0 \text{ and} \\ 2x + y - 3z &= 0, \end{aligned}$$

$$\begin{aligned} \therefore \frac{x}{6-3} &= \frac{y}{6-3} = \frac{z}{-1+4} \Rightarrow \frac{x}{3} = \frac{y}{3} = \frac{z}{3} \\ \Rightarrow x : y : z &= 1 : 1 : 1 \end{aligned}$$

Exercise 3 :

PREVIOUS YEARS MCQs

- (d) $|\text{Adj. } A| = |A|^2$
- (b) $|A| = 7, \text{adj}A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}, A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$
 $\therefore 14A^{-1} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$
- (a) $F(\alpha) \cdot F(-\alpha)$

$$\begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha + 0 & \cos \alpha \sin \alpha - \cos \alpha \sin \alpha + 0 & 0+0+0 \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha + 0 & \sin^2 \alpha + \cos^2 \alpha + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad [\because \cos^2 \alpha + \sin^2 \alpha = 1]$$

$$F(\alpha) \cdot F(-\alpha) = I \quad \therefore [F(\alpha)]^{-1} = F(-\alpha)$$

- (a) Let $A = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

$$\begin{aligned} \Rightarrow x(x^2 - 12) - 3(2x - 14) + 7(12 - 7x) &= 0 \\ \Rightarrow x^3 - 12x - 6x + 42 + 84 - 49x &= 0 \\ \Rightarrow x^3 - 67x + 126 &= 0 \end{aligned}$$

If $(x+9)$ is a factor of the given equation then
 $(x+9)(x^2 - 9x + 14) = 0 \Rightarrow x^2 - 9x + 14 = 0$
 Thus $(x-7)(x-2) = 0$ is the other factor.

- (a) Given determinant is $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

$$\text{We have } M_{31} = \begin{vmatrix} -3 & 5 \\ 0 & 4 \end{vmatrix} = -12 - 0 = -12$$

$$\Rightarrow A_{31} = M_{31} = -12$$

$$M_{32} = \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = 8 - 30 = -22$$

$$\Rightarrow A_{32} = -M_{32} = 22$$

$$M_{33} = \begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix} = 0 + 18 = 18 \Rightarrow A_{33} = M_{33} = 18$$

$$\begin{aligned} \therefore a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} \\ = (2)(-12) + (-3)(22) + (5)(18) \\ = -24 - 66 + 90 = -90 + 90 = 0 \end{aligned}$$

- (c) If $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and

$$A^{-1} = \frac{1}{k} \text{adj}(A) \quad \dots(i)$$

$$\text{Also, we know } A^{-1} = \frac{\text{adj}(A)}{|A|} \quad \dots(ii)$$

\therefore By comparing (i) and (ii)

$$|A| = k \Rightarrow |A| = \begin{vmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 3(2+1) + 2(1+0) + 4(1-0) = 9 + 2 + 4 = 15$$

- (a) We know that $\text{adj}(\text{adj } A) = |A|^{n-2} A$, if $|A| \neq 0$, provided order of A is n .
 $\therefore \text{adj}(\text{adj } A) = |A| A$ (as $n = 3$)
 $\therefore \det(\text{adj}(\text{adj } A)) = |A|^3 \det A = |A|^4$

$$\text{But } |A| = \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 14$$

$$\therefore \det(\text{adj}(\text{adj } A)) = (14)^4$$

- (a) The system is $0x_1 + x_2 - x_3 = 1$
 $-x_1 + 0x_2 + 2x_3 = 2$
 $x_1 - 2x_2 + 0x_3 = 3$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ or } AX = B$$

Clearly $|A| = 0$

$$\text{Now Adj } A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

$\therefore (\text{Adj } A) B \neq 0 \Rightarrow$ system is inconsistent

9. (a) $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$|A| = 0 - 1 = -1$

$\therefore A^{-1} = -1 \begin{bmatrix} 0 & -1 \\ -1 & x \end{bmatrix}$

$\because A = A^{-1} \Rightarrow x = 0$

10. (d) $\because (A - 2I)(A - 4I) = 0$
 $\Rightarrow A^2 - 4A - 2A + 8I = 0$
 $\Rightarrow A^2 - 6A + 8I = 0$

Multiply A^{-1} both sides we get :

$A^{-1} \cdot A \cdot A - 6A^{-1} \cdot A + 8A^{-1} \cdot I = A^{-1} \cdot 0$

$\Rightarrow IA - 6I + 8A^{-1} = 0$

$\Rightarrow A - 6I + 8A^{-1} = 0$

$\Rightarrow A + 8A^{-1} = 6I$

11. (b) $(A^2 - 5A)A^{-1} = A^2A^{-1} - 5AA^{-1} = A \cdot AA^{-1} - 5I$
 $= A - 5I$

$$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 2 & 3 \\ -1 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$$

12. (c) Here $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$

we know that,

$$a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33} = |A|$$

$$= +1(7-20) - 2(7-10) + 3(4-2)$$

$$= -13 + 6 + 6 = -1$$

13. (b) Consider $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

So, $|A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix}$

$= 1(3 \times (-1) - 0) - 0(3 \times (-1) - 0 - 0) + 0(3 \times 2 - 5 \times 3)$
 $= 1 \times (-3) - 0 - 0 = -3$

Now, $\text{adj } A$

$$= \begin{bmatrix} (3 \times (-1) - 0) & -3 \times (-1) - 0 & (3 \times 2 - 5 \times 3) \\ -(0 - 0) & (1 \times (-1) - 0) & -(2 \times 1 - 5 \times 0) \\ (3 \times 0 - 0) & -(1 \times 0 - 0) & (3 \times 1 - 0) \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-3} \begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

14. (d) Consider, $A = \begin{bmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{bmatrix}$

So,

$$|A| = \begin{vmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{vmatrix} = \alpha(9-2) - 14(6-6) - 1(4-18)$$

$= 7\alpha + 14$

As, inverse of matrix A does not exist.

Therefore, $|A| = 0$

$\Rightarrow 7\alpha + 14 = 0$

Hence, $\alpha = -2$

15. (b) Given, $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 5 \\ 1 & 2 & 1 \end{bmatrix}$

Since, the sum of the product of element other than the corresponding cofactor is zero.

$\therefore a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$

16. (a) It is given that $A = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

As, $(B^{-1}A^{-1})^{-1} = (A^{-1})^{-1}(B^{-1})^{-1} = AB$

$[\because (AB)^{-1} = B^{-1}A^{-1}]$

$[\because (A^{-1})^{-1} = A^{-1}]$

$$= \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+2 & -2+0 \\ 0+2 & 3+0 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$$

17. (c) Given, $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

and $AX = I$

$\Rightarrow X = A^{-1}I \Rightarrow X = A^{-1}$

Then, $A^{-1} = \frac{1}{|A|} \begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix}$

$$= \frac{1}{3-8} \begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}$$

18. (a) Given $|B| = K$ and $AB = I$ ($\therefore A^{-1} = B$)

$$|AB| = 1 \Rightarrow |A| = \frac{1}{|B|} = \frac{1}{K}$$

$$\text{adj}(\text{adj} A) = |A|^{3-2} A = |A| A = \frac{1}{K} A$$

$$(\text{adj}(\text{adj} A))^{-1} = \left| \frac{1}{K} A \right| = KA^{-1} = KB \left(\because (KA)^{-1} = \frac{1}{C} A^{-1} \right)$$

19. (c) $\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & a \\ -1 & -3 & b \end{vmatrix} = 0$

$$\Rightarrow 2a + b - 6 = 0 \quad \dots(i)$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & a+1 \\ -1 & -3 & 2b \end{vmatrix} = 0$$

$$\Rightarrow a + b - 8 = 0 \quad \dots(ii)$$

By adding eqs. (i) and (ii) we get $a = -2, b = 10$

$$\Rightarrow 7a + 3b = 7(-2) + 3(10) = 16$$

20. (c) Given, $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ and $P = \text{adj} A$

$$\Rightarrow |P| = 2\alpha - 6$$

We know, $|\text{adj} A| = |A|^{n-1}$

$$\therefore |P| = |A|^{3-1} \Rightarrow 2\alpha - 6 = (4)^2 \Rightarrow \alpha = 11$$

21. (c) For Infinitely many solutions,

$$\Delta = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$$

$$\Rightarrow 2(2\mu + 5) + \lambda(-4 - 3\mu) + 3(7) = 0$$

$$\Rightarrow 4\mu - 3\lambda\mu - 4\lambda + 31 = 0 \quad \dots(i)$$

$$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0$$

$$\Rightarrow 2(-17) + \lambda(1) + 5(7) = 0$$

$$\Rightarrow \lambda = -1$$

Similarly, $\Delta_2 = \Delta_1 = 0$

From equation (i),

$$4\mu + 3\mu + 4 + 31 = 0 \Rightarrow \mu = -5$$

$$\therefore \lambda^2 + \mu^2 = 26.$$

22. (b) $\text{Det}(A) = 6$

The given equation is

$$|\text{adj}(\text{adj} A)|(\text{adj} A)^{-1} = kA$$

$$|\text{adj}(\text{adj} A)| = |A|^{(3-1)^2} = |A|^4 \quad [A \text{ is } 3 \times 3 \text{ matrix}]$$

Also,

$$(\text{adj} A)^{-1} = \frac{\text{adj}(\text{adj} A)}{|\text{adj} A|} = \frac{|A|^{3-2} A}{|A|^{3-1}} = \frac{|A|}{|A|^2} A = \frac{A}{|A|}$$

$$\therefore |A|^4 \cdot \frac{A}{|A|} = kA$$

$$\text{We get } |A|^3 A = kA \Rightarrow k = |A|^3 = 6^3 = 216$$

23. (b) $\Delta = \begin{vmatrix} 3 & 1 & \beta \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$$3\alpha + 4\beta - \alpha\beta + 3 = 0$$

$$\Delta_3 = \begin{vmatrix} 3 & 1 & 3 \\ 2 & \alpha & -3 \\ 1 & 2 & 4 \end{vmatrix} = 0$$

$$9\alpha + 19 = 0 \Rightarrow \alpha = \frac{-19}{9}, \beta = \frac{6}{11}$$

$$\Rightarrow 22\beta - 9\alpha = 31$$

24. (a) $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & -1 & 2 \\ -1 & 1 & -2 \end{bmatrix} \Rightarrow |A| = 2$

And co-factor matrix of $A = \begin{bmatrix} 0 & 2 & 1 \\ 2 & -4 & -3 \\ 2 & -6 & -5 \end{bmatrix}$

$\Rightarrow \text{Adj}(A) = \begin{bmatrix} 0 & 2 & 2 \\ 2 & -4 & -6 \\ 1 & -3 & -5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \cdot \text{Adj}(A) \Rightarrow 2A^{-1} = \begin{bmatrix} 0 & 2 & 2 \\ 2 & -4 & -6 \\ 1 & -3 & -5 \end{bmatrix}$

$A + 2A^{-1} = \begin{bmatrix} 1 & 4 & 0 \\ 4 & -5 & -4 \\ 0 & -2 & -7 \end{bmatrix}$

25. (a) $|5 \cdot \text{adj } A| = 5 \Rightarrow 5^3 \cdot |A|^{3-1} = 5$
 $\Rightarrow 125 |A|^2 = 5 \Rightarrow |A| = \pm \frac{1}{5}$

26. (b) Given, $\text{Adj } A = \begin{bmatrix} 0 & 4 & -6 \\ 10 & 8 & 0 \\ 2 & 4 & -4 \end{bmatrix} \Rightarrow |\text{Adj } A| = 16,$

Hence, $|A| = 4$

Also, $\text{Adj}(\text{Adj } A) = \begin{bmatrix} -32 & -8 & 48 \\ 40 & 12 & -60 \\ 24 & 8 & -40 \end{bmatrix}$

$\therefore A = |A| [\text{Adj}(A)]^{-1} = \frac{|A|}{|\text{Adj}(A)|} \cdot \text{Adj}(\text{Adj}(A))$

$\Rightarrow A = \frac{4}{16} \cdot \text{Adj}(\text{Adj}(A)) = \begin{bmatrix} -8 & -2 & 12 \\ 10 & 3 & -15 \\ 6 & 2 & -10 \end{bmatrix}$

On comparison, we get

$a = -8, b = -2, c = 12, d = 10, e = 3, f = -15$

$\ell = 6, m = 2, n = -10$

Now, $\frac{cd}{fb} + \frac{ln}{em} = \frac{(12)(10)}{(-15)(-2)} + \frac{(6)(-10)}{(2)(3)} = -6$

also, $a + m = -8 + 2 = -6$

27. (b) $|A| = \begin{vmatrix} 1 & -3 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -1 \end{vmatrix} = -42$

Now, $A^2 \text{adj } A = A \cdot A \text{adj } A = A |A| = -42A$

28. (c) $|A| = \begin{vmatrix} 2 & -3 & 2 \\ 3 & 1 & -1 \\ 1 & -3 & -3 \end{vmatrix}$
 $= 2(-3-3) + 3(-9+1) + 2(-9-1)$
 $= -12 - 24 - 20 = -56.$

$\text{adj } A = \begin{bmatrix} -6 & -15 & 1 \\ 8 & -8 & 8 \\ -10 & 3 & 11 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj}(A) = \frac{1}{-56} \begin{bmatrix} -6 & -15 & 1 \\ 8 & -8 & 8 \\ -10 & 3 & 11 \end{bmatrix}$

$X = A^{-1} B$

$= \frac{1}{-56} \begin{bmatrix} -6 & -15 & 1 \\ 8 & -8 & 8 \\ -10 & 3 & 11 \end{bmatrix} \begin{bmatrix} -15 \\ -2 \\ -8 \end{bmatrix}$

$= \frac{1}{-56} \begin{bmatrix} 90 + 30 - 8 \\ -120 + 16 - 64 \\ 150 - 6 - 88 \end{bmatrix} = \frac{1}{-56} \begin{bmatrix} 112 \\ -168 \\ 56 \end{bmatrix}$

$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$

$\therefore 2\alpha + \beta = -4 + 3 = -1 = \gamma$

Exercise 1 :

WARM-UP
Topic-wise MCQs

1. (b) $\pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$

2. (a) $\frac{\pi}{6}, \pi - \frac{\pi}{6}$

3. (c) $n\pi + \frac{\pi}{3}$

4. (b) So, $\cos x = \cos\left(\frac{2\pi}{3}\right)$

General sol $x = 2n\pi \pm \frac{2\pi}{3}$

5. (c) Given $\tan^2 9x = -2\sin^2 x$

$$\Rightarrow 2\sin^2 x + \tan^2 9x = 0$$

Since, sum of two positive quantities = 0

So, each = 0, $\sqrt{2}\sin x = 0$, $\tan 9x = 0$

$$\Rightarrow x = 0 \text{ or } n\pi, 9x = 0 \text{ or } x = \frac{n\pi}{9}$$

Hence, common value of $x = n\pi$, $n \in \mathbf{I}$

6. (c) $\tan x = \tan \frac{5\pi}{6}$

General sol $x = n\pi + \frac{5\pi}{6}$

7. (a) $\cos x = \frac{\sqrt{3}}{2} = \cos \frac{\pi}{6}$

General sol $x = n\pi \pm \frac{\pi}{6}$

8. (b) Given, $\tan(\cot x) = \cot(\tan x) = \tan\left(\frac{\pi}{2} - \tan x\right)$

$$\Rightarrow \cot x = n\pi + \frac{\pi}{2} - \tan x \Rightarrow \cot x + \tan x = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \frac{1}{\sin x \cos x} = n\pi + \frac{\pi}{2} \Rightarrow \frac{1}{\sin 2x} = \frac{n\pi}{2} + \frac{\pi}{4}$$

$$\Rightarrow \sin 2x = \frac{1}{\frac{n\pi}{2} + \frac{\pi}{4}} = \frac{4}{(2n+1)\pi}$$

9. (b) $\tan x = 1 = \tan \frac{\pi}{4}$

General sol $x = n\pi \pm \frac{\pi}{4}$

10. (c) $\cos \theta = \frac{1}{\sqrt{2}} = \cos \frac{7\pi}{4} \Rightarrow \tan \theta = -1 = \tan \frac{7\pi}{4}$

11. (a) $\sin \theta = \frac{1}{2} = \sin \frac{\pi}{6}$ and $\cos \theta = \frac{\sqrt{3}}{2} = \cos \frac{\pi}{6}$

General sol $\theta = 2n\pi + \frac{\pi}{6}$

12. (d) Since $x + y = \frac{2\pi}{3}$... (i)

And, $\cos x + \cos y = \frac{3}{2}$... (ii)

From (ii), $2\cos \frac{x+y}{2} \cos \frac{x-y}{2} = \frac{3}{2}$

or $2\cos \frac{\pi}{3} \cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$ [By (i)]

or $\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$

This is not possible

Hence the given system of equations has no solution.

13. (a) $\cos \theta = -\frac{1}{\sqrt{2}} = \cos \frac{5\pi}{4}$, $\tan \theta = 1 = \cos \frac{5\pi}{4}$

General sol $\theta = 2n\pi + \frac{5\pi}{4}$, $n \in \mathbf{I}$

14. (a)

15. (b) We have, $y + \cos x = \sin x \Rightarrow y = \sin x - \cos x$

As $-\sqrt{2} \leq \sin x - \cos x \leq \sqrt{2}$

so, $-\sqrt{2} \leq y \leq \sqrt{2}$

16. (c)

17. (d) $5\cos 2\theta + 2\cos^2 \frac{\pi}{2} + 1 = 0$, $-\pi < \pi < \pi$

$$\Rightarrow 10\cos^2 \theta + \cos \theta - 3 = 0$$

$$\cos \theta = -\frac{3}{5} \text{ or } \cos \theta = \frac{1}{2} = \cos \frac{\pi}{3}$$

18. (b) We have, $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$$\Rightarrow 1 = \frac{1}{1 - \tan x \tan y}$$
 [Using given equations]

$$\Rightarrow \tan x \tan y = 0 \Rightarrow \text{Either } \tan x = 0 \text{ or } \tan y = 0$$

$$\therefore \text{Either } x = n\pi \text{ and } y = \frac{\pi}{4} - x = \frac{\pi}{4} - n\pi$$

or $y = m\pi$ and $x = \frac{\pi}{4} - m\pi$

19. (a)

20. (c) $1 + \cot \theta = \operatorname{cosec} \theta$

$$\Rightarrow \frac{1}{\sin \theta} = 1 + \frac{\cos \theta}{\sin \theta} \Rightarrow \sin \theta + \cos \theta = 1$$

$$\sin \theta \sin \frac{\pi}{4} + \cos \theta \cos \frac{\pi}{4} = \cos \frac{\pi}{4}$$

$$\theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}. \text{ Hence, } \theta = 2n\pi \text{ or } \theta = 2n\pi + \frac{\pi}{2}$$

But, $\theta = 2n\pi$ is ruled out

21. (d) $\tan \theta = -\frac{1}{\sqrt{3}} = \tan \left(-\frac{\pi}{6}\right) \therefore \theta = n\pi - \frac{\pi}{6}$

22. (a) $2 \tan^2 \theta = \sec^2 \theta = 1 + \tan^2 \theta \Rightarrow \tan^2 \theta = 1 = (1)^2$
 $= \tan^2 \frac{\pi}{4}$

$$\theta = n\pi \pm \frac{\pi}{4}, n \in \mathbb{I}$$

23. (b) $\sin 3\alpha = 4 \sin \alpha \sin(x + \alpha) \sin(x - \alpha)$
 $\therefore \sin 3\alpha = 4 \sin \alpha (\sin^2 x \cos^2 \alpha - \cos^2 x \sin^2 \alpha)$
 $\therefore 3 \sin \alpha - 4 \sin^3 \alpha = 4 \sin \alpha (\sin^2 x - \sin^2 \alpha)$

$$\therefore \sin^2 x = \left(\frac{3}{4}\right) \Rightarrow \sin^2 x = \sin^2 \frac{\pi}{3} \therefore x = n\pi \pm \frac{\pi}{3}$$

24. (b) $\tan \theta + \tan\left(\frac{\pi}{2} - \theta\right) = 2 \Rightarrow \tan \theta + \frac{1}{\tan \theta} = 2$

$$\Rightarrow \tan^2 \theta - 2 \tan \theta + 1 = 0$$

$$\Rightarrow \tan \theta = 1 = \tan \frac{\pi}{4} \Rightarrow \theta = n\pi + \frac{\pi}{4}$$

25. (b) $\sin^2 \theta \sec \theta + \sqrt{3} \tan \theta = 0$

$$\therefore (\sin^2 \theta + \sqrt{3} \sin \theta) \sec \theta = 0$$

$$\therefore \sin \theta (\sin \theta + \sqrt{3}) \sec \theta = 0 \Rightarrow \sin \theta = 0$$

$$\therefore \theta = n\pi, n \in \mathbb{I} \left[\because \sin \theta \neq -\sqrt{3}, \sec \theta \neq 0 \right]$$

26. (c) $\sec 4\theta - \sec 2\theta = 2$

$$\therefore \cos 2\theta - \cos 4\theta = 2 \cos 4\theta \cos 2\theta$$

$$\therefore -\cos 4\theta = \cos 6\theta \Rightarrow 2 \cos 5\theta \cos \theta = 0$$

$$\therefore \theta = n\pi + \frac{\pi}{2} \text{ or } \frac{n\pi}{5} + \frac{\pi}{10}$$

27. (d) $\tan \theta \tan 2\theta = 1 \therefore \tan \theta \frac{2 \tan \theta}{1 - \tan^2 \theta} = 1$

$$\therefore 2 \tan^2 \theta = 1 - \tan^2 \theta \therefore 3 \tan^2 \theta = 1 \therefore \theta = n\pi \pm \frac{\pi}{6}$$

28. (a) $4 \sin^2 \theta + 2(\sqrt{3} + 1) \cos \theta = 4 + \sqrt{3}$

$$\Rightarrow 4 - 4 \cos^2 \theta + 2(\sqrt{3} + 1) \cos \theta = 4 + \sqrt{3}$$

$$\Rightarrow 4 \cos^2 \theta - 2(\sqrt{3} + 1) \cos \theta + \sqrt{3} = 0$$

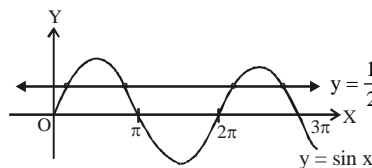
$$\Rightarrow \cos \theta = \frac{2(\sqrt{3} + 1) \pm \sqrt{4(\sqrt{3} + 1)^2 - 16\sqrt{3}}}{8}$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \text{ or } \frac{1}{2} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{6} \text{ or } 2n\pi \pm \frac{\pi}{3}$$

29. (d) We have $\sin^2 \theta - \sin \theta - 2 = 0 \Rightarrow (\sin \theta + 1)(\sin \theta - 2) = 0$

$$\text{As } \sin \theta \neq 2 \therefore \sin \theta = -1 = \sin \frac{3\pi}{2}$$

$$\therefore \theta = \frac{3\pi}{2} = \frac{6\pi}{4} \in \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$$



30. (a)

$$2 \sin^2 x + 5 \sin x - 3 = 0 \Rightarrow (\sin x + 3)(2 \sin x - 1) = 0$$

$$\Rightarrow \sin x = \frac{1}{2} \text{ and } \sin x \neq -3 \therefore \text{In } [0, 3\pi], x \text{ has 4 values.}$$

31. (d) $\cot \theta + \cot\left(\frac{\pi}{4} + \theta\right) = 2$

$$\therefore \frac{\cos \theta}{\sin \theta} + \frac{\cos\left(\frac{\pi}{4} + \theta\right)}{\sin\left(\frac{\pi}{4} + \theta\right)} = 2$$

$$\therefore \sin\left(\frac{\pi}{4} + 2\theta\right) = 2 \sin \theta \sin\left(\frac{\pi}{4} + \theta\right)$$

$$= \cos\left(\theta - \frac{\pi}{4} - \theta\right) - \cos\left(\theta + \frac{\pi}{4} + \theta\right)$$

$$\therefore \sin\left(\frac{\pi}{4} + 2\theta\right) = \cos\left(\frac{-\pi}{4}\right) - \cos\left(2\theta + \frac{\pi}{4}\right)$$

$$\Rightarrow \sin\left(\frac{\pi}{4} + 2\theta\right) + \cos\left(\frac{\pi}{4} + 2\theta\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}} \cos 2\theta + \frac{1}{\sqrt{2}} \sin 2\theta\right)$$

$$+ \left(\frac{1}{\sqrt{2}} \cos 2\theta - \frac{1}{\sqrt{2}} \sin 2\theta\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{2}{\sqrt{2}} \cos 2\theta = \frac{1}{\sqrt{2}} \Rightarrow \cos 2\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$\Rightarrow 2\theta = 2n\pi \pm \frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$$

32. (c) $\cot \theta + \tan \theta = 2 \operatorname{cosec} \theta$

$$\Rightarrow \frac{1}{\sin \theta \cos \theta} = \frac{2}{\sin \theta} \Rightarrow \cos \theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$$

33. (c) $\cot \theta = -\sqrt{3} = -\cot \frac{\pi}{6} = \cot\left(\pi - \frac{\pi}{6}\right) = \cot \frac{5\pi}{6}$

$$\theta = \frac{5\pi}{6} \text{ or } -\frac{\pi}{6} \quad \dots(i)$$

$$\operatorname{cosec} \theta = -2 = -\operatorname{cosec} \frac{\pi}{6} = \operatorname{cosec}\left(-\frac{\pi}{6}\right)$$

$$\theta = -\frac{\pi}{6} \text{ or } -\frac{5\pi}{6} \quad \dots(ii)$$

$$\text{Common value} = -\frac{\pi}{6}$$

34. (a) We have, $\sin x = -\frac{\sqrt{3}}{2} = -\sin \frac{\pi}{3}$
 Hence, $\sin x = \sin \frac{4\pi}{3}$, which gives
 $x = n\pi + (-1)^n \frac{4\pi}{3}$, where $n \in \mathbb{Z}$.
35. (b) Let $A = \{\theta : \sin \theta = \tan \theta\}$ and $B = \{\theta : \cos \theta = 1\}$
 Now, $A = \left\{ \theta : \sin \theta = \frac{\sin \theta}{\cos \theta} \right\} = \{\theta : \sin \theta (\cos \theta - 1) = 0\}$
 $= \{\theta = 0, \pi, 2\pi, 3\pi, \dots\}$
 For $B : \cos \theta = 1 \Rightarrow \theta = \pi, 2\pi, 4\pi, \dots$
 This shows that A is not contained in B . i.e. $A \not\subset B$. but $B \subset A$.
36. (d) $\sin 4\theta + 2\sin 4\theta \cos 3\theta = 0 \Rightarrow \sin 4\theta(1 + 2\cos 3\theta) = 0$
 $\sin 4\theta = 0$ or $\cos 3\theta = -\frac{1}{2}$
 $4\theta = n\pi; n \in I$ or $3\theta = 2n\pi \pm \frac{2\pi}{3}, n \in I$
 $\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ or $\theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}$ [$\because \theta \in (0, \pi)$]
37. (d) $\tan(A - B) = 1 \Rightarrow A - B = 45^\circ$ or 225°
 $\sec(A + B) = \frac{2}{\sqrt{3}} \Rightarrow A + B = 30^\circ$ or 330°
 $A + B = 330^\circ = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$... (i)
 and $A - B = 225^\circ = \frac{5\pi}{4}$... (ii)
 Solving (i) and (ii), we get
 $2B = \frac{11\pi}{6} - \frac{5\pi}{4} \Rightarrow 2B = \frac{7\pi}{12} \Rightarrow B = \frac{7\pi}{24}$.
38. (c) Given equation is $\cos x - \sin x = \frac{1}{\sqrt{2}}$
 Dividing equation by $\sqrt{2}$,
 $\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x = \frac{1}{2} \Rightarrow \cos\left(\frac{\pi}{4} + x\right) = \cos \frac{\pi}{3}$
 $\Rightarrow \frac{\pi}{4} + x = 2n\pi \pm \frac{\pi}{3}; x = 2n\pi + \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi + \frac{\pi}{12}$
 or $x = 2n\pi - \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi - \frac{7\pi}{12}$.
39. (c) We have, $\sec \theta + \tan \theta = \sqrt{3}$... (i)
 $\Rightarrow \sec \theta - \tan \theta = \frac{1}{\sqrt{3}}$... (ii)
 [$\because \sec^2 \theta - \tan^2 \theta = 1$]
 By solving (i) and (ii), we get
 $\tan \theta = \frac{1}{2} \left(\sqrt{3} - \frac{1}{\sqrt{3}} \right) = \frac{1}{\sqrt{3}} \therefore \tan \theta = \tan \left(\frac{\pi}{6} \right)$
 $\Rightarrow \theta = n\pi + \frac{\pi}{6}$

- \therefore Solutions for $0 \leq \theta \leq 2\pi$ are $\frac{\pi}{6}$ and $\frac{7\pi}{6}$.
 Hence, there are two solutions.
40. (b) We have $\sec^2 \theta = \frac{4}{3} \Rightarrow \cos^2 \theta = \frac{3}{4}$
 $\Rightarrow \cos^2 \theta = \cos^2 \left(\frac{\pi}{6} \right) \Rightarrow \theta = n\pi \pm \frac{\pi}{6} \dots \dots \dots$
41. (b) The given equation is $\tan x + \sec x = 2 \cos x$;
 $\Rightarrow \sin x + 1 = 2\cos^2 x \Rightarrow \sin x + 1 = 2(1 - \sin^2 x)$;
 $\Rightarrow 2\sin^2 x + \sin x - 1 = 0 \Rightarrow (2\sin x - 1)(\sin x + 1) = 0$
 $\Rightarrow \sin x = \frac{1}{2}, -1 \Rightarrow x = 30^\circ, 150^\circ, 270^\circ$.
42. (d) Since, $\cos \theta = x + \frac{1}{x} \Rightarrow \cos \theta = \frac{x^2 + 1}{x}$
 $\Rightarrow x^2 - x \cos \theta + 1 = 0$
 For real values of x , $(-\cos \theta)^2 - 4 \times 1 \times 1 \geq 0$
 $\cos^2 \theta \geq 4$; $\cos \theta \geq 2$ and $\cos \theta \leq -2$ [$-1 \leq \cos \theta \leq 1$]
 which is not possible So, no value of θ is possible.
43. (c) $x + \frac{1}{x} = 2 \cos \theta$ (given) ... (i)
 $\left(x + \frac{1}{x} \right)^3 = x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} \left(x + \frac{1}{x} \right)$ By using (i)
 $8\cos^3 \theta = x^3 + \frac{1}{x^3} + 3(2 \cos \theta)$
 $x^3 + \frac{1}{x^3} = 8\cos^3 \theta - 6 \cos \theta$
 $= 2(4\cos^3 \theta - 3\cos \theta) = 2 \cos 3\theta$
44. (d) $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$
 $\Rightarrow \frac{\cos A}{2R \sin A} = \frac{\cos B}{2R \sin B} = \frac{\cos C}{2R \sin C}$
 $\Rightarrow \cot A = \cot B = \cot C$
 $\Rightarrow A = B = C = 60^\circ \Rightarrow \Delta ABC$ is equilateral
 Hence, $\Delta = \frac{\sqrt{3}}{4} a^2 = \sqrt{3}$.
45. (b) Since, $A + B + C = \pi$. Therefore,
 $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$
 $\Rightarrow xy + yz + zx = 1$... (i)
 where $x = \tan \frac{A}{2}, y = \tan \frac{B}{2}, z = \tan \frac{C}{2}$.
 We know that $(x - y)^2 + (y - z)^2 + (z - x)^2 \geq 0$
 $\Rightarrow 2\Sigma x^2 \geq 2\Sigma xy \Rightarrow \Sigma x^2 \geq \Sigma xy$
 $\Rightarrow \Sigma x^2 \geq 1$ [$\because \Sigma xy = 1$ (From(i))]
 $\Rightarrow \tan^2 \frac{A}{2} + \tan^2 \frac{B}{2} + \tan^2 \frac{C}{2} \geq 1$

46. (c)
$$\frac{2 \cos A}{a} + \frac{\cos B}{b} + \frac{2 \cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$$

$$\Rightarrow \frac{2(b^2 + c^2 - a^2)}{2abc} + \frac{a^2 + c^2 - b^2}{2abc} + \frac{2(a^2 + b^2 - c^2)}{2abc}$$

$$= \frac{a}{bc} + \frac{b}{ca}$$

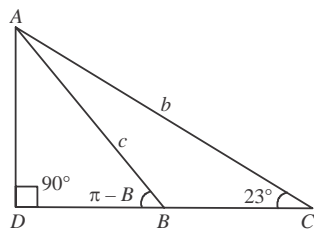
$$\Rightarrow \frac{3b^2 + c^2 + a^2}{2abc} = \frac{a}{bc} + \frac{b}{ca} = \frac{a^2 + b^2}{abc}$$

$$\Rightarrow 3b^2 + c^2 + a^2 = 2a^2 + 2b^2$$

$$\Rightarrow b^2 + c^2 = a^2. \text{ Hence } \angle A = 90^\circ.$$

47. (c)
$$AD = c \sin B = \frac{\sin A \sin B}{\sin^2 B - \sin^2 C}$$

$\therefore \sin(B + C) \sin(B - C) = \sin A$
 $\sin(B - C) = 1 = \sin 90^\circ \quad [\because A + B + C = \pi]$
 $B - C = 90^\circ$
 $B = 90^\circ + C = 90^\circ + 23^\circ$
 $\angle B = 113^\circ$



48. (c) Let $a = \alpha - \beta, b = \alpha + \beta, c = \sqrt{3\alpha^2 + \beta^2}$
 $\therefore \cos C = \frac{a^2 + b^2 - c^2}{2ab}$
 $\Rightarrow \cos C = \frac{\alpha^2 + \beta^2 - 2\alpha\beta + \alpha^2 + \beta^2 + 2\alpha\beta - 3\alpha^2 - \beta^2}{2(\alpha^2 - \beta^2)}$
 $\Rightarrow \cos C = -\frac{(\alpha^2 - \beta^2)}{2(\alpha^2 - \beta^2)} = \cos\left(\frac{2\pi}{3}\right)$
 $\Rightarrow \angle C = \frac{2\pi}{3}$ (largest angle).

49. (d) We know by sine law in $\triangle ABC$ as

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin(\pi - A - B)} = 2R$$

$$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin(A + B)} = 2R$$

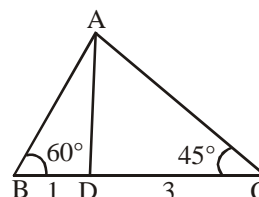
- (a) If we know $a, \sin A, \sin B$ we can find b, c ; values of $\angle's A, B$ and C all.
- (b) Using a, b, c , we can find $\angle A, \angle B, \angle C$ using cosine law.
- (c) $a, \sin B, R$ are given then $\sin A, b$ and hence $\sin(A + B)$ and then C can be found.
- (d) If we know $a, \sin A, R$ then we know only the ratio $\frac{b}{\sin B}$ or $\frac{c}{\sin(A + B)}$, we can not determine the values of $b, c, \sin B, \sin C$ separately.
 $\therefore \Delta$ can not be determined in this case.

50. (a) By sine rule, the sine of the angles are proportional to the lengths of opposite sides.

i.e.,
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Given a $\triangle ABC$ in which $\angle ABD = 60^\circ$

and $\angle ACD = 45^\circ$, also $\frac{BD}{DC} = \frac{1}{3}$



From $\triangle ABD$, (By sine Rule)

$$\frac{\sin \angle BAD}{BD} = \frac{\sin 60^\circ}{AD} \quad \dots(i)$$

Now, in $\triangle ADC$;

$$\frac{\sin \angle CAD}{DC} = \frac{\sin 45^\circ}{AD} \quad \dots(ii)$$

Eqⁿ (i) divided by (ii)

$$\frac{\sin \angle BAD}{\sin \angle CAD} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{1} \times \frac{1}{3} = \frac{\sqrt{3}}{\sqrt{2} \times 3} = \frac{1}{\sqrt{6}}$$

51. (c)
$$\frac{b - c}{a} = \frac{\sin B - \sin C}{\sin A} = \frac{2 \sin \frac{B - C}{2} \cos \frac{B + C}{2}}{2 \sin \frac{A}{2} \cos \frac{A}{2}}$$

$$= \frac{\sin \frac{B - C}{2}}{\cos \frac{A}{2}}$$

$$\Rightarrow (b - c) \cos \frac{A}{2} = a \sin \frac{B - C}{2}$$

52. (a) $(a + c)^2 - b^2 = 3ac \Rightarrow a^2 + c^2 - b^2 = ac$

But $\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2} \Rightarrow B = \frac{\pi}{3}$

53. (a) From the given relation

$$\sin C = \frac{1 - \cos A \cos B}{\sin A \sin B} \leq 1 \quad \dots(i)$$

$$\Rightarrow 1 \leq \cos A \cos B + \sin A \sin B$$

$$\Rightarrow \cos(A - B) \geq 1 \quad \because \cos \theta \leq 1 \quad \dots(ii)$$

$$\therefore A - B = 0 \text{ or } A = B$$

Hence from (i), $\sin C = \frac{1 - \cos^2 A}{\sin^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1$

$\therefore C = 90^\circ \Rightarrow A + B = 90^\circ \text{ or } A = B = 45^\circ \quad [\text{By (ii)}]$

Hence $a : b : c = \sin A : \sin B : \sin C = 1 : 1 : \sqrt{2}$

54. (b) $\sin^2 A + \sin^2 B + \sin^2 C$
 $= 1 - \cos^2 A + 1 - \cos^2 B + \sin^2 C$
 $= 2 - \cos^2 A - \cos(B+C)\cos(B-C)$
 $= 2 - \cos A[\cos A - \cos(B-C)]$
 $= 2 - \cos A[-\cos(B+C) - \cos(B-C)]$
 $= 2 + \cos A \cdot 2 \cos B \cos C$
 $\therefore \sin^2 A + \sin^2 B + \sin^2 C - 2 \cos A \cos B \cos C = 2$

55. (d) $a^2 + a^2 + 4b^2 - 4ab = 2ac - c^2$
 $\Rightarrow (a-2b)^2 + (a-c)^2 = 0$
 which is possible only when
 $a-2b=0$ and $a-c=0$

or $\frac{a}{1} = \frac{b}{1/2} = \frac{c}{1} = \lambda$ (say)
 $\therefore a = \lambda, b = \lambda/2, c = \lambda$

$\therefore \cos B = \frac{a^2 + c^2 - b^2}{2ac}$
 $= \frac{\lambda^2 + \lambda^2 - \frac{\lambda^2}{4}}{2\lambda^2} = 1 - \frac{1}{8} = \frac{7}{8}$

56. (a) $\frac{\cot A + \cot B}{\cot C} = \frac{\sin(A+B)}{\sin A \sin B} \cdot \frac{\sin C}{\cos C}$
 $= \frac{\sin^2 C}{\sin A \sin B \cos C} = \frac{c^2}{ab} \cdot \frac{2ab}{a^2 + b^2 - c^2}$
 $= \frac{2c^2}{a^2 + b^2 - c^2} = \frac{2c^2}{17c^2 - c^2} = \frac{9}{4} = \frac{m}{n}$

$\Rightarrow (m+n) = 9+4 = 13$

57. (b) If $A+B+C = \pi$,
 then $\cos mA + \cos mB + \cos mC$

$= 1 - 4 \sin \frac{mA}{2} \sin \frac{mB}{2} \sin \frac{mC}{2}$

\therefore For $m=2$

$\cos 2A + \cos 2B + \cos 2C = 1 - 4 \sin A \sin B \sin C$
 $\Rightarrow \cos 2A + \cos 2B + \cos 2C + 4 \sin A \sin B \sin C = 1$

58. (d) We have, $\frac{\tan\left(\frac{B}{2}\right)}{\cot\left(\frac{C-A}{2}\right)} = \frac{\sin \frac{B}{2} \sin\left(\frac{C-A}{2}\right)}{\cos \frac{B}{2} \cos\left(\frac{C-A}{2}\right)}$
 $= \frac{\sin C - \sin A}{\sin C + \sin A} = \frac{kc - ka}{kc + ka} = \frac{c-a}{c+a} = \frac{a}{3a} = \frac{1}{3} (\because c=2a)$

59. (c) $\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2 - b^2}{a^2 + b^2}$
 $\Rightarrow \frac{\sin(A-B)}{\sin(A+B)} = \frac{\sin^2 A - \sin^2 B}{\sin^2 A + \sin^2 B}$

$\Rightarrow \frac{\sin(A-B)}{\sin C} = \frac{\sin(A-B) \sin(A+B)}{\sin^2 A + \sin^2 B}$

$\Rightarrow \sin(A-B) \left[\frac{1}{\sin C} - \frac{\sin C}{\sin^2 A + \sin^2 B} \right] = 0$

Either $\sin(A-B) = 0 \Rightarrow A = B$ i.e. isosceles or $\sin^2 A + \sin^2 B = \sin^2 C$ or $a^2 + b^2 = c^2$ i.e., right angled triangle.

60. (d) Here $\tan \frac{A}{2} \tan \frac{C}{2} = \frac{s-b}{s}$

$\frac{5}{6} \cdot \frac{2}{5} = \frac{s-b}{s} \Rightarrow 3s - 3b = s \Rightarrow 2s = 3b$
 $\Rightarrow a + b + c = 3b$ or $a + c = 2b$.

$\therefore a, b, c$ are in A.P., also $\sin A, \sin B, \sin C$ are in A.P.

61. (b) $(a+b+c)(a+b-c) = 3ab$

$\Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2} \Rightarrow \cos C = \cos \frac{\pi}{3} \Rightarrow \angle C = \pi/3$

62. (c) $ab^2 \cos A + ba^2 \cos B + ac^2 \cos A + ca^2 \cos C$
 $+ bc^2 \cos B + b^2 c \cos C$
 $= ab(b \cos A + a \cos B) + ac(c \cos A + a \cos C)$
 $+ bc(c \cos B + b \cos C)$
 $= abc + abc + abc = 3abc$.

63. (a) $8R^2 = a^2 + b^2 + c^2 = 4R^2(\sin^2 A + \sin^2 B + \sin^2 C)$
 $\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C = 2$
 $\Rightarrow (\cos^2 A - \sin^2 C) + \cos^2 B = 0$
 $\Rightarrow \cos(A-C) \cos(A+C) + \cos^2 B = 0$
 $\Rightarrow 2 \cos A \cos B \cos C = 0$
 So that, $\cos A = 0$ or $\cos B = 0$ or $\cos C = 0$

$\Rightarrow A = \frac{\pi}{2}$ or $B = \frac{\pi}{2}$ or $C = \frac{\pi}{2}$.

64. (b) $s-a=3 \Rightarrow b+c-a=6$... (i)
 $s-c=2 \Rightarrow a+b-c=4$... (ii)

Adding these two equations, we get $b=5$

Since B is a right angle

$\therefore b^2 = a^2 + c^2 \Rightarrow a^2 + c^2 = 25$... (iii)

Solving, we get $a=3, c=4$.

65. (c) It is given that area of $\Delta ABC = \text{Area of } \Delta DEF$

$\Rightarrow \frac{1}{2} (AB)(AC) \sin A = \frac{1}{2} (DE)(EF) \sin E$

$\Rightarrow \sin A = \sin E \Rightarrow \sin 2E = \sin E \Rightarrow 2E = \pi - E$

$\Rightarrow E = \frac{\pi}{3} \Rightarrow A = 2E = \frac{2\pi}{3}$.

66. (b) We have $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow b^2 - 2bc \cos A + (c^2 - a^2) = 0$

It is given that b_1 and b_2 are roots of this equation.

Therefore $b_1 + b_2 = 2c \cos A$ and $b_1 b_2 = c^2 - a^2$

$\Rightarrow 3b_1 = 2c \cos A, 2b_1^2 = c^2 - a^2, (\because b_2 = 2b_1 \text{ given})$

$$\Rightarrow 2\left(\frac{2c}{3} \cos A\right)^2 = c^2 - a^2$$

$$\Rightarrow 8c^2(1 - \sin^2 A) = 9c^2 - 9a^2$$

$$\Rightarrow \sin A = \sqrt{\frac{9a^2 - c^2}{8c^2}}$$

67. (a) We have, $\cos A = \frac{c^2 + b^2 - a^2}{2bc}$
 $\Rightarrow c^2 - 2bc \cos A + (b^2 - a^2) = 0$
 It is given that c_1 and c_2 are roots of this equation.
 Let C_1 and C_2 be the angles corresponding to the sides c_1 and c_2 respectively

Therefore $c_1 + c_2 = 2b \cos A$ and $c_1 c_2 = b^2 - a^2$
 $\Rightarrow k(\sin C_1 + \sin C_2) = 2k \sin B \cos A$
 $\Rightarrow \sin C_1 + \sin C_2 = 2 \sin B \cos A$
 Now sum of the areas of two triangles

$$= \frac{1}{2} ab \sin C_1 + \frac{1}{2} ab \sin C_2 = \frac{1}{2} ab (\sin C_1 + \sin C_2)$$

$$= \frac{1}{2} ab (2 \sin B \cos A) = ab \sin B \cos A$$

$$= b (a \sin B) \cos A = b (b \sin A) \cos A = \frac{1}{2} b^2 \sin 2A.$$

68. (b) $\Sigma a^3 \cos(B - C) = \Sigma a^2 [k \sin A \cos(B - C)]$
 $= k \Sigma a^2 \sin(B + C) \cos(B - C)$
 $= k \Sigma a^2 (\sin 2B + \sin 2C)$
 $= k \Sigma a^2 (2 \sin B \cos B + 2 \sin C \cos C)$
 $= \Sigma a^2 (b \cos B + c \cos C)$
 $= a^2 (b \cos B + c \cos C) + b^2 (c \cos C + a \cos A)$
 $\qquad\qquad\qquad + c^2 (a \cos A + b \cos B)$
 $= ab (a \cos B + b \cos A) + ac (a \cos C + c \cos A)$
 $\qquad\qquad\qquad + bc (b \cos C + c \cos A)$
 $= abc + abc + abc = 3abc$

69. (a) $\Sigma a^2 (\cos^2 B - \cos^2 C) = \Sigma a^2 (\sin^2 C - \sin^2 B)$
 $= k^2 \Sigma a^2 (c^2 - b^2) = 0.$

70. (a) $2a^2 b^2 + 2b^2 c^2 = a^4 + b^4 + c^4$
 Also, $(a^2 - b^2 + c^2)^2$
 $= a^4 + b^4 + c^4 - 2(a^2 b^2 + b^2 c^2 - c^2 a^2)$
 $\Rightarrow (a^2 - b^2 + c^2)^2 = 2c^2 a^2$
 $\Rightarrow \frac{a^2 - b^2 + c^2}{2ca} = \pm \frac{1}{\sqrt{2}} = \cos B$

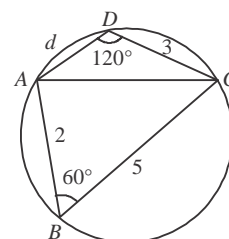
$$\Rightarrow B = 45^\circ \text{ or } 135^\circ.$$

71. (b) Since A, B, C are in A.P., therefore $B = 60^\circ$
 $[\because A + B + C = 180 \text{ and } A + C = 2B]$

Now, $\sin(2A + B) = \frac{1}{2}$ (given)
 $\Rightarrow 2A + B = 30^\circ \text{ or } 150^\circ$
 But as $B = 60^\circ$, $2A + B \neq 30^\circ$.
 Hence $2A + B = 150^\circ \Rightarrow A = 45^\circ$
 Hence $A = 45^\circ, B = 60^\circ, C = 75^\circ$

72. (a) Let the fourth side be of 'd' length.
 We know that $AB^2 + BC^2 - 2AB \cdot BC \cos 60^\circ = AC^2$
 $= CD^2 + DA^2 - 2CD \cdot DA \cos 120^\circ$ [By cosine rule]

or $4 + 25 - 2 \cdot 2 \cdot 5 \cdot \frac{1}{2} = 9 + d^2 + 3d$
 $\Rightarrow d^2 + 3d - 10 = 0$
 $\Rightarrow d = -5 \text{ or } d = 2$
 $\therefore d = 2.$



73. (c) Since $\cos 3A + \cos 3B + \cos 3C = 1$
 $\Rightarrow 4 \sin \frac{3A}{2} \sin \frac{3B}{2} \sin \frac{3C}{2} = 0$
 Either $\frac{3A}{2} = 180^\circ$ or $\frac{3B}{2} = 180^\circ$ or $\frac{3C}{2} = 180^\circ$
 Either $A = 120^\circ$ or $B = 120^\circ$ or $C = 120^\circ$

74. (c) Consider a triangle ABC.
 Given, angles of a triangle are in the ratio 4 : 1 : 1.
 Angles are $4x, x$ and x .
 i.e., $\angle A = 4x, \angle B = x, \angle C = x$
 Now, by angle sum property of Δ , we have
 $\angle A + \angle B + \angle C = 180^\circ$
 $\Rightarrow 4x + x + x = 180^\circ \Rightarrow x = \frac{180^\circ}{6} = 30^\circ$
 $\therefore \angle A = 120^\circ, \angle B = 30^\circ, \angle C = 30^\circ$

We know, ratio of sides of ΔABC is given by
 $\sin A : \sin B : \sin C = \sin 120^\circ : \sin 30^\circ : \sin 30^\circ$
 $= \frac{\sqrt{3}}{2} : \frac{1}{2} : \frac{1}{2} = \sqrt{3} : 1 : 1$

Required ratio = $\frac{\sqrt{3}}{1+1+\sqrt{3}} = \frac{\sqrt{3}}{2+\sqrt{3}}$.

75. (b) $A + B = 180^\circ - C = 90^\circ$
 $a = 2R \sin A, b = 2R \sin B, c = 2R \sin C$

$$\therefore \frac{a^2 - b^2}{a^2 + b^2} = \frac{\sin^2 A - \sin^2 B}{\sin^2 A + \sin^2 B}$$

$$= \frac{\sin(A+B) \sin(A-B)}{\sin^2 A + \sin^2(90^\circ - A)} \quad [\because A+B=90^\circ]$$

$$= \frac{\sin 90^\circ \sin(A-B)}{\sin^2 A + \cos^2 A} = \sin(A-B)$$

76. (b) From cosine and sine formula, we have

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \text{ and } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k.$$

Given, in any ΔABC ,

$$\cos A = \frac{\sin B}{2 \sin C}$$

From the above given formula, we have
 $\sin B = bk, \sin C = ck$

$$\therefore \cos A = \frac{bk}{2ck} = \frac{b}{2c}$$

Put the value of $\cos A$ in the formula, which gives

$$\frac{b}{2c} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow c^2 = a^2 \Rightarrow c = a$$

77. (a) Since $A + B + C = \pi$

$$\therefore \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2} \Rightarrow \frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$$

$$\therefore \tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cot \frac{C}{2}$$

$$\Rightarrow \frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \tan \frac{B}{2}} = \frac{1}{\tan \frac{C}{2}}$$

$$\Rightarrow \tan \frac{A}{2} \tan \frac{C}{2} + \tan \frac{B}{2} \tan \frac{C}{2} = 1 - \tan \frac{A}{2} \tan \frac{B}{2}$$

$$\Rightarrow \tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$$

78. (d) Use $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$ and $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$

79. (c) $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$ and $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$

80. (c) $0 < 3 < \pi$, $\pi < 5 < 2\pi$, $2\pi < 7 < 3\pi$

81. (a) $\frac{3 \sin 2\theta}{4 + 5 \cos 2\theta} = \frac{6 \tan \theta}{9 + \tan^2 \theta} = \frac{2t}{1+t^2}$, where $t = \frac{\tan \theta}{3}$

82. (a) We have

$$\alpha + \beta = \left(\sin^{-1} \frac{\sqrt{3}}{2} + \cos^{-1} \frac{\sqrt{3}}{2} \right) + \left(\sin^{-1} \frac{1}{3} + \cos^{-1} \frac{1}{3} \right)$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

Since $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for all x

$$\text{Also, } \alpha = \frac{\pi}{3} + \sin^{-1} \frac{1}{3} < \frac{\pi}{3} + \sin^{-1} \frac{1}{2}$$

as $\sin \theta$ is increasing in $\left[0, \frac{\pi}{2}\right]$

$$\therefore \alpha < \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2} \Rightarrow \beta > \frac{\pi}{2} > \alpha \Rightarrow \alpha < \beta$$

83. (b) $\sin^{-1} \frac{1+x^2}{2x}$ is defined if $\left| \frac{1+x^2}{2x} \right| \leq 1 \Rightarrow x = \pm 1$

$$84. (b) \frac{1}{1+r+r^2} = \frac{1}{1+r(r+1)} = \frac{\frac{1}{r(r+1)}}{1 + \frac{1}{r(r+1)}} = \frac{\frac{1}{r} - \frac{1}{r+1}}{1 + \frac{1}{r} \left(\frac{1}{r+1} \right)}$$

$$\therefore \tan^{-1} \left(\frac{1}{1+r+r^2} \right) = \tan^{-1} \frac{1}{r} - \tan^{-1} \frac{1}{r+1}$$

$$\therefore \sum_{r=1}^{\infty} \tan^{-1} \left(\frac{1}{1+r+r^2} \right) = \tan^{-1} 1 = \frac{\pi}{4}$$

85. (a) $\tan^{-1} \left(\frac{x}{3} \right) + \tan^{-1} \left(\frac{x}{2} \right) = \tan^{-1} x$

$$\text{or } \tan^{-1} \left(\frac{x/3 + x/2}{1 - x^2/6} \right) = \tan^{-1} x$$

where $x > 0$ and $x^2/6 < 1 \Rightarrow -\sqrt{6} < x < \sqrt{6}$

$$\text{Now, } \left(\frac{5x}{6-x^2} \right) = x \Rightarrow x = 0 \text{ or } x^2 - 1 = 0 \Rightarrow x = \pm 1.$$

Therefore, $x = \{-1, 0, 1\} \Rightarrow 3$ solutions.

86. (a) $2 \cos^{-1} x = \cot^{-1} \left(\frac{2x^2 - 1}{2x\sqrt{1-x^2}} \right)$

Put $x = \cos \theta$: L.H.S. = 2θ ;

$$0 \leq \theta \leq \pi \text{ and } -1 \leq x \leq 1 \quad \dots(i)$$

R.H.S.

$$= \cot^{-1} \left(\frac{\cos 2\theta}{2 \cos \theta |\sin \theta|} \right) = \cot^{-1} (\cot 2\theta) = 2\theta$$

$$\text{if } 0 < 2\theta < \pi \quad \dots(ii)$$

From (i) and (ii), we get $0 < \theta < \pi/2$

$$\therefore x \in (0, 1)$$

87. (d) $\cos^{-1} \left(\sqrt{\frac{2}{3}} \right) - \cos^{-1} \left(\frac{\sqrt{6}+1}{2\sqrt{3}} \right)$

$$= \tan^{-1} \left(\frac{1}{\sqrt{2}} \right) - \tan^{-1} \left(\frac{\sqrt{3}-\sqrt{2}}{1+\sqrt{6}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{1}{\sqrt{2}} - \frac{\sqrt{3}-\sqrt{2}}{1+\sqrt{6}}}{1 + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}(1+\sqrt{6})}} \right) = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = \frac{\pi}{6}.$$

88. (a) Let $\sin^{-1} \frac{1}{a} = \theta$; $\sin^{-1} \frac{1}{b} = \phi$ then $\sin^{-1} \frac{1}{x} = \theta + \phi$

$$\Rightarrow \sin \sin^{-1} \frac{1}{x} = \sin(\theta + \phi)$$

$$\Rightarrow \frac{1}{x} = \sin \theta \cos \phi + \cos \theta \sin \phi$$

$$= \frac{1}{a} \sqrt{1 - \frac{1}{b^2}} + \sqrt{1 - \frac{1}{a^2}} \cdot \frac{1}{b} = \frac{\sqrt{b^2 - 1}}{ab} + \frac{\sqrt{a^2 - 1}}{ab}$$

$$\Rightarrow x = \frac{ab}{\sqrt{a^2 - 1} + \sqrt{b^2 - 1}}$$

89. (d) We have

$$\cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) + \cos^{-1} \left(\frac{1-y^2}{1+y^2} \right) = \frac{\pi}{2}$$

Put $x = \tan \theta$ and $y = \tan \phi$, we get

$$\cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) + \cos^{-1} \left(\frac{1 - \tan^2 \phi}{1 + \tan^2 \phi} \right) = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} (\cos 2\theta) + \cos^{-1} (\cos 2\phi) = \frac{\pi}{2}$$

$$\Rightarrow 2(\theta + \phi) = \frac{\pi}{2} \Rightarrow \theta + \phi = \frac{\pi}{4}$$

So, $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1} \left(\frac{x+y}{1-xy} \right) = \tan^{-1} 1$$

$$\Rightarrow x + y = 1 - xy \Rightarrow x + y + xy = 1$$

90. (a) $\cot^{-1} (\sqrt{\cos \alpha}) - \tan^{-1} (\sqrt{\cos \alpha}) = x$

$$\tan^{-1} \left(\frac{1}{\sqrt{\cos \alpha}} \right) - \tan^{-1} (\sqrt{\cos \alpha}) = x$$

$$\Rightarrow \tan^{-1} \frac{\frac{1}{\sqrt{\cos \alpha}} - \sqrt{\cos \alpha}}{1 + \frac{1}{\sqrt{\cos \alpha}} \cdot \sqrt{\cos \alpha}} = x$$

$$\Rightarrow \tan^{-1} \frac{1 - \cos \alpha}{2\sqrt{\cos \alpha}} = x$$

$$\Rightarrow \tan x = \frac{1 - \cos \alpha}{2\sqrt{\cos \alpha}} \text{ or } \cot x = \frac{2\sqrt{\cos \alpha}}{1 - \cos \alpha}$$

[Considering a Δ with perpendicular

$$= (1 - \cos \alpha) \text{ and base } = 2\sqrt{\cos \alpha}]$$

$$\Rightarrow \sin x = \frac{1 - \cos \alpha}{1 + \cos \alpha} = \frac{1 - (1 - 2 \sin^2 \alpha / 2)}{1 + 2 \cos^2 \alpha / 2 - 1}$$

$$\text{or } \sin x = \tan^2 \frac{\alpha}{2}$$

91. (c) $\cos^{-1} (\cos(-5)) + \sin^{-1} (\sin(6)) - \tan^{-1} (\tan(12))$

$$[\because 1 \text{ radian} = 57.27^\circ]$$

$$= (2\pi - 5) + (6 - 2\pi) - (12 - 4\pi) = 4\pi - 11$$

92. (b) $\cos^{-1} \left(\frac{3}{\sqrt{9x^2 - 12x + 22}} \right)$

Let $\frac{3}{\sqrt{9x^2 - 12x + 22}} = u$

$$9x^2 - 12x + 22 = 9 \left(x^2 - \frac{4}{3}x + \left(\frac{2}{3} \right)^2 - \left(\frac{2}{3} \right)^2 \right) + 22$$

$$= 9 \left(x - \frac{2}{3} \right)^2 + 18$$

Minimum value of $\sqrt{9x^2 - 12x + 22}$ is $\sqrt{18} = 3\sqrt{2}$

$$\Rightarrow \sqrt{9x^2 - 12x + 22} \geq 3\sqrt{2} \Rightarrow \frac{1}{\sqrt{9x^2 - 12x + 22}} \leq \frac{1}{3\sqrt{2}}$$

So, $u \leq \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$

Apply \cos^{-1}

$$\cos^{-1} \left(\frac{1}{\sqrt{2}} \right) \leq \cos^{-1}(u) < \cos^{-1}(0)$$

[$\because \cos^{-1}(u)$ is decreasing function]

$$\frac{\pi}{4} \leq \cos^{-1}(u) < \frac{\pi}{2}$$

Range of $f(x)$ is $\left[\frac{\pi}{4}, \frac{\pi}{2} \right)$

93. (b) Doman of $\cos^{-1}(n)$ is $[-1, 1]$

Domain of $\cos^{-1}(1-n)$ is $[0, 2]$

\therefore Domain of $\cos^{-1}(1-n) - 2\cos^{-1}n$ is $[0, 1]$

Put $n = 0 \Rightarrow \cos^{-1}(1) - 2\cos^{-1}0 = 0 - \pi = -\pi \neq \frac{\pi}{2}$

Put $n = 1 \Rightarrow \cos^{-1}(0) - 2\cos^{-1}1 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$\therefore n = 1$ is solution.

Since $\cos^{-1}(1-n)$ and $2\cos^{-1}(n)$ both are decreasing function in $(0, 1)$. So strictly decreasing function can

intersect a constart value (like $\frac{\pi}{2}$) at most once.

Hence only one solution $n = 1$

94. (c) $\tan \left(2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} \right)$

$$= \tan \left[\tan^{-1} \frac{2 \cdot \frac{1}{3}}{1 - \left(\frac{1}{3} \right)^2} + \tan^{-1} \frac{1}{7} \right]$$

$$= \tan \left[\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7} \right]$$

$$= \tan \left[\tan^{-1} \frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \cdot \frac{1}{7}} \right] = \tan[\tan^{-1} 1] = 1$$

95. (c) $\theta = \tan^{-1} \left(\frac{1}{3} \right) + \tan^{-1} \left(\frac{1}{7} \right) + \tan^{-1} \left(\frac{1}{13} \right) +$

$$\tan^{-1} \left(\frac{1}{21} \right) + \tan^{-1} \left(\frac{1}{31} \right)$$

General term

$$\begin{aligned}
 &= \tan^{-1}\left(\frac{1}{1+n(n+1)}\right) = \tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right) \\
 &= \tan^{-1}(n+1) - \tan^{-1}n \\
 \therefore \theta &= \tan^{-1}(2) - \tan^{-1}(1) + \tan^{-1}3 - \tan^{-1}2 + \tan^{-1}4 \\
 &\quad - \tan^{-1}3 + \tan^{-1}5 - \tan^{-1}4 + \tan^{-1}6 - \tan^{-1}5 \\
 &= \tan^{-1}6 - \tan^{-1}1 = \tan^{-1}\frac{6-1}{1+6} = \tan^{-1}\frac{5}{7} \\
 \therefore \tan \theta &= \frac{5}{7}.
 \end{aligned}$$

96. (c) $\therefore \sin^{-1}\sqrt{x^2+x+1} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore -1 \leq \sqrt{x^2+x+1} \leq 1 \Rightarrow 0 \leq x^2+x+1 \leq 1$$

Case I: $x^2+x+1 \geq 0, \Delta = 1^2 - 4(1)(1) = -3$

Since, the discriminant is negative and the coefficient of x^2 is positive than x^2+x+1 is always positive for all real x .

Case II: $x^2+x+1 \geq 1 \Rightarrow x(x+1) \geq 0$

$$\Rightarrow x \in [-1, 0] \Rightarrow A = [-1, 0]$$

$$x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\therefore -1 \leq x \leq 0 \Rightarrow 0 \leq \left(x+\frac{1}{2}\right)^2 \leq \frac{1}{4}$$

$$\Rightarrow \frac{3}{4} \leq \left(x+\frac{1}{2}\right)^2 + \frac{3}{4} \leq 1 \Rightarrow \frac{\sqrt{3}}{2} \leq \sqrt{x^2+x+1} \leq 1$$

$$y \in \left[\sin^{-1}\frac{\sqrt{3}}{2}, \sin^{-1}1\right] \Rightarrow y \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right] = B$$

Now, $A^C \cap B = \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

97. (c) $f(x) = \sqrt{\log_e\left(\frac{1}{x^2-4x+4}\right)} + \sin^{-1}(x^2-2)$

For domain of $\sqrt{\log_e\left(\frac{1}{x^2-4x+4}\right)}$

$$\Rightarrow \log_e \frac{1}{x^2-4x+4} \geq 0$$

$$\frac{1}{x^2-4x+4} \geq 1 \Rightarrow x^2-4x+3 \leq 0$$

$$x \in [1, 3] - \{2\}$$

For domain of $\sin^{-1}(x^2-2)$

$$-1 \leq x^2-2 \leq 1 \Rightarrow 1 \leq x^2 \leq 3$$

$$x \in [-\sqrt{3}, -1] \cup [1, \sqrt{3}]$$

... (i)

... (ii)

Domain of $f(x)$ is intersection of (i) & (ii),

i.e., $[1, \sqrt{3}]$

98. (a) $\cot(\cos^{-1}x) = \sec\left(\tan^{-1}\frac{9}{\sqrt{b^2-a^2}}\right)$
 $\Rightarrow \cot(\cos^{-1}x) = \sec\left(\tan^{-1}\frac{a/b}{\sqrt{1-\left(\frac{a}{b}\right)^2}}\right)$

Let $\frac{a}{b} = \sin \alpha$

$$\Rightarrow \cot(\cos^{-1}x) = \sec \alpha$$

$$\Rightarrow \cos^{-1}x = \cot^{-1}(\sec \alpha) = \cos^{-1}\left(\frac{\sec \alpha}{\sqrt{1+\sec^2 \alpha}}\right)$$

$$\Rightarrow x = \frac{\sec \alpha}{\sqrt{1+\sec^2 \alpha}} = \frac{\frac{b}{\sqrt{b^2-a^2}}}{\sqrt{1+\frac{b^2}{b^2-a^2}}} = \frac{b}{\sqrt{2b^2-a^2}}$$

99. (d) $\tan^{-1}1 + \frac{1}{2}\cos^{-1}x^2 - \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right) = 0$

$$\Rightarrow \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2 - \tan^{-1}\left(\frac{1 + \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}}{1 - \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}}\right) = 0$$

$$\Rightarrow \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2 - \tan^{-1}(1) - \tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right) = 0$$

$$\Rightarrow \frac{1}{2}\cos^{-1}x^2 = \tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)$$

$$\Rightarrow \frac{1}{2}\cos^{-1}x^2 = \frac{1}{2}\cos^{-1}x^2$$

Since, the equation simplifies to $0 = 0$, it is true for all valid value of x .

The valid domain for x is $[-1, 1]$.

Therefore, there are infinitely many solutions with this domain.

100. (d) $2\cot^{-1}(x^2+2x+k) = \pi - 3\tan^{-1}(x^2+2x+k)$

$$2\cot^{-1}x(x^2+2x+k) + 2\tan^{-1}(x^2+2x+k)$$

$$+ \tan^{-1}(x^2+2x+k) = \pi$$

$$\pi + \tan^{-1}(x^2+2x+k) = \pi$$

$$\tan^{-1}(x^2+2x+k) = 0$$

$$x^2+2x+k=0 \text{ has 2 distinct roots if } D > 0$$

$$4-4k > 0$$

$$(k-1) < 0 \Rightarrow k \in (-\infty, 1)$$

Exercise 2 :

ACCELERATOR

Topic-wise MCQs

1. (d) Given equation is

$$\cos\left(\frac{\pi}{3} + x\right) \cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4} \cos^2 2x, \quad x \in [-3\pi, 3\pi]$$

$$\Rightarrow 4\left(\cos^2\left(\frac{\pi}{3}\right) - \sin^2 x\right) = \cos^2 2x$$

$$\Rightarrow 4\left(\frac{1}{4} - \sin^2 x\right) = \cos^2 2x \Rightarrow 1 - 4 \sin^2 x = \cos^2 2x$$

$$1 - 2(1 - \cos 2x) = \cos^2 2x \quad \left\{ \sin^2 x = \frac{1 - \cos 2x}{2} \right\}$$

Assume $\cos 2x = t$

$$-1 + 2 \cos 2x = \cos^2 2x$$

$$\Rightarrow t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0 \Rightarrow t = 1$$

Put the value of t .Then, $\cos 2x = 1$.

$$2x = 2n\pi$$

$$\Rightarrow x = n\pi \Rightarrow n = -3, -2, -1, 0, 1, 2, 3$$

Therefore, there are 7 solutions.

2. (a)
- $\tan \theta(1 + \sqrt{5} \tan 2\theta) = \sqrt{5} - \tan 2\theta$

$$\tan \theta + \sqrt{5} \tan 2\theta \tan \theta = \sqrt{5} - \tan 2\theta$$

$$\Rightarrow \tan 3\theta = \sqrt{5}$$

$$\Rightarrow \theta = \frac{n\pi}{3} + \frac{\alpha}{3}; \quad \tan \alpha = \sqrt{5}$$

Five solution

3. (b)
- $\sin \theta + \cos \theta = 1$
- . Dividing by
- $\sqrt{1^2 + 1^2} = \sqrt{2}$
- ,

$$\frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4} \Rightarrow \theta + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$$

$$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$$

4. (a)
- $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$$\Rightarrow 2 \sin 4\theta \cos 2\theta + \sin 4\theta = 0$$

$$\Rightarrow \sin 4\theta (2 \cos 2\theta + 1) = 0$$

$$\Rightarrow 2 \cos 2\theta = -1 \Rightarrow \cos 2\theta = -\frac{1}{2} = \cos\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow 2\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$$

$$\text{and } \sin 4\theta = 0 \Rightarrow 4\theta = n\pi \Rightarrow \theta = \frac{n\pi}{4}$$

$$\theta = \frac{n\pi}{4} \text{ or } n\pi \pm \frac{\pi}{3}$$

5. (c)
- $\sqrt{2} \sec \theta + \tan \theta = 1 \Rightarrow \frac{\sqrt{2}}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = 1$

$$\Rightarrow \sin \theta - \cos \theta = -\sqrt{2}$$

Dividing by $\sqrt{2}$ on both sides, we get

$$\frac{1}{\sqrt{2}} \sin \theta - \frac{1}{\sqrt{2}} \cos \theta = -1$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cos \theta - \frac{1}{\sqrt{2}} \sin \theta = 1$$

$$\Rightarrow \cos\left(\theta + \frac{\pi}{4}\right) = \cos(0)$$

$$\Rightarrow \theta + \frac{\pi}{4} = 2n\pi \pm 0 \Rightarrow \theta = 2n\pi - \frac{\pi}{4}$$

6. (a) We have
- $\tan 5\theta = \cot 2\theta$

$$\Rightarrow \tan 5\theta = \tan\left(\frac{\pi}{2} - 2\theta\right) \dots \left[\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \right]$$

$$\Rightarrow 5\theta = n\pi + \frac{\pi}{2} - 2\theta \Rightarrow 7\theta = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{n\pi}{7} + \frac{\pi}{14}$$

7. (a) Given,
- $3 \tan(\theta - 15^\circ) = \tan(\theta + 15^\circ)$
- .
- $\frac{\tan A}{\tan B} = \frac{3}{1}$
- ,

where $A = \theta + 15^\circ$, $B = \theta - 15^\circ$

On applying componendo and dividendo, we get

$$\Rightarrow \frac{\tan A + \tan B}{\tan A - \tan B} = \frac{3+1}{3-1} \Rightarrow \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}} = 2$$

$$\Rightarrow \frac{\sin(A+B)}{\sin(A-B)} = 2 \Rightarrow \sin 2\theta = 2 \sin 30^\circ$$

$$\Rightarrow \sin 2\theta = 2 \cdot \frac{1}{2} = 1 = \sin \frac{\pi}{2}$$

$$\Rightarrow 2\theta = n\pi + (-1)^n \frac{\pi}{2} \Rightarrow \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{4}$$

8. (b)
- $\sin^4 \theta + \cos^4 \theta = -\lambda$

$$\Rightarrow (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cdot \cos^2 \theta = -\lambda$$

$$\Rightarrow 1 - 2 \sin^2 \theta \cos^2 \theta = -\lambda \Rightarrow \lambda = \frac{(\sin 2\theta)^2}{2} - 1$$

$$\Rightarrow \text{as } \sin^2 2\theta \in [0, 1] \Rightarrow \lambda \in \left[-1, \frac{-1}{2}\right]$$

9. (b) Given:
- $\tan x + \sec x = 2 \cos x$

$$\Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2 \cos x$$

$$\Rightarrow \sin x + 1 = 2 \cos^2 x = 2(1 - \sin^2 x)$$

$$\Rightarrow \sin x + 1 = 2 - 2 \sin^2 x$$

$$\Rightarrow 2 \sin^2 x + \sin x - 1 = 0$$

$$\Rightarrow 2 \sin^2 x + 2 \sin x - \sin x - 1 = 0$$

$$\Rightarrow 2 \sin x (\sin x + 1) - 1(\sin x + 1) = 0$$

$$= (2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x + 1 \neq 0 \quad \left(\because 0 \leq x \leq \frac{\pi}{2} \right)$$

$$\therefore 2\sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2} = \sin \frac{\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{6}$$

\therefore Only one value of x satisfy the given equation.

10. (d) Given equation is, $\cos 2x + \alpha \sin x = 2\alpha - 7$

$$1 - 2\sin^2 x + \alpha \sin x = 2\alpha - 7$$

$$2\sin^2 x - \alpha \sin x + (2\alpha - 8) = 0$$

$$\Rightarrow \sin x = \frac{\alpha \pm \sqrt{\alpha^2 - 8(2\alpha - 8)}}{4}$$

$$\Rightarrow \sin x = \frac{\alpha \pm (\alpha - 8)}{4} \Rightarrow \sin x = \frac{\alpha - 4}{2}$$

[$\sin x = 2$ (rejected)]

$$\therefore \text{equation has solution, then } \frac{\alpha - 4}{2} \in [-1, 1]$$

$$\Rightarrow \alpha \in [2, 6]$$

11. (a) Let, the functions is,

$$f(\theta) = 3\cos\theta + 5\sin\theta \cdot \cos \frac{\pi}{6} - 5\sin \frac{\pi}{6} \cos\theta$$

$$= 3\cos\theta + 5 \times \frac{\sqrt{3}}{2} \sin\theta - 5 \times \frac{1}{2} \cos\theta$$

$$= \left(3 - \frac{5}{2}\right) \cos\theta + 5 \times \frac{\sqrt{3}}{2} \sin\theta$$

$$= \frac{1}{2} \cos\theta + \frac{5\sqrt{3}}{2} \sin\theta$$

$$\max f(\theta) = \sqrt{\frac{1}{4} + \frac{25}{4}} \times 3 = \sqrt{\frac{76}{4}} = \sqrt{19}$$

12. (a) We have $|4\sin x - 1| < \sqrt{5}$

$$\Rightarrow -\sqrt{5} < 4\sin x - 1 < \sqrt{5}$$

$$\Rightarrow -\left(\frac{\sqrt{5}-1}{4}\right) < \sin x < \frac{\sqrt{5}+1}{4}$$

$$\Rightarrow \sin\left(\frac{-\pi}{10}\right) < \sin x < \sin\left(\frac{3\pi}{10}\right)$$

$$\Rightarrow x \in \left(-\frac{\pi}{10}, \frac{3\pi}{10}\right) \quad [\because x \in (-\pi, \pi)]$$

13. (d) $\cos 7\theta = \cos\theta - \sin 4\theta \Rightarrow \sin 4\theta = \cos\theta - \cos 7\theta$

$$\Rightarrow \sin 4\theta = 2\sin 4\theta \sin 3\theta \Rightarrow \sin 4\theta(1 - 2\sin 3\theta) = 0$$

$$\therefore \sin 4\theta = 0 \text{ or } \sin 3\theta = \frac{1}{2}$$

$$\Rightarrow 4\theta = n\pi \text{ or } 3\theta = n\pi + (-1)^n \frac{\pi}{6}$$

$$\Rightarrow \theta = \frac{n\pi}{4} \text{ or } \frac{n\pi}{3} + (-1)^n \frac{\pi}{18}$$

14. (b) The given equation can be rewritten as

$$8^{1+|\cos x|+|\cos x|^2+\dots\text{to } \infty} = 64$$

$$\Rightarrow 8^{1/(1-|\cos x|)} = 8^2$$

[since, $1 + |\cos x| + |\cos x|^2 + \dots$ to ∞ is an infinite G.P.]

$$\Rightarrow \frac{1}{1-|\cos x|} = 2 \Rightarrow 1-|\cos x| = \frac{1}{2}; \Rightarrow |\cos x| = \frac{1}{2}$$

$$\Rightarrow \cos x = \pm \frac{1}{2}, \text{ but } x \in (-\pi, \pi)$$

$$\Rightarrow x = \pm \frac{\pi}{3}, \pm \frac{2\pi}{3}$$

15. (a) Let $81^{\sin^2 \theta} = x$, then equation is

$$x + \frac{81}{x} = 30 \Rightarrow x^2 - 30x + 81 = 0$$

$$\therefore x = 3 \text{ or } 27 \Rightarrow 81^{\sin^2 \theta} = 3 \text{ or } 27$$

$$\therefore \sin^2 \theta = \frac{1}{4} \text{ or } \frac{3}{4} \Rightarrow \sin \theta = \pm \frac{1}{2}, \pm \frac{\sqrt{3}}{2}$$

$$\text{for } 0 \leq \theta \leq \pi, \theta = \frac{\pi}{6} \text{ or } \frac{\pi}{3}, \frac{2\pi}{3} \text{ or } \frac{5\pi}{6}$$

16. (c) We have $\cos x + \cos 2x + \cos 3x = 0$

$$\text{or } (\cos 3x + \cos x) + \cos 2x = 0$$

$$\text{or } 2\cos 2x \cdot \cos x + \cos 2x = 0$$

$$\text{or } \cos 2x(2\cos x + 1) = 0$$

We have, either $\cos 2x = 0$ or $2\cos x + 1 = 0$

$$\text{If } \cos 2x = 0, \text{ then } 2x = (2m+1)\frac{\pi}{2}$$

$$\text{or } x = (2m+1)\frac{\pi}{4}, m \in I$$

$$\text{If } 2\cos x + 1 = 0, \text{ then } \cos x = -\frac{1}{2} = \cos \frac{2\pi}{3}$$

$$\Rightarrow x = 2n\pi \pm \frac{2\pi}{3}, n \in I$$

Hence the required general solution are

$$x = (2m+1)\frac{\pi}{4} \text{ and } x = 2n\pi \pm \frac{2\pi}{3}, m, n \in I$$

17. (b) $16^{\sin^2 x} + 16^{1-\sin^2 x} = 10$

$$\text{If } 16^{\sin^2 x} = t, \text{ then } t + \frac{16}{t} = 10 \Rightarrow t^2 - 10t + 16 = 0$$

$$\Rightarrow t = 2, 8 \Rightarrow 16^{\sin^2 x} = 16^{1/4} \text{ or } 16^{3/4}$$

$$\Rightarrow \sin x = \pm \frac{1}{2}, \pm \frac{\sqrt{3}}{2}$$

$$\text{Now, } \sin x = \frac{1}{2}, \text{ then } x = \frac{\pi}{2}, \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\sin x = -\frac{1}{2}, \text{ then } x = \pi + \frac{\pi}{6} = \frac{7\pi}{6} \text{ or } 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

Thus there will be four solutions corresponding to

$$\sin x = \pm \frac{1}{2}. \text{ Similarly four solutions for } \sin x = \pm \frac{\sqrt{3}}{2}.$$

Hence, there will be 8 solutions in all.

18. (d) We have, $2\sin^2 x + 3\sin x - 2 > 0$

$$\Rightarrow (2\sin x - 1)(\sin x + 2) > 0$$

$$\Rightarrow 2\sin x - 1 > 0 \quad [\because \sin x + 2 > 0 \forall x \in \mathbb{R}]$$

$$\Rightarrow \sin x > 1/2 \Rightarrow x \in (\pi/6, 5\pi/6)$$

$$\text{Also, } x^2 - x - 2 < 0$$

$$\Rightarrow (x-2)(x+1) < 0 \Rightarrow -1 < x < 2$$

$$\text{As } 2 < \frac{5\pi}{6}, \text{ we obtain that } x \text{ must lie in } \left(\frac{\pi}{6}, 2\right)$$

19. (c) We have, $8\tan^2 \frac{x}{2} = 1 + \sec x$

$$\Rightarrow 8\left(\frac{1-\cos x}{1+\cos x}\right) = 1 + \frac{1}{\cos x} = \frac{1+\cos x}{\cos x}$$

$$\Rightarrow 8\cos x - 8\cos^2 x = (1+\cos x)^2$$

$$\Rightarrow 8\cos x - 8\cos^2 x = 1 + \cos^2 x + 2\cos x$$

$$\Rightarrow 9\cos^2 x - 6\cos x + 1 = 0$$

$$\Rightarrow (3\cos x - 1)^2 = 0 \Rightarrow 3\cos x - 1 = 0$$

$$\Rightarrow \cos x = \frac{1}{3} = \cos \alpha \text{ (say)} \Rightarrow x = 2n\pi \pm \alpha$$

$$\therefore x = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right), \text{ where } n \in \mathbb{I}$$

20. (b) Consider equation, $1 + \sin^4 x = \cos^2 3x$

$$\text{L.H.S.} = 1 + \sin^4 x \text{ and } \text{R.H.S.} = \cos^2 3x$$

$$\because \text{L.H.S.} \geq 1 \text{ and } \text{R.H.S.} \leq 1 \Rightarrow \text{L.H.S.} = \text{R.H.S.} = 1$$

$$\sin^4 x = 0, \text{ and } \cos^2 3x = 1$$

$$\Rightarrow \sin x = 0 \text{ and } (4\cos^2 x - 3)^2 \cos^2 x = 1$$

$$\Rightarrow \sin x = 0 \text{ and } \cos^2 x = 1 \Rightarrow x = 0, \pm\pi, \pm 2\pi$$

Hence, total number of solutions is 5.

21. (d) $\sin 3x = \cos 2x$

$$\Rightarrow 3\sin x - 4\sin^3 x = 1 - 2\sin^2 x$$

$$\Rightarrow 4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$$

$$\Rightarrow \sin x = 1, \frac{-2 \pm 2\sqrt{5}}{8}$$

$$\text{In the interval } \left(\frac{\pi}{2}, \pi\right), \sin x = \frac{-2 + 2\sqrt{5}}{8}$$

So, there is only one solution.

22. (d) $\left| \sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x} \right| = 1$

$$\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x} = \pm 1$$

On squaring above eqn, we get,

$$2(\sin^4(x) + \cos^4(x)) + 18(\cos^2(x) + \sin^2(x)) - 1$$

$$= 2\left(\sqrt{2\sin^4 x + 18\cos^2 x}\right)\left(\sqrt{2\cos^4 x + 18\sin^2 x}\right)$$

$$\Rightarrow 19 - 4\sin^2(x)\cos^2(x)$$

$$= 2\left(\sqrt{2\sin^4(x) + 18\cos^2(x)}\right)\left(\sqrt{2\cos^4(x) + 18\sin^2(x)}\right)$$

Again squaring, we get

$$361 = 144(1 - 3\sin^2(x)\cos^2(x)) + 144\sin^2(x)\cos^2(x)$$

On solving, we get

$$\sin 2x = \pm\sqrt{\frac{217}{254}}; \text{ Hence, } x = \frac{\theta}{2}, \frac{\pi-\theta}{2}, \frac{2\pi+\theta}{2}, \frac{3\pi-\theta}{2}$$

$$\text{and } \frac{\pi+\theta}{2}, \frac{2\pi-\theta}{2}, \frac{3\pi+\theta}{2}, \frac{4\pi-\theta}{2}$$

So total number of solutions = 8

23. (c) $2\sin^3 \alpha - 7\sin^2 \alpha + 7\sin \alpha - 2 = 0$

$$\Rightarrow 2\sin^2 \alpha (\sin \alpha - 1) - 5\sin \alpha (\sin \alpha - 1) + 2(\sin \alpha - 1) = 0$$

$$\Rightarrow (\sin \alpha - 1)(2\sin^2 \alpha - 5\sin \alpha + 2) = 0$$

$$\Rightarrow \sin \alpha - 1 = 0 \text{ or } 2\sin^2 \alpha - 5\sin \alpha + 2 = 0$$

$$\sin \alpha = 1 \text{ or } \sin \alpha = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4}$$

$$\alpha = \frac{\pi}{2} \text{ or } \sin \alpha = \frac{1}{2}, 2$$

$$\text{Now, } \sin \alpha \neq 2 \text{ for, } \sin \alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}, \frac{2\pi}{3}$$

There are three values of α between $[0, 2\pi]$

24. (b) Given equation is $e^{\sin x} - e^{-\sin x} - 4 = 0$

Put $e^{\sin x} = t$ in the given equation, we get

$$t^2 - 4t - 1 = 0$$

$$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$$\Rightarrow e^{\sin x} = 2 \pm \sqrt{5} \quad (\because t = e^{\sin x})$$

$$\Rightarrow e^{\sin x} = 2 - \sqrt{5} \text{ and } e^{\sin x} = 2 + \sqrt{5}$$

$$\Rightarrow e^{\sin x} = 2 - \sqrt{5} < 0 \text{ and } \sin x = \ln(2 + \sqrt{5}) > 1$$

So, rejected.

Hence, given equation has no solution.

\therefore The equation has no real roots.

and $\cos 4x = 1 \Rightarrow 4x = 2m\pi \Rightarrow x = m\frac{\pi}{2}$

$\therefore \frac{\pi}{4} + n\pi = \frac{m\pi}{2}$

This equation has no solution for integer n, m such that $x \in [-\pi, \pi]$.

32. (d) $\cos \theta + \cos 2\theta - \sqrt{3}(\sin \theta + \sin 2\theta) + 1 = 0$
 $\Rightarrow \cos \theta + 2\cos^2 \theta - 1 - \sqrt{3}(\sin \theta + 2\sin \theta \cos \theta) + 1 = 0$
 $\Rightarrow \cos \theta (1 + 2\cos \theta) - \sqrt{3} \sin \theta (1 + 2\cos \theta) = 0$

$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$ and $\cos \theta = -\frac{1}{2}$

$\Rightarrow \theta = 30^\circ, 210^\circ$ and $\theta = 120^\circ, 240^\circ$
 \Rightarrow Number of solution = 4.

33. (b) $2\sin^2 \theta - 3\cos^2 \theta = \sin \theta \cos \theta; \theta \in (-\pi, \pi)$
 $2\sin^2 \theta - 3(1 - \sin^2 \theta) = \sin \theta \cos \theta$

$5\sin^2 \theta - 3 - \sin \theta \cos \theta = 0$

Divide by $\cos^2 \theta$

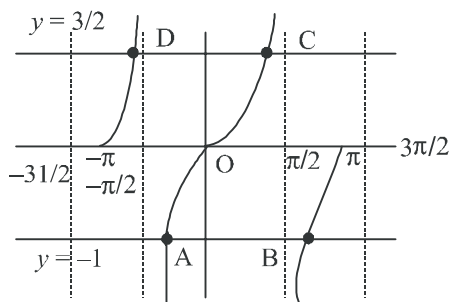
$5\tan^2 \theta - \tan \theta - 3\sec^2 \theta = 0$

$\Rightarrow 2\tan^2 \theta - \tan \theta - \tan \theta - 3 = 0$

Divide by $\cos^2 \theta$

$5\tan^2 \theta - \tan \theta - 3\sec^2 \theta = 0 \Rightarrow 2\tan^2 \theta - \tan \theta - 3 = 0$

Let $\tan \theta = x$



$2x^2 - x - 3 = 0$
 $(x + 1)(2x - 3) = 0$

$x = -1, x = \frac{3}{2}$

$\tan x = -1, \tan x = \frac{3}{2}$

So, there are 4 solution

34. (b) We have, $\tan^2 x + 3\cot^2 x = 2\sec^2 x$

$\tan^2 x + \frac{3}{\tan^2 x} = 2(1 + \tan^2 x)$

$\Rightarrow \tan^4 x + 2\tan^2 x - 3 = 0$

Let $y = \tan^2 x$

$\Rightarrow y^2 + 2y - 3 = 0 \Rightarrow (y + 3)(y - 1) = 0$

$\Rightarrow y = -3$ or $y = 1$

$\Rightarrow y = \tan^2 x \neq -3$, we have, $\tan^2 x = 1$
 i.e. $\tan x = 1$ or $\tan x = -1$

For $\tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

For $\tan x = -1 \Rightarrow \frac{3\pi}{4}, \frac{7\pi}{4}$

There are 4 solutions for the given equation in the interval $[0, 2\pi]$.

35. (c) We have $\sin 5x (2\sin x \sin 3x + \cos 4x) = 0$
 $\sin 5x (\cos(x - 3x) - \cos(x + 3x) + \cos 4x) = 0$
 $\sin 5x \cos(2x) = 0$
 $\therefore \sin 5x = 0$

$\Rightarrow 5x = n\pi \Rightarrow x = \frac{n\pi}{5}$

For $x \in (-\pi, \pi)$ possible value for n are $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

This gives 9 solutions :

$\frac{-4\pi}{5}, \frac{-3\pi}{5}, \frac{-2\pi}{5}, \frac{-\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$

Set $\cos 2x = 0 \Rightarrow 2x = \frac{\pi}{2} + n\pi \Rightarrow x = \frac{\pi}{4} + \frac{n\pi}{2}$

For $x \in (-\pi, \pi)$ possible value for n are $\{-2, -1, 0, 1\}$

This gives 4 solutions : $\frac{-3\pi}{4}, \frac{-\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

Total number of distinct solutions $9 + 4 = 13$

36. (b) Number of solutions of the eqn.

$\sin^2 \theta + 2\cos^2 \theta - \sqrt{3}\sin \theta \cos \theta = 2; \theta \in (-\pi, \pi)$

Adding $\sin^2 \theta$ both sides

$2\sin^2 \theta + 2\cos^2 \theta - \sqrt{3}\sin \theta \cos \theta = 2 + \sin^2 \theta$

$\Rightarrow 2(\sin^2 \theta + \cos^2 \theta) = 2 + \sin^2 \theta + \sqrt{3}\sin \theta \cos \theta$

$\Rightarrow 0 = \sin \theta (\sin \theta + \sqrt{3}\cos \theta)$

Case-I: $\sin \theta = 0 \Rightarrow \theta = 0^\circ$ ($\because \theta \in (-\pi, \pi)$)

Case-II: $\sin \theta + \sqrt{3}\cos \theta = 0$

$\Rightarrow \tan \theta = -\sqrt{3} \Rightarrow \theta = \frac{2\pi}{3}, \frac{-\pi}{3}$ ($\because \theta \in (-\pi, \pi)$)

Hence, a total of three solutions are there.

37. (c) $1 + \cos x + \cos^2 x + \cos^3 x + \dots \infty = 4 + 2\sqrt{3}$

$S = \frac{1}{1 - \cos x} = 4 + 2\sqrt{3}$

$\Rightarrow 1 - \cos x = \frac{4 - 2\sqrt{3}}{4} = 1 - \frac{\sqrt{3}}{2} \Rightarrow \cos x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{6} \Rightarrow x = (12n \pm 1)\frac{\pi}{6}$

38. (c) $\sqrt{6-5\cos x+7\sin^2 x}-\cos x=0$

$$\Rightarrow \sqrt{6-5\cos x+7\sin^2 x}=\cos x$$

Squaring both sides

$$\Rightarrow 6-5\cos x+7\sin^2 x=\cos^2 x$$

$$\Rightarrow 13-5\cos x-8\cos^2 x=0$$

$$\Rightarrow \cos x=1 \text{ or } \frac{-13}{8} \text{ (Not possible)}$$

So, $\cos x=1 \Rightarrow x=2n\pi$

Now, check all the options

$$\tan x + \cot x = 0 + \infty \neq 2$$

$$\cot x + \operatorname{cosec} x = \infty + \infty \neq 1$$

$$\tan x + \sec x = 0 + 1 = 1$$

$$\sec x + \operatorname{cosec} x = 1 + \infty \neq 2$$

39. (a) $3\sin\theta=4\cos\theta-1 \Rightarrow 9\sin^2\theta=16\cos^2\theta-8\cos\theta+1$

$$\Rightarrow 9(1-\cos^2\theta)=16\cos^2\theta-8\cos\theta+1$$

$$\Rightarrow 25\cos^2\theta-8\cos\theta-8=0$$

$$\therefore \cos\theta = \frac{8 \pm \sqrt{64+800}}{50} = \frac{8 \pm 12\sqrt{6}}{50}$$

$$= \frac{4+6\sqrt{6}}{25} = \frac{2(3\sqrt{6}+2)}{25} \quad \left[\because \theta \in \left[0, \frac{\pi}{4}\right] \right]$$

$$= \frac{2(3\sqrt{6}+2)}{25} \times \frac{(3\sqrt{6}-2)}{3\sqrt{6}-2} = \frac{4}{3\sqrt{6}-2}$$

40. (d) $B=45^\circ, a=2(\sqrt{3}+1)$

$$\text{Area of } \triangle ABC = 6+2\sqrt{3}$$

$$\frac{1}{2}ac\sin B = 6+2\sqrt{3}$$

$$C = \frac{(6+2\sqrt{3}) \times 2}{2(\sqrt{3}+1)} = 2\sqrt{6}$$

$$\therefore b^2 = a^2 + c^2 - 2ac \cos B$$

$$= (2(\sqrt{3}+1))^2 + (2\sqrt{6})^2 - 2 \times (2\sqrt{3}+1) \times (2\sqrt{6}) \cos 45^\circ$$

$$b^2 = 16 + 8\sqrt{3} + 24 - 24 - 8\sqrt{3}$$

$$b^2 = 16 \Rightarrow b = 4$$

41. (b) $\frac{b}{c} = \frac{(\sqrt{3}+1)^2 + 2(\sqrt{2}-1)}{(\sqrt{3}+1)^2 - 2(\sqrt{2}-1)} = \frac{1+\sqrt{3}+\sqrt{2}}{3+\sqrt{3}-\sqrt{2}}$

$$\frac{\sin B}{\sin C} = \frac{1+1.732+1.414}{3+1.732+1.414} = 1.249$$

$$\frac{\sin B}{\sin(150-B)} = \frac{\sin(97.5^\circ)}{\sin(52.5^\circ)} = \frac{0.9914}{0.7933} \quad [\text{Let } B=97.5^\circ]$$

$$= 1.249.$$

42. (c) $\because \cos c = \cos 120^\circ = \cos(180^\circ - 60^\circ)$

$$= -\cos 60^\circ = \frac{-1}{2}$$

$$\therefore c^2 = a^2 + b^2 - 2ab \cos c$$

$$19 = a^2 + 9 + 3a \Rightarrow a^2 + 3a - 10 = 0$$

$$\Rightarrow (a+5)(a-2) = 0 \Rightarrow a = 2 \quad (\because a \neq -5)$$

43. (c) $\tan \frac{A-B}{2} = \sqrt{\frac{1-\cos(A-B)}{1+\cos(A-B)}} = \sqrt{\frac{1-\frac{31}{32}}{1+\frac{31}{32}}} = \frac{1}{3\sqrt{7}}$

$$\therefore \tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{c}{2}$$

$$\Rightarrow \frac{1}{3\sqrt{7}} = \frac{5-4}{5+9} \cot \frac{c}{2} \Rightarrow \cot \frac{c}{2} = \frac{3}{\sqrt{7}} \Rightarrow \tan \frac{c}{2} = \frac{\sqrt{7}}{3}$$

$$\cos c = \frac{1 - \tan^2 \frac{c}{2}}{1 + \tan^2 \frac{c}{2}} = \frac{1 - \frac{7}{9}}{1 + \frac{7}{9}} = \frac{1}{8}$$

$$\text{Now, } c^2 = a^2 + b^2 - 2ab \cos c$$

$$= 25 + 16 - 40 \times \frac{1}{8} = 36 \Rightarrow c = 6.$$

44. (c) $\because a, b, c$ are in AP $\Rightarrow 2b = a + c$

$$\Rightarrow 2 \sin B = \sin A + \sin C \text{ (By sine formula)}$$

$$\because A+B+C=180^\circ \Rightarrow B=180^\circ-3C \quad [\because A=2C]$$

$$2 \sin(180^\circ-3C) = \sin 2C + \sin C$$

$$2(3 \sin C - 4 \sin^3 C) = 2 \sin C \cdot \cos C + \sin C$$

$$\Rightarrow 8 \cos^2 C - 2 \cos C - 3 = 0 \quad (\because \cos C \neq 0)$$

$$(2 \cos C + 1)(4 \cos C - 3) = 0$$

$$\Rightarrow \cos C = \frac{3}{4} \quad [\because C \text{ is acute angle}]$$

$$\cos A = \cos 2C = 2 \cos^2 C - 1 = \frac{1}{8}$$

$$\cos B = \cos(180^\circ - 3C) = -\cos 3C = \frac{9}{16}$$

$$\therefore \cos A : \cos B : \cos C = 2 : 9 : 12$$

45. (d) $\because A, B, C$ are in AP $\Rightarrow 2B = A + C$

$$\because A+B+C=180^\circ \Rightarrow B=60^\circ$$

$$\text{Given that } rr_3 = r_1 r_2$$

$$(4R \sin A/2 \cdot \sin B/2 \cdot \sin C/2) \left(4R \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \sin \frac{C}{2} \right)$$

$$= \left(4R \sin \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2} \right) \left(4R \cos \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \cos \frac{C}{2} \right)$$

$$\Rightarrow \sin^2 \frac{C}{2} = \cos^2 \frac{C}{2} \Rightarrow \tan^2 \frac{C}{2} = 1 \Rightarrow C = 90^\circ \text{ and } A = 30^\circ$$

$$\frac{a}{\sin 30^\circ} = \frac{10}{\sin 90^\circ} \Rightarrow a = 5$$

$$\frac{b}{\sin 60^\circ} = \frac{10}{\sin 90^\circ} \Rightarrow b = 5\sqrt{3}$$

$$\text{Now, } a^2 + b^2 + c^2 = 25 + 75 + 100 = 200$$

46. (b) We know that

$$r_1 = \frac{\Delta}{s-a} \text{ and } r_3 = \frac{\Delta}{s-c}$$

$$\therefore r_1 + r_3 = \Delta \left(\frac{1}{s-a} + \frac{1}{s-c} \right) = \frac{\Delta b}{(s-a)(s-c)}$$

$$\therefore 1 + \cos B = 2 \cos^2 B = \frac{2.5(s-b)}{ac}$$

$$\text{Now, } \frac{2(r_1 + r_3)}{ac(1 + \cos B)} = \frac{2 \cdot \Delta b}{(s-a)(s-c)ac} \times \frac{ac}{2s(s-b)}$$

$$= \frac{\Delta b}{\Delta^2} = \frac{b}{\Delta}$$

47. (b) Given, $a = 8, b = 10, c = 12$

$$s = \frac{8+10+12}{2} = 15$$

$$\therefore \Delta = \sqrt{15(15-8)(15-10)(15-12)} = 15\sqrt{7}$$

$$\Rightarrow r = \frac{\Delta}{s} = \frac{15\sqrt{7}}{15} = \sqrt{7}$$

$$\text{So, } R = \frac{abc}{4\Delta} = \frac{960}{4 \times 15\sqrt{7}} = \frac{16}{\sqrt{7}}$$

$$\text{Hence, } \frac{r}{R} = \frac{7}{16}$$

48. (d) Given, $a = 13, b = 8, c = 7$

$$\text{We know, } \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \cos A = \frac{8^2 + 7^2 - 13^2}{2 \times 8 \times 7} = -\frac{1}{2}$$

$$\text{So, } \cos(B + C) = \cos(\pi - A) = -\cos A = \frac{1}{2}$$

49. (b) We know, $r_1 = \frac{\Delta}{S-a}, r_2 = \frac{\Delta}{S-b}, r_3 = \frac{\Delta}{S-c}$

$$\text{Given, } (r_1 - r_3)(r_1 - r_2) - 2r_2r_3 = 0$$

$$\Rightarrow (r_1 - r_3)(r_1 - r_2) = 2r_2r_3$$

$$\Rightarrow \Delta^2 \left(\frac{1}{(S-a)} - \frac{1}{(S-c)} \right) \left(\frac{1}{(S-a)} - \frac{1}{(S-b)} \right)$$

$$= \frac{2\Delta^2}{(S-b)(S-c)} \Rightarrow \frac{(a-c)(a-b)}{(S-a)^2} = 2$$

$$\Rightarrow \frac{4(a-c)(a-b)}{(b+c-a)^2} = 2$$

$$\Rightarrow 2(a-c)(a-b) = (b+c-a)^2$$

$$\Rightarrow 2a^2 - 2ab - 2ac + 2bc = a^2 + b^2 + c^2 + 2bc - 2ab - 2ac$$

$$\Rightarrow a^2 - b^2 - c^2 = 0 \Rightarrow a^2 - b^2 = c^2$$

50. (c) $a : b : c = 4 : 5 : 6 \Rightarrow a = 4k, b = 5k, c = 6k$

$$\frac{\cos A + 3 \cos C}{\cos B} = \frac{\frac{b^2 + c^2 - a^2}{2bc} + \frac{3(a^2 + b^2 - c^2)}{2ab}}{\frac{a^2 + c^2 - b^2}{2ac}}$$

$$= \frac{ab^2 + ac^2 - a^3 + 3a^2c + 3cb^2 - 3c^3}{b(a^2 + c^2 - b^2)}$$

$$= \frac{k^3(100 + 144 - 64 + 3 \times 96 + 3 \times 6 \times 25 - 3 \times 216)}{k^3(5 \times (16 + 36 - 25))}$$

$$= \frac{244 + 288 + 450 - 712}{5 \times 27} = \frac{270}{5 \times 27} = 2$$

51. (a) $a = 6, b = 8, c = 10$

$$\frac{2r_2r_3}{rr_1} = \frac{2 \times \frac{\Delta}{s-b} \times \frac{\Delta}{s-c}}{\frac{\Delta}{s} \times \frac{\Delta}{s-a}} = \frac{2 \times s(s-a)}{(s-b)(s-c)}$$

$$= \frac{2 \times (a+b+c) \times (-a+b+c)}{(a-b+c) \times (a+b-c)} = \frac{2 \times 24 \times 12}{8 \times 4} = 18$$

$$\therefore \frac{2r_2r_3}{\pi r_1} = 18 = b + c$$

52. (b) Given that a, b, c are in H.P. $\Rightarrow b = \frac{2ac}{a+c}$

We know that

$$\operatorname{cosec}^2 \frac{A}{2} = \frac{bc}{(s-b)(s-c)}; \operatorname{cosec}^2 \frac{B}{2} = \frac{ca}{(s-a)(s-c)}$$

$$\text{and } \operatorname{cosec}^2 \frac{C}{2} = \frac{ab}{(s-a)(s-b)}$$

Now,

$$\operatorname{cosec}^2 \frac{A}{2} + \operatorname{cosec}^2 \frac{C}{2} = \frac{bc}{(s-b)(s-c)} + \frac{ab}{(s-a)(s-b)}$$

$$= \frac{bc(s-a) + ab(s-c)}{(s-a)(s-b)(s-c)} = \frac{b[s(a+c) - 2ac]}{(s-a)(s-b)(s-c)}$$

$$= \frac{b[s(a+c) - b(a+c)]}{(s-a)(s-b)(s-c)} = \frac{b(a+c)(s-b)}{(s-a)(s-b)(s-c)}$$

$$= \frac{2ac}{(s-a)(s-c)} = 2 \operatorname{cosec}^2 \frac{B}{2}$$

$$\therefore \operatorname{cosec}^2 \frac{A}{2}, \operatorname{cosec}^2 \frac{B}{2}, \operatorname{cosec}^2 \frac{C}{2} \text{ are in A.P.}$$

53. (a) $\therefore \frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = \frac{a+b+c}{abc}$... (i)

$$\text{We know semi perimeter } s = \frac{a+b+c}{2}$$

Area $\Delta = rs$

$$\text{So, } a+b+c = 2s = \frac{2\Delta}{r} \text{ ... (ii)}$$

$$\text{and Area } \Delta = \frac{abc}{4R} \Rightarrow abc = 4\Delta R \text{ ... (iii)}$$

By using eq (i), (ii) and (iii), we get

$$\frac{a+b+c}{abc} = \frac{\frac{2\Delta}{r}}{4\Delta R} = \frac{1}{2rR} = \frac{1}{30} \quad [\because r = 3, R = 5]$$

54. (c) Given ΔABC ,

$$\therefore A + B + C = 180^\circ \Rightarrow (120^\circ + B) + B + C = 180^\circ$$

$$[A - B = 120^\circ \text{ (given)}]$$

$$\Rightarrow B = 30^\circ - \frac{C}{2}$$

$$\Rightarrow A = B + 120^\circ = 30^\circ - \frac{C}{2} + 120^\circ = 150^\circ - \frac{C}{2}$$

$$\text{We know } r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$\Rightarrow \frac{R}{8} = 4R \sin \left(75^\circ - \frac{C}{4} \right) \sin \left(15^\circ - \frac{C}{4} \right) \sin \frac{C}{2} \quad [\therefore R = 8r]$$

$$\Rightarrow \frac{1}{16} = \left[\cos 60^\circ - \cos \left(90^\circ - \frac{C}{2} \right) \right] \sin \frac{C}{2}$$

$$\frac{1}{16} = \frac{1}{2} \sin \frac{C}{2} - \sin^2 \frac{C}{2}$$

$$\Rightarrow 16 \sin^2 \frac{C}{2} - 8 \sin \frac{C}{2} + 1 = 0 \Rightarrow \sin \frac{C}{2} = \frac{1}{4} \Rightarrow \cos C = \frac{7}{8}$$

$$\therefore \frac{1 + \cos C}{1 - \cos C} = 15$$

55. (d)
$$\sqrt{\frac{r r_2}{r_3 r_1}} = \sqrt{\frac{\frac{\Delta}{s} \times \frac{\Delta}{s-b}}{\frac{\Delta}{s-c} \times \frac{\Delta}{s-a}}} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$$

$$= \sqrt{\frac{(s-c)^2 (s-a)^2}{s(s-a)(s-b)(s-c)}} = \frac{(s-c)(s-a)}{\Delta}$$

$$\text{Now, } r_1 + r_3 = \frac{\Delta}{s-a} + \frac{\Delta}{s-c} = \frac{\Delta b}{(s-c)(s-a)}$$

$$\Rightarrow \frac{b}{r_1 + r_3} = \frac{(s-c)(s-a)}{\Delta} = \sqrt{\frac{r r_2}{r_3 r_1}}$$

56. (b) $a = 2, c = 5, \sin \frac{A}{2} = \frac{1}{4} \sqrt{\frac{3}{5}}$

$$\cos A = 1 - 2 \cdot \frac{1}{16} \cdot \frac{3}{5} = \frac{37}{40}$$

$$\therefore a^2 = b^2 + c^2 + 2bc \cos A$$

$$\Rightarrow 4 = b^2 + 25 + 2 \cdot b \cdot 5 \cdot \frac{37}{40} \Rightarrow b = 4, \frac{21}{4} \text{ (Invalid)}$$

$$\Rightarrow b = 4$$

$$S = \frac{a+b+c}{2} = \frac{11}{2}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{11}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{7}{2}} = \frac{1}{4} \sqrt{231}$$

57. (c) $s = \frac{a+b+c}{2} = \frac{5b+b}{2} = 3b$

$$\cot \frac{A}{2} \cdot \cot \frac{C}{2} = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} \cdot \sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = \frac{s}{s-b}$$

$$= \frac{3b}{3b-b} = \frac{3}{2}$$

58. (a) $r_1 = 3, r_2 = 4, r_3 = 6$

$$\Rightarrow \Delta = 3(s-a), \Delta = 4(s-b), \Delta = 6(s-c)$$

$$\text{Now, } 3(s-a) = 4(s-b) \Rightarrow s = 4b - 3a$$

$$\text{and } 4(s-b) = 6(s-c) \Rightarrow s = 3c - 2b$$

$$\text{So, } 4b - 3a = 3c - 2b \Rightarrow 2b = a + c$$

$$\Rightarrow s = \frac{a+b+c}{2} = \frac{3b}{2}$$

$$\text{Since } \Delta^2 = r_1 r_2 r_3 = \frac{\Delta}{s} \cdot 3 \cdot \frac{\Delta}{s-b} \cdot 6$$

$$\Rightarrow s(s-b) = 3.6 \Rightarrow \frac{3b}{2} \cdot \frac{b}{2} = 3.6 \Rightarrow b = 2\sqrt{6}$$

59. (a) Let the length of 3 sides ΔABC be p, q, r .

$$p + q = x$$

$$pq = y$$

$$r = c$$

$$x^2 - c^2 = y$$

$$(p+q)^2 - c^2 = pq$$

$$p^2 + q^2 + 2pq - c^2 = pq$$

$$p^2 + q^2 - c^2 = -pq$$

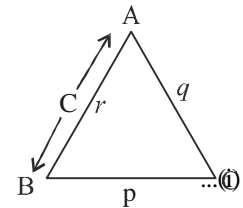
Using Cosine rule

$$\cos C = \frac{p^2 + q^2 - c^2}{2pq}$$

$$\cos C = \frac{-pq}{2pq} = \frac{-1}{2} \Rightarrow \angle C = \frac{2\pi}{3}$$

$$\text{Circumradius } 2R = \frac{c}{\sin C}$$

$$\therefore R = \frac{c}{2 \times \sin \left(\frac{\pi}{3} \right)} = \frac{c}{\sqrt{3}}$$



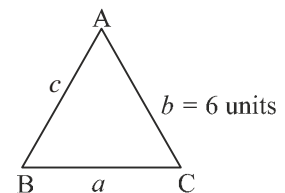
60. (c) $\Delta = 4\sqrt{5}, b = 6$ units

We know,

$$\tan \left(\frac{\beta}{2} \right) = \frac{\Delta}{s(s-b)}$$

$$\frac{\sqrt{5}}{4} = \frac{4\sqrt{5}}{s(s-b)}$$

$$s(s-6) = 16; S = \frac{a+b+c}{2}$$



($\therefore b = 6$ units)

$$\frac{(a+b+c)}{2} \left(\frac{a+b+c}{2} - 6 \right) = 16$$

$$(a+b+6)(a+b-6) = 64$$

$$(a+b)^2 = 100$$

$$a+b = 100$$

$$c = 6$$

Possible length of a, b, c are 3, 7, 6 or 7, 3, 6

So, smallest side is of length 3 units.

61. (d) $r_1 = 2r_2 = 3r_3$.

We know,

$$r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$$

$$\therefore \frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c} = k$$

$$s-a = \frac{\Delta}{k} = \lambda \quad \dots(i)$$

$$(\because \lambda = \frac{\Delta}{k}, \text{ be a constant})$$

$$s-b = \frac{2\Delta}{k} = 2\lambda \quad \dots(ii)$$

$$s-b = \frac{3\Delta}{k} = 3\lambda \quad \dots(iii)$$

Adding (i), (ii) and (iii)

$$3s - (a+b+c) = \lambda + 2\lambda + 3\lambda$$

$$3s - 2s = 6\lambda$$

$$s = 6\lambda$$

From (i), (ii)

$$a = 5\lambda, b = 4\lambda$$

$$a : b = 5 : 4$$

62. (c) Semi perimeter $s = \frac{3+5+7}{2} = \frac{15}{2}$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{15}{2} \left(\frac{15}{2} - 3 \right) \left(\frac{15}{2} - 5 \right) \left(\frac{15}{2} - 7 \right)}$$

$$= \frac{15\sqrt{3}}{4}$$

$$\therefore \cot A + \cot B + \cot C$$

$$= \frac{a^2 + b^2 + c^2}{4\Delta} = \frac{(9+25+49)}{4 \times 15\sqrt{3}} \times 4 = \frac{83}{15\sqrt{3}}$$

63. (d) $c^2 - a^2 = b(\sqrt{3}c - b)$

$$\Rightarrow c^2 - a^2 = \sqrt{3}bc - b^2 \Rightarrow \frac{c^2 + b^2 - a^2}{2bc} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos A = \frac{\sqrt{3}}{2} \text{ or } \angle A = 30^\circ$$

$$b^2 - a^2 = c(c-a) \Rightarrow b^2 - a^2 = c^2 - ac$$

$$\Rightarrow \frac{c^2 + a^2 - b^2}{2ac} = \frac{1}{2}$$

$$\Rightarrow \cos B = \frac{1}{2} \text{ or } \angle B = 60^\circ$$

$$\text{Hence, } \angle C \text{ or } \angle ACB = 180^\circ - (30 + 60) = 90^\circ$$

64. (b) We know, $P = \frac{2\Delta}{(\text{side})}$

$$\text{So, } \frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{1}{4\Delta^2} (a^2 + b^2 + c^2)$$

$$= \frac{25 + 36 + 16}{4 \left(\frac{15}{2} \left(\frac{15}{2} - 4 \right) \left(\frac{15}{2} - 5 \right) \left(\frac{15}{2} - 6 \right) \right)} = \frac{44}{225}$$

65. (b) Given, A, B, C are in A.P., $2B = A + C$

$$A + B + C = \pi \Rightarrow B = \frac{\pi}{3}$$

$$\text{law of cosines, } b^2 = a^2 + c^2 - 2ac \cos B$$

$$\Rightarrow b^2 = a^2 + c^2 - 2ac \left(\frac{1}{2} \right)$$

$$\Rightarrow b^2 = a^2 + c^2 - ac$$

$$\therefore \sqrt{a^2 - ac + c^2} \cdot \cos \left(\frac{A-C}{2} \right) = \sqrt{b^2} \left(\frac{a+c}{2b} \right) = \frac{a+c}{2}$$

66. (d) Domain of $f(x) = \sin^{-1} \left(\frac{x-1}{2x+3} \right)$ is

$$2x+3 \neq 0 \text{ \& } x \neq \frac{-3}{2} \text{ and } \left| \frac{x-1}{2x+3} \right| \leq 1$$

$$|x-1| \leq |2x+3|$$

$$\text{For } |2x+3| \geq |x-1|$$

On solving for x

$$x \in (-\infty, -4] \cup \left[-\frac{2}{3}, \infty \right)$$

$$\alpha = -4 \text{ \& } \beta = -\frac{2}{3} \therefore 12\alpha\beta = 32$$

67. (a) $\sin^{-1} \left(\frac{3x-22}{2x-19} \right) + \log_e \left(\frac{3x^2-8x+5}{x^2-3x-10} \right)$

$$-1 \leq \frac{3x-22}{2x-19} \leq 1$$

$$\frac{3x-22}{2x-19} + 1 \geq 0 \text{ and } \frac{3x-22}{2x-19} - 1 \leq 0$$

$$\Rightarrow \frac{5x-41}{2x-19} \geq 0 \text{ and } \frac{x-3}{2x-19} \leq 0$$

$$x \in \left(-\infty, \frac{41}{5} \right] \cup \left(\frac{19}{2}, \infty \right) \text{ and } x \in \left[3, \frac{19}{2} \right)$$

$$\Rightarrow x \in \left[3, \frac{41}{5} \right] \quad \dots(i)$$

$$\text{and, } \frac{3x^2 - 8x + 5}{x^2 - 3x - 10} > 0$$

$$\frac{(3x-5)(x-1)}{(x-5)(x-2)} > 0$$

$$\Rightarrow x \in (-\infty, -2) \cup \left[1, \frac{5}{3} \right] \cup (5, \infty) \quad \dots(ii)$$

Taking intersection from (i) and (ii)

$$x \in \left(5, \frac{41}{5} \right]$$

$$\Rightarrow 3\alpha + 10\beta = 15 + 82 = 97.$$

68. (b) $\cos^{-1} x - \left(\frac{\pi}{2} - \cos^{-1} y \right) = \alpha$

$$\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2} + \alpha$$

$$\alpha \in \left[-\frac{\pi}{2}, \pi \right], \frac{\pi}{2} + \alpha \in \left[0, \frac{3\pi}{2} \right]$$

$$\cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}) = \frac{\pi}{2} + \alpha$$

Taking cos on both sides.

$$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\sin \alpha$$

$$(xy + \sin \alpha)^2 = (1-x^2)(1-y^2)$$

$$x^2y^2 + 2xy \sin \alpha + \sin^2 \alpha = 1 - x^2 - y^2 + x^2y^2$$

$$x^2 + y^2 + 2xy \sin \alpha = 1 - \sin^2 \alpha$$

$$x^2 + y^2 + 2xy \sin \alpha = \cos^2 \alpha$$

$$\text{Min. value of } \cos^2 \alpha = 0 \text{ at } \alpha = \frac{\pi}{2}$$

69. (c) The given function is defined if

$$-1 \leq \frac{2-|x|}{4} \leq 1 \Rightarrow \left| \frac{2-|x|}{4} \right| \leq 1$$

$$\Rightarrow -4 \leq 2-|x| \leq 4 \Rightarrow -2 \leq |x| \leq 6$$

$$|x| \leq 6$$

$$\Rightarrow x \in [-6, 6] \quad \dots(i)$$

Now, $3-x \neq 1$

and $x \neq 2$

and $3-x > 0 \Rightarrow x < 3$

From (i), (ii) and (iii)

$$\Rightarrow x \in [-6, 3) - \{2\}$$

$$\alpha = 6, \beta = 3, \gamma = 2$$

$$\alpha + \beta + \gamma = 11$$

70. (b) For domain of $\frac{2x+3}{4x^2+x-3} > 0$ and $-1 \leq \frac{2x-1}{x+2} \leq 1$

$$\frac{2x+3}{(4x-3)(x+1)} > 0, \frac{3x+1}{x+2} \geq 0 \& \frac{x-3}{x+2} \leq 0$$

$$\begin{array}{ccccccc} & - & + & - & + & & \\ & \frac{1}{4} & -1 & -\frac{3}{2} & 3 & & \\ -\infty & & & & & & \infty \end{array}$$

$$x \in \left(\frac{-3}{2}, -1 \right) \cup (3, \infty) \quad \dots(i)$$

$$\begin{array}{ccccccc} & + & - & & + & & \\ & -2 & & -\frac{1}{3} & & & \infty \end{array}$$

$$x \in (-\infty, -2) \cup \left[-\frac{1}{3}, \infty \right) \quad \dots(ii)$$

$$\begin{array}{ccccccc} & + & - & & + & & \\ & -2 & & -3 & & & \infty \end{array}$$

$$x \in (-2, 3] \quad \dots(iii)$$

From (i), (ii) & (iii)

$$x \in \left(\frac{3}{4}, 3 \right]$$

$$\therefore \alpha = \frac{3}{4}, \beta = 3$$

$$\Rightarrow 5\beta - 4\alpha = 15 - 3 = 12$$

71. (c) We know,

$$\sin^{-1} x \in \left[\frac{-\pi}{2}, \frac{\pi}{2} \right], \cos^{-1} x \in [0, \pi] \Rightarrow \tan^{-1} x \in \left(\frac{-\pi}{2}, \frac{\pi}{2} \right)$$

$$\sin^{-1}(-\cos 2) = \sin^{-1} \left(-\sin \left(\frac{\pi}{2} - 2 \right) \right) = - \left(\frac{\pi}{2} - 2 \right) = 2 - \frac{\pi}{2}$$

$$\left[\frac{\pi}{2} - 2 \approx -0.4292 \in \left[\frac{-\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$\cos^{-1}(\sin 3) = \cos^{-1} \left(\cos \left(\frac{\pi}{2} - 3 \right) \right)$$

$$= \cos^{-1} \left(\cos \left(3 - \frac{\pi}{2} \right) \right) = 3 - \frac{\pi}{2} \quad \left[3 - \frac{\pi}{2} \approx 1.4292 \in [0, \pi] \right]$$

$$\tan^{-1}(\cot 5) = \tan^{-1} \left(\tan \left(\frac{\pi}{2} - 5 \right) \right)$$

$$\left[\frac{\pi}{5} - 5 \notin \left(\frac{-\pi}{2}, \frac{\pi}{2} \right) \right], \text{ So we add } \pi \text{ to } \frac{\pi}{2} - 5$$

$$\text{i.e. } \frac{\pi}{2} - 5 + \pi = \frac{3\pi}{2} - 5 \approx -0.2876 \in \left(\frac{-\pi}{2}, \frac{\pi}{2} \right)$$

$$\therefore \tan^{-1}(\cot 5) = \tan^{-1} \left(\tan \left(\frac{3\pi}{2} - 5 \right) \right) = \frac{3\pi}{2} - 5$$

$$\begin{aligned} \therefore \sin^{-1}(-\cos 2) + \cos^{-1}(\sin 3) + \tan^{-1}(\cot 5) \\ = 2 - \frac{\pi}{2} + 3 - \frac{\pi}{2} + \frac{3\pi}{2} - 5 = \frac{\pi}{2} \end{aligned}$$

72. (b) Since, $0 \leq \frac{x^2}{1+x^2} < 1 \Rightarrow 0 \leq \sin^{-1} \left(\frac{x^2}{1+x^2} \right) < \frac{\pi}{2}$
 $\Rightarrow 0 \leq 4 \sin^{-1} \left(\frac{x^2}{1+x^2} \right) \leq 2\pi \Rightarrow \text{Range of } f(x) = [0, 2\pi]$

73. (a) Given equation is

$$\sin^{-1} \left(\sin \frac{2\pi}{3} \right) + \cos^{-1} \left(\cos \frac{7\pi}{6} \right) + \tan^{-1} \left(\tan \frac{3\pi}{4} \right)$$

$$\sin^{-1} \left(\sin \frac{2\pi}{3} \right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\cos^{-1} \left(\cos \frac{7\pi}{6} \right) = 2\pi - \frac{7\pi}{6} = \frac{5\pi}{6}$$

$$\tan^{-1} \left(\tan \frac{3\pi}{4} \right) = \tan^{-1}(-1) = \tan^{-1} \left(\tan \left(-\frac{\pi}{4} \right) \right) = -\frac{\pi}{4}$$

$$\Rightarrow \sin^{-1} \left(\sin \frac{2\pi}{3} \right) + \cos^{-1} \left(\cos \frac{7\pi}{6} \right) + \tan^{-1} \left(\tan \frac{3\pi}{4} \right)$$

$$= \frac{\pi}{3} + \frac{5\pi}{6} - \frac{\pi}{4} = \frac{4\pi + 10\pi - 3\pi}{12} = \frac{11\pi}{12}$$

74. (b) $\frac{6 + 2 \log_3 x}{-5x} > 0 \& x > 0 \& x \neq \frac{1}{3}$

This gives $x \in \left(0, \frac{1}{27} \right)$... (i)

For $\sin^{-1} x$, $x \in [-1, 1]$

$$-1 \leq \log_3 x \left(\frac{6 + 2 \log_3 x}{-5x} \right) \leq 1; 3x \leq \frac{6 + 2 \log_3 x}{-5x} \leq \frac{1}{3x}$$

$$\begin{aligned} \downarrow \qquad \qquad \qquad \downarrow \\ 15x^2 + 6 + 2 \log_3 x \geq 0 \qquad 6 + 2 \log_3 x + \frac{5}{3} \geq 0 \end{aligned}$$

$$\in \left(0, \frac{1}{27} \right) \qquad \dots \text{(ii)}$$

$$x \geq 3^{-\frac{23}{6}} \qquad \dots \text{(iii)}$$

from (i), (ii) & (iii)

$$x \in \left(3^{-\frac{23}{6}}, \frac{1}{27} \right)$$

$\therefore \alpha$ is small positive quantity.

$$\& \beta = \frac{1}{27}$$

$$\therefore \alpha^2 + \frac{5}{\beta} = (3^{-\frac{23}{6}})^2 + 5 \times 27 = 135 + \frac{1}{3^{\frac{23}{3}}}$$

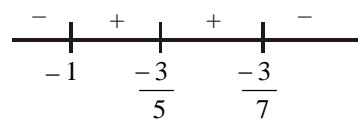
$\therefore \alpha^2 + \frac{5}{\beta}$ is just greater than 135.

75. (a) $f(x) = \sec^{-1} \frac{2x}{5x+3}$

$$\Rightarrow \left| \frac{2x}{5x+3} \right| \geq 1 \Rightarrow |2x| \geq |5x+3|$$

$$\Rightarrow (2x)^2 - (5x+3)^2 \geq 0$$

$$\Rightarrow (7x+3)(-3x-3) \geq 0$$



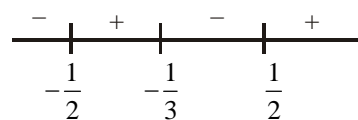
$$\therefore \text{domain} \left[-1, -\frac{3}{5} \right) \cup \left(-\frac{3}{7}, -\frac{3}{5} \right]$$

So, $\alpha = -1, \beta = -\frac{3}{5}, \gamma = -\frac{3}{5}, \delta = -\frac{3}{7}$

$$\Rightarrow |3\alpha + 10(\beta + \gamma) + 21\delta|$$

$$\Rightarrow \left| -3 + 10 \left(-\frac{6}{5} \right) + \left(-\frac{3}{7} \right) 21 \right| = |-24| = 24$$

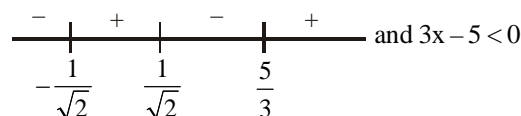
76. (b) $\frac{6x^2 + 5x + 1}{2x - 1} > 0 \Rightarrow \frac{(3x+1)(2x+1)}{2x-1} > 0$



$$x \in \left(-\frac{1}{2}, -\frac{1}{3} \right) \cup \left(\frac{1}{2}, \infty \right) \qquad \dots \text{(i)}$$

$$\text{and } -1 \leq \frac{2x^2 - 3x + 4}{3x - 5} \leq 1$$

$$\frac{2x^2 - 1}{3x - 5} \geq 0 \text{ and } \frac{2x^2 - 6x + 9}{3x - 5} \leq 0$$



$$\Rightarrow x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right] \cup \left(\frac{5}{3}, \infty \right) \quad \dots(ii)$$

$$\Rightarrow x < \frac{5}{3} \quad \dots(iii)$$

$$(i) \cap (ii) \cap (iii) \equiv \left(\frac{-1}{2}, \frac{1}{3} \right) \cup \left(\frac{1}{2}, \frac{1}{\sqrt{2}} \right]$$

$$\text{So } 18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2) = 18 \left(\frac{1}{4} + \frac{1}{9} + \frac{1}{4} + \frac{1}{2} \right) = 20$$

77. (b) We have $0 < x < 1$.

$$\text{Now, } 2 \tan^{-1} \left(\frac{1-x}{1+x} \right) = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

$$\text{Let } x = \tan \theta, \theta \in \left(0, \frac{\pi}{4} \right).$$

$$\text{So, } 2 \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \theta \right) \right) = \cos^{-1}(\cos 2\theta)$$

$$2 \left(\frac{\pi}{4} - \theta \right) = 2\theta \Rightarrow 4\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{8}$$

$$x = \tan \frac{\pi}{8} \therefore x = \sqrt{2} - 1 \approx 0.414$$

78. (c) $\tan^{-1} \left(\frac{1+\sqrt{3}}{3+\sqrt{3}} \right) + \sec^{-1} \left(\frac{\sqrt{8+4\sqrt{3}}}{\sqrt{6+3\sqrt{3}}} \right)$
 $= \tan^{-1} \left(\frac{1+\sqrt{3}}{\sqrt{3}(1+\sqrt{3})} \right) + \sec^{-1} \left(\frac{2\sqrt{2+\sqrt{3}}}{\sqrt{3}(\sqrt{2+\sqrt{3}})} \right)$
 $= \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) + \sec^{-1} \left(\frac{2}{\sqrt{3}} \right) = \left(\frac{\pi}{6} + \frac{\pi}{6} \right) = \frac{\pi}{3}$

79. (c) $f(x) = \sin^{-1} \left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \right)$

Now, Take domain of \sin^{-1}

$$\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \geq -1 \text{ and } \frac{x^2 - 3x + 2}{x^2 + 2x + 7} \leq 1$$

$$2x^2 - x + 9 \geq 0 \Rightarrow x \in \mathbb{R} \text{ and } 5x \geq -5 \Rightarrow x \geq -1$$

Hence, Domain $x \in [-1, \infty)$.

80. (b) $\left| \frac{x^2 - 4x + 2}{x^2 + 3} \right| \leq 1 \Rightarrow (x^2 - 4x + 2)^2 \leq (x^2 + 3)^2$

$$\Rightarrow (x^2 - 4x + 2)^2 - (x^2 + 3)^2 \leq 0$$

$$\Rightarrow (2x^2 - 4x + 5)(-4x - 1) \leq 0 \Rightarrow -4x - 1 \leq 0 \Rightarrow x \geq \frac{-1}{4}$$

81. (c) Given functions is

$$f(x) = \sin^{-1}[2x^2 - 3] + \log_2 \left(\log_{\frac{1}{2}}(x^2 - 5x + 5) \right)$$

$$-1 \leq [2x^2 - 3] \leq 1 \Rightarrow -1 \leq 2x^2 - 3 < 2$$

$$\Rightarrow 2 \leq 2x^2 < 5 \Rightarrow 1 \leq x^2 < \frac{5}{2}$$

$$\Rightarrow x \in \left(-\sqrt{\frac{5}{2}}, -1 \right) \cup \left[1, \sqrt{\frac{5}{2}} \right) \quad \dots(i)$$

Similarly, $x^2 - 5x + 5 > 0$

$$\Rightarrow \left(x - \left(\frac{5-\sqrt{5}}{2} \right) \right) \left(x - \left(\frac{5+\sqrt{5}}{2} \right) \right) > 0 \quad \dots(ii)$$

$$\text{and, } \log_{\frac{1}{2}}(x^2 - 5x + 5) > 0 \Rightarrow x^2 - 5x + 5 < 1$$

$$\Rightarrow x^2 - 5x + 4 < 0 \Rightarrow x \in (1, 4) \quad \dots(iii)$$

by equation (i), (ii) & (iii) we get

$$x \in \left(1, \frac{5-\sqrt{5}}{2} \right)$$

82. (d) $-1 \leq \frac{2 \sin^{-1} \left(\frac{1}{4x^2 - 1} \right)}{\pi} \leq 1$

$$\Rightarrow -\pi/2 \leq \sin^{-1} \frac{1}{4x^2 - 1} \leq \pi/2$$

On solving inequalities we get; $-1 \leq \frac{1}{4x^2 - 1} \leq 1$

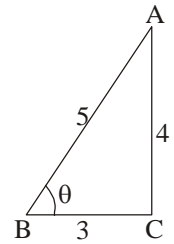
$$x \in \left(-\infty, \frac{-1}{\sqrt{2}} \right] \cup \left[\frac{1}{\sqrt{2}}, \infty \right) \cup \{0\}$$

83. (c) Let $\tan^{-1} \frac{4}{3} = \theta \Rightarrow \tan \theta = \frac{4}{3}$

$$\Rightarrow \cos^{-1} \left(\frac{3}{10} \cos \theta + \frac{2}{5} \sin \theta \right)$$

$$= \cos^{-1} \left(\frac{3}{10} \times \frac{3}{5} + \frac{2}{5} \cdot \frac{4}{5} \right)$$

$$= \cos^{-1} \left(\frac{9}{50} + \frac{8}{25} \right) = \cos^{-1} \left(\frac{25}{50} \right) = \cos^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{3}$$



84. (d) Given function domain is $[-1, 1]$.

$$-1 \leq \frac{x^2 - 5x + 6}{x^2 - 9} \leq 1$$

Take maximum value and subtract 1 both sides.

$$\frac{x^2 - 5x + 6}{x^2 - 9} - 1 \leq 0, \frac{1}{x+3} \geq 0, x \neq 3$$

$$x \in (-3, \infty) - \{3\} \quad \dots(i)$$

Take minimum value and add 1 both sides.

$$\frac{x^2 - 5x + 6}{x^2 - 9} + 1 \geq 0, \frac{2x+1}{x+3} \geq 0, x \neq 3$$

$$x \in (-\infty, -3) \cup \left[-\frac{1}{2}, \infty \right) - \{3\} \quad \dots(ii)$$

Now, take the intersection of two equations (i) and (ii)

$$x \in \left[-\frac{1}{2}, \infty\right) - \{3\}$$

Now, take $x^2 - 3x + 2 > 0$, $x \in (-\infty, 1) \cup (2, \infty)$

$$x^2 - 3x + 2 \neq 1, x \neq \frac{3 \pm \sqrt{5}}{2}$$

Take intersection of all the solutions.

$$\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2}\right\}$$

Note: 3 is not in the domain.

85. (b) Given relation is $(x * 1) * 1 = x * (1 * 1)$
 Apply $x * y = x^2 + y^3 \Rightarrow (x^2 + 1) * 1 = x * (2)$
 $(x^2 + 1)^2 + 1 = x^2 + 8 \Rightarrow x^4 + x^2 - 6 = 0$
 $\Rightarrow (x^2 + 3)(x^2 - 2) = 0 \Rightarrow x^2 = 2 \Rightarrow x^4 = 4$

Take, $2 \sin^{-1} \left(\frac{x^4 + x^2 - 2}{x^4 + x^2 + 2} \right)$

Put the value of x^2 and x^4 in the given function

$$\Rightarrow 2 \sin^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{3}$$

86. (c) $-1 \leq \frac{x-1}{x+1} \leq 1 \Rightarrow \frac{x-1}{x+1} \leq 1$... (i)
 $\Rightarrow \frac{2}{x+1} \geq 0 \Rightarrow x \in (-1, \infty)$... (i)
 and $\frac{x-1}{x+1} \geq -1 \Rightarrow \frac{2x}{x+1} \geq 0 \Rightarrow x \in (-\infty, -1) \cup [0, \infty)$... (ii)
 from (i) and (ii) $x \in [0, \infty)$... (iii)

Now, $-1 \leq \frac{3x^2 + x - 1}{(x-1)^2} \leq 1 \Rightarrow \frac{3x^2 + x - 1}{(x-1)^2} \leq 1$

$$\Rightarrow \frac{2x^2 + 3x - 2}{(x-1)^2} \leq 0$$

$$\Rightarrow \frac{(2x-1)(x+2)}{(x-1)^2} \leq 0 \Rightarrow x \in \left[-2, \frac{1}{2}\right]$$
 ... (iv)

and $\frac{3x^2 + x - 1}{(x-1)^2} \geq -1 \Rightarrow \frac{x(4x-1)}{(x-1)^2} \geq 0$

$$\Rightarrow x \in (-\infty, 0] \cup \left[\frac{1}{4}, \infty\right)$$
 ... (v)

from (iv) and (v); $x \in [-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$... (vi)

from (iii) and (vi); $x \in \{0\} \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

87. (a) $0 \leq x^2 - x + 1 \leq 1$
 $\Rightarrow x^2 - x \leq 0 \Rightarrow x \in [0, 1]$... (i)

Also, $0 < \sin^{-1} \left(\frac{2x-1}{2} \right) \leq \frac{\pi}{2}$

$$\Rightarrow 0 < \frac{2x-1}{2} \leq 1 \Rightarrow 0 < 2x-1 \leq 2$$

$$\Rightarrow 1 < 2x \leq 3 \Rightarrow \frac{1}{2} < x \leq \frac{3}{2}$$
 ... (ii)

From (i) and (ii), we get

$$x \in \left(\frac{1}{2}, 1\right] \Rightarrow \alpha = \frac{1}{2}, \beta = 1 \text{ So, } \alpha + \beta = \frac{3}{2}$$

88. (a) Let $\frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8} = q \Rightarrow \sin 4q = \frac{\sqrt{63}}{8}$

or $\cos 4q = \frac{1}{8}$ [$\because \cos q = \sqrt{1 - \sin^2 q}$]

$$\Rightarrow 2 \cos^2 2q - 1 = \frac{1}{8}$$
 [$\because \cos 2q = 2 \cos^2 q - 1$]

$$\Rightarrow \cos^2 2q = \frac{9}{16} \Rightarrow \cos 2q = \frac{3}{4}$$

$$\Rightarrow 2 \cos^2 q - 1 = \frac{3}{4} \Rightarrow \cos^2 q = \frac{7}{8} \Rightarrow \cos q = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\searrow \tan q = \frac{1}{\sqrt{7}}$$
 [$\because \sin q = \sqrt{\sec^2 q - 1}$]

89. (a) $2 \sin^{-1} x + 3 \cos^{-1} x = \frac{2\pi}{5}$
 $\left[\because 2(\sin^{-1} x + \cos^{-1} x) = 2 \times \frac{\pi}{2} = \pi \right]$

$$\Rightarrow \pi + \cos^{-1} x = \frac{2\pi}{5}$$

$$\Rightarrow \cos^{-1} x = \frac{-3\pi}{5} \left(\because \cos^{-1} x \in [0, \pi] \right)$$

Not possible

90. (b) Given that, $\cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5 + \cot^{-1} n = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{n} = \frac{\pi}{4}$$

$$\left\{ \cot^{-1} x = \tan^{-1} \frac{1}{x} \right\}$$

$$\Rightarrow \tan^{-1} \left(\frac{46}{48} \right) + \tan^{-1} \frac{1}{n} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{23}{24} \right) + \tan^{-1} \frac{1}{n} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \frac{1}{n} = \tan^{-1} 1 - \tan^{-1} \frac{23}{24}$$

$$\Rightarrow \tan^{-1} \frac{1}{n} = \tan^{-1} \left(\frac{1 - \frac{23}{24}}{1 + \frac{23}{24}} \right) = \tan^{-1} \left(\frac{1}{47} \right)$$

$$\Rightarrow \tan^{-1} \frac{1}{n} = \tan^{-1} \frac{1}{47} \Rightarrow n = 47$$

91. (d) Let $\sin^{-1}\alpha = A$, $\sin^{-1}\beta = B$, $\sin^{-1}\gamma = C$

$$A + B + C = \pi$$

$$\text{Given } (\alpha + \beta + \gamma)(\alpha + \beta - \gamma) = 3\alpha\beta$$

$$(\alpha + \beta)^2 - \gamma^2 = 3\alpha\beta$$

$$\alpha^2 + \beta^2 - \gamma^2 = \alpha\beta$$

$$\frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta} = \frac{1}{2}$$

[\because A, B, C are the sides of triangle, so

$$\cos C = \frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta}]$$

$$\Rightarrow \cos C = \frac{1}{2}, \sin C = \gamma, \cos C = \sqrt{1 - \gamma^2} = \frac{1}{2}$$

$$\gamma = \frac{\sqrt{3}}{2}$$

92. (b) $a = \sin^{-1}(\sin 5) = 5 - 2\pi$

$$\text{and } b = \cos^{-1}(\cos 5) = 2\pi - 5$$

$$\therefore a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2$$

$$= 8\pi^2 - 40\pi + 50$$

93. (c) Give $\cos^{-1}(2x) - 2\cos^{-1}\sqrt{1-x^2} = \pi$

$$\cos^{-1}(2x) - \cos^{-1}(2(1-x^2)-1) = \pi$$

$$\cos^{-1}(2x) - \cos^{-1}(1-2x^2) = \pi$$

$$-\cos^{-1}(1-2x^2) = \pi - \cos^{-1}(2x)$$

Taking cos both sides we get

$$\cos(-\cos^{-1}(1-2x^2)) = \cos(\pi - \cos^{-1}(2x))$$

$$1 - 2x^2 = -2x \Rightarrow 2x^2 - 2x - 1 = 0$$

$$\text{On solving, } x = \frac{1 \pm \sqrt{3}}{2}$$

$$\text{As } x \in [-1/2, 1/2], x = \frac{1 + \sqrt{3}}{2} = \text{rejected}$$

$$\text{So } x = \frac{1 - \sqrt{3}}{2} \Rightarrow x^2 - 1 = -\sqrt{3}/2$$

$$= 2\sin^{-1}(x^2 - 1) = 2\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{-2\pi}{3}$$

94. (a) Since, $\cos^{-1}\frac{4}{5} = \tan^{-1}\frac{3}{4}$

$$\Rightarrow \sin^{-1}\frac{\alpha}{17} = \tan^{-1}\frac{77}{36} - \tan^{-1}\frac{3}{4} = \tan^{-1}\left(\frac{\frac{77}{36} - \frac{3}{4}}{1 + \frac{77}{36} \cdot \frac{3}{4}}\right)$$

$$\Rightarrow \sin^{-1}\frac{\alpha}{17} = \tan^{-1}\frac{8}{15} = \sin^{-1}\frac{8}{17} \Rightarrow \frac{\alpha}{17} = \frac{8}{17} \Rightarrow \alpha = 8$$

$$\text{Now, } \sin^{-1}(\sin 8) + \cos^{-1}(\cos 8)$$

$$= 3\pi - 8 + 8 - 2\pi = \pi$$

95. (c) By given condition

$$a_2 - a_1 = a_3 - a_2 = \dots = a_{2022} - a_{2021} = 1$$

$$\therefore \tan^{-1}\left(\frac{a_2 - a_1}{1 + a_1 a_2}\right) + \tan^{-1}\left(\frac{a_3 - a_2}{1 + a_2 a_3}\right) + \dots$$

$$+ \tan^{-1}\left(\frac{a_{2022} - a_{2021}}{1 + a_{2021} a_{2022}}\right)$$

$$= \left[\tan^{-1} a_2 - \tan^{-1} a_1\right] + \left[\tan^{-1} a_3 - \tan^{-1} a_2\right] + \dots$$

$$+ \left[\tan^{-1} a_{2022} - \tan^{-1} a_{2021}\right]$$

$$= \tan^{-1} a_{2022} - \tan^{-1} a_1$$

$$\text{Since, } a_{2022} = 2022 \Rightarrow a_1 = 1$$

$$= \tan^{-1}(2022) - \tan^{-1} 1 = \tan^{-1} 2022 - \frac{\pi}{4} \text{ (option c)}$$

96. (d) Given an equation $\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1$

$$\Rightarrow \frac{\ln \cos x - \ln \sin x}{\ln \cos x} + 4 \frac{\ln \sin x - \ln \cos x}{\ln \sin x} = 1$$

$$\Rightarrow (\ln \sin x)^2 - 4(\ln \sin x)(\ln \cos x) + 4(\ln \cos x)^2 = 1$$

$$\Rightarrow \ln \sin x = 2 \ln \cos x$$

$$\Rightarrow \ln \sin x = \ln(1 - \sin^2 x)$$

$$\Rightarrow \sin^2 x + \sin x - 1 = 0 \Rightarrow \sin x = \frac{-1 + \sqrt{5}}{2}$$

$$\text{So, } \alpha + \beta = 4$$

97. (a) **Case I:** $x > 0$

$$\text{We know that } \tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x.$$

$$\text{Then, } \tan^{-1}\frac{2x}{1-x^2} + \tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$$

$$x = 2 - \sqrt{3}$$

$$\text{Case II: } x < 0$$

$$\text{Then, } \cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right) + \pi.$$

$$\tan^{-1}\frac{2x}{1-x^2} + \tan^{-1}\frac{2x}{1-x^2} + \pi = \frac{\pi}{3}$$

$$x = \frac{-1}{\sqrt{3}} \Rightarrow \alpha = 2$$

98. (b) $\cos(\sin^{-1} x) = \cos(\cos^{-1} \sqrt{1-x^2}) = \sqrt{1-x^2}$

$$\cot\left(\tan^{-1} \sqrt{1-x^2}\right) = \cot \cot^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right) = \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}} = k \Rightarrow 1-2x^2 = k^2(1-x^2)$$

$$\Rightarrow (k^2-2)x^2 = k^2-1; x^2 = \frac{k^2-1}{k^2-2}$$

\therefore solution of $x^2 - bx - 5 = 0$ with roots

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} \text{ and } \frac{\alpha}{\beta} \text{ So } \alpha^2 = \frac{k^2-1}{k^2-2}, \beta^2 = \frac{k^2-1}{k^2-2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 2 \left(\frac{k^2-2}{k^2-1} \right) \& \frac{\alpha}{\beta} = -1$$

$$\text{Sum of roots} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{\alpha}{\beta} = b$$

$$\Rightarrow \frac{2(k^2-2)}{k^2-1} - 1 = b \quad \dots(i)$$

$$\text{Product of roots} = \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} \right) \frac{\alpha}{\beta} = -5$$

$$\Rightarrow \frac{2(k^2-2)}{k^2-1} (-1) = -5 \Rightarrow 2k^2-4 = 5k^2-5$$

$$\Rightarrow 3k^2 = 1 \Rightarrow k^2 = \frac{1}{3} \dots \text{Put in (i)}$$

$$\Rightarrow b = \frac{2(k^2-2)}{k^2-1} - 1 = 5 - 1 = 4 \Rightarrow \frac{b}{k^2} = \frac{4}{\frac{1}{3}} = 12$$

99. (b) Given expression

$$\tan \left(2 \tan^{-1} \frac{1}{5} + \sec^{-1} \frac{\sqrt{5}}{2} + 2 \tan^{-1} \frac{1}{8} \right)$$

$$\tan \left(2 \left(\tan^{-1} \left(\frac{1}{5} \right) + \tan^{-1} \left(\frac{1}{8} \right) \right) + \sec^{-1} \frac{\sqrt{5}}{2} \right)$$

$$= \tan \left(2 \tan^{-1} \left(\frac{\frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{40}} \right) + \sec^{-1} \frac{\sqrt{5}}{2} \right)$$

$$\text{Let } \sec^{-1} \frac{\sqrt{5}}{2} = \theta, \text{ Then, } \tan \theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1} \frac{1}{2}$$

$$\text{So, } \tan \left(2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} \right)$$

$$= \tan \left(\tan^{-1} \left(\frac{\frac{2}{3}}{\frac{8}{9}} \right) + \tan^{-1} \frac{1}{2} \right)$$

$$= \tan \left(\tan^{-1} \left(\frac{3}{4} \right) + \tan^{-1} \left(\frac{1}{2} \right) \right)$$

$$= \tan \left(\tan^{-1} \left(\frac{\frac{3}{4} + \frac{1}{2}}{1 - \frac{3}{4} \times \frac{1}{2}} \right) \right) = \tan \left(\tan^{-1} (2) \right) = 2$$

100. (b) Given $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta} = k$

$$\sin^{-1} x = k\alpha \text{ and } \cos^{-1} x = k\beta \Rightarrow k = \frac{\pi}{2(\alpha + \beta)}$$

$$\sin \left(\frac{2\pi\alpha}{\alpha + \beta} \right) = \sin (4 \sin^{-1} x)$$

$$= 2 \sin (2 \sin^{-1} x) \cos (2 \sin^{-1} x) = 4x \sqrt{1-x^2} (1-2x^2)$$

Exercise 3 :

PREVIOUS YEARS MCQs

1. (b) $\cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow \frac{2}{3} = \frac{8^2 + c^2 - 7^2}{2 \times 8 \times c}$

$$c = \alpha = 9 \Rightarrow \cos C = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8} = \frac{2}{7}$$

$$49 \cos 3C + 42$$

$$= 49(4 \cos^3 C - 3 \cos C) + 42$$

$$[\because \cos 3x = 4 \cos^3 x - 3 \cos x]$$

$$= 49 \left(4 \left(\frac{2}{7} \right)^3 - 3 \left(\frac{2}{7} \right) \right) + 42 = \frac{32}{7}$$

$$m + n = 32 + 7 = 39$$

2. (a) $\cos 2x + a \sin x = 2a - 7$

$$\Rightarrow 1 - 2 \sin^2 x + a \sin x = 2a - 7$$

$$a(\sin x - 2) = 2(\sin x - 2)(\sin x + 2)$$

$$\sin x \neq 2 \text{ then } a = 2(\sin x + 2) \text{ and } -1 \leq \sin x \leq 1$$

$$\Rightarrow a \in [2, 6] \Rightarrow p = 2 \quad q = 6$$

$$r = \tan 9^\circ + \cot 9^\circ - \tan 27^\circ - \cot 27^\circ$$

$$r = \frac{1}{\sin 9^\circ \cdot \cos 9^\circ} - \frac{1}{\sin 27^\circ \cdot \cos 27^\circ} = 2 \left[\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1} \right]$$

$$\left[\because \sin 18^\circ = \frac{\sqrt{5}-1}{4}, \sin 32^\circ = \frac{\sqrt{5}+1}{4} \right]$$

$$r = 4 \Rightarrow p \cdot q \cdot r = 2 \times 6 \times 4 = 48.$$

3. (b) $4 + 5 \tan \theta = \sec \theta$

$$\text{Squaring: } 24 \tan^2 \theta + 40 \tan \theta + 15 = 0$$

$$\tan \theta = \frac{-10 \pm \sqrt{10}}{12} \text{ and } \tan \theta = -\left(\frac{10 + \sqrt{10}}{12}\right)$$

$$\therefore \tan \theta = \frac{\sqrt{10} - 10}{12}$$

4. (d) Given that

$$\lambda = \cos^2 2x - 2 \sin^4 x - 2 \cos^2 x$$

$$\lambda = (2 \cos^2 x - 1)^2 - 2(1 - \cos^2 x) - 2 \cos^2 x$$

$$= 4 \cos^4 x - 4 \cos^2 x + 1 - 2(1 - 2 \cos^2 x + \cos^4 x) - 2 \cos^2 x$$

$$= 2 \cos^4 x - 2 \cos^2 x + 1 - 2$$

$$= 2 \left[\cos^4 x - \cos^2 x - \frac{1}{2} \right]$$

$$= 2 \left[\left(\cos^2 x - \frac{1}{2} \right)^2 - \frac{3}{4} \right]$$

$$\lambda_{\max} = 2 \left[\frac{1}{4} - \frac{3}{4} \right] = 2 \times \left(-\frac{2}{4} \right) = -1 \text{ (when } \cos x = 1)$$

$$\lambda_{\max} = 2 \left[0 - \frac{3}{4} \right] = -\frac{3}{2} \text{ (when } \cos^2 x - \frac{1}{2} = 0)$$

$$\text{So, Range} = \left[-\frac{3}{2}, -1 \right]$$

5. (c) We have $f(x) = \sin^{-1}(x - \sqrt{x})$

$$= \sin^{-1} \left(\left(\sqrt{x} - \frac{1}{2} \right)^2 - \frac{1}{4} \right)$$

$$\therefore \text{Range of } f(x) \text{ is } \left[\sin^{-1} \left(\frac{-1}{4} \right), \sin^{-1} 1 \right]$$

$$= \left[-\sin^{-1} \frac{1}{4}, \frac{\pi}{2} \right]$$

6. (d) $\sin \left(\tan^{-1} \frac{4}{5} + \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{1}{9} - \tan^{-1} \frac{1}{7} \right)$

$$= \sin \left[\left(\tan^{-1} \frac{4}{5} + \tan^{-1} \frac{1}{9} \right) + \left(\tan^{-1} \frac{4}{3} - \tan^{-1} \frac{1}{7} \right) \right]$$

$$= \sin \left[\tan^{-1} \left(\frac{\frac{4}{5} + \frac{1}{9}}{1 - \frac{4}{5} \cdot \frac{1}{9}} \right) + \tan^{-1} \left(\frac{\frac{4}{3} - \frac{1}{7}}{1 + \frac{4}{3} \cdot \frac{1}{7}} \right) \right]$$

$$= \sin \left[\tan^{-1} \left(\frac{36 + 5}{45 - 4} \right) + \tan^{-1} \left(\frac{28 - 3}{21 + 4} \right) \right]$$

$$= \sin \left[\tan^{-1} \frac{41}{41} + \tan^{-1} \frac{25}{25} \right]$$

$$= \sin [\tan^{-1}(1) + \tan^{-1}(1)] = \sin \left(\frac{\pi}{4} + \frac{\pi}{4} \right) = \sin \frac{\pi}{2} = 1$$

7. (a) $\cos^{-1} \left(\frac{1}{2} \right) + \sin^{-1}(1) + \tan^{-1} \frac{1}{\sqrt{3}}$

$$= \frac{\pi}{3} + \frac{\pi}{2} + \frac{\pi}{6} = \frac{6\pi}{6} = \pi$$

8. (a) Let $\tan^{-1} \left(\frac{a}{x} \right) + \tan^{-1} \left(\frac{b}{x} \right) = \frac{\pi}{2}$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{a}{x} + \frac{b}{x}}{1 - \frac{ab}{x^2}} \right) = \frac{\pi}{2} \Rightarrow \frac{\frac{a+b}{x}}{1 - \frac{ab}{x^2}} = \tan \frac{\pi}{2}$$

$$\Rightarrow 1 - \frac{ab}{x^2} = 0 \Rightarrow x^2 = ab \Rightarrow x = \sqrt{ab}$$

9. (a) Let $\operatorname{cosec}^{-1} \left(\frac{-2}{\sqrt{3}} \right) = \theta$

$$\Rightarrow \operatorname{cosec} \theta = \frac{-2}{\sqrt{3}} = -\operatorname{cosec} \frac{\pi}{3} = \operatorname{cosec} \left(\frac{-\pi}{3} \right)$$

$$\Rightarrow \theta = \frac{-\pi}{3} \in \left[\frac{-\pi}{3}, \frac{\pi}{2} \right] - \{0\}$$

\therefore Principal value of $\operatorname{cosec}^{-1} \left(\frac{-2}{\sqrt{3}} \right)$ is $\left(\frac{-\pi}{3} \right)$

10. (d) We have, $\sin \left(\sin^{-1} \frac{1}{5} + \cos^{-1} x \right) = 1$

$$\Rightarrow \sin^{-1} \frac{1}{5} + \cos^{-1} x = \sin^{-1}(1)$$

$$\Rightarrow \sin^{-1} \frac{1}{5} + \frac{\pi}{2} - \sin^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} \frac{1}{5} - \sin^{-1} x = 0$$

$$\Rightarrow \sin^{-1} \frac{1}{5} = \sin^{-1} x \Rightarrow x = \sin \left(\sin^{-1} \frac{1}{5} \right) \Rightarrow x = \frac{1}{5}$$

11. (a) $\sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) = \left[\sin^{-1} \sin \left(-\frac{\pi}{3} \right) \right] = -\frac{\pi}{3}$

12. (c) Given, $\cos(2 \sin^{-1} x) = \frac{1}{9}$

Assume $\sin^{-1} x = \theta$;

$$\cos(2\theta) = \frac{1}{9}, \sin \theta = \pm \frac{2}{3}$$

as m and n are co-prime natural numbers, $x = \frac{2}{3}$

i.e. $m = 2, n = 3$ (on comparing with $x = \frac{m}{n}$)

So, the quadratic equation becomes $2x^2 - 3x + 1 = 0$, whose

roots are $\alpha = 1, \beta = \frac{1}{2}$

$\left(1, \frac{1}{2} \right)$ lies on $5x + 8y = 9$

13. (b) $\tan^{-1} x + \tan^{-1} 2x = \frac{\pi}{4}; x > 0$

$$\tan^{-1} \frac{x+2x}{1-2x^2} = \frac{\pi}{4} \Rightarrow \frac{x+2x}{1-2x^2} = 1$$

$$\Rightarrow 2x^2 + 3x - 1 = 0$$

$$x = \frac{-3 \pm \sqrt{9+8}}{8} = \frac{-3 \pm \sqrt{17}}{8}$$

Only possible $x = \frac{-3 + \sqrt{17}}{8}$

14. (b) Given, $f(x) = \ln(4x^2 + 11x + 6) + \sin^{-1}(4x + 3)$

$$+ \cos^{-1}\left(\frac{10x+6}{3}\right)$$

Now, (i) $4x^2 + 11x + 6 > 0 \Rightarrow (4x + 3)(x + 2) > 0$

$$x \in (-\infty, -2) \cup \left(-\frac{3}{4}, \infty\right)$$

(ii) $4x + 3 \in [-1, 1] \Rightarrow x \in [-1, -1/2]$

(iii) $\frac{10x+6}{3} \in [-1, 1]$

$$x \in \left[-\frac{9}{10}, -\frac{3}{10}\right]$$

$$x \in \left[-\frac{3}{4}, -\frac{1}{2}\right] \quad \text{So, } \alpha = -\frac{3}{4}, \beta = -\frac{1}{2}$$

$$\alpha + \beta = -\frac{5}{4} \Rightarrow 36|\alpha + \beta| = 45$$

15. (a) $\left(\cos \frac{x}{4} - 2 \sin x\right) \sin x + \left(1 + \sin \frac{x}{4} - 2 \cos x\right) \cos x = 0$

$$\Rightarrow \left(\sin x \cos \frac{x}{4} + \cos x \sin \frac{x}{4}\right) + \cos x - 2(\sin^2 x + \cos^2 x) = 0$$

$$\Rightarrow \sin\left(x + \frac{x}{4}\right) + \cos x - 2(1) = 0 \Rightarrow \sin \frac{5x}{4} + \cos x = 2$$

$$\Rightarrow \sin \frac{5x}{4} = \cos x = 1$$

$$\Rightarrow \sin \frac{5x}{4} = 1 \Rightarrow \frac{5x}{4} = 2n\pi + \frac{\pi}{2} \Rightarrow x = \frac{8n\pi}{5} + \frac{2\pi}{5}$$

$$\& \cos x = 1 \Rightarrow x = 2m\pi$$

$$\text{Thus we have } \frac{8n\pi}{5} + \frac{2\pi}{5} = 2m\pi \Rightarrow m = \frac{4n+1}{5}$$

$$\therefore n \in I, \text{ so } m \text{ must be of the form } m = 5k+1$$

Hence the solution of the equation is $x = 2(5k+1)\pi, k \in I$

16. (a) $\sin 2x - \sin 4x + \sin 6x = 0$

$$\Rightarrow (\sin 2x + \sin 6x) - \sin 4x = 0$$

$$\Rightarrow 2 \sin 4x \cos 2x - \sin 4x = 0$$

$$\sin 4x = 0 \Rightarrow 4x = n\pi \Rightarrow x = \frac{n\pi}{4}$$

$$2 \cos 2x - 1 = 0 \Rightarrow \cos 2x = \frac{1}{2}$$

$$\Rightarrow 2x = 2n\pi \pm \frac{\pi}{3} \text{ or } x = n\pi \pm \frac{\pi}{6}$$

17. (c) Let $x^2 = \cos 2\theta \Rightarrow 0 \leq 2\theta < \frac{\pi}{2} (\because x^2 > 0)$

$$\text{The expression} = \tan^{-1} \left(\frac{\sqrt{2} \cos \theta + \sqrt{2} \sin \theta}{\sqrt{2} \cos \theta - \sqrt{2} \sin \theta} \right)$$

$$= \tan^{-1} \left(\frac{1 + \tan \theta}{1 - \tan \theta} \right) = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2} \cos^{-1}(x^2)$$

18. (a) $\cos \left[2 \cos^{-1} x + \sin^{-1} x \right]$

$$= \cos \left[\cos^{-1} x + \cos^{-1} x + \sin^{-1} x \right]$$

$$= \cos \left[\cos^{-1} x + \pi/2 \right] = -\sin \cos^{-1} x$$

$$= -\sin \sin^{-1} \sqrt{1-x^2} = -\sqrt{1-x^2}$$

$$= -\sqrt{1 - \left(\frac{1}{5}\right)^2} = -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}$$

19. (b) $\because \sin x \cos x = \frac{1}{4} \Rightarrow 2 \sin x \cos x = \frac{1}{2}$

$$\Rightarrow \sin 2x = \frac{1}{2} \Rightarrow 2x = n\pi + (-1)^n \frac{\pi}{6}, n \in I.$$

For $n=0, x = \frac{\pi}{12}$.

For $n=1, x = \frac{5\pi}{12}$.

$$\therefore x \text{ has only 2 values in } \left(0, \frac{\pi}{2}\right).$$

20. (b) $\cos \theta = \sqrt{2}$ has no solution, since value of $\cos \theta$ lies in $[-1, 1]$

21. (b) $\sin x + \sin 3x + \sin 5x = 0$

$$\sin 5x + \sin x + \sin 3x = 0$$

$$2 \sin 3x \cos 2x + \sin 3x = 0$$

$$\therefore \sin 3x [2 \cos 2x + 1] = 0$$

$$\therefore \sin 3x = 0 \text{ or } 2 \cos 2x + 1 = 0$$

$$\Rightarrow \sin 3x = \sin n\pi$$

$$\Rightarrow 3x = n\pi \Rightarrow x = \frac{n\pi}{3}$$

also $2 \cos 2x = -1$

$$\Rightarrow \cos 2x = -1/2 \Rightarrow \cos 2x = -\cos \pi/3$$

$$\cos 2x = \cos(\pi - \pi/3)$$

$$\Rightarrow \cos 2x = \cos \frac{2\pi}{3} \Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}$$

$$x = \frac{n\pi}{3}, x = n\pi \pm \frac{\pi}{3}$$

$$x \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \text{ gives}$$

$$x = \pi, \frac{2\pi}{3} \& \frac{4\pi}{3}$$

22. (b) We have, $\tan 2\theta = 1$.

$$\Rightarrow \tan 2\theta = \tan \frac{\pi}{4} \Rightarrow 2\theta = n\pi + \frac{\pi}{4}$$

$$\Rightarrow \theta = \frac{n}{2}\pi + \frac{\pi}{8}$$

Also, the value of $\tan 2\theta$ is positive. So, θ lies in 1st and 3rd quadrants.

$\therefore \theta = \frac{\pi}{8} \& \frac{9\pi}{8}$, are the two principal solutions.

23. (b) Here, $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

$$\Rightarrow \tan^{-1} \frac{2 \cos x}{1 - \cos^2 x} = \tan^{-1} \left(\frac{2}{\sin x} \right)$$

$$\Rightarrow \frac{2 \cos x}{1 - \cos^2 x} = \frac{2}{\sin x} \Rightarrow \frac{\cos x}{\sin^2 x} = \frac{1}{\sin x}$$

$$\Rightarrow \frac{\cos x}{\sin x} = 1 \quad [\because \sin x \neq 0]$$

$$\Rightarrow \cot x = 1 \Rightarrow x = \frac{\pi}{4}$$

$$\text{Hence, } \sin x + \cos x = \sin \frac{\pi}{4} + \cos \frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

$$24. \text{ (b) } \frac{\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)}{\operatorname{cosec}^{-1}(-\sqrt{2}) + \cos^{-1}\left(-\frac{1}{2}\right)}$$

$$= \frac{\tan^{-1}(\sqrt{3}) - (\pi - \sec^{-1}(2))}{-\operatorname{cosec}^{-1}(\sqrt{2}) + \pi - \cos^{-1}\left(\frac{1}{2}\right)}$$

$$= \frac{\frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right)}{-\frac{\pi}{4} + \pi - \frac{\pi}{3}} = \frac{-\frac{\pi}{3}}{\frac{12\pi - 3\pi - 4\pi}{12}} = \frac{-\frac{\pi}{3}}{\frac{5\pi}{12}} = -\frac{4}{5}$$

25. (c) The given equation is $\sin 2x + \cos 2x = 0$
 $\Rightarrow \sin 2x = -\cos 2x \Rightarrow \tan 2x = -1$

$$[\because \pi < x < 2\pi \Rightarrow 2\pi < 2x < 4\pi]$$

$$\Rightarrow 2x = 2\pi + \frac{3\pi}{4}, 2\pi + \left(\frac{3\pi}{2} + \frac{\pi}{4}\right)$$

$$\Rightarrow 2x = \frac{11\pi}{8}, \frac{15\pi}{4} \Rightarrow x = \frac{11\pi}{8}, \frac{15\pi}{8}$$

Exercise 1 :

WARM-UP
Topic-wise MCQs

1. (d) Coefficient of x^2 + Coefficient of y^2
 $= \cos^2\theta - 1 + \sin^2\theta = 1 - 1 = 0$
- So, lines are perpendicular i.e., $\theta = \frac{\pi}{2}$
2. (c) $m_1 + m_2 = -\frac{2h}{b^2} \Rightarrow 4m_2 = -\frac{2h}{b^2} \Rightarrow m_2 = -\frac{2h}{4b^2}$
- and $m_1 \cdot m_2 = \frac{a^2}{b^2} \Rightarrow 3m_2^2 = \frac{a^2}{b^2} \Rightarrow h = \pm \frac{2ab}{\sqrt{3}}$
3. (a) $\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right| = \sqrt{p^2 - 1} \Rightarrow \sec \theta = p$
4. (b) $a + 2hm_1 + bm_1^2 = 0 \dots(i)$
- $a_1 + 2hm_1m_2 + b_1m_2^2 = 0 \quad (\because m_2 = -\frac{1}{m_1})$
- $\Rightarrow a_1m_1^2 - 2hm_1 + b_1 = 0 \dots(ii)$
- $= \frac{1}{2ha_1 + 2hb_1} = \frac{-m_1}{aa_1 - bb_1} = \frac{m_1^2}{-2h_1a - 2hb_1}$
- $\Rightarrow (bb_1 - aa_1)^2 + 4(h_1a + b_1h)(ha_1 + h_1b) = 0$
5. (d) Equation of pair of lines through origin and perpendicular to $ax^2 + 2hxy + by^2 = 0$ is $bx^2 - 2hxy + ay^2 = 0$.
6. (c) The lines represented by two pairs are equally inclined.
7. (a) Put $\left(1 + \frac{r}{\sqrt{2}}, 2 + \frac{r}{\sqrt{2}}\right)$ in $x^2 + 4xy + y^2 = 0$, we get $3r^2 + 9\sqrt{2}r + 13 = 0$ for which product of the roots is $13/3$.
8. (b) $x^2 - 5xy + 6y^2 = 0 \Rightarrow x^2 - 2xy - 3xy + 6y^2 = 0$
9. (a)
10. (c)
11. (a) The given equation is

$$x^2(\tan^2 \theta + \cos^2 \theta) - 2xy \tan \theta + y^2 \sin^2 \theta = 0 \quad \dots(i)$$

Let the equation (i) represent two straight lines $y = m_1x$ and $y = m_2x$ where m_1 and m_2 are their slopes since (i) represents two straight lines passing through origin.

$$\text{Hence } m_1 + m_2 = \frac{2 \tan \theta}{\sin^2 \theta} = \frac{2}{\sin \theta \cos \theta} \text{ and}$$

$$m_1 m_2 = \frac{\tan^2 \theta + \cos^2 \theta}{\sin^2 \theta} = \frac{\sin^2 \theta + \cos^4 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$\text{Therefore } (m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1 m_2$$

$$= \frac{4}{\sin^2 \theta \cos^2 \theta} - \frac{4(\sin^2 \theta + \cos^4 \theta)}{\sin^2 \theta \cos^2 \theta}$$

$$= 4 \left[\frac{1 - \sin^2 \theta - \cos^4 \theta}{\sin^2 \theta \cos^2 \theta} \right] = 4 \left[\frac{\cos^2 \theta - \cos^4 \theta}{\sin^2 \theta \cos^2 \theta} \right]$$

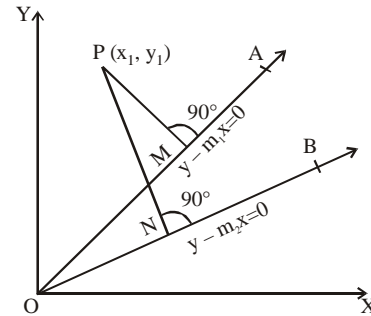
$$= \frac{4 \cos^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta \sin^2 \theta} = \frac{4 \cos^2 \theta \sin^2 \theta}{\cos^2 \theta \sin^2 \theta}$$

$$\text{Hence } (m_1 - m_2)^2 = 4 \text{ or } |m_1 - m_2| = 2$$

$$\text{or } |\tan \theta_1 - \tan \theta_2| = 2$$

where θ_1 and θ_2 are the inclination of the lines represented by (i) from the x-axis.

12. (a) Let the lines represented by $ax^2 + 2hxy + by^2 = 0$ be OA and OB whose equations are respectively



$$y - m_1x = 0 \text{ and } y - m_2x = 0$$

$$\text{Then } m_1 + m_2 = -\frac{2h}{b} \text{ and } m_1 m_2 = \frac{a}{b}.$$

From $P(x_1, y_1)$ draw perpendiculars PM and PN to OA and OB respectively. Let $PM = p_1$ and $PN = p_2$.

$$\therefore p_1 = \frac{y_1 - m_1 x_1}{\sqrt{1 + m_1^2}} \text{ and } p_2 = \frac{y_1 - m_2 x_1}{\sqrt{1 + m_2^2}}$$

$$\therefore p_1 p_2 = \frac{(y_1 - m_1 x_1)(y_1 - m_2 x_1)}{\sqrt{(1 + m_1^2)(1 + m_2^2)}}$$

$$= \frac{y_1^2 - (m_1 + m_2)x_1 y_1 + m_1 m_2 x_1^2}{\sqrt{1 + m_1^2 + m_2^2 + m_1^2 m_2^2}}$$

$$= \frac{y_1^2 - (m_1 + m_2)x_1 y_1 + m_1 m_2 x_1^2}{\sqrt{1 + (m_1 + m_2)^2 - 2m_1 m_2 + m_1^2 m_2^2}}$$

$$= \frac{y_1^2 + \frac{2h}{b} x_1 y_1 + \frac{a}{b} x_1^2}{\sqrt{1 + \frac{4h^2}{b^2} - \frac{2a}{b} + \frac{a^2}{b^2}}} = \frac{by_1^2 + 2hx_1 y_1 + ax_1^2}{\sqrt{(a-b)^2 + 4h^2}}$$

13. (d) The given equation can be rewritten as
 $(x - y)(x^2 + xy + 2y^2) = 0$
 $x - y = 0$ represents a straight line passing through origin.
 But the equation $x^2 + xy + 2y^2 = 0$, represents two imaginary straight line as

$$h^2 - ab = \frac{1}{4} - 1.2 = \frac{-7}{4} < 0$$

14. (c)

15. (a) Given equation of pair of lines is

$$(x^2 + y^2)\sqrt{3} = 4xy$$

$$\therefore a = \sqrt{3}, h = -2, b = \sqrt{3}$$

$$\therefore \tan \theta = \left| \frac{2\sqrt{4-3}}{2\sqrt{3}} \right| = \frac{1}{\sqrt{3}} \Rightarrow \theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = \frac{\pi}{6}$$

16. (c) We have $6x^2 - xy - 12y^2 = 0$

$$\text{or } (2x - 3y)(3x + 4y) = 0 \quad \dots(i)$$

$$\text{and } 15x^2 + 14xy - 8y^2 = 0$$

$$\text{or } (5x - 2y)(3x + 4y) = 0 \quad \dots(ii)$$

Equation of the line common to (i) and (ii) is

$$3x + 4y = 0$$

Equation of any line parallel to (iii) is

$$3x + 4y = k$$

Since its distance from (iii) is 7, we have

$$\left| \frac{k}{\sqrt{3^2 + 4^2}} \right| = 7 \text{ or } k = \pm 35.$$

17. (b) We have the equation $2x^2 - xy - y^2 = 0$

$$\Rightarrow (2x + y)(x - y) = 0$$

If (h, k) be the point then remaining pair is

$$(2x + y + h)(x - y + k) = 0$$

Where, $2x + y + h = 0$ and $x - y + k = 0$

It passes through the point $(1, 0)$

$$\therefore 2 \times 1 + 0 + h = 0 \Rightarrow 2 + h = 0 \Rightarrow h = -2$$

$$\text{and } 1 - 0 + k = 0 \Rightarrow 1 + k = 0 \Rightarrow k = -1$$

$$\therefore \text{Required pair is } (2x + y - 2)(x - y - 1) = 0$$

$$\Rightarrow 2x^2 - 2xy - 2x + xy - y^2 - y - 2x + 2y + 2 = 0$$

$$\therefore 2x^2 - xy - y^2 - 4x + y + 2 = 0$$

18. (b) Given equations are $x^2 + y^2 = 9$ and $x + y = 3$

Consider $x + y = 3$

Squaring on both side, we get

$$(x + y)^2 = 9 \Rightarrow x^2 + y^2 + 2xy = 9$$

Put the value of $x^2 + y^2 = 9$ in above eqⁿ.

$$9 + 2xy = 9 \Rightarrow 2xy = 0$$

$\therefore xy = 0$ which is the required equation of pair of lines.

19. (d) $y^2 + xy - 12x^2 = 0 \Rightarrow (y + 4x)(y - 3x) = 0$

$$\therefore \frac{y}{x} = 3, -4,$$

The two pairs will have a line common if

$$3 \text{ or } -4 \text{ will be a root of } b \left(\frac{y}{x} \right)^2 + 2h \left(\frac{y}{x} \right) + a = 0$$

$$\therefore 9b + 6h + a = 0 \text{ or } 16b - 8h + a = 0$$

20. (b) Joint equation of pair of lines having slopes m_1 and m_2 and passing through the origin is

$$y^2 - (m_1 + m_2)xy + m_1m_2x^2 = 0$$

$$\Rightarrow 3x^2 - 4xy + y^2 = 0$$

21. (b) The required equation is

$$y^2 - (m_1 + m_2)xy + m_1m_2x^2 = 0$$

$$\Rightarrow y^2 - \left(\frac{8}{3} \right)xy - x^2 = 0$$

$$\Rightarrow 3x^2 + 8xy - 3y^2 = 0$$

22. (d) $x^2 + xy - 12y^2 = 0$

$$\Rightarrow x^2 + 4xy - 3xy - 12y^2 = 0$$

$$\Rightarrow x(x + 4y) - 3y(x + 4y) = 0 \Rightarrow (x - 3y)(x + 4y) = 0$$

$$\Rightarrow x - 3y = 0 \text{ and } x + 4y = 0$$

23. (a) $3x^2 - 10xy - 8y^2 = 0$

$$\Rightarrow 3x^2 - 12xy + 2xy - 8y^2 = 0$$

$$\Rightarrow 3x(x - 4y) + 2y(x - 4y) = 0$$

$$\Rightarrow (3x + 2y)(x - 4y) = 0$$

$$\Rightarrow 3x + 2y = 0 \text{ and } x - 4y = 0$$

24. (c) It is a homogeneous equation of degree 2 in x and y .

\therefore Option (c) is the correct answer.

25. (b) $L_1 = x^2 - y^2 = 0$ represents pair of straight lines passing through the origin.

The equation of pair of straight lines parallel to L_1 and passing through $(3, 4)$, is

$$(x - 3)^2 - (y - 4)^2 = 0$$

$$\Rightarrow x^2 - y^2 - 6x + 8y - 7 = 0$$

26. (a) The required equation is

$$-3x^2 + 7xy + 5y^2 = 0$$

$$\text{i.e. } 3x^2 - 7xy - 5y^2 = 0$$

27. (d) Given equation of pair of lines is

$$ax^2 - 6xy + y^2 = 0$$

$$\therefore A = a, H = -3, B = 1$$

Given that, $m_1 = 2m_2$

$$\therefore m_1 : m_2 = 2 : 1$$

$$\frac{H^2}{AB} = \frac{(2+1)^2}{4 \times 2 \times 1}$$

$$\therefore \frac{9}{a} = \frac{9}{8} \Rightarrow a = 8$$

28. (d) Given equation of pair of lines is

$$ax^2 + 10xy + y^2 = 0$$

$$\therefore A = a, H = 5, B = 1$$

Let the slopes of the lines be m_1 and m_2

$$m_1 + m_2 = \frac{-2H}{B} \text{ and } m_1m_2 = \frac{A}{B}$$

Given that $m_2 = 4m_1$

$$\therefore m_1 + 4m_1 = \frac{-2H}{B} = -10 \Rightarrow m_1 = -2$$

and $m_1 \times 4m_1 = \frac{A}{B} = a$

$$\Rightarrow 4m_1^2 = a \Rightarrow a = 16$$

29. (c) Given equation of pair of lines is

$$ax^2 + 4xy + y^2 = 0$$

$$\therefore A = a, H = 2, B = 1$$

$$m_1 + m_2 = -4 \text{ and } m_1 m_2 = a$$

Given that $m_1 = 3m_2$

$$\therefore 3m_2 + m_2 = -4 \Rightarrow m_2 = -1$$

Hence, $m_1 = -3$

$$\therefore a = (-1)(-3) = 3$$

30. (d) Given equation of pair of lines is

$$ax^2 + (3a + 1)xy + 3y^2 = 0$$

$$\therefore A = a, H = \frac{3a+1}{2}, B = 3$$

Given that $m_1 = \frac{1}{m_2} \Rightarrow m_1 m_2 = 1$

Now $m_1 m_2 = \frac{a}{3} \Rightarrow \frac{a}{3} = 1 \Rightarrow a = 3$

Also $m_1 + m_2 = -\left(\frac{3a+1}{3}\right) = \frac{-10}{3}$

$$\Rightarrow m_1 + \frac{1}{m_1} = \frac{-10}{3}$$

$$\Rightarrow 3m_1^2 + 10m_1 + 3 = 0$$

$$\therefore m_1 = \frac{-1}{3} \text{ or } -3.$$

31. (a) Given equation of pair of lines is

$$6x^2 + 41xy - 7y^2 = 0$$

$$\therefore a = 6, h = \frac{41}{2}, b = -7$$

α and β are angles made by the two lines with X-axis.

\therefore their slopes m_1 and m_2 respectively are $m_1 = \tan \alpha$ and $m_2 = \tan \beta$

$$\tan \alpha \cdot \tan \beta = m_1 m_2 = -\frac{6}{7}$$

32. (a) Given equation of pair of lines is

$$6x^2 - xy - y^2 = 0$$

$$\therefore a = 6, h = -\frac{1}{2}, b = -1$$

If θ is the acute angle between the pair of lines, then

$$\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{2\sqrt{\frac{1}{4} + 6}}{5} \right| = 1$$

$$\Rightarrow \theta = \tan^{-1}(1) = 45^\circ$$

33. (c) Given equation of pair of lines is $\sqrt{3}xy - y^2 = 0$

$$\therefore a = 0, h = \frac{\sqrt{3}}{2}, b = -1$$

$$\therefore \tan \theta = \left| \frac{2\sqrt{\frac{3}{4} - 0}}{0 - 1} \right| = \left| \frac{2 \times \frac{\sqrt{3}}{2}}{-1} \right| = \sqrt{3}$$

$$\Rightarrow \theta = \tan^{-1}(\sqrt{3}) = 60^\circ$$

34. (b) Given equation of pair of lines is

$$11y^2 - 24xy + 4x^2 = 0$$

i.e. $4x^2 - 24xy + 11y^2 = 0$

$$\therefore a = 4, h = -12, b = 11$$

$$\therefore \tan \theta = \left| \frac{2\sqrt{144 - 44}}{4 + 11} \right| = \frac{2(10)}{15} = \frac{4}{3}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

35. (c) Given equation of pair of lines is

$$2x^2 - 3xy + y^2 = 0$$

$$\therefore a = 2, h = \frac{-3}{2}, b = 1$$

$$\therefore \tan \theta = \left| \frac{2\sqrt{\frac{9}{4} - 2}}{3} \right| = \left| \frac{\sqrt{9 - 8}}{3} \right| = \frac{1}{3}$$

$$\therefore \cot \theta = 3 \Rightarrow \theta = \cot^{-1}(3)$$

36. (c) Given equation of pair of lines is

$$x^2 (\cos \theta - \sin \theta) + 2xy \cos \theta$$

$$+ y^2 (\cos \theta + \sin \theta) = 0$$

$$\therefore a = \cos \theta - \sin \theta, h = \cos \theta, b = \cos \theta + \sin \theta$$

The acute angle α between the pair of lines is given by

$$\tan \alpha = \left| \frac{2\sqrt{\cos^2 \theta - (\cos^2 \theta - \sin^2 \theta)}}{2 \cos \theta} \right|$$

$$\Rightarrow \tan \alpha = \tan \theta \Rightarrow \alpha = \theta$$

37. (c) Given equation of pair of lines is

$$x^2 - 4hxy + 3y^2 = 0$$

$$\therefore A = 1, H = -2h, B = 3$$

Now, $\theta = 60^\circ \Rightarrow \tan \theta = \sqrt{3}$

$$\therefore \tan \theta = \left| \frac{2\sqrt{H^2 - AB}}{A + B} \right|$$

$$\Rightarrow \sqrt{3} = \left| \frac{2\sqrt{4h^2 - 3}}{4} \right| \Rightarrow h = \pm \frac{\sqrt{15}}{2}$$

38. (c) Given equation of pair of lines is $3x^2 + 18xy + by^2 = 0$

$$\therefore a = 3, h = 9, b = b$$

$$\text{Now } \theta = \pi \Rightarrow \tan \theta = 0$$

$$\therefore \tan \theta = \left| \frac{2\sqrt{81 - 3b}}{3 + b} \right|$$

$$\Rightarrow 0 = \left| \frac{2\sqrt{81 - 3b}}{3 + b} \right| \Rightarrow 81 = 3b \Rightarrow b = 27$$

39. (c) Given equation of pair of lines is $3x^2 + 10xy + 8y^2 = 0$

$$\therefore a = 3, h = 5, b = 8$$

$$\text{Now, } \theta = \tan^{-1}(p) \Rightarrow \tan \theta = p$$

$$\therefore \tan \theta = \left| \frac{2\sqrt{25 - 24}}{11} \right|$$

$$\Rightarrow p = \left| \frac{2}{11} \right| = \frac{2}{11}$$

40. (c) Given equation of pair of lines is $\lambda y^2 + (1 - \lambda^2)xy - \lambda x^2 = 0$

$$\therefore a = -\lambda, b = \lambda$$

$$\text{Now, } a + b = 0$$

\therefore The lines are perpendicular.

\therefore Angle between the lines is 90° .

41. (c) Given equation of pair of lines is $xy = 0$

$$\therefore a = 0, h = \frac{1}{2}, b = 0$$

$$\text{Now, } a + b = 0$$

\therefore The lines are perpendicular to each other.

\therefore Angle between the pair of lines is 90° .

42. (c) Given equation of pair of lines is $x^2 + ky^2 + 4xy = 0$

$$\therefore a = 1, h = \frac{k}{2}, b = 4$$

The pair of lines are coincident if $h^2 - ab = 0$

$$\Rightarrow h^2 = ab \Rightarrow \frac{k^2}{4} = 4(1)$$

$$\Rightarrow k = \pm 4$$

43. (d) Given equation of pair of lines is $3x^2 + 2hxy + y^2 = 0$

$$\therefore a = 3, h = h, b = 1$$

The two lines are real and coincident if

$$h^2 - ab = 0$$

$$\therefore h^2 - ab = h^2 - 3$$

For these lines to be real and coincident,

$$h^2 - 3 \geq 0 \Rightarrow h^2 \geq 3$$

44. (a) Given equation of pair of lines is

$$9x^2 - 12xy + 4y^2 = 0$$

$$a = 9, h = -6, b = 4$$

$$\text{Now, } h^2 - ab = (6)^2 - 9 \times 4 = 0$$

\therefore The lines are coincident.

45. (a) The condition for a pair of straight lines to be real and coincident is $h^2 - ab = 0$

Consider the equation,

$$4x^2 - 4xy + y^2 = 0$$

$$\therefore a = 4, h = -2, b = 1$$

$$h^2 - ab = (-2)^2 - (4)(1) = 0$$

\therefore Option (a) is the correct answer.

46. (c) Given equation of pair of lines is

$$6x^2 + hxy + 12y^2 = 0$$

$$\therefore A = 6, H = \frac{h}{2}, B = 12$$

Since, lines are parallel,

$$H^2 - AB = 0$$

$$\Rightarrow \frac{h^2}{4} = 6(12) \Rightarrow h^2 = (24)(12)$$

$$\Rightarrow h = \pm 12\sqrt{2}$$

47. (b) Given equation of pair of lines is

$$4x^2 + hxy + y^2 = 0$$

The lines are coincident

$$\therefore H^2 = AB \Rightarrow \frac{h^2}{4} = 4(1) \Rightarrow h = \pm 4$$

48. (d) Given equation of pair of lines is $x^2 + xy + y^2 = 0$

$$\therefore a = 1, h = \frac{1}{2}, b = 1$$

$$\text{Here, } h^2 - ab = \frac{-3}{4} < 0$$

Hence, the lines are imaginary.

49. (a) The condition for a pair of straight lines to be perpendicular is $a + b = 0$.

Consider the equation $2x^2 = 2y(2x + y)$

$$\text{i.e. } 2x^2 - 4xy - 2y^2 = 0$$

$$\therefore a = 2, b = -2$$

$$\therefore a + b = 2 + (-2) = 0$$

\therefore Option (a) is the correct answer.

50. (c) It is a homogeneous equation of degree 2 in x and y .

Hence, it represents a pair of lines and $a + b = 0$

\therefore Lines are perpendicular.

51. (c) Given equation of pair of lines is

$$a^2x^2 + bcy^2 = a(b+c)xy$$

$$\therefore A = a^2, B = bc$$

Since the lines are mutually perpendicular,

$$A + B = 0 \Rightarrow a^2 + bc = 0$$

52. (c) Given equation of pair of lines is

$$3y^2 + 9xy + kx^2 = 0$$

i.e. $kx^2 + 9xy + 3y^2 = 0$

$$\therefore a = k, b = 3$$

The lines are perpendicular

$$\therefore a + b = 0$$

$$\Rightarrow k + 3 = 0 \Rightarrow k = -3$$

53. (a) $\Delta = 0 \Rightarrow 6(12)(20)$

$$+ 2\left(\frac{31}{2}\right)(11)(h) - 6\left(\frac{31}{2}\right)^2 - 12(11)^2 - 20h^2 = 0$$

$$\Rightarrow 40h^2 - 682h + 2907 = 0$$

$$\Rightarrow h = \frac{171}{20} \text{ or } \frac{17}{2}$$

54. (c) (i) $\Delta = 0 \Rightarrow c = 0$ or $a + b = 0$

(ii) $\Delta = 0 \Rightarrow c = 0$ or $bd = ac$

55. (a) $\frac{\partial f}{\partial x} = 2x - 5y + 1 = 0$... (i)

$$\frac{\partial f}{\partial y} = -5x + 12y - 3 = 0$$
 ... (ii)

On solving (i) and (ii) we get $x = -3, y = -1$.

56. (a)

57. (b) $(x - 2)(y - 3) = 0 \Rightarrow xy - 3x - 2y + 6 = 0$

58. (a)

59. (a) The two curves are given by

$$ax^2 + 2hxy + by^2 + 2gx = 0$$
 ... (i)

$$a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$$
 ... (ii)

To get the combined equation of the straight lines joining the point of intersection of (i) and (ii) to the origin, we

make (ii) homogeneous with the help of (i), i.e., we eliminate the terms in first degree from (i) and (ii).

Hence multiplying (i) by g' and (ii) by g and subtracting, we get

$$(g'ax^2 + 2g'hxy + g'by^2 + 2gg'x)$$

$$-(ga'x^2 + 2h'gxy + gb'y^2 + 2gg'x) = 0$$

$$\text{or } x^2(ag' - ga') + 2(g'h - gh')xy + (g'b - gb')y^2 = 0$$
 ... (iii)

which is the required equation.

If the lines represented by (iii) be at right angles, then

$$(ag' - ga') + (g'b - gb') = 0 \text{ or } g'(a + b) - g(a' + b') = 0$$

$$\text{or } g(a' + b') - g'(a + b) = 0$$

60. (c)

61. (d) $\Delta = 0 \Rightarrow ch^2 + fgh = 0 \Rightarrow fg = -ch$

62. (b) We have $a = 1, h = -\sqrt{3}, b = 3, g = -\frac{3}{2}, f = \frac{3\sqrt{3}}{2}, c = -4$.

$$\text{Thus } abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

Hence the equation represents a pair of straight lines.

$$\text{Again } \frac{a}{h} = \frac{h}{b} = \frac{g}{f} = -\frac{1}{\sqrt{3}}$$

\therefore the lines are parallel. The distance between them

$$= 2\sqrt{\frac{g^2 - ac}{a(a+b)}} = 2\sqrt{\frac{\frac{9}{4} + 4}{1(1+3)}} = \frac{5}{2}$$

63. (c) We have the equation

$$y^2 + xy + px^2 - x - 2y + p = 0$$

We know any general equation

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$
 ... (i)

represents two straight lines if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$
 ... (ii)

On comparing given equation with (i), we get

$$a = p, b = 1, h = \frac{1}{2}, g = -\frac{1}{2}, f = -1, c = p$$

Put these value in equation (ii)

$$p \times 1 \times p + 2 \times -1 \times -\frac{1}{2} \times \frac{1}{2} - p \times (-1)^2 - 1$$

$$\times \left(-\frac{1}{2}\right)^2 - p \times \left(\frac{1}{2}\right)^2 = 0$$

$$\Rightarrow p^2 + \frac{1}{2} - p - \frac{1}{4} - \frac{p}{4} = 0 \Rightarrow p^2 - \frac{5p}{4} + \frac{1}{4} = 0$$

$$\Rightarrow 4p^2 - 5p + 1 = 0 \Rightarrow (4p - 1)(p - 1) = 0$$

$$\Rightarrow p = 1, \frac{1}{4}$$

64. (a) $9x^2 - 6xy + y^2 + 18x - 6y + 8 = 0$

$$\Rightarrow ((3x)^2 - 2 \times (3x) \times y + y^2) + 6(3x - y) + 8 = 0$$

$$\Rightarrow (3x - y)^2 + 6(3x - y) + 8 = 0$$

$$\text{Let } 3x - y = z$$

$$\therefore z^2 + 6z + 8 = 0$$

$$\Rightarrow z^2 + 4z + 2z + 8 = 0$$

$$\Rightarrow z(z + 4) + 2(z + 4) = 0$$

$$\Rightarrow (z + 2)(z + 4) = 0$$

$$\Rightarrow z = -2, z = -4$$

$$3x - y + 2 = 0$$
 ... (i)

$$\text{or } 3x - y + 4 = 0$$
 ... (ii)

If P_1 be the distance of line (i) from the origin, then

$$P_1 = \frac{2}{\sqrt{9+1}} = \frac{2}{\sqrt{10}}$$

Also, if P_2 be the distance of line (ii) from the origin,

$$\text{then } P_2 = \frac{4}{\sqrt{10}}$$

So, distance between lines

$$P = P_2 - P_1 = \frac{4}{\sqrt{10}} - \frac{2}{\sqrt{10}} = \frac{2}{\sqrt{10}}$$

65. (b) Given equation of pair of lines is $ax^2 - y^2 + 4x - y = 0$

$$\therefore A = a, B = -1, C = 0, F = \frac{-1}{2}, G = 2, H = 0$$

The given equation represents a pair of straight lines,

$$\therefore ABC + 2FGH - AF^2 - BG^2 - CH^2 = 0$$

$$\Rightarrow 0 - 0 - a\left(\frac{1}{4}\right) - (-1)(4) = 0$$

$$\Rightarrow -\frac{a}{4} + 4 = 0 \Rightarrow a = 16$$

66. (a) Consider $2x^2 + 3xy - 2y^2 + 5x + 5y + 3 = 0$

Comparing the given equation with

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, \text{ we get}$$

$$a = 2, b = -2, c = 3, f = \frac{5}{2}, g = \frac{5}{2}, h = \frac{3}{2}$$

Condition for equation to represent pair of lines is

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\therefore 2(-2)(3) + 2\left(\frac{5}{2}\right)\left(\frac{5}{2}\right)\left(\frac{3}{2}\right) - 2\left(\frac{5}{2}\right)^2 - (-2)\left(\frac{5}{2}\right)^2 - 3\left(\frac{3}{2}\right)^2$$

$$= -12 + \frac{75}{4} - \frac{50}{4} + \frac{50}{4} - \frac{27}{4} = 0$$

\therefore Condition is satisfied.

\therefore Option (a) is the correct answer.

67. (c) Given equation of pair of lines is $y^2 + xy + px^2 - x - 2y = 0$

$$\therefore a = p, b = 1, c = 0, f = -1, g = \frac{-1}{2}, h = \frac{1}{2}$$

The given equation represents pair of straight lines if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\Rightarrow p(1)(0) + 2(-1)\left(\frac{-1}{2}\right)\left(\frac{1}{2}\right) - p(-1)^2 - 1\left(\frac{-1}{2}\right)^2 - 0 = 0$$

$$\Rightarrow \frac{1}{2} - p - \frac{1}{4} = 0 \Rightarrow p = \frac{1}{4}$$

68. (a) Given equation of pair of lines is $6x^2 + 11xy - 10y^2 + x + 31y + k = 0$

$$\therefore a = 6, b = -10, c = k, f = \frac{31}{2}, g = \frac{1}{2}, h = \frac{11}{2}$$

$$\text{Now } abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\Rightarrow -6(10)k + 2\left(\frac{31}{2}\right)\left(\frac{1}{2}\right)\left(\frac{11}{2}\right) - 6\left(\frac{31}{2}\right)^2 + 10\left(\frac{1}{2}\right)^2 - k\left(\frac{11}{2}\right)^2 = 0$$

$$\Rightarrow -k \frac{361}{4} = \frac{5415}{4} \Rightarrow k = -15$$

69. (a) Given equation of pair of lines is $x^2 - y^2 - x - \lambda y - 2 = 0$

$$\therefore a = 1, b = -1, c = -2, f = \frac{-\lambda}{2}, g = \frac{-1}{2}, h = 0$$

$$\text{Now, } abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\therefore 2 - \frac{\lambda^2}{4} + \frac{1}{4} = 0 \Rightarrow \frac{\lambda^2}{4} = \frac{9}{4}$$

$$\Rightarrow \lambda^2 = 9 \Rightarrow \lambda = \pm 3$$

70. (c) Given equation of pair of lines is

$$2x^2 - 5xy + 3y^2 + 8x - 9y + 6 = 0$$

$$\therefore a = 2, b = 3, c = 6, f = \frac{-9}{2}, g = 4, h = \frac{-5}{2}$$

The point of intersection is given by

$$\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$$

$$\equiv \left(\frac{\left(\frac{-5}{2}\right)\left(\frac{-9}{2}\right) - 3(4)}{2(3) - \left(\frac{5}{2}\right)^2}, \frac{4\left(\frac{-5}{2}\right) - 2\left(\frac{-9}{2}\right)}{2(3) - \left(\frac{5}{2}\right)^2}\right) \equiv (3, 4)$$

71. (c) Given equation of pair of lines is

$$3x^2 + 10xy + 3y^2 - 15x - 21y + 18 = 0$$

$$a = 3, b = 3, c = 18, f = \frac{-21}{2}, g = \frac{-15}{2}, h = 5$$

The point of intersection is

$$\left(\frac{(5)\left(\frac{-21}{2}\right) - (3)\left(\frac{-15}{2}\right)}{(3)(3) - (5)^2}, \frac{\left(\frac{-15}{2}\right)(5) - (3)\left(\frac{-21}{2}\right)}{(3)(3) - (5)^2}\right)$$

$$\equiv \left(\frac{15}{8}, \frac{3}{8}\right)$$

72. (a) Given equation of pair of lines is

$$6x^2 - xy - 12y^2 - 8x + 29y - 14 = 0$$

$$a = 6, b = -12, h = \frac{-1}{2}$$

$$\therefore \tan \theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right| = \frac{17}{6} \Rightarrow \theta = \tan^{-1}\left(\frac{17}{6}\right)$$

73. (a) Given equation of pair of lines is
 $3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$
 $\therefore a = -3, b = 3$
 Now, $a + b = -3 + 3 = 0$
 \therefore The lines are perpendicular to each other.

74. (a) Given equation of pair of lines is
 $x^2 - y^2 - 2y - 1 = 0$
 $\therefore a = 1, b = -1$
 Now $a + b = 1 + (-1) = 0$
 \therefore The lines are perpendicular to each other.

75. (c) Given equation of pair of lines is
 $3xy - 4y = 0$
 $\therefore a = b = 0$
 Now, $a + b = 0$
 \therefore The lines are perpendicular to each other.

76. (b) Given equation of pair of lines is
 $x^2 + 2\sqrt{3}xy + 3y^2 - 3x - 3\sqrt{3}y - 4 = 0$
 $\therefore a = 1, h = \sqrt{3}, b = 3$

Now, $h^2 - ab = (\sqrt{3})^2 - (1)(3) = 0$

\therefore The lines are parallel.

77. (b) Given equation of pair of lines is
 $4x^2 + 2pxy + 25y^2 + 2x + 5y - 1 = 0$
 $\therefore a = 4, b = 25, h = p, g = 1, f = \frac{5}{2}, c = -1$

The lines are parallel

$\therefore h^2 - ab = 0 \Rightarrow h^2 = ab$

$\Rightarrow p^2 = 4(25) = 100 \Rightarrow p = 10$

In this case, if we take $p = -10$, then $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0$

$\therefore p$ cannot be -10 .

78. (c) Given equation of pair of lines is
 $px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$
 $a = p, b = 3, c = q, f = 1, g = 7, h = -4$
 This lines are perpendicular if $a + b = 0$
 $\Rightarrow p + 3 = 0 \Rightarrow p = -3$
 Since, the equation represents a pair of lines
 $\therefore abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
 $\Rightarrow -9q - 56 + 3 - 147 - 16q = 0$
 $\Rightarrow -25q - 200 = 0$
 $\Rightarrow q = -8$

79. (a) Given equation of pair of lines is
 $2x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$
 $\therefore a = 2, b = 12, c = -3, f = -8, g = \frac{5}{2}, h = -5$

Equation of perpendicular drawn from origin on

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is
 $bx^2 - 2hxy + ay^2 = 0$

$\therefore 12x^2 + 10xy + 2y^2 = 0$

i.e., $6x^2 + 5xy + y^2 = 0$

80. (b) Given equation of pair of lines is
 $ax^2 + 6xy + by^2 - 10x + 10y - 6 = 0$
 $A = a, B = b, C = -6, F = -5, G = 5, H = 3$

The lines are perpendicular

$\therefore a + b = 0 \Rightarrow a = -b$

Also these lines satisfy the condition

$ABC + 2FGH - AF^2 - BG^2 - CH^2 = 0$

$\Rightarrow 6a^2 + 2(-75) - 25a + 25a + 54 = 0$

$\Rightarrow 6a^2 - 96 = 0$

$\Rightarrow a^2 - 16 = 0$

$\Rightarrow a = \pm 4$

Exercise 2 :

ACCELERATOR
 Topic-wise MCQs

1. (a) Let $ax^2 + 2hxy + by^2 = 0$ represents the lines OA and OB respectively $y - m_1x = 0$ and $y - m_2x = 0$.

Then

$m_1 + m_2 = -\frac{2h}{b}$ and $m_1m_2 = \frac{a}{b}$

To get A, solve the equation $y = m_1x$ and $lx + my + n = 0$

\therefore A is $\left(\frac{n}{l + mm_1}, -\frac{m_1n}{l + mm_1}\right) = (x_1, y_1)$ say.

Similarly,

\therefore B is $\left(\frac{n}{l + mm_2}, -\frac{m_2n}{l + mm_2}\right) = (x_2, y_2)$, say.

Now, $\Delta OAE = \frac{1}{2}(x_1y_2 - x_2y_1)$

$= \frac{1}{2} \left[\frac{m_2n^2}{(l + mm_1)(l + mm_2)} - \frac{m_1n^2}{(l + mm_1)(l + mm_2)} \right]$

$= \frac{1}{2} n^2 \frac{\sqrt{(m_2 - m_1)^2}}{l^2 + lm(m_1 + m_2) + m^2m_1m_2}$

$= \frac{1}{2} n^2 \frac{\sqrt{(m_2 + m_1)^2 - 4m_1m_2}}{l^2 - \frac{2h}{b}lm + \frac{a}{b}m^2}$

$= \frac{1}{2} n^2 \frac{\sqrt{\frac{4h^2}{b^2} - \frac{4a}{b}}}{l^2 - \frac{2h}{b}lm + \frac{a}{b}m^2} = \frac{n^2 \sqrt{h^2 - ab}}{am^2 - 2h/m + bl^2}$

2. (b) Let m and m^2 be the slopes of the lines represented by $ax^2 + 2hxy + by^2 = 0$. Then, $m + m^2 = -\frac{2h}{b}$

and $mm^2 = \frac{a}{b}$ or $m^3 = \frac{a}{b} \therefore (m + m^2)^3 = \left(-\frac{2h}{b}\right)^2$

$$\therefore m^3 + m^6 + 3mm^2(m + m^2) = -\frac{8h^3}{b^3}$$

$$\therefore \frac{a}{b^2}(a+b) + \frac{8h^3}{b^3} = \frac{6ah}{b^2} \therefore \frac{a+b}{h} + \frac{8h^2}{ab} = 6$$

These are the set of parallel lines and the distance between parallel lines are equal. So, the figure is a rhombus.

3. (d) Given equations is $ax^2 + 2hxy + by^2 = 0$... (i)
Let slope of the lines be m and λm , then

$$m + \lambda m = -\frac{2h}{b} \quad \text{and} \quad m(\lambda m) = \frac{a}{b}$$

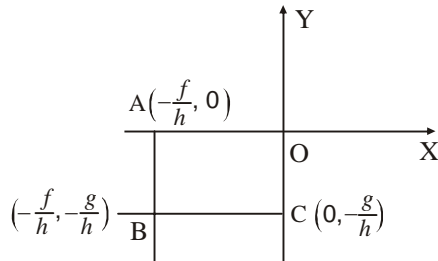
Eliminating m we get, $\frac{(1+\lambda)^2}{\lambda} = \frac{4h^2/b^2}{a/b} = \frac{4h^2}{ab}$

or $ab(1+\lambda)^2 = 4h^2\lambda$, which is the required condition.

4. (b) The equation $2hxy + 2gx + 2fy + c = 0$ represents a pair of straight line if

$$0 + 2fgh - 0 - 0c^2 = 0 \Rightarrow c = \frac{2fg}{h}$$

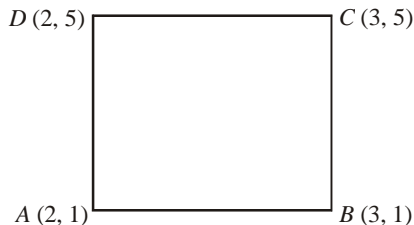
$$\therefore 2hxy + 2gx + 2fy + \frac{2fg}{h} = 0$$



$$\Rightarrow (hx + f)(hy + g) = 0 \Rightarrow x = -\frac{f}{h}, y = -\frac{g}{h}$$

$$\therefore \text{Required area of rectangle} = \left| -\frac{f}{h} \right| \left| -\frac{g}{h} \right| = \frac{|fg|}{h^2}$$

5. (c) The sides of parallelogram are $x = 2, x = 3, y = 1, y = 5$.



$$\therefore \text{Diagonal AC is } \frac{y-1}{5-1} = \frac{x-2}{3-2} \text{ or } y = 4x - 7$$

$$\text{Equation diagonal BD is } \frac{x-2}{3-2} = \frac{y-5}{1-5} \text{ or } 4x + y = 13$$

6. (c) For any point $P(x, y)$ that is equidistant from the given line, we have

$$x + y - \sqrt{2} = -(x + y - 2\sqrt{2})$$

$$\text{or } 2x + 2y - 3\sqrt{2} = 0$$

7. (d) Given equation of pair of lines is

$$4xy + 2x + 6y + 3 = 0$$

$$\Rightarrow 2x(2y + 1) + 3(2y + 1) = 0$$

$$\Rightarrow (2y + 1)(2x + 3) = 0$$

\therefore Separate equations of lines are

$$2x + 3 = 0 \text{ and } 2y + 1 = 0$$

$$\text{i.e. } x = \frac{-3}{2} \text{ and } y = \frac{-1}{2}$$

The equation of line passing through $(2, 1)$ and

perpendicular to $x = \frac{-3}{2}$ is $y = 1$ i.e. $y - 1 = 0$.

The equation of line passing through $(2, 1)$ and

perpendicular to $y = \frac{-1}{2}$ is $x = 2$ i.e. $x - 2 = 0$.

\therefore Combined equation of pair of lines is

$$(x - 2)(y - 1) = 0$$

$$\Rightarrow xy - x - 2y + 2 = 0$$

8. (d) One of the lines is $3x + 4y = 0$

$$\text{i.e. } \frac{y}{x} = -\frac{3}{4}$$

The given joint equation is $6x^2 - xy + 4cy^2 = 0$

$$\Rightarrow 4c\left(\frac{y}{x}\right)^2 - \left(\frac{y}{x}\right) + 6 = 0 \quad \dots(i)$$

Substituting value of $\frac{y}{x}$ in equation (i), we get

$$4c\left(\frac{-3}{4}\right)^2 - \left(\frac{-3}{4}\right) + 6 = 0$$

$$\Rightarrow 4c \times \frac{9}{16} + \frac{3}{4} + 6 = 0$$

$$\Rightarrow \frac{9c}{4} + \frac{3+24}{4} = 0 \Rightarrow 9c + 27 = 0$$

$$\Rightarrow c = -3$$

9. (d) Given line $2x - y = 0 \Rightarrow$ Slope = 2

Let the slope of required line be m

$$\therefore \tan 30^\circ = \left| \frac{m-2}{1+2m} \right|$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \left| \frac{m-2}{1+2m} \right|$$

$$\Rightarrow m^2 + 16m - 11 = 0 \quad \dots(i)$$

Since, the line passes through origin, its equation is

$$y = mx \Rightarrow m = \frac{y}{x}$$

Substituting the value of m in equation (i), we get

$$\left(\frac{y}{x}\right)^2 + 16\left(\frac{y}{x}\right) - 11 = 0 \Rightarrow 11x^2 - 16xy - y^2 = 0$$

10. (b) The combined equation of the pair of lines through the origin and forming an equilateral triangle with

$$\text{the line } x + y + \sqrt{3} = 0$$

$$((1)x + (1)y)^2 - 3((1)y - (1)x)^2 = 0$$

$$\text{i.e., } x^2 + 2xy + y^2 - 3x^2 + 6xy - 3y^2 = 0$$

$$\text{i.e., } x^2 - 4xy + y^2 = 0$$

11. (a) We have, $pq(x^2 - y^2) + (p^2 - q^2)xy = 0$

$$\Rightarrow pqx^2 - pqy^2 + p^2xy - q^2xy = 0$$

$$\Rightarrow pqx^2 + p^2xy - pqy^2 - q^2xy = 0$$

$$\Rightarrow px(qx + py) - qy(py + qx) = 0$$

$$\Rightarrow (px - qy)(qx + py) = 0 \text{ or } L_1L_2 = 0$$

$$\text{Hence one of the line is } px - qy = 0$$

12. (b) $\sqrt{(x-2)^2 + y^2} + \sqrt{(x+2)^2 + y^2} = 4$

$$\text{i.e. } \sqrt{(x-2)^2 + y^2} = 4 - \sqrt{(x+2)^2 + y^2}$$

Squaring both sides, we get

$$(x-2)^2 + y^2 = 16 - 8\sqrt{(x+2)^2 + y^2} + (x+2)^2 + y^2$$

$$\Rightarrow x^2 - 4x + 4 + y^2 = 16 + x^2 + 4x + 4 + y^2$$

$$-8\sqrt{(x+2)^2 + y^2}$$

$$\Rightarrow x + 2 = \sqrt{(x+2)^2 + y^2}$$

Again squaring both sides, we get

$$(x+2)^2 = (x+2)^2 + y^2$$

$$\Rightarrow y^2 = 0$$

This is an equation of pair of two coincident straight lines.

13. (d) $x^2 + 6xy = 0 \Rightarrow x(x + 6y) = 0$

$\Rightarrow x = 0$ and $x + 6y = 0$ are two straight lines $x = 0$ represents Y-axis.

14. (d) Given equation of pair of lines is

$$ax^2 + xy - by^2 = 0$$

Comparing the equations, we get

$$Ax^2 + 2Hxy + By^2 = 0$$

$$\therefore A = a, H = \frac{1}{2} \text{ and } B = -a$$

\therefore The equation represents a pair of straight lines for all real values of 'a'.

15. (a) The combined equation of pair of straight lines passing through origin and perpendicular to $3x^2 + xy - 2y^2 = 0$ is given by $-2x^2 - xy + 3y^2 = 0$ i.e. $2x^2 + xy - 3y^2 = 0$

Since the required lines pass through $(2, -3)$.

\therefore By shifting the origin to $(2, -3)$, we get

$$2(x-2)^2 + (x-2)(y+3) - 3(y+3)^2 = 0$$

$$\Rightarrow 2x^2 + xy - 3y^2 - 5x - 20y - 25 = 0$$

16. (b) Separate equation of lines represented by

$$3x^2 - 8xy + 5y^2 = 0 \text{ are}$$

$$x - y = 0 \text{ and } 3x - 5y = 0$$

Line perpendicular to $x - y = 0$ i.e. $y = x$ and passing through $(1, 2)$ is

$$(y-2) = -1(x-1)$$

$$\text{i.e. } x + y - 3 = 0$$

...(i)

Line perpendicular to $3x - 5y = 0$

i.e. $y = \frac{3}{5}x$ and passing through $(1, 2)$ is

$$(y-2) = \frac{-5}{3}(x-1)$$

$$\text{i.e. } 5x + 3y - 11 = 0$$

...(ii)

\therefore combined equation is

$$(x + y - 3)(5x + 3y - 11) = 0$$

17. (c) Slope of the line $4x + 3y = 0$ is $m = -\frac{4}{3}$

$$kx^2 - 5xy - 6y^2 = 0$$

$$\Rightarrow -6m^2 - 5m + k = 0$$

$$\Rightarrow -6\left(-\frac{4}{3}\right)^2 - 5\left(-\frac{4}{3}\right) + k = 0$$

$$\Rightarrow k - \frac{32}{3} + \frac{20}{3} = 0$$

$$\Rightarrow k = \frac{12}{3} \Rightarrow k = 4$$

18. (a) Substituting the value of y in the equation

$$ax^2 + 2hxy + by^2 = 0$$

$$\Rightarrow ax^2 - 2hx(mx) + b(mx)^2 = 0$$

$$\Rightarrow a + 2hm + bm^2 = 0$$

19. (b) Let $y = mx$ be a line common to the given pair of lines,

It satisfies the given equations

$$\therefore am^2 + 2m + 1 = 0 \text{ and} \quad \dots(i)$$

$$m^2 + 2m + a = 0 \quad \dots(ii)$$

On solving (i) and (ii), we get

$$\frac{m^2}{2(1-a)} = \frac{m}{a^2-1} = \frac{1}{2(1-a)}$$

$$\therefore m^2 = 1 \text{ and } m = -\left(\frac{a+1}{2}\right)$$

$$\therefore (a+1)^2 = 4 \Rightarrow a = 1 \text{ or } -3$$

But for $a = 1$ the two pair have both the lines common.

So $a = -3$ and the slope m of the line common to both the pairs is 1.

$$\text{Now } x^2 + 2xy + ay^2 = x^2 + 2xy - 3y^2 \\ = (x-y)(x+3y)$$

$$\text{and } ax^2 + 2xy + y^2 = -3x^2 + 2xy + y^2 \\ = -(x-y)(3x+y)$$

Thus, required equation is $(x+3y)(3x+y) = 0$
i.e., $3x^2 + 10xy + 3y^2 = 0$

20. (a) The equation of given lines are

$$ax^2 + 2hxy + by^2 = 0 \quad \dots(i)$$

$$a'x^2 + 2h'xy + b'y^2 = 0 \quad \dots(ii)$$

Let the line common to both be $y = mx$.

It will satisfy both the above equations.

$$\text{Hence, } a + 2mh + bm^2 = 0 \quad \dots(iii)$$

$$\text{and } a' + 2mh' + b'm^2 = 0 \quad \dots(iv)$$

Now eliminating 'm' from the equations (iii) and (iv), we get

$$\frac{m^2}{2ha' - 2h'a} = \frac{-m}{ba' - b'a} = \frac{1}{2bh' - 2b'h}$$

$$\Rightarrow m^2 = \frac{ha' - h'a}{bh' - b'h} \quad \dots(v)$$

$$\text{and } m^2 = \frac{(ab' - ba')^2}{4(bh' - b'h)^2} \quad \dots(vi)$$

From (v) and (vi), we get the required condition.

21. (a) $6x^2 + xy - y^2 = 0$

$$\Rightarrow 6x^2 + 3xy - 2xy - y^2 = 0$$

$$\Rightarrow 2x + y = 0 \text{ and } 3x - y = 0$$

Consider option (a), let $a = \frac{1}{2}$

\therefore equation $3x^2 - axy - y^2 = 0$ becomes

$$3x^2 - \frac{1}{2}xy - y^2 = 0$$

$$\Rightarrow 6x^2 - xy - 2y^2 = 0$$

$$\Rightarrow 3x - 2y = 0 \text{ and } 2x + y = 0$$

\therefore Given pair of lines have common line $2x + y = 0$

\therefore Option (a) is the correct answer.

22. (c) Given equation of pair of lines is

$$x^2 + 4xy + y^2 = 0$$

$$\therefore a = 1, h = 2, b = 1$$

$$\therefore \tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right| = \left| \frac{2\sqrt{(2)^2 - (1)(1)}}{1+1} \right| = \sqrt{3}$$

$$\Rightarrow \theta = \tan^{-1}(\sqrt{3}) = 60^\circ$$

23. (a) Given equation of pair of lines is

$$x^2 + hxy + 2y^2 = 0$$

$$\therefore A = 1, H = \frac{h}{2}, B = 2$$

$$\text{Given that } m_1 = 2m_2$$

$$\therefore m_1 : m_2 = 2 : 1$$

$$\frac{H^2}{AB} = \frac{(2+1)^2}{4 \times 2 \times 1} \Rightarrow \frac{h^2}{8} = \frac{9}{8}$$

$$\therefore h = \pm 3$$

24. (c) If the gradients of two lines are in ratio $m : n$,

$$\text{then } \frac{h^2}{ab} = \frac{(m+n)^2}{4mn} = \frac{(3+1)^2}{4 \times 3} = \frac{4}{3}$$

25. (d) Given equation of pair of lines is $ax^2 + 2hxy + by^2 = 0$

$$\text{Given that } m_1 = 5m_2$$

$$\therefore m_1 : m_2 = 5 : 1$$

$$\frac{h^2}{ab} = \frac{(5+1)^2}{4 \times 5 \times 1}$$

$$\therefore 20h^2 = 36ab$$

$$\therefore 5h^2 = 9ab$$

26. (c) Given equation of pair of lines is

$$ax^2 + 2hxy + by^2 = 0$$

$$\text{given that } m_2 = \lambda m_1$$

$$\therefore m_1 : m_2 = 1 : \lambda$$

$$\frac{h^2}{ab} = \frac{(1+\lambda)^2}{4 \times 1 \times \lambda}$$

$$\therefore 4\lambda h^2 = ab(1+\lambda)^2$$

27. (a) Given equation of pair of lines is

$$ax^2 + 2hxy + by^2 = 0$$

$$\text{Given that } m_1 = m_2^2$$

$$m_1 m_2 = m_2^2 m_2 = \frac{a}{b}$$

$$\therefore m_2 = \left(\frac{a}{b}\right)^{\frac{1}{3}}$$

$$\text{Also, } m_1 + m_2 = m_2^2 + m_2 = \frac{-2h}{b}$$

$$\therefore \left\{ \left(\frac{a}{b}\right)^{\frac{1}{3}} \right\}^2 + \left(\frac{a}{b}\right)^{\frac{1}{3}} = \frac{-2h}{b}$$

Cubing both sides, we get

$$\left(\frac{a}{b}\right)^2 + \frac{a}{b} + 3\left(\frac{a}{b}\right)^{\frac{2}{3}} \cdot \left(\frac{a}{b}\right)^{\frac{1}{3}} \left\{ \left(\frac{a}{b}\right)^{\frac{2}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}} \right\} = \frac{-8h^3}{b^3}$$

$$\therefore \left(\frac{a}{b}\right)^2 + \frac{a}{b} - \frac{6ah}{b^2} = \frac{-8h^3}{b^3} \dots \therefore \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}} = \frac{-2h}{b}$$

$$\therefore ab(a+b) - 6abh + 8h^3 = 0$$

28. (a) Given equation of pair of lines is

$$2x^2 - 5xy + 3y^2 = 0$$

$$\therefore a = 2, h = \frac{-5}{2}, b = 3$$

$$\therefore m_1 + m_2 = \frac{5}{6} \text{ and } m_1 \cdot m_2 = \frac{2}{3} \quad \dots(i)$$

$$\text{Slopes of lines} = \frac{1}{m_1} \text{ and } \frac{1}{m_2}$$

\(\therefore\) Required equation of pair of lines is

$$y^2 - \left(\frac{1}{m_1} + \frac{1}{m_2} \right) xy + \frac{1}{m_1 m_2} x^2 = 0$$

$$\Rightarrow y^2 - \left(\frac{m_1 + m_2}{m_1 m_2} \right) xy + \frac{1}{m_1 m_2} x^2 = 0$$

$$\Rightarrow y^2 - \left(\frac{\frac{5}{6}}{\frac{2}{3}} \right) xy + \frac{1}{\left(\frac{2}{3} \right)} x^2 = 0$$

$$\Rightarrow 2y^2 - 5xy + 3x^2 = 0$$

29. (d) The line makes angles α and β with X-axis

$$\therefore m_1 = \tan \alpha \text{ and } m_2 = \tan \beta$$

$$\Rightarrow \cot \alpha = \frac{1}{m_1} \text{ and } \cot \beta = \frac{1}{m_2}$$

Given equation of pair of lines is $2x^2 - 3xy + y^2 = 0$

$$\therefore a = 2, h = \frac{-3}{2}, b = 1$$

$$\text{Now, } m_1 + m_2 = 3 \text{ and } m_1 m_2 = 2$$

$$\therefore \cot^2 \alpha + \cot^2 \beta = \frac{1}{m_1^2} + \frac{1}{m_2^2} = \frac{m_1^2 + m_2^2}{(m_1 m_2)^2}$$

$$= \frac{(m_1 + m_2)^2 - 2m_1 m_2}{(m_1 m_2)^2}$$

$$= \frac{(3)^2 - 2(2)}{(2)^2} = \frac{5}{4}$$

30. (a) Let m, m_1 and m_2 be the slopes of the lines L, L_1 and L_2 respectively.

$$m + m_1 = \frac{-a}{3}, m \cdot m_1 = \frac{2}{3}$$

$$\text{and } m + m_2 = \frac{b}{3}, m \cdot m_2 = -\frac{2}{3}$$

$$(m \cdot m_1)(m \cdot m_2) = \left(\frac{2}{3} \right) \left(-\frac{2}{3} \right)$$

$$\Rightarrow m^2 m_1 m_2 = -\frac{4}{9}$$

$$\Rightarrow m^2 (-1) = -\frac{4}{9} \quad \dots[m_1 \cdot m_2 = -1]$$

$$\Rightarrow m^2 = \frac{4}{9}$$

$$\Rightarrow m = \pm \frac{2}{3}$$

By solving, we get

$$m_1 = 1, m_2 = -1$$

Substituting the values of m, m_1, m_2 in $m + m_1 = \frac{-a}{3}$

and $m + m_2 = \frac{b}{3}$, we get

$$a^2 = 25 \text{ and } b^2 = 1$$

$$\therefore a^2 + b^2 = 26$$

31. (c) Given equation of pair of lines is

$$4x^2 - 24xy + 11y^2 = 0$$

$$\therefore a = 4, h = -12, b = 11$$

$$\therefore \tan \theta = \pm 2 \frac{\sqrt{h^2 - ab}}{a + b} = \pm 2 \frac{\sqrt{144 - 44}}{15} = \pm \frac{4}{3}$$

$$\Rightarrow \theta = \tan^{-1} \left(\pm \frac{4}{3} \right)$$

32. (a) Let m_1 and m_2 be the slopes of the lines given by $x^2 + 4xy + y^2 = 0$

$$\therefore m_1 + m_2 = -4 \Rightarrow m_2 = -4 - m_1$$

$$\text{and } m_1 \cdot m_2 = 1 \Rightarrow m_1(-4 - m_1) = 1$$

$$\Rightarrow m_1^2 + 4m_1 + 1 = 0$$

$$\therefore m_1, m_2 = -2 \pm \sqrt{3}$$

Slope of line $x - y = 4$ is $m_3 = 1$

\(\therefore\) Angle between first two lines,

$$\tan^{-1} \theta_{12} = \left| \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right| = \left| \frac{(-2 + \sqrt{3}) - (-2 - \sqrt{3})}{1 + (-2 + \sqrt{3})(-2 - \sqrt{3})} \right|$$

$$\Rightarrow \theta_{12} = \tan^{-1}(\sqrt{3}) = 60^\circ$$

Angle between second and third line

$$\Rightarrow \theta_{23} = \tan^{-1} \left(\frac{-2 - \sqrt{3} - 1}{1 + (-2 - \sqrt{3})1} \right) = \tan^{-1}(\sqrt{3}) = 60^\circ$$

Similarly, we have, $\theta_{31} = 60^\circ$

\(\therefore\) The triangle formed by the lines is equilateral triangle.

33. (a) Given equation of pair of lines is

$$x^2 - 2pxy + y^2 = 0$$

$$\therefore a = 1, h = -p, b = 1$$

$$\therefore \tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$$

$$\Rightarrow \tan \theta = \frac{\pm 2\sqrt{p^2 - 1}}{1+1} = \pm \sqrt{p^2 - 1}$$

$$\Rightarrow \tan^2 \theta = p^2 - 1$$

$$\Rightarrow \sec^2 \theta - 1 = p^2 - 1$$

$$\Rightarrow \theta = \sec^{-1} p$$

34. (c) Given equation of pair of lines is $(x^2 + y^2) \sin \theta + 2xy = 0$

$$\therefore a = b = \sin \theta, h = 1$$

$$\therefore \tan \theta = \left(\frac{2\sqrt{1 - \sin^2 \theta}}{2 \sin \theta} \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{\cos \theta}{\sin \theta} \right) = \tan^{-1} (\cot \theta)$$

$$\Rightarrow \theta = \tan^{-1} \left\{ \tan \left(\frac{\pi}{2} - \theta \right) \right\} = \frac{\pi}{2} - \theta$$

35. (b) Given equation of pair of lines is $ax^2 + xy + by^2 = 0$

$$\therefore A = a, H = \frac{1}{2}, B = b$$

$$\text{Now, } \theta = 45^\circ \Rightarrow \tan \theta = 1$$

$$\therefore \tan 45^\circ = \left| \frac{2\sqrt{\frac{1}{4} - ab}}{a+b} \right|$$

$$\Rightarrow (a+b)^2 = (1 - 4ab)$$

$$\Rightarrow a^2 + b^2 + 6ab - 1 = 0$$

The above equation is satisfied by $a = 1$ and $b = -6$.

36. (a) Given equation of pair of lines is

$$\therefore a = -\tan^2 A, h = \frac{k}{2}, b = 1$$

$$\therefore \tan 2A = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$$

$$\tan 2A = \frac{2\sqrt{\frac{k^2}{4} + \tan^2 A}}{1 - \tan^2 A}$$

$$\Rightarrow \frac{2 \tan A}{1 - \tan^2 A} = \frac{2\sqrt{\frac{k^2}{4} + \tan^2 A}}{1 - \tan^2 A}$$

$$\Rightarrow \frac{k^2}{4} + \tan^2 A = \tan^2 A \Rightarrow k = 0$$

37. (b) Here, $a_1 = a, h_1 = h, b_1 = b$

$$a_2 = 2, h_2 = \frac{-5}{2}, b_2 = 3$$

Given that $\theta_1 = \theta_2$

$$\Rightarrow \tan \theta_1 = \tan \theta_2$$

$$\Rightarrow \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right| = \left| \frac{2\sqrt{\frac{25}{4} - 6}}{5} \right|$$

$$\Rightarrow \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right| = \left| \frac{1}{5} \right|$$

Squaring both sides, we get

$$4 \times 25(h^2 - ab) = (a+b)^2$$

$$100(h^2 - ab) = (a+b)^2$$

Comparing with given condition,

$$k(h^2 - ab) = (a+b)^2, \text{ we get}$$

$$k = 100$$

38. (a) Comparing the given equations with $ax^2 + 2hxy + by^2 = 0$, we get

$$a_1 = 3, h_1 = \frac{-7}{2}, b_1 = 4$$

$$a_2 = 6, h_2 = \frac{-5}{2}, b_2 = 1$$

If θ_1 and θ_2 are acute angle between the two pairs of lines, then

$$\tan \theta_1 = \left(\frac{2\sqrt{\frac{49}{4} - 12}}{3+4} \right) = \frac{1}{7}$$

$$\Rightarrow \theta_1 = \tan^{-1} \left(\frac{1}{7} \right)$$

$$\tan \theta_2 = \left(\frac{2\sqrt{\frac{25}{4} - 6}}{6+1} \right) = \left(\frac{1}{7} \right) \Rightarrow \theta_2 = \tan^{-1} \left(\frac{1}{7} \right)$$

Hence, $\theta_1 = \theta_2$.

39. (a) Given equation of pair of lines is $a^2x^2 + bcy^2 = a(b+c)xy$

$$\therefore A = a^2, H = \frac{-a(b+c)}{2}, B = bc$$

Since the lines are coincident

$$\therefore H^2 - AB = 0$$

$$\Rightarrow \left\{ \frac{-a(b+c)}{2} \right\}^2 - a^2(bc) = 0$$

$$\Rightarrow a^2(b-c)^2 = 0$$

$$\Rightarrow a = 0 \text{ or } b = c$$

40. (d) Given equation of pair of lines is
 $(p - q)x^2 + 2(p + q)xy + (q - p)y^2 = 0$
 $\therefore a = p - q, h = p + q, b = q - p$
 Since, the lines are mutually perpendicular
 $\therefore a + b = 0$
 $\Rightarrow (p - q) + (q - p) = 0$
 The above equation is true for all values of p and q .

41. (a) Given equation of pair of lines is
 $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$
 $\therefore A = 3a, H = \frac{5}{2}, B = a^2 - 2$
 Since, the lines are perpendicular
 $\therefore A + B = 0$
 $\Rightarrow 3a + (a^2 - 2) = 0$
 $\Rightarrow a^2 + 3a - 2 = 0$
 Since, the equation is a quadratic equation in ' a ' and $B^2 - 4AC > 0$,
 The roots of ' a ' are real and distinct.
 \therefore Lines are perpendicular to each other for two values of ' a '.

42. (c) Given equation of pair of lines is
 $ay^2 + (-1 - \lambda^2)xy - ax^2 = 0$
 $\therefore A = -a, H = \frac{-1 - \lambda^2}{2}, B = a$
 $A + B = (-a) + a = 0$
 \Rightarrow Angle between the given lines is 90°
 Now, consider $xy = 0$. Here, $A = B = 0$
 $\Rightarrow A + B = 0$
 \therefore The angle between the lines is 90°

43. (a) Given equation of pair of lines is
 $4x^2 + 12xy + 9y^2 = 0$
 $a = 4, h = 6, b = 9$
 Here, $h^2 - ab = (6)^2 - (4)(9) = 36 - 36 = 0$
 Hence, the lines are real and coincident.

44. (d) The equation of the given straight line is
 $y - 3x = 2 \Rightarrow \frac{y - 3x}{2} = 1$ (i)

The equation of the given curve is
 $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ (ii)

Making the equation (ii) homogeneous of the second degree in x and y by means of (i), we get

$$x^2 + 2xy + 3y^2 + 4x\left(\frac{y - 3x}{2}\right) + 8y\left(\frac{y - 3x}{2}\right) - 11\left(\frac{y - 3x}{2}\right)^2 = 0$$

$$\Rightarrow 119x^2 - 34xy - 17y^2 = 0 \Rightarrow 7x^2 - 2xy - y^2 = 0$$
(iii)

this is the equation of the lines joining the origin to the points of intersection (i) and (ii).

It is of the form $ax^2 + 2hxy + by^2 = 0$ (iv)

From (iii) and (iv), $a = 7, 2h = -2$ i.e. $h = -1$ and $b = -1$
 If θ be the required angle,

$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a + b} = \frac{2\sqrt{1 + 7}}{7 - 1} = \frac{2\sqrt{8}}{6} = \frac{2 \times 2\sqrt{2}}{6}$$

$$\therefore \theta = \tan^{-1} \frac{2\sqrt{2}}{3}$$

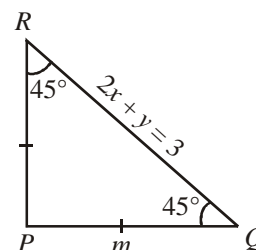
45. (b) Let m be the slope of PQ then

$$\tan 45^\circ = \left| \frac{m - (-2)}{1 + m(-2)} \right|$$

$$\Rightarrow 1 = \left| \frac{m + 2}{1 - 2m} \right| \Rightarrow \pm 1 = \frac{m + 2}{1 - 2m}$$

$$\Rightarrow m + 2 = 1 - 2m \text{ or } -1 + 2m = m + 2$$

$$\Rightarrow m = -1/3 \text{ or } m = 3$$



As PR also makes $\angle 45^\circ$ with RQ .
 \therefore The above two values of m are for PQ and PR .

$$\therefore \text{Equation of } PQ, y - 1 = -\frac{1}{3}(x - 2)$$

$$\Rightarrow 3y - 3 = -x + 2 \Rightarrow x + 3y - 5 = 0$$

and equation of PR is $3x - y - 5 = 0$

\therefore Combined equation of PQ and PR is

$$(x - 3y - 5)(3x - y - 5) = 0$$

$$\Rightarrow 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$$

46. (b) The given equation can be written as
 $(x^2 + y^2)(\cos^2 \theta \sin^2 \alpha + \sin^2 \theta)$
 $= x^2 \tan^2 \alpha - 2xy \tan \alpha \sin \theta + y^2 \sin^2 \theta$
 or $(\cos^2 \theta \sin^2 \alpha + \sin^2 \theta - \tan^2 \alpha)x^2 + 2(\tan \alpha \sin \theta)xy + \cos^2 \theta \sin^2 \alpha y^2 = 0$

Since the slope of these lines are given as $\tan \theta_1$ and $\tan \theta_2$

$$\text{Sum of the slopes} = \frac{-2 \tan \alpha \sin \theta}{\cos^2 \theta \sin^2 \alpha} \quad \left(\because \theta = \frac{\pi}{6} \right)$$

$$\Rightarrow \tan \theta_1 + \tan \theta_2 = \frac{-2 \tan \alpha \times \frac{1}{2}}{\frac{3}{4} \times \sin^2 \alpha} = -\frac{8}{3} \operatorname{cosec} 2\alpha$$

47. (b) The equation of lines joining the origin to the points of intersection will be obtained by making one equation homogeneous with the help of the other. we have

$$x^2 + y^2 = a^2 \quad \text{and} \quad x^2 + y^2 + 2(gx + fy) = 0$$

From both the equations, $2(gx + fy) = -a^2$

squaring both the sides, we get

$$4(gx + fy)^2 = a^4 = a^2(x^2 + y^2)$$

$\Rightarrow a^2(x^2 + y^2) - 4(gx + fy)^2 = 0$, which is the required equation.

48. (b) Given equation is $(x^2 + y^2) \sin^2 \alpha = (x \cos \beta - y \sin \beta)^2$
 $\Rightarrow x^2(\sin^2 \alpha - \cos^2 \beta) + 2xy \sin \beta \cos \beta$
 $+ y^2(\sin^2 \alpha - \sin^2 \beta) = 0 \quad \dots(i)$

Let the angle between the lines representing by (i) is θ

$$\therefore \tan \theta = 2 \left| \frac{\sqrt{h^2 - ab}}{a + b} \right|$$

$$= 2 \frac{\sqrt{\sin^2 \beta \cos^2 \beta - (\sin^2 \alpha - \cos^2 \beta)(\sin^2 \alpha - \sin^2 \beta)}}{|\sin^2 \alpha - \cos^2 \beta + \sin^2 \alpha - \sin^2 \beta|}$$

$$= 2 \frac{\sqrt{\{\sin^2 \beta \cos^2 \beta - \sin^4 \alpha + \sin^2 \alpha \sin^2 \beta + \sin^2 \alpha \cos^2 \beta - \sin^2 \beta \cos^2 \beta\}}}{|(2 \sin^2 \alpha - 1)|}$$

$$= 2 \frac{\sqrt{\sin \alpha (1 - \sin^2 \alpha)}}{|-\cos 2\alpha|} = \frac{2 \sin \alpha \cos \alpha}{|-\cos 2\alpha|} = \tan 2\alpha$$

$$\Rightarrow \theta = 2\alpha$$

49. (d) We have $ax^2 + 2\lambda xy + by^2 + 2Kx + 2Ky + 2K = 0$

$$h = \lambda, g = K, c = 2K, f = K,$$

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$ab(2K) + 2\lambda K^2 + aK^2 - bK^2 - 2\lambda^2 K = 0$$

$$2K\lambda^2 - 2K^2\lambda + (a + b)K^2 - 2abK = 0$$

For real λ , $B^2 - 4AC \geq 0$

$$4K^4 - 4 \cdot 2K[(a + b)K^2 - 2aK] \geq 0$$

$$K^2 - 2(a + b)K + 4ab \geq 0, (K - 2a)(K - 2b) \geq 0$$

$$K \leq 2a \text{ or } K \geq 2b.$$

50. (b) The distance between the parallel straight lines given by

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \text{ is } 2\sqrt{\frac{g^2 - ac}{a(a+b)}}$$

Here, $a = 8, b = 2, c = 15, g = 13$.

$$\text{So, req. distance} = 2\sqrt{\frac{169 - 120}{80}} = 2 \times \frac{7}{4\sqrt{5}} = \frac{7}{2\sqrt{5}}$$

51. (b) Let the given equation represents the straight lines $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$

$$\text{Then } ax^2 + 2hxy + by^2 + 2gx + 2fy + c$$

$$= (l_1x + m_1y + n_1)(l_2x + m_2y + n_2)$$

Comparing the coefficient, we get

$$l_1l_2 = a, m_1m_2 = b, n_1n_2 = c$$

$$l_1m_2 + l_2m_1 = 2h, l_1n_2 + l_2n_1 = 2g$$

$$= 2g \text{ and } m_1n_2 + m_2n_1 = 2f$$

The product of perpendiculars drawn from the origin to the lines is

$$= \frac{|n_1|}{\sqrt{l_1^2 + m_1^2}} \times \frac{|n_2|}{\sqrt{l_2^2 + m_2^2}}$$

$$= \frac{|n_1n_2|}{\sqrt{l_1^2l_1^2 + l_1^2m_2^2 + l_2^2m_1^2 + m_1^2m_2^2}}$$

$$= \frac{|n_1n_2|}{\sqrt{l_1^2l_1^2 + (l_1m_2 + l_2m_1)^2 - 2l_1l_2m_1m_2 + m_1^2m_2^2}}$$

$$= \frac{|c|}{\sqrt{a^2 + 4h^2 - 2ab + b^2}} = \frac{|c|}{\sqrt{(a-b)^2 + 4h^2}}$$

52. (a)

53. (b) Given equation of pair of lines is

$$x^2 + kxy + y^2 - 5x - 7y + 6 = 0$$

$$\therefore a = 1, b = 1, c = 6, f = \frac{-7}{2}, g = \frac{-5}{2}, h = \frac{k}{2}$$

$$\text{Now, } abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\Rightarrow (1)(1)(6) + 2\left(\frac{-7}{2}\right)\left(\frac{-5}{2}\right)\left(\frac{k}{2}\right) - 1\left(\frac{-7}{2}\right)^2$$

$$- 1\left(\frac{-5}{2}\right)^2 - 6\left(\frac{k}{2}\right)^2 = 0$$

$$\Rightarrow 6 + \frac{35k}{4} - \frac{49}{4} - \frac{25}{4} - \frac{6k^2}{4} = 0$$

$$\Rightarrow -6k^2 + 35k - 50 = 0 \Rightarrow (2k - 5)(3k - 10) = 0$$

$$\Rightarrow k = \frac{5}{2} \text{ or } k = \frac{10}{3}$$

54. (c) Given equation of pair of lines is

$$ax^2 + by^2 + cx + cy = 0$$

$$\therefore A = a, B = b, C = 0, F = \frac{c}{2}, G = \frac{c}{2}, H = 0$$

$$\text{Now, } ABC + 2FGH - AF^2 - BG^2 - CH^2 = 0$$

$$\Rightarrow ab(0) + 2\left(\frac{c}{2}\right)\left(\frac{c}{2}\right)(0) - a\left(\frac{c}{2}\right)^2$$

$$- b\left(\frac{c}{2}\right)^2 - 0(0)^2 = 0$$

$$\Rightarrow ac^2 + bc^2 = 0 \Rightarrow c^2(a + b) = 0$$

$$\Rightarrow c(a + b) = 0$$

55. (b) Given equation of pair of lines is

$$hxy + gx + fy + c = 0$$

$$A = B = 0, C = c, F = \frac{f}{2}, G = \frac{g}{2}, H = \frac{h}{2}$$

$$\text{Now, } ABC + 2FGH - AF^2 - BG^2 - CH^2 = 0$$

$$\Rightarrow 0 + 2\left(\frac{f}{2}\right)\left(\frac{g}{2}\right)\left(\frac{h}{2}\right) - 0 - 0 - c\left(\frac{h}{2}\right)^2 = 0$$

$$\Rightarrow \frac{fgh}{4} - \frac{ch^2}{4} = 0$$

$$\Rightarrow fg = ch$$

56. (a) Given equation of pair of lines is

$$3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0$$

$$a = 3, b = 2, c = 2, f = \frac{5}{2}, g = \frac{5}{2}, h = \frac{7}{2}$$

$$\text{Consider, } abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= (3)(2)(2) + 2\left(\frac{5}{2}\right)\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)$$

$$- 3\left(\frac{5}{2}\right)^2 - 2\left(\frac{5}{2}\right)^2 - 2\left(\frac{7}{2}\right)^2 = 0$$

\(\therefore\) The given equation represents a pair of straight lines.

57. (c) Given equation of pair of lines is

$$2x^2 - 10xy + 2\lambda y^2 + 5x - 16y - 3 = 0$$

$$\therefore a = 2, b = 2\lambda, c = -3, f = -8, g = \frac{5}{2}, h = -5$$

Since the equation represents pair of lines,

$$\therefore abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\Rightarrow 2(2\lambda)(-3) + 2(-8)\left(\frac{5}{2}\right)(-5) - 2(64)$$

$$- 2\lambda\left(\frac{25}{4}\right) + 3(25) = 0$$

$$\Rightarrow \frac{49\lambda}{2} = 147 \Rightarrow \lambda = 6$$

\(\therefore\) Point of intersection of the lines is

$$\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$$

$$\equiv \left(\frac{(-5)(-8) - 2(6)\left(\frac{5}{2}\right)}{2(12) - (-5)^2}, \frac{\frac{5}{2}(-5) - 2(-8)}{2(12) - (-5)^2}\right)$$

$$\equiv \left(-10, \frac{-7}{2}\right)$$

58. (a) Given equation of pair of lines is

$$2x^2 + 5xy + 3y^2 + 6x + 7y + 4 = 0$$

$$a = 2, b = 3, h = \frac{5}{2}$$

$$\theta = \tan^{-1} m \Rightarrow \tan \theta = m$$

$$\tan \theta = \left| \frac{2\sqrt{\frac{25}{4} - 6}}{2 + 3} \right| \Rightarrow m = \frac{1}{5}$$

59. (b) Given equation of pair of lines is

$$x^2 + y^2 - 2x - 1 = 0$$

...(i)

$x + y = 1$ intersects the above pair of lines

\(\therefore\) It satisfies equation (i)

$$\therefore x^2 + y^2 - 2x(x + y) - (x + y)^2 = 0$$

$$\Rightarrow 2x^2 + 4xy = 0 \Rightarrow x^2 + 2xy = 0$$

$$\therefore a = 1, b = 0, h = 1$$

$$\therefore \tan \theta = \frac{2\sqrt{1^2 - 0}}{1} \Rightarrow \theta = \tan^{-1}(2)$$

60. (a) Given equation of pair of lines is

$$2x^2 + 5xy + 2y^2 + 3x + 3y + 1 = 0$$

$$a = 2, b = 2, c = 1, f = \frac{3}{2}, g = \frac{3}{2}, h = \frac{5}{2}$$

$$\therefore \tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right| = \left| \frac{2\sqrt{\left(\frac{25}{4}\right) - 4}}{2 + 2} \right| = \frac{3}{4}$$

$$\therefore \cos \theta = \frac{4}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{5}\right)$$

61. (b) Given equation of pair of lines is

$$2x^2 - 3xy - 2y^2 + 10x + 5y = 0$$

$$\therefore a = 2, b = -2, c = 0, f = \frac{5}{2}, g = 5, h = \frac{-3}{2}$$

\(\therefore\) Point of intersection of the lines is

$$\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right) \equiv (-1, 2)$$

Slope of line joining origin and $(-1, 2)$ $m = -2$

Slope of $kx + y + 3 = 0$ is $-k$

$$\text{Now, } (-k)(-2) = -1 \Rightarrow k = \frac{-1}{2}$$

62. (d) Given equation of pair of lines is

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$\theta = \frac{\pi}{4} \Rightarrow \tan \theta = 1$$

$$\therefore 1 = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$$

$$\begin{aligned} \Rightarrow 4(h^2 - ab) &= (a+b)^2 \\ \Rightarrow 4h^2 - 4ab &= a^2 + 2ab + b^2 \\ \Rightarrow a^2 + 6ab + b^2 &= 4h^2 \end{aligned}$$

63. (a) The joint equation of the pair of straight lines joining the origin to the points of intersection of the line

$$lx + my + n = 0 \text{ and } ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \text{ is}$$

$$ax^2 + 2hxy + by^2 + 2g\left(\frac{lx+my}{-n}\right)x + 2f\left(\frac{lx+my}{-n}\right)y + c\left(\frac{lx+my}{-n}\right)^2 = 0$$

$$\text{Here, } l = 2, m = 1, n = -1 \text{ and } a = 3, b = 0, c = 1, f = 0, g = -2, h = 2$$

$$\begin{aligned} \therefore 3x^2 + 4xy - 4x(2x+y) + (2x+y)^2 &= 0 \\ \Rightarrow 3x^2 + 4xy - 8x^2 - 4xy + 4x^2 + y^2 + 4xy &= 0 \\ \Rightarrow x^2 - 4xy - y^2 &= 0 \end{aligned}$$

$$\therefore A = 1, B = -1, H = -2$$

$$\therefore \tan \theta = \frac{2\sqrt{4+1}}{0} = \infty \Rightarrow \theta = \frac{\pi}{2}$$

64. (b) Given equation of pair of lines is

$$9x^2 + y^2 + 6xy - 4 = 0$$

$$\therefore a = 9, b = 1, h = 3$$

$$h^2 - ab = 3^2 - 9(1) = 0$$

$$\therefore \text{The lines are parallel}$$

$$\text{Now, } 9x^2 + 6xy + y^2 = 4$$

$$\Rightarrow (3x + y)^2 = 4 \Rightarrow 3x + y = \pm 2$$

Hence, the lines are parallel and not coincident.

65. (c) Given equation of pair of lines is

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$\therefore A = a, B = b, H = h$$

The lines are parallel

$$\therefore H^2 = AB$$

$$\Rightarrow h = \sqrt{ab}$$

$$\text{Now, } ABC + 2FGH - AF^2 - BG^2 - CH^2 = 0$$

$$\Rightarrow abc + 2fg\sqrt{ab} - af^2 - bg^2 - abc = 0$$

$$\Rightarrow (\sqrt{a}f - \sqrt{b}g)^2 = 0 \Rightarrow af^2 = bg^2$$

66. (c) Given equation of pair of lines is

$$x^2 + k_1y^2 + 2k_2y = a^2$$

$$a = 1, b = k_1, c = -a^2, f = k_2, g = 0, h = 0$$

The lines are perpendicular

$$\therefore a + b = 0 \Rightarrow k_1 = -1$$

Substituting value of k_1 in the given equation of lines, we get

$$x^2 - y^2 + 2k_2y - a^2 = 0$$

$$\Rightarrow a^2 - k_2^2 = 0 \Rightarrow k_2 = \pm a$$

67. (a) $(x^2 + y^2)(h^2 + k^2 - a^2) = (hx + ky)^2$

$$\Rightarrow x^2(h^2 + k^2 - a^2) + y^2(h^2 + k^2 - a^2)$$

$$= h^2x^2 + k^2y^2 + 2hkxy$$

$$\Rightarrow x^2(k^2 - a^2) + y^2(h^2 - a^2) - 2hkxy = 0$$

$$\therefore A = k^2 - a^2, B = h^2 - a^2$$

The lines are perpendicular

$$\therefore A + B = 0$$

$$\Rightarrow k^2 - a^2 + h^2 - a^2 = 0 \Rightarrow h^2 + k^2 = 2a^2$$

68. (c) Given equation of pair of lines is

$$2x^2 - 4xy - py^2 + 4x + qy + 1 = 0$$

$$a = 2, b = -p, c = 1, f = \frac{q}{2}, g = 2, h = -2$$

The lines are perpendicular

$$\therefore a + b = 0$$

$$\Rightarrow 2 - p = 0 \Rightarrow p = 2$$

The equations represents pair of lines

$$\therefore 2(-2)(1) + 2\left(\frac{q}{2}\right)(2)(2) - 2\left(\frac{q}{2}\right)^2 + 2(2)^2 - 1(2)^2 = 0$$

$$\Rightarrow q^2 - 8q = 0 \Rightarrow q = 0 \text{ or } 8$$

69. (c) Given equation of pair of lines is

$$12x^2 + 7xy + by^2 + gx + 7y - 1 = 0$$

$$\therefore A = 12, B = b, C = -1, F = \frac{7}{2}, G = \frac{g}{2}, H = \frac{7}{2}$$

The lines are perpendicular

$$\therefore A + B = 0 \Rightarrow 12 + b = 0 \Rightarrow b = -12$$

$$\text{Also, } ABC + 2FGH - AF^2 - BG^2 - CH^2 = 0$$

$$\Rightarrow (12)(-12)(-1) + 2\left(\frac{7}{2}\right)\left(\frac{g}{2}\right)\left(\frac{7}{2}\right)$$

$$- (12)\left(\frac{7}{2}\right)^2 - (-12)\left(\frac{g}{2}\right)^2 - (-1)\left(\frac{7}{2}\right)^2 = 0$$

$$\Rightarrow 12g^2 + 49g + 37 = 0$$

$$\Rightarrow (g + 1)(12g + 37) = 0 \Rightarrow g = -1 \text{ or } -\frac{37}{12}$$

70. (c) Given $ax^2 + 2hxy + by^2 = -2gx$

$$a_1x^2 + 2h_1xy + b_1y^2 = -2g_1x$$

$$\therefore 2\frac{ax^2 + 2hxy + by^2}{a_1x^2 + 2h_1xy + b_1y^2} = \frac{g}{g_1}$$

We have,

$$(ag_1 - a_1g)x^2 + 2(hg_1 - h_1g)xy + (bg_1 - b_1g)y^2 = 0$$

$$\therefore A = (ag_1 - a_1g), B = (bg_1 - b_1g)$$

The lines are perpendicular

$$\therefore A + B = 0$$

$$\Rightarrow (ag_1 - a_1g) + (bg_1 - b_1g) = 0$$

$$\Rightarrow (a + b)g_1 = (a_1 + b_1)g$$

Exercise 3 :

PREVIOUS YEARS MCQs

1. (d) We know angle between the pair of lines

$ax^2 + by^2 + 2hxy = 0$ is given by

$$\theta = \tan^{-1} \left(\pm \frac{2\sqrt{h^2 - ab}}{a+b} \right)$$

On comparing with given equation

$ax^2 + by^2 + 2hxy = 0$, we get

$$a = 2, b = 3, h = \frac{5}{2}$$

$$\therefore \theta = \tan^{-1} \left(\pm \frac{2\sqrt{\frac{25}{4} - 6}}{5} \right) = \tan^{-1} \left(\pm \frac{1}{5} \right)$$

2. (a) $3x + 4y = 0$ is one of the lines of the pair

$$6x^2 - xy + 4cy^2 = 0.$$

Put $y = -\frac{3}{4}x$, we get

$$6x^2 + \frac{3}{4}x^2 + 4c\left(-\frac{3}{4}x\right)^2 = 0 \Rightarrow 6 + \frac{3}{4} + \frac{9c}{4} = 0 \Rightarrow c = -3$$

3. (a) $3x + 4y = 0$ is one of the line of the pair equations of lines

$$6x^2 - xy + 4cy^2 = 0, \text{ Put } y = -\frac{3}{4}x,$$

$$\text{we get, } 6x^2 + \frac{3}{4}x^2 + 4c\left(-\frac{3}{4}x\right)^2 = 0$$

$$\Rightarrow 6 + \frac{3}{4} + \frac{9c}{4} = 0 \Rightarrow c = -3$$

4. (b) Given equation of pair of lines is

$$x^2 + 2hxy + 2y^2 = 0$$

$$\therefore A = 1, B = 2, H = h$$

Let the slopes of the lines be m_1, m_2 .

$$\therefore m_1 + m_2 = \frac{-2h}{2}, m_1 m_2 = \frac{1}{2}$$

$$\text{Given, } \frac{m_1}{m_2} = \frac{1}{2} \Rightarrow m_1 = \frac{1}{2}m_2$$

$$\Rightarrow \frac{1}{2}m_2 \times m_2 = \frac{1}{2} \Rightarrow m_2^2 = 1 \Rightarrow m_2 = \pm 1$$

$$\therefore m_1 = \pm \frac{1}{2}$$

$$\text{Now, } m_1 + m_2 = \frac{-2h}{2}$$

$$\therefore \text{ when } m_1 = \frac{1}{2}, m_2 = 1$$

$$m_1 + m_2 = \frac{-2h}{2}$$

$$\therefore h = \frac{-3}{2}$$

When $m_1 = \frac{-1}{2}, m_2 = -1$

$$\therefore m_1 + m_2 = \frac{-2h}{2} \Rightarrow \frac{-1}{2} - 1 = \frac{-2h}{2}$$

$$\therefore h = \frac{3}{2}$$

5. (a) The slope of line $3x + 2y - 8 = 0$ is $m_1 = \frac{-3}{2}$

Let m be the slope of one of the lines making an angle

$$\frac{\pi}{4} \text{ with } 3x + 2y - 8 = 0$$

$$\therefore \tan \frac{\pi}{4} = \frac{|m - m_1|}{|1 + mm_1|} \Rightarrow 1 = \frac{\left| m - \left(\frac{-3}{2} \right) \right|}{\left| 1 + m \left(\frac{-3}{2} \right) \right|}$$

$$\Rightarrow 1 = \frac{|2m + 3|}{|2 - 3m|}$$

Squaring on both sides, we get

$$(2 - 3m)^2 = (2m + 3)^2 \Rightarrow 5m^2 - 24m - 5 = 0$$

This is the auxiliary equation of two lines and their

joint equation is obtained by putting $m = \frac{y}{x}$.

- \therefore The joint equation of the lines is

$$5\left(\frac{y}{x}\right)^2 - 24\left(\frac{y}{x}\right) - 5 = 0$$

$$\text{i.e., } 5x^2 + 24xy - 5y^2 = 0$$

6. (a) Given equation of pair of lines is

$$ax^2 + (2a + 1)xy + 2y^2 = 0$$

$$A = a, H = \frac{2a+1}{2}, B = 2$$

$$\text{Given condition, } m_1 = \frac{1}{m_2}$$

$$\therefore m_1 \cdot m_2 = 1$$

$$\text{Product of slopes} = \frac{A}{B} = \frac{a}{2}$$

$$\therefore m_1 \cdot m_2 = 1 = \frac{a}{2}$$

$$\therefore a = 2$$

$$\text{Also, sum of slopes} = \frac{-2H}{B} = -\left(\frac{2a+1}{2}\right) = \frac{-5}{2}$$

$$\text{Using } (m_1 + m_2)^2 = m_1^2 + m_2^2 + 2m_1m_2$$

$$\left(\frac{-5}{2}\right)^2 = m_1^2 + m_2^2 + 2 \times 1$$

$$\therefore m_1^2 + m_2^2 = \frac{25}{4} - 2$$

$$\therefore m_1^2 + m_2^2 = \frac{17}{4}$$

7. (b) Substituting $x = 10 - 3y$ in equation $6x^2 + xy - y^2 = 0$, we get
- $$6(10 - 3y)^2 + (10 - 3y)y - y^2 = 0$$
- $$\therefore 6(100 - 60y + 9y^2) + 10y - 3y^2 - y^2 = 0$$
- $$\therefore 600 - 360y + 54y^2 + 10y - 3y^2 - y^2 = 0$$
- $$\therefore 50y^2 - 350y + 600 = 0$$
- $$\therefore y^2 - 7y + 12 = 0$$
- $$\therefore y = 4 \text{ or } y = 3$$
- $$\Rightarrow x = -2 \text{ or } x = 1$$
- \therefore Co-ordinates of the triangle are $(0, 0)$, $(-2, 4)$ and $(1, 3)$.

$$\therefore \text{Centroid of the triangle is } \left(\frac{-1}{3}, \frac{7}{3}\right).$$

8. (a) Given equation of pair of straight lines is

$$kx^2 - 4xy + y^2 = 0$$

$$\therefore a = k, h = -2, b = 1$$

$$\therefore \tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$$

$$\therefore \frac{1}{2} = \left| \frac{2\sqrt{4 - k}}{k + 1} \right|$$

$$\therefore \frac{1}{4} = \frac{4(4 - k)}{(k + 1)^2}$$

$$\therefore k^2 + 2k + 1 = 16(4 - k)$$

$$\therefore k^2 + 2k + 1 = 64 - 16k$$

$$\therefore k^2 + 18k - 63 = 0$$

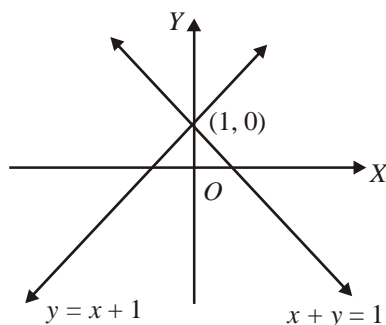
$$\therefore k^2 + 21k - 3k - 63 = 0$$

$$\therefore k(k + 21) - 3(k + 21) = 0$$

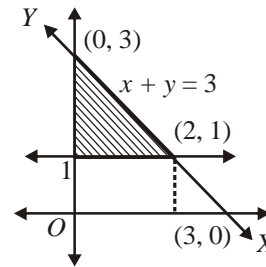
$$\therefore (k - 3)(k + 21) = 0$$

$$\therefore k = 3 \text{ or } k = -21.$$

9. (a) $x^2 - y^2 + 2y = 1$ or $x = \pm(y - 1)$



The bisectors of the above lines are $x = 0$ and $y = 1$.



So, the area between $x = 0$, $y = 1$, and $x + y = 3$ is the shaded region shown in the figure. The area is given by $(1/2) \times 2 \times 2 = 2$ sq. units.

10. (b) Given equation of pair of lines is

$$x^2 - 3xy - 4y^2 = 0$$

$$\therefore a = 1, h = \frac{-3}{2}, b = -4$$

Using *Shortcut 9*, we get the equation of the angle bisectors of the above pair of lines is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h} \text{ i.e. } \frac{x^2 - y^2}{1 + 4} = \frac{xy}{\frac{-3}{2}}$$

$$\therefore \frac{x^2 - y^2}{5} = \frac{-2xy}{3}$$

$$\therefore 3x^2 + 10xy - 3y^2 = 0$$

Comparing with the given equation of bisectors, we get $k = -10$

11. (c) From the diagram, the required lines are

$$y = \frac{x}{\sqrt{3}}, \text{ i.e.,}$$

$$\sqrt{3}y - x = 0 \text{ and}$$

$$y = \frac{-x}{\sqrt{3}}, \text{ i.e., } \sqrt{3}y + x = 0$$

$$\therefore \text{Combined equation is}$$

$$(\sqrt{3}y - x)(\sqrt{3}y + x) = 0; \text{ i.e., } 3y^2 - x^2 = 0$$

12. (b) Here, $a = 1, h = \operatorname{cosec} \alpha, b = 1$

$$\therefore \tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right| = \left| \frac{2\sqrt{\operatorname{cosec}^2 \alpha - 1}}{1 + 1} \right| = \left| \sqrt{\cot^2 \alpha} \right|$$

$$\therefore \tan \theta = \cot \alpha$$

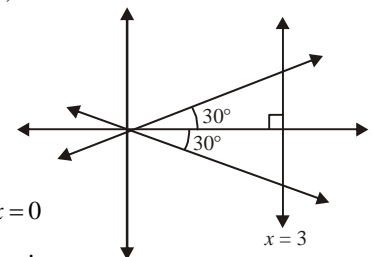
$$\therefore \tan \theta = \tan \left(\frac{\pi}{2} - \alpha \right)$$

$$\therefore \theta = \frac{\pi}{2} - \alpha$$

13. (d) Given equation of pair of lines is

$$x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$$

$$a = 1, b = \lambda, c = 2, f = \frac{-5}{2}, g = \frac{3}{2}, h = \frac{-3}{2}$$



$$\theta = \tan^{-1}\left(\frac{1}{3}\right) \Rightarrow \tan \theta = \frac{1}{3}$$

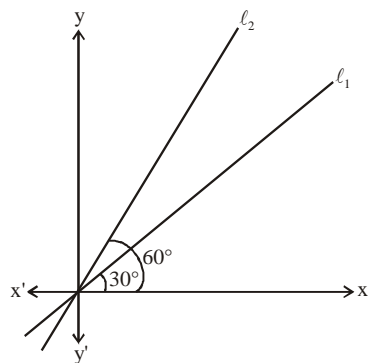
$$\text{Since, } \tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$$

$$\Rightarrow \frac{1}{3} = \left| \frac{2\sqrt{\left(\frac{-3}{2}\right)^2 - \lambda}}{\lambda + 1} \right|$$

$$\Rightarrow (\lambda + 1)^2 = 9(9 - 4\lambda) \Rightarrow \lambda^2 + 38\lambda - 80 = 0$$

$$\Rightarrow (\lambda + 40)(\lambda - 2) = 0 \Rightarrow \lambda = -40, 2$$

14. (a) Let ℓ_1 and ℓ_2 are the two lines, which trisects the first quadrant (as shown in the figure)



$$\text{slope of } \ell_1 = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{and slope of } \ell_2 = \tan 60^\circ = \sqrt{3}$$

\therefore equation of ℓ_1 is :

$$x = \frac{y}{\sqrt{3}} \quad (\because \ell_1 \text{ passes through centre})$$

& equation of ℓ_2 is :

$$x = \sqrt{3}y \quad (\because \ell_2 \text{ passes through centre})$$

\therefore joint equation is :

$$\left(x - \frac{y}{\sqrt{3}}\right)(x - \sqrt{3}y) = 0; \quad x^2 - \sqrt{3}xy - \frac{xy}{\sqrt{3}} + y^2 = 0$$

$$\Rightarrow \frac{\sqrt{3}x^2 - 3xy - xy + \sqrt{3}y^2}{\sqrt{3}} = 0$$

$$\Rightarrow \sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$$

15. (a) Comparing the given equation with $ax^2 + 2hxy + by^2 = 0$, we get
 $a = \sin^2 \theta - 1 = -\cos^2 \theta$, $b = \cos^2 \theta$
 Here, $a + b = -\cos^2 \theta + \cos^2 \theta = 0$
 \therefore The lines are perpendicular

$$\therefore \theta = \frac{\pi}{2}$$

16. (b) Comparing the given equation with $ax^2 + 2hxy + by^2 = 0$, we get
 $a = 2$, $2h = -3$, $b = 1$

$$m_1 + m_2 = \frac{-2h}{b} = 3,$$

$$m_1 \cdot m_2 = \frac{a}{b} = 2$$

$$(m_1)^3 + (m_2)^3 = (m_1 + m_2)(m_1^2 - m_1m_2 + m_2^2)$$

$$= 3 [(m_1 + m_2)^2 - 3m_1m_2]$$

$$= 3 [(3)^2 - 3(2)] = 3(3) = 9$$

17. (c) $5x + y - 1 = 0$ coincides
 $5x^2 + xy - kx - 2y + 2 = 0$

$$\therefore a = 5, b = 0, h = \frac{1}{2}, g = -\frac{k}{2}, f = -1, c = 2$$

If the above equation represents a pair of straight lines, then

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 5 & \frac{1}{2} & -\frac{k}{2} \\ \frac{1}{2} & 0 & -1 \\ -\frac{k}{2} & -1 & 2 \end{vmatrix} = 0$$

$$\therefore 5(0-1) - \frac{1}{2}\left(1 - \frac{k}{2}\right) - \frac{k}{2}\left(\frac{-1}{2} - 0\right) = 0$$

$$\therefore -5 - \frac{1}{2} + \frac{k}{4} + \frac{k}{4} = 0 \Rightarrow \frac{k}{2} = \frac{11}{2} \Rightarrow k = 11$$

18. (a) $\because ax^2 + 2hxy + 2by^2 = 0$
 Let the slope of one line is m .
 In slope of other line = $2m$.
 We know that

$$m + 2m = \frac{-2h}{b} \text{ \& } m \times 2m = \frac{a}{b}.$$

$$\Rightarrow 3m = \frac{-2h}{b} \text{ \& } 2m^2 = \frac{a}{b}. \Rightarrow m = \frac{-2h}{b} \Rightarrow m^2 = \frac{4h^2}{9b^2}.$$

$$\therefore 2\left(\frac{4h^2}{9b^2}\right) = \frac{a}{b}. \Rightarrow 8h^2 = 9ab$$

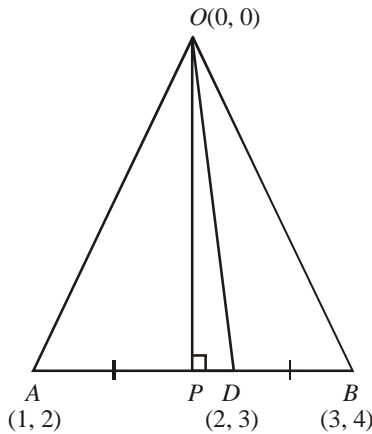
19. (c) $x^2 - y^2 + x + 3y - 2 = 0$
 Comparing the above equation with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
 we get

$$a = 1, h = 0, b = -1, g = \frac{1}{2}, f = \frac{3}{2}, c = -2$$

\therefore req. point of intersection is:

$$\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right) \equiv \left(\frac{0 + \frac{1}{2}, 0 - \frac{3}{2}}{-1, -1}\right) \equiv \left(-\frac{1}{2}, \frac{3}{2}\right)$$

20. (d)



Since, $O(0, 0)$, $A(1, 2)$ and $B(3, 4)$ are the vertices of ΔOAB .

Consider that OP and OD are altitude and median of ΔOAB , respectively.

Then, Coordinates of D

$$= \left(\frac{1+3}{2}, \frac{2+4}{2} \right) = (2, 3)$$

So, equation of OD is $(y - 0) = \left(\frac{3-0}{2-0} \right) (x - 0)$

Hence, $y = \frac{3}{2}x \Rightarrow 3x - 2y = 0$

Now, slope of $OP = \frac{-1}{\text{Slope of } AB}$

$$\frac{-1}{\left(\frac{3-1}{4-2} \right)} = -1 \text{ [As, } OP \perp AB] = \frac{1}{\left(\frac{3-1}{4-2} \right)} = -1$$

\therefore Equation of OP is $(y - 0) = -1(x - 0)$

$\therefore y = -x \Rightarrow x + y = 0$

Hence, joint equation of OP and OD is:

$$(x + y)(3x - 2y) = 0$$

$\Rightarrow 3x^2 + xy - 2y^2 = 0$... (i)

After comparing eq. (i) with $ax^2 + 2hxy + by^2 = 0$,

we get $a = p$, $b = -q$, $h = 0$

As slopes of pair of lines represented by

$$ax^2 + 2hxy + by^2 = 0$$

are real and distinct iff $h^2 - ab > 0$

So, $0 + pq > 0$

Hence, $pq > 0$

22. (c) We have, pair of lines

$$kx^2 + 5xy + y^2 = 0$$
 ... (i)

After comparing eq. (i) with

$$ax^2 + 2hx + by^2 = 0, \text{ we get}$$

$$a = k, b = 1 \text{ and } 2h = 5$$

Suppose m_1 and m_2 are two slopes of pair of lines.

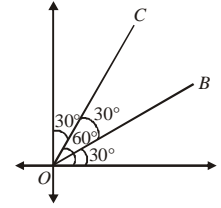
Then $m_1 + m_2 = \frac{-2h}{b} = -5$ and $m_1 m_2 = \frac{a}{b} = k$

As, $(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1 m_2$

$$\Rightarrow (1)^2 = (-5)^2 - 4k$$

$$\Rightarrow 4k = 24 \Rightarrow k = 6$$

23. (c) In a trisection of lines in quadrant, angle 90° is divided into three parts and each part contain 30° , as shown in figure



\therefore Equation of line OB is

$$y = \tan 30^\circ x \Rightarrow y = \frac{1}{\sqrt{3}}x$$

$$x - \sqrt{3}y = 0$$

And equation of line OC is

$$y = \tan 60^\circ x \Rightarrow y = \sqrt{3}x ; (\sqrt{3}x - y) = 0$$

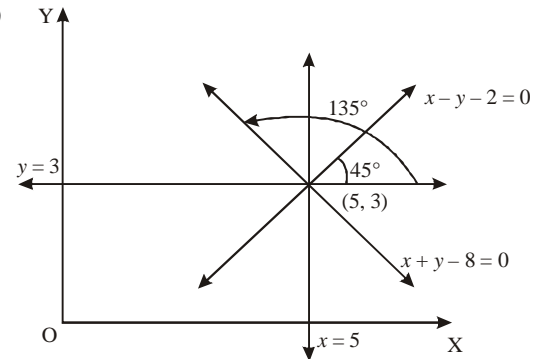
\therefore Their combined equation is

$$(x - \sqrt{3}y)(\sqrt{3}x - y) = 0$$

$$\Rightarrow \sqrt{3}x^2 - xy - 3xy + \sqrt{3}y^2 = 0$$

$$\Rightarrow \sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$$

24. (b)



The equation of bisectors are

$$y - 3 = (1)(x - 5) \text{ and } y - 3 = (-1)(x - 5)$$

$$\Rightarrow x - y - 2 = 0 \text{ and } x + y - 8 = 0$$

\therefore The joint equation of the bisectors is

$$(x - y - 2)(x + y - 8) = 0$$

$$\Rightarrow x^2 - y^2 - 10x + 6y + 16 = 0$$

25. (c) (a) $x^2 - x = 0 \Rightarrow x(x - 1) = 0$

$$\Rightarrow x = 0 \text{ and } x = 1,$$

This represent a pair of lines.

(b) $xy - x = 0 \Rightarrow x(y - 1) = 0$

$$\Rightarrow x = 0, y = 1,$$

This represent a pair of lines.

(c) $y^2 - x + 1 = 0$

$$\Rightarrow y^2 = (x - 1),$$

This represent a parabola.

Which does not represent a pair of lines.

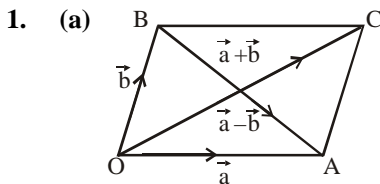
(d) $xy + x + y + 1 = 0$

$$\Rightarrow x(y + 1) + (y + 1) = 0 \Rightarrow (y + 1)(x + 1) = 0$$

$$\Rightarrow y = -1 \text{ and } x = -1$$

This represent a pair of lines.

Exercise 1 :

WARM-UP
Topic-wise MCQs

Let $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$. Complete the parallelogram OACB.

$$\vec{a} + \vec{b} = \overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC} \Rightarrow |\vec{a} + \vec{b}| = OC$$

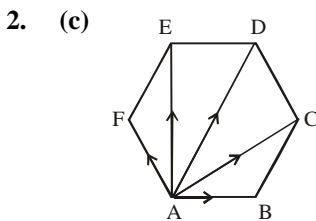
$$\text{Again } \vec{a} - \vec{b} = \overrightarrow{OA} - \overrightarrow{OB} = \overrightarrow{BA} \Rightarrow |\vec{a} - \vec{b}| = BA$$

$$\text{Given } |\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| \Rightarrow OC = BA$$

\therefore Diagonals of the parallelogram OACB are equal.

\therefore OACB is a rectangle.

$\therefore \vec{a}$ and \vec{b} are adjacent sides of a rectangle.

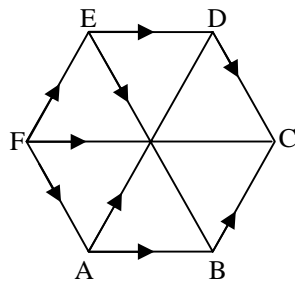


$$\begin{aligned} \overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} &= \overrightarrow{ED} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{CD} \\ &= (\overrightarrow{AC} + \overrightarrow{CD}) + \overrightarrow{AD} + (\overrightarrow{AE} + \overrightarrow{ED}) = \overrightarrow{AD} + \overrightarrow{AD} + \overrightarrow{AD} = 3\overrightarrow{AD} \\ \therefore n &= 3 \end{aligned}$$

3. (d) Given ABCDEF is a regular hexagon. We know from the hexagon that \overrightarrow{AD} is parallel to \overrightarrow{BC}
 $\Rightarrow \overrightarrow{AD} = 2\overrightarrow{BC}$

Similarly, \overrightarrow{EB} is parallel to \overrightarrow{FA} .
 $\Rightarrow \overrightarrow{EB} = 2\overrightarrow{FA}$ and
 \overrightarrow{FC} is parallel to \overrightarrow{AB}
 $\Rightarrow \overrightarrow{FC} = 2\overrightarrow{AB}$

$$\begin{aligned} \text{Thus, } \overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC} &= 2\overrightarrow{BC} + 2\overrightarrow{FA} + 2\overrightarrow{AB} \\ &= 2(\overrightarrow{FA} + \overrightarrow{AB} + \overrightarrow{BC}) = 2(\overrightarrow{FC}) = 4(\overrightarrow{AB}) \end{aligned}$$



4. (a) Let $\vec{r}_1 = \hat{i} + \hat{j} + \hat{k}$ and $\vec{r}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

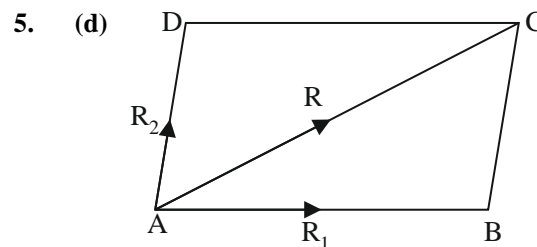
$$\text{Let } \vec{r} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\text{Then, } |\vec{r}| = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

Thus, the unit vector in the direction of the sum of vectors

$$\text{is } \frac{\vec{r}_1 + \vec{r}_2}{|\vec{r}_1 + \vec{r}_2|} \text{ i.e.}$$

$$\frac{\vec{r}}{|\vec{r}|} = \frac{3\hat{i} + 4\hat{j} + 5\hat{k}}{5\sqrt{2}} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{4}{5\sqrt{2}}\hat{j} + \frac{5}{5\sqrt{2}}\hat{k}$$



$$\text{Let } \vec{R}_1 = 2\hat{i} + 4\hat{j} - 5\hat{k} \text{ and } \vec{R}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\therefore \vec{R} \text{ (along } \overrightarrow{AC}) = \vec{R}_1 + \vec{R}_2 = 3\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\therefore \vec{a} \text{ (unit vector along } \overrightarrow{AC}) = \frac{\vec{R}}{|\vec{R}|}$$

$$= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{9 + 36 + 4}} = \frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$$

6. (d) Given, $\vec{a} = 3\hat{i} + 5\hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} - 5\hat{k}$
and $\vec{c} = 5\hat{i} + 2\hat{j} - 3\hat{k}$

$$\text{Clearly, } \vec{a} + \vec{b} = 5\hat{i} + 2\hat{j} - 3\hat{k} = \vec{c}$$

$\therefore \vec{a} + \vec{b} = \vec{c} \Rightarrow$ The vectors $\vec{a}, \vec{b}, \vec{c}$ are represented by three sides of a triangle.

$$\text{Now, } |\vec{a}| = |3\hat{i} + 5\hat{j} + 2\hat{k}| = \sqrt{38}$$

$$|\vec{b}| = |2\hat{i} - 3\hat{j} - 5\hat{k}| = \sqrt{38}$$

$$|\vec{c}| = |5\hat{i} + 2\hat{j} - 3\hat{k}| = \sqrt{38}$$

$$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}|$$

Hence, $\vec{a}, \vec{b}, \vec{c}$ forms the sides of an equilateral triangle.

7. (d) If $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent vectors, then \vec{c} should be a linear combination of \vec{a} and \vec{b} .

$$\text{Let } \vec{c} = p\vec{a} + q\vec{b}$$

$$\text{i.e., } \hat{i} + \alpha\hat{j} + \beta\hat{k} = p(\hat{i} + \hat{j} + \hat{k}) + q(4\hat{i} + 3\hat{j} + 4\hat{k})$$

Equating coefficients of $\hat{i}, \hat{j}, \hat{k}$ on both sides, we get

$$1 = p + 4q, \alpha = p + 3q, \beta = p + 4q$$

From first and third, $\beta = 1$

$$\text{Now, } |\vec{c}| = \sqrt{3} \text{ [Given]} \Rightarrow 1 + \alpha^2 + \beta^2 = 3$$

$$\text{Hence, } \alpha = \pm 1, \beta = 1$$

8. (d) Since \vec{a} and \vec{b} are collinear.

$$\therefore \vec{a} = \lambda \vec{b} \Rightarrow (x\hat{i} - 2\hat{j} + 5\hat{k}) = \lambda(\hat{i} + y\hat{j} - z\hat{k})$$

On comparing, we get

$$x = \lambda, -2 = \lambda y \text{ and } 5 = -\lambda z$$

For $\lambda = 1$, we have

$$x = 1, y = -2 \text{ and } z = -5$$

For $\lambda = \frac{1}{2}$, we have

$$x = \frac{1}{2}, y = -4 \text{ and } z = -10$$

For $\lambda = -\frac{1}{2}$, we have

$$x = -\frac{1}{2}, y = 4 \text{ and } z = 10$$

9. (d) If $\vec{a} + 2\vec{b}$ is collinear with \vec{c} , then

$$\vec{a} + 2\vec{b} = t\vec{c}$$

Also, if $\vec{b} + 3\vec{c}$ is collinear with \vec{a} , then

$$\vec{b} + 3\vec{c} = \lambda\vec{a}$$

$$\Rightarrow \vec{b} = \lambda\vec{a} - 3\vec{c}$$

Putting the value of \vec{b} in (i), we get

$$\vec{a} + 2(\lambda\vec{a} - 3\vec{c}) = t\vec{c} \Rightarrow \vec{a} + 2\lambda\vec{a} - 6\vec{c} = t\vec{c}$$

$$\Rightarrow (\vec{a} - 6\vec{c}) = t\vec{c} - 2\lambda\vec{a}$$

On comparing, we get

$$1 = -2\lambda \Rightarrow \lambda = -\frac{1}{2} \text{ and } -6 = t$$

$$\Rightarrow t = -6$$

From, (i) we get

$$\vec{a} + 2\vec{b} = -6\vec{c} \Rightarrow \vec{a} + 2\vec{b} + 6\vec{c} = \vec{0}$$

10. (a) According to question

$$2|\hat{i} + x\hat{j} + 3\hat{k}| = |4\hat{i} + (4x-2)\hat{j} + 2\hat{k}|$$

$$\Rightarrow 2\sqrt{1+x^2+9} = \sqrt{4^2+(4x-2)^2+2^2}$$

$$\Rightarrow 4(x^2+10) = 16+16x^2+4-16x+4$$

$$\Rightarrow 12x^2-16x-16=0 \Rightarrow 3x^2-4x-4=0$$

$$\Rightarrow (3x+2)(x-2)=0 \Rightarrow x=2, -\frac{2}{3}$$

11. (c) Let A(2, -1, 3), B(3, -5, 1) and C(-1, 11, 9) be three points respectively, then

$$\vec{OA} = 2\hat{i} - \hat{j} + 3\hat{k}, \vec{OB} = 3\hat{i} - 5\hat{j} + \hat{k} \text{ and}$$

$$\vec{OC} = -\hat{i} + 11\hat{j} + 9\hat{k}$$

$$\therefore \vec{AB} = \vec{OB} - \vec{OA} = (3\hat{i} - 5\hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k})$$

$$= \hat{i} - 4\hat{j} - 2\hat{k}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = (-\hat{i} + 11\hat{j} + 9\hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k})$$

$$= -3\hat{i} + 12\hat{j} + 6\hat{k} \Rightarrow \vec{AC} = -3\vec{AB}$$

Thus, the vector \vec{AB} and \vec{AC} are parallel having the same initial point A.

Hence, the points A, B, C are collinear.

12. (a) Let $\vec{OA} = 60\hat{i} + 3\hat{j}, \vec{OB} = 40\hat{i} - 8\hat{j}$

$$\text{and } \vec{OC} = a\hat{i} - 52\hat{j}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = -20\hat{i} - 11\hat{j}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = (a-60)\hat{i} - 55\hat{j}$$

Now, $\vec{AB} \propto \vec{AC}$ as A, B, C are collinear.

$$\text{i.e., } (-20\hat{i} - 11\hat{j}) = \lambda\{(a-60)\hat{i} - 55\hat{j}\}$$

On comparing, we get

$$-20 = (a-60)\lambda \text{ and } -11 = -55\lambda$$

$$\Rightarrow \lambda = \frac{1}{5}; \therefore -20 = (a-60)\left(\frac{1}{5}\right)$$

$$\Rightarrow a-60 = -100 \Rightarrow a = -100 + 60 \Rightarrow a = -40$$

13. (a) Given that, $\vec{OA} = \hat{i} + x\hat{j} + 3\hat{k}$

$$\vec{OB} = 3\hat{i} + 4\hat{j} + 7\hat{k} \text{ and } \vec{OC} = y\hat{i} - 2\hat{j} - 5\hat{k}$$

Since, A, B, C are collinear.

$$\text{Then, } \vec{AB} = \lambda\vec{BC}$$

$$\Rightarrow 2\hat{i} + (4-x)\hat{j} + 4\hat{k} = \lambda [(y-3)\hat{i} - 6\hat{j} - 12\hat{k}]$$

On comparing the coefficients of \hat{i}, \hat{j} and \hat{k} , we get

$$2 = (y-3)\lambda \quad \dots(i)$$

$$4-x = -6\lambda \quad \dots(ii)$$

$$\text{and } 4 = -12\lambda \Rightarrow \lambda = -\frac{1}{3}$$

On putting the value of λ in (i) and (ii), we get

$$y = -3 \text{ and } x = 2$$

14. (a) Given that, $\vec{OA} = 2\hat{i} + \hat{j} - \hat{k}, \vec{OB} = 3\hat{i} - 2\hat{j} + \hat{k}$

$$\text{and } \vec{OC} = \hat{i} + 4\hat{j} - 3\hat{k}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{BC} = \vec{OC} - \vec{OB} = -2\hat{i} + 6\hat{j} - 4\hat{k}$$

$$|\vec{AB}| = \sqrt{1^2 + (-3)^2 + 2^2} = \sqrt{1+9+4} = \sqrt{14}$$

$$|\vec{BC}| = \sqrt{(-2)^2 + (6)^2 + (-4)^2} = \sqrt{4+36+16} = \sqrt{56}$$

$$\vec{CA} = \vec{OA} - \vec{OC} = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$|\vec{CA}| = \sqrt{1^2 + (-3)^2 + (2)^2} = \sqrt{1+9+4} = \sqrt{14}$$

It is clear that two sides of a triangle are equal, Therefore, points A, B, C form an isosceles triangle.

15. (d) Let $\vec{P} = \hat{i} + \hat{j} - \hat{k}, \vec{Q} = 2\hat{i} + 3\hat{j}, \vec{R} = 5\hat{j} - 2\hat{k}$

$$\text{and } \vec{S} = -\hat{j} + \hat{k}$$

$$\therefore \vec{PQ} = \hat{i} + 2\hat{j} + \hat{k} \Rightarrow |\vec{PQ}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6},$$

$$\vec{QR} = -2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\Rightarrow |\overline{QR}| = \sqrt{(-2)^2 + 2^2 + (-2)^2} = 2\sqrt{3}$$

$$\text{and } \overline{RS} = -6\hat{j} + 3\hat{k}$$

$$\Rightarrow |\overline{RS}| = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$$

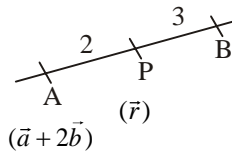
$$\text{Similarly, } \overline{SP} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\Rightarrow |\overline{SP}| = \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{9} = 3$$

Which does not satisfy the conditions of square, rectangle and parallelogram.

16. (c) Let position vector of B be \vec{r}

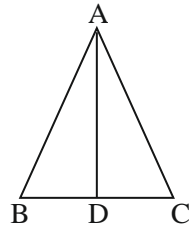
$$\therefore \vec{a} = \frac{2\vec{r} + 3(\vec{a} + 2\vec{b})}{2+3} \text{ (Section formula)}$$



$$\Rightarrow 5\vec{a} = 2\vec{r} + 3\vec{a} + 6\vec{b} \Rightarrow 2\vec{r} = 2\vec{a} - 6\vec{b} \Rightarrow \vec{r} = \vec{a} - 3\vec{b}$$

17. (c) Let D is the mid point of BC.

$$\begin{aligned} \therefore \overline{AD} &= \frac{\overline{AB} + \overline{AC}}{2} \\ &= \frac{(3\hat{i} + 4\hat{k}) + (5\hat{i} - 2\hat{j} + 4\hat{k})}{2} \\ &= 4\hat{i} - \hat{j} + 4\hat{k} \end{aligned}$$



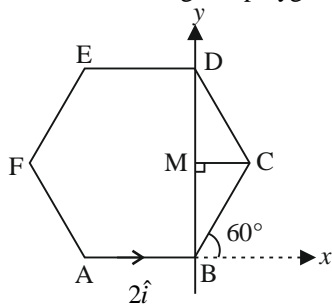
$$\begin{aligned} \therefore \text{Length of } \overline{AD} &= |\overline{AD}| \\ &= \sqrt{4^2 + (-1)^2 + 4^2} = \sqrt{16+1+16} = \sqrt{33} \end{aligned}$$

18. (b) $\overline{PQ} = \overline{OQ} - \overline{OP} = 4\hat{i} - 5\hat{j} + 11\hat{k}$

$$\begin{aligned} \therefore \frac{\overline{PQ}}{|\overline{PQ}|} &= \frac{4}{\sqrt{162}}\hat{i} - \frac{5}{\sqrt{162}}\hat{j} + \frac{11}{\sqrt{162}}\hat{k} \\ \therefore \cos \gamma &= \frac{11}{\sqrt{162}}, \end{aligned}$$

where γ is the angle of \overline{PQ} with z-axis.

19. (c) Given, ABCDEF a regular polygon



\overline{AB} is along the x-axis and BD is along the y-axis.

$$\overline{AB} = 2\hat{i} \Rightarrow AB = BC = CD = \dots = 2$$

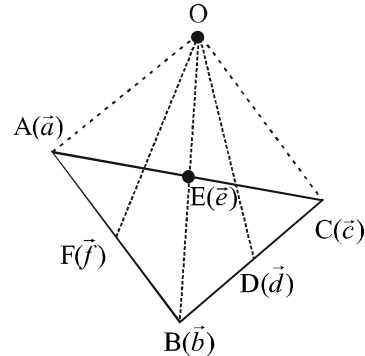
From the figure, $BM = BC \sin 60^\circ = 2 \sin 60^\circ = \sqrt{3}$

$$\therefore \overline{BD} = 2\sqrt{3}\hat{j}$$

$$\overline{BC} = BC \cos 60^\circ \hat{i} + BC \sin 60^\circ \hat{j} = \hat{i} + \sqrt{3}\hat{j}$$

$$\therefore \overline{CD} = \overline{BD} - \overline{BC} = 2\sqrt{3}\hat{j} - (\hat{i} + \sqrt{3}\hat{j}) = -\hat{i} + \sqrt{3}\hat{j}$$

20. (a) Let the point O as origin



$$\overline{AD} + \frac{2}{3}\overline{BE} + \frac{1}{3}\overline{CF}$$

$$= (\vec{d} - \vec{a}) + \frac{2}{3}(\vec{e} - \vec{b}) + \frac{1}{3}(\vec{f} - \vec{c}) = \frac{1}{2}(\vec{c} - \vec{a})$$

$$\left(\text{Putting } \vec{d} = \frac{\vec{b} + \vec{c}}{2}, \vec{e} = \frac{\vec{a} + \vec{c}}{2}, \vec{f} = \frac{\vec{a} + \vec{b}}{2} \right)$$

$$= \frac{1}{2}(\overline{AC})$$

21. (a) Let \vec{R} be the resultant of the given vectors.

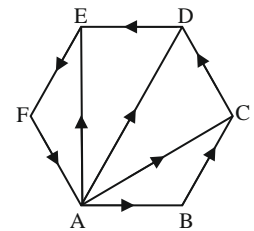
$$\begin{aligned} \text{Then, } \vec{R} &= (2\hat{i} + 3\hat{j} - \hat{k}) + (4\hat{i} - 3\hat{j} + 2\hat{k}) \\ &= 6\hat{i} + 0\hat{j} + \hat{k} \end{aligned}$$

The required unit vector parallel to \vec{R}

$$= \frac{\vec{R}}{|\vec{R}|} = \frac{6\hat{i} + 0\hat{j} + \hat{k}}{\sqrt{6^2 + 0^2 + 1^2}} = \frac{6\hat{i} + 0\hat{j} + \hat{k}}{\sqrt{37}} = \frac{1}{\sqrt{37}}(6\hat{i} + \hat{k})$$

22. (c) From above figure

$$\begin{aligned} \overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} \\ &= \overline{ED} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{CD} \\ & \quad [\because \overline{AB} = \overline{ED}, \overline{AF} = \overline{CD}] \\ &= (\overline{AC} + \overline{CD}) + (\overline{AE} + \overline{ED}) + \overline{AD} \\ &= \overline{AD} + \overline{AD} + \overline{AD} = 3\overline{AD} \end{aligned}$$



23. (d) $A = (7, -4, 7), B = (1, -6, 10), C = (-1, -3, 4)$ and $D = (5, -1, 5)$

$$\begin{aligned} AB &= \sqrt{(7-1)^2 + (-4+6)^2 + (7-10)^2} \\ &= \sqrt{36+4+9} = 7 \end{aligned}$$

$$\text{Similarly } BC = 7, CD = \sqrt{41}, DA = \sqrt{17}$$

\therefore None of the options is satisfied

24. (d) Given, $\overline{OA} = \hat{i} + 2\hat{j} - \hat{k}, \overline{OB} = \hat{i} + \hat{j} + \hat{k},$

$$\overline{OC} = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

Position vector of B w.r.t. origin at A

$$\overline{AB} = \overline{OB} - \overline{OA} = -\hat{j} + 2\hat{k}$$

Position vector of C w.r.t. origin at A

$$\overline{AC} = \overline{OC} - \overline{OA} = \hat{i} + \hat{j} + 3\hat{k}$$

25. (b) We know for an equilateral triangle, centroid is the same as orthocenter

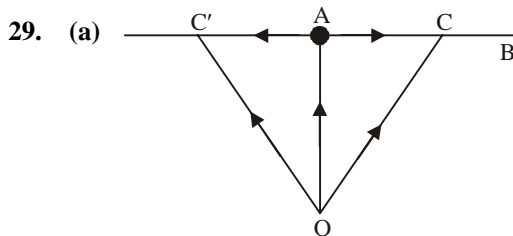
$$\therefore \frac{\vec{OA} + \vec{OB} + \vec{OC}}{3} = \vec{0}; \therefore \vec{OA} + \vec{OB} + \vec{OC} = \vec{0}$$

$$\begin{aligned} \text{Now, } \vec{AB} + 2\vec{BC} + 3\vec{CA} &= \vec{OB} - \vec{OA} + 2\vec{OC} - 2\vec{OB} + 3\vec{OA} - 3\vec{OC} \\ &= -\vec{OB} + 2\vec{OA} - \vec{OC} \\ &= -(\vec{OB} + \vec{OA} + \vec{OC}) + 3\vec{OA} = 3\vec{OA} = 3\vec{a} \end{aligned}$$

26. (c) Since, \vec{a} and \vec{b} vectors are in the same direction & \vec{b} and \vec{c} are in the opposite direction.
 $\Rightarrow \vec{a}$ and \vec{c} are in opposite directions.

27. (c) Given, \hat{j} and \hat{k} are unit vectors along Y and Z-axis, the unit vector bisecting \vec{OY} and \vec{OZ} is $\frac{\hat{j} + \hat{k}}{\sqrt{2}}$.

28. (d) We have $\vec{AC} = 2015\vec{AB}$
 $\therefore \vec{c} - \vec{a} = 2015(\vec{b} - \vec{a})$
 $\therefore \vec{c} = 2015\vec{b} - 2014\vec{a}$



We have $A(\hat{i} + \hat{j} - 2\hat{k})$ and $B(\hat{i} - 3\hat{j} + \hat{k})$
 On line AB points C and C' are at distance 1 unit from A.
 $\vec{OC} = \vec{OA} + \vec{AC}$, where \vec{AC} is unit vector in direction of \vec{AB}

$$\text{Similarly, } \vec{OC}' = \vec{OA} + \frac{\vec{AB}}{|\vec{AB}|} = \frac{5\hat{i} + \hat{j} - 7\hat{k}}{5}$$

30. (d) If a, b, c are sides of a triangle then length of median bisecting the vertex c is $\frac{1}{2}\sqrt{2b^2 + 2a^2 - c^2}$.
 \Rightarrow Required median length
 $= \frac{1}{2}\sqrt{2(26) + 2(14) - 56} = \sqrt{6}$

31. (b) Let $\vec{OA} = \vec{a}, \vec{OB} = \vec{b}$

$$\begin{aligned} \therefore \vec{OM} &= \frac{\vec{a} + \vec{b}}{2} \\ \therefore \vec{OC} &= \frac{\vec{a} + \vec{b}}{6}, \vec{OX} = \frac{2}{3}\vec{b} \end{aligned}$$

$$\text{Let } \frac{OY}{YA} = \lambda$$

$$\therefore \vec{OY} = \frac{\lambda}{\lambda + 1}\vec{a}$$

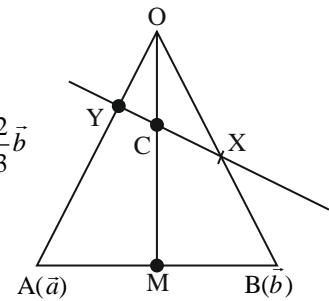
Now points Y, C and X are collinear

$$\begin{aligned} \therefore \vec{YC} &= m\vec{CX} \\ \therefore \frac{\vec{a} + \vec{b}}{6} - \frac{\lambda}{\lambda + 1}\vec{a} &= m\left(\frac{2\vec{b}}{3} - m\frac{\vec{a} + \vec{b}}{6}\right) \end{aligned}$$

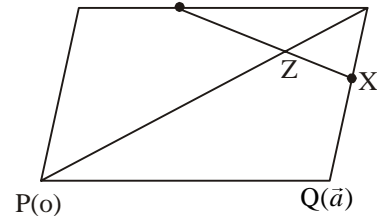
Comparing coefficients of \vec{a} and \vec{b}

$$\therefore \frac{1}{6} - \frac{\lambda}{\lambda + 1} = -\frac{m}{6} \text{ and } \frac{1}{6} = \frac{2m}{3} - \frac{m}{6}$$

$$\therefore m = \frac{1}{3} \text{ and } \lambda = \frac{2}{7}$$



32. (a) $S(\vec{b}), Y, R(\vec{a} + \vec{b})$



$$\text{P.V. of X is } \frac{4(\vec{a} + \vec{b}) + \vec{a}}{5} = \frac{5\vec{a} + 4\vec{b}}{5}$$

$$\text{P.V. of Y is } \frac{4\vec{b} + \vec{a} + \vec{b}}{5} = \frac{\vec{a} + 5\vec{b}}{5}$$

$$\text{Now, } \vec{PZ} = m\vec{PR}$$

$$\vec{PZ} = m(\vec{a} + \vec{b})$$

Let Z divides YX in the ratio $\lambda : 1$

$$\text{P.V. of Y} = \frac{\lambda\vec{OX} + \vec{OY}}{\lambda + 1}$$

$$\therefore \vec{PZ} = \frac{\lambda\left(\frac{5\vec{a} + 4\vec{b}}{5}\right) + \frac{\vec{a} + 5\vec{b}}{5}}{\lambda + 1} = m(\vec{a} + \vec{b})$$

Comparing coefficients of \vec{a} and \vec{b}

$$m = \frac{5\lambda + 1}{5(\lambda + 1)} \text{ and } m = \frac{4\lambda + 5}{5(\lambda + 1)}$$

$$\lambda = 4$$

$$\therefore \vec{PZ} = \frac{4\left(\frac{5\vec{a} + 4\vec{b}}{5}\right) + \frac{\vec{a} + 5\vec{b}}{5}}{4 + 1} = \frac{21(\vec{a} + \vec{b})}{25} = \frac{21}{25}\vec{PR}$$

33. (b) It is given that the point P, Q and R are collinear

$$\Rightarrow \vec{PQ} = \lambda\vec{QR} \Rightarrow 15t(4\hat{j} - 3\hat{i}) = \lambda[(1-t)(\hat{i} + \hat{j}) - 60t\hat{j}]$$

$$= \lambda[(1-t)\hat{i} + (1-6t)\hat{j}]$$

$$\Rightarrow \frac{45t}{t-1} = \frac{60t}{1-6t} \Rightarrow 3 - 183t = 4t - 4 \Rightarrow t = \frac{7}{187}$$

34. (c) As per question

$$\vec{a} + 3\vec{b} = \lambda\vec{c} \quad \dots(i)$$

$$\vec{b} + 2\vec{c} = \mu\vec{a} \quad \dots(ii)$$

On solving equations (i) and (ii)

$$(1+3\mu)\vec{a} - (\lambda+6)\vec{c} = 0$$

As \vec{a} and \vec{c} are non collinear,

$$\therefore 1+3\mu = 0 \text{ and } \lambda+6 = 0$$

$$\text{From (i), } \vec{a} + 3\vec{b} + 6\vec{c} = \vec{0}$$

35. (a) Let $\vec{\alpha}$ and $\vec{\beta}$ are collinear for same k i.e., $\vec{\alpha} = k\vec{\beta}$

$$\Rightarrow (\lambda - 2)\vec{a} + \vec{b} = k(4\lambda - 2)\vec{a} + 3\vec{b}$$

$$\Rightarrow (\lambda - 2)\vec{a} + \vec{b} = k(4\lambda - 2)\vec{a} + 3k\vec{b}$$

$$\Rightarrow (\lambda - 2 - k(4\lambda - 2))\vec{a} + \vec{b}(1 - 3k) = 0$$

But \vec{a} and \vec{b} are non-collinear, then

$$\lambda - 2 - k(4\lambda - 2) = 0, 1 - 3k = 0$$

$$\Rightarrow k = \frac{1}{3} \text{ and } \lambda - 2 - \frac{1}{3}(4\lambda - 2) = 0$$

$$\Rightarrow 3\lambda - 6 - 4\lambda + 2 = 0 \Rightarrow \lambda = -4$$

36. (a) Since, \vec{a}_1 and \vec{a}_2 are collinear, therefore $\vec{a}_2 = \lambda\vec{a}_1$

$$\Rightarrow \hat{i} + y\hat{j} + z\hat{k} = \lambda(x\hat{i} - \hat{j} + \hat{k}) \Rightarrow 1 = \lambda x, y = -\lambda, z = \lambda$$

$$\text{Now, } x\hat{i} + y\hat{j} + z\hat{k} = \frac{1}{\lambda}\hat{i} - \lambda\hat{j} + \lambda\hat{k}$$

$$\text{Unit vector parallel to } x\hat{i} + y\hat{j} + z\hat{k} = \frac{\frac{1}{\lambda}\hat{i} - \lambda\hat{j} + \lambda\hat{k}}{\sqrt{\frac{1}{\lambda^2} + \lambda^2 + \lambda^2}}$$

$$= \frac{\hat{i} - \lambda^2\hat{j} + \lambda^2\hat{k}}{\sqrt{1 + 2\lambda^4}}$$

$$\text{Let } \lambda = 1, \text{ possible unit vector} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$$

37. (a) Vectors along to sides are $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and

$$\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$$

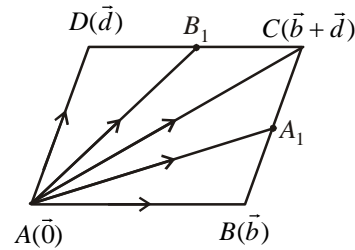
Clearly the vector along the longer diagonal is

$$\vec{a} + \vec{b} = 3\hat{i} + 6\hat{j} + 2\hat{k}$$

Hence, length of the longer diagonal is

$$|\vec{a} + \vec{b}| = |3\hat{i} + 6\hat{j} + 2\hat{k}| = 7.$$

38. (b) Let position vectors of A, B, D be $\vec{0}, \vec{b}$ and \vec{d} respectively.



$$\therefore \vec{AC} = \vec{AB} + \vec{BC}$$

$$= \vec{AB} + \vec{AD} = \vec{b} + \vec{d} \quad \dots(i)$$

$$\text{Now, } \vec{AA}_1 = \frac{\vec{AB} + \vec{AC}}{2}$$

[Since A_1 is the mid point of side BC]

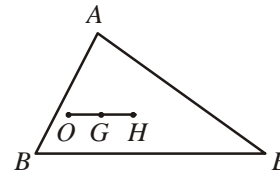
$$\text{So, } \vec{AA}_1 = \frac{\vec{b} + \vec{b} + \vec{d}}{2} = \vec{b} + \frac{\vec{d}}{2} \quad \text{[Using (i)]}$$

$$\text{Similarly, } \vec{AB}_1 = \frac{\vec{AD} + \vec{AC}}{2} = \vec{d} + \frac{\vec{b}}{2}$$

$$\therefore \vec{AA}_1 + \vec{AB}_1 = \frac{3}{2}(\vec{b} + \vec{d}) = \frac{3}{2}\vec{AC}$$

$$\therefore \lambda = \frac{3}{2} = 1.5$$

39. (a)



Here, O is circum centre = o , G is centroid = g , H is orthocentre = p

$$\text{Since, } \frac{OG}{GH} = \frac{1}{2} \Rightarrow \frac{GH + OG}{OG} = 3$$

$$\Rightarrow \vec{p} = 3\vec{g} \Rightarrow k = 3.$$

40. (a) Let P be origin as

$$|\vec{a} + \vec{b} + \vec{c}|^2 + |\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$$

$$= 3(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)$$

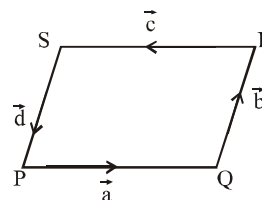
$$\text{So, } 9(GP)^2 + (AB)^2 + (BC)^2 + (CA)^2$$

$$= 3((PA)^2 + (PB)^2 + (PC)^2)$$

$$\Rightarrow 9 \times 2^2 + (a^2 + b^2 + c^2) = 3(1^2 + 3^2 + 4^2)$$

$$\Rightarrow a^2 + b^2 + c^2 = 42.$$

41. (c)



Let PQRS be the quadrilateral and

$$\overline{PQ} = \vec{a}, \overline{QR} = \vec{b}, \overline{RS} = \vec{c} \text{ and } \overline{SP} = \vec{d}.$$

Let PQRS be a parallelogram.

$$\therefore PQ = SR \text{ and } PQ \parallel SR$$

$$\Rightarrow \overline{PQ} = \overline{SR} \Rightarrow \vec{a} = -\vec{c} \Rightarrow \vec{a} + \vec{c} = \vec{0}.$$

Conversely, let $\vec{a} + \vec{c} = \vec{0}$.

$$\Rightarrow \overline{PQ} + \overline{RS} = \vec{0} \Rightarrow \overline{PQ} = -\overline{RS} \Rightarrow \overline{PQ} = \overline{SR}$$

$$\therefore PQ = SR \text{ and } PQ \parallel SR$$

\therefore PQRS is a parallelogram.

Further, Let $\vec{a} + \vec{c} = \vec{0}$ We have

$$\vec{a} + \vec{b} + \vec{c} + \vec{d} = (\overline{PQ} + \overline{QR}) + (\overline{RS} + \overline{SP}) = \overline{PR} + \overline{RP} = \vec{0}$$

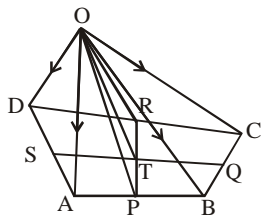
$$\vec{a} + \vec{b} + \vec{c} + \vec{d} = \vec{0}$$

$$\Rightarrow (\vec{a} + \vec{c}) + (\vec{b} + \vec{d}) = \vec{0} \Rightarrow \vec{0} + (\vec{b} + \vec{d}) = \vec{0}$$

$$\Rightarrow (\vec{b} + \vec{d}) = \vec{0}$$

\therefore The quadrilateral PQRS is a parallelogram if and only if $\vec{a} + \vec{c} = \vec{0}$, which implies that $\vec{b} + \vec{d} = \vec{0}$.

42. (a)



Let P, Q, R, S be the mid-points of the sides AB, BC, CD and DA respectively.

Let T be the point of intersection of the lines PR and SQ.

$$\text{Let } \overline{OA} + \overline{OB} + \overline{OC} + \overline{OD} = \vec{0}$$

$$\Rightarrow (\overline{OP} + \overline{PA}) + (\overline{OP} + \overline{PB}) + (\overline{OR} + \overline{RC}) + (\overline{OR} + \overline{RD}) = \vec{0}$$

$$\Rightarrow 2\overline{OP} + (\overline{PA} + \overline{PB}) + 2\overline{OR} + (\overline{RC} + \overline{RD}) = \vec{0}$$

$$\Rightarrow 2(\overline{OP} + \overline{OR}) = \vec{0} \Rightarrow \overline{OP} + \overline{OR} = \vec{0}$$

$$\Rightarrow (\overline{OT} + \overline{TP}) + (\overline{OT} + \overline{TR}) = \vec{0}$$

$$\Rightarrow 2\overline{OT} + (\overline{TP} + \overline{TR}) = \vec{0}$$

$$\Rightarrow 2\overline{OT} + \vec{0} = \vec{0} \Rightarrow \overline{OT} = \vec{0} \quad (\because PQRS \text{ is a } \parallel\text{gm})$$

\Rightarrow The point O coincides with T.

[Note that the line joining the mid-points of diagonals of ABCD is also the same point and is the centroid].

43. (d) Let the given position vectors represent points P, Q and R respectively. Then

$$\overline{PQ} = \text{P.V. of } Q - \text{P.V. of } P$$

$$= (-2\vec{a} + 3\vec{b} + 2\vec{c}) - (\vec{a} - 2\vec{b} + 3\vec{c}) = -3\vec{a} + 5\vec{b} - \vec{c}$$

$$\overline{QR} = \text{P.V. of } R - \text{P.V. of } Q$$

$$= (-8\vec{a} + 13\vec{b}) - (-2\vec{a} + 3\vec{b} + 2\vec{c}) = -6\vec{a} + 10\vec{b} - 2\vec{c}$$

$$= 2(-3\vec{a} + 5\vec{b} - \vec{c}) = 2\overline{PQ}$$

$\therefore \overline{QR}$ and \overline{PQ} are collinear vectors.

\therefore P, Q and R are collinear points.

44. (d) Given, $\overline{OA} = 7\hat{j} + 10\hat{k}$, $\overline{OB} = -\hat{i} + 6\hat{j} + 6\hat{k}$,

$$\overline{OC} = -4\hat{i} + 9\hat{j} + 6\hat{k} \Rightarrow \overline{AB} = \overline{OB} - \overline{OA} = -\hat{i} - \hat{j} - 4\hat{k}$$

$$\overline{BC} = \overline{OC} - \overline{OB} = -3\hat{i} + 3\hat{j};$$

$$\overline{CA} = \overline{OA} - \overline{OC} = 4\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\Rightarrow |\overline{AB}| = \sqrt{1^2 + 1^2 + 4^2} = \sqrt{18} = 3\sqrt{2};$$

$$\Rightarrow |\overline{BC}| = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2};$$

$$\Rightarrow |\overline{CA}| = \sqrt{4^2 + 2^2 + 4^2} = \sqrt{36} = 6$$

$3\sqrt{2}, 3\sqrt{2}$ & 6 are sides of a right angled Δ .

$$\therefore (3\sqrt{2})^2 + (3\sqrt{2})^2 = 36 = 6^2$$

Hence the ΔABC is a right-angled and isosceles also.

45. (b) P.V. of A = $2\hat{i} + \hat{j} + 3\hat{k}$; P.V. of B = $4\hat{i} + \hat{j} + 3\hat{k}$;

$$\text{P.V. of } C = 2\hat{j} + \hat{k}.$$

$$\therefore \overline{BA} = \text{P.V. of } A - \text{P.V. of } B = -2\hat{i}$$

$$\text{Similarly, } \overline{AC} = -2\hat{i} + \hat{j} - 2\hat{k}$$

$$\therefore \cos(\angle BAC) = \frac{\overline{BA} \cdot \overline{AC}}{|\overline{BA}| \cdot |\overline{AC}|} = \frac{4}{\sqrt{4} \cdot \sqrt{9}} = \frac{4}{2 \cdot 3} = \frac{2}{3}$$

$$\Rightarrow \angle BAC = \cos^{-1}\left(\frac{2}{3}\right).$$

46. (c) We have $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 5$. It is given that

$$\vec{a} \perp (\vec{b} + \vec{c}), \vec{b} \perp (\vec{c} + \vec{a}) \text{ and } \vec{c} \perp (\vec{a} + \vec{b})$$

$$\Rightarrow \vec{a} \cdot (\vec{b} + \vec{c}) = 0, \vec{b} \cdot (\vec{c} + \vec{a}) = 0, \vec{c} \cdot (\vec{a} + \vec{b}) = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0, \vec{b} \cdot \vec{c} + \vec{b} \cdot \vec{a} = 0, \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} = 0$$

$$\text{Adding all these, we get, } 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0$$

$$\text{Now, } |\vec{a} + \vec{b} + \vec{c}|^2$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$= 3^2 + 4^2 + 5^2 + 0 = 50$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{50} = 5\sqrt{2}.$$

47. (a) Given, $|\vec{a}| = |\vec{b}|$
 $\therefore |\vec{a} - \vec{b}| = \sqrt{\vec{a}^2 + \vec{a}^2 - 2\vec{a}^2 \cos \theta}$
 $= \sqrt{2\vec{a}^2(1 - \cos \theta)} = \sqrt{2\vec{a}^2 \cdot 2\sin^2 \frac{\theta}{2}} = 2a \sin \frac{\theta}{2}$
 $\Rightarrow |\vec{a} - \vec{b}| = 2a \sin \frac{\theta}{2} \Rightarrow a \sin \frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{2}$

48. (d) Given $|\vec{a} + \vec{b}| = 6 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 36$... (i)
 Similarly, $|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} = 64$... (ii)
 and $|\vec{c}|^2 + |\vec{a}|^2 + 2\vec{c} \cdot \vec{a} = 100$... (iii)
 Adding (i), (ii) and (iii), we get
 $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 100$
 $\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = 100$... (iv)
 $(\because \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0)$

Now,
 $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$
 $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$
 $\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 100$ [Using (iv)]
 $\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = 10$

49. (a) Given $|\vec{a} - \vec{b}| = |\vec{a}| = |\vec{b}| = 1$
 $\Rightarrow |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$
 $\Rightarrow 1 = 1 + 1 - 2|\vec{a}||\vec{b}| \cos \theta$
 (where θ is angle between \vec{a} and \vec{b})
 $\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

50. (a) Given two vectors also lie in xy -plane. Let a vector is in xy -plane, $\vec{a} = x\hat{i} + y\hat{j}$
 $\because \vec{a} \perp (\hat{i} - \hat{j}) \Rightarrow \vec{a} \cdot (\hat{i} - \hat{j}) = 0$
 $\Rightarrow (x\hat{i} + y\hat{j}) \cdot (\hat{i} - \hat{j}) = 0 \Rightarrow x - y = 0 \Rightarrow x = y$
 $\therefore \vec{a} = x\hat{i} + x\hat{j}$ and $|\vec{a}| = \sqrt{x^2 + x^2} = x\sqrt{2}$
 \therefore Required unit vector $= \frac{\vec{a}}{|\vec{a}|} = \frac{x(\hat{i} + \hat{j})}{x\sqrt{2}} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$

51. (b) Given, angle between \vec{a} and $\vec{b} = \frac{\pi}{2}$
 $\Rightarrow \vec{a} \cdot \vec{b} = 0$
 $\therefore |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 1 + 1 + 0 = 2$
 $\Rightarrow |\vec{a} + \vec{b}| = \sqrt{2}$

52. (c) Given that $|\vec{a}| = 2\sqrt{2}, |\vec{b}| = 3$
 One diagonal is $5\vec{a} + 2\vec{b} + \vec{a} - 3\vec{b} = 6\vec{a} - \vec{b}$
 Its length $= |6\vec{a} - \vec{b}|$
 $= \sqrt{36\vec{a}^2 + \vec{b}^2 - 2 \times 6|\vec{a}||\vec{b}| \cos 45^\circ}$

$$= \sqrt{36 \times 8 + 9 - 12 \times 2\sqrt{2} \times 3 \times \frac{1}{\sqrt{2}}} = 15$$

\therefore Other diagonal is $5\vec{a} + 2\vec{b} - \vec{a} + 3\vec{b} = 4\vec{a} + 5\vec{b}$
 Its length $= \sqrt{16 \times 8 + 25 \times 9 + 40 \times 6} = \sqrt{593}$

53. (b) We can write, $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$
 $= 2[|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2] - 2\Sigma(\vec{a} \cdot \vec{b})$
 $= 2(1 + 1 + 1) - 2\Sigma(\vec{a} \cdot \vec{b})$... (i)
 $= 6 - 2\Sigma(\vec{a} \cdot \vec{b})$

But $(\vec{a} + \vec{b} + \vec{c})^2 \geq 0$
 $\Rightarrow (1 + 1 + 1) + 2\Sigma(\vec{a} \cdot \vec{b}) \geq 0$
 $\therefore 3 \geq -2\Sigma(\vec{a} \cdot \vec{b})$... (ii)
 From (i) and (ii), we get

$$|\vec{a} - \vec{b}| + |\vec{b} - \vec{c}| + |\vec{c} - \vec{a}| \leq 6 + 3 = 9$$

54. (b) $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = \vec{a} \cdot \vec{a} - \vec{b} \cdot \vec{b}$
 $= |\vec{a}|^2 - |\vec{b}|^2 = \{1^2 + 2^2 + (-3)^2\} - \{3^2 + (-1)^2 + 2^2\} = 0$
 $\Rightarrow (\vec{a} + \vec{b}) \perp (\vec{a} - \vec{b})$

55. (a) Let the required vector be $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$
 $\vec{v} \cdot (\hat{i} + \hat{j} - 3\hat{k}) = 0$
 $\Rightarrow x + y - 3z = 0$... (i)
 $\vec{v} \cdot (\hat{i} + 3\hat{j} - 2\hat{k}) = 5$
 $\Rightarrow x + 3y - 2z = 5$... (ii)
 and $\vec{v} \cdot (2\hat{i} + \hat{j} + 4\hat{k}) = 8$
 $\Rightarrow 2x + y + 4z = 8$... (iii)

On solving (i), (ii), and (iii), we get
 $x = 1, y = 2, z = 1$
 Hence, the required vector is

$$\vec{v} = \hat{i} + 2\hat{j} + \hat{k}$$

56. (a) We are given that $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors
 $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$... (i)
 Let $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$ (say) ... (ii)
 $\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}\lambda$... (iii)

Let θ be the angle between \vec{a} and $\vec{a} + \vec{b} + \vec{c}$, then

$$\cos \theta = \frac{\vec{a} \cdot (\vec{a} + \vec{b} + \vec{c})}{|\vec{a}||\vec{a} + \vec{b} + \vec{c}|} = \frac{\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}}{\lambda(\sqrt{3}\lambda)}$$

[using (i), (ii) & (iii)]

$$= \frac{|\vec{a}|^2 + 0 + 0}{\sqrt{3}\lambda^2} = \frac{\lambda^2}{\sqrt{3}\lambda^2} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

57. (c) Given, $|\vec{u}|=1, |\vec{v}|=2, |\vec{w}|=3$

The projection of \vec{v} along $\vec{u} = \frac{\vec{v}\cdot\vec{u}}{|\vec{u}|}$ and the projection of

$$\vec{w} \text{ along } \vec{u} = \frac{\vec{w}\cdot\vec{u}}{|\vec{u}|}$$

$$\text{Given, } \frac{\vec{v}\cdot\vec{u}}{|\vec{u}|} = \frac{\vec{w}\cdot\vec{u}}{|\vec{u}|} = \vec{v}\cdot\vec{u} = \vec{w}\cdot\vec{u}$$

and \vec{v}, \vec{w} are perpendicular to each other.

$$\therefore \vec{v}\cdot\vec{w} = 0$$

$$\text{Now, } |\vec{u} - \vec{v} + \vec{w}|^2$$

$$= |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 - 2\vec{u}\cdot\vec{v} + 2\vec{u}\cdot\vec{w} - 2\vec{v}\cdot\vec{w}$$

$$= 1 + 4 + 9 - 2\vec{u}\cdot\vec{v} + 2\vec{u}\cdot\vec{w} \Rightarrow |\vec{u} - \vec{v} + \vec{w}|^2 = 14$$

$$\Rightarrow |\vec{u} - \vec{v} + \vec{w}| = \sqrt{14}$$

58. (b) $\vec{a}\cdot\vec{b} = 4(0) + 6(3) + 0(4) = 18$

$$\text{and } |\vec{b}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

\therefore Vector form of component of \vec{a} along \vec{b}

$$= \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2} \right) \vec{b} = \frac{18}{25} (3\hat{j} + 4\hat{k})$$

59. (a) Since, $\vec{a}\cdot\vec{\beta} = \frac{b}{a} + \frac{4a}{b} + 1 \geq 5$; So, $\left(\frac{10}{5 + \vec{a}\cdot\vec{\beta}} \right)_{\max} = 1$

60. (b) Given, $|\vec{p} + \vec{q} + \vec{r}| = 1$

On squaring, we get

$$\Rightarrow |\vec{p}|^2 + |\vec{q}|^2 + |\vec{r}|^2 + 2\vec{p}\cdot\vec{q} + 2\vec{r}\cdot\vec{p} + 2\vec{q}\cdot\vec{r} = 1$$

$$1 + 1 + 1 + 0 + 0 + 2\vec{q}\cdot\vec{r} = 1$$

$$\Rightarrow \vec{q}\cdot\vec{r} = -1 \Rightarrow \cos \theta = -1 \Rightarrow \theta = \pi$$

61. (a) $|\overline{OP}| = |\hat{a} \cos t + \hat{b} \sin t|$

$$= (\cos^2 t + \sin^2 t + 2 \cos t \sin t \hat{a}\cdot\hat{b})^{1/2}$$

$$= (1 + 2 \cos t \sin t \hat{a}\cdot\hat{b})^{1/2}$$

$$= (1 + \sin 2t \hat{a}\cdot\hat{b})^{1/2}$$

$$\therefore |\overline{OP}|_{\max} = (1 + \hat{a}\cdot\hat{b})^{1/2} \text{ when } t = \pi/4$$

$$\hat{u} = \frac{\hat{a} + \hat{b}}{\sqrt{2}|\hat{a} + \hat{b}|} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$$

62. (b) Given, $\vec{a} = \cos \theta \hat{i} + \sin \theta \hat{j}$

$$\vec{b} = -\sin \theta \hat{i} + \cos \theta \hat{j} \text{ and } \vec{c} = \hat{k}$$

$$|\vec{a}| = |\vec{b}| = |\vec{c}| \text{ and } \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a}$$

$$\therefore \text{ Required angle is } \cos^{-1} \left(\frac{1}{\sqrt{3}} \right).$$

63. (a) Let $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{u} = \hat{i} - 2\hat{j} + 3\hat{k}; \vec{v} = 2\hat{i} + \hat{j} + 4\hat{k}; \vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$$

$$(\vec{u}\cdot\vec{R} - 15)\hat{i} + (\vec{v}\cdot\vec{R} - 30)\hat{j} + (\vec{w}\cdot\vec{R} - 25)\hat{k} = \vec{0} \quad (\text{Given})$$

$$\text{So } \vec{u}\cdot\vec{R} = 15 \Rightarrow x - 2y + 3z = 15 \quad \dots(i)$$

$$\vec{v}\cdot\vec{R} = 30 \Rightarrow 2x + y + 4z = 30 \quad \dots(ii)$$

$$\vec{w}\cdot\vec{R} = 25 \Rightarrow x + 3y + 3z = 25 \quad \dots(iii)$$

Solving, we get

$$x = 4$$

$$y = 2$$

$$z = 5$$

$$|\vec{R}|^2 = 45 \Rightarrow |\vec{R}| = \sqrt{45} \text{ \& } 6 < \sqrt{45} < 7$$

$$[|\vec{R}|] = 6$$

64. (b) Let $\vec{c} = \alpha\hat{i} + \beta\hat{j}$

Given that $\vec{b} \perp \vec{c}$, $\therefore \vec{b}\cdot\vec{c} = 0$

$$\Rightarrow (4\hat{i} + 3\hat{j})\cdot(\alpha\hat{i} + \beta\hat{j}) = 0$$

$$\text{or } 4\alpha + 3\beta = 0$$

$$\text{or } \frac{\alpha}{3} = \frac{\beta}{-4} = \lambda$$

$$\text{or } \alpha = 3\lambda, \beta = -4\lambda$$

... (i)

Now, let $\vec{a} = x\hat{i} + y\hat{j}$ be the required vector.

Projection of \vec{a} along \vec{b} gives

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{4x + 3y}{\sqrt{4^2 + 3^2}} = 1$$

$$\text{or } 4x + 3y = 5$$

... (ii)

Also projection of \vec{a} along \vec{c} is 2

$$\Rightarrow \frac{\alpha x + \beta y}{\sqrt{\alpha^2 + \beta^2}} = 2$$

$$\text{or } 3\lambda x - 4\lambda y = 10\lambda$$

$$\text{or } 3x - 4y = 10$$

... (iii)

Solving (ii) and (iii), we get

$$x = 2, y = -1$$

Therefore, the required vector is $2\hat{i} - \hat{j}$

$$\therefore \text{ required dot product} = 8 - 3 = 5$$

65. (a) Projection of \vec{V} on \vec{r} is $\frac{\vec{V}\cdot\vec{r}}{|\vec{r}|} = \frac{74}{\sqrt{6}}$

$$\Rightarrow \frac{(\vec{p}\cdot\vec{r}) + \lambda(\vec{q}\cdot\vec{r})}{|\vec{r}|} = \frac{74}{\sqrt{6}} \Rightarrow \frac{-1 + \lambda(5)}{\sqrt{6}} = \frac{74}{\sqrt{6}}$$

$$\Rightarrow \lambda = 15$$

66. (d) We have, $p\vec{x} + q\vec{y} = \vec{a}$

$$|p\vec{x} - q\vec{y}|^2 = |p\vec{x} + q\vec{y}|^2 - 4pq\vec{x}\cdot\vec{y} \geq 0 \Rightarrow \vec{x}\cdot\vec{y} \leq \frac{|\vec{a}|^2}{4pq}$$

$$\text{but } \vec{x}\cdot\vec{y} \geq \frac{|\vec{a}|^2}{4pq} \Rightarrow |\vec{a}|^2 = 4pq\vec{x}\cdot\vec{y} \Rightarrow |p\vec{x} - q\vec{y}| = 0$$

$$\Rightarrow p\vec{x} - q\vec{y} = 0 \Rightarrow \vec{x} = \frac{\vec{a}}{2p}, \vec{y} = \frac{\vec{a}}{2q} \Rightarrow \frac{|\vec{x}|}{|\vec{y}|} = \frac{q}{p}$$

67. (c) $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$,

$$\vec{a}\cdot\vec{b} = 0, \vec{c}\cdot\vec{a} = \cos \alpha, \vec{c}\cdot\vec{b} = \cos \beta$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c})\cdot(\vec{a} + \vec{b} + \vec{c})$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$$

$$= 3 + 2(0 + \cos \beta + \cos \alpha) \Rightarrow 1 = 3 + 2(\cos \beta + \cos \alpha)$$

$$\therefore \cos \alpha + \cos \beta = -1.$$

68. (b) Let $\vec{d} = \vec{b} + \lambda \vec{c}$

$$\therefore \vec{d} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$$

$$= (1 + \lambda)\hat{i} + (2 + \lambda)\hat{j} - (1 + 2\lambda)\hat{k}$$

If θ be the angle between \vec{d} and \vec{a} , then projection of \vec{d} or $(\vec{b} + \lambda \vec{c})$ on \vec{a}

$$= |\vec{d}| \cos \theta = \left| \vec{d} \cdot \frac{\vec{a}}{|\vec{a}|} \right| = \frac{\vec{d} \cdot \vec{a}}{|\vec{a}|}$$

$$= \frac{2(\lambda + 1) - (\lambda + 2) - (2\lambda + 1)}{\sqrt{4 + 1 + 1}} = \frac{-\lambda - 1}{\sqrt{6}}$$

But projection of \vec{d} on $\vec{a} = \sqrt{\frac{2}{3}}$

$$\therefore -\frac{\lambda + 1}{\sqrt{6}} = \sqrt{\frac{2}{3}} \Rightarrow \frac{\lambda^2 + 2\lambda + 1}{6} = \frac{2}{3}$$

$$\Rightarrow \lambda^2 + 2\lambda - 3 = 0 \Rightarrow \lambda^2 + 3\lambda - \lambda - 3 = 0$$

$$\Rightarrow \lambda(\lambda + 3) - 1(\lambda + 3) = 0, \Rightarrow \lambda = 1, -3$$

when $\lambda = 1$, then $\vec{b} + \lambda \vec{c} = 2\hat{i} + 3\hat{j} - 3\hat{k}$

when $\lambda = -3$, then $\vec{b} + \lambda \vec{c} = -2\hat{i} - \hat{j} + 5\hat{k}$

69. (b) As $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$

$$= 3(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2) - |\vec{a} + \vec{b} + \vec{c}|^2$$

$$\Rightarrow 3 \times 3 - |\vec{a} + \vec{b} + \vec{c}|^2 = 9$$

or $|\vec{a} + \vec{b} + \vec{c}|^2 = 0$

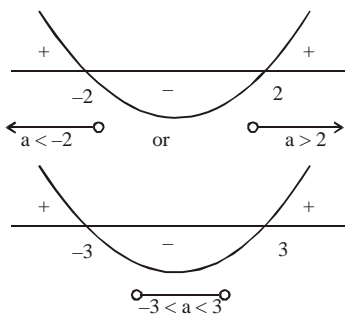
or $\vec{a} + \vec{b} + \vec{c} = 0$

or $\vec{b} + \vec{c} = -\vec{a}$

$$\Rightarrow |2\vec{a} + 5(\vec{b} + \vec{c})| = |-3\vec{a}| = 3|\vec{a}| = 3$$

70. (b) Required value = $\frac{\vec{b} \cdot \vec{a}}{|\vec{b}| |\vec{a}|} = \frac{|\vec{a}|}{|\vec{b}|} = \frac{7}{3}$.

71. (c) Given, $\vec{r} = (a^2 - 4)\hat{i} + 2\hat{j} - (a^2 - 9)\hat{k}$... (i)



For θ to be acute, $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$ is +ve,

i.e., $\cos \theta > 0 \Rightarrow \vec{a} \cdot \vec{b} > 0$.

\therefore For vector \vec{r} to be inclined at acute angles with the coordinate axes, we must have

$$\vec{r} \cdot \hat{i} > 0, \vec{r} \cdot \hat{j} > 0 \text{ and } \vec{r} \cdot \hat{k} > 0 \text{ since } \vec{r} \cdot \hat{j} = 2 > 0$$

$$\Rightarrow \vec{r} \cdot \hat{i} > 0, \text{ and } \vec{r} \cdot \hat{k} > 0$$

Now, $\vec{r} \cdot \hat{i} = [(a^2 - 4)\hat{i} + 2\hat{j} - (a^2 - 9)\hat{k}] \cdot \hat{i} = a^2 - 4$

$$\vec{r} \cdot \hat{k} = [(a^2 - 4)\hat{i} + 2\hat{j} - (a^2 - 9)\hat{k}] \cdot \hat{k} = -(a^2 - 9)$$

$$\therefore \text{From (i), } (a^2 - 4) > 0 \text{ and } -(a^2 - 9) > 0$$

i.e., $a^2 - 9 < 0$

$$\Rightarrow (a - 2)(a + 2) > 0 \text{ and } (a + 3)(a - 3) < 0$$

$$\Rightarrow a < -2 \text{ or } a > 2 \text{ and } -3 < a < 3$$

$$\Rightarrow a \in (-3, -2) \cup (2, 3).$$

72. (b) Given $\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a}$ is perpendicular to \vec{b} .

$$\vec{a} \cdot \vec{c} = 0 \Rightarrow \vec{a} \text{ is perpendicular to } \vec{c}.$$

$\therefore \vec{a}$ is perpendicular to the plane of \vec{b} and \vec{c} .

Also \vec{a} is a unit vector.

Therefore, $\vec{a} = \pm \frac{\vec{b} \times \vec{c}}{|\vec{b} \times \vec{c}|}$... (i)

But $|\vec{b} \times \vec{c}| = |\vec{b}| |\vec{c}| \sin \frac{\pi}{6} = 1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2}$.

\therefore From (i) we have $\vec{a} = \pm 2(\vec{b} \times \vec{c})$. $\therefore n = \pm 2$.

73. (d) Let O be the origin. Then $\vec{OA} = 3\hat{i} - \hat{j} + 2\hat{k}$,

$$\vec{OB} = \hat{i} - \hat{j} - 3\hat{k} \text{ and } \vec{OC} = 4\hat{i} - 3\hat{j} + \hat{k}$$

$$\therefore \vec{AB} = \vec{OB} - \vec{OA} = (\hat{i} - \hat{j} - 3\hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = -2\hat{i} + 0\hat{j} - 5\hat{k}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = (4\hat{i} - 3\hat{j} + \hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = \hat{i} - 2\hat{j} - \hat{k}$$

Now, $\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix}$

$$= (0 - 10)\hat{i} - (2 + 5)\hat{j} + (4 - 0)\hat{k} = -10\hat{i} - 7\hat{j} + 4\hat{k}$$

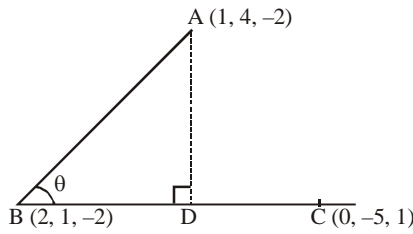
$$\Rightarrow |\vec{AB} \times \vec{AC}| = \sqrt{(-10)^2 + (-7)^2 + (4)^2}$$

$$= \sqrt{100 + 49 + 16} = \sqrt{165}$$

A unit vector perpendicular to the plane of ΔABC is perpendicular to both \vec{AB} and \vec{AC} . Hence, a unit vector perpendicular, to the plane of ΔABC

$$= \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} = \frac{-10\hat{i} - 7\hat{j} + 4\hat{k}}{\sqrt{165}} = -\frac{1}{\sqrt{165}}(10\hat{i} + 7\hat{j} - 4\hat{k}).$$

74. (c)



$$AD = AB \sin \theta = AB \cdot \frac{|\overline{BC} \times \overline{BA}|}{|\overline{BC}| \cdot |\overline{BA}|} = \frac{|\overline{BC} \times \overline{BA}|}{|\overline{BC}|}$$

[∵ $|\overline{BA}| = BA = AB$]

$$\text{Now } \overline{BC} = -2\hat{i} - 6\hat{j} + 3\hat{k} \text{ and } \overline{BA} = -\hat{i} + 3\hat{j}$$

$$\therefore \overline{BC} \times \overline{BA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -6 & 3 \\ -1 & 3 & 0 \end{vmatrix} = -9\hat{i} - 3\hat{j} - 12\hat{k}$$

$$|\overline{BC} \times \overline{BA}| = \sqrt{9^2 + 3^2 + (12)^2} = 3\sqrt{26} \text{ and}$$

$$|\overline{BC}| = \sqrt{4 + 36 + 9} = 7$$

$$\therefore AD = \frac{3\sqrt{26}}{7}$$

75. (c) $\vec{p} + \vec{q} = 3\hat{i} + 5\hat{j} + 2\hat{k}$ and $\vec{p} - \vec{q} = \hat{i} + \hat{j}$
 \vec{r} is lar to $(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$

$$\text{Now, } (\vec{p} + \vec{q}) \times (\vec{p} - \vec{q}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 2 \\ 1 & 1 & 0 \end{vmatrix} = -2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\therefore \vec{r} = \lambda(-2\hat{i} + 2\hat{j} - 2\hat{k}) \text{ and } |\vec{r}| = \pm 2\sqrt{3}\lambda = \sqrt{3} \Rightarrow \lambda = \pm \frac{1}{2}$$

$$\therefore |\alpha| + |\beta| + |\gamma| = 1 + 1 + 1 = 3$$

76. (d) $|\vec{a}| = 1, |\vec{b}| = 4, \vec{a} \cdot \vec{b} = 2; \vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$

$$\text{or } \vec{c} + 3\vec{b} = 2\vec{a} \times \vec{b} \Rightarrow |\vec{a}| \cdot |\vec{b}| \cos \theta = 2$$

$$\text{or } \cos \theta = \frac{2}{|\vec{a}| \cdot |\vec{b}|} = \frac{2}{4} = \frac{1}{2} \text{ or } \theta = \frac{\pi}{3}$$

$$\Rightarrow |\vec{c} + 3\vec{b}|^2 = |2\vec{a} \times \vec{b}|^2$$

$$\text{or } |\vec{c}|^2 + 9|\vec{b}|^2 + 2\vec{c} \cdot 3\vec{b} = 4|\vec{a}|^2|\vec{b}|^2 \sin^2 \theta$$

$$\text{or } |\vec{c}|^2 + 144 + 6\vec{b} \cdot \vec{c} = 48$$

$$\text{or } |\vec{c}|^2 + 96 + 6(\vec{b} \cdot \vec{c}) = 0$$

... (i)

$$\text{or } \vec{c} = 2\vec{a} \times \vec{b} - 3\vec{b} \Rightarrow \vec{b} \cdot \vec{c} = 0 - 3 \times 16 = -48.$$

Putting value of $\vec{b} \cdot \vec{c}$ in Eq. (i), we have

$$|\vec{c}|^2 + 96 - 6 \times 48 = 0, |\vec{c}|^2 = 48 \times 4 = 192$$

Again, putting the value of $|\vec{c}|$ in Eq. (i), we have

$$192 + 96 + 6|\vec{b}| \cdot |\vec{c}| \cos \alpha = 0$$

$$\text{or } 6 \times 4 \times 8\sqrt{3} \cos \alpha = -288$$

$$\text{or } \cos \alpha = -\frac{288}{6 \times 4 \times 8\sqrt{3}} = -\frac{\sqrt{3}}{2} \text{ or } \alpha = \frac{5\pi}{6}$$

77. (a) Let $\vec{b} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}, \vec{a} \times \vec{b} = \hat{j} - \hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \alpha & \beta & \gamma \end{vmatrix} = \hat{j} - \hat{k}$$

$$\Rightarrow \beta - \gamma = 0, \alpha - \gamma = 1, \alpha - \beta = 1$$

$$\Rightarrow \beta = \gamma, \alpha = 1 + \gamma, \alpha = 1 + \beta, \vec{a} \cdot \vec{b} = 1$$

$$\Rightarrow \alpha + \beta + \gamma = 1,$$

$$\Rightarrow \beta + 1 + \beta + \beta = 1 \Rightarrow \beta = 0$$

$$\therefore \alpha = 1, \gamma = 0,$$

$$\therefore \vec{b} = \hat{i}.$$

78. (c) $(\vec{a} - \vec{c}) \cdot (\vec{b} + \vec{c}) = 0$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{c}(\vec{a} - \vec{b}) - |\vec{c}|^2 = 0$$

$$\Rightarrow (\vec{a} - \vec{b}) \cdot \vec{c} = 1$$

$$\Rightarrow (\vec{a} - \vec{b}) \cdot (\lambda\vec{a} + \mu\vec{b} + \omega(\vec{a} \times \vec{b})) = 1$$

$$\Rightarrow \lambda - \mu = 1 \Rightarrow \lambda = \mu + 1$$

Now $\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a}, \vec{b}, \vec{a} \times \vec{b}$ are mutually perpendicular.

$$|\vec{c}| = 1 \Rightarrow \lambda^2 + \mu^2 + \omega^2 = 1$$

$$\Rightarrow (\mu + 1)^2 + \mu^2 + \omega^2 = 1.$$

79. (a) $|\vec{\beta}|^2 = |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - |\vec{a} \cdot \vec{b}|^2$

$$= 1 - \cos^2 \theta = \sin^2 \theta$$

$$\vec{\alpha}^2 = \vec{a}^2 + (\vec{a} \cdot \vec{b})^2(\vec{b})^2 - 2(\vec{a} \cdot \vec{b})^2$$

$$= 1 + \cos^2 \theta - 2 \cos^2 \theta$$

$$= 1 - \cos^2 \theta = \sin^2 \theta.$$

$$\Rightarrow |\vec{\alpha}| = |\vec{\beta}|.$$

80. (c) To determine the area of rectangle, use the relation that if a and b are the adjacent sides of a parallelogram,

then area of parallelogram = $|a \times b|$.

The position vectors of vertices A, B, C and D of rectangle ABCD are given.

First, we compute vectors AB and AD.

Now, AB = PV of B - PV of A

$$= \left(\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k} \right) - \left(-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k} \right)$$

$$= [1 - (-1)]\hat{i} + \left(\frac{1}{2} - \frac{1}{2} \right)\hat{j} + (4 - 4)\hat{k} = 2\hat{i}$$

and AD = PV of D - PV of A

$$= \left(-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k} \right) - \left(-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k} \right)$$

$$= [1 - (-1)]\hat{i} + \left(-\frac{1}{2} - \frac{1}{2} \right)\hat{j} + (4 - 4)\hat{k} = -\hat{j}$$

$$\therefore \overline{AB} \times \overline{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = -2\hat{k}$$

Area of rectangle ABCD = $|\overline{AB} \times \overline{AD}| = \sqrt{(-2)^2} = 2$ sq. units

Hence, the area of the rectangle is $|\overline{AB} \times \overline{AD}| = 2$ sq. units

81. (c) If other adjacent sides is \vec{a} , then
Area of parallelogram = $|(3\hat{i} + 4\hat{j}) \times \vec{a}| = 4\sqrt{29}$
82. (a) Given, $\vec{b} \times \vec{a} = \vec{0}$
 $\Rightarrow \vec{b} \parallel \vec{a} \Rightarrow \frac{p}{2} = 1 = \frac{q}{3} \Rightarrow p = 2, q = 3$
83. (d) Area of $\Delta = \frac{1}{2} |(\hat{j} + \lambda\hat{k}) \times (\hat{i} + \lambda\hat{k})|$
 $= \frac{1}{2} |-\hat{k} + \lambda\hat{i} + \lambda\hat{j}| = \frac{1}{2} \sqrt{2\lambda^2 + 1}$
 $\Rightarrow \frac{9}{4} \leq \frac{1}{4} (2\lambda^2 + 1) \leq \frac{33}{4} \Rightarrow 4 \leq \lambda^2 \leq 16 \Rightarrow 2 \leq |\lambda| \leq 4$

84. (b) A(-1, 1, 2), B(1, 2, 3) and C(t, 1, 1)
 $\Rightarrow \overline{AB} = 2\hat{i} + \hat{j} + \hat{k} \Rightarrow \overline{AC} = (t+1)\hat{i} + 0\hat{j} - \hat{k}$
- $$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ t+1 & 0 & -1 \end{vmatrix} = -\hat{i} + (t+3)\hat{j} - (t+1)\hat{k}$$
- $$|\overline{AB} \times \overline{AC}| = \sqrt{1 + (t+3)^2 + (t+1)^2}$$
- $$= \sqrt{2t^2 + 8t + 11}$$
- Area of $\Delta ABC = \frac{1}{2} |\overline{AB} \times \overline{AC}|$
 $= \frac{1}{2} \sqrt{2t^2 + 8t + 11} = \frac{1}{2} \sqrt{2(t+2)^2 + 3} \geq \frac{\sqrt{3}}{2}$

85. (c) $\vec{p} \times \vec{q} = (\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$
 $\Rightarrow |\vec{p} \times \vec{q}|^2 = 4(|\vec{b}|^2 |\vec{a}|^2 - (\vec{b} \cdot \vec{a})^2)$
 $\Rightarrow |\vec{p} \times \vec{q}|^2 = 4\{625 - (\vec{b} \cdot \vec{a})^2\}$
 $\Rightarrow |\vec{p} \times \vec{q}|^2 = 2\{625 - (\vec{a} \cdot \vec{b})^2\}^{1/2} \Rightarrow \lambda = 625$
86. (d) $\vec{u} \times \vec{v} = (2\hat{i} - \hat{j} + \hat{k}) \times (-3\hat{j} + 2\hat{k}) = \hat{i} - 4\hat{j} - 6\hat{k}$

Let $\vec{w} = a\hat{i} + b\hat{j}$

We have, $a^2 + b^2 = 1$

So let $a = \cos \theta, b = \sin \theta$

Now, $(\vec{u} \times \vec{v}) \cdot \vec{w} = a - 4b = \cos \theta - 4 \sin \theta$

Max. value = $\sqrt{1^2 + (-4)^2} = \sqrt{17}$

87. (a) Let $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$
 $\therefore \vec{a} \cdot \vec{b} = 3 \Rightarrow b_1 - b_2 + 2b_3 = 3$

...(i)

$$\vec{a} \times \vec{b} = 2\hat{i} - \hat{k} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= (-b_3 - 2b_2)\hat{i} - \hat{j}(b_3 - 2b_1) + \hat{k}(b_2 + b_1)$$

$$\Rightarrow -b_3 - 2b_2 = 2 \quad \dots(ii)$$

$$b_3 - 2b_1 = 0 \quad \dots(iii)$$

$$b_2 + b_1 = -1 \quad \dots(iv)$$

$bz = -1 - b_1, b_3 = 2b_1$ [equating coefficients]
put in (i) we get

$$b_1 + 1 + b_1 + 4b_1 = 3$$

$$6b_1 = 2 \Rightarrow b_1 = \frac{1}{3}$$

$$b_2 = -1 - \frac{1}{3} = -\frac{4}{3} \Rightarrow b_3 = 2 \times \frac{1}{3} = \frac{2}{3}$$

So, $\vec{b} = \frac{1}{3}\hat{i} - \frac{4}{3}\hat{j} + \frac{2}{3}\hat{k}$

Projection of \vec{b} on vector $\vec{a} - \vec{b}$ is $\Rightarrow \frac{[\vec{b} \cdot (\vec{a} - \vec{b})]}{|\vec{a} - \vec{b}|}$

$$= \frac{\frac{2}{9} - \frac{4}{9} + \frac{8}{9}}{\sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{4}{3}\right)^2}} = \frac{\frac{2}{9}}{\sqrt{\frac{21}{9}}}$$

$$= \frac{2}{3} \times \frac{\sqrt{3}}{\sqrt{7}} = \frac{2}{\sqrt{21}}$$

88. (c) Given, $\vec{c} \times \vec{a} = \vec{b} \Rightarrow |\vec{c} \times \vec{a}| = |\vec{b}| \Rightarrow |\vec{c}| |\vec{a}| \sin \theta = 3$

$$\Rightarrow |\vec{c}| = \frac{3}{2 \sin \theta} \Rightarrow |\vec{c} - \vec{a}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a}$$

$$= |\vec{c}|^2 + 4 - 2|\vec{c}| \cdot |\vec{a}| \cos \theta$$

$$= \frac{9}{4 \sin^2 \theta} + 4 - 2 \cdot \frac{3}{2 \sin \theta} \cdot 2 \cdot \cos \theta$$

$$= 4 + \frac{9}{4} \operatorname{cosec}^2 \theta - 6 \cot \theta = \frac{9}{4} + \left(\frac{3}{2} \cot \theta - 2 \right)^2$$

$$\Rightarrow |\vec{c} - \vec{a}|^2 \geq \frac{9}{4} \Rightarrow |\vec{c} - \vec{a}| \geq \frac{3}{2} \Rightarrow 2|\vec{c} - \vec{a}| \geq 3$$

\therefore Min of $2|\vec{c} - \vec{a}| = 3$

89. (c) $|\vec{a} + \vec{b} + \vec{a} \times \vec{b}|^2 + |\vec{a} + \vec{b} - \vec{a} \times \vec{b}|^2$

$$= 2(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 + 2\vec{a} \cdot \vec{b})$$

$$= 2\{1 + |\vec{a}|^2 + |\vec{b}|^2 + |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 + 2\vec{a} \cdot \vec{b} - 1\}$$

$$= 2\{(1 + |\vec{a}|^2)(1 + |\vec{b}|^2) - (\vec{a} \cdot \vec{b})^2\}$$

90. (b) $\vec{r}_1 = a\hat{i} + b\hat{j} + c\hat{k}, \vec{r}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$$|\vec{r}_1 \times \vec{r}_2|^2 \leq |\vec{r}_1|^2 |\vec{r}_2|^2$$

...(i)

$$\text{Now, } \vec{r}_1 \times \vec{r}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}(5b-4c) + \hat{j}(3c-5a) + \hat{k}(4a-3b)$$

So, from (i)

$$(5b-4c)^2 + (3c-5a)^2 + (4a-3b)^2 \leq 50$$

91. (c) Given, $\vec{z} + (\vec{z} \times \vec{x}) = \vec{y} \Rightarrow |\vec{z} + (\vec{z} \times \vec{x})|^2 = |\vec{y}|^2$
 $\Rightarrow |\vec{z}|^2 + |\vec{z}|^2 |\vec{x}|^2 \sin^2 \theta = 1 \Rightarrow |\vec{z}| = \frac{1}{\sqrt{1+\sin^2 \theta}} = \frac{2}{\sqrt{7}}$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2}$$

92. (d) Given, $\vec{a} \times \vec{b} = 3(\vec{a} \times \vec{c}) \Rightarrow \vec{a} \times (\vec{b} - 3\vec{c}) = \vec{0}$
 $\Rightarrow \vec{a} \parallel \vec{b} - 3\vec{c} \Rightarrow \vec{b} - 3\vec{c} = \lambda \vec{a} \Rightarrow |\vec{b} - 3\vec{c}|^2 = \lambda^2 |\vec{a}|^2$
 $\Rightarrow |\vec{b}|^2 + 9|\vec{c}|^2 - 6(\vec{b} \cdot \vec{c}) = \lambda^2 |\vec{a}|^2$
 $\Rightarrow 2 - 6 \times 1 \times \frac{1}{3} \cos 60^\circ = \lambda^2 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$

93. (d) Given, $\vec{c} = x\vec{a} + y\vec{b} + \vec{a} \times \vec{b}$
 and $\vec{a} \cdot \vec{b} = 0, |\vec{c}| = 2, |\vec{a}| = |\vec{b}| = 1$... (i)

Let vector \vec{c} make angle α with vector \vec{a} and \vec{b}
 $\therefore \vec{c} \cdot \vec{a} = \vec{c} \cdot \vec{b} = 2 \cos \alpha$

Taking dot product of equation (i) with \vec{a} and \vec{b} , we have

$$\Rightarrow x = y = 2 \cos \alpha$$

$$\text{Now, } |\vec{c}|^2 = x^2 + y^2 + |\vec{a} \times \vec{b}|^2 = 2(4 \cos^2 \alpha) + 1$$

$$\Rightarrow 4 = 8 \cos^2 \alpha + 1 \Rightarrow 8 \cos^2 \alpha = 3$$

94. (a) Given $\vec{a} \cdot \vec{b} = 0, \vec{a}$ is perpendicular to \vec{b}
 $\vec{a} \cdot \vec{c} = 0 \Rightarrow \vec{a}$ is perpendicular to \vec{c}
 $\therefore \vec{a}$ is perpendicular to the plane of \vec{b} and \vec{c} .
 Also \vec{a} is a unit vector.
 $\therefore \vec{a} = \pm \frac{\vec{b} \times \vec{c}}{|\vec{b} \times \vec{c}|}$... (i)

$$\text{But } |\vec{b} \times \vec{c}| = |\vec{b}| |\vec{c}| \sin \frac{\pi}{6} = 1.1 \cdot \frac{1}{2}$$

$$\therefore \text{From (i), we have } \vec{a} = \pm 2(\vec{b} \times \vec{c})$$

$$\therefore |\lambda| = 2$$

95. (c) Given, $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$... (i)
 $(\vec{r} - \vec{c}) \times \vec{b} = \vec{0}$
 $\Rightarrow \vec{r} - \vec{c} = \lambda \vec{b} \Rightarrow \vec{r} \cdot \vec{a} = \vec{a} \cdot \vec{c} + \lambda \vec{a} \cdot \vec{b}$
 $\Rightarrow \vec{r} \cdot \vec{b} = \vec{c} \cdot \vec{b} - \frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} (|\vec{b}|)^2$ (2)
 $= 1 - \frac{-4}{1} = 9$

96. (a) q = Area of parallelogram with \vec{OA} and \vec{OC} as adjacent sides
 $= |\vec{OA} \times \vec{OC}| = |\vec{a} \times \vec{b}|$

p = Area of quadrilateral OABC

$$= \frac{1}{2} |\vec{OA} \times \vec{OB}| + \frac{1}{2} |\vec{OB} \times \vec{OC}|$$

$$= \frac{1}{2} [|\vec{a} \times (10\vec{a} + 2\vec{b})| + |(10\vec{a} + 2\vec{b}) \times \vec{b}|]$$

$$= 6 |\vec{a} \times \vec{b}| \Rightarrow k = 6$$

97. (c) Let the position vectors of points A, B, C, D be $\vec{a}, \vec{b}, \vec{c}$ and \vec{d} , respectively, with respect to some origin.

$$|\vec{AB} \times \vec{CD} + \vec{BC} \times \vec{AD} + \vec{CA} \times \vec{BD}|$$

$$= |(\vec{b} - \vec{a}) \times (\vec{d} - \vec{c}) + (\vec{c} - \vec{b}) \times (\vec{d} - \vec{a}) + (\vec{a} - \vec{c}) \times (\vec{d} - \vec{b})|$$

$$= 2|\vec{b} \times \vec{a} + \vec{c} \times \vec{b} + \vec{a} \times \vec{c}| \dots (i)$$

$$= 4 \times \frac{1}{2} |(\vec{b} - \vec{a}) \times (\vec{b} - \vec{c})| = 4 \times (\text{area of } \Delta ABC)$$

$$\Rightarrow k = 4$$

98. (d) We have given that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$
 $\Rightarrow \vec{c} = -\vec{a} - \vec{b}$

Given that

$$\frac{\vec{a} \cdot (\vec{c} - \vec{b})}{\vec{c} \cdot (\vec{a} - \vec{b})} = \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}$$

$$\Rightarrow \frac{\vec{a} \cdot (-\vec{a} - 2\vec{b})}{(-\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})} = \frac{3}{7}$$

$$\Rightarrow \frac{9 + 2\vec{a} \cdot \vec{b}}{9 - 16} = \frac{3}{7}$$

$$\Rightarrow \vec{a} \cdot \vec{b} = -6 \Rightarrow |\vec{a} \times \vec{b}|^2 = a^2 b^2 - (\vec{a} \cdot \vec{b})^2$$

$$= 9 \times 16 - 36 = 108$$

99. (c) Given, vector equation is $\hat{b} = \vec{c} + 2(\vec{c} \times \vec{a})$.
 Take modulus and square both sides.

$$|\hat{b}|^2 = |\vec{c} + 2(\vec{c} \times \vec{a})|^2$$

$$\Rightarrow |\hat{b}|^2 = |\vec{c}|^2 + 4|\vec{c} \times \vec{a}|^2 + 4\vec{c} \cdot (\vec{c} \times \vec{a})$$

Here, $(\vec{c} \times \vec{a}) = |\vec{c}| |\vec{a}| \sin \theta$

$$1 = |\vec{c}|^2 + 4|\vec{c}|^2 \sin^2 \frac{\pi}{12} + 0$$

$$\Rightarrow 1 = |\vec{c}|^2 + 4|\vec{c}|^2 \left(\frac{\sqrt{3}-1}{2\sqrt{2}} \right)^2$$

$$\Rightarrow |\vec{c}|^2 = \frac{1}{3\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{3+\sqrt{3}}{6}$$

Take, $6^2 |\vec{c}|^2 = 6(3 + \sqrt{3})$.

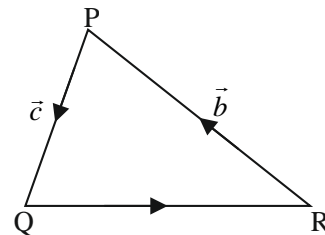
100. (d) $|\vec{a} + \vec{b} + \vec{c}|^2 = 0$

$$3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = \frac{-3}{2} \Rightarrow \lambda = \frac{-3}{2}$$

$$\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times (-\vec{a} - \vec{b}) + (-\vec{a} - \vec{b}) \times \vec{a} \quad [\because \vec{c} = -\vec{a} - \vec{b}]$$

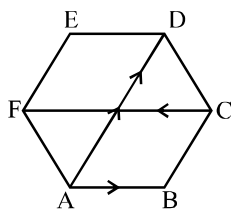
$$= \vec{a} \times \vec{b} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} \Rightarrow \vec{d} = 3(\vec{a} \times \vec{b})$$



Exercise 2 :

ACCELERATOR
Topic-wise MCQs

1. (a)



In a regular hexagon,

$$\overline{AB} = 2\overline{FC}$$

$$\Rightarrow \overline{AB} = -2\overline{CF}$$

$$\therefore n = -2$$

Also, $\overline{AD} = 2\overline{BC}$

$$\therefore m = 2$$

Now, $mn = 2(-2) = -4$

2. (c) Let \vec{a}, \vec{b} be the sides and \vec{d}_1, \vec{d}_2 be the diagonals of the parallelogram.

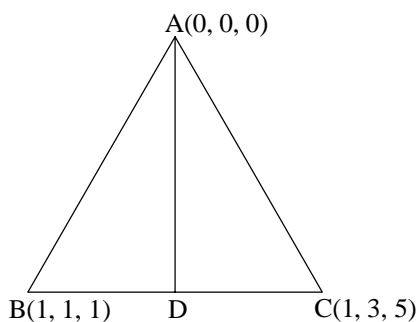
Given $\vec{d}_1 = 3\hat{i} + 6\hat{j} - 2\hat{k}, \vec{d}_2 = -\hat{i} - 2\hat{j} - 8\hat{k}$

Hence, $\vec{a} = \frac{\vec{d}_1 + \vec{d}_2}{2} = \hat{i} + 2\hat{j} - 5\hat{k} \Rightarrow |\vec{a}| = \sqrt{30}$

and $\vec{b} = \frac{\vec{d}_1 - \vec{d}_2}{2} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$\Rightarrow |\vec{b}| = \sqrt{29}$ (shorter side)

3. (c)



$\vec{AB} = \hat{i} + \hat{j} + \hat{k}, \vec{AC} = \hat{i} + 3\hat{j} + 5\hat{k}$

Since D is the midpoint of B and C

Hence, $D \equiv \left(\frac{1+1}{2}, \frac{1+3}{2}, \frac{1+5}{2}\right) \equiv (1, 2, 3)$

Length of median AD.

$$AD = \sqrt{(1-0)^2 + (2-0)^2 + (3-0)^2} = \sqrt{14}$$

4. (a) $\vec{a} = \hat{i} + \lambda\hat{j} + 2\hat{k}, \vec{b} = \mu\hat{i} + \hat{j} - \hat{k}$

\vec{a} and \vec{b} are orthogonal Hence, $\vec{a} \cdot \vec{b} = 0$
 $\Rightarrow \lambda + \mu = 2$... (i)

Since, $|\vec{a}| = |\vec{b}| \Rightarrow \sqrt{1 + \lambda^2 + 4} = \sqrt{\mu^2 + 1 + 1}$
 $\Rightarrow \mu^2 - \lambda^2 = 3 \Rightarrow (\mu - \lambda)(\mu + \lambda) = 3$
 $\Rightarrow \mu - \lambda = \frac{3}{2}$... (ii)

Solving (i) and (ii) we get $(\lambda, \mu) = \left(\frac{1}{4}, \frac{7}{4}\right)$

5. (c) Centroid of BCD = $\frac{\vec{b} + \vec{c} + \vec{d}}{3}$

$$\Rightarrow -\hat{i} + 2\hat{j} - 3\hat{k} = \frac{(-2\hat{i} + \hat{j} + 3\hat{k}) + (3\hat{i} + 2\hat{j} - \hat{k}) + \vec{d}}{3}$$

$$\Rightarrow \vec{d} = -4\hat{i} + 3\hat{j} - 11\hat{k}$$

\therefore Centroid of tetrahedron $\vec{g} = \frac{\vec{a} + \vec{b} + \vec{c} + \vec{d}}{4}$

$$\vec{g} = \frac{-1\hat{i} + \hat{j} - 3\hat{k}}{2}$$

$$\vec{GD} = \vec{d} - \vec{g} = (-4\hat{i} + 3\hat{j} - 11\hat{k}) - \left(\frac{-1\hat{i} + \hat{j} - 3\hat{k}}{2}\right)$$

$$= \frac{-7\hat{i} + 2\hat{j} - 19\hat{k}}{2}$$

$$|\vec{GD}| = \sqrt{\left(\frac{-7}{2}\right)^2 + (2)^2 + \left(\frac{-19}{2}\right)^2} = \frac{\sqrt{213}}{\sqrt{2}}$$

6. (c) $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}, \vec{b} = 6\hat{i} + 3\hat{j} - 2\hat{k}, \vec{c} = -4\hat{i} + 3\hat{i} + 12\hat{k}$

$$|\vec{a}| = \sqrt{1+4+4} = 3, |\vec{b}| = \sqrt{36+9+4} = 7,$$

$$|\vec{c}| = \sqrt{16+9+144} = 13$$

$$\vec{a} + \vec{b} + \vec{c} = 3\hat{i} + 4\hat{j} + 12\hat{k} \Rightarrow |\vec{a} + \vec{b} + \vec{c}| = 13$$

$$\therefore \sqrt{(|\vec{a}| + |\vec{b}| + |\vec{c}|) + |\vec{a} + \vec{b} + \vec{c}|} = \sqrt{3+7+13+13}$$

$$= \sqrt{36} = 6$$

7. (b) Given, $\vec{BC} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{CA} = 6\hat{i} + 3\hat{j} - 2\hat{k}$

In a triangle $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$

$$\vec{AB} = -(\vec{BC} + \vec{CA})$$

$$\vec{AB} = -\hat{i} + 2\hat{j} - 2\hat{k} - 6\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{AB} = -7\hat{i} - \hat{j}$$

$$|\vec{BC}| = \sqrt{1^2 + (-2)^2 + (2)^2} = 3$$

$$|\vec{CA}| = \sqrt{(6)^2 + (3)^2 + (-2)^2} = 7$$

$$|\vec{AB}| = \sqrt{(-7)^2 + (1)^2} = 5\sqrt{2}$$

Perimeter of triangle = $|\vec{AB}| + |\vec{BC}| + |\vec{CA}|$

$$5\sqrt{2} + 3 + 7 = 5(2 + \sqrt{2})$$

8. (a) The centroid of face BCD is given as $\frac{2}{3}(\hat{i} + \hat{j} + \hat{k})$

$$\frac{(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}) + (a_2\hat{i} + b_2\hat{j} + c_2\hat{k}) + (a_3\hat{i} + b_3\hat{j} + c_3\hat{k})}{3}$$

$$= \frac{2}{3}(\hat{i} + \hat{j} + \hat{k})$$

$$\Rightarrow a_1 + a_2 + a_3 = 2$$

$$b_1 + b_2 + b_3 = 2$$

$$c_1 + c_2 + c_3 = 2$$

The centroid of the tetrahedron is given as $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$$\text{So, } \frac{(\hat{i} + \hat{j} + \hat{k}) + (a_1\hat{i} + b_1\hat{j} + c_1\hat{k}) + (a_2\hat{i} + b_2\hat{j} + c_2\hat{k}) + (a_3\hat{i} + b_3\hat{j} + c_3\hat{k})}{4}$$

$$= \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$$

$$\Rightarrow \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} = \frac{(1 + a_1 + a_2 + a_3)\hat{i} + (1 + b_1 + b_2 + b_3)\hat{j} + (1 + c_1 + c_2 + c_3)\hat{k}}{4}$$

$$\Rightarrow \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} = \frac{(1+2)\hat{i} + (1+2)\hat{j} + (1+2)\hat{k}}{4}$$

$$= \frac{3}{4}(\hat{i} + \hat{j} + \hat{k})$$

$$\therefore \alpha = \frac{3}{4}, \beta = \frac{3}{4}, \gamma = \frac{3}{4}$$

$$\Rightarrow 2\alpha + \beta + \gamma = 2\left(\frac{3}{4}\right) + \frac{3}{4} + \frac{3}{4} = \frac{12}{4} = 3.$$

9. (c) $\vec{a} = (x + 2y - 3)\hat{i} + (2x - y + 3)\hat{j}$

$$\vec{b} = (3x - 2y)\hat{i} + (x - y + 1)\hat{j}$$

$$\therefore \vec{a} = 2\vec{b}$$

$$\Rightarrow (x + 2y - 3) = 2(3x - 2y)$$

$$\Rightarrow x + 2y - 3 = 6x - 4y$$

$$\Rightarrow 5x - 6y = -3$$

$$\Rightarrow 5x + 6 = -3 \text{ using}$$

$$(y = -1)$$

$$\Rightarrow 5x = -9$$

$$\text{and } (2x - y + 3)$$

$$= 2(x - y + 1)$$

$$\Rightarrow 2x - y + 3$$

$$= 2x - 2y + 2$$

$$\Rightarrow y = -1$$

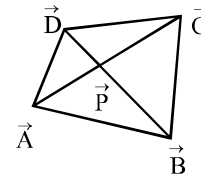
Finally, $y - 5x = -1 - (-9) = -1 + 9 = 8$

10. (b) We have, $\vec{A} = 7\hat{i} - 4\hat{j} + 7\hat{k}$

$$\vec{B} = \hat{i} - 6\hat{j} + 10\hat{k}$$

$$\vec{C} = -\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{D} = 5\hat{i} - \hat{j} + \hat{k}$$



It can be observed that:

$$\vec{A} + \vec{C} = \vec{B} + \vec{D}$$

$$\Rightarrow \frac{\vec{A} + \vec{C}}{2} = \frac{\vec{B} + \vec{D}}{2}$$

Or Diagonals midpoint coincide, or alternatively diagonals bisect each other.

Therefore, quadrilateral is ||gm.

Now, the p.v. of intersection of diagonals = mid-point of diagonals

$$p\hat{i} + q\hat{j} + r\hat{k} = \frac{\vec{A} + \vec{C}}{2} \text{ or } \frac{\vec{B} + \vec{D}}{2}$$

$$p\hat{i} + q\hat{j} + r\hat{k} = 3\hat{i} - 3.5\hat{j} + 5.5\hat{k}$$

$$\Rightarrow p = 3; q = -3.5; r = 5.5$$

Finally, $p + q + r = 5$

11. (d) Let position vectors

A, B, C, D be $\vec{a}, \vec{b}, \vec{c}, \vec{d}$

So, $\vec{AB} = \vec{b} - \vec{a}, \vec{CB} = \vec{b} - \vec{c}, \vec{CD} = \vec{d} - \vec{c}$

$$\vec{AD} = \vec{d} - \vec{a}$$

∴ Midpoint of AC :

$$\vec{E} = \frac{\vec{a} + \vec{c}}{2}$$

Midpoint of BD:

$$\vec{F} = \frac{\vec{b} + \vec{d}}{2}$$

$$\therefore \vec{EF} = \vec{F} - \vec{E} = \frac{\vec{b} + \vec{d} - \vec{a} - \vec{c}}{2}$$

So, $\vec{AB} + \vec{CB} + \vec{CD} + \vec{AD}$

$$= \vec{b} - \vec{a} + \vec{b} - \vec{c} + \vec{d} - \vec{c} + \vec{d} - \vec{a}$$

$$= 2\vec{b} + 2\vec{d} - 2\vec{c} - 2\vec{a}$$

$$= 2 \times 2 \vec{EF} = 4 \vec{EF}$$

12. (a) $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{B} = 7\hat{i} - \hat{k}$

$$\vec{P} = -2\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\vec{P} = \vec{A} + t(\vec{B} - \vec{A})$$

$$-2\hat{i} + 3\hat{j} + 5\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k} + t(6\hat{i} - 2\hat{j} - 4\hat{k})$$

Equating components, we get

$$t = -\frac{1}{2} \text{ (So P divides AB externally in ratio } t = -\frac{1}{2}\text{)}$$

Harmonic conjugate \vec{Q} : $t' = \frac{-t}{1-2t}$

$$t' = \frac{-\left(-\frac{1}{2}\right)}{1-2\left(-\frac{1}{2}\right)} = \frac{1}{4}$$

$$\therefore \vec{Q} = \vec{A} + t'(\vec{B} - \vec{A})$$

$$\Rightarrow \vec{Q} = (\hat{i} + 2\hat{j} + 3\hat{k}) - \frac{1}{4}(6\hat{i} - 2\hat{j} - 4\hat{k})$$

$$\Rightarrow \vec{Q} = \left(\frac{-1}{2}\hat{i} + \frac{5}{2}\hat{j} + 4\hat{k}\right)$$

Thus, sum of scalar components of position vector (\vec{Q})

$$= \frac{-1}{2} + \frac{5}{2} + 4 = 6$$

13. (c) $(2\hat{i} - 3\hat{j} + \hat{k})$ $(\hat{i} + 2\hat{j} - 3\hat{k})$

Position vector of $\vec{C} = \frac{7}{5}\hat{i} + 0\hat{j} - \frac{7}{5}\hat{k}$

$$\vec{CD} = (3\hat{i} - \hat{j} + 2\hat{k}) - \left(\frac{7}{5}\hat{i} + 0\hat{j} - \frac{7}{5}\hat{k}\right)$$

$$= \frac{8}{5}\hat{i} - \hat{j} + \frac{17}{5}\hat{k} = \frac{(8\hat{i} - 5\hat{j} + 17\hat{k})}{5}$$

$$|\vec{CD}| = \frac{(8\hat{i} - 5\hat{j} + 17\hat{k})}{\sqrt{64 + 25 + 289}} = \frac{(8\hat{i} - 5\hat{j} + 17\hat{k})}{3\sqrt{42}}$$

14. (a) $(2\hat{i} - 5\hat{j} + 3\hat{k})$ $(4\hat{i} + \hat{j} - 6\hat{k})$

PQ trisect AB ; and R divides PQ in the ratio $2 : 3$.

Let's assume AB line segment of length 3 units.

$$PR \text{ and } RQ \equiv \frac{2}{5} \text{ and } \frac{3}{5} \Rightarrow AR = 1 + \frac{2}{5}, RB = 1 + \frac{3}{5}$$

$$\Rightarrow AR = \frac{7}{5}, RB = \frac{8}{5} \Rightarrow AR : RB = 7 : 8$$

∴ R divides AB in the ratio $7 : 8$.

Using section formula : Position vector of point R

$$\vec{R} = \frac{8 \times 2 + 7 \times 4}{15} \hat{i} + \frac{(8 \times -5) + (7 \times 1)}{15} \hat{j}$$

$$+ \frac{(8 \times 3) + (7 \times -6)}{15} \hat{k}$$

$$\Rightarrow \vec{R} = \frac{44}{15} \hat{i} - \frac{33}{15} \hat{j} - \frac{18}{15} \hat{k}$$

15. (c) Line joining is $\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{j} - 3\hat{k}) \quad \dots(i)$$

Line joining $2\hat{i} + \hat{j} - 6\hat{k}$ and $3\hat{i} - \hat{j} - 7\hat{k}$ is

$$\vec{r} = 2\hat{i} + \hat{j} - 6\hat{k} + \mu(\hat{i} - 2\hat{j} - \hat{k}) \quad \dots(ii)$$

From (i) and (ii)

$$\hat{i} + (2\lambda - 1)\hat{j} + (1 - 3\lambda)\hat{k} = (2 + \mu)\hat{i} + (1 - 2\mu)\hat{j} - (6 + \mu)\hat{k}$$

On comparing we get, $2 + \mu = 1 \Rightarrow \mu = -1$

Putting in (ii) we get

Position vector of intersection point, $\hat{i} + 3\hat{j} - 5\hat{k}$.

16. (d) We know that if $\vec{a}, \vec{b}, \vec{c}$ are collinear then

$$(\vec{a} - \vec{b}) = k(\vec{a} - \vec{c})$$

$$\therefore (\lambda - 2)\vec{a} + (\lambda - 2)\vec{b} + 3\vec{c} = k[(\lambda - 4)\vec{a} - 9\vec{b} + 9\vec{c}]$$

$$\Rightarrow (\lambda - 2)\vec{a} + (\lambda - 2)\vec{b} + 3\vec{c} = k(\lambda - 4)\vec{a} - 9k\vec{b} + 9k\vec{c}$$

On comparing both sides, we get $3 = 9k \Rightarrow k = \frac{1}{3}$

$$\lambda - 2 = k(\lambda - 4) \Rightarrow 3\lambda - 6 = \lambda - 4 \Rightarrow 2\lambda = 6 - 4 \Rightarrow \lambda = 1$$

And $(\lambda - 2) = k(\lambda - 4) \Rightarrow (1 - 2) = \frac{1}{3}(1 - 4)$

$$\Rightarrow -1 = -1 \text{ satisfy. } \therefore \lambda = 1.$$

17. (b) $\vec{D} \equiv \frac{3\hat{i}}{2} - \frac{3\hat{k}}{2}$; $\vec{E} \equiv \hat{i} - \frac{3}{2}\hat{j} - \frac{\hat{k}}{2}$; $\vec{DE} \equiv \frac{-\hat{i}}{2} - \frac{3}{2}\hat{j} + \hat{k}$

Vector along \vec{DE} is $-\hat{i} - 3\hat{j} + 2\hat{k}$

$$\text{Unit vector along } \vec{DE} = \frac{(-\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{9+4+1}} = \frac{-\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{14}}$$

18. (d) Given: $\vec{a} = (2x + y)\hat{i} + 3\hat{j} + 9\hat{k}$

$\vec{b} = 2\hat{i} + \hat{j} - (x - y)\hat{k}$ $\therefore \vec{a}$ and \vec{b} are co-linear vectors

$$\therefore \frac{2x + y}{2} = \frac{3}{1} = \frac{9}{-(x - y)}$$

$$\Rightarrow \frac{2x + y}{2} = \frac{3}{1} \Rightarrow 2x + y = 6 \quad \dots(i)$$

$$\text{and } \frac{3}{1} = \frac{9}{y - x} \Rightarrow y - x = 3 \quad \dots(ii)$$

Solving eqns. (i) and (ii), we get

$$x = 1 \text{ and } y = 4; \text{ Now, } x^3 + 27y^3 = 1 + 27 \times 64 = 1729.$$

19. (a) \therefore Points $(k, 1, 5)$, $(1, 0, 3)$ & $(7, -2, m)$ are collinear.

$A \equiv (k, 1, 5)$, $B \equiv (1, 0, 3)$, $C \equiv (7, -2, m)$

$$\vec{AB} = (1 - k)\hat{i} + (0 - 1)\hat{j} - 2\hat{k}$$

$$\vec{AC} = (7 - k)\hat{i} - 3\hat{j} + (m - 5)\hat{k}; \vec{AB} = \lambda \vec{AC}$$

$$\Rightarrow (1 - k)\hat{i} - \hat{j} - 2\hat{k} = \lambda[(7 - k)\hat{i} - 3\hat{j} + (m - 5)\hat{k}]$$

Comparing both sides, we get

$$1 - k = \lambda(7 - k) \dots (i); \quad -1 = -3\lambda \Rightarrow \lambda = \frac{1}{3} \quad \dots(ii)$$

$$-2 = \lambda(m - 5) \Rightarrow -6 = m - 5 \Rightarrow m = -1$$

$$\text{from eqn. (i)} \Rightarrow 1 - k = \frac{1}{3}(7 - k)$$

$$\Rightarrow 3 - 3k = 7 - k \Rightarrow 2k = -4 \Rightarrow k = -2$$

$$\therefore (k, m) = (-2, -1)$$

20. (a) Given points are $A = (1, 2, 0)$, $B = (2, 0, -1)$,

$C = (0, -2, 3)$ and $D = (-1, 2, -3)$

Here $G_1 = (1, 0, 2/3)$ and $G_2 = (1/2, 1/2, -1/4)$

Now P divides $G_1 G_2$ in the ratio 4:3 internally

So, $P = (5/7, 2/7, 1/7)$

21. (d) Here we are given that $\vec{OA} = \vec{a}, \vec{OB} = \vec{b}, \vec{OC} = \vec{c}$

$$\text{Now P.V of D i.e } \vec{OD} = \frac{1 \times \vec{OB} + 3 \times \vec{OC}}{1 + 3}; \vec{OD} = \frac{\vec{b} + 3\vec{c}}{4}$$

$$\vec{OE} = \frac{4\vec{OD} + \vec{OA}}{4 + 1} = \frac{4(\frac{\vec{b} + 3\vec{c}}{4}) + \vec{a}}{5} \Rightarrow \vec{OE} = \frac{\vec{a} + \vec{b} + 3\vec{c}}{5}$$

$$\text{Now, } \vec{OE} = \frac{2\vec{OB} + 3\vec{OF}}{2 + 3} \Rightarrow \vec{OF} = \frac{5\vec{OE} - 2\vec{OB}}{3}$$

$$\Rightarrow \vec{OF} = \frac{5(\frac{\vec{a} + \vec{b} + 3\vec{c}}{5}) - 2\vec{b}}{3} \Rightarrow \vec{OF} = \frac{\vec{a} - \vec{b} + 3\vec{c}}{3}$$

$$\Rightarrow \text{P.V. of F is } \frac{\vec{a} - \vec{b} + 3\vec{c}}{3}$$

22. (a) Given that D divides BC in the ratio 2 : 3, E divides CA in the ratio 2 : 1 and P divides DE in the ratio 3:5. According to question,

$$\vec{d} = \frac{2\vec{c} + 3\vec{b}}{5} \Rightarrow 5\vec{d} = 2\vec{c} + 3\vec{b} \quad \dots(i)$$

$$\vec{e} = \frac{2\vec{a} + \vec{c}}{3} \Rightarrow 2\vec{a} + \vec{c} = 3\vec{e} \quad \dots(ii)$$

Multiply equation (ii) by 2, we get

$$4\vec{a} + 2\vec{c} = 6\vec{e} \Rightarrow 2\vec{c} = 6\vec{e} - 4\vec{a}$$

Solve (i) and (ii), $5\vec{d} - 3\vec{b} = 6\vec{e} - 4\vec{a}$

$$\Rightarrow 5\vec{d} + 3\vec{e} = 3\vec{b} - 4\vec{a} + 9\vec{e} \quad \dots(iii)$$

$$\text{Now, } \vec{p} = \frac{3\vec{e} + 5\vec{d}}{3 + 5} = \frac{1}{8}(3\vec{b} - 4\vec{a} + 9\vec{e}) \quad \{\text{from (iii)}\}$$

$$\vec{p} = \frac{1}{8}[(3\vec{b} - 4\vec{a} + 3(2\vec{a} + \vec{c}))] \quad \{\text{from (ii)}\}$$

$$\vec{p} = \frac{1}{8}(2\vec{a} + 3\vec{b} + 3\vec{c})$$

23. (b) Given three position vectors of the vertices of triangle ABC are $2\hat{i} + 4\hat{j} - 5\hat{k}$, $\hat{i} + \hat{j} + \hat{k}$ and $\hat{j} + 2\hat{k}$ respectively.

$$\vec{BA} = (2 - 1)\hat{i} + (4 - 1)\hat{j} + (-5 - 1)\hat{k} = \hat{i} + 3\hat{j} - 6\hat{k}$$

$$\vec{CA} = (2 - 0)\hat{i} + (4 - 1)\hat{j} + (-5 - 2)\hat{k} = 2\hat{i} + 3\hat{j} - 7\hat{k}$$

$$\text{Position vector } \vec{AD} = \frac{1}{2}(\vec{BA} + \vec{CA})$$

$$= \frac{1}{2}(3\hat{i} + 6\hat{j} - 13\hat{k})$$

Unit vector along $\widehat{AD} = \frac{\overline{AD}}{|\overline{AD}|}$

$$\widehat{AD} = \frac{\frac{1}{2}(3\hat{i} + 6\hat{j} - 13\hat{k})}{\frac{\sqrt{9+36+169}}{2}} = \frac{3\hat{i} + 6\hat{j} - 13\hat{k}}{\sqrt{214}}$$

24. (b) Given vectors are $\overline{AB} = 2\hat{i} + 2\hat{j} + \hat{k}$
and $\overline{AC} = 2\hat{i} + 4\hat{j} + 4\hat{k}$
Centroid G of triangle has position vector \overline{AG} can be obtained by putting A as O, B as $2\hat{i} + 2\hat{j} + \hat{k}$ and C as $2\hat{i} + 4\hat{j} + 4\hat{k}$.

So, $\overline{AG} = \frac{\vec{O} + \vec{B} + \vec{C}}{3} = \frac{0 + 2\hat{i} + 2\hat{j} + \hat{k} + 2\hat{i} + 4\hat{j} + 4\hat{k}}{3}$

$\overline{AG} = \frac{4\hat{i} + 6\hat{j} + 5\hat{k}}{3}$, Now, $\frac{22}{7} \times |\overline{AG}|^2 + 5 = 33 + 5 = 38$

25. (a) We have, $OA = a$, $OB = b$ be two non-collinear vectors,

$\overline{OP} = x_1\vec{a} + y_1\vec{b}$, $\overline{OQ} = x_2\vec{a} + y_2\vec{b}$

where, $x_1 = \frac{-3}{4}$, $x_2 = \frac{1}{3}$, $y_1 = \frac{7}{4}$, $y_2 = \frac{5}{3}$

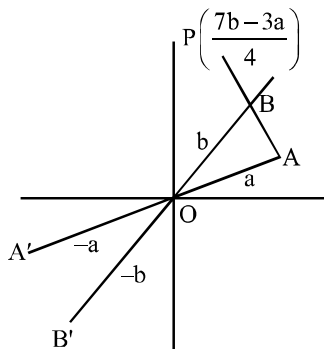
$\overline{OP} = \frac{-3}{4}\vec{a} + \frac{7}{4}\vec{b} = \frac{-3\vec{a} + 7\vec{b}}{4} = \frac{7\vec{b} - 3\vec{a}}{7-3}$

$\therefore OP$ divides AB externally in ratio $m : n = 7 : 3$.

$\overline{OQ} = \frac{1}{3}\vec{a} + \frac{5}{3}\vec{b} = \frac{\vec{a} + 5\vec{b}}{3}$

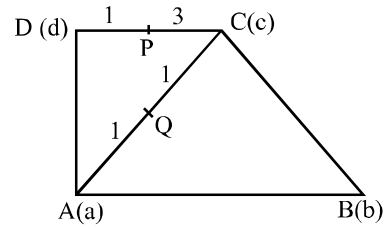
Also,

$\overline{A'O} = \overline{OA} = \vec{a}$, $\overline{OA'} = -\vec{a}$; $\overline{B'O} = \overline{OB} = \vec{b}$, $\overline{OB'} = -\vec{b}$



Clearly we can observe from the figure that, P lies inside the $\Delta A'OB$ and Q lies outside the ΔAOB .

26. (d) In quadrilateral $ABCD$, the point P divides DC in the ratio 1 : 3 internally & Q is the mid point of AC .



We have, $\overline{OP} = \frac{c+3d}{4}$ and $\overline{OQ} = \frac{a+c}{2}$

It is given that $AB + AD + BC - 2DC = \lambda PQ$

$(b-a) + (d-a) + (c-b) - 2(c-d)$

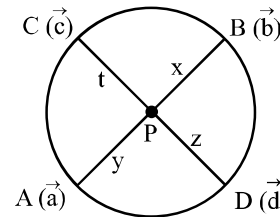
$= \lambda \left[\frac{a+c}{2} - \frac{c-3d}{4} \right] - 2a - c + 3d = \frac{\lambda}{4} [2a + c - 3d]$

$\therefore \frac{2\lambda}{4} = -2 \Rightarrow \lambda = -4$

27. (b) Given, $A(\vec{a}), B(\vec{b}), C(\vec{c}), D(\vec{d})$ are concyclic points and $x\vec{a} + y\vec{b} + z\vec{c} + t\vec{d} = 0$

$x + y + z + t = 0 \Rightarrow x + y = -(z + t)$; $\frac{x\vec{a} + y\vec{b}}{x + y} = -\frac{(z\vec{c} + t\vec{d})}{x + y}$

Here x, y, z, t not all zero at a time



$\frac{x\vec{a} + y\vec{b}}{x + y} = \frac{(z\vec{c} + t\vec{d})}{z + t}$; $P = \frac{x\vec{a} + y\vec{b}}{x + y}$ or $P = \frac{z\vec{c} + t\vec{d}}{z + t}$

$AP = \frac{x\vec{a} + y\vec{b}}{x + y} - \vec{a} = x(\vec{a} - \vec{b})$; $BP = \frac{x\vec{a} + y\vec{b}}{x + y} - \vec{b} = y(\vec{a} - \vec{b})$

$CP = \frac{z\vec{c} + t\vec{d}}{z + t} - \vec{c} = t(\vec{d} - \vec{c})$; $DP = \frac{z\vec{c} + t\vec{d}}{z + t} - \vec{d} = z(\vec{d} - \vec{c})$

$\therefore AP \times BP = CP \times DP$ [By similarity]

$|x(\vec{a} - \vec{b})| |y(\vec{a} - \vec{b})| = |t(\vec{d} - \vec{c})| |z(\vec{d} - \vec{c})|$

$|xy| |\vec{a} - \vec{b}|^2 = |zt| |\vec{d} - \vec{c}|^2$

28. (b) Let vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; Let $\vec{b} = 6\hat{i} + 8\hat{j}$

Hence, $\lambda \left(\frac{\vec{a}}{|\vec{a}|} + \frac{6\hat{i} + 8\hat{j}}{\sqrt{6^2 + 8^2}} \right) = \lambda \left(\frac{\vec{a}}{|\vec{a}|} + \frac{6\hat{i} + 8\hat{j}}{10} \right)$

$= 19\hat{i} + 22\hat{j} + 5\hat{k} \Rightarrow \frac{\vec{a}}{|\vec{a}|} = \frac{19\hat{i} + 22\hat{j} + 5\hat{k}}{\lambda} - \left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j} \right)$

$$= \left(\frac{19}{\lambda} - \frac{3}{5}\right)\hat{i} + \left(\frac{22}{\lambda} - \frac{4}{5}\right)\hat{j} + \frac{5}{\lambda}\hat{k}$$

$\therefore \frac{\vec{a}}{|\vec{a}|}$ is unit vector along \vec{a} , so

$$\left(\frac{19}{\lambda} - \frac{3}{5}\right)^2 + \left(\frac{22}{\lambda} - \frac{4}{5}\right)^2 + \left(\frac{5}{\lambda}\right)^2 = 1$$

$$\frac{361}{\lambda^2} + \frac{484}{\lambda^2} + \frac{25}{\lambda^2} + \frac{9}{25} + \frac{16}{25} - \frac{114}{5\lambda} - \frac{176}{5\lambda} = 1$$

$$\Rightarrow \frac{870}{\lambda^2} - \frac{290}{5\lambda} = 0 \Rightarrow \lambda = 15$$

$$\text{So, } \frac{\vec{a}}{|\vec{a}|} = \left(\frac{19}{15} - \frac{3}{5}\right)\hat{i} + \left(\frac{22}{15} - \frac{4}{5}\right)\hat{j} + \frac{\hat{k}}{3}$$

$$= \frac{10}{15}\hat{i} + \frac{10}{15}\hat{j} + \frac{\hat{k}}{3} = \frac{1}{3}(2\hat{i} + 2\hat{j} + \hat{k})$$

Hence, option (b) is correct.

29. (a) Given that $\vec{OA} = \vec{a} + \vec{b} + \vec{c}$ and $\vec{OB} = \vec{a} - 2\vec{b} + 3\vec{c}$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= -3\vec{b} + 2\vec{c}, |\vec{AB}| = \sqrt{a+x} = \sqrt{13}$$

$$\vec{OP} = \frac{m(\vec{OB}) + n(\vec{OA})}{m+n}$$

$$\vec{OP} = \frac{1(\vec{a} - 2\vec{b} + 3\vec{c}) + 3(\vec{a} + \vec{b} + \vec{c})}{4} = \frac{4\vec{a} + \vec{b} + 6\vec{c}}{4}$$

$$\vec{OQ} = \frac{1(\vec{a} - 2\vec{b} + 3\vec{c}) - 3(\vec{a} + \vec{b} + \vec{c})}{-2} = \frac{-2\vec{a} - 5\vec{b}}{-2}$$

$$\vec{OQ} = \frac{4\vec{a} + 10\vec{b}}{4}$$

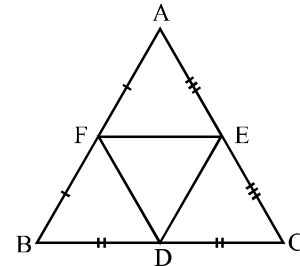
$$\vec{PQ} = \vec{OQ} - \vec{OP} = \frac{4\vec{a} + 10\vec{b}}{4} - \frac{4\vec{a} + \vec{b} + 6\vec{c}}{4} = \frac{9\vec{b} - 6\vec{c}}{4}$$

$$|\vec{PQ}| = \sqrt{\left(\frac{9}{4}\right)^2 + \left(-\frac{6}{4}\right)^2} = \sqrt{\frac{81}{16} + \frac{36}{16}} = \frac{\sqrt{117}}{4}$$

$$|\vec{PQ}| = \frac{3\sqrt{13}}{4} = \frac{3}{4}|\vec{AB}| \Rightarrow 3|\vec{AB}| = 4|\vec{PQ}|$$

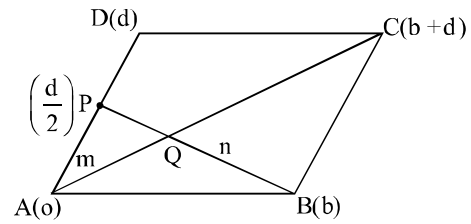
30. (b) Given that, (1, 0, 3), (2, 1, 5), (-2, 3, 6) are the midpoint of the sides of triangle.

As we know that, centroid of ΔABC is also the centroid of ΔDEF .



$$\therefore \text{Centroid, } G = \left(\frac{1+2-2}{3}, \frac{0+1+3}{3}, \frac{3+5+6}{3}\right) = \left(\frac{1}{3}, \frac{4}{3}, \frac{14}{3}\right)$$

31. (a) ABCD is a parallelogram.



Let Q divides AC and BP in the ratio $m : 1$ and $n : 1$ respectively.

Now, point

$$Q = \frac{m(b+d) + 1(0)}{m+1} = \frac{m(b+d)}{m+1} = \frac{m}{m+1}b + \frac{m}{m+1}d \quad \dots(i)$$

$$\text{and } Q = \frac{n\left(\frac{d}{2}\right) + b}{n+1} = \frac{1}{n+1}b + \frac{n}{2(n+1)}d \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{m}{m+1} = \frac{1}{n+1} \text{ and } \frac{m}{m+1} = \frac{n}{2(n+1)}$$

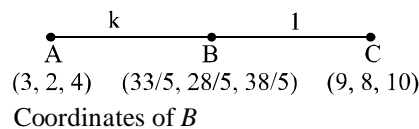
$$\Rightarrow \frac{1}{n+1} = \frac{n}{2(n+1)} \Rightarrow n = 2; \therefore \frac{m}{m+1} = \frac{1}{2+1} = \frac{1}{3}$$

$$3m = m + 1; 2m = 1 \Rightarrow m = \frac{1}{2}$$

Hence, the required ratio

$$= AQ : QC = m : 1 = \frac{1}{2} : 1 = 1 : 2.$$

32. (d) Let point $B\left(\frac{33}{5}, \frac{28}{5}, \frac{38}{5}\right)$ divides the line segment \vec{AC} in the ratio $k : 1$, where points $A(3, 2, 4)$ and $C(9, 8, 10)$.



$$= \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}, \frac{kz_2 + z_1}{k+1} \right)$$

$$\left(\frac{33}{5}, \frac{28}{5}, \frac{38}{5} \right) = \left(\frac{9k+3}{k+1}, \frac{8k+2}{k+1}, \frac{10k+4}{k+1} \right)$$

Now, comparing the respective x, y and z components, we get

$$\therefore \frac{9k+3}{k+1} = \frac{33}{5} \Rightarrow 45k+15 = 33k+33$$

$$\Rightarrow 12k = 18 \Rightarrow k = \frac{3}{2} \quad \dots(i)$$

and $\frac{8k+2}{k+1} = \frac{28}{5} \Rightarrow 40k+10 = 28k+28$

$$\Rightarrow 12k = 18 \Rightarrow k = \frac{3}{2} \quad \dots(ii)$$

and $\frac{10k+4}{k+1} = \frac{38}{5} \Rightarrow 50k+20 = 38k+38$

$$\Rightarrow 12k = 18 \Rightarrow k = \frac{3}{2} \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we get $k = \frac{3}{2}$

Hence, $k : 1 = \frac{3}{2} : 1 = 3 : 2$

33. (a) According to question from fig.

$$\Delta AQP \sim \Delta CQB$$

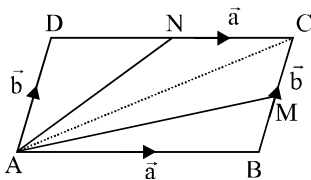
$$\Rightarrow \frac{AQ}{CQ} = \frac{QP}{QB} = \frac{AP}{CB}$$

$$\frac{AP}{AD} = \frac{AP}{CB} = \frac{3}{4} \quad (AD = CB)$$

$$\therefore \frac{AQ}{CQ} = \frac{3}{4}$$

34. (c) Given, M and N are the mid-points of BC and CD.

$$\therefore BM = \frac{1}{2}\vec{b} \quad \text{and} \quad DN = \frac{1}{2}\vec{a}$$



Now, in ΔABM , $AM = AB + BM = \vec{a} + \frac{1}{2}\vec{b}$

and in ΔADN , $AN = AD + DN = \vec{b} + \frac{1}{2}\vec{a}$

$$\therefore \vec{AM} + \vec{AN} = \vec{a} + \frac{1}{2}\vec{b} + \vec{b} + \frac{1}{2}\vec{a}$$

$$= \frac{3}{2}\vec{a} + \frac{3}{2}\vec{b} = \frac{3}{2}(\vec{a} + \vec{b})$$

$$\Rightarrow \vec{AM} + \vec{AN} = \frac{3}{2}\vec{AC} \quad [\because AC = \vec{a} + \vec{b}]$$

35. (c) $\therefore \vec{AB} = x\vec{AC} + y\vec{AD}$

$$(\lambda - 2)\hat{i} + 4\hat{j} + 5\hat{k} = x(-6\hat{i} + 2\hat{j} + 3\hat{k}) + y(-3\hat{i} - 3\hat{j} + 4\hat{k})$$

$$(\lambda - 2) = -6x - 3y \dots(i); 4 = 2x - 3y \dots(ii); 5 = 3x + 4y \dots(iii)$$

Solving equations (ii) and (iii), we get

$$x = \frac{31}{17} \quad \text{and} \quad y = \frac{-2}{17}$$

Putting in (i) we get $\lambda = \frac{-146}{17}$

$$\text{Now, } 17(\lambda + 9) = 17 \left[\frac{-146}{17} + 9 \right] = 7$$

36. (c) Let position vector of point P is

$$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}; \quad \vec{OA} = \hat{i} + 2\hat{j} - 2\hat{k} \quad (\text{Given})$$

$$\text{Since, } \vec{AP} = \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$$

$$\Rightarrow (x-1)\hat{i} + (y-2)\hat{j} + (z+2)\hat{k} = \lambda(2\hat{i} + 3\hat{j} - 3\hat{k})$$

On comparing we get

$$x - 1 = 2\lambda, \quad y - 2 = -3\lambda, \quad z + 2 = -6\lambda$$

$$\text{Given } AP = 21; \quad \sqrt{(x-1)^2 + (y-2)^2 + (z+2)^2} = 21$$

$$\sqrt{4\lambda^2 + 9\lambda^2 + 36\lambda^2} = 21 \Rightarrow \lambda = \pm 3 \Rightarrow x = \pm 6 + 1 = 7 \text{ or } -5$$

$$y = \pm 9 + 2 = -7 \text{ or } 11; \quad z = \pm 18 - 2 = -20 \text{ or } 16$$

$$\vec{OP} = 7\hat{i} - 7\hat{j} - 20\hat{k} \quad \text{or} \quad -5\hat{i} + 11\hat{j} + 16\hat{k}$$

37. (a) $\vec{a} = 4\hat{i} + 5\hat{j} - 3\hat{k}$, $\vec{b} = 6\hat{i} - 2\hat{j} - 2\hat{k}$

Magnitude of component of \vec{b} parallel to $\vec{a} = \frac{|\vec{b} \cdot \vec{a}|}{|\vec{a}|}$

$$= \frac{|(4\hat{i} + 5\hat{j} - 3\hat{k}) \cdot (6\hat{i} - 2\hat{j} - 2\hat{k})|}{\sqrt{50}} = \frac{20}{\sqrt{50}} = 2\sqrt{2}$$

38. (b) $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2 \Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$$

$$\begin{aligned}
 &= (\vec{a} - \vec{b})(\vec{a} - \vec{b}) \\
 &\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} \\
 &4\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \cdot \vec{b} = 0 \\
 &\therefore \vec{a} \text{ and } \vec{b} \text{ are perpendicular.}
 \end{aligned}$$

39. (c) Given that $|\vec{a}| = |\vec{b}| = 1$.

$$\begin{aligned}
 \therefore |\vec{a} + \vec{b}|^2 &= (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} \\
 1 &= 1 + 1 + 2|\vec{a}||\vec{b}|\cos\theta \\
 \Rightarrow -1 &= 2\cos\theta \Rightarrow \cos\theta = -\frac{1}{2} = \cos\left(\pi - \frac{\pi}{3}\right) \Rightarrow \theta = \frac{2\pi}{3}
 \end{aligned}$$

40. (b) $\vec{a} + \lambda\vec{b} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$

$$= \hat{i}(1 + \lambda) + (2 - \lambda)\hat{j} + (1 + 4\lambda)\hat{k}$$

$\therefore \vec{a} + \lambda\vec{b}$ is perpendicular to \vec{c} .

$$\therefore (\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$$

$$\begin{aligned}
 \Rightarrow [(1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (1 + 4\lambda)\hat{k}] \cdot [\hat{i} + \hat{j} + \hat{k}] &= 0 \\
 \Rightarrow 1 + \lambda + 2 - \lambda + 1 + 4\lambda &= 0 \Rightarrow \lambda = -1
 \end{aligned}$$

41. (c) The component of \hat{i} in the direction of the vector $\hat{i} + \hat{j} + 2\hat{k}$

$$= \frac{1}{\sqrt{1+1+4}} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

42. (c) Given $|\vec{a}| = 2, |\vec{b}| = 3, \theta = 120^\circ$

$$\text{Now, } \left[\frac{\vec{a}}{2} - \frac{\vec{b}}{3} \right]^2 = \frac{1}{4}|\vec{a}|^2 + \frac{|\vec{b}|^2}{9} - \frac{2(\vec{a} \cdot \vec{b})}{6}$$

$$= 1 + 1 - \frac{1}{3}(|\vec{a}||\vec{b}|\cos\theta)$$

$$= 1 + 1 - \frac{1}{3}(6)\left(-\frac{1}{2}\right) = 3$$

43. (b) Given $\vec{\alpha} = \hat{i} - 3\hat{j}, \vec{\beta} = \hat{i} + 2\hat{j} - \hat{k}$

$$\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$$

$$\text{Since } \vec{\beta}_1 \parallel \vec{\alpha} \Rightarrow \vec{\beta}_1 = m\vec{\alpha} = m(\hat{i} - 3\hat{j})$$

Where $m = \text{constant}$.

$$\text{Hence, } \vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1 = (1 - m)\hat{i} + (2 + 3m)\hat{j} - \hat{k}$$

$$\text{Since, } \vec{\beta}_2 \perp \vec{\alpha} \Rightarrow \vec{\beta}_2 \cdot \vec{\alpha} = 0$$

$$\Rightarrow (1 - m) \cdot 1 + (-3)(2 + 3m) = 0$$

$$\Rightarrow m = -\frac{1}{2}$$

$$\Rightarrow \vec{\beta}_1 = -\frac{1}{2}(\hat{i} - 3\hat{j}) \Rightarrow \text{No option is correct}$$

44. (b) Let \vec{a} make equal angle α with co-ordinate axes.
Hence, $l = m = n = \cos\alpha$

$$\Rightarrow l^2 + m^2 + n^2 = 1 \Rightarrow 3l^2 = 1 \Rightarrow l = \frac{1}{\sqrt{3}}$$

$$\text{Hence, } \hat{a} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$$

Now, the projection of $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ on \vec{a} is $\vec{b} \cdot \hat{a}$

$$= (5\hat{i} + 7\hat{j} - \hat{k}) \cdot \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k}) = \frac{11}{\sqrt{3}}$$

45. (d) $\vec{a} \cdot \vec{b} = 0$ given

$$\text{Now } (\vec{a} + \vec{b}) \cdot (\vec{a}) = |\vec{a} + \vec{b}| \cdot |\vec{a}| \cos 60^\circ$$

$$\Rightarrow |\vec{a}|^2 + \vec{a} \cdot \vec{b} = \frac{1}{2}|\vec{a} + \vec{b}| \cdot |\vec{a}|$$

$$\Rightarrow |\vec{a}| \|\vec{a}\| = \frac{1}{2}|\vec{a} + \vec{b}|$$

$$\Rightarrow 2|\vec{a}| = |\vec{a} + \vec{b}|$$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = 4|\vec{a}|^2$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a} \cdot \vec{b}) = 4|\vec{a}|^2$$

$$\Rightarrow 3|\vec{a}|^2 = |\vec{b}|^2 \Rightarrow |\vec{b}| = \sqrt{3}|\vec{a}|$$

46. (b) Since \vec{a} and \vec{b} are unit vectors

$$\text{Hence, } |\vec{a}| = |\vec{b}| = 1$$

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a} \cdot \vec{b})$$

$$= 1 + 1 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$|\vec{a} - \vec{b}|^2 = 2(1 - \cos\theta) = 4\sin^2\frac{\theta}{2}$$

$$\Rightarrow \sin\frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{2} = \left| \frac{\vec{a} - \vec{b}}{2} \right|$$

47. (a) $\theta = \frac{2\pi}{3}$

The projection of \vec{a} in the direction

$$\vec{b} = -2$$

$$\Rightarrow \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = -2 \Rightarrow \frac{|\vec{a}| |\vec{b}| \cos \frac{2\pi}{3}}{|\vec{a}| |\vec{b}|} = -2$$

$$\Rightarrow |\vec{a}| = 4$$

48. (b) Given $|\vec{a}| = |\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 0$

$$\text{Now, } (3\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 6\vec{b})$$

$$= 15|\vec{a}|^2 - 18(\vec{a} \cdot \vec{b}) + 10(\vec{b} \cdot \vec{a}) - 12|\vec{b}|^2$$

$$= 15 - 12 = 3$$

49. (c) Since \vec{a} and \vec{b} are orthogonal

$$\text{Hence, } \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow 2 + 2\lambda + 3 = 0 \Rightarrow \lambda = -\frac{5}{2}$$

50. (d) $2\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}|$

$$\Rightarrow 2|\vec{a}| |\vec{b}| \cos \theta = |\vec{a}| |\vec{b}|$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

51. (a) Given $|\vec{a}| = 1, |\vec{b}| = 1$

$$(\sqrt{3}\vec{a} - \vec{b}) \cdot (\sqrt{3}\vec{a} - \vec{b}) = 1$$

(since $(\sqrt{3}\vec{a} - \vec{b})$ is a unit vector)

$$\Rightarrow 3|\vec{a}|^2 + |\vec{b}|^2 - 2\sqrt{3}(\vec{a} \cdot \vec{b}) = 1$$

$$\Rightarrow 2\sqrt{3}|\vec{a}| |\vec{b}| \cos \theta = 3$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$$

52. (c) $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$

$$\Rightarrow (\vec{a} + \vec{b}) = -\vec{c}$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = \vec{c} \cdot \vec{c}$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}| |\vec{b}| \cos \theta = |\vec{c}|^2$$

$$\Rightarrow 9 + 25 + 30 \cos \theta = 49$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

53. (c) Given, $|\vec{a}| = a = 3, |\vec{b}| = b = 4, |\vec{c}| = c = 5$

Also given

$$\vec{a} \cdot (\vec{b} + \vec{c}) = 0 \Rightarrow \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0 \quad \dots(i)$$

$$\vec{b} \cdot (\vec{a} + \vec{c}) = 0 \Rightarrow \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{c} = 0 \quad \dots(ii)$$

$$\vec{c} \cdot (\vec{a} + \vec{b}) = 0 \Rightarrow \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} = 0 \quad \dots(iii)$$

(i) + (ii) + (iii) we get,

$$2[\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}] = 0$$

$$\text{Now, } |\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2[\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}]$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 9 + 16 + 25 + 0 = 50$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$$

54. (d) Given, $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{b} = 2\sqrt{3}\hat{i} - 2\sqrt{3}\hat{j} + \sqrt{3}\hat{k}$$

$$\therefore |\vec{a}| = \sqrt{2^2 + 1^2 + (-2)^2} = 3$$

$$|\vec{b}| = \sqrt{(2\sqrt{3})^2 + (-2\sqrt{3})^2 + (\sqrt{3})^2} = 3\sqrt{3}$$

$$\vec{a} \cdot \vec{b} = 2(2\sqrt{3}) + 1(-2\sqrt{3}) + (-2)(\sqrt{3}) = 0$$

$$\therefore \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{0}{(3)(3\sqrt{3})} = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

$$\therefore \vec{c} = \vec{b} - \vec{a} = (2 - 2\sqrt{3})\hat{i} + (1 + 2\sqrt{3})\hat{j} + (-2 - \sqrt{3})\hat{k}$$

$$|\vec{c}| = 6$$

\therefore Using law of cosines

$$a^2 = b^2 + c^2 - 2bc \cos \theta$$

$$\Rightarrow (3)^2 = (3\sqrt{3})^2 + 6^2 - 2(3\sqrt{3})(6) \cos \theta$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$$

$$\therefore \text{Hence, perimeter} = |\vec{a}| + |\vec{b}| + |\vec{c}|$$

$$= 3 + 3\sqrt{3} + 6 = 9 + 3\sqrt{3}$$

55. (d) We have $|\vec{a} + 2\vec{b}| = |2\vec{a} - \vec{b}|$

$$\Rightarrow |\vec{a} + 2\vec{b}|^2 = |2\vec{a} - \vec{b}|^2$$

$$\Rightarrow (\vec{a} + 2\vec{b}) \cdot (\vec{a} + 2\vec{b}) = (2\vec{a} - \vec{b}) \cdot (2\vec{a} - \vec{b})$$

$$\Rightarrow |\vec{a}|^2 + 4(\vec{a} \cdot \vec{b}) + 4|\vec{b}|^2 = 4|\vec{a}|^2 - 4(\vec{a} \cdot \vec{b}) + |\vec{b}|^2$$

Given $|\vec{a}| = |\vec{b}|$

$$\Rightarrow |\vec{a}|^2 + 4(\vec{a} \cdot \vec{b}) + 4|\vec{a}|^2 = 4|\vec{a}|^2 - 4(\vec{a} \cdot \vec{b}) + |\vec{a}|^2$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0$$

Since, $\vec{a} \cdot \vec{b} = 0$ & $\vec{a}, \vec{b} \neq 0$, the angle between them is 90° .

Given \vec{c} is parallel to vector \vec{a}

\therefore The angle between \vec{b} and \vec{c} is the same as the angle between \vec{b} and \vec{a}

56. (b) $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ & $\vec{b} = 9\hat{i} + 6\hat{j} - 18\hat{k}$

$$\text{Projection of } \vec{b} \text{ on } \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{-39}{3} = -13$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{-39}{21} = \frac{-13}{7}$$

57. (d) Let $\vec{r} = x\hat{i} + 4\hat{j} + z\hat{k}$

$$\vec{r} \cdot \vec{a} = 0 \Rightarrow x + 2y + 3z = 0 \quad \dots(i)$$

$$\vec{r} \cdot \vec{b} = -2 \Rightarrow 2x - 3y + z = -2 \quad \dots(ii)$$

$$\vec{r} \cdot \vec{c} = 6 \Rightarrow 3x + y - 2z = 6 \quad \dots(iii)$$

From eq. (i), (ii) and (iii), we get
 $x = 1, y = 1, z = -1$

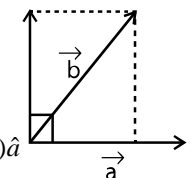
$$\vec{r} = \hat{i} + \hat{j} - \hat{k}$$

$$\begin{aligned} \therefore \vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) &= (\hat{i} + \hat{j} - \hat{k}) \cdot (3\hat{i} + \hat{j} + \hat{k}) \\ &= (3 + 1 - 1) = 3 \end{aligned}$$

58. (c) $\vec{a} = \hat{i} + \sqrt{11}\hat{j} - 2\hat{k}$

$$\vec{b} = \hat{i} + \sqrt{11}\hat{j} - 10\hat{k}$$

Component of $\vec{b} \perp \vec{a} = \vec{b} - (\vec{b} \cdot \vec{a})\hat{a}$

$$= \vec{b} - \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} \cdot \frac{\vec{a}}{|\vec{a}|} = \vec{b} - \frac{(\vec{b} \cdot \vec{a})\vec{a}}{|\vec{a}|^2}$$


component of \vec{b} along direction of \vec{a}

Component of \vec{b} along $\vec{a} = (\vec{b} \cdot \hat{a})\hat{a}$

$$= \frac{(\vec{b} \cdot \vec{a})\vec{a}}{|\vec{a}|^2}$$

$$\therefore \text{Component of } \vec{b} \perp \vec{a} = \vec{b} - \frac{(\vec{b} \cdot \vec{a})\vec{a}}{|\vec{a}|^2}$$

$$= (\hat{i} + \sqrt{11}\hat{j} - 10\hat{k}) - \frac{(1 + 11 + 20)}{(\sqrt{1^2 + 11 + 4})^2} (\hat{i} + \sqrt{11}\hat{j} - 2\hat{k})$$

$$= \hat{i} + \sqrt{11}\hat{j} - 10\hat{k} - \frac{32}{16} (\hat{i} + \sqrt{11}\hat{j} - 2\hat{k})$$

$$= \hat{i} + \sqrt{11}\hat{j} - 10\hat{k} - 2\hat{i} - 2\sqrt{11}\hat{j} + 4\hat{k}$$

$$= -\hat{i} - \sqrt{11}\hat{j} - 6\hat{k} = -(\hat{i} + \sqrt{11}\hat{j} + 6\hat{k})$$

59. (b) $(\vec{a}, \vec{b}) = 60^\circ$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\Rightarrow \cos 60^\circ = \frac{(\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j} + p\hat{k})}{\left| \sqrt{1^2 + 2^2 + 2^2} \right| \left| \sqrt{2^2 + (-1)^2 + (p)^2} \right|}$$

$$\Rightarrow \frac{1}{2} = \frac{2 - 2 + 2p}{3\sqrt{5 + p^2}} \Rightarrow \frac{1}{2} = \frac{2p}{3\sqrt{5 + p^2}}$$

$$\Rightarrow 9(5 + p^2) = (4p)^2$$

$$\Rightarrow 9p^2 + 45 = 16p^2$$

$$\Rightarrow 7p^2 = 45$$

$$\Rightarrow p = \pm \sqrt{\frac{45}{7}} = \pm \frac{3\sqrt{5}}{\sqrt{7}}$$

60. (b) $a = |\vec{a}|, b = |\vec{b}|$

$$\left(\frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2} \right)^2 = \left(\frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2} \right) \cdot \left(\frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2} \right)$$

$$\frac{\vec{a} \cdot \vec{b}}{a^4} + \frac{\vec{b} \cdot \vec{b}}{b^4} - \frac{2\vec{a} \cdot \vec{b}}{a^2 b^2} = \frac{a^2}{a^4} + \frac{b^2}{b^4} - \frac{2ab \cos \theta}{a^2 b^2}$$

$$= \frac{1}{a^2} + \frac{1}{b^2} - \frac{2 \cos \theta}{ab}$$

Now, $\left(\frac{\vec{a} - \vec{b}}{ab}\right)^2$

$$= \frac{(\vec{a} - \vec{b})(\vec{a} - \vec{b})}{a^2 b^2} = \frac{a^2 + b^2 - 2ab \cos \theta}{a^2 b^2}$$

$$= \frac{1}{a^2} + \frac{1}{b^2} - \frac{2 \cos \theta}{ab}$$

61. (a) Given, $|\vec{a}| = 5, |\vec{b}| = 12$

$$|\vec{a} - \vec{b}| = 13$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos \theta = 169$$

$$\Rightarrow \cos \theta = 0 \Rightarrow \theta = 90^\circ$$

So, $|2\vec{a} + \vec{b}|^2$

$$= 4|\vec{a}|^2 + |\vec{b}|^2 + 4|\vec{a}||\vec{b}|\cos \theta$$

$$= 100 + 144 + 0$$

$$|2\vec{a} + \vec{b}|^2 = 244$$

Hence, $|2\vec{a} + \vec{b}| = \sqrt{244} = 2\sqrt{61}$

62. (c) $\therefore |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$

$$\therefore 37 = 9 + 16 + 2\vec{a} \cdot \vec{b}$$

$$\Rightarrow 12 = 2\vec{a} \cdot \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 6 \Rightarrow |\vec{a}||\vec{b}|\cos \theta = 6$$

$$\Rightarrow 3 \cdot 4 \cos \theta = 6 \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

$$\therefore |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

$$\Rightarrow k^2 = 9 + 16 - 12 = 13$$

Now, $\frac{4}{13}(k \sin \theta)^2 = \frac{4}{13} \cdot k^2 \sin^2 \theta$

$$= \frac{4}{13} \cdot 13 \sin^2 60^\circ = 4 \cdot \frac{3}{4} = 3.$$

63. (c) $\cos \theta = \frac{(4\hat{i} - \hat{j} + 2\hat{k}) \cdot (\hat{i} + 3\hat{j} - 2\hat{k})}{\sqrt{21} \times \sqrt{14}} = \frac{-3}{7\sqrt{6}}$

$$\sin \theta = \sqrt{1 - \left(\frac{-3}{7\sqrt{6}}\right)^2} = \sqrt{1 - \frac{9}{49 \times 6}} = \frac{\sqrt{285}}{7\sqrt{6}}$$

$$\cos \theta < 0 \Rightarrow \theta \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow \sin \theta > 0$$

$$\sin 2\theta = 2 \cos \theta \cdot \sin \theta = 2 \times \left(\frac{-3}{7\sqrt{6}}\right) \times \frac{\sqrt{285}}{7\sqrt{6}} = \frac{-\sqrt{285}}{49}.$$

64. (c) $\vec{a} \perp \vec{b}, \vec{a} \perp \vec{c}$ angle b/w \vec{b} & $\vec{c} = \frac{2\pi}{3}$

$$|\vec{a} + 3\vec{b} - 4\vec{c}|^2 = (\vec{a} + 3\vec{b} - 4\vec{c}) \cdot (\vec{a} + 3\vec{b} - 4\vec{c})$$

$$= \left| \vec{a} \right|^2 + 3\vec{a} \cdot \vec{b} - 4\vec{a} \cdot \vec{c} + 3\vec{b} \cdot \vec{a} + 9\left| \vec{b} \right|^2 - 12\vec{b} \cdot \vec{c} - 4\vec{c} \cdot \vec{a} - 12\vec{b} \cdot \vec{c} + 16\left| \vec{c} \right|^2$$

$$= 1 + 9 + 16 - 24(1)(1) \cos \frac{2\pi}{3} = 38$$

65. (a) Equation of vector passing through \vec{a} and parallel to \vec{b} is $\vec{r} = \vec{a} + \lambda \vec{b}$;

$$\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$$

Projection of \vec{r} on $\vec{c} = \frac{\vec{r} \cdot \vec{c}}{|\vec{c}|} = \frac{9}{\sqrt{6}}$

$$\frac{(2 + \lambda) + (2\lambda - 1) - 2(1 - \lambda)}{\sqrt{6}} = \frac{9}{\sqrt{6}}$$

$$\Rightarrow 5\lambda - 1 = 9 \Rightarrow \lambda = 2 \therefore \vec{r} = 4\hat{i} + 3\hat{j} - \hat{k}$$

$$|\vec{r}| = \sqrt{4^2 + 3^2 + (-1)^2} = \sqrt{26}$$

66. (d) $|\vec{a}| = 4, |\vec{b}| = 5; |\vec{a} - \vec{b}| = 3 \Rightarrow |\vec{a} - \vec{b}|^2 = 9$

$$(\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) = 9; |\vec{a}|^2 - 2|\vec{a}||\vec{b}|\cos \theta + |\vec{b}|^2 = 9$$

$$16 + 25 - 2(4)(5)\cos \theta = 9$$

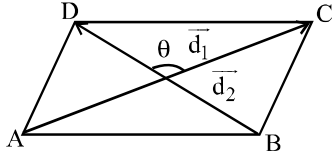
$$\cos \theta = \frac{4}{5} \Rightarrow \cot \theta = \frac{4}{3}; \cot^2 \theta = \frac{16}{9}$$

67. (a) Projection of \vec{b} on $\vec{a} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\right) \vec{a} = \frac{-4}{3} \vec{a}$

$$\vec{p} = \frac{-4}{3}(\hat{i} - 2\hat{j} + 2\hat{k}); \text{ Projection of } \vec{c} \text{ on } \vec{a} = \left(\frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2}\right) \vec{a}$$

$$\vec{q} = \frac{-13}{3} \vec{a} \therefore 13\vec{p} = 4\left(\frac{-13}{3}\right) \vec{a} = 4\vec{q}.$$

68. (b) The diagonals of parallelogram are represented by $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where \vec{a}, \vec{b} are adjacent sides of parallelogram



$$\therefore \vec{d}_1 = \vec{BC} + \vec{CD} = 3\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{d}_2 = \vec{BC} - \vec{CD} = \hat{i} - \hat{j} + 3\hat{k}$$

$$\therefore \cos \theta = \frac{\vec{d}_1 \cdot \vec{d}_2}{|\vec{d}_1| \cdot |\vec{d}_2|} = \frac{3-3-3}{\sqrt{19} \cdot \sqrt{11}} = \frac{-3}{\sqrt{209}}$$

$$\therefore \sin \theta = \frac{\sqrt{200}}{\sqrt{209}} \Rightarrow \tan \theta = \frac{-10\sqrt{2}}{3}$$

69. (a) $|\vec{a}| = 8, |\vec{b}| = 3; \vec{a} \cdot \vec{b} = 0$
 $|\vec{a} - 2\vec{b}|^2 = (\vec{a} - 2\vec{b}) \cdot (\vec{a} - 2\vec{b}) = |\vec{a}|^2 + 4|\vec{b}|^2 - 4\vec{a} \cdot \vec{b}$
 $= 64 + 36 = 100 \therefore |\vec{a} - 2\vec{b}| = 10.$

70. (d) $|\vec{a} - \vec{b}|^2 + |\vec{a} + 2\vec{b}|^2 = 20$
 $\Rightarrow (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) + (\vec{a} + 2\vec{b}) \cdot (\vec{a} + 2\vec{b}) = 20$
 $\Rightarrow |\vec{a}|^2 - 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 + |\vec{a}|^2 + 4\vec{a} \cdot \vec{b} + 4|\vec{b}|^2 = 20$
 $\Rightarrow 2|\vec{a}|^2 + 5|\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 20$
 $\Rightarrow 2(1)^2 + 5(2)^2 + 2(1)(2) \cos \theta = 20$
 $\Rightarrow 22 + 4 \cos \theta = 20 \Rightarrow \cos \theta = \frac{-1}{2} \therefore \theta = \frac{2\pi}{3}$

71. (a) Given $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}, \vec{b} = \hat{i} - \hat{j} + 3\hat{k}$

$$\text{Then, } \vec{x} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right) \cdot \vec{b}$$

$$\vec{x} = \left(\frac{(2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{j} + 3\hat{k})}{\left(\sqrt{(1)^2 + (-1)^2 + (3)^2} \right)^2} \right) \cdot (\hat{i} - \hat{j} + 3\hat{k})$$

$$\vec{x} = \left(\frac{2-1-3}{(1+1+9)} \right) (\hat{i} - \hat{j} + 3\hat{k}); \vec{x} = \frac{-2}{11} (\hat{i} - \hat{j} + 3\hat{k})$$

$$\vec{y} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \right) \cdot \vec{a} \Rightarrow \vec{y} = \left(\frac{(2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{j} + 3\hat{k})}{\left((2)^2 + (1)^2 + (-1)^2 \right)} \right) \cdot (2\hat{i} + \hat{j} - \hat{k})$$

$$\vec{y} = \left(\frac{2-1-3}{4+1+1} \right) \cdot (2\hat{i} + \hat{j} - \hat{k})$$

$$\vec{y} = \frac{-2}{6} (2\hat{i} + \hat{j} - \hat{k}) = \frac{-1}{3} (2\hat{i} + \hat{j} - \hat{k}).$$

$$\text{Angle between } \vec{a} \text{ and } \vec{b} \text{ is } \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

$$\cos \theta = \frac{(2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{j} + 3\hat{k})}{\sqrt{6} \sqrt{11}} = \frac{2-1-3}{\sqrt{66}} = \frac{-2}{\sqrt{66}}$$

$$\text{Take square both sides } \cos^2 \theta = \frac{4}{66} \quad \dots(i)$$

$$\text{Now, } x^2 + y^2 = \frac{4}{11^2} (1+1+9) + \frac{1}{9} (4+1+1)$$

$$x^2 + y^2 = \frac{34 \times 2}{33 \times 2} = \frac{68}{66}$$

$$x^2 + y^2 = 17 \times \frac{4}{66} = 17 \cos^2 \theta \quad \{\text{from (i)}\}$$

72. (a) Given $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ now component of \vec{b} perpendicular to \vec{a} is given as

$$= \vec{b} - \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \right) \vec{a}$$

$$= (2\hat{i} - 3\hat{j} + \hat{k}) - \left(\frac{(\hat{i} - 2\hat{j} - 2\hat{k}) \cdot (2\hat{i} - 3\hat{j} + \hat{k})}{(\sqrt{1+4+4})^2} \right) (\hat{i} - 2\hat{j} - 2\hat{k})$$

$$= 2\hat{i} - 3\hat{j} + \hat{k} - \frac{2}{3} (\hat{i} - 2\hat{j} - 2\hat{k})$$

$$= \frac{1}{3} (4\hat{i} - 5\hat{j} + 7\hat{k})$$

73. (d) Given vectors are $2\hat{i} - \hat{j} + 2\hat{k}$ and $a\hat{i} + 4\hat{j} + b\hat{k}$.

$$\text{Take, } \cos \theta = \frac{(2\hat{i} - \hat{j} + 2\hat{k}) \cdot (a\hat{i} + 4\hat{j} + b\hat{k})}{\sqrt{2^2 + (-1)^2 + 2^2} \sqrt{a^2 + 4^2 + b^2}}$$

$$\Rightarrow \frac{2}{3} = \frac{2(a-2+b)}{3\sqrt{a^2 + b^2 + 16}} \Rightarrow \sqrt{a^2 + b^2 + 16} = ((a+b) - 2)$$

$$a^2 + b^2 + 16 = (a+b)^2 + 4 - 4(a+b)$$

$$\Rightarrow 4a + 4b + 12 = 2ab \Rightarrow 2(a+b+3) = ab$$

74. (d) Now $\overline{AB} = \overline{OB} - \overline{OA} = 4\hat{i} + 7\hat{j} - 6\hat{k}$
 $\overline{CD} = \overline{OD} - \overline{OC} = -8\hat{i} - 14\hat{j} + 12\hat{k}$
 Now $\overline{AB} \cdot \overline{CD} = \sqrt{16+49+36} \cdot \sqrt{64+196+144} \cos \theta$
 $\Rightarrow -202 = 2 \times 101 \cos \theta \Rightarrow \cos \theta = -1 = \cos \pi \Rightarrow \theta = \pi$

75. (c) We have $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \Rightarrow 1 = \sqrt{3} |\vec{b}| \frac{1}{3} \Rightarrow |\vec{b}| = \sqrt{3}$
 Since components of \vec{b} w.r.t $(\hat{i}, \hat{j}, \hat{k})$ be integers
 $\therefore \vec{b} = \pm \hat{i} \pm \hat{j} \pm \hat{k}$

Also $\vec{a} \cdot \vec{b} = 1$ therefore take one negative sign.

So, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ or $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ or $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$

Hence, three possible vectors.

76. (b) $\therefore l = \vec{b} \cdot \vec{c} = 1 - 6 - 2 = -7; m = \vec{c} \cdot \vec{a} = 1 + 3 - 2 = 2$
 Now, $m\vec{b} + l\vec{a} = 2(\hat{i} - 2\hat{j} + \hat{k}) + (-7)(\hat{i} + \hat{j} + \hat{k})$
 $= -5\hat{i} - 11\hat{j} - 5\hat{k}$

Then $\begin{bmatrix} m\vec{b} + l\vec{a} & \vec{b} & \vec{c} & \vec{d} \end{bmatrix} = \begin{vmatrix} -5 & -11 & -5 \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -63$

77. (b) $\cos \theta = \frac{0.1 + (-3)(0) + 2(-2)}{\sqrt{0^2 + (-3)^2 + 2^2} \sqrt{1^2 + 0^2 + (-2)^2}} = \frac{-4}{\sqrt{65}}$

$\theta = \cos^{-1} \left(\frac{-4}{\sqrt{65}} \right)$

78. (b) Given $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \{(\vec{a})\}^2 = \vec{a} \cdot \vec{a}$
 $\Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = (\vec{a} + \vec{b} + \vec{c})(\vec{a} + \vec{b} + \vec{c})$
 $\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$
 $\Rightarrow 3^2 + 4^2 + 2^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$
 $\Rightarrow 29 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$
 $\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-29}{2}$
 $\therefore \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} + 2|\vec{a}| + |\vec{b}| + |\vec{c}|$
 $= \frac{-29}{2} + 2 \cdot (3 + 4 + 2) = \frac{-29 + 36}{2} = \frac{7}{2}$

79. (c) The orthogonal projection of \vec{a} on \vec{b} is $\vec{x} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$

And the orthogonal projection of \vec{b} on \vec{a} is $\vec{y} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$

$\therefore \vec{x} - \vec{y} = \vec{a} \cdot \vec{b} \left(\frac{\vec{b}}{|\vec{b}|^2} - \frac{\vec{a}}{|\vec{a}|^2} \right)$

$\therefore \vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$

$\therefore \vec{a} - \vec{b} = -\hat{i} + 3\hat{j}$ and $\vec{a} \cdot \vec{b} = 2 - 2 + 4 = 4$

and $|\vec{a}|^2 = 9 = |\vec{b}|^2 \therefore \vec{x} - \vec{y} = (-\hat{i} + 3\hat{j}) \frac{4}{9} = \frac{4}{9}(-\hat{i} + 3\hat{j})$

So, $|\vec{x} - \vec{y}| = \frac{4}{9} \sqrt{10}$

80. (b) \vec{d} is perpendicular to both \vec{a} and \vec{b} , then \vec{d} must be parallel to $\vec{a} \times \vec{b}$

So, $\vec{d} = \lambda(\vec{a} \times \vec{b})$

$(\vec{a} \times \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & -2 & -2 \end{vmatrix} = 4\hat{i} + 3\hat{j} - \hat{k}$

Given $|\vec{d} \times \vec{c}| = 14$

$\Rightarrow |\lambda| \cdot |(4, 3, -1) \times (6, 3, -2)| = 14$

$|\lambda| \cdot |(-3, 2, -6)| = 14$

$|\lambda| \cdot \sqrt{9+4+36} = 14$

$|\lambda| = 2$

$\therefore \vec{d} = \pm 2(4, 3, -1) = (8, 6, -2)$ or $(-8, -6, 2)$

$\therefore \vec{d} \cdot \vec{c} = (8, 6, -2) \cdot (6, 3, -2) = 70$

or $\vec{d} \cdot \vec{c} = (-8, -6, -2) \cdot (6, 3, -2) = -70$

$|\vec{d} \cdot \vec{c}| = 70$

81. (b) $A = 2\vec{a} + 3\vec{b} - \vec{c}, B = \vec{a} - 2\vec{b} + 3\vec{c}$,

$C = 3\vec{a} + 4\vec{b} - 2\vec{c}, D = \vec{a} - 6\vec{b} + 6\vec{c}$

$\vec{AB} = -\vec{a} - 5\vec{b} + 4\vec{c}, \vec{AC} = \vec{a} + \vec{b} - \vec{c}$

$\vec{AD} = -\vec{a} - 9\vec{b} + 7\vec{c}$

$[\vec{AB} \ \vec{AC} \ \vec{AD}] = \begin{vmatrix} -1 & -5 & 4 \\ 1 & 1 & -1 \\ -1 & -9 & 7 \end{vmatrix}$

$= -1[7-9] + 5[7-1] + 4[-9+1]$
 $= 2 + 30 - 32 = 0$

82. (d) $\vec{a} + 2\vec{b} = (\hat{i} - 2\hat{j} - 2\hat{k}) + 2(2\hat{i} + \hat{j} + 2\hat{k})$
 $= 5\hat{i} + 2\hat{k}$

$$3\vec{a} - \vec{b} = 3(\hat{i} - 2\hat{j} - 2\hat{k}) - (2\hat{i} + \hat{j} + 2\hat{k}) = \hat{i} - 7\hat{j} - 8\hat{k}$$

$$\text{So, } (\vec{a} + 2\vec{b}) \times (3\vec{a} - \vec{b})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 0 & 2 \\ 1 & -7 & -8 \end{vmatrix}$$

$$= 14\hat{i} + 42\hat{j} - 35\hat{k}$$

$$83. \text{ (b) } \vec{a} = \hat{i} + 2\hat{j} + \hat{k}; \vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$$

$$\text{Let } \vec{c} = x\hat{i} + y\hat{j} + z\hat{k}. \text{ Given } \vec{a} \times \vec{c} = \vec{b}$$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ x & y & z \end{vmatrix} = 3(\hat{i} - \hat{j} + \hat{k})$$

$$\text{On comparing } 2z - y = 3 \dots \text{(i); } z - x = 3 \dots \text{(ii)}$$

$$y - 2x = 3 \dots \text{(iii); Also, } \vec{a} \cdot \vec{c} = 3$$

$$\Rightarrow x + 2y + z = 3 \dots \text{(iv)}$$

$$\text{From (ii) and (iv) } x + y = 0 \dots \text{(v)}$$

$$\text{From (iii) and (iv)}$$

$$x = -1, y = 1, z = 2; \therefore \vec{c} = -\hat{i} + \hat{j} + 2\hat{k}$$

$$\text{Now, } \vec{c} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ 3 & -3 & 3 \end{vmatrix} = 9\hat{i} + 9\hat{j}$$

$$\vec{a} \cdot (\vec{c} \times \vec{b} - \vec{b} - \vec{c}) = (\hat{i} + 2\hat{j} + \hat{k}) \cdot (9\hat{i} + 9\hat{j} - 3\hat{i} + 3\hat{j} - 3\hat{k} + \hat{i} - \hat{j} - 2\hat{k})$$

$$= (\hat{i} + 2\hat{j} + \hat{k}) \cdot (7\hat{i} + 11\hat{j} - 5\hat{k}) = 7 + 22 - 5 = 24$$

$$84. \text{ (c) Here } \vec{a} \cdot \vec{b} = 4 \Rightarrow 4 = \sqrt{8+p^2} \times 7 \cos \theta$$

$$\Rightarrow \cos \theta = \frac{4}{7\sqrt{8+p^2}} \Rightarrow \sin \theta = \frac{\sqrt{376+49p^2}}{7\sqrt{8+p^2}}$$

$$\text{Now } |\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| |\sin \theta|$$

$$\Rightarrow 5\sqrt{17} = \sqrt{8+p^2} \times 7 \times \frac{\sqrt{376+49p^2}}{7\sqrt{8+p^2}}$$

$$\Rightarrow 25 \times 17 = 376 + 49p^2 \Rightarrow p^2 = 1 \Rightarrow p = \pm 1$$

$$85. \text{ (c) It is given that, } \vec{p} = 2\hat{i} - 3\hat{j} + \hat{k}, \vec{q} = \hat{i} + \hat{j} - \hat{k}$$

$$\therefore \vec{p} \cdot \vec{q} = (2\hat{i} - 3\hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) = 2 - 3 - 1 = -2$$

$$|\vec{p}| = \sqrt{(2)^2 + (-3)^2 + (1)^2} = \sqrt{4+9+1} = \sqrt{14}$$

$$|\vec{q}| = \sqrt{(1)^2 + (1)^2 + (-1)^2} = \sqrt{1+1+1} = \sqrt{3}$$

Now, \vec{a} = orthogonal projections of \vec{p} on \vec{q}

$$= \frac{\vec{p} \cdot \vec{q}}{|\vec{q}|^2} \vec{q} = \frac{-2}{3} (\hat{i} + \hat{j} - \hat{k})$$

\vec{b} = orthogonal projections of \vec{q} on \vec{p}

$$= \frac{\vec{q} \cdot \vec{p}}{|\vec{p}|^2} \vec{p} = \frac{-2}{14} (2\hat{i} - 3\hat{j} + \hat{k})$$

$$\text{Now, } \vec{a} \times \vec{b} = \frac{4}{42} (\hat{i} + \hat{j} - \hat{k}) \times (2\hat{i} - 3\hat{j} + \hat{k})$$

$$= \frac{4}{42} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -3 & 1 \end{vmatrix} = \frac{4}{42} [\hat{i}(1-3) - \hat{j}(1+\hat{i}) + \hat{k}(-3-2)]$$

$$= \frac{4}{42} (-2\hat{i} - 3\hat{j} - 5\hat{k}) \text{ and } \vec{a} \cdot \vec{b} = \frac{4}{42} (\hat{i} + \hat{j} - \hat{k}) \cdot (2\hat{i} - 3\hat{j} + \hat{k})$$

$$= \frac{4}{42} [(1,1,-1) \cdot (2,-3,1)] = \frac{4}{42} (2-3-1) = \frac{-8}{42}$$

$$\therefore \frac{\vec{a} \times \vec{b}}{\vec{a} \cdot \vec{b}} = \frac{\frac{4(-2\hat{i} - 3\hat{j} - 5\hat{k})}{42}}{\frac{-8}{42}} = \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2}$$

$$86. \text{ (a) We have,}$$

$$(\vec{a} + \vec{b}) \cdot \vec{p} = (\vec{a} + \vec{b}) \cdot \frac{\vec{b} \times \vec{c}}{\vec{a} \cdot (\vec{b} \times \vec{c})}$$

$$= \frac{\vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{b} \times \vec{c})}{\vec{a} \cdot (\vec{b} \times \vec{c})} = \frac{[\vec{a} \vec{b} \vec{c}] + [\vec{b} \vec{b} \vec{c}]}{[\vec{a} \vec{b} \vec{c}]}$$

$$= [\vec{a} \vec{b} \vec{c}] = 1 \quad [\because [\vec{b} \vec{b} \vec{c}] = 0]$$

$$\text{Similarly, } (\vec{b} + \vec{c}) \cdot \vec{q} = (\vec{c} + \vec{a}) \cdot \vec{r} = 1$$

$$\therefore \alpha = 1 + 1 + 1 = 3$$

$$\text{Now, } (\vec{a} + \vec{b}) \cdot (\vec{b} + \vec{c}) \times (\vec{a} + \vec{b} + \vec{c})$$

$$= [\vec{a} + \vec{b} \vec{b} + \vec{c} \vec{a} + \vec{b} + \vec{c}]$$

$$= \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} [\vec{a} \vec{b} \vec{c}]$$

$$= [1(1-1) - 1(0-1)] [\vec{a} \vec{b} \vec{c}] = [\vec{a} \vec{b} \vec{c}]$$

$$\text{and } \vec{b} \cdot (\vec{a} \times \vec{c}) = [\vec{b} \vec{a} \vec{c}] = -[\vec{a} \vec{b} \vec{c}]$$

$$\therefore \beta = \frac{[\vec{a} \vec{b} \vec{c}]}{-[\vec{a} \vec{b} \vec{c}]} = -1 \therefore \alpha + \beta = 3 - 1 = 2$$

$$87. \text{ (c) It is given that, } \vec{a} + l\vec{b} + l^2\vec{c} = 0$$

$$\therefore \vec{a} \times \vec{b} = l^2(\vec{b} \times \vec{c}) \text{ and } \vec{c} \times \vec{a} = l(\vec{b} \times \vec{c})$$

$$\text{and } \vec{a} \times \vec{b} = l(\vec{c} \times \vec{a}) \text{ and it is also given that,}$$

$$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 3(\vec{b} \times \vec{c})$$

$$\Rightarrow \vec{a} \times \vec{b} + \vec{c} \times \vec{a} = 2(\vec{b} \times \vec{c})$$

$$\Rightarrow l^2(\vec{b} \times \vec{c}) + l(\vec{b} \times \vec{c}) = 2(\vec{b} \times \vec{c}) \Rightarrow l^2 + l - 2 = 0$$

Here minimum value of $l = \frac{-D}{4a} = -\frac{9}{4}$

88. (d) Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ Let vector $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a} \cdot \vec{b} = b_1 + b_2 + b_3 = 1$ (given) ... (i)

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

Hence, $\vec{a} \times \vec{b} = (b_3 - b_2)\hat{i} + (b_1 - b_3)\hat{j} + (b_2 - b_1)\hat{k} = \hat{j} - \hat{k}$ (Given)

So, $b_2 = b_3, b_1 = b_3 + 1$ and $b_1 = b_2 + 1$... (ii)

From Eqs. (i) and (ii), we get $(b_3 + 1) + b_3 + b_3 = 1$

$\Rightarrow b_3 = 0 = b_2$ and $b_1 = 1$. So, $\vec{b} = \hat{i}$

Hence, option (d) is correct.

89. (e) Given vectors are $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$

Now, find cross product of \vec{a} and \vec{b} .

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 3\hat{i} - 2\hat{j} - \hat{k}$$

Cross triple product is

$$[\vec{a} \vec{b} \vec{p}] = \vec{p} \cdot (\vec{a} \times \vec{b}) = \vec{p} \cdot (3\hat{i} - 2\hat{j} - \hat{k})$$

$$[\vec{a} \vec{b} \vec{p}] = |\vec{p}| |\vec{a} \times \vec{b}| \cos \theta \quad \dots (i)$$

$[\vec{a} \vec{b} \vec{c}]$ is maximum when $\cos \theta = 1$

From (i), $\vec{p} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}; \vec{p} = \frac{1}{\sqrt{14}}(3\hat{i} - 2\hat{j} - \hat{k})$

90. (b) Given that, $\mathbf{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$$(\mathbf{a} \times \hat{i})(\hat{i} + \hat{j}) + (\mathbf{a} \times \hat{j})(\hat{j} + \hat{k}) + (\mathbf{a} \times \hat{k})(\hat{k} + \hat{i})$$

$$= (\mathbf{a} \times \hat{i})\hat{i} + (\mathbf{a} \times \hat{i})\hat{j} + (\mathbf{a} \times \hat{j})\hat{j} + (\mathbf{a} \times \hat{j})\hat{k} + (\mathbf{a} \times \hat{k})\hat{k} + (\mathbf{a} \times \hat{k})\hat{i}$$

$$= [\mathbf{a} \hat{i} \hat{i}] + [\mathbf{a} \hat{i} \hat{j}] + [\mathbf{a} \hat{j} \hat{j}] + [\mathbf{a} \hat{j} \hat{k}] + [\mathbf{a} \hat{k} \hat{k}] + [\mathbf{a} \hat{k} \hat{i}]$$

$$(\because (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = [\mathbf{a} \mathbf{b} \mathbf{c}])$$

$$= [\mathbf{a} \hat{i} \hat{j}] + [\mathbf{a} \hat{j} \hat{k}] + [\mathbf{a} \hat{k} \hat{i}] \quad [:\mathbf{a} \hat{i} \hat{i}] = [\mathbf{a} \hat{j} \hat{j}] = [\mathbf{a} \hat{k} \hat{k}] = 0]$$

$$= \mathbf{a} \cdot (\hat{i} \times \hat{j}) + \mathbf{a} \cdot (\hat{j} \times \hat{k}) + \mathbf{a} \cdot (\hat{k} \times \hat{i})$$

$$= \mathbf{a} \cdot \hat{k} + \mathbf{a} \cdot \hat{i} + \mathbf{a} \cdot \hat{j} = \mathbf{a} \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$= (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = x + y + z.$$

91. (c) Given that, $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are unit vector

$$\Rightarrow |\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1 \text{ and } \mathbf{a} + \mathbf{b} + \mathbf{c} = 0, \theta = \frac{\pi}{3}$$

Now, $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0; \mathbf{a} \times (\mathbf{a} + \mathbf{b} + \mathbf{c}) = 0$

$$\mathbf{a} \times \mathbf{a} + \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c} = 0; \mathbf{a} \times \mathbf{b} = \mathbf{c} \times \mathbf{a}$$

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{c} \times \mathbf{a}|$$

Similarly, $|\mathbf{a} \times \mathbf{b}| = |\mathbf{b} \times \mathbf{c}|$

$$\therefore |\mathbf{a} \times \mathbf{b}| + |\mathbf{b} \times \mathbf{c}| + |\mathbf{c} \times \mathbf{a}| = 3|\mathbf{a} \times \mathbf{b}|$$

$$= 3|\mathbf{a}||\mathbf{b}|\sin \theta = 3 \times 1 \times 1 \times \sin \frac{\pi}{3} = \frac{3\sqrt{3}}{2}$$

92. (d) We have, $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}, \mathbf{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\mathbf{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$

Since, the given vectors are coplanar, therefore

$$[\mathbf{a} \mathbf{b} \mathbf{c}] = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$$

$$1(1-2x+4) - 1(-1-2x) + 1(x-2+x) = 0;$$

$$5 - 2x + 1 + 2x + 2x - 2 = 0; 2x + 4 = 0 \quad \therefore x = -2.$$

93. (c) $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{12}{10 \times 2} = \frac{3}{5}$

$$\therefore \sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} \Rightarrow \frac{4}{5} = \frac{|\vec{a} \times \vec{b}|}{10 \times 2} \Rightarrow |\vec{a} \times \vec{b}| = \frac{4}{5} \times 20 = 16$$

94. (b) We know that volume of parallelepiped

$$= \vec{a} \cdot (\vec{b} \times \vec{c})$$

$$= \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0(0-1) - 1(0-1) + 1(1-0)$$

$$= 0 + 1 + 1 = 2 \text{ cu. units.}$$

95. (c) $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0} \quad \dots (i)$

$$(\vec{a} + 2\vec{b} + 3\vec{c}) \times \vec{b} = \vec{0}$$

$$\Rightarrow \vec{a} \times \vec{b} + \vec{0} + 3(\vec{c} \times \vec{b}) = 0$$

$$\vec{a} \times \vec{b} = 3(\vec{b} \times \vec{c}) \quad \dots (ii)$$

from eqn (i)

$$(\vec{a} + 2\vec{b} + 3\vec{c}) \times \vec{c} = \vec{0}$$

$$\vec{a} \times \vec{c} + 2(\vec{b} \times \vec{c}) + \vec{0} = \vec{0}$$

$$\Rightarrow \vec{c} \times \vec{a} = 2(\vec{b} \times \vec{c}) \quad \dots(\text{iii})$$

$$\text{Now, } (\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a})$$

$$= 3(\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}) + 2(\vec{b} \times \vec{c})$$

$$\Rightarrow (\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) = 6(\vec{b} \times \vec{c}) \quad \dots(\text{iv})$$

$$\text{Also, given } (\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a})$$

$$= \lambda(\vec{b} \times \vec{c}) \quad \dots(\text{v})$$

from equation (iv) & (v), we get :

$$\lambda = 6$$

$$96. \text{ (d) } |\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$$

$$|\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta + |\vec{a}|^2 |\vec{b}|^2 \cos^2 \theta = 144$$

$$= |\vec{a}|^2 |\vec{b}|^2 (\sin^2 \theta + \cos^2 \theta) = 144$$

$$\Rightarrow 16|\vec{b}|^2 = 144 \Rightarrow |\vec{b}|^2 = 9$$

$$\Rightarrow |\vec{b}| = 3.$$

$$97. \text{ (d) } |\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 36.$$

$$\Rightarrow (|\vec{a}| |\vec{b}| \sin \theta)^2 + (|\vec{a}| |\vec{b}| \cos \theta)^2 = 36$$

$$\Rightarrow (|\vec{a}| |\vec{b}|)^2 (\sin^2 \theta + \cos^2 \theta) = 36$$

$$\Rightarrow (3|\vec{b}|)^2 = 36 \Rightarrow |\vec{b}| = 2$$

$$98. \text{ (c) Given area} = 15 \Rightarrow |\vec{a} \times \vec{b}|^2 = 15$$

$$\text{Now, } |(3\vec{a} + 2\vec{b}) \times (\vec{a} + 3\vec{b})|$$

$$= |9(\vec{a} \times \vec{b}) + 2(\vec{b} \times \vec{a})|.$$

$$= |9(\vec{a} \times \vec{b}) - 2(\vec{a} \times \vec{b})| = |7(\vec{a} \times \vec{b})|$$

$$= 7(15) = 105$$

$$99. \text{ (d) } |\vec{a}| = 16, |\vec{b}| = 4$$

$$\sqrt{|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2}$$

$$= \sqrt{|\vec{a}|^2 |\vec{b}|^2 (\sin^2 \theta + \cos^2 \theta)}$$

$$= |\vec{a}| |\vec{b}| = 64$$

$$100. \text{ (b) } \vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = -2\hat{i} + \hat{j} + 3\hat{k}$$

Unit vector perpendicular to \vec{a} and \vec{b} both

$$\hat{n} = \frac{\pm(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ -2 & 1 & 3 \end{vmatrix} = 5\hat{i} - 5\hat{j} + 5\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{25 + 25 + 25} = 5\sqrt{3}$$

$$\hat{n} = \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \pm \frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$$

Exercise 3 :

PREVIOUS YEARS MCQs

- (c) $(\vec{a} + \vec{b} + \vec{c}) \cdot [(\vec{a} - \vec{c}) \times (\vec{a} - \vec{b})] = -3[\vec{a} \vec{b} \vec{c}]$
- (d) $\vec{a} \cdot (\vec{b} \times \vec{c}) \neq 0$ for any value of m
- (a) Let A, B, C are the vertices of a Δ whose position vectors are \vec{a}, \vec{b} and \vec{c} respectively. Let G be the centroid

of ΔABC . \therefore Centroid of triangle $(G) = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$

Consider, $\vec{GA} + \vec{GB} + \vec{GC}$

$$= \left(\vec{a} - \frac{\vec{a} + \vec{b} + \vec{c}}{3} \right) + \left(\vec{b} - \frac{\vec{a} + \vec{b} + \vec{c}}{3} \right) + \left(\vec{c} - \frac{\vec{a} + \vec{b} + \vec{c}}{3} \right)$$

$$= \frac{1}{3}[2\vec{a} - \vec{b} - \vec{c} + 2\vec{b} - \vec{a} - \vec{c} + 2\vec{c} - \vec{a} - \vec{b}] = 0.$$

- (b) $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} - 5\hat{k}$
 $\vec{a} - \vec{b} = -\hat{i} + 5\hat{j} + 2\hat{k} \Rightarrow |\vec{a} - \vec{b}| = \sqrt{1 + 25 + 4} = \sqrt{30}$
 $|\vec{a}| = \sqrt{1 + 4 + 9} = \sqrt{14} \quad |\vec{b}| = \sqrt{4 + 9 + 25} = \sqrt{38}$
 $\therefore |\vec{a} - \vec{b}| > |\vec{b}| - |\vec{a}|$

- (a) A vector in the plane of \vec{a} and \vec{b} is
 $\vec{u} = \vec{a} + \lambda\vec{b} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (1 + \lambda)\hat{k}.$

$$\text{Projection of } \vec{u} \text{ on } \vec{c} = \frac{1}{\sqrt{3}} \Rightarrow \frac{\vec{u} \cdot \vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \vec{u} \cdot \vec{c} = 1 \Rightarrow |1 + \lambda + 2 - \lambda - 1 - \lambda| = 1$$

$$\Rightarrow |2 - \lambda| = 1 \Rightarrow \lambda = 1 \text{ or } 3$$

$$\Rightarrow \vec{u} = 2\hat{i} + \hat{j} + 2\hat{k} \text{ or } 4\hat{i} - \hat{j} + 4\hat{k}.$$

- (a) Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$$

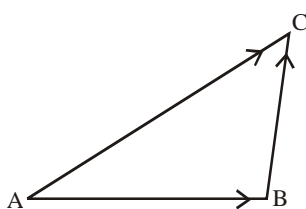
$$\vec{c} = 7\hat{i} + 9\hat{j} + 11\hat{k}$$

$$\text{Then, } \vec{a} + \vec{b} = 2\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\text{and } \vec{b} + \vec{c} = 8\hat{i} + 12\hat{j} + 16\hat{k}$$

$$\begin{aligned} \text{Area of parallelogram} &= \frac{1}{2} |(\vec{a} + \vec{b}) \times (\vec{b} + \vec{c})| \\ &= \frac{1}{2} |2\hat{i} + 2\hat{j} + 3\hat{k} \times 4(2\hat{i} + 3\hat{j} + 4\hat{k})| \\ &= \frac{8}{2} |(\hat{i} + 2\hat{j} + 3\hat{k}) \times (2\hat{i} + 3\hat{j} + 4\hat{k})| \\ &= 4 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 4 |-\hat{i} + 2\hat{j} - \hat{k}| \\ &= 4\sqrt{1+4+1} = 4\sqrt{6} \text{ sq. units.} \end{aligned}$$

7. (a) By triangle law of vector addition $\vec{AB} + \vec{BC} = \vec{AC}$



or $\vec{AB} + \vec{BC} = -\vec{CA}$

or $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$.

8. (b) If $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{a} - \lambda\vec{b}$, then

$$(\vec{a} + \lambda\vec{b}) \cdot (\vec{a} - \lambda\vec{b}) = |\vec{a} + \lambda\vec{b}| |\vec{a} - \lambda\vec{b}| \cdot \cos 90^\circ$$

$$\Rightarrow (\vec{a} + \lambda\vec{b}) \cdot (\vec{a} - \lambda\vec{b}) = 0$$

$$\Rightarrow \vec{a} \cdot \vec{a} - \lambda \cdot \vec{a} \cdot \vec{b} + \lambda \cdot \vec{b} \cdot \vec{a} - \lambda^2 \cdot \vec{b} \cdot \vec{b} = 0$$

$$\Rightarrow a^2 - \lambda^2 b^2 = 0 \Rightarrow \lambda^2 = \frac{a^2}{b^2} \Rightarrow \lambda^2 = \frac{3^2}{4^2}$$

$$\Rightarrow \lambda = \frac{3}{4}$$

9. (a) Equating the components in

$$\alpha(\hat{i} + 2\hat{j} + 3\hat{k}) + \beta(2\hat{i} + 3\hat{j} + \hat{k}) + \gamma(3\hat{i} + \hat{j} + 2\hat{k})$$

$$= -3(\hat{i} - \hat{k}), \text{ we have}$$

$$\alpha + 2\beta + 3\gamma = -3 \quad \dots\text{(i)}$$

$$2\alpha + 3\beta + \gamma = 0 \quad \dots\text{(ii)}$$

$$3\alpha + \beta + 2\gamma = 3 \quad \dots\text{(iii)}$$

Solving the equations (i), (ii), & (iii), we get

$$\alpha = 2, \beta = -1, \gamma = -1.$$

10. (a) We have, $[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}]$

$$= (\vec{a} \times \vec{b}) \cdot \{(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})\}$$

$$= (\vec{a} \times \vec{b}) \cdot \{(\vec{m} \cdot \vec{a})\vec{c} - (\vec{m} \cdot \vec{c})\vec{a}\}$$

(where $\vec{m} = \vec{b} \times \vec{c}$)

$$= \{(\vec{a} \times \vec{b}) \cdot \vec{c}\} \cdot \{(\vec{a} \cdot (\vec{b} \times \vec{c}))\} = [\vec{a} \quad \vec{b} \quad \vec{c}]^2 = 4^2 = 16$$

11. (a) By definition of scalar triple product

$$\vec{a} \cdot (\vec{b} \times \vec{c}) \text{ can be written as } [\vec{a} \quad \vec{b} \quad \vec{c}]$$

$$\frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{(\vec{c} \times \vec{a}) \cdot \vec{b}} + \frac{\vec{b} \cdot (\vec{a} \times \vec{c})}{\vec{c} \cdot (\vec{a} \times \vec{b})} = \frac{[\vec{a} \quad \vec{b} \quad \vec{c}]}{[\vec{c} \quad \vec{a} \quad \vec{b}]} + \frac{[\vec{b} \quad \vec{a} \quad \vec{c}]}{[\vec{c} \quad \vec{a} \quad \vec{b}]}$$

$$= \frac{[\vec{a} \quad \vec{b} \quad \vec{c}]}{[\vec{a} \quad \vec{b} \quad \vec{c}]} - \frac{[\vec{a} \quad \vec{b} \quad \vec{c}]}{[\vec{a} \quad \vec{b} \quad \vec{c}]} = 1 - 1 = 0$$

$$\therefore [\vec{a} \quad \vec{b} \quad \vec{c}] = [\vec{b} \quad \vec{c} \quad \vec{a}] = [\vec{c} \quad \vec{a} \quad \vec{b}]$$

$$\text{but } [\vec{b} \quad \vec{a} \quad \vec{c}] = -[\vec{a} \quad \vec{b} \quad \vec{c}]$$

12. (b) Let $\vec{x} = \sqrt{3}(\vec{a} \times \vec{b})$ and $\vec{y} = \vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$

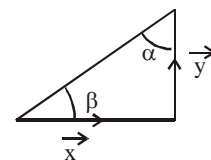
clearly $\vec{x} \cdot \vec{y} = 0 \Rightarrow \vec{x}$ and \vec{y} are perpendicular

So, one angle is $\frac{\pi}{2}$. Also $|\vec{x}| = \sqrt{3} |\vec{b} \sin \theta|$, where θ is

angle between vectors \vec{a} and \vec{b} ($|\vec{a}| = 1$)

$$|\vec{y}| = \sqrt{\{\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}\}^2} = \sqrt{b^2 - (\vec{a} \cdot \vec{b})^2}$$

$$= \sqrt{b^2 - b^2 \cos^2 \theta} = b \sin \theta$$



$$\therefore \frac{|\vec{x}|}{|\vec{y}|} = \sqrt{3} = \tan \alpha \Rightarrow \alpha = \frac{\pi}{3}. \text{ So, } \beta = \frac{\pi}{6}$$

13. (d) We can write the given expression

$$= \{\hat{i} \cdot (\vec{p} \times \vec{q})\}\hat{i} + \{\hat{j} \cdot (\vec{p} \times \vec{q})\}\hat{j} + \{\hat{k} \cdot (\vec{p} \times \vec{q})\}\hat{k}$$

$$= \vec{p} \times \vec{q}$$

Since for any vector \vec{a} ,

$$\vec{a} = (\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$$

14. (c) $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$$\vec{a} \cdot \vec{b} = (1)(1)\cos \theta = \cos \theta \text{ and } \vec{c} \cdot \vec{a} = \cos \theta, \vec{b} \cdot \vec{c} = \cos \theta$$

$$[\vec{a} \quad \vec{b} \quad \vec{c}]^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix} = \begin{vmatrix} 1 & \cos \theta & \cos \theta \\ \cos \theta & 1 & \cos \theta \\ \cos \theta & \cos \theta & 1 \end{vmatrix}$$

Operate $R_1 \rightarrow R_1 + R_2 + R_3$

$$= (1 + 2\cos \theta) \begin{vmatrix} 1 & 1 & 1 \\ \cos \theta & 1 & \cos \theta \\ \cos \theta & \cos \theta & 1 \end{vmatrix}$$

Operate $C_2 \rightarrow C_2 - C_1; C_3 \rightarrow C_3 - C_1$

$$= (1+2\cos\theta) \begin{vmatrix} 1 & 0 & 0 \\ \cos\theta & 1-\cos\theta & 0 \\ \cos\theta & 0 & 1-\cos\theta \end{vmatrix}$$

$$= (1+2\cos\theta)(1-\cos\theta)^2$$

$$\therefore [\vec{a} \vec{b} \vec{c}] = (1-\cos\theta)\sqrt{1+2\cos\theta}$$

15. (c) $\because \vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coterminous edges of a parallelepiped.

$$\text{Then, its volume } (v) = [\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}]$$

We know, scalar triple product

$$[\vec{a} \vec{b} \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) \equiv (\vec{a} \times \vec{b}) \cdot \vec{c}$$

$$\text{Consider } [\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}]$$

$$= (\vec{a}+\vec{b}) \cdot \{(\vec{b}+\vec{c}) \times (\vec{c}+\vec{a})\}$$

$$= (\vec{a}+\vec{b}) \cdot \{(\vec{b} \times \vec{c}) + (\vec{b} \times \vec{a}) + (\vec{c} \times \vec{c}) + (\vec{c} \times \vec{a})\}$$

$$= (\vec{a}+\vec{b}) \cdot \{(\vec{b} \times \vec{c}) + (\vec{b} \times \vec{a}) + (\vec{c} \times \vec{a})\} (\because \vec{c} \times \vec{c} = 0)$$

$$= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{a})$$

$$+ \vec{b} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{b} \times \vec{a}) + \vec{b} \cdot (\vec{c} \times \vec{a})$$

$$= [\vec{a} \vec{b} \vec{c}] + [\vec{a} \vec{b} \vec{a}] + [\vec{a} \vec{c} \vec{a}] + [\vec{b} \vec{b} \vec{c}] + [\vec{b} \vec{b} \vec{a}] + [\vec{b} \vec{c} \vec{a}]$$

$$= [\vec{a} \vec{b} \vec{c}] + [\vec{b} \vec{c} \vec{a}] = 2[\vec{a} \vec{b} \vec{c}]$$

16. (c) $[\vec{p}+\vec{q}-\vec{r} \quad \vec{p}-\vec{q} \quad \vec{q}-\vec{r}]$
 $= (\vec{p}+\vec{q}-\vec{r}) \cdot [(\vec{p}-\vec{q}) \times (\vec{q}-\vec{r})]$
 $= (\vec{p}+\vec{q}-\vec{r}) \cdot [\vec{p} \times \vec{q} - \vec{p} \times \vec{r} - \vec{q} \times \vec{q} + \vec{q} \times \vec{r}]$
 $= [\vec{p}+\vec{q}-\vec{r}] \cdot [\vec{p} \times \vec{q} - \vec{p} \times \vec{r} + \vec{q} \times \vec{r}] (\because \vec{a} \times \vec{a} = 0)$
 $= [\vec{p} \vec{p} \vec{q}] - [\vec{p} \vec{p} \vec{r}] + [\vec{p} \vec{q} \vec{r}] + [\vec{q} \vec{p} \vec{q}] - [\vec{q} \vec{p} \vec{r}]$
 $+ [\vec{q} \vec{q} \vec{r}] - [\vec{r} \vec{p} \vec{q}] + [\vec{r} \vec{p} \vec{r}] - [\vec{r} \vec{q} \vec{r}]$
 $= 0 - 0 + [\vec{p} \vec{q} \vec{r}] - 0 - [\vec{q} \vec{p} \vec{r}] + 0 - [\vec{r} \vec{p} \vec{q}] + 0 - 0$

$$[\because [\vec{a} \vec{a} \vec{b}] = 0]$$

$$= [\vec{p} \vec{q} \vec{r}] + [\vec{p} \vec{q} \vec{r}] - [\vec{p} \vec{q} \vec{r}]$$

$$\therefore [\vec{a} \vec{b} \vec{c}] = [\vec{b} \vec{c} \vec{a}] = -[\vec{b} \vec{a} \vec{c}]$$

$$= [\vec{p} \vec{q} \vec{r}]$$

17. (c) $|\vec{a}|=1, |\vec{b}|=2, |\vec{c}|=3$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{c} = 0$$

$$\text{Now, } [\vec{a}+\vec{b}+\vec{c} \quad \vec{b}-\vec{a} \quad \vec{c}]$$

$$= (\vec{a}+\vec{b}+\vec{c}) \cdot [(\vec{b}-\vec{a}) \times \vec{c}] = (\vec{a}+\vec{b}+\vec{c}) \cdot (\vec{b} \times \vec{c} - \vec{a} \times \vec{c})$$

$$= [\vec{a} \vec{b} \vec{c}] - [\vec{b} \vec{a} \vec{c}] = 2[\vec{a} \vec{b} \vec{c}] = 2\vec{a} \cdot (\vec{b} \times \vec{c})$$

$$= 2|\vec{a}| \cdot |\vec{b} \times \vec{c}| \cos 0^\circ (\because \vec{a} \text{ \& } (\vec{b} \times \vec{c}) \text{ are parallel})$$

$$= 2|\vec{a}| \cdot |\vec{b} \times \vec{c}| = 2|\vec{a}| |\vec{b}| |\vec{c}| \sin 90^\circ$$

$$= 2(1)(2)(3) = 12$$

18. (d) $\overline{PQ} = (-1, y-5, 4-x)$

$$\overline{QR} = (2, 8-y, -4)$$

$\therefore P, Q, R$ are collinear

$$\therefore \frac{-1}{2} = \frac{y-5}{8-y} = \frac{4-x}{-4}$$

$$\Rightarrow -8 + y = 2y - 10 \text{ \& } 4 = 8 - 2x$$

$$\Rightarrow y = 2 \text{ \& } x = 2$$

$$\therefore \boxed{x+y=4}$$

19. (c) We have, $O(0, 0, 0), P(2, 3, 4), Q(1, 2, 3)$ and $R(x, y, z)$ are coplanar.

$$\text{So, } \begin{vmatrix} x-0 & y-0 & z-0 \\ 2-0 & 3-0 & 4-0 \\ 1-0 & 2-0 & 3-0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & y & z \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow x(9-8) - y(6-4) + z(4-3) = 0$$

$$\Rightarrow x - 2y + z = 0$$

20. (c) Given, $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}, \vec{c} = 3\hat{i} - \hat{k}$
and $\vec{c} = m\vec{a} + n\vec{b}$

$$\text{As, } 3\hat{i} - \hat{k} = m(\hat{i} + \hat{j} - 2\hat{k}) + n(2\hat{i} - \hat{j} + \hat{k})$$

$$\Rightarrow 3\hat{i} - \hat{k} = (m+2n)\hat{i} + (m-n)\hat{j} + (-2m+n)\hat{k}$$

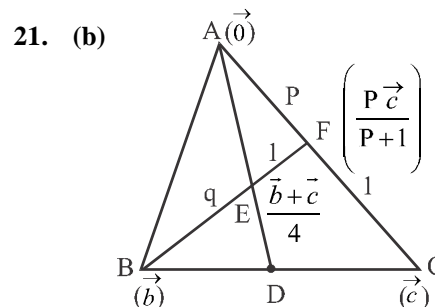
After equating the coefficient of \hat{i}, \hat{j} and \hat{k} , on both sides respectively, we get

$$3 = m + 2n, 0 = m - n \text{ and } -1 = -2m + n$$

$$\Rightarrow 3 = n + 2n \Rightarrow n = 1$$

$$\Rightarrow m = 1 \text{ and } n = 1$$

$$\Rightarrow m + n = 1 + 1 = 2$$



$$\text{Let } A(\vec{0}), B(\vec{b}), C(\vec{c})$$

$$\text{The position vector of D is } \frac{\vec{b} + \vec{c}}{2}$$

E is mid point of AD, then position vector of E is

$$\frac{\vec{b} + \vec{c}}{4} \quad \dots (i)$$

Let F divides AC in ratio P : 1.

Position vector of F is $\frac{P\vec{c}}{P+1}$.

Let E divides BF in the ratio q : 1

Position vector of E is $\frac{\frac{qP\vec{c}}{P+1} + \vec{b}}{q+1}$... (ii)

Equating position vector of E of (i) and (ii),

$$\frac{\vec{b} + \vec{c}}{4} = \frac{qP\vec{c} + \vec{b}(P+1)}{1(P+1)(q+1)}$$

We get $q = 3, P = \frac{1}{2}$

Then $AF : FC \Rightarrow 1 : 2 \Rightarrow \frac{AF}{AC} = \frac{1}{2}$

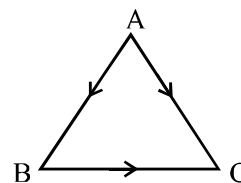
$$\frac{FC}{AP} = \frac{2}{1} \Rightarrow \frac{AC}{AF} = \frac{3}{1} \Rightarrow \frac{AF}{AC} = \frac{1}{3}$$

22. (b) $(\vec{a} - \vec{b}) = 3\hat{i} - 6\hat{j} + 2\hat{k}$

Then, the required vector in the direction of $(\vec{a} - \vec{b})$ is

$$28 \cdot \frac{(\vec{a} - \vec{b})}{|\vec{a} - \vec{b}|} = \frac{28}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}) = 12\hat{i} - 24\hat{j} + 8\hat{k}$$

23. (b) $\vec{AB} = 2\hat{i} + 3\hat{j} - 6\hat{k}; \vec{BC} = 6\hat{i} - 2\hat{j} + 3\hat{k}$
 $\vec{AC} = \vec{AB} + \vec{BC} = 8\hat{i} + \hat{j} - 3\hat{k}$



Perimeter of ΔABC

$$= |\vec{AB}| + |\vec{BC}| + |\vec{AC}| = 14 + \sqrt{74}$$

24. (a) $a\hat{i} + \hat{j} + 3\hat{k}, 4\hat{i} + 5\hat{j} + \hat{k}$ and $4\hat{i} + 2\hat{j} + 6\hat{k}$ are coplanar then

$$\begin{vmatrix} a & 1 & 3 \\ 4 & 5 & 1 \\ 4 & 2 & 6 \end{vmatrix} = 0 \Rightarrow 18a - 36 = 0 \Rightarrow a = 2$$

25. (c) $\vec{AB} = \hat{i} + 5\hat{j} - 4\hat{k}, \vec{BC} = -5\hat{i} - 4\hat{j} - \hat{k}$
 $\vec{AC} = -4\hat{i} + \hat{j} - 5\hat{k} \therefore |\vec{AB}| = |\vec{BC}| = |\vec{AC}| = \sqrt{42}$
 $\therefore A, B, C$ form an equilateral triangle.

Exercise 1 :

WARM-UP
Topic-wise MCQs

1. (b) Position vector of line segment = $2\hat{i} + 3\hat{j} + 6\hat{k}$

$$\text{length} = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7 \text{ units}$$

2. (d) Direction ratios are $\langle a + b, b + c, c + a \rangle$

Then, direction cosine,

$$l = \frac{(a+b)}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

$$m = \frac{(b+c)}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

$$n = \frac{(c+a)}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

Sum of squares of direction cosines

$$l^2 + m^2 + n^2 = \frac{(a+b)^2 + (b+c)^2 + (c+a)^2}{(a+b)^2 + (b+c)^2 + (c+a)^2} = 1.$$

3. (b) As $\frac{a}{(1/bc)} = \frac{b}{(1/ca)} = \frac{c}{(1/ab)}$, Hence lines are

parallel.

4. (d) (k, k, k) is direction cosines of a line. Then,
 $k^2 + k^2 + k^2 = 1$

$$\Rightarrow 3k^2 = 1 \Rightarrow k = \pm \frac{1}{\sqrt{3}}$$

5. (d) $l = \cos 90^\circ = 0, m = \cos 60^\circ = \frac{1}{2}$

$$\therefore l^2 + m^2 + n^2 = 1 \Rightarrow \frac{1}{4} + n^2 = 1 \Rightarrow n = \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2} = \cos \frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{6}$$

6. (d) Let the locus point $P(x, y, z)$

\therefore Distance of $P(x, y, z)$ from XY -plane is $|z|$

Distance of $P(x, y, z)$ from the Z -axis is $\sqrt{x^2 + y^2}$

According to question,

$$|z| = 2\sqrt{x^2 + y^2} \Rightarrow 4x^2 + 4y^2 - z^2 = 0$$

7. (c) Let $A(-4, 9, K), B(-1, 6, K)$ and $C(0, 7, 10)$

$$AB^2 = (-1+4)^2 + (6-9)^2 + (K-K)^2 = 18$$

$$BC^2 = (0+1)^2 + (7-6)^2 + (10-K)^2 = 2 + (10-K)^2$$

$$AC^2 = (0+4)^2 + (7-9)^2 + (10-K)^2 = 20 + (10-K)^2$$

Case 1 : $AB = BC$ and $AC^2 = AB^2 + BC^2$

$$18 = 2 + (10-K)^2 \Rightarrow K = 6 \text{ and } 14$$

$K = 6$ and $K = 14$ satisfy $AC^2 = AB^2 + BC^2$

Case 2 : $AB = AC$ and $BC^2 = AB^2 + AC^2$

$$18 = 20 + (10-K)^2 \Rightarrow (10-K)^2 = -2 \text{ (Not possible)}$$

Case 3 : $BC \neq AC$ as $2 + (10-K)^2 \neq 20 + (10-K)^2$

\therefore Two values of K are 6 and 14.

8. (a) $l = \cos 60^\circ = \frac{1}{2}; m = \cos 45^\circ = \frac{1}{\sqrt{2}}, n = \cos \theta$

$$\therefore l^2 + m^2 + n^2 = 1 \Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \theta = 1$$

$$\cos^2 \theta = 1 - \frac{3}{4} = \frac{1}{4} \Rightarrow \cos \theta = \frac{1}{2} \quad (\because \theta \text{ is acute})$$

$$\Rightarrow \theta = 60^\circ \Rightarrow \tan 60^\circ = \sqrt{3}$$

9. (d) $AB = \sqrt{(2+1)^2 + (-3-2)^2 + (1-3)^2} = \sqrt{38}$

$$BC = \sqrt{(3-2)^2 + (1+3)^2 + (-2-1)^2} = \sqrt{26}$$

$$AC = \sqrt{(3+1)^2 + (1-2)^2 + (-2-3)^2} = \sqrt{42}$$

Since sum any two side is greater than third side and all sides are different.

So, points form scalene triangle.

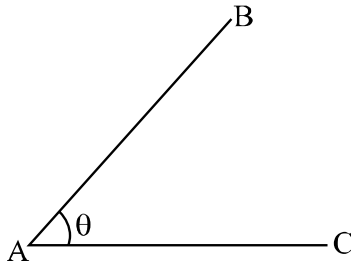
10. (a) $l = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}; m = \cos \frac{\pi}{3} = \frac{1}{2}$

$$\therefore l^2 + m^2 + n^2 = 1 \Rightarrow \frac{1}{2} + \frac{1}{4} + n^2 = 1$$

$$\Rightarrow n^2 = \frac{1}{4} \Rightarrow n = \frac{1}{2}$$

\therefore Direction cosines are $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$.

11. (d) $A = (0, 4, -3)$, $B = (5, 0, 12)$, $C = (7, 24, 0)$



$$\overline{AB} = 5\hat{i} - 4\hat{j} + 15\hat{k}$$

$$\overline{AC} = 7\hat{i} + 20\hat{j} + 3\hat{k}$$

$$\cos \theta = \frac{\overline{AB} \cdot \overline{AC}}{|\overline{AB}| \cdot |\overline{AC}|} = \frac{35 - 80 + 45}{|\overline{AB}| |\overline{AC}|} = 0$$

$$\theta = 90^\circ$$

12. (a) Give lines are $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$ and

$$\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$$

Lines pass through the points $\vec{a}_1 = (3, 2, 1)$ and

$$\vec{a}_2 = (-3, 6, 5), \vec{b}_1 = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{b}_2 = 2\hat{i} + \hat{j} - 3\hat{k}, \vec{a}_2 - \vec{a}_1 = 6\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\text{Shortest Distance} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix} = 10\hat{i} - 8\hat{j} - 4\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 60 + 32 + 16 = 108$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{100 + 64 + 16} = \sqrt{180}$$

$$S.D = \frac{108}{\sqrt{180}} = \frac{108}{6\sqrt{5}} = \frac{18}{\sqrt{5}}$$

13. (c) The direction cosines of the line are $\cos \theta, \cos \beta, \cos \theta$
 $\therefore \cos^2 \theta + \cos^2 \beta + \cos^2 \theta = 1 \Rightarrow 2\cos^2 \theta = \sin^2 \beta = 3\sin^2 \theta$

$$\Rightarrow 2\cos^2 \theta = 3 - 3\cos^2 \theta \therefore \cos^2 \theta = \frac{3}{5}$$

14. (a) Given d.c's of two lines are ℓ, m, n connected by the relations $\ell + m + n = 0$ and $\ell m = 0$

$$\text{Now, } \ell + m + n = 0 \Rightarrow \ell = -m - n \Rightarrow \ell = -(m + n)$$

$$\text{and } \ell m = 0 \Rightarrow -(m + n)m = 0 \Rightarrow -mm - mn = 0$$

$$mm = -mn; \text{ Therefore } m \text{ and } m + n = 0$$

Then $\frac{\ell_1}{-1} = \frac{m_1}{0} = \frac{n_1}{1}$ and if $\ell + m + n = 0$ then

$$\frac{\ell_2}{0} = \frac{m_2}{-1} = \frac{n_2}{1}$$

$(\ell_1, m_1, n_1) = (-1, 0, 1)$ and $(\ell_2, m_2, n_2) = (0, -1, 1)$

We know that angle between them

$$\cos \theta = \frac{0 + 0 + 1}{\sqrt{1+0+1}\sqrt{0+1+1}} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

$$\cos \theta = \frac{1}{2} = \cos 60^\circ \Rightarrow \theta = 60^\circ \Rightarrow \theta = \frac{\pi}{3}$$

15. (c) Let D.R.'s of $\overline{AB} = \langle 1, 1, 2 \rangle$;

$$\overline{AC} = \langle \sqrt{3}-1, -\sqrt{3}-1, 4 \rangle;$$

$$\overline{BC} = \langle -\sqrt{3}-1, \sqrt{3}-1, 4 \rangle;$$

$$\text{So } \cos A = \frac{\sqrt{3}-1-\sqrt{3}-1+8}{\sqrt{6}\sqrt{24}} = \frac{1}{2} \Rightarrow A = 60^\circ$$

$$\text{and } \cos C = \frac{-3+1-3+1+16}{\sqrt{24}\sqrt{24}} = \frac{12}{24} \Rightarrow C = 60^\circ$$

Obviously $B = 60^\circ$

\therefore Triangle will be an equilateral.

16. (b) We know that dr's of z-axis are $(0, 0, 1)$

So, dr's of the required line are $0, 0$ and 1

Now, equation of the line passing through (a, b, c) and

$$\text{having dr's } 0, 0 \text{ and } 1 \text{ is } \frac{x-a}{0} = \frac{y-b}{0} = \frac{z-c}{1}.$$

17. (b) We know that, $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \dots (i)$

$$\text{Now, } \cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta$$

$$= \cos^2 \alpha + \cos^2 \beta - 1 = 1 - \cos^2 \gamma - 1 = -\cos^2 \gamma$$

18. (a) We have,

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \dots (i)$$

$$\Rightarrow 2\cos^2 \alpha + 2\cos^2 \beta + 2\cos^2 \gamma = 2$$

$$\Rightarrow 2\cos^2 \alpha - 1 + 2\cos^2 \beta - 1 + 2\cos^2 \gamma - 1 = 2 - 3$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$$

Hence statement I is correct.

and now from (i),

$$1 - \sin^2 \alpha + 1 - \sin^2 \beta + 1 - \sin^2 \gamma = 1$$

$$\Rightarrow \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$$

Hence, only statement I is correct.

19. (c) A (0, 0, 0), B (a, 0, 0), C (0, b, 0), D (0, 0, c)

Let the equidistant point be P(x, y, z)

i.e, AP = BP, AP = CP, AP = DP

$$\Rightarrow \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$$

$$= \sqrt{(x-a)^2 + y^2 + z^2}$$

$$\Rightarrow x^2 + y^2 + z^2 = (x-a)^2 + y^2 + z^2$$

$$\Rightarrow x^2 = x^2 - 2ax + a^2$$

$$\Rightarrow a^2 - 2ax = 0$$

$$\Rightarrow a(a - 2x) = 0$$

Since $a \neq 0$, $a = 2x \Rightarrow x = \frac{a}{2}$

Similarly, we will get $y = \frac{b}{2}$, $z = \frac{c}{2}$

20. (b) Let $x - 1 = (y - 3) = 1 - z = k$ (Say)

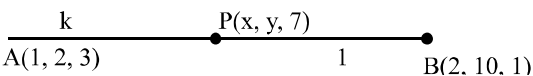
$$\therefore l = k, m = \frac{k}{2} \text{ and } n = k$$

From $l^2 + m^2 + n^2 = 1$

$$k^2 + \left(\frac{k}{2}\right)^2 + k^2 = 1 \Rightarrow k = \frac{2}{3}$$

$$\therefore l = \frac{2}{3}, m = \frac{1}{3} \text{ and } n = \frac{2}{3} \therefore l^4 + m^4 + n^4$$

$$= \left(\frac{2}{3}\right)^4 + \left(\frac{1}{3}\right)^4 + \left(\frac{2}{3}\right)^4 = \frac{11}{27}$$

21. (a) 

Let AP : BP = k : 1

$$7 = \frac{k+3}{k+1} \Rightarrow k = \frac{-2}{3}$$

So ratio is 2 : 3 externally.

$$\text{Now, } x = \frac{2 \times 2 - 3 \times 1}{2 - 3} = -1$$

$$y = \frac{2 \times 10 - 3 \times 2}{2 - 3} = -14$$

$$\Rightarrow x + y = -15$$

22. (b) Given $l + 2m + n = 0$... (i)

and $2l - 2m + 3n = 0$... (ii)

From (i)

$$l = -2m - n$$
 ... (iii)

Substitute (iii) in (ii)

$$2(-2m - n) - 2m + 3n = 0$$

$$-4m - 2n - 2m + 3n = 0 \Rightarrow -6m + n = 0 \Rightarrow n = 6m$$

From (iii) we get, $l = -2m - n = -2m - 6m = -8m$

$$\frac{l}{-8m} = \frac{m}{m} = \frac{n}{6m}$$

$$\therefore \frac{l}{-8} = \frac{m}{1} = \frac{n}{6} = \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{(-8)^2 + (1)^2 + (6)^2}} = \frac{1}{\sqrt{101}}$$

$$\therefore l = -\frac{8}{\sqrt{101}}, n = \frac{1}{\sqrt{101}}, n = \frac{6}{\sqrt{101}}$$

$$\text{Hence, } l^2 + m^2 + n^2 = \frac{64}{101} + \frac{1}{101} + \frac{36}{101} = \frac{29}{101}$$

23. (a) If $a_1a_2 + b_1b_2 + c_1c_2 = 0$ then both lines are perpendicular.

$$\therefore (-3)(2) + 4(3) + 3(-2) = -6 + 12 - 6 = 0$$

So, option (a) is satisfy the condition.

24. (d) The YZ-plane divides the line segment AB in the ratio 2 : 3. X coordinate is:

$$0 = \frac{3\alpha + 2(3)}{2+3} \Rightarrow 3\alpha = -6 \Rightarrow \alpha = -2$$

The ZX-plane divides the line segment AB in the ratio 4 : 5.

$$\text{Y coordinates is: } 0 = \frac{5(4) + 4\beta}{4+5} \Rightarrow \beta = -5$$

\therefore Point C divides AB externally in the ratio -2 : -5

Coordinates of point C:

$$\left(\frac{5(-2) - 2(3)}{5-2}, \frac{5(4) - 2(-5)}{5-2}, \frac{5(7) - 2(8)}{5-2} \right)$$

$$\text{Coordinates of point C } \left(\frac{-16}{3}, 10, \frac{19}{3} \right)$$

25. (a) $\frac{8}{5} = \frac{mp + 2n}{m + n} \Rightarrow \frac{m}{n} = \frac{2}{8 - 5p}$... (i)

and $\frac{-1}{5} = \frac{m(-2) + np}{m + n} \Rightarrow \frac{m}{n} = \frac{1 + 5p}{5}$... (ii)

From (i) and (ii),

$$\frac{2}{8 - 5p} = \frac{1 + 5p}{9} \Rightarrow 5p^2 - 7p + 2 = 0$$

$$\Rightarrow p = 1 \text{ or } p = \frac{2}{5} \quad (\because p \text{ is integer})$$

$$\therefore \frac{m}{n} = \frac{2}{8 - 5} = \frac{2}{3} \therefore \frac{3m + n}{3n} = 1 = p.$$

26. (a) $2l + m - n = 0 \Rightarrow n = 2l + m$

Substitute in $l^2 - 2m^2 + n^2 = 0$

$$l^2 - 2m^2 + (2l + m)^2 = 0$$

$$l^2 - 2m^2 + 4l^2 + m^2 + 4lm = 0$$

$$5l^2 - m^2 + 4lm = 0$$

Divide by m^2

$$5\left(\frac{l}{m}\right)^2 + 4\left(\frac{l}{m}\right) - 1 = 0 \left(\text{let } \frac{l}{m} = x\right)$$

$$5x^2 + 4x - 1 = 0$$

$$5x^2 + (5 - 1)x - 1 = 0$$

$$5x(x + 1) - 1(x + 1) = 0$$

$$x = -1, x = \frac{1}{5}$$

When $\frac{l}{m} = -1$

$$l = -m, -2m + m - n = 0 \Rightarrow n = -m$$

So Dr's of l, m, n are $-1, 1, -1$ or $1, -1, 1$

When $\frac{l}{m} = \frac{1}{5}$

$$l = \frac{m}{5}, \frac{2m}{5} + m = n \Rightarrow n = \frac{7m}{5}$$

Dr's of l, m, n is $\frac{1}{5}, 1, \frac{7}{5}$ i.e. $1, 5, 7$

$$\cos \theta = \frac{(1 \times 1 + (-1) \times 5 + 1 \times 7)}{\sqrt{3} \times \sqrt{49 + 25 + 1}} = \frac{(1 - 5 + 7)}{\sqrt{3} \sqrt{75}} = \frac{1}{5}$$

27. (d) $AB = \sqrt{(2-1)^2 + (-1+3)^2 + (1+5)^2} = \sqrt{41}$

$$BC = \sqrt{(3-1)^2 + (-4+3)^2 + (-4+5)^2} = \sqrt{6}$$

$$AC = \sqrt{(2-3)^2 + (-1+4)^2 + (1+4)^2} = \sqrt{35}$$

$$\therefore AB^2 = BC^2 + AC^2$$

So ΔABC is right triangle

$$\therefore \text{Circum radius} = \frac{AB}{2} = \frac{\sqrt{41}}{2}$$

28. (c) Mid-point of $BC = \frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}$

Direction ratios of median

$$= \left(\frac{\lambda-1}{2} - 2, 4 - 3, \frac{\mu+2}{2} - 5 \right) = \left(\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2} \right)$$

Since median equally inclined to the coordinate axes So, direction ratio are $(1, 1, 1)$.

$$\frac{\lambda-5}{2} = 1 \Rightarrow \lambda = 7 \text{ and } \frac{\mu-8}{2} = 1 \Rightarrow \mu = 10$$

$$\therefore 10\lambda - 7\mu = 70 - 70 = 0$$

29. (a) Let, the point in the xy -plane be $P(x, y, 0)$.

Since, $PA^2 = PB^2$

$$(x-2)^2 + (y-0)^2 + (0-3)^2 = (x-0)^2 + (y-3)^2 + (0-2)^2$$

$$\Rightarrow x^2 - 4x + 4 + y^2 + 9 = x^2 + y^2 - 6y + 9 + 4$$

$$\Rightarrow -4x + 13 = -6y + 13$$

$$\Rightarrow 2x = 3y$$

$$\therefore PB^2 = PC^2$$

$$(x-0)^2 + (y-3)^2 + (0-2)^2 = (x-0)^2 + (y-0)^2 + (0-1)^2$$

$$\Rightarrow x^2 + y^2 - 6y + 9 + 4 = x^2 + y^2 + 1$$

$$\Rightarrow -6y + 13 = 1 \Rightarrow y = 2$$

Also, $x = 3$

Hence, co-ordinates of the point are $(3, 2, 0)$.

30. (c) Let, $\vec{d}_1 = (1, -2, 2)$

$$\vec{d}_2 = (-2, 3, -6)$$

$$\therefore \vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ -2 & 3 & -6 \end{vmatrix}$$

$$= 6\hat{i} + 2\hat{j} - \hat{k}$$

Hence, direction ratios of the line perpendicular to l_1 and l_2 are $(6, 2, -1)$.

31. (c) For the X-axis: $\vec{a} = (1, 0, 0), |\vec{a}| = 1$

$$\text{So, } \frac{\vec{a}}{|\vec{a}|} = (1, 0, 0)$$

For the other line: $\vec{b} = (3, -1, 5), |\vec{b}| = \sqrt{35}$

$$\text{So, } \frac{\vec{b}}{|\vec{b}|} = \left(\frac{3}{\sqrt{35}}, \frac{-1}{\sqrt{35}}, \frac{5}{\sqrt{35}} \right)$$

We know, the direction vector of the external bisector is

$$\vec{d} = \left(\frac{\vec{a}}{|\vec{a}|} - \frac{\vec{b}}{|\vec{b}|} \right)$$

$$\Rightarrow \vec{d} = \left(1 - \frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}}, \frac{-5}{\sqrt{35}} \right)$$

$$\Rightarrow \vec{d} = \left(\frac{\sqrt{35}-3}{\sqrt{5}}, \frac{1}{\sqrt{5}}, -\sqrt{5} \right)$$

32. (a) $A(0, 1, 2), B(2, -1, 3), C(1, -3, 1)$

Here, $AC = 3\sqrt{2}, AB = 3, BC = 3$

$$\therefore (AB)^2 + (BC)^2 = (AC)^2$$

$\Rightarrow \Delta ABC$ is right angled triangle.

\therefore Now, circumcentre = Midpoint of A and C

$$= \left(\frac{0+1}{2}, \frac{1-3}{2}, \frac{2+1}{2} \right) = \left(\frac{1}{2}, -1, \frac{3}{2} \right)$$

and orthocentre = point B = B (2, -1, 3)

Therefore, distance between circumcentre and orthocentre

$$= \sqrt{\frac{9}{4} + 0 + \frac{9}{4}} = \frac{3}{\sqrt{2}}$$

33. (b) Given, $l - 2m + n = 0 \Rightarrow l = 2m - n$

$$\therefore lm + 10mn - 2nl = 0$$

$$\Rightarrow (2m - n)m + 10mn - 2n(2m - n) = 0$$

$$\Rightarrow 2m^2 + 5mn + 2n^2 = 0$$

$$\Rightarrow 2\left(\frac{m}{n}\right)^2 + 5\left(\frac{m}{n}\right) + 2 = 0$$

$$\Rightarrow 2x^2 + 5x + 2 = 0$$

$$\Rightarrow (2x + 1)(x + 2) = 0 \quad \left(\text{Let } x = \frac{m}{n} \right)$$

$$\Rightarrow x = -2m \text{ or } m = -2n.$$

Case I: If $n = -2m$ then d.r.'s (4, 1, -2)

Case II: If $m = -2n$ then d.r.'s (-5, -2, 1)

$$\text{Now, } \cos \theta = \frac{|4(-5) + 1(-2) + (-2)(1)|}{\sqrt{16+1+4} \sqrt{25+4+1}} = \frac{8}{\sqrt{70}}$$

34. (d) \therefore The line make an obtuse angle with the x -axis. This means l is negative but $a = 3$ is positive so take negative sign for all l, m and n .

$$\therefore l = -\frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{-3}{7}; m = \frac{2}{7} \text{ and } n = \frac{-6}{7}$$

The components are :

$$x = \text{length} \times l = -27; y = 18 \text{ and } z = -54.$$

35. (a) Let $P(0, -7, 1)$ divides the line segment joining $(2, -5, 3)$ and $(-1, -8, 0)$ in $\lambda : 1$ ratio internally.

$$\therefore 0 = \frac{-\lambda + 2}{\lambda + 1} \Rightarrow \lambda = 2$$

$\therefore Q(\alpha, \beta, \gamma)$ divides the line segment in $\lambda : 1$ externally

$$Q(\alpha, \beta, \gamma) = \left(\frac{-4}{2-1}, \frac{-11}{2-1}, \frac{-3}{2-1} \right) = (-4, -11, -3)$$

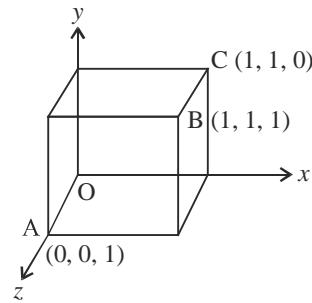
$$\Rightarrow \alpha - \beta + \gamma = -4 + 11 - 3 = 4$$

36. (c) Let point B divide the line segment joining A and C in the ratio $\lambda : 1$.

$$\therefore x_1 + 4k\ell = \frac{\lambda(x_1 + k\ell) + x_1}{\lambda + 1}$$

$$\Rightarrow 3k\ell\lambda + 4k\ell = 0 \Rightarrow \lambda = -4/3$$

37. (b)



$$\vec{AB} = \text{Diagonal of one face which is cotermines} = \hat{i} + \hat{j}$$

$$\vec{AC} = \text{Diagonal of cube} = \hat{i} + \hat{j} - \hat{k}$$

$$\cos \theta = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|}$$

$$\cos \theta = \frac{2}{\sqrt{2} \times \sqrt{3}} \Rightarrow \theta = \cos^{-1} \left(\frac{\sqrt{2}}{\sqrt{3}} \right)$$

38. (d) Let the coordinates of third vertex C is (a, b, c)

$$A(1, -4, 2), B(3, 1, 0), G(1, 0, 1)$$

$$\text{Then } 1 = \frac{1+3+a}{3}, 0 = \frac{-4+1+b}{3}, 1 = \frac{2+0+c}{3}$$

$$a = -1, b = 3, c = 1$$

$$c(-1, 3, 1)$$

$$AG^2 + CG^2 = 17 + 13 = 30$$

$$BG^2 = 6$$

$$\therefore AG^2 = 5 BG^2$$

39. (b) $P(3, 4, \alpha)$

$$\text{Distance of P from x-axis} = \sqrt{16 + \alpha^2}$$

$$\text{Distance of P from y-axis} = \sqrt{9 + \alpha^2}$$

$$\text{Distance of P from z-axis} = \sqrt{25} = 5$$

$$Z = \sqrt{16 + \alpha^2} + \sqrt{9 + \alpha^2} + 5$$

For max & min.

$$\frac{dZ}{d\alpha} = \frac{2\alpha}{2\sqrt{16 + \alpha^2}} + \frac{2\alpha}{2\sqrt{9 + \alpha^2}}$$

$$\frac{dZ}{d\alpha} = \alpha \left[\frac{1}{\sqrt{16 + \alpha^2}} + \frac{1}{\sqrt{9 + \alpha^2}} \right] > 0$$

For minimum of $z, \alpha = 0$

$$\sec 0 = 1$$

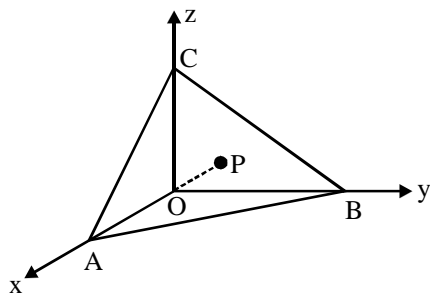
40. (d) Here $OP = \sqrt{h^2 + k^2 + l^2} = p$

\therefore DRs of OP are :

$$\frac{h}{\sqrt{h^2 + k^2 + l^2}}, \frac{k}{\sqrt{h^2 + k^2 + l^2}}, \frac{l}{\sqrt{h^2 + k^2 + l^2}}$$

or $\frac{h}{p}, \frac{k}{p}, \frac{l}{p}$

Since OP is normal to the plane, therefore, equation of plane is



$$\frac{h}{p}x + \frac{k}{p}y + \frac{l}{p}z = p \quad \text{or} \quad hx + ky + lz = p^2$$

$$\therefore A\left(\frac{p^2}{h}, 0, 0\right), B\left(0, \frac{p^2}{k}, 0\right), C\left(0, 0, \frac{p^2}{l}\right)$$

Now, Area of ΔABC , $\Delta = \sqrt{A_{xy}^2 + A_{yz}^2 + A_{zx}^2}$

Where, A_{xy}^2 is area of projection of ΔABC on xy plane
= area of ΔAOB

$$\text{Now, } A_{xy} = \frac{1}{2} \begin{vmatrix} p^2/h & 0 & 1 \\ 0 & p^2/k & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{p^4}{2|hk|}$$

Similarly, $A_{yz} = \frac{p^4}{2|kl|}$ and $A_{zx} = \frac{p^4}{2|lh|}$

$$\therefore \Delta^2 = A_{xy}^2 + A_{yz}^2 + A_{zx}^2, \quad \Delta = \frac{p^5}{2hkl}$$

41. (d) Given $A(1, 2, 3)$, $B(-1, -2, -1)$ and $C(2, 3, 2)$. Let D be (α, β, γ) . Since $ABCD$ is a parallelogram, diagonals AC and BD bisect each other i.e., mid-point of segment AC is same as mid-point of segment BD .

$$\Rightarrow \left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right) = \left(\frac{\alpha-1}{2}, \frac{\beta-2}{2}, \frac{\gamma-1}{2}\right)$$

$$\Rightarrow \alpha - 1 = 3, \beta - 2 = 5, \gamma - 1 = 5$$

$$\Rightarrow \alpha = 4, \beta = 7, \gamma = 6$$

Hence, the point D is $(4, 7, 6)$.

We have $C(2, 3, 2)$ and $D(4, 7, 6)$.

\therefore Equation of line CD is

$$\frac{x-2}{4-2} = \frac{y-3}{7-3} = \frac{z-2}{6-2}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-3}{4} = \frac{z-2}{4}$$

i.e., $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-2}{2}$ is the required equation of line.

42. (d) $AB = \sqrt{(2-2)^2 + (5+1)^2 + (1-1)^2} = 6$

$$AC = \sqrt{(2-0)^2 + (-1+2)^2 + (1-3)^2} = 3$$

D divides BC in ratio $\frac{AB}{AC} = \frac{6}{3} = \frac{2}{1}$

$$\therefore D = \frac{2 \cdot (0) + 1(2)}{2+1}, \frac{2(-2) + 1(5)}{2+1}, \frac{2(3) + 1(1)}{2+1}$$

$$D = \left(\frac{2}{3}, \frac{1}{3}, \frac{7}{3}\right)$$

$$\therefore AD = \sqrt{\left(\frac{2}{3}-2\right)^2 + \left(\frac{1}{3}-1\right)^2 + \left(\frac{7}{3}-1\right)^2} = \frac{4}{\sqrt{3}}$$

43. (a) From the first relation, $l = 5m - 3n$. Putting this value of l in second relation:

$$7(5m - 3n)^2 + 5m^2 - 3n^2 = 0$$

$$\Rightarrow 180m^2 - 210mn + 60n^2 = 0$$

$$\text{or } 6m^2 - 7mn + 2n^2 = 0$$

Note that it, being quadratic in m, n , gives two sets of values of m, n , and hence gives the d.r.s. of two lines.

Now, factorising it, we get

$$6m^2 - 3mn + 4mn + 2n^2 = 0 \quad \text{or} \quad (2m - n)(3m - 2n) = 0$$

$$\Rightarrow \text{either } 2m - n = 0, \text{ or } 3m - 2n = 0$$

Taking $2m - n = 0$ we get $2m = n$.

Also putting $m = n/2$ in $l = 5m - 3n$, we get

$$l = (5n/2) - 3n \Rightarrow l = -n/2 \Rightarrow n = -2l$$

Thus, we get, $-2l = 2m = n$ or $\frac{l}{-1} = \frac{m}{1} = \frac{n}{2}$

$$\Rightarrow \text{d.r.s. of one line are } -1, 1, 2.$$

Hence, the d.c.s. of one line are

$$\left[\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right] \quad \text{or} \quad \left[\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right]$$

Taking $3m - 2n = 0$, we get $3m = 2n$ or $m = \frac{2n}{3}$.

Putting this value in $l = 5m - 3n$, we obtain

$$l = 5 \times \frac{2n}{3} - 3n = \frac{n}{3} \quad \text{or} \quad n = 3l$$

Thus $3l = \frac{3m}{2} = n \Rightarrow \frac{l}{1} = \frac{m}{2} = \frac{n}{3}$

⇒ the d.r.'s of the second line are 1, 2, 3; and hence d.c.s.

of second line are $\left[\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right]$ or $\left[\frac{-1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}\right]$

44. (b) Let the line makes an angle θ with the positive direction of z-axis. Given that lines makes angle $\frac{\pi}{4}$ with x-axis and y-axis.

$$\therefore l = \cos \frac{\pi}{4}, m = \cos \frac{\pi}{4}, n = \cos \theta$$

We know that, $l^2 + m^2 + n^2 = 1$

$$\therefore \cos^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} + \cos^2 \theta = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

Hence, angle with positive direction of the z-axis is $\frac{\pi}{2}$.

45. (d) Distance of the point (α, β, γ) from y-axis

$$= \sqrt{(\alpha-0)^2 + (\beta-\beta)^2 + (\gamma-0)^2} = \sqrt{\alpha^2 + \gamma^2}$$

46. (c) It makes θ with x and y-axes.

$$l = \cos \theta, m = \cos \theta, n = \cos(\pi - 2\theta)$$

$$\text{we have } l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \cos^2 \theta + \cos^2 \theta + \cos^2(\pi - 2\theta) = 1$$

$$\Rightarrow 2 \cos^2 \theta + (-\cos 2\theta)^2 = 1$$

$$\Rightarrow 2 \cos^2 \theta - 1 + \cos^2 2\theta = 0$$

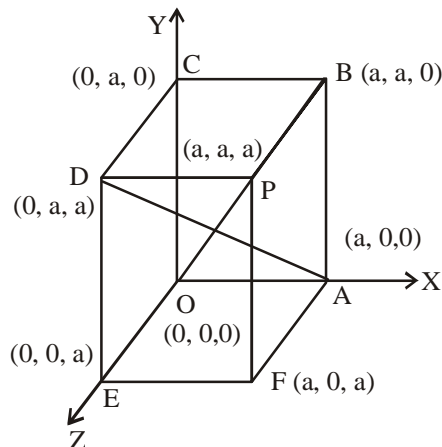
$$\Rightarrow \cos 2\theta - [1 + \cos 2\theta] = 0$$

$$\Rightarrow \cos 2\theta = 0 \text{ or } \cos 2\theta = -1$$

$$\Rightarrow 2\theta = \pi/2 \text{ or } 2\theta = \pi$$

$$\Rightarrow \theta = \pi/4 \text{ or } \theta = \frac{\pi}{2} \Rightarrow \theta = \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$$

47. (d)



diagonals are OP and AD and Acute angle = θ

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{a(-a) + (a)(a) + (a)(a)}{\sqrt{a^2 + a^2 + a^2} \sqrt{a^2 + a^2 + a^2}}$$

$$= \frac{-a^2 + a^2 + a^2}{\sqrt{3a^2} \sqrt{3a^2}} = \frac{a^2}{3a^2} = \frac{1}{3}$$

$$\Rightarrow 3 \cos \theta = 1$$

48. (d) Since equation of line is

$$\frac{x-1}{3} = \frac{y+1}{-2} = \frac{z-1}{6} = \lambda \text{ (let)}$$

$$\Rightarrow (x, y, z) = (3\lambda + 1, -2\lambda - 1, 6\lambda + 1)$$

Now,

$$\sqrt{(3\lambda + 1 - 1)^2 + (-2\lambda - 1 + 1)^2 + (6\lambda + 1 - 1)^2} = 2$$

$$= \sqrt{9\lambda^2 + 4\lambda^2 + 36\lambda^2} = 2 \Rightarrow \sqrt{49\lambda^2} = 2$$

$$\Rightarrow \pm 7\lambda = 2 \Rightarrow \lambda = \pm \frac{2}{7}$$

$$\text{So, } (x, y, z) = \left(\frac{13}{7}, \frac{-11}{7}, \frac{19}{7}\right) \text{ or } \left(\frac{1}{7}, \frac{-3}{7}, \frac{-5}{7}\right)$$

49. (c) As three points are collinear.

$$\text{So, } \Delta = \begin{vmatrix} k & 1 & 3 \\ 1 & -2 & k+1 \\ 15 & 2 & -4 \end{vmatrix} = 0$$

$$\Rightarrow k(8 - 2(k+1)) + 1(15(k+1) + 4) + 3(2 + 30) = 0$$

$$\Rightarrow k(6 - 2k) + (15k + 19) + 96 = 0$$

On solving, we will get two different values of k .

50. (c) Let the side of the cube be a .

$$\therefore \text{The length of a diagonal} = a\sqrt{3}$$

$$\text{The length of a diagonal of face} = a\sqrt{2}$$

Let any two diagonal of cube be

$$\vec{d}_1 = a\hat{i} + a\hat{j} + a\hat{k} \text{ and } \vec{d}_2 = a\hat{i} + a\hat{j} - a\hat{k}$$

$$\therefore \vec{d}_1 \cdot \vec{d}_2 = |\vec{d}_1| |\vec{d}_2| \cos \alpha$$

$$\Rightarrow a^2 + a^2 - a^2 = (a\sqrt{3})(a\sqrt{3}) \cos \alpha \Rightarrow \cos \alpha = \frac{1}{3}$$

Let the diagonal of a face be

$$\vec{F} = a\hat{i} + a\hat{j} \text{ correspond to } \vec{d}_1$$

$$\therefore \vec{d}_1 \cdot \vec{F} = |\vec{d}_1| |\vec{F}| \cos \beta$$

$$a^2 + a^2 = (a\sqrt{3})(a\sqrt{2}) \cos \beta \Rightarrow \cos \beta = \frac{2}{\sqrt{6}}$$

$$\text{Now, } \cos \alpha + \cos^2 \beta = \frac{1}{3} + \frac{4}{6} = 1.$$

51. (c) O(0, 0, 0), A(3, 1, 4), B(1, 3, 2) and C(0, 4, -2)

Centroid of tetrahedron

$$= \left(\frac{0+3+1+0}{4}, \frac{0+1+3+4}{4}, \frac{0+4+2-2}{4} \right)$$

$$= \left(\frac{4}{4}, \frac{8}{4}, \frac{4}{4} \right)$$

$$G = (1, 2, 1)$$

Centroid of ΔABC

$$= G_1 = \left(\frac{3+1+0}{3}, \frac{1+3+4}{3}, \frac{4+2-2}{3} \right) = \left(\frac{4}{3}, \frac{8}{3}, \frac{4}{3} \right)$$

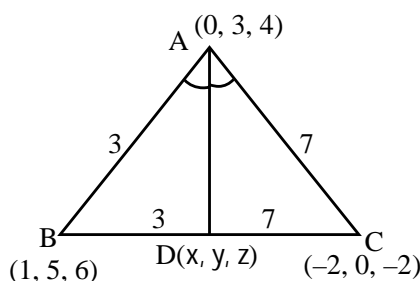
$$(1, 2, 1) G \quad G_1 \bullet \xrightarrow{1} A \xrightarrow{2} \bullet \left(\frac{4}{3}, \frac{8}{3}, \frac{4}{3} \right)$$

Let A divides GG_1 in ratio 1 : 2

$$A = \left(\frac{\frac{4}{3} + 2 \cdot \frac{8}{3}}{3}, \frac{\frac{8}{3} + 2 \cdot \frac{4}{3}}{3}, \frac{\frac{4}{3} + 2 \cdot \frac{4}{3}}{3} \right)$$

$$A = \left(\frac{10}{3}, \frac{20}{9}, \frac{10}{9} \right)$$

52. (b) A(0, 3, 4), B(1, 5, 6), C(-2, 0, -2)



AB = 3 unit, AC = 7 unit

AD is angle bisector of $\angle A$.

BD : DC = AB : AC = 3 : 7

Using section formula,

$$x = \frac{-6+7}{10}, y = \frac{0+35}{10}, z = \frac{-6+42}{10}$$

$$x = \frac{1}{10}, y = \frac{35}{10}, z = \frac{36}{10}$$

$$AD = \sqrt{\left(\frac{1}{10}\right)^2 + \left(\frac{5}{10}\right)^2 + \left(\frac{4}{10}\right)^2} = \sqrt{\frac{1+25+16}{10^2}}$$

$$AD = \frac{\sqrt{42}}{10}$$

53. (d) Two planes $ax + by + cz + d = 0$ and $ax + by + cz + d_1 = 0$ are parallel to each other.

\therefore They have no common point.

54. (c) The equation of plane which contains z-axis is $3x + 2y = 0$ as z is absent in this equation.

55. (d) Since, $xy + yz = 0$

$$xy = -yz$$

Hence, the locus is a pair of perpendicular planes.

56. (d) Line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$

passes through (4, 2, k) and lies in plane $2x - 4y + z = 7$

so it must satisfy

$$2(4) - 4(2) + k = 7 \Rightarrow k = 7$$

57. (d) Since, the required plane passing through (1, -1, -1)

therefore only equation given in option 'd' satisfied by the point (1, -1, -1).

Hence, Required equation of plane is

$$14x - 5y + 3z = 16$$

58. (c) The equation of a plane parallel to the plane

$3x + 4y - 5z = 0$ is given by,

$$3x + 4y - 5z = d \quad \dots (i)$$

Since plane (i) passes through (1, 2, 3) then, $3 + 8 - 15 = d$

$$\Rightarrow d = -4$$

\therefore from (i),

$$3x + 4y - 5z + 4 = 0$$

59. (b) Angle between two planes,

$$\cos \theta = \frac{(2, -1, 1) \cdot (1, 1, 1)}{\sqrt{4+1+1} \sqrt{1+1+1}} = \frac{2-1+1}{\sqrt{6} \sqrt{3}} = \frac{\sqrt{2}}{3}$$

60. (c) Equation plane is $\frac{x}{2} + \frac{y}{2} + \frac{z}{1} = 1$

$$\Rightarrow x + y + 2z - 2 = 0$$

\therefore direction ratio's of normal are $\langle 1, 1, 2 \rangle$

So, Direction cosines of normal are $\left\langle \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle$

61. (c) Let $P_1 : x - 3z - 4 = 0$ and $P_2 : y - 2z + 3 = 0$ be two planes. Let $ax + by + cz = d$ be the equation of line.

Since, the line of intersection will be perpendicular to the normal of both the planes

$$\therefore a(1) + b(0) + c(-3) = 0$$

$$\Rightarrow a - 3c = 0 \quad \dots (i)$$

$$\text{and } a(0) + b(1) + c(-2) = 0$$

$$\Rightarrow b - 2c = 0 \quad \dots (ii)$$

From (i) and (ii) we have

$$\frac{a}{3} = \frac{b}{2} = \frac{c}{1}$$

Hence, d.Rs = $\langle 3, 2, 1 \rangle$.

62. (d) Eqn. of line

$$\Rightarrow \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\text{i.e., } \frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+1}{3} = K (\text{say})$$

$$\Rightarrow x - 1 = 2K; y - 2 = -3K; z + 1 = 3K$$

$$\Rightarrow x = 2K + 1; y = -3K + 2; z = 3K - 1$$

Since the line meets yz plane, $x = 0$

$$\therefore 2K + 1 = 0 \Rightarrow K = -\frac{1}{2}$$

$$\therefore y = -3\left(-\frac{1}{2}\right) + 2 = \frac{3}{2} + 2 = \frac{7}{2}$$

$$z = 3\left(-\frac{1}{2}\right) - 1 = -\frac{3}{2} - 1 = -\frac{5}{2}$$

63. (a) We know, the equation of plane passing through 3 points (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$$

So, the plane passing through points $(-2, 6, -6)$, $(-3, 10, -9)$ and $(-5, 0, -6)$ is

$$\begin{vmatrix} x + 2 & y - 6 & z + 6 \\ -1 & 4 & -3 \\ -3 & -6 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (x + 2)(-18) - (y - 6)(-9) + (z + 6)(6 + 12) = 10$$

$$\Rightarrow -18x - 36 + 9y - 54 + 18z + 108 = 0$$

$$\Rightarrow -18x + 9y + 18z + 18 = 0$$

$$\Rightarrow 2x - y - 2z - 2 = 0$$

$$\Rightarrow 2x - y - 2z = 2.$$

64. (d) drs of line is 2, 3, 4
going through option,

$$2(1) + 3(2) + 4(-2) = 0$$

$$2(4) + 3(4) - 4(5) = 0$$

65. (c) Angle between planes

$$= \cos^{-1}\left(\frac{2-1+2}{6}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Distance between planes

$$= \frac{\left|\frac{2}{3} - 4\right|}{\sqrt{9}} = \frac{10}{3 \times 3} = \frac{10}{9}$$

66. (d) Given that $\frac{x-1}{\lambda} = \frac{y-2}{1} = \frac{z-3}{2}$

and $\frac{x+26}{-2} = \frac{y+18}{3} = \frac{z+28}{\lambda}$ are coplanar

$$\therefore \begin{vmatrix} 27 & 20 & 31 \\ \lambda & 1 & 2 \\ -2 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

Now, normal of plane P, which contains both lines

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ -2 & 3 & 3 \end{vmatrix} = -3\hat{i} - 13\hat{j} + 11\hat{k}$$

\Rightarrow Equation of required plane P :

$$3x + 13y - 11z + 4 = 0$$

$(0, 4, 5)$ does not lie on plane P.

67. (b) Let $P(\vec{r})$ be any point on the plane.

Clearly $\vec{r} - \vec{a}$ will be in linear combination of $\vec{b} - \vec{a}$ and $\vec{c} - \vec{a}$

$\Rightarrow \vec{r} - \vec{a}, \vec{b} - \vec{a}, \vec{c} - \vec{a}$ will be coplanar

$$\Rightarrow (\vec{r} - \vec{a}) \cdot \{(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})\} = 0$$

$$\Rightarrow (\vec{r} - \vec{a}) \cdot \{\vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a}\} = 0$$

$$\Rightarrow \vec{r} \cdot \{\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}\} = [\vec{a} \vec{b} \vec{c}]$$

68. (a) Given equation of plane is

$$x - 3y + 5z = d$$

Since, it passes through $(1, 2, 4)$, then

$$1 - 3(2) + 5(4) = d$$

$$\Rightarrow 1 - 6 + 20 = d$$

$$d = 15$$

Therefore, the equation of the plane will be

$$x - 3y + 5z = 15$$

$$\Rightarrow \frac{x}{15} - \frac{3y}{15} + \frac{5z}{15} = 1$$

$$\Rightarrow \frac{x}{15} + \frac{y}{-5} + \frac{z}{3} = 1$$

Hence, the intercept cut by the plane on axes X, Y, Z are 15, -5 and 3, respectively.

69. (a) \therefore The line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane

$$x + 3y - \alpha z + \beta = 0$$

\therefore Pt (2, 1, -2) lies on the plane i.e. $2 + 3 + 2\alpha + \beta = 0$

$$\text{or } 2\alpha + \beta + 5 = 0 \quad \dots(i)$$

Also normal to plane will be perpendicular to line,

$$\therefore 3 \times 1 - 5 \times 3 + 2 \times (-\alpha) = 0 \Rightarrow \alpha = -6$$

From equation (i) then, $\beta = 7 \therefore (\alpha, \beta) = (-6, 7)$

70. (d) Mid point of PQ is $= \left(\frac{3}{2}, \frac{7}{2}, \frac{-9}{2}\right)$

DR of the normal is $= (4 - (-1), 5 - 2, -10 - 1) = 5, 3, -11$

$$\therefore \text{Eqn. of plane is } 5\left(x - \frac{3}{2}\right) + 3\left(y - \frac{7}{2}\right) - 11\left(z + \frac{9}{2}\right) = 0$$

$$\Rightarrow 5x + 3y - 11z = \frac{135}{2} \Rightarrow r \cdot (5\hat{i} + 3\hat{j} - 11\hat{k}) = \frac{135}{2}$$

71. (c) Equation of straight line passes through P(3, 2, -1) and parallel to given line is

$$\frac{x-3}{-2} = \frac{y-2}{2} = \frac{z+1}{1} = \lambda \quad \dots(i)$$

\therefore Q(-2 λ + 3, 2 λ + 2, λ - 1) be any point in line (i) and it is also lies on given plane.

$$\therefore 3(-2\lambda + 3) - 2(2\lambda + 2) + 4(\lambda - 1) + 1 = 0$$

$$\Rightarrow -4\lambda + 4 = 0 \Rightarrow \lambda = -1$$

$$\therefore Q(5, 0, -2)$$

$$\text{Now, distance PQ} = \sqrt{(5-3)^2 + (0-2)^2 + (-2+1)^2} = 3$$

72. (c) Normal of plane E

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = -3\hat{i} - 3\hat{j}$$

$$\therefore \vec{n} = \hat{i} + \hat{j}$$

Equation plane E

$$1(x-1) + 1(y+1) + 0(y-1) = 0$$

$$x + y = 0$$

Distance from Q(a, a, 2) is

$$d = \frac{|a+a|}{\sqrt{1+1}} \Rightarrow \frac{2a}{\sqrt{2}} = 3\sqrt{2}$$

$$a = \pm 3$$

$$\therefore PQ^2 = (2)^2 + 4^2 + 1 = 21$$

73. (b) (2, -1, -3) lies on the plane $px - qy + z = 5$ so it will satisfy the given plane.

$$\text{So } 2p + q = 8 \quad \dots(i)$$

Also given line is perpendicular to normal plane so

$$3p + 2q - 1 = 0 \quad \dots(ii)$$

$$\Rightarrow p = 15, q = -22$$

$$\text{Eq. of plane } 15x - 22y + z - 5 = 0$$

$$\text{its distance from origin} = \frac{6}{\sqrt{710}} = \sqrt{\frac{5}{142}}$$

74. (d) If θ be the angle between the given line and plane, then

$$\sin \theta = \frac{1 \times 1 + 2 \times 2 + \lambda \times 3}{\sqrt{1^2 + 2^2 + \lambda^2} \cdot \sqrt{1^2 + 2^2 + 3^2}} = \frac{5 + 3\lambda}{\sqrt{14} \cdot \sqrt{5 + \lambda^2}}$$

$$\Rightarrow \cos \theta = \sqrt{1 - \frac{(5 + 3\lambda)^2}{14(5 + \lambda^2)}} \quad \therefore \theta = \cos^{-1} \sqrt{1 - \frac{(5 + 3\lambda)^2}{14(5 + \lambda^2)}}$$

$$\text{But it is given that } \theta = \cos^{-1} \sqrt{\frac{5}{14}}$$

$$\therefore \sqrt{1 - \frac{(5 + 3\lambda)^2}{14(5 + \lambda^2)}} = \sqrt{\frac{5}{14}} \Rightarrow \lambda = \frac{2}{3}$$

75. (c) Since, $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1} a$ with X-axis

$$\text{Suppose } \vec{a} = (1\hat{i} + 0\hat{j} + 0\hat{k})$$

$$\text{and } \vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$\text{Hence, } \alpha = \sin(\sin^{-1} \alpha) = \frac{|\vec{a} \cdot \vec{n}|}{|\vec{a}|}$$

$$= \frac{|(1\hat{i}) \cdot (2\hat{i} - 3\hat{j} + 6\hat{k})|}{\sqrt{1} \sqrt{4 + 9 + 36}} = \frac{2}{7}$$

76. (c) Given equation of plane $x + y + z = 3$

Let point P is (0, 0, 0) and point Q is the foot of perpendicular drawn from point P on the plane.

Since, PQ is perpendicular to the plane, so direction ratio of line PQ $< 1, 1, 1 >$.

\therefore Equation of line PQ,

$$\frac{x-0}{1} = \frac{y-0}{1}$$

$$= \frac{z-0}{1} = \lambda$$

$$\therefore (x, y, z) = (\lambda, \lambda, \lambda)$$

As point Q lies on the given plane.

$$\therefore \lambda + \lambda + \lambda = 3 \Rightarrow \lambda = 1$$

Hence, co-ordinate of point Q = (1, 1, 1).

77. (b) Equation of plane is $ax + by + cz = d$
 Given the plane equation is $2x - 3y + 6z + 4 = 0$
 On comparing we get $a = 2, b = -3, c = 6, d = -4$
 Here we need to find the direction cosine of a normal to the plane i.e $\langle l, m, n \rangle$
 Direction cosines of normal to plane

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{2}{\sqrt{49}} = \frac{2}{7}$$

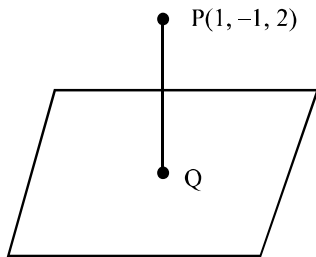
$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{-3}{\sqrt{49}} = \frac{-3}{7}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{6}{\sqrt{49}} = \frac{6}{7}$$

$$\therefore \langle l, m, n \rangle = \left\langle \frac{2}{7}, \frac{-3}{7}, \frac{6}{7} \right\rangle$$

$$\begin{aligned} \therefore 49(7l^2 + m^2 - n^2) &= 49 \left(7 \left(\frac{2}{7} \right)^2 + \left(\frac{-3}{7} \right)^2 - \left(\frac{6}{7} \right)^2 \right) \\ &= 49 \left(\frac{28}{49} + \frac{9}{49} - \frac{36}{49} \right) = 1 \end{aligned}$$

78. (c)



Assume the line from (1, -1, 2) meet the plane at Q
 Direction ratios of the line from the point (1, -1, 2) to the given plane is $\langle 3, 2, 2 \rangle$
 So the equation of the line passing through P and with direction ratios will be:

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{2} = \lambda$$

$$x = 3\lambda + 1, y = 2\lambda - 1, z = 2\lambda + 2$$

Now, since Q lies on the plane so it must satisfy the equation of the plane.

$$\text{i.e } x + 2y + 3z = 18$$

$$\therefore 3\lambda + 1 + 4\lambda - 2 + 6\lambda + 6 = 18$$

$$13\lambda + 5 = 18 \Rightarrow \lambda = 1$$

Therefore, coordinates of Q are

$$(3 + 1, 2 - 1, 2 + 2) = (4, 1, 4)$$

79. (d) Given planes are

$$x - 2y + z = 1 \quad \dots(i)$$

$$\text{and } -3x + 6y - 3z = -2$$

$$\equiv x - 2y + z = \frac{2}{3} \quad \dots(ii)$$

Since both planes are parallel and

$$a = 1, b = -2, c = 1$$

$$\text{and } d_1 = -1, d_2 = \frac{-2}{3}$$

$$\therefore \text{Distance} = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$\text{Distance} = \left| \frac{1 - \frac{2}{3}}{\sqrt{1 + 4 + 1}} \right| = \frac{1}{3\sqrt{6}}$$

80. (d) Given equation of plane are

$$2x - y - 2z + 1 = 0$$

$$\Rightarrow a_1 = 2, b_1 = -1, c_1 = -2, d_1 = 1$$

$$\text{and } 3x - 4y + 5z - 3 = 0$$

$$\Rightarrow a_2 = 3, b_2 = -4, c_2 = 5, d_2 = -3$$

\therefore Required angle is

$$\cos \theta = \pm \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\Rightarrow \cos \theta = \frac{2(3) + (-1)(-4) + 5(-2)}{\sqrt{2^2 + 1^2 + 2^2} \sqrt{3^2 + 4^2 + 5^2}} = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

81. (a) Given planes, $p_1 : x + y + z = 1$

$$p_2 : 2x + 3y + 4z = 7$$

So, equation of plane passing through intersection of planes p_1 and p_2 is

$$x + y + z - 1 + k(2x + 3y + 4z - 7) = 0.$$

$$\Rightarrow x + y + z - 1 + 2kx + 3ky + 4kz - 7k = 0$$

$$\Rightarrow x(1+2k) + y(1+3k) + z(1+4k) - 1 - 7k = 0.$$

This is perpendicular to $x - 5y + 3z = 5$.

$$\Rightarrow x - 5y + 3z - 5 = 0.$$

$$\Rightarrow 1(1+2k) - 5(1+3k) + 3(1+4k) = 0$$

$$\Rightarrow 1 + 2k - 5 - 15k + 3 + 12k = 0$$

$$\Rightarrow -k - 1 = 0 \Rightarrow k = -1$$

\therefore Equation of plane is $x + y + z - 1 - 1(2x + 3y + 4z - 7) = 0$

$$\Rightarrow x + y + z - 1 - 2x - 3y - 4z + 7 = 0$$

$$\Rightarrow -x - 2y - 3z + 6 = 0$$

$$\Rightarrow x + 2y + 3z - 6 = 0.$$

82. (a) The equation of plane passing through the intersection of planes $2x + y + 2z = 9$ and $4x - 5y - 4z = 1$ is

$$(2x + y + 2z - 9) + \lambda(4x - 5y - 4z - 1) = 0$$

Given that this plane passes through $(3, 2, 1)$

$$\Rightarrow 2(3) + 2 + 2(1) - 9 + \lambda[4(3) - 5(2) - 4(1) - 1] = 0$$

$$\Rightarrow 1 + \lambda(-3) = 0 \Rightarrow \lambda = \frac{1}{3}$$

$$\therefore \text{Equation is } (2x + y + 2z - 9) + \frac{1}{3}(4x - 5y - 4z - 1) = 0$$

$$\Rightarrow 6x + 3y + 6z - 27 + 4x - 5y - 4z - 1 = 0$$

$$\Rightarrow 10x - 2y + 2z - 28 = 0$$

$$\Rightarrow 10x - 2y + 2z = 28$$

83. (b) Normal vector to the intersection of planes

$$ax + by = 3 \text{ and } ax + by + cz = 0$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ a & b & c \end{vmatrix} = b\hat{c}\hat{i} - ac\hat{j}$$

Direction ratios of line are $b, -a, 0$

Direction ratios of normal of the plane are $0, 1, -1$

$$\cos 60^\circ = \frac{-a}{\sqrt{2}\sqrt{b^2 + a^2}} = \frac{1}{2}$$

$$\Rightarrow \frac{a}{\sqrt{a^2 + b^2}} = \frac{1}{\sqrt{2}} \Rightarrow b = \pm a$$

So, D.R.'s can be $(\pm a, -a, 0)$

$$\text{Therefore, D.C.'s can be } \pm \left(\frac{\pm 1}{\sqrt{2}}, -\frac{1}{2}, 0 \right)$$

84. (b) Here $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{j} - \hat{k})$. Let (L_1) say,

$$\vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{i} - 3\hat{k}). \text{ Let } (L_2) \text{ say,}$$

$\therefore L_1$ and L_2 are coplanar, so direction ratios of two fix points $(1, -1, 1)$ and $(\alpha, 0, -1)$ will be $(1 - \alpha), 0, 1$

$$\text{Thus, } \begin{vmatrix} 0 & 3 & -1 \\ 2 & 0 & -3 \\ (1-\alpha) & 0 & 1 \end{vmatrix} = 0 \Rightarrow \alpha = \frac{5}{3}$$

$$\text{Now, } \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix} = \hat{i}(-9) - \hat{j}(2) + \hat{k}(-6)$$

$$= (9, 2, 6)$$

Equation of plane :

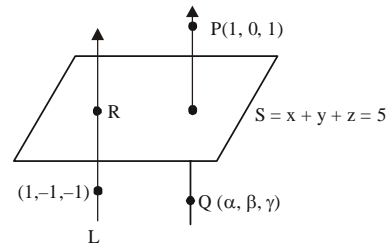
$$9(x - 1) + 2(y + 1) + 6(z - 1) = 0$$

$$9x + 2y + 6z - 13 = 0$$

Perpendicular distance from $(\alpha, 0, 0)$

$$= \frac{\left| \left(9 \cdot \frac{5}{3} + 0 + 0 - 13 \right) \right|}{\sqrt{81 + 36 + 4}} = \frac{2}{\sqrt{121}} = \frac{2}{11}$$

85. (b)



Given, a line L is parallel to PQ direction then ratio's of line L is $\langle 1, 1, 1 \rangle$.

Apply formula for image of point on the plane

$$\frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

$$\frac{\alpha - 1}{1} = \frac{\beta - 0}{1} = \frac{\gamma - 1}{1} = \frac{-2(2 - 5)}{3}$$

$$\alpha = 3, \beta = 2, \gamma = 3; Q \equiv (3, 2, 3)$$

Here, line $L \parallel PQ$.

Equation of line L is shown below

$$\frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{1} = \mu$$

Here point $R, \mu + 1, \mu - 1, \mu - 1$ is intersection point of plane $x + y + z = 5$

$$3\mu - 1 = 5 \Rightarrow \mu = 2$$

So, point R is $(3, 1, 1)$; $QR^2 = 5$

86. (d) Points $A(1, 1, \lambda)$ & $B(-3, 0, 1)$ are equidistant from plane $3x + 4y - 12z + 13 = 0$

$$\frac{|3 + 4 - 12\lambda + 13|}{13} = \frac{|-9 - 12 + 13|}{13}$$

$$|-12\lambda + 20| = |-8|$$

$$|-12\lambda + 20| = 8$$

$$-12\lambda + 20 = 8 \text{ and } 12\lambda - 20 = 8$$

$$\lambda = 1 \text{ and } \lambda = \frac{28}{12} = \frac{7}{3} \Rightarrow \lambda = 1, \frac{7}{3}$$

87. (c) Dr's of line passing through (2, 1, 2) and (1, 2, 1) is $\langle 1, -1, 1 \rangle$

$$\hat{n} \text{ of plane is } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & -1 & 2 \end{vmatrix} \Rightarrow \hat{n} = -\hat{i} - \hat{j} + \hat{k}$$

$$\hat{n} = \langle A, B, C \rangle \\ = \langle -1, -1, 1 \rangle$$

Then the equation of plane is $ax + by + cz + d = 0$

i.e. $-x - y + z + d = 0$

(2, 1, 2) point passes through equation (i)

$$-2 + 1 + 2 + d = 0 \Rightarrow d = -1$$

From (i),

$$-x - y + z + 1 = 0 \Rightarrow x + y - z - 1 = 0$$

$$a = 1, b = 1, c = -1, d = -1$$

$$\frac{a+b}{c+d} = \frac{2}{-2} = -1$$

88. (c) $\vec{d} = \lambda(\vec{a} \times \vec{b}); \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix} = -3\hat{i} - 3\hat{j} - 3\hat{k}$

$$\vec{d} = -3\lambda(\hat{i} + \hat{j} + \hat{k})$$

$$\vec{d} \cdot \vec{c} = 2 \Rightarrow -3\lambda(2+1-1) = 2 \Rightarrow \lambda = -\frac{1}{3}$$

$$\vec{d} = \hat{i} + \hat{j} + \hat{k} \Rightarrow |\vec{d}| = \sqrt{3}$$

89. (c) Let, A : $\hat{i} - 2\hat{j} - 3\hat{k}$; B : $3\hat{i} - \hat{j} + 4\hat{k}$; C : $-3\hat{i} + 2\hat{j} - 5\hat{k}$
 $\therefore \vec{AB} = 2\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{AC} = -4\hat{i} + 4\hat{j} - 2\hat{k}$

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 7 \\ -4 & 4 & -2 \end{vmatrix} = 6(-5\hat{i} - 4\hat{j} + 2\hat{k})$$

$$\text{Equation of plane } -5(x-1) - 4(y+2) + 2(z+3) = 0$$

$$\Rightarrow 5x + 4y - 2z - 3 = 0 \quad \dots(i)$$

The point $(-\hat{i} + 9\hat{j} + 14\hat{k})$ i.e. $(-1, 9, 14)$ satisfy equation (i)

90. (d) Given planes are $\vec{r} \cdot (1\hat{i} - 2\hat{j} - \alpha\hat{k}) = 7$

and $\vec{r} \cdot (2\hat{i} + 4\hat{j} - 2\hat{k}) = 5$

$$\Rightarrow \vec{n}_1 \cdot \vec{n}_2 = 0 \Rightarrow 22 - 8 - 2\alpha = 0 \Rightarrow \alpha = 7$$

91. (a) Given direction ratio's $(-1, 2, 1)$ and $(1, 3, 2)$ and passes through $(2, 1, k)$.

Now, the normal which is perpendicular to both the lines will be given by,

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix} \Rightarrow \vec{n} = \hat{i}(4-3) - \hat{j}(-2-1) + \hat{k}(-3-2)$$

$$\vec{n} = \hat{i} + 3\hat{j} - 5\hat{k}$$

Equation of plane passes through $(3, -1, 4)$ is,

$$(\hat{i} + 3\hat{j} - 5\hat{k}) \cdot [(x-3)\hat{i} + (y+1)\hat{j} + (z-4)\hat{k}] = 0$$

$$(x-3) + 3(y+1) - 5(z-4) = 0$$

$$x + 3y - 5z - 3 + 3 + 20 = 0$$

$$x + 3y - 5z = -20 \quad \dots(i)$$

Satisfy the point $(2, 1, k)$; $x + 3y - 5z = -20$

$$\Rightarrow (2) + 3(1) - 5(k) = -20 \Rightarrow 25 = 5k \Rightarrow k = 5$$

92. (b) Let P $\equiv (x, y, z)$ a general point on the plane and

A $\equiv (1, -2, 3)$ is a point given on the plane.

$$\Rightarrow \vec{AP} = (x-1)\hat{i} + (y+2)\hat{j} + (z-3)\hat{k}$$

$$\text{Given } \vec{AP} \perp \vec{B} \quad (\because \vec{B} = -\hat{i} + 2\hat{j} - 3\hat{k})$$

$$\Rightarrow x - 2y + 3z = 14 \text{ is equation of the plane.}$$

93. (a) Here \vec{n}_1 for the plane $x - y + z + 2 = 0$ is $\vec{n}_1 = \hat{i} - \hat{j} + \hat{k}$

and \vec{n}_2 for the plane $2x + y - 2z + 5 = 0$ is $\vec{n}_2 = 2\hat{i} + \hat{j} - 2\hat{k}$

Now DRs. of the line common to the planes

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & -2 \end{vmatrix} = \hat{i} + 4\hat{j} + 3\hat{k}$$

$$\text{Hence, DCs are } \frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}, \frac{3}{\sqrt{26}}$$

94. (a) Required plane passes through $(2, -1, 3)$.

Let a, b and c are Dr's of normal to required plane.

Since required plane perpendicular to both given planes

$$\therefore \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -3\hat{i} - 2\hat{j} + 5\hat{k}$$

\therefore Equation of required plane

$$-3(x-2) - 2(y+1) + 5(z-3) = 0$$

$$\Rightarrow x + \frac{2}{3}y - \frac{5}{3}z + \frac{11}{3} = 0$$

By comparing $ax + by + cz + d = 0$, we get $d = \frac{11}{3}$

95. (c) Direction ratio of line = $[2, 5, 1]$

Direction ratio of normal of plane = $[8, 2, -1]$

Angle between line and plane

$$\sin \theta = \frac{\left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|}{1}$$

$$= \frac{16 + 10 - 1}{\sqrt{4 + 25 + 1} \sqrt{64 + 4 + 1}} = \frac{25}{\sqrt{30} \sqrt{69}} = \frac{25}{\sqrt{2070}}$$

$$\therefore \theta = \sin^{-1} \left(\frac{25}{\sqrt{2070}} \right)$$

96. (c) $2x - y + z - 3 = 0 \dots(i)$; $x + 2y + z - 4 = 0 \dots(ii)$

Solving (i) and (ii); $x = -1, y = 0, z = 5$

Line of intersection is parallel to $\vec{r}_1 \times \vec{r}_2$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = (3)\hat{i} - \hat{j} + 5\hat{k}$$

Equation of line of intersection of plane.

$$\frac{x+1}{-3} = \frac{y}{-1} = \frac{z-5}{5}; \text{ D.C. are } \pm \left(\frac{-3}{\sqrt{35}}, \frac{-1}{\sqrt{35}}, \frac{5}{\sqrt{35}} \right)$$

97. (c) Given equation of the line,

$$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-1}{-1} = r \text{ (say)} \quad \dots(i)$$

Let the point P(2r+1, r-1, 1-r)

Lies on the above line and the point P is the intersection of line in Eq. (i) and the plane $x+2y+3z=4$, also, then $2r+1+2r-2+3-3r=4 \Rightarrow r=2$ Thus, point P(5, 1, -1)

$$\text{Now, } (2\hat{i}-3\hat{j}) \times (\hat{i}+2\hat{j}-\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 0 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= \hat{i}(3) - \hat{j}(-2) + \hat{k}(4+3) = 3\hat{i} + 2\hat{j} + 7\hat{k}$$

\therefore Equation of required line which is parallel to

$$3\hat{i} + 2\hat{j} + 7\hat{k}$$

$$\frac{x-5}{3} = \frac{y-1}{2} = \frac{z+1}{7} \text{ or } \frac{x-5}{-3} = \frac{y-1}{-2} = \frac{z+1}{-7}$$

98. (c) Since, normal makes equal angles with co-ordinate axis. So, it intercept with all the axis will be same. So, equation of plane will be :

$$\frac{x}{a} + \frac{y}{a} + \frac{z}{a} = 1 \Rightarrow x + y + z = a \quad \dots(i)$$

Now, it passes through (-1, 2, 3). So, we get $a = 4$

Putting the values of a in equation (i), we get

$$x + y + z - 4 = 0$$

99. (c) The given equation of plane is: $56x + 4y + 9z = 2016$

$$\Rightarrow \frac{56x}{2016} + \frac{y}{504} + \frac{z}{224} = 1 \Rightarrow \frac{x}{36} + \frac{y}{504} + \frac{z}{224} = 1$$

Then co-ordinate of A, B and C are (36, 0, 0), (0, 504, 0) and (0, 0, 224). ; Now, the centroid is

$$\left(\frac{36+0+0}{3}, \frac{0+504+0}{3}, \frac{0+0+224}{3} \right)$$

$$\text{Centroid} = \left(12, 168, \frac{224}{3} \right).$$

100. (d) $\gamma = (1 + \lambda - \mu)\hat{i} + (2 - \lambda)\hat{j} + (3 - 2\lambda + 2\mu)\hat{k}$

$$= \hat{i} + \hat{i} - \mu\hat{i} + 2\hat{j} - \hat{j} + 3\hat{k} - 2\hat{k} + 2\mu\hat{k}$$

$$= (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} - 2\hat{k}) + \mu(-\hat{i} + 2\hat{k})$$

Equation of plane is given by

$$\{r - (\hat{i} + 2\hat{j} + 3\hat{k})\} \cdot [(\hat{i} - \hat{j} - 2\hat{k}) \times (-\hat{i} + 2\hat{k})] = 0$$

$$\Rightarrow [r - (\hat{i} + 2\hat{j} + 3\hat{k})] \cdot [(-2\hat{i} - \hat{k})] = 0$$

$$\Rightarrow r \cdot (2\hat{i} + \hat{k}) - 5 = 0 \Rightarrow r \cdot (2\hat{i} + \hat{k}) = 5$$

Now, equation of plane in cartesian form

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{j} + \hat{k}) = 5; \therefore 2x + z = 5$$

Exercise 2 :

ACCELERATOR

Topic-wise MCQs

1. (d) $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ have vector form

$$= (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$$

Required equation in vector form is

$$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \mu(3\hat{i} + 7\hat{j} + 2\hat{k})$$

2. (a) The given line is

$$\frac{x-2}{2} = \frac{2y-5}{-3} = z+1,$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-\frac{5}{2}}{-\frac{3}{2}} = \frac{z+1}{0}$$

This shows that the given line passes through the point

$$\left(2, \frac{5}{2}, -1 \right) \text{ and has direction ratios } \left(2, \frac{-3}{2}, 0 \right).$$

Thus, given line passes through the point having position vector

$$\vec{a} = 2\hat{i} + \frac{5}{2}\hat{j} - \hat{k} \text{ and is parallel to the vector}$$

$$\vec{b} = \left(2\hat{i} - \frac{3}{2}\hat{j} - 0\hat{k} \right). \text{ So, its vector equation is}$$

$$\vec{r} = \left(2\hat{i} + \frac{5}{2}\hat{j} - \hat{k} \right) + \lambda \left(2\hat{i} - \frac{3}{2}\hat{j} - 0\hat{k} \right).$$

Hence, $p = 0$

3. (d) Given $\alpha = 45^\circ$, $\beta = 60^\circ$, $\gamma = ?$

$$\therefore \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\therefore \cos^2 \gamma = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \Rightarrow \gamma = 60^\circ \text{ or } 120^\circ$$

4. (d) The projection of the segment on the coordinates axes are -9, 12, -8. Thus the direction ratios of the segment PQ are -9, 12, -8. Hence the direction cosines are

$$-\frac{9}{17}, \frac{12}{17}, \frac{-8}{17}$$

5. (a) The lines intersect if

$$\begin{vmatrix} 2-1 & 3-2 & 1-3 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$$\Rightarrow 2k^2 + 5k - 25 = 0$$

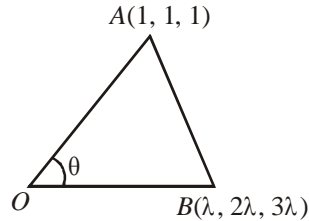
$$\Rightarrow k = -5, \frac{5}{2}$$

6. (b) Let any point on the second line be $(\lambda, 2\lambda, 3\lambda)$

$$\cos \theta = \frac{6}{\sqrt{42}}, \sin \theta = \frac{\sqrt{6}}{\sqrt{42}}$$

$$\Delta_{OAB} = \frac{1}{2} (OA) OB \sin \theta$$

$$= \frac{1}{2} \sqrt{3} \lambda \sqrt{14} \times \frac{\sqrt{6}}{\sqrt{42}} = \sqrt{6} \text{ or } \lambda = 2$$



So, B is $(2, 4, 6)$.

7. (b) Given equation of line is

$$\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$$

\Rightarrow DR's of the given line are 2, 1, 1

$$\Rightarrow \text{DC's of the given line are } \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$$

Since, required lines make an angle $\frac{\pi}{3}$ with the given line

The DC's of the required lines are

$$\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}} \text{ and } \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}} \text{ respectively.}$$

Also, both the required lines pass through the origin.

\therefore Equation of required lines are

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{-1} \text{ and } \frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$$

8. (b) The angle θ between the two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{a_2} = \frac{z-z_1}{a_3}$$

$$\text{and } \frac{x-x_2}{b_1} = \frac{y-y_2}{b_2} = \frac{z-z_2}{b_3} \text{ is given by:}$$

$$\cos \theta = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Now in the given equation: $a_1 = 2, a_2 = 2, a_3 = -1$
 $b_1 = 1, b_2 = 2, b_3 = 2$

$$\therefore \cos \theta = \frac{2 \cdot 1 + 2 \cdot 2 + (-1) \cdot 2}{\sqrt{4+4+1} \sqrt{4+4+1}} \Rightarrow \theta = \cos^{-1} \frac{4}{9}$$

9. (b) Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be the initial and final points of the vector whose projections on the three coordinate axes are 6, -3, 2 then

$$x_2 - x_1 = 6; y_2 - y_1 = -3; z_2 - z_1 = 2$$

So that direction ratios of \overline{PQ} are 6, -3, 2

\therefore Direction cosines of \overline{PQ} are

$$\frac{6}{\sqrt{6^2 + (-3)^2 + 2^2}}, \frac{-3}{\sqrt{6^2 + (-3)^2 + 2^2}},$$

$$\frac{2}{\sqrt{6^2 + (-3)^2 + 2^2}} = \frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$$

10. (c) Let feet of perpendicular is $(2\alpha, 3\alpha + 2, 4\alpha + 3)$

\Rightarrow Direction ratio of the \perp line is

$$2\alpha - 3, 3\alpha + 3, 4\alpha - 8. \text{ and}$$

\Rightarrow Direction ratio of the line are 2, 3, 4

$$\Rightarrow 2(2\alpha - 3) + 3(3\alpha + 3) + 4(4\alpha - 8) = 0$$

$$\Rightarrow \alpha = 1 \Rightarrow \text{Feet of } \perp \text{ is } (2, 5, 7)$$

$$\Rightarrow \text{Length } \perp \text{ is } \sqrt{1^2 + 6^2 + 4^2} = \sqrt{53}$$

11. (a) Let $a_1 = 2x, b_1 = 2x, c_1 = x$

$$\text{and } a_2 = 7 - 3 = 4, b_2 = 2 - 1 = 1, c_2 = 12 - 4 = 8$$

$$\therefore \cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{2x \times 4 + 2x \times 1 + x \times 8}{\sqrt{4x^2 + 4x^2 + x^2} \sqrt{16 + 1 + 64}}$$

$$= \frac{18x}{3x \times 9} = \frac{2}{3} \Rightarrow \theta = \cos^{-1} \left(\frac{2}{3} \right)$$

12. (c) Given, $l + m + n = 0$ and $l^2 = m^2 + n^2$

$$\text{Now, } (-m - n)^2 = m^2 + n^2$$

$$\Rightarrow mn = 0 \Rightarrow m = 0 \text{ or } n = 0$$

If $m = 0$ then $l = -n$

$$\text{We know } l^2 + m^2 + n^2 = 1 \Rightarrow n = \pm \frac{1}{\sqrt{2}}$$

$$\text{i.e. } (l_1, m_1, n_1) = \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right)$$

If $n = 0$ then $l = -m$

$$l^2 + m^2 + n^2 = 1 \Rightarrow 2m^2 = 1$$

$$\Rightarrow m = \pm \frac{1}{\sqrt{2}}$$

$$\text{Let } m = \frac{1}{\sqrt{2}} \Rightarrow l = -\frac{1}{\sqrt{2}} \text{ and } n = 0$$

$$(l_2, m_2, n_2) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right)$$

$$\therefore \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

13. (b) Equation of plane through $(1, 0, 0)$ is

$$a(x - 1) + by + cz = 0$$

...(i)

(i) passes through $(0, 1, 0)$.

$$-a + b = 0 \Rightarrow b = a;$$

$$\text{Also, } \cos 45^\circ = \frac{a+a}{\sqrt{2(2a^2+c^2)}}$$

$$\Rightarrow 2a = \sqrt{2a^2+c^2} \Rightarrow 2a^2 = c^2$$

$$\Rightarrow c = \sqrt{2}a.$$

So, d.r of normal are a, a, $\sqrt{2}a$ i.e. 1, 1, $\sqrt{2}$.

14. (c) Equation of line L_1 is

$$\frac{x-1}{2} = \frac{y-1}{1} = \frac{z}{-1} = \lambda$$

$$\Rightarrow x = 2\lambda + 1, y = \lambda + 1, z = -\lambda$$

Equation of line L_2 is

$$\frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{1} = \mu \Rightarrow x = \mu, y = \mu + 1, z = \mu + 2$$

On comparing we get $\mu = \lambda = -1$

\therefore Point of intersection $(-1, 0, 1)$

$$\Rightarrow x = -1, y = 0, z = 1$$

$$y - x = 0 + 1 = 1 = z$$

15. (c) $\vec{b}_1 = \vec{AB} = -2\hat{i} + \hat{j} + 2\hat{k}$ pass through

$$\vec{OA} = \hat{i} + \hat{j} - \hat{k} = \vec{a}_1, \vec{b}_2 = \vec{CD} = 2\hat{i} - 2\hat{j} \text{ pass through}$$

$$\vec{OC} = \hat{j} + 2\hat{k} = \vec{a}_2$$

Shortest distance between \vec{AB} and \vec{CD}

$$= \frac{|(a_2 - a_1) \cdot (b_1 \times b_2)|}{|b_1 \times b_2|}$$

$$= \frac{|(-\hat{i} + 3\hat{k}) \cdot (-2\hat{i} + \hat{j} + 2\hat{k}) \times (2\hat{i} - 2\hat{j})|}{|b_1 \times b_2|}$$

$$\frac{(-\hat{i} + 3\hat{k}) \cdot (4\hat{i} + 4\hat{j} + 2\hat{k})}{|4\hat{i} + 4\hat{j} + 2\hat{k}|} = \frac{|-4 + 6|}{6} = \frac{1}{3}$$

16. (d) Given points can be written as,

$$\vec{r} = (1+t)\hat{i} + (-6+2t)\hat{j} + (2+t)\hat{k} \quad \dots(i)$$

$$\text{and } \vec{r} = (2s)\hat{i} + (4+s)\hat{j} + (1+2s)\hat{k} \quad \dots(ii)$$

For point of intersection, we get

$$2s = 1 + t \text{ and } 4 + s = -6 + 2t$$

Solving above we get, $s = 4, t = 7$

Putting $s = 4$ in Eq. (ii), we get

$$\vec{r} = 8\hat{i} + 8\hat{j} + 9\hat{k}$$

17. (b) We know, shortest distance

$$= \frac{|(b_1 \times b_2) \cdot (a_2 - a_1)|}{|b_1 \times b_2|}$$

$$\therefore d = \frac{|[4\hat{i} + 3\hat{j} - \hat{k}] \times (6\hat{i} + 3\hat{j} - 2\hat{k}) \cdot ((\hat{i} + 2\hat{j} - 4\hat{k}) - (3\hat{i} - 5\hat{j} - 2\hat{k}))|}{|(4\hat{i} + 3\hat{j} - 2\hat{k}) \times (6\hat{i} + 3\hat{j} - 2\hat{k})|}$$

$$d = \frac{|(-3\hat{i} + 2\hat{j} - 6\hat{k}) \cdot (-2\hat{i} + 7\hat{j} - 6\hat{k})|}{|(1 - 3\hat{i} + 2\hat{j} - 6\hat{k})|} = \frac{56}{7} = 8$$

18. (a) A(1, 0, 2), B(2, 1, 0), C(2, -5, 3), D(0, 3, 2)

$$\text{Eq. of line AB is } \frac{x-1}{1} = \frac{y}{1} = \frac{z-2}{-2} = \lambda \quad \dots(i)$$

$$\text{Eq. of line CD is } \frac{x-2}{-2} = \frac{y+5}{8} = \frac{z-3}{-1} = \mu \quad \dots(ii)$$

$$\Rightarrow x = \lambda + 1, y = \lambda, z = -2\lambda + 2 \quad [\text{From (i)}]$$

$$\text{and } x = -2\mu + 2, y = 8\mu - 5, z = -\mu + 3 \quad [\text{From (ii)}]$$

$$\text{Solving for } \mu \text{ and } \lambda \text{ we get } \mu = \frac{3}{5}, \lambda = \frac{7}{5}$$

$$\therefore x = \frac{12}{5}, y = \frac{7}{5}, z = \frac{-4}{5}$$

$$P(a, b, c) = \left(\frac{12}{5}, \frac{7}{5}, \frac{-4}{5}\right) \Rightarrow a + b + c = 3$$

19. (c) Given, $\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + t(3\hat{i} - 2\hat{j} - 2\hat{k}) = \vec{a}_1 + t\vec{b}_1$

$$\text{and } \vec{r} = (7\hat{i} + 4\hat{k}) + s(\hat{i} - 2\hat{j} + 2\hat{k}) = \vec{a}_2 + s\vec{b}_2$$

$$\text{Now, } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & -2 \\ 1 & -2 & 2 \end{vmatrix} = -8\hat{i} - 8\hat{j} - 4\hat{k}$$

Since, required shortest distance

$$= \frac{|(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{|(-8\hat{i} - 8\hat{j} - 4\hat{k}) \cdot (8\hat{i} + 2\hat{j} + 7\hat{k})|}{|-8\hat{i} - 8\hat{j} - 4\hat{k}|}$$

$$= \frac{|-64 - 16 - 28|}{12} = \frac{108}{12} = 9.$$

20. (c) Since, equation of line AB is $\frac{x-0}{3} = \frac{y-2}{-2} = \frac{z-4}{-3}$

$$\Rightarrow \frac{x}{3} = \frac{y-2}{-2} = \frac{z-4}{-3} = k \quad (\text{Let})$$

$$\Rightarrow (x, y, z) = (3k, -2k + 2, -3k + 4)$$

Now, direction ratio of PC is $(3k + 1, -2k + 1, -3k + 4)$

$$\text{So, } 3(3k + 1) + (-2)(-2k + 1) - 3(-3k + 4) = 0$$

$$\Rightarrow 22k = 11 \Rightarrow k = \frac{1}{2}$$

$$\text{So, } C\left(\frac{3}{2}, 1, \frac{5}{2}\right) \therefore PC = \sqrt{\frac{25}{4} + 0 + \frac{25}{4}} = \frac{5}{\sqrt{2}}$$

21. (b) Equation of line through $(5, 1, a)$ and $(3, b, 1)$ is

$$\frac{x-5}{5-3} = \frac{y-1}{1-b} = \frac{z-a}{a-1} = \lambda \text{ (say)}$$

$$\text{Since this line passes through } \left(0, \frac{17}{2}, \frac{-13}{2}\right)$$

$$\text{Hence } \frac{0-5}{5-3} = \lambda \Rightarrow \lambda = -\frac{5}{2} \text{ and } \frac{\frac{17}{2}-1}{1-b} = \lambda = -\frac{5}{2}$$

$$\Rightarrow b = 4$$

$$\text{and } \frac{-\frac{13}{2}-a}{a-1} = \lambda = -\frac{5}{2} \Rightarrow a = 6$$

$$\text{Therefore } a + b = 6 + 4 = 10$$

22. (a) Given,

$$\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + t(\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\text{and } \vec{r} = (-4\hat{i} - \hat{k}) + s(3\hat{i} - 2\hat{j} - 2\hat{k})$$

$$\text{Here } \vec{a} = 6\hat{i} + 2\hat{j} + 2\hat{k}, b = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{c} = -4\hat{i} - \hat{k}, d = 3\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\therefore \text{Shortest distance} = \frac{|(\vec{a} - \vec{c}) \cdot (\vec{b} \times \vec{d})|}{|\vec{b} \times \vec{d}|} \quad \dots(i)$$

$$\text{Now, } \vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$$= \hat{i}(4+4) - \hat{j}(-2-6) + \hat{k}(-2+6) = 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$$\text{By Eq. (i), S.D.} = \frac{|(10\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (8\hat{i} + 8\hat{j} + 4\hat{k})|}{\sqrt{64+64+16}}$$

$$= \frac{|80+16+12|}{\sqrt{144}} = \frac{108}{12} = 9$$

23. (b) The lines are $\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$

$$\text{and } \frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}$$

$$\text{Here, } \vec{a}_1 = -2\hat{j} + \hat{k}, b_1 + 6\hat{i} + 6\hat{j} + \hat{k}, \vec{a}_2 = -\hat{i},$$

$$\vec{b}_2 = 12\hat{i} + 6\hat{j} - \hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 6 & 1 \\ 12 & 6 & -1 \end{vmatrix} = -12\hat{i} + 18\hat{j} - 36\hat{k}$$

$$\text{Shortest distance} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 - \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \frac{|(-\hat{i} + 2\hat{j} - \hat{k}) \cdot (-12\hat{i} + 18\hat{j} - 36\hat{k})|}{\sqrt{(-12)^2 + (18)^2 + (-36)^2}}$$

$$= \frac{|+12+36+36|}{\sqrt{1764}} = \frac{84}{42} = 2$$

24. (b) The given lines are parallel and

$$\vec{a}_1 = \hat{i} + \hat{j}, \vec{a}_2 = 2\hat{i} - 3\hat{k}$$

$$\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\text{Now, } \vec{a}_2 - \vec{a}_1 = (2\hat{i} - 3\hat{k}) - (\hat{i} + \hat{j}) = \hat{i} - \hat{j} - 3\hat{k}$$

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 1 & -1 & -3 \end{vmatrix}$$

$$= \hat{i}(6+3) - \hat{j}(-3-3) + \hat{k}(-1+2)$$

$$= 9\hat{i} + 6\hat{j} + \hat{k}$$

$$|\vec{b}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$\text{Distance, } d = \frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|} = \frac{|9\hat{i} + 6\hat{j} + \hat{k}|}{\sqrt{14}}$$

$$= \frac{1}{\sqrt{14}} \sqrt{(9)^2 + (6)^2 + (1)^2} = \sqrt{\frac{59}{7}}$$

25. (a) Any line passing through the point $(1, 1, 1)$ is

$$\frac{x-1}{a} = \frac{y-1}{b} = \frac{z-1}{c} \quad \dots(i)$$

$$\text{This line intersects the line } \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

$$\text{If } a : b : c \neq 2 : 3 : 4 \text{ and } \begin{vmatrix} 1-1 & 2-1 & 3-1 \\ a & b & c \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\Rightarrow a - 2b + c = 0 \quad \dots(ii)$$

$$\text{Again, line (i) intersects line } \frac{x-(-2)}{1} = \frac{y-3}{2} = \frac{z-(-1)}{4}$$

$$\text{If } a : b : c \neq 2 : 3 : 4 \text{ and } \begin{vmatrix} -2-1 & 3-1 & -1-1 \\ a & b & c \\ 1 & 2 & 4 \end{vmatrix} = 0$$

$$\Rightarrow 6a + 5b - 4c = 0$$

From (ii) and (iii) by cross multiplication, we have ... (iii)

$$\frac{a}{8-5} = \frac{b}{6+4} = \frac{c}{5+12} \text{ or } \frac{a}{3} = \frac{b}{10} = \frac{c}{17}$$

$$\text{So, the required line is } \frac{x-1}{3} = \frac{y-1}{10} = \frac{z-1}{17}.$$

26. (a) Given equation of line is

$$x+5 = \frac{1}{4}(y+3) = -\frac{1}{9}(z-6)$$

$$\text{or } \frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda (\text{say})$$

$$x = \lambda - 5, y = 4\lambda - 3, z = -9\lambda + 6$$

$$(x, y, z) \equiv (\lambda - 5, 4\lambda - 3, -9\lambda + 6) \quad \dots (i)$$

Let it is foot of perpendicular

$$\text{So, d.r.'s of } \perp \text{ line is } (\lambda - 5 - 2, 4\lambda - 3 - 4, -9\lambda + 6 + 1)$$

$$\equiv (\lambda - 7, 4\lambda - 7, -9\lambda + 7)$$

D.r.'s of given line is (1, 4, -9) and both lines are \perp

$$\therefore (\lambda - 7) \cdot 1 + (4\lambda - 7) \cdot 4 + (-9\lambda + 7) \cdot (-9) = 0$$

$$\Rightarrow 98\lambda = 98 \Rightarrow \lambda = 1 \therefore \text{Point is } (-4, 1, -3).$$

27. (b) Given

$$l + 3m + 5n = 0 \quad \dots (i)$$

$$\text{and } 5lm - 2mn + 6nl = 0 \quad \dots (ii)$$

From eq. (i) we have

$$l = -3m - 5n$$

Put the value of l in eq. (ii), we get;

$$5(-3m - 5n)m - 2mn + 6n(-3m - 5n) = 0$$

$$\Rightarrow 15m^2 + 45mn + 30n^2 = 0 \Rightarrow m^2 + 3mn + 2n^2 = 0$$

$$\Rightarrow m^2 + 2mn + mn + 2n^2 = 0 \Rightarrow (m+n)(m+2n) = 0$$

$$\therefore m = -n \text{ or } m = -2n$$

$$\text{For } m = -n, l = -2n$$

$$\text{And for } m = -2n, l = n$$

$$\therefore (l, m, n) = (-2n, -n, n) \text{ Or } (l, m, n) = (n, -2n, n)$$

$$\Rightarrow (l, m, n) = (-2, -1, 1) \text{ Or } (l, m, n) = (1, -2, 1)$$

Therefore, angle between the lines is given as:

$$\cos(\theta) = \frac{(-2)(1) + (-1)(-2) + (1)(1)}{\sqrt{6} \cdot \sqrt{6}}$$

$$\Rightarrow \cos(\theta) = \frac{1}{6} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6}\right)$$

28. (a) Given points and direction ratios are shown below.

$$a_1 = (1, 2, 3), a_2 = (2, 4, 5), \vec{b}_2 = 2\hat{i} + 3\hat{j} + \lambda\hat{k}$$

$$\vec{b}_2 = \hat{i} + 4\hat{j} + 5\hat{k}$$

Apply shortest distance formula,

$$\text{Shortest distance} = \frac{|(a_2 - a_1) \cdot (b_1 \times b_2)|}{|b_1 \times b_2|}$$

$$\text{S.D.} = \frac{|((2-1)\hat{i} + (4-2)\hat{j} + (5-3)\hat{k}) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|b_1 \times b_2|} \quad \dots (i)$$

$$\text{Take, } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & \lambda \\ 1 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}(15 - 4\lambda) + \hat{j}(\lambda - 10) + \hat{k}(5)$$

$$= (15 - 4\lambda)\hat{i} + (\lambda - 10)\hat{j} + 5\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(15 - 4\lambda)^2 + (\lambda - 10)^2 + 25}$$

From equation (i),

$$\text{S.D.} = \frac{|(\hat{i} + 2\hat{j} + 2\hat{k}) \cdot [(15 - 4\lambda)\hat{i} + (\lambda - 10)\hat{j} + 5\hat{k}]|}{\sqrt{(15 - 4\lambda)^2 + (\lambda - 10)^2 + 25}}$$

$$\frac{|15 - 4\lambda + 2\lambda - 20 + 10|}{\sqrt{(15 - 4\lambda)^2 + (\lambda - 10)^2 + 25}} = \frac{1}{\sqrt{3}}$$

Take square both sides,

$$3(5 - 2\lambda)^2 = 225 + 16\lambda^2 - 120\lambda + \lambda^2 + 100 - 20\lambda + 25$$

$$12\lambda^2 + 75 - 60\lambda = 17\lambda^2 - 140\lambda + 350$$

$$5\lambda^2 - 80\lambda + 275 = 0 \Rightarrow \lambda^2 - 16\lambda + 55 = 0$$

$$(\lambda - 5)(\lambda - 11) = 0 \Rightarrow \lambda = 5, 11$$

$$\text{Sum of values of } \lambda = 5 + 11 = 16$$

29. (a) Solving the equations of the planes we get $y = -2$.

Put $z = 0$ and $y = -2$ in any of the plane we get $x = 5$ so

$(5, -2, 0)$ is a point on the line of intersection. If the line has direction cosines proportional to l, m, n , then

$$l + m + 2n = 0$$

$$\text{and } 2l + 3m + 4n = 0$$

On solving we get, $\frac{l}{-2} = \frac{m}{0} = \frac{n}{1}$ and so the equation of

$$\text{line is } \frac{x-5}{-2} = \frac{y+2}{0} = \frac{z}{1}$$

$$\text{Equation of z-axis is } \frac{x}{0} = \frac{y}{0} = \frac{z}{1}$$

\therefore shortest distance

$$= \frac{\begin{vmatrix} 5 & -2 & 0 \\ -2 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix}}{\sqrt{(0-0)^2 + (0+2)^2 + (0-0)^2}} = \frac{4}{2} = 2$$

30. (c) Line along $\vec{a} = \hat{i} + \hat{j}$ and passes through P is

$$\frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{0} = \lambda$$

$$\therefore L(\lambda+1, \lambda-1, -1)$$

Line along $\vec{b} = \hat{j} - \hat{k}$ and passes through Q is

$$\frac{x+1}{0} = \frac{y-1}{1} = \frac{z-1}{-1} = \mu \therefore M(-1, \mu+1, -\mu+1)$$

D.r.s. of LM = $(\lambda+2, \lambda-\mu-2, \mu-2)$

Since LM and in same direction,

$$\therefore \frac{\lambda+2}{1} = \frac{\lambda-\mu-2}{-1} = \frac{\mu-2}{1} \Rightarrow \text{we get } \lambda = \mu \text{ and } \mu = 8$$

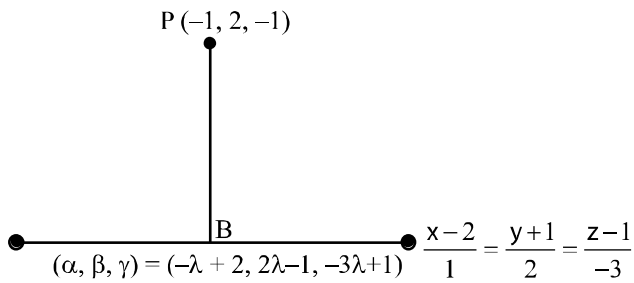
$$\therefore M(-1, 9, -7)$$

$$\text{Now, } \vec{PM} = -2\hat{i} + 10\hat{j} - 6\hat{k}$$

31. (b) Equation of line joining the points

$(2, -1, 1)$ and $(1, 1, -2)$ is

$$\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-1}{-3}$$



$$\text{Let } \frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-1}{-3} = \lambda$$

$$x = -\lambda + 2, y = 2\lambda - 1, z = -3\lambda + 1$$

$$\alpha = -\lambda + 2, \beta = 2\lambda - 1, \gamma = -3\lambda + 1$$

$$\text{Dr's of PB } (-\lambda + 3, 2\lambda - 3, -3\lambda + 2)$$

Since, $PB \perp$ line

$$-1(-\lambda + 3) + 2(2\lambda - 3) - 3(-3\lambda + 2) = 0$$

$$\lambda = \frac{15}{14}$$

$$B\left(\frac{13}{14}, \frac{16}{14}, \frac{-31}{14}\right)$$

$$\alpha + \beta + \gamma = \frac{13}{14} + \frac{16}{14} - \frac{31}{14} = \frac{-2}{14} = \frac{-1}{7}$$

32. (d) Equation of plane passing through the line of intersection is

$$\vec{r} \cdot [(\hat{i} - 2\hat{k}) + \lambda(2\hat{j} + \hat{k})] = 3 + \lambda 5$$

$$\Rightarrow \vec{r} \cdot [\hat{i} + 2\lambda\hat{j} + (\lambda - 2)\hat{k}] = 3 + \lambda 5$$

Since it passes through point $(\hat{i} + 2\hat{j} + 3\hat{k})$

$$(\hat{i} + 2\hat{j} + 3\hat{k}) \cdot [\hat{i} + 2\lambda\hat{j} + (\lambda - 2)\hat{k}] = 3 + \lambda 5$$

$$1 + 4\lambda + 3\lambda - 6 = 3 + 5\lambda \Rightarrow \lambda = 4$$

\therefore Equation of plane is

$$\vec{r} \cdot (\hat{i} + 8\hat{j} + 2\hat{k}) = 23.$$

33. (a) Given, $A = \hat{i} - 3\hat{j}$

$$P = (1, -3, 0) + t(0, 1, -2)$$

$$= (1, -3 + t, -2t)$$

Distance point P from plane $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 5\hat{k}) = 0$

$$= \frac{|2(1) + 3(-3 + t) + 5(-2t)|}{\sqrt{2^2 + 3^2 + 5^2}}$$

$$= \frac{|-7 - 7t|}{\sqrt{38}}$$

Min. distance occurs when numerator = 0

$$1 + t = 0 \Rightarrow t = -1$$

$$\text{So, } P = (1, -4, 2)$$

$$\therefore AP = \vec{P} - \vec{A} = (0, -1, 2)$$

Equation of plane

$$0(x-1) - 1(y+4) + 2(z-2) = 0$$

$$-y + 2z = 8$$

$$\vec{r} \cdot (-\hat{j} + 2\hat{k}) = 8$$

34. (c) If the plane $ax + by + cz + d = 0$ to be perpendicular to xy - plane, the coefficient of z is equal to zero.

$$\Rightarrow c = 0.$$

35. (c) Direction ratios. of given line are 1, -2, 3 and the d.r. of normal to the given plane are 1, 2, 1.

Since $1 \times 1 + (-2) \times 2 + 3 \times 1 = 0$, therefore, the line is parallel to the plane.

Also, the base point of the line $(1, 2, 1)$ lies in the given plane.

$$(1 + 2 \times 2 + 1 = 6 \text{ is true})$$

Hence, the given line lies in the given plane.

36. (a) The planes $x + y = 0$ i.e. $x = -y$ and $y + z = 0$

$$\text{i.e. } z = -y \text{ meet in the line } \frac{x}{1} = \frac{y}{-1} = \frac{z}{1}.$$

Any point on this line is $(t, -t, t)$. This point lies in the plane $x + z = 0$ if $t + t = 0 \Rightarrow t = 0$.

So the three planes meet in a unique point $(0, 0, 0)$.

37. (c) Required equation of plane $P_1 + \lambda P_2 = 0$

$$\Rightarrow (x + 3y - z - 5) + \lambda(2x - y + z - 3) = 0$$

$$x(1 + 2\lambda) + y(3 - \lambda) + z(\lambda - 1) = (3\lambda + 5)$$

Satisfy $(2, 1, -2)$ in the above equation.

$$\text{So, } \lambda = 1 \Rightarrow \text{Plane is } 3x + 2y - 8 = 0$$

38. (a) Dr's of line are $\langle 1, -1, 2 \rangle$

Dr's of normal of plane $\langle 2, 3, \lambda \rangle$

Line is perpendicular to normal of plane

$$\vec{d} \cdot \vec{n} = 0 \Rightarrow 2 - 3 + 2\lambda = 0$$

$$\therefore \lambda = \frac{1}{2}$$

39. (d) The required plane is

$$(2 + 3\lambda)x + (-8 - 5\lambda)y + (4 + 4\lambda)z + P - 10\lambda = 0$$

Compare the coefficients with the plane

$$\text{We get, } 4 + 4\lambda = 0 \Rightarrow \lambda = -1$$

$$x + 3y + 0z + 13 = 0$$

Then we get $p = 3$.

40. (c) Since, OP is perpendicular to the plane

$$\therefore \vec{N} = \vec{OP} = \hat{i} + 2\hat{j} - 3\hat{k}$$

Plane is passing through P

$$\therefore \vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$$

Therefore, equation of the plane is

$$r \cdot \vec{N} = \vec{a} \cdot \vec{N}$$

$$\Rightarrow r \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = (\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})$$

$$x + 2y - 3z = 14$$

41. (c) Given lines will be coplanar

$$\text{If } \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow -1(1 + 2k) - (1 + k^2) + 1(2 - k) = 0$$

$$\Rightarrow k = 0, -3$$

42. (d) Obviously the line perpendicular to the plane because

$$\frac{a}{a} = \frac{b}{b} = \frac{c}{c} \text{ i.e., their direction ratios are proportional.}$$

43. (a) The equation of the given line is $6x = 4y = 3z$

which is written in symmetric form as

$$\frac{x-0}{1/6} = \frac{y-0}{1/4} = \frac{z-0}{1/3}$$

Direction ratios of this line are $\left(\frac{1}{6}, \frac{1}{4}, \frac{1}{3}\right)$ and equation of the plane is $3x + 2y - 3z - 4 = 0$

If θ be the angle between line and plane, then direction ratios of the normal to this plane is $(3, 2, -3)$

$$\sin \theta = \frac{\left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|}{\left| \frac{\frac{1}{6} \times 3 + \frac{1}{4} \times 2 + \frac{1}{3} \times (-3)}{\sqrt{\frac{1}{36} + \frac{1}{16} + \frac{1}{9}} \sqrt{9 + 4 + 9}} \right|} = 0$$

$$\Rightarrow \theta = 0^\circ$$

44. (a) Equation of the line is $\frac{x-2}{1} = \frac{y+2}{-3} = \frac{z-5}{2} = \lambda \dots (i)$

Hence, any point on the line (i) can be taken as

$$x = \lambda + 2$$

$$y = -(3\lambda + 2)$$

$$z = (2\lambda + 5)$$

From some λ point lies on the plane

$$2x - 3y + 4z = 163$$

$$2(\lambda + 2) + 3(3\lambda + 2) + 4(2\lambda + 5) = 163$$

$$\Rightarrow 19\lambda = 133 \Rightarrow \lambda = 7$$

Hence, $P \equiv (9, -23, 19)$

Also, Eq. (i) intersect YZ-plane, i.e., $x = 0$

$$\lambda + 2 = 0$$

Hence, $\lambda = -2$

$$\therefore Q(0, 4, 1)$$

$$\text{So, } PQ = \sqrt{9^2 + 27^2 + 18^2} = 9\sqrt{1 + 3^2 + 2^2} = 9\sqrt{14}$$

$$\Rightarrow a = 9 \text{ and } b = 14$$

Hence, $a + b = 9 + 14 = 23$.

45. (c) Let the direction cosines of line L be l, m, n , then

$$2l + 3m + n = 0 \quad \dots (i)$$

$$\text{and } l + 3m + 2n = 0 \quad \dots (ii)$$

on solving equations (i) and (ii), we get

$$\frac{l}{6-3} = \frac{m}{1-4} = \frac{n}{6-3} \Rightarrow \frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$$

$$\text{Now } \frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{3^2 + (-3)^2 + 3^2}}$$

$$\therefore l^2 + m^2 + n^2 = 1$$

$$\therefore \frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{1}{\sqrt{27}}$$

$$\Rightarrow l = \frac{3}{\sqrt{27}} = \frac{1}{\sqrt{3}}, m = -\frac{1}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}$$

Line L , makes an angle α with +ve x -axis

$$\therefore l = \cos \alpha \Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

46. (d) Equation of plane bisecting the angle containing origin is (making constant term of same sign)

$$\frac{-3x+6y-2z-5}{\sqrt{3^2+6^2+2^2}} = + \left[\frac{4x-12y+3z-3}{\sqrt{4^2+12^2+3^2}} \right]$$

$$\text{or } \frac{-3x+6y-2z-5}{7} = \frac{4x-12y+3z-3}{13}$$

$$\text{or } 67x-162y+47z+44=0.$$

47. (a) Two given planes are coplanar, if

$$\begin{vmatrix} -2-(-1) & k-1 & 0-(-1) \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & k-1 & 1 \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\Rightarrow (-1)(4-9) - (k-1)(8-6) + 6-2 = 0 \Rightarrow k = \frac{11}{2}$$

48. (d) The given lines are

$$x-1 = \frac{y+3}{-\lambda} = \frac{z-1}{\lambda} = s \quad \dots(i)$$

$$\text{and } 2x = y-1 = \frac{z-2}{-1} = t \quad \dots(ii)$$

The lines are coplanar, if

$$\begin{vmatrix} 0-1 & 1-(-3) & 2-1 \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & 4 & 1 \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix} = 0$$

$$\text{Apply } c_2 \rightarrow c_2 + c_3; \begin{vmatrix} -1 & 5 & 1 \\ 1 & 0 & \lambda \\ \frac{1}{2} & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow -5 \left(-1 - \frac{\lambda}{2} \right) = 0 \Rightarrow \lambda = -2$$

49. (d) Two planes are coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

Applying $C_2 \rightarrow C_2 + C_1, C_3 \rightarrow C_3 + C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1-k \\ k & k+2 & 1+k \end{vmatrix} = 0$$

$$\Rightarrow 1[2+2k-(k+2)(1-k)] = 0$$

$$\Rightarrow 2+2k-(-k^2-k+2) = 0$$

$$k^2+3k=0 \Rightarrow k(k+3)=0$$

$$\text{or } k=0 \text{ or } -3$$

50. (c) Since the lines are passing through (0, 0, 0)

$$a(x-3)+b(y+4)+c(z-7)=0$$

$$P: 9a-b-5c=0$$

$$-11a-b+5c=0$$

Here we get direction ratios as (1, -1, 2) equation of plane

$$= x-y+2z=21$$

$$d = \frac{8}{\sqrt{6}}$$

$$d^2 = \frac{32}{3}$$

51. (c) $2l+3m+4n=0$ and $3l+4m+5n=0$

$$\Rightarrow \frac{l}{-1} = \frac{m}{2} = \frac{n}{-1}$$

Equation of plane will be

$$a(x-1)+b(y-2)+c(z-3)=0$$

$$\Rightarrow -l(x-1)+2(y-2)-1(z-3)=0$$

$$\Rightarrow -x+1+2y-4-z+3=0 \Rightarrow -x+2y-z=0$$

$$\Rightarrow x-2y+z=0$$

$$\therefore \frac{|d|}{\sqrt{6}} = \sqrt{6} \text{ or } d=6$$

52. (c) $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -4 & 1 \end{vmatrix} = -2\hat{i} - 4\hat{j} - 10\hat{k}$

Equation of plane is

$$\vec{r} \cdot (-2\hat{i} - 4\hat{j} - 10\hat{k}) = 0 \Rightarrow x+2y+5z=0$$

53. (b) We know that, the image (α, β, γ) of a point (x_1, y_1, z_1) in a plane $ax+by+cz+d=0$ is given by

$$\frac{\alpha-x_1}{a} = \frac{\beta-y_1}{b} = \frac{\gamma-z_1}{c} = \frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$

$$\therefore -2 \frac{(ax_1+by_1+(cz_1+d))}{a^2+b^2+c^2} = \frac{-2(4+2+4+1)}{4^2+2^2+4^2} = \frac{-11}{36}$$

$$\therefore \frac{\alpha-x_1}{a} = \frac{-11}{36} \Rightarrow \frac{\alpha-1}{4} = \frac{-11}{36} \Rightarrow \alpha = \frac{-13}{9}$$

$$\therefore \frac{\beta-y_1}{b} = \frac{-11}{36} \Rightarrow \frac{\beta-1}{4} = \frac{-11}{36} \Rightarrow \beta = \frac{-2}{9}$$

$$\therefore \frac{\gamma - z_1}{c} = \frac{-11}{36} \Rightarrow \frac{\gamma - 1}{4} = \frac{-11}{36} \Rightarrow \gamma = \frac{-13}{9}$$

$$\text{Hence, } \alpha + \beta + \gamma = \frac{-13}{9} - \frac{2}{9} - \frac{13}{9} = -\frac{28}{9}$$

54. (b) $\vec{n} = 2\hat{i} - \hat{j} + 3\hat{k}$

$$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$$

Equation of plane is $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$$2x - y + 3z = 4 + 1 + 9$$

$$\Rightarrow 2x - y + 3z - 14 = 0$$

55. (d) p is positive real number.

Perpendicular distance from point $(-\hat{i} + p\hat{i} - 3\hat{k})$ to

plane $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) - 7 = 0$ is 6

$$\therefore \frac{|(-\hat{i} + p\hat{i} - 3\hat{k}) \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) - 7|}{|2\hat{i} - 3\hat{j} + 6\hat{k}|} = 6$$

$$\frac{|-2 - 3p - 18 - 7|}{7} = 6$$

$$|-3p - 27| = 42$$

$$\Rightarrow 3p + 27 = 42 \quad \text{or} \quad 3p - 27 = -42$$

$$3p = 15 \quad 3p = -15$$

$$p = 5 \quad p = -5 \quad (\text{neglect})$$

56. (c) Given, $V_1 = \hat{i} + \hat{j}$, $V_2 = \hat{i} + 2\hat{j}$,

$$V_3 = 2\hat{i} - \hat{j}, V_4 = 3\hat{i} + 2\hat{k}$$

$$\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k} \quad \therefore \vec{n}_1 = V_1 \times V_2 = \hat{k}$$

$$\therefore \vec{n}_2 = V_3 \times V_4 = -2\hat{i} - 4\hat{j} + 3\hat{k}$$

$$\text{So, } \vec{a} = \vec{n}_1 \times \vec{n}_2$$

$$= \hat{k} \times (-2\hat{i} - 4\hat{j} + 3\hat{k}) = 4\hat{i} - 2\hat{j}$$

$$\text{Thus } \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\cos \theta = \frac{(4\hat{i} - 2\hat{j}) \cdot (\hat{i} - 2\hat{j} + 2\hat{k})}{(\sqrt{4^2 + (-2)^2}) (\sqrt{1^2 + (-2)^2 + 2^2})}$$

$$\cos \theta = \frac{4}{3\sqrt{5}}$$

57. (c) The normals are proportional: $(-4, -2, 2)$
 $= -2(2, 1, -1)$.

So the planes are parallel.

$$d = \frac{|\alpha - (-2)|}{\sqrt{(-4)^2 + (-2)^2 + 2^2}} = \frac{|\alpha + 2|}{2\sqrt{6}}$$

$$\text{So, } 2 = \frac{|\alpha + 2|}{2\sqrt{6}} \Rightarrow \alpha = -2 \pm 4\sqrt{6}$$

$$\alpha = (-2 + 4\sqrt{6})(-2 - 4\sqrt{6}) = -92$$

58. (a) $\vec{n}_1 = a\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{n}_2 = 3\hat{i} + a\hat{j} + \hat{k}$

$$\cos \frac{\pi}{3} = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1| |\vec{n}_2|} = \frac{|3a - a + 3|}{\sqrt{a^2 + 1 + 9} \sqrt{9 + a^2 + 1}}$$

$$\frac{1}{2} = \frac{|2a + 3|}{a^2 + 10} \Rightarrow a = 2$$

\therefore Equation of plane is $(a + 2)x + (a - 4)y + 2az = a$
 $\Rightarrow 2x - y + 2z = 1$

\therefore Direction ratios of normal are $(2, -1, 2)$.

59. (d) Two given planes mutually perpendicular
 $2(3k) + k(-k) + (-5)1 = 0$

$$k = 1, 5$$

but $k < 3$, so $k = 1$

Plane passing through these is

Hence family of planes is $P_1 + \lambda P_2 = 0$

$$2x + y - 5z - 1 + \lambda(3x - y + z - 5) = 0$$

$$(2 + 3\lambda)x + (1 - \lambda)y + (\lambda - 5)z - (1 + 5\lambda) = 0$$

$$\frac{x}{\frac{5\lambda + 1}{2 + 3\lambda}} + \frac{y}{\frac{5\lambda + 1}{1 - \lambda}} + \frac{z}{\frac{5\lambda + 1}{\lambda - 5}} = 1$$

$$\text{Given } \frac{5\lambda + 1}{2 + 3\lambda} = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$\text{So intercept on y-axis} = \frac{5\lambda + 1}{1 - \lambda} = 7$$

60. (b) Equation of plane are $P_1: \vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 6$ and

$$P_2: \vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) = 7$$

Equation of plane through intersection of two plane
 $P_1 + \lambda P_2 = 0$

$$(\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) - 6) + \lambda(\vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) - 7) = 0 \dots\dots(i)$$

and it passes through point $\left(2, 3, \frac{1}{2}\right)$

$$\Rightarrow \left(2 + 9 - \frac{1}{2} - 6\right) + \lambda \left(-12 + 15 - \frac{1}{2} - 7\right) = 0$$

$$\Rightarrow \lambda = 1$$

Put the value of λ in equation (i),

Equation of plane is $\vec{r} \cdot (-5\hat{i} + 8\hat{j} - 2\hat{k}) = 13$

$$|\vec{a}|^2 = 25 + 64 + 4 = 93; d = 13$$

$$\text{Value of } \frac{|13\vec{a}|^2}{d^2} = 93$$

61. (d) D.r.'s of the line is (3, 4, 5) and d.r.'s of the normal of the plane is (2, -2, 1).

Let θ be the angle between line and plane, then $(90 - \theta)$ be the angle between the line and normal of the plane.

$$\therefore \cos(90 - \theta) = \frac{3 \times 2 + 4(-2) + 5 \times 1}{\sqrt{3^2 + 4^2 + 5^2} \sqrt{2^2 + (-2)^2 + 1^2}}$$

$$\Rightarrow \sin \theta = \frac{6 - 8 + 5}{\sqrt{50} \sqrt{9}} \Rightarrow \sin \theta = \frac{3}{5\sqrt{2} \times 3}$$

$$\Rightarrow \sin \theta = \frac{1}{5\sqrt{2}} \Rightarrow \sin \theta = \frac{\sqrt{2}}{10}$$

62. (a) Angle between line and normal to plane is

$$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{2 - 2 + 2\sqrt{\lambda}}{3 \times \sqrt{5 + \lambda}}$$

Where θ is angle between line and plane

$$\Rightarrow \sin \theta = \frac{2\sqrt{\lambda}}{3\sqrt{5 + \lambda}} = \frac{1}{3} \Rightarrow \lambda = \frac{5}{3}$$

63. (a) Given equation of line is

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$$

$$(\hat{x}\hat{i} + \hat{y}\hat{j} + \hat{z}\hat{k}) = (2 + 3\lambda)\hat{i} + (-1 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}$$

Any point on the line is

$$(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda)$$

Since it also lie on the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k})$

$$\text{So, } [(2 + 3\lambda)\hat{i} + (-1 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}] \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

$$\Rightarrow 2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5$$

$$\Rightarrow \lambda = 0$$

Therefore, coordinate of the point of intersection of line and plane is (2, -1, 2).

$$\therefore \text{Distance } d = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13$$

64. (b) Equation of any plane passing through (-2, -2, 2) is $a(x+2) + b(y+2) + c(z-2) = 0$ since it contains the line joining the points (1, 1, 1) and (1, -1, 2), it contains these points as well so that

$$3a + 3b - c = 0 \text{ and } 3a + b + 0 = 0$$

on solving we get $\frac{a}{1} = \frac{b}{-3} = \frac{c}{-6}$ and thus the equation of the plane is

$$x + 2 - 3(y + 2) - 6(z - 2) = 0$$

$$\text{or } \frac{x}{-8} + \frac{y}{8/3} + \frac{z}{8/6} = 1$$

$$\text{The reqd. sum} = -8 + \frac{8}{3} + \frac{8}{6} = -4.$$

65. (c) Equation of a plane through the line of intersection of given plane is

$$ax + by + cz + d + \lambda(a'x + b'y + c'z + d') = 0$$

$$\Rightarrow (a + \lambda a')x + (b + \lambda b')y + (c + \lambda c')z + (d + \lambda d') = 0$$

It is parallel to $y = 0, z = 0$

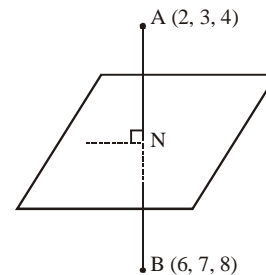
i.e., x-axis whose direction ratios are 1, 0, 0

$$\therefore 1(a + \lambda a') + 0(b + \lambda b') + 0(c + \lambda c') = 0 \Rightarrow \lambda = -\frac{a}{a'}$$

Hence, the required plane is

$$y(a'b - ab') + z(a'c - ac') + (a'd - ad') = 0$$

66. (b) If the given points be A (2, 3, 4) and B (6, 7, 8), then their mid-point N(4, 5, 6) must lie on the plane. The direction ratios of AB are 4, 4, 4, i.e. 1, 1, 1.



\therefore The required plane passes through N (4, 5, 6) and is normal to AB. Thus its equation is

$$1(x - 4) + 1(y - 5) + 1(z - 6) = 0 \Rightarrow x + y + z = 15$$

67. (b) We shall find the equation of a plane passing through any three out of the given four points and show that the fourth point satisfies the equation.

Now, any plane passing through (0, -1, -1) is:

$$a(x - 0) + b(y + 1) + c(z + 1) = 0 \quad \dots(i)$$

If it passes through (-4, 4, 4); we have

$$a(-4) + b(5) + c(5) = 0 \quad \dots(ii)$$

Also, if plane passes through (4, 5, 1) we have

$$a(4) + b(6) + c(2) = 0$$

$$\Rightarrow 2a + 3b + c = 0 \quad \dots(iii)$$

Solving (ii) and (iii) by cross multiplication method, we obtain

$$\frac{a}{-5} = \frac{b}{7} = \frac{c}{-11} = k$$

Putting in (i), we get

$$-5kx + 7k(y+1) - 11k(z+1) = 0$$

(Required equation of plane)

Clearly the fourth point namely (3, 9, 4) satisfies this equation hence the given points are coplanar and the equation of plane containing those points is

68. (a) The given lines are

$$\frac{x+1}{a} = y = \frac{z-1}{a}$$

$$\frac{x+2}{3} = y = \frac{z}{3/b}$$

Since, lines are co-planar, then

$$\begin{vmatrix} a & 1 & a \\ 3 & 1 & \frac{3}{b} \\ -1 & 0 & -1 \end{vmatrix} = 0 \Rightarrow -\left(\frac{3}{b} - a\right) - 1(a-3) = 0$$

$$\Rightarrow a - \frac{3}{b} - a + 3 = 0$$

$$\Rightarrow b = 1, a \in \mathbb{R} - \{0\}$$

69. (b) Let $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Then,

$$\hat{n} = \frac{\vec{n}}{|\hat{n}|} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{4+9+16}} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$$

Hence, the required equation of the plane is

$$\vec{r} \cdot \left(\frac{2}{\sqrt{29}}\hat{i} + \frac{-3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k} \right) = \frac{6}{\sqrt{29}}$$

70. (c) Equation of plane passing through (1, 1, 1) and perpendicular to line

$$\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4} \text{ is } 3x + 4z - 7 = 0$$

$$\text{Distance of plane from the origin} = \frac{|0+0-7|}{\sqrt{9+16}} = \frac{7}{5}$$

71. (b) Length of perpendicular from the origin to the plane

$$ax + by + \sqrt{2ab}z - 1 = 0 \text{ is}$$

$$= \frac{|0+0+0-1|}{\sqrt{a^2+b^2+2ab}} = \frac{1}{\sqrt{(a+b)^2}} = \frac{1}{a+b}$$

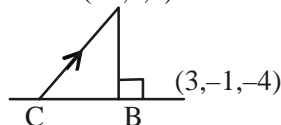
72. (b) Here, $a = 2\hat{i} + 5\hat{j} - 3\hat{k}$, $N = 6\hat{i} - 3\hat{j} + 2\hat{k}$ and $d = 4$.

Therefore, the distance of the point (2, 5, -3) from the

$$\text{given plane is } \frac{|(2\hat{i} + 5\hat{j} - 3\hat{k}) \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) - 4|}{|6\hat{i} - 3\hat{j} + 2\hat{k}|}$$

$$= \frac{|12 - 15 - 6 - 4|}{\sqrt{36+9+4}} = \frac{13}{7} \quad \left(\because \text{distance} = \left| \frac{a \cdot N - d}{N} \right| \right)$$

73. (b) $A(-1, 4, 3)$



Let B be foot of \perp coordinates of B = $\left(-2, \frac{7}{2}, \frac{3}{2}\right)$

\therefore Direction ratio of line AB is $\langle 2, 1, 3 \rangle$ so $m = 1, n = 3$

So equation of AC is $\frac{x+1}{3} = \frac{y-4}{-1} = \frac{z-3}{-4} = \lambda$

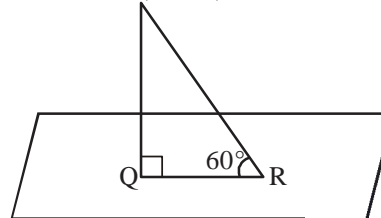
\Rightarrow point C is $(3\lambda - 1, -\lambda + 4, -4\lambda + 3)$. \therefore C lies on the plane.

$$\therefore 6\lambda - 2 - \lambda + 4 - 12\lambda + 9 = 4$$

$$\Rightarrow \lambda = 1 \Rightarrow C(2, 3, -1)$$

$$\Rightarrow AC = \sqrt{26}$$

74. (b) $P(1, 2, 3)$



$$x + 2y + z = 14$$

$$\text{Length of perpendicular PQ} = \left| \frac{1+4+3-14}{\sqrt{6}} \right| = \sqrt{6}$$

$$QR = (PQ) \cot 60^\circ = \sqrt{2}$$

$$\therefore \text{Area of } \Delta PQR = \frac{1}{2}(PQ)(QR) = \sqrt{3}$$

75. (c) The foot of the perpendicular

F(1, -2, 0) lies on the plane $ax + by - 3z + d = 0$

$$a(1) + b(-2) - 3(0) + d = 0$$

$$a - 2b + d = 0$$

...(i)

$$\therefore \vec{PF} = \vec{F} - \vec{P} = (-1, -2, 3)$$

The normal vector of the plane

$$ax + by - 3z + d = 0 \text{ is } \vec{n} = (a, b, -3)$$

$$\therefore \vec{PF} \parallel \vec{n}$$

$$\text{So, } \frac{-1}{a} = \frac{-2}{b} = \frac{3}{-3}$$

We get, $a = 1, b = 2$

Substitute $a = 1$ and $b = 2$ in eqn. (i)

$$1 - 2 \times 2 + d = 0 \Rightarrow d = 3$$

Hence, $a + b + d = 3 + 1 + 2 = 6$.

76. (a) π_1 plane is determined by the vectors $\hat{i} + \hat{j}, \hat{i} + \hat{k}$
 \Rightarrow Both these vectors lie in the plane, or in other words normal to the plane is perpendicular to both these vectors.

Let \vec{n}_1 be normal to π_1

$$\Rightarrow \vec{n}_1 = (\hat{i} + \hat{j}) \times (\hat{i} + \hat{k}) = \hat{i} - \hat{j} - \hat{k}$$

Similarly, for π_2 , we can have

$$\vec{n}_2 = (\hat{j} - \hat{k}) \times (\hat{k} + \hat{i}) = \hat{i} + \hat{j} + \hat{k}$$

Now, \vec{a} is \parallel line of intersection of π_1 & π_2

$\Rightarrow \vec{a}$ (or the line of intersection of π_1 & π_2) is \perp^r to both normal n_1 & n_2

Therefore $\vec{a} = \vec{n}_1 \times \vec{n}_2 = (\hat{i} - \hat{j} - \hat{k}) \times (\hat{i} + \hat{j} + \hat{k})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 0\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\text{Now, angle between } \vec{a} \text{ \& } \vec{b} = \cos^{-1} \left| \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \right|$$

$$= \cos^{-1} \left| \frac{(-2\hat{j} + 2\hat{k})(\hat{i} + \hat{j} - \hat{k})}{2\sqrt{2} \cdot \sqrt{3}} \right|$$

$$= \cos^{-1} \left| \frac{-2-2}{2\sqrt{2}\sqrt{3}} \right| = \cos^{-1} \left| \frac{-\sqrt{2}}{\sqrt{3}} \right| = \cos^{-1} \left(\frac{\sqrt{2}}{\sqrt{3}} \right)$$

Numerical value of which corresponds to option (a).

77. (d) Let $\vec{AB} = B - A = (4, -4, 3)$

$$\vec{AC} = C - A = (-5, 11, 5)$$

Normal vector $\vec{n} = \vec{AB} \times \vec{AC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -4 & 3 \\ -5 & 11 & 5 \end{vmatrix} = -53\hat{i} - 35\hat{j} + 24\hat{k}$$

Normal vector is $(-53, -35, 24)$

The given plane equation is $53x + by + cz + d = 0$

Normal vector $(53, b, c)$

We will use the normal vector $(53, 35, -24)$ by multiplying by -1

$$\therefore b = 35 \text{ and } c = -24$$

Substitute one of the points i.e. $A(2, 1, -1)$ into the plane equation

$$53x + 35y - 24z + d = 0$$

$$53(2) + 35(1) - 24(-1) + d = 0 \Rightarrow d = -165$$

$$\therefore \frac{d}{b+c} = \frac{-165}{35-24} = -15$$

78. (b) $P(1, 2, 1)$ image in plane $x + 2y + 2z = 16$

$$\Rightarrow \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-1}{2} = \frac{-2(1+2 \times 2 + 2 \times 1 - 16)}{1^2 + 2^2 + 2^2}$$

$$\Rightarrow Q(3, 6, 5)$$

$$\text{Given } \vec{r} = -\hat{k} + 6(\hat{i} + \hat{j} + 2\hat{k})$$

$$\text{So } AQ = 3\hat{i} + 6\hat{j} + 6\hat{k}$$

$$= 3(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\text{Now } \vec{n} = (\hat{i} + 2\hat{j} + 2\hat{k}) \times (\hat{i} + \hat{j} + 2\hat{k})$$

$$\Rightarrow 2\hat{i} + 0\hat{j} + \hat{k}$$

Now equation of plane = $2x - z = 1$

\Rightarrow point lying on plane from the option is $(1, 2, 1)$.

79. (c) \therefore Equation of rotated plane will pass through family of the planes $P_1 + \lambda P_2 = 0$

$$(2x + y - 5z) + \lambda(3x - y + 4z - 7) = 0$$

$$\text{or, } (2 + 3\lambda)x + (1 - \lambda)y + (-5 + 4\lambda)z - 7\lambda = 0$$

$$2x + y - 5z = 0$$

Since, it is rotated by $\frac{\pi}{2}$

$$2(2 + 3\lambda) + (1 - \lambda) - 5(-5 + 4\lambda) = 0$$

$$\Rightarrow 4 + 6\lambda + 1 - \lambda + 25 - 20\lambda = 0$$

$$\lambda = 2$$

Hence, required plane : $8x - y + 3z - 14 = 0$

Putting x, y, z from the given options we get option (c) is correct.

80. (b) Given planes are $ax + by = 0$... (i)

$$\text{and } z = 0 \text{ ... (ii)}$$

\therefore Equation of any plane passing through the line of intersection of planes (i) and (ii) may be taken as;

$$ax + by + kz = 0 \text{ ... (iii)}$$

The direction cosines of normal to the plane (iii) are :

$$\left\langle \frac{a}{\sqrt{a^2 + b^2 + k^2}}, \frac{b}{\sqrt{a^2 + b^2 + k^2}}, \frac{k}{\sqrt{a^2 + b^2 + k^2}} \right\rangle$$

The direction cosines of normal to the plane (i) are :

$$\left\langle \frac{a}{\sqrt{a^2 + b^2}}, \frac{b}{\sqrt{a^2 + b^2}}, 0 \right\rangle$$

Since the angle between the planes (i) and (iii) is α ,

$$\Rightarrow \cos \alpha = \frac{a \cdot a + b \cdot b + k \cdot 0}{\sqrt{a^2 + b^2 + k^2} \sqrt{a^2 + b^2}} = \frac{a^2 + b^2}{\sqrt{a^2 + b^2 + k^2} \sqrt{a^2 + b^2}}$$

$$\Rightarrow k^2 \cos^2 \alpha = a^2(1 - \cos^2 \alpha) + b^2(1 - \cos^2 \alpha)$$

$$\Rightarrow k^2 = \frac{(a^2 + b^2) \sin^2 \alpha}{\cos^2 \alpha} \Rightarrow k = \pm \sqrt{a^2 + b^2} \tan \alpha$$

Putting in (iii) we get, equation of plane as :

$$ax + by \pm z\sqrt{a^2 + b^2} \tan \alpha = 0.$$

81. (c) The equation of line passing through (1, 2, 3) and parallel to \mathbf{b} is given by

$$\mathbf{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\mathbf{b}_1\hat{i} + \mathbf{b}_2\hat{j} + \mathbf{b}_3\hat{k}) \quad \dots(i)$$

The equations of the given planes are

$$r \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5 \quad \dots(ii)$$

$$\text{and } r \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6 \quad \dots(iii)$$

The line in eq. (i) and plane in eq. (ii) are parallel. Therefore, the normal to the plane of eq. (ii) and the given line are perpendicular.

$$\therefore (\hat{i} - \hat{j} + 2\hat{k}) \cdot \lambda(\mathbf{b}_1\hat{i} + \mathbf{b}_2\hat{j} + \mathbf{b}_3\hat{k}) = 0$$

$$\Rightarrow \lambda(\mathbf{b}_1 - \mathbf{b}_2 + 2\mathbf{b}_3) = 0$$

$$\Rightarrow (\mathbf{b}_1 - \mathbf{b}_2 + 2\mathbf{b}_3) = 0 \quad \dots(iv)$$

$$\text{Similarly, } (3\hat{i} + \hat{j} + \hat{k}) \cdot \lambda(\mathbf{b}_1\hat{i} + \mathbf{b}_2\hat{j} + \mathbf{b}_3\hat{k}) = 0$$

$$\Rightarrow \lambda(3\mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3) = 0 \quad \dots(v)$$

From eqs. (iv) and (v), we obtain

$$\frac{\mathbf{b}_1}{(-1) \times 1 - 1 \times 2} = \frac{\mathbf{b}_2}{2 \times 3 - 1 \times 1} = \frac{\mathbf{b}_3}{1 \times 1 - 3(-1)}$$

$$\Rightarrow \frac{\mathbf{b}_1}{-3} = \frac{\mathbf{b}_2}{5} = \frac{\mathbf{b}_3}{4}$$

Therefore, the direction ratios of \mathbf{b} are -3, 5 and 4.

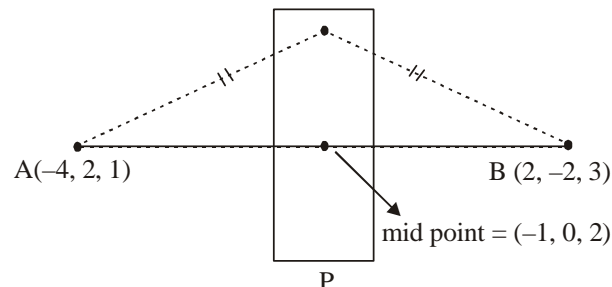
$$\therefore \mathbf{b} = \mathbf{b}_1\hat{i} + \mathbf{b}_2\hat{j} + \mathbf{b}_3\hat{k} = -3\hat{i} + 5\hat{j} + 4\hat{k}$$

Substituting the value of \mathbf{b} in eq. (i), we obtain

$$\mathbf{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$$

This is the equation of the required line.

82. (c)



Given co-ordinates A (-4, 2, 1) and B (2, -2, 3), O is the origin.

$$\text{Normal vector} = \vec{AB} = (\vec{OB} - \vec{OA}) = (6\hat{i} - 4\hat{j} + 2\hat{k})$$

Mid-point of AB lies on the plane P with co-ordinate (-1, 0, 2).

Now, apply equation of plane

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

$$P = 6(x + 1) - 4(y) + 2(z - 1) = 0$$

$$\text{Required plane } P \equiv 3x - 2y + z + 1 = 0$$

$$L \equiv 2x + y + 3z - 1 = 0$$

$$\text{Angle between } P \text{ and } L = \left| \frac{\hat{n}_1 \cdot \hat{n}_2}{|\hat{n}_1| |\hat{n}_2|} \right| = \cos \theta$$

$$\text{Here, } \hat{n}_1 = 3\hat{i} - 2\hat{j} + \hat{k}; \hat{n}_2 = 2\hat{i} + \hat{j} + 3\hat{k}$$

$$\theta = \cos^{-1} \left(\frac{6 - 2 + 3}{\sqrt{14} \times \sqrt{14}} \right)$$

$$\theta = \cos^{-1} \left(\frac{7}{14} \right) = \cos^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{3}$$

83. (b) The distance of point (1, 1, p) from the plane

$$r \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0 \text{ or}$$

(in cartesian form) $3x + 4y - 12z + 13 = 0$, is

$$d_1 = \frac{|3 \times 1 + 4 \times 1 - 12 \times p + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}}$$

$$= \frac{|3 + 4 - 12p + 13|}{\sqrt{169}} = \frac{|20 - 12p|}{13}$$

...(i)

The distance of the point (-3, 0, 1) from the plane

$$3x + 4y - 12z + 13 = 0 \text{ is}$$

$$d_2 = \frac{|3 \times (-3) + 4 \times 0 - 12 \times 1 + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}} = \frac{8}{13}$$

According to the given condition,

$$d_1 = d_2$$

$$\Rightarrow \frac{|20 - 12p|}{13} = \frac{8}{13} \Rightarrow \frac{20 - 12p}{13} = \pm \frac{8}{13}$$

Taking +ve sign, we get

$$\frac{20 - 12p}{13} = \frac{8}{13} \Rightarrow p = 1$$

Taking -ve sign, we get

$$\frac{20 - 12p}{13} = \frac{8}{13} \Rightarrow p = \frac{28}{12} = \frac{7}{3}$$

84. (c) Any plane passing through the line of intersection of the given planes is

$$(x + 2y + 3z - 5) + \lambda(3x - 2y - z + 1) = 0$$

$$\Rightarrow (1 + 3\lambda)x + (2 - 2\lambda)y + (3 - \lambda)z + (-5 + \lambda) = 0 \quad \dots(i)$$

$$\text{Intercept on X-axis is } \frac{5 - \lambda}{1 + 3\lambda}$$

[Put $y = z = 0$]

Intercept on Z-axis is $\frac{5-\lambda}{3-\lambda}$.

$$\text{Given } \frac{5-\lambda}{1+3\lambda} = \frac{5-\lambda}{3-\lambda} \Rightarrow \lambda = \frac{1}{2}$$

$[\lambda = 5$ is not admissible as in that case each intercept is zero]. Put $\lambda = \frac{1}{2}$ in (i), we get the equation of the required plane as $5x + 2y + 5z - 9 = 0$.

85. (b) Direction cosines of \vec{n} are $\frac{1}{2}, \frac{1}{4}, \frac{1}{2}$.

Equation of the plane :

$$\frac{1}{2}(x-\sqrt{2}) + \frac{1}{4}(y+1) + \frac{1}{2}(z-1) = 0$$

$$\Rightarrow 2(x-\sqrt{2}) + (y+1) + 2(z-1) = 0$$

$$\Rightarrow 2x + y + 2z = 2\sqrt{2} - 1 + 2 \Rightarrow 2x + y + 2z = 2\sqrt{2} + 1$$

86. (b) Let direction ratios of the line be $\langle a, b, c \rangle$, then

$$2a - b + c = 0$$

$$a - b - 2c = 0$$

$$\text{i.e., } \frac{a}{3} = \frac{b}{5} = \frac{c}{-1}$$

\therefore direction ratios of the line are $(3, 5, -1)$

Any point on the given line is $(2 + \lambda, 2 - \lambda, 3 - 2\lambda)$. It lies on the plane π if

$$2(2 + \lambda) - (2 - \lambda) + (3 - 2\lambda) = 4$$

$$\text{i.e., } 4 + 2\lambda - 2 + \lambda + 3 - 2\lambda = 4$$

$$\text{i.e., } \lambda = -1$$

\therefore the point of intersection of the line and the plane is $(1, 3, 5)$

$$\therefore \text{ equation of the required line is } \frac{x-1}{3} = \frac{y-3}{5} = \frac{z-5}{-1}$$

87. (a) Equation of line is

$$\vec{r} = \vec{a}_1 + \hat{i}(\vec{a}_2 - \vec{a}_1) = \hat{i} + 2\hat{j} + \lambda(-\hat{i} - \hat{j} - 2\hat{k})$$

$$\Rightarrow \frac{x-1}{1} = \frac{y-2}{1} = \frac{z}{2} = k \Rightarrow P(k+1, k+2, 2k)$$

$$\vec{n} = (\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & 3 \\ -4 & 1 & 1 \end{vmatrix} = -10\hat{j} + 10\hat{k}$$

Equation of plane is

$$\vec{n} \cdot (-10\hat{j} + 10\hat{k}) = (2\hat{i} - \hat{j}) \cdot (-10\hat{j} + 10\hat{k})$$

$$\vec{n} \cdot (-\hat{j} + \hat{k}) = (2\hat{i} - \hat{j}) \cdot (-\hat{j} + \hat{k})$$

$$-y + z = 1 \Rightarrow -k - 2 + 2k = 1 \Rightarrow k = 3$$

$$\therefore \text{ Intersection point } \vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\text{Now, } \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4 + 5 + 6 = 15.$$

88. (b) Normal vector of required plane is

$$\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 5 \\ 5 & 2 & -3 \end{vmatrix} = -\hat{i} + 31\hat{j} + 19\hat{k}$$

Equation of plane

$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$$

$$-1(x-3) + 31(y-2) + 19(z-5) = 0$$

$$\Rightarrow x - 31y - 19z + 154 = 0$$

$$\therefore b = -31, c = -19 \text{ and } d = 154$$

$$\text{Now, } 2b + 3c + d = 2(-31) + 3(-19) + 154 = 35$$

89. (c) $\vec{AB} = -\hat{i} - 2\hat{j} + \hat{k}$; $\vec{AC} = -\hat{i} + 2\hat{j}$

Normal of plane passing through A, B, C is

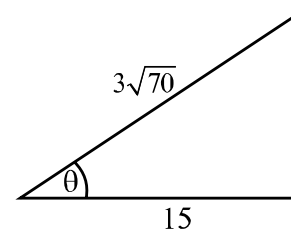
$$\vec{n}_1 = \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & 1 \\ -1 & 2 & 0 \end{vmatrix} = -2\hat{i} - \hat{j} - 4\hat{k}$$

$$\vec{AD} = -2\hat{i} - \hat{j} - \hat{k}$$

Normal of plane passing through A, C, D is

$$\vec{n}_2 = \vec{AC} \times \vec{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -2 & -1 & -1 \end{vmatrix} = -2\hat{i} - \hat{j} + 5\hat{k}$$

$$\cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \|\vec{n}_2\|} = \frac{4 + 1 - 20}{\sqrt{4+1+16} \sqrt{4+1+25}} = \frac{15}{3\sqrt{70}}$$



$$p^2 = 630 - 225 = 405$$

$$p = 9\sqrt{5}$$

$$\tan \theta = \frac{9\sqrt{5}}{15} = \frac{3}{\sqrt{5}}$$

$$90. \quad (d) \quad \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 3 & 1 & -1 \end{vmatrix} = 2\hat{i} + 5\hat{j} + 11\hat{k}$$

\therefore Dr. of plane π is (2, 5, 11)

So, $a = 2, b = 5$

\therefore Equation of π is $2x + 5y + 11z + d = 0$

\therefore Distance from origin is $\sqrt{6}$

$$\therefore \sqrt{6} = \frac{|0+0+0+d|}{\sqrt{4+25+121}} \Rightarrow |d| = 30$$

Intercept on the coordinate and in $\left(\frac{-d}{2}, \frac{-d}{5}, \frac{-d}{11}\right)$ but

given that all are positive so $d = -30 = -3ab$.

91. (a) Plane $2x + 3y + z = 1$, normal vector

$$\vec{n}_1 = 2\hat{i} + 3\hat{j} + \hat{k}$$

Plane $x + 3y + 2z = 2$, normal vector $\vec{n}_2 = \hat{i} + 3\hat{j} + 2\hat{k}$

The direction vector of the line $\vec{n} = \vec{n}_1 \times \vec{n}_2$

$$\Rightarrow \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix} = 3\hat{i} - 3\hat{j} + 3\hat{k}$$

Direction vector of positive x -axis $\vec{b} = \hat{i}$,

$$\therefore \cos \alpha = \frac{\vec{n} \cdot \vec{b}}{|\vec{n}| |\vec{b}|} = \frac{3}{3\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}}$$

92. (a) The equation of the line is

$$\vec{r} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} + \hat{k})$$

$$\vec{r} = (1 + 2\lambda)\hat{i} - 2\hat{j} + \lambda\hat{k} \quad \dots(i)$$

The equation of the plane is

$$\vec{r} = (\hat{i} + 2\hat{j}) + t(2\hat{j} - \hat{k}) + s(\hat{i} + 2\hat{k})$$

$$\vec{r} = (1 + s)\hat{i} + (2 + 2t)\hat{j} + (-t + 2s)\hat{k} \quad \dots(ii)$$

From (i) and (ii), we get

$$1 + 2\lambda = 1 + s \quad \dots(iii)$$

$$-2 = 2 + 2t \quad \dots(iv)$$

$$\lambda = -t + 2s \quad \dots(v)$$

From the eq. (ii), (iv) and (v) we get

$$t = -2, s = \frac{-4}{3}, \lambda = \frac{-2}{3}$$

Put $\lambda = \frac{-2}{3}$ into the line of equation, we get

$$\vec{r} = \frac{-1}{3}\hat{i} - 2\hat{j} - \frac{2}{3}\hat{k}$$

The point of intersection is $\frac{-1}{3}(\hat{i} + 6\hat{j} + 2\hat{k})$

93. (a) A(1, -2, 3) and B(6, 4, 5) lies on plane π

Dr's of AB is $5\hat{i} + 6\hat{j} + 2\hat{k}$, Dr's normal of $3x - y + z = 2$ are (3, -1, 1)

Normal unit vector of π ,

$$\hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & 2 \\ 3 & -1 & 1 \end{vmatrix} = 8\hat{i} + \hat{j} - 2\hat{k}$$

The eqⁿ. of plane passing through A(1, -2, 3)

and normal unit vector \hat{n} is

$$8(x - 1) + 1(y + 2) - 23(z - 3) = 0$$

$$8x + y - 23z + 63 = 0$$

$$\perp^r \text{ distance of plane } \pi \text{ from origin} = \frac{|63|}{\sqrt{594}}$$

94. (b) $\vec{OA} = \hat{i} + 2\hat{j} + \hat{k}; \vec{OB} = 2\hat{i} + \hat{j} + 3\hat{k}$

$$\vec{BC} = -3\hat{i} - \hat{k}$$

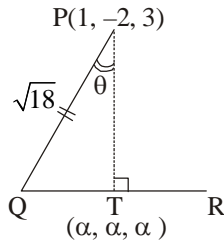
Normal vector of face OAB

$$\vec{n} = \vec{OA} \times \vec{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5\hat{i} - \hat{j} - 3\hat{k}$$

Angle between face OAB and BC is

$$\sin \theta = \frac{|\vec{n} \cdot \vec{BC}|}{|\vec{n}| |\vec{BC}|} = \frac{|-15 + 0 + 3|}{\sqrt{25 + 1 + 9} \sqrt{9 + 1}} = \frac{12}{\sqrt{350}} = \sqrt{\frac{144}{350}} = \sqrt{\frac{72}{175}} \Rightarrow \theta = \sin^{-1} \left(\frac{6\sqrt{2}}{5\sqrt{7}} \right)$$

95. (b)



Given the planes $-x + 2y - z = 0$ & $3x - 5y + 5z$ whose intersection point is P.

Now, find normal from point P to line QR.

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(1) + \hat{k}(-1) \quad \vec{n} = -\hat{i} - \hat{j} - \hat{k}$$

$$\text{Equation of L is } \frac{x}{1} = \frac{y}{1} = \frac{z}{1}.$$

$$\text{Take } \frac{x}{1} = \frac{y}{1} = \frac{z}{1} = \lambda, \Rightarrow \frac{x}{1} = \lambda, x = \lambda$$

Then, $y = \lambda, z = \lambda$

So, the coordinati of T is $(\lambda, \lambda, \lambda)$

DR: of PT $\rightarrow \lambda - 1, \lambda + 2, \lambda - 3,$

DR: of QR $\rightarrow 1, 1, 1$

$$\Rightarrow (\lambda - 1) \times 1 + (\lambda + 2) \times 1 + (\lambda - 3) \times 1 = 0$$

$$3\lambda = 2, \lambda = \frac{2}{3}$$

$$PT^2 = \frac{1}{9} + \frac{64}{9} + \frac{49}{9}$$

$$PT^2 = \frac{114}{9}$$

$$PT = \frac{\sqrt{114}}{3}$$

In ΔPTQ

$$\cos \theta = \frac{PT}{PQ}$$

$$\cos \theta = \frac{\sqrt{114}}{3} \times \frac{1}{3\sqrt{2}} = \frac{\sqrt{57}}{9} = \frac{\sqrt{19 \times 3}}{3 \times 3} = \frac{\sqrt{19}}{3\sqrt{3}}$$

$$\cos 2\theta = \frac{2 \times 19}{27} - 1 = \frac{11}{27}$$

$$\sin 2\theta = \sqrt{1 - \left(\frac{11}{27}\right)^2} = \frac{\sqrt{38 \times 16}}{27} = \frac{4}{27} \sqrt{38}$$

Area of ΔPQR

$$= \frac{1}{2} ab \sin 2\theta$$

$$\text{Area} = \frac{1}{2} \times \sqrt{18} \times \sqrt{18} \times \frac{4}{27} \sqrt{38}$$

$$\frac{18}{2} \times \frac{4}{27} \sqrt{38} = \frac{36}{27} \sqrt{38} = \frac{4}{3} \sqrt{38} \text{ square units.}$$

$$96. \text{ (c) } \frac{x_1 - 2}{1} = \frac{y_1 + 2}{-2} = \frac{z_1 - 1}{1} = \frac{-(2 + 4 + 1 - 1)}{6}$$

$$x_1 = 1, y_1 = 0, z_1 = 0 \Rightarrow l = 1 + 0 + 0 = 1$$

$$d = \left| \frac{2 + 4 + 1 - 1}{\sqrt{6}} \right| = \sqrt{6} \Rightarrow l + 3d^2 = 19.$$

$$97. \text{ (a) } P(2, \beta, \alpha) \text{ lies on } x + 2y - z - 2 = 0$$

$$\Rightarrow 2 + 2\beta - \alpha - 2 = 0 \Rightarrow \alpha = 2\beta \quad \dots(i)$$

$$Q(\alpha, -1, \beta) \text{ lies on } 2x - y + 3z + 6 = 0$$

$$\Rightarrow 2\alpha + 3\beta + 7 = 0 \quad \dots(ii)$$

$$\text{Solving (i) and (ii), } \alpha = -2, \beta = -1 \quad \overline{PQ} = -4\hat{i} + \hat{k}$$

$$\text{So, direction cosines are } \left(\frac{-4}{\sqrt{17}}, 0, \frac{1}{\sqrt{17}} \right)$$

$$98. \text{ (c) Equation of } L_1 \text{ will be } \frac{x}{3} = \frac{y}{1} = \frac{z}{-5}$$

$$\text{Equation of } L_2 \text{ will be } \frac{x}{2} = \frac{y}{3} = \frac{z}{-1}$$

$$\vec{r}_1 = 3\hat{i} + \hat{j} - 5\hat{k}, \quad \vec{r}_2 = 2\hat{i} + 3\hat{j} - \hat{k} \quad \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -5 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= (-1 + 15)\hat{i} - (-3 + 10)\hat{j} + 7\hat{k} = 14\hat{i} - 7\hat{j} + 7\hat{k}$$

\therefore Equation of plane is

$$14(x - 0) - 7(y - 0) + 7(z - 0) = 0 \Rightarrow 2x - y + z = 0$$

$$99. \text{ (d) Since equation of line L is } \frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{2}$$

$$\text{and equation of the plane is } \begin{vmatrix} x-2 & y-1 & z+2 \\ -2-2 & -3-1 & 6+2 \\ 0-2 & 2-1 & -1+2 \end{vmatrix} = 0$$

$$\Rightarrow x + y + z = 1$$

Now, angle between line and plane is given by

$$\sin \theta = \left| \frac{1 \times 2 + 1 \times 1 + 2 \times 1}{\sqrt{1+1+1} \cdot \sqrt{2^2+1+2^2}} \right| = \frac{5}{\sqrt{3} \times 3}$$

$$\Rightarrow \sin^2 \theta = \frac{25}{27} \Rightarrow \cos^2 \theta = \frac{2}{27}$$

$$\Rightarrow 27 \cos^2 \theta = 2.$$

$$100. \text{ (d) Given two planes are } 2x + y + z + 1 = 0 \text{ and}$$

$2x + y + z + \alpha = 0$; Since, these are parallel planes

So, distance between them

$$= \frac{|1-\alpha|}{\sqrt{4+1+1}} \Rightarrow \frac{|1-\alpha|}{\sqrt{6}} = 3$$

$$\Rightarrow |1-\alpha| = 3\sqrt{6} \Rightarrow 1-\alpha = \pm 3\sqrt{6} \Rightarrow \alpha = 1-3\sqrt{6}, 1+3\sqrt{6}$$

Now, product of all value of α

$$= (1-3\sqrt{6})(1+3\sqrt{6}) = 1-9 \times 6 = -53$$

Exercise 3 :

PREVIOUS YEARS MCQs

1. (a) Given equations are

$$l+m-n=0 \Rightarrow n=l+m$$

Now, put the value of n in another equation then,

$$3l^2+m^2+cl(l+m)=0$$

$$3l^2+m^2+cl^2+clm=0$$

$$(3+c)l^2+clm+m^2=0$$

$$(3+c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0$$

...(i)

Given that lines are parallel.

Then, roots of (i) must be equal $\Rightarrow D=0$

$$c^2-4(3+c)=0 \Rightarrow c^3-4c-12=0$$

$$(c-6)(c+2)=0 \Rightarrow c=6 \text{ or } c=-2.$$

Therefore, +ve value of $c=6$.

2. (b) Given lines l_1, l_2 and l_3 are

$$l_1: \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$$

$$l_2: \frac{x-1}{1} = \frac{y+\frac{3}{2}}{\alpha} = \frac{z+5}{2}$$

$$l_3: \frac{x-1}{-3} = \frac{y-\frac{1}{2}}{-2} = \frac{z-0}{4}$$

$$l_1 \perp l_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}}+4} = 0 \Rightarrow \alpha = 3$$

angle between l_2 & l_3

$$\cos \theta = \frac{|1 \times (-3) + (-2) \left(\frac{\alpha}{2}\right) + 2 \times 4|}{\sqrt{1+4+\frac{\alpha^2}{4}} \sqrt{9+16+4}}$$

$$\cos \theta = \frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}} \sqrt{29}}$$

$$\text{put } \alpha = 3 \Rightarrow \cos \theta = \frac{2}{\sqrt{\frac{29}{4}} \sqrt{29}} = \frac{4}{29}$$

$$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$$

3. (a) Given equation of plane is

$$x-3y+5z=d$$

Since, it passes through $(1, 2, 4)$, then

$$1-3(2)+5(4)=d$$

$$\Rightarrow 1-6+20=d$$

$$d=15$$

Therefore, the equation of the plane will be

$$x-3y+5z=15$$

$$\Rightarrow \frac{x}{15} - \frac{3y}{15} + \frac{5z}{15} = 1 \Rightarrow \frac{x}{15} + \frac{y}{-5} + \frac{z}{3} = 1$$

Hence, the intercept cut by the plane on axes X, Y, Z are 15, -5 and 3, respectively.

4. (c) For $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}, k\right)$ to represent direction cosines, we should have

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + k^2 = 1$$

$$\text{or, } \frac{1}{2} + \frac{1}{4} + k^2 = 1 \Rightarrow k = \pm \frac{1}{2}$$

5. (d) The equations of the given lines are not in symmetrical form. We first put them in symmetrical form.

Equations of the first line are $x = ay + b, z = cy + d$.

These equation can be written as

$$\frac{x-b}{a} = y = \frac{z-b}{c} \text{ or } \frac{x-b}{a} = \frac{y-0}{1} = \frac{z-b}{c} \quad \dots(i)$$

Similarly, equation of second line

$x = a'y + b', z = c'y + d'$ can be written as

$$\frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-b'}{c'} \quad \dots(ii)$$

Clearly, d.r.'s of the first line are $a, 1, c$ and those of the second line are $a', 1, c'$. Now lines (i) and (ii) are perpendicular if $aa' + cc' + 1 = 0$

6. (a) Let the coordinate of foot of perpendicular be (x, y, z)

$$\text{Take, } \frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = \lambda$$

$$(x, y, z) = (4\lambda - 2, 2\lambda + 1, 3\lambda - 1)$$

Equation of line from point A to point

$$P = (4\lambda - 3)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 5)\hat{k}$$

Line passes through the vector \vec{b} . Then, $b = 4\hat{i} + 2\hat{j} + 3\hat{k}$

$$\text{So, } \overrightarrow{AP} \cdot \vec{b} = 0$$

$$4(4\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 5) = 0$$

$$29\lambda = 12 + 2 + 15 = 29 \Rightarrow \lambda = 1$$

Now, put the value of λ in the value of point P.

Then, $P = (2, 3, 2)$

Distance of point P from the plane $3x + 4y + 12z + 23 = 0$.

Required distance

$$d = \frac{|6+12+24+23|}{\sqrt{9+16+144}}, d = \frac{|65|}{13} = 5$$

7. (d) $2\hat{i} - p\hat{j} + 5\hat{k}$ and $3\hat{i} + p\hat{j} + p\hat{k}$ are perpendicular

$$\Rightarrow 2 \times 3 + p(-p) + 5(p) = 0$$

$$\Rightarrow p = -1 \text{ or } p = 6$$

Hence for $p = 6$, the lines are perpendicular.

8. (a) The given line is

$$\frac{x-2}{2} = \frac{2y-5}{-3} = z+1, \Rightarrow \frac{x-2}{2} = \frac{y-\frac{5}{2}}{-\frac{3}{2}} = \frac{z+1}{0}$$

This shows that the given line passes through the point

$\left(2, \frac{5}{2}, -1\right)$ and has direction ratios $\left(2, \frac{-3}{2}, 0\right)$. Thus, given line passes through the point having position vector

$\vec{a} = 2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}$ and is parallel to the vector

$\vec{b} = \left(2\hat{i} - \frac{3}{2}\hat{j} - 0\hat{k}\right)$. So, its vector equation is

$$\vec{r} = \left(2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda \left(2\hat{i} - \frac{3}{2}\hat{j} - 0\hat{k}\right).$$

Hence, $p = 0$

9. (d) Given points are

A(2, 3, 9); B(5, 2, 1); C(1, λ , 8); D(λ , 2, 3)

$$[\overline{AB \ AC \ AD}] = 0$$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda - 3 & -1 \\ \lambda - 2 & -1 & -6 \end{vmatrix} = 0$$

$$\Rightarrow [-6(\lambda - 3) - 1] - 8(1 - (\lambda - 3)(\lambda - 2)) + (6 + (\lambda - 2)) = 0$$

$$3(-6\lambda + 17) - 8(-\lambda^2 + 5\lambda - 5) + (\lambda + 4) = 8$$

$$8\lambda^2 - 57\lambda + 95 = 0$$

Apply the rule of product whose roots are $\alpha\beta$.

$$\alpha\beta = \frac{95}{8}$$

10. (a) Here, $\vec{a}_1 = -7\hat{i} + 6\hat{j}$, $\vec{b}_1 = -6\hat{i} + 7\hat{j} + \hat{k}$

$$\vec{a}_2 = 7\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{b}_2 = -2\hat{i} + \hat{j} + \hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 14\hat{i} - 4\hat{j} + 6\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 7 & 1 \\ -2 & 1 & 1 \end{vmatrix} = 6\hat{i} + 4\hat{j} + 8\hat{k}$$

$$\begin{aligned} \text{Distance} &= \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} = \frac{116}{\sqrt{116}} \\ &= \sqrt{116} = 2\sqrt{29} \end{aligned}$$

11. (b) Let (x, y, z) be the point which divides line segment joining A(1, -2, 3) and B(3, 4, -5) internally in the ratio 2 : 3. Therefore

$$x = \frac{2(3) + 3(1)}{2+3} = \frac{9}{5}, y = \frac{2(4) + 3(-2)}{2+3} = \frac{2}{5},$$

$$z = \frac{2(-5) + 3(3)}{2+3} = \frac{-1}{5}.$$

Thus, the required point is $\left(\frac{9}{5}, \frac{2}{5}, \frac{-1}{5}\right)$

12. (d) Since d. c. of line are $\left\{\frac{1}{c}, \frac{1}{c}, \frac{1}{c}\right\}$

$$\therefore \frac{1}{c^2} + \frac{1}{c^2} + \frac{1}{c^2} = 1 \Rightarrow c^2 = 3 \Rightarrow c = \pm\sqrt{3}$$

13. (a) It is given that, the foot of perpendicular from point Q(a, b, c) to the yz plane is A(0, b, c) and the foot of perpendicular from point Q to the zx plane is B(a, 0, c).

Let the equation of plane passing through the point (0, 0, 0) be $Ax + By + Cz = 0$ (i)

As it is passing through the point A(0, b, c) and B(a, 0, c).

So, $0 + Bb + Cc = 0$ and $Aa + 0 + Cc = 0$

$$\Rightarrow Cc = Bb \text{ and } Cc = -Aa$$

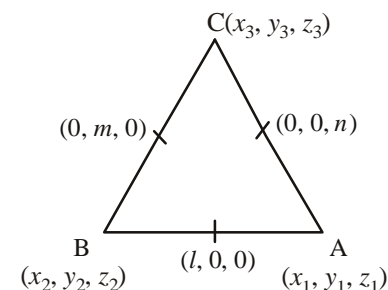
Therefore, $-Aa = -Bb = Cc = k$

$$\Rightarrow A = -\frac{k}{a}, B = -\frac{k}{b} \text{ and } C = \frac{k}{c}$$

$$-\frac{k}{a}x - \frac{k}{b}y + \frac{k}{c}z = 0 \text{ [From Eq. (i)]}$$

$$\Rightarrow -\frac{x}{a} - \frac{y}{b} + \frac{z}{c} = 0 \text{ or } \frac{x}{a} + \frac{y}{b} - \frac{z}{c} = 0$$

14. (c) From the figure



$$x_1 + x_2 = 2l, y_1 + y_2 = 0, z_1 + z_2 = 0$$

$$x_2 + x_3 = 0, y_2 + y_3 = 2m, z_1 + z_3 = 0$$

$$\text{and } x_1 + x_3 = 0, y_1 + y_3 = 0, z_1 + z_3 = 2n$$

On solving, we get $x_1 = l, x_2 = l, x_3 = -l$

$$y_1 = -m, y_2 = m, y_3 = m \text{ and } z_1 = n, z_2 = -n, z_3 = n$$

\therefore Coordinates are A(l, -m, n), B(l, m, -n) and C(-l, m, n)

$$\begin{aligned} &\therefore \frac{AB^2 + BC^2 + CA^2}{l^2 + m^2 + n^2} \\ &= \frac{(4m^2 + 4n^2) + (4l^2 + 4n^2) + (4l^2 + 4m^2)}{l^2 + m^2 + n^2} = 8 \end{aligned}$$

15. (a) Let the line be $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$... (i)

If line (i) intersects with the line $\frac{x-1}{2}$

$$= \frac{y+3}{4} = \frac{z-5}{3}, \text{ then}$$

$$\begin{vmatrix} a & b & c \\ 2 & 4 & 3 \\ 4 & -3 & 14 \end{vmatrix} = 0 \Rightarrow 9a - 7b - 10c = 0 \quad \dots(ii)$$

From (i) and (ii), we have $\frac{a}{1} = \frac{b}{-3} = \frac{c}{5}$

$$\therefore \text{The line is } \frac{x}{1} = \frac{y}{-3} = \frac{z}{5}$$

16. (d) Equation of plane passing through $(1, -3, 1)$ and whose normal $(1, -3, 1)$ is

$$1(x-1) - 3(y+3) + 1(z-1) = 0$$

$$\Rightarrow x - 3y + z - 11 = 0$$

$$\Rightarrow \frac{x}{11} - \frac{y}{11/3} + \frac{z}{11} = 0$$

The above plane intercept the x -axis at 11.

17. (b) Let $D(x, y, z)$ be the required point. Then, the mid-point of diagonal BD is $\left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$

Also, the mid-point of diagonal AC is

$$\left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) \text{ i.e., } (1, 0, 2).$$

But, the mid-points of the diagonals of a parallelogram always coincide.

$$\therefore \frac{x+1}{2} = 1, \frac{y+2}{2} = 0 \text{ and } \frac{z-4}{2} = 2$$

So, $x = 1, y = -2,$ and $z = 8.$

Hence, the required point is $D(1, -2, 8).$

18. (c) Any plane containing $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ is

$$a(x+1) + b(y-3) + c(z+2) = 0 \quad \dots(i)$$

$$\text{where } -3a + 2b + c = 0 \quad \dots(ii)$$

If the plane passes through $(0, 7, -7).$

$$\therefore a + 4b - 5c = 0 \quad \dots(iii)$$

From Eqs. (ii) and (iii),

$$\frac{a}{-10-4} = \frac{b}{1-15} = \frac{c}{-12-2}$$

$$\Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{1}$$

Therefore, the plane (i) becomes

$$(x+1) + (y-3) + (z+2) = 0$$

$$\Rightarrow x + y + z = 0$$

19. (a) The equations of line BC are (using two point form)

$$\frac{x-1}{5-1} = \frac{y-4}{4-4} = \frac{z-6}{4-6} \text{ i.e., } \frac{x-1}{2} = \frac{y-4}{0} = \frac{z-6}{-1}$$

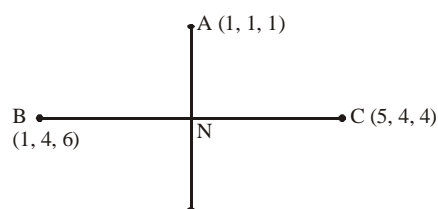
Any point on this line is $(2t+1, 4, -t+6).$ If this is the foot of perpendicular from A on the line $BC,$ then d.n. of this perpendicular are

$$\langle 2t+1-1, 4-4, -t+6-1 \rangle \text{ i.e. } \langle 2t, 3, -t+5 \rangle$$

Using condition of perpendicularity, we have

$$(2t)(2) + 3 \times 0 + (-t+5)(-1) = 0 \Rightarrow 5t - 5 = 0 \Rightarrow t = 1.$$

\therefore Required foot of perpendicular is $(2+1, 4, -1+6) = (3, 4, 5).$



20. (a) Equation of the line through $(1, -2, 3)$ parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{-6}$ is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r \text{ (say)} \quad \dots(i)$$

Then, any point on Eq. (i) is

$$(2r+1, 3r-2, -6r+3).$$

If this point lies on the plane $x - y + z = 5,$ then

$$(2r+1) - (3r-2) + (-6r+3) = 5$$

$$\Rightarrow -7r + 6 = 5 \Rightarrow r = \frac{1}{7}$$

Since, the point is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right).$

Distance between $(1, -2, 3)$ and $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$

$$= \sqrt{\left(\frac{4}{49} + \frac{9}{49} + \frac{36}{49}\right)} = \sqrt{\left(\frac{49}{49}\right)} = 1$$

21. (a) Any plane through the given line $2x - y + 3z + 1 + \lambda(x + y + z + 3) = 0$ (From $S + \lambda S' = 0$)

If this plane is parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3},$ then the

normal to the plane is also perpendicular to the above line.

$$\therefore (2+\lambda)1 + (\lambda-1)2 + (3+\lambda)3 = 0$$

$$(\because l_1 l_2 + m_1 m_2 + n_1 n_2 = 0)$$

$$\Rightarrow \lambda = -\frac{3}{2}$$

and the required plane is $x - 5y + 3z - 7 = 0.$

22. (d) Here; $\frac{7+p+q+1}{3} = 3 \Rightarrow p+q=1$... (i)

$\frac{-8+q+5p}{3} = -5 \Rightarrow 5p+q=-7$... (ii)

and $\frac{1+5+0}{3} = r \Rightarrow r=2$

Subtract (ii) from (i), we get:

$p+q-5p-q=1+7$
 $\Rightarrow -4p=8 \Rightarrow p=-2.$

from (i) we get,

$-2+q=1 \Rightarrow q=3.$

$\therefore p=-2, q=3 \text{ \& } r=2.$

23. (d) Angle between the lines :

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$. is :

$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$

\therefore angle between two given lines is :

$\cos \theta = \frac{(2)(1) + (-2)(2) + (1)(2)}{\sqrt{4+4+1} \sqrt{1+4+4}}$
 $= \frac{|2-4+2|}{9}$

$\Rightarrow \cos \theta = 0 \Rightarrow \theta = 90^\circ.$

24. (c) Here the given three points P (6, -1, 2), Q(8, -7, 2λ) and R (5, 2, 4) are collinear.

we know that if three points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) are collinear, then

$\frac{x_1-x_2}{x_2-x_3} = \frac{y_1-y_2}{y_2-y_3} = \frac{z_1-z_2}{z_2-z_3}$.

$\therefore \frac{6-8}{8-5} = \frac{-1+7}{-7-2} = \frac{2-2\lambda}{2\lambda-4}$

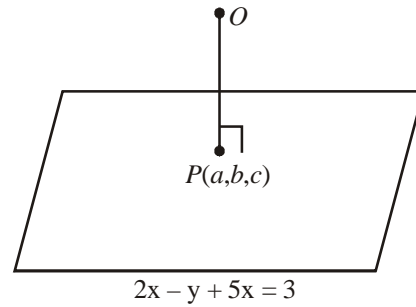
$\Rightarrow \frac{-2}{3} = \frac{2-2\lambda}{2\lambda-4} \Rightarrow -4\lambda+8=6-6\lambda$

$\Rightarrow 2\lambda = -2 \Rightarrow \lambda = -1.$

25. (d) let the co-ordinates of foot of perpendicular from the origin (O) to the plane $2x - y + 5x = 3$ is $P(a, b, c)$.

\therefore direction ratios of OP are $\langle a, b, c \rangle$

which is also the direction ratios of normal to the given plane.



$\therefore \frac{a}{2} = \frac{b}{-1} = \frac{c}{5} = k.$

$\Rightarrow a = 2k, b = -k, c = 5k.$

$\therefore P(a, b, c)$ passes the given plane

$\therefore 2(2k) - (-k) + 5(5k) = 3$

$\Rightarrow 4k + k + 25k = 3$

$\Rightarrow k = \frac{3}{30} = \frac{1}{10}.$

$\therefore a = \frac{2}{10} = \frac{1}{5}; b = -\frac{1}{10} \text{ and } c = \frac{5}{10} = \frac{1}{2}.$

26. (a) We know that equation of plane passing through a point with position vector \vec{a} and normal to the vector \vec{r} is :

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

\therefore the plane passes through $(-1, 2, 1)$

$\therefore \vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$

Also plane is perpendicular to the line containing $(-3, 1, 2)$ and $(2, 3, 4)$

$\therefore \vec{n} = 5\hat{i} + 2\hat{j} + 2\hat{k}$

\therefore required equation is :

$[\vec{r} - (-\hat{i} + 2\hat{j} + \hat{k})] \cdot (5\hat{i} + 2\hat{j} + 2\hat{k}) = 0$

$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} + 2\hat{k}) - [(-\hat{i} + 2\hat{j} + \hat{k}) \cdot (5\hat{i} + 2\hat{j} + 2\hat{k})] = 0$

$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} + 2\hat{k}) - [-5 + 4 + 2] = 0$

$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} + 2\hat{k}) = 1$

27. (c) Let a, b, c be d. rs of desired line which is also perpendicular to the given lines.

$\therefore 2a - 2b + c = 0$

$a - 2b + 2c = 0$

$\Rightarrow \begin{vmatrix} a & -b & c \\ -2 & 1 & 1 \\ -2 & 2 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & -2 & 2 \end{vmatrix}$

$$\Rightarrow \frac{a}{-4+2} = \frac{-b}{4-1} = \frac{c}{-4+2}$$

$$\Rightarrow \frac{a}{-2} = \frac{b}{-3} = \frac{c}{-2}$$

\therefore direction ratios are $\langle -2, -3, -2 \rangle$

Hence equation of desired line is

$$\frac{x-3}{-2} = \frac{y+1}{-3} = \frac{z-2}{-2} \text{ or } \frac{x-3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$$

28. (b) Consider $(x_1, y_1, z_1) \equiv (-3, 2, -5)$

As, the line is equally inclined to coordinate axes.

$$l = -1, m = 1 \text{ and } n = -1$$

Since, the equation of line passing through (x_1, y_1, z_1) and direction cosines l, m, n is

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

$$\text{Hence, } \frac{x+3}{-1} = \frac{y-2}{1} = \frac{z+5}{-1}$$

29. (a) The coordinates of given point is $(2, 3, \lambda)$. So, equation of the plane is $\vec{r} \cdot (3\hat{i} + 2\hat{j} + 6\hat{k}) = 13$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} + 2\hat{j} + 6\hat{k}) = 13$$

$$\Rightarrow 3x + 2y + 6z - 13 = 0$$

Therefore, distance of the plane from the given point $(2, 3, \lambda)$ will be

$$\left| \frac{3 \times 2 + 2 \times 3 + 6 \times \lambda - 13}{\sqrt{3^2 + 2^2 + 6^2}} \right| = 5 \text{ [Given]}$$

$$\Rightarrow \pm 5 = \frac{6\lambda - 1}{\sqrt{49}}$$

$$\Rightarrow \pm 35 = 6\lambda - 1$$

$$\Rightarrow 35 = 6\lambda - 1 \text{ or } -35 = 6\lambda - 1$$

$$\text{Hence, } \lambda = 6, -\frac{17}{3}$$

30. (b) Since, $\frac{x-1}{2} = \frac{y+1}{2} = \frac{z-1}{4} = \lambda \dots(i)$

$$\text{and } \frac{x-3}{1} = \frac{y-6}{2} = \frac{z}{1} \dots(ii)$$

Now, any point on the line (i) is $P(2\lambda+1, 2\lambda-1, 4\lambda+1)$

$$\therefore \frac{2\lambda+1-3}{1} = \frac{2\lambda-1-6}{2} = \frac{4\lambda+1}{1} \text{ [from (ii)]}$$

$$\text{So, } 4\lambda - 4 = 2\lambda - 7 \Rightarrow 2\lambda = -3$$

Hence, point of intersection P is

$$= \left(2 \times \left(-\frac{3}{2} \right) + 1, 2 \times \left(-\frac{3}{2} \right) - 1, 4 \times \left(-\frac{3}{2} \right) + 1 \right)$$

$$\equiv (-2, -4, -5)$$

31. (a) Equation of given line is

$$\frac{x+2}{2} = \frac{2y-5}{3}, z+1=0$$

$$\text{or } \frac{x+2}{2} = \frac{y-\frac{5}{2}}{\frac{3}{2}}, z+1=0$$

So, DR of given line are $\langle 2, \frac{3}{2}, 0 \rangle$

$$\text{As, } \sqrt{2^2 + \left(\frac{3}{2}\right)^2} + 0 = \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2}$$

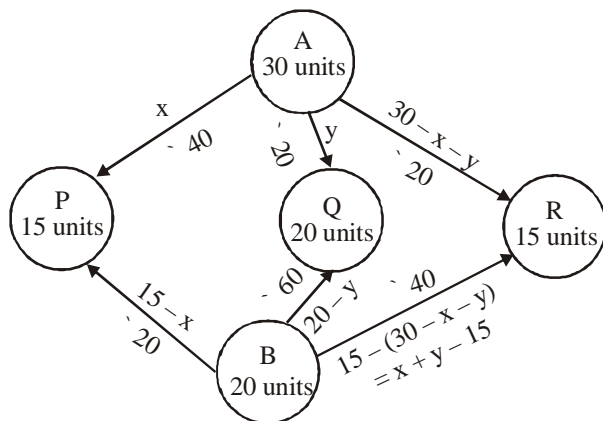
$$\therefore \text{DC of given line are } \langle \frac{2}{5/2}, \frac{3/2}{5/2}, 0 \rangle \text{ or } \langle \frac{4}{5}, \frac{3}{5}, 0 \rangle$$

Exercise 1 :

WARM-UP
Topic-wise MCQs

- (c) Optimum value of objective function.
- (d) Two or more variables.
- (b) The half plane containing the point (h, k) and the points on $ax + by = 4$.
- (a) The half plane that contains the positive x-axis.
- (c) In linear programming problem, concave region is not used. Convex region is used in linear programming.
- (c) Two constraints are $x \geq 0, y \geq 0$ and the third one will be of the type $ax + by \leq c$.
- (c) $x + y \leq 100; 400x + 900y \leq 30000$
or $4x + 9y \leq 300$ and $100x + 120y = c$.
- (e) The given information can be expressed as given in the diagram:

In order to simply, we assume that 1 unit = 1000 bricks
Suppose that depot A supplies x units to P and y units to Q, so that depot A supplies $(30 - x - y)$ bricks to builder R. Now, as P requires a total of 15000 bricks, it requires $(15 - x)$ units from depot B.
Similarly, Q requires $(20 - y)$ units from B and R requires $15 - (30 - x - y) = x + y - 15$ units from B.
Using the transportation cost given in table, total transportation cost.
 $Z = 40x + 20y + 20(30 - x - y) + 20(15 - x) + 60(20 - y) + 40(x + y - 15)$
 $= 40x - 20y + 1500$
Obviously the constraints are that all quantities of bricks supplied from A and B to P, Q, R are non-negative.



$\therefore x \geq 0, y \geq 0, 30 - x - y \geq 0, 15 - x \geq 0, 20 - y \geq 0, x + y - 15 \geq 0.$

Since, 1500 is a constant, hence instead of minimizing $Z = 40x - 20y + 1500$, we can minimize $Z = 40x - 20y$.

Hence, mathematical formulation of the given LPP is
Minimize $Z = 40x - 20y$,

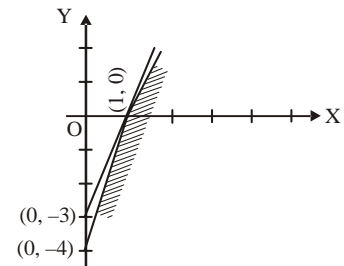
subject to the constraints:

$x + y \geq 15, x + y \leq 30,$

$x \leq 15, y \leq 20, x \geq 0, y \geq 0$

- (b) The shaded region represented by $4x - 2y \leq -3$.
- (c) Given by corner points of the feasible region.
- (a) The problem is to be re-evaluated.
- (c) If a L.P.P admits two optimal solution, it has an infinite number of optimal solutions.
- (b) The given shaded region is the solution of the following inequalities—
 $x + y \leq 7 \Rightarrow 2x - 3y + 6 \geq 0 \Rightarrow x \geq 0, y \geq 0$
- (c) Obviously, solution set of constraints included the point (3, 4).
- (a) The linear constraints are $x \geq 0, y \geq 0, 3x + 2y \geq 12, x + 3y \geq 11$.
- (c) Z is 325.5 maximum at $(\frac{21}{2}, \frac{69}{2})$.
- (a) Following figure will be obtained on drawing the graphs of given inequations.

From $3x - y \geq 3$,



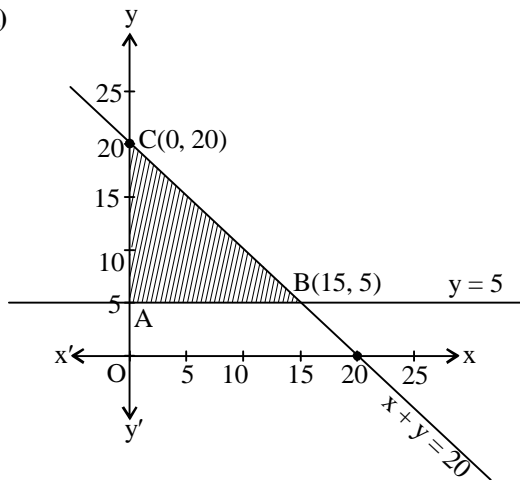
$\Rightarrow \frac{x}{1} + \frac{y}{-3} = 1$

From $4x - y > 4, \Rightarrow \frac{x}{1} + \frac{y}{-4} = 1.$

Clearly the common region of both is true for positive value of (x, y).

- (a) The equations, corresponding to inequalities $3x + 2y \leq 6$ and $6x + 4y \geq 20$, are $3x + 2y = 6$ and $6x + 4y = 20$. So the lines represented by these equation are parallel. Hence the graphs are disjoint.

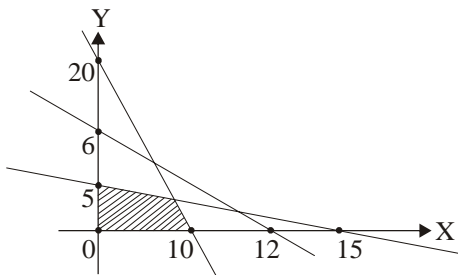
19. (d)



Corner points	$Z = 7x - 8y$
A(0,5)	$Z = -40$
B(15,5)	$Z = 65$
C(0,20)	$Z = -160$ (Minimum)

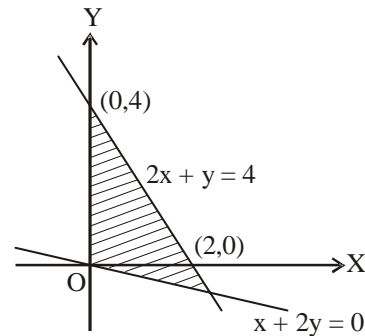
Hence minimum at (0, 20).

20. (c) In given all equations, the origin is present in shaded area, option (c) satisfy this condition.
21. (c) Let the manufacturer produce x units of type A circuit and y units of type B circuits.
 Maximise $Z = 50x + 60y$, subject to $2x + y \leq 20$, $x + 2y \leq 12$, $x + 3y \leq 15$, $x \geq 0$, $y \geq 0$.

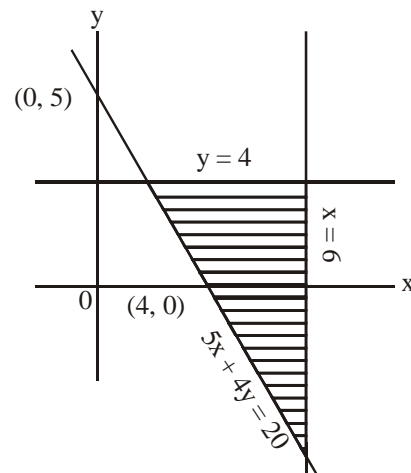


Corner Points	$Z = 50x + 60y$
O(0,0)	$Z = 0$
A(10,0)	$Z = 500$
B(9,2)	$Z = 570$ Maximum
C(0,5)	$Z = 300$

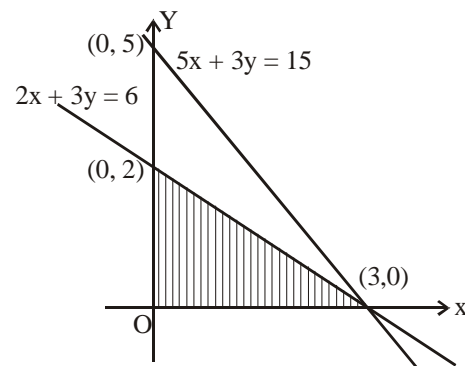
22. (d)



23. (c) Common region is triangle.



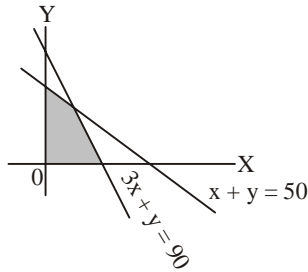
24. (d) Here (0, 2), (0, 0) and (3, 0) all are vertices of feasible region. Hence option (d) is correct.



25. (a)

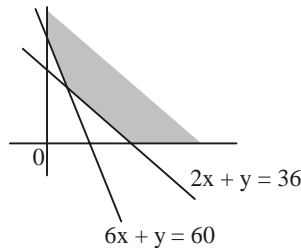
Corner Points	$Z = 5x + 7y$
(0, 0)	$Z = 0$
(0, 2)	$Z = 14$
(7, 0)	$Z = 35$
(3, 4)	$Z = 43 \leftarrow$ Maximum

26. (c)



Corner Points	$Z = 4x + y$
(0, 0)	$Z = 0$
(30, 0)	$Z = 120 \leftarrow$ Maximum
(20, 30)	$Z = 110$
(0, 50)	$Z = 50$

27. (a)



Corner Points	$Z = 20x + 9y$
(18, 0)	$Z = 360$
(6, 24)	$Z = 336 \leftarrow$ Minimum
(0, 60)	$Z = 540$

28. (b) Table of values of the objective function:

Corner Points	Value of $Z = 4x + 3y$
(0, 0)	$4 \times 0 + 3 \times 0 = 0$
(0, 40)	$4 \times 0 + 3 \times 40 = 120$
(20, 40)	$4 \times 20 + 3 \times 40 = 200$
(60, 20)	$4 \times 60 + 3 \times 20 = 300 \leftarrow$ Maximum
(60, 0)	$4 \times 60 + 3 \times 0 = 240$

29. (b) Table of values of the objective function:

Corner Points	Value of $Z = 3x - 4y$
(0, 0)	$3 \times 0 - 4 \times 0 = 0$
(5, 0)	$3 \times 5 - 4 \times 0 = 15 \leftarrow$ Maximum
(6, 5)	$3 \times 6 - 4 \times 5 = -2$
(6, 8)	$3 \times 6 - 4 \times 8 = -14$
(4, 10)	$3 \times 4 - 4 \times 10 = -28$
(0, 8)	$3 \times 0 - 4 \times 8 = -32 \leftarrow$ Minimum

Minimum of $Z = -32$ at (0, 8).

30. (a) Table of values of the objective function F:

Corner Points	Value of $F = 3x - 4y$
(0, 0)	$3 \times 0 - 4 \times 0 = 0 \leftarrow$ Maximum
(6, 12)	$3 \times 6 - 4 \times 12 = -30$
(6, 16)	$3 \times 6 - 4 \times 16 = -46 \leftarrow$ Minimum
(0, 4)	$3 \times 0 - 4 \times 4 = -16$

Hence, maximum of $F = 0$

31. (d) Table of values of objective function:

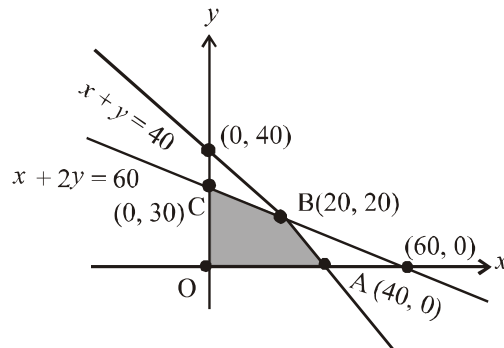
Corner Points	Value of $F = 4x - 6y$
(0, 2)	$4 \times 0 + 6 \times 2 = 12 \leftarrow$ Minimum
(3, 0)	$4 \times 3 + 6 \times 0 = 12 \leftarrow$ Minimum
(6, 0)	$4 \times 6 + 6 \times 0 = 24$
(6, 8)	$4 \times 6 + 6 \times 8 = 72$
(0, 5)	$4 \times 0 + 6 \times 5 = 30$

Since the minimum value (F) = 12 occurs at two distinct corner points, it occurs at every point of the segment joining these two points.

32. (b) We must have value of Z at (3, 0) = value of Z at (1, 1)

$$\Rightarrow 3p + 0.q = 1.p + 1.q \Rightarrow 3p = p + q \Rightarrow p = \frac{1}{2}q$$

33. (b) $\frac{x}{40} + \frac{y}{40} \leq 1$



$$\text{and } \frac{x}{60} + \frac{y}{30} \leq 1$$

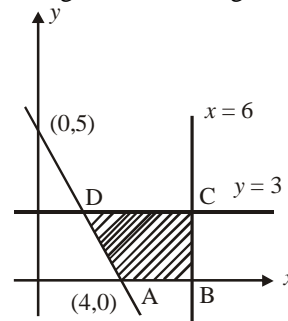
$$\text{and } z = 3x + 4y$$

Here OABC is a feasible region

$$O(0, 0) = 0, B(20, 20) = 140 \text{ (Maximum)}$$

$$A(40, 0) = 120, C(0, 30) = 120$$

34. (c) The given shaded region is



$$\text{Equation of line } AB \text{ is } y = 0$$

$$\text{Equation of line } BC \text{ is } x = 6$$

$$\text{Equation of line } CD \text{ is } y = 3$$

$$\text{Equation of line } AD \text{ is } 5x + 4y = 20$$

So the required inequalities are

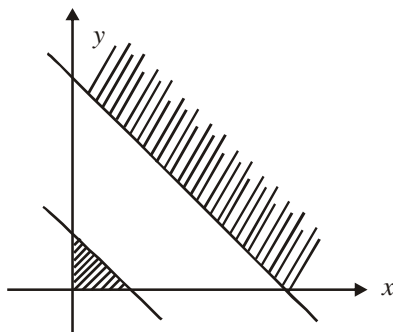
$$5x + 4y \geq 20$$

$$x \leq 6 \Rightarrow y \leq 3 \Rightarrow y \geq 0, x \geq 0$$

35. (d) Given constraints,

$$x + 2y \leq 2, x + 2y \geq 8, x, y \geq 0$$

The region formed by above inequalities are



So, from graph, it is clear that there is no common region which can satisfy all the constraints.

So, there is no feasible region.

⇒ Z has no feasible solution.

36. (b) Z is 7 minimum at $(\frac{3}{2}, \frac{1}{2})$.

37. (b) Let number of bags of cattle feed of brand P = x

And number of bags of cattle feed of brand Q = y

Therefore, the above L.P.P. is given as :

Minimise, $Z = 250x + 200y$, subject to the constraints :

$3x + 1.5y \geq 80, 2.5x + 11.25y \geq 45, 2x + 3y \geq 24, x, y \geq 0.$

Corner points	Z = 250 x + 200 y
C(0,12)	2400
B(18,0)	4500
D(3,6)	1950 (Min.)
A(9,2)	2650

Here Z = 1950 is minimum.

i.e. 3 bags of brand P and 6 bags of brand Q;

Minimum cost of the mixture = ₹ 1950.

38. (c) Let number of necklaces and bracelets produced by firm per day be x and y, respectively.

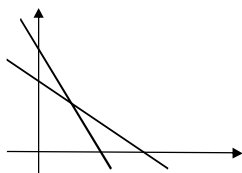
Maximise $z = 100x + 300y$,

subject to the constraints,

$x + y \geq 24$

$2x + y \geq 32$

$x, y \geq 0$



Corner points	Z = 100 x + 300 y
(0,0)	Z=0
(16,0)	Z= 1600
(8,16)	Z= 5600
(0, 24)	Z= 7200 (Max.)

39. (b) Let x machines of type A and y machines of type B be bought Maximise $Z = 60x + 40y$. Subject to constraints $5x + 6y < 45, 3x + 2y < 18$ s.t. $x > 0, y > 0$

Corner Point	Z = 60x + 40y
O(0,0)	Z = 0
A(6,0)	Z = 360 Maximum
B($\frac{9}{4}, \frac{45}{8}$)	Z = 360 Maximum
C($0, \frac{15}{2}$)	Z = $\frac{15}{2}$ 300

Hence, either 6 machine of type A and no machine of type B or 2 machine of type A and 6 machine of type B be used to have maximum output.

40. (b)

Corner Points	Z = 3x + 4y
(0, 38)	Z = 152
(52, 0)	Z = 156
(44, 16)	Z = 196 ← Maximum
(0, 0)	Z = 0

Exercise 2 :

ACCELERATOR

Topic-wise MCQs

- (b) The constraint are short in number.
- (b) 'p' is a linear inequality and 'q' is a non-linear inequality.
- (a) Minimize $Z = 5x + 7y$
- (a) Let the number toys A and B sold by the dealer be x and y respectively.
∴ $x \geq 0, y \geq 0$ [∵ Number of toys cannot be negative]
∴ Objective function is Maximize $Z = 10x + 15y$.
Also constraints are $x + y \leq 40, 75x + 90y \leq 580$
- (c) The constraints are $5x + 4y \leq 20, 6x + 3y \leq 12, x \geq 0, y \geq 0$.
- (b) Let x = number of table clothes produced in a day, and y = number of curtains produced in a day.
∴ $x \geq 0, y \geq 0$ [∵ Both items cannot be negative]
Representing the given information in tabular form, we get

	Table cloth (x)	Curtain (y)	Minimum requirement
Money earned (₹)	50	250	500

- ∴ $50x + 250y \geq 500$
∴ Total hours = $Z = x + 3y$
∴ Required LPP is formulated as
Minimize, $Z = x + 3y$, subject to $50x + 250y \geq 500, x \geq 0, y \geq 0$
- (b) (xy) will be maximum at $\frac{x}{1} = \frac{y}{1} = \frac{8}{1+1} \Rightarrow x = 4, y = 4$
∴ Maximum value of $xy = 4 \times 4 = 16$.
 - (a) As Z is maximum at (30, 30) and (0, 40)
⇒ $30a + 30b = 40b \Rightarrow b - 3a = 0$
 - (d) P → linear, Q → convex, R → bounded, S → maximum
 - (b) If an LPP admits optimal solutions at two consecutive vertices of a feasible region, then the optimal solution occurs at every point on the line joining these two points.

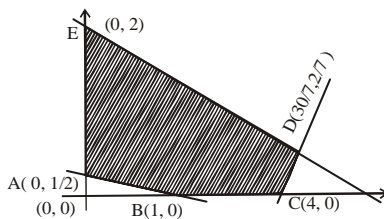
11. (c) We observe, (0, 0) does not satisfy the inequality $x - y \geq 1$.
So, the half-plane represented by the above inequality will not contain origin therefore, it will not contain the shaded feasible region.
12. (b) The strict inequality represents an open half plane and it contains the origin as (0, 0) satisfies it.
13. (c) The objective function maximizes the combined profit earned from selling X and Y.
14. (a) Assume that x and y take arbitrary large values. So the objective function can be made as large as we want. Hence, the problem has unbounded solution.
15. (b) The feasible region is unbounded, x and y can take arbitrary large values. Hence, the problem has unbounded solution.

16. (a) Condition (i)
 $i = 1, x_{11} + x_{12} + x_{13} + \dots + x_{1n}$
 $i = 2, x_{21} + x_{22} + x_{23} + \dots + x_{2n}$
 $i = 3, x_{31} + x_{32} + x_{33} + \dots + x_{3n}$

 $i = m, x_{m1} + x_{m2} + x_{m3} + \dots + x_{mn} \rightarrow m \text{ constraints}$
 Condition (ii)
 $j = 1, x_{11} + x_{21} + x_{31} + \dots + x_{m1}$
 $j = 2, x_{12} + x_{22} + x_{32} + \dots + x_{m2}$
 $j = n, x_{1n} + x_{2n} + x_{3n} + \dots + x_{mn} \rightarrow n \text{ constraints}$
 $\therefore \text{total constraints} = m + n$

17. (c) We find that the feasible region is on the same side of the line $2x + 5y = 10$ as the origin, on the same side of the line $x - y = 4$ as the origin and on the opposite side of the line $x + 2y = 1$ from the origin. Moreover, the lines meet the coordinate axes at (5, 0), (0, 2); (1, 0), (0, 1/2) and (4, 0). The

lines $x - y = 4$ and $2x + 5y = 10$ intersect at $(\frac{30}{7}, \frac{2}{7})$.

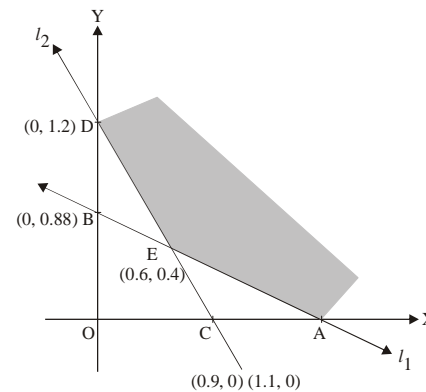


The values of the objective function at the vertices of the pentagon are:

- (i) $Z = 0 + \frac{5}{2} = \frac{5}{2}$ (ii) $Z = 2 + 0 = 2$
 (iii) $Z = 8 + 0 = 8$ (iv) $Z = \frac{60}{7} + \frac{10}{7} = 10$
 (v) $Z = 0 + 10 = 10$

The maximum value 10 occurs at the points D(30/7, 2/7) and E(0, 2). Since D and E are adjacent vertices, the objective function has the same maximum value 10 at all the points on the lines DE.

18. (d) Maximum value of $Z = px + qy$ occurs at (3, 4) and (0, 5),
 At (3, 4), $Z = px + qy = 3p + 4q$
 At (0, 5), $Z = 0 + q \cdot 5 = 5q$
19. (b) Minimize $Z = 5x + 4y$
 Subject to constraints;
 $20x + 25y \geq 22, 20x + 15y \geq 18, x \geq 0, y \geq 0$



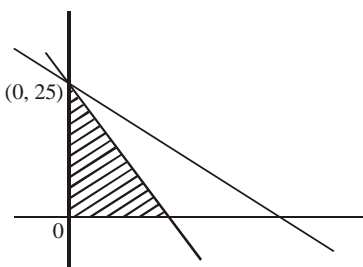
Lines l_1 and l_2 meet at E(0.6, 0.4).
 The feasible region has been shaded and it is an unbounded region with vertices A, E and D.

Minimize $Z = 5x + 4y$
 At A(1.1, 0), $Z = 5 \times 1.1 + 0 = 5.5$
 At E(0.6, 0.4), $Z = 5 \times 0.6 + 4 \times 0.4 = 4.6$
 At D(0, 1.2), $Z = 5 \times 0 + 4 \times 1.2 = 4.8$
 \therefore The minimum cost of producing this cereal is ₹ 4.6

20. (d) Clearly point (2000, 0) is outside.
21. (d) 36 at (0, 6).
22. (a) Given, $Z = 3x + 9y$
 At (0, 20), $Z = 3 \times 0 + 9 \times 20 = 180$
 At (15, 15), $Z = 3 \times 15 + 9 \times 15 = 180$
 At (5, 5), $Z = 3 \times 5 + 9 \times 5 = 60$ (minimum)
 At (0, 10), $Z = 3 \times 0 + 9 \times 10 = 90$
 So, minimum value of Z occurs at (5, 5).

23. (c) First draw the graph of the given inequalities
So, the shaded region is the feasible region.

Let maximum value of $6xy + y^2$ be k , then $6xy + y^2 = k$
For a solution with feasible region, $6xy + y^2 = k$ and $4x + 3y = 75$ must have at least one positive solution.



$$y^2 + 6y\left(\frac{75-3y}{4}\right) - k = 0$$

$$\Rightarrow \frac{7}{2}y^2 - \frac{225}{2}y + k = 0 \Rightarrow \left(\frac{225}{2}\right)^2 \geq 4 \cdot \frac{7}{2}k$$

$$\Rightarrow k \leq \frac{225^2}{56} \approx 904$$

Hence, the required maximum value of z is 904.

24. (c)

Corner Points	Value of $Z = 4x + y$
A(0, 50)	$Z = 50$
B(20, 30)	$Z = 80 + 30 = 110$
C(30, 0)	$Z = 120$ (Maximum)
O(0, 0)	$Z = 0$

25. (b) At $P\left(\frac{3}{13}, \frac{24}{13}\right)$, $Z = \frac{3}{13} + 2\left(\frac{24}{13}\right) = \frac{51}{13} = 3.923$

At $Q\left(\frac{3}{2}, \frac{15}{4}\right)$, $Z = \frac{3}{2} + 2\left(\frac{15}{4}\right) = 9$

At $R\left(\frac{7}{2}, \frac{3}{4}\right)$, $Z = \frac{7}{2} + 2\left(\frac{3}{4}\right) = 5$

At $S\left(\frac{18}{7}, \frac{2}{7}\right)$, $Z = \frac{18}{7} + 2\left(\frac{2}{7}\right) = \frac{22}{7} = 3.143$

\therefore Maximum value of Z is 9, and minimum value of Z is $\frac{22}{7}$.

26. (b) The corner points of the feasible region are

$O(0, 0), E(12, 0), H(18, 2), G(5, 15), D\left(0, \frac{35}{2}\right)$

\therefore The maximum value of $4x + 5y$ is at $G(5, 15)$

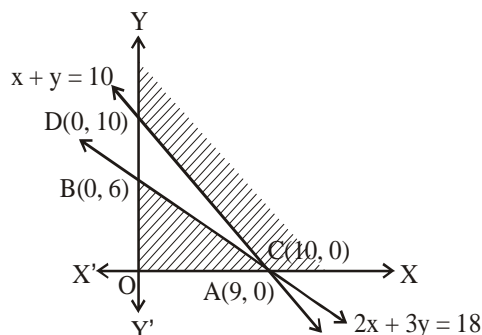
\therefore Maximum $4x + 5y = 4(5) + 5(15) = 95$

27. (c) Shaded region lies on origin side of $3x + 10y = 30$ and $x = 6$, and on non-origin side of $4x + 5y = 20$
 $\therefore 4x + 5y \geq 20, 3x + 10y \leq 30, x \leq 6, x, y \geq 0$

28. (d) Since there are two disjoint feasible regions, the LPP has no solution.

29. (a) The feasible region is disjoint.
 \therefore There is no solution.

30. (d)

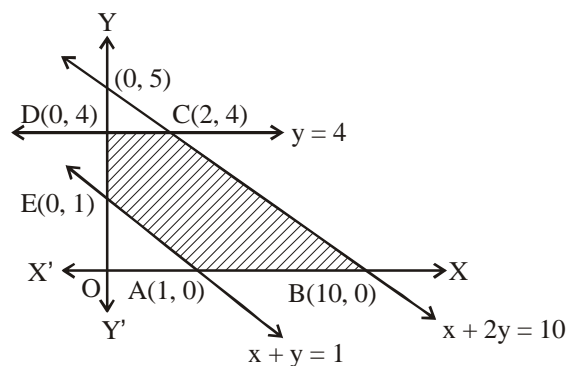


The feasible region is disjoint.

\therefore There is no point common to all inequations.

\therefore There is no maximum value of Z .

31. (c)



The corner points of feasible region are $A(1, 0), B(10, 0), C(2, 4), D(0, 4)$ and $E(0, 1)$

At $A(1, 0), Z = 1 + 0 = 1$

At $B(10, 0), Z = 10 + 0 = 10$

At $C(2, 4), Z = 2 + 4 = 6$

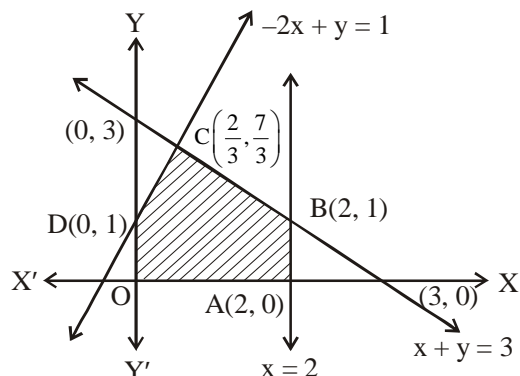
At $D(0, 4), Z = 0 + 4 = 4$

At $E(0, 1), Z = 0 + 1 = 1$

Z has minimum value at both $A(1, 0)$ and $E(0, 1)$.

$\therefore Z$ has infinite solutions on seg AE .

32. (b)



The feasible region lies on origin side of all the lines and in first quadrant.

\therefore The corner points of feasible region are

$O(0,0)$, $A(2,0)$, $B(2,1)$, $C\left(\frac{2}{3}, \frac{7}{3}\right)$ and $D(0,1)$

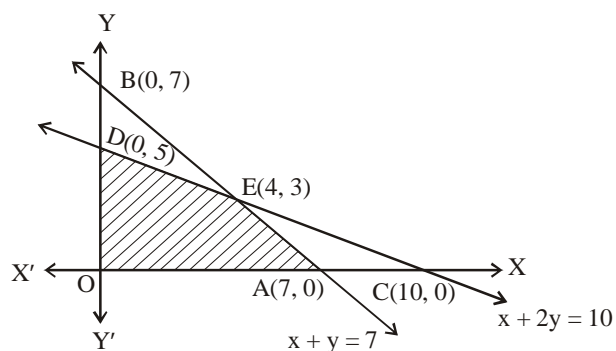
Maximum value of $Z = 3x + 2y$ is at $B(2,1)$

\therefore Maximum $Z = 3(2) + 2(1) = 8$

33. (b) Shaded region lies on origin side of $x + 2y = 8$ and $x - y = 1$, and on non-origin side of $2x + y = 2$.

$\therefore x + 2y \leq 8$, $x - y \leq 1$, $2x + y \geq 2$

34. (c)



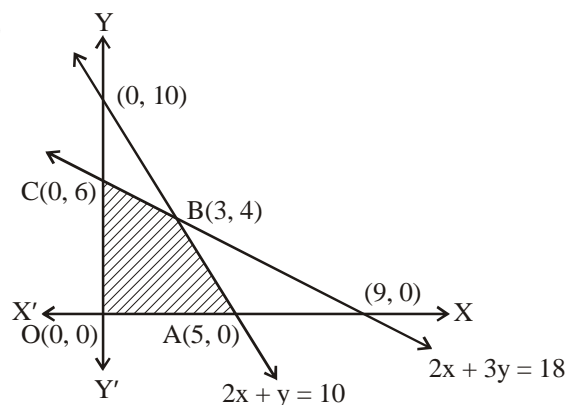
Feasible region lies on origin side of $x + y = 7$ and $x + 2y = 10$, and in first quadrant.

The corner points of feasible region are $O(0,0)$, $A(7,0)$, $E(4,3)$ and $D(0,5)$

\therefore Maximum $Z = 5x + 2y$ is at $A(7,0)$

\therefore Maximum $Z = 5(7) + 2(0) = 35$

35. (d)



Feasible region lies on the origin side of $2x + y = 10$ and $2x + 3y = 18$ in first quadrant.

The corner points of the feasible region are $O(0,0)$, $A(5,0)$, $B(3,4)$ and $C(0,6)$

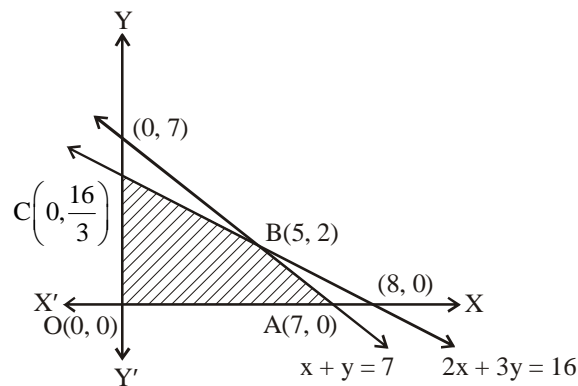
At $A(5,0)$, $Z = 45$

At $B(3,4)$, $Z = 79$

At $C(0,6)$, $Z = 78$

\therefore Maximum value of Z is 79.

36. (c)



Feasible region lies on the origin side of $x + y = 7$ and $2x + 3y = 16$, in first quadrant.

The corner points of the feasible region are $O(0,0)$,

$A(7,0)$, $B(5,2)$ and $C\left(0, \frac{16}{3}\right)$

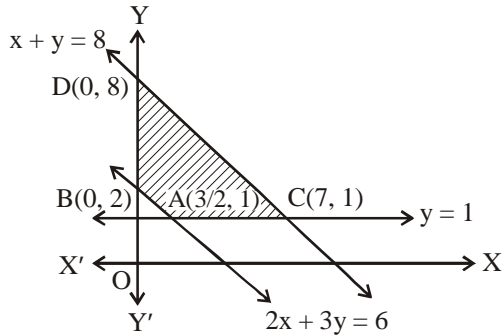
At $A(7, 0), Z = 21$

At $B(5, 2), Z = 19$

At $C\left(0, \frac{16}{3}\right), Z = \frac{32}{3} = 10.67$

\therefore Maximum value of Z is 21.

37. (b)



The feasible region lies on the non-origin side of $2x + 3y = 6$ and $y = 1$ and on origin side of $x + y = 8$
The corner points of feasible region are

$A\left(\frac{3}{2}, 1\right), B(0, 2), C(7, 1)$ and $D(0, 8)$

Substituting above points in $Z = 4x + 6y$, we get

Min, $Z = 12$ at $A\left(\frac{3}{2}, 1\right)$ and $B(0, 2)$.

38. (b) The given data may be put in the following tabular form

Refinery	High grade	Medium grade	Low grade	Cost per day
A	100	300	200	400
B	200	400	100	300
Minimum Requirement	12000	20000	15000	

Suppose refineries A and B should run for x and y days respectively to minimize the total cost.

The mathematical form of the above is

Minimize $Z = 400x + 300y$

Subject to

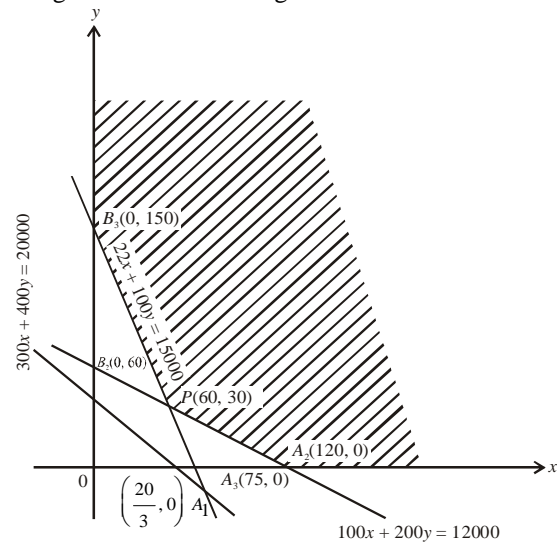
$$100x + 200y \geq 12000$$

$$300x + 400y \geq 20000$$

$$200x + 100y \geq 15000$$

and $x, y \geq 0$

The feasible region of the above LPP is represented by the shaded region in the given figure. The corner points of the feasible region are $A_2(120, 0), P(60, 30)$ and $B_3(0, 150)$. The value of the objective function at these points are given in the following table



Point (x, y)	Value of the objective function $Z = 400x + 300y$
$A_2(120, 0)$	$Z = 400 \times 120 + 300 \times 0 = 48000$
$P(60, 30)$	$Z = 400 \times 60 + 300 \times 30 = 33000$
$B_3(0, 150)$	$Z = 400 \times 0 + 300 \times 150 = 45000$

Clearly, Z is minimum when $x = 60, y = 30$. Hence, the machine A should run for 60 days and the machine B should run for 30 days to minimize the cost while satisfying the constraints.

39. (a) The given information can be summarized in the following tabular form

Machine	Time required to produce products		Max. Machine hours available
	Nut	Bolt	
A	1	3	12
B	3	1	12
Profit (in ₹)	2.50	1.00	

Let the manufacturer produce x packages of nuts and y packages of bolts each day.

Since, machine A takes one hour to produce one package of nuts and 3 h to produce one package of bolts. Therefore, the total time required by machine A to produce x packages of nuts and y packages of bolts is $(x + 3y)$ h. But machine A operates for at most 12 h.

$\therefore x + 3y \leq 12$

Similarly, the total time required by machine B to produce x packages of nuts and y packages of bolts is (3x + y) h. But machine B operates for at most 12 h.

$\therefore 3x + y \leq 12$

Since, the profit one package of nuts is ₹ 2.50 and on one package of bolts the profits is ₹ 1. Therefore, profit on x packages of nuts and y packages of bolts is of ₹ (2.50x + y). Let Z denote the total profit. Then, $Z = 2.50x + y$

Clearly, $x \geq 0$ and $y \geq 0$.

Thus, the above LPP can be stated mathematically as follows

Maximize $Z = 2.50x + y$

Subject to $x + 3y \leq 12$

$3x + y \leq 12$

and, $x, y \geq 0$

To solve this LPP graphically, we first convert the inequations into equations to obtain the following equations.

$x + 3y = 12,$

$3x + y = 12,$

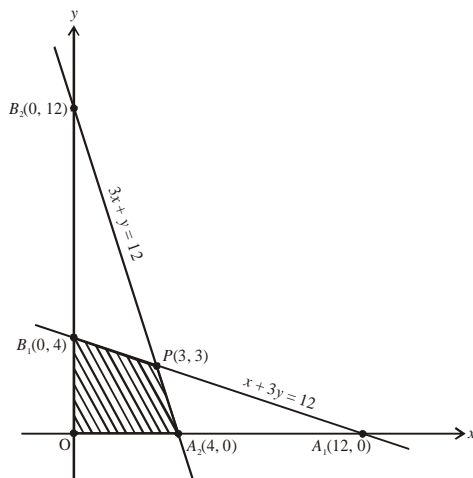
$x = 0, y = 0$

The line $x + 3y = 12$ meets the coordinate axes at $A_1(12, 0)$ and $B_1(0, 4)$. Join these two points to obtain the line represented by $x + 3y = 12$. The region represented by the inequation $x + 3y \leq 12$ is the region containing the origin as $x = 0, y = 0$ satisfies the inequation

$x + 3y \leq 12.$

Clearly, $x \geq 0$ and $y \geq 0$ represent all points in first quadrant.

Thus, the shaded region OA_2PB_1 in given figure represents the feasible region of the given LPP.



The coordinates of the corner-points of the feasible region OA_2PB_1 are $O(0, 0)$, $P(3, 3)$ and $B_1(0, 4)$. These points are obtained by solving the corresponding intersecting lines simultaneously.

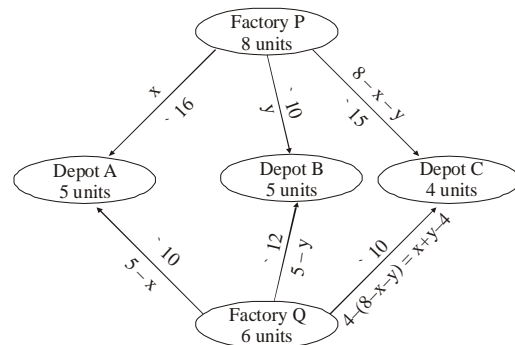
The values of the objective function at the corner-points of the feasible region are given in the following table

Point (x, y)	Value of the objective function $Z = 2.50x + y$
$O(0, 0)$	$Z = 2.50 \times 0 + 1 \times 0 = 0$
$A_2(4, 0)$	$Z = 2.50 \times 4 + 1 \times 0 = 10$
$P(3, 3)$	$Z = 2.50 \times 3 + 1 \times 3 = 10.50$
$B_1(0, 4)$	$Z = 2.50 \times 0 + 1 \times 4 = 4$

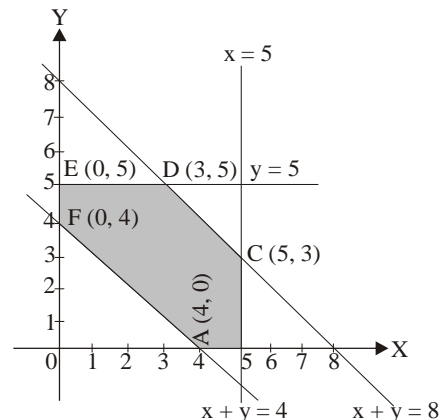
Clearly, Z is maximum at $x = 3, y = 3$ and the maximum value of Z is 10.50.

Hence, the optimal production strategy for the manufacturer will be to manufacture 3 packages each of nuts and bolts daily and in this case his maximum profit will be ₹ 10.50. Hence, (a) is the correct answer.

40. (c) The given data can be represented as given in the following diagram:



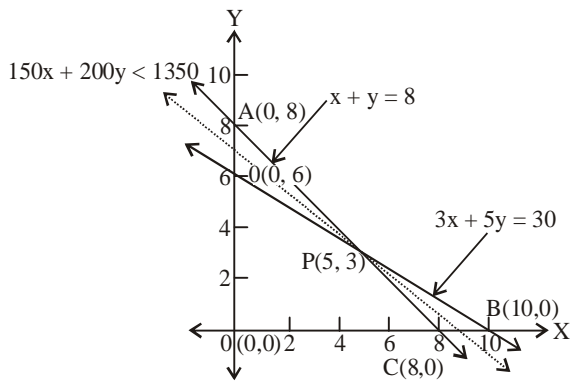
Minimize $Z = x - 7y + 190$ subject to constraints $x + y \geq 4, x + y \leq 8, x \leq 5, y < 5, x \geq 0, y \geq 0$



- $\therefore Z = x - 7y + 190$
- \therefore At A(4, 0), $Z = 194$
- At B(5, 0), $Z = 195$
- At C(5, 3), $Z = 174$
- At D(3, 5), $Z = 158$
- At E(0, 5), $Z = 155$
- At F(0, 4), $Z = 162$

Thus Z is minimum at E(0, 5) i.e., when $x = 0, y = 5$. Thus, for minimum transportation cost, factory P should supply 0, 5, 3 units to depots A, B, C respectively and factory Q should supply 5, 0, 1 units respectively to depots A, B, C.

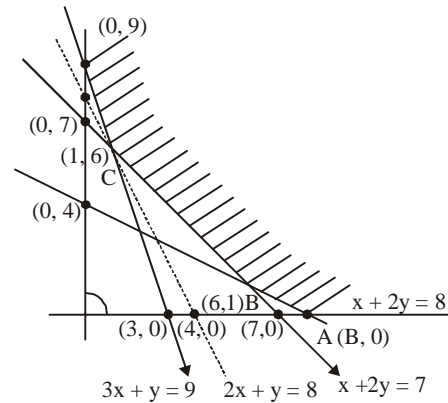
41. (a) Let tailors A and B work for x and y days, respectively.
 Minimise $Z = 150x + 200y$ Subject to
 $3x + 5y \geq 30, x + y \geq 8$ and $x \geq 0, y \geq 0$



Corner points	Value of $Z = 150x + 200y$
A(0, 8)	$Z = 150(0) + 200(8) = 1600$
P(5, 3)	$Z = 150(5) + 200(3) = 1350$ (minimum)
B(10, 0)	$Z = 150(10) + 200(0) = 1500$

Since the feasible region is unbounded and open half of $150x + 200y < 1350$ has no points in common with feasible region. Therefore the minimum labour cost is ` 1350.

42. (c) Let the person takes x units of tablet X and y unit of tablet Y.
 Minimise $Z = 2x + y$ Subject to $3x + y \geq 9,$
 $x + y \geq 7, x + 2y \geq 8, x \geq 0, y \geq 0.$
 [On solving $x + 2y = 8$ and $x + y = 7$ we get $x = 6, y = 1$ and on solving $3x + y = 9$ and $x + y = 7,$ we get $x = 1, y = 6$]



Corner Points	Values of $Z = 2x + y$
(8, 0)	16
(6, 1)	13
(1, 6)	8 (minimum)
(0, 9)	9

Since the feasible region is unbounded and open half of $2x + y < 8$ has no points in common with feasible region. Therefore $Z = 2x + y$ has 8 as minimum cost.

43. (a) Suppose x grams of wheat and y grams of rice are mixed in the daily diet.
 Since every grams of wheat provides 0.1 g of proteins and every gram of rice gives 0.05 g of proteins. Therefore, x gms of wheat and y grams of rice will provide $0.1x + 0.05y$ g of proteins. But the minimum daily requirement of proteins is of 50 g.

$$\therefore 0.1x + 0.05y \geq 50 \Rightarrow \frac{x}{10} + \frac{y}{20} \geq 50$$

Similarly, x grams of wheat and y grams of rice will provide $0.25x + 0.5y$ g of carbohydrates and the minimum daily requirement of carbohydrates is of 200 g.

$$\therefore 0.25x + 0.5y \geq 200 \Rightarrow \frac{x}{4} + \frac{y}{2} \geq 200$$

Since, the quantities of wheat and rice cannot be negative. Therefore, $x \geq 0, y \geq 0$

It is given that wheat costs ` 4 per kg and rice ` 6 per kg. So, x grams of wheat and y grams of rice will cost

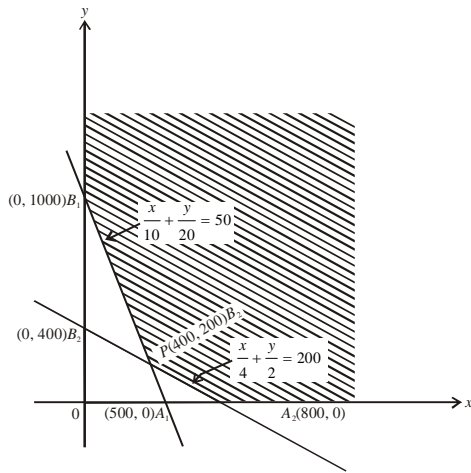
$$\text{` } \frac{4x}{1000} + \frac{6y}{1000}$$

Minimize $z = \frac{4x}{1000} + \frac{6y}{1000}$

Subject to the constraints

$\frac{x}{10} + \frac{y}{20} \geq 50, \frac{x}{4} + \frac{y}{2} \geq 200, \text{ and } x \geq 0, y \geq 0$

The solution set of the linear constraints is shaded in figure. The vertices of the shaded region are $A_2(800, 0), P(400, 200)$ and $B_1(0, 1000)$.

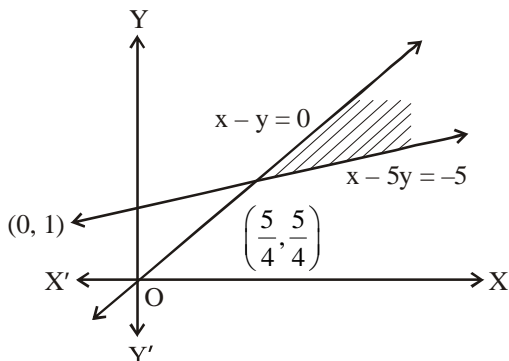


The values of the objective function at these points are given in the following table.

Point (x, y)	Value of objective function $Z = \frac{4x}{1000} + \frac{6y}{1000}$
$A_2(800, 0)$	$Z = \frac{4}{1000} \times 800 + \frac{6}{1000} \times 0 = 3.2$
$P(400, 200)$	$Z = \frac{4}{1000} \times 400 + \frac{6}{1000} \times 200 = 2.8$
$B_1(0, 1000)$	$Z = \frac{4}{1000} \times 0 + \frac{6}{1000} \times 1000 = 6$

Clearly, Z is minimum for $x = 400$ and $y = 200$. The minimum diet cost is 2.8.

44. (b)



The feasible region is unbounded whose vertex

is $(\frac{5}{4}, \frac{5}{4})$.

\therefore Minimum $Z = 2x + 10y$ is at $(\frac{5}{4}, \frac{5}{4})$

$\therefore Z = 2(\frac{5}{4}) + 10(\frac{5}{4}) = 15$

Exercise 3 :

PREVIOUS YEARS MCQs

1. (d) Corner point

At (0, 0); $P = 0$; At (0, 10); $P = 30$

At (10, 0); $P = 10$;

At $(\frac{20}{3}, \frac{20}{3})$; $P = \frac{80}{3} = 26.67$

2. (a) Feasible region of inequalities gives optimal solution of LPP.

3. (c) Given $Z = ax + by$... (i)

Now $Z_{(4, 6)} = 4a + 6b$

$\Rightarrow 42 = 4a + 6b$... (ii)

and $Z_{(3, 2)} = 3a + 2b$

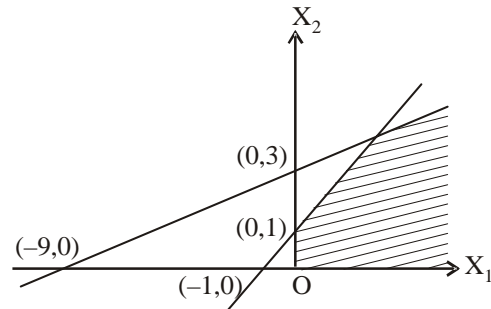
$3 \times 19 = 3 \times (3a + 2b)$

$\Rightarrow 57 = 9a + 6b$... (iii)

Subtract equation (iii) from (ii)

$\Rightarrow 5a = 15 \Rightarrow a = 3, b = 5$

4. (b) It is clear from the graph, the constraints define the unbounded feasible space.



5. (d) From the graph,

$-7x + 4y \leq 14, x - 6y \leq 3$

$3x + 4y \leq 18, 2x + 3y \leq 3$

6. (a) Let Shamali invest `x in saving certificate and `y in PPF.

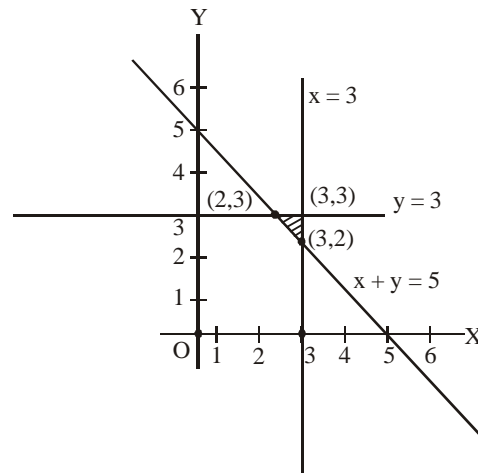
$$\therefore x + y \leq 50000, x \geq 15000 \text{ and } y \geq 20000$$

$$\text{Total income} = \frac{8}{100}x + \frac{9}{100}y$$

\therefore Given problem can be formulated as

$$\text{Maximize } Z = 0.08x + 0.09y$$

$$\text{Subject to, } x + y \leq 50000, x \geq 15000, y \geq 20000.$$



\therefore Corner points of the bounded region are :

$$(3, 2), (2, 3) \text{ \& } (3, 3)$$

$$Z = 10x + 25y.$$

$$\text{At } (3, 2), Z = 30 + 50 = 80 \text{ (Minimum).}$$

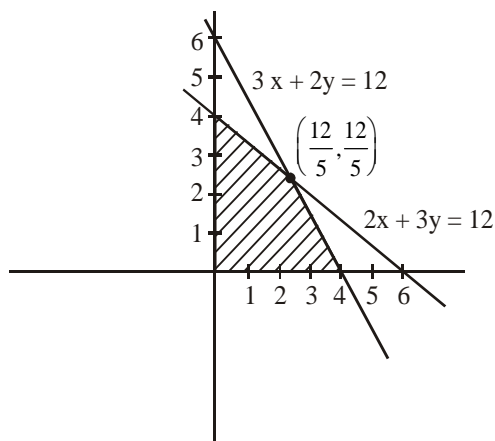
$$\text{At } (2, 3), Z = 20 + 75 = 95$$

$$\text{At } (3, 3), Z = 30 + 75 = 105.$$

7. (d) $\therefore 3x + 2y \leq 12$ or $\frac{x}{4} + \frac{y}{6} \leq 1$

$$2x + 3y \leq 12 \text{ or } \frac{x}{6} + \frac{y}{4} \leq 1.$$

$$\text{and } x \geq 0, y \geq 0$$



\therefore Corner points are : (0, 0), (0, 4), (4, 0) and $(\frac{12}{5}, \frac{12}{5})$.

$$\therefore Z = 9x + 11y.$$

$$\text{At } (0, 0), Z = 0.$$

$$\text{At } (0, 4), Z = 44.$$

$$\text{At } (4, 0), Z = 36.$$

$$\text{At } (\frac{12}{5}, \frac{12}{5}), Z = \frac{108 + 132}{5} = \frac{240}{5} = 48.$$

\therefore maximum value of Z is 48.

8. (a) $Z = 10x + 25y$ subject to :

$$x + y \geq 5; x \leq 3; y \leq 3; x \geq 0; y \geq 0$$

9. (a) The objective function is given as, minimize,

$$Z = 4x_1 + 5x_2$$

$$\text{Subject to constraints, } 2x_1 + x_2 \geq 7,$$

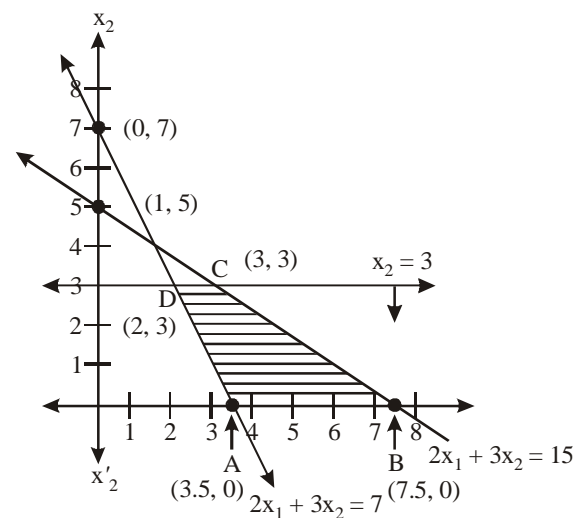
$$2x_1 + 3x_2 \leq 15, x_2 \leq 3 \text{ and } x_1, x_2 \geq 0$$

$$\text{For line } 2x_1 + x_2 = 7$$

x_1	0	1	2	3
x_2	7	5	3	1

$$\text{For line } 2x_1 + 3x_2 = 15$$

x_1	0	3	6
x_2	5	3	1



Now, the value of Z at corner points are calculated as:

Corner points	$Z = 4x_1 + 5x_2$
A(3.5, 0)	$Z = 4 \times 3.5 + 5 \times 0 = 14$ (minimum)
B(7.5, 0)	$Z = 4 \times 7.5 + 5 \times 0 = 30$
C(3, 3)	$Z = 4 \times 3 + 5 \times 3 = 27$
D(2, 3)	$Z = 4 \times 2 + 5 \times 3 = 23$

Hence, the minimum value of Z is 14 at point (3.5, 0) which lies on X -axis.

10. (c) Objective function of a LPP defined over convex set attains its optimum value at atleast one of the corner points.

11. (b) In the given figure, OC line passes through (0, 0) and (3, 3).

So, equation of line is $y = x$.

Also, shaded portion of this line is towards the X -axis, so $x - y \geq 0$.

In the given figure, line AB is parallel to Y -axis.

\therefore Equation of line AB is $x = 5$, also shaded portion of this line is towards the origin, so

$x - 5 \leq 0$ or $x \leq 5$.

In the given figure, line BC is parallel to X -axis.

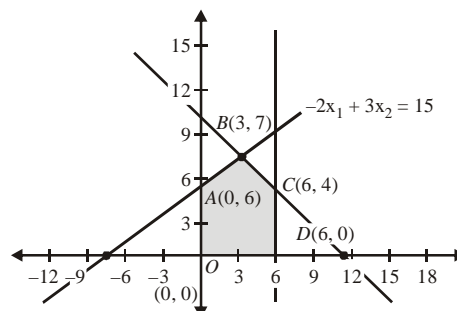
Therefore, equation of line BC is $y = 3$. Also, shaded portion of this line is towards the origin, so $y - 3 \leq 0$ or $y \leq 3$.

Also, the shaded region lies in first quadrant, so $x \geq 0$ and $y \geq 0$.

Hence, constraints of given shaded region are $x, y \geq 0$, $x - y \geq 0$, $x \leq 5$, $y \leq 3$.

12. (c) Since, objective function is $Z = x_1 + x_2$ and given constraints are

$$x_1 + x_2 \leq 10, -2x_1 + 3x_2 \leq 15, x_1 \leq 6, x_1, x_2 \geq 0.$$



Now, the point of intersection of lines $x_1 + x_2 = 10$ and $-2x_1 + 3x_2 = 15$ is $B(3, 7)$ and point of intersection of lines $x_1 = 6$ and $x_1 + x_2 = 10$ is

$$C(6, 4)$$

Here, the feasible region is $OABCD$. The corner points of the feasible region are $O(0, 0)$, $A(0, 6)$, $B(3, 7)$, $C(6, 4)$ and $D(6, 0)$.

Corner points	$Z = x_1 + x_2$
$O(0, 0)$	$Z = 0 + 0 = 0$
$A(0, 6)$	$Z = 0 + 6 = 6$
$B(3, 7)$	$Z = 3 + 7 = 10$ maximum
$C(6, 4)$	$Z = 6 + 4 = 10$ maximum
$D(6, 0)$	$Z = 6 + 0 = 6$

Hence, Z is maximum at each point of the segment joining two points $B(3, 7)$ and $C(6, 4)$