Hints & Solutions

1 Rotational Dynamics

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(a)** **2.** **(a)**

3. **(b)** $v_{max} = \sqrt{\mu_s rg} = \sqrt{0.5 \times 40 \times 9.8} = 14$ m/s

4. **(d)** $\tan\theta = \dfrac{v^2}{rg} = \dfrac{(14\sqrt{3})^2}{20\sqrt{3} \times 9.8} = \sqrt{3}$

or $\theta = 60°$

5. **(a)** $mg - N = \dfrac{mv^2}{R}$

or $N = mg - \dfrac{mv^2}{R}$

As $R_B > R_A$, and so $N_B > N_A$.

6. **(a)** $\omega = \dfrac{\theta}{t} = \dfrac{2\pi}{60} = \dfrac{\pi}{30}$ rad/s.

7. **(d)** $\tan\theta = v^2/rg$, $\tan\theta = H/1.5$, $r = 200$ m, $b = 1.5$ m

$v = 36$ km/hour $= 36 \times (5/18) = 10$ m/s.

Putting these values, we get H = 0.075 m.

8. **(a)** There is no change in the angular velocity, when speed is constant.

9. **(d)** Frequency of revolution is

$\nu = \dfrac{18}{60}$

As, $\omega = 2\pi\nu = 2\pi \times \dfrac{18}{60}$

We know that, $\nu = R\omega$

$= 0.1\text{m} \times \dfrac{18 \times 2\pi}{60}\text{s}^{-1} = 6\pi \times 10^{-2}\text{ms}^{-1}$

Hence speed of particle is $6\pi \times 10^{-2}$ ms^{-1}

10. **(c)** $\upsilon = r\omega \Rightarrow \omega = \dfrac{\upsilon}{r} =$ constant [As υ and r are constant]

11. **(a)** $a = \dfrac{\upsilon^2}{r} = \dfrac{(400)^2}{160} = 10^3 \text{m/s}^2 = 1\text{km/s}^2$

12. **(a)** $\omega = \dfrac{\upsilon}{r} = \dfrac{100}{100} = 1$ rad/s

13. **(b)** $\upsilon = r\omega = 0.5 \times 70 = 35$ m/s

14. **(b)** $\upsilon = r\omega = 20 \times 10$ cm/s $= 2$ m/s

15. **(b)** Average velocity

$= \dfrac{\text{Total displacement}}{\text{Total time}} = \dfrac{2m}{1s} = 2\,ms^{-1}$

16. **(a)** Here, $\nu_0 = 420$ rpm $= 7$ rps

$\therefore \omega_0 = 2\pi\nu_0 = 2 \times \dfrac{22}{7} \times 7 = 44\text{ rad s}^{-1}$

$\omega = 0$, $\alpha = -2$ rad s^{-2}

$t = \dfrac{\omega - \omega_0}{\alpha} = \dfrac{-44}{-2} = 22$s

17. **(d)** Angular acceleration,

$\alpha = \dfrac{\omega - \omega_0}{t} = \dfrac{2\pi(\nu - \nu_0)}{t}$

Given, $\nu = 4500 \text{ rpm} = \dfrac{4500}{60}\text{s}^{-1}$

$\nu_0 = 1200 \text{ rpm} = \dfrac{1200}{60}\text{s}^{-1}$

t = 10 s

Substituing the given values, we get

$\alpha = \dfrac{\omega - \omega_0}{t} = \dfrac{\frac{2\pi}{60}(4500 - 1200)}{10}$

$= 11\pi\text{rads}^{-2} = \dfrac{11\pi \times 180}{\pi} = 1980 \text{ degs}^{-2}$

18. **(d)** As $\alpha = \dfrac{\omega - \omega_0}{t} = \dfrac{600\pi - 200\pi}{10} = 40\pi$ rad/s^2

Now, $\theta = \dfrac{\omega^2 - \omega_0^2}{2\alpha} = \dfrac{(600\pi)^2 - (200\pi)^2}{2 \times 40\pi}$

$\theta = 4000\pi$

$\therefore$ Number of revolution, $n = \dfrac{\theta}{2\pi} = \dfrac{4000\pi}{2\pi} = 2000$

19. **(b)** Angular velocity is axial vector.

20. **(c)** Angular speed, $\omega = \dfrac{120 \times 2\pi}{60} = 4\pi$ rad/sec

21. **(a)** $\omega_i = \dfrac{900}{60} \times 2\pi = 30\pi$ rad/s, $\omega_f = 0$, $t = 60$s

$\omega_f = \omega_i + \alpha t \quad \therefore \alpha = \dfrac{\omega_f - \omega_i}{t} = \dfrac{0 - 30\pi}{60} = -\dfrac{\pi}{2}$ rad/s^2

Here, –ve sign shows angular retardation.

22. **(a)** $a = \alpha R$

23. **(a)** $w_0 = 600\text{ rev/min} = \frac{600 \times 2\pi}{60} = 20\pi \text{ rad/s}$

$\alpha = -2 \text{ rad/s}^2, \omega = 0$

using, $\omega^2 = \omega_0^2 + 2\alpha\theta$

$\Rightarrow 0 = (20\pi)^2 - 2 \times 2 \times \theta$

$\theta = 100\pi^2$

$\therefore$ Number of revolutions $= \frac{\theta}{2\pi} = \frac{100\pi^2}{2\pi} = 157$

24. **(c)** $\omega^2 = \omega_0^2 - 2\alpha\theta \Rightarrow 0 = 4\pi^2 n^2 - 2\alpha\theta$

$$\theta = \frac{4\pi^2\left(\frac{1200}{60}\right)^2}{2 \times 4} = 200\pi^2 rad$$

$\therefore 2\pi n = 200\pi^2 \Rightarrow n = 100\pi = 314$ revolutions

25. **(c)** Given, no. of rotation $n = 1800$ rpm = 1800 rps

Time, $t = 2$ minutes = 120s

$\therefore$ Initial angular speed $\omega_0 = \frac{2\pi \times 1800}{60} \text{rad s}^{-1}$

$= 60\pi \text{ rad s}^{-1}$

Final angular speed (as wheel comes to rest)

$\omega = 0$

$\therefore$ Angular retardation $= \frac{\omega_0 - \omega}{t} = \frac{60\pi - 0}{120} = \frac{\pi}{2} \text{ rad s}^{-2}$

26. **(b)**

27. **(a)** From figure,

$N \sin\theta = \frac{mv^2}{r}$ .... (i)

$N \cos\theta = mg$ ... (ii)

Dividing, we get

$\tan\theta = \frac{v^2}{rg}$ or $\theta = \tan^{-1}\frac{v^2}{rg}$

28. **(b)** Since surface (ice) is frictionless, so the centripetal force required for skating will be provided by inclination of boy with the vertical and that angle is given as

$\tan\theta = \frac{v^2}{rg}$ where v is speed of skating & r is radius of circle in which he moves.

29. **(b)** For banking $\tan\theta = \frac{V^2}{Rg}$

$\tan 45 = \frac{V^2}{90 \times 10} = 1$

$V = 30$ m/s

30. **(b)** For circular motion of aircraft,

$r = 9$ km, $v = 540 \text{ km h}^{-1} = 540 \times \frac{5}{18} = 150 \text{ ms}^{-1}$

Banked angle, $\tan\theta = \frac{v^2}{rg} = \frac{150 \times 150}{9 \times 10^3 \times 10} = \frac{1}{4}$

$\therefore \theta = \cot^{-1}(4)$

31. **(d)** Angle of banking is $\tan\theta = \frac{v^2}{rg} = \frac{20^2}{40\sqrt{3} \times 10} = \frac{1}{\sqrt{3}}$

32. **(b)**

33. **(b)** For negotiating a circular curve on a levelled road, the maximum velocity of the car is $v_{max} = \sqrt{\mu rg}$

Here $\mu = 0.6$, $r = 150$ m, $g = 9.8$

$\therefore v_{max} = \sqrt{0.6 \times 150 \times 9.8} \simeq 30 \text{m/s}$

34. **(b)** $F = \mu(mg)$

Centripetal force $F = mv^2/r$

$\therefore \mu mg = (mv^{2/r})$ or $= v^2/\mu g$

or $r = \frac{(12)^2}{0.4 \times 10} = 36\text{m}$

35. **(c)** $v_{max} = \sqrt{\mu rg} = \sqrt{0.75 \times 60 \times 9.8} = 21$ m/s

36. **(b)** Here, $v = 20 \text{ ms}^{-1}$, $r = 100$ m, $a_T = 3 \text{ ms}^{-2}$

Centripetal acceleration, $a_C = \frac{v^2}{r} = \frac{20 \times 20}{100} = 4\text{ms}^{-2}$

The resultant acceleration $= \sqrt{a_C^2 + a_T^2}$

$= \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5\text{ms}^{-2}$

37. **(b)** Due to centrifugal force, the inner wheel will be left up when car is taking a circular turn. Due to this, the reaction on outer wheel is more than that on inner wheel.

38. **(b)**

39. **(d)** Centripetal acceleration, is always directed towards the centre of the circle. The direction of centripetal acceleration changes continuously. Therefore, a centripetal acceleration is not a constant vector.

40. **(b)** Centripetal acceleration $a_c = v^2/r$

It acts along the radius and directed towards the centre of the circular path.

41. **(a)** $F = \frac{mv^2}{r}$. If m and v are constants then $F \propto \frac{1}{r}$

$\therefore \frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)$

42. **(b)** $F = \frac{mv^2}{r}$. For same mass and same speed if radius is doubled then force should be halved.

43. **(a)** $\frac{a_R}{a_r} = \frac{\omega_R^2 \times R}{\omega_r^2 \times r} = \frac{T_r^2}{T_R^2} \times \frac{R}{r} = \frac{R}{r}$ [As $T_r = T_R$]

44. **(c)** They have same ω
Centripetal acceleration = $\omega^2 r$

$$\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$$

45. **(d)** $m\omega^2 r = mg$

or $\omega = \sqrt{\frac{g}{r}}$

$\therefore$ $T = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{4}{9.8}} = 4\text{s}$

46. **(b)** At the bottom of the circle

$$T = mg + \frac{mv^2}{r} = 2\times 10 + \frac{2\times 4^2}{1} = 52 \text{ N.}$$

47. **(d)** In a circular motion,

$$a = \frac{v^2}{r} \Rightarrow \frac{a_2}{a_1} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{2v_1}{v_1}\right)^2 = 4$$

48. **(b)** Net acceleration in non-uniform circular motion.

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(2)^2 + \left(\frac{900}{500}\right)^2} = 2.7 \text{ m/s}^2$$

a_t = tangential acceleration

a_c = centripetal acceleration = $\frac{v^2}{r}$

49. **(b)**

50. **(c)** Radial acceleration is given by,

$$a_r = \frac{v^2}{r} = \frac{(5)^2}{10} = 2.5 \text{ ms}^{-2}$$

Net acceleration, $a = \sqrt{a_r^2 + a_t^2} = \sqrt{(2)^2 + (2.5)^2} = 3.2 \text{ ms}^{-2}$

51. **(c)** Centripetal force, $F = \frac{mv^2}{r}$

$\Rightarrow v^2 \propto r$ (For same m & F)

$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \frac{\sqrt{3}}{2}$ [$\because r_1 : r_2 = 3:4$]

52. **(d)** Given, mass of child, m = 5 kg
Radius of merry-go-round, R = 2 m

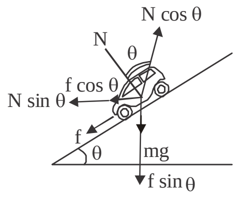
Angular velocity, $\omega = \frac{2\pi}{3.14} = 2\text{rad/s}$

The centrifugal force on the child will be
$F = M\omega^2 R = 5 \times 2^2 \times 2 = 40$ N

53. **(a)** Centripetal force $F_c = \frac{mv^2}{r}$ and $v = r \times \omega$

$\therefore F_c = m\omega^2 r = 200 \times (0.2)^2 \times 70 = 560$ N

54. **(a)**



Clearly form the figure, N sinθ and f cosθ contribute to the centripetal force.

$$\therefore \text{N} \sin\theta + \text{f} \cos\theta = \frac{mv^2}{R}$$

55. **(a)** $a_c = \frac{v^2}{r} = \frac{(250)^2}{10^3} = 62.5 \text{ m/s}^2 \Rightarrow a_c/g = \frac{62.5}{9.8} = 6.38$

56. **(b)**

57. **(d)** As body covers equal angle in equal time intervals. Its angular velocity and hence magnitude of linear velocity is constant.

58. **(c)** From equation $\omega = \omega_0 + \alpha t$

or $\omega = \alpha t \Rightarrow \alpha = \frac{\omega}{t} = \frac{10}{5} = 2 \text{ rad s}^{-2}$

Now, using displacement, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\therefore \theta = \frac{1}{2} \times 2 \times (5)^2 = 25 \text{rad}$

59. **(c)** Speed, v = constant (from question)

Centripetal acceleration, $a = \frac{V^2}{r}$

ra = constant
Hence graph (c) correctly describes relation between acceleration and radius.

60. **(a)** Acceleration = $\frac{v^2}{r}$ towards the centre.

61. **(c)** In circular motion,

$a_c = \frac{v^2}{r}$ $\therefore$ $v = \sqrt{a_c . r} = \sqrt{18 \times 50 \times 10^{-2}} = 3 \text{m/s}$

Also, $w = \frac{v}{r} = \frac{3}{50 \times 10^{-2}} = 6 \text{ rad/s}$

$\therefore$ $\theta = \omega t = 6 \times \frac{\pi}{18} = \frac{\pi}{3}$

$\therefore$ $\Delta v = 2v\sin\frac{\theta}{2} = 2\times 3\sin\frac{\pi}{6} = 2\times 3\times\frac{1}{2} = 3$ m/s

62. **(d)** Velocity of car, V = 30 m/s
Radius of road, R = 300 m.
Tangential acceleration, $a_T = 4\ m/s^2$

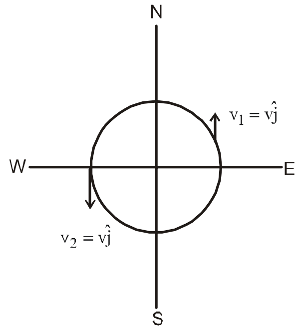
Centripetal acceleration, $a_C = \frac{V^2}{R}$

$\therefore\ a_C = \frac{30 \times 30}{300} = 3\ m/s^2$

Net acceleration is given as:

$a_T = \sqrt{a_T{}^2 + a_C{}^2} = \sqrt{(3)^2 + (4)^2} = 5 m/s^2$

63. **(c)** Speed of body, v = 20 m/s
Change in velocity in half revolution



$\Delta v = \vec{v}_2 - \vec{v}_1 = -v\hat{j} - v\hat{j} = -2v\hat{j}$

$|\Delta v| = 2v = 2 \times 20 = 40$ m/s

64. **(b)** Centripetal force

$F = \frac{mv^2}{R}$ or $v = \sqrt{\frac{FR}{m}}$ or , $v \propto \sqrt{R}$ as mass 'm' and force 'F' is constant. $\frac{R_1}{R_2} = \frac{1}{2}$

$\therefore \frac{V_1}{V_2} = \sqrt{\frac{R_1}{R_2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

65. **(b)** Radial acceleration, $a_r = \frac{V^2}{r}$

Tangential acceleration, $a_t = a$

Resultant acceleration, $a_R = \sqrt{a_r^2 + a_t^2}$

$= \sqrt{\left(\frac{V^2}{r}\right)^2 + a^2} = \sqrt{\frac{V^4}{r^2} + a^2}$

66. **(b)** Linear speed is given as
$v = \omega r \quad \because \quad v \propto r$

$\therefore \quad v \propto r\ \frac{v_A}{v_B} = \frac{r_A}{r_B}$

67. **(b)** $\frac{v_1}{v_2} = \frac{r_1\omega}{r_2\omega} = \frac{1}{2}\ [v = r\omega]$

68. **(a)** As time periods are same and so

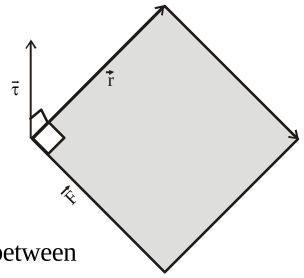
$\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2} = \frac{2}{4} = \frac{1}{2}$

69. **(a)**

70. **(b)** Torque, $\vec{\tau} = \vec{r} \times \vec{F}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 5 & 2 & -5 \end{vmatrix} = 8\hat{i} + 10\hat{j} + 12\hat{k}$

71. **(d)** We know that $\vec{\tau} = \vec{r} \times \vec{F}$



The angle between $\vec{\tau}$ and $\vec{r}$ is 90° and the angle between $\vec{\tau}$ and $\vec{F}$ is also 90°. We also know that the dot product of two vectors which have an angle of 90° between them is zero.
Therefore (d) is the correct option.

72. **(b)** Moment of inertia of disc about the axis along its diameter is $\frac{MR^2}{4}$

Radius of gyration is $I = Mk^2$

on comparing $k = \frac{R}{2}$.

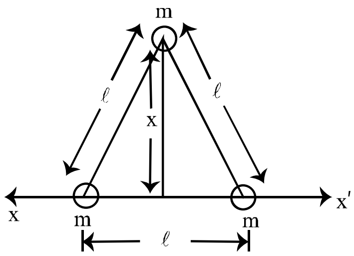
73. **(c)** For solid sphere, $I = \frac{2}{5} mr^2 = 20$

$\therefore \quad mr^2 = 20 \times \frac{5}{2} = 50$

For thin spherical shell,

$I = \frac{2}{3} mr^2 = \frac{2}{3} \times 50 = 33.3\ kg\ m^2$

74. **(c)**



The moment of inertia of the system of particle about XX′ is

$I = mx^2 = m\,(l \sin 60°)^2 = \frac{3}{4} ml^2$

75. **(c)** m = 20 kg, d = 20 cm $\therefore$ r = 10 cm

$\therefore$ The moment of inertia of a solid sphere about its tangent is

$$I=\frac{2}{5}mr^2+mr^2=\frac{7}{5}mr^2=\frac{7}{5}\times 20\times 10\times 10\times 10^{-4}$$

$= 0.28\ kg\ m^2$

76. **(a)** MOI of a solid cylinder, $I_1=\frac{1}{2}mr^2$

MOI of a hollow cylinder, $I_2 = mr^2$

$\therefore$ $I_1 < I_2$

77. **(d)** $I=Mk^2 \Rightarrow k=\sqrt{\frac{I}{M}}$

78. **(c)** Torque = rate of change of angular momentum

79. **(b)** Moment of inertia of a uniform circular disc about its diameter $I=\frac{1}{4}MR^2$

Using perpendicular axis theorem, MI of the disc about an axis perpendicular to its plane and passing through the centre, $I' = 2I$

$\therefore$ MI of the disc about an axis perpendicular to its plane and passing through a point on the rim.

$I'' = I' + MR^2 = 2I + MR^2$

$$=2\times\frac{1}{4}MR^2+MR^2=\frac{3}{2}MR^2=3\times 2\left(\frac{1}{4}MR^2\right)$$

$\therefore$ $I'' = 6I$

80. **(a)** Moment of inertia of hollow cylinder about its axis is

$$I_1=\frac{M}{2}=\left(R_1^2+R_2^2\right)$$

where, R_1 = inner radius and R_2 = outer radius.

Moment of inertia of thin hollow cylinder of radius R about its axis is

$I_2 = MR^2$

Given, $I_1 = I_2$ and both cylinders have same mass (M).

So, we have

$$\frac{M}{2}\left(R_1^2+R_2^2\right)=MR^2$$

$(10^2 + 20^2) / 2 = R^2$

$R \approx 16\ cm$

81. **(a)** Power of the engine,

$P = \tau\omega = 100 \times 100 = 10 \times 10^3\ W = 10kW$

82. **(a)** Work done $=\Delta KE=\frac{1}{2}I\left(\omega_1^2-\omega_2^2\right)$

In this case, $\omega_2 = 0$,

Work done $=\frac{1}{2}\times 10\times 50^2 = 12500\ J$

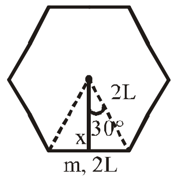
So, average power $=\frac{\text{Work done}}{\text{Time}}=\frac{12500}{10}=1250\ W$

83. **(d)** As, $I_2\,\omega_2 = I_1\,\omega_1$

$$\therefore\quad \omega_2=\frac{I_1}{I_2}\times\omega_1=\frac{I_1}{\frac{75I_1}{100}}\times 3\pi=\frac{4}{3}\times 3\pi=4\pi\ rads^{-1}$$

84. **(a)** As the acrobat bends his body, then moment of inertia I will decrease and hence ω of acrobat will increase as no external torque is acting on the acrobat.

85. **(a)** $x = 2L\cos 30° = L\sqrt{3}$



$$I=6\left(\frac{m(2L)^2}{12}+mx^2\right)=20mL^2$$

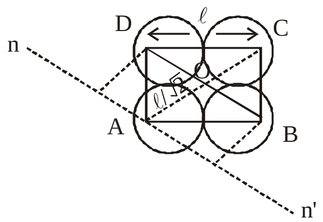
86. **(c)** About the diameter moment of inertia of the loop = moment of inertia of the ring $I=\frac{1}{2}mR^2$ where R = radius of the loop

Here, $\rho=\frac{m}{l}$ or, $\rho=\frac{m}{2\pi R}\Rightarrow R=\frac{m}{2\pi\rho}$

$$\therefore\ I=\frac{1}{2}m\left(\frac{m}{2\pi\rho}\right)^2=\frac{m^3}{8\pi^2\rho^2}$$

87. **(d)**

88. **(c)**

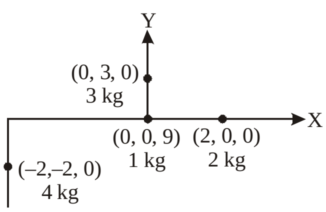


$I_{nn'}$ = M.I due to the point mass at B + M.I due to the point mass at D + M.I due to the point mass at C.

$$=2\times m\left(\frac{\ell}{\sqrt{2}}\right)^2+m(\sqrt{2}\ell)^2=m\ell^2+2m\ell^2=3m\ell^2$$

89. **(d)**

90. **(a)** Moment of inertia of the whole system about the axis of rotation will be equal to the sum of the moments of inertia of all the particles.

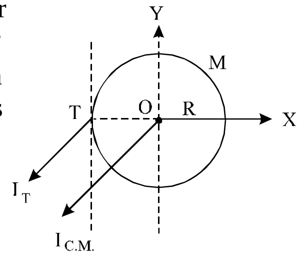


$\therefore$ $I = I_1 + I_2 + I_3 + I_4 = 0 + 0 + 27 + 16 = 43\ kg\ m^2$

91. **(d)** $K=\frac{L^2}{2I}\Rightarrow L^2=2KI\Rightarrow L=\sqrt{2KI}$

$$\frac{L_1}{L_2}=\sqrt{\frac{K_1}{K_2}\cdot\frac{I_1}{I_2}}=\sqrt{\frac{K}{K}\cdot\frac{I}{2I}}=\frac{1}{\sqrt{2}}\Rightarrow L_1:L_2=1:\sqrt{2}$$

92. **(c)** M.I. of a uniform circular disc of radius 'R' and mass 'M' about an axis passing through C.M. and normal to the disc is



$I_{C.M.} = \frac{1}{2}\ MR^2$

From parallel axis theorem

$I_T = I_{C.M.} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

93. **(d)** Moment of inertia of a hollow cylinder of mass M and radius r about its own axis is Mr^2.

94. **(b)** Because the entire mass of a ring is at its periphery *i.e.*, at maximum distance from the centre and $I = Mr^2$

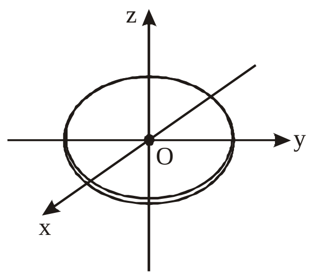
95. **(a)** According to parallel axis theorem of the moment of Inertia $I = I_{cm} + md^2$
d is maximum for point B so I_{max} about B.

96. **(b)** The distribution of mass about axis EF is minimum so radius of gyration is minimum and therefore moment of inertia is minimum about EF.

97. **(c)** In a hollow sphere, the mass is distributed away from the axis of rotation. So, its moment of inertia is greater than that of a solid sphere.

98. **(b)** By bending his body, he decreases his moment of inertia. This would increase his angular velocity.

99. **(a)** Moment of inertia of the disc about an axis perpendicular to it and through its centre is $\frac{MR^2}{2}$.



By the theorem of perpendicular axes, $I_z = I_x + I_y$
As x and y axes are along two diameters of the disc. By symmetry, the moment of inertia of the disc is the same about any diametre. Hence,

$I_x = I_y \qquad \therefore\ I_z = 2I_x$

$\because\ I_z = \frac{1}{2}MR^2 \qquad \therefore\ I_x = \frac{I_z}{2} = \frac{1}{4}MR^2$

100. **(d)** $I = MK^2 = 160 \Rightarrow K^2 = \frac{160}{M} = \frac{160}{10} = 16$

$\Rightarrow K = 4\ \text{m}$

101. **(d)** Radius of gyration of circular disc $k_{disc} = \frac{R}{\sqrt{2}}$

Radius of gyration of circular ring $k_{ring} = R$

Ratio $= \frac{k_{disc}}{k_{ring}} = \frac{1}{\sqrt{2}}$.

102. **(b)** $K = \frac{1}{2}I\omega^2$

$K' = \frac{1}{2}\left(\frac{1}{2}\right)(2\omega)^2 = 2K$

103. **(c)** $E = \frac{L^2}{2I} \Rightarrow L = \sqrt{2EI}$

104. **(a)** Kinetic energy $= \frac{1}{2}I\omega^2$ and for ring about its axis passing through its centre and perpendicular to its plane $I = mr^2$

Hence, $KE = \frac{1}{2}mr^2\omega^2$

105. **(d)** $\frac{1}{2}I\omega^2 = \frac{1}{2}mv^2$

$\frac{1}{2}\times 3\times(2)^2 = \frac{1}{2}\times 12\times v^2 \Rightarrow v = 1 m/s$

106. **(b)** Kinetic energy, $K = \frac{1}{2}I\omega^2$,

Angular momentum, $L = I\omega$

$\therefore\ K = \frac{L^2}{2I} \ \Rightarrow\ I = \frac{L^2}{2K}$

107. **(d)** Sphere possesses both translational and rotational kinetic energy.

108. **(d)** $K_R = K_T \Rightarrow \frac{1}{2}m\upsilon^2\left(\frac{K^2}{R^2}\right) = \frac{1}{2}m\upsilon^2 \quad \therefore\ \frac{K^2}{R^2} = 1$

i.e., the body is ring.

109. **(c)** $\frac{1}{2}I\omega^2 = 360 \Rightarrow I = \frac{2\times 360}{(30)^2} = \frac{2\times 360}{30\times 30} = 0.8\ \text{kg}\times m^2$

110. **(d)** Kinetic energy $E = \frac{L^2}{2I}$

If angular momenta are equal then $E \propto \frac{1}{I}$

Kinetic energy $E = K$ [Given in the problem]

If $I_A > I_B$ then $K_A < K_B$

111. **(b)** **112.** **(c)** **113.** **(a)** **114.** **(b)**

115. **(d)** Torque about the CM is caused by friction because the lever arm of the weight force is zero:

$\tau = fR = I\alpha$

$f = \mu n = \mu mg\cos\theta$

$\mu = \frac{f}{mg\cos\theta} = \frac{I\alpha/R}{mg\cos\theta}$

$= \frac{\left(\frac{2}{3}g\sin\theta\right)\left(\frac{1}{2}mR^2\right)}{R^2 mg\cos\theta} = \left(\frac{1}{2}\tan\theta\right)$

As there is no slipping between any point of contact hence distance moved by the man is 2L.

116. (c) $a = \dfrac{g \sin\theta}{1 + \dfrac{I}{MR^2}}$

a is minimum, if I is maximum.

$I_{max} = MR^2$ [for ring]

$\Rightarrow a_{min} = \dfrac{g\sin\theta}{1+\dfrac{MR^2}{MR^2}} = \dfrac{g\sin\theta}{2}$

117. (c) $I_A\omega_A = I_B\omega_B$ (Given)

$\therefore \dfrac{\omega_A}{\omega_B} = \dfrac{I_B}{I_A}$

Kinetic energy $= \dfrac{1}{2}I\omega^2$

$\therefore \dfrac{(K.E)_A}{(K.E)_B} = \dfrac{\frac{1}{2}I_A\omega_A^2}{\frac{1}{2}I_B\omega_B^2} = \dfrac{I_A}{I_B}\times\left(\dfrac{I_B}{I_A}\right)^2$ (Given (i))

$= \dfrac{I_B}{I_A}$ As $I_A < I_B$ (Given) $\therefore (K.E.)_A < (K.E.)_B$

118. (d) Rotational kinetic energy is

$K_R = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}Mk^2\left(\dfrac{v}{R}\right)^2$ $(\because I = Mk^2 \text{ and } v = R\omega)$

$= \dfrac{1}{2}Mv^2\left(\dfrac{k^2}{R^2}\right)$

Translational kinetic energy is $K_T = \dfrac{1}{2}Mv^2$

As per question, $K_R = 40\%\ K_T$

$\therefore \dfrac{1}{2}mv^2\left(\dfrac{k^2}{R^2}\right) = 40\%\dfrac{1}{2}Mv^2$ or $\dfrac{k^2}{R^2} = \dfrac{40}{100} = \dfrac{2}{5}$

For solid sphere, $\dfrac{k^2}{R^2} = \dfrac{2}{5}$

Hence, the body is solid ball.

119. (a) For hollow sphere,

$I = \dfrac{2}{3}mR^2 = mK^2 \Rightarrow \dfrac{R^2}{k^2} = \dfrac{3}{2}$

$\therefore \dfrac{(KE)_R}{(KE)_{Total}} = \dfrac{1}{\left(1+\dfrac{R^2}{k^2}\right)} = \dfrac{1}{1+\dfrac{3}{2}} = \dfrac{2}{5} = \dfrac{x}{5}$ (given)

120. (c) From the work energy theorem

Given, Mass of disc, m = 50 kg

Speed of centre of mass v = 0.4 m/s

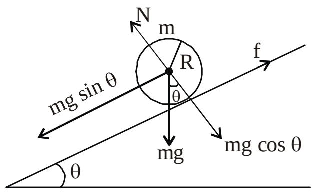
W = Change in Kinetic energy $= 0 - \left(\dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2\right)$

$\Rightarrow W = 0 - \dfrac{1}{2}mv^2\left(1+\dfrac{K^2}{R^2}\right) = \dfrac{-3}{4}mv^2$

$= -\dfrac{3}{4}\times 50\times 0.4^2 = -6J$

Absolute work, |W| = 6J

121. (b)



For rolling on inclined plane,

$a = \dfrac{g\sin\theta}{1+\dfrac{I}{MR^2}} = \dfrac{10\sin 60^\circ}{1+\dfrac{1}{2}} = \dfrac{10}{\sqrt{3}} m/s^2$

$= \dfrac{x}{\sqrt{3}} m/s^2$ (given) $\therefore$ x = 10

122. (b) For ring, $\dfrac{K^2}{R^2} = 1$

For disc, $\dfrac{K^2}{R^2} = \dfrac{1}{2}$

$\dfrac{(K.E)_{ring}}{(K.E)_{disc}} = \dfrac{\left[\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)\right]_{ring}}{\left[\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)\right]_{disc}}$

$= \dfrac{(1+1)}{\left(1+\dfrac{1}{2}\right)} = \dfrac{2\times 2}{3} = 1.33$

123. (d) The speed of cylinder at the bottom of inclined plane is

$v = \sqrt{\dfrac{2gh}{1+\dfrac{K^2}{R^2}}} = \sqrt{\dfrac{2gh}{\left(1+\dfrac{1}{2}\right)}} = \sqrt{\dfrac{4gh}{3}}$

124. (c) For solid cylinder, $\dfrac{K^2}{R^2} = \dfrac{1}{2}$

$(K.E)_{Total} = \dfrac{1}{2}mv^2\left(1+\dfrac{K^2}{R^2}\right)$

$= (K.E)_T\left(1+\dfrac{K^2}{R^2}\right) = 140\left(1+\dfrac{1}{2}\right) = 210\,J$

125. (c)

126. (a) $a = \dfrac{g\sin\theta}{1+\dfrac{K^2}{R^2}} = \dfrac{g\sin\theta}{1+\dfrac{2}{5}} = \dfrac{5}{7}g\sin\theta$

127. (d) As the inclined plane is frictionless therefore all the bodies will slide down along the inclined plane with same acceleration $g\sin\theta$.

128. (b) Time of descent $\propto \dfrac{K^2}{R^2}$

Order of value of $\dfrac{K^2}{R^2}$, $\underset{(0.4)}{\text{Sphere}} < \underset{(0.5)}{\text{Disc}} < \underset{(1.0)}{\text{Ring}}$

It means sphere will reach the ground first and at last ring will reach the bottom.

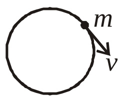
Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. (c) Centripetal acceleration, $a_c = \omega^2 R$

$= \dfrac{(2\pi)^2 \times 1.5\times 10^{11}}{(365\times 86400)^2} \approx 6\times 10^{-3}\ \text{ms}^{-2}$

$\dfrac{h}{b} = \dfrac{v^2}{Rg} \Rightarrow h = \dfrac{v^2 b}{Rg}$

2. (a) $\dfrac{v^2}{r} = a$, the centripetal acceleration [Given].



If v is doubled, $a' = \dfrac{4v^2}{r} = 4a$

3. (c) Minimum angular velocity $\omega_{\min} = \sqrt{g/R}$

$\therefore\ T_{\min} = \dfrac{2\pi}{\omega_{\min}} = 2\pi\sqrt{\dfrac{R}{g}} = 2\pi\sqrt{\dfrac{2}{10}} \cong 3s$

4. (b) $v = \sqrt{gr} = \sqrt{10\times 40} = 20\,\text{ms}^{-1}$

5. (a) By doing so component of weight of vehicle provides centripetal force.

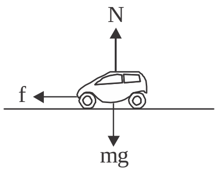
6. (a) The value of frictional force should be equal or more than required centripetal force. i.e. $\mu mg \geq \dfrac{mv^2}{r}$

7. (c) $F = m\omega^2 R$ $\quad\therefore\quad F \propto \omega^2$ (m and R are constant)
If angular velocity is doubled force will becomes four times.

8. (c) $F = m\omega^2 R$ $\quad\therefore\quad F \propto R$ (m and ω are constant)

If radius of the path is halved, then force will also become half.

9. (c) Normal reaction N = weight mg thus the centripetal force required by the car for circular motion is provided by the component of the force of friction b/w the road and the car tyres.



10. (c) Angle traced in first one second

$\theta_1 = \omega_0 + \dfrac{\alpha}{2}(2\times 1-1) = \dfrac{\alpha}{2}$ (Since $\omega_0 = 0$)

and in the next one second $\theta_2 = \dfrac{\alpha}{2}(2\times 2-1) = \dfrac{3\alpha}{2}$

$\therefore\ \dfrac{\theta_1}{\theta_2} = 3$

11. (c) For the block P,

$N = m\,\omega_A^2 R_A = m\,\omega_B^2 R_B = m\omega_C^2 R_C$

$\Rightarrow R_A : R_B : R_C = \dfrac{1}{\omega_A^2} : \dfrac{1}{\omega_B^2} : \dfrac{1}{\omega_C^2}$

As, $R_A < R_B < R_C \Rightarrow \dfrac{1}{\omega_A^2} < \dfrac{1}{\omega_B^2} < \dfrac{1}{\omega_C^2}$

$\therefore\ \omega_A > \omega_B > \omega_C$

12. (d) From right hand thumb rule, the direction of angular velocity is $-\hat{k}$.

13. (a) The velocity should be such that the centripetal acceleration is equal to the acceleration due to gravity

$\dfrac{v^2}{R} = g$ or $v = \sqrt{gR}$

14. (d) As speed of car on circular track increases, so besides the centripetal acceleration (a_R), a tangential acceleration (a_t) also works at right angles to a_R.
Centripetal acceleration,

$a_c = \dfrac{v^2}{r} = \dfrac{(30)^2}{500} = 1.8\,\text{ms}^{-2}$

Tangential acceleration, $a_t = 2\,\text{ms}^{-2}$

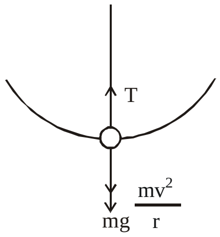
$\therefore$ Resultant acceleration $a = \sqrt{a_t^2 + a_c^2}$

$= \sqrt{(1.8)^2 + (2)^2} = 2.7\ \text{ms}^{-2}$

15. (b) Circumference $= 2\pi r = 2\pi \times \dfrac{20}{\pi} = 40\text{m}$

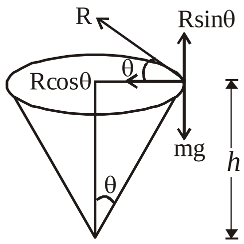
Distance travelled in 2 revolutions = 2 × 40 = 80 m
Initial velocity, u = 0
Final velocity v = 80m/sec
Applying the formula, $v^2 = u^2 + 2as$
$(80)^2 = 0^2 + 2\times a\times 80 \Rightarrow a = 40\ \text{m/sec}^2$

16. (a)



At the lowest point, as shown in the figure both mg and centrifugal force $\frac{mv^2}{r}$ will act in the same direction So, $T = mg + \frac{mv^2}{r}$

17. (d) The particle is moving in circular path



From the figure, $mg = R\sin\theta$ ...(i)

$\frac{mv^2}{r} = R\cos\theta$ ...(ii)

From equations (i) and (ii) we get

$\tan\theta = \frac{rg}{v^2}$ but $\tan\theta = \frac{r}{h}$

$\therefore h = \frac{v^2}{g} = \frac{(0.5)^2}{10} = 0.025\text{m} = 2.5\text{cm}$

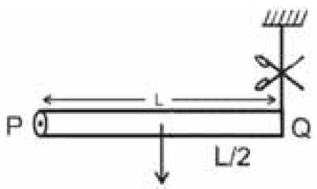
18. (a) Since water does not fall down, therefore the velocity of revolution should be just sufficient to provide centripetal acceleration at the top of vertical circle. So,

$v = \sqrt{(g\,r)} = \sqrt{\{10 \times (1.6)\}} = \sqrt{(16)} = 4\,\text{m/sec.}$

19. (a) We know that $\tan\theta = \frac{v^2}{Rg}$ and $\tan\theta = \frac{h}{b}$

Hence $\frac{h}{b} = \frac{v^2}{Rg} \Rightarrow h = \frac{v^2 b}{Rg}$

20. (d)



Weight of the rod will produce the torque

$\tau = mg\frac{L}{2} = I\alpha = \frac{mL^2}{3}\alpha \quad \left[\because I_{rod} = \frac{ML^2}{3}\right]$

Hence, angular acceleration $\alpha = \frac{3g}{2L}$

21. (a) Torque about point (1, 0, 3) is given by :

$\tau = r \times F = \left(\hat{i} + 3\hat{k}\right) \times \left(2\hat{i} + 3\hat{j}\right) = 3\hat{k} + 6\left(+\hat{j}\right) + 9\left(-\hat{i}\right)$

The vector in XY plane is $r = \hat{i} + 3\hat{j}$

The components of torque along $r = \frac{\tau.r}{|r|}$

$= \frac{\left(-9\hat{i} + 6\hat{j} + 3\hat{k}\right).\left(\hat{i} + 3\hat{j}\right)}{\sqrt{1^2 + 3^2}} = \frac{9}{\sqrt{10}}\,N - m$

22. (a) $\vec{L} = m\left(\vec{r} \times \vec{v}\right) = \frac{100}{1000}\left[\frac{9\hat{i} + 5\hat{k}}{100}\right] \times \frac{5\hat{i}}{100}$

$= 25 \times 10^{-5}\left(-\hat{k}\right)\text{kgm}^2\,\text{s}^{-1}$

23. (a) Angular momentum $L = \vec{r} \times \vec{p}$

$\because$ $\vec{r}$ and $\vec{p}$ are constant

$\therefore$ L is constant.

24. (c) According to problem disc is melted and recasted into a solid sphere so their volume will be same.

$V_{Disc} = V_{Sphere} \Rightarrow \pi R_{Disc}^2 t = \frac{4}{3}\pi R_{Sphere}^3$

$\Rightarrow \pi R_{Disc}^2\left(\frac{R_{Disc}}{6}\right) = \frac{4}{3}\pi R_{Sphere}^3 \quad \left[t = \frac{R_{Disc}}{6},\ \text{given}\right]$

$\Rightarrow R_{Disc}^3 = 8R_{Sphere}^3 \Rightarrow R_{Sphere} = \frac{R_{Disc}}{2}$

Moment of inertia of disc

$I_{Disc} = \frac{1}{2}MR_{Disc}^2 = I$ (given) $\therefore$ $M(R_{Disc})^2 = 2I$

Moment of inertia of sphere $I_{Sphere} = \frac{2}{5}MR_{Sphere}^2$

$= \frac{2}{5}M\left(\frac{R_{Disc}}{2}\right)^2 = \frac{M}{10}(R_{Disc})^2 = \frac{2I}{10} = \frac{I}{5}$

25. (b) Total moment of inertia is given by :

$I = M_1 r_1^2 + M_2 r_2^2$

$I = \frac{M}{2}\left(\frac{l}{2}\right)^2 + \frac{M}{2}\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{8} + \frac{Ml^2}{8} = \frac{Ml^2}{4}$

26. (d) Net moment of inertia is given by

$I = I_1 + I_2 = \frac{Ml^2}{12} + \frac{Ml^2}{12} = \frac{Ml^2}{6}$

27. **(b)** Moment of inertia of rod is $I_{rod} = \frac{ML^2}{12}$

Radius of gyration is $I = Mk^2$

On comparing radius of gyration is $k = \frac{L}{\sqrt{12}}$

28. **(d)** $I_1\omega_1 = I_2\omega_2$

$$\omega_2 = \frac{I\omega \times 100}{60I} = \frac{5\omega}{3}$$

29. **(c)** Let p be the density of the discs and t is the thickness of discs.

Moment of inertia of disc is given by

$$I = \frac{MR^2}{2} = \frac{[\rho(\pi R^2)t]R^2}{2}$$

$I \propto R^4$ (As ρ and t are same)

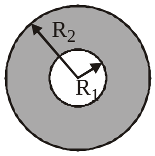
$$\frac{I_2}{I_1} = \left(\frac{R_2}{R_1}\right)^4 \Rightarrow \frac{16}{1} = \alpha^4 \Rightarrow \alpha = 2$$

30. **(a)**

31. **(b)** Use theorem of parallel axes.

32. **(a)** $I = \frac{\frac{m}{2}\left(\frac{\ell}{2}\right)^2}{3} + \frac{m}{2}\left(\frac{\ell}{2}\right)^2 = \frac{m\ell^2}{6}$

33. **(a)** $\rho = \frac{M}{\frac{4}{3}\pi(R_2^3 - R_1^3)}$



$$I_{shell} = \frac{2}{5}M_2R_2^2 - \frac{2}{5}M_1R_1^2 \quad .... (1)$$

$$M_2 = \rho \times \frac{4}{3}\pi R_2^3 = \frac{MR_2^3}{(R_2^3 - R_1^3)}; \; M_1 = \frac{MR_1^3}{R_2^3 - R_1^3}$$

Putting values of M_1 and M_2 in eq. (1),

$$I_{shell} = \frac{2M}{5}\frac{(R_2^5 - R_1^5)}{(R_2^3 - R_1^3)}$$

34. **(b)** $I = 2 \times 5 \times (0.2)^2 + 2 \times 2 \times (0.4)^2 = 1 kg \times m^2$

35. **(a)** Moment of inertia for ring, $I = mR_1^2 = mK_1^2$

$\therefore$ Radius of gyration $K_1 = R_1$

Moment of inertia for solid sphere

$I' = \frac{2}{5}m'R_2^2 \quad \therefore$ Radius of gyration $K_2 = \sqrt{\frac{I'}{m'}} = \sqrt{\frac{2}{5}}R_2$

$\because K_1 = K_2$ (given) $\qquad \therefore R_1 = \sqrt{\frac{2}{5}}R_2$

$\therefore \frac{R_1}{R_2} = \sqrt{\frac{2}{5}} \qquad \therefore x = 5$

36. **(d)** Moment of Inertia, $I = I_1 + I_2$

$$= M_1r_1^2 + M_2r_2^2 = 3\left(\frac{2}{10}\right)^2 + 5 \times \left(\frac{7}{10}\right)^2$$

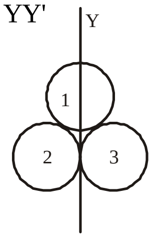
$$= \frac{12}{100} + \frac{5 \times 49}{100} = 0.12 + 2.45 = 2.57 \text{ kg m}^2.$$

37. **(d)** Moment of inertia of system about YY'

$I = I_1 + I_2 + I_3$

$= \frac{1}{2}MR^2 + \frac{3}{2}MR^2 + \frac{3}{2}MR^2$

$= \frac{7}{2}MR^2$



38. **(a)** When angular acceleration (α) is zero then torque on the wheel becomes zero.

$\theta(t) = 2t^3 - 6t^2$

$\Rightarrow \frac{d\theta}{dt} = 6t^2 - 12t$

$\Rightarrow \alpha = \frac{d^2\theta}{dt^2} = 12t - 12 = 0$

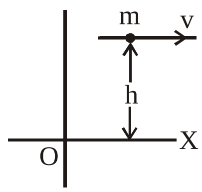
$\therefore t = 1$ sec.

39. **(d)** Torque due to gravitational force $= mg \times \frac{l}{2}$

Angular acceleration $= \frac{\text{Torque}}{\text{Moment of Inertia}}$

$$= \frac{mg\, l/2}{ml^2/3} = \frac{3}{2}\frac{g}{l}$$

40. **(b)** Angular momentum of body moving parallel to x-axis is given,



$L = h \times (mv)$; as 'h' is constant so, angular momentum remains constant.

41. **(c)** $E_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}\frac{L_1^2}{I_1}$ $(\because L_1 = I_1\omega_1)$

Similarly $E_2 = \frac{1}{2}\frac{L_2^2}{I_2}$

So $\frac{E_1}{E_2} = \frac{L_1^2}{I_1} \times \frac{I_2}{L_2^2} = \frac{I_2}{I_1}$ $(\because L_1 = L_2)$

$\Rightarrow E_1 < E_2 \because I_1 > I_2$

42. **(c)** The disc may be assumed as combination of two semi circular parts.

Let I be the moment of inertia of the uniform semicircular disc

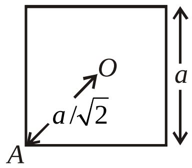
$\Rightarrow 2I = \frac{2Mr^2}{2} \Rightarrow I = \frac{Mr^2}{2}$

43. (c) M.I. of the plate about an axis perpendicular to its plane and passing through its centre

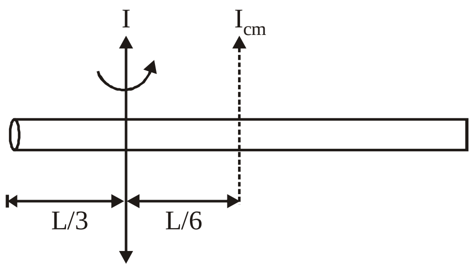
$I_0 = \frac{Ma^2}{6}$

By parallel axes theorem

$I_A = I_0 + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{2}{3}ma^2$



44. (b) $I_{cm} = \frac{ML^2}{12}$ (about middle point)



$\therefore\ I = I_{cm} = Mx^2 = \frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 = \frac{ML^2}{9}$

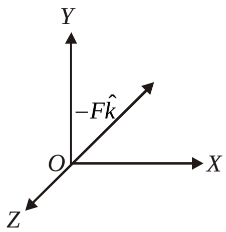
45. (b) $\vec{F} = -F\hat{k}$

$\vec{r} = (\hat{i} - \hat{j})$

$\vec{r} = \vec{r} \times \vec{F} = (\hat{i} - \hat{j})(-F\hat{k})$

$= -F(\hat{i} \times \hat{k}) + F(\hat{j} \times \hat{k})$

$\Rightarrow -F(-\hat{j}) + F(\hat{i}) \Rightarrow F\hat{j} + F\hat{i} = F(\hat{i} + \hat{j})$



46. (a) Given, $r = 0.4$ m, $a = 8$ rad s^{-2}

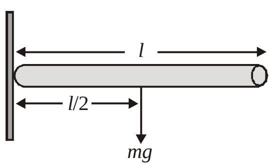
$m = 4$ kg, $I = ?$

Torque, $\tau = Ia = mgr \Rightarrow 4 \times 10 \times 0.4 = I \times 8$

$\Rightarrow\ I = \frac{16}{8} = 2\,kg - m^2$

47. (d) Weight of the rod will produce the torque

$\tau = Ia \Rightarrow mg \times \frac{1}{2} = \frac{ml^2}{3} \times a$



Angular acceleration

$a = \frac{3g}{2l}$

48. (b) Angular momentum, $L = I\omega$

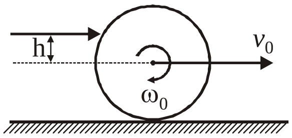
Rotational kinetic energy, $K = \frac{1}{2}I\omega^2$

$\frac{L}{K} = \frac{I\omega}{\frac{1}{2}I\omega^2} = \frac{2}{\omega}$

$\therefore L = \frac{2K}{\omega}$

$\therefore\ \frac{L'}{L} = \frac{K'}{K} \times \frac{\omega}{\omega'} = \left(\frac{K}{2K}\right)\left(\frac{\omega}{2\omega}\right)\ \ L' = \frac{L}{4}$

49. (d) When the ball is hit by a cue, the linear impulse imparted to the ball = change in momentum = mv_0



Angular momentum = Moment of momentum

$I\omega_0 = (mv_0)h$

$\frac{2}{5}mr^2\omega_0 = mv_0h$ or $\omega_0 = \frac{5v_0h}{2r^2}$

50. (a) Here, $L = 1.8$ kg m^2 s^{-1}, $M = 1.5$ kg, $\omega = 0.3$ rad s^{-1}

Angular momentum, $L = I\omega$

$L = k^2 M\omega$ $(\because I = MK^2)$

or $1.8 = k^2 \times 1.5 \times (0.3)$

$\Rightarrow k^2 = \frac{1.8}{1.5 \times 0.3} = 4$

$\Rightarrow k = 2$ m.

51. (a) Angular momentum $L = m(v \times r)$

$= 2\,kg\left(\frac{dr}{dt} \times r\right) = 2\,kg(4t\,j \times 5i - 2t^2\hat{j})$

$= 2\,kg\,(-20\,t\,\hat{k}) = 2\,kg \times -20 \times 2\,m^{-2}\,s^{-1}\,\hat{k} = -80\,\hat{k}$

52. (a) According to parallel axis theorem of the moment of Inertia

$I = I_{cm} + md^2$

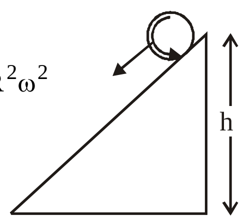
d is maximum for point B so I_{max} about B.

53. (c) $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

or, $mgh = \frac{1}{2}m\omega^2R^2 + \frac{1}{2}mR^2\omega^2$

or, $mgh = m\omega^2R^2$

$\therefore\ \omega = \frac{\sqrt{gh}}{R}$

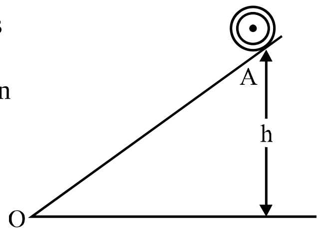


54. **(a)** In pure rolling work done by friction, $\omega_{friction} = 0$. Therefore, PE = kE. Initially, $PE_{ring} = PE_{sphere}$ so finally, $KE_{ring} = KE_{sphere}$

$\therefore$ Ratio of kinetic energy $= \dfrac{KE_{ring}}{KE_{sphere}} = 1$

$\therefore \dfrac{7}{x} = 1 \quad \therefore \quad x = 7$

55. **(c)** The given situation is shown in figure.
According to conservation of mechanical energy.
$M{\cdot}E_i = M{\cdot}E_f$
$\Rightarrow U_i + (K_r)_{initial} + K_i$
$= U_f + (K_r)_f + K_f$



$$\Rightarrow mgh + 0 + 0 = 0 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

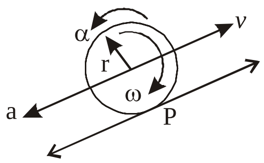
$$\Rightarrow mgh + \frac{1}{2}\pm\omega^2 + \frac{1}{2}mv^2$$

$$\therefore mgh = \frac{1}{2}\times\frac{1}{2}mR^2\times\frac{v^2}{R^2} + \frac{1}{2}mv^2 \left[\text{For disc } I = \frac{1}{2}mR^2\right]$$

$$\Rightarrow v = \sqrt{\frac{4}{3}gh} \quad \therefore v \propto \sqrt{h}$$

$\therefore$ So speed depends on height of incline plane.

56. **(a)** Since the body rolls without sliding,
So $a_p = 0, u_p = 0$ and $v = \omega r, \alpha = \alpha r$
Since the body's acceleration a is downward parallel to the plane, therefore angular acceleration of the body must be in anticlockwise sense.



That means the friction must act up the plane to produce an anticlockwise torque to produce anticlockwise angular acceleration.

57. **(b)** Here, R = 2m, M = 100 kg, $v = 200$ cm s^{-1}
$= 20 \times 10^{-2}$ ms^{-1}
Total kinetic of the loop $= K_T + K_R$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \qquad [\because \text{ for a hoop, } I = MR^2]$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}MR^2\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 \qquad [\because v = R\omega]$$

$= Mv^2$
Work required to stop the hoop = Total kinetic energy of the hoop
$Mv^2 = (100\text{ kg})(20 \times 10^{-2}\text{ ms}^{-1}) = 4J$

58. **(b)** Acceleration of a rolling body down an inclined plane is

$$a = \frac{g\sin\theta}{1 + \dfrac{k^2}{R^2}}$$

For solid cylinder, $k^2 = \dfrac{R^2}{2}$

$$\therefore a = \frac{g\sin\theta}{1 + \dfrac{1}{2}} = \frac{2}{3}g\sin\theta$$

59. **(d)** $E = E_t + E_r = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\left(\frac{2}{5}mr^2\right)\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

$$\therefore \frac{E_r}{E} = \frac{\frac{1}{2}mv^2}{\frac{7}{10}mv^2} = \frac{5}{7}$$

60. **(d)** Translational K.E. = Rotational K.E.

$\dfrac{1}{2}mv^2 = \dfrac{1}{2}I\omega^2$ As, $v = R\omega$

$\Rightarrow \dfrac{1}{2}mR^2\omega^2 = \dfrac{1}{2}I\omega^2$

$\Rightarrow I = mR^2$

which is the moment of inertia of hollow cylinder about its axis.

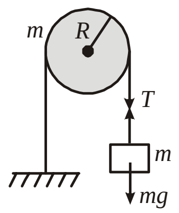
61. **(b)** $K_T = \dfrac{1}{2}mv^2\left(1 + \dfrac{K^2}{R^2}\right) \left[\because \dfrac{K^2}{R^2} = \dfrac{1}{2} \text{ for disc}\right]$

$$= \frac{1}{2}\times 0.41 \times (2)^2 \times \left(\frac{3}{2}\right) = 1.23J$$

62. **(b)** For translational motion,
$mg - T = ma$ .....(1)
For rotational motion,

$T.R = I\alpha = I\dfrac{\alpha}{R}$ ....(2)

Solving (1) & (2),



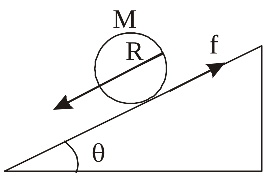
$$a = \frac{mg}{\left(m + \dfrac{I}{R^2}\right)} = \frac{mg}{m + \dfrac{mR^2}{2R^2}} = \frac{2mg}{3m} = \frac{2g}{3}$$

63. **(d)** From conservation of mechanical energy

$$\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v^2}{R^2}\right) = mg\left(\frac{3v^2}{4g}\right)$$

after solving $I = \dfrac{mR^2}{2}$ Which is for disc.

64. **(d)** Net work done by frictional force when drum rolls down without slipping is zero.



$W_{net} = 0$, $W_{trans.} + W_{rot.} = 0$
$\Delta K_{trans.} + \Delta K_{rot.} = 0$
$\Delta K_{trans} = -\Delta K_{rot}$.
i.e., converts translation energy to rotational energy.

65. (b) When a solid sphere is rotated in free space, there will be no external torque.
i.e., $\tau = 0$

But $\tau = \frac{dL}{dt}$ where L is the angular momentum.

Hence $\tau = \frac{dL}{dt} = 0 \Rightarrow L = \text{constant}$.

66. (d) Rotational energy $= \frac{1}{2}I(\omega)^2 = \frac{1}{2}(mK^2)\omega^2$

Linear kinetic energy $= \frac{1}{2}m\omega^2R^2$

$\therefore$ Required fraction

$$= \frac{\frac{1}{2}(mK^2)\omega^2}{\frac{1}{2}(mK^2)\omega^2 + \frac{1}{2}m\omega^2R^2} = \frac{K^2}{K^2+R^2}$$

67. (c) $E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\times\frac{2}{5}MR^2\times\frac{v^2}{R^2} = \frac{7}{10}Mv^2 = 0.7\text{ J}$$

68. (c) When hollow cylinder sides with out rolling, it posses only translational kinetic energy, $K_T = \frac{1}{2}m\upsilon^2$

When it rolls without slipping, it possess both types of kinetic energy, $K_N = \frac{1}{2}m\upsilon^2\left(1+\frac{K^2}{R^2}\right)$

$$\therefore \frac{K_T}{K_N} = \frac{1}{\left(1+\frac{K^2}{R^2}\right)} = \frac{1}{2}$$

$\left[\text{For hollow cylinder } \frac{K^2}{R^2} = 1\right]$

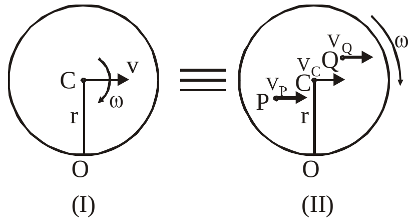
69. (a) Time of descent $t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{K^2}{R^2}\right)}$

For solid sphere $\frac{K^2}{R^2} = \frac{2}{5}$

For hollow sphere $\frac{K^2}{R^2} = \frac{2}{3}$

As $\left(\frac{K^2}{R^2}\right)_{Hollow} > \left(\frac{K^2}{R^2}\right)_{solid}$

70. (a)



From Fig. (I), we have OC = r (radius)
Therefore, $v = r\omega$
Since, ω = constant, therefore $v \propto r$
Now, form Fig (II), it is clear that the distance, OP < OC < OQ $\Rightarrow V_P < V_C < V_Q$ or $V_Q > V_C > V_P$.

71. (b) $mgh = \frac{1}{2}mv^2 + \frac{1}{2}lw^2$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{MR^2}{2}\right)\left(\frac{V}{R}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

Rotation K.E. $= \frac{1}{4}mv^2 = \frac{1}{4}\times\frac{4}{3}mgh$

$$= \frac{mgh}{3} = 2\times9.8\times(3/3) = 19.6\text{J}$$

72. (d) K.E. of rotation $= \frac{1}{2}I\omega^2$

$$= \frac{1}{2}\times\left(\frac{1}{2}mr^2\right)\omega^2$$

$$= \frac{1}{4}\times20\times(0.25)^2\times100\times100 = 3125\text{ J}$$

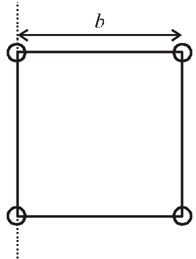
Exercise 3 : PREVIOUS YEARS MCQs

1. (c) Child is at the internal part of the system, so velocity of centre of mass will not change due to his movement.
From law of conservation of angular momentum
$I_1\omega_1 + I_2\omega_2 = (I_1+I_2)\omega$
Angular velocity of system $\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1+I_2}$

$\therefore$ Rotaional kinetic energy $= \frac{1}{2}(I_1+I_2)\omega^2$

$$= \frac{1}{2}(I_1+I_2)\left(\frac{I_1\omega_1+I_2\omega_2}{I_1+I_2}\right)^2 = \frac{(I_1\omega_1+I_2\omega_2)^2}{2(I_1+I_2)}$$

2. (c)



Moment of inertia of a sphere $= \frac{2}{5}ma^2$

Using the parallel axis theorem of moment of inertia, we have

$$I = 2\times\frac{2}{5}ma^2 + 2\left(\frac{2}{5}ma^2 + mb^2\right)$$

$$\Rightarrow \quad I = \frac{2}{5}ma^2 \times 4 + 2\times mb^2 = \frac{8}{5}ma^2 + 2mb^2$$

3. **(a)** Since water does not fall down, therefore the velocity of revolution should be just sufficient to provide centripetal acceleration at the top of vertical circle. So,

$$v = \sqrt{(g\,r)} = \sqrt{\{10\times(1.6)\}} = \sqrt{(16)} = 4 \text{ m/sec.}$$

4. **(b)** For negotiating a circular curve on a levelled road, the maximum velocity of the car is

$v_{max} = \sqrt{\mu rg}$

Here $\mu = 0.6, r = 150$ m, $g = 9.8$

$\therefore\ v_{max} = \sqrt{0.6\times150\times9.8} \simeq 30\text{m/s}$

5. **(b)** We have $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

Here, $\omega_0 = 0, \theta = 5rad$ and $t = 1$ sec

$$\Rightarrow 5 = \frac{1}{2}\alpha\times1^2 \ \Rightarrow \alpha = 10 \text{ rad/s}^2$$

Now, $\theta_n = \frac{\alpha}{2}(2n-1)+\omega_0$

$$\theta_2 = \frac{10}{2}(2\times2-1)+0 = 15 \text{ rad/sec.}$$

6. **(c)** The tension T_1 at the topmost point is given by

$$T_1 = \frac{m v_1^2}{20} - mg$$

Centrifugal force acting outward while weight acting downward.

The tension T_2 at the lowest point

$$T_2 = \frac{m v_2^2}{20} + m g$$

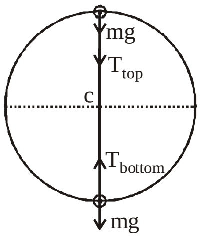
Centrifugal force and weight (both) acting downward

$$T_2 - T_1 = \frac{m v_2^2 - m v_1^2}{20} + 2 m g$$

$$v_1^2 = v_2^2 - 2gh \text{ or } v_2^2 - v_1^2 = 2g(40) = 80g$$

$$\therefore \quad T_2 - T_1 = \frac{80 m g}{20} + 2 m g = 6 m g$$

7. **(b)** $T_{top} = \frac{mv^2}{r} - mg \qquad$ ...(i)



$T_{bottom} = \frac{mv^2}{r} + mg \qquad$ ...(ii)

Solving (i) and (ii) we get :

$$\frac{T_{top}}{T_{bottom}} = \frac{v^2 - rg}{v^2 + rg} = \frac{79}{81}$$

8. **(c)** Tangential acceleration $= \alpha r$

Radial acceleration $= \frac{v^2}{r}$

$\therefore$ Ratio $= \frac{\alpha r}{v^2/r} = \frac{\alpha r^2}{v^2}$

9. **(b)** Speed at top most point, $v = \sqrt{3rg}$

Centripetal acceleration, $a_c = \frac{v^2}{T} = \frac{3rg}{r} = 3g$

10. **(a)** The total mechanical energy remains conserved, kinetic energy changes into potential energy and vice-versa. At the highest point potential energy is maximum and at the lowest point its velocity and hence kinetic energy is maximum.

11. **(d)** We know in vertical circle velocity of a particle at lowest point, $v_l = \sqrt{5gr}$

velocity of particle at highest point, $v_h = \sqrt{rg}$

So, KE at highest point of the vertical circle

$$K_h = \frac{1}{2}m\,rg = \frac{mgr}{2}$$

and KE at lowest point of the vertical circle

$$(K_l) = \frac{1}{2}m\,5gr = \frac{5mgr}{2}$$

So, required ratio $\frac{K_h}{K_l} = \frac{\frac{mgr}{2}}{\frac{5mgr}{2}} = \frac{1}{5} = 0.2$

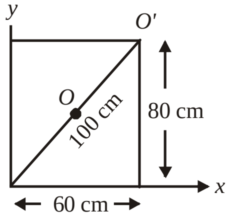
12. **(c)** Using law of conservation of angular momentum,

$I_1\omega = I_2\omega_2$

$$\frac{MR^2}{2}\omega = \frac{3}{2}\left(\frac{MR^2}{2}\right)\omega_2 \Rightarrow \omega_2 = \frac{2}{3}\omega$$

13. **(b)** Moment of inertia of rectangular sheet about an axis passing through O,

$$I_O = \frac{M}{12}(a^2+b^2) = \frac{M}{12}[(80)^2+(60)^2]$$



From the parallel axis theorem, moment of inertia about O',

$$I_{O'} = I_O + M(50)^2$$

$$\frac{I_O}{I_{O'}} = \frac{\frac{M}{12}(80^2+60^2)}{\frac{M}{12}(80^2+60^2)+M(50)^2} = \frac{1}{4}$$

14. (d) Ratio of M.I is

$$\frac{M_A r^2}{M_B(2r)^2} = \frac{I_A}{I_B} = \frac{1}{4} \quad [\because M_A = M_B]$$

or, $I_A = \frac{I_B}{4}$

15. (b) $\frac{\text{Rotational KE}}{\text{Total KE}} = \frac{\frac{1}{2}mv^2\left(\frac{K^2}{R^2}\right)}{\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)} = \frac{K^2}{K^2+R^2}$.

16. (d)

17. (d) $I = \frac{2}{5}mr^2$

$\Rightarrow \quad I \propto r^2 \quad \Rightarrow \quad I_1 : I_2$

$= r_1^2 : (2r_1)^2 = 1:4 \qquad [\text{as } r_2 = 2r_1]$

18. (a) $I = \frac{3MR^2}{2} = \frac{3}{8\pi^2}.Q.L^3$

19. (b) Torque, $\tau = I\alpha$

$$F \times R = \frac{MR^2}{2}\times\frac{\omega}{t}$$

$\therefore$ Tangential force, $F = \frac{MR\omega}{2t}$

20. (b) Conservation of angular momentum,

$I_1\omega_1 = I_2\omega_2$

or, $I\omega = 2I\omega_1 \Rightarrow \omega_1 = \frac{\omega}{2}$

Original $KE = 2\frac{1}{2}I\omega^2$

New $KE = 2\frac{1}{2}I\omega_1^2 = \frac{1}{2}2I\left(\frac{\omega}{2}\right)^2 = \frac{I\omega^2}{4}$

Change in $KE = \frac{1}{2}I\omega^2 - \frac{I\omega^2}{4} = \frac{I\omega^2}{4}$

21. (c) According to question, $I = 2\text{kg m}^2$

$\omega_0 = 60\text{ rad/s}, \ \omega = 0$

$t = 5\text{ min} = 5\times 60 = 300\text{ s}$

using, $\omega = \omega_0 + \alpha t \Rightarrow \alpha = \frac{\omega-\omega_0}{t}$

$= \frac{0-60}{300} = \frac{-60}{300} = \frac{-1}{5}\text{ rad/s}^2$

For $t = 2$ min

$\omega = \omega_0 + \alpha t$

$= 60 - \frac{1}{5}\times 120 = 60 - 24 \Rightarrow \omega = 36\text{ rad/s}$

Angular momentum,

$L = 1\omega = 2\times 36 = 72\text{ kg m}^2/\text{s}$

22. (d) From conservation of angular momentum, as net torque on the system is zero

$I_1\omega_1 = (I_1+I_2)\omega_2 \Rightarrow \frac{\omega_2}{\omega_1} = \frac{I_1}{I_1+I_2}$

Energy lost $\Delta E = E_1 - E_2$

$= \frac{1}{2}I_1\omega_1^2 - \frac{1}{2}(I_1+I_2)\omega_2^2$

$= \frac{1}{2}\omega_1^2\left[I_1 - (I_1+L_2)\frac{\omega_2^2}{\omega_1^2}\right]$

$= \frac{1}{2}\omega_1^2\left[I_1 - (I_1+I_2)\frac{I_1^2}{(I_1+I_2)^2}\right] \quad \left[\because \frac{\omega_2}{\omega_1} = \frac{I_1}{I_1+I_2}\right]$

$= \frac{1}{2}\omega_1^2\left[\frac{I_1^2+I_1I_2-I_1^2}{I_1+I_2}\right]$

or, $\Delta E = \frac{1}{2}\left[\frac{I_1I_2}{I_1+I_2}\right]\omega_1^2$

23. (a) The radius of gyration $K = \sqrt{\frac{I}{M}}$

Moment of inertia of a thin rod about an axis perpendicular to its length and passing through one end

$I = \frac{ML^2}{3} \qquad \therefore K = \sqrt{\frac{ML^2}{3M}} = \frac{L}{\sqrt{3}}$

$\therefore K : L = \frac{L}{\sqrt{3}}/L = 1:\sqrt{3}$

24. (a)

25. (d) If the polar ice melts,a part of water will shift from poles towards the equatorial region and this water will shift away from the axis of rotation of earth. Hence, the moment of inertion will increase.

From the conservation of angular momentum,

L = Iω = constant. As I increases, ω will decreases. The duration of day increases.

26. **(b)** Given mass m = 2 kg, radius of sphere R = 0.5

$$\frac{K_{rot}}{K_{terms}} = ?\text{ or, } K_{rot} : K_{trans} = ?$$

$$\frac{K_{rot}}{K_{trans}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \frac{\frac{2}{5}mR^2\omega^2}{mR^2\omega^2} = \frac{2}{5} \quad \therefore K_{rot} : K_{trans} = 2:5$$

27. **(a)** Since linear density is ρ, and total mass is m, we have $m = \rho(2\pi R)$

$$R = \frac{m}{2\pi\rho}$$

Moment of inertia about tangent to the ring and parallel to diameter. This is offset from the diameter by a distance R, so use the parallel axis theorem.

$$I = U_{diameter} + MR^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$$

Substitute $R = \frac{m}{2\pi\rho}$

$$I = \frac{3}{2}m\frac{m^2}{4\pi^2\rho^2} \Rightarrow I = \frac{3m^3}{8\pi^2\rho^2}$$

28. **(a)** Given,

$$\frac{K_{ring}}{K_{disc}} = \frac{\sqrt{12}}{\sqrt{K}}$$

$$I_{ring} = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

$$K_{ring} = R\sqrt{3/2}$$

$$I_{disc} = \frac{1}{2}MR^2 + MR^2 = \frac{5}{R}MR^2 \Rightarrow K_{disc} = R\sqrt{5/4}$$

$$\frac{K_{ring}}{K_{disc}} = \frac{R\sqrt{\frac{3}{2}}}{R\sqrt{\frac{5}{4}}} = \sqrt{\frac{6}{5}} \Rightarrow \frac{\sqrt{12}}{\sqrt{K}} = \frac{\sqrt{6}}{\sqrt{5}} \Rightarrow K = 10$$

2 Mechanical Properties of Fluids

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(d)** Hydraulic machines & lifts are based on

$P_1 = P_2$; $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

2. **(d)** Pressure applied to enclosed fluid is transmitted equally in all direction according to Pascal law.

3. **(b)**

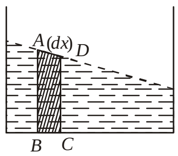
4. **(a)** Total cross-sectional area of the thigh bones

$A = 2\,(10 \times 10^{-4}) = 2 \times 10^{-3}\,m^2$

Force acting on the bones = $mg = 40 \times 10 = 400$ N

$\therefore \quad P_{av} = \frac{F}{A} = \frac{400}{2\times 10^{-3}} = 2 \times 10^5\ N/m^2$

5. **(c)** Let us consider a small dotted segment of thickness dx for observation.



Since, this segment is accelerated towards right, a net force is acting in this segment towards right from the liquid towards the left of *ABCD*. According to Newton's third law, the segment *ABCD* will also apply a force on the previous section creating a pressure on it which makes the liquid rise.

6. **(c)** Liquid pressure depends upon the height of liquid column and is independent of the shape of liquid surface and the area of liquid surface. The liquid at rest exerts equal pressure in all directions.

7. **(d)** Given, $P_{h/2} = \frac{3}{4}(p_{bottom})$

$\Rightarrow \quad p_0 + \rho g \frac{h}{2} = \frac{3}{4}(p_0 + \rho g h)$

$\Rightarrow \quad h = \frac{p_0}{2\rho g} = \frac{10^5}{2\times 10^3 \times 10} = 5m$

8. **(a)** Equating pressure,

$\frac{F}{\pi R^2} = \frac{1200}{\pi [20R]^2}$ or $F = 3$ kgf

9. **(a)** $P \times \frac{\pi D^2}{4} = mg$ or $P = \frac{4mg}{\pi D^2}$ pascal

or $\quad P = \frac{4mg}{\pi D^2 \times 10^5}$ atmosphere

10. **(c)** Here, m = 50 kg, D = 1 cm = 10^{-2} m, g = 10 ms^{-1}

$\therefore$ Pressure exerted by the heel on the horizontal floor is

$p = \frac{F}{A} = \frac{mg}{\pi (D/2)^2} = \frac{4mg}{\pi D^2} = \frac{4 \times 50kg \times 10ms^{-2}}{3.14 \times (10^{-2}m)^2}$

$= 6.4 \times 10^6$ Pa

11. **(b)** The maximum force, which the bigger piston can bear, m = 3000 kg, F = 3000 × 9.8 N

Area of piston, A = 425 cm^2 = $425 \times 10^{-4}\ m^2$

$\therefore$ Maximum pressure on the bigger piston,

$P = \frac{F}{A} = \frac{3000 \times 9.8}{425 \times 10^{-4}} = 6.92 \times 10^5$ Pa

$\therefore$ The maximum pressure the smaller piston can bear is 6.92×10^5 Pa.

12. **(b)** The gauge pressure at a depth of 50 m,

$P = \rho g h = 1025 \times 10 \times 50 = 512500$ Pa

13. **(d)** The total pressure at the given depth.

$= P_0 + \Delta p = P_0 + \rho g h = 1.01 \times 10^5 + 1000 \times 10 \times 10$

$= 1.01 \times 10^5 + 10^5 = 2.01 \times 10^5\ Nm^{-2}$

$\simeq 2 \times 10^5\ Nm^{-2}$

14. **(b)** $p_0 + \frac{h}{3} dg = \frac{1}{2}\left(hdg + p_0\right)$

$p_0 + \frac{hdg}{3} = \frac{p_0}{2} + \frac{hdg}{2}$

$\Rightarrow \frac{Hdg}{2} = hdg\left(\frac{1}{6}\right)^2$

h = 3H = 30 m

15. **(d)** According to Pascal's law, $\frac{F_2}{\pi r_2^2} = \frac{w}{\pi r_1^2}$

$\Rightarrow \quad F_2 = w\left(\frac{r_2}{r_1}\right)^2 = 5 \times 10^3 \left(\frac{0.1}{1}\right)^2 = 50$ N

16. **(d)** $\because \quad P = h\rho g \Rightarrow P \propto h$, $\therefore$ P is same for all.

17. **(c)** $F = PA = \rho g h A$

$= 10^3 \times 10 \times 10 \times 3 \times 10^{-4} = 30$ N

18. **(a)** When a highly soluble salt (like sodium chloride) is dissolved in water, the surface tension of water increases.

19. **(a)**

20. **(a)**

21. **(b)** When radius is doubled, increase in surface area

$= 2 \times 4\pi\,(4R^2 - R^2) = 2 \times 4\pi \times 3R^2 = 24\pi R^2$

Work done = Increase in area × T $= 24\pi R^2$ T.

22. **(c)** Work done, $W = T\Delta A = \sigma.4\pi r^2 = 4\pi r^2 \sigma$.

23. **(a)** Energy needed = Increment in surface energy

= (surface energy of *n* small drops) – (surface energy of one big drop)

$= n4\pi r^2 T - 4\pi R^2 T = 4\pi r T(nr^2 - R^2)$

24. **(c)** Work done to increase the diameter of bubble from d to $W = 2\pi(D^2 - d^2)\ T = 2\pi[(2D)^2 - (D)^2]\ T$

$= 6\pi D^2 T$

25. (c) $W = 8\pi R^2 T \quad \therefore \; W \propto R^2$ [T is constant]
If radius becomes double then work done will become four times

26. (b) Surface energy of combined drop will be lowered, so excess surface energy will raise the temperature of the drop.

27. (b) Surface energy = surface tension × increment in area = T × A

28. (c) Because energy is liberated

29. (a) Surface tension $= \frac{\text{Surface energy}}{\text{Area}}$

or $T = \frac{E}{A}$

30. (d) $\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$ [As volume remains constant]

$R^3 = 1000r^3 \Rightarrow R = 10r \Rightarrow r = \frac{R}{10}$

31. (a) We know that angle of contact is the angle between the tangent to liquid surface at the point of contact and solid surface inside the liquid. In case of pure water and pure glass, the angle of contact is zero.

32. (c) Wetability of a surface by a liquid primarily depends on angle of contact between the surface and liquid.

If angle of contact is acute liquids wet the solid and vice-versa.

33. (c) Tangent drawn at point of contact makes 90° with wall of container.

34. (b) Both liquids water and alcohol have same nature (i.e. wet the solid). Hence angle of contact for both is acute.

35. (b) Since for such liquid (Non-wetting) angle of contact is obtuse.

36. (d) **37. (c)** **38. (a)** **39. (c)**

40. (b) **41. (a)**

42. (d) The water rises to height h due to capilarity.

43. (d) Rise of liquid in a capillary tube,

$h = \frac{2S\cos\theta}{r\rho g}, \quad h \propto \frac{1}{r} \quad \therefore \frac{h_1}{h_2} = \frac{r_2}{r_1} = \frac{2}{1}$

44. (b) When lift is accelerated downwards, the observed weight of body in a lift decreases. Hence to counter balance the upward pull due to S.T. on the liquid meniscus, the height through which the liquid rises must increase.

45. (a) $h = \frac{2\,T\cos\theta}{r\rho g}$; The liquid will rise i.e., h is positive

if $\cos\theta$ is +ve; It is so if $\theta < 90^\circ$ or θ is acute.

46. (c) $r \propto \frac{1}{h} \Rightarrow \frac{r_1}{r_2} = \frac{h_2}{h_1} = \frac{6.6}{2.2} = \frac{3}{1}$

47. (b) $h \propto \frac{1}{r} \therefore r_1 h_1 = r_2 h_2 \Rightarrow h_2 = \frac{r_1 h_1}{r_2} = 2.4mm$

48. (b) $l = \frac{h}{\cos\theta} = \frac{2}{\cos 60^\circ} = 4.0cm$

49. (a) $h \propto \frac{1}{R}$

50. (b) $h \propto \frac{1}{r}$

51. (a) Water rises in plant fibres due to capillarity.

52. (a) $h = \frac{2T\cos\theta}{rdg} \Rightarrow rh = \text{constant}.$

53. (d) As the liquid molecule inside the liquid is surrounded equally from all sides, so net force on a molecule of surface B will be zero.

Thus, the statement given in option (d) is correct rest are incorrect.

54. (a)

55. (b) Work done $= 8\pi T[r_2^2 - r_1^2]$

$= 8\pi T\left[\frac{D^2}{4} - \frac{d^2}{4}\right] = 2\pi[D^2 - d^2]\,T$

56. (a)

57. (c) $W = 2 \times 4\pi R^2 \times \sigma$; R is increased by a factor of 2. So, W is increased by a factor of 4.

58. (d) Surface tension of a liquid is due to force of attraction between like molecules of a liquid i.e., cohesive force between the molecules.

59. (c) Angle of contact is acute.

60. (b) The surface tension of oil is less than that of water, so the oil spreads as a thin layer.

61. (a) $h = \frac{2\,T\cos\theta}{r\rho g}$; The liquid will rise i.e., h is positive

if $\cos\theta$ is +ve; It is so if $\theta < 90^\circ$ or θ is acute.

62. (c) Excess pressure inside the soap bubble

$p = \frac{4s}{r} \quad \therefore \quad p \propto \frac{1}{r}$

63. (c) Excess of pressure in a soap bubble, P = 4T/r *i.e.*, $P \propto \frac{1}{r}$ therefore pressure in a smaller bubble is more than that of a bigger bubble. When two bubbles of different radii are in communication, then the air flows from higher pressure to lower pressure *i.e.*, from smaller bubble into larger one.

64. (a) When two drops merge together to form one drop, the surface area of drop will decrease, due to which the energy of bigger drop is less than the sum of the energy of two smaller drops. Due to it, the energy is released.

65. (d)

66. (d) In the satellite , the weight of the liquid column is zero. So the liquid will rise up to the top of the tube.

67. (b) $h = \frac{2T}{rdg} = \frac{2 \times 6 \times 10^{-2}}{5 \times 10^{-4} \times 10^3 \times 10} = 2.4 \times 10^{-2}\,m = 2.4cm$

68. (d) Since $h = \frac{2s\cos\theta}{h\rho g}$

if θ is obtuse, $\cos\theta$ is negative. Hence h is negative and water is depressed in the tube.

69. (b) For capillary rise, according, to Zurin's law

$h_1 r_1 = h_2 r_2$

$6 \times 1 = h_2 \times 2$

$\Rightarrow h_2 = 3$ cm

70. (c) $h = \frac{2\sigma\cos\theta}{r\rho g} \Rightarrow \sigma \propto \frac{h\rho}{\cos\theta}$

$\Rightarrow \frac{\sigma_w}{\sigma_m} = \frac{h_w\rho_w}{\cos\theta_w} \times \frac{\cos\theta_m}{h_m\rho_m} = \frac{10\times1}{\cos 0°} \times \frac{\cos 135°}{-3.1\times13.6}$

$= \frac{10\times(-0.707)}{-3.1\times13.6} \approx \frac{1}{6}$

71. (c)

72. (a) Surface tension = workdone per unit area in increasing the surface area of a liquid under isothermal condition.

73. (b) Over a small temperature ranges, S.T. of water decreases linearly with rise of temperature.

74. (d) Surface tension = 0.075 N/m;

diameter = 30 cm = 0.30 m

$\therefore$ Force = $0.075 \times 0.30 = 0.0225$ N $= 2.25 \times 10^{-2}$N.

75. (b)

76. (a) Because film tries to cover minimum surface area.

77. (d) Surface tension of a liquid is due to force of attraction between like molecules of a liquid i.e., cohesive force between the molecules.

78. (a) The surface tension of liquid at critical temperature is zero.

79. (b) When a sparingly soluble salt (like detergent) added to water, the surface tension of water decreases.

80. (a) Most viscous liquid comes to rest quickly due to dissipation of energy at a larger rate.

Hence, most viscous liquid comes to rest at the earliest.

81. (d) According to Stokes' law, the retarding force is proportional to velocity. Initial velocity will increase. When viscous force plus buoyant force become equal to the force of gravity, the net force and hence acceleration become zero. The spherical body then moves with a constant velocity called terminal velocity.

82. (d)

83. (b) Terminal velocity, $V_T = \frac{2a^2(\rho-\sigma)g}{9\eta}$

$V_T = \frac{2\times(2\times10^{-3})^2 \times (8.9-1.5)\times10^3\times9.8}{9\times6.5\times10^{-2}}$

$= 9.9 \times 10^{-1}$ kg m s^{-1}

84. (c) $F = 6\pi\eta v$, $F' = 6\pi\eta(2r)(2v) = 4F$

85. (d) Since the steel ball is small therefore upthrust may be neglected.

Now, $6\pi\eta r v_0 = mg$ or $v_0 \propto \frac{mg}{\eta r}$

86. (b) Initially the terminal velocity V of sphere of radius a is

$W_{eff} = 6\pi\eta aV$ ...(i)

(W_{eff} = weight – Buoyant force)

As the radius is doubled, mass is increased to 8 times and new terminal velocity will be

$8W_{eff} = 6\pi\eta 2aV'$ ...(ii)

from (i) and (ii) $V' = 4V$

87. (d) **88. (b)**

89. (c) We know that for air bubble in liquid

$v_T = v = \frac{2r^2 g}{9\eta}(\rho_f - \rho) \Rightarrow \eta = \frac{2r^2 g}{9v}(\rho_f - \rho)$

$\Rightarrow \eta = \frac{2r^2 g}{9v}(\sigma)$ [$\because \rho_f = \sigma$ and $\rho \approx o$]

90. (b) $v \propto \frac{\rho - \rho_0}{\eta}$

$\therefore \frac{v_2}{v_1} = \frac{\rho-\rho_{02}}{\rho-\rho_{01}} \times \frac{\eta_1}{\eta_2} = \frac{7.8-1.2}{7.8-1} \times \frac{8.5\times10^{-4}\times10}{13.2}$

$= 6.25 \times 10^{-4}$ cms^{-1}

91. (d) From stoke's law, $F = 6\pi\eta R_1 v$, and $V = \frac{4}{3}\pi R^3$

$F' = 6\pi\eta R_2 v$, $\left(\text{volume } 8V = \frac{4}{3}\pi(2R)^3\right)$

$= 6\pi\eta(2R)v = 2F$

92. (c) If ρ is the density of the ball and ρ' that of the another ball, m for the balls are the same, but $r' = 2r$

$\therefore$ $mg = 6\pi r\eta v$ (by Stoke's law)

or, $6\pi r\eta v = 6\pi 2r\eta v'$ So, $v' = \frac{v}{2}$

93. (a) As the temperature rises the atoms of the liquid become more mobile and the coefficient of viscosity falls.

94. (b) Let r be the radius of small drops of water.

R = radius of big drop formed

as volume remain same.

$\therefore 8.\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R^3 \Rightarrow R = 2r$

Terminal velocity,

$v_T = \frac{2}{9\eta}(\rho-\sigma)r^2 g$

$\therefore v_T \propto r^2$ $\therefore \frac{v_1}{v_2} = \left(\frac{r}{R}\right)^2$

$\Rightarrow \frac{10}{v_2} = \left(\frac{1}{2}\right)^2$ ($\because v_1 = 10$ cm/s given)

$\Rightarrow v_2 = 40$ cm/s

95. (b) $\eta = 10^{-2}$ poise

$v = 18$ km/h $= \frac{18000}{3600} = 5$ m/s

$l = 5$ m

Strain rate $= \frac{v}{l}$

Coefficient of viscosity, $\eta = \frac{\text{shearing stress}}{\text{strain rate}}$

$\therefore$ Shearing stress = $\eta \times$ strain rate

$= 10^{-2} \times \frac{5}{5} = 10^{-2}$ Nm^{-2}

96. **(b)** When terminal velocity is reached then body moves with constant velocity hence, accelesation is zero.

97. **(c)**

98. **(a)** Stoke's Law

$F = 6\pi\eta rv$

$\Rightarrow F = 6 \times \pi \times (2 \times 10^{-5}) \times (0.5 \times 10^{-3}) \times 0.7$

$\Rightarrow F \approx 13.195 \times 10^{-8}$ N

99. **(c)** Terminal velocity

$$V_T = \frac{2}{9} r^2 \frac{(\rho - \sigma) g}{\eta}$$

If ρ, σ, η are constant then $V_T \propto r^2$

$$\text{Density} = \frac{\text{mass}(w)}{\text{volume}(v)} = \frac{m}{\frac{4}{m}\pi r^3}$$

$$\frac{8}{r^3} = \frac{64}{(r')3} \text{ or, } (r') = \frac{64}{8} r^3 \therefore r' = 2r$$

$$\frac{V_T}{V'_T} = \frac{r^2}{(r')^2} = \frac{r^2}{(2r)^2} = \frac{1}{4}$$

$\therefore V'_T = 4\, V_T = 4 \times 3\, \text{cms}^{-1} = 12\, \text{cms}^{-1}$

100. **(c)** **101.** **(d)** **102.** **(a)** **103.** **(b)**

104. **(a)**

105. **(b)** $\because A_1V_1 = A_2V_2$

$$\therefore \frac{V_1}{V_2} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{3.75}{2.5}\right)^2 = \frac{3}{2}$$

106. **(c)** In steady state, $A_1v_1 = A_2v_2$

$$\pi \times 2^2 \times 1 = \pi \times 1^2 \times \sqrt{2gh} \Rightarrow h = \frac{16}{20} = 0.8\text{m}$$

107. **(d)** $x = \sqrt{2gh_1} \times \sqrt{\frac{2h_2}{g}}$ or $x = 2\sqrt{h_1h_2}$

Now, imagine a hole at a depth h_2 below the free surface of the liquid. The height of this hole will be h_1. Clearly, x remains the same.

108. **(a)** $v = \sqrt{2gh}$

But $p = h\rho g$ or $\frac{p}{\rho} = gh$

$$\therefore v\sqrt{\frac{2p}{\rho}} = \sqrt{\frac{2 \times 2 \times 10^5}{10^3}}\ \text{ms}^{-1} = 20\ \text{ms}^{-1}$$

R^2v = constant

109. **(d)** $v = \sqrt{2gh}$

But $h\rho g = p$

$$\therefore hg = \frac{p}{\rho}$$

$$\therefore v = \sqrt{\frac{2p}{\rho}} = \sqrt{\frac{2.6.4 \times 10^5}{800}}\ \text{ms}^{-1} = 40\ \text{ms}^{-1}$$

110. **(a)** The height of water in the tank becomes maximum when the volume of water flowing into the tank per second becomes equal to the volume flowing out per second.

Volume of water flowing out per second = $Av = A\sqrt{2gh}$

and volume of water flowing in per second = 80 cm^3/sec.

$\therefore A\sqrt{2gh} = 80 \Rightarrow 3 \times \sqrt{2 \times 980 \times h} = 80$

$\Rightarrow 3 \times \sqrt{2 \times 980 \times h} = 80$

$$h = \frac{6400}{1960 \times 3} \simeq 1.1\ \text{cm.}$$

111. **(c)** $R = 2\sqrt{h(H-h)}$

Now, $3.3[H - 3.3] = 4.7\,[H - 4.7]$

or $(4.7 - 3.3)H = 4.7 \times 4.7 - 3.3 \times 3.3$

or $H = \frac{22.09 - 10.89}{1.4}$ cm $= \frac{11.2}{1.4}$ cm = 8cm

112. **(b)**

113. **(c)** According to equation of continuity

$A_1V_1 = A_2V_2 + A_3V_3$

$\Rightarrow 4 \times 0.2 = 2 \times 0.2 + 0.4 \times V_3$

$\Rightarrow V_3 = 1$ m/s.

114. **(b)**

115. **(c)** $P_a + \frac{1}{2}\rho_1 v_1^2 + 0 = Pa + \frac{1}{2}\rho_2 v_2^2 + (\rho_1 g h_1 + \rho_2 g h_2)$

As $v_2 << v_1$, $\therefore v_1 = \sqrt{2g(h_1 + h_2 \frac{\rho_2}{\rho_1})}$.

116. **(b)**

117. **(c)** Velocity of efflux

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = 2\ \text{ms}^{-1}$

118. **(b)** Beyond the critical speed, the flow of fluids becomes turbulent as the flow loses its steadiness.

119. **(a)** **120.** **(b)**

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(a)** $P_1 = P_0 + \rho g d_1$

$P_2 = P_0 + \rho g d_2$

$\Delta P = P_2 - P_1 = \rho g \Delta d$

$3.03 \times 10^6 = 10^3 \times 10 \times \Delta d \Rightarrow \Delta d \simeq 300$ m

2. **(d)** Depth, h = 3m

density, $\rho = 10^3$ kg/m^3

Acceleration due to gravity, g = 10 m/s^2

Pressure at bottom is given by

$P = \rho gh$

$P = 10^3 \times 10 \times 3$

$P = 30 \times 10^3$ Pa

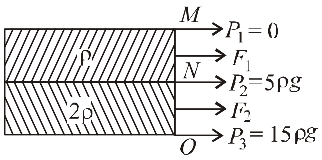
3. **(d)** Let P_1, P_2 and P_3 be the pressure at points M, N and O respectively.

Pressure is given by $P = \rho gh$

Now, $P_1 = 0$ ($\because$ $h = 0$)

$P_2 = \rho g(5)$

$P_3 = \rho g(15) = 15\rho g$



Force on upper part,

$$F_1 = \frac{(P_1 + P_2)}{2} A$$

Force on lower part, $F_2 = \dfrac{(P_2 + P_3)}{2} A$

$$\therefore \frac{F_1}{F_2} = \frac{5\rho g}{20\rho g} = \frac{5}{20} = \frac{1}{4}$$

4. **(a)**

5. **(a)** Mass of liquid in horizontal portion of U-tube = $Ad\rho$

Pseudo force on this mass = $Ad\rho a$

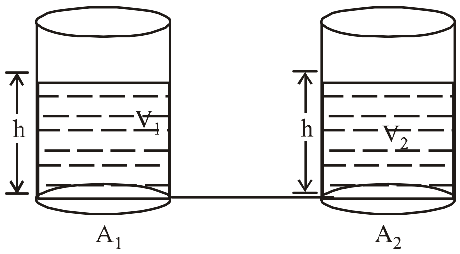
Force due to pressure difference in the two limbs

$= (h_1\rho g - h_2\rho g) A$

Equating both the forces

$$(h_1 - h_2)\,\rho gA = Ad\rho a \Rightarrow (h_1 - h_2) = \frac{Ad\rho a}{\rho gA} = \frac{ad}{g}$$

6. **(a)**



$V_1 = 3V_2 \Rightarrow A_1 h = 3A_2 h$

$\therefore$ $A_1 = 3A_2$

$\therefore$ Pressure at the bottom of the vessel is

$P = P_0 + \delta gh$ = constant

$$\therefore \frac{P_1}{P_2} = \frac{1}{1} = 1:1$$

7. **(a)**

8. **(a)** Mass of liquid in horizontal portion of U-tube = $Ad\rho$

Pseudo force on this mass = $Ad\rho a$

Force due to pressure difference in the two limbs

$= (h_1\rho g - h_2\rho g) A$

Equating both the forces

$$(h_1 - h_2)\,\rho gA = Ad\rho a \Rightarrow (h_1 - h_2) = \frac{Ad\rho a}{\rho gA} = \frac{ad}{g}$$

9. **(c)** Since pressure is transmitted undiminished throughout the fluid (Pascal's law)

$$F_1 = \frac{A_1}{A_2} F_2 = \frac{\pi\,(5 \times 5)}{\pi\,(15 \times 15)}(1350 \times 9.81) \approx 1.5 \times 10^3 \text{ N}$$

The air pressure that will produce this force is

$$P = \frac{F_1}{A_1} = \frac{1.5 \times 10^3}{\pi\,(5 \times 10^{-2}\,\text{m})^2} = 1.9 \times 10^5\,\text{Pa}$$

10. **(d)** The pressure outside the submarine is

$P = P_a + \rho gh$

Pressure inside the submarine is P_a.

Net pressure acting on the window is

$P_g = P - P_a = \rho gh$

$= 10^{-3}\,\text{kgm}^{-3} \times 10\,\text{ms}^{-2} \times 2000\,\text{m} = 2 \times 10^7\,\text{Pa}$

Area of window is

$A = 50\,\text{cm} \times 50\,\text{cm} = 2500 \times 10^{-4}\,\text{m}^2$

Force on the window is

$F = P_g A = 2 \times 10^7\,\text{Pa} \times 2500 \times 10^{-4}\,\text{m}^2 = 5 \times 10^6\,\text{N}$

11. **(b)** A liquid film has two surfaces,

so upward force = 2Tl

According to question,

Weight of the body hanged from wire (mg)

= Upward force due to surface tension (2Tl)

$$\Rightarrow \quad m = \frac{2Tl}{g}$$

12. **(a)** Surface tension,

$$T = \frac{F}{2l} = \frac{2 \times 10^{-2}}{2 \times 10 \times 10^{-2}} = 0.1\,\text{Nm}^{-1}$$

13. **(d)** Force on one side of the stick

$F_1 = T_1 \times L = 0.07 \times 2 = 0.14$ N

and force on other side of the stick

$F_2 = T_2 \times L = 0.06 \times 2 = 0.12$ N

So, net force on the stick = $F_1 - F_2 = 0.14 - 0.12 = 0.02$ N

14. **(b)** Excess pressure inside an air bubble just below the surface of water, $p_1 = \dfrac{2T}{r}$, due to only one free surface and excess pressure inside a drop,

$$p_2 = \frac{2T}{r} \qquad \therefore \quad p_1 = p_2$$

15. **(c)** Pressure inside the bubble is

$p_i = p_o + 2S/r = 1.1 \times 10^5 + (2 \times 7.30 \times 10^{-2}/10^{-3})$

$= 1.01146 \times 10^5$ Pa

16. **(c)** The total pressure inside the air bubble is

$$p_t = p + p_o + h\rho g = \frac{2\sigma}{r} + p_o + h\rho g \quad \left(\because p = \frac{2\sigma}{r}\right)$$

17. **(b)** The height of capillary rise is

$$h \propto \frac{1}{r} \Rightarrow \frac{h_1}{h_2} = \left(\frac{r_2}{r_1}\right) = \frac{0.4}{0.3} = \frac{4}{3}$$

18. **(c)** For capillary tube,

$h_A = 1.8$ cm, $d_B = 0.9d_A$

The height of capillary rise is given by

$$h = \frac{2S\cos\theta}{\rho rg} \Rightarrow h \propto \frac{1}{r} \propto \frac{1}{d}$$

$$\therefore \quad \frac{h_B}{h_A} = \left(\frac{d_A}{d_B}\right) = \left(\frac{d_A}{0.9d_A}\right) = \frac{10}{9}$$

$$\therefore \quad h_B = \frac{10}{9} \times h_A = \frac{10 \times 1.8}{9} = 2\,\text{cm}$$

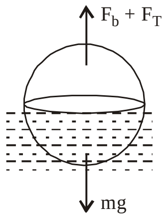
19. **(b)**

20. **(a)** As liquid drop is in equilibrium.

So $F_{net} = 0$

Boyant force + surface tension = mg

$$\sigma \frac{V}{2} g + 2\pi RT = \rho V g$$



$$\Rightarrow 2\pi RT = \frac{(2\rho - \sigma)}{2} \frac{4}{3} \pi R^3 g \left[\because V = \frac{4}{3}\pi R^3\right]$$

$$\Rightarrow R^2 = \frac{3T}{(2\rho - \sigma) g} \Rightarrow R = \sqrt{\frac{3 \times 7.5 \times 10^{-2}\ N - m^{-1}}{(2\rho - \sigma) \times 10}}$$

$$\Rightarrow R = \frac{3}{20\sqrt{(2\rho - \sigma)}} m = \frac{15}{\sqrt{2\rho - \sigma}} cm$$

21. **(c)** Work done, W = S [2 × (Change in area)]

[$\because$ there are two free surface]

$$\text{Surface tension, } S = \frac{W}{2 \times (\text{change in area})}$$

= 3 × 10^{-2} N/m

$$= \frac{3 \times 10^{-4}}{2 \times 10 (11 - 6) \times (10^{-2})^2}$$

22. **(a)** $W = T\Delta A = 4\pi R^2 T (n^{1/3} - 1)$

$= 4 \times 3.14 \times (10^{-2})^2 \times 460 \times 10^{-3} [(10^6)^{1/3} - 1] = 0.057$

23. **(c)** Work done = Change in surface energy

$\Rightarrow W = 2T \times 4\pi (R_2^2 - R_1^2)$

$= 2 \times 0.03 \times 4\pi[(5)^2 - (3)^2] \times 10^{-4} J = 0.4\ \pi$ mJ

24. **(a)** As volume remain same i.e volume of two smaller drops will be equal to volume of one big drop.

$$2 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow \quad 2r^3 = R^3$$

$$\Rightarrow 2 = \left(\frac{R}{r}\right)^3 \Rightarrow \frac{R}{r} = 2^{\frac{1}{3}}$$

$$\frac{U_i}{U_f} = \frac{T \times 2 \times (4\pi r^2)}{T \times 4\pi R^2} = \frac{2r^2}{2^{\frac{2}{3}} R^2} = 2^{\frac{1}{3}} : 1$$

25. **(b)** $\frac{2T}{r} = \frac{2 \times 0.07}{0.14 \times 10^{-3}} = 10^3\ N/m^2$

Pressure applied, $= P_a - \frac{2T}{r} = 10^5 - 10^3$

$= 99 \times 10^3\ N/m^2$

26. **(c)** Water fills the tube entirely in gravity less condition *i.e.*, 20 cm.

27. **(a)** Weight of the liquid column = T cosθ × 2πr.

For water θ = 0°. Here weight of liquid column W = 7.5 × 10^{-4}N and T = 6 × 10^{-2}N/m. Then circumference, 2πr = W/T = 1.25 × 10^{-2}m

28. **(b)** $T = \frac{F}{2\pi r} = \frac{6.28 \times 10^{-4}}{2 \times 3.14 \times 2 \times 10^{-3}} = 5 \times 10^{-2}\ N/m$

29. **(c)** Work done = Surface tension × increase in area of the film

W = S × ΔA

Increase in area = Final area – initial area

= 10 × (0.5 + 0.1) – 10 × 0.5 = 1 cm^2

∴ W = 72 × 2 × 1 = 144 erg

[$\because$ There are 2 free surfaces; ∴ ΔA = 2 × 1].

30. **(d)** F_{req} = mg + 2 [T (2 π R)] [T = 75 × 10^{-3} N/m]

= 0.1 + 2 [75 × 10^{-3} (0.2)] = 0.130 N

31. **(b)** **32.** **(c)**

33. **(b)** Given,

Angle of contact θ = 30°

Surface tension, T = 0.05 Nm^{-1}

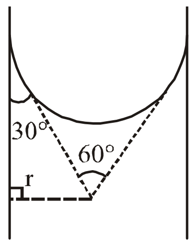
Radius of capillary tube, r = 0.15 mm = 0.15 × 10^{-3}m

Density of methylene iodide,

ρ = 667 kg m^{-3}

Capillary rise, $h = \frac{2T\cos\theta}{\rho g r}$

$$= \frac{2 \times 0.05 \times \frac{\sqrt{3}}{2}}{667 \times 10 \times 0.15 \times 10^{-3}} = 0.087\ m$$



34. **(c)** The height of liquid column in a capillary tube,

$$h = \frac{2S\cos\theta}{r\rho g} \therefore \frac{h_1}{h_2} = \frac{S_1}{S_2}\frac{\rho_2}{\rho_1} \quad (\because S_2 = 2S_1\ \rho_2 = 2\rho_1)$$

$$\frac{5}{h_2} = \left[\frac{1}{2}\right]\left[\frac{2}{1}\right] \Rightarrow h_2 = 5\ cm = 0.05\ m$$

35. **(b)**

36. **(b)** When the bubble gets detached,

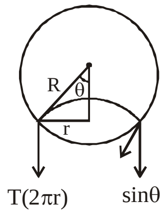
Bouyant force = force due to surface tension

$$2\pi r T \sin\theta = \frac{4}{3}\pi R^3 \rho_\omega g$$

$$\Rightarrow T \times \frac{r}{R} \times 2\pi r = \frac{4}{3}\pi R^3 \rho_\omega g$$

$$\text{or, } \frac{2T}{R}(\pi r^2) = \frac{4\pi R^3}{3T}\rho_w g$$

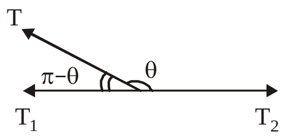
$$\Rightarrow r = R^2\sqrt{\frac{2\rho_w g}{3T}}$$



37. **(d)** $T_1 + T\cos(\pi - \theta) = T_2$

$$\therefore \cos(\pi - \theta) = \frac{T_2 - T_1}{T}$$

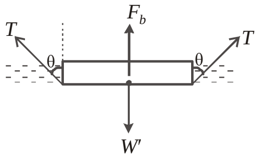
$$\Rightarrow \cos\theta = \frac{T_1 - T_2}{T}$$



38. (c) For floating disc, $F_{net} = 0$

or $F_b + 2\pi r T \cos\theta = W'$

or $W + 2\pi r T\cos\theta = W'$.



39. (b) $h = \dfrac{2T\cos\theta}{rdg}$, Height in capillary depends upon the surface tension of the liquid and surface tension of soap water solution is less than water. So, height will be less in second case.

40. (d) At equilibrium, weight of the given block is balanced by force due to surface tension, i.e.,

2L. S = W

or $S = \dfrac{W}{2L} = \dfrac{1.5\times10^{-2}\,N}{2\times0.3\,m} = 0.025\,Nm^{-1}$

41. (d) Excess pressure inside the soap bubble = $\dfrac{4S}{r}$

So the pressure inside the soap bubble = $P_{atm} + \dfrac{4S}{r}$

From ideal gas equation $PV = nRT$

$$\frac{P_AV_A}{P_BV_B} = \frac{n_A}{n_B} \Rightarrow \frac{\left(8+\dfrac{4S}{r_A}\right)\dfrac{4}{3}\pi(r_A)^3}{\left(8+\dfrac{4S}{r_B}\right)\dfrac{4}{3}\pi(r_B)^3} = \frac{n_A}{n_B}$$

Substituting S = 0.04 N/m, r_A = 2cm, r_B = 4 cm.

$\dfrac{n_A}{n_B} = \dfrac{1}{6}$ ∴ $\dfrac{n_B}{n_A} = 6$.

42. (d) The excess pressure inside a bubble of soap is given by $p = \dfrac{4T}{r}$. It means excess pressure inside the bubbles A and C is more than inside B. So, air will go towards B from A and C. So, A and C will start collapsing with the increasing volume of B.

43. (c) $h = \dfrac{2T\cos\theta}{r\rho g} \Rightarrow h \propto \dfrac{1}{r} \Rightarrow \dfrac{h_2}{h_1} = \dfrac{r_1}{r_2} = \dfrac{2}{3}$

$\left(\because r_1 = r,\ \ r_2 = r + 50\% \text{ of } r = \dfrac{3}{2}r\right)$

New mass $m_2 = \pi r_2^2 h_2\rho = \pi\left(\dfrac{3}{2}r_1\right)^2\left(\dfrac{2}{3}h_1\right)\rho$

$= \dfrac{3}{2}\left(\pi r_1^2 h_1\right)\rho = \dfrac{3}{2}m$

44. (d) When radius is decreased by ΔR,

$4\pi R^2\Delta R\rho L = 4\pi T[R^2 - (R-\Delta R)^2]$

$\Rightarrow \rho R^2\Delta RL = T[R^2 - R^2 + 2R\Delta R - \Delta R^2]$

$\Rightarrow \rho R^2\Delta RL = T2R\Delta R$ [ΔR is very small]

$\Rightarrow R = \dfrac{2T}{\rho L}$

45. (c)

46. (b) When a capillary tube is broken at a height of 6 cm, the height of water column will be 6 cm.

As $h = \dfrac{25\cos\theta}{\rho rg}$ or $\dfrac{h}{\cos\theta}$ = constant

∴ $\dfrac{8}{\cos0°} = \dfrac{6}{\cos\theta}$ or $\cos\theta = \dfrac{6\cos0°}{8} = \dfrac{3}{4}$

$\theta = \cos^{-1}\left(\dfrac{3}{4}\right)$

47. (d) Ascent formula $h\dfrac{2T\cos\theta}{rdg}$

$\Rightarrow \dfrac{h_1}{h_2} = \dfrac{T_1}{T_2}\times\dfrac{d_2}{d_1}$ [r, θ and g and constant]

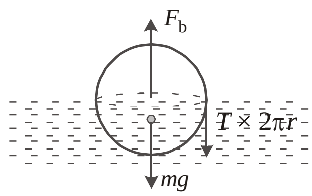
$= \dfrac{60}{50}\times\dfrac{0.6}{0.8} = \dfrac{9}{10}$

48. (a) $\Delta P = \dfrac{4T}{r}$ ∴ $\Delta P \propto \dfrac{1}{r}$

As radius of soap bubble increases with time

$\therefore \Delta P \propto \dfrac{1}{t}$

49. (c) $T\times 2\pi r + mg = F_b$



or $T\times2\pi r + \rho\dfrac{4}{3}\pi r^3 g = \left[\dfrac{\frac{4}{3}\pi r^3}{2}\right]\sigma g$

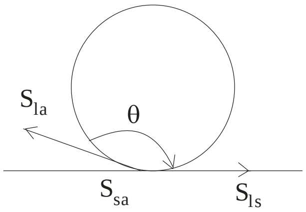
∴ $r = \sqrt{\dfrac{3T}{g(2\rho-\sigma)}}$

50. (a) $h_1 = \dfrac{2T}{r_1\rho g}$ and $h_2 = \dfrac{2T}{r_2\rho g}$

∴ $h = h_2 - h_1 = \dfrac{2T}{\rho g}\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right)$.

51. (d) $h \propto \dfrac{1}{r}$

52. (d) According to question, the situation can be depicted as below,



From, this we have, $S_{la}\cos\theta + S_{is} = S_{sa}$

∵ θ > 90° and cos θ is negative.

This implies that $S_{is} > S_{la}$. Hence, liquid does not spread on solid surface.

Thus, the statement given in option (d) is correct, rest are incorrect.

53. **(d)** Given, radius of mercury drop, $r = 10^{-3}$ m
Initial surface energy E_1 = surface tension × Area
$E_1 = 0.45 \times 4\pi r^2 = 0.45 \times 4\pi\,(10^{-3})^2$
Initial volume = n × final volume

$$\frac{4}{3}\pi(10^{-3})^3 = 125 \times \frac{4\pi}{3} R_{new}^3$$

$\therefore\ 10^{-3} = 5\,R_{new}$ $\qquad \therefore\ R_{new} = \frac{10^{-3}}{5}$ m

So, final surface energy, $E_2 = 0.45 \times 125 \times 4\pi\left(\frac{10^{-3}}{5}\right)^2$

Increase in energy = $E_2 - E_1$

Increase in energy = $0.45 \times 4\pi \times (10^{-3})^2\left[\frac{125}{25} - 1\right]$

$= 4 \times 0.45 \times 4\pi \times 10^{-6} = 2.26 \times 10^{-5}$ J

54. **(d)** We know that terminal velocity is given by

$$V_T = \frac{2gr^2}{g\eta}(\rho - \rho_\ell)$$

Here, we have no involvement of buoyant force. So remove ρ_ℓ.

Then, $v_T = \frac{2gr^2\rho}{9\eta} = \frac{2 \times 10 \times 10^{-12} \times 10^3}{9 \times 1.8 \times 10^{-5}}$
$= 123.4 \times 10^{-6}$ m/s.

55. **(a)** Using, $v^2 - u^2 = 2gh \Rightarrow v^2 - 0^2 = 2gh \Rightarrow v = \sqrt{2gh}$

Terminal velocity, $V_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$

After falling through h the velocity should be equal to terminal velocity

$$\therefore \sqrt{2gh} = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta} \Rightarrow 2gh = \frac{4}{81}\frac{r^4 g^2(\rho-\sigma)^2}{\eta^2}$$

$$\Rightarrow h = \frac{2r^4 g(\rho-\sigma)^2}{81\eta^2} \Rightarrow h \propto r^4$$

56. **(a)** Terminal velocity is given by

$$v_T = \frac{2}{9}\frac{r^2}{\eta}(d - \rho)g$$

$$\frac{v_P}{v_Q} = \frac{r_P^2}{r_Q^2} \times \frac{\eta_Q}{\eta_P} \times \frac{(d - \rho_P)}{(d - \rho_Q)}$$

$$\Rightarrow \frac{v_P}{v_Q} = \left(\frac{1}{0.5}\right)^2 \times \left(\frac{2}{3}\right) \times \frac{(8 - 0.8)}{(8 - 1.6)} \Rightarrow v_P : v_Q = 3 : 1$$

57. **(c)** When two drops of equal radius r combine to form a big drop, the radius of big drop.

$$\frac{4}{3}\pi R^3 = \frac{4\pi}{3}r^3 + \frac{4\pi}{3}r^3 \quad \text{or} \quad R^3 = 2r^3 \text{ or } R = 2^{1/3} r$$

Now $\frac{V_R}{V_r} = \left(\frac{R}{r}\right)^2 = 2^{\frac{2}{3}} = 4^{\frac{1}{3}}$ $\therefore$ $V_R = 5 \times 4^{1/3}$ cm/s

58. **(b)** Given, $v_{t_1} = v_{t_2}$

$$\frac{2}{9}g\frac{r^2}{n_w}[\sigma_1 - \rho_w] = \frac{2}{9}g\frac{r^2}{n_l}(\sigma_2 - \rho_l)$$

$$\Rightarrow \frac{n_w}{n_l} = \frac{\sigma_1 - \rho_w}{\sigma_2 - \rho_1} = \frac{3.2 - 1}{6 - 1.6} = \frac{1}{2}$$

59. **(b)** At equilibrium

$$mg = 6\pi\eta r v_0 \text{ or } \rho\frac{4\pi}{3}r^3 g = 6\pi\eta r v$$

$\therefore\ \frac{v_r}{v_{2r}} = \frac{(r)^2}{(2r)^2}$ or $v_2 r = (v_r) \times 4 = 4$ cm/s

60. **(c)**

61. **(b)** Using Bernoullie's equation

$$P + \frac{1}{2}(v_1^2 - v_2^2) + \rho gh = P$$

$$\Rightarrow v_2^2 = v_1^2 + 2gh \Rightarrow v_2 = \sqrt{v_1^2 + 2gh}$$

Equation of continuity, $A_1v_1 = A_2v_2$

$$\Rightarrow (1\text{ cm}^2)(1\text{ m/s}) = (A_2)\left(\sqrt{(1)^2 + 2 \times 10 \times \frac{15}{100}}\right)$$

$$\Rightarrow 10^{-4} \times 1 = A_2 \times 2 \Rightarrow A_2 = \frac{10^{-4}}{2} = 5 \times 10^{-5}\text{ m}^2$$

62. **(d)** Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho\upsilon_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho\upsilon_2^2 + \rho gh_2$$

For horizontal pipe, $h_1 = 0$ and $h_2 = 0$ and taking

$P_1 = P,\ P_2 = \frac{P}{2}$, we get $\Rightarrow P + \frac{1}{2}\rho\upsilon^2 = \frac{P}{2} + \frac{1}{2}\rho V^2$

$$\Rightarrow \frac{P}{2} + \frac{1}{2}\rho\upsilon^2 = \frac{1}{2}\rho V^2 \Rightarrow V = \sqrt{\upsilon^2 + \frac{P}{\rho}}$$

63. **(a)** From the equation of continuity
$A_1v_1 = A_2v_2$
Here, v_1 and v_2 are the velocities at two ends of pipe.
A_1 and A_2 are the area of pipe at two ends

$$\Rightarrow \frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi(4.8)^2}{\pi(6.4)^2} = \frac{9}{16}$$

64. **(a)** From theorem of continuity $A_A V_A = A_B V_B$
$1.5 \times V_A = 25 \times 10^{-2} \times 60$

$$\therefore V_A = \frac{25 \times 60 \times 10^{-2} \times 10}{1.5} = 10\text{ cm/s}$$

By Bernoulli's theorem, $P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$

$$P_A + \frac{1}{2} \times 1000 \times (0.1)^2 = P_B + \frac{1}{2} \times 1000 \times (0.6)^2$$

$$\Rightarrow P_A + 5 = P_B + \frac{1}{2} \times 1000 \times 36 \times 10^{-2}$$

$P_A + 5 = P_B + 180 \quad \therefore P_A - P_B = 175$ Pa

65. **(b)** Time taken for the level to fall from H to H',

$$t = \frac{A}{A_o}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H'}\right]$$

66. **(b)** $\Delta m_P = \Delta m_Q = \Delta m_R$, we have

$A_P v_P \rho_p \Delta t = A_Q v_Q \rho_Q \Delta t = A_R V_R \rho_R \Delta t$ ...(i)

For flow of incompressible fluids,

we have, $\rho_p = \rho_R = \rho_Q$

Thus, Eq. (i) reduces to,

$\Rightarrow \; A_P v_P = A_R V_R = A_Q v_Q$

Also, from the given figure we can also say that,

$A_R > A_Q > A_P$

67. **(a)** From the Bernoulli's theorem,

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$\Rightarrow \; p_1 - p_2 = \frac{1}{2} \times 1.3 \times [(120)^2 - (90^2)] = 4095 \text{ Pa}$$

68. **(b)** $V_2^2 = V_0^2 + 2gh$

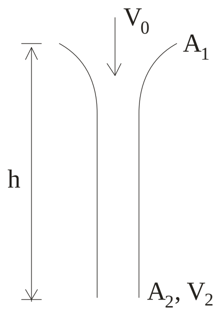
and $A_1V_0 = A_2V_2$

Solving, $\frac{A_2}{A_1} = \frac{V_0}{\sqrt{V_0^2 + 2gh}}$

$$A_2 / A_1 = \frac{1}{2} = \frac{V_0}{\sqrt{V_0^2 + 2gh}}$$

$$4V_0^2 = V_0^2 + 2gh$$

$$h = \frac{3V_0^2}{2g}$$



69. **(c)** $t = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H_1} - \sqrt{H_2}\right]$

Now, $T_1 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{\frac{H}{\eta}}\right]$

and $T_2 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{H}{\eta}} - \sqrt{0}\right]$

According to problem $T_1 = T_2$

$$\therefore \quad \sqrt{H} - \sqrt{\frac{H}{\eta}} = \sqrt{\frac{H}{\eta}} - 0 \Rightarrow \sqrt{H} = 2\sqrt{\frac{H}{\eta}} \Rightarrow \eta = 4$$

70. **(a)**

Exercise 3 : PREVIOUS YEARS MCQs

1. **(c)** Velocity of efflux

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = 2 \text{ ms}^{-1}$

2. **(c)** Work done, W = S [2 × (Change in area)]

[∵ there are two free surface]

Surface tension, $S = \frac{W}{2 \times (\text{change in area})}$

$$= \frac{3 \times 10^{-4}}{2 \times 10(11-6) \times (10^{-2})^2} = 3 \times 10^{-2} \text{ N/m}$$

3. **(d)** If angle of contact is greater than 90°, then liquid does not wet the solid surface.

4. **(a)** Change in pressure in soap bubble is,

$$\Delta P = 2\left(\frac{2S}{R}\right) = \frac{4S}{R}$$

5. **(b)** Since volume conserved, so $V_B = V_S$

$$\frac{4}{3}\pi R^3 = 1000\frac{4}{3}\pi r^3 \Rightarrow R = 10\, r$$

Here,

r = Radius of small drop

R = Radius of big drop

Initial energy $E_i = 1000\,(4\pi r^2 S)$

final energy $E_f = 4\pi R^2 S = 100\,(4\pi r^2 S)$

$$\therefore \frac{E_f}{E_i} = \frac{1}{10} \Rightarrow E_f = \frac{1}{10}E_i$$

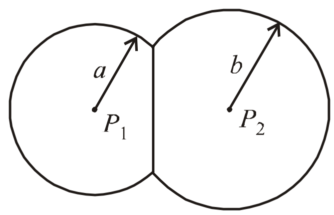
6. **(b)** From the Bernoulli's theorem

$$P + \rho gh + \frac{1}{2}\rho V^2 = \text{constant}$$

7. **(a)** Let *R* be the radius of curvature of common surface

$P_1 = P_0 + \frac{4T}{a}$ and $P_2 = P_0 + \frac{4T}{b}$

And $P_1 - P_2 = \frac{4T}{R}$



$$\left(P_0 + \frac{4T}{a}\right) - \left(P_0 + \frac{4T}{b}\right) = \frac{4T}{R}$$

$$\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{R} \qquad \therefore \; R = \frac{ab}{(b-a)}$$

8. **(b)** Since volume conserved, so $V_B = V_S$

$$\frac{4}{3}\pi R^3 = 1000\frac{4}{3}\pi r^3 \Rightarrow R = 10\,r$$

Here,

r = Radius of small drop

R = Radius of big drop

Initial energy $E_i = 1000\,(4\pi r^2 S)$

final energy $E_f = 4\pi R^2 S = 100\,(4\pi r^2 S)$

$$\therefore \frac{E_f}{E_i} = \frac{1}{10} \Rightarrow E_f = \frac{1}{10}E_i$$

9. **(d)** We have, $h = \dfrac{2T\cos\theta}{r\rho g}$

Mass of the water in the capillary

$$m = \rho V = \rho \times \pi r^2 h = \rho \times \pi r^2 \times \frac{2T\cos\theta}{r\rho g} \Rightarrow m \propto r$$

$$\therefore \frac{m_1}{m_2} = \frac{r}{2r} \quad \text{or } m_2 = 2m_1 = 2\text{ M.}$$

10. **(c)** We know that $h = \dfrac{2T\cos\theta}{r\rho g}$

In a moving lift acceleration becomes

= (g + a) = (g + 2g) = 3 g

$$\therefore h' = \frac{2T\cos\theta}{r\rho(3g)} = \frac{h}{3}$$

11. **(c)** $V \propto r^2$

$$\therefore \frac{V_R}{V_{3R}} = \frac{1}{9}$$

12. **(a)** $E_i = 0^- A_i 0^-$ = surface energy per unit area

$= T\,a_i\ [\because 0^- = T] = T.4\pi r_i^2$

Now, $V_i = V_f \Rightarrow \frac{4}{3}\pi r_i^3 = 64\times\frac{4}{3}\pi r_f^3$

$\Rightarrow r_i^3 = 64 r_f^3 \Rightarrow r_i = 4r_F$

So, $E_f = 0^- A_f = T \times 64 \times$

$$4\pi r_f^2 = 256T\times\pi\frac{r_i^2}{16} = 16\pi T r_i^2$$

So, $\Delta E = E_f - E_i = 12\pi + T r_i^2$

$= 12\pi \times 0.075 \times 0.1^2$

$= 2.82 \times 10^{-4}$ J

13. **(c)** Reynold's number is given as $R_e = \dfrac{\rho V d}{\eta}$

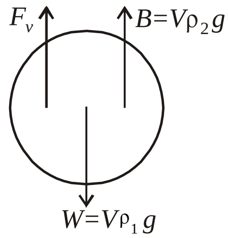
14. **(b)** For capillary rise, according, to Zurin's law

$h_1 r_1 = h_2 r_2$

$6 \times 1 = h_2 \times 2 \Rightarrow h_2 = 3$ cm

15. **(a)** The condition for terminal speed (v_t) is

Weight = Buoyant force + Viscous force



$$\therefore V\rho_1 g = V\rho_2 g + kv_t^2$$

$$\therefore v_t = \sqrt{\frac{Vg(\rho_1-\rho_2)}{k}}$$

16. **(c)** $m \times 10 = 2 \times 3 \times 10^{-2} \times \dfrac{10}{100}$

or $m = 6 \times 10^{-4}$ kg $= 6 \times 10^{-4} \times 10^3$ g = 0.6 g

17. **(a)** With the increase in temperature, the surface tension of liquid decreases and angle of contact also decreases.

18. **(b)** Terminal velocity, $v_0 = \dfrac{2\,r^2(\rho-\rho_0)g}{9\eta}$

$$= \frac{2\times(2\times10^{-3})^2\times(8-1.3)\times10^3\times9.8}{9\times0.83}$$

$= 0.07$ ms^{-1}

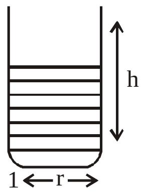
19. **(c)** R_{big} single drop $= 2^{\frac{1}{3}}\ r_{\text{small drop}}$

$U = T \times A$

$$\text{So, } \frac{U_{initially}}{U_{finally}} = \frac{2\times T\times 4\pi r^2}{T\times 4\pi R^2}$$

$$= \frac{2r^2}{\left(2^{1/3}r\right)^2} = 2^{\left(1-\frac{2}{3}\right)} = 2^{1/3}:1$$

20. **(d)**



Height in the capillary

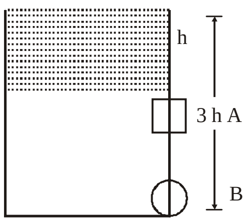
$$h = \frac{2s\cos\theta}{\rho g r} \quad \therefore \quad h \propto \frac{1}{r}$$

21. **(c)** Terminal velocity $v \propto \rho_s - \rho_l$

$$\therefore \frac{v_2}{v_1} = \frac{e_2-\sigma}{e_1-\sigma}$$

$$\therefore v_2 = \left[\frac{e_2-\sigma}{e_1-\sigma}\right]v$$

22. **(c)** Using equation of continuity



$A_1V_1 = A_2V_2$

$L^2\sqrt{2gh} = \pi r^2\sqrt{6gh}$

$L^4 gh = \pi^2 r^4 6gh$

$$\therefore L = (\pi)^{\frac{1}{2}}(r)(3)^{\frac{1}{4}}$$

23. (b) From Jurin's law, $h \propto \frac{1}{r}$ or, rh = constant

$r_1h_1 = r_2h_2$ $\quad A_1 = \pi r_1^2$

$\frac{r_1}{r_2} = \frac{h_2}{h_1}$ $\quad A_2 = \pi r_2^2$

$3 = \frac{h_2}{h_1} = \frac{h_2}{h_1}$ $\quad \frac{\pi r_1^2}{9} = \pi r_2^2$

$h_2 = 3h_1 = 3h$ $\quad \frac{r_1^2}{r_2^2} = 9 \Rightarrow \frac{r_1}{r_2} = 3h$

24. (a) According to question,

force F = 150 dyne 105×10^{-5} N and

Surface tension $T = 7 \times 10^{-2}$ N/m

∵ Circumference of the capillary × surface tension = upward force

$\therefore 2\pi rT = F$

or, $2\pi r = \frac{F}{T} = \frac{105 \times 10^{-5}}{7 \times 10^{-2}} = 15 \times 10^{-3}$ m

$= 1.5 \times 10^{-2} = 1.5$ cm

25. (d) Let radius of big drop = R
and of small drop = r
Volume of big drop = n(Volume of small drop)

$\frac{4}{3}\pi R^3 = n.\frac{4}{3}\pi r^3$

$R^3 = nr^3 \Rightarrow R = n^{1/3}.r$

Surface energy of n drops,

$E_2 = n \times 4\pi r^2 \times T$

Surface energy of big drop,

$E_1 = 4\pi R^2 T$

$\therefore \frac{E_2}{E_1} = \frac{nr^2}{R^2} = \frac{nr^2}{\left(n^{1/3}.r\right)^2}$

$= \frac{nr^2}{n^{2/3}.r^2} = n^{1/3}$ $[\because R = n^{1/3}.r]$

or, ratio of energy, $E_2 : E_1 = \sqrt[3]{n} : 1$

26. (c) Surface area of the liquid drop A = $4\pi R^2$
Let E be the surface energy of liquid drop.
When the drop splits into 512 droplets, the surface area becomes
$A_2 = 512 \times 4\pi r^2$ [r = radius of smaller drop]
Comparing the volumes of bigger and all smaller droplets, we get

i.e. $\frac{4}{3}\pi R^3 = 512 \times \frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{8}$

Total area of smaller droplets is

$A_1 = 512 \times 4 \times \pi \times \left(\frac{R}{8}\right)^2 = 8A$

Change in surface area $A_2 - A_1$

$= 4\pi\left(\frac{512 \times R^2}{64} - R^2\right)$

$= 4\pi\left(8R^2 - R^2\right) = 7R^2$

Surface energy, E = AT [A = area, T = tension]

So, $\frac{E_n}{E_0} = \frac{A_1 \times T}{A \times T} = \frac{8A}{A} = 8$

$\therefore$ $E_n = 8E$

27. (c) Rise of water in the capillary tube (h) is given by

$h = \frac{2T\cos\theta}{\rho g(R - r)}$

In the given case,

$\cos\theta = 1$ as $\theta = 0°$

$\therefore h = \frac{2T}{\rho g(R - r)}$

3 Kinetic Theory of Gases and Radiation

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(c)**
2. **(d)** Pressure exerted by a gas is given by
$P=\frac{1}{3}\frac{mn}{V}v^2$ or $P=\frac{1}{3}\rho v^{-2}$ $\therefore$ $P\propto\rho$
Therefore, pressure exerted by a gas is directly proportional to the density of the gas.
3. **(d)** Since it hits the plane wall parallel to y z – plane and it rebounds with same velocity, its y and z components of velocity do not change, but the x-component reverses the sign.
$\therefore$ Velocity after collision is ($-v_x$, v_y and v_z).
The change in momentum is
$-mv_x - mv_x = -2mv_x$
4. **(b)** As the temperature increases, the average velocity increases. So, the collisions are faster.
5. **(b)** $P_0=\frac{1}{3}\frac{mnc^2}{V}$ and $P'=\frac{1}{3}\left(\frac{m}{2}\right)\times n\times(2c)^2 = 2P_0$.
6. **(None)** Rate of change of momentum during collision
$=\frac{mv-(-mv)}{\Delta t} = \frac{2mv}{\Delta t}N$
so pressure
$P=\frac{N\times(2mv)}{\Delta t\times A} = \frac{10^{22}\times2\times10^{-26}\times10^4}{1\times1}=2\,N/m^2$
7. **(a)** The rms speed of the gas molecule is
$v=\sqrt{\frac{3RT}{M}}\Rightarrow v\propto\sqrt{T}$
$\therefore\ \frac{v_2}{v_1}=\sqrt{\frac{T_2}{T_1}}=\sqrt{\frac{(273+90)}{(273+27)}}=\frac{11}{10}$
$\therefore$ % change in rms speed is
$\%\Delta v=\frac{v_2-v_1}{v_1}\times100=\frac{\frac{11}{10}v_1-v_1}{v_1}\times100=10\%$
8. **(d)** The rms velocity of a gas molecule of mass m is
$v_{rms}=\sqrt{\frac{3KT}{m}}\Rightarrow v_{rms}\propto\frac{1}{\sqrt{m}}$
9. **(a)** RMS speed, $v_{rms}=\sqrt{\frac{3RT}{M}}\Rightarrow v_{rms}\propto\sqrt{\frac{T}{M}}$
$v_{rms,o}$ = 75% of $v_{rms,N}=\frac{3}{4}v_{rms,N}$
$\therefore\ \frac{v_{rms,o}}{v_{rms,N}}=\sqrt{\frac{T_o}{T_N}\times\frac{M_N}{M_o}}=\sqrt{\frac{T_o}{(273+287)}\times\frac{28}{32}}$
$\Rightarrow\left(\frac{3}{4}\right)^2=\frac{T_o}{20\times32}\Rightarrow T_o=360\,K=(360-273)°C=87°C$
10. **(d)** RMS speed of gas, $V_{rms}=\sqrt{\frac{3RT}{M}}$ $\therefore$ $V_{rms}\,\alpha\sqrt{T}$
Initial temperature , $T_i = 27+273=300k$
Final temperature, $T_f = 159+273=432$ k
% increase in RMS speed $=\frac{\sqrt{432}-\sqrt{300}}{\sqrt{300}}\times100$
$=\frac{20.78-17.32}{17.32}\times100 = 19.78\,H\approx20$
11. **(a)** rms speed is given by
$V_{rms}=\sqrt{\frac{3RT}{M}}$
$M_{He}=4$ $M_{O_2}=32$
$\sqrt{\frac{3RT_{He}}{M_{He}}}=\sqrt{\frac{3RT_{O_2}}{M_{O_2}}}$
$\frac{T_{He}}{T_{O_2}}=\frac{M_{He}}{M_{O_2}}=\frac{4}{3^2}=\frac{1}{8}$
12. **(d)** $V_{rms}=\sqrt{\frac{3RT}{M}}$ At $27°C=27+273=300K$
$\therefore V_{rms_1}=\sqrt{\frac{3R(300)}{M}}$ and $V_{rms_2}=\sqrt{\frac{3R(432)}{M}}$
at T' = 159 + 273 = 432
$\therefore$ Percentage increase in the r ms speed
$=\left(\frac{V_{rms_2}-V_{rms_1}}{V_{rms_1}}\right)\times100$
$=\frac{\sqrt{\frac{3R}{M}}\left(\sqrt{432}-\sqrt{300}\right)}{\sqrt{\frac{3R}{M}}\left(\sqrt{300}\right)}\times100=\left(\frac{20.78-17.32}{17.32}\right)\times100\simeq20\%$
13. **(b)** We have
$V_{rms}=\sqrt{\frac{3RT}{M_0}}$, where M_0 = molecular mass
$V_{rms}\propto\frac{1}{\sqrt{M_0}}$

$(V_{rms})_{H_2} = (V_{rms})_{0.2} \times \left(\frac{2}{32}\right)^{1/2} = 500 \times 4 = 2000$ m/s

14. **(b)** r.m.s velocity $= \frac{\sqrt{3RT}}{M}$

Equating r.m.s velocity expression for oxygen and hydrogen molecular.

$$\sqrt{\frac{3R \times 20}{2}} = \sqrt{\frac{3R \times T}{3_z}}$$

T = 320 K

15. **(a)** $V_1 = 1, V_2 = 2, V_3 = 3, V_4 = 4, V_5 = 5$

Root mean square speed is given by

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + V_4^2 + V_5^2}{5}} \quad ...(i)$$

$$= \frac{\sqrt{1^2 + 2^2 + 3^3 + 4^4 + 5^5}}{5}$$

average speed,

$$V_{avg} = \frac{V_1 + V_2 + V_3 + V_4 + V_5}{5} \quad ...(ii)$$

$$= \frac{1+2+3+4+5}{5}$$

$$\frac{V_{rms}}{V_{avg}} = \sqrt{\frac{1^1 + 2^2 + 3^2 + 4^2 + 5^2}{1+2+3+4+5}} \times \frac{5}{\sqrt{5}} = \frac{\sqrt{11}}{3} \text{ or } \sqrt{11:3}$$

16. **(a)** Pressure of an ideal gas

$$p = \frac{1}{3}\frac{mN}{V}(v^2)$$

where m = mass and V = speed

$$p' = \frac{1}{3}\frac{(2m)N}{V} \cdot \left(\frac{v}{2}\right)^2 = \frac{1}{2}\left\{\frac{1}{3}\frac{mN}{V}(v)^2\right\} = \frac{1}{2}p'$$

$$\therefore \quad \frac{p}{p'} = \frac{2}{1} \Rightarrow p : p' = 2 : 1$$

17. **(b)** $V_{rms} \propto \frac{1}{\sqrt{M}}$ [$\because$ T = const.]

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}} \Rightarrow \frac{0.5}{v_2} = \sqrt{\frac{2}{32}} \Rightarrow \frac{0.5}{v_2} = \frac{1}{4}$$

$\Rightarrow v_2 = 2$ km/s.

18. **(b)** $v_{rms} = \sqrt{\frac{3RT}{M_0}}$; $v_{rms} \propto \sqrt{\frac{T}{M_0}}$

as, T becomes double and M_0 becomes half

So, v_{rms} will become $\sqrt{4}$ times i.e. 2 times.

19. **(a)**

20. **(a)** Root mean square speed is given by

$$v_{rms} = \sqrt{\frac{3KT}{M}} \Rightarrow v_{rms} \propto \frac{1}{M}$$

$$\therefore \frac{(v_{rms})_{O_2}}{(v_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} \Rightarrow \frac{(v_{rms})_{O_2}}{(v_{rms})_{H_2}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

$$\Rightarrow (v_{rms})_{H_2} = 4 \times (v_{rms})_{O_2} \quad \left(\because (v_{rms})_{O_2} = 160\, m/s\right)$$

$= 4 \times 160 = 640$ m/s

21. **(c)** Lighter the molecule, higher the average speed.

22. **(c)** In equilibrium, the average kinetic energy of molecules of different gases will be equal. That is

$$\frac{1}{2}m_1\bar{v}_1^2 = \frac{1}{2}m_2\bar{v}_2^2 = \left(\frac{3}{2}k_BT\right)$$

23. **(b)** Average kinetic energy of gas molecues depends on the temperature of the gas as

$$\frac{1}{2}mv^2 = \frac{3}{2}k_BT$$

24. **(c)** **25.** **(a)**

26. **(a)** rms speed is given by

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$M_{He} = 4 = 32$

$$\sqrt{\frac{3RT_{He}}{M_{He}}} = \sqrt{\frac{3RT_{O_2}}{M_{O_2}}}$$

$$\frac{T_{He}}{T_{O_2}} = \frac{M_{He}}{M_{O_2}} = \frac{4}{3^2} = \frac{1}{8}$$

27. **(c)** The centre of mass of the gas molecules moves with uniform speed along with lorry. As there is no change in relative motion, the translational kinetic energy and hence the temperature of the gas molecules will remain same.

28. **(a)** $V_{rms} = \sqrt{\frac{3RT}{M}}$ M is least for hydrogen among the hydrogen, oxygen, nitrogen and carbon dioxide.

29. **(c)**

30. **(d)** Translational kinetic enegy $E = \frac{3}{2}k_BNT$

$$\therefore \quad PV = \frac{2}{3}E \qquad [\because k_B NT = PV]$$

31. **(c)** $(v_{rms})_{O_2} = (v_{rms})_{H_2} \therefore \sqrt{\frac{3RT_{O_2}}{M_{O_2}}} = \sqrt{\frac{3R(300)}{M_{H_2}}}$

$$T_{O_2} = 300 \times \frac{M_{O_2}}{M_{H_2}} = 300 \times \frac{32}{2} = 4800 \text{ K}$$

32. **(a)**

33. **(c)** $v_{oxg.} = \sqrt{\frac{3R \times 289}{32}} \left(v_{rms} = \sqrt{\frac{3RT}{M}}\right)$

$v_H = \sqrt{\frac{3R \times 400}{2}}$ so $v_H = 2230.59$ m/sec

34. **(b)** Root-mean square-velocity is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}} \text{ i.e., } v_{rms} \propto \sqrt{\left(\frac{T}{M}\right)}$$

$\therefore \frac{(v_{rms})O_2}{(v_{rms})H_2} = \sqrt{\left[\frac{T_{O_2}}{T_{H_2}} \times \frac{M_{H_2}}{M_{O_2}}\right]} = \frac{1}{2}$

$\therefore (v_{rms})O_2 = (v_{rms})H_2 \times \frac{1}{2} = \frac{1930}{2} = 965\,m/s$

35. **(a)**

36. **(a)** The average kinetic energy per molecule of any ideal gas is always equal to $\left(\frac{3}{2}\right)k_BT$. It depends only on the temperature and is independent of the mass and nature of the gas.

37. **(a)** $\frac{3}{2}k_BT = K_{av}$ where K_{av} is the average kinetic energy of the proton.

$\therefore \frac{2K_{av}}{3k_B} = \frac{2\times 4.14\times 10^{-14}\,J}{3\times 1.38\times 10^{-23}JK^{-1}} = 2\times 10^9 K.$

38. **(a)** Volume $= \frac{\text{mass}}{\text{density}} = \frac{1}{4}\,m^3$

$K.E = \frac{5}{2}PV = \frac{5}{2}\times 8\times 10^4 \times \frac{1}{4} = 5\times 10^4\,J$

39. **(d)** The kinetic energy of a gas molecule is

$K = K.E/\text{molecule} = \frac{f}{2}k_BT$

For diatomic gas, $f_1 = 7$ (At high temperature)
$f_2 = 5$ (At low temperature)

$\therefore \frac{k_1}{k_2} = \frac{f_1}{f_2} = \frac{7}{5}$

40. **(c)** For ideal gas,
$V = 3m^3, P = 3\times 10^5$ pa
$\therefore$ The energy of the gas is

$E = \frac{3}{2}pV = \frac{3}{2}\times 3\times 10^5 \times 3 = 13.5\times 10^5\,J$

41. **(c)** Temperature, T = 77°C = 273 + 77 = 350 K
Boltzman constant, $K_B = 1.38\times 10^{-23}$ J/K

Energy is given by, $E = \frac{3}{2}K_BT$

$E = \frac{3}{2}\times 1.38\times 10^{-23}\times 350$

$E(eV) = \frac{1.5\times 1.38\times 350\times 10^{-23}}{1.6\times 10^{-19}}$

$E(eV) = 4.52\times 10^{-2}\,eV$

42. **(c)** Average kinetic energy of a gas is given by

$E = \frac{1}{2}KT$

$\frac{E_2}{E_1} = \frac{T_2}{T_1} \Rightarrow E_2 = \frac{T_2 E_1}{T_1} = \frac{(127+273)\times 3.3\times 10^{-20}}{(27+273)}$

$= \frac{400\times 3.3\times 10^{-20}}{300} = 4.4\times 10^{-20}\,J$

43. **(d)** Kinetic energy is given by

$K.E = \frac{3}{2}RT$

since, PV = n RT
Here n = 1

$E = \frac{3}{2}PV \Rightarrow P = \frac{2E}{3V}$

44. **(b)** Pressure, $P = \frac{2}{3}\frac{E}{V} = \frac{2}{3}\times\frac{4.5\times 10^5}{10\times 10^{-3}} = 30\times 10^6\,Nm^{-2}$

45. **(c)** As we know that,

Kinetic energy, per unit volume, $E = \frac{1}{2}\rho V^2$ ...(i)

and pressure per unit volume, $p = \frac{1}{3}\rho V^2$ ...(ii)

From Eqs. (i) and (ii), we get

$\therefore p = \frac{1}{3}2E \Rightarrow p = \frac{2}{3}E$

Hence, $p \propto E$.

46. **(a)** Average kinetic energy

$E = \frac{3}{2}K_BT \quad \therefore E \propto T$

Therefore, at same temperature average kinetic energy of O_2 molecule will be same as E.

47. **(d)** Given, Energy 0.69 $eV = 0.69\times 1.6\times 10^{-19}$ V

Also average translational kinetic energy $= \frac{3}{2}kT$

$= \frac{3}{2}\times 1.38\times 10^{-23}T = 0.6\times 1.6\times 10^{-19}$

$\Rightarrow T = 5333\,K = (5333-273)°C = 5060°C.$

48. **(a)** The average kinetic energy of molecules is given by

$KE_{av} = \frac{3}{2}KT$. So it depends only on temperature

$KE_1 : KE_2 = 1:1$

49. **(b)** For monoatomic gas, f = 3

$\therefore (C_p)_1 = \left(1+\frac{f}{2}\right)R = \left(1+\frac{3}{2}\right)R = \frac{5R}{3}$

For diatomic gas, f = 5

$\therefore (C_p)_2 = \left(1+\frac{f}{2}\right)R = \left(1+\frac{5}{2}\right)R = \frac{7R}{2}$

$\therefore \frac{(C_p)_1}{(C_p)_2} = \frac{\frac{5R}{2}}{\frac{7R}{2}} = 5:7$

50. **(d)**

51. **(c)** Specific heat capacity at constant volume = x % x specific heat capacity at constant pressure
$C_V = x\,\%\times C_P$

$\frac{3}{2}R = x\%\times\frac{5}{2}R \Rightarrow x = 60$

52. **(c)** The number of rotational degrees of freedom of a monoatomic molecule is zero.

53. **(a)** Given, $\gamma = \frac{5}{3}$

$\frac{C_p}{C_v} = \frac{5}{3}$

Percentage of heat energy needed to increase the internal energy

$= \frac{\Delta U}{Q} \times 100 = \frac{nC_v \Delta T}{nC_p\, dT} \times 100 = \frac{C_v}{C_p} \times 100$

$= \frac{3}{5} \times 100 = 60\%$

54. **(a)** 1 additional vibrational mode adds two DOF

So, $f_{net} = 5 + 2 = 7$

So, $\gamma = 1 + \frac{2}{7} = \frac{9}{7} \Rightarrow \frac{C_P}{C_V} = \frac{9}{7} \Rightarrow 49C_P^2 = 81C_V^2$.

55. **(b)** A diatomic molecule has 1 vibrational DOF.

56. **(b)** A simple pendulum, has 2 degree of freedom :
1 translational due to motion of bob.
and 1 rotational about string of pendulum.

57. **(a)** For a gas with n degree of freedom.

Specific heat at constant volume, $C_v = \frac{n}{2}R$

Specific heat at constant pressure,

$C_p = \left(\frac{n}{2}+1\right).R \quad \therefore \quad \frac{C_p}{C_v} = \frac{\left(\frac{n}{2}+1\right)R}{\frac{n}{2}.R} = \left(\frac{n+2}{n}\right)$

58. **(c)** As, $v = \sqrt{\frac{\gamma P}{\rho}}$

$\Rightarrow \gamma = \frac{v^2\rho}{p} = \frac{330 \times 330}{10^5} \times \frac{1400}{1089}$

$\gamma = 1.4$

So, $\gamma = \frac{C_P}{C_V} = 1.4$

$\left[\because \gamma = 1 + \frac{2}{f};\quad 1.4 = 1 + \frac{2}{f};\quad \therefore f = 5\right]$

Since, for diatomic gas, the volume of γ is 1.4.
Hence, the degree of freedom for diatomic gas is equal to 5

59. **(d)** Translational degrees of freedom = 3
Rotational degrees of freedom = 2

60. **(a)** $C_v = \frac{nR}{2}, C_p = \frac{(n+2)R}{2} \quad (\because C_p - C_v = R)$

$\frac{C_v}{C_p} = \frac{n}{n+2}$

61. **(b)** Law of equipartition of energy states that the energy for each degree of freedom in thermal equilibrium is $\frac{1}{2}k_BT$. Thus each vibrational mode gives 2 degrees of freedom (kinetic and potential energy modes) corresponding to the energy $2 \times \frac{1}{2}k_BT = k_BT$

62. **(c)** A fly moving in a room has three degrees of freedom, because it is free to move in space.

63. **(d)** We know that $C_V = \frac{f}{2}R$

Here, $f = 5 + 2 = 7$ [$\because$ 1 Vib.mode = 2DOF]

So, $C_V = \frac{7}{2}R$.

64. **(c)**

65. **(c)** No. of degree of freedom = $3K - N$
where K is no. of atom and N is the number of relations between atoms. For triatomic gas,
$K = 3, N = 3$

66. **(c)** Moles of He = $\frac{2}{4} = \frac{1}{2}$

Molecules = $\frac{1}{2} \times 6.02 \times 10^{23} = 3.01 \times 10^{23}$

As there are 3 degrees of freedom corresponding to 1 molecule of a monatomic gas.

$\therefore$ Total degrees of freedom = $3 \times 3.01 \times 10^{23} = 9.03 \times 10^{23}$

67. **(a)** As degree of freedom is defined as the number of independent variables required to define body's motion completely. Here f = 2(1 Translational + 1 Rotational).

68. **(c)** $C = \frac{Q}{m\Delta T}$; If $\Delta T = 0, C = \infty$ and if $Q = 0$, then $C = 0$

69. **(a)** Work done is to be done is expanding the gas at constant pressure.

70. **(c)** A rigid diatomic molecule has 5 degrees of freedom total internal energy of one mole of rigid diatomic gas is

$U = \frac{5}{2}k_BT \times N_A = \frac{5}{2}RT \quad (\because R = k_BN_A)$

71. **(d)** Both are diatomic gases and $C_p - C_v = R$ for all gases.

72. **(b)** For a monoatomic gas, the average energy of a molecule at temperature T is $\frac{3}{2}k_BT$.

$\therefore$ Internal energy $U = \frac{3}{2}RT$

$C_V = \frac{dU}{dT} = \frac{3}{2}R$

For an ideal gas, $C_P - C_V = R$

$\therefore \quad C_P = \frac{5}{2}R$ and $\gamma = \frac{C_P}{C_V} = \frac{5}{3}$

73. **(d)** $\frac{C_P}{C_V} = \gamma = 1 + \frac{2}{n}$

74. **(c)** P-V diagram of the gas is a straight line passing through origin. Hence $P \propto V$ or PV^{-1} = constant
Molar heat capacity in the process PV^x = constant is

$C = \frac{R}{\gamma - 1} + \frac{R}{1-x}$; Here $\gamma = 1.4$ (For diatomic gas)

$\Rightarrow C = \frac{R}{1.4 - 1} + \frac{R}{1+1} \Rightarrow C = 3R$

75. **(c)**

76. **(c)** We know that ratio of specific heats,

$\gamma = 1 + \frac{2}{n}$ or $n = \frac{2}{\gamma - 1}$ [where n = Degree of freedom]

77. **(b)** Number of translational degrees of freedom are same for all types of gases.

78. **(c)** Since $\frac{R}{C_V} = 0.67 \Rightarrow C_V = \frac{3}{2}R$, hence gas is monoatomic.

79. **(b)** $\gamma = \frac{C_P}{C_V} = \frac{15}{10} = \frac{3}{2} \Rightarrow C_V = \frac{2}{3}C_P$

$C_P - C_V = \frac{R}{J} \Rightarrow C_P - \frac{2}{3}C_P = \frac{R}{J}$

$\Rightarrow \frac{C_P}{3} = \frac{R}{J} \Rightarrow C_P = \frac{3R}{J}$

80. **(c)** **81.** **(c)** **82. (c)** **83.** **(a)**

84. **(c)** Water has highest specific heat capacity and hence it is used as a coolant in car radiators as well as heater in hot water bags.

85. **(a)** Initially black body absorbs all the incident energy and so it is the darkest one. Black body radiates maximum energy if conditions are same.

86. **(b)** According to Wein's displacement law, λT = const.

87. **(c)** Temperature of surface of the sun was found out to be 5791 k approx by using Stefan's law i.e.

$E = \sigma T^4$

where, E = amount of heat energy radiated/ sec area.

σ = Stefan - Boltzmann constant.

88. **(b)** Black body has maximum radiated energy at same temperature.

89. **(c)** When light incident on pin hole, enters into the box and suffers successive reflection at the inner wall. At each reflection some energy is absorbed. Hence the ray one it enters the box can never come out and pin hole acts like a perfect black body.

90. **(a)**

91. **(a)**

92. **(c)** $E \propto T^4$ (Stefan's law)

93. **(a)** $\frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 \Rightarrow \frac{E}{E_2} = \left(\frac{273+0}{273+273}\right)^4 \Rightarrow E_2 = 16E.$

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(d)** Pressure on the wall due to force exerted by molecule on walls due to its rate of transfer of momentum to wall. In an ideal gas, when a molecule collides elastically with a wall, the momentum transferred to each molecule will be twice the magnitude of its normal momentum is 2 *mv*. For the wall *EFGH*, absorbs those molecules. Which strike to it so rate of change in momentum to it become only *mv* so the pressure of *EFGH* would half of *ABCD*.

2. **(b)** As the relative velocity of molecule with respect to the walls of container does not change in rocket, due to the mass of a molecule is negligible with respect to the mass of whole system and system of gas moves as a whole and (g = 0) on molecule energy where.

Hence pressure of the gas inside the vessel, as observed by us, on the ground remain the same.

3. **(c)** $v_{rms} = \sqrt{\frac{3RT}{M_0}}$ and, $v_{rms_f} = 2v_{rms_i} \Rightarrow \sqrt{\frac{T_f}{T_i}} = 2$

$\Rightarrow T_f = 4T_i = 1200$ k

Now, $T_f = 1200K, T_i = 300K, n = \frac{14}{28} = \frac{1}{2}$

So, $Q = nC_v\Delta T = \frac{1}{2} \times \frac{5R}{2} \times 900$

Q = 9360 J

4. **(d)** RMS speed of gas molecule at NTP

$V_{rms} = \sqrt{\frac{3RT}{M}}$

Average speed of gas molecule at NTP

$V_{avg} = \sqrt{\frac{8RT}{\pi M}}$

The required ratio

$\Rightarrow \frac{V_{rms}}{V_{avg}} = \sqrt{\frac{3\pi}{8}}$

5. **(b)** Mean free path $\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$ $\therefore \lambda \propto T$

$\frac{1500d}{\lambda} = \frac{273}{373} \therefore \lambda = 2049d$

6. **(c)** $v_{rms} = v_e$

$\sqrt{\frac{3RT}{M}} = 11.2 \times 10^3 \Rightarrow \sqrt{\frac{3 \times 8.314 \times T}{2 \times 10^{-3}}} = 11200$

$\Rightarrow$ T = 10058 K ~ 10^4K

7. **(a)** $V_{rms} = \sqrt{\frac{3RT}{M}}$ and $V_{av} = \sqrt{\frac{8RT}{\pi M}}$

$\therefore \sqrt{\frac{3RT}{M}} = \left(1 + \frac{5}{x}\right)^{\frac{1}{2}} \sqrt{\frac{8RT}{\pi M}} \Rightarrow \frac{3 \times 22}{7 \times 8} = 1 + \frac{5}{x}$

$\therefore$ x = 28

8. **(d)** Given, temperature of oxygen molecules, $T = 27°C$

Pressure of oxygen

$l_{mean} = \frac{RT}{\sqrt{2}\pi d^2 N_A P} = \frac{1.38 \times 300 \times 10^{-23}}{\sqrt{2} \times 3.14 \times (0.3 \times 10^{-9})^2 \times 1.01 \times 10^5}$

$= 102 \times 10^{-9}$ m $= 102$ nm

9. **(None)**

10. **(a)** At 0 K, molecular motion stops. Hence, kinetic energy of molecules becomes zero.

11. **(d)** $v_{rms} \propto \sqrt{T}$

As temperature increases from 300 K to 1200 K that is four times, so, v_{rms} will be doubled.

12. **(c)** Given,

R.M.S. speed of chlorine molecule,

$V_{C1} = 490$ m/s

R.M.S. Speed,

$$v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{rms} \propto \sqrt{\frac{1}{M}}$$

$$\therefore \frac{v_{Ar}}{v_{Cl}} = \sqrt{\frac{M_{Cl}}{M_{Ar}}}$$

$$\Rightarrow \frac{v_{Ar}}{490} = \sqrt{\frac{70.9}{39.9}} = 1.33$$

$\Rightarrow v_{Ar} = 1.33 \times 490 = 651.7$ m/s

13. **(b)** Using, $\tau = \frac{1}{2n\pi d^2 V_{avg}}$, where $V_{av} = \sqrt{\frac{3RT}{M_0}}$

$$\therefore t \propto \frac{\sqrt{T}}{P} \quad \left[\therefore n = \frac{\text{no.of molecules}}{\text{Volume}}\right]$$

or, $\frac{t_1}{6\times10^{-8}} = \frac{\sqrt{500}}{2P} \times \frac{P}{\sqrt{300}} \approx 4\times10^{-8}$

14. **(c)** As $V_1 = 1.5 V_2$

$$\sqrt{\frac{3RT}{m_1}} = \sqrt{\frac{8RT}{\pi m_2}} \times 1.5$$

$$\frac{3}{m_1} = \frac{8}{\pi}\frac{1}{m_2} \times 2.25$$

$$\frac{m_1}{m_2} = \frac{3\pi}{2.25\times8} = 0.52$$

15. **(c)** $(V_{rms})_{Neon} = (V_{rms})_{He}$

$$\Rightarrow \sqrt{\frac{3R\times T}{20.2}} = \sqrt{\frac{3R\times240}{4}} \Rightarrow \frac{T}{20.2} = 60 \Rightarrow T = 1212\,K$$

16. **(b)**

17. **(c)** Given,

Mass of first uranium isotope, $M_1 = 238$ units

Mass of second uranium isotope, $M_2 = 235$ units

RMS speed of gas molecules,

$$v_{rms} = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v_{rms} \propto \frac{1}{\sqrt{M}}$$

The required percentage,

$$\frac{v_{rms_2} - v_{rms_1}}{v_{rms_2}} \times 100 = \frac{\sqrt{M_1} - \sqrt{M_2}}{\sqrt{M_1}} \times 100$$

$$= \frac{\sqrt{238} - \sqrt{235}}{\sqrt{238}} \times 100 = \frac{15.427 - 15.329}{15.427} \times 100$$

$$= \frac{0.097}{15.427} \times 100 = 6.34\times10^{-3} \times 100 = 0.634 \approx 0.64.$$

18. **(c)** $v_{rms} = \sqrt{\frac{3\rho v}{\text{mass of the gas}}}$

19. **(d)** RMS speed of gas molecule at NTP

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

Average speed of gas molecule at NTP

$$V_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

The required ratio

$$\Rightarrow \frac{V_{rms}}{V_{avg}} = \sqrt{\frac{3\pi}{8}}$$

20. **(a)**

21. **(a)** Ratio of the diameter of the molecules

$$\frac{d_1}{d_2} = \frac{1}{2}$$

Mean free path is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 h}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{d_2^2}{d_1^2} = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$$

22. **(a)** Given, temperature, T = 300 K

Pressure, $p = 1\text{atm} = 1.01\times10^5\ \text{N/m}^2$

Radius, $r = \frac{\text{diameter}}{2} = 0.6\times10^{-10}$ m,

$k = 1.38\times10^{-23}\ \text{JK}^{-1}$ is Boltzmann constant.

Mean free path is given by

$$\lambda = \frac{k}{\sqrt{2}\,\pi d^2 p}$$

$$\lambda = \frac{1.38\times10^{-23}\times300}{1.414\pi\times(1.2\times10^{-10})^2\times1.01\times10^5} = \frac{0.2}{\pi}\times10^{-5}\ \text{m}$$

23. **(d)** Given, we know that mean free path is given by,

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 \cdot p} \Rightarrow \lambda \propto \frac{T}{p} \qquad \therefore \frac{\lambda_1}{\lambda_2} = \frac{T_1}{T_2} \times \frac{p_2}{p_1}$$

$$\Rightarrow \frac{10^{-7}}{\lambda_2} = \frac{300}{400} \times \frac{200}{600} \Rightarrow \lambda_2 = 10^{-7} \times \frac{4\times6}{3\times2} = 4\times10^{-7}\,m$$

24. **(b)** Mean free path

$$L = \frac{kT}{\sqrt{2}\pi d^2 P}$$

T = Temperature

d = Diameter

P = Pressure

As radius gets doubled. So is diameter.

New mean free path will be

$$L = \frac{K2T}{\sqrt{2}\pi(2d)^2 2P} = \frac{kT}{4\times\sqrt{2}\pi d^2 P} = \frac{L}{4}$$

The mean free path = $\frac{L}{4}$.

25. **(a)** Mean free path,

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2},$$

n = number density, d = diameter

$$\Rightarrow d^2 = \frac{1}{\sqrt{2}\pi n\lambda} = \frac{1\times\pi}{\sqrt{2}\times\pi\times 2\sqrt{2}\times 10^8\times 10^{-2}}$$

$$\Rightarrow d = \frac{1}{2}\times 10^{-3}\text{cm} \Rightarrow d = 5\times 10^{-4}\text{cm}$$

26. **(b)** Mean free path in a gas is 100 times the interatomic distance.

27. **(a)** Mean free path, $\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

where,
n = number of molecules per unit volume,
d = diameter of the molecules

28. **(d)**

29. **(d)** Given, temperature of oxygen molecules, $T = 27°C$
Pressure of oxygen

$$l_{mean} = \frac{RT}{\sqrt{2}\pi d^2 N_A P} = \frac{1.38\times 300\times 10^{-23}}{\sqrt{2}\times 3.14\times(0.3\times 10^{-9})^2\times 1.01\times 10^5}$$

$$= 102\times 10^{-9}\text{ m} = 102\text{ nm}$$

30. **(a)** Average KE per molecule, K.E = $\frac{3}{2}$kT

Average Kinetic energy per molecule of the gas is independent of mass.

$$\frac{K.E_{Ar}}{K.E_H} = \frac{1}{1}$$

31. **(b)** Total degree of freedom,
f = 3 + 3 + (4 × 2) [∵ 1 vibrational mode is equal to 2DOF]
⇒ f = 14
Work done in adiabatic process is given by

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{nR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{1\times 8.314\times(-10)}{\left(\frac{8}{7}-1\right)} = -582\text{ J} \qquad \left(\because \gamma = 1 + \frac{2}{f} = \frac{8}{7}\right)$$

32. **(d)** Here, initial internal energy = Final internal energy

$$\frac{F_1}{2}n_1RT_1 + \frac{F_2}{2}n_2RT_2 = \frac{F_1}{2}n_1RT + \frac{F_2}{2}n_2RT$$

or $$T = \frac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}$$

33. **(d)** Here degree of freedom, $f = 3 + 3 = 6$ for triatomic non-linear molecule.
Internal energy of a mole of the gas at temperature T,

$$U = \frac{f}{2}nRT = \frac{6}{2}RT = 3RT$$

34. **(c)** $V = 25\times 10^{-3}\text{ m}^3$, N = 1 mole of O_2
T = 300 K
V_{rms} = 200 m/s

$$\therefore \lambda = \frac{1}{\sqrt{2}N\pi r^2}$$

Average time $\frac{1}{\tau} = \frac{\langle V\rangle}{\lambda} = 200.N\pi r^2.\sqrt{2}$

$$= \frac{\sqrt{2}\times 200\times 6.023\times 10^{23}}{25\times 10^{-3}}.\pi\times 10^{-18}\times 0.09$$

The closest value in the given option is = 10^{10}

35. **(a)** Here, $nC_v\Delta T = \frac{1}{2}mv^2$

For monoatomic gas, $C_v = \frac{3}{2}R$

$$\Rightarrow \Delta T = \frac{1}{2}\left(\frac{m}{n}\right)\cdot\frac{2}{3R}(30)^2$$

Lets assume mass per mole = 4 kg/mol

$$= \frac{1}{2}\times 4\times\frac{2}{3R}\times 30\times 30 = \frac{3600}{3R} \qquad \therefore X = 3600.$$

36. **(a)** According to question VT = K and, TV^{x-1} = cons.
$\therefore x - 1 = 1 \Rightarrow 1 - x = -1$

$\because C = \frac{R}{1-x} + C_V$ (For polytropic process)

$$C = \frac{R}{-1} + \frac{3R}{2} = \frac{R}{2} \quad \therefore \Delta Q = nC\,\Delta T = \frac{R}{2}\times\Delta T$$

[here, n = 1 mole]

37. **(c)** Heat transferred,
$Q = nC_v\Delta T$ as gas in closed vessel
To double the rms speed, temperature should be 4 times
i.e., T' = 4T as $v_{rms} = \sqrt{3RT/M}$

$$\therefore Q = \frac{15}{28}\times\frac{5\times R}{2}\times(4T - T)$$

$$\left[\therefore \frac{CP}{CV} = \gamma_{diatomic} = \frac{7}{5} \,\&\, C_p - C_v = R\right]$$

or, Q = 10000 J = 10 kJ

38. **(b)** Let S_p and S_v be the specific heat per unit mass of the gas at constant pressure and volume.
At constant pressure, heat required

$$\Delta Q_1 = mS_p\times\Delta T \Rightarrow 160 = mS_p\times 50 \qquad ...(i)$$

At constant volume, heat required

$$\Delta Q_2 = mS_v\Delta T \Rightarrow 240 = mS_v\times 100 \qquad ...(ii)$$

Dividing (i) by (ii), we get

$$\frac{160}{240} = \frac{S_p}{S_v} \Rightarrow \frac{S_p}{S_v} = \frac{4}{3} \Rightarrow \frac{C_p}{C_v} = \frac{4}{3} \qquad \left[\because S = \frac{C}{M_0}\ S\,\alpha\,C\right]$$

$$\gamma = \frac{C_p}{C_v} = \frac{4}{3} = 1 + \frac{2}{f} \qquad \text{(Here, } f = \text{degree of freedom)}$$

$$\Rightarrow f = 6.$$

39. **(c)** Amount of heat required (Q) to raise the temperature at constant volume

$Q = nC_v\Delta T$ ...(i)

Amount of heat required (Q_1) at constant pressure

$Q_1 = nC_P\Delta T$ ...(ii)

Dividing equation (ii) by (i), we get

$$\therefore \frac{Q_1}{Q} = \frac{C_p}{C_v} \Rightarrow Q_1 = (Q)\left(\frac{7}{5}\right) \left(\because \gamma = \frac{C_p}{C_v} = \frac{7}{5}\right)$$

40. **(a)** $C_p - C_v = R \Rightarrow C_p = C_v + R$

$$\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = \frac{C_v}{C_v} + \frac{R}{C_v}$$

$$\Rightarrow \gamma = 1 + \frac{R}{C_v} \Rightarrow \frac{R}{C_v} = \gamma - 1 \Rightarrow C_v = \frac{R}{\gamma - 1}$$

41. **(c)** From Mayer's formula,

$C_p - C_v = R$

$MC_p - MC_v = R$

$C_p - C_v = \frac{R}{M}$

42. **(d)** $C_P = \frac{7}{2}R;\ C_V = C_P - R = \frac{7}{2}R - R = \frac{5}{2}R$

$$\frac{C_P}{C_V} = \frac{7/2\,R}{5/2\,R} = \frac{7}{5}$$

43. **(a)** At constant pressure,

$$dU = nC_V dT = n\frac{5}{2}RdT$$

$$dQ = nC_P dT = n\left(\frac{7R}{2}\right)dT$$

and $dW = nRdT$

$$\therefore dU : dQ : dW = \frac{5}{2} : \frac{7}{2} : 1 = 5 : 7 : 2.$$

44. **(c)** Given, $U = 3PV + 4$

$$\because \quad U = \frac{f}{2}nRT = \frac{f}{2}PV$$

$$\therefore \quad \frac{f}{2}PV = 3PV + 4 \quad \text{or,} \quad f = 6 + \frac{8}{PV}$$

So $f_{min} = 6$

Hence the gas is polyatomic.

45. **(a)** $(C_V)_{mix}$ of two mixed is given as,

$$(C_V)_{mix} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

$$= \frac{1.\frac{3R}{2} + 3.\frac{5R}{2}}{1+3} = \frac{9R}{4} = \frac{\alpha^2}{4}R \Rightarrow \alpha = 3$$

46. **(c)** $m_{H_2} = m_{N_2} = 2:3, T_{mixture} = T_{H_2} = T_{N_2} = 30°C$

$$\therefore \quad \langle KE \rangle / \text{molecule} = \frac{f}{2}K_B T$$

$$\therefore \quad \frac{\langle KE \rangle_{H_2}/\text{molecule}}{\langle KE \rangle_{N_2}/\text{molecule}} = \frac{\frac{3}{2}K_B T_{H_2}}{\frac{3}{2}K_B T_{N_2}} = 1:1$$

47. **(d)** According to question, the translation KE of a molecule of gas = $\frac{3}{2}kt$

According to the question,

$$\frac{3}{2}kt = eV \Rightarrow \frac{3}{2}kt = 10e$$

$$\Rightarrow \quad T = \frac{20e}{3K_B} = \frac{20 \times 1.6 \times 10^{-19}}{3 \times 1.38 \times 10^{-23}} = 77.3 \times 10^3 K$$

48. **(c)** $n_1 = 1$ mole, $n_2 = 1$ mole, $\delta_1 = \frac{7}{5}, \delta_2 = \frac{4}{3}$

$$\therefore \frac{n_1 + n_2}{\delta_{mix} - 1} = \frac{n_1}{\delta_1 - 1} + \frac{n_2}{\delta_2 - 1} \Rightarrow \frac{1+1}{\delta_{mix} - 1} = \frac{1}{\frac{7}{5} - 1} + \frac{1}{\frac{4}{3} - 1}$$

$$\Rightarrow \frac{2}{\delta_{mix} - 1} = \frac{5}{2} + \frac{3}{1} = \frac{11}{2} \quad \therefore \delta_{mix} = \frac{15}{11}$$

49. **(a)**

50. **(b)** $n_1 = 2$ moles, $T_1 = 27°c = 300K$

For monoatomic gas, $f_1 = 3$

$$\therefore \quad U = \frac{n_1 f_1 R T_1}{2} = \frac{2 \times 3 \times R \times 300}{2} = 900R$$

For diatomic gas, $f_2 = 5$

$n_2 = 3$moles, $T_2 = 127°c = 400$ K

$$U' = \frac{n_2 f_2 R T_2}{2} = \frac{3 \times 5 \times R \times 400}{2} = 3000R$$

$$\therefore \quad \frac{U'}{U} = \frac{3000R}{900R} = \frac{10}{3} \qquad \therefore \quad U' = \frac{10U}{3}$$

51. **(c)** The mean kinetic energy of a polyatomic gas per molecule per degree of freedom is <K.E> /dof/molecule $= \frac{1}{2}KT$

$$\therefore \quad \langle K.E \rangle/\text{molecule} = \left(\frac{1}{2}KT\right)f = \frac{nKT}{2}$$

52. **(c)** For a gas, $Q = 100J$, $\Delta U = 60J$

$$\therefore \frac{Q}{\Delta U} = \frac{100}{60} = \frac{5}{3} \Rightarrow \frac{nc_p \Delta T}{nc_v \Delta T} = \frac{5}{3} \Rightarrow \gamma = \frac{5}{3} = 1.66$$

$\therefore$ The gas is monoatomic.

53. **(b)** For diatomic gas, $f_1 = 5$

$$C_{v_1} = \frac{f_1 R}{2} = \frac{5R}{2}$$

$$C_{p_1} = C_v + R = \frac{5R}{2} + R\frac{7R}{2} = C$$

For monoatomic gas, $f_2 = 3$

$$C_{v_2} = \frac{f_2 R}{2} = \frac{3R}{2}$$

$$\text{Now, } \frac{C}{C_{v_2}} = \frac{\frac{7R}{2}}{\frac{3R}{2}} = \frac{7}{3} \qquad \therefore C_{v_2} = \frac{3C}{7}$$

54. **(c)** Total internal energy $u = n\frac{f}{2}RT$ for diatomic gas

$f = 5$, $n = 4$ moles and $T = 27°C = 27 + 273 = 300$ K given

Therefore $u = 4 \times \frac{5}{2} \times 8.31 \times 300 = 24.39 \times 10^3 J$

or, $u = 24.39$ kJ

55. **(a)** From Wein's displacement law, $\lambda_m \times T$ = constant

P – max. intensity is at violet $\Rightarrow \lambda_m$ is minimum $\Rightarrow$ temp maximum

R – max. intensity is at green ⇒ λ_m is moderate ⇒ temp moderate
Q – max. intensity is at red ⇒ λ_m is maximum ⇒ temp minimum
i.e., $T_p > T_R > T_Q$

56. **(c)**

57. **(d)** From Wien's law, $\lambda_m T$ = constant, where T is the temperature of black body and λ_m is the wavelength corresponding to maximum energy of emission. Energy distribution of black body radiation is given below:

(i) U_1 and U_2 are not zero because a black body emits radiations of nearly all wavelengths.

(ii) Since U_1 corresponding to lower wavelength, U_3 corresponds to higher wavelength and U_2 corresponds to medium wave length, hence $U_2 > U_1$.

58. **(c)** According to Stefan's Law, the rate of loss of heat is

$$\frac{Q}{t} = \sigma A(T_1^4 - T_2^4) \times e$$

here $\sigma = 5.67 \times 10^{-8} J/m^2 \times sec.K^2$,
$T_1 = 527 + 273 = 800K$,
$T_2 = 27 + 273 = 300K$ & $A = 200 \times 10^{-4} m^2$

So, $\frac{Q}{t} = 5.67 \times 10^{-8} \times 2 \times 10^{-2}$
$[(800)^4 - (300)^4] \times 0.4$
$\cong$ 182 joule

59. **(d)** $E = \sigma \times$ area $\times T^4$; T increases by a factor $\frac{3}{2}$.

Area increases by a factor $\frac{1}{4}$.

60. **(c)** Power radiated by the sun at t°C
$= \sigma(t + 273)^4 4\pi r^2$
Power received by a unit surface

$$= \frac{\sigma(t+273)^4 4\pi r^2}{4\pi R^2} = \frac{r^2 \sigma(t+273)^4}{R^2}$$

Exercise 3 : PREVIOUS YEARS MCQs

1. **(a)** We have $PV = nRT \Rightarrow V = \frac{nR}{P}T$

$V = mT$, where m = slope of $v - T_{curve} = \frac{nR}{P}$

Now, as $m_2 > m_1 \Rightarrow \frac{nR}{P_2} > \frac{nR}{P_1} \Rightarrow P_2 < P_1$

2. **(d)** We know that $C_v = \frac{f}{2}R$

Here, $f = 5 + 2$ [$\because$ 1 Vib.mode = 2DOF]
$= 7$

So, $C_v = \frac{7}{2}R$

3. **(a)** Mean free path, $\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

where,
n = number of molecules per unit volume,
d = diameter of the molecules

4. **(d)** $T_{O_2} = 47 + 273 = 320$ K

$v_{H_2} = v_{O_2}$

$$\sqrt{\frac{3RT_{H_2}}{m_{H_2}}} = \sqrt{\frac{3RT_{O_2}}{m_{O_2}}}$$

$$\because v_{rms} = \sqrt{\frac{3RT}{m}} \Rightarrow \sqrt{\frac{T_{H_2}}{2}} = \sqrt{\frac{320}{32}}$$

$$\Rightarrow T_{H_2} = \frac{320}{16} = 20K$$

5. **(b)** $n_A = \frac{1}{2}$ mol, $n_B = \frac{1}{32}$ mol

By ideal gas equation,
$PV = nRT \Rightarrow P \propto n$

$$\therefore \frac{P_A}{P_B} = \frac{n_A}{n_B} = \frac{32}{2} = 16$$

6. **(c)** The average distance between two successive collisions of molecule is

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

7. **(c)** For monoatomic gas, $C_v = \frac{3}{2}R$

For diatomic gas, $C'_v = \frac{5}{2}R$

$$\therefore \text{Ratio} = \frac{C_v}{C'_v} = \frac{\frac{3}{2}R}{\frac{5}{2}R} = \frac{3}{5}$$

8. **(a)** Each degree of rotation of diatomic molecule has energy $\frac{RT}{2}$ per mole.

9. **(b)** $v_{rms} = \sqrt{\frac{3RT}{M_0}}$; $v_{rms} \propto \sqrt{\frac{T}{M_0}}$

as, T becomes double and M_0 becomes half

So, v_{rms} will become $\sqrt{4}$ times i.e. 2 times.

10. **(None)** Average kinetic energy per molecule $= \frac{f}{2}KT$

$$\frac{Av.K.E_{Ar}}{Av.K.E_{O_2}} = \frac{\frac{3}{2}KT}{\frac{5}{2}KT} = \frac{3}{5}$$

11. **(d)** $V_A = 2V_B$; $T_A = 2T_B$; $P_A = 2P_B$

$$\frac{P_A V_A}{TA} = \frac{P_B V_B}{T_B} = n_A R = n_B R$$

$$\therefore \frac{\eta_A}{\eta_B} = \frac{P_A V_A T_B}{P_B V_B T_A} = \frac{(2P_B)(2V_B)(T_B)}{P_B V_B (2T_B)} = 2$$

12. **(a)** $C_p - C_v = R \Rightarrow C_p = C_v + R$

$\because \gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = \frac{C_v}{C_v} + \frac{R}{C_v}$

$\Rightarrow \gamma = 1 + \frac{R}{C_v} \Rightarrow \frac{R}{C_v} = \gamma - 1 \Rightarrow C_v = \frac{R}{\gamma - 1}$

13. **(b)** $\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

or $\frac{2}{\gamma - 1} = \frac{1}{\frac{5}{3} - 1} + \frac{1}{\frac{7}{5} - 1}$

$\therefore \quad \gamma = \frac{3}{2}$.

14. **(a)** $C_p - C_v = R$ ...(i)

$\frac{C_p}{C_v} = \gamma$ ...(ii)

Solving (i) and (ii)

We get : $C_p = \frac{R\gamma}{\gamma - 1}$

15. **(d)** $C_P - C_V = R$, $\frac{C_P}{C_V} = \gamma \Rightarrow C_P = \gamma C_V$

$\gamma C_V - C_V = R \Rightarrow C_V(r-1) = R$

$\therefore C_V = \frac{R}{(\gamma - 1)}$

16. **(b)** Heat energy incident on the surface,
$Q_i = 1000\ J/m$
Coefficient of absorption, $a = 0.8$ coefficient of reflection, $r = 0.1$ Heat energy transmitted in 5 minutes, $Q_t = ?$
$\because 1 = r + a + t \Rightarrow t = 1 - 0.1 - 0.8 = 0.1$
$Q_t = Q_i \times t \times T$ (t = cofficient of transmitance)
$Q_t = 0.1 \times 1000 \times 5 = 500\ J$

17. **(c)** For rigid diatomic molecule, $\frac{C_P}{C_V} = \frac{7}{5}$

$\therefore C_V = \frac{5}{7} C_P$ ...(i)

Also, $C_p - C_v = R$

or, $C_P - \frac{5}{7} C_P = R \Rightarrow \frac{2}{7} C_P = R$

$n = \frac{2}{7} = 0.2857$

18. **(a)** Ideal gas equation, $pV = nRT$

$pV = \frac{m'}{M} RT$ here, m' is the mass of the gas

and M molecular weight $p = \frac{m'}{V} \frac{RT}{M}$

$\therefore p = \frac{\rho RT}{M}$

$\therefore \rho = \frac{m'}{V}$ density of the gas

$\rho = \frac{pM}{RT} = \frac{pM}{NkT}$, N is Avogadro, number

$\rho = \frac{pm}{KT}$, where $m = \frac{M}{N}$ mass of each molecule.

19. **(a)** We have given

$\frac{R}{C_v} = 0.4$ ...(i)

Here, R = universal gas constant
C_v = molar specific heat at constant volume
We know that, $C_p - C_v = R$

$\therefore \frac{C_p - C_v}{C_v} = 0.4 \Rightarrow \frac{C_p}{C_v} = 0.4 + 1 = 1.4$

i.e. $\Upsilon = 1.4$
The gas is diatomic in nature.

20. **(b)** RMS velocity is $V_{rms} = \sqrt{\frac{3RT}{M}}$

$V_{rms} \propto \frac{1}{\sqrt{M}}$

Ratio of RMS velocity of helium to oxygen is:

$\frac{V_H}{V_O} = \sqrt{\frac{32}{4}} = \frac{2\sqrt{2}}{1}$

21. **(c)** For monoatomic gas, $C_v = \frac{3}{2} R$

For diatomic gas, $C'_v = \frac{5}{2} R$

$\therefore$ Ratio $= \frac{C_v}{C'_v} = \frac{\frac{3}{2}R}{\frac{5}{2}R} = \frac{3}{5}$

22. **(c)** Given,

$\frac{\rho_1}{\rho_2} = \frac{4}{25}$

RMS speed $\propto \frac{1}{\sqrt{\rho}}$

$\therefore$ Ratio of rms velocities $= \sqrt{\frac{\rho_2}{\rho_1}} = \frac{5}{2}$

23. **(c)** Collision frequency $= \frac{\text{average speed}}{\text{mean free path}}$

frequency $= \frac{1}{T} = \frac{V_{avg}}{\lambda} = \frac{600}{3 \times 10^{-7}} = 2 \times 10^9\ \text{sec}^{-1}$.

24. **(b)** Given, initial temperature = 127°C = 400K
Final temperature = 527°C + 273 = 800K
The rms speed of gas molecules,

$$v'_{rms} = \sqrt{\frac{3K_BT}{m}} = \sqrt{\frac{3K_B}{m}(400)}$$

$$v''_{rms} = \sqrt{\frac{3K_B}{m}(800)}$$

$$\frac{v'_{rms}}{v''_{rms}} = \sqrt{\frac{400}{800}} = \frac{1}{\sqrt{2}}$$

$$v''_{rms} = \sqrt{2}\, v'_{rms}$$

25. **(c)** $(V_{rms})_1 = 50m/s$ at $T_1 = 77°C = 350K$

$T_2 = 150.5°c = 423.5K$

$$V_{rms} \propto \sqrt{T} \Rightarrow \frac{(V_{rms})_2}{(V_{rms})_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\Rightarrow \frac{(V_{rms})_2}{50} = \sqrt{\frac{423.5}{350}} = \sqrt{1.21} \approx 1.1$$

$(V_{rms})_2 = 50 \times 1.1 = 55m/s$

26. **(c)**

27. **(b)** Internal energy of an ideal gas:

$$U = \frac{f}{2}nRT$$

Substitutes the given values

$$2250 = \frac{f}{2} \times 3R \times 300$$

$$f = 5$$

28. **(b)** For a rigid diatomic gas,

f = 5, n = 1 mole

$\therefore$ Internal energy $U = \frac{f}{2}nRT = \frac{5}{2} \times 1 \times RT = \frac{5}{2}RT$

29. **(b)** For the given gas molecule, F = 6

T = 48°C = (47 + 273) K = 320 K

$\therefore$ Total internal energy of the gas molecule is

$$\Delta U = \frac{f}{2}kT = \frac{6}{2} \times 1.38 \times 10^{-23} \times 320J$$

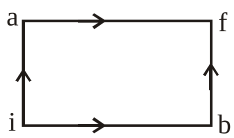
$= 1324.8 \times 10^{-23}$ J $= \frac{1324.8 \times 10^{-23}}{1.6 \times 10^{-19}}eV$

$= 828 \times 10^{-4}$ eV

4 Thermodynamics

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(d)** Pressure, volume, temperature and mass are all macroscopic variables which can be measured.
2. **(d)** Thermodynamics concerned with ΔH, ΔU and and ΔW.
3. **(d)**
4. **(a)** Zeroth law defines temperature and first law defines internal energy.
5. **(c)** Heat and work are not state variables. They are energy, transfer to a system which change the internal energy of a system, which is a state variable.
6. **(c)** Internal energy of an ideal gas depends only on the temperature.
7. **(a)** Work is energy transfer brought about by moving piston of a cylinder containing the gas, by raising or lowering some weight connected to it.
8. **(b)**
9. **(a)** As we know,
   $\Delta Q = \Delta u + \Delta w$ (Ist law of thermodynamics)
   $\Rightarrow \quad \Delta Q = \Delta u + P\Delta v$
   or $\quad 150 = \Delta u + 100(1-2) = \Delta u - 100$
   $\therefore \quad \Delta u\ 150 + 100 = 250$ J
10. **(b)** For path iaf,
   Q = 50 cal
   W = 20 cal
   By first law of thermodynamics,
   $\Delta U = Q - W = 50 - 20 = 30$ cal.



   For path ibf
   Q' = 36 cal
   W' = ?
   or, $W' = Q' - \Delta U'$
   Since, the change in internal energy does not depend on the path, therefore
   $\Delta U' = 30$ cal
   $\therefore \quad W' = Q' - \Delta U' = 36 - 30 = 6$ cal.
11. **(c)** Heat and work depends on the path taken to reach a specific value. Hence, heat and work are path functions.
12. **(d)** Heat always refers to energy transmitted from one body to another because of temperature difference.
13. **(b)**
14. **(b)** $\Delta Q = \Delta U + \Delta W$
15. **(c)**
16. **(b)** According to first law of thermodynamics
   $Q = \Delta U + W$
   Given : Q = 2 kcal = 2000 × 4.2 = 8400 J
   W = 400 J
   $\therefore \quad \Delta U = Q - W = 8400 - 400 = 8000$ J
17. **(d)** According to first law of thermodynamics
   $\Delta Q = \Delta U + \Delta W$
   $\Delta U = \Delta Q - \Delta W$
   $\Delta Q = 35J, \Delta W = -15J$
   $\therefore \quad \Delta U = 35J - (-15J) = 50J$
18. **(a)**
19. **(d)** The given statement is zeroth law of thermodynamics.
   It was formulated by R. H. Fowler in 1931.
20. **(d)** For the gas, Q = 18J, W = – 12J
   $\therefore$ Change in internal energy,
   $\Delta U = \Delta Q - W = 18 - (-12) = 30J$
21. **(a)** In thermal equilibrium the macroscopic variables like pressure, volume, temperature, mass and composition do not change with time.
22. **(a)**
23. **(c)**
24. **(c)** For monoatomic gas, $\gamma = \frac{5}{3}$
   For an adiabatic process, PV^{γ} = constant
   For an adiabatic process, $PV^{\gamma} = P'(V')^{\gamma}$
   $$\Rightarrow \quad PV^{5/3} = P'\left(\frac{V}{8}\right)^{5/3} \qquad \left(\text{Given, } V' = \frac{V}{8}\right)$$
   $$\Rightarrow \quad P' = (8)^{5/3} P = (2)^5 P = 32\ P$$
25. **(a)**
26. **(a)**
27. **(b)** In adiabatic process, no heat is taken or given by the system
   i.e., $\Delta Q = 0 \Rightarrow \Delta U = -\Delta W$
   If ΔW is negative (work done on system), then ΔU increases & temperature increases and vice-versa. So work done in adiabatic change in particular gas (ideal gas) depends on change in temperature.

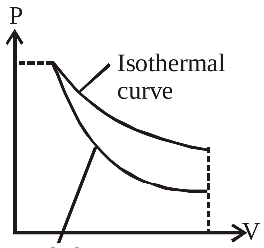
28. **(a)**

29. **(b)**

30. **(a)**

31. **(c)** $\frac{\text{Slope of adiabatic curve}}{\text{Slope of isothermal curve}} = \frac{(dP/dV)_{adi}}{(dP/dV)_{iso}} = +\gamma$

So slope to adiabatic curve is $\gamma\left(=\frac{C_P}{C_V}\right)$ times of isothermal curve, as clear also from figure.



32. **(c)** $T_1 = 273 + 27 = 300K$

$T_2 = 273 + 927 = 1200K$

For adiabatic process, $P^{1-\gamma} T^{\gamma}$ = constant

$\Rightarrow P_1^{1-\gamma} T_1^{\gamma} = P_2^{1-\gamma} T_2^{\gamma}$

$$\Rightarrow \left(\frac{P_1}{T_2}\right)^{1-\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma} \Rightarrow \left(\frac{P_1}{P_2}\right)^{1-1.4} = \left(\frac{1200}{300}\right)^{1.4}$$

$$P_2 = P_1 4^{\left(\frac{1.4}{0.4}\right)} = P_1 4^{\left(\frac{7}{2}\right)} = P_1 (2^7) = 2 \times 128 = 256 \text{ atm}$$

33. **(b)** Initial temperature $(T_1) = 18°C = 291$ K

Let Initial volume $(V_1) = V$ Final volume $(V_2) = \frac{V}{8}$

According to adiabatic process, $TV^{\gamma-1}$ = constant

According to question, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\Rightarrow T_2 = 293(8)^{\frac{7}{5}-1} = 293 \times 2.297 = 668.4K$$

$$\left[\text{For diatomic gas } \gamma = \frac{C_p}{C_v} = \frac{7}{5}\right]$$

34. **(c)** Curve A, B shows expansion. For expansion of a gas,

$W_{isothermal} > W_{adiabatic}$

$P_{isothermal} > P_{adiabatic}$

$T_{isothermal} > T_{adiabatic}$

$\Rightarrow$ Slope of curve for isothermal change < slope of curve for adiabatic change.

So, curve B shows isothermal change and curve A shows adiabatic change.

35. **(b)** Work done in any process is given as

$$w = \frac{P_2V_2 - P_1V_1}{1-x}$$

In adiabatic process,

$P_2V_2^{\gamma} = P_1V_1^{\gamma}$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 75\left(\frac{1200}{150}\right)^{\frac{5}{3}} = 75(8)^{\frac{5}{3}}$$

= 75 × 32 = 2400 kPa

Here, $x = \gamma$

$$\text{So, } w = \frac{P_2V_2 - P_1V_1}{1-\gamma} = \left(\frac{2400\times150 - 75\times1200}{1-\frac{5}{3}}\right)\times10^{-3}$$

$= 405000 \times 10^{-3} = 405$ J

36. **(d)** Under adiabatic change

$$\frac{T_2}{T_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}} \text{ or } T_2 = T_1 (P_1/P_2)^{\frac{1-\gamma}{\gamma}}$$

$$\therefore \quad T_2 = 300\,(4/1)^{\frac{1-(7/5)}{(7/5)}}; \quad \gamma = 1.4 = 7/5 \text{ for air}$$

$$\text{or} \quad T_2 = 300\,(4)^{-2/7}$$

37. **(a)** From first law of thermodynamics,

$\Delta H = \Delta u + w$

In adiabatic process $\Delta H = 0$

$\therefore \quad \Delta u = -w$

38. **(c)** In an isochoric process, no work is done on or by the gas. V is constant.

39. **(a)** For adiabatic process, $V \propto T^{\frac{1}{1-\gamma}}$

And for isothermal process, temperature is constant.

Let $\gamma = 1.5$

then, $V \propto \frac{1}{T^2}$

Hence, (c) is the correct V-T graph.

Also for adiabatic process, $P \propto T^{\frac{r}{\gamma-1}}$

Let $\gamma = 1.5$

Then, $P \propto T^3$

Hence, (d) is the correct P-T graph.

40. **(d)** Isobaric compression is represented by curve AO

Work done = area under AD

$= 2 \times 10^2 \times (3 - 1)$

$= 4 \times 10^2 = 400$ J.

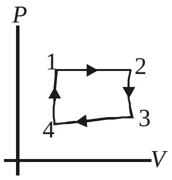
41. **(a)** For adiabatic process Q = 0.
By first law of thermodynamics,
$Q = \Delta E + W \Rightarrow \Delta E_{int} = -W$.

42. **(b)**

43. **(b)** Bursting of helium balloon is irreversible and in this process $\Delta Q = 0$, so adiabatic.

44. **(d)** From C to D, V is constant. So process is isochoric. From D to A, the curve represents constant temperature. So the process is isothermal.
From A to B, pressure is constant . So, the process is isobaric.
BC represents constant entropy.

45. **(b)** Heat engine is device by which a system is made to undergo cyclic process that result in conversion of heat into work.
When gas (system) in heat engine undergoes process $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$, then work done by gas = area enclosed by figure formed by joining 1, 2 , 3 , 4.



Work is positive if arrows move clockwise.

46. **(b)**

47. **(b)**

48. (b)

49. **(c)**

50. **(a)**

51. **(c)** The working of an air coditioner is similar to the working of a refrigerator. An air conditioner removes heat from the room, does some work and rejects the heat to the surroundings. As air conditioner is put in the middle of the room then due to continuous, external work the room will become slightly warmer.

52. **(d)** The coefficient of performance of a refrigerator is given by

$$\alpha = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

Substituting the given values, we get

$$\frac{1}{3} = \frac{Q_2}{200 - Q_2}$$

$$\Rightarrow 200 - Q_2 = 3Q_2 \Rightarrow 4Q_2 = 200$$

or $Q_2 = \frac{200}{4} J = 50J$

$\therefore$ $W = Q_1 - Q_2 = 200\,J - 50\,J = 150\,J$

53. **(b)** Here, Coefficient of performance (β) = 5
$T_1 = 27°C = (27 + 273)K = 300\,K$

As, $\beta = \frac{T_2}{T_1 - T_2} \Rightarrow 5 = \frac{T_2}{300 - T_2}$

or $1500 - 5T_2 = T_2$ or $6T_2 = 1500$

$\therefore$ $T_2 = \frac{1500}{6} = 250\,K$

54. **(a)** External amount of work must be done in order to flow heat from lower temperature to higher temperature. This is according to second law of thermodynamics.

55. **(c)** For process to be reversible it must be quasi-static. For quasi static process all changes take place infinitely slowly. Isothermal process occur very slowly so it is quasi-static and hence it is reversible.

56. **(a)** Slow isothermal expansion or compression of an ideal gas is reversible process, while the other given processes are irreversible in nature.

57. **(a)** First operation in carnot cycle is isothermal expansion.

58. **(a)**

59. **(d)** $\eta = 1 - \frac{T_2}{T_1}$ So for η be high T_1 must be high and T_2 must be low.

60. **(d)** $\eta = 1 - \frac{T_1}{T_2}$

$T_1 = -23°C = 250\,K$, $T_2 = 100°C = 373K$

$$\eta = 1 - \frac{250}{373} = \frac{373 - 250}{373}$$

61. **(a)** $\eta = 1 - \frac{300}{400} = \frac{100}{400} = \frac{1}{4}$

$$\eta = \frac{1}{4} \times 100 = 25\%$$

Hence, it is not possible to have efficiency more than 25%.

62. **(c)** Absolute zero temperature is practically not reachable.

63. **(d)** Efficiency, $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$

$$\eta' = 1 - \frac{(T_2 - 100)}{(T_1 - 100)} \times 100 = \frac{(T_1 - 100 - T_2 + 100)}{T_1 - 100} \times 100$$

$$\eta' = \left(\frac{T_1 - T_2}{T_1 - 100}\right) \times 100.$$

Comparing with η we get, the efficiency increases.

64. **(b)** $\eta = \frac{(627 + 273) - (273 + 27)}{627 + 273}$

$$= \frac{900 - 300}{900} = \frac{600}{900} = \frac{2}{3}$$

work = (η) × Heat = $\frac{2}{3} \times 3 \times 10^6 \times 4.2$ J = 8.4×10^6 J

65. **(d)** $\frac{W}{Q} = \left(1 - \frac{T_1}{T_2}\right) \Rightarrow \frac{1200}{Q} = \left(1 - \frac{400}{800}\right)$

$\therefore$ Amount of heat energy supplied to the engine from the source, $Q = 2400$ J.

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(d)** $dU = dQ - dW = (8 \times 10^5 - 6.5 \times 10^5) = 1.5 \times 10^5$ J
$dW = dQ - dU = 10^5 - 1.5 \times 10^5 = -0.5 \times 10^5$ J
–ve sign indicates that work done on the gas is 0.5×10^5 J.

2. **(c)** Given that dQ = – 30 J and dW = – 10 J
$E_i = 40$ J and let final internal energy = E_f
So, $dQ = E_f - E_i + dW$ $-30 = E_f - 40 - 10$; $E_f = 20$ J

3. **(c)** $Q = mL = 1 \times L = L$; $W = P(V_2 - V_1)$
Now $Q = \Delta U + W$
or $L = \Delta U + P(V_2 - V_1)$ $\therefore$ $\Delta U = L - P(V_2 - V_1)$

4. **(b)** $dW = P\Delta V = 1.01 \times 10^5 [1671 - 1] \times 10^{-6}$ Joule
$= \frac{1.01 \times 167}{4.2}$ cal. = 40 cal. nearly
$\Delta Q = mL = 1 \times 540$,
$\Delta Q = \Delta W + \Delta U$ or $\Delta U = 540 - 40 = 500$ cal.

5. **(a)** $W = P(dV) = 0.01 \times 10^5 (1671 - 1) \times 10^{-6} = 167$ J
$Q = \Delta U + W$
or $\Delta U = Q - W = mL - 167 = 540 \times 4.2 - 167$
$= 2099$ J

6. **(c)** $W = \frac{\pi r_1 r_2}{2} = \frac{\pi \times 1 \times 1}{2} = \pi/2$ J

7. **(c)** Heat absorbed in a thermodynamic process is given by $\Delta Q = \Delta U + \Delta W$.
Here ΔU is same for all the three processes as it depends only on initial and final states.
But $\Delta W_I = +$ Ve, $\Delta W_{II} = 0$, $\Delta W_{III} = -$ ve
$\therefore$ $\Delta Q_I > \Delta Q_{II}$

8. **(c)** $-20 = \Delta U + 50 \Rightarrow \Delta U = -70$.
From $2 \to 1$, $\Delta U = 70$ kJ.
Now $10 = 70 + W \Rightarrow W = -60$ kJ.

9. **(c)** As $\Delta U = 0$ in a cyclic process,
$\Delta Q = \Delta W$ = area of circle = πr^2
or $\Delta W = 10^2 \pi$J

10. **(d)** Work done = Area under the P – V curve
$W = (80 \text{ kPa}) (250 \times 10^{-6})$ kt 1/2 = 10 J
Since the arrow is anticlockwise,
$\therefore$ Work done = – 10 J

11. **(b)** Work done = Area under the curve
$\Rightarrow W = \frac{1}{2} \times (4-2) \times (400-100) = \frac{1}{2}(2) \times 300$
W = 300J

12. **(d)** In each cyclic process, $\Delta U = U_{final} - U_{initial} = 0$

13. **(a)** In an isochoric process volume remains constant whereas pressure remains constant in isobaric process.

14. **(d)**

15. **(b)** By first law of thermodynamics,
dq = dw + du

16. **(b)** In an isobaric process, $\frac{V}{T}$ = constant
Work done by gas, $W = p(V_2 - V_1) = \mu(T_2 - T_1)$.
Since, temperature changes, so does internal energy. Thus, from the first law of thermodynamics, we can say that, heat absorbed goes party to increase internal energy and partly to do work.

17. **(d)** VT^3 = constant
$\Rightarrow$ $T(V)^{\frac{1}{3}}$ = constant ...(i)
For adiabatic process,
$TV^{\gamma-1}$ = constant ...(ii)
Comparing Eqs. (i) and (ii),
$\gamma - 1 = \frac{1}{3} \Rightarrow \gamma = \frac{4}{3}$

18. **(a)**

19. **(c)** For isothermal expansion process,
$pV = p' \times 2V$ $[\because V' = 2V]$
$\Rightarrow$ $p' = \frac{P}{2}$
For adiabatic expansion
$pV\gamma$ = constant $\Rightarrow$ $p' V'^{\gamma} = p'' V''^{\gamma}$
$\Rightarrow$ $\frac{p}{2}(2V)^{\frac{5}{3}} = p''(16V)^{\frac{5}{3}}$
$\Rightarrow$ $p'' = \frac{p}{2}\left[\frac{2V}{16V}\right]^{\frac{5}{3}} = \frac{p}{64}$

20. **(c)** When heat is given to a gas in an isothermal change, the result will be external work done by the gas, because when there is an isothermal change, the temperature remains the same.
Hence, $\Delta U = 0$

21. **(b)** In isothermal process of an ideal gas, $\Delta U = 0$
So, $\Delta Q = \Delta W$ (from first law of thermodynamics)
As the area under the curve represents the amount of the work done, i.e.
$\Delta W = +20$ J $\Rightarrow$ $\Delta Q = +20$ J

22. **(a)** In isothermal process, temperature is constant. From ideal gas equation,
$PV = \mu RT$
At constant temperature,
PV = constant

So, with decreasing pressure P, V must increase to keep product PV constant. This inverse relation between p and V is depicted correctly by curve A, where P decreases with increasing V.

23. **(d)** For an isothermal process, PV = constant

Slope is given by

$$\left(\frac{dP}{dV}\right)_{iso} = -\frac{P}{V} \quad ...(i)$$

For an adiabatic process, PV^γ = constant

$\therefore$ Slope is given by

$$\left(\frac{dP}{dV}\right)_{ad} = \frac{-\gamma P}{V} \quad ...(ii)$$

From Eqs. (i) and (ii),

$$\left(\frac{dP}{dV}\right)_{ad} = \gamma\left(\frac{dP}{dV}\right)_{iso}$$

24. **(c)** In isothermal expansion, $V_2 > V_1$

As, work done in an isothermal process is given by

$$\Delta W = \mu RT \ln\frac{V_2}{V_1}$$

In this case, $\Delta W_1 > 0 \Rightarrow \Delta W_1 = +ve$

Similarly, in isothermal compression, $V_1 > V_2 \Rightarrow \Delta W_2 < 0 \Rightarrow \Delta W_2 = -ve$

25. **(c)** In adiabatic process, heat remains constant. So, from first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U + \Delta W = 0$

When work is done on system,

$\Delta W = -ve \quad \Rightarrow \quad \Delta U = -\Delta W = +ve$

As, $\Delta U = \mu C_v \Delta T = +ve \Rightarrow \Delta T = +ve$

$\Rightarrow$ T increases or $T_f > T_i$.

26. **(d)** For adiabatic process, $pV^\gamma = TV^{\gamma-1}$ = constant

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

Here, $T_2 = 2T_1$ and $V_2 = \frac{V_1}{2}$

$$\left(\frac{1}{2}\right)^{\gamma-1} = \left(\frac{1}{2}\right) \Rightarrow \gamma - 1 = 1 \text{ or } \gamma = 2$$

$\Rightarrow pV^\gamma = pV^2$ = constant

27. **(c)** Slope of isothermal curve $= -\tan\phi = \frac{dp}{dV} = -\frac{p}{V}$

[$\because$ for isothermal process, pV = constant $\Rightarrow$ pdV + vdp = 0 or $\frac{dp}{dV} = -\frac{p}{V}$]

Similarly, slope of adiabatic curve,

$$-\tan\phi = \frac{d}{dV}\left(\frac{\text{constant}}{V^\gamma}\right) = -\gamma\left(\frac{p}{V}\right)$$

[$\because$ for adiabatic process, pV^γ = constant]

As we know, $\gamma > 1$

Thus, $(\text{Slope})_{adiabatic} > (\text{Slope})_{isothermal}$

28. **(c)** $\therefore$ No. of moles of oxygen,

$$\mu = \frac{128}{32} = 4 \text{ mol}$$

For an isothermal process,

$$W = \mu RT \log_e \frac{V_2}{V_1}$$

$$= 2.3 \times 4 \times RT \times \log_{10}\frac{150}{75}$$

$$= 2.3 \times 1192\ R \log_{10} 2$$

29. **(b)**

30. **(a)** Ideal gas equation at constant pressure is given by

$p\,\Delta V = nR\Delta T \quad ...(i)$

$\therefore$ Work done, $W = p\,\Delta V$

$W = nR\,\Delta T \quad ...(ii)$

At constant pressure, heat given to gas is given by

$Q = nC_p\Delta T \quad ...(iii)$

From Eqs. (ii) and (iii), we get

$$\frac{W}{Q} = \frac{nR\Delta T}{nC_p\Delta T} = \frac{R}{C_p} = \frac{R}{\left(\frac{7}{2}\right)R} = \frac{2}{7}$$

Given, W = 10 J

$$\Rightarrow \frac{10}{Q} = \frac{2}{7} \Rightarrow Q = 35\text{ J}$$

31. **(d)** Using relation,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_2}{T_1} = \left(\frac{8}{1}\right)^{\frac{1.5-1}{1.5}} = \left(\frac{8}{1}\right)^{\frac{1}{3}} = 2$$

$\Rightarrow T_2 = 2T_1 \Rightarrow T_2 = 2(273 + 27) = 600\text{ K} = 327°C$

32. **(b)**

33. **(c)** Since, the given process is adiabatic, hence $\Delta Q = 0$.

From first law of thermodynamics, we get

$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W$

The process is adiabatic compression, therefore volume decreases. Hence,

$\Delta W = -ve \quad \Rightarrow \quad \Delta W = +ve$

Thus, internal energy of the system increases positively with change in temperature as gas is compressed adiabatically.

34. **(d)** The process in which heat remains constant is called adiabatic process.

In adiabatic process relation between temperature and volume is given as:

$TV^{\gamma-1}$ = constant

35. **(b)** The relation between temperature and pressure is given by

PV^{γ} = constant (PV = nRT)

$$\Rightarrow V = \frac{nRT}{P} \Rightarrow p\left(\frac{nRT}{p}\right)^{\gamma} = \text{constant}$$

$p^{1-\gamma}T^{\gamma}$ = constant

36. **(b)** Work done on the system, W = 166.28 J

Increase in temperature, $\Delta T = T_2 - T_1 = 8° C$

Work done in adiabatic process,

$$W = \frac{nR(T_2 - T_1)}{\gamma - 1}$$

$$166.28 = \frac{8.314 \times 8}{\gamma - 1}$$

$$\Rightarrow \gamma - 1 = \frac{8.314 \times 8}{166.28} = 0.4 \Rightarrow \gamma = 1.4$$

The gas is diatomic in nature.

37. **(c)** The work done by a gas is maximum when it expands isobarically.

38. **(d)** For mono-atomic, f = 3

$$\gamma = 1 + \frac{2}{f} = \frac{5}{3}$$

For adiabatic PV^{γ} = constant (∴PV = n RT)

$$\left(\frac{nRT}{V}\right)V^{\gamma} = \text{constant}$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow \frac{630(V)^{\gamma-1}}{(27V)^{\gamma-1}} = T_2$$

Final temperature of gas,

$$T_2 = \frac{630}{(27)^{\gamma-1}} = \frac{630}{(3)^{3\times\frac{5}{3}-1}} = 70 \text{ K.}$$

39. **(c)** $P_1 = P: V_1 = V ; V_2 = 27 V$

For isothermally

$P_1 V_1 = P_2 V_2$

$$P_2 = \frac{P_1V_1}{V_2} = \frac{P \times V}{27V} = \frac{P}{27}$$

For adiabatic

$$P_2V_2^{\gamma} = P_3V_3^{\gamma}$$

$$\frac{P}{27} \times (27V)^{\frac{5}{3}} = P_3 \times V^{\frac{5}{3}} \Rightarrow \frac{P}{27} \times (3)^{3\times\frac{5}{3}} = P_3$$

The final pressure of the gas

P_3 = 9P

40. **(b)** The condition for adiabatic process is

$TV^{\gamma-1}$= constant

Before compression, $T_1V_1^{\gamma-1}$ = const. ...(i)

After compression, $T_2 V_2^{\gamma-1}$ = const. ...(ii)

Dividing (ii) by (i):

$T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1}(T_1)$$

$$T_2 = \left(\frac{V \times 27}{8V}\right)^{\frac{5}{3}-1}[400] = \frac{9}{4} \times 400 = 900 \text{ K.}$$

Change in temperature,

$\Delta T = T_2 - T_1$ = 900 – 400 = 500 K

41. **(d)** $\eta_A = \frac{T_1 - T_2}{T_1} = \frac{W_A}{Q_1}$ and, $\eta_B = \frac{T_2 - T_3}{T_2} = \frac{W_B}{Q_2}$

According to question, $W_A = W_B$

$$\therefore \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \times \frac{T_2 - T_3}{T_1 - T_2} = \frac{T_1}{T_2}$$

$$\therefore T_2 = \frac{T_1 + T_3}{2} = \frac{600 + 400}{2} = 500K$$

42. **(b)** Initially, η = 0.25

$$1 - \frac{T_{sink}}{T_{source}} = 0.25 \Rightarrow 0.75 = \frac{T_{sink}}{T_{source}}$$

$$T_{source} = \frac{T_{sink}}{0.75} = \frac{300}{0.75} = 400 \text{ K}$$

Finally, η = 0.25 + 100% of 0.25

η = 0.5

$$\Rightarrow 1 - \frac{T_{sink}}{T'_{source}} = 0.5 \Rightarrow 0.5 = \frac{300}{T'_{source}}$$

$$\Rightarrow T'_{source} = 600 \text{ K}$$

∴ ΔT = (600 – 400) K = 200 K = 200 °C

43. **(d)** $\frac{W}{Q} = \left(1 - \frac{T_1}{T_2}\right) \Rightarrow \frac{1200}{Q} = \left(1 - \frac{400}{800}\right)$

∴ Amount of heat energy supplied to the engine from the source, Q = 2400 J.

44. **(a)** Efficiency of heat engine

$\eta = 1-\frac{T_s}{T} \Rightarrow \frac{1}{4} = 1-\frac{T_s}{T}$ [$T_s \to$ sink temperature]

$\Rightarrow 4T_s = 3T$ or, $T_s = \frac{3}{4}T$ ...(i)

$\Rightarrow \frac{1}{2} = 1-\frac{T_s - 58}{T}$ ...(ii)

From equation (i) & (ii)

$\frac{3}{4} = \frac{58}{T} + \frac{1}{2} \Rightarrow \frac{1}{4} = \frac{58}{T} \Rightarrow T = 232$

$\therefore T_s = \frac{3}{4}T = 174°C$

45. **(c)** Efficiency, $\eta = \frac{\text{Work done}}{\text{Heat absorbed}} = \frac{W}{\Sigma Q}$

$= \frac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_3} = 0.5$

Here, Q_1 = 1915 J, Q_2 = – 40 J and Q_3 = 125 J

$\therefore \frac{1915 - 40 + 125 + Q_4}{1915 + 125} = 0.5$

$\Rightarrow 1915 - 40 + 125 + Q_4 = 1020 \Rightarrow Q_4 = 1020 - 2000$

$\Rightarrow Q_4 = -Q = -980$ J $\Rightarrow Q = 980$ J

46. **(b)** According to question, $\eta_1 = \eta_2 = \eta_3$

$\therefore 1-\frac{T_2}{T_1} = 1-\frac{T_3}{T_2} = 1-\frac{T_4}{T_3}$

[$\because$ Three engines are equally efficient]

$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$

$\Rightarrow T_2 = \sqrt{T_1 T_3}$ ...(i)

$T_3 = \sqrt{T_2 T_4}$ ...(ii)

From (i) and (ii)

$T_2 = (T_1^2 T_4)^{1/3}$; $T_3 = (T_1 T_4^2)^{1/3}$

47. **(d)** Efficiency of engine *A*, $\eta_1 = 1-\frac{T}{T_1}$,

Efficiency of engine *B*, $\eta_2 = 1-\frac{T_2}{T}$

Here, $\eta_1 = \eta_2$

$\therefore \frac{T}{T_1} = \frac{T_2}{T} \Rightarrow T = \sqrt{T_1 T_2}$

48. **(b)** Q_1 = 1000 J, Q_2 = 600 J T_1 = 127°C = 400 K

Efficiency of carnot engine,

$\eta = \frac{W}{Q_1} \times 100\%$ or, $\eta = \frac{Q_2 - Q_1}{Q_1} \times 100\%$

or, $\eta = \frac{1000-600}{1000} \times 100\% = 40\%$

Now, for carnot cycle $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$\frac{600}{1000} = \frac{T_2}{400} \Rightarrow T_2 = \frac{600 \times 400}{1000} = 240$ K

$= 240 - 273 \Rightarrow -33°C$

49. **(a)** $W_{AB} = 0$, $W_{BC} = P\Delta V = nR\Delta T = -nRT_0$

$W_{CA} = nRT \ell n\frac{V_f}{V_i} = nR(2T_0)\ell n2$

$Q_{BC} = nC_p\Delta T = \left(\frac{nR\gamma}{\gamma - 1}\right)T_0$

Efficiency, $\eta = \frac{W}{Q} = \left[\frac{2\ell n2 - 1}{\gamma/(\gamma - 1)}\right]$

50. **(a)** As we know $\eta = \frac{W}{Q_1} = 1-\frac{T_2}{T_1}$

$\Rightarrow \eta = 1-\frac{300\text{K}}{1200\text{K}} = \frac{3}{4}$

$\frac{3}{4} = \frac{W}{Q_1} \Rightarrow Q_1 = W \times \frac{4}{3} \Rightarrow Q_1 = 12.6 \times 10^6 \times \frac{4}{3}$

$Q_1 = 16.8 \times 10^6$ J.

Exercise 3 : PREVIOUS YEARS MCQs

1. **(a)**
2. **(c)** Internal energy depends only on initial and final state
   So, $\Delta U_A = \Delta U_B$
   Also $\Delta Q = \Delta U + W$
   As $W_A > W_B \Rightarrow \Delta Q_A > \Delta Q_B$
3. **(a)**
4. **(b)**
5. **(c)**
6. **(c)**
7. **(d)** $|\Delta W|$ = Area under P-V diagram

$= \frac{1}{2} \times (50 + 10) \times 10^3 \times 150 \times 10^{-6}$

$= 30 \times 150 \times 10^{-3} = 4500 \times 10^{-3}$

= 4.5 J. As volume is decreasing. So $\Delta W = -4.5$ J

By Ist law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

$\Rightarrow 0 = \Delta U - 4.5$

$\Rightarrow \Delta U = +4.5$ J

8. **(a)**

9. **(a)**

10. **(b)** Efficiency of heat engine is given as $\eta = 1 - \frac{T_L}{T_H}$

For $T_1 = 447$ °C and $T_2 = 147$ °C

$$\eta_1 = 1 - \frac{147+273}{447+273} = 1 - \frac{420}{720} \Rightarrow \eta_1 = \frac{300}{720}$$

For $T_1 = 947$ °C and $T_2 = 47$ °C

$$\eta_2 = 1 - \frac{47+273}{947+273} = 1 - \frac{320}{1220}$$

$$\Rightarrow \eta_2 = \frac{900}{1220}$$

so, $\frac{\eta_1}{\eta_2} = \frac{300}{720} \times \frac{1220}{900} = \frac{122}{72 \times 3} \Rightarrow \frac{\eta_1}{\eta_2} = 0.56$

11. **(a)**

12. **(b)**

13. **(d)**

14. **(d)** Isochoric proceess dV = 0

W = 0 proceess 1

Isobaric : $W = P\,\Delta V = nR\Delta T$

Adiabatic $|W| = \frac{nR\Delta T}{\gamma - 1}$ $0 < \gamma - 1 < 1$

As workdone in case of adiabatic process is more so process 3 is adiabatic and process 2 is isobaric.

15. **(b)** When efficiency of carnot engine, $\eta = 0.2$

Efficiency of a Carnot engine,

$$\eta = 1 - \frac{T_2}{T_1} \text{ or, } 0.2 = 1 - \frac{T_2}{T_1}$$

or $\frac{T_2}{T_1} = 0.8$ ...(i)

When T_2 is reduced by 50 K, its efficiency becomes 0.4

$\therefore$ $0.4 = 1 - \frac{T_2 - 50}{T_1}$

or $\frac{T_2 - 50}{T_1} = 0.6$ ...(ii)

Dividing eqn. (i) by (ii)

$$\frac{T_2}{T_2 - 50} = \frac{0.8}{0.6} = \frac{4}{3}$$

$\Rightarrow$ $3T_2 = 4T_2 - 200$ or $T_2 = 200$ K

From eqn. (ii), $T_1 = \frac{T_2 - 50}{0.6} = \frac{200 - 50}{0.6} = 250$K

16. **(b)**

17. **(d)** $T_2 = 7°C = (7 + 273) = 280$ K

$$\eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = 1 - \eta$$

$$= 1 - \frac{50}{100} = \frac{50}{100} = \frac{1}{2}$$

$\therefore$ $T_1 = 2 \times T_2 = 2 \times 280 = 560$ K

New efficiency, $\eta' = 70\%$

$\therefore$ $\frac{T_2}{T_1} = 1 - \eta' = 1 - \frac{70}{100} = \frac{30}{100} = \frac{3}{10}$

$\therefore$ $T_1' = \frac{10}{3} \times 280 = \frac{2800}{3} = 933.3K$

$\therefore$ Increase in the temperature of high temp. reservoir = 933.3 – 560 = 373.3 K = 380 K

18. **(d)**

19. **(b)**

20. **(b)**

21. **(b)**

22. **(d)** Coefficient of performance,

$$\text{Cop} = \frac{T_2}{T_1 - T_2}$$

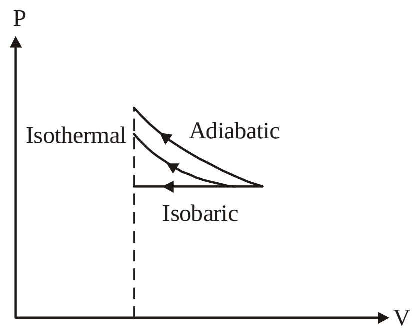
$$5 = \frac{273 - 20}{T_1 - (273 - 20)} = \frac{253}{T_1 - 253}$$

$5T_1 - (5 \times 253) = 253$

$5T_1 = 253 + (5 \times 253) = 1518 \therefore T_1 = \frac{1518}{5} = 303.6$

or, $T_1 = 303.6 - 273 = 30.6 \cong 31°C$

23. **(d)**



Since area under the curve is maximum for adiabatic process so, work done (W = PdV) on the gas will be maximum for adiabatic process.

5 Oscillations

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(b)** **2.** **(a)**

3. **(c)** Displacement, $y = r \sin \omega t$

$$V = \frac{dy}{dt} = r\omega \cos \omega t$$

$$a = \frac{dV}{dt} = -\omega^2 r \sin \omega t$$

$a = -\omega^2 y \qquad \therefore a \propto y$

4. **(d)** $x(t) = A\cos(\omega t + \phi)$

$$V(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a(f) = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi)$$

5. **(c)** The motion of particle is circular or elliptic, when two S.H.M. which are perpendicular to each other superimpose on the particle. The particle moves on a

(i) Ellipse if amplitudes of two S.H.M. are different

(ii) Circle, if amplitudes of two S.H.M. are same.

6. **(d)** **7.** **(b)**

8. **(c)** For a particle executing SHM,

$x = k\sin^2 \omega t$

On differentiation

$$\frac{dx}{dt} = 2k\omega(\sin \omega t)(\cos \omega t)$$

Again on differentiation

$$a = \frac{d^2x}{dt^2} = 2k\omega^2 \cos 2\omega t$$

$a = 2k\omega^2 \cos 2\omega t$

The given equation does not satisfy the SHM condition.

9. **(d)** Simple harmonic motion is represented by a sine function or a cosine function or a linear combination of both.

10. **(d)**

11. **(d)** Displacement equation is given as

$x = A\sin \omega t$

$$v = \frac{dx}{dt} = A\omega \cos \omega t$$

$$= A\omega \sin\left(\omega t = \frac{\pi}{2}\right)$$

Acceleration of the particle is,

$$a = \frac{dv}{dt} = -A\omega^2 \sin \omega t = A\omega^2 \sin(\omega t + \pi)$$

Phase difference between acceleration and velocity is

$$\pi - \frac{\pi}{2} = 0.5\pi$$

12. **(c)** The motion of particle will not be SHM but that of its projection on a diameter will be SHM. However the motion of particle will be periodic because it has constant speed.

13. **(c)** Equation of motion $y = a\cos \omega t$

$$\Rightarrow \frac{a}{2} = a\cos \omega t \Rightarrow \cos \omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{3}$$

$$\Rightarrow \frac{2\pi t}{T} = \frac{\pi}{3} \Rightarrow t = \frac{\frac{\pi}{3} \times T}{2\pi} = \frac{4}{3 \times 2} = \frac{2}{3}\text{sec}$$

14. **(a)** Equation of two simple harmonic motion

$y = A\sin(\omega t + \phi)$ ....(1)

$x = A\sin\left(\omega t + \phi + \frac{\pi}{2}\right) \Rightarrow x = A\cos(\omega t + \phi)$ ....(2)

On squaring and adding equations (1) and (2),

$x^2 + y^2 = A^2$

This is an equation of a circle. Hence, resulting motion will be a circular motion.

15. **(c)** Displacement equation for SHM is

$y = a\sin \omega t$

$$2 = 4\sin \omega t \Rightarrow \frac{1}{2} = \sin \omega t$$

$$\Rightarrow \sin\frac{\pi}{6} = \sin \omega t \Rightarrow \omega t = \frac{\pi}{6} \Rightarrow \frac{2\pi}{T}.t = \frac{\pi}{6}$$

$$\Rightarrow t = \frac{T}{12} = \frac{1}{12}\text{s}$$

16. **(c)** Given, $x(t) = 20\cos \omega t$

$$x(t) = 20\cos\frac{2\pi}{T}.t$$

$$\therefore x(t) = 20\cos\frac{2\pi}{4}.1 = 0$$

17. **(d)** $x = 4(\cos \pi t + \sin \pi t)$

$$= 4\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos \pi t + \frac{1}{\sqrt{2}}\sin \pi t\right]$$

$$= 4\sqrt{2}\left[\sin\frac{\pi}{4}\cos \pi t + \cos\frac{\pi}{4}\sin \pi t\right]$$

$$= 4\sqrt{2}\sin\left(\pi t + \frac{\pi}{4}\right)$$

Standard equation of displacement is:

$x = a\sin(\omega t + \phi)$

Comparing the given equation with standard equation

$a = 4\sqrt{2}$

18. **(b)**

19. **(a)** Resultant displacement will be the vector sum of two displacements:

$$S = \sqrt{S_1^2 + S_2^2 + 2S_1S_2 \cos 37°}$$

$$= \sqrt{a^2 \sin^2 \omega t + b^2 \sin^2 \omega t + 2ab \sin^2 \omega t \times \frac{4}{5}}$$

$$S = \sqrt{a^2 + b^2 + \frac{8ab}{5} \sin \omega t}$$

Which shows that the particle will perform SHM.

20. **(c)**

21. **(b)** The maximum amplitude is given by
$\omega^2 A = (2\pi \times 0.5)^2 \times A = g$

$$\therefore A = \frac{10}{(2\pi \times 0.5)^2} = 1\text{m}$$

22. **(b)** For a particle executing SHM,
At mean position,
$v \to$ maximum, $a \to$ minimum.

23. **(a)** Comparing given equation with standard equation of S.H.M.
$x = A\sin(\omega t + \phi)$
Amplitude
$A = 3$ m, $\omega = 2\pi \, s^{-1}$
$\therefore$ Maximum speed,
$v_{max} = A\omega = 3 \text{ m} \times 2\pi \text{ s}^{-1} = 6\pi \text{ m s}^{-1}$

24. **(a)** In SHM,
$a_{max} = A\omega^2$ and $v_{max} = A\omega$

$$\omega = \frac{a_{max}}{v_{max}} = \frac{1}{0.5} = 2 \text{ rad s}^{-1}$$

25. **(a)** t = 0, v maximum. The motion begins from mean position. So it represents S.H.M.

26. **(a)** At point 2, the acceleration of the particle is maximum, which is at the extreme position. At extreme position, the velocity of the particle will be zero.

27. **(d)** $v^2 = \omega^2(A^2 - x^2)$ ...(i)
and $a^2 = (\omega^2 x^2) = \omega^4 x^2$ ...(ii)
From above equations, we have

$$v^2 = -\frac{a^2}{\omega^2} + \omega^2 A^2 \Rightarrow y = mx + c$$

It represents straight line with negative slope.

28. **(a)** Velocity in SHM is given by

$$v = \omega\sqrt{a^2 - y^2}$$

At $y = 4$ cm $= 0.04$ m, $v = 3$ m/s

$$\therefore 3 = \omega\sqrt{a^2 - (0.04)^2} \quad \text{....(i)}$$

At $y = 3$ cm $= 0.03$ m, $v = 4$ m/s

$$\therefore 4 = \omega\sqrt{a^2 - (0.03)^2} \quad \text{....(ii)}$$

Dividing (ii) by (i), we get a = 0.05 = 5 cm

29. **(c)** Given, $x = 10 \sin\left(2t - \frac{\pi}{6}\right)$
$A = 10$ and $\omega = 2$ Hz

$$\therefore \upsilon = \omega\sqrt{A^2 - d^2} = 2\sqrt{(10)^2 - (6)^2}$$

$$= 2\sqrt{100 - 36} = 2 \times 8 = 16 \text{m s}^{-1}$$

30. **(c)**

31. **(a)** Maximum velocity,
$v_{max} = a\omega$

$$v_{max} = a \times \frac{2\pi}{T} \Rightarrow T = \frac{2\pi a}{v_{max}} = \frac{2 \times 3.14 \times 7 \times 10^{-3}}{4.4} \approx 0.01 \text{ s}$$

32. **(b)** Using $v^2 = \omega^2 (a^2 - y^2)$ we have

$10^2 = \omega^2 (a^2 - 4^2)$ and $8^2 = \omega^2 (a^2 - 5^2)$;
so $10^2 - 8^2 = \omega^2 (5^2 - 4^2) = (3\omega^2)$ or $6 = 3\omega$ or $\omega = 2$
or $T = 2\pi/\omega = 2\pi/2 = \pi$ s.

33. **(b)** Here, $x = 2 \times 10^{-2} \cos \pi t$
Speed is given by

$$v = \frac{dx}{dt} = -2 \times 10^{-2} \pi \sin \pi t$$

For the first time, the speed to be maximum,
$\sin \pi t = 1$

or, $\sin \pi t = \sin \frac{\pi}{2} \Rightarrow \pi t = \frac{\pi}{2}$ or $t = \frac{1}{2} = 0.5$ sec.

34. **(b)** At $t = 1$ sec

$$x = \sin \pi\left(1 + \frac{1}{3}\right) = \sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$$

So, $V = \omega\sqrt{A^2 - x^2} = \pi\sqrt{1 - \frac{3}{4}} = \frac{\pi}{2} m/s = 157 \text{ cm/s}^{-1}$

35. **(c)** Max. force = mass × max. acceleration
$= m\, 4\pi^2 v^2 a = 1 \times 4 \times \pi^2 \times (60)^2 \times 0.02 = 288\pi^2$

36. **(a)** The motion of the particle is periodic, (not oscillatory), beacuse it returns to its starting point after a fixed time.

37. **(b)** Elasticity brings the particle towards mean position and inertia needed to cross mean position.

38. **(c)** The motion of a planet around the sun is periodic motion but not a simple harmonic motions.

39. **(d)**

40. **(d)** Initially $t = 0$
$x = a \cos\pi t = -a \cos 0° = a$
Finally at $t = 3$
$x = a \cos 3\pi = -3a$
Total displacement = 2a

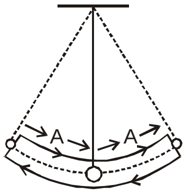
41. **(c)** Given $t = 1$s

$$\therefore x = 5 \cos\left(2\pi + \frac{\pi}{4}\right) = 5 \cos\frac{\pi}{4} = \frac{5}{\sqrt{2}} \text{ m}$$

i.e., displacement at $t = 1$s is $\frac{5}{\sqrt{2}}$ m

42. **(c)** phase at time, $t = 2\pi nt + \alpha$

43. **(d)** As seen from figure after one time period the bob return to its equilibrium position, so displacement of the particle is zero, but distance covered by the particle in one time period is 4A (where A is amplitude of bob, when it does S.H.M.)



44. **(b)**

45. **(a)** For $x = (-A)$, we have

$-A = A\sin(\omega \times 0 + \phi_0)$ or $\phi_0 = -\frac{\pi}{2}$.

So for $x < (-A)$, $\phi_0 < (-\pi/2)$.

46. **(d)** $\frac{T}{2} = 0.5$ sec $\Rightarrow T = 1$ sec

$\nu = \frac{1}{T} = 1\text{ Hz}$

47. **(a)** At extreme position

$y = A\sin(\omega t + \frac{\pi}{2}) = A\cos\omega t$

Now $y = A/2$ then $A/2 = A\cos\omega t \Rightarrow \omega t = \cos^{-1}(1/2)$

$\frac{2\pi}{T}t = \frac{\pi}{3} \Rightarrow t = T/6$

48. **(a)**

49. **(d)** According to equation of SHM

$x = A\sin\omega t$

Here, $x = \frac{A}{2}$ and $t = 1$

$\therefore \frac{A}{2} = A\sin\omega \times 1 = A\sin\omega$

or, $\sin\omega = \frac{1}{2}, \omega = \frac{\pi}{6}$

$\therefore \frac{2\pi}{T} = \frac{\pi}{6}$ or $T = 12$ s

50. **(c)** As phase difference $= \frac{\pi}{4}$, the resultant path of particle is an ellipse.

51. **(c)** $x = A\sin\omega t\cos\omega t = \frac{A}{2}\sin 2\omega t$

52. **(a)** The amplitude is a maximum displacement from the mean position.

53. **(c)** At centre $v_{max} = a\omega = a.\frac{2\pi}{T} = \frac{0.2 \times 2\pi}{0.01} = 40\pi$

54. **(d)** **55.** **(d)**

56. **(a)** For simple harmonic motion, $F = -kx$. Here, $k = Ak$.

57. **(c)** $y = A\sin PT + B\cos PT$

Let $A = r\cos\theta, B = r\sin\theta$

$\Rightarrow y = r\sin(PT + \theta)$ which is the equation of SHM.

58. **(c)** If $y_1 = a_1\sin\omega t$ and $y_2 = a_2\sin(\omega t + 0) = a_2\sin\omega t$

$\Rightarrow \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2} - \frac{2y_1y_2}{a_1a_2} = 0 \Rightarrow y_2 = -\frac{a_2}{a_1}y_1$

This is the equation of straight line.

59. **(c)** $y = a(\cos\omega t + \sin\omega t)$

$= a\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\omega t + \frac{1}{\sqrt{2}}\sin\omega t\right]$

$= a\sqrt{2}[\sin 45° \cos\omega t + \cos 45° \sin\omega t]$

$= a\sqrt{2}\sin(\omega t + 45°) \Rightarrow$ Amplitude $= a\sqrt{2}$

60. **(b)**

61. **(b)** Total energy of the system is given by:

$E = KE + PE = \frac{1}{2}kA^2$ ...(i)

If KE = PE, then

$\frac{1}{2}kA^2 = 2 \times PE = 2\left(\frac{1}{2}kx^2\right)$

$\frac{1}{2}kx^2 = \frac{1}{2}\left[\frac{1}{2}kA^2\right] \Rightarrow x = \frac{A}{\sqrt{2}}$

62. **(b)** Velocity is given by:

$v = -A\omega\sin(\omega t + \phi)$ ...(i)

$\Rightarrow T_1 = \frac{2\pi}{\omega}$

Kinetic energy is given by:

$KE = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \phi)$

$= mA^2\omega^2[1 - \cos(2\omega + 2\phi)]$

$\Rightarrow T_2 = \frac{2\pi}{2\omega}$ $T_2 = \frac{T_1}{2} \Rightarrow T_1 = 2T_2$

63. **(b)** Kinetic energy = Potential energy

$\frac{1}{2}m\omega^2(a^2 - y^2) = \frac{1}{2}m\omega^2y^2$

$a^2 - y^2 = y^2$

$2y^2 = a^2 \Rightarrow y = \pm\frac{a}{\sqrt{2}}$

64. **(d)**

65. **(a)** $E = \frac{1}{2}m\omega^2A^2$ and K. E $= \frac{1}{2}m\omega^2(A^2 - x^2)$

Putting $x = \frac{A}{2}$ or $x^2 = \frac{A^4}{4}$

$K.E. = \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right) = \frac{3}{4}\left[\frac{1}{2}m\omega^2A^2\right] = \frac{3E}{4}$

66. **(c)** $v = \omega\sqrt{(A^2 - x^2)} \Rightarrow v \propto \sqrt{(A^2 - x^2)}$

At $x = A/3$

$\sqrt{(A^2 - x^2)} = \sqrt{\left(A^2 - \frac{4A^2}{9}\right)} = A\sqrt{\frac{8}{9}}$

At $x = 2A/3$

$\sqrt{(A^2 - x^2)} = \sqrt{\left(A^2 - \frac{4A^2}{9}\right)} = A\sqrt{\frac{5}{9}}$

$K' = \frac{5}{9} \times \frac{9}{8}K = \frac{5}{8}K$

67. **(c)** $T=\frac{\pi}{2}$

But $T=2\pi\left(\left|\frac{x}{a}\right|\right)^{1/2}=\frac{\pi}{2}$ (given)

$\left|\frac{x}{a}\right|=\frac{1}{16}$

A = –16 x = – 16 sin x

For small oscillation, sin x = x

F = ma = –16 sin x (since m = 1 kg)

$U=-\int F dx=-16\cos x$

68. **(a)** m = 2g, $x=8\cos\left(50t+\frac{\pi}{12}\right)$ m

$V_{max}=A\omega=8\times50=400$ m/s

$\therefore\ (K.E)_{max}=\frac{1}{2}mv^2_{max}=\frac{1}{2}\times2\times10^{-3}\times400\times400=160$ J

69. **(c)** $T=\frac{\pi}{2}$ s, m = 1kg, x = 0.2 m

$\therefore\ \omega=\frac{2\pi}{T}=\frac{2\pi\times2}{\pi}=4$ rad/s $\therefore$ Potential energy,

$U=\frac{1}{2}kx^2=\frac{1}{2}m\omega^2x^2=\frac{1}{2}\times1\times4^2\times0.2\times0.2=0.32$ J

70. **(d)** m = log = 10×10^{-3} kg, U = $(50x^2+100)$ J

$\therefore$ Restoring force, $F=-\frac{du}{dx}$

$\Rightarrow F=-\frac{d}{dx}(50x^2+100)\Rightarrow ma=-(100x)$

$\Rightarrow a=-\frac{100}{10\times10^{-3}}x=-10^4x=-\omega^2x$

$\Rightarrow \omega^2=10^4\Rightarrow\omega=100$

$\therefore$ Frequency, $f=\frac{w}{2\pi}=\frac{100}{2\pi}=\frac{50}{\pi}s^{-1}$

71. **(d)** Here, m = 4 kg; k = 800 Nm^{-1}; E = 4 J

In SHM, $E=\frac{1}{2}kA^2\ \therefore\ 4=\frac{1}{2}\times800\times A^2$

$A^2=\frac{8}{800}=\frac{1}{100}\Rightarrow A=0.1$ m

Maximum acceleration, $a_{max}=\omega^2A$

$=\frac{k}{m}A$

$\Rightarrow \omega=\sqrt{\frac{k}{m}}=\frac{800\ Nm^{-1}}{4\ kg}\times0.1\ m=20\ ms^{-2}$

72. **(b)** The kinetic energy of a particle executing S.H.M. is given by

$K=\frac{1}{2}ma^2\omega^2\sin^2\omega t$

where, m = mass of particle
a = amplitude
ω = angular frequency
t = time

Now, average K.E. = $\langle K\rangle$

$=\left\langle\frac{1}{2}m\omega^2a^2\sin^2\omega t\right\rangle=\frac{1}{2}m\omega^2a^2\langle\sin^2\omega t\rangle$

$=\frac{1}{2}m\omega^2a^2\left(\frac{1}{2}\right)=\frac{1}{4}m\omega^2a^2\ \left(\because\ \langle\sin^2\theta\rangle=\frac{1}{2}\right)$

$=\frac{1}{4}ma^2(2\pi\nu)^2$ $(\because\ \omega=2\pi\nu)$

or, $\langle K\rangle=\pi^2ma^2\nu^2$

73. **(b)** In SHM, Total energy, $E_{total}=\frac{1}{2}m\omega^2A^2$

and, Kinetic energy, $E_K=\frac{1}{2}m\omega^2(A^2-x^2)$

where x is the distance from the mean position.

At $x=0.707A$

$E_K=\frac{1}{2}m\omega^2(A^2-(0.707A^2)=\frac{1}{2}m\omega^2(0.5A^2)$

As per question, E_{total} = 100 J

$\therefore\ E_K=0.5\left(\frac{1}{2}m\omega^2A^2\right)=0.5\times100J=50J$

74. **(d)** Kinetic energy, $k=\frac{1}{2}m\omega^2A^2\cos^2\omega t$

Potential energy, $U=\frac{1}{2}m\omega^2A^2\sin^2\omega t$

$\frac{k}{U}=\cot^2\omega t=\cot^2\frac{\pi}{90}(210)=\frac{1}{3}$

75. **(b)** General equation for displacement is $x=A\sin(\omega t+\phi)$

Comparing with given equation, $y=A\cos(30°)$

$\omega t+\phi=30°$

$\therefore x=40\times\frac{\sqrt{3}}{2}\Rightarrow20\sqrt{3}\ cm$ and $A=40\ cm$

Kinetic energy, $K.E=\frac{1}{2}k\left(A^2-x^2\right)=200$

$200=\frac{1}{2}k\left(\frac{1600-1200}{100\times100}\right)$

$\Rightarrow 400\times100\times100=k\times400\Rightarrow k=10^4$ $\therefore x=4$

76. **(d)** $E=\frac{1}{2}m\omega^2a^2\Rightarrow E\propto a^2$

77. **(a)** P.E. of body in S.H.M. at an instant,

$U=\frac{1}{2}m\omega^2y^2=\frac{1}{2}ky^2$

If the displacement, y = (a – x) then

$U=\frac{1}{2}k(a-x)^2=\frac{1}{2}k(x-a)^2$

78. **(a)** If displacement of particle is y, then

$KE=\frac{1}{2}m\omega^2(a^2-y^2)$ & $P.E.=\frac{1}{2}m\omega^2y^2$

If KE = PE $\frac{1}{2}m\omega^2y^2=\frac{1}{2}m\omega^2a^2-\frac{1}{2}m\omega^2y^2$

$2y^2=a^2\quad\therefore y=\frac{a}{\sqrt{2}}$

79. **(d)** Total mechanical energy is constant throughout the motion and equals $\frac{1}{2}m\omega^2A^2$.

80. **(c)** Kinetic energy, $K = \frac{1}{2}mv^2$

$= \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi)$

Potential energy, $U = \frac{1}{2}m\omega^2A^2\cos^2(\omega t+\phi)$

$\therefore$ Total mech. energy $= \frac{1}{2}m\omega^2A^2$

81. **(a)** K.E. of a body undergoing SHM is given by,

$K.E. = \frac{1}{2}ma^2\omega^2\cos^2\omega t$ and $T.E. = \frac{1}{2}ma^2\omega^2$

Given K.E. = 0.75 T.E.

$\Rightarrow 0.75 = \cos^2\omega t \Rightarrow \omega t = \frac{\pi}{6}$

$\Rightarrow t = \frac{\pi}{6\times\omega} \Rightarrow t = \frac{\pi\times 2}{6\times 2\pi} \Rightarrow t = \frac{1}{6}\,s$

82. **(d)** Potential energy of an object executing SHM is given as

$U(x) = \frac{1}{2}kx^2 \Rightarrow U(x) = \frac{1}{2}kA^2\cos^2\omega t$

At $t = \frac{T}{4}$

$U(x) = \frac{1}{2}kA^2\cos^2\left(\frac{2\pi}{T}\times\frac{T}{4}\right) = 0$

83. **(d)** P.E. is maximum at extreme position and minimum at mean position.
Time to go from extreme position to mean position is,

$t = \frac{T}{4}$; where T is time period of SHM

$5s = \frac{T}{4} \Rightarrow T = 20s.$

84. **(a)**

85. **(d)** $P.E. = \frac{1}{2}Kx^2 = E$

At half way $P.E. = \frac{1}{2}K\left(\frac{x}{2}\right)^2 = \frac{\frac{1}{2}Kx^2}{4} = \frac{E}{4}$

86. **(c)** P.E. changes from zero to maximum twice in each vibration so its time period is T/2

87. **(a)** $E = \frac{1}{2}m\omega^2a^2$

88. **(b)** $\frac{Potential\ energy\ (U)}{Total\ energy\ (E)} = \frac{\frac{1}{2}m\omega^2y^2}{\frac{1}{2}m\omega^2a^2} = \frac{y^2}{a^2}$

So, $\frac{2.5}{E} = \frac{\left(\frac{a}{2}\right)^2}{a^2} \Rightarrow E = 10\,J$

89. **(c)** In S.H.M., frequency of K.E. and P.E.
= 2 × (Frequency of oscillating particle)

90. **(b)** So $a = 6$ cm, $\omega = 100$ *rad/s*

$K_{max} = \frac{1}{2}m\omega^2a^2 = \frac{1}{2}\times1\times(100)^2\times(6\times10^{-2})^2 = 18J$

91. **(b)** Total energy $U = \frac{1}{2}ka^2$

92. **(d)**

93. **(c)** $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, $n' = \frac{1}{2\pi}\sqrt{\frac{k}{4m}} = \frac{1}{2}\times\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

On putting the value of n we get $n' = \frac{n}{2}$

94. **(d)** $T = 2\pi\sqrt{\frac{\ell}{g}}$ $\qquad T \propto \sqrt{\ell}$

If ℓ is increased by 4 times, time period will increase by two times.

95. **(b)** Amplitude, A = 10 cm = 0.1 m
Maximum acceleration of the block is

$a_{max} = \omega^2A = \frac{kA}{m} \qquad \left(\because \omega = \sqrt{\frac{k}{m}}\right)$

$= \frac{1000\,N\,m^{-1}\times 0.1m}{10\,kg} = 10\,m\,s^{-2}$

96. **(c)** $K_e = \frac{(2k)(k)}{2k+k} = \frac{2k}{3}$

$\Rightarrow$ Time period, $T = 2\pi\sqrt{\frac{m}{k_e}} = 2\pi\sqrt{\frac{3}{2\frac{k}{3}}} = \frac{6\pi}{\sqrt{2\times9\pi^2}}$

$= \sqrt{2} = 1.414s$

97. **(d)** The effective spring constant of two springs in series;

$K = \frac{k_1k_2}{k_1+k_2}$.

Time period, $T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$

98. **(d)** The effective value of acceleration due to gravity is $\sqrt{(a^2+g^2)}$

99. **(a)** $\omega^2 = \frac{acceleration}{displacement} = \frac{2.0}{0.02}$

$\omega^2 = 100$ or $\omega = 10$ rad/s

100. **(c)** We know that $T = 2\pi\sqrt{\frac{l}{g}}$

$\frac{\Delta T}{T}\times100 = \frac{1}{2}\frac{\Delta l}{l}\times100$

If length is increased by 2%., time period increases by 1%.

101. (d) $T = 2\pi\sqrt{\frac{m}{K}}$ $\therefore \frac{T_1}{T_2} = \sqrt{\frac{M_1}{M_2}}$

$\therefore T_2 = T_1\sqrt{\frac{M_2}{M_1}} = T_1\sqrt{\frac{2M}{M}}$

$T_2 = T_1\sqrt{2} = \sqrt{2}\,T$ (where T_1 = T)

102. (b) At resonance, amplitude of oscillation is maximum

$\Rightarrow 2\omega^2 - 36\omega + 9$ is minimum

$\Rightarrow 4\omega - 36 = 0$ (derivative is zero)

$\Rightarrow \omega = 9$

103. (d)

104. (b) $T = 2\pi\sqrt{l_{eff}/g}$; l_{eff} decreases when the child stands up.

105. (a) Frequency ν' of driving force < frequency ν of damped oscillator.

106. (d) The time period T of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}} = \frac{\ell}{\text{frequency}(n)}$$

Since, n × ℓ

$$\therefore \frac{n_1}{n_2} = \sqrt{\frac{\ell_2}{\ell_1}}$$

$$\Rightarrow \frac{\ell_1}{\ell_2} = \left(\frac{n_2}{n_1}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

Exercise 2 : ACCELERATOR

Topic-wise MCQs

1. (d) At mean position velocity is maximum

i.e., $v_{max} = \omega a \Rightarrow \omega = \frac{v_{max}}{a} = \frac{16}{4} = 4$

$\therefore v = \omega\sqrt{a^2 - y^2} \Rightarrow 8\sqrt{3} = 4\sqrt{4^2 - y^2}$

$\Rightarrow 192 = 16(16 - y^2) \Rightarrow 12 = 16 - y^2 \Rightarrow y = 2cm.$

2. (a) Restoration force, $F = m\omega^2 A$

$$\therefore \frac{F}{m} = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A$$

Point A covers 30° $= \frac{\pi}{6}$ in 0.1 s

$\therefore$ 2π covered in $\frac{2\pi \times 0.1}{\pi/6} = 1.2$ s or $T = 1.2$ s

$\therefore$ Restoration force per unit mass,

$$\frac{F}{m} = \frac{4\pi^2}{(1.2)^2} \times 0.36 = \pi^2 = (3.14)^2 \simeq 9.87 \text{ N}$$

3. (b) Let equation of simple harmonic motion is

$x = A\sin(\omega t + \delta)$

It is given, A = 0.5 m and

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4}s^{-1} = \frac{\pi}{2}s^{-1}$$

At t = 0.5 s, x = 0.3m so $0.3 = 0.5\sin(\omega t + \delta)$

$$\Rightarrow \sin\left(\frac{\pi}{2} \times \frac{1}{2} + \delta\right) = \frac{3}{5} \Rightarrow \frac{\pi}{4} + \delta = 37^\circ$$

$\Rightarrow \delta = 37^\circ - 45^\circ = -8^\circ$

So, equation of motion is

$$x = (0.5\text{ m})\sin\left[\frac{\pi t}{2} - 8^\circ\right]$$

4. (c) Velocity, $v = \omega\sqrt{A^2 - x^2}$ and acceleration = $\omega^2 x$

Now given,

$\omega^2 x = \omega\sqrt{A^2 - x^2} \Rightarrow \omega^2.1 = \omega\sqrt{2^2 - 1^2}$

$\Rightarrow \omega = \sqrt{3} \therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}$

5. (d) At t = 0, $x = 5 = \frac{A}{2}$

$\Rightarrow$ Initial phase, $\phi = 30^\circ = \frac{\pi}{6}$

$\Rightarrow x = A\sin(\omega t + \phi)$

$= 10\sin\left(\frac{2\pi}{T}t + \frac{\pi}{6}\right) = 10\sin\left(\pi t + \frac{\pi}{6}\right)$

6. (a) $y_1 = 4\sin(4\pi t + \frac{\pi}{2}) = 4\cos 4\pi t$

$y_2 = 3\cos(4\pi t) = 3\cos 4\pi t$

The phase difference = 0, both are along the same line

$\therefore A^2 = 4^2 + 3^2 + 2 \times 4 \times 3\cos 0^\circ$

$A^2 = (4+3)^2 \Rightarrow A = 7.$

The resultant amplitude is 7 units.

7. (c) Equation of SHM

$$x(t) = A\sin^2(\alpha t) = A\left[\frac{1 - \cos 2\alpha t}{2}\right] = \frac{A}{2} - \frac{A}{2}\cos 2\alpha t$$

$$\left[\text{using } \sin^2\theta = \frac{1 - \cos^2\theta}{2}\right]$$

Comparing = cos cot

$$\alpha = \frac{\omega}{2} = \frac{2\pi}{2T}\left[\because \omega = \frac{2\pi}{T}\right] = \frac{\pi}{(0.2)}\text{rad/s}[0s\ T = 0.2s]$$

$\therefore \alpha = 5\pi$ rad/s

8. (b) Acceleration in simple harmonic motion is given by

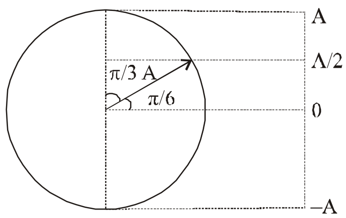
$|a| = \omega^2 x$

$\Rightarrow 16 = \omega^2(4) \Rightarrow \omega = 2$ rad/s

Velocity, $v = \omega\sqrt{A^2 - x^2}$

$$\Rightarrow A = \sqrt{\frac{v^2}{\omega^2} + x^2} \Rightarrow A = \sqrt{\frac{4}{4} + 16} \Rightarrow A = \sqrt{17}\text{ m}$$

9. **(d)**



Let time from 0 to A/2 is t_1
and from A/2 to A is t_2
From the standard equation of SHM,
$x = A_0 \sin(\omega t)$

$$\Rightarrow \frac{A}{2} = A\sin(\omega t_1)$$

$$\Rightarrow \omega t_1 = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \quad ...(i)$$

then $\omega t_1 = \pi/6$
Using $x = A_0 \sin\omega t$ again
$A = A \sin \omega(t_1 + t_2)$

$$\omega(t_1 + t_2) = \sin^{-1}(1) = \frac{\pi}{2}$$

Using (i)

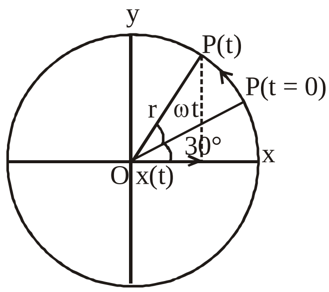
$$\omega t_2 = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3} \quad ...(ii)$$

Dividing equation (i) by (ii) we get

$$\frac{t_1}{t_2} = \frac{1}{2}$$

$$\Rightarrow \quad t_2 = 2t_1 = 2 \times 2 = 4 \text{ sec}$$

10. **(a)**



$x(t) = r\cos(\omega t + 30°) = r\cos(\omega t + \pi/6)$

11. **(b)** Equation of SHM is given by
$x = A\sin(\omega t + \delta)$
$(\omega t + \delta)$ is called phase.

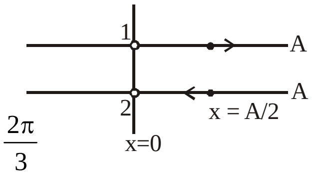
When $x = \frac{A}{2}$, then

$$\sin(\omega t + \delta) = \frac{1}{2}$$

$$\Rightarrow \omega t + \delta = \frac{\pi}{6}$$

$$\text{or } \phi_1 = \frac{\pi}{6}$$

For second particle,



$$\phi_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\therefore \phi = \phi_2 - \phi_1 = \frac{4\pi}{6} = \frac{2\pi}{3}$$

12. **(d)** $y = \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} \Rightarrow$ Period, $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

The given function is not satisfying the standard differential equation of S.H.M. $\frac{d^2y}{dx^2} = -\omega^2 y$. Hence it represents periodic motion but not S.H.M.

13. **(b)** We know that if $t \to \infty$, $\log \omega t$ diverges to ∞. Hence, it cannot represents any kind of repeated motion.

14. **(b)** Function $(\sin\omega t + \cos\omega t)$ represents a periodic function with period $2\pi/\omega$.
In Option (a), $A\sin^3(\omega t)^2$ is not periodic function due to cube of sine value and square of t value.
In Option (c), $\tan(\omega t)^3$ is not periodic function due to cube of t value.
In Option (d), the function $e^{\omega t}$ is not periodic, as it increases with decreasing time.

15. **(c)** Displacement equation for SHM is,

$$y = a\sin\omega t = a\sin\frac{2\pi}{T}t$$

If $y = \frac{A}{2}$ then,

$$\frac{a}{2} = a\sin\frac{2\pi t}{3} \quad \sin\frac{\pi}{6} = \sin\frac{2\pi t}{3}$$

$$\Rightarrow \quad t = \frac{1}{4}s$$

16. **(c)** $\frac{aT}{x} = \frac{\omega^2 xT}{x} = \omega^2 T = \frac{4\pi^2}{T^2}T = \frac{4\pi^2}{T}$ = constant

17. **(c)** Maximum particle velocity, is given by:
$v_{max} = A\omega = 3 \times 50 = 150 \text{ ms}^{-1}$

18. **(b)** Amplitude of SHM,

$$A = \frac{\text{Stroke length}}{2} = \frac{1}{2} = 1\text{m}$$

Maximum velocity is given as
$v_{max} = \omega A = 100 \times 1 = 100 \text{ m min}^{-1}$

19. **(b)** $\frac{(a_{max})_1}{(a_{max})_2} = \frac{\omega_1^2}{\omega_2^2} = \frac{(100)^2}{(1000)^2} = 1 : 10^2$

20. **(a)**

21. **(b)** $v = \frac{dx}{dt} = -4\pi \times \sin(2\pi t)$

Velocity is maximum when $\sin(2\pi t) = \sin 90°$

$$2\pi t = \frac{\pi}{2} \Rightarrow t = \frac{1}{4} = 0.25 \text{ s}$$

22. **(c)** For SHM, $y = A\sin(2t + \phi)$

$\therefore$ $v = 2A\cos(2t + \phi)$

At $t = 0$, $y = 2m$, $v = 4\ ms^{-1}$

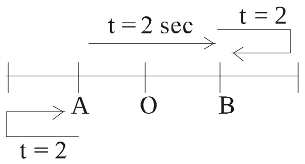
$\therefore$ $2 = a\sin(2 \times 0 + \phi) \Rightarrow \sin\phi = \frac{2}{A}$ ...(i)

Also, $4 = 2A\cos(0 + \phi) \Rightarrow \cos\phi = \frac{2}{A}$ ...(ii)

From eqs (i) and (ii), we get

$\sin\phi = \cos\phi \Rightarrow \phi = 45°$

23. **(d)** From the given information it can be inferred that points A and B are equidistant from mean position. Hence from diagram it is clear that time period of oscillation is = 2 + 2 × 2 + 2 = 8 seconds.



24. **(b)** Let the equation of simple harmonic motion be,

$x = A\sin\omega t$

Then, the required ratio is $\dfrac{\int_0^{T/4} \omega^2 A\sin\omega t\, dt}{\frac{T}{4} \times \omega^2 A} = \frac{2}{\pi}$

25. **(d)** $y_1 = 4\sin(10t + \phi)$

$y_2 = 5\cos 10t$

$v_1 = \frac{dy_1}{dt} = 40\cos(10t + \phi)$

$v_2 = \frac{dy_2}{dt} = -50\sin 10t = 50\cos(10t + \pi/2)$

Phase difference between v_1 and $v_2 = \left(\phi - \frac{\pi}{2}\right)$

26. **(d)** $x = A\sin t\frac{2\pi}{T}$; for $x = \frac{A}{2} \Rightarrow \frac{A}{2} = A\sin\frac{2\pi}{T}t$

Solving, $t = \frac{T}{6}$.

27. **(c)** $y = 0.2\sin(10\pi t + 1.5\pi)\cos(10\pi t + 1.5\pi)$

$= 0.1\sin 2(10\pi t + 1.5\pi)$

$= 0.1\sin(20\pi t + 3.0\pi)$ [$\because \sin 2A = 2\sin A\cos A$]

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = \frac{1}{10} = 0.1\text{sec}$

28. **(b)** $x = 12\sin\omega t - 16\sin^3\omega t = 4[3\sin\omega t - 4\sin^3\omega t]$

$= 4[\sin 3\omega t]$ (By using $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$)

$\therefore$ maximum acceleration $A_{max} = 3(\omega)^2 \times 4 = 36\omega^2$

29. **(c)** Given, $x = a\cos\omega t$ ... (i)

$y = a\sin\omega t$ ... (ii)

Squaring and adding Eqs. (i) and (ii),

$\Rightarrow$ $x^2 + y^2 = a^2$ [$\because \cos^2\omega t + \sin^2\omega t = 1$]

This is the equation, of a circle

Clearly, the locus is a circular of constant radius a.

30. **(c)** Given $4v^2 = 25 - x^2$

Differentiating with respect to *t* on both sides,

$8va = -2xv \Rightarrow$ $a = -\frac{x}{4}$

Motion is simple harmonic:

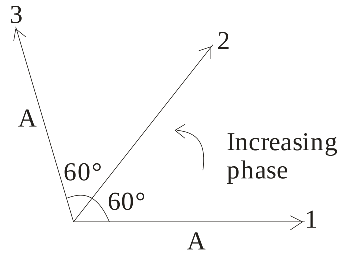
So, $\omega^2 = \frac{1}{4}$ and $T = \frac{2\pi}{\frac{1}{2}} = 4\pi$

31. **(b)** $x = 3\sin(5\pi t + \pi/3) + \cos(5\pi t + \pi/3)$

Amplitude = $x_{max} = \sqrt{3^2 + 1^2} = \sqrt{10}, \omega = 5\pi$

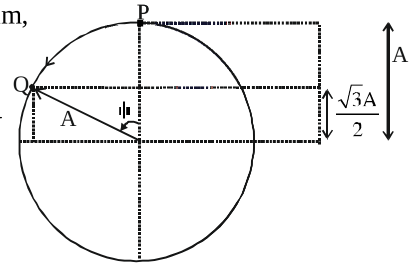
$T = \frac{2\pi}{\omega} = \frac{2\pi}{5\pi} = 0.4s$

32. **(d)** The resultant of three vectors shown in the figure is 2A (along the direction of 2).



33. **(a)** The phasor diagram of SHM is

From phasor diagram,



$\cos\phi = \dfrac{\frac{\sqrt{3}A}{2}}{A} = \frac{\sqrt{3}}{2}$

$\therefore \phi = \omega t = \frac{\pi}{6}$

$\Rightarrow \frac{2\pi}{T}t = \frac{\pi}{6}$

$\therefore t = \frac{\pi}{2}s = \frac{\pi}{x}s$ (given) [$\because$ $T = 6\pi$ s]

$\therefore x = 2$

34. **(d)** Given, mass of particle, m = 0.5 kg

Force, $F = -50(x) = ma$

$\Rightarrow 0.5\,a = -50x \Rightarrow a = (-100x)$

Comparing with $a = \omega^2 x$ we get

$\omega^2 = 100$

$\therefore$ Time period $T = \frac{2\pi}{10} = \left(\frac{\pi}{5}\right) = \frac{22}{7 \times 5} = \left(\frac{22}{35}\right) = \frac{x}{35}$

$\Rightarrow x = 22$

35. **(d)** According to equation of SHM

$x = A\sin\omega t$

Here, $x = \frac{A}{2}$ and $t = 1$

$\therefore \frac{A}{2} = A\sin\omega \times 1$

$= A\sin\omega$

or, $\sin \omega = \frac{1}{2}, \omega = \frac{\pi}{6}$

$\therefore \frac{2\pi}{T} = \frac{\pi}{6}$ or T = 12 s

36. **(a)** Given : T/2 = 0.5 s

$\therefore$ T = 1s

Frequency,

$f = \frac{1}{T} = \frac{1}{1} = 1\text{hz}$

If A is the amplitude, then

2A = 50 cm $\Rightarrow$ A = 25 cm.

37. **(b)** The block and the piston will separate from each other when the maximum restoring force just exceeds the weight of the block, that is

$m\omega^2 x \geq mg$

$\Rightarrow x \geq \frac{g}{\omega^2}$

$\Rightarrow x \geq \frac{g}{\left(\frac{2\pi}{T}\right)^2}$

$\therefore$ Minimum value of $x = \frac{gT^2}{4\pi^2}$

$= \frac{10 \times 1^2}{4 \times (3.14)^2} = 0.25\text{ m}$

38. **(a)** Let $x_1 = A\sin(\omega t + \phi_1)$ and $x_2 = A\sin(\omega t + \phi_2)$

$x_2 - x_1 = A[\sin(\omega t + \phi_2) - \sin(\omega t - \phi_1)]$

$= 2A\cos\left(\frac{2\omega t + \phi_1 + \phi_2}{1}\right)\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$

The resultant motion can be treated as a simple harmonic motion with amplitude $2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$.

Given, maximum distance between the particles

$\therefore$ Amplitude of resultant SHM

$= X_0 + A - X_0 = A$

$\therefore \quad 2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right) = A.$

$\Rightarrow \quad \phi_2 - \phi_1 = \pi/3.$

39. **(a)** Equation of two simple harmonic motion

$y = A\sin(\omega t + \phi)$ ....(1)

$x = A\sin\left(\omega t + \phi + \frac{\pi}{2}\right) \Rightarrow x = A\cos(\omega t + \phi)$ ....(2)

On squaring and adding equations (1) and (2),

$x^2 + y^2 = A^2$

This is an equation of a circle. Hence, resulting motion will be a circular motion.

40. **(a)** The given velocity-position graph depicts that the motion of the particle is SHM.

In SHM, at t = 0, v = 0 and $x = x_{max}$

So, option (a) is correct.

41. **(b)** $K = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2 x^2$, at $x = \pm\frac{A}{\sqrt{2}}$.

42. **(c)** Kinetic energy $K = \frac{1}{2}m\omega^2(a^2 - y^2)$

$= \frac{1}{2} \times 10 \times \left(\frac{2\pi}{2}\right)^2 [10^2 - 5^2] = 375\,\pi^2\ ergs$

43. **(d)** $\frac{U}{E} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} = \frac{y^2}{a^2} \Rightarrow \frac{U}{80} = \frac{\left(\frac{3}{4}a\right)^2}{a^2} = \frac{9}{16} \Rightarrow U = 45J$

44. **(c)**

45. **(c)** $E = \frac{1}{2}m\omega^2 a^2 \Rightarrow \frac{E'}{E} = \frac{a'^2}{a^2} \Rightarrow \frac{E'}{E} = \frac{\left(\frac{3}{4}a\right)^2}{a^2} \left(\because a' = \frac{3}{4}a\right)$

$\Rightarrow E' = \frac{9}{16}E$

46. **(a)**

47. **(c)**

48. **(c)** At maximum energy of the particle, velocity resonance takes place, which occurs when frequency of external periodic force is equal to natural frequency of undamped vibrations, i.e. $\omega_2 = \omega_0$.

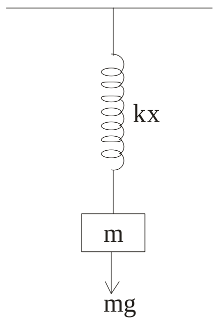
Further amplitude resonance takes place at a frequency of external force which is less than the frequency of undamped natural vibrations, i.e. $\omega_1 \neq \omega_0$.

49. **(c)** At equilibrium

Mg = Kx $\Rightarrow$ x = mg/k

Now, with the extra $\frac{mg}{k}$ stretch, the net stretch become

$= \frac{mg}{k} + \frac{mg}{k} = \frac{2mg}{k}$



50. **(a)** We know that $T = 2\pi\sqrt{\frac{M}{k}}$

From first case, $2 = 2\pi\sqrt{\frac{M}{k}}$ .... (1)

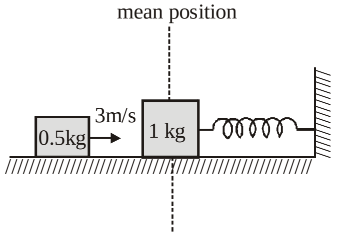
In second case, $4 = 2\pi\sqrt{\frac{M+2}{k}}$ .... (2)

From eq. (1) and eq. (2)

$\frac{4}{2} = \sqrt{\frac{M+2}{M}} \Rightarrow 4 = 1 + \frac{2}{M}$

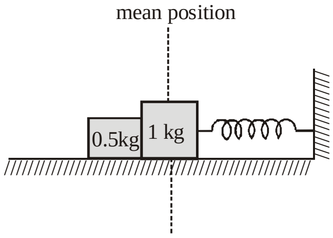
$\frac{2}{M} = 3 \Rightarrow M = \frac{2}{3}\text{kg}$

51. (a) Applying linear momentum conservation,
$0.5 \times 3 = (1 + 0.5)\,v$ or $v = 1$ m/s
By conservation of energy,



After collision

$$\frac{1}{2}(1+0.5)\,v^2 = \frac{1}{2}kA^2 \Rightarrow A = \sqrt{\frac{1.5}{k}} \times v$$

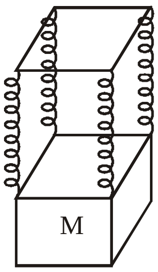


$$\Rightarrow A = \sqrt{\frac{1.5}{600}} \times 1 = \frac{1}{20}\,m = 0.05m$$

$A = 5$ cm.
Time period of oscillation,

$$T = 2\pi\sqrt{\frac{m_1 + m_2}{k}} = 2\pi\sqrt{\frac{1.5}{600}} = \frac{2\pi}{20} = \frac{\pi}{10}\,s$$

52. (a) The springs are in parallel.
Spring constant = 4×4000 Nm^{-1}
$M = 40$ kg.
$\therefore$ Period of oscillation,



$$T = 2\pi\sqrt{\frac{40}{16000}}$$

$$T = \frac{2\pi}{20} = \frac{\pi}{10} \Rightarrow T = 0.314\ s$$

53. (a) Given: mass of the body, $m = 500$ g $= 500 \times 10^{-3}$ kg
spring constant, $k = 8\pi^2$ N m^{-1}
The frequency of oscillation is

$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{8\pi^2 Nm^{-1}}{500 \times 10^{-3} kg}} = 2\ Hz$$

54. (b) $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$ and $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

In series, effective spring constant, $k = \frac{k_1 k_2}{k_1 + k_2}$

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$ ...(ii)

Now, $t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2}$

$t_1^2 + t_2^2 = T^2$ [Using equation (ii)]

55. (d) Time period of pendulum doesn't depends upon mass but it depends upon length (distance between point of suspension and centre of mass). In first three cases length are same so $T = T_1 = T_2$ but in last case centre of mass lowers which in turn increases the length. So in this case time period will be more than the other cases.

56. (b) Distance covered by lift is given by
$y = t^2$
$\therefore$ Acceleration of lift upwards

$$= \frac{d^2y}{dt^2} = \frac{d}{dt}(2t) = 2\ m/s^2 = \frac{g}{5}$$

$$T' = 2\pi\sqrt{\frac{\ell}{g + \frac{g}{5}}} = 2\pi\sqrt{\frac{\ell}{\frac{6}{5}g}} = \sqrt{\frac{5}{6}}T.$$

57. (a) In damped harmonic oscillator amplitude falls exponentially.

After 100 oscillations amplitude falls to $\frac{1}{3}$ times.
$\therefore$ After next 100 oscillations i.e., after 200 oscillations amplitude falls to $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ times.

58. (d) In damped oscillation, amplitude goes on decaying exponentially.
$a = a_0 e^{-bt}$ where b = damping coefficient.

Initially, $\frac{a_0}{3} = a_0 e^{-b \times 100T}$, T = time of one oscillation

or $\frac{1}{3} = e^{-100bT}$
Finally, $a = a_0 e^{-b \times 200T}$ or $a = a_0[e^{-100bT}]^2$

or $a = a_0 \times \left[\frac{1}{3}\right]^2$ [from (*i*)]
or $a = a_0/9$.

59. (c) $\because A = A_0 e^{-\frac{bt}{2m}}$ (where, A_0 = maximum amplitude)
According to the questions, after 5 seconds,

$$0.9A_0 = A_0 e^{-\frac{b(5)}{2m}} \quad \ldots(i)$$

After 10 more seconds,

$$A = A_0 e^{-\frac{b(15)}{2m}} \quad \ldots(ii)$$

From equations (i) and (ii)
$A = 0.729\,A_0$
$\therefore\ \alpha = 0.729$

60. (a) Let T_1 and T_2 be the time period of the two pendulums

$$T_1 = 2\pi\sqrt{\frac{1}{g}} \text{ and } T_2 = 2\pi\sqrt{\frac{4}{g}}$$

As $\ell_1 < \ell_2$ therefore $T_1 < T_2$
Let at t = 0 they start swinging together. Since their time periods are different, the swinging will not be in unison always. Only when number of completed oscillations differ by an integer, the two pendulums will again begin to swing together

Let longer length pendulum complete n oscillation and shorter length pendulum complete (n + 1) oscillation. For unison swinging

$$(n+1)T_1 = nT_2$$

$$(n+1)\times 2\pi\sqrt{\frac{1}{g}} = (n)\times 2\pi\sqrt{\frac{4}{g}} \Rightarrow n = 1$$

$$\therefore n+1 = 1+1 = 2$$

Exercise 3 : PREVIOUS YEARS MCQs

1. **(a)** Speed $= v = \omega\sqrt{a^2 - x^2}$

$$= \frac{2\pi}{T}\sqrt{a^2 - a^2/4} = \frac{\sqrt{3}a\pi}{T}$$

2. **(d)** Total energy in SHM $= \frac{1}{2}kA^2$

Here, k = force constant
A = amplitude of S.H.M.

Potential energy, $= \frac{1}{2}kx^2$

x = displacement from mean position

Kinetic energy $= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$$\therefore \frac{P.E}{K.E} = \frac{x^2}{A^2 - x^2} = \frac{\left(\frac{A}{2}\right)^2}{A^2 - \left(\frac{A}{2}\right)^2} = \frac{A^2}{4\left(\frac{3A^2}{4}\right)} = \frac{1}{3}$$

3. **(b)** We know that $V = \omega\sqrt{A^2 - x^2}$

Initially $V = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

Finally $3V = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}$

Where A'= final amplitude (Given at $x = \frac{2A}{3}$, velocity to trebled)

On dividing we get

$$\frac{3}{1} = \frac{\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}}{\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}} \Rightarrow 9\left[A^2 - \frac{4A^2}{9}\right] = A'^2 - \frac{4A^2}{9}$$

$$\Rightarrow A' = \frac{7A}{3}$$

4. **(a)** As we know,

kinetic energy $= \frac{1}{2}m\omega^2(A^2 - x^2)$

Potential energy $= \frac{1}{2}m\omega^2 x^2$

$$\frac{\frac{1}{2}m\omega^2(A^2 - x^2)}{\frac{1}{2}m\omega^2 x^2} = \frac{1}{4} \Rightarrow \frac{A^2 - x^2}{x^2} = \frac{1}{4}$$

$$4A^2 - 4x^2 = x^2 \Rightarrow x^2 = \frac{4}{5}A^2 \Rightarrow x = \frac{2}{\sqrt{5}}A.$$

5. **(b)** $v_1 = \frac{dy_1}{dt} = 0.1\times 100\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$

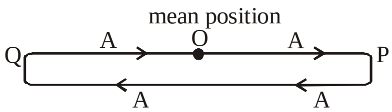
$$v_2 = \frac{dy_2}{dt} = -0.1\pi\sin\pi t = 0.1\pi\cos\left(\pi t + \frac{\pi}{2}\right)$$

$$\therefore \text{Phase diff.} = \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6} = -\frac{\pi}{6}$$

6. **(a)** In $x = A\cos\omega t$, the particle starts oscillating from extreme position. So at $t = 0$, its potential energy is maximum.

7. **(b)**

8. **(d)**



in one time period total distance travelled = A + A + A + A = 4A [as in each quarter starting from mean position it travelles A distance as shown]

9. **(c)** Given : Path length = 16 *cm*

$$\therefore \text{Amplitude } a = \frac{16}{2} = 8\,cm$$

Time period $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{\pi^2}} = 2\pi\times\frac{1}{\pi} = 2s$

Maximum velocity $V_{max} = a\omega$

$$= a\times\frac{2\pi}{T} = 8\times\frac{2\pi}{2} = 8\pi\,cm/s$$

10. **(b)** Distance travelled in one oscillation = 4a

Average velocity $= \frac{total\ distance}{Time}$

$$= \frac{4a}{T} = 4an\ [\because n = \frac{1}{T}]$$

11. **(d)** Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

and, frequency, $n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$25 = \frac{1}{4\pi^2}\frac{k}{m} \Rightarrow k = 100\pi^2 m$$

$$kA = mg \Rightarrow A = \frac{mg}{k}$$

$$V_{max} = \omega A = \frac{2\pi}{T}A = 2\pi nA$$

$$= \frac{2\pi 5\times mg}{k} = \frac{10\pi\times m\times 10}{100\pi^2 m} = \frac{1}{\pi}$$

12. **(d)** Given, particle velocity, $v = \pi$ m/s and time period $T = 16$ s

Displacement of the particle, $x = A\sin\omega t$

Velocity of the particle,

$$v = \frac{dx}{dt} = A\omega\cos\omega t \quad ...(i)$$

Angular velocity,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{16} = \frac{\pi}{8} = \text{rad/s}$$

Now from eq. (i),

$$\pi = A \times \frac{\pi}{8} \times \cos\frac{\pi}{8} \times 2$$

$$1 = \frac{A}{8}\cos\frac{\pi}{4} = \frac{A}{8}.\frac{1}{\sqrt{2}}$$

$$\therefore\ A = 8\sqrt{2}\text{ m}$$

13. (c) In SHM, $KE = \frac{1}{2}m\omega^2(A^2 - Y^2)$

KE when $Y = \frac{A}{2} = \frac{1}{2}m\omega^2\left[A^2\left(\frac{A}{2}\right)^2\right] = \frac{1}{2}m\omega^2\left(\frac{3}{4}A^2\right)$

K.E when $Y = \frac{A}{2} = \frac{1}{2}m\omega^2\left[A^2\left(\frac{A}{2}\right)^2\right] = \frac{1}{2}m\omega^2\left(\frac{3}{4}A^2\right)$

$$\therefore\ \frac{KE_{Y=A/4}}{KE_{Y=A/2}} = \frac{\frac{1}{2}m\omega^2\left(\frac{15}{16}A^2\right)}{\frac{1}{2}m\omega^2\left(\frac{3}{164}A^2\right)} = \frac{5}{4}$$

$\therefore\ KE_{Y=A/4} : KE_{Y=A/2} = 5:4$

14. (b) Given: Time period on the surface = T

Height = $h = \frac{R}{2}$

New acceleration due to gravity at height h is

$$g' = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R+\frac{R}{2}}\right)^2 = 9\left(\frac{2}{3}\right)^2 = \frac{4}{9}g$$

Time period formula :

$$T' = 2\pi\sqrt{\frac{\ell}{g'}} = 2\pi\sqrt{\frac{\ell}{\frac{4}{9}g}} = 2\pi\sqrt{\frac{9\ell}{4g}} = \frac{3}{2}\times 2\pi\sqrt{\frac{\ell}{g}} = \frac{3T}{2}$$

15. (d)

16. (b) Angular frequency $w = 2\pi n$ and

$$n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

Given $K = 2.5\ Nm^{-1}$, $m = 0.1$ Kg $\omega = ?$

$$n = \frac{1}{2\pi}\sqrt{\frac{2.5}{0.1}} = \frac{5}{2\pi}$$

$$\therefore \omega = 2\pi \times \frac{5}{2\pi} = 5\text{ rad s}^{-1}$$

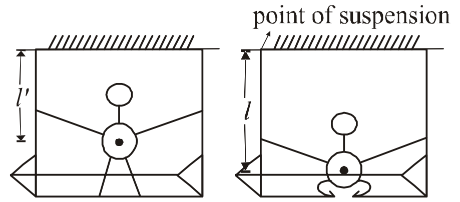
17. (a)

18. (b) The time period $T = 2\pi\sqrt{\frac{\ell}{g}}$ where ℓ = distance between the point of suspension and the centre of mass of the child.

As the child stands up, her centre of mass is raised. The distance between point of suspension and centre of mass decreases ie length ℓ decreases.

$\therefore\ \ell' < \ell$

$\therefore\ T' < T$ *i.e.*, the period decreases.



Case (ii) child standing Case (i) child sitting

19. (d) Original time period,

$$T_0 = 2\pi\sqrt{\frac{\ell}{g}}$$

New time period $T = 2\pi\sqrt{\frac{\ell/16}{g}} = \frac{2\pi}{4}\sqrt{\frac{\ell}{g}}\ \therefore\ T = \frac{T_0}{4}$

20. (a) At any instant the total energy in SHM is

$\frac{1}{2}kA_0^2$ = constant,

where A_0 = amplitude

k = spring constant

hence total energy is independent of x.

21. (a) In stationary lift, $T = 2\pi\sqrt{\frac{l}{g}}$

In upward moving lift, $T' = 2\pi\sqrt{\frac{l}{g+a}}$

(Here, a = acceleration of lift)

$$\frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{g}{g+\frac{g}{2}}} \Rightarrow T' = \sqrt{\frac{2}{3}}T$$

22. (d)

23. (b) At $t = 1$ sec

$$x = \sin\pi\left(1+\frac{1}{3}\right) = \sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$$

So, $V = \omega\sqrt{A^2 - x^2} = \pi\sqrt{1-\frac{3}{4}} = \frac{\pi}{2} m/s = 157$ cm/s

24. (c) The kinetic energy (K. E.) of particle in SHM is given by,

$K.E = \frac{1}{2}k(A^2 - x^2)$;

Potential energy of particle in SHM is $U = \frac{1}{2}kx^2$

Where A = amplitude and $k = m\omega^2$

x = displacement from the mean position

At the mean position $x = 0$

$\therefore$ K.E. $= \frac{1}{2}kA^2$ = Maximum, and U = 0

25. (a) The equation of SHM, $\frac{d^2x}{dt^2} + \alpha x = 0$

$$\frac{d^2x}{dt^2} = -\alpha x = -\omega^2 x$$

$$\Rightarrow\ \omega = \sqrt{\alpha}\ \Rightarrow\ T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$$

6 Superposition of Waves

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(b)** $v = \frac{\omega}{k} = 5\pi \text{ cm/s}$

2. **(b)** Equation of wave is, $y = 2\cos 2\pi(330t - x)$ m
$y = A\cos(\omega t - kx)$
On comparing $\omega = 2\pi \times 330$ $[\because \omega = 2\pi F]$
$2\pi f = 2\pi \times 330 \Rightarrow f = 330$ Hz

3. **(a)** Comparing the given equation
$y = 10^{-3}\sin(50t + 2x)$ with standard equation,
$y = a\sin(\omega t - kx)$
$\Rightarrow$ wave is moving along –ve x-axis with speed
$$v = \frac{\omega}{K} \Rightarrow v = \frac{50}{2} = 25 \text{ m/sec.}$$

4. **(d)**

5. **(c)** The equation of progressive wave propagating in the positive direction of X-axis is
$$y = a\sin\frac{2\pi}{\lambda}(vt - x) \text{ or}$$
$y = a\sin(\omega t - kx)$

6. **(d)** Consider the two waves of equal amplitude A and equal frequency travelling in the same direction
$y_1(x, t) = A\sin(kx - \omega t)$
$y_2(x, t) = A\sin(kx - \omega t + \phi)$
$y(x, t) = A\sin(kx - \omega t) + A\sin(kx - \omega t + \phi)$
$$= A\left[2\sin\left[\frac{(kx - \omega t) + (kx - \omega t + \phi)}{2}\right]\cos\frac{\phi}{2}\right]$$
$$= 2A\cos\frac{\phi}{2}\left(kx - \omega t + \frac{\phi}{2}\right)$$
The resultant amplitude is $2A\cos\frac{\phi}{2}$
Magnitude of wave varies between 0 to 2A because the given cosine function will vary from 0 to π.

7. **(c)** The reflected wave has the same shape as the incident wave but it suffers a phase change of π or 180° on reflection. This is due to the fact that the boundary is rigid and the disturbance must have zero displacement at all times at the boundary.

8. **(a)** Since, the boundary is not completely rigid, then a part of the incident wave is reflected and a part is transmitted into the second medium.

9. **(d)** Path difference $(\Delta x) = 50 \text{ cm} = \frac{1}{2}\text{m}$
$\therefore$ Phase difference $\Delta\varphi = \frac{2\pi}{\lambda} \times \Delta x \Rightarrow \varphi = \frac{2\pi}{1} \times \frac{1}{2} = \pi$
Total phase difference $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$
$$\Rightarrow A = \sqrt{a^2 + a^2 + 2a^2\cos(2\pi/3)} = a$$

10. **(b)** Since the sources are incoherent there will be no sustained interference.
Hence at the given point,
Average intensity $= I_0 = I_1 + I_2 + I_3 + ... + I_{10} = 10I$

11. **(c)** $\frac{a_1 + a_2}{a_1 - a_2} = 5 \Rightarrow a_1 + a_2 = 5(a_1 - a_2)$ $\quad \frac{a_1}{a_2} = \frac{3}{2}$
$$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{9}{4}$$

12. **(d)** Reflecting pulse will be inverted as it is reflected by a denser medium. The wall exerts force in downward direction.

13. **(b)** On reflection at a denser medium, change of phase $= \pi$ (radian)

14. **(d)** *yz* A sin $(kx - \omega t)$ for the wave progressing along the *x*-axis and for the reflected wave,
$y' = A\sin(kx + \omega t)$.
But the position of the rigid wall is at $x = 0$.
$\therefore$ For the given wave, its reflected wave
$y' = -A\sin(kx + \omega t)$.

15. **(b)** Given, $y = 0.3\sin(0.157x)\cos(200\pi t)$
So $k = 0.157$ and $w = 200\pi$
or $f = 100$ Hz, $v = \frac{w}{k} = \frac{200\pi}{0.157} = 4000$ m/s
Now, using $f = \frac{nv}{2l} = \frac{4v}{2l} = \frac{2v}{l}$
$$\therefore l = \frac{2v}{f} = \frac{2 \times 4000}{100} = 80m$$

16. **(d)**

17. **(b)** After 2 s, the each wave travels a distance $= 2 \times 2 = 4$ m.
The wave shape is shown in figure.
Thus energy is purely kinetic.

18. **(d)** Here, $A_1 = A, A_2 = A, \phi = 120°$

The amplitude of the resultant wave is

$$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$$

$$= \sqrt{A^2 + A^2 + 2AA\cos 120°}$$

$$= \sqrt{A^2 + A^2 - A^2} \quad \left(\because \cos 120° = -\frac{1}{2}\right)$$

$= A_R = A$

19. **(c)** If two waves of nearly equal frequency superpose, they give beats if they both travel in straight line and $I_{min} = 0$ if they have equal amplitudes.

20. **(a)** The resultant amplitude is given by

$$A_R = \sqrt{A^2 + A^2 + 2AA\cos\theta} = \sqrt{2A^2(1+\cos\theta)}$$

$= 2\ A \cos \theta/2$

21. **(b)** Superposition of waves does not alter the frequency of resultant wave and resultant amplitude

$\Rightarrow$ $a^2 = a^2 + a^2 + 2a^2 \cos\varphi = 2a^2(1 + \cos\varphi)$

$\Rightarrow$ $\cos\varphi = -1/2 = \cos 2\pi/3 \therefore \varphi = 2\pi/3$

22. **(b)**

23. **(d)** $y = \frac{1}{\sqrt{a}}\sin\omega t \pm \frac{1}{\sqrt{b}}\sin\left(\omega t + \frac{\pi}{2}\right)$

Here phase difference $= \frac{\pi}{2}$

Resultant amp. $= \sqrt{\left(\frac{1}{\sqrt{a}}\right)^2 + \left(\frac{1}{\sqrt{b}}\right)^2} = \sqrt{\frac{1}{a} + \frac{1}{b}} = \sqrt{\frac{a+b}{ab}}$

24. **(c)** Two waves are said to be coherent, if they have same frequency and phase.

25. **(a)**

26. **(b)** On reflection from rigid boundary, change of phase = π. and from open boundary, no change of phase occurs.

27. **(d)** $y = A\sin(kx - \omega t)$ for the wave progressing along the x-axis and for the reflected wave,

$y' = A\sin(kx + \omega t)$.

But the position of the rigid wall is at $x = 0$.

$\therefore$ For the given wave, its reflected wave

$y' = -A\sin(kx + \omega t)$.

28. **(a)** When a sound wave gets reflected from a rigid boundary, the particles at the boundary are unable to vibrate. Thus, a reflected wave is generated which interferes with the incoming wave to produce zero displacement at the rigid boundary. At these points (zero displacement), the pressure variation is maximum. Thus, a reflected pressure wave has the same phase as the incident wave.

29. **(d)** Wave number $K = \frac{2\pi}{\lambda} = 0.6\ cm^{-1}$

$\therefore\ \frac{\lambda}{2} = \frac{\pi}{0.6} cm \ \therefore\ \ell = \frac{3\lambda}{2} = 3\left(\frac{\pi}{0.6}\right) cm = 15.7 cm$

30. **(a)** $v = \frac{1}{\ell(3D)}\sqrt{\frac{2T}{\pi\rho}}; v' = \frac{1}{\ell D}\sqrt{\frac{T}{\pi(2\pi)}}$

$$\frac{v}{v'} = \frac{\sqrt{2}}{3} \times \sqrt{2} = \frac{2}{3}$$

31. **(a)** Let ℓ_1 and ℓ_2 be the lengths of closed and open pipes respectively. (Neglecting end correction)

$$\ell_1 = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4\ell_1 \text{ and } \ell_2 = \frac{\lambda_2}{2} \Rightarrow \lambda_2 = 2\ell_2$$

Given $n_1 = n_2$ so $\frac{v}{\lambda_1} = \frac{v}{\lambda_2} \Rightarrow \frac{v}{4\ell_1} = \frac{v}{2\ell_2} = \frac{\ell_1}{\ell_2} = \frac{1}{2}$

32. **(d)** Because the tuning fork is an resonance with air column in the pipe closed at one end, the frequency is $n = \frac{(2N-1)v}{4\ell}$ where, N = 1, 2. 3 .... corresponds to different mode of vibration putting n = 340 Hz, v = 340 m/s, the length of air column in the pipe can be

$$\ell = \frac{(2N-1)340}{4\times340} = \frac{(2N-1)}{4} m = \frac{(2N-1)\times100}{4} cm$$

For N - 1, 2, 3, ... we get ℓ = 25 cm, 75 cm, 125 cm...

As the tube is only 120 cm long, length of air column after water is poured in it may be 25 cm or 75 cm only, 125 cm is not possible, the corresponding length of water column in the tube will be (120 – 25) cm = 95 cm or (120 – 75) cm = 45 cm.

Thus minimum length of water column is 45 cm.

33. **(c)** Critical hearing frequency for a person is 20,000 Hz. If a closed pipe vibration in Nth mode then frequency of vibration $n = \frac{(2N-1)v}{4l} = (2N-1)n_1$

(where n_1 = fundamental frequency of vibration)

Hence $20{,}000 = (2N-1) \times 1500 \Rightarrow N = 7.1 = 7$

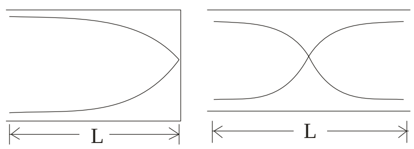
Also, in closed pipe

Number of over tones

= (No. of mode of vibration) – 1 = 7 – 1 = 6.

34. **(b)** $f_0 = f_c \Rightarrow \frac{v}{2L} = \frac{v}{4L_c} \Rightarrow L_c = L/2$

35. **(a)** The fundamental modes of vibration of a pipe closed at one end and open at both ends (of same length) are shown in figure.



(a) (b)

The wavelength in figure (b) is half of that in figure (a). Hence the fundamental frequency in figure (b) is double that in figure (a).

$\therefore$ $f_{open} = 2 \times 512 = 1024$ Hz

36. **(d)** For producing beats, their must be small difference in frequency.

37. **(b)** $3\times\frac{v}{4\ell_c}=4\times\frac{v}{2\ell o}$ or $\frac{\ell_c}{\ell o}=\frac{3v}{4}\times\frac{2}{4v}=\frac{3}{8}$

38. **(d)** $5\times\frac{v}{4\ell}-\frac{v}{\ell}=100$ or $\frac{v}{\ell}\left(\frac{5}{4}-1\right)=100$

or $\frac{v}{4\ell}=100$ o r $\frac{v}{2\ell}$ = 200 or v = 200 Hz

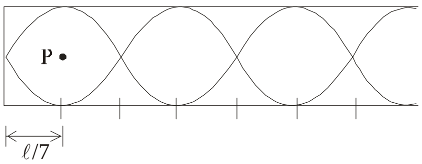
39. **(c)** $\frac{\lambda}{2}=\ell_2-\ell_1=29.5-10.5=19$ cm

Third resonance will be observed at

$\ell_3=\ell_2+\frac{\lambda}{2}=29.5+19=48.5$ cm

40. **(a)** The figure shows variation of displacement of particles in a closed organ pipe for 3rd overtone.

For third overtone $\ell=\frac{7\lambda}{4}$ or $\lambda=\frac{4\ell}{7}$ or $\frac{\lambda}{4}=\frac{\ell}{7}$



Hence the amplitude at P at a distance $\frac{\ell}{7}$ from closed end is 'a' because there is an antinode at that point.

41. **(d)** $L=(2n-1)\frac{\lambda}{4}$

$\lambda=\frac{4L}{(2n-1)}\Rightarrow\lambda_1=\frac{4L}{2(1)-1}=4L;$

$\lambda_2=\frac{4L}{4-1}=\frac{4L}{3}$ and $\lambda_3=\frac{4L}{6-1}=\frac{4L}{5}$

42. **(a)** Equation of standing wave

y = 2A sin kx cos ωt

y = A as amplitude is 2A

A = 2A sin kx

$\frac{2\pi}{\lambda}x=\frac{\pi}{6}\Rightarrow x_1=\frac{1}{4}m$ and $\frac{2\pi}{\lambda}\cdot x=\frac{\pi}{2}+\frac{\pi}{3}$

$\Rightarrow x_1=1.25\text{ m}\Rightarrow x_2-x_1=1\text{ m}$

43. **(b)** Second overtone of open pipe $=\frac{3V}{2\ell_1}$

Second overtone of closed pipe $=\frac{5V}{4\ell_2}$

Since, ratio of frequency are same

$\frac{3V}{2\ell_1}=\frac{5V}{4\ell_2}\Rightarrow\frac{\ell_1}{\ell_2}=\frac{4\times3}{2\times5}=\frac{6}{5}$

Now, the ratio of fundamental frequencies

$$\frac{f_1}{f_2}=\frac{\frac{V}{2\ell_1}}{\frac{V}{4\ell_2}}\Rightarrow\frac{2\ell_2}{\ell_1}=10:6=5:3$$

44. **(b)** In stationary wave all the particles in one particular segment (i.e., between two modes) vibrates in the same phase.

45. **(a)** If $y_{incident}$ = a sin (ωt – kx) and $y_{stationary}$ = a sin (ωt) cos kx then it is clear that frequency of both is same (ω).

46. **(d)** Particles have kinetic energy maximum at mean position.

47. **(a)** Waves A and B satisfied the conditions required for a standing wave.

48. **(a)** Comparing given equation with standard equation

$y=2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}$ gives us

$\frac{2\pi}{\lambda}=\frac{\pi}{15}\Rightarrow\lambda=30$

Distance between nearest node and antinodes

$=\frac{\lambda}{4}=\frac{30}{4}=7.5$

49. **(b)** Since the point x = 0 is a node and reflection is taking place from point x = 0. This means that reflection must be taking place from the fixed end and hence the reflected ray must suffer an additional phase change of π or a path change of $\frac{\lambda}{2}$.

So, if $y_{incident}$ = a cos (kx – ωt)

$\Rightarrow$ $y_{reflected}$ = a cos(–kx – ωt + π) = – cos (ωt + kx)

50. **(d)**

51. **(b)** Distance between the consecutive node $=\frac{\lambda}{2}$,

but $\lambda=\frac{v}{n}=\frac{20}{n}$ so $\frac{\lambda}{2}=\frac{10}{n}$

52. **(b)** 2A = 6 or A = 3 cm

53. **(b)** Equation of the component waves are:

y = A sin (ωt – kx) and y = A sin (ωt + kx)

where ωt – kx = constant or ωt + kx = constnat

Differentiating w.r.t. 't', we get

$\omega-k\frac{dx}{dt}=0$ and $\omega+k\frac{dx}{dt}=0$

$\Rightarrow v=\frac{dx}{dt}=\frac{\omega}{k}$ and $v=-\frac{\omega}{k}$

i.e., the speed of component waves is $\frac{\omega}{k}$.

54. **(c)** For string $\lambda = \frac{2\ell}{p}$

where p = No. of loops = Order of vibration

Hence for forth mode p = 4 $\Rightarrow \lambda = \frac{\ell}{2}$

Hence v = nλ = 500 × $\frac{2}{2}$ = 500 Hz

55. **(c)** Let fundamental frequency be f.
Min. frequency of n^{th} overtone = (n + 1)f
Given, 2f = – 320 ⇒ f = 160 Hz

56. **(b)** $f \propto \frac{1}{L}$ or $f = \frac{L'}{L} \Rightarrow L' = fL$

57. **(b)** Two consecutive frequencies are 420 Hz and 480 Hz.
so the fundamental frequency will be 60 Hz.

$\therefore\ 60 = \frac{1}{2\times\ell}\sqrt{\frac{450}{5\times10^{-3}}} \Rightarrow \ell = 2.1\,m$

Hence (b).

58. **(b)** The equation of stationary wave is

$y = 2\sin\left(\frac{\pi x}{15}\right)\cos(48\pi t)$

Compair with
$y = A\sin(Kx)\cos(\omega t)$

$k = \frac{2\pi}{\lambda} = \frac{\pi}{15} \Rightarrow \lambda = 30$

Distance between node and next antinode $x = \frac{\lambda}{4}$

$\therefore x = \frac{\lambda}{4} = \frac{30}{4} = 7.5\,\text{units}$

59. **(a)** First overtone (3rd harmonic) of the closed pipe of length l_1 is $l_1 = \frac{3v}{4f_c}$

First overtone (2nd harmonic) frequency of the open pipe of length l_2 is

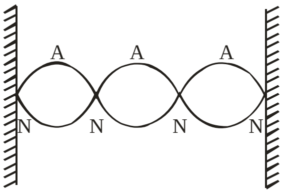
$l_2 = \frac{v}{f_0}$

As per question $f_c = f_0$

$\therefore\ \frac{l_1}{l_2} = \frac{3}{4}$

60. **(b)** The waves set up in a closed pipe are longitudinal and stationary.

61. **(c)** The vibrations of second overtone or third harmonic of a stretched string is shown in the figure.



From figure, number of nodes and antinodes will be 4 and 3 respectively.

62. **(c)** Speed of sound, v = 330 m s^{-1}
Length of pipe, L = 30 cm = 30 × 10^{-2} m
In an open pipe, the frequency of its n^{th} harmonic is given by

$\upsilon_n = \frac{nv}{2L}$ where n = 1, 2, 3, ...

$\therefore\ n = \frac{2L\upsilon_n}{v} = \frac{2\times30\times10^{-2}\times1.1\times10^3}{330} = 2$

63. **(d)**

64. **(b)** For open organ pipe,
l = 30 cm, f = 1.65 KHz

$\therefore\ l = \frac{n\lambda}{2} \Rightarrow n = \frac{2l}{\frac{v}{f}} = \frac{2\times30\times10^{-2}\times1.65\times10^3}{330}$ $\therefore$ n = 3

65. **(d)** Third overtone has a frequency 7 n, which means $L = \frac{7\lambda}{4}$ = three full loops + one half loop, which would make four nodes and four antinodes.

66. **(b)** $\frac{3\lambda}{2} = 2$ or $\lambda = \frac{4}{3}m$

Velocity, $v = f\lambda = 240\times\frac{4}{3} = 320\,\text{m/sec}$

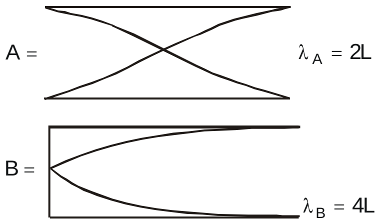
Also $f_1 = \frac{240}{3} = 80\,\text{Hz}$

67. **(a)** Total length of string $\ell = \ell_1 + \ell_2 + \ell_3$
(As string is divided into three segments)

But frequency $\propto \frac{1}{\text{length}}$ $\left(\because f = \frac{1}{2\ell}\sqrt{\frac{T}{m}}\right)$

so $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

68. **(c)** $\frac{\lambda_A}{\lambda_B} = \frac{1}{2} \Rightarrow \frac{n_A}{n_B} = \frac{2}{1}$



69. **(d)** $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}\ \therefore \frac{n_2}{n_1} = \frac{\ell_1}{\ell_2}\sqrt{\frac{T_2}{T_1}}$

$= \frac{\ell_1}{\left[\ell_1 - \frac{40}{100}\ell_1\right]}\frac{\sqrt{T_1 + \left(\frac{44}{100}\right)T_1}}{T_1} = \frac{100}{60}\times\frac{12}{10} = 2:1$

70. **(d)** In case of closed organ pipe frequency,

$$f_n = (2n+1)\frac{v}{4l}$$

for $n = 0, f_0 = 100$ Hz
$n = 1, f_1 = 300$ Hz
$n = 2, f_2 = 500$ Hz
$n = 3, f_3 = 700$ Hz
$n = 4, f_4 = 900$ Hz
$n = 5, f_5 = 1100$ Hz
$n = 6, f_6 = 1300$ Hz

Hence possible natural oscillation whose frequencies < 1250 Hz = 6(n = 0, 1, 2, 3, 4, 5)

71. **(b)** Fundamental frequency of closed organ pipe

$$V_c = \frac{V}{4l_c}$$

Fundamental frequency of open organ pipe

$$V_0 = \frac{V}{2l_0}$$

Second overtone frequency of open organ pipe $= \frac{3V}{2l_0}$

From question,

$$\frac{V}{4l_c} = \frac{3V}{2l_0} \Rightarrow l_0 = 6l_c = 6 \times 20 = 120 \text{ cm}$$

72. **(b)** $3 \times \frac{v}{4l_c} = 4 \times \frac{v}{2l_0}$ or $\frac{l_c}{l_0} = \frac{3v}{4} \times \frac{2}{4v} = \frac{3}{8}$

73. **(b)** Frequency $\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$ $\quad \because V \propto \frac{1}{l}$

$$\therefore \; l_1 : l_2 : l_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$$

74. **(d)**

75. **(a)** $L_0 = 60$ cm $\qquad \nu_0 = 256$ Hz.

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} \quad \therefore \; \nu \propto \frac{1}{L}$$

$$\frac{\nu_1}{\nu_0} = \frac{L_0}{L_1} \Rightarrow \nu_1 = \nu_0 \frac{L_0}{L_1} = 256 \times \frac{60}{15} = 1024 \text{ Hz.}$$

76. **(a)** For open pipe, $n = \frac{v}{2\ell}$, where n_0 is the fundamental frequency of open pipe.

$$\therefore \; \ell = \frac{v}{2n} = \frac{330}{2 \times 300} = \frac{11}{20}$$

As freq. of 1st overtone of open pipe = freq. of 1st overtone of closed pipe

$$\therefore \; 2\frac{v}{2\ell} = 3\frac{v}{4\ell'} \Rightarrow \ell' = \frac{3\ell}{4} = \frac{3}{4} \times \frac{11}{20} = 41.25 \text{ cm}$$

77. **(a)**

78. **(c)** Given,
Density of wire $\rho = 8 \times 10^3$ kg/m^3
Extension developed in the wire, $\Delta L = 3.2 \times 10^{-4}$ m
Length of wire, L = 0.5 m
Young modulus of wire $Y = 8 \times 10^{10}$ N/m^2
Fundamental frequency,

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{YA\Delta L}{\rho L}} = \frac{1}{2 \times 0.5}\sqrt{\frac{8 \times 10^{10} \times 3.2 \times 10^{-4}}{8 \times 10^3 \times 0.5}}$$

$\Rightarrow \; f = 80$ Hz

79. **(c)** **80.** **(b)**

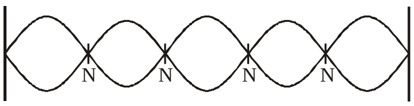
81. **(a)** Frequency does not depend upon radius. As length is doubled, fundamental frequency becomes half.

82. **(a)**

83. **(a)** $\upsilon \propto \sqrt{T}$

84. **(d)**

85. **(a)**



Total no. of nodes = 4

86. **(b)** n_A = Known frequency = 256 Hz, n_B = ?
x = 4 bps, which is decreasing after loading (i.e., $x\downarrow$) also known tuning fork is loaded so $n_A \downarrow$
Hence $n_A \downarrow - n_B = x\downarrow$ ... (i) Correct
$n_B - n_A \downarrow = x\downarrow$ ... (ii) Wrong
$\Rightarrow \; n_B = n_A - x = 256 - 4 = 252$ Hz

87. **(c)** Let n be the frequency of fork C then

$$n_A = n + \frac{3n}{100} = \frac{103n}{100} \text{ and } n_B = n - \frac{2n}{100} = \frac{98}{100}$$

but $n_A - n_B = 5 \Rightarrow \frac{5n}{100} = 5 \Rightarrow n = 100$Hz

$$\therefore \; n_A = \frac{(103)(100)}{100} = 103\text{Hz}$$

88. **(b)** As $y = A_b \sin(2\pi n_{av} t)$
where $A_b = 2A\cos(\pi n_A t)$

where $n_A = \frac{n_1 - n_2}{2}$

Hence, the amplitude of vibration at any point changes simple harmonically with a frequency equal to the difference in the frequencies of the two waves.

89. **(d)** Number of beats/s = $n_1 - n_2$

$$\Delta n = n_1 - n_2 = \frac{v}{2}\left(\frac{1}{l_1} - \frac{1}{l_2}\right)$$

$$4 = \frac{v}{2}\left(\frac{1}{100} - \frac{1}{102.5}\right)$$

$$v = \frac{4 \times 2 \times 100 \times 102.5}{2.5} = 32800 \text{ ms}^{-1}$$

90. **(c)** $n = n_2 \sim n_1$

$\therefore \quad n_1 = n_2 \pm n$

91. **(a)**

92. **(b)** A tuning fork produces 4 beats/sec with another tuning fork of frequency 288 cps. From this information we can conclude that the frequency of unknown fork is 288 + 4 cps or 288 – 4 cps i.e. 292 cps or 284 cps. When a little wax is placed on the unknown fork, it produces 2 beats/sec. When a little wax is placed on the unknown fork, its frequency decreases and simultaneously the beat frequency decreases confirming that the frequency of the unknown fork is 292 cps.

93. **(d)**

94. **(d)** For producing beats, their must be small difference in frequency.

95. **(c)**

96. **(d)** As number of beats/sec = diff. in frequencies has to be less than 10, therefore $0 < (n_1 - n_2) < 10$

97. **(b)** Frequency of vibration of a string in nth harmonic is given as $f_n = n.\frac{v}{2l}$

$$240 = 3 \times \frac{v}{2 \times 2} \Rightarrow v = \frac{4 \times 240}{3} = 320 ms^{-1}$$

Fundamental frequency is given by:

$$f = \frac{f_n}{n} = \frac{f_3}{3} = \frac{240}{3} = 80 Hz$$

98. **(b)**

99. **(d)** For first overtone, n = 2

$$f_o = \frac{nv}{2l} = \frac{2v}{2 \times 50} = \frac{v}{50}$$

Length of closed organ pipe, $\ell_2 = \ell$

Fundamental frequency,

$$f_c = \frac{v}{4l}$$

Since, $f_o = f_c \Rightarrow \frac{v}{50} = \frac{v}{4l} \Rightarrow \ell = \frac{50}{4} = 12.5$ cm

100. **(c)** In case of octave means, the frequency is in the ratio of 2 : 1.

$$\frac{f'}{f} = \frac{2}{1}$$

Also, $f \propto \sqrt{T}$

$$\frac{f'}{f} = \sqrt{\frac{T'}{T}} = \frac{2}{1} \Rightarrow T' = 4T$$

$T' = 4 \times 4 = 16$ kg –wt

101. **(c)** Frequency of nth harmonic in an organ pipe open on both ends is given by:

$$\nu = \frac{nv}{2L} \Rightarrow n = \frac{2\nu L}{v}$$

$$n = \frac{2 \times 1000 \times 16.6 \times 10^{-2}}{332} = 1$$

102. **(d)** Fundamental frequency for an open organ pipe is,

$$\nu_1 = \frac{v}{2L} = \frac{330}{2 \times 1} = 165 Hz$$

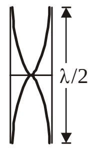
Number of tones present in the open organ pipe

$$n = \frac{\nu}{\nu_1} = \frac{1000}{165} = 6.06 \simeq 6$$

103. **(b)** $\lambda = \frac{2\ell}{n}$ (n = number of loops)

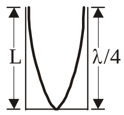
104. **(a)**

105. **(c)** The fundamental frequency of an organ pipe open at both ends is



$$\nu_0 = \frac{v}{2L} \quad ....(i)$$

The fundamental frequency of an organ pipe closed at one end is



$$\nu_c = \frac{v}{4L} \quad ....(ii)$$

Dividing equation (i) by (ii)

$$\frac{\nu_0}{\nu_c} = \frac{v}{2L} \times \frac{4L}{v} = \frac{2}{1}$$

106. **(c)** Harmonics in closed organ pipe :

$v_1 : v_2 : v_3 .... = 1 : 3 : 5 : ......$

So, only odd harmonics are present. ⇒ II correct

For open organ pipe

Natural frequencies = $v = \frac{nv}{2L}$; $n = 1, 2, 3, ...$

Thus, even and odd i.e., all the harmonics are present.

107. **(c)** Beat frequency is given as $n_2 - n_1$

$$2 = \frac{v}{\lambda_2} - \frac{v}{\lambda_1}$$

$$2 = v\left(\frac{1}{2.02} - \frac{1}{2}\right) \Rightarrow 2 = v\left(\frac{200 - 202}{404}\right)$$

v = 404 m/s

108. **(c)**

109. **(b)** From the given equations of progressive wave, $\omega_1 = 500\pi$ and $\omega_2 = 506\pi$

Using $n = \frac{\omega}{2\pi}$

$n_1 = 250$ and $n_2 = 253$

Beat frequency = $n_2 - n_1 = 253 - 250 = 3$ beats per second

Number of beats per min = 60 × 3 = 180

110. **(d)** Beats are observed when intensity at a point varies with time and beat frequency is equal to the frequency of oscillations of intensity at the point.

Intensity at a point not only depends upon the frequency of medium particle also. Therefore, beats are observed when amplitude of oscillation of medium particles varies with time. If the beat frequency at a point is equal to n, it means, at that point amplitude of oscillation of medium particles varies with frequency n.

Amplitude of vibrations changes not only at the point of observation, but at all the points. Therefore, (a) and (b) are wrong. Since the frequency of variation of intensity is observed n times per second therefore, the maximum of intensity is observed n times per second and the intensity become zero n times per second. Hence, (c) is also wrong. Obviously, only option (d) is correct.

111. (a) Since source of frequency x gives 8 beats per second with frequency 250 Hz, its possible frequency are 258 or 242. As source of frequency x gives 12 beats per second with a frequency 270 Hz, its possible frequencies 282 or 258 Hz. The only possible frequency of x which gives 8 beats with frequency 250 Hz also 12 beats per second with 258 Hz.

112. (a) Given : Wavelength of first wave (λ_1) = 50 cm = 0.5 m

Wavelength of second wave (λ_2) = 51 cm = 0.51m

frequency of beats per sec (n) = 12.

We know that the frequency of beats,

$$n = 12 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2} \Rightarrow 12 = v\left[\frac{1}{0.5} - \frac{1}{0.51}\right]$$

$$= v[2 - 1.9608] = v \times 0.0392$$

or, $v = \frac{12}{0.0392} = 306$ m/s [where, v = velocity of sound]

113. (b) For production of beats different frequencies are essential. The different amplitudes affect the minimum and maximum amplitude of the beats and different phases affect the time of occurrence of minimum and maximum.

114. (c) Beats are produced. Frequency of beats will be $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$. Hence time period = 12 s.

115. (d) The time interval between two successive beats

$$T = \frac{1}{\text{Beat frequency}} = \frac{1}{f_1 - f_2}$$

116. (b) Tunning fork A, $f_A = 250\ H_z$

Tunning fork B, $f_B = x$

$|f_A - f_B| = 5$

$250 - x = 5$

$x = 245$ or 255

When it is loaded, it's beat frequency is decreasing by 3 beat/sec.

Beat is decreased = frequency increased to 250Hz. So, it initial frequency must be less than 250Hz

$\because x = 245$ Hz

Exercise 2 : ACCELERATOR

Topic-wise MCQs

1. (a) $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

where ϕ = phase difference

$= [(x - vt + 1.5) - (x - vt)] \times 2\pi = 2\pi \times 1.5 = 3\pi$

So, $A = \sqrt{5^2 + 3^2 + 2.5.3\cos\pi} = \sqrt{4} = 2$ cm.

2. (a)

3. (c) Compairing the given $y = 0.5\sin\left(\frac{2\pi}{\lambda}400t - \frac{2\pi}{\lambda}x\right)$ with standard equation

$$\omega = \frac{2\pi}{\lambda}400 \text{ and } k = \frac{2\pi}{\lambda}$$

$\therefore$ velocity of the wave $v = \frac{\omega}{k} = \frac{\frac{2\pi}{\lambda}400}{\frac{2\pi}{\lambda}}$ $\therefore v = 400$ m/s

4. (a)

A. Reflected wave from a rigid boundary suffers a phase change of π radians.

$\therefore y_{r_1}(x, t) = a\sin(kx - \omega t + \pi) = -a\sin(kx - \omega t)$

B. Reflected wave from an open boundary will be in same phase with the incident wave.

$\therefore y_{r_2}(x, t) = a\sin(kx - \omega t)$

C. At the rigid boundary, the resultant displacement is zero.

D. At the open boundary, the resultant displacement

$= y(x, t) + y_{r_2}(x, t) = a\sin(kx - \omega t) + a\sin(kx - \omega t)$

$= 2a\sin(kx - \omega t)$

5. (d) $\mu = \frac{1.2}{2} = 0.6$ kg / m

$f = 5$ Hz, $\lambda = 2\ell = 4$m

$v = n\lambda = 5 \times 4 = 20$ m/s

Using $v = \sqrt{\frac{T}{\mu}} \Rightarrow T = 20^2 \times 0.6 = 240$ N

6. (b) $n_1 = n_2$

$T \to$ Same

$r \to$ Same

$l \to$ Same

Frequency of vibration

$$n = \frac{p}{2l}\sqrt{\frac{T}{\pi r^2 \rho}}$$

As T, r, and l are same for both the wires

$n_1 = n_2$

$$\frac{p_1}{\sqrt{\rho_1}} = \frac{p_2}{\sqrt{\rho_2}} \Rightarrow \frac{p_1}{p_2} = \frac{1}{2} \quad \because \quad \rho_2 = 4\rho_1$$

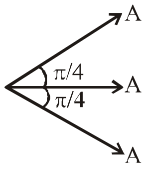
7. **(c)** At nodes pressure change is maximum.

8. **(a)** In case of independent sources, the phase difference between them does not remain constant.

9. **(a)** $A_{res} = \sqrt{2}A + A$

$= (\sqrt{2}+1)A$ as $I \propto A^2$

$I_{res} = (\sqrt{2}+1)^2 I_o = 5.8 I_o$



10. **(c)**

11. **(b)** Intensity for coherent wave

$I_2 = I_A + I_B + 2\sqrt{I_A I_B}\cos 60^o$

$\Rightarrow \quad I_2 = I_0 + 9I_0 + 2\sqrt{9I_0^2}.\frac{1}{2} = 13I_0$

Intensity for incoherent wave

$I_1 = I_A + I_B = I_0 + 9I_0 \Rightarrow I_1 = 10I_0$

$\therefore \frac{I_1}{I_2} = \frac{10}{13} = \frac{10}{x}$ (given) $\therefore$ x = 13

12. **(a)** $y_1 = A_1 \sin k(x - vt)$

$y_1 = 12 \sin 6.28\,(x - vt)$

$y_2 = 5 \sin 6.28\,(x - vt + 3.5)$

Phase difference, $\Delta\phi = k(\Delta x) = 6.28 \times 3.5 = 7\pi$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$\Rightarrow A = \sqrt{(12)^2 + (5)^2 + 2(12)(5)\cos(7\pi)}$$

$$= \sqrt{144 + 25 - 120} = \sqrt{49} = 7 \text{ mm}$$

13. **(c)** The wavelength of sound source $= \frac{330}{110} = 3$ metre.

The phase difference between interfering waves at P is

$$= \Delta\phi = \frac{2\pi}{\lambda}(S_2P - S_1P) = \frac{2\pi}{3}(5-4) = \frac{2\pi}{3}$$

$\therefore$ Resultant intensity at

$$P = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\frac{2\pi}{3} = 3I_0$$

14. **(c)** Path differences, $\Delta x = S_2D - S_1D = 5 - 4 = 1$ m

$\therefore$ The corresponding phase differences will be

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{4}.1 = \frac{\pi}{2}$$

Using $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$.

$$I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\frac{\pi}{2} = 2I_0$$

15. **(a)** **16.** **(a)** **17.** **(a)**

18. **(b)** Fundamental harmonic produced by open pipe,

$$f_{\text{open pipe}} = \frac{v}{2l}$$

And by closed pipe, $f_{\text{closed pipe}} = \frac{v}{4l}$

$$\therefore \frac{f_0}{f_c} = \frac{v}{2l} \times \frac{4l}{v} = \frac{2}{1} \qquad \therefore f_0 : f_c = 2:1$$

19. **(a)** Frequency of string, $f = \frac{1}{2l}\sqrt{\frac{T}{m}}$

Frequency $\propto \sqrt{\text{Tension}}$

Difference of f_A and f_B is 6 Hz.

If tension decreases, f_B decreases and becomes f'_B.

Now, difference of f_A and f'_B = 7 Hz (increases)

So, $f_A > f_B$

$f_A - f_B = 6$ Hz

$\Rightarrow f_A = 530$ Hz $\Rightarrow f_B = 524$ Hz (original)

20. **(a)** For closed organ pipe, third harmonic

$$n = \frac{(2N-1)V}{4\ell} = \frac{3V}{4\ell} \quad (\because N = 2)$$

For open organ pipe, fundamental frequenty

$$n = \frac{NV}{2\ell} = \frac{V}{2\ell'} \quad (\because N = 1)$$

According to question, $\frac{3V}{4\ell} = \frac{V}{2\ell'}$

$$\Rightarrow \ell' = \frac{4\ell}{3\times 2} = \frac{2\ell}{3} = \frac{2\times 20}{3} = 13.33 \text{ cm}$$

21. **(a)** Difference in two successive frequencies of closed pipe

$$\frac{2v}{4l} = 260 - 220 = 40 \text{ Hz} \Rightarrow \frac{v}{4\ell} = 20\text{Hz}$$

Which is the fundamental frequency of system of closed organ pipe.

22. **(a)** Sound produced by an open organ pipe is richer because (i) It contains all harmonics. No harmonic is missing. (ii) Frequency of fundamental note in an open organ pipe is twice the fundamental frequency is a closed organ pipe of same length. Fundamental frequency of open pipe, $f_0 = \frac{v}{2L}$, fundamental frequency in closed pipe,

$$f_c = \frac{v}{4L} \qquad \therefore f_0 = 2f_c$$

23. **(a)** From formula, $f = \frac{1}{x}\sqrt{\frac{T}{m}} \Rightarrow \frac{1}{f} \propto l$

$\therefore l_1 : l_2 : l_3 = \frac{1}{f_1} : \frac{1}{f_2} : \frac{1}{f_3} = f_2 f_3 : f_1 f_3 : f_1 f_2$

[Given: $f_1 : f_2 : f_3 = 1 : 3 : 5$]

= 15 : 5 : 3

Therefore the positions of two bridges below the wire are

$\frac{15 \times 100}{15+5+3}$ cm and $\frac{15 \times 100 + 5 \times 100}{15+5+3}$ cm

i.e., $\frac{1500}{23}$ cm, $\frac{2000}{23}$ cm

24. **(c)** The difference in frequency in open organ pipe is given by:

$f = \frac{nv}{2L}$

Change in frequency,

$\Delta f = f_1 - f_2 = \frac{n_1 v}{2L_1} - \frac{n_2 v}{2L_2} \Rightarrow \Delta f = \frac{6v}{2 \times 0.6} - \frac{5v}{2 \times 0.9}$

$\Delta f = \frac{v}{2}\left[\frac{6}{0.6} - \frac{5}{0.9}\right] \Rightarrow \Delta f = \frac{333}{2}\left(\frac{6}{0.6} - \frac{5}{0.9}\right) = 740$ Hz

25. **(a)** Beat frequency is given by:

$v_{beat} = |v_1 - v_2| \Rightarrow v_{beat} = |11 - 9| = 2$ Hz

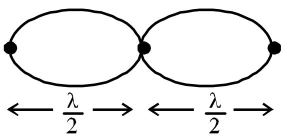
26. **(a)**

27. **(c)** $f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$, where n = nth harmonic and is equal to number of antinodes.

$f_A = \frac{p}{2l}\sqrt{\frac{T}{\rho A_o}}$ $\quad f_B = \frac{q}{2l}\sqrt{\frac{T}{4\rho A_o}}$

$\therefore \frac{f_A}{f_B} = \frac{2p}{q} \Rightarrow \frac{p}{q} = \frac{1}{2}$

28. **(a)** Wavelength of the standing wave is given by



$\Rightarrow \frac{\lambda}{2} + \frac{\lambda}{2} = \lambda = 1.21\,\mathring{A}$

29. **(d)** Velocity of wave on string

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{8}{5} \times 1000} = 40$ m/s

Here, T = tension and μ = mass/length

Wavelength of wave $\lambda = \frac{v}{n} = \frac{40}{100}$ m

Separation b/w successive nodes,

$\frac{\lambda}{2} = \frac{40}{2 \times 100} = \frac{20}{100}$ m = 20 cm

30. **(c)** It is given that tuning fork of frequency 256 Hz makes 5 beats/second with the vibrating string of a piano. Therefore, possible frequency of the piano are (256 ± 5) Hz. i.e., either 261Hz or 251 Hz. When the tension in the piano string increases, its frequency will increases. As the original frequency was 261Hz, the beat frequency should decreases, we can conclude that the frequency of piano string is 251Hz

31. **(b)** Tunning fork A, $f_A = 250\ H_z$

Tunning fork B, $f_B = x$

$|f_A - f_B| = 5$

$250 - x = 5$

x = 245 or 255

When it is loaded, it's beat frequency is decreasing by 3 beat/sec.

Beat is decreased = frequency increased to 250Hz. So, it initial frequency must be less than 250Hz

$\because x = 245$ Hz

32. **(a)** For open organ pipe,

$l_o = 72$ cm, $f = \frac{3v}{2l_o}$

For closed organ pipe, $f = \frac{5v}{4l_c}$

$\therefore \frac{3v}{2l_o} = \frac{5v}{4l_c} \Rightarrow l_c = \frac{5l_o}{6} = \frac{5 \times 72}{6} = 60$ cm

33. **(c)**

34. **(c)** For metal wire, ε = 1%, δ = 8000 kg m^{-3},

Y = 2 × 10^{11} Nm^{-2}, ℓ = 1m

The fundamental frequency of the transverse wave,

$f_o = \frac{1}{2\ell}\sqrt{\frac{Y\varepsilon}{\delta}} = \frac{1}{2 \times 1}\sqrt{\frac{2 \times 10^{11} \times 0.01}{8000}} = 250$Hz

35. **(a)** For closed organ pipe,

Fundamental frequency, $f = \frac{v}{4\ell}$ $\quad \therefore f_2 - f_1 = 6$

$\Rightarrow \frac{v}{4l_1} - \frac{v}{4l_2} = 6 \Rightarrow \frac{336}{4 \times 150 \times 10^{-2}} - \frac{336}{4\ell_2} = 6$

$\therefore \ell_2 = \frac{42}{25}$ m $= \frac{42}{25} \times 100 = 168$ cm

36. **(a)** For open organ pipe, $f_o = \frac{v}{2l}$

For closed organ pipe, $f_c = \frac{v}{4l}$

$\therefore \quad \frac{v}{2l} - \frac{v}{4l} = 100 \Rightarrow \frac{v}{4l} = 100$

$\therefore \quad f_{2,0} - f_{3,c} = \frac{2v}{2l} - \frac{3v}{4l} = \frac{v}{4l} = 100\,\text{Hz}$

37. **(d)** Frequency $n = \frac{V}{\lambda} = \sqrt{\frac{T}{\pi r^2 d}}$ where d = density

$\therefore \quad V = \lambda\sqrt{\frac{T}{\pi r^2 d}}$ so here $V \propto \frac{1}{\sqrt{d}}$

If the density of the material of the string B is 2% more than that of A then

$V' \propto \frac{1}{\sqrt{d + d \times \frac{2}{100}}}$ or $V' \propto \frac{1}{\sqrt{d(1+0.2)}}$

$\therefore \quad \frac{V'}{V} = \sqrt{\frac{1}{d(1.02)}} \Bigg/ \sqrt{\frac{1}{d}} \quad \therefore V' = \frac{V}{\sqrt{1.02}}$

38. **(c)** We have given, $y = 0.03 \sin(450\,t - 9x)$

Comparing it with standard equation of wave, we get $\omega = 450\; k = 9$

$\therefore \quad v = \frac{\omega}{k} = \frac{450}{9} = 50\,\text{m/s}$

Velocity of travelling wave on a stretched string is given by

$v = \sqrt{\frac{T}{\mu}} \Rightarrow \frac{T}{\mu} = 2500$

μ = linear mass density

$\Rightarrow T = 2500 \times 5 \times 10^{-3} \Rightarrow 12.5\,\text{N}$

39. **(b)** $\frac{3\lambda}{2} = 2$ or $\lambda = \frac{4}{3}m$

Velocity, $v = f_\lambda = 240 \times \frac{4}{3} = 320\,\text{m/sec}$

Also $f_1 = \frac{240}{3} = 80\,\text{Hz}$

40. **(b)** Given, $y = 0.3 \sin(0.157\,x)\cos(200\,\pi t)$

So $k = 0.157$ and $w = 200\pi$

or f = 100 Hz, $v = \frac{w}{k} = \frac{200\pi}{0.157} = 4000\text{m/s}$

Now, using $f = \frac{nv}{2l} = \frac{4v}{2l} = \frac{2v}{l}$

$\therefore l = \frac{2v}{f} = \frac{2 \times 4000}{100} = 80m$

41. **(a)** If a closed pipe vibration in N^{th} mode then frequency of vibration $n = \frac{(2N-1)v}{4l} = (2N-1)n_1$

(where n_1 = fundamental frequency of vibration)

Hence, $20{,}000 = (2N-1) \times 1500 \Rightarrow N = 7.1 \approx 7$

$\therefore$ Number of over tones = (No. of mode of vibration) – 1 = 7 – 1 = 6

42. **(d)** Beat frequency = $f_1 - f_2$

$\frac{40}{12} = \frac{v}{4.08} - \frac{v}{4.16} \Rightarrow \frac{10}{3} = v\left(\frac{0.08}{4.08 \times 4.16}\right)$

$\Rightarrow v = 707.2$ m/s

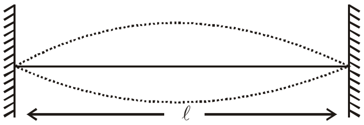
43. **(a)** $f_A = 340 \pm 5 = 335$ or 340 Hz

When fork A is filled, then beat frequency decreases to 2 beats/s.

It is possible only when

$f_A = 335$ Hz

44. **(d)**



Fundamental frequency, f = 50 Hz

linear mass density μ = mass/ length = 20 g/m

mass of string, m = 18g

length of string, $\ell = \frac{m}{\mu} = \frac{18}{20}m = \frac{9}{10}m$

from diagram $\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell = \frac{9}{5}m$

The speed of the transverse waves in the string

$v = f\lambda = 50 \times \frac{9}{5} = 90$ m/s

45. **(b)** In organ pipe, for 5^{th} harmonic

frequency $f_5 = \frac{5v}{4\ell}$

Here,

v = Speed of sound

l = length of organ pipe

On substituting respective values,

we get $405 = \frac{5 \times 324}{4\ell} \Rightarrow \ell = 1\text{m}$

46. **(d)** For second harmonic of open organ pipe,

the length given as $L = \frac{n\lambda}{2}; n = 2$

So frequency of vibration is $f = \frac{v}{\lambda}$

$f = \frac{v}{\lambda} = \frac{v}{L} = \frac{360}{\frac{40}{100}} = 900\text{Hz}$

47. **(d)** Fundamental frequency,

$f = \frac{nv}{2\ell}$, for fundamental mode n = 1

$\Rightarrow \quad f = \frac{v}{2\ell} \Rightarrow f \propto \frac{1}{\ell} \quad \therefore \quad \frac{f_1}{f_2} = \frac{\ell_2}{\ell_1} \Rightarrow \frac{120}{180} = \frac{\ell_2}{90}$

Hence, $\ell_2 = 60$ cm

48. (b) Area of cross-section, $A = 2$ cm^2
Speed of sound, $v = 330$ m/s

$\frac{v}{4\ell_1} = \frac{330}{4\ell_1} = 30 \Rightarrow \ell_1 = \frac{11}{4}$ m

$\frac{v}{4\ell_2} = \frac{330}{4\ell_2} = 110 \Rightarrow \ell_2 = \frac{3}{4}$ m

$\Delta\ell = \ell_1 - \ell_2 = 2$ m

Change in volume $= A\Delta\ell = 400$ cm^3

Mass of water $= \rho A(\ell_1 - \ell_2)$ $(\because \rho = 1 g/cm^3)$
$= 1 \times 4 \times 10^{-4} = 0.4$ kg $= 400$ g

49. **(c)** Frequency of tuning fork $f = \frac{1}{2L}\sqrt{\frac{T}{m}}$

For T = 6N and L = 1m

$f_1 = \frac{1}{2}\sqrt{\frac{6}{m}}$ ...(i)

For T = 54N and L = 1m

$f_2 = \frac{1}{2}\sqrt{\frac{54}{m}}$ ...(ii)

$\frac{f_1}{f_2} = \frac{1}{3}$, Given $f_2 - f_1 = 12$ $\therefore f_1 = 6$ Hz

50. **(b)** Frequency of closed organ pipe

$f_c = (2n+1)\frac{v}{4\ell}$

For seventh overtones, n = 7

$\therefore f_c = \frac{(2 \times 7 + 1)v}{4\ell} = \frac{15v}{4\ell}$

Frequincy of open organ pipe $= (n+1)\frac{v}{2\ell}$

For n = 7 $\quad f_0 = \frac{8v}{2\ell} \Rightarrow \frac{f_c}{f_0} = \frac{15}{16} = \frac{a-1}{a} \Rightarrow a = 16$

51. **(a)** Velocity $v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1}{k\rho}}$

$[\because$ Compressibility, $K = \frac{1}{B \text{ (Bulk modulus)}}]$

And both closed and open organ pipes vibrating in their first overtone with same frequency

$\therefore f_{closed} = f_{open}$

$\frac{3v_1}{4L} = \frac{2v_2}{2L'} \Rightarrow L' = \frac{4}{3}L\left(\frac{v_2}{v_1}\right)$

$L' = \frac{4}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ [$\because$ compressibility are equal]

$\therefore x = 4$

52. **(c)** In a closed organ pipe, two waves travelling in opposite direction (one incident and other reflected wave from boundary) superimpose with each other to develop a wave pattern which is standing or stationary.
Harmonics in closed organ pipe :
$v_1 : v_2 : v_3$ .... = 1 : 3 : 5 : ......
So, only odd harmonics are present. $\Rightarrow$ II correct

Natural frequencies = $v = \frac{nv}{2L}$; $n = 1, 2, 3, ...$

Thus, even and odd i.e., all the harmonics are present.

53. **(b)** Fundamental frequency,

$f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{A\rho}} \left[\because v = \sqrt{\frac{T}{\mu}} \text{ and } \mu = \frac{m}{\ell}\right]$

Also, $Y = \frac{T\ell}{A\Delta\ell} \Rightarrow \frac{T}{A} = \frac{Y\Delta\ell}{\ell} \Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{\gamma\Delta\ell}{\ell\rho}}$ ....(i)

Putting the value of $\ell, \frac{\Delta\ell}{\ell}, \rho$ and γ in eq^n. (i) we get,

$f = \sqrt{\frac{2}{7}} \times \frac{10^3}{3}$ or, $f \approx 178.2$ Hz

54. **(a)** Given $\frac{nv}{2\ell} = 315$ and $(n+1)\frac{v}{2\ell} = 420$

$\Rightarrow \frac{n+1}{n} = \frac{420}{315} \Rightarrow n = 3$

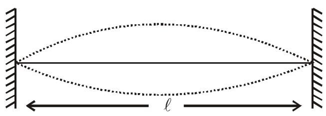
Hence $3 \times \frac{v}{2\ell} = 315 \Rightarrow \frac{v}{2\ell} = 105$ Hz

The lowest resonant frequency is when n = 1
Therefore lowest resonant frequency = 105 Hz.

55. **(a)** $\sqrt{T/l}$ = constant; Tension decreases by a factor $(8-1)/8$, length decreases by square root of this *i.e.*, 0.77.

56. **(a)** Both Statement I and Statement II are correct. According to the property of persistence of hearing the impression of a sound heard persists on our mind for $\frac{1}{10}$s. Therefore, the number of beats per second should be less than 10.
Hence the difference in frequency of the two sources must be less than 10.

57. **(b)**



Fundamental frequency, f = 50 Hz
linear mass density μ = mass/ length = 20 g/m
mass of string, m = 18g

length of string, $\ell = \frac{m}{\mu} = \frac{18}{20} m = \frac{9}{10} m$

from diagram $\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell = \frac{9}{5} m$

The speed of the transverse waves in the string

$v = f\lambda = 50 \times \frac{9}{5} = 90$ m/s

58. **(b)** Frequency, $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$ $(T = \text{Tension})$

$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}}$ or, $\left(\frac{f_2}{f_1}\right)^2 = \frac{T_2}{T_1}$

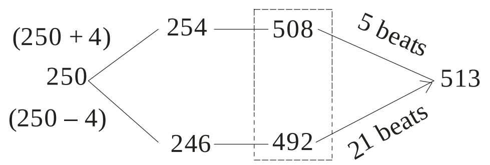
$\Rightarrow \left(\frac{50}{30}\right)^2 = \frac{mg}{180g} \Rightarrow m = \frac{25}{9} \times 180 = 500g$

59. **(d)** n_A = Known frequency = 341 Hz, n_B = ?
x = 6 bps, which is decreasing (i.e., x↓) after loading (from 6 to 1 bps)
x = 6 bps, which remains the same after loading.
Unknown tuning fork F_2 is loaded so $n_B\downarrow$
Hence $n_A - n_B\downarrow = x$ ... (i) Wrong
$n_B\downarrow - n_A = x$ ... (ii) Correct
$\Rightarrow$ $n_B = n_A + x = 256 + 6 = 262$ Hz.

60. **(a)** If the original frequency of B (v_B) were greater than that of A(v_A), A(v_A), further increase in v_B should have resulted in an increase in the beat frequency. But the beat frequency is found to decrease.
Given, $v_A - v_B = 5$ Hz
427 Hz $- V_B = 5 \Rightarrow v_B = 422$ Hz.

61. **(a)** According to question there are two possible cases

Second harmonics



Therefore, unknown frequency is 254 Hz.

62. **(c)** The frequency of fork 2
= frequency of fork 1 ± beat frequency
= 200 ± 4 = 196 or 204 Hz
Since, on attaching the tape on the prong of fork 2, its frequency decreases, but now the number of beats per second is 6. It is possible only when before attaching the tape, the frequency of fork 2 is less than the frequency of tuning fork 1. Therefore, the original frequency of fork 2 was 196 Hz.

63. **(a)** Let the frequency of the fork A be υ_A.
$\therefore$ $\upsilon_A = \upsilon_B \pm 4 = (384 \pm 4)$ Hz = 388 Hz or 380 Hz
When one of the prongs of A is filed, its frequency increases.
If $\upsilon_A = 380$ Hz, further increase in υ_A will result in decrease in the beat frequency when sounded with B.
If $\upsilon_A = 388$ Hz, further increase in υ_A will result in increase in the beat frequency when sounded with B.
Thus, the frequency of the fork A is 388 Hz.

64. **(c)** $n_A \sim n_B = 4$
Now, if tension in string A increased, then frequency of string A increased and beats produced increased.
$\therefore$ $n_A > n_B$
$\therefore$ $n_B = 480\ H_2$
$\therefore$ $n_A = n_B + 4 = 480 + 4 = 484$ Hz

65. **(d)** $y_1 = 5\sin 400\pi t$, $y_2 = 8\sin 408\pi t$
$\therefore$ $\omega_1 = 400p$, $\omega_2 = 408\pi$
$\therefore$ Number of beats per minute is

$$60|f_1 - f_2| = 60\left|\frac{\omega_1}{2\pi} - \frac{\omega_2}{2\pi}\right| = 60\left|\frac{400\pi}{2\pi} - \frac{408\pi}{2\pi}\right|$$

$= 240$ cycle / min

66. **(a)**

67. **(b)** $\lambda_1 = 50$ cm.
$\lambda_2 = 51$ cm.

$v \propto \sqrt{T}$ $\Rightarrow$ $\frac{v_1}{v_2} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{273+20}{273}}$

$\Rightarrow$ $v_2 = 319.23$.

$\nu_1 = \frac{v_2}{\lambda_1} = \frac{319.23}{0.50} = 640$ Hz.

$\nu_2 = \frac{v_2}{\lambda_2} = \frac{319.23}{51 \times 10^{-2}} = 625.94 = 626$ Hz.

No. of beats = $\nu_2 - \nu_1 = 14$ Hz

68. **(c)**

69. **(d)** The frequency of the piano string $= 512 \pm 4 = 516$ or 508. When the tension is increased, beat frequency decreases to 2, it means that frequency of the string is 508 as frequency of string increases with tension.

70. **(d)** Let $f_1 = f_0$. Then
$f_2 = f_0 + 4$
$f_3 = f_0 + 2 \times 4$
$f_4 = f_0 + 3 \times 4$
⋮ ⋮
$f_{20} = f_0 + 19 \times 4$
Now, $f_{20} = 2f_1 \Rightarrow f_0 + 19 \times 4 = 2f_0$
$\Rightarrow f_0 = 76$ Hz
So, $f_{20} = 76 + 19 \times 4 = 152$ Hz.

Exercise 3 : PREVIOUS YEARS MCQs

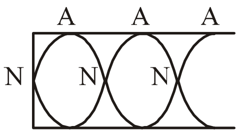
1. **(d)** Given: Wave equation: y = A sin (kx) cos (ωt)

$k = \frac{7\pi}{4}$ and $\omega = 350\,\pi$

wave speed $V = \frac{\omega}{k} = \frac{350\pi}{\frac{7\pi}{4}} = 200\ \text{m/s}$

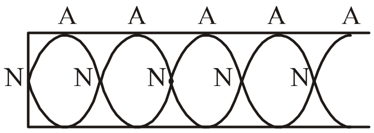
2. **(b)** In a closed organ pipe,

Fifth Harmonics



Number of nodes = 3

Ninth Harmonic



Number of nodes = 5

3. **(d)** Identify the angular frequencies

$\omega_1 = 250\pi = 2\pi f_1 \Rightarrow f_1 = \frac{250\pi}{2\pi} = 125\text{Hz}$

$\omega_2 = 260\pi = 2\pi f_2 \Rightarrow f_2 = \frac{260\pi}{2\pi} = 130\text{Hz}$

Beat frequency:

$f_{beat} = |f_1 - f_2| = |130 - 125| = 5\text{Hz}$

$T_{beat} = \frac{1}{f_{beat}} = \frac{1}{5} = 0.2\,\text{s}$

4. **(c)**

5. **(b)** Let ℓ_1 and ℓ_2 be the length of open and closed organ pipe. We have given $\frac{\ell_1}{\ell_2} = \frac{2}{3}$

Frequency of third harmonic of the open organ pipe $= \frac{3v}{2\ell_1}$

Frequency of fifth harmonic of the closed organ pipe $= \frac{5v}{4\ell_2}$

$\therefore$ Ratio $= \frac{3V}{2\ell_1} \times \frac{4\ell_2}{5V} = \frac{6}{5} \times \frac{3}{2} = \frac{9}{5}$

6. **(a)** $f_1 = 320$ Hz, $f_2 = 323$ Hz

$f'_{beat} = |f_2 - f_1| = |323 - 320| = 3\text{Hz}$

3 beats per second = 3 cycles of loud (max) and soft (min) second

Each beat cycle per second

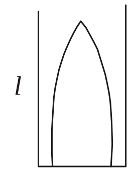
(maximum to next maximum) lasts:

$T = \frac{1}{f_{beat}} = \frac{1}{3}\text{second} \Rightarrow \frac{T}{2} = \frac{1}{2\times 3} = \frac{1}{6}\text{ second}$

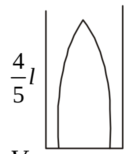
7. **(a)** Resultant intensity $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$= I + I + 2\sqrt{I \times I}\cos 90° = 2I + 0 = 2I$

8. **(a)** $\lambda_1 = 4l,\ f_1 = \frac{v}{4l}$



when a closed organ pipe is filled with water up to $\frac{1}{5}$ of its volume, the effective length of the air column decreases to $\frac{4}{5}$ of its original length.

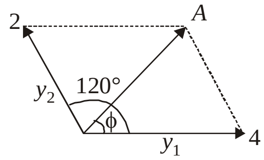


$\lambda_2 = \frac{16l}{5},\ f_2 = \frac{5V}{16l}$

$\Delta f = f_2 - f_1 \Rightarrow \Delta F = \frac{5V}{16l} - \frac{4V}{16l} \Rightarrow \Delta F = \frac{V}{16l}$

$\therefore \frac{\Delta F}{f} = \frac{\frac{V}{l}\left(\frac{1}{16}\right)}{\frac{V}{4l}} \times 100 = 25\%.$

9. **(d)** Resultant amplitude, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$



Given, $y_1(x, t) = 4\sin(Kx - \omega t)$

and $y_2(x, t) = 2\sin\left(Kx - \omega t + \frac{2\pi}{3}\right)$

$\therefore A = \sqrt{2^2 + 4^2 + 2\times 2\times 4\times\cos 120°} = \sqrt{12} = 2\sqrt{3}$

And $\tan\phi = \frac{2\sin 120°}{4 + 2\cos 120°} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$\therefore$ Phase $\phi = \frac{\pi}{6}$

10. **(d)** Third overtone has a frequency 7n, which means L = three full loops + one half loop,
which would make four nodes and four antinodes.

11. **(c)** For closed organ pipe

$n_1 : n_2 : n_3 \ldots\ldots = 1 : 3 : 5 : \ldots\ldots$

12. **(a)** Here, $A_1 = A, A_2 = A, \phi = 120^o$

The amplitude of the resultant wave is

$$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$= \sqrt{A^2 + A^2 + 2AA\cos 120^o}$$

$$= \sqrt{A^2 + A^2 - A^2} \qquad \left[\because \cos 120^o = -\frac{1}{2}\right]$$

$\therefore A_R = A$

13. **(a)** The time interval between successive maximum intensities will be $= \dfrac{1}{n_1 \sim n_2} \Rightarrow \dfrac{1}{454 - 450} = \dfrac{1}{4}$ s

14. **(d)** When two waves of almost equal frequencies f_1 and f_2 reach at a point simultaneously, beats are produced.

Beat frequency, $f_{beat} = (f_1 - f_2)$

Time interval between successive maxima

$$= \frac{1}{f_{beat}} = \frac{1}{(f_1 - f_2)}$$

15. **(a)** The wave equation is $y = A\sin(\omega t)\cos(kx)$;

$v = \omega/k = 100/0.01 = 10^4$ m/s.

16. **(b)**

17. **(a)** Loudness (dB) = $10 \log_{10}\left(\dfrac{I_2}{I_{ref}}\right)$

or $120 = 10\log_{10}\left(\dfrac{I}{10^{-12}}\right) \Rightarrow I = 1$

Also $I = \dfrac{P}{4\pi r^2} = \dfrac{2}{4\pi r^2} \Rightarrow 1 = \dfrac{2}{4\pi r^2} \Rightarrow r = \sqrt{\dfrac{1}{2\pi}}$

$\Rightarrow r = 40$ cm

18. **(c)** Here $\dfrac{\lambda}{2} = 5.0\ cm \Rightarrow \lambda = 10\ cm$

Hence $n = \dfrac{v}{\lambda} = \dfrac{200}{10} = 20\ Hz$

19. **(b)** $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$\therefore$ Sum of maximum and minimum intensities $= 2(I_1 + I_2)$

20. **(c)**

21. **(a)** Fundamental frequency of open pipe

$$n_1 = \frac{v}{2l} = \frac{350}{2 \times 0.5} = 350 \text{ Hz}$$

22. **(a)** For closed pipe $n = \dfrac{v}{4l} \Rightarrow n = \dfrac{332}{4 \times 42} = 2Hz.$

23. **(a)** Fundamental frequency of open tube is, $v = \dfrac{v}{2L}$

where v is the velocity of sound in air and L is the length of the tube

$$\therefore v = \frac{330ms^{-1}}{2 \times 0.25m} = 660Hz$$

The emitted frequencies are $v, 2v, 3v, 4v, ...$

i.e., 660 Hz, 1320 Hz, 1980Hz, 2640Hz,...

24. **(a)** In open organ pipe both even and odd harmonics are produced.

25. **(c)** In case of open pipe, $n = \dfrac{N_v}{2l}$ where N = order of harmonics = order of mode of vibration $\Rightarrow N = \dfrac{n \times 2l}{v}$

$= \dfrac{480}{330} \times 2 \times 1 \cong 3$ [Here v = 330 m/s]

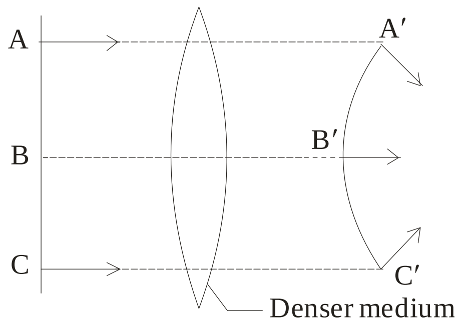
7 Wave Optics

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(c)** Primary wave font and wave lets are equal because light does not change medium. It is travelling only in one homogenous medium.
2. **(a)** By drawing sphere of radius vt at each point of primary wavefront and then drawing a common tangent at each sphere, new position of wavefront can be obtained.
3. **(b)** When ABC wavefront passes through glass lens, its velocity is reduced.

$$v_{glass} = \frac{v_{air}}{n_{glass}}$$

where, v_{glass} and v_{air} are velocities in glass and air, respectively and n_{glass} is refractive index from air to glass.



At points A and C waves remain in glass for a short duration, so they travel a larger distance while wave at B covers a small distance as it remains in glass for a longer duration (middle portion of glass in thick) and finally A' B' C' is the position of new wavefront. It is concave in shape as shown in the figure.

4. **(a)** $\frac{1}{\mu} = \tan 30°;\ \mu = \cot 30°$

Using Brewster's law :

$\tan i_\beta = \mu = \cot 30° = \tan 60°$

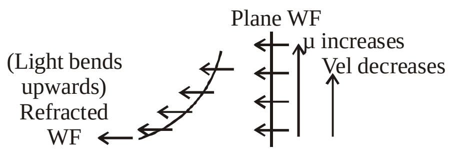
$i_\beta = 60°$

5. **(b)** $I' = I\cos^2\theta = I\cos^2(45°) = I\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I}{2}.$

6. **(c)** $\mu = \tan i$

$\Rightarrow\ i = \tan^{-1}(\mu) = \tan^{-1}(\sqrt{3}) = 60°.$

7. **(a)**
8. **(c)** A slit would give divergent; a biprism would give double; a glass slab would give a parallel wavefront. Edge is downward.
9. **(b)**



10. **(b)** Phase reversal occurs i.e. phase change = π takes place on reflection, because glass is much denser than water.
11. **(a)**
12. **(b)** Given, refractive index of medium μ = 1

Using Brewster's law, $\tan i_p = \mu$

$\tan i_p = 1$

$i_p = \tan^{-1}(1) = 45°$

As $i_p + r = 90°$

$r = 90° - I_p$

$= 90° - 45°$

$\Rightarrow\ r = 45°$

13. **(a)** According to Brewster's law, when a beam of ordinary light (i.e., unpolarisded) is reflected from a transparent medium (like glass), the reflected light is completely plane polarised at certain angle of incidence called the angle of polarisation.
14. **(c)** Using Malus' law :

$I = I_0\cos^2\theta$

$\Rightarrow I_0/2 = I_0\cos^2\theta$

$\Rightarrow \cos^2\theta = \frac{1}{2}$

$\Rightarrow \theta = 45°$

15. **(d)** Angle of polarisation, $i_p = 60°$

$\therefore\ \mu = \tan i_p = \tan 60° = \sqrt{3}$

$\therefore$ Critical angle, $C = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

16. **(c)** Brewster's angle ,

$i_p = \tan^{-1}(\mu) = \tan^{-1}(1.5)$

$= \tan^{-1}\left(\frac{3}{2}\right)$

17. (a) From Brewster's law

$\mu = \tan i_p$

$\Rightarrow \mu = \tan 60° = \sqrt{3}$

Using Snell's law of refraction $\mu = \dfrac{\sin i}{\sin r}$

$$\Rightarrow \sin r = \frac{\sin i}{\mu} = \frac{\sin 60°}{\sqrt{3}} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}$$

$$\Rightarrow r = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

18. (d)

19. (b) Angle between plane of vibration and plane of polarisation is 90°.

20. (d) The angle of incidence for total polarization is given by $\tan\theta = n \Rightarrow \theta = \tan^{-1} n$

21. (b) $I = I_0 \cos^2\theta$

Intensity of polarized light $= \dfrac{I_0}{2}$

$\Rightarrow$ Intensity of untransmitted light $= I_0 - \dfrac{I_0}{2} = \dfrac{I_0}{2}$

22. (b) Intensity of light transmitted by polariser is half of intensity of unpolarised light.

23. (b) ${}^a\mu_g = \tan\theta_P$ where θ_P = polarising angle.

or, ${}^a\mu_g = \tan 60°$

or, $\dfrac{c}{v_g} = \sqrt{3}$

or, $v_g = \dfrac{c}{\sqrt{3}} = \dfrac{3\times 10^8}{\sqrt{3}} = \sqrt{3}\times 10^8\,ms^{-1}$

24. (c) Huygen's wave theory fails to explain the particle nature of light (i.e., photoelectric effect).

25. (b) If a plane wave of light travelling along the y-direciton electric field may be along any direction in x-z plane (i.e. y = c), hence wavefront may be represented by y = c.

26. (c) The locus of all particles in a medium vibrating in the same phase is called wave front.

27. (b) On polarisation by reflection, the reflected and refracted waves are at 90° to each other.

28. (b) $I' = I\cos^2\theta = I\cos^2(45°)$

$$= I\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I}{2}.$$

29. (c) $\mu = \tan i$

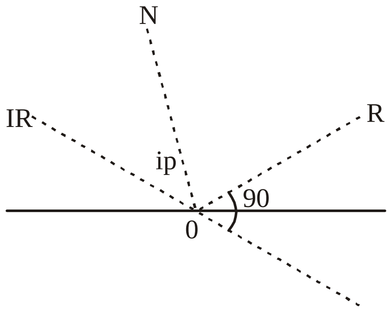
$\Rightarrow i = \tan^{-1}(\mu) = \tan^{-1}(\sqrt{3}) = 60°.$

30. (a) $\mu = \tan i_p = \tan 60° = \sqrt{3}$

$\sin i_C = \dfrac{1}{\mu} = \dfrac{1}{\sqrt{3}} \Rightarrow i_C = \sin^{-1}\dfrac{1}{\sqrt{3}}$

31. (d) Required angle

$= 2\times 57.5 + 90 = 205°$



32. (d) For minima, path difference $\Delta = (2n-1)\dfrac{\lambda}{2}$

For third minima $n = 3 \Rightarrow \Delta = (2\times 3 - 1)\dfrac{\lambda}{2} = \dfrac{5\lambda}{2}$

33. (d) $I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$

Put $a_1^2 + a_2^2 = A$ and $a_1a_2 = B \therefore I = A + B\cos\phi$

34. (a) $I = 4I_0\cos^2\dfrac{\phi}{2}$

At central position $I_1 = 4I_0$ ... (i)

Since the phase difference between two successive fringes is 2x, the phase difference between two points separated by a distance equal to one quarter of the distance between the two, successive fringes is equal to

$\delta = (2\pi)\left(\dfrac{1}{4}\right) = \dfrac{\pi}{2}$ radian

$\Rightarrow I_2 = 4I_0\cos^2\left(\dfrac{\pi/2}{2}\right) = 2I_0$ ... (ii)

Using (i) and (ii), $\dfrac{I_1}{I_2} = \dfrac{4I_0}{2I_0} = 2$

35. (a) In YDSE, $I_1 = I, \Delta x_1 = \lambda, \Delta x_2 = \dfrac{\lambda}{3}$

$\therefore \phi_1 = \dfrac{2\pi}{\lambda}\Delta x_1 = \dfrac{2\pi}{\lambda}.\lambda = 2\pi$

$\phi_2 = \dfrac{2\pi}{\lambda}\Delta x_2 = \dfrac{2\pi}{\lambda}.\dfrac{\lambda}{3} = \dfrac{2\pi}{3}$

Intensity, $I \propto \cos^2\dfrac{\phi}{2}$

$\therefore \frac{I_2}{I_1} = \left(\frac{\cos\frac{\phi_2}{2}}{\cos\frac{\phi_1}{2}}\right)^2 = \left(\frac{\cos\frac{\pi}{3}}{\cos\pi}\right)^2 \Rightarrow I_2 = \frac{I_1}{4} = \frac{I}{4}$

36. **(d)** Intensity $I \propto 1 + \cos\phi$ where ϕ = phase difference

$\therefore \frac{I_P}{I_Q} = \frac{1+\cos 0°}{1+\cos\frac{\pi}{2}} = \frac{1+1}{1+0} = \frac{2}{1}$

37. **(c)** $\Delta x_{max} = d = 5000$ Å. Given $\lambda = 3000$ Å
As $\lambda < d < 2\lambda$ $\therefore n = 3$.

38. **(d)** $x \propto D$

$\therefore$ If d becomes thrice, then X becomes $\frac{1}{3}$ times.

39. **(c)** $\beta = \beta'$

or $\frac{D\lambda}{d} = \frac{D'\lambda}{(2d)}$ $\therefore D' = 2D$

40. **(c)** Distance of nth maxima, $x = n\lambda\frac{D}{d} \propto \lambda$

As $\lambda_b < \lambda_g$ $\therefore x_{blue} < x_{green}$

41. **(a)** $I_{min} \propto (a - a)^2$
$I'_{min} \propto (2a - a)^2$
Clearly, the intensity of minima increases.
Again $I_{max} \propto (a + a)^2$
$I'_{max} \propto (2a + a)^2$
Clearly, the intensity of maxima increases.

42. **(c)** $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{9}{1}$ or $\frac{a_1}{a_2} = \frac{3}{1}$

$\therefore \frac{I_{max}}{I_{min}} = \frac{(3+1)^2}{(3-1)^2} = \frac{16}{4} = \frac{4}{1}$

43. **(d)**

44. **(b)** Given,
$\lambda_1 = 400 \text{ nm} = 400 \times 10^{-9}$ m
$\lambda_2 = 5000$ Å $= 5000 \times 10^{-10}$ m
$= 500 \times 10^{-9}$ m
Distance between two consecutive fringes for wavelength λ_1 is β_1 and that for wavelength λ_2 is β_2.
$\therefore n_1\beta_1 = n_2\beta_2$

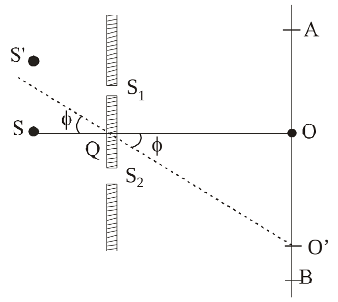
$\Rightarrow n_1\frac{\lambda_1 D}{d} = n_2\frac{\lambda_2 D}{d}$

$\Rightarrow 5 \times \frac{400 \times 10^{-9} \times D}{d}$

$= n_2 \times \frac{500 \times 10^{-9} \times D}{d}$

$\Rightarrow n_2 = \frac{5 \times 400}{500} = 4$

45. **(b)** Since, when the source S is on the perpendicular bisector, then the central bright fringe occurs at O, which is also on the perpendicular bisector.

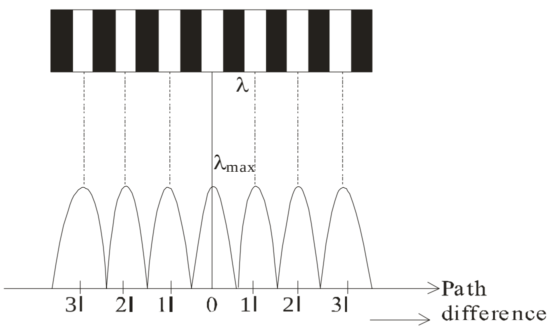


If S is shifted by an angle ϕ to point S', then the central fringe appears at a point O' at an angle $-\phi$ (shown in figure above), which means that it is shifted by the same angle on the other side of the bisector. i.e. towards end B of screen.

46. **(c)** The contrast interference will occur when there is absolute darkness at the dark band due to destructive interference, i.e. $I_R = I_{min} = 0$ and there is complete (max) brightness at the bright band due to constructive interference, i.e. $I_R = I_{max} = 4I_0$, which is possible only when individual intensities are same.
So, $I_1 = I_2 = I_0$

47. **(a)**

48. **(d)** The fringes are straight lines although S_1 and S_2 are point source. If slits are used instead of point sources, each pair of points would have produced straight line fringes with increased intensities as shown below.



49. **(c)** If a is the amplitude of wave, then

$I = 4a^2$, then $I_0 = a^2 = \frac{I}{4}$.

50. **(d)** For maxima $d\sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{d} = \frac{8\lambda}{10\lambda} \Rightarrow \sin\theta = \frac{4}{5} \Rightarrow \tan\theta = \frac{4}{3}$$

Also $\tan\theta = \frac{y}{D}$ $\therefore$ $y = \frac{4D}{3}$

51. **(a)** Let θ be the angular width in water. We know angular width = $\frac{\lambda}{d}$

$\Rightarrow$ Angular width $\propto \lambda$

$$\frac{\theta}{0.4^\circ} = \frac{\lambda_w}{\lambda_a} \quad \text{...... (i)}$$

Now, ${}_a\mu_w = \frac{\lambda_a}{\lambda_w} \Rightarrow \frac{\lambda_a}{\lambda_w} = \frac{4}{3}$

Hence from eq. (1), we have

$$\frac{\theta}{0.4^\circ} = \frac{3}{4} \Rightarrow \theta = 0.3^\circ$$

52. **(a)** Fringe width $\omega = \frac{\lambda D}{d}$.

When the apparatus is immersed in a liquid, then λ will decrease μ times and hence ω is reduced μ (refractive index) times. $10\omega' = (5.5)\,\omega$

or $10\lambda'\left(\frac{D}{d}\right) = (5.5)\frac{\lambda D}{d}$ or $\frac{\lambda}{\lambda'} = \frac{10}{5.5} = \mu$ or $\mu = 1.8$

53. **(c)** Intensity at a point in Young's double slit experiment is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Here $I_1 = I_2 = I_0$ (say)

At P

$$\therefore I_p = I_0 + I_0 + 2I_0\cos\frac{\pi}{3} = 2I_0 + 2I_0 \times \frac{1}{2} = 3I_0$$

At Q

$$I_Q = I_0 + I_0 + 2I_0\cos 90^\circ = 2I_0$$

$$\frac{I_P}{I_Q} = \frac{3}{2}$$

54. **(c)** Fringe width, $\beta_1 = 2\text{mm} = 2 \times 10^{-3}$ m

$\lambda_1 = 400$ nm ; $\lambda_2 = 600$ nm

$= 400 \times 10^{-9}$ m $= 600 \times 10^{-9}$ m

Fringe width $(\beta) = \frac{D\lambda}{d} \Rightarrow \frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1}$

$$\Rightarrow \frac{\beta_2}{2 \times 10^{-3}} = \frac{600 \times 10^{-9}}{400 \times 10^{-9}} = \frac{3}{2} \Rightarrow \beta_2 = 3\text{mm}$$

55. **(d)** $\frac{I_{max}}{I_{min}} = \frac{25}{9}$ or $\left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \frac{25}{9}$

where a denotes amplitude.

$$\frac{a_1 + a_2}{a_1 - a_2} = \frac{5}{3} \text{ or } 5a_1 - 5a_2 = 3a_1 + 3a_2$$

or, $5a_1 - 5a_2 = 3a_1 + 3a_2$ or $2a_1 = 8a_2$

or, $\frac{a_1}{a_2} = 4$ or $\left(\frac{a_1}{a_2}\right)^2 = 16 = \frac{I_1}{I_2}$.

56. **(b)** We have given, $\frac{I_1}{I_2} = \frac{1}{4}$

$\Rightarrow I_2 = 4I_1$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 9I_1$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = I_1$$

$$\therefore \frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{9I_1 + I_1}{9I_1 - I_1} = \frac{10}{8} = \frac{5}{4} = \frac{2\alpha + 1}{\beta + 3}$$

$\alpha = 2 \quad \beta = 1$

$$\therefore \frac{\alpha}{\beta} = \frac{2}{1} = 2$$

57. **(a)**

58. **(b)** Path difference $= 171.5\lambda = \frac{343}{2}\lambda$

= odd multiple of half wavelength .

It means dark fringe is observed.

According to question, $0.01029 = \frac{343}{2}\lambda$

$$\Rightarrow \lambda = \frac{0.01029 \times 2}{343} = 6 \times 10^{-5}\text{ cm}$$

$\Rightarrow \lambda = 6000$ Å.

59. **(a)** For strong reflection, the least optical path difference introduced by the film should by λ/2. The optical path difference between the waves reflected from the two surfaces of the film is 2 μd.

Thus, for strong reflection, $2\mu d = \lambda/2$.

$$d = \frac{\lambda}{4\mu} = \frac{589}{4 \times 1.25} = 118\text{ nm}.$$

60. **(b)** Phase diff. $= \frac{2\pi}{\lambda}x$

$$\text{Path difference} = \frac{2\pi \times 2.1 \times 10^{-6}}{5460 \times 10^{-10}} = 7.692\,\pi\text{ radian}.$$

61. **(d)** It will be concentric circles.

62. **(b)** For minima, phase diff. = odd integral multiple of $\pi = (2n - 1)\pi$.

63. **(c)** $\sin\theta = \frac{\lambda}{d} = \frac{589 \times 10^{-9}}{0.589 \times 10^{-3}} = 10^{-3} = \frac{1}{1000} = 0.0001$

64. **(d)**

65. **(b)** Introducing a converging lens in the path of parallel beam does not introduce any extra path differences in a parallel beam. Rather it gives a more intense pattern on the screen.

66. **(c)** Here, $I_1 = I_0$ and $I_2 = 4I_0$

$\because \quad I = I_1 + I_2 + 2\sqrt{I_1 + I_2}\cos\phi$

$= I_0 + 4I_0 + 2 \times 2I_0\left(-\frac{1}{2}\right)$

$= 5\,I_0 - 2I_0 = 3\,I_0$

67. **(a)** For interference phase difference must be constant.

68. **(a)** As $\beta = \frac{\lambda D}{d}$ and $\lambda_b < \lambda_y$,

$\therefore$ fringe width β will decrease

69. **(d)** $\beta = \frac{\lambda D}{d} = \frac{5000 \times 10^{-10} \times 0.9}{3 \times 10^{-3}}\,m$

$= 1.5 \times 10^{-4}\,m = 0.15\,mm$

70. **(b)** Fringe width, $\beta = \frac{\lambda D}{d}$

71. **(a)** $\beta \propto \frac{\lambda D}{d}$ as $d \to \frac{d}{3}$ so $\beta \to 3\beta \therefore n = 3$

72. **(c)** $\beta \propto \frac{\lambda}{d}$

73. **(a)** Distance between two consective dark fringes

$$\beta = \frac{\lambda D}{d} = \frac{5000 \times 10^{-10} \times 1}{0.2 \times 10^{-2}} = 0.25\,mm$$

74. **(d)** $\beta = \frac{\lambda D}{d}$

$\overrightarrow{VIBGYOR}$ λ increase

$\lambda_R > \lambda_G > \lambda_B$

So, $\beta_R > \beta_G > \beta_B$

75. **(b)** $\beta = \frac{\lambda D}{d} = \frac{6000 \times 10^{-10} \times 2}{4 \times 10^{-3}} = 3 \times 10^{-4}\,m = 0.3\,mm$

76. **(b)** $\beta = \frac{\lambda D}{d} \Rightarrow \beta \propto \lambda$

77. **(b)** First diffraction minima is given by, $n\lambda = a \sin\theta$

$1 \times 6500 = a \sin 30°$

$a = 13000$ Å $= 1.3$ micron

78. **(d)** If the width of diffraction bands increases the wavelength of light also increases.

Hence, if yellow light is replaced by blue light, diffraction bands become narrower.

79. **(b)** The angular width of principal maximum is given by

$2\theta = \frac{2\lambda}{a}$

80. **(d)** Width of central maximum

$$= \frac{2\lambda D}{d} = \frac{2 \times 600 \times 10^{-9} \times 2}{1 \times 10^{-3}} = 2.4\,mm$$

81. **(b)** If a converging lens of focal length f is placed after the slit and the fringe are obtained on screen, then separation of the central maximum from the first null of the diffraction pattern is λ/a.

If f is the focal length of the lens, then the size of screen will be $f\lambda/a$.

82. **(d)** For first dark fringe on either side,

$$d \sin\theta = \lambda \text{ or } \frac{dy}{D} = \lambda$$

$\therefore \quad y = \frac{\lambda D}{d}$

Therefore, distance between two dark fringes on either side

$= 2y = \frac{2\lambda D}{d}$

$= \frac{2(600 \times 10^{-6}\,mm)(2 \times 10^{3}\,mm)}{(1.0\,mm)}$

$= 2.4\,mm$

83. **(c)**

84. **(a)** Angular width of central maxima is given by :

$\theta = \frac{2\lambda}{a} = \frac{2 \times 6 \times 10^{-7}}{2 \times 10^{-4}} = 6 \times 10^{-3}$ rad

85. **(a)** Condition for minima is $d \sin\theta = n\lambda$

For Ist minima $d \sin\theta = \lambda$

$\sin\theta = \frac{\lambda}{a} = \frac{2}{4} = \frac{1}{2} \qquad \therefore \theta = 30°$

Angular spread $= 2\theta = 60°$

86. **(c)** Because both source & screen are effectively at infinite distance from the diffractive device.

87. **(b)** Condition for diffraction is

Slit width $\le$ wavelength of light

$\Rightarrow \; a \Rightarrow \lambda \qquad \therefore \frac{a}{\lambda} \le 1$

88. **(b)** Given,

Diameter of the objective,

$d = 3.6\,m$

wavelength of light,

$\lambda = 540 \times 10^{-9}$ m

Resolving limit, $d\theta = \frac{1.22\lambda}{d}$

$$d\theta = \frac{1.22 \times 540 \times 10^{-9}}{3.6} = 183 \times 10^{-9}$$

$= 1.83 \times 10^{-7}$ rad.

89. **(c)** Limit of resolution $= \frac{1.22\lambda}{d}$

$$= \frac{1.22 \times 600 \times 10^{-9}}{250 \times 10^{-2}} = 3 \times 10^{-7} \text{ rad}$$

90. **(b)** In diffraction, width of central maxima $\propto \lambda$.

$\therefore$ Wavelength of X rays is less than that of yellow light, so the width decreases.

91. **(d)** Resolving power $\alpha\ (1/\lambda)$.

Hence, $\frac{(R.P)_1}{(R.P)_2} = \frac{\lambda_2}{\lambda_1} = \frac{5}{4}$.

92. **(b)** $\lambda_{Blue} < \lambda_{Red}$. Therefore finge pattern will contract because fringe width $\propto \lambda$.

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(a)**

2. **(d)** As $i > i_c$, there cannot be any refracted wave. So, for all angles of incidence greater than the critical angle, we will not have any wavefront in medium 2. Hence, it is formed in medium 1 (on same side of AB).

3. **(a)** A plane wave AB incident at an angle i on a reflecting surface MN, then t be the time taken by the wavefront to advance from point B to C. So, from the given figure the distance BC = vt

where, v is the speed of light.

For obtaining reflected wavefront, a sphere of radius vt should be drawn from the point A (see figure given in question). The tangent CE drawn on this sphere represents the reflected wavefront of AB.

4. **(d)** $\frac{I}{4} = \frac{I}{2} \cos^2\theta$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \cos\theta = \cos 45°$$

$\theta = 45°$

5. **(c)** According to Malus' law

Here, $\theta = 60°$

$$\therefore I = I_0 \cos^2 60° = I_0 \left(\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

6. **(d)** We know that when light strike the interface of two media at Brewster's angle, then reflected light will be plane-polarized with its $\vec{E}$ vector vibrating in a single plane. Now, if there will be no electric field vector then there will no '$\vec{E}$' vibrating in reflected light. So there will be no reflected light.

7. **(c)** Speed of light is different in different media and different medium has different refractive index.

$$^1\mu_2 = \frac{\text{Speed of light in medium 1}}{\text{Speed of light in medium 2}}$$

8. **(c)** Given, refractive index, $\mu = \frac{4}{3}$

According to Brewster's law when unpolarised light strikes at polarising angle i_p on an interface then reflected and refracted rays are normal to each other and is given by :

$\tan i_p = \mu$

$$\therefore i_p = \tan^{-1}\left(\frac{4}{3}\right)$$

9. **(c)** Here Angle of incidence, i = 57

$\tan 57° = 1.54$

$u_{glass} = \tan i$

It means, Here Breswster's law is followed and the reflected ray is completely polarised.

Now, when reflected ray is analysed through a polaroid then intensity of light is given by malus law.

i.e. $I = I_0 \cos^2\theta$

on rotating polaroid 'θ' changes. Due to which intensity first decreases and then increases.

10. **(c)** At Brewster's angle, only the reflected light is plane polarised, but transmitted light is partially polarised.

11. **(c)** **12.** **(a)**

13. **(b)** $\mu = \tan i_p = \tan 54.74° = \sqrt{2}$

$$\therefore \sqrt{2} = \frac{\sin 45°}{\sin r} \Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30°$$

14. **(d)** Given: $\frac{I_1}{I_2} = \frac{49}{1} = \frac{A_1^2}{A_2^2}$

$$\Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{49}{1}} = \frac{7}{1}$$

or $A_1 = 7A_2$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

$$= \frac{(7+1)^2}{(7-1)^2} = \frac{64}{36} = \frac{16}{9}.$$

15. (d) $\frac{\text{(Maximum intensity) coherent interference}}{\text{(Maximum intensity) coherent interference}}$

$= \frac{n^2 I_0}{n I_0} = n$

16. (c) $\frac{a_1}{a_2} = \frac{3}{5}$

$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(3+5)^2}{(3-5)^2} = \frac{16}{1}$

17. (c) For constructive interference path difference is even multiple of $\frac{\lambda}{2}$.

18. (c)

19. (b) $\phi = \pi/3, a_1 = 4, a_2 = 3$

So, $A = \sqrt{a_1^2 + a_2^2 + 2a_1.a_2 \cos\phi} \Rightarrow A \approx 6$

20. (d) Resultant intensity $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

For maximum I_R, $\phi = 0°$

$\Rightarrow \quad I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

21. (c) The fringe width is given by, $\beta = \frac{\lambda D}{d}$

The angular width of fringe is given by

$\frac{d}{D} = \frac{\lambda}{\beta} = \frac{6\times10^{-7}}{0.12\times10^{-3}} = 5 \times 10^{-3}$ rad.

22. (b) In an interference experiment, the spacing between successive maxima or minima is called fringe width

i.e., fringe width, $\beta = \frac{\lambda D}{d}$.

23 (b) $\beta' = \frac{\beta}{\mu} = \frac{0.133}{1.33} = 0.1$ cm

24. (d) The nearest white spot will be at the central maxima.

$\therefore \quad y = \frac{3d}{4} - \frac{d}{2} = \frac{d}{4}$

So, the value of A is 4.

25. (c) Given, D = 200 cm = 2m

d = 1 mm = 10^{-3} m

Path difference

= 6.5 m = 6.5×10^{-3} m

We know that, path difference

$= \frac{7\lambda D}{d} - \frac{(2\times1-1)\lambda_D}{2d}$

$6.5 \times 10^{-3} = \frac{13\lambda D}{2d} = \frac{13\times\lambda\times200\times10^{-2}}{2\times1\times10^{-3}}$

$\lambda = 0.5 \times 10^{-3} \times 10^{-3}$

$= 5 \times 10^{-7}$ m = 5000 Å

26. (d) Using relation, $d \sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{d}$

For n = 3, $\sin\theta = \frac{3\lambda}{d} = \frac{3\times589\times10^{-9}}{0.589}$

$= 3 \times 10^{-6}$ or $\theta = \sin^{-1}(3 \times 10^{-6})$

27. (b) Position of nth maxima from central maxima is given by $x_n = \frac{n\lambda D}{d}$

$\Rightarrow \quad x_n \propto n\lambda \Rightarrow \frac{d_1}{d_2} = \frac{n_1\lambda_1}{n_2\lambda_2} = \frac{8\lambda_1}{6\lambda_2} = \frac{4}{3}\left(\frac{\lambda_1}{\lambda_2}\right)$

28. (d) Intensity of each source $= I_0 = \frac{100}{4} = 25$unit

If the intensity of one of the source is reduced by 36% then $I_1 = 25$ unit and

$I_2 = 25 - 25\times\frac{36}{100} = 16(\text{unit})$

Hence resultant intensity at the same point will be

$I = I_1 + I_2 + 2\sqrt{I_1 I_2} = 25 + 16 + 2\sqrt{25\times16} = 81$ unit

29. (b) Angular separation $= \frac{\lambda}{d}$

For angular separation to be 10% greater λ should be 10% greater.

Now wavelength is $\left(589 + \frac{589}{10}\right)$ nm

or (589 + 58.9) nm, i.e., 647.9 nm, i.e., 648 nm

30. (d)

31. (c) Wavelength for which maximum obtained at the hole has the maximum intensity on passing. So,

$x = \frac{n\lambda D}{d}$

$\lambda = \frac{xd}{nD} = \frac{1\times10^{-3}\times0.5\times10^{-3}}{n\times50\times10^{-2}} = \frac{1\times10^{-6}}{n} = \frac{1000\text{nm}}{n}$

n = 1, λ = 1000 nm → Not in the given range

n = 2, λ = 500nm

32. (a) Path difference between first and 11th bright fringe

= S_1B (10 bright fringes) = 10 λ

= 10 × (6000 × 10^{-7})m = 6×10^{-6} m

33. (c) Here $A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

$\because a_1 = a_2 = a$

$\therefore \quad A^2 = 2a^2(1+\cos\delta) = 2a^2\left(1 + 2\cos^2\frac{\delta}{2} - 1\right)$

$\Rightarrow A^2 \propto \cos^2\frac{\delta}{2}$

Now, $I \propto A^2 \quad \therefore\ I \propto A^2 \propto \cos^2\frac{\delta}{2}$

$\therefore I \propto \cos^2\frac{\delta}{2}$.

34. (b) Let nth fringe of 2500 Å coincide with (n – 2)th fringe of 3500Å.

$\therefore\ 3500(n-2) = 2500 \times n$

$1000\,n = 7000, n = 7$

$\therefore$ 7th order fringe of 1st source will coincide with 5th order fringe of 2nd source.

35. (a) $K = I + I + \sqrt{II}\cos 2\pi = 4I$,

$K' = I + I + 2\sqrt{II}\cos 2\pi = I$,

$= \frac{K}{4}$

36. (d) As fringe width $\beta = \frac{D}{d}\lambda$

$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ...$

$\mu_{Vacuum} < \mu_{Air}$ so $\lambda_{Vacuum} > \lambda_{Air}$

Therefore when chamber is evacuated fringe width β slightly increases.

37. (b) Fringe width, $\beta = \frac{\lambda D}{d}$

Angular separation of the fringes is

$\theta = \frac{\beta}{D} = \frac{\lambda D}{dD} = \frac{\lambda}{d}$

38. (c) $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{I} + \sqrt{4I})^2 = 9I$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 = (\sqrt{I} - \sqrt{4I})^2 = I$

39. (d) For 100th max., $d\sin\theta = 100\lambda$

$\sin\theta = \frac{100 \times 5000 \times 10^{-9}}{1 \times 10^{-3}} = \frac{5 \times 10^{-4}}{10^{-3}} = 0.5 = \frac{1}{2}$

$\therefore\ y = D\sin\theta = 1 \times \frac{1}{2} = \frac{1}{2} = \frac{1}{\sqrt{4}}$

40. (d) $I < I_{avg}$

If each source have equal intensity

$4I_0\cos^2\phi/2 < 2I_0$

$\cos^2\phi/2 < \frac{1}{2}$ i.e. $\frac{\pi}{2} < \phi < \frac{3\pi}{2}$

41. (b) $I = 4I_0\cos^2\frac{\phi}{2}$

or $I = 4I_0\cos^2\left(\frac{\pi y}{\beta}\right) \quad \left(\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}d\frac{y}{D}\right)$

where $\beta = \frac{\lambda D}{d}$ (fringe width)

Putting $I = 2I_0$ we get

$y = \frac{\beta}{4} = \frac{\lambda D}{4d} = 1.25 \times 10^{-4}$ m

42. (d) Given $d = .5$ mm $= 5 \times 10^{-2}$ cm

D = 100 cm.

$X_n = X_{11} - X_1 = 9.72$ mm.

$\therefore\ X_n = \frac{n\lambda D}{d}$

$\Rightarrow \lambda = \frac{X_n d}{nD} = \frac{0.972 \times 5 \times 10^{-2}}{10 \times 100} = 4.86 \times 10^{-5}$ cm.

43. (d) $\frac{I_{max}}{I_{min}} = \left(\frac{\frac{a_1}{a_2}+1}{\frac{a_1}{a_2}-1}\right)^2 \Rightarrow \frac{a_1 + a_2}{a_1 - a_2} = 6$

$\frac{7}{5} = \frac{a_1}{a_2}$.

44. (d) Angular fringe width $\theta = \frac{\lambda}{d} \Rightarrow \propto \lambda$

$\lambda_w = \frac{\lambda_a}{\mu_w}$

So $\theta_w = \frac{\theta_{air}}{\mu_w} = \frac{0.40}{\frac{4}{3}} = 0.30°$

45. (d) For minima, path difference $\Delta = (2n-1)\frac{\lambda}{2}$

For third minima n = 3

$\Rightarrow \Delta = (2 \times 3 - 1)\frac{\lambda}{2} = \frac{5\lambda}{2}$

46. (a) $\sin\theta_2 = \frac{2\lambda}{d}$

or $\theta_2 = \sin^{-1}\left(\frac{2\lambda}{d}\right)$

Given $y = D(2\theta_2) = 8 \times 10^{-2}$

or $1.5 \times 2 \times \sin^{-1}\left(\frac{2\lambda}{d}\right) = 8 \times 10^{-2} \Rightarrow d \simeq 0.005$ cm

47. **(b)** The width of the central maximum is given by

$$\beta = \frac{2\lambda D}{d}$$

$\Rightarrow$ If $d \to 2d$, then β decreases.

Also, intensity $I = I_0\left[\frac{\sin\alpha}{\alpha}\right]^2$

where $\alpha = \frac{\pi d \sin\theta}{\lambda}$

$\therefore$ I increases as d increases

$\therefore$ The central maximum will become narrower and brighter.

48. **(c)** In Fraunhoffer diffraction, for minimum intensity,

$$\Delta x = m\frac{\lambda}{2}$$

For first minimum, m = 1

$$\therefore \quad \Delta x = \frac{\lambda}{2}$$

49. **(a)** We know that for maxima

$$b\sin\theta = (2n+1)\frac{\lambda}{2}$$

or $\sin\theta = \frac{2n+1}{2}\left(\frac{\lambda}{b}\right)$

So on decreasing the slit width, 'b', keeping λ same, $\sin\theta$ and hence θ increases.

50. **(b)** separation of first minima $= 2\frac{D}{d}\lambda$

Givn $D = 1m, d = 2mm = 2\times10^{-3}m, \lambda = 500\ nm$ $= 500\times10^{-9}m$

First minima is separated by a distance $= 2\frac{D}{d}\lambda$

$$= \frac{2\times1\times500\times10^{-9}}{2\times10^{-3}} = 0.5mm$$

51. **(a)** Angular width of central maxima is given by,

Q = 2 λ a

Where,

λ = Wavelength

a = Width of slit,

It does not depend on distance between slit and source.

52. **(b)** Angular width $\theta = 2\sin^{-1}\left(\frac{\lambda}{d}\right)$ or, $\theta \propto \lambda$

$\therefore \theta_1 = \theta = k\lambda_1$ ... (i)

and $\theta_2 = \theta - \theta\times\frac{30}{100} = k\lambda_2$

or, $\theta_2 = \frac{7\theta}{10} = k\lambda_2$ ... (ii)

Dividing eq. (ii) by (i)

$$\frac{k\lambda_2}{k\lambda_1} = \frac{\frac{7\theta}{10}}{\theta} \Rightarrow \frac{\lambda_2}{6000} = \frac{7}{10} \therefore \lambda_2 = \frac{7}{10}\times6000 = 4200Å$$

Exercise 3 : PREVIOUS YEARS MCQs

1. **(c)** Given, $d_1 = 0.2\ mm, d_2 = 0.4\ mm$

$$\beta = \frac{D\lambda}{d} \propto \frac{1}{d}$$

$$\beta_{new} = \frac{\beta}{2}$$

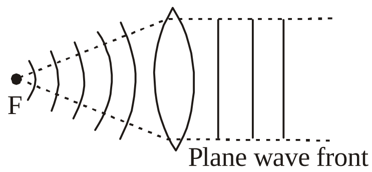
Hence, β is half so 50% decrement.

2. **(a)** Here, $x_1 = 2d$ and $x_2 = \sqrt{5}d$

For, first minima, $\Delta x = \frac{\lambda}{2}$

$\therefore \Delta x = x_2 - x_1 = \sqrt{5}d - 2d = \frac{\lambda}{2}$ $\Rightarrow d = \frac{\lambda}{2(\sqrt{5}-2)}$

3. **(d)**



As, the light emerges parallel. So, plane wavefront is formed.

4. **(c)** $\frac{I_0}{4} = I_0\cos^2(\phi/2) \Rightarrow \phi = 2\pi/3$

$\Rightarrow \Delta x\times(2\pi/\lambda) = 2\pi/3 \Rightarrow \Delta x = \lambda/3$

$\sin\theta = \Delta x/d \Rightarrow \sin\theta = \lambda/3d \Rightarrow \theta = \sin^{-1}(\lambda/3d)$

5. **(b)** We have

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2} = \frac{(\sqrt{4}-\sqrt{1})^2}{(\sqrt{4}-\sqrt{1})^2} = \frac{9}{1}$$

By componendo and dividendo

$$\frac{I_{max}+I_{min}}{I_{max}-I_{min}} = \frac{9+1}{9-1} = \frac{10}{8} = \frac{5}{4}$$

So, $x = 4$

6. **(b)** Frings width, $\beta = 12$ mm

Refractive index of water, $\mu = \frac{4}{3}$

The frings width is given by,

$$\beta = \frac{D\lambda}{d} \quad ..... (i)$$

Here, λ is wavelength of light.

D is distance between screen and source.

d is distance between coherent source.

If the entire arrangement is placed in water then fringes width becomes $\beta' = \frac{D\lambda'}{d}$ ..... (ii)

Dividing equation (ii) by (i), we have

$$\Rightarrow \frac{\beta'}{\beta} = \frac{\lambda'}{\lambda}$$

$$\Rightarrow \beta' = \frac{12 \times 3}{4} \quad \left(\because \mu = \frac{\lambda}{\lambda'}\right)$$

$$\Rightarrow \beta' = 9 \text{ mm}$$

7. **(c)**

8. **(d)** It is a one of Fraunhoffer diffraction from single slit. so for bright fringe where a is the width of slit.

$$a \sin\theta = (2n+1)\frac{\lambda}{2}$$

$$\lambda = \frac{2a\sin\theta}{2n+1} = \frac{2\times1.2\times10^{-5}\times0.0906}{2\times1+1} = 7248\text{Å}.$$

9. **(b)** Fringe width $\beta = \frac{\lambda D}{d}$

10. **(d)** Resolving power $\propto$, d

Resolving power of a telescope is proportional to the diameter.

11. **(None)**

12. **(b)** $I_{max} = (a_1 + a_2)^2$ and $I_{min} = (a_1 - a_2)^2$

$$I_{max} + I_{min} = a_1^2 + a_2^2 + a_1^2 + a_2^2$$

$$= 2\left(a_1^2 + a_2^2\right) = 2(I_1 + I_2)$$

13. **(c)** $d = \frac{\lambda}{2\mu \sin\alpha} = \frac{\lambda}{2N.A}$

N.A. limit of resolution is decrease (c).

14. **(b)** Given $d = 0.2 \times 10^{-3}$m, $D = 2$m and $\lambda = 5 \times 10^{-7}$m

From $B = \frac{\lambda D}{d} = \frac{5\times10^{-7}\times2}{0.2\times10^{-3}} = \frac{5\times10^{-7}}{10^{-4}}$

$\therefore B = 510^{-3}$m

Distance between 1st minima on either side

$$= 5\times10^{-3} + 5\times10^{-3} = 10\times10^{-3} = 10^{-2}\text{m}$$

15. **(b)** Polarising angle, $\tan\theta = \mu$

Also, $M = \frac{C}{V}$

or, $\cot\theta = \frac{v}{c}$ $\therefore \theta = \cot^{-1}\frac{v}{c}$

16. **(b)** Resultant intensity for two coherent sources,

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

For two identical light waves, $I_1 = I_2 = I$

$$\therefore I_R = 4I\cos^2\frac{\phi}{2} \text{ or, } 1_R \propto \cos^2\frac{\phi}{2}$$

17. **(c)** Resultant intensity of interferring wave at 'P'

$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2 \cos\phi}$$

For $\phi = \frac{\pi}{2}, I_p = I + 9I = 10I$

Again resultant intensity at 'Q'

$$I_Q = I_1 + I_2 + 2\sqrt{I_1 I_2 \cos\phi}$$

For $\phi = \pi, I_Q = I + 9I + (-2\sqrt{9(1)^2})$

$= 10I - 6I = 4I$

$\therefore$ Difference between the resultant intensity

$$\Delta I = I_P - I_Q = 10I - 4I = 6I$$

18. **(a)** According to Brewster's law, $\tan i_p = \mu$

Clearly, Polarising angle depends on wavelength and wavelength is different for different colours of light.

19. **(a)** Resolving power of a telescope is given by

$$RP = \frac{d}{1.22\lambda}$$

As $RP \propto \frac{1}{\lambda}$

$\therefore$ On decreasing the wavelength of light, resolving power of a telescope increases.

20. **(b)** Linear width is given as:

$$W = \frac{2\lambda}{a}f$$

Substituting values

$$W = \frac{2\times6000\times10^{-10}\times20\times10^{-2}}{0.01\times10^{-3}} = 24\times10^{-3} = 24\text{mm}$$

21. **(a)** Condition for minima is $d\sin\theta = n\lambda$

For Ist minima $d\sin\theta = \lambda$

$$\sin\theta = \frac{\lambda}{a} = \frac{2}{4} = \frac{1}{2} \quad \therefore \theta = 30°$$

Angular spread $= 2\theta = 60°$

22. **(c)** In Young's double slit experiment, fringe width β is is given by: $\beta = \frac{\lambda D}{d}$

Given: d' = 3d

Let initial fringe width $= \beta_1 = \frac{\lambda D}{d}$

Final fringe width $= \beta_2 = \frac{\lambda D}{3d}$

$$\frac{\beta_1}{\beta_2} = \frac{\lambda D / d}{\lambda D / 3d} = \frac{1}{1/3} = 3$$

23. **(d)** In YDSE, $\beta = \frac{\lambda D}{d1} = 0.3\text{mm}$

Now, $\lambda' = \lambda + 20\%$ of $\lambda = 1.2\lambda$.

$d' = d - 25\%$ of $d = 0.75d$

$\therefore$ New fringe width is

$$\beta = \frac{\lambda' D}{d'} = \frac{1.2\lambda D}{0.75d} = \frac{1.2 \times 0.3}{0.75} = 0.48 \text{ mm}$$

24. **(d)** Intensity of unpolarized I_o

$I = 37.5\%$ of I_o

$I = 0.375\, I_o$

After first Polaroid, intensity

$$(I_1) = \frac{I_o}{2}$$

Intensity offer the second Polaroid

$I = I_1 \cos^2\theta$

$$0.375 I_o = \left(\frac{I_o}{2}\right)\cos^2\theta$$

$$\cos^2\theta = 0.75 \Rightarrow \cos\theta = \sqrt{0.75}$$

$$\cos\theta = \frac{\sqrt{3}}{2}, \theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{\theta = 30^\circ}$$

25. **(c)** Malus's Law-

$I = I_o \cos^2\theta \Rightarrow I_1 = \frac{I_o}{2}$

light is now polarized along the axis of first sheet

Second polarizer (30° from first)

$$I_2 = I_1 \cos^2(30^\circ)$$

$$= \frac{I_o}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{I_o}{2} \times \frac{3}{4} = \frac{3I_o}{8}$$

Third polarizer (again 30° from second)

$$I_3 = I_2 \cos^2(30^\circ) = \frac{3I_o}{8} \times \frac{3}{4} = \frac{9I_o}{32}$$

So,the ratio of intensities from the second and third sheets.

$$\frac{I_2}{I_3} = \frac{3I_o / 8}{9I_o / 32} = \frac{3}{8} \times \frac{32}{9} = \frac{96}{72} = \frac{4}{3}$$

answer is (c)

8 Electrostatics

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(a)** Gauss's law is valid for any closed surface, no matter what its shape or size.

2. **(c)** By Gauss's theorem, $\phi = \frac{Q_{in}}{\epsilon_0}$

Thus, the net flux depends only on the charge enclosed by the surface. Hence, there will be no effect on the net flux if the radius of the surface is doubled.

3. **(a)**

4. **(b)** The surfaces encloses the total charge
= 4C + 6C – 3C = 7C
So TNEI = 7C

5. **(b)** In case of cylinder for x > r

$$E = \frac{1}{2\pi k\varepsilon_0}\cdot\frac{1}{l}\cdot\frac{1}{x}$$

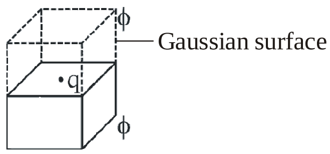
$$E \propto \frac{1}{x} \quad \text{or} \quad \frac{E_1}{E_2} = \frac{x_2}{x_1}$$

6. **(c)** Energy density $u = \frac{1}{2}k\varepsilon_0 E^2$

$$u_1 = \frac{1}{2}k\varepsilon_0(2E)^2$$

$$\frac{u_1}{u} = 4 \quad \therefore u_1 = 4\mu$$

7. **(b)**



Total flux emerges from the system $\phi_T = \frac{q}{\varepsilon_0}$

Therefore the flux linked with the cube $\phi_{cube} = \frac{q}{2\varepsilon_0}$

8. **(a)** $\vec{E} = \left(6\hat{i} + 5\hat{j} + 3\hat{k}\right) N/C$

$\vec{A} = 30\hat{i}\, m^2$

Electric flux, $\phi = \vec{E}.\vec{A}$

$= \left(6\hat{i} + 5\hat{j} + 3\hat{k}\right).\left(30\hat{i}\right) = 6 \times 30 = 180$ V-m

9. **(d)**

10. **(d)** Electric field due to uniformly charged solid non-conducting sphere is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r \text{ (for } r < R) \qquad \therefore E \propto r$$

So, electric field increases as *r* increases.

11. **(a)** According to Gauss' law, $\phi = \frac{q}{\varepsilon_0}$

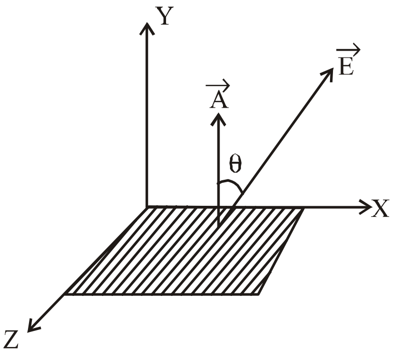
As, $\phi_{sphere} = \phi_{cube}$ $\therefore \phi_{sphere} : \phi_{cube} = 1 : 1$

12. **(a)** By Gauss theorem,

$$\text{Total electric flux} = \frac{\text{Total charge inside cube}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

13. **(b)**

14. **(a)**



$\vec{E} = \left(8\hat{i} + 13\hat{j}\right) Nc^{-1}, \vec{A} = 3\hat{j}m^2$

$\therefore$ Electric flux, $\phi = \vec{E}\cdot\vec{A}$

$\therefore \phi = \left(8\hat{i} + 13\hat{j}\right)\cdot\left(3\hat{j}\right) = 13 \times 13 = 39$ Wb

15. **(d)** Net flux through the surface = $\phi_2 - \phi_1$

By Gauss's law, $\phi_2 - \phi_1 = \frac{q_{enc}}{\varepsilon_0}$

$q_{enc} = (\phi_2 - \phi_1)\varepsilon_0$

16. **(c)** Electric field between the sheets

$= \frac{(\sigma_1 - \sigma_2)}{2\epsilon_0}$ (from Gauss's theorem)

Here, $\sigma_1 = \sigma_2$ E = 0

17. **(a)** The electric field at a distance r from a uniform straight wire is

$$E = \frac{2k\lambda}{r} = \frac{2 \times 9 \times 10^9 \times 0.2 \times 10^{-6}}{3} = 1.2 \times 10^3\ Vm^{-1}$$

18. **(a)** By Gauss' theorem,

Electric flux, $\phi = \frac{q_i}{\varepsilon_0} = \frac{+q - q}{\varepsilon_0} = 0$

19. **(c)** For plane sheet of charge, $E = \sigma/(2\varepsilon_0)$

20. **(d)** If there is only one type of charge in the universe then it will produce electric field. Hence Gauss's law is valid.

$\oint \vec{E}.d\vec{s} = 0$ if charge is outside

$\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$ if charge is inside.

21. **(a)** Gaussian surface cannot pass through any discrete charge because electric field due to a system of discrete charges is not well defined at the location of the charges. But the Gaussian surface can pass through a continuous charge distribution.

22. **(d)** All other statements except (iv) are in correct. The electric field over the Gaussian surface remains continuous and uniform at every point.

23. **(a)** According to Gauss's law total electric flux through a closed surface is $\frac{1}{\varepsilon_0}$ times the total charge inside that surface.

Electric flux, $\phi_E = \frac{q}{\varepsilon_0}$

Charge on α-particle = 2e $\therefore \phi_E = \frac{2e}{\varepsilon_0}$

24. **(c)** $\oint \vec{E}\cdot d\vec{A} = 0$, represents charge inside close surface is zero. Electric field as any point on the surface may be zero.

25. **(a)** Electric charge resides only on the surface of a spherical shell. According to Gauss's theorem the total electric flux over a closed surface is equal to the $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the closed surface.

26. **(a)** The flux is zero according to Gauss' Law because it is a open surface which enclosed a charge *q*.

27. **(d)** According to Gauss' Law

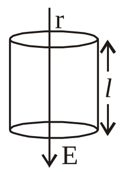
$\oint E.ds = \frac{Q_{\text{enclosed by closed surface}}}{\varepsilon_o} = \text{flux}$

so total flux = Q/ε_o

Since cube has six face, so flux coming out through one wall or one face is $Q/6\varepsilon_o$.

28. **(c)** If electric dipole, the flux coming out from positive charge is equal to the flux coming in at negative charge *i.e.* total charge on sphere = 0. From Gauss law, total flux passing through the sphere = 0.

29. **(a)** Since –ve electric flux = + ve flux electric flux enclosed with a cylinder here
∴ Total Electric Flux = 0.



30. **(c)** By Gauss's theorem, $\phi = \frac{Q_{in}}{\epsilon_0}$

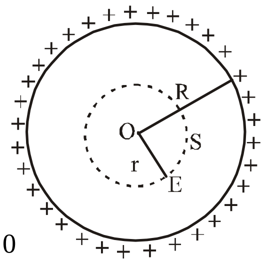
Thus, the net flux depends only on the charge enclosed by the surface. Hence, there will be no effect on the net flux if the radius of the surface is doubled.

31. **(c)** $\phi_{Total} = \frac{q_{enclosed}}{\varepsilon_0}$

$\phi_{Total} = \frac{0}{\varepsilon_0} = 0$

32. **(a)** Charge resides on the outer surface of a conducting hollow sphere of radius R. We consider a spherical surface of radius $r < R$.

By Gauss's theorem,



$\oint \vec{E}.d\vec{s} = \frac{1}{\varepsilon_0} \times$ charge enclosed

or, $E.4\pi r^2 = \frac{1}{\varepsilon_0} \times 0 \Rightarrow E = 0$

33. **(d)** Here, $D = 2r = 4.4\ m$, or $= r = 2.2\ m$

$\sigma = 60\ \mu C\ m^{-2}$

Charge on the sphere, $q = \sigma \times 4\pi r^2$

$= 60 \times 10^{-6} \times 4 \times \frac{22}{7} \times (2.2)^2 = 3.7 \times 10^{-3}$ C

34. **(a)** For the curved surface, θ = 90º

$\therefore \phi = E\ ds \cos 90° = 0.$

35. **(a)** 36. **(d)**

37. **(a)** $\phi = \vec{E}.\vec{A} = 4\hat{i}.(2\hat{i} + 3\hat{j}) = 8$ V-m

38. **(a)** Here, E must be perpendicular to Y-Z plane, i.e., area must be parallel to X-plane,

so $d\vec{s} = 20\hat{i}$ units

∴ electric flux = $\vec{E}.d\vec{s} = (5\hat{i} + 4\hat{j} + 9\hat{k}).(20\hat{i}) = 100$ units

39. **(d)** $C = \frac{\epsilon_0 A}{d}$; when $d \to \infty$, $C \to 0$

40. **(d)** If C' is the capacitance of each capacitor, then

$\frac{C'}{2} = 2C,$

$\therefore C' = 4C.$

41. **(a)** The maximum potential difference C can withstand is

$V = \frac{Q}{C}$

The maximum potential difference 2C can withstand is

$V' = \frac{Q}{4C}$

So, in parallel combination, we can apply a maximum potential difference of $\frac{Q}{4C}$

Total capacitance in parallel is C + 2C = 3C

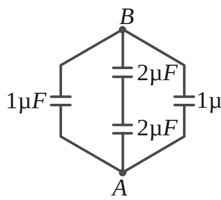
$\therefore$ Total charge $q = (3C) \times \left(\frac{Q}{4C}\right) = \frac{3Q}{4}$

42. **(a)** Due to insertion of a dielectric slab capacitance increase by K times. The potential difference, the electric field and the stored energy decreases by $\frac{1}{K}$ times.

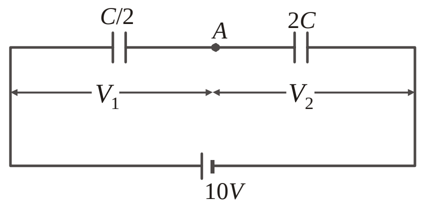
43. **(d)** **44.** **(a)**

45. **(a)** The equivalent circuit is shown in figure.

$C_{AB} = 3\mu F$.



46. **(a)** $V_1 + V_2 = 10$ and $\frac{C}{2}V_1 = 2CV_2$

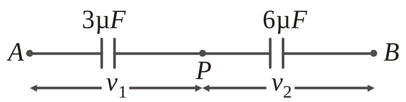


On solving, above equations, we get

$V_1 = 8V$ and $V_2 = 2V$.

47. **(c)** $V_1 + V_2 = 1200$

and $3V_1 = 6V_2$



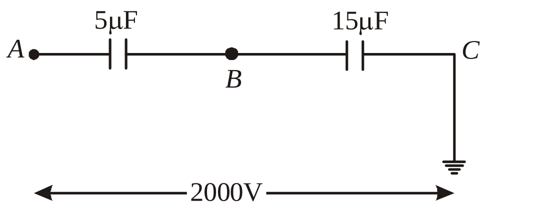
On solving above equations, we have

$V_1 = 800V$ and $V_2 = 400V$

Now $V_P - V_B = 400$;

As $V_B = 0$, $\therefore V_P = 400$ V

48. **(c)** The given circuit can be redrawn as follows



$$(V_A - V_B) = \left(\frac{15}{5+15}\right) \times 2000 \Rightarrow V_A - V_B = 1500V$$

$\Rightarrow$ $2000 - V_B = 1500V \Rightarrow V_B = 500V$

49. **(b)** Charge on each plate of each capacitor

$$Q = \pm CV = \pm 25 \times 10^6 \times 200 = \pm 5 \times 10^{-3} C$$

50. **(a)** Capacitance of spherical conductor $= 4\pi\varepsilon_0 a$ where a is radius of conductor.

Therefore, $C = \frac{1}{9 \times 10^9} \times 1 = \frac{1}{9} \times 10^{-9}$

$$= 0.11 \times 10^{-9} \text{ F} = 1.1 \times 10^{-10} \text{ F}$$

51. **(d)** $C \propto \frac{1}{d}$

52. **(a)** If d be the distance between the plates, then

$$d = \frac{V}{E} = \frac{30}{30 \times 10^6} = 10^{-6} \text{ m}$$

Capacitance of capacitor,

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$15 \times 10^{-9} = \frac{8.85 \times 10^{-12} \times 2.5 \times A}{10^{-6}}$$

$\Rightarrow$ $A = 6.7 \times 10^{-4}$ m^2

53. **(a)** Capacitance of spherical conductor

$C = 4\pi\varepsilon_0 r$ ...(i)

$8V_S = V_B$.

$\Rightarrow$ $8 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ $\Rightarrow$ $R = 2r$

Let capacitance of the bigger drop $= C_B$.

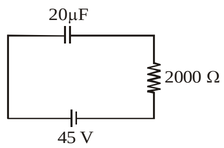
$\therefore$ $\frac{C_B'}{C} = \frac{R}{r}$ (Using (i))

Substitute $R = 2r$.

or $\frac{C_B'}{C} = \frac{2r}{r}$ $\therefore$ $C_B' = 2C$

54. **(a)** Final charge on the capacitor,

$Q = CV = 20 \times 10^{-6} \times 45 = 9 \times 10^{-4}$C.



55. **(d)** Since capacitance $C = \frac{\varepsilon_0 A}{d}$, as d decreases capacitance increases.

56. **(d)**

57. **(b)** As the capacitor is isolated after charging, charge Q will remains constant. When plate separation d increases, then capacitance decreases as $C = \frac{\varepsilon_0 A}{d}$ and hence, potential increases as $V = \frac{Q}{C}$

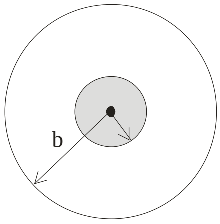
58. **(d)** When battery is disconnected, charge remains constant. On introducing glass slab, capacity increases C $= \frac{K\varepsilon_0 A}{d}$ where K = dielectric constant. Potential difference $V = \frac{Q}{C}$ and energy stored $u = \frac{1}{2}QV$ decreases.

59. **(d)** Given : $(b - a) = 1 \times 10^{-3}$ m ... (i)

and $C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) = 1\times10^{-6}$

$\Rightarrow 1\times10^{-6} = \frac{1}{9\times10^9}\left(\frac{ab}{10^{-3}}\right)$

$\Rightarrow$ ab = 9 ... (ii)



From equations (i) and (ii)

$b - \frac{9}{b} = \frac{1}{1000} \Rightarrow 1000\,b^2 - b - 9000 = 0$

$\Rightarrow b = \frac{1\pm\sqrt{(-1)^2 - 4(1000)(-9000)}}{2\times1000}$

{Solving of quadratic equation}

$\Rightarrow b = \frac{1\pm\sqrt{36\times10^6}}{2000} = \frac{\sqrt{36\times10^6}}{2000} = 3\text{m}$

60. **(a)** High K means good insulating property and high x means able to withstand electric field gradient to a higher value.

61. **(c)** In this process, capacity increases, so battery supplies additional charge to capacitor.

62. **(b)** $C_{medium} = K \times C_{air}$

63. **(b)**

64. **(a)** For parallel combination of capacitors,

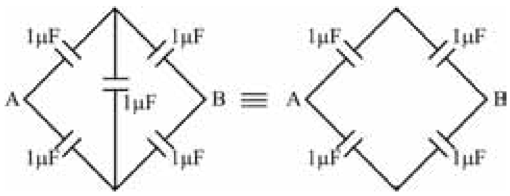
$C_p = C_1 + C_2 + C_3 = 4 + 6 + 12 = 22\ \mu F$

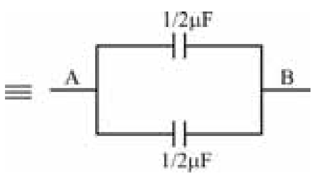
For series combination of capacitors,

$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{1}{2}$ $\therefore$ $C_s = 2\ \mu F$

$\therefore \frac{C_s}{C_p} = \frac{2}{22} = 1:11$

65. **(a)** The equivalent figure will be





$C_{eq} = \frac{1}{2} + \frac{1}{2} = 1\ \mu F$

66. **(a)** Let the capacitance of each capacitor be C.

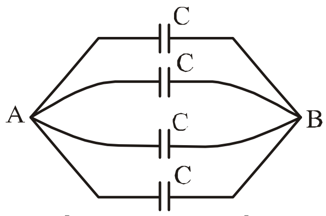
Equivalent capacitance of the two capacitor connected in parallel, $C_p = C + C = 2C$

Equivalent capacitance of the two capacitor connected in series, $C_s = \frac{CC}{C+C} = \frac{C}{2}$

Given : $C_p - C_p = C_s = 6\ \mu F$

i.e., $2C - \frac{C}{2} = 6\times10^{-6}\ \text{F} \Rightarrow C = 4\mu F$

67. **(a)** The equivalent circuit can be redrawn as



$\therefore$ The equivalent capacitance between A and B is $C_{eq} = 4C = 4 \times 8 = 32\ \mu F$

68. **(a)** $C_1 = 5\ \mu F$, $C_2 = (4 + 8 + 4) = 16\ \mu F$

$\therefore V_1 = \left(\frac{C_2}{C_1+C_2}\right)V = \left(\frac{16}{16+5}\right)63 = 48V$

69. **(a)** In parallel grouping of capacitors

$C_{eq} = C_1 + C_2 + \dots\dots\dots\dots C_n$

70. **(d)** To get effective capacitance of 6 μF two capacitors of 4 μF each connected in sereies and one of 4 μF capacitor in parallel with them.

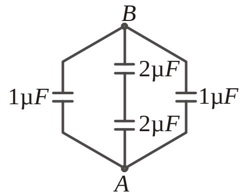
Two capacitances in series

$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

1 capacitor in parallel

$\therefore C_{eq} = C_3 + C = 4 + 2 = 6\mu F$

71. **(a)** The equivalent circuit is shown in figure.



$C_{AB} = 1 + \frac{2\times2}{2+2} + 1$

$C_{AB} = 3\mu F.$

72. **(b)** $C_1 = 1$ pF, $C_2 = 2$ pF and $C_3 = 4$ pF

To obtain least combination of capacitance when all the capacitors are connected in series combination. The effective capacitance C' will be,

$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$ or $C' = \frac{4}{7}$pF

73. **(a)** Charge will be distributed between them in proportion to their capacity as they have equal potential. Hence, net voltage drop across two capacitors when connected in parallel can be given as

$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{Q+0}{C+2C} = \frac{Q}{3C}$

$Q_1 = CV = C \times \frac{Q}{3C} = \frac{Q}{3}$

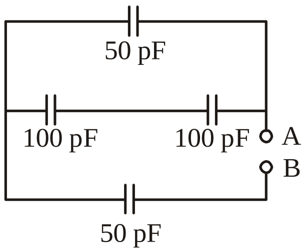
$Q_2 = 2CV = 2C \times \frac{Q}{3C} = \frac{2Q}{3}$

74. **(b)** Common potential is given as

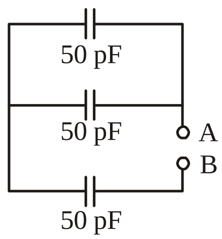
$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$\therefore V = \frac{10 \times 200 + 20 \times 100}{10 + 20} = 133.3V$$

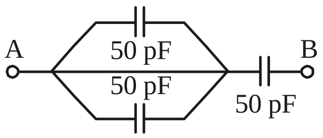
75. **(d)** The equivalent capacitance given as



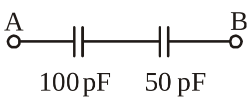
(I)



(II)



(III)



(IV)

Now 100 pF and 50 pF are in series,

$$\therefore C_{AB} = \frac{100 \times 50}{100 + 50} = \frac{100 \times 50}{150} = \frac{100}{3} \text{pF}$$

76. **(b)** $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

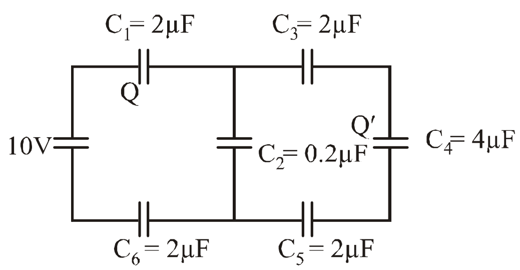
$\frac{1}{C_{series}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

$\frac{3+2+1}{6} = \frac{6}{6}$

$C_{series} = 1\ \mu F$

77. **(a)**

78. **(b)**



Equivalent capacitance $C_{eq} = 0.5\ \mu F$

Charge given by battery

$Q = C_{eq} V = 0.5 \times 10 = 5\ \mu C$

Charge on C_4,

$$Q' = \frac{5\mu C \times 0.8}{0.8 + 0.2} = 4\mu C$$

79. **(b)** $C_1 < C_2$

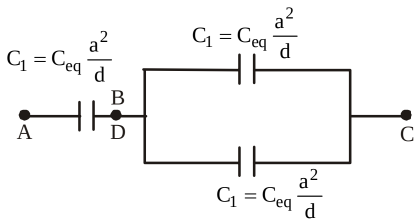
$\therefore \frac{C_1}{C_1 + C_2} < \frac{1}{2}$ and $\frac{C_2}{C_1 + C_2} > \frac{1}{2}$

$C = \frac{C_1C_2}{C_1 + C_2} = C_1 \cdot \frac{C_2}{C_1 + C_2} > \frac{C_1}{2}$

Similarly, $C < \frac{C_2}{2}$

80. **(c)** Equivalent circuit is shown

$\therefore C_{eq} = \frac{3\varepsilon_0 a^2}{5d}$



81. **(a)** $\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{15} + \frac{1}{20} = \frac{6+4+3}{60} = \frac{60}{13} \mu F$

So, $Q = C_{eq}V = \frac{60}{13} \times 13\mu C = 60\mu C$

In series, charge across each capacitor is same and is equal to net charge.

So, $Q_{15\mu F} = 60\mu C$

82. **(d)** Let 'n' such capacitors are in series and such 'm' such branch are in parallel.

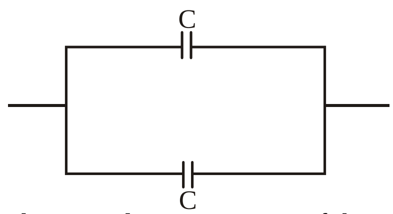
$\therefore$ 250 × n = 1000 $\therefore$ n = 4 … (i)

Also $\frac{8}{n} \times m = 16 \Rightarrow m = \frac{16 \times n}{8} = 8$ … (ii)

$\therefore$ No. of capacitor $= 8 \times 4 = 32$

83. **(b)**

84. **(c)** The circuit can be reduced to



The equivalent capacitance of the combination is

$C_{eq} = C + C = 2C$

85. **(a)**

86. **(a)** In series $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ and charge on each capacitor is same.

87. **(b)** Potential difference across the branch de is 6 V. Net capacitance of de branch is 2.1 μF

So, q = CV = 2.1 × 6 μC = 12.6 μ C

Potential across 3 μF capacitance is

$$V = \frac{12.6}{3} = 4.2 \text{ volt}$$

Potential across 2 and 5 combination in parallel is $6 - 4.2 = 1.8$ V

So, $q' = (1.8)(5) = 9\ \mu C$

88. (b) Work done = Change in energy

$= \frac{1}{2}\left(C + \frac{C}{2}\right)V^2 = \frac{1}{2}\left(\frac{3C}{2}\right)V^2 = \frac{3CV^2}{4}$

89. (a) Work = increase in potential energy

$= Kq_1q_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$= 9 \times 10^9 \times 8 \times 10^{-6} \times 12 \times 10^{-6}\left(\frac{1}{6\times10^{-2}} - \frac{1}{10\times10^{-2}}\right)$

$= 5.8$ J

90. (d) $U = \frac{1}{2}QV$ = Area of triangle OAB

91. (c) As battery is disconnected, total charge Q is shared equally by two capacitors.

Energy of each capacitor $= \frac{(Q/2)^2}{2C} = \frac{1}{4}\frac{Q^2}{2C} = \frac{1}{4}U.$

92. (d) When capacitor in removed from the battery its change becomes constant. Its energy $U = \frac{Q^2}{2C}$. As C decreases with increase in separation of plates and so U will increase.

93. (c) Work done = energy stored $= \frac{1}{2}CV^2.$

94. (a) $E = \frac{1}{2}CV^2 = \frac{1}{2}\times1\times10^{-6}\times(4000)^2 = 8$ J.

95. (b) Here, $C_1 = 2\ \mu F$, $C_2 = 4\ \mu F$

Potential difference applied, $V = 6$ V

In series combination, the equivalent capacitance will be,

$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \left(\frac{2\times4}{2+4}\right)\mu F = \frac{4}{3}\mu F$

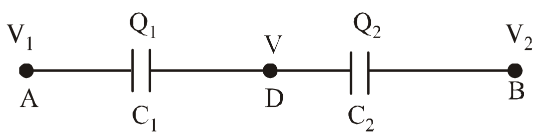
Electric charged stored,

$Q = CV = \left(\frac{4}{3}\times6\right)\mu C = 8\ \mu C$

Energy stored in the system, $U = \frac{1}{2}CV^2$

$= \frac{1}{2}\times\frac{4}{3}\times10^{-6}\times(6)^2 J = 24\mu J$

96. (c)



In series combination, $Q_1 = Q_2$

$\Rightarrow C_1(V_1 - V) = C_2(V - V_2)$

$\Rightarrow V(C_1 + C_2) = C_1V_1 + C_2V_2 \Rightarrow V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

97. (c)

98. (c) $\frac{1}{2}C_1V_1^2 = \frac{1}{2}C_2V_2^2$

because total energy is transferred (given).

$\therefore \frac{1}{2}\times900\times10^{-6}\times100^2 = \frac{1}{2}\times100\times10^{-6}\times V^2$

$\therefore V^2 = 90000 \Rightarrow V = 300$ V.

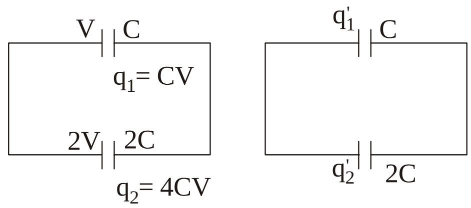
99. (b) According to energy conservation, energy remains the same.

$\Rightarrow U_{parallel} = U_{series} \Rightarrow \frac{1}{2}(nC)V^2 = \frac{1}{2}\left(\frac{C}{n}\right)V'^2$

$\Rightarrow V' = nV$

(V' = potential difference across series combination)

100. (c)



Initial state Final state

From the change conservation

$q_1' + q_2' = q_1 + q_2$

$q_1' + q_2' = 5CV$ ..... (1)

$\frac{q_1'}{C} = \frac{q_2'}{2C}$ ..... (2)

From equation (1) and (2)

$q_1' = \frac{5}{3}CV,\ q_2' = \frac{10}{3}CV$

Loss of energy, $\Delta E = \frac{\Delta q_1^2}{2C_1} + \frac{\Delta q_2^2}{2C_2}$

$= \frac{\left(CV - \frac{5}{3}CV\right)^2}{2C} + \frac{\left(4CV - \frac{10}{3}CV\right)^2}{2(2C)}$

$= \frac{4}{18}CV^2 + \frac{2}{18}CV^4 = \frac{1}{3}CV^2 = \frac{2}{3}E = \frac{x}{3}E$ (given)

$\therefore x = 2$

101. (d)

102. (b) By inserting the dielectric slab, capacitance (i.e., ability to hold the charge) increases. In the presence of battery more charge is supplied from battery.

103. (b) Energy will be lost during transfer of charge (heating effect).

104. (b) $u = \int_0^v CV\ dV = \frac{1}{2}CV^2$

105. (c) Work done $= \frac{1}{2}\frac{q^2}{C} = \frac{(8\times10^{-18})^2}{2\times100\times10^{-6}} = 32\times10^{-32}$ J

106. (b) $U = U_f - U_i = \frac{q^2}{2}\left(\frac{1}{C_f} - \frac{1}{C_i}\right)$

$= \frac{(5\times10)^2}{2}\left(\frac{1}{2} - \frac{1}{5}\right)\times10^6 = 3.75\times10^{-6}$J

107. (d) $\vec{E} = \left(30\hat{i} + 40\hat{j}\right)NC^{-1},\ V(0,0) = 0$

Now, $dv = -\vec{E}.d\vec{r} = -\left(30\hat{i} + 40\hat{j}\right)\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$

$\Rightarrow \int_{(0,0)}^{(1,2)} dv = -30\int_0^1 dx - 40\int_0^2 dy$

$\Rightarrow$ V(1, 2) – V(0, 0) = – 30 × 1 – 40 × 2

$\therefore$ V(1, 2) = – 110V

108. (d) Surface of metallic cube is an equipotential surface. Therefore, electric field is normal to the surface of the cube.

109. (b) Electric field is always zero inside a conductor.
If there is any excess of charge on a hollow conductor it always resides on the outer surface of conductor. Therefore inside a hollow conductor there is no charge and hence charge density is zero.

110. (a) Here, $V(x) = \frac{20}{x^2-4}$ volt

We know that $E = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{20}{x^2-4}\right)$

or, $E = +\frac{40x}{(x^2-4)^2}$

At x = 4 μm,

$E = +\frac{40\times 4}{(4^2-4)^2} = +\frac{160}{144} = +\frac{10}{9}$ volt / μm.

Positive sign indicates that $\vec{E}$ is in +ve x-direction.

111. (b) As we move towards a positive charge distribution V increases i.e., $\frac{dV}{dr}$ is positive. The increase in potential is steepest when we move exactly towards charge distribution. But E is in a direction exactly away from charge distribution, therefore E is in exactly opposite direction in which increase in potential is steepest. Hence $E = -\frac{dV}{dr}$.

112. (b) $\vec{E} = \frac{\partial v}{\partial x}\hat{i} + \frac{\partial v}{\partial y}\hat{j}$

$\therefore |\vec{E}| = k(\sqrt{x^2+y^2}) = kr$

Given $v = -kxy$

$E \propto r$

$\therefore \vec{E} = ky\hat{i} + kx\hat{j}$

113. (d) We have

$V = 3x^2$ Volt

$E_x = \frac{-\partial V}{\partial x} = -6x \Rightarrow E_{x/x=1} = -6$

$E_y = -\frac{\partial V}{\partial y} = 0$

$E_z = -\frac{\partial V}{\partial z} = 0$

So, $|\vec{E}| = \sqrt{-6^2 + 0^2 + 0^2} = 6$

In vector form, $\vec{E} = -6\hat{i}\,V/m$

114. (d)

115. (c)

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. (a)

2. (b) $F = \frac{\sigma^2 ds}{2\epsilon_0 k} = \frac{q^2}{2\epsilon_0 kds}$

$= \frac{\left(\sqrt{8.85\times10^{-6}}\right)^2}{2\times8.85\times10^{-12}\times1\times1} = 0.5N$

3. (a)

4. (b) $E = \frac{F}{q} = \frac{2.25}{15\times10^{-4}} = 1500 N/C$

5. (b)

6. (d) $E = \frac{\sigma}{\epsilon_0 k} = \frac{120\times10^{-6}}{8.85\times10^{-12}\times4}$ = 3.389 × 10^6 N/C

7. (a) $f = \frac{\sigma^2}{2\epsilon_0 k} = \frac{q^2}{2\epsilon_0 kds^2}$

$= \frac{q^2}{32\epsilon_0 k\pi^2 R^4}$ $\left(\because ds = 4\pi R^2\right)$

$= \frac{\left(12\times10^{-6}\right)^2}{32\times8.85\times10^{-12}\times1\times9.87\times\left(10^{-1}\right)^4}$

= 5.151 × 10^2 N/M^2

8. (d)

9. (d) By Gauss's law $\phi = \frac{1}{\varepsilon_0}(Q_{enclosed})$

$\Rightarrow Q_{enclosed} = \phi\varepsilon_0 = \left(-8\times10^3 + 4\times10^3\right)\varepsilon_0$

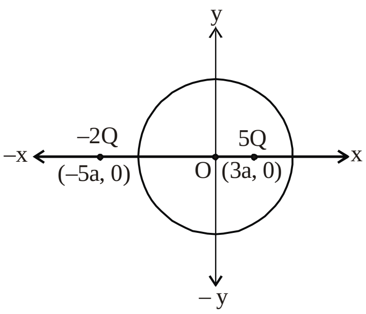
$= -4\times10^3\varepsilon_0$ coulomb

10. (c)

11. (a) The net force acting on charge q should be zero.

$F = \frac{1}{4\pi\epsilon_0}\left[\frac{4q\times q}{1^2} + \frac{Qq}{(1/2)^2}\right] = 0$

12. (b)



Using Gauss law,

Electric flux, $\phi = \frac{q_{enclosed}}{\varepsilon_0} = \frac{5Q}{\varepsilon_0}$

13. (a) Given,

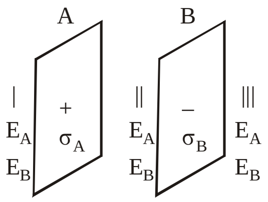
Area of surface, A = 4m^2

Electric flux, $\phi = \vec{E}\cdot\vec{A} = \vec{E}.\hat{n}(A)$

$= \left(\frac{2\hat{i}+6\hat{j}+8\hat{k}}{\sqrt{6}}\right)\cdot 4\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$

$= 4 \times \frac{(4+6+8)}{6} = 12$ Vm

14. (d)

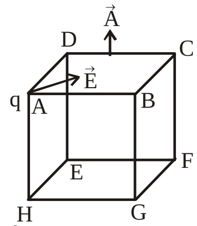


$\boldsymbol{E_1} = \boldsymbol{E_A} + \boldsymbol{E_B} = \frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$

15. (b) The x-component of electric field is constant and does not contribute to net flux through the closed surface. Due to y-component,
Incoming flux = –6 units ($\because$ y = 0)
Outgoing flux = 9 units ($\because$ y = 1)

By Gauss' law, $\phi_{net} = \frac{q}{\varepsilon_0} \Rightarrow (9-6) = \frac{q}{\varepsilon_0}$

16. (c) Since the charge is placed at the corner *A*, the electric field direction will be parallel to the planes *ABCD*, *ABGH* and *ADEH*.



$\therefore$ Flux through face *ABCD*,

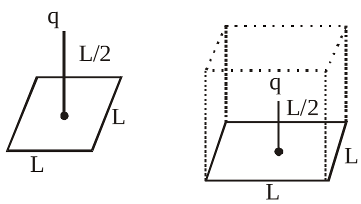
$\phi = \vec{E}\cdot d\vec{A}$

[As area vector is perpendicular to the plane]
$= EdA \cos 90° = 0$

17. (c)

18. (c) $\phi = E(ds)\cos\theta = E(2\pi r^2)\cos 0° = 2\pi r^2 E$.

19. (c)



The given square of side L may be considered as one of the faces of a cube with edge L. Then given charge q will be considered to be placed at the centre of the cube. Then according to Gauss's theorem, the magnitude of the electric flux through the faces (six) of the cube is given by

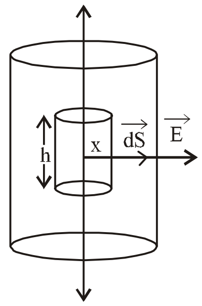
$\phi = q/\varepsilon_0$

Hence, electric flux through one face of the cube for the given square will be

$\phi' = \frac{1}{6}\phi = \frac{q}{6\varepsilon_0}$

20. (a) $\phi = EA\cos 0° = E \times \frac{\pi d^2}{4}$, $\therefore E = \frac{4\phi}{\pi d^2}$.

21. (a) By Gauss law



$\int E ds \cos 0 = \frac{q}{\varepsilon_0}$

$\Rightarrow E.2\pi xh = \frac{\rho\times\pi x^2 h}{\varepsilon_0}$

$\Rightarrow E = \frac{\rho\pi}{2\varepsilon_0}$

$\Rightarrow E = \frac{\rho}{2\varepsilon_0}\times\frac{2\varepsilon_0}{\rho} = 1$

22. (c) Flux, $\phi = \vec{E}\cdot\vec{A} = \left[\frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}\right]\cdot\vec{A}$

$= \frac{E_0}{5}(2\hat{i}+3\hat{j})\cdot(0.4\hat{i}) = \frac{4000}{5}(2\times 0.4) = 640$ Nm^2C^{-1}

23. (c) Here, $\ell = 2.4$ *m*, $r = 4.6$ *mm* $= 4.6 \times 10^{-3}$ *m*
$q = -4.2 \times 10^{-7}$ C

Linear charge density, $\lambda = \frac{q}{\ell}$

$= \frac{-4.2\times 10^{-7}}{2.4} = -1.75 \times 10^{-7}$ C m^{-1}

Electric field, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

$= \frac{-1.75\times 10^{-7}}{2\times 3.14\times 8.854\times 10^{-12}\times 4.6\times 10^{-3}} = -6.7\times 10^5$ N C^{-1}

24. (a) Here, $q = 1$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2N^{-1}m^{-2}$
Number of lines of force = Electric force

$= \frac{q}{\varepsilon_0} = \frac{1}{8.85\times 10^{-12}} = 1.13 \times 10^{11}$

25. (a) $\vec{E} = E_0\hat{i} + 2E_0\hat{j}$

Given, E_0 = 100N/c So, $\vec{E} = 100\hat{i} + 200\hat{j}$

Radius of circular surface = 0.02 m

Area = $\pi r^2 = \frac{22}{7}\times 0.02\times 0.02$

$= 1.25\times 10^{-3}\hat{i}\ m^2$ [Loop is parallel to Y-Z plane]

Now, flux (ϕ) = EA cosθ

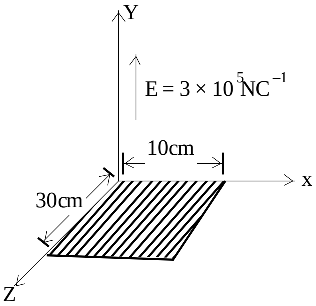
$= (100\hat{i} + 200\hat{j}).1.25\times 10^{-3}\hat{i}\cos\theta°$ [$\theta = 0°$]

$= 125 \times 10^{-3}$ Nm^2/C = 0.125 Nm^2/C

26. (b) $E = \frac{q}{4\pi\varepsilon_0 r^2} \Rightarrow Ar = \frac{q}{4\pi\epsilon_0 r^2}$

$\Rightarrow q = 4\pi\varepsilon_0 Ar^3$

27. (b)



$\vec{E} = (3\times10^5)\hat{j}$

$\vec{A} = (10\times30\times10^{-4})\hat{j}$ ∴ Electric flux,

$\phi = \vec{E}.\vec{A} = (3\times10^5)\hat{j}.(10\times30\times10^{-4})\hat{j} = 9\times10^3$ Vm

28. (d) By Gauss's law $\phi = \frac{1}{\varepsilon_0}(Q_{enclosed})$

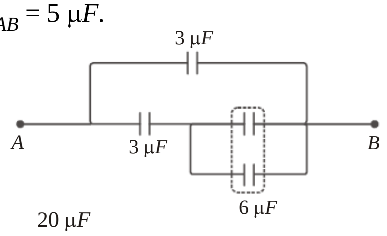
$\Rightarrow Q_{enclosed} = \phi\varepsilon_0 = (-8\times10^3 + 4\times10^3)\varepsilon_0$

$= -4\times10^3\,\varepsilon_0$ coulomb.

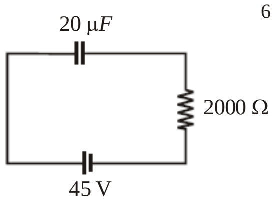
29. (d) As source is disconnected hence Q = $q_1 + q_2$, after disconnecting from the source they are connected in parallel, hence net potential difference ≠ $V_1 + V_2$, when charged capacitor at different potentials are connected together their always occurs a loss of energy in the form of heat.

30. (a) **31. (b)**

32. (d) $C_{AB} = 5\ \mu F$.



33. (a)



Final charge on the capacitor,

Q = CV = 20 × 10^{-6} × 45 = 9 × 10^{-4}C.

34. (d) When oil is placed between space of plates

$C = \frac{2A\varepsilon_0}{d}$ ...(1) $\left[\because C = \frac{KA\varepsilon_0}{d}, \text{if } K = 2\right]$

When oil is removed $C' = \frac{A\varepsilon_0}{d}$ ...(2)

On comparing both equations, we get C' = C/2

35. (c) Capacitance will increase but not 5 times (because dielectric is not filled completely). Hence, new capacitance may be 200 μF.

36. (a) Equivalent capacitance of two parallel capacitors 10 μF and 6 μF = (10 + 6)μF = 16 μF

This 16 μF capacitor is in series combination with 4μF capacitor,

∴ Equivalent capacitance of the entire combination

$= \frac{16\times4}{16+4} = \frac{64}{20} = 3.2\mu F$

37. (d) As we know,

Common potential = $\frac{\text{Total charge}}{\text{Total capacity}}$

$Q_1 = C_0V_1$, $Q_2 = 0$, therefore

$V_2 = \frac{C_0V_1 + 0}{C_0 + kC_0} = \frac{V_1}{1+k}$

$1 + k = \frac{V_1}{V_2}$ or $k = \frac{V_1}{V_2} - 1 = \frac{V_1 - V_2}{V_2}$

38. (b) In series combinatiion of capacitors

V_{eff} = V + V + V = 3V

$\frac{1}{C_{eff}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \Rightarrow C_{eff} = \frac{C}{3}$

Thus, the capacitance and breakdown voltage of the combinatioin will be $\frac{C}{3}$ and 3V.

39. (b) Using the formula, $C = \frac{\varepsilon_0 A}{d}$ Capacitance of the 1st part (K_1 = 8)

$C_1 = \frac{8\varepsilon_0 A/2}{d} = 4\frac{\varepsilon_0 A}{d}$

= 4 × C $\left[\because \text{Original capacitance } = C = \frac{\varepsilon_0 A}{d} = 1\right]$

= 4 μF

Capacitance of the 2nd part

$C_2 = \frac{4\epsilon_0(A/2)}{d} = 2\frac{\epsilon_0 A}{d} = 2C = 2\mu F$

They are in parallel .

∴ Total capacitance = 4 + 2 = 6μF

So, capacitance will increase 6 times

40. (b) By using

$V = V_0 e^{-t/CR} \Rightarrow 40 = 50e^{-1/CR} \Rightarrow e^{-1/CR} = 4/5$

Potential difference after 2 sec

$V' = V_0 e^{-2/CR} = 50(e^{-1/CR})^2 = 50\left(\frac{4}{5}\right)^2 = 32V$

Fraction of energy after 1 sec $= \frac{\frac{1}{2}C(V_f)^2}{\frac{1}{2}C(V_i)^2} = \left(\frac{40}{50}\right)^2 = \frac{16}{25}$

41. (c) $C = \frac{2\times2}{2+2} + 2 = 3\mu F$

42. **(b)** From figure, equivalent capacity of 3μF, 6 μF and 3 μF condensers is C_p =12 μF.
Now, C_p and condenser of 2 μF are in series. Thus, p.d. across 2 μF condenser is,

$$V = \left(\frac{VC_p}{C_1 + C_p}\right) = \frac{70 \times 12 \times 10^{-6}}{14 \times 10^{-6}} = 60V$$

43. **(a)** The charge on condenser $C_1 = Q_1 = C_1V$
If it is connected across uncharged condenser of capacity C_2, then p.d. across C_2 is given by,

$$\frac{Q_1}{C_2} = V\frac{C_1}{C_1 + C_2}$$

44. **(d)**

45. **(b)** $Q = C_s v = \left(\frac{C_1C_2}{C_1 + C_2}\right)V = \left(\frac{5 \times 10}{5 + 10}\right) \times 10^{-6} \times 200$

$= 6.667 \times 10^{-4}C$

46. **(a)** $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$= \frac{CK \times 0 + CV_0}{CK + C} = \frac{CV_0}{C(K+1)} = \frac{V_0}{K+1}$

$K = \frac{V_0 - V}{V}$

47. **(d)** **48.** **(c)**

49. **(d)** $C = \frac{Ak\varepsilon_0}{d} = \frac{(5 \times 10^{-4}) \times 5 \times 8.85 \times 10^{-12}}{2 \times 10^{-3}}$

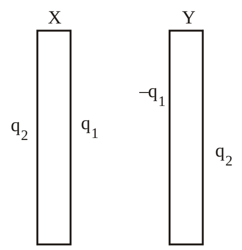
$= 1.10 \times 10^{-11} = 11 \times 10–12$ F = 11pF

50. **(b)** In parallel, potential is same, say V

$$\frac{Q_1}{Q_2} = \frac{C_1V}{C_2V} = \frac{C_1}{C_2}$$

51. **(b)** $q_1 + q_2 = 3q$
$-q_1 + q_2 = q \Rightarrow q_1 = q$

Capacitance of the system $= \frac{\varepsilon_0 A}{d}$



Potential difference between the plates $= \frac{qd}{\varepsilon_0 A}$

52. **(c)** C = 10 μF ; d = 8 cm
C' = ? ; d' = 4 cm

$C = \frac{A \epsilon_0}{d} \Rightarrow C \alpha \frac{1}{d}$

If d is halved then C will be doubled.
Hence C' = 2C = 2 × 10 μF = 20 μF

53. **(c)** Initially, $C_0 = \frac{\varepsilon_0 A}{d} = 18$ μF ...(i)
When $d' = 3d$ and a dielectric medium is introduced,

$C = \frac{K\varepsilon_0 A}{3d} = 72$ μF
...(ii)
On dividing Eq. (ii) by Eq.(i), we get

$$\frac{K}{3} = \frac{72}{18} = 4$$

∴ Dielectric constant, $K = 12$

54. **(a)** Initial capacitance, $C_1 = \frac{\varepsilon_0 A}{d}$

Final capacitance, $C_2 = \frac{K\varepsilon_0 A}{\frac{d}{2}} = \frac{2 \times 10\varepsilon_0 A}{d} = \frac{20\varepsilon_0 A}{d}$

$\therefore \frac{C_2}{C_1} = \frac{20\varepsilon_0 A}{d} \times \frac{d}{\varepsilon_0 A} = 20$

55. **(c)** When the battery is removed,
Q = constant
$\Rightarrow C_1V = C_2V \Rightarrow C_1 = C_2$

$$\Rightarrow \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{(d + 1.6) - t\left(1 - \frac{1}{k}\right)}$$

$\therefore d = (d + 1.6) - 2\left(1 - \frac{1}{k}\right)$ ∴ K = 5

56. **(d)**

57. **(d)** Capacity of parallel plate capacitor

$C = \frac{k\varepsilon_0 A}{d}$ (For air $k_r = 1$)

So, $\frac{\varepsilon_0 A}{d} = 8 \times 10^{-12}$

If $d \rightarrow \frac{d}{2}$ and $k_r \rightarrow 6$ then new capacitance

$C' = 6 \times \frac{\varepsilon_0 A}{d/2} = 12\frac{\varepsilon_0 A}{d} = 12 \times 8$ pF = 96 pF

58. **(c)** $q_1 = C_1V = 10 \times 12 = 120\mu C$

$q_2 = C_2V = KC_1 \times V = 5 \times 10 \times 12 = 600\mu C$
Additional charge that flows
$= q_2 - q_1 = 600 - 120 = 480\mu C.$

59. **(d)** When a dielectric slab is introduced between its plates, its potential difference will become

$$V = \frac{V_0}{K}$$

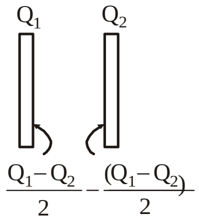
where K is the dielectric constant of the slab.
Here V_0 be the potential difference between the plates of an air filled parallel plate capacitor.

$\therefore K = \frac{V_0}{V}$

Here, $V_0 = 4$ V, V = 2V $\quad \therefore K = \frac{4V}{2V} = 2$

60. **(b)** $V = \frac{Q}{C}$

By Gauss law



Charge on inner plates = $\left|\frac{Q_1 - Q_2}{2}\right|$

So, $V = \left(\frac{Q_1 - Q_2}{2C}\right) = \left(\frac{4-2}{2\times1}\right) = 1$ V

61. **(c)** If dielectric is removed, capacitance decreases but since capacitor is still connected to the cell. Potential difference will remain same.

$\therefore \quad C_1 > C_2$ and $V_1 = V_2$

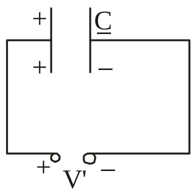
62. **(b)** Potential of both spheres will be same.

$\therefore \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ or $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$

63. **(d)** When there is no battery, charge remains same while potential difference and electric field decreases.

i.e., $Q' = Q_0 V' = \frac{V_0 \times 3}{9} = \frac{V_0}{3}$ and $E' = \frac{E_0 \times 3}{9} = \frac{E_0}{3}$

64. **(c)**



Battery is disconnected.

Q = Charge remains context

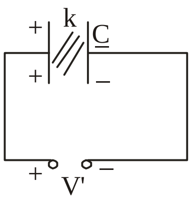
C' = K C

Q' = C' V'

Q = C' V'

Q = K C V'

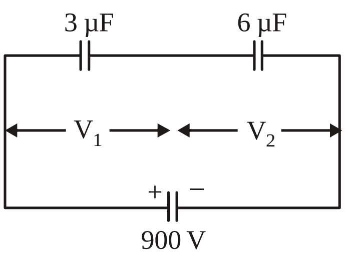
$V' = \frac{Q}{KC} = \frac{V}{K}$



65. **(d)** Let V_1 and V_2 be the voltages across 3 μF and 6 μF capacitors respectively. So,

$V_1 + V_2 = 900$ V ...(i)

As 3 μF and 6 μF capacitors are series, charges on each is the same.



$\therefore C_1 V_1 = C_1 V_2$ or $\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{6\,\mu F}{3\,\mu F} = 2$

or $V_1 = 2V_2$

Substituting this value of V_1 in eqn. (i), we get

$2V_2 + V_2 = 900$ V or $V_2 = 300$ V

From eqn. (i)

$V_1 = 900\text{ V} - V_2 = 900\text{ V} - 300\text{ V} = 600$ V

Now they are reconnected in parallel.

$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{(3\mu F)(600\,V) + (6\mu F)(300\,V)}{3\mu F + 6\mu F}$

= 400 V

66. **(d)** Start with $C_3$and C_4 in parallel, then C_2 in series, then C_5 in parallel, then C_1 in series and finally C_6 in parallel.

Let $C_1 = C_2 = C_3 = 2x$ and $C_4 = C_5 = C_6 = x$ where x = 200 pF

Then, $C_{eq} = \frac{43}{21}x$

so, $C_{eq} = \frac{43}{21} \times 200\text{ pF} = 409.5\text{ pF}$

67. **(c)** C_{eq} between P and R $= \frac{C}{3} + \frac{C}{2} = \frac{5C}{6}$

C_{eq} between P and Q $= C + \frac{C}{4} = \frac{5C}{4}$

ratio = $\frac{4}{6} = \frac{2}{3}$

68. **(d)** $C_{eq} = C + 2C + 3C + ... + nC = \frac{(n+1)n}{2}C$

69. **(c)** $q_1 = C_1V = 10\times12 = 120\,\mu C$

$q_2 = C_2V = KC_1 \times V = 5\times10\times12 = 600\,\mu C$

Additional charge that flows

$= q_2 - q_1 = 600 - 120 = 480\,\mu C.$

70. **(c)** The charge on the capacitor is $Q = CV$

$= 900 \times 10^{-12}\,F \times 100\text{ V} = 9 \times 10^{-8}$ C

The energy stored by the capacitor is

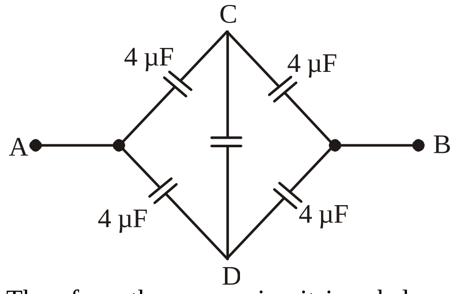
$= \left(\frac{1}{2}\right)CV^2 = \frac{1}{2}QV = \frac{1}{2} \times 9 \times 10^{-8}\,C \times 100\,V$

$= 4.5 \times 10^{-6}$ J

In the steady situation, the two capacitors have their positive plates at the same potential and their negative plates at the same potential. Let the common potential difference b v.' The charge on each capacitor is Q = CV.' By charge conservation law, Q'= Q, this implies V' = V/2. The total energy of the system is

$\frac{1}{2}Q'V' = \frac{1}{4}Q.V = 2.25\times10^{-6}J$

71. (a) The given circuit can be redrawn as



Therefore, the green circuit is a balanced Wheatstone bridge and the capacitance in arm *CD* is ineffective.

As $\frac{4\mu F}{4\mu F} = \frac{4\mu F}{4\mu F}$

So their equivalent capacitance C_1 is

$$C_1 = \frac{(4\mu F)(4\mu F)}{4\mu F + 4\mu F} = 2\,\mu F$$

So their equivalent capacitance C_2 is

$$C_2 = \frac{(4\mu F)(4\mu F)}{4\mu F + 4\mu F} = 2\,\mu F$$

The effective capacitance between the points *A* and *B* is

$C_{AB} = C_1 + C_2 = 2\ \mu F + 2\ \mu F = 4\ \mu F$

72. (a) In Ist case when capacitor C attached with battery charged with the energy.

$U_1 = U$ (stored energy on capacitor).

In IInd case after disconnect of battery similar capacitor is attached in parallel with Ist capacitor then

$C_{eq.} = C' = 2C$.

Now, $\frac{U_1}{U_2} = \frac{\frac{1}{2}\frac{q^2}{C}}{\frac{1}{2}\frac{q^2}{C'}} = \frac{C'}{C} = \frac{2C}{C}$ $\quad (\because C' = 2C)$

$U_2 = \frac{U}{2}$

73. (a) **74. (a)**

75. (b) $E = \frac{1}{2}C_s V^2 = \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)V^2$

$= \frac{32\times10^{-6}\times10^4}{2\times121} = \frac{4}{3}\times10^{-2}\,J$

76. (b) $V_1 = \frac{Q}{C_1} = \frac{VC_2}{C_1+C_2} = \frac{6\times6}{3+6} = 4V$

$E_1 = \frac{1}{2}C_1V_1^2 = \frac{3\times10^{-6}\times16}{2} = 24\times10^{-6}\,J$

$V_2 = \frac{Q}{C_2} = \frac{VC_1}{C_1+C_2} = \frac{6\times3}{3+6} = 2V$

$E_2 = \frac{1}{2}C_2V_1^2 = \frac{6\times10^{-6}\times4}{2} = 12\times10^{-6}\,J$

77. (b) The loss of energy is given by

$$\Delta E = \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1 - V_2)^2 = 0.0375J$$

78. (b) From figure, the equivalent capacity of the combination is,

$\frac{1}{C_s} = \frac{1}{4} + \frac{1}{(4+4)} + \frac{1}{4} = \frac{5}{8}$

$\therefore\ C_s = \frac{8}{5}\mu F$

Thus, $E = \frac{1}{2}C_sV^2 = \frac{1}{2}\times\frac{8}{5}\times10^{-6}\times225$

$= 1.8 \times 10^{-7}$ J $= 1.8$ erg

79. (d) $U = \frac{1}{2}CV^2$

$U = \frac{1}{2}\left(\frac{A\,\epsilon_0}{d}\right)(Ed)^2 = \frac{1}{2}A\,\epsilon_0\,E^2d$

80. (c) Energy of given to conductor, $U = \frac{1}{2}CV^2$

or $U = \frac{1}{2}\times5\times10^{-6}\times(800)^2 = 1.6$ joule

81. (d)

82. (a) $C_1 = 1\mu F$, $C_2 = 2\mu F$, $V_1 = 6kV$, $V_2 = 4kV$

$\therefore\ Q_1 = C_1V_1 = 1\times10^{-6}\times6\times10^3 = 6\times10^{-3}\,C$

$Q_2 = C_2V_2 = 2\times10^{-6}\times4\times10^3 = 8\times10^{-3}C$

As, $Q_1 < Q_2$

Maximum charge that the capacitors withstand in series combination is Q,

$\therefore\ Q_{max} = Q_1 = 6\times10^{-3}C$

For series Combination, $C = \frac{C_1C_2}{C_1+C_2} = \frac{1\times2}{1+2} = \frac{2}{3}\mu F$

$\therefore\ V_{max} = \frac{Q_{max}}{C} = \frac{6\times10^{-3}}{\frac{2}{3}\times10^{-6}} = 9kV$

83. (d) $Q_1 = CV$ and $Q_2 = CV$

Applying charge conservation, $CV_1 + CV_2 = Q_1 + Q_2$

$CV_1 + CV_2 = 2CV \Rightarrow V_1 + V_2 = 2V$

84. (d) Equivalent capacitance, $\frac{1}{C_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3}$

$\Rightarrow\ C_{eq} = \frac{6}{11}\mu F$

Charge supplied from battery, $Q = \frac{6}{11}\times11 = 6\mu C$

Hence potential difference across 1 μF capacitor,

$= \frac{6}{1} = 6V$

85. (a) $V_c = \frac{Q_{net}}{C_{net}} = \frac{C_1V_1 + C_2V_2}{C_1+C_2} = \frac{CV + 2CV}{2C} = \frac{3V}{2}$

Decrease in energy $\Delta E = E_2 - E_1$

$\Delta E = \frac{1}{2}CV^2 + \frac{1}{2}C(2V)^2 - \frac{1}{2}2C\left(\frac{3V}{2}\right)^2 = \frac{CV^2}{4}$

86. (a) For parallel combination, $E = \frac{1}{2}(C_1 + C_2 + C_3)V^2$

$\therefore\ E = \frac{1}{2}(25+30+45)(100)^2$ ...(i)

For series combination, $\frac{9}{x}E = \frac{1}{2}C_{eq}V^2$

$$\therefore \quad \frac{9}{x}E = \frac{1}{2}\frac{1}{\left(\frac{1}{25}+\frac{1}{30}+\frac{1}{45}\right)}(100)^2 \qquad ...(ii)$$

From eqn(i) and (ii), we get $x = 89$

87. **(b)** The statement given in option (b) is correct but rest are incorrect and these can be corrected as, When battery is disconnected, charge remains conserved.
The plates are pulled apart, so capacity will decrease, so there is an increase of voltage.
Now, capacity decreases, hence energy will increase as charge is constant $\left(E = \frac{Q^2}{2C}\right)$.

88. **(a)** Let $E = \frac{1}{2}C_0V_0^2$, then $E_1 = 2E$ and $E_2 = \frac{E}{2}$

so $\frac{E_1}{E_2} = \frac{4}{1}$

89. **(b)** Field inside a hollow conducting sphere is zero but potential inside is constant, being equal to potential on the surface of the conductor.

90. **(c)** **91.** **(b)** **92.** **(a)**

93. **(b)** Potential inside the hollow sphere is same as that on the surface.

94. **(c)** Electric potential inside a conductor is constant and it is equal to that on the surface of conductor.

95. **(b)** The given point is inside the larger sphere. So, potential at this point is the same as on the surface of the sphere. The value is $\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$.
The given point is outside the smaller sphere. So, the charge on the smaller sphere would behave as if concentrated at the centre. The potential due to smaller sphere is $\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r}$.
Applying principle of superposition of potentials, the total potential is $\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1}+\frac{q_2}{r}\right]$.

96. **(a)** Electric field at a point is equal to the negative gradient of the electrostatic potential at that point.
Potential gradient relates with electric field according to the following relation:

$$E = \frac{-dV}{dr}$$

$$\vec{E} = -\frac{\partial V}{\partial r} = \left[-\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial x}\hat{k}\right]$$

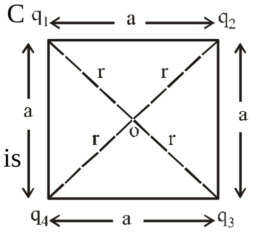
$$= \hat{i}\left(2xy + z^3\right) + \hat{j}\,x^2 + \hat{k}\,3xz^2$$

97. **(c)** Using, $V_f - V_i = -\int_i^f E.ds$

$\Rightarrow \quad V_f - V_i = -Ed \Rightarrow V_C - V_A = -5E$

$\Rightarrow \quad V_A - V_C = 5E$

98. **(c)** $q_1 = +1 \times 10^{-8}C$, $q_2 = -2 \times 10^{-8}$ C
$q_3 = +3 \times 10^{-8}C$, $q_4 = +2 \times 10^{-8}$ C
$a = 1m$



$$\therefore \quad r = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}} = \frac{1}{\sqrt{2}}m$$

The electric potential at centre 0 is

$$V_0 = \frac{k}{r}(q_1 + q_2 + q_3 + q_4)$$

$$\frac{9\times10^9}{\frac{1}{\sqrt{2}}}(1 - 2 + 3 + 2) \times 10^{-8} = 509 \text{ V} \Rightarrow 510\text{V}$$

99. **(c)** $\frac{kq}{r} = 60mV$

$$V_i = V_f \Rightarrow \left(\frac{4}{3}\pi r^3\right)\times125 = \frac{4}{3}\pi R^3 \Rightarrow R = 5r$$

Also, $Q = 125q$

$\therefore$ Potential on big drop, $V = \frac{KQ}{R} = \frac{k(125q)}{5r}$

$$= 25\left(\frac{kg}{r}\right) = 25 \times 60 \times 10^{-3} = 1.5V$$

100. **(b)**

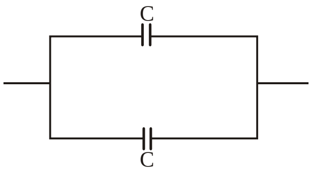
Exercise 3 : PREVIOUS YEARS MCQs

1. **(a)**

2. **(a)**

3. **(a)**
$$f = \frac{\sigma^2}{2\in_0 k} = \frac{q^2}{2\in_0 kds^2}$$
$$= \frac{q^2}{32\in_0 k\pi^2R^4} \quad \left(\therefore ds = 4\pi R^2\right)$$
$$= \frac{\left(12\times10^{-6}\right)^2}{32\times8.85\times10^{-12}\times1\times9.87\times\left(10^{-1}\right)^4}$$
$= 5.151 \times 10^2$ N/M^2

4. **(c)** The circuit can be reduced to



The equivalent capacitance of the combination is

$C_{eq} = C + C = 2C$

5. **(c)** Electric field between plates given by,

$$E = \frac{q_1 - q_2}{2A\in_0}$$

(Here, $q_1 > q_2$)
The, the potential difference will be

$$V = Ed = \frac{q_1 - q_2}{2A\in_0}d = \frac{q_1 - q_2}{2C} \quad \left(\because C = \frac{\in_0 A}{d}\right)$$

6. **(c)** Electric field due to infinite sheet is given by $E = \frac{\sigma}{2\epsilon_0}$,

clearly $|\vec{E}|$ is independent of distance

So, $E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$

7. **(d)** All other statements except (iv) are in correct.
The electric field over the Gaussian surface remains continuous and uniform at every point.

8. **(c)** Here, $\ell = 2.4\ m$, $r = 4.6\ mm = 4.6 \times 10^{-3}\ m$
$q = -4.2 \times 10^{-7}$ C

Linear charge density, $\lambda = \frac{q}{\ell}$

$= \frac{-4.2 \times 10^{-7}}{2.4} = -1.75 \times 10^{-7}\ \text{C m}^{-1}$

Electric field, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

$= \frac{-1.75 \times 10^{-7}}{2 \times 3.14 \times 8.854 \times 10^{-12} \times 4.6 \times 10^{-3}} = -6.7 \times 10^5\ \text{N C}^{-1}$

9. **(d)**

10. **(b)** Using Gauss's law, $\phi = \frac{q}{\epsilon_0}$

Here, q = charge inside the closed surface

$\therefore \phi = \frac{q + (-2q) + 5q}{\epsilon_0} \Rightarrow \phi = \frac{4q}{\epsilon_0}$

11. **(c)** Using Gauss law

$\int \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$EdA = \frac{\sigma \times dA}{\varepsilon_0}$ $\quad [\because Q = \sigma dA]$

$\Rightarrow E = \frac{\sigma}{\varepsilon_0}$

12. **(a)** $U = \frac{q^2}{2c}$. As C = constant, $U \propto q^2$

So, $\frac{U_2}{U_1} = \left(\frac{q_2}{q_1}\right)^2 \Rightarrow q_1^2 = \frac{U_1}{U_2} q_2^2$

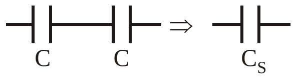
$\Rightarrow q_1^2 = \frac{U}{1.44U} \times (q_1 + 2)^2 . \left(\frac{q_1}{q+2}\right)^2 = \frac{1}{1.44}$

$\Rightarrow \frac{q_1}{q_1 + 2} = \frac{1}{1.2} \Rightarrow 1.2q_1 = q_1 + 2 \Rightarrow 0.2\ q_1 = 2$

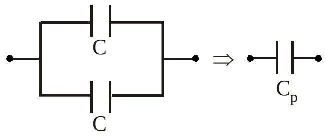
$\Rightarrow q_1 = 10$ C

13. **(c)** For series combination,

$\frac{1}{C_S} = \frac{1}{C} + \frac{1}{C} \Rightarrow \boxed{C_S = \frac{C}{2}}$



For parallel combination,



$C_p = C + C \Rightarrow C_p = 2C \Rightarrow \frac{C_S}{C_p} = \frac{\left(\frac{C}{2}\right)}{2C} = \frac{1}{4} = 1:4$

14. **(d)** When two capacitors with capacitance C_1 and C_2 at potential V_1 and V_2 connected to each other by wire, charge begins to flow from higher to lower potential till they acquire common potential. Here, some loss of energy takes place which is given by.

Heat loss, $H = \frac{C_1 C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

In the equation, put $V_2 = 0$, $V_1 = V_0$

$C_1 = C,\ C_2 = \frac{C}{2}$

Loss of heat $= \frac{C \times \frac{C}{2}}{2\left(C + \frac{C}{2}\right)}(V_0 - 0)^2 = \frac{C}{6}V_0^2$

$\Delta H = \frac{1}{6}CV_0^2$

15. **(b)** In series combination of capacitors, the charge on each capacitor is same.

$\therefore\ Q = C_1V_1 = C_2V_2$

$\therefore\ \frac{V_2}{V_1} = \frac{C_1}{C_2} = \frac{4}{1}$

16. **(b)** When two similar capacitors are joined in series, then effective capacity is

$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$

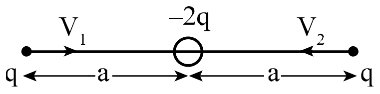
$\therefore\ C_s = \frac{C}{2}$

When joined in parallel, $C_p = C + C = 2C$
Since, $C_p - C_s = 6\mu F$

$\therefore\ 2C - \frac{C}{2} = 6$

$\Rightarrow 4C - C = 12$
or $C = 4\mu F$

17. **(d)** Potential energy of the system,



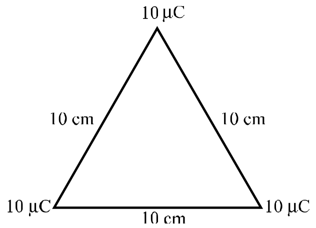
$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q(-2q)}{a} + \frac{q(-2q)}{a} + \frac{qq}{2a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{2q^2}{a} - \frac{2q^2}{a} + \frac{q^2}{2a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-4q^2}{a} + \frac{q^2}{2a}\right] = \frac{-7q^2}{8\pi\varepsilon_0 a}$$

18. **(c)**



Electrostatic PE of the system will be

$$U = 3 \times \frac{K \times 10 \times 10 \times 10^{-12}}{10 \times 10^{-2}}$$

$$= \frac{3 \times 9 \times 10^9 \times 10^2 \times 10^{-12} \times 10^1}{10^2 \times 10^{-2}}$$

$U = 3 \times 9 \times 10^9 \times 10^{-9} = 3 \times 9 = 27$ J

19. **(c)** We know that,

$$E = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{q}{r^2} \quad \text{...(i)}$$

and $\sigma = \dfrac{q}{4\pi R^2}$

$\therefore\ q = \sigma \times (4\pi R^2)$ ...(ii)

From Eqs. (i) and (ii), we get

$$E = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{\sigma(4\pi R^2)}{r^2} = \frac{\sigma R^2}{\varepsilon_0 K r^2}$$

20. **(c)** Given, $q_1 = 0.1$ C, $q_2 = 0.5$ C, $C = 48 \times 10^{-6}$ F

Increase in energy stored

$$= \frac{1}{2C}\left[q_2^2 - q_1^2\right]$$

$$= \frac{1}{2 \times 48 \times 10^{-6}}\left[(0.5)^2 - (0.1)^2\right]$$

= 2500 J

21. **(d)** Since, q is the charge enclosed by the surface, then the electric flux, $\phi = \dfrac{q}{\varepsilon_0}$.

If charge q is placed at a corner of cube, it will be divided into 8 such cubes. Therefore, electric flux through the cube is

$$\phi' = \frac{1}{8}\left(\frac{q}{\varepsilon_0}\right)$$

22. **(a)**