

Class-XI

MHT-CET

Study Material

PHYSICS

Booklet-2

(Solutions)

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Hints & Solutions

5 Sound

Exercise 1 : WARM-UP
Topic-wise MCQs

1. (c) **2.** (b) **3.** (c) **4.** (c)

5. **(b)** Sound wave propagate so fast in a gas that there is no time for the exchange of energy with the medium (gas). Hence, it is quite close to an adiabatic process.

6. **(c)** The restoring force that produces gravity waves is the pull of gravity, which tends to keep the water surface at its lowest level.

Hence statement given in option (c) is incorrect.

7. **(a)** Equation of motion is given by:

$$y = a\sin(kx - \omega t) = \sin\left(\frac{2\pi}{2\pi} \times x - 2 \times \frac{1}{\pi}x\right)$$

$$\left(\text{We know that } k = \frac{2\pi}{\lambda} \text{ and } \omega = 2\pi v\right)$$

$$= \sin(x - 2t)$$

8. **(d)** Mass per unit length of the wire,

$$\mu = \frac{M}{L} = \frac{10^{-2}}{20} = 5 \times 10^{-4}\,\text{kg/m}$$

Speed of the transverse wave on the wire is given by:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{125}{5 \times 10^{-4}}} = 500\,\text{m/s}$$

9. **(b)** Since, the frequency of the wave remains same in the two media

$$v = \frac{v_A}{\lambda_A} = \frac{v_B}{\lambda_B} \Rightarrow v_B = \frac{\lambda_B}{\lambda_A} \times v_A = \frac{0.25}{0.10} \times 0.80 = 2\ \text{ms}^{-1}$$

10. **(a)** **11.** **(b)**

12. **(b)** Equation of wave is, $y = 2\cos 2\pi(330t - x)$ m

$y = A\cos(\omega t - kx)$

On comparing $\omega = 2\pi \times 330$ $[\because \omega = 2\pi F]$

$2\pi f = 2\pi \times 330 \Rightarrow f = 330$ Hz

13. **(a)** At STP

Temperature, $T = 273$K

Molecular mass of oxygen, $M = 32 \times 10^{-3}$kg

Speed of sound is given by

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.3 \times 273}{32 \times 10^{-3}}} = 314.8541 \simeq 315\,\text{m/s}$$

14. **(d)** The displacement of the points *B* and *F* are equal in magnitude and sign. So these points are in same phase.

15. **(c)**

16. **(b)** Time period, $T = \frac{2\pi}{\omega}$; from given eqn.

$$\omega = 3.0\ \text{s}^{-1} \quad \text{or} \quad T = \frac{2\pi}{3} = 2.09\ \text{s}$$

17. **(a)** $y = 2\sin(\omega t - kx)$

Maximum particle velocity, $v_m = A\omega$

Wave velocity, $v_p = \frac{\omega}{k}$

$v_p = v_m$

$$\frac{\omega}{k} = A\omega$$

$$k = \frac{1}{A} = \frac{2\pi}{\lambda}$$

$\lambda = 2\pi A = 2\pi \times 2 = 4\pi$

18. **(b)** Travelling wave $Y(x, t) = A\sin(Kx - \omega t)$

Here, $A = \frac{0.06}{2} = 0.03$, $\omega = 2\pi f = 2\pi \times 245 = 1.5 \times 10^3$

$$K = \frac{\omega}{v} = \frac{1.5 \times 10^3}{300} \simeq 5.1$$

$\therefore Y(x, t) = 0.03\sin[5.1x - (1.5 \times 10^3)t]$

19. **(a)** At $t = 0$; $y = \frac{1}{1 + x^2}$

We know that displacement of wave is a function of $(x - vt)$.

$$\therefore \quad y = \frac{1}{1 + (x - vt)^2}$$

at $t = t$; $y = \frac{1}{1 + (x - vt)^2}$

We have given, at $t = 1$, $y = \frac{1}{1+(x-2)^2}$

By comparing

$V = 2m/s$

20. **(a)** Comparing the given equation

$y = 10^{-3}\sin(50t + 2x)$ with standard equation,

$y = a\sin(\omega t - kx)$

$\Rightarrow$ wave is moving along –ve x-axis with speed

$v = \frac{\omega}{K} \Rightarrow v = \frac{50}{2} = 25$ m/sec.

21. **(d)**

22. **(a)** $\frac{\text{Speed of body}}{\text{Speed of sound}} =$ mach number

23. **(b)** Standard equation

$$y(x,t) = A\cos\left(\frac{\omega}{V}x - \omega t\right)$$

From any of the displacement equation

Say y_1

$\frac{\omega}{V} = 0.50\pi$ and $\omega = 100\pi$

$\therefore \quad \frac{100\pi}{V} = 0.5\pi \quad \therefore V = \frac{100\pi}{0.5\pi} = 200$ m/s

24. **(b)** $y(x, t) = f(x - vt)$

$y = (x, 0) = \frac{4\times10^{-3}}{8 - x^2}$

For a travelling wave in the x-direction

$$y(x,t) = \frac{4\times10^{-3}}{8-(x-5t)^2}$$

25. **(d)** $T = \mu v^2 = \mu\frac{\omega^2}{k^2} = 0.04\frac{(2\pi/0.004)^2}{(2\pi/0.50)^2} = 6.25$ N

26. **(a)** $\frac{v_A}{v_B} = \frac{D_B}{D_A} = \frac{1}{2}$

27. **(a)** Velocity of sound $= \sqrt{\frac{\gamma RT}{M}}$

When water vapour are present in air, average molecular weight of air decreases and hence velocity increases.

28. **(c)** From $\Delta x = \frac{\lambda}{2\pi}\Delta\phi$, $\lambda = 2\pi\frac{\Delta x}{\Delta\phi} = \frac{2\pi(0.4)}{1.6\pi} = 0.5$ m

$\nu = \frac{v}{\lambda} = \frac{330}{0.5} = 660$ Hz

29. **(b)** If $\rho_H = 1$, then $\rho_{mix} = \frac{4\times1+1\times16}{(4+1)} = 4$

$\frac{v_{mix}}{v_H} = \sqrt{\frac{\rho_H}{\rho_{mix}}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$v_{mix} = \frac{v_H}{2} = \frac{1224}{2} = 612$ m/s

30. **(d)** $y(x, t) = 5\sin(6t + 0.003x)$

Compair with $y(x, t) = y.\sin(\omega t + kx)$

$k = 0.003$ cm^{-1}, $\omega = 6$ rad/s,

wave velocity, $v = \frac{\omega}{k} \Rightarrow \frac{6}{0.003\times10^2} = 20$ ms^{-1}

31. **(a)** The speed of transverse wave in the wire is

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{70}{7\times10^{-3}}} = 100$ m/s

32. **(d)** In solid, speed of sound is given as

$V = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{3.2\times10^{11}}{8\times10^3}} = \sqrt{0.4\times10^8}$

$= \sqrt{40\times10^6} = 6.32\times10^3$ ms^{-1}

33. **(c)** Phase difference $\Delta\phi = 60\times\frac{\pi}{180} = \frac{\pi}{3}$

The phase difference is given by

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$\frac{\pi}{3} = \frac{2\pi}{\lambda}(6\text{ m})$

$\Rightarrow \lambda = 36$ m ($\because$ frequency, $f = 500$ Hz)

$v = f\lambda = (500\text{ Hz})(36\text{ m}) = 18000$ m/s $= 18$ km/s

34. **(c)** Velocity of wave is

$V = \frac{\omega}{K} = \frac{4}{8} = \frac{1}{2}$ m/s

35. **(a)** Frequency of wave is a function of the source of waves. Therefore, it remains unchanged.

36. **(c)**

37. **(d)** Longitudinal waves like sound require material medium.

38. **(c)** Sound can travel longitudinally as well as transversly in solids.

39. **(b)** At $t = 0, x = 0, y = 0$

$\phi = \pi$ rad

40. (b) $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{E}{\rho}}$

41. (c) The equation of progressive wave propagating in the positive direction of X-axis is

$y = a \sin \frac{2\pi}{\lambda}(vt - x)$ or

$y = a \sin(\omega t - kx)$

42. (c) $y = A \sin(\omega t - kx)$

Particle velocity,

$v_p = \frac{dy}{dt} = A\,\omega \cos(\omega t - kx)$

$\therefore \quad v_{p\,max} = A\,\omega$

wave velocity $= \frac{\omega}{k}$

$\therefore \quad A\,\omega = \frac{\omega}{k}$

i. e., $A = \frac{1}{k}$ But $k = \frac{2\pi}{\lambda}$

$\therefore \quad \lambda = 2\pi A$

43. (a) The intensity of wave, $I = 2\pi^2 f^2 A^2 \rho v$, so $I \propto f^2$ and $I \propto A^2$.

44. (d)

45. (a) $V_{max} = a\omega = 0.1 \times 2\pi \times 300 = 60\pi$ cm s^{-1}

46. (b) From equation, $\omega = 100$

$\therefore \frac{2\pi}{T} = 100 \Rightarrow \nu = \frac{100}{2\pi}$

$\frac{2\pi}{\lambda} = 20 \Rightarrow \lambda = \frac{2\pi}{20}$

$v = \lambda\nu = \frac{2\pi}{20} \times \frac{100}{2\pi} = 5 m/s$

47. (a) Here, $y_1 = a \sin(\omega t + kx + 0.57)$

and $y_2 = a \cos(\omega t + kx) = a \sin\left[\frac{\pi}{2} + (\omega t + kx)\right]$

Phase difference, $\Delta\phi = \phi_2 - \phi_1$

$= \frac{\pi}{2} - 0.57 = \frac{3.14}{2} - 0.57 = 1.57 - 0.57 = 1$ radian

48. (c) 49. (b) 50. (b) 51. (a) 52. (b)

53. (c) 54. (d) 55. (c)

56. (c) Velocity of recession (v) is given by $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

$\Rightarrow \quad v = \left(\frac{\Delta\lambda}{\lambda}\right) c = \left(\frac{0.1\,Å}{6000\,Å}\right) \times 3 \times 10^8\ (m/s)$

$= 5 \times 10^3$ m/s $= 5$ km/s.

57. (c) Frequency is not affected by blowing of wind so long as source and listener are stationary.

58. (b) $\frac{f_{approach} - f_{recede}}{f} = \frac{\Delta f}{f} = \frac{v}{v - v_s} - \frac{v}{v + v_s}$

$\therefore \frac{\Delta f}{f} = \frac{v(v + v_s - v + v_s)}{v^2 - v_s^2} = \frac{2vv_s}{v^2 - v_s^2}$

But $\frac{\Delta f}{f} \times 100 = 2\%$

$\Rightarrow 0.02 = \frac{2(300)v_s}{(300)^2 - v_s^2}$

$\Rightarrow 0.02 = \frac{2(300)v_s}{(300)^2} = \frac{2}{300} v_s$

$\therefore v_s = (0.01)300 = 3$m/s

59. (c) $40 = 10 \log_{10}\left(\frac{I_I}{I_0}\right)$

$\therefore \frac{I_I}{I_0} = 10^4$ ...(i)

Also,

$20 = 10 \log_{10}\left(\frac{I_2}{I_0}\right)$

$\Rightarrow \frac{I_2}{I_0} = 10^2$ ...(ii)

$\therefore \frac{I_2}{I_0} = 10^{-2} = \frac{r_I^2}{r_2^2}$

$\therefore r_2 = 100 r_I^2 \quad \Rightarrow r_2 = 10$m $\quad (\therefore r_1 = 1m)$

60. (c) The frequency that the observer receives directly from the source has frequency n_1 = 500 Hz. As the observer and source both move towards the fixed wall with velocity u, the apparent frequency of the reflected wave coming from the wall to the observer will have frequency

$n_2 = \left(\frac{V}{V - u}\right) 500\,Hz$

where V is the velocity of sound wave in air. The apparent frequency of this reflected wave as heard by the observer will then be

$$n_3=\left(\frac{V+u}{V}\right)n_2=\left(\frac{V+u}{V}\right)\left(\frac{V}{V-u}\right)500=\left(\frac{V+u}{V-u}\right)500$$

It is given, that the number of beat per second is $n_3 - n_1 = 10$

$$\therefore\ (n_3 - n_1) = 10 = \left(\frac{V+u}{V-u}\right)500-500$$

$$\Rightarrow 10 = 500\left[\frac{V+u}{V-u}-1\right]\Rightarrow 10 = \frac{2\times u\times 500}{V-u}$$

Hence,

$$10V = 1000u + 10u = 1010u$$

Putting $u = 4$ m/s,

we have $V=\frac{1}{10}[4040]=404\,m/s\simeq 400\,m/s$

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(d)** $y = 5(\sin 4\pi t + \sqrt{3}\cos 4\pi t)$

$= [5\sin 4\pi t + 5\sqrt{3}\cos 4\pi t]$ ...(i)

General equation for its motion is for a left travelling wave

$y = A\sin(kx + \omega t) + B\cos(kx + \omega t)$ ...(ii)

$= a\sin(kx + \omega t + \phi)$

Here, $a = \sqrt{A^2 + B^2}$ and $\phi = \tan^{-1}(B/A)$

On comparing we get, $A = 5$ and $B = 5\sqrt{3}$

Amplitude, $a = \sqrt{5^2 + \left(5\sqrt{3}\right)^2} = 10$

2. **(a)** Given, amplitude $a = 10$ cm

wave velocity = 2 × maximum particle velocity

i.e, $\frac{\omega\lambda}{2\pi} = 2a\omega$ or, $\lambda = 4\pi a = 4\times 10 = 40$ cm $\left(\because a=\frac{10}{\pi}\right)$

3. **(c)** Compare the given equation with standard form

$$y = r\sin\left[\frac{2\pi x}{\lambda}-\frac{2\pi t}{T}\right]$$

$\frac{2\pi}{\lambda} = 3, \lambda = \frac{2\pi}{3}$ and $\frac{2\pi}{T} = 15 \Rightarrow T = \frac{2\pi}{15}$

Speed of propagation, $v = \frac{\lambda}{T} = \frac{2\pi/3}{2\pi/15} = 5$

4. **(d)** $y = \frac{1}{\sqrt{a}}\sin\omega t \pm \frac{1}{\sqrt{b}}\sin\left(\omega t + \frac{\pi}{2}\right)$

Here phase difference = $\frac{\pi}{2}$

Resultant amp. $= \sqrt{\left(\frac{1}{\sqrt{a}}\right)^2 + \left(\frac{1}{\sqrt{b}}\right)^2} = \sqrt{\frac{1}{a}+\frac{1}{b}} = \sqrt{\frac{a+b}{ab}}$

5. **(b)** The velocity of a transverse wave in a stretched wire is given by

$$v = \sqrt{\frac{T}{\mu}}$$

Where,

T = Tension in the wire

μ = linear density of wire $\quad (\because V \propto T)$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{v}{v}\times 2 = \sqrt{\frac{2.06\times 10^4}{T_2}}$$

$$\Rightarrow T_2 = \frac{2.06\times 10^4}{4} = 0.515\times 10^4\,N$$

$$\Rightarrow T_2 = 5.15\times 10^3\,N$$

6. **(a)** **7.** **(a)** **8.** **(c)** **9.** **(a)** **10.** **(d)** **11.** **(b)**

12. **(b)** $n_1 = n_2$

$T \rightarrow$ Same

$r \rightarrow$ Same

$l \rightarrow$ Same

Frequency of vibration

$$n = \frac{p}{2l}\sqrt{\frac{T}{\pi r^2\rho}}$$

As T, r, and l are same for both the wires

$n_1 = n_2$

$$\frac{p_1}{\sqrt{\rho_1}} = \frac{p_2}{\sqrt{\rho_2}} \Rightarrow \frac{p_1}{p_2} = \frac{1}{2} \quad \because \quad \rho_2 = 4\rho_1$$

13. **(d)** Velocity of wave on string

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{8}{5}\times 1000} = 40\text{m/s}$$

Here, T = tension and μ = mass/length

Wavelength of wave $\lambda = \frac{v}{n} = \frac{40}{100}$ m

Separation b/w successive nodes,

$$\frac{\lambda}{2} = \frac{40}{2\times100} = \frac{20}{100}\text{m} = 20\text{ cm}$$

14. (d) We have

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{4} = 0.5\pi\,\text{cm}^{-1}$$

$\omega = 2\pi f = 2\pi \times 100 = 200\,\pi\text{s}^{-1}$

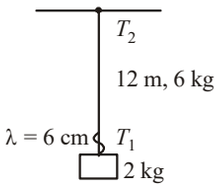
15. (c) As tension is different at every point of rope. So velocity of wave will be different at different point.

$$f = \text{constant} \Rightarrow \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} \Rightarrow \lambda_2 = \frac{v_2}{v_1}\lambda_1$$

$$\lambda_2 = \sqrt{\frac{T_2}{T_1}}\ \lambda_1\ [\because v \propto \sqrt{T}]$$

$$= \sqrt{\frac{8g}{4g}}.\lambda_1 = 2\times6\text{ cm}$$

$= 12$ cm



16. (b)

17. (b) Total length of sonometer wire,

$l = 110\text{ cm} = 1.1$ m

Length of wire is in ratio,

6 : 3 : 2 i.e. 60 cm, 30 cm, 20 cm.

Tension in the wire, T = 400 N

Mass per unit length, m = 0.01 kg

Minimum common frequency = ?

As we know,

Frequency, $\nu = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1000}{11}$ Hz

Similarly, $\nu_1 = \frac{1000}{6}$ Hz ; $\nu_2 = \frac{1000}{3}$ Hz ; $\nu_3 = \frac{1000}{2}$ Hz

Hence common frequency = 1000 Hz

18. (b) $V = \frac{8}{0.4} = 20$ cm/s and $\lambda = 4\times2 = 8$ cm

and, $\omega = VK = 20\times\frac{2\pi}{8} = 5\pi$ rad/sec

So, $(V_{max})_{particle} = \omega A = 5\pi \times \frac{1\text{ cm}}{2} = \frac{5\pi}{2}$ cm/s

19. (c) For sound wave, $\Delta x = 50$cm, $\Delta\phi = 1.8\,\pi$

$\therefore$ Path difference, $\Delta x = \frac{\lambda}{2\pi}\cdot\Delta\phi$

$\Rightarrow 50 = \frac{\lambda}{2\pi}\times1.8\pi \therefore \lambda = \frac{500}{9}$ cm

$\therefore$ Frequency, $f = \frac{v}{\lambda} = \frac{340\times9}{500\times10^{-2}} = 612$Hz

20. (c) Compairing the given $y = 0.5\sin\left(\frac{2\pi}{\lambda}400t - \frac{2\pi}{\lambda}x\right)$ with standard equation

$\omega = \frac{2\pi}{\lambda}400$ and $k = \frac{2\pi}{\lambda}$

$\therefore$ velocity of the wave $v = \frac{\omega}{k} = \frac{2\pi}{\lambda}\frac{400}{\frac{2\pi}{\lambda}}$ $\therefore v = 400$ m/s

21. (a) Given, $l = 60$ cm, $m = 6$ g, $A = 1$ mm^2, $v = 90$ m/s and $Y = 16\times10^{11}$ Nm^{-2}

Using, $v = \sqrt{\frac{T}{m}\times l} \Rightarrow T = \frac{mv^2}{I}$

Again from, $Y = \frac{T}{A}\Delta L / L_0$

$$\Delta L = \frac{Tl}{YA} = \frac{mv^2\times l}{l(\text{YA})} = \frac{6\times10^{-3}\times90^2}{16\times10^{11}\times10^{-6}} = 3\times10^{-4}m$$

$= 0.03$ mm

22. (c) We have given, $y = 0.03\sin(450\,t - 9x)$

Comparing it with standard equation of wave, we get $\omega = 450$ k $= 9$

$\therefore\ v = \frac{\omega}{k} = \frac{450}{9} = 50$ m/s

Velocity of travelling wave on a stretched string is given by

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow \frac{T}{\mu} = 2500$$

μ = linear mass density

$\Rightarrow T = 2500\times5\times10^{-3} \Rightarrow 12.5$ N

23. (a) $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

where ϕ = phase difference

$= [(x - vt + 1.5) - (x - vt)]\times 2\pi = 2\pi\times1.5 = 3\pi$

So, $A = \sqrt{5^2 + 3^2 + 2.5.3\cos\pi} = \sqrt{4} = 2$ cm.

24. (a) Using $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$,

where, T = tension and $\mu = \frac{\text{mass}}{\text{length}}$

$$f_x = \frac{1}{2\ell}\sqrt{\frac{T_x}{\mu}} \text{ and } f_z = \frac{1}{2\ell}\sqrt{\frac{T_z}{\mu}}$$

$$\frac{f_x}{f_z} = \frac{450}{300} = \sqrt{\frac{T_x}{T_z}} \quad \therefore \frac{T_x}{T_z} = \frac{9}{4} = 2.25.$$

25. (c) From the equation $\omega = 160$ and $k = 0.5$

Speed of wave

$$v = \frac{\omega}{k} = \frac{160}{0.5} \times \frac{18}{5} \text{km/h} = 1152 \text{km}$$

26. (d) Frequency $n = \frac{V}{\lambda} = \sqrt{\frac{T}{\pi r^2 d}}$ where d = density

$$\therefore \quad V = \lambda\sqrt{\frac{T}{\pi r^2 d}} \text{ so here } V \propto \frac{1}{\sqrt{d}}$$

If the density of the material of the string B is 2% more than that of A then

$$V' \propto \frac{1}{\sqrt{d + d \times \frac{2}{100}}} \text{ or } V' \propto \frac{1}{\sqrt{d(1+0.2)}}$$

$$\therefore \quad \frac{V'}{V} = \sqrt{\frac{1}{d(1.02)}}\Big|\sqrt{\frac{1}{d}} \quad \therefore V' = \frac{V}{\sqrt{1.02}}$$

27. (d) $\frac{R_1}{R_2} = \frac{1}{2}; \frac{\rho_1}{\rho_2} = \frac{1}{4}$

$$v = \sqrt{\frac{T}{\mu}}\left(\text{where } \mu = \frac{\text{mass}}{\text{Length}} = \frac{\rho V}{L} = \rho A = \rho \pi R^2\right)$$

$$v = \sqrt{\frac{T}{\rho \pi R^2}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_2}{\rho_1 \pi R_1^2}} \times \sqrt{\frac{\rho_2 \pi R_2^2}{T}} = \sqrt{\frac{\rho_2}{\rho_1} \times \frac{R_2^2}{R_1^2}} = \sqrt{4 \times (2)^2} = \frac{4}{1}$$

28. (b)

29. (a) $y_1 = A_1 \sin k(x - vt)$

$y_1 = 12 \sin 6.28 (x - vt)$

$y_2 = 5 \sin 6.28 (x - vt + 3.5)$

Phase difference, $\Delta\phi = k(\Delta x) = 6.28 \times 3.5 = 7\pi$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi$$

$$\Rightarrow A = \sqrt{(12)^2 + (5)^2 + 2(12)(5)\cos(7\pi)}$$

$$= \sqrt{144 + 25 - 120} = \sqrt{49} = 7 \text{ mm}$$

30. (c) 31. (c) 32. (d) 33. (b)

34. (c) The equation of progressive wave propagating in the positive direction of X-axis is

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \text{ or}$$

$$y = a \sin(\omega t - kx)$$

35. (c) $y = A \sin(\omega t - kx)$

Particle velocity,

$$v_p = \frac{dy}{dt} = A\,\omega \cos(\omega t - kx)$$

$$\therefore \quad v_{p\,\max} = A\,\omega$$

$$\text{wave velocity} = \frac{\omega}{k}$$

$$\therefore \quad A\,\omega = \frac{\omega}{k}$$

$$\text{i. e.,} \quad A = \frac{1}{k} \text{ But } k = \frac{2\pi}{\lambda}$$

$$\therefore \quad \lambda = 2\pi A$$

36. (a) The intensity of wave, $I = 2\pi^2 f^2 A^2 \rho v$, so $I \propto f^2$ and $I \propto A^2$.

37. (a) Here, $y_1 = a \sin(\omega t + kx + 0.57)$

$$\text{and} \quad y_2 = a \cos(\omega t + kx) = a \sin\left[\frac{\pi}{2} + (\omega t + kx)\right]$$

Phase difference, $\Delta\phi = \phi_2 - \phi_1$

$$= \frac{\pi}{2} - 0.57 = \frac{3.14}{2} - 0.57 = 1.57 - 0.57 = 1 \text{ radian}$$

38. (d) Consider the two waves of equal amplitude A and equal frequency travelling in the same direction

$y_1(x, t) = A \sin(kx - \omega t)$

$y_2(x, t) = A \sin(kx - \omega t + \phi)$

$y(x, t) = A \sin(kx - \omega t) + A \sin(kx - \omega t + \phi)$

$$= A\left[2 \sin\left[\frac{(kx - \omega t) + (kx - \omega t + \phi)}{2}\right]\cos\frac{\phi}{2}\right]$$

$$= 2A \cos\frac{\phi}{2}\left(kx - \omega t + \frac{\phi}{2}\right)$$

The resultant amplitude is $2A \cos\frac{\phi}{2}$

Magnitude of wave varies between 0 to 2A because the given cosine function will vary from 0 to π.

39. (a) $\lambda = \frac{v}{n} = \frac{340}{170} = 2m, \quad n' = \frac{340}{340-17} \times 170$

$n' = 178.9\,Hz$

Now $\lambda' = \frac{v}{n'} = \frac{340}{178.9} = 1.9$

$\Rightarrow \lambda - \lambda' = 2 - 1.9 = 0.1\text{m}$

40. (d) $\nu' = \nu\left(\frac{v + v_o}{v - v_S}\right)$

Here, $\nu = 600$ Hz, $\nu_o = 15$ m/s

$v_s = 20$ m/s, $v = 340$ m/s

$\therefore \quad \nu' = 600\left(\frac{355}{320}\right) \approx 666$ Hz

Exercise 3 : PREVIOUS YEARS MCQs

1. (a) Velocity of sound = $\sqrt{\frac{\gamma RT}{M}}$

When water vapour are present in air, average molecular weight of air decreases and hence velocity increases.

2. (b)

3. (a) f = 500Hz, Distance between points X and Y = 600 m

t = 2 s

$V = \frac{600}{2} = 300\text{ m/s}$

Find the wavelength $\lambda = \frac{V}{f} = \frac{300}{500} = 0.6\text{m}$

number of waves between X and Y

number of waves $= \frac{\text{Distance}}{\lambda} = \frac{600}{0.6} = 1000$ waves

4. (d) Given wave equation:

$y(x, t) = 0.5 \sin(70.1x - 10\pi t)$

This is a standard form of travelling wave:

$y(x, t) = A \sin(kx - \omega t)$

Identify angular frequency:

$\omega = 10\,\pi$ rod/s

Use the relation between angular frequency and frequency

$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi}$

$f = \frac{10\pi}{2\pi} = 5\text{Hz}$

5. (d) Given: Wave equation: $y = A \sin(kx) \cos(\omega t)$

$k = \frac{7\pi}{4}$ and $\omega = 350\,\pi$

wave speed $V = \frac{\omega}{k} = \frac{350\pi}{\frac{7\pi}{4}} = 200\text{ m/s}$

6. (c) $f' = \left(f \times \frac{v + v_o}{v - v_s}\right), v_o = 0.1v, v_s = 0.1v$

$f' = f \times \frac{v + 0.1v}{v - 0.1v} = f \times \frac{1.1v}{0.9v} = f \times \frac{1.1}{0.9}$

$f' = f \times 1.222$ .....

$\frac{f' - f}{f} \times 100 = \left(\frac{1.222 - 1}{1}\right) \times 100 = 22.2\%$

Approximately 22% increase in frequency

7. (c) Using velocity of wave in a string $V = \sqrt{\frac{T}{M}}$

Where T = tension, m = mass per unit length

Give mass = $3g = 3 \times 10^{-3}$ kg length $l = 80$ cm

$= 80 \times 10^{-2}$m

$\therefore \quad m = \frac{3 \times 10^{-3}}{80 \times 10^{-2}} = \frac{3}{8} 10^{-2}\text{ kg/m}$

Given equation $y = 1.5 \sin\{(5 \times 10^{-3} x) + 20t\}$

comparing with standard equation

$K = \frac{2\pi}{\pi} = 5 \times 10^{-3} \therefore \quad \pi = \frac{2\pi}{2 \times 10^{-3}}$

and $w = 2\pi n = 20 \quad \therefore \quad n = \frac{20}{2\pi}$

$V = n\lambda = \sqrt{\frac{T}{m}}$ or, $\frac{20}{2\pi} \times \frac{2\pi}{5 \times 10^{-3}} = \sqrt{\frac{T \times 8}{3 \times 10^{-2}}}$

or, $(4000)^2 = \frac{800}{3} T$

$\therefore \quad T = \frac{4000 \times 4000 \times 3 \times 10^{-4}}{800} = 6\text{N}$

[Since x and y are in cm]

8. (c) $V = \sqrt{\frac{T}{\mu}}, \mu = \frac{m}{\ell} = \frac{5 \times 10^{-3}}{0.81} \approx 6.17 \times 10^{-3}\text{Kg/m}$

$V = \sqrt{\frac{50}{6.17 \times 10^{-3}}} \approx \sqrt{8100}$

$\boxed{V = 90\ m/s}$

Hence, option (c) is correct

9. **(b)** Given,

Distance moved by a wave between two extreme point = 10 cm

$\therefore$ Amplitude $= \frac{10}{2}cm = 5cm = 0.05m$

Angular frequency, $\omega = 2\pi f = 2\pi \times 210$

$= 2 \times \frac{22}{7} \times 210 = 1320$

Wave number, $K = \frac{\omega}{v} = \frac{1320}{330} = 4$

Substituting the values in standard

Equation of wave $S(x, t) = A \sin (Kx - \omega t)$

we get

$S(x, t) = 0.05 \sin[4x - 1320t]m$

10. **(a)** Given, $\Delta f = f_{echo} - n = 0.1n$

Since the source is moving towards the stationary cliff

$$f' = n.\frac{v}{v - v_s}$$

The cliff acts like a new source emitting f' and the observer is moving toward it.

$$f_{echo} = f' \frac{v + v_s}{u}$$

Substitute f'

$$f_{echo} = \left(\frac{n\,.\,v}{v - v_s}\right).\left(\frac{v + v_s}{v}\right) \Rightarrow f_{echo} = n\left(\frac{v + v_s}{v - v_s}\right)$$

$\therefore \quad f_{echo} - n = 0.1n$

$$\Rightarrow n\left[\frac{v + v_s}{v - v_s} - 1\right] = 0.1n \Rightarrow \frac{v + v_s}{v - v_s} = 1.1$$

$$\Rightarrow v + v_s = 1.1v - 1.1v_s \Rightarrow v - 1.1v = -v_s - 1.1v_s$$

$$\Rightarrow -0.1v = -2.14v_s$$

$$\Rightarrow v_s = \frac{0.1 \times 336}{2.1}$$

$$\Rightarrow v_s = 16\ m/s$$

6 Optics

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(a)** The distances measured in the direction of incident light are taken as positive. Hence option (a) is correct.

2. **(d)** Convex mirror always forms, virtual, erect and smaller image.

3. **(b)** $f = -15\text{cm}$, $m = +2$ (Positive because image is virtual)

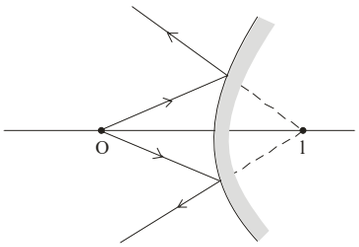
$\because \quad m = -\frac{v}{u} \Rightarrow v = -2u.$ By using mirror formula

$$\frac{1}{-15} = \frac{1}{(-2u)} + \frac{1}{u} \Rightarrow u = -7.5\text{cm}$$

4. **(d)** $u = -30\text{cm}$, $f = +30$ cm, by using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{+30} = \frac{1}{v} + \frac{1}{(-30)}$$

$v = 15$ cm, (Behind the mirror)



5. **(a)** $m = \pm 3$ and $f = -6$ cm

Now $m = \frac{f}{f-u} \Rightarrow \pm 3 = \frac{-6}{-6-u}$

For real image $-3 = \frac{-6}{-6-u} \Rightarrow u = -8\text{cm}$

For virtual image $3 = \frac{-6}{-6-u} \Rightarrow u = -4\text{cm}$

6. **(a)** For real image

$u = -u_1$, $v = -2u_1$, $f = -20$ cm

Substituting in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

We get $\frac{1}{-2u_1} - \frac{1}{-u_1} = -\frac{1}{20}$ or $u_1 = 30$ cm

For virtual image

$u = -u_2$, $v = 2u_2$, $f = -20$ cm

$\therefore \quad \frac{1}{-2u_2} - \frac{1}{-u_2} = -\frac{1}{20}$ or $u_2 = 10$ cm

Thus, distance between two positions of the object are $u_1 - u_2$ or $30\text{ cm} - 10\text{cm} = 20$ cm.

7. **(a)** For convex mirror, $f = +15\text{cm}$

$m = -\frac{v}{u} = +\frac{1}{2} \Rightarrow v = -\frac{u}{2}$

By mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{2}{-u} + \frac{1}{u} = \frac{1}{f}$$

$u = -f = -15$ cm

8. **(c)**

9. **(c)** If the object is placed at focal point of convex mirror, the image will be at infinity.

10. **(c)** Using mirror formula :

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{-20}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{-10} + \frac{1}{20} = -\frac{1}{20} \Rightarrow v = -20\text{ cm}$$

11. **(c)** Here focal length = f and $u = -f$

On putting these values in

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{f} = -\frac{1}{f} + \frac{1}{v} \Rightarrow v = \frac{f}{2}$$

12. **(b)** $f = \frac{R}{2} = \frac{-15}{2} = -7.5\text{ cm}$

Mirror formula is given by :

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{(-7.5)} - \frac{1}{(-10)}$$

$$\frac{1}{v} = \frac{-25}{750} \text{ or } v = -30\text{ cm}$$

Magnification is given by :

$$m = \frac{-v}{u} = -\frac{-30}{-10} = -3$$

Hence the image formed is real, magnified and inverted.

13. **(b)** The focal length of the mirror is

$$f = \frac{-R}{2} = \frac{-30\text{ cm}}{2} = -15\text{ cm}$$

As the person wants his real enlarged image, so magnification, $m = -\frac{v}{u} = -3$ or $v = 3u$

According to mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\therefore \frac{1}{3u} + \frac{1}{u} = \frac{1}{-15\ cm} \Rightarrow u = \frac{-60\text{ cm}}{3} = -20\text{ cm}$$

–ve sign shows that the person should stand infront of the mirror.

14. **(b)** As image is real

$\therefore$ Magnification, $m = -\frac{v}{u} = -3$ or $v = 3u$

Here, $u = -20$ cm. $\therefore v = -60$ cm

According to mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{-60} + \frac{1}{-20}$$

$f = -15$ cm. $|f| = 15$ cm

15. **(d)**

16. **(d)** $m = 4 = -\frac{v}{4} \Rightarrow v = -4u$

Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-4u} + \frac{1}{u} = \frac{-1}{0.6}$

$4u = -0.45$ m

17. **(d)**

18. **(a)** Rectilinear propagation of light means light travels in straight line.

19. **(b)**

20. **(c)** Because they form nearly point image of point source.

21. **(d)** **22.** **(a)** **23.** **(c)**

24. **(b)** Erect and enlarged image can produced by concave mirror.

$$\frac{I}{O} = \frac{f}{f-u} \Rightarrow \frac{+3}{+1} = \frac{f}{f-(-4)} \Rightarrow f = -6\text{ cm}$$

$\Rightarrow R = 2f = -12$ cm

25. **(d)**

26. **(c)** As $i > i_c$

At $i = i_c$ angle of refraction

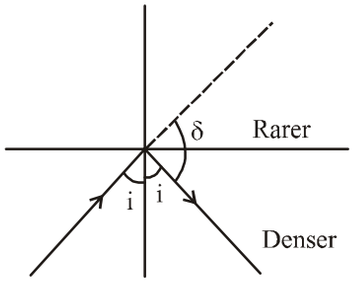
'r' $= 90° \therefore \frac{\sin i_c}{\sin 90°} = \mu = 1$

27. **(a)** When light ray goes from one medium to other, then frequency of the light remains unchanged.

28. **(d)** Real depth = 100 cm, $\mu = \frac{4}{3}$

$\therefore$ Apparent depth $= \frac{\text{Real depth}}{\mu} = \frac{100 \times 3}{4} = 75$ cm

29. **(c)**



The deviation is given by

$\delta = \pi - 2i$

At $i = C$, $\delta = \delta_{max}$ $\therefore$ $\delta_{max} = \pi - 2C$

30. **(b)**

31. **(a)** Critical angle is given by $\sin\theta_c = \frac{\mu_r}{\mu_d}$

$$\therefore \sin\theta_c = \frac{\mu_r}{\mu_d} = \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{\mu_r}{\mu_d} \Rightarrow \frac{\mu_d}{\mu_r} = \frac{\sqrt{2}}{1}$$

32. **(c)** As, critical angle, $i_c = \sin^{-1}\left(\frac{n_r}{n_d}\right)$

For total internal reflection at surface AB, $\theta > i_c$

$$\Rightarrow \quad \theta > \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) \qquad ...(i)$$

Also, for total internal reflection at surface CD,

$$\theta > \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \qquad ...(ii)$$

For both Eqs. (i) and (ii) to be correct, $\theta > \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\because \quad \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) > \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

Hence, for total internal reflection at both surfaces AB and CD is possible, when

$\Rightarrow \theta > 60°$ $\therefore$ $\theta_{min} = 60°$

33. **(b)** Difference between apparent and real depth of a pond is due to the refraction of light, not due to the total internal reflection. Other three phenomena are due to the total internal reflection.

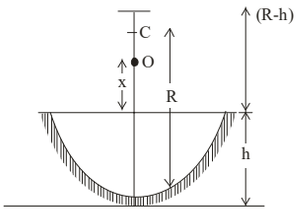
34. **(a)** Optional path = μt

In medium (1), optical path = $\mu_1 d_1$

In medium (2), optical path = $\mu_2 d_2$

$\therefore$ Total path = $\mu_1 d_1 + \mu_2 d_2$

35. **(b)**



$$\text{Since, } \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{\mu}$$

$$\frac{x}{R-h} = \frac{1}{\mu} \quad \therefore \quad x = \frac{R-h}{\mu}$$

36. **(a)** Given, $n_{oil} = 2$, $n_{glass} = 3/2$

Velocity of light in oil is given by

$$v_{oil} = \frac{\text{Velocity of light } (c)}{n_{oil}} = \frac{c}{n_{oil}}$$

Similarly, $v_{glass} = \frac{c}{n_{glass}}$

$$\therefore \quad \frac{v_{glass}}{v_{oil}} = \frac{c/n_{glass}}{c/n_{oil}} = \frac{n_{oil}}{n_{glass}} = \frac{2}{\frac{3}{2}} = \frac{4}{3}$$

37. **(d)**

38. **(c)** We know that, $\sin\theta = \frac{n_2}{n_1}$

$$\Rightarrow \sin\theta = \frac{v}{v'} \Rightarrow v' = \frac{v}{\sin\theta}$$

39. **(b)**

40. **(d)** We know that critical angle is given by

$$\sin C = \frac{\mu_r}{\mu_d} \text{ and } \mu \propto \frac{1}{V} \Rightarrow \frac{\mu_r}{\mu_d} = \frac{v_d}{v_r} = \frac{1.5\times10^{10}}{2\times10^{10}} = \frac{3}{4}$$

$$\text{So, } \sin C = \frac{3}{4} \Rightarrow C = \sin^{-1}\left(\frac{3}{4}\right)$$

Therefore, for TIR

$$\theta > C \Rightarrow \theta > \sin^{-1}\left(\frac{3}{4}\right)$$

41. **(d)**

42. **(b)** For critical angle θ_c, $\sin\theta_c = \frac{1}{\mu}$

For greater wavelength or lesser frequency μ is less. So, critical angle would be more. So, they will not suffer reflection and come out at angles less than 90°.

43. **(a)** $t_2 - t_1 = 5 \times 10^{-10}$

$$\Rightarrow \quad \frac{d}{v_B} - \frac{d}{v_A} = 5\times10^{-10} \Rightarrow \frac{d\mu_B}{C} - \frac{d\mu_A}{C} = 5\times10^{-10}$$

$$\Rightarrow \quad d\left(\frac{\mu_B}{C} - \frac{\mu_A}{C}\right) = 5\times10^{-10}$$

$$\Rightarrow \quad d\left(\frac{2\mu_A}{C} - \frac{\mu_A}{C}\right) = 5\times10^{-10}$$

$$\Rightarrow \quad d = \frac{5\times10^{-10}}{\frac{\mu_A}{C}} = 5\times10^{-10} v_A m$$

44. **(c)** For total internal reflection

$\sin i > \sin C$

where,

i = angle of incidence, C = critical angle

$$\text{But, } \sin C = \frac{1}{\mu} \quad \therefore \sin i > \frac{1}{\mu} \text{ or } \mu > \frac{1}{\sin i}$$

$$\mu > \frac{1}{\sin 45°} \qquad (i = 45° \text{ (Given)})$$

$$\mu > \sqrt{2}$$

45. **(d)** Total thickness = t; Refrative index = μ

$$\text{Speed of light in Glass plate} = \frac{c}{\mu}$$

$$\left[\because v = \frac{\text{Speed of light in vacuum}}{\text{R.I. of Medium}}\right]$$

$$\text{Time taken} = \frac{t}{\frac{c}{\mu}} = \frac{\mu t}{c} \quad \text{(where t = thickness of glass plate)}$$

46. **(c)** Let x = Apparent position of the silvered surface in cm.

Since the image is formed due to relfection at the silvered face and by the property of mirror image distance of object from the mirror = Distance of image from the mirror

or $x + 8 = 12 + 6 - x$ or $x = 5$ cm.

Therefore refractive index of glass

$$= \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{6}{5} = 1.2$$

47. **(a)** Given that $_{w}\mu_{g} = \frac{5}{4}$ and $_{a}\mu_{w} = \frac{4}{3}$

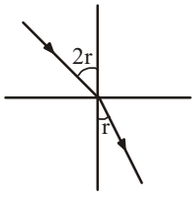
$\therefore\ _{a}\mu_{g} = {}_{w}\mu_{g} \times {}_{a}\mu_{w} = \frac{5}{4}\times\frac{4}{3} = \frac{5}{3}$

48. **(b)** $^{a}\mu_{g} = 1.5$

$\therefore$ 1.5 = cosecC or C = 42°. Critical angle for glass = 42°. Hence a ray of light incident at 50° in glass medium undergoes total internal reflection. δ denotes the deviation of the ray.

$\therefore\ \delta = 180° - (50° + 50°)$ or $\delta = 80°$.

49. **(a)** $\mu = \frac{\sin i}{\sin r} \quad \because\ i = 2r$



$\Rightarrow\ \mu = \frac{\sin 2r}{\sin r} = \frac{2\sin r.\cos r}{\sin r}$

$r = \cos^{-1}\left(\frac{\mu}{2}\right),\ \therefore\ i = 2\cos^{-1}\left(\frac{\mu}{2}\right)$

50. **(a)**

51. **(a)** $n_d \sin i_c = n_r \sin 90°$ ($\because$ From Snell's law)

$\sin i_c = \frac{n_r}{n_d} = \frac{v_d}{v_r} \quad \left(\therefore v = \frac{c}{n}\right)$

$\sin i_c = \frac{1.5\times10^{8}}{2\times10^{8}} = \frac{1.5}{2}$

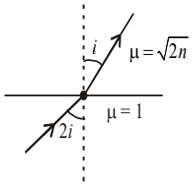
$\sin i_c = \frac{3}{4}$

$\tan i_c = \frac{3}{\sqrt{4^2-3^2}} \Rightarrow \frac{3}{\sqrt{7}}$

The critical angle between them, $i_c = \tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$

52. **(d)** Let $2i$ be angle of incidence

By Snell's law



$\Rightarrow 1 \sin 2i = \sqrt{2n}\ \sin i$

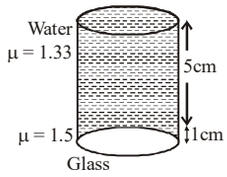
$\Rightarrow 2 \sin i \cos i = \sqrt{2n}\ \sin i$

$\Rightarrow \cos i = \sqrt{\frac{n}{2}} \Rightarrow i = \cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

$\Rightarrow 2i = 2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

53. **(c)** Real depth = 5 cm + 1cm = 6 cm

Apparent depth = $\frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \ldots.$

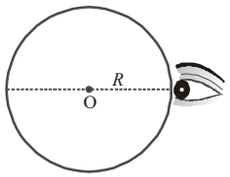


$= \frac{5}{1.33} + \frac{1}{1.5}$

$\simeq 3.8 + 0.7 \simeq 4.5$ cm

$\therefore$ Shift = 6 cm – 4.5 cm $\cong$ 1.5 cm

54. **(c)** $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



or $\frac{1}{v} - \frac{1.5}{-R} = \frac{1-1.5}{-R}$

$\therefore\quad v = -R = -6$ cm

55. **(a)** $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{16} = (1.5-1)\left(\frac{1}{R} - \frac{1}{\infty}\right) \Rightarrow \frac{1}{16} = 0.5\times\frac{1}{R} \Rightarrow R = 8$ cm

56. **(d)** $P_2 = P - P_1 = \frac{100}{80} - \frac{100}{20} = -3.75$ D

57. **(d)**

58. **(b)** If $R_1 = R, R_2 = -2R$

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{6} = (1.5-1)\left(\frac{1}{R} + \frac{1}{2R}\right) = \frac{0.5\times3}{2R}$

R = 4.5 cm

59. **(c)**

60. **(a)** For Ist lens given $f_1 = 10, u = -30, v = ?$

using, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow v = \frac{uf}{u+f} = \frac{-30\times10}{-30+10} = 15$

For 2nd lens given $f_1 = -10, u = 10, v = ?$

using, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{uf}{u+f} = \frac{10\times-10}{10-10} = \infty$

for 3rd lens given $f = 30, u = -\infty, v = ?$

using, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ So v will be 30.

The final image formed 30 cm to the right of the third lens

61. **(c)** Using lens makers formula,

$$\frac{1}{f}=\left(\frac{\mu_1}{\mu_a}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{20}=(\mu-1)\left(\frac{1}{15}-\frac{1}{(-30)}\right) \Rightarrow \mu-1=\frac{1}{2} \Rightarrow \mu=1.5$$

62. **(a)** Equivalent Power, $P_{eq}=P_1+P_2+P_3+.....$

Since all lenses are identical

$$\therefore 5P=25D \Rightarrow P=5D \quad \therefore \frac{1}{f}=5 \Rightarrow f=\frac{1}{5}m=20\ cm$$

63. **(c)** In first liquid, $\mu_1=1.25$

$$\therefore \quad \frac{1}{f_1}=\left(\frac{\mu_2}{1.25}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

In second liquid, $\mu_1=1.5$

$$\therefore \quad \frac{1}{f_2}=\left(\frac{\mu_2}{1.5}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\therefore \quad \frac{f_1}{f_2}=\frac{\left(\frac{\mu_2}{1.5}-1\right)}{\left(\frac{\mu_2}{1.25}-1\right)}=\frac{5}{16} \qquad \therefore \quad \mu_2=1.65$$

64. **(b)** When convex lens placed in air,

$$\frac{1}{f_a}=(\mu_1-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(i)$$

When convex lens immersed in medium,

$$\frac{1}{f_m}=\left(\frac{\mu_1}{\mu_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(ii)$$

Dividing eq(i) by eq(ii), we get

$$\frac{f_m}{f_a}=\frac{(\mu_1-1)}{\left(\frac{\mu_1}{\mu_m}-1\right)}=\frac{\left(\frac{5}{4}\mu_m-1\right)}{\left(\frac{5}{4}-1\right)}$$

$$\Rightarrow \quad \frac{2f_a}{f_a}=4\left(\frac{5}{4}\mu_m-1\right)=5\mu_m-4 \qquad \therefore \ \mu_m=1.2$$

65. **(a)** In air, $\frac{1}{f_1}=(\mu_2-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ ...(i)

In medium, $P_2=-1D \Rightarrow f_2=\frac{100}{P_2}\ cm=-100\ cm$

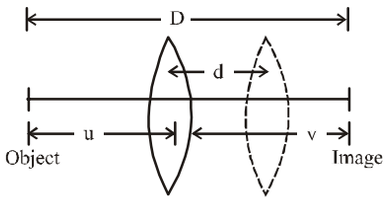
$$\frac{1}{f_2}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(ii)$$

From eqns (i) and (ii), we get

$$\frac{f_2}{f_1}=\frac{(\mu_2-1)}{\left(\frac{\mu_2}{\mu_1}-1\right)} \Rightarrow \frac{-100}{20}=\frac{(1.5-1)}{\left(\frac{1.5}{\mu_1}-1\right)}=\frac{0.5}{\left(\frac{1.5}{\mu_1}-1\right)}$$

$$\therefore \mu_1=\frac{5}{3}$$

66. **(b)**



D = 90 cm, d = 30 cm

By displacement method,

$$\text{Focal length, } f=\frac{D^2-d^2}{4D}=\frac{(90)^2-(30)^2}{4\times 90}=20\ cm$$

67. **(c)** For thin convex lens, P = + 4D, $\mu_\ell=\frac{3}{2}\ \mu_m=\frac{5}{3}$

$$\therefore \quad P=\frac{1}{f_a} \Rightarrow f_a=\frac{1}{4}m=25cm$$

In air,

$$\frac{1}{f_a}=(\mu_\ell-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(i)$$

In medium,

$$\frac{1}{f_m}=\left(\frac{\mu_\ell}{\mu_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(ii)$$

Dividing eq (ii) by (i)

$$\frac{f_m}{f_a}=\frac{(\mu_\ell-1)}{\left(\frac{\mu_\ell}{\mu_m}-1\right)}=\frac{\left(\frac{3}{2}-1\right)}{\left(\frac{3/2}{5/3}-1\right)}=-5$$

$$\therefore \quad f_m=-5\times f_a=-5\times 25=-125\ cm$$

$\therefore$ The lens behaves like a concave lens of focal length 125 cm.

68. **(b)** $R_1=60$ cm, $R_2=\infty$, $\mu=1.6$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow \frac{1}{f}=(1.6-1)\left(\frac{1}{60}\right)$$

$\Rightarrow f=100$ cm.

69. **(d)** $P = \frac{1}{f} = (\mu_1 - \mu_2)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(μ_1 is refractive index of lens and μ_2 is of surrounding medium)

$\Rightarrow 1.25 = (1.5 - \mu_2)\left(\frac{1}{0.2} + \frac{1}{0.4}\right)$

$\Rightarrow \frac{1.25 \times 0.08}{0.6} = (1.5 - \mu_2) \Rightarrow \mu_2 = \frac{4}{3}$

70. **(a)**

71. **(a)** $\frac{|P_1|}{|P_2|} = \frac{2}{3} \Rightarrow \frac{f_2}{f_1} = \frac{2}{3}$ ...(i)

Focal length of their combination

$\frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_2} \Rightarrow \frac{1}{30} = \frac{1}{f_1} - \frac{1 \times 3}{2f_1}$

from (i)

$\Rightarrow \frac{1}{30} = \frac{1}{f_1}\left[1 - \frac{3}{2}\right] = \frac{1}{f_1} \times \left(-\frac{1}{2}\right) \therefore f_1 = -15\text{ cm}$

$\therefore f_2 = \frac{2}{3} \times f_1 = \frac{2}{3} \times 15 = 10\text{ cm}$

72. **(b)**

73. **(a)** Let the distance between the lenses be d.

Then, equivalent power is

$P = P_1 + P_2 - d P_1 P_2$

Given $P_1 = P_2 = +5$ D $\quad \therefore P = (10 - 25d)$ D

For P to be –ve, $10 - 25d < 0 \Rightarrow d > \frac{2}{5}$ m

or, $d > 0.4$ m or $d > 40$ cm

74. **(b)** For biconcave lens, $R_1 = -R$, $R_2 = +R$

$\frac{1}{f} = (\mu - 1)\left[\frac{-1}{R} - \frac{1}{R}\right] = \frac{-2(\mu - 1)}{R}$

For planoconcave lens, $R_1 = \infty$ $R_2 = +R$

$\frac{1}{f} = (\mu - 1)\left[\frac{1}{\infty} - \frac{1}{R}\right] = \frac{-(\mu - 1)}{R}$

$\therefore \frac{1}{f} = \frac{2}{f'} \Rightarrow f' = 2f$

So, power will be halved.

75. **(d)**

76. **(d)**

77. **(b)** $u = -50$ cm, $\mu_1 = 1$, $\mu_2 = 1.5$,

$R = 10$ cm $\quad \therefore \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{1.5}{v} - \frac{1}{-50} = \frac{1.5 - 1}{10} \quad \therefore v = 50$ cm

78. **(a)** From the formula

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f} = (1.5 - 1)\left(\frac{2}{40}\right) = \frac{1}{40}$

focal length, f = 40 cm

79. **(c)**

80. **(c)** From the formula,

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{25} - \frac{1}{25} = 0$

Power of combination $= \frac{1}{f} = 0$

81. **(a)** When lens is cut in two half then power of one part will be same. i.e., *P*, as focal length remains same.

82. **(a)** In the position of minimum deviation, $i_1 = i_2$.

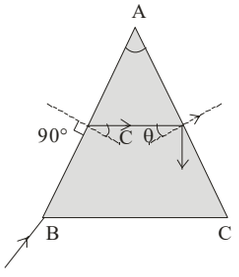
83. **(c)** In the position of minimum deviation

$i = e = \frac{A + \delta m}{2} = \frac{60 + 30}{2} = 45°$

84. **(c)** From ray diagram

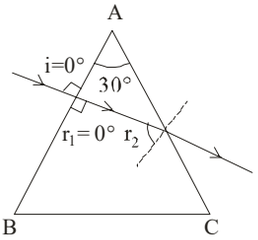
A = C + 0 for TIR at AC

$\theta > C$ so $A > 2C$



85. **(d)**

86. **(c)** The given prism can be shown as below.



Given, $A = 30°, i = 0° \Rightarrow r_1 = 0°$

As, the angle of prism, $A = r_1 + r_2$

$\Rightarrow 30° = 0 + r_2 \Rightarrow r_2 = 30°$

87. **(d)** $r' \le C$

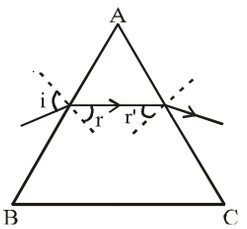
$(r + r' = A)$

$(A - r) \le C$

$A \le (r + C)$

$r_{max} = C$ if $i = 90°$

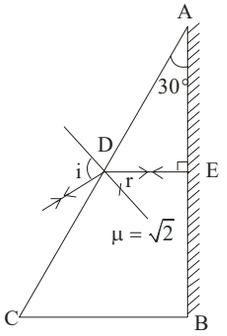
$A \le 2C$



$$\frac{A}{2} \le \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\mu \le \sqrt{1+\cot^2\left(\frac{A}{2}\right)} \Rightarrow \mu_{max} = \sqrt{1+\cot^2\left(\frac{A}{2}\right)}$$

88. **(d)**



Then, the angle of refraction,

$r = 90° - 60° = 30°$

From Snell's law, $n_1 \sin i = n_2 \sin r$

$1 \times \sin i = \sqrt{2} \sin 30°$

$i = 45°$

89. **(c)** Refraction index of the material of the prism

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \cot\frac{A}{2} = \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} \quad \left[\because \mu = \cot\frac{A}{2}\right]$$

$$\Rightarrow \sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right) \Rightarrow \frac{\pi}{2} - \frac{A}{2} = \frac{A}{2} + \frac{\delta_m}{2}$$

$\Rightarrow \delta_m = \pi - 2A$

90. **(b)** Given, refractive index, $\mu = \sqrt{2}$

As $\mu = \dfrac{\sin\left(\frac{\delta_{min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$ (Here δ_{min} = minimum deviation)

$$\sqrt{2} = \frac{\sin\left(\frac{\delta_{min} + 60}{2}\right)}{\sin\left(\frac{60}{2}\right)} \Rightarrow \frac{1}{\sqrt{2}} = \sin\left(\frac{\delta_{min} + 60}{2}\right)$$

$$\therefore \frac{\delta_{min} + 60}{2} = 45 \Rightarrow \delta_{min} = 30$$

91. **(d)** As we know $\mu = \dfrac{1}{\sin C}$

$$\mu = \frac{1}{\sin 60^o} = \frac{2}{\sqrt{3}}$$

For total internal reflection to take place

$$\sin C = \frac{1}{\mu} = \frac{2}{3}, \quad C = \sin^{-1}\frac{2}{3} \cong 42^o$$

$\theta < 45^o$

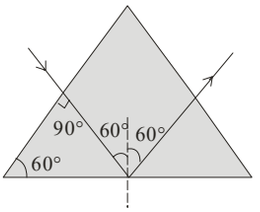
Angle of incidence > critical angle, hence TIR takes place

92. **(b)** $i = \dfrac{A + \delta_m}{2} = \dfrac{60 + 30}{2} = 45°$

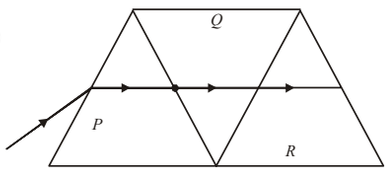
93. **(a)** $\mu = \dfrac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin(A/2)} = \dfrac{\sin 45°}{\sin 26.5°} = 1.58$

94. **(a)** When the angle of prism is small, $\delta = (\mu - 1)A$

95. **(b)**



96. **(c)**



There will be no refraction from P to Q and then from Q to R (all being identical). Hence the ray will now have the same deviation.

97. **(c)** For the prism as the angle of incidence (i) increases, the angle of deviation (δ) first decreases goes to minimum value and then increases.

98. **(d)**

99. **(b)**

100. **(a)** Angular magnification for telescope,

$$|m| = \left|\frac{f_o}{f_e}\right| = \left|\frac{F_1}{F_2}\right| = \frac{F_1}{F_2}$$

101. **(d)** For compound microscope,

$f_o = 1.5$ cm, $f_e = 6.25$ cm

$u_o = -2$ cm

$v_e = -25$ cm

For objective,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} \Rightarrow \frac{1}{1.5} = \frac{1}{v_o} - \frac{1}{-2}$$

$\therefore v_o = 6$ cm

For eyepiece,

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e} \Rightarrow \frac{1}{6.25} = \frac{1}{-25} - \frac{1}{u_e}$$

$\therefore \quad u_e = -5$ cm

$\therefore$ The distance between two lenses is

$|v_o| + |v_e| = 6 + |-5| = 11$ cm

102. **(a)** For compound microscope,

$D = 25$ cm, $f_o = 8$ mm $= 0.8$ cm

$f_e = 2.5$ cm, $u_o = -9$ mm $= -0.9$ cm,

$v_e = -D - 25$ cm

For objective,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} \Rightarrow \frac{1}{0.8} = \frac{1}{v_o} - \frac{1}{-0.9}$$

$\therefore \quad v_o = 7.2$ cm

For eyepiece, $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e} \Rightarrow \frac{1}{2.5} = \frac{1}{-25} - \frac{1}{u_e}$

$\therefore \quad u_e = -2.27$ cm

$\therefore$ The distance between two lenses is

$|v_o| + |u_e| = 7.2 + |-2.27| = 9.47$ cm

103. **(d)** For astronomical telescope, $f_o = 50$ cm, $f_e = 5$ cm, $D = 25$ cm

The length of the telescope when final image is formed at the least distance of distinct vision is given by

$$L = f_o + \frac{f_e D}{f_e + D} = 50 + \frac{5 \times 25}{5 + 25} = \frac{325}{6} \text{cm}$$

104. **(a)**

105. **(b)** Length of tube of telescope,

$L = f_o + f_e \Rightarrow L = 1.5 + 0.05 = 1.55$ m

106. **(a)** For a telescope,

Angular magnification, $m = \frac{\beta}{\alpha} = \frac{f_o}{f_e}$

$$\therefore \quad \frac{\beta}{0.5°} = \frac{0.3}{0.03} \Rightarrow \beta = 5°$$

107. **(c)** $f_o + f_e = 80$ and $\frac{f_o}{f_e} = 19 \Rightarrow f_o = 76$ cm

and $f_e = 4$ cm.

108. **(a)** If final image is formed at infinity, then the distance between the two lenses of telescope is equal to length to tube $= f_e + f_o = 0.3 + 0.05 = 0.35$ m

109. **(d)**

110. **(b)** In normal adjustment,

$$M = \frac{f_0}{f_e} = 20, \; f_e = \frac{f_0}{20} = \frac{60}{20} = 3 \text{ cm}$$

111. **(d)**

112. **(b)** One should increase the power of lens i.e., decrease the focal length of a lens.

113. **(a)** $\because f_0 + f_e = 30$

magnification, $m = \frac{f_0}{f_e}$

$$2 = \frac{f_0}{f_e} \Rightarrow f_0 = 2f_e \Rightarrow f_0 + \frac{f_0}{2} = 30 \; \therefore \; f_0 = 20 \text{ cm}$$

114. **(c)** Magnifying power of telescope,

$$MP = \frac{\beta \text{ (angle subtended by image at eye piece)}}{\alpha \text{ (angle subtended by object on objective)}}$$

Also, $MP = \frac{f_o}{f_e} = \frac{150}{5} = 30, \qquad \alpha = \frac{50}{1000} = \frac{1}{20}$ rad

$$\therefore \quad \beta = \theta = MP \times \alpha = 30 \times \frac{1}{20} = \frac{3}{2} = 1.5 \text{ rad}$$

115. **(c)** Given : Length of compound microscope, $L = 10$ cm

Focal length of objective $f_0 = 1$ cm and of eye-piece, $f_e = 5$ cm

$u_0 = f_e = 5$ cm

Final image formed at infinity (∞), $v_e = \infty$

$v_0 = 10 - 5 = 5$

Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}\Rightarrow\frac{1}{5}-\frac{1}{u_0}=\frac{1}{1}\Rightarrow u_0=-\frac{5}{4}\text{ cm}$$

$$\Rightarrow\frac{5}{4}=\frac{n}{40}\Rightarrow n=\frac{200}{4}=50\text{ cm.}$$

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(c)** For A, u = –15 cm, f = – 20 cm

So, $\frac{1}{V_A}+\frac{1}{-15}=\frac{1}{-20}\Rightarrow\frac{1}{V_A}=\frac{1}{15}-\frac{1}{20}$

$$\Rightarrow\frac{1}{V_A}=\frac{20-15}{300}\Rightarrow\frac{1}{V_A}=\frac{5}{300}\Rightarrow V_A=\frac{300}{5}=60\text{ cm}$$

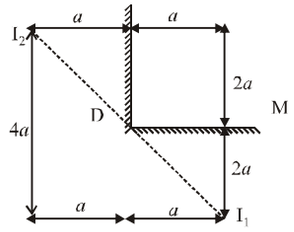
For B, u = – 25 cm, f = – 20 cm

So, $\frac{1}{V_B}+\frac{1}{-25}=\frac{1}{-20}\Rightarrow\frac{1}{V_B}=-\frac{1}{20}+\frac{1}{25}$

$$\Rightarrow\frac{1}{V_B}=\frac{-5}{500}\Rightarrow V_B=-100\text{ cm}$$

So, distance between image of A and B are (100 – (–60)) cm i.e. 160 cm

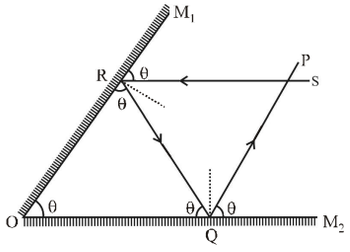
2. **(b)** From figure shortest distance between the images



$$D=\sqrt{(4a^2)+(2a)^2}=2\sqrt{5a^2}=4.6\,a$$

3. **(c)** **4.** **(b)**

5. **(b)**



Let angle between the two mirrors be θ.

Ray PQ ∥ mirror M_1 and Rs ∥ mirror M_2

∴ $M_1Rs=\angle ORQ=\angle M_1OM_2=\theta$

Similarly, $\angle M_2QP=\angle OQR=\angle M_2OM_1=\theta$

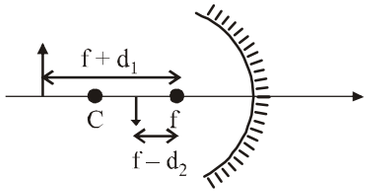
∴ In ΔORQ, $3\theta=180^\circ\Rightarrow\theta=\frac{180^\circ}{3}=60^\circ$

6. **(a)** From Newton's formula

$(f+d_1)(f-d_2)=f^2\Rightarrow f^2+fd_1-fd_2-d_1d_2=f^2$

$$f=\frac{d_1d_2}{d_1-d_2}$$

$$\therefore\ R=2f=\frac{2d_1d_2}{d_1-d_2}$$



7. **(d)**



Mirror will be convex mirror

$V_i=-m^2v_0$

V_i is velocity of image w.r.t mirror

V_0 is velocity of object w.r.t mirror

$V_0=40$ m/s

Magnification $m=\frac{f}{f-u}=\frac{10}{10+190}=\frac{1}{20}$

$$v_i=-\frac{1}{20^2}\times40=-0.1\text{ m/s}$$

8. **(a)** Magnification (m) is given by

$$m=\frac{\text{height of the image}}{\text{height of the object}}=\frac{-25\text{ cm}}{-100\text{ cm}}=\frac{1}{4}$$

Image is diminished and magnification is positive.

∴ It is possible if object is placed in front of convex mirror.

9. **(b)** From mirror formula,

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f'}-\frac{1}{u^2}\frac{du}{dt}-\frac{1}{v^2}\frac{dv}{dt}=0$$

$$\therefore\quad\frac{dv}{dt}=\frac{-v^2}{u^2}\left(\frac{du}{dt}\right)=-\left(\frac{10}{30}\right)^2\times9=-1\text{ m/s}$$

$$\Rightarrow\quad\left|\frac{dv}{dt}\right|=1\text{ m/s}$$

10. **(a)** As, linear magnification, $m=\frac{h_2}{h_1}=\frac{-v}{u}$

where, v is image distance,

$$\Rightarrow\quad v=\frac{-h_2}{h_1}\times u=-\frac{(-3)}{2}\times(-16)=-24\text{ cm}$$

Now, using mirror formula,

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}=-\frac{1}{24}-\frac{1}{16}=\frac{-2-3}{48}=\frac{-5}{48}$$

$$\Rightarrow\quad f=\frac{-48}{5}=-9.6\text{ cm}$$

11. **(c)**

12. **(b)** Object distance $u = -40$ cm

Focal length $f = -20$ cm

According to mirror formula

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \text{ or } \frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

$$\text{or } \frac{1}{v}=\frac{1}{-20}-\frac{1}{(-40)}=\frac{1}{-20}+\frac{1}{40} \text{ or } v=-40\,\text{cm}.$$

Negative sign shows that image is infront of concave mirror. The image is real.

Also, object is at centre of curvature hence, image is also at centre of curvature.

$$\text{Magnification, } m=-\frac{v}{u}=-\frac{(-40)}{(-40)}=-1$$

The image is of the same size and inverted.

13. **(a)** Here, $u = f + x_1, v = f + x_2$

$$\text{use } f=\frac{uv}{u+v} \text{ and solve to get } f=\sqrt{x_1x_2}$$

14. **(c)** $m=-\frac{v}{u}=-n \Rightarrow v=nu$

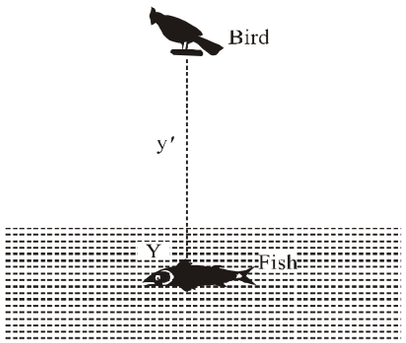
$$\text{using, } \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$f=\frac{nu}{(n+1)}$$

$$\frac{f(n+1)}{n}=u$$

15. **(a)** Effective height of the bird as seen by the fish,

$Y = y + \mu y'$



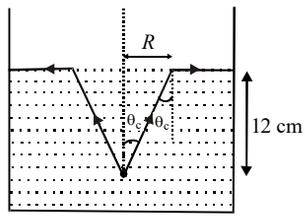
Differentiating both sides w.r.t.t

$$\frac{dY}{dt}=\frac{dy}{dt}+\mu\frac{dy'}{dt}$$

Given $\frac{dY}{dt}=12\,\text{ms}^{-1}; \frac{dy}{dt}=8\,\text{ms}^{-1}$

$$\therefore 12=8+\frac{4}{3}\frac{dy}{dt} \qquad \therefore \frac{dy'}{dt}=\frac{4\times3}{4}=3\,\text{ms}^{-1}$$

16. **(a)** $\sin\theta_c=\frac{1}{\mu}=\frac{3}{4}$



$$\text{or } \tan\theta_c=\frac{3}{\sqrt{16-9}}$$

$$=\frac{3}{\sqrt{7}}=\frac{R}{12}$$

$$\Rightarrow R=\frac{36}{\sqrt{7}}\text{ cm}$$

17. **(a)** Refractive index, $\frac{\mu_2}{\mu_{air}}=\frac{c}{v_2}$

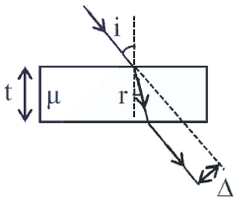
$$\text{or, } \therefore \frac{\sqrt{\mu_{r_2}\varepsilon_{r_2}}}{(1)}=\frac{c}{v_2} \qquad \therefore \sqrt{(1)(9)}=\frac{c}{v_2} \qquad \therefore v_2=\frac{c}{3}$$

18. **(b)**

19. **(a)** Given, thickness of glass slab

$t = 4\sqrt{3}$ cm, $\mu = \sqrt{2}$ and

$i = \theta_c$



Lateral displacement

$$\Rightarrow i=\sin^{-1}\left(\frac{1}{\mu}\right)=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$\therefore i = 45°$

And according to Snell's law

$\mu_1 \sin i = \mu_2 \sin r$

$1 \sin 45° = \sqrt{2} \sin r \ \therefore r = 30°$

$$\text{Lateral displacement } \Delta=\frac{t\sin(i-r)}{\cos r}$$

$$\Rightarrow \Delta=\frac{4\sqrt{3}\times\sin15°}{\cos30°}=\frac{4\sqrt{3}\times0.25}{\frac{\sqrt{3}}{2}} \qquad \therefore \Delta=2\text{cm}$$

20. **(d)**

21. **(b)** From the Snell's law of refraction

$\mu_1 \sin 45° = \mu_2 \sin 30°$

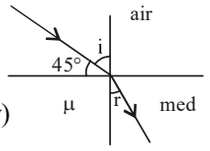
Here, μ_1 and μ_2 are the refractive index of two medium.

$$\therefore \mu_1\times\frac{1}{\sqrt{2}}=\mu_2\times\frac{1}{2} \Rightarrow \frac{\mu_1}{\mu_2}=\frac{1}{\sqrt{2}} \ \because \mu\propto\frac{1}{\lambda}$$

$$\therefore \frac{\mu_1}{\mu_2}=\frac{\lambda_2}{\lambda_1}=\frac{1}{\sqrt{2}} \Rightarrow \lambda_2=\frac{\lambda_1}{\sqrt{2}}$$

Frequency of light wave doesn't change with medium.

22. **(c)** $i=45°, D=15°$



$D=i-r$

$15°=45°-r \Rightarrow r=30°$

$n_1 \sin 1 = n_2 \sin r$ (from snells's law)

$1 \sin 45° = \mu \sin 30°$

$$\frac{1}{\sqrt{2}}=\mu\frac{1}{2} \Rightarrow \mu=\sqrt{2}=1.414$$

23. **(d)** Apparent rise $= d\left(1-\frac{1}{a\mu_w}\right)=12\times\left(1-\frac{3}{4}\right)=3$ cm.

24. **(a)**

25. **(c)** Critical angle $=\sin^{-1}\left(\frac{1}{\mu}\right)$

$$\therefore \quad \theta \sin^{-1}\left(\frac{1}{\mu_{\lambda_1}}\right) \text{ and } \theta' = \sin^{-1}\left(\frac{1}{\mu_{\lambda_2}}\right)$$

Since $\mu_{\lambda_2} > \mu_{\lambda_1}$, hence $\theta' < \theta$

26. **(c)** $$^{a}n_g=\frac{\sin i}{\sin r}=\frac{\sin 50°}{\sin 40°} \quad ...(i)$$

Similarly, $$^{a}n_w=\frac{\sin 50°}{\sin 45°} \quad ...(ii)$$

$$^{w}n_g=\frac{\sin 45°}{\sin \theta} \quad ...(iii)$$

As, $^{a}n_w \times {}^{w}n_g = {}^{a}n_g$

$$\Rightarrow \quad \frac{\sin 50°}{\sin 45°}\times\frac{\sin 45°}{\sin\theta}=\frac{\sin 50°}{\sin 40°}$$

[using Eqs. (i), (ii) and (iii)]

$\sin\theta = \sin 40°$ or $\theta = 40°$

27. **(a)**
$$\left.\begin{aligned}\mu_g &= \frac{c}{v_g} \Rightarrow \frac{3}{2}=\frac{c}{v_g}\\ \mu_\omega &= \frac{c}{v_\omega} \Rightarrow \frac{4}{3}=\frac{c}{v_\omega}\end{aligned}\right\}$$

Dividing, $\frac{v_\omega}{v_g}=9:8$ or $v_g : v_\omega = 8:9$

28. **(d)** $\sin C=\frac{1}{\mu}=\frac{1}{\sqrt{2}}$

$$\therefore \ C=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=45°$$

Now $\frac{\sin C}{\sin r}=\frac{1}{\mu}$ or $\frac{\sin 45°}{\sin r}=\frac{1}{\sqrt{2}}$

$\sin r = 1$ or $r = 90°$

29. **(a)** The critical angle for total internal reflection from glass to water,

$$\sin i_c=\frac{\mu_{water}}{\mu_{glass}}=\frac{1.33}{1.55}=\frac{8}{9} \Rightarrow \ i_c=\sin^{-1}\left(\frac{8}{9}\right)$$

30. **(c)** $R = 20$ cm, $h_0 = 2$, $u = -30$ cm

We have, $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$=\left(\frac{3}{2}-1\right)\left[\frac{1}{20}-\left(-\frac{1}{20}\right)\right] \ \therefore f=20 \text{ cm}$$

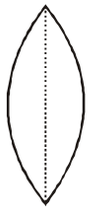
$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{1}{20}=\frac{1}{v}+\frac{1}{30}$$

$$\frac{1}{v}=\frac{1}{20}-\frac{1}{30} \Rightarrow v=60 \text{ cm}$$

$$\Rightarrow h_i=\frac{v}{u}\times h_0=\frac{60}{30}\times 2=-4 \text{ cm}$$

So, image is inverted.

31. **(b)** Power of lens, $P=\frac{1}{f}$



Let power of each part is P_1

$$\therefore \quad P_1+P_1=P=\frac{1}{f} \Rightarrow 2P_1=\frac{1}{0.1}=10$$

$$\Rightarrow \quad P_1=5D$$

Hence, the power of each lens after cut is 5D.

32. **(c)**

33. **(b)** From lens maker's formula,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)=\frac{1}{2f_2}$$

Similarly, for plano-concave lens

$$\frac{1}{f_2}=(\mu_2-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)=-\frac{(\mu_2-1)}{R}$$

We will ignore – ve sign as it does hold too much importance at this point of time.

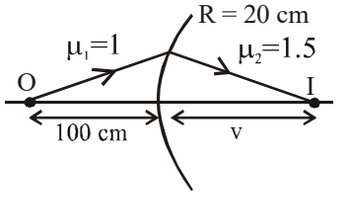
Dividing $\frac{1}{f_1}$ by $\frac{1}{f_2}$ we get,

$$\frac{(\mu_1-1)}{R}=\frac{(\mu_2-1)}{2R}\Rightarrow 2\mu_1-\mu_2=1$$

34. **(d)** Given,

Radius of convex curved surface, R = 20 cm

Distance of object from curved surface, u = 100 cm



$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\Rightarrow\frac{1.5}{v}-\frac{1}{-100}=\frac{1.5-1}{20}\Rightarrow\frac{1.5}{v}=\frac{1}{40}-\frac{1}{100}\Rightarrow v=100\text{ cm}$$

Distance from object = 100 + 100 = 200 cm

35. **(b)** Using lens formula,

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{-16}+\frac{1}{12}\Rightarrow v=48\text{ cm}$$

36. **(d)**

37. **(d)**

38. **(b)** From the lens maker's formula

$$\frac{1}{f}=\left(\frac{\mu_\ell}{\mu_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Here $R_1=R_2=R=20$ cm

$$\therefore P=\frac{1}{f}=\left(\frac{\mu_\ell}{\mu_m}-1\right)\left(\frac{1}{R}-\frac{1}{-R}\right)$$

When lens is in air

$\mu_\ell=1.8$, $\mu_{air}=1$

$$\therefore P=(1.8-1)\left(\frac{1}{20}+\frac{1}{20}\right)$$

When lens is immersed in liquid

$\mu_\ell=1.8$, $\mu_m=1.5$

$$\therefore P'=\left(\frac{1.8}{1.5}-1\right)\left(\frac{1}{20}+\frac{1}{20}\right);$$

Dividing $\frac{P}{P'}=\frac{0.8}{1.2-1}=4$

39. **(b)** Using lens maker's formula

$$\frac{1}{f}=\left(\frac{\mu_g}{\mu_a}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

Here, μ_g and μ_a are the refractive index of glass and air respectively

$$\Rightarrow\quad\frac{1}{f}=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\qquad\text{...(i)}$$

When immersed in liquid

$$\frac{1}{f_l}=\left(\frac{\mu_g}{\mu_l}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

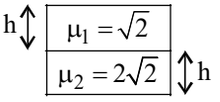
[Here, μ_l = refractive index of liquid]

$$\Rightarrow\frac{1}{f_l}=\left(\frac{1.5}{1.42}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\qquad\text{...(ii)}$$

Dividing (i) by (ii)

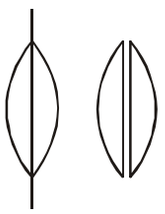
$$\Rightarrow\quad\frac{f_l}{f}=\frac{(1.5-1)1.42}{0.08}=\frac{1.42}{0.16}=\frac{142}{16}\approx 9$$

40. **(d)** Apparent depth,



$$D=\frac{t_1}{\mu_1}+\frac{t_2}{\mu_2}=\frac{h}{\sqrt{2}}+\frac{h}{2\sqrt{2}}=\frac{3h}{2\sqrt{2}}=\frac{3h\sqrt{2}}{4}$$

41. **(d)** $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$



This is the combination of two lenses of equal focal lengths

$$\therefore\ \frac{1}{f}=\frac{1}{f'}+\frac{1}{f'}=\frac{2}{f'}\Rightarrow f'=2f.$$

42. **(c)** According to lens maker's formula,

$$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

If lens is in water

$$\frac{1}{f_w}=\left(\frac{\mu_g}{\mu_w}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\qquad\text{...(i)}$$

If lens in air,

$$\frac{1}{f_a}=\left(\mu_g-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\qquad\text{...(ii)}$$

Dividing eqn (ii) by (i),

$$\frac{f_w}{f_a}=\left(\frac{\mu_g-1}{\frac{\mu_g}{\mu_w}-1}\right)=\left(\frac{\frac{3}{2}-1}{\frac{3\times3}{2\times4}-1}\right)=\frac{1}{4}$$

43. **(b)** Let object distance = u

$\therefore\ u+v=x$ ...(i)

Magnification, $m=\frac{f}{f-u}=\frac{f-v}{f}$ ...(ii)

As image is real, magnification is negative.

$$-m=\frac{f}{f-u}\Rightarrow u=\frac{(m+1)f}{f}$$

Again, $-m=\frac{f-v}{f}\ \Rightarrow\ v=(m+1)f$

Put this value of v in (i), $x=\frac{(m+1)}{m}f+(m+1)f$

On solving, we get, $f=\frac{mx}{(m+1)^2}$

44. **(b)** From Lens maker's formula,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

For equiconvex lens, $R_1=+R$ and $R_2=-R$

$$\therefore\quad -4.5=(1.6-1)\left(\frac{1}{R}+\frac{1}{R}\right)\ \text{or}\ 4.5=0.6\times\frac{2}{R}$$

or $R=\frac{-1.2}{4.5}=-0.266\,\text{m}=-26.6\,\text{cm}$

45. **(a)** Using thin lens formula :

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{+75}-\frac{1}{-25}=\frac{1}{75}+\frac{1}{25}=\frac{4}{75}$$

$$\Rightarrow f=\frac{75}{4}=+18.75\,\text{cm}$$

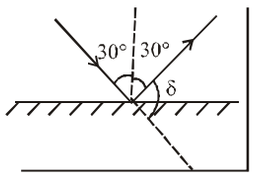
The lens is a convex lens.

46. **(d)** u = – 20 cm, f = 10 cm

$$\frac{1}{f}=\frac{-1}{u}+\frac{1}{v}$$

$$\Rightarrow\frac{1}{v}=\frac{1}{10}-\frac{1}{20}\ \Rightarrow\quad v=20\,\text{cm}.$$

47. **(b)**



From the figure, it is clear

$\delta=180°-2\times30°=120°$

48. **(c)** Angle of prism A = 60°

By prism formula

$$\mu=\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}\ \Rightarrow\text{or}\ \sqrt{3}=\frac{\sin\left(\frac{60°+\delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$\frac{\sqrt{3}}{2}=\sin\left(\frac{60°+\delta_m}{2}\right)\Rightarrow\sin60°=\sin\left(\frac{60°+\delta m}{2}\right)$$

$$60°=\frac{60°+\delta_m}{2}\Rightarrow\delta_m=60°$$

$$\theta=\frac{\delta_m+A}{2}=\frac{60°+60°}{2}=60°$$

49. **(a)** Given $i=60°$

$A=\delta=e$

$\delta=i+e-A\ \Rightarrow\ \delta=i\quad(\because e=A)$

$$\mu=\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

Here angle of deviation is min.($\because$ i = e)

$$\therefore\ \mu=\frac{\sin\left(\frac{60°+60°}{2}\right)}{\sin\frac{60°}{2}}=1.73$$

50. **(c)** Given $i=2r_1=A$

And at minimum deviation $r_1=r_2=\frac{A}{2}$

From Snell's law,

$\mu_1\sin i=\mu_2\sin r_1\ \therefore 1.\sin i=\sqrt{3}\sin r_1$

$$\Rightarrow 1\sin A=\sqrt{3}\sin\frac{A}{2}\Rightarrow 2\sin\frac{A}{2}\cos\frac{A}{2}=\sqrt{3}\sin\frac{A}{2}$$

$$\Rightarrow\cos\frac{A}{2}=\frac{\sqrt{3}}{2}\Rightarrow\frac{A}{2}=30°\ \text{or, A = 60°}$$

Hence angle of prism = 60°

51. **(a)** According to prism formula,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Here, δ_m = angle of minimum deviation

A = angle of prism

We have given, $\delta_m = A$

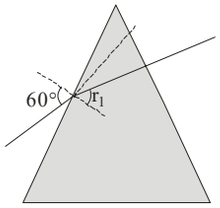
$$\therefore \quad \mu = \frac{\sin A}{\sin A/2} \Rightarrow \mu = 2\cos\frac{A}{2} \Rightarrow A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$$

52. **(b)** **53.** **(b)**

54. **(d)** From figure it is clear that

$\angle e = \angle r_2 = 0$

From $A = r_1 + r_2 \Rightarrow r_2 = A = 45°$



$$\therefore \quad \mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60°}{\sin 45°} = \sqrt{\frac{3}{2}}$$

Also from

$i + e = A + \delta \Rightarrow 60 + 0 = 45 + \delta \Rightarrow \delta = 15$

55. **(a)** Deviation produced by a thin prism

$\delta = A(\mu - 1)$

In air, $\delta_1 = A({}_a\mu_g - 1) = A\left(\frac{3}{2} - 1\right) = \frac{A}{2}$

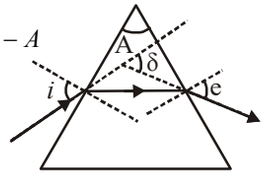
In water, ${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_\omega} = \frac{3/2}{4/3} = \frac{9}{8}$

$$\delta_2 = A({}_w\mu_g - 1) = A\left(\frac{9}{8} - 1\right) = \frac{A}{8}$$

$$\frac{\delta_1}{\delta_2} = \frac{\frac{A}{2}}{\frac{A}{8}} = \frac{8}{2} = \frac{4}{1}$$

56. **(b)** From the fig.

Angle of deviation, $\delta = i + e - A$



Here, $e = i$ and $e = \frac{3}{4}A$

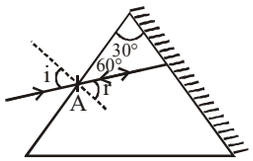
$$\therefore \ \delta = \frac{3}{4}A + \frac{3}{4}A - A = \frac{A}{2}$$

For equilateral prism, $A = 60°$

$$\therefore \ \delta = \frac{60°}{2} = 30°$$

57. **(d)** The incident ray will retrace its path, only when it strikes the surface of prism at 90°.

As the normal ray on a plane mirror retraces its path. Applying Snell's law at A,



$$\frac{\sin i}{\sin 30°} = \sqrt{2} = \mu$$

$$\Rightarrow \sin i = \sqrt{2} \times \sin 30°$$

$$= \sqrt{2} \times \frac{1}{2} \quad = \frac{1}{\sqrt{2}} = \sin 45° \Rightarrow i = 45°$$

58. **(b)** $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \Rightarrow \sqrt{2}\mu = \frac{\sin\left(\frac{60+\delta_m}{2}\right)}{\sin\left(\frac{60}{2}\right)}$

$$\Rightarrow \quad \sqrt{2} \times \sin 30 = \sin\left(\frac{60+\delta_m}{2}\right) \Rightarrow \sin 45°$$

$$= \sin\left(\frac{60+\delta_m}{2}\right) \Rightarrow \delta_m = 30°$$

59. **(c)** For least distance of distinct vision, the magnifying power

$$M = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right) = -\frac{60}{5}\left(1 + \frac{5}{25}\right) = -14.4$$

Now, $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$

v_e and u_e are negative

$$\therefore \quad \frac{1}{f_e} = -\frac{1}{25} - \left(-\frac{1}{u_e}\right)$$

$$\Rightarrow \frac{1}{u_e} = \frac{1}{25} + \frac{1}{5} \text{ or, } u_e = 4.17 \text{ cm}$$

The length of telescope in this position

$L = f_0 + u_e = 60 + 4.17 = 64.17$ cm

60. **(a)**

61. **(d)** $|M| = \frac{f_0}{f_e} = 5$, $L = f_0 + f_e = 36$

$\therefore f_e = 6$ cm, $f_0 = 30$ cm

62. **(a)** According question, $M = 375$

$L = 150$ mm, $f_0 = 5$ mm and $f_e = ?$

Using, magnification, $M \simeq \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$

$$\Rightarrow 375 = \frac{150}{5}\left(1+\frac{250}{f_e}\right) \quad (\because D = 25\,\text{cm} = 250\,\text{mm})$$

$$\Rightarrow 12.5 = 1+\frac{250}{f_e} \Rightarrow f_e = \frac{250}{11.5} = 21.7 \approx 22\,mm$$

63. **(c)** Magnification will be done by compound microscope only when $f_o < f_e$

64. **(a)** $f_o + f_e = 54$ and $\frac{f_o}{f_e} = m = 8 \Rightarrow f_o = 8f_e$

$$\Rightarrow \quad 8f_e + f_e = 54 \Rightarrow f_e = \frac{54}{9} = 6$$

$$\Rightarrow \quad f_o = 8f_e = 8 \times 6 = 48$$

65. **(a)** (i) $M = -\frac{f_0}{f_e}\left(1+\frac{f_e}{d}\right) = -\frac{200}{5}\left(1+\frac{5}{25}\right) = -48$

(since least distance d = 25cm)

(ii) $M = -\frac{f_0}{f_e} = -\frac{200}{5} = -40$

Exercise 3 : PREVIOUS YEARS MCQs

1. **(b)** Minimum deviation of the prism when it is dipped in water = $\delta_m' = ({}_w\mu_g - 1)A$

$$= \left(\frac{{}_a\mu_g}{{}_a\mu_\omega} - 1\right)A = \left(\frac{\frac{3}{2}}{\frac{4}{3}} - 1\right)A = \frac{1}{8}A$$

2. **(c)** The magnifying power of telescope in normal adjustment is given by, $M = \frac{f_0}{f_e}$

$$\Rightarrow \quad 10 = \frac{f_0}{20} \Rightarrow f_0 = 200\,\text{cm}.$$

3. **(None)** Mirror formula $\frac{1}{v}+\frac{1}{u} = \frac{1}{f}$

For convex mirror V & f positive and u negative

$$\frac{h_i}{h_o} = \frac{v}{u} \text{ or, } n = \frac{v}{u} \therefore v = nu$$

$$\frac{1}{v}+\frac{1}{u} = \frac{1}{f} \text{ or } \frac{1}{nu}+\frac{1}{-u} = \frac{1}{f}$$

$$\text{or, } \frac{1}{f} = \frac{u-nu}{(nu)u} = \frac{1-n}{nu} \text{ or, } u = f\left(\frac{1-n}{n}\right)$$

$$\text{or, } u = \frac{f}{n} - f$$

4. **(a)**

5. **(a)** Use lens maker's formula:

$$\frac{1}{f} = \left(\frac{\mu_{less}}{\mu_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Let $C = \left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

Given: $f_{air} : f_{liquid} = 1 : 2$

$$f_{liquid} = 2f_{air} \Rightarrow \frac{1}{f_{liquid}} = \frac{1}{2f_{air}}$$

$$\frac{1}{f_{liquid}} = \left(\frac{1.5}{\mu} - 1\right)C = \frac{1}{2}\left(\frac{1}{f_{air}}\right) = \frac{1}{2} \times 0.5C = 0.25C$$

$$\left(\frac{1.5}{\mu} - 1\right)C = 0.25C \Rightarrow \frac{1.5}{\mu} - 1 = 0.25 \Rightarrow \frac{1.5}{\mu} = 1.25$$

$$\mu = \frac{1.5}{1.25} = 1.20$$

6. **(c)** Use the less formula:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Given: v = –35cm

u = –25cm

$$\frac{1}{f} = \frac{1}{-3s} - \frac{1}{-2s} = \frac{-1}{35} + \frac{1}{25}$$

$$\frac{1}{f} = \frac{-10}{875}$$

f = +87.5 cm

The focal length is positive, the lens must be convex (converging)

7. **(d)** For a convex lens,

$u_1 = -20$ cm, $u_2 = -10$cm

The magnification of a lens is

$$m = \frac{f}{u+f} \Rightarrow m_1 = \frac{f}{u_1+f} \text{ and } m_2 = \frac{f}{u_2+f}$$

For the same size of images formed by the convex lens, $m_1 = -m_2$

$$\Rightarrow \frac{f}{u_1+f} = -\frac{f}{u_2+f}$$

$$\Rightarrow \frac{1}{-20+f} = -\frac{1}{-10+f}$$

$\Rightarrow 2f = 10 + 20$

$\therefore f = 15$ cm

8. **(d)** Let f_1 and f_2 be the focal lengths

Formula: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

and $f = 4$ cm, $f_1 + f_2 = 18$

Given: $f_1 = 6$cm, $f_2 = 12$ cm

$$\frac{1}{6} = \frac{1}{12} + \frac{2+1}{12} = \frac{3}{12}$$

$f = 4$

Less of low power has longer focal length = 12 cm

9. **(d)** Power of a lens $P = \frac{1}{f(\text{in metre})}$

To see very distant objects need concave lens so f is negative.

$$P = \frac{1}{-\frac{400}{100}} = -\frac{100}{400} = -0.25\ D$$

10. **(d)** $i = 60°$

Prism angle, $A = 30°$

when second face is silvered, and the ray retraces its path

$\Rightarrow$ ray hits the second face normally ($\angle i = 0$)

$r = A = 30°$

on applying shell's law of first face

$\mu_{air} \sin i = \mu_{prism} \sin r$

$1 \times \sin 60° = \mu \times \sin 30°$

$$\frac{\sqrt{3}}{2} = \mu \times \frac{1}{2}$$

$$\boxed{\mu = \sqrt{3}}$$

11. **(d)** Given, length of rod = 6 cm.

Focal length = – 9 cm

One end of the rod is at $u_1 = -15$cm

Using Mirror formula

$$\frac{1}{f} = \frac{1}{v_1} + \frac{1}{u_1} \Rightarrow -\frac{1}{9} = \frac{1}{v_1} - \frac{1}{15}$$

$$\Rightarrow \frac{1}{v_1} = -\frac{1}{9} + \frac{1}{15} = \frac{-5+3}{45} = \frac{-2}{45}$$

Rod is 6cm long $\rightarrow$ second end is at

$u_2 = -15 - 6 = -21$ cm

Using mirror formula again

$$\frac{1}{f} = \frac{1}{v_2} + \frac{1}{u_2} \Rightarrow -\frac{1}{9} = \frac{1}{v_2} - \frac{1}{21}$$

$$\Rightarrow \frac{1}{v_2} = -\frac{1}{9} + \frac{1}{21} = \frac{-7+3}{63} = \frac{-4}{63}$$

$\Rightarrow v_2 = -15.75$ cm

Image length $|v_1 - v_2| = |-22.5 - (-15.75)| = 6.75$ cm

12. **(b)** Given,

Object distance from principal focus = 16 cm.

Image distance from principal focus = 9 cm.

Using newton's formula

$$f = \sqrt{x_1 x_2} = \sqrt{16 \times 9} = \sqrt{144}$$

f = 12 cm.

13. **(a)**

14. **(b)** For concave mirror,

$u = -24$ cm, $v = -12$cm, $v_o = 12 ms^{-1}$

$\therefore$ speed of the image is

$$v_i = -m^2 v_o = -\left(-\frac{v}{u}\right)^2 v_o = -\left(\frac{-12}{-24}\right)^2 \times 12$$

$= -3$ m/s

15. **(c)** $R_1 = +6$ cm, $R_2 = 12$ cm

$\mu_{lens} = 1.5$, $\mu_{liquid} = 1.3$

Using maker's formula in a medium

$$\frac{1}{f} = \left(\frac{\mu_{lens}}{\mu_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f} = \left(\frac{2}{13} \times \frac{1}{4}\right) \Rightarrow \boxed{f = 26\ cm}$$

16. **(c)** Angle of incidence on first face, i = 9.3°

Refracting angle of the prism, A = 6°

The ray emerges normally from the second face

Angle of emergence e = 0°

Angle of refraction at second surface $r_2 = 0°$

From prism geometry

$r_1 + r_2 = A \Rightarrow r_1 + 0 = 6 \Rightarrow r_1 = 6°$

Now apply snell's law at first surface

$$\eta = \frac{\sin i}{\sin r_1} = \frac{\sin 9.3°}{\sin 6°} = \frac{0.1616}{0.1045} \Rightarrow n = 1.546$$

17. **(c)**

18. **(d)**

19. **(a)**

20. **(b)** Dispersion is the fundamental property of light that causes coloured rays to separate.

21. **(a)** **22.** **(b)**

23. **(a)** Magnification produced by compound microscope

$$M = m_0 \times \left(1 + \frac{D}{fe}\right) \Rightarrow 24 = m_0 \times \left(1 + \frac{25}{5}\right)$$

$$\therefore m_0 = \frac{24}{6} = 4$$

7 Electrostatics

Exercise 1: WARM-UP
Topic-wise MCQs

1. **(a)** Number of protons $= \frac{1C}{1.66\times10^{-19}C} \approx 6\times10^{18}$

2. **(b)**

3. **(c)** In case of metallic sphere, either solid or hollow, the charge will reside on the surface of the sphere. Since both spheres have same surface area, so they can hold equal maximum charge.

4. **(b)** On touching the metal knob with a positively charged rod some electrons from the gold leaves get transferred to the rod making gold leaves positively charged and they get separated. When a negatively charged rod is touched with metal knob some negative charge flows to the gold leaves lessening the positive charge there and the separation between the leaves decreases.

5. **(d)** The net charge on the object = 1C – ne $[\because Q = ne]$
$= 1C - 5\times10^{18}\times1.6\times10^{-19}C = +0.20\,C$

6. **(a)** In case of spherical metal conductor, the charges quickly spread uniformly over the entire surface because of which charges stay for longer time on the spherical surface. While in case of non-spherical surface, the charge concentration is different at different points due to which the charges do not stay on the surface for longer time.

7. **(a)** When a point positive charge is brought near an isolated conducting plane, some negative charge develops on the surface of the plane towards the charge and an equal positive charge develops on opposite side of the plane. This process is called charging by induction. So, the field at a point P on the other side of the plane is away from the plane and field lines are always perpendicular to the surface of conductor.

8. **(b)** Here, $q_1 = 1\times10^{-7}C$, q_2 and 2×10^{-7} C, $r = 20$ cm $= 20\times10^{-2}$ m

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9\times1\times10^{-7}\times2\times10^{-7}}{(20\times10^{-2})^2} = 4.5\times10^{-3}\,N$$

9. **(d)** From Coulomb's law $F = \frac{Kq_1q_2}{r^2}$ i.e., $F \propto \frac{1}{r^2}$ which is correctly shown by graph (d).

10. **(b)** Electrostatic force is given by $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

Here, charge and distance are same. So, force between two protons will be same.

11. **(c)** $\frac{q_1q_2}{r^2} = \frac{q_1q_2}{Kr'^2}$ or $Kr'^2 \Rightarrow r^2r' = \frac{r}{\sqrt{K}}$

12. **(a)** Here, $r = 3.2\times10^{-15}$m
Charge on α-particles, $q_1 = q_2 = 2\times1.6\times10^{-19}$C
Now,

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2} = 9\times10^9\frac{(2\times1.6\times10^{-19})^2}{(3.2\times10^{-15})^2} = 90N$$

13. **(b)** Let the charge q be placed between the given two charges and at a distance r from the charge 9e.

$$\frac{K(9e)q}{r^2} = \frac{Keq}{(16-r)^2} \Rightarrow \frac{3}{r} = \frac{1}{16-r} \Rightarrow r = 12cm$$

14. **(d)** They will not experience any force if $|\vec{F}_G| = |\vec{F}_e|$

$$\Rightarrow\ G\frac{m^2}{(16\times10^{-2})^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{(16\times10^{-2})^2} \Rightarrow \frac{q}{m} = \sqrt{4\pi\varepsilon_0 G}$$

15. **(b)** Here $q_1 = q_2 = 3.2\times10^{-7}$ C, $r = 60$ cm $= 0.6$ m

Electrostatic force, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

$$= \frac{9\times10^9(3.2\times10^{-7})^2}{(0.6)^2} = 2.56\times10^{-3}\,N$$

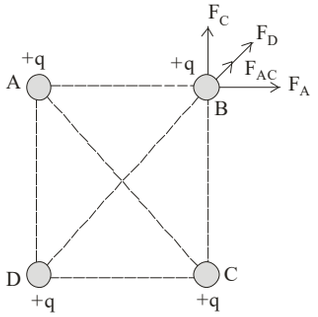
16. **(d)**

17. **(c)** When a soap bubble is charged, the charge gets uniformly distributed over its surface which results in repulsion between nearby charges. This causes, the bubble to expand.

18. **(c)** Force on each charge is zero. But if any of the charge is displaced, net force starts acting on all of them.

19. **(c)** $F_{net} = F_{AC} + F_D = \sqrt{F_A^2 + F_C^2} + F_D$

Since $F_A = F_C = \frac{kq^2}{a^2}$ and $F_D = \frac{kq^2}{(a\sqrt{2})^2}$



$$= F_{net} = \frac{\sqrt{2}kq^2}{a^2} + \frac{kq^2}{2a^2} = \frac{kq^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$= \frac{q^2}{4\pi\varepsilon_0 a^2}\left(\frac{1+2\sqrt{2}}{2}\right)$$

20. **(a)** According to coulomb's law, the force between two charges q_1 and q_2 is

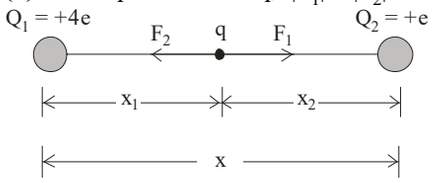
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1(2Q - q_1)}{r^2} \quad (\because q_1 + q_2 = 2Q)$$

For F to be maximum $\frac{dF}{dq_1} = 0$ as Q and r are constant.

$$\therefore \frac{d}{dq_1}\left[\frac{1}{4\pi\varepsilon_0 r^2}\left(2Qq_1 - q_1^2\right)\right] = 0$$

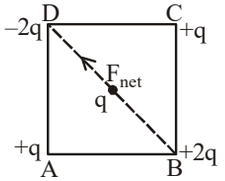
$2Q - 2q_1 = 0$; Hence, $\frac{Q}{q_1} = 1$

21. **(c)** For equilibrium of q, $|F_1| = |F_2|$



$$\Rightarrow \quad x_2 = \frac{x}{\sqrt{\frac{Q_1}{Q2}} + 1} = \frac{x}{\sqrt{\frac{4e}{e}} + 1} = \frac{x}{3}$$

22. **(d)** Polarities of the charges at *B* and *D* are opposite. So, force on charge kept at the centre is along the diagnol *BD*.



But due to same polarities of charges at A and C forces at O cancel each other.

23. **(a)** Q_1 ——— Q_2 $\quad K = 5 \quad F = \frac{1}{4\pi\varepsilon_0 k}\frac{Q_1 Q_2}{r^2}$

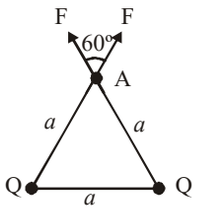
Q_1 ——— Q_2 force in the charges in the air is

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2} = K\,F = 5\,F$$

24. **(b)** $\therefore F = \frac{kQ^2}{a^2}$

$$F_{net} = \sqrt{F^2 + F^2 + 2F^2\cos 60^\circ}$$

$$= \sqrt{3}F = \frac{\sqrt{3}kQ^2}{a^2}$$



25. **(d)** $Q_1 + Q_2 = Q$ … (i) and $F = k\frac{Q_1 Q_2}{r^2}$ … (ii)

From (i) and (ii) $F = \frac{kQ_1(Q - Q_1)}{r^2}$

For F to be maximum $\frac{dF}{dQ_1} = 0 \Rightarrow Q_1 = Q_2 = \frac{Q}{2}$

26. **(b)** $T_{AB} = \frac{kq \cdot 2q}{d^2} + \frac{kq \cdot 4q}{(2d)^2} = \frac{3kq^2}{d}$

$$T_{BC} = \frac{k \cdot 4q \cdot 2q}{d^2} + \frac{kq \cdot 4q}{(2d)^2} = \frac{9kq^2}{d^2}$$

27. **(d)** Charges (–e) on electron and (e) on proton exert a force of attraction given by

$$\text{Force} = (K)\frac{(-e)(e)}{r^2}\hat{r} = \frac{-Ke^2}{r^3}\vec{r} \quad \left(\because \hat{r} = \frac{\vec{r}}{|r|}\right)$$

28. **(b)** Let distance between the two divided charges be r. From Coulomb's law, force between two charge,

$$F = \frac{Kq(4-q)}{r^2}$$

For F to be maximum, $\frac{dF}{dq} = \frac{K}{r^2}[4 - 2q] = 0 \Rightarrow q = 2$

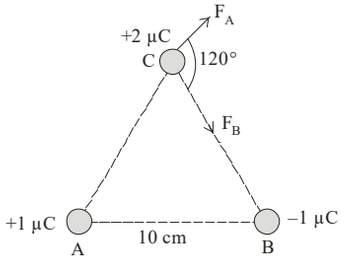
29. **(d)**

30. **(b)** F_A = Force on C due to charge placed at A

$$= 9\times10^9 \times \frac{10^{-6}\times 2\times10^{-6}}{(10\times10^{-2})^2} = 1.8\text{N}$$

F_B = Force on C due to charge placed at B

$$= 9\times10^9 \times \frac{10^{-6}\times 2\times10^{-6}}{(0.1)^2} = 1.8\text{N}$$



Net force on C,

$$F_{net} = \sqrt{(F_A)^2 + (F_B)^2 + 2F_A F_B\cos 120^\circ} = 1.8\text{N}$$

31. **(b)**

32. **(d)** Let two sphere carrying charges q_1 and q_2. According to Coulomb's law, the force of repulsion between q_1 and q_2 separated by a distance d is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(+6\mu C)(+9\mu C)}{d^2} \quad \text{... (i)}$$

When a charge of –3 μC is given to both the spheres.

New charge on $q_1 = +6\ \mu C - 3\ \mu C = +3\ \mu C$
New charge on $q_2 = +9\ \mu C - 3\ \mu C = +6\ \mu C$
From the Coulomb's law

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{(+3\,\mu C)(+6\,\mu C)}{d^2} \quad \ldots(ii)$$

Dividing eqn. (ii) by eqn. (i), we get

$$\frac{F'}{F} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{(+3\mu C)(+6\,\mu C)}{d^2}}{\frac{1}{4\pi\varepsilon_0}\frac{(+6\mu C)(+9\mu C)}{d^2}} = \frac{1}{3} \text{ or } F' = \frac{F}{3}$$

33. (d)

34. (b) Force on charge q_1 due to q_2 is $F_{12} = k\frac{q_1q_2}{b^2}$

Force on charge q_1 due to q_3 is $F_{13} = k\frac{q_1q_3}{a^2}$

The X - component of the force (F_x) on

q_1 *is* $F_{12} + F_{13}\sin\theta \quad \therefore\ F_x = k\frac{q_1q_2}{b^2} + k\frac{q_1q_2}{a^2}\sin\theta$

$$\therefore\ F_x \propto \frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$$

35. (c) In vacuum, $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$ …(i)

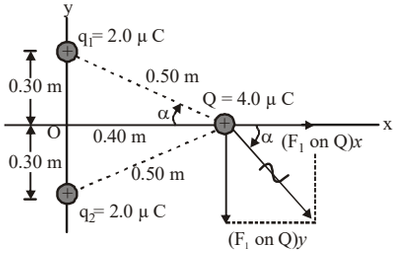
Suppose, force between the chrages is same when charges are r' distance apart in dielectric.

$$\therefore\ F' = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{kr'^2} \quad \ldots(ii)$$

From (i) and (ii), $kr'^2 = r^2$ or, $r = \sqrt{kr'}$
In the given situation, force between the charges would be

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\left(\frac{r}{2}+\sqrt{4}\frac{r}{2}\right)^2} = \frac{4}{9}\frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{4F}{9}$$

36. (b) $F_{net} = 2|F_{31}|\cos\alpha = 2\times\frac{1}{4\pi\varepsilon_0}\times\frac{2\times4\times10^{-12}}{(0.5)^2}\times\frac{4}{5} = 0.46\ N$



37. (b) Here, $\vec{r}_1 = \hat{i}+\hat{j}+\hat{k}$

$\vec{r}_2 = 2\hat{i}+3\hat{j}+\hat{k}$

$\therefore\ \vec{r} = \vec{r}_2 - \vec{r}_1 = (2\hat{i}+3\hat{j}+\hat{k}) - (\hat{i}+\hat{j}+\hat{k}) = \hat{i}+2\hat{j}$

$|\vec{r}| = \sqrt{(1)^2+(2)^2} = \sqrt{5}$

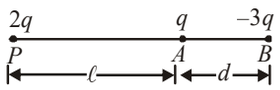
By Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = \frac{9\times10^9\times3\times10^{-6}\times3\times10^{-6}}{(\sqrt{5})^2}$$

$$= \frac{81}{5}\times10^{-3}\ N \text{ Nearest answer is } 16\times10^{-3}\ N.$$

38. (c)

39. (b)



Let a charge $2q$ be placed at P, at a distance I from A where charge q is placed, as shown in figure.
The charge $2q$ will not experience any force, when force, when force of repulsion on it due to q is balanced by force of attraction on it due to $-3q$ at B where AB = d

or $\frac{(2q)(q)}{4\pi\varepsilon_0\ell^2} = \frac{(2q)(-3q)}{4\pi\varepsilon_0(\ell+d)^2}$

$(\ell+d)^2 = 3\ell^2 \quad \text{or} \quad 2\ell^2 - 2\ell d - d^2 = 0$

$$\therefore\ \ell = \frac{2d \pm \sqrt{4d^2+2d^2}}{4} = \frac{d}{2} \pm \frac{\sqrt{3}d}{2}$$

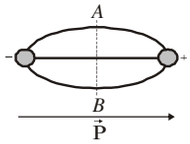
$$\ell = \frac{d+\sqrt{3}d}{2}$$

40. (a)

41. (b) In non–uniform field, force and torque are created.

42. (b) **Given :** Dipole moment of the dipole = $\vec{p}$ and uniform electric field = $\vec{E}$. We know that dipole moment (p) = q.a (where q is the charge and a is dipole length). And when a dipole of dipole moment $\vec{p}$ is placed in uniform electric field $\vec{E}$, then Torque (τ) = Either force × perpendicular distance between the two forces = qa E sin θ or $\tau = pE\sin\theta$ or $\vec{\tau} = \vec{p}\times\vec{E}$ (vector form)

43. (b) The direction of electric field at equatorial point A or B will be in opposite direction, as that of direction of dipole moment.



44. (d) **45. (c)**

46. (a) The electric field lines do not form closed loop.

47. (b) The electric field due to a infinitely long wire is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow E \propto \frac{1}{r}$$

48. (a)

49. (b) We have $E_a = \frac{2kp}{r^3}$ and $E_e = \frac{kp}{r^3} \Rightarrow E_a = 2E_e$

50. **(a)** $-eE = mg, \vec{E} = -\dfrac{9.1\times10^{-31}\times10}{1.6\times10^{-19}}$

$= -5.6\times10^{-11}\,N/C$

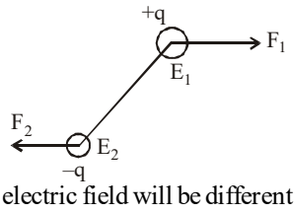
51. **(d)** The angle between the $\vec{p}$ and $\vec{E}$ is 180° because the direction of electric field $(\vec{E})$ at any point on the equatorial line is opposite to the dipole moment $(\vec{p})$.

52. **(c)**

53. **(d)**

54. **(d)**

55. **(c)**



The electric field will be different at the location of force on the two charges. Therefore the two charges will be unequal. This will result in a force as well as torque.

56. **(c)**

57. **(c)** Charges (q) = 2×10^{-6} C, Distance (d) = 3 cm $= 3 \times 10^{-2}$ m and electric field (E) = 2×10^5 N/C. Torque (τ) = q.d.

$E = (2 \times 10^{-6}) \times (3 \times 10^{-2}) \times (2 \times 10^5) = 12 \times 10^{-3}$ N–m.

58. **(d)** Given, $\sigma_1 = \sigma_2$

$$\frac{Q}{4\pi r_1^2} = \frac{Q}{4\pi r_2^2} \Rightarrow \frac{Q}{4\pi \in_0 r_1^2} = \frac{Q}{4\pi \in_0 r_2^2}$$

$\because E_1 = E_2 \quad \therefore \dfrac{E_1}{E_2} = \dfrac{1}{1}$

59. **(b)**

60. **(c)** $\dfrac{E_1}{E_2} = \dfrac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{\left(\frac{a}{\sqrt{2}}\right)^2}}{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{\left(\frac{a}{2}\right)^2}} = \dfrac{1}{2}$

61. **(c)** Force F = ma = qE $\therefore\ a = \dfrac{qE}{m}$

$$v = u + at = 0 + \left(\frac{qE}{m}\right)t = \frac{qE}{m}t$$

Kinetic energy of the particle after time t

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qEt}{m}\right)^2 = \frac{q^2E^2t^2}{2m}$$

62. **(c)** Using work – energy theorem, W = ΔKE.

$Fx = \Delta KE \Rightarrow QE \times x = \dfrac{1}{2}mv^2$

$\Rightarrow\ Q \times 300 \times 0.5 = 0.12 \Rightarrow Q = 8 \times 10^{-4}\ C = 800\ \mu C$

63. **(a)** Force due to circular motion:

$F_c = mx\omega^2$

$\because$ Electrostatic force is given by $F_e = eE$

$F_c = F_e$

$$mx\omega^2 = eE \Rightarrow E = \frac{mx\omega^2}{e}$$

64. **(b)** $E = \dfrac{1}{4\pi\varepsilon_0}\cdot q\left[1 - \frac{1}{2^2} + \frac{1}{4^2} - \frac{1}{8^2} + \ldots\right]$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{\left[1-\left(-\frac{1}{4}\right)\right]} = 36V/m$$

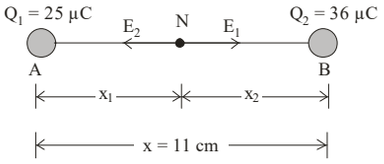
65. **(c)**

66. **(c)** Electric field at a point on the axial, $\vec{E}_{ax} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\vec{p}}{r^3}$;

Electric field at a point on the equatorial,

$$\vec{E}_{eq} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3} = \frac{\vec{E}_{ax}}{2} \quad \therefore \vec{E}_{ax} = -2\vec{E}_{eq}$$

67. **(a)** Suppose electric field is zero at point N in the figure. Then at N,



$|E_1| = |E_2|$

which gives $x_1 = \dfrac{x}{\sqrt{\frac{Q_2}{Q_1}}+1} = \dfrac{11}{\sqrt{\frac{36}{25}}+1} = 5\,cm$

68. **(a)** The magnitude of electric field due to a point charge q at a distance r from the charge is $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$

$$\therefore 2\,N\,C^{-1} = \frac{\left(9\times10^9\,N\,m^2\,C^{-2}\right)q}{(0.3\,m)^2}$$

or $q = \dfrac{\left(2\,N\,C^{-1}\right)\left(0.09\,m^2\right)}{\left(9\times10^9\,N\,m^2\,C^{-2}\right)} = 2\times10^{-11}\,C$

Thus, the magnitude of point charge will be 2×10^{-11} C.

69. **(d)** Electric field at equatorial point

$$E_{eq} = \frac{p}{4\pi\varepsilon_o(r^2+l^2)^{3/2}}$$

$$E = \frac{p}{4\pi\varepsilon_0 r^3} \quad [\text{For } r >> l]$$

Apparently, $E \propto p$ and . $E \propto \dfrac{1}{r^3} \propto r^{-3}$

70. **(c)** $\tau = p \cdot E$ (As, $\theta = 90^o$)

For maximum, $\tau = p \cdot E$

$\tau = q \times 2aE = 1 \times 10^{-6} \times 2 \times 10^{-2} \times 10^6 = 20 \times 10^{-3}$ N-m

71. **(c)** Electric field $\propto$ Number of field lines per unit area.

$$\therefore \frac{E_A}{E_B}=\frac{2}{1}\Rightarrow E_B=\frac{E_A}{2}$$

Force on 20μC at A

$F_A = qE_A = 20\times10^{-6}\times40 = 800\times10^{-6} = 8\times10^{-4}$ N

Force on 20 μC charge at *B*,

$$F_B = qE_B = \frac{qE_A}{2} = \frac{1}{2}\times8\times10^{-4} = 4\times10^{-4}\ \text{V m}^{-1}$$

72. **(d)** **73.** **(d)**

74. **(b)** The electric field around a charge propagates with the speed of light away from the charge. Therefore the required time $= \frac{\text{distance}}{\text{speed}} = \frac{OP}{c}$.

75. **(c)**

76. **(b)** Torque acting on the dipole, $|\tau| = pE\sin\theta$

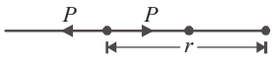
$= 4\times10^{-9}\times5\times10^{-4}\times\sin30° = 10^{-4}$ N m

77. **(a)** Electric field is given as

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \frac{9\times10^9\times3\times10^{-9}}{\left(3\times10^{-2}\right)^2} = 3\times10^4\ \text{V m}^{-1}$$

78. **(c)** $E = \frac{1}{4\pi\in_0}\left[\frac{2p}{(r-\ell/2)^3}-\frac{2p}{(r+\ell/2)^3}\right]$

$= \frac{1}{4\pi\in_0}\frac{3Q}{r^4}.$



79. **(c)** $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(1.2-0)\hat{i}+(-1.6-0)\hat{j}}{\sqrt{(1.2)^2+(1.6)^2}} = \frac{1}{2}\left(1.2\hat{i}-1.6\hat{j}\right)$$

$$\vec{E} = 9\times10^9\times\frac{\left(-8\times10^{-9}\right)}{2^2}\times\left[\frac{1}{2}\left(1.2\hat{i}-1.6\hat{j}\right)\right] = -10.8\hat{i}+14.4\hat{j}$$

80. **(b)** Since $\tau = pE\sin\theta$ on decreasing the distance between the two charges, and on decreasing angle θ between the dipole and electric field, sin θ decreases therefore torque decreases.

81. **(d)** Torque, $\vec{\tau} = \vec{p}\times\vec{E} = pE\sin\theta$

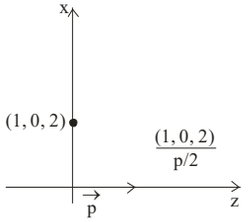
$4 = p\times2\times10^5\times\sin30°$

or, $p = \frac{4}{2\times10^5\times\sin30°} = 4\times10^{-5}$ Cm

Dipole moment, $p = q\times l$

$$q = \frac{p}{l} = \frac{4\times10^{-5}}{0.02} = 2\times10^{-3}\text{C} = 2\text{mC}$$

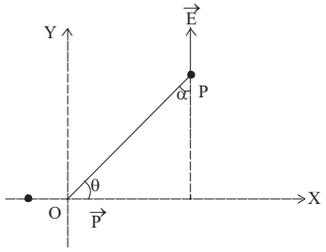
82. **(b)**



The given point is at the axis of $\frac{\vec{p}}{2}$ dipole and at equatorial line of $\vec{p}$ dipole. So, the total field at given point is

$$\vec{E} = \frac{k\vec{p}}{(1)^3}+\frac{2k\left(\vec{p}/2\right)}{(2)^3} = k\vec{p}\left(-1+\frac{1}{8}\right) = \frac{-7p\hat{k}}{32\pi\varepsilon_0}$$

83. **(b)** We know that in this case electric field makes an angle θ + α with the direction of dipole.



Also, $\tan\alpha = \frac{1}{2}\tan\theta$

Here, $\theta+\alpha = 90°\Rightarrow\alpha = 90°-\theta$

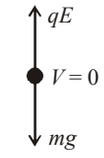
Hence $\tan(90°-\theta) = \frac{1}{2}\tan\theta$

$\Rightarrow \cot\theta = \frac{1}{2}\tan\theta \Rightarrow \tan^2\theta = 2 \Rightarrow \tan\theta = \sqrt{2}$

84. **(b)** $\tau = \left(2.57\times10^{-17}\,\text{Cm}\right)\left(3.0\times10^4\,\frac{\text{N}}{\text{C}}\right)\left(\frac{1}{2}\right)$

$= 3.855\times10^{-13}$ Nm.

85. **(b)** As water droplet is at rest



So, $\vec{F}_{\text{net}} = 0 \Rightarrow mg = qE \Rightarrow q = \frac{mg}{E}$

$$\Rightarrow q = \frac{0.1\times10^{-3}\times9.8}{4.9\times10^5} \Rightarrow q = 2\times10^{-9}\,C$$

86. **(a)** Because of the presence of positive test charge q_0 in front of positively charged ball, charge on the ball will be redistributed, less charge on the front half surface and more charge on the back half surface. As a result of this force F between ball and point charge will decrease i.e., actual electric field will be greater than F/q_0.

87. **(b)** The cork floats motionless if the weight of the cork is equal to electrostatic force due to uniformly charged plane near the earth's surface, i.e., $mg = F_e = QE$

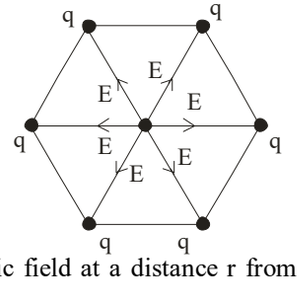
Now, electric field E due to large plane is

$$E = \frac{\sigma}{2\varepsilon_0} \Rightarrow mg = \frac{\sigma}{2\varepsilon_0}Q$$

$$\sigma = \frac{2\varepsilon_0 mg}{Q} = \frac{2\times8.85\times10^{-12}\times10^{-3}\times9.8}{1\times10^{-6}}$$

$= 17.3\times10^{-8}$ c/m^2

88. **(c)** Since there are identical charges at each corner of the regular polygon, the electric field at the centre will be zero. (Note : We conclude from here that the field due to one charge is equal in magnitude to the combined field of all the remaining charges).



89. **(b)** Electric field at a distance r from an infinity long conductor, $E_1 = \frac{\lambda}{2\pi\varepsilon_0 r}$

For semi-circle, $E_2 = \frac{\lambda}{2\pi\varepsilon_0 r}\sin\left(\frac{\theta}{2}\right)$

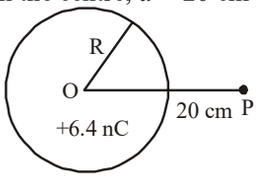
Here, $\theta = \pi = \frac{\lambda}{2\pi\varepsilon_0 r}\sin\frac{\pi}{2}$

$E_2 = \frac{\lambda}{2\pi\varepsilon_0 r}$ $\therefore$ $E_1 = E_2$

90. **(d)** Number of electrons removed, $n = 4 \times 10^{10}$

Charge on this sphere, $q = nC = 4 \times 10^{10} \times 1.6 \times 10^{-19}$C $= 6.4 \times 10^{-9}$ C

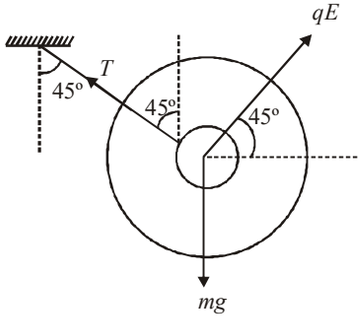
Distance from the centre, $d = 20$ cm $= 0.2$ m



Electric field at point P, $E = \frac{q}{4\pi\varepsilon_0 d^2}$

$= \left(\frac{9\times10^9 \times 6.4\times10^{-9}}{(0.2)^2}\right)$ N C^{-1} = 1440 N C^{-1}

91. **(c)**



$E = \hat{i} + \hat{j}$

$E = |E| = \frac{\sqrt{2}N}{C}$

For equilibrium,

$T = \cos 45° + qE \sin 45° = mg$

$T \sin 45° = qE \cos 45°$ $\Rightarrow T = qE$

$qE \cos 45° + qE \sin 45° = mg$

$\frac{qE}{\sqrt{2}} + \frac{qE}{\sqrt{2}} = mg$

$\sqrt{2}qE = mg \Rightarrow q = \frac{20}{2} = 10C$

92. **(d)** For distances far away from centre of dipole

$E_{axis} = E_a = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$, $E_{equa} = E_e = \frac{1}{4\pi\varepsilon_0} - \frac{p}{r^3}$

$\frac{d}{dr}(E_a) = \frac{1}{4\pi\varepsilon_0}2p\frac{d}{dr}(r^{-3}) = -6\cdot\frac{1}{4\pi\varepsilon_0}\frac{p}{r^4}$ ... (i)

$\frac{d}{dr}(E_e) = \frac{1}{4\pi\varepsilon_0}p\frac{d}{dr}(r^{-3}) = -3\frac{1}{4\pi\varepsilon_0}\frac{p}{r^4}$ ... (ii)

From equations (i) and (ii) the magnitude of change in electric field w.r.t. distance is more in case of axis of dipole as compared to equatorial plane.

93. **(a)** Given : Length of the dipole $(2l) = 10$cm $= 0.1$m or $l = 0.05$ m

Charge on the dipole (q) $= 500\ \mu C = 500 \times 10^{-6}$C

and distance of the point on the axis from the mid-point of the dipole $(r) = 20 + 5 = 25$ cm $= 0.25$ m. We know that the electric field intensity due to dipole on the given point

$(E) = \frac{1}{4\pi\varepsilon_0} \times \frac{2(q.2l)r}{(r^2 - l^2)^2}$

$= 9\times10^9 \times \frac{2(500\times10^{-6}\times0.1)\times0.25}{[(0.25)^2 - (0.05)^2]^2}$

$= \frac{225\times10^3}{3.6\times10^{-3}} = 6.25\times10^7\ N/C$

94. **(a)** From Gauss law, electric field due to an infinitely long wire, $E = \frac{2}{4\pi\varepsilon_0}\cdot\frac{\lambda}{x}$

or $E = 2\times9\times10^9 \times \frac{1}{4}\times10^{-2} \times \frac{1}{20\times10^{-2}}$

$\therefore$ $E = 2.25 \times 10^8$ NC^{-1}

95. **(c)** Given, $\lambda = (3x^2 + 5x + 2)$ C/m.

$l = 3$m

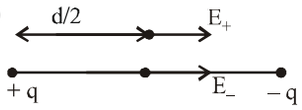
The net charge present on the rod is

$q = \int_0^l \lambda dx = \int_0^3 (3x + 5x + 2)dx$

$= \left[3\cdot\frac{x^3}{3} + 5\cdot\frac{x^2}{2} + 5x\right]_0^3 = 55.5$ C

96. **(b)** Electric field is directly proportional to the magnitude of charge and inversely proportional to the square of the distance from the charge. Therefore charge $+Q$ produce a comparatively stronger electric field than $+q$ which get at cancelled with each other at a point closer to $+q$.

97. **(b)**



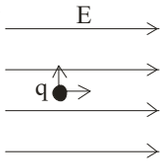
So, $E_{net} = E_+ + E_-$

$$= 2\frac{kq}{\left(\frac{d}{2}\right)^2} = \frac{8kq}{d^2} \Rightarrow 6.4\times10^4 = 8\times9\times10^9\times\frac{8\times10^{-6}}{d^2}$$

$$\Rightarrow d^2 = \frac{72\times8\times10^{+3}}{6.4\times10^4} \Rightarrow d^2 = 9 \Rightarrow d = 3 \text{ m}$$

98. **(d)**

99. **(b)** E points along the tangent to the lines of force. If initial velocity is zero then due to electric force, charge particle, if positive, always moves in the direction of E and opposite to direction of E, if negative. But let's take a case where the initial velocity is not zero, i.e., let say charge particle is released with some velocity making an angle 90° with lines of force. Then in this case two velocities act on the particle as shown in figure (one due to electrostatic force and other the initial velocity). So, the net velocity will not be in the direction of E. Hence, charge will not travel along the electric lines of force.



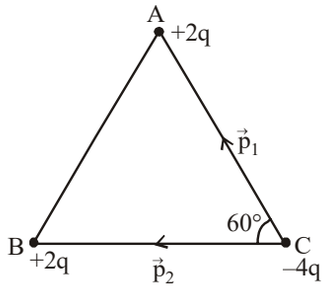
100. **(a)** Net dipole of the system will be

$$\vec{p} = \vec{p}_1 + \vec{p}_2$$

$$p = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\theta} \qquad (\because p_1 = p_2 = 2qx)$$

$$= \sqrt{4q^2x^2 + 4q^2x^2 + 2\times4q^2x^2\times\cos60}$$

$$= \sqrt{12q^2x^2} = 2\sqrt{3}\,qx$$



Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(c)** Electrostatic force $F = \frac{K Q_1 Q_2}{r^2}$

$$F = \frac{9\times10^9\times(Q)\times\left(36\times10^{-6}-Q\right)}{(4)^2}$$

$$\text{or,}\quad 0.18 = \frac{9\times10^9\times Q\times\left(36\times10^{-6}-Q\right)}{16}$$

$$\Rightarrow \frac{18\times10^{-2}\times16}{9\times10^9} = Q\times(36\times10^{-6}-Q)$$

$$\Rightarrow 32\times10^{-11} = Q\times(36\times10^{-6}-Q)$$

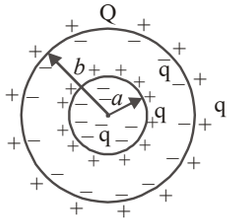
Solving we get $Q = 20\times10^{-6} = 20\ \mu C$

and $36\times10^{-6} - 20\times10^{-6} = 16\times10^{-6} = 16\ \mu C$

Therefore the bigger charge is 20 μ C

2. **(c)** The total charge on the outer surface will be $Q + q$.

So charge density will be $\left[\frac{Q+q}{4\pi b^2}\right]$



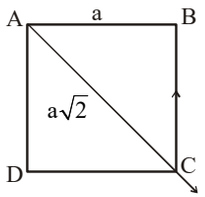
3. **(d)** It is given that $\vec{F}_A + \vec{F}_B + \vec{F}_D = 0$

Where $\vec{F}_A$, $\vec{F}_B$ and $\vec{F}_D$ are the forces applied by charges placed at A, B and D on the charge placed at C.

$$\Rightarrow \vec{F}_B + \vec{F}_D = -\vec{F}_A$$

$$|\vec{F}_B + \vec{F}_D| = \sqrt{2}\frac{KqQ}{a^2}$$

$$\Rightarrow |\vec{F}_A| = \frac{KQ^2}{2a^2}$$



$$\therefore \sqrt{2}\frac{KqQ}{a^2} = -\frac{KQ^2}{2a^2} \Rightarrow \frac{Q}{q} = -2\sqrt{2}$$

4. **(a)**

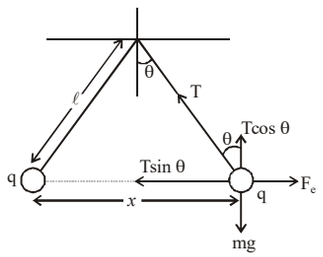
5. **(d)** In equilibrium, $F_e = T\sin\theta$

$$mg = T\cos\theta$$

$$\tan\theta = \frac{F_e}{mg}$$

$$= \frac{q^2}{4\pi\in_0 x^2\times mg}$$

also $\tan\theta \approx \sin\theta = \frac{x/2}{\ell}$



Hence, $\frac{x}{2\ell} = \frac{q^2}{4\pi\in_0 x^2\times mg}$

$$\Rightarrow x^3 = \frac{2q^2\ell}{4\pi\in_0 mg} \quad \therefore \quad x = \left(\frac{q^2\ell}{2\pi\in_0 mg}\right)^{1/3}$$

Therefore, $x \propto \ell^{1/3}$

6. **(c)** $y = ut + \frac{1}{2}at^2 \Rightarrow y = \frac{1}{2}at^2 \qquad (\because u = 0)$

$F = ma = qE$

$$\Rightarrow \quad a = \frac{qE}{m} \quad \therefore \quad y = \frac{1}{2}\frac{qE}{m}\frac{x^2}{v^2} \qquad (\because x = vt)$$

$$\Rightarrow \quad x^2 = \frac{2mv^2}{qE}y$$

Compairing with $x^2 = 4ay$

So, the trajectory of the particle will be parabolic.

7. **(b)** The number of field lines crossing an area is directly related to electric flux passing through surface. In first case, the flux crossing the surface is more than the flux crossing through surface in second case, so $n_1 > n_2$.

8. **(a)** As, $F_{air} = F_{med}$

$$\Rightarrow \frac{1}{4\pi\in_0}\frac{q_1q_2}{d_{air}^2} = \frac{1}{K}\frac{1}{4\pi\in_0}\frac{q_1q_2}{d^2} \Rightarrow d_{air} = d\sqrt{K}$$

9. **(a)**

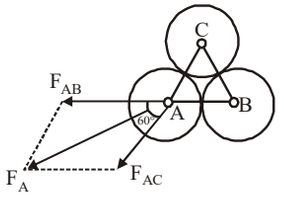
10. **(b)**

11. **(c)** For external points, a charged sphere behave as if the whole of its charge is concentrated at its center.

Force on A due to B,

$$F_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2}\text{along } \overrightarrow{BA}$$



And force on A due to C,

$$F_{AC} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2}\text{along } \overrightarrow{CA}$$

Now as angle between BA and CA is 60° and

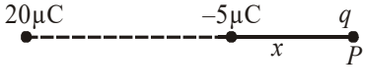
$|F_{AB}| = |F_{AC}| = F$

$$F_A = \sqrt{F^2 + F^2 + 2FF\cos 60} = \sqrt{3}F = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{3}}{4}\left(\frac{q}{R}\right)^2$$

12. **(b)** $20\mu C$ $\quad$ $-5\mu C$

Let, charge q be placed at P.

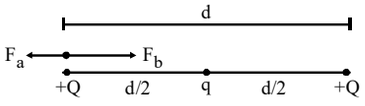
At point P forces due to 20 μC & $-5\mu C$ should be in opp-osite direction



For net force $\vec{F} = 0$ & from coulomb's law force

$$F = \frac{k\,q_1\,q_2}{r^2} \Rightarrow k\frac{20q}{(5+x)^2} = \frac{k5q}{x^2} \Rightarrow x = 5\text{ cm}$$

13. **(a)**



Force due to charge + Q, $F_a = \frac{KQQ}{d^2}$

Force due to charge q, $F_b = \frac{KQq}{\left(\frac{d}{2}\right)^2}$

For equilibrium, $\vec{F}_a + \vec{F}_b = 0$

$$\Rightarrow \frac{kQQ}{d^2} + \frac{kQq}{(d/2)^2} = 0 \quad \therefore q = -\frac{Q}{4}$$

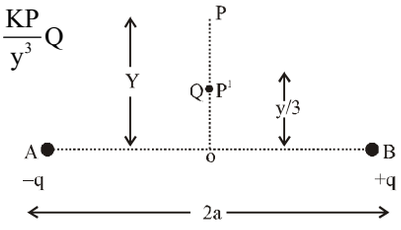
14. **(d)** Electric field of equitorial plane of dipole $= -\frac{K\vec{P}}{r^3}$

$\therefore$ At point P, $F = +\frac{KP}{y^3}Q$

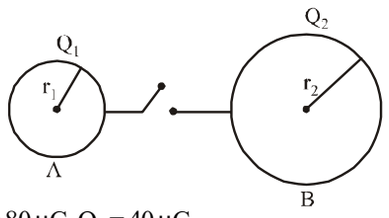
At Point P^1,

$$F^1 = +\frac{KPQ}{(y/3)^3}$$

$= 27$ F.



15. **(b)**



$Q_1 = 80\,\mu C, Q_2 = 40\,\mu C$

$r_1 = 4$ cm, $r_2 = 6$ cm $\quad\therefore$ After connecting wire,

$$Q_1' = \left(\frac{r_1}{r_1 + r_2}\right)(Q_1 + Q_2) = \left(\frac{4}{4+6}\right)(80+40) = 48\,\mu C$$

$\therefore$ Charge flow from A to B is

$\Delta Q = Q_1 - Q_1' = 80 - 48 = 32\ \mu C$

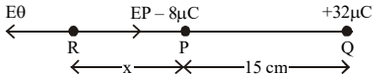
16. **(b)** Electric field infinity long wire, $E = \frac{\lambda}{2\pi\varepsilon_0\pi}$

$$F = qE = \frac{\lambda.q}{2\pi\varepsilon_0 r} = m\omega^2 r \qquad \left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$$

$$\frac{2k\lambda q}{r} = m\omega^2 r \Rightarrow \omega^2 = \frac{2k\lambda q}{mr^2}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{2k\lambda q}{mr^2} \Rightarrow T = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$$

17. **(a)** Let the charge be placed at a distance x from – 8μc at which resultant electric field becomes zero



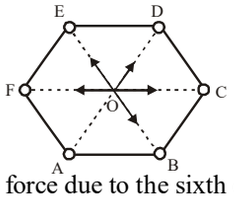
At point R, $E_R = 0 \quad\therefore \quad E_P = E_\theta$

$$\therefore \quad \frac{8}{4\pi\varepsilon_o x^2} = \frac{32}{4\pi\varepsilon_o(15+x)^2} \Rightarrow \frac{x+15}{x} = 2$$

On solving x = 15

18. **(b)** $q_1q_2 < 0$

19. **(d)** If there had been a sixth charge +q at the remaining vertex of haxagon, force due to all the six charges on – q at O will be zero.

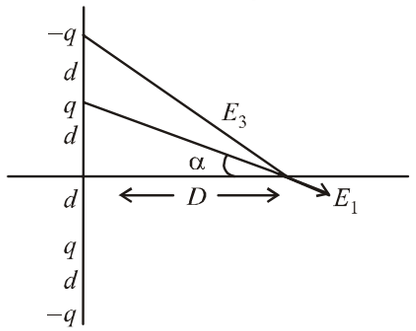


Now if f is the force due to the sixth charge and F due to the remaining five charge then,

$\vec{F} + \vec{f} = 0$ i.e., $\vec{F} = -\vec{f}$

$$|\vec{F}| = |\vec{f}| = \frac{1}{4\pi\varepsilon_0}\frac{q\times q}{L^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$$

20. **(d)** $\vec{E} = (\vec{E_1} + \vec{E_2}) + (\vec{E_3} + \vec{E_4})$ or $E = 2E\cos\alpha - 2E\cos\beta$



$$= \frac{2kq}{(D^2+d^2)} \times \frac{D}{\sqrt{D^2+d^2}} - \frac{2kq}{(D^2+(2d)^2} \times \frac{D}{\sqrt{D^2+(2d)^2}}$$

$$= \frac{2kqD}{(D^2+d^2)^{3/2}} - \frac{2kqD}{[D^2+(2d)^2]^{3/2}}$$

For d << D

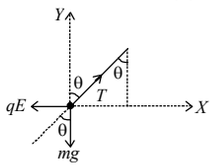
$$E \propto \frac{D}{D^3} \propto \frac{1}{D^2}$$

21. **(d)** At equilibrium resultant force on bob must be zero, so

$T\cos\theta = mg$ ..... (i)

$T\sin\theta = qE$ ..... (ii)

Solving (i) and (ii) we get



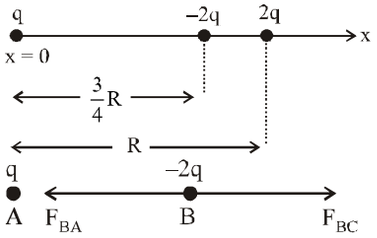
$$\tan\theta = \frac{qE}{mg}$$

$$\tan\theta = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10} = \frac{1}{2}$$

[Here, $q = 5\times10^{-6}$ C, E = 2000 v/m, m = 2×10^{-3} kg]

$$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right)$$

22. **(b)** From coulomb's law



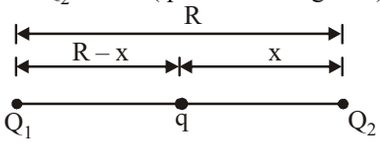
$$F_{BA} = \frac{Kq(2q)}{\left(\frac{3}{4}R\right)^2} = \frac{32Kq^2}{9R^2} \text{ and } F_{BC} = \frac{K(2q)(2q)}{\left(\frac{R}{4}\right)^2} = \frac{64Kq^2}{R^2}$$

$$F_B = F_{BC} - F_{BA} = \frac{544Kq^2}{9R^2}$$

$$= \frac{544\times9\times10^9\times\left(2\times10^{-6}\right)^2}{9\times\left(2\times10^{-2}\right)^2} = 5440 \text{ N}$$

23. **(c)** Net force on each of the charge due to the other charges is zero. However, disturbance in any direction other than along the line on which the charges lie, will not make the charges return.

24. **(c)** Force on Q_2 is zero (q should be negative)



$$\frac{kQ_1Q_2}{R^2} = \frac{kqQ_2}{x^2} \text{ or } \frac{x}{R} = \sqrt{\frac{q}{Q_1}} = \frac{\sqrt{q}}{\sqrt{Q_1}}$$

Force on q is zero,

$$\frac{kQ_1q}{(R-x)^2} = \frac{kqQ_2}{x^2} \text{ or } \frac{R-x}{x} = \frac{\sqrt{Q_1}}{\sqrt{Q_2}}$$

$$\text{or } \frac{R}{x} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}} \text{ or } \frac{\sqrt{Q_1}}{\sqrt{q}} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}}$$

$$\text{or } q = \frac{Q_1Q_2}{\left(\sqrt{Q_1}+\sqrt{Q_2}\right)^2}$$

25. **(b)** From law of conservation of charge $q_i = q_f$

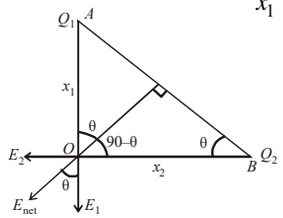
$$\Rightarrow 64q = Q \Rightarrow \frac{Q}{q} = 64 \text{ and, } \frac{4}{3}\pi R^3 = 64\times\frac{4}{3}\pi r^3$$

$$\Rightarrow R = 4r \Rightarrow \frac{r}{R} = \frac{1}{4}$$

$$\text{So, } \frac{\sigma_{bigger}}{\sigma_{smaller}} = \frac{\frac{Q}{4\pi R^2}}{\frac{q}{4\pi r^2}} = \frac{Q}{q}\times\left(\frac{r}{R}\right)^2 = 64\times\frac{1}{16} = \frac{4}{1}$$

26. **(c)** Electric field due charge Q_2, $E_2 = \frac{kQ_2}{x_2^2}$

Electric field due charge Q_1, $E_1 = \frac{kQ_1}{x_1^2}$



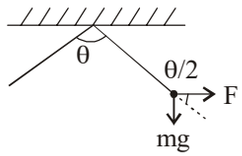
From figure,

$$\tan\theta = \frac{E_2}{E_1} = \frac{x_1}{x_2} \Rightarrow \frac{kQ_2}{x_2^2\times\frac{kQ_1}{x_1^2}} = \frac{x_1}{x_2}$$

$$\Rightarrow \frac{Q_2x_1^2}{Q_1x_2^2} = \frac{x_1}{x_2} \Rightarrow \frac{Q_2}{Q_1} = \frac{x_2}{x_1} \text{ or, } \frac{Q_1}{Q_2} = \frac{x_1}{x_2}.$$

27. **(a)** If spheres are in air,

$$\tan\frac{\theta}{2} = \frac{F}{mg} = \frac{q^2}{4\pi\in_0 r^2mg}$$



If spheres are in water,

$$\tan\frac{\theta}{2} = \frac{F'}{mg'} = \frac{q^2}{4\pi k\in_0 r^2mg'}$$

Since angle remains same.

$$\Rightarrow \frac{q^2}{4\pi\in_0 r^2mg} = \frac{q^2}{4\pi k\in_0 r^2mg'} \Rightarrow g = kg' = g\left(1-\frac{1}{1.5}\right)k$$

$\therefore$ k = 3

28. **(a)** Acceleration of charge particle in an electric field is

$a = \frac{qE}{m}$ $\therefore v = u + at = \left(\frac{qE}{m}\right)t \Rightarrow v \propto \frac{q}{m}$

$\therefore$ Work done by electric field,

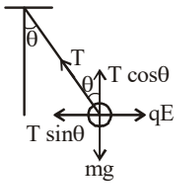
$W = \Delta K = \frac{1}{2}mv^2 \Rightarrow W \propto mv^2 \propto m\left(\frac{q}{m}\right)^2$ $\therefore$ $W \propto \frac{q^2}{m}$

$\therefore$ $\frac{W_p}{W_\alpha} = \left(\frac{q_p}{q_\alpha}\right)^2\left(\frac{m_\alpha}{m_p}\right) = \left(\frac{1}{2}\right)^2\left(\frac{4}{1}\right) = 1:1$

29. **(b)** In equilibrium,

$T\cos\theta = mg$ ....(1)

$T\sin\theta = qE$ ....(2)



From (1), $T = \frac{mg}{\cos\theta}$

From (2), $T = \frac{qE}{\sin\theta}$

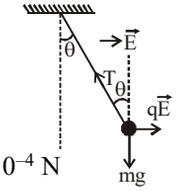
30. **(c)** At equilibrium

$T\cos\theta = mg$ and $T\sin\theta = qE$

$mg = 30.7 \times 10^{-6} \times 9.8 = 3 \times 10^{-4}$ N

$qE = 2 \times 10^{-8} \times 20{,}000 = 4 \times 10^{-4}$ N

$\therefore T = \sqrt{(3\times10^{-4})^2 + (4\times10^{-4})^2} = 5 \times 10^{-4}$ N



31. **(b)** $F = qE = mg$ $(q = 6e = 6\times 1.6 \times 10^{-19})$

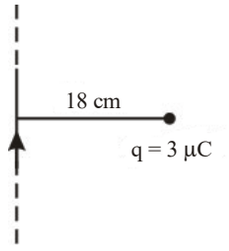
Density (d) $= \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi r^3}$ or $r^3 = \frac{m}{\frac{4}{3}\pi d}$

Putting the value of d and m $\left(= \frac{qE}{g}\right)$ and solving we get

$r = 7.8 \times 10^{-7}$m

32. **(d)**

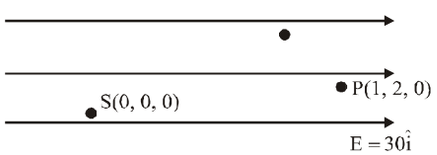
33. **(b)**



$E = \frac{\lambda}{2\pi\varepsilon_0 . r} = \frac{2\lambda}{4\pi\varepsilon_0 . r} = \frac{9\times10^{-9}\times2\times1}{3\times18\times10^{-2}} = \frac{1}{3}\times10^{11}$ N/C

So, $F = qE = 3\times10^{-6}\times\frac{1}{3}\times10^{11} = 10^5$ N

34. **(c)**



$W = \vec{F}\cdot\vec{S} \Rightarrow W = q\vec{E}\cdot\vec{S}$

$W = 2 \times 10^{-2}\left[30\hat{i}\cdot(-\hat{i})\right] = 2 \times 10^{-2}(-30) = -600$ mJ

35. **(b)** $\tau = -PE\sin\theta$

$I\alpha = -PE(\theta) \Rightarrow \alpha = \frac{PE}{I}(-\theta)$

On comparing with

$\alpha = -\omega^2\theta$

$\omega = \sqrt{\frac{PE}{I}} = \sqrt{\frac{qdE}{2m\left(\frac{d}{2}\right)^2}} = \sqrt{\frac{2qE}{md}}$

36. **(c)** Let n be the number of electrons missing.

$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{d^2} \Rightarrow q = \sqrt{4\pi\varepsilon_0 d^2 F} = ne$

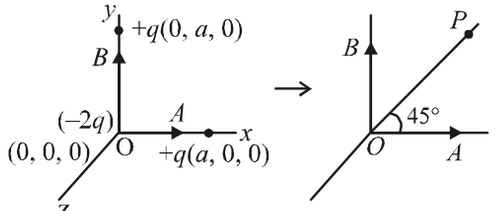
$\therefore$ $n = \sqrt{\frac{4\pi\varepsilon_0 F d^2}{e^2}}$

37. **(a)** Three point charges $+q$, $-2q$ and $+q$ are placed at points B $(x = 0, y = a, z = 0)$,

O $(x = 0, y = 0, z = 0)$ and $A(x = a, y = 0, z = 0)$

The system consists of two dipole moment vectors due to ($+q$ and $-q$) and again due to ($+q$ and $-q$) charges having equal magnitudes qa units – one along $\overrightarrow{OA}$ and other along $\overrightarrow{OB}$. Hence, net dipole moment,

$p_{net} = \sqrt{(qa)^2 + (qa)^2} = \sqrt{2}qa$ along $\overrightarrow{OP}$ at an angle 45° with positive X-axis.



38. **(b)** $\rho = Cr^2$

$q = \int_0^r 4\pi r^2 dr\rho = \int_0^r 4\pi C r^4 dr = \frac{4}{5}\pi C r^5$

$E_{r=2R} = \frac{kq_{(r=R)}}{(2R)^2} = \frac{k(4/5)\pi CR^5}{4R^2}$

$E_r = \frac{R}{2} = \frac{kq_{(r=R)}}{(R/2)^2} = \frac{k(4/5)\pi C(R/2)^5}{(R/2)^2}$

Now solve to get $\frac{E_{r=2R}}{E_{r=R/2}} = 2$

39. **(d)** Here $a = \frac{qE}{m} = \frac{40\times10^{-6}\times10^5}{100\times10^{-6}} = 40000$ m/s^2

As charge particle is moving opposite to direction of $\vec{E}$

So $\vec{E}$ will declerate the charge and charge will come to rest.

Now, $v^2 = u^2 + 2as$

$0 = 200^2 + 2 \times (-40 \times 10^3) \times s$

$80 \times 10^3 s = 4 \times 10^4$

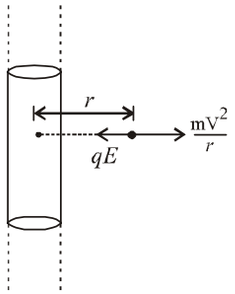
$\therefore$ $s = \frac{4\times10^4}{80\times10^3} = \frac{40}{80} = 0.5$ m.

40. **(a)** As charge particle is moving in circular path. So,

$$qE = \frac{mv^2}{r}$$

$$\Rightarrow mv^2 = qEr$$

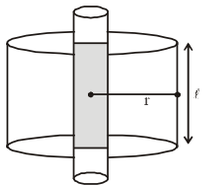
$$\Rightarrow \frac{mv^2}{2} = \frac{1}{2}qEr \quad ...(i)$$



Now, by Gauss's law

$$E \times 2\pi rl = \frac{q_{in}}{\epsilon_0}$$

$$E \times 2\pi rl = \frac{\rho \times \pi R^2 l}{\epsilon_0}$$



$$E = \frac{\rho R^2}{2\epsilon_0 r} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{1}{2}mv^2 = \frac{1}{2}q \times \frac{\rho R^2}{2\epsilon_0 r} \times r$$

$$\Rightarrow K.E. = \frac{\rho q R^2}{4\epsilon_0}$$

Exercise 3 : PREVIOUS YEARS MCQs

1. **(a)** If $R >> L_1$ it will behave like a dipole and in dipole we have, $E \propto \frac{1}{r^3}$

2. **(b)** Electrostatic force acting between two point charges in vacuum

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

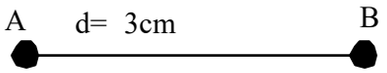
Electrostatic force acting between two point charges in a medium of dielectric constant K is

$$F' = \frac{1}{4\pi(K\epsilon_0)}\frac{q_1 q_2}{(r')^2} = \frac{25}{4\pi(5\epsilon_0)}\frac{q_1 q_2}{(r)^2}$$

$$\Rightarrow F' = 5F$$

3. **(a)**

4. **(d)**



1.6×10^{-19} C $\qquad$ 3.2×10^{-19} C

Coulomb's Law

$$F = \frac{K Q_1 Q_2}{d^2} \Rightarrow \frac{9\times10^9 \times 1.6\times10^{-19} \times 3.2\times10^{-19}}{(3\times10^{-2})^2}$$

$$F = \frac{46.08\times10^{-29}}{9\times10^{-4}} N$$

$F = 5.12 \times 10^{-25}$ N

5. **(c)** According to coulomb's law force $F = \frac{kq_1 q_2}{r^2}$

For same distance and medium $F \propto q_1 q_2$

When spheres are connected by a conducting wire and after some time the wire is removed then

$$q_1^1 = q_2^1 = +2uc$$

$$\therefore \frac{F}{F^1} = \frac{q_1 q_2}{q_1^1 q_2^1} = \frac{8\times4}{2\times2} \therefore F^1 = \frac{F}{8}$$

6. **(a)** Given: Total charge : $q_1 + q_2 = 25\ \mu C = 25 \times 10^{-6} C$

Distance between charges: r = 1.5 M

Force between them: F = 0.6N

$k = 9 \times 10^9\ Nm^2/C^2$

Use coulomb's law: $F = \frac{kq_1 q_2}{r^2}$

Let $q_1 = x$ then $q_2 = 25 \times 10^{-6} - x$

$$F = \frac{9\times10^9 \times x \times (25\times10^{-6} - x)}{(1.5)^2}$$

$$0.6 = \frac{9\times10^9 \times (25\times10^{-6}x - x^2)}{2.25}$$

$x = 15\ \mu C, 10\mu C$

The difference between the two charges is 15 – 10 = 5μC

7. **(a)** Force, $F = \frac{Kq_1 q_2}{r^2}$ when dielectric is introduced the

$$F' = \frac{F}{K_{eff}}$$

Given F = 98 N, d = 9 cm = 9×10^{-2} m,

$t_1 = 6$ cm = 6×10^{-2} m

$K_1 = 4$, $t_2 = 3$ cm = 3×10^{-2} m, $K_2 = 9$, F'= ?

$$\frac{d}{K_{eff}} = \frac{t_1}{K_1} + \frac{t_2}{K_2} \Rightarrow \frac{9\times10^{-2}}{K_{eff}} = \frac{6\times10^{-2}}{4} = \frac{3\times10^{-2}}{9}$$

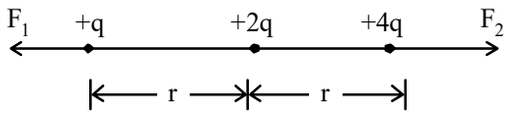
$$\Rightarrow \frac{9}{K_{eff}} = \frac{6}{4} + \frac{3}{9} \Rightarrow \frac{9}{K_{eff}} = \frac{54+12}{36}$$

$$\Rightarrow K_{eff} = \frac{9\times36}{66} \simeq 4.9$$

$$\therefore F' = \frac{98}{4.9} = 20$$

Nearest option is (a) 18N

8. **(d)**

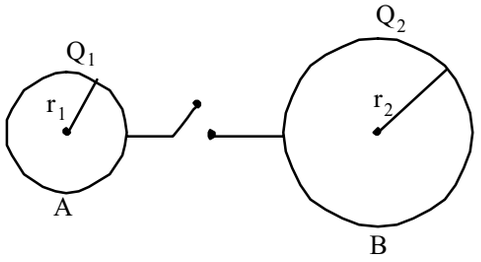


$$F_1 = \frac{k(q)(2q)}{r^2} + \frac{k(q)(4q)}{4r^2} = \frac{12kq^2}{4r^2}$$

$$F_2 = \frac{k(4q)(2q)}{r^2} + \frac{k(4q)(q)}{4r^2} = \frac{36kq^2}{4r^2}$$

$$\therefore \frac{F_1}{F_2} = \frac{12}{36} = 1:3$$

9. **(b)**

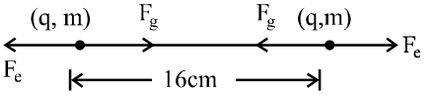


$Q_1 = 80\ \mu C$, $Q_2 = 40\ \mu C$
$r_1 = 4$ cm, $r_2 = 6$ cm
$\therefore$ After connecting wire,

$$Q_1' = \left(\frac{r_1}{r_1 + r_2}\right)(Q_1 + Q_2) = \left(\frac{4}{4+6}\right)(80+40) = 48\ \mu C$$

$\therefore$ Charge flow from A to B is
$\Delta Q = Q_1 - Q_1' = 80 - 48 = 32\ \mu C$

10. **(a)**



At equilibrium, $F_g = F_e$

$$\Rightarrow \frac{Gm^2}{(16)^2} = \frac{kq^2}{(16)^2}$$

$$\Rightarrow \left(\frac{q}{m}\right)^2 = \frac{G}{k} = \frac{G}{\frac{1}{4\pi\varepsilon_0}} = 4\pi\varepsilon_0 G$$

$$\therefore \frac{q}{m} = \sqrt{4\pi\varepsilon_0 G}$$

11. **(d)** For two charged conducting spheres,

$r_1 = 5$ cm, $r_2 = 10$ cm, $E_1 = E = \frac{1}{4\pi\varepsilon_o}.\frac{Q_1}{r_1^2}$

Also, $\sigma_1 = \sigma_2 \Rightarrow \frac{Q_1}{4\pi r_1^2} = \frac{Q_2}{4\pi r_2^2}$

$$\Rightarrow \frac{Q_1}{4\pi\varepsilon_o r_1^2} = \frac{Q_2}{4\pi\varepsilon_o r_2^2} \Rightarrow E_2 = E$$

12. **(d)** For charged particle,
$m = 0.5$g, $q = 10\ \mu$c, $E = 8 NC^{-1}$, $t = 5$s

$\therefore$ Acceleration, $a = \frac{qE}{m}$

$\therefore$ Velocity after time t is

$$v = u + at = 0 + \left(\frac{qE}{m}\right).t$$

$$= \frac{10\times10^{-6}\times8\times5}{0.5\times10^{-3}} = 0.8\ ms^{-1}$$

13. **(a)** $m = 3.2\times10^{-27}$kg, $q = e = 1.6\times10^{-19}$ C
At equilibrium, $mg = q^E$

$$\Rightarrow E = \frac{mg}{q} = \frac{3.2\times10^{-27}\times9.8}{1.6\times10^{-19}} = 19.6\times10^{-8}\ NC^{-1}$$

14. **(a)** $\tau = pE\sin\theta$.
$\tau = 2\times10^{-10}\times10^4\times\sin 30°$
$\tau = 10^{-10}\times10^4$

$$\boxed{\tau = 10^{-6}\ N\ m}$$

15. **(b)** Dipole moment, $p = 5\times10^{-30}$ C-m
The charge on neutral NH_3 is $q = 7 + 1\times3 = 10e$
$\therefore\ p = q(l)$
$\Rightarrow\ 5\times10^{-30} = 10\times1.6\times10^{-19}\times l$

$$\therefore\ l = \frac{5\times10^{-30}}{16\times10^{-19}} = 3.125\times10^{-12}\ m$$

16. **(a)** The electric field at a distance r from a uniform straight wire is

$$E = \frac{2k\lambda}{r} = \frac{2\times9\times10^9\times0.2\times10^{-6}}{3} = 1.2\times10^3\ Vm^{-1}$$

17. **(c)** The electric field at equatorial line is

$$\vec{E} = \frac{-K\vec{P}}{r^3} \Rightarrow \vec{E} \text{ and } \vec{P} \text{ are anti parallel}$$

$\therefore\ \theta =$ Angle between

18. **(c)** Give, electrostatic force in air = F
30% of the space is filled with a dielectric medium (with dielectric constant K)

Effective distance $d' = 0.7d + 0.3d\sqrt{k}$

Electrostatic force reduces to F' = $\frac{F}{2.56}$

Force in air, $F = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{d^2}$

Force in the dielectric – filled case $F' = \frac{F}{2.56}$

Apply Coulomb's law

$$\Rightarrow \frac{F}{2.56} = \frac{1}{4\pi\varepsilon_o}\frac{q_1\,q_2}{(d')^2} \Rightarrow \frac{1}{2.56} = \frac{1}{(0.7+0.3\sqrt{k})^2}$$

$$\Rightarrow\ 0.7 + 0.3\sqrt{2} = 1.6 \Rightarrow 0.3\sqrt{k} = 0.9 \Rightarrow k = 9$$

19. **(b)** Using Coulomb's law, force between, 2μC and Q is,

$$F_1 = \frac{1}{4\pi\in_o}\frac{2\times\theta}{(1)^2}$$

Force between charge 2μC and 4μC is

$$F_2 = \frac{1}{4\pi\in_o}\frac{2\times4}{(2)^2}$$

Force between charge 2μC and 12μC is

$$F_3 = \frac{1}{4\pi\in_o}\frac{2\times12}{(4)^2}$$

Given, $F_1 + F_2 + F_3 = 0$

$$\frac{1}{4\pi\epsilon_o}\left[\frac{2Q}{(1)^2}+\frac{8}{(2)^2}+\frac{24}{(4)^2}\right]=0$$

$$\Rightarrow \frac{2Q}{1}+2+\frac{3}{2}=0 \Rightarrow Q=\frac{7}{4}\mu C=-1.75\mu C$$

20. **(c)** Electrostatic force $F=\frac{K Q_1 Q_2}{r^2}$

$$F=\frac{9\times10^9\times(Q)\times\left(36\times10^{-6}-Q\right)}{(4)^2}$$

or, $0.18=\frac{9\times10^9\times Q\times\left(36\times10^{-6}-Q\right)}{16}$

$$\Rightarrow \frac{18\times10^{-2}\times16}{9\times10^9}=Q\times(36\times10^{-6}-Q)$$

$\Rightarrow 32\times10^{-11}=Q\times(36\times10^{-6}-Q)$

Solving we get $Q=20\times10^{-6}=20\,\mu C$

and $36\times10^{-6}-20\times10^{-6}=16\times10^{-6}=16\,\mu C$

Therefore the bigger charge is $20\,\mu C$

21. **(b)** By law of conservation of mechanical energy

$$\Rightarrow \frac{-9\times10^9\times32\times10^{-28}}{4}+\frac{9\times10^9\times32\times10^{-28}}{2}$$

$$=\frac{1}{2}\times9.1\times10^{-31}V^2$$

$\Rightarrow -7.2\times10^{-18}+14.4\times10^{-18}=4.55\times10^{-31}V^2$

$\Rightarrow -7.2\times10^{-18}=4.55\times10^{-31}V^2$

$\Rightarrow V=3.977\times10^6\,m/s$

22. **(c)** $s=\frac{1}{2}at^2 \Rightarrow t\propto\frac{1}{\sqrt{a}}$

$$\begin{cases}q_p=e, m_p=m\\ q_\alpha=2e, m_\alpha=4m\end{cases} \quad \therefore \frac{a_\propto}{a_p}=\frac{q_\propto m_p}{q_p m_\propto}$$

$$\frac{a_\alpha}{a_p}=\frac{2e\times m}{e\times4m}=\frac{2}{4}=\frac{1}{2}$$

$$\frac{t_p}{t_\alpha}=\sqrt{\frac{a_\alpha}{a_p}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$$

23. **(b)** $F_e=-eE$ (electron)

$F_p=+eE$ (positron)

$$a_e=-\frac{eE}{m}, a_p=\frac{+eE}{m}$$

Displacement of electron

$$(s_e)=\frac{1}{2}a_e t^2=\frac{1}{2}\left(-\frac{eE}{m}\right)t^2=-\frac{eEt^2}{2m}$$

Displacement of positron

$$(s_p)=\frac{1}{2}a_p t^2=\frac{1}{2}\times\left(+\frac{eE}{m}\right)t^2=\frac{eEt^2}{2m}$$

Distance of separation between them in direction of the field:

$$\text{As } \Delta s=s_p-s_e=\frac{eEt^2}{2m}-\left(-\frac{eEt^2}{2m}\right)=\frac{eEt^2}{m}$$

24. **(a)** Acceleration of charge particle in an electric field is

$$a=\frac{qE}{m} \therefore v=u+at=\left(\frac{qE}{m}\right)t \Rightarrow v\propto\frac{q}{m}$$

$\therefore$ Work done by electric field,

$$W=\Delta K=\frac{1}{2}mv^2 \Rightarrow W\propto mv^2\propto m\left(\frac{q}{m}\right)^2$$

$$\therefore \frac{W_p}{W_\alpha}=\left(\frac{q_p}{q_\alpha}\right)^2\left(\frac{m_\alpha}{m_p}\right)=\left(\frac{1}{2}\right)^2\left(\frac{4}{1}\right)=1:1$$

25. **(c)**

26. **(a)**

8 Semiconductors

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(c)** A p-type semiconductor is uncharged or neutral, because during the formation of p-type semiconductor, there is no addition of charged particle into the semiconductor.

2. **(d)** A positive hole is a vacancy created when an electron leaves bond.

3. **(d)** At room temperature resistivity of a semiconductor is in between 10^{-3} to $10^{8}\ \Omega$ cm.

4. **(c)** The atom is electrically neutral.

5. **(c)** The number of electrons made available for conduction by dopant atoms depends strongly upon the doping level and is independent of any increase in ambient temperature. However, number of free electrons. generated in semiconductor atom increases weakly with temperature.

6. **(a)** Thermally generated electrons ($n_i \sim 10^{16}\ m^{-3}$) are negligibly small as compared to those produced by doping.

Therefore, $n_e \approx N_D$

Given, total number of atoms

$= 5 \times 10^{28}\ m^{-3}$

$n_i = 1.5 \times 10^{16} m^{-3}$

For every 10^6 atoms, 1 As is doped.

1 As contributely $1e^-$

$n_e = 5 \times 10^{28} / 10^6$

$= 5 \times 10^{22}$

Since, $n_e n_h = n_i^2$

The number of holes

$$n_h = \frac{n_i^2}{n_e} = \frac{(1.5\times10^6)^2}{5\times10^{22}} = \frac{2.25\times10^{32}}{5\times10^{22}}$$

$= 4.5 \times 10^9\ m^{-3}$

7. **(a)**

8. **(c)** Electric conduction, in a semi conductors occurs due to both electrons & holes.

9. **(a)**

10. **(c)** When small amount of antimony (pentavalent) is added to germanium crystal then crystal becomes n-type semi conductor. Therefore, there will be more free electrons than holes in the semiconductor.

11. **(c)** As the temperature increases, the bond between atom break and a large number of charge carrier emitted while the effect of relaxation time is much less.

12. **(d)** In extrinsic semi conductor the number of holes are not equal to number of electrons i.e.,

$n_h \neq n_e$

In p - type $n_h > n_e$

In n - type $n_e > n_h$

13. **(b)**

14. **(d)** $n_i^2 = n_e n_h$

$(1.5 \times 10^{16})^2 = n_e (4.5 \times 10^{22})$

$\Rightarrow \quad n_e = 0.5 \times 10^{10}$

or $\quad n_e = 5 \times 10^9$

Given $\quad n_h = 4.5 \times 10^{22}$

$\Rightarrow n_h >> n_e$

$\therefore$ Semiconductor is p-type and

$n_e = 5 \times 10^9\ m^{-3}$.

15. **(d)** Here, $n_i = 10^{16}\ m^{-3}$, $n_h = 5 \times 10^{22}\ m^{-3}$

As $n_e n_h = n_i^2$

$$\therefore \quad n_e = \frac{n_i^2}{n_h} = \frac{(10^{16}\ m^{-3})^2}{5\times10^{22}\ m^{-3}} = 2\times10^9\ m^{-3}$$

16. **(c)** In semiconductors,

$n_e\, n_h = n_i^2$

$$n_e = \frac{n_i^2}{n_h} = \frac{(1.2\times10^{15})^2}{4\times10^{20}} = \frac{1.44\times10^{30}}{4\times10^{20}}$$

$= 0.36 \times 10^{10} = 3.6 \times 10^9\ m^{-3}$

17. **(a)** At absolute zero kelvin temperature, covalent bonds are very strong and there are no free electrons and hence semiconductor behaves as perfect insulator.

18. **(d)**

19. **(c)** Copper is a conductor so its resistance decreases on decreasing temperature as thermal agitation decreases whereas germanium is semiconductor therefore on decreasing temperature resistance increases.

20. **(d)** For semiconductor, $n = AT^{3/2} e^{-\frac{E_g}{2KT}}$;

so $n \propto T^{3/2}$

21. **(a)** At absolute zero kelvin temperature, covalent bonds are very strong and there are no free electrons and hence semiconductor behaves as perfect insulator.

22. **(d)**

23. **(c)** A crystal structure is composed of a unit cell, a set of atoms arranged in a particular way; which is periodically repeated in three dimensions on a lattice. The spacing between unit cells in various directions is called its lattice parameters or constants. Increasing these lattice constants will increase or widen the band-gap (E_g), which means more energy would be required by electrons to reach the conduction band from the valence band. Automatically E_c and E_v decreases.

24. **(d)** For a p-type semiconductor, the acceptor energy level, as shown in the diagram, is slightly above the top E_v of the volume band. With very small supply of energy an electron from the valence band can jump to the level E_A and ionise acceptor negatively

25. **(c)** Electric conduction, in a semi conductors occurs due to both electrons & holes.

26. **(a)**

27. **(c)** When small amount of antimony (pentavalent) is added to germanium crystal then crystal becomes n-type semi conductor. Therefore, there will be more free electrons than holes in the semiconductor.

28. **(c)** By doping, the band gap reduce from 1eV to 0.3 to 0.7 eV & electron can achieve this energy (0.3eV to 0.7eV) at room temperature & reach in C.B (conduction band).

29. **(d)** In extrinsic semi conductor the number of holes are not equal to number of electrons i.e.,

$n_h \neq n_e$

In p - type $n_h > n_e$

In n - type $n_e > n_h$

30. **(b)**

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(a)**

2. **(d)**

3. **(c)** *N*-type semiconductors are neutral because neutral atoms are added during doping.

4. **(d)** In *P*-type semiconductors, holes are majority charge carriers and electrons are minority charge carriers.

5. **(a)** In *P*-type semiconductors, holes are majority charge carriers.

6. **(a)** Charge carriers inside the *P*-type semiconductor are holes (mainly). Inside the conductor charge carriers are electrons and for cell ions are the charge carriers.

7. **(d)** $\text{Ge} + \underset{\text{impurity}}{\text{Trivalent}} \rightarrow \underset{\text{semiconductor}}{P-\text{type}}$

8. **(b)** $\text{Ge} + \underset{\text{impurity}}{\text{Pantavalent}} \rightarrow \underset{\text{semiconductor}}{N-\text{type}}$

9. **(d)** Extrinsic $\rightarrow$ P-type ($N_p >> N_e$)

S . C. $\rightarrow$ N-type ($N_e >> N_p$)

10. **(b)** The depletion region of an unbiased semiconductor diode contains no free or mobile charge carriers, but only fixed and immobile ions.

11. **(a)** At 0K, motion of free electrons stop. Hence conductivity becomes zero. Therefore, at 0K intrinsic semiconductor becomes insulator.

12. **(b)** Atom of semiconductor are bounded by covalent bonds between the atoms of same or different type. The concept of hole describes the lack of an electron at a position where one could exist in an atom or atomic lattice. If an electron is excited into a higher state, it leaves a hole in its old state. So, hole can be defined as a vacancy created when an electron leaves a covalent bond.

13. **(a)** Donor impurity gives extra electron to the atom which is present in its outer shell.

Thus when these type of impurity is added, the concentration of electron increases while that of holes decreases.

Thus option (a) is correct answer.

14. **(b)** Majority carriers in an n-type semiconductor are electrons.

15. **(c)** In p-region of p-n junction

holes concentration > electrons concentration and in n-region of p-n junction.

electrons concentration > holes concentration.

16. **(d)** When an n-type semiconductor is heated, number of holes and electrons increases equally as when a free electron is produced, simultaneously a hole is also produced.

Exercise 3 : PREVIOUS YEARS MCQs

1. **(c)** Since current conduction requires free electrons and holes, and neither are present at OK, the material cannot conduct electricity.

So, it behaves as insulator.

2. **(d)** $n_e = n_h = 1.5 \times 10^{16}\ m^{-3}$

$\therefore$ Number of intrinsic charge carriers,

$$n_i^2 = n_e n_h = \left(1.5 \times 10^{16}\right)^2$$

Again,

$$\left(1.5 \times 10^{16}\right)^2 = n_e (3 \times 10^{22})$$

$\therefore \quad n_e = 7.5 \times 10^9\ m^{-3}$

3. **(a)** If energy gap, $E_g > 5$ eV, then the substance is insulator.

4. **(c)** At thermal equilibrium,

$n_i^2 = n_e n_h \Rightarrow n_i = \sqrt{n_e n_h}$

5. **(a)** Here, $n_i = 1.4 \times 10^{16}\ m^{-3}$

$n_h = 4 \times 10^{22}\ m^{-3}$

$n^2_i = n_h n_e$

Electron concentration, $n_e = \frac{n_i^2}{n_h}$

$= \frac{\left(1.4\times10^{16}\right)^2}{4\times10^{22}} = 0.49 \times 10^{10}\ m^{-3}$

6. **(d)** At absolute zero temperature, a semiconductor has no free electrons in the conduction bond, so it behaves like on insulator.

7. **(c)** Any n-type or p-type semiconductors are neutral.

8. **(c)** using $= n_e n_h$ Given $n_i = 6 \times 10^{15} m^{-3}$,

$n_e = 4 \times 10^{22}\ m^{-3}$, $n_h = ?$

$(6 \times 10^{15})^2 = 4 \times 10^{22} \times n_h$

$= 9 \times 10^8\ m^{-3}$

9. **(a)** Donor impurity gives extra electron to the atom which is present in its outer shell.
Thus when these type of impurity is added, the concentration of electron increases while that of holes decreases.
Thus option (a) is correct answer.

10. **(c)**

(A) *n* type semiconductor holes are minority carries and e^- are majority carriers.

(B) Dopant are pentavalent atom.

(C) $n_e \cdot n_h = n_I^2$ for intrinsic semiconductor

(E) In *n* type semiconductor primary source of holes generation are thermal excitation.

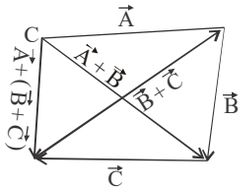
11. **(c)** In insulators, valence band is completely filled while conduction band is empty. The energy band gap is maximum in insulators.

9 Vectors and Error Analysis

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(c)** A vector quantity is defined as the quantity which has magnitude and direction and for which all the mathematical operations are possible only through vector laws of algebra.

2. **(a)**



It illustrates the associative law of addition.

3. **(c)** The resultant of $\vec{A}\times 0$ is a vector of zero magnitude. The product of a vector with a scalar gives a vector.

4. **(c)**

5. **(a)** Using, Pythagoras theorem $(5)^2 = (3)^2 + (4)^2$

Thus, A and B are perpendicular.

6. **(c)** Given, $A + B + C = 0$

$\Rightarrow \quad A = -(B + C) = -1(B + C)$

Hence, vector A is anti-parallel to vector (B + C).

7. **(d)** $-3A = -3 \times 7$ units (North) = 21 units

The direction of vector is towards south.

8. **(d)** **9.** **(a)**

10. **(c)** Vector product of two parallel vectors

$$\vec{A}.\vec{B} = AB\cos\theta\ ;\ \theta = \frac{\pi}{2}$$

$$\therefore \vec{A}.\vec{B} = AB\cos\left(\frac{\pi}{2}\right) = 0$$

11. **(b)** The magnitude of the resultant R will be same but direction will get changed.

12. **(b)** $|Q|^2 = Q_x^2 + Q_y^2$

$$(5)^2 = Q_x^2 + 4^2 \Rightarrow Q_x = \sqrt{25-16} = 3$$

13. **(c)** Using, $\tan\theta = \dfrac{A_y}{A_x} \Rightarrow A_y = A_x \tan\theta$

$$\Rightarrow \quad A_y = 50\tan 60° = 50 \times \sqrt{3} \simeq 87\text{N}$$

14. **(b)** $\left|\vec{P}+\vec{Q}\right| = \left|\vec{P}\right| = \left|\vec{Q}\right|$

$$\Rightarrow \quad P^2 + Q^2 + 2PQ\cos\theta = P^2 = Q^2$$

$$\Rightarrow \quad P^2 + P^2 + 2P^2\cos\theta = P^2 \ \Rightarrow\ 2P^2(1+\cos\theta) = P^2$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2} = \cos 120° \quad \therefore \quad \theta = 120°$$

15. **(b)**

16. **(d)** Resultant of vectors is given by:

$R^2 = a^2 + b^2 + 2ab\cos\theta.$

Given $R = a = b = x$

$x^2 = x^2 + x^2 + 2x^2\cos\theta$

$$-\frac{1}{2} = \cos\theta \ \Rightarrow\ \theta = 120°$$

17. **(c)** $\vec{R} = \vec{A} + \vec{B}$

$$R = \frac{\left|\vec{B}\right|}{2}$$

$$\frac{\left|\vec{B}\right|}{2} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$A^2 + B^2 + 2AB\cos\theta = \frac{B^2}{4}$$

$$A^2 + 2AB\cos\theta + \frac{3B^2}{4} = 0 \qquad ...(1)$$

$\vec{R} \perp$ to $\vec{A}$

$$\vec{A}\cdot\left(\vec{A}+\vec{B}\right) = 0$$

$A^2 + AB\cos\theta = 0$

$AB\cos\theta = -A^2$ ...(2)

put this value in equation (1)

$$A^2 - 2A^2 + \frac{3B^2}{4} = 0$$

$$A^2 - \frac{3B^2}{4} = 0 \Rightarrow A = \frac{\sqrt{3}B}{2}$$

from equation (2),

$$\cos\theta = -\frac{A}{B} = \frac{\sqrt{\frac{3}{2}}B}{B}$$

$\cos\theta = \cos 150 \Rightarrow \theta = 150°$

18. (c) $Q = 6\hat{i} - 8\hat{j} = 2\left(3\hat{i} - 4\hat{j}\right) = 2P \quad [\because P = 3\hat{i} - 4\hat{j}]$

$$R = \frac{3}{4}\hat{i} - \hat{j} = \frac{1}{4}\left(3\hat{i} - 4\hat{j}\right) = \frac{P}{4}$$

Thus, P, Q and R are parallel with unequal magnitude.

19. (d) All the three unit vectors have the magnitude as unity

$\therefore \quad \hat{i} = \hat{j} = \hat{k} = 1$

20. (d) $\sqrt{(0.4)^2 + (0.7)^2 + c^2} = 1$

$c^2 = 1 - (0.16 + 0.49) \Rightarrow c^2 = 1 - 0.65 = 0.35 \Rightarrow c = \sqrt{0.35}$

21. (a) We have, $\hat{n}_1 \cdot \hat{n}_2$

$$= \frac{1}{13}\left(5\hat{i} + 12\hat{j}\right) \cdot \frac{1}{5}\left(3\hat{i} + 4\hat{j}\right) = \frac{1}{65}(15 + 48) = \frac{63}{65}$$

22. (c) $\vec{A} = Ax\hat{i} + Ay\hat{j} + Az\hat{k}$; $|\vec{A}| = \sqrt{Ax^2 + Ay^2 + Az^2}$

$\therefore$ Even if one component is non – zero the sum $Ax^2 + Ay^2 + Az^2$ can't be zero.

$\therefore$ for $|\vec{A}| = 0, Ax = Ay = Az = 0.$

23. (d) We have, $\vec{P}.\vec{Q} = 0$

$$\Rightarrow \quad \left(\hat{i} + 2m\hat{j} + m\hat{k}\right).\left(4\hat{i} - 2\hat{j} + m\hat{k}\right) = 0$$

$$\Rightarrow \quad 4 - 4m + m^2 = 0 \Rightarrow (m-2)^2 = 0 \Rightarrow m = 2$$

24. (b) $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos 120°}$ $(\theta = 120°)$

$$= \sqrt{A^2 + B^2 + \cancel{2}AB\left(\frac{-1}{\cancel{2}}\right)} \left(\cos 120° = -\frac{1}{2}\right)$$

$$= \sqrt{A^2 + B^2 - A(A)} = \sqrt{B^2} = B \quad (\therefore A = B)$$

25. (c) Given, $|A + B| = |A - B|$

$$|A + B| = \sqrt{|A|^2 + |B|^2 - 2|A||B|\cos\theta}$$

$\therefore \quad |A|\,|B|\cos\theta = 0$

$\therefore \quad |A| = 0$ or $|B| = 0$

or $\cos\theta = 0$ i.e., $\theta = 90°$

$\Delta r = r(P') - r(P)$

26. (b) Resultant,

$\vec{R} = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) + \left(2\hat{i} - 7\hat{j} - 4\hat{k}\right)$

$\therefore$ Angle made by $\vec{R}$ with x axis is

$$\cos\theta = \frac{R_x}{R} = \frac{4}{\sqrt{(4)^2 + (-4)^2}} = \frac{1}{\sqrt{2}} = \cos 45°$$

$\therefore \quad \theta = 45°$

27. (a) Using, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$4F^2 + 9F^2 + 12F^2\cos\theta = R^2$

When forces Q is doubled,

$4F^2 + 36F^2 + 24F^2\cos\theta = 4R^2$

$4F^2 + 36F^2 + 24F^2\cos\theta$

$= 4(13F^2 + 12F^2\cos\theta) = 52F^2 + 48F^2\cos\theta$

$$\therefore \cos\theta = -\frac{12F^2}{24F^2} = -\frac{1}{2} \Rightarrow \theta = 120^o$$

28. (a) We have

$$\vec{r_1} \times \vec{r_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$= \hat{i}(2) + \hat{j}(0) + \hat{k}(-2) = 2\hat{i} - 2\hat{k}$

So, unit vector in the direction of

$$\left(\vec{r_1} \times \vec{r_2}\right) = \frac{2\hat{i} - 2\hat{k}}{\sqrt{2^2 + (-2)^2}} = \frac{\hat{i}}{\sqrt{2}} - \frac{\hat{k}}{\sqrt{2}}$$

29. **(c)** Let $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ be the direction cosines of a vector

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \sin^2\alpha + \cos^2\alpha = 1 + 1$

$\sin^2\alpha + 2\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2$

30. **(b)** Given $\vec{A}$ and $\vec{B}$ lie in a plane and vector $\vec{C}$ lies outside this plane.

Resultant vector of $\vec{A}$ and $\vec{B}$ lies in the same plane as that vectors $\vec{A}$ and $\vec{B}$.

Resultant vector of $\vec{A}, \vec{B}$ and $\vec{C}$ in non.

coplanar vector therefore, their resultant can never be zero.

31. **(c)**

32. **(d)** Here $A_y = 2\sqrt{3}$

$$A_y = A\cos 30° = 2\sqrt{3} \Rightarrow A\frac{\sqrt{3}}{2} = 2\sqrt{3} \Rightarrow A = 4$$

$$\text{Now } A_x = A\sin 30° = 4 \times \frac{1}{2} = 2$$

33. **(c)** $\text{Unit vector} = \dfrac{\vec{A}+\vec{B}}{|\vec{A}+\vec{B}|} = \dfrac{3\hat{i}+5\hat{k}}{\sqrt{3^2+5^2}}$

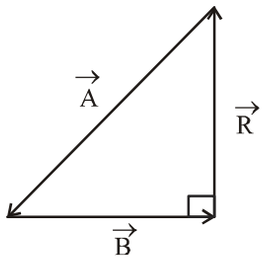
34. **(c)** The resultant of any three vectors will be cancel out by fourth vector.

35. **(a)** Let two vector be $\vec{A}$ and $\vec{B}$

$|\vec{A}| + |\vec{B}| = 18$

$|\vec{R}| = 12$

Let $|\vec{A}|$ be x,



then $|\vec{B}| = 18 - x$

$$|\vec{R}| = \sqrt{|\vec{A}|^2 - |\vec{B}|^2}$$

$$12 = \sqrt{x^2 - (18-x)^2}$$

$144 = x^2 - (324 + x^2 - 36x)$

$144 = x^2 - 324 - x^2 + 36x$

$36x = 468 \quad \therefore |\vec{A}| = x = 13$

$|\vec{B}| = 18 - x = 18 - 13 = 5$

36. **(c)** As cross product of two vectors is vector only

So, $(\vec{A}\times\vec{C})\times(\vec{B}\times\vec{C})$ is vector only.

37. **(c)** We have,

$$a_x = \frac{dV_x}{dt} = \frac{d}{dt}(6+2t) = 2$$

$$a_y = \frac{dV_y}{dt} = \frac{d}{dt}(4+2\sqrt{3}t) = 2\sqrt{3}$$

$$\text{So, } \vec{a} = a_x\hat{i} + a_y\hat{j} = 2\hat{i} + 2\sqrt{3}\hat{j}$$

38. **(c)** Resultant of two vector $\vec{A}$ and $\vec{B}$ having angle θ between them $R' = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

It is given $A = B = R$

$$\therefore \quad R' = \sqrt{R^2 + R^2 + 2R^2\cos\theta}$$

$$= R\sqrt{2}\sqrt{1+\cos\theta} = \sqrt{2}R\sqrt{2\cos^2\frac{\theta}{2}} = 2R\cos\frac{\theta}{2}$$

39. **(c)** Since, $(A + B)$ makes an angle θ_1 with A

$$\tan\theta_1 = \frac{B\sin\alpha}{A + B\cos\alpha}$$

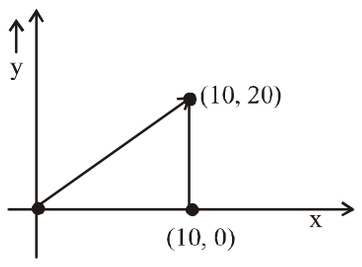
$A - B$ makes an angle Q_2 with A

$$\tan\theta_2 = \frac{B\sin\alpha}{A - B\cos\alpha}$$

$$\therefore \tan\theta_1 - \tan\theta_2 = \frac{B\sin\alpha}{A + B\cos\alpha} - \frac{B\sin\alpha}{A - B\cos\alpha}$$

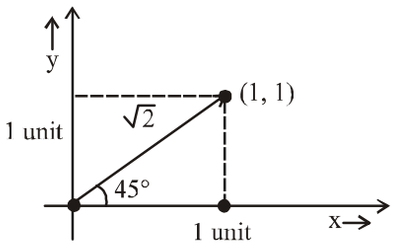
$$= \frac{-2B^2\sin\alpha\cos\alpha}{A^2 - B^2\cos^2\alpha} = \frac{-B^2\sin 2\alpha}{A^2 - B^2\cos^2\alpha}$$

40. **(a)**



Let $\vec{S_1}$ be ant's displacement vector. Then,

$\vec{S_1} = 10\hat{i} + 20\hat{j}$



Let $\vec{S_2}$ be the positive vector

Then, from diagram

$\vec{S_2} = \hat{i} + \hat{j}$

So, $\vec{S_1} . \vec{S_2} = \left(10\hat{i} + 20\hat{j}\right).\left(\hat{i} + \hat{j}\right) = 30$ cm.

41. **(c)**

42. **(d)** Taking large no. of observations and their mean value, we can eliminate random errors.

43. **(d)**

44. **(a)** Least count is the smallest value that can be measured by the measuring instrument.

45. **(d)** Instrumental errors, imperfection in experimental technique, personal error are source of systematic errors.

46. **(a)** Absolute error, $\Delta a_n = a_n - a_{\text{mean}}$

47. **(d)** If z = AB then, $\frac{\Delta z}{z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

48. **(a)** Fractional error $= \frac{\Delta M}{M}$ as $\Delta M_A = \Delta M_B$

as $M_B > M_A \Rightarrow \frac{\Delta M_A}{M_A} > \frac{\Delta M_B}{M_B}$

49. **(a)** Percentage error in length = fractional error × 100

$= \frac{\Delta l}{l} \times 100 = \frac{0.05}{5} \times 100 = 1\%$

50. **(c)** $\frac{\Delta Z}{Z} = 3\left(\frac{\Delta A}{A}\right)$

51. **(None)** $\delta = \frac{M}{V} = \frac{M}{l^3} = Ml^{-3}$

$\frac{\Delta\delta}{\delta} = \frac{\Delta M}{M} + 3\frac{\Delta l}{l} = \frac{0.10}{10.00} + 3\left(\frac{0.01}{0.10}\right) = 0.31\ \text{kg/m}^3$

52. **(a)**

53. **(d)** $P = \frac{a^3b^2}{cd}$, $\frac{\Delta P}{P} \times 100\% = 3\frac{\Delta a}{a} \times 100\% + 2\frac{\Delta b}{b} \times$

$100\% + \frac{\Delta c}{c} \times 100\% + \frac{\Delta d}{d} \times 100\%.$

$= 3 \times 1\% + 2 \times 2\% + 3\% + 4\% = 14\%$

54. **(c)** Given, $P = a^{1/2} b^2 c^2 d^{-4}$,

Maximum relative error, $\frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}$

$= \frac{1}{2} \times 2 + 2 \times 1 + 3 \times 3 + 4 \times 5 = 32\%$

55. **(a)** $X = M^a L^b T^c$;

$\frac{\Delta X}{X} \times 100 = \left(\frac{a\ \Delta M}{M} + \frac{b\ \Delta L}{L} + \frac{c\ \Delta T}{T}\right) \times 100$

$= (a\alpha + b\beta + c\gamma)\%$

56. **(c)** We have, $S = \frac{\sqrt{a}\ b}{c^3 d^4}$

or, $\frac{\Delta S}{S} = \frac{1}{2}\frac{\Delta a}{a} + \frac{\Delta b}{b} - \frac{3\Delta c}{c} - \frac{4\Delta d}{d}$

$\left|\frac{\Delta S}{S} \times 100\right| = \left|\frac{1}{2} \times 2\%\right| + |1\%| + |3 \times 1\%| + |4 \times 1\%| = 9\%$

57. **(c)** $V = (30 \pm 0.3)V, I = (5 \pm 0.1)A$

Resistance, $R = \frac{V}{I}$

$\therefore \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{0.3}{30} + \frac{0.1}{5} = 0.03$

$\therefore \quad \%\,\Delta R = \left(\frac{\Delta R}{R} \times 100\right) = 0.03 \times 100 = 3\%$

58. **(a)** $\frac{\Delta m}{m}\times 100 = 3\%, \frac{\Delta v}{v}\times 100 = 4\%$

Kinetic energy, $k = \frac{1}{2}mu^2$

$\therefore$ Percentage error in kinetic energy,

$$\frac{\Delta k}{k}\times 100 = \left(\frac{\Delta m}{m}\times 100\right) + 2\left(\frac{\Delta u}{v}\times 100\right)$$

$= 3\% + 2(4\%) = 11\%$

59. **(d)** In simple pendulum experiment, $\frac{\Delta \ell}{\ell} = 1\%, \frac{\Delta T}{T} = 2\%$

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g = \frac{4\pi^2\ell}{T^2}$

$\therefore \frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T} = 1\% + 2(2\%) = 5\%$

60. **(d)** $C = (4.0 \pm 0.2)\mu F, V = (10.0 \pm 0.1)V$

The charge on capacitor is

$Q = CV = 4 \times 10 = 40\ \mu C$

Also, $\frac{\Delta Q}{Q} = \frac{\Delta C}{C} + \frac{\Delta V}{V} = \frac{0.2}{4} + \frac{0.1}{10} = 0.06$

$\therefore \quad \%\Delta Q = \frac{\Delta Q}{Q} \times 100 = 0.06 \times 100 = 6\%$

$\therefore$ Charge, $(Q \pm \Delta Q\%) = 40\ \mu C \pm 6\%$

61. **(d)** Since $R = \frac{V}{I}$ so, error in the measurement of resistance

$$\frac{\Delta R}{R}\times 100 = \frac{\Delta V}{V}\times 100 + \frac{\Delta I}{I}\times 100$$

Given current $= (10 \pm 0.5)$ A and potential difference $= (100 \pm 6)$v

$\therefore$ Error in the measurement of resistance

$$\frac{\Delta R}{R}\times 100 = \frac{6}{100}\times 100 + \frac{0.5}{10}\times 100 = 6 + 5 = 11\%$$

62. **(b)** Given, $V + \Delta V = (50 \pm 3)$ V

$I + \Delta I = (5 \pm 0.1)$ A

from ohm's law

$V = IR$

$\ln V = \ln I + \ln R$

$$\frac{\Delta V}{V} = \frac{\Delta I}{I} + \frac{\Delta R}{R}$$

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{3}{50} - \frac{0.1}{5}$$

$$\left|\frac{\Delta R}{R}\right| \times 100 = \frac{4}{100} \times 100 = 4\%$$

63. **(b)** Area of rectangle, $A = l \times b$

$\therefore \ln A = \ln l + \ln b$ or, $\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$

Here, $\frac{\Delta l}{l} = \frac{\Delta b}{b} = 1\% = 0.01$

$\therefore \Delta A = A\left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) \quad [\because A = l \times b = 1 \times 0.5]$

$= 1 \times 0.5(0.01 + 0.01) = 0.01$

$\therefore$ Area of the table including error $A' = A \pm \Delta A$

or, $A' = (0.5 \pm 0.01)\ m^2$.

64. **(a)** We have, $\frac{\Delta m}{m} = 1\%$ and, $\frac{\Delta \partial}{d} = 2\%$

Now, density, $\partial = \frac{\text{mass}}{\text{volume}} \Rightarrow \partial = \frac{m}{v}$

$V = l^3$

Then, $\partial = \frac{m}{1^3} \Rightarrow 1 = \left(\frac{m}{\partial}\right)^{\frac{1}{3}}$

Then the error in measurement of length,

$$\frac{\Delta l}{1}\times 100 = \frac{1}{3}\left(\frac{\Delta m}{m}\times 100 + \frac{\Delta \partial}{\partial}\times 100\right)$$

$$\frac{\Delta l}{1}\times 100 = \frac{1}{3}(1+2)\times 100 = 1\%$$

65. **(b)** Pressure, $P = \frac{F}{L^2}$ $[\because$ Area of square $= L^2]$

$$\frac{\Delta P}{P} = \frac{\Delta F}{F} + \frac{2\Delta L}{L}$$

$$\left(\frac{\Delta P}{P}\times 100\right)=\left(\frac{\Delta F}{F}\times 100\right)+2\left(\frac{\Delta L}{L}\times 100\right)$$

$=4\%+2(3\%)=10\%.$

66. **(b)** $Z=\dfrac{A^4B^{1/3}}{CD^{3/2}}$

$$\frac{\Delta Z}{Z}=4\left(\frac{\Delta A}{A}\right)+\frac{1}{3}\left(\frac{\Delta B}{B}\right)+\left(\frac{\Delta C}{C}\right)+\frac{3}{2}\left(\frac{\Delta D}{D}\right)$$

67. **(d)** Given : $Z=\dfrac{a^2b^{2/3}}{\sqrt{c}d^3}$

Percentage error in Z,

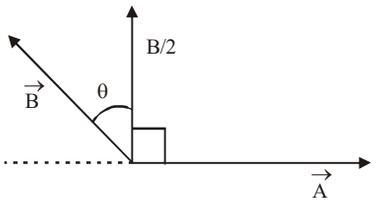
$$=\frac{\Delta Z}{Z}=\frac{2\Delta a}{a}+\frac{2}{3}\frac{\Delta b}{b}+\frac{1}{2}\frac{\Delta c}{c}+\frac{3\Delta d}{d}$$

$$=2\times 2+\frac{2}{3}\times 1.5+\frac{1}{2}\times 4+3\times 2.5=14.5\%.$$

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(c)** Among the given quantities potential energy is a scalar quantity.

2. **(d)**



From the figure

$B\cos\theta=\dfrac{B}{2}$

$\Rightarrow \theta=60°$

So, angle between two vectors is $90°+60°=150°$

3. **(d)** The resultant of two vectors $\vec{A}$ and $\vec{B}$ is given by

$R_{min}\le R\le R_{max} \Rightarrow (A-B)\le R\le (A+B)$

For $A=8$ units, $B=4$ units

$4\le R\le 12$

4. **(a)** A vector has a maximum value along its own direction only. Hence the component of a vector $\vec{r}$ is along positive x–axis.

5. **(a)** $\vec{N}=|\vec{A}|\hat{B}=|\vec{A}|\dfrac{\hat{B}}{|B|}=\dfrac{\left(\sqrt{3^2+4^2}\right)\left(4\hat{i}+3\hat{j}\right)}{\sqrt{4^2+3^2}}$

$$=\frac{5(4\hat{i}+3\hat{j})}{5}=4\hat{i}+3\hat{j} \quad \therefore x=4$$

6. **(c)** Resultant of two vector $\vec{A}$ and $\vec{B}$ having angle θ between them $R'=\sqrt{A^2+B^2+2AB\cos\theta}$

It is given $A=B=R$

$\therefore \quad R'=\sqrt{R^2+R^2+2R^2\cos\theta}$

$$=R\sqrt{2}\sqrt{1+\cos\theta}=\sqrt{2}R\sqrt{2\cos^2\frac{\theta}{2}}=2R\cos\frac{\theta}{2}$$

7. **(b)** $\vec{B}\times\vec{C}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 4 & 3\\ -8 & -1 & 3\end{vmatrix}$

$\Rightarrow \hat{i}(12+3)-\hat{j}(-3+24)+\hat{k}(1+32)=15\hat{i}-21\hat{j}+33\hat{k}$

$\Rightarrow \vec{A}\cdot(\vec{B}\times\vec{C})=(-x\hat{i}-6\hat{j}-2\hat{k})\cdot(15\hat{i}-21\hat{j}+33\hat{k})$

$\Rightarrow 0=-15x+126-66\Rightarrow x=4$

8. **(a)** Given, Angle between two vectors, $\theta=\cos^{-1}\left(\dfrac{5}{9}\right)$

$\therefore \quad \cos\theta=\dfrac{5}{9}$

Given, $|\vec{a}+\vec{b}|=\sqrt{2}\,|\vec{a}-\vec{b}|$

$\Rightarrow \quad a^2+b^2+2\vec{a}\cdot\vec{b}=2a^2+2b^2-4\vec{a}\cdot\vec{b}$

$\Rightarrow \quad 6\vec{a}\cdot\vec{b}=a^2+b^2$

$\Rightarrow \quad 6ab\cos\theta=a^2+b^2$

$\Rightarrow \quad 6\times\dfrac{5}{9}ab=a^2+b^2$

$\Rightarrow \frac{10}{3}ab = a^2 + b^2$

$\Rightarrow \quad \frac{10}{3}nb^2 = n^2b^2 + b^2 \qquad (\because \quad a = nb \text{ given})$

$\Rightarrow \quad 3n^2 - 10n + 3 = 0$

On Solving $n = \frac{1}{3}$ and $n = 3$

Integer value $n = 3$

9. **(a)** We have given,

$|\vec{X} - \vec{Y}| = n|\vec{X} + \vec{Y}|$

As $|\vec{A} + \vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta$

and $|\vec{A} - \vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cos\theta$

$\therefore \quad |\vec{X}|^2 + |\vec{Y}|^2 - 2|\vec{X}||\vec{Y}|\cos\theta$

$= n^2\left[|\vec{X}|^2 + |\vec{Y}|^2 + 2|\vec{X}||\vec{Y}|\cos\theta\right]$

Given, $|\vec{X}| = |\vec{Y}|$

$\Rightarrow \quad 2|\vec{X}|^2 - 2|\vec{X}|^2\cos\theta = 2n^2|\vec{X}|^2 + 2n^2|\vec{X}|^2\cos\theta$

$\Rightarrow \quad 1 - \cos\theta = n^2 + n^2\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{1-n^2}{1+n^2} \Rightarrow \theta = \cos^{-1}\frac{1-n^2}{1+n^2}$

10. **(c)** $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$

$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 3 - 1 = 2$

11. **(d)** $|B| = \sqrt{7^2 + (24)^2} = \sqrt{625} = 25$

Unit vector in the direction of A will be $\hat{A} = \frac{3\hat{i} + 4\hat{j}}{5}$

So, required vector $= 25\left(\frac{3\hat{i} + 4\hat{j}}{5}\right) = 15\hat{i} + 20\hat{j}$

12. **(d)** Using,

$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\theta$

$5^2 = 3^2 + 5^2 + 2 \times 3 \times 5\cos\theta$ or $\cos\theta = -0.3$

$\left(2\vec{A_1} + 3\vec{A_2}\right).\left(3\vec{A_1} - 2\vec{A_2}\right) = 2A_1 \times 3A_1$

$+ (3A_2)(3A_1)\cos\theta - (2A_1)(2A_2)\cos\theta - 3A_2 \times 2A_2$

$= 6A_1^2 + 9A_1A_2\cos\theta - 4A_1A_2\cos\theta - 6A_2^2$

$= 6A_1^2 - 6A_2^2 + 5A_1A_2\cos\theta$

$= 6 \times 3^2 - 6 \times 5^2 + 5 \times 3 \times 5(-0.3) = -118.5$

13. **(b)** $\frac{B}{2} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ ..... (i)

$\therefore \quad \tan 90° = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow A + B\cos\theta = 0$

$\therefore \quad \cos\theta = -\frac{A}{B}$

Hence, from (i) $\frac{B^2}{4} = A^2 + B^2 - 2A^2 \Rightarrow A = \sqrt{3}\frac{B}{2}$

$\Rightarrow \quad \cos\theta = -\frac{A}{B} = -\frac{\sqrt{3}}{2} \quad \therefore \theta = 150°$

14. **(b)** The unit vector in the direction of $\vec{P} \times \vec{Q}$,

$\hat{\eta} = \frac{\vec{P} \times \vec{Q}}{|\vec{P} \times \vec{Q}|}$

$\vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & \sqrt{3} & 2 \\ 4 & \sqrt{3} & 2.5 \end{vmatrix} = \frac{\sqrt{3}}{2}\hat{i} + \frac{\hat{j}}{2} - \sqrt{3}\hat{k}$

$|\vec{P} \times \vec{Q}| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (-\sqrt{3})^2}$

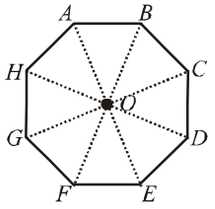
$= \sqrt{\frac{3}{4} + \frac{1}{4} + 3} = \sqrt{\frac{3+1+12}{4}} = \sqrt{\frac{16}{4}} = 2$

$\Rightarrow \quad \frac{\vec{P} \times \vec{Q}}{|\vec{P} \times \vec{Q}|} = \frac{1}{2}\left(\sqrt{3}\frac{\hat{i}}{2} + \frac{\hat{j}}{2} - \sqrt{3}\hat{k}\right)$

$= \frac{1}{4}(\sqrt{3}\,\hat{i} + \hat{j} - 2\sqrt{3}\hat{k})$

x = 4

15. (c)



From figure,

$\overline{AO} + \overline{OB} = \overline{AB}$ $\overline{AO} + \overline{OC} = \overline{AC}$ $\overline{AO} + \overline{OD} = \overline{AD}$

$\overline{AO} + \overline{OE} = \overline{AE}$ $\overline{AO} + \overline{OF} = \overline{AF}$ $\overline{AO} + \overline{OG} = \overline{AG}$

$\overline{AO} + \overline{OH} = \overline{AH}$

Now, adding we get

$8\overline{AO} = (\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} + \overline{AG} + \overline{AH})$

$\Rightarrow 8(2\hat{i} + 3\hat{j} - 4\hat{k})$
$= \overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} + \overline{AG} + \overline{AH}$

$\therefore \overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} + \overline{AG} + \overline{AH}$

$= 16\hat{i} + 24\hat{j} - 32\hat{k}$

16. (b) Density, $D = \frac{\text{Mass(M)}}{\text{Volume(V)}}$

$\therefore \frac{\Delta D}{D} = \frac{\Delta M}{M} + \frac{\Delta V}{V} = \left(\frac{0.01}{22.42} + \frac{0.1}{4.7}\right) \times 100 = 2\%$

17. (c) $d = \frac{M}{V} = \frac{M}{L^3} = ML^{-3}$

$\frac{\Delta d}{d} = \frac{\Delta M}{V} + 3\frac{\Delta L}{L} = \frac{0.10}{10.00} + 3\left(\frac{0.01}{0.10}\right) = 0.31 \text{ kgm}^{-3}$

18. (b) $z = a^2x^3y^{1/2}$

$\frac{\Delta z}{z} = 2\frac{\Delta a}{a} + \frac{3\Delta x}{x} + \frac{1}{2}\frac{\Delta y}{y}$

a is constant

So, $\frac{\Delta z}{z} \times 100\% = \left(3\frac{\Delta x}{x} + \frac{1}{2}\frac{\Delta y}{y}\right) \times 100\%$

$= 3 \times 4 + \frac{1}{2} \times 12 = 18\%$

19. (a) Actual $R = \frac{8}{2} = 4$

% error in R = % error in V + % error in I

$= \frac{0.5}{8} \times 100\% + \frac{0.2}{2} \times 100\% = 16.25\%$

$R = 4\Omega \pm 16.25\%$

20. (a) We have

$\rho = \frac{m}{v} = \frac{m}{\frac{4}{3}\pi r^2 h} \Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta h}{h}$

$= \frac{0.006}{0.6} + \frac{2 \times 0.005}{0.5} + \frac{0.04}{4}$

$= 0.01 + 2 \times 0.01 + 0.01$

$= 0.01 + 0.02 + 0.01 = 0.04$

So, $\frac{\Delta\rho}{\rho} \times 100 = 4\%$

21. (d) Maximum percentage error in A

= 3(% error in P) + 2(% error in Q)

$+ \frac{1}{2}$(% error in R) + 1(% error in S)

$= 3 \times 0.5 + 2 \times 1 + \frac{1}{2} \times 3 + 1 \times 1.5$

=1.5+2+1.5+1.5=6.5%

22. (a) Given, $T = 2\pi\sqrt{\frac{l}{g}}$

Squaring on both sides and rearranging, we get

$g = \frac{4\pi^2 l}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$

(Here, $\Delta l = 1$ mm, $l = 1$ m, $\Delta T = 0.01$ s, $T = 1.95$ s)

$$\therefore \frac{\Delta g}{g} = \frac{1\times10^{-3}}{1} + 2\times\frac{0.01}{1.95} \Rightarrow \frac{\Delta g}{g} = 0.0113$$

Percentage error in g

$$\frac{\Delta g}{g}\times 100 = 1.13\%$$

23. **(d)** Surface tension, $T = \frac{rhg}{2}\times10^3$

Relative error in surface tension, $\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + 0$

($\because$ g, 2 and 10^3 are constant)

Percentage error

$$100\times\frac{\Delta T}{T} = \left(\frac{10^{-2}\times0.01}{1.25\times10^{-2}} + \frac{10^{-2}\times0.01}{1.45\times10^{-2}}\right)100$$

$= (0.8 + 0.689) = (1.489) = 1.489\% \cong 1.5\%$

24. **(a)** Given $f = \frac{p}{2\ell}\sqrt{\frac{T}{\mu}}$. Taking log of both sides

$$\log f = \log\left(\frac{p}{2}\right) - \log\ell + \frac{1}{2}\log T - \frac{1}{2}\log\mu$$

Differentiating partially on both sides,

$$\frac{df}{f} = 0 - \frac{d\ell}{\ell} + \frac{1}{2}\frac{dT}{T} - \frac{1}{2}\frac{d\mu}{\mu}$$

$$\text{or} \quad \frac{df}{f}\times100 = \left(-\frac{d\ell}{\ell}\times100\right) + \left(\frac{1}{2}\frac{dT}{T}\times100\right) - \left(\frac{1}{2}\frac{d\mu}{\mu}\times100\right)$$

$$= (-1) + \frac{1}{2}(-2) - \frac{1}{2}(4) = -1 - 1 - 2 = -4\%$$

25. **(a)** The current voltage relation of diode is

$I = (e^{1000\,V/T} - 1)$ mA (given)

When, $I = 5mA$, $e^{1000\,V/T} = 6mA$

Also, $dI = (e^{1000\,V/T})\times\frac{1000}{T}.\,dV$

Error $= \pm 0.01$ (By exponential function)

$$= (6\,mA)\times\frac{1000}{300}\times(0.01) = 0.2\,\text{mA}$$

26. **(c)** $B - A = 4.19 - 3.25$

$= \pm(0.01 + 0.01) = 0.94 \pm 0.02$

27. **(b)** Relative error in 1.02 g

$= (\pm\,0.01/1.02)\times100\% \cong \pm1\%$

The relative error in 9.89 g

$= (\pm\,0.01/9.89)\times100\% = \pm0.1\%$

28. **(c)** Resistance, $R = \frac{\rho l}{A}$

$$\Rightarrow R = \frac{\rho l}{\pi\frac{d^2}{4}} = \frac{4\rho\ell}{\pi d^2}$$

$$\therefore \frac{\Delta R}{R} = \frac{\Delta l}{l} + \frac{2\Delta d}{d} = 0.1 + 2(0.1)$$

$$\Rightarrow \frac{\Delta R}{R} = 0.3\%$$

29. **(d)** $I = MR^2$

On differentiation.

$$\frac{\Delta I}{I}\times100 = \pm\left(\frac{\Delta M}{M}\times100 + \frac{2\Delta R}{R}\times100\right)$$

$= \pm(1 + 2\times0.5) = \pm2\%$

30. **(a)** $V = 200$ V, $\Delta V = 5$V

$I = 20$ A, $\Delta I = 0.2$ A

$$R = \frac{V}{I}$$

Using error analysis

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{5}{200} + \frac{0.2}{20} = \frac{7}{200}$$

$$\therefore \%\text{ error} = \frac{\Delta R}{R}\times100 = \frac{7}{200}\times100 = 3.5\%$$

31. **(b)** $\rho = \frac{RA}{l}$

$$\Rightarrow \frac{R\pi(d/2)^2}{l} = \frac{\pi R d^2}{4l}$$

On differentiation

$$\frac{\Delta\rho}{\rho}=\frac{\Delta R}{R}+2\frac{\Delta d}{d}+\frac{\Delta l}{l}$$

$$\frac{\Delta\rho}{\rho}\times 100$$

$$=\left[\frac{1}{65}+2\left(\frac{0.5\times10^{-3}}{10\times10^{-3}}\right)+\frac{0.1\times10^{-3}}{5\times10^{-3}}\times100\right]\approx 13\%$$

32. **(b)** Volume, $V=\frac{\pi D^2 h}{4}$

$$=\frac{\pi}{4}\times(12.6)^2\times(34.2)=4262.22\,cm^3$$

$$\frac{\Delta V}{V}=2\left(\frac{\Delta D}{D}\right)+\frac{\Delta h}{h}$$

$$\Delta V=\left(\frac{2\times0.1}{12.6}+\frac{0.1}{34.2}\right)\times4262.22\simeq 80\,cm^3$$

33. **(b)** Resistivity is given by,

$$\rho=R\frac{A}{\ell}=R\frac{\pi r^2}{\ell}$$

Taking log and differentiate

$$\frac{\Delta\rho}{\rho}=\frac{\Delta R}{R}+2\frac{\Delta r}{r}+\frac{\Delta\ell}{\ell}$$

$$\frac{\Delta\rho}{\rho}\times100=\left(\frac{10}{100}+2\times\frac{0.05}{0.35}+\frac{0.2}{15}\right)\times100$$

$$=\left(\frac{1}{10}+\frac{2}{7}+\frac{1}{75}\right)\times100=39.93\%$$

34. **(d)** Here, $A=1.0\ m\pm0.2\ m, B=2.0\pm0.2\ m$

$AB=(1.0\,m)(2.0\,m)=2.0\,m^2$

$\sqrt{AB}=\sqrt{2.0}m=1.414\,m=1.414\,m$

Rounding off to two significant figures, we get

$\sqrt{AB}=1.4\,m$

$$\frac{\Delta\sqrt{AB}}{\sqrt{AB}}=\frac{1}{2}\left(\frac{\Delta A}{A}+\frac{\Delta A}{B}\right)=\frac{1}{2}\left(\frac{0.2}{1.0}+\frac{0.2}{2.0}\right)=\frac{0.3}{2}$$

$$\Delta\sqrt{AB}=\frac{0.3}{2}\times\sqrt{AB}=\frac{0.3}{2}\times1.414=0.212\,m$$

Rounding off to one significant figure, we get

$\Delta\sqrt{AB}=0.2\,m$

The correct value for is $\sqrt{AB}$ is $1.4\,m\pm0.2\,m$.

35. **(d)** Time period of spring is $T=2\pi\sqrt{\frac{m}{k}}$

Squaring both sides, $T^2\propto\frac{m}{k}$

on differentiation $\frac{2\Delta T}{T}=\frac{\Delta m}{m}-\frac{\Delta k}{k}$

$$\frac{\Delta k}{k}=\frac{\Delta m}{m}-\frac{2\Delta T}{T}=\frac{\Delta k}{k}\times100=\left(\frac{\Delta m}{m}-\frac{2\Delta T}{T}\right)\times100$$

$$\frac{\Delta k}{k}\times100=(-1)\%-2(2)\%=5\%$$

36. **(c)** $Q=\frac{a^4b^3}{c^2}$

By error analysis,

$$\frac{\Delta Q}{Q}=4\left(\frac{\Delta a}{a}\right)+3\left(\frac{\Delta b}{b}\right)+2\left(\frac{\Delta c}{c}\right)$$

$$\frac{\Delta Q}{Q}\times100=4\left(\frac{\Delta a}{a}\times100\right)+3\left(\frac{\Delta b}{b}\times100\right)+2\left(\frac{\Delta c}{c}\times100\right)$$

% error in Q $=4\times3\%+3\times4\%+2\times5\%$

$=12\%+12\%+10\%=34\%$

Exercise 3 : PREVIOUS YEARS MCQs

1. **(b)**

2. **(a)** Resultant of two vectors is maximum at 0°.

3. **(d)** Vector $\vec{A}$ is perpendicular to $\vec{B}$.

or $\left(2\hat{i}+3\hat{j}+8\hat{k}\right)\cdot\left(-4\hat{i}+4\hat{j}+\alpha\hat{k}\right)=0$

$-8+12+8\alpha=0$ or $\alpha=-1/2$

4. **(c)** Pressure $P=\frac{\text{Force (F)}}{\text{area (A)}}$ or $P=\frac{f}{\ell\times\ell}$

Error in F = 3% and error in side ℓ = 1% given

$\therefore$ Error in pressure = 3% + 1% + 1% = 5%

5. (d) $|\vec{A}+\vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2\vec{A}.\vec{B}$

$= A^2 + B^2 + 2AB\cos\theta$

$|\vec{A}-\vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 - 2\vec{A}.\vec{B}$

$= A^2 + B^2 - 2AB\cos\theta$

So, $A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$4AB\cos\theta = 0 \Rightarrow \cos\theta = 0 \quad \therefore \quad \theta = 90^\circ$

So, angle between A & B is 90°.

6. (a) Component of $\vec{a}$ along $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

7. (d) Projection of vector $\vec{A}$ on vector $\vec{B}$ is given by

$$(A\cos\theta)\hat{B} = A\left(\frac{\vec{A}.\vec{B}}{AB}\right)\hat{B} = \frac{\vec{A}.\vec{B}}{B}\hat{B}$$

$$= \frac{2}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \hat{i}+\hat{j}$$

8. (b) $|\vec{A}\times\vec{B}| = AB\sin\theta$, $\vec{A}.\vec{B} = AB\cos\theta$

$|\vec{A}+\vec{B}| = \sqrt{3}\,\vec{A}.\vec{B} \Rightarrow AB\sin\theta = \sqrt{3}\,AB\cos\theta$

or, $\tan\theta = \sqrt{3}, \quad \therefore \theta = 60^\circ$

$\therefore |\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos 60^\circ}$

$= \sqrt{A^2 + B^2 + AB}$

9. (b) Unit vector along y axis = $\hat{j}$, so the required vector

$= \hat{j} - [(\hat{i} - 3\hat{j} + 2\hat{k}) + (3\hat{i} + 6\hat{j} + 7\hat{k})]$

$= -4\hat{i} - 2\hat{j} - 9\hat{k}$

10. (c)

11. (d) Given number of observations, $n = 5$

$a_1 = 2.4m, a_2 = 2.5m, a_3 = 2.6m, a_4 = 2.8m, a_5 = 3.0m$

Average value of observations,

$$a_m = \frac{a_1 + a_2 + a_3 + a_4 + a_5}{5}$$

$$= \frac{2.4 + 2.5 + 2.6 + 2.8 + 3.0}{5} = \frac{13.3}{5} = 2.66$$

Absolute errors in individual observed value

$|\Delta a_1| = |a_1 - \bar{a}| = |2.4 - 2.66| = |-0.26| = 0.26$

$|\Delta a_2| = |a_2 - \bar{a}| = |2.5 - 2.66| = |-0.16| = 0.16$

$|\Delta a_3| = |a_3 - \bar{a}| = |2.6 - 2.66| = |-0.06| = 0.06$

$|\Delta a_4| = |a_4 - \bar{a}| = |2.8 - 2.66| = 0.14$

$|\Delta a_5| = |a_5 - \bar{a}| = |3.0 - 2.66| = 0.34$

Mean absolute error,

$$\Delta\bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4| + |\Delta a_5|}{n}$$

$$= \frac{0.26 + 0.16 + 0.06 + 0.14 + 0.34}{5} = \frac{0.96}{5}$$

$\Rightarrow \Delta\bar{a} = 0.192$

$$\text{Relative error} = \frac{\text{Mean absolute error}}{\text{Mean value}}$$

$$= \frac{\Delta\bar{a}}{a_m} = \frac{0.192}{2.66} = 0.072$$