

Class-XI

MHT-CET

Study Material

PHYSICS

Booklet-1

(Solutions)

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Hints & Solutions

1 Motion in a Plane

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(a)** When location of a particle has changed, it must have covered some distance and undergone some displacement.

2. **(c)**

3. **(d)** $\frac{\text{Displacement}}{\text{distance}} \le 1$

4. **(b)**

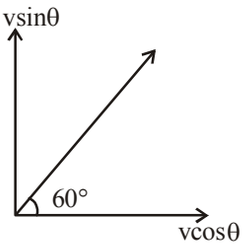
5. **(a)** Total time of motion is 3 min 20 sec = 20 sec.
As time period of circular motion is 40 sec so in 20 sec athlete will complete 5 revolution *i.e.*, he will be at starting point *i.e.*, displacemet = zero.

6. **(c)**

7. **(a)** $v_{av} = \frac{v_1 t/3 + v_2(2t/3)}{t} = \frac{v_1}{3} + \frac{2v_2}{3}$

8. **(c)**

9. **(d)**



$\vec{v} = v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j} = 10\cos 60\hat{i} + 10\sin 60\hat{j}$

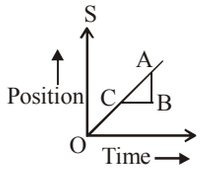
$= 10\times\frac{1}{2}\hat{i} + 10\times\frac{\sqrt{3}}{2}\hat{j} = 5\hat{i} + 5\sqrt{3}\,\hat{j}$

10. **(d)** When $s \propto t$, so $\frac{s}{t} = \text{constant}$.

11. **(c)** The velocity-time graph for a uniform motion is a straight. line parallel to time axis. Its slope is zero.

12. **(b)** $\frac{|\text{Average velocity}|}{|\text{Average speed}|} = \frac{|\text{displacement}|}{|\text{distance}|}$
because displacement will either be equal or less than distance. It can never be greater than distance travelled.

13. **(c)** The slope of the tangent drawn on position-time graph at any instant gives instantaneous velocity.



$AC = \frac{AB}{BC} = V_{inst}$

14. **(d)** $v_A = \tan 30°$ and $v_B = \tan 60°$

$\therefore \quad \frac{v_A}{v_B} = \frac{\tan 30°}{\tan 60°} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$

15. **(b)** The speed in general $\ge$ the magnitude of velocity.

16. **(c)** **17.** **(a)**

18. **(d)** The nature of the path is decided by the direction of velocity, and the direction of acceleration. The trajectory can be a straight line, circle or a parabola depending on these factors.

19. **(c)** Slope of velocity-time graph shows acceleration.

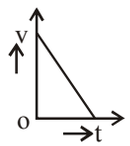
20. **(c)** Because acceleration is a vector quantity.

21. **(d)** **22.** **(b)**

23. **(b)** For a particle moving with uniform acceleration the displacement-time graph is a parabola.



24. **(d)** Velocity time curve will be a straight line as shown:



25. **(b)** At the highest point v = 0.

26. **(a)** Let s be the distance travelled by the vehicle before it stops.
Final velocity $\upsilon = 0$, initial velocity = u
Using equation of motion $v^2 - u^2 = 2aS$
$0^2 - u^2 = 2aS$

Stopping distance, $S = -\frac{u^2}{2a}$

27. **(b)** Let the initial velocity of ball be u $\therefore$ Time of rise

$t_1 = \frac{u}{g+a}$ and height reached $= \frac{u^2}{2(g+a)}$

Time of fall t_2 is given by

$\frac{1}{2}(g-a)t_2^2 = \frac{u^2}{2(g+a)}$

$t_2 = \frac{u}{\sqrt{(g+a)(g-a)}} = \frac{u}{(g+a)}\sqrt{\frac{g+a}{g-a}}$

$\therefore \quad t_2 > t_1$ because $\frac{1}{g+a} < \frac{1}{g-a}$

28. **(d)**

29. **(d)** Because acceleration due to gravity is constant so the slope of line will be constant i.e. velocity time curve for a body projected vertically upwards is straight line.

30. **(b)**

31. **(b)** Free fall of an object (in vacuum) is a case of motion with uniform acceleration.

32. **(c)** Both balls ball-1 drops from the top of a tower horizontally and ball-2 is dropped vertically downward from the tower at the same time will reach at the same time on the ground as both balls will move under the effect of same value of gravitational acceleration *g*.

33. **(a)** Given,

We have $x^2 + y^2 = a^2\cos^2 t + a^2\sin^2 t$

$\Rightarrow \quad x^2 + y^2 = a^2$ which is the equation of circle,

and $\frac{dz}{dt} = 1 = \text{const.}$

It means particle moves along *z* direction with constant speed.

Thus, the particle will move along *z* direction with constant speed tracing a circular path in *xyz* plane.

Hence, path is helix.

34. **(c)**

35. **(d)** $v(t) = \frac{dr}{dt}\frac{d}{dt}(3t\hat{i} + 2t^2\hat{j} + 5\hat{k}) = 3\hat{i} + 4t\hat{j}$

Now at t = 1s, $v = 3\hat{i} + 4\hat{j}$

Hence, its direction is $\theta = \tan^{-1}\left[\frac{4}{3}\right] \cong 53°$ with X-axis

36. **(c)** $\vec{r} = 2t^2\hat{i} + 3t\hat{j} + 4\hat{k}$

$\therefore \quad \vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt} = (2t^2\hat{i} + 3t\hat{j} + 4\hat{k}) = 4t\hat{i} + 3\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(4t\hat{i} + 3\hat{j}) = 4\hat{i} \quad \therefore \vec{a} = 4\,ms^{-2}$ along x-direction

37. **(a)** $a_x = \frac{6t}{3}, a_y = \frac{4t}{3}$

so $u_x = \int_o^t a_x dt = t^2 \Rightarrow (u_x)_{t=3} = 9\text{m/sec}$

and $u_y = \int_o^t a_y dt = \frac{2t^2}{3} \Rightarrow (u_y)_{t=3} = 6\text{m/sec}$

(because u_x & $u_y = 0$ at t = 0 sec)

38. **(a)**

39. **(d)** Along same straight line, velocity & acceleration can be in the same direction, opposite to each other or perpendicular as in circular motion with uniform speed. Thus θ can be anywhere between 0 & 180°.

40. **(d)**

41. **(a)** According to the question, at any instant *t*, $x = 4t^2, y = 3t^2$

$\therefore v_x = \frac{dx}{dt} = \frac{d}{dt}(4t^2) = 8t$ and $v_y = \frac{dy}{dt} = \frac{d}{dt}(3t^2) = 6t$

The speed of the particle at instant *t*.

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(8t)^2 + (6t)^2} = 10t$

42. **(d)** The equation of motion for projectile is

$x = x_0 + U_x t + \frac{1}{2}a_x t^2$

$\therefore$ The shape of the trajectory depends on the initial position, initial velocity and acceleration.

43. **(a)** $\vec{u} = 3\hat{i} + 4\hat{j} \quad \Rightarrow ux = 3$ and $uy = 4$

$\vec{a} = 0.4\hat{i} + 0.3\hat{j} \Rightarrow ax = 0.4$ and $ay = 0.3$

So, $V_x = ux + a_x t = 3 + 0.4 \times 10 = 3 + 4 = 7$

$V_y = uy + a_y t = 4 + 0.3 \times 10 = 4 + 3 = 7$

$\vec{V} = 7\hat{i} + 7\hat{j} \Rightarrow |\vec{V}| = \sqrt{7^2 + 7^2} = 7\sqrt{2}$ units

44. **(c)** $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 80 - 65 = 15$ km/hr

[$\because$ both are moving in the same direction]

45. **(c)** $V_y = u\sin\theta - gt_m = 0$

$\therefore t_m = \frac{u_y \sin\theta}{g}$ (time to reach the maximum height)

Total time of flight $T_f = \frac{2(u\sin\theta)}{g} \quad \therefore T_f = 2t_m$

46. **(d)** Horizontal range $= \frac{u^2\sin 2\theta}{g}$

For maximum range $\theta = 45°$

$\therefore R_{max} = \frac{u^2\sin 90°}{g} = \frac{u^2}{g} \qquad (\because \sin 90° = 1)$

47. **(b)** If air resistance is ignored, then there is no acceleration in horizontal direction in projectile motion. Hence the particle move with constant velocity in horizontal direction.

48. **(d)** $T = \frac{2u\sin(\theta - \alpha)}{g\cos\alpha}$

49. **(c)** If the angle of projection is $\frac{\pi}{4}$, then range $= \frac{v_0^2}{g}\sin(\pi/2)$

$\Rightarrow \quad (R)_{max} = \frac{v_0^2}{g} \qquad [\because \{\sin(\pi/2)\}_{max} = 1]$

50. **(d)** At the highest point of trajectory, the acceleration is equal to g.

51. **(a)** **52.** **(a)**

53. **(d)** $\frac{H_1}{H_2}=\frac{u^2\sin^2\theta/2g}{u^2\sin^2(90°-\theta)/2g}=\tan^2\theta$

54. **(c)** We know that, $y_m = H = \frac{(u\sin\theta)^2}{2g} = \frac{u^2\sin^2\theta}{2g}$

$\therefore \quad \frac{\Delta H}{H}=\frac{2\Delta u}{u}$. Given $\frac{\Delta u}{u}=2\%$

$\therefore \quad \frac{\Delta H}{H}=2\times 2=4\%$

55. **(c)** $y = x\tan\theta-\frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

$y=50\tan 60°-\frac{10\times 50\times 50}{2\times 25\times 25\times\cos^2 60°}=5\,m$

56. **(a)** For maximum range, the angle of projection, $\theta = 45°$

$\therefore R=\frac{u^2\sin 2\theta}{g}=\frac{(20)^2\sin(2\times 45°)}{10}=\frac{400\times 1}{10}=40m.$

57. **(a)** $t_1=\frac{2u\sin\theta}{g}$ and

$t_2=\frac{2u\sin(90-\theta)}{g}=\frac{2u\cos\theta}{g}$

$\therefore \quad t_1 t_2=\frac{4u^2\cos\theta\sin\theta}{g^2}=\frac{2}{g}\left[\frac{u^2\sin 2\theta}{g}\right]=\frac{2}{g}R,$

where R is the range.

Hence $t_1 t_2 \propto R$

58. **(b)** For same range, the angle of projections are : θ and $90° - \theta$. So,

$h_1=\frac{u^2\sin^2\theta}{2g}$ and $h_2=\frac{u^2\sin^2(90°-\theta)}{2g}=\frac{u^2\cos^2\theta}{2g}$

Also, $R=\frac{u^2\sin 2\theta}{g}$

$h_1 h_2=\frac{u^2\sin^2\theta}{2g}\times\frac{u^2\cos^2\theta}{2g}$

$=\frac{u^2}{16}\frac{u^2(2\sin\theta\cos\theta)^2}{g^2}=\frac{R^2}{16}$ or $R^2 = 16\, h_1 h_2$

59. **(a)** As we know, range $R=\frac{u^2\sin 2\theta}{g}$

and, area $A=\pi R^2$

$\therefore \quad A\propto R^2$ or, $A\propto u^4 \quad \therefore \quad \frac{A_1}{A_2}=\frac{u_1^4}{u_2^4}=\left[\frac{1}{2}\right]^4=\frac{1}{16}$

60. **(c)** As $v^2 = u^2 + 2as$

$\Rightarrow 0 = u^2 - 2gH \Rightarrow u^2 = 2gH \quad \Rightarrow u=\sqrt{2\times 10\times 136}$

$R_{max}=\frac{u^2}{g}=\frac{2\times 10\times 136}{10}=272\ m$

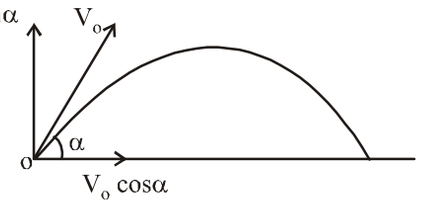
61. **(c)** As, $H_{max}=\frac{u^2\sin^2\theta}{2g}\Rightarrow u^2\propto\frac{1}{\sin^2}\theta$

$\Rightarrow u\propto\frac{1}{\sin\theta}\Rightarrow\frac{\sin\theta_2}{\sin\theta_1}=\frac{u_1}{u_2}$

$\Rightarrow \sin\theta_1=\frac{u_2}{u_1}\times\sin\theta_2=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{2}}$

So, $\theta_1=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=45°$

62. **(a)**



To cover a range R, the boy should run with a constant velocity of $V_0\cos\alpha$

63. **(a)** $H=\frac{u^2\sin^2\theta}{2g}, T=\frac{2u\sin\theta}{g}$

$\Rightarrow \frac{H}{T}=\frac{u^2\sin^2\theta}{2g}\times\frac{g}{2u\sin\theta}=\frac{1}{4}u\sin\theta$

Also, $u \propto T$

$\therefore \quad \frac{H}{T}\propto T\Rightarrow H\propto T^2\Rightarrow\left(\frac{T_2}{T_1}\right)=\sqrt{\frac{1.1H_1}{H_1}}=\sqrt{1.1}$

$\therefore \quad T_2=\sqrt{1.1}\ T_1=1,05\ T_1 \quad \therefore \quad \Delta T\%=5\%$

64. **(c)** $R = 50m, \theta = 45°$

By equation of trajectory

$y=x\tan\theta\left(1-\frac{x}{R}\right)$

At $x = 20m, y = h$

$\therefore \quad h=20\tan 45°\left(1-\frac{20}{50}\right)=20\times\frac{3}{5}=12m$

65. **(b)** $u = 20\ ms^{-1}$

$H_2=H_1+10\Rightarrow\frac{u_2^2\sin^2(90°-\theta)}{2g}=\frac{u_1^2\sin^2\theta}{2g}+10$

$\Rightarrow \frac{(20)^2\cos^2\theta}{20} = \frac{(20)^2\sin^2\theta}{20} + 10$

$\Rightarrow \cos 2\theta = \frac{1}{2} = \cos 60° \quad \therefore \quad \theta = 30°$

66. **(d)** For same range, $R_1 = R_2 = R$
$\theta_1 = \theta, \theta_2 = (90° - \theta)$

$\therefore T_1 = \frac{2u\sin\theta_1}{g} = \frac{2u\sin\theta}{g}$

$\therefore T_2 = \frac{2u\sin\theta_2}{g} = \frac{2u\sin(90°-\theta)}{g} = \frac{2u\cos\theta}{g}$

$\therefore T_1.T_2 = \left(\frac{2u\sin\theta}{g}\right)\left(\frac{2u\cos\theta}{g}\right) = \frac{2}{g}\left(\frac{u^2\sin 2\theta}{g}\right) = \frac{2R}{g}$

$\therefore \quad T_1 T_2 \propto R$

67. **(d)** For vertical upward projection, $T_1 = \frac{2u_1}{g}$

For projectile motion, $T_2 = \frac{2u_2\sin\theta}{g}$

$T_1 = T_2 \Rightarrow \frac{2u_1}{g} = \frac{2u_2\sin\theta}{g} \Rightarrow u_1 = u_2\sin\theta$

$\therefore \frac{H_1}{H_2} = \frac{u_1^2/2g}{\frac{u_2^2\sin^2\theta}{2g}} = \left(\frac{u_1}{u_2\sin\theta}\right) = 1:1$

68. **(b)** $R_{max} = \frac{u^2}{g} = 80 \Rightarrow u = \sqrt{800}$ m/s

$x = (u\cos\theta)t = (u\cos 45°) \times t = \frac{3}{\sqrt{2}} \times \sqrt{800} = 60$ m

69. **(b)** For first object, if range is maximum, then
$\theta = 45°$

$\therefore H_1 = \frac{u_1^2\sin^2\theta}{2g} = \frac{u_1^2\sin^2 45°}{2g} = \frac{u_1^2}{4g}$

For second object, $H_2 = \frac{u_2^2}{2g}$

$\therefore H_1 = H_2 \Rightarrow \frac{u_1^2}{4g} = \frac{u_2^2}{2g} \Rightarrow u_1 : u_2 = \sqrt{2} : 1$

70. **(a)** $R_{max} = \frac{u^2}{g} = 32 \Rightarrow u = \sqrt{320}$ m/s

When the ball thrown horizontally,

$\therefore \quad R = u\left(\sqrt{\frac{2H}{g}}\right) = (\sqrt{320})\sqrt{\frac{2\times 25}{10}} = 40$m

71. **(b)** Horizontal range, $R = \frac{u^2\sin 2\theta}{g}$

Maximum height, $H = \frac{u^2\sin^2\theta}{2g}$

$\frac{H_P}{H_Q} = \frac{1}{6}$ or, $H_P : H_Q = 1 : 6$

72. **(c)** $x = 4t$

$V_x = \frac{dx}{dt} = 4$

$Y = 3t - \frac{gt^2}{2}$

$V_y = \frac{dy}{dt} = 3 - gt$

at $t = 0$ $V_y = 3$

$V_i = \sqrt{V_x^2 + V_y^2} = \sqrt{4^2 + 3^2} = 5$ m/s

73. **(b)** Time of flight $t = \frac{2u\sin\theta}{g}$ or, $8 = \frac{2\times u\times 1}{10}$

$\Rightarrow u = 40$ m/s
$\therefore$ The position of the stone after 6s from the ground.

$h = ut + \frac{1}{2}gt^2 = 40\times 6 + \frac{1}{2}(-10)\times 6^2$

$= 240 - 180 = 60$m

74. **(c)** Only in case of parabolic motion, the direction and magnitude of the velocity changes, acceleration remains same. Morever, in case of uniform circular motion, the direction changes.

75. **(d)** $V_h = V\cos\theta$
where *h* is the maximum height

$V_h \times h = (\upsilon\cos\theta)\left(\frac{\upsilon^2\sin^2\theta}{2g}\right) = \frac{\upsilon^3\sin^2\theta\cos\theta}{2g} = \frac{\sqrt{3}\,\upsilon^3}{16g}$

76. **(c)**

77. **(b)** As we know, $R = u\cos\theta \times t$
Given, $R = 300$ m, $t = 6$ s

$\therefore u\cos\theta = \frac{R}{t} = \frac{300}{6} = 50 ms^{-1}$

78. **(b)** Horizontal range $R = \frac{u^2\sin 2\theta}{g}$.

Range is same for angle of projection θ and $(90° - \theta)$.

79. **(d)** For complementary angles of projection $(45° + \alpha)$ and $(45° - \alpha)$ with same initial velocity u, range R is same.
$\theta_1 + \theta_2 = (45° + \alpha) + (45° - \alpha) = 90°$

80. **(a)** Given,
Initial velocity of projectile, u = 40 m/s
Angle, $\theta = 30°$
Time of flight

$$T = \frac{2u\sin\theta}{g} = \frac{2\times 40\times 1}{10\times 2} = 45 \quad (\because\ g = 10\ m/s^2)$$

It means projectile is at maximum height at t = 2s. At maximum height vertical component of velocity is zero.

Velocity at t = 2s = $V_x = u\cos\theta = 40\cos 30° = 20\sqrt{3} ms^{-1}$.

81. **(b)** Comparing the given equation with the equation of trajectory of a projectile,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \text{ we get, } \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$$

$$\text{and } 2u^2\cos^2\theta = 20 \Rightarrow u^2 = \frac{20}{2\cos^2\theta} = \frac{40}{3}$$

$$\text{Now, } R_{max} = \frac{u^2}{g} = \frac{40}{3\times 10} = \frac{4}{3} m$$

82. **(c)** the horizontal velocity of the stone will be the same as that of the train. In this way, the horizontal motion will be uniform. The vertical motion will be controlled by the force of gravity. Hence it is accelerated motion. The resultant motion is a parabolic trajectory.

83. **(a)**

84. **(d)** Equation of trajectory of projectile

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta} = x\tan 45° - \frac{1}{2}\frac{10x^2}{u^2\cos^2 45°}$$

$$\Rightarrow \quad y = x - \frac{10x^2}{2u^2\left(\frac{1}{2}\right)} \Rightarrow 10 = 20 - \frac{(10)(100)}{u^2} \Rightarrow u = 20$$

$$\text{Time period, } T = \frac{2u\sin\theta}{g} = \frac{(2)(20)}{\sqrt{2}(10)} = 2\sqrt{2}$$

$$\vec{v} = u\cos\theta\hat{i} + (\mu\sin\theta - gt)\hat{j} = [10\sqrt{2}\hat{i} + (10\sqrt{2} - 10(2)]\hat{j}$$

$$\text{Momentum } \vec{P} = M\vec{v} = 100\sqrt{2}\hat{i} + (100\sqrt{2} - 200)\hat{j}$$

85. **(a)** The trajectory of projectile projected. From the ground is given by

$$y = x - \frac{x^2}{20}$$

For maximum height,

$$\frac{dy}{dx} = 0 \Rightarrow 1 - \frac{2x}{20} = 0 \qquad \therefore\ x = 10$$

Hence, the maximum height attained by the projectile,

$$y_{max} = 10 - \frac{100}{20} = 5m$$

86. **(c)** The bullet performs a horizontal journey of 100 cm with constant velocity of 1500 m/s. The bullet also performs a vertical journey of h with zero initial velocity and downward acceleration g.

$$\therefore \text{ For horizontal journey, time (t)} = \frac{\text{Distance}}{\text{Velocity}}$$

$$\therefore\ t = \frac{100}{1500} = \frac{1}{15}\sec \qquad \ldots(1)$$

The bullet performs vertical journey for this time.

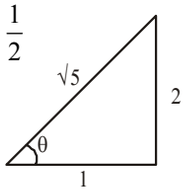
$$\text{For vertical journey, } h = ut + \frac{1}{2}gt^2$$

$$h = 0 + \frac{1}{2}\times 10\times\left(\frac{1}{15}\right)^2 \text{ or, } h = \frac{20}{9} cm = 2.2cm$$

87. **(c)**

88. **(a)** We know, $R = 4H\cot\theta \Rightarrow \cot\theta = \frac{1}{2}$

From triangle we can say that



$$\sin\theta = \frac{2}{\sqrt{5}}, \cos\theta = \frac{1}{\sqrt{5}}$$

$$\therefore \text{ Range of projectile } R = \frac{2v^2\sin\theta\cos\theta}{g}$$

$$= \frac{2v^2}{g}\times\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$$

89. **(a)** Maximum height $H_{max} = \frac{u^2\sin^2\theta}{2g}$

and time of flight, $T = \frac{2u\sin\theta}{g}$

Given, $(H_{max})_1 = 8\times(H_{max})_2$

$$\Rightarrow \frac{u^2\sin^2\theta_1}{2g} = 8\times\frac{u^2\sin^2\theta_2}{2g}$$

$$\Rightarrow \sin\theta_1 = 2\sqrt{2}\sin\theta_2$$

$$\therefore \frac{T_1}{T_2} = \frac{\frac{2u\sin\theta_1}{g}}{\frac{2u\sin\theta_2}{g}} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{2\sqrt{2}\sin\theta_2}{\sin\theta_2} = 2\sqrt{2}$$

90. **(d)** For two projectiles,

$\theta_1 = 45 + \alpha;\ \theta_2 = 45 - \alpha$

Time of flight, $T = \frac{2u\sin\theta}{g} \Rightarrow T \propto \sin\theta$

$$\therefore \frac{T_1}{T_2} = \frac{\sin(45+\alpha)}{\sin(45-\alpha)} = \frac{\frac{1}{\sqrt{2}}\cos\alpha + \frac{1}{\sqrt{2}}\sin\alpha}{\frac{1}{\sqrt{2}}\cos\alpha - \frac{1}{\sqrt{2}}\sin\alpha}$$

$$____ = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1+\tan\alpha}{1-\tan\alpha}.$$

91. **(d)** For the projectile,
Range, $R = 3H_{max}$

$$\Rightarrow \quad \frac{u^2\sin 2\theta}{g} = \frac{3u^2\sin^2\theta}{2g}$$

$$\Rightarrow \quad 2\sin\theta\cos\theta = \frac{3}{2}\sin^2\theta$$

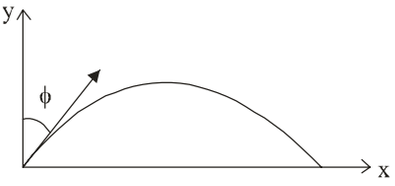
$$\Rightarrow \quad \tan\theta = \frac{4}{3} \Rightarrow \theta = 53°$$

$$\therefore \quad R = \frac{u^2(2\sin\theta.\cos\theta)}{g} = \frac{u^2\left(2\times\frac{4}{5}\times\frac{3}{5}\right)}{g} = \frac{24\mu^2}{25g}$$

$$= \frac{nu^2}{25g} \text{ (given)}$$

$$\therefore \quad n = 24$$

92. **(c)**



The maximum height is

$$h_m = \frac{u^2\cos^2\phi}{2g} \Rightarrow h_m \propto \cos^2\phi$$

93. **(c)** For the projectile, u = constant

$$H_{max} = \frac{(u\sin\theta)^2}{2g} \quad \Rightarrow \quad H_{max} \propto \sin^2\theta$$

$$\therefore \frac{(H_{max})_1}{(H_{max})_2} = \frac{\sin^2(45°-\alpha)}{\sin^2(45°+\alpha)}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\cos\alpha - \frac{1}{\sqrt{2}}\sin\alpha\right)^2}{\left(\frac{1}{\sqrt{2}}\cos\alpha + \frac{1}{\sqrt{2}}\sin\alpha\right)^2} = \frac{1-\sin 2\alpha}{1+\sin 2\alpha}$$

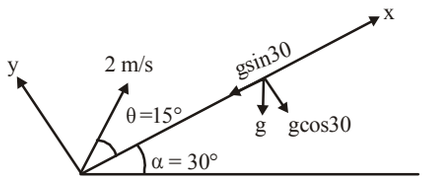
94. **(a)** On an inclined plane, time of flight (T) is given by

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

Substituting the values, we get

$$T = \frac{(2)(2\sin 15°)}{g\cos 30°} = \frac{4\sin 15°}{10\cos 30°}$$

Distance, $S = (2\cos 15°)T - \frac{1}{2}g\sin 30°(T)^2$



$$= (2\cos 15°)\frac{4}{10}\frac{\sin 15°}{10\cos 30°} - \left(\frac{1}{2}\times 10\sin 30°\right)\frac{16\sin^2 15°}{100\cos^2 30°}$$

$$= \frac{16\sqrt{3}-16}{60} \simeq 0.1952\text{m} \simeq 20\text{cm}$$

95. **(b)** From equation, $\vec{u} = \hat{i} + 2\hat{j} \Rightarrow u_x = 1m/s, V_y = 2m/s$

$$\Rightarrow x = u_x t = 1\times t = t \qquad \ldots(i)$$

$$y = v_y t - \frac{1}{2}gt^2 = 2t - \frac{1}{2}(10t^2) \qquad \ldots(ii)$$

From (i) and (ii), $y = 2x - 5x^2$

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(a)** Since displacement is zero.

2. **(d)** Let the total distance be d. Then for first half distance,
time = $\frac{d}{2v_0}$, next distance. = $v_1 t$ and last half distance = $v_2 t$

$$\therefore \quad v_1 t + v_2 t = \frac{d}{2}; \quad t = \frac{d}{2(v_1+v_2)}$$

Now average speed

$$t = \frac{d}{\frac{d}{2v_0} + \frac{d}{2(v_1+v_2)} + \frac{d}{2(v_1+v_2)}} = \frac{2v_0(v_1+v_2)}{(v_1+v_2)+2v_0}$$

3. **(c)** On differentiating, acceleration = $0.2t$
$\Rightarrow a = f(t)$

4. **(d)** From displacement-time graph, it is clear that in equal intervals of time displacements are not equal infact, decreases and after 40s displacement constant *i.e.*, the particle stops.

5. **(a)** $x = x_0 + (ut + \frac{1}{2}at^2)$ At $t = 0, x = -2, \therefore -2 = x_0 + 0$

or $x_0 = -2$

Thus, $0 = -2 + (u \times 1 + \frac{1}{2} \times a \times 1^2)$ ...(i)

and $6 = -2 + (u \times 2 + \frac{1}{2} \times a \times 2^2)$ ...(ii)

After solving equations, we get $u = 0$, $a = 4$ m/s^2.
Now for $t = 3$,

$x = -2 + (u \times 3 + \frac{1}{2} \times 4 \times 3^2) = 16$ m.

Clearly it reprisrts motion with congtont acceleration.

6. **(c)** The distance covered in n^{th} second is

$S_n = u + \frac{1}{2}(2n-1)a$

where u is initial velocity & a is acceleration

then $26 = u + \frac{19a}{2}$ ....(i)

$28 = u + \frac{21a}{2}$ ....(ii)

$30 = u + \frac{23a}{2}$ ....(iii)

$32 = u + \frac{25a}{2}$ ....(iv)

From eqs. (i) and (ii) we get u = 7m/sec, a=2m/sec^2
$\therefore$ The body starts with initial velocity u =7m/sec
and moves with uniform acceleration a = 2m/sec^2

7. **(a)** Time $= \frac{\text{total length}}{\text{relative velocity}} = \frac{X+X}{20+20} = \frac{X}{40}$s $= 1$s

8. **(c)**

9. **(c)** During OA, acceleration = tan 30° $= \frac{1}{\sqrt{3}}$ m/s^2

During AB, acceleration = –tan60° $= -\sqrt{3}$ m/s^2

Required ratio $= \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$

10. **(c)**

11. **(d)** For uniform motion with zero acceleration, *v-t* graph is a straight line parallel to the time axis.

12. **(c)** Average speed $= \frac{100+300}{10+\frac{300}{20}} = \frac{400}{25} = 16$ m/s

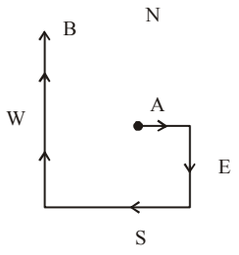
13. **(b)** Displacement = Final position – Initial position
$\Delta x = x_2 - x_1$
Then,
(i) $\Delta x = 7\text{ m} - (-3\text{ m}) = 10$ m
(ii) $\Delta x = -3\text{ m} - 7\text{ m} = -10$ m
(iii) $\Delta x = 3\text{ m} - (-7\text{ m}) = 10$ m

14. **(b)** The slope of the graph $\frac{ds}{dt}$ is maximum at C and hence the instantaneous velocity is maximum at C.

15. **(c)** Acceleration $a = \tan\theta$, where θ is the angle of tangent drawn on the velocity-time graph with the time axis.

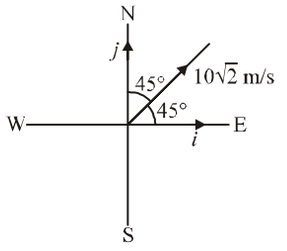
16. **(c)**

17. **(a)** Let us assume the person start with position A and ends at position B.
The direction of B wrt. A is North-West.



18. **(b)** $\vec{V} = \frac{d\vec{r}}{dt} = \frac{d\vec{x}}{dt} + \frac{d\vec{y}}{dt} = (4+2t)\hat{i} + (2+t)\hat{j}$

19. **(a)**



Initial velocity

$u = 10\sqrt{2}\cos 45°\,\hat{i} + 10\sqrt{2}\cos 45°\,\hat{j}$

Acceleration applied

$as = -2\hat{j}$

$\therefore$ Velocity after 5 s is
$v = u + a_s t$

$= 10(\hat{i} + \hat{j}) + (-2\hat{j}) \times 5 = 10\hat{i} + 10\hat{j} - 2\hat{j} = 10\hat{i}$

Hence, body will move towards east with the velocity 10 ms^{-1}.

20. **(d)** $\Delta r = (5\hat{i} + 6\hat{j}) - 2(2\hat{i} + 3\hat{j}) = 3\hat{i} + 3\hat{j}$

21. **(d)** Velocity, $\vec{v} = \frac{\vec{ds}}{dt} = \frac{d(2t^2\hat{j} + 5\hat{k})}{dt} = 4t\hat{j}$

In y – z plane

So, at t = 1 sec $\vec{v} = 4\hat{j}$

22. **(c)** Using $v^2 - u^2 = 2as$
$u_x = 5$ m/s, $a_x = 3$ m/s^2 and x = 84 m

$v_x^2 - u_x^2 = 2ax \Rightarrow v_x^2 - 25 = 2(3)(84)$

$\therefore V_x = 23$ m/s

$v_x - u_x = a_x t \Rightarrow t = \frac{23-5}{3} = 6$s

$v_y = 0 + a_y t = 0 + 2 \times (6) = 12$ m/s

$v^2 = v_x^2 + v_y^2 = 23^2 + 12^2 = 673$

$\Rightarrow$ $v = \sqrt{673}$ m/s $= \sqrt{\alpha}$ m/s (given) $\therefore$ $\alpha = 673$

23. (b) $y = bx^2$. Differentiating w.r.t to t an both sides, we get

$$\frac{dy}{dx} = b2x\frac{dx}{dt} \quad \Rightarrow v_y = 2bxv_x$$

Again differentiating w.r.t to t on both sides we get

$$\frac{dv_y}{dt} = 2bv_x\frac{dx}{dt} + 2bx\frac{dv_x}{dt} = 2bv_x^2 + 0$$

[$\frac{dv_x}{dt} = 0$, because the particle has constant acceleration along y-direction]

Now, $\frac{dv_y}{dt} = a = 2bv_x^2$; $v_x^2 = \frac{a}{2b}$ $\Rightarrow$ $v_x = \sqrt{\frac{a}{2b}}$

24. (b) **25. (b)** **26. (d)**

27. (d) Acceleration throughout the projectile motion remains constant and equal to value of g.

28. (b)

29. (a) Time of flight $= \frac{2u\sin\theta}{g} = \frac{2u_y}{g} = \frac{2 \times u_{vertical}}{g}$

30. (a) Direction of velocity is always tangent to the path so, at the top of trajectory, it is in horizontal direction and acceleration due to gravity is always in vertically downward direction. It means angle between $\vec{v}$ and $\vec{g}$ are perpendicular to each other.

31. (c)

32. (a) For both cases $t = \sqrt{\frac{2h}{g}}$ = constant

33. (d) $R = 4H\cot\theta$ if $\theta = 45°$ then $R = 4H\cot(45°) = 4H$

34. (d) Kinetic energy at the highest point

35. (c) Given, horizontal range, $R = 2H = 2 \times \frac{v^2\sin^2\theta}{2g}$

For maximum height, $\theta = 90° = 2 \times \frac{v^2\sin^2 90}{2g}$

$R = \frac{v^2}{g}$

36. (a) $V = \hat{i} + 2\hat{j}$

$x = t$ .....(i)

$y = 2t - \frac{1}{2}(10t^2)$ .....(ii)

From eq. (i) and (ii)

$y = 2x - 5x^2$.

37. (b) Horizontal distance covered should be same for the time of collision.

$400\cos\theta = 200$ or $\cos\theta = \frac{1}{2}$ or $\theta = 60°$

38. (a) Horizontal range $= \frac{u^2\sin 2\theta}{g}$ so $g \propto u^2$

39. (b) Horizontal range, $R = \frac{u^2\sin 2\theta}{g}$ ....(1)

Maximum height, $H = \frac{u^2\sin^2\theta}{2g}$ ....(2)

According to the condition,

$R = H$

$\Rightarrow$ $\frac{u^2\sin 2\theta}{g} = \frac{u^2\sin^2\theta}{2g} \Rightarrow 2\sin\theta\cos\theta = \frac{\sin^2\theta}{2}$

$2\cos\theta = \frac{\sin\theta}{2} \Rightarrow \cot\theta = \frac{1}{4}$

$\Rightarrow$ $\tan\theta = 4 \Rightarrow \theta = [\tan^{-1}(4)]$

40. (b) Horizontal range is same when angle of projection with the horizonatal is θ and $(90° - \theta)$.

41. (b) Maximum height,

$H = \frac{u^2\sin^2 45°}{2g} = \frac{u^2}{4g}$ ... (1)

Range, $R = \frac{u^2\sin 90°}{g} = \frac{u^2}{g}$

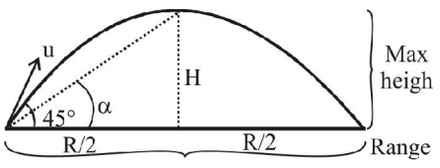
$\therefore$ $\frac{R}{2} = \frac{u^2}{2g}$ ... (2)

According to the required condition,

$\therefore$ $\tan\alpha = \frac{H}{R/2}$

Put the values from equation (1) and (2)

$\Rightarrow$ $\frac{\left(\frac{u^2}{4g}\right)}{\left(\frac{u^2}{2g}\right)} = \tan\alpha$ $\therefore \alpha = \tan^{-1}\left(\frac{1}{2}\right)$



42. (d)

43. **(d)** Tower height and final height is same so, required range is:

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \sin(2\times 45°)}{10} = 40 \text{ m}$$

44. **(c)** Range of projectile

$$R = \frac{v^2 \sin 20}{g} \quad (\because R \propto \sin(2\theta))$$

$$\frac{R_1}{R_2} = \frac{\sin(2\theta_1)}{\sin(2\theta_2)} = \frac{\sin(2\times 15)}{\sin(2\times 45)} = \frac{\sin 30°}{\sin 90°}$$

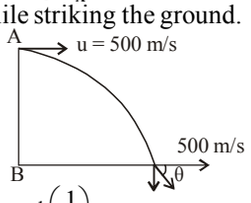
$$\Rightarrow \frac{50}{R_2} = \frac{1}{2} \Rightarrow R_2 = 100\text{m}$$

or $$\frac{g_{planet}}{g_{earth}} = \frac{(u_{planet})^2}{(u_{earth})^2}$$

Therefore $g_{planet} = \left(\frac{3}{5}\right)^2 (9.8 \text{ m/s}^2) = 3.5 \text{ m/s}^2$

45. **(a)** Horizontal component of velocity v_x = 500 m/s and vertical component of velocity while striking the ground.

$u_v = 0 + 10 \times 10 = 100$ m/s



$\therefore$ Angle with which it strikes the ground

$$\theta = \tan^{-1}\left(\frac{u_v}{u_x}\right) = \tan^{-1}\left(\frac{100}{500}\right) = \tan^{-1}\left(\frac{1}{5}\right)$$

46. **(c)** For bomb

$v_x = v$, $v_y = -gt$ $\therefore v_B = v_x + v_y = v\hat{i} - gt\,\hat{j}$

For observer

$v_x = v$, $v_y = 0$, $\therefore v_A = v_x + v_y = v\hat{i}$

Relative velocity of bomb w.r.t. observer = $v_B - v_A$

$= v\hat{i} - gt\,\hat{j} - v\hat{i} = -gt\,\hat{j}$

So, bomb will fall vertically down only.

47. **(a)** Range of a projectile is maximum when it is projected at an angle of 45° and is given by

$R_{max} = \frac{u^2}{g}$, where u is the velocity of projection

$$\Rightarrow R = \frac{u^2}{g} \quad \therefore u^2 = Rg \qquad \ldots(i)$$

Now, to hit a target at a distance (R/2) from the gun, we must have

$$\frac{R}{2} = \frac{u^2 \sin 2\theta}{g},$$

where θ is the angle of projection.

$$\Rightarrow \frac{R}{2} = \frac{Rg \sin 2\theta}{g}; \text{ from (i)}$$

$$\Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow \sin 2\theta = \sin 30°$$

$\Rightarrow 2\theta = 30° \quad \therefore \theta = 15°$

48. **(a)** $x = (u\cos\theta)\,t = 10\cos 30° \times 1 = 8.66$ m

Vertical component is,

$y = ut\sin\theta - 1/2gt^2 = 10\sin 30° \times 1 - 1/2 \times 9.8 \times 1^2$

$= 0.1$ m

Thus, the position coordinate after 1s is (8.66 m, 01. m).

49. **(c)** Given, $y = 2x - 9x^2$

On comparing with,

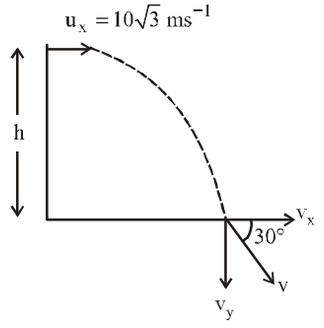
$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta},$$

We have, $\tan\theta = 2$ or $\cos\theta = \frac{1}{\sqrt{5}}$

and $$\frac{g}{2u^2\cos^2\theta} = 9 \text{ or } \frac{10}{2u^2(1/\sqrt{5})^2} = 9$$

$\therefore \quad u = 5/3$ m/s

50. **(d)**



$u_x = v_x = 10\sqrt{3}$ ms^{-1}

$$t = \sqrt{\frac{2h}{g}} \quad \therefore v_y = u_y + gt = 0 + g\sqrt{\frac{2h}{g}}$$

$$\therefore \tan 30° = \frac{v_y}{v_x} = \frac{g\sqrt{\frac{2h}{g}}}{10\sqrt{3}} = \frac{\sqrt{2gh}}{10\sqrt{3}} \quad \therefore h = 5\text{m}$$

51. **(a)** For a projectile, $y = 3x - 0.8x^2$ .....(i)

At x = R, y = 0,

$\therefore \quad 0 = 3R - 0.8R^2 \Rightarrow R(0.8R - 3) = 0$

$$\therefore R = \frac{3}{0.8} = \frac{15}{4}\text{m}$$

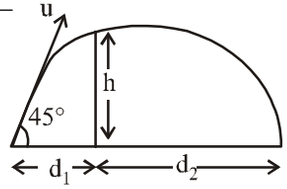
Also, $y = x\tan\theta - \frac{gn^2}{2u_x^2}$ ...(ii)

Comparing eq(i) with eq(ii), we get

$$\frac{g}{2u_x^2} = 0.8 \Rightarrow u_x = \sqrt{\frac{10}{2\times 0.8}} = \frac{5}{2}\text{m/s}$$

Now, $R = u_x . T = T = \dfrac{R}{u_x} = \dfrac{\frac{15}{4}}{\frac{5}{2}} = 1.5\text{s}$

52. **(b)** $R = d_1 + d_2 = \dfrac{u^2 \sin 90°}{g}$



$$\therefore \frac{u^2}{g} = d_1 + d_2$$

Using, $y = x\tan\theta - \dfrac{gx^2}{24^2\cos^2\theta}$

$$\therefore\ h = d_1 \tan 45° - \frac{d_1^2}{2(d_1+d_2)\times\frac{1}{2}} = \frac{d_1 d_2}{d_1+d_2}$$

53. **(a)** When projectile travels horizontally, then $t = \dfrac{T}{2}$

$$\therefore \frac{T}{2} = 3 \Rightarrow \frac{u\sin\theta}{g} = 3$$

$u, = u\sin\theta = 30$

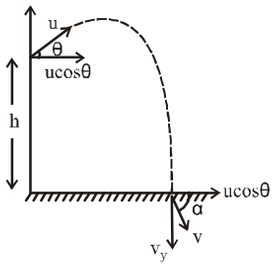
After time, t = 1s, $\tan 45° = \dfrac{v_y}{v_x} \Rightarrow v_y = v_x$

$\Rightarrow$ $u\sin\theta - gt = u\cos\theta$ $\Rightarrow$ $30 - 10\times 1 = u\cos\theta$

$u_x = u\cos\theta = 20$

$$\therefore\ u = \sqrt{u_x^2 + u_y^2} = \sqrt{(30)^2 + (20)^2} = 10\sqrt{13}\text{ m/s}$$

54. **(d)** $u = 40\text{ ms}^{-1}$, $v = 1.25 \times 40 = 50\text{ ms}^{-1}$



$v_y^2 = u_y^2 + 2gh = (-u\sin\theta)^2 + 2\times 10\times h$

$= (40\times\sin 30°)^2 + 20h = 400 + 20h$

$\therefore$ $v^2 = v_x^2 + u_y^2 \Rightarrow (50)^2 = (u\cos\theta)^2 + 400 + 20h$

$\Rightarrow$ $2500 = (40\times\cos 30°)^2 + 400 + 20h$ $\therefore$ $h = 45\text{ m}$

55. **(d)** For projectile motion,

$\theta = \tan^{-1}(\sqrt{7}) \Rightarrow \tan\theta = \sqrt{7}$

At $y = \dfrac{H}{2}$, $v_y^2 = u_y^2 - 2g\left(\dfrac{H}{2}\right)$

$\therefore$ $v_y^2 = u^2\sin^2\theta - gH$, $v_x^2 = u_x^2 = u^2\cos^2\theta$

$\therefore$ $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2(\sin^2\theta + \cos^2\theta) - gH}$

$= \sqrt{u^2 - gH}$

Also, $v = nu \Rightarrow v^2 = n^2 u^2$

$\Rightarrow$ $u^2 - gH = n^2 u^2$

$$\Rightarrow\ n^2 = 1 - \frac{gH}{u^2} = 1 - \frac{\frac{u^2\sin^2\theta}{2}}{u^2}$$

$$= 1 - \frac{1}{2}\left(1 - \frac{1}{1+\tan^2\theta}\right) = 1 - \frac{1}{2}\left(1 - \frac{1}{1+7}\right) = \frac{9}{16}$$

$\therefore$ $n = \dfrac{3}{4}$

56. **(a)** Here vertical component of velocity $u = -100\sin 30° = -50$ m/s

Let t be the time taken to reach

$$h = ut + \frac{1}{2}gt^2 \Rightarrow 375 = -50t + \frac{1}{2}\times 10\, t^2$$

or, $t^2 - 10t - 75 = 0 \Rightarrow t^2 + 5t - 15t - 75 = 0$

$t(t+5) - 15(t+5) = 0$

$\therefore$ $t = 15\text{s}$

Therefore the horizontal distance between the cannon and the target $x = (v\cos\theta)\,t = 100\times\cos 30°\times 15$

$$= 100\times\frac{\sqrt{3}}{2}\times 15 = 750\sqrt{3}\text{ m}$$

57. **(d)** At maximum height inclination with horizontal becomes zero

So, $t = \dfrac{u\sin\theta}{g} \Rightarrow u = \dfrac{gt}{\sin\theta}$ ...(i)

Now, $R = u\cos\theta\times T \Rightarrow R = u\cos\theta\times 2t$

$$\Rightarrow\ \cos\theta = \frac{R}{2ut} \Rightarrow \cos\theta = \frac{R\sin\theta}{2t.gt}\ \text{[From (i)]}$$

$$\Rightarrow \tan\theta = \frac{2gt^2}{R} \Rightarrow \tan\theta = \frac{20t^2}{R} \Rightarrow \cot\theta = \frac{R}{20t^2}$$

58. **(b)** Two bodies will collide at the highest point if both cover the same vertical height in the same time.

So $\dfrac{V_1^2\sin^2 30°}{2g} = \dfrac{V_2^2}{2g} \Rightarrow \dfrac{V_2}{V_1} = \sin 30° = \dfrac{1}{2}$

$\therefore$ $V_2 = \dfrac{1}{2}V_1$

59. **(b)** **60.** **(c)**

61. **(d)** For projectile A,

$u_y = u_A \cos 45^\circ = \frac{u_A}{\sqrt{2}}$, H = –400m

$\therefore\ H = u_y t - \frac{1}{2}gt^2 \Rightarrow -400 = \left(\frac{u_A}{\sqrt{2}}\right)t - \frac{1}{2}\times 10\, t^2$

$\Rightarrow 5t^2 - \left(\frac{u_A}{\sqrt{2}}\right)t - 400 = 0$

$\therefore$ Time of flight $= t_1 + t_2 = \frac{-\left(-\frac{u_A}{\sqrt{2}}\right)}{2\times 5} = \frac{u_A}{10\sqrt{2}}$ ...(i)

For projetile B

$u_y = u_B \cos 60^\circ = \frac{u_B}{2}$, H = – 400 m

$\therefore H = u_y t - \frac{1}{2}gt^2 \Rightarrow -400 = \left(\frac{u_B}{2}\right) - \frac{1}{2}\times 10 \times t^2$

$\Rightarrow 5t^2 - \left(\frac{u_B}{2}\right)t - 400 = 0$

$\therefore$ Time of flight $= t_1 + t_2 = \frac{-\left(-\frac{u_A}{2}\right)}{2\times 5} = \frac{u_B}{20}$ ...(ii)

From equation (i) and (ii)

$\frac{u_A}{10\sqrt{2}} = \frac{u_B}{20}$ $\therefore$ $u_A : u_B = 1:\sqrt{2}$

62. **(a)** Maximum height is given by:

$H_{max} = \frac{u^2 \sin^2\theta}{2g}$

substitute $H_{max} = 4R$ and $u = \frac{2\pi R}{T}$

$\Rightarrow 4R = \frac{4\pi^2 R^2}{T^2 2g}\sin^2\theta \Rightarrow \theta = \sin^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{1/2}$

63. **(c)** Maximum height, $H = \frac{u^2 \sin^2\theta}{2g}$

$H_1 = \frac{u^2 \sin^2\theta}{2g}$ ... (i)

$H_2 = \frac{u^2 \sin^2(90^\circ - \theta)}{2g} = \frac{u^2 \cos^2\theta}{2g}$ ... (ii)

Dividing (i) by (ii), we get $\frac{H_1}{H_2} = \frac{\tan^2\theta}{1}$

64. **(d)** Equation of trajectory of projectile

$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta} = x\tan 45^\circ - \frac{1}{2}\frac{10x^2}{u^2\cos^2 45^\circ}$

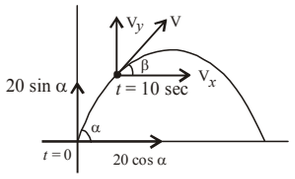
$\Rightarrow \quad y = x - \frac{10x^2}{2u^2\left(\frac{1}{2}\right)} \Rightarrow 10 = 20 - \frac{(10)(100)}{u^2} \Rightarrow u = 20$

Time period, $T = \frac{2u\sin\theta}{g} = \frac{(2)(20)}{\sqrt{2}(10)} = 2\sqrt{2}$

$\vec{v} = u\cos\theta\hat{i} + (\mu\sin\theta - gt)\hat{j} = [10\sqrt{2}\hat{i} + (10\sqrt{2} - 10(2)]\hat{j}$

Momentum $\vec{P} = M\vec{v} = 100\sqrt{2}\hat{i} + (100\sqrt{2} - 200)\hat{j}$

65. **(b)** $v_x = 20\cos\alpha$ $[\because a_x = 0]$



$v_y = 20\sin\alpha - g\times t$ $[\because a_y = -g]$
$= 20\sin\alpha - 10\times 10 = 20\sin\alpha - 100$

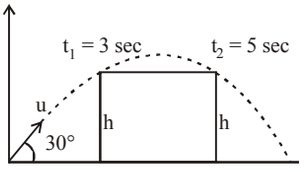
So, $\tan\beta = \frac{v_y}{v_x} = \frac{20\sin\alpha - 100}{20\cos\alpha} = \tan\alpha - 5\sec\alpha$

66. **(b)** At highest point, only horizontal component of velocity exists

So, $u\cos\theta = \frac{\sqrt{3}}{2}u \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$

$T = \frac{2u\sin\theta}{g} = \frac{2\times u\times \sin 30^\circ}{g} = \frac{u}{g}$

67. **(b)** Time of flight, T = 3 + 5 = 8 sec



$T = \frac{2u\sin\theta}{g} \Rightarrow 8 = \frac{2u\sin(30^\circ)}{10}$ $[\because \theta = 30^\circ$ given]

$\Rightarrow u = 80$ m/s

68. **(d)** Given : $y = \alpha x - \beta x^2$

For maximum height, maximum value of y

$\frac{dy}{dx} = 0 \Rightarrow \frac{d(\alpha x - \beta x^2)}{dx} = 0 \Rightarrow \alpha - 2\beta x = 0$

$\therefore \quad x = \frac{\alpha}{2\beta}$

$\therefore \quad y = \alpha\left(\frac{\alpha}{2\beta}\right) - \beta\left(\frac{\alpha^2}{4\beta^2}\right) \Rightarrow \left(\frac{\alpha^2}{2\beta}\right) - \left(\frac{\alpha^2}{4\beta}\right) \Rightarrow \frac{\alpha^2}{4\beta}$

So, $H_{max} = \frac{\alpha^2}{4\beta}$

Hence option (d) is correct answer.

69. (b) 70. (d) 71. (b) 72. (c) 73. (a) 74. (b)

75. (c)

Exercise 3 : PREVIOUS YEARS MCQs

1. (a) Horizontal range is same when angle of projection is θ or $(90° - \theta)$.

2. (a) Formula: Range of a projectile

$$R = \frac{u^2 \sin^2\theta}{g}$$

Substitute the given values

$$180\sqrt{3} = \frac{60^2 \sin^2\theta}{10}$$

$$\sin 2\theta = \frac{180\sqrt{3}}{360} = \frac{\sqrt{3}}{2}$$

$2\theta = 60°$ or $2\theta = 180° - 60° = 120°$

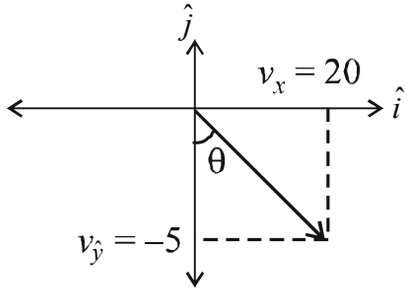
$\theta = 30°$ or $\theta = 60°$

3. (b)

4. (c) For the moving body,

$\vec{r} = 5t^2\hat{i} - 5t\hat{j} \quad \therefore \vec{v} = \frac{d\vec{r}}{dt} = 10t\hat{i} - 5\hat{j}$

$\therefore$ At $t = 2s$, $\vec{v} = 20\hat{i} - 5\hat{j}$



$\therefore v = \sqrt{(-5)^2 + (20)^2} = 5\sqrt{17}\, m/s$

$\therefore \tan\theta = \frac{20}{5} = 4 \; ; \theta = \tan^{-1} 4$ with -ve y-axis

5. (d) Time of flight $(T) = \frac{2u\sin\theta}{g}$

Time of flight of A: $T_1 = \frac{2u\sin\theta_1}{g}$

Time of flight of B: $T_2 = \frac{2U\sin\theta_2}{g}$

$\frac{T_1}{T_2} = \frac{\sin\theta_1}{\sin\theta_2}$, since $\theta_1 + \theta_2 = 90°$

$\sin\theta_1 = \cos\theta_2$ using the identify

$$\frac{T_1}{T_2} = \frac{\cos\theta_2}{\sin\theta_2} \Rightarrow \boxed{\frac{T_1}{T_2} = \tan\theta_1}$$

6. (d) Horizontal range $R = \frac{u^2 \sin^2\theta}{g}$

Maximum height $H = \frac{u^2 \sin^2\theta}{2g}$

Given $R = 3H \Rightarrow \frac{u^2 \sin^2\theta}{g} = 3\left(\frac{u^2 \sin^2\theta}{2g}\right)$

$\Rightarrow \sin 2\theta = \frac{3}{2}\sin^2\theta \Rightarrow 2\sin\theta.\cos\theta = \frac{3}{2}\sin^2\theta$

$\Rightarrow \; 4\sin\theta\cos\theta = 3\sin^2\theta \Rightarrow 4\cos\theta = 3\sin\theta$

$\therefore \frac{\sin\theta}{\cos\theta} = \frac{4}{3}$

$\therefore \tan\theta = \frac{4}{3} \quad \therefore \sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$

Range $R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \times 2 \times \sin\theta \times \cos\theta}{g}$

or $R = u^2 \times 2 \times \frac{4}{5} \times \frac{3}{5} \times \frac{1}{g} = \frac{24u^2}{25g}$

7. (b) Maximum height $H_{max} = \frac{u^2 \sin^2\theta}{2g}$

And at maximum height of a projectile there is only horizontal component of velocity i.e., $v = u\cos\theta$.

Given, at maximum height $v = 20$ m/s, $\theta = 45°$, $H_{max} = ?$

$v = u\cos\theta \Rightarrow 20 = u \times \cos 45°$

$\therefore \; u = \frac{20}{1/\sqrt{2}} = 20\sqrt{2}$

$$H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{(20\sqrt{2})^2 \times (1/\sqrt{2})^2}{2 \times 10} = \frac{800 \times 1}{2 \times 10 \times 2}$$

$= 20$ m

8. **(c)** When a particle returns to its starting point its displacement is zero.

9. **(d)** The expression of the height of a projectile at time t is:

$$h = u\sin\theta t - \frac{1}{2}gt^2$$

substitute the values

$$60 = u\sin\theta \times 2 - \frac{1}{2}\times 10 \times 2^2$$

$$60 = 2u\sin\theta - \frac{1}{2}\times 10\times 4 = 2\,u\sin\theta - 2\theta$$

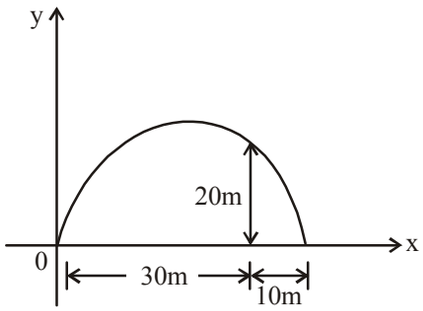
$2u\sin\theta = 80$

$u\sin\theta = 40$

Time of flight

$$T = \frac{2u\sin\theta}{g} = \frac{2.40}{10} = 8s$$

10. **(b)**



Here, $x = 30m$, $y = 20m$, $R = 30 + 10 = 40m$

By equation of trajectory,

$$y = x\tan\theta\left[1 - \frac{x}{R}\right] \Rightarrow 20 = 30\tan\theta\left[1 - \frac{30}{40}\right]$$

$$\Rightarrow \frac{2}{3} = \frac{1}{4}\tan\theta \qquad \therefore \theta = \tan^{-1}\left(\frac{8}{3}\right)$$

11. **(b)**

12. **(a)** The equation of motion of a projectile is

$y = Ax - Bx^2$

At $x = R$, $y = 0$

$\therefore 0 = AR - BR^2 \Rightarrow R(A - BR) = 0$

$$\therefore \text{Range of projectile, } R = \frac{A}{B}$$

$$\text{Again at } x = \frac{R}{2} = \frac{A}{2B},\ y = H$$

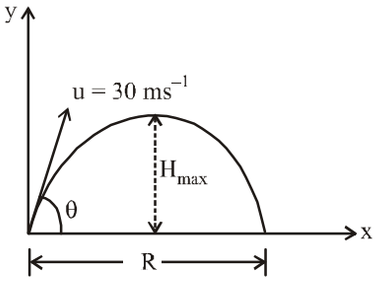
$$\therefore H = A\left(\frac{A}{2B}\right) - B\left(\frac{A}{2B}\right)^2$$

$$\frac{A^2}{2B} - \frac{A^2}{4B} = \frac{A^2}{4B}$$

$$\therefore \text{Maximum Height, } H = \frac{A^2}{AB}$$

$$\therefore \frac{H}{R} = \frac{A^2/AB}{A/B} = \frac{A}{4}$$

13. **(a)** Height of auditorium,



$H_{max} = 30\,m$

Speed, $u = 30\ ms^{-1}$

$$\therefore H_{max} = \frac{4^2\sin^2\theta}{2g}$$

$$\Rightarrow 30 = \frac{(30)^2\sin^2\theta}{2\times 10} \Rightarrow \sin\theta = \sqrt{\frac{2}{3}}$$

$$\therefore \cot^2\theta = \text{cosec}^2\theta - 1 = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\therefore \cot\theta = \sqrt{\frac{1}{2}}$$

$\therefore$ The length of the auditorium is given by

$$R = 4H_{max}\cot\theta = 4\times 30\times\sqrt{\frac{1}{2}} = 60\sqrt{2}\ m$$

14. **(d)**

15. **(c)** As distance travelled in successive seconds differ by 2 m each, therefore acceleration is constant = 2 m s^{-2}

$$\text{From } D_n = u + \frac{a}{2}(2n-1) \therefore D_8 = u + \frac{2}{2}(2\times 8 - 1)$$

$$20 = u + 15 \text{ or } u = 5\,m\,s^{-1}$$

16. **(c)** $s = vt + \frac{1}{2}at^2$

It is not a kinematic equation of motion.

All others are kinematic equations of motion.

17. **(a)**

18. **(b)** Area covered by the bullets on the ground $= \pi R_{max}^2$

$$R_{max} = \frac{v^2 \sin 2\theta}{g} = \frac{(10)^2 \sin(2\times 45°)}{g} = \frac{100}{10} = 10\text{m}$$

$\therefore$ Area covered $= \pi R_{max}^2 = 3.14 \times 10^2 = 314\ m^2$

19. **(a)** Here horizontal distance travelled by ball = horizontal distance travelled by person in same time.

$$\therefore \frac{x}{v_o \cos\theta} = \frac{x}{0.5\ v_o} \Rightarrow \cos\theta = 0.5 \ \therefore\ \theta = 60°$$

20. **(b)** Standard equation is

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

Given, $y = \sqrt{3}x - 0.2x^2$, time of flight t = ?

Comparing given equation with standard equation,

$\tan\theta = \sqrt{3} \therefore \theta = 60°$

And $0.2 = \frac{g}{2} \times \frac{1}{u^2\cos^2\theta}$ or, $u^2\cos^2(60°) = 25$

$$\Rightarrow u^2 \times \left(\frac{1}{2}\right)^2 = 25 \Rightarrow u^2 = 100$$

$\therefore u = \sqrt{100} = 10\ m/s$

$\therefore$ Time of flight, $t = \frac{24\sin\theta}{g} = \frac{2\times 10\times \sin 60°}{10}$

$$= 2\times\frac{\sqrt{3}}{2} = \sqrt{3}s$$

21. **(a)** Given, acceleration due to gravity, $g = 10 m/s^2$

According to the question, total time of flight

$$T = \frac{2u}{g}\sin\theta$$

So, Maximum height,

$$T = \frac{2u}{g} \Rightarrow \ u = \frac{T.g}{2}$$

$$= \frac{(2.3+5.7)\times 10}{2} = \frac{8\times 10}{2} = 40\ m/s$$

Now use the equation for height when vertical velocity becomes zero.

$$H = \frac{u^2}{2g} = \frac{40\times 40}{2\times 10} = 80\ m$$

22. **(c)**

23. **(b)** Given,

Initial velocity, $u = 19.6\ ms^{-1}$

Maximum height, $h_{max} = 9.8\ m$

$$h_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{(19.6)^2\times\sin^2\theta}{2\times 9.8}$$

$$\Rightarrow 9.8 = \frac{(19.6)^2\times\sin^2\theta}{2\times 9.8} \Rightarrow \sin^2 = \frac{1}{2} \Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \ \theta = 45°$$

$$\text{Range, } R = \frac{u^2\sin 2\theta}{g} = \frac{u^2\times 2\sin\theta\cos\theta}{g}$$

$$\text{Range, } R = \frac{u^2\sin 2\theta}{g} = \frac{u^2\times 2\sin\theta\cos\theta}{g}$$

$$\Rightarrow R = \frac{(19.6)^2\times 2\times\sin 45°\cos 45°}{9.8}$$

$$= 2\times 19.6 = 39.2m$$

24. **(d)** For body projected at an angle (1), time taken to reach maximum height.

$v = 4 + at \Rightarrow 0 = u\sin\theta - gt_1$

$$\Rightarrow t_1 = \frac{u\sin\theta}{g}$$

For body projected at an angle $90 - \theta$, time taken

$v = u + at \ \Rightarrow \ 0 = u\sin(90-0) - gt_2$

$$\Rightarrow t_2 = \frac{u\cos\theta}{g}$$

Horizontal distance travelled by first body

$x_1 = 4\cos\theta \times t_1$.

$$x_1 = u\cos\theta\times\frac{u\sin\theta}{g} = \frac{u^2\sin\theta\cos\theta}{g}$$

Horizontal distance travelled by second body

$x_2 = 4\cos(90-\theta) \times t_2$

$$x_2 = u\sin\theta\times\frac{4\cos\theta}{g} = \frac{u^2\sin\theta\cos\theta}{g}$$

Total distance between their positions $= |x_1| + |x_2|$

$$= \frac{u^2\sin\theta\cos\theta}{g} + \frac{u^2\sin\theta\cos\theta}{g}$$

$$= \frac{u^2\sin 2\theta}{g} = \frac{u^2\sin 2(90-\theta)}{g}$$

25. **(b)** Given maximum height = horizontal range.

$$\frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin 2\theta}{g}$$

$4\sin\theta\cos\theta = \sin^2\theta \Rightarrow \tan\theta = 4$

26. **(a)** $v = u + at \Rightarrow v = 0 + 5\times 10 = 50\ m/s$

2 Laws of Motion

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(a)** No force is required for an object moving in straight line with constant velocity or for non acceleration motion.

2. **(c)** From Newton's second law if $\Sigma F_i = 0$ then the body is in translational equilibrium.

3. **(d)** Inertia is defined as the ability of a body to oppose any change in its state of rest or of uniform motion.

4. **(c)** Newton's first law of motion defines the inertia of body. It states that every body has a tendency to remain in its state (either rest or motion) due to its inertia.

5. **(c)** Newton's first law of motion is related to the physical independence of force.

6. **(a)**

7. **(d)** From principle of momentum conservation,
$M_1V_1 = M_2V_2$
or, $1000 \times 6 = 1200 \times v \Rightarrow v = 5 m/s$

8. **(c)** According to newton's second law, when external force acts on a body it produces an acceleration of the body in the direction of the force.

9. **(a)** Mass, m = 10 kg
Initial velocity, u = 10 m/s
Time, t = 10 sec
final velocity, V = 0
using Newton's IInd law

$$F = ma = m\left(\frac{V-u}{t}\right) = \frac{10\times10}{10} = 10$$

F = 10 N

10. **(c)** Force is the mutual interaction between the bodies is according to newton's third law of motion.

11. **(c)** Given,
Velocity of the bullet, v = 1000 ms^{-1}
Momentum of bullet after firing, p = 20 kg ms^{-1}
Linear momentum, $p = mv$

$$\Rightarrow \quad m = \frac{p}{v} = \frac{20}{1000} kg$$

$$\Rightarrow \quad m = \frac{20}{1000} \times 1000 \text{ g} \Rightarrow m = 20g$$

12. **(b)** As we know, $|a| = \left|\frac{v^2 - u^2}{2S}\right|$

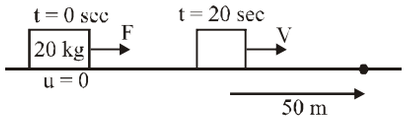
$= \frac{400}{50}$ = 8 m/s^2. So force required = F = ma
= 800 × 8 = 6400 N

13. **(c)**

14. **(c)** The gun applied a force F_{12} on the bullet in forward direction & according to Newton's third law bullet applies a reaction force on gun F_{21} in backward direction. But the recoil speed of gun is very low in comparison to bullet due to large mass.

15. **(b)** $m = \frac{\sqrt{8^2 + (-6)^2 + (-10)^2}}{1} = 10\sqrt{2} kg$

16. **(b)**



Assume surface to be frictionless
Then, 50 = V × 10 ⇒ V = 5 m/s
As v = u + at ⇒ V = 0 + a × 20

$$\Rightarrow \quad 5 = a \times 20 \Rightarrow a = \frac{1}{4} m/s^2$$

$$\text{So, } F = ma = 20 \times \frac{1}{4} = 5N$$

17. **(d)** By Newton's second law of motion
$F = n(mv) = nmv$

18. **(a)**

19. **(c)** Given $y = ut + gt^2$

Equation of motion is $y = ut + \frac{1}{2}at^2$

On comparing a = 2g
Thus, F = ma = 2mg

20. **(b)** **21.** **(d)**

22. **(d)** Force acting on the stone,
F = ma = 2 × 4 = 8 N

23. **(b)** Change in momentum is given by,
$\Delta p = F \cdot \Delta t$
or $F \cdot \Delta t = p_f - p_i$
$10 \times 0.25 = p_f - mv_i \Rightarrow 2.5 = p_f - 0.5 \times 0 \Rightarrow p_f = 2.5$ N–s

24. **(a)**

25. **(b)** When force retards motion *i.e.*, F –(ve) so, work done –(ve)

26. **(c)** When a man pushes a wall and fails to displace it, then displacement of wall = 0
$\therefore$ Work done by man = $F \times 0 = 0$
Therefore, man does no work at all.

27. **(a)** When a person carrying load on his head moves over a horizontal road, work done against gravitational force is zero.

28. **(d)** $W = FS\cos\theta$
$\therefore$ If $F = 0$; $W = 0$
If $S = 0$; $W = 0$
& if $\theta = 90°$; $\cos 90° = 0$ $\therefore$ $W = 0$.

29. **(c)** Displacement of the particle when it takes a complete round the circular path is zero.
$\therefore$ Work done = force × displacement
$W = F \times 0 = 0$
Therefore, work done by the force is zero.

30. **(b)** From work-energy theorem,
$W_{Porter} + W_{mg} = \Delta K.E. = 0$ ($\because$ velocity constant)
or, $W_{Porter} = -W_{mg} = -mgh$

$$\therefore \quad W_{Porter} = -80 \times 9.8 \times \frac{80}{100} = -627.2J$$

31. **(c)** Motion without slipping implies pure rolling. During pure rolling work done by friction force is zero.

32. **(d)** Given : $\vec{F} = 3\hat{i} + \hat{j}$

$$\vec{r_1} = \left(2\hat{i} + \hat{k}\right), \ \vec{r_2} = \left(4\hat{i} + 3\hat{j} - \vec{k}\right)$$

$$\vec{r} = \vec{r_2} - \vec{r_1} = \left(4\hat{i} + 3\hat{j} - \vec{k}\right) - \left(2\hat{i} + \hat{k}\right)$$

or $\vec{r} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

So work done by the given force $w = \vec{f}.\vec{r}$

$$= \left(3\hat{i} + \hat{j}\right).\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right) = 6 + 3 = 9J$$

33. **(c)** $W = F\,s\cos\theta = 10 \times 2\cos 60° = 10$ J.

34. **(c)** $W = F\,s\cos\theta$, $\cos\theta = \frac{W}{Fs} = \frac{25}{5\times 10} = \frac{1}{2}$, $\theta = 60°$.

35. **(a)** Conservative force is negative gradient of potential
$$F(x) = \frac{-dV(x)}{dx}$$

36. **(c)** The law of conservation of momentum is true in all type of collisions, but kinetic energy is conserved only in elastic collision. The kinetic energy is not conserved in inelastic collision but the total energy is conserved in all type of collisions.

37. **(c)** In a perfectly inelastic collision, the two bodies move together as one body.

38. **(b)** In an inelastic collision, momentum remains conserved, but K.E is changed.

39. **(c)**

40. **(a)** During elastic collision between two equal masses, the velocities get exchanged. Hence energy transfer is maximum when $m_1 = m_2$.

41. **(a)** In an elastic collision

$$V_1 = \frac{(m_1 - m_2)}{m_1 + m_2}u_1 \ ; \ V_2 = \frac{2m_1u_1}{m_1 + m_2}$$

$$\therefore \text{ if } m_1 = m_2, \text{ then } V_1 = 0; \text{ and } V_2 = \frac{2\cancel{m_1}v_1}{\cancel{2m_1}} = u_1$$

42. **(c)**

43. **(c)** Apply conservation of momentum,

$$m_1v_1 = (m_1 + m_2)v; \ v = \frac{m_1\,v_1}{(m_1 + m_2)}$$

Here $v_1 = 36$ km/hr $= 10$ m/s, $m_1 = 2$ kg, $m_2 = 3$ kg

$$v = \frac{10 \times 2}{5} = 4 \text{ m/s}$$

$$\text{K.E. (initial)} = \frac{1}{2} \times 2 \times (10)^2 = 100\,J$$

$$\text{K.E. (Final)} = \frac{1}{2} \times (3+2) \times (4)^2 = 40 \text{ J}$$

Loss in K.E. = 100 – 40 = 60 J
Alternatively use the formula

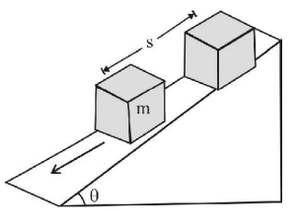
$$-\Delta E_k = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2$$

44. **(d)** As the body moves in the direction of force therefore work done by gravitational force will be positive.
$W = Fs = mgh = 10 \times 9.8 \times 10 = 980$ J

45. **(a)** Work done by the applied force is zero in (i) because in case (i) both the force and displacement are mutually perpendicular.

46. **(d)** $W = FS\cos\theta = 10 \times 4 \times \cos 60° = 20$ joule

47. **(b)** $W = mg\sin\theta \times s = 2 \times 10^3 \times \sin 15° \times 10 = 5.17$ kJ



48. **(c)** Work done by variable force

$$\text{Work done, } W = \int_{y_i}^{y_f} F\,dy \Rightarrow \int_{y=0}^{y_f=1} F.dy$$

where, $F = 5 + 10\,y$

$$\therefore \quad W = \int_0^1 (5 + 10y)\,dy = \left[5y + \frac{10y^2}{2}\right]_0^1 = 10 \text{ J}$$

49. **(d)** Friction is a non- conservative force.

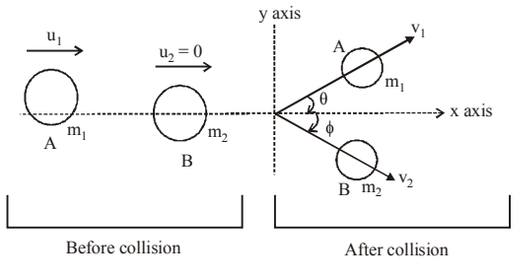
50. **(b)** When the identical balls collide head-on, their velocities are exchanged.

51. **(d)** Due to the same mass of A and B as well as due to elastic collision, velocities of spheres get interchanged after the collision.

52. **(a)** In an elastic head-on collision, of two equal masses their kinetic energies or velocities are exchanged. Hence when the first ball collides with the second ball at rest,

the second ball attains the speed of 0.4 m/s and the first ball comes to rest. This process continues. Thus the velocity of the last ball is 0.4 ms^{-1}.

53. (b)



Given $m_1 = m_2$

We will apply the principle of conservation of momentum in the mutually perpendicular dir^n.

Along x-axis, $m_1u_1 = m_1v_1 \cos\theta + m_2 v_2 \cos\phi$

or $u_1 = v_1 \cos\theta + v_2 \cos\phi$ ...(i)

Along y-axis, $0 = m_1v_1 \sin\theta - m_2 v_2 \sin\phi$

or $0 = v_1 \sin\theta - v_2 \cos\phi$ ...(ii)

Again for elastic collision, kinetic energy is conserved

$$\Rightarrow \frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $u_1^2 = v_1^2 + v_2^2$ ...(iii)

Squaring and adding (i) & (ii), we get

$u_1^2 = v_1^2 (\cos^2\theta + \sin^2\theta) + v_2^2 (\cos^2\phi + \sin^2\phi)$

$+ 2v_1v_2 \cos\theta \cos\phi - 2v_1v_2 \sin\theta \sin\phi$

or $u_1^2 = v_1^2 + v_2^2 + 2v_1v_2 \cos(\theta + \phi)$ ...(iv)

Using (iii) & (iv), we get

$$\cos(\theta+\phi) = 0 = \cos\frac{\pi}{2}$$

$$\Rightarrow \theta + \phi = \frac{\pi}{2}$$

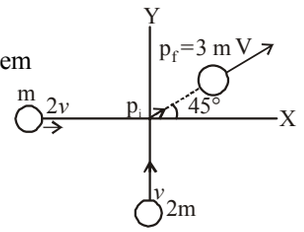
Note : This is a standard case of oblique collision.

54. (a) Initial momentum of the system

$$(\vec{p}_i) = m(2V)\hat{i} + 2m(V)\hat{j}$$

Final momentum of the system

$$(\vec{p}_f) = 3m\vec{V}_f$$



By conservation of linear momentum $\vec{p}_i = \vec{p}_f$

$$2mV\hat{i} + 2mV\hat{j} = 3m\vec{V}_f$$

$$\vec{V}_f = \frac{2}{3}V\hat{i} + \frac{2V\hat{j}}{3}$$

$$\left|\vec{V}_f\right| = \frac{2\sqrt{2}}{3}V$$

Loss in K.E. of system = $K_{initial} - K_{final}$

$$= \frac{1}{2}m(2V)^2 + \frac{1}{2}2m(V)^2 - \frac{1}{2}\times 3m \times \left(\frac{2\sqrt{2}}{3}V\right)^2$$

$$= 2mV^2 + mV^2 - \frac{4}{3}mV^2 = \frac{5}{3}mV^2$$

$$\% \text{ Loss in K.E.} = \frac{\text{Loss in K.E. of system}}{K_{initial}} \times 100$$

$$= \frac{\frac{5}{3}mV^2}{3mV^2} \times 100 \approx 56\%$$

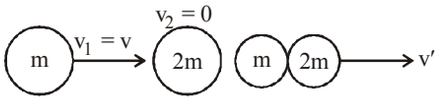
55. (c) From momentum conservation in first collision velocity interchanges

For second collision, $2 \times 20 = 4 \times v$; $v = 10$ m/s

56. (b) If the collision is perfectly inelastic then after the collision ball will move along the incline.

57. (a) $e = |v_1 - v_2| / |u_1 - u_2|$

58. (c)



Applying conservation of linear momentum

$\Rightarrow \vec{P}_i = \vec{P}_f$ ($\because P = mv$)

$mv_1 + 2mv_2 = (m + 2m)\, v'$

$mv + 2m \times 0 = (3m)\, v'$

$\Rightarrow mv = 3mv'$

$$\Rightarrow v' = \frac{v}{3}$$

59. (b) From law of conservation of momentum, before collision and after collision linear momentum (P) will be same.

Initial momentum = final momentum.

$\Rightarrow 0 = m_1v_1 - m_2v_2$

$\Rightarrow m_1v_1 = m_2v_2$

$p_1 = p_2$

Now, $E = \frac{p^2}{2m}$

$$\therefore \frac{E_1}{E_2} = \frac{p_1^2}{2m_1} \times \frac{2m_2}{P_2^2} \Rightarrow \frac{E_1}{E_2} = \frac{m_2}{m_1} \quad [p_1 = p_2]$$

60. (a) Clearly $v_1 = 2\ ms^{-1}$, $v_2 = 0$

$m_1 = m$ (say), $m_2 = 2m$

$v_1' = ?$, $v_2' = ?$

$$e = \frac{v_2' - v_1'}{v_1 - v_2} \quad \text{....(i)}$$

By conservation of momentum,

$2m = mv_1' + 2mv_2'$ ... (ii)

From (i), $0.5 = \frac{v_2' - v_1'}{2}$

$\therefore \quad v_2' = 1 + v_1'$

From (ii), $2 = v_1' + 2 + 2v_1'$

$\Rightarrow \quad v_1' = 0$ and $v_2' = 1\ ms^{-1}$

61. **(b)** Here,

Mass of the gun, $M = 100$ kg

Mass of the ball, $m = 1$ kg

Height of the cliff, $h = 500$ m

$g = 10\ ms^{-2}$

Time taken by the ball to reach the ground is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 500m}{10ms^{-2}}} = 10s$$

Horizontal distance covered = ut

$\therefore \quad 400 = u \times 10$

where u is the velocity of the ball.

$u = 40\ ms^{-1}$

According to law of conservation of linear momentum, we get

$0 = Mv + mu$

$$v = -\frac{mu}{M} = -\frac{(1kg)(40ms^{-1})}{100kg} = -0.4ms^{-1}$$

–ve sign shows that the direction

–ve sign shows that the direction of recoli

62. **(a)** Impulse = change in momentum = $m\vec{v}_2 - m\vec{v}_1$

63. **(b)** If a large force F acts for a short time dt the impulse imparted I is

$$I = F \cdot dt = \frac{dp}{dt} \cdot dt$$

$I = dp$ = change in momentum

64. **(a)**

65. **(a)** As we know, |impulse| = |change in momentum|

$= |p_2 - p_1| = |0 - mv_1| = |0 - 3 \times 2| = 6$ Ns

66. **(d)**

67. **(a)** As the ball, m = 10 g = 0.01 kg rebounds after striking the wall

$\therefore$ Change in momentum = $mv - (-mv) = 2mv$

Inpulse = Change in momentum = $2mv$

$$\therefore v = \frac{\text{Impulse}}{2m} = \frac{0.54\ N\ s}{2 \times 0.01\ kg} = 27\ m\ s^{-1}$$

68. **(d)**

69. **(c)** Mass = 150 gm = $\frac{150}{1000} kg$

$$\text{Force} = \text{Mass} \times \text{acceleration} = \frac{150}{1000} \times 20N = 3N$$

Impulsive force = $F.\Delta T = 3 \times 0.1 = 0.3$ N

70. **(a)** **71.** **(b)**

72. **(c)** $v_1 = 10$ m/s ; $m_1 = 10$ kg

$v_2 = 0$; $m_2 = 9$ kg

$v_3 = v$; $m_3 = 1$ kg

According to conservation of momentum

$m_1v_1 = m_2v_2 + m_3v_3$

$10 \times 10 = 9 \times 0 + 1 \times v$; $v = 100$ m/s.

73. **(a)** If m_1, m_2 are masses and u_1, u_2 are velocity then by conservation of momentum $m_1u_1 + m_2u_2 = 0$ or $|m_1u_1| = |m_2u_2|$

74. **(b)** Centre of mass depends on the distribution of mass in the body.

75. **(a)** Position of COM of a mass – system is given as,

$$\vec{r}_{com} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1 + m_2} = \frac{1(\hat{i} + 2\hat{j} + \hat{k}) + 3(-3\hat{i} - 2\hat{j} + \hat{k})}{1+3}$$

$$= -2\hat{i} - \hat{j} + \hat{k}$$

$$|\vec{r}_{cm}| = |-2\hat{i} - \hat{j} + \hat{k}| = \sqrt{(2)^2 + (1)^2 + (1)^2} = \sqrt{6}$$

Only option (a) magnitude is $\sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$

So option (a) is correct.

76. **(d)** The position of centre of mass of a system depends upon mass, position and symmetry of the body.

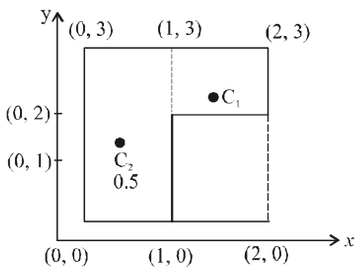
$$R_{CM} = \frac{\sum m_i r_i}{\sum m_i}$$

77. **(a)** $X_{cm} = \frac{\int_0^L (dm)x}{\int_0^L (dm)} = \frac{\int_0^L (\lambda x dx)x}{\int_0^L (\lambda x dx)} = \frac{2L}{3}$

78. **(a)** **79.** **(a)**

80. **(b)** $x_{cm} = \frac{1 \times 0 + 1 \times PQ + 1 \times PR}{1+1+1} = \frac{PQ + PR}{3}$ and $y_{cm} = 0$

81. **(b)**



For given Lamina

$x \quad y$

$m_1 = 1$, $C_1 = (1.5, 2.5)$

$m_2 = 3$, $C_2 = (0.5, 1.5)$

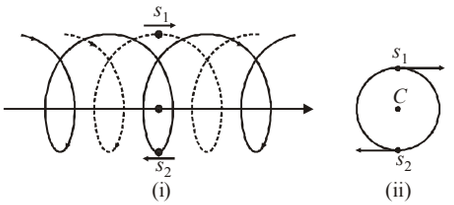
$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1.5 + 1.5}{4} = 0.75$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{2.5 + 4.5}{4} = 1.75$$

$\therefore$ Coordinate of centre of mass of flag shaped lamina (0.75, 1.75)

82. **(c)** **83.** **(a)** **84.** **(a)**

85. **(a)** When no external force acts on the binary star, its CM will move like a free particle [Fig. (a)]. From the CM frame, the two stars will seems to move in a circle about the CM with diametrically opposite positions.



(i) Trajectories of two stars. S_1 (dotted line) and S_2 (solid line) forming a binary system with their centre of mass C in uniform motion

(ii) The same binary system, with the centre of mass C at rest.

So, to understand the motion of a complicated system, we can separate the motion of the system into two parts. So, the combination of the motion of the CM and motion about the CM could described the motion of the system.

86. **(c)** $m_1 d = m_2\ d_2 \Rightarrow d_2 = \dfrac{m_1 d}{m_2}$

87. **(a)** $X_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$$X_{cm} = \frac{300 \times (0) + 500(40) + 400 \times 70}{300 + 500 + 400}$$

$$X_{cm} = \frac{500 \times 40 + 400 \times 70}{1200} = 40 \text{ cm}$$

88. **(b)** The coordinates of C.M of three particle are

$x = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$ & $y = \dfrac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$

here $m_1 = m_2 = m_3 = m$

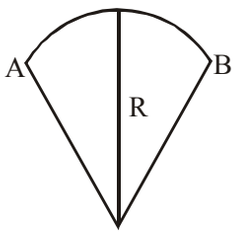
so $x = \dfrac{(x_1 + x_2 + x_3)m}{m + m + m} = 2$, $y = \dfrac{(y_1 + y_2 + y_3)m}{m + m + m} = 2$

so coordinates of C.M. of three particle are (2, 2)

89. **(b)** **90.** **(d)** **91.** **(a)**

92. **(b)** $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \dfrac{2 \times 2 + 4 \times 10}{2 + 4} = 7.3 m/s$

93. **(d)** If Arc AB $\to 0$ centre of mass is at a distance R from the origin as the Arc length AB increases, centre of mass starts moving down.

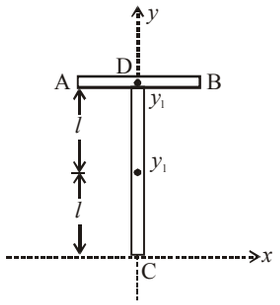


94. **(b)** Centre of mass is closer to massive part of the body therefore the bottom piece of bat has larger mass.

95. **(a)** For translatory motion the force should be applied on the centre of mass of the body. So we have to calculate the location of centre of mass of 'T' shaped object.

Let mass of rod AB is m so the mass of rod CD will be 2m.

Let y_1 is the centre of mass of rod AB and y_2 is the centre of mass of rod CD. We can consider that whole mass of the rod is placed at their respective centre of mass i.e., mass m is placed at y_1 and mass 2m is placed at y_2.



Taking point 'C' at the origin positon vector of point y_1 and y_2 can be written as $\vec{r}_1 = 2\ell\,\hat{j}, r_2 = l\hat{j}$,

and $m_1 = m$ and $m_2 = 2m$

Position vector of centre of mass of the system

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{m2l\,\hat{j} + 2ml\,\hat{j}}{m + 2m} = \frac{4ml\,\hat{j}}{3m} = \frac{4}{3}l\,\hat{j}$$

Hence the distance of cente of mass from C= $\dfrac{4}{3}l$.

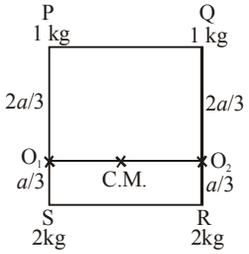
96. **(c)** Centre of mass lies always on the line that joins the two particles.

For the combination cd and ab this line does not pass through the origin.

For combination bd, initially it passes through the origin but later on it moves toward negative x-axis.

But for combination ac it will always pass through origin. So we can say that centre of mass of this combination will remain at origin.

97. **(d)**



Centre of mass of P and S will be at O_1, Q and R will be at O_2.

$\therefore$ The centre of mass of the system is at the midpoint of O_1 and O_2 which is farthest from P and Q.

98. **(a)** Centre of mass of solid hemisphere of radius R lies at a distance $\frac{3R}{8}$ above the centre of flat side of hemisphere.

$\therefore h_{cm} = \frac{3R}{8} = \frac{3\times 8}{8} = 3$ cm

99. **(b)** COM of ring is at centre of ring and of semi-circular ring is at $\frac{2R}{\pi}$ as shown in figure.

Hence value of x = 2

100. **(a)** $\vec{T} = \vec{r} \times \vec{F}$

$\vec{T}$ is $\perp$ to both $\vec{r}$ and $\vec{F}$

so $\vec{r} \cdot \vec{T} = 0$ and $\vec{F} \cdot \vec{T} = 0$

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(d)** As the rain drop is falling down with a constant speed, its acceleration, $a = 0$. Hence, net force on the drop = ma = 0. As the cork is floating on the surface of water, its weight is balanced by the upthrust. Hence, net force on the cork is zero.
The force exerted by the engine is balanced by the friction due to rough road. As the car is moving with constant velocity, its acceleration a = 0. Hence net force on the car, $F = ma = 0$.
Whenever a body is thrown vertically upwards gravitational pull of earth gives it a uniform acceleration, $a = g$ in downward direction.
Hence, net force on the pebble = $mg = 0.05 \times 10 = 0.5$ N vertically downwards.

2. **(a)**

3. **(c)** $F = \frac{dp}{dt} = \frac{mdv}{dt}$

$$\left(4t^2\hat{i} + 2t\,\hat{j}\right) = \frac{4dv}{dt}$$

Rearranging and intigrating both sides,

$$\Rightarrow \quad dv = \frac{1}{4}\int_0^3 \left(4t^2\,\hat{i} + 2t\,\hat{j}\right)dt$$

$$\Rightarrow \quad v = \frac{1}{4}\left[\frac{4t^3}{3}\hat{i} + \frac{2t^2}{2}\hat{j}\right]_0^3 = 9\hat{i} + 2.25\hat{j}$$

4. **(a)** Using Newton's second law, F = ma
In component form,

$a_x = \frac{F_x}{m} = \frac{2}{1} = 2\,m/s^2$

$a_y = \frac{F_y}{m} = \frac{-3}{1} = -3\,m/s^2$

Displacement is given as,

$s = ut + \frac{1}{2}at^2$

$s_x = u_x t + \frac{1}{2}a_x t^2 = 18$ m

$s_y = u_y t + \frac{1}{2}a_y t^2 = -1.5$ m

x and y - coordinates of point = (18, – 1.5)

5. **(b)** Here force applied on the block increases linearly with time, so

$F = ma \Rightarrow a = \frac{F}{m} = \frac{kt}{m}$ or, $a \propto t$

Hence a vs t graph will be a straight line passing through origin.

6. **(a)** Using Newton's IInd law, $F = \frac{\Delta p}{\Delta t} = \frac{mv}{t}$

$F = \frac{0.12 \times 25}{0.1} = 30$N

7. **(d)** Here, $u_x = 6$ ms^{-1}, $v_x = 0$,

$a_x = -\frac{3}{5}$ ms^{-2}, $t = ?$

Using $v_x = u_x + a_x t$

$\Rightarrow t = \frac{v_x - u_x}{a_x} = \frac{0-6}{-3/5} = \frac{30}{3} = 10$s

8. **(a)** The magnitude of net force on the stone is
F = mg = (0.05 kg) (9.8 m s^{-2}) = 0.49 N
Its direction is vertically downwards.

9. **(d)** Mass, m = 20g = 20 × 10^{-3} kg
velocity, u = 500 m/s
distance, d = 1 cm = 0.0/m
final velocity, v = 0
Using Equation of Motion, $v^2 - u^2 = 2ad$
$-(500)^2 = 2 \times a \times 0.01$

$a = -\frac{500\times 50000}{2\times 0.01} = -1.25 \times 10^7$ m/s^2

using newton's second law,
F = m a
$F = 20 \times 10^{-3} \times 1.25 \times 10^7$

$F = 250\times 10^3$ N

10. **(b)** Mass of body = 6 kg
Initial velocity, u = 4 m/s
Final velocity, v = 6 m/s.
Force, F = 12 N
Using Newtones IInd law, F = ma

$$a = \frac{F}{m} = \frac{12}{6} = 2\,m/s^2$$

Using Eq^n of Motion,

$$v^2 - u^2 = 2as;\ s = \frac{v^2 - u^2}{2a}$$

$$s = \frac{(6)^2 - (4)^2}{2\times 2} = \frac{36-16}{4} = \frac{20}{4} = 5$$

11. **(d)** From equation of motion

$$v^2 = u^2 + 2as \qquad \Rightarrow a = \frac{v^2 - u^2}{2s}$$

$$= -\frac{(7)^2 + (0)^2}{2\times 10} = -\frac{49}{20} = -2.45\ m/s^2 = -\frac{g}{4}\ m/s^2$$

We have $g = 9.8\ m/s^2$

$$a = -\frac{g}{4} \therefore ma = -\frac{mg}{4} \quad \therefore R = -\frac{W}{4}$$

12. **(b)**

13. **(d)** $v^2 = u^2 - 2gh$ or $v = \sqrt{u^2 - 2gh}$

Momentum, $P = mv = m\sqrt{u^2 - 2gh}$

At h = 0, P = mu and at $h = \frac{u^2}{f}, P = 0$

upward direction is positive and downward direction is negative.

14. **(c)** Mass (m) = 0.3 kg $\Rightarrow$ F = m.a = – 15 x

$$a = -\frac{15}{0.3}x = \frac{-150}{3}x = -50x\ ;\ a = -50\times 0.2 = 10\,m/s^2$$

15. **(b)** Here $u = 10\ ms^{-1}$, $v = -2\ ms^{-1}$, t = 4 s, a = ?

Using $a = \frac{v-u}{t} = \frac{-2-10}{4} = -3\ m/s^2$

$\therefore$ Force, F = ma = 10 × (–3) = –30 N

16. **(d)** Here m = 0.5 kg; u = – 10 m/s ; t = 1/50 s ; $v = +15\ ms^{-1}$

Force = m (v– u)/t = 0.5 (10 + 15) × 50 = 625 N

17. **(c)** Work done, $W = \int \vec{F}\cdot\vec{ds} = \left(-x\hat{i} + y\hat{j}\right)\cdot\left(d\times\hat{i} + dy\hat{j}\right)$

$$\Rightarrow W = -\int_1^0 x\,dx + \int_0^1 y\,dy = \left(0 + \frac{1}{2}\right) + \frac{1}{2} = 1J$$

18. **(b)** (A)→(3); (B)→(1); C→(2,3); (D)→(1)

19. **(c)** $W = \vec{F}.\vec{s} = (5\hat{i} + 2\hat{j}).(2\hat{i} + 3\hat{j}) = 10 + 6 = 16$ J.

20. **(a)** W = F s cos 90° = zero

21. **(a)** $x = 3t - 4t^2 + t^3 \quad \frac{dx}{dt} = 3 - 8t + 3t^2$

$$\text{Acceleration} = \frac{d^2x}{dt^2} = -8 + 6t$$

Acceleration after 4 sec = –8 + 6 × 4 = 16

Displacement in 4 sec = 3 ×4 – 4 × 4^2 + 4^3 = 12 m

$\therefore$ Work = Force × displacement

= Mass × acc. × disp. = 3 × 10^{-3} × 16 × 12 = 576 mJ

22. **(c)** As the cord is trying to hold the motion of the block, work done by the cord is negative.

$$W = -M(g-a)\,d = -M\left(g - \frac{g}{4}\right)d = \frac{-3Mgd}{4}$$

23. **(d)** Though an equal and opposite force acts on the road but since road does not undergo any displacement, hence no work is done on the road.

24. **(b)** $W = \int_0^{x_1} F\,dx = \int_0^{x_1} c\,x\,dx = \left[\frac{1}{2}cx^2\right]_0^{x_1}$

$$= \frac{1}{2}c(x_1^2 - 0) = \frac{1}{2}cx_1^2$$

25. **(b)** By work-energy thorem

$W = \Delta K$

$$\Rightarrow W = \frac{1}{2}m(v_f^2 - v_i^2) \Rightarrow W = \frac{1}{2}\times 0.5\times(16^2 - 4^2)$$

$$\Rightarrow W = \frac{1}{4}\times 240 \Rightarrow W = 60\ J$$

26. **(d)** When work is done upon a system by a conservative force then its potential energy increases.

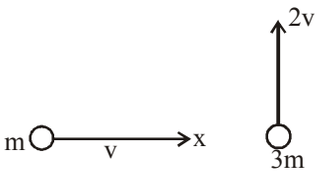
27. **(a)**

28. **(b)** $m_1v_1 + m_2v_2 = (m_1 + m_2)v_{sys.}$

$$20\times 10 + 5\times 0 = (20+5)v_{sys} \Rightarrow v_{sys} = 8m/s$$

K. E. of composite mass $= \frac{1}{2}(20+5)\times(8)^2 = 800J$

29. **(a)** As the two masses stick together after collision, hence it is inelastic collision. Therefore, only momentum is conserved.



$$\therefore mv\hat{i} + 3m(2v)\hat{j} = (4m)\vec{v}$$

$$\vec{v} = \frac{v}{4}\hat{i} + \frac{6}{4}v\hat{j} = \frac{v}{4}\hat{i} + \frac{3}{2}v\hat{j}$$

30. **(d)** K. E. of colliding body before collision $= \frac{1}{2}mv^2$

After collision its velocity becomes

$$v' = \frac{(m_1 - m_2)}{(m_1 + m_2)}v = \frac{m}{3m}v = \frac{v}{3}$$

$\therefore$ K. E. after collision $= \frac{1}{2}\frac{mv^2}{9}$

$$\text{Ratio of kinetic energy} = \frac{K.E._{\cdot before}}{K.E._{\cdot after}} = \frac{\frac{1}{2}mv^2}{\frac{1mv^2}{2\times 9}} = 9:1$$

31. **(d)** For the object of mass 2.0 kg.

$$\frac{\Delta k}{k} = \frac{k - k/4}{k} = \frac{3}{4}$$

Kinetic energy transferred

$$\frac{\Delta k}{k} = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

Here, $m_1 = 2.0$ kg, $m_2 = M$

$$\therefore \frac{3}{4} = \frac{4\times 2M}{(2+M)^2} \Rightarrow M = \frac{2}{3}\text{kg or } 6\text{kg}$$

32. **(a)** Initial, K.E.$= \frac{1}{2}mv^2 = \frac{1}{2}\times\frac{20}{1000}\times 600\times 600 = 3600$ J

Change in K.E. = P.E.

$$\frac{1}{2}m(v^2 - v'^2) = mgh$$

$$\Rightarrow 3600 - \frac{1}{2}\times\frac{20}{1000}\times v_1^2 = 4\times 10\times 80$$

$$\Rightarrow v_1 = 200\,m/s$$

33. **(c)** Initial kinetic energy of the system

$$K.E_i = \frac{1}{2}mu^2 + \frac{1}{2}M(0)^2 = \frac{1}{2}\times 0.5\times 2\times 2 + 0 = 1J$$

For collision, applying conservation of linear momentum

$m \times u = (m + M) \times v$

$\therefore \quad 0.5\times 2 = (0.5+1)\times v$

$$\Rightarrow v = \frac{2}{3}m/s$$

Final kinetic energy of the system is

$$K.E_f = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(0.5+1)\times\frac{2}{3}\times\frac{2}{3} = \frac{1}{3}J$$

$$\therefore \text{ Energy loss during collision} = \left(1 - \frac{1}{3}\right)J = 0.67J$$

34. **(a)**

35. **(a)** As the floor exerts a force on the ball along the normal, & no force parallel to the surface, therefore the velocity component along the parallel to the floor remains constant. Hence $V \sin\theta = V^1 \sin\theta^1$.

36. **(b)** The law of conservation of energy is valid at any instant & in all circumstances.

37. **(b)** As no external force is acting on system so, $P_i = P_f$

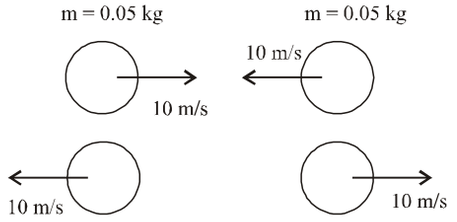
$0.2 \times 10 = 10 \times v \Rightarrow v = 0.2$ m/sec

$$\text{Loss in K.E.} = \frac{1}{2}\times(0.2)\times 10^2 - \frac{1}{2}\times 10(0.2)^2$$

$$= \frac{1}{2}\times 10\times(0.2)[10 - 0.2] = 9.8\text{ J}$$

38. **(b)**

39. **(b)**



Chnage in momentum of any one ball

$$|\Delta\vec{P}| = 2\times 0.05\times 10 = 1$$

$$|\vec{F}_{av}| = \frac{|\Delta\vec{P}|}{\Delta t} = \frac{1}{0.005} = \frac{1000}{5} = 200\text{ N}$$

40. **(c)** $F = 600 - 2\times 10^5\, t = 0 \Rightarrow t = 3\times 10^{-3}$ sec

$$\text{Impulse } I = \int_0^t F\,dt = \int_0^{3\times 10^{-3}}\left(600 - 2\times 10^3 t\right)dt$$

$$= \left[600t - 10^5 t^2\right]_0^{3\times 10^{-3}} = 0.9\text{ N} \times \text{sec}$$

41. **(b)** Velocity between $t = 0$ and $t = 2$ sec

$$\Rightarrow v_1 = \frac{dx}{dt} = \frac{4}{2} = 2\text{ m/s}$$

Velocity at $t = 2$ sec, $v_f = 0$

Impulse = Change in momentum = $mv\,(v_f - v_t)$

$0.1\,(0 - 2) = -0.2$ kg m sec^{-1}

42. **(c)** Impulse = Change in momentum = $m\,(v_2 - v_1)$ ... (i)

Again impulse = Area between the graph and time axis

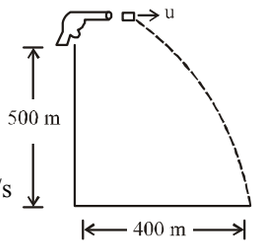
$$= \frac{1}{2}\times 2\times 4 + 2\times 4 + \frac{1}{2}(4 + 2.5)\times 0.5 + 2\times 2.5$$

$= 4 + 8 + 1.625 + 5 = 18.625$ ... (ii)

From (i) and (ii), $m\,(v_2 - v_1) = 18.625$

$$\Rightarrow v_2 = \frac{18.625}{m} + v_1 = \frac{18.625}{2} + 5 = 14.25\,m/s$$

43. **(d)** Time of flight,



$$T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times 500}{10}} = 10s$$

$$\therefore\ R = u.\ T \Rightarrow\ u = \frac{400}{10} = 40\ m/s$$

$\therefore$ Recoil velocity of gun,

$$\therefore\ V = \left(\frac{m}{M}\right)u = \frac{1\times 40}{100} = 0.4\ m/s$$

44. **(b)** From law of conservation of momentum

$MV = m_1v_1 + m_2v_2$

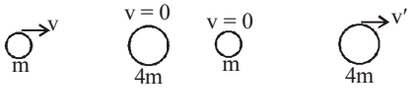
Here, $M = 100$ kg, $v = 10^4$ m s^{-1}

$m_1 = 10$ kg, $v_1 = 0$; $m_2 = 90$ kg, $v_2 = ?$

$$\therefore\ 100 \times 10^4 = 10 \times 0 + 90 \times v_2\quad \therefore\ v_2 = \frac{100\times 10^4}{90}$$

$v_2 = 11.11 \times 10^3$ m s^{-1}.

45. **(b)**



Before Collision After Collision

According to law of conservation of linear momentum,

$$mv + 4m \times 0 = 4\,mv' + 0 \Rightarrow v' = \frac{v}{4}$$

Coefficient of restitution,

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{\frac{v}{4}}{v}$$

or, $e = \frac{1}{4} = 0.25$

46. **(b)** $(A) \rightarrow v_1\ (B)_{\text{rest}} \Rightarrow (A)(B) \rightarrow v_2$

(Before collision) (After collision)

By conservation of linear momentum,

$P_i = P_f$

$\Rightarrow\ mv_1 = mv_2 + mv_2 \Rightarrow\ mv_1 = 2\,mv_2$

$$\therefore\ \frac{v_1}{v_2} = \frac{2}{1}$$

47. **(d)** From, $F = ma$

$$a = \frac{F}{m} = \frac{0.1x}{10} = 0.01x = V\frac{dV}{dx}$$

So, $\int_{v_1}^{v_2} vdV = \int_{20}^{30} \frac{x}{100}dx$

$$-\frac{V^2}{2}\Big|_{V_1}^{V_2} = \frac{x^2}{200}\Big|_{20}^{30} = \frac{30\times 30}{200} - \frac{20\times 20}{200} = 4.5 - 2 = 2.5$$

$$\frac{1}{2}m\left(V_2^2 - V_1^2\right) = 10 \times 2.5\ J = -25J$$

Final K.E. $= \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - 25 = \frac{1}{2}\times 10\times 10\times 10 - 25$

$= 500 - 25 = 475$ J

48. **(c)** From work-energy theorem,

$W_g + W_a = \Delta K.E$ or, $mgh + W_a = \frac{1}{2}mv^2 - 0$

$$10^{-3}\times 10\times 10^3 + W_a = \frac{1}{2}\times 10^{-3}\times (50)^2 \Rightarrow W_a = -8.75\ J$$

which is the work done due to air resistance

Work done due to gravity = mgh

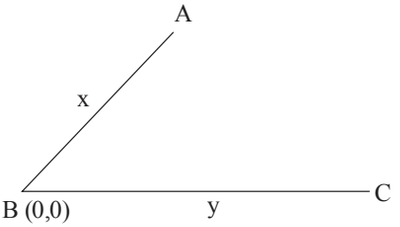
$= 10^{-3} \times 10 \times 10^3 = 10$ J

49. **(c)** Centre of mass $x_{cm} = \frac{x}{2}\frac{(\rho x)\left(\frac{x}{2}\right)\frac{1}{2} + \rho y\left(\frac{y}{2}\right)}{\rho(x+y)}$

$$\Rightarrow \frac{1}{2} + \frac{y}{x} = \frac{y^2}{x^2}$$

$$\therefore \frac{BC}{AB} = \frac{y}{x}$$

$$= \frac{1+\sqrt{3}}{2} = 1.37$$



50. **(c)** Let M be the mass of circular disk.

The density of desk; $\sigma = \frac{M}{\pi a^2}$

Now let M' be the mass of cut out disk.

$$\therefore M' = \sigma \times \pi\left(\frac{a}{2}\right)^2 = \frac{M}{\pi a^2} \times \pi\frac{a^2}{4} = \frac{M}{4}$$

$$X_{com} = \frac{M\times a + \frac{M}{4}\times\frac{3}{2}a}{M - \frac{M}{4}} = \frac{5}{6}a.$$

51. **(d)** $X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$$= \frac{1\times 0 + 1\times 3 + 1\times 0}{1+1+1} = 1$$

$$Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times 0 + 1\times 0 + 1\times 4}{1+1+1} = \frac{4}{3}$$

Therefore the coordinates of centre of mass are $\left(1, \frac{4}{3}\right)$.

52. **(b)** Centre of mass of the rod is given by:

$$x_{cm} = \frac{\int_0^L (ax + \frac{bx^2}{L})dx}{\int_0^L (a + \frac{bx}{L})dx} = \frac{\frac{aL^2}{2} + \frac{bL^2}{3}}{aL + \frac{bL}{2}} = \frac{L\left(\frac{a}{2} + \frac{b}{3}\right)}{a + \frac{b}{2}}$$

Now $\frac{7L}{12} = \frac{\frac{a}{2} + \frac{b}{3}}{a + \frac{b}{2}} \Rightarrow b = 2a$

53. **(c)**

54. **(a)** Acceleration of centre of mass (a_{cm}) is given by

$$\therefore \vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...}$$

$$= \frac{(2m)a\hat{j} + 3m \times a\hat{i} + ma(-\hat{i}) + 4m \times a(-\hat{j})}{2m + 3m + 4m + m}$$

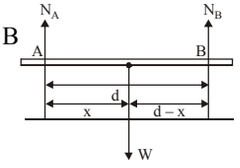
$$= \frac{2a\hat{i} - 2a\hat{j}}{10} = \frac{a}{5}(\hat{i} - \hat{j})$$

55. **(c)**

56. **(a)** Does not shift as no external force acts. The centre of mass of the system continues its original path. It is only the internal forces which comes into play while breaking.

57. **(c)**

58. **(c)** By torque balancing about B



$N_A (d) = W (d - x)$

$$N_A = \frac{W(d-x)}{d}$$

59. **(b)** Given :

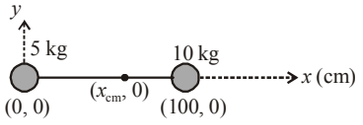
Force, $\vec{F} = 3\hat{j}$ N

Position vector, $\vec{r} = 2\hat{k}$ m

Torque, $\vec{\tau} = \vec{r} \times \vec{F} = 2\hat{k} \times 3\hat{j} = 6(\hat{k} \times \hat{j}) = 6(-\hat{i})$

$\Rightarrow \vec{\tau} = -6\hat{i}$ Nm

60. **(b)**



Centre of mass of a system of two particles of masses m_1 and m_2 having position vectors $\vec{x_1}$ and $\vec{x_2}$ is

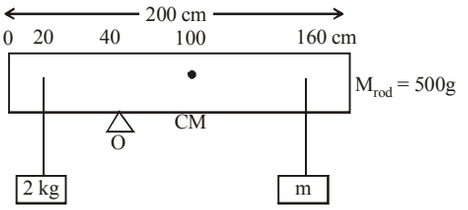
$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

Taking particle of mass 5 kg at origin.

$$\Rightarrow x_{cm} = \frac{5 \times 0 + 100 \times 10}{5 + 10} = \frac{200}{3} = 66.66 \text{ cm}$$

$\Rightarrow x_{cm} = 67$ cm

61. **(a)**



Rod will be in equilibrium, if $\vec{\tau}_{net} = 0$ about point O.

Torque at point O due to 2 kg mass

$$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta(\hat{n})$$

$$\tau_1 = 20 \times 20 \times 10^{-2} \times \sin 90°(\hat{k}) = 4\text{Nm}(\hat{k})$$

Torque due to mass of rod

$$\tau_2 = 5 \times 60 \times 10^{-2} \times \sin 90°(-\hat{k}) = 3\text{Nm}(-\hat{k})$$

Torque due to mass m

$$\tau_3 = mg \times 120 \times 10^{-2} \times \sin 90°(-\hat{k})$$

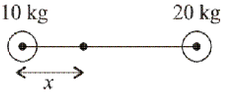
$$= 12\text{m Nm}(-\hat{k})$$

So $\vec{\tau_1} + \vec{\tau_2} + \vec{\tau_3} = 0 (\because \vec{\tau}_{net} = 0)$

$\Rightarrow 4 - 3 - 12m = 0 \Rightarrow 12m = 1$

$$\therefore m = \frac{1}{12}\text{kg}$$

62. **(a)**



As, distance of COM from

$$m_1 = \frac{m_2 r}{m_1 + m_2}$$

Here, $m_1 = 10$ kg, $m_2 = 20$ kg and $r = 10$ m

So, required distance $= \frac{20 \times 10}{20 + 10} = \frac{20}{3} m$

Exercise 3 : PREVIOUS YEARS MCQs

1. **(a)** From law of conservation of linear momentum

$P_i = P_f \Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$ ...(i)

$2 \times 4 + 0 = 2 \times 1 + m_2 v_2$

In elastic collision coefficient of restitution, e = 1 and

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$\Rightarrow v_2 - v_1 = e(u_1 - u_2) \Rightarrow v_2 - 1 = 1(4 - 0)$ or, $v_2 = 5$

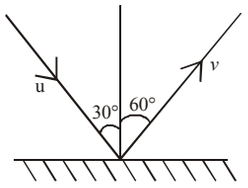
Now from eq. (i)

$$8 = 2 + m_2 \times 5 \Rightarrow m_2 = \frac{6}{5}$$

Velocity of centre of mass

$$V_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{2\times1 + \frac{6}{5}\times5}{2+\frac{6}{5}} = 2.5 \text{ m/s}$$

2. **(b)** Parallel to surface velocity does not change



$u \sin 30° = v \sin 60°$

$$\frac{u}{2} = \frac{\sqrt{3}v}{2} \Rightarrow v = \frac{u}{\sqrt{3}}$$

$$\text{Restitution, } e = \frac{v\cos 60°}{u\cos 30°} = \frac{\frac{u}{\sqrt{3}}\times\frac{1}{2}}{u\times\frac{\sqrt{3}}{2}} = \frac{1}{3}$$

3. **(b)** Given :

Force, $\vec{F} = 3\hat{j}$ N

Position vector, $\vec{r} = 2\hat{k}$ m

Torque, $\vec{\tau} = \vec{r}\times\vec{F} = 2\hat{k}\times3\hat{j} = 6(\hat{k}\times\hat{j}) = 6(-\hat{i})$

$\Rightarrow \vec{\tau} = -6\hat{i}$ Nm

4. **(d)** Evaluate the integral for work done

$$W = \int_2^4 \left(6x^2 - 4x\right) dx$$

$$W = \left[6.\frac{x^{2+1}}{2+1} - 4.\frac{x^{1+1}}{1+1}\right]_2^4$$

W = 88J

5. **(a)** The speed of the particle is maximum where the force is zero.

$$F(x) = \frac{-du}{dx} - \frac{du[5x(x-4)]}{dx} = -5(2x-4)$$

For F = 0, x = 2 m.

6. **(b)** Here



Before collision After collision

$P_i = mv$ $P_f = 3\,mv'$

$KE_i = \frac{1}{2}mv^2$ $KE_f = \frac{1}{2}(3\,m)(v)^2$

7. **(b)** $m_1 = 2kg$, $\vec{v_1} = 20$ m/s north

$m_2 = 3kg$, $\vec{v_2} = 10$ m/s east

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

$$\vec{v}_{cm} = \frac{2\left(0\hat{i} + 20\hat{j}\right) + 3\left(10\hat{i} + 0\hat{j}\right)}{(2+3)}$$

$$\vec{v}_{cm} = \frac{30\hat{i} + 40\hat{j}}{5} = 6\hat{i} + 8\hat{j}$$

$$\left|\vec{v}_{cm}\right| = \sqrt{6^2 + 8^2} = \sqrt{100}$$

$$\boxed{\left|\vec{v}_{cm}\right| = 10 \text{ m/s}}$$

8. **(c)** Given: $M_1 = 2kg$ $\vec{V_1} = 20$ m/s along x – axis

$M_2 = 3kg$, $\vec{V_2} = 10$ m/s along y – axis

Use vector formula for center of mass velocity:

$$\vec{V}_{CM} = \frac{M_1\vec{V}_1 + M_2\vec{V}_2}{M_1 + M_2}$$

$$\vec{V}_{CM} = \frac{2\times20\hat{i} + 3\times10\hat{j}}{2+3} = 8\hat{i} + 6\hat{j} \text{ m/s}$$

$$\left|\vec{V}_{CM}\right| = \sqrt{8^2 + 6^2} = \sqrt{100} = 10 \text{ m/s}$$

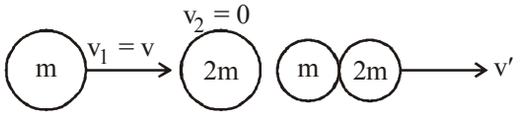
9. **(c)** A non-conservative force is one for which work depends on the path.

10. **(b)** In inelastic collision, there is always loss of kinetic energy.

$\therefore$ $k_i > k_f$

11. **(c)** In elastic collision, both kinetic energy and momentum are conserved. Total energy is also conserved.

12. **(c)**



Applying conservation of linear momentum

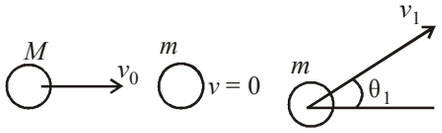
$\Rightarrow \vec{P}_i = \vec{P}_f$ ($\because$ P = mv)

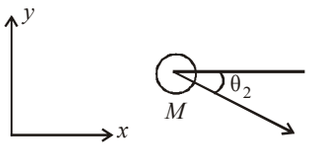
$mv_1 + 2mv_2 = (m + 2m)\,v'$

$mv + 2m \times 0 = (3m)\,v'$

$\Rightarrow mv = 3mv' \Rightarrow v' = \frac{v}{3}$

13. (c)





Let $\theta_1 = \theta_2 = \theta$. Then,

$Mv_0 = mv_1 \cos\theta + Mv_2 \cos\theta$ ...(i)

and, $0 = mv_1 \sin\theta - Mv_2 \sin\theta$

$\Rightarrow v_2 = \frac{mv_1}{M}$ ...(ii)

from (i) & (ii), we get

$$Mv_0 = mv_1 \cos\theta + M\left(\frac{mv_1}{M}\right)\cos\theta$$

$Mv_0 = 2mv_1 \cos\theta$ ....(iii)

By conservation of K.E.

$$\frac{1}{2}Mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$$

$$\Rightarrow \frac{1}{2}M\left(\frac{2mv_1\cos\theta}{M}\right)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}M.\frac{m^2v_1^2}{M^2}$$

$$\Rightarrow \frac{1}{2}\left(\frac{4m^2v_1^2\cos^2\theta}{M}\right)^2 = \frac{1}{2}mv_1^2 + \frac{m^2v_1^2}{2M}$$

$$\Rightarrow \frac{4m\cos^2\theta}{M} = 1 + \frac{m}{M} \Rightarrow 4\cos^2\theta = \frac{M}{m} + 1$$

For largest $\frac{M}{m}, \cos\theta = 1$ So, $\frac{M}{m} = 3$

14. (d)

15. (a) Impulse $= \Delta\vec{P} = \vec{P}_f - \vec{P}_i = mv - (-mv) = 2mv$

$= 2 \times 0.4 \times 15 = 12$ Ns

16. (d) $T = \frac{2m_1m_2}{m_1 + m_2}g = \frac{2\times10\times6}{10+6} \times 9.8 = 73.5$ N

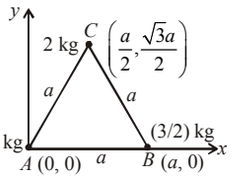
17. (a) **18. (a)**

19. (a) Let the masses 1 kg, $\frac{3}{2}$ kg and 2 kg are located at the vertices A, B and C as shown in figure. The coordinates of points A, B and C are (0,0), (a, 0), $\left(\frac{a}{2}, \frac{\sqrt{3a}}{2}\right)$ respectively.

The coordinates of centre of mass are

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{\left[1\times0 + \frac{3}{2}\times a + 2\times\frac{a}{2}\right]}{\left[1 + \frac{3}{2} + 2\right]} = \frac{5a}{9}$$



$$Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{\left[1\times0 + \frac{3}{2}\times0 + 2\times\frac{\sqrt{3a}}{2}\right]}{\left[1 + \frac{3}{2} + 2\right]} = \frac{2\sqrt{3a}}{9} = \frac{2a}{3\sqrt{3}}$$

20. (b) For equilibrium

$$\frac{dU}{dr} = 0 \Rightarrow \frac{-2A}{r^3} + \frac{B}{r^2} = 0 ; r = \frac{2A}{B}$$

For stable equilibrium

$\frac{d^2U}{dr^2}$ should be positive at $r = \frac{2A}{B}$.

Here $\frac{d^2U}{dr^2} = \frac{6A}{r^4} - \frac{2B}{r^3}$ is +ve value for $r = \frac{2A}{B}$

21. (a) **22. (b)**

3 Gravitation

Exercise 1 : WARM-UP
Topic-wise MCQs

1. **(b)** By Kepler's period law, $T^2 \mu r^3$

$$\therefore \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3 = \left(\frac{R+3R}{R}\right)^3 = 64$$

$$\Rightarrow \frac{T_2}{T_1} = 8 \Rightarrow T_2 = 8T_1 = 8 \times 80 = 640 \text{ min}$$

2. **(c)** When a planet revolves around the sun in general for the planet angular momentum about the sun and aerial velocity of the planet are constant.

3. **(c)**

4. **(b)** According to Kepler's law, the areal velocity of a planet around the sun always remains constant.

SCD : $A_1 - t_1$ (areal velocity constant)

SAB : $A_2 - t_2$

$$\frac{A_1}{t_1} = \frac{A_2}{t_2},\ t_1 = t_2 . \frac{A_1}{A_2},\ (\text{given } A_1 = 2A_2)$$

$$= t_2 . \frac{2A_2}{A_2} \qquad \therefore t_1 = 2t_2$$

5. **(d)** From Kepler's law of periods,

$$T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 365\left(\frac{R/2}{R}\right)^{3/2}$$

$$= 365 \times \frac{1}{2\sqrt{2}} = 129 \text{ days.}$$

6. **(d)** Time period does not depend on the mass. Also,

$$T^2 \propto r^3 \text{ or } \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{R}{2R}\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2} = \frac{1}{2\sqrt{2}}$$

7. **(a)** **8.** **(a)**

9. **(d)** Kepler's third law states that the square of time period of revolution of a planet around the sun is directly proportional to the cube of semi-major axis of its elliptical orbit.

10. **(b)** In planetary motion, there is no external torque. Hence from the equation $\frac{\overrightarrow{dL}}{dt} = \vec{\tau}_{ext.}$, if $\vec{\tau}_{ext.} = 0 \Rightarrow L = \text{constant}$

11. **(b)**

12. **(d)** Angular momentum of the planet about S is conserved. So, mvr = constant.

v is maximum when r is minimum.

So, v is maximum at point P_4. Hence K.E. is maximum at P_4.

13. **(d)** $\frac{L}{2m} = \frac{dA}{dt}$ (L = angular momentum)

$$\frac{mv_{max}r_{min}}{2m} = \frac{dA}{dt};\ v_{max} = \frac{2\,dA/dt}{r_{min}} = 40$$

14. **(c)** $\frac{dA}{dt} = \frac{L}{2m} \Rightarrow \frac{dA}{dt} \propto vr \propto \omega r^2$

15. **(c)** $\frac{T_p}{T_e} = \frac{1}{3} \Rightarrow \left(\frac{T_p}{T_e}\right)^2 = \left(\frac{r_p}{r_e}\right)^3 \Rightarrow r_e^3 = 9r_p^3$

16. **(d)** Time period does not depend upon the mass of satellite.

17. **(c)** $T^2 \propto R^3$ (Kepler's law)

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} = \left(\frac{10^{13}}{10^{12}}\right)^3 \Rightarrow \frac{T_1}{T_2} = 10\sqrt{10}$$

18. **(b)** $\frac{T^2}{r^3} = \text{constant} \Rightarrow T^2 r^{-3} = \text{constant}$

19. **(a)** By law of conservation of angular momentum, mvr = constant

$$v_{min} \times r_{max} = v_{max} \times r_{min}$$

$$\therefore v_{max} = \frac{60 \times 1.6 \times 10^{12}}{8 \times 10^{12}} = \frac{60}{5} = 12\,m/s$$

20. **(b)** $\frac{T^2}{R^3} = \frac{T^2}{d^3} = \frac{1}{n^2 d^3} = \text{constant}$

$$\therefore n_1^2 d_1^3 = n_2^2 d_2^3 \qquad \text{[where n = frequency]}$$

21. **(b)** According to Keepler's third law of planetary motion, i.e., laws of periods

$T^2 \propto R^3$ or $T \propto R^{3/2}$

$$\therefore \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R}{4R}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2} = \frac{1}{8}$$

Therefore the ratio of their respective periods is 1 : 8

22. **(b)** $\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$

23. **(d)** For central force, torque is zero.

$$\because = \frac{dL}{dt} = 0 \Rightarrow L = \text{constant}$$

i.e., Angular momentum is constant.

24. **(c)** Angular momentum of the earth around the sun is

$$L = M_E v_0 r$$

$$\Rightarrow \quad L = M_E \sqrt{\frac{GM_s}{r}}\, r \qquad \left(\because \upsilon_0 = \sqrt{\frac{GM_s}{r}}\right)$$

$$\Rightarrow \quad L = \left[M_E^2 G M_s r\right]^{1/2}$$

where, M_E = Mass of the earth

M_S = Mass of the sun

r = Distance between the sun and the earth

$\therefore\ L \propto \sqrt{r}$

25. **(b)** Since $T^2 \propto r^3 \quad \therefore \left(\frac{T'}{T}\right)^2 = \left(\frac{1}{4}\right)^3 = T' = \frac{1}{8}T$

26. **(c)** Let the point mass M is placed at a distance of × m from m_1.

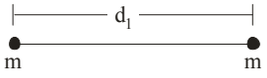
Also, $\frac{m_1}{m_2} = \frac{4}{3}$

As given $F_{13} = \frac{1}{3}F_{23}$

$$\frac{Gm_1M}{x^2} = \frac{1}{3}\frac{Gm_2M}{(1-x)^2}$$

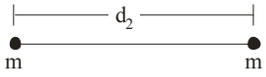
$$\Rightarrow \frac{m_1}{m_2} = \frac{x^2}{3(1-x)^2} \Rightarrow \frac{4}{3} = \frac{x^2}{3(1-x)^2} \quad \therefore \quad x = \frac{2}{3}\text{m}$$

27. **(b)**



Gravitational force, $F_1 = \frac{Gm.m}{d_1^2} = \frac{Gm^2}{d_1^2}$

Similarly,



Gravitational force, $F_2 = \frac{Gm^2}{d_2^2}$

$$\because \quad \frac{F_1}{F_2} = \left(\frac{d_2}{d_1}\right)^2 \Rightarrow F_2 = F_1\left(\frac{d_1}{d_2}\right)^2$$

28. **(c)** The gravitational force is independent of the intervening medium. In other words, the force between two masses remains the same whether they are in air, vacuum, water or separated by a brick wall.

Hence, (a) is an incorrect statement.

The gravitational force is a conservation force.

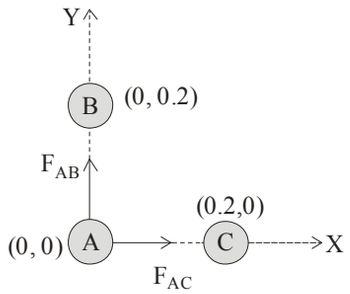
Hence, (b) is an incorrect statement.

The gravitational force obeys Newton's third law of motion. Thus, it form action-reaction pair.

Hence, (c) is a correct statement.

The gravitational force is a central force. Hence, (d) is an incorrect statement.

29. **(a)** Let particle A lies at origin, particle B and C on y and x-axis respectively



$$\vec{F}_{AC} = \frac{Gm_A m_B}{r_{AB}^2}\hat{i} = \frac{6.67\times10^{-11}\times1\times1}{(0.2)^2}\hat{i} = 1.67\times10^{-9}\hat{i}\text{N}$$

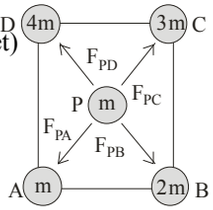
Similarly $\vec{F}_{AB} = 1.67\times10^{-9}\hat{j}\text{N}$

$\therefore$ Net force on particle A

$$\vec{F} = \vec{F}_{AC} + \vec{F}_{AB} = 1.67\times10^{-9}\left(\hat{i}+\hat{j}\right)\text{N}$$

30. **(c)** If two particles of mass m are placed x distance apart then force of attraction $\frac{Gmm}{x^2} = F$ (Let)

Now according to problem particle of mass m is placed at the centre (P) of square. Then it will experience four forces



F_{PA} = force at point P due to particle $A = \frac{Gmm}{x^2} = F$

Similarly, $F_{PB} = \frac{G2mm}{x^2} = 2F, F_{PC} = \frac{G3mm}{x^2} = 3F$

and $F_{PD} = \frac{G4mm}{x^2} = 4F$

Hence the net force on P

$$\vec{F}_{net} = \vec{F}_{PA} + \vec{F}_{PB} + \vec{F}_{PC} + \vec{F}_{PD} = 2\sqrt{2}F$$

$$\therefore \quad F_{net} = 2\sqrt{2}\frac{Gmm}{x^2} = 2\sqrt{2}\frac{Gm^2}{\left(a/\sqrt{2}\right)^2}$$

$\left[x = \frac{a}{\sqrt{2}} = \text{half of the diagonal of the square} \right]$

$= \frac{4\sqrt{2}Gm^2}{a^2}$

31. **(d)** G is a universal gravitational constant as the value of G is same for all pairs of bodies situated anywhere in the universe.

32. **(b)** It will remain the same as the gravitational force is independent of the medium separating the masses.

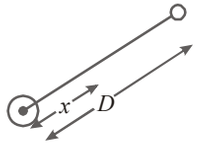
33. **(a)** $F \propto xM \times (1-x)M = xM^2(1-x)$

For maximum force, $\frac{dF}{dx} = 0$

$\Rightarrow \frac{dF}{dx} = M^2 - 2xM^2 = 0 \Rightarrow x = 1/2$

34. **(d)** $\frac{Gm_e}{x^2} = \frac{Gm_m}{(D-x)^2}$

or $\frac{G(81m)}{x^2} = \frac{m}{(D-x)^2}$

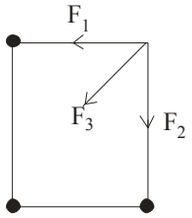


$\therefore \; x = \frac{9D}{10}$.

35. **(a)** $F = \frac{Gm_1m_2}{r^2} = \frac{Gm_1m_2}{(r_1+r_2)^2}$.

36. **(d)** $F_1 = F_2 = \frac{GM}{a^2}$

Resultant of F_1 and F_2 is $\sqrt{2}\frac{GM}{a^2}$



Now, $F_3 = \frac{GM}{(\sqrt{2}a)^2} = \frac{GM}{2a^2}$

Now, $\frac{\sqrt{2}GM}{a^2}$ and $\frac{GM}{2a^2}$ act in the same direction.

Their resultant is $\frac{\sqrt{2}GM}{a^2} + \frac{GM}{2a^2}$ or $\frac{GM}{a^2}\left[\sqrt{2} + \frac{1}{2}\right]$

37. **(d)** $M = \rho \times \frac{4}{3}\pi R^3$

So, $F = \frac{GM^2}{4R^2} = \frac{G\left(\rho \times \frac{4}{3}\pi R^3\right)^2}{4R^2} = \frac{4G\rho^2\pi^2R^4}{9} \; \therefore F \propto R^4$

38. **(c)** $F = \frac{GMm}{a^3} = \frac{mv^2}{a}$

Orbital speed, $v = \frac{\sqrt{GM}}{a}$

Time period of revolution of a planet, $T = \frac{2\pi r}{v}$

$T = \frac{2\pi a}{\frac{\sqrt{GM}}{a}} = \frac{2\pi a^2}{\sqrt{GM}} \Rightarrow T \propto a^2$

Hence, the orbit of the planet around the sun will not be elliptical because for elliptical orbit $T^2 \propto a^3$.

As force $F = \left(\frac{GM}{a^3}\right)m = g'm$.

Since, g' = acceleration due to gravity is constant, hence path followed by a projectile will be approximately parabolic (as $T \propto a^2$).

39. **(b)** Given that at some height h, acceleration due to gravity,

$g_h = 4.9 \text{ m/s}^2 \approx \frac{g}{2}$ ...(i)

$\therefore$ The ratio of acceleration due to gravity at earth's surface and at some altitude h is

$\left(1 + \frac{h}{R_e}\right) = \sqrt{\frac{g}{g_h}} = \sqrt{2}\left[\text{from Eq.(i)}\right] \; \therefore \; \frac{h}{R_e} = \sqrt{2} - 1$

or $h = 0.414 \times R_e$

$h = 0.414 \times 6400 \text{ km} = 2.6 \times 10^6 \text{ m}$

40. **(c)** $w' = mg'$ ...(i)

$g' = g\left(\frac{R_E}{R_E + h}\right)^2$

As per question

$h\left(\frac{R_E}{2}\right) \quad \therefore \quad g' = g\left[\frac{R_E}{\frac{3R_E}{2}}\right] = \frac{4}{9}g$ ...(ii)

On substituting Eq. (ii) in Eq. (i), we get

$w' = \frac{4mg}{9} = \frac{4}{9}w$

41. **(c)** $g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{g}{2} = g\left(\frac{R}{R+h}\right)^2$

$\Rightarrow \quad h = (\sqrt{2} - 1)R$

42. **(c)** $mg' = mg\left(1 - \frac{d}{R}\right)$

Thus, the weight of the body at half way down $\left(\text{i.e. } d = \frac{R}{2}\right)$ to the centre of the earth is

$mg' = 200 \times \left(1 - \frac{R/2}{R}\right) = 200\left(1 - \frac{1}{2}\right) = 200 \times \frac{1}{2} = 100 \text{ N}$

43. **(c)** $F = \dfrac{GMm}{(R+h)^2}$

Then, as per question,

$$\frac{GMm}{(R+h)^2} = \frac{1}{16}\frac{GMm}{R^2}$$

$h = 3R$

44. **(b)** Gravitational pull depends upon the acceleration due to gravity on that planet.

$$\therefore \quad M_m = \frac{1}{81}M_e, g_m = \frac{1}{6}g_e$$

$$g = \frac{Gm}{R^2} \Rightarrow \frac{R_e}{R_m} = \left(\frac{M_e}{M_m} \times \frac{g_m}{g_e}\right)^{1/2} = \left(81 \times \frac{1}{6}\right)^{1/2}$$

$$\therefore \quad R_e = \frac{9}{\sqrt{6}}R_m$$

45. **(d)** For scientist A which goes down in mine

$$g' = g\left(1 - \frac{d}{R}\right)$$

For scientist B, which goes up in air $g' = g\left(1 - \dfrac{2h}{R}\right)$

So, it is clear that value of g measured by each will decrease at different rates.

46. **(b)** Acceleration due to gravity at height h is given by

$$g' = g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad \frac{g}{100} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{10} \Rightarrow h = 9R$$

47. **(b)** $g' = g\left(1 - \dfrac{d}{R}\right) \Rightarrow \dfrac{g}{4} = g\left(1 - \dfrac{d}{R}\right) \Rightarrow d = \dfrac{3R}{4}$

48. **(c)** Acceleration due to gravity $g = \dfrac{GM}{R^2}$

$$\therefore \quad \frac{g_{planet}}{g_{earth}} = \frac{M_{planet}}{M_{earth}}\left(\frac{R_{earth}}{R_{planet}}\right)^2 = \frac{1}{10} \times \left(\frac{3}{1}\right)^2 = \frac{9}{10}$$

If a stone is thrown with velocity u from the surface of the planet then maximum height.

$$H = \frac{u^2}{2g} \Rightarrow \frac{H_{planet}}{H_{earth}} = \frac{g_{earth}}{g_{planet}}$$

$$\Rightarrow \quad H_{planet} = \frac{10}{9} \times H_{earth} = \frac{10}{9} \times 90 = 100 \text{ metre}$$

49. **(c)** Weight of the body at height R,

$$W' = W\left(\frac{R}{R+h}\right)^2 = W\left(\frac{R}{R+\frac{R}{2}}\right)^2 = W\left(\frac{2}{3}\right)^2$$

$$= \frac{4}{9}W = \frac{4}{9} \times 72 = 32\text{N}$$

50. **(d)** On surface of earth, mg = 300 N

At depth $\dfrac{R}{4}$ acceleration due to gravity is,

$$g' = g\left[1 - \frac{d}{R}\right] \Rightarrow g' = g\left[1 - \frac{R}{4R}\right] = \frac{3g}{4}$$

Weight at depth $\dfrac{R}{4} = mg' = m \times \dfrac{3g}{4} = \dfrac{3}{4} \times 300 = 225$ N

51. **(d)** Gravitational pull = mg

Value of g varies with height 'h' as

$$g_s\left(1 + \frac{h}{R}\right)^{-2} = g_s(1+2)^{-2} = \frac{g_s}{9}$$

$$\therefore \text{Gravitational pull} = mg = 90 \times \frac{g_s}{9} = 90 \times \frac{10}{9} = 100\text{N}$$

52. **(d)** Using $W_{(R+x)} = W_{(R-x)}$

$$\frac{GMm}{(R+x)^2} = \frac{GMm(R-x)}{R^3} \qquad [\because W = mg]$$

$$\Rightarrow R^3 = (R+x)^2(R-x) \Rightarrow R^3 = (R^2 - x^2)(R+x)$$

$$\Rightarrow x^2 + Rx - R^2 = 0$$

$$\therefore \ x = \frac{-R \pm \sqrt{R + 4R^2}}{2}; \ x = \frac{\sqrt{5}R - R}{2}$$

53. **(d)** Acceleration due to gravity $g = \dfrac{GM}{R^2} \Rightarrow g \propto \dfrac{1}{R^2}$

$$\therefore \quad \frac{g_2}{g_1} = \frac{R_1^2}{R_2^2} = \frac{R_1^2}{\left(\frac{R_1}{2}\right)^2} \qquad \left(\because R_2 = \frac{R_1}{2}\right)$$

Therefore, $g_2 = 4g_1 = 4g$

54. **(a)** Acceleration due to gravity at height h is

$$g_h = \frac{g}{(R+h)^2} \Rightarrow g_h \propto \frac{1}{(R+h)^2}$$

$$\frac{g_{h_2}}{g_{h_1}} = \left(\frac{R+h_1}{R+h_2}\right)^2 \Rightarrow \frac{\frac{g}{4}}{g} = \left(\frac{6400+0}{6400+h_2}\right)^2$$

$$\Rightarrow \frac{1}{2} = \frac{6400}{6400+h_2} \Rightarrow h_2 = 6400 \text{ km}$$

55. **(d)** For plant and earth,

$r_1 : r_2 = 1 : 2, \delta_1 : \delta_2 = 4 : 1, g_2 = 9.8\ ms^{-2}$

$\therefore$ Acceleration due to gravity, $g = \frac{4}{3}\pi\delta Gr \ \therefore g \propto \delta r$

$\therefore \ \frac{g_1}{g_2} = \frac{\delta_1}{\delta_2}\cdot\frac{r_1}{r_2} = \frac{4}{1}\times\frac{1}{2} = 2$

$\therefore \ g_1 = 2g_2 = 2\times 9.8 = 19.6\ ms^{-2}$

56. **(a)** Acceleration due to gravity at height h is given by

$$g' = g\left(\frac{R}{R+h}\right)^2$$

At h = 1280

$$g_1 = g\left(\frac{6400}{6400+1280}\right)^2 = \frac{25}{36}g$$

at h = 3200

$$g_2 = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{6400}{3200+6400}\right)^2 = \frac{4g}{9}$$

$$\therefore \ \frac{g_1}{g_2} = \frac{27}{5}\frac{25}{36}g\times\frac{9}{4g} = \frac{25}{16}$$

57. **(a)** $h = \frac{R}{2}$

The acceleration due to gravity at height h is

$$g_h = \frac{GM}{(R+h)^2} = \frac{GM}{\left(R+\frac{R}{2}\right)^2} = \frac{4}{9}\left(\frac{GM}{R^2}\right)$$

$$= \frac{4}{9}g = \frac{4}{9}\times 9.8 = 4.4\ ms^{-2}$$

58. **(d)** Given $g(h) = x$, $h = 10$ km

Since $h << R$

$$g(h) = g\left(1-\frac{2h}{R}\right)$$

Given that, $g(h) = g(d)$

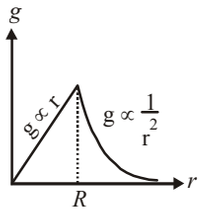
$$\Rightarrow g\left(1-\frac{2h}{R}\right) = g\left(1-\frac{d}{R}\right)$$

$$1-\frac{2\times 10}{R} = 1-\frac{d}{R} \Rightarrow d = 20\,km$$

59. **(b)** Acceleration due to gravity for $r < R$,

$$g = \frac{GMr}{R^3} \text{ or } g \propto r$$

Acceleration due to gravity for $r \geq R$, $g = \frac{GM}{r^2}$ or $g \propto \frac{1}{r^2}$



60. **(b)** Weight on the surface of earth, w = mg = 100 N

Variation in acceleration due to gravity with height,

$$g' = \frac{gR^2}{(r+h)^2} = \frac{gR^2}{\left(R+\frac{R}{4}\right)^2} = \frac{16g}{25} \quad \left[\because h = \frac{R}{4}\right]$$

$\therefore$ Weight on height $\frac{R}{4} = mg' = \frac{16}{25}mg = \frac{16}{25}\times 100 = 64N$

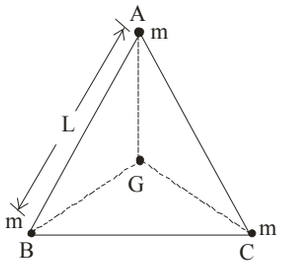
61. **(c)** Gravitational intensity at point P, $I = \frac{GM}{r^2}$ and gravitational potential $V = -\frac{GM}{r}$

$V = I\times r = 6\ N/kg \times 8000\ km = 4.8\times 10^7 \ \frac{Joule}{kg}$

62. **(c)** Gravitational potential on the surface of the shell is V = Gravitational potential due to particle (V_1) + Gravitational potential due to shell itself (V_2)

$$= -\frac{Gm}{R} + \left(\frac{-G3m}{R}\right) = -\frac{4Gm}{R}$$

63. **(c)**



$V_T = V_A + V_B + V_C$

$$\Rightarrow \ V_T = -\frac{Gm}{AG} - \frac{Gm}{BG} - \frac{Gm}{CG}$$

Also, $AG = BG = CG = \frac{2}{3}\times\left(\frac{\sqrt{3}L}{2}\right) = \frac{\sqrt{3}L}{3}$

$$\Rightarrow \ V_T = -Gm\left(\frac{1}{AG}+\frac{1}{BG}+\frac{1}{CG}\right)$$

$$= -Gm\left(\frac{3\times 3}{\sqrt{3}L}\right) = \frac{-3\sqrt{3}Gm}{L}$$

64. **(a)** $E = \frac{-GMm}{R}$, $h = 1.5\ R$

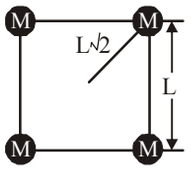
$\therefore \quad R' = (R + h) = (R + 1.5\ R) = 2.5\ R$

$\therefore \quad E' = -\frac{GMm}{R'} = -\frac{GMm}{2.5R} = \frac{1}{2.5}\left(-\frac{GMm}{R}\right)$

$\therefore \quad E' = \frac{E}{2.5} = 0.4E$

65. **(a)** Potential at the centre due to single mass $= \frac{-GM}{L/\sqrt{2}}$

Potential at the centre due to all four masses.



$= -4\frac{GM}{L/\sqrt{2}} = -4\sqrt{2}\frac{GM}{L} = -\sqrt{32} \times \frac{GM}{L}$

66. **(b)** Gravitational potential energy (GPE) on the surface of earth, $E_1 = -\frac{GMm}{R}$

GPE at 3R, $E_2 = -\frac{GMm}{(R + 3R)} = -\frac{GMm}{4R}$

$\therefore$ Change in GPE

$= E_2 - E_1 = -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3GMm}{4R} = \frac{3g\,R^2 m}{4R}$

$\left(\because g = \frac{GM}{R^2}\right) = \frac{3}{4}mg\,R$

67. **(a)** Potential at the given point = Potential at the point due to the shell + Potential due to the particle

$= -\frac{GM}{a} - \frac{2GM}{a} = -\frac{3GM}{a}$

68. **(a)** Energy of the satellite in initial orbit

$E_i = -\frac{1}{2}\frac{GMm}{(2R)}$

Enery of the satellite in final orbit

$E_f = -\frac{1}{2}\frac{GMm}{(3R)}$

Thus required energy $= E_f - E_i$

$= -\frac{1}{2}\left(\frac{GMm}{(3R)}\right) - \left[-\frac{1}{2}\left(\frac{GMm}{(2R)}\right)\right]$

$= -\frac{GMm}{6R} + \frac{GMm}{4R} = \frac{GMm}{12R}$

69. **(b)** Potential energy of the 1 kg mass which is placed at the earth surface $= -\frac{GM}{R}$

Its potential energy at infinite = 0

$\therefore$ Work done = change in potential energy $= \frac{GM}{R}$

70. **(d)** Potential energy of a satellite in an orbit is equal to twice the total energy.

$\therefore \quad P.E = 2E_0$

71. **(b)** As we know, $T.E = K + U = \left(\frac{GMm}{2r}\right) + \left(\frac{-GMm}{r}\right)$

$TE = \frac{-GMm}{2r}$

Hence, total energy is negative and kinetic energy is positive.

72. **(b)** Kinetic energy = – (Total energy)

Negative energy means the satellite is bound to the earth by an attractive force. Hence to remove satellite. The energy required (+ K.E.)

73. **(d)**

74. **(b)** Kinetic energy = – (Total energy)

Negative energy means the satellite is bound to the earth by an attractive force. Hence to remove satellite. The energy required (+ K.E.)

75. **(a)** K.E. $= \frac{GMm}{2R}$

76. **(a)** Velocity of satellite at a distance R from the centre of the earth,

$v = \sqrt{\frac{GM}{R}}$

Now, velocity at an altitude $\frac{R}{2}$ will be

$v' = \sqrt{\frac{GM}{R + \frac{R}{2}}} = \sqrt{\frac{2}{3}\frac{GM}{R}} = \sqrt{\frac{2}{3}}v$

77. **(b)**

78. **(c)** KE of satellite, $K = \frac{1}{2}mv_o^2$

$= \frac{1}{2}m\left(\sqrt{\frac{GM_E}{(R_E + h)}}\right)^2 = \frac{1}{2}\frac{GmM_E}{(R_E + h)}$

79. **(d)** Let m = mass of satellite,

M = mass of earth

Potential energy of satellite

$$P.E = -\frac{GM\,m}{R}$$

Here, G = gravitational constant

Kinetic energy of satellite

$$K.E = +\frac{GM\,m}{2R}$$

Total energy of satellite

$$T.E = -\frac{GM\,m}{2R}$$

Speed of satellite $v = \sqrt{\frac{GM}{R}}$

It is clear that only speed of satellite does not depend on the mass of satellite. So, speed is same for both satellite.

80. (a)

81. (d) The line period of satellite is

$$T = 2\pi\sqrt{\frac{R^3}{CM}} \Rightarrow R \propto T^{\frac{2}{3}}$$

$\therefore$ Kinetic energy,

$$K = \frac{CM_m}{R} \Rightarrow k \propto \frac{1}{R} \propto \frac{1}{T^{\frac{2}{3}}} \propto T^{-\frac{2}{3}}$$

82. (c) $v_i = \frac{1}{2}v_e = \frac{1}{2}\sqrt{\frac{2GM}{R}}, U_f = 0$

Applying conservation of mechanical energy,

$K_i + V_i = K_f + U_f$

$$\Rightarrow \frac{1}{2}mv_i^2 + \frac{-GM_m}{R} = 0 + \frac{-GM_m}{R+h}$$

$$\Rightarrow \frac{1}{2}m\left(\frac{1}{2}\frac{2GM_m}{R}\right) - \frac{GM_m}{R} = -\frac{GM_m}{R+h}$$

$$\Rightarrow \frac{3}{4R} = \frac{1}{R+h} \qquad \therefore\ h = \frac{R}{3}$$

83. (b)

84. (a) Escape speed is given by

$$v_{escape} = \sqrt{\frac{2GM}{R}}$$

For planet $\Rightarrow v_{Planet} = \sqrt{\frac{2GM_p}{R_p}} = v$

For moon

$$v_{Moon} = \sqrt{\frac{2GM_m}{R_m}} = \sqrt{\frac{2GM_p \times 16}{144\,R_p}} = \frac{1}{3}\sqrt{\frac{2GM_p}{R_p}}$$

$$\therefore\ v_{Moon} = \frac{v_{Planet}}{3} = \frac{v}{3}$$

85. (b) Using law of conservation of energy

$$KE = \frac{1}{2}mv_e^2 = \frac{GMm}{R_E} \qquad \left[\text{Using } g = \frac{GM}{R_E^2}\right]$$

$KE = mgR_E$

86. (d)

87. (c) $v = R\sqrt{\frac{8}{3}\pi pG} \Rightarrow \frac{v_p}{v_e} = \frac{R_p}{R_e}\sqrt{\frac{\rho_p}{\rho_e}} = 2\sqrt{\frac{1}{4}} = 1$

88. (a)

89. (b) Escape velocity $V = \sqrt{\frac{2GM}{R}}$

$$\therefore\ V_p = \sqrt{\frac{2GM_p}{R_p}} \text{ and } V_E = \sqrt{\frac{2GM_E}{R_E}}$$

$$\therefore\ \frac{V_P}{V_E} = \frac{\sqrt{\frac{2GM_P}{R_P}}}{\sqrt{\frac{2GM_E}{R_E}}} = \sqrt{\frac{R_E}{R_P} \times \frac{M_P}{M_E}}$$

$$\Rightarrow V_P = \sqrt{\frac{1}{4} \times 9} \times V_E = \frac{3}{2}V_E$$

$$\therefore\ V_P = \frac{3}{2} \times 11.2\ \text{km/s} = 16.8\ \text{km/s}$$

90. (c) Orbital speed, $v = \sqrt{\frac{GM}{r}}$

$$v \propto \frac{1}{\sqrt{r}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{3r}{r}} = \sqrt{3} : 1$$

91. (a) Since, escape velocity $\left(V_e = \sqrt{2gR_e}\right)$ is independent of angle of projection, so it will not change.

92. (b) The orbital speed of satellite is independent of mass of satellite, so the ball will behave as a satellite and will continue to move with the same speed in the original orbit.

93. (b) Escape velocity is the minimum velocity with which a body is projected to escape from earth gravitation field

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R_e} \Rightarrow v_e = \sqrt{\frac{2GM}{R_e}}$$

94. (b) Due to inertia of motion it will move tangentially to the original orbit with same velocity.

95. **(a)** Escape velocity, $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}} \Rightarrow v_e \propto m^0$

In this expression the mass of the body (m) is not present hence escape velocity is independent of the mass.

96. **(a)** $v_e = \sqrt{\frac{2GM}{R}} \quad \therefore v_e \propto \sqrt{\frac{M}{R}}$

If mass and radius of the planet are three times than that of earth then escape velocity will be same.

97. **(a)** $v_0 = \sqrt{gR}$ i.e., v_o independent of mass of the satellite

98. **(d)** $v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_0 = \sqrt{\frac{GM}{R}}$

$v_e = \sqrt{2}v_0$

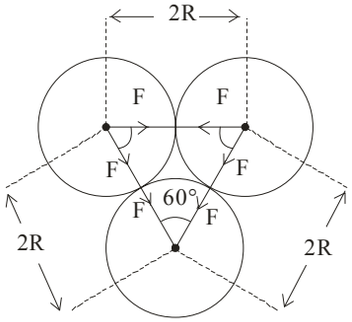
99. **(d)** The escape velocity for an object from the surface of earth is $\sqrt{2gR}$ or $\sqrt{2GM/R}$.

100. **(c)** The velocity of the spoon will be equal to the orbital velocity when dropped out of the space-ship.

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(d)**

2. **(a)**



$$F = G.\frac{M.M}{(2R)^2} = \frac{GM^2}{4R^2}$$

$$F_{net} = \sqrt{F^2 + F^2 + 2F^2\cos 60^\circ} = \sqrt{3}F$$

$$\therefore F_{net} = \frac{\sqrt{3}}{4}\frac{GM^2}{R^2}$$

3. **(d)** As per the question,

$$\text{force} = \frac{GMm}{r^{3/2}} = m\omega^2 r \Rightarrow \frac{GMm}{r^{3/2}} = \frac{4\pi^2 mr}{T^2}$$

$$\Rightarrow T^2 = \left(\frac{4\pi^2}{GM}\right).r^{5/2} \Rightarrow T^2 \propto r^{5/2}$$

4. **(d)** Maximum force, $F_{max} = F_1 + F_2$

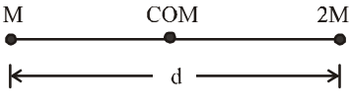
$$= \frac{CM_m}{(2R_E)^2} + \frac{1.5\,GM_m}{(3R_E)^2} = \frac{5}{12}\frac{GM_m}{R_E^2}$$

Minimum force, $F_{min} = F_1 - F_2$

$$= \frac{GM_m}{(2R_E)^2} - \frac{1.5CM_m}{(3R_E)^2} = \frac{1}{12}.\frac{GM_m}{R_E^2}$$

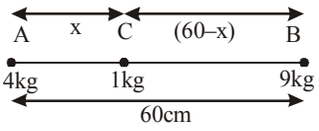
$$\therefore \frac{F_{min}}{F_{max}} = \frac{CM_m}{12R_E^2} \times \frac{12R_E^2}{5GM_m} = \frac{1}{5} = 1:5$$

5. **(d)** $F_g = M\omega^2 d_1$



$$\Rightarrow \frac{G(M)(2M)}{d^2} = M\omega^2\left(\frac{2M}{3M}\right)d \quad \therefore \quad \omega = \sqrt{\frac{3GM}{d^3}}$$

6. **(d)**



$$\therefore \quad G\frac{4\times 1}{x^2} = G\frac{9\times 1}{(60-x)^2}$$

or $$\frac{2}{3} = \frac{x}{(60-x)} \Rightarrow x = 24\text{ cm}$$

7. **(c)** $$\%\text{ change} = \frac{\left[\frac{G(2m_1)(2m_2)}{(3r)^2} - \frac{Gm_1m_2}{r^2}\right]}{\frac{Gm_1m_2}{r^2}} \times 100$$

$$= \frac{\frac{4}{9} - 1}{1} \times 100 = -56\%,$$

–ve sign indicates that force of attraction decreases.

8. **(a)** **9.** **(d)**

10. **(c)** It is applicable to both small and big bodies.

11. **(b)** Various regions of spherical shell attract the point mass inside it in various directions. These forces cancel each other completely. Therefore the gravitational force on the particle is zero.

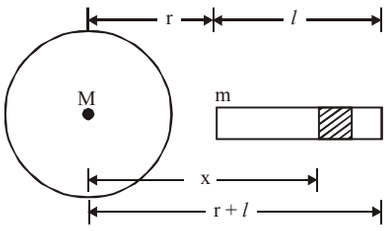
12. **(a)** The gravitational force of attraction on a body of mass m is given by

$$F = \frac{GMm}{R^2} \text{ Therefore, } F \propto \frac{1}{R^2}$$

The radius of earth is maximum at equator, therefore, gravitational force of attraction is least at equator.

13. **(b)**

14. **(a)** The force of attraction between sphere and shaded position $dF = GM\dfrac{\left(\dfrac{m}{l}dx\right)}{x^2}$



$$F = \int_r^{r+l} \frac{GMm}{lx^2}dx = \frac{GMm}{l}\int_r^{r+l}\frac{1}{x^2}dx$$

$$= \frac{GMm}{l}\int_r^{r+l} x^{-2}dx = \frac{GMm}{l}\left[\frac{x^{-2+1}}{-2+1}\right]_r^{r+l}$$

$$= -\frac{GMm}{l}\left[x^{-1}\right]_r^{r+l} = -\frac{GMm}{l}\left[\frac{1}{x}\right]_r^{r+l} = \frac{GMm}{r(r+l)}$$

15. **(c)** As m_2 attracts m_1 towards itself, $\therefore$ force is along r^3.

16. **(a)** k represents gravitational constant which depends only on the system of units.

17. **(c)** $F = \dfrac{G\times m\times m}{(2R)^2} = \dfrac{G\times\left(\frac{4}{3}\pi R^3\rho\right)^2}{4R^2} = \dfrac{4}{9}\pi^2\rho^2R^4$

18. **(a)** Same force acts on both masses

Hence $a \propto \dfrac{1}{m}$ $(F = ma)$

In absence of external force (remember mutual gravitational force is an internal force for the system) total energy remains constant.

19. **(d)** $k\,F \propto \dfrac{1}{r^2}$. If r becomes double then F reduces to $\dfrac{F}{4}$.

20. **(a)**

21. **(c)** By Kepler's law of area, $\dfrac{dA}{dt} = \text{constant}$

$$\therefore \quad \frac{A_1}{t_1} = \frac{A_2}{t_2} = \frac{A_3}{t_3} \Rightarrow \frac{A_1}{2} = \frac{A_2}{3} = \frac{A_3}{6}$$

22. **(b)**

23. **(a)** $\dfrac{T^2}{a^3} = \dfrac{(84)^2}{\left(287\times10^{10}\right)^3}$

$= 2.98\times10^{-34}\ yr^2\ m^{-3} \approx 3.00\times10^{-34}\ yr^2\ m^{-3}$

24. **(c)** From Kepler's law of periods,

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3} = \text{constant}$$

or $\left(\dfrac{T_1}{T_2}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^3$

25. **(b)** If time period for the mars and the earth are T_m and T_e, respectively. Then, according to Kepler's third law,

$$\frac{T_m^2}{T_e^2} = \frac{R_{ms}^3}{R_{es}^3} \Rightarrow T_m = \left(\frac{R_{ms}}{R_{es}}\right)^{3/2}.T_e$$

$\therefore \quad T_m = (1.52)^{3/2}\times 365$ days

26. **(c)** According to law of conservation of angular momentum $mv_1d_1 = mv_2d_2$

$mv_1d_1 = mv_2d_2 \quad \therefore \quad v_2 = \dfrac{d_1v_1}{d_2}$

27. **(b)** By Kepler's period law, $T^2 \propto r^3$

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

$T_1 = 200$ days, $r_2 = \dfrac{r}{4}$

$$\frac{(200)^2}{r^3} = \frac{T_2^2}{\left(\frac{r}{4}\right)^3} \Rightarrow T_2 = \frac{200}{4\times2} = 25 \text{ days}$$

28. **(c)** Acceleration due to gravity varies with depth as,

$$g_d = g\left(\frac{R_e - d}{R_e}\right) = g\left(1 - \frac{d}{R_e}\right) = 9.8\left(1 - \frac{1600}{6400}\right)$$

$= 9.8\times0.75 = 7.35\ m\,s^{-2}$

29. **(a)** The mass of the body on the surface of the moon is also M because it doesn't depend on the acceleration due to gravity and remains the same at all places.

30. **(d)** $g = \dfrac{4}{3}\pi\rho GR \quad \therefore \dfrac{g_1}{g_2} = \dfrac{R_1\rho_1}{R_2\rho_2}$

31. **(b)** $W_{eff} = \dfrac{mg}{\left(1+\frac{h}{R}\right)^2} = \dfrac{mg}{\left(1+\frac{9R}{R}\right)^2} = \dfrac{mg}{100} = \dfrac{W}{100}$

32. **(a)** We know that,

$$g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2} = \frac{4}{3}\pi GR\rho$$

$$\frac{g'}{g} = \frac{R'}{R} = \frac{0.2R}{R} = 0.2 \quad \therefore \quad g' = 0.2g$$

33. (d)

34. (c) Acceleration due to gravity g varies with dpth as

$$g' = g\left(1 - \frac{d}{R_E}\right)$$

35. (b) 36. (d) 37. (a)

38. (a) As we know, $g' = g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$

$\therefore \frac{2h}{R} = \frac{d}{R} \quad \therefore h = \frac{d}{2}$ or $d = 2h$.

$\therefore d = 2 \times 50 = 100$ km.

39. (b) Acceleration due to gravity (g) is given by

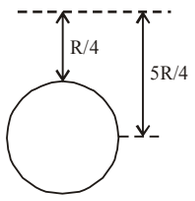
$$g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$$

As one moves from the equator to the poles, the radius of the earth decreases, hence g increases.

40. (b) $g = \frac{GM_E}{R_E^2}$

41. (a) Effective value of 'g' at 'h' height is given as

$$g_{eff} = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$



$$\Rightarrow \quad g_{eff} = \frac{g}{\left(1 + \frac{1}{4}\right)^2} = \frac{16g}{25}$$

$$\text{Change in 'g'} = \frac{g_{eff} - g}{g} \times 100 = \frac{\frac{16}{25} - 1}{1} \times 100$$

$$= \frac{-9}{25} \times 100 = -36\%$$

Hence % decrease in the weight = 36%

42. (a) The weight (= mg) of the body at the centre of the earth is zero, because the value of g at centre is zero.

43. (a)

44. (b) At earth surface, *P.E.* of system is $-\frac{GmM}{R}$

At a distance R from the earth's surface, *P.E* .of system is $-\frac{GmM}{2R}$

$$\therefore \Delta U = \frac{-GmM}{2R} + \frac{GmM}{R}; \quad \Delta U = \frac{GmM}{2R}$$

Now $\frac{GM}{R^2} = g; \ \therefore \frac{GM}{R} = gR$

$$\therefore \Delta U = \frac{1}{2}mgR$$

45. (c) $V_{in} = \frac{-GM}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right]$,

$$V_{surface} = \frac{-GM}{R}, V_{out} = \frac{-GM}{r}$$

46. (c) P.E. at 2R from the surface of the earth = P.E. at 3R from the centre

Work done in taking a body from centre up to 3R

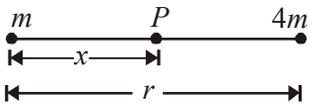
$$= -\frac{GMm}{3R}$$

Work done up to the surface $= -\frac{GMm}{R}$

$$\therefore \quad \text{Work done} = \frac{GMm}{R}\left(-1 + \frac{1}{3}\right) = -\frac{2}{3}mgR$$

47. (c) Let the gravitational field at P, distant x from mass m, be zero.

$$\therefore \quad \frac{Gm}{x^2} = \frac{4Gm}{(r-x)^2}$$



$$\Rightarrow \frac{1}{x} = \frac{2}{r-x}$$

$$\therefore \quad r - x = 2x$$

$$x = \frac{r}{3}$$

Gravitational potential at P,

$$V = -\frac{Gm}{\frac{r}{3}} - \frac{4Gm}{\frac{2r}{3}} = -\frac{9Gm}{r}$$

48. (d)

49. (c) Acceleration due to gravity g varies with dpth as

$$g' = g\left(1 - \frac{d}{R_E}\right)$$

50. (a) $v_e = 11.1$ km/h

Also, $v_e = \sqrt{2gR_E}$ or $v_e = \sqrt{\frac{2GM_E}{R_E}}$

$\therefore$ Escape velocity on the surface of moon,

$$v_m = \sqrt{\frac{2GM_m}{R_m}} = \sqrt{\frac{2G\frac{M_E}{81}}{\frac{R_E}{4}}}$$

$$= \frac{2}{9}v_e = \frac{2}{9} \times 11.1 = 2.46 \text{ km/h}$$

51. **(a)** Binding energy = – kinetic energy
And if this amount of energy (E_k) given to satellite then it will escape into outer space.

52. **(c)** $$\frac{(v_e)_p}{(v_e)_e} = \frac{\sqrt{\frac{2GM_p}{R_p}}}{\sqrt{\frac{2GM_e}{R_e}}} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$$

$$= \sqrt{\frac{10M_e}{M_e} \times \frac{R_e}{R_e/10}} = 10$$

$$\therefore (v_e)_p = 10 \times (v_e)_e = 10 \times 11 = 110\ km/s$$

53. **(c)** According to Kepler's law of period $T^2 \propto R^3$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} = \frac{(6R)^3}{(3R)^3} = 8$$

$$\frac{24 \times 24}{T_2^2} = 8$$

$$T_2^2 = \frac{24 \times 24}{8} = 72 = 36 \times 2$$

$$T_2 = 6\sqrt{2}$$

54. **(b)** $v_e = 11.2$ km/s

$v_e = \sqrt{2gR}$

For the planet $g' = 2g$, $d' = 2d$

$$\therefore\ R' = \frac{d'}{2} = \frac{2d}{2} = 2R$$

$$v_e' = \sqrt{2g'R'} = \sqrt{2 \times 2g \times 2R}$$

$$= 2\sqrt{2gR} = 2 \times 11.2 = 22.4 \text{ km/s.}$$

55. **(c)** Escape velocity of the planet is

$$v_P = \sqrt{\frac{2GM_P}{R_P}}$$

As, $v_P = 3v_e$ and $R_P = 4R_e$

v_e and R_e are escape velocity and radius of earth respectively

$$\therefore \sqrt{\frac{2GM_P}{4R_e}} = 3\sqrt{\frac{2GM_e}{R_e}}$$

Squaring both sides, $\frac{M_P}{4R_e} = \frac{9M_e}{R_e}$

$$M_P = 36\,M_e = 36 \times 6 \times 10^{24} \text{ kg}$$
$$= 216 \times 10^{24} \text{ kg} = 2.16 \times 10^{26} \text{ kg}$$

56. **(a)** Time period of satellite is given by :

$$= \frac{\text{circumference of an orbit}}{\text{Velocity in orbit}} = \frac{2\pi R}{v_0}$$

$$= \frac{2\pi R}{\sqrt{\frac{GM_m}{R}}} = \frac{2\pi R^{3/2}}{\sqrt{GM_m}}$$

Squaring both sides, we get

$$T^2 = \frac{4\pi^2 R^3}{GM_m}$$

57. **(b)** Kinetic energy of the satellite moving in an orbit around the earth

$$\text{K.E.} = \frac{1}{2}mV^2 = \frac{GM_E m}{2(R_E + h)}$$

And potential energy $\text{P.E.} = \frac{GM_E m}{(R_E + h)}$

$$\therefore \quad \frac{\text{K.E.}}{\text{P.E.}} = \frac{\frac{GM_E m}{2(R_E + h)}}{\frac{GM_E m}{(R_E + h)}} = \frac{1}{2}$$

58. **(d)** Orbital radius of satellites $r_1 = R + R = 2R$
$r_2 = R + 7R = 8R$

$$U_1 = \frac{-GMm}{r_1} \text{ and } U_2 = \frac{-GMm}{r_2}$$

$$K_1 = \frac{GMm}{2r_1} \text{ and } K_2 = \frac{GMm}{2r_2}$$

$$E_1 = \frac{GMm}{2r_1} \text{ and } E_2 = \frac{GMm}{2r_1}$$

$$\therefore \frac{U_1}{U_2} = \frac{K_1}{K_2} = \frac{E_1}{E_2} = 4$$

59. **(b)** $v_e\sqrt{2}v_0 = 1.414v_0$

Fractional increase in orbital velocity $\left(\frac{\Delta v}{v}\right)$

$$= \frac{v_e - v_0}{v_0} = 0.414$$

$\therefore$ Percentage increase = 41.4%

60. **(c)** $v_0 = \frac{v_e}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$ km/s

61. **(d)** Orbital velocity $v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}}$ and $v_0 = r\omega$

This gives $r^3 = \frac{R^2 g}{\omega^2}$

62. **(c)** $\upsilon \propto \frac{1}{\sqrt{r}}$, If $r = R$ then $\upsilon = V_0$

If $r = R + h = R + 3R = 4R$ then $\upsilon = \frac{V_0}{2} = 0.5V_0$

63. **(b)** 6 R from the surface of earth and 7 R from the centre.

64. **(a)**

Exercise 3 : PREVIOUS YEARS MCQs

1. **(a)** $g_{eff} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$\Rightarrow \frac{g}{2} = \frac{g}{\left(1+\frac{h}{R}\right)^2} \Rightarrow \sqrt{2} = 1 + \frac{h}{R} \Rightarrow \frac{h}{R} = \sqrt{2} - 1$

$\Rightarrow h = (\sqrt{2} - 1) \times 6400 \times 10^3 \text{ m} = 2.6 \times 10^6 \text{ m}$

2. **(c)** Radius of circle, r = a

Centrifugal force, $F = m\omega^2 r$

$\Rightarrow \frac{Gmm}{(2a)^2} = m\omega^2 a \qquad \left(\because F = \frac{G M_1 M_2}{d}\right)$

Here $M_1 = m$; $M_2 = m$ and $d = 2a$

$\Rightarrow$ angular speed, $\omega = \sqrt{\frac{Gm}{4a^3}}$

3. **(b)** Escape velocity $V = \sqrt{\frac{2GM}{R}}$

$\therefore V_p = \sqrt{\frac{2GM_p}{R_p}}$ and $V_E = \sqrt{\frac{2GM_E}{R_E}}$

$\therefore \frac{V_P}{V_E} = \frac{\sqrt{\frac{2GM_P}{R_P}}}{\sqrt{\frac{2GM_E}{R_E}}} = \sqrt{\frac{R_E}{R_P} \times \frac{M_P}{M_E}}$

$\Rightarrow V_P = \sqrt{\frac{1}{4} \times 9} \times V_E = \frac{3}{2} V_E$

$\therefore V_P = \frac{3}{2} \times 11.2 \text{ km/s} = 16.8 \text{ km/s}$

4. **(c)** Mass of the satellite = m and height of satellite from earth (h) = 6.4×10^6 m.

We know that gravitational potential energy of the satellite at height

$h = -\frac{GM_e m}{R_e + h} = -\frac{gR_e^2 m}{2R_e}$

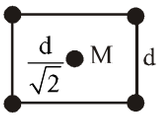
$= -\frac{gR_e m}{2} = -0.5 mgR_e$

(where, $GM_e = gR_e^2$ and $h = R_e$)

5. **(c)** By kepler's law $T^2 \propto r^3$

$\Rightarrow \left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3 \Rightarrow 2^2 = \left(\frac{r_A}{r_B}\right)^3 \Rightarrow r_A^3 = 4r_B^3$

6. **(a)**



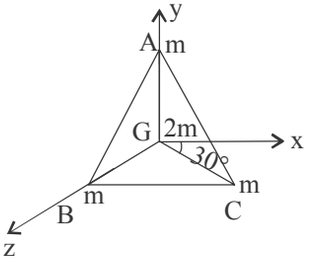
$U_{net} = -\frac{Gmm}{d} \times 4 - \frac{GMm}{d} \times 4\sqrt{2}$

$-\frac{Gmm}{\sqrt{2}d} \times 2 = -\frac{Gm}{d}\left[(4 + \sqrt{2})\,m + 4\sqrt{2}M\right]$

7. **(c)** $F = \frac{GM(3m)}{d^2} = \frac{3GMm}{d^2}$.

8. **(c)** It is applicable to both small & big bodies.

9. **(c)**



$F_{GA} = \frac{Gm(2\,m)}{1}\hat{j}$

$F_{GB} = \frac{Gm(2m)}{1}(-\hat{i}\cos 30° - \hat{j}\sin 30°)$

$F_{GC} = \frac{Gm(2m)}{1}(\hat{i}\cos 30° - \hat{j}\sin 30°)$

$\therefore$ Resultant force on (2m) is F_R

$= F_{GA} + F_{GB} + F_{GC}$

$= 2Gm^2\hat{j} + 2Gm^2\hat{i}(-\cancel{\cos 30°} + \cancel{\cos 30°})$

$+ 2Gm^2\,\hat{j}(-\sin 30° - \sin 30°)$

$= 2Gm^2\hat{j}.2Gm^2\hat{j}\left(-\cancel{2} \times \frac{1}{\cancel{2}}\right)$

$= 2Gm^2\hat{j} - 2Gm^2\hat{j} = 0$.

10. (a) Potential at the given point = Potential at the point due to the shell + Potential due to the particle

$$= -\frac{GM}{a} - \frac{2GM}{a} = -\frac{3GM}{a}$$

11. (c)

12. (c)

13. (b) Using conservation of energy

Total mechanical energy at surface = total mechanical energy at height h

Using this, we have

$$\frac{1}{R} - \frac{1}{r} = \frac{1}{4R}$$

$$\therefore \quad \frac{1}{r} = \frac{1}{R} - \frac{1}{4R}$$

$$= \frac{1}{R} \cdot \frac{3}{4}$$

$$\therefore \quad r = 4/3R$$

$$\therefore \quad h = R/3$$

14. (d) According to question, speed of satellite in its orbit.

$$v_c = \frac{1}{4} v_e .$$

(v_e = escape speed)

$$\sqrt{\frac{GM}{R+h}} = \frac{1}{4}\sqrt{\frac{2GM}{(R)}} \; ; \sqrt{\frac{GM}{(R+h)}} = \frac{1}{16} \times \frac{2Gm}{(R)}$$

$R + h = 8(R)$

$R + h = 8R$

$7R = h$

15. (a) Binding energy on the surface of the earth

$$E_1 = \frac{GMm}{R} \qquad \text{..... (i)}$$

Binding energy of revolving satellite at a height h from the earth surface,

$$E_2 = \frac{GMm}{2(R+h)} \qquad \text{.... (ii)}$$

From eqs. (i) and (ii),

$$\frac{E_1}{E_2} = \frac{2(R+h)}{R}$$

16. (c) Acceleration due to gravity varies with depth d as,

$$g' = g\left(1 - \frac{d}{R}\right)$$

And according to question, $g' = \frac{g}{n}$

$$g' = \frac{g}{n} = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{1}{n} = 1 - \frac{d}{R}$$

or, $\frac{d}{R} = 1 - \frac{1}{n} = \frac{n-1}{n} \therefore d = R\left(\frac{n-1}{n}\right)$

17. (a) The value of acceleration due to gravity g at a height h is given by

$$g' = g\left(\frac{R_e}{R_e + h}\right)^2$$

Here, Re = radius of earth

g = acceleration due to gravity at earth surface.

$$g' = \frac{g}{4} \text{ and } R_e = R$$

So, $\frac{g}{4} = g\left(\frac{R}{R+h}\right)^2$

$$\frac{1}{2} = \frac{R}{R+h}$$

$$\Rightarrow \quad 2R = R + h \quad \Rightarrow \quad R = h$$

4 Thermal Properties of Matter

Exercise 1: WARM-UP
Topic-wise MCQs

1. **(a)** Heat energy is associated with the kinetic energy of random motion of molecules.

2. **(a)** Temperature on celsius scale (T_C), kelvin scale (T_K) and Fahrenheit scale (T_F) are related as

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_K - 273.15}{100}$$

3. **(a)** $V + \Delta V = (L + \Delta L)^3 = (L + \alpha L \Delta T)^3$

$= L^3 + (1 + 3\alpha\Delta T + 3\alpha^2\Delta T^2 + \alpha^3\Delta T^3)$

$\Rightarrow$ α^2 and α^3 terms are neglected.

$\therefore$ $V(1 + \gamma\Delta T) = V(1 + 3\alpha\Delta T)$

$1 + \gamma\Delta T = 1 + 3\alpha\Delta T$

$\therefore$ $\gamma = 3\alpha$.

4. **(c)** Temperature on Fahrenheight $F = \frac{9}{5}C + 32$

Given $F = 2C$ or $C = F/2$

$$\therefore \quad F = \frac{9}{5} \times \frac{F}{2} + 32 \Rightarrow F - 0.9F = 32 \therefore F = 320°F$$

5. **(b)** $$\frac{\text{Reading on new scale} - \text{LFP}}{\text{UFP} - \text{LFP}} = \frac{C - 0}{100}$$

$$\frac{z°X - 20°X}{110°X - 20°X} = \frac{40 - 0}{100} \Rightarrow z°X - 20°X$$

$$= \frac{40}{100} \times 90°X = 36°X$$

$\therefore$ Reading on new scale.

$z°X = 36°X + 20°X = 56°X$

6. **(c)** Given, $\Delta T = T_b - T_f = 100$

Temperature of liquid, $T = \left(\frac{3}{5}\right)$th of distance between lower and upper fixed point

$$\Rightarrow \quad T = \frac{3}{5} \times 100 = 60°C$$

On Kelvin scale, $T_K = 273.15 + 60 = 333.15$ K

7. **(d)** For a thermometer

$$\frac{\text{Reading on any scale} - \text{Lower fixed point}}{\text{Upper fixed point} - \text{Lower fixed point}} = \text{constant}$$

i.e. $$\frac{X - LFP}{UFP - LFP} = \frac{C - 0}{100 - 0}$$

$$\frac{C - 0}{100 - 0} = \frac{64 - 5}{95 - 5} = \frac{59}{90} = 0.655 \Rightarrow C = 65.5\,°C$$

8. **(c)** $$\frac{X - (-125)}{500} = \frac{Y - (-70)}{40}$$

For $Y = 50$

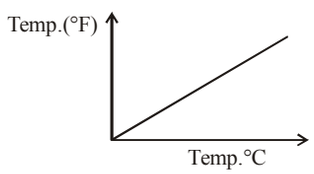
$X = 1375.0°X$

9. **(c)** $$\frac{T_F - 32}{180} = \frac{T_K - 273}{100}$$

$$= \frac{\theta - 32}{180} = \frac{\theta - 273}{100} \Rightarrow 5\theta - 32 \times 5 = 9\theta - 273 \times 9$$

$= \theta = 574.25$

10. **(c)**



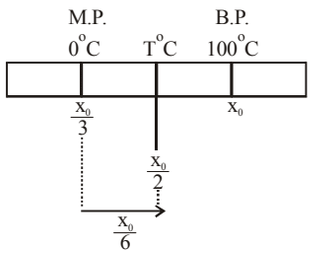
11. **(a)** Let required temperature = T°C

$$\Rightarrow T°C = \frac{x_0}{2} - \frac{x_0}{3} = \frac{x_0}{6}$$

$$\& \left(x_0 - \frac{x_0}{3}\right) = (100 - 0°C)$$

$$\Rightarrow \frac{2x_0}{3} = 100 \Rightarrow x_0 = \frac{300}{2}$$

$$\Rightarrow T°C = \frac{x_0}{6} = \frac{150}{6} = 25°C$$



12. **(b)** For different temperature scales, we have

$$\frac{x - \text{L.F.P}}{\text{U.F.P.} - \text{L.F.P.}} = \text{constant}$$

Where L.F.P $\Rightarrow$ Lower Fixed point

U.H.F. $\Rightarrow$ Upper fixed point

where x is the measurement at that scale. Here, if C and W be the measurements on Celsius and W scale then,

$$\frac{C-0}{100-0}=\frac{W-39}{239-39} \quad (\therefore C=39°C)$$

$$\Rightarrow W=\frac{39\times 200}{100}+39=78+39=117° \text{ W}$$

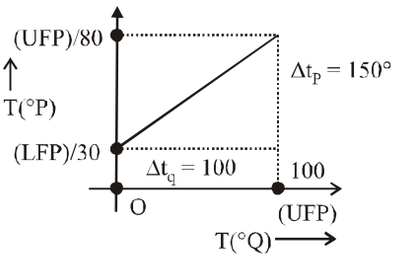
13. **(c)** Mercury thermometer is based on the principle of change of volume with rise of temperature and can measure temperatures ranging from –30°C to 357°C.

14. **(a)** Let the readings of two thermometers coincide at $C=F=x$

As $\frac{C}{5}=\frac{F-32}{9}$ $\quad\therefore \frac{x}{5}=\frac{x-32}{9}$

or $9x=5x-160$ $\quad x=-40°C$

15. **(b)**



As, $\frac{\text{Reading}-\text{LFP}}{\text{UFP}-\text{LFP}}=\text{Constant}$

$$\frac{t_P-30}{180-30}=\frac{t_Q-0}{100-0}\Rightarrow\frac{t_P-30}{150}=\frac{t_Q}{100}$$

16. **(b)** Change in L_A = change in L_B

i.e., $\Delta L_A\ L_B \Rightarrow \alpha_A\Delta TL_A=\alpha_B\Delta TL_B$

or $\alpha_A L_A=\alpha_B L_B$

17. **(c)** Due to the Expansion of the rod due to heating it, the rod develops a strain, trying to resist the change in length. Thus the strain tends to reduce the length of the rod.

$$\text{Strain} \Rightarrow \frac{\Delta L}{L}=\frac{L\alpha\Delta\theta}{L}\Rightarrow\alpha\Delta\theta$$

$\text{Strain} \Rightarrow -12\times10^{-6}\times50 \Rightarrow -6\times10^{-4}$

18. **(b)** Change in temperature of each rod,

$\Delta T=250-40=210°C$

Clearly, change in length of the brass bar

$\Delta L_b=\alpha_0 L\Delta T=(2.0\times10–5)\times50\times210=0.21$ cm

and change in length of steel rod

$\Delta L_s=\alpha_s L\Delta T=(1.2\times10^{-5})\times50\times210=0.126$ cm

Since the ends of the rod are free to expand, change in the length of the combined rod

$\Delta L=\Delta L_b+\Delta L_s=0.21+0.13=0.34$ cm

19. **(b)** As the wire is not free to contract, the thermal stress is developed at the wire.

The change in temperature,

$\Delta T=27°C-(-39°C)=66°C$

Let ΔL be the change in length of the wire.

$\Delta L=\alpha L\Delta T=(2\times10^{-5})\times1.8\times66=2.376\times10^{-3}$m

As $Y=\frac{FL}{A\Delta L}=\frac{FL}{(\pi D^2/4)\Delta L}=\frac{4FL}{\pi D^2\Delta L}$

F (tension in the wire) $=\frac{Y\pi D^2\Delta L}{4L}$

$$=\frac{(0.91\times10^{11})\times3.142\times(2\times10^{-3})^2\times(2.376\times10^{-3})}{4\times1.8}$$

$=3.8\times10^2$ N

20. **(a)** The steel tape gives correct reading only at the temperature 27°C at which it has been calibrated. At any other temperature 45°C the scale will expand and give less reading than the true value.

Hence length of the steel rod at 27°C, i.e., L = 63 cm

Let ΔL be the increase in the length of the steel tape when temperature rises from 27°C to 45°C, i.e., $\Delta T=45°C-27°C=180°C=18$ K

Clearly, $\Delta L=\alpha L\Delta T$

$=(1.20\times10^{-5}/K)\times(63\text{ cm})\times18\text{ K}=0.0136$ cm

Actual length of the rod at 45°C

$=63\text{ cm}+0.0136\text{ cm}=63.0136$ cm

21. **(a)** Given, decreases in temperature

$(\Delta t)=57-37=20°C$

Coefficient of cubical expansion of copper

$\gamma_{Cu}=3\alpha_{Cu}=5.1\times10^{-5}/°C$

Let initial volume of the cavity be V and its volume increases by ΔV due to increase in temperature.

$$\therefore\quad \Delta V=\gamma V\Delta t \quad\Rightarrow\quad \frac{\Delta V}{V}=\gamma\Delta t$$

Thermal stress produced = B × Volumetric strain

$$=B\times\frac{\Delta V}{V}=B\times\gamma\Delta t$$

$=140\times10^9\times(1.5\times10^{-5}\times20)=42\times10^6$N/m^2

This is about 10^3 times of atmospheric pressure.

22. **(c)** $PV=nRT$

$PdV=nRdT$

$$\gamma=\frac{1}{V}\frac{dV}{dT}\text{ and }\frac{dV}{dT}=\frac{nR}{P}\cdot\gamma=\frac{1}{T}$$

For given temperature T_0, $\gamma=\frac{1}{T_0}$

23. **(a)** Let T be the rise in temperature from correct value, then

$T=35°C-20°C=15°C$

Using the relation,

Measured length = Correct length × $(1+\alpha_1 T)$

$\Rightarrow\quad 26=l(1+1.2\times10^{-5}\times15)$

$$\Rightarrow\quad l=\frac{26}{1.00018}=25.9953\text{ m}$$

24. **(c)** Since, the elongation is same for both the rods,
$(\Delta l)_{Cu} = (\Delta l)_{Al}$
$\Rightarrow L_{Cu}\alpha_{Cu}\Delta T = L_{Al}\alpha_{Al}\Delta T$ ...(i)
Substituting the given values,

$$L_{Al} = \frac{88 \times 1.7 \times 10^{-5}}{2.2 \times 10^{-5}} = 68 \text{ cm}$$

25. **(c)** $\Delta T = \frac{\Delta L}{L\alpha} = \frac{1}{100 \times 0.00002} = 500°C$

26. **(c)** $\Delta l_1 = \Delta l_2$ or $l_1\alpha_a t = l_2\alpha_s t$

$$\therefore \quad \frac{l_1}{l_2} = \frac{\alpha_s}{\alpha_a} \text{ or } \frac{l_1}{l_1 + l_2} = \frac{\alpha_s}{\alpha_a + \alpha_s}$$

27. **(c)**

28. **(a)** Thermal strain in the wire,

$$\frac{\Delta l}{l} = \alpha \Delta T$$

The corresponding stress is,
$Y \times \text{strain} = Y\alpha\Delta T$
The tension F developed in the wire is
$F = \text{Stress} \times \text{Cross-sectional area} = Y\alpha\Delta T \times A = YA\alpha\Delta T$
Here, $A = \pi r^2 = 3.14\,(1.0 \times 10^{-3}\text{ m})^2 = 3.14 \times 10^{-6}\text{ m}^2$
and $\Delta T = 27° - (-39°) = 66°C$
$\therefore \quad F = 3.77 \times 10^2\text{ N}$

29. **(a)** The contraction in the length of the wire due to change in temperature is given as
$\Delta L_1 = \alpha L \Delta T = 1.2 \times 10^{-5} \times 3 \times (-170 - 30) = -7.2 \times 10^{-3}\text{ m}$

$$A = 3.14 \times \left(\frac{1}{2} \times 10^{-3}\right) = 0.785 \times 10^{-6}\text{ m}^2$$

The expansion in the length,

$$\Delta L_2 = \frac{FL}{AY} = \frac{mgL}{AY} = \frac{(10 \times 10) \times 3}{\left(0.785 \times 10^{-6}\right)\left(2 \times 10^{11}\right)}$$

$$= 1.91 \times 10^{-3}\text{ m}$$

Resultant change in length
$= \Delta L_1 + \Delta L_2 \simeq -5.3 \times 10^{-3} = -5.3\text{ mm}$

30. **(b)** $\Delta m = \rho\Delta m$
Volume expansion coefficient is given by

$$\gamma = \frac{\Delta V}{V\Delta T} = \frac{\Delta m}{m\Delta T} = \frac{\frac{x}{50}}{x(353 - 273)}$$

$\Rightarrow \quad \gamma = 2.5 \times 10^{-4}/°C$

31. **(d)**

32. **(c)** $\Delta T = 50°C, \alpha = 20 \times 10^{-5}C^{-1}$
$\therefore$ By conservation of angular momentum,
$I_1\omega_1 = I_2\omega_2$

$$\Rightarrow \quad \left(\frac{2}{5}mr_1^2\right)\omega_0 = \left(\frac{2}{5}mr_1^2\right)(1 + \alpha\Delta T)^2\,\omega_2$$

$$\Rightarrow \quad \omega_0 = (1 + 20 \times 10^{-5} \times 50)^2\,\omega_2$$

$$\therefore \quad \omega_2 = 0.98\,\omega_0$$

33. **(a)** Let L′ be the length of steel rod.
$L' = L + L\alpha\Delta T = 300 + 300 \times 1.2 \times 10^{-5} \times (50 - 27)$
$= 300 + 0.0828 = 300.0828\text{ m}$
The actual length of steel rod at 50°C is

$$L_2 = \frac{300.0828}{300} \times 110 = 110.03\text{ cm}$$

34. **(d)** L at $10°C = L_0(1 + 10\alpha)$
L at $40°C = L_0(1 + 40\alpha)$
$L_{40°C} = 0.05 + L_{10°C}$
$L_0 + L_0\alpha \times 40 = 0.05 + L_0 + 10L_0\alpha$
$30\,l_0\alpha = 0.05$

$$L_0 = \frac{0.05}{30\alpha}$$

The length of the rod at 0°C

$$= \frac{0.05}{30 \times 1.5 \times 10^{-5}} = 111.1\text{ cm}$$

35. **(c)** Coefficient of linean expansion,

$$\alpha = \frac{\Delta L}{L\,\Delta T} = \frac{0.027}{30 \times (105 - 30)} = \frac{0.027}{30 \times 75} = 12 \times 10^{-6}/^{o}C$$

36. **(d)** Ideal gas at a temperature, T = 27 + 273 = 300K
From ideal gas
pv = n RT
pdv = nRd

$$\frac{dv}{dT} = \frac{nR}{p}$$

The coefficient of volume expansion

$$\gamma = \frac{1}{v}\frac{dv}{dT} = \frac{1}{v} \times \frac{nR}{p} = \frac{nR}{pv}$$

$$= \frac{nR}{nRT} = \frac{1}{T} = \frac{1}{300} = 33 \times 10^{-4}\text{ K}^{-1}$$

37. **(b)** $\rho = \frac{m}{V}$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta V}{V} = \frac{V_0 Y\Delta T}{V_0} = Y\Delta T$$

$$\text{So, } \frac{\Delta\rho}{\rho} \times 100 = Y\Delta T \times 100$$

$= 18.2 \times 10^{-5} \times 50 \times 100 = 0.91\ \%$

38. **(c)** $\Delta l = \Delta l_A + \Delta l_B = l_A\alpha_A\Delta T + l_B\,\alpha_B\Delta T$
$= 0.5 \times 200 \times (2 \times 10^{-5} + 1 \times 10^{-5})$
$= 100 \times 3 \times 10^{-5} = 3 \times 10^{-3}\text{m} = 3\text{ mm}$

39. **(b)** $\Delta\ell = \ell_0\alpha\Delta T$
$= 5 \times 10^{-2} \times 2 \times 10^{-5} \times 200 = 2 \times 10^{-4}\text{ m} = 0.02\text{ cm}$
So, $\ell = \ell_0 + \Delta\ell = (5 + 0.02)\text{ cm} = 5.02\text{ cm}$

40. **(a)** We have

$(\Delta \ell)_s = (\Delta \ell)_c$ [$\because$ Difference in length is always same]

$\Rightarrow \ell_s \alpha_s \Delta T = \ell_c \alpha_c \Delta T$

$\Rightarrow \ell_s \times 1.1 \times 10^{-5} = \ell_c \times 1.7 \times 10^{-5}$

$\Rightarrow \frac{\ell_s}{\ell_c} = \frac{17}{11}$

41. **(c)** $L = L_0(1 + \alpha\Delta\theta) \Rightarrow \frac{L_1}{L_2} = \frac{1+\alpha(\Delta\theta)_1}{1+\alpha(\Delta\theta)_2}$

$\Rightarrow \frac{10}{L_2} = \frac{1+11\times10^{-6}\times20}{1+11\times10^{-6}\times19} \Rightarrow L_2 = 9.99989$

$\Rightarrow$ Length is shorten by

$10 - 9.99989 = 0.00011 = 11 \times 10^{-5}$ cm.

42. **(c)** Time lost/gained per day $= \frac{1}{2}\alpha\Delta\theta \times 86400$ second

$12 = \frac{1}{2}\alpha(40-\theta)\times86400$ .... (i)

$4 = \frac{1}{2}\alpha(\theta-20)\times86400$ ....(ii)

On dividing we get, $3 = \frac{40-\theta}{\theta-20}$

$3\theta - 60 = 40 - \theta$

$4\theta = 100 \Rightarrow \theta = 25°C$

43. **(b)**

44. **(b)** We have $\Delta V = V_0 \lambda\Delta T$

$\Delta V = a^3.(3\alpha)\,\Delta T$

Now, $6a^2 = 24$ [$\because$ Total surface area of cube $= 6a^2$]

$\Rightarrow a^2 = 4 \Rightarrow a = 2$

So, $\Delta V = 2^3(3\times5\times10^{-4})\times10 = 1200\times10^{-4}\ m^3$

$= 1200\times10^2\ cm^3 = 1.2\times10^5\ cm^3$

45. **(a)** $\gamma_{real} = \gamma_{app.} + \gamma_{vessel}$

So $(\gamma_{app.} + \gamma_{vessel})_{glass} = (\gamma_{app} + \gamma_{vessel})_{steel}$

$\Rightarrow 153\times10^{-6} + (\gamma_{vessel})_{glass} = 144\times10^{-6} + (\gamma_{vessel})_{steel}$

Further,

$(\gamma_{vessel})_{steel} = 3\alpha = 3\times(12\times10^{-6}) = 36\times10^{-6}/°C$

$\Rightarrow 153\times10^{-6} + (\gamma_{vessel})_{glass} = 144\times10^{-6} + 36\times10^{-6}$

$\Rightarrow (\gamma_{vessel})_{glass} = 3\alpha = 27\times10^{-6}/°C \Rightarrow \alpha = 9\times10^{-6}/°C$

46. **(a)** From question,

Rise in temperature $\Delta t = 40°C$

Fractional change in the density $\frac{\Delta\rho}{\rho_0} = ?$

Coefficient of volume expansion

$\gamma = 5\times10^{-4}K^{-1}$ $\rho = \rho_0(1-\gamma\Delta t)$

$\Rightarrow \frac{\Delta\rho}{\rho_0} = \gamma\Delta T = (5\times10^{-4})(40) = 0.02$

47. **(c)** $\alpha = \frac{\Delta L}{L\Delta T}$; It found greatest for rod (c).

48. **(a)** $\Delta l_{net} = L_1\alpha_1\Delta T + L_2\alpha_2\Delta T$

$(L_1 + L_2)\,\alpha\Delta T = (L_1\alpha_1 + L_2\alpha_2)\Delta T$

$\alpha = \frac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}$

49. **(c)** Solids, liquids and gases all expand on being heated, as a result density (= mass/volume) decreases.

50. **(a)** Among glass, wood and metals, metals expand more for same rise in temperature.

51. **(c)** As coefficient of cubical expansion of liquid equals coefficient of cubical expansion of vessel, the level of liquid will not change on heating.

52. **(c)** Initial diameter of tyre = $(1000 - 6)\ mm = 994\ mm$, so

initial radius of tyre $R = \frac{994}{2} = 497\,mm$

and change in diameter $\Delta D = 6\ mm$ so $\Delta R = \frac{6}{2} = 3\ mm$

After increasing temperature by $\Delta\theta$ tyre will fit onto wheel

Increment in the length (circumference) of the iron tyre

$\Delta L = L\times\alpha\times\Delta\theta = L\times\frac{\gamma}{3}\times\Delta\theta$ [As $\alpha = \frac{\gamma}{3}$]

$2\pi\Delta R = 2\pi R\left(\frac{\gamma}{3}\right)\Delta\theta \Rightarrow \Delta\theta = \frac{3}{\gamma}\frac{\Delta R}{R} = \frac{3\times3}{3.6\times10^{-5}\times497}$

$\Rightarrow \Delta\theta \simeq 500°C$

53. **(a)** The area of circular hole increases when we heat the metal sheet & expansion of metal sheet will be independent of shape & size of the hole.

54. **(a)** Since metal *A* has higher coefficient of expansion compared to that of metal *B*, so *A* will contract more than *B*, and hence it will bend towards left.

55. **(c)** Due to volume expansion of both mercury and flask, the change in volume of mercury relative to flask is given

by $\Delta V = V_0\left[\gamma_L - \gamma_g\right]\Delta\theta = V\left[\gamma_L - 3\alpha_g\right]\Delta\theta$

$= 50\,[180\times10^{-6} - 3\times9\times10^{-6}](38-18) = 0.153$ cc

56. **(d)**

57. **(a)** The latent heat of vaporization is always greater than latent heat of fusion because in liquid to vapour phase change there is a large increase in volume. Hence more heat is required as compared to solid to liquid phase change.

58. **(c)** By principle of calorimetry,

Heat lost = Heat gained

$\Rightarrow m_1 c_1 (70 - T) = m_2 c_2 (T - 0) + m_2 L$

$\Rightarrow 74\times1\times(70-T) = 37\times1\times T + 37\times80$

$\therefore T = 20°C$

59. **(b)**

60. **(d)** Mass of gas, m = 20g
Change in temperature, $\Delta T = 35°C - 25°C = 10°C$
Specific heat capacity of the gas. $C_v = 0.2$ cal g^{-1} $°C^{-1}$
The change in internal energy is given by
$Q = \Delta U = mC_v \Delta T$
$(\Delta V = 0 ; W = 0) = 20 \times 0.2 \times 10 \times 4.200 = 168$ J

61. **(b)** Mass of ice, m = 50g
Temperature, T = – 10°C
Added mass, m′ = 200g
Temperature at which mass added, T′ = 30°C
final temperature of mixture, T_f = ?
Heat lost = heat gained
$m' C_w \Delta T_w = m C_i \Delta T_i + m L$
$200 \times 1 \times (30 - T_f) = 50 \times 0.5 \times (T_f + 10) + 50 \times 80$
$T_f = 7°C$

62. **(d)** Mass, m = 2kg
Temperature, $\Delta T = 10°C$
Heat capacity, c = 4200 $JKg^{-1}K^{-1}$
Heat energy is given by
$Q = m c \Delta T = 2 \times 10 \times 4200$
Q = 84000J

63. **(a)** From the conservation of energy
Change in potential energy = Heat energy
$\Rightarrow \quad mgh = mc\Delta T$

$$\Rightarrow \Delta T = \frac{gh}{c} = \frac{10 \times 63}{4200 J/kgC} = 0.147°C$$

64. **(b)** BO = Sublimation Curve.
AO = Fusion Curve
CO = Vaporization Curve

65. **(c)** Specific heat capacity of copper is 386.4 J kg^{-1} K^{-1} therefore specific heat capacity will remain same.
It is independent of mass of the substance.

66. **(a)** Heat gain by water = Heat lost by steam
$ms(\Delta\theta) = m \times L + ms(\Delta\theta)$
$150 \times 1 \times (40 - 20) = m[(540 + 1 \times (100 - 40)]$

$$\Rightarrow 150 \times 20 = m(540 + 60) \therefore m = \frac{150 \times 20}{600} = 5g$$

$\therefore$ Total mass of water = 150 + 5 = 155g.

67. **(a)** Mass of ice, m = 10 kg
Specific heat capacity of ices
= 0.5 cal g^{-1} = 2093.4 Joule/kg
Latent heat of fusion of ice,
L = 80 cal g^{-1} = 334944 J/kg
Heat energy = $mL + ms\Delta T$
$= 10 \times 334944 + 10 \times 2093.4 \times [0 - (-10)]$
$= 3349440 + 209340 \simeq 357 \times 10^4$ J

68. **(d)** Apply principle of calorimetry
$100 \times 1 \times (90 - 24) = 540\ m + m \times 1 \times (100 - 90)$
$6600 = 540\ m + 10\ m$

$$6600 = 550\ m \Rightarrow m = \frac{6600}{550} = 12g$$

69. **(a)** We have
$\Delta Q = ms_i[-10° - 0°] - mL_f + mS_w[0° - 30°]$
$= -mS_i(+10°) - mL_f - mS_w(30°)$
$= -0.2 \times 2100 \times 10 - 0.2 \times 3.35 \times 10^5 - 0.2 \times 4186 \times 30$
$= -96316$ J
–ve sign indicate that heat is absorbed
So, $\Delta Q_{absorbed} = 96316$J

70. **(a)** $\Delta Q = nC\Delta T$

$$\Rightarrow \quad 3600 = \left(\frac{176}{44}\right) C \times 30 \quad \Rightarrow \quad 3600 = 120C$$

$$\Rightarrow \quad C = 30 J/mol\ K$$

71. **(b)** Latent heat of ether = 90 cal/g
Latent heat of water = 80 cal/g
Let required mass of ether be 'm'
So, heat loss by water = Heat gain by ether
$\Rightarrow \quad 5 \times 80 = m \times 90$

$$\Rightarrow \quad m = \frac{400}{90} = 4.44\ gm \simeq 4\ gm$$

72. **(a)** $\Delta Q = mS\Delta T$

$$m = \frac{\Delta Q}{S\Delta T} = \frac{2100}{900 \times 2} = \frac{2100}{1800} = \frac{7}{6} = 1.16\ kg$$

73. **(c)**

74. **(d)** $$\theta_{mix} = \frac{m_W \theta_W - \frac{m_i L_i}{S_W}}{m_i + m_W}$$

$$= \frac{300 \times 25 - \frac{100 \times 80}{1}}{100 + 300} = -1.25°C$$

Which is not possible. Hence $\theta_{mix} = 0°C$

75. **(a)** Initially, on heating temperature rises from – 100°C to 0°C. then ice melts and temperature does not rise. After the whole ice has melted, temperature beigins to rise until it reaches 100°C. then it becomes constant, as at the boiling point will not rise.

76. **(a)** OA refers to change of state from ice to water without change of temperature.

77. **(c)** The heat lost in condensation = $x \times 540$ cal.

$$\therefore \quad x \times 540 = y \times 80 + y \times 1 \times (100 - 0) \text{ or } \frac{x}{y} = \frac{1}{3}.$$

78. **(c)** Specific heat of water = 4200 J/kg-K
Latent heat of fusion = 3.36×10^5 J/kg
Latent heat of vapourisation = 22.68×10^5 J/kg
$x \times 10^{-3} \times 22.68 \times 10^5 \text{ J} = y \times 10^{-3} \times 3.36 \times 10^5 \text{ J} + y \times 10^{-3} \times 4200 \times 100$

$$\therefore \quad \frac{x}{y} = \frac{7.56}{22.68} = \frac{1}{3}$$

79. **(c)** Let x kg of ice can melt
Using law of Calorimetry,
Heat lost by copper = Heat gained by ice
$2 \times 400 \times (500 - 0) = x \times 3.5 \times 10^5$

$$\text{or} \quad x = \frac{2 \times 400 \times 500}{3.5 \times 10^5} = \frac{8}{7}\text{kg}$$

80. **(d)** Water equivalent is the quantity of water whose thermal capacity is same as the heat capacity of the body.

81. **(b)** Work done to raise the temperature of 100 gm water through 10°C is
$W = JQ = 4.2 \times (100 \times 10^{-3} \times 1000 \times 10) = 4200$ J

82. **(a)** Heat produced = $ms\Delta T = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$

$$\Rightarrow \Delta T = \frac{v^2}{4s} = \frac{(200)^2}{4 \times 125} = \frac{4 \times 10^4}{4 \times 125} = 80°C$$

83. **(a)** $W = JQ \Rightarrow \frac{1}{2}\left(\frac{1}{2}Mv^2\right) = J(m.c.\Delta\theta)$

$$\Rightarrow \frac{1}{4} \times 1 \times (50)^2 = 4.2[200 \times 0.105 \times \Delta\theta] \Rightarrow \Delta\theta = 7.1°C$$

84. **(a)**

85. **(b)** Let the final temperature be T.
Then $200 \times 1 \times (T - 20) + 20 \times (T - 20) = 440\,(92 - T)$
Solving it, we get T = 68°C.

86. **(d)** $K_2 : K_1 = 1 : 4$,
$T_1 = 0°C$, $T_2 = 100°C$

$$H = \frac{k_1 A(100 - T)}{x}$$

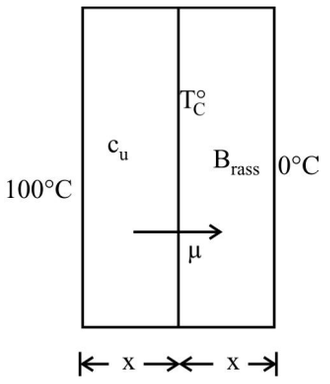
$$= \frac{k_2 A(T - 0)}{x}$$

$$\Rightarrow \frac{k_2}{k_1} = \frac{100 - T}{T}$$

$$\Rightarrow \frac{100 - T}{T} = \frac{1}{4}$$

$$\Rightarrow 400 - 4T = T$$

$$\therefore \quad T = 80°C$$



87. **(b)** Rate of heat flow for one rod

$$= \frac{K_1 A_1 (T_1 - T_2)}{d} \quad [d \to \text{Length}]$$

Rate of heat flow for other rod = $\frac{K_2 A_2 (T_1 - T_2)}{d}$

In steady state, $\frac{K_1 A_1 (T_1 - T_2)}{d}$

$$= \frac{K_2 A_2 (T_1 - T_2)}{d} \Rightarrow K_1 A_1 = K_2 A_2$$

88. **(a)** Equivalent thermal resistance,
$R_{eq} = R_1 + R_2$

$$\Rightarrow \quad \frac{2\ell}{K_{eq} \times A} = \frac{\ell}{K_1 A} + \frac{\ell}{K_2 A} \quad \left[\because R = \frac{1}{K}\frac{\ell}{A}\right]$$

$$\Rightarrow \quad \frac{2}{K_{eq}} = \frac{K_1 + K_2}{K_1 K_2} \Rightarrow K_{eq} = \frac{2K_1 K_2}{K_1 + K_2}$$

89. **(b)** **90.** **(a)**

91. **(d)** Thermal resistance R = l/KA
Where l = 20cm. & A(cylindrical rod)
$= \pi r^2 = 40\pi \text{cm}^2$

So $R = \frac{20}{0.5 \times 40\pi} = 0.318 \frac{\text{cal}}{\text{sec} \times {}^\circ C}$

92. **(c)** Temperature difference between the end points A and D = 200 – 20 = 180°C
As the resistances for the three parts are equal, the temperature difference must be distribuited equally in the three parts (= 180/3 = 60°C)
∴ Temperature of B = 200°C – 60° = 140°C.

93. **(c)** Water has highest specific heat capacity and hence it is used as a coolant in car radiators as well as heater in hot water bags.

94. **(b)** As 1g of steam at 100°C melts 8g of ice at 0°C.
10 g of steam will melt 8 × 10 g of ice at 0°C
Water in calorimeter = 500 + 80 + 10g = 590g

95. **(b)**

96. **(c)** $\frac{\Delta Q_1}{\Delta t} = \frac{\Delta Q_2}{\Delta t} \Rightarrow K_1 A_1 \frac{T_1 - T_2}{L_1} = K_2 A_2 \frac{T_1 - T_2}{L_2}$

$(L_1 = L_2) \qquad \therefore \quad \frac{A_1}{A_2} = \frac{K_2}{K_1}$

97. **(b)**

98. **(b)** When hot water temperature (T) and surrounding temparature (T_0) readings are noted, and $\log(T - T_0)$ is plotted versus time, we get a straight line having a negative slope; as a proof of Newton's law of cooling.

99. **(b)** We have $\theta - \theta_s = (\theta_0 - \theta_s)\, e^{-kt}$ .....(i)
where θ_0 = initial temperature of body = 40°C
θ = temperature of body after time t
θ_s = temperature of surrounding
Since body cools from 40°C to 38°C in 10 min, we have
$38° - 30° = (40° - 30°)\, e^{-10k}$ ..... (ii)
Let after 10 min, the body temp. be θ.
$\theta - 30° = (38° - 30°)\, e^{-10k}$ ..... (iii)
Dividing equ. (ii) by equ. (iii) gives,

$$\frac{8°}{\theta - 30°} = \frac{10°}{8°} \Rightarrow \theta - 30° = 6.4° \Rightarrow \theta = 36.4°$$

100. **(a)**

Exercise 2 : ACCELERATOR
Topic-wise MCQs

1. **(d)** As, $\Delta\ell = \ell_0 \alpha \Delta T$
let final heat coefficient be α''
$\therefore$ $(L + 2L)\, \alpha'' \Delta T = L\, \alpha\, \Delta T + 2L \,.\, 2\alpha.\Delta T$
$\Rightarrow 3\alpha'' = \alpha + 4\alpha = 5\alpha \Rightarrow \alpha'' = 5\alpha/3$

2. **(b)** $PV = nRT$
On differentiation, $P\Delta V = nR\Delta T$

$$\Rightarrow \quad \frac{\Delta V}{\Delta T} = \frac{nR}{P}$$

Let γ be the coefficient of volume expansion

$$\gamma = \frac{\Delta V}{V \Delta T} = \frac{nR}{PV} = \frac{1}{T}$$

3. **(b)** $\text{Stress} = Y \times \text{strain} = Y \times \dfrac{\Delta\ell}{\ell}$

$$= Y \times \frac{\ell \alpha \Delta T}{\ell} = Y\alpha\Delta T \quad (\because \Delta\ell = \ell\alpha\Delta T)$$

Compressive Tension = Stress × Area of cross section
$= YA\alpha\Delta T = 2 \times 10^{11} \times 10^{-4} \times 10^{-5} \times (200 - 0) = 4 \times 10^4 N$

4. **(c)** Coefficient of linear expansion,

$$\alpha = \frac{\Delta L}{L\, \Delta T} = \frac{0.027}{30 \times (105 - 30)} = \frac{0.027}{30 \times 75} = 12 \times 10^{-6}/°C$$

5. **(b)** Given
$T_1 = 20° C, \quad T_2 = 40° C$
$\rho_{20} = 998\ kg/m^3, \quad \rho_{40} = 992\ kg/m^3$

$$\text{As } \rho_{T2} = \frac{\rho_{T1}}{1 + \gamma \Delta T} = \frac{\rho_{T1}}{1 + \gamma (T_2 - T_1)}$$

$$\therefore \quad 992 = \frac{998}{1 + \gamma (40 - 20)} \Rightarrow 992 = \frac{998}{1 + 20\gamma}$$

$$\Rightarrow \quad 20\gamma = \frac{998}{992} - 1 = \frac{6}{992}$$

$$\Rightarrow \quad \gamma = \frac{6}{992} \times \frac{1}{20} = 3 \times 10^{-4}/°C$$

6. **(d)** Ideal gas at a temperature, $T = 27 + 273 = 300K$
From ideal gas
$pv = n\ RT$
$pdv = nRd$

$$\frac{dv}{dT} = \frac{nR}{p}$$

The coefficient of volume expansion

$$\gamma = \frac{1}{v}\frac{dv}{dT} = \frac{1}{v} \times \frac{nR}{p} = \frac{nR}{pv}$$

$$= \frac{nR}{nRT} = \frac{1}{T} = \frac{1}{300} = 33 \times 10^{-4}\, K^{-1}$$

7. **(a)** Let L' be the length of steel rod.
$L' = L + L\, \alpha\, \Delta T = 300 + 300 \times 1.2 \times 10^{-5} \times (50 - 27)$
$= 300 + 0.0828 = 300.0828\ m$
The actual length of steel rod at 50°C is

$$L_2 = \frac{300.0828}{300} \times 110 = 110.03\ cm$$

8. **(d)** $L \text{ at } 10°C = L_0 (1 + 10\alpha)$
$L \text{ at } 40°C = L_0 (1 + 40\alpha)$
$L_{40°C} = 0.05 + L_{10°C}$
$L_0 + L_0\alpha \times 40 = 0.05 + L_0 + 10\, L_0\alpha$
$30\, l_0\alpha = 0.05$

$$L_0 = \frac{0.05}{30\alpha}$$

The length of the rod at 0°C

$$= \frac{0.05}{30 \times 1.5 \times 10^{-5}} = 111.1\ cm$$

9. **(b)** $\dfrac{\text{Reading on new scale} - LFP}{UFP - LFP} = \dfrac{C - 0}{100}$

$$\frac{z°X - 20°X}{110°X - 20°X} = \frac{40 - 0}{100} \Rightarrow z°X - 20°X$$

$$= \frac{40}{100} \times 90°X = 36°X$$

$\therefore$ Reading on new scale.
$z°X = 36°X + 20°X = 56°X$

10. **(a)** Change in length in both rods are same i.e.

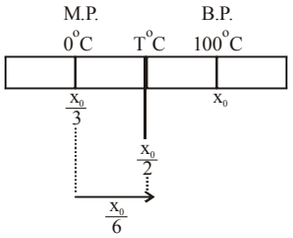
$$\Delta\ell_1 = \Delta\ell_2$$

$$\ell\alpha_1\Delta\theta_1 = \ell\alpha_2\Delta\theta_2$$

$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta\theta_2}{\Delta\theta_1} \qquad \left[\because \frac{\alpha_1}{\alpha_2} = \frac{4}{3}\right]$$

$$\frac{4}{3} = \frac{\theta - 30}{180 - 30} \Rightarrow \theta = 230°C$$

11. **(a)** Let required temperature = T°C



$$\Rightarrow T°C = \frac{x_0}{2} - \frac{x_0}{3} = \frac{x_0}{6}$$

$$\& \left(x_0 - \frac{x_0}{3} \right) = (100 - 0°C)$$

$$\Rightarrow \frac{2x_0}{3} = 100 \Rightarrow x_0 = \frac{300}{2}$$

$$\Rightarrow T°C = \frac{x_0}{6} = \frac{150}{6} = 25°C$$

12. **(b)**

13. **(a)** $E = \frac{F/A}{\Delta\ell/\ell} = \frac{\text{stress}}{\text{strain}}$ where $\Delta\ell = (\ell' - \ell) = \ell\alpha t$ so $F = EA\alpha t$

14. **(b)** Due to volume expansion of both liquid and vessel, the change in volume of liquid relative to container is given by

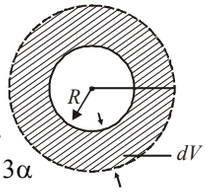
$$\Delta V = V_0[\gamma_L - \gamma_g]\Delta\theta$$

Given $V_0 = 1000cc, \alpha_g = 0.1\times10^{-4}/°C$

$$\therefore \gamma_g = 3\alpha_g = 3\times0.1\times10^{-4}/°C = 0.3\times10^{-4}/°C$$

$$\therefore \Delta V = 1000[1.82\times10^{-4} - 0.3\times10^{-4}]\times100 = 15.2cc$$

15. **(d)** As the temperature increases radius of the sphere increases as shown.



Original volume $V = \frac{4}{3}\pi R^3$

Coefficient of linear expansion = α

Coefficient of volume expansion = 3α

As we know that $\gamma = \frac{\Delta V}{V\Delta t}$

By putting the value of V, increase in the volume

$$\Rightarrow \Delta V = 3V\alpha\Delta t = 4\pi R^3\alpha\Delta t$$

16. **(c)**

17. **(b)** Loss of weight at 27°C is

$= 46 - 30 = 16 = V_1 \times 1.24\,\rho_1 \times g$ ... (i)

Loss of weight at 42°C is

$= 46 - 30.5 = 15.5 = V_2 \times 1.2\,\rho_1 \times g$ ... (ii)

Now dividing (i) by (ii), we get $\frac{16}{15.5} = \frac{V_1}{V_2} \times \frac{1.24}{1.2}$

But $\frac{V_2}{V_1} = 1 + 3\alpha(t_2 - t_1) = \frac{15.5\times1.24}{16\times1.2} = 1.001042$

$$\Rightarrow 3\alpha(42° - 27°) = 0.001042 \Rightarrow \alpha = 2.316\times10^{-5}/°C$$

18. **(a)** Using $\alpha = \frac{\Delta L}{L_0.\Delta T}$

$$\alpha_1 = \frac{0.0650}{30\times100}; \alpha_2 = \frac{0.0350}{30\times100}$$

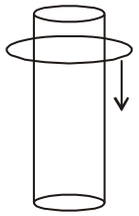
$L_{01}.\alpha_1.\Delta T + L_{02}.\alpha_2.\Delta T = 0.0580$ and $L_{01} + L_{02} = 30$

Solving $L_{01} = 23$ cm

19. **(b)** Temperature to which the iron ring must be heated so to fit the wheel $T' = \frac{d_w - d_i}{d_i\alpha} + T_1$

$$= \frac{5.012 - 5}{5\times1.2\times10^{-5}} + 27 = \frac{1.2\times10^{-2}}{5\times1.2\times10^{-5}} + 27°C = 227°C$$

20. **(a)** For slipping of ring on steel rod



$A_{ring} = A_{rod}$

$\Rightarrow A_o(1 + \beta\Delta T) = A_o'(1 + \beta'\Delta T)$

$\Rightarrow (\beta A_o - \beta' A_o')\Delta T = A'_o - A_o$

$$\Rightarrow \Delta T = \frac{A'_o - A_o}{\beta A_o - \beta' A'_o}$$

$$= \frac{10 - 9.98}{2\left[17\times10^{-6}\times9.98 - 11\times10^{-6}\times10\right]} = 167.6°C$$

21. **(a)**

22. **(a)** $\gamma = \frac{V_2 - V_1}{V_1(T_2 - T_1)}; \alpha = \frac{\ell_2 - \ell_1}{\ell_1(T_2 - T_1)}$

$T_1 = 0°C, T_2 = 100°C$

$\gamma = \frac{V_2 - V_1}{100V_1}; \alpha = \frac{\ell_2 - \ell_1}{100\ell_1}$

$\ell_1 = 10$ cm; $\ell_2 - \ell_1 = 0.08$ mm $= 0.008$ cm

$\alpha = \frac{0.008}{10\times100} = 8\times10^{-6}/°C; \gamma = 3\alpha = 24\times10^{-6}/°C$

$\therefore\ 24\times10^{-6} = \frac{V_2 - 1000}{1000\times100}$

$V_2 - 1000 = 24\times10^{-6}\times10^5 = 2.4 \quad \therefore V_2 = 1002.4$ cc

23. **(b)** When there is no change in liquid level in vessel then $\gamma'_{real} = \gamma'_{vessel}$

Change in volume in liquid relative to vessel

$$\Delta V_{app} = V\gamma'_{app}\Delta\theta = V(\gamma'_{real} - \gamma'_{vessel})$$

24. **(b)** As, $\frac{dL}{L_0} = \alpha(T)dT \quad \int_{L_0}^{L} dL = L_0\int_{T_0}^{T}\alpha(T)dT$

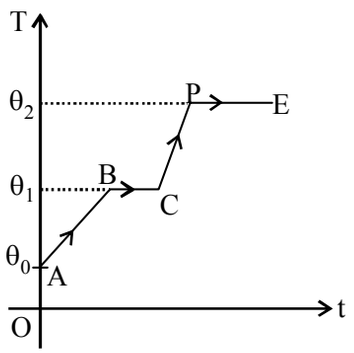
$$L - L_0 = L_0\int_{T_0}^{T}\alpha(T)dT \quad L = L_0\left[1 + \int_{T_0}^{T}\alpha(T)dT\right]$$

25. **(b)**

26. **(b)** Power absorbed P = 100 W
Mass m = 0.1 kg
and, $\Delta T = (127 - 126)°C = 1°C$
Specific heat of heater,
$s = 1\ J/(g°C) = 1 \times 10^3\ J/(kg°C)$
$\therefore$ Heat rejected is given as
$Q = ms\ \Delta\ T = 0.1 \times 1 \times 10^3 \times 1 = 100$ J

$$\therefore \text{ Time taken} = \frac{\text{Heat rejected}}{\text{Power}} = \frac{100}{100} = 1s$$

27. **(c)** The temperature-time graph for the solid cube is



The heat supplied to the cube is
$Q = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DE}$
$= mS_1(\theta_1 - \theta_0) + m\,L_f + m\,S_2(\theta_2 - \theta_1) + m\,L_v$

28. **(d)** Rate of melting ice $\propto$ rate of heat transfer (dQ/dt)

$$\text{Further } \frac{dQ}{dt} = \frac{\text{temperature difference}}{(l/KA)}$$

$$\text{or } \frac{dQ}{dt} = \frac{\text{temp. diff}}{l} \times A$$

If temp diff. A and l are all doubled, then dQ/dt, and hence rate of melting of ice, are doubled.

29. **(b)** Given that latent heat of fusion of ice L, = 3.4×10^5 J kg^{-1}
Thermal conductivity of insulation wall, K = 0.05 Wm^{-1} C^{-1}
Change in temperature, $\Delta T = 40°C$
Area of ice cube, $A = 2(0.6 \times 0.5 + 0.5 \times 0.2 + 0.2 \times 0.6)$
$= 2(0.3 + 0.1 + 0.12) = 1.04\ m^2$
We have

$$\frac{dQ}{dt} = \frac{KA\Delta T}{\ell} = \frac{0.05 \times 1.04 \times 40}{10^{-2}} = 2.08 \times 10^2\ J/s$$

$$\frac{dQ}{dt} = mL$$

$$2.08 \times 10^2 = m \times 3.4 \times 10^5$$

$$m = \frac{2.08}{3.4 \times 10^3} \Rightarrow 0.61 \times 10^{-3}\ kg/s = 61 \times 10^{-5}\ Kg/s$$

30. **(a)** Initially liquid oxygen will gain the temperature up to its boiling temperature then it change its state to gas. After this again its temperature will increase, so corresponding graph will be graph (a).

31. **(a)** For water, $m_1 = 360$ g, $T_1 = 40°C$, $T_2 = 100°C$
$\therefore$ Heat required, $H_1 = m_1C_1(T_2 - T_1)$
$= 360 \times 1 \times (100 - 40) = 21600$ cal
At equilibrium,
Heat released by steam = Heat required by water
$\therefore$ Mass of steam converted into water,

$$m_2 = \frac{H_1}{L} = \frac{21600}{540} = 40g$$

$\therefore$ Mass of steam left, $m_s = 60 - 40 = 20g$
Mass of water, $m_w = 360 + 40 = 400g$

$$\therefore \frac{m_s}{m_w} = \frac{20}{400} = 1:20$$

32. **(d)** Let m gram of ice is added.
From principal of calorimeter
heat gained (by ice) = heat lost (by water)
$\therefore 20 \times 2.1 \times m + (m - 20) \times 334 = 50 \times 4.2 \times 40$
$\Rightarrow 376\,m = 8400 + 6680 \Rightarrow m = 40.1$

33. **(a)** Heat gain by water = Heat lost by steam
$ms\ (\Delta\theta) = m \times L + ms\ (\Delta\theta)$
$150 \times 1 \times (40 - 20) = m[(540 + 1 \times (100 - 40)]$
$\Rightarrow 150 \times 20 = m\ (540 + 60)$

$$\therefore m = \frac{150 \times 20}{600} = 5g$$

$\therefore$ Total mass of water = 150 + 5 = 155g.

34. **(a)** $M_1C_{ice} \times (10) + M_1L = M_2C_{\omega}\ (50)$
or $M_1 \times C_{ice}\ (= 0.5) \times 10 + M_1L = M_2 \times 1 \times 50$

$$\Rightarrow \quad L = \frac{50M_2}{M_1} - 5$$

35. **(c)** Heat loss = Heat gain = $mS\Delta\theta$
So, $m_AS_A\Delta\theta_A = m_BS_B\Delta\theta_B$
$\Rightarrow 100 \times S_A \times (100 - 90) = 50 \times S_B \times (90 - 75)$

$$2S_A = 1.5S_B \Rightarrow S_A = \frac{3}{4}S_B$$

Now, $100 \times S_A \times (100 - \theta) = 50 \times S_B \times (\theta - 50)$

$$2 \times \left(\frac{3}{4}\right) \times (100 - \theta) = (\theta - 50)$$

$\Rightarrow 300 - 3\theta = 2\theta - 100$
$\Rightarrow 400 = 5\theta \Rightarrow \theta = 80°C$

36. **(d)** Assume final temperature = T°C
Heat loss = Heat gain = $ms\Delta T$
$\Rightarrow m_B\,s_B\,\Delta T_B = m_w\,s_w\Delta T_w$
$0.1 \times 400 \times (500 - T) = 0.5 \times 4200 \times (T - 30) + 800\,(T - 30)$
$\Rightarrow 40\,(500 - T) = (T - 30)\,(2100 + 800)$

$\Rightarrow \quad 20000 - 40T = 2900\,T - 30 \times 2900$

$\Rightarrow \quad 20000 + 30 \times 2900 = T(2940)$

$T = 30.4°C$

$$\frac{\Delta T}{T} \times 100 = \frac{6.4}{30} \times 100 = 21\%,$$

so the closest answer is 20%.

37. **(d)** $Q = (m_1 s_1 + m_2 s_2 + m_3 s_3)\,\Delta T$

$\Rightarrow \quad 44.4 = (2.4 \times 0.216 + 1.6 \times 0.0917 + 0.8 \times 0.0931)\Delta T$

$\Rightarrow \quad \Delta T = 60 \Rightarrow T - 20 = 60$

$\therefore \quad T = 80°c.$

38. **(d)** Apply the conservation of energy

$mg(h_1 - h_2) = mc\,\Delta T$

$10 \times (20 - 10.8) = 460 \times \Delta T$

$$\frac{92}{460} = \Delta T \Rightarrow \Delta T = 2°C$$

39. **(c)** Heat gain = heat lost

$$C_A(16-12) = C_B(19-16) \Rightarrow \frac{C_A}{C_B} = \frac{3}{4}$$

and $$C_B(23-19) = C_c(28-23) \Rightarrow \frac{C_B}{C_C} = \frac{5}{4}$$

$$\Rightarrow \frac{C_A}{C_C} = \frac{15}{16} \qquad \text{...(i)}$$

If θ is the temperature when A and C are mixed then,

$$C_A(\theta - 12) = C_C(28 - \theta) \Rightarrow \frac{C_A}{C_C} = \frac{28-\theta}{\theta - 12} \qquad \text{...(ii)}$$

On solving equations (i) and (ii) $\theta = 20.2°C$

40. **(b)** From, $Q = nC\Delta T \Rightarrow \frac{dQ}{dt} = nC\frac{dT}{dt}$

Rate of heat is same for both gases

$$\text{i.e., } \frac{dQ_1}{dt} = \frac{dQ_2}{dt} \text{ or, } n_1C_1\left(\frac{dT}{dt}\right)_1 = n_2C_2\left(\frac{dT}{dt}\right)_2$$

$$\therefore \frac{C_1}{C_2} = \frac{\left(\frac{dT}{dt}\right)_2}{\left(\frac{dT}{dt}\right)_1} = \frac{\frac{90}{6}}{\frac{120}{3}} = \frac{90 \times 3}{120 \times 6} = \frac{3}{8}$$

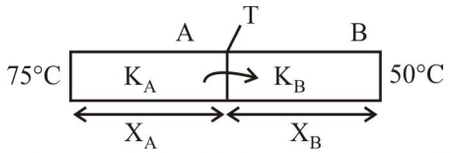
41. **(c)** Since specific heat = 0.6 kcal/g × °C = 0.6 cal/g × °C

From graph it is clear that in a minute, the temperature is raised from 0°C to 50°C.

$\Rightarrow$ Heat required for a minute = 50 × 0.6 × 50 = 1500 cal.

Also from graph, Boiling point of wax is 200°C.

42. **(c)** The given situation is shown below



In steady state, both slab have same rate of heat flow

$$\therefore \quad \frac{dQ}{dt} = \frac{K_A(75°C - T)}{X_A} = \frac{K_B(T - 50°C)}{X_B}$$

$\because \quad K_B = 2K_A$ and $X_A = 2X_B$

$$\therefore \quad \frac{K_B(75°C - T)}{2 \times 2X_B} = \frac{K_B(T - 50°C)}{X_B}$$

$\Rightarrow 75°C - T = 4T - 200°C \Rightarrow 5T = 275°C \Rightarrow T = 55°C$

43. **(c)** Temperature of interface

$$\theta = \frac{K_1\theta_1 l_2 + K_2\theta_2 l_1}{K_1 l_2 + K_2 l_1} = \frac{K \times 0 \times 2 + 3 \times 100 \times 1}{K \times 2 + 3K \times 1}$$

$$= \frac{300K}{5K} = 60°C$$

44. **(a)** The top surface of the ice is at $T_C = -5.0°C$ and the bottom of the body of water is at $T_H = 40°C$ and the interface between the ice and the water is at $T_X = 0.0°C$. The rate of heat flow through both layers should be equal

$$\frac{k_{water}A(T_H - T_X)}{L - L_{ice}} = \frac{k_{ice}A(T_X - T_C)}{L_{ice}}$$

We cancel the area A and solve for thickness of the ice layer: $L_{ice} = 1.1$ m.

45. **(d)**

46. **(d)** $$\frac{Q}{At} = \frac{K(\theta_1 - \theta_2)}{d} = \text{constant}$$

$$\therefore \quad K_A\left(\frac{\theta_1 - \theta}{d}\right) = K_B\left(\frac{\theta - \theta_2}{d}\right)$$

$$\frac{K_A}{K_B} = \frac{\theta - \theta_2}{\theta_1 - \theta} \text{ or } 3 = \frac{\theta - \theta_2}{\theta_1 - \theta}$$

or $\quad 3\theta_1 + \theta_2 = 4\theta \quad (1)$

Given $\theta_1 - \theta_2 = 40 \quad (2)$

Solving (1) and (2) we have, $\theta - \theta_2 = 15°C$

$\therefore \quad \theta_1 - \theta = \theta_1 - \theta_2 + \theta_2 - \theta$

$= \quad (\theta_1 - \theta_2) - (\theta - \theta_2) = 20°C - 15°C = 5°C$

47. **(b)** $$\frac{dQ}{dt} = \frac{dQ1}{dt} + \frac{dQ_2}{dt}$$

$$K\frac{4\pi R^2(\theta_1 - \theta_2)}{L} = K_1\frac{\pi R^2(\theta_1 - \theta_2)}{L}$$

$$= K_2\frac{3\pi R^2(\theta_1 - \theta_2)}{L} \quad \text{or} \quad 4K = K_1 + 3K_2$$

or $\quad K = (K_1 + 3K_2)/4$

48. **(b)**

49. **(b)**

50. **(c)** In series $\frac{dQ}{dt}$ = constant

$$\Rightarrow \quad \frac{T_1 - T_2}{R} = \text{constant}$$

$$\Rightarrow \quad \frac{450 - T}{\frac{l_s}{K_s A_s}} = \frac{T - 0}{\frac{l_c}{K_C A_C}} \Rightarrow \frac{450 - T}{\frac{l_s}{l_c} \cdot \frac{K_c}{K_s} \cdot \frac{A_c}{A_s}} = T$$

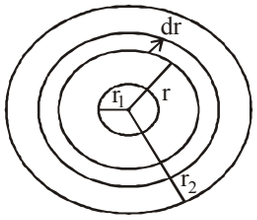
$$\Rightarrow \quad \frac{450 - T}{2.9.\frac{1}{2}} = T \quad \Rightarrow 450 - T = 9T$$

$\Rightarrow 450 = 10T \Rightarrow T = 45\,°C$

51. **(a)** Resistance of spherical shell of thickness dr and radius r is

$$dR = \frac{dr}{K(4\pi r^2)} \quad \left[\text{using } R = \frac{1}{K}\frac{\ell}{A}\right]$$

Total resistance



$$R = \int_{r_1}^{r_2} \frac{dr}{K(4\pi r^2)}$$

$$\Rightarrow \quad R = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\text{current} = \frac{\theta_2 - \theta_1}{R} = \frac{4\pi K r_1 r_2}{r_2 - r_1}(\theta_2 - \theta_1)$$

52. **(d)** For slab in series, we have

$$R_{eq} = R_1 + R_2 = \frac{x}{KA} + \frac{4x}{2KA} = \frac{3x}{KA}$$

Now, in a steady state rate of heat transfer through the slab is given by

$$\frac{dQ}{dt} = \frac{T_2 - T_1}{R_{eq}} = \frac{(T_2 - T_1)}{3x} KA \qquad \ldots(i)$$

$$\text{Given } \frac{dQ}{dt} = \left(\frac{A(T_2 - T_1)K}{x}\right) f \qquad \ldots(ii)$$

Comparing (i) and (ii), we get f = 1/3

53. **(d)** Mass of ice melted = (volume of sphere) × (density of ice)

Let K_1 and K_2 be the thermal conductivities of larger and smaller sphere.

For bigger sphere,

$$\frac{K_1 4\pi (2r)^2 \times 100}{t/4} = \frac{\frac{4}{3}\pi(2r)^3 \rho L}{25 \times 60}$$

For smaller sphere,

$$\frac{K_2 \times 4\pi r^2 \times 100}{t} = \frac{\frac{4}{3}\pi r^3 \rho L}{16 \times 60} \quad \therefore \frac{K_1}{K_2} = \frac{8}{25}$$

54. **(a)**

55. **(d)** Here, length of the rod, $\Delta x = 20\text{ cm} = 20 \times 10^{-2}\text{ m}$

diameter = 2cm $\therefore$ radius, $r = 1\text{ cm} = 10^{-2}\text{ m}$

area of cross section, $a = \pi r^2 = \pi(10^{-2})^2$ sq. m

$\Delta T = 100 - 0 = 100°C$

Mass of ice melted, m = 25g

and $L_{ice} = 80\text{ cal. g}^{-1}$

$\Delta Q = mL = 25 \times 80 = 2000\text{ cal} = 2000 \times 4.2\text{ J}$

$\Delta t = 5\text{ min} = 300\text{ s}$

$$\text{From } \frac{\Delta Q}{\Delta t} = KA\,\frac{\Delta T}{\Delta x}$$

Putting the value and solving we get

$K = 1.78 Js^{-1}m^{-1}°C^{-1}$

56. **(d)**

57. **(c)** For rod A, $\Delta T_1 = 40°C$

For rod B, $\Delta T_2 = 60°C$

$$\therefore \frac{Q_A}{Q_B} = \frac{3}{8} \Rightarrow \frac{A_1 \frac{\Delta T_1}{l_1}}{A_2 \frac{\Delta T_2}{t_2}} = \frac{3}{8} \qquad \ldots(i)$$

Also, $m_1 = m_2 \Rightarrow \delta V_1 = \delta V_2 \Rightarrow V_1 = V_2$

$$\Rightarrow \quad A_1 l_1 = A_2 l_2 \quad \frac{A_1}{A_2} = \frac{l_2}{l_1} \qquad \ldots(ii)$$

From eq(i) and (ii), we get

$$\left(\frac{\Delta T_1}{\Delta T_2}\right)\left(\frac{l_2}{l_1}\right)^2 = \frac{3}{8} \Rightarrow \left(\frac{40}{60}\right)\left(\frac{l_2}{l_1}\right)^2 = \frac{3}{8}$$

$\therefore \quad l_1 : l_2 = 4 : 3$

58. **(a)** $l = 20\text{cm} = 0.2\text{m}, A = 4 \times 10^{-4}\text{ m}^2, \theta_1 = 0°\text{c}, \theta_2 = 100°\text{c}$

$$\therefore H = \frac{kA(\theta_2 - \theta_1)}{l} \Rightarrow \frac{mL}{t} = \frac{kA(\theta_2 - \theta_1)}{l}$$

$$\Rightarrow \quad \frac{5 \times 80 \times 4.2}{60} = \frac{k \times 4 \times 10^{-4} \times 100}{0.2}$$

$\therefore \quad k = 140\ W_m^{-1} k^{-1}.$

59. **(c)** **60. (a)**

Exercise 3 : PREVIOUS YEARS MCQs

1. **(c)** $F/C = \frac{13}{5}F = \frac{9}{5}C + 32$

$$F = \frac{13}{5}c$$

$$\frac{13c}{5} = \frac{9c}{5} + 32 \Rightarrow \frac{4c}{5} = 32$$

$\boxed{c = 40°C}$

Hence the option (c) is correct

2. **(a)** $Q = \frac{KA\Delta T}{L} \Rightarrow Q_{copper} = \frac{K_{copper}A(100-T)}{L}$

$Q_{Brass} = \frac{K_{Brass}A(T-0)}{L}$

Steady state condition

$Q_{copper} = Q_{Brass}$

$\frac{K_{copper}A(100-T)}{L} = \frac{K_{Brass}AT}{L}$

$K_{copper}(100-T) = K_{Brass}T$

$\frac{K_{copper}}{K_{Brass}} = 4 \Rightarrow 4(100-T) = T$

$\boxed{T = 80°C}$

3. **(b)**

4. **(b)** Suppose, M is the mass of steam required.

By principle of calorimetry,

Heat lost by steam = Heat gained by water

$m_s L = mc\Delta T$

Heat needed by water: $Q = mc\Delta T = 100 \times 4.18 \times 66$

$= 275\,88$ J

Heat given by steam:

$Q = m_s \times L = m_s \times 2260 \times 10^3$ J/kg

$m_s \times 2260 = 27588$

$m_s \approx 12$ g

5. **(b)**

6. **(c)** Given, length of scale, ℓ = 1m = 1000 mm

Accuracy = 0.5 mm

Coefficient of linear expansion = 10^{-5}/°C

Change in length due to temperature change is

$\Delta L \geq \alpha L\Delta T$

$0.5 \times 10^{-3} \geq 10^{-5} \times 1 \times \Delta T$

$\Rightarrow \quad \frac{0.5 \times 10^{-3}}{10^{-5}} \geq \Delta T \Rightarrow \Delta T \leq 50°C$

The scale works accurately within ±50°C of its manufacturing temperature.

7. **(b)** Apply Temperature Conversion formula

$°C = \frac{5}{9}(°F - 32)$

$°C = \frac{5}{9}(122-32) = \frac{5}{9} \times 90 = 50$

Faulty Celsius reading = 49°C

Correct clesius temperature = 50°C

Correction = Correct – Faulty = 50 – 49 = 1°C

8. **(c)** $F = \frac{9}{5}C + 32$ ...(i)

F = 1.9C (given)

$\therefore \quad \frac{9}{5}C + 32 = 1.9C$

$\Rightarrow$ C = 320, putting in equation ...(i)

$F = \left(\frac{9}{5} \times 320 + 32\right) = 608°F$

9. **(c)**

10. **(a)** From Newton's Law of cooling

$\frac{T_1 - T_2}{t} = k\left(\frac{T_1 + T_2}{2} - T_s\right)$

When

$T_1 = 60°C$

$T_2 = 50°C$

t = 10 min

$\Rightarrow \frac{60-50}{10} = k\left(\frac{60+50}{2} - T_s\right)$

$\Rightarrow$ 1 k(55 – T_s) ...(i)

When, T_1 = 50°C, T_2 = 40°C, t = 15 min

$\frac{50-40}{15} = k\left(\frac{50+40}{2} - T_s\right)$

$\Rightarrow \frac{2}{3} = k(45 - T_s)$ ...(ii)

Solving (i) and (ii) we get

$T_s = 25, k = \frac{1}{30}$

When body cool from 40°C

$\frac{40-30}{t} = \frac{1}{30}\left(\frac{40+30}{2} - 25\right) \Rightarrow \frac{10}{t} = \frac{10}{30}$

11. **(d)** Energy is conserved, so $mgh = ms\Delta T$

$\therefore \Delta T = \frac{gh}{s} = \frac{10 \times 200}{4200} K = \frac{10}{21}$ K = 0.48 K.

12. **(d)** By the principle of mixture, heat given out by water = heat absorbed by the melted ice.

$mS(\theta - 0) = ML$

$M = \frac{mS\theta}{L}$

13. **(a)** For water, m = 114g, T_1 = 30°c, T_2 = 70°c,

S_ω = 1 cal g^{-1} °c^{-1}

Heat required to increase the temperature of water from T_1 to T_2 is

$Q = m S_\omega (T_2 - T_1) = 114 \times 1 \times (70-30) = 114 \times 40$ cal

$\therefore$ Mass of steam converted into water is

$$m_S = \frac{Q}{L_v} = \frac{114 \times 40}{540} = 8.4g \approx 8g$$

$\therefore$ Total mass of water present in the mixture of steam and water is

$m_{s,mixture} = 114 + 8 = 122g$

14. **(d)** For a closed vessel, volume = constant

$\therefore$ Work done, $W = p\Delta V = 0$

By 1st law of thermodynamics,

$\Delta Q = \Delta U + W = \Delta U$

$\therefore$ Increase in internal energy is

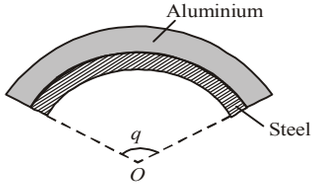
$\Delta U = \Delta Q = mS\Delta T$

Here, $m = 5kg$, $s = 4200\ J\ kg^{-1}k^{-1}$

$\Delta T = (30 - 20)^\circ C = 10^\circ C$

$\therefore \Delta U = 5 \times 4200 \times 10 = 210\ kJ$

15. **(d)** If strips of aluminium and steel are fixed together on metallic strip and both are heated then ($\alpha_{Al} > \alpha_{steel}$) aluminium will exapnd more because the metallic strip with higher coefficient of linear expansion (α_{Al}) will expand more Thus it should have larger radius of curvature. Hence, aluminium will be on convex side.



16. **(b)** The compressive force developed in the metallic bar is $F = YA\alpha\Delta T$

$= 0.5 \times 10^{11} \times 10^{-3} \times 10^{-5} \times 100 = 50 \times 10^3 N$

17. **(c)** Since $\frac{F-32}{9} = \frac{C}{5}$

$$\Rightarrow \Delta C = \frac{5}{9}\Delta F \Rightarrow 40 = \frac{5}{9}\Delta F \Rightarrow \Delta F = 72^\circ F$$

18. **(d)** Given,

Boiling point of water, = UFP 65°

Freezing point of water, = LFP – 15°

$$\frac{X - LFP}{UFP - LFP} = \frac{T_F - 32}{212 - 32}$$

$$\Rightarrow \frac{-95 - (-15)}{65 - (-15)} = \frac{T_F - 32}{180}$$

$$\Rightarrow \frac{-80}{80} = \frac{T_F - 32}{180} \Rightarrow T_F = -180 + 32$$

$$\Rightarrow T_F = -148^\circ F$$

19. **(d)** The volume expansion coefficient of the gas,

$$\gamma = \frac{dv}{vdT} \quad ...(i)$$

$PT^2 =$ constant, using $PV = nRT$

$$P = \frac{nRT}{V}$$

$$PT^2 = \frac{nRT}{V} \times T^2 = \text{constant}$$

$$\Rightarrow T^3 = KV \quad ...(ii)$$

$$\text{So, } \frac{d}{dT}(KV) = 3T^2 \Rightarrow \frac{KdV}{dT} = 3T^2$$

$$\frac{dV}{dT} = \frac{3T^2}{K}$$

Put this value in equation (i)

$$\Rightarrow \gamma = \frac{3T^2}{KV} = \frac{3T^2}{T^3} = \frac{3}{T}$$

20. **(a)** Young's modulus $Y = \frac{\text{stress}}{\text{strain}}$

$$= \frac{F/A}{A(\Delta\ell/\ell)}$$

Using, coefficient of linear expansion,

$$\alpha = \frac{\Delta\ell}{\ell\Delta T} \Rightarrow \frac{\Delta\ell}{\ell} = \alpha\Delta T$$

$$\therefore Y = \frac{F}{A(\alpha\Delta T)}$$