Airmen Group -X Solved Paper 2019

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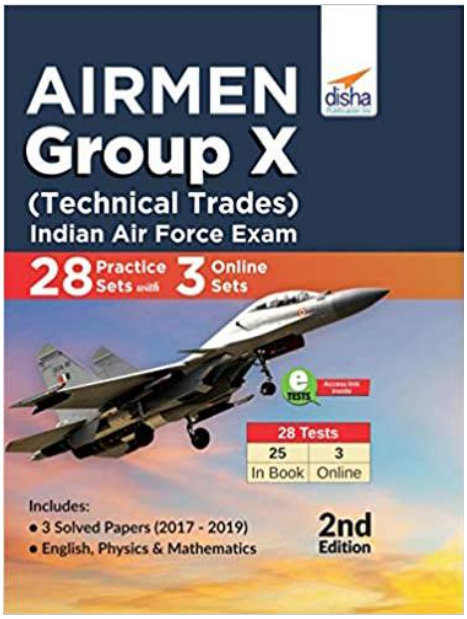

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INDIAN AIR FORCE AIRMEN GROUP X
SOLVED PAPER - 2019
(Memory Based)

ENGLISH LANGUAGE

1. Find the one which best expresses the meaning of the given word.
Wriggle
(a) Shirk (b) Wise
(c) Inform (d) Insist

2. Find the word opposite in meaning to the given word.
Lout
(a) Rowdy (b) Hooligan
(c) Gentleman (d) Brute

DIRECTIONS (Qs. 3-4) : *In the following questions, out of the four alternatives choose the one which can be substituted for the given words/sentences.*

3. A sleeping room for many persons
(a) boarding (b) hostel
(c) dormitory (d) dwelling

4. One who murders one's mother
(a) genocide (b) germicide
(c) matricide (d) patricide

5. Find the correctly spelt word.
(a) Dependance (b) Accomodation
(c) Dissapoint (d) Aberration

6. Find the meaning of the Idiom/Phrase
Fine and Dandy
(a) Miserable (b) Repetition
(c) Extraneous (d) Excellent

7. The following sentence has been given in Active/Passive voice. From the given alternatives, choose the one which best expresses the given sentence in Passive/Active voice.
You will be taken care of by me.
(a) I will be taking care of you.
(b) I was take care of you.
(c) I will take care of you.
(d) I will being take care of you.

8. The following sentence has been given in direct/ Indirect Speech. Out of the four alternatives suggested, select the one which best expresses the same sentence in Indirect/ Direct speech.
The Swami said to the villager, "Please bring me a glass of water"
(a) The swami ordered the villager to bring him a glass of water.
(b) The swami requested to the villager to bring him a glass of water.
(c) The swami requested told the villager to bring me a glass of water.
(d) The swami requested the villager to bring him a glass of water

DIRECTIONS (Qs. 9-10) : *In the following question, some part of the sentence may have errors. Find out which part of the sentence has an error and select the appropriate option. If there is 'No Error', mark (D) as your answer.*

9. If the opulence of gold leaf decoration (A)/was not enough, the Catherine Palace built (B)/ in 1717 by Catherine I, has an Amber room. (C)/ No Error (D)
(a) A (b) B
(c) C (d) D

10. Surprisingly there was free seating and (A)/ therefore one have to be (B)/ at the venue early. (C)/ No Error (D)
(a) A (b) B
(c) C (d) D

DIRECTIONS (Qs. 11-13) : *You have a brief passage with 10 questions. Read the passage carefully and choose the best answer to each question out of the four alternatives.*

Just as some men like to play football or cricket, similarly some men like to climb mountains. This is often very difficult to do, for mountains are not just big hills. Paths are usually very steep. Some mountain sides are straight up and down, so it may take many hours to climb as little as one hundred feet. There is always the danger that you may fall off and be killed or injured. Men talk about conquering a mountain. It is a wonderful feeling to reach the top of a mountain

after climbing for hours and may be, even days. You look down and see the whole country below you. You feel godlike. Two Italian prisoners of war escaped from a prison camp in Kenya during the war. They did not try to get back to their own country, for they knew that was impossible. Instead, they climbed to the top of Mount Kenya, and then they came down again and gave themselves up. They had wanted to get that feeling of freedom that one has, after climbing a difficult mountain.

11. Some men like to climb mountains because
    (a) they do not like to play football or cricket
    (b) they want to have a wonderful feeling
    (c) they know the trick of climbing
    (d) they like to face danger
12. To climb mountains is often difficult because
    (a) mountains are big hills
    (b) it consumes more time
    (c) prisoners often escape from camps and settle there
    (d) paths are steep and uneven
13. It is a wonderful feeling _____. 'It' refers to:
    (a) the steep path (b) the mountain
    (c) the prisoner (d) mountaineering
14. Find the Adjective of the word
    Help
    (a) Helping (b) Helpful
    (c) Help (d) Helpee
15. Find the plural of the word
    Half
    (a) Halfs (b) Halves
    (c) Half (d) Halfes
16. Find the part of speech of the given word
    Hurray
    (a) Adjective (b) Verb
    (c) Interjection (d) Conjunction

DIRECTIONS (Qs. 17-19) : *Pick out the most effective word from the given words to fill in the blank to make the sentence meaningfully complete.*

17. As a last _____ the students have appealed to the Principal.
    (a) course (b) method
    (c) chance (d) resort
18. The police _____ into action on receiving telephone call yesterday.
    (a) swang (b) swung
    (c) swing (d) run
19. The twins are so alike that I cannot _____ one from the other.
    (a) discern (b) tell
    (c) say (d) notice
20. It is time to buy books.
    It is time _____.
    (a) all the books to be bought
    (b) for the books are being bought
    (c) for the books to be bought
    (d) for the books have been bought.

PHYSICS

21. A rifle man, who together with his rifle has a mass of 100 kg, stands on a smooth surface and fires 10 shots horizontally. Each bullet has a mass 10 g and a muzzle velocity of 800 ms^{-1}. The velocity which the rifle man attains after firing 10 shots is
    (a) 8 ms^{-1} (b) 0.8 ms^{-1}
    (c) 0.08 ms^{-1} (d) $-0.8\,ms^{-1}$
22. The force between two charges 0.06 m apart is 5 N. If each charge is moved towards the other by 0.01 m, then the force between them will become
    (a) 7.20 N (b) 11.25 N
    (c) 22.50 N (d) 45.00 N
23. Which of the following circular rods, (given radius r and length *l*) each made of the same material and whose ends are maintained at the same temperature will conduct most heat?
    (a) $r = 2r_0; l = 2l_0$ (b) $r = 2r_0; l = l_0$
    (c) $r = r_0; l = 2l_0$ (d) $r = r_0; l = l_0$
24. The dimensions of torque are
    (a) $[MLT^{-2}]$ (b) $[ML^2T^{-2}]$
    (c) $[ML^2T^{-1}]$ (d) $[M^2L^2T^{-2}]$
25. A body executes S.H.M with an amplitude A. At what displacement from the mean position is the potential energy of the body is one fourth of its total energy?
    (a) A/4 (b) A/2
    (c) 3A/4 (d) $\frac{2}{3}A$
26. The photoelectric work function for a metal surface is 4.125 eV. The cut-off wavelength for this surface is
    (a) 4125 Å (b) 3000 Å
    (c) 6000 Å (d) 2062 Å

27. The door of a working refrigerator is left open in a well insulated room. The temperature of air in the room will
(a) decrease
(b) increase in winters and decrease in summers
(c) remain the same
(d) increase

28. At magnetic poles, the angle of dip is
(a) 45° (b) 30°
(c) zero (d) 90°

29. The nuclei of which one of the following pairs of nuclei are isotones?
(a) ${}_{34}Se^{74}$, ${}_{31}Ga^{71}$ (b) ${}_{38}Sr^{84}$, ${}_{38}Sr^{86}$
(c) ${}_{42}Mo^{92}$, ${}_{40}Zr^{92}$ (d) ${}_{20}Ca^{40}$, ${}_{16}S^{32}$

30. Two electric bulbs marked 25W – 220 V and 100W – 220V are connected in series to a 440 V supply. Which of the bulbs will fuse?
(a) Both (b) 100 W
(c) 25 W (d) Neither

31. A charged particle with velocity 2×10^3 m/s passes undeflected through electric and magnetic field. Magnetic field is 1.5 tesla. The electric field intensity would be
(a) 2×10^3 N/C (b) 1.5×10^3 N/C
(c) 3×10^3 N/C (d) $4/3 \times 10^{-3}$ N/C

32. An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rpm, the acceleration of a point on the tip of the blade is
(a) 1600 ms^{-2} (b) 47.4 ms^{-2}
(c) 23.7 ms^{-2} (d) 50.55 ms^{-2}

33. If a spring extends by x on loading, then the energy stored by the spring is (if T is tension in the spring and k is spring constant)
(a) $\frac{T^2}{2x}$ (b) $\frac{T^2}{2k}$
(c) $\frac{2x}{T^2}$ (d) $\frac{2T^2}{k}$

34. If a source of power 4kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called
(a) X -rays (b) ultraviolet rays
(c) microwaves (d) γ -rays

35. A transistor is essentially
(a) a current operated device
(b) power driven device
(c) a voltage operated device
(d) resistance operated device

36. A stick is thrown in the air and lands on the ground at some distance from the thrower. The centre of mass of the stick will move along a parabolic path
(a) in all cases
(b) only if the stick is uniform
(c) only if the stick has linear motion but no rotational motion
(d) only if the stick has a shape such that its centre of mass is located at some point on it and not outside it

37. A.C. power is transmitted from a power house at a high voltage as
(a) the rate of transmission is faster at high voltages
(b) it is more economical due to less power loss
(c) power cannot be transmitted at low voltages
(d) a precaution against theft of transmission lines

38. A coil having 500 square loops each of side 10 cm is placed normal to a magnetic field which increases at the rate of 1 Wb/m^2. The induced e.m.f. is
(a) 0.1 V (b) 5.0 V
(c) 0.5 V (d) 1.0 V

39. The colours seen in the reflected white light from a thin oil film are due to
(a) diffraction (b) interference
(c) polarisation (d) dispersion

40. The moon has a mass of 1/81 that of the earth and a radius of 1/4 that of the earth. The escape speed from the surface of the earth is 11.2 km/s. The escape speed from the surface of the moon is:
(a) 1.25 km/s (b) 2.49 km/s
(c) 3.7 km/s (d) 5.6 km/s

41. Kerosene oil rises up in a wick of a lantern because of
(a) diffusion of the oil through the wick
(b) capillary action
(c) buoyant force of air
(d) the gravitational pull of the wick

42. When light falls on a given plate at angle of incidence of 60°, the reflected and refracted rays are found to be normal to each other. The refractive index of the matertial of the plate is then
(a) 0.866 (b) 1.5
(c) 1.732 (d) 2

43. A ball is dropped downwards, after 1 sec another ball is dropped downwards from the same point. What is the distance between them after 3 sec?
(a) 25m (b) 20m
(c) 50m (d) 9.8m

44. The range of wavelength of visible light is
(a) 10 Å to 100 Å
(b) 4000 Å to 8000 Å
(c) 8000 Å to 10,000 Å
(d) 10,000 Å to 15,000 Å

45. The service area of space wave communication increases by
(a) increasing the height of transmitting antenna
(b) decreasing the height of receiving antenna
(c) increasing the height of both transmitting and receiving antenna
(d) decreasing the distance between transmitting and receiving antenna

MATHEMATICS

46. On the set N of all natural numbers define the relation R by aRb if and only if the GCD of a and b is 2, then R is
(a) reflexive, but not symmetric
(b) symmetric only
(c) reflexive and transitive
(d) reflexive, symmetric and transitive

47. If $\sin\theta + \cos\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$, then
(a) $2n = m(n^2 - 1)$ (b) $2m = n(m^2 - 1)$
(c) $2n = m(m^2 - 1)$ (d) None of these

48. $\tan^{-1}\left(\frac{a_1x - y}{a_1y + x}\right) + \tan^{-1}\left(\frac{a_2 - a_1}{a_1a_2 + 1}\right)$

$+ \tan^{-1}\left(\frac{a_3 - a_2}{a_2a_3 + 1}\right) + \ldots + \tan^{-1}\left(\frac{a_n - a_{n-1}}{a_{n-1}a_n + 1}\right)$

$+ \tan^{-1}\frac{1}{a_n}$ can be equal to
(a) $\tan^{-1} xy$ (b) $\tan^{-1}\frac{x}{y}$
(c) $\tan^{-1}\frac{y}{x}$ (d) None of these

49. The foci of the ellipse $25(x+1)^2 + 9(y+2)^2 = 225$ are at :
(a) (–1, 2) and (–1, –6)
(b) (–2, 1) and (–2, 6)
(c) (–1, –2) and (–2, –1)
(d) (–1, –2) and (–1, –6)

50. A triangle ABC is right angled at A has points A and B as (2, 3) and (0, –1) respectively. If BC = 5, then point C may be
(a) (–4, 2) (b) (4, –2)
(c) (0, 4) (d) (0, –4)

51. The direction cosines of two lines are related by $l + m + n = 0$ and $al^2 + bm^2 + cn^2 = 0$. The lines are parallel if
(a) $a + b + c = 0$ (b) $a^{-1} + b^{-1} + c^{-1} = 0$
(c) $a = b = c$ (d) None of these

52. If the vectors $\vec{a}$ $2\hat{i}$ $\hat{j}$ $4\hat{k}$, $\vec{b}$ $4\hat{i} - 2\hat{j}$ $3\hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} - \lambda\hat{k}$ are coplanar, then the value of λ is equal to
(a) 2 (b) 1
(c) 3 (d) –1

53. If ${}^{n}C_{r-1}$ ${}^{n\ 1}C_{r-1}$ ${}^{n\ 2}C_{r-1}$ .... ${}^{2n}C_{r-1}$ $= {}^{2n\ 1}C_{r^2-132} - {}^{n}C_{r}$, then the value of r and the minimum value of n are
(a) 10 (b) 11
(c) 12 (d) 13

54. If the p^{th}, q^{th}, and r^{th} terms of an H.P. be respectively a, b and c, then
$(q - r)\,bc + (r - p)\,ca + (p - q)\,ab =$
(a) 0 (b) $ap + bq + cr$
(c) $\frac{a}{p}\ \frac{b}{q}\ \frac{c}{r}$ (d) None of these

55. If the second term in the expansion $\left(\sqrt[13]{a} + \frac{a}{\sqrt{a^{-1}}}\right)^n$ is $14a^{5/2}$, then $\frac{{}^{n}C_3}{{}^{n}C_2} =$
(a) 4 (b) 3
(c) 12 (d) 6

56. If $A=\begin{pmatrix} p & q \\ 0 & 1 \end{pmatrix}$, then $A^8=\begin{pmatrix} p^8 & q\left(\frac{p^8-1}{p-1}\right) \\ 0 & k \end{pmatrix}$.

The value of k is

(a) 1 (b) 0
(c) 2 (d) –1

57. Let $A=\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ and

$10B=\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverse of matrix A, then α is

(a) 5 (b) –1
(c) 2 (d) –2

58. Let ω be a complex number such that $2\omega+1=z$ where $z=\sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix}=3k$, then k is equal to :

(a) 1 (b) $-z$
(c) z (d) -1

59. If $y=\sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x=1$ is equal to :

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$
(c) 1 (d) $\sqrt{2}$

60. If $f(x)=\begin{cases} (1+|\sin x|)^{a/|\sin x|}, & -\pi/6<x<0 \\ b & x=0 \\ e^{\tan 2x/\tan 3x} & 0<x<\pi/6 \end{cases}$ is a continuous function on $(-\pi/6, \pi/6)$, then

(a) $a=2/3, b=e^2$ (b) $a=1/3, b=e^{1/3}$
(c) $a=2/3, b=e^{2/3}$ (d) None of these

61. If $y=e^{a\cos^{-1}x}$, $-1\le x\le 1$, then $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y$ is equal to

(a) 0 (b) 1
(c) –1 (d) a

62. The curve $y=ax^3+bx^2+cx+5$ touches the x-axis at $P(-2, 0)$ and cuts the y-axis at a point Q where its gradient is 3. Then the value of b/ac is

(a) 9/2 (b) 1/2
(c) 2 (d) 3

63. A box, constructed from a rectangular metal sheet, is 21 cm by 16 cm by cutting equal squares of sides x from the corners of the sheet and then turning up the projected portions. The value of x so that volume of the box is maximum is

(a) 1 (b) 2
(c) 3 (d) 4

64. The mean income of a group of workers is $\overline{X}$ and that of another group is $\overline{Y}$. If the number of workers in the second group is 10 times the number of workers in the first group, then the mean income of combined group is

(a) $\frac{\overline{X}+10\overline{Y}}{3}$ (b) $\frac{\overline{X}+10\overline{Y}}{11}$
(c) $\frac{10\overline{X}+10\overline{Y}}{Y}$ (d) $\frac{X+10\overline{Y}}{9}$

65. A book has 999 pages. If a page is opened at random the probability that the sum of the digits in its number is 9, is

(a) $\frac{55}{999}$ (b) $\frac{22}{333}$
(c) $\frac{117}{999}$ (d) $\frac{4}{37}$

66. If the vector $p\hat{i}\ \hat{j}\ \hat{k}, \hat{i}\ q\hat{j}\ \hat{k}$ and $\hat{i}\ \hat{j}\ r\hat{k}$ $p\neq q\neq r\neq 1$ are coplanar, then the value of $pqr-(p+q+r)$ is

(a) 2 (b) 0
(c) -1 (d) -2

67. $\int \frac{3\ 2\cos x}{2\ 3\cos x^2}dx$ is equal to :

(a) $\left(\frac{\sin x}{3\cos x+2}\right)+c$ (b) $\left(\frac{2\cos x}{3\sin x+2}\right)+c$
(c) $\left(\frac{2\cos x}{3\cos x+2}\right)+c$ (d) $\left(\frac{2\sin x}{3\sin x+2}\right)+c$

68. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 \quad 2^x} dx$ is :

(a) $\frac{\pi}{2}$ (b) 4π

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

69. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is :

(a) 9 (b) 36

(c) 18 (d) $\frac{27}{4}$

70. The general solution of the differential equation $\frac{dy}{dx} \quad \sin(x \quad y) \quad \sin(x - y)$ is

(a) $\log \tan y + \sin x = C$

(b) $\log \tan \frac{y}{2} \quad 2 \sin x \quad C$

(c) $\tan \frac{y}{2} \quad \log \sin x \quad C$

(d) None of these

Hints & Explanations

1. (a) Wriggle means 'avoid (something) by devious means'. Shirk means the same.
2. (c) Lout means 'an uncouth and aggressive man or boy'.
3. (c) Dormitory -a large bedroom for a number of people in a school or institution.
4. (c) Matricide-the killing of one's mother.
5. (d) 6. (d) 7. (c) 8. (d)
9. (a) Replace 'if' with 'as if'.

   As if: used to describe how a situation seems to be; used when you are giving a possible explanation for something or saying that something appears to be the case when it is not.

   For e.g. They stared at me as if I was crazy.
10. (b) Replace 'have to be' with 'had to be'. The sentence is in 'Past Indefinite Tense' which will take past form of the verb with it.
11. (c) Some men like to climb mountains because they know the trick of climbing.
12. (d) To climb mountains is often difficult because paths are steep and uneven.
13. (b) 14. (b) 15. (b) 16. (c)
17. (d) 'a last resort' means 'the only option'.
18. (b) 'swung' is a 'V2 form' of 'swing'. 'swing into action' means 'to start doing something very quickly'.
19. (b) 'Tell one from the other' means 'to distinguish'.
20. (c) The passive voice of the given Sentence is - it is time for the books to be bought.
21. (b) According to law of conservation of momentum,

$$100v = -\frac{10}{1000}\times 10\times 800$$

i.e., $v = 0.8\ ms^{-1}$.

22. (b)
23. (b) We know that $Q = \frac{T_H - T_L}{R}$

Also Thermal resistance, $R = \frac{\ell}{KA}$ $\frac{\ell}{K\pi r^2}$

Heat flow will be maximum when thermal resistance is minimum. From given option

(i) $r = 2r_0, \ell = 2\ell_0$

$\therefore$ R $\frac{2\ell_0}{K\pi(2r_0)^2}$ $\frac{\ell_0}{2K\pi r_0^2}$

(ii) $r = 2r_0, \ell = \ell_0$

$\therefore$ R $\frac{\ell_0}{K\pi(2r_0)^2}$ $\frac{\ell_0}{4K\pi r_0^2}$

(iii) $r = r_0, \ell = 2\ell_0$

$\therefore$ R $\frac{2\ell_0}{K\pi r_0^2}$ $\frac{2\ell_0}{K\pi r_0^2}$

(iv) $r = r_0, \ell = \ell_0$

$\therefore$ R $\frac{\ell_0}{K\pi r_0^2}$ $\frac{\ell_0}{K\pi r_0^2}$

It is clear that for option (b) resistance is minimum, hence heat flow will be maximum.

24. (b) [Torque] = [Force] [distance] = ML^2T^{-2}
25. (b) $P.E = \frac{1}{2}M\omega^2x^2$,

Total energy $E = \frac{1}{2}M\omega^2A^2$

26. (b) Since work function for a metal surface is

$$W = \frac{hc}{\lambda_0}$$

where λ_0 is threshold wavelength or cut-off wavelength for a metal surface.

here $W = 4.125$ eV

$= 4.125 \times 1.6 \times 10^{-19}$ Joule

so λ_0 $\frac{6.6\times10^{-34}\times3\times10^8}{4.125\times1.6\times10^{-19}}$ 3000Å

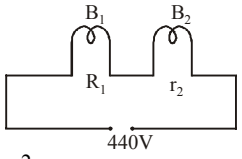
27. (d) In a refrigerator, the heat dissipated in the atmosphere is more than that taken from the cooling chamber, therefore the room is heated. If the door of a refrigerator is kept open.
28. (d) At poles, $\delta = 90°$.
29. (a) Isotones means equal number of neutrons i.e., $(A - Z) = 74 - 34 = 71 - 31 = 40$.

30. (c) The current upto which bulb of marked 25W -220V, will not fuse $I_1 = \frac{W_1}{V_1} = \frac{25}{220}$ Amp

Similarly, $I_2 = \frac{W_2}{V_2} = \frac{100}{220}$ Amp

The current flowing through the circuit

$I = \frac{440}{R_{eff}}$



$R_{eff} = R_1 + R_2$

$R_1 = \frac{V_1^2}{P_1} = \frac{(220)^2}{25}$;

$R_2 = \frac{V_2^2}{P} = \frac{(220)^2}{100}$

$I = \frac{440}{\frac{(220)^2}{25} + \frac{(220)^2}{100}} = \frac{440}{(220)^2\left[\frac{1}{25} + \frac{1}{100}\right]}$

$= \frac{40}{220}$Amp

$\because I_1\left(=\frac{25}{220}A\right) < I\left(=\frac{40}{220}A\right) < I_2\left(=\frac{100}{200}A\right)$

Thus the bulb marked 25W-220 will fuse.

31. (c) $E = vB = 2 \times 10^3 \times 1.5 = 3 \times 10^3$ N/C

32. (c) Centripetal acc. $= \omega^2 r = 4\pi^2 v^2 r$

$= 4 \times (3.14)^2 \times \frac{120}{60} \times \frac{30}{100} \quad 23.7 \text{ ms}^{-2}$

$[\because \omega = 2\pi v]$

33. (b) U $\frac{F^2}{2K}$ $\frac{T^2}{2K}$

34. (a) Power, $P = nh\nu$, n = no., of photons per second

$\Rightarrow \nu = \frac{P}{nh} = \frac{4 \times 10^3}{10^{20} \times 6.63 \times 10^{-34}}$

$= 6 \times 10^{16}$ Hz

35. (d) A transistor is a current operating device in which the emitter current controls the collector current

36. (a) We may consider the entire mass of the stick to be concentrated as a point mass at the centre of mass of the stick. The centre of mass moves as a projectile, it will move along a parabolic path.

37. (b)

38. (b) $e \quad \frac{d\phi}{dt} \quad \frac{d}{dt}$ NBA NA $\frac{dB}{dt}$

$= 500 \times 10^{-2} \times 1 = 5.0$ V

39. (b) 40. (b)

41. (b) Kerosene oil rises up in wick of a lantern because of capillary action. If the surface tension of oil is zero, then it will not rise, so oil rises up in a wick of a lantern due to surface tension.

42. (c) Here $i = 60°$. As the angle between reflected and refracted ray is 90°, then $i + r = 90$

or $r = 30°$

Now $\mu \quad \frac{\sin i}{\sin r} \quad \frac{\sin 60}{\sin 30} \quad \frac{\sqrt{3}/2}{1/2}$

$\sqrt{3} \quad 1.732$

The angle for which $i + r = 90°$, called Brewster' Angle.

43. (a) $S = ut + \frac{1}{2}at^2$ here $a = g$

For first body $u_1 = 0 \Rightarrow S_1 = \frac{1}{2}g \times 9$

For second body $u_2 = 0 \Rightarrow S_2 = \frac{1}{2}g \times 4$

So difference between them after 3 sec.

$= S_1 - S_2 = \frac{1}{2}g \times 5$

If $g = 10$ m/sec^2 then $S_1 - S_2 = 25$ m.

44. (b) The range of visible radiations is 4000Å to 8000 Å

45. (c) Maximum range of space wave propagation

$d = \frac{4}{3} \times 1.23\left[\sqrt{H_t} \quad \sqrt{H_r}\right]$

$\Rightarrow \quad d \propto H_t; \quad d \propto H_r$

$\therefore$ d increases if H_t and H_r i.e. height of transmitting and receiving antenna increases.

46. (b) The relation R is defined by aRb, if and only if the GCD of a and b is 2.

(i) aRa, then GCD of a and a is a $\therefore$ R is not reflexive

(ii) $aRb \Rightarrow bRa$

If GCD of a and b is 2, then GCD of b and a is 2.

$\therefore$ R is symmetric.

(iii) a R b, bR$c \Rightarrow c$Ra
If GCD of a and b is 2 and GCD of b and c is 2, then it need, not to be GCD of c and a is 2. $\therefore$ R is not transitive

47. (b) We have, $\sin\theta + \cos\theta = m$ ....(i)
and $\sec\theta + \text{cosec}\,\theta = n$

$$\Rightarrow \quad \frac{1}{\cos\theta} \quad \frac{1}{\sin\theta} \quad n \Rightarrow \frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta} \quad n$$

$$\Rightarrow \quad \frac{m}{\cos\theta\sin\theta} \quad n \qquad [\text{Using (i)}]$$

$$\Rightarrow \quad \cos\theta\sin\theta \quad \frac{m}{n} \qquad ...(ii)$$

Squaring (i), we get
$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = m^2$

$$\Rightarrow \quad \frac{2m}{n} = m^2 - 1 \;\Rightarrow\; 2m = n(m^2-1)$$

48. (b) The given expression

$$\tan^{-1}\frac{a_1 - \frac{y}{x}}{1 \quad a_1\frac{y}{x}} \quad (\tan^{-1}a_2 - \tan^{-1}a_1)$$

$$+(\tan^{-1}a_3 - \tan^{-1}a_2) + ... +$$

$$(\tan^{-1}a_n - \tan^{-1}a_{n-1} \quad \tan^{-1}\frac{1}{a_n}$$

$$= \tan^{-1}a_1 - \tan^{-1}\frac{y}{x} \quad \tan^{-1}a_n$$

$$-\tan^{-1}a_1 \quad \tan^{-1}\frac{1}{a_n}$$

$$(\tan^{-1}a_n \quad \cot^{-1}a_n) - \tan^{-1}\frac{y}{x}$$

$$= \frac{\pi}{2} - \tan^{-1}\frac{y}{x} \quad \cot^{-1}\frac{y}{x} \quad \tan^{-1}\frac{x}{y}$$

49. (a) The given eq. is
$25(x+1)^2 + 9(y+2)^2 = 225$

$$\Rightarrow \quad \frac{(x \quad 1)^2}{9} \quad \frac{(y \quad 2)^2}{25} \quad 1$$

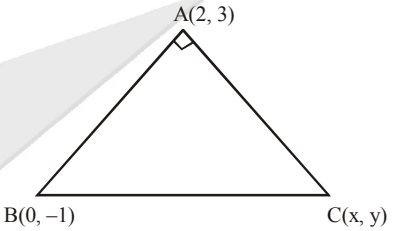
centre of the ellipse is (–1, –2) and $a = 3$, $b = 5$, so that $a < b$.

$$\Rightarrow \quad 3 = 5\sqrt{1-e^2} \Rightarrow e^2 = 1 - \frac{9}{25} \quad \frac{16}{25}$$

$$\Rightarrow \quad e \quad \frac{4}{5}$$

Hence, foci are $\left(-1, -2-5\times\frac{4}{5}\right)$ and $\left(-1, -2+5\times\frac{4}{5}\right)$, *i.e.,* foci are (–1, –6) and (–1, 2).

50. (c) Slope of AB $= 2 \Rightarrow$ Slope of AC $= -\frac{1}{2}$



$$\Rightarrow \quad \frac{y-3}{x-2} = -\frac{1}{2} \Rightarrow x + 2y - 8 \quad 0 \quad ....(i)$$

Also, $x^2 + (y+1)^2 = 25$
$\Rightarrow \quad (8-2y)^2 + (y+1)^2 = 25 \qquad [\text{from (i)}]$
$\Rightarrow \quad y = 2$ or 4 and correspondingly $x = 4$ and $x = 0$. Hence, C is (0, 4) or (4, 2).

51. (b) For $n = -(l+m)$, the second relation gives
$al^2 + bm^2 + c(l+m)^2 = 0$
or $\quad (a+c)l^2 + 2clm + (b+c)m^2 = 0.$
For parallel lines, the two roots must be equal
$\Rightarrow \quad 4c^2 - 4(b+c)(a+c) = 0$
$\Rightarrow \quad ab + bc + ca = 0$

52. (b) $\begin{vmatrix} 2 & 1 & 4 \\ 4 & -2 & 3 \\ 2 & -3 & -\lambda \end{vmatrix} = 0$

$\Rightarrow 2(2\lambda+9) - 1(-4\lambda-6) + 4(-12+4) = 0$
$\Rightarrow \lambda = 1$

53. (c) Given,
${}^nC_r + {}^nC_{r-1} + {}^{n+1}C_{r-1} + {}^{n+2}C_{r-1} + ... + {}^{2n}C_{r-1}$
$= {}^{2n\ 1}C_{r^2-132}$

$$\Rightarrow \quad {}^{n\ 1}C_r \quad {}^{n\ 1}C_{r-1} \quad ..... \quad {}^{2n}C_{r-1}$$

$${}^{2n\ 1}C_{r^2-132}$$

....

$$\Rightarrow \quad {}^{2n}C_r \quad {}^{2n}C_{r-1} \quad {}^{2n\ 1}C_{r^2-132}$$

$\Rightarrow \quad {}^{2n-1}C_r = {}^{2n-1}C_{r^2-132}$

$\Rightarrow \quad r^2 - r - 132 = 0$

$\Rightarrow \quad (r-12)(r+11) = 0$

$\Rightarrow \quad r = 12 \Rightarrow n \geq 12$

So, minimum value of $n = 12$.

54. (a) The p^{th}, q^{th}, r^{th} tems of the corresponding A.P. are $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$.

If A be the first term and D be the common difference of the corresponding A.P. then

$$A + (p-1)D = \frac{1}{a} \quad ...(i)$$

$$A + (q-1)D = \frac{1}{b} \quad ...(ii)$$

$$A + (r-1)D = \frac{1}{c} \quad ...(iii)$$

Subtracting (ii) from (ii) we get

$$(p-q)D = \frac{1}{a} - \frac{1}{b} \quad \frac{b-a}{ab}$$

or $\quad (p-q)\,ab \quad \frac{b-a}{D}$

Similarly, $(q-r)\,bc \quad \frac{c-b}{D}$

and $(r-p)\,ca \quad \frac{a-c}{D}$

$\therefore \quad (q-r)bc + (r-p)ca + (p-q)ab$

$\frac{c-b+a-c+b-a}{D} \quad 0$

55. (a) We have, $T_2 = 14\,a^{\frac{5}{2}}$

$\Rightarrow \quad {}^{n}C_1 (a^{\frac{1}{13}})^{n-1} (a^{\frac{3}{2}}) \quad 14a^{\frac{5}{2}}$

$\Rightarrow \quad na^{\frac{n-1}{13}} \quad \frac{3}{2} \quad 14a^{\frac{5}{2}} \Rightarrow n = 14$

$\Rightarrow \quad \frac{{}^{n}C_3}{{}^{n}C_2} \quad \frac{{}^{14}C_3}{{}^{14}C_2} \quad \frac{12}{3} \quad 4$

56. (a) $A^2 = \begin{pmatrix} p & q \\ 0 & 1 \end{pmatrix}\begin{pmatrix} p & q \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} p^2 & pq+q \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} p^2 & q(p+1) \\ 0 & 1 \end{pmatrix}$

$A^3 = \begin{pmatrix} p & q \\ 0 & 1 \end{pmatrix}\begin{pmatrix} p^2 & pq+q \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} p^3 & q(p^2+p+1) \\ 0 & 1 \end{pmatrix}$

Similarly, $A^4 = \begin{pmatrix} p^4 & q(p^3+p^2+p+1) \\ 0 & 1 \end{pmatrix}$

and so on.

$\therefore \quad A^8 = \begin{pmatrix} p^8 & q(p^7+p^8+...+1) \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} p^8 & q\left(\frac{p^8-1}{p-1}\right) \\ 0 & 1 \end{pmatrix}$

57. (a) Given that $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$\Rightarrow \quad B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

Also since, $B = A^{-1} \Rightarrow AB = I$

$\Rightarrow \quad \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \quad \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-2 \\ 0 & 10 & -5+\alpha \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \quad \frac{5-\alpha}{10} = 0 \Rightarrow \alpha = 5$

58. (b) Given $2\omega + 1 = z; \quad z = \sqrt{3}i$

$\Rightarrow \quad \omega = \frac{\sqrt{3}i - 1}{2}$

$\Rightarrow \quad \omega$ is complex cube root of unity

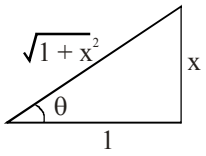
Applying $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 3 & 0 & 0 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$$

$= 3(-1-\omega-\omega) = -3(1+2\omega) = -3z$

$\Rightarrow \quad k = -z$

59. (a) Let $y = \sec(\tan^{-1} x)$ and $\tan^{-1} x = \theta$.



$\Rightarrow \quad x = \tan\theta$

Thus, we have $y = \sec\theta$

$\Rightarrow \quad y = \sqrt{1 \quad x^2}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{1 \quad x^2}} \cdot 2x$

At $x = 1, \quad \dfrac{dy}{dx} \quad \dfrac{1}{\sqrt{2}}$ .D

60. (c) $b = f(0) = \lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0} e^{\tan 2x / \tan 3x}$

$= \lim\limits_{x \to 0} e^{\frac{\tan 2x}{2x} \times \frac{3x}{\tan 3x} \times \frac{2}{3}} = e^{2/3}.$

Also, $b = f(0) = \lim\limits_{x \to 0^-} f(x)$

$= \lim\limits_{x \to 0^-} (1 + |\sin x|)^{\frac{1}{|\sin x|} \times a} = e^a.$

Therefore, $a = 2/3$ and $b = e^{2/3}$.

61. (a) $y = e^{a \cos^{-1} x}$

$\therefore \quad \dfrac{dy}{dx} \quad e^{a\cos^{-1} x} \dfrac{-a}{\sqrt{1-x^2}} \quad \dfrac{-ay}{\sqrt{1-x^2}}$

or $\quad (1-x^2)\left(\dfrac{dy}{dx}\right)^2 = a^2 y^2$

Differentiating both sides with respect to x, we get

$$\left(\frac{dy}{dx}\right)^2 (-2x) + (1-x^2) \times 2 \frac{dy}{dx} \frac{d^2y}{dx^2}$$

$$a^2 \quad 2y \frac{dy}{dx}$$

or $\quad -x\left(\dfrac{dy}{dx}\right)^2 + (1-x^2)\dfrac{d^2y}{dx^2}$

$$a^2 \quad y\left[\frac{dy}{dx} \neq 0\right]$$

or $\quad (1-x^2)\dfrac{d^2y}{dx^2} - \left(\dfrac{dy}{dx}\right)^2 - a^2 y = 0.$

62. (b) Since the curve $y = ax^3 + bx^2 + cx + 5$ touches x-axis at $P(-2, 0)$, the x-axis is the tangent at $(-2, 0)$. The curve meets the y-axis in $(0, 5)$. We have

$$\frac{dy}{dx} = 3ax^2 + 2bx + c$$

or $\quad \left.\dfrac{dy}{dx}\right|_{(0,\ 5)} = 0 + 0 + c = 3 \qquad$ (Given)

or $\quad c = 3 \qquad ...(1)$

and $\quad \left.\dfrac{dy}{dx}\right|_{(-2,\ 0)} = 0$

or $\quad 12a - 4b + c = 0$

or $\quad 12a - 4b + 3 = 0 \qquad$ [From (1)] $\quad ....(2)$

and $(-2, 0)$ lies on the curve. Then

$0 = -8a + 4b - 2c + 5$

or $\quad 0 = -8a + 4b - 1 \qquad (\because c = 3)$

or $\quad 8a - 4b + 1 = 0 \qquad ...(3)$

From (2) and (3), we get $a = -\dfrac{1}{2}, \ b = -\dfrac{3}{4}$

Hence, $a = -\dfrac{1}{2}, \ b = -\dfrac{3}{4}$, and $c = 3$.

63. (c) The dimensions of the box after cutting equal squares of side x on the corner will be $21 - 2x$, $16 - 2x$, and height x.

$V = x(21-2x)(16-2x)$

$= x(336 - 74x + 4x^2) = 4x^3 + 336x - 74x^2$

$\therefore \quad \dfrac{dV}{dx} \quad 12x^2 \quad 336 - 148x$

$\dfrac{dV}{dx} = 0$ gives $x = 3$ for which $\dfrac{d^2V}{dx^2}$ is –ve and hence, maximum.

64. (b) Let the number of workers in two groups be n and $10\,n$ then the mean income

$$\frac{n\bar{x} \quad 10n\bar{Y}}{n \quad 10n} \quad \frac{\bar{x} \quad 10\bar{Y}}{11}$$

65. (a) Total no. of cases = 999
Favourable cases.
The page number can be with all identical digits,
i.e. $333 \longrightarrow 1$
The identical and one different like 009, 117, 225 and
$441 \longrightarrow 4 \times \frac{3!}{2!} = 12$
All different digits like 018, 027, 036, 045, 126, 135, 234 $\longrightarrow 7 \times 3! = 42$
So favourable cases = 55
Desired probability = $\frac{55}{999}$

66. (d) The given vectors are collinear if

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} \quad 0$$

$\Rightarrow \quad p(qr-1)+1(1-r)+1(1-q)=0$
$\Rightarrow \quad pqr-p+1-r+1-q=0$
$\Rightarrow \quad pqr-(p+q+r)=-2$

67. (a) $I = \int \frac{3 \quad 2\cos x}{2 \quad 3\cos x^{\,2}} dx$

Multiplying the numerator and denominator by $\text{cosec}^2 x$

$$I = \int \frac{3\text{cosec}^2 x \quad 2\cot x \,\text{cosec}\, x}{2\,\text{cosec}\, x \quad 3\cot x^{\,2}} dx$$

$$= -\int \frac{-3\text{cosec}^2 x - 2\cot x \,\text{cosec}\, x}{2\,\text{cosec}\, x \quad 3\cot x^{\,2}} dx$$

Put $2\text{cosec}\, x + 3\cot x = t$
$[-2\cot x \,\text{cosec}\, x - 3\text{cosec}^2 x]\, dx = dt$

$$\therefore \quad I = -\int \frac{dt}{t^2} = -\left(\frac{-1}{t}\right) \quad c$$

$$\frac{1}{2\cos ec\, x \quad 3\cot x} \quad c \quad \frac{\sin x}{2 \quad 3\cos x} \quad c$$

68. (c) Let, $I \quad \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 \quad 2^x} dx$ ...(i)

Using, $\int_a^b f(x)dx \quad \int_a^b f(a \quad b-x)dx$, we get:

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 \quad 2^{-x}} dx \qquad ...(ii)$$

Adding (i) and (ii), we get;

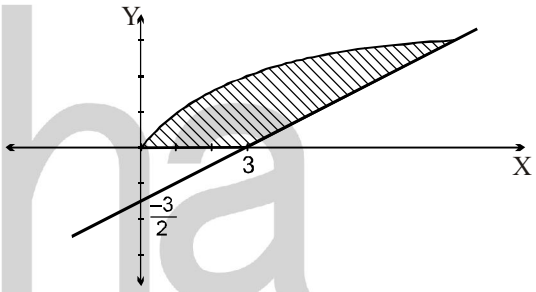
$$2I \quad \int_{-\pi/2}^{\pi/2} \sin^2 x \, dx$$

$$\Rightarrow \quad 2I \quad 2 \int_0^{x/2} \sin^2 x \, dx$$

$$\Rightarrow \quad 2I = 2 \times \frac{\pi}{4} \Rightarrow I = \frac{\pi}{4}$$

69. (a) Given curves are

$y = \sqrt{x}$ ...(1)
and $2y - x + 3 = 0$ ...(2)
On solving both we get $y = -1, 3$



$$\text{Required area} = \int_0^3 (2y+3) - y^2 \quad dy$$

$$= y^2 + 3y - \frac{y^3}{3}\Big|_0^3 \quad 9.$$

70. (b) The equation is,

$$\frac{dy}{dx} = \sin(x-y) - \sin(x \quad y)$$
$$= 2\cos x \sin(-y)$$

$$\Rightarrow \quad \frac{dy}{\sin y} \quad 2\cos x dx \quad 0$$

$$\Rightarrow \quad \int \cos ec\, y \, dy \quad 2\int \cos x \, dx \quad C$$

$$\Rightarrow \quad \log \tan \frac{y}{2} \quad 2\sin x \quad C$$