JEE Main - 2016

(Held on 3rd April, 2016)

Time : 3 Hours • **Each correct answer has + 4 marks • Each wrong answer has – 1 mark.** **Max. Marks : 360**

Section - 1

PHYSICS

1. A particle performs simple harmonic mition with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is :

(1) $A\sqrt{3}$ (2) $\frac{7A}{3}$

(3) $\frac{A}{3}\sqrt{41}$ (4) 3A

2. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is :

(1) $\alpha = \frac{\beta}{1+\beta}$ (2) $\alpha = \frac{\beta^2}{1+\beta^2}$

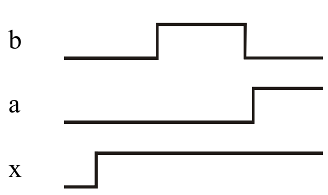
(3) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$ (4) $\alpha = \frac{\beta}{1-\beta}$

3. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s, and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be:

(1) 92 ± 1.8 s (2) 92 ± 3s

(3) 92 ± 2 s (4) 92 ± 5.0 s

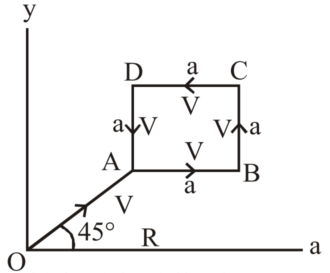
4. If a, b, c, d are inputs to a gate and x is its output, then, as per the following time graph, the gate is :



(1) OR (2) NAND

(3) NOT (4) AND

5. A particle of mass *m* is moving along the side of a square of side '*a*', with a uniform speed *v* in the *x*-*y* plane as shown in the figure :



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

(1) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

(2) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A.

(3) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B.

(4) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

6. Choose the correct statement :

(1) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(2) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

(3) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(4) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

7. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be:

(1) $= v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (2) $= v\left(\frac{3}{4}\right)^{\frac{1}{2}}$

(3) $> v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (4) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

8. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is:

(1) $\frac{\pi^2}{16}$ (2) $\frac{\pi^2}{8\sqrt{2}}$

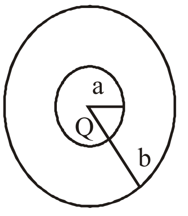
(3) $\frac{\pi^2}{8}$ (4) $\frac{\pi^2}{16\sqrt{2}}$

9. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now :

(1) 2f (2) f

(3) $\frac{f}{2}$ (4) $\frac{3f}{4}$

10. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), have volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is :



(1) $\frac{2Q}{\pi\left(a^2-b^2\right)}$ (2) $\frac{2Q}{\pi a^2}$

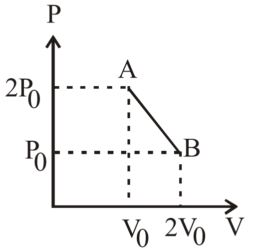
(3) $\frac{Q}{2\pi a^2}$ (4) $\frac{Q}{2\pi\left(b^2-a^2\right)}$

11. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to :

(1) 0.044 H (2) 0.065 H

(3) 80 H (4) 0.08 H

12. 'n' moles of an ideal gas undergoes a process A $\rightarrow$ B as shown in the figure. The maximum temperature of the gas during the process will be :



(1) $\frac{9P_0V_0}{2nR}$ (2) $\frac{9P_0V_0}{nR}$

(3) $\frac{9P_0V_0}{4nR}$ (4) $\frac{3P_0V_0}{2nR}$

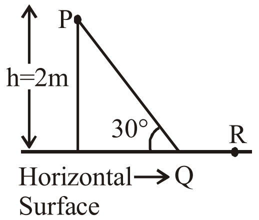
13. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8 \text{ ms}^{-2}$:

(1) 9.89×10^{-3} kg (2) 12.89×10^{-3} kg

(3) 2.45×10^{-3} kg (4) 6.45×10^{-3} kg

14. A point particle of mass m, moves long the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.

The value of the coefficient of friction μ and the distance x (= QR), are, respectively close to :



(1) 0.29 and 3.5 m (2) 0.29 and 6.5 m

(3) 0.2 and 6.5 m (4) 0.2 and 3.5 m

15. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by :

(1) Linear increase for Cu, exponential decrease of Si.

(2) Linear decrease for Cu, linear decrease for Si.

(3) Linear increase for Cu, linear increase for Si.

(4) Linear increase for Cu, exponential increase for Si.

16. Arrange the following electromagnetic radiations per quantum in the order of increasing energy :

A : Blue light B : Yellow light

C : X-ray D : Radiowave.

(1) C, A, B, D (2) B, A, D, C

(3) D, B, A, C (4) A, B, D, C

17. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a currect of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is :

(1) 0.1 Ω (2) 3Ω

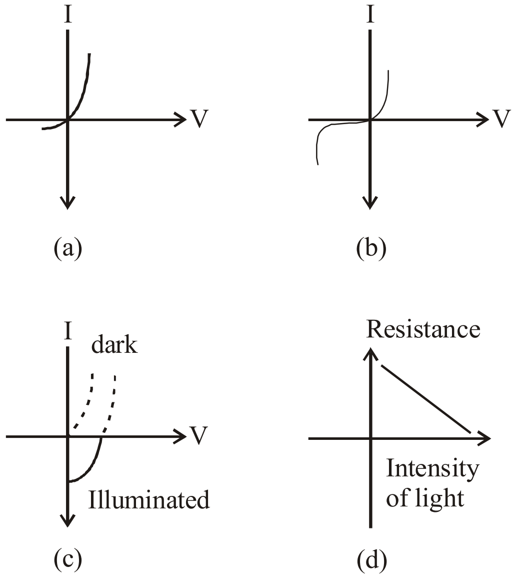
(3) 0.01Ω (4) 2Ω

18. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed number of A and B nuclei will be :

(1) 1 : 4 (2) 5 : 4

(3) 1 : 16 (4) 4 : 1

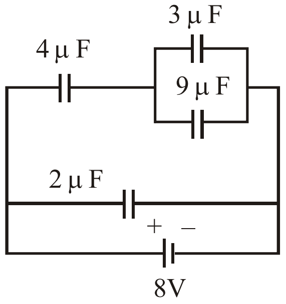
19. Identify the semiconductor devices whose characteristics are given below, in the order (a), (b), (c), (d) :



(a) (b) (c) (d)

(1) Solar cell, Light dependent resistance, Zener diode, simple diode
(2) Zener diode, Solar cell, simple diode, Light dependent resistance
(3) Simple diode, Zener diode, Solar cell, Light dependent resistance
(4) Zener diode, Simple diode, Light dependent resistance, Solar cell

20. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μ F and 9 μ F capacitors), at a point distance 30 m from it, would equal :



(1) 420 N/C (2) 480 N/C
(3) 240 N/C (4) 360 N/C

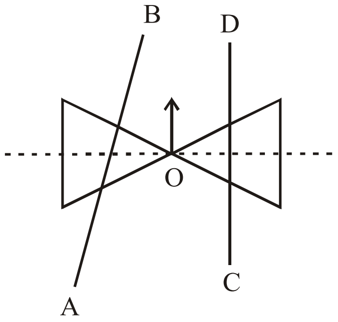
21. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; h < < R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.)

(1) $\sqrt{gR/2}$ (2) $\sqrt{gR}\left(\sqrt{2}-1\right)$
(3) $\sqrt{2gR}$ (4) $\sqrt{gR}$

22. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that wen the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line?

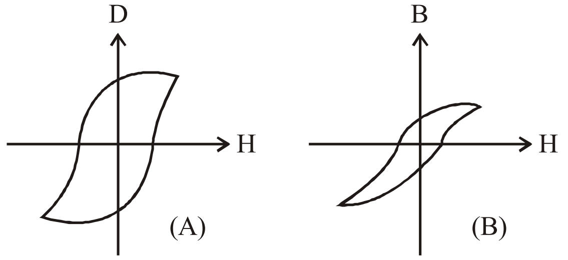
(1) 0.70 mm (2) 0.50 mm
(3) 0.75 mm (4) 0.80 mm

23. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD, which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and Cd (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to :



(1) go straight.
(2) turn left and right alternately.
(3) turn left.
(4) turn right.

24. Hysteresis loops for two magnetic materials A and B are given below :



(A) (B)

These materials are used to make magnets for elecric generators, transformer core and electromagnet core. Then it is proper to use :

(1) A for transformers and B for electric generators.
(2) B for electromagnets and transformers.
(3) A for electric generators and trasformers.
(4) A for electromagnets and B for electric generators.

25. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when :

(1) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

(2) $a = \dfrac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$

(3) $a = \dfrac{\lambda^2}{L}$ and $b_{min} = \left(\dfrac{2\lambda^2}{L}\right)$

(4) $a = \sqrt{\lambda l}$ and $b_{min} = \left(\dfrac{2\lambda^2}{L}\right)$

26. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the supports is :
(take $g = 10\ ms^{-2}$)

(1) $2\sqrt{2}s$ (2) $\sqrt{2}\,s$

(3) $2\pi\sqrt{2}\,s$ (4) 2 s

27. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) :

(1) $n = \dfrac{C_P - C}{C - C_V}$ (2) $n = \dfrac{C - C_V}{C - C_P}$

(3) $n = \dfrac{C_P}{C_V}$ (4) $n = \dfrac{C - C_P}{C - C_V}$

28. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears :

(1) 20 times taller (2) 20 times nearer

(3) 10 times taller (4) 10 times nearer

29. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot it was found thata ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index?

(1) 1.7 (2) 1.8

(3) 1.5 (4) 1.6

30. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20° C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively :

(1) 30°C; $\alpha = 1.85 \times 10^{-3}/°C$

(2) 55°C; $\alpha = 1.85 \times 10^{-2}/°C$

(3) 25°C; $\alpha = 1.85 \times 10^{-5}/°C$

(4) 60°C; $\alpha = 1.85 \times 10^{-4}/°C$

Section - 2

MATHEMATICS

31. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x$ and $x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is :

(1) $\pi - \dfrac{4\sqrt{2}}{3}$ (2) $\dfrac{\pi}{2} - \dfrac{2\sqrt{2}}{3}$

(3) $\pi - \dfrac{4}{3}$ (4) $\pi - \dfrac{8}{3}$

32. If $f(x) + 2f\left(\dfrac{1}{x}\right) = 3x\ , x \neq 0$ and

$S = \{x \in R : f(x) = f(-x)\}$; then S:

(1) contains exactly two elements.

(2) contains more than two elements.

(3) is an empty set.

(4) contains exactly one element.

33. The integral $\displaystyle\int \frac{2x^{12} + 5x^9}{\left(x^5 + x^3 + 1\right)^3} dx$ is equal to :

(1) $\dfrac{x^5}{2\left(x^5 + x^3 + 1\right)^2} + C$ (2) $\dfrac{-x^{10}}{2\left(x^5 + x^3 + 1\right)^2} + C$

(3) $\dfrac{-x^5}{\left(x^5 + x^3 + 1\right)^2} + C$ (4) $\dfrac{x^{10}}{2\left(x^5 + x^3 + 1\right)^2} + C$

where C is an arbitrary constant.

34. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then :

(1) $g'(0) = -\cos(\log 2)$

(2) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(3) g is not differentiable at $x = 0$

(4) $g'(0) = \cos(\log 2)$

35. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on:

(1) a hyperbola (2) a parabola

(3) a circle (4) an ellipse which is not a circle

36. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is :

(1) 6 (2) 5

(3) 3 (4) – 4

37. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is :

(1) 1 (2) $\dfrac{7}{4}$

(3) $\dfrac{8}{5}$ (4) $\dfrac{4}{3}$

38. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\sqrt{3}$

(3) $\frac{4}{3}$ (4) $\frac{4}{\sqrt{3}}$

39. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is :

(1) 243 (2) 729

(3) 64 (4) 2187

40. The Boolean Expression $(p\wedge \sim q)\vee q\vee(\sim p\wedge q)$ is equivalent to:

(1) $p\vee q$ (2) $p\vee \sim q$

(3) $\sim p\wedge q$ (4) $p\wedge q$

41. Consider

$$f(x)=\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x\in\left(0,\frac{\pi}{2}\right).$$

A normal to y = f(x) at $x=\frac{\pi}{6}$ also passes through the point :

(1) $\left(\frac{\pi}{6},0\right)$ (2) $\left(\frac{\pi}{4},0\right)$

(3) $(0,0)$ (4) $\left(0,\frac{2\pi}{3}\right)$

42. $\lim\limits_{n\to\infty}\left(\frac{(n+1)(n+2)\ldots 3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to:

(1) $\frac{9}{e^2}$ (2) $3\log 3-2$

(3) $\frac{18}{e^4}$ (4) $\frac{27}{e^2}$

43. If one of the diameters of the circle, given by the equation, $x^2+y^2-4x+6y-12=0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is:

(1) 5 (2) 10

(3) $5\sqrt{2}$ (4) $5\sqrt{3}$

44. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true ?

(1) E_1 and E_3 are independent.

(2) E_1, E_2 and E_3 are independent.

(3) E_1 and E_2 are independent.

(4) E_2 and E_3 are independent.

45. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is:

(1) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (2) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

46. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\ldots\ldots.,$ is $\frac{16}{5}m$, then m is equal to :

(1) 100 (2) 99

(3) 102 (4) 101

47. The system of linear equations

$x+\lambda y-z=0$

$\lambda x-y-z=0$

$x+y-\lambda z=0$

has a non-trivial solution for:

(1) exactly two values of λ.

(2) exactly three values of λ.

(3) infinitely many values of λ.

(4) exactly one value of λ.

48. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $lx+my-z=9$, then l^2+m^2 is equal to :

(1) 5 (2) 2

(3) 26 (4) 18

49. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is :

(1) 52nd (2) 58th

(3) 46th (4) 59th

50. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(1) $3a^2-34a+91=0$

(2) $3a^2-23a+44=0$

(3) $3a^2-26a+55=0$

(4) $3a^2-32a+84=0$

51. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then:

(1) $x=2r$ (2) $2x=r$

(3) $2x=(\pi+4)r$ (4) $(4-\pi)x=\pi r$

52. Let $p=\lim\limits_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) 2 (4) 1

53. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is:

(1) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(2) $x^2 + y^2 - 4x + 9y + 18 = 0$

(3) $x^2 + y^2 - 4x + 8y + 12 = 0$

(4) $x^2 + y^2 - x + 4y - 12 = 0$

54. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)\,dx = x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to :

(1) $\frac{2}{5}$ (2) $\frac{4}{5}$

(3) $-\frac{2}{5}$ (4) $-\frac{4}{5}$

55. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\sqrt{3}}{2}\left(\vec{b} + \vec{c}\right)$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(1) $\frac{2\pi}{3}$ (2) $\frac{5\pi}{6}$

(3) $\frac{3\pi}{4}$ (4) $\frac{\pi}{2}$

56. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = A A^T, then 5a + b is equal to :

(1) 4 (2) 13

(3) –1 (4) 5

57. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is:

(1) 20 (2) 5

(3) 6 (4) 10

58. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along the line $x = y = z$ is :

(1) $\frac{10}{\sqrt{3}}$ (2) $\frac{20}{3}$

(3) $3\sqrt{10}$ (4) $10\sqrt{3}$

59. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus?

(1) $\left(\frac{1}{3}, -\frac{8}{3}\right)$ (2) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$

(3) (–3, –9) (4) (–3, –8)

60. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is:

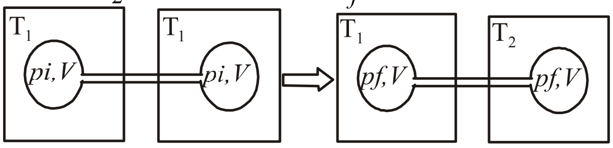
(1) 7 (2) 9

(3) 3 (4) 5

Section - 3

CHEMISTRY

61. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure p_i and temperature T_1 are connected through a narrow tube of negligible volume as shown in the figure below. The temperature of one of the bulbs is then raised to T_2. The final pressure p_f is :



(1) $2p_i\left(\frac{T_2}{T_1 + T_2}\right)$ (2) $2p_i\left(\frac{T_1T_2}{T_1 + T_2}\right)$

(3) $p_i\left(\frac{T_1T_2}{T_1 + T_2}\right)$ (4) $2p_i\left(\frac{T_1}{T_1 + T_2}\right)$

62. Which one of the following statements about water is **FALSE** ?

(1) There is extensive intramolecular hydrogen bonding in the condensed phase.

(2) Ice formed by heavy water sinks in normal water.

(3) Water is oxidized to oxygen during photosynthesis.

(4) Water can act both as an acid and as a base.

63. In the Hofmann bromamide degradation reaction, the number of moles of NaOH and Br_2 used per mole of amine produced are :

(1) Two moles of NaOH and two moles of Br_2.

(2) Four moles of NaOH and one mole of Br_2.

(3) One mole of NaOH and one mole of Br_2.

(4) Four moles of NaOH and two moles of Br_2.

64. Which of the following atoms has the highest first ionization energy?

(1) K (2) Sc

(3) Rb (4) Na

65. The concentration of fluoride, lead, nitrate and iron in a water sample from an underground lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of :

(1) Nitrate (2) Iron

(3) Fluoride (4) Lead

66. The heats of combustion of carbon and carbon monoxide are –393.5 and –283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is :

(1) –676.5 (2) – 110.5

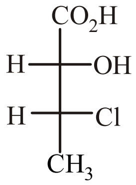
(3) 110.5 (4) 676.5

67. The equilibrium constant at 298 K for a reaction $A + B \rightleftharpoons C + D$ is 100. If the initial concentration of all the four species were 1 M each, then equilibrium concentration of D (in mol L^{-1}) will be :

(1) 1.818 (2) 1.182

(3) 0.182 (4) 0.818

68. The absolute configuration of



is :

(1) (2S, 3S) (2) (2R, 3R)

(3) (2R, 3S) (4) (2S, 3R)

69. For a linear plot of log (x/m) versus log p in a Freundlich adsorption isotherm, which of the following statements is correct? (k and n are constants)

(1) Only $1/n$ appears as the slope.

(2) $\log(1/n)$ appears as the intercept.

(3) Both k and $1/n$ appear in the slope term.

(4) $1/n$ appears as the intercept.

70. The distillation technique most suited for separating glycerol from spent-lye in the soap industry is :

(1) Steam distillation.

(2) Distillation under reduced pressure.

(3) Simple distillation

(4) Fractional distillation

71. Which of the following is an anionic detergent?

(1) Cetyltrimethyl ammonium bromide.

(2) Glyceryl oleate.

(3) Sodium stearate.

(4) Sodium lauryl sulphate.

72. The species in which the N atom is in a state of *sp* hybridization is :

(1) NO_3^- (2) NO_2

(3) NO_2^+ (4) NO_2^-

73. Thiol group is present in :

(1) Cysteine (2) Methionine

(3) Cytosine (4) Cystine

74. Which one of the following ores is best concentrated by froth floatation method?

(1) Galena (2) Malachite

(3) Magnetite (4) Siderite

75. Which of the following statements about low density polythene is **FALSE**?

(1) Its synthesis requires dioxygen or a peroxide initiator as a catalyst.

(2) It is used in the manufacture of buckets, dust-bins etc.

(3) Its synthesis requires high pressure.

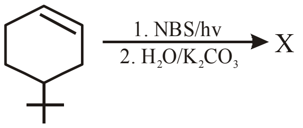
(4) It is a poor conductor of electricity.

76. Which of the following compounds is metallic and ferromagnetic?

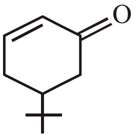
(1) VO_2 (2) MnO_2

(3) TiO_2 (4) CrO_2

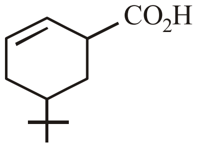
77. The product of the reaction given below is:



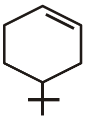
(1)



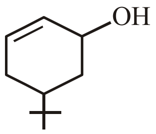
(2)



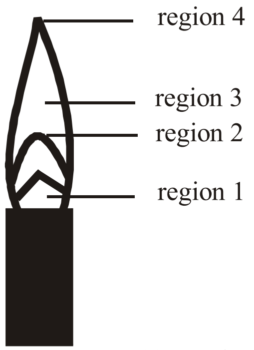
(3)



(4)



78. The hottest region of Bunsen flame shown in the figure below is :



(1) region 3 (2) region 4

(3) region 1 (4) region 2

79. At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion the gases occupy 330 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is:

(1) C_4H_8 (2) C_4H_{10}

(3) C_3H_6 (4) C_3H_8

80. The pair in which phosphorous atoms have a formal oxidation state of + 3 is :

(1) Orthophosphorous and hypophosphoric acids

(2) Pyrophosphorous and pyrophosphoric acids

(3) Orthophosphorous and pyrophosphorous acids

(4) Pyrophosphorous and hypophosphoric acids

81. The reaction of propene with HOCl ($Cl_2 + H_2O$) proceeds through the intermediate:

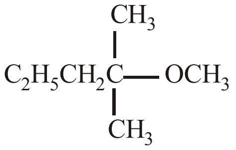
(1) $CH_3-CH(OH)-CH_2^+$

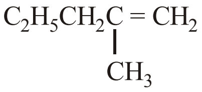
(2) $CH_3-CHCl-CH_2^+$

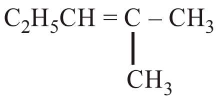
(3) $CH_3-CH^+-CH_2-OH$

(4) $CH_3-CH^+-CH_2-Cl$

82. 2-chloro-2-methylpentane on reaction with sodium methoxide in methanol yields:

(i) 
$C_2H_5CH_2C(CH_3)_2-OCH_3$

(ii) 
$C_2H_5CH_2C(CH_3)=CH_2$

(iii) 
$C_2H_5CH=C(CH_3)-CH_3$

(1) (iii) only	(2) (i) and (ii)
(3) All of these	(4) (i) and (iii)

83. Which one of the following complexes shows optical isomerism?

(1) *trans* $[Co(en)_2Cl_2]Cl$	(2) $[Co(NH_3)_4Cl_2]Cl$
(3) $[Co(NH_3)_3Cl_3]$	(4) *cis*$[Co(en)_2Cl_2]Cl$

(en = ethylenediamine)

84. The main oxides formed on combustion of Li, Na and K in excess of air are, respectively:

(1) Li_2O_2, Na_2O_2 and KO_2	(2) Li_2O, Na_2O_2 and KO_2
(3) Li_2O, Na_2O and KO_2	(4) LiO_2, Na_2O_2 and K_2O

85. 18 g glucose ($C_6H_{12}O_6$) is added to 178.2 g water. The vapour pressure of water (in torr) for this aqueous solution is:

(1) 752.4	(2) 759.0
(3) 7.6	(4) 76.0

86. The reaction of zinc with dilute and concentrated nitric acid, respectively, produces:

(1) NO and N_2O	(2) NO_2 and N_2O
(3) N_2O and NO_2	(4) NO_2 and NO

87. Decomposition of H_2O_2 follows a first order reaction. In fifty minutes the concentration of H_2O_2 decreases from 0.5 to 0.125 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be:

(1) 2.66 L min^{-1} at STP

(2) 1.34×10^{-2} mol min^{-1}

(3) 6.96×10^{-2} mol min^{-1}

(4) 6.93×10^{-4} mol min^{-1}

88. The pair having the same magnetic moment is:
[At. No.: Cr = 24, Mn = 25, Fe = 26, Co = 27]

(1) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$

(2) $[CoCl_4]^{2-}$ and $[Fe(H_2O)_6]^{2+}$

(3) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

(4) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

89. Galvanization is applying a coating of:

(1) Cu	(2) Zn
(3) Pb	(4) Cr

90. A stream of electrons from a heated filaments was passed two charged plates kept at a potential difference *V* esu. If e and *m* are charge and mass of an electron, respectively, then the value of h/λ (where λ is wavelength associated with electron wave) is given by:

(1) $\sqrt{meV}$	(2) $\sqrt{2meV}$
(3) meV	(4) $2meV$

Hints and Solutions

SECTION-1 : PHYSICS

1. **(2)** We know that $V = \sqrt[\omega]{A^2 - x^2}$

Initially $V = \sqrt[\omega]{A^2 - \left(\frac{2A}{3}\right)^2}$

Finally $3v = \sqrt[\omega]{A'^2 - \left(\frac{2A}{3}\right)^2}$

Where A'= final amplitude (Given at $x = \frac{2A}{3}$, velocity to trebled)

On dividing we get $\frac{3}{1} = \frac{\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}}{\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}}$

$9\left[A^2 - \frac{4A^2}{9}\right] = A'^2 - \frac{4A^2}{9}$ $\quad\therefore\; A' = \frac{7A}{3}$

2. **(2, 4)** We know that $\alpha = \frac{I_c}{I_e}$ and $\beta = \frac{I_c}{I_b}$

Also $I_e = I_b + I_c$

$$\therefore\; \alpha = \frac{Ic}{I_b + I_c} = \frac{\frac{I_c}{I_b}}{1 + \frac{I_c}{I_b}} = \frac{\beta}{1+\beta}$$

Option (2) and (4) are therefore correct.

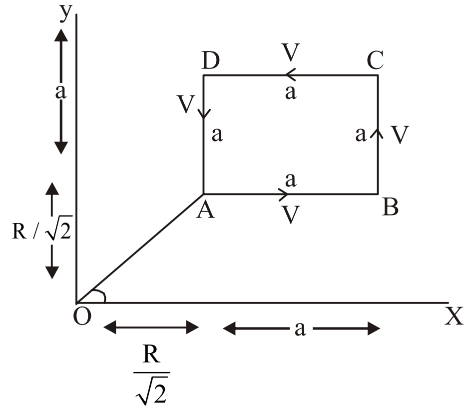
3. **(3)** $\Delta T = \frac{|\Delta T_1| + |\Delta T_2| + |\Delta T_3| + |\Delta T_4|}{4}$

$= \frac{2+1+3+0}{4} = 1.5$

As the resolution of measuring clock is 1.5 therefore the mean time should be 92 ± 1.5

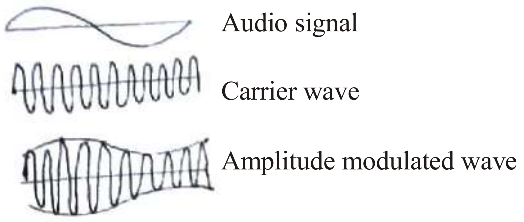
4. **(1)** In case of an 'OR' gate the input is zero when all inputs are zero. If any one input is ' 1', then the output is '1'.

5. **(1)** We know that $|L| = mvr_\perp$



In none of the cases, the perpendicular distance $r_\perp$ is $\left(\frac{R}{\sqrt{2}} - a\right)$

6. **(3)** In amplitude modulation, the amplitude of the high frequency carrier wave made to vary in proportional to the amplitude of audio signal.



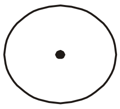
7. **(3)** $h\nu_0{}^2 - h\nu_0 = \frac{1}{2}mv^2$

$\therefore\; \frac{4}{3}h\nu_0 - h\nu_0 = \frac{1}{2}mv'^2$

$\therefore\; \frac{v'^2}{v^2} = \frac{\frac{4}{3}\nu - \nu_0}{\nu - \nu_0}$ $\qquad \therefore\; v' = v\sqrt{\frac{\frac{4}{3}\nu - \nu_0}{\nu - \nu_0}}$

$\therefore\; v' > \sqrt[v]{\frac{4}{3}}$

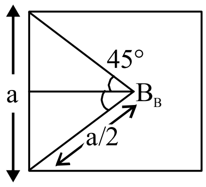
8. **(2)** **Case (a) :**



$\beta_A = \frac{\mu_0}{4\pi}\frac{I}{R} \times 2\pi = \frac{\mu_0}{4\pi}\frac{I}{\ell/2\pi} \times 2\pi$ $\qquad (2\pi R = \ell)$

$= \frac{\mu_0}{4\pi}\frac{I}{\ell} \times (2\pi)^2$

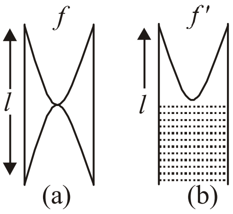
Case (b) :



$B_B = 4 \times \frac{\mu_0}{4\pi}\frac{I}{a/2}[\sin 45° + \sin 45°]$

$= 4 \times \frac{\mu_0}{4\pi} \times \frac{I}{\ell/8} \times \frac{2}{\sqrt{2}} = \frac{\mu_0}{4\pi}\frac{I}{\ell} \times {}^{32}\sqrt{2}$ $\qquad [4a = l]$

9. **(2)**



The fundamental frequency in case (a) is $f = \dfrac{v}{2\ell}$

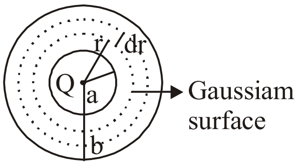
The fundamental frequency in case (b) is

$$f' = \frac{v}{4(\ell/2)} = \frac{u}{2\ell} = f$$

10. **(3)** Applying Gauss's law

$$\oint_S \vec{E}.\vec{ds} = \frac{Q}{\epsilon_0}$$

$$\therefore\ E \times 4\pi r^2 = \frac{Q + 2\pi ar^2 - 2\pi Aa^2}{\epsilon_0}$$



$$\rho = \frac{dr}{dv}$$

$$Q = \rho 4\pi r^2$$

$$Q = \int_a \frac{A}{r} 4\pi r^2 dr = 2\pi A\,[r^2 - a^2]$$

$$E = \frac{1}{4\pi \epsilon_0}\left[\frac{Q - 2\pi Aa^2}{r^2} + 2\pi A\right]$$

For E to be independent of 'r'

$Q - 2\pi Aa^2 = 0$

$$\therefore\ A = \frac{Q}{2\pi a^2}$$

11. **(2)** Here

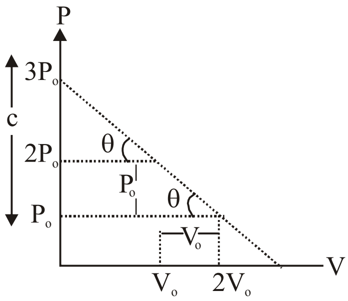
$$i = \frac{e}{\sqrt{R^2 + X_L^2}} = \frac{e}{\sqrt{R^2 + \omega^2 L^2}} = \frac{e}{\sqrt{R^2 + 4\pi^2 v^2 L^2}}$$

$$10 = \frac{220}{\sqrt{64 + 4\pi^2 (50)^2 L}} \quad [\because R = \frac{V}{I} = \frac{80}{10} = 8]$$

On solving we get

$L = 0.065\,H$

12. **(3)** The equation for the line is



$$P = \frac{-P_0}{V_0}V + 3P$$

$$[\text{slope} = \frac{-P_0}{V_0},\ c = 3P_0]$$

$$PV_0 + P_0 V = 3P_0 V_0 \qquad ...(i)$$

But $\quad pv = nRT$

$$\therefore\ p = \frac{nRT}{v} \qquad ...(ii)$$

From (i) & (ii) $\dfrac{nRT}{v}V_0 + P_0 V = 3P_0 V_0$

$$\therefore\ nRT\,V_0 + P_0 V^2 = 3P_0 V_0 \qquad ...(iii)$$

For temperature to be maximum $\dfrac{dT}{dv} = 0$

Differentiating e.q. (iii) by 'v' we get

$$nRV_0 \frac{dT}{dv} + P_0(2v) = 3P_0 V_0$$

$$\therefore\ nRV_0 \frac{dT}{dv} = 3P_0 V_0 - 2P_0 V$$

$$\frac{dT}{dv} = \frac{3P_0 V_0 - 2P_0 V}{nRV_0} = 0$$

$$V = \frac{3V_0}{2} \qquad \therefore\ p = \frac{3P_0}{2} \qquad \text{[From (i)]}$$

$$\therefore\ T_{max} = \frac{9P_o V_o}{4nR} \quad \text{[From (iii)]}$$

13. **(2)** $n = \dfrac{W}{\text{input}} = \dfrac{mgh \times 1000}{\text{input}} = \dfrac{10 \times 9.8 \times 1 \times 1000}{\text{input}}$

$$\text{Input} = \frac{98000}{0.2} = 49 \times 10^4 J$$

$$\text{Fat used} = \frac{49 \times 10^4}{3.8 \times 10^7} = 12.89 \times 10^{-3} kg.$$

14. **(1)** Loss in P.E. = Work done against friction from p → Q + work done against friction from Q → R

$mgh = \mu(mg\cos\theta)\,PQ + \mu mg\,(QR)$

$h = \mu \cos\theta \times PQ + \mu(QR)$

$$2 = \mu \times \frac{\sqrt{3}}{2} \times \frac{2}{\sin 30°} + \mu x$$

$$2 = 2\sqrt{3}\,\mu + \mu x \qquad \text{---}(i)$$

$$[\sin 30° = \frac{2}{PQ}]$$

Also work done P → Q = work done Q → R

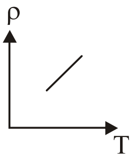
$\therefore\ 2\sqrt{3}\,\mu = \mu x$

$\therefore\ x \approx 3.5m$

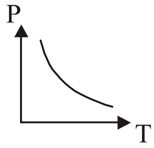
From (i) $2 = 2\sqrt{3}\,\mu + 2\sqrt{3}\,\mu = 4\sqrt{3}\,\mu$

$$\mu = \frac{2}{4\sqrt{3}} = \frac{1}{2\times 1.732} = 0.29$$

15. **(1)**



Metal (for limited range of temperature)



Semiconductor $\rho = \rho_o\, e^{\frac{-Eg}{K_B T}}$

16. **(3)** $\xrightarrow{\text{E, Decreases}}$

γ-rays X-rays uv-rays Visible rays VIBGYOR IR rays Microwaves Radio waves

Radio wave < yellow light < blue light < X-rays (Increasing order of energy)

17. **(3)** $Ig\,G = (I - Ig)s$

$\therefore\ 10^{-3} \times 100 = (10 - 10^{-3}) \times S$

$\therefore\ S \approx 0.01\,\Omega$

18. **(2)** For $A_{t_{1/2}} = 20$ min, t = 80 min, number of half lifes n = 4

$\therefore$ Nuclei remaining $= \frac{N_o}{2^4}$. Therefore nuclei decayed

$= N_0 - \frac{N_o}{2^4}$

For $B_{t_{1/2}} = 40$ min., t = 80 min, number of half lifes n = 2

$\therefore$ Nuclei remaining $= \frac{N_o}{2^2}$. Therefore nuclei decayed

$= N_0 - \frac{N_o}{2^2}$

$$\therefore \text{Required ratio} = \frac{No - \frac{No}{2^4}}{No - \frac{No}{2^2}} = \frac{1-\frac{1}{16}}{1-\frac{1}{4}} = \frac{15}{16}\times\frac{4}{3} = \frac{5}{4}$$

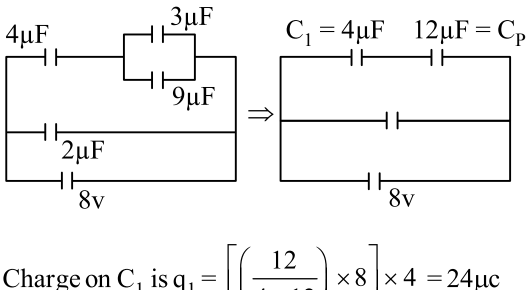
19. **(3)** Graph (a) is for a simple diode.

Graph (b) is showing the V Break down used for zener diode.

Graph (c) is for solar cell which shows cut-off voltage and open circuit current.

Graph (d) shows the variation of resistance h and hence current with intensity of light.

20. **(1)**



Charge on C_1 is $q_1 = \left[\left(\frac{12}{4+12}\right)\times 8\right]\times 4 = 24\mu c$

The voltage across C_P is $V_P = \frac{4}{4+12}\times 8 = 2v$

$\therefore$ Voltage across 9μF is also 2V

$\therefore$ Charge on 9μF capacitor = 9 × 2 = 18μC

$\therefore$ Total charge on 4 μF and 9μF = 42μc

$$\therefore\ E = \frac{KQ}{r^2} = 9\times 10^9 \times \frac{42\times 10^{-6}}{30\times 30} = 420\,\text{Nc}^{-1}$$

21. **(2)** For h << R, the orbital velocity is $\sqrt{gR}$

Escape velocity $= \sqrt{2gR}$

$\therefore$ The minimum increase in its orbital velocity

$= \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}\,(\sqrt{2} - 1)$

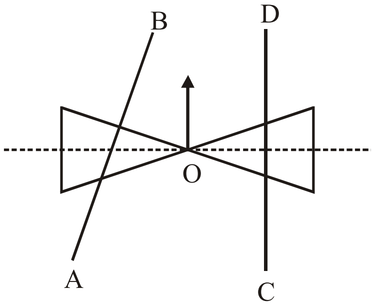
22. **(4)** L.C. $= \frac{0.5}{50} = 0.01$ mm

Zero error = 5 × 0.01 = 0.05 mm (Negative)

Reading = (0.5 + 25 × 0.01) + 0.05 = 0.80 mm

23. **(3)** As shown in the diagram, the normal reaction of AB on roller will shift towards O.

This will lead to tending of the system of cones to turn left.



24. **(2)** Graph [A] is for material used for making permanent magnets (high coercivity)

Graph [B] is for making electromagnets and transformers.

25. **(1)** Given geometrical spread = a

Diffraction spread $= \frac{\lambda}{a}\times L = \frac{\lambda L}{a}$

The sum $b = a + \frac{\lambda L}{a}$

For b to be minimum $\frac{db}{da} = 0$ $\quad \frac{d}{da}\left(a + \frac{\lambda L}{a}\right) = 0$

$$a = \sqrt{\lambda L}$$

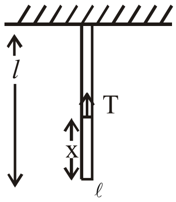
$$b\,\text{min} = \sqrt{\lambda L} + \sqrt{\lambda L} = 2\sqrt{\lambda L} = \sqrt{4\lambda L}$$

26. **(1)** We know that velocity in string is given by

$$v = \sqrt{\frac{T}{\mu}} \qquad \text{...(I)}$$

where $\mu = \frac{m}{l} = \frac{\text{mass of string}}{\text{length of string}}$

The tension $T = \frac{m}{\ell} \times x \times g$ ..(II)



From (1) and (2)

$$\frac{dx}{dt} = \sqrt{gx}$$

$$x^{-1/2}dx = \sqrt{g}\,dt \quad \therefore \int_0^\ell x^{-1/2}dx - \sqrt{g}\int_0^\ell dt$$

$$2\sqrt{l} = \sqrt{g} \times t \quad \therefore \ t = 2\sqrt{\frac{\ell}{g}} = 2\sqrt{\frac{20}{10}} = 2\sqrt{2}$$

27. **(4)** For a polytropic process

$$C = C_v + \frac{R}{1-n} \quad \therefore \ C - C_v = \frac{R}{1-n}$$

$$\therefore \ 1-n = \frac{R}{C-C_v} \quad \therefore \ 1 - \frac{R}{C-C_v} = n$$

$$\therefore \ n = \frac{C-C_v-R}{C-C_v} = \frac{C-C_v-C_p+C_v}{C-C_v}$$

$$= \frac{C-C_p}{C-C_v} \ (\because C_p - C_{v=R})$$

28. **(2)** A telescope magnifies by making the object appearing closer.

29. **(3)** We know that $i + e - A = \delta$

$35° + 79° - A = 40° \quad \therefore \ A = 74°$

$$\text{But } \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2} = \frac{\sin\left(\frac{74+\delta_m}{2}\right)}{\sin\frac{74}{2}}$$

$$= \frac{5}{3}\sin\left(37° + \frac{\delta_m}{2}\right)$$

μ_{max} can be $\frac{5}{3}$. That is μ_{max} is less than $\frac{5}{3} = 1.67$

But δ_m will be less than 40° so

$$\mu < \frac{5}{3}\sin 57° < \frac{5}{3}\sin 60° \ \Rightarrow \mu = 1.45$$

30. **(3)** Time lost/gained per day $= \frac{1}{2} \propto \Delta\theta \times 86400$ second

$12 = \frac{1}{2}\alpha(40-\theta) \times 86400$ .... (i)

$4 = \frac{1}{2}\alpha(\theta-20) \times 86400$ ....(ii)

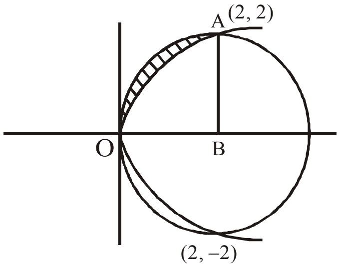
On dividing we get, $3 = \frac{40-\theta}{\theta-20}$

$3\theta - 60 = 40 - \theta$

$4\theta = 100 \Rightarrow \theta = 25°C$

SECTION-2 : MATHEMATICS

31. **(4)**



Points of intersection of the two curves are (0, 0), (2, 2) and (2, –2)

Area = Area (OAB) – area under parabola (0 to 2)

$$= \frac{\pi \times (2)^2}{4} - \int_0^2 \sqrt{2}\sqrt{x}\ dx$$

$$= \pi - \frac{8}{3}$$

32. **(1)** $f(x) + 2f\left(\frac{1}{x}\right) = 3x$ ....(1)

$f(\frac{1}{x}) + 2f(x) = \frac{3}{x}$ ....(2)

Adding (1) and (2) $\Rightarrow f(x) + f\left(\frac{1}{x}\right) = x + \frac{1}{x}$

Substracting (1) from (2) $\Rightarrow f(x) - f\left(\frac{1}{x}\right) = \frac{3}{x} - 3x$

On adding the above equations

$$\Rightarrow f(x) = \frac{2}{x} - x$$

$$f(x) = f(-x) \Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x \Rightarrow x = \frac{2}{x}$$

$x^2 = 2$ or $x = \sqrt{2}, -\sqrt{2}$.

33. **(4)** $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3}dx$

Dividing by x^{15} in numerator and denominator

$$\int \frac{\frac{2}{x^3} + \frac{5}{x^6}\ dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

Substitute $1+\frac{1}{x^2}+\frac{1}{x^5}=t$

$$\Rightarrow \left(\frac{-2}{x^3}-\frac{5}{x^6}\right)dx=dt$$

$$\Rightarrow \left(\frac{2}{x^3}+\frac{5}{x^6}\right)dx=-dt$$

This gives,

$$\int\frac{\frac{2}{x^3}+\frac{5}{x^6}dx}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}=\int\frac{-dt}{t^3}$$

$$=\frac{1}{2t^2}+C$$

$$=\frac{1}{2\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2}+C$$

$$=\frac{x^{10}}{2(x^5+x^3+1)^2}+C$$

34. **(4)** $g(x)=f(f(x))$

In the neighbourhood of $x=0$,

$f(x)=|\log 2-\sin x|=(\log 2-\sin x)$

$\therefore\ g(x)=|\log 2-\sin|\log 2-\sin x||$

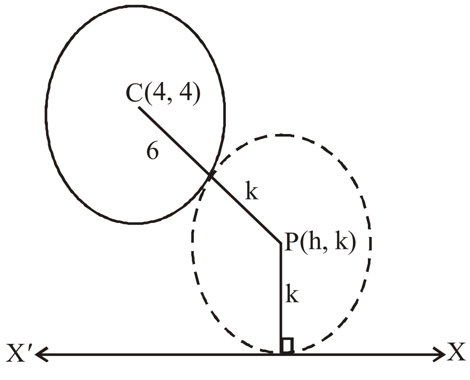
$=(\log 2-\sin(\log 2-\sin x))$

$\therefore\ g(x)$ is differentiable

and $g'(x)=-\cos(\log 2-\sin x)(-\cos x)$

$\Rightarrow g'(0)=\cos(\log 2)$

35 **(2)**



For the given circle,

centre : (4, 4)

radius = 6

$$6+k=\sqrt{(h-4)^2+(k-4)^2}$$

$(h-4)^2=20k+20$

$\therefore$ locus of (h, k) is

$(x-4)^2=20(y+1)$,

which is a parabola.

36. **(3)** $(x^2-5x+5)^{x^2+4x-60}=1$

Case I

$x^2-5x+5=1$ and $x^2+4x-60$ can be any real number

$\Rightarrow x=1, 4$

Case II

$x^2-5x+5=-1$ and $x^2+4x-60$ has to be an even number

$\Rightarrow x=2, 3$

where 3 is rejected because for $x=3$, $x^2+4x-60$ is odd.

Case III

x^2-5x+5 can be any real number and $x^2+4x-60=0$

$\Rightarrow x=-10, 6$

$\Rightarrow$ Sum of all values of $x=-10+6+2+1+4=3$

37. **(4)** Let the GP be a, ar and ar^2 then $a=A+d$; $ar=A+4d$; $ar^2=A+8d$

$$\Rightarrow \frac{ar^2-ar}{ar-a}=\frac{(A+8d)-(A+4d)}{(A+4d)-(A+d)}$$

$$r=\frac{4}{3}$$

38. **(1)** $\frac{2b^2}{a}=8$ and $2b=\frac{1}{2}(2ae)$

$$\Rightarrow 4b^2=a^2e^2 \Rightarrow 4a^2(e^2-1)=a^2e^2$$

$$\Rightarrow 3e^2=4 \Rightarrow e=\frac{2}{\sqrt{3}}$$

39. **(2)** Total number of terms $= {}^{n+2}C_2=28$

$(n+2)(n+1)=56$

$x=6$

Sum of coefficients $=(1-2+4)^n=3^6=729$

40. **(1)** $(p\wedge\sim q)\vee q\vee(\sim p\wedge q)$

$\Rightarrow \{(p\vee q)\wedge(\sim q\vee q)\}\vee(\sim p\wedge q)$

$\Rightarrow \{(p\vee q)\wedge T\}\vee(\sim p\wedge q)$

$\Rightarrow (p\vee q)\vee(\sim p\wedge q)$

$\Rightarrow \{(p\vee q)\vee\sim p\}\wedge(p\vee q\vee q)$

$\Rightarrow T\wedge(p\vee q)$

$\Rightarrow p\vee q$

41. **(4)** $f(x)=\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$

$$=\tan^{-1}\left(\sqrt{\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{\left(\sin\frac{x}{x}-\cos\frac{x}{2}\right)^2}}\right)=\tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$$

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$$

$$\Rightarrow\ y=\frac{\pi}{4}+\frac{x}{2}$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{1}{2}$$

Slope of normal $=\dfrac{-1}{\left(\dfrac{dy}{dx}\right)}=-2$

At $\left(\dfrac{\pi}{6},\dfrac{\pi}{4}+\dfrac{\pi}{12}\right)$

$$y-\left(\frac{\pi}{4}+\frac{\pi}{12}\right)=-2\left(x-\frac{\pi}{6}\right)$$

$$y-\frac{4\pi}{12}=-2x+\frac{2\pi}{6}$$

$$y-\frac{\pi}{3}=-2x+\frac{\pi}{3}$$

$$y=-2x+\frac{2\pi}{3}$$

This equation is satisfied only by the point $\left(0,\dfrac{2\pi}{3}\right)$

42. **(4)** $\mathrm{y}=\lim\limits_{n\to\infty}\left(\dfrac{(n+1)(n+2)...3n}{n^{2n}}\right)^{\frac{1}{n}}$

$$\ln \mathrm{y}=\lim_{n\to\infty}\frac{1}{n}\ln\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)....\left(1+\frac{2n}{n}\right)$$

$$\ln y=\lim_{n\to\infty}\frac{1}{n}\left[\ln\left(1+\frac{1}{n}\right)+\ln\left(1+\frac{2}{n}\right)+....+\ln\left(1+\frac{2n}{n}\right)\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\ln\left(1+\frac{r}{n}\right)=\int_0^2\ln(1+x)dx$$

Let $1+x=t\Rightarrow dx=dt$

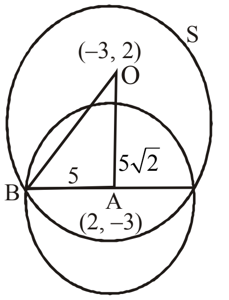
when $x=0,\ t=1$

$x=2,\ t=3$

$$\ln y=\int_1^3\ln t\ d\ t=[t\ln t-t]_1^3=\ln\left(\frac{3^3}{e^2}\right)=\ln\left(\frac{27}{e^2}\right)$$

$$\Rightarrow\ y=\frac{27}{e^2}$$

43. **(4)**



Centre of S : O (–3, 2) centre of given circle A(2, –3)

$\Rightarrow$ OA $=5\sqrt{2}$

Also AB = 5 ($\because$ AB = r of the given circle)

$\Rightarrow$ Using pythagoras theorem in ΔOAB

$\mathrm{r}=5\sqrt{3}$

44. **(2)** $\mathrm{P(E_1)}=\dfrac{1}{6}$; $\mathrm{P(E_2)}=\dfrac{1}{6}$; $\mathrm{P(E_3)}=\dfrac{1}{2}$

$$\mathrm{P(E_1\cap E_2)}=\frac{1}{36},\mathrm{P(E_2\cap E_3)}=\frac{1}{12},\mathrm{P(E_1\cap E_3)}=\frac{1}{12}$$

And $\mathrm{P(E_1\cap E_2\cap E_3)}=0\neq\mathrm{P(E_1).P(E_2).P(E_3)}$

$\Rightarrow$ $\mathrm{E_1, E_2, E_3}$ are not independent.

45. **(2)** Rationalizing the given expression

$$\frac{(2+3i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$$

For the given expression to be purely imaginary, real part of the above expression should be equal to zero.

$$\Rightarrow\frac{2-6\sin^2\theta}{1+4\sin^2\theta}=0\qquad\Rightarrow\sin^2\theta=\frac{1}{3}$$

$$\Rightarrow\sin\theta=\pm\frac{1}{\sqrt{3}}$$

46. **(4)** $\left(\dfrac{8}{5}\right)^2+\left(\dfrac{12}{5}\right)^2+\left(\dfrac{16}{5}\right)^2+\left(\dfrac{20}{5}\right)^2...+\left(\dfrac{44}{5}\right)^2$

$$\mathrm{S}=\frac{16}{25}\left(2^2+3^2+4^2+...+11^2\right)$$

$$=\frac{16}{25}\left(\frac{11(11+1)(22+1)}{6}-1\right)$$

$$=\frac{16}{25}\times505=\frac{16}{5}\times101$$

$$\Rightarrow\frac{16}{5}\mathrm{m}=\frac{16}{5}\times101$$

$\Rightarrow$ m = 101.

47. **(2)** For trivial solution,

$$\begin{vmatrix}1&\lambda&-1\\\lambda&-1&-1\\1&1&-\lambda\end{vmatrix}=0$$

$$\Rightarrow-\lambda(\lambda+1)(\lambda-1)=0$$

$$\Rightarrow\lambda=0,+1,-1$$

48. **(2)** Line lies in the plane $\Rightarrow$ (3, –2, –4) lie in the plane

$\Rightarrow 3\ell-2\mathrm{m}+4=9$ or $3\ell-2\mathrm{m}=5$ ..... (1)

Also, ℓ, m,–1 are dr's of line perpendicular to plane and 2, –1, 3 are dr's of line lying in the plane

$\Rightarrow 2\ell-\mathrm{m}-3=0$ or $2\ell-\mathrm{m}=3$ .....(2)

Solving (1) and (2) we get $\ell=1$ and m = –1

$\Rightarrow \ell^2+\mathrm{m}^2=2.$

49. **(2)** ALLMS

No. of words starting with

$$\text{A : } \underline{\text{A}}\,\_\,\_\,\_\,\_\,\_\ \frac{4!}{2!} = 12$$

$$\text{L : } \underline{\text{L}}\,\_\,\_\,\_\,\_\,\_\ 4! = 24$$

$$\text{M : } \underline{\text{M}}\,\_\,\_\,\_\,\_\,\_\ \frac{4!}{2!} = 12$$

$$\text{S : } \underline{\text{S}}\ \underline{\text{A}}\,\_\,\_\,\_\,\_\,\_\ \frac{3!}{2!} = 3$$

$$\text{: } \underline{\text{S}}\ \underline{\text{L}}\,\_\,\_\,\_\,3! = 6$$

SMALL → 58th word

50. **(4)** $\bar{x} = \frac{2+3+a+11}{4} = \frac{a}{4} + 4$

$$\sigma = \sqrt{\sum \frac{x_i^2}{n} - \left(\bar{x}\right)^2}$$

$$\Rightarrow 3.5 = \sqrt{\frac{4+9+a^2+121}{4} - \left(\frac{a}{4}+4\right)^2}$$

$$\Rightarrow \frac{49}{4} = \frac{4(134+a^2) - (a^2+256+32a)}{16}$$

$$\Rightarrow 3a^2 - 32a + 84 = 0$$

51. **(1)** $4x + 2\pi r = 2 \quad \Rightarrow 2x + \pi r = 1$

$S = x^2 + \pi r^2$

$$S = \left(\frac{1-\pi r}{2}\right)^2 + \pi r^2$$

$$\frac{dS}{dr} = 2\left(\frac{1-\pi r}{2}\right)\left(\frac{-\pi}{2}\right) + 2\pi r$$

$$\Rightarrow \frac{-\pi}{2} + \frac{\pi^2 r}{2} + 2\pi r = 0 \quad \Rightarrow r = \frac{1}{\pi + 4}$$

$$\Rightarrow x = \frac{2}{\pi + 4} \quad \Rightarrow x = 2r$$

52. **(1)** $\ln P = \lim_{x \to 0^+} \frac{1}{2x} \ln(1 + \tan^2 \sqrt{x})$

$$\lim_{x \to 0^+} \frac{1}{x} \ln\left(\sec\sqrt{x}\right)$$

Applying L hospital's rule :

$$= \lim_{x \to 0^+} \frac{\sec\sqrt{x}\tan\sqrt{x}}{\sec\sqrt{x} \cdot 2\sqrt{x}}$$

$$= \lim_{x \to 0^+} \frac{\tan\sqrt{x}}{2\sqrt{x}} = \frac{1}{2}$$

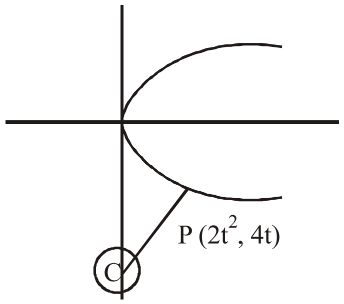
53. **(3)** Minimum distance ⇒ perpendicular distance

Eq^n of normal at $p(2t^2, 4t)$

$y = -tx + 4t + 2t^3$

It passes through $C(0, -6)$

$\Rightarrow t^3 + 2t + 3 = 0 \Rightarrow t = -1$



Centre of new circle $= P(2t^2, 4t)$

$= P(2, -4)$

Radius = PC

$= \sqrt{(2-0)^2 + (-4+6)^2}$

$= 2\sqrt{2}$

∴ Equation of circle is :

$$(x-2)^2 + (y+4) = \left(2\sqrt{2}\right)^2$$

$$\Rightarrow \quad x^2 + y^2 - 4x + 8y + 12 = 0$$

54. **(2)** $y(1 + xy)dx = xdy$

$$\frac{xdy - ydx}{y^2} = xdx$$

$$\Rightarrow \int -d\left(\frac{x}{y}\right) = \int xdx$$

$$-\frac{x}{y} = \frac{x^2}{2} + C \text{ as } y(1) = -1 \Rightarrow C = \frac{1}{2}$$

Hence, $y = \frac{-2x}{x^2+1} \Rightarrow f\left(\frac{-1}{2}\right) = \frac{4}{5}$

55. **(2)** $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$

$$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$$

On comparing both sides

$$\vec{a} \cdot \vec{b} = -\frac{\sqrt{3}}{2} \Rightarrow \cos\theta = -\frac{\sqrt{3}}{2}$$

[∵ $\vec{a}$ and $\vec{b}$ are unit vectors]

where θ is the angle between $\vec{a}$ and $\vec{b}$

$$\theta = \frac{5\pi}{6}$$

56. **(4)** $A(\text{adj } A) = A\,A^T$

$$\Rightarrow A^{-1}A\,(\text{adj } A) = A^{-1}A\,A^T$$

$\text{adj } A = A^T$

$$\Rightarrow \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$$\Rightarrow a = \frac{2}{5} \text{ and } b = 3$$

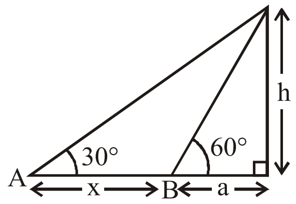
$$\Rightarrow 5a + b = 5$$

57. **(2)** $\tan 30° = \frac{h}{x+a}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+a} \Rightarrow \sqrt{3}h = x + a \qquad ...(1)$$

$$\tan 60° = \frac{h}{a} \Rightarrow \sqrt{3} = \frac{h}{a}$$

$$\Rightarrow h = \sqrt{3a} \qquad ...(2)$$



From (1) and (2)

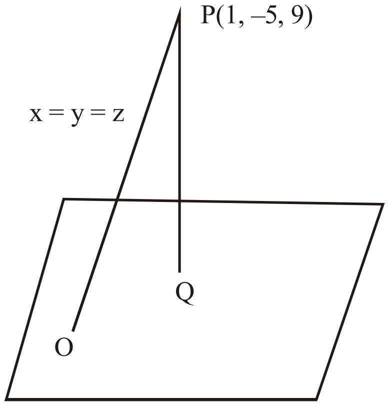
$3a = x + a \Rightarrow x = 2a$

Here, the speed is uniform

So, time taken to cover x = 2 (time taken to cover a)

$\therefore$ Time taken to cover $a = \frac{10}{2}$ minutes = 5 minutes

58. **(4)**



eq^n of PO : $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$

$\Rightarrow x = \lambda + 1;\ y = \lambda - 5; z = \lambda + 9.$

Putting these in eq^n of plane :-

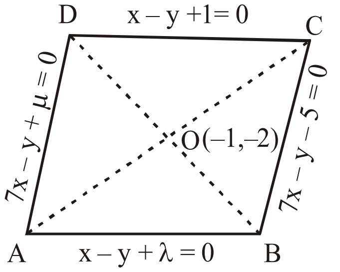
$\lambda + 1 - \lambda + 5 + \lambda + 9 = 5$

$\Rightarrow \lambda = -10$

$\Rightarrow$ O is (–9, –15, –1)

$\Rightarrow$ distance OP = $10\sqrt{3}$

59. **(1)**



Let other two sides of rhombus are

$x - y + \lambda = 0$

and $7x - y + \mu = 0$

then O is equidistant from AB and DC and from AD and BC

$\therefore |-1+2+1| = |-1+2+\lambda| \Rightarrow \lambda = -3$

and $|-7+2-5| = |-7+2+\mu| \Rightarrow \mu = 15$

$\therefore$ Other two sides are $x - y - 3 = 0$ and $7x - y + 15 = 0$

On solving the eq^{n}s of sides pairwise, we get

the vertices as $\left(\frac{1}{3}, \frac{-8}{3}\right), (1, 2), \left(\frac{-7}{3}, \frac{-4}{3}\right), (-3, -6)$

60. **(1)** $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$\Rightarrow 2 \cos 2x \cos x + 2 \cos 3x \cos x = 0$

$$\Rightarrow 2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right) = 0$$

$$\cos x = 0,\ \cos\frac{5x}{2} = 0,\ \cos\frac{x}{2} = 0$$

$$x = \pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$$

SECTION-3 : CHEMISTRY

61. **(1)** For a given mass of an ideal gas, the volume and amount (moles) of the gas are directly proportional if the temperature and pressure are constant. i.e

$V \propto n$

Hence in the given case.

Initial moles and final moles are equal $(n_T)_i = (n_T)_f$

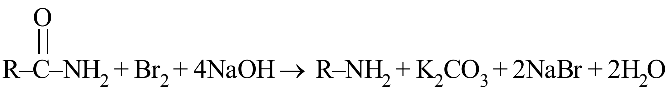
$$\frac{P_iV}{RT_1} + \frac{P_iV}{RT_1} = \frac{P_fV}{RT_1} + \frac{P_fV}{RT_2}$$

$$2\frac{P_i}{T_1} = \frac{P_f}{T_1} + \frac{P_f}{T_2}$$

$$P_f = \frac{2\,P_iT_2}{T_1 + T_2}$$

62. **(1)** There is extensive intermolecular hydrogen bonding in the condensed phase instead of intramolecular H-bonding.

63. **(2)** 4 moles of NaOH and one mole of Br_2 is required during production of one mole of amine during Hoffmann's bromamide degradation reaction.



$$R\text{–}\overset{\overset{\displaystyle O}{||}}{C}\text{–}NH_2 + Br_2 + 4NaOH \rightarrow R\text{–}NH_2 + K_2CO_3 + 2NaBr + 2H_2O$$

64. (2) Alkali metals have the lowest ionization energy in each period on the other hand Sc is a d - block element. Transition metals have smaller atomic radii and higher nuclear charge leading to high ionisation energy.

65. (1) The maximum limit of nitrate in drinking water is 50 ppm. Excess nitrate in drinking water can cause disease such as methemoglobinemia ('blue baby' syndrome).

66. (2) Given

$$C(s) + O_2(g) \rightarrow CO_2(g); \quad \Delta H = -393.5 \text{ kJ mol}^{-1} \quad \ldots(i)$$

$$CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g); \ \Delta H = -283.5 \text{ kJ mol}^{-1} \ldots(ii)$$

$\therefore$ Heat of formation of CO = eqn(i) – eqn(ii)

$= -393.5 - (-283.5)$

$= -110$ kJ

67. (1) Given,

$$A + B \rightleftharpoons C + D$$

	A	B	C	D
No. of moles initially	1	1	1	1
At equilibrium	1–a	1–a	1+a	1+a

$$\therefore K_c = \left(\frac{1+a}{1-a}\right)^2 = 100$$

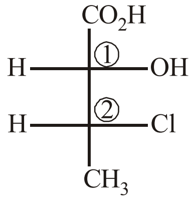
$$\therefore \frac{1+a}{1-a} = 10$$

On solving

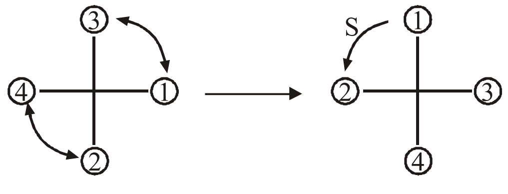
$a = 0.81$

$[D]_{\text{At eq}} = 1 + a = 1 + 0.81 = 1.81$

68. (4)

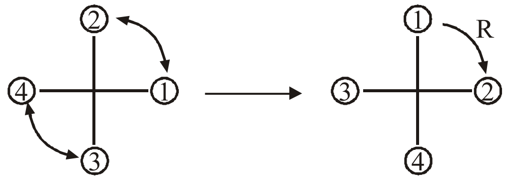


At (1),



It is 'S'configurated

At. (2),

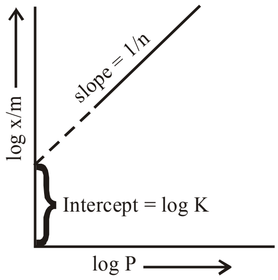


It is 'R' configurated.

69. (1) According to Freundlich adsorption isotherm

$$\log \frac{x}{m} = \log K + \frac{1}{n} \log P$$

Thus if a graph is plotted between log(x/m) and log P, a straight line will be obtained



The slope of the line is equal to 1/n and the intercept on log x/m axis will correspond to log K.

70. (2) Spent-lye and glycerol are separated by distillation under reduced pressure.

Under the reduced pressure the liquid boil at low temperature and the temperature of decomposition will not reach. e.g. glycerol boils at 290°C with decomposition but at reduced pressure it boils at 180° C without decomposition.

71. (4) Sodium lauryl sulphate ($C_{11}H_{23}CH_2OSO_3^-Na^+$) is an anionic detergent. Glyceryl oleate is a glyceryl ester of oleic acid. Sodium stearate ($C_{17}H_{35}COO^-Na^+$) is a soap. Cetyltrimethyl ammonium bromide

$$\left[CH_3(CH_2)_{15}\overset{+}{N}(CH_3)_3\right]Br^- \text{ is a cationic detergent.}$$

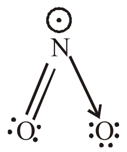
72. (3) Hybridization (H) $= \frac{1}{2}$[no. of valence electrons of central atom + no. of Monovalent atoms attached to it + (–ve charge if any) – (+ve charge if any)]

$$NO_2^+ = \frac{1}{2}[5+0+0-1] = 2 \text{ i.e. sp hybridisation}$$

$$NO_2^- = \frac{1}{2}[5+0+1-0] = 3 \text{ i.e. sp}^2 \text{ hybridisation}$$

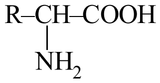
$$NO_3^- = \frac{1}{2}[5+0+1-0] = 3 \text{ i.e. sp}^2 \text{ hybridisation}$$

The lewis structure of NO_2 shows a bent molecular geometry with trigonal planar electron pair geometry hence the hybridization will be sp^2



73. (1) Among 20 naturally occuring amino acids "Cysteine" has '– SH' or thiol functional group.

$\Rightarrow$ General formula of amino acid $\rightarrow$



$$\text{R–}\underset{\substack{|\\ NH_2}}{CH}\text{–COOH}$$

$\Rightarrow$ Value of R = $-CH_2-SH$ in Cysteine.

74. **(1)** Froth floatation method is mainly applicable for sulphide ores.

(1) Malachite ore : $Cu(OH)_2 . CuCO_3$

(2) Magnetite ore : Fe_3O_4

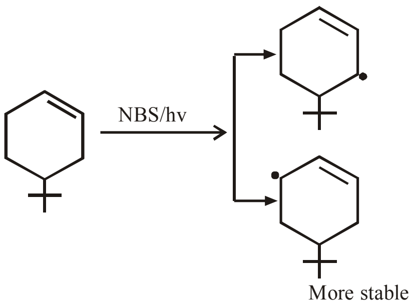
(3) Siderite ore : $FeCO_3$

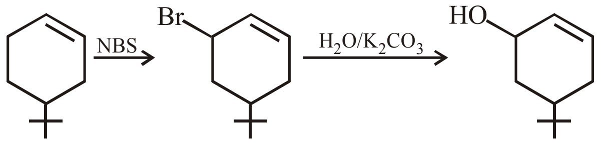
(4) Galena ore : PbS (Sulphide Ore)

75. **(2)** High density polythene is used in the manufacture of housewares like buckets, dustbins, bottles, pipes etc. Low density polythene is used for insulating electric wires and in the manufacture of flexible pipes, toys, coats, bottles etc.

76. **(4)** Out of all the four given metallic oxides CrO_2 is attracted by magnetic field very strongly. The effect persists even when the magnetic field is removed. Thus CrO_2 is metallic and ferromagnetic in nature

77. **(4)** N – bromosuccinimide results into bromination at allylic and benzylic positions





78. **(4)** Region 2 (blue flame) will be the hottest region of Bunsen flame shown in given figure

79. **(4)** $C_xH_{y(g)} + \left(\frac{4x+y}{4}\right) O_{2(g)} \rightarrow xCO_{2(g)} + \frac{y}{2} H_2O(l)$

Volume of O_2 used $= 375 \times \frac{20}{100} = 75$ ml

$\therefore$ From the reaction of combustion

1 ml C_xH_y requires $= \frac{4x+y}{4}$ ml O_2

$15\text{ ml} = 15\left(\frac{4x+y}{4}\right) = 75$

So, $4x + y = 20$

$x = 3$

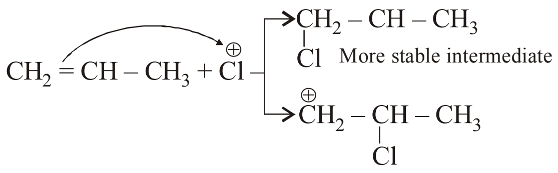
$y = 8$

$\boxed{C_3H_8}$

80. **(3)** Phosphorous acid contain P in +3 oxidation state.

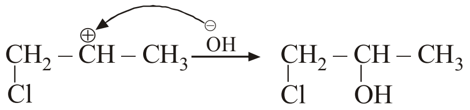
Acid	Formula	Oxidation state of Phosphorous
Pyrophosphorous acid	$H_4P_2O_5$	+3
Pyrophosphoric acid	$H_4P_2O_7$	+5
Orthophosphorous acid	H_3PO_3	+3
Hypophosphoric acid	$H_4P_2O_6$	+4

81. **(4)**



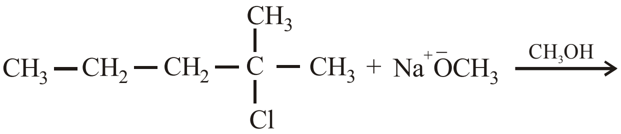
$$CH_2 = CH - CH_3 + \overset{\oplus}{Cl} \rightarrow \begin{cases} \overset{\oplus}{C}H_2 - CH - CH_3 \text{ (Cl on CH)} \\ \underset{Cl}{CH_2} - \overset{\oplus}{C}H - CH_3 \end{cases}$$

More stable intermediate



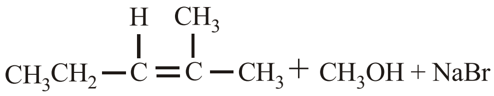
$$\underset{Cl}{CH_2} - \overset{\oplus}{C}H - CH_3 \xrightarrow{\overset{\ominus}{O}H} \underset{Cl}{CH_2} - \underset{OH}{CH} - CH_3$$

82. **(1)** When *tert* -alkyl halides are used in Williamson synthesis elimination occurs rather than substitution resulting into formation of alkene. Here alkoxide ion abstract one of the β-hydrogen atom along with acting as a nucleophile.



$$CH_3 - CH_2 - CH_2 - \overset{CH_3}{\underset{Cl}{C}} - CH_3 + Na^+\overset{-}{O}CH_3 \xrightarrow{CH_3OH}$$

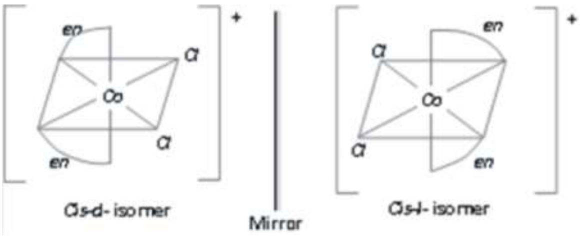
2-Chloro-2-methylpentane



$$CH_3CH_2 - \overset{H}{C} = \overset{CH_3}{C} - CH_3 + CH_3OH + NaBr$$

2-Methyl-pent-2-ene

83. **(3)** Optical isomerism occurs when a molecule is non-super imposable with its mirror image hence the complex *cis*-$[Co(en)_2Cl_2]Cl$ is optically active.



84. **(2)** On heating with excess of air Li, Na and K forms following oxides

$4\,Li + O_2 \longrightarrow 2Li_2O$

Lithium monoxide

$2Na + O_2 \xrightarrow{575\,K} Na_2O_2$

Sodium peroxide

$K + O_2 \longrightarrow KO_2$

Potassium superoxide

85. (1) According to Raoult's Law

$$\frac{P^\circ - P_s}{P_s} = \frac{W_B \times M_A}{M_B \times W_A} \qquad \ldots(i)$$

Here P° = Vapour pressure of pure solvent,
P_s = Vapour pressure of solution
W_B = Mass of solute, W_A = Mass of solvent
M_B = Molar mass of solute, M_A = Molar Mass of solvent
Vapour pressure of pure water at 100° C (by assumption = 760 torr)
By substituting values in equation (i) we get,

$$\frac{760 - P_s}{P_s} = \frac{18 \times 18}{180 \times 178.2} \qquad \ldots(ii)$$

On solving (ii) we get
$P_s = 752.4$ torr

86. (3) Reaction of Zn with dil. HNO_3
$4Zn + 10HNO_3(dil) \longrightarrow 4Zn(NO_3)_2 + 5H_2O + N_2O$
(Zn reacts differently with very dilute HNO_3)
Reaction of Zn with conc. HNO_3
$Zn + 4HNO_3(conc.) \longrightarrow Zn(NO_3)_2 + 2H_2O + 2NO_2$

87. (4) $H_2O_2(aq) \to H_2O(aq) + \frac{1}{2}O_2(g)$

For a first order reaction

$$k = \frac{2.303}{t}\log\frac{a}{(a-x)}$$

Given $a = 0.5, (a-x) = 0.125, t = 50$ min

$$\therefore \quad k = \frac{2.303}{50}\log\frac{0.5}{0.125}$$

$= 2.78 \times 10^{-2}\ \text{min}^{-1}$
$r = k[H_2O_2] = 2.78 \times 10^{-2} \times 0.05$
$= 1.386 \times 10^{-3}\ \text{mol min}^{-1}$

Now

$$-\frac{d[H_2O_2]}{dt} = \frac{d[H_2O]}{dt} = \frac{2d[O_2]}{dt}$$

$$\therefore \frac{2d[O_2]}{dt} = -\frac{d[H_2O_2]}{dt}$$

$$\therefore \frac{d[O_2]}{dt} = \frac{1}{2} \times \frac{d[H_2O_2]}{dt}$$

$$= \frac{1.386 \times 10^{-3}}{2} = 6.93 \times 10^{-4}\ \text{mol min}^{-1}$$

88. (4)

	Complex	Metal ion	Configuration	Magnetic moment $\mu = \sqrt{n(n+2)}$
(a)	$[Cr(H_2O)_6]^{2+}$	Cr^{+2}	d^4	$\sqrt{24}$
(b)	$[Fe(H_2O)_6]^{2+}$	Fe^{2+}	d^6	$\sqrt{24}$
(c)	$[CoCl_4]^{2-}$	Co^{2+}	d^7	$\sqrt{15}$
(d)	$[Mn(H_2O)_6]^{2+}$	Mn^{2+}	d^5	$\sqrt{35}$

Since (a) and (b), each has 4 unpaired electron they will have same magnetic moment

89. (2) **Galvanization** is the process by which zinc is coated over corrosive (easily rusted) metals to prevent them from corrosion.

90. (2) As electron of charge 'e' is passed through 'V' volt, kinetic energy of electron will be eV

$$\text{Wavelength of electron wave } (\lambda) = \frac{h}{\sqrt{2m.K.E}}$$

$$\lambda = \frac{h}{\sqrt{2meV}} \Rightarrow \therefore \frac{h}{\lambda} = \sqrt{2meV}$$

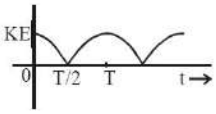
JEE Main - 2017

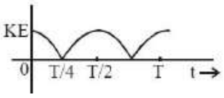
Time : 3 Hours • Each correct answer has + 4 marks • Each wrong answer has – 1 mark. **Max. Marks : 360**

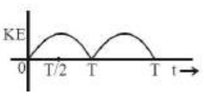
Section - 1

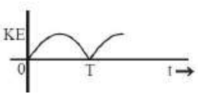
PHYSICS

1. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like:

(1) 

(2) 

(3) 

(4) 

2. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1 × 10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be :

(1) 2.5×10^{25} (2) -2.5×10^{25}

(3) -1.61×10^{23} (4) 1.38×10^{23}

3. Which of the following statements is false ?

(1) A rheostat can be used as a potential divider

(2) Kirchhoff's second law represents energy conservation

(3) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

(4) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

4. The following observations were taken for determining surface tensiton T of water by capillary method :

Diameter of capilary, D = 1.25 × 10^{-2} m

rise of water, h = 1.45 × 10^{-2} m

Using g = 9.80 m/s^2 and the simplified relation

$T = \frac{rhg}{2}\times10^3$ N/m, the possible error in surface tension is closest to :

(1) 2.4 % (2) 10 % (3) 0.15% (4) 1.5%

5. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m < \omega_c$. Which of the following frequencies is not contained in the modulated wave ?

(1) $\omega_m + \omega_c$ (2) $\omega_c - \omega_m$ (3) ω_m (4) ω_c

6. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is :

(1) real and at a distance of 40 cm from the divergent lens

(2) real and at a distance of 6 cm from the convergent lens

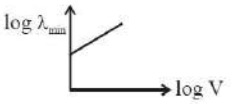
(3) real and at a distance of 40 cm from convergent lens

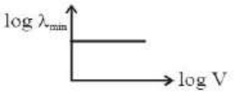
(4) virtual and at a distance of 40 cm from convergent lens.

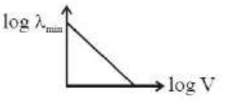
7. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum ?

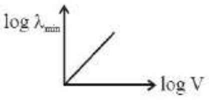
(1) 1 (2) $\frac{3}{\sqrt{2}}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\frac{\sqrt{3}}{2}$

8. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays.If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in :

(1) 

(2) 

(3) 

(4) 

9. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by

(1) $t = T\log(1.3)$ (2) $t = \frac{T}{\log(1.3)}$

(3) $t = T\frac{\log 2}{\log 1.3}$ (4) $t = \frac{\log 1.3}{\log 2}$

10. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E_1} = E\hat{i}$, it experiences a torque $\vec{T_1} = \tau\hat{i}$. When subjected to another electric field $\vec{E_2} = \sqrt{3E_1}\hat{j}$ it experiences torque $\vec{T_2} = -\vec{T_1}$. The angle θ is :

(1) 60° (2) 90° (3) 30° (4) 45°

11. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be :

(1) 135° (2) 180° (3) 45° (4) 90°

12. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that
$C_p - C_v = a$ for hydrogen gas
$C_p - C_v = b$ for nitrogen gas
The correct relation between a and b is :

(1) a = 14 b (2) a = 28 b (3) $a = \frac{1}{14}b$ (4) a = b

13. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by (Given : room temperature = 30° C, specific heat of copper = 0.1 cal/gm°C

(1) 1250°C (2) 825°C (3) 800°C (4) 885° C

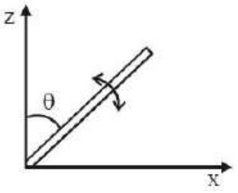
14. A body of mass m = 10^{-2} kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its intial speed is $v_0 = 10$ ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be:

(1) 10^{-4} kg m^{-1} (2) 10^{-1} kg m^{-1} s^{-1}
(3) 10^{-3} kg m^{-1} (4) 10^{-3} kg s^{-1}

15. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into to voltmeter of range 0 – 10 V is

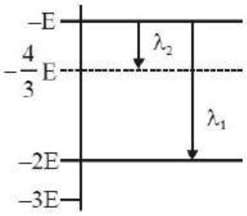
(1) $2.535 \times 10^3 \Omega$ (2) $4.005 \times 10^3 \Omega$
(3) $1.985 \times 10^3 \Omega$ (4) $2.045 \times 10^3 \Omega$

16. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is



(1) $\frac{3g}{2\ell}\cos\theta$ (2) $\frac{2g}{3\ell}\cos\theta$ (3) $\frac{3g}{2\ell}\sin\theta$ (4) $\frac{2g}{2\ell}\sin\theta$

17. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1 / \lambda_2$, is given by

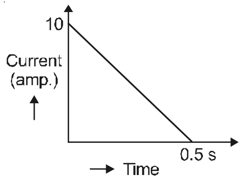


(1) $r = \frac{3}{4}$ (2) $r = \frac{1}{3}$ (3) $r = \frac{4}{3}$ (4) $r = \frac{2}{3}$

18. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of

(1) 81 (2) $\frac{1}{81}$ (3) 9 (4) $\frac{1}{9}$

19. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is



(1) 250 Wb
(2) 275 Wb
(3) 200 Wb
(4) 225 Wb

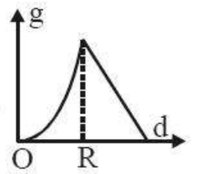
20. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is :

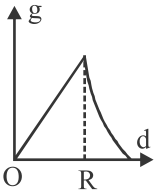
(1) 9.75 mm (2) 15.6 mm (3) 1.56 mm (4) 7.8 mm

21. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is :

(1) 6.98 s (2) 8.76 s (3) 6.65 s (4) 8. 89 s

22. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius):

(1)


(2)


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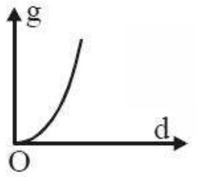

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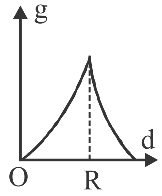
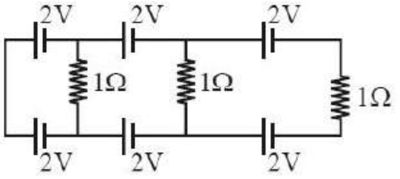


23.



In the above circuit the current in each resistance is

(1) 0.5A (2) 0 A (3) 1 A (4) 0.25 A

24. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is

(1) $\frac{\lambda_A}{\lambda_B}=\frac{2}{3}$ (2) $\frac{\lambda_A}{\lambda_B}=\frac{1}{2}$

(3) $\frac{\lambda_A}{\lambda_B}=\frac{1}{3}$ (4) $\frac{\lambda_A}{\lambda_B}=2$

25. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by :

(1) $\frac{3\alpha}{PK}$ (2) $3PK\alpha$

(3) $\frac{P}{3\alpha K}$ (4) $\frac{P}{\alpha K}$

26. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 secand will be

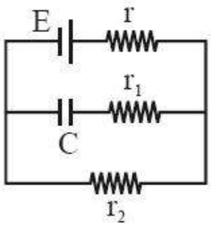
(1) 9 J (2) 18 J (3) 4.5 J (4) 22 J

27. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1})

(1) 17.3 GHz (2) 15.3 GHz

(3) 10.1 GHz (4) 12.1 GHz

28. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be :



(1) $CE\frac{r_2}{(r+r_2)}$

(2) $CE\frac{r_1}{(r_1+r)}$

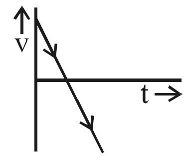
(3) CE

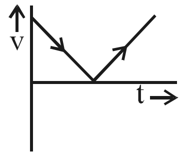
(4) $CE\frac{r_1}{(r_2+r)}$

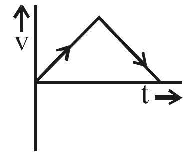
29. A capacitance of 2μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

(1) 24 (2) 32 (3) 2 (4) 16

30. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?

(1)


(2)


(3)


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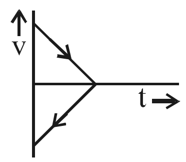


Section - 2

MATHEMATICS

31. Let k be an integer such that triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point :

(1) $\left(2,\frac{1}{2}\right)$ (2) $\left(2,-\frac{1}{2}\right)$

(3) $\left(1,\frac{3}{4}\right)$ (4) $\left(1,-\frac{3}{4}\right)$

32. If, for a positive integer n, the quadratic equation,

$$x(x+1)+(x+1)(x+2)+.....+(x+\overline{n-1})(x+n)=10n$$

has two consecutive integral solutions, then n is equal to :

(1) 11 (2) 12 (3) 9 (4) 10

33. The function $f:R\to\left[-\frac{1}{2},\frac{1}{2}\right]$ defined as $f(x)=\frac{x}{1+x^2}$, is :

(1) neither injective nor surjective

(2) invertible

(3) injective but not surjective

(4) surjective but not injective

34. The following statement

$(p\to q)\to[(\sim p\to q)\to q]$ is :

(1) a fallacy (2) a tautology

(3) equivalent to $\sim p\to q$ (4) equivalent to $p\to\sim q$

35. If S is the set of distinct values of 'b' for which the following system of linear equations

$x+y+z=1$

$x+ay+z=1$

$ax+by+z=0$

has no solution, then S is :

(1) a singleton (2) an empty set

(3) an infinite set

(4) a finite set containing two or more elements

36. The area (in sq. units) of the region

$\{(x,y): x\ge 0, x+y\le 3, x^2\le 4y \text{ and } y\le 1+\sqrt{x}\}$ is :

(1) $\frac{5}{2}$ (2) $\frac{59}{12}$ (3) $\frac{3}{2}$ (4) $\frac{7}{3}$

37. For any three positive real numbers a, b and c,

$9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c)$. Then :

(1) a, b and c are in G.P. (2) b, c and a are in G.P.

(3) b, c and a are in A.P. (4) a, b and c are in A.P.

38. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is :

(1) 484 (2) 485 (3) 468 (4) 469

39. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the y-axis passes through the point:

(1) $\left(\frac{1}{2},\frac{1}{3}\right)$ (2) $\left(-\frac{1}{2},-\frac{1}{2}\right)$

(3) $\left(\frac{1}{2},\frac{1}{2}\right)$ (4) $\left(\frac{1}{2},-\frac{1}{3}\right)$

40. A hyperbola passes through the point $P\left(\sqrt{2},\sqrt{3}\right)$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point :

(1) $\left(-\sqrt{2},-\sqrt{3}\right)$ (2) $\left(3\sqrt{2},2\sqrt{3}\right)$

(3) $\left(2\sqrt{2},3\sqrt{3}\right)$ (4) $\left(\sqrt{3},\sqrt{2}\right)$

41. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy, \forall\, x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to :

(1) 255 (2) 330 (3) 165 (4) 190

42. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30°. Then $\vec{a}.\vec{c}$ is equal to :

(1) $\frac{1}{8}$ (2) $\frac{25}{8}$ (3) 2 (4) 5

43. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan \beta$ is equal to :

(1) $\frac{4}{9}$ (2) $\frac{6}{7}$ (3) $\frac{1}{4}$ (4) $\frac{2}{9}$

44. Twenty metres of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is :

(1) 30 (2) 12.5 (3) 10 (4) 25

45. The integral $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$ is equal to :

(1) –1 (2) –2 (3) 2 (4) 4

46. If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $\frac{4}{3}$ (2) $\frac{1}{3}$ (3) $-\frac{2}{3}$ (4) $-\frac{1}{3}$

47. Let $I_n = \int \tan^n x \, dx, (n > 1)$. $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is constant of integration, then the ordered pair (a, b) is equal to :

(1) $\left(-\frac{1}{5},0\right)$ (2) $\left(-\frac{1}{5},1\right)$

(3) $\left(\frac{1}{5},0\right)$ (4) $\left(\frac{1}{5},-1\right)$

48. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to :

(1) 1 (2) –z (3) z (4) –1

49. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + ... + ({}^{21}C_{10} - {}^{10}C_{10})$ is :

(1) $2^{20} - 2^{10}$ (2) $2^{21} - 2^{11}$

(3) $2^{21} - 2^{10}$ (4) $2^{20} - 2^{9}$

50. $\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals :

(1) $\frac{1}{4}$ (2) $\frac{1}{24}$ (3) $\frac{1}{16}$ (4) $\frac{1}{8}$

51. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is :

(1) $-\frac{7}{9}$ (2) $-\frac{3}{5}$ (3) $\frac{1}{3}$ (4) $\frac{2}{9}$

52. If the image of the point P(1, –2, 3) in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to line, $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to :

(1) $6\sqrt{5}$ (2) $3\sqrt{5}$ (3) $2\sqrt{42}$ (4) $\sqrt{42}$

53. The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines

$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$ and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$, is :

(1) $\frac{10}{\sqrt{74}}$ (2) $\frac{20}{\sqrt{74}}$ (3) $\frac{10}{\sqrt{83}}$ (4) $\frac{5}{\sqrt{83}}$

54. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}.g(x)$, then g(x) equals :

(1) $\frac{3}{1+9x^3}$ (2) $\frac{9}{1+9x^3}$

(3) $\frac{3x\sqrt{x}}{1-9x^3}$ (4) $\frac{3x}{1-9x^3}$

55. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is :

(1) $4\left(\sqrt{2}+1\right)$ (2) $2\left(\sqrt{2}+1\right)$

(3) $2\left(\sqrt{2}-1\right)$ (4) $4\left(\sqrt{2}-1\right)$

56. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is :

(1) $\frac{6}{25}$ (2) $\frac{12}{5}$ (3) 6 (4) 4

57. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is :

(1) $x + 2y = 4$ (2) $2y - x = 2$
(3) $4x - 2y = 1$ (4) $4x + 2y = 7$

58. If two different numbers are taken from the set (0, 1, 2, 3, ..., 10), then the probability that their sum as well as absolute difference are both multiple of 4, is :

(1) $\frac{7}{55}$ (2) $\frac{6}{55}$ (3) $\frac{12}{55}$ (4) $\frac{14}{55}$

59. For three events A, B and C,
P(Exactly one of A or B occurs)
= P(Exactly one of B or C occurs)
= P(Exactly one of C or A occurs) = $\frac{1}{4}$ and
P(All the three events occur simultaneously) = $\frac{1}{16}$.
Then the probability that at least one of the events occurs, is :

(1) $\frac{3}{16}$ (2) $\frac{7}{32}$ (3) $\frac{7}{16}$ (4) $\frac{7}{64}$

60. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to :

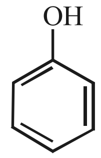
(1) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (2) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

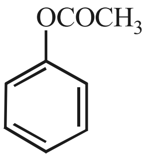
(3) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (4) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

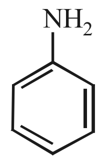
Section - 3

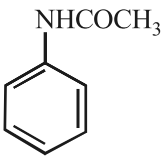
CHEMISTRY

61. Which of the following compounds will form significant amount of meta product during mono-nitration reaction ?

(1) 

(2) 

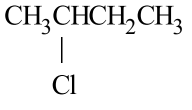
(3) 

(4) 

62. ΔU is equal to

(1) Isochoric work (2) Isobaric work
(3) Adiabatic work (4) Isothermal work

63. The increasing order of the reactivity of the following halides for the S_N1 reaction is



$CH_3\underset{\underset{Cl}{|}}{C}HCH_2CH_3$

(I)

$CH_3CH_2CH_2Cl$

(II)

p-$H_3CO-C_6H_4-CH_2Cl$

(III)

(1) (III) < (II) < (I) (2) (II) < (I) < (III)
(3) (I) < (III) < (II) (4) (II) < (III) < (I)

64. The radius of the second Bohr orbit for hydrogen atom is :
(Plank's const. $h = 6.6262 \times 10^{-34}$ Js ; mass of electron $= 9.1091 \times 10^{-31}$ kg ; charge of electron $e = 1.60210 \times 10^{-19}$ C ; permittivity of vaccum $\in_0 = 8.854185 \times 10^{-12}$ kg^{-1} m^{-3} A^2)

(1) 1.65Å (2) 4.76Å
(3) 0.529Å (4) 2.12Å

65. pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is

(1) 7.2 (2) 6.9
(3) 7.0 (4) 1.0

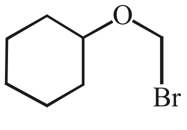
66. The formation of which of the following polymers involves hydrolysis reaction?

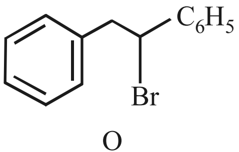
(1) Nylon 6 (2) Bakelite
(3) Nylon 6, 6 (4) Terylene

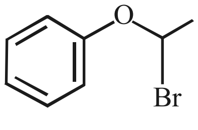
67. The most abundant elements by mass in the body of a healthy human adult are :
Oxygen (61.4%) ; Carbon (22.9%), Hydrogen (10.0%) ; and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^1H atoms are replaced by ^2H atoms is

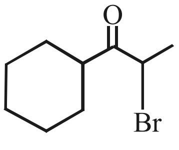
(1) 15 kg (2) 37.5 kg
(3) 7.5 kg (4) 10 kg

68. Which of the following , upon treatment with tert-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?

(1) 

(2) 

(3) 

(4) 

69. In the following reactions, ZnO is respectively acting as a/an:

(A) $ZnO + Na_2O \rightarrow Na_2ZnO_2$
(B) $ZnO + CO_2 \rightarrow ZnCO_3$

(1) base and acid (2) base and base
(3) acid and acid (4) acid and base

70. Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect is :

(1) Both form basic carbonates

(2) Both form soluble bicarbonates

(3) Both form nitrides

(4) Nitrates of both Li and Mg yield NO_2 and O_2 on heating

71. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is :

(1) Six (2) Zero

(3) Two (4) Four

72. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be :

(1) 2a (2) $2\sqrt{2}$ a

(3) $\sqrt{2}$ a (4) $\frac{a}{\sqrt{2}}$

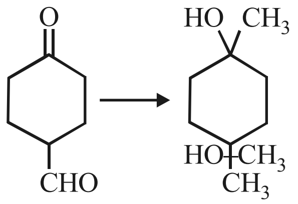
73. Two reactions R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then $\ln(k_2/k_1)$ is equal to :

$(R = 8.314\ J\ mol^{-1}K^{-1})$

(1) 8 (2) 12

(3) 6 (4) 4

74. The correct sequence of reagents for the following conversion will be :



(1) $[Ag(NH_3)_2]^+ OH^-$, H^+/CH_3OH, CH_3MgBr

(2) CH_3MgBr, H^+/CH_3OH, $[Ag(NH_3)_2]^+ OH^-$

(3) CH_3MgBr, $[Ag(NH_3)_2]^+ OH^-$, H^+/CH_3OH

(4) $[Ag(NH_3)_2]^+ OH^-$, CH_3MgBr, H^+/CH_3OH

75. The Tyndall effect is observed only when following conditions are satisfied :

(a) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.

(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.

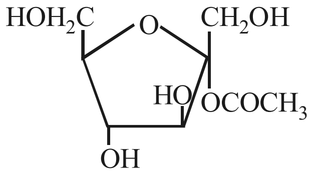
(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

(1) (a) and (d) (2) (b) and (d)

(3) (a) and (c) (4) (b) and (c)

76. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?

(1)



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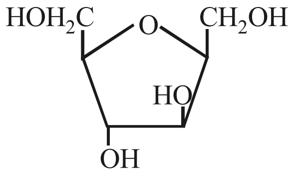

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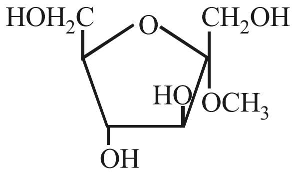

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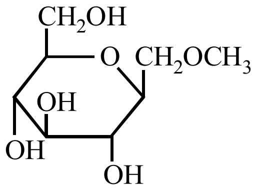


77. Given

$C_{(graphite)} + O_2(g) \rightarrow CO_2(g)$; $\Delta_r H° = -393.5\ kJ\ mol^{-1}$

$H_2(g) + \frac{1}{2} O_2(g) \rightarrow H_2O(l)$; $\Delta_r H° = -285.8\ kJ\ mol^{-1}$

$CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g); \Delta_r H° = +890.3\ kJ\ mol^{-1}$

Based on the above thermochemical equations, the value of $\Delta_r H°$ at 298 K for the reaction

$C_{(graphite)} + 2H_2(g) \rightarrow CH_4(g)$ will be :

(1) $+74.8\ kJ\ mol^{-1}$ (2) $+144.0\ kJ\ mol^{-1}$

(3) $-74.8\ kJ\ mol^{-1}$ (4) $-144.0\ kJ\ mol^{-1}$

78. Which of the following reactions is an example of a redox reaction?

(1) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

(2) $XeF_2 + PF_5 \rightarrow [XeF]^+ PF_6^-$

(3) $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$

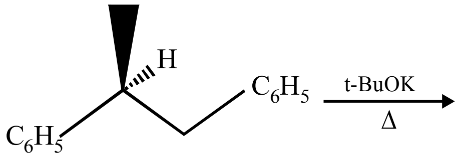
(4) $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$

79. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are :

(1) ClO^- and ClO_3^- (2) ClO_2^- and ClO_3^-

(3) Cl^- and ClO^- (4) Cl^- and ClO_2^-

80. The major product obtained in the following reaction is :



(1) $(\pm)C_6H_5CH(O^tBu)CH_2C_6H_5$

(2) $C_6H_5CH = CHC_6H_5$

(3) $(+)C_6H_5CH(O^tBu)CH_2C_6H_5$

(4) $(-)C_6H_5CH(O^tBu)CH_2C_6H_5$

81. Sodium salt of an organic acid 'X' produces effervescences with conc. H_2SO_4. 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. 'X' is :

(1) C_6H_5COONa (2) HCOONa

(3) CH_3COONa (4) $Na_2C_2O_4$

82. Which of the following species is not paramagnetic ?

(1) NO (2) CO

(3) O_2 (4) B_2

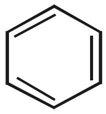
83. The freezing point of benzene decreases by 0.45°C when 0.2g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be :

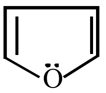
(K_f for benzene = 5.12 K kg mol^{-1})

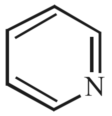
(1) 64.6% (2) 80.4%

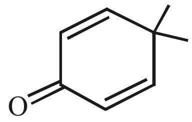
(3) 74.6% (4) 94.6%

84. Which of the following molecules is least resonance stabilized?

(1) 

(2) 

(3) 

(4) 

85. On treatment of 100 mL of 0.1 M solution of $CoCl_3 . 6H_2O$ with excess $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is :

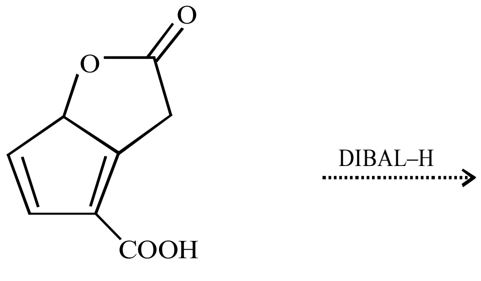
(1) $[Co(H_2O)_4 Cl_2]Cl.2H_2O$

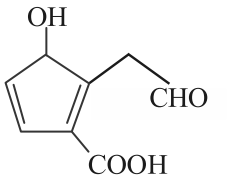
(2) $[Co(H_2O)_3Cl_3].3H_2O$

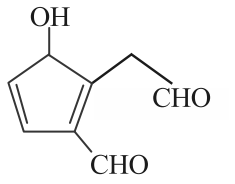
(3) $[Co(H_2O)_6]Cl_3$

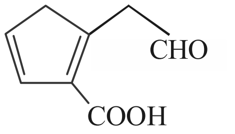
(4) $[Co(H_2O)_5Cl]Cl_2.H_2O$

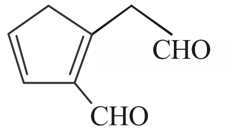
86. The major product obtained in the following reaction is :



(1) 

(2) 

(3) 

(4) 

87. A water sample has ppm level concentration of following anions

$F^- = 10; SO_4^{2-} = 100; NO_3^- = 50$

the anion/anions that make/makes the water sample unsuitable for drinking is/are :

(1) only NO_3^- (2) both SO_4^{2-} and NO_3^-

(3) only F^- (4) only SO_4^{2-}

88. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is :

(1) 1186 (2) 84.3

(3) 118.6 (4) 11.86

89. Given

$$E^o_{Cl_2/Cl^-} = 1.36V, \quad E^o_{Cr^{3+}/Cr} = -0.74V,$$

$$E^o_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33V, E^o_{MnO_4^-/Mn^{2+}} = 1.51V.$$

Among the following, the strongest reducing agent is

(1) Cr (2) Mn^{2+}

(3) Cr^{3+} (4) Cl^-

90. The group having isoelectronic species is :

(1) $O^{2-}, F^-, Na^+, Mg^{2+}$

(2) O^-, F^-, Na, Mg^+

(3) O^{2-}, F^-, Na, Mg^{2+}

(4) O^-, F^-, Na^+, Mg^{2+}

Hints and Solutions

SECTION-1 : PHYSICS

1. **(2)** For a particle executing SHM

At mean position; $t = 0, \omega t = 0, y = 0, V = V_{max} = a\omega$

$\therefore \quad K.E. = KE_{max} = \frac{1}{2} m\omega^2 a^2$

At extreme position : $t = \frac{T}{4}, \omega t = \frac{\pi}{2}, y = A, V = V_{min} = 0$

$\therefore \quad K.E. = KE_{min} = 0$

Kinetic energy in SHM, $KE = \frac{1}{2} m\omega^2 (a^2 - y^2)$

$= \frac{1}{2} m\omega^2 a^2 \cos^2 \omega t$

Hence graph (2) correctly depicts kinetic energy time graph.

2. **(2)** Given: Temperature $T_i = 17 + 273 = 290\,K$

Temperature $T_f = 27 + 273 = 300\,K$

Atmospheric pressure, $P_0 = 1 \times 10^5\,Pa$

Volume of room, $V_0 = 30\,m^3$

Difference in number of molecules, $N_f - N_i = ?$

The number of molecules

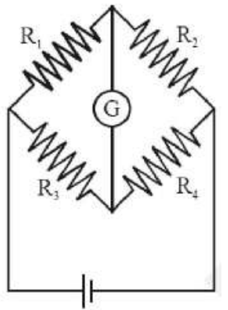
$$\Rightarrow \quad N = \frac{PV}{RT}(\mathrm{N}_0)$$

$$\therefore \quad N_f - N_i = \frac{P_0 V_0}{R}\left(\frac{1}{T_f} - \frac{1}{T_i}\right)\mathrm{N}_0$$

$$= \frac{1 \times 10^5 \times 30}{8.314} \times 6.023 \times 10^{23}\left(\frac{1}{300} - \frac{1}{290}\right)$$

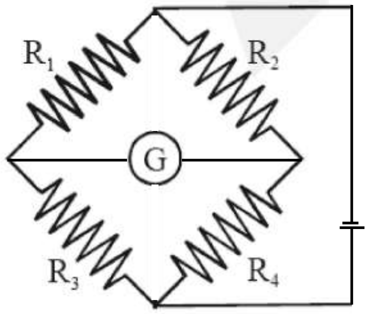
$$= -2.5 \times 10^{25}$$

3. **(4)** There is no change in null point, if the cell and the galvanometer are exchanged in a balanced wheatstone bridge.



On balancing condition $\frac{R_1}{R_3} = \frac{R_2}{R_4}$

After exchange



On balancing condition $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

4. **(4)** Surface tension, $T = \frac{rhg}{2} \times 10^3$

Relative error in surface tension, $\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + 0$

($\because$ g, 2 and 10^3 are constant)

Percentage error

$$100 \times \frac{\Delta T}{T} = \left(\frac{10^{-2} \times 0.01}{1.25 \times 10^{-2}} + \frac{10^{-2} \times 0.01}{1.45 \times 10^{-2}}\right)100$$

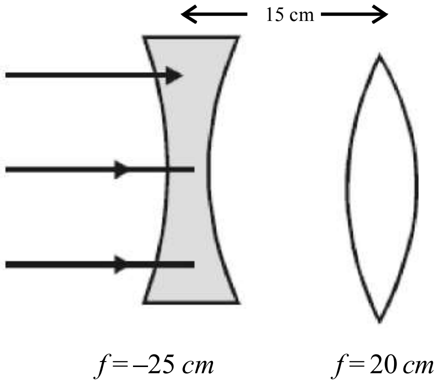
$= (0.8 + 0.689)$

$= (1.489) = 1.489\% \cong 1.5\%$

5. **(3)** Modulated carrier wave contains frequency w_c and $w_c \pm w_m$

6. **(3)** As parallel beam incident on diverging lens will form image at focus.

$\therefore \quad v = -25\,cm$



The image formed by diverging lens is used as an object for converging lens,

So for converging lens $u = -25 - 15 = -40\,cm$, $f = 20\,cm$

$\therefore$ Final image formed by converging lens

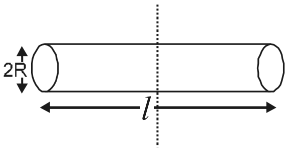
$$\frac{1}{V} - \frac{1}{-40} = \frac{1}{20}$$

or, $V = 40$ cm from converging lens real and inverted.

7. **(3)** As we know, moment of inertia of a solid cylinder about an axis which is perpendicular bisector

$$I = \frac{mR^2}{4} + \frac{ml^2}{12}$$

$$I = \frac{m}{4}\left[R^2 + \frac{l^2}{3}\right]$$



Let V = Volume of cylinder = $\pi R^2 l$

$$= \frac{m}{4}\left[\frac{V}{\pi l} + \frac{l^2}{3}\right] \quad \Rightarrow \quad \frac{dI}{dl} = \frac{m}{4}\left[\frac{-V}{\pi l^2} + \frac{2l}{3}\right] = 0$$

$$\frac{V}{\pi l^2} = \frac{2l}{3} \quad \Rightarrow V = \frac{2\pi l^3}{3}$$

$$\pi R^2 l = \frac{2\pi l^3}{3} \Rightarrow \frac{l^2}{R^2} = \frac{3}{2} \text{ or, } \frac{l}{R} = \sqrt{\frac{3}{2}}$$

8. **(3)** In X-ray tube, $\lambda_{\min} = \frac{hc}{eV}$

$$\text{In } \lambda_{\min} = In\left(\frac{hc}{e}\right) - InV$$

Clearly, log $\lambda_{\min}$ versus log V graph
slope is negative hence option (3) correctly depicts.

9. **(4)** Let initially there are total N_0 number of nuclei

At time t $\frac{N_B}{N_A} = 0.3\text{(given)}$

$\Rightarrow \quad N_B = 0.3N_A$

$N_0 = N_A + N_B = N_A + 0.3N_A$

$\therefore \quad N_A = \frac{N_0}{1.3}$

As we know $N_t = N_0 e^{-\lambda t}$

or, $\frac{N_0}{1.3} = N_0 e^{-\lambda t}$

$$\frac{1}{1.3} = e^{-\lambda t} \Rightarrow ln(1.3) = \lambda t$$

$$\text{or, } t = \frac{ln(1.3)}{\lambda} \quad \Rightarrow \quad t = \frac{ln(1.3)}{\frac{ln(2)}{T}} = \frac{ln(1.3)}{ln(2)}T$$

10. **(1)** T = PE sinθ Torque experienced by the dipole in an electric field, $\vec{T} = \vec{P} \times \vec{E}$

$\vec{p} = p\cos\theta\ \hat{i} + p\sin\theta\ \hat{j}$

$\vec{E}_1 = E\hat{i}$

$\vec{T}_1 = \vec{p} \times \vec{E}_1 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}) \times E(\hat{i})$

$\tau\hat{k} = pE\sin\theta(-\hat{k})$ ...(i)

$\vec{E}_2 = \sqrt{3}\,E_1\hat{j}$

$\vec{T}_2 = p\cos\theta\hat{i} + p\sin\theta\hat{j}) \times \sqrt{3}\,E_1\hat{j}$

$\tau\hat{k} = \sqrt{3}\,pE_1\cos\theta\hat{k}$ ...(ii)

From eqns. (*i*) and (*ii*)

$pE\sin\theta = \sqrt{3}\,pE\cos\theta$

$\tan\theta = \sqrt{3} \quad \therefore \quad \theta = 60°$

11. **(2)** In common emitter configuration for *n-p-n* transistor input and output signals are 180° out of phase *i.e.,* phase difference between output and input voltage is 180°.

12. **(1)** As we know, $C_p - C_v = R$ where C_p and C_v are molar specific heat capacities

or, $C_p - C_v = \frac{R}{M}$

For hydrogen ($M = 2$) $C_p - C_v = a = \frac{R}{2}$

For nitrogen ($M = 28$) $C_p - C_v = b = \frac{R}{28}$

$\therefore \quad \frac{a}{b} = 14 \quad \text{or,} \quad a = 14b$

13. **(4)** According to principle of calorimetry,
Heat lost = Heat gain

$100 \times 0.1(-75) = 100 \times 0.1 \times 45 + 170 \times 1 \times 45$

$10 - 750 = 450 + 7650$

$10 = 1200 + 7650 = 8850$

$T = 885°C$

14. **(1)** Let V_f is the final speed of the body.
From questions,

$$\frac{1}{2}mV_f^2 = \frac{1}{8}mV_0^2 \qquad \Rightarrow \quad V_f = \frac{V_0}{2} = 5\,m/s$$

$$F = m\left(\frac{dV}{dt}\right) = -kV^2 \qquad \therefore \quad (10^{-2})\frac{dV}{dt} = -kV^2$$

$$\int_{10}^{5}\frac{dV}{V^2} = -100K\int_{0}^{10} dt$$

$$\frac{1}{5} - \frac{1}{10} = 100K(10) \qquad \text{or,} \quad K = 10^{-4}\,kgm^{-1}$$

15. **(3)** Given : Current through the galvanometer,
$i_g = 5 \times 10^{-3}\,A$

Galvanometer resistance, $G = 15\,\Omega$

Let resistance R to be put in series with the galvanometer to convert it into a voltmeter.

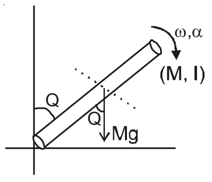
$V = i_g(R + G)$

$10 = 5 \times 10^{-3}(R + 15)$

$\therefore \quad R = 2000 - 15 = 1985$

$= 1.985 \times 10^3\,\Omega$

16. **(3)** Torque at angle θ



$$\tau = Mg\sin\theta.\frac{l}{2}$$

Also $\tau = I\alpha$

$\therefore \quad I\alpha = Mg\sin\theta\frac{l}{2}$

$$\frac{Ml^2}{3}.\alpha = Mg\sin\theta\frac{l}{2} \quad \left[\because I_{rod} = \frac{Ml^2}{3}\right]$$

$$\Rightarrow \quad \frac{l\alpha}{3} = g\frac{\sin\theta}{2} \qquad \therefore \quad \alpha = \frac{3g\sin\theta}{2l}$$

17. **(2)** From energy level diagram, using $\Delta E = \frac{hc}{\lambda}$

For wavelength λ_1 $\Delta E = -E - (-2E) = \frac{hc}{\lambda_1}$

$\therefore \quad \lambda_1 = \frac{hc}{E}$

For wavelength λ_2 $\Delta E = -E - \left(-\frac{4E}{3}\right) = \frac{hc}{\lambda_2}$

$\therefore \quad \lambda_2 = \frac{hc}{\left(\frac{E}{3}\right)} \qquad \therefore \quad r = \frac{\lambda_1}{\lambda_2} = \frac{1}{3}$

18. **(3)** As linear dimension increases by a factor of 9

$\therefore \quad \frac{v_f}{v_i} = 9^3$

$\because$ Density remains same

So, mass $\propto$ Volume

$\frac{m_f}{m_i} = 9^3 \quad \Rightarrow \quad \frac{(Area)_f}{(Area)_i} = 9^2$

Stress $(\sigma) = \frac{force}{area} = \frac{(mass) \times g}{area}$

$\frac{\sigma_2}{\sigma_1} = \left(\frac{m_f}{m_i}\right)\left(\frac{A_i}{A_f}\right) = \frac{9^3}{9^2} = 9$

19. **(1)** According to Faraday's law of electromagnetic induction, $\varepsilon = \frac{d\phi}{dt}$

Also, $\varepsilon = iR$

$\therefore \quad iR = \frac{d\phi}{dt} \quad \Rightarrow \quad \int d\phi = R\int idt$

Magnitude of change in flux $(d\phi) = R \times$ area under current vs time graph

or, $d\phi = 100 \times \frac{1}{2} \times \frac{1}{2} \times 10 = 250 \text{ Wb}$

20. **(4)** For common maxima, $n_1\lambda_1 = n_2\lambda_2$

$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{520 \times 10^{-9}}{650 \times 10^{-9}} = \frac{4}{5}$

For λ_1

$y = \frac{n_1\lambda_1 D}{d}, \lambda_1 = 650 \text{ nm}$

$y = \frac{4 \times 650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}}$ or, $y = 7.8$ mn

21. **(3)** Given : Magnetic moment, $M = 6.7 \times 10^{-2} Am^2$

Magnetic field, $B = 0.01\ T$

Moment of inertia, $I = 7.5 \times 10^{-6}$ Kgm^2

Using, $T = 2\pi\sqrt{\frac{I}{MB}}$

$$= 2\pi\sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = \frac{2\pi}{10} \times 1.06\, s$$

Time taken for 10 complete oscillations

$t = 10T = 2\pi \times 1.06$

$= 6.6568 \approx 6.65$ s

22. **(2)** Variation of acceleration due to gravity, g with distance 'd' from centre of the earth

If $\quad d < R, g = \frac{Gm}{R^2}.d \qquad$ *i.e.*, $g \propto d$ (straight line)

If $\quad d = R, g_s = \frac{Gm}{R^2}$

If $\quad d > R, g = \frac{Gm}{d^2} \qquad$ *i.e.*, $g \propto \frac{1}{d^2}$

23. **(2)** The potential difference in each loop is zero.

$\therefore$ No current will flow or current in each resistance is Zero.

24. **(4)** From question, $m_A = M; m_B = \frac{m}{2}$

$u_A = V \quad u_B = 0$

Let after collision velocity of $A = V_1$ and

velocity of $B = V_2$

Applying law of conservation of momentum,

$mu \ = m\text{v}_1 + \left(\frac{m}{2}\right)\text{v}_2$

or, $24 = 2\text{v}_1 + \text{v}_2 \qquad ....(i)$

By law of collision

$e = \frac{v_2 - v_1}{u - 0}$

or, $u = \text{v}_2 - \text{v}_1 \qquad ....(ii)$

[$\because$ collision is elastic, $e = 1$]

using eqns (i) and (ii)

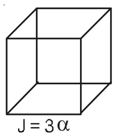
$\text{v}_1 = \frac{4}{3}$ and $\text{v}_2 = \frac{4}{3}u$

de-Broglie wavelength $\lambda = \frac{h}{p}$

$\therefore \quad \frac{\lambda_A}{\lambda_B} = \frac{P_B}{P_A} = \frac{\frac{m}{2} \times \frac{4}{3}u}{m \times \frac{4}{3}} = 2$

25. **(3)** As we know, Bulk modulus

$K = \frac{\Delta P}{\left(\frac{-\Delta V}{V}\right)} \quad \Rightarrow \quad \frac{\Delta V}{V} = \frac{P}{K}$



$V = V_0(1 + \gamma\Delta t)$

$\frac{\Delta V}{V_0} = \gamma\Delta t$

$\therefore \quad \frac{P}{K} = \gamma\Delta t \ \Rightarrow \Delta t = \frac{P}{\gamma K} = \frac{P}{3\alpha K}$

26. (3) Using, $F = ma = m\dfrac{dV}{dt}$

$$6t = 1.\frac{dV}{dt} \quad [\because \quad m = 1 \text{ kg given}]$$

$$\int_0^v dV = \int 6t\, dt$$

$$V = 6\left[\frac{t^2}{2}\right]_0^1 = 3\,\text{ms}^{-1} \quad [\because \quad t = 1 \text{ sec given}]$$

From work-energy theorem,

$$W = \Delta KE = \frac{1}{2}m\left(V^2 - u^2\right) = \frac{1}{2} \times 1 \times 9 = 4.5 \text{ J}$$

27. **(1)** Use relativistic doppler's effect as velocity of observer is not small as compared to light

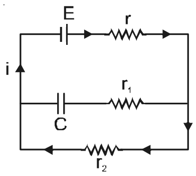
$$f = f_0\sqrt{\frac{c+\text{v}}{c+\text{v}}}\ ; \quad V = \text{relative speed of approach}$$

$f_0 = 10$ GHz

$$\therefore \quad f = 10\sqrt{\frac{c+\frac{c}{2}}{c-\frac{c}{2}}} = 10\sqrt{3} = 17.3 \text{ GHz}$$

28. **(1)** In steady state, flow fo current through capacitor will be zero.

Current through the circuit,



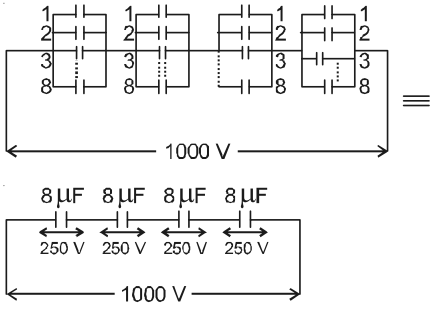
$$i = \frac{E}{r + r_2}$$

Potential difference through capacitor

$$V_c = \frac{Q}{C} = E - ir = E - \left(\frac{E}{r + r_2}\right)r$$

$$\therefore \quad Q = CE\frac{r_2}{r + r_2}$$

29. **(2)** To get a capacitance of 2 μF arrangement of capacitors of capacitance 1 μF as shown in figure 8 capacitors of 1 μF in parallel with four such branches in series i.e., 32 such capacitors are required.



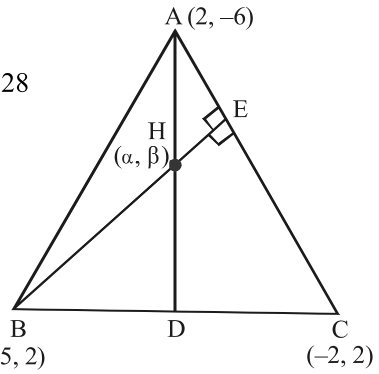
$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \qquad \therefore \ C_{eq} = 2\ \mu F$$

30. **(1)** For a body thrown vertically upwards acceleration remains constant ($a = -g$) and velocity at anytime t is given by $V = u - gt$

During rise velocity decreases linearly and during fall velocity increases linearly and direction is opposite to each other. Hence graph (1) correctly depicts velocity versus time.

SECTION-2 : MATHEMATICS

31. **(1)** We have



$$\frac{1}{2}\left\|\begin{matrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{matrix}\right\| = 28$$

$\Rightarrow 5k^2 + 13k - 46 = 0$

or $5k^2 + 13k + 66 = 0$

Now, $5k^2 + 13k - 46 = 0$

$$\Rightarrow \quad k = \frac{-13 \pm \sqrt{1089}}{10}$$

$$\therefore \quad k = \frac{-23}{5}; k = 2$$

since k is an integer, ∴ k = 2

Also $5k^2 + 13k + 66 = 0$

$$\Rightarrow k = \frac{-13 \pm \sqrt{-1151}}{10}$$

So no real solution exist

For orthocentre

$BH \perp AC$

$$\therefore \quad \left(\frac{\beta - 2}{\alpha - 5}\right)\left(\frac{8}{-4}\right) = -1$$

$\Rightarrow \quad \alpha - 2\beta = 1$ ...(1)

Also $CH \perp AB$

$$\therefore \quad \left(\frac{\beta - 2}{\alpha + 2}\right)\left(\frac{8}{3}\right) = -1$$

$\Rightarrow \quad 3\alpha + 8\beta = 10$ ... (2)

Solving (1) and (2), we get

$$\alpha = 2, \beta = \frac{1}{2}$$

orthocentre is $\left(2, \dfrac{1}{2}\right)$

32. **(1)** We have

$$\sum_{r=1}^{n} (x + r - 1)(x + r) = 10n$$

$$\sum_{r=1}^{n} (x^2 + xr + (r-1)x + r^2 - r = 10n$$

$$\Rightarrow \sum_{r=1}^{n} (x^2 + (2r-1)x + r(r-1) = 10n$$

$\Rightarrow nx^2 + \{1+3+5+....+(2n-1)\}x + \{1.2+2.3+....+(n-1)$ $n\} = 10\,n$

$$\Rightarrow nx^2 + n^2x + \frac{(n-1)n(n+1)}{3} = 10n$$

$$\Rightarrow x^2 + nx + \frac{n^2-31}{3} = 0$$

Let α and $\alpha + 1$ be its two solutions

($\because$ it has two consequtive integral solutions)

$\Rightarrow \alpha + (\alpha + 1) = -n$

$$\Rightarrow \alpha = \frac{-n-1}{2} \qquad ...(1)$$

$$\text{Also } \alpha(\alpha+1) = \frac{n^2-31}{3} \qquad ...(2)$$

Putting value of (1) in (2), we get

$$-\left(\frac{n+1}{2}\right)\left(\frac{1-n}{2}\right) = \frac{n^2-31}{3}$$

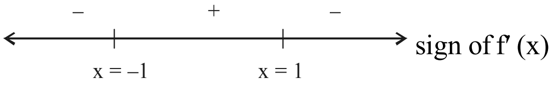
$\Rightarrow n^2 = 121$

$\Rightarrow n = 11$

33. **(4)** we have $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$,

$$f(x) = \frac{x}{1+x^2}\ \forall x \in R$$

$$\Rightarrow f'(x) = \frac{(1+x^2).1 - x.2x}{(1+x^2)^2} = \frac{-(x+1)(x-1)}{(1+x^2)^2}$$



$\Rightarrow f'(x)$ changes sign in different intervals.

$\therefore$ Not injective

$$\text{Now } y = \frac{x}{1+x^2}$$

$\Rightarrow y + yx^2 = x$

$\Rightarrow yx^2 - x + y = 0$

For $y \neq 0$, $D = 1 - 4y^2 \geq 0$

$$\Rightarrow y \in \left[\frac{-1}{2}, \frac{1}{2}\right] - \{0\}$$

For $y = 0 \Rightarrow x = 0$

$$\therefore \text{Range is } \left[\frac{-1}{2}, \frac{1}{2}\right]$$

$\Rightarrow$ Surjective but not injective

34. **(2)** We have

p	q	~ p	p → q	~ p → q	(~ p → q) → q	(p → q) → ((~p→q)→ q)
T	F	F	F	T	F	T
T	T	F	T	T	T	T
F	F	T	T	F	T	T
F	T	T	T	T	T	T

$\therefore$ It is tautology.

35. **(1)**
$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$$

$\Rightarrow 1\,[a-b] - 1\,[1-a] + 1\,[b-a^2] = 0 \Rightarrow (a-1)^2 = 0$

$\Rightarrow a = 1$

For a = 1, First two equations are identical

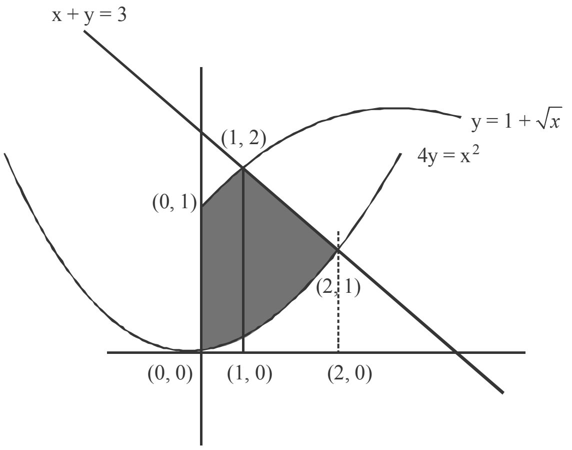
ie. $x + y + z = 1$

To have no solution with $x + by + z = 0$

$b = 1$

So $b = \{1\} \Rightarrow$ It is singleton set.

36. **(1)**



Area of shaded region

$$= \int_0^1 (1+\sqrt{x})dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$$

$$= x\Big]_0^1 + \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Bigg]_0^1 + 3x\Big]_1^2 - \frac{x^2}{2}\Bigg]_1^2 - \frac{x^3}{12}\Bigg]_0^2 = \frac{5}{2}\text{sq.units}$$

37. **(3)** We have

$9(25a^2 + b^2) + 25\,(c^2 - 3ac) = 15b\,(3\,a + c)$

$\Rightarrow 225a^2 + 9b^2 + 25c^2 - 75ac = 45ab + 15bc$

$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - 75ac - 45ab - 15\,bc = 0$

$$\frac{1}{2}\,[(15\,a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2\,] = 0$$

it is possible when $15a - 3b = 0$, $3b - 5\,c = 0$ and $5c - 15a = 0$

$\Rightarrow 15a = 3b = 5$

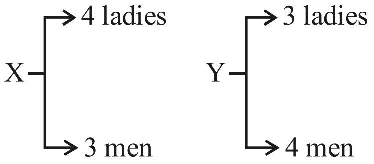
$$\Rightarrow b = \frac{5c}{3}, a = \frac{c}{3}$$

$$\Rightarrow a + b = \frac{c}{3} + \frac{5c}{3} = \frac{6c}{3}$$

$\Rightarrow a + b = 2c$

$\Rightarrow$ b, c, a are in A.P.

38. (2)



Possible cases for X are
(1) 3 ladies, 0 man
(2) 2 ladies, 1 man
(3) 1 lady, 2 men
(4) 0 ladies, 3 men
Possible cases for Y are
(1) 0 ladies, 3 men
(2) 1 lady, 2 men
(3) 2 ladies, 1 man
(4) 3 ladies, 0 man

No. of ways $= {}^4C_3 \,.\, {}^4C_3 + ({}^4C_2 \,.\, {}^3C_1)^2 + ({}^4C_1 \,.\, {}^3C_2)^2 + ({}^3C_3)^2$

$= 16 + 324 + 144 + 1 = 485$

39. (3) We have $y = \dfrac{x+6}{(x-2)(x-3)}$

At y-axis, $x = 0 \Rightarrow y = 1$
On differentiating, we get

$$\frac{dy}{dx} = \frac{(x^2-5x+6)\,(1)-(x+6)\,(2x-5)}{(x^2-5x+6)^2}$$

$\dfrac{dy}{dx} = 1$ at point (0, 1)

$\therefore$ Slope of normal $= -1$
Now equation of normal is $y - 1 = -1\,(x - 0)$
$\Rightarrow y - 1 = -x$
$x + y = 1$

$\therefore \left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ satisfy it.

40. (3) Equation of hyperbola is $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

foci is $(\pm 2, 0) \Rightarrow ae = 2 \Rightarrow a^2e^2 = 4$
Since $b^2 = a^2\,(e^2 - 1)$
$b^2 = a^2\,e^2 - a^2$
$\therefore \quad a^2 + b^2 = 4$ ...(1)

Hyperbola passes through $(\sqrt{2}, \sqrt{3})$

$\therefore \dfrac{2}{a^2} - \dfrac{3}{b^2} = 1$ ...(2)

$$\frac{2}{4-b^2}\,\frac{-3}{b^2} = 1$$

$\Rightarrow b^4 + b^2 - 12 = 0$
$\Rightarrow (b^2 - 3)\,(b^2 + 4) = 0$
$\Rightarrow b^2 = 3$
$b^2 = -4$ (Not possible)
For $b^2 = 3$
$\Rightarrow a^2 = 1$

$\therefore \dfrac{x^2}{1} - \dfrac{y^2}{3} = 1$

Equation of tangent is $\dfrac{\sqrt{2}x}{1} - \dfrac{\sqrt{3}y}{3} = 1$

Clearly $(2\sqrt{2}, 3\sqrt{3})$ satisfies it.

41. (2) $f(x) = ax^2 + bx + c$
$f(1) = a + b + c = 3 \Rightarrow f(1) = 3$
Now $f(x + y) = f(x) + f(y) + xy$ ...(1)
Put $x = y = 1$ in eqn (1)
$f(2) = f(1) + f(1) + 1$
$= 2f(1) + 1$
$f(2) = 7$
$\Rightarrow f(3) = 12$
Now, $S_n = 3 + 7 + 12 + \ldots\ldots\ldots t_n$ ...(1)
$S_n = 3 + 7 + \ldots\ldots\ldots t_{n-1} + t_n$ ...(2)
Subtract (2) from (1)
$t_n = 3 + 4 + 5 + \ldots.$ upto n terms

$$t_n = \frac{(n^2+5n)}{2}$$

$$S_n = \sum t_n = \sum \frac{(n^2+5n)}{2}$$

$$S_n = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2}\right] = \frac{n(n+1)(n+8)}{6}$$

$$S_{10} = \frac{10\times 11\times 18}{6} = 330$$

42. (3) Given :

$\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k},\ \ \vec{b} = \hat{i} + \hat{j}$

$\Rightarrow |\vec{a}| = 3$

$\therefore\ \vec{a}\times\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{2^2 + 2^2 + 1^2} = 3$

We have $(\vec{a}\times\vec{b})\times\vec{c} = |\vec{a}\times\vec{b}\,||\vec{c}\,|\ \sin 30\ n$

$\Rightarrow |(\vec{a}\times\vec{b})\times\vec{c}\,| = 3\,|\vec{c}\,|.\dfrac{1}{2} \Rightarrow 3 = 3\,|\vec{c}\,|.\dfrac{1}{2}$

$\therefore\ |\vec{c}\,| = 2$

Now $|\vec{c} - \vec{a}\,| = 3$

On squaring, we get

$\Rightarrow c^2 + a^2 - 2 - \vec{c}.\vec{a} = 9$

$\Rightarrow 4+9-2-\vec{a}.\vec{c}=9$

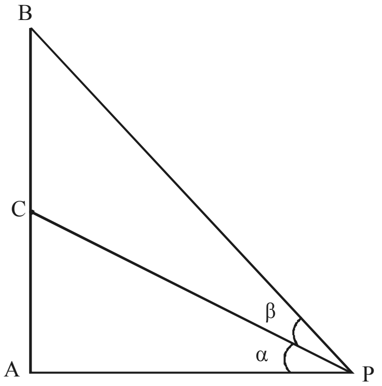
$\Rightarrow \vec{a}.\vec{c}=2$ $\quad [\because \vec{c}.\vec{a}=\vec{a}.\vec{c}\,]$

43. **(4)** Since AP = 2AB $\Rightarrow \dfrac{AB}{AP}=\dfrac{1}{2}$ ...(1)

Let $\angle APC=\alpha$

$\tan\alpha=\dfrac{AC}{AP}=\dfrac{1}{2}\dfrac{AB}{AP}=\dfrac{1}{4}$ $\quad$ ($\because$ C is the mid point $\therefore AC=\dfrac{1}{2}AB$)

$\Rightarrow \tan\alpha=\dfrac{1}{4}$



As $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$\Rightarrow \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{1}{2}\left[\begin{array}{l}\because \tan(\alpha+\beta)=\dfrac{AB}{AP}\\ \tan(\alpha+\beta)=\dfrac{1}{2}\ \ [\text{From}(1)]\end{array}\right]$$

$$\Rightarrow \frac{\frac{1}{4}+\tan\beta}{1-\frac{1}{4}\tan\beta}=\frac{1}{2}$$

$\therefore \tan\beta=\dfrac{2}{9}$

44. **(4)** We have

Total length $=r+r+r\theta=20$

$\Rightarrow 2r+r\theta=20$

$\Rightarrow \theta=\dfrac{20-2r}{r}$ ...(1)

$A=\text{Area}=\dfrac{\theta}{2\pi}\times\pi r^2=\dfrac{1}{2}r^2\theta=\dfrac{1}{2}r^2\left(\dfrac{20-2r}{r}\right)$

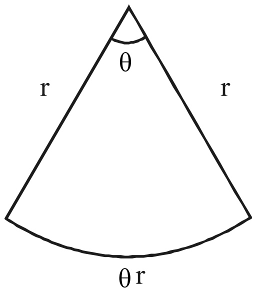
$A=10r-r^2$

For A to be maximum

$\dfrac{dA}{dr}=0 \Rightarrow 10-2r=0$

$\Rightarrow r=5$

$\dfrac{d^2A}{dr^2}=-2<0$



$\therefore$ For r = 5 A is maximum

From (1)

$\theta=\dfrac{20-2(5)}{5}=\dfrac{10}{5}=2$

$A=\dfrac{2}{2\pi}\times\pi(5)^2=25$ sq. m.

45. **(3)** $I=\displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{dx}{1+\cos x}$ ...(i)

$I=\displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{dx}{1-\cos x}$ ...(ii)

Using $\displaystyle\int_a^b f(x)dx=\int_a^b f(a+b-x)\,dx$

Adding (i) and (ii)

$$2I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{2}{\sin^2 x}dx$$

$$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\operatorname{cosec}^2 x\,dx$$

$$I=-\ (\cot x)_{\pi/4}^{3\pi/4}=-\left[\cot\frac{3\pi}{4}-\cot\frac{\pi}{4}\right]=2$$

46. **(2)** We have $(2+\sin x)\dfrac{dy}{dx}+(y+1)\cos x=0$

$\Rightarrow \dfrac{d}{dx}(2+\sin x)(y+1)=0$

On integrating, we get

$(2+\sin x)(y+1)=C$

At x = 0, y = 1 we have

$(2+\sin 0)(1+1)=C$

$\Rightarrow C=4$

$\Rightarrow y+1=\dfrac{4}{2+\sin x}$

$y=\dfrac{4}{2+\sin x}-1$

Now $y\left(\dfrac{\pi}{2}\right)=\dfrac{4}{2+\sin\frac{\pi}{2}}-1$

$=\dfrac{4}{3}-1=\dfrac{1}{3}$

47. **(3)** $I_n = \int \tan^n x\,dx, n > 1$

Let $I = I_4 + I_6$

$= \int (\tan^4 x + \tan^6 x)dx = \int \tan^4 x \sec^2 x\,dx$

Let $\tan x = t$

$\Rightarrow \sec^2 x\,dx = dt$

$\therefore I = \int t^4 dt$

$= \frac{t^5}{5} + C$

$= \frac{1}{5}\tan^5 x + C \Rightarrow$ On comparing, we have

$a = \frac{1}{5}, b = 0$

48. **(2)** Given $2\omega + 1 = z$;

$z = \sqrt{3}i$

$\Rightarrow \omega = \frac{\sqrt{3}i - 1}{2}$

$\Rightarrow \omega$ is complex cube root of unity

Applying $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 3 & 0 & 0 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$$

$= 3(-1 - \omega - \omega) = -3(1 + 2\omega) = -3z$

$\Rightarrow k = -z$

49. **(1)** We have $({}^{21}C_1 + {}^{21}C_2 \ \ldots\ldots + {}^{21}C_{10})$
$- ({}^{10}C_1 + {}^{10}C_2 \ldots\ldots {}^{10}C_{10})$

$= \frac{1}{2}[({}^{21}C_1 + \ldots. + {}^{21}C_{10}) + ({}^{21}C_{11} + \ldots. {}^{21}C_{20})] - (2^{10} - 1)$

$(\because {}^{10}C_1 + {}^{10}C_2 + \ldots. + {}^{10}C_{10} = 2^{10} - 1)$

$= \frac{1}{2}[2^{21} - 2] - (2^{10} - 1)$

$= (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10}$

50. **(3)** $\lim\limits_{x \to \frac{\pi}{2}} \frac{\cot x(1 - \sin x)}{-8\left(x - \frac{\pi}{2}\right)^3} = \lim\limits_{x \to \frac{\pi}{2}} \frac{\cot x(1 - \sin x)}{8\left(\frac{\pi}{2} - x\right)^3}$

Put $\frac{\pi}{2} - x = t \Rightarrow$ as $x \to \frac{\pi}{2} \Rightarrow t \to 0$

$= \lim\limits_{t \to 0} \frac{\cot\left(\frac{\pi}{2} - t\right)\left(1 - \sin\left(\frac{\pi}{2} - t\right)\right)}{8t^3}$

$= \lim\limits_{t \to 0} \frac{\tan t(1 - \cos t)}{8t^3}$

$= \lim\limits_{t \to 0} \frac{\tan t}{8t} \cdot \frac{1 - \cos t}{t^2}$

$= \frac{1}{8} . 1 . \frac{1}{2} = \frac{1}{16}$

51. **(1)** We have

$5\tan^2 x - 5\cos^2 x = 2(2\cos^2 x - 1) + 9$

$\Rightarrow 5\tan^2 x - 5\cos^2 x = 4\cos^2 x - 2 + 9$

$\Rightarrow 5\tan^2 x = 9\cos^2 x + 7$

$\Rightarrow 5(\sec^2 x - 1) = 9\cos^2 x + 7$

Let $\cos^2 x = t$

$\Rightarrow \frac{5}{t} - 9t - 12 = 0$

$\Rightarrow 9t^2 + 12t - 5 = 0$

$\Rightarrow 9t^2 + 15t - 3t - 5 = 0$

$\Rightarrow (3t - 1)(3t + 5) = 0$

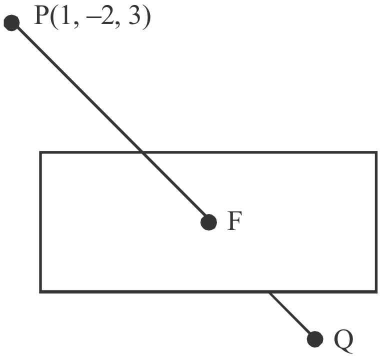
$\Rightarrow t = \frac{1}{3}$ as $t \neq -\frac{5}{3}$.

$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{1}{3}\right) - 1 = -\frac{1}{3}$

$\cos 4x = 2\cos^2 2x - 1 = 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$

52. **(3)** Equation of line PQ is $\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5}$

Let F be $(\lambda + 1, 4\lambda - 2, 5\lambda + 3)$



Since F lies on the plane

$\therefore 2(\lambda + 1) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$

$2\lambda + 2 + 12\lambda - 6 - 20\lambda - 12 + 22 = 0$

$\Rightarrow -6\lambda + 6 = 0 \Rightarrow \lambda = 1$

$\therefore$ F is (2, 2, 8)

$PQ = 2\,PF = 2\sqrt{1^2 + 4^2 + 5^2} = 2\sqrt{42}$

53. **(3)** Let the plane be

$a(x - 1) + b(y + 1) + c(z + 1) = 0$

Normal vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i}+7\hat{j}+3\hat{k}$$

So plane is $5(x-1)+7(y+1)+3(z+1)=0$

$\Rightarrow 5x+7y+3z+5=0$

Distance of point (1, 3, –7) from the plane is

$$\frac{5+21-21+5}{\sqrt{25+49+9}}=\frac{10}{\sqrt{83}}$$

54. **(2)** Let $F(x)=\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ where $x\in\left(0,\frac{1}{4}\right)$.

$$=\tan^{-1}\left(\frac{2.(3x^{3/2})}{1-(3x^{3/2})^2}\right)=2\tan^{-1}(3x^{3/2})$$

As $3x^{3/2}\in\left(0,\frac{3}{8}\right)$

$$\left[\because 0<x<\frac{1}{4}\Rightarrow 0<x^{3/2}<\frac{1}{8}\Rightarrow 0<3x^{3/2}<\frac{3}{8}\right]$$

So $\frac{dF(x)}{dx}=2\times\frac{1}{1+9x^3}\times 3\times\frac{3}{2}\times x^{1/2}$

$$=\frac{9}{1+9x^3}\sqrt{x}$$

On comparing

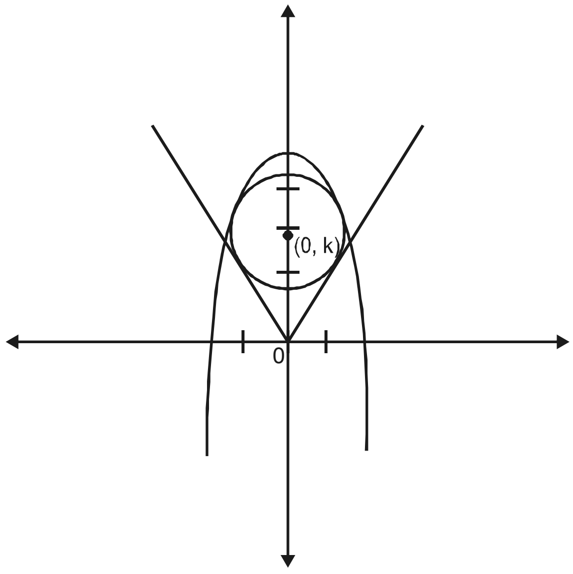
$\therefore g(x)=\frac{9}{1+9x^3}$

55. **(None)**

(Let the equation of circle be

$x^2+(y-k)^2=r^2$

It touches $x-y=0$



$$\Rightarrow \left|\frac{0-k}{\sqrt{2}}\right|=r$$

$\Rightarrow k=r\sqrt{2}$

$\therefore$ Equation of circle becomes

$$x^2+(y-k)^2=\frac{k^2}{2} \qquad ...(i)$$

It touches $y=4-x^2$ as well

$\therefore$ Solving the two equations

$$\Rightarrow 4-y+(y-k)^2=\frac{k^2}{2}$$

$$\Rightarrow y^2-y(2k+1)+\frac{k^2}{2}+4=0$$

It will give equal roots $\therefore D=0$

$$\Rightarrow (2k+1)^2=4\left(\frac{k^2}{2}+4\right)$$

$$\Rightarrow 2k^2+4k-15=0$$

$$\Rightarrow k=\frac{-2+\sqrt{34}}{2}$$

$$\therefore r=\frac{k}{\sqrt{2}}=\frac{-2+\sqrt{34}}{2\sqrt{2}}$$

Which is not matching with any of the option given here.

56. **(2)** We can apply binomial probability distribution

We have n = 10

$p=$ Probability of drawing a green ball $=\frac{15}{25}=\frac{3}{5}$

Also $q=1-\frac{3}{5}=\frac{2}{5}$

Variance = npq

$$=10\times\frac{3}{5}\times\frac{2}{5}=\frac{12}{5}$$

57. **(3)** Eccentricity of ellipse $=\frac{1}{2}$

Now, $-\frac{a}{e}=-4\Rightarrow a=4\times\frac{1}{2}=2\Rightarrow a=2$

We have $b^2=a^2(1-e^2)=a^2\left(1-\frac{1}{4}\right)=4\times\frac{3}{4}=3$

$\therefore$ Equation of ellipse is

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

Now differentiating, we get

$$\Rightarrow \frac{x}{2}+\frac{2y}{3}\times y'=0\Rightarrow y'=-\frac{3x}{4y}$$

$$y'\Big|_{(1,3/2)}\Big|=-\frac{3}{4}\times\frac{2}{3}=-\frac{1}{2}$$

Slope of normal = 2

$\therefore$ Equation of normal at $\left(1, \frac{3}{2}\right)$ is

$$y - \frac{3}{2} = 2(x-1) \Rightarrow 2y - 3 = 4x - 4$$

$\therefore 4x - 2y = 1$

58. **(2)** Let $A \equiv \{0, 1, 2, 3, 4, \ldots\ldots., 10\}$

$n(S) = {}^{11}C_2 = 55$ where 'S' denotes sample space

Let E be the given event

$\therefore E \equiv \{(0,4), (0,8), (2,6), (2,10), (4,8), (6,10)\}$

$\Rightarrow n(E) = 6$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{55}$$

59. **(3)** P (exactly one of A or B occurs)

$$= P(A) + P(B) - 2P(A \cap B) = \frac{1}{4} \qquad \ldots(1)$$

P (Exactly one of B or C occurs)

$$= P(B) + P(C) - 2P(B \cap C) = \frac{1}{4} \qquad \ldots(2)$$

P (Exactly one of C or A occurs)

$$= P(C) + P(A) - 2P(C \cap A) = \frac{1}{4} \qquad \ldots(3)$$

Adding (1), (2) and (3),we get

$$2\Sigma P(A) - 2\Sigma P(A \cap B) = \frac{3}{4}$$

$$\therefore \Sigma P(A) - \Sigma P(A \cap B) = \frac{3}{8}$$

$$\text{Now, } P(A \cap B \cap C) = \frac{1}{16}$$

$\therefore P(A \cup B \cup C)$

$= \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$

$$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$$

60. **(3)** We have $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$$\Rightarrow A^2 = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$$

$$\Rightarrow 3A^2 = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix}$$

$$\text{Also } 12A = \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$$

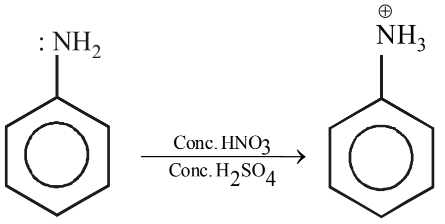
$$\therefore 3A^2 + 12A = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix} = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\text{adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

SECTION-3 : CHEMISTRY

61. **(3)** Nitration takes place in presence of concentrated HNO_3 + concentrated H_2SO_4.

In strongly acidic nitration medium, the amine is converted into anilinium ion ($-NH_3^+$); substitution is thus controlled not by $-NH_2$ group but by $-NH_3^+$ group which, because of its positive charge, directs the entering group to the meta-position instead of ortho, and para.



$-NH_2$ gp : *o*, *p*-directing $\quad -NH_3^+$ gp : *m*-directing

62. **(3)** From 1st law of thermodynamics

$\Delta U = q + w$

For adiabatic process :

$q = 0$

$\therefore \Delta U = w$

i.e. change in internal energy (ΔU) is equal to adiabatic work.

63. **(2)** Since S_N1 reactions involve the formation of carbocation as intermediate in the rate determining step, more is the stability of carbocation higher will be the reactivity of alkyl halides towards S_N1 route.

Since stability of carbocation follows order.

$$\underset{1°}{CH_3 - CH_2 - \overset{\oplus}{C}H_2}$$

$$< \underset{2°}{CH_3 - \overset{\oplus}{C}H - CH_2 - CH_3}$$

$$< \underset{\text{benzylic}}{p - H_3CO - C_6H_4 - \overset{\oplus}{C}H_2}$$

Hence correct order is II < I < III

64. **(4)** Radius of nth Bohr orbit in H-atom

$= 0.53\, n^2 Å$

Radius of 2nd Bohr orbit $= 0.53 \times (2)^2$

$= 2.12$ Å

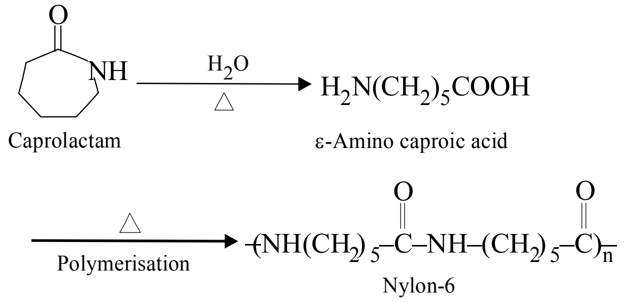
65. **(2)** The salt (AB) given is a salt is of weak acid and weak base. Hence its pH can be calculated by the formula

$$\therefore \quad pH = 7 + \frac{1}{2}pK_a - \frac{1}{2}pK_b$$

$$= 7 + \frac{1}{2}(3.2) - \frac{1}{2}(3.4)$$

$$= 6.9$$

66. **(1)** Formation of nylon-6 involves hydrolysis of caprolactum, (its monomer) in initial state.



67. **(3)** Percentage (by mass) of elements given in the body of a healthy human adult is :-

Oxygen = 61.4%, Carbon = 22.9%,

Hydrogen = 10.0% and Nitrogen = 2.6%

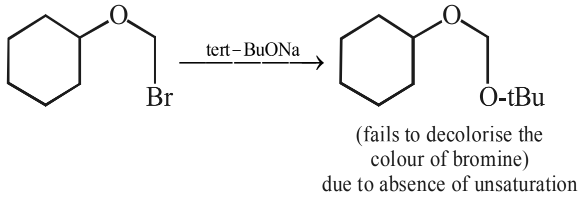
$\because$ Weight of person = 75 kg

$\therefore$ Mass due to ^1H is $= 75 \times \frac{10}{100} = 7.5\,kg$

If ^1H atoms are replaced by ^2H atoms, mass gain by person would be = 7.5 kg

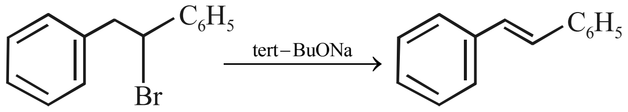
68. **(1)**

(1)



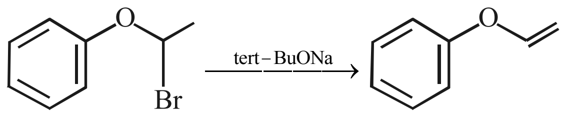
(fails to decolorise the colour of bromine) due to absence of unsaturation

(2)



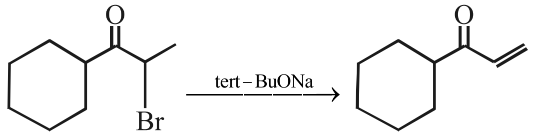
(it decolorises bromine solution)

(3)



(it decolorises bromine solution)

(4)



(it decolorises bromine solution)

69. **(4)** Although ZnO is an amphoteric oxide but in given reaction.

(A) $\underset{\text{acid}}{ZnO} + \underset{\text{base}}{Na_2O} \longrightarrow \underset{\text{salt}}{Na_2ZnO_2}$

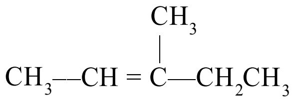
(B) $\underset{\text{base}}{ZnO} + \underset{\text{acid}}{CO_2} \longrightarrow \underset{\text{salt}}{ZnCO_3}$

70. **(1)** Mg can form basic carbonate like

$3MgCO_3 \,.\, Mg(OH)_2 \,.\, 3H_2O$

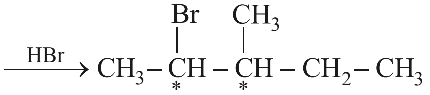
While Li can form only carbonate (Li_2CO_3), not basic carbonate.

71. **(4)**



$CH_3-CH=\overset{CH_3}{\overset{|}{C}}-CH_2CH_3$

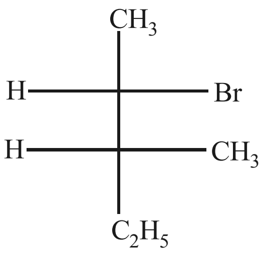
3–Methyl pent–2–ene



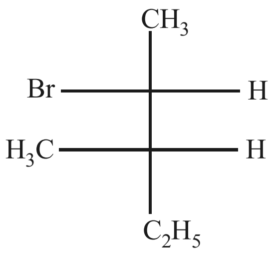
$\xrightarrow{HBr} CH_3-\overset{Br}{\overset{|}{\underset{*}{CH}}}-\overset{CH_3}{\overset{|}{\underset{*}{CH}}}-CH_2-CH_3$

2–Bromo–3-methyl pentane
(2 chiral centre)

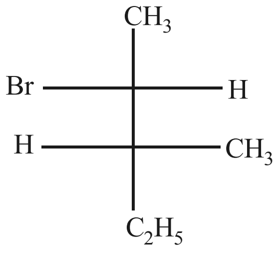
A compound having two chiral centres can exist in 4 stereoisomeric forms (2^n).



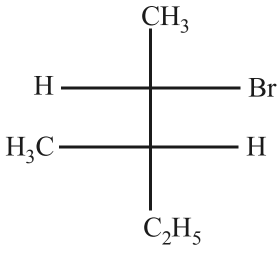
(I)



(II)



(III)



(IV)

72. **(4)** For a fcc unit cell

$$r = \frac{\sqrt{2}\,a}{4}$$

$\therefore$ Closest distance (2r) $= \frac{\sqrt{2}\,a}{4} = \frac{a}{\sqrt{2}}$

73. **(4)** From arrhenius equation,

$$k = A.e^{\frac{-Ea}{RT}}$$

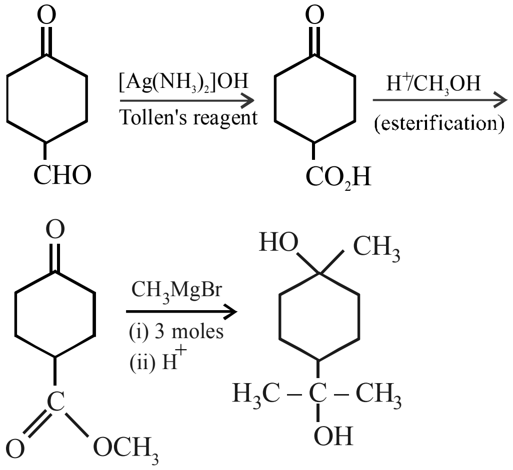
so, $k_1 = A.e^{-E_{a_1}/RT}$ .....(1)

$k_2 = A.e^{-E_{a_2}/RT}$ .....(2)

On dividing equation (2)/(1) $\Rightarrow \frac{k_2}{k_1} = e^{\frac{(E_{a_1} - E_{a_2})}{RT}}$

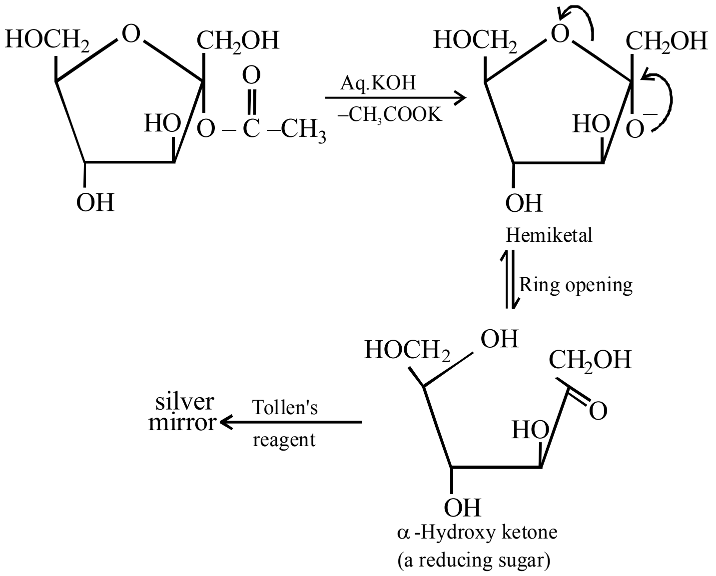
$$\ln\left(\frac{k_2}{k_1}\right)=\frac{E_{a_1}-E_{a_2}}{RT}=\frac{10,000}{8.314\times 300}=4$$

74. **(1)**



75. **(2)** **Theory based**

76. **(1)**



77. **(3)** Given

$CO_2(g)+2H_2O(\ell)\rightarrow CH_4(g)+2O_2(g);\Delta_rH^\circ=890.3$...(iii)

$C_{(graphite)}+O_2(g)\rightarrow CO_2(g)$;

$\Delta_rH^\circ=-393.5\text{ kJ mol}^{-1}$ ...(i)

$H_2(g)+\frac{1}{2}O_2(g)\rightarrow H_2O(\ell)$;

$\Delta_rH^\circ=-285.8\text{ kJ mol}^{-1}$ ...(ii)

$H_2(g)+\frac{1}{2}O_2(g)\rightarrow H_2O(\ell)$;

$\Delta_rH^\circ=-285.8\text{ kJ mol}^{-1}$ ...(iii)

$$\Delta_rH^\circ=\sum(\Delta_rH^\circ)_{products}-\sum(\Delta_fH^\circ)_{Reactants}$$

$$=(\Delta H^o_{CH_4}+2\times\Delta H^o_{O_2})-(\Delta H^o_{CO_2}+2\times\Delta H^o_{H_2O})$$

$$890.3=\left[1\times(\Delta_f H^\circ)_{CH_4}+2\times 0\right]-\left[1\times(-393.5)+2(-285.8)\right]$$

$$(\Delta_f H^\circ)_{CH_4}=890.3-965.1=-74.8\text{ kJ / mol}$$

78. **(1)** In the reaction

Oxidation

$$\overset{+4}{Xe}F_4+\overset{+1}{O_2}F_2\rightarrow\overset{+6}{Xe}F_6+\overset{0}{O_2}$$

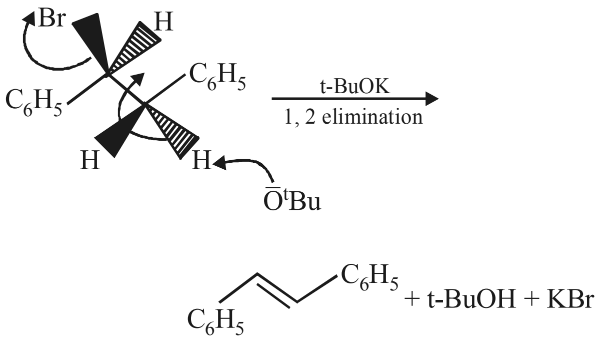
Reduction

79. **(3)** $Cl_2+NaOH\rightarrow NaCl+NaClO+H_2O$

[cold and dilute]

80. **(2)** *t*–BuOK is a bulky strong base and causes elimination (dehydrohlogenation) reaction

The given reaction proceeds via biomolecular elimination.



81. **(4)** $\underset{'x'}{Na_2C_2O_4}+\underset{(conc.)}{H_2SO_4}\rightarrow Na_2SO_4+\underset{(oxalic\ acid)}{H_2C_2O_4}$

$$\underset{\text{Oxalic acid}}{H_2C_2O_4}\xrightarrow{\text{conc. }H_2SO_4}\underset{\text{effervescens}}{CO\uparrow+CO_2\uparrow+H_2O}$$

$$\underset{'x'}{Na_2C_2O_4}+CaCl_2\rightarrow\underset{\text{(white ppt.)}}{CaC_2O_4\downarrow}+2NaCl$$

$$5CaC_2O_4\downarrow+\underset{\text{(purple)}}{2KMnO_4}+8H_2SO_4\rightarrow$$

$$K_2SO_4+5CaSO_4+\underset{\text{(colourless)}}{2MnSO_4}+10CO_2+8H_2O$$

82. **(2)**

1. NO (15) → one unpaired electron is present in π* molecular orbit hence **paramagnetic**.
2. $CO(14)\rightarrow\sigma^2_{1s},\sigma^{*2}_{1s},\sigma^2_{2s},\sigma^{*2}_{2s},\pi^2_{2p_x},\pi^2_{2p_y},\sigma^2_{2p_z}$

   No unpaired electron, hence **diamagnetic.**
3. $O_2(16)\rightarrow\sigma^2_{1s},\sigma^{*2}_{1s},\sigma^2_{2s},\sigma^{*2}_{2s},\sigma^2_{2p_z},\pi^2_{2p_x},\pi^2_{2p_y},\pi^{*1}_{2p_x},\pi^{*1}_{2p_y}$

   Two unpaired electrons, hence **paramagnetic**.
4. $B_2(10)\rightarrow\sigma^2_{1s},\sigma^{*2}_{1s},\sigma^2_{2s},\sigma^{*2}_{2s},\pi^1_{2p_x},\pi^1_{2p_y}$

   B_2 contains two unpaired electrons, hence **paramagnetic**

83. **(4)** In benzene

$$2CH_3COOH\rightleftharpoons(CH_3COOH)_2$$

$$1-\alpha\qquad\qquad\alpha/2$$

$$i=1-\alpha+\alpha/2=1-\alpha/2$$

Here α is degree of association

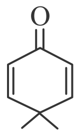
$$\Delta T_f=iK_fm$$

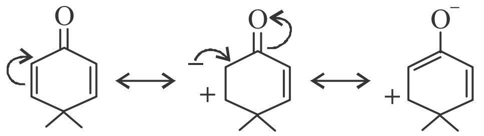
$$0.45=\left(1-\frac{\alpha}{2}\right)(5.12)\frac{\left(\frac{0.2}{60}\right)}{\frac{20}{1000}}$$

$$1-\frac{\alpha}{2}=0.527$$

$$\alpha=0.945$$

% degree of association = 94.6%

84. **(4)**  is non-aromatic and hence least resonance stabilized,



whereas other three are aromatic and resonance stabilized.

85. **(4)** Moles of complex $=\frac{\text{Molarity}\times\text{Volume(mL)}}{1000}$

$$=\frac{100\times0.1}{1000}=0.01\text{ mole}$$

Moles of ions precipitated with excess of $AgNO_3$

$$=\frac{1.2\times10^{22}}{6.02\times10^{23}}=0.02\text{ moles}$$

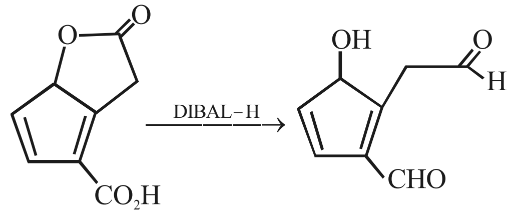
$0.01\times n=0.02$

$\therefore\ n=2$

It means $2Cl^-$ ions present in ionization sphere

$\therefore$ Complex is $[Co(H_2O)_5Cl]Cl_2.H_2O$

86. **(2)** DIBAL-H is a bulkier reducing agent. It reduces both ester and carboxylic group into an aldehyde (partial reduction) at low temperature.



87. **(3)** Above 2 ppm concentration of F^- in drinking water causes brown mottling of teeth.

88. **(2)** Given chemical eq^n

$$\underset{1gm}{M_2CO_3}+2HCl\rightarrow2MCl+H_2O+\underset{0.01186\text{ mol}}{CO_2}$$

From the above chemical eq^n.

$nM_2CO_3=nCO_2$

$$\frac{1}{\text{Molar mass of }M_2CO_3}=0.01186$$

$$\therefore\ \text{Molar mass of }M_2CO_3=\frac{1}{0.01186}$$

M = 84.3 gm/mol

89. **(1)** $E^\circ_{MnO_4^-/Mn^{2+}}=1.51V$

$E^\circ_{Cl_2/Cl^-}=1.36V$

$E^\circ_{Cr_2O_7^{2-}/Cr^{3+}}=1.33V$

$E^\circ_{Cr^{3+}/Cr}=-0.74$

More negative the E° value of the species, more stronger is the reducing agent. Since Cr^{3+} is having least reducing potential, so Cr is the best reducing agent.

90. **(1)** Isoelectronic species have same no. of electrons.

ions	O^{2-}	F^-	Na^+	Mg^{2+}
	8+2	9+1	11–1	12–2
No. of e^- =	10	10	10	10

Therefore O^{2-}, F^-, Na^+, Mg^{+2} are isoelectronic

JEE Main - 2018

Time : 3 Hours • Each correct answer has + 4 marks • Each wrong answer has – 1 mark. **Max. Marks : 360**

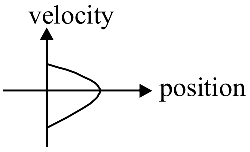
Section - 1

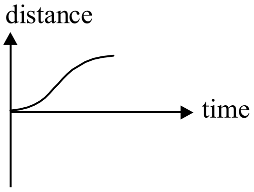
PHYSICS

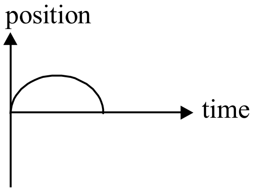
1. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is:

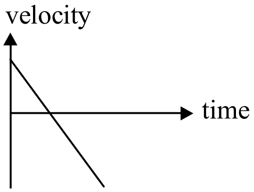
(1) 2.5% (2) 3.5%
(3) 4.5% (4) 6%

2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

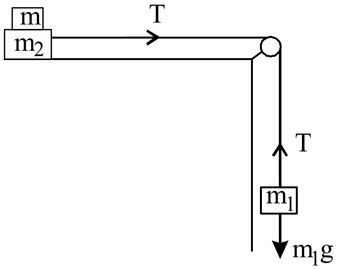
(1)


(2)


(3)


(4)


3. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is:



(1) 18.3 kg (2) 27.3 kg
(3) 43.3 kg (4) 10.3 kg

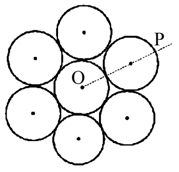
4. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is:

(1) $-\frac{k}{4a^2}$ (2) $\frac{k}{2a^2}$
(3) zero (4) $-\frac{3}{2}\frac{k}{a^2}$

5. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is:

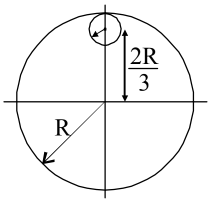
(1) $\frac{v_0}{4}$ (2) $\sqrt{2}v_0$
(3) $\frac{v_0}{2}$ (4) $\frac{v_0}{\sqrt{2}}$

6. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is:



(1) $\frac{19}{2}MR^2$ (2) $\frac{55}{2}MR^2$
(3) $\frac{73}{2}MR^2$ (4) $\frac{181}{2}MR^2$

7. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing, through centre of disc is :



(1) $4\,MR^2$ (2) $\frac{40}{9}\,MR^2$

(3) $10\,MR^2$ (4) $\frac{37}{9}\,MR^2$

8. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then:

(1) $T \propto R^{3/2}$ for any n. (2) $T \propto R^{n/2+1}$

(3) $T \propto R^{(n+1)/2}$ (4) $T \propto R^{n/2}$

9. A solid sphere of radius *r* made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass *m* is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere $\left(\frac{dr}{r}\right)$, is :

(1) $\frac{Ka}{mg}$ (2) $\frac{Ka}{3mg}$

(3) $\frac{mg}{3Ka}$ (4) $\frac{mg}{Ka}$

10. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 189 K (b) 2.7 kJ

(2) (a) 195 K (b) –2.7 kJ

(3) (a) 189 K (b) –2.7 kJ

(4) (a) 195 K (b) 2.7 kJ

11. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly:

(1) $2.35 \times 10^3\ N/m^2$ (2) $4.70 \times 10^3\ N/m^2$

(3) $2.35 \times 10^2\ N/m^2$ (4) $4.70 \times 10^2\ N/m^2$

12. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = 6.02 $\times 10^{23}$ gm mole^{-1})

(1) 6.4 N/m (2) 7.1 N/m

(3) 2.2 N/m (4) 5.5 N/m

13. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3\ kg/m^3$ and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?

(1) 5 kHz (2) 2.5 kHz

(3) 10 kHz (4) 7.5 kHz

14. Three concentric metal shells A, B and C of respective radii *a*, *b* and *c* (*a* < *b* < *c*) have surface charge densities +σ, –σ and +σ respectively. The potential of shell B is:

(1) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (2) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

(3) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$ (4) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

15. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $k = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be:

(1) 1.2 n C (2) 0.3 n C

(3) 2.4 n C (4) 0.9 n C

16. In an a.c. circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30\,t$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively:

(1) 50W, 10A (2) $\frac{1000}{\sqrt{2}}$ W, 10A

(3) $\frac{50}{\sqrt{2}}$ W, 0 (4) 50W, 0

17. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between:

(1) 11.6 V and 11.7 V (2) 11.5 V and 11.6 V

(3) 11.4 V and 11.5 V (4) 11.7 V and 11.8 V

18. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is :

(1) $r_e > r_p = r_\alpha$ (2) $r_e < r_p = r_\alpha$

(3) $r_e < r_p < r_\alpha$ (4) $r_e < r_\alpha < r_p$

19. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is:

(1) 2 (2) $\sqrt{3}$

(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

20. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by:

(1) $\frac{\omega_0 L}{R}$ (2) $\frac{\omega_0 R}{L}$

(3) $\frac{R}{(\omega_0 C)}$ (4) $\frac{CR}{\omega_0}$

21. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\frac{z}{c}-t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos[k(2z-ct)]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is nonmagnetic. If $\in_{r_1}$ and $\in_{r_2}$ refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) $\frac{\in_{r_1}}{\in_{r_2}} = 4$ (2) $\frac{\in_{r_1}}{\in_{r_2}} = 2$

(3) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{4}$ (4) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{2}$

22. Unpolarized light of intensity I passes through an ideal polarizer A. Another indentical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is:

(1) 0° (2) 30°

(3) 45° (4) 60°

23. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?

(i.e. distance between the centres of each slit.)

(1) 25 μm (2) 50 μm

(3) 75 μm (4) 100 μm

24. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the nth state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the nth state to the ground state. For large n, (A, B are constants)

(1) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (2) $\Lambda_n \approx A + B\lambda_n$

(3) $\Lambda_n^2 \approx A + B\lambda_n^2$ (4) $\Lambda_n^2 \approx \lambda$

25. If the series limit frequency of the Lyman series is ν_1, then the series limit frequency of the P-fund series is :

(1) 25 ν_L (2) 16 ν_L

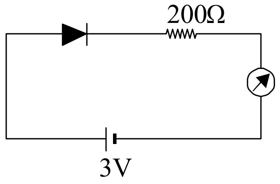
(3) $\nu_L/16$ (4) $\nu_L/25$

26. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively:

(1) $(\cdot 89, \cdot 28)$ (2) $(\cdot 28, \cdot 89)$

(3) (0, 0) (4) (0, 1)

27. The reading of the ammeter for a silicon diode in the given circuit is :



(1) 0 (2) 15 mA

(3) 11.5 mA (4) 13.5 mA

28. A telephonic communication service is working at carrier frequency of 10 GHz. Only10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(1) 2×10^3 (2) 2×10^4

(3) 2×10^5 (4) 2×10^6

29. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 1 Ω (2) 1.5 Ω

(3) 2 Ω (4) 2.5 Ω

30. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1kΩ. How much was the resistance on the left slot before interchanging the resistances?

(1) 990 Ω (2) 505 Ω

(3) 550 Ω (4) 910 Ω

Section - 2

CHEMISTRY

31. The ratio of mass percent of C and H of an organic compound ($C_XH_YO_Z$) is 6 : 1. If one molecule of the above compound ($C_XH_YO_Z$) contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_XH_YO_Z$ is :

(1) $C_3H_6O_3$
(2) C_2H_4O
(3) $C_3H_4O_2$
(4) $C_2H_4O_3$

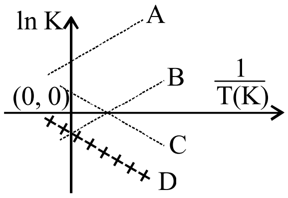
32. Which type of 'defect' has the presence of cations in the interstitial sites?

(1) Schottky defect
(2) Vacancy defect
(3) Frenkel defect
(4) Metal deficiency defect

33. According to molecular orbital theory, which of the following will not be a viable molecule?

(1) He_2^{2+}
(2) He_2^+
(3) H_2^-
(4) H_2^{2-}

34. Which of the following lines correctly show the temperature dependence of equilibrium constant, K, for an exothermic reaction?



(1) A and B
(2) B and C
(3) C and D
(4) A and D

35. The combustion of benzene (l) gives CO_2 (*g*) and H_2O (*l*). Given that heat of combustion of benzene at constant volume is –3263.9 kJ mol^{-1} at 25°C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be : (R= 8.314 JK^{-1} mol^{-1})

(1) 4152.6
(2) –452.46
(3) 3260
(4) –3267.6

36. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

(1) $[Co(H_2O)_6]Cl_3$
(2) $[Co(H_2O)_5Cl]Cl_2.H_2O$
(3) $[Co(H_2O)_4Cl_2]Cl. 2H_2O$
(4) $[Co(H_2O)_3Cl_3].3H_2O$

37. An aqueous solution contains 0.10 MH_2S and 0.20 M HCl. If the equilibrium constants for the formation of HS^- from H_2S is 1.0 × 10^{-7} and that of S^{2-} from HS^- ions is 1.2 × 10^{-13} then the concentration of S^{2-} ions in aqueous solution is :

(1) 5 × 10^{-8}
(2) 3 × 10^{-20}
(3) 6 × 10^{-21}
(4) 5 × 10^{-19}

38. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1 × 10^{-10}. What is the original concentration of Ba^{2+}?

(1) 5 × 10^{-9} M
(2) 2 × 10^{-9} M
(3) 1.1 × 10^{-9} M
(4) 1.0 × 10^{-10} M

39. At 518°C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is :

(1) 2
(2) 3
(3) 1
(4) 0

40. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane? (Atomic weight of B = 10.8 u)

(1) 6.4 hours
(2) 0.8 hours
(3) 3.2 hours
(4) 1.6 hours

41. The recommended concentration of fluoride ion in drinking water is up to 1ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2. Ca(OH)_2]$ to :

(1) $[CaF_2]$
(2) $[3(CaF_2).Ca(OH)_2]$
(3) $[3Ca_3(PO_4)_2.CaF_2]$
(4) $[3\{(Ca(OH)_2\}.CaF_2]$

42. Which of the following compounds contain(s) no covalent bond(s)? KCl, PH_3, O_2, B_2H_6, H_2SO_4

(1) KCl, B_2H_6, PH_3
(2) KCl, H_2SO_4
(3) KCl
(4) KCl, B_2H_6

43. Which of the following are Lewis acids?

(1) PH_3 and BCl_3
(2) $AlCl_3$ and $SiCl_4$
(3) PH_3 and $SiCl_4$
(4) BCl_3 and $AlCl_3$

44. Total number of lone pair of electrons in I_3^- ion is :

(1) 3
(2) 6
(3) 9
(4) 12

45. Which of the following salts is the most basic in aqueous solution?

(1) $Al(CN)_3$
(2) CH_3COOK
(3) $FeCl_3$
(4) $Pb(CH_3COO)_2$

46. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are respectively:

(1) $(H_2O + O_2)$ and H_2O
(2) $(H_2O + O_2)$ and $(H_2O + OH^-)$
(3) H_2O and $(H_2O + O_2)$
(4) H_2O and $(H_2O + OH^-)$

47. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN)_2(O)_2(O)_2(NH_3)]$ respectively are :

(1) +3, +4, and +6
(2) +3, +2, and +4
(3) +3, 0, and +6
(4) +3, 0, and + 4

48. The compound that **does not** produce nitrogen gas by the thermal decomposition is :

(1) $Ba(N_3)_2$
(2) $(NH_4)_2Cr_2O_7$
(3) NH_4NO_2
(4) $(NH_4)_2SO_4$

49. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is :

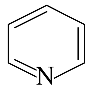
(1) Zn (2) Ca
(3) Al (4) Fe

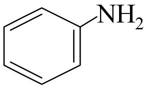
50. Consider the following reaction and statements:
$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$
(I) Two isomers are produced if the reactant complex ion is a *cis*-isomer.
(II) Two isomers are produced if the reactant complex ion is a *trans* -isomer
(III) Only one isomer is produced if the reactant complex ion is a *trans* -isomer
(IV) Only one isomer is produced if the reactant complex ion is a *cis*-isomer.
The correct statements are:
(1) (I) and (II) (2) (I) and (III)
(3) (III) and (IV) (4) (II) and (IV)

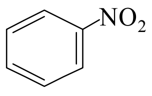
51. Glucose on prolonged heating with HI gives :
(1) *n*-Hexane (2) 1- Hexene
(3) Hexanoic acid (4) 6-iodohexanal

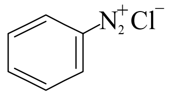
52. The *trans*-alkenes are formed by the reduction of alkynes with:
(1) H_2-Pd/C, $BaSO_4$ (2) $NaBH_4$
(3) Na/liq. NH_3 (4) Sn - HCl

53. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?

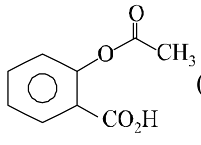
(1) 

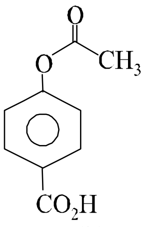
(2) 

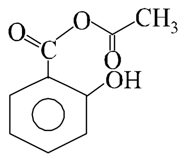
(3) 

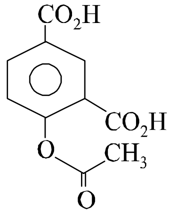
(4) 

54. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces :

(1) 

(2) 

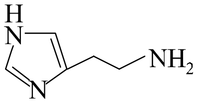
(3) 

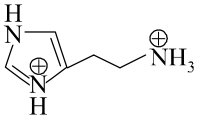
(4) 

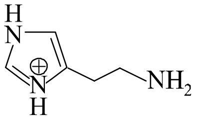
55. An alkali is titrated against an acid with methyl orange as indicator, which of the following in a correct combination?

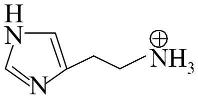
	Base	**Acid**	**End point**
(1)	Weak	Strong	Colourless to pink
(2)	Strong	Strong	Pinkish red to yellow
(3)	Weak	Strong	Yellow to Pinkish red
(4)	Strong	Strong	Pink to colourless

56. The predominant form of histamine present in human blood is (pK_a, Histidine – 6.0)

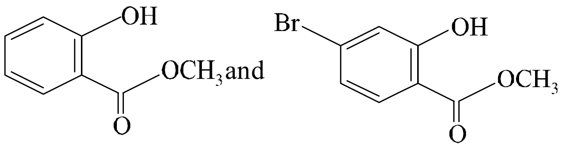
(1) 

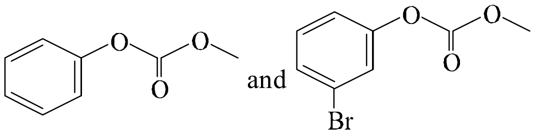
(2) 

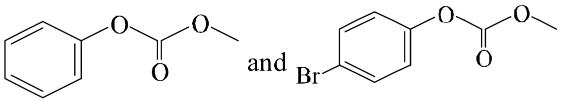
(3) 

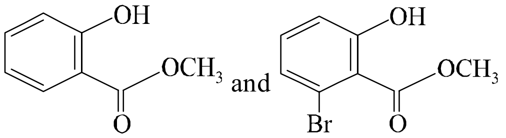
(4) 

57. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively :

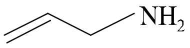
(1) 

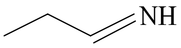
(2) 

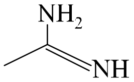
(3) 

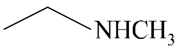
(4) 

58. The increasing order of basicity of the following compounds is

(a) 

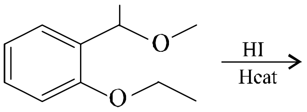
(b) 

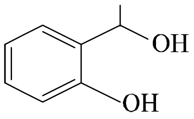
(c) 

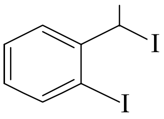
(d) 

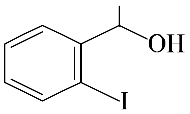
(1) (a) < (b) < (c) < (d) (2) (b) < (a) < (c) < (d)
(3) (b) < (a) < (d) < (c) (4) (d) < (b) < (a) < (c)

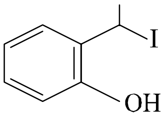
59. The major product formed in the following reaction is :



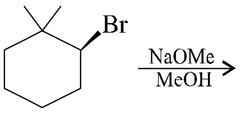
(1) 

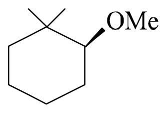
(2) 

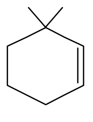
(3) 

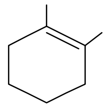
(4) 

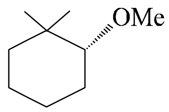
60. The major product of the following reaction is :



(1) 

(2) 

(3) 

(4) 

Section - 3

MATHEMATICS

61. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x+\cos^3 x \sin^2 x+\sin^3 x \cos^2 x+\cos^5 x)^2}dx \text{ is equal to :}$$

(1) $\frac{-1}{3(1+\tan^3 x)}+C$ (2) $\frac{1}{1+\cot^3 x}+C$

(3) $\frac{-1}{1+\cot^3 x}+C$ (4) $\frac{1}{3(1+\tan^3 x)}+C$

(where C is a constant of integration)

62. Tangents are drawn to the hyperbola $4x^2-y^2=36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is :

(1) $54\sqrt{3}$ (2) $60\sqrt{3}$

(3) $36\sqrt{5}$ (4) $45\sqrt{5}$

63. Tangent and normal are drawn at P(16, 16) on the parabola $y^2=16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB=\theta$, then a value of $\tan\theta$ is :

(1) 2 (2) 3

(3) $\frac{4}{3}$ (4) $\frac{1}{2}$

64. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b}-24$, then $|\vec{u}|^2$ is equal to :

(1) 315 (2) 256

(3) 84 (4) 336

65. If $\alpha,\beta \in C$ are the distinct roots, of the equation $x^2-x+1=0$, then $\alpha^{101}+\beta^{107}$ is equal to :

(1) 0 (2) 1

(3) 2 (4) – 1

66. Let $g(x)=\cos x^2, f(x)=\sqrt{x}$, and $\alpha, \beta\ (\alpha<\beta)$ be the roots of the quadratic equation $18x^2-9\pi x+\pi^2=0$. Then the area (in sq. units) bounded by the curve $y=(gof)(x)$ and the lines $x=\alpha, x=\beta$ and $y=0$, is :

(1) $\frac{1}{2}(\sqrt{3}+1)$ (2) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

(3) $\frac{1}{2}(\sqrt{2}-1)$ (4) $\frac{1}{2}(\sqrt{3}-1)$

67. The sum of the co-efficients of all odd degree terms in the expansion of

$(x+\sqrt{x^3-1})^5+(x-\sqrt{x^3-1})^5, (x>1)$ is :

(1) 0 (2) 1

(3) 2 (4) – 1

68. Let $a_1, a_2, a_3, ..., a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1}=416$ and $a_9+a_{43}=66$. If $a_1^2+a_2^2+...+a_{17}^2=140m$, then m is equal to :

(1) 68 (2) 34

(3) 33 (4) 66

69. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is :

(1) 4 (2) 2

(3) 3 (4) 9

70. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is :

(1) 50 (2) $100\sqrt{3}$

(3) $50\sqrt{2}$ (4) 100

71. Two sets A and B are as under :

$A=\{(a, b)\in R\times R : |a-5|<1 \text{ and } |b-5|<1\};$

$B=\{(a,b)\in R\times R : 4(a-6)^2+9(b-5)^2\le 36\}$. Then :

(1) $A\subset B$

(2) $A\cap B=\phi$ (an empty set)

(3) neither $A\subset B$ nor $B\subset A$

(4) $B\subset A$

72. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :

(1) less than 500
(2) at least 500 but less than 750
(3) at least 750 but less than 1000
(4) at least 1000

73. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of h(x) is :

(1) -3 (2) $-2\sqrt{2}$
(3) $2\sqrt{2}$ (4) 3

74. For each $t \in R$, let [t] be the greatest integer less than or equal to t. Then

$$\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right)$$

(1) is equal to 15. (2) is equal to 120.
(3) does not exist (in R). (4) is equal to 0.

75. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + 2^x} dx$ is :

(1) $\frac{\pi}{2}$ (2) 4π
(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{8}$

76. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$
(3) $\frac{3}{4}$ (4) $\frac{3}{10}$

77. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x + y + z = 7$ is:

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$
(3) $\sqrt{\frac{2}{3}}$ (4) $\frac{2}{\sqrt{3}}$

78. If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6} + x\right) \cdot \cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2}\right) - 1$ in [0, π] is kπ, then k is equal to :

(1) $\frac{13}{9}$ (2) $\frac{8}{9}$
(3) $\frac{20}{9}$ (4) $\frac{2}{3}$

79. A straight the through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is:

(1) $2x + 3y = xy$ (2) $3x + 2y = xy$
(3) $3x + 2y = 6xy$ (4) $3x + 2y = 6$

80. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$$1^2 + 2 \cdot 2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$$

If $B - 2A = 100\lambda$, then λ is equal to :

(1) 248 (2) 464
(3) 496 (4) 232

81. If the curves $y^2 = 6x, 9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :

(1) $\frac{7}{2}$ (2) 4
(3) $\frac{9}{2}$ (4) 6

82. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is :

(1) $2\sqrt{10}$ (2) $3\sqrt{\frac{5}{2}}$
(3) $\frac{3\sqrt{5}}{2}$ (4) $\sqrt{10}$

83. Let $S = \{t \in R : f(x) = |x - \pi|(e^{|x|} - 1)\sin|x|$ is not differentiable at t}. Then the set S is equal to :

(1) $\{0\}$ (2) $\{\pi\}$
(3) $\{0, \pi\}$ (4) ϕ (an empty set)

84. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to :

(1) (–4, 3) (2) (–4, 5)
(3) (4, 5) (4) (–4, –5)

85. The Boolean expression

$\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to :

(1) p (2) q
(3) ~q (4) ~p

86. If the system of linear equations

$x + ky + 3z = 0$
$3x + ky - 2z = 0$
$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) 10 (2) -30
(3) 30 (4) -10

87. Let $S = \{x \in R : x \geq 0$ and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$. Then S :

(1) contains exactly one element.
(2) contains exactly two elements.
(3) contains exactly four elements.
(4) is an empty set.

88. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is :

(1) 185
(2) 85
(3) 95
(4) 195

89. Let $y - y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $\frac{-8}{9\sqrt{3}}\pi^2$
(2) $-\frac{8}{9}\pi^2$
(3) $-\frac{4}{9}\pi^2$
(4) $\frac{4}{9\sqrt{3}}\pi^2$

90. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is :

(1) $\frac{1}{3\sqrt{2}}$
(2) $\frac{1}{2\sqrt{2}}$
(3) $\frac{1}{\sqrt{2}}$
(4) $\frac{1}{4\sqrt{2}}$

Hints and Solutions

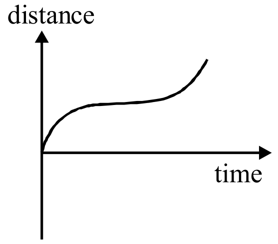
SECTION-1 : PHYSICS

1. (3) $\text{Density}(d) = \dfrac{\text{Mass}(M)}{\text{Volume}(V)} = \dfrac{M}{L^3}$

$\therefore$ Error in density, $\dfrac{\Delta d}{d} = \dfrac{\Delta M}{M} + \dfrac{3\Delta L}{L}$

$= 1.5\% + 3(1\%) = 4.5\%$

2. (2) Graphs in option (3) position-time and option (1) velocity-position are corresponding to velocity-time graph option (4) and its distance-time graph is as given below. Hence distance-time graph option (2) is incorrect.



3 (2) Given : $m_1 = 5\text{kg}$; $m_2 = 10\text{kg}$; $\mu = 0.15$

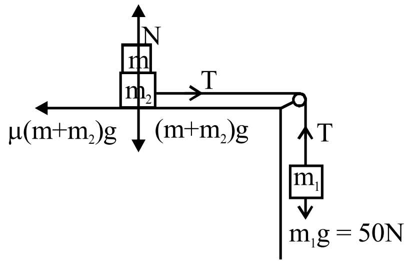
For m_1, $m_1 g - T = m_1 a$

$\Rightarrow 50 - T = 5 \times a$

and, $T - 0.15\,(m + 10)\,g = (10 + m)a$

For rest $a = 0$

or, $50 = 0.15\,(m + 10)\,10$



$$\Rightarrow \quad 5 = \frac{3}{20}(m + 10)$$

$$\frac{100}{3} = m + 10 \quad \therefore \; m = 23.3\text{kg} \;\; ; \text{ close to option (2)}$$

4. (3) $F = -\dfrac{\partial u}{\partial r}\hat{r} = \dfrac{K}{r^3}\hat{r}$

Since particle is moving in circular path

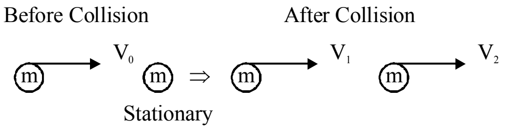
$$F = \frac{mv^2}{r} = \frac{K}{r^3} \Rightarrow mv^2 = \frac{K}{r^2}$$

$$\therefore \; \text{K.E.} = \frac{1}{2}mv^2 = \frac{K}{2r^2}$$

Total energy = P.E. + K.E.

$$= -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{Zero} \quad (\because \text{P.E.} = -\frac{K}{2r^2} \text{ given})$$

5. (2)



$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow \; v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \qquad \text{....(i)}$$

From momentum conservation

$mv_0 = m(v_1 + v_2)$ ....(ii)

Squarring both sides,

$(v_1 + v_2)^2 = v_0{}^2$

$\Rightarrow \; v_1{}^2 + v_2{}^2 + 2v_1v_2 = v_0{}^2$

$$2v_1v_2 = -\frac{v_0^2}{2}$$

$$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 = \frac{3}{2}v_0^2 + \frac{v_0^2}{2}$$

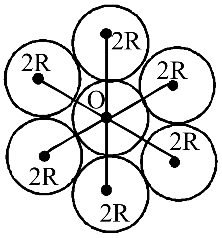
Solving we get relative velocity between the two particles

$v_1 - v_2 = \sqrt{2}v_0$

6. (4) Using parallel axes theorem, moment of inertia about 'O'

$I_o = I_{cm} + md^2$

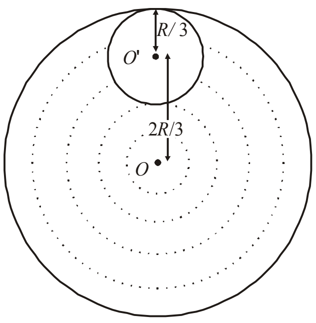
$$= \frac{7MR^2}{2} + 6(M \times (2R)^2) = \frac{55MR^2}{2}$$



Again, moment of inertia about point P, $I_p = I_o + md^2$

$$= \frac{55MR^2}{2} + 7M(3R)^2 = \frac{181}{2}MR^2$$

7. (1) Let σ be the mass per unit area.



The total mass of the disc = $\sigma \times \pi R^2 = 9M$

The mass of the circular disc cut

$$= \sigma \times \pi \left(\frac{R}{3}\right)^2 = \sigma \times \frac{\pi R^2}{9} = M$$

Let us consider the above system as a complete disc of mass $9M$ and a negative mass M super imposed on it. Moment of inertia (I_1) of the complete disc = $\frac{1}{2}9MR^2$ about an axis passing through O and perpendicular to the plane of the disc.

M.I. of the cut out portion about an axis passing through O' and perpendicular to the plane of disc

$$= \frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2$$

∴ *M.I.* (I_2) of the cut out portion about an axis passing through O and perpendicular to the plane of disc

$$= \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$$

[Using perpendicular axis theorem]

∴ The total *M.I.* of the system about an axis passing through O and perpendicular to the plane of the disc is $I = I_1 + I_2$

$$= \frac{1}{2}9MR^2 - \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$$

$$= \frac{9MR^2}{2} - \frac{9MR^2}{18} = \frac{(9-1)MR^2}{2} = 4MR^2$$

8. **(3)** $m\omega^2 R = \text{Force} \propto \frac{1}{R^n}$ $\quad$ (Force $= \frac{mv^2}{R}$)

$$\Rightarrow \omega^2 \propto \frac{1}{R^{n+1}} \Rightarrow \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$$

Time period $T = \frac{2\pi}{\omega}$

Time period, $T \propto R^{\frac{n+1}{2}}$

9. **(3)** Bulk modulus, $K = \frac{\text{volumetric stress}}{\text{volumetric strain}}$

$$K = \frac{mg}{a\left(\frac{dV}{V}\right)} \Rightarrow \frac{dV}{V} = \frac{mg}{Ka} \quad \text{....(i)}$$

volume of sphere, $V = \frac{4}{3}\pi R^3$

Fractional change in volume $\frac{dV}{V} = \frac{3dr}{r}$ ...(ii)

Using eq. (i) & (ii) $\frac{3dr}{r} = \frac{mg}{Ka}$

$\therefore \frac{dr}{r} = \frac{mg}{3Ka}$ (fractional decrement in radius)

10. **(3)** In an adiabatic process

$TV^{\gamma-1}$ = Constant $\quad$ or, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

For monoatomic gas $\gamma = \frac{5}{3}$

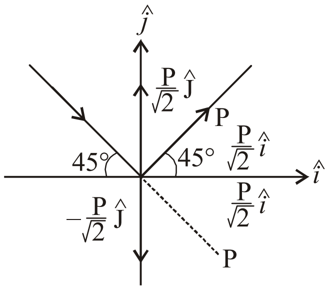
$$(300)V^{2/3} = T_2(2V)^{2/3} \Rightarrow T_2 = \frac{300}{(2)^{2/3}}$$

T_2 = 189 K (final temperature)

Change in internal energy $\Delta U = n\frac{f}{2}R\ \Delta T$

$$= 2\left(\frac{3}{2}\right)\left(\frac{25}{3}\right)(-111) = -2.7\,\text{kJ}$$

11. **(1)** Change in momentum



$$\Delta P = \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{i} - \frac{P}{\sqrt{2}}\hat{i}$$

$$\Delta P = \frac{2P}{\sqrt{2}}\hat{j} = I_H \text{ molecule}$$

$$\Rightarrow I_{wall} = -\frac{2P}{\sqrt{2}}\hat{j}$$

Pressure, P

$$= \frac{F}{A} = \frac{\sqrt{2}P}{A}n \quad (\because n = \text{no. of particles})$$

$$= \frac{\sqrt{2} \times 3.32 \times 10^{-27} \times 10^3 \times 10^{23}}{2 \times 10^{-4}}$$

$= 2.35 \times 10^3 \text{N/m}^2$

12. **(2)** As we know, frequency in SHM

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 10^{12}$ where m = mass of one atom

Mass of one atom of silver, $= \frac{108}{\left(6.02 \times 10^{23}\right)} \times 10^{-3}$ kg

$$\frac{1}{2\pi}\sqrt{\frac{k}{108 \times 10^{-3}} \times 6.02 \times 10^{23}} = 10^{12}$$

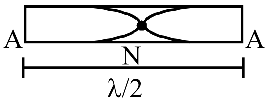
Solving we get, spring constant, K = 7.1N/m

13. **(1)** In solids, Velocity of wave $V = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}}$

$v = 5.85 \times 10^3$ m/sec

Since rod is clamped at middle fundamental wave shape is as follow

$$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L$$



$\lambda = 1.2\text{m}$ (∵ $L = 60\text{ cm} = 0.6\text{m}$ (given)
Using $v = f\lambda$

$$\Rightarrow \quad f = \frac{v}{\lambda} = \frac{5.85\times10^3}{1.2}$$

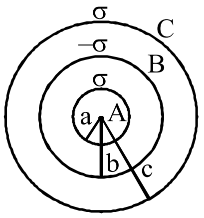
$= 4.88 \times 10^3\text{ Hz} \simeq 5\text{ KHz}$

14. **(2)** Potential outside the shell, $V_{\text{outside}} = \frac{KQ}{r}$

where r is distance of point from the centre of shell

Potential inside the shell, $V_{\text{inside}} = \frac{KQ}{R}$

where 'R" is radius of the shell



$$V_B = \frac{Kq_A}{r_b} + \frac{Kq_B}{r_b} + \frac{Kq_C}{r_c}$$

$$V_B = \frac{1}{4\pi\in_0}\left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$V_B = \frac{\sigma}{\in_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

15. **(1)** Charge on Capacitor, $Q_i = CV$
After inserting dielectric of dielectric constant = K $Q_f = (kC)V$
Induced charges on dielectric

$$Q_{\text{ind}} = Q_f - Q_i = KCV - CV$$

$$= (K-1)CV = \left(\frac{5}{3} - 1\right) \times 90\text{pF} \times 2\text{V} = 1.2\text{nc}$$

16. **(2)** As we know, average power $P_{avg} = V_{rms} I_{rms} \cos\theta$

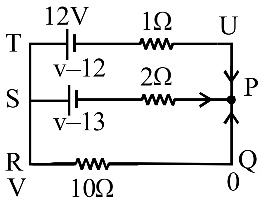
$$= \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\theta = \left(\frac{100}{\sqrt{2}}\right)\left(\frac{20}{\sqrt{2}}\right)\cos 45° \quad (\because \theta = 45°)$$

$$P_{avg} = \frac{1000}{\sqrt{2}}\text{ watt}$$

Wattless current $I = I_{rms} \sin\theta$

$$= \frac{I_0}{\sqrt{2}}\sin\theta = \frac{20}{\sqrt{2}}\sin 45° = 10\text{A}$$

17. **(2)**



Using Kirchhoff's law at P we get

$$\frac{V-12}{1} + \frac{V-13}{2} + \frac{V-0}{10} = 0$$

[Let potential at P, Q, U = 0 and at R = V

$$\Rightarrow \frac{V}{1} + \frac{V}{2} + \frac{V}{10} = \frac{12}{1} + \frac{13}{2} + \frac{0}{10}$$

$$\Rightarrow \frac{10+5+1}{10}V = \frac{24+13}{2}$$

$$\Rightarrow V\left(\frac{16}{10}\right) = \frac{37}{2}$$

$$\Rightarrow V = \frac{37\times10}{16\times2} = \frac{370}{32} = 11.56\text{ volt}$$

18. **(2)** As we know, radius of circular path in magnetic field

$$r = \frac{\sqrt{2Km}}{qB}$$

For electron, $r_e = \frac{\sqrt{2Km_e}}{eB}$ ....(i)

For proton, $r_p = \frac{\sqrt{2Km_p}}{eB}$ ....(ii)

For α particle,

$$r_\alpha = \frac{\sqrt{2Km_a}}{q_\alpha B} = \frac{\sqrt{2K4m_p}}{2eB} = \frac{\sqrt{2Km_p}}{eB} \text{ ...(iii)}$$

∴ $r_e < r_p = r_\alpha$ (∵ $m_e < m_p$)

19. **(3)** Magnetic field at the centre of loop, $B_1 = \frac{\mu_0 I}{2R}$

Dipole moment of circular loop is m = IA
$m_1 = I.A = I.\pi R^2$ {R = Radius of the loop}
If moment is doubled (keeping current constant)
R becomes $\sqrt{2}R$

$$m_2 = I.\pi\left(\sqrt{2}R\right)^2 = 2.I\pi R^2 = 2m_1$$

$$B_2 = \frac{\mu_0 I}{2\left(\sqrt{2}R\right)}$$

$$\therefore \frac{B_1}{B_2} = \frac{\frac{\mu_0 I}{2R}}{\frac{\mu_0 I}{2\left(\sqrt{2}R\right)}} = \sqrt{2}$$

20. **(1)** Quality factor $Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$

21. **(3)** Velocity of EM wave is given by $v = \frac{1}{\sqrt{\mu\in}}$

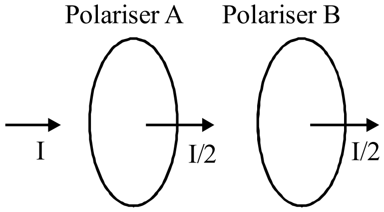
Velocity in air $= \frac{\omega}{k} = C$

Velocity in medium $= \frac{C}{2}$

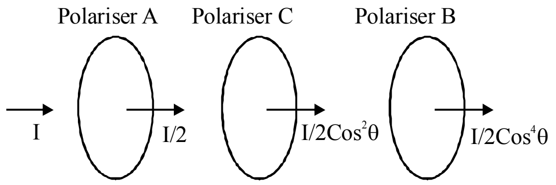
Here, $\mu_1 = \mu_2 = 1$ as medium is non-magnetic

$$\therefore \frac{\frac{1}{\sqrt{\epsilon_{r_1}}}}{\frac{1}{\sqrt{\epsilon_{r_2}}}} = \frac{C}{\left(\frac{C}{2}\right)} = 2 \quad \Rightarrow \quad \frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$$

22. **(3)** Axis of transmission of A & B are parallel.



After introducing polariser C between A and B,



$$\frac{I}{2}\cos^4\theta = \frac{I}{8} \Rightarrow \cos^4\theta = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \text{ or, } \theta = 45°$$

23. **(1)** Angular width of central maxima $= \frac{2\lambda}{d}$

or, $\lambda = \frac{d}{2}$; Fringe width, $\beta = \frac{\lambda \times D}{d'}$

$$10^{-2} = \frac{d}{2} \times \frac{50 \times 10^{-2}}{d'} = \frac{10^{-6} \times 50 \times 10^{-2}}{2 \times d'}$$

Therefore, slit separation distance, d' = 25μm

24. **(1)** Wavelength of emitted photon from n^{th} state to the ground state, $\frac{1}{\Lambda_n} = RZ^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$

$$\Lambda_n = \frac{1}{RZ^2}\left(1 - \frac{1}{n^2}\right)^{-1}$$

Since n is very large, using binomial theorem

$$\Lambda_n = \frac{1}{RZ^2}\left(1 + \frac{1}{n^2}\right)$$

$$\Lambda_n = \frac{1}{RZ^2} + \frac{1}{RZ^2}\left(\frac{1}{n^2}\right)$$

As we know, $\lambda_n = \frac{2\pi r}{n} = 2\pi\left(\frac{n^2h^2}{4\pi^2 mZe^2}\right)\frac{1}{n} \propto n$

$$\Lambda_n \approx A + \frac{B}{\lambda_n^2}$$

25. **(4)** $h\nu_L = E_\infty - E_1$ ...(i)

$h\nu_f = E_\infty - E_5$ ...(ii)

$$E \propto \frac{z^2}{n^2} \Rightarrow \frac{E_5}{E_1} = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$$

Eqn (i) / (ii) $\Rightarrow \frac{h\nu_L}{h\nu_f} = \frac{E_1}{E_5}$

$$\Rightarrow \frac{\nu_L}{\nu_f} = \frac{25}{1} \Rightarrow \nu_f = \frac{\nu_L}{25}$$

26. **(1)** For collision of neutron with deuterium:

Applying conservation of momentum :

$mv + 0 = mv_1 + 2mv_2$ .....(i)

$v_2 - v_1 = v$ .....(ii)

$\because$ Collision is elastic, e = 1

From eqn (i) and eqn (ii) $v_1 = -\frac{v}{3}$

$$P_d = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{8}{9} = 0.89$$

Now, For collision of neutron with carbon nucleus

Applying Conservation of momentum

$mv + 0 = mv_1 + 12mv_2$ ....(iii)

$v = v_2 - v_1$ ....(iv)

From eqn (iii) and eqn (iv)

$$v_1 = -\frac{11}{13}v$$

$$P_c = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{11}{13}v\right)^2}{\frac{1}{2}mv^2} = \frac{48}{169} \approx 0.28$$

27. **(3)** Clearly from fig. given in question, Silicon diode is in forward bias.

$\therefore$ Potential barrier across diode

$\Delta V = 0.7$ volts

Current, $I = \frac{V - \Delta V}{R} = \frac{3 - 0.7}{200} = \frac{2.3}{200} = 11.5\text{mA}$

28. **(3)** If n = no. of channels

10% of 10 GHz = n × 5 KHz

or, $\frac{10}{100} \times 10 \times 10^9 = n \times 5 \times 10^3 \Rightarrow n = 2 \times 10^5$

29. **(2)** Using formula, internal resistance,

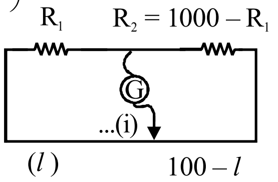
$$r = \left(\frac{l_1 - l_2}{l_2}\right)s = \left(\frac{52 - 40}{40}\right) \times 5 = 1.5\Omega$$

30. **(3)** $R_1 + R_2 = 1000$

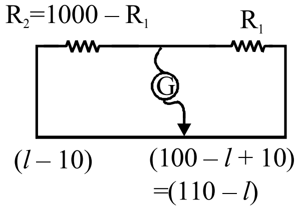
$\Rightarrow R_2 = 1000 - R_1$

On balancing condition

$R_1(100 - l) = (1000 - R_1)l$ ...(i)



On Interchanging resistance balance point shifts left by 10 cm



On balancing condition

$(1000 - R_1)(110 - l) = R_1(l - 10)$

or, $R_1(l - 10) = (1000 - R_1)(110 - l)$ ...(ii)

Dividing eqn (i) by (ii)

$$\frac{100-l}{l-10} = \frac{l}{110-l}$$

$\Rightarrow$ $(100 - l)(110 - l) = l(l - 10)$

$\Rightarrow$ $11000 - 100l - 110l + l^2 = l^2 - 10l$

$\Rightarrow$ $11000 = 200l$

or, $l = 55$

Putting the value of 'l' in eqn (i)

$R_1(100 - 55) = (1000 - R_1)\,55$

$\Rightarrow$ $R_1(45) = (1000 - R_1)\,55$

$\Rightarrow$ $R_1(9) = (1000 - R_1)\,11$

$\Rightarrow$ $20\,R_1 = 11000$

$\therefore$ $R_1 = 550 K\Omega$

SECTION-2 : CHEMISTRY

31. **(4)**

Element	Relative mass	Relative mole	Simplest whole number ratio
C	6	$\frac{6}{12} = 0.5$	1
H	1	$\frac{1}{1} = 1$	2

So, X = 1, Y = 2

Equation for combustion of C_XH_Y

$$C_XH_Y + \left(X + \frac{Y}{4}\right)O_2 \longrightarrow XCO_2 + \frac{Y}{2}H_2O$$

Oxygen atoms required $= 2\left(X + \frac{Y}{4}\right)$

As mentioned,

$$2\left(X + \frac{Y}{4}\right) = 2Z \Rightarrow \left(1 + \frac{2}{4}\right) = Z \Rightarrow Z = 1.5$$

$\therefore$ molecule can be written as

$C_XH_YO_Z$

$C_1H_2O_{3/2}$

$\Rightarrow$ $C_2H_4O_3$

32. **(3)** In Frenkel defect some of ion (usually cation due to their small size) missing from their normal position and occupies position in interstitial position.

33. **(4)**

Electronic configuration			Bond order
He_2^+	(Z = 3)	$\sigma_{1s^2}\overset{*}{\sigma}_{1s^1}$	$\frac{2-1}{2} = 0.5$
H_2^-	(Z = 3)	$\sigma_{1s^2}\overset{*}{\sigma}_{1s^1}$	$\frac{2-1}{2} = 0.5$
H_2^{2-}	(Z = 4)	$\sigma_{1s^2}\overset{*}{\sigma}_{1s^2}$	$\frac{2-2}{2} = 0$
He_2^{2+}	(Z = 2)	σ_{1s^2}	$\frac{2-0}{2} = 1$

Molecule having zero bond order will not be a viable molecule.

34. **(1)** From thermodynamics

$$\ln K = \frac{-\Delta H°}{RT} + \frac{\Delta S°}{R}$$

for exothermic reaction,

$\Delta H = -ve$

$$\text{slope} = \frac{-\Delta H°}{R} = +ve$$

So from graph, line should be A & B.

35. **(4)** $C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$

$$\Delta n_g = 6 - \frac{15}{2} = -\frac{3}{2}$$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -3263.9 + \left(-\frac{3}{2}\right) \times 8.314 \times 298 \times 10^{-3}$$

$= -3263.9 + (-3.71) = -3267.6$ kJ mol^{-1}

36. **(4)** Number of particles (i)

(1)	$[Co(H_2O)_6]Cl_3$	1
(2)	$[Co(H_2O)_5Cl]Cl_2.H_2O$	3
(3)	$[Co(H_2O)_4Cl_2]Cl.2H_2O$	2
(4)	$[Co(H_2O)_3Cl_3].3H_2O$	4

$\Delta T_f \propto i$; where $\Delta T_f = (T_f - T_f')$

Remember, the greater the no. of particles, the lower will be the freezing point. Compound (d) will have the highest freezing point due to least no of particles.

37. **(2)** In presence of external H^+,

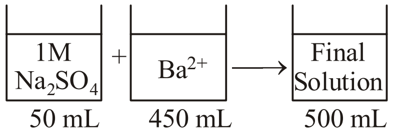
$$H_2S \rightleftharpoons 2H^+ + S^{2-},\ K_{a_1}.K_{a_2} = K_{eq}$$

$$\therefore \quad \frac{[H^+]^2[S^{2-}]}{[H_2S]} = 1 \times 10^{-7} \times 1.2 \times 10^{-13}$$

$$\frac{[0.2]^2[S^{2-}]}{[0.1]} = 1.2 \times 10^{-20}$$

$$[S^{2-}] = 3 \times 10^{-20}$$

38. (3)



Concentration of SO_4^{2-} in $BaSO_4$ solution

$M_1V_1 = M_2V_2$

$1 \times 50 = M_2 \times 500$

$$M_2 = \frac{1}{10}$$

For just precipitation

Ionic product = K_{sp}

$[Ba^{2+}]\,[SO_4^{\ 2-}] = K_{sp}(BaSO_4)$

$$[Ba^{2+}] \times \frac{1}{10} = 10^{-10}$$

$[Ba^{2+}] = 10^{-9}$ M in 500 mL solution

Thus $[Ba^{2+}]$ in original solution (450 mL)

$\Rightarrow M_1 \times 450 = 10^{-9} \times 500$

[where M_1 = molarity of original solution]

$$M_1 = \frac{500}{450} \times 10^{-9} = 1.11 \times 10^{-9}\ M$$

39. (1) $CH_3CHO \longrightarrow CH_4 + CO$

Generally $r \propto (a - x)m$

$r_1 = 1$ torr s^{-1}, when 5% reacted

$r_2 = 0.5$ torr s^{-1}, when 33 % reacted

$(a - x_1) = 0.95$(unreacted)

$(a - x_2) = 0.67$(unreacted)

$$\frac{r_1}{r_2} = \left[\frac{(a-x_1)}{(a-x_2)}\right]^m ; \frac{1}{0.5} = \left(\frac{0.95}{0.67}\right)^m$$

$2 = (1.41)^m \Rightarrow 2 = (\sqrt{2})^m$

$\Rightarrow m = 2$

40. (3) $B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

27.66 g of B_2H_6 (1 mole) requires 3 moles of oxygen (O_2) for complete burning.

Required O_2 (3 moles) is obtained by electrolysis of 6 moles of H_2O

On electrolysis : $6H_2O \longrightarrow 6H_2 + 3O_2$

Number of Faradays = 12 = Amount of charge

$12 \times 96500 = i \times t$

$12 \times 96500 = 100 \times t$

$$t = \frac{12 \times 96500}{100}\text{second} = \frac{12 \times 96500}{100 \times 3600}\text{hour}$$

= 3.2 hours

41. (3)
$$\underset{\text{Hydroxyapatite}}{[3Ca_3(PO_4)_2.Ca(OH)_2]} + \underset{\substack{\text{(drinking}\\ \text{water}\\ \text{upto 1ppm)}}}{2F^-} \longrightarrow$$
$$\underset{\substack{\text{Fluorapatite}\\ \text{(Harder teeth}\\ \text{enamel)}}}{[3Ca_3(PO_4)_2.CaF_2]} + 2OH^-$$

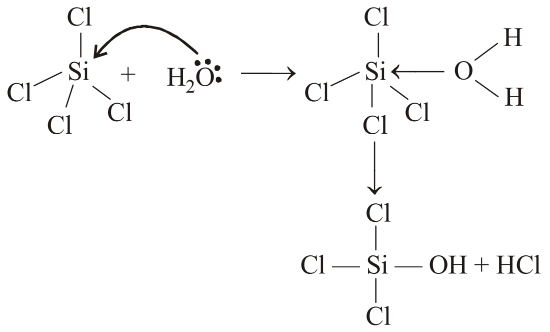
42. (3) KCl is an ionic compound while other (PH_3, O_2, B_2H_6, H_2SO_4) are covalent compounds.

43. (2, 4) BCl_3 and $AlCl_3$, both have vacant *p*-orbital and incomplete octet thus they behave as Lewis acids.

$SiCl_4$ can accept lone pair of electron in *d*-orbital of silicon hence it can act as Lewis acid.

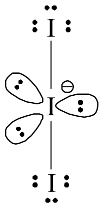
Although the most suitable answer is (c). However, both options (c) and (a) can be considered as correct answers.

e.g. hydrolysis of $SiCl_4$



i.e., option (a) $AlCl_3$ and $SiCl_4$ is also correct.

44. (3)



∴ Total number of lone pair of electrons is 9.

45. (2) CH_3COOK is a salt of weak acid (CH_3COOH) and strong base (KOH).

$FeCl_3$ is a salt of weak base $[Fe(OH)_3]$ and strong acid (HCl).

Pb $(CH_3COO)_2$ is a salt of weak base $Pb(OH)_2$ and weak acid (CH_3COOH)

$Al(CN)_3$ is a salt of weak base [Al $(OH)_3$] and weak acid (HCN).

46. (3) $[Fe(CN)_6]^{4-} + \frac{1}{2}H_2O_2 + H^+ \longrightarrow [Fe(CN)_6]^{3-} + H_2O$

$$[Fe(CN)_6]^{3-} + \frac{1}{2}H_2O_2 + OH^- \longrightarrow [Fe(CN)_6]^{4-} + H_2O + \frac{1}{2}O_2$$

47. (3) $[Cr(H_2O)_6]Cl_3$

$\Rightarrow x + 6 \times 0 + (-1) \times 3 = 0$

$\Rightarrow x = +3$

$[Cr(C_6H_6)_2]$

$x + 2 \times 0 = 0 ; x = 0$

$K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$

$2 \times 1 + x + 2 \times (-1) + 2 \times (-2) + (-2) + 0 = 0$

$x = +6$

48. (4)
(1) $Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

(2) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} Cr_2O_3 + N_2 + 4H_2O$

(3) $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

(4) $(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$

NH_3 is evolved in reaction (c).

49. (3)

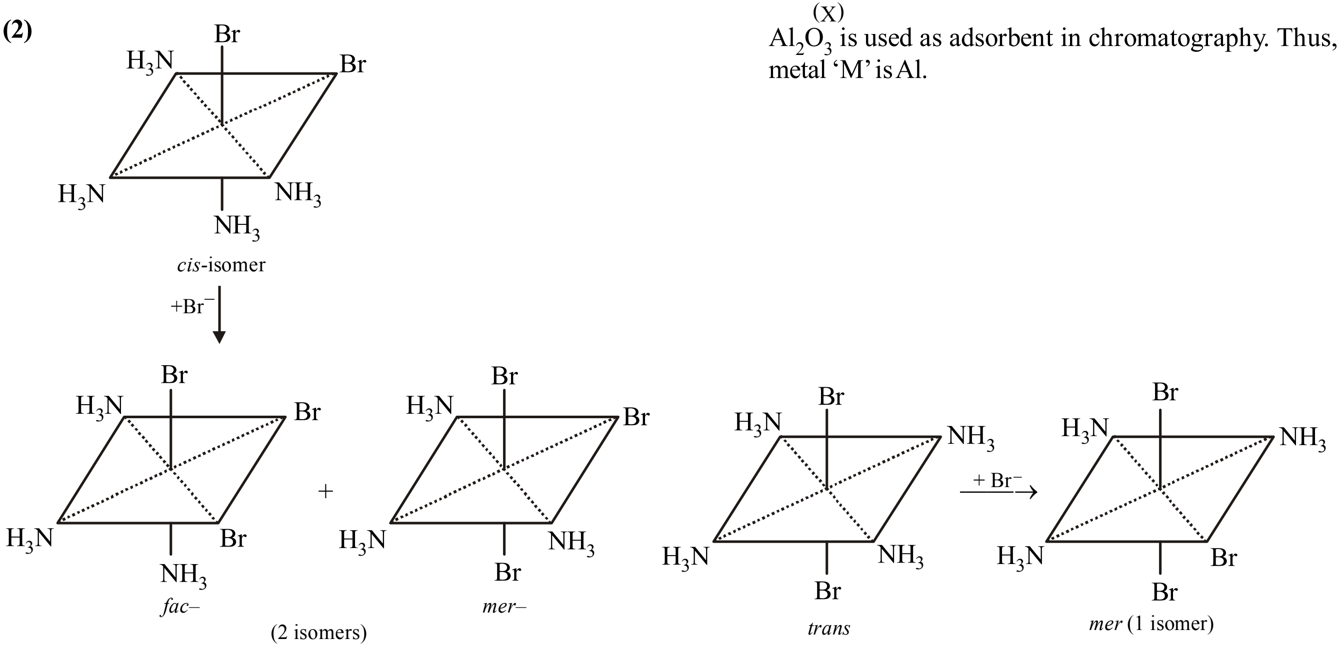
$$Al + 3H_2O \xrightarrow{NaOH} \underset{\substack{\text{(X)}\\ \text{White gelatinous ppt.}}}{Al(OH)_3} \downarrow + 3/2H_2(g)$$

$$Al(OH)_3 \xrightarrow{\text{excess of NaOH}} \underset{\text{Soluble}}{Na[Al(OH)_4]}$$

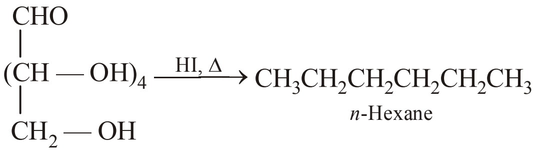
$$\underset{\text{(X)}}{2Al(OH)_3} \xrightarrow{\Delta} Al_2O_3 + 3H_2O$$

Al_2O_3 is used as adsorbent in chromatography. Thus, metal 'M' is Al.

50. (2)

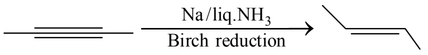


51. (1)



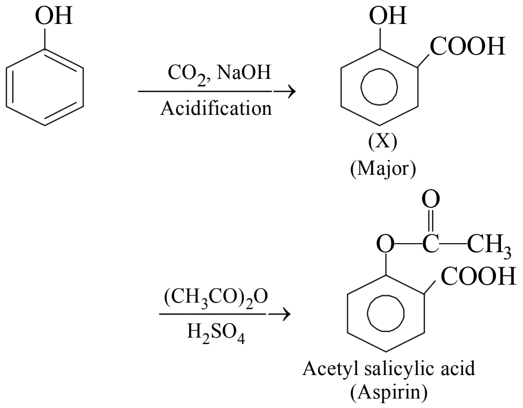
$$\begin{matrix} CHO \\ | \\ (CH-OH)_4 \\ | \\ CH_2-OH \end{matrix} \xrightarrow{HI,\ \Delta} \underset{n\text{-Hexane}}{CH_3CH_2CH_2CH_2CH_2CH_3}$$

52. (3)



53. (1) Kjeldahl's method is not applicable for compounds containing nitrogen in nitro and azo groups and nitrogen in ring, as N of these compounds does not change to ammonium sulphate under these conditions.

54. (1)



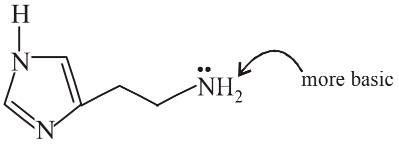
55. (3) pH range for methyl orange is

$$\xleftarrow[\text{Pinkish red}]{\qquad} 3.9 - 4.5 \xrightarrow[\text{Yellow}]{\qquad}$$

Generally, weak bases have pH greater than 7. When methyl orange is added to a weak base solution, solution becomes yellow. This solution is then titrated by a strong acid and at the end point pH will be less than 3.1.

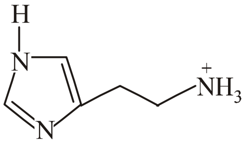
∴ Solution becomes pinkish red.

56. (4) Structure of histamine

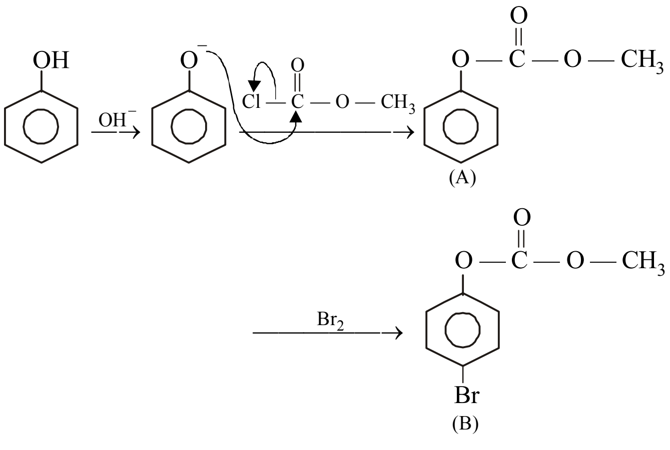


Blood is slightly basic in nature (7.35 pH). At this pH, terminal NH_2 will get protonated due to more basic nature.

∴ Predominant structure of histamine is

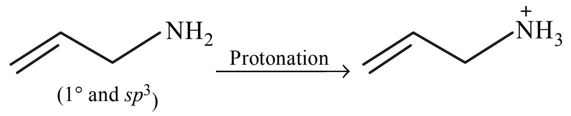


57. (3)

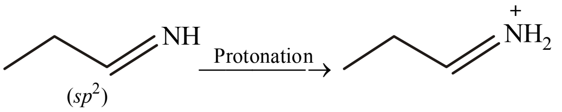


58. (3)

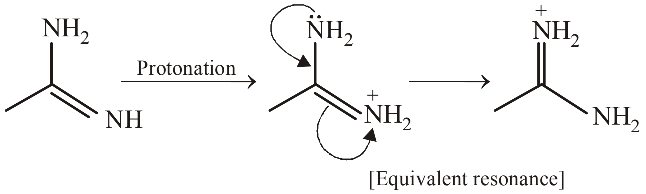
(a)



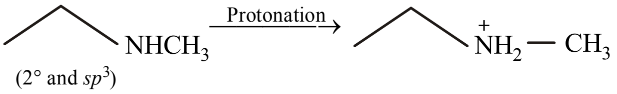
(b)



(c)

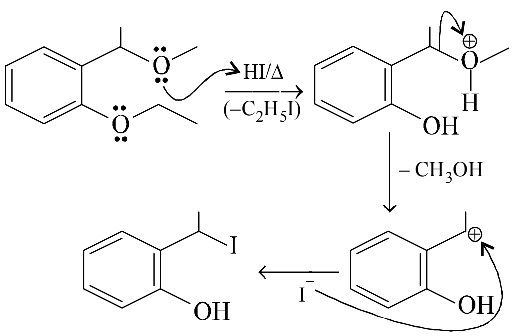


(d)



Hence, correct order of basicity will be : b < a < d < c.

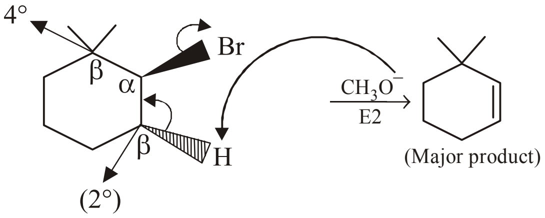
59. (4)



60. (2) CH_3O^- is a strong base and strong nucleophile, so favourable condition is $S_N2/E2$.

The given alkyl halide is 2° and β carbons are 4° and 2°, so sufficiently hindered, thus E2 dominates over S_N2.

Also, polarity of CH_3OH (solvent) is not as high as H_2O, so E1 is also dominated by E2.



SECTION-3 : MATHEMATICS

61. (1) Let I

$$= \int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

$$= \int \frac{\sin^2 x \cos^2 x}{[(\sin^2 x + \cos^2 x)(\sin^3 x + \cos^3 x)]^2} dx$$

$$= \int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx \ = \int \frac{\tan^2 x \cdot \sec^2 x}{(1 + \tan^3 x)^2} dx$$

Now, put $(1 + \tan^3 x) = t$

$\Rightarrow$ $3 \tan^2 x \sec^2 x \, dx = dt$

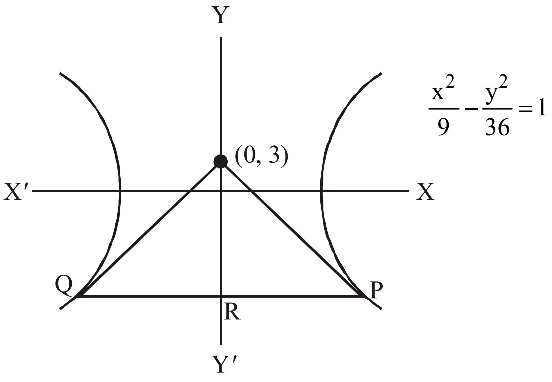
$\therefore$ $I = \frac{1}{3}\int \frac{dt}{t^2} = -\frac{1}{3t} + C \ = \frac{-1}{3(1 + \tan^3 x)} + C$

62. (4) Here equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{36} = 1$

Now, PQ is the chord of contant

$\therefore$ Equation of PQ is : $\frac{x(0)}{9} - \frac{y(3)}{36} = 1$

$\Rightarrow$ $y = -12$



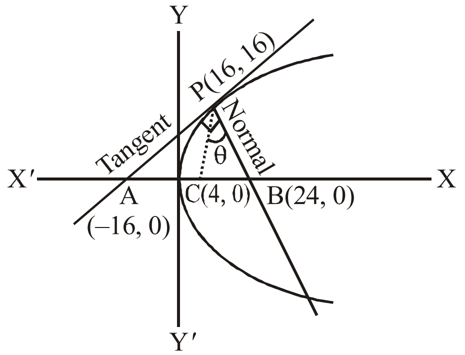
$\therefore$ Area of $\Delta PQT = \frac{1}{2} \times TR \times PQ$

$\because$ $P \equiv (3\sqrt{5}, -12)$ $\therefore$ $TR = 3 + 12 = 15,$

$\therefore$ Area of $\Delta PQT = \frac{1}{2} \times 15 \times 6\sqrt{5} = 45\sqrt{5}$ sq. units

63. (1) Equation of tnagent at P(16, 16) is given as:

$x - 2y + 16 = 0$



Slope of PC $(m_1) = \frac{4}{3}$

Slope of PB $(m_2) = -2$

Hence, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right| = \left|\frac{\frac{4}{3} + 2}{1 - \frac{4}{3} \cdot 2}\right|$

$\Rightarrow$ $\tan\theta = 2$

64. (4) $\because \vec{u}, \vec{a}$ & $\vec{b}$ are coplanar

$$\therefore \vec{u}=\lambda(\vec{a}\times\vec{b})\times\vec{a} \quad =\lambda\{\vec{a}^2.\vec{b}-(\vec{a}.\vec{b})\vec{a}\}$$

$$=\lambda\{-4\hat{i}+8\hat{j}+16\hat{k}\}=\lambda'\{-\hat{i}+2\hat{j}+4\hat{k}\}.$$

Also, $\vec{u}.\vec{b}=24 \Rightarrow \lambda'=4$

$\therefore$ $\vec{u}=-4\hat{i}+8\hat{j}+16\hat{k} \Rightarrow |\vec{u}|^2=336$

65. (2) α, β are roots of $x^2-x+1=0$

$\therefore$ $\alpha=-\omega$ and $\beta=-\omega^2$

where ω is cube root of unity

$\therefore$ $\alpha^{101}+\beta^{107}=(-\omega)^{101}+(-\omega)^{107}$

$=-[\omega^2+\omega]=-[-1]=1$

66. (4) Here, $18x^2-9\pi x+\pi^2=0$

$\Rightarrow$ $(3x-\pi)(6x-\pi)=0$

$\Rightarrow$ $\alpha=\frac{\pi}{6}, \beta=\frac{\pi}{3}$

Also, gof(x) = cosx

$\therefore$ Req. area = $\int_{\pi/6}^{\pi/3}\cos x\,dx = \frac{\sqrt{3}-1}{2}$

67. (3) Since we know that,

$(x+a)^5+(x-a)^5$

$=2[{}^5C_0x^5+{}^5C_2x^3\cdot a^2+{}^5C_4x\cdot a^4]$

$\therefore$ $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$

$=2[{}^5C_0x^5+{}^5C_2x^3(x^3-1)+{}^5C_4x(x^3-1)^2]$

$\Rightarrow$ $2[x^5+10x^6-10x^3+5x^7-10x^4+5x]$

$\therefore$ Sum of coefficients of odd degree terms = 2.

68. (2) $\because \sum_{k=0}^{12}a_{4k+1}=416 \Rightarrow \frac{13}{2}[2a_1+48d]=416$

$\Rightarrow$ $a_1+24d=32$ ...(1)

Now, $a_9+a_{43}=66 \Rightarrow 2a_1+50d=66$ ...(2)

From eq. (1) & (2) we get; d = 1 and $a_1=8$

Also, $\sum_{r=1}^{17}a_r^2=\sum_{r=1}^{17}[8+(r-1)1]^2=140\ m$

$\Rightarrow$ $\sum_{r=1}^{17}(r+7)^2=140\ m$

$\Rightarrow$ $\sum_{r=1}^{17}(r^2+14r+49)=140\ m$

$\Rightarrow$ $\left(\frac{17\times18\times35}{6}\right)+14\left(\frac{17\times18}{2}\right)+(49\times17)=140$

$\Rightarrow$ $m=34$

69. (2) Given $\sum_{i=1}^{9}(x_i-5)=9 \Rightarrow \sum_{i=1}^{9}x_i=54$ ...(i)

Also, $\sum_{i=1}^{9}(x_i-5)^2=45$

$\Rightarrow$ $\sum_{i=1}^{9}x_i^2-10\sum_{i=1}^{9}x_i+9(25)=45$ ...(ii)

From (i) and (ii) we get,

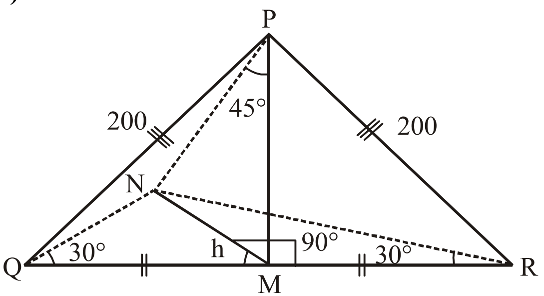
$\sum_{i=1}^{9}x_i^2=360$

Since, variance = $\frac{\sum x_i^2}{9}-\left(\frac{\sum x_i}{9}\right)^2$

$=\frac{360}{9}-\left(\frac{54}{9}\right)^2=40-36=4$

$\therefore$ Standard deviation $=\sqrt{\text{Variance}}=2$

70. (4)



Let height of tower MN = h

In ΔQMN we have

$\tan 30°=\frac{MN}{QM}$

$\therefore$ $QM=\sqrt{3}h=MR$ ...(1)

Now in ΔMNP

MN = PM ...(2)

In ΔPMQ we have :

$MP=\sqrt{(200)^2-(\sqrt{3}h)^2}$

$\therefore$ From (2), we get :

$\sqrt{(200)^2-(\sqrt{3}h)^2}=h \Rightarrow h=100m$

71. (1) $A=\{(a,b)\in R\times R:|a-5|<1,|b-5|<1\}$

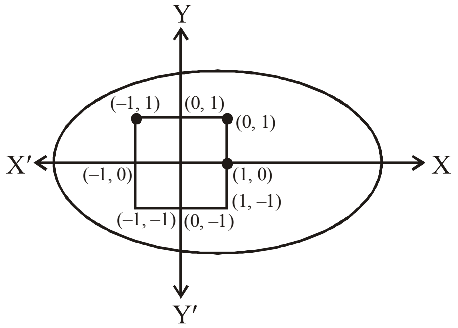
Let $a-5,=x, b-5=y$

Set A contains all points inside $|x|<1,|y|<1$

$B=\{(a,b)\in R\times R:4(a-6)^2+9(B-5)^2\le 36\}$

Set B contains all points inside or on

$\frac{(x-1)^2}{9}+\frac{y^2}{4}=1$



$\therefore$ $(\pm1, \pm1)$ lies inside the ellipse.

Hence, $A \subset B$.

72. **(4)** $\therefore$ Required number of ways $= {}^6C_4 \times {}^3C_1 \times 4!$
$= 15 \times 3 \times 24 = 1080$

73. **(3)** Here, $h(x) = \dfrac{x^2 + \dfrac{1}{x^2}}{x - \dfrac{1}{x}} = \left(x - \dfrac{1}{x}\right) + \dfrac{2}{x - \dfrac{1}{x}}$

When $x - \dfrac{1}{x} < 0$

$\therefore$ $x - \dfrac{1}{x} + \dfrac{2}{x - \dfrac{1}{x}} \le -2\sqrt{2}$

Hence, $-2\sqrt{2}$ will be local maximum value of h(x).

When $x - \dfrac{1}{x} > 0$

$\therefore$ $x - \dfrac{1}{x} + \dfrac{2}{x - \dfrac{1}{x}} \ge 2\sqrt{2}$

Hence, $2\sqrt{2}$ will be local minimum value of h(x).

74. **(2)** Since, $\lim\limits_{x \to 0^+} x\left(\left[\dfrac{1}{x}\right] + \left[\dfrac{2}{x}\right] + ... + \left[\dfrac{15}{x}\right]\right)$

$= \lim\limits_{x \to 0^+} x\left(\dfrac{1+2+3+...+15}{x}\right) - \left(\left\{\dfrac{1}{x}\right\} + \left\{\dfrac{2}{x}\right\} + ... + \left\{\dfrac{15}{x}\right\}\right)$

$\because$ $0 \le \left\{\dfrac{r}{x}\right\} < 1 \quad \Rightarrow \quad 0 \le x\left\{\dfrac{r}{x}\right\} < x$

$\therefore$ $\lim\limits_{x \to 0^+} x\left(\dfrac{1+2+3+...+15}{x}\right) = \dfrac{15 \times 16}{2} = 120$

75. **(3)** Let, $I = \displaystyle\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx$ ...(i)

Using, $\displaystyle\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get :

$I = \displaystyle\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{-x}} dx$ ...(ii)

Adding (i) and (ii), we get;

$2I = \displaystyle\int_{-\pi/2}^{\pi/2} \sin^2 x \, dx \Rightarrow 2I = 2 \cdot \int_0^{x/2} \sin^2 x \, dx$

$\Rightarrow$ $2I = 2 \times \dfrac{\pi}{4} \Rightarrow I = \dfrac{\pi}{4}$

76. **(1)** Let R_t be the even of drawing red ball in t^{th} draw and B_t be the event of drawing black ball in t^{th} draw.
Now, in the given bag there are 4 red and 6 black balls.

$\therefore$ $P(R_1) = \dfrac{4}{10}$ and $P(B_1) = \dfrac{6}{10}$

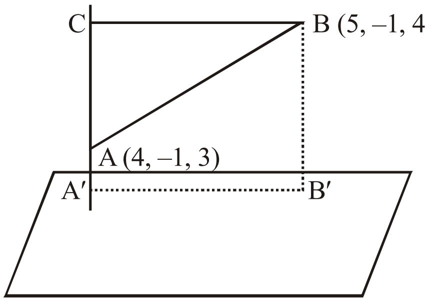
And, $P\left(\dfrac{R_2}{R_1}\right) = \dfrac{6}{12}$ and $P\left(\dfrac{R_2}{B_1}\right) = \dfrac{4}{12}$

Now, required probability

$= P(R_1) \times P\left(\dfrac{R_2}{R_1}\right) + P(B_1) \times P\left(\dfrac{R_2}{B_1}\right)$

$= \left(\dfrac{4}{10} \times \dfrac{6}{12}\right) + \left(\dfrac{6}{10} \times \dfrac{4}{12}\right) = \dfrac{2}{5}$

77. **(3)**



$AC = \overrightarrow{AB}.\widehat{AC} \ = \left(\hat{i} + \hat{k}\right).\dfrac{\left(\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}$

Now, $A'B' = BC = \sqrt{AB^2 - AC^2} = \sqrt{2 - \dfrac{4}{3}} = \sqrt{\dfrac{2}{3}}$

$\therefore$ Length of projection $= \sqrt{\dfrac{2}{3}}$

78. **(1)** $\because$ $8\cos x\left(\cos^2 \dfrac{\pi}{6} - \sin^2 x - \dfrac{1}{2}\right) = 1$

$\Rightarrow$ $8\cos x\left(\dfrac{3}{4} - \dfrac{1}{2} - \sin^2 x\right) = 1$

$\Rightarrow$ $8\cos x\left(\dfrac{1}{4} - (1 - \cos^2 x)\right) = 1$

$\Rightarrow$ $8\cos x\left(\dfrac{1}{4} - 1 + \cos^2 x\right) = 1$

$\Rightarrow$ $8\cos x\left(\cos^2 x - \dfrac{3}{4}\right) = 1$

$\Rightarrow$ $8\left(\dfrac{4\cos^3 x - 3\cos x}{4}\right) = 1$

$\Rightarrow$ $2(4\cos^3 x - 3\cos x) = 1$

$\Rightarrow$ $2\cos 3x = 1 \Rightarrow \cos 3x = \dfrac{1}{2}$

$\therefore \quad 3x = 2n\pi \pm \frac{\pi}{3}, n \in 1$

$\Rightarrow \quad x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$

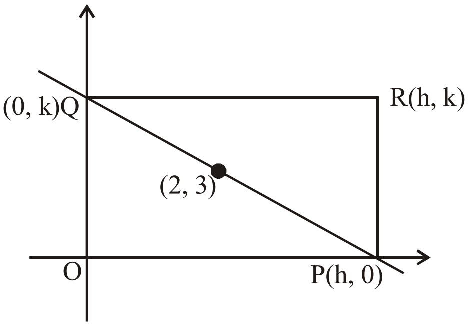
In $x \in [0, \pi] : x = \frac{\pi}{9}, \frac{2\pi}{3} + \frac{\pi}{9}, \frac{2\pi}{3} - \frac{\pi}{9}$, only

Sum of all the solutions of the equation

$$= \left(\frac{1}{9} + \frac{2}{3} + \frac{1}{9} + \frac{2}{3} - \frac{1}{9}\right)\pi = \frac{13}{9}\pi$$

79. **(2)** Equation of PQ is

$$\frac{x}{h} + \frac{y}{k} = 1 \qquad \text{...(1)}$$



Since, (1) passes through the fixed point (2, 3) Then,

$$\frac{2}{h} + \frac{3}{k} = 1$$

Then, the locus of R is $\frac{2}{x} + \frac{3}{y} = 1$ or $3x + 2y = xy$.

80. **(1)** Here, $B - 2A =$

$$\sum_{n=1}^{40} a_n - 2\sum_{n=1}^{20} a_n = \sum_{n=21}^{40} a_n - 2\sum_{n=1}^{20} a_n$$

$$B - 2A = (21^2 + 2.22^2 + 23^2 + 2.24^2 + ... + 40^2) - (1^2 + 2.2^2 + 3^2 + 2.4^2 ..... + 20^2)$$

$$= 20\,[22 + 2.24 + 26 + 2.28 + ... + 60]$$

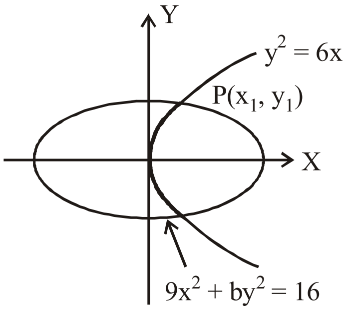
$$= 20\left[\underbrace{(22 + 24 + 26....60)}_{20 \text{ terms}} + \underbrace{(24 + 28 + ... + 60)}_{10 \text{ terms}}\right]$$

$$20\left[\frac{20}{2}(22 + 60) + \frac{10}{2}(24 + 60)\right]$$

$$= 10[20.82 + 10.84]$$

$$= 100[164 + 84] = 100.248$$

81. **(3)** Let curve intersect each other at point $P(x_1, y_1)$



Since, point of intersection is on both the curves, then

$$y_1^2 = 6x_1 \qquad \text{...(i)}$$

and $9x_1^2 + by_1^2 = 16 \qquad$ ...(ii)

Now, find the slope of tangent to both the curves at the point of intersection $P(x_1, y_1)$

For slope of curves:

curve (i):

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = m_1 = \frac{3}{y_1}$$

curve (ii):

and $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = m_2 = -\frac{9x_1}{by_1}$

Since, both the curves intersect each other at right angle then,

$$m_1 m_2 = -1 \Rightarrow \frac{27x_1}{by_1^2} = 1 \Rightarrow b = 27\frac{x_1}{y_1^2}$$

$\therefore$ from equation (i), $b = 27 \times \frac{1}{6} = \frac{9}{2}$

82. **(2)** Since Orthocentre of the triangle is A(–3, 5) and centriod of the triangle is B(3, 3), then

$AB = \sqrt{40} = 2\sqrt{10}$

Centroid divides orthocentre and circumcentre of the triangle in ratio 2 : 1

$\therefore AB : BC = 2 : 1$

Now, $AB = \frac{2}{3}AC$

$\Rightarrow \quad AC = \frac{3}{2}AB = \frac{3}{2}(2\sqrt{10}) \Rightarrow AC = 3\sqrt{10}$

$\therefore$ Radius of circle with AC as diametre

$$= \frac{AC}{2} = \frac{3}{2}\sqrt{10} = 3\sqrt{\frac{5}{2}}$$

83. **(4)** $f(x) = |x - \pi|(e^{|x|} - 1)\sin|x|$

Check differentiability of f(x) at $x = \pi$ and $x = 0$

at $x = \pi$: $\quad$ R.H.D. $= \lim\limits_{h \to 0} \frac{|\pi + h - \pi|(e^{|x+h|} - 1)\sin|\pi + h| - 0}{h}$

$$\text{L.H.D} = \lim_{h \to 0} \frac{|\pi - h - \pi|(e^{|\pi - h|} - 1)\sin|\pi - h| - 0}{-h}$$

$= 0$

$\because$ RHD = LHD

Therefore, function is differentiable at $x = \pi$

at $x = 0$:

$$\text{R.H.D.} = \lim_{h \to 0} \frac{|h - \pi|(e^{|h|} - 1)\sin|h| - 0}{h} = 0$$

$$\text{L.H.D.} = \lim_{h \to 0} \frac{|-h - \pi|(e^{|-h|} - 1)\sin|-h| - 0}{-h} = 0$$

$\therefore$ RHD = LHD

Therefore, function is differentiable.
at $x = 0$.
Since, the function f(x) is differentiable at all the points including π and 0.
i.e., f(x) is every where differentiable .
Therefore, there is no element in the set S.
$\Rightarrow S = \phi$ (an empty set)

84. (2) Here, $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$

Put $x = 0 \Rightarrow \begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3 \Rightarrow A^3 = (-4)^3$

$\Rightarrow A = -4$

$\Rightarrow \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (Bx-4)(x+4)^2$

Now take x common from both the sides

$$\therefore \begin{vmatrix} 1-\frac{4}{x} & 2x & 2x \\ 2x & 1-\frac{4}{x} & 2x \\ 2x & 2x & 1-\frac{4}{x} \end{vmatrix} = (B-\frac{4}{x})(1+\frac{4}{x})^2$$

Now take $x \to \infty$, then $\frac{1}{x} \to 0$

$\Rightarrow \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B = 5$

$\therefore$ ordered pair (A, B) is (–4, 5)

85. (4) $\sim(p \vee q) \vee (\sim p \wedge q)$

$(\sim p \wedge \sim q) \vee (\sim p \wedge q) \Rightarrow \sim p \wedge (\sim q \vee q)$

$\Rightarrow \sim p \wedge t \equiv \sim p$

86. (1) For non zero solution of the system of linear equations;

$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$

$\Rightarrow k = 11$
Now equations become
$x + 11y + 3z = 0$ ...(1)
$3x + 11y - 2z = 0$ ...(2)
$2x + 4y - 3z = 0$ ...(3)
Adding equations (1) & (3) we get
$3x + 15y = 0 \Rightarrow x = -5y$
Now put $x = -5y$ in equation (1), we get
$-5y + 11y + 3z = 0 \Rightarrow z = -2y$

$\therefore \frac{xz}{y^2} = \frac{(-5y)(-2y)}{y^2} = 10$

87. (2) Case-I: $x \in [0,9]$

$2(3-\sqrt{x}) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 8\sqrt{x} + 12 = 0 \Rightarrow \sqrt{x} = 4, 2 \Rightarrow x = 16, 4$

Since $x \in [0,9]$
$\therefore$ $x = 4$

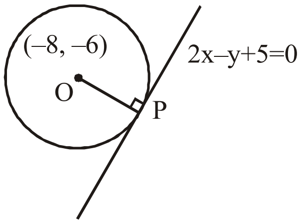
Case-II: $x \in [9, \infty]$

$2(\sqrt{x}-3) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 4\sqrt{x} = 0 \Rightarrow x = 16, 0$

Since $x \in [9, \infty]$
$\therefore x = 16$
Hence, $x = 4$ & 16

88. (3) Equation of tangent at (1, 7) to $x^2 = y - 6$ is
$2x - y + 5 = 0$.



Now, perpendicular from centre O(–8, –6) to
$2x - y + 5 = 0$ should be equal to radius of the circle

$\therefore \left|\frac{-16+6+5}{\sqrt{5}}\right| = \sqrt{64+36-C}$

$\Rightarrow \sqrt{5} = \sqrt{100-c} \Rightarrow c = 95$

89. (2) Consider the given differential equation the
$\sin x dy + y \cos x dx = 4x dx$
$\Rightarrow$ $d(y.\sin x) = 4x dx$
Integrate both sides
$\Rightarrow$ $y.\sin x = 2x^2 + C$ ...(1)

$\Rightarrow$ $y(x) = \frac{2x^2}{\sin x} + c$ ...(2)

$\because$ eq. (2) passes through $\left(\frac{\pi}{2}, 0\right)$

$\Rightarrow 0 = \frac{\pi^2}{2} + C \Rightarrow C = -\frac{\pi^2}{2}$

Now, put the value of C in (1)

Then, $y \sin x = 2x^2 - \frac{\pi^2}{2}$ is the solution

$\therefore y\left(\frac{\pi}{6}\right) = \left(2.\frac{\pi^2}{36} - \frac{\pi^2}{2}\right)2 = -\frac{8\pi^2}{9}$

90. (1) Equation of plane passing through the line of intersection of first two planes is:

$(2x - 2y + 3z - 2) + \lambda(x - y + z + 1) = 0$

or $x(\lambda+2) - y(2+\lambda) + z(\lambda+3) + (\lambda-2) = 0$ ...(i)
is having infinite number of solution with
$x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$, then

$\begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$

$\Rightarrow \lambda = 5$
Now put $\lambda = 5$ in (i), we get
$7x - 7y + 8z + 3 = 0$
Now perpendicular distance from (0, 0, 0) to the place

containing L_1 and $L_2 = \frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$

JEE Main - 2019

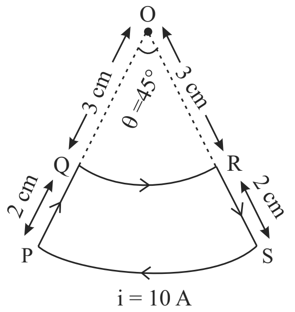
9 JANUARY 2019 (MORNING SHIFT)

Time : 3 Hours **• Each correct answer has + 4 marks • Each wrong answer has – 1 mark.** **Max. Marks : 360**

Section - 1

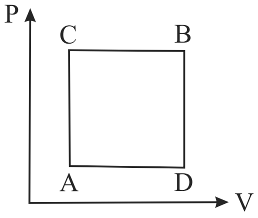
PHYSICS

1. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to:



(1) 1.0×10^{-7} T (2) 1.5×10^{-7} T
(3) 1.5×10^{-5} T (4) 1.0×10^{-5} T

2. A gas can be taken from A to B via two different processes ACB and ADB.



When path ACB is used 60 J of heat flows into the system and 30J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is :

(1) 40 J (2) 80 J (3) 100 J (4) 20 J

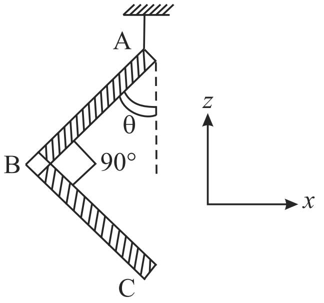
3. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\,\hat{j}$ V / m. The corresponding magnetic field $\vec{B}$, at that point will be:

(1) $18.9 \times 10^{-8}\ \hat{k}$T (2) $2.1 \times 10^{-8}\ \hat{k}$T
(3) $6.3 \times 10^{-8}\ \hat{k}$T (4) $18.9 \times 10^{8}\ \hat{k}$T

4. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio:

(1) 16 : 9 (2) 25 : 9 (3) 4 : 1 (4) 5 : 3

5. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then:



(1) $\tan\theta = \frac{1}{2\sqrt{3}}$ (2) $\tan\theta = \frac{1}{2}$

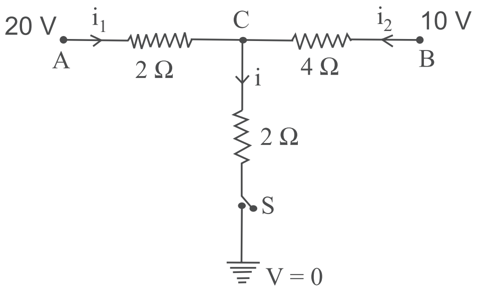
(3) $\tan\theta = \frac{2}{\sqrt{3}}$ (4) $\tan\theta = \frac{1}{3}$

6. A mixture of 2 moles of helium gas (atomic mass = 4u), and 1 mole of argon gas (atomic mass = 40u) is kept at 300 K in a container. The ratio of their rms speeds

$\left[\frac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right]$ is close to :

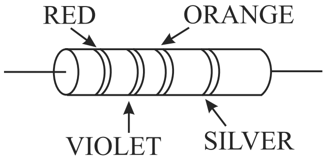
(1) 3.16 (2) 0.32 (3) 0.45 (4) 2.24

7. When the switch S, in the circuit shown, is closed then the valued of current i will be:



(1) 3A (2) 5A (3) 4A (4) 2A

8. A resistance is shown in the figure. Its value and tolerance are given respectively by:



(1) 270 Ω, 10% (2) 27 kΩ, 10%
(3) 27 kΩ, 20% (4) 270 Ω, 5%

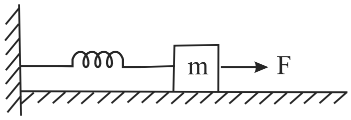
9. A bar magnet is demagnetized by inserthing it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is:

(1) 285 A/m (2) 2600 A/m
(3) 520 A/m (4) 1200 A/m

10. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is:

(1) $\frac{F}{A\alpha\Delta T}$ (2) $\frac{F}{A\alpha(\Delta T - 273)}$
(3) $\frac{F}{2A\alpha\Delta T}$ (4) $\frac{2F}{A\alpha\Delta T}$

11. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is:



(1) $\frac{2F}{\sqrt{mk}}$ (2) $\frac{F}{\pi\sqrt{mk}}$
(3) $\frac{\pi F}{\sqrt{mk}}$ (4) $\frac{F}{\sqrt{mk}}$

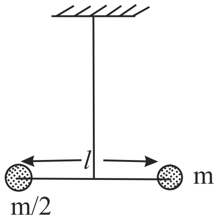
12. Three charges + Q, q, + Q are placed respectively, at distance, d/2 and d from the origin, on the *x*-axis. If the net force experienced by + Q, placed at $x = 0$, is zero, then value of q is:

(1) – Q/4 (2) + Q/2
(3) + Q/4 (4) – Q/2

13. A conducting circular loop made of a thin wire, has area 3.5 × 10^{-3}m^2 and resistance 10Ω. It is placed perpendicular to a time dependent magnetic field B (t) = (0.4T) sin (50πt). The the net charge flowing through the loop during t = 0 s and t = 10 ms is close to:

(1) 14 mC (2) 7 mC
(3) 21 mC (4) 6 mC

14. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length *l*. The rod is suspended by a thin wire of torsional constant *k* at the centre of mass of the rod-mass system (see figure). Because of torsional constant *k*, the restoring toruque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be:

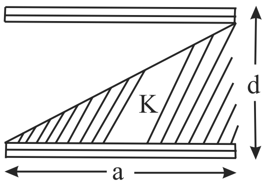


(1) $\frac{3k\theta_0^2}{l}$ (2) $\frac{2k\theta_0^2}{l}$
(3) $\frac{k\theta_0^2}{l}$ (4) $\frac{k\theta_0^2}{2l}$

15. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is:

(1) 2.0% (2) 2.5% (3) 1.0% (4) 0.5%

16. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is:



(1) $\frac{K\epsilon_0 a^2}{2d(K+1)}$ (2) $\frac{K\epsilon_0 a^2}{d(K-1)}\text{In K}$
(3) $\frac{K\epsilon_0 a^2}{d}\text{In K}$ (4) $\frac{1}{2}\frac{K\epsilon_0 a^2}{d}$

17. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field.

If, for an n-type semiconductor, the density of electrons is 10^{19} m $^{-3}$ and their mobility is 1.6m^2/(V.s) then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to:

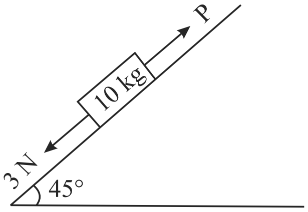
(1) 2 Ωm (2) 4 Ωm

(3) 0.4 Ωm (4) 0.2 Ωm

18. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is:

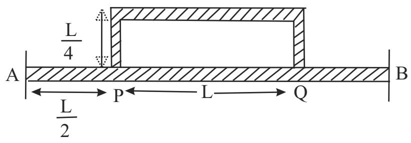
(1) $\frac{L}{m}$ (2) $\frac{4L}{m}$ (3) $\frac{L}{2m}$ (4) $\frac{2L}{m}$

19. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward? (take g = 10 ms^{-2})



(1) 32 N (2) 18 N (3) 23 N (4) 25 N

20. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to:



(1) 45°C (2) 75°C (3) 60°C (4) 35°C

21. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speeed of transverse waves in the string is 60 ms^{-1}. when the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to:

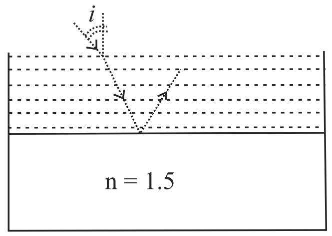
(1) $\frac{g}{30}$ (2) $\frac{g}{5}$ (3) $\frac{g}{10}$ (4) $\frac{g}{20}$

22. A sample of radioactive material A, that has an activity of 10 mCi (1 Ci = 3.7 × 10^{10} decays/s), has twice the number of nuclei as another sample of a different radioactive material B which has an acitvity of 20 mCi. The correct choices for half-lives of A and B would then be respectively:

(1) 5 days and 10 days (2) 10 days and 40 days

(3) 20 days and 5 days (4) 20 days and 10 days

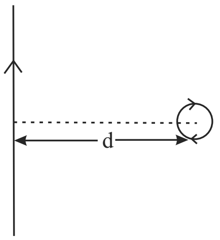
23. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle *i* (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is:



(1) $\sqrt{\frac{5}{3}}$ (2) $\frac{3}{\sqrt{5}}$

(3) $\frac{5}{\sqrt{3}}$ (4) $\frac{4}{3}$

24. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The redius of the loop is a and distance of its centre from the wire is d (d>>a). If the loop applies a force F on the wire then:



(1) F = 0 (2) $F \propto \left(\frac{a}{d}\right)$

(3) $F \propto \left(\frac{a^2}{d^3}\right)$ (4) $F \propto \left(\frac{a}{d}\right)^2$

25. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then, by light of wavelength λ_2 = 540 nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of (2) The work function of the metal (in eV) is close to:

(Energy of photon = $\frac{1240}{\lambda(\text{in nm})}$eV)

(1) 1.8 (2) 2.5 (3) 5.6 (4) 1.4

26. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is:

(1) $y = x^2$ + constant (2) $y^2 = x$ + constant

(3) $y^2 = x^2$ + constant (4) xy = constant

27. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screem is shifted by a distance d. Then d is:

(1) 1.1 cm away from the lens

(2) 0

(3) 0.55 cm towards the lens

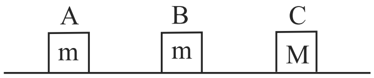
(4) 0.55 cm away from the lens

28. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is:

(1) $\frac{R}{\sqrt{5}}$ (2) $\frac{R}{\sqrt{2}}$

(3) R (4) $R\sqrt{2}$

29. There block A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an inital speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m?



(1) 5 (2) 2 (3) 4 (4) 3

30. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm^2, is v. If the electron density in copper is 9×10^{28}/m^3 the value of v in mm/s close to (Take charge of electron to be = 1.6×10^{-19}C)

(1) 0.02 (2) 3 (3) 2 (4) 0.2

Section - 2

CHEMISTRY

31. Two complexes $[Cr(H_2O)_6]Cl_3$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured, respectively. The incorrect statement regarding them is:

(1) Δ_0 values of (A) and (B) are calculated from the energies of violet and yellow light, respectively.

(2) both are paramagnetic with three unpaired electrons.

(3) both absorb energies corresponding to their complementary colors.

(4) Δ_0 value for (A) is less than that of (B).

32. The correct decreasing order for acid strength is:

(1) $NO_2CH_2COOH > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$

(2) $FCH_2COOH > CNCH_2COOH > NO_2CH_2COOH > ClCH_2COOH$

(3) $CNCH_2COOH > NO_2CH_2COOH > FCH_2COOH > ClCH_2COOH$

(4) $NO_2CH_2COOH > CNCH_2COOH > FCH_2COOH > ClCH_2COOH$

33. The major product of following reaction is:

$$R-C\equiv N \xrightarrow[\text{(ii) } H_2O]{\text{(i) AlH (i – Bu)}_2}$$

(1) RCOOH (2) $RCONH_2$

(3) RCHO (4) RCH_2NH_2

34. The highest value of the calculated spin only magnetic moment (in BM) among all the transition metal complexes is :

(1) 5.92 (2) 6.93

(3) 3.87 (4) 4.90

35. 0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a container of volume 10 m^3 at 1000 K. Given R is the gas constant in JK^{-1} mol^{-1}, x is:

(1) $\frac{2R}{4+R}$ (2) $\frac{2R}{4-R}$

(3) $\frac{4+R}{2R}$ (4) $\frac{4-R}{2R}$

36. The one that is extensively used as a piezoelectric material is:

(1) tridymite (2) amorphous silica

(3) quartz (4) mica

37. Correct statements among 'A' to 'D' regarding silicones are:

(A) They are polymers with hydrophobic character.

(B) They are biocompatible.

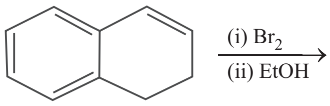
(C) In general, they have high thermal stability and low dielectric strength.

(D) Usually, they are resistant to oxidation and used as greases.

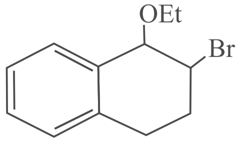
(1) (A), (B), (C) and (D) (2) (A), (B) and (C) only

(3) (A) and (B) only (4) (A), (B) and (D) only

38. The major product of the following reaction is:



(1)



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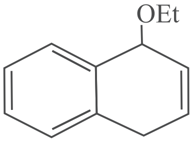
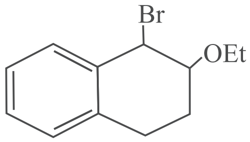


(3)



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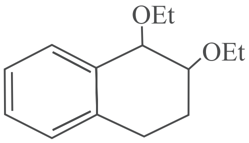


39. In general, the properties that decrease and increase down a group in the periodic table, respectively, are:

(1) atomic radius and electronegativity.

(2) electron gain enthalpy and electronegativity.

(3) electronegativity and atomic radius.

(4) electronegativity and electron gain enthalpy.

40. A solution of sodium sulfate contains 92 g of Na^+ ions per kilogram of water. The molality of Na^+ ions in that solution in mol kg^{-1} is:

(1) 12 (2) 4 (3) 8 (4) 16

41. The correct match between Item-I and Item-II is:

	Item-I (Drug)		Item-II (Test)
A	Chloroxylenol	P	Carbylamine test
B	Norethindrone	Q	Sodium hydrogen carbonate test
C	Sulphapyridine	R	Ferric chloride test
D	Penicillin	S	Bayer's test

(1) A → R ; B → P ; C → S ; D → Q

(2) A → Q ; B → S ; C → P ; D → R

(3) A → R ; B → S ; C → P ; D → Q

(4) A → Q ; B → P ; C → S ; D → R

42. A water sample has ppm level concentration of the following metals: Fe = 0.2; Mn = 5.0; Cu = 3.0; Zn = 5.0. The metal that makes the water sample unsuitable for drinking is:

(1) Cu (2) Mn (3) Fe (4) Zn

43. The anodic half-cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is : (Molar mass of $PbSO_4$ = 303 g mol^{-1})

(1) 22.8 (2) 15.2 (3) 7.6 (4) 11.4

44. Which one of the following statements regarding Henry's law is not correct?

(1) Higher the value of K_H at a given pressure, higher is the solubility of the gas in liquids.

(2) Different gases have different K_H (Henry's law constant) values at the same temperature.

(3) The partial pressure of the gas in vapour phase is proportional to the mole fraction of the gas in the solution.

(4) The value of K_H increases with increase of temperature and K_H is function of the nature of the gas.

45. The following results were obtained during kinetic studies of the reaction;

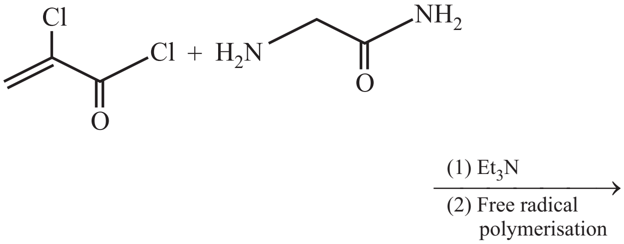
2A + B → Products

Experiment	[A] (in mol L^{-1})	[B] (in mol L^{-1})	Initial Rate of reaction (in mol L^{-1} min^{-1})
I	0.10	0.20	6.93×10^{-3}
II	0.10	0.25	6.93×10^{-3}
III	0.20	0.30	1.386×10^{-2}

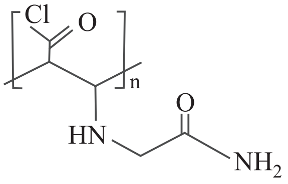
The time (in minutes) required to consume half of A is:

(1) 5 (2) 10 (3) 1 (4) 100

46. Major product of the following reaction is:



(1)



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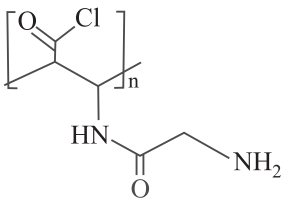

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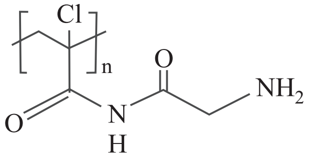

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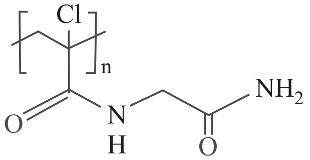


47. The alkaline earth metal nitrate that does not crystallise with water molecules, is:

(1) $Mg(NO_3)_2$ (2) $Sr(NO_3)_2$

(3) $Ca(NO_3)_2$ (4) $Ba(NO_3)_2$

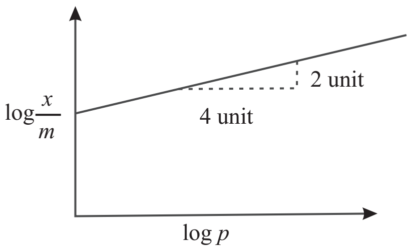
48. 20 mL of 0.1 M H_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resultant mixture is:

[pK_b of $NH_4OH = 4.7$].

(1) 5.2 (2) 9.0

(3) 5.0 (4) 9.4

49. Adsorption of a gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure p. $\frac{x}{m}$ is proportional to:



(1) p^2 (2) $p^{1/4}$

(3) $p^{1/2}$ (4) p

50. Which amongst the following is the strongest acid?

(1) $CHBr_3$ (2) CHI_3

(3) $CH(CN)_3$ (4) $CHCl_3$

51. The ore that contains both iron and copper is:

(1) copper pyrites (2) malachite

(3) dolomite (4) azurite

52. For emission line of atomic hydrogen from $n_i = 8$ to $n_f = n$, the plot of wave number ($\bar{\nu}$) against $\left(\frac{1}{n^2}\right)$ will be (The Rydberg constant, R_H is in wave number unit)

(1) Linear with intercept – R_H

(2) Non linear

(3) Linear with slope R_H

(4) Linear with slope – R_H

53. The isotopes of hydrogen are:

(1) Tritium and protium only

(2) Protium and deuterium only

(3) Protium, deuterium and tritium

(4) Deuterium and tritium only

54. According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^-?

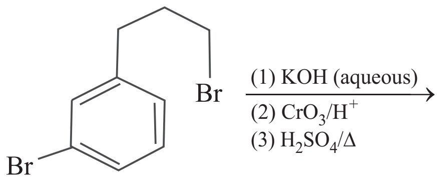
(1) Li_2^+ is unstable and Li_2^- is stable

(2) Li_2^+ is stable and Li_2^- is unstable

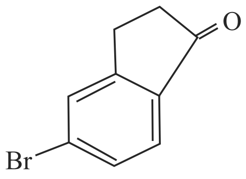
(3) Both are stable

(4) Both are unstable

55. The major product of the following reaction is:



(1)



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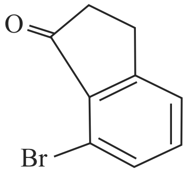

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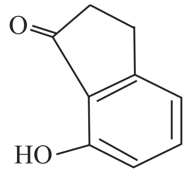

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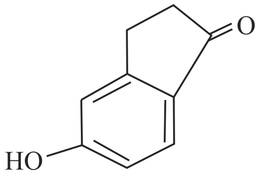


56. Aluminium is usually found in +3 oxidation state. In contrast, thallium exists in +1 and + 3 oxidation states. This is due to:

(1) inert pair effect (2) diagonal relationship

(3) lattice effect (4) lanthanoid contraction

57. The increasing order of pKa of the following amino acids in aqueous solution is:

Gly Asp Lys Arg

(1) Asp < Gly < Arg < Lys

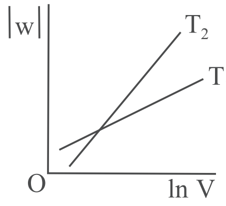
(2) Gly < Asp < Arg < Lys

(3) Asp < Gly < Lys < Arg

(4) Arg < Lys < Gly < Asp

58. Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and T_2 ($T_1 < T_2$). The correct graphical depiction of the dependence of work done (w) on the final volume (V) is:

(1)



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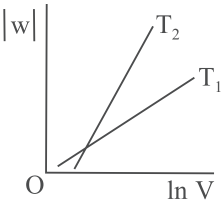

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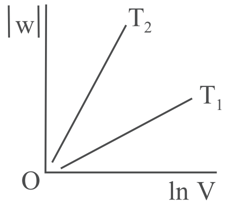

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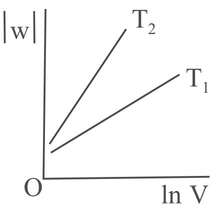
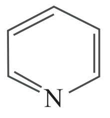
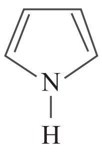


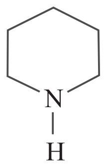
59. Arrange the following amines in the decreasing order of basicity :



I



II

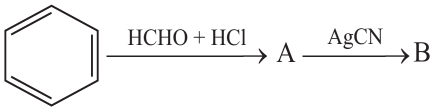


III

(1) I > II > III (2) III > I > II

(3) III > II > I (4) I > III > II

60. The compounds A and B in the following reaction are, respectively :



(1) A = Benzyl alcohol, B = Benzyl cyanide

(2) A = Benzyl chloride, B = Benzyl cyanide

(3) A = Benzyl alcohol, B = Benzyl isocyanide

(4) A = Benzyl chloride, B = Benzyl isocyanide

Section - 3

MATHEMATICS

61. The value of $\int_0^{\pi} |\cos x|^3 \, dx$ is:

(1) 0 (2) $\frac{4}{3}$

(3) $\frac{2}{3}$ (4) $-\frac{4}{3}$

62. The maximum volume (in cu.m) of the right circular cone having slant height 3 m is:

(1) 6π (2) $3\sqrt{3}\pi$

(3) $\frac{4}{3}\pi$ (4) $2\sqrt{3}\pi$

63. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral

$$\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2\left(x^2-1\right)}{2\sin(x^2-1)+\sin 2\left(x^2-1\right)}}\,dx \text{ is equal to:}$$

(1) $\log_e\left|\frac{1}{2}\sec^2\left(x^2-1\right)\right|+c$

(2) $\frac{1}{2}\log_e|\sec(x^2-1)|+c$

(3) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(4) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$

(where c is a constant of integration)

64. If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx}+2y=x^2$ satisfying $y(1)=1$, then $y\left(\frac{1}{2}\right)$ is equal to:

(1) $\frac{7}{64}$ (2) $\frac{1}{4}$

(3) $\frac{49}{16}$ (4) $\frac{13}{16}$

65. Axis of a parabola lies along x-axis. If its vertex and focus are at distance 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

(1) $(5, 2\sqrt{6})$ (2) (8, 6)

(3) $(6, 4\sqrt{2})$ (4) (4, – 4)

66. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola

$\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta}=1$ is greater than 2, then the length of

its latus rectum lies in the interval:

(1) $(3, \infty)$ (2) (3/2, 2]

(3) (2, 3] (4) (1, 3/2]

67. For $x \in \mathbf{R} - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$,

$f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2oJof_1)(x) = f_3(x)$ then J(x) is equal to:

(1) $f_3(x)$ (2) $\frac{1}{x}f_3(x)$

(3) $f_2(x)$ (4) $f_1(x)$

68. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$, then $|\vec{c}|^2$ is equal to:

(1) $\frac{19}{2}$ (2) 9

(3) 8 (4) $\frac{17}{2}$

69. If a, b and c be three distinct real numbers in G.P. and a + b + c = xb, then x cannot be:

(1) – 2 (2) – 3

(3) 4 (4) 2

70. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$,then x is equal to:

(1) $\frac{\sqrt{145}}{12}$ (2) $\frac{\sqrt{145}}{10}$

(3) $\frac{\sqrt{146}}{12}$ (4) $\frac{\sqrt{145}}{11}$

71. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is :

(1) $2\sqrt{3}y = 12x + 1$ (2) $\sqrt{3}y = x + 3$

(3) $2\sqrt{3}y = -x - 12$ (4) $\sqrt{3}y = 3x + 1$

72. The system of linear equations

$x + y + z = 2$
$2x + 3y + 2z = 5$
$2x + 3y + (a^2 - 1)z = a + 1$

(1) is inconsistent when a = 4

(2) has a unique solution for |a| = $\sqrt{3}$

(3) has infinitely many solutions for a = 4

(4) is inconsistent when |a| = $\sqrt{3}$

73. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to:

(1) 6 (2) 8

(3) 4 (4) 14

74. The equation of the line passing through (–4, 3, 1), parallel to the plane x + 2y – z – 5 = 0 and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

(1) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

(2) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(3) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(4) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

75. Consider the set of all lines px + qy + r = 0 such that 3p + 2q + 4r = 0. Which one of the following statements is true?

(1) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

(2) Each line passes through the origin.

(3) The lines are all parallel .

(4) The lines are not concurrent.

76. $\lim\limits_{y \to 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

(1) exists and equals $\frac{1}{4\sqrt{2}}$

(2) exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

(3) exists and equals $\frac{1}{2\sqrt{2}}$

(4) does not exist

77. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point:

(1) (– 3, 0, – 1) (2) (– 3, 1, 1)

(3) (3, 3, – 1) (4) (3, 2, 1)

78. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then |tan θ| is equal to:

(1) $\frac{4}{9}$ (2) $\frac{8}{15}$ (3) $\frac{7}{17}$ (4) $\frac{8}{17}$

79. If A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when θ = $\frac{\pi}{12}$, is equal to:

(1) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

80. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$ then the ordered pair $(\oplus, \odot)$ is:

(1) $(\vee, \wedge)$ (2) $(\vee, \vee)$

(3) $(\wedge, \vee)$ (4) $(\wedge, \wedge)$

81. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is:

(1) 16 (2) 22
(3) 20 (4) 18

82. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the expression

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals:

(1) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
(2) $13 - 4\cos^6\theta$
(3) $13 - 4\cos^2\theta + 6\cos^4\theta$
(4) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

83. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is:

(1) $\frac{8}{3}$ (2) $\frac{32}{3}$ (3) $\frac{56}{3}$ (4) $\frac{14}{3}$

84. Let $a_1, a_2, \ldots\ldots, a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$.

If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to:

(1) 52 (2) 57 (3) 47 (4) 42

85. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined as

$$f(x) = \begin{cases} 5, & \text{if} \quad x \le 1 \\ a + bx, & \text{if} \quad 1 < x < 3 \\ b + 5x, & \text{if} \quad 3 \le x < 5 \\ 30, & \text{if} \quad x \ge 5 \end{cases}$$

Then, f is :

(1) continuous if a = 5 and b = 5
(2) continuous if a = – 5 and b = 10
(3) continous if a = 0 and b = 5
(4) not continuous for any values of a and b

86. Let $A = \left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3 + 2i\sin\theta}{1 - 2i\sin\theta} \text{ is purely imaginary}\right\}$

Then the sum of the elements in A is:

(1) $\frac{5\pi}{6}$ (2) π (3) $\frac{3\pi}{4}$ (4) $\frac{2\pi}{3}$

87. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:

(1) 500 (2) 200 (3) 300 (4) 350

88. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to:

(1) – 256 (2) 512 (3) – 512 (4) 256

89. Three circles of radii a, b, c (a < b < c) touch each other externally. If they have x-axis as a common tangent, then:

(1) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$ (2) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$
(3) a, b, c are in A.P (4) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

90. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals:

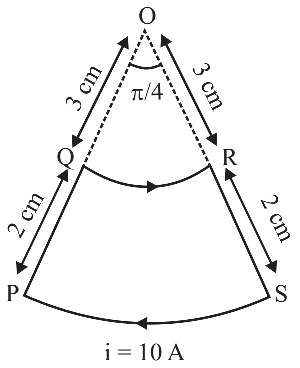
(1) 49/169 (2) 52/169
(3) 24/169 (4) 25/169

Hints and Solutions

SECTION-1 : PHYSICS

1. **(4)** There will be no magnetic field at O due to wire PQ and RS

Magnetic field at 'O' due to arc QR $= \frac{\mu_0}{4\pi}\frac{(10)}{\left(3\times10^{-2}\right)}\times\frac{\pi}{4}$ (Perpendicular outwards)



Magnetic field at 'O' due to arc PS

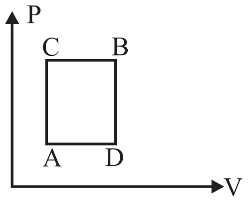
$= \frac{\mu_0}{4\pi}\times\frac{(10)}{\left(5\times10^{-2}\right)}\times\frac{\pi}{4}$ (Perpendicular inwards)

∴ Net magnetic field at 'O'

$$B = \frac{\mu_0}{4\pi}\times10\left[\frac{1}{\left(3\times10^{-2}\right)}-\frac{1}{\left(5\times10^{-2}\right)}\right]\times\frac{\pi}{4}$$

$\Rightarrow B = \frac{\pi}{3}\times10^{-5}\,T \approx 1\times10^{-5}\,T$ (Perpendicular outwards)

2. **(1)**



$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$

$\Rightarrow 60\text{ J} = 30\text{ J} + \Delta U_{ACB}$

$\Rightarrow U_{ACB} = 30\text{ J}$

$\Delta Q_{ADB} = \Delta U_{ADB} + \Delta W_{ADB}$

$= 10\text{ J} + 30\text{ J} = 40\text{ J}$ $\quad [\because\ \Delta U_{ADB} = \Delta U_{ACB} = 30\text{ J}]$

3. **(2)** As we know,

$$|B| = \frac{|E|}{C} = \frac{6.3}{3\times10^{8}} = 2.1\times10^{-8}\,T$$

As $\vec{V}\perp\vec{E}\perp\vec{B}$ therefore direction of $\vec{B}$ is in z direction

$\vec{B} = 2.1\times10^{-8}\hat{k}\,T$

4. **(2)** $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{\frac{I_1}{I_2}}+1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}}-1\right)^2} = 16\,(\text{given})$

$$\therefore\ \sqrt{\frac{I_1}{I_2}}+1 = 4\left(\sqrt{\frac{I_1}{I_2}}-1\right)$$

$$\therefore\ \sqrt{\frac{I_1}{I_2}} = \frac{5}{3}\ \therefore\ \frac{I_1}{I_2} = \frac{25}{9}$$

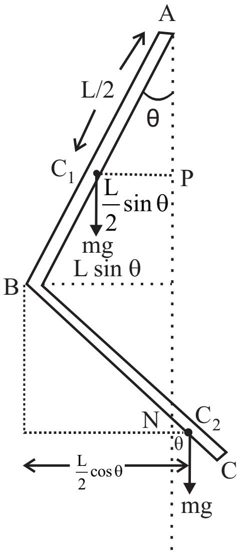
5. **(4)** Given that, the rod is of uniform mass density and AB = BC

Let mass of one rod is m.

Balancing torque about hinge point.

$mg\,(C_1P) = mg\,(C_2N)$

$$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$



$$\Rightarrow \frac{3}{2}\text{mgL}\sin\theta = \frac{\text{mgL}}{2}\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{3}$$

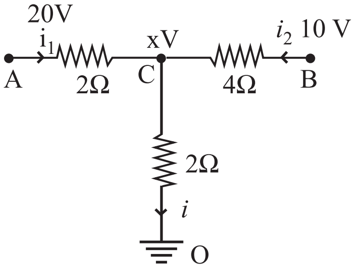
or, $\tan\theta = \frac{1}{3}$

6. **(1)** Using $V_{\text{rms}} = \sqrt{\frac{\gamma RT}{M}}$ we have

$$\frac{V_{\text{rms}}(\text{He})}{V_{\text{rms}}(\text{Ar})} = \sqrt{\frac{M_{\text{Ar}}}{M_{\text{He}}}} = \sqrt{\frac{40}{4}}$$

= 3.16

7. **(2)**



Let voltage at C = x volt

From kirchhoff's current law,

KCL : $i_1 + i_2 = i$

$$\frac{20-x}{2} + \frac{10-x}{4} = \frac{x-0}{2} \Rightarrow x = 10$$

$$\therefore\ i = \frac{V}{R} = \frac{x}{R} = \frac{10}{2} = 5\text{A}$$

8. **(2)** Using colour code we have
$R = 27 \times 10^3\ \Omega \pm 10\%$
$= 27\ \text{k}\Omega \pm 10\%$

9. **(2)** Corecivity, $H = \frac{B}{\mu_0}$ and $B = \mu_0 ni \left(n = \frac{N}{\ell}\right)$

or, $H = \frac{N}{\ell}i = \frac{100}{0.2} \times 5.2 = 2600$ A/m

10. **(1)** Young's modulus $Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{(\Delta\ell/\ell)}$

Using, coefficient of linear expansion,

$$\frac{\Delta\ell}{\ell} = \alpha\Delta T$$

$$\therefore Y = \frac{F}{A(\alpha\Delta T)}$$

11. **(4)** Maximum speed is at equilibrium position where

$$F = kx \Rightarrow x = \frac{F}{k}$$

From work-energy theorem,

$$W_F + W_{sp} = \Delta KE$$

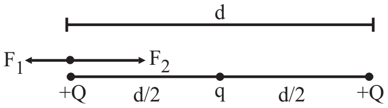
$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}\frac{F^2}{K} = \frac{1}{2}mv^2$$

or, $v_{\max} = \frac{F}{\sqrt{mk}}$

12. **(1)**



Force due to charge + Q,

$$F_1 = \frac{KQQ}{d^2}$$

Force due to charge q,

$$F_2 = \frac{KQq}{\left(\frac{d}{2}\right)^2}$$

For equilibrium,

$$\frac{kQQ}{d^2} + \frac{kQq}{(d/2)^2} = 0 \qquad \therefore q = -\frac{Q}{4}$$

13. **[Bonus]**

Net charge $Q = \frac{\Delta\phi}{R} = \frac{1}{10}A(B_f - B_i) = \frac{1}{10} \times 3.5 \times 10^{-3}$

$$\left(0.4\sin\frac{\pi}{2} - 0\right)$$

$$= \frac{1}{10}(3.5 \times 10^{-3})(0.4 - 0)$$

$= 1.4 \times 10^{-4}$

No option matches, so it should be a bonus.

14. **(3)** Distance of c.m from m/2

$$= \frac{\frac{m}{2} \times 0 + m \times \ell}{\frac{m}{2} + m} = \frac{2\ell}{3}$$

$$I_{cm} = \frac{m}{2}\left(\frac{2\ell}{3}\right)^2 + m\left(\frac{\ell}{3}\right)^2 = \frac{1}{3}m\ell^2$$

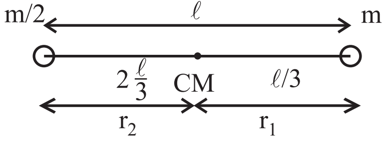
At the mean position

$$\frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}k\theta_0^2$$

$$\therefore \ \omega^2 = \frac{k}{I_{cm}}\theta_0^2$$

$$\omega^2 = \frac{3k}{m\ell^2}\theta_0^2$$

As we know, $\omega = \sqrt{\frac{k}{I_{cm}}}$



Tension in the rod when it passes through the mean position,

$$= m\omega^2\frac{\ell}{3} = m\left[\frac{3k}{m\ell^2}\theta_0^2\right]\frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$$

15. **(3)** Resistance, $R = \frac{\rho\ell}{A}$

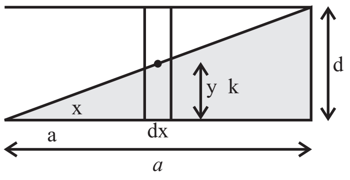
$$R = \rho\frac{\ell}{A}\times\frac{\ell}{\ell} = \frac{\rho\ell^2}{V}$$

[∵ Volume (V) = A ℓ]

Since resistivity and volume remains constant therefore % change in resistance

$$\frac{\Delta R}{R} = \frac{2\Delta\ell}{\ell} = 2\times(0.5) = 1\%$$

16. **(2)**



From figure, $\frac{y}{x} = \frac{d}{a} \Rightarrow y = \frac{d}{a}x$

$$dy = \frac{d}{a}(dx) \Rightarrow \frac{1}{dC} = \frac{y}{K\varepsilon_0 adx} + \frac{(d-y)}{\varepsilon_0 adx}$$

$$\frac{1}{dC} = \frac{y}{\varepsilon_0 adx}\left(\frac{y}{k} + d - y\right)$$

$$\int dC = \int\frac{\varepsilon_0 adx}{\frac{y}{k} + d - y}$$

$$\text{or, } C = \varepsilon_0 a.\frac{a}{d}\int_0^d \frac{dy}{d + y\left(\frac{1}{k} - 1\right)} \qquad \left[\because dy = \frac{d}{a}dx\right]$$

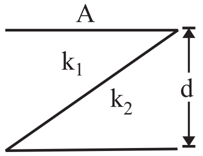
$$= \frac{\varepsilon_0 a^2}{\left(\frac{1}{k} - 1\right)d}\left[\ell n\left(d + y\left(\frac{1}{k} - 1\right)\right)\right]_0^d$$

$$= \frac{k\in_0 a^2}{(1-k)d}\ell n\left(\frac{d + d\left(\frac{1}{k} - 1\right)}{d}\right)$$

$$= \frac{k\in_0 a^2}{(1-k)d}\ell n\left(\frac{1}{k}\right) = \frac{k\in_0 a^2\ell nk}{(k-1)d}$$

Alternatively remember

$$C = \frac{k_1k_3A\,\varepsilon_0}{d(k_1 - k_2)}\log_e\frac{k_1}{k_2}$$



Here $k_1 = 1$, $k_2 = k$, $A = a^2$

$$\therefore \ C = \frac{k\,a^2\varepsilon_0}{d(1-k)}\log_e\frac{1}{k} = \frac{ka^2\varepsilon_0}{d(k-1)}\log_e k$$

17. **(3)** $\rho = \frac{1}{\sigma} = \frac{1}{n_e e\mu_e}$

$$\left[\begin{array}{l}\because \ \sigma = e(n_e\mu_e + n_h\mu_h) \\ \text{Here } n_h\mu_h \text{is neglected}\end{array}\right]$$

$$= \frac{1}{10^{19}\times1.6\times10^{-19}\times1.6}$$

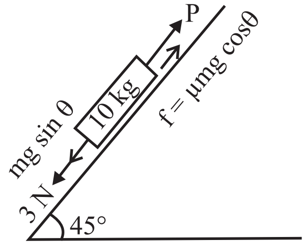
or $\rho = 0.4\ \Omega m$

18. **(3)** Areal velocity $= \frac{\pi R^2}{T} = \frac{\pi R^2}{(2\pi R/v)} = \frac{vR}{2}$

$$\therefore \frac{dA}{dt} = \frac{R}{2}\times\frac{L}{mR} \qquad [\because \ L = mvR]$$

$$\therefore \ \frac{dA}{dt} = \frac{1}{2}\frac{L}{m}$$

19. **(1)**



For equilibrium

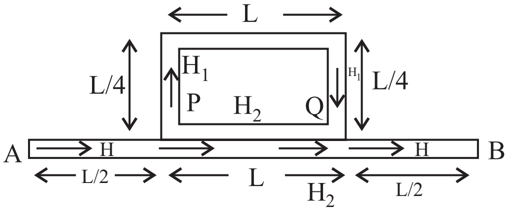
$$3 + mg\sin\theta = P + \mu mg\cos\theta$$

$$\Rightarrow 3+10\times10\times\frac{1}{\sqrt{2}}$$

$$= P + 0.6 \times 10 \times 10 \times \cos45°$$

$$\therefore\ P = 31.28 \approx 32\text{ N}$$

20. **(1)**



At P,

$$H = H_1 + H_{2s}$$

$$\frac{kA\left(T_A - T_P\right)}{L/2}$$

$$= \frac{kA\left(T_P - T_Q\right)}{3L/2} + \frac{kA\left(T_P - T_Q\right)}{L}$$

$$\therefore\ 2\ (T_A - T_P) = \frac{2}{3}\left(T_P - T_Q\right) + \left(T_P - T_Q\right)$$

$$\therefore 2\ (T_A - T_P)\ = \frac{5}{3}\left(T_P - T_Q\right) \qquad \ldots\text{(i)}$$

At, Q

$$H_1 + H_2 = H$$

$$\therefore\ \frac{kA\left(T_P - T_Q\right)}{3L/2} + \frac{kA\left(T_P - T_Q\right)}{L}$$

$$= \frac{kA\left(T_Q - T_B\right)}{L/2}$$

$$\therefore\ 2\left(T_Q - T_P\right) = \frac{5}{3}\left(T_P - T_Q\right) \qquad \ldots\text{(ii)}$$

From (i) & (ii)

$$2\ (T_A - T_P) + 2\ (T_Q - T_B)\ = \frac{10}{3}\left(T_P - T_Q\right)$$

$$T_A - T_B = \frac{8}{3}\left(T_P - T_Q\right)$$

$$\therefore\ T_P - T_Q = \frac{3}{8}\times120 = 45°C$$

21. **(2)** Wave speed $V = \sqrt{\frac{T}{\mu}}$

when car is at rest a = 0

$$\therefore 60 = \sqrt{\frac{Mg}{\mu}}$$

Similarly when the car is moving with acceleration a,

$$60.5 = \sqrt{\frac{M\left(g^2 + a^2\right)^{1/2}}{\mu}}$$

on solving we get

$$a = \frac{g}{\sqrt{30}} \qquad \text{[which is closest to g/5]}$$

22. **(3)** Activity $A = \lambda N$

For material, A

$$10 = (2\ N_0)\lambda_A$$

For material, B $\quad 20 = N_0\lambda_B$

$$\Rightarrow \lambda_B = 4\lambda_A$$

$$\therefore\ T_{½A} = 4T_{½B}\left[\because T_{½} = \frac{0.693}{\lambda}\right]$$

i.e. 20 days half-lives for A and 5 days $\left(T_{½}\right)_B$ for material B.

23. **(2)** For i ≈ 90° at air liquid interface we have by Snell's law

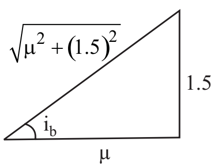
$$\mu = \frac{\sin 90°}{\sin r} \quad \therefore\ \sin r = \frac{1}{\mu}$$

According to Brewster's law, refractive index of liquid (μ) is equal to tangent of polarising angle

$$\because \tan i_b = \frac{1.5}{\mu}$$

$$\therefore\ \sin i_b = \frac{1.5}{\sqrt{\mu^2 + 1.5^2}}$$

Here $\sin r < \sin i_b$



$$\therefore \frac{1}{\mu} \leq \frac{1.5}{\sqrt{\mu^2+(1.5)^2}}$$

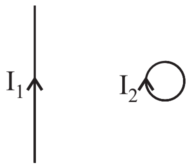
or, $\sqrt{\mu^2+(1.5)^2} \leq 1.5 \times \mu$

$$\Rightarrow \mu^2 + (1.5)^2 \leq (\mu \times 1.5)^2$$

$\Rightarrow \mu \geq \frac{3}{\sqrt{5}}$ i.e. minimum

value of μ should be $\frac{3}{\sqrt{5}}$

24. **(4)** We know that $F = -\frac{dv}{dr}$ where r = distance of the loop from straight current carrying wire



Here $U = -\vec{m}.\vec{B} = -I_2 \pi a^2 \times \frac{\mu_0}{4\pi}\frac{I_1}{r} \times 2 \times \cos 0$

$$= -\frac{\mu_0 \, I_1 \, I_2 \, a^2}{2r}$$

$$\therefore \; F = -\frac{d}{d(r)}\left[-\frac{\mu_0 \; I_1 I_2 a^2}{2r}\right] = -\frac{\mu_0 \; I_1 I_2 a^2}{r^2}$$

Here r = d

$\therefore \; F \propto \frac{a^2}{d^2}$ (attractive)

25. **(1)** From Einstein's photoelectric equation,

$$\frac{hc}{\lambda_1} - \phi = \frac{1}{2}m(2v)^2 \qquad ...(i)$$

and $\frac{hc}{\lambda_2} - \phi = \frac{1}{2}mv^2$ ....(ii)

From eqn. (i) & (ii)

$$\Rightarrow \frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = 4 \Rightarrow \frac{hc}{\lambda_1} - \phi = \frac{4hc}{\lambda_2} - 4\phi$$

$$\Rightarrow \frac{4hc}{\lambda_2} - \frac{hc}{\lambda_1} = 3\phi \;\Rightarrow \phi = \frac{1}{3}hc\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$= \frac{1}{3} \times 1240\left(\frac{4 \times 350 - 540}{350 \times 540}\right)$$

= 1.8 eV

26. **(3)** From given equation,

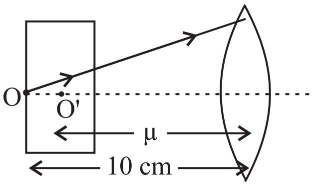
$$\vec{v} = k\left(y\hat{i} + x\hat{j}\right) = ky\hat{i} + kx\hat{j} = V_x\hat{i} + V_y\hat{j}$$

$$\frac{dx}{dt} = ky \text{ and } \frac{dy}{dt} = kx$$

Now, $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y} = \frac{dy}{dx} \Rightarrow ydy = xdx$

Integrating both sides we get $y^2 = x^2 + c$

27. **(4)**



As the object and image distance is same, object is placed at 2f. Therefore 2f = 10 or f = 5 cm.

Shift due to slab, $d = t\left(1 - \frac{1}{\mu}\right)$

in the direction of incident ray

$$\Rightarrow d = 1.5\left(1 - \frac{2}{3}\right) = 0.5 \text{ cm}$$

Now, u = – 9.5 cm

Again using lens formulas $\frac{1}{v} - \frac{1}{-9.5} = \frac{1}{5}$

$\Rightarrow$ v = 10.55 cm

Thus, screen is shifted by a distance d = 10.55 – 10 = 0.55 cm away from the lens.

28. **(2)** Electric field on the axis of a ring of radius R at a distance h from the centre,

$$E = \frac{kQh}{\left(h^2 + R^2\right)^{3/2}}$$

Condition for maximum electric field

we have $\frac{dE}{dh} = 0$

$$\Rightarrow \frac{d}{dh}\left[\frac{kQh}{\left(R^2 + h^2\right)^{3/2}}\right] = 0$$

On solving we get, $h = \frac{R}{\sqrt{2}}$

29. **(3)** Kinetic energy of block A

$$k_1 = \frac{1}{2}mv_0^2$$

$\therefore$ From principle of linear momentum conservation

$$mv_0 = (2m+M)v_f \Rightarrow v_f = \frac{mv_0}{2m+M}$$

$$K.E_f = \frac{1}{6}K.E_i \qquad \text{(given)}$$

$$\frac{1}{2}(2m+M)v_f^2 = \frac{1}{6}\times\frac{1}{2}mv_0^2$$

$$6(2m+M)\frac{m^2v_0^2}{(2m+M)^2} = mv_0^2$$

$\Rightarrow 6m = 2m + M$

$\Rightarrow 4m = M$

$$\therefore \frac{M}{m} = \frac{4}{1}$$

30. **(1)** Using, $I = neAv_d$

$$\therefore \text{ Drift speed } v_d = \frac{I}{neA}$$

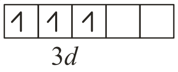
$$\frac{1.5}{9\times10^{28}\times1.6\times10^{-19}\times5\times10^{-6}}$$

$= 0.02 \times 10^{-3}$ ms^{-1}

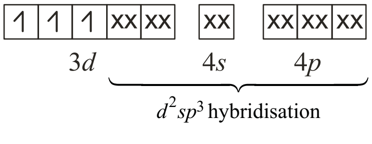
$= 0.02$ mms^{-1}

SECTION-2 : CHEMISTRY

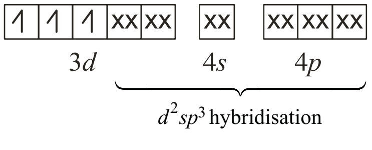
31. **(1)** E.C. of Cr^{3+} ($3d^3$):



For complex A $[Cr(H_2O)_2]^{3+}$:



For complex B $[Cr(NH_3)_6]^{3+}$:



Here, both the complexes (A) and (B) are paramagnetic with 3 unpaired electrons each. Also H_2O is a weak field ligand which causes lesser splitting than NH_3 which is comparatively stronger field ligand. Hence, the (Δ_0) value of (A) and (B) are calculated from the wavelengths of light absorbed and not from the wavelengths of light emitted.

32. **(4)** The acidic strength of a compound or an acid depends on the inductive effect (–I). Higher the (–I) effect of a substituent higher will be acidic strength. Now, the decreasing order of (–I) effect of the given substituents is $NO_2 > CN > F > Cl$.

$\therefore$ The correct decreasing order of acidic strength amongst the given carboxylic acids is:
$NO_2CH_2COOH > CNCH_2COOH > FCH_2COOH > ClCH_2COOH$

33. **(3)** $R{-}C{\equiv}N \xrightarrow[\text{(ii) } H_2O]{\text{(i) } AlH(i\text{-}Bu)_2} R{-}CHO$

The reduction of nitriles to aldehydes can be done using DIBAL-H[$AlH(i\text{-}Bu)_2$].

34. **(1)** Magnetic moment, $\mu = \sqrt{n(n+2)}$BM (where, n = no. of unpaired electrons)
As transition metal atom/ion in a complex may have unpaired electrons ranging from zero to 5. So, maximum number of unpaired electrons that may be present in a complex is 5.
$\therefore$ Maximum value of magnetic moment among all the transition metal complexes is

$$= \sqrt{5(5+2)} = \sqrt{35} = 5.92 \text{ BM}$$

35. **(4)** Ideal gas equation: $PV = nRT$

After putting the values we get,

$200 \times 10 = (0.5 + x) \times R \times 1000$
(total no. of moles are $0.5 + x$)

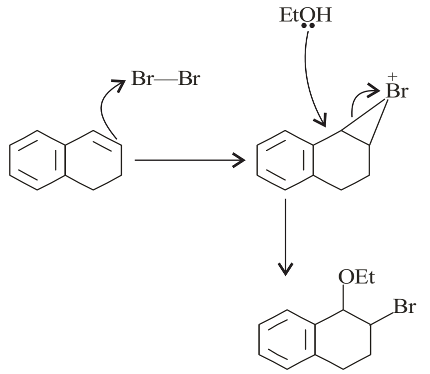
$$x = \frac{4-R}{2R}$$

36. **(3)** Quartz exhibits piezoelectricity and thus can be used as a piezoelectric material.

37. **(4)** Silicones are polymers containing Si—O—Si linkages with strong hydrophobic character.

Generally, they exhibit high thermal stability with high dielectric strength. Silicon greases are resistant to oxidation which are commonly used for greasing purposes.

38. **(1)** Mechanism involved for the given reaction is:



(major product)

39. **(3)** Generally, electronegativity decreases down the group as the size increases. This can also be formulated as:

$$\text{Electronegativity} \propto \frac{1}{\text{size}}$$

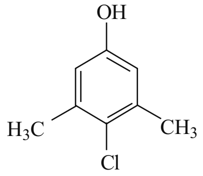
40. **(2)** Number of moles in 92 g of $Na^+ = \frac{92}{23}$

= 4 moles

$$\text{Molality } (m) = \frac{\text{Number of moles}}{\text{Mass of solvent (in kg)}}$$

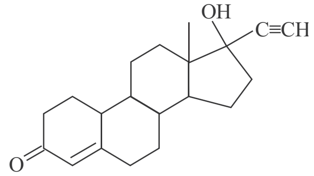
$$\therefore \quad m = \frac{4}{1} = 4 \text{ mol kg}^{-1}$$

41. **(3)** As chloroxylenol contains phenolic group so it gives positive ferric chloride test.



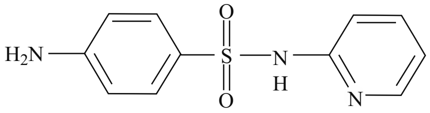
Chloroxylenol

Norethindrone has double bond, thus it will give Bayer's test.



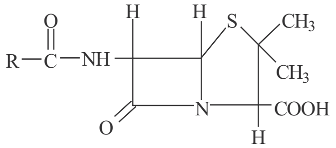
Norethindrone

Sulphapyridine contains $-NH_2$ group so it gives carbylamine test.



Sulphapyridine

Penicillin contains –COOH group so it will give sodium hydrogen carbonate ($NaHCO_3$) test.



Pencillin

42. **(2)** The water sample containing Mn = 5 ppm is unsuitable for drinking as the prescribed level for Mn in drinking water is 0.5 ppm.

43. **(3)** Half cell reaction: $PbSO_4 \rightarrow Pb^{4+} + 2e^-$

According to the reaction:

$PbSO_4 \rightarrow Pb^{4+} + 2e^-$

We require 2F for the electrolysis of 1 mol or 303 g of $PbSO_4$

$\therefore$ Amount of $PbSO_4$ electrolysed by 0.05F = $\frac{303}{2} \times .05 = 7.575 \text{ g} \approx 7.6 \text{ g}$

44. **(1)** The solubility of the gas in liquids decreases with the increase in value of K_H at a given pressure.

45. **(1)** From experiment 1 and II, it is observed that order of reaction w.r.t. (c) is zero.

From experiment II and III, α can be calculated as:

$$\frac{1.386 \times 10^{-2}}{6.93 \times 10^{-3}} = \left(\frac{0.2}{0.1}\right)^{\alpha}$$

$\therefore \alpha = 1$

Now, Rate = $K[A]^1$

or, $6.93 \times 10^{-3} = K(0.1)$

$K = 6.93 \times 10^{-2}$

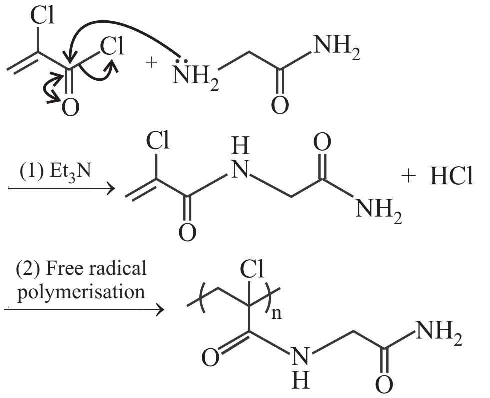
For the reaction, $2A + B \rightarrow$ Products

$$2Kt = \ln\frac{[A]_0}{[A]}$$

$$\therefore \quad t_{1/2} = \frac{0.693}{2K} = \frac{0.693}{06.93 \times 10^{-2} \times 2}$$

$t_{1/2} = 5$

46. **(4)** Mechanism for the formation of major product is as follows:



47. (4) The chances of formation of hydrate decreases with the decrease in the charge density down the group. This is why, $Ba(NO_3)_2$ does not crystallise with water molecules.

48. (2) m. mol of $H_2SO_4 = 20 \times 0.1 = 2$

m. mol of $NH_4OH = 30 \times 0.2 = 6$

$$H_2SO_4 + 2NH_4OH \rightarrow (NH_4)_2\,SO_4 + 2H_2O$$

Initial	2 m mol	6 m mol	0
Final	(2-2) = 0 m mol	(6 – 2 × 2) = 2 m mol	2 m mol

$[NH_4OH]_{left} = 2$ m mol

$[(NH_4)_2\,SO_4] = 2$ m mol

$[\overset{+}{N}H_4] = 2 \times 2 = 4$ m mol

Total Volume = 30 + 20 = 50 mL

$$pOH = pK_b + \log\left[\frac{\text{Salt}}{\text{Base}}\right]$$

$$= 4.7 + \log\frac{4/50}{2/50}$$

$$= 4.7 + \log 2 = 5$$

$$pH = 14 - pOH$$

$$pH = 14 - 5 = 9$$

49. (3) In Freundlich adsorption isotherm the extent of adsorption (x/m) of a gas on the surface of a solid is related to the pressure of the gas (P) which can be formulated as:

$$\frac{x}{m} = k(p)^{1/n}$$

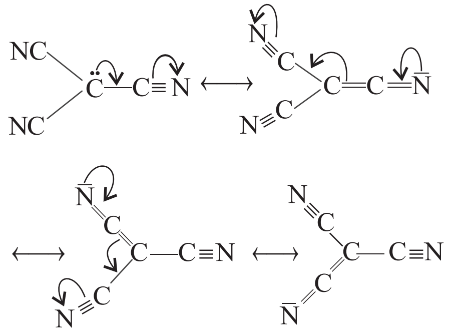
$$\Rightarrow \log\frac{x}{m} = \log k + \frac{1}{n}\log p$$

In the given plot, the slope between $\log\frac{x}{m}$ versus $\log P = \frac{2}{4} = \frac{1}{2}$

$$\therefore \frac{x}{m} \propto p^{1/2}$$

50. (3) Due to the resonance stabilisation of the conjugate base, $CH(CN)_3$ is the strongest acid amongst the given compounds.

$$CH(CN)_3 \rightleftharpoons \overset{+}{H} + \bar{C}\,(CN)_3$$



The conjugate bases of $CHBr_3$ and CHI_3 are stabilised by inductive effect of halogens. This is why, they are less stable. Also, the conjugate base of $CHCl_3$ involves back-bonding between 2*p* and 3*p* orbitals.

51. (1) Amongst the given ores, copper pyrite ($CuFeS_2$), dolomite ($MgCO_3.CaCO_3$), malachite [$CuCO_3.Cu(OH)_2$], azurite [$2CuCO_3.Cu(OH)_2$], copper pyrite contains both copper and iron.

52. (4) As we know,

$$\bar{v} = -R_H\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)Z^2 \text{ (where, } Z = 1)$$

After putting the values, we get

$$\bar{v} = -R_H\left(\frac{1}{n^2} - \frac{1}{8^2}\right)$$

$$\Rightarrow \bar{v} = \frac{R_H}{64} - \frac{R_H}{n^2}$$

Comparing to $y = mx + c$, we get

$$x = \frac{1}{n^2} \text{ and } m = -R_H \text{ (slope)}$$

53. (3) Hydrogen has three isotopes:

Protium (${}_1H^1$), deuterium (${}_1H^2$) and tritium (${}_1H^3$).

54. (3) Electronic configuratios of Li_2^+ and Li_2^-:

Li_2^+: $\sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^1$

Li_2^-: $\sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^1$

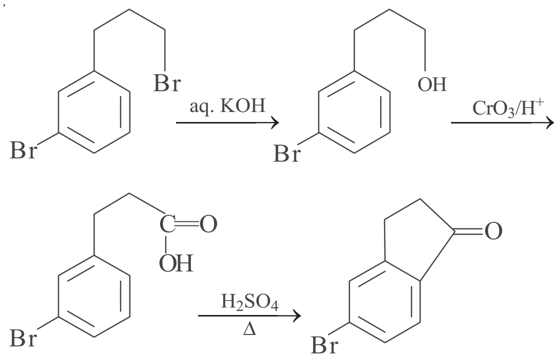
Now,

Bond order of $Li_2^+ = \frac{1}{2}(3-2) = \frac{1}{2}$

Bond order of $Li_2^- = \frac{1}{2}(4-3) = \frac{1}{2}$

Here, both Li_2^+ and Li_2^- have positive bond order, thus both are stable.

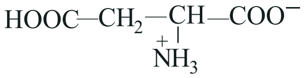
55. **(1)** For the given reaction condition, the major product is:



56. **(1)** Due to the inert pair effect, thallium exists in more than one oxidation state. Also, for thallium + 1 oxidation state is more stable than +3 oxidation state.

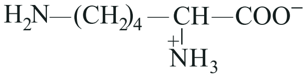
57. **(3)** Structure of the given α-amino acids are:

$HN_3—CH_2—COO^-$
Glycine (Gly)



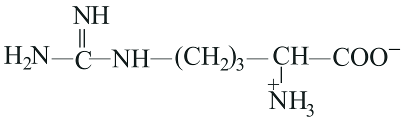
$HOOC—CH_2—\underset{\overset{+}{N}H_3}{CH}—COO^-$

Aspartic acid (Asp)



$H_2N—(CH_2)_4—\underset{\overset{+}{N}H_3}{CH}—COO^-$

Lysine (Lys)



$H_2N—\overset{\overset{NH}{\|}}{C}—NH—(CH_2)_3—\underset{\overset{+}{N}H_3}{CH}—COO^-$

Arginine (Arg)

Here, aspartic acid is an acidic and glycine is a neutral amino acid while lysine and arginine are basic amino acids. Also, arginine is more basic due to the stronger basic functional groups.

∴ The order of *pKa* value is directly proportional to the basic strength of amino acids, i.e. Arg > Lys > Gly > Asp.

58. **(2)** For reversible isothermal expansion,

$$w = -nRT\ln\frac{V_2}{V_1}$$

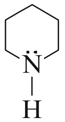
$$\Rightarrow \quad |w| = nRT\ln\frac{V_2}{V_1}$$

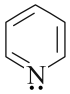
$|w| = \text{nRT}(\ln V_2 - \ln V_1)$

$|w| = \text{nRT}\ln V_2 - \text{nRT}\, V_1$

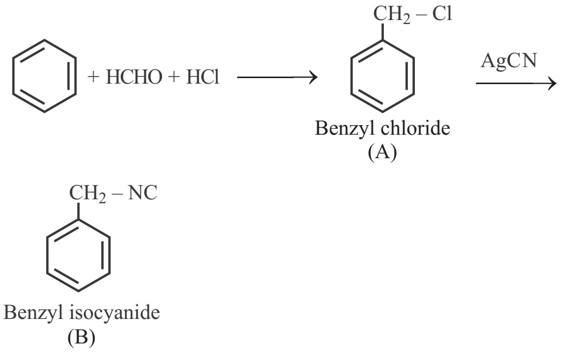
$y = mx + c$

So, slope of curve 2 is more than curve 1 and intercept of curve 2 is more negative than curve 1.

59. **(2)** Compound, III  is most basic as the lone pair of nitrogen is easily available for the donation.

In case of compound (I)  lone pair is not involved in resonance but nitrogen atom is sp^2 hybridised whereas in compound II the lone pair of nitrogen is involved in aromaticity which makes it least basic.

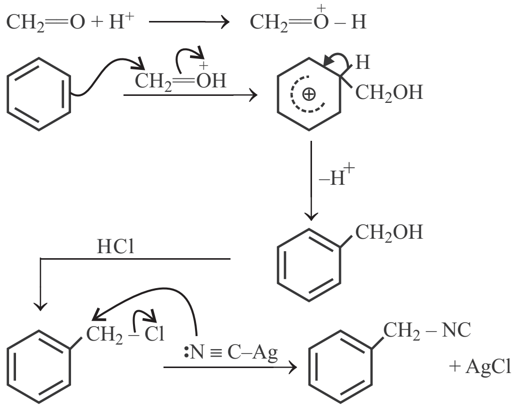
60. **(4)**



Benzyl chloride (A)

Benzyl isocyanide (B)

Mechanism:



SECTION-3 : MATHEMATICS

61. **(2)** $\text{I} = \int_0^{\pi} |\cos x|^3\, dx$

$$= 2\int_0^{\pi/2} \cos^3 x\, dx$$

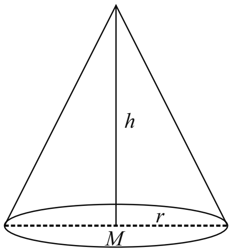
$$= \frac{2}{4}\int_0^{\pi/2} (3\cos x + \cos 3x)dx$$

$[\because \ \cos 3\theta = 4\cos^3\theta - 3\cos\theta]$

$$=\frac{1}{2}\left[3\sin x+\frac{\sin 3x}{3}\right]_0^{\pi/2}$$

$$=\frac{1}{2}\left(3-\frac{1}{3}\right)=\frac{4}{3}$$

62. **(4)**



$h^2 + r^2 = \ell^2 = 9$ ...(1)

Volume of cone

$V = \frac{1}{3}\pi r^2 h$ ...(2)

From (1) and (2),

$$\Rightarrow \; V=\frac{1}{3}\pi(9-h^2)h$$

$$\Rightarrow \; V=\frac{1}{3}\pi(9h-h^3)$$

$$\Rightarrow \; \frac{dv}{dh}=\frac{1}{3}\pi\left(9-3h^2\right)$$

For maxima/minima,

$$\frac{dV}{dh}=0 \Rightarrow \frac{1}{3}\pi(9-3h^2)=0$$

$\Rightarrow \; h=\pm\sqrt{3} \Rightarrow h=\sqrt{3}$ $\quad (\because h>0)$

Now; $\frac{d^2V}{dh^2}=\frac{1}{3}\pi(-6h)$

Here, $\left(\frac{d^2V}{dh^2}\right)_{\text{at } h=\sqrt{3}} < 0$

Then, $h=\sqrt{3}$ is point of maxima

Hence, the required maximum volume is,

$$V=\frac{1}{3}\pi(9-3)\sqrt{3}=2\sqrt{3}\pi$$

63. **(3, 4)** Consider the given integral

$$I=\int x\sqrt{\frac{2\sin(x^2-1)-2\sin(x^2-1)\cos(x^2-1)}{2\sin(x^2-1)+2\sin(x^2-1)\cos(x^2-1)}}dx$$

$(\therefore \sin 2\theta = 2\sin\theta\cos\theta)$

$$\Rightarrow \; I=\int x\sqrt{\frac{1-\cos(x^2-1)}{1+\cos(x^2-1)}}dx$$

$$\Rightarrow \; I=\int x\left|\tan\left(\frac{x^2-1}{2}\right)\right|dx,$$

Now let $\frac{x^2-1}{2}=t \quad \Rightarrow \quad \frac{2x}{2}dx=dt$

$$\therefore \; I=\int|\tan(t)|\,dt=\ln|\sec t|+C$$

or $I=\ln\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c=\frac{1}{2}\ln\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

64. **(3)** Since, $x\frac{dy}{dx} \quad y=x^2$

$$\Rightarrow \; \frac{dy}{dx}+\frac{2}{x}y=x$$

I.F. $=e^{\int\frac{2}{x}dx}=e^{2\ln x}=e^{\ln x^2}=x^2.$

Solution of differential equation is:

$$y\cdot x^2=\int x\cdot x^2dx$$

$y\cdot x^2=\frac{x^4}{4}+C$ ...(1)

$\because \; y(1)=1$

$\therefore \; C=\frac{3}{4}$

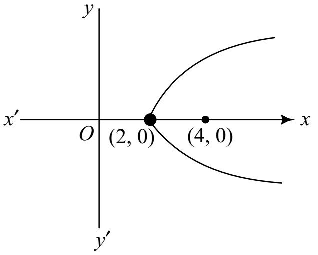
Then, from equation (1)

$$y\cdot x^2=\frac{x^4}{4}+\frac{3}{4}$$

$$\therefore \; y=\frac{x^2}{4}+\frac{3}{4x^2}$$

$$\therefore \; y\left(\frac{1}{2}\right)=\frac{1}{16}+3=\frac{49}{16}$$

65. **(2)** Since, vertex and focus of given parabola is (2, 0) and (4, 0) respectively



Then, equation of parabola is

$(y-0)^2 = 4 \times 2(x-2)$

$\Rightarrow \; y^2 = 8x - 16$

Hence, the point (8, 6) does not lie on given parabola.

66. **(1)** $\because a^2 = \cos^2\theta, b^2 = \sin^2\theta$

and $e > 2 \Rightarrow e^2 > 4 \Rightarrow 1 + b^2/a^2 > 4$

$\Rightarrow 1 + \tan^2\theta > 4$

$\Rightarrow \sec^2\theta > 4 \Rightarrow \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Latus rectum,

$$LR = \frac{2b^2}{a} = \frac{2\sin^2\theta}{\cos\theta} = 2(\sec\theta - \cos\theta)$$

$$\Rightarrow \frac{d(LR)}{d\theta} = 2(\sec\theta\tan\theta + \sin\theta) > 0 \ \forall\ \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

$$\therefore \min(LR) = 2\left(\sec\frac{\pi}{3} - \cos\frac{\pi}{3}\right) = 2\left(2 - \frac{1}{2}\right) = 3$$

max (LR) tends to infinity as $\theta \to \frac{\pi}{2}$

Hence, length of latus rectum lies in the interval $(3, \infty)$

67. **(1)** The given relation is

$$(f_2 o\, J o\, f_1)(x) = f_3(x) = \frac{1}{1-x}$$

$$\Rightarrow (f_2 o J)(f_1(x)) = \frac{1}{1-x}$$

$$\Rightarrow (f_2 o\, J)\left(\frac{1}{x}\right) = \frac{1}{1-x} \qquad \left[\because f_1(x) = \frac{1}{x}\right]$$

$$\Rightarrow f_2\left(J\left(\frac{1}{x}\right)\right) = \frac{1}{1-x}$$

$$\Rightarrow (f_2\, J(x)) = \frac{1}{1-\frac{1}{x}} = \frac{x}{x-1}$$

$$\left[\frac{1}{x} \text{ is replaced by } x\right]$$

$$\Rightarrow 1 - J(x) = \frac{x}{x-1} \qquad [\because f_2(x) = 1 - x]$$

$$\therefore J(x) = 1 - \frac{x}{x-1} = \frac{1}{1-x} = f_3(x)$$

68. **(1)** $\because |\vec{a} \times \vec{c}|^2 = |\vec{a}|^2 |\vec{c}|^2 - (\vec{a}.\vec{c})^2$

$\Rightarrow |-\vec{b}|^2 = 2|\vec{c}|^2 - 16$

$\Rightarrow 3 = 2|\vec{c}|^2 - 16$

$\Rightarrow |\vec{c}|^2 = \frac{19}{2}$

69. **(4)** $\because a, b, c$, are in G.P.

$\Rightarrow b^2 = ac$

Since, $a + b + c = xb$

$\Rightarrow a + c = (x - 1)b$

Take square on both sides, we get

$a^2 + c^2 + 2ac = (x - 1)^2 b^2$

$\Rightarrow a^2 + c^2 = (x - 1)^2 ac - 2ac \ [\because b^2 = ac]$

$\Rightarrow a^2 + c^2 = ac[(x - 1)^2 - 2]$

$\Rightarrow a^2 + c^2 = ac[x^2 - 2x - 1]$

$\because a^2 + c^2$ are positive and $b^2 = ac$ which is also positive. Then, $x^2 - 2x - 1$ would be positive but for $x = 2$, $x^2 - 2x - 1$ is negative.

Hence, x cannot be taken as 2.

70. **(1)** $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}; \left(x > \frac{3}{4}\right)$

$$\Rightarrow \cos^{-1}\left(\frac{2}{3x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{3}{4x}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{2}{3x}\right) = \sin^{-1}\left(\frac{3}{4x}\right)$$

$$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

Put $\sin^{-1}\left(\frac{3}{4x}\right) = \theta \Rightarrow \sin\theta = \frac{3}{4x}$

$$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{9}{16x^2}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{16x^2 - 9}}{4x}\right)$$

$$\therefore \cos^{-1}\left(\frac{2}{3x}\right) = \cos^{-1}\left(\frac{\sqrt{16x^2 - 9}}{4x}\right)$$

$$\Rightarrow \frac{2}{3x} = \frac{\sqrt{16x^2 - 9}}{4x} \Rightarrow x^2 = \frac{64 + 81}{9 \times 16}$$

$$\Rightarrow x = \pm\sqrt{\frac{145}{144}}$$

$$\Rightarrow x = \frac{\sqrt{145}}{12} \qquad \left(\because x > \frac{3}{4}\right)$$

71. (2) Since, the equation of tangent to parabola $y^2 = 4x$ is

$$y = mx + \frac{1}{m} \qquad ...(1)$$

The line (1) is also the tangent to circle

$x^2 + y^2 - 6x = 0$

Then centre of circle = (3, 0)

radius of circle = 3

The perpendicular distance from centre to tangent is equal to the radius of circle

$$\therefore \quad \frac{\left|3m+\frac{1}{m}\right|}{\sqrt{1+m^2}} = 3 \quad \Rightarrow \quad \left(3m+\frac{1}{m}\right)^2 = 9\left(1+m^2\right)$$

$$\Rightarrow \quad m = \pm\frac{1}{\sqrt{3}}$$

Then, from equation (1): $y = \pm\frac{1}{\sqrt{3}}x \pm \sqrt{3}$

Hence, $\sqrt{3}y = x+3$ is one of the required common tangent.

72. (4) Since the system of linear equations are

$$x + y + z = 2 \qquad ...(1)$$

$$2x + 3y + 2z = 5 \qquad ...(2)$$

$$2x + 3y + (a^2 - 1)z = a + 1 \qquad ...(3)$$

If $a^2 = 3$, then plane represented by eqn (2) and eqn (3) are $2x + 3y + 2z = 5$ and

$2x + 3y + 2z = \pm\sqrt{3} + 1$ which are parallel, i.e., have no solution.

Hence, the given system of equations is inconsistent, for $|a| = \sqrt{3}$

73. (2) $2^{403} = 2^{400} \cdot 2^3$

$= 2^{4 \times 100} \cdot 2^3$

$= (2^4)^{100} \cdot 8$

$= 8(2^4)^{100} = 8(16)^{100}$

$= 8\,(1 + 15)^{100}$

$= 8 + 15\,\mu$

When 2^{403} is divided by 15, then remainder is 8.

Hence, fractional part of the number is $\frac{8}{15}$

Therefore value of k is 8

74. (3) Let any point on the intersecting line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1} = \lambda \text{ (say)}$$

is $(-3\lambda - 1, 2\lambda + 3, -\lambda + 2)$

Since, the above point lies on a line which passes through the point (–4, 3, 1)

Then, direction ratio of the required line

$= <-3\lambda - 1 + 4, 2\lambda + 3 - 3, -\lambda + 2 - 1>$

or $<-3\lambda + 3, 2\lambda, -\lambda + 1>$

Since, line is parallel to the plane

$x + 2y - z - 5 = 0$

Then, perpendicular vector to the line is $\hat{i} + 2\hat{j} - \hat{k}$

Now $(-3\lambda + 3)(1) + (2\lambda)(2) + (-\lambda + 1)(-1) = 0$

$\Rightarrow \lambda = -1$

Now direction ratio of the required line

$= <6, -2, 2>$ or $<3, -1, 1>$

Hence required equation of the line is

$$\frac{(x+4)}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

75. (1) The given equations of the set of all lines

$$px + qy + r = 0 \qquad ...(1)$$

and given condition is :

$3p + 2q + 4r = 0$

$$\Rightarrow \quad \frac{3}{4}p + \frac{2}{4}q + r = 0 \qquad ...(2)$$

From (1) & (2) we get :

$$\therefore \quad x = \frac{3}{4}, y = \frac{1}{2}$$

Hence the set of lines are concurrent and passing through the fixed point $\left(\frac{3}{4}, \frac{1}{2}\right)$

76. (1) $L = \lim\limits_{y \to 0} \dfrac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

$$= \lim_{y \to 0} \frac{\left(\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}\right)\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)}$$

$$= \lim_{y \to 0} \frac{1+\sqrt{1+y^4} - 2}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)}$$

$$= \lim_{y \to 0} \frac{\left(\sqrt{1+y^4} - 1\right)\left(\sqrt{1+y^4} + 1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)\left(\sqrt{1+y^4} + 1\right)}$$

$$= \lim_{y \to 0} \frac{1 + y^4 - 1}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)\left(\sqrt{1+y^4} + 1\right)}$$

$$= \frac{1}{2\sqrt{2} \times 2} = \frac{1}{4\sqrt{2}}$$

77. **(4)** Since, equation of plane through intersection of planes

$x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$(2x + 3y - z + 4) + \lambda(x + y + z - 1) = 0$

$(2 + \lambda)x + (3 + \lambda)y + (-1 + \lambda)z + (4 - \lambda) = 0$ ...(1)

But, the above plane is parallel to y-axis then

$(2 + \lambda) \times 0 + (3 + \lambda) \times 1 + (-1 + \lambda) \times 0 = 0$

$\Rightarrow \lambda = -3$

Hence, the equation of required plane is

$-x - 4z + 7 = 0$

$\Rightarrow x + 4z - 7 = 0$

Therefore, (3, 2, 1) the passes through the point.

78. **(2)** Since, the equation of curves are

$y = 10 - x^2$ ...(1)

$y = 2 + x^2$ ...(2)

Adding eqn (1) and (2), we get

$2y = 12 \quad \Rightarrow \quad y = 6$

Then, from eqn (1)

$x = \pm 2$

Differentiate equation (1) with respect to x

$$\frac{dy}{dx} = -2x \Rightarrow \left(\frac{dy}{dx}\right)_{(2,6)} = -4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)} = 4$$

Differentiate equation (2) with respect to x

$$\frac{dy}{dx} = 2x \Rightarrow \left(\frac{dy}{dx}\right)_{(2,6)} = 4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)} = -4$$

$$\text{At } (2, 6)\ \tan\theta = \left(\frac{(-4)-(4)}{1+(-4)\times(4)}\right) = \frac{-8}{15}$$

$$\text{At } (-2, 6),\ \tan\theta = \frac{(4)-(-4)}{1+(4)(-4)} = \frac{8}{-15}$$

$$\therefore |\tan\theta| = \frac{8}{15}$$

79. **(3)** $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow |A| = 1$

$$\text{adj}(A) = \begin{bmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & +\cos\theta \end{bmatrix}^T$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = B$$

$$B^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\Rightarrow B^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$

$$\Rightarrow A^{-50} = B^{50} = \begin{bmatrix} \cos(50\theta) & \sin(50\theta) \\ -\sin(50\theta) & \cos(50\theta) \end{bmatrix}$$

$$(A^{-50})_{\theta=\frac{\pi}{12}} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\left[\because \cos\left(\frac{50\pi}{12}\right) = \cos\left(4\pi + \frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\right]$$

80. **(3)** Check each option

(1) $(p \vee q) \wedge (\sim p \wedge q) = (\sim p \wedge q)$

(2) $(p \vee q) \wedge (\sim p \vee q) = (p \wedge \sim p) \vee q = F \vee q = q$

(3) $(p \wedge q) \wedge (\sim p \vee q) = (p \wedge q \wedge \sim p) \vee (p \wedge q) \wedge q$

$= F \vee (p \wedge q) = p \wedge q$

(4) $(p \wedge q) \wedge (\sim p \wedge q) = (p \wedge \sim p) \wedge q = F \sim q = F$

81. **(3)** $\because$ Variance $= \sigma^2 = \dfrac{\Sigma x_i^2}{N} - (\overline{x})^2$

$$\Rightarrow 18 = \frac{\Sigma x_i^2}{5} - (150)^2$$

$$\Rightarrow \Sigma x_i^2 = 90 + 112590 = 112590$$

Then, variance of the height of six students

$$V' = \frac{112590 + (156)^2}{6} - \left(\frac{750 + 156}{6}\right)^2$$

$= 22821 - 22801$

$= 20$

82. **(2)** $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3(1 - 2\sin\theta\cos\theta)^2 + 6$
$(1 + 2\sin\theta\cos\theta) + 4\sin^6\theta$

$= 3(1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta) + 6$
$+ 12\sin\theta\cos\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta\cos^2\theta + 4\sin^6\theta$

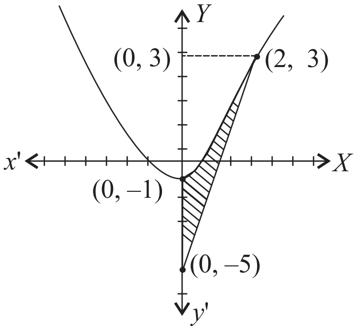
$= 9 + 12\cos^2\theta(1 - \cos^2\theta) + 4(1 - \cos^2\theta)^3$

$= 9 + 12\cos^2\theta - 12\cos^4\theta$
$+ 4(1 - \cos^6\theta - 3\cos^2\theta + 3\cos^4\theta)$

$= 9 + 4 - 4\cos^6\theta$

$= 13 - 4\cos^6\theta$

83. (1)



$\because$ Curve is given as :
$y = x^2 - 1$

$$\Rightarrow \frac{dy}{dx} = 2x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)} = 4$$

$\therefore$ equation of tangent at (2, 3)

$(y - 3) = 4(x - 2)$

$\Rightarrow y = 4x - 5$

but $x = 0$

$\Rightarrow y = -5$

Here the curve cuts Y–axis
$\therefore$ required area

$$= \frac{1}{4}\int_{-5}^{3}(y+5)dy - \int_{-1}^{3}\sqrt{y+1}\,dy$$

$$= \frac{1}{4}\left[\frac{y^2}{2}+5y\right]_{-5}^{3} \frac{-2}{3}\left[(y+1)^{3/2}\right]_{-1}^{3}$$

$$= \frac{1}{4}\left[\frac{9}{2}+15-\frac{25}{2}+25\right] -\frac{2}{3}\left[4^{3/2}-0\right]$$

$$= \frac{32}{4}-\frac{16}{3}=\frac{8}{3} \text{ sq-units.}$$

84. (1) $S = \sum_{i=1}^{30} a_i = \frac{30}{2}[2a_1 + 29d]$

$$T = \sum_{i=1}^{15} a_{(2i-1)} = \frac{15}{2}[2a_1 + 28d]$$

Since, $S - 2T = 75$

$\Rightarrow 30\,a_1 + 435d - 30a_1 - 420\text{d} = 75$

$\Rightarrow d = 5$

Also, $a_5 = 27 \quad \Rightarrow \quad a_1 + 4d = 27$

$\Rightarrow a_1 = 7,$

Hence, $\quad a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

85. (4) Let $f(x)$ is continuous at $x = 1$, then

$f(1^-) = f(1) = f(1^+)$

$\Rightarrow 5 = a + b$ . ..(1)

Let $f(x)$ is continuous at $x = 3$, then

$f(3^-) = f(3) = f(3^+)$

$\Rightarrow a + 3b = b + 15 \quad \Rightarrow \text{a} + 2\text{b} = 15$ ...(2)

Solving (i) & (2) we get b = 10, a = –5

Now $f(x)$ is continuous at $x = 5$, then

$f(5^-) = f(5) = f(5^+)$

$\Rightarrow b + 25 = 30$

Which is not satisfied by a = –5 and b = 10.

Hence, $f(x)$ is not continuous for any values of a and b

86. (4) Suppose $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$

Since, z is purely imaginary, then $z + \bar{z} = 0$

$$\Rightarrow \frac{3+2i\sin\theta}{1-2i\sin\theta}+\frac{3-2i\sin\theta}{1+2i\sin\theta}=0$$

$$\Rightarrow \frac{(3+2i\sin\theta)(1+2i\sin\theta)+(3-2i\sin\theta)(1-2i\sin\theta)}{1+4\sin^2\theta}=0$$

$$\Rightarrow \sin^2\theta = \frac{3}{4} \quad \Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

Now, the sum of elements in A

$$= -\frac{\pi}{3}+\frac{\pi}{3}+\frac{2\pi}{3}=\frac{2\pi}{3}$$

87. (3) Since, the number of ways to select 2 girls is ${}^5C_2$.

Now, 3 boys can be selected in 3 ways.

(1) Selection of A and selection of any 2 other boys (except B) in ${}^5C_2$ ways

(2) Selection of B and selection of any 2 two other boys (except A) in ${}^5C_2$ ways

(3) Selection of 3 boys (except A and B) in ${}^5C_3$ ways

Hence, required number of different teams
$= {}^5C_2\,({}^5C_2 + {}^5C_2 + {}^5C_3) = 300$

88. (1) Consider the equation

$x^2 + 2x + 2 = 0$

$$x = \frac{-2\pm\sqrt{4-8}}{2} = -1\pm i$$

Let $\alpha = -1 + i,\ \beta = -1 - i$

$$\alpha^{15} + \beta^{15} = (-1 + i)^{15} + (-1 - i)^{15}$$

$$= \left(\sqrt{2}e^{i\frac{3\pi}{4}}\right)^{15} + \left(\sqrt{2}e^{-i\frac{3\pi}{4}}\right)^{15}$$

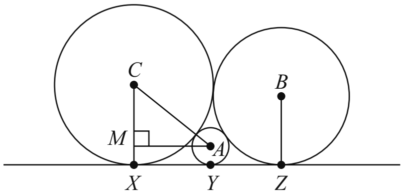
$$= \left(\sqrt{2}\right)^{15}\left[e^{\frac{i45\pi}{4}} + e^{\frac{-i45\pi}{4}}\right]$$

$$= \left(\sqrt{2}\right)^{15}.2\cos\frac{45\pi}{4} = \left(\sqrt{2}\right)^{15}.2\cos\frac{3\pi}{4}$$

$$= = \frac{-2}{\sqrt{2}}\left(\sqrt{2}\right)^{15}$$

$$= -2\left(\sqrt{2}\right)^{14} = -256$$

89. **(1)**



$$AM^2 = AC^2 - MC^2$$

$$= (a + c)^2 - (a - c)^2 = 4ac$$

$$\Rightarrow AM^2 = XY^2 = 4ac$$

$$\Rightarrow XY = 2\sqrt{ac}$$

Similarly, $YZ = 2\sqrt{ba}$ and $XZ = 2\sqrt{bc}$

Then, $XZ = XY + YZ$

$$\Rightarrow 2\sqrt{bc} = 2\sqrt{ac} + 2\sqrt{ba}$$

$$\Rightarrow \frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$$

90. **(4)** X = number of aces drawn

$\therefore$ $P(X = 1) + P(X = 2)$

$$= \left\{\frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52}\right\} + \left\{\frac{4}{52} \times \frac{4}{52}\right\}$$

$$= \frac{24}{169} + \frac{1}{169} = \frac{25}{169}$$

JEE Main - 2019

9 APRIL 2019 (MORNING SHIFT)

Time : 3 Hours • **Each correct answer has + 4 marks • Each wrong answer has – 1 mark.** **Max. Marks : 360**

Section - 1

PHYSICS

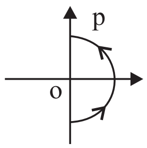
1. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is:

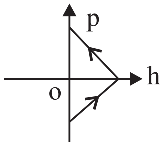
(1) 0.01 kg/m^3 (2) 0.10 kg/m^3
(3) 0.31 kg/m^3 (4) 0.07 kg/m^3

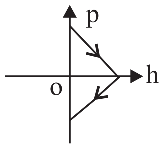
2. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?

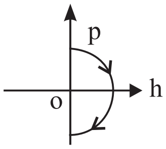
(1) 90° (2) 150° (3) 120° (4) 60°

3. A ball is thrown vertically up (taken as + z-axis) from the ground. The correct momentum-height (p-h) diagram is:

(1)


(2)


(3)


(4)


4. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be:

(1) $\frac{MgL}{2n^2}$ (2) $\frac{MgL}{n^2}$

(3) $\frac{2MgL}{n^2}$ (4) nMgL

5. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body?

(1) 1.0 kg (2) 1.5 kg
(3) 1.8 kg (4) 1.2 kg

6. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as kθ^2. If its moment of inertia is I then the angular acceleration of the disc is

(1) $\frac{k}{4I}\theta$ (2) $\frac{k}{I}\theta$ (3) $\frac{k}{2I}\theta$ (4) $\frac{2k}{I}\theta$

7. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is :

(1) 4 : 3 : 2 (2) 10 : 15 : 7
(3) 14 : 15 : 20 (4) 2 : 3 : 4

8. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be:

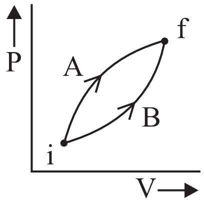
(1) $\frac{2GM}{9a^2}$ (2) $\frac{GM}{9a^2}$

(3) $\frac{GM}{3a^2}$ (4) $\frac{2GM}{3a^2}$

9. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is :

(1) M (2) $\frac{M}{2}$ (3) 4 M (4) 2 M

10. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then:



(1) $\Delta Q_A < \Delta Q_B, \Delta U_A < \Delta U_B$
(2) $\Delta Q_A > \Delta Q_B, \Delta U_A > \Delta U_B$
(3) $\Delta Q_A > \Delta Q_B, \Delta U_A = \Delta U_B$
(4) $\Delta Q_A = \Delta Q_B; \Delta U_A = \Delta U_B$

11. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be:

(1) 100 m/s (2) $80\sqrt{5}$ m/s

(3) $100\sqrt{5}$ m/s (4) 80 m/s

12. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be:

(1) $\dfrac{m\bar{v}^2}{6k_B}$ (2) $\dfrac{m\bar{v}^2}{3k_B}$

(3) $\dfrac{m\bar{v}^2}{7k_B}$ (4) $\dfrac{m\bar{v}^2}{5k_B}$

13. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(1) $2T\sqrt{\frac{1}{10}}$ (2) $2T\sqrt{\frac{1}{14}}$

(3) $4T\sqrt{\frac{1}{15}}$ (4) $4T\sqrt{\frac{1}{14}}$

14. The pressure wave,

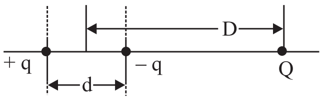
P = 0.01 sin[1000t – 3x] Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}. Approximate value of T is :

(1) 4°C (2) 11°C (3) 12°C (4) 15°C

15. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is Y = 0.3 sin(0.157x) cos(200πt). The length of the string is: (All quantities are in SI units.)

(1) 20 m (2) 80 m (3) 40 m (4) 60 m

16. A system of three charges are placed as shown in the figure:

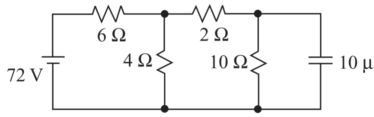


If D >> d, the potential energy of the system is best given by

(1) $\dfrac{1}{4\pi\epsilon_0}\left[\dfrac{-q^2}{d}\dfrac{-qQd}{2D^2}\right]$ (2) $\dfrac{1}{4\pi\epsilon_0}\left[\dfrac{-q^2}{d}+\dfrac{2qQd}{D^2}\right]$

(3) $\dfrac{1}{4\pi\epsilon_0}\left[+\dfrac{q^2}{d}+\dfrac{qQd}{D^2}\right]$ (4) $\dfrac{1}{4\pi\epsilon_0}\left[-\dfrac{q^2}{d}-\dfrac{qQd}{D^2}\right]$

17. Determine the charge on the capacitor in the following circuit:



(1) 60 μC (2) 2 μC

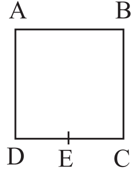
(3) 10 μC (4) 200 μC

18. A capacitor with capacitance 5 μF is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 μF, how much work is done?

(1) 6.25×10^{-6} J (2) 3.75×10^{-6} J

(3) 2.16×10^{-6} J (4) 2.55×10^{-6} J

19. A wire of resistance R is bent to form a square ABCD as shown in the figure. The effective resistance between E and C is: (E is mid-point of arm CD)



(1) R (2) $\frac{7}{64}$R

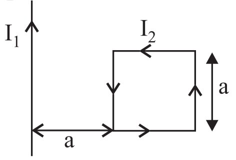
(3) $\frac{3}{4}$R (4) $\frac{1}{16}$R

20. A rectangular coil (Dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction, is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is:

(1) 0.38 Nm (2) 0.55 Nm

(3) 0.42 Nm (4) 0.27 Nm

21. A rigid square of loop of side 'a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be:



(1) Repulsive and equal to $\dfrac{I_1I_2}{}$

(2) Attractive and equal to $\dfrac{\mu_o I_1 I_2}{3\pi}$

(3) Repulsive and equal to $\dfrac{\mu_o I_1 I_2}{4\pi}$

(4) Zero

22. A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 k Ω resistance. The maximum voltage, that can be measured using this voltmeter, will be close to:

(1) 40 V (2) 15 V (3) 20 V (4) 10 V

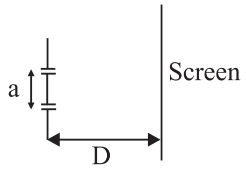
23. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to:
(1) L (2) L^2 (3) $1/L^2$ (4) 1/L

24. The magnetic field of a plane electromagnetic wave is given by:
$\vec{B} = B_0\,\hat{i}[\cos(kz-\omega t)] + B_1\,\hat{j}\cos(kz+\omega t)$
Where $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T.
The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at z = 0 is closest to:
(1) 0.6 N (2) 0.1 N
(3) 0.9 N (4) 3×10^{-2} N

25. A signal Acosωt is transmitted using v_0 sin ω modulated (AM) signal is:
(1) $v_0 \sin\omega_0\ t + \frac{A}{2}\sin (\omega_0 - \omega)t + \frac{A}{2}(\omega_0 + \omega)\ t$
(2) $v_0 \sin[\omega_0(1 + 0.01\ A\sin\omega t)t]$
(3) $v_0 \sin\omega_0 t + A\cos\omega t$
(4) $(v_0 + A) \cos\omega t \sin\omega_0 t$

26. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is:
(1) 0.24 m (2) 1.60 m
(3) 0.32 m (4) 0.16 m

27. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be:



(1) $\frac{2nD\lambda}{a(\mu-1)}$ (2) $\frac{nD\lambda}{a(\mu-1)}$
(3) $\frac{D\lambda}{a(\mu-1)}$ (4) $\frac{2D\lambda}{a(\mu-1)}$

28. The electric field of light wave is given as
$\vec{E} = 10^3 \cos$
$\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi \times 6\times10^{14} t\right)\hat{x}\frac{N}{C}$
This light falls on a metal plate of work function 2eV. The stopping potential of the photo-electrons is:
Given, E (in eV) = $\frac{12375}{\lambda\,(\text{in Å})}$
(1) 2.0 V (2) 0.72 V
(3) 0.48 V (4) 2.48 V

29. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2^{nd} Balmer line (n = 4 to n = 2) will be;
(1) 889.2 nm (2) 488.9 nm
(3) 642.7 nm (4) 388.9 nm

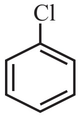
30. An NPN transistor is used in common emitter configuration as an amplifier with 1 k Ω load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15 μA change in the base current of the amplifier. The input resistance and voltage gain are:
(1) 0.33 kΩ, 1.5 (2) 0.67 kΩ, 300
(3) 0.67 kΩ, 200 (4) 0.33 kΩ, 300

Section - 2

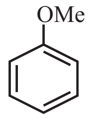
CHEMISTRY

31. The element having greatest difference between its first and second ionization energies, is:
(1) Ca (2) Sc
(3) Ba (4) K

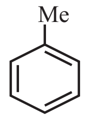
32. The increasing order of reactivity of the following compounds towards aromatic electrophilic substitution reaction is:



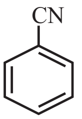
A



B



C



D

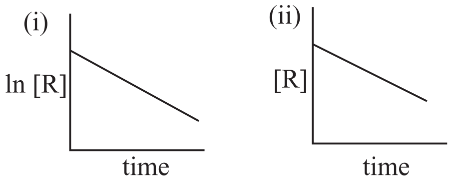
(1) D < A < C < B (2) B < C < A < D (3)
A < B < C < D (4) D < B < A < C

33. Consider the van der Waals constants, a and b, for the following gases,

Gas	Ar	Ne	Kr	Xe
a/ (atm dm^6 mol^{-2})	1.3	0.2	5.1	4.1
b/ (10^{-2} dm^3 mol^{-1})	3.2	1.7	1.0	5.0

Which gas is expected to have the highest critical temperature?
(1) Kr (2) Ne (3) Xe (4) Ar

34. The given plots represents the variation of the concentration of a reactant R with time for two different reactions (i) and (ii). The respective orders of the reactions are:



(1) 1, 0 (2) 1, 1
(3) 0, 1 (4) 0, 2

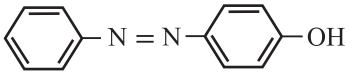
35. Among the following, the set of parameters that represents path functions, is:
(A) q + w (B) q
(C) w (D) H – TS

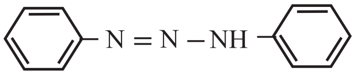
(1) (B) and (C) (2) (B), (C) and (D)
(3) (A) and (D) (4) (A), (B) and (C)

36. The ore that contains the metal in the form of fluoride is:
(1) cryolite (2) malachite
(3) magnetite (4) sphalerite

37. Excessive release of CO_2 into the atmosphere results in:
(1) global warming
(2) polar vortex
(3) formation of smog
(4) depletion of ozone

38. Aniline dissolved in dilute HCl is reacted with sodium nitrate at 0°C. This solution was added dropwise to a solution containing equimolar mixture of aniline and phenol in dil. HCl. The structure of the major product is:

(1)


(2)


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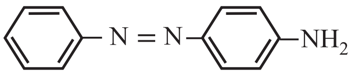
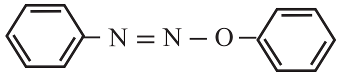


(4)


39. Among the following, the molecule expected to be stabilized by anion formation is:
C_2, O_2, NO, F_2
(1) C_2 (2) F_2 (3) NO (4) O_2

40. The correct order of the oxidation states of nitrogen in NO, N_2O, NO_2 and N_2O_3 is:
(1) $NO_2 < NO < N_2O_3 < N_2O$
(2) $NO_2 < N_2O_3 < NO < N_2O$
(3) $N_2O < N_2O_3 < NO < NO_2$
(4) $N_2O < NO < N_2O_3 < NO_2$

41. Liquid 'M' and liquid 'N' form an ideal solution. The vapour pressures of pure liquids 'M' and 'N' are 450 and 700 mmHg, respectively, at the same temperature. Then correct statement is:
(x_M = Mole fraction of 'M' in solution;
x_N = Mole fraction of 'N' in solution;
y_M = Mole fraction of 'M' in vapour phase;
y_N = Mole fraction of 'N' in vapour phase)

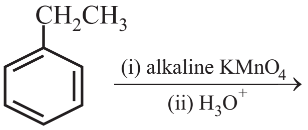
(1) $\frac{x_M}{x_N} = \frac{y_M}{y_N}$ (2) $(x_M - y_M) < (x_N - y_N)$
(3) $\frac{x_M}{x_N} < \frac{y_M}{y_N}$ (4) $\frac{x_M}{x_N} > \frac{y_M}{y_N}$

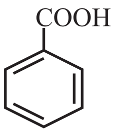
42. The osmotic pressure of a dilute solution of an ionic compound XY in water is four times that of a solution of 0.01 M $BaCl_2$ in water. Assuming complete dissociation of the given ionic compounds in water, the concentration of XY (in mol L^{-1}) in solution is:
(1) 4×10^{-2} (2) 6×10^{-2}
(3) 4×10^{-4} (4) 16×10^{-4}

43. The number of water molecules(s) not coordinated to copper ion directly in $CuSO_4 \cdot 5H_2O$, is:
(1) 2 (2) 3 (3) 1 (4) 4

44. The standard Gibbs energy for the given cell reaction in kJ mol^{-1} at 298 K is:
$Zn(s) + Cu^{2+}(aq) \rightarrow Zn^{2+}(aq) + Cu(s)$,
$E^{\circ} = 2$ V at 298 K
(Faraday's constant, F = 96000 C mol^{-1})
(1) – 384 (2) 384 (3) 192 (4) – 192

45. The major product of the following reaction is:



(1)


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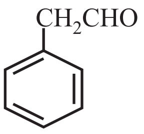
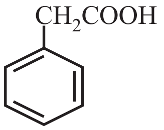


(3)


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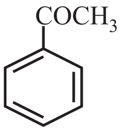
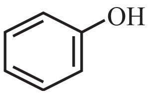
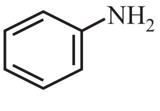


46. For any given series of spectral lines of atomic hydrogen, let $\Delta\bar{v} = \bar{v}_{max} - \bar{v}_{min}$ be the difference in maximum and minimum frequencies in cm^{-1}. The ratio $\Delta\bar{v}_{Lyman} / \Delta\bar{v}_{Balmer}$ is :
(1) 4 : 1 (2) 9 : 4
(3) 5 : 4 (4) 27 : 5

47. The organic compound that gives following qualitative analysis is:

	Test	**Inference**
(a)	Dil. HCl	Insoluble
(b)	NaOH solution	soluble
(c)	Br_2/water	Decolourization

(1)


(2)


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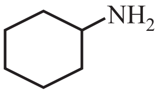

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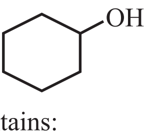
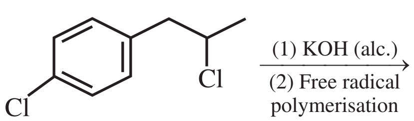
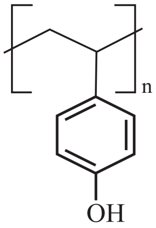
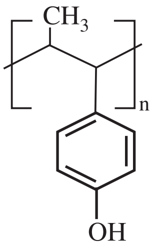


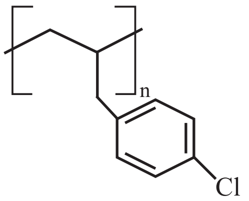
48. C_{60}, an allotrope of carbon cantains:
(1) 12 hexagons and 20 pentagons.
(2) 18 hexagons and 14 pentagons.
(3) 16 hexagons and 16 pentagons.
(4) 20 hexagons and 12 pentagons.

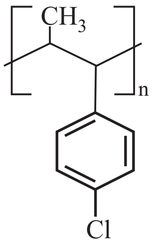
49. The major product of the following reaction is:



(1) 

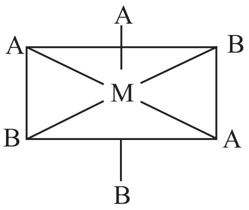
(2) 

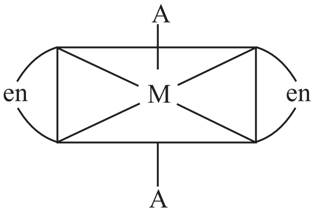
(3) 

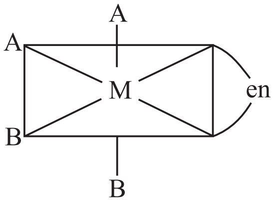
(4) 

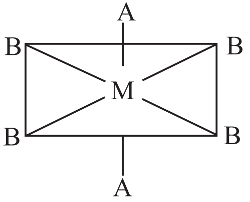
50. The one that will show optical activity is:

(en = ethane 1, 2-diamine)

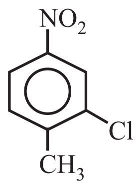
(1) 

(2) 

(3) 

(4) 

51. The correct IUPAC name of the following compound is:



(1) 5-chloro-4-methyl-1-nitrobenzene

(2) 2-chloro-1-methyl-4-nitrobenzene

(3) 3-chloro-4-methyl-1-nitrobenzene

(4) 2-methyl-5-nitro-1-chlorobenzene

52. Match the catalysts **(Column I)** with products **(Column II)**.

Column I		**Column II**
Catalyst		**Product**
(A) V_2O_5	(i)	Polyethylene
(B) $TiCl_4/\ Al(Me)_3$	(ii)	ethanol
(C) $PdCl_2$	(iii)	H_2SO_4
(D) Iron Oxide	(iv)	NH_3

(1) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii)

(2) (A)-(ii); (B)-(iii); (C)-(i); (D)-(iv)

(3) (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)

(4) (A)-(iv); (B)-(iii); (C)-(ii); (D)-(i)

53. Which of the following statements is not true about sucrose?

(1) It is a non reducing sugar.

(2) The glycosidic linkage is present between C_1 of α-glucose and C_1 of β-fructose.

(3) It is also named as invert sugar.

(4) On hydrolysis, it produces glucose and fructose.

54. Magnesium powder burns in air to give:

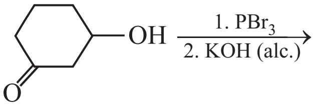
(1) $Mg(NO_3)_2$ and Mg_3N_2

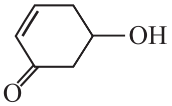
(2) MgO and Mg_3N_2

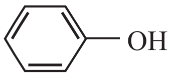
(3) MgO only

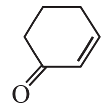
(4) MgO and $Mg(NO_3)_2$

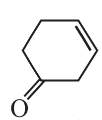
55. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

56. The major product of the following reaction is:

$CH_3C \equiv CH \xrightarrow[\text{(ii) DI}]{\text{(i) DCl (1equiv.)}}$

(1) $CH_3CD(I)CHD(Cl)$ (2) $CH_3CD(Cl)CHD(I)$

(3) $CH_3CD_2CH(Cl)(I)$ (4) $CH_3C(I)(Cl)CHD_2$

57. The major product of the following reaction is:

$CH_3CH = CHCO_2CH_3 \xrightarrow{LiAlH_4}$

(1) $CH_3CH_2CH_2CO_2CH_3$ (2) $CH_3CH = CHCH_2OH$

(3) $CH_3CH_2CH_2CH_2OH$ (4) $CH_3CH_2CH_2CHO$

58. The degenerate orbitals of $[Cr(H_2O)_6]^{3+}$ are:

(1) d_{xz} and d_{yz} (2) d_{yz} and dz^2

(3) d_{z^2} and d_{xz} (4) $d_{x^2-y^2}$ and d_{xy}

59. The aerosol is a kind of colloid in which:

(1) solid is dispersed in gas

(2) gas is dispersed in solid

(3) gas is dispersed in liquid

(4) liquid is dispersed in water

60. For a reaction,
$N_2(g) + 3H_2(g) \rightarrow 2\ NH_3(g)$; identify dihydrogen ($H_2$) as a limiting reagent in the following reaction mixtures.

(1) 56 g of N_2 + 10 g of H_2

(2) 35 g of N_2 + 8 g of H_2

(3) 28 g of N_2 + 6 g of H_2

(4) 14 g of N_2 + 4 g of H_2

Section - 3

MATHEMATICS

61. Slope of a line passing through P(2, 3) and intersecting the line x + y = 7 at a distance of 4 units from P, is:

(1) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ (2) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

(3) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (4) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

62. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to:

(1) $2\sqrt{6}$ (2) $2\sqrt{\frac{10}{3}}$

(3) $4\sqrt{\frac{5}{3}}$ (4) $\sqrt{6}$

63. If f(x) is a non-zero polynomial of degree four, having local extreme points at x = –1, 0, 1; then the set
S = {x ∈ R : f(x) = f(0)}
contains exactly:
(1) four irrational numbers.
(2) four rational numbers.
(3) two irrational and two rational numbers.
(4) two irrational and one rational number.

64. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx$ is equal to:

(1) $-3 \tan^{-1/3} x + C$ (2) $-\frac{3}{4} \tan^{-4/3} x + C$

(3) $-3 \cot^{-1/3} x + C$ (4) $3 \tan^{-1/3} x + C$

(Here C is a constant of integration)

65. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is:

(1) $\frac{25}{192}$ (2) $\frac{7}{32}$

(3) $\frac{1}{192}$ (4) $\frac{25}{32}$

66. If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, x + 2y + 3z = 15 at a point P, then the distance of P from the origin is:

(1) $\sqrt{5}/2$ (2) $2\sqrt{5}$ (3) 9/2 (4) 7/2

67. If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, –5) is perpendicular to the line, – x + y + 4 = 0, then which one of the following points lies on the curve?

(1) (–2, 1) (2) (–2, 2)

(3) (2, –1) (4) (2, –2)

68. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is:

(1) $\frac{\pi-2}{8}$ (2) $\frac{\pi-1}{4}$

(3) $\frac{\pi-2}{4}$ (4) $\frac{\pi-1}{2}$

69. The value of
$\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is :

(1) $\frac{3}{4} + \cos 20°$ (2) 3/4

(3) $\frac{3}{2}(1 + \cos 20°)$ (4) 3/2

70. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is :

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{\sqrt{15}}{2}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{3}{\sqrt{5}}$

71. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ (x ≠ 0) with y(1) = 1, is:

(1) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$ (2) $y = \frac{x^3}{5} + \frac{1}{5x^2}$

(3) $y = \frac{x^2}{4} + \frac{3}{4x^2}$ (4) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$

72. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is:

(1) $\sim p \wedge \sim q$ (2) $p \wedge q$

(3) $p \leftrightarrow q$ (4) $\sim p \vee \sim q$

73. All the points in the set
$S = \left\{\frac{\alpha + i}{\alpha - i} : \alpha \in R\right\} (i = \sqrt{-1})$ lie on a:

(1) straight line whose slope is 1.

(2) circle whose radius is 1.

(3) circle whose radius is $\sqrt{2}$.

(4) straight line whose slope is –1.

74. If the fourth term in the Binomial expansion of $\left(\frac{2}{x} + x^{\log_8 x}\right)^6$ (x > 0) is 20×8^7, then a value of x is:

(1) 8^3 (2) 8^2 (3) 8 (4) 8^{-2}

75. If the function f defined on $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ by

$$f(x)=\begin{cases}\dfrac{\sqrt{2}\cos x-1}{\cot x-1}, & x\neq\dfrac{\pi}{4}\\ k, & x=\dfrac{\pi}{4}\end{cases}$$

is continuous, then k is equal to:

(1) 2 (2) $\frac{1}{2}$ (3) 1 (4) $\frac{1}{\sqrt{2}}$

76. If the function $f : R - \{1, -1\} \to A$ defined by $f(x)=\frac{x^2}{1-x^2}$, is surjective, then A is equal to:

(1) $R - \{-1\}$ (2) $[0, \infty)$

(3) $R - [-1, 0)$ (4) $R - (-1, 0)$

77. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$, also passes through the point:

(1) $(-\sqrt{2}, 1, -4)$ (2) $(\sqrt{2}, -1, 4)$

(3) $(-\sqrt{2}, -1, -4)$ (4) $(\sqrt{2}, 1, 4)$

78. Let the sum of the first n terms of a non-constant A.P., a_1, a_2, a_3, ………….. be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to:

(1) (50, 50 + 46A) (2) (50, 50 + 45A)

(3) (A, 50 + 45A) (4) (A, 50 + 46A)

79. Let $S = \{\theta \epsilon [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$.

Then the sum of the elements of S is:

(1) $\frac{13\pi}{6}$ (2) $\frac{5\pi}{3}$ (3) 2π (4) π

80. Let p, q ϵ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then:

(1) $p^2 - 4q + 12 = 0$ (2) $q^2 - 4p - 16 = 0$

(3) $q^2 + 4p + 14 = 0$ (4) $p^2 - 4q - 12 = 0$

81. Let $f(x) = 15 - |x - 10|$; x ϵ R. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is:

(1) {5, 10, 15} (2) {10, 15}

(3) {5, 10, 15, 20} (4) {10}

82. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, f(1)) and (– 1, f(– 1)), then S is equal to:

(1) $\left\{\frac{1}{3}, 1\right\}$ (2) $\left\{-\frac{1}{3}, -1\right\}$

(3) $\left\{\frac{1}{3}, -1\right\}$ (4) $\left\{-\frac{1}{3}, 1\right\}$

83. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is:

(1) $x^2 + y^2 - 4x^2y^2 = 0$ (2) $x^2 + y^2 - 2xy = 0$

(3) $x^2 + y^2 - 16x^2y^2 = 0$ (4) $x^2 + y^2 - 2x^2y^2 = 0$

84. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$.

If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to:

(1) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (2) $3\hat{i} - 9\hat{j} - 5\hat{k}$

(3) $\frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$ (4) $\frac{1}{2}\left(3\hat{i} - 9\hat{j} + 5\hat{k}\right)$

85. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is:

(1) $\frac{10}{3}$ (2) $\frac{9}{2}$ (3) $\frac{31}{6}$ (4) $\frac{13}{6}$

86. If

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\cdots\cdots\cdots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix},$$

then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is:

(1) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

(3) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$

87. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$, where the function f satisfies $f(x + y) = f(x) f(y)$ for all natural numbers x, y and $f(1) = 2$. Then the natural number 'a' is:

(1) 2 (2) 16 (3) 4 (4) 3

88. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then:

(1) m + n = 68 (2) m = n = 78

(3) n = m – 8 (4) m = n = 68

89. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R,

$$\begin{vmatrix}y+1 & \alpha & \beta\\ \alpha & y+\beta & 1\\ \beta & 1 & y+\alpha\end{vmatrix}$$ is equal to:

(1) $y(y^2 - 1)$ (2) $y(y^2 - 3)$

(3) y^3 (4) $y^3 - 1$

90. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is:

(1) 25 (2) 22 (3) 24 (4) 20

Hints and Solutions

SECTION-1 : PHYSICS

1. **(Bonus)** $d=\frac{M}{V}=\frac{M}{L^3}=ML^{-3}$

$$\frac{\Delta d}{d}=\frac{\Delta M}{M}+3\frac{\Delta L}{L}$$

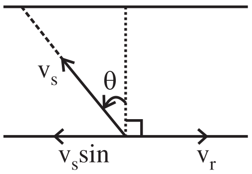
$$=\frac{0.10}{10.00}+3\left(\frac{0.01}{0.10}\right)=0.31\text{kgm}^{-3}$$

2. **(3)** To cross the river straight

$V_s\sin\theta=V_r\ \therefore\ \sin\theta=\frac{v_r}{v_s}=\frac{2}{4}=\frac{1}{2}$

$\therefore\ \theta=30°$

Direction of swimmer with respect to flow = 90° + 30° = 120°

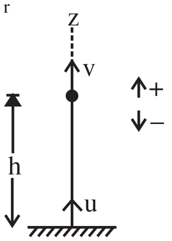


3. **(4)** $v^2=u^2-2gh$

or $v=\sqrt{u^2-2gh}$

Momentum, p = mv

$\therefore\ p=m\sqrt{u^2-2gh}$

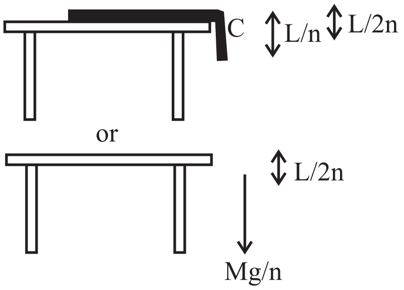


Therefore graph between p and h cannot have straight line.

(2) and (3) are not possible.

During upward journey as h increases, p decreases and in downward journey as h decreases p increases. Therefore 4 is the correct option.

4. **(2)** Length of hanging part = L/n
Mass of hanging part = M/n
Weight of hanging part = Mg/n
Let 'C' be the centre of mass of the hanging part.



The hanging part can be assumed to be a particle of weight Mg/n at a distance L/n below the table top. The work done in lifting it to the table top is equal to increase in its potential energy.

$$\therefore\ W=\left(\frac{Mg}{n}\right)\left(\frac{L}{n}\right)$$

$$\therefore\ W=\frac{MgL}{n^2}$$

5. **(4)** For head on elastic collision we have

$$V_1=\frac{(m_1-m_2)u_1}{m_1+m_2}+\frac{2m_2u_2}{m_1+m_2}$$

Here m_1 = 2kg, u_1 = x, u_2 = 0,
v_1 = x/4

$$\therefore\ \frac{x}{4}=\frac{(2-m_2)x}{2+m_2}\Rightarrow m_2=1.2\text{kg}$$

6. **(4)** Work done by torque is responsible for change in kinetic energy.

$$\therefore\ \tau=\frac{dE}{d\theta}\ \therefore I\alpha=2K\theta\ \therefore\ \alpha=\frac{2k\theta}{I}$$

7. **(3)** $mgh=\frac{1}{2}mv_{cm}^2+\frac{1}{2}I_{cm}\omega^2$

$$=\frac{1}{2}mv_{cm}^2+\frac{1}{2}I_{cm}\left(\frac{v_{cm}}{R}\right)^2$$

$$=\frac{1}{2}\left(m+\frac{I_{cm}}{R^2}\right)v_{cm}^2=\frac{1m}{2}\left[1+\frac{K^2}{R^2}\right]$$

$\therefore h\propto 1+\frac{K^2}{R^2}$

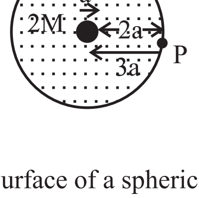
For ring : $h\propto 2$ ($\because$ K = R)

For solid cylisnder, $h\propto\frac{3}{2}$ $\left(\because K=\frac{R}{\sqrt{2}}\right)$

For solid sphere, $h\propto\frac{7}{5}$ $\left(\because K=\sqrt{\frac{2}{5}}R\right)$

Ratio of heights $2:\frac{3}{2}:\frac{7}{5}\Rightarrow 20:15:14$

8. **(3)** $E_P=\frac{GM}{(3a)^2}+\frac{G(2M)}{(3a)^2}=\frac{GM}{3a^2}$



For a part on the surface of a spherical uniform charge distribution the whole mass acts as a point mass kept at the centre.

9. **(4)** We have, $h = \frac{2T\cos\theta}{r\rho g}$

Mass of the water in the capillary

$m = \rho V = \rho \times \pi r^2 h = \rho \times \pi r^2 \times \frac{2T\cos\theta}{r\rho g}$

$\Rightarrow m \propto r$

$\therefore \frac{m_1}{m_2} = \frac{r}{2r}$

or, $m_2 = 2m_1 = 2m$

10. **(3)** Internal energy depends only on initial and final state

So, $\Delta U_A = \Delta U_B$

Also $\Delta Q = \Delta U + \Delta W$

$\therefore W_A > W_B \Rightarrow \Delta Q_A > \Delta Q_B$

[Area under P-V graph gives the work done.]

11. **(3)** $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{(273+127)}{(273+227)}}$$

$$= \sqrt{\frac{400}{500}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\therefore v_2 = \frac{\sqrt{5}}{2}v_1 = \frac{\sqrt{5}}{2}\times 200 = 100\sqrt{5}\text{ m/s.}$$

12. **(1)** In this case the total degree of freedom is 6.

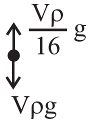
According to law of equipartition of energy,

$$\frac{1}{2}mv^{-2} = 6\left(\frac{1}{2}k_BT\right)$$

$$\therefore \frac{1}{2}mv^{-2} = 3k_BT$$

or, $T = \frac{mv^{-2}}{6k_B}$

13. **(3)** $T = 2\pi\sqrt{\frac{l}{g}}$



$$V\rho\, g_{eff} = V\rho g - \frac{V\rho}{16}g$$

$$g_{eff} = \left(g - \frac{g}{16}\right) = \frac{15g}{16}$$

Now $T' = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{\frac{15g}{16}}} = \frac{4}{\sqrt{15}}T$

14. **(1)** On comparing with $P = P_0 \sin(\omega t - kx)$, we have

$\omega = 1000$ rad/s, $K = 3\text{ m}^{-1}$

$$\therefore v_0 = \frac{\omega}{k} = \frac{1000}{3} = 333.3\text{m/s}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

or, $\frac{333.3}{336} = \sqrt{\frac{273+0}{273+t}}$ $\therefore t = 4°C$

15. **(2)** Given, $y = 0.3 \sin(0.157\,x)\cos(200\,\pi t)$

So, $k = 0.157$ and $\omega = 200\pi = 2\pi f$

$\therefore f = 100$ Hz and $v = \frac{\omega}{k} = \frac{200\pi}{0.157} = 4000$m/s

Now, using $f = \frac{nv}{2l} = \frac{4v}{2l} = \frac{2v}{l}$ [here n = 4]

$$\therefore l = \frac{2v}{f} = \frac{2\times 4000}{100} = 80m$$

16. **(4)** $U = \frac{1}{4\pi\in_0}\left[\frac{q(-q)}{d} + \frac{qQ}{\left(D+\frac{d}{2}\right)} + \frac{(-q)Q}{\left(D-\frac{d}{2}\right)}\right]$

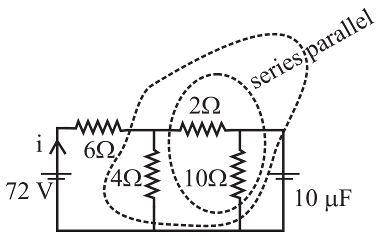
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-q^2}{d} + \frac{qQ\left(D-\frac{d}{2}\right) - qQ\left(D+\frac{d}{2}\right)}{D^2 - \frac{d^2}{4}}\right]$$

$$= \frac{1}{4\pi\in_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right], \because \frac{d^2}{4} << D$$

17. **(4)** At steady state, there is no current in capacitor.

2Ω and 10Ω are in series. There equivalent resistance is 12Ω. This 12Ω is in parallel with 4Ω and there combined resistance is 12 × 4/(12 + 4). This resistance is in series with 6Ω. Therefore, current drawn from battery

$$i = \frac{V}{R} = \left(\frac{72}{6+\frac{12\times 4}{12+4}}\right) = 8A$$



Current in 10Ω resistor

$$i' = \left(\frac{4}{4+12}\right)8 = 2A$$

Pd across capacitor, $V = i'R = 2 \times 10 = 20V$

$\therefore$ Charge on the capacitor, $q = CV$

$= 10 \times 20 = 200\ \mu C$.

18. **(2)** $W = U_f - U_i = \frac{q^2}{2}\left(\frac{1}{C_f}-\frac{1}{C_i}\right) \quad \left(\because U=\frac{q^2}{2C}\right)$

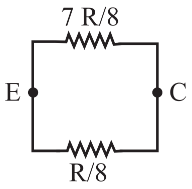
$= \frac{(5\times10^{-6})^2}{2}\left(\frac{1}{2}-\frac{1}{5}\right)\times10^6$

$= 3.75 \times 10^{-6}$J

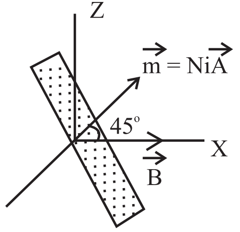
19. **(2)** Here $R_{DA} = R_{AB} = R_{BC} = R/4$
and $R_{DE} = R_{EC} = R/8$
Now $R_{ED,}$ $R_{DA,}$ $R_{AB,}$ R_{BC} are in series.

$$\therefore R_s = \frac{R}{8}+\frac{R}{4}+\frac{R}{4}+\frac{R}{4} = \frac{R+2R+2R+2R}{8} = \frac{7R}{8}$$

$$\therefore R_{eq} = \frac{\left(\frac{7R}{8}\right)\left(\frac{R}{8}\right)}{R} = \frac{7R}{64}$$



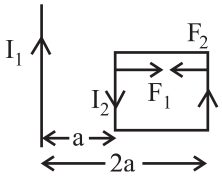
20. **(4)** τ = mB sin45° = N (iA) B sin 45°



$$= 100\times3(5\times2.5)\times10^{-4}\times1\times\frac{1}{\sqrt{2}} = 0.27 \text{ Nm}$$

21. **(3)** $F = F_1 - F_2 = \frac{\mu_0}{2\pi}\left(\frac{I_1I_2}{a}-\frac{I_1I_2}{2a}\right)\times a$

$= \frac{\mu_0 i_1 i_2}{4\pi}$ (Repulsive)



22. **(3)** $V = i_g (G + R) = 4 \times 10^{-3}$
(50 + 5000) = 20V

23. **(4)** Inductance = $\frac{\mu_0 N^2 A}{L}$

24. **(1)** $B_0 = \sqrt{B_0^2 + B_1^2} = \sqrt{30^2+2^2}\times10^{-6}$

$\approx 30 \times 10^{-6}$T

$\therefore E_0 = cB = 3\times10^8\times30\times10^{-6}$

$= 9 \times 10^3$ V/m

$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{9}{\sqrt{2}}\times10^3 V/m$

Force on the charge,

$F = E_{rms}Q = \frac{9}{\sqrt{2}}\times10^3\times10^{-4} \simeq 0.64N$

25. **(1)** The equation of amplitude modulated wave

$m = (v_0 + A\cos\omega t)\sin\omega t$

$= v_0\sin\omega_0 t + A\cos\omega t \sin\omega_0 t$

$= v_0\sin\omega_0 t + \frac{A}{2}[\sin(\omega_0-\omega)t + \sin(\omega_0+\omega)t]$

26. **(3)** $+5 = -\frac{v}{u} \Rightarrow v = -5u$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{-5u}+\frac{1}{u}=\frac{-1}{0.4}$

∴ u = – 0.32 m.

27. **(Bonus)** Shift = nβ (given)

$\therefore D\frac{(\mu-1)t}{a} = \frac{n\lambda D}{a} \quad \left[\because \text{Shift} = \frac{D(\mu-1)t}{a}\right]$

or $t = \frac{n\lambda}{(\mu-1)}$

28. **(3)** Here $\omega = 2\pi \times 6 \times 10^{14}$

$\Rightarrow f = 6 \times 10^{14}$ Hz

Wavelength

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^{14}} = 0.5\times10^{-6} m = 5000\text{Å}$$

Given $E = \frac{12375}{5000} = 2.48 eV$

Using $E = W + eV_s$
$\Rightarrow 2.48 = 2 + eV_s$
or $V_s = 0.48$ V

29. **(2)** $\frac{1}{\lambda_1} = R\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = \frac{5R}{36}$

$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = \frac{3R}{16}$

$\therefore \frac{\lambda_2}{\lambda_1} = \frac{80}{108}$

$\lambda_2 = \frac{80}{108}\lambda_1 = \frac{80}{108}\times660 = 488.9\text{nm}.$

30. **(2)** Given $\Delta V_i = 10 \times 10^{-3} V$

$\Delta I_c = 3 \times 10^{-3}$A

$\Delta I_b = 15 \times 10^{-6}$A

$R_i = \frac{\Delta V_i}{\Delta I_b} = \frac{10\times10^{-3}}{15\times10^{-6}} = 0.67k\Omega$

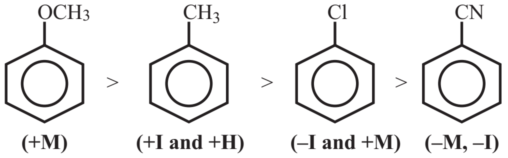
$\therefore$ Voltage gain $= \frac{\Delta I_c}{\Delta I_b}\times\frac{R_0}{R_i}$

$= \left(\frac{3\times10^{-3}}{15\times10^{-6}}\right)\times\frac{1000}{670} \quad = 200\times\frac{1000}{670} \approx 300$

SECTION-2 : CHEMISTRY

31. **(4)** Alkali metals have high difference in the first and second ionisation energy as they achieve stable noble gas configuration after first ionisation.

32. **(1)**



33. **(1)** Critical temperature $= \frac{8a}{27Rb}$

Value of $\frac{a}{b}$ is highest for Kr. Therefore, Kr has greatest value of critical temperature.

34. **(1)** In graph (i), ln [Reactant vs time is linear with positive intercept and negative slope. Hence it is 1st order In graph (ii), [Reactant] vs time is linear with positive intercept and negative slope. Hence, it is zero order.

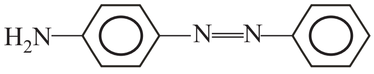
35. **(1)** We know that heat and work are not state functions but $q + w = \Delta U$ is a state function. $H - TS$ (i.e. G) is also a state function.

36. **(1)**

Magnetite	Fe_3O_4
Sphalerite	ZnS
Cryolite	Na_3AlF_6
Malachite	$CuCO_3.Cu(OH)_2$

37. **(1)** Global warming is caused by the emission of green house gases. 72% of the totally emitted green house gases is CO_2. Therefore, excessive release of CO_2 is the main cause of global warming.

38. **(3)** In acidic medium aniline is more reactive than phenol that's why electrophilic aromatic substitution of $Ph - \overset{+}{N_2}$ takes place with aniline.



39. **(1)** Configuration of C_2

$= \sigma 1s^2 \sigma^* 1s^2\, \sigma 2s^2 \sigma^* 2s^2\, \pi 2p_x^2 = \pi 2p_y^2$

Configuration of C_2^-

$= \sigma 1s^2\, \sigma^* 1s^2\, \sigma 2s^2\, \sigma^* 2s^2\, \pi 2p_x^2 = \pi 2p_y^2\ \ \sigma 2p_z^1$

Bond order

$$= \frac{\text{No.of bonding e}^- - \text{No.of antibonding e}^-}{2}$$

C_2 has *s-p* mixing and the HOMO is $\pi 2p_x = \pi 2p_y$ and LUMO is $\sigma 2p_z$. So, the extra electron will occupy bonding molecular orbital and this will lead to an increase in bond order.

C_2^- has more bond order than C_2.

40. **(4)**

(Oxide)	(Oxidation state)
N_2O	+1
NO	+2
N_2O_3	+3
NO_2	+4

So, $N_2O < NO < N_2O_3 < NO_2$

41. **(4)** $P_M^o = 450$ mmHg, $P_N^o = 700$ mmHg

$P_M = P_M^o x_M = y_M P_T$

$\Rightarrow P_M^\circ = \frac{y_M}{x_M}(P_T)$

Similarly, $P_N^o = \frac{y_N}{x_N}(P_T)$

Given, $P_M^o < P_N^\circ$

$\Rightarrow \frac{y_M}{x_M} < \frac{y_N}{x_N}$

$\Rightarrow \frac{y_M}{y_N} < \frac{x_M}{x_N}$

42. **(2)** We know, $\pi = iCRT$; $\pi_{xy} = 4\pi_{BaCl_2}$

$\therefore\ 2[XY] = 4 \times (0.01) \times 3$

$[XY] = 0.06$

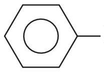
$= 6 \times 10^{-2}$ mol/L

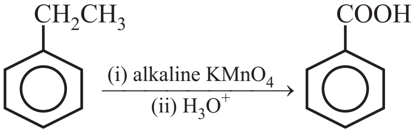
43. **(3)** In $CuSO_4 \cdot 5H_2O$, four H_2O molecules are directly coordinated to the central metal ion while one H_2O molecule is hydrogen bonded.

44. **(1)** $\Delta G^\circ = -nFE^\circ_{cell}$

$= -2 \times (96000) \times 2\ V = -384000$ J/mol

$= -384$ kJ/mol

45. **(1)** Alkaline $KMnO_4$ converts  —R with a benzylic hydrogen into benzoic acid.



46. **(2)** $\bar{v} \propto \Delta E$

For H-atom

$\bar{v} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For Lyman series,

$\bar{v}\,(\max) = 13.6\left(1 - \frac{1}{\infty}\right)$

$\bar{v}\,(\min) = 13.6\left(1 - \frac{1}{4}\right)$

$\therefore\ \bar{v}_{\max} - \bar{v}_{\min} = 13.6\left(\frac{1}{4}\right)$

For Balmer series,

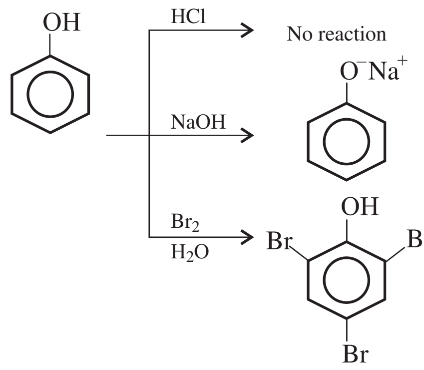
$\bar{v}\,(\max) = 13.6\left(\frac{1}{4} - \frac{1}{\infty}\right)$

$\bar{v}\,(\min) = 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$

$\therefore\ \bar{v}_{\max} - \bar{v}_{\min} = 13.6\left(\frac{1}{9}\right)$

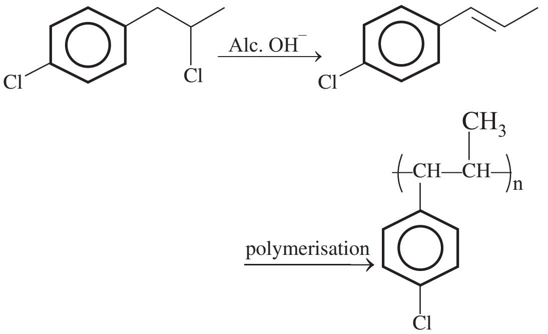
$\frac{\Delta\bar{v}_{Lyman}}{\Delta\bar{v}_{Balmer}} = \frac{9}{4}$

47. (1)

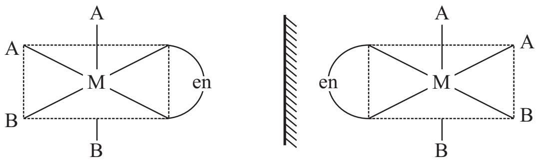


48. (4) Fullerene (C_{60}) contains 20 hexagons (six membered) rings and 12 pentagons (five membered rings):

49. (4)

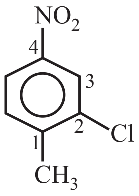


50. (3)



No plane of symmetry or centre of symmetry
Hence it is optically active.

51. (2)



2-Chloro-1-methyl-4-nitrobenzene

52. (3) (A) $V_2O_5 \rightarrow$ Preparation of H_2SO_4 in contact process

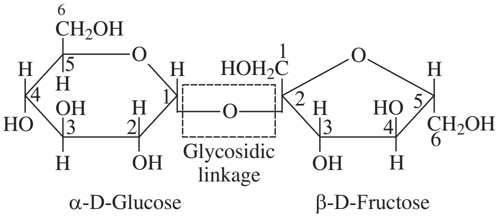
(B) $TiCl_4$+ $Al(Me)_3 \rightarrow$ Polyethylene (Ziegler-Natta catalyst)

(C) $PdCl_2 \rightarrow$ Ethanal (Wacker's process)

(D) Iron oxide $\rightarrow NH_3$ in (Haber's process)

53. (2) Sucrose contains glycosidic link between C_1 of α-D glucose and C_2 of β-D-fructose.

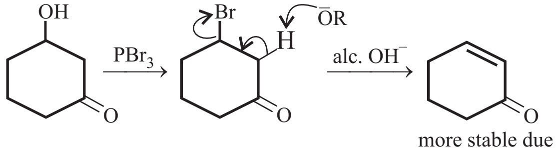
$$C_{12}H_{22}O_{11} + H_2O \longrightarrow \text{Glucose} + \text{Fructose}$$



Sucrose

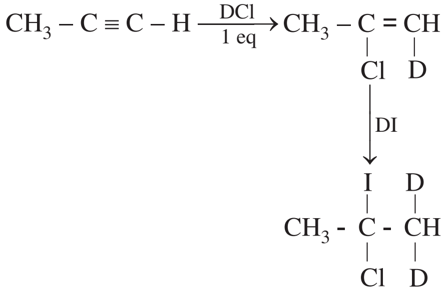
54. (2) Mg burns in air and produces a mixture of nitride and oxide.

55. (3)



more stable due to conjugation

56. (4)



$$CH_3-C\equiv C-H \xrightarrow[1\ eq]{DCl} CH_3-\underset{Cl}{\underset{|}{C}}=\underset{D}{\underset{|}{CH}}$$

$$\Big\downarrow DI$$

$$CH_3-\overset{I}{\overset{|}{\underset{Cl}{\underset{|}{C}}}}-\overset{D}{\overset{|}{\underset{D}{\underset{|}{CH}}}}$$

Both additions follow Markovnikov's rule.

57. (2) $LiAlH_4$ reduces esters to alcohols but does not reduce $C = C$.

$$CH_3CH = CH - CO_2Me \xrightarrow{LiAlH_4}$$

$$CH_3CH = CHCH_2OH$$

58. (1) Cr^{3+} has d^3 configuration and forms an octahedral inner orbitals complex.

The set of degenerate orbitals are

(d_{xy} , d_{yz} and d_{xz}) and ($d_{x^2-y^2}$ and d_{z^2}).

59. (1) In aerosol, the dispersion medium is gas while the dispersed phase can be both solid or liquid.

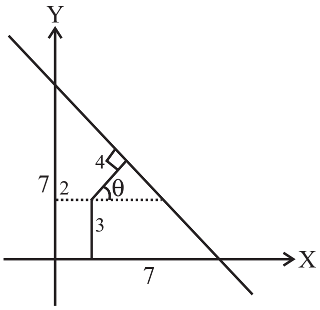
60. (1) According to the stoichiometry of balanced equation 28 g N_2 react with 6 g H_2

$$\underset{\substack{1\text{ mole}\\28\text{ g}}}{N_2} + \underset{\substack{3\text{ mole}\\6\text{ g}}}{3H_2} \longrightarrow 2NH_3$$

∴ For 56 g of N_2, 12 g of H_2 is required.

SECTION-3 : MATHEMATICS

61. (2)



Since point at 4 units from P (2, 3) will be A (4 cosθ + 2, 4 sin (θ + 3) and this point will satisfy the equation of line x + y = 7

$$\Rightarrow \cos\theta + \sin\theta = \frac{1}{2}$$

On squaring

$$\Rightarrow \sin 2\theta - \frac{3}{4} \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta} = -\frac{3}{4}$$

$\Rightarrow 3\tan^2\theta + 8\tan\theta + 3 = 0$

$$\Rightarrow \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6} \quad \text{(ignoring –ve sign)}$$

$$\Rightarrow \tan\theta = \frac{-8+2\sqrt{7}}{6} = \frac{1-\sqrt{7}}{1+\sqrt{7}}$$

62. (1) $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$$\Rightarrow \frac{k^2+2}{4} - \left(\frac{k}{4}\right)^2 = 5$$

$\Rightarrow 4k^2 + 8 - k^2 = 80$

$\Rightarrow 3k^2 = 72$

$\Rightarrow k = 2\sqrt{6}$

63. (4) Since, function f(x) have local extreem points at x = –1, 0, 1. Then

f(x) = K (x + 1) x (x – 1)

= K (x³ – x)

$$\Rightarrow f(x) = K\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + C$$

(using integration)

$\Rightarrow f(0) = C$

$$\because f(x) = f(0) \Rightarrow K\left(\frac{x^4}{4} - \frac{x^2}{2}\right) = 0$$

$$\Rightarrow \frac{x^2}{2}\left(\frac{x^2}{2} - 1\right) = 0 \Rightarrow x = 0, 0, \sqrt{2}, -\sqrt{2}$$

$$\therefore S = \{0, -\sqrt{2}, \sqrt{2}\}$$

64. (1) $I = \int \sec^{\frac{2}{3}} x.\text{cosec}^{\frac{4}{3}} dx$

$$I = \int \frac{\sec^2 x dx}{\tan^{\frac{4}{3}} x}$$

Put tan x = z

$\Rightarrow \sec^2 x\, dx = dz$

$$\Rightarrow I = \int z^{-\frac{4}{3}} \cdot dz = \frac{z^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)} + C$$

$$\Rightarrow I = -3(\tan x)^{\frac{-1}{3}} + C$$

65. (4) P (at least one hit the target) = 1 – P (none of them hit the target)

$$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{8}\right)$$

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{7}{8} = 1 - \frac{7}{32} = \frac{25}{32}$$

66. (3) Let point on line be P (2k + 1, 3k – 1, 4k + 2)

Since, point P lies on the plane x + 2y + 3z = 15

∴ 2k + 1 + 6k – 2 + 12k + 6 = 15

$$\Rightarrow k = \frac{1}{2} \qquad \therefore P \equiv \left(2, \frac{1}{2}, 4\right)$$

Then the distance of the point P from the origin is

$$OP = \sqrt{4 + \frac{1}{4} + 16} = \frac{9}{2}$$

67. (4) y = x³ + ax – b

Since, the point (1, –5) lies on the curve.

⇒ 1 + a – b = – 5

⇒ a – b = –6 ...(1)

$$\text{Now, } \frac{dy}{dx} = 3x^2 + a \Rightarrow \left(\frac{dy}{dx}\right)_{\text{at } x=1} = 3 + a$$

Since, required line is perpendicular to y = x – 4, then slope of tangent at the point P (1, –5) = –1

∴ 3 + a = – 1⇒ a = – 4 ⇒ b = 2

∴ the equation of the curve is y = x³ – 4x – 2

⇒ (2, –2) lies on the curve.

68. (2) Let $I = \int_0^{\pi/2} \frac{\sin^3 x dx}{\sin x + \cos x}$ ...(1)

Use the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^3 x\, dx}{\sin x + \cos x} \qquad ...(2)$$

Adding equations (1) and (2), we get

$$2I = \int_0^{\pi/2}\left(1-\frac{1}{2}\sin(2x)\right)dx$$

$$\Rightarrow I = \frac{1}{2}\left[x+\frac{1}{4}\cos 2x\right]_0^{\pi/2} \Rightarrow \; I = \frac{\pi-1}{4}$$

69. (2) $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$

$$= \left(\frac{1+\cos 20°}{2}\right)+\left(\frac{1+\cos 100°}{2}\right)$$

$$-\frac{1}{2}(2\cos 10° \cos 50°)$$

$$= 1+\frac{1}{2}(\cos 20° + \cos 100°) - \frac{1}{2}[\cos 60° + \cos 40°]$$

$$= \left(1-\frac{1}{4}\right)+\frac{1}{2}[\cos 20° + \cos 100° - \cos 40°]$$

$$= \frac{3}{4}+\frac{1}{2}[2\cos 60° \times \cos 40° - \cos 40°]$$

–

70. (3) Since, lx + my + n = 0 is a normal to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,$

then $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}$

but it is given that $mx-y+7\sqrt{3}$ is normal to hyperbola

$$\frac{x^2}{24}-\frac{y^2}{18}=1$$

then $\frac{24}{m^2}-\frac{18}{(-1)^2}=\frac{(24+18)^2}{(7\sqrt{3})^2} \Rightarrow m = \frac{2}{\sqrt{5}}$

71. (3) $\frac{dy}{dx}+\frac{2}{x}y = x$ and y(1) = 1 (given)

Since, the above differential equation is the linear differential equation, then

$$I.F = e^{\int \frac{2}{x}dx} = x^2$$

Now, the solution of the linear differential equation

$$y \times x^2 = \int x^3 dx \;\Rightarrow yx^2 = \frac{x^4}{4}+C$$

$$\because y(1)=1 \qquad \therefore 1\times 1 = \frac{1}{4}+C \quad \Rightarrow C = \frac{3}{4}$$

$\therefore$ solution becomes

$$y = \frac{x^2}{4}+\frac{3}{4x^2}$$

72. (4) $\sim (p \vee (\sim p \wedge q))$

$= \sim ((p \vee \sim p) \wedge (p \vee q)) = \sim (t \wedge (p \vee q))$

$= \sim (p \vee q) = \sim p \wedge \sim q$

73. (2) Let z∈ S then $z = \frac{\alpha+i}{\alpha-i}$

Since, z is a complex number and let

z = x + iy

Then, $x+iy = \frac{(\alpha+i)^2}{\alpha^2+1}$ (by rationalisation)

$$\Rightarrow x+iy = \frac{(\alpha^2-1)}{\alpha^2+1}+\frac{i(2\alpha)}{\alpha^2+1}$$

Then compare both sides

$$x = \frac{\alpha^2-1}{\alpha^2+1} \qquad ...(1)$$

$$y = \frac{2\alpha}{\alpha^2+1} \qquad ...(2)$$

Now squaring and adding equations (1) and (2)

$$\Rightarrow x^2+y^2 = \frac{(\alpha^2-1)^2}{(\alpha^2+1)^2}+\frac{4\alpha^2}{(\alpha^2+1)^2}=1$$

74. (2) $\because T_4 = 20 \times 8^7$

$$\Rightarrow {}^6C_3\left(\frac{2}{x}\right)^3 \times (x^{\log_8 x})^3 = 20\times 8^7$$

$$\Rightarrow 8\times 20\times\left(\frac{x^{\log_8 x}}{x}\right)^3 = 20\times 8^7$$

$$\Rightarrow \frac{x^{\log_8 x}}{x} = 64$$

Now, take $\log_8$ on both sides, we get

$(\log_8 x)^2 - (\log_8 x) = 2$

$\Rightarrow \log_8 x = -1$ or $\log_8 x = 2$

$\Rightarrow x = \frac{1}{8}$ or $x = 8^2$

75. (2) Since, f(x) is continuous, then

$$\lim_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right)$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k$$

Now by L- hospital's rule

$$\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\sin x}{\operatorname{cosec}^2 x} = k \Rightarrow \frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)}{(\sqrt{2})^2} = k \quad \Rightarrow k = \frac{1}{2}$$

76. (3) $f(x) = \frac{x^2}{1-x^2}$

$$\Rightarrow f(-x) = \frac{x^2}{1-x^2} = f(x)$$

$$f'(-x) = \frac{2x}{(1-x^2)^2}$$

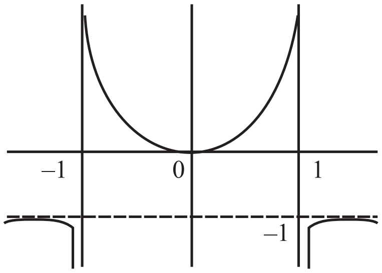
$\therefore$ f(x) increases in x ∈ (10, ∞)

Also f(0) = 0 and

$\lim_{x\to\pm\infty} f(x) = -1$ and f(x) is even function

$\therefore$ Set A = R – [–1, 0)

And the graph of function f(x) is



Alternative

For f to be subjective A = Range of f.

$$\frac{x^2}{1-x^2} = y \Rightarrow x^2 = y - x^2 y$$

$$\Rightarrow x = \pm\sqrt{\frac{y}{1+y}} \Rightarrow y(1+y) \geq 0 \text{ and } y \neq -1$$

$$\Rightarrow y \in (-\infty, -1) \cup [0, \infty) \Rightarrow y \in R - [-1, 0)$$

$$\Rightarrow A = R - [-1, 0)$$

77. **(4)** Let the required plane passing through the points (0, –1, 0) and (0, 0, 1) be

$\frac{x}{\lambda} + \frac{y}{-1} + \frac{z}{1} = 1$ and the given plane is

$y - z + 5 = 0$

$$\therefore \cos\frac{\pi}{4} = \frac{-1-1}{\sqrt{\left(\frac{1}{\lambda^2}+1+1\right)}\sqrt{2}}$$

$$\Rightarrow \lambda^2 = \frac{1}{2} \Rightarrow \frac{1}{\lambda} = \pm\sqrt{2}$$

Then, the equation of plane is

$\pm\sqrt{2}x - y + z = 1$

Then the point $(\sqrt{2}, 1, 4)$ satisfies the equation of plane $-\sqrt{2}x - y + z = 1$

78. **(4)** $\because S_n = \left(50 - \frac{7A}{2}\right)n + n^2 \times \frac{A}{2}$

$\Rightarrow a_1 = 50 - 3A$

$\therefore d = a_2 - a_1 = (S_2 - S_1) - S_1$

$\Rightarrow d = \frac{A}{2} \times 2 = A$

Now, $a_{50} = a_1 + 49 \times d$

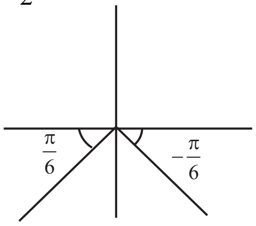
$= (50 - 3A) + 49\,A = 50 + 46\,A$

So, $(d, a_{50}) = (A, 50 + 46\,A)$

79. **(3)** $2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}$ or $\sin\theta = 2 \rightarrow$ Not possibe



The required sum of all solutions in [–2π , 2π] is

$$= \left(\pi + \frac{\pi}{6}\right) + \left(2\pi - \frac{\pi}{6}\right) + \left(-\frac{\pi}{6}\right) + \left(-\pi + \frac{\pi}{6}\right) = 2\pi$$

80. **(4)** Since $2 - \sqrt{3}$ is a root of the quadratic equation $x^2 + px + q = 0$

$\therefore 2 + \sqrt{3}$ is the other root

$\Rightarrow$ Sum of roots = 4, Product of roots = 1

$\Rightarrow p = -4, q = 1$

$\Rightarrow p^2 - 4q - 12 = 16 - 4 - 12 = 0$

81. **(1)** Since, $f(x) = 15 - |(10 - x)|$

$\therefore g(x) = f(f(x)) = 15 - |10 - [15 - |10 - x|]|$

$= 15 - ||10 - x| - 5|$

$\therefore$ Then, the points where function g(x) is Non-differentiable are

$10 - x = 0$ and $|10 - x| = 5$

$\Rightarrow x = 10$ and $x - 10 = \pm 5$

$\Rightarrow x = 10$ and $x = 15, 5$

82. **(4)** $y = f(x) = x^3 - x^2 - 2x$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 2x - 2$$

$f(1) = 1 - 1 - 2 = -2, \quad f(-1) = -1 - 1 + 2 = 0$

Since the tangent to the curve is parallel to the line segment joining the points (1, –2) and (–1, 0)

And their slopes are equal.

$$\Rightarrow 3x^2 - 2x - 2 = \frac{-2-0}{2} \Rightarrow x = 1, \frac{-1}{3}$$

Hence, the required set $S = \left\{\frac{-1}{3}, 1\right\}$

83. **(1)** Let any tangent to circle $x^2 + y^2 = 1$ is

$x\cos\theta + y\sin\theta = 1$

Since, P and Q are the point of intersection on the co-ordinate axes.

Then $P \equiv \left(\frac{1}{\cos\theta}, 0\right)$ & $Q \equiv \left(0, \frac{1}{\sin\theta}\right)$

$\therefore$ mid-point of PQ be

$$M \equiv \left(\frac{1}{2\cos\theta}, \frac{1}{2\sin\theta}\right) \equiv (h, k)$$

$$\Rightarrow \cos\theta = \frac{1}{2h} \quad \text{...(1)}$$

$$\sin\theta = \frac{1}{2k} \quad \text{...(2)}$$

Now squaring and adding equation (1) and (2)

$$\frac{1}{h^2} + \frac{1}{k^2} = 4$$

$\Rightarrow h^2 + k^2 = 4h^2k^2$

$\therefore$ locus of M is : $x^2 + y^2 - 4x^2y^2 = 0$

84. **(3)** $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$ ...(1)

Since, $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

$\therefore \vec{\beta}_2 . \vec{\alpha} = 0$

Since, $\vec{\beta}_1$ is parallel to $\vec{a}$.

then $\vec{\beta}_1 = \lambda\vec{\alpha}$ (say)

$\vec{a}.\vec{\beta} = \vec{a}.\vec{\beta}_1 - \vec{\alpha}.\vec{\beta}_2$

$\Rightarrow 5 = \lambda\alpha^2 \Rightarrow 5 = \lambda \times 10$ $(\because |\vec{\alpha}| = \sqrt{10})$.

$$\Rightarrow \lambda = \frac{1}{2}$$

$$\therefore \vec{\beta}_1 = \frac{\vec{\alpha}}{2}$$

Cross product with $\vec{\beta}_1$ in equation (1)

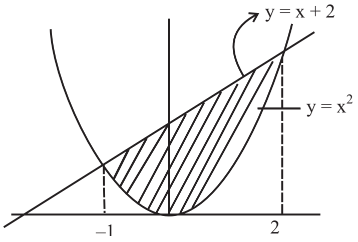
$$\Rightarrow \vec{\beta} \times \vec{\beta}_1 = -\vec{\beta}_2 \times \vec{\beta}_1$$

$$\Rightarrow \vec{\beta} \times \vec{\beta}_1 = \vec{\beta}_1 \times \vec{\beta}_2 \;\; \Rightarrow \vec{\beta}_1 \times \vec{\beta}_2 = \frac{-\vec{\beta} \times \vec{\alpha}}{2}$$

$$\Rightarrow \vec{\beta}_1 \times \vec{\beta}_2 = \frac{1}{2}\begin{vmatrix} \hat{i} & j & k \\ 2 & -1 & 3 \\ 3 & 1 & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left[-3\hat{i} - \hat{j}(-9) + \hat{k}(5)\right] = \frac{1}{2}\left[-3\hat{i} + 9\hat{j} + 5\hat{k}\right]$$

85. (2)



Required area is equal to the area under the curves $y \geq x^2$ and $y \leq x + 2$

$$\therefore \text{ requried area } \int_{-1}^{2} ((x+2) - x^2)dx$$

$$= \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)_{-1}^{2} = \left(2 + 4 - \frac{8}{3}\right) - \left(+\frac{1}{2} - 2 + \frac{1}{3}\right) = \frac{9}{2}$$

86. (2) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 1+2+3+\ldots+(n-1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{(n-1)n}{2} = 78 \;\; \Rightarrow n^2 - n - 15 = 0 \Rightarrow n = 13$$

Now, the matrix $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix}$

Then, the required inverse of

$$\begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$$

87. (4) $\because$ f(x + y) = f(x) . f(y)

$\Rightarrow$ Let $f(x) = t^x$

$\because$ f(1) = 2 $\therefore$ t = 2 $\Rightarrow f(x) = 2^x$

Since, $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$

Then, $\sum_{k=1}^{10} 2^{a+k} = 16(2^{10} - 1)$

$$\Rightarrow 2^a \sum_{k=1}^{10} 2^k = 16(2^{10} - 1)$$

$$\Rightarrow 2^a \times \frac{((2^{10}) - 1) \times 2}{(2-1)} = 16 \times (2^{10} - 1)$$

$\Rightarrow 2.2^a = 16 \Rightarrow a = 3$

88. (2) Since, m = number of ways the committee is formed with at least 6 males

$= {}^8C_6 \,.\, {}^5C_5 + {}^8C_7 \,.\, {}^5C_4 + {}^8C_8 \,.\, {}^5C_3 = 78$

and n = number of ways the committee is formed with at least 3 females

$= {}^5C_3 \,.\, {}^8C_8 + {}^5C_4 \,.\, {}^8C_7 + {}^5C_5 \,.\, {}^8C_6 = 78$

Hence, m = n = 78

89. (3) Let $\alpha = \omega$ and $\beta = \omega^2$ are roots of $x^2 + x + 1 = 0$

$$\& \textit{ Let } \Delta = \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix} = \Delta$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} y+1+\omega+\omega^2 & \omega & \omega^2 \\ y+1+\omega+\omega^2 & y+\omega^2 & 1 \\ 1+\omega+\omega^2+y & 1 & y+\omega \end{vmatrix}$$

$$\Delta = \begin{vmatrix} y & \omega & \omega^2 \\ y & y+\omega^2 & 1 \\ y & 1 & y+\omega \end{vmatrix} (\because 1+\omega+\omega^2 = 0)$$

$$\Delta = y\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & y+\omega^2 & 1 \\ 1 & 1 & y+\omega \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$, we get

$$\Delta = y\begin{vmatrix} y+\omega^2-\omega & 1-\omega^2 \\ 1-\omega & y+\omega-\omega^2 \end{vmatrix}$$

$\Rightarrow \Delta = y$

$$\left[y - (\omega - \omega^2)(y + (\omega - \omega^2) - (1-\omega)(1-\omega^2)\right]$$

$$\Rightarrow \Delta = y\left[y^2 - (\omega - \omega^2)^2 - 1 + \omega^2 + \omega - \omega^3\right]$$

$$\Rightarrow \Delta = y\left[y^2 - \omega^2 - \omega^4 + 2\omega^3 - 1 + \omega^2 + \omega^4 - \omega^3\right]$$

$(\because \omega^4 = \omega)$

$\Rightarrow \Delta = y(y^2) = y^3$

90. (1) $\because y^2 = 16x \Rightarrow a = 4$

One end of focal of the parabola is at (1, 4)

$\because$ y – coordinate of focal chord is $2at$

$\therefore 2 \; at = 4$

$$\Rightarrow t = \frac{1}{2}$$

Hence, the required length of focal chord

$$= a\left(t + \frac{1}{t}\right)^2 = 4 \times \left(2 + \frac{1}{2}\right)^2 = 25$$

JEE MAIN 2020

(Held on 07-01-2020 Morning Shift)

PHYSICS

1. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If $\gamma = 1.40$, the work done by air is: ($3^{1.4} = 4.6555$) [Take air to be an ideal gas]

(1) 60.7 J (2) 90.5 J

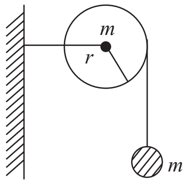
(3) 100.8 J (4) 48 J

2. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, g = 10 ms^{-2})

(1) 1.7 ms^{-1} (2) 1.9 ms^{-1}

(3) 1.5 ms^{-1} (4) 2.0 ms^{-1}

3.



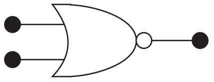
As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be:

(1) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (2) $r\sqrt{\frac{3}{2gh}}$

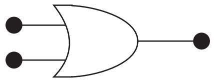
(3) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ (4) $r\sqrt{\frac{3}{4gh}}$

4. Which of the following gives a reversible operation?

(1)



(2)



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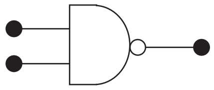

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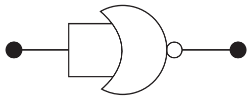


5. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area of the circular coil area is given by ϕ_0. Which of the following option is correct?

(1) $\phi_{i}. = \phi_0$ (2) $\phi_{i}. > \phi_0$

(3) $\phi_{i}. < \phi_0$ (4) $\phi_{i}. = -\phi_0$

6. A polarizer - analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer - analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero, is:

(1) 71.6° (2) 18.4°

(3) 90° (4) 45°

7. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be:

(1) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

(2) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

(3) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

(4) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

8. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth):

(1) $\frac{m}{20}\left(u^2 + \frac{113}{200}\frac{GM}{R}\right)$

(2) $5m\left(u^2 - \frac{119}{200}\frac{GM}{R}\right)$

(3) $\frac{3m}{8}\left(u + \sqrt{\frac{5GM}{6R}}\right)^2$

(4) $\frac{m}{20}\left(u - \sqrt{\frac{2GM}{3R}}\right)^2$

9. A long solenoid of radius R carries a time (t) - dependent current $I(t) = I_0t(1 - t)$. A ring of radius $2R$ is placed coaxially near its middle. During the time interval $0 \le t \le 1$, the induced current (I_R) and the induced $EMF(V_R)$ in the ring change as:

(1) Direction of I_R remains unchanged and V_R is maximum at $t = 0.5$
(2) At $t = 0.25$ direction of I_R reverses and V_R is maximum
(3) Direction of I_R remains unchanged and V_R is zero at $t = 0.25$
(4) At $t = 0.5$ direction of I_R reverses and V_R is zero

10. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the Young's modulus of wire is 16×10^{11} Nm^{-2} the extension of wire over its natural length is:

(1) 0.03 mm (2) 0.02 mm
(3) 0.04 mm (4) 0.01 mm

11. Two moles of an ideal gas with $\frac{C_p}{C_V} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_p}{C_V} = \frac{4}{3}$. The value of $\frac{C_p}{C_V}$ for the mixture is:

(1) 1. 45 (2) 1.50
(3) 1.47 (4) 1.42

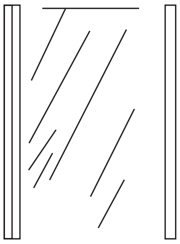
12. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to:

(1) 22 mm (2) 12 mm
(3) 2 mm (4) 33 mm

13. The time period of revolution of electron in its ground state orbit in a hydrogen atom is 1.6×10^{-16} s. The frequency of revolution of the electron in its first excited state (in s^{-1}) is:

(1) 1.6×10^{14} (2) 7.8×10^{14}
(3) 6.2×10^{15} (4) 5.6×10^{12}

14.



A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression:

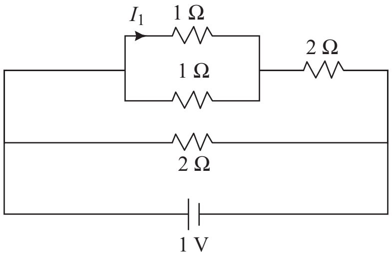
(1) $\frac{AK \epsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$

(2) $\frac{A \epsilon_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$

(3) $\frac{A \epsilon_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$

(4) $\frac{AK \epsilon_0}{d}(1+\alpha d)$

15. The current I_1 (in A) flowing through 1 Ω resistor in the following circuit is:

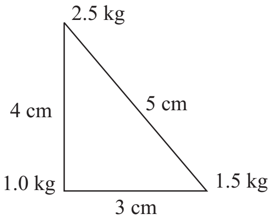


(1) 0.4 (2) 0.5
(3) 0.2 (4) 0.25

16. Visible light of wavelength 6000×10^{-8} cm falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60° from the central maximum. If the first minimum is produced at θ_1, then θ_1 is close to:

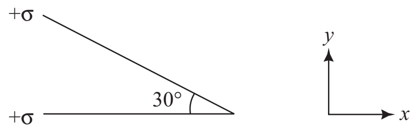
(1) 20° (2) 30°
(3) 25° (4) 45°

17. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point:



(1) 0.6 cm right and 2.0 cm above 1 kg mass
(2) 1.5 cm right and 1.2 cm above 1 kg mass
(3) 2.0 cm right and 0.9 cm above 1 kg mass
(4) 0.9 cm right and 2.0 cm above 1 kg mass

18. Two infinite planes each with uniform surface charge density $+\sigma$ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by:



(1) $\frac{\sigma}{2\in_0}\left[(1+\sqrt{3})\hat{y}-\frac{\hat{x}}{2}\right]$

(2) $\frac{\sigma}{2\in_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

(3) $\frac{\sigma}{\in_0}\left[\left(1+\sqrt{3}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

(4) $\frac{\sigma}{2\in_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

19. If the magnetic field in a plane electromagnetic wave is given by $\vec{B} = 3\times10^{-8}\sin(1.6\times10^3x+48\times10^{10}t)\ \hat{j}$ T, then what will be expression for electric field?

(1) $\vec{E} = (60\sin(1.6\times10^3x+48\times10^{10}t)\,\hat{k}$ v/m)

(2) $\vec{E} = (9\sin(1.6\times10^3x+48\times10^{10}t)\,\hat{k}$ v/m)

(3) $\vec{E} = (3\times10^{-8}\sin(1.6\times10^3x+48\times10^{10}t)\,\hat{j}$ v/m)

(4) $\vec{E} = (3\times10^{-8}\sin(1.6\times10^3x+48\times10^{10}t)\,\hat{i}$ v/m)

20. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is:

(1) $\frac{1}{4}l$
(2) $\frac{1}{8}l$
(3) $\sqrt{\frac{7}{48}}l$
(4) $\sqrt{\frac{3}{8}}l$

21. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is ________.

Given 600

Answer:

22. A non-isotropic solid metal cube has coefficients of linear expansion as: 5×10^{-5}/°C along the x-axis and 5×10^{-6}/°C along the y and the z-axis. If the coefficient of volume expansion of the solid is $C\times10^{-6}$/°C then the value of C is ________.

Given 15

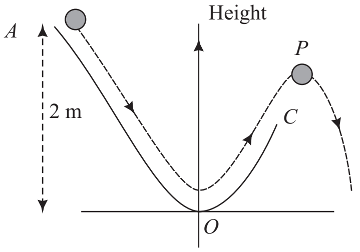
Answer:

23. A loop ABCDEFA of straight edges has six corner points $A(0, 0, 0)$, $B\{5, 0, 0)$, $C(5, 5, 0)$, $D(0, 5, 0)$, $E(0, 5, 5)$ and $F(0, 0, 5)$. The magnetic field in this region is $\vec{B} = (3\hat{i} + 4\hat{k})$T. The quantity of flux through the loop *ABCDEFA* (in Wb) is ________.

Given 25

Answer:

24. A particle (m = 1 kg) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is: (Figure drawn is schematic and not to scale; take $g = 10$ ms^{-2}) ________.



Given 0

Answer:

25. A beam of electromagnetic radiation of intensity 6.4×10^{-5} W/cm^2 is comprised of wavelength, $\lambda = 310$ nm. It falls normally on a metal (work function φ = 2eV) of surface area of 1 cm^2. If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x. (hc = 1240 eVnm, 1 eV = 1.6×10^{-19} J), then x is ________.

Given 20

Answer:

CHEMISTRY

1. The number of orbitals associated with quantum numbers $n = 5$, $m_s = +\frac{1}{2}$ is:

(1) 11
(2) 25
(3) 50
(4) 15

2. Given that the standard potentials (E°) of Cu^{2+}/Cu and Cu^{+}/Cu are 0.34 V and 0.522 V respectively, the E° of Cu^{2+}/Cu^{+} is:

(1) 0.182 V
(2) + 0.158 V
(3) – 0.182 V
(4) – 0.158 V

3. In comparison to the zeolite process for the removal of permanent hardness, the synthetic resins method is:

(1) less efficient as it exchanges only anions

(2) more efficient as it can exchange both cations as well as anions

(3) less efficient as the resins cannot be regenerated
(4) more efficient as it can exchange only cations

4. Match the following:

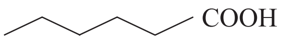
(i) Riboflavin	(a) Beriberi
(ii) Thiamine	(b) Scurvy
(iii) Pyridoxine	(c) Cheilosis
(iv) Ascorbic acid	(d) Convulsions

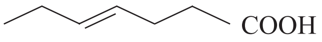
(1) (i) – (a), (ii) – (d), (iii) – (c), (iv) – (b)
(2) (i) – (c), (ii) – (d), (iii) – (a), (iv) – (b)
(3) (i) – (c), (ii) – (a), (iii) – (d), (iv) – (b)
(4) (i) – (d), (ii) – (b), (iii) – (a), (iv) – (c)

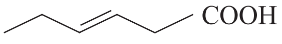
5. At 35°C, the vapour pressure of CS_2 is 512 mm Hg and that of acetone is 344 mm Hg. A solution of CS_2 in acetone has a total vapour pressure of 600 mm Hg. The false statement amongst the following is:
(1) Raoult's law is not obeyed by this system
(2) a mixture of 100 mL CS_2 and 100 mL acetone has a volume < 200 mL
(3) CS_2 and acetone are less attracted to each other than to themselves
(4) heat must be absorbed in order to produce the solution at 35°C

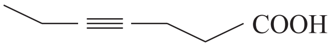
6. A solution of m-chloroaniline, m-chlorophenol and m-chlorobenzoic acid in ethyl acetate was extracted initially with a saturated solution of $NaHCO_3$ to give fraction *A*. The left over organic phase was extracted with dilute NaOH solution to give fraction *B*. The final organic layer was labelled as fraction *C*. Fractions *A*, *B* and *C*, contain respectively:
(1) m-chlorobenzoic acid, m-chloroaniline and m-chlorophenol
(2) m-chlorobenzoic acid, m-chlorophenol and m-chloroaniline
(3) m-chlorophenol, m-chlorobenzoic acid and m-chloroaniline
(4) m-chloroaniline, m-chlorobenzoic acid and m-chlorophenol

7. What is the product of following reaction?
Hex-3-ynal
(i) $\xrightarrow{NaBH_4}$?
(ii) PBr_3
(iii) Mg/ether
(iv) CO_2/H_3O^+

(1) 

(2) 

(3) 

(4) 

8. Amongst the following statements, that which was not proposed by Dalton was:
(1) chemical reactions involve reorganization of atoms. These are neither created nor destroyed in a chemical reaction.
(2) all the atoms of a given element have identical properties including identical mass. Atoms of different elements differ in mass.
(3) when gases combine or reproduced in a chemical reaction they do so in a simple ratio by volume provided all gases are at the same T & P.
(4) matter consists of indivisible atoms.

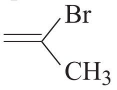
9. The dipole moments of CCl_4, $CHCl_3$ and CH_4 are in the order:
(1) $CHCl_3 < CH_4 = CCl_4$ (2) $CCl_4 < CH_4 < CHCl_3$
(3) $CH_4 < CCl_4 < CHCl_3$ (4) $CH_4 = CCl_4 < CHCl_3$

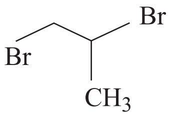
10. The IUPAC name of the complex $[Pt\ (NH_3)_2Cl(NH_2CH_3)]$ CI is:
(1) Diamminechlorido (methanamine) platinum (II) chloride
(2) Diammine (methanamine) chlorido platinum (II) chloride
(3) Diamminechlorido (aminomethane) platinum (II) chloride
(4) Bisammine (methanamine) chlorido platinum (II) chloride

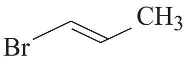
11. The purest form of commercial iron is:
(1) pig iron (2) wrought iron
(3) cast iron (4) scrap iron and pig iron

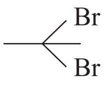
12. The electron gain enthalpy (in kj/mol) of fluorine, chlorine, bromine and iodine, respectively, are:
(1) –296, –325, –333 and –349
(2) –349, –333, –325 and –296
(3) –333, –349, –325 and –296
(4) –333, –325, –349 and –296

13. 1-methyl ethylene oxide when treated with an excess of HBr produces:

(1) 

(2) 

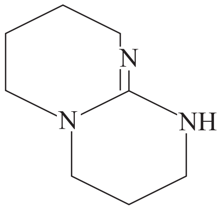
(3) 

(4) 

14. The increasing order of pK_b for the following compounds will be:

$NH_2 - CH = NH$,

(A)



(B)

CH_3NHCH_3

(C)

(1) (B) < (C) < (A)
(2) (A) < (B) < (C)
(3) (C) < (A) < (B)
(4) (B) < (A) < (C)

15. Oxidation number of potassium in K_2O, K_2O_2 and KO_2, respectively, is:

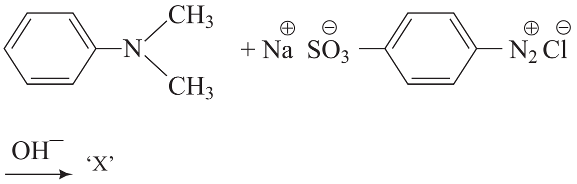
(1) $+2, +1$ and $+\frac{1}{2}$

(2) +1, +1 and +1

(3) +1, +4 and +2

(4) +1, +2 and +4

16. Consider the following reaction:



The product '*X*' is used:

(1) in protein estimation as an alternative to ninhydrin
(2) in acid base titration as an indicator
(3) as food grade colourant
(4) in laboratory test for phenols

17. The atomic radius of Ag is closest to:

(1) Au
(2) Ni
(3) Cu
(4) Hg

18. The theory that can completely/properly explain the nature of bonding in $[Ni(Co)_4]$ is:

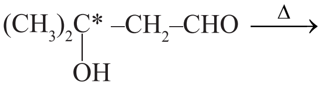
(1) Werner's theory
(2) Molecular orbital theory
(3) Crystal field theory
(4) Valence bond theory

19. Consider the following reactions:

(a) $(CH_3)_3CCH(OH)CH_3 \xrightarrow{\text{conc.}H_2SO_4}$

(b) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{\text{alc.KOH}}$

(c) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{(CH_3)_3O^*K^{\oplus}}$

(d) 

$(CH_3)_2\underset{\substack{| \\ OH}}{C^*} - CH_2 - CHO \xrightarrow{\Delta}$

Which of these reaction(s) will not produce Saytzeff product?

(1) (a), (c) and (d)
(2) (d) only
(3) (c) only
(4) (b) and (d)

20. The relative strength of interionic/ intermolecular forces in decreasing order is:

(1) dipole-dipole > ion-dipole > ion-ion
(2) ion-dipole > ion-ion > dipole-dipole
(3) ion-dipole > dipole-dipole > ion-ion
(4) ion-ion > ion-dipole > dipole-dipole

21. Chlorine reacts with hot and concentrated NaOH and produces compounds (*X*) and (*Y*). Compound (*X*) gives white precipitate with silver nitrate solution. The average bond order between Cl and O atoms in (*Y*) is ———.

Given 2

Answer:

22. The number of chiral carbons in chloramphenicol is ———.

Given 6

Answer:

23. During the nuclear explosion, one of the products is ^{90}Sr with half life of 6.93 years. If 1 µg of ^{90}Sr was absorbed in the bones of a newly born baby in place of Ca, how much time, in years, is required to reduce it by 90% if it is not lost metabolically ———.

Given 5.36

Answer:

24. For the reaction ;

$A(l) \rightarrow 2B(g)$

$\Delta U = 2.1$ kcal, $\Delta S = 20$ cal K^{-1} at 300 K.

Hence ΔG in kcal is ———.

Given –3.9

Answer:

25. Two solutions, *A* and *B*, each of 100 L was made by dissolving 4g of NaOH and 9.8g of H_2SO_4 in water, respectively. The pH of the resultant solution obtained from mixing 40 L of solution A and 10 L of solution. B is ———.

Given 7

Answer:

MATHEMATICS

1. If $y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1+\tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}, \alpha \in \left(\frac{3\pi}{4}, \pi\right)$,

then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is:

(1) 4 (2) $\frac{4}{3}$

(3) –4 (4) $-\frac{1}{4}$

2. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is:

(1) 27 (2) 7

(3) $\frac{21}{2}$ (4) 16

3. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to:

(1) $\frac{3}{2}$ (2) $-\frac{1}{2}$

(3) $\frac{1}{2}$ (4) $-\frac{3}{2}$

4. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is:

(1) $\frac{1}{2}(6!)$ (2) 6!

(3) 5^6 (4) $\frac{5}{2}(6!)$

5. A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ $(\alpha, \beta \in R)$ lies in the plane of the vectors, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then:

(1) $\vec{a} \cdot \hat{i} + 3 = 0$

(2) $\vec{a} \cdot \hat{i} + 1 = 0$

(3) $\vec{a} \cdot \hat{k} + 2 = 0$

(4) $\vec{a} \cdot \hat{k} + 4 = 0$

6. Let $x^k + y^k = a^k$, $(a, k > 0)$ and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is:

(1) $\frac{3}{2}$ (2) $\frac{4}{3}$

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

7. Let α and β be two real roots of the equation $(k+1)\tan^2x - \sqrt{2}\,.\,\lambda\tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is:

(1) $10\sqrt{2}$ (2) 10

(3) 5 (4) $5\sqrt{2}$

8. If $f(a + b + 1 - x) = f(x)$, for all x, where a and b are fixed positive real numbers,

then $\frac{1}{a+b}\int_a^b x(f(x) + f(x + 1))dx$ is equal to:

(1) $\int_{a+1}^{b+1} f(x)dx$

(2) $\int_{a-1}^{b-1} f(x)dx$

(3) $\int_{a-1}^{b-1} f(x+1)dx$

(4) $\int_{a+1}^{b+1} f(x+1)dx$

9. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is:

(1) $\frac{1}{6}(24\pi - 1)$

(2) $\frac{1}{3}(6\pi - 1)$

(3) $\frac{1}{3}(12\pi - 1)$

(4) $\frac{1}{6}(12\pi - 1)$

10. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0$,

where $a, b, c \in$ R are non-zero and distinct; has a non-zero solution, then:

(1) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(2) a, b, c are in G.P.

(3) $a + b + c = 0$

(4) a, b, c are in A.P.

11. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is:

(1) (6, 5, 2) (2) (6, 5, –2)

(3) (4, 3, 2) (4) (3, 4, –2)

12. The logical statement

$(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to:

(1) p (2) q

(3) $\sim P$ (4) $\sim q$

13. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is:

(1) $\sqrt{3}$ (2) $3\sqrt{2}$

(3) $\frac{3}{\sqrt{2}}$ (4) $2\sqrt{3}$

14. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5, otherwise X takes the value –1. Then the expected value of X, is:

(1) $\frac{3}{16}$ (2) $\frac{1}{8}$

(3) $-\frac{3}{16}$ (4) $-\frac{1}{8}$

15. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to:

(1) –32 (2) –64

(3) –128 (4) 128

16. The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + ... + 49^2 + 49 + 1$, is:

(1) 32 (2) 63

(3) 60 (4) 65

17. If Re $\left(\frac{z-1}{2z+i}\right)$ = 1, where $z = x + iy$, then the point (x, y) lies on a:

(1) circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$.

(2) straight line whose slope is $-\frac{2}{3}$.

(3) straight line whose slope is $\frac{3}{2}$.

(4) circle whose diameter is $\frac{\sqrt{5}}{2}$.

18. Let α be a root of the equation $x^2 + x + 1 = 0$

and the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$,

then the matrix A^{31} is equal to:

(1) A (2) I_3

(3) A^2 (4) A^3

19. If $y = y(x)$ is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that $y(0) = 0$, then $y(1)$ is equal to:

(1) $1 + \log_e 2$

(2) $2 + \log_e 2$

(3) $2e$

(4) $\log_e 2$

20. Let the function, f: [–7, 0] → R be continuous on [–7, 0] and differentiable on (–7, 0). If $f(-7) = -3$ and $f'(x) \leq 2$, for all $x \in (-7, 0)$, then for all such functions f, $f'(-1) + f(0)$ lies in the interval:

(1) $(-\infty, 20]$

(2) $[-3, 11]$

(3) $(-\infty, 11]$

(4) $[-6, 20]$

21. $\lim\limits_{x \to 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ is equal to ———.

Given 0

Answer:

22. If the sum of the coefficients of all even powers of x in the product

$(1 + x + x^2 + ... + x^{2n})(1 - x + x^2 - x^3 + ... + x^{2n})$ is 61, then n is equal to ———.

Given

Answer:

23. Let S be the set of points where the function, $f(x) = |2 - |x - 3||$, $x \in$ R, is not differentiable.

Then $\sum\limits_{x \in S} f(f(x))$ is equal to ———.

Given 9

Answer:

24. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then $m + n$ is equal to ———.

Given --

Answer:

25. Let A(1, 0)), B(6, 2) and C$\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment

PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is ———.

Given --

Answer:

HINTS AND SOLUTIONS

JEE Main 2020 (07-01-2020) Morning Shift

PHYSICS

1. **(2)** Given, $V_1 = 1$ litre, $P_1 = 1$ atm

$V_2 = 3$ litre, $\gamma = 1.40$,

Using, PV^r = constant $\Rightarrow P_1V_1^\gamma = P_2V_2^\gamma$

$$\Rightarrow P_2 = P_1 \times \left(\frac{1}{3}\right)^{1.4} = \frac{1}{4.6555}\,atm$$

$$\therefore \text{ Work done, } W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$= \frac{\left(1\times1 - \frac{1}{4.6555}\times3\right)1.01325\times10^5\times10^{-3}}{0.4} = 90.1\,J$$

Closest value of $W = 90.5\,J$

2. **(2)** Total force required to lift maximum load capacity against frictional force = 400 N

$F_{total} = Mg$ + friction

$= 2000 \times 10 + 4000$

$= 20,000 + 4000 = 24000\,N$

Using power, $P = F \times v$

$60 \times 746 = 24000 \times v$

$\Rightarrow v = 1.86$ m/s $\approx 1.9\,m/s$

Hence speed of the elevator at full load is close to 1.9 ms^{-1}

3. **(1)** When the bob covered a distance 'h'

$$\text{Using } mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

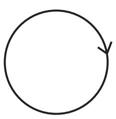
$$= \frac{1}{2}\text{m}(\omega\text{r})^2 + \frac{1}{2}\times\frac{mr^2}{2}\times\omega^2 \;(\because v = \omega r \textit{ no slipping})$$

$$\Rightarrow \quad mgh = \frac{3}{4}m\omega^2r^2$$

$$\Rightarrow \omega = \sqrt{\frac{4gh}{3r^2}} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$$

4. **(4)** A logic gate is reversible if we can recover input data from the output. Hence NOT gate.

5. **(4)** As magnetic field lines form close loop, hence every magnetic field line creating magnetic flux through the inner region (ϕ_i) must be passing through the outer region. Since flux in two regions are in opposite region.



$\therefore \phi_i = -\phi_0$

6. **(2)** According to question, the intensity of light coming out of the analyser is just 10% of the original intensity (I_0)

Using, $I = I_0\cos^2\theta$

$$\Rightarrow \frac{}{10} = \quad \cos \quad \theta \qquad \Rightarrow \frac{1}{10} = \cos^2\theta$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{10}} = 0.316 \Rightarrow \theta \approx 71.6°$$

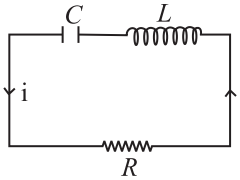
Therefore, the angle by which the analyser need to be rotated further to reduced the output intensity to be zero

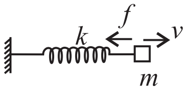
$\phi = 90° - \theta = 90° - 71.6° = 18.4°$

7. **(4)** In damped harmonic oscillation,

$$\frac{md^2x}{dt^2} = -kx - bv$$

$$\Rightarrow \frac{md^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \qquad \text{....(i)}$$





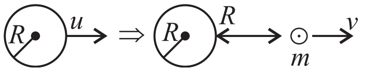
$$\text{In LCR circuit, } \frac{-q}{C} - iR - \frac{Ldi}{dt} = 0$$

$$L\frac{d^2}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \quad \text{...(ii)}$$

Comparing equations (i) & (ii)

$$L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$$

8. **(2)**



$$\frac{1}{2}mu^2 + \frac{-GMm}{R} = \frac{1}{2}mv^2 + \frac{-GMm}{2R}$$

$$\Rightarrow \frac{1}{2}m(v^2 - u^2) = \frac{-GMm}{2R}$$

$$\Rightarrow \text{V} = \sqrt{\text{V} = \text{u}^2 - \frac{\text{GM}}{\text{R}}} \qquad \text{...(i)}$$

$$v_0 = \sqrt{\frac{GM}{2R}} \quad \therefore v_{rad} = \frac{m \times v}{\left(\frac{m}{10}\right)} = 10\,v$$

Ejecting a rocket of mass $\frac{m}{10}$

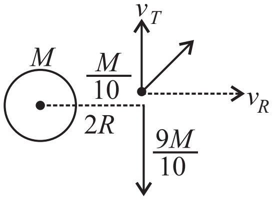
$$\therefore \frac{9m}{10}\times\sqrt{\frac{GM}{2R}}=\frac{m}{10}\times v_{\tau}\Rightarrow V_{\tau}^{2}=81\frac{GM}{2R}$$

Kinetic energy of rocket,

$$\text{KE}_{rocket}=\frac{1}{2}\frac{M}{10}\left(V_T^2+V_r^2\right)=\frac{1}{2}\times\frac{m}{10}\times\left((u^2-\frac{GM}{R})100+81\frac{GM}{R}\right)$$

$$=\frac{m}{20}\times100\left(u^2-\frac{GM}{R}+\frac{81}{200}\frac{GM}{R}\right)$$

$$=5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$$



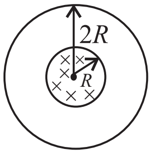
9. **(4)** According to question, $I(t) = I_0t(1-t)$

$\therefore I = I_0t - I_0t^2$

$\phi = B.A$

$\phi = (\mu_0\, nI)\times(\pi R^2)$

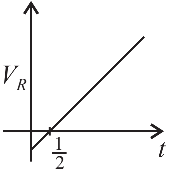
$(\because\ B=\mu_0\,\text{nI}\ and\ A=\pi R^2)$



$$V_R=\frac{-d\phi}{dt}$$

$$V_R=\mu_0 n\pi R^2(I_0-2I_0t)$$

$$\Rightarrow V_R=0 \text{ at } t=\frac{1}{2}s$$



10. **(1)** Given, $l = 60$ cm, $m = 6$ g, $A = 1\ \text{mm}^2$, $v = 90$ m/s and $Y = 16\times10^{11}\ \text{Nm}^{-2}$

Using, $v=\sqrt{\frac{T}{m}\times l}\Rightarrow T=\frac{mv^2}{I}$

Again from, $Y=\frac{T}{A}\Delta L/L_0$

$$\Delta L=\frac{Tl}{YA}=\frac{mv^2\times l}{l(\text{YA})}$$

$$=\frac{6\times10^{-3}\times90^2}{16\times10^{11}\times10^{-6}}=3\times10^{-4}m$$

$= 0.03$ mm

11. **(4)** Using, $\gamma_{\text{mixture}}=\frac{n_1C_{p_1}+n_2C_{p_2}}{n_1C_{v_1}+n_2C_{v_2}}$

$$\Rightarrow\frac{n_1}{\gamma_1-1}+\frac{n_2}{\gamma_2-1}=\frac{n_1+n_2}{\gamma_m-1}$$

$$\Rightarrow\frac{3}{\frac{4}{3}-1}+\frac{2}{\frac{5}{3}-1}=\frac{5}{\gamma_m-1}$$

$$\Rightarrow\frac{9}{1}+\frac{2\times3}{2}=\frac{5}{\gamma_m-1}$$

$$\Rightarrow\gamma_\text{m}-1=\frac{5}{12}$$

$$\Rightarrow\gamma_m=\frac{17}{12}=1.42$$

12. **(1)** According question, $M = 375$

$L = 150$ mm, $f_0 = 5$ mm and $f_e = ?$

Using, magnification, $M\simeq\frac{L}{f_0}\left(1+\frac{D}{f_e}\right)$

$$\Rightarrow375=\frac{150}{5}\left(1+\frac{250}{f_e}\right)\quad(\because D = 25\text{ cm} = 250\text{ mm})$$

$$\Rightarrow12.5=1+\frac{250}{f_e}$$

$$\Rightarrow f_e=\frac{250}{11.5}=21.7\approx22\,mm$$

13. **(2)** For first excited state $n' = 3$

Time period $T\propto\frac{n^3}{z^2}$

$$\Rightarrow\frac{T_2}{T_1}=\frac{n'^3}{n^3}$$

$\therefore T_2 = 8T_1 = 8\times1.6\times10^{-16}s$

$\therefore$ Frequency, $\nu=\frac{1}{T_2}=\frac{1}{8\times1.6\times10^{-16}}$

$\approx 7.8\times10^{14}\ Hz$

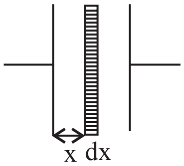
14. **(1)** Given, $K(x) = K(1+\alpha x)$

Capacitance of element, $C_{el}=\frac{K\varepsilon_0A}{dx}$

$$\Rightarrow C_{el}=\frac{\varepsilon_0K(1+\alpha x)A}{dx}$$

$$\therefore\int d\left(\frac{1}{C}\right)=\frac{1}{C_{el}}=\int_0^d\left(\frac{dx}{\varepsilon_0KA(1+\alpha x)}\right)$$

$$\Rightarrow\frac{1}{C}=\frac{1}{\varepsilon_0KA\alpha}[ln(1+\alpha x)]_0^d$$



$$\Rightarrow\frac{1}{C}=\frac{1}{\varepsilon_0KA\alpha}ln(1+\alpha d)[\alpha\,\text{d}<<1]$$

$$=\frac{1}{\varepsilon_0KA\alpha}\left[\alpha d-\frac{\alpha^2d^2}{2}\right]$$

$$=\frac{1}{\varepsilon_0KA}\left[1-\frac{\alpha d}{2}\right]$$

$$\therefore C=\frac{\varepsilon_0 KA}{d\left(1-\frac{\alpha d}{2}\right)}\Rightarrow C=\frac{\varepsilon_0 KA}{d}\left(1+\frac{\alpha d}{2}\right)$$

15. **(3)**

16. **(3)** Given, $\lambda = 6000 \times 10^{-8}$ cm

Second diffraction minimum at 60° i.e., $\theta_2 = 60°$

Using, $d\sin\theta = n\lambda$

$d\sin\theta_2 = 2\lambda$ (for 2nd minima)

$\Rightarrow$ $d\sin 60° = 2\lambda$

$$\Rightarrow \mathrm{d}\times\left(\frac{\sqrt{3}}{2}\right)=2\lambda \qquad ...(i)$$

$$\Rightarrow \frac{\lambda}{d}=\frac{\sqrt{3}}{4}$$

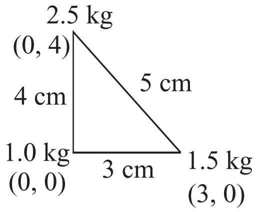
For first minima,

$d\sin\theta_1 = \lambda$

$$\Rightarrow \sin\theta_1=\frac{\lambda}{d}=\frac{\sqrt{3}}{4}=0.43\Rightarrow\theta_1<30°$$

Hence closest option, $\theta_1 \approx 25°$

17. **(4)**



$$X_{\text{cm}}=\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}$$

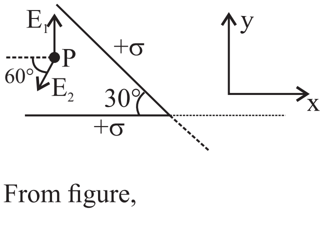
$$X_{cm}=\frac{1\times0+1.5\times3+2.5\times0}{1+1.5+2.5}=\frac{1.5\times3}{5}=0.9cm$$

$$Y_{\text{cm}}=\frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}$$

$$Y_{cm}=\frac{1\times0+1.5\times0+2.5\times4}{1+1.5+2.5}=\frac{2.5\times4}{5}=2cm$$

Hence, centre of mass of system is at point (0.9, 2)

18. **(4)**



From figure,

$$\vec{E}_1=\frac{\sigma}{2\varepsilon_0}\hat{y} \text{ and } \vec{E}_2=\frac{\sigma}{2\varepsilon_0}(-\cos 60°\hat{x}-\sin 60°\hat{y})$$

$$=\frac{\sigma}{2\varepsilon_0}\left(-\frac{1}{2}\hat{x}-\frac{\sqrt{3}}{2}\hat{y}\right)$$

Electric field in the region shown in figure (P)

$$\vec{E}_P=\vec{E}_1+\vec{E}_2=\frac{\sigma}{2\varepsilon_0}\left[-\frac{1}{2}\hat{x}+\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}\right]$$

$$\text{or, } \vec{E}_P=\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$$

19. **(2)** Given, $\vec{B}=3\times10^{-8}\sin(1.6\times10^{3}x+48\times10^{10}\,\mathrm{t})\ \hat{j}T$

Using, $E_0=B_0\times C=3\times10^{-8}\times3\times10^{8}=9\ V/m$

$\therefore$ Electricfield, $\vec{E}=9\sin(1.6\times10^{3}x+48\times10^{10}t)\hat{\mathrm{k}}\ V/m$

20. **(3)** Moment inertia of the rod passing through a point $\frac{\ell}{4}$ away from the centre of the rod

$I = Ig + \mathrm{m}\ell^2$

$$\Rightarrow I=\frac{MI^2}{12}+M\times\left(\frac{I^2}{16}\right)=\frac{7MI^2}{48}$$

Using $I=MK^2=\frac{7MI^2}{48}$ (K = radius of gyration)

$$\Rightarrow K=\sqrt{\frac{7}{48}}I$$

21. **(600.00)** Given; $T_1 = 900\ K$, $T_2 = 300K$, $W = 1200\ J$

$$\text{Using, } 1-\frac{T_2}{T_1}=\frac{W}{Q_1}$$

$$\Rightarrow 1-\frac{300}{900}=\frac{1200}{Q_1}$$

$$\Rightarrow \frac{2}{3}=\frac{1200}{Q_1}\Rightarrow Q_1=1800$$

Therefore heat energy delivered by the engine to the low temperature reservoir, $Q_2 = Q_1 - W = 1800 - 1200 = 600.00\ J$

22. **(60.00)** Volume, $V = lbh$

$$\therefore \gamma=\frac{\Delta V}{V}=\frac{\Delta\ell}{\ell}+\frac{\Delta b}{b}+\frac{\Delta h}{h}$$

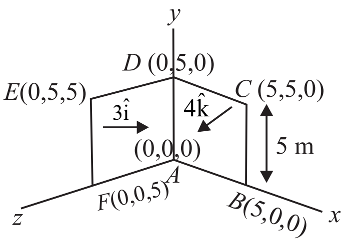
(γ = coefficient of volume expansion)

$\Rightarrow$ $\gamma = 5\times10^{-5} + 5\times10^{-6} + 5\times10^{-6}$

$= 60\times10^{-6}/°C$

$\therefore$ Value of $C = 60.00$

23. **(175.00)**



Flux through the loop ABCDEFA,

$\phi = \vec{B}.\vec{A} = (3\hat{i} + 4\hat{k}).(25\hat{i} + 25\hat{k})$

$\Rightarrow \phi = (3 \times 25) + (4 \times 25) = 175$ weber

24. **(10.00)**

Kinetic energy = change in potential energy of the particle,

KE = mgΔh

Given, $m = 1$ kg,

$\Delta h = h_2 - h_1 = 2 - 1 = 1m$

$\therefore KE = 1 \times 10 \times 1 = 10\ J$

25. **(11.00)** Energy of proton

$$E = \frac{hc}{\lambda} = \frac{1240}{310} = 4eV > 2eV\ [= \phi]$$

(so emission of photoelectron will take place)

$= 4 \times 1.6 \times 10^{-19} = 6.4 \times 10^{-19}$ joule

$$N = \frac{6.4 \times 10^{-5} \times 1}{4 \times 6.4 \times 10^{-19}} = 10^{14}$$

No. of photoelectrons emitted per second

$$= \frac{10^{14}}{10^{3}} = 10^{11}$$ ($\because$ 1 in 10^3 photons ejects an electron)

$\therefore$ Value of $X = 11.00$

CHEMISTRY

26. **(2)** The possible number of orbitals in a shell in term of 'n' is n^2

$\therefore n = 5 ; n^2 = 25$

27. **(2)** $Cu^{2+} + 2e^- \longrightarrow Cu, \Delta G_1^\circ = -2F(0.34)$ ...(i)

$Cu^+ + e^- \longrightarrow Cu, \Delta G_2^\circ = -F(0.522)$ ...(ii)

Subtract (ii) from (i)

$Cu^{2+} + e^- \longrightarrow Cu^+, \quad \Delta G_3^\circ = -F(E^0)$

$\therefore \Delta G_1^\circ - \Delta G_2^\circ = G_3^\circ$

$\Rightarrow -FE^\circ = -2F(0.34) + F(0.522)$

$\Rightarrow E^\circ = 0.68 - 0.522 = 0.158$ V

28. **(2)** Synthetic resin method is more efficient than zeolite process as it can exchange both cations as well as anions.

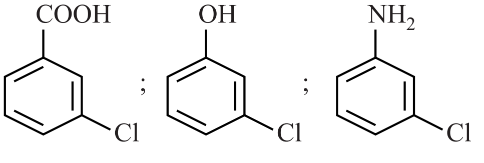
29. **(3)**

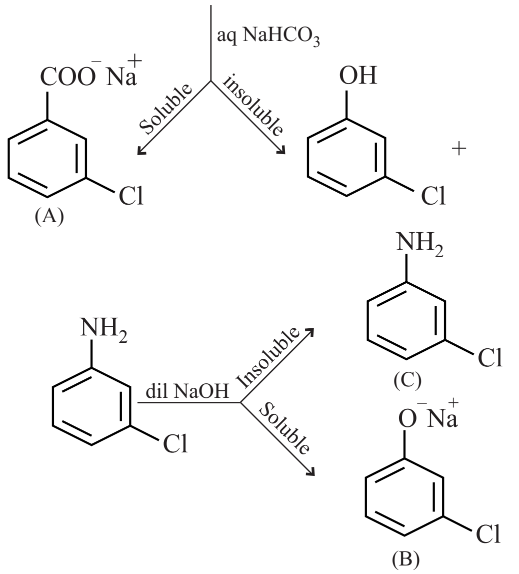
Vitamins	Deficiency Diseases
Vitamin B_1 (thiamine)	Beri Beri
Vitamin B_2 (riboflavin)	Cheilosis
Vitamin B_6 (pyridoxine)	Convulsions
Vitamin C (ascorbic acid)	Scurvy

30. **(2)** Mixture of carbon disulphide and acetone will show positive deviation from Raoult's Law.

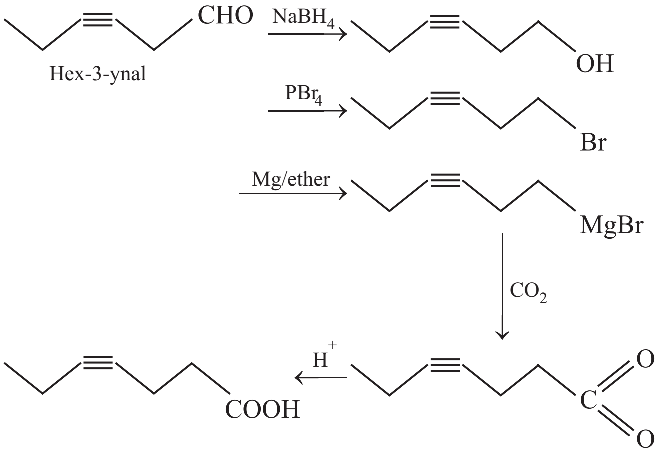
The dipolar interaction between solute (CS_2) solvent (acetone) molecules in solution are weaker. So the vapour pressure of solution will be greater than the individual vapoure pressure of pure components.

31. **(2)**





32. **(4)**



33. (3) Except (3) all postulates was given by the Dalton.

34. (4) $\mu_{CCl_4} = \mu_{CH_4} = 0$ due to symmetrical structure but $\mu_{CHCl_3} \neq 0$. So dipole moment order is :

$CHCl_3 > CH_4 = CCl_4$

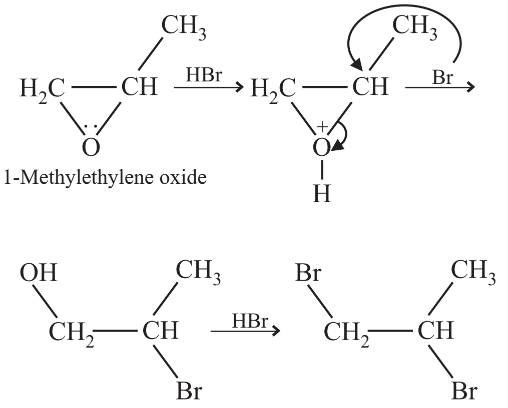
35. (1) $[Pt\,(NH_3)_2Cl(NH_2CH_3)]Cl$

Diamine chlorido (methanamine) platinum (II) chloride

36. (2) Wrought iron is purest form of commercial iron.

37. (3) Chlorine has highest electron gain enthalpy (most negative) among the given elements, the electron gain enthalpy decreases down the group i.e., moves to least negative.

38. (2)



39. (4) Conjugate acid of guanadine(B) is resonance stabilised and have 2 resonance structure.
Similarly conjugate acid of (A) is also resonance stabilised and have one resonance structure. (C) does not exhibit resonance structure.

therefore the basic order is, k_b : (B) > (A) > (C)

$\therefore$ pk_b : (B) < (A) < (C)

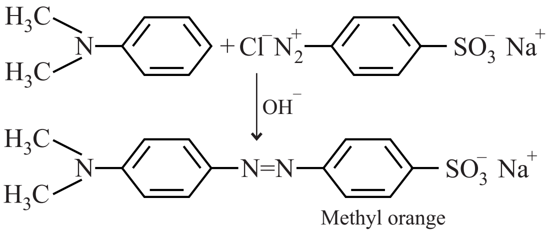
40. (2) $K_2O : 2x - 2 = 0 \Rightarrow x = +1$

$K_2O_2 : 2x - 2 = 0 \Rightarrow x = +1$

$KO_2 : x - 1 = 0 \Rightarrow x = +1$

Thus, potassium shows +1 state in all its oxides, superoxides and peroxides.

41. (2)

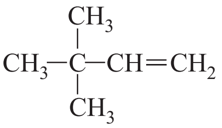


Methyl orange is used as an indicator in acid base titrations.

42. (1) Atomic size of elements of 4*d* and 5*d* transition series are nearly same due to lanthanide contraction.

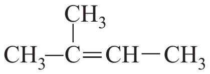
43. (2) The covalant character of the bonding (M – C σ and M – C π bonding) which exists between the metal and the carbon atom of the CO can only be explained by the molecular orbital theory.

44. (3) (A) $(CH_3)_2CCH(OH)CH_3 \xrightarrow{conc.\,H_2SO_4}$



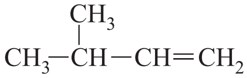
$CH_3-C(CH_3)_2-CH=CH_2$

(B) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{alc.KOH}$



$CH_3-C(CH_3)=CH-CH_3$

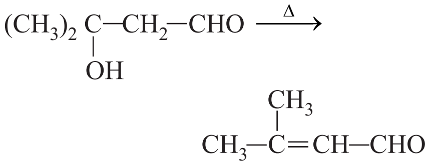
(C) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{t\text{-}BuO^-K^+}$



$CH_3-CH(CH_3)-CH=CH_2$

Due to bulky nature of tertiary butoxide, the least hindered hydrogen is eliminated. Therefore, Hoffman product is formed.

(D)



$(CH_3)_2C(OH)-CH_2-CHO \xrightarrow{\Delta} CH_3-C(CH_3)=CH-CHO$

45. (4) Among given intermolecular forces, ionic interactions are stronger as compared to van der Waal interaction Thus, correct order is ion-ion > ion-dipole > dipole-dipole

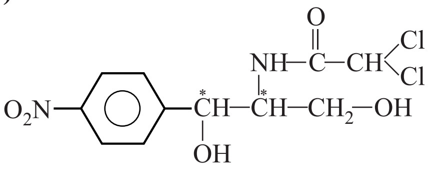
46. (1.67)

$$\underset{\text{Hot \& conc.}}{3Cl_2 + 6NaOH} \longrightarrow \underset{(X)}{5NaCl} + \underset{(Y)}{NaClO_3} + 3H_2O$$

$$\underset{(X)}{NaCl} + AgNO_3 \longrightarrow \underset{\text{(white ppt.)}}{AgCl} + NaNO_3$$

Average bond order between Cl and O atom in $NaClO_3 = \frac{5}{3} = 1.67$

47. (2.00)



48. (23.03) $t_{1/2} = 6.93$ years,
$a = 10^{-6}$ g

$$t_{1/2} = \frac{0.693}{K}$$

$$\Rightarrow \quad K = \frac{0.693}{t_{1/2}} = \frac{0.693}{6.93} = 0.1$$

For Ist order reaction,

$$K = \frac{2.303}{t}\log\frac{a}{a-x}$$

$$t = \frac{2.303}{K}\log\frac{a}{a-x}$$

$$= \frac{2.303}{0.1} \log \frac{10^{-6}}{10^{-7}}$$

$$= \frac{2.303}{0.1} = 23.03 \text{ years}$$

49. **(–2.70)**

$\Delta U = 2.1 \text{ kcal} = 2.1 \times 10^3 \text{ cal}$

$\Delta n_g = 2$

$\Delta H = \Delta U + \Delta n_g RT$

$= 2.1 \times 10^3 + 2 \times 2 \times 300$

$= 2100 + 1200$

$= 3300 \text{ cal}$

$\Delta G = \Delta H - T\Delta S$

$= 3300 - 300 \times 20$

$= 3300 - 6000$

$= -2700 \text{ cals}$

$= -2.7 \text{ kcal}$

50. **(10.60)** $M_{H_2SO_4} = \frac{9.8}{98 \times 100} = 10^{-3} \text{ M}$

$M_{NaOH} = \frac{4}{40 \times 100} = 10^{-3} \text{ M}$

After neutralisation [OH^-] can be calculated as

$$[OH^-] = \frac{\left(40 \times 10^{-3}\right) - \left(2 \times 10^{-3} \times 10\right)}{50}$$

$$= \frac{20}{50} \times 10^{-3}$$

$$[OH^-] = \frac{2}{5} \times 10^{-3}$$

$pOH = 3.397$

$pH = 14 - pOH$

$= 14 - 3.397 = 10.603$

MATHEMATICS

51. **(1)** $y(\alpha) = \sqrt{\dfrac{\dfrac{2\sin\alpha}{\cos\alpha} + \dfrac{\cos\alpha}{\sin\alpha}}{\sec^2\alpha}} = \sqrt{\dfrac{2\cos^2\alpha}{\sin\alpha\cos\alpha} + \dfrac{1}{\sin^2\alpha}}$

$= \sqrt{2\cot\alpha + \text{cosec}^2\alpha} = \sqrt{2\cot\alpha + 1 + \cot^2\alpha}$

$= |1 + \cot\alpha| = -1 - \cot\alpha \qquad \left[\because \alpha \in \left(\frac{3\pi}{4}, \pi\right)\right]$

$\frac{dy}{d\alpha} = \text{cosec}^2\alpha \quad \Rightarrow \quad \left(\frac{dy}{d\alpha}\right)_{\alpha = \frac{5\pi}{6}} = 4$

52. **(4)** Let 5 terms of A.P. be

$a - 2d, a - d, a, a + d, a + 2d$.

Sum = 25 $\Rightarrow$ $5a = 25$ $\Rightarrow$ $a = 5$

Product = 2520

$(5 - 2d)(5 - d)5(5 + d)(5 + 2d) = 2520$

$\Rightarrow$ $(25 - 4d^2)(25 - d^2) = 504$

$\Rightarrow$ $625 - 100d^2 - 25d^2 + 4d^4 = 504$

$\Rightarrow$ $4d^4 - 125d^2 + 625 - 504 = 0$

$\Rightarrow$ $4d^4 - 125d^2 + 121 = 0$

$\Rightarrow$ $4d^4 - 121d^2 - 4d^2 + 121 = 0$

$\Rightarrow$ $(d^2 - 1)(4d^2 - 121) = 0$

$\Rightarrow$ $d = \pm 1, d = \pm \frac{11}{2}$

$d = \pm 1$ and $d = -\frac{11}{2}$, does not give $\frac{-1}{2}$ as a term

$\therefore$ $d = \frac{11}{2}$

$\therefore$ Largest term = $5 + 2d = 5 + 11 = 16$

53. **(2)** $(gof)(x) = g(f(x)) = f^2(x) + f(x) - 1$

$$g\left(f\left(\frac{5}{4}\right)\right) = 4\left(\frac{5}{4}\right)^2 - 10.\frac{5}{4} + 5 = -\frac{5}{4}$$

$[\because g(f(x)) = 4x^2 - 10x + 5]$

$$g\left(f\left(\frac{5}{4}\right)\right) = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$$

$$-\frac{5}{4} = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$$

$$f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) + \frac{1}{4} = 0$$

$$\left(f\left(\frac{5}{4}\right) + \frac{1}{2}\right)^2 = 0$$

$$t\left(\frac{5}{4}\right) = -\frac{1}{2}$$

54. **(4)** Five digits numbers be 1, 3, 5, 7, 9

For selection of one digit, we have ${}^5C_1$ choice.

And six digits can be arrange in $\frac{6!}{2!}$ ways.

Hence, total such numbers = $\frac{5.6!}{2!} = \frac{5}{2}.6!$

55. **(3)** Angle bisector between $\vec{b}$ and $\vec{c}$ can be

$$\vec{a} = \lambda(\hat{b} + \hat{c}) \quad \text{or} \quad \vec{a} = \mu(\hat{b} - \hat{c})$$

If $\vec{a} = \lambda\left(\dfrac{\hat{i} + \hat{j}}{\sqrt{2}} + \dfrac{\hat{i} - \hat{j} + 4\hat{k}}{3\sqrt{2}}\right)$

$$= \frac{\lambda}{3\sqrt{2}}[3\hat{i} + 3\hat{j} + \hat{i} - \hat{j} + 4\hat{k}]$$

$$= \frac{\lambda}{3\sqrt{2}}[4\hat{i}+2\hat{j}+4\hat{k}]$$

Compare with $\vec{a} = \alpha\hat{i}+2\hat{j}+\beta\hat{k}$

$$\frac{2\lambda}{3\sqrt{2}} = 2 \quad \Rightarrow \quad \lambda = 3\sqrt{2}$$

$$\vec{a} = 4\hat{i}+2\hat{j}+4\hat{k}$$

Not satisfy any option

Now consider $\vec{a} = \mu\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} - \frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$$\vec{a} = \frac{\mu}{3\sqrt{2}}(3\hat{i}+3\hat{j}-\hat{i}+\hat{j}-4\hat{k})$$

$$= \frac{\mu}{3\sqrt{2}}(2\hat{i}+4\hat{j}-4\hat{k})$$

Compare with $\vec{a} = \alpha\hat{i}+2\hat{j}+\beta\hat{k}$

$$\frac{4\mu}{3\sqrt{2}} = 2 \quad \Rightarrow \quad \mu = \frac{3\sqrt{2}}{2}$$

$$\vec{a} = \hat{i}+2\hat{j}-2\hat{k}$$

$\therefore \quad \vec{a}\,.\,\vec{k}+2=0$

$-2+2=0$

56. **(3)** $k.x^{k-1}+k.y^{k-1}\frac{dy}{dx}=0$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1}$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{x}{y}\right)^{k-1} = 0$$

$$\Rightarrow \quad k-1 = -\frac{1}{3}$$

$$\Rightarrow \quad k = 1-\frac{1}{3} = \frac{2}{3}$$

57. **(2)** $(k+1)\tan^2 x - \sqrt{2}\lambda\tan x + (k-1) = 0$

$$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1} \qquad \text{[Sum of roots]}$$

$$\tan\alpha\cdot\tan\beta = \frac{k-1}{k+1} \qquad \text{[Product of roots]}$$

$$\therefore \quad \tan(\alpha+\beta) = \frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}} = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}$$

$$\tan^2(\alpha+\beta) = \frac{\lambda^2}{2} = 50$$

$\lambda = 10.$

58. **(3)** $I = \frac{1}{(a+b)}\int_a^b x[f(x)+f(x+1)]dx$ ...(i)

$x \to a+b-x$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)[f(a+b-x)+f(a+b+1-x)]dx$$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)[f(x+1)+f(x)]dx \qquad \text{...(ii)}$$

[$\because$ put $x \to x+1$ in $f(a+b+1-x) = f(x)$]

Add (i) and (ii)

$$2I = \int_a^b [f(x+1)+f(x)]dx$$

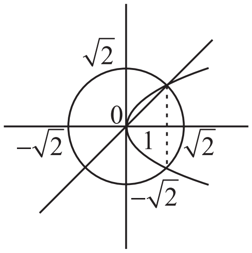
$$2I = \int_a^b f(x+1)dx + \int_a^b f(x)dx$$

$$= \int_a^b f(a+b+1-x)dx + \int_a^b f(x)dx$$

$$2I = 2\int_a^b f(x)dx$$

$$\therefore \quad \int_{a-1}^{b-1} f(x+1)dx \qquad [\because \text{ Put } x \to x+1]$$

59. **(4)** Total area – enclosed area between line and parabola



$$= 2\pi - \int_0^1 \sqrt{x} - x dx$$

$$= 2\pi - \left(\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right)_0^1$$

$$= 2\pi - \left(\frac{2}{3}-\frac{1}{2}\right) = 2\pi - \left(\frac{1}{6}\right) = \frac{12\pi-1}{6}$$

60. **(1)** For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$\Rightarrow \quad (3bc-4bc)-(2ac-4ac)+(2ab-3ab)=0$

$\Rightarrow \quad -bc+2ac-ab=0$

$\Rightarrow \quad ab+bc+2ac$

$$\Rightarrow \frac{2}{b}=\frac{1}{a}+\frac{1}{c}$$

$$\Rightarrow \frac{1}{a},\frac{1}{b},\frac{1}{c} \text{ in A.P.}$$

61. **(2)** Equation of plane is $x + y - 2z = 3$

$$\Rightarrow \frac{x-2}{1}=\frac{y-1}{1}=\frac{z-6}{-2}=\frac{-2(2+1-12-3)}{6}$$

$\Rightarrow (x, y, z) = (6, 5, -2)$

62. **(3)**

p	q	$p \Rightarrow q$	$\sim p$	$q \Rightarrow \sim p$	$(p \Rightarrow q) \wedge (p \Rightarrow \sim q)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Clearly $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to $\sim p$

63. **(2)** $2ae = 6$ and $\frac{2a}{e}=12$

$\Rightarrow ae = 3$ ...(i)

and $\frac{a}{e}=6 \Rightarrow e=\frac{a}{6}$ ...(ii)

$\Rightarrow a^2 = 18$ [From (i) and (ii)]

$\Rightarrow b^2 = a^2 - a^2e^2 = 18 - 9 = 9$

$\therefore$ Latus rectum $= \frac{2b^2}{a}=\frac{2\times 9}{3\sqrt{2}}=3\sqrt{2}$

64. **(2)**

k	0	1	2	3	4	5
$P(k)$	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

k = No. of times head occur consecutively

Now expectation

$$=\sum xP(k)=(-1)\times\frac{1}{32}+(-1)\times\frac{12}{32}+(-1)\times\frac{11}{32}$$
$$+3\times\frac{5}{32}+4\times\frac{2}{32}+5\times\frac{1}{32}=\frac{1}{8}$$

65. **(3)** $y = mx + 4$ ...(i)

Tangent of $y^2 = 4x$ is

$\Rightarrow y = mx + \frac{1}{m}$ ...(ii)

[$\because$ Equation of tangent of $y^2 = 4ax$ is $y = mx + \frac{a}{m}$]

From (i) and (ii)

$4=\frac{1}{m} \Rightarrow m=\frac{1}{4}$

So, line $y = \frac{1}{4}x+4$ is also tangent to parabola $x^2 = 2by$, so solve both equations.

$$x^2 = 2b\left(\frac{x+16}{4}\right)$$

$\Rightarrow 2x^2 - bx - 16b = 0$

$\Rightarrow D = 0$ [For tangent]

$\Rightarrow b^2 - 4 \times 2 \times (-16b) = 0$

$\Rightarrow b^2 + 32 \times 4b = 0$

$b = -128$, $b = 0$ (not possible)

66. **(2)** $\frac{(49)^{126}-1}{48}=\frac{((49)^{63}+1)(49^{63}-1)}{48}\left[\because S_n=\frac{a(r^n-1)}{r-1}\right]$

$\therefore$ K = 63

67. **(4)** $\because$ $z = x + iy$

$$\left(\frac{z-1}{2z+i}\right)=\frac{(x-1)+iy}{2(x+iy)+i}$$
$$=\frac{(x-1)+iy}{2x+(2y+1)i}\times\frac{2x-(2y+1)i}{2x-(2y+1)i}$$

$$\text{Re}\left(\frac{z+1}{2z+i}\right)=\frac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2}=1$$

$$\Rightarrow \left(x+\frac{1}{2}\right)^2+\left(y+\frac{3}{4}\right)^2=\left(\frac{\sqrt{5}}{4}\right)^2.$$

68. **(4)** Solution of $x^2 + x + 1 = 0$ is ω, ω^2

So, $\alpha = \omega$ and

$\omega^4 = \omega^3.\omega = 1.\omega = \omega$

$$A^2=\frac{1}{3}\begin{bmatrix}1&1&1\\1&\omega&\omega^2\\1&\omega^2&\omega\end{bmatrix}\begin{bmatrix}1&1&1\\1&\omega&\omega^2\\1&\omega^2&\omega\end{bmatrix}=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$$

$\Rightarrow A^4 = I$

$\Rightarrow A^{30} = A^{28} \times A^3 = A^3$

69. **(1)** Let $e^y = t$

$$e^y\frac{dy}{dx}=\frac{dt}{dx}$$

$\therefore \frac{dt}{dx}-t=e^x$ $\left[\because e^y\frac{dy}{dx}-e^y=e^x\right]$

I.F. $= e^{\int -1.dx} = e^{-x}$

$t(e^{-x})=\int e^x.e^{-x}dx \Rightarrow e^{y-x} = x + c$

Put $x = 0$, $y = 0$, then we get $c = 1$

$e^{y-x} = x + 1$

$y = x + \log_e(x + 1)$

Put $x = 1$ $\therefore$ $y = 1 + \log_e 2$

70. **(1)** From, LMVT for $x \in [-7, -1]$

$$\frac{f(-1)-f(-7)}{(-1+7)}\le 2 \Rightarrow \frac{f(-1)+3}{6}\le 2 \Rightarrow f(-1)\le 9$$

From, LMVT for $x \in [-7, 0]$

$$\frac{f(0)-f(-7)}{(0+7)}\le 2$$

$$\frac{f(0)+3}{7}\le 2 \Rightarrow f(0)\le 11$$

$\therefore f(0) + f(-1) \le 20$

71. **(36)** Let $3^x = t^2$

$$\lim_{t\to 3}\frac{t^2+\frac{27}{t^2}-12}{\frac{1}{t}-\frac{3}{t^2}}$$

$$= \lim_{t\to 3}\frac{t^4-12t^2+27}{t-3}$$

$$= \lim_{t\to 3}\frac{(t^2-3)(t+3)(t-3)}{t-3}$$

$= (3^2 - 3)\ (3 + 3) = 36.$

72. **(30)** Let $(1 - x + x^2 \ldots\ldots x^{2n})\ (1 + x + x^2 \ldots\ldots x^{2n})$

$= a_0 + a_1x + a_2x^2 + \ldots\ldots$

put $x = 1$

$1(2n + 1) = a_0 + a_1 + a_2 + \ldots\ldots a_{2n}$ ...(i)

put $x = -1$

$(2n + 1) \times 1 = a_0 - a_1 + a_2 + \ldots\ldots a_{2n}$ ...(ii)

Adding (i) and (ii), we get,

$4n + 2 = 2(a_0 + a_2 + \ldots\ldots) = 2 \times 61$

$\Rightarrow\ 2n + 1 = 61\ \Rightarrow\ n = 30.$

73. **(3)** $\because$ $f(x)$ is non differentiable at $x = 1, 3, 5$

[$\because$ $|x - 3|$ is not differentiable at $x = 3$]

$\Sigma f(f(x)) = f(f(1) + f(f(3)) + f(f(5))$

$= 1 + 1 + 1 = 3$

74. **(18)** Var $(1, 2, \ldots\ldots, n) = 10$

$$\Rightarrow\ \frac{1^2+2^2+\ldots\ldots+n^2}{n}-\left(\frac{1+2+\ldots\ldots+n}{n}\right)^2=10$$

$$\Rightarrow\ \frac{(n+1)(2n+1)}{6}-\left(\frac{n+1}{2}\right)^2=10$$

$\Rightarrow\ n^2 - 1 = 120\ \Rightarrow\ n = 11$

Var $(2, 4, 6, \ldots\ldots, 2m) = 16 \Rightarrow$ Var $(1, 2, \ldots\ldots, m) = 4$

$\Rightarrow\ m^2 - 1 = 48 \Rightarrow m = 7$

$\Rightarrow\ m + n = 18$

75. **(5)** P will be centroid of ΔABC

$$P\left(\frac{17}{6},\frac{8}{3}\right)\ \Rightarrow\ PQ=\sqrt{(4)^2+(3)^2}=5$$

JEE ADVANCED 2015

1. The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of three sections.
2. **Section 1** contains 8 questions. The answer to each of the questions is a single-digit integer ranging from 0 to 9 (both inclusive).
3. **Section 2** contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONE OR MORE THAN ONE** are correct.
4. **Section 3** contains 2 multiple matching questions. One or more entries in **Column - I** may match with one or more entries in **Column - II**.

PAPER - 1

PHYSICS

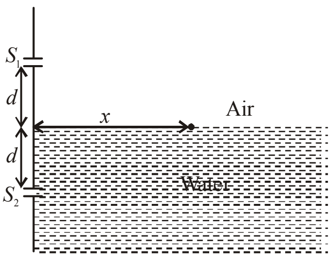
SECTION - I

This section contains **8 questions.** Each question, when worked out will result in one integer from 0 to 9 *(both inclusive).*

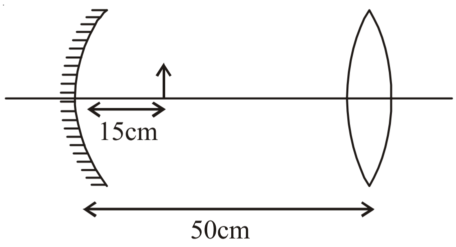
1. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

2. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is

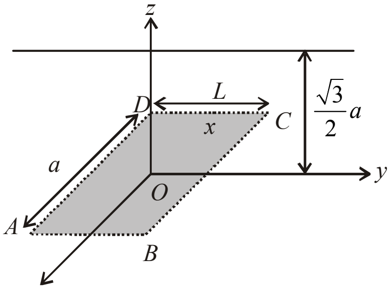
3. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index $= \frac{4}{3}$) as shown in the figure. The positions of maximum on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), $2d$ is the separation between the slits and m is an integer. The value of p is



4. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index $\frac{7}{6}$, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



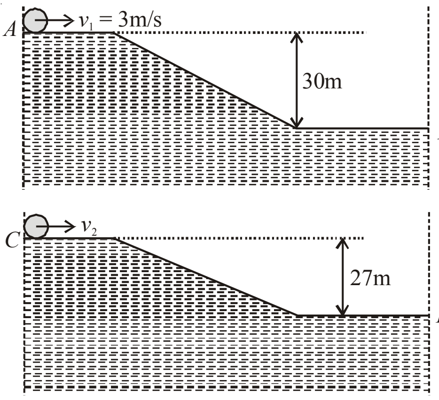
5. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the $y-z$ plane at $z = \frac{\sqrt{3}}{2}$ a (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the $x-y$ plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is



6. Consider a hydrogen atom with its electron in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is (hc = 1242 eV nm)

7. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $\frac{1}{4}^{th}$ of its value of the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

8. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and v_1 = 3 m/s then v_2 in m/s is (g = 10 m/s^2)



SECTION - II

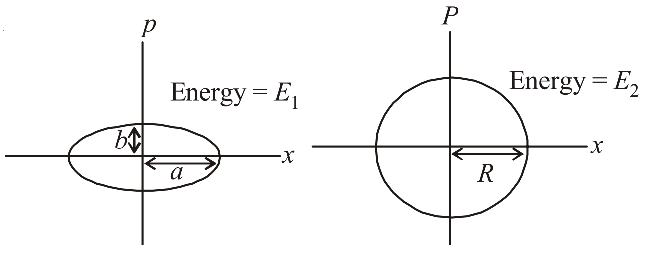
This section contains **10 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

9. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is(are)

(a) $M \propto \sqrt{c}$ (b) $M \propto \sqrt{G}$

(c) $L \propto \sqrt{h}$ (d) $L \propto \sqrt{G}$

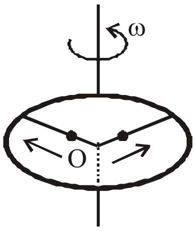
10. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation(s) is(are)



(a) $E_1\omega_1 = E_2\omega_2$ (b) $\frac{\omega_2}{\omega_1} = n^2$

(c) $\omega_1\omega_2 = n^2$ (d) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

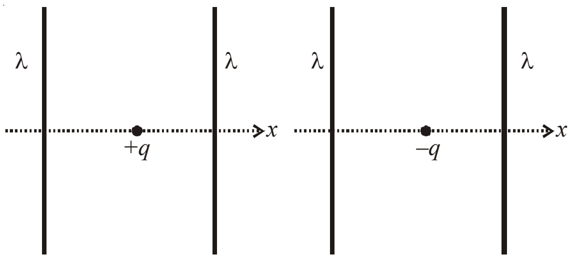
11. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}$R from O. At this instant the distance of the other mass from O is



(a) $\frac{2}{3}$R (b) $\frac{1}{3}$R

(c) $\frac{3}{5}$R (d) $\frac{4}{5}$R

12. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and –q are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are)



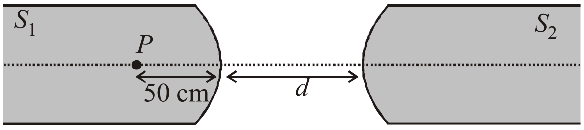
(a) Both charges execute simple harmonic motion

(b) Both charges will continue moving in the direction of their displacement

(c) Charge +q executes simple harmonic motion while charge –q continues moving in the direction of its displacement

(d) Charge –q executes simple harmonic motion while charge +q continues moving in the direction of its displacement

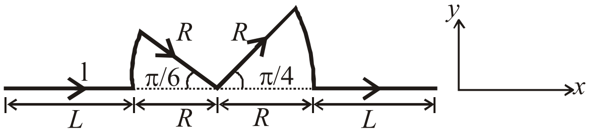
13. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is



(a) 60 cm (b) 70 cm

(c) 80 cm (d) 90 cm

14. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is(are)



(a) If $\vec{B}$ is along $\hat{z}$, $F \propto (L+R)$

(b) If $\vec{B}$ is along $\hat{x}$, $F = 0$

(c) If $\vec{B}$ is along $\hat{y}$, $F \propto (L+R)$

(d) If $\vec{B}$ is along $\hat{z}$, $F = 0$

15. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are)

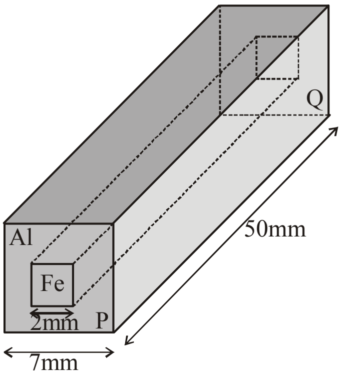
(a) The average energy per mole of the gas mixture is $2RT$

(b) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$

(c) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2

(d) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $\frac{1}{\sqrt{2}}$

16. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are 2.7×10^{-8} Ω m and 1.0×10^{-7} Ω m, respectively. The electrical resistance between the two faces P and Q of the composite bar is

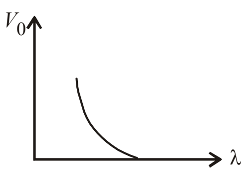


(a) $\frac{2475}{64}\mu\Omega$ (b) $\frac{1875}{64}\mu\Omega$

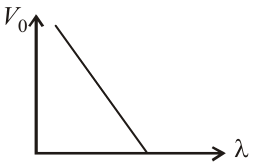
(c) $\frac{1875}{49}\mu\Omega$ (d) $\frac{2475}{132}\mu\Omega$

17. For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and 1/λ.

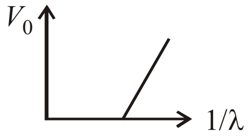
(a)



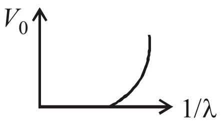
(b)



(c)



(d)



18. Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then :

(a) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm

(b) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm

(c) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm

(d) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm

SECTION - III

This section contains **TWO questions.** Each question contains two columns, **Column I and Column II**. **Column I** has **four** entries (A), (B), (C) and (D). **Column II** has **five entries** (P), (Q), (R), (S) and (T). Match the entries in **Column I** with the entries in **Column II**. One or more entries in **Column I** may match with one or more entries in **Column II**.

19. Match the nuclear processes given in column I with the appropriate option(s) in column II.

	Column I		**Column II**
(A)	Nuclear fusion	(P)	Absorption of thermal neutrons by $^{235}_{92}U$
(B)	Fission in a nuclear reactor	(Q)	$^{60}_{27}Co$ nucleus
(C)	β-decay	(R)	Energy production in stars via hydrogen conversion to helium
(D)	γ-ray emission	(S)	Heavy water
		(T)	Neutrino emission

20. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 constants). Match the potential energies in column I to the corresponding statement(s) in column II.

	Column I		**Column II**
(A)	$U_1(x)=\frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P)	The force acting on the particle is zero at $x = a$
(B)	$U_2(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q)	The force acting on the particle is zero at $x = 0$
(C)	$U_3(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(R)	The force acting on the particle is zero at $x = -a$
(D)	$U_4(x)=\frac{U_0}{2}\left[\frac{x}{a}-\frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(S)	The particle experiences an attractive force towards $x = 0$ in the region $\|x\| < a$
		(T)	The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$

CHEMISTRY

SECTION - I

This section contains **8 questions.** Each question, when worked out will result in one integer from 0 to 9 *(both inclusive).*

1. Among the triatomic molecules/ions, $BeCl_2$, N_3^-, N_2O, NO_2^+, O_3, SCl_2, ICl_2^-, I_3^- and XeF_2, the total number of linear molecule(s)/ion(s) where the hybridization of the central atom does not have contribution from the d-orbital(s) is

[Atomic number : S = 16, Cl = 17, I = 53 and Xe = 54]

2. Not considering the electronic spin, the degeneracy of the second excited state (n = 3) of H atom is 9, while the degeneracy of the second excited state of H^- is

3. All the energy released from the reaction $X \rightarrow Y$, $\Delta_r G° = -193$ kJ mol^{-1} is used for oxidizing M^+ as $M^+ \rightarrow M^{3+} + 2e^-$, $E° = -0.25$ V

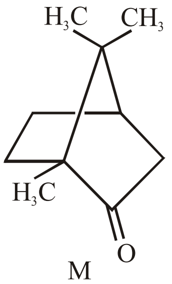
Under standard conditions, the number of moles of M^+ oxidized when one mole of X is converted to Y is

[F = 96500 C mol^{-1}]

4. If the freezing point of a 0.01 molal aqueous solution of a cobalt(III) chloride-ammonia complex (which behaves as a strong electrolyte) is –0.0558°C, the number of chloride(s) in the coordination sphere of the complex is

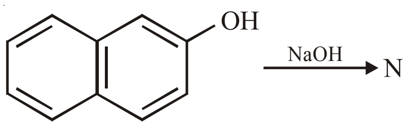
[K_f of water = 1.86 K kg mol^{-1}]

5. The total number of stereoisomers that can exist for M is



M

6. The number of resonance structures for N is



7. The total number of lone pairs of electrons in N_2O_3 is

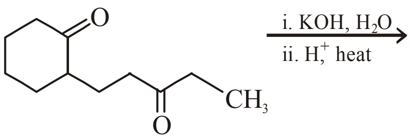
8. For the octahedral complexes of Fe^{3+} in SCN^- (thiocyanato-S) and in CN^- ligand environments, the difference between the spin-only magnetic moments in Bohr magnetons (when approximated to the nearest integer) is

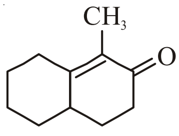
[Atomic number of Fe = 26]

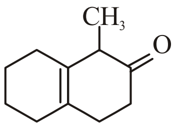
SECTION - II

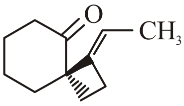
This section contains **10 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

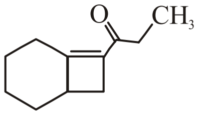
9. The major product of the following reaction is



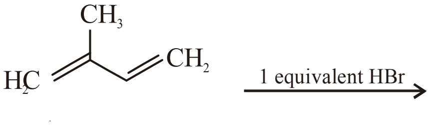
(a) 

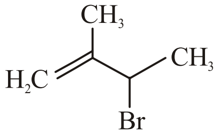
(b) 

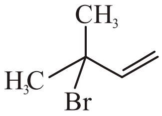
(c) 

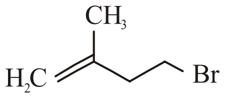
(d) 

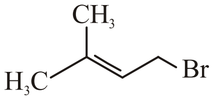
10. In the following reaction, the major product is



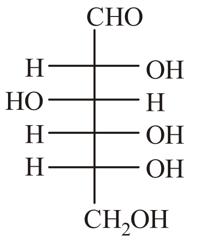
(a) 

(b) 

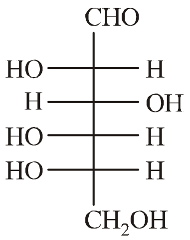
(c) 

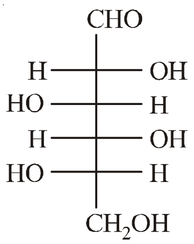
(d) 

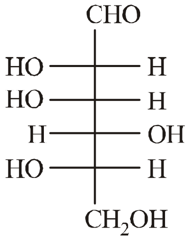
11. The structure of D-(+)-glucose is

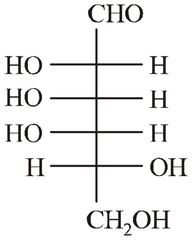


The structure of L-(–)-glucose is

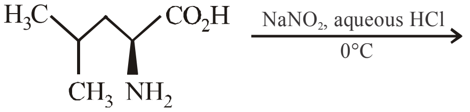
(a) 

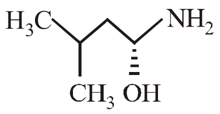
(b) 

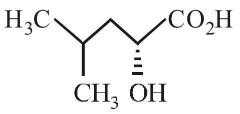
(c) 

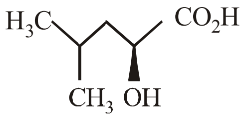
(d) 

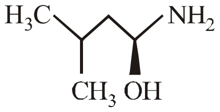
12. The major product of the reaction is



(a) 

(b) 

(c) 

(d) 

13. The correct statement(s) about Cr^{2+} and Mn^{3+} is(are)
[Atomic numbers of Cr = 24 and Mn = 25]

(a) Cr^{2+} is a reducing agent
(b) Mn^{3+} is an oxidizing agent
(c) Both Cr^{2+} and Mn^{3+} exhibit d^4 electronic configuration
(d) When Cr^{2+} is used as a reducing agent, the chromium ion attains d^5 electronic configuration

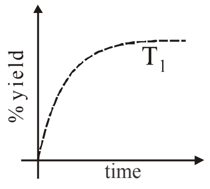
14. Copper is purified by electrolytic refining of blister copper. The correct statement(s) about this process is(are)

(a) Impure Cu strip is used as cathode
(b) Acidified aqueous $CuSO_4$ is used as electrolyte
(c) Pure Cu deposits at cathode
(d) Impurities settle as anode-mud

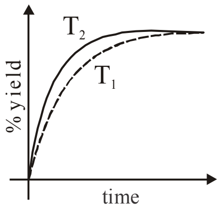
15. Fe^{3+} is reduced to Fe^{2+} by using

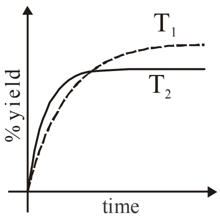
(a) H_2O_2 in presence of NaOH
(b) Na_2O_2 in water
(c) H_2O_2 in presence of H_2SO_4
(d) Na_2O_2 in presence of H_2SO_4

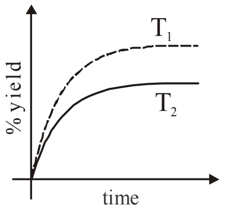
16. The %yield of ammonia as a function of time in the reaction $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g), \Delta H < 0$ at (P, T_1) is given below

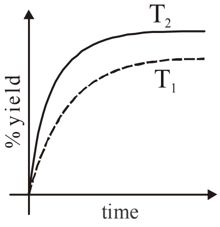


If this reaction is conducted at (P, T_2), with $T_2 > T_1$, the %yield of ammonia as a function of time is represented by

(a) 

(b) 

(c) 

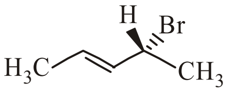
(d) 

17. If the unit cell of a mineral has cubic close packed (*ccp*) array of oxygen atoms with *m* fraction of octahedral holes occupied by aluminium ions and *n* fraction of tetrahedral holes occupied by magnesium ions, *m* and *n*, respectively, are

(a) $\frac{1}{2}, \frac{1}{8}$ (b) $1, \frac{1}{4}$

(c) $\frac{1}{2}, \frac{1}{2}$ (d) $\frac{1}{4}, \frac{1}{8}$

18. Compound(s) that on hydrogenation produce(s) optically inactive compound(s) is (are)

(a)


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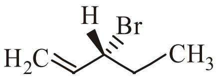

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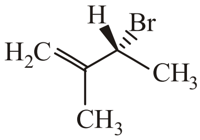

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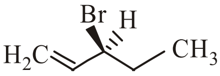

SECTION - III

This section contains **TWO questions.** Each question contains two columns, **Column I and Column II**. **Column I** has **four** entries (A), (B), (C) and (D). **Column II** has **five entries** (P), (Q), (R), (S) and (T). Match the entries in **Column I** with the entries in **Column II**. One or more entries in **Column I** may match with one or more entries in **Column II**.

19. Match the anionic species given in Column-I that are present in the ore(s) given in Column-II.

	Column-I		**Column-II**
(A)	Carbonate	(P)	Siderite
(B)	Sulphide	(Q)	Malachite
(C)	Hydroxide	(R)	Bauxite
(D)	Oxide	(S)	Calamine
		(T)	Argentite

20. Match the thermodynamic processes given under Column-I with the expressions given under Column-II.

	Column-I		**Column-II**
(A)	Freezing of water at 273 K and 1 atm	(P)	q = 0
(B)	Expansion of 1 mol of an ideal gas into a vacuum under isolated conditions	(Q)	w = 0
(C)	Mixing of equal volumes of two ideal gases at constant temperature and pressure in an isolated container	(R)	$\Delta S_{sys} < 0$
(D)	Reversible heating of $H_2(g)$ at 1 atm from 300 K to 600 K, followed by reversible cooling to 300 K at 1 atm	(S)	$\Delta U = 0$
		(T)	$\Delta G = 0$

MATHEMATICS

SECTION - I

This section contains **8 questions.** Each question, when worked out will result in one integer from 0 to 9 *(both inclusive).*

1. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is

2. Let $f: R \to R$ be a function defined by $f(x) = \begin{cases} [x], & x \le 2 \\ 0, & x > 2 \end{cases}$

where $[x]$ is the greatest integer less than or equal to x, if

$I = \int_{-1}^{2} \frac{xf(x^2)}{2 + f(x+1)} dx$, then the value of $(4I - 1)$ is

3. A cylindrical container is to be made from certain solid material with the following constraints: It has a fixed inner volume of V mm^3, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container.

If the volume of the material used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\frac{V}{250\pi}$ is

4. Let $F(x) = \int_{x}^{x^2 + \frac{\pi}{6}} 2\cos^2 t(dt)$ for all $x \in R$ and $f: \left[0, \frac{1}{2}\right] \to [0, \infty)$ be a continuous function. For $a \in \left[0, \frac{1}{2}\right]$, if $F'(a) + 2$ is the area of the region bounded by $x = 0, y = 0, y = \text{f}(x)$ and $x = a$, then $f(0)$ is

5. The number of distinct solutions of the equation

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

in the interval $[0, 2\pi]$ is

6. Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$, then the distance between A and B is

7. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is

8. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is

SECTION - II

This section contains **10 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

9. Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha ?$$

(a) -4 (b) 9

(c) -9 (d) 4

10. In R^3, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be the plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true?

(a) $2\alpha + \beta + 2\gamma + 2 = 0$ (b) $2\alpha - \beta + 2\gamma + 4 = 0$

(c) $2\alpha + \beta - 2\gamma - 10 = 0$ (d) $2\alpha - \beta + 2\gamma - 8 = 0$

11. In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie (s) on M?

(a) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$ (b) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

(c) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$ (d) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

12. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P?

(a) $(4, 2\sqrt{2})$ (b) $(9, 3\sqrt{2})$

(c) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (d) $(1, \sqrt{2})$

13. Let $y(x)$ be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. If $y(0) = 2$, then which of the following statement is (are) true?

(a) $y(-4) = 0$ (b) $y(-2) = 0$

(c) $y(x)$ has a critical point in the interval $(-1, 0)$

(d) $y(x)$ has no critical point in the interval $(-1, 0)$

14. Consider the family of all circles whose centers lie on the straight line $y = x$. If this family of circle is represented by the differential equation $Py'' + Qy' + 1 = 0$, where P, Q are functions of x, y and y' $\left(\text{here } y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}\right)$, then which of the following statements is (are) true?

(a) $P = y + x$ (b) $P = y - x$

(c) $P + Q = 1 - x + y + y' + (y')^2$ (d) $P - Q = x + y - y' - (y')^2$

15. Let g : $R \rightarrow R$ be a differentiable function with $g(0) = 0$, $g'(0) = 0$ and $g'(1) \neq 0$. Let $f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

and $h(x) = e^{|x|}$ for all $x \in R$. Let $(foh)(x)$ denote $f(h(x))$ and $(hof)(x)$ denote $h(f(x))$. Then which of the following is (are) true?

(a) f is differentiable at $x = 0$

(b) h is differentiable at $x = 0$

(c) foh is differentiable at $x = 0$

(d) hof is differentiable at $x = 0$

16. Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x \in R$ and $g(x) = \frac{\pi}{2}$ $\sin x$ for all $x \in R$. Let $(fog)(x)$ denote $f(g(x))$ and $(gof)(x)$ denote $g(f(x))$. Then which of the following is (are) true?

(a) Range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(b) Range of fog is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(c) $\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$

(d) There is an $x \in R$ such that $(gof)(x) = 1$

17. Let ΔPQR be a triangle. Let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$, $\vec{b}.\vec{c} = 24$, then which of the following is (are) true?

(a) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$ (b) $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$

(c) $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$ (d) $\vec{a}.\vec{b} = -72$

18. Let X and Y be two arbitrary, 3 × 3, non-zero, skew-symmetric matrices and Z be an arbitrary 3 × 3, non zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

(a) $Y^3Z^4 - Z^4Y^3$ (b) $X^{44} + Y^{44}$

(c) $X^4Z^3 - Z^3X^4$ (d) $X^{23} + Y^{23}$

SECTION - III

This section contains **TWO questions.** Each question contains two columns, **Column I and Column II**. **Column I** has **four** entries (A), (B), (C) and (D). **Column II** has **five entries** (P), (Q), (R), (S) and (T). Match the entries in **Column I** with the entries in **Column II**. One or more entries in **Column I** may match with one or more entries in **Column II**.

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19. Match the following

Column I	**Column II**
(A) In R^2, if the magnitude of the projection vector of the vector $\alpha\hat{i}+\beta\hat{j}$ on $\sqrt{3}\hat{i}+\hat{j}$ is $\sqrt{3}$ and if $\alpha = 2+\sqrt{3}\beta$, then possible value of $\lvert\alpha\rvert$ is/are	(P) 1
(B) Let *a* and *b* be real numbers such that the function $f(x)=\begin{cases}-3ax^2-2, & x<1\\ bx+a^2, & x\ge 1\end{cases}$ if differentiable for all $x \in R$ Then possible value of *a* is (are)	(Q) 2
(C) Let $\omega \neq 1$ be a complex cube root of unity. If $(3-3\omega+2\omega^2)^{4n+3}+(2+3\omega-3\omega^2)^{4n+3}$ $+(-3+2\omega+3\omega^2)^{4n+3}=0,$ then possible value (s) of n is (are)	(R) 3
(D) Let the harmonic mean of two positive real numbers *a* and *b* be 4. If *q* is a positive real number such that a, 5, *q*, *b* is an arithmetic progression, then the value(s) of $\lvert q-a \rvert$ is (are)	(S) 4
	(T) 5

20. Match the following.

Column I	**Column II**
(A) In a triangle ΔXYZ, let a, b, and c be the lengths of the sides opposite to the angles *X*, *Y* and *Z*, respectively. If $2(a^2-b^2)=c^2$ and $\lambda = \frac{\sin(X-Y)}{\sin Z}$, then possible values of n for which $\cos(n\pi\lambda)=0$ is (are)	(P) 1
(B) In a triangle ΔXYZ, let *a*, *b* and *c* be the lengths of the sides opposite to the angles *X*, *Y*, and *Z* respectively. If $1+\cos 2X-2\cos 2Y$ $=2\sin X \sin Y$, then possible value (s) of $\frac{a}{b}$ is (are)	(Q) 2
(C) In R^2, let $\sqrt{3}i+\hat{j},\hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ be the position vectors of *X*, *Y*, and *Z* with respect to the origin O, respectively. If the distance of *Z* from the bisector of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\frac{3}{\sqrt{2}}$, then possible value(s) of $\lvert\beta\rvert$ is (are)	(R) 3
(D) Suppose that $F(\alpha)$ denotes the area of the region bounded by $x=0, x=2, y^2=4x$ and $y=\lvert \alpha x-1\rvert+\lvert \alpha x-2\rvert+\alpha x$, where $\alpha \in \{0, 1\}$. Then the value(s) of $F(\alpha)+\frac{8}{3}\sqrt{2}$, when $\alpha=0$ and $\alpha=1$, is (are)	(S) 5
	(T) 6

PAPER - 2

1. The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of three sections.
2. **Section 1** contains 8 questions. The answer to each of the questions is a single-digit integer ranging from 0 to 9 (both inclusive).
3. **Section 2** contains 8 multiple choice questions. Each question has four choice (a), (b), (c) and (d) out of which **ONE OR MORE THAN ONE** are correct.
4. **Section 3** contains 2 paragraphs each describing theory, experiment and data etc. four questions relate to two paragraphs with two questions on each paragraph. Each question pertaining to a particular passage should have one or more correct answer among the four given choices (a), (b), (c) and (d).

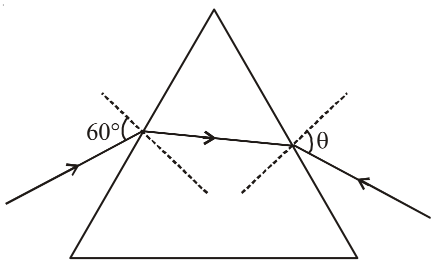
PHYSICS

SECTION - I

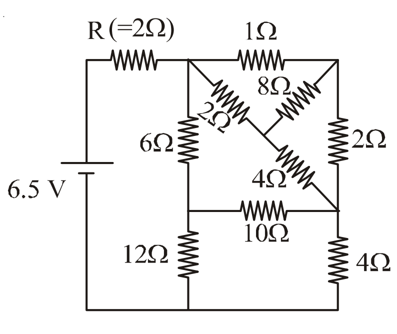
This section contains **8 questions.** Each question, when worked out will result in one integer from 0 to 9 *(both inclusive).*

1. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where N(t) is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rates of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is

2. The monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is

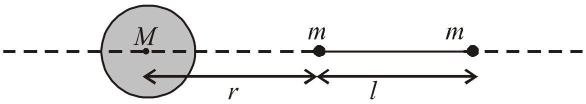


3. In the following circuit, the current through the resistor R (= 2 Ω) is I amperes. The value of I is



4. An electron is an excited state of Li^{2+} ion has angular momentum $3h/2\pi$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is

5. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is



6. The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t,)$ where $\alpha = 0.2\ s^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of $E(t)$ at $t = 5$ s is

7. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If , $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is

8. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\frac{\pi}{3}, \frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is

SECTION - II

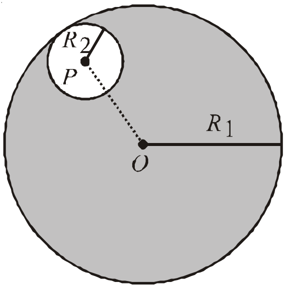
This section contains **8 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

9. In terms of potential difference *V*, electric current *I*, permittivity ε_0, permeability μ_0 and speed of light *c*, the dimensionally correct equation(s) is(are)

(a) $\mu_0 I^2 = \varepsilon_0 V^2$ (b) $\mu_0 I = \mu_0 V$

(c) $I = \varepsilon_0 c V$ (d) $\mu_0 c I = \varepsilon_0 V$

10. Consider a uniform spherical charge distribution of radius R_1 centred at the origin *O*. In this distribution, a spherical cavity of radius R_2, centred at *P* with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are)



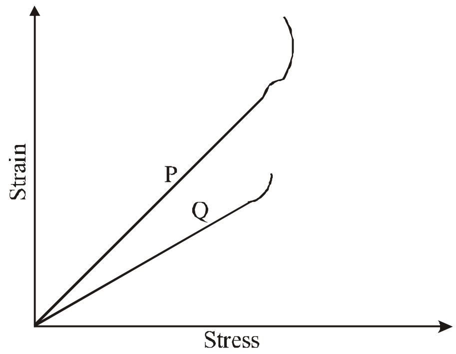
(a) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

(b) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

(c) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(d) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

11. In plotting stress versus strain curves for two materials *P* and *Q*,a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is (are)



(a) *P* has more tensile strength than *Q*

(b) *P* is more ductile than *Q*

(c) *P* is more brittle than *Q*

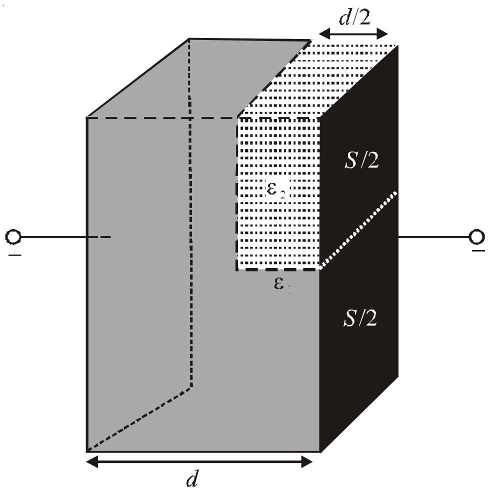
(d) The Young's modulus of *P* is more than that of *Q*

12. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If *P*(*r*) is the pressure at $r(r < R)$, then the correct option(s) is (are)

(a) $P(r=0)=0$ (b) $\dfrac{P(r=3R/4)}{P(r=2R/3)} = \dfrac{63}{80}$

(c) $\dfrac{P(r=3R/5)}{P(r=2R/5)} = \dfrac{16}{21}$ (d) $\dfrac{P(r=R/2)}{P(r=R/3)} = \dfrac{20}{27}$

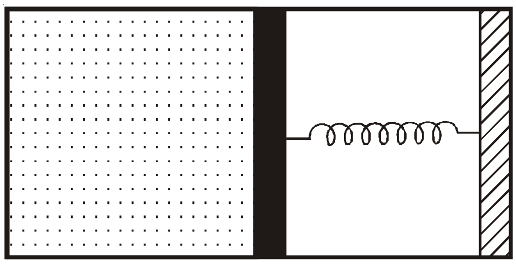
13. A parallel plate capacitor having plates of area *S* and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative primitivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\dfrac{C_2}{C_1}$ is



(a) 6/5 (b) 5/3

(c) 7/5 (d) 7/3

14. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance *x*. Ignoring the friction between the piston and the cylinder, the correct statement(s) is (are)



(a) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\dfrac{1}{4} P_1 V_1$

(b) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(c) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3} P_1V_1$

(d) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6} P_1V_1$

15. A fission reaction is given by ${}^{236}_{92}\text{U} \rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + x + y$, where x and y are two particles. Considering ${}^{236}_{92}\text{U}$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_x(2 MeV) and K_y(2 MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}\text{U}$, ${}^{140}_{54}\text{Xe}$ and ${}^{94}_{38}\text{Sr}$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct option(s) is(are)

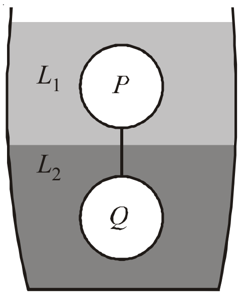
(a) $x = n, y = n$, $K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV

(b) $x = p, y = e^-$, $K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV

(c) $x = p, y = n$, $K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV

(d) $x = n, y = n$, $K_{Sr} = 86$ MeV, $K_{Xe} = 129$ MeV

16. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then



(a) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$

(b) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$

(c) $\vec{V}_P \cdot \vec{V}_Q > 0$

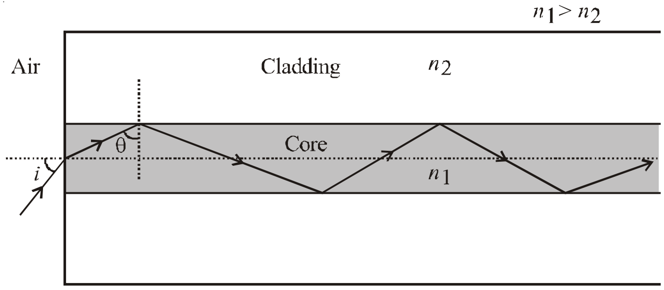
(d) $\vec{V}_P \cdot \vec{V}_Q < 0$

SECTION - III

This section contains 2 paragraphs, each describing theory, experiments, data etc. four questions related to the two paragraphs with two questions on each paragraph. Each question has one or more than one correct answer(s) among the four given options (a), (b), (c) and (d).

PARAGRAPH 1

Light guidance in an optical fibre can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as sin i_m.



17. For two structure namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is(are)

(a) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(b) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(c) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

(d) NA of S_1 placed in air is the same as that of S_2 placed in water

18. If two structure of same cross-sectional area, but different numerical apertures NA_1 and NA_2($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

(a) $\frac{NA_1\, NA_2}{NA_1 + NA_2}$

(b) $NA_1 + NA_2$

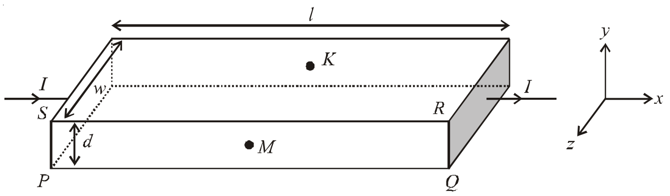
(c) NA_1

(d) NA_2

PARAGRAPH 2

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are ℓ, w and d, respectively.

A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface $PQRS$ and appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross-section of the strip and carried by electrons.



19. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2 respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is(are)

(a) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$

(b) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

(c) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$

(d) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

20. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are)

(a) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

(b) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(c) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

(d) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

CHEMISTRY

SECTION - I

This section contains **8 questions.** Each question, when worked out will result in one integer from 0 to 9 *(both inclusive).*

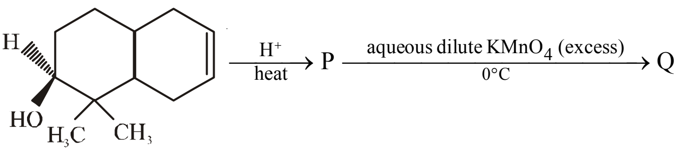
1. Three moles of B_2H_6 are completely reacted with methanol. The number of moles of boron containing product formed is

2. The molar conductivity of a solution of a weak acid HX (0.01 M) is 10 times smaller than the molar conductivity of a solution of a weak acid HY (0.10 M). If $\lambda^0_{x^-} \approx \lambda^0_{y^-}$ the difference in their pK_a values, $pK_a(HX) - pK_a(HY)$, is (consider degree of ionization of both acids to be <<1)

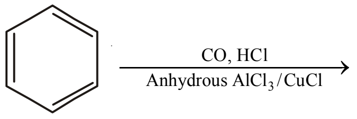
3. A closed vessel with rigid walls contains 1 mol of $^{238}_{92}U$ and 1 mol of air at 298 K. Considering complete decay of $^{238}_{92}U$ to $^{206}_{82}Pb$, the ratio of the final pressure to the initial pressure of the system at 298 K is

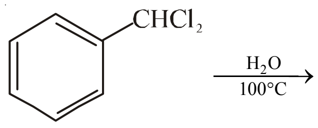
4. In dilute aqueous H_2SO_4, the complex diaquodioxalatoferrate(II) is oxidized by MnO_4^-. For this reaction, the ratio of the rate of change of $[H^+]$ to the rate of change of $[MnO_4^-]$ is

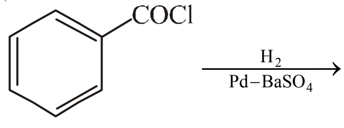
5. The number of hydroxyl group(s) in Q is

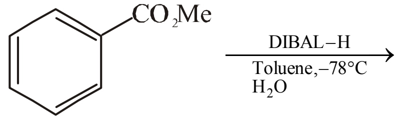


6. Among the following, the number of reaction(s) that produce(s) benzaldehyde is

I.


II.


III.


IV.


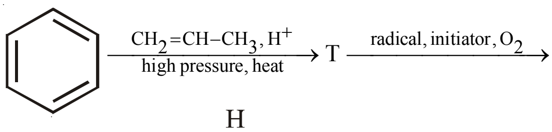
7. In the complex acetylbromidodicarbonylbis (triethylphosphine) iron (II), the number of Fe–C bond(s) is

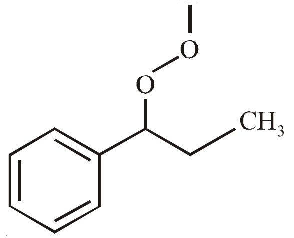
8. Among the complex ions, $[Co(NH_2{-}CH_2{-}CH_2{-}NH_2)_2Cl_2]^+$, $[CrCl_2(C_2O_4)_2]^{3-}$, $[Fe(H_2O)_4(OH)_2]^+$, $[Fe(NH_3)_2(CN)_4]^-$, $[Co(NH_2{-}CH_2{-}CH_2{-}NH_2)_2(NH_3)Cl]^{2+}$ and $[Co(NH_3)_4(H_2O)Cl]^{2+}$, the number of complex ion(s) that show(s) *cis-trans* isomerism is

SECTION - II

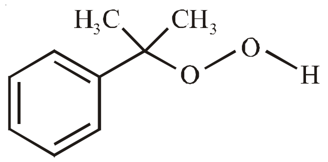
This section contains **8 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

9. The major product U in the following reactions is

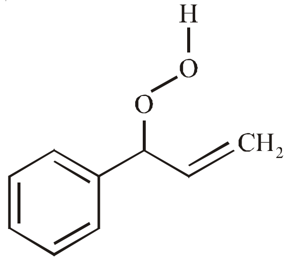


(a)


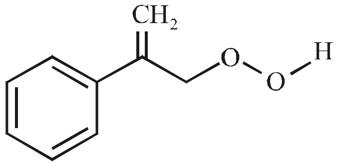
(b)



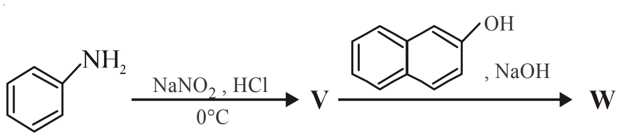
(c)



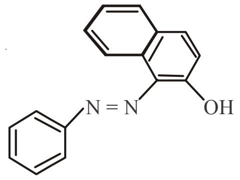
(d)



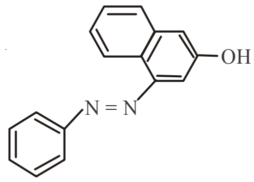
10. In the following reactions, the major product W is



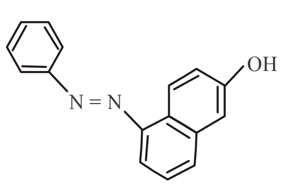
(a)



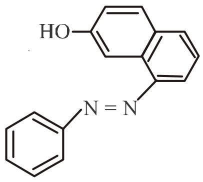
(b)



(c)



(d)



11. The correct statement(s) regarding, (i) HClO, (ii) $HClO_2$, (iii) $HClO_3$ and (iv) $HClO_4$, is(are)

- (a) The number of Cl=O bonds in (ii) and (iii) together is two
- (b) The number of lone pairs of electrons on Cl in (ii) and (iii) together is three
- (c) The hybridization of Cl in (iv) is sp^3
- (d) Amongst (i) to (iv), the strongest acid is (i)

12. The pair(s) of ions where BOTH the ions are precipitated upon passing H_2S gas in presence of dilute HCl, is(are)

- (a) Ba^{2+}, Zn^{2+}
- (b) Bi^{3+}, Fe^{3+}
- (c) Cu^{2+}, Pb^{2+}
- (d) Hg^{2+}, Bi^{3+}

13. Under hydrolytic conditions, the compounds used for preparation of linear polymer and for chain termination, respectively, are

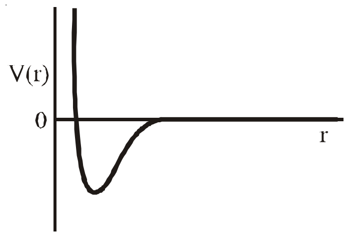
- (a) CH_3SiCl_3 and $Si(CH_3)_4$
- (b) $(CH_3)_2SiCl_2$ and $(CH_3)_3SiCl$
- (c) $(CH_3)_2SiCl_2$ and CH_3SiCl_3
- (d) $SiCl_4$ and $(CH_3)_3SiCl$

14. When O_2 is adsorbed on a metallic surface, electron transfer occurs from the metal to O_2. The true statement(s) regarding this adsorption is(are)

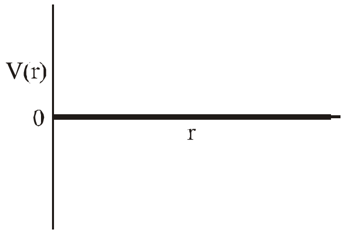
- (a) O_2 is physisorbed
- (b) Heat is released
- (c) Occupancy of π^*_{2p} of O_2 is increased
- (d) Bond length of O_2 is increased

15. One mole of a monoatomic real gas satisfies the equation p(V – b) = RT where b is a constant. The relationship of interatomic potential V(r) and interatomic distance r for the gas is given by

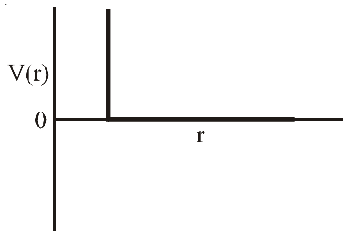
(a)



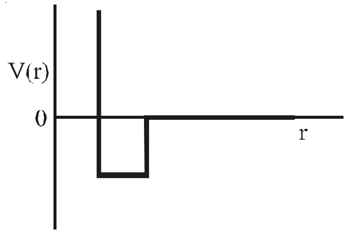
(b)



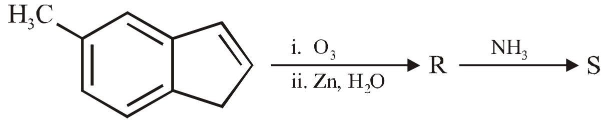
(c)

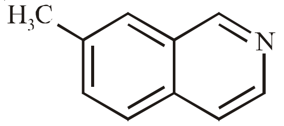


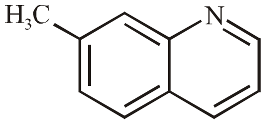
(d)

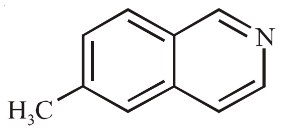


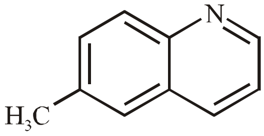
16. In the following reactions, the product S is



(a)


(b)


(c)


(d)


SECTION - III

This section contains 2 paragraphs, each describing theory, experiments, data etc. four questions related to the two paragraphs with two questions on each paragraph. Each question has one or more than one correct answer(s) among the four given options (a), (b), (c) and (d).

PARAGRAPH 1

When 100 mL of 1.0 M HCl was mixed with 100 mL of 1.0 M NaOH in an insulated beaker at constant pressure, a temperature increase of 5.7 °C was measured for the beaker and its contents (Expt.1). Because the enthalpy of neutralization of a strong acid with a strong base is a constant (–57.0 kJ mol^{-1}), this experiment could be used to measure the calorimeter constant. In a second experiment (Expt. 2), 100 mL of 2.0 M acetic acid ($K_a = 2.0 \times 10^{-5}$) was mixed with 100 mL of 1.0 M NaOH (under identical conditions to Expt. 1) where a temperature rise of 5.6 °C was measured.

(Consider heat capacity of all solutions as 4.2 J g^{-1} K^{-1} and density of all solutions as 1.0 g mL^{-1})

17. Enthalpy of dissociation (in kJ mol^{-1}) of acetic acid obtained from the Expt.2 is

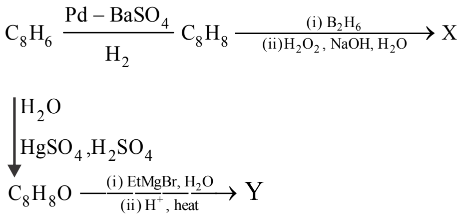
(a) 1.0 (b) 10.0
(c) 24.5 (d) 51.4

18. The pH of the solution after Expt. 2 is

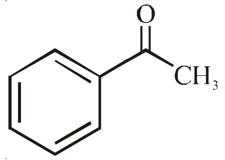
(a) 2.8 (b) 4.7
(c) 5.0 (d) 7.0

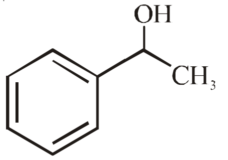
PARAGRAPH 2

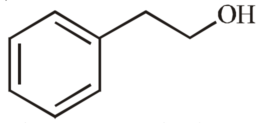
In the following reactions

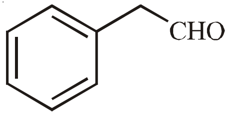


19. Compound X is

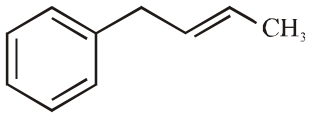
(a)


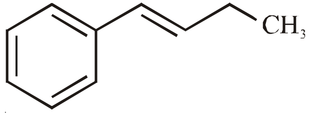
(b)


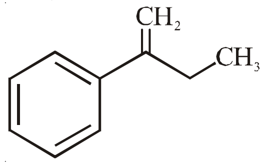
(c)


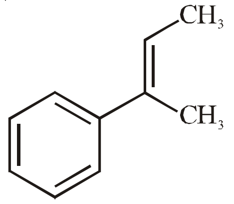
(d)


20. The major compound Y is

(a)


(b)


(c)


(d)


MATHEMATICS

SECTION - I

This section contains **8 questions.** Each question, when worked out will result in one integer from 0 to 9 *(both inclusive).*

1. Let m and n be two positive integers greater than 1. If $\lim\limits_{\alpha\to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right) = -\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is

2. If $\alpha = \int_0^1 (e^{9x+3\tan^{-1}x})\left(\frac{12+9x^2}{1+x^2}\right)dx$ where $\tan^{-1}x$ takes only principal values, then the value of $\left(\log_e|1+\alpha| - \frac{3\pi}{4}\right)$ is

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^{x} f(t)dt$ for all $x \in [-1, 2]$ and $G(x) = \int_{-1}^{x} t\,|f(f(t))|\,dt$ for all $x \in [-1, 2]$. If $\lim_{x \to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is

4. Suppose that $\vec{p}, \vec{q}$ and $\vec{r}$ are three non-coplanar vectors in $\mathbb{R}^3$. Let the components of a vector $\vec{s}$ along $\vec{p}, \vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p}+\vec{q}+\vec{r}), (\vec{p}-\vec{q}+\vec{r})$ and $(-\vec{p}-\vec{q}+\vec{r})$ are x, y and z, respectively, then the value of $2x + y + z$ is

5. For any integer k, let $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i \sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression $\dfrac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}|}$ is

6. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

7. The coefficient of x^9 in the expansion of $(1 + x)(1 + x^2)(1 + x^3) \ldots (1 + x^{100})$ is

8. Suppose that the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1 > 0$ and $f_2 < 0$. Let P_1 and P_2 be two parabolas with a common vertex at (0, 0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangent to P_2 which passes through $(f_1, 0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2} + m_2^2\right)$ is

SECTION - II

This section contains **8 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

9. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is (are)

(a) $\cos\beta > 0$ (b) $\sin\beta < 0$

(c) $\cos(\alpha + \beta) > 0$ (d) $\cos\alpha < 0$

10. Let E_1 and E_2 be two ellipses whose centers are at the origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y - 1)^2 = 2$. The straight line $x + y = 3$ touches the curves S, E_1 and E_2 at P, Q and R respectively. Suppose that PQ = PR = $\frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2, respectively, then the correct expression(s) is (are)

(a) $e_1^2 + e_2^2 = \frac{43}{40}$ (b) $e_1e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

(c) $\left|e_1^2 - e_2^2\right| = \frac{5}{8}$ (d) $e_1e_2 = \frac{\sqrt{3}}{4}$

11. Consider the hyperbola $H : x^2 - y^2 = 1$ and a circle S with center $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the x-axis at point M. If (l, m) is the centroid of the triangle PMN, then the correct expression(s) is(are)

(a) $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$ for $x_1 > 1$

(b) $\frac{dm}{dx_1} = \frac{x_1}{3\left(\sqrt{x_1^2 - 1}\right)}$ for $x_1 > 1$

(c) $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}$ for $x_1 > 1$

(d) $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

12. The option(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} e^t\left(\sin^6 at + \cos^4 at\right)dt}{\int_0^{\pi} e^t\left(\sin^6 at + \cos^4 at\right)dt} = L?$$

(a) $a = 2, \mathrm{L} = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$ (b) $a = 2, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

(c) $a = 4, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$ (d) $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

13. Let f, g: $[-1, 2] \to \mathbb{R}$ be continuous functions which are twice differentiable on the interval $(-1, 2)$. Let the values of f and g at the points -1, 0 and 2 be as given in the following table:

	$x = -1$	$x = 0$	$x = 2$
$f(x)$	3	6	0
$g(x)$	0	1	-1

In each of the intervals $(-1, 0)$ and $(0, 2)$ the function $(f - 3g)''$ never vanishes. Then the correct statement(s) is(are)

(a) $f'(x) - 3g'(x) = 0$ has exactly three solutions in $(-1, 0) \cup (0, 2)$

(b) $f'(x) - 3g'(x) = 0$ has exactly one solution in $(-1, 0)$

(c) $f'(x) - 3g'(x) = 0$ has exactly one solution in $(0, 2)$

(d) $f'(x) - 3g'(x) = 0$ has exactly two solutions in $(-1, 0)$ and exactly two solutions in $(0, 2)$

14. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Then the correct expression(s) is(are)

(a) $\int_0^{\pi/4} xf(x)dx = \frac{1}{12}$ (b) $\int_0^{\pi/4} f(x)dx = 0$

(c) $\int_0^{\pi/4} xf(x)dx = \frac{1}{6}$ (d) $\int_0^{\pi/4} f(x)dx = 1$

15. Let $f'(x) = \frac{192x^3}{2 + \sin^4 \pi x}$ for all $x \in \mathbb{R}$ with $f\left(\frac{1}{2}\right) = 0$.

If $m \leq \int_{1/2}^{1} f(x)dx \leq M$, then the possible values of m and M are

(a) $m = 13, M = 24$ (b) $m = \frac{1}{4}, M = \frac{1}{2}$

(c) $m = -11, M = 0$ (d) $m = 1, M = 12$

16. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) *a* subset(s) of S?

(a) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (b) $\left(-\frac{1}{\sqrt{5}}, 0\right)$

(c) $\left(0, \frac{1}{\sqrt{5}}\right)$ (d) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

SECTION - III

This section contains 2 paragraphs, each describing theory, experiments, data etc. four questions related to the two paragraphs with two questions on each paragraph. Each question has one or more than one correct answer(s) among the four given options (a), (b), (c) and (d).

PARAGRAPH 1

Let F : $\mathbb{R} \to \mathbb{R}$ be a thrice differentiable function. Suppose that $F(1) = 0$, $F(3) = -4$ and $F(x) < 0$ for all $x \in \left(\frac{1}{2}, 3\right)$. Let $f(x) = xF(x)$ for all $x \in \mathbb{R}$.

17. The correct statement(s) is(are)

(a) $f'(1) < 0$

(b) $f(2) < 0$

(c) $f'(x) \neq 0$ for any $x \in (1, 3)$

(d) $f'(x) = 0$ for some $x \in (1, 3)$

18. If $\int_1^3 x^2 F'(x)dx = -12$ and $\int_1^3 x^3 F''(x)dx = 40$, then the correct expression(s) is (are)

(a) $9f'(3) + f'(1) - 32 = 0$ (b) $\int_1^3 f(x)dx = 12$

(c) $9f'(3) - f'(1) + 32 = 0$ (d) $\int_1^3 f(x)dx = -12$

PARAGRAPH 2

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

19. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

(a) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$
(b) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$
(c) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$
(d) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

20. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)

(a) $n_1 = 4$ and $n_2 = 6$
(b) $n_1 = 2$ and $n_2 = 3$
(c) $n_1 = 10$ and $n_2 = 20$
(d) $n_1 = 3$ and $n_2 = 6$

SOLUTIONS

Paper - 1

PHYSICS

1. (2) $\frac{P_A}{P_B}=\frac{A_A}{A_B}\frac{T_A^4}{T_B^4}=\frac{A_A}{A_B}\times\frac{\lambda_B^4}{\lambda_A^4}$

$\therefore \frac{\lambda_A}{\lambda_B}\left[\frac{A_A}{A_B}\times\frac{P_B}{P_A}\right]^{\frac{1}{4}}=\left[\frac{R_A^2}{R_B^2}\times\frac{P_B}{P_A}\right]^{\frac{1}{4}}=\left[\frac{400\times400}{10^4}\right]^{\frac{1}{4}}$

$\therefore \frac{\lambda_A}{\lambda_B}=2$

2. (3) $N_o \xrightarrow{T} \frac{N_o}{2} \xrightarrow{T} \frac{N_o}{4} \xrightarrow{T} \frac{N_o}{8}$

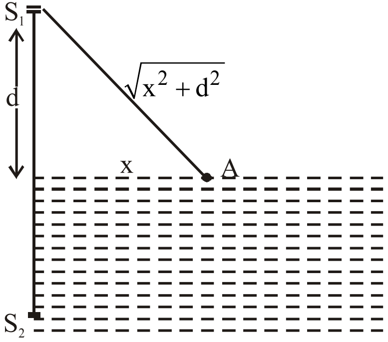
100% 50% 25% 12.5%

Three half life are required. Therefore $n = 3$

3. (3) For maxima

Path defference = $m\lambda$

$\therefore \quad S_2A - S_1A = m\lambda$



$\therefore \quad \left[(n-1)\sqrt{d^2+x^2}+\sqrt{d^2+x^2}\right]-\sqrt{d^2-x^2}=m\lambda$

$\therefore \quad (n-1)\sqrt{(d^2+x^2)}=m\lambda$

$\therefore \quad \left(\frac{4}{3}-1\right)\sqrt{d^2+x^2}=m\lambda$

$\therefore \quad \sqrt{d^2+x^2}=3m\lambda$

$\therefore \quad d^2+x^2=9m^2\lambda^2$

$\therefore \quad x^2=9m^2\lambda^2-d^2$

$\therefore \quad p^2=9 \quad \Rightarrow \quad p=3$

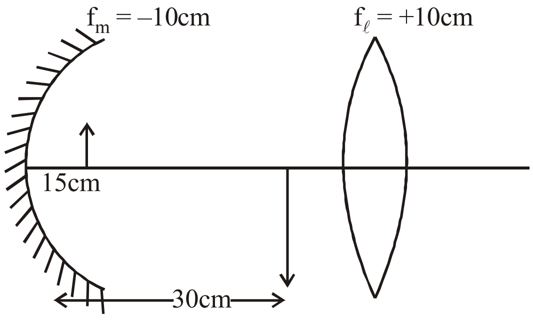
4. (7) Applying mirror formula

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-10}+\frac{1}{15}$

$\therefore \quad \frac{1}{v}=\frac{-15+10}{150}=\frac{-5}{150}=\frac{-1}{30}$

$\therefore \quad v=-30\text{cm}$



For convex lens $u = |2f_\ell|$

Therefore image will have a magnification of *1*.

When the set – up is kept in a medium

The focal length of the lens will change

$$\frac{\frac{1}{f_\ell}}{\frac{1}{f'_\ell}}=\frac{\left(\frac{n_\ell}{n_s}-1\right)}{\left(\frac{n_\ell}{n'_s}-1\right)}\Rightarrow\frac{f'_\ell}{10}=\frac{\left[\frac{1.5}{1}-1\right]}{\left[\frac{1.5}{7/6}-1\right]}$$

$\Rightarrow \quad f'_\ell = 17.5$ cm.

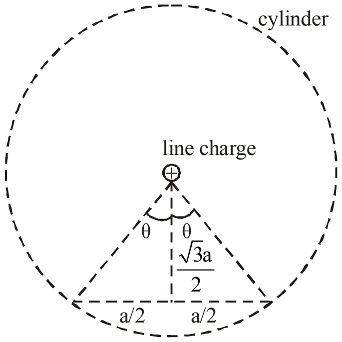
Applying lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f'_\ell}$

$\therefore \quad \frac{1}{v}-\frac{1}{-20}=\frac{1}{17.5} \quad \Rightarrow \quad v = 140$ cm.

M'_ℓ = Magnification by lens = $\frac{v}{u}=\frac{140}{-20}=-7$

Now $\left|\frac{M_2}{M_1}\right|=\left|\frac{\text{Mmirror}\times M'_\ell}{\text{Mmirror}\times M_\ell}\right|=7$

5. (6)



$\tan\theta=\frac{a/2}{\sqrt{3}a/2}=\frac{1}{\sqrt{3}}$

$\therefore \quad \theta = 30°$

The flux through the dotted cylinder by Gauss's law is

$$\phi \text{ cylinder} = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

∴ For 360° angle the flux is $\frac{\lambda L}{\varepsilon_0}$

∴ For 60° angle the flux will be $\frac{\lambda L}{6\varepsilon_0}$

Therefore n = 6

6. (2) $\frac{hc}{\lambda} = \frac{13.6}{n^2} + 10.2$

∴ $\frac{1242}{90} = \frac{13.6}{n^2} + 10.2$

∴ $n^2 = 4$

∴ $n = 2$

7. (2) Let h be the height to which the bullet rises

then, $g^1 = g\left(1 + \frac{h}{R}\right)^{-2}$

⇒ $\frac{g}{4} = \text{g}\left(1 + \frac{h}{R}\right)^{-2}$

⇒ $h = R$

We know that $v_e = \sqrt{\frac{2GM}{R}} = v\sqrt{N}$ (given) ...(i)

Now applying conservation of energy for the throw
Loss of kinetic energy = Gain in gravitaional potential energy

∴ $\frac{1}{2}mv^2 = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$

∴ $v = \sqrt{\frac{GM}{R}}$ ...(ii)

Comparing (i) & (ii) $N = 2$

8. (7) Total kinetic energy of a rolling disc = $\frac{1}{2}\text{mv}^2 + \frac{1}{2}\text{I}\omega^2$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right)$$

$$K.E = \frac{3}{4}mv^2$$

For surface AB

$k.E_i$ + loss in gravitational potential energy = $K.E_f$

$\frac{3}{4}m(3)^2 + mg(30) = \frac{3}{4}mV_B^2$ ...(i)

For surface CD

$\frac{3}{4}m(v_2)^2 + mg(27) = \frac{3}{4}mV_D^2$ ...(ii)

Given $V_B = V_D$. Therefore from (i) and (ii)

$$\frac{3}{4}m(3)^2 + mg \times 30 = \frac{3}{4}m(v_2)^2 + mg \times 27$$

∴ $V_2 = 7$

9. (a, c, d)

$L \propto h^x c^y G^z$

Dimensionally

$$[M^0L^1T^0] = [ML^2T^{-1}]^x[LT^{-1}]^y[M^{-1}L^3T^{-2}]^z$$

$$M^0L^1T^0 = M^{x-z}L^{2x+y+3z}T^{-x-y-2z}$$

∴ $x - z = 0$ ⇒ $x = z$

∴ $2x + y + 3z = 1$ and $-x - y - 2z = 0$

On solving we get

$x = \frac{1}{2}, y = -\frac{3}{2}, z = \frac{1}{2}$

∴ $L \propto \sqrt{h}$

$L \propto \sqrt{G}$

C, D are correct options

$M \propto h^x c^y G^z$

$M'L^\circ T^\circ \propto [ML^2T^{-1}]^x[LT^{-1}]^y[M^{-1}L^3T^{-2}]^Z$

∴ $M'L^\circ T^\circ \propto M^{x-z}L^{2x+y+3z}T^{-x-y-2z}$

∴ $x - z = 1$

$2x + y + 3z = 0$

$-x - y - 2z = 0$

On solving we get

$x = \frac{1}{2}, y = \frac{1}{2}, z = -\frac{1}{2}$

∴ $M \propto \sqrt{C}$

A is the correct option.

10. (b, d)

Maximum linear momentum in case 1 is $(\text{p}_1)_{\text{max}} = \text{m}\nu_{\text{max}}$

$\text{b} = \text{m}[\text{aw}_1]$ ...(i)

Maximum linear momentum in case 2 is $(p_2)_{\text{max}} = \text{m}\nu_{\text{max}}$

$R = m[R\omega_2]$

∴ $1 = \text{m}\omega_2$ ...(ii)

Dividing (i) & (ii) $\frac{b}{1} = \frac{a\omega_1}{\omega_2}$

∴ $\frac{\omega_1}{\omega_2} = \frac{b}{a} = \frac{1}{n^2}$ ∴ B is a correct option.

Also $E_1 = \frac{1}{2}m\,\omega_1^2a^2$

$E_2 = \frac{1}{2}m\,\omega_2^2R^2$

$$\therefore \frac{E_1}{E_2} = \frac{\omega_1^2}{\omega_2^2} \times \frac{a^2}{R^2} = \frac{\omega_1^2}{\omega_2^2} \times \frac{1}{n} = \frac{\omega_1^2}{\omega_2^2} \times \frac{\omega_2}{\omega_1} = \frac{\omega_1}{\omega_2}$$

∴ $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$ D is the correct option

11. (d) Applying conservation of angular mumentum about the axis

$$MR^2 \times \omega = MR^2 \times \frac{8\omega}{9} + \frac{M}{8} \times \frac{9R^2}{25} \times \frac{8\omega}{9} + \frac{M}{8}r^2 \times \frac{8\omega}{9}$$

$$\Rightarrow \quad r = \frac{4R}{5}$$

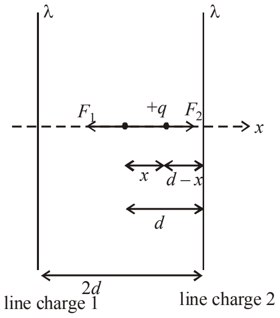
D is the correct option

12. (c) Force on change q when it is given a small displacement x is $F_{net} = F_1 - F_2$

$$F_{net} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d-x} - \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d+x}$$

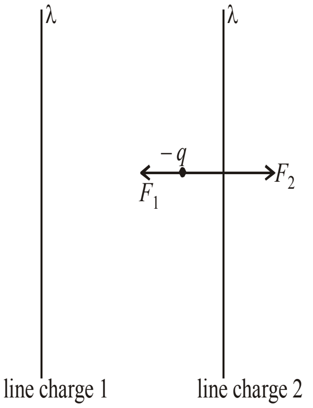
$$\therefore F_{net} = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{d+x-d+x}{d^2-x^2}\right]$$

$$\therefore F_{net} = \frac{\lambda}{2\pi\varepsilon_0}\frac{2x}{d^2-x^2}$$



When $x << d$ then

$$F_{net} = \frac{\lambda}{\pi\varepsilon_0}x$$ and is directed towards the mean position therefore the charge +q will execute SHM.



In case of charge ($-q$)

$F_2 > F_1$ therefore the charge $-q$ continues to move in the direction of its displacement.

[C] is the correct option.

13. (b) For refraction in S_1

$$-\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$-\frac{1.5}{-50} + \frac{1}{V} = \frac{1-1.5}{-10}$$

$\Rightarrow \quad v = 50$ cm.

For refraction in S_2

$$-\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$-\frac{1}{-(d-50)} + \frac{1.5}{\infty} = \frac{1.5-1}{10}$$

$$\therefore \quad \frac{1}{d-50} = \frac{1}{20}$$

$\therefore \quad d = 70$ cm.

B is the correct option.

14. (a, b, c)

$$\vec{F} = I\left[\left(\int \vec{dl}\right) \times \vec{B}\right]$$

If $\vec{B}$ is along $\vec{z}$ then $\vec{F} = I\left[(2L+2R)\hat{i} \times B\hat{x}\right]$

option [A] is correct

If $\vec{B}$ is along $\vec{x}$ then $\vec{F} = I\left[(2L+2R)\hat{i} \times B\hat{i}\right] = 0$

If $\vec{B}$ is along $\vec{y}$ then $\vec{F} = I\left[(2L+2R)\hat{i} \times \hat{j}\right]$

Option (b) and (c) are also correct

15. (a, b, d)

$$\text{Total energy} = \frac{3}{2}RT + \frac{5}{2}RT = 4RT$$

$$\therefore \text{ Average energy per mole} = \frac{4RT}{2} = 2RT$$

We know that $V_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{n_1+n_2}{\gamma_{\text{mix}}-1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$$\Rightarrow \frac{2}{\gamma_{\text{mix}}-1} = \frac{1}{\frac{5}{3}-1} + \frac{1}{\frac{7}{5}-1}$$

$$\frac{2}{\gamma_{\text{mix}}-1} = \frac{3}{2} + \frac{5}{2} = 4$$

$$\therefore \gamma_{\text{mix}} - 1 = \frac{1}{2} \qquad \therefore \gamma_{\text{mix}} = \frac{3}{2}$$

$$\frac{(V_s)_{mix}}{(V_s)_{He}} = \sqrt{\frac{\gamma_{mix}}{M_{mix}} \times \frac{M_{He}}{\gamma_{He}}}$$

$$= \sqrt{\frac{\frac{3}{2}\times 4}{3\times\frac{5}{3}}} \qquad \left[\because M_{mix} = \frac{1\times 2 + 1\times 4}{2} = 3\right]$$

$$= \sqrt{\frac{6}{5}}$$

We know that $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\therefore \frac{(V_{rms})_{He}}{(V_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{H_E}}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

$\therefore$ options [A], [B] and [C] are correct.

16. (b) $R_{Fe} = \frac{\rho_{Fe}\times l_{Fe}}{A_{Fe}} = \frac{10^{-7}\times 50\times 10^{-3}}{4\times 10^{-6}} = \frac{25}{2}\times 10^{-4}$

$$R_{Al} = \frac{\rho_{Al}\times l_{Al}}{A_{Al}} = \frac{2.7\times 10^{-8}\times 50\times 10^{-3}}{(49-4)\times 10^{-6}} = \frac{2.7\times 50}{45}\times 10^{-5}$$

$= 0.3\times 10^{-4}$

$$R_{total} = \frac{R_{Fe}\times R_{Al}}{R_{Fe}+R_{Al}} = \frac{12.5\times 10^{-4}\times 0.3\times 10^{-4}}{12.8\times 10^{-4}} \approx 29\mu\Omega$$

(B) is the correct option.

17. (a, c)

We know that $\frac{hC}{\lambda} - W = eV_0 \Rightarrow \frac{hc}{e\lambda} - \frac{W}{e} = V_0$

For V_0 versus $\frac{1}{\lambda}$ we should get a straight line with negative slope and positive intercept.

For V_0 vesus λ, we will get a hyperbola. As λ decreases V_0 increases.

(a) and (c) are the correct options

18. (b, c)

Vernier callipers

$1\text{ MSD} = \frac{1\text{cm}}{8} = 0.125\text{cm}$

$5\text{ VSD} = 4\text{MSD}$

$\therefore 5\text{VSD} = 4\times \frac{1}{8}\text{cm} = 0.5\text{cm}$

$\therefore 1\text{ VSD} = 0.1\text{cm}$

$\text{L.C} = 1\text{MSD} - 1\text{VSD}$

$= 0.125\text{cm} - 0.1\text{cm}$

$= 0.025\text{cm}$

Screw gauge

One complete revolution = 2M.S.D

If the pitch of screw gauge is twice the L.C of vernier callipers then pitch = 2 × 0.025 = 0.05cm.

L.C of screw Gauge

$$= \frac{\text{pitch}}{\text{Total no. of divisions of circular scale}}$$

$= \frac{0.05}{100}\text{cm} = 0.0005\text{cm} = 0.005\text{ mm}.$

(b) is a correct option

Now if the least count of the linear scale of the screw gauge is twice the least count of venier callipers then.

L.C of linear scale of screw gauge = 2 × 0.025 = 0.05cm.

Then pitch = 2 × 0.05 = 0.1cm.

Then L.C of screw gauge = $\frac{0.1}{100}$ cm = 0.001cm = 0.01mm.

(c) is a correct option.

19. A → R, T ; B → P, S ; C → P, Q, R, T; D → P, Q, R, T

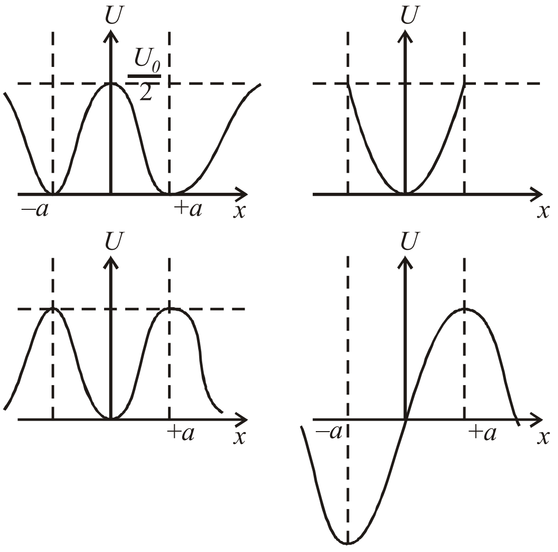
Based on facts

20. A → P, Q, R, T; B → Q, S; C → P, Q, R, S; D → P, R, T

For A

$$\text{Fx} = -\frac{dU}{dx} = \frac{-d}{dx}\left[\frac{U_0}{2}\left(1-\left(\frac{x}{a}\right)^2\right)\right]^2$$

$$= \frac{-2U_0}{a^3}(x-a)x(x+a)$$



For B $F_x = \frac{-dU}{dx} = -U_0\left(\frac{x}{a}\right)$

For C $F_x = \frac{-dU}{dx} = U_0\frac{e^{-x^2/a^2}}{a^3}x(x-a)(x+a)$

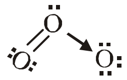
For D $F_x = -\frac{dU}{dx} = -\frac{U_0}{2a^3}[(x-a)(x+a)]$

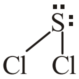
CHEMISTRY

1. (4) $Cl-Be-Cl$
Hybridization *sp*
Structure linear

N_3^-
Hybridisation *sp*
Structure linear

$O=\overset{+}{N}\rightarrow O$
Hybridisation *sp*
Structure Linear


Hybridisation sp^2
Structure Trigonal planar


Hybridisation sp^3
Structure Angular

$[Cl-\ddot{I}-Cl]^-$
Hybridisation sp^3d
Structure linear

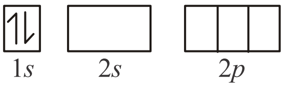
$[I-\ddot{I}-I]^-$
Hybridisation sp^3d
Structure Linear

$F-\ddot{Xe}-F$
Hybridisation sp^3d
Structure Linear

$\overset{-}{N}=\overset{+}{N}=O$
Hybridisation *sp*
Structure Linear

Only $BeCl_2$, N_3^-, N_2O and NO_2 are linear with *sp*-hybridisation.

2. (3) Ground state configuration:

H^-: 

in second excited state, electron will jump from 1s to 2p, so degeneracy of second excited state of H^- is 3.

3. (4) $X \longrightarrow Y$; $\Delta G° = -193\ kJ\ mol^{-1}$

$M^+ \longrightarrow M^{3+} + 2e^-$ $E° = -0.25V$

Hence $\Delta G°$ for oxidation will be

$\Delta G° = -nFE°$
$= -2 \times 96500 \times (-0.25)$
$= 48250\ J = 48.25\ kJ$

48.25 kJ energy oxidises one mole M^+

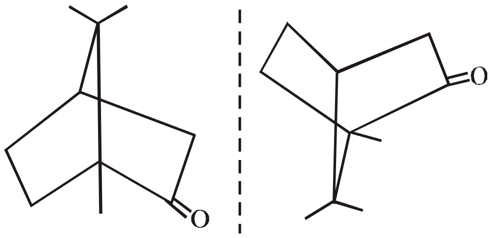
$\therefore$ 193 kJ energy oxidises $\frac{193}{48.25}$ mole $M^+ = 4$ mole M^+

4. (1) Given $\Delta T_f = 0.0558°C$
as we know, $\Delta T_f = i \times K_f \times m$
$\Rightarrow 0.0558 = i \times 1.86 \times 0.01$

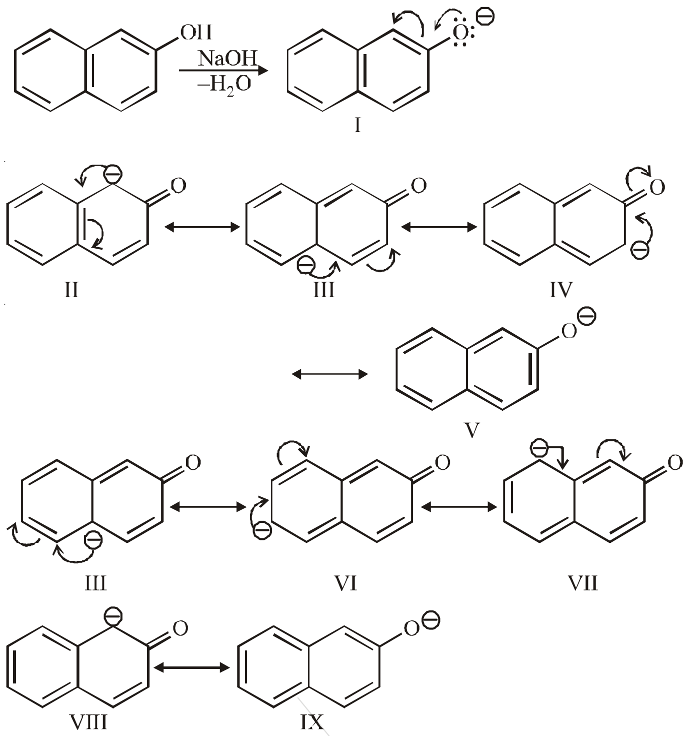
$\boxed{i = 3}$

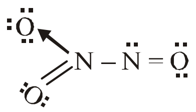
Therefore the complex will be $[Co(NH_3)_5Cl]Cl_2$
Hence number of chloride in co-ordination sphere is 1.

5. (2) The molecule cannot show geometrical isomerism, so only its mirror image will be the other stereoisomer.



6. (9)



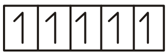
7. (8) 

Number of lone pairs = 8

8. (4) $Fe(26) \longrightarrow [Ar]_{18}\ 3d^6 4s^2$

$Fe^{3+} \longrightarrow [Ar]_{18}\ 3d^5 4s^0$

SCN^- is weak field ligand hence pairing will not occur.

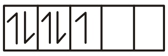
$\therefore$ $Fe^{3+} \longrightarrow$ 

Unpaired electrons = 5

Magnetic moment = $\sqrt{5(5+2)}$ B.M.

$= \sqrt{35}$ B.M. = 5.92 B.M.

CN^- is strong field ligand hence pairing will take place.

$\therefore Fe^{3+} \longrightarrow$ 

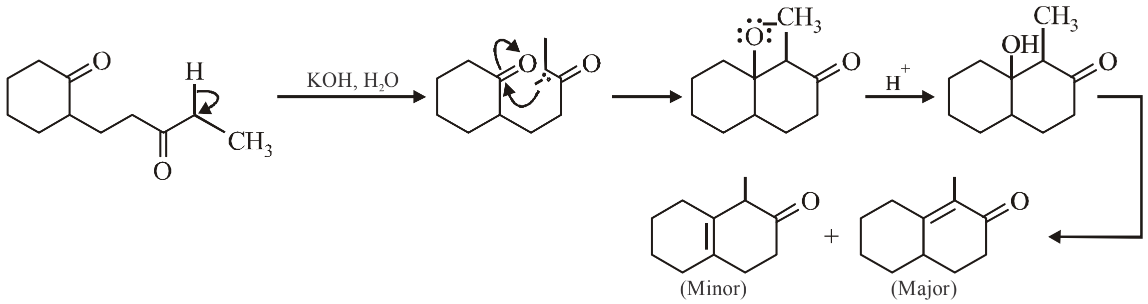
Unpaired electrons = 1

Magnetic moment = $\sqrt{1(1+2)}$B.M. $= \sqrt{3}$ B.M. = 1.732

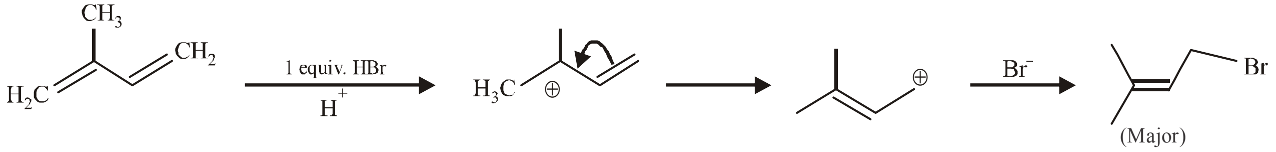
Difference = 5.92 – 1.732 = 4.188

Hence answer is (4).

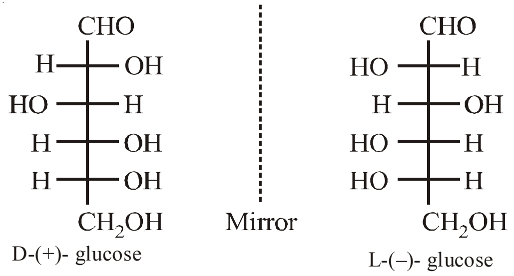
9. (a)



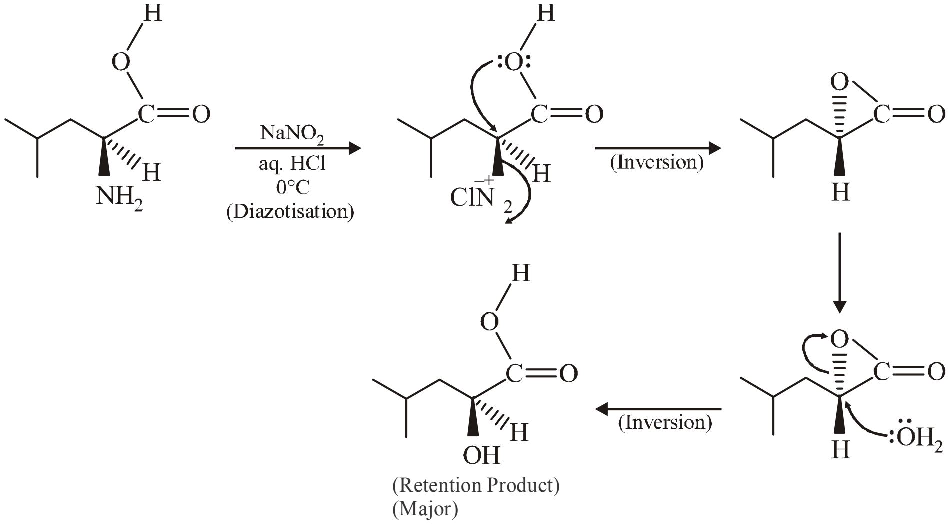
10. (d)



11. (a)



12. (c)



13. (a, b, c)

Cr^{2+} is a reducing agent and Mn^{3+} is an oxidizing agent and both have electronic configuration d^4.

$E^{\circ}_{Cr^{3+}/Cr^{2+}} = -0.41V$ $\qquad$ $E^{\circ}_{Mn^{3+}/Mn^{2+}} = 1.51V$

Above E° values explains reducing nature of Cr^{2+} and oxidizing behaviour of Mn^{3+}.

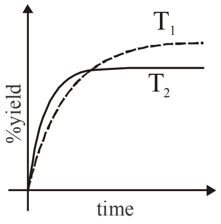
14. (b, c, d)

In electrolytic refining of blister Cu, acidified $CuSO_4$ is used as electrolyte, pure Cu deposits at cathode and impurities settle as anode mud.

15. (c, d)

Fe^{3+} is reduced to Fe^{2+} by H_2O_2 and Na_2O_2 in acidic medium.

16. (b) Initially on increasing temperature rate of reaction will increase, so % yield will also increase with time. But at equilibrium % yield at high temperature (T_2) would be less than at T_1 as reaction is exothermic so the graph is



17. (a) In *ccp*, O^{2-} ions are 4.

Hence total negative charge = –8

Let Al^{3+} ions be x, and Mg^{2+} ions be y.

Total positive charge = $3x + 2y$

$\Rightarrow 3x + 2y = 8$

This relation is satisfied only by $x = 2$ and $y = 1$.

Hence number of $Al^{3+} = 2$.

and number of $Mg^{2+} = 1$.

$\Rightarrow n$ = fraction of octahedral holes occupied by Al^{3+}

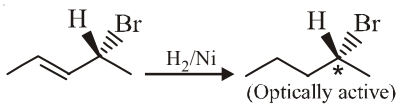
$$= \frac{2}{4} = \frac{1}{2}$$

and m = fraction of tetrahedral holes occupied by Mg^{2+}

$$= \frac{1}{8}$$

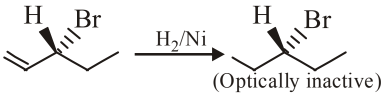
18. (b, d)

(A)



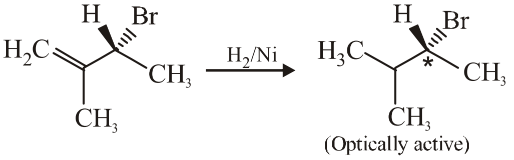
(Optically active)

(B)



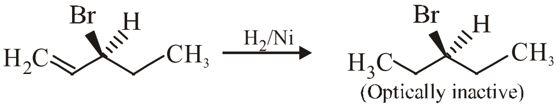
(Optically inactive)

(C)



(Optically active)

(D)



(Optically inactive)

19. A-(P, Q, S), B-(T), C-(Q, R), D-(R)

Siderite	→	$FeCO_3$
Malachite	→	$CuCO_3 \cdot Cu(OH)_2$
Bauxite	→	$Al_2O_3.xH_2O$ or $AlO_x(OH)_{3-2x}$ $0 < x < 1$
Calamine	→	$ZnCO_3$
Argentite	→	Ag_2S

20. A- (R, T), B -(P, Q, S), C -(P, Q, S), D- (P, Q, S, T)

(A) → R, T

$$H_2O(\ell) \rightleftharpoons H_2O(s)$$

It is at equilibrium at 273 K and 1 atm

So ΔS_{sys} is negative

As it is equilibrium process so $\Delta G = 0$

(B) → P, Q, S

Expansion of 1 mole of an ideal gas in vacuum under isolated condition

Hence, w = 0

and $q_p = C_p dT$ $(\because dT = 0)$

$\Rightarrow q = 0$

$\Delta U = C_v dT$ $(\because dT = 0)$

$\Delta U = 0$

(C) → P, Q, S

Mixing of two ideal gases at constant temperature

Hence, $\Delta T = 0$

$\therefore q = 0$

$\Delta U = 0$

also w = 0 $(\Delta U = q + w)$

(D) → P, Q, S, T

Reversible heating and cooling of gas follows same path also initial and final position is same.

$$\text{Hence, } \left.\begin{matrix} q = 0 \\ w = 0 \end{matrix}\right\} \text{Path same}$$

$$\left.\begin{matrix} \Delta U = 0 \\ \Delta G = 0 \end{matrix}\right\} \text{State function}$$

MATHEMATICS

1. (2) End points of latus rectum of $y^2 = 4x$ are $(1, \underline{+}2)$

Equation of normal to $y^2 = 4x$ at (1, 2) is

$y - 2 = -1(x - 1)$ or $x + y - 3 = 0$

As it is tangent to circle $(x - 3)^2 + (y + 2)^2 = r^2$

$$\therefore \quad \left|\frac{3 + (-2) - 3}{\sqrt{2}}\right| = r \Rightarrow r^2 = 2$$

2. (0) $I = \displaystyle\int_{-1}^{2} \frac{xf(x^2)}{2 + f(x+1)} dx$

$-1 < x < 2 \Rightarrow 0 < x^2 < 4$

Also $0 < x^2 < 1 \Rightarrow f(x^2) = [x^2] = 0$

$1 \le x^2 < 2 \Rightarrow f(x^2) = [x^2] = 1$

$2 \le x^2 < 3 \Rightarrow f(x^2) = 0$ (using definition of f)

$3 \le x^2 < 4 \Rightarrow f(x^2) = 0$ (using definition of f)

Also $1 \le x^2 < 2 \Rightarrow 1 \le x < \sqrt{2}$

$\Rightarrow 2 \le x + 1 < \sqrt{2} + 1$

$\Rightarrow f(x + 1) = 0$

$$\therefore \quad I = \int_{1}^{\sqrt{2}} \frac{x \times 1}{2 + 0} dx = \left[\frac{x^2}{4}\right]_1^{\sqrt{2}} = \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$$

$\Rightarrow 4I = 1$ or $4I - 1 = 0$

3. (4) Let r be the internal radius and R be the external radius. Let h be the internal height of the cylinder.

$$\therefore \quad \pi r^2 h = V \Rightarrow h = \frac{V}{\pi r^2}$$

Also Vol. of material = $M = \pi[(r + 2)^2 - r^2]h + \pi(r + 2)^2 \times 2$

or $$M = 4\pi(r+1).\frac{V}{\pi r^2} + 2\pi(r+2)^2$$

$$\Rightarrow \quad M = 4V\left[\frac{1}{r}+\frac{1}{r^2}\right] + 2\pi(r+2)^2$$

$$\frac{dM}{dr} = 4V\left[\frac{-1}{r^2}-\frac{2}{r^3}\right] + 4\pi(r+2)$$

For min. value of M, $\frac{dM}{dr} = 0$

$$\Rightarrow \quad \frac{-4V}{r^3}(r+2) + 4\pi(r+2) = 0$$

$$\Rightarrow \quad \frac{4V}{r^3} = 4\pi \text{ or } r^3 = \frac{V}{\pi} = 1000$$

$\therefore \quad V = 1000\pi$

$$\therefore \quad \frac{V}{250\pi} = 4$$

4. (3) $F(x) = \int_x^{x^2+\pi/6} 2\cos^2 t \, dt$

$$F'(\alpha) = 2\cos^2\left(\alpha^2+\frac{\pi}{6}\right).2\alpha - 2\cos^2\alpha$$

$$F'(\alpha) + 2 = \int_0^{\alpha} f(x)dx$$

$\Rightarrow \quad F''(\alpha) = f(\alpha)$

$$\therefore \quad f(\alpha) = 4\alpha.2\cos\left(\alpha^2+\frac{\pi}{6}\right).\left[-\sin\left(\alpha^2+\frac{\pi}{6}\right)\right].2\alpha$$

$$+ 4\cos^2\left(\alpha^2+\frac{\pi}{6}\right) - 4\cos\alpha\,(-\sin\alpha)$$

$$\therefore \quad f(0) = 4\cos^2\frac{\pi}{6} = 4 \times \frac{3}{4} = 3$$

5. (8) $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$$\Rightarrow \quad \frac{5}{4}\cos^2 2x + 1 - \frac{1}{2}\sin^2 2x + 1 - \frac{3}{4}\sin^2 2x = 2$$

$$\Rightarrow \quad \frac{5}{4}(\cos^2 2x - \sin^2 2x) = 0 \Rightarrow \cos 4x = 0$$

$$\Rightarrow \quad 4x = (2n+1)\frac{\pi}{2} \text{ or } \quad x = (2n+1)\frac{\pi}{8}$$

For $x \in [0, 2\pi]$, n can take values 0 to 7

$\therefore$ 8 solutions.

6. (4) Let $(t^2, 2t)$ be any point on $y^2 = 4x$. Let (h, k) be the image of $(t^2, 2t)$ in the line $x + y + 4 = 0$. Then

$$\frac{h-t^2}{1} = \frac{k-2t}{1} = \frac{-2(t^2+2t+4)}{2}$$

$\Rightarrow \quad h = -(2t+4)$ and $k = -(t^2+4)$

For its intersection with, $y = -5$, we have

$-(t^2+4) = -5 \quad \Rightarrow t = \pm 1$

$\therefore \quad A(-6, -5)$ and $B(-2, -5)$

$\therefore \quad AB = 4.$

7. (8) $P(x \geq 2) \geq 0.96$

$\Rightarrow \quad 1 - P(x=0) - P(x=1) \geq 0.96$

$\Rightarrow \quad P(x=0) + P(x=1) \leq 0.04$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^n \leq 0.04$$

$$\Rightarrow \quad \frac{n+1}{2^n} \leq \frac{1}{25}$$

$$\Rightarrow \quad \frac{2^n}{n+1} \geq 25$$

$\Rightarrow$ minimum value of n is 8.

8. (5) $n = 5! \times 6!$

For second arrangement,

5 boys can be made to stand in a row in 5! ways, creating 6 alternate space for girls. A group of 4 girls can be selected in ${}^5C_4$ ways. A group of 4 and single girl can be arranged at 2 places out of 6 in ${}^6P_2$ ways. Also 4 girls can arrange themselves in 4! ways.

$\therefore \quad m = 5! \times {}^6P_2 \times {}^5C_4 \times 4!$

$$\frac{m}{n} = \frac{5! \times 6 \times 5 \times 5 \times 4!}{5! \times 6!} = 5$$

9. (b, c) $\quad R_2 - R_1, \quad R_3 - R_2$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 2\alpha+3 & 4\alpha+3 & 6\alpha+3 \\ 2\alpha+5 & 4\alpha+5 & 6\alpha+5 \end{vmatrix} = -648\alpha$$

$$R_3 - R_2$$

$$2\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 2\alpha+3 & 4\alpha+3 & 6\alpha+3 \\ 1 & 1 & 1 \end{vmatrix} = -648\alpha$$

$$C_2 - C_1, \quad C_3 - C_2$$

$$\begin{vmatrix} (1+\alpha)^2 & \alpha(3\alpha+2) & \alpha(5\alpha+2) \\ 2\alpha+3 & 2\alpha & 2\alpha \\ 1 & 0 & 0 \end{vmatrix} = -324\alpha$$

$\Rightarrow \quad 2\alpha^2(-2\alpha) = -324\alpha$

$\Rightarrow \quad \alpha^3 - 81\alpha = 0$

$\Rightarrow \quad \alpha = 0, 9, -9$

10. (b, d) $P_3 : x+\lambda y+z-1=0$

Also $\left|\dfrac{\lambda-1}{\sqrt{2+\lambda^2}}\right| = 1 \Rightarrow \lambda^2-2\lambda+1=\lambda^2+2$

$\Rightarrow \quad \lambda = \dfrac{-1}{2}$

And $\left|\dfrac{\alpha+\lambda\beta+\gamma-1}{\sqrt{2+\lambda^2}}\right| = 2 \Rightarrow \dfrac{\alpha-\frac{1}{2}\beta+\gamma-1}{\frac{3}{2}} = \pm 2$

$\alpha-\frac{1}{2}\beta+\gamma-1=\pm 3$

$\Rightarrow \quad 2\alpha-\beta+2\gamma-2=\pm 6$

$\Rightarrow \quad 2\alpha-\beta+2\gamma-8=0$ or $2\alpha-\beta+2\gamma+4=0$

11. (a, b) $L : \dfrac{x}{a}=\dfrac{y}{b}=\dfrac{z}{c}=\lambda$

Where $\left.\begin{array}{l} a+2b-c=0 \\ 2a-b+c=0 \end{array}\right\}$ *As L is parallel to both P_1 and P_2.*

$\Rightarrow \quad \dfrac{a}{1}=\dfrac{b}{-3}=\dfrac{c}{-5}$

$\therefore$ Any point on line L is $(\lambda, -3\lambda, -5\lambda)$

Equation of line perpendicular to P_1 drawn from any point on L is

$$\frac{x-\lambda}{1}=\frac{y+3\lambda}{2}=\frac{z+5\lambda}{-1}=\mu$$

$\therefore \quad M(\mu+\lambda, 2\mu-3\lambda, -\mu-5\lambda)$

But M lies on P_1,

$\therefore \quad \mu+\lambda+4\mu-6\lambda+\mu+5\lambda+1=0$

$\Rightarrow \quad \mu=\dfrac{-1}{6}$

$\therefore \quad M\left(\lambda-\dfrac{1}{6}, -3\lambda\dfrac{-1}{3}, -5\lambda+\dfrac{1}{6}\right)$

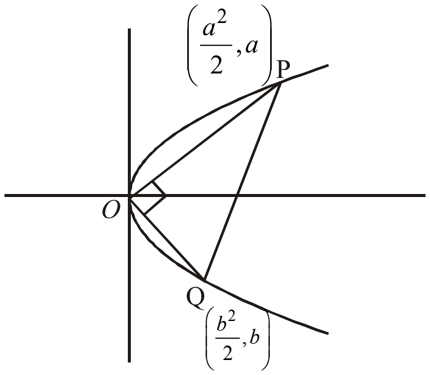
For locus of M,

$$x=\lambda-\frac{1}{6}, y=-3\lambda-\frac{1}{3}, z=5\lambda+\frac{1}{6}$$

$\Rightarrow \quad \dfrac{x+1/6}{1}=\dfrac{y+1/3}{-3}=\dfrac{z-1/6}{-5}=\lambda$

On checking the given point, we find $\left(0, \dfrac{-5}{6}, \dfrac{-2}{3}\right)$ and $\left(\dfrac{-1}{6}, \dfrac{-1}{3}, \dfrac{1}{6}\right)$ satisfy the above eqn.

12. (a, d) Let point P in first quadrant, lying on parabola $y^2=2x$ be $\left(\dfrac{a^2}{2}, a\right)$. Let Q be the point $\left(\dfrac{b^2}{2}, b\right)$. Clearly $a>0$.



$\because \quad PQ$ is the diameter of circle through P, O, Q

$\therefore \quad \angle POQ = 90° \Rightarrow \dfrac{a}{a^2/2}\times\dfrac{b}{b^2/2}=-1 \Rightarrow ab=-4$

$\Rightarrow \quad b$ is negative.

Also ar. $\Delta POQ = 3\sqrt{2}$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{a^2}{2} & a & 1 \\ \frac{b^2}{2} & b & 1 \end{vmatrix} = 3\sqrt{2}$$

$\Rightarrow \quad \dfrac{1}{4}ab(a-b)=\pm 3\sqrt{2}$

$\Rightarrow \quad a-b=\pm 3\sqrt{2}$ (using $ab=-4$)

As a is positive and b is negative, we have $a-b = 3\sqrt{2}$

$a+\dfrac{4}{\text{a}} = 3\sqrt{2}$ (using $ab=-4$)

$\Rightarrow \quad a^2-3\sqrt{2}\,a+4=0$

$\Rightarrow \quad a^2-2\sqrt{2}\,a-\sqrt{2}\,a+4=0$

$\Rightarrow \quad (a-2\sqrt{2})(a-\sqrt{2})=0$

$\Rightarrow \quad a=2\sqrt{2}, \sqrt{2}$

$\therefore \quad$ Point P can be $\left(\dfrac{(2\sqrt{2})^2}{2}, 2\sqrt{2}\right)$ or $\left(\dfrac{(\sqrt{2})^2}{2}, \sqrt{2}\right)$

i.e. $(4, 2\sqrt{2})$ or $(1, \sqrt{2})$

13. (a, c) $\dfrac{dy}{dx}+\dfrac{e^x}{1+e^x}y=\dfrac{1}{1+e^x}$

I.F. $=1+e^x$

$\therefore$ Sol^n: $y(1+e^x)=x+c$

$y(0)=2 \Rightarrow c=4$

$$\therefore \quad y = \frac{x+4}{e^x+1}$$

$$\therefore \quad y(-4) = 0$$

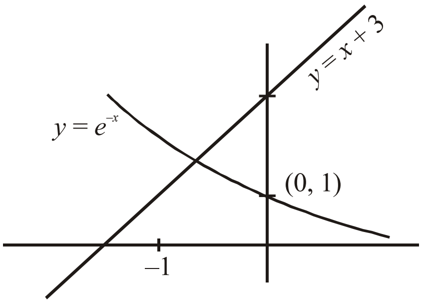
$$\text{Also } \frac{dy}{dx} = \frac{(e^x+1) - e^x(x+4)}{(e^x+1)^2}$$

For critical point $\frac{dy}{dx} = 0$

$\Rightarrow \quad e^x(x+3) = 1$

$\Rightarrow \quad x + 3 = e^{-x}$

Its solution will be intersection point of $y = x + 3$ and $y = e^{-x}$



Clearly there is a critical point in (–1, 0).

14. (b, c) Let the equation of circle be

$$x^2 + y^2 + 2gx + 2gy + c = 0$$

$\Rightarrow \quad 2x + 2yy' + 2g + 2gy' = 0$

$\Rightarrow \quad x + yy' + g + gy' = 0$ ...(i)

Differentiating again,

$$1 + yy'' + (y')^2 + gy'' = 0$$

$$\Rightarrow \quad g = -\left[\frac{1+(y')^2+yy''}{y''}\right]$$

Substituting value of g in eqn. (i)

$$x + yy' - \frac{1+(y')^2+yy''}{y''} - \left(\frac{1+(y')^2+yy''}{y''}\right)y' = 0$$

$\Rightarrow \quad xy'' + yy'y'' - 1 - (y')^2 - yy'' - y' - (y')^3 - yy'y'' = 0$

$\Rightarrow \quad (x - y)y'' - y'(1 + y' + (y')^2) = 1$

or $\quad (y - x)y'' + [1 + y' + (y')^2]\,y' + 1 = 0$

$\quad Py'' + Qy' + 1 = 0$

$\therefore \quad P = y - x, Q = 1 + y' + (y')^2$

Also $P + Q = 1 - x + y + y' + (y')^2$

15. (a, d) $f(x) = \begin{cases} \frac{x}{|x|}g(x), x \neq 0 \\ 0 \quad\quad ,x = 0 \end{cases}$

$$= \begin{cases} -g(x) & , \quad x < 0 \\ 0 & , \quad x = 0 \\ g(x) & , \quad x > 0 \end{cases}$$

$$f'(x) = \begin{cases} -g'(x) & , \quad x < 0 \\ 0 & , \quad x = 0 \\ g'(x) & , \quad x > 0 \end{cases}$$

$\therefore \quad Lf'(0) = -g'(0) = 0$

$\quad Rf'(0) = g'(0) = 0$

$\therefore \quad f$ is differentiable at $x = 0$

$$h(x) = e^{|x|} = \begin{cases} e^{-x} & , \quad x < 0 \\ e^x & , \quad x \geq 0 \end{cases}$$

$$h'(x) = \begin{cases} -e^{-x} & , \quad x < 0 \\ e^x & , \quad x \geq 0 \end{cases} \Rightarrow Lh'(0) = -1, Rh'(0) = 1$$

$\therefore \quad h$ is not differentiable at $x = 0$

$\quad f\text{oh}(x) = f(h(\text{x})) = g(e^{|x|})$ as $e^{|x|} > 0$

$$= \begin{cases} g(e^{-x}) & \text{if} \quad x < 0 \\ g(1) & \text{if} \quad x = 0 \\ g(e^x) & \text{if} \quad x > 0 \end{cases}$$

$$f'[h(x)] = \begin{cases} -g'(e^{-x}).e^{-x} & , \quad x < 0 \\ 0 & , \quad x = 0 \\ g'(e^x)e^x & , \quad x > 0 \end{cases}$$

$Lf'(h(0)) = -g'(1)$

$Rf'(h(0)) = g'(1)$

$\because \quad g'(1) \neq 0, \therefore Lf'(h(0)) \neq Rf'(h(0))$

$\therefore \quad foh$ is not differentiable at $x = 0$.

$$hof(x) = \begin{cases} e^{|f(x)|} & , \quad x \neq 0 \\ 1 & , \quad x = 0 \end{cases}$$

$$Lh'(f(0)) = \lim_{k \to 0} \frac{h(f(0)) - h(f(0-k))}{k}$$

$$= \lim_{k \to 0} \frac{1 - e^{|g(-k)|}}{k} = \lim_{k \to 0} \frac{1 - e^{|g(-k)|}}{|g(-k)|} \times \frac{|g(-k)|}{k}$$

$$= 1 \times 0 = 0 \left(\because g'(0) = 0 \Rightarrow \lim_{k \to 0} \frac{g(-k)}{k} = \lim_{k \to 0} \frac{g(k)}{k} = 0\right)$$

$$Rh'(f(0)) = \lim_{k \to 0} \frac{h(f(0+k)) - h(f(0))}{k}$$

$$= \lim_{k \to 0} \frac{e^{|g(k)|} - 1}{k} = \lim_{k \to 0} \frac{e^{|g(k)|} - 1}{|g(k)|} \times \frac{|g(k)|}{k} = 0$$

$\therefore \quad hof$ is differentiable at $x = 0$.

16. (a, b, c)

$$f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$$

$$-1 \le \sin x \le 1 \Rightarrow -\frac{\pi}{2} \le \frac{\pi}{2}\sin x \le \frac{\pi}{2}$$

$$\Rightarrow \quad -1 \le \sin\left(\frac{\pi}{2}\sin x\right) \le 1$$

$$\Rightarrow \quad \frac{-\pi}{6} \le \frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right) \le \frac{\pi}{6}$$

$$\Rightarrow \quad \frac{-1}{2} \le \sin\left[\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right] \le \frac{1}{2}$$

$$\therefore \quad \text{Range of} f = \left[\frac{-1}{2}, \frac{1}{2}\right]$$

$$fog(x) = \sin\left[\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right]$$

$$\text{Range of} fog = \left[\frac{-1}{2}, \frac{1}{2}\right]$$

$$\lim_{x\to 0} \frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{2}\sin x}$$

$$= \lim_{x\to 0} \frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)} \times \frac{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)}{\frac{\pi}{2}\sin x}$$

$$= \pi/6$$

$$gof(x) = \frac{\pi}{2}\sin\left(\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right)$$

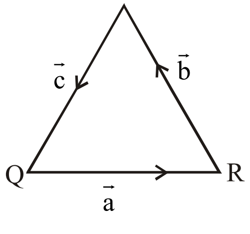
$$-\frac{\pi}{2}\sin\left(\frac{1}{2}\right) \le g(f(x)) \le \frac{\pi}{2}\sin\left(\frac{1}{2}\right)$$

$-0.73 \le g(f(x)) \le 0.73$

$\therefore$ $gof(x) \ne 1$ for any $x \in R$.

17. (a, c, d)

$\vec{a} + \vec{b} + \vec{c} = \vec{o}$



$$\Rightarrow \quad \left|\vec{b} + \vec{c}\right|^2 = \left|-\vec{a}\right|^2$$

$$\Rightarrow \quad \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\vec{b}.\vec{c} = \left|\vec{a}\right|^2$$

$$\Rightarrow \quad 48 + \left|\vec{c}\right|^2 + 48 = 144 \Rightarrow \left|\vec{c}\right|^2 = 48 \Rightarrow \left|\vec{c}\right| = 4\sqrt{3}$$

$$\therefore \quad \frac{\left|\vec{c}\right|^2}{2} - \left|\vec{a}\right| = \frac{48}{2} - 12 = 12$$

$$\frac{\left|\vec{c}\right|^2}{2} + \left|\vec{a}\right| = 24 \ne 30$$

Also $\left|\vec{b}\right| = \left|\vec{c}\right| \Rightarrow \angle Q = \angle R$

and $\cos(180 - P) = \dfrac{\vec{b}.\vec{c}}{\left|\vec{b}\right|\left|\vec{c}\right|} = \dfrac{1}{2}$

$\Rightarrow \quad \angle P = 120° \ \therefore \angle Q = \angle R = 30°$

Again $\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{0} \Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$

$$\therefore \quad \left|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}\right| = 2\left|\vec{a} \times \vec{b}\right| = 2 \times 12 \times 4\sqrt{3} \times \sin 150 = 48\sqrt{3}$$

And $\vec{a}.\vec{b} = 12 \times 4\sqrt{3} \times \cos 150 = -72$

18. (c, d)

$X' = -X, Y' = -Y, Z' = Z$

$(Y^3Z^4 - Z^4Y^3)' = (Z^4)'(Y^3)' - (Y^3)'(Z^4)'$

$= (Z')^4(Y')^3 - (Y')^3(Z')^4$

$= -Z^4Y^3 + Y^3Z^4 = Y^3Z^4 - Z^4Y^3$

$\therefore$ Symmetric matrix.

Similarly $X^{44} + Y^{44}$ is symmetric matrix and $X^4Z^3 - Z^3X^4$ and $X^{23} + Y^{23}$ are skew symmetric matrices.

19. [A → Q; B → P, Q; C → P, Q, S, T; D → Q, T]

(A) $\dfrac{\sqrt{3}\alpha + \beta}{2} = \sqrt{3} \Rightarrow \alpha = \dfrac{2\sqrt{3} - \beta}{\sqrt{3}}$

$\therefore \quad \dfrac{2\sqrt{3} - \beta}{\sqrt{3}} = 2 + \sqrt{3}\beta \Rightarrow \beta = 0 \Rightarrow \alpha = 2$

(B) $Lf'(1) = -6a$ and $Rf'(1) = b$

$-6a = b$ ...(i)

Also f is continuous at $x = 1$,

$\therefore \quad -3a - 2 = b + a^2$

$\Rightarrow \quad a^2 - 3a + 2 = 0$ (using (i))

$\Rightarrow \quad a = 1, 2$

(C) $(3 - 3\omega + 2\omega^2)^{4n+3} + (2 + 3\omega - 3\omega^2)^{4n+3} + (-3 + 2\omega + 3\omega^2)^{4n+3} = 0$

$$\Rightarrow \quad (3 - 3\omega + 2\omega^2)^{4n+3} + \left(\frac{2\omega^2 + 3 - 3\omega}{\omega^2}\right)^{4n+3} + \left(\frac{-3\omega + 2\omega^2 + 3}{\omega}\right)^{4n+3} = 0$$

$$\Rightarrow \quad (3 - 3\omega + 2\omega^2)^{4n+3}\left[1 + \omega^{4n+3} + (\omega^2)^{4n+3}\right] = 0$$

$\Rightarrow$ $4n + 3$ should be an integer other than multiple of 3.

$\therefore$ $n = 1, 2, 4, 5$

(D) $\dfrac{2ab}{a+b} = 4 \Rightarrow ab = 2a + 2b$ ...(i)

Also $a + q = 10$ or $a = 10 - q$

and $b + 5 = 2q$ or $b = 2q - 5$

Putting values of a and b in eq^n(i)

$q = 4$ or $\dfrac{15}{2} \Rightarrow a = 6$ or $\dfrac{5}{2}$

$\therefore$ $|q - a| = 2$ or 5.

20. [A→P, R, S; B → P; C → P, Q; D→S, T]

(A) $2(a^2 - b^2) = c^2$

$\Rightarrow$ $2(\sin^2 x - \sin^2 y) = \sin^2 z$

$\Rightarrow$ $2\sin(x + y)\sin(x - y) = \sin^2 z$

$\Rightarrow$ $2\sin(x - y) = \sin z$ $(\because \sin(x + y) = \sin z)$

$\Rightarrow$ $\dfrac{\sin(x-y)}{\sin z} = \dfrac{1}{2} = \lambda$

$\therefore$ $\cos(n\pi\lambda) = 0 \Rightarrow \cos\dfrac{n\pi}{2} = 0 \Rightarrow n = 1, 3, 5$

(B) $1 + \cos 2X - 2\cos 2Y = 2\sin X\sin Y$

$\Rightarrow$ $2\cos^2 X - 2\cos 2Y = 2\sin X\sin Y$

$\Rightarrow$ $1 - \sin^2 X - 1 + 2\sin^2 Y = \sin X\sin Y$

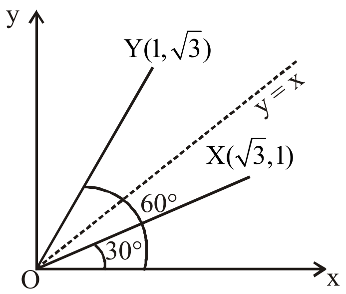
$\Rightarrow$ $\sin^2 X + \sin X\sin Y - 2\sin^2 Y = 0$

$\Rightarrow$ $(\sin X - \sin Y)(\sin X + 2\sin Y) = 0$

$\Rightarrow$ $\dfrac{\sin X}{\sin Y} = 1$ or -2

$\therefore$ $\dfrac{a}{b} = 1.$

(C) $X(\sqrt{3}, 1)$, $Y(1, \sqrt{3})$, $Z(\beta, 1 - \beta)$



By symmetry, acute angle bisector of $\angle XOY$ is $y = x$.

$\therefore$ Distance of Z from bisector

$$= \left|\frac{\beta - 1 + \beta}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}} \Rightarrow 2\beta - 1 = \pm 3 \text{ or } \beta = 2 \text{ or } -1$$

$\therefore$ $|\beta| = 1, 2$

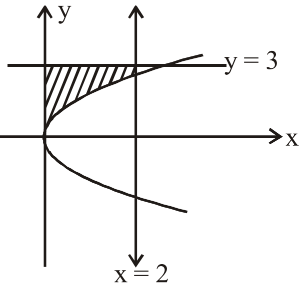
(D) For $\alpha = 0, y = 3$

For $\alpha = 1, y = |x - 1| + |x - 2| + x$

Case I

F(α) is the area bounded by $x = 0, x = 2, y^2 = 4x$ and $y = 3$

$\therefore$ $F(\alpha) = \int_0^2 (3 - 2\sqrt{x})dx$



$$= \left|3x - \frac{4x\sqrt{x}}{3}\right|_0^2 = 6 - \frac{8\sqrt{2}}{3}$$

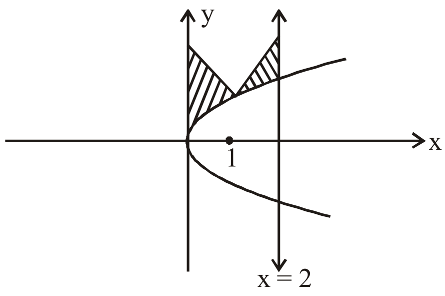
$\therefore$ $F(\alpha) + \dfrac{8}{3}\sqrt{2} = 6$

Case II

F(α) is the area bounded by $x = 0$, $x = 2$, $y^2 = 4x$ and $y = |x - 1| + |x - 2| + x$

$$= \begin{cases} 3 - x, 0 \le x < 1 \\ x + 1, 1 \le x \le 2 \end{cases}$$

$\therefore$ $F(\alpha) = \int_0^1 (3 - x - 2\sqrt{x})dx + \int_1^2 (x + 1 - 2\sqrt{x})dx$



$$= \left(3x - \frac{x^2}{2} - \frac{4x}{3}\sqrt{x}\right)_0^1 + \left(\frac{x^2}{2} + x - \frac{4}{3}x\sqrt{x}\right)_1^2$$

$$= 3 - \frac{1}{2} - \frac{4}{3} + 2 + 2 - \frac{8\sqrt{2}}{3} - \frac{1}{2} - 1 + \frac{4}{3} = 5 - \frac{8\sqrt{2}}{3}$$

$F(\alpha) + \dfrac{8\sqrt{2}}{3} = 5$

Paper - 2

PHYSICS

1. (2) $R = -\frac{dA}{dt} = -\frac{d}{dt}\left[-\frac{dN}{dt}\right] = \frac{d^2N}{dt^2} = \frac{d^2\left(N_o e^{-\lambda t}\right)}{dt^2}$

$\therefore \quad R = N_0 \lambda^2 e^{-\lambda t} = (N_0\lambda)\, \lambda\, e^{-\lambda t} = A_0 \lambda e^{-\lambda t}$

$[\because A_0 = N_0\lambda]$

$$\therefore \quad \frac{R_P}{R_Q} = \frac{\lambda_P\, e^{-\lambda_P t}}{\lambda_Q\, e^{-\lambda_Q t}} = \frac{\lambda P}{\lambda Q} \times \frac{e^{\lambda_Q t}}{e^{\lambda_P t}} = \frac{2\tau}{\tau}\, \frac{e^{\frac{2\tau}{2\tau}}}{e^{\frac{2\tau}{\tau}}} = \frac{2}{e}$$

$\therefore \quad n = 2$

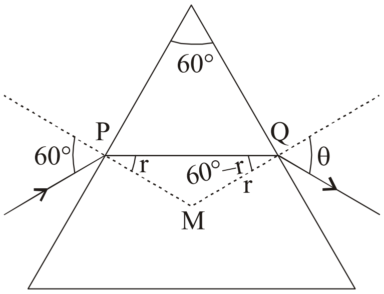
2. (2) Here $\angle MPQ + \angle MQP = 60°$. If $\angle MPQ = r$ then $\angle MQP = 60 - r$

Applying Snell's law at P

$\sin 60° = n \sin r$ ...(i)

Differentiating w.r.t 'n' we get

$O = \sin r + n \cos r \times \frac{dr}{dn}$ ...(ii)



Applying Snell's law at Q

$\sin \theta = n \sin (60° - r)$ ...(iii)

Differentiating the above equation w.r.t 'n' we get

$$\cos\theta \frac{d\theta}{dn} = \sin(60° - r) + n\cos(60° - r)\left[-\frac{dr}{dn}\right]$$

$$\therefore \cos\theta \frac{d\theta}{dn} = \sin(60° - r) - n\cos(60° - r)\left[-\frac{\tan r}{n}\right]$$

[from (ii)]

$$\therefore \quad \frac{d\theta}{dn} = \frac{1}{\cos\theta}[\sin(60° - r) + \cos(60° - r)\tan r] \text{ ...(iv)}$$

From eq. (i), substituting $n = \sqrt{3}$ we get $r = 30°$

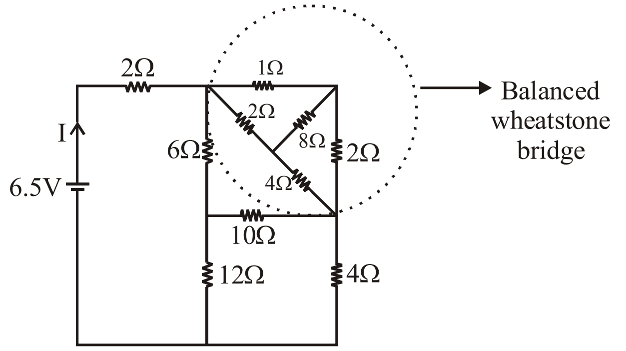
From eq (iii), substituting $n = \sqrt{3}$, $r = 30°$ we get $\theta = 60°$

On substituting the values of r and θ in eq (iv) we get

$$\frac{d\theta}{dn} = \frac{1}{\cos 60°}[\sin 30° + \cos 30° \tan 30°] = 2$$

3. (1) The equivalent resistance of balanced wheatstone bridge is

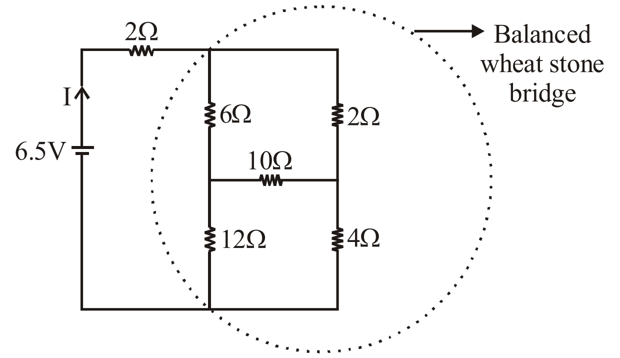
$$R_{eq} = \frac{3 \times 6}{3 + 6} = 2\Omega$$



The equivalent resistance of balanced wheat stone bridge is

$$R_{eq} = \frac{6 \times 18}{24} = \frac{9}{2}\Omega$$

$$\therefore \quad I = \frac{6.5}{2 + 4.5} = 1\text{A}$$



4. (2) Given $mvr = \frac{3h}{2\pi} \Rightarrow n = 3$

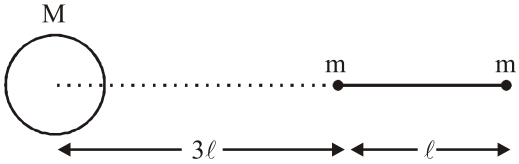
$\therefore \quad \frac{h\, r}{\lambda} = \frac{3\, h}{2\pi}$ $\left[\because \lambda = \frac{h}{mv}\right]$

$\therefore \quad \lambda = \frac{2\pi r}{3} = \frac{2}{3}\pi\left[a_0 \frac{n^2}{z}\right]$ $\left[\because r = a_0 \frac{n^2}{z}\right]$

$\therefore \quad \lambda = \frac{2}{3}\pi a_0 \left[\frac{3 \times 3}{3}\right] = 2\pi a_0$

$\therefore \quad$ p = 2

5. (7) For the tension in the rod to be zero, the force on both the masses m and m should be equal in magnitude and direction. Therefore



$$\frac{GMm}{(4\ell)^2} + \frac{Gmm}{\ell^2} = \frac{GMm}{(3\ell)^2} - \frac{Gmm}{\ell^2}$$

$$\therefore \quad 2m = M\left[\frac{1}{9} - \frac{1}{16}\right]$$

$$\therefore \quad m = \frac{7M}{288}$$

$K = 7$

6. (4) $E = A^2 e^{-0.2t}$

$\therefore \quad \log_e E = 2\log_e A - 0.2t$

On differentiating we get

$$\frac{dE}{E} = 2\frac{dA}{A} - 0.2\frac{dt}{t} \times t$$

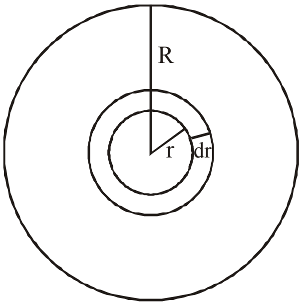
As errors always add up therefore

$$\frac{dE}{E} \times 100 = 2\left(\frac{dA}{A} \times 100\right) + 0.2t\left(\frac{dt}{t} \times 100\right)$$

$$\therefore \quad \frac{dE}{E} \times 100 = 2 \times 1.25\% + 0.2 \times 5 \times 1.5\%$$

$$\therefore \quad \frac{dE}{E} \times 100 = 4\%$$

7. (6) $I = \int_0^R (dm)r^2$



$$\therefore \quad I = \int_0^R \rho \times 4\pi r^2\, dr \times r^2$$

$$\therefore \quad I = 4\pi \int_0^R \rho\, r^4\, dr$$

$$\therefore \quad I_A = 4\pi \int_0^R k\,\frac{r}{R} \times r^4\, dr = \frac{4\pi K}{R}\int_0^R r^5\, dr$$

$$= \frac{4\pi K}{R}\left(\frac{R^6}{6}\right) = 4\pi K\,\frac{R^5}{6}$$

$$I_B = 4\pi \int_0^R K\left(\frac{r}{R}\right)^5 r^4\, dr$$

$$= \frac{4\pi K}{R^5} \times \frac{R^{10}}{10} = 4\pi K\,\frac{R^5}{10}$$

$$\therefore \quad \frac{I_B}{I_A} = \frac{6}{10} \Rightarrow \text{n} = 6$$

8. (3) $y = \sqrt{I_0}\left[\sin O + \sin\frac{\pi}{3} + \sin\frac{2\pi}{3} + \sin\pi\right]$

$$y = \sqrt{I_0}\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] = \sqrt{3}\,\sqrt{I_0}$$

$\therefore \quad I_r = y^2 = 3I_0 \quad \Rightarrow \quad n = 3$

9. (a, c) We know that

$$C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \text{ and } R = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Now, $\mu_0 I^2 = \varepsilon_0 V^2$

$\therefore \quad \frac{\mu_0}{\varepsilon_0} = \frac{V^2}{I^2} = R^2 \quad \Rightarrow \quad$ Option A is correct

Now, $\varepsilon_0 I = \mu_0 V$

$\therefore \quad \frac{\mu_0}{\varepsilon_0} = \frac{I}{V} = \frac{1}{R} \quad \Rightarrow \quad$ Option B is incorrect

Now, $I = \varepsilon_0 C V$

$$\therefore \quad \frac{1}{\varepsilon_0 C} = \frac{V}{I} = R$$

$$\therefore \quad \frac{1}{\varepsilon_0 \frac{1}{\sqrt{\mu_0\varepsilon_0}}} = R$$

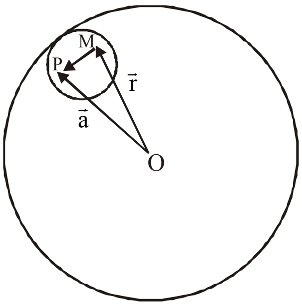
$\therefore \quad \sqrt{\frac{\mu_0}{\varepsilon_0}} = R \quad \Rightarrow \quad$ Option C is correct

Now, $\mu_0 C I = \varepsilon_0 V$

$$\therefore \quad \frac{\mu_0}{\varepsilon_0} = \frac{V}{I_C} = \frac{R}{C} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \times \frac{1}{\frac{1}{\sqrt{\mu_0\varepsilon_0}}} = \mu_0$$

$\Rightarrow$ Option (d) is incorrect

10. (d) Assume the cavity to contain similar charge distribution of positive and negative charge as the rest of sphere. Electric field at M due to uniformly distributed charge of the whole sphere of radius R_1



$$\vec{E} = \frac{\rho}{3\varepsilon}\vec{r}$$

Electric field at M due the negative charge distribution in the cavity

$$\vec{E}_2 = \frac{\rho}{3\varepsilon}\overrightarrow{MP}$$

∴ The total electric field at M is

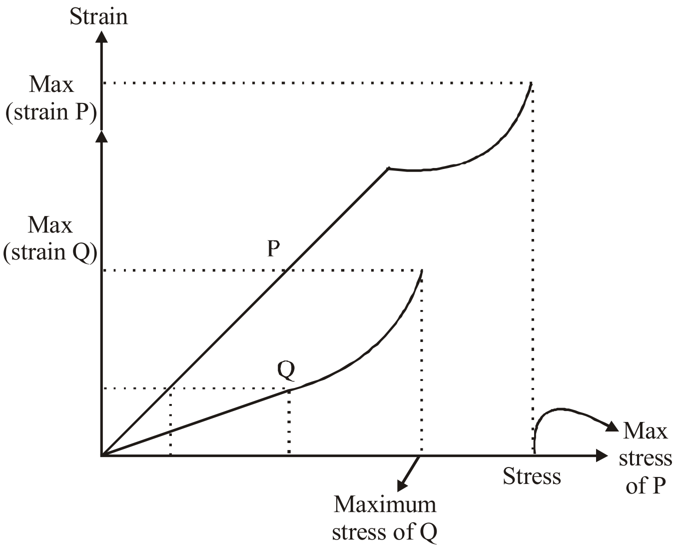
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}\overrightarrow{MP}$$

$$\therefore \quad \vec{E} = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}\left(\vec{a} - \vec{r}\right)\left[\because \ \vec{r} + \overrightarrow{MP} = \vec{a}\right]$$

$$\therefore \quad \vec{E} = \frac{\rho}{3\varepsilon}\vec{a}$$

(d) is the correct option

11. (a, b) The maximum stress that P can withstand before breaking is greater than Q. Therefore (A) is a correct option.

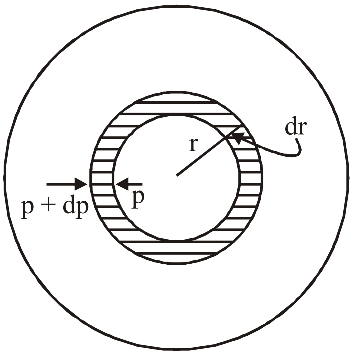


The strain of P is more than Q therefore P is more ductile. Therefore (B) is a correct option.

$$\text{Y} = \frac{\text{stress}}{\text{strain}}$$

For a given strain, stress is more for Q. Therefore $Y_Q > Y_P$.

12. (b, c) Let us consider an elemental mass dm shown in the shaded portion.



Here $P\,4\pi r^2 - (P + dP)\,4\pi r^2 = \frac{GMr}{R^3}\,\rho\,(4\pi r^2)\,dr$

$$\therefore \quad -\int_0^P dp = \frac{GM\rho}{R^3}\int_R^r r\,dr$$

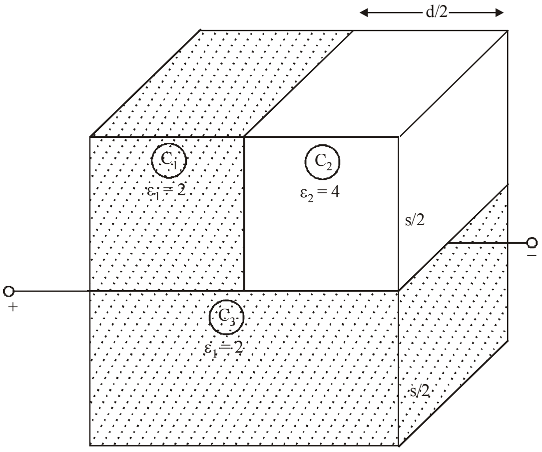
$$\therefore \quad P = \frac{GM\rho}{2R^3}\left[R^2 - r^2\right]$$

$$\therefore \quad \frac{P\,(\text{r} = 3\,\text{R}/4)}{P\,(\text{r} = 2\,\text{R}/3)} = \frac{\left[R^2 - \frac{9R^2}{16}\right]}{\left[R^2 - \frac{4R^2}{9}\right]} = \frac{\frac{7R^2}{16}}{\frac{5R^2}{9}} = \frac{63}{80}$$

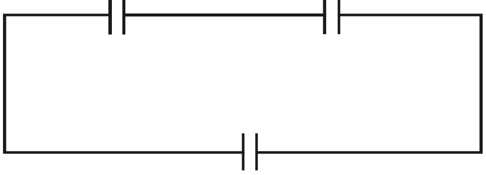
$$\text{and} \quad \frac{P\,(\text{r} = 3\,\text{R}/5)}{P\,(\text{r} = 2\,\text{R}/5)} = \frac{\left[R^2 - \frac{9R^2}{25}\right]}{\left[R^2 - \frac{4R^2}{25}\right]} = \frac{16}{21}$$

B and C are correct options.

13. (d)



$$\text{C}_1 = \frac{2\,\varepsilon_0\ \text{s}/2}{\text{d}/2} \qquad \text{C}_2 = \frac{4\,\varepsilon_0\ \text{s}/2}{\text{d}/2}$$



$$\text{C}_3 = \frac{2\,\varepsilon_0\ \text{s}/2}{\text{d}} = \frac{\varepsilon_0 \text{s}}{\text{d}}$$

$$C_{\text{eq}} = \frac{C_1 \times C_2}{C_1 + C_2} + C_3 = \frac{\frac{2\varepsilon_0 s}{d} \times \frac{4\,\varepsilon_0 s}{d}}{\frac{6\varepsilon_0 s}{d}} + \frac{\varepsilon_0 s}{d}$$

$$= \frac{4}{3}\frac{\varepsilon_0 \text{s}}{\text{d}} + \frac{\varepsilon_0 \text{s}}{\text{d}}$$

$$\therefore \quad C_{\text{eq}} = \frac{7}{3}\frac{\varepsilon_0 s}{d} = \frac{7}{3}C_1 \qquad \left[\because C_1 = \frac{\varepsilon_0 s}{d}\right]$$

14. (b) Applying combined gas law

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

If $V_2 = 2\,V_1$ and $T_2 = 3T_1$ then

$$\frac{P_1V_1}{T_1}=\frac{P_2\times 2V_1}{3T_1} \Rightarrow P_1=\frac{2}{3}P_2$$

Now change in internal energy

$$\Delta U=\frac{f}{2}\,[nR\,(T_2-T_1)]=\frac{f}{2}\,[P_2V_2-P_1V_1]$$

For monoatomic gas f = 3

$$\Delta U=\frac{3}{2}\left[\frac{3}{2}P_1\times 2V_1-P_1V_1\right]=3P_1\,\mathrm{V}_1$$

∴ (b) is the correct option.

Now assuming that the pressure on the piston on the right hand side (not considering the affect of spring) remains the same throughout the motion of the piston then,

Pressure of gas = $P_1+\frac{kx}{A} \Rightarrow P_2=P_1+\frac{kx}{A}$

where k is spring constant and A = area of piston

Energy stored = $\frac{1}{2}kx^2$

$$P_2=P_1+\frac{kx}{A}$$

$$\frac{3}{2}P_1=P_1+\frac{kx}{A}$$

$$\frac{P_1}{2}=\frac{kx}{A}$$

$$\therefore \quad kx=\frac{P_1A}{2}$$

Also,

$V_2 = V_1 + Ax$

$V_1 = Ax$

$$\therefore \quad x=\frac{V_1}{A}$$

$$\therefore \quad \text{Energy}=\frac{1}{2}\frac{P_1A}{2}\times\frac{V_1}{A}=\frac{1}{4}P_1V_1$$

∴ A is correct

Now

$$W=\int PdV=\int\left(P_1+\frac{kx}{A}\right)dV=\int P_1dV+\int\frac{kx}{A}dV$$

$$\therefore \quad W=\int P_1dV+\int\frac{kx}{A}\times(\mathrm{dx})\,\mathrm{A}$$

$$\therefore \quad W=P_1\,(\mathrm{V}_2-\mathrm{V}_1)+\frac{kx^2}{2}$$

$$\left[\begin{array}{l}\text{Here on applying } \dfrac{P_1V_1}{T_1}=\dfrac{P_2V_2}{T_2} \text{ we get } P_2=\dfrac{4P_1}{3}\\ \text{and } V_2=V_1+Ax\Rightarrow x=\dfrac{2V_1}{A}\,[\because V_2=3V_1]\end{array}\right]$$

$$\therefore \ \mathrm{W}=2P_1V_1+\frac{1}{2}\times\frac{P_1A}{3}\times\frac{2V_1}{A}=\frac{7}{3}P_1V_1$$

C is correct option

Heat supplied

$Q = W + \Delta U$

$$=\frac{7\,P_1V_1}{3}+\frac{3}{2}(P_2V_2-P_1V_1)$$

$$=\frac{7P_1V_1}{3}+\frac{3}{2}\left[\frac{4}{3}P_1\,3V_1-P_1V_1\right]=\frac{41}{6}P_1V_1$$

15. (a) ${}^{236}_{92}\mathrm{U}\rightarrow{}^{140}_{54}\mathrm{Xe}+{}^{94}_{38}\mathrm{Sr}+x+y$

The number of proton in reactants is equal to the products (leaving x and y) and mass number of product (leaving x and y) is two less than reactants

∴ $x = p, y = e^-$ is ruled out [B] is incorrect

and $x = p, y = n$ is ruled out [C] is incorrect

Total energy loss = (236 × 7.5) – [140 × 8.5 + 94 × 8.5] = 219 MeV

The energies of kx and ky together is 4MeV

The energy remain is distributed by Sr and Xe which is equal to 219 – 4 = 215 MeV

∴ A is the correct option

Also momentum is conserved

$\therefore K.E.\propto\frac{1}{m}$. Therefore $\mathrm{K.E_{sr}} > \mathrm{K.E_{xe}}$

16. (a, d) From the figure it is clear that

(a) $\sigma_2 > \sigma_1$

(b) $\rho_2 > \sigma_2$ [As the string is taut]

(c) $\rho_1 < \sigma_1$ [As the string is taut]

∴ $\rho_1 < \sigma_1 < \sigma_2 < \rho_2$

When P alone is in L_2

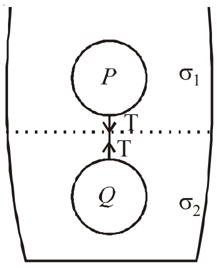
$$V_P=\frac{2\pi r^2\,(\rho_1-\sigma_2)\,\mathrm{g}}{9\eta_2} \text{ is negative as } \rho_1<\sigma_2$$

Where r is radius of sphere.

When Q alone is in L_1

$$V_Q=\frac{2\pi r^2\,(\rho_2-\sigma_1)\,\mathrm{g}}{9\eta_1} \text{ is positive as } \rho_2>\sigma_1$$

Therefore $\vec{V}_P.\vec{V}_Q<O$ option (d) is correct



Also $\frac{V_P}{V_Q} = \frac{\rho_1 - \sigma_2}{\rho_2 - \sigma_1} \times \frac{\eta_1}{\eta_2}$ ...(i)

For equilibrium of Q

$T + \frac{4}{3}\pi r^3 \, \sigma_2 g = \frac{4}{3}\pi r^3 \, \rho_2 g$ ...(ii)

For equilibrium of P

$T + \frac{4}{3}\pi r^3 \, \rho_1 g = \frac{4}{3}\pi r^3 \, \sigma_1 g$ ...(iii)

(iii) – (ii) gives

$\rho_1 - \sigma_2 = \sigma_1 - \rho_2$ ...(iv)

From (i) and (iv)

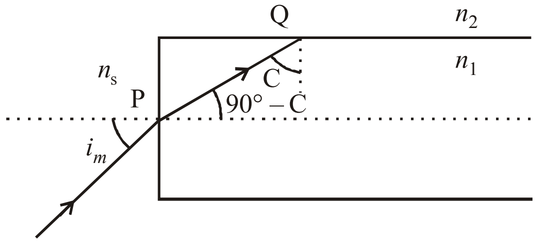
$\frac{V_P}{V_Q} = -\frac{\eta_1}{\eta_2}$ $\quad \therefore \quad$ $\frac{|V_P|}{|V_Q|} = \frac{\eta_1}{\eta_2}$

$\therefore$ A is also a correct option

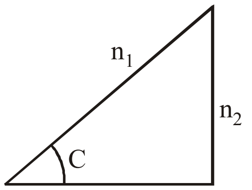
17. (a, c)

Applying Snell's law at P; $n_s \sin i_m = n_1 \sin (90° - C)$

n_s = Refractive index of surrounding



Also $\sin C = \frac{n_2}{n_1}$



Now

$$NA = \sin i_m = \frac{n_1}{n_s}\cos C = \frac{n_1}{n_s}\sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\therefore \quad NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_s}$$

For S_1 (in air)

$$NA = \sqrt{\frac{45}{16} - \frac{9}{4}} = \frac{3}{4}$$

For S_1 (in water)

$$NA = \frac{3}{4}\sqrt{\frac{45}{16} - \frac{9}{4}} = \frac{9}{16}$$

For s_1 $\left(\text{in } n_s = \frac{6}{\sqrt{15}}\right)$

$$NA = \frac{\sqrt{15}}{6}\sqrt{\frac{45}{16} - \frac{9}{4}} = \frac{3\sqrt{15}}{24}$$

For S_2 (in water)

$$NA = \frac{3}{4}\sqrt{\frac{64}{25} - \frac{49}{25}} = \frac{3}{4}\frac{\sqrt{15}}{5}$$

For S_2 (in air)

$$NA = \sqrt{\frac{64}{25} - \frac{49}{25}} = \frac{\sqrt{15}}{5}$$

For S_2 (in $n_s = \frac{4}{\sqrt{15}}$)

$$NA = \frac{\sqrt{15}}{4}\sqrt{\frac{64}{25} - \frac{49}{25}} = \frac{3}{4}$$

For S_2 $\left(\text{in } n_s = \frac{16}{3\sqrt{15}}\right)$

$$NA = \frac{3\sqrt{15}}{16}\sqrt{\frac{64}{25} - \frac{49}{25}} = \frac{9}{16}$$

(a), (c) are correct options

18. (d) $NA = \frac{1}{n_s}\sqrt{n_1^2 - n_2^2}$

Here

$NA_2 < NA_1$

$\therefore$ the NA of combined structure is equal to the smaller value of the two numerical apertures.

(d) is the correct option.

19. (a, d) When megnetic force balances electric force

$F_B = F_E$

$q\, v_d B = q\, E$

$\therefore \quad v_d B = \frac{V}{w}$ $\qquad [\because V = E \times w]$

$\therefore \quad V = wv_d B = w\left[\frac{I}{newd}\right] \times B$

$\left[v_d = \frac{I}{neA} = \frac{I}{newd}\right]$

$\therefore \quad V = \frac{I}{ned} \times B$

$\therefore \quad V \propto \frac{1}{d} \quad \Rightarrow \quad V_1 d_1 = V_2 d_2$

when $d_1 = 2d_2$, $V_2 = 2V_1$

and when $d_1 = d_2$, $V_2 = V_1$

(a), (d) are correct options

20. (a, d) Here

$V \propto \frac{B}{n} \Rightarrow \frac{V_1 n_1}{B_1} = \frac{V_2 n_2}{B_2}$

If $B_1 = B_2$ and $n_1 = 2n_2 \Rightarrow V_2 = 2V_1$

and of $B_1 = 2B_2$ and $n_1 = n_2 \Rightarrow V_2 = 0.5V_1$

A and C are the correct options.

CHEMISTRY

1. (6) $3B_2H_6 + 18CH_3OH \rightarrow 6B(OCH_3)_3 + 18H_2$

2. (3) $1 \rightarrow HX \qquad 2 \rightarrow HY$

$$\alpha_1 = \frac{(\lambda_m)_{HX}}{\overset{\circ}{\lambda}_m} \qquad \alpha_2 = \frac{(\lambda_m)_{HY}}{\overset{\circ}{\lambda}_m}$$

$$K_{a_1} = C_1\alpha_1^2 \qquad K_{a_2} = C_2\alpha_2^2$$

$$= 0.01\frac{(\lambda_m)^2_{HX}}{\left(\overset{\circ}{\lambda}_m\right)^2} \qquad = 0.1\frac{(\lambda_m)^2_{HY}}{\left(\overset{\circ}{\lambda}_m\right)^2}$$

$$\therefore \quad \frac{K_{a_1}}{K_{a_2}} = \frac{0.01(\lambda_m)^2_{HX}}{0.1(\lambda_m)^2_{HY}} = 0.1\left(\frac{(\lambda_m)_{HX}}{(\lambda_m)_{HY}}\right)^2$$

$$= 0.1\left(\frac{1}{10}\right)^2 = 10^{-3}$$

$$pK_a(HX) - pK_a(HY) = -\log\frac{K_{a_1}}{K_{a_2}} = -\log 10^{-3} = 3$$

3. (9) Number of moles in gas phase, at start $(n_i) = 1$

$${}^{238}_{92}U \rightarrow {}^{206}_{82}Pb + 8\,{}^{4}_{2}He + 6\,{}^{0}_{-}\beta$$

Now number of moles in gas phase, after decomposition (n_F) $= 1 + 8 = 9$ mole

at constant temperature and pressure

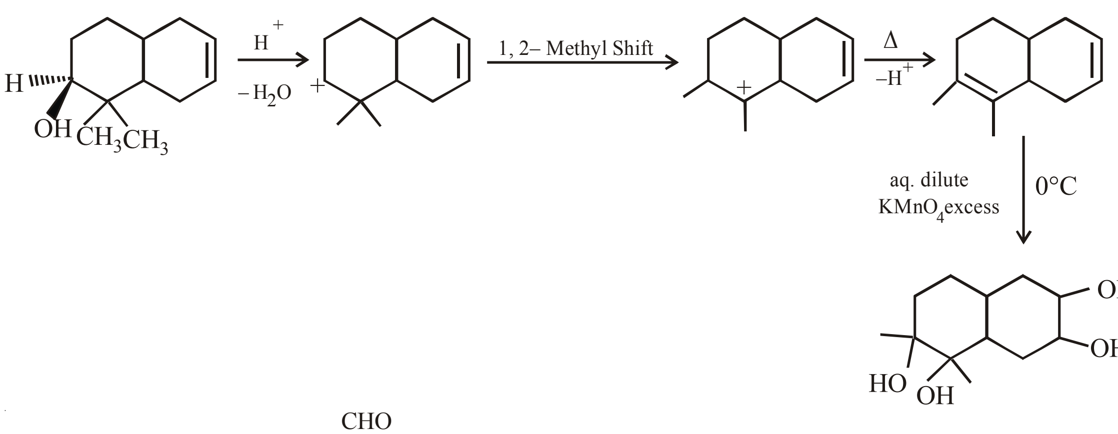
$$\frac{P_F}{P_{in}} = \frac{n_F}{n_{in}} = \frac{9}{1} = 9$$

4. (8) $8H^+ + 5[Fe(H_2O)_2(OX)_2]^{2-} + MnO_4^- \rightarrow Mn^{2+} + 5\,[Fe(H_2O)_2(OX)_2]^- + 4H_2O$

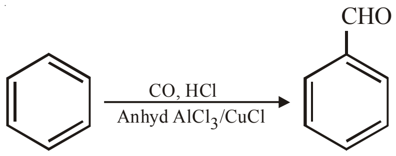
$$\text{Rate} = \frac{1}{8}\frac{d[H^+]}{dt} = -\frac{d[MnO_4^-]}{dt}$$

$$\text{Hence, } \frac{\text{rate of } [H^+]\,\text{decay}}{\text{rate of } [MnO_4^-]\text{ decay}} = 8$$

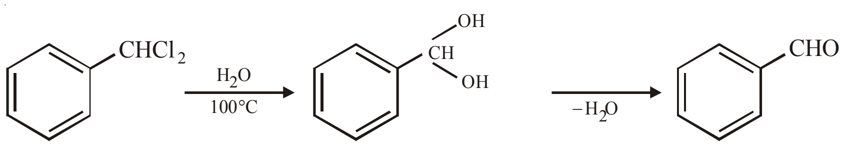
5. (4)

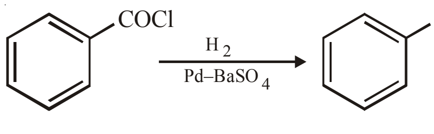


6. (4)

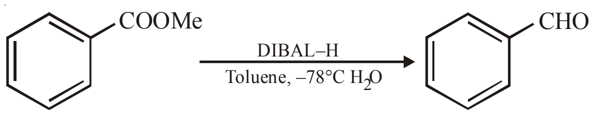


(Gatterman Koch Reaction)

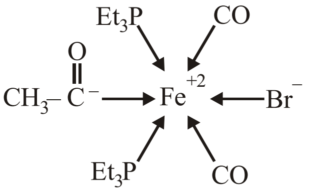




(Rosenmund Reduction)

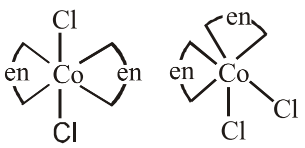


7. (3)

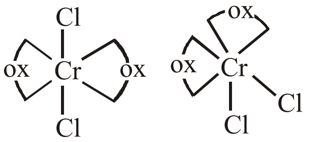


8. (6) All the complexes given show cis-trans isomerism

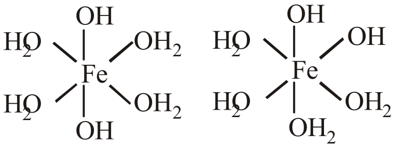
$[Co(NH_2—CH_2—CH_2—NH_2)Cl_2]^+$



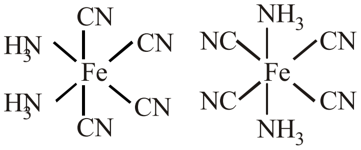
$[CrCl_2(C_2O_4)_2]^{3-}$



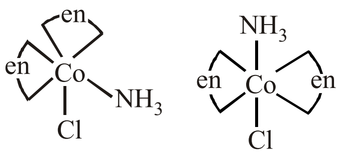
$[Fe(H_2O)_4(OH)_2]^+$



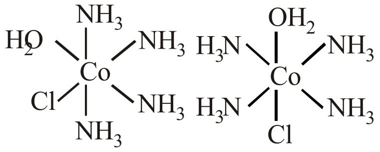
$[Fe(NH_3)_2(CN)_4]^-$



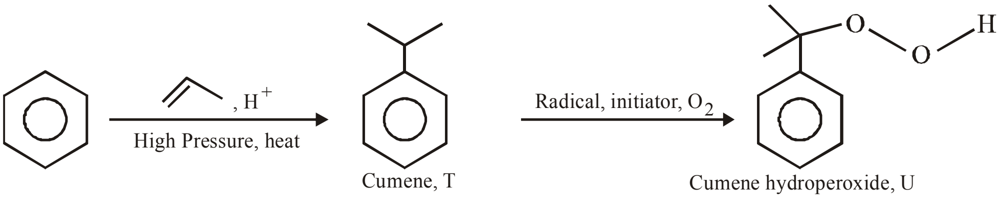
$[Co(NH_2–CH_2–CH_2–NH_2)_2(NH_3)Cl]^{2+}$



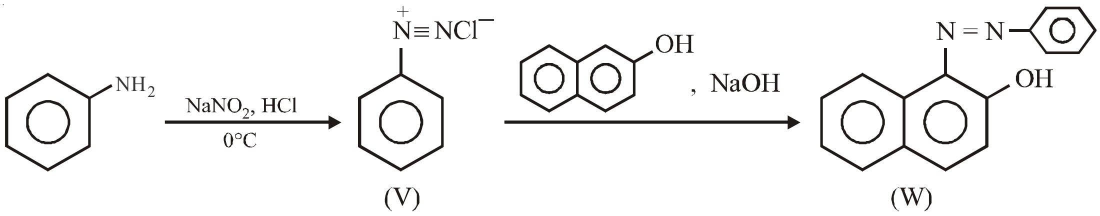
$(Co(NH_3)_4(H_2O)Cl]^{2+}$



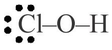
9. (b)



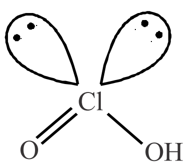
10. (a)



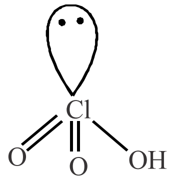
11. (b, c)



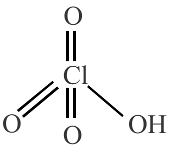
(i)



(ii)



(iii)



(iv)

Number of Cl = O bonds in (ii) and (iii) together is 3
Number of lone pairs on Cl in (ii) and (iii) together is 3
Hybridisation of Cl in all the four is sp^3
Strongest acid is $HClO_4$ (iv)

12. (c, d) Only group II cations precipitate as sulphide with H_2S in acidic medium that is (Cu^{2+}, Pb^{2+}) and (Hg^{2+}, Bi^{3+})

13. (b) $(CH_3)_2SiCl_2$ form linear polymer on hydrolysis and $(CH_3)_3SiCl$ is a chain terminator.

14. (b, c, d) Reaction on metal surface

$$M \longrightarrow M^+ + e^-$$

$$O_2 + e^- \longrightarrow O_2^-$$

This is an example of chemisorption.

15. (c) $P(V–b) = RT$

$\Rightarrow \quad PV - Pb = RT$

$\Rightarrow \quad \dfrac{PV}{RT} = \dfrac{Pb}{RT} + 1$

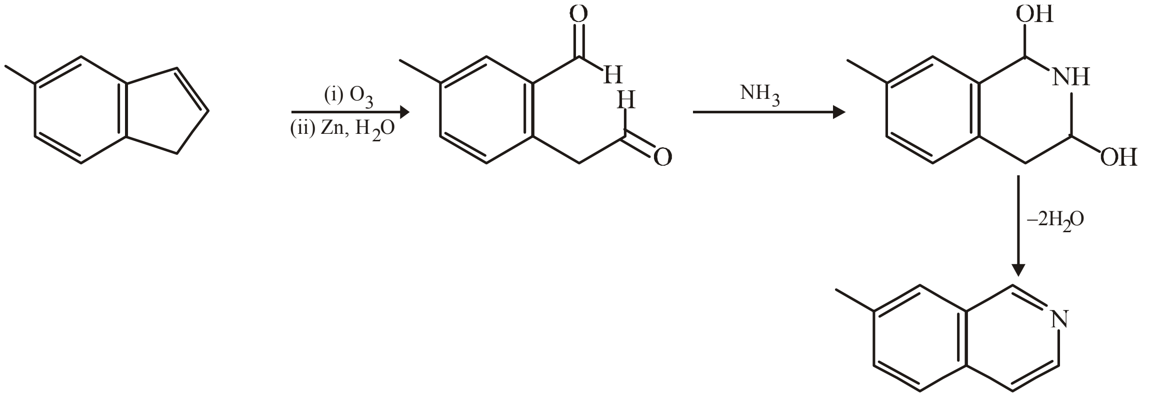
$\Rightarrow \quad Z = 1 + \dfrac{Pb}{RT}$

Hence Z > 1 at all pressures.

This means , repulsive tendencies will be dominant when interatomic distance are small.

This means, interatomic potential is never negative but becomes positive at small interatomic distances.

16. (a)



17. (a) Let the heat capacity of insulated beaker be C.

Mass of aqueous content in expt. 1 = (100 + 100) × 1 = 200 g

$\Rightarrow$ ± Total heat capacity = (C + 200 × 4.2) J/K

Moles of acid, base neutralised in expt. 1 = 0.1 × 1 = 0.1

$\Rightarrow$ Heat released in expt. 1 = 0.1 × 57 = 5.7 KJ = 5.7 × 1000 J

$\Rightarrow \quad 5.7 \times 1000 = (C + 200 \times 4.2) \times \Delta T.$

$5.7 \times 1000 = (C + 200 + 4.2) \times 5.7$

$\Rightarrow \quad (C + 200 \times 4.2) = 1000$

In second experiment,

$n_{CH_3COOH} = 0.2,\ n_{NaOH} = -\ 0.1$

Total mass of aqueous content = 200 g

$\Rightarrow$ Total heat capacity = (C + 200 × 4.2) = 1000

$\Rightarrow$ Heat released = 1000 × 5.6 = 5600 J.

Overall, only 0.1 mol of CH_3COOH undergo neutralization.

$\Rightarrow \quad \Delta H_{\text{neutralization}}$ of $CH_3COOH = \dfrac{-5600}{0.1}$

$= -\ 56000$ J/mol

$= -\ 56$ KJ/mol.

$\Rightarrow \quad \Delta H_{\text{neutralization}}$ of $CH_3COOH = 57 - 56 = 1$ KJ/mol

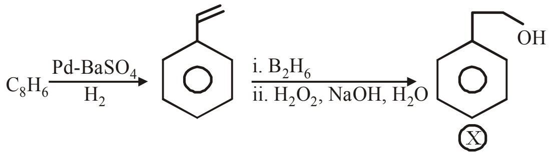
18. (b) Final solution contain 0.1 mole of CH_3COOH and CH_3COONa each.

Hence it is a buffer solution.

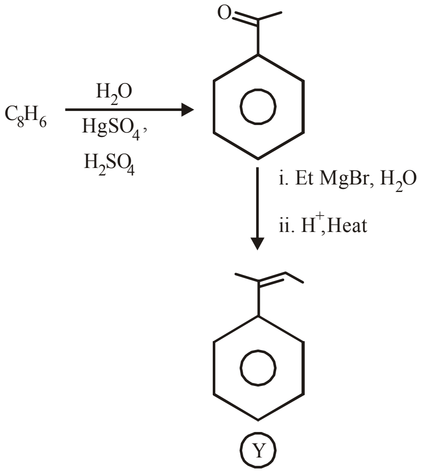
$$pH = pK_a \log \frac{[CH_3COO^-]}{[CH_3COOH]}$$

$$= 5–\log 2 + \log\frac{0.1}{0.1} = 4.7$$

19. (c)



20. (d)



MATHEMATICS

1. (2) $\lim\limits_{\alpha\to 0}\dfrac{e^{\cos\alpha^n}-e}{\alpha^m}=\dfrac{-e}{2}$

$$\Rightarrow \lim_{\alpha\to 0}\frac{e\left[e^{\cos\alpha^n-1}-1\right]}{\cos\alpha^n-1}\times\frac{\cos\alpha^n-1}{\alpha^m}=\frac{-e}{2}$$

$$\Rightarrow e\lim_{\alpha\to 0}\frac{-2\sin^2\dfrac{\alpha^n}{2}}{\left(\dfrac{\alpha^n}{2}\right)^2}\times\frac{\left(\dfrac{\alpha^n}{2}\right)^2}{\alpha^m}=\frac{-e}{2}$$

$$\Rightarrow \frac{-e}{2}\alpha^{2n-m}=\frac{-e}{2}\text{ or }\alpha^{2n-m}=1$$

$$\Rightarrow 2n-m=0\Rightarrow\frac{m}{n}=2$$

2. (9) $\alpha=\int_0^1 e^{(9x+3\tan^{-1}x)}\left(\dfrac{12+9x^2}{1+x^2}\right)dx$

Let $9x+3\tan^{-1}x=t\Rightarrow\dfrac{12+9x^2}{1+x^2}dx=dt$

$$\therefore\quad \alpha=\int_0^{9+\frac{3\pi}{4}}e^t dt=e^{9+\frac{3\pi}{4}}-1$$

$$\therefore\quad \log_e\left|1+e^{9+\frac{3\pi}{4}}-1\right|-\frac{3\pi}{4}=9$$

3. (7) $\lim\limits_{x\to 1}\dfrac{F(x)}{G(x)}=\dfrac{1}{14}\Rightarrow\lim\limits_{x\to 1}\dfrac{\int_{-1}^{x}f(t)dt}{\int_{-1}^{x}t|f(f(t))|dt}$

$\because\quad \int_{-1}^{1}f(t)dt=0$ and $\int_{-1}^{1}t|f(f(t))|dt=0$

$f(t)$ being odd function

$\therefore$ Using L Hospital's rule, we get

$$\lim_{x\to 1}\frac{f(x)}{x|f(f(x))|}=\frac{1}{14}$$

$$\Rightarrow\frac{f(1)}{|f(f(1))|}=\frac{1}{14}\Rightarrow\frac{1/2}{\left|f\left(\frac{1}{2}\right)\right|}=\frac{1}{14}$$

$$\Rightarrow\left|f\left(\frac{1}{2}\right)\right|=7\Rightarrow f\left(\frac{1}{2}\right)=7$$

4. (9) $\vec{s}=4\vec{p}+3\vec{q}+5\vec{r}$

$$\vec{s}=x(-\vec{p}+\vec{q}+\vec{r})+y(\vec{p}-\vec{q}+\vec{r})+z(-\vec{p}-\vec{q}+\vec{r})$$

$$\Rightarrow -x+y-z=4$$
$$x-y-z=3$$
$$x+y+z=5$$

Solving above equations $x=4, y=\dfrac{9}{2}, z=\dfrac{-7}{2}$

$\therefore\quad 2x+y+z=9$

5. (4) $\alpha_k=\cos\dfrac{k\pi}{7}+i\sin\dfrac{k\pi}{7}=e^{\frac{i\pi k}{7}}$

$$\alpha_{k+1}-\alpha_k=e^{\frac{i\pi(k+1)}{7}}-e^{\frac{i\pi k}{7}}=e^{\frac{i\pi k}{7}}(e^{i\pi/7}-1)$$

$$|\alpha_{k+1}-\alpha_k|=|e^{i\pi/7}-1|$$

$$\Rightarrow\sum_{k=1}^{12}|\alpha_{k+1}-\alpha_k|=12|e^{i\pi/7}-1|$$

Similarly $\sum\limits_{k=1}^{3}|\alpha_{4k-1}-\alpha_{4k-2}|=3|e^{i\pi/7}-1|$

$$\therefore\quad\frac{\sum\limits_{k=1}^{12}|\alpha_{k+1}-\alpha_k|}{\sum\limits_{k=1}^{3}|\alpha_{4k-1}-\alpha_{4k-2}|}=4$$

6. (9) $\dfrac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]}=\dfrac{6}{11}\Rightarrow a=9d$

$a_7=a+6d=15d$

$\because\quad 130<15d<140\Rightarrow d=9$

($\because$ All terms are natural numbers $\therefore$ $d\in N$)

7. (8) In expansion of $(1+x)(1+x^2)(1+x^3)\ldots.(1+x^{100})$

x^9 can be found in the following ways

$x^9, x^{1+8}, x^{2+7}, x^{3+6}, x^{4+5}, x^{1+2+6}, x^{1+3+5}, x^{2+3+4}$

The coefficient of x^9 in each of the above 8 cases is 1.

$\therefore$ Required coefficient = 8.

8. (4) Ellipse: $\dfrac{x^2}{9}+\dfrac{y^2}{5}=1$

$\Rightarrow\quad a=3, b=\sqrt{5}$ and $e=\dfrac{2}{3}$

$\therefore\quad f_1=2$ and $f_2=-2$

$P_1: y^2 = 8x$ and $P_2: y^2 = -16x$

$$T_1: y = m_1x + \frac{2}{m_1}$$

It passes through (–4, 0),

$$0 = -4m_1 + \frac{2}{m_1} \Rightarrow m_1^2 = \frac{1}{2}$$

$$T_2: y = m_2x - \frac{4}{m_2}$$

It passes through (2, 0)

$$0 = 2m_2 - \frac{4}{m_2} \Rightarrow m_2^2 = 2$$

$$\therefore \quad \frac{1}{m_1^2} + m_2^2 = 4$$

9. (b, c, d)

$$\alpha = 3\sin^{-1}\frac{6}{11} > 3\sin^{-1}\frac{1}{2} \text{ or } \alpha > \frac{\pi}{2}$$

$\therefore \quad \cos\alpha < 0$

$$\beta = 3\cos^{-1}\frac{4}{9} > 3\cos^{-1}\frac{1}{2} \text{ or } \beta > \pi$$

$\therefore \quad \cos\beta < 0$ and $\sin\beta < 0$

Also $\alpha + \beta > \frac{3\pi}{2} \quad \therefore \cos(\alpha + \beta) > 0.$

10. (a, b)

Let $E_1: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b$

and $E_2: \frac{x^2}{c^2} + \frac{y^2}{d^2} = 1$ where $c < d$

Also $S: x^2 + (y - 1)^2 = 2$

Tangent at $P(x_1, y_1)$ to S is $x + y = 3$

To find point of contact put $x = 3 - y$ in S. We get P(1, 2)

Writing eq^n of tangent in parametric form

$$\frac{x-1}{\frac{-1}{\sqrt{2}}} = \frac{y-2}{\frac{1}{\sqrt{2}}} = \pm\frac{2\sqrt{2}}{3}$$

$$x = \frac{-2}{3} + 1 \text{ or } \frac{2}{3} + 1 \text{ and } y = \frac{2}{3} + 2 \text{ or } \frac{-2}{3} + 2$$

$$\Rightarrow \quad x = \frac{1}{3} \text{ or } \frac{5}{3} \text{ and } y = \frac{8}{3} \text{ or } \frac{4}{3}$$

$$\therefore \quad Q\left(\frac{5}{3}, \frac{4}{3}\right) \text{ and } R\left(\frac{1}{3}, \frac{8}{3}\right)$$

eq^n of tangent to E_1 at Q is

$$\frac{5x}{3a^2} + \frac{4y}{3b^2} = 1 \text{ which is identical to } \frac{x}{3} + \frac{y}{3} = 1$$

$$\Rightarrow \quad a^2 = 5 \text{ and } b^2 = 4 \Rightarrow e_1^2 = 1 - \frac{4}{5} = \frac{1}{5}$$

eq^n of tangent to E_2 at R is

$$\frac{x}{3c^2} + \frac{8y}{3d^2} = 1 \text{ identical to } \frac{x}{3} + \frac{y}{3} = 1$$

$$\Rightarrow \quad c^2 = 1, d^2 = 8 \Rightarrow e_2^2 = 1 - \frac{1}{8} = \frac{7}{8}$$

$$\therefore \quad e_1^2 + e_2^2 = \frac{43}{40}, e_1e_2 = \frac{\sqrt{7}}{2\sqrt{10}}, \left|e_1^2 - e_2^2\right| = \frac{27}{40}$$

11. (a, b, d) $H: x^2 - y^2 = 1$ S : Circle with centre $N(x_2, 0)$

Common tangent to H and S at $P(x_1, y_1)$ is

$$xx_1 - yy_1 = 1$$

$$\Rightarrow \quad m_1 = \frac{x_1}{y_1}$$

Also radius of circle S with centre $N(x_2, 0)$ through point of contact (x_1, y_1) is perpendicular to tangent

$$\therefore \quad m_1m_2 = -1 \Rightarrow \frac{x_1}{y_1} \times \frac{0 - y_1}{x_2 - x_1} = -1$$

$\Rightarrow \quad x_1 = x_2 - x_1$ or $x_2 = 2x_1$

M is the point of intersection of tangent at P and x-axis

$$\therefore \quad M\left(\frac{1}{x_1}, 0\right)$$

$\because \quad$ Centroid of ΔPMN is (ℓ, m)

$$\therefore \quad x_1 + \frac{1}{x_1} + x_2 = 3\ell \text{ and } y_1 = 3m$$

Using $x_2 = 2x_1$,

$$\Rightarrow \quad \frac{1}{3}\left(3x_1 + \frac{1}{x_1}\right) = l \text{ and } \frac{y_1}{3} = m$$

$$\therefore \quad \frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}, \frac{dm}{dy_1} = \frac{1}{3}$$

Also (x_1, y_1) lies on H, $\therefore \; x_1^2 - y_1^2 = 1$

or $\quad y_1 = \sqrt{x_1^2 - 1}$

$$\therefore \quad m = \frac{1}{3}\sqrt{x_1^2 - 1}$$

$$\therefore \quad \frac{dm}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}}$$

12. (a, c) Let $F(t) = e^t(\sin^6 at + \cos^6 at)$

Then $F(k\pi + t) = e^{k\pi + t}[\sin^6(k\pi + t)a + \cos^6(k\pi + t)a]$

$= e^{k\pi}.e^t[\sin^6 at + \cos^6 at]$ for even values of a.

$\therefore \quad F(k\pi + t) = e^{k\pi}F(t)$ ...(i)

$$\text{Now } \int_0^{4\pi} F(t)dt = \int_0^{\pi} F(t)dt + \int_{\pi}^{2\pi} F(t)dt + \int_{2\pi}^{3\pi} F(t)dt + \int_{3\pi}^{4\pi} F(t)dt$$

$$\text{Also } \int_{\pi}^{2\pi} F(t)dt = \int_0^{\pi} F(\pi + x)dx \text{ (putting } t = \pi + x)$$

$$= \int_0^{\pi} e^{\pi} F(x)dx \text{ using eq}^n\text{(i)}$$

$$= e^{\pi}\int_0^{\pi} F(t)dt$$

$$\text{Similarly } \int_{2\pi}^{3\pi} F(t)dt = e^{2\pi}\int_0^{\pi} F(t)dt$$

$$\int_{3\pi}^{4\pi} F(t)dt = e^{3\pi}\int_0^{\pi} F(t)dt$$

$$\therefore \quad \int_0^{4\pi} F(t)dt = (1 + e^{\pi} + e^{2\pi} + e^{3\pi})\int_0^{\pi} F(t)dt$$

$$\Rightarrow \frac{\int_0^{4\pi} F(t)dt}{\int_0^{\pi} F(t)dt} = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$$, where *'a'* can take any even value.

13. (b, c) Let $h(x) = f(x) - 3g(x)$

$h(-1) = h(0) = h(2) = 3$

$\therefore$ By Rolle's theorem $h'(x) = 0$ has atleast one solution in $(-1, 0)$ and atleast one solution in $(0, 2)$ But $h''(x)$ never vanishes in $(-1, 0)$ and $(0, 2)$ therefore $h'(x) = 0$ should have exactly one solution in each interval.

14. (a, b) $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$

$= (7\tan^4 x - 3)(\tan^4 x + \tan^2 x)$

$= (7\tan^6 x - 3\tan^2 x)\sec^2 x$

$$\int_0^{\pi/4} f(x)dx = \left[\tan^7 x - \tan^3 x\right]_0^{\pi/4}$$

$$= 1 - 1 = 0$$

$$\therefore \quad \int_0^{\pi/4} xf(x)dx = \left[x\left(\tan^7 x - \tan^3 x\right)\right]_0^{\pi/4}$$

$$-\int_0^{\pi/4}\left(\tan^7 x - \tan^3 x\right)dx$$

$$= \int_0^{\pi/4} \tan^3 x(1 - \tan^2 x)\ \sec^2 x\, dx = \left[\frac{\tan^4 x}{4} - \frac{\tan^6 x}{6}\right]_0^{\pi/4}$$

$$= \frac{1}{12}$$

15. (d) $f'(x) = \dfrac{192x^3}{2 + \sin^4 \pi x}$

$$\Rightarrow \frac{192x^3}{3} \le f'(x) \le \frac{192x^3}{2}$$

$$\Rightarrow 64x^3 \le f'(x) \le 96x^3$$

$$\Rightarrow \int_{1/2}^{x} 64x^3 dx \le \int_{1/2}^{x} f'(x)dx \le \int_{1/2}^{x} 96x^3 dx$$

$$\Rightarrow \frac{64x^4}{4} - \frac{64}{4}\times\frac{1}{16} \le \int_{1/2}^{x} f'(x)dx \le \frac{96x^4}{4} - \frac{96}{4\times 16}$$

$$\Rightarrow 16x^4 - 1 \le \int_{1/2}^{x} f'(x) \le 24x^4 - \frac{3}{2}$$

$$\Rightarrow 16x^4 - 1 \le f(x) \le 24x^4 - \frac{3}{2}$$

$$\Rightarrow \int_{1/2}^{1}(16x^4 - 1)dx \le \int_{1/2}^{1} f(x)dx \le \int_{1/2}^{1}\left(24x^4 - \frac{3}{2}\right)dx$$

$$\Rightarrow \left(\frac{16x^5}{5} - x\right)_{\frac{1}{2}}^{1} \le \int_{1/2}^{1} f(x)dx \le \left[\frac{24x^5}{5} - \frac{3}{2}x\right]_{\frac{1}{2}}^{1}$$

$$\Rightarrow 2.6 \le \int_{1/2}^{1} f(x)dx \le 3.9$$

$\therefore$ Only (d) is the correct option.

16. (a, d) $\alpha x^2 - x + \alpha = 0$ has distinct real roots.

$\therefore \quad D > 0 \Rightarrow 1 - 4\alpha^2 > 0$

$\Rightarrow \alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ ...(i)

Also $|x_1 - x_2| < 1$

$\Rightarrow (x_1 - x_2)^2 < 1$

$\Rightarrow (x_1 + x_2)^2 - 4x_1x_2 < 1$

$\Rightarrow \dfrac{1}{\alpha^2} - 4 < 1$

$\Rightarrow \dfrac{1}{\alpha^2} < 5$ or $\alpha^2 > \dfrac{1}{5}$

$\Rightarrow \alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$ ...(ii)

Combining (i) and (ii)

$$S = \left(-\frac{1}{2}, \frac{-1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$$

$\therefore$ Subsets of S can be $\left(-\frac{1}{2}, \frac{-1}{\sqrt{5}}\right)$ and $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$.

17. (a, b, c) $f(x) = xF(x) \Rightarrow f'(x) = F(x) + xF'(x)$

$$\therefore \quad f'(1) = F(1) + F'(1) = F'(1) < 0 \left(\because F'(x) < 0, x \in \left(\frac{1}{2}, 3\right) \right)$$

$$f(2) = 2F(2) < 0,$$

$\left(\because F'(x) < 0 \Rightarrow F\right.$ is decreasing on $\left(\frac{1}{2}, 3\right)$ and $F'(1) = 0$,

$\left. F(3) = -4 \right)$

$f'(x) = F(x) + xF'(x)$

For the same reason given above and $F'(x) < 0$ given.

$F(x) < 0 \ \forall \ x \in (1, 3)$

$\therefore \quad f'(x) \neq 0, x \in (1, 3).$

18. (c,d) $\int_1^3 x^2 F'(x)dx = -12$

$$\Rightarrow \quad \left[x^2 F(x)\right]_1^3 - \int_1^3 2x\, F(x)\, dx = -12$$

$$\Rightarrow \quad 9\,F(3) - F(1) - 2\int_1^3 xF(x)dx = -12$$

$$\Rightarrow \quad \int_1^3 xF(x)dx = -12 \Rightarrow \int_1^3 f(x)dx = -12 \qquad ...(i)$$

Also $\int_1^3 x^3 F''(x)dx = 40$

$$\Rightarrow \quad \left[x^3 F'(x)\right]_1^3 - 3\int_1^3 x^2 F'(x)dx = 40$$

$$\Rightarrow \quad \left[x^2 \left(f'(x) - F(x)\right)\right]_1^3 - 3 \times (-12) = 40$$

$$\left\{ \begin{array}{l} \text{Using } xF'(x) = f'(x) - F(x) \\ \text{and} \int_1^3 x^2 F'(x)dx = -12 \end{array} \right.$$

$$\Rightarrow \quad 9(f'(3) - F(3)) - (f'(1) - F(1)) = 4$$

$$\Rightarrow \quad 9f'(3) - 9 \times (-4) - f'(1) + 0 = 4$$

$$\Rightarrow \quad 9f'(3) - f'(1) + 32 = 0$$

19. (a,b) Let $E_1 \equiv$ box I is selected

$E_2 \equiv$ box II is selected

$E \equiv$ ball drawn is red

$$P(E_2/E) = \frac{\frac{n_3}{n_3 + n_4} \times \frac{1}{2}}{\frac{n_1}{n_1 + n_2} \times \frac{1}{2} + \frac{n_3}{n_3 + n_4} \times \frac{1}{2}} = \frac{1}{3}$$

$$\text{or} \quad \frac{\frac{n_3}{n_3 + n_4}}{\frac{n_1}{n_1 + n_2} + \frac{n_3}{n_3 + n_4}} = \frac{1}{3}$$

On checking the options we find A and B are the correct options.

20. (c, d) $E_1 \equiv$ Red ball is selected from box I

$E_2 \equiv$ Black ball is selected from box I

$E \equiv$ Second ball drawn from box I is red

$\therefore \quad P(E) = P(E_1)\, P(E/E_1) + P(E_2)\, P(E/E_2)$

$$= \frac{n_1}{n_1 + n_2} \times \frac{n_1 - 1}{n_1 + n_2 - 1} + \frac{n_2}{n_1 + n_2} \times \frac{n_1}{n_1 + n_2 - 1}$$

On checking the options, we find C and D have the correct values.

JEE ADVANCED 2016

1. The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of three sections.
2. **Section 1** contains 5 questions. Each question has four choices (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
4. **Section 3** contains 5 questions. The answer to each of the questions is a single-digit integer ranging from 0 to 9 (both inclusive).
3. **Section 2** contains 8 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONE OR MORE THAN ONE** are correct.

PAPER - 1

PHYSICS

SECTION - I

This section contains **5 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** option is correct.

1. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below :

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8 \text{m s}^{-1}$ and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of J s) found from such an experiment is

(a) 6.0×10^{-34} (b) 6.4×10^{-34}

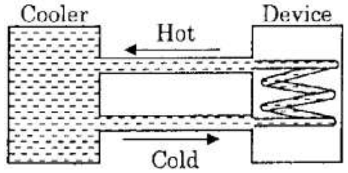
(c) 6.6×10^{-34} (d) 6.8×10^{-34}

2. A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth, vertical wall of height h(< l) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/l and the frictional force f at the bottom of the stick are ($g = 10 \text{ m s}^{-2}$)

(a) $\frac{h}{l} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}\text{N}$ (b) $\frac{h}{l} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}\text{N}$

(c) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}\text{N}$ (d) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}\text{N}$

3. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is

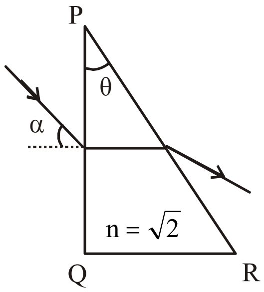


(Specific heat of water is 4.2 kJ $\text{kg}^{-1}\text{K}^{-1}$ and the density of water is 1000 kg m^{-3})

(a) 1600 (b) 2067

(c) 2533 (d) 3933

4. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is

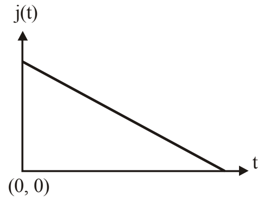


(a) 15° (b) 22.5°

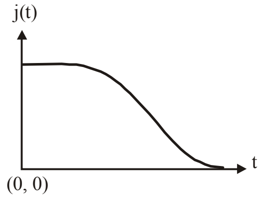
(c) 30° (d) 45°

5. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material?

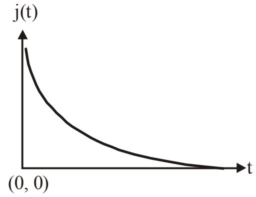
(a)



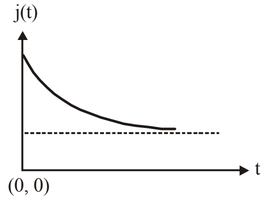
(b)



(c)



(d)



SECTION - II

This section contains **8 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

6. Highly excited states for hydrogen–like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where n>>1. Which of the following statement(s) is(are) true?

(a) Relative change in the radii of two consecutive orbitals does not depend on Z

(b) Relative change in the radii of two consecutive orbitals varies as 1/n

(c) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

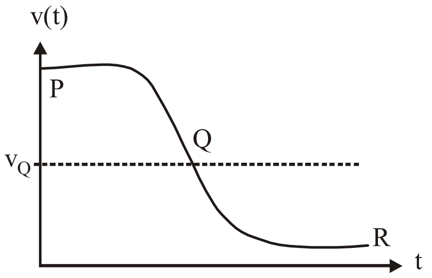
(d) Relative change in the angular momenta of two consecutive orbitals varies as 1/n

7. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let v(t) represent the beat frequency measured by a person sitting in the car at time t. Let v_P, v_Q and v_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person?

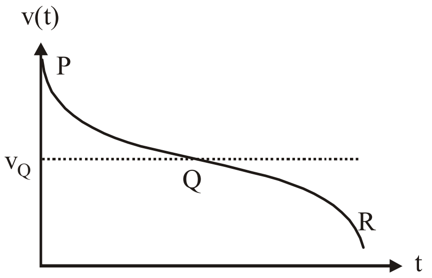
(a) $v_P + v_R = 2\,v_Q$

(b) The rate of change in beat frequency is maximum when the car passes through Q

(c) The plot below represents schematically the variation of beat frequency with time



(d) The plot below represents schematically the variation of beat frequency with time



8. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black–body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non–uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true?

(a) The temperature distribution over the filament is uniform

(b) The resistance over small sections of the filament decreases with time

(c) The filament emits more light at higher band of frequencies before it breaks up

(d) The filament consumes less electrical power towards the end of the life of the bulb

9. A plano–convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true?

(a) The refractive index of the lens is 2.5

(b) The radius of curvature of the convex surface is 45 cm

(c) The faint image is erect and real

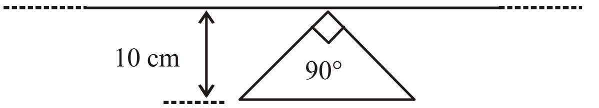
(d) The focal length of the lens is 20 cm

10. A length–scale (l) depends on the permittivity (ε) of a dielectric material. Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is(are) dimensionally correct?

(a) $l = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (b) $l = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$

(c) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (d) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

11. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 A s^{-1}. Which of the following statement(s) is(are) true?



(a) The magnitude of induced *emf* in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(b) If the loop is rotated at a constant angular speed about the wire, an additional *emf* of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(c) The induced current in the wire is in opposite direction to the current along the hypotenuse

(d) There is a repulsive force between the wire and the loop

12. The position vector $\vec{r}$ of a particle of mass m is given by the following equation

$$\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j},$$

where $\alpha = 10/3$ ms^{-3}, $\beta = 5$ ms^{-2} and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle?

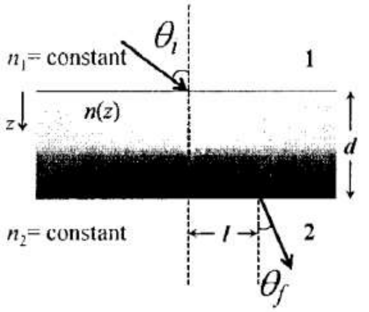
(a) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})$ ms^{-1}

(b) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -5/3)\ \hat{k}$ N m s

(c) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$ N

(d) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -(20/3)\ \hat{k}$ Nm

13. A transparent slab of thickness d has a refractive index n(z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 ($> n_1$), as shown in the figure. A ray of light is incident with angle θ_i, from medium 1 and emerges in medium 2 with refraction angle θ_t with a lateral displacement *l*.



Which of the following statement(s) is(are) true?

(a) $n_1 \sin\theta_i = n_2 \sin\theta_f$

(b) $n_1 \sin \theta_i = (n_2 - n_1) \sin\theta_f$

(c) *l* is independent of n_2

(d) *l* is dependent on n(z)

SECTION - III

This section contains **5 questions.** Each question, when worked out will result in one integer from 0 to 9 *(both inclusive).*

14. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2$, (P/P_0), where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C?

15. The isotope $^{12}_{5}B$ having a mass 12.014 u undergoes β–decay to $^{12}_{6}C$. $^{12}_{6}C$ has an excited state of the nucleus $\left(^{12}_{6}C^*\right)$ at 4.041 MeV above its ground state. If $^{12}_{5}E$ decays to $^{12}_{6}C^*$, the maximum kinetic energy of the β–particle in units of MeV is (1 u = 931.5 MeV/c^2, where c is the speed of light in vacuum).

16. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking hc/e = 1.237×10^{-6} eV m and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is

17. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is

18. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is

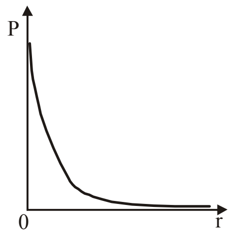
CHEMISTRY

SECTION - I

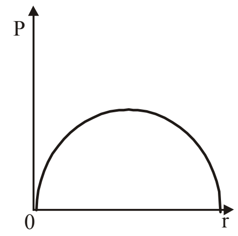
This section contains **5 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** option is correct.

19. P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness, dr, at a distance r from the nucleus. The volume of this shell is $4\pi r^2 dr$. The qualitative sketch of the dependence of P on r is

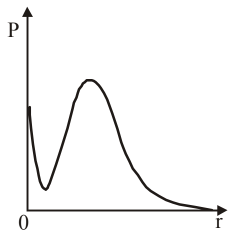
(a)



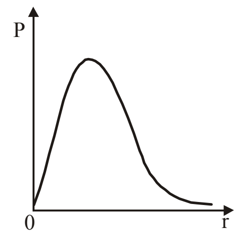
(b)



(c)



(d)



20. One mole of an ideal gas at 300 K in thermal contact with surroundings expands isothermally from 1.0 L to 2.0 L against a constant pressure of 3.0 atm. In this process, the change in entropy of surroundings (ΔS_{surr}) in JK^{-1} is
(1 L atm = 101.3 J)

(a) 5.763 (b) 1.013
(c) –1.013 (d) –5.763

21. The increasing order of atomic radii of the following Group 13 elements is

(a) $A\ell < Ga < In < T\ell$ (b) $Ga < A\ell < In < T\ell$
(c) $A\ell < In < Ga < T\ell$ (d) $A\ell < Ga < T\ell < In$

22. Among $[Ni(CO)_4]$, $[NiCl_4]^{2-}$, $[Co(NH_3)_4C\ell_2]C\ell$, $Na_3[CoF_6]$, Na_2O_2 and CsO_2, the total number of paramagnetic compounds is

(a) 2 (b) 3
(c) 4 (d) 5

23. On complete hydrogenation, natural rubber produces

(a) ethylene–propylene copolymer
(b) vulcanised rubber
(c) polypropylene
(d) polybutylene

SECTION - II

This section contains **8 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

24. According to the Arrhenius equation,

(a) a high activation energy usually implies a fast reaction.
(b) rate constant increases with increase in temperature. This is due to a greater number of collisions whose energy exceeds the activation energy.
(c) higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant.
(d) the pre–exponential factor is a measure of the rate at which collisions occur, irrespective of their energy.

25. A plot of the number of neutrons (N) against the number of protons (P) of stable nuclei exhibits upward deviation from linearity for atomic number, Z > 20. For an unstable nucleus having N/P ratio less than 1, the possible mode(s) of decay is(are)

(a) β^- –decay (β emission)
(b) orbital or K–electron capture
(c) neutron emission
(d) β^-–decay (positron emission)

26. The crystalline form of borax has

(a) tetranuclear $[B_4O_5(OH)_4]^{2-}$ unit
(b) all boron atoms in the same plane
(c) equal number of sp^2 and sp^3 hybridized boron atoms
(d) one terminal hydroxide per boron atom

27. The compound(s) with TWO lone pairs of electrons on the central atom is(are)

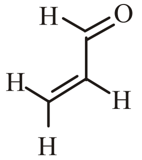
(a) BrF_5 (b) ClF_3
(c) XeF_4 (d) SF_4

28. The reagent(s) that can selectively precipitate S^{2-} from a mixture of S^{2-} and SO_4^{2-} in aqueous solution is(are)

(a) $CuCl_2$ (b) $BaC\ell_2$
(c) $Pb(OOCCH_3)_2$ (d) $Na_2[Fe(CN)_5NO]$

29. Positive Tollen's test is observed for

(a)



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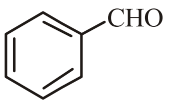

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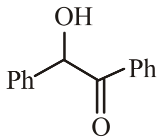
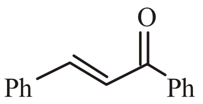
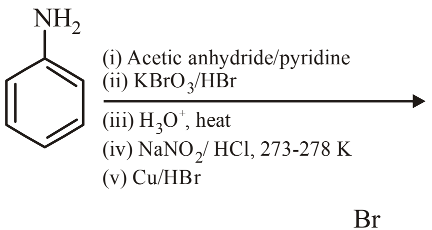


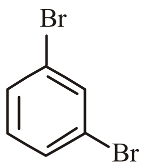
(d)



30. The product(s) of the following reaction sequence is(are)

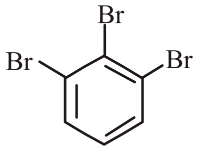


(a)

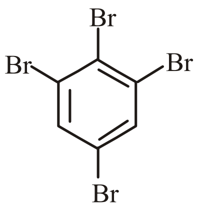


(b)

(c)



(d)



31. The correct statement(s) about the following reaction sequence is(are)

$$\text{Cumene}(C_9H_{12})\xrightarrow[\text{(ii)}H_3O^+]{\text{(i)}O_2}\mathbf{P}\xrightarrow{CHCl_3/NaOH}$$

$$\mathbf{Q}\,(\text{major})+\mathbf{R}\,(\text{minor})$$

$$\mathbf{Q}\xrightarrow[PhCH_2Br]{NaOH}\mathbf{S}$$

(a) **R** is steam Volatile

(b) **Q** gives dark violet coloration with 1% aqueous $FeCl_3$ solution

(c) **S** gives yellow precipitate with 2, 4–dinitrophenylhydrazine

(d) **S** gives dark violet coloration with 1% aqueous $FeCl_3$ solution

SECTION - III

This section contains **5 questions.** Each question, when worked out will result in one integer from 0 to 9 *(both inclusive).*

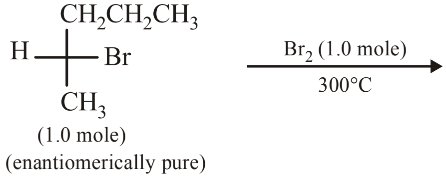
32. The mole fraction of a solute in a solution is 0.1. At 298 K, molarity of this solution is the same as its molality. Density of this solution at 298 K is 2.0 g cm^{-3} . The ratio of the molecular weights of the solute and solvent, $\left(\frac{MW_{solute}}{MW_{solvent}}\right)$, is

33. The diffusion coefficient of an ideal gas is proportional to its mean free path and mean speed. The absolute temperature of an ideal gas is increased 4 times and its pressure is increased 2 times. As a result, the diffusion coefficient of this gas increases x times. The value of x is

34. In neutral or faintly alkaline solution, 8 moles of permanganate anion quantitatively oxidize thiosulphate anions to produce X moles of a sulphur containing product. The magnitude of X is

35. The number of geometric isomers possible for the complex $[CoL_2Cl_2]^-$ ($L = H_2NCH_2CH_2O^-$) is

36. In the following monobromination reaction, the number of possible chiral products is



MATHEMATICS

SECTION - I

This section contains **5 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** option is correct.

37. Let $-\frac{\pi}{6}<\theta<-\frac{\pi}{12}$. Suppose α_1 and $B_1$are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

(a) $2(\sec\theta - \tan\theta)$ (b) $2\sec\theta$

(c) $-2\tan\theta$ (d) 0

38. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

(a) 380 (b) 320

(c) 260 (d) 95

39. Let S = $\left\{x\in(-\pi,\pi): x\neq 0,\pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

(a) $-\frac{7\pi}{9}$ (b) $-\frac{2\pi}{9}$

(c) 0 (d) $\frac{5\pi}{9}$

40. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that
P (computer turns out to be defective given that it is produced in plant T_1)
= 10P
(computer turns out to be defective given that it is produced in plant T_2),
where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

(a) $\frac{36}{73}$ (b) $\frac{47}{79}$

(c) $\frac{78}{93}$ (d) $\frac{75}{83}$

41. The least value of $\alpha\in\mathbb{R}$ for which $4\alpha x^2+\frac{1}{x}\geq 1$, for all $x > 0$, is

(a) $\frac{1}{64}$ (b) $\frac{1}{32}$

(c) $\frac{1}{27}$ (d) $\frac{1}{25}$

SECTION - II

This section contains **8 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

42. Consider a pyramid OPQRS located in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) with O as origin, and OP and OR along the x–axis and the y–axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point T of diagonal OQ such that TS = 3. Then

(a) the acute angle between OQ and OS is $\frac{\pi}{3}$

(b) the equation of the plane containing the triangle OQS is $x - y = 0$

(c) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(d) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

43. Let $f : (0, \infty) \to \mathbb{R}$ be a differentiable function such that $f'(x) = 2 - \frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) \neq 1$. Then

(a) $\lim\limits_{x \to 0+} f'\left(\frac{1}{x}\right) = 1$

(b) $\lim\limits_{x \to 0+} xf\left(\frac{1}{x}\right) = 2$

(c) $\lim\limits_{x \to 0+} x^2 f'(x) = 0$

(d) $|f(x)| \leq 2$ for all $x \in (0, 2)$

44. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in \mathbb{R}$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then

(a) $a = 0, k = 8$

(b) $4a - k + 8 = 0$

(c) $\det(P \operatorname{adj}(Q)) = 2^9$

(d) $\det(Q \operatorname{adj}(P)) = 2^{13}$

45. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X,Y, Z, respectively, and $2s = x + y + z$. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then

(a) area of the triangle XYZ is $6\sqrt{6}$

(b) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(c) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2} = \frac{4}{35}$

(d) $\sin^2\left(\frac{X+Y}{2}\right) = \frac{3}{5}$

46. A solution curve of the differential equation $(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0$, $x > 0$, passes through the point (1, 3). Then the solution curve

(a) intersects $y = x + 2$ exactly at one point

(b) intersects $y = x + 2$ exactly at two points

(c) intersects $y = (x + 2)^2$

(d) does NOT intersect $y = (x + 3)^2$

47. Let $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \mathbb{R}$. Then

(a) $g'(2) = \frac{1}{15}$

(b) $h'(1) = 666$

(c) $h(0) = 16$

(d) $h(g(3)) = 36$

48. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y–axis, then

(a) $Q_2Q_3 = 12$

(b) $R_2R_3 = 4\sqrt{6}$

(c) area of the triangle OR_2R_3 is $6\sqrt{2}$

(d) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

49. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s)

(a) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$

(b) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$

(d) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

SECTION - III

This section contains **5 questions.** Each question, when worked out will result in one integer from 0 to 9 *(both inclusive).*

50. The total number of distinct $x \in \mathbb{R}$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

51. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1 + x)^2 + (1 + x)^3 + ... + (1 + x)^{49} + (1 + mx)^{50}$ is $(3n + 1)\ {}^{51}C_3$ for some positive integer n. Then the value of n is

52. The total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$ is

53. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x \to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals.

54. Let $z = \frac{-1+\sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and $r, s \in \{1, 2, 3\}$. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is

PAPER - 2

1. The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of three sections.
2. **Section 1** contains 6 questions. Each question has four options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
3. **Section 2** contains 8 multiple choice questions. Each question has four choice (a), (b), (c) and (d) out of which **ONE OR MORE THAN ONE** are correct.
4. **Section 3** contains 2 paragraphs each describing theory, experiment and data etc. four questions relate to two paragraphs with two questions on each paragraph. Each question pertaining to a particular passage should have **ONLY ONE** correct answer among the four given choices (a), (b), (c) and (d).

PHYSICS

SECTION - I

This section contains **6 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** option is correct.

1. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E=\frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

The measured masses of the neutron ^1_1H, $^{15}_7N$ and $^{15}_8O$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the $^{15}_7N$ and $^{15}_8O$ nuclei are same, 1 u = 931.5 Me V/c^2 (c is the speed of light) and $e^2/(4\pi\varepsilon_0)$ = 1.44 MeV fm. Assuming that the difference between the binding energies of $^{15}_7N$ and $^{15}_8O$ is purely due to the electrostatic energy, the radius of either of the nuclei is

(1 fm = 10^{-15} m)

(a) 2.85 fm
(b) 3.03 fm
(c) 3.42 fm
(d) 3.80 fm

2. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?

(a) 64
(b) 90
(c) 108
(d) 120

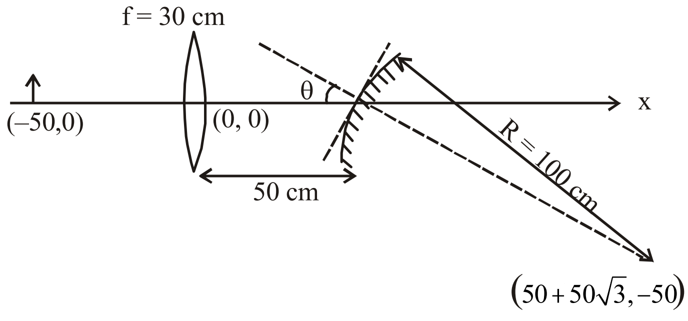
3. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = (1/32) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately

(a) 112 J
(b) 294 J
(c) 588 J
(d) 813 J

4. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400 °C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is 1.2×10^{-5} K^{-1}, the change in length of the wire PQ is

(a) 0.78 mm
(b) 0.90 mm
(c) 1.56 mm
(d) 2.34 mm

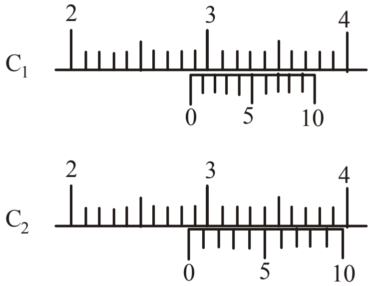
5. A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle θ = 30° to the axis of the lens, as shown in the figure.



If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are

(a) (0, 0)
(b) $(50-25\sqrt{3},25)$
(c) $(25,25\sqrt{3})$
(d) $(125/3,25\sqrt{3})$

6. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2, respectively, are

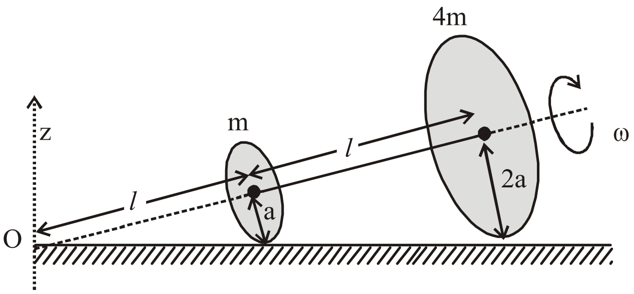


(a) 2.85 and 2.82 (b) 2.87 and 2.83
(c) 2.87 and 2.86 (d) 2.87 and 2.87

SECTION - II

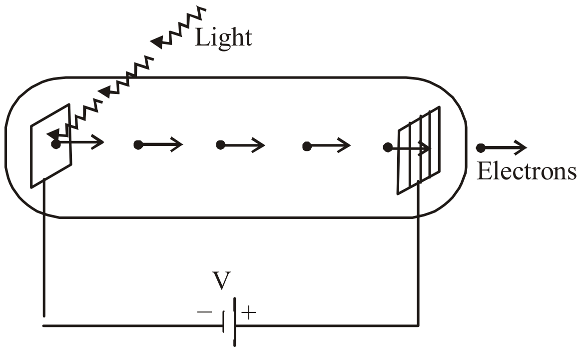
This section contains **8 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

7. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}a$ through their centres. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is (are) true?



(a) The centre of mass of the assembly rotates about the z-axis with an angular speed of ω/5
(b) The magnitude of angular momentum of center of mass of the assembly about the point O is 81 $ma^2\omega$
(c) The magnitude of angular momentum of the assembly about its center of mass is 17 $ma^2\omega/2$.
(d) The magnitude of the z-component of $\vec{L}$ is 55 $ma^2\omega$.

8. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statement(s) is (are) true?



(a) λ_e decreases with increase in ϕ and λ_{ph}
(b) λ_e is approximately halved, if d is doubled
(c) For large potential difference ($V >> \phi/e$), λ_e is approximately halved if V is made four times
(d) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < hc/\phi$

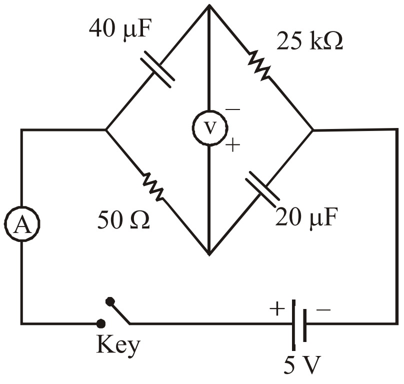
9. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52s, 0.56s, 0.57s, 0.54s and 0.59s. The least count of the watch used for the measurement of time period is 0.01s. Which of the following statement(s) is (are) true?

(a) The error in the measurement of r is 10%
(b) The error in the measurement of T is 3.75%
(c) The error in the measurement of T is 2%
(d) The error in the determined value of g is 11%

10. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_C < R/2$, which of the following statement(s) about any one of the galvanometers is (are) true?

(a) The maximum voltage range is obtained when all the components are connected in series
(b) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer
(c) The maximum current range is obtained when all the components are connected in parallel
(d) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors

11. In the circuit shown below, the key is pressed at time t = 0. Which of the following statement(s) is(are) true?



(a) The voltmeter displays –5V as soon as the key is pressed, and displays +5V after a long time

(b) The voltmeter will display 0V at time t = ln 2 seconds

(c) The current in the ammeter becomes 1/e of the initial value after 1 second

(d) The current in the ammeter becomes zero after a long time.

12. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases: (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m(<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is (are) true about the motion after the mass m is placed on the mass M?

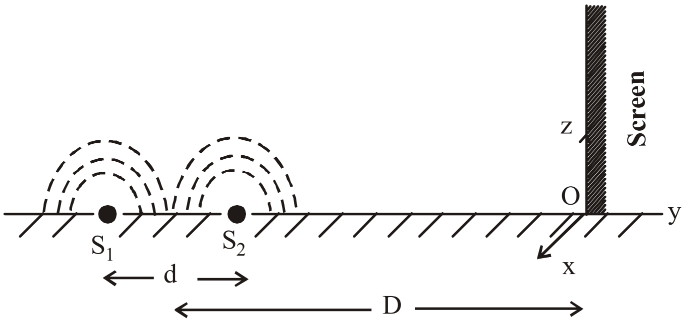
(a) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged.

(b) The final time period of oscillation in both the cases is same.

(c) The total energy decreases in both the cases.

(d) The instantaneous speed at x_0 of the combined masses decreases in both the cases

13. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for z > 0) at a distance D = 3 m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the sources d = 0.6003 mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is(are) true of the intensity pattern on the screen?



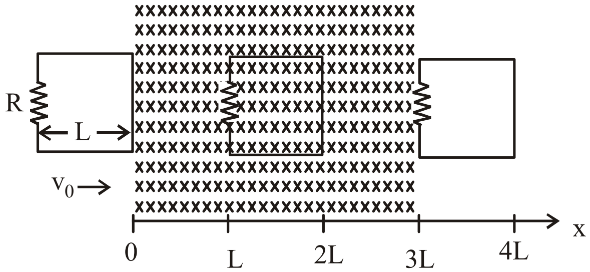
(a) Straight bright and dark bands parallel to the x-axis

(b) The region very close to the point O will be dark

(c) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

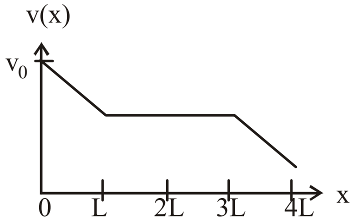
(d) Semi circular bright and dark bands centered at point.

14. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At t = 0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0. the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive.

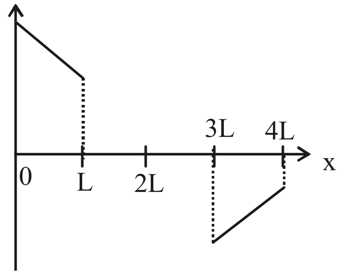


Which of the following schematic plot(s) is (are) correct? (Ignore gravity)

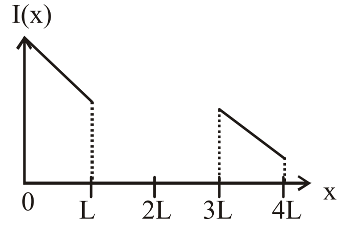
(a)



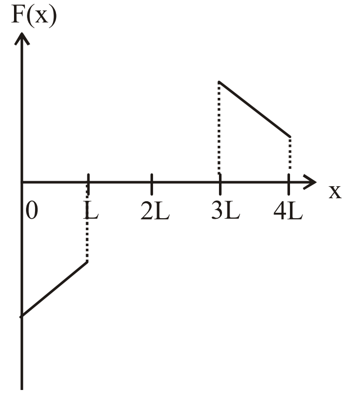
(b)



(c)



(d)



SECTION - II

This section contains 2 paragraphs, each describing theory, experiments, data etc. four questions related to the two paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (a), (b), (c) and (d).

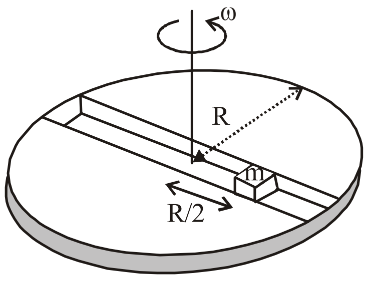
PARAGRAPH 1

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m\left(\vec{v}_{rot} \times \vec{\omega}\right) + m\left(\vec{\omega} \times \vec{r}\right) \times \vec{\omega}.$$

where $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $\left(\vec{\omega} = \omega\hat{k}\right)$. A small block of mass m is gently placed in the slot at $\vec{r}\left(R/2\right)\hat{i}$ at $t = 0$ and is constrained to move only along the slot.



15. The distance r of the block at time t is

(a) $\frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$ (b) $\frac{R}{2}\cos\omega t$

(c) $\frac{R}{4}\left(e^{2\omega t} + e^{-2\omega t}\right)$ (d) $\frac{R}{2}\cos 2\omega t$

16. The net reaction of the disc on the block is

(a) $\frac{1}{2}m\omega^2R\left(e^{2\omega t} - e^{-2\omega t}\right)\hat{j} + mg\hat{k}$

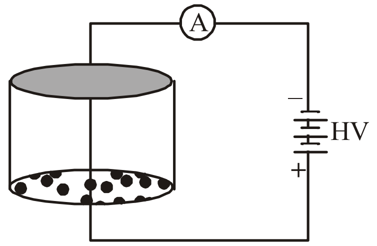
(b) $\frac{1}{2}m\omega^2R\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$

(c) $-m\omega^2R\cos\omega t\,\hat{j} - mg\hat{k}$

(d) $m\omega^2R\,\text{sins}\,\omega t\,\hat{j} - mg\hat{k}$

PARAGRAPH 2

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius r <<h. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)



17. Which one of the following statements is correct?

(a) The balls will stick to the top plate and remain there

(b) The balls will bounce back to the bottom plate carrying the same charge they went up with

(c) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(d) The balls will execute simple harmonic motion between the two plates

18. The average current in the steady state registered by the ammeter in the circuit will be

(a) zero
(b) proportional to the potential V_0
(c) proportional to $V_0^{1/2}$
(d) proportional to V_0^2

CHEMISTRY

SECTION - I

This section contains **6 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** option is correct.

19. For the following electrochemical cell at 298 K,
$Pt(s)|H_2(g, 1\ bar)|\ H^+(aq, 1\ M)\ ||\ M^{4+}(aq), M^{2+}(aq)|Pt(s)$

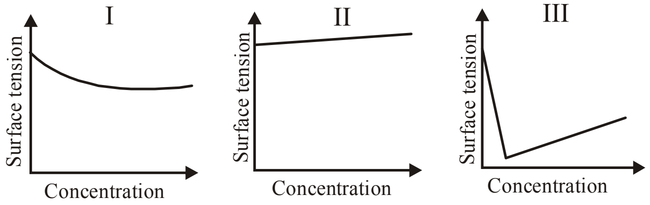
$$E_{cell} = 0.092\ V \text{ when } \frac{\left[M^{2+}(aq)\right]}{\left[M^{4+}(aq)\right]} = 10^x.$$

$$\text{Given}: E^\circ_{M^{4-}/M^{2+}} = 0.151\ V;\ 2.303\frac{RT}{F} = 0.059V$$

The value of x is

(a) –2 (b) –1
(c) 1 (d) 2

20. The qualitative sketches I, II and III given below show the variation of surface tension with molar concentration of three different aqueous solutions of KCl, CH_3OH and $CH_3(CH_2)_{11}OSO_3^-\ Na^+$ at room temperature. The correct assignment of the sketches is



(a) I : KCl II : CH_3OH III : $CH_3(CH_2)_{11}OSO_3^-Na^+$
(b) I : $CH_3(CH_2)_{11}OSO_3^-Na^+$ II : CH_3OH III : KCl
(c) I : KCl II : $CH_3(CH_2)_{11}OSO_3^-Na^+$ III : CH_3OH
(d) I : CH_3OH II : KCl III : $CH_3(CH_2)_{11}OSO_3^-Na^+$

21. In the following reaction sequence in aqueous solution, the species X, Y and Z, respectively, are

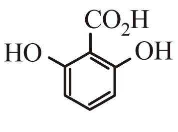
$$S_2O_3^{2-} \xrightarrow{Ag^+} \underset{\text{clear solution}}{X} \xrightarrow{Ag^+} \underset{\text{white precipitate}}{Y} \xrightarrow{\text{with time}} \underset{\text{black precipitate}}{Z}$$

(a) $[Ag(S_2O_3)_2]^{3-}$, $Ag_2S_2O_3$, Ag_2S
(b) $[Ag(S_2O_3)_3]^{5-}$, Ag_2SO_3, Ag_2S
(c) $[Ag(SO_3)_2]^{3-}$, $Ag_2S_2O_3$, Ag
(d) $[Ag(SO_3)_3]^{3-}$, Ag_2SO_4, Ag

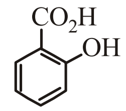
22. The geometries of the ammonia complexes of Ni^{2+}, Pt^{2+} and Zn^{2+} respectively, are

(a) octahedral, square planar and tetrahedral
(b) square planar, octahedral and tetrahedral
(c) tetrahedral, square planar and octahedral
(d) octahedral, tetrahedral and square planar

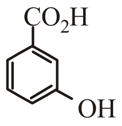
23. The correct order of acidity for the following compounds is



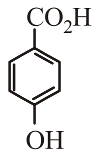
I



II



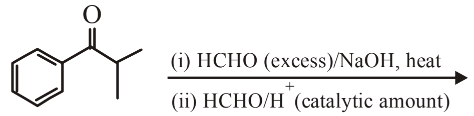
III

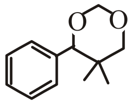


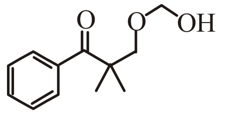
IV

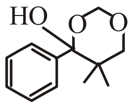
(a) I > II > III > IV (b) III> I > II > IV
(c) III > IV > II > I (d) I > III > IV > II

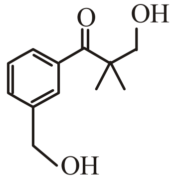
24. The major product of the following reaction sequence is



(a)


(b)


(c)


(d)


SECTION - II

This section contains **8 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

25. According to Molecular Orbital Theory,

(a) C_2^{2-} is expected to be diamagnetic
(b) O_2^{2+} is expected to have a longer bond length than O_2
(c) N_2^+ and N_2^- have the same bond order
(d) He_2^+ has the same energy as two isolated He atoms

26. Mixture(s) showing positive deviation from Raoult's law at 35 °C is (are)

(a) carbon tetrachloride + methanol
(b) carbon disulphide + acetone
(c) benzene + toluene
(d) phenol + aniline

27. The CORRECT statement(s) for cubic close packed (ccp) three dimensional structure is (are)

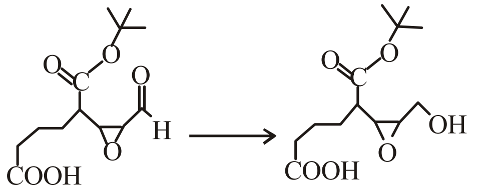
(a) The number of the nearest neighbours of an atom present in the topmost layer is 12
(b) The efficiency of atom packing is 74%
(c) The number of octahedral and tetrahedral voids per atom are 1 and 2, respectively
(d) The unit cell edge length is $2\sqrt{2}$ times the radius of the atom

28. Extraction of copper from copper pyrite ($CuFeS_2$) involves
(a) crushing followed by concentration of the ore by froth-flotation
(b) removal of iron as slag
(c) self-reduction step to produce 'blister copper' following evolution of SO_2
(d) refining of 'blister copper' by carbon reduction

29. The nitrogen containing compound produced in the reaction of HNO_3 with P_4O_{10}
(a) can also be prepared by reaction of P_4 and HNO_3
(b) is diamagnetic
(c) contains one N-N bond
(d) reacts with Na metal producing a brown gas

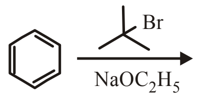
30. For 'invert sugar', the correct statement(s) is(are)
(Given : specific rotations of (+) -sucrose, (+)-maltose, L-(-)-glucose and L-(+) fructose in aqueous solution are + 66°, +140°, –52° and +92°, respectively)
(a) 'invert sugar' is prepared by acid catalyzed hydrolysis of maltose
(b) 'invert sugar' is an equimolar mixture of D-(+)-glucose and D-(-)-fructose
(c) specific rotation of 'invert sugar' is –20°
(d) on reaction with Br_2 water, 'invert sugar' forms saccharic acid as one of the products

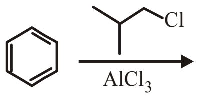
31. Reagent(s) which can be used to bring about the following transformation is (are)

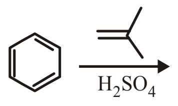


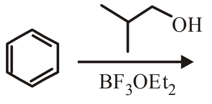
(a) $LiAlH_4$ in $(C_2H_5)_2O$
(b) BH_3 in THF
(c) $NaBH_4$ in C_2H_5OH
(d) Raney Ni/H_2 in THF

32. Among the following, reaction(s) which gives(give) tert-butyl benzene as the major product is(are)

(a) 

(b) 

(c) 

(d) 

SECTION - III

This section contains 2 paragraphs, each describing theory, experiments, data etc. four questions related to the two paragraphs with two questions on each paragraph. Each question has one or more than one correct answer(s) among the four given options (a), (b), (c) and (d).

PARAGRAPH 1

Thermal decomposition of gaseous X_2 to gaseous X at 298 K takes place according to the following equation:

$$X_2(g) \rightleftharpoons 2X(g)$$

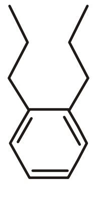
The standard reaction Gibbs energy, $\Delta_rG°$, of this reaction is positive. At the start of the reaction, there is one mole of X_2 and no X. As the reaction proceeds, the number of moles of X formed is given by β. Thus, $\beta_{\text{equilibrium}}$ is the number of moles of X formed at equilibrium. The reaction is carried out at a constant total pressure of 2 bar. Consider the gases to behave ideally.
(Given R = 0.083 L bar K^{-1} mol^{-1})

33. The equilibrium constant K_p for this reaction at 298 K, in terms of $\beta_{\text{equilibrium}}$, is
(a) $\frac{8\beta^2_{\text{equilibrium}}}{2-\beta_{\text{equilibrium}}}$
(b) $\frac{8\beta^2_{\text{equilibrium}}}{4-\beta^2_{\text{equilibrium}}}$
(c) $\frac{4\beta^2_{\text{equilibrium}}}{2-\beta_{\text{equilibrium}}}$
(d) $\frac{4\beta^2_{\text{equilibrium}}}{4-\beta^2_{\text{equilibrium}}}$

34. The INCORRECT statement among the following, for this reaction, is
(a) Decrease in the total pressure will result in formation of more moles of gaseous X
(b) At the start of the reaction, dissociation of gaseous X_2 takes place spontaneously
(c) $\beta_{\text{equilibrium}} = 0.7$
(d) $K_C < 1$

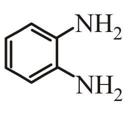
PARAGRAPH 2

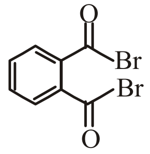
Treatment of compound O with $KMnO_4/H^+$ gave P, which on heating with ammonia gave Q. The compound Q on treatment with Br_2/NaOH produced R. On strong heating, Q gave S, which on further treatment with ethyl 2-bromopropanoate in the presence of KOH followed by acidification, gave a compound T.

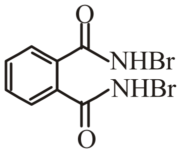


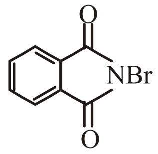
(O)

35. The compound R is

(a) 

(b) 

(c) 

(d) 

36. The compound T is
(a) glycine
(b) alanine
(c) valine
(d) serine

MATHEMATICS

SECTION - I

This section contains **6 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** option is correct.

37. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and *I* be the identity matrix of order 3.

If Q = $[q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\dfrac{q_{31} + q_{32}}{q_{21}}$ equals

(a) 52 (b) 103
(c) 201 (d) 205

38. Let $b_i > 1$ for i = 1, 2, ..., 101. Suppose $\log_e b_1, \log_e b_2, ..., \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, ..., a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + ... + b_{51}$ and $s = a_1 + a_2 + ... + a_{51}$, then

(a) $s > t$ and $a_{101} > b_{101}$ (b) $s > t$ and $a_{101} < b_{101}$
(c) $s < t$ and $a_{101} > b_{101}$ (d) $s < t$ and $a_{101} < b_{101}$

39. The value of $\displaystyle\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to

(a) $3 - \sqrt{3}$ (b) $2\left(3 - \sqrt{3}\right)$
(c) $2\left(\sqrt{3} - 1\right)$ (d) $2\left(2 - \sqrt{3}\right)$

40. The value of $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1 + e^x} dx$ is equal to

(a) $\dfrac{\pi^2}{4} - 2$ (b) $\dfrac{\pi^2}{4} + 2$
(c) $\pi^2 - e^{\frac{\pi}{2}}$ (d) $\pi^2 + e^{\frac{\pi}{2}}$

41. Area of the region $\left\{(x, y) \in \mathbb{R}^2 : y \ge \sqrt{|x+3|}, 5y \le x + 9 \le 15\right\}$ is equal to

(a) $\dfrac{1}{6}$ (b) $\dfrac{4}{3}$
(c) $\dfrac{3}{2}$ (d) $\dfrac{5}{3}$

42. Let *P* be the image of the point (3,1,7) with respect to the plane x–y+z = 3. Then the equation of the plane passing through *P* and containing the straight line $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{1}$ is.

(a) $x + y - 3z = 0$ (b) $3x + z = 0$
(c) $x - 4y + 7z = 0$ (d) $2x - y = 0$

SECTION - II

This section contains **8 multiple choice questions.** Each question has 4 choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE are correct.**

43. Let $f(x) = \lim\limits_{n \to \infty} \left(\dfrac{n^n (x+n)\left(x + \frac{n}{2}\right)...\left(x + \frac{n}{n}\right)}{n!\left(x^2 + n^2\right)\left(x^2 + \frac{n^2}{4}\right)...\left(x^2 + \frac{n^2}{n^2}\right)} \right)^{\frac{x}{n}}$, for all x > 0. Then

(a) $f\left(\frac{1}{2}\right) \ge f(1)$ (b) $f\left(\frac{1}{3}\right) \le f\left(\frac{2}{3}\right)$
(c) $f'(2) \le 0$ (d) $\dfrac{f'(3)}{f(3)} \ge \dfrac{f'(2)}{f(2)}$

44. Let a, b ∈ $\mathbb{R}$ and f : $\mathbb{R} \to \mathbb{R}$ be defined by $f(x) = a\cos(|x^3 - x|) + b|x|\sin(|x^3 + x|)$.
Then f is.

(a) differentiable at x = 0 if a = 0 and b = 1
(b) differentiable at x = 1 if a = 1 and b = 0
(c) NOT differentiable at x = 0 if a = 1, b = 0
(d) NOT differentiable at x = 1 if a = 1 and b = 1

45. Let f: $\mathbb{R} \to (0, \infty)$ and g : $\mathbb{R} \to \mathbb{R}$ be twice differentiable functions such that f" and g" are continuous functions on $\mathbb{R}$. Suppose f '(2) = g(2) = 0, f"(2) ≠ 0 and g'(2) ≠ 0. If $\lim\limits_{x \to 2} \dfrac{f(x)g(x)}{f'(x)g'(x)} = 1$, then.

(a) *f* has a local minimum at x = 2
(b) *f* has a local maximum at x = 2
(c) $f''(2) > f(2)$
(d) $f(x) - f''(x) = 0$ for at least one x ∈ R

46. Let $f : \left[-\frac{1}{2}, 2\right] \to \mathbb{R}$ and $g : \left[-\frac{1}{2}, 2\right] \to \mathbb{R}$ be functions defined by $f(x) = [x^2 - 3]$ and $g(x) = |x| f(x) + |4x - 7| f(x)$, where [y] denotes the greatest integer less than or equal to y for y ∈ $\mathbb{R}$. Then

(a) *f* is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$
(b) *f* is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$
(c) *g* is NOT differentiable exactly at four points in $\left(-\frac{1}{2}, 2\right)$
(d) *g* is NOT differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$

47. Let a, b $\in \mathbb{R}$ and $a^2 + b^2 \neq 0$.

Suppose $S = \left\{ Z \in C : Z = \frac{1}{a+ibt}, + \in \mathbb{R}, t \neq 0 \right\}$, where $i = \sqrt{-1}$. If z = x+ iy and z $\in$ S, then (x, y) lies on

(a) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0$, $b \neq 0$

(b) the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for $a < 0$, $b \neq 0$

(c) the x-axis for $a \neq 0$, $b = 0$

(d) the y-axis for $a = 0$, $b \neq 0$

48. Let *P* be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$.
Let Q be the point on the circle dividing the line segment SP internally. Then

(a) SP = $2\sqrt{5}$

(b) SQ : QP = $\left(\sqrt{5}+1\right) : 2$

(c) the x-intercept of the normal to the parabola at P is 6

(d) the slope of the tangent to the circle at Q is $\frac{1}{2}$

49. Let $a, \lambda, \mu, \in \mathbb{R}$. Consider the system of linear equations

$ax + 2y = \lambda$

$3x - 2y = \mu$

Which of the following statement(s) is (are) correct?

(a) If a = –3, then the system has infinitely many solutions for all values of λ and μ

(b) If $a \neq -3$, then the system has a unique solution for all values of λ and μ

(c) If $\lambda + \mu = 0$, then the system has infinitely many solutions for a = –3

(d) If $\lambda + \mu \neq 0$, then the system has no solution for a = –3

50. Let $\hat{u} = u_1 i + u_2 \hat{j} + u_3 \hat{k}$ be a unit vector in $\mathbb{R}^3$ and $\hat{w} = \frac{1}{\sqrt{6}}\left(\hat{i} + \hat{j} + 2\hat{k}\right)$. Given that there exists a vector $\vec{v}$ in $\mathbb{R}^3$ such that $\left|\hat{u} \times \vec{v}\right| = 1$ and $\hat{w} \cdot \left(\hat{u} \times \vec{v}\right) = 1$. Which of the following statement(s) is (are) correct?

(a) There is exactly one choice for such $\vec{v}$

(b) There are infinitely many choices for such $\vec{v}$

(c) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$

(d) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$

SECTION - III

This section contains 2 paragraphs, each describing theory, experiments, data etc. four questions related to the two paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (a), (b), (c) and (d).

PARAGRAPH 1

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2 respectively after two games.

51. *P (X > Y)* is

(a) $\frac{1}{4}$ (b) $\frac{5}{12}$

(c) $\frac{1}{2}$ (d) $\frac{7}{12}$

52. *P (X = Y)* is

(a) $\frac{11}{36}$ (b) $\frac{1}{3}$

(c) $\frac{13}{36}$ (d) $\frac{1}{2}$

PARAGRAPH 2

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$ for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

53. The orthocentre of the triangle F_1MN is

(a) $\left(-\frac{9}{10}, 0\right)$ (b) $\left(\frac{2}{3}, 0\right)$

(c) $\left(\frac{9}{10}, 0\right)$ (d) $\left(\frac{2}{3}, \sqrt{6}\right)$

54. If the tangents to the ellipse at *M* and *N* meet at *R* and the normal to the parabola at *M* meets the x-axis at Q, then the ratio of area of the triangle *MQR* to area of the quadrilateral MF_1NF_2 is

(a) 3 : 4 (b) 4 : 5

(c) 5 : 8 (d) 2 : 3

SOLUTIONS

Paper - 1

PHYSICS

1. **(b)** $\frac{hc}{e\lambda_1}-\frac{\phi}{e}=V_{0_1}$ and $\frac{hc}{e\lambda_2}-\frac{\phi}{e}=V_{0_2}$

$$\therefore \frac{hc}{e}\left[\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right]=V_{0_1}-V_{0_2}$$

$$\therefore h=\frac{e(V_{0_1}-V_{0_2})\lambda_1\lambda_2}{(\lambda_2-\lambda_1)\mathrm{c}}$$

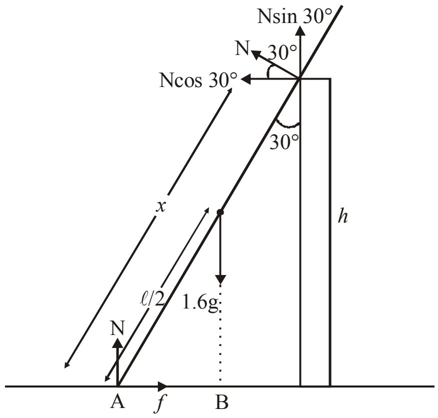
From the first two values given in data

$$h=\frac{1.6\times10^{-19}[2-1]\times0.4\times0.3\times10^{-6}}{0.1\times3\times10^{8}}$$

$h=0.64\times10^{-33}=6.4\times10^{-34}$ J-s

Similarly if we calculate h for the last two values of data $h=6.4\times10^{-34}$J-s

2. **(d)** Considering the normal reaction of the floor and wall to be N and with reference to the figure.



By vertical equilibrium.

$$\mathrm{N}+\mathrm{N}\sin30°=1.6\text{ g}\Rightarrow\mathrm{N}=\frac{3.2g}{3}\quad\ldots\text{(i)}$$

By horizontal equilibrium

$$\mathrm{f}=\mathrm{N}\cos30°=\frac{\sqrt{3}}{2}N=\frac{16\sqrt{3}}{3}\text{ From (i)}$$

Taking torque about A we get

$1.6\mathrm{g}\times\mathrm{AB}=\mathrm{N}\times\mathrm{x}$

$$1.6\text{ g}\times\frac{\ell}{2}\cos60°=\frac{3.2g}{3}\times x\;\;\therefore\frac{3\ell}{8}=x\quad\ldots\text{(ii)}$$

But $\cos30°=\frac{h}{x}$ $\quad\therefore\; x=\frac{h}{\cos30°}$ … (iii)

From (ii) and (iii) $\frac{h}{\cos30°}=\frac{3\ell}{8}\;\;\therefore\;\;\frac{h}{\ell}=\frac{3\sqrt{3}}{16}$

3. **(b)** $\mathrm{P_{heater}}-\mathrm{P_{cooler}}=\frac{mC\Delta T}{t}=\frac{V\rho C\Delta T}{t}$

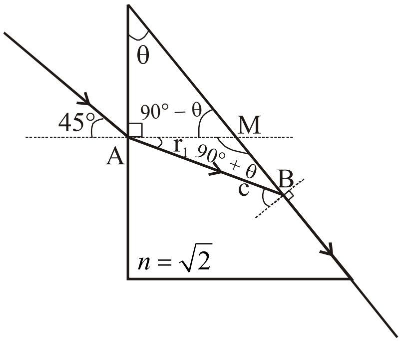
$$\therefore(3000-\mathrm{P_{cooler}})=\frac{0.12\times1000\times4.2\times10^{3}\times20}{3\times60\times60}$$

$\therefore\mathrm{P_{cooler}}=2067\mathrm{W}$

4. **(a)** Applying Snell's law at A

$1\times\sin45°=\sqrt{2}\times\sin r_1$

$\therefore\mathrm{r_1}=30°$ …(i)



Applying Snell's law at B

$\sqrt{2}\;\sin\mathrm{C}=1\times\sin90°$

$\therefore\;\;\mathrm{C}=45°$ …(ii)

In ΔAMB, $90°+\theta+\mathrm{r_1}+(90°-\mathrm{C})=180°$ (From fig.)

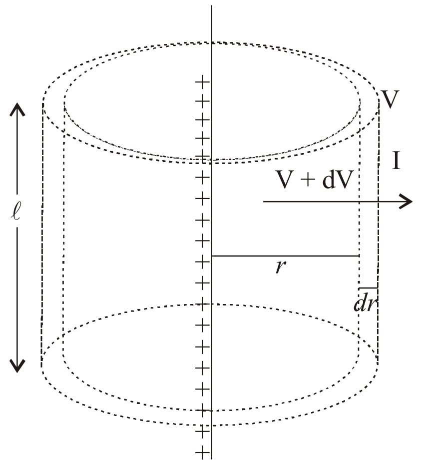
$\therefore\;\theta=15°$

5. **(c)** $J=\frac{I}{2\pi r\ell}=\frac{dV/dR}{2\pi r\ell}$ …(i)

$$dR=\rho\frac{dr}{2\pi r\ell}=\frac{1}{\sigma}\times\frac{dr}{2\pi r\ell}\quad\ldots\text{(ii)}$$

Now $E=-\frac{dV}{dr}$

$$\therefore\;\mathrm{dV}=-\mathrm{Edr}=-\frac{\lambda}{2\pi\in r}dr\quad\ldots\text{(iii)}$$



From (i), (ii), and (iii)

$$J = \frac{1}{2\pi r\ell}\left[\frac{\lambda dr}{2\lambda \in r} \times \frac{\sigma 2\pi r\ell}{dr}\right] = \frac{\lambda\sigma}{2\pi \in r} \quad \ldots \text{(iv)}$$

$$\text{Also } I = \frac{dV}{dR} = \frac{-\lambda}{2\pi \in r} dr \times \frac{\sigma \times 2\pi r\ell}{dr} = \frac{-\lambda\sigma\ell}{\in} \quad \ldots\text{(v)}$$

Here negative sign signifies that the current is decreasing

$$\text{But } I = \frac{d(q)}{dt} = \frac{d(\lambda\ell)}{dt} = \ell\frac{d\lambda}{dt} \quad \ldots \text{(vi)}$$

From (v) and (vi)

$$\ell\frac{d\lambda}{dt} = -\frac{\lambda\sigma\ell}{\in} \Rightarrow \frac{d\lambda}{\lambda} = \frac{-\sigma}{\in\ell}dt$$

On integrating

$$\int_{\lambda_0}^{\lambda} \frac{d\lambda}{\lambda} = -\frac{\sigma}{\in}\int_0^t dt$$

$$\therefore \log_e \frac{\lambda}{\lambda_0} = -\frac{\sigma t}{\in} \quad \therefore \lambda = \lambda_0 e^{-\frac{\sigma}{\in}t}$$

Substituting this value in (iv) we get

$$J = \frac{\sigma\lambda_0}{2\pi \in r} e^{-\frac{\sigma}{\in}t}$$

6. (a, b, d)

We know that $r = r_0\frac{n^2}{z}$, $E_n = -\frac{13.6Z^2}{n^2}$, $L_n = \frac{nh}{2\pi}$

Relative change in the radii of two consecutive orbitals

$$\frac{r_n - r_{n-1}}{r_n} = 1 - \frac{r_{n-1}}{r_n} = 1 - \frac{(n-1)^2}{n^2} \quad \text{does not depend on Z}$$

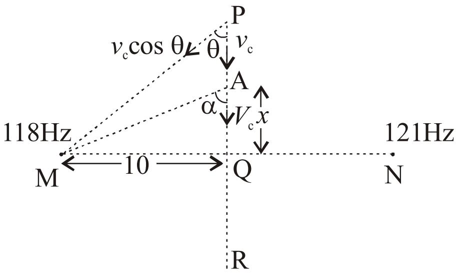
$$= \frac{2n-1}{n^2} \approx \frac{2}{n} \quad (\because \text{ n} >> 1)$$

Relative change in the energy of two consecutive orbitals

$$\frac{E_n - E_{n-1}}{E_n} = 1 - \frac{E_{n-1}}{E_n} = 1 - \frac{n^2}{(n-1)^2} = \frac{-2n+1}{(n-1)^2} \approx \frac{-2}{n}$$

$$\frac{L_n - L_{n-1}}{L_n} = 1 - \frac{L_{n-1}}{L_n} = 1 - \frac{(n-1)}{n} = \frac{1}{n}$$

7. (a,b, c)



$$\nu_P = 121 - 118 = \left[\frac{v + v\cos\theta}{v}\right]$$

$$\nu_\theta = 121 - 118 = 3$$

$$\nu_R = (121-118)\left[\frac{v - v_c\cos\theta}{v}\right]$$

$\therefore \nu_P + \nu_R = 2\nu_Q \Rightarrow (A)$ is correct option

In general when the car is passing through A

$$\nu = 3\left[\frac{v + v_c\cos\alpha}{v}\right] \quad \ldots\text{(i)}$$

$$\therefore \frac{d\nu}{d\alpha} = -3\left[\frac{v_c\sin\alpha}{v}\right] \quad \left|\frac{d\nu}{d\alpha}\right| \text{ is max when } \sin\alpha = 1$$

i.e., $\alpha = 90°$ (at Q)

$\Rightarrow$ (b) is correct option.

$$\text{From (i) } \frac{d\nu}{dt} = \frac{3v_c}{v}(-\sin\alpha)\frac{d\alpha}{dt} \quad \ldots\text{(ii)}$$

$$\text{Also } \tan\alpha = \frac{10}{x} \therefore \sec^2\alpha\frac{d\alpha}{dt} = -\frac{10}{x^2}\frac{dx}{dt}$$

$$\therefore \frac{d\alpha}{dt} = \frac{-10v}{x^2\sec^2\alpha} \quad \ldots\text{(iii)}$$

From (ii) & (iii)

$$\frac{d\nu}{dt} = -\frac{3v_c}{v}\sin\alpha \times \left(\frac{-10v}{x^2\sec^2\alpha}\right) = \frac{30V_c\sin\alpha}{x^2\sec^2\alpha}$$

$$\therefore \frac{d\nu}{dt} = \frac{30v_c\sin\alpha}{(10\cot\alpha)^2\sec^2\alpha} = 0.3v_c\sin^3\alpha \text{. At } \alpha = 90°$$

$$\frac{d\nu}{dt} = max$$

$\therefore$ (c) is the correct option

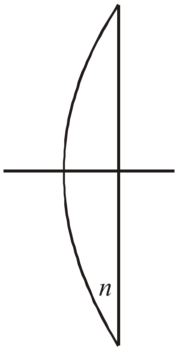
8. **(c, d)** With the use of filament and the evaporation involved, the filament will become thinner thereby decreasing the area of cross-section and increasing the resistance. Therefore the filament will consume less power towards the end of life.

As the evaporation is non-uniform, the area of cross-section will be different at different cross-section. Therefore temperature distribution will be non-uniform. The filament will break at the point where the temperature is maximum.

When the filament temperature is higher $\left(\lambda_n \propto \frac{1}{T}\right)$, it emits light of lower wavelength or higher band of frequencies.

9. **(a, d)** **For lens**

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{-30}=\frac{1}{f}$$



Also, $m=\frac{v}{u} \Rightarrow -2=\frac{v}{u}$

On solving we get $f = +20$ cm and $v = 60$ cm.

For reflection

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R} \Rightarrow \frac{1}{10}+\frac{1}{-30}=\frac{2}{R}$$

$\Rightarrow R = 30\,\text{cm}$

The image formed by convex side is faint erect and virtual.

By lens maker formula

$$\frac{1}{f}=\left(\frac{n_l}{n_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\therefore\ \frac{1}{20}=\left(\frac{n_l}{1}-1\right)\left(\frac{1}{30}\right)\ \therefore n_l = 2.5$$

10. **(b, d)** We know that, dimensionally $\in=\frac{q^2}{\ell^2 F}$,

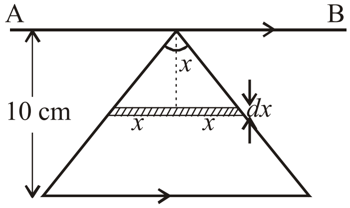
$$k_B T=\frac{RT}{N_A}=PV=F\times\ell$$

Now $\sqrt{\frac{\varepsilon k_B T}{nq^2}}=\left[\frac{q^2}{\ell^2 F}\times\frac{F\times\ell}{\ell^{-3}q^2}\right]^{1/2}=\ell$

Also $\sqrt{\frac{q^2}{\in n^{1/3}k_B T}}=\left[\frac{\ell^2 F\times q^2}{q^2\ell^{-1}\times F\times\ell}\right]^{1/2}=\ell$

11. **(a, d)** The flux passing through the triangular wire if i current flows through the inifinitely long conducting wire

$$=\int_0^{0.1}\frac{\mu_0 i}{2\pi x}\times 2\pi dx$$



$$\phi=\frac{\mu_0 i}{10\pi}=Mi$$

$$\therefore\ M=\frac{\mu_0}{10\pi}$$

Induced emf in the wire

$$=M\frac{di}{dt}=\frac{\mu_0}{10\pi}\times 10=\frac{\mu_0}{\pi} \qquad \frac{dI}{dt}=10AS^{-1}$$

As the current in the triangular wire is decreasing the induced current in AB is in the same direction as the current in the hypotenuse of the triangular wire. Therefore force will be repulsive.

12. **(a, b)** $\vec{r}=\alpha t^3\hat{i}+\beta t^2\hat{j}$

$$\vec{r}=\frac{10}{3}t^3\hat{i}+5t^2\hat{j}m$$

$$\vec{v}=\frac{d\vec{r}}{dt}=10t^2\hat{i}+10t\hat{j}\ ms^{-1}$$

$$\vec{a}=\frac{d\vec{v}}{dt}=20t\hat{i}+10\hat{j}ms^{-2}$$

At t = 1s

$$\vec{r}_{t=1}=\frac{10}{3}\hat{i}+5\hat{j}m\ ;$$

$$\vec{v}_{t=1}=10\hat{i}+10\hat{j}\ ms^{-1}$$

$$\vec{p}_{t=1}=\hat{i}+\hat{j}\ \ kgms^{-1}$$

$$\vec{a}_{t=1}=20\hat{i}+10\hat{j}\ ms^{-2}$$

$$\vec{L}=\vec{r}\times\vec{p}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{10}{3} & 5 & 0\\ 1 & 1 & 0\end{vmatrix}=\hat{k}\left[\frac{10}{3}-5\right]=-\frac{5}{3}\hat{k}\ \text{kgms}^{-1}$$

$$\vec{F}=m\vec{a}=(2\hat{i}+\hat{j})N$$

$$\vec{\tau}=\vec{r}\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{10}{3} & 5 & 0\\ 2 & 1 & 0\end{vmatrix}=\hat{k}\left[\frac{10}{3}-10\right]=\frac{-20}{3}\hat{k}\ \text{N–m}$$

13. **(a, c, d)**

$n_1\sin\theta_i=n_2\sin\theta_f$ [$\because$ 1 and 2 interfaces are parallel]

l depends on the refractive index of transparent slab but not on n_2. In fact θ_f depends on n_2.

14. **(9)** Here $P\propto T^4$ or $P=P_0T^4$

$$\therefore\ \log_2 P=\log_2 P_0+\log_2 T^4\ \ \therefore\ \log_2\frac{P}{P_0}=4\log_2 T$$

$$\text{At T} = 487°\text{C} = 760\,\text{K},\ \log_2 \frac{P}{P_0} = 4\log_2 760 = 1 \ \ldots (1)$$

At T = 2767°C = 3040K,

$$\log_e \frac{\rho}{\rho_0} = 4\log_2 3040 = 4\log_2 (760 \times 4)$$

$$= 4\left[\log_2 760 + \log_2 2^2\right]$$

$$= 4\log_2 760 + 8 = 1 + 8 = 9$$

15. **(9)** Maximum kinetic energy of β-particle

$$= [\text{mass of } {}^{12}_{5}B \ - \text{mass of } {}^{12}_{6}C\,] \times 931.5 - 4.041$$

$$= [12.014 - 12] \times 931.5 - 4.041] = 9\text{MeV}$$

16. **(6)** $E = \frac{hc}{\lambda} = \frac{1.237 \times 10^{-6}}{970 \times 10^{-10}} eV = 12.75 eV$

∴ The energy of electron after absorbing this photon $= -13.6 + 12.75 = -0.85\text{eV}$

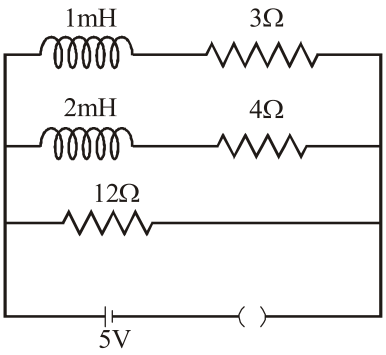
This corresponds to n = 4

$$\text{Number of spectral line} = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

17. **(3)** $$\frac{V_P}{V_Q} = \frac{\dfrac{2r_1^2(\sigma - \rho_1)g}{9\eta_1}}{\dfrac{2r_2^2(\sigma - \rho_2)g}{9\eta_2}} = \frac{r_1^2(\sigma - \rho_1)}{r_2^2(\sigma - \rho_2)} \times \frac{\eta_2}{\eta_1}$$

$$= \frac{1^2}{(0.5)^2}\frac{[8 - 0.8]}{[8 - 1.6]} \times \frac{2}{3} = 3$$

18. **(8)**



$$\text{At t} = 0\ \ I_{\min} = \frac{5}{12}$$

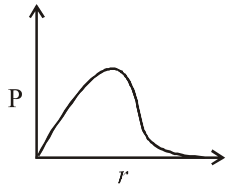
$$\text{At t} = \alpha\ \ I_{\max} = \frac{5}{R_{eq}} = \frac{5}{3/2} = \frac{10}{3}$$

$$\left[\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{8}{12}\right]$$

$$\therefore \frac{I_{\max}}{I_{\min}} = \frac{10}{3} \times \frac{12}{5} = 8$$

CHEMISTRY

19. **(d)** Radial probability function curve for 1*s* is (D).
Here P is $4\pi r^2 R^2$.



20. **(c)** From 1st law of thermodynamics

$$q_{sys} = \Delta U - w = 0 - [-P_{ext}.\Delta V]$$
$$= 3.0\text{ atm} \times (2.0\text{ L} - 1.0\text{ L}) = 3.0\text{ L-atm}$$

$$\therefore\ \ \Delta S_{surr} = \frac{(q_{rev})_{surr}}{T} = -\frac{q_{sys}}{T}$$

$$= -\frac{3.0 \times 101.3\,\text{J}}{300\,\text{K}} = -1.013\ \text{J/K}$$

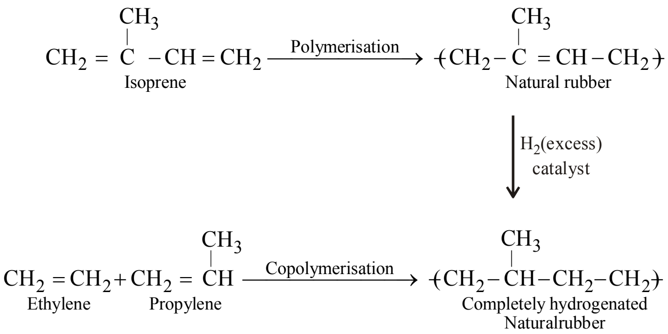
21. **(b)** Atomic radii increases on moving down a group. However due to poor shielding effect of *d* orbit, atomic radius of Ga is smaller than Al (**anomaly**). Thus the correct order is Ga < Al < In < Tl

22. **(b)**

	Compound/Ion	**Magnetic nature of compound**
1.	$[Ni(CO)_4]$	Diamagnetic
2.	$[NiCl_4]^{2-}$	Paramagnetic
3.	$[Co(NH_3)_4Cl_2]Cl$	Diamagnetic
4.	$Na_3[CoF_6]$	Paramagnetic
5.	Na_2O_2	Diamagnetic
6.	CsO_2	Paramagnetic

So total number of paramagnetic compounds is 3.

23. **(a)**



$$CH_2 = \overset{CH_3}{\overset{|}{C}} - CH = CH_2 \xrightarrow{\text{Polymerisation}} -\!(CH_2 - \overset{CH_3}{\overset{|}{C}} = CH - CH_2)\!-$$

Isoprene — Natural rubber

$$\downarrow \text{H}_2\text{(excess) catalyst}$$

$$CH_2 = CH_2 + CH_2 = \overset{CH_3}{\overset{|}{CH}} \xrightarrow{\text{Copolymerisation}} -\!(CH_2 - \overset{CH_3}{\overset{|}{CH}} - CH_2 - CH_2)\!-$$

Ethylene Propylene Completely hydrogenated Naturalrubber

24. **(b, c, d)**

(A) High activation energy usually implies a **slow reaction**.

(B) Rate constant of a reaction increases with increase in temperature due to increase in number of collisions whose energy exceeds the activation energy.

(C) $k = P \times Z \times e^{-E_a/RT}$

(D) So, pre-exponential factor (*A*) = *P* × *Z* and it is independent of activation energy or energy of molecules.

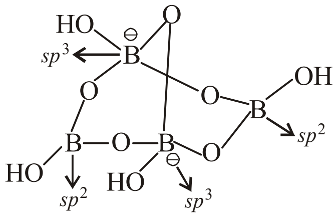
25. **(b, d)** When N/P ratio is less than one, then proton changes into neutron.

Position emission : ${}^{1}_{1}\text{H} \longrightarrow {}^{1}_{0}\text{n} + {}^{\ 0}_{+1}\beta$

k-electron capture : ${}^{1}_{1}\text{H} + {}^{\ 0}_{-1}\text{e} \longrightarrow {}^{1}_{0}\text{n} + \text{X-rays}$

26. (a, c, d)

Structure of borax



Correct formula of borax is $Na_2[B_4O_5(OH)_4].8H_2O$

(A) Borax has tetranuclear. $[B_4O_5(OH)_4]^{2-}$ unit

(B) Only two 'B' atom lie in same plane

(C) two Boron are sp^2 and two are sp^3 hybridised.

(D) one terminal hydroxide per boran atom.

27. (b, c)

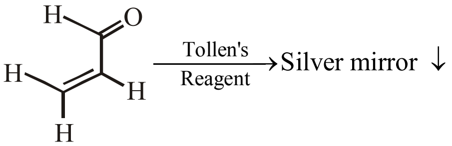
Compound		**Number of lone pairs on central atom**
BrF_5	→	1
ClF_3	→	2
XeF_4	→	2
SF_4	→	1

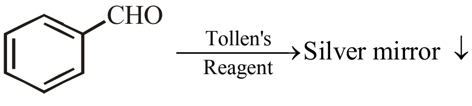
28. (a or a, c)

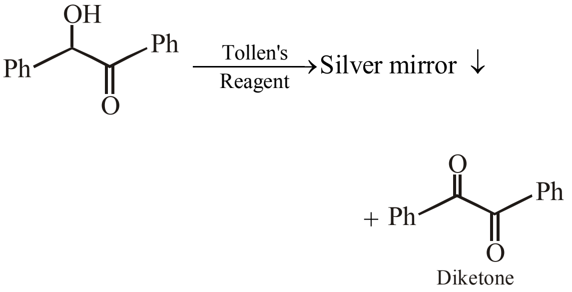
	S^{2-}	SO_4^{2-}
Cu^{2+}	CuS (ppt)	$CuSO_4$ (Soluble)
Ba^{2+}	BaS (Soluble)	$BaSO_4$ (ppt)
$Pb(OAc)_2$	PbS (ppt)	$PbSO_4$ (ppt)
$Na_2[Fe(CN)_5NO]$	$Na_4[Fe(CN)_5(NOS)]$ Colour (not a ppt)	____

29. (a, b, c)

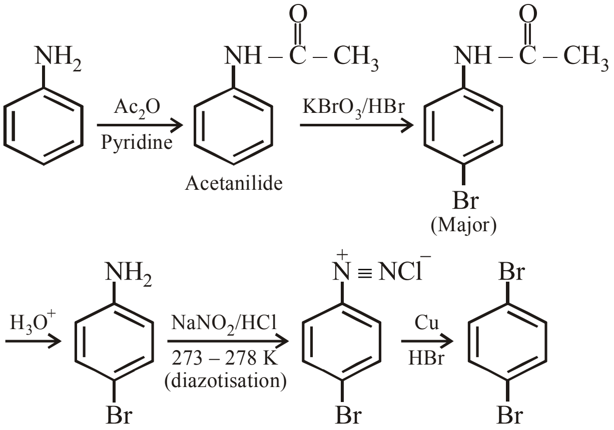
Aldehydes and α-Hydroxyketones show positive Tollen's test.



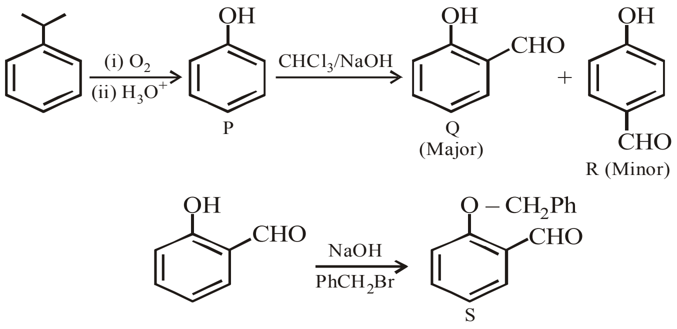




30. (b)



31. (b, c)



Q is steam volatile not R.

Q and R show positive test with 1% aqueous $FeCl_3$ solution.

Q, R, S give yellow precipitate with 2, 4-dinitrophenyl hydrazine.

32. (9) 1 mole solution has 0.1 mole solute and 0.9 mole solvent.

Let M_1 = Molar mass solute

M_2 = Molar mass solvent

$$\text{Molality, } m = \frac{0.1}{0.9M_2} \times 1000 \qquad ....(1)$$

$$\text{Molarity, } M = \frac{0.1}{0.1M_1 + 0.9M_2} \times 2 \times 1000 \qquad ....(2)$$

$$\because \quad m = M$$

$$\Rightarrow \frac{0.1 \times 1000}{0.9M_2} = \frac{200}{0.1M_1 + 0.9M_2}$$

$$\Rightarrow \frac{M_1}{M_2} = 9$$

33. (4) Diffusion coefficient $\propto \lambda\mu$

Since $\lambda \propto \frac{T}{P}$ and $\mu \propto \sqrt{T}$

$\therefore$ Diffusion coefficient $\propto \frac{T\sqrt{T}}{P}$

Thus $\frac{D_i}{D_f} = \frac{\frac{T\sqrt{T}}{P}}{\frac{4T\sqrt{4T}}{2P}} = \frac{1}{(4\times 2)/2} = \frac{1}{4}$

or $\frac{D_f}{D_i} = \frac{4}{1}$

34. (6) $8MnO_4^- + 3S_2O_3^{2-} + H_2O \longrightarrow$

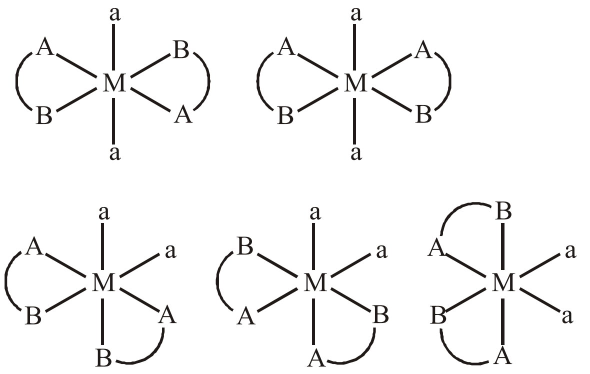
$8MnO_2 + 6SO_4^{2-} + 2OH^-$

$\therefore$ 8 moles of MnO_4^- form 6 moles of SO_4^{2-}

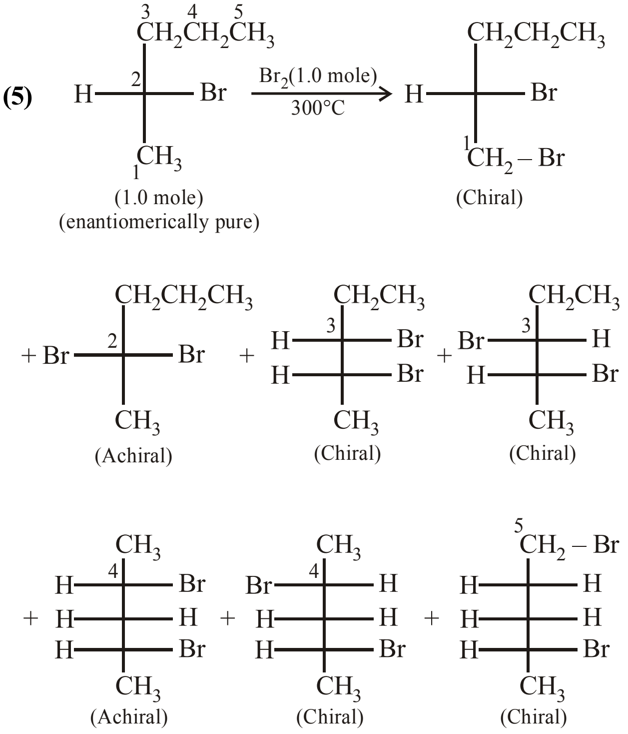
35. (5) $[CoL_2Cl_2]^-$ $(L = H_2NCH_2CH_2O^-)$

L is unsymmetrical didentate ligand.

So the complete is equivalent to $[M(AB)_2a_2]$ Possible G.I. are



36. (5)



MATHEMATICS

37. (c) $x^2 - 2x\sec\theta + 1 = 0 \Rightarrow x = \sec\theta \pm \tan\theta$

and $x^2 + 2x\tan\theta - 1 = 0 \Rightarrow x = -\tan\theta \pm \sec\theta$

$\because \ -\frac{\pi}{6} < \theta < -\frac{\pi}{12}$

$\Rightarrow \sec\frac{\pi}{6} > \sec\theta > \sec\frac{\pi}{12}$

and $-\tan\frac{\pi}{6} < \tan\theta < -\frac{\tan\pi}{12}$

also $\tan\frac{\pi}{12} < -\tan\theta < \tan\frac{\pi}{6}$

α_1, β_1 are roots of $x^2 - 2x\sec\theta + 1 = 0$

and $\alpha_1 > \beta_1$

$\therefore\ \alpha_1 = \sec\theta - \tan\theta$ and $\beta_1 = \sec\theta + \tan\theta$

α_2, β_2 are roots of $x^2 + 2x\tan\theta - 1 = 0$ and $\alpha_2 > \beta_2$

$\therefore\ \alpha_2 = -\tan\theta + \sec\theta,\ \beta_2 = -\tan\theta - \sec\theta$

$\therefore\ \alpha_1 + \beta_2 = \sec\theta - \tan\theta - \tan\theta - \sec\theta = -2\tan\theta$

38. (a) Either one boy will be selected or no boy will be selected. Also out of four members one captain is to be selected.

$\therefore$ Required number of ways $= ({}^4C_1 \times {}^6C_3 + {}^6C_4) \times {}^4C_1$

$= (80 + 15) \times 4 = 380$

39. (c) $\sqrt{3}\ \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$

$\Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \cos^2 x - \sin^2 x$

$\Rightarrow \cos\left(x - \frac{\pi}{3}\right) = \cos 2x$

$\Rightarrow x - \frac{\pi}{3} = 2n\pi \pm 2x$

$\Rightarrow x = \frac{2n\pi}{3} + \frac{\pi}{9}$ or $x = -2n\pi - \frac{\pi}{3}$

For $x \in S,\ n = 0 \Rightarrow x = \frac{\pi}{9}, -\frac{\pi}{3}$

$n = 1 \Rightarrow x = \frac{7\pi}{9}$

$n = -1 \Rightarrow x = \frac{-5\pi}{9}$

$\therefore$ Sum of all values of $x = \frac{\pi}{9} - \frac{\pi}{3} + \frac{7\pi}{9} - \frac{5\pi}{9} = 0$

40. (c) $P(T_1) = \frac{20}{100},\ P(T_2) = \frac{80}{100},\ P(D) = \frac{7}{100}$

Let $P\left(\frac{D}{T_2}\right) = x$, then $P\left(\frac{D}{T_1}\right) = 10x$

Also $P(D)=P(T_1)\;P\left(\frac{D}{T_1}\right)+P(T_2)\,P\left(\frac{D}{T_2}\right)$

$$\Rightarrow \frac{7}{100}=\frac{20}{100}\times 10x+\frac{80}{100}\times x$$

$$\Rightarrow \frac{7}{280}=x \text{ or } x=\frac{1}{40}$$

$$P\left(\frac{D}{T_1}\right)=\frac{10}{40} \text{ and } P\left(\frac{D}{T_2}\right)=\frac{1}{40}$$

$$\Rightarrow P\left(\frac{\bar{D}}{T_1}\right)=\frac{30}{40} \text{ and } P\left(\frac{\bar{D}}{T_2}\right)=\frac{39}{40}$$

$$P\left(\frac{T_2}{\bar{D}}\right)=\frac{P\left(\frac{\bar{D}}{T_2}\right)P(T_2)}{P\left(\frac{\bar{D}}{T_1}\right)P(T_1)+P\left(\frac{\bar{D}}{T_2}\right)P(T_2)}$$

$$=\frac{\frac{80}{100}\times\frac{39}{40}}{\frac{20}{100}\times\frac{30}{40}+\frac{80}{100}\times\frac{39}{40}}=\frac{156}{186}=\frac{26}{31}$$

Also $\frac{78}{93}=\frac{26}{31}$

41. (c) Let $f(x)=4\alpha x^2+\frac{1}{x}$

For $x>0$, $f_{min}=1$

$$f'(x)=8\alpha x-\frac{1}{x^2}=0\Rightarrow x=\frac{1}{2\alpha^{1/3}}$$

$$f_{min}=1\Rightarrow 4\alpha\left(\frac{1}{2\alpha^{1/3}}\right)^2+2\alpha^{1/3}=1$$

$$\Rightarrow 3\alpha^{\frac{1}{3}}=1 \text{ or } \alpha=\frac{1}{27}$$

42. (b, c, d)

The coordinates of vertices of pyramid OPQRS will be

$O(0,0,0),\ P(3,0,0),\ Q(3,3,0),\ R(0,3,0),\ S\left(\frac{3}{2},\frac{3}{2},3\right)$

dr's of OQ = 1, 1, 0

dr's of OS = 1, 1, 2

$\therefore$ acute angle between OQ and OS

$$=\cos^{-1}\left(\frac{2}{\sqrt{2}\times\sqrt{6}}\right)=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\neq\frac{\pi}{3}$$

$$\text{Eqn of plane OQS}=\begin{vmatrix} x & y & z\\ 1 & 1 & 0\\ 1 & 1 & 2\end{vmatrix}=0$$

$\Rightarrow 2x-2y=0$ or $x-y=0$

length of perpendicular from P (3, 0, 0) to plane $x-y=0$

$$\text{is}=\left|\frac{3-0}{\sqrt{2}}\right|=\frac{3}{\sqrt{2}}$$

$$\text{Eqn of RS}: \frac{x}{\frac{3}{2}}=\frac{y-3}{\frac{-3}{2}}=\frac{z}{3} \text{ or } \frac{x}{1}=\frac{y-3}{-1}=\frac{z}{2}=\lambda$$

If ON is perpendicular to RS, then N $(\lambda, -\lambda+3, 2\lambda)$

$\because$ ON $\perp$ RS $\Rightarrow 1\times\lambda-1(-\lambda+3)+2\times 2\lambda=0$

$$\Rightarrow\lambda=\frac{1}{2}\Rightarrow N\left(\frac{1}{2},\frac{5}{2},1\right)$$

$$\therefore ON=\sqrt{\frac{1}{4}+\frac{25}{4}+1}=\sqrt{\frac{15}{2}}$$

43. (a) $f'(x)=2-\frac{f(x)}{x}$

$$\Rightarrow f'(x)+\frac{1}{x}f(x)=2$$

If $=e^{\log x}=x$

$$\therefore f(x).x=\int 2x\,dx=x^2+C$$

or $f(x)=x+\frac{C}{x}, C\neq 0$ as $f(x)\neq 1$

(a) $\lim\limits_{x\to 0^+} f'\left(\frac{1}{x}\right)=\lim\limits_{x\to 0^+}(1-Cx^2)=1$

(b) $\lim\limits_{x\to 0^+} xf\left(\frac{1}{x}\right)=\lim\limits_{x\to 0^+} x\left(\frac{1}{x}+Cx\right)\lim\limits_{x\to 0^+} 1+Cx^2=1$

(c) $\lim\limits_{x\to 0^+} x^2f'x$

$$=\lim_{x\to 0^+} x^2\left(1-\frac{C}{x^2}\right)=\lim_{x\to 0^+} x^2-C=-C$$

(d) for $C\neq 0$, f(x) is unbounded as $0<x<2$

$$\Rightarrow\frac{C}{2}<\frac{C}{x}<\infty\Rightarrow\frac{C}{2}<x+\frac{C}{x}<\infty$$

44. (b,c) $PQ=kI\Rightarrow\frac{P.Q}{k}=I\Rightarrow P^{-1}=\frac{Q}{k}$

Also $|P|=12\alpha+20$

Comparing the third elements of 2^{nd} low on both sides, we get

$$-\left(\frac{3\alpha+4}{12\alpha+20}\right)=\frac{1}{k}\times\frac{-k}{8}$$

$\Rightarrow 24\alpha+32=12\alpha+20\Rightarrow\alpha=-1$

$\therefore |P|=8$

Also $PQ=kI\Rightarrow|P|\,|Q|=k^3$

$$\Rightarrow 8\times\frac{k^2}{2}=k^3\Rightarrow k=4\Rightarrow|Q|=\frac{k^2}{2}=8$$

(b) $4\alpha - k + 8 = 4 \times (-1) - 4 + 8 = 0$

(c) Now det (P adj Q) = |P| adj Q|

$= |P|\,|Q|^2 = 8 \times 8^2 = 2^9$

(d) $|Q \text{ adj } P| = |Q|\,|P|^2 = 2^9$

45. **(a, c, d)** $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2} = \frac{s-x+s-y+s-z}{4+3+2} = \frac{s}{9}$

$\therefore x = \frac{5s}{9}, y = \frac{6s}{9}, z = \frac{7s}{9}$

Area of incircle $= \pi r^2 = \pi \frac{\Delta^2}{s^2} = \frac{8\pi}{3}$

$\Rightarrow \frac{s(s-x)(s-y)(s-z)}{s^2} = \frac{8}{3}$

$\Rightarrow \frac{4 \times 3 \times 2s^3}{9 \times 9 \times 9s} = \frac{8}{3}$ or $s = 9$

$\therefore x = 5, y = 6, z = 7$

(a) Area of $\Delta xyz = \sqrt{s(s-x)(s-y)(s-z)}$

$\sqrt{9 \times 4 \times 3 \times 2} = 6\sqrt{6}$

(b) Radius of circumcircle, $R = \frac{xyz}{4\Delta}$

$= \frac{5 \times 6 \times 7}{4 \times 6\sqrt{6}} = \frac{35\sqrt{6}}{24} = \frac{4}{35}$

(c) $r = 4R \sin\frac{x}{2} \sin\frac{y}{2} \sin\frac{z}{2} \Rightarrow \sin\frac{x}{2} \sin\frac{y}{2} \sin\frac{z}{2} = \frac{r}{4R}$

$= \frac{2\sqrt{2} \times 24}{\sqrt{3} \times 4 \times 35\sqrt{6}}$

(d) $\sin^2 \frac{x+y}{2} = \cos^2 \frac{z}{2}$

$= \frac{s(s-z)}{xy} = \frac{9 \times 2}{5 \times 6} = \frac{3}{5}$

46. **(a, d)** $[(x+2)^2 + y(x+2)] \frac{dy}{dx} = y^2$

$\Rightarrow y^2 \frac{dx}{dy} - (x+2)y = (x+2)^2$

$\Rightarrow \frac{1}{(x+2)^2} \frac{dx}{dy} - \frac{1}{(x+2)y} = \frac{1}{y^2}$

Let $\frac{-1}{x+2} = u \Rightarrow \frac{1}{(x+2)^2} \frac{dx}{dy} = \frac{du}{dy}$

$\therefore$ Eqn becomes

$\frac{du}{dy} + \frac{1}{y} u = \frac{1}{y^2}$

If $= e^{\log = y} = y$

$\therefore$ Solution is : $u \times y = \int y \times \frac{1}{y^2} dy = \log y + C$

$\Rightarrow \frac{-y}{x+2} = \log y + C$

As it passes through $(1, 3) \Rightarrow C = -1 - \log 3$

$\therefore \frac{-y}{x+2} = \log y - 1 - \log 3$

$\Rightarrow \log\frac{y}{3} = 1 - \frac{y}{x+2}$ ...(1)

Intersection of (1) and $y = x + 2$

$\log\frac{y}{3} = 0 \Rightarrow y = 3 \Rightarrow x = 1$

$\therefore$ (1, 3) is the only intersection point.

Intersection of (1) and $y = (x+2)^2$

$\log\frac{(x+2)^2}{3} = 1 - (x+2)$ or $\log\frac{(x+2)^2}{3} + (x+2) = 1$

$\because \frac{(x+2)^2}{3} > \frac{4}{3} > 1, \forall x > 0$

$\therefore$ LHS > 2, $\forall$ $x > 0 \Rightarrow$ no solution.

47. **(b, c)** $f(x) = x^3 + 3x + 2 \Rightarrow f'(x) = 3x^2 + 3$

Also $f(0) = 2, f(1) = 6, f(2) = 16, f(3) = 38, f(6) = 236$

And $g(f(x)) = x \Rightarrow g(2) = 0, g(6) = 1, g(16) = 2, g(38) = 2, g(236) = 6$

(a) $g(f(x)) = x \Rightarrow g'(f(x)).\, f'(x) = 1$

For $g'(2)$, $f(x) = 2 \Rightarrow x = 0$

$\therefore$ Putting $x = 0$, we get $g'(f(0))\, f'(0) = 1$

$\Rightarrow g'(2) = \frac{1}{3}$

(b) $h(g(g(x))) = x \Rightarrow h'(g(g(x))).\, g'(g(x)).\, g'(x) = 1$

For $h'(1)$, we need $g(g(x)) = 1$

$\Rightarrow g(x) = 6 \Rightarrow x = 236$

$\therefore$ Putting $x = 236$, we get

$h'[g(g(236))] = \frac{1}{g'(g(236)).g'(236)}$

$\Rightarrow h'(g(6)) = \frac{1}{g'(6)\, g'(236)}$

$\Rightarrow h'(1) = \frac{1}{g'(f(1))\, g'(f(6))} = f'(1)\, f'(6)$

$= 6 \times 111 = 666$

(c) $h[g(g(x))] = x$

For $h(0)$, $g(g(x))\ \ 0 \Rightarrow g(x) = 2 \Rightarrow x = 16$

$\therefore$ Putting $x = 16$, we get

$h(g(g(16))) = 16$

$\Rightarrow h(0) = 16$

(d) $h[g(g(x))] = x$

For $h(g(3))$, we need $g(x) = 3 \Rightarrow x = 38$

$\therefore$ Putting $x = 38$, we get

$h[g(g(38))] = 38 \Rightarrow h(g(3)) = 38$

48. (a, b, c) $C_1 : x^2 + y^2 = 3$ ..(i)

parabola : $x^2 = 2y$ ...(ii)

Intersection point of (i) and (ii) in first quadrant

$y^2 + 2y - 3 = 0 \Rightarrow y = 1 \qquad (\because y \neq -3)$

$\therefore x = \sqrt{2}$

$P(\sqrt{2}, 1)$

Equation of tangent to circle C_1 at P is $\sqrt{2}x + y - 3 = 0$

Let centre of circle C_2 be $(0, k)$; $r = 2\sqrt{3}$

$$\therefore \left|\frac{k-3}{\sqrt{3}}\right| = 2\sqrt{3} \Rightarrow K = 9 \text{ or } -3$$

$\therefore Q_2\,(0, 9), Q_3\,(0, -3)$

(a) $Q_2Q_3 = 12$

(b) R_2R_3 = length of transverse common tangent

$$= \sqrt{(Q_2Q_3)^2 - (r_1 + r_2)^2} = \sqrt{(12)^2 - (4\sqrt{3})^2} = 4\sqrt{6}$$

(c) Area $(\Delta OR_2R_3) = \frac{1}{2} \times R_2R_3 \times$ length of $\perp$ from origin to tangent

$$= \frac{1}{2} \times 4\sqrt{6} \times \sqrt{3} = 6\sqrt{2}$$

(d) Area $(\Delta PQ_2Q_3) = \frac{1}{2} \times Q_2Q_3 \perp$ distance of P from y–axis $= \frac{1}{2} \times 12 \times \sqrt{2} = 6\sqrt{2}$

49. (a, c) Circle : $x^2 + y^2 = 1$

Equation of tangent at $P(\cos\theta, \sin\theta)$

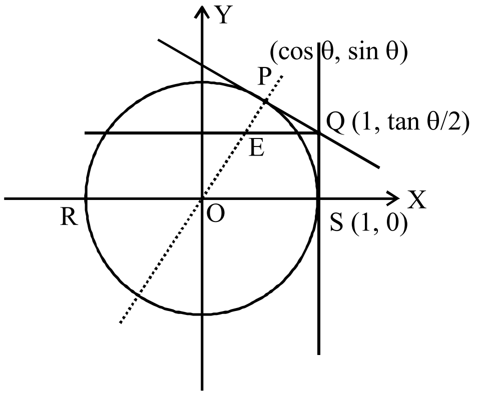
$x\cos\theta + y\sin\theta = 1$ ...(1)

Equation of normal at P

$y = x\tan\theta$ ...(2)

Equation of tangent at S is $x = 1$

$$\therefore Q\left(1, \frac{1-\cos\theta}{\sin\theta}\right) = Q\left(1, \frac{\tan\theta}{2}\right)$$



$\therefore$ Equation of line through Q and parallel to RS is $y = \frac{\tan\theta}{2}$

$\therefore$ Intersection point E of normal and $y = \tan\theta$

$$\frac{\tan\theta}{2} = x\tan\theta \Rightarrow x = \frac{1 - \tan^2\theta/2}{2}$$

$$\therefore \text{Locus of E} : x = \frac{1-y^2}{2} \text{ or } y^2 = 1 - 2x$$

It is satisfied by the points $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{3}, \frac{-1}{\sqrt{3}}\right)$

50. (2)
$$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$$

$$\Rightarrow x^3\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} + x^6\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} = 10$$

Operating $C_2 - C_1$, $C_3 - C_1$ for both the determinants, we get

$$\Rightarrow x^3\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 3 & 6 & -2 \end{vmatrix} + x^6\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 6 \\ 3 & 6 & 24 \end{vmatrix} = 10$$

$\Rightarrow x^3\,(-4 + 6) + x^6\,(48 - 36) = 10$

$\Rightarrow 2x^3 + 12x^6 = 10$

$\Rightarrow 6x^6 + x^3 - 5 = 0$

$$\Rightarrow (6x^3 - 5)\,(x^3 + 1) = 0 \Rightarrow x = \left(\frac{5}{6}\right)^{\frac{1}{3}}, -1$$

51. (5) $(1 + x)^2 + (1 + x)^3 + ... + (1 + x)^{49} + (1 + mx)^{50}$

$$= (1 + x)^2\left[\frac{(1+x)^{48} - 1}{(1+x) - 1}\right] + (1 + mx)^{50}$$

$$= \frac{1}{x}\left[(1+x)^{50} - (1+x)^2\right] + (1 + mx)^{50}$$

Coeff. of x^2 in the above expansion

= Coeff. of x^3 in $(1 + x)^{50}$ + Coeff. of x^2 in $(1 + mx)^{50}$

$\Rightarrow {}^{50}C_3 + {}^{50}C_2\,m^2$

$\therefore (3n + 1)\,{}^{51}C_3 = {}^{50}C_3 + {}^{50}C_2\,m^2$

$$\Rightarrow (3n + 1) = \frac{{}^{50}C_3}{{}^{51}C_3} + \frac{{}^{50}C_2}{{}^{51}C_3}m^2$$

$$\Rightarrow 3n + 1 = \frac{16}{17} + \frac{1}{17}m^2 \Rightarrow n = \frac{m^2 - 1}{51}$$

Least positive integer m for which n is an integer is m = 16 and then n = 5

52. (1) Let $f(x) = \int_0^x \frac{t^2}{1+t^4}dt - 2x + 1$

$$\Rightarrow f'(x) = \frac{x^2}{1+x^4} - 2 < 0 \ \forall\ x \in [0, 1]$$

∴ f is decreasing on [0, 1]

Also f(0) = 1

and $f(1) = \int_0^1 \frac{t^2}{1+t^4}dt - 1$

For $0 \le t \le 1 \Rightarrow 0 \le \frac{t^2}{1+t^4} < \frac{1}{2}$

$\therefore \int_0^1 \frac{t^2}{1+t^4}dt < \frac{1}{2}$

⇒ f(1) < 0

∴ f(x) crosses *x*-axis exactly once in [0, 1]

∴ f(x) = 0 has exactly one root in [0, 1]

53. **(7)** $\lim\limits_{x\to 0} \frac{x^2 \sin \beta x}{\alpha x - \sin x} = 1$

$$\Rightarrow \lim_{x\to 0} \frac{x^3\beta}{\alpha x - \sin x} = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{x^3\beta}{\alpha x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...\infty\right)} = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{x^3\beta}{(\alpha - 1)x + \frac{x^3}{3!} - \frac{x^5}{5!} + ...\infty} = 1$$

For above to be possible, we should have

$\alpha - 1 = 0$ and $\beta = \frac{1}{3!}$

$\Rightarrow \alpha = 1$ and $\beta = \frac{1}{6}$

$\therefore 6(\alpha + \beta) = 6\left(1 + \frac{1}{6}\right) = 7$

54. **(1)** $z = \frac{-1 + i\sqrt{3}}{2} \Rightarrow z^3 = 1$ and $1 + z + z^2 = 0$

$$P^2 = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}\begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$$

$$= \begin{bmatrix} z^{2r} + z^{4s} & z^{2s}\left((-z)^5 + z^r\right) \\ z^{2s}\left((-z)^r + z^r\right) & z^{4s} + z^{2r} \end{bmatrix}$$

For $P^2 = -I$ we should have

$z^{2r} + z^{4s} = 1$ and $z^{2s}((-z)^r + z^r) = 0$

$\Rightarrow z^{2r} + z^s + 1 = 0$

⇒ r is odd and s = r but not a multiple of 3.

Which is possible when s = r = 1

∴ only one pair is there.

Paper - 2

PHYSICS

1. **(c)** Binding energy of nitrogen atom

$= [8 \times 1.008665 + 7 \times 1.007825 - 15.000109 \times 931]$

Binding energy of oxygen atom

$= [7 \times 1.008665 + 8 \times 1.007825 - 15.003065] \times 931$

∴ Difference = 0.0037960 × 931 MeV ...(I)

Also $E_O = \frac{3}{2} \times \frac{8 \times 7}{R} \times \frac{e^2}{4\pi \in_0} = \frac{3}{5} \times \frac{56}{R} \times 1.44\text{MeV}$

$E_N = \frac{3}{5} \times \frac{7 \times 6}{R} \times \frac{e^2}{4\pi \in_0} = \frac{3}{5} \times \frac{42}{R} \times 1.44\text{MeV}$

$\therefore E_O - E_N = \frac{3}{5} \times \frac{14}{R} \times 1.44\text{MeV}$ ...(II)

From (i) & (ii)

$\frac{3}{5} \times \frac{14}{R} \times 1.44 = 0.0037960 \times 931$

∴ R = 3.42 fm

2. **(c)** $\frac{A}{A_0} = \frac{1}{2^n}$

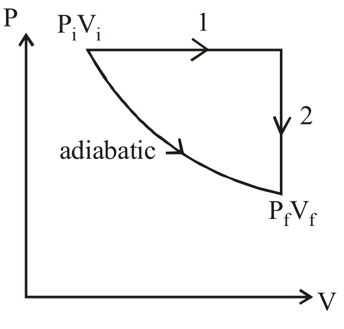
$\therefore 2^n = \frac{A_0}{A} = \frac{64}{1} = 2^6 \Rightarrow n = 6$

$\therefore$ time $= 6 \times t_{\frac{1}{2}} = 6 \times 18 = 108$

3. **(c)** $P^3V^5 = \text{constant} \Rightarrow PV^{5/3} = \text{constant} \Rightarrow \gamma = \frac{5}{3}$

⇒ monoatomic gas

For adiabatic process



$$W = \frac{P_f V_f - P_i V_i}{1-\gamma} = \frac{\frac{1}{32}\times 10^5 \times 8 \times 10^{-3} - 10^5 \times 10^{-3}}{1-\frac{5}{3}}$$

$$\therefore \; W = \frac{25-100}{(3-5)/3} = \frac{75\times 3}{2} = 112.5\text{J}$$

From first law of thermodynamics $q = \Delta u + w$

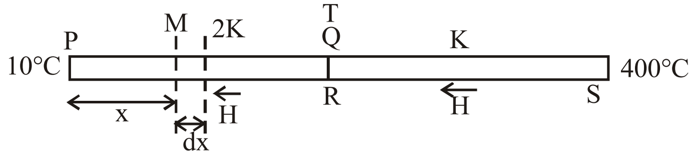
$\therefore \Delta u = -w$

$\therefore \Delta u = -112.5$ J

Now applying first law of thermodynamics for process 1 & 2 and adding $q_1 + q_2 = \Delta U + P_i(V_f - V_i)$

$= -112.5 + 10^5 (8-1) \times 10^{-3} = 587.55$

4. **(a)** The heat flow rate is same



$$\therefore \; \frac{KA(400-T)}{\ell} = \frac{2KA(T-10)}{\ell}$$

$\therefore$ T = 140°C

The temperature gradient access Pd is

$$\frac{dT}{dx} = \frac{140-10}{1} \qquad \therefore dt = 130\,dx$$

Therefore change temperature at a cross-section M distant 'x' from P is

$\Delta T = 130x$

Extension in a small elemental length 'dx' is

$dl = dx \propto \Delta T = dx \propto (130x)$

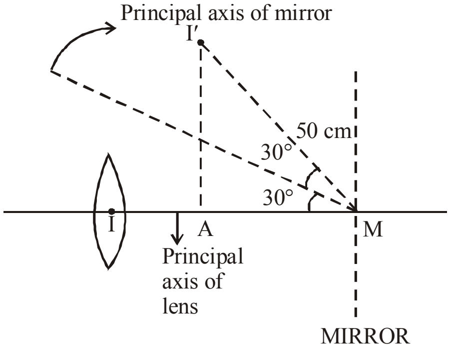
$$\therefore \int dl = 130 \propto \int_0^1 x dx$$

$$\therefore \Delta l = 130 \times 1.2 \times 10{-}5 \times \frac{1}{2} = 78 \times 10^{-5}\,\text{m}$$

5. **(c)** For lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} - \frac{1}{-50} = \frac{1}{30}$$

$\therefore \quad v = 75$ cm



Therefore object distance for mirror is 25 cm and object is virtual

For minor $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \; \therefore \; \frac{1}{v} + \frac{1}{25} = \frac{1}{50} \; \therefore v = -50$ cm

The image I would have formed as shown had the mirror have been straight. But here the mirror is tilted by 30°. Therefore the image will be tilted by 60° and will be formed at A.

Here MA = 50 cos 60° = 25 cm

and I′A = 50 sin 60° = $25\sqrt{3}$ cm

6. **(b)** For C_1

L.C. = 1MSD − 1VSD

= 1mm − 0.9 mm = 0.1 mm = 0.01 cm [10 VS D = 9 mm]

Reading = MSR + L.C × Verni scale division coinciding

Main scale division = 2.8 + (0.01) × 7 = 2.87 cm

For C_2

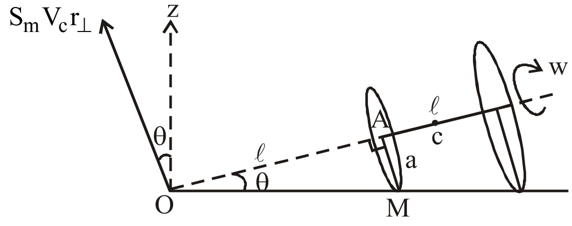
L.C = 1 mm − 1.1 mm [10 VSD = 11 mm]

L.C = − 0.1 mm = − 0.01 cm

Reading = 2.8 + (10 − 7) × 0.01 = 2.83 cm

7. **(a, c)** In ΔOAM, $OM = \sqrt{l^2 + a^2} = \sqrt{2ha^2 + a^2} = 5a$

The circumference of a circle of radius OM will be 2π(5a) = 10πa.



For completing this circle once, the smaller disc will have to take $\frac{10\pi a}{2\pi a}$ = 5 rounds.

Therefore the C.M. of the assembly rotates about z-axis with an angular speed of w/5.

The angular momentum about the C.M. of the system

$$L_c = I_c w = \left[\frac{1}{2}ma^2\right]\omega + \left[\frac{1}{2}\times 4m \times (2a)^2\right]\times\omega = \frac{17ma^2\omega}{2}$$

$$\text{Now } v_c = \frac{m\times\omega a + 4m\times 2\omega a}{5m} = \frac{9\omega a}{5}$$

$$\text{and } r_l = \frac{ml + 4m\times 2l}{5m} = \frac{9l}{5}$$

$$\text{L of C.M} = \frac{5m\times 9\omega a}{5}\times\frac{9l}{5} = 81m\omega a^2 \times \frac{\sqrt{24}}{5}$$

$$L_Z = \frac{81m\omega a^2\sqrt{24}}{5}\cos\theta - I_c\omega\sin\theta$$

$$= 81m\omega a^2\sqrt{\frac{24}{5}}\times\sqrt{\frac{24}{5}} - \frac{17ma^2\omega}{10} = \frac{1134}{50}m\omega a^2$$

8. **(c)** The wavelength of emitted photoelectron as per de Broglie is

$$\lambda e = \frac{h}{p} = \frac{h}{\sqrt{2m(K.E)}}$$

When ϕ increases, K.E. decreases and therefore λ_e increases

When λ_{ph} increases, N_{ph} decreases, K.E decreases and therefore λe increases.

λe is independent of the distance d.

Also $\frac{hc}{\lambda_{ph}} + eV - \phi = \frac{h^2}{2m\lambda_e^2} \quad \left[\lambda_e = \frac{h}{\sqrt{2\text{ mk. E}}}\right]$

$$\therefore \quad \frac{hc}{e\lambda_{ph}} + V - \frac{\phi}{e} = \frac{h^2}{2me\lambda_e^2} \qquad ...(1)$$

For $V >> \frac{\phi}{e}$, $\phi << eV$

Also $\frac{hc}{e\lambda_{Ph}} << V$. Then from eq (1).

$$\lambda e \propto \frac{1}{\sqrt{V}}$$

Therefore if V is made from times, λe is approximately half.

9. **(a, b, d)**

% error in measurement of 'r' = $\frac{1}{10} \times 100 = 10\%$

$$T_{mean} = \frac{0.52+0.56+0.57+0.54+0.59}{6} = 0.556 \approx 0.56 \text{ S}$$

$$\Delta_T = \frac{0.04+0+0.01+0.02+0.03}{6} = 0.016 \approx 0.02\text{S}$$

$\therefore$ % error in the measurement of 'T'

$$= \frac{0.02}{0.56} \times 100 = 3.57\%$$

% error in the value of g

$$= 2\frac{\Delta T}{T} \times 100 + \left(\frac{\Delta R + \Delta r}{R - r}\right) \times 100$$

$$= 2(3.57) + \left(\frac{1+1}{60-10}\right) \times 100 \approx 11\%$$

10. **(a, c)** The range of voltmeter 'V' is given by the expression

$$V = I_g\ [R_c + (R_c + R + R)]$$

V is max in this case as RHS is maximum. Thus (A) is correct.

The range of ammeter I is given by the expression

$$I = \frac{I_g R_c}{R_{eq}} + I_g \qquad \text{Where } \frac{1}{R_{eq}} = \frac{1}{R_c} + \frac{1}{R} + \frac{1}{R}$$

Here R_{eq} is minimum and therefore I is maximum. Thus (c) is the correct option.

11. **(a, b, c, d)**

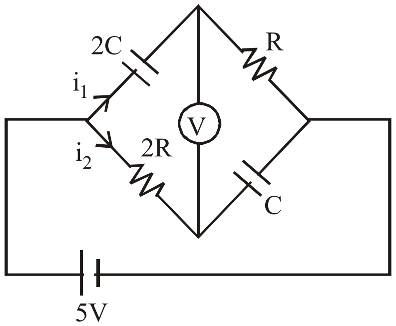
At t=0, Capacitors act as short circuit and voltmeter display –5v

At t = α, Capacitor acts as open circuit and no current flows through voltmeter ($\because$ very high resistance of voltmeter)

so it display +5v. (A) is the correct option

$$q_1 = \quad 2Cv\left(1 - e^{-t/2CR}\right), \quad i_1 = \frac{V}{R}e^{-\frac{t}{2CR}}$$

$$q_2 = \quad Cv\left(1 - e^{-t/2CR}\right), \quad i_2 = \frac{V}{2R}e^{-t\frac{t}{2CR}}$$



$$\therefore \ \Delta V = -i_2 \times 2R + \frac{2_1}{2_C} = V\left[1 - 2e^{-t/2CR}\right] = 0$$

(b) is the correct option

$$\text{At } \tau = 1\text{sec}, \ i = \frac{i_o}{e} \qquad \left[\because i = io\ e^{-t/\tau}\right]$$

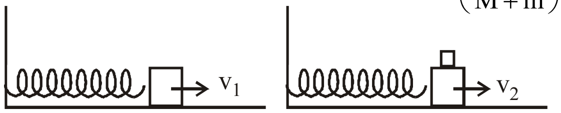
$\therefore$ (c) is the correct option

After a long time no current flows since both capcitor and voltmeter does not allow current to flow.

12. **(a, b, d)**

Case (i) : Applying conservation of linear momentum.

$$MV_1 = (M+m)\,V_2 \qquad -(1) \therefore \ V_2 = \left(\frac{M}{M+m}\right)V_1$$



From (1)

$M\ (A_1 \times \omega_1) = (M+m)\ (A_2 \times \omega_2)$

$$\therefore \quad MA_1 \times \sqrt{\frac{K}{M}} = (M+m)\,A_2 \times \sqrt{\frac{K}{M+m}}$$

$$\therefore \quad A_2 = \sqrt{\frac{M}{M+m}} = A_1$$

Also $E_1 = \frac{1}{2}\ MV_1^2$

and $\quad E_2 = \frac{1}{2}(M+m)V_2^2 = \frac{1}{2}(M+m)$

$$\times \frac{M^2V_1^2}{(M+m)^2} = \frac{1}{2}\left(\frac{M}{M+m}\right)^2 V_1^2$$

Clearly $E_2 < E_1$

The new time Period $T_2 = 2\sqrt{\frac{m+M}{K}}$

Case (ii) : The new time Period $T_2 = 2\sqrt{\frac{m+M}{K}}$

Also $A_2 = A_1$

Here $E_2 = E_1$

The instantaneous value of speed at X_o of the combined masses decreases in both the cases.

13. (b, d) Path difference at O = d = 0.6003mm

$$\text{Now, } \frac{\lambda}{2} = \frac{600 \times 10^{-6}}{2}\ \text{mm} = 300 \times 10^{-6}\ \text{mm}$$

$$\text{For} \quad n\frac{\lambda}{2} = d \text{ we get } n = \frac{0.6003}{300 \times 10^{-6}} = 2001$$

As n is a whole number, the condition for minima is satisfied.

Therefore 'O' will be dark.

Also, as the screen is perpendicular to the plane containing the slits, therefore fringes obtained will be semi-circular (Top half of the screen is available)

14. (a, b) $i = \frac{BLV}{R}$ ...(i)

[Counter clockwise direction while entering, Zero when completely inside and clockwise while exiting]

$$F = iLB = \frac{B^2L^2V}{R} \quad \text{....(ii)}$$

[Toward left while entering and exiting and zero when completely inside]

$$\therefore \quad -mV\frac{dv}{dx} = \frac{B^2L^2V}{R}$$

$$\therefore \quad \int_{v_0}^{v} dV = -\frac{B^2L^2}{mR}\int_{0}^{x} dx$$

$$V - V_0 = -\frac{B^2L^2}{mR}x$$

$$\therefore \quad V = V_0 - \frac{B^2L^2x}{mR} \quad \text{...(iii)}$$

[V decreases from x=0 to x=L, remains constant for x = L to x= 3L again decreases from x=3L to x=4L]

From (i) and (iii)

$$i = \frac{BL}{R}\left[V_0 - \frac{B^2L^2x}{mR}\right]$$

[i decreases from x=0 to x=Li becomes zero from x = L to x = 3L i changes direction and decreases from x = 3L to x = 4L]

These characteristics are shown in graph (a) and (b) only.

15. (a) Force on the block along flot $= m\, r\omega^2 = m\, v\frac{dv}{dr}$

$$\therefore \int_{o}^{v} vdv = \int_{R/2}^{r} \omega^2 rdr \Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt}$$

$$\therefore \int_{R/4}^{r} \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_{o}^{t} \omega\, dt$$

On solving we get

$$r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2}e^{wt}$$

$$\text{or } r^2 - \frac{R^2}{4} = \frac{R^2}{4}e^{2wt} + r^2 - 2r\frac{R}{I}e^{wt}$$

$$\therefore \quad r = \frac{R}{4}\left(e^{wt} + e^{-wt}\right)$$

16. (b) $\vec{F}rot = \vec{F}_{in} + 2m\left(V_{rot}\,\hat{i}\right) \times w\,\hat{k} + m\left(w\,\hat{k} \times r\,\hat{i}\right) \times w\,\hat{k}$

$$\therefore m\, r\omega^2\,\hat{i} = \vec{F}_{in} + 2m\, V_{rot}\,\omega\left(-\hat{j}\right) + m\omega^2 r\,\hat{i}$$

$$\vec{F}_{in} = mrV_{rot}\ \omega\,\hat{j} \quad \text{...(i)}$$

$$\text{But } r = \frac{R}{4}\left[e^{wt} + e^{-wt}\right]$$

$$\therefore \quad \frac{dr}{dt} = V_r = \frac{R}{4}\left[\omega e^{wt} - \omega e^{-wt}\right] \quad \text{...(ii)}$$

From (i) and (ii)

$$\vec{F}_{in} = 2m\frac{R\omega}{4}\left(e^{wt} - e^{-wt}\right)\omega\,\hat{j}$$

$$\therefore \quad \vec{F}_{in} = \frac{mR\omega^2}{2}\left(e^{wt} - e^{-wt}\right)\hat{j}$$

$$\therefore \quad \vec{F}_{reaction} = \frac{mR\omega^2}{2}\left(e^{wt} - e^{-wt}\right)\hat{j} + mg\,\hat{K}$$

17. (c) After colliding the top plate, the ball will gain negative charge and get repelled by the top plate and bounce back to the bottom plate.

18. (d) $I_{av}\ \alpha\,\frac{Q}{t}$ ...(i)

Here $Q \propto V_0$ ...(ii)

Also $S = ut + \frac{1}{2}at^2$

$$h = \frac{1}{2}\frac{QE}{m}t^2 = \frac{1}{2}\left(\frac{Q \times 2V_0}{mh}\right) \times t^2$$

$$\therefore \quad t \, \alpha \frac{1}{V^0} \quad -(iii) \qquad [\because Q \, \alpha \, V_0]$$

From (i), (ii) and (iii)

$$I_{av} \propto \frac{V_0}{1/V_0} = I_{av} \propto V_0^{\;2}$$

CHEMISTRY

19. **(d)** At anode : $H_2(g) \rightleftharpoons 2H^+(aq) + 2e^-$

At cathode : $M^{4+}(aq) + 2e^- \rightleftharpoons M^{2+}(aq)$

Net cell reaction : $H_2(g) + M^{4+}(aq) \rightleftharpoons 2H^+(aq) + M^{2+}(aq)$

$$\text{Now, } E_{cell} = \left(E^{\circ}_{M^{4+}/M^{2+}} - E^{\circ}_{H^+/H_2}\right)$$

$$- \frac{0.059}{n}.\log\frac{\left[H^+\right]^2\left[M^{2+}\right]}{P_{H_2}.\left[M^{4+}\right]}$$

$$\text{or, } 0.092 = (0.151 - 0) - \frac{0.059}{2}.\log\frac{1^2 \times \left[M^{2+}\right]}{1 \times \left[M^{4+}\right]}$$

$$\therefore \frac{\left[M^{2+}\right]}{\left[M^{4+}\right]} = 10^2 \Rightarrow x = 2$$

20. **(d)**

- A solution of CH_3OH and water shows positive deviation from Raoult's law, it means by adding CH_3OH intermolecular force of attraction decreases and hence surface tension decreases.
- By adding KCl in water, intermolecular force of attraction bit increases, so surface tension increases by small value.
- By adding surfactant like $CH_3(CH_2)_{11}OSO_3^-Na^+$, surface tension decreases rapidly and after forming micelle it slightly increases.

21. **(a)**

$$S_2O_3^{2-} \xrightarrow{Ag^+} \underset{\substack{(X)\\ \text{clear solution}}}{\left[Ag(S_2O_3)_2\right]^{3-}} \xrightarrow{Ag^+}$$

$$\underset{\substack{(Y)\\ \text{white precipitate}}}{Ag_2S_2O_3\downarrow} \xrightarrow{\text{on standing}} \underset{\substack{(Z)\\ \text{black precipitate}}}{Ag_2S\downarrow}$$

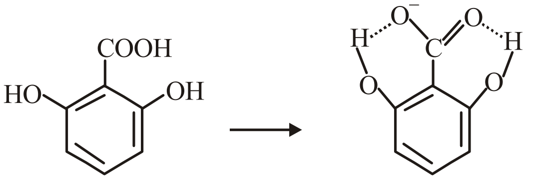
So, X, Y and Z are $[Ag(S_2O_3)_2]^{3-}$, $Ag_2S_2O_3$ and Ag_2S respectively.

22. **(a)** Ni^{2+} with NH_3 shows CN = 6 forming $[Ni(NH_3)_6]^{2+}$ (Octahedral)

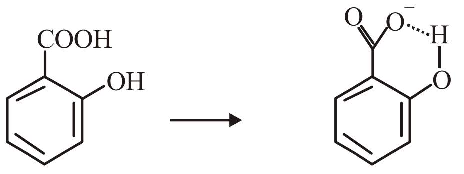
Pt^{2+} with NH_3 shows CN = 4 forming $[Pt(NH_3)_4]^{2+}$ (5d series CMA, square planner)

Zn^{2+} with NH_3 shows CN = 4 forming $[Zn(NH_3)_4]^{2+}$ ($3d^{10}$ configuration, tetrahedral)

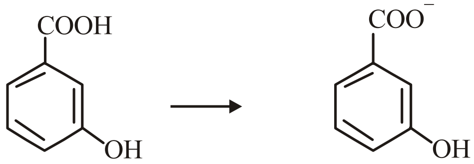
23. **(a)**



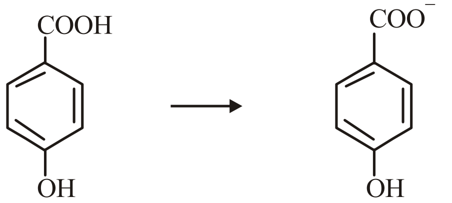
(I) (stabilizes by more H-bonding)



(II) (stabilizes by H-bonding)



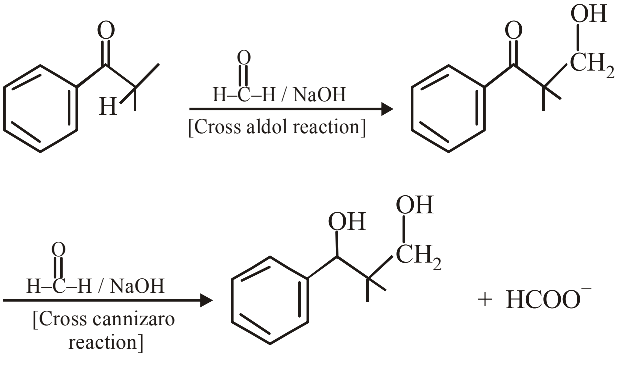
(III) (stabilizes by –I effect)

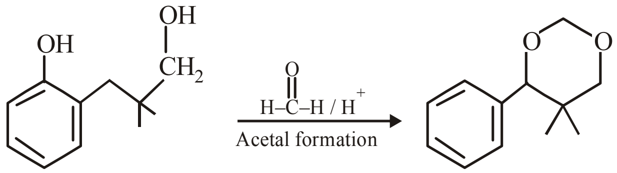


(IV) (destabilizes by +M effect)

$\therefore$ acidity order is I > II > III > IV

24. **(a)**





25. (a, c)

(A) The molecular orbital energy configuration of C_2^{2-} is

$\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2 = \pi_{2p_y}^2, \sigma_{2p_z}^2$

In the MO of C_2^{2-} there is no unpaired electron hence it is diamagnetic

(B) Bond order of O_2^{2+} is 3 and O_2 is 2 therefore bond length of O_2 is greater than O_2^{2+}

(C) The molecular orbital energy configuration of N_2^+ is

$\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2 = \pi_{2p_y}^2, \sigma_{2p_z}^1$

Bond order of $N_2^+ = \frac{1}{2}(9-4) = 2.5$

The molecular orbital energy configuration of N_2^- is

$\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2 = \pi_{2p_y}^2, \sigma_{2p_z}^2, \pi_{2p_x}^{*1} = \pi_{2p_y}^{*}$

Bond order of $N_2^- = \frac{1}{2}(10-5) = 2.5$

(D) He_2^+ has less energy in comparison to two isolated He atoms because some energy is released during the formation of He_2^+ from 2 He atoms.

26. (a, b)

(A) H-bonding of methanol breaks when CCl_4 is added so bonds become weaker, resulting positive deviation.

(B) Mixing of polar and non-polar liquids will produce a solution of weaker interaction, resulting positive deviation

(C) Ideal solution

(D) –ve deviation because stronger H-bond is formed.

27. (b, c, d)

CCP is ABC ABC ...... type packing

(A) In topmost layer, each atom is in contact with 6 atoms in same layer and 3 atoms below this layer

(B) Packing fraction = $\dfrac{4 \times \frac{4}{3}\pi r^3}{\left(\frac{4r}{\sqrt{2}}\right)^3} = (0.74)$

(C) Each FCC unit has effective no of atoms = 4
Octahedral void = 4
Tetrahedral void = 8

(D) $4r = a\sqrt{2}$

28. (a, b, c)

Copper pyrite $[CuFeS_2]$

$\downarrow$ Crushing into fine powder

Concentrated by froth floatation process

$\downarrow$

Roasting take place in reverberatory furnace

$$\begin{cases} 2CuFeS_2 + \underset{(air)}{O_2} \xrightarrow{\Delta} Cu_2S + 2FeS + SO_2 \\ FeS + O_2 \xrightarrow{\Delta} FeO + SO_2 \end{cases}$$

$\downarrow$ Smelting $\left[\begin{array}{l} \text{Iron is removed as slag of } FeSiO_3 \\ FeO + SiO_2\ (flux) \xrightarrow{\Delta} FeSiO_3(\ell)(slag) \end{array}\right]$

Copper matte $(Cu_2S + FeS)$

$\downarrow$ Self reduction $\left[\begin{array}{l} \text{Self reduction occurs in Bessemer converter} \\ 2Cu_2S + 3O_2 \longrightarrow 2Cu_2S + 2SO_2(\uparrow) \\ Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2(\uparrow) \end{array}\right]$

(Blister copper)

$\downarrow$

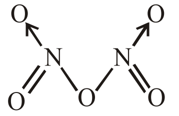
Refining of blister copper is done by poling followed by electrorefining but not by carbon reduction method.

29. (b, d) $P_4O_{10} + 4HNO_3 \xrightarrow{\text{dehydration of } HNO_3} 4(HPO_3) + 2\underset{\text{(required product)}}{N_2O_5}$

(A) $P_4 + 20HNO_3 \longrightarrow 4H_3PO_4 + 20NO_2\uparrow + 4H_2O$

(B) N_2O_5 is diamagnetic in nature

(C) $N_2O_5 \rightarrow$



N_2O_5 contains one N–O–N bond not N–N bond.

(D) $Na + N_2O_5 \rightarrow NaNO_3 + \underset{\text{(Brown gas)}}{NO_2\uparrow}$

30. **(b,c)** Invert sugar is an equimolar mixture of D-(+) glucose and D(–) glucose.

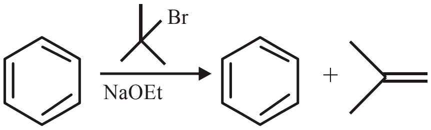
$$\underset{+\text{ sucrose}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{H^+} \underbrace{\underset{\substack{D(+)\text{–glucose}\\(+52°)}}{C_6H_{12}O_6} + H_2O \xrightarrow{+} \underset{\substack{D(+)\text{–glucose}\\(-92°)}}{C_6H_{12}O_6}}_{\text{Invert sugar}}$$

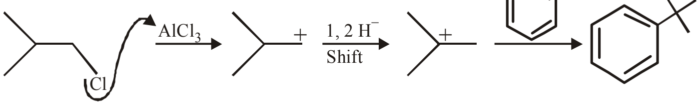
- Specific rotation of invert sugar $= \dfrac{-92° + 52°}{2} = -20°$
- D-glucose on oxidation with Br_2-water produces gluconic acid and not saccharic acid.

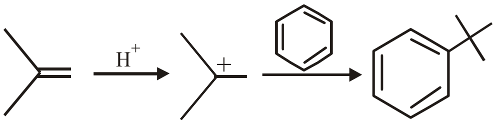
31. **(c, d)**

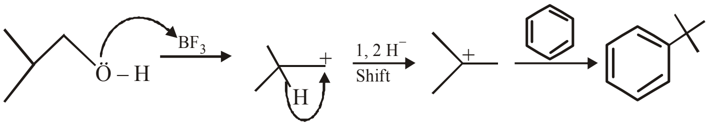
$LiAlH_4/(C_2H_5)_2O$: Reduces to esters, carboxylic acid, epoxides and aldehydes and ketones.
BH_3 in T.H.F : Reduces to –COOH and aldehydes into alcohols but do not reduce to esters and epoxides.
$NaBH_4$ in C_2H_5OH : Reduces only aldehydes and ketones into alcohols but not to others.
Raney Ni in T.H.F. : Do not reduce to –COOH, –COOR and epoxide but it can reduce aldehyde into alcohols.

32. **(b, c, d)**









33. **(b)**

$$X_2(g) \rightleftharpoons 2X(g)$$

Initial mole : $1 \qquad 0$

moles at equilibrium : $\left(1 - \dfrac{\beta_{eqbm}}{2}\right) \qquad \beta_{eqbm}$

Partial pressure : $\dfrac{1 - \dfrac{\beta_{eqbm}}{2}}{\left(1 + \dfrac{\beta_{eqbm}}{2}\right)} \times P \qquad \dfrac{\beta_{eqbm}}{\left(1 + \dfrac{\beta_{eqbm}}{2}\right)} P$

$$\therefore \quad K_p = \frac{(P_x)^2}{P_{x_2}} = \frac{\beta^2_{eqbm} P}{\left(1 - \dfrac{\beta^2_{eqbm}}{4}\right)}$$

$$\therefore \quad K_p = \frac{4\beta^2_{eqbm} P}{\left(4 - \beta^2_{eqbm}\right)}$$

Since P = 2 bar

So, $\boxed{K_p = \dfrac{8\beta^2_{eqbm}}{\left(4 - \beta^2_{eqbm}\right)}}$

34. (c) (A) Correct statement.

As on decrease in pressure reaction move indirection where no. of gaseous molecules increase.

(B) Correct statement

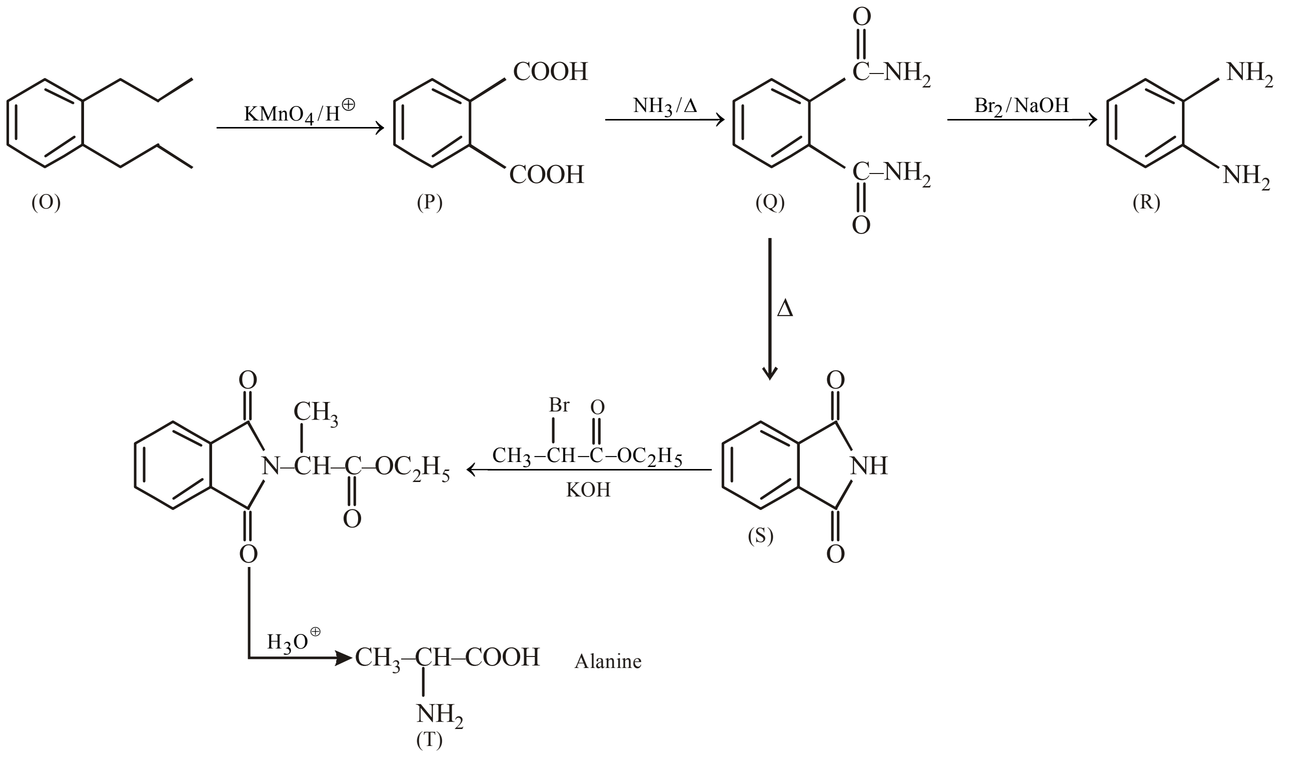
At the start of reaction $Q_P < K_P$ so dissociation of X_2 take place spontaneousely.

(C) Incorrect statement as

$$K_P = \frac{8\beta_{eq}^2}{4-\beta_{eq}^2} = \frac{8\times(0.7)^2}{4-(0.7)^2} > 1$$

35. (a)

36. (b)



MATHEMATICS

37. (b) $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = I + \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 16 & 4 & 0 \end{bmatrix} = I + A$

$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 16 & 0 & 0 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \quad A^n = O, \forall\, n \geq 3$

Now $P^{50} = (I + A)^{50} = {}^{50}C_0\, I^{50} + {}^{50}C_1\, I^{49} A + {}^{50}C_2\, I^{48} A^2 + O$

$= I + 50A + 25 \times 49\, A^2.$

$\therefore \quad Q = P^{50} - I = 50A + 25 \times 49\, A^2.$

$\Rightarrow \quad q_{21} = 50 \times 4 = 200$

$\Rightarrow \quad q_{31} = 50 \times 16 + 25 \times 49 \times 16 = 20400$

$\Rightarrow \quad q_{32} = 50 \times 4 = 200$

$\therefore \quad \frac{q_{31} + q_{32}}{q_{21}} = \frac{20600}{200} = 103$

38. (b) $\log_c b_1, \log_c b_2, \ldots, \log_c b_{101}$ are in A.P.

$\Rightarrow \quad b_1, b_2, \ldots, b_{101}$ are in G.P.

Also $a_1, a_2, \ldots, a_{101}$ are in A.P.

where $a_1 = b_1$ are $a_{51} = b_{51}$.

$\therefore$ $b_2, b_3, \ldots, b_{50}$ are GM's and $a_2, a_3, \ldots, a_{50}$ are AM's between b_1 and b_{51}.

$\because \quad GM < AM \Rightarrow b_2 < a_2, b_3 < a_3, \ldots, b_{50} < a_{50}$

$\because \quad b_1 + b_2 + \ldots + b_{51} < a_1 + a_2 + \ldots + a_{51}$

$\Rightarrow \quad t < s$

Also a_1, a_{51}, a_{101} are in AP

b_1, b_{51}, b_{101} are in GP

$\because \quad a_1 = b_1$ and $a_{51} = b_{51}$

$\therefore \quad b_{101} > a_{101}$

39. (c) $$\sum_{k=1}^{13}\frac{1}{\sin\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$$

$$=\sum_{k=1}^{13}\frac{1}{\sin\frac{\pi}{6}}\left[\frac{\sin\left\{\frac{\pi}{4}+\frac{k\pi}{6}-\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\right\}}{\sin\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}\right]$$

$$\sum_{k=1}^{13}2\left[\cot\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\right]$$

$$=2\left[\left\{\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right\}+\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{2\pi}{6}\right)\right.$$

$$\left.+\ldots\ldots+\left\{\cot\left(\frac{\pi}{4}+\frac{12\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right\}\right]$$

$$=2\left[\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right]=2\left[1-\cot\frac{5\pi}{12}\right]$$

$$=2\left[1-\frac{\sqrt{3}-1}{\sqrt{3}+1}\right]=2\left[1-\left(2-\sqrt{3}\right)\right]=2\left(\sqrt{3}-1\right)$$

40. (a) $$I=\int_{-\pi/2}^{\pi/2}\frac{x^2\cos x}{1+e^x}dx \qquad ....(i)$$

Applying $\int_a^b f(x)\,dx$

$=\int_a^b f(a+b-x)\,dx$, we get

$$I=\int_{-\pi/2}^{\pi/2}\frac{e^x x^2\cos x}{1+e^x}dx \qquad ....(ii)$$

Adding (i) and (ii)

$$2I=\int_{-\pi/2}^{\pi/2}x^2\cos x\,dx=2\int_0^{\pi/2}x^2\cos x\,dx$$

$$I=\left[x^2\sin x+2x\cos x-2\sin x\right]_0^{\frac{\pi}{2}}=\frac{\pi^2}{4}-2$$

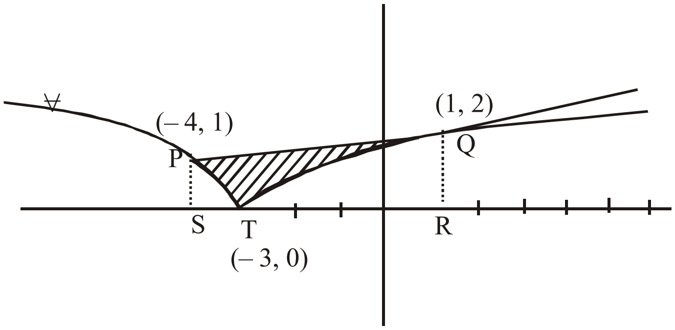
41. (c) $y\ge\sqrt{|x+3|}\Rightarrow y^2=|x+3|$

$$\Rightarrow\quad y^2=\begin{cases}-(x+3)\text{ if }x<-3\\ (x+3)\text{ if }x\ge-3\end{cases} \qquad ....(i)$$

Also $y\le\frac{x+9}{5}$ and $x\le 6$ ....(ii)

Solving (i) and (ii) we get intersection points as $(1,2)$, $(6,3)$, $(-4,1)$, $(-39,-6)$

The graph of given region is as follows-



Required area = Area (trap PQRS) – Area (PST + TQR)

$$=\frac{1}{2}\times(1+2)\times5-\left[\int_{-4}^{-3}\sqrt{-x-3}\,dx+\int_{-3}^{1}\sqrt{x+3}\,dx\right]$$

$$=\frac{15}{2}-\left[\left(\frac{2(-x-3)^{3/2}}{-3}\right)_{-4}^{-3}+\left(\frac{2(x+3)^{3/2}}{3}\right)_{-3}^{1}\right]$$

$$=\frac{15}{2}-\left[\frac{2}{3}+\frac{16}{3}\right]=\frac{15}{2}-6=\frac{3}{2}\text{ sq.units}$$

42. (c) P, the image of point (3, 1, 7) in the plane $x-y+z=3$ is given by

$$\frac{x-3}{1}=\frac{y-1}{-1}=\frac{z-7}{1}=\frac{-2(3-1+7-3)}{1^2+1^2+1^2}$$

$$\Rightarrow\quad\frac{x-3}{1}=\frac{y-1}{-1}=\frac{z-7}{1}=-4$$

$\Rightarrow\quad x=-1, y=5, z=3$

$\therefore\quad P(-1, 5, 3)$

Now equation of plane through (–1, 5, 3) and containing the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ is

$$\begin{vmatrix}x & y & z\\ -1 & 5 & 3\\ 1 & 2 & 1\end{vmatrix}=0\Rightarrow -x+4y-7z=0$$

or $\quad x-4y+7z=0$

43. (b, c)

$$f(x)=\lim_{n\to\infty}\left[\frac{n^n(x+n)\left(x+\frac{n}{2}\right)...\left(x+\frac{n}{n}\right)}{n!\left(x^2+n^2\right)\left(x^2+\frac{n^2}{4}\right)...\left(x^2+\frac{n^2}{n^2}\right)}\right]^{x/n}$$

$$=\lim_{n\to\infty}\left[\frac{n^{2n}\left(\frac{x}{n}+1\right)\left(\frac{x}{n}+\frac{1}{2}\right)....\left(\frac{x}{n}+\frac{1}{n}\right)}{n^{2n}\cdot n!\left(\frac{x^2}{n^2}+1\right)\left(\frac{x^2}{n^2}+\frac{1}{2^2}\right)....\left(\frac{x^2}{n^2}+\frac{1}{n^2}\right)}\right]^{x/n}$$

$$= \lim_{n\to\infty}\left[\frac{\left(\frac{x}{n}+1\right)\left(\frac{x}{n}+\frac{1}{2}\right)....\left(\frac{x}{n}+\frac{1}{n}\right)}{\left(1.\frac{x^2}{n^2}+1\right)\left(2.\frac{x^2}{n^2}+\frac{1}{2}\right)....\left(n.\frac{x^2}{n^2}+\frac{1}{n}\right)}\right]^{x/n}$$

$$\Rightarrow \quad \ln f(x) = \lim_{n\to\infty}\frac{x}{n}\left[\sum_{r=1}^{n}\ln\left(\frac{x}{n}+\frac{1}{r}\right)-\sum_{r=1}^{n}\ln\left(\frac{rx^2}{n^2}+\frac{1}{r}\right)\right]$$

$$= \lim_{n\to\infty}\frac{x}{n}\left[\sum_{r=1}^{n}\left\{\ln\frac{1}{r}+\ln\left(\frac{rx}{n}+1\right)\right\}-\left\{\ln\frac{1}{r}+\ln\left(\frac{r^2x^2}{n^2}+1\right)\right\}\right]$$

$$= \lim_{n\to\infty}\frac{x}{n}\left[\sum_{r=1}^{n}\ln\left(1+\frac{rx}{n}\right)-\ln\left(1+\frac{r^2x^2}{n^2}\right)\right]$$

$$= x\int_0^1 \ln(1+xy)\,dy - x\int_0^1 \ln\left(1+x^2y^2\right)\,dy$$

Let $xy = t \Rightarrow x\,dy = dt$

$$\therefore \quad \ln f(x) = \int_0^x \ln(1+t)\,dt - \int_0^x \ln\left(1+t^2\right)\,dt$$

$$\ln f(x) = \int_0^x \ln\left(\frac{1+t}{1+t^2}\right)dt$$

$$\Rightarrow \quad \frac{f'(x)}{f(x)} = \ln\left(\frac{1+x}{1+x^2}\right)$$

$$\Rightarrow \quad \frac{f'(2)}{f(2)} = \ln\left(\frac{3}{5}\right) < 0$$

$\Rightarrow \quad f'(2) < 0 \;\; \therefore$ (c) is correct

and $\frac{f'(3)}{f(3)} = \ln\left(\frac{2}{5}\right) < \frac{f'(2)}{f(2)} \;\; \therefore$ (d) is not correct

Also $f'(x) = f(x)\ \ln\left(\frac{1+x}{1+x^2}\right) > 0,\ ,\ \forall x \in (0, 1)$

$\therefore$ f is an increasing function.

$\therefore \quad \frac{1}{2} < 1 \Rightarrow f\left(\frac{1}{2}\right) \le f(1)$

$\therefore$ (a) is not correct

and $\frac{1}{3} < \frac{2}{3} \Rightarrow f\left(\frac{1}{3}\right) \le f\left(\frac{2}{3}\right)$

$\therefore$ (b) is correct

Hence (b) and (c) are the correct options.

44. **(a, b)** $f(x) = a\cos^3(|x^3 - x|) + b|x|\sin(|x^3 + x|)$

(a) If $a = 0, b = 1$

$\Rightarrow f(x) = |x|\sin|x^3 + x|$

$= x\sin(x^3 + x), x \in R$

$\therefore$ f is differentiable every where

(b), (c) If $a = 1, b = 0 \Rightarrow f(x) = \cos^3(|x^3 - x|)$

$= \cos^3(x^3 - x)$

which is differentiable every where.

(d) when $a = 1, b = 1, f(x) = \cos(x^3 - x) + x\sin(x^3 + x)$

which is differentiable at $x = 1$

$\therefore$ Only a and b are the correct options.

45. **(a, d)** $\lim_{x\to 2}\frac{f(x)g(x)}{f'(x)g'(x)} = 1 \;\left[\frac{0}{0}\text{form}\right]$

$$\Rightarrow \lim_{x\to 2}\frac{f'(x)g(x)+f(x)g'(x)}{f''(x)g'(x)+f'(x)g''(x)} = 1$$

$$\Rightarrow \frac{g'(2)f(2)}{f''(2)g'(2)} = 1 \Rightarrow f(2) = f''(2)$$

$\therefore \;\; f(x) - f''(x) = 0$ for atleast one $x \in R$.

$\because$ Range of f(x) is $(0, \infty)$

$\therefore \quad f(x) > 0, \forall x \in R$

$\Rightarrow \quad f(2) > 0 \Rightarrow f''(2) > 0$

$\Rightarrow \quad$ f has a local minimum at $x = 2$

46. **(b, c)** $f(x) = [x^2 - 3]$ is discontinuous at all integral points in $\left[-\frac{1}{2}, 2\right]$

Which happens when $x = 1, \sqrt{2}, \sqrt{3}, 2$

$\therefore$ f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$

Also $g(x) = (|x| + |4x - 7|)f(x)$

Here f is not differentiable at $x = 1, \sqrt{2}, \sqrt{3} \in \left(-\frac{1}{2}, 2\right)$

and $|x| + |4x - 7|$ is not differentiable at O and $\frac{7}{4}$

But $f(x) = 0, \forall x \in \left[\sqrt{3}, 2\right]$

$\therefore \quad$ g(x) becomes differentiable at $x = \frac{7}{4}$

Hence g(x) is non-differentiable at four points i.e., $0, 1, \sqrt{2}, \sqrt{3}$

47. (a, c, d) $z = \dfrac{1}{a + ibt} = x + iy$

$$\Rightarrow \quad x + iy = \frac{a - ibt}{a^2 + b^2t^2}$$

$$\Rightarrow \quad x = \frac{a}{a^2 + b^2t^2},\ y = \frac{-bt}{a^2 + b^2t^2}$$

$$\Rightarrow \quad x^2 + y^2 = \frac{1}{a^2 + b^2t^2} = \frac{x}{a}$$

$$\Rightarrow \quad x^2 + y^2 - \frac{x}{a} = 0$$

$\therefore$ Locus of z is a circle with centre $\left(\dfrac{1}{2a}, 0\right)$ and radius $= \dfrac{1}{2|a|}$ irrespective of 'a' +ve or –ve

Also for b = 0, a ≠ 0, we get, y = 0

$\therefore$ locus is x-axis

and for a = 0, b ≠ 0 we get x = 0

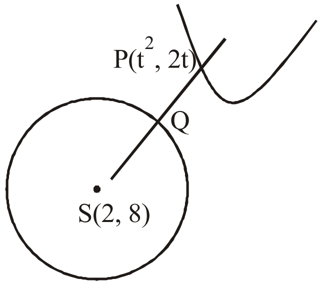
$\Rightarrow$ locus is y-axis.

$\therefore$ a, c, d are the correct options.

48. (a, c, d)

Let point P on parabola $y^2 = 4x$ be $(t^2, 2t)$

$\because$ PS is shortest distance, therefore PS should be the normal to parabola.



Equation of normal to $y^2 = 4x$ at P $(t^2, 2t)$ is

$y - 2t = -t(x - t^2)$

It passes through S(2, 8)

$\therefore$ $8 - 2t = -t(2 - t^2) \Rightarrow t^3 = 8$ or $t = 2$

$\therefore$ P(4, 4)

Also slope of tangent to circle at $Q = \dfrac{-1}{\text{Slope of PS}} = \dfrac{1}{2}$

Equation of normal at t = 2 is 2x + y = 12

Clearly x-intercept = 6

$SP = 2\sqrt{5}$ and SQ = r = 2

$\therefore$ Q divides SP in the ratio SQ : PQ

$= 2 : 2\left(\sqrt{5} - 1\right)$ or $\left(\sqrt{5} + 1\right) : 4$

Hence a, c , d are the correct options.

49. (b, c, d)

$ax + 2y = \lambda$

$3x - 2y = \mu$

For unique solution, $\dfrac{a}{3} \neq \dfrac{2}{-2} \Rightarrow a \neq -3$

$\therefore$ (b) is the correct option.

For infinite many solutions and a = – 3

$\dfrac{-3}{3} = \dfrac{2}{-2} = \dfrac{\lambda}{\mu} \Rightarrow \dfrac{\lambda}{\mu} = -1$ or $\lambda + \mu = 0$

$\therefore$ (c) is the correct option.

Also if $\lambda + \mu \neq 0$, then $\dfrac{-3}{3} = \dfrac{2}{-2} \neq \dfrac{\lambda}{\mu}$

$\Rightarrow$ system has no solution.

$\therefore$ (d) is the correct option.

50. (b, c) $\left|\hat{u} \times \vec{v}\right| = 1 \Rightarrow |v| \sin\theta = 1$ ...(i)

$\hat{w}.\left(\hat{u} \times \vec{v}\right) = 1 \Rightarrow |v| \sin\theta \cos\alpha = 1$ ...(ii)

where α is the angle between $\hat{w}$ and a vector ⊥ lat to $\vec{u}$ & $\vec{v}$.

From (i) and (ii) $\cos\alpha = 1 \Rightarrow \alpha = 0°$

$\Rightarrow$ $\hat{w}$ is perpendicular to the plane containing $\vec{u}$ & $\vec{v}$

$\Rightarrow$ $\hat{w}$ is perpendicular to $\vec{u}$

Clearly there can be infinite many choices for $\vec{v}$.

Also if $\hat{u}$ lies in xy plane i.e., $\hat{u} = u_1\hat{i} + u_2\hat{j}$ then

$\hat{w}.\vec{u} = 0$

$\Rightarrow u_1 + u_2 = 0 \Rightarrow |u_1| = |u_2|$

Also if $\hat{u}$ lies in xz plane, i.e., $\hat{u} = u_1\hat{i} + u_3\hat{k}$ then

$\hat{w}.\vec{u} = 0 \Rightarrow u_1 + 2u_3 = 0 \Rightarrow |u_1| = 2|u_3|$

Hence (b) and (c) are the correct options.

For (Qs. 51-52)

(X, Y) = {(6, 0), (4, 1), (3, 3), (2, 2), (4, 4), (0, 6)}

51. (b) $P(X > Y) = P(T_1$ wins 2 games or T_1 win one game other is a draw)

$$= \frac{1}{2} \times \frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{2}\right) = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$$

52. (c) $P(X = Y) = P(T_1$ wins 1 game loses other game or both the games draw)

$$= \left(\frac{1}{2} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{2}\right) + \frac{1}{6} \times \frac{1}{6} = \frac{1}{3} + \frac{1}{36} = \frac{13}{36}$$

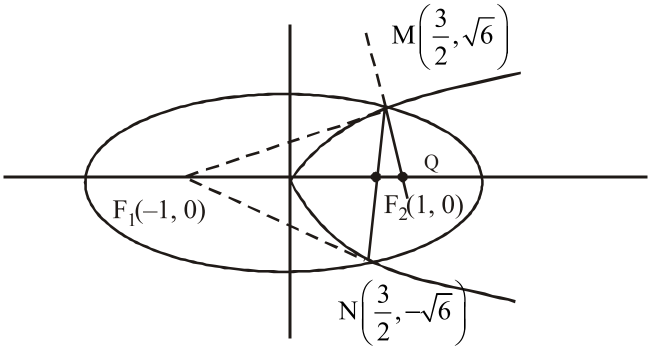
For (Qs. 53-54)

For ellipse $\frac{x^2}{9}+\frac{y^2}{8}=1$, $e=\sqrt{1-\frac{8}{9}}=\frac{1}{3}$

$\therefore$ $F_1(-1, 0)$ and $F_2(1, 0)$

Parabola with vertex at (0, 0) and focus at $F_2(1, 0)$ is $y^2 = 4x$.

Intersection points of ellipse and parabola are $M\left(\frac{3}{2},\sqrt{6}\right)$ and $N\left(\frac{3}{2},-\sqrt{6}\right)$



53. (a) For orthocentre of ΔF_1MN, clearly one altitude is x-axis i.e. y = 0 and altitude from M to F_1N is $y-\sqrt{6}=\frac{5}{2\sqrt{6}}\left(x-\frac{3}{2}\right)$

Putting y = 0 in above equation, we get

$$x=-\frac{9}{10}$$

$\therefore$ Orthocentre $\left(-\frac{9}{10},0\right)$

54. (c) Tangents to ellipse at M and N are

$$\frac{x}{6}+\frac{y\sqrt{6}}{8}=1 \text{ and } \frac{x}{6}-\frac{y\sqrt{6}}{8}=1$$

Their intersection point is R (6, 0)

Also normal to parabola at $M\left(\frac{3}{2},\sqrt{6}\right)$ is

$$y-\sqrt{6}=-\frac{\sqrt{6}}{2}\left(x-\frac{3}{2}\right)$$

Its intersection with x-axis is $Q\left(\frac{7}{2},0\right)$

Now ar $(\Delta MQR)=\frac{1}{2}\times\frac{5}{2}\times\sqrt{6}=\frac{5\sqrt{6}}{4}$

Also area $(MF_1NF_2) = 2 \times \text{Ar}(F_1MF_2)$

$$=2\times\frac{1}{2}\times 2\times\sqrt{6}=2\sqrt{6}$$

$$\therefore \quad \frac{ar(\Delta MQR)}{ar(MF_1NF_2)}=\frac{5\sqrt{6}}{4\times 2\sqrt{6}}=5:8$$

JEE ADVANCED 2017

1. The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of three sections.
2. **Section I** contains 7 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONE OR MORE THAN ONE** is/are correct.
3. **Section II** contains 5 questions. The answer to each of the questions is a single-digit integer ranging from 0 to 9 (both inclusive).
4. **Section III** contains 6 questions of Matching type, contains two tables each having 3 columns and 4 rows. Each question has four choices (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.

PAPER - 1

PHYSICS

SECTION - I

This section contains **7** questions. Each question has 4 options (A), (B), (C) and (D). **ONE or MORE THAN ONE** of these four options is (are) correct.

1. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules.
Which of the following options is/are true?

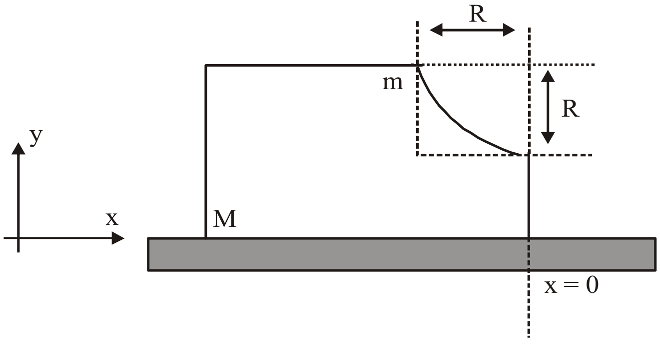
(A) The pressure difference between the leading and trailing faces of the plate is proportional to uv

(B) The resistive force experienced by the plate is proportional to v

(C) The plate will continue to move with constant non-zero acceleration, at all times

(D) At a later time the external force F balances the resistive force

2. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down.
When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct?



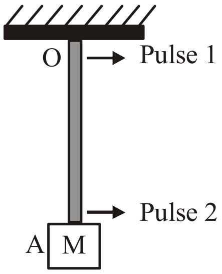
(A) The position of the point mass m is: $x = -\sqrt{2}\dfrac{mR}{M+m}$

(B) The velocity of the point mass m is: $v = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$

(C) The x component of displacement of the center of mass of the block M is: $-\dfrac{mR}{M+m}$

(D) The velocity of the block M is: $V = -\dfrac{m}{M}\sqrt{2gR}$

3. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct?



(A) The time $T_{AO} = T_{OA}$

(B) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope

(C) The wavelength of Pulse 1 becomes longer when it reaches point A

(D) The velocity of any pulse along the rope is independent of its frequency and wavelength

4. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be $T_0 = 300$ K. For $T_0 = 300$ K, the value of $\sigma T_0^4 = 460$ Wm^{-2} (where σ is the Stefan-Boltzmann constant). Which of the following options is/are correct?

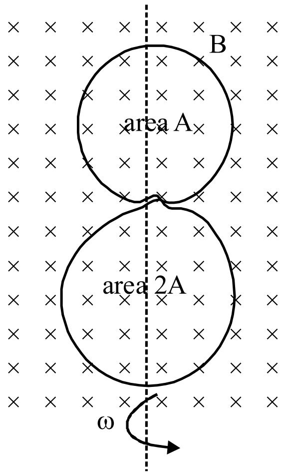
(A) The amount of energy radiated by the body in 1 second is close to 60 joules

(B) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time

(C) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

(D) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

5. A circular insulated copper wire loop is twisted to form two loops of area A and 2A as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(A) The emf induced in the loop is proportional to the sum of the areas of the two loops

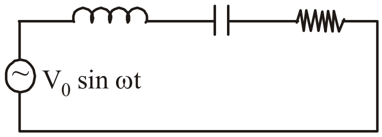
(B) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

(C) The net emf induced due to both the loops is proportional to cos ωt

(D) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

6. In the circuit shown, L = 1 μH, C = 1 μF and R = 1 kΩ. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?

L = 1μH C = 1μF R = 1kΩ



(A) The current will be in phase with the voltage if $\omega = 10^4$ $rad.s^{-1}$

(B) The frequency at which the current will be in phase with the voltage is independent of R

(C) At ω ~ 0 the current flowing through the circuit becomes nearly zero

(D) At $\omega >> 10^6$ rad. s^{-1}, the circuit behaves like a capacitor

7. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$.
Which of the following options is/are correct?

(A) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

(B) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(C) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

(D) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is

$$i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2}-1} - \cos A\right]$$

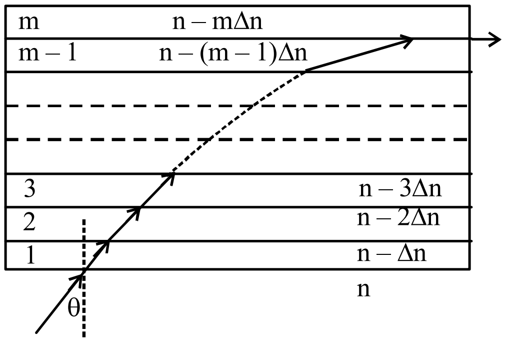
SECTION - II

This section contains **5** questions. The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

8. A drop of liquid of radius $R = 10^{-2}$ m having surface tension $S = \frac{0.1}{4\pi} Nm^{-1}$ divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$ then the value of α is

9. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.25$, then the smallest possible n_f is

10. A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle θ = 30°. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m?



11. A stationary source emits sound of frequency $f_0 = 492$ Hz. The sound is reflected by a large car approaching the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original.
What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received).

12. ^{131}I is an isotope of Iodine that B decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from person's body, and gives an activity of 115 Bq. The total volume of blood in the person¡¦s body, in liters is approximately
(you may use $e^x \approx 1 + x$ for $|x| << 1$ and $\ln 2 \approx 0.7$).

SECTION - III

This section contains **6** questions of **MATCHING TYPE**, contains two tables each having 3 columns and 4 rows. Based on each table, there are three questions. Each question has four options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.

Answer (Qs. 13-15) : By appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin (x = 0, y = 0, z = 0) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

	Column 1	**Column 2**	**Column 3**
(I)	Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0\hat{z}$	(P) $\vec{B} = -B_0\hat{x}$
(II)	Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(Q) $\vec{B} = B_0\hat{x}$
(III)	Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(R) $\vec{B} = B_0\hat{y}$
(IV)	Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(S) $\vec{B} = B_0\hat{z}$

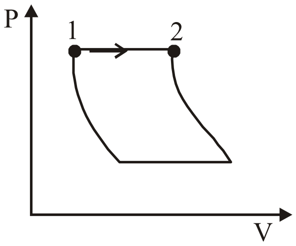
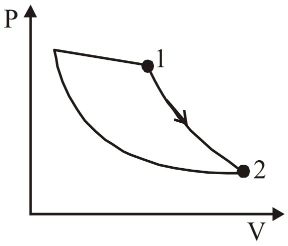
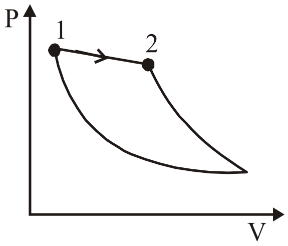
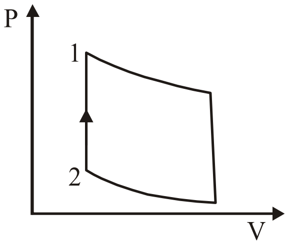
13. In which case will the particle move in a straight line with constant velocity?
(A) (III) (ii) (R) (B) (IV) (i) (S) (C) (III) (iii) (P) (D) (II) (iii) (S)

14. In which case will the particle describe a helical path with axis along the positive z direction?
(A) (IV) (i) (S) (B) (II) (ii) (R) (C) (III) (iii) (P) (D) (IV) (ii) (R)

15. In which case would the particle move in a straight line along the negative direction of y-axis (i.e., move along $-\hat{y}$)?
(A) (II) (iii) (Q) (B) (III) (ii) (R) (C) (IV) (ii) (S) (D) (III) (ii) (P)

Answer (Qs. 16-18) : By appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding P–V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here Y is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

Column 1	Column 2	Column 3
(I) $W_{1\rightarrow2}=\frac{1}{\gamma-1}(P_2V_2-P_1V_1)$	(i) Isothermal	(P) 
(II) $W_{1\rightarrow2}=-PV_2+PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1\rightarrow2}=0$	(iii) Isobaric	(R) 
(IV) $W_{1\rightarrow2}=-nRT\ \ln\left(\frac{V_2}{V_1}\right)$	(iv) Adiabatic	(S) 

16. Which of the following options is the only correct representation of a process in which $\Delta U=\Delta Q-P\Delta V$?

(A) (II) (iv) (R) (B) (III) (iii) (P) (C) (II) (iii) (S) (D) (II) (iii) (P)

17. Which one of the following options is the correct combination?

(A) (IV) (ii) (S) (B) (III) (ii) (S) (C) (II) (iv) (P) (D) (II) (iv) (R)

18. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

(A) (I) (ii) (Q) (B) (IV) (ii) (R) (C) (III) (iv) (R) (D) (I) (iv) (Q)

CHEMISTRY

SECTION - I

This section contains **7** questions. Each question has 4 options (A), (B), (C) and (D). **ONE or MORE THAN ONE** of these four options is (are) correct.

19. An ideal gas is expanded from (p_1, V_1, T_1) to (p_2, V_2, T_2) under different conditions. The correct statement(s) among the following is(are)

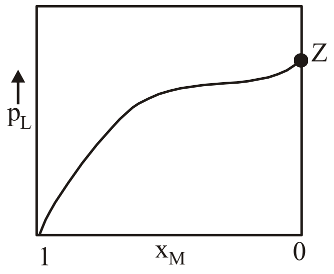
(A) The work done on the gas is maximum when it is compressed irreversibly from (p_2, V_2) to (p_1, V_1) against constant pressure p_1

(B) If the expansion is carried out freely, it is simultaneously both isothermal as well as adiabatic

(C) The work done by the gas is less when it is expanded reversibly from V_1 to V_2 under adiabatic conditions as compared to that when expanded reversibly from V_1 to V_2 under isothermal conditions

(D) The change in internal energy of the gas is (i) zero, if it is expanded reversibly with $T_1 = T_2$, and (ii) positive, if it is expanded reversibly under adiabatic conditions with $T_1 \neq T_2$

20. For a solution formed by mixing liquids L and M, the vapour pressure of L plotted against the mole fraction of M in solution is shown in the following figure. Here x_L and x_M represent mole fractions of L and M, respectively, in the solution. The correct statement(s) applicable to this system is (are)



(A) The point Z represents vapour pressure of pure liquid M and Raoult's law is obeyed from $x_L = 0$ to $x_L = 1$

(B) The point Z represents vapour pressure of pure liquid L and Raoult's law is obeyed when $x_L \to 1$

(C) The point Z represents vapour pressure of pure liquid M and Raoult's law is obeyed when $x_L \to 0$

(D) Attractive intermolecular interactions between L-L in pure liquid L and M-M in pure liquid M are stronger than those between L-M when mixed in solution.

21. The correct statement(s) about the oxoacids, $HClO_4$ and HClO, is(are)

(A) The central atom in both $HClO_4$ and HClO is sp^3 hybridized

(B) $HClO_4$ is more acidic than HClO because of the resonance stabilization of its anion

(C) $HClO_4$ is formed in the reaction between Cl_2 and H_2O

(D) The conjugate base of $HClO_4$ is weaker base than H_2O

22. The colour of the X_2 molecules of group 17 elements changes gradually from yellow to violet down the group. This is due to

(A) The physical state of X_2 at room temperature changes from gas to solid down the group

(B) Decrease in ionization energy down the group

(C) Decrease in π^* - σ^* gap down the group

(D) Decrease in HOMO-LUMO gap down the group

23. Addition of excess aqueous ammonia to a pink coloured aqueous solution of $MCl_2.6H_2O(X)$ and NH_4Cl gives an octahedral complex Y in the presence of air. In aqueous solution, complex Y behaves as 1:3 electrolyte. The reaction of X with excess HCl at room temperature results in the formation of a blue coloured complex Z. The calculated spin only magnetic moment of X and Z is 3.87 B.M., whereas it is zero for complex Y. Among the following options, which statement(s) is(are) correct?

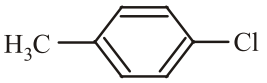
(A) Addition of silver nitrate to Y gives only two equivalents of silver chloride

(B) The hybridization of the central metal ion in Y is d^2sp^3

(C) Z is a tetrahedral complex

(D) When X and Z are in equilibrium at 0°C, the colour of the solution is pink

24. The IUPAC name(s) of the following compound is(are)



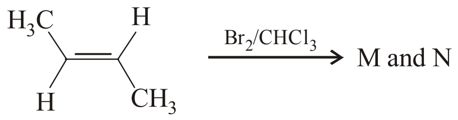
(A) 1-chloro-4-methylbenzene

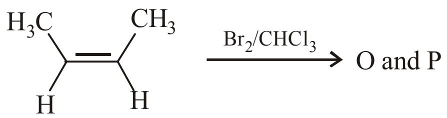
(B) 4-chlorotoluene

(C) 4-methylchlorobenzene

(D) 1-methyl-4-chlorobenzene

25. The correct statement(s) for the following addition reactions is(are)

(i)


(ii)


(A) O and P are identical molecules

(B) (M and O) and (N and P) are two pairs of diastereomers

(C) (M and O) and (N and P) are two pairs of enantiomers

(D) Bromination proceeds through *trans*-addition in both the reactions

SECTION - II

This section contains **5** questions. The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

26. A crystalline solid of a pure substance has a face-centred cubic structure with a cell edge of 400 pm. If the density of the substance in the crystal is 8 g cm^{-3}, then the number of atoms present in 256 g of the crystal is N × 10^{24}. The value of N is

27. The conductance of a 0.0015 M aqueous solution of a weak monobasic acid was determined by using a conductivity cell consisting of platinized Pt electrodes. The distance between the electrodes is 120 cm with an area of cross section of 1 cm^2. The conductance of this solution was found to be 5×10^{-7}S. The pH of the solution is 4. The value of limiting molar conductivity $\left(\Lambda_m^o\right)$ of this weak monobasic acid in aqueous solution is $Z \times 10^2$ S cm^{-1} mol^{-1}. The value of Z is

28. The sum of the number of lone pairs of electrons on each central atom in the following species is

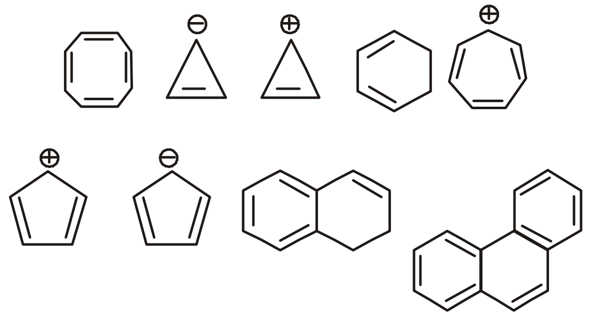
$[TeBr_6]^{2-}$, $[BrF_2]^+$, SNF_3 and $[XeF_3]^-$

(Atomic numbers: N = 7, F = 9, S = 16, Br = 35, Te = 52, Xe = 54)

29. Among H_2, He_2^+, Li_2, Be_2, B_2, C_2, N_2, O_2^- and F_2, the number of diamagnetic species is

(Atomic numbers: H = 1, He = 2, Li = 3, Be = 4, B = 5, C = 6, N = 7, O = 8, F = 9)

30. Among the following, the number of aromatic compound(s) is

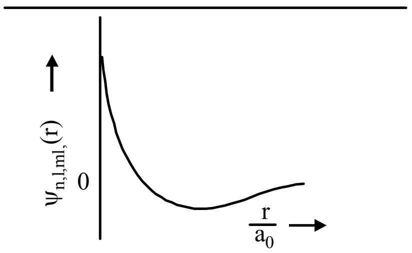


SECTION - III

This section contains **6** questions of **MATCHING TYPE**, contains two tables each having 3 columns and 4 rows. Based on each table, there are three questions. Each question has four options (A), (B), (C) and (D) **ONLY ONE** of these four option is correct.

(Qs. 31-33) : By appropriately matching the information given in the three columns of the following table.

The wave function, ψ_{n,l,m_l} is a mathematical function whose value depends upon spherical polar coordinates (r, θ, ϕ) of the electron and characterized by the quantum numbers n, l and m_l. Here r is distance from nucleus, θ is colatitude and ϕ is azimuth. In the mathematical functions given in the table, Z is atomic number and a_0 is Bohr radius.

Column 1	Column 2	Column 3
(i) 1s orbital	(i) $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\left(\frac{Zr}{a_0}\right)}$	(P) 
(ii) 2s orbital	(ii) One radial node	(Q) Probability density at nucleus $\propto \frac{1}{a_0^3}$
(iii) $2p_z$ orbital	(iii) $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{5}{2}} re^{-\left(\frac{Zr}{2a_0}\right)} \cos\theta$	(R) Probability density is maximum at nucleus
(iv) $3d_{z^2}$ orbital	(iv) xy-plane is a nodal plane	(S) Energy needed to excite electron from n = 2 state to n = 4 state is $\frac{27}{32}$ times the energy needed to excite electron from n = 2 state to n = 6 state

31. For the given orbital in Column 1, the only CORRECT combination for any hydrogen-like species is

(A) (I) (ii) (S)

(B) (IV) (iv) (R)

(C) (II) (ii) (P)

(D) (III) (iii) (P)

32. For hydrogen atom, the only CORRECT combination is
(A) (I) (i) (S)
(B) (II) (i) (Q)
(C) (I) (i) (P)
(D) (I) (iv) (R)

33. For He^+ ion, the only INCORRECT combination is
(A) (I) (i) (R)
(B) (II) (ii) (Q)
(C) (I) (iii) (R)
(D) (I) (i) (S)

(Qs. 34-36) : By appropriately matching the information given in the three columns of the following table.

Columns 1, 2 and 3 contain starting materials, reaction conditions, and type of reactions, respectively.

	Column 1		Column 2		Column 3
(I)	Toluene	(i)	$NaOH/Br_2$	(P)	Condensation
(II)	Acetophenone	(ii)	Br_2/hv	(Q)	Carboxylation
(III)	Benzaldehyde	(iii)	$(CH_3CO)_2O/$ CH_3COOK	(R)	Substitution
(IV)	Phenol	(iv)	$NaOH/CO_2$	(S)	Haloform

34. For the synthesis of benzoic acid, the only CORRECT combination is
(A) (II) (i) (S)
(B) (IV) (ii) (P)
(C) (I) (iv) (Q)
(D) (III) (iv) (R)

35. The only CORRECT combination that gives two different carboxylic acids is
(A) (II) (iv) (R)
(B) (IV) (iii) (Q)
(C) (III) (iii) (P)
(D) (I) (i) (S)

36. The only CORRECT combination in which the reaction proceeds through radical mechanism is
(A) (III) (ii) (P)
(B) (IV) (i) (Q)
(C) (II) (iii) (R)
(D) (I) (ii) (R)

MATHEMATICS

SECTION - I

This section contains **7** questions. Each question has 4 options (A), (B), (C) and (D). **ONE or MORE THAN ONE** of these four options is (are) correct.

37. If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following cannot be sides of a right angled triangle?
(A) a, 4, 1
(B) a, 4, 2
(C) 2a, 8, 1
(D) 2a, 4, 1

38. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k ?
(A) $p = -2, h = 2, k = -4$
(B) $p = -1, h = 1, k = -3$
(C) $p = 2, h = 3, k = -4$
(D) $p = 5, h = 4, k = -3$

39. Let [x] be the greatest integer less than or equals to x. Then, at which of the following point(s) the function $f(x) = x \cos(\pi(x + [x]))$ is discontinuous?
(A) $x = -1$
(B) $x = 0$
(C) $x = 1$
(D) $x = 2$

40. Let $f : R \to (0,1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1)?
(A) $x^9 - f(x)$
(B) $x - \int_0^{\frac{\pi}{2} - x} f(t) \cos t \, dt$
(C) $e^x - \int_0^x f(t) \sin t \, dt$
(D) $f(x) + \int_0^{\frac{\pi}{2}} f(t) \sin t \, dt$

41. Which of the following is(are) not the square of a 3 × 3 matrix with real entries?
(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

42. Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az + b}{z + 1}\right) = y$, then which of the following is(are) possible value(s) of x ?
(A) $-1 + \sqrt{1 - y^2}$
(B) $-1 - \sqrt{1 - y^2}$
(C) $1 + \sqrt{1 + y^2}$
(D) $1 - \sqrt{1 + y^2}$

43. Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{2}{5}$. Then
(A) $P(Y) = \frac{4}{15}$
(B) $P(X' \mid Y) = \frac{1}{2}$
(C) $P(X \cap Y) = \frac{1}{5}$
(D) $P(X \cup Y) = \frac{2}{5}$

SECTION - II

This section contains **5** questions. The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.

44. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points?

45. Let $f : R \to R$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$.

If $g(x) = \int_x^{\frac{\pi}{2}} [f'(t)\text{cosec } t - \cot\ t \text{cosec } t\ f(t)]\,dt$ for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim\limits_{x \to 0} g(x) =$

46. For a real number α, if the system $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ of linear equations, has infinitely many solutions, then $1 + \alpha + a^2 =$

47. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$

48. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

SECTION - III

This section contains 6 questions of matching type. This section contains two tables each having 3 columns and 4 rows. Based on each table, there are three questions. Each question has four options (A), (B), (C) and (D) only **ONE OF** these four option is correct.

(Qs. 49-51) : By appropriately matching the information given in the three columns of the following table. Column 1, 2, and 3 contain conics, equations of tangents to the conics and points of contact, respectively.

Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2 + 1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2 - 1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2 + 1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

49. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (– 1, 1), then which of the following options is the only correct combination for obtaining its equation?

(A) (I) (i) (P) (B) (I) (ii) (Q) (C) (II) (ii) (Q) (D) (III) (i) (P)

50. If a tangent to a suitable conic (column 1) is found to be $y = x + 8$ and its point of contact is (8, 16), then which of the following options is the only correct combination?

(A) (I) (ii) (Q) (B) (II) (iv) (R) (C) (III) (i) (P) (D) (III) (ii) (Q)

51. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only correct combination?

(A) (IV) (iii) (S) (B) (IV) (iv) (S) (C) (II) (iii) (R) (D) (II) (iv) (R)

(Qs. 52-54) : By appropriately matching the information given in the three columns of the following table.

Let $f(x) = x + \log_e x - x \log_e x, x \in (0, \infty)$

Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.

Column 2 contains information about the limiting behaviour of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.

Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

	Column 1		**Column 2**		**Column 3**
(I)	$f(x) = 0$ for some $x \in (1, e^2)$	(i)	$\lim_{x \to \infty} f(x) = 0$	(P)	f is increasing in (0, 1)
(II)	$f'(x) = 0$ for some $x \in (1, e)$	(ii)	$\lim_{x \to \infty} f(x) = -\infty$	(Q)	f is increasing in (e, e^2)
(III)	$f'(x) = 0$ for some $x \in (0, 1)$	(iii)	$\lim_{x \to \infty} f'(x) = -\infty$	(R)	f' is increasing in (0, 1)
(IV)	$f''(x) = 0$ for some $x \in (1, e)$	(iv)	$\lim_{x \to \infty} f''(x) = 0$	(S)	f' is decreasing in (e, e^2)

52. Which of the following options is the only correct combination?

(A) (I) (i) (P) (B) (II) (ii) (Q) (C) (III) (iii) (R) (D) (IV) (iv) (S)

53. Which of the following options is the only correct combination?

(A) (I) (ii) (R) (B) (II) (iii) (S) (C) (III) (iv) (P) (D) (IV) (i) (S)

54. Which of the following options is the only incorrect combination?

(A) (I) (iii) (P) (B) (II) (iv) (Q) (C) (III) (i) (R) (D) (II) (iii) (P)

PAPER - 2

1. The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of three sections.
2. **Section I** contains 7 questions. Each question has four options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
3. **Section II** contains 7 multiple choice questions. Each question has four choice (a), (b), (c) and (d) out of which **ONE OR MORE THAN ONE** is/are correct.
4. **Section III** contains 2 paragraphs each describing theory, experiment and data etc. 4 questions relate to two paragraphs with two questions on each paragraph. Each question pertaining to a particular passage should have **ONLY ONE** correct answer among the four given choices (a), (b), (c) and (d).

PHYSICS

SECTION - I

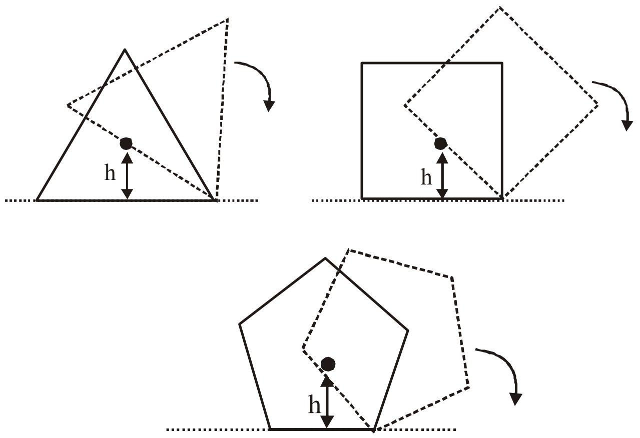
This section contains **7** questions. Each question has 4 options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

1. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to

(A) R (B) R^3

(C) $\frac{1}{R}$ (D) $R^{2/3}$

2. Consider regular polygons with number of sides n = 3, 4, 5.... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as



(A) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$ (B) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

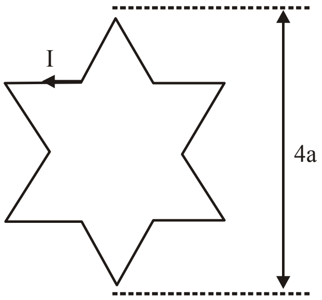
(C) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$ (D) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

3. A photoelectric material having work–function ϕ_0 is illuminated with light of wavelength $\lambda\left(\lambda < \frac{hc}{\phi_0}\right)$. The fastest photoelectron has a de–Broglie wavelength λ_d. A change in wavelength of the incident light by Δλ result in a change $\Delta\lambda_d$ in λ_d. Then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to

(A) λ_d/λ (B) λ_d^2/λ^2

(C) λ_d^3/λ (D) λ_d^3/λ^2

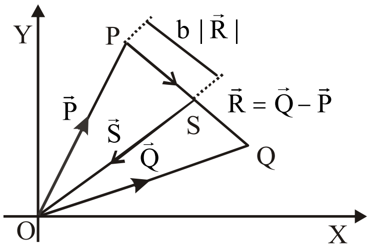
4. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is 4a. The magnitude of the magnetic field at the center of the loop is



(A) $\frac{\mu_0 l}{4\pi a}6[\sqrt{3}-1]$ (B) $\frac{\mu_0 l}{4\pi a}6[\sqrt{3}+1]$

(C) $\frac{\mu_0 l}{4\pi a}3[\sqrt{3}-1]$ (D) $\frac{\mu_0 l}{4\pi a}3[2-\sqrt{3}]$

5. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b\,|\vec{R}|$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is



(A) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$

(B) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

(C) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

(D) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$

6. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2\ \text{km s}^{-1}$. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun–Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet)

(A) $v_s = 22\ \text{km s}^{-1}$
(B) $v_s = 42\ \text{km s}^{-1}$
(C) $v_s = 62\ \text{km s}^{-1}$
(D) $v_s = 72\ \text{km s}^{-1}$

7. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be $L = 20$ meters. Take the acceleration due to gravity $g = 10\ \text{ms}^{-2}$ and the velocity of sound is $300\ \text{ms}^{-1}$. Then the fractional error in the measurement, $\delta L/L$, is closest to

(A) 0.2%
(B) 1%
(C) 3%
(D) 5%

SECTION - II

This section contains **7** questions. Each question has 4 options (A), (B), (C) and (D). **ONE or MORE THAN ONE** of these four options is (are) correct.

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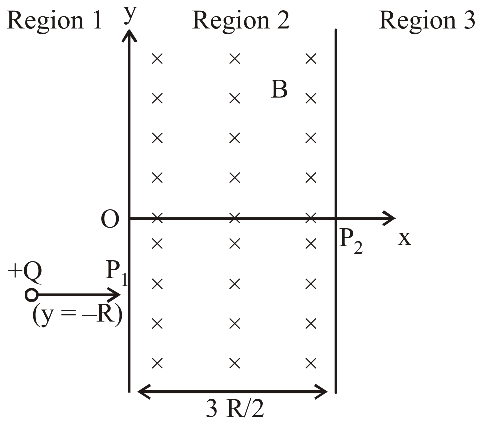
8. A uniform magnetic field B exists in the region between $x = 0$ and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge +Q and momentum p directed along x–axis enters region 2 from region 1 at point P_1 ($y = -R$). Which of the following option(s) is/are correct?



(A) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re–enter region 1

(B) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x–axis

(C) When the particle re–enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from y–axis is $p/\sqrt{2}$

(D) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re–entry into region 1 is inversely proportional to the mass of the particle

9. The instantaneous voltages at three terminals marked X, Y and Z are given by

$V_x = V_0 \sin \omega t$,

$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$ and

$V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

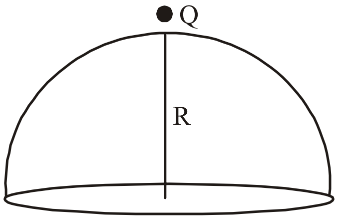
(A) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

(B) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(C) $V_{XY}^{rms} = V_0$

(D) Independent of the choice of the two terminals

10. A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



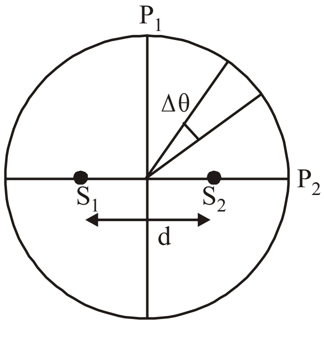
(A) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

(B) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$

(C) The component of the electric field normal to the flat surface is constant over the surface

(D) The circumference of the flat surface is an equipotential

11. Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the centre of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?



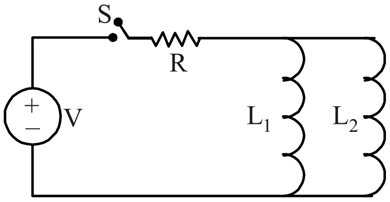
(A) A dark spot will be formed at the point P_2

(B) At P_2 the order of the fringe will be maximum

(C) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

(D) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant

12. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At t = 0, the switch is closed and current begins to flow. Which of the following options is/are correct?



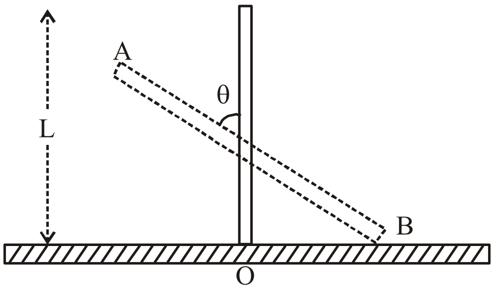
(A) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

(B) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(C) The ratio of the currents through L_1 and L_2 is fixed at all times (t > 0)

(D) At t = 0, the current through the resistance R is $\frac{V}{R}$

13. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct?



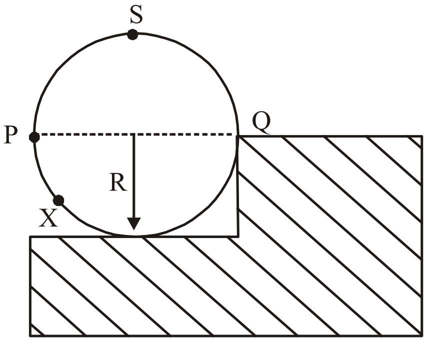
(A) The midpoint of the bar will fall vertically downward

(B) The trajectory of the point A is a parabola

(C) Instantaneous torque about the point in contact with the floor is proportional to sinθ

(D) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to (1 – cosθ)

14. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?



(A) If the force is applied at point P tangentially then decreases continuously as the wheel climbs

(B) If the force is applied normal to the circumference at point X then τ is constant

(C) If the force is applied normal to the circumference at point P then τ is zero

(D) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

SECTION - III

This section contains 2 paragraphs, each describing theory, experiments, data etc. four questions related to the two paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (A), (B), (C) and (D).

PARAGRAPH 1

Consider a simple RC circuit as shown in Figure 1.

Process 1: In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time T >> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2: In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time T >> RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in Figure 2.

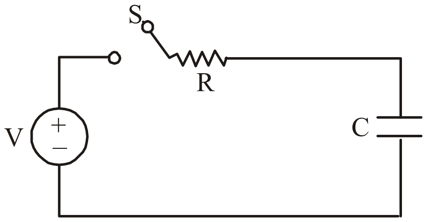


Figure 1

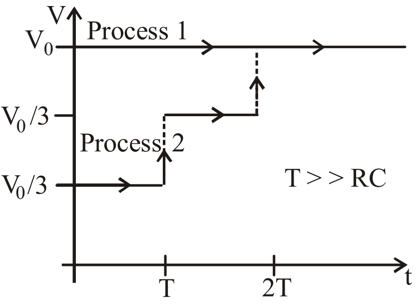


Figure 2

15. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by :

(A) $E_C = E_D$　(B) $E_C = E_D \text{ In } 2$

(C) $E_C = \frac{1}{2}E_D$　(D) $E_C = 2E_D$

16. In Process 2, total energy dissipated across the resistance E_D is :

(A) $E_D = \frac{1}{2}CV_0^2$　(B) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$

(C) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$　(D) $E_D = 3CV_0^2$

PARAGRAPH 2

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

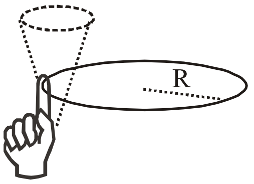


Figure 1

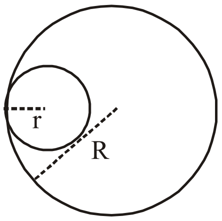


Figure 2

17. The total kinetic energy of the ring is

(A) $M\omega_0^2R^2$　(B) $\frac{1}{2}M\omega_0^2(R-r)^2$

(C) $M\omega_0^2(R-r)^2$　(D) $\frac{3}{2}M\omega_0^2(R-r)^2$

18. The minimum value of ω_0 below which the ring will drop down is

(A) $\sqrt{\frac{g}{\mu(R-r)}}$ (B) $\sqrt{\frac{2g}{\mu(R-r)}}$

(C) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (D) $\sqrt{\frac{g}{2\mu(R-r)}}$

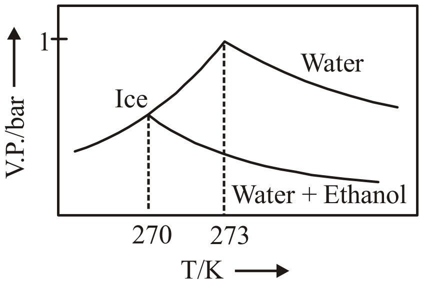
CHEMISTRY

SECTION - I

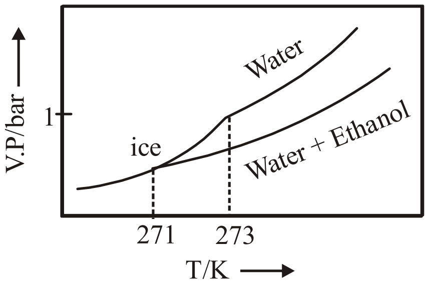
This section contains **7** questions. Each question has 4 options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

19. Pure water freezes at 273 K and 1 bar. The addition of 34.5 g of ethanol to 500 g of water changes the freezing point of the solution. Use the freezing point depression constant of water as 2 K kg mol^{-1}. The figures shown below represent plots of vapour pressure (V.P.) versus temperature (T). [molecular weight of ethanol is 46 g mol^{-1}] Among the following, the option representing change in the freezing point is

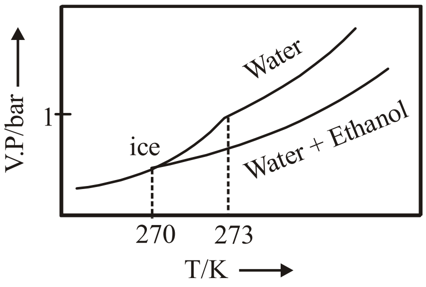
(A)



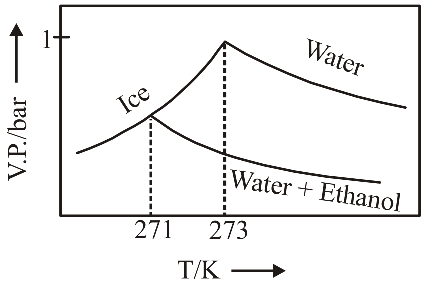
(B)



(C)



(D)



20. For the following cell,

Zn(s) | $ZnSO_4$(aq) || $CuSO_4$(aq) | Cu(s)

when the concentration of Zn^{2+} is 10 times the concentration of Cu^{2+}, the expression for ΔG (in J mol^{-1}) is [F is Faraday constant; R is gas constant; T is temperature; E° (cell) = 1.1 V]

(A) 1.1F (B) 2.303RT – 2.2F

(C) 2.303RT + 1.1F (D) –2.2F

21. The standard state Gibbs free energies of formation of C(graphite) and C(diamond) at T = 298 K are

$\Delta_f G°$ [C(graphite)] = 0 kJ mol^{-1}

$\Delta_f G°$ [C(diamond)] = 2.9 kJ mol^{-1}

The standard state means that the pressure should be 1 bar, and substance should be pure at a given temperature. The conversion of graphite [C(graphite)] to diamond [C(diamond)] reduces its volume by 2×10^{-6} m^3 mol^{-1}. If C(graphite) is converted to C(diamond) isothermally at T = 298 K, the pressure at which C(graphite) is in equilibrium with C(diamond), is

[Useful information : 1 J = 1 kg m^{2}s^{-2}; 1 Pa = 1 kg m^{-1} s^{-2}; 1 bar = 10^5 Pa]

(A) 14501 bar (B) 58001 bar

(C) 1450 bar (D) 29001 bar

22. Which of the following combination will produce H_2 gas?

(A) Fe Metal and conc. HNO_3

(B) Cu metal and conc. HNO_3

(C) Zn metal and NaOH(aq)

(D) Au metal and NaCN(aq) in the presence of air

23. The order of the oxidation state of the phosphorus atom in H_3PO_2, H_3PO_4, H_3PO_3 and $H_4P_2O_6$ is

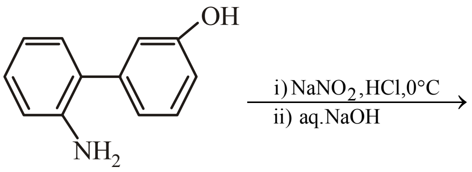
(A) $H_3PO_3 > H_3PO_2 > H_3PO_4 > H_4P_2O_6$

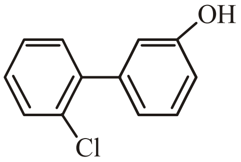
(B) $H_3PO_4 > H_3PO_2 > H_3PO_3 > H_4P_2O_6$

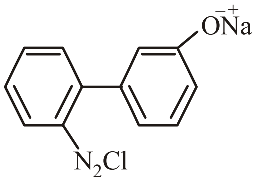
(C) $H_3PO_4 > H_4P_2O_6 > H_3PO_3 > H_3PO_2$

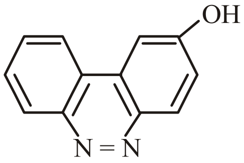
(D) $H_3PO_2 > H_3PO_3 > H_4P_2O_6 > H_3PO_4$

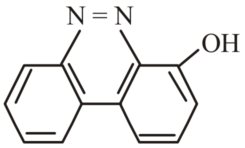
24. The major product of the following reaction is



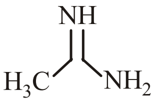
(A) 

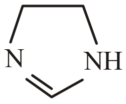
(B) 

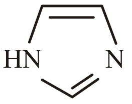
(C) 

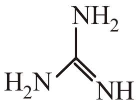
(D) 

25. The order of basicity among the following compounds is


I


II


III


IV

(A) II > I > IV > III (B) IV > II > III > I

(C) IV > I > II > III (D) I > IV > III > II

SECTION - II

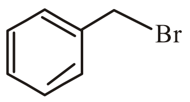
This section contains **7** questions. Each question has 4 options (A), (B), (C) and (D). **ONE or MORE THAN ONE** of these four options is (are) correct.

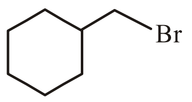
26. The correct statement(s) about surface properties is(are)
- (A) Adsorption is accompanied by decrease in enthalpy and decrease in entropy of the system
- (B) The critical temperatures of ethane and nitrogen are 563 K and 126 K, respectively. The adsorption of ethane will be more than that of nitrogen on same amount of activated charcoal at a given temperature
- (C) Cloud is an emulsion type of colloid in which liquid is dispersed phase and gas is dispersion medium
- (D) Brownian motion of colloidal particles does not depend on the size of the particles but depends on viscosity of the solution

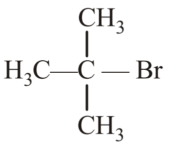
27. For a reaction taking place in a container in equilibrium with its surroundings, the effect of temperature on its equilibrium constant K in terms of change in entropy is described by
- (A) With increase in temperature, the value of K for exothermic reaction decreases because the entropy change of the system is positive
- (B) With increase in temperature, the value of K for endothermic reaction increases because unfavourable change in entropy of the surroundings decreases
- (C) With increase in temperature, the value of K for endothermic reaction increases because the entropy change of the system is negative
- (D) With increase in temperature, the value of K for exothermic reaction decreases because favourable change in entropy of the surroundings decreases

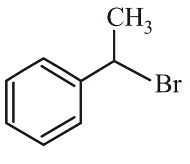
28. In a bimolecular reaction, the steric factor P was experimentally determined to be 4.5. The correct option(s) among the following is(are)
- (A) The activation energy of the reaction is unaffected by the value of the steric factor
- (B) Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation
- (C) Since P = 4.5, the reaction will not proceed unless an effective catalyst is used
- (D) The value of frequency factor predicted by Arrhenius equation is higher than that determined experimentally

29. For the following compounds, the correct statement(s) with respect to nucleophilic substitution reaction is(are)


(I)


(II)


(III)


(IV)

- (A) I and III follow S_N1 mechanism
- (B) I and II follow S_N2 mechanism
- (C) Compound IV undergoes inversion of configuration
- (D) The order of reactivity for I, III and IV is : IV > I > III

30. Among the following, the correct statement(s) is(are)
- (A) $Al(CH_3)_3$ has the three–centre two–electron bonds in its dimeric structure
- (B) BH_3 has the three–centre two–electron bonds in its dimeric structure
- (C) $AlCl_3$ has the three–centre two–electron bonds in its dimeric structure
- (D) The Lewis acidity of BCl_3 is greater than that of $AlCl_3$

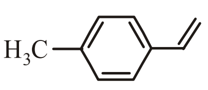
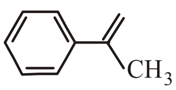
31. The option(s) with only amphoteric oxides is(are)
- (A) Cr_2O_3, BeO, SnO, SnO_2
- (B) Cr_2O_3, CrO, SnO, PbO
- (C) NO, B_2O_3, PbO, SnO_2
- (D) ZnO, Al_2O_3, PbO, PbO_2

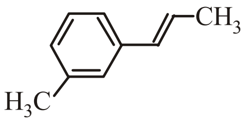
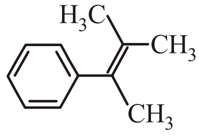
32. Compounds P and R upon ozonolysis produce Q and S, respectively. The molecular formula of Q and S is C_8H_8O. Q undergoes Cannizzaro reaction but not haloform reaction, whereas S undergoes haloform reaction but not Cannizzaro reaction

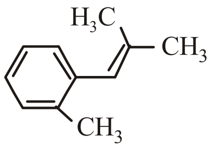
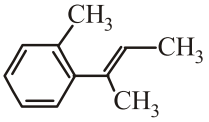
(i) $P \xrightarrow[\text{ii) } Zn/H_2O]{\text{i) } O_3/CH_2Cl_2} \underset{(C_8H_8O)}{Q}$

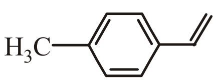
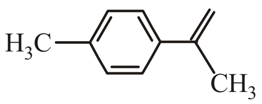
(ii) $R \xrightarrow[\text{ii) } Zn/H_2O]{\text{i) } O_3/CH_2Cl_2} \underset{(C_8H_8O)}{S}$

The option(s) with suitable combination of P and R, respectively, is(are)

(A)  and 

(B)  and 

(C)  and 

(D)  and 

SECTION - III

This section contains **2** paragraphs, each describing theory, experiments, data etc. four questions related to the two paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (A), (B), (C) and (D).

PARAGRAPH 1

Upon heating $KClO_3$ in the presence of catalytic amount of MnO_2, a gas W is formed. Excess amount of W reacts with white phosphorus to give X. The reaction of X with pure HNO_3 gives Y and Z.

33. W and X are, respectively

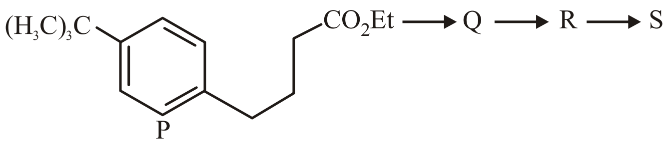
(A) O_3 and P_4O_6 (B) O_2 and P_4O_6
(C) O_2 and P_4O_{10} (D) O_3 and P_4O_{10}

34. Y and Z are, respectively

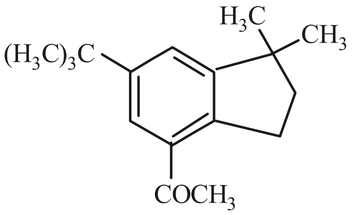
(A) N_2O_3 and H_3PO_4 (B) N_2O_5 and HPO_3
(C) N_2O_4 and HPO_3 (D) N_2O_4 and H_3PO_3

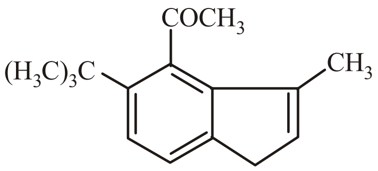
PARAGRAPH 2

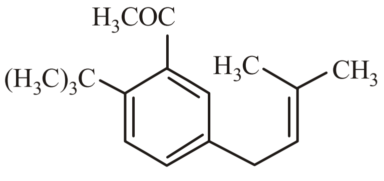
The reaction of compound P with CH_3MgBr (excess) in $(C_2H_5)_2O$ followed by addition of H_2O gives Q. The compound Q on treatment with H_2SO_4 at 0ºC gives R. The reaction of R with CH_3COCl in the presence of anhydrous $AlCl_3$ in CH_2Cl_2 followed by treatment with H_2O produces compound S. [Et in compound P is ethyl group]

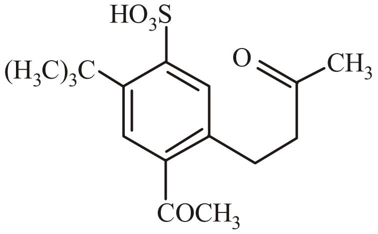


35. The product S is

(A) 

(B) 

(C) 

(D) 

36. The reactions, Q to R and R to S, are

(A) Dehydration and Friedel–Crafts acylation
(B) Aromatic sulfonation and Friedel–Crafts acylation
(C) Friedel–Crafts alkylation, dehydration and Friedel–Crafts acylation
(D) Friedel–Crafts alkylation and Friedel–Crafts acylation

MATHEMATICS

SECTION - I

This section contains **7** questions. Each question has 4 options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

37. The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is

(A) $14x + 2y - 15z = 1$
(B) $14x - 2y + 15z = 27$
(C) $14x + 2y + 15z = 31$
(D) $-14x + 2y + 15z = 3$

38. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overrightarrow{OP}.\overrightarrow{OQ} + \overrightarrow{OR}.\overrightarrow{OS} = \overrightarrow{OR}.\overrightarrow{OP} + \overrightarrow{OQ}.\overrightarrow{OS} = \overrightarrow{OQ}.\overrightarrow{OR} + \overrightarrow{OP}.\overrightarrow{OS}$$

Then the triangle PQR has S as its

(A) Centroid (B) Circumcentre
(C) Incentre (D) Orthocenter

39. If $y = y(x)$ satisfies the differential equation

$$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx, x > 0 \text{ and}$$

$y(0) = \sqrt{7}$, then $y(256) =$

(A) 3 (B) 9
(C) 16 (D) 80

40. If $f : R \to R$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in R$, and $f\ f\left(\frac{1}{2}\right) = \frac{1}{2}$, $f(1) = 1$, then

(A) $f'(1) \le 0$ (B) $0 < f'(1) \le \frac{1}{2}$

(C) $\frac{1}{2} < f'(1) \le 1$ (D) $f'(1) > 1$

41. How many 3×3 matrices M with entries from $\{0, 1, 2\}$ are there, for which the sum of the diagonal entries of $M^T M$ is 5?

(A) 126
(B) 198
(C) 162
(D) 135

42. Let $S = \{1, 2, 3, ..., 9\}$. For $k = 1, 2, ..., 5$, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

(A) 210
(B) 252
(C) 125
(D) 126

43. Three randomly chosen non–negative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is

(A) $\frac{36}{55}$
(B) $\frac{6}{11}$
(C) $\frac{1}{2}$
(D) $\frac{5}{11}$

SECTION - II

This section contains **7** questions. Each question has 4 options (A), (B), (C) and (D). **ONE or MORE THAN ONE** of these four options is (are) correct.

44. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)dt$, then

(A) $g'\left(\frac{\pi}{2}\right) = -2\pi$
(B) $g'\left(-\frac{\pi}{2}\right) = 2\pi$
(C) $g'\left(\frac{\pi}{2}\right) = 2\pi$
(D) $g'\left(-\frac{\pi}{2}\right) = -2\pi$

45. Let α and β be non–zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true?

(A) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$
(B) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$
(C) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$
(D) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

46. If $f : R \to R$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in R$, and $f(0) = 1$, then

(A) f(x) is increasing in $(0,\infty)$
(B) f(x) is decreasing in $(0, \infty)$
(C) $f(x) > e^{2x}$ in $(0, \infty)$
(D) $f'(x) < e^{2x}$ in $(0, \infty)$

47. Let $f(x) = \frac{1 - x(1 + |1 - x|)}{|1 - x|}\cos\left(\frac{1}{1 - x}\right)$ for $x \neq 1$. Then

(A) $\lim_{x \to 1^-} f(x) = 0$
(B) $\lim_{x \to 1^-} f(x)$ does not exist
(C) $\lim_{x \to 1^+} f(x) = 0$
(D) $\lim_{x \to 1^+} f(x)$ does not exist

48. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

(A) $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$
(B) $f'(x) = 0$ at more than three points in $(-\pi, \pi)$
(C) f(x) attains its maximum at $x = 0$
(D) f(x) attains its minimum at $x = 0$

49. If the line $sx = \alpha$ divides the area of region $R = \left\{(x, y) \in R^2 : x^3 \le y \le x, 0 \le x \le 1\right\}$ into two equal parts, then

(A) $0 < \alpha \le \frac{1}{2}$
(B) $\frac{1}{2} < \alpha < 1$
(C) $2\alpha^4 - 4\alpha^2 + 1 = 0$
(D) $\alpha^4 + 4\alpha^2 - 1 = 0$

50. If $I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)}dx$, then

(A) $1 > \log_e 99$
(B) $1 < \log_e 99$
(C) $1 < \frac{49}{50}$
(D) $1 > \frac{49}{50}$

SECTION - III

This section contains **2** paragraphs, Based on each paragraph, there are 2 questions. Each question has four options (A), (B), (C) and (D) ONLY ONE of these four options is correct.

PARAGRAPH 1

Let O be the origin, and $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}, \overrightarrow{RP}, \overrightarrow{PQ}$ respectively, of a triangle PQR.

51. $|\overrightarrow{OX} \times \overrightarrow{OY}| =$

(A) $\sin(P + Q)$
(B) $\sin 2R$
(C) $\sin(P + R)$
(D) $\sin(Q + R)$

52. If the triangle PQR varies, then the minimum value of $\cos(P + Q) + \cos(Q + R) + \cos(R + P)$ is

(A) $-\frac{5}{3}$
(B) $-\frac{3}{2}$
(C) $\frac{3}{2}$
(D) $\frac{5}{3}$

PARAGRAPH–2

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, ...$, let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.s

53. $a_{12} =$

(A) $a_{11} - a_{10}$
(B) $a_{11} + a_{10}$
(C) $2a_{11} + a_{10}$
(D) $a_{11} + 2a_{10}$

54. If $a_4 = 28$, then $p + 2q =$

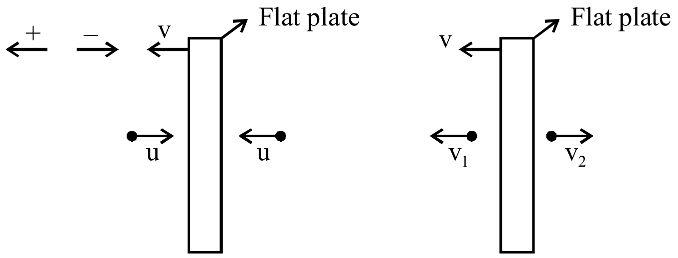
(A) 21
(B) 14
(C) 7
(D) 12

SOLUTIONS

Paper - 1

PHYSICS

1. (A, B, D)



Before collision Just after collision

$$1=\frac{v_1-v}{v+u} \qquad 1=\frac{v+v_2}{u-v}$$

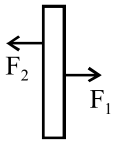
$$\therefore\ v_1=u+2v \qquad \therefore\ v_2=u-2v$$

$$\therefore\ \Delta v_1=2u+2v \quad \text{and } \Delta v_2=2u-2v$$

$$\text{Now } F_1=\frac{dp_1}{dt}=\rho A(u+v)(2u+2v)$$

$$\text{and } F_2=\frac{dp_2}{dt}=\rho A(\text{u}-\text{v})(2\,\text{u}-2\,\text{v})$$

$$\therefore F_1=2\rho A(u+v)^2 \quad \text{and } F_2=2\rho A(u-v)^2$$



$$\Delta F=F_1-F_2=8\rho Auv \quad \therefore\ P=\frac{\Delta F}{A}=8\rho uv$$

The net force $F_{net}=F-\Delta F=ma$

$$\therefore\ F-8\rho Auv=ma$$

2. (B, C) Let the block be displaced by x. If initially the centre of mass of the system is at origin then

$$\text{O}=\frac{M\times x+m(x+R)}{M+m}$$

$$\text{O}=Mx+mx+mR \quad \therefore\ x=\frac{-mR}{m+M}$$

$\therefore$ 'C' is the correct option

If v is the velocity of mass 'm' as it leaves the block and V is the velocity of block at that instant then according to conservation of linear momentum

mv = MV

By energy conservation

$$\text{mgR}=\frac{1}{2}mv^2+\frac{1}{2}MV^2$$

On solving we get, $\text{v}=\sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$

$$\therefore\ \text{V}=\frac{m}{M}\sqrt{\frac{2gR}{1+\dfrac{m}{M}}}$$

$\therefore$ (B) is the correct option.

3. (A, D) We know that $\lambda=\dfrac{v}{f}=\dfrac{1}{f}\sqrt{\dfrac{T}{\mu}}$

Where T = tension of string.

Here $T_o > T_A$ $\therefore\ \lambda_o > \lambda_A$ so option (C) is wrong.

Velocity being a vector quantity has direction. The velocities of the two pulses cannot be same at midpoint.

$T_{Ao}=T_{oA}$ because speed (or velocity) of wave depends on mediums (and not on the wavelength or frequency of wave)

(A), (D) are the correct options

4. (C)

Energy radiated = $\sigma A(T^4-T_0^{\,4})\,\text{t}$

[For a black body e = 1]

$$=\sigma A[(\text{T}_0+10)^4-T_0^{\,4}]t$$

$$=\sigma AT_0^{\,4}\left[\left(1+\frac{10}{T_0}\right)^4-1\right]t$$

$$=\sigma AT_0^{\,4}\left[\frac{40}{T_0}\right]\times t=460\times1\times\frac{40}{300}\times1=61.33J$$

$$P=\frac{\text{Energy radiated}}{\text{time}}=\sigma AT^4-\sigma AT_0^{\,4}$$

$$\therefore\ \left|\frac{dp}{dT_0}\right|=\sigma A(4T_0^{\,3}) \quad \therefore\ |dp|=\sigma A(4T_0^{\,3})dT_0$$

$$\therefore\ |\Delta P|=4\sigma AT_0^{\,3}$$

A, B are not correct options as human body is not a black body.

Energy radiated $\propto$ A where A is the surface area of the body

$\therefore$ 'C' is the correct option

5. **(B, D)** For smaller loop $\phi = BA\cos\omega t$
The rate of change of flux

$$\frac{d\phi}{dt} = -BA\omega\sin\omega t$$

For $\frac{d\phi}{dt}$ to be maximum, sin ωt should be maximum and this will happen when ωt = 90° i.e., the plane of loop is perpendicular to the plane of paper.
Option (D) is correct.
The emf produced will oppose each other. The net emf will also be proportional to sin ωt.
$e_{net} = B(2A)\omega\sin\omega t - BA\omega\sin\omega t = BA\omega\sin\omega t$
∴ option (B) is also correct.

6. **(B, C)** The angular frequency at which the current and voltage will be at same phase is

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{(10^{-6}\times10^{-6})^{1/2}} = 10^6\,\text{rad s}^{-1}$$

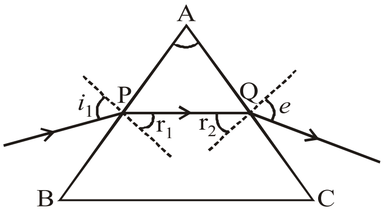
This value is independent of 'R' So (B) is correct option.

At $\omega \approx 0$, the current $i = \dfrac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

$i = 0$ (The circuit behaves as d.c circuit)
∴ C is a correct option
If $\omega >> \omega_0$, circuit behaves as an inductor.

7. **(A, C, D)**
For minimum deviation (when $i_1 = A$)
$i_1 = e$

$$r_1 = r_2 = r\ (\text{say}) = \frac{A}{2}$$



$$\delta_m = 2i_1 - A,$$

Here $\delta_m = A$ ∴ $i_1 = A$ ∴ $e = A$

$$\therefore\ r_1 = \frac{i_1}{2}$$

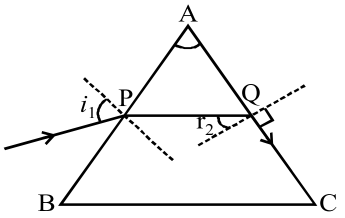
option (C) is correct

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin A}{\sin A/2}$$

$$= \frac{2\sin A/2\cos A/2}{\sin A/2} = 2\cos A/2$$

Applying Snell's law at Q

$$\mu = \frac{\sin 90°}{\sin r_2} = \frac{1}{\sin r_2} \Rightarrow r_2 = \sin^{-1}\left(\frac{1}{\mu}\right)$$



Emergent ray tangential
to the surface

But $r_1 + r_2 = A$
∴ $r_1 = A - r_2$

$$\therefore\ r_1 = A - \sin^{-1}\left(\frac{1}{\mu}\right)$$

Applying Snell's law at 'P' we get

$$\mu = \frac{\sin i_1}{\sin r_1} \qquad \therefore\ i_1 = \sin^{-1}\left[\mu\sin(A - \sin^{-1}\frac{1}{\mu})\right]$$

(D) is correct option.
For minimum deviation PQ || BC
∴ (A) is also a correct option.

8. **(6)** $\Delta U = S[k\times4\pi r^2 - 4\pi R^2]$

$$\left[\text{where } \frac{4}{3}\pi R^3 = k\times\frac{4}{3}\pi r^3\right]$$

$$\therefore\ R = K^{1/3}r$$

$$\therefore \Delta U = 4\pi s\left[k\times\frac{R^2}{k^{2/3}} - R^2\right] = 4\pi SR^2\left[k^{1/3} - 1\right]$$

$$\therefore \Delta U = 4\pi SR^2\left[10^{\alpha/3} - 1\right]$$

$$\therefore 10^{-3} = 4\pi\times\frac{0.1}{4\pi}\times(10^{-2})^2\left[10^{\alpha/3} - 1\right]$$

$$\therefore 10^2 = 10^{\alpha/3} - 1$$

Neglecting 1 we get $10^2 = 10^{\alpha/3}$ ∴ $\frac{\alpha}{3} = 2$ ∴ $\alpha = 6$

9. **(5)** Here $\frac{U_i}{U_f} = \frac{n_f^2}{n_i^2} = 6.25$ ∴ $\frac{n_f}{n_i} = 2.5$
If $n_i = 2$ then $n_f = 5$

10. **(8)** Here n × sin 30° = [n – m × 0.1] sin 90°
∴ 1.8 × sin 30° = 1.8 – m × 0.1 ∴ m = 8

11. **(6)** Frequency perceived by reflector $= f_1 = 492\left[\frac{332}{330}\right]$
Frequency perceived by the source f_2

$$= 492\left[\frac{332}{330}\right]\times\left[\frac{330}{328}\right] = 498\text{Hz}$$

∴ Beat frequency = 498 – 492 = 6Hz

12. (5) $A = A_0 e^{-\lambda t} \quad \therefore \quad A_0 = Ae^{\lambda t} = 115\,(1 + \lambda t)$

$$\therefore A_0 = 115\left[1 + \frac{\ln 2}{t_{1/2}} \times 11.5\right]$$

$$= 115\left[1 + \frac{0.7}{8 \times 24} \times 11.5\right]$$

$$A_0 \approx 120 Bq$$

120 Bq activity level is in 2.5 ml

$\therefore$ 2.4×10^5 Bq activity level will be in

$$\frac{2.5 \times 2.4 \times 10^5}{120} \text{ml}$$

$$\frac{2.5 \times 2.4 \times 10^5}{120 \times 1000} l = 5l$$

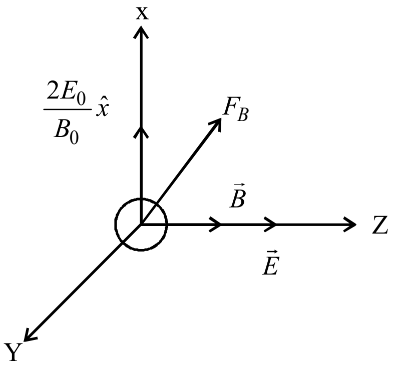
13. (D) For the particle to move in straight line, electric force should be equal and opposite to the magnetic force. (D) is the correct option.

$$\vec{F}_E = -e\vec{E} = -e(-E_0\hat{x}) = eE_0\hat{x}$$

$$\vec{F}_B = q(v \times \vec{B}) = -e\left[\frac{E_0}{B_0}\hat{y} \times B_0\hat{z}\right]$$

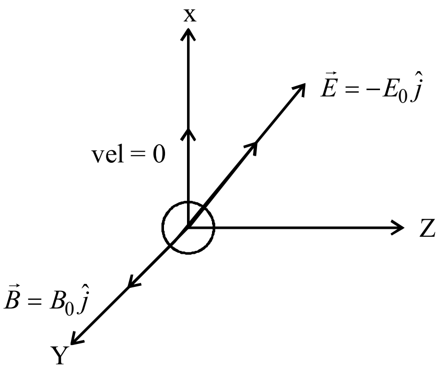
$$\vec{F}_B = -eE_0\hat{x}$$

14. (A)



The force due to magnetic field F_B will provide the necessary centripetal force for circular motion which will be in X-Y plane. The force due to electric field will accelerate proton in Z-direction. Thus the path will be helical with increasing pitch.

15. (B)



The electric field will apply a force on –Y axis thereby accelerating the charge along –Y axis.

$F_B = qv\,B \sin\theta$

Here $\theta = 180°$ therefore, $F_B = 0$

16. (D) $\Delta U = \Delta Q - P\Delta V$

show that work done = $P\Delta V$

which is the formula for isobaric process.

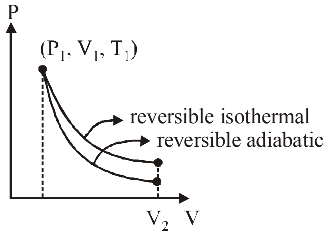
17. (B) Work done in isochoric process is zero for which we get a vertical line in P-V graph.

18. (D) Laplace's correction of the speed of sound in ideal gas is related to adiabatic process.

CHEMISTRY

19. (A, B, C)

(A) During adiabatic expansion, the final temperature is less than the initial temperature. Therefore the final volume in adiabatic expansion will also be less than the final volume in isothermal expansion. This can be graphically shown as



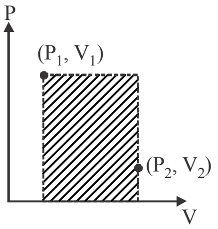
The magnitude of work done by the gas is equal to the area under the curve. As seen from the figure the area under curve in reversible isothermal is more. Hence, the magnitude of work done is lesser in adiabatic reversible expansion as compared to the corresponding work in isothermal expansion.

(B) In free expansion, $P_{ext} = 0 \therefore W = 0$

If carried out isothermally $(\Delta U = 0) \Rightarrow q = 0$ (Adiabatic) ; From I law

If carried out adiabatically $(q = 0) \Rightarrow \Delta U = 0$ (Isothermal) ; From I law

(C)



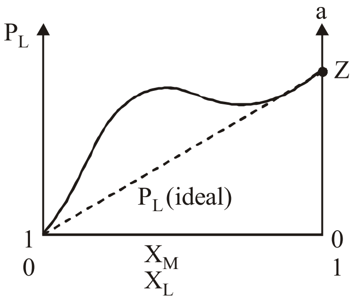
During irreversible compression, maximum work is done on the gas (corresponding to shaded area)

(D) When $T_1 = T_2 \Rightarrow \Delta U = nC_V\Delta T = 0$

In reversible adiabatic expansion, $T_2 < T_1$.

$\therefore \quad \Delta T = -ve \therefore \Delta U = -ve$

20. (B, D)



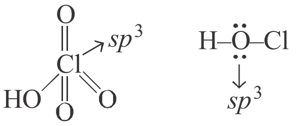
The graph representing +ve deviation from Raoult's law therefore M – L < M – M or L – L

$P_L \geq P^\circ_L X_L$

but when $X_L = 1$, mixture has almost pure liquid L so $P_L = P^\circ_L$

21. (A, B, D)

(A) In both the acids central atom is sp^3 hybridized.



(B) ClO_4^- is resonance stabilized anion

$HClO_4 \longrightarrow ClO_4^- + H^+$

$HClO \longrightarrow ClO^- + H^+$

Hence $HClO_4$ is more acidic than HClO.

(C) $Cl_2 + H_2O \rightarrow HCl + HOCl$

(D) $\underset{\text{Acid 1}}{HClO_4} + \underset{\text{Base 1}}{H_2O} \rightarrow \underset{\text{Base 1}}{ClO_4^-} + \underset{\text{acid 1}}{H_3O^+}$

Since H_2O is accepting H^+ from $HClO_4$ so H_2O is stronger base compared to ClO_4^-.

22. (C, D)

The colour of X_2 molecules of halogens is due to absorption of light in the visible region. The energy acquired in this manner excites the valence electron from the highest occupied molecular orbital (HOMO) to the lowest unoccupied molecular orbital (LUMO), i.e., transition from π^* to σ^* molecular orbital.

$$X_2 : \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2$$

$$= \pi 2p_y^2, \underset{(\text{HOMO})}{\pi^* 2p_x^2 = \pi^* 2p_y^2}, \quad \underset{(\text{LUMO})}{\sigma^* 2p_z^0}$$

23. (B, C, D)

$$\underset{(X)}{[Co(H_2O)_6]Cl_2} \xrightarrow[NH_4Cl + Air]{\text{Excess } NH_3 \text{ (aq.)}} \underset{(Y)}{[Co(NH_3)_6]Cl_3}$$

Pink Solution

$\downarrow$ HCl (excess) room temperature

$[CoCl_4]^{2-}$ (Z)

Blue colour (tetrahedral: sp^3 as Cl^- is a weak field ligand)

1 : 3 electrolyte (d^2sp^3 hybridisation of central metal atom as NH_3 is a strong field ligand)

$\downarrow$ 3 $AgNO_3$ (aq)

3 AgCl (3 eq of AgCl)

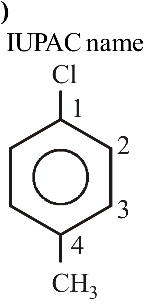
$$\underset{(\text{Blue})}{[CoCl_4]^{2-}} + 6H_2O \rightleftharpoons \underset{(\text{Pink})}{Co(H_2O)_6^{2+}} + 4Cl^-$$

X and Z in equilibrium at 0°C ⇒ then equilibrium is shifted towards X, making colour of solution pink.

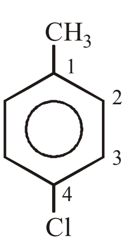
24. (A, B)

IUPAC name : 1-chloro-4-methylbenzence

(A)



(B)

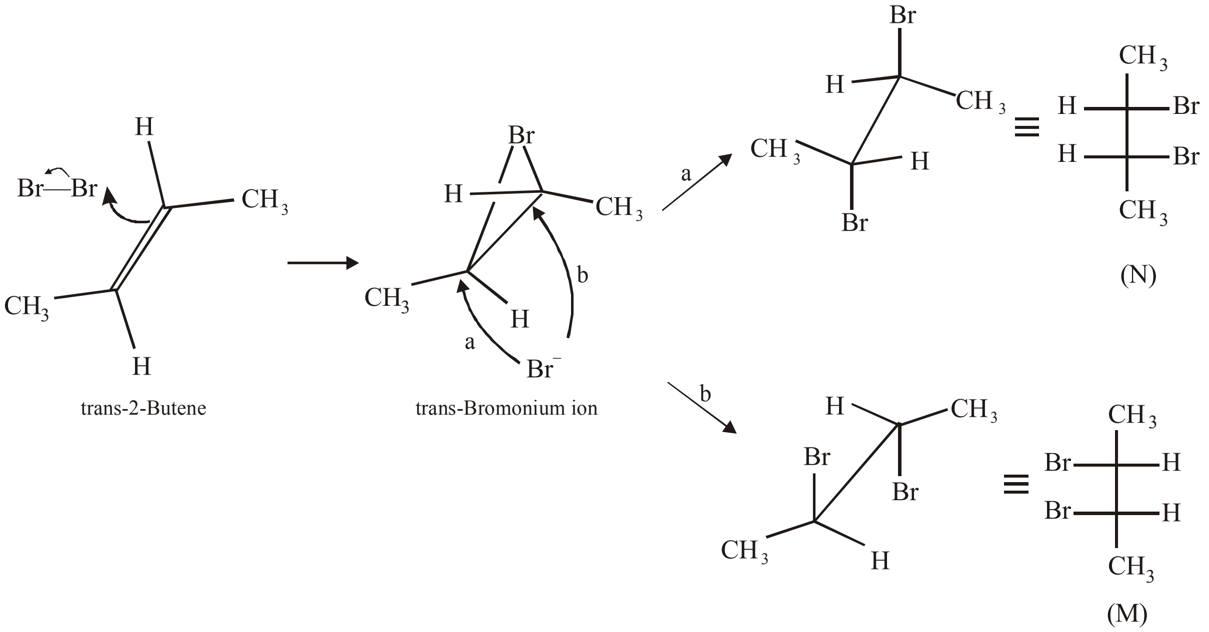


4-chlorotoluene

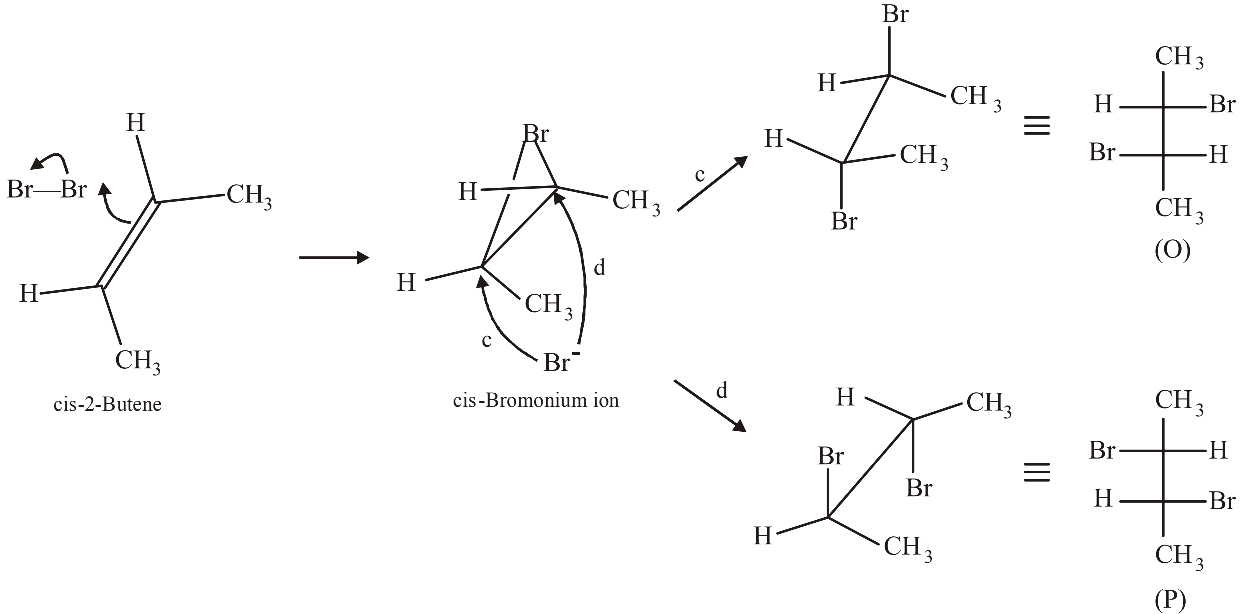
25. (B, D)

(B) Bromination proceeds through trans-addition in both the reactions.
M and N are identical, hence, M and O and N and P are two set of diastereomers.

(D)



M and N are same
(meso-2, 3-Dibromobutane)



II and III are enantiomers
rac-2, 3-Dibromobutane

26. **(2)** Density (d) $= \dfrac{Z \times M}{a^3 \times N_A}$ (d = density)

For FCC, Z = 4

Given a = 4×10^{-8} cm

$$8 = \frac{4 \times M}{\left(4 \times 10^{-10}\right)^3 \times 6 \times 10^{23}}$$

$$M = \frac{8 \times \left(4 \times 10^{-8}\right)^3 \times 6 \times 10^{23}}{4}$$

$$= \frac{8 \times 6 \times 10^{23} \times 64 \times 10^{-24}}{4}$$

No. of atoms $= \dfrac{\text{wt}}{\text{Molar mass}} \times N_A$

$$\frac{256 \times 10 \times 6 \times 10^{23}}{8 \times 6 \times 16} = 2 \times 10^{24}$$

∴ Value of N = 2

27. **(6)** The formula for conductance is $G = \kappa \times \dfrac{a}{\ell}$

$$5 \times 10^{-7} = \kappa \times \frac{1}{120}$$

$\kappa = 6 \times 10^{-5}$ s cm^{-1}

$$\Lambda_m^c = \frac{\kappa \times 1000}{M} = \frac{6 \times 10^{-5} \times 1000}{0.0015} = 40$$

∵ pH = 4

∴ $[H^+] = 10^{-4} = c\alpha = 0.0015\,\alpha$

$$\alpha = \frac{10^{-4}}{0.0015}$$

$$\alpha = \frac{\Lambda_m^c}{\Lambda_m^o} \Rightarrow \frac{10^{-4}}{0.0015} = 40$$

$$\Lambda_m^o = 6 \times 10^2 \text{ s cm}^2 \text{ mole}^{-1}$$

i.e., z = 6

28. **(6)**

	Species	Number of lone pairs on central atom
(i)	$[TeBr_6]^{2-}$	1
(ii)	$[BrF_2]^+$	2
(iii)	SNF_3	0
(iv)	$[XeF_3]^-$	3

Total number of lone pairs = 1 + 2 + 0 + 3 = 6

29. **(6)** (H_2, Cl_2, Be_2, C_2, N_2, F_2)

H_2 : $\sigma 1s^2$ (Diamagnetic)

He_2^+ : $\sigma 1s^2, \sigma^* 1s^1$ (Paramagnetic)

Li_2 : $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2$ (Diamagnetic)

Be_2 : $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2$ (Diamagnetic)

B_2 : $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^1 = \pi 2p_y^1$ (Paramagnetic)

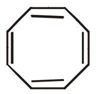
C_2 : $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^2 = \pi 2p_y^2$ (Diamagnetic)

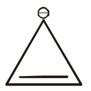
N_2 : $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^2$ (Diamagnetic)

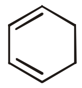
O_2^- : $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^2 = \pi^* 2p_y^1$ (Paramagnetic)

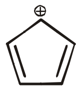
F_2 : $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^2 = \pi^* 2p_y^2$ (Diamagnetic)

30. **(5)**

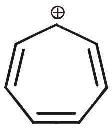

Non-aromatic

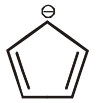

Anti-aromatic

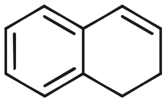

Non-aromatic

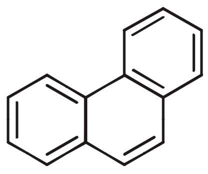

Anti-aromatic


Aromatic


Aromatic


Aromatic


Aromatic


Aromatic

31. **(C)** 1s wave function for He$^+$ is given by

$$\Psi_{(1s)} = \Psi_{n,\ell,m_\ell} = \frac{Z}{\sqrt{\pi} a_0^{3/2}} \exp^{-\left(\frac{Zr}{a_0}\right)}$$

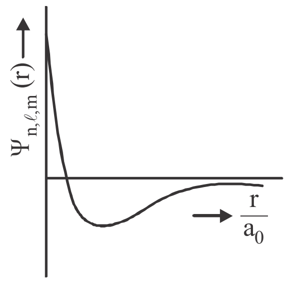
$$\text{or } \propto \left(\frac{Z}{a_0}\right)^{3/2} \exp^{-\left[\frac{Zr}{a_0}\right]}$$

i.e., it is independent of cos θ.

The probability of finding an electron at zero distance from the nucleus is zero. The probability increases gradually as the distance increases, goes to maximum and then begins to decrease.

32. **(A)** For a given orbital with principal quantum number, n and azimuthal quantum numbr, ℓ.

Number of radial nodes = (n – ℓ – 1)



33. **(C)** Refer ans 31.

Energy needed to excite from n = 2 to n = 4

$$\Delta E_{2-4} = 13.6\, Z^2 \times \frac{3}{16}\,\text{eV}$$

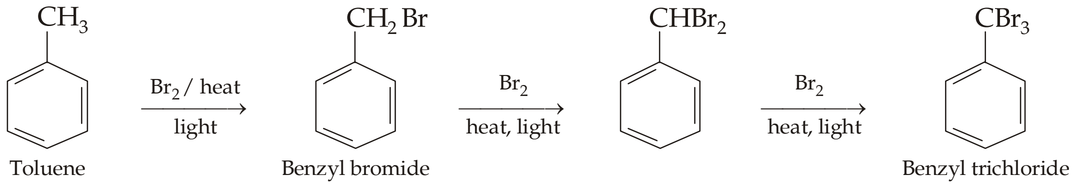
Similarly,

$$\Delta E_{2-6} = 13.6\, Z^2 \times \frac{8}{36}\,\text{eV}$$

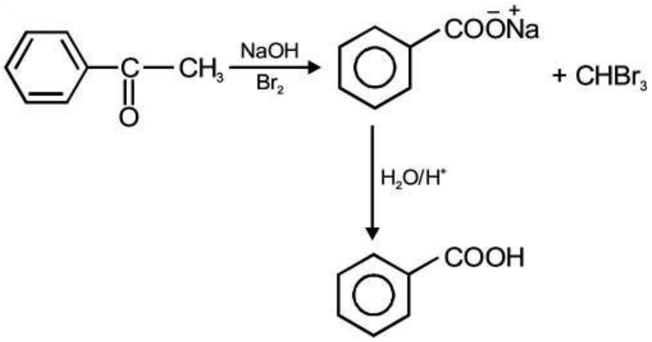
on dividing

$$\frac{\Delta E_{2-4}}{\Delta E_{2-6}} = \frac{3}{16} \times \frac{36}{8} = \frac{27}{32}$$

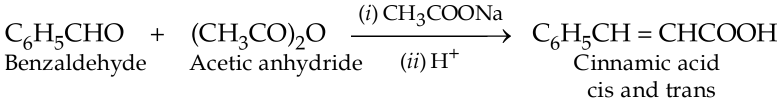
34. **(A)** Alkylbenzenes when treated with Br_2 at high temperature, in the presence of sunlight and absence of halogen carrier undergo **halogenation in the side chain.** Thus



35. **(C)**



36. **(D)** Perkin condensation of benzaldehyde with $(CH_3CO)_2O/CH_3COOK$ yields cis and trans form of cinnamic acid.



$$\underset{\text{Benzaldehyde}}{C_6H_5CHO} + \underset{\text{Acetic anhydride}}{(CH_3CO)_2O} \xrightarrow[(ii)\,H^+]{(i)\,CH_3COONa} \underset{\substack{\text{Cinnamic acid}\\ \text{cis and trans}}}{C_6H_5CH = CHCOOH}$$

MATHEMATICS

37. **(A, B, C)**

$\therefore$ 2x – y + 1 = 0 i.e. y = 2x + 1 is a tangent to hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{16} = 1$$

$\therefore \quad c^2 = a^2m^2 - b^2$

$1^2 = a^2 \times 2^2 - 16$

$$\Rightarrow \quad a^2 = \frac{17}{4}$$

$$\Rightarrow \quad a = \frac{\sqrt{17}}{2}$$

$\therefore \quad$ a, 4, 1; a, 4, 2; 2a, 8, 1

i.e. $\frac{\sqrt{17}}{2}, 4, 1;\ \frac{\sqrt{17}}{2}, 4, 2;\ \sqrt{17}, 8, 1$

cannot be the sides of a right triangle.

38. **(C)** If (h, k) is the mid point of chord of parabola $y^2 = 16x$, then equation of chord will be given by

$T = S_1$

$\Rightarrow \quad yk - 8(x + h) = k^2 - 16h$

$\Rightarrow \quad 8x - ky = 8h - k^2 \quad \ldots(1)$

But given, the equation of chord is

$2x + y = p \quad \ldots(2)$

$\therefore \quad$ (1) and (2) are identical lines

$$\Rightarrow \quad \frac{8}{2} = \frac{-k}{1} = \frac{8h - k^2}{p}$$

$\Rightarrow \quad k = -4$ and $8h - 16 = 4p$

$\Rightarrow \quad k = -4$ and $p = 2h - 4$

which are satisfied by option (C).

39. **(A, C, D)**

Let x = n be any integer not equal to zero. Then

$$\lim_{x \to n^-} x\cos(\pi(x + [x]))$$

$= n\cos(\pi(n + n - 1))$

$= n\cos(2n - 1)\pi = -n$

$$\lim_{x \to n^+} x\cos(\pi(x + [x]))$$

$= n\cos(\pi(n + [n]))$

$= n\cos(\pi(n + n))$

$= n \cos 2n\pi = n$

LHL ≠ RHL $\Rightarrow$ limit does not exist at any non zero integer n.

$\therefore$ f is discontinuous at $x = -1, 1, 2$

At $x = 0$, LHL = RHL = 0 = f(0)

$\therefore$ f is continuous at $x = 0$.

40. (A, B) Let us check the given options one by one.

(A) Let $g(x) = x^9 - f(x)$

$\Rightarrow \quad g(0) = -f(0) < 0 \qquad \therefore \quad f(x) \in (0, 1)$

Also $g(1) = 1 - f(1) > 0$

$\therefore \quad x^9 - f(x) = 0$ for some $x \in (0, 1)$

(B) Let $h(x) = x\int_0^{\frac{\pi}{2}-x} f(t)\cos t\,dt$

$$h(0) = -\int_0^{\frac{\pi}{2}} f(t)\cos t\,dt < 0$$

and $\quad h(1) = 1 - \int_0^{\frac{\pi}{2}-1} f(t)\cos t\,dt > 0$

$\therefore \quad h(x) = x - \int_0^{\frac{\pi}{2}-x} - f(t)\cos t\,dt = 0$

at some $x \in (0, 1)$

(C) $e^x - \int_0^x - f(t)\sin t\,dt$

$\because \quad x \in (0, 1) \Rightarrow e^x \in (1, e)$

and $0 < f(t) < 1$ and $0 < \sin t < \forall\ x \in (0, 1)$

$\therefore \quad 0 < \int_0^x f(t)\sin t\,dt < 1$

$\therefore \quad e^x - \int_0^x f(t)\sin t\,dt \neq 0$ for any $x \in (0, 1)$

(D) $f(x) + \int_0^{\frac{\pi}{2}} f(t)\sin t\,dt > 0 \ \forall x \in (0, 1)$

41. (B, D) In options (A) and (C) $|A^2| = 1$

and in option (B) and (D) $|A^2| = -1$

We know $|A^2| = |A|^2$

and $|A|^2 \neq -1 \Rightarrow$ matrices given in options B & D cannot be the squares of any 3 × 3 matrix with real entries.

42. (A, B) $a - b = 1, y \neq 0$

$$\operatorname{Im}\left(\frac{az+b}{z+1}\right) = y$$

$$\Rightarrow \quad \operatorname{Im}\left[\frac{a(x+iy)+b}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}\right] = y$$

$$\Rightarrow \quad \frac{-(ax+b)y + ay(x+1)}{(x+1)^2 + y^2} = y$$

$$\Rightarrow \quad \frac{-axy - by + axy + ay}{(x+1)^2 + y^2} = y$$

$\Rightarrow \quad a - b = (x+1)^2 + y^2$

$\Rightarrow \quad 1 = (x+1)^2 + y^2$

$\Rightarrow \quad x = -1 \pm \sqrt{1 - y^2}$

43. (A, B) $P(X) = \dfrac{1}{3},\ P(X/Y) = \dfrac{1}{2},\ P(Y/X) = \dfrac{2}{5}$

$$P(X \cap Y) = P(Y/X)\,P(X) = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$$

$$P(Y) = \frac{P(X \cap Y)}{P(X/Y)} = \frac{\frac{2}{15}}{\frac{1}{2}} = \frac{4}{15}$$

$$P(X'/Y) = \frac{P(X' \cap Y)}{P(Y)} = \frac{P(Y) - P(X \cap Y)}{P(Y)}$$

$= 1 - P(X/Y)$

$= \dfrac{1}{2}$

$$P(X \cup Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$$

$\therefore$ A and B are the correct options.

44. (2) Centre $(-1, -2)$

Geometrically, circle will have exactly 3 common points with axes in the cases

(i) Passing through origin $\Rightarrow$ p = 0

(ii) Touching x-axis and intersecting y-axis at two points i.e. $f^2 > C$ and $g^2 = C$.

i.e. $4 > -p$ and $1 = -p$

$\Rightarrow \quad p > -4$ and $p = -1$

$\Rightarrow \quad p = -1$

(iii) Touching y-axis and intersecting x-axis at two points i.e. $f^2 = c$ and $g^2 > C$

$\Rightarrow \quad 4 = -p$ and $1 > -p$

$\Rightarrow \quad p = -4$ and $p > -1$

which is not possible.

$\therefore$ Only two values of p are possible.

45. (2) Given $f(0) = 0,\ f\left(\dfrac{\pi}{2}\right) = 3,\ f'(0) = 1$

$$g(x) = \int_x^{\frac{\pi}{2}} \left[f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t\, f(t)\right] dt$$

$$g(x) = \lim_{x\to 0} \int_x^{\frac{\pi}{2}} \frac{d}{dt}\left(f(t)\operatorname{cosec} t\right) dt$$

$$= f\left(\frac{\pi}{2}\right)\operatorname{cosec}\frac{\pi}{2} - f(x)\operatorname{cosec} x$$

$$= 3 - f(x)\operatorname{cosec} x = 3 - \frac{f(x)}{\sin x}$$

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} 3 - \frac{f(x)}{\sin x} = 3 - \lim_{x\to 0} \frac{f(x)}{\sin x}$$

$$= 3 - \lim_{x\to 0} \frac{f'(x)}{\cos x} = 3 - f'(0) = 3 - 1 = 2$$

46. (1) For infinite many solutions

$$\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix} = 0 \Rightarrow (1-\alpha^2)^2 = 0 \Rightarrow \alpha = \pm 1$$

For $\alpha = 1$, the system will have no solution and for $\alpha = -1$, all three equations reduce to $x - y + z = 1$ giving infinite many dependent solutions.

$\therefore \quad 1 + \alpha + \alpha^2 = 1 - 1 + 1 = 1$

47. (5) $x = 10!$ and $y = {}^{10}C_1 \times \dfrac{10!}{2!} = 50 \times 9!$

$$\therefore \quad \frac{y}{9x} = \frac{50 \times 9!}{10!} = 5$$

48. **(6)** Let the sides be a – d, a, a + d where d is positive.

Using Pythagoras theorem,

$(a + d)^2 = (a - d)^2 + a^2$

$\Rightarrow$ a = 4d

$\therefore$ Sides are 3d, 4d, 5d

Area = 24 $\Rightarrow \frac{1}{2} \times 3d \times 4d = 24$

$\Rightarrow$ d = 2

$\therefore$ Sides are 6, 8, 10.

$\therefore$ Smallest side = 6.

49. **(B)** For a = $\sqrt{2}$ and point of contact (–1, 1).

Equation of circle is satisfied

$x^2 + y^2 = 2$

then eqn. of tangent is

$-x + y = 2 \Rightarrow m = 1$

and point of contact

$$\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right) = \left(\frac{-\sqrt{2}}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{2}}\right) = (-1, 1)$$

$\therefore$ (I) (ii), (Q) is the correct combination.

50. **(C)** Tangent $y = x + 8 \Rightarrow m = 1$

point (8, 16)

$\therefore$ both the coordinates as well as m, are positive,

the only possibility of point is $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = (8, 16)$

$\Rightarrow$ a = 8

Also it satisfies the equation of curve.

$y^2 = 4ax$ for the point (8, 16)

And equation of tangent $my = m^2x + a$ is satisfied by m = 1 and a = 8

$\therefore$ (III), (i), (P) is the correct combination.

51. **(D)** Point of contact $\left(\sqrt{3}, \frac{1}{2}\right)$ and tangent $\sqrt{3}x + 2y = 4$.

$\therefore$ $m = -\frac{\sqrt{3}}{2}$

$\therefore$ Both the coordinates are positive and m is negative the possibilities for points. are

$$Q\left(-\frac{ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right) \quad \text{OR}$$

$$R\left(-\frac{a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$$

For point Q $\left(\frac{\sqrt{3}a}{\sqrt{7}}, \frac{2a}{\sqrt{7}}\right) = \left(\sqrt{3}, \frac{1}{2}\right)$

We get a = $\sqrt{7}$ and a = $\frac{\sqrt{7}}{4}$

which is not possible.

For point R $\left(\frac{a^2\sqrt{3}}{\sqrt{3a^2+4}}, \frac{2}{\sqrt{3a^2+4}}\right) = \left(\sqrt{3}, \frac{1}{2}\right)$

$\Rightarrow \frac{a^2}{\sqrt{3a^2+4}} = 1$ and $\frac{2}{\sqrt{3a^2+4}} = \frac{1}{2}$

$\Rightarrow a^4 - 3a^2 - 4 = 0$ and $3a^2 = 12$

$\Rightarrow a^2 = 4$

Also for $a^2 = 4$ equation of ellipse

$x^2 + a^2y^2 = a^2$ is satisfied for the point $\left(\sqrt{3}, \frac{1}{2}\right)$

$\therefore$ II, (iv), R is the correct combination.

(For questions 52-54) : We observe the following, in the given table.

$f(x) = x + \log_e x - x\log_e x,\ x \in (0, \infty)$

$\Rightarrow$ $f'(x) = \frac{1}{x} - \log_e x$ and $f''(x) = -\frac{1+x}{x^2}$

$f(1) = 1 > 0$ and $f(e^2) = e^2 + 2 - 2e^2 = 2 - e^2 < 0$

$\therefore$ f(x) = 0 for some $x \in (1, e^2)$

$\therefore$ (I) is true.

$f'(1) = 1 > 0$ and $f'(e) = \frac{1}{e} - 1 < 0$

$\therefore$ f' (x) = 0 for some $x \in (1, e)$

$\therefore$ (II) is true.

If $x \in (0, 1)$, $\frac{1}{x} > 0$ and $\log_e x < 0$

$\therefore$ $f'(x) = \frac{1}{x} - \log_e x > 0 \Rightarrow$ f is increasing on (0, 1)

$\therefore$ $f'(x) \neq 0$ for some $x \in (0, 1)$

$\therefore$ (III) is false.

If $x \in (1, e)$, $f''(x) < 0 \Rightarrow$ f' is decreasing on (i, e)

$\therefore$ $f''(x) \neq 0$ for some $x \in (1, e)$

$\therefore$ (IV) is false.

Also $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} x + (1 - x)\log_e x = -\infty$

$\therefore$ (i) is false and (ii) is true.

$\lim\limits_{x\to\infty} f'(x) = \lim\limits_{x\to\infty} \frac{1}{x} - \log_e x = -\infty$

$\therefore$ (iii) is true

$\lim\limits_{x\to\infty} f''(x) = \lim\limits_{x\to\infty} -\frac{1}{x^2} - \frac{1}{x} = 0$

$\therefore$ (iv) is true.

f is increasing on (0, 1) already discussed

$\therefore$ (P) is true.

If $x \in (e, e^2)$ then

$f'(x) = \frac{1}{x} - \log_e x < 0$

$\Rightarrow$ f is decreasing in (e, e^2)

$\therefore$ (Q) is true.

For $x \in (0, 1)$, $f''(x) < 0$

$\Rightarrow$ f' is decreasing in (0, 1)

$\therefore$ R is false.

For $x \in (e, e^2)$ $f''(x) < 0$

$\Rightarrow$ f' decreasing in (e, e^2)

$\therefore$ (S) is true.

52. **(B)** The only correct combination is (II), (ii), (Q)

53. **(B)** The only correct combination is (II), (iii), (S)

54. **(C)** The only incorrect combination is (III), (i), (R).

Paper - 2

PHYSICS

1. **(A)** $\frac{1}{\rho}\frac{d\rho}{dt}$ = constant

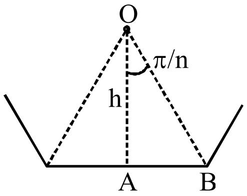
∴ $\frac{4\pi R^3}{3m}\frac{d}{dt}\left[\frac{m}{\frac{4}{3}\pi R^3}\right]$ = constant

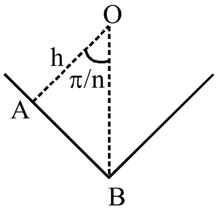
∴ $R^3\frac{d}{dt}(R^{-3})$ = constant

∴ $R^3(-3R^{-4})\frac{dR}{dt}$ = constant

∴ $\left|\frac{dR}{dt}\right|\propto R$

2. **(B)**





In ΔOAB $\cos\frac{\pi}{n}=\frac{OA}{OB}$ ∴ $OB=\frac{h}{\cos\frac{\pi}{n}}$

$$\Delta=\frac{h}{\cos\frac{\pi}{n}}-h=h\left[\frac{1}{\cos\frac{\pi}{n}}-1\right]$$

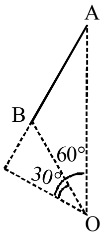
3. **(D)** $\frac{hC}{\lambda}-\phi_0=K.\text{E}=\frac{p^2}{2m_e}=\frac{h^2}{2m_e\lambda_d^2}$

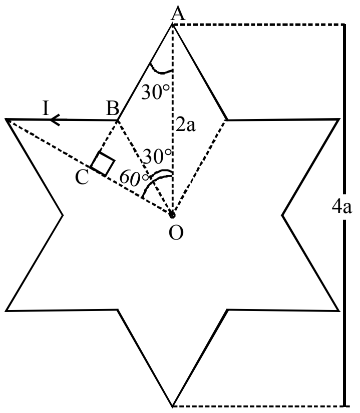
Differentiating on both sides

$$\frac{-hC}{\lambda^2}d\lambda=\frac{h^2}{2m_e}\left(\frac{-2}{\lambda_d^3}\times d\lambda_d\right)$$

∴ $\frac{d\lambda_d}{d\lambda}\propto\frac{\lambda_d^3}{\lambda^2}$

4. **(A)**





In Δ OAC cos 60° = $\frac{OC}{OA}$

∴ OC = $2a\times\frac{1}{2}=a$

The magnetic field at 'O' due to

AB = $\frac{\mu_0}{4\pi}\frac{I}{a}[\sin 60°-\sin 30°]$

$$=\frac{\mu_0}{4\pi}\frac{I}{a}\left[\frac{\sqrt{3}}{2}-\frac{1}{2}\right]=\frac{\mu_0 I}{4\pi a}\times\frac{1}{2}(\sqrt{3}-1)$$

The total magnetic field due to all the straight segments of the star is

$$=\left[\frac{\mu_0}{4\pi}\frac{I}{a}\times\frac{1}{2}(\sqrt{3}-1)\right]\times 12=\frac{\mu_0}{4\pi}\frac{I}{a}\times 6(\sqrt{3}-1)$$

5. **(A)** Here $\vec{P}+b\vec{R}=\vec{S}$ ∴ $\vec{R}=\frac{\vec{S}-\vec{P}}{b}$

Also $\vec{R}=\vec{Q}-\vec{P}$

∴ $\frac{\vec{S}-\vec{P}}{b}=\vec{Q}-\vec{P}$ ∴ $\vec{S}-\vec{P}=b\vec{Q}-b\vec{P}$

∴ $\vec{S}=b\vec{Q}+(1-b)\vec{P}$

6. **(B)** $\frac{1}{2}mV_e^2-\frac{GM_e m}{R_e}-\frac{GM_e m\times 3\times 10^5}{2.5\times 10^4 R_e}=0$

$$\frac{V_e^2}{2}=\frac{GM_e}{R_e}\left[1+\frac{3\times 10^5}{2.5\times 10^4}\right]$$

$$V_e=\sqrt{13\left(\frac{2GM_e}{R_e}\right)}=\sqrt{13}\times 11.2\approx 42$$

7. **(B)** $\text{T}=\sqrt{\frac{2L}{g}}+\frac{L}{v}$

with error limits

$$\text{T}+\delta\text{T}=\sqrt{\frac{2(L+\delta L)}{g}}+\frac{L+\delta L}{v}$$

$$\therefore\ \text{T}+\delta\text{T}=\sqrt{\frac{2L}{g}\left(1+\frac{\delta L}{L}\right)}+\frac{L}{v}\left(1+\frac{\delta L}{L}\right)$$

$$\therefore\ \text{T}+\delta\text{T}=\sqrt{\frac{2L}{g}}\times\left(1+\frac{\delta L}{2L}\right)+\frac{L}{v}\left(1+\frac{\delta L}{L}\right)$$

$$\therefore\ \text{T}+\delta\text{T}=\sqrt{\frac{2L}{g}}+\sqrt{\frac{2L}{g}}\frac{\delta L}{2L}+\frac{L}{v}+\frac{L}{v}\frac{\delta L}{L}$$

$$\mathrm{T}+\delta\mathrm{T}=T+\sqrt{\frac{2L}{g}}\frac{\delta L}{2L}+\frac{L}{v}\frac{\delta L}{L}$$

$$\delta\mathrm{T}=\frac{\delta L}{L}\left[\frac{1}{2}\sqrt{\frac{2L}{g}}+\frac{L}{v}\right]$$

Substituting $\delta\mathrm{T}=0.015$, $\mathrm{L}=20\,\mathrm{m}$, $\mathrm{g}=10\,\mathrm{ms}^{-2}$, $\mathrm{v}=300\,\mathrm{ms}^{-1}$

We get

$$\frac{\delta L}{L}=\frac{15}{1600}$$

$$\therefore\quad \frac{\delta L}{L}\times 100=\frac{15}{1600}\times 100=\frac{15}{16}\%\approx 1\%$$

8. **(A, B)** For the charge +Q to return region 1, the radius of the circular path taken by charge should by $\frac{3R}{2}$

$$\frac{mv^2}{(3R/2)}=QvB \quad \therefore\ \frac{2p}{3R}=QB$$

$$\therefore\ B=\frac{2p}{3QR}$$

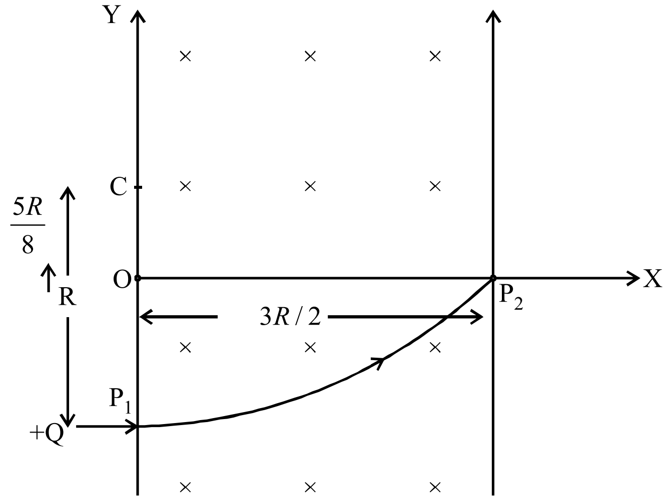
i.e., B should be equal or greater than $\frac{2p}{2QR}$

'A' is the correct option.

When $B=\frac{8p}{13QR}$

$$\frac{mv^2}{r}=Qv\left(\frac{8p}{13QR}\right) \quad \therefore\ r=\frac{13R}{8}$$

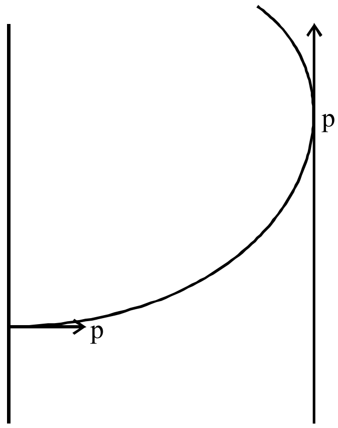
Thus 'C' is the of the centre of circular path of radius $\frac{13R}{8}$



Also $CP_2=\sqrt{CO^2+OP_2^{\,2}}$

$$=\sqrt{\left(\frac{5R}{8}\right)^2+\left(\frac{3R}{2}\right)^2}$$

$$\mathrm{CP}_2=\frac{13R}{8}$$



Thus the particle will enter region 3 through the point P_1 on X-axis

'B' is the correct option.

Change in momentum = $\sqrt{2}p$

Thus 'C' is incorrect

Further $\frac{mv^2}{r}=qvB \ \therefore\ r=\frac{mv}{qB} \quad \therefore\quad r\propto m$

$\therefore$ 'D' is incorrect.

9. **(A, D)** The potential difference between X and Y is

$$V_{XY}=V_X-V_Y$$

$$V_{XY}=(V_{XY})_0\sin(\omega t+\theta_1)$$

where $(V_{XY})_0=\sqrt{V_0^{\,2}+V_0^{\,2}-2V_0^{\,2}\cos\frac{2\pi}{3}}=\sqrt{3}V_0$

and $(V_{XY})_{rms}=\frac{(V_{XY})_0}{\sqrt{2}}=\sqrt{\frac{3}{2}}V_0$

(A) is the correct option

Now the potential difference between Y and Z is

$$V_{YZ}=V_Y-V_Z$$

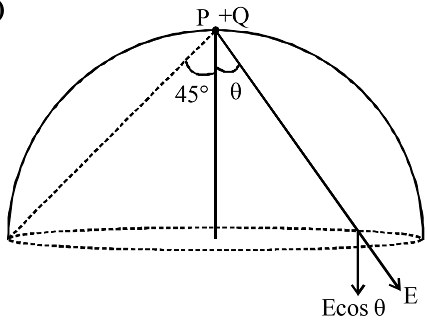
$$V_{YZ}=(V_{YZ})_0\sin(\omega t+\theta_2)$$

Where $(V_{YZ})_0=\sqrt{V_0^{\,2}+V_0^{\,2}-2V_0^{\,2}\cos\frac{2\pi}{3}}=\sqrt{3}V_0$

and $(V_{YZ})_{rms}=\frac{(V_{YZ})_0}{\sqrt{2}}=\sqrt{\frac{3}{2}}V_0$

Thus (D) is the correct option.

10. (A, D)



The circumference of the flat surface is an equipotential because the distance of each point on the circumference is equal from + Q

(D) is the correct option.

The component of electric field normal to the flat surface is Ecosθ.

Here E as well as θ changes for different point on the flat surface. Therefore (C) is incorrect.

The total flux through the curved and flat surface should be less than $\frac{Q}{\varepsilon_0}$. Therefore (B) is incorrect.

The solid angle subtended by the flat surface at

$$P = 2\pi\left(1-\frac{1}{\sqrt{2}}\right)$$

∴ Flux passing through curved surface

$$= -\frac{Q'}{\varepsilon_0}\frac{2\pi\left(1-\frac{1}{\sqrt{2}}\right)}{4\pi} = -\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right).$$

(A) is the correct option.

11. (B, C)

Path difference at P_2 is

$p = S_1P_2 - S_2P_2 = d = 1.8 \text{ mm} = 1.8 \times 10^{-3}\text{m}$

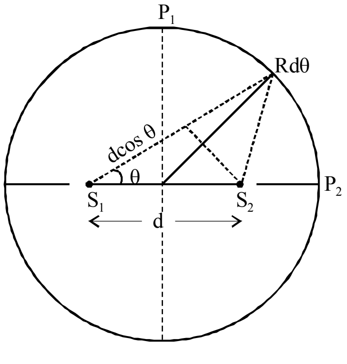
$= 3000 \times 600 \times 10^{-9}$ m

$p = 3000\,\lambda$.

As the path difference is an integral multiple of λ, P_2 should be a bright fringe with 300th maxima. (A) is incorrect.

Further at P_1, path difference = 0. Therefore a bright fringe will be present at P_1 also. Therefore total number of fringes between P_1 and P_2 is 3000. (C) is a correct option.

Obviously at P_2 the order of the fringe will be maximum. Thus (B) is a correct option.



Now path difference

$p = d\cos\theta = n\lambda$ (for bright fringe)

$$\therefore \cos\theta = \frac{n\lambda}{d}$$

$$\therefore -\sin\theta\,\Delta\theta = (\Delta n)\frac{\lambda}{d}$$

$$\text{or } \Delta\theta = -\frac{\Delta n\lambda}{d\sin\theta}$$

As we move from p_1 to p_2, θ decreases and therefore $\Delta\theta$ increases. Therefore (D) is incorrect.

12. (A, B, C)

After a long time the current through the resistor is constant I will divide into two parts L_1 and L_2 which are in parallel

$\therefore I_1L_1 = I_2L_2$

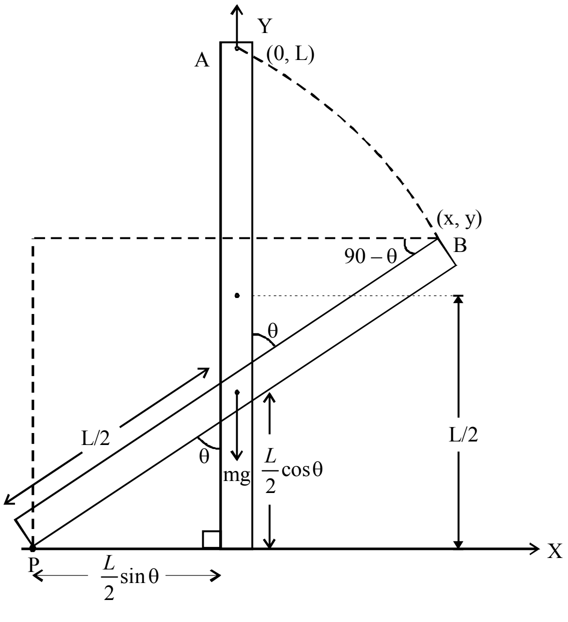
Further $I_1 = \frac{V}{R}\left[\frac{L_2}{L_1+L}\right]$

and $I_2 = \frac{V}{R}\left[\frac{L_1}{L_1+L_2}\right]$

Also the ratio of currents through L_1 and L_2 is fixed at all times At t = 0, $I \approx 0$

13. (A, C, D)

As $F_x = 0$, $a_x = 0$. Therefore the force acting in vertical direction will move the mid point of the bar fall vertically downwards. (A) is correct option.



When the bar makes an angle θ with the vertical, the displacement of its mid point from the initial position is $\frac{L}{2} - \frac{L}{2}\cos\theta$

(D) is a correct option.

Instantaneous torque about the point of contact P is

$$\tau = mg \times \frac{L}{2}\sin\theta$$

(C) is a correct option.

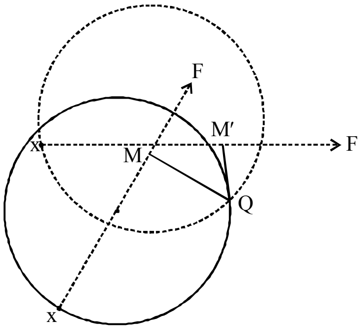
Now $x = \frac{L}{2}\sin\theta$, $y = L\sin(90-\theta) = L\cos\theta$

$$\therefore \left(\frac{2x}{L}\right)^2 + \left(\frac{y}{L}\right)^2 = 1 \text{ or } \frac{4x^2}{L^2} + \frac{y^2}{L^2} = 1$$

This is equation of ellipse. Therefore B is incorrect

14. (C) If the force is applied at P tangential than the I remains constant and is equal to F × 2R where F is the applied force.

If force is applied normal to X, then as the wheels climbs, then the perpendicular distance of force from Q will go on changing initially the perpendicular is Q_M, later it becomes QM'.



If the force is applied normal to the circumference at point P then I is zero. So (C) is correct.

If the force is applied tangentially at point S then $\tau = F \times R$ and the wheel will climb.

15. (A) Work done by battery = q × V

$\therefore$ $W = CV_o \times V_o = CV_o^2$

Energy stored in the battery = $\frac{1}{2}CV_0^2$

$\therefore$ Energy dissipated

$$E_D = W - E_C = CV_0^2 - \frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2$$

$\therefore$ $E_C = E_D$

16. (C) Let V_i and V_f be the initial and final voltage in each process. Then

Energy dissipated = $W_{\text{battery}} - \Delta U$

$$= C(V_f - V_i)V_f - \frac{1}{2}C(V_f - V_i)^2 = \frac{1}{2}C(V_f - V_i)^2$$

$\therefore$ Total heat dissipated

$$E_D = \frac{1}{2}C\left[\left(\frac{V_o}{3}-0\right)^2 + \left(\frac{2V_o}{3}-\frac{V_o}{3}\right)^2 + \left(V_o - \frac{2V_o}{3}\right)^2\right]$$

$$= \frac{1}{6}CV_o^2$$

17. (C) Here $\omega_0(R-r) = \omega R$ $\therefore$ $\omega = \omega_o\left(\frac{R-r}{R}\right)$

Now total kinetic energy of the ring (Kinetic rotational + kinetic translational)

$$K.E_{total} = \frac{1}{2}(2MR^2)\omega^2 = M\omega_0^2(R-r)^2$$

18. (A) $\mu M\omega_{\min}^2(R-r) = Mg$

$$\therefore \omega_{\min} = \sqrt{\frac{g}{\mu(R-r)}}$$

CHEMISTRY

19. (C) As T increase, V.P. increases

$\Delta T_f = K_f \times m$

$$273 - T_f' = 2 \times \frac{34.5 \times 1000}{46 \times 500}$$

$\therefore$ $T'_f = 270$ K

20. (B) $Zn(s) + Cu^{2+}_{(aq)} \rightarrow Zn^{2+}_{(aq)} + Cu(s)$

$$\Delta G = \Delta G^\circ + 2.303\ RT\log_{10}Q;\ Q = \frac{\left[Zn^{2+}\right]}{\left[Cu^{2+}\right]}$$

$[\Delta G^\circ = -nFE^\circ] = -2 \times F \times 1.1$

Given $[Zn^{2+}] = 10[Cu^{2+}]$

$\therefore$ $\Delta G = -2F(1.1) + 2.303\ RT\log_{10}10$

$= 2.303\ RT - 2.2F$

21. (A) At eq. $\Delta G = 0$

$\Delta G^\circ = dp(\Delta V)$

$2.9\ kJ\ mol^{-1} = (P_2 - 1) \times 2 \times 10^{-6}\ m^3\ mol^{-1}$

or

$$2.9 \times 10^3 \frac{kg\ m^2}{s^2 mol^{-1}} = (P_2 - 1)(2 \times 10^{-6})\frac{m^3}{mol^{-1}}$$

$$(P_2 - 1) = \frac{2.9 \times 10^3 \times 10^6}{2}\frac{kg}{ms^2}$$

$$= 1.45 \times 10^9 \frac{kg}{ms^2} = 1.45 \times 10^9\ Pa$$

$\therefore$ $P_2 = 1.45 \times 10^9 + 1Pa$

$= 14500 \times 10^5 + 1 = 14501 \times 10^5\ Pa = 14501$ bar

22. (C) $Fe + conc.\ HNO_3 \rightarrow Fe_2O_3$

$Cu + conc.\ HNO_3 \rightarrow CuNO_3 + NO_2 + H_2O$

$4Au + 8NaCN + O_2 + 2H_2O \rightarrow 4Na[Au(CN)_2] + 4NaOH$

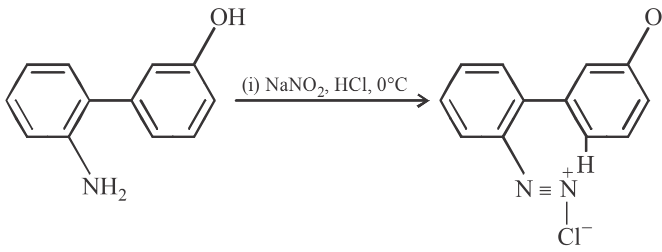
$Zn + NaOH \rightarrow Na_2ZnO_2 + H_2$

23. (C) Correct order :

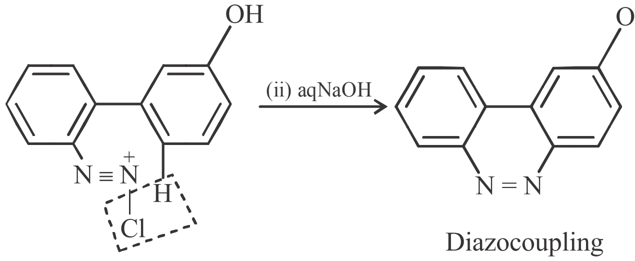
$$\underset{(+5)}{H_3PO_4} > \underset{(+4)}{H_4P_2O_6} > \underset{(+3)}{H_3PO_3} > \underset{(+1)}{H_3PO_2}$$

24. (C)

Step 1 :

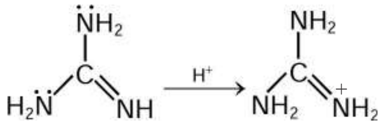


Step 2 :



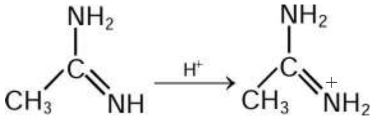
Diazocoupling

25. (C)



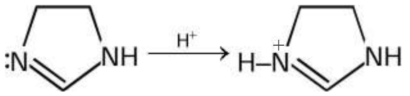
(IV)

The conjugate acid is stabilized by resonance with two different –NH$_2$ group. Hence electron density increases on N of = NH



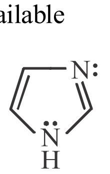
(I)

The conjugate acid is stabilized by resonance with one –NH$_2$ group. Hence as compared to IV lesser increase of electron density on N of = NH



(II)

Lone pair is not involvd in aromaticity. Hence more available



(III)

Lone pair is involved in aromatic sextet. Hence not available.

Hence the correct order of basic strength is

IV > I > II > III

26. (A, B)

(A) As adsorption is spontaneous, ΔG for the process is –ve. Adsorption is accompained by decrease in randomness. Therefore ΔS and TΔS for the process is also negative. As ΔS for the process is –ve and the process is spontaneous, ΔH for the process has to be –ve i.e, enthalpy of the system decreases.

(B) Under a given set of conditions of temperature and pressure the easily liquefiable gases e.g. C_2H_6, NH_3 and HCl are adsorbed more than the gases like N_2, H_2 and CO. The ease with which a gas can be liquefied is determined by its critical temperature.

Critical temperature is the minimum temperature above which a gas can be liquified. This implies that gases with high critical temperature values can be easily liquified as compared to gases with low critical temperature value.

27. (B, D)

28. (A, B) The Arrhenius equation is

$k = Ae^{-Ea/RT}$

where A = Pre-exponential factor

A is not directly related with temperature and activation energy.

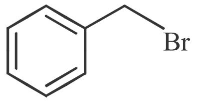
Where $A = \underset{(\text{Frequency factor})}{Z} \times \underset{(\text{Steric factor})}{P}$

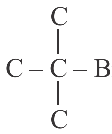
Hence we can say the E_a is independent of steric factor

$$P = \frac{K_{\text{actual}}}{K_{\text{theoretical}}}$$

So, $A_{\text{actual}} > A_{\text{theoretical}}$

29. (A, B, C)

(A) I is  (1° benzylic halide) and

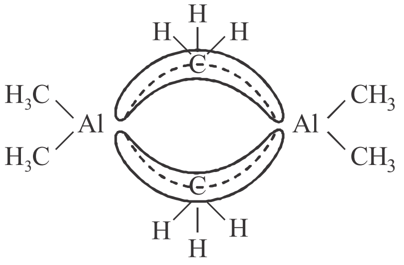

$C-\overset{C}{\underset{C}{C}}-Br$ (3° alkylhalide). Follow S_N1.

(B) I and II follow S_N2 also, as both are 1° halide.

(C) Compound (IV) undergoes inversion of configuration due to presence of chiral carbon atom.

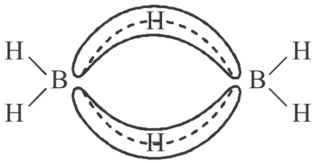
30. (A, B, D)

(A) Structure of $Al_2(CH_3)_6$



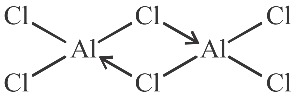
3C-2e⁻ Bond

(B) Structure of B_2H_6



3C-2e⁻ Bond

(C) Structure of Al_2Cl_6



(D) BCl_3 is stronger lewis acid as the bond formed with the base will involve 2p orbital overlap which is stronger than 3p orbital overlap in the case of $AlCl_3$.

31. (A, D) NO ⇒ Neutral

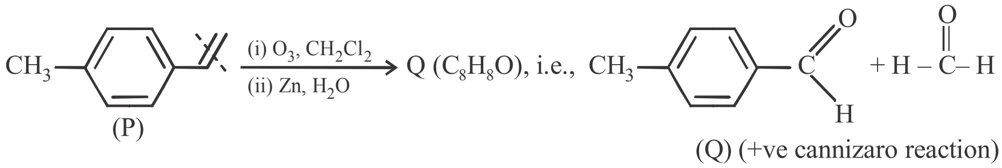
B_2O_3 ⇒ Acidic

CrO ⇒ Basic

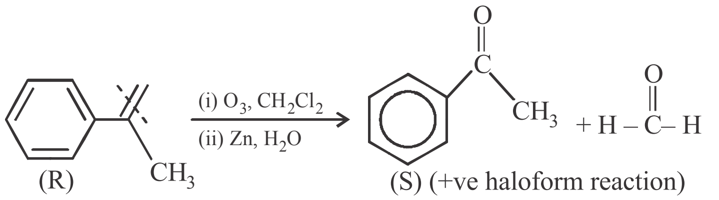
All other oxides are amphoteric

32. (A, B)

(C)

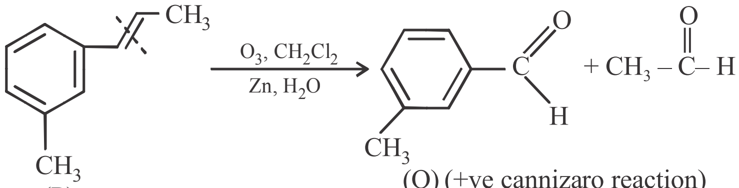


(Q) (+ve cannizaro reaction)



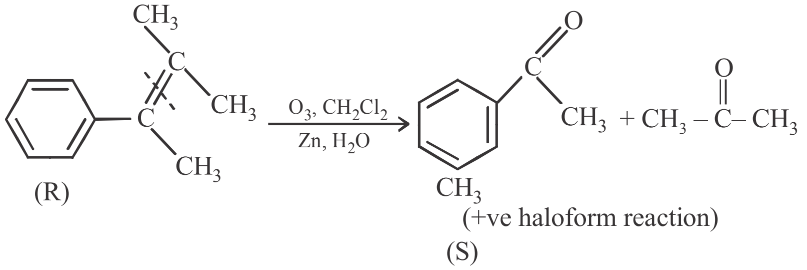
(S) (+ve haloform reaction)

(D)



(P)

(Q) (+ve cannizaro reaction)



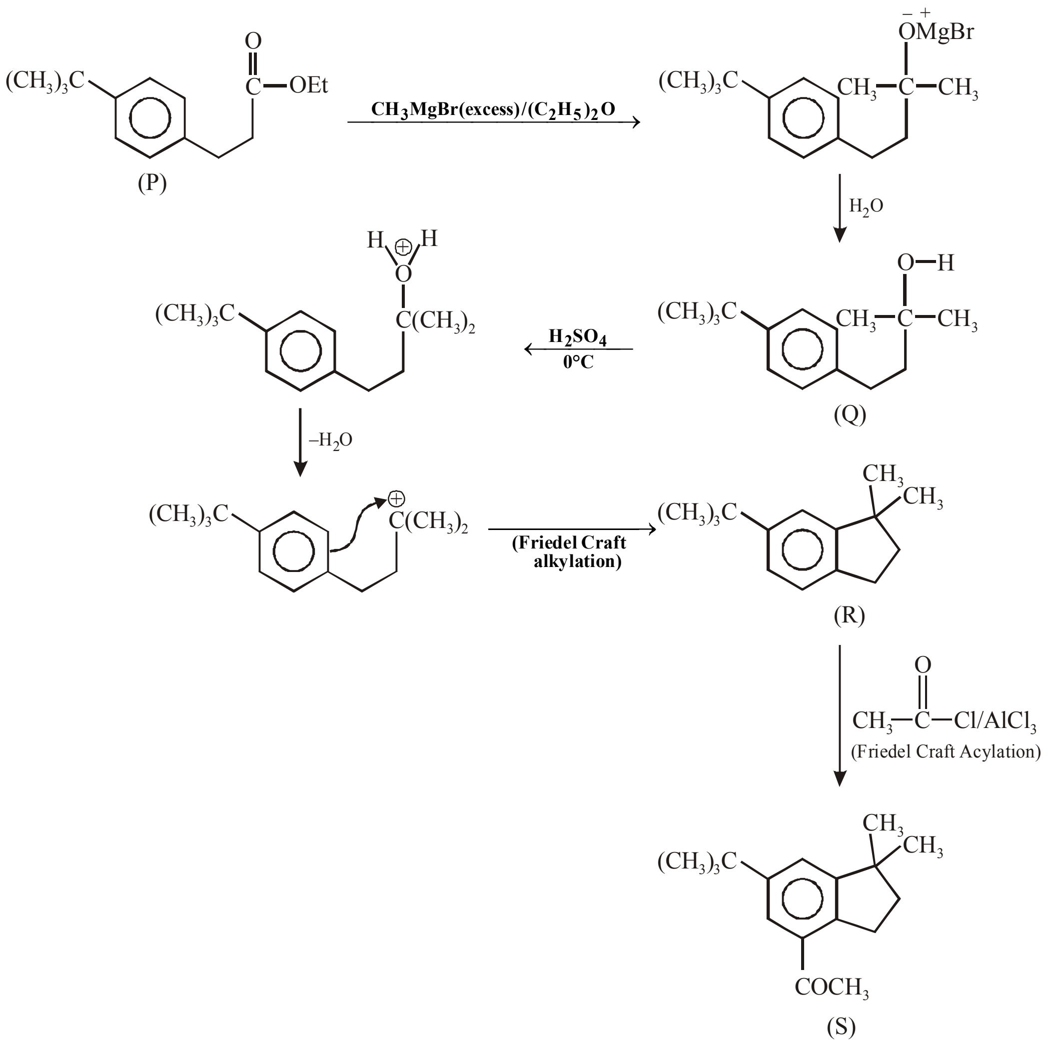
33. (C) & 34. (B)

$$KClO_3 \xrightarrow[MnO_2]{\Delta} KCl + \underset{(W)}{O_2}$$

$$\underset{(white)}{P_4} + \underset{(excess)}{O_2} \rightarrow \underset{(X)}{P_4O_{10}}$$

$$P_4O_{10} + HNO_3 \rightarrow \underset{(Y)}{N_2O_5} + \underset{(Z)}{HPO_3}$$

35. (A) & 36. (D)



MATHEMATICS

37. **(C)** The required equation of plane is given by

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = 0$$

$\Rightarrow\ (x-1)(-14)-(y-1)(2)+(z-1)(-15)=0$

$\Rightarrow\ 14x-14+2y-2+15z-15=0$

$\Rightarrow\ 14x+2y+15z=31$

38. **(D)** $\overrightarrow{OP}\cdot\overrightarrow{OQ}+\overrightarrow{OR}\cdot\overrightarrow{OS}=\overrightarrow{OR}\cdot\overrightarrow{OP}+\overrightarrow{OQ}\cdot\overrightarrow{OS}$

$\Rightarrow\ \left(\overrightarrow{OQ}-\overrightarrow{OR}\right)\cdot\overrightarrow{OP}-\left(\overrightarrow{OQ}-\overrightarrow{OR}\right)\cdot\overrightarrow{OS}=0$

$\Rightarrow\ \left(\overrightarrow{OQ}-\overrightarrow{OR}\right)\cdot\left(\overrightarrow{OP}-\overrightarrow{OS}\right)=0$

$\Rightarrow\ \overrightarrow{RQ}\cdot\overrightarrow{SP}=0$

$\Rightarrow\ RQ\perp SP$ ...(1)

Also $\overrightarrow{OR}\cdot\overrightarrow{OP}+\overrightarrow{OQ}\cdot\overrightarrow{OS}=\overrightarrow{OQ}\cdot\overrightarrow{OR}+\overrightarrow{OP}\cdot\overrightarrow{OS}$

$\Rightarrow\ \overrightarrow{OR}\cdot\left(\overrightarrow{OP}-\overrightarrow{OQ}\right)-\overrightarrow{OS}\cdot\left(\overrightarrow{OP}-\overrightarrow{OQ}\right)=0$

$\Rightarrow\ \left(\overrightarrow{OP}-\overrightarrow{OQ}\right)\cdot\left(\overrightarrow{OR}-\overrightarrow{OS}\right)=0$

$\Rightarrow\ \overrightarrow{QP}\cdot\overrightarrow{SR}=0$

$\Rightarrow\ QP\perp SR$ ...(2)

From (1) and (2) S should be the orthocentre of ΔPQR.

39. **(A)** Given DE can be written as

$$\int dy=\int\frac{1}{\left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)\left(\sqrt{9+\sqrt{x}}\right)8\sqrt{x}}dx$$

Putting $\sqrt{4+\sqrt{9+\sqrt{x}}}=t$

We get

$$\frac{1}{2\sqrt{4+\sqrt{9+\sqrt{x}}}\cdot 2\sqrt{9+\sqrt{x}}\cdot 2\sqrt{x}}dx=dt$$

$\therefore\quad \int dy=\int dt\quad\Rightarrow\quad y=t+c$

or $\quad y=\sqrt{4+\sqrt{9+\sqrt{x}}}+C$

$y(0)=\sqrt{7}\quad\Rightarrow\quad C=0$

$\therefore\quad y=\sqrt{4+\sqrt{9+\sqrt{x}}}$

$\therefore\quad y(256)=3$

40. **(D)** $f''(x)>0,\ \forall x\in R$

$f\left(\frac{1}{2}\right)=\frac{1}{2},\ f(1)=1$

$\therefore$ f' is an increasing function on R.

By Lagrange's Mean Value theorem.

$$f'(\alpha)=\frac{f(1)-f\left(\frac{1}{2}\right)}{1-\frac{1}{2}},\ \alpha\in\left(\frac{1}{2},1\right)$$

$\Rightarrow\ f'(\alpha)=1$ for some $\alpha\in\left(\frac{1}{2},1\right)$

$\Rightarrow\ f'(1)>1$

41. **(B)** Let $M=\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$ where $a_i\in\{0,1,2\}$

Then $M^TM=\begin{bmatrix} a_1 & a_4 & a_7 \\ a_2 & a_5 & a_8 \\ a_3 & a_6 & a_9 \end{bmatrix}\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$

Sum of the diagonal entries in $M^TM=5$

$\Rightarrow\ (a_1^2+a_4^2+a_7^2)+(a_2^2+a_5^2+a_8^2)$
$+(a_3^2+a_6^2+a_9^2)=5$

It is possible when

Case I: 5 a_is are 1 and 4 a_i's are zero

Which can be done in

${}^9C_4$ ways $=\frac{9\times8\times7\times6}{4\times3\times2\times1}=126$

Case II: 1 a_i is 1 and 1a_i is 2 and rest.

7a_i's are zero

It can be done in ${}^9C_1\times{}^8C_1=9\times8=72$ ways

$\therefore$ Total no. of ways = 126 + 72 = 198.

42. **(D)** $N_1={}^5C_1\times{}^4C_4=5$

$N_2={}^5C_2\times{}^4C_3=40$

$N_3={}^5C_3\times{}^4C_2=60$

$N_4={}^5C_4\times{}^4C_1=20$

$N_5={}^5C_5=1$

$\therefore\quad N_1+N_2+N_3+N_4+N_5=126$

43. **(B)** Total number of non negative solutions of x + y + z = 10 are $={}^{12}C_2=66$ (using ${}^{n+r-1}C_{r-1}$)

If z is even then there can be following cases:

$z=0\ \Rightarrow$ No. of ways of solving $x+y=10\ \Rightarrow\ {}^{11}C_1$

$z=2\ \Rightarrow$ No. of ways of solving $x+y=8\ \Rightarrow\ {}^{9}C_1$

$z=4\ \Rightarrow$ No. of ways of solving $x+y=6\ \Rightarrow\ {}^{7}C_1$

$z=6\ \Rightarrow$ No. of ways of solving $x+y=4\ \Rightarrow\ {}^{5}C_1$

$z=8\ \Rightarrow$ No. of ways of solving $x+y=2\ \Rightarrow\ {}^{3}C_1$

$z=10\ \Rightarrow$ No. of ways of solving $x+y=0\ \Rightarrow\ 1$

$\therefore$ Total ways when z is even = 11 + 9 + 7 + 5 + 3 + 1 = 36

$\therefore$ Required probability $=\frac{36}{66}=\frac{6}{11}$

44. $g(x)=\int_{\sin x}^{\sin 2x}\sin^{-1}(t)\,dt$

$\Rightarrow g'(x)=\sin^{-1}(\sin 2x)\cdot 2\cos 2x-\sin^{-1}(\sin x)\cdot\cos x$

$g'\left(\frac{\pi}{2}\right)=\sin^{-1}(\sin\pi)\cdot 2\cos\pi-\sin^{-1}\left(\sin\frac{\pi}{2}\right)\cos\frac{\pi}{2}$

$= 0$

$$g'\left(-\frac{\pi}{2}\right) = \sin^{-1}(\sin(-\pi)) \cdot 2\cos(-\pi) -$$

$$\sin^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right) \cdot \cos\left(\frac{-\pi}{2}\right) = 0$$

∴ None of the options are matching here.

45. (A, C) If we consider $\tan \alpha/2 = x$ and $\tan \beta/2 = y$, then $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$

$$\Rightarrow\ 2\left[\frac{1-y^2}{1+y^2} - \frac{1-x^2}{1+x^2}\right] = 1 - \frac{(1-x^2)(1-y^2)}{(1+x^2)(1+y^2)}$$

$$\Rightarrow\ 2[(1+x^2)(1-y^2) - (1-x^2)(1+y^2)] = (1+x^2)(1+y^2) - (1-x^2)(1-y^2)$$

$$\Rightarrow\ 4(x^2 - y^2) = 2(x^2 + y^2)$$

$$\Rightarrow\ x^2 = 3y^2$$

$$\Rightarrow\ x = \pm\sqrt{3}\,y$$

$$\Rightarrow\ \tan\frac{\alpha}{2} \pm \sqrt{3}\tan\frac{\beta}{2} = 0$$

46. (A, C) $f'(x) - 2f(x) > 0$

$$\Rightarrow\ e^{-2x}\frac{df(x)}{dx} - 2e^{-2x}f(x) > 0$$

$$\Rightarrow\ \frac{d}{dx}\left(e^{-2x}f(x)\right) > 0$$

$\Rightarrow$ $e^{-2x}f(x)$ is an increasing function.

∴ for $x > 0$

$f(x) > f(0)$

$\Rightarrow$ $e^{-2x}f(x) > 1$

$\Rightarrow$ $f(x) > e^{2x}$ in $(0, \infty)$

Also $f'(x) > 2e^{2x} > 0$

∴ f is an increasing function in $(0, \infty)$

47. (A, D) $$\lim_{x\to 1^-} f(x) = \lim_{h\to 0}\frac{1-(1-h)[1+h]}{h}\cos\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0}\frac{1-1+h^2}{h}\cos\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} h\cos\left(\frac{1}{h}\right) = 0$$

$$\lim_{x\to 1^+} f(x) = \lim_{h\to 0}\frac{1-(1+h)(1+h)}{h}\cos\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0}\frac{-2h-h^2}{h}\cos\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0}(-2-h)\cos\left(\frac{1}{h}\right)$$

$= -2 \times$ (Some value oscillating between -1 and 1)

∴ does not exist.

48. (B, C) $$f(x) = \begin{vmatrix} \cos 2x & \cos 2x & \sin 2x \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$$

Operating $C_1 \to C_1 - C_2$

$$\Rightarrow\ f(x) = \begin{vmatrix} 0 & \cos 2x & \sin 2x \\ -2\cos x & \cos x & -\sin x \\ 0 & \sin x & \cos x \end{vmatrix}$$

$$\Rightarrow\ f(x) = 2\cos 3x\cos x$$

$$\Rightarrow\ f(x) = \cos 4x + \cos 2x$$

$$f_{\max} = 2 \text{ at } x = 0$$

$$f'(x) = -4\sin 4x - 2\sin 2x$$

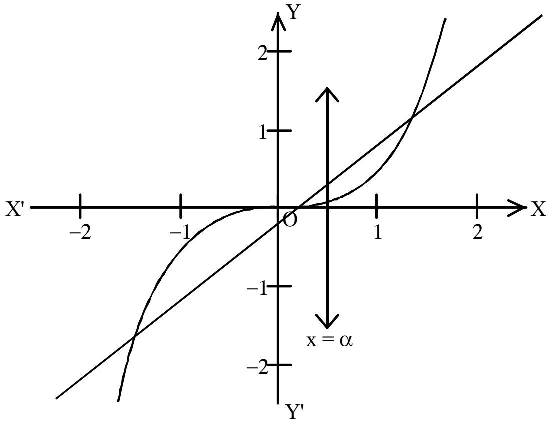
$$= -2\sin 2x\,[4\cos 2x + 1]$$

$$f'(x) = 0\ \Rightarrow\ \sin 2x = 0 \text{ or } \cos 2x = \frac{-1}{4}$$

$\Rightarrow$ $x = -\frac{\pi}{2}, 0, \frac{\pi}{2}$ which is true for some $x \in (-\pi, \pi)$

∴ $f'(x) = 0$ at more than three points in $(-\pi, \pi)$

49. (B, C) $$\int_0^\alpha (x - x^3)dx = \frac{1}{2}\int_0^1 (x - x^3)dx$$



$$\Rightarrow\ \left(\frac{x^2}{2} - \frac{x^4}{4}\right)_0^\alpha = \frac{1}{2}\left(\frac{x^2}{2} - \frac{x^4}{4}\right)_0^1$$

$$\Rightarrow\ \frac{\alpha^2}{2} - \frac{\alpha^4}{4} = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$\Rightarrow\ \frac{2\alpha^2 - \alpha^4}{4} = \frac{1}{8} \quad \text{or} \quad 4\alpha^2 - 2\alpha^4 = 1$$

$$\Rightarrow\ 2\alpha^4 - 4\alpha^2 + 1 = 0$$

$$\Rightarrow \quad \alpha^2 = \frac{4 \pm \sqrt{16-8}}{4} = 1 \pm \frac{1}{\sqrt{2}}$$

$$\therefore \quad 0 < \alpha < 1 \quad \Rightarrow \quad \alpha^2 = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \alpha = \sqrt{1-0.71} = \sqrt{0.29} > \sqrt{0.25} = \frac{1}{2} \text{ also } \alpha < 1$$

$$\Rightarrow \quad \frac{1}{2} < \alpha < 1$$

50. (B, D) $I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)} dx$

Let $x - k = t \Rightarrow dx = dt$

$$\therefore \quad I = \sum_{k=1}^{98} \int_0^1 \frac{k+1}{(t+k)(t+k+1)} dt$$

$$\Rightarrow \quad I > \sum_{k=1}^{98} \int_0^1 \frac{k+1}{(t+k+1)^2} dt$$

$$\Rightarrow \quad I > \sum_{k=1}^{98} (k+1)\left(\frac{-1}{t+k+1}\right)_0^1$$

$$\Rightarrow \quad I > \sum_{k=1}^{98} (k+1)\left(\frac{1}{k+1} - \frac{1}{k+2}\right)$$

$$\Rightarrow \quad I > \sum_{k=1}^{98} \frac{1}{k+2} = \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{100}$$

$$\Rightarrow \quad I > \frac{1}{100} + \frac{1}{100} + \ldots + \frac{1}{100} = \frac{98}{100} = \frac{49}{50}$$

$$\Rightarrow \quad I > \frac{49}{50}$$

Also $\frac{k+1}{x(x+1)} < \frac{k+1}{x(k+1)}$ [∵ least value of $x + 1$ is $k + 1$]

$$\Rightarrow \quad \frac{k+1}{x(x+1)} < \frac{1}{x}$$

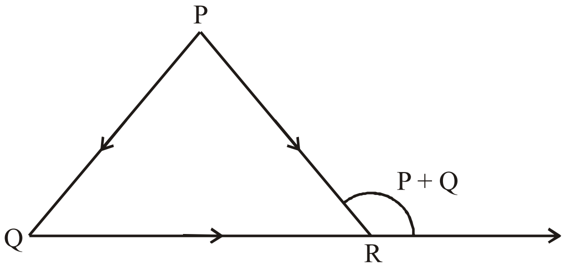
$$\therefore \quad I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)} dx < \sum_{k=1}^{98} \int_k^{k+1} \frac{1}{x} dx$$

$$\Rightarrow \quad I < \sum_{k=1}^{98} \log_e\left(\frac{k+1}{k}\right)$$

$$\Rightarrow \quad I < \log_e\left(\frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \ldots \times \frac{99}{98}\right)$$

$$\Rightarrow \quad I < \log_e 99$$

51. (A) $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ are unit vectors in the directions of sides $\overrightarrow{OR}, \overrightarrow{RP}$ and $\overrightarrow{PQ}$ respectively,



$$\therefore \quad \overrightarrow{OX} = \frac{\overrightarrow{QR}}{|\overrightarrow{QR}|}, \overrightarrow{OY} = \frac{\overrightarrow{RP}}{|\overrightarrow{RP}|}, \overrightarrow{OZ} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}$$

$$\therefore \quad |\overrightarrow{OX} \times \overrightarrow{OY}| = \frac{|\overrightarrow{OR} \times \overrightarrow{RP}|}{|\overrightarrow{QR}||\overrightarrow{RP}|}$$

$$= \frac{|\overrightarrow{QR}||\overrightarrow{RP}| \sin(P+Q)}{|\overrightarrow{QR}||\overrightarrow{RP}|}$$

$= \sin (P + Q)$

52. (B) $\cos (P + Q) + \cos (Q + R) + \cos (R + P)$
$= \cos (180 - R) + \cos (180 - P) + \cos (180 - Q)$
$= -[\cos P + \cos Q + \cos R]$

In any ΔPQR, $\cos P + \cos Q + \cos R \le \frac{3}{2}$

$$\Rightarrow \quad -(\cos P + \cos Q + \cos R) \ge -\frac{3}{2}$$

$\therefore$ Required minimum value $= -\frac{3}{2}$

53. (B) ∵ α, β are roots of $x^2 - x - 1 = 0$
$\therefore \alpha^2 - \alpha - 1 = 0, \beta^2 - \beta - 1 = 0$
$\Rightarrow \alpha^2 = \alpha + 1$ and $\beta^2 = \beta + 1$
Also $a_n = p\alpha^n + q\beta^n$
$\Rightarrow a_0 = p + q$
$a_1 = p\alpha + p\beta$
$a_2 = p\alpha^2 + q\beta^2 = p(\alpha + 1) + q(\beta + 1)$
$= (p\alpha + q\beta) + (p + q) = a_1 + a_0$
$a_3 = p\alpha^3 + q\beta^3 = p\alpha(\alpha + 1) + q\beta(\beta + 1)$
$= (p\alpha^2 + q\beta^2) + (p\alpha + q\beta)$
$= a_2 + a_1$
Proceeding in the same manner, we get
$a_{12} = a_{11} + a_{10}$

54. (D) $a_4 = a_3 + a_2 = a_2 + a_1 + a_2 = 2a_2 + a_1$
$= 2a_1 + 2a_0 + a_1 = 3a_1 + 2a_0$
$= 3(p\alpha + q\beta) + 2(p + q)$

$$= 3\left[p\left(\frac{1+\sqrt{5}}{2}\right) + q\left(\frac{1-\sqrt{5}}{2}\right)\right] + 2(p+q)$$

$$= \frac{7}{2}(p+q) + \frac{3}{2}(p-q)\sqrt{5} = 28$$

$\Rightarrow p = q = 4 \quad \therefore \quad p + 2q = 12$

JEE ADVANCED 2018

PAPER - 1

PHYSICS

SECTION - I (MAXIMUM MARKS: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.

  Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.

  Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.

  Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.

  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : **–2** In all other cases.
- **For Example:** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

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1. The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true?

(A) $v = \sqrt{\dfrac{k}{2m}}R$ (B) $v = \sqrt{\dfrac{k}{m}}R$

(C) $L = \sqrt{mk}R^2$ (D) $L = \sqrt{\dfrac{mk}{2}}R^2$

2. Consider a body of mass 1.0 kg at rest at the origin at time $t = 0$. A force $\vec{F} = (\alpha t\,\hat{i} + \beta\hat{j})$ is applied on the body, where $\alpha = 1.0\ \text{Ns}^{-1}$ and $\beta = 1.0\ N$. The torque acting on the body about the origin at time $t = 1.0\,s$ is $\vec{\tau}$. Which of the following statements is (are) true?

(A) $|\vec{\tau}| = \dfrac{1}{3}Nm$

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at $t = 1\,s$ is $\vec{v} = \dfrac{1}{2}(\hat{i} + 2\hat{j})ms^{-1}$

(D) The magnitude of displacement of the body at $t = 1s$ is $\dfrac{1}{6}m$

3. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true?

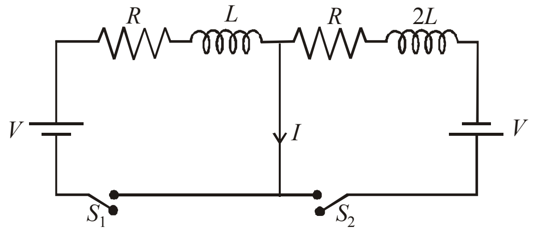
(A) For a given material of the capillary tube, h decreases with increase in r

(B) For a given material of the capillary tube, h is independent of σ

(C) If this experiment is performed in a lift going up with a constant acceleration, then h decreases

(D) h is proportional to contact angle θ

4. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude $I_{\max}$ at time $t = \tau$. Which of the following statements is (are) true?



(A) $I_{\max} = \dfrac{V}{2R}$ (B) $I_{\max} = \dfrac{V}{4R}$

(C) $\tau = \dfrac{L}{R}\ln 2$ (D) $\tau = \dfrac{2L}{R}\ln 2$

5. Two infinitely long straight wires lie in the xy-plane along the lines $x = \pm R$. The wire located at $x = +R$ carries a constant current I_1 and the wire located at $x = -R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true?

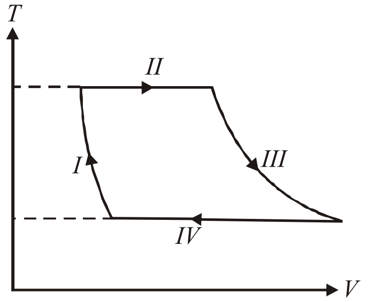
(A) If $I_1 = I_2$, then $\vec{B}$ **cannot** be equal to zero at the origin (0, 0, 0)

(B) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(C) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(D) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

6. One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (where ***V*** is the volume and ***T*** is the temperature). Which of the statements below is (are) true?



(A) Process I is an isochoric process

(B) In process II, gas absorbs heat

(C) In process IV, gas releases heat

(D) Processes I and III are **not** isobaric

SECTION - II (MAXIMUM MARKS: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If ONLY the correct numerical value is entered as answer.
  Zero Marks : **0** In all other cases.

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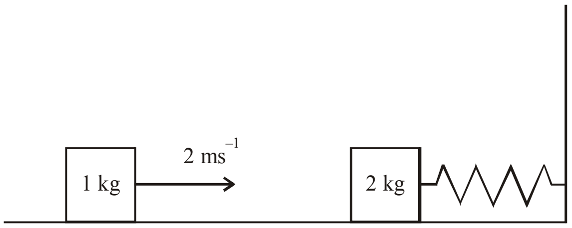
7. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a$ (cos $\omega t\,\hat{i}$ + sin $\omega t\,\hat{j}$), where a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A}+\vec{B}| = \sqrt{3}\,|\vec{A}-\vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in *seconds*, is __________.

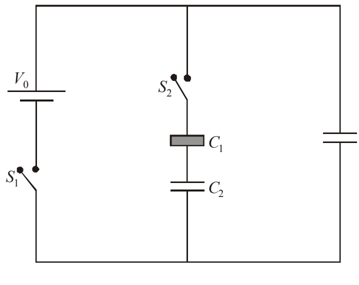
8. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 ms^{-1} and the man behind walks at a speed 2.0 ms^{-1}. A third man is standing at a height 12 *m* above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 ms^{-1}. At the instant, when the moving men are 10 *m* apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is __________.

9. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is __________. Take $g = 10\ ms^{-2}$.

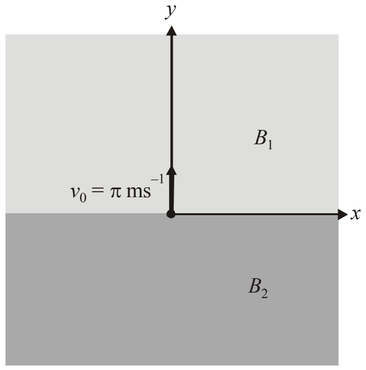
10. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 Nm^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 ms^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is _________.



11. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5 μC. The value of ε_r = ____________.

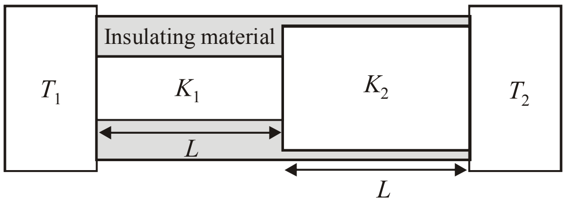


12. In the xy-plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{k}$ and the region $y < 0$ has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi \text{ ms}^{-1}$ at $t = 0$, as shown in the figure. Neglect gravity in this problem. Let $t = T$ be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is __________.



13. Sunlight of intensity 1.3 kW m^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is __________.

14. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures $T_1 = 300$ K and $T_2 = 100$ K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then K_1/K_2 = __________.



SECTION-III (MAXIMUM MARKS: 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If ONLY the correct option is chosen.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : –1 In all other cases.

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, $[E]$ and $[B]$ stand for dimensions of electric and magnetic fields respectively, while $[\varepsilon_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. $[L]$ and $[T]$ are dimensions of length and time respectively. All the quantities are given in SI units.

15. The relation between $[E]$ and $[B]$ is

(A) $[E] = [B][L][T]$ (B) $[E] = [B]\,[L]^{-1}\,[T]$

(C) $[E] = [B]\,[L]\,[T]^{-1}$ (D) $[E] = [B]\,[L]^{-1}\,[T]^{-1}$

16. The relation between $[\varepsilon_0]$ and $[\mu_0]$ is

(A) $[\mu_0] = [\varepsilon_0]\,[L]^2\,[T]^{-2}$ (B) $[\mu_0] = [\varepsilon_0]\,[L]^{-2}\,[T]^2$

(C) $[\mu_0] = [\varepsilon_0]^{-1}\,[L]^2\,[T]^{-2}$ (D) $[\mu_0] = [\varepsilon_0]^{-1}\,[L]^{-2}\,[T]^2$

PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = x/y$. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp (\Delta y/y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\Delta x/x \ll 1$, $\Delta y/y \ll 1$. Therefore, the higher powers of these quantities are neglected.

17. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa ($\Delta a/a \ll 1$, then what is the error Δr in determining r?

(A) $\frac{\Delta a}{(1+a)^2}$ (B) $\frac{2\Delta a}{(1+a)^2}$ (C) $\frac{2\Delta a}{(1-a^2)}$ (D) $\frac{2a\Delta a}{(1-a^2)}$

18. In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For $|x| \ll 1$, $\ln(1 + x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ, in s^{-1}, is

(A) 0.04 (B) 0.03 (C) 0.02 (D) 0.01

CHEMISTRY

SECTION - I (MAXIMUM MARKS: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.

  Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.

  Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.

  Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.

  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : **–2** In all other cases.
- **For Example:** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

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19. The compound(s) which generate(s) N_2 gas upon thermal decomposition below 300 °C is (are)

(A) NH_4NO_3 (B) $(NH_4)_2Cr_2O_7$

(C) $Ba(N_3)_2$ (D) Mg_3N_2

20. The correct statement(s) regarding the binary transition metal carbonyl compounds is (are) (Atomic numbers: Fe = 26, Ni = 28)

(A) Total number of valence shell electrons at metal centre in $Fe(CO)_5$ or $Ni(CO)_4$ is 16

(B) These are predominantly low spin in nature

(C) Metal–carbon bond strengthens when the oxidation state of the metal is lowered

(D) The carbonyl C–O bond weakens when the oxidation state of the metal is increased

21. Based on the compounds of group 15 elements, the correct statement(s) is (are)

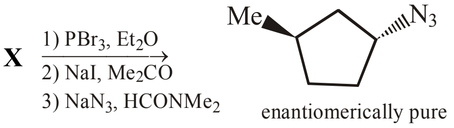
(A) Bi_2O_5 is more basic than N_2O_5

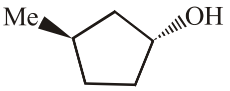
(B) NF_3 is more covalent than BiF_3

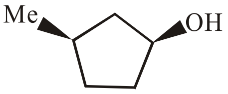
(C) PH_3 boils at lower temperature than NH_3

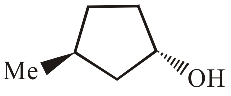
(D) The N–N single bond is stronger than the P–P single bond

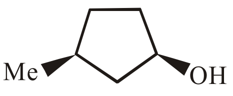
22. In the following reaction sequence, the correct structure(s) of **X** is (are)



(A)


(B)


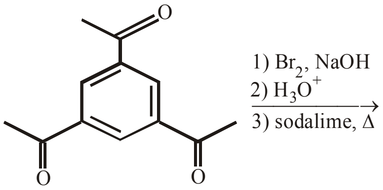
(C)


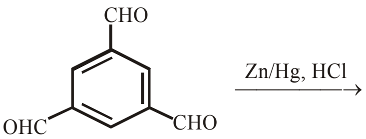
(D)


23. The reaction (s) leading to the formation of 1,3,5-trimethylbenzene is (are)

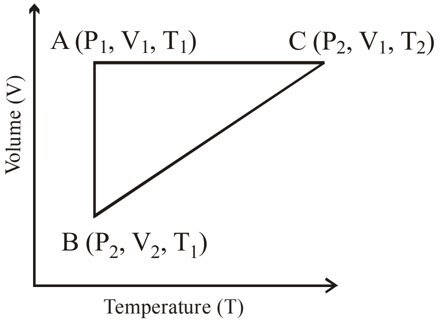
(A)


(B)


(C)


(D)


24. A reversible cyclic process for an ideal gas is shown below. Here, P, V, and T are pressure, volume and temperature, respectively. The thermodynamic parameters q, w, H and U are heat, work, enthalpy and internal energy, respectively.



The correct option(s) is (are)

(A) $q_{Ac} = \Delta U_{BC}$ and $w_{AB} = P_2(V_2 - V_1)$

(B) $w_{BC} = P_2(V_2 - V_1)$ and $q_{BC} = \Delta H_{AC}$

(C) $\Delta H_{CA} < \Delta U_{CA}$ and $q_{AC} = \Delta U_{BC}$

(D) $q_{BC} = \Delta H_{AC}$ and $\Delta H_{CA} > \Delta U_{CA}$

SECTION - II (MAXIMUM MARKS: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If ONLY the correct numerical value is entered as answer.
  Zero Marks : **0** In all other cases.

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25. Among the species given below, the total number of diamagnetic species is_______.

H atom, NO_2 monomer, O_2^-(superoxide), dimeric sulphur in vapour phase, Mn_3O_4, $(NH_4)_2[FeCl_4]$,$(NH_4)_2[NiCl_4]$, K_2MnO_4, K_2CrO_4

26. The ammonia prepared by treating ammonium sulphate with calcium hydroxide is completely used by $NiCl_2.6H_2O$ to form a stable coordination compound. Assume that both the reactions are 100% complete. If 1584 g of ammonium sulphate and 952 g of $NiCl_2.6H_2O$ are used in the preparation, the combined weight (in grams) of gypsum and the nickel-ammonia coordination compound thus produced is________.

(Atomic weights in g mol^{-1}: H = 1, N = 14, O = 16, S = 32, Cl = 35.5, Ca = 40, Ni = 59)

27. Consider an ionic solid **MX** with NaCl structure. Construct a new structure **(Z)** whose unit cell is constructed from the unit cell of **MX** following the sequential instructions given below. Neglect the charge balance.

(i) Remove all the anions **(X)** except the central one

(ii) Replace all the face centered cations **(M)** by anions **(X)**

(iii) Remove all the corner cations **(M)**

(iv) Replace the central anion **(X)** with cation **(M)**

The value of $\left(\frac{\text{number of anions}}{\text{number of cations}}\right)$ in **Z** is____.

28. For the electrochemical cell,

$Mg(s) \mid Mg^{2+}(aq, 1\ M) \parallel Cu^{2+}(aq, 1\ M) \mid Cu(s)$

the standard emf of the cell is 2.70 V at 300 K. When the concentration of Mg^{2+} is changed to x M, the cell potential changes to 2.67 V at 300 K. The value of x is_______.

(given, $\frac{F}{R} = 11500\,K\,V^{-1}$, where F is the Faraday constant and R is the gas constant, ln (10=2.30)

29. A closed tank has two compartments **A** and **B**, both filled with oxygen (assumed to be ideal gas). The partition separating the two compartments is fixed and is a perfect heat insulator (Figure 1). If the old partition is replaced by a new partition which can slide and conduct heat but does **NOT** allow the gas to leak across (Figure 2), the volume (in m^3) of the compartment **A** after the system attains equilibrium is_______.

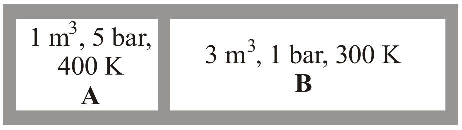


Figure 1

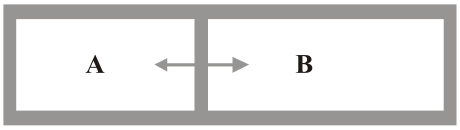


Figure 2

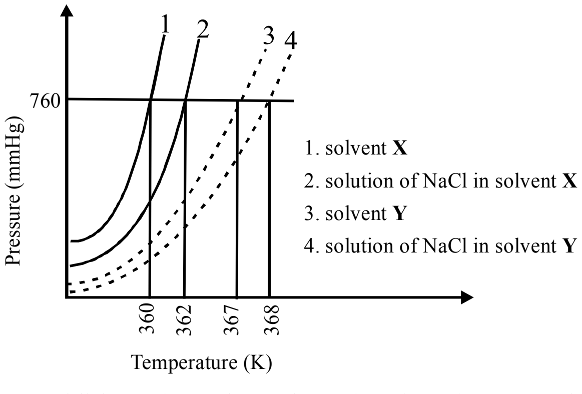
30. Liquids **A** and **B** form ideal solution over the entire range of composition. At temperature T, equimolar binary solution of liquids **A** and **B** has vapour pressure 45 Torr. At the same temperature, a new solution of A and **B** having mole fractions x_A and x_B, respectively, has vapour pressure of 22.5 Torr. The value of x_A/x_B in the new solution is________.

(given that the vapour pressure of pure liquid **A** is 20 Torr at temperature T)

31. The solubility of a salt of weak acid (**AB**) at pH 3 is $\mathbf{Y}\times10^{-3}$ mol L^{-1}. The value of **Y** is________.

(Given that the value of solubility product of **AB** (K_{sp}) = 2×10^{-10} and the value of ionization constant of **HB** (K_a) = 1×10^{-8})

32. The plot given below shows P — T curves (where P is the pressure and T is the temperature) for two solvents **X** and **Y** and isomolal solutions of NaCl in these solvents. NaCl completely dissociates in both the solvents.



On addition of equal number of moles of a non-volatile solute S in equal amount (in kg) of these solvents, the elevation of boiling point of solvent **X** is three times that of solvent **Y**. Solute **S** is known to undergo dimerization in these solvents. If the degree of dimerization is 0.7 in solvent Y, the degree of dimerization in solvent **X** is______.

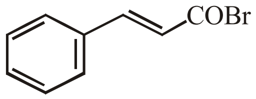
SECTION - III (MAXIMUM MARKS: 12)

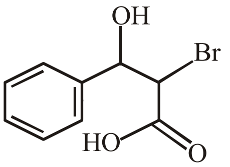
- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If ONLY the correct option is chosen.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : –1 In all other cases.

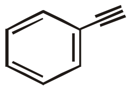
PARAGRAPH "X"

Treatment of benzene with CO/HCl in the presence of anhydrous $AlCl_3$/CuCl followed by reaction with Ac_2O/NaOAc gives compound **X** as the major product. Compound **X** upon reaction with Br_2/Na_2CO_3, followed by heating at 473 K with moist KOH furnishes **Y** as the major product. Reaction of **X** with H_2/Pd-C, followed by H_3PO_4 treatment gives **Z** as the major product.

33. The compound **Y** is

(A)


(B)


(C)


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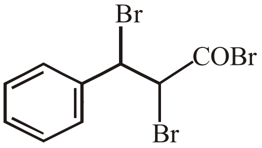
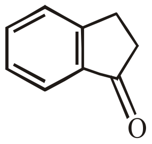


34. The compound **Z** is

(A)


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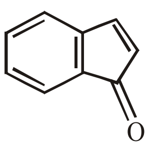

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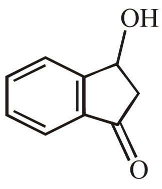

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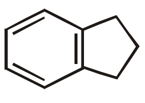
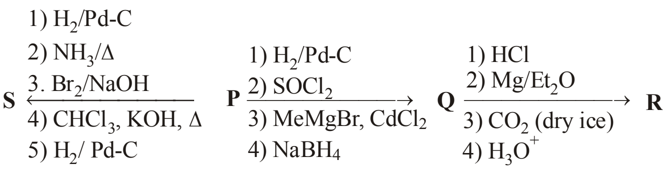


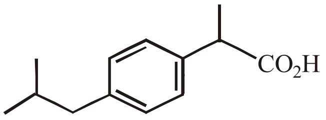
PARAGRAPH "A"

An organic acid **P** ($C_{11}H_{12}O_2$) can easily be oxidized to a dibasic acid which reacts with ethylene glycol to produce a polymer dacron. Upon ozonolysis, **P** gives an aliphatic ketone as one of the products. **P** undergoes the following reaction sequences to furnish R *via* **Q**. The compound **P** also undergoes another set of reactions to produce **S**.



$$\mathbf{S} \xleftarrow[\substack{4)\ CHCl_3,\ KOH,\ \Delta\\ 5)\ H_2/\ Pd\text{-}C}]{\substack{1)\ H_2/Pd\text{-}C\\ 2)\ NH_3/\Delta\\ 3.\ Br_2/NaOH}} \mathbf{P} \xrightarrow[\substack{3)\ MeMgBr,\ CdCl_2\\ 4)\ NaBH_4}]{\substack{1)\ H_2/Pd\text{-}C\\ 2)\ SOCl_2}} \mathbf{Q} \xrightarrow[\substack{3)\ CO_2\ (dry\ ice)\\ 4)\ H_3O^+}]{\substack{1)\ HCl\\ 2)\ Mg/Et_2O}} \mathbf{R}$$

35. The compound **R** is

(A)


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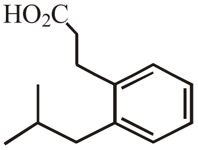

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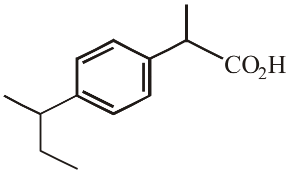

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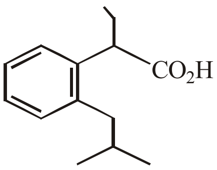
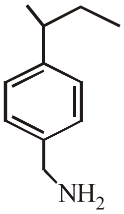
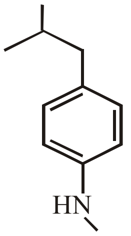
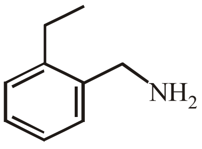


36. The compound **S** is

(A)


(B)


(C)


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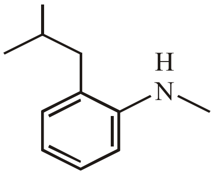

MATHEMATICS

SECTION - I (MAXIMUM MARKS: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
  Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.
  Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.

Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : **–2** In all other cases.

- **For Example:** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

---

37. For a non-zero complex number z, let arg(z) denote the principal argument with $-\pi < \arg(z) \le \pi$. Then, which of the following statement (s) is (are) **FALSE?**

(A) $\arg(-1-i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(B) The function $f:\mathbb{R} \to (-\pi, \pi]$, defined by $f(t) = \arg(-1+it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$

(C) For any two non-zero complex numbers z_1 and z_2,
$$\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$$
is an integer multiple of 2π

(D) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition
$$\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$$, lies on a straight line

38. In a triangle PQR, let ∠PQR = 30° and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE?

(A) ∠QPR = 45°

(B) The area of the triangle PQR is $25\sqrt{3}$ and ∠QRP = 120°

(C) The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$

(D) The area of the circumcircle of the triangle PQR is 100π.

39. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE?

(A) The line of intersection of P_1 and P_2 has direction ratios 1, 2, – 1

(B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$
is perpendicular to the line of intersection of P_1 and P_2

(C) The acute angle between P_1 and P_2 is 60°.

(D) If P_3 is the plane passing through the point (4, 2, – 2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$

40. For every twice differentiable function $f:\mathbb{R} \to [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE?

(A) There exist $r, s \in \mathbb{R}$, where $r < s$, such that f is one-one on the open interval (r, s)

(B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \le 1$

(C) $\lim_{x \to \infty} f(x) = 1$

(D) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \ne 0$

41. Let $f:\mathbb{R} \to \mathbb{R}$ and $g:\mathbb{R} \to \mathbb{R}$ be two non-constant differentiable functions. If
$$f'(x) = (e^{(f(x)-g(x))})g'(x) \text{ for all } x \in \mathbb{R},$$
and $f(1) = g(2) = 1$, then which of the following statement (s) is (are) TRUE ?

(A) $f(2) < 1 - \log_e 2$
(B) $f(2) > 1 - \log_e 2$
(C) $g(1) > 1 - \log_e 2$
(D) $g(1) < 1 - \log_e 2$

42. Let $f:[0, \infty) \to \mathbb{R}$ be a continuous function such that
$$f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\,dt$$
for all $x \in [0, \infty)$. Then, which of the following statement (s) is (are) TRUE?

(A) The curve $y = f(x)$ passes through the point (1, 2)

(B) The curve $y = f(x)$ passes through the point (2, –1)

(C) The area of the region
$$\left\{(x, y) \in [0,1] \times \mathbb{R} : f(x) \le y \le \sqrt{1-x^2}\right\} \text{ is } \frac{\pi - 2}{4}$$

(D) The area of the region
$$\left\{(x, y)\} \in [0,1] \times \mathbb{R} : f(x) \le y \le \sqrt{1-x^2}\right\} \text{ is } \frac{\pi - 1}{4}$$

SECTION - II (MAXIMUM MARKS: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : **+3** If ONLY the correct numerical value is entered as answer.
Zero Marks : **0** In all other cases.

43. The value of $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is __________.

44. The number of 5 digit numbers which are divisible by 4, with digits from the set $\{1, 2, 3, 4, 5\}$ and the repetition of digits is allowed, is _________.

45. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ..., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is ____.

46. The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right)$$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is ______.

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$, respectively.)

47. For each positive integer n, let

$$y_n = \frac{1}{n}(n+1)(n+2)...(n+n)^{\frac{1}{n}}$$

For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of $[L]$ is ______.

48. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}\cdot\vec{b} = 0$. For some $x, y \in \mathbb{R}$, let $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a}\times\vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is ________.

49. Let a, b, c be three non-zero real numbers such that the equation : $\sqrt{3}a\cos x + 2b\sin x = c, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is _______.

50. A farmer F_1 has a land in the shape of a triangle with vertices at P(0, 0), Q(1, 1) and R(2, 0). From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n (n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is ______.

SECTION - III (MAXIMUM MARKS: 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If ONLY the correct option is chosen.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : –1 In all other cases.

PARAGRAPH - X

Let S be the circle in the xy-plane defined by the equation

$x^2 + y^2 = 4.$

51. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope –1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3, and G_3 lie on the curve

(A) $x + y = 4$
(B) $(x-4)^2 + (y-4)^2 = 16$
(C) $(x-4)(y-4) = 4$
(D) $xy = 4$

52. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

(A) $(x+y)^2 = 3xy$
(B) $x^{2/3} + y^{2/3} = 2^{4/3}$
(C) $x^2 + y^2 = 2xy$
(D) $x^2 + y^2 = x^2y^2$

PARAGRAPH - A

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

53. The probability that, on examination day, the student S_1 gets the previously allotted seat R_1, and **NONE** of the remaining students gets the seat previously allotted to him/her is

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$ (C) $\frac{7}{40}$ (D) $\frac{1}{5}$

54. For i = 1, 2, 3, 4, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

PAPER - 2

PHYSICS

SECTION - I (MAXIMUM MARKS: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
  Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.
  Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.
  Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : **–2** In all other cases.
- **For Example:** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

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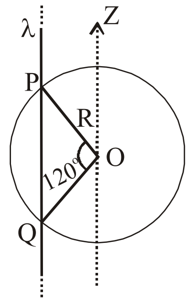
1. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true?

(A) The force applied on the particle is constant
(B) The speed of the particle is proportional to time
(C) The distance of the particle from the origin increases linearly with time
(D) The force is conservative

2. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity μ_0. Which of the following statements is (are) true?

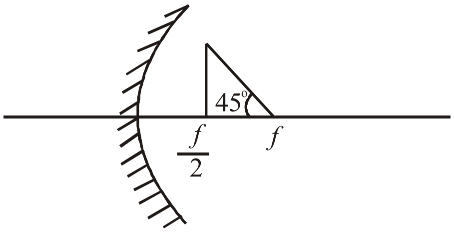
(A) The resistive force of liquid on the plate is inversely proportional to h
(B) The resistive force of liquid on the plate is independent of the area of the plate
(C) The tangential (shear) stress on the floor of the tank increases with μ_0
(D) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid

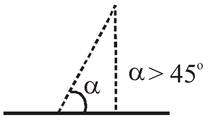
3. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is $\in_0$. Which of the following statements is (are) true?

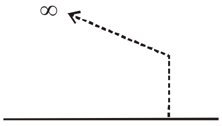


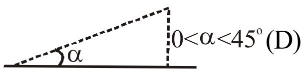
(A) The electric flux through the shell is $\sqrt{3}R\lambda/\in_0$
(B) The z-component of the electric field is zero at all the points on the surface of the shell
(C) The electric flux through the shell is $\sqrt{2}R\lambda/\in_0$
(D) The electric field is normal to the surface of the shell at all points

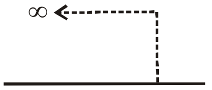
4. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.)



(A) 

(B) 

(C) 

(D) 

5. In a radioactive decay chain, ${}^{232}_{90}\text{Th}$ nucleus decays to ${}^{212}_{82}\text{Pb}$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true?

(A) $N_\alpha = 5$ (B) $N_\alpha = 6$ (C) $N_\beta = 2$ (D) $N_\beta = 4$

6. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 *Hz* is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 *cm* and 83.9 *cm.* Which of the following statements is (are) true?

(A) The speed of sound determined from this experiment is 332 ms^{-1}

(B) The end correction in this experiment is 0.9 *cm*

(C) The wavelength of the sound wave is 66.4 *cm*

(D) The resonance at 50.7 *cm* corresponds to the fundamental harmonic

SECTION - II (MAXIMUM MARKS: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+3** If ONLY the correct numerical value is entered as answer.

  Zero Marks : **0** In all other cases.

7. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4$ *kg* is at rest on this surface. An impulse of 1.0 *Ns* is applied to the block at time $t = 0$ so that it starts moving along the x -axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4s$. The displacement of the block, in *metres*, at $t = \tau$ is __________. Take $e^{-1} = 0.37$.

8. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 *m* and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in *metres*, is ___________.

9. A particle, of mass 10^{-3} *kg* and charge 1.0 *C*, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t \hat{i}$, where $E_0 = 1.0$ NC^{-1} and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is ____________.

10. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} $N\,m\,rad^{-1}$. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 –1.0 *A*. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in *ohms*, is __________.

11. A steel wire of diameter 0.5 *mm* and Young's modulus 2×10^{11} $N\,m^{-2}$ carries a load of mass *M*. The length of the wire with the load is 1.0 *m*. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 *mm*, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 *kg*, the vernier scale division which coincides with a main scale division is __________. Take $g = 10$ $m\,s^{-2}$ and $\pi = 3.2$.

12. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 *K* and the universal gas constant $R = 8.0$ $J\,mol^{-1}\,K^{-1}$, the decrease in its internal energy, in *Joule*, is__________.

13. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 *W* is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 *V* is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of *n* is __________. Mass of the electron $m_e = 9 \times 10^{-31}$ *kg* and 1.0 eV $= 1.6 \times 10^{-19}$ *J*.

14. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of *Z* is __________.

SECTION - III (MAXIMUM MARKS: 12)

- This section contains **FOUR (04)** questions.
- Each question has **TWO (02)** matching lists: **LIST-I** and **LIST-II**.
- **FOUR** options are given representing matching of elements from **LIST-I** and **LIST-II**. **ONLY ONE** of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme:

  Full Marks : **+3** If ONLY the option corresponding to the correct matching chosen.

  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : –1 In all other cases.

15. The electric field E is measured at a point P (0, 0, d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

LIST–I	**LIST–II**
P. E is independent of d	**1.** A point charge Q at the origin
Q. E $\propto 1/d$	**2.** A small dipole with point charges Q at $(0, 0, l)$ and $-Q$ at $(0, 0, -l)$. Take $2l << d$
R. E $\propto 1/d^2$	**3.** An infinite line charge coincident with the x-axis, with uniform linear charge density λ
S. E $\propto 1/d^3$	**4.** Two infinite wires carrying uniform linear charge density parallel to the x - axis. The one along $(y = 0, z = l)$ has a charge density $+\lambda$ and the one along $(y = 0, z = -l)$ has a charge density $-\lambda$Take $2l << d$
	5. Infinite plane charge coincident with the xy-plane with uniform surface charge density

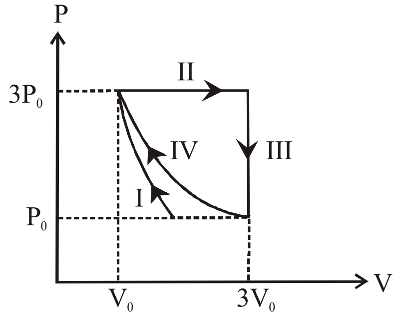
(A) $P \to 5; Q \to 3, 4; R \to 1; S \to 2$
(B) $P \to 5; Q \to 3; R \to 1, 4; S \to 2$
(C) $P \to 5; Q \to 3; R \to 1, 2; S \to 4$
(D) $P \to 4; Q \to 2, 3; R \to 1; S \to 5$

16. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively, Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2, and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II.

LIST–I	**LIST–II**
P. v_1/v_2	**1.** 1/8
Q. L_1/L_2	**2.** 1
R. K_1/K_2	**3.** 2
S. T_1/T_2	**4.** 8

(A) $P \to 4; Q \to 2; R \to 1; S \to 3$
(B) $P \to 3; Q \to 2; R \to 4; S \to 1$
(C) $P \to 2; Q \to 3; R \to 1; S \to 4$
(D) $P \to 2; Q \to 3; R \to 4; S \to 1$

17. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List-II.



LIST–I	**LIST–II**
P. In process I	**1.** Work done by the gas is zero
Q. In process II	**2.** Temperature of the gas remains unchanged
R. In process III	**3.** No heat is exchanged between the gas and its surroundings
S. In process IV	**4.** Work done by the gas is $6 P_0 V0$

(A) $P \to 4; Q \to 3; R \to 1; S \to 2$
(B) $P \to 1; Q \to 3; R \to 2; S \to 4$
(C) $P \to 3; Q \to 4; R \to 1; S \to 2$
(D) $P \to 3; Q \to 4; R \to 2; S \to 1$

18. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq \beta$ In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are conserved for that path.

LIST–I	**LIST–II**
P. $\vec{r}(t) = \alpha t \hat{i} + \beta t \hat{j}$	**1.** $\vec{p}$
Q. $\vec{r}(t) = \alpha \cos \omega t \ \hat{i} + \beta \sin \omega t \ \hat{j}$	**2.** $\vec{L}$
R. $\vec{r}(t) = \alpha(\cos \omega t \ \hat{i} + \sin \omega t \ \hat{j})$	**3.** K
S. $\vec{r}(t) = \alpha t \ \hat{i} + \frac{\beta}{2} t^2 \ \hat{j}$	**4.** U
	5. E

(A) $P \to 1, 2, 3, 4, 5; Q \to 2, 5; R \to 2, 3, 4, 5; S \to 5$
(B) $P \to 1, 2, 3, 4, 5; Q \to 3, 5; R \to 2, 3, 4, 5; S \to 2, 5$
(C) $P \to 2, 3, 4; Q \to 5; R \to 1, 2, 4; S \to 2, 5$
(D) $P \to 1, 2, 3, 5; Q \to 2, 5; R \to 2, 3, 4, 5; S \to 2, 5$

CHEMISTRY

SECTION - I (MAXIMUM MARKS: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
  Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.
  Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.
  Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : **–2** In all other cases.

- **For Example:** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

---

19. The correct option(s) regarding the complex $[Co(en)(NH_3)_3(H_2O)]^{3+}$ (en = $H_2NCH_2CH_2NH_2$) is (are)

(A) It has two geometrical isomers

(B) It will have three geometrical isomers if bidentate 'en' is replaced by two cyanide ligands

(C) It is paramagnetic

(D) It absorbs light at longer wavelength as compared to $[Co(en)(NH_3)_4]^{3+}$

20. The correct option(s) to distinguish nitrate salts of Mn^{2+} and Cu^{2+} taken separately is (are)

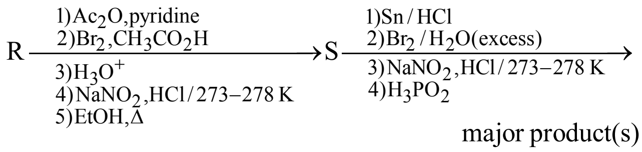
(A) Mn^{2+} shows the characteristic green colour in the flame test

(B) Only Cu^{2+} shows the formation of precipitate by passing H_2S in acidic medium

(C) Only Mn^{2+} shows the formation of precipitate by passing H_2S in faintly basic medium

(D) Cu^{2+}/Cu has higher reduction potential than Mn^{2+}/Mn (measured under similar conditions)

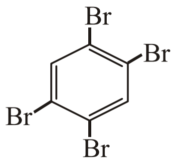
21. Aniline reacts with mixed acid (conc. HNO_3 and conc. H_2SO_4) at 288 K to give **P** (51 %),**Q** (47%) and **R** (2%). The major product(s) of the following reaction sequence is (are)



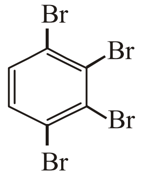
$$R \xrightarrow[\substack{3) H_3O^+ \\ 4) NaNO_2, HCl/273-278\ K \\ 5) EtOH, \Delta}]{\substack{1) Ac_2O, pyridine \\ 2) Br_2, CH_3CO_2H}} S \xrightarrow[\substack{3) NaNO_2, HCl/273-278\ K \\ 4) H_3PO_2}]{\substack{1) Sn/HCl \\ 2) Br_2/H_2O(excess)}}$$

major product(s)

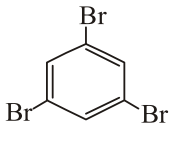
(A)



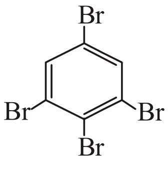
(B)



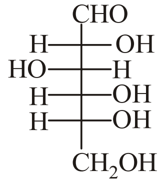
(C)



(D)



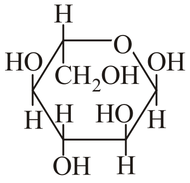
22. The Fischer presentation of D-glucose is given below.



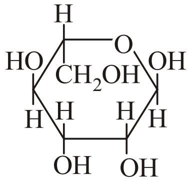
D-glucose

The correct structure(s) of β-L-glucopyranose is (are)

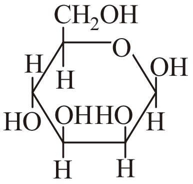
(A)



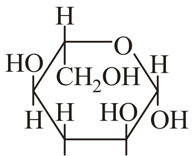
(B)



(C)

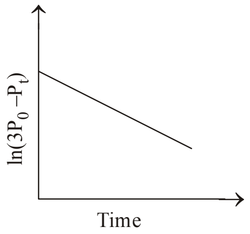


(D)

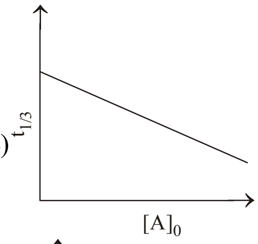


23 . For a first order reaction A(g) → 2B(g) + C(g) at constant volume and 300 K, the total pressure at the beginning (t = 0) and at time t are P_0 and P_t, respectively. Initially, only A is present with concentration $[A]_0$, and $t_{1/3}$ is the time required for the partial pressure of A to reach $1/3^{rd}$ of its initial value. The correct option(s) is (are) (Assume that all these gases behave as ideal gases)

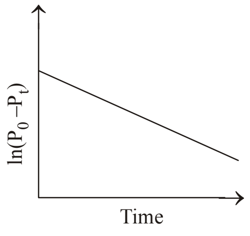
(A)



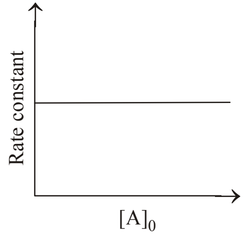
(B)



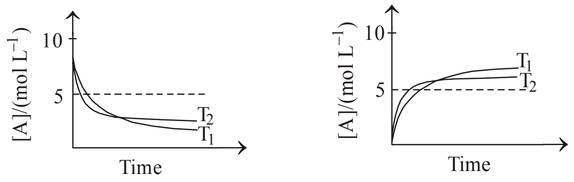
(C)



(D)



24. For a reaction, A ⇌ P, the plots of [A] and [P] with time at temperatures T_1 and T_2 are given below.



If $T_2 > T_1$, the correct statement(s) is (are)
(Assume $\Delta H°$ and $\Delta S°$ are independent of temperature and ratio of lnK at T_1 to lnK at T_2 is greater than $\frac{T_2}{T_1}$. Here H, S,G and K are enthalpy, entropy, Gibbs energy and equilibrium constant, respectively.)

(A) $\Delta H° < 0, \Delta S° < 0$ (B) $\Delta G° < 0, \Delta H° > 0$

(C) $\Delta G° < 0, \Delta S° < 0$ (D) $\Delta G° < 0, \Delta S° > 0$

SECTION - II (MAXIMUM MARKS: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+3** If ONLY the correct numerical value is entered as answer.

  Zero Marks : **0** In all other cases.

---

25. The total number of compounds having at least one bridging oxo group among the molecules given below is______. N_2O_3, N_2O_5, P_4O_6, P_4O_7, $H_4P_2O_5$, $H_5P_3O_{10}$, $H_2S_2O_3$, $H_2S_2O_5$

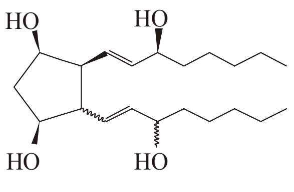
26. Galena (an ore) is partially oxidized by passing air through it at high temperature. After some time, the passage of air is stopped, but the heating is continued in a closed furnace such that the contents undergo self-reduction. The weight (in kg) of Pb produced per kg of O_2 consumed is______ .
(Atomic weights in g mol^{-1}: O = 16, S = 32, Pb = 207)

27. To measure the quantity of $MnCl_2$ dissolved in an aqueous solution, it was completely converted to $KMnO_4$ using the reaction,

$MnCl_2 + K_2S_2O_8 + H_2O \rightarrow KMnO_4 + H_2SO_4 + HCl$
(equation not balanced).

Few drops of concentrated HCl were added to this solution and gently warmed. Further, oxalic acid (225 mg) was added in portions till the colour of the permanganate ion disappeared. The quantity of $MnCl_2$ (in mg) present in the initial solution is ______.
(Atomic weights in g mol^{-1}: Mn = 55, Cl = 35.5)

28. For the given compound **X**, the total number of optically active stereoisomers is ______.



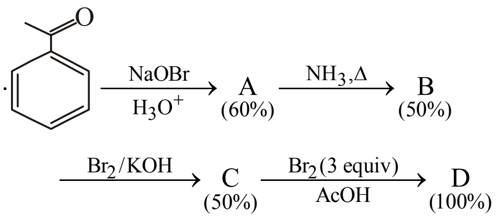
X

This type of bond indicates that the configuration at the specific carbon and the geometry of the double bond is fixed

This type of bond indicates that the configuration at the specific carbon and the geometry of the double bond is **NOT** fixed

29. In the following reaction sequence, the amount of **D** (in g) formed from 10 moles of acetophenone is ______.

(Atomic weights in g mol^{-1}: H = 1, C = 12, N = 14, O = 16, Br = 80. The yield (%) corresponding to the product in each step is given in the parenthesis)



30. The surface of copper gets tarnished by the formation of copper oxide. N_2 gas was passed to prevent the oxide formation during heating of copper at 1250 K. However, the N_2 gas contains 1 mole % of water vapour as impurity. The water vapour oxidises copper as per the reaction given below: $2Cu(s) + H_2O(g) \rightarrow Cu_2O(s) + H_2(g)$

p_{H_2} is the minimum partial pressure of H_2 (in bar) needed to prevent the oxidation at 1250 K. The value of $\ln(p_{H_2})$ is_____.

(Given: total pressure = 1 bar, R (universal gas constant) = 8 J K^{-1} mol^{-1}, ln (10) = 2.3. Cu (s) and Cu_2O (s) are mutually immiscible.

At 1250 K : $2\,Cu(s) + ½O_2(g) \rightarrow Cu_2O(s)$;
$\Delta G° = –78{,}000\ J\ mol^{-1}$

$H_2(g) + ½O_2(g) \rightarrow H_2O(g)$; $\Delta G° = –1{,}78{,}000 J\ mol^{-1}$;
(G is the Gibbs energy)

31. Consider the following reversible reaction,

$A(g) + B(g) \rightarrow AB(g)$.

The activation energy of the backward reaction exceeds that of the forward reaction by 2RT (in J mol^{-1}). If the pre-exponential factor of the forward reaction is 4 times that of the reverse reaction, the absolute value of $\Delta G°$ (in J mol^{-1}) for the reaction at 300 K is_____.

(Given; ln(2) = 0.7, RT = 2500 J mol^{-1} at 300 K and G is the Gibbs energy)

32. Consider an electrochemical cell:

$A(s)\,|\,A^{n+}\,(aq, 2\ M)\,||B^{2n+}\,(aq, 1\ M)\,|\,B(s)$.

The value of $\Delta H°$ for the cell reaction is twice that of $\Delta G°$ at 300 K. If the emf of the cell is zero, the $\Delta S°$ (in J K^{-1} mol^{-1}) of the cell reaction per mole of B formed at 300 K is______.

(Given: ln(2) = 0.7, R (universal gas constant) = 8.3 J K^{-1} mol^{-1}. H, S and G are enthalpy, entropy and Gibbs energy, respectively.)

SECTION - III (MAXIMUM MARKS: 12)

- This section contains **FOUR (04)** questions.
- Each question has **TWO (02)** matching lists: **LIST-I** and **LIST-II**.
- **FOUR** options are given representing matching of elements from **LIST-I** and **LIST-II**. **ONLY ONE** of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme:

  Full Marks : **+3** If ONLY the option corresponding to the correct matching chosen.

  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : –1 In all other cases.

33. Match each set of hybrid orbitals from LIST-I with complex(es) given in LIST-II

	List - I		List- II
P.	dsp^2	**1.**	$[FeF_6]^{4-}$
Q.	sp^3	**2.**	$[Ti(H_2O)_3Cl_3]$
R.	sp^3d^2	**3.**	$[Cr(NH_3)_6]^{3+}$
S.	d^2sp^3	**4.**	$[FeCl_4]^{2-}$
		5.	$Ni(CO)_4$
		6.	$[Ni(CN)_4]^{2-}$

The correct option is

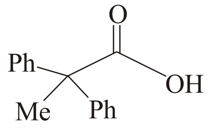
(A) P-5; Q-4, 6 ; R-2, 3; S-1

(B) P-5, 6; Q-4; R- 3; S-1, 2

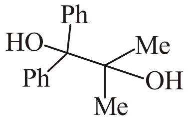
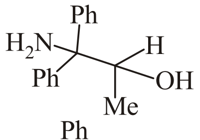
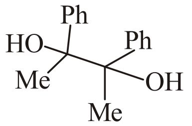
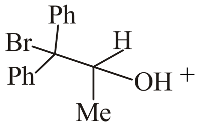
(C) P-6; Q-4, 5 ; R-1; S-2, 3

(D) P-4, 6; Q-5, 6 ; R-1, 2; S-3

34. The desired product **X** can be prepared by reacting the major product of the reactions in LIST-I with one or more appropriate reagents in LIST-II. (given, order of migratory aptitude: aryl > alkyl > hydrogen)



X

	List - I		List- II
P.	 $+ H_2SO_4$	**1.**	I_2, NaOH
Q.	 $+ HNO_2$	**2.**	$[Ag(NH_3)_2]OH$
R.	 $+ H_2SO_4$	**3.**	Fehling solution
S.	 $+ AgNO_3$	**4.**	HCHO, NaOH
		5.	NaOBr

The correct option is

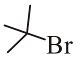
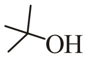
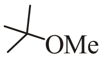
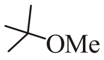
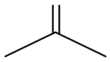
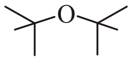
(A) P-1; Q-2, 3 ; R-1, 4; S-2, 4

(B) P-1, 5 ; Q-3, 4 ; R-4, 5; S-3

(C) P-1, 5; Q-3, 4 ; R-5; S-2, 4

(D) P-1, 5; Q-2, 3 ; R-1, 5; S-2, 3

35. LIST-I contains reactions and LIST-II contains major products.

	List - I			List- II
P.	 + 	⟶	**1.**	
Q.	 + HBr	⟶	**2.**	
R.	 + NaOMe	⟶	**3.**	
S.	 + MeBr	⟶	**4.**	
			5.	

Match the reaction in LIST-I with one or more products in LIST-II and choose the correct option.

(A) P-1, 5; Q-2 ; R-3; S-4

(B) P-1, 4 ; Q-2 ; R-4; S-3

(C) P-1, 4; Q-1, 2 ; R-3, 4; S-4

(D) P-4, 5; Q-4 ; R-4 ; S-3, 4

36 . Dilution processes of different aqueous solutions, with water, are given in LIST-I. The effects of dilution of the solutions on $[H^+]$ are given in LIST-II.

(Note: Degree of dissociation (a) of weak acid and weak base is << 1; degree of hydrolysis of salt <<1; $[H^+]$ represents the concentration of H^+ ions)

	LIST-I		LIST-II
P.	(10 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 60 mL	1.	the value of $[H^+]$ does not change on dilution
Q.	(20 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 80 mL	2.	the value of $[H^+]$ changes to half of its initial value on dilution
R.	(20 mL of 0.1 M HCl + 20 mL of 0.1 M ammonia solution) diluted to 80 mL	3.	the value of $[H^+]$ changes to two times of its initial value on dilution
S.	10 mL saturated solution of $Ni(OH)_2$ in equilibrium with excess solid $Ni(OH)_2$ is diluted to 20 mL (solid $Ni(OH)_2$ is still present after dilution).	4.	the value of $[H^+]$ changes to $\frac{1}{\sqrt{2}}$ times of its initial value on dilution
		5.	the value of $[H^+]$ changes to $\sqrt{2}$ times of its initial value on dilution

MATHEMATICS

SECTION - I (MAXIMUM MARKS: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
  Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.
  Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.
  Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : **–2** In all other cases.
- **For Example:** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

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37. For any positive integer n, define $f_n : (0, \infty) \to \mathbb{R}$ as

$$f_n\ (x)\ =\ \sum_{j=1}^{n}\ \tan^{-1}\left(\frac{1}{1+(x+j)\,(x+j-1)}\right) \text{ for all } x \in (0, \infty).$$

$\left(\begin{array}{l}\text{Here, the inverse trigonometric function } \tan^{-1} x \text{ assumes}\\ \text{values in} \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).\end{array}\right)$

Then, which of the following statement(s) is (are) TRUE?

(A) $\sum_{j=1}^{5} \tan^2(f_j(0)) = 55$

(B) $\sum_{j=1}^{10} (1 + f_j'(0)) \sec^2(f_j(0)) = 10$

(C) For any fixed positive integer n, $\lim\limits_{x\to\infty} \tan(f_n(x)) = \frac{1}{n}$

(D) For any fixed positive integer n, $\lim\limits_{x\to\infty} \sec^2(f_n(x)) = 1$

38. Let T be the line passing through the points $P(-2, 7)$ and $Q(2, -5)$. Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point $R(1, 1)$ be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE?

(A) The point $(-2, 7)$ lies in E_1

(B) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie in E_2

(C) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2

(D) The point $\left(0, \frac{3}{2}\right)$ does **NOT** lie in E_1

39. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that b_1, $b_2, b_3, \in \mathbb{R}$ and the system of equations (in real variables)

$$-x + 2y + 5z = b_1$$
$$2x - 4y + 3z = b_2$$
$$x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

(A) $x + 2y + 3z = b_1, 4y + 5z = b_2$ and $x + 2y + 6z = b_3$

(B) $x + y + 3z = b_1, 5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$

(C) $-x + 2y - 5z = b_1, 2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$

(D) $x + 2y + 5z = b_1, 2x + 3z = b_2$ and $x + 4y - 5z = b_3$

40. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin $O(0, 0)$ and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE?

(A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(C) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$

(D) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and x = 1 is $\frac{1}{16}(\pi - 2)$

41. Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy$ $(x, y, \in \mathbb{R}, i = \sqrt{-1})$ of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE?

(A) If L has exactly one element, then $|s| \neq |t|$

(B) If $|s| = |t|$, then L has infinitely many elements

(C) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(D) If L has more than one element, then L has infinitely many elements

42. Let $f : (0, \pi) \to \mathbb{R}$ be a twice differentiable function such that

$$\lim_{t \to x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x \text{ for all } x \in (0, \pi).$$

If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE?

(A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$

(B) $f(x) < \frac{x^4}{6} - x^2$ for all $x \in (0, \pi)$

(C) There exists $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$

(D) $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

SECTION - II (MAXIMUM MARKS: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+3** If ONLY the correct numerical value is entered as answer.

  Zero Marks : **0** In all other cases.

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43. The value of the integral

$$\int_0^{\frac{1}{2}} \frac{1 + \sqrt{3}}{((x+1)^2 (1-x)^6)^{\frac{1}{4}}} dx \quad \text{is \_\_\_\_\_ .}$$

44. Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1, 0, 1\}$. Then, the maximum possible value of the determinant of P is _____ .

45. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is _____ .

46. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = (2 + 5y)(5y - 2),$$

then the value of $\lim\limits_{x \to -\infty} f(x)$ is _____.

47. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 1$ and satisfying the equation

$$f(x + y) = f(x) f'(y) + f'(x) f(y) \text{ for all } x, y \in \mathbb{R}$$

Then, the value of $\log_e(f(4))$ is _____.

48. Let P be a point in the first octant, whose image Q in the plane $x + y = 3$ (that is, the line segment PQ is perpendicular to the plane $x + y = 3$ and the mid-point of PQ lies in the plane $x + y = 3$) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is _____ .

49. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where $O(0, 0, 0)$ is the origin. Let $S\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p} = \overrightarrow{SP}$, $\vec{q} = \overrightarrow{SQ}$, $\vec{r} = \overrightarrow{SR}$ and $\vec{t} = \overrightarrow{ST}$, then the value of $|(\vec{p} \times \vec{q}) \times (\vec{r} \times \vec{t})|$ is _____ .

50. Let

$$X = ({}^{10}C_1)^2 + 2({}^{10}C_2)^2 + 3({}^{10}C_3)^2 + \cdots + 10({}^{10}C_{10})^2,$$

where ${}^{10}C_r$, $r \in \{1, 2, \cdots, 10\}$ denote binomial coefficients.

Then, the value of $\frac{1}{1430} X$ is _____ .

SECTION - III (MAXIMUM MARKS: 12)

- This section contains **FOUR (04)** questions.
- Each question has **TWO (02)** matching lists: **LIST-I** and **LIST-II**.
- **FOUR** options are given representing matching of elements from **LIST-I** and **LIST-II**. **ONLY ONE** of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme:

  Full Marks : **+3** If ONLY the option corresponding to the correct matching chosen.

  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : –1 In all other cases.

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51. Let $E_1 = \left\{x \in \mathbb{R} : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$

and $E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{ is a real number}\right\}$.

(Here, the inverse trigonometric function $\sin^{-1} x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$).

Let $f : E_1 \rightarrow \mathbb{R}$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$ and $g : E_2 \rightarrow \mathbb{R}$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$.

	LIST - I		LIST - II
P.	The range of f is	**1.**	$\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
Q.	The range of g contains	**2.**	$(0, 1)$
R.	The domain of f contains	**3.**	$\left[-\frac{1}{2}, \frac{1}{2}\right]$
S.	The domain of g is	**4.**	$(-\infty, 0) \cup (0, \infty)$
		5.	$\left(-\infty, \frac{e}{e-1}\right]$
		6.	$(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is:

(A) $P \rightarrow 4;\ Q \rightarrow 2;\ R \rightarrow 1;\ S \rightarrow 1$
(B) $P \rightarrow 3;\ Q \rightarrow 3;\ R \rightarrow 6;\ S \rightarrow 5$
(C) $P \rightarrow 4;\ Q \rightarrow 2;\ R \rightarrow 1;\ S \rightarrow 6$
(D) $P \rightarrow 4;\ Q \rightarrow 3;\ R \rightarrow 6;\ S \rightarrow 5$

52. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

	LIST - I		LIST - II
P.	The value of α_1 is	**1.**	136
Q.	The value of α_2 is	**2.**	189
R.	The value of α_3 is	**3.**	192
S.	The value of α_4 is	**4.**	200
		5.	381
		6.	461

The correct option is:

(A) $P \rightarrow 4;\ Q \rightarrow 6;\ R \rightarrow 2;\ S \rightarrow 1$
(B) $P \rightarrow 1;\ Q \rightarrow 4;\ R \rightarrow 2;\ S \rightarrow 3$
(C) $P \rightarrow 4;\ Q \rightarrow 6;\ R \rightarrow 5;\ S \rightarrow 2$
(D) $P \rightarrow 4;\ Q \rightarrow 2;\ R \rightarrow 3;\ S \rightarrow 1$

53. Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a > b > 0$, be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of 60° at one of its vertices N. Let the area of the triangle LMN be $4\sqrt{3}$.

	LIST - I		LIST - II
P.	The length of the conjugate axis of H is	**1.**	8
Q.	The eccentricity of H is	**2.**	$\frac{4}{\sqrt{3}}$
R.	The distance between the foci of H is	**3.**	$\frac{2}{\sqrt{3}}$
S.	The length of the latus rectum of H is	**4.**	4

The correct option is:

(A) $P \rightarrow 4;\ Q \rightarrow 2;\ R \rightarrow 1;\ S \rightarrow 3$
(B) $P \rightarrow 4;\ Q \rightarrow 3;\ R \rightarrow 1;\ S \rightarrow 2$
(C) $P \rightarrow 4;\ Q \rightarrow 1;\ R \rightarrow 3;\ S \rightarrow 2$
(D) $P \rightarrow 3;\ Q \rightarrow 4;\ R \rightarrow 2;\ S \rightarrow 1$

54. Let $f_1 : \mathbb{R} \rightarrow \mathbb{R}$, $f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$, $f_3 : \left(-1, e^{\frac{\pi}{2}} - 2\right) \rightarrow \mathbb{R}$ and $f_4 : \mathbb{R} \rightarrow \mathbb{R}$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right)$,

(ii) $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

(iii) $f_3(x) = [\sin(\log_e(x + 2))]$, where, for $t \in \mathbb{R}$, $[t]$ denotes the greatest integer less than or equal to t,

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

	LIST - I		LIST - II
P.	The function f_1 is	**1.**	**NOT** continuous at $x = 0$
Q.	The function f_2 is	**2.**	continuous at $x = 0$ and **NOT** differentiable at $x = 0$
R.	The function f_3 is	**3.**	differentiable at $x = 0$ and its derivative is **NOT** continuous at $x = 0$
S.	The function f_4 is	**4.**	differentiable at $x = 0$ and its derivative is continuous at $x = 0$

The correct option is:

(A) $P \rightarrow 2;\ Q \rightarrow 3;\ R \rightarrow 1;\ S \rightarrow 4$
(B) $P \rightarrow 4;\ Q \rightarrow 1;\ R \rightarrow 2;\ S \rightarrow 3$
(C) $P \rightarrow 4;\ Q \rightarrow 2;\ R \rightarrow 1;\ S \rightarrow 3$
(D) $P \rightarrow 2;\ Q \rightarrow 1;\ R \rightarrow 4;\ S \rightarrow 3$

SOLUTIONS

Paper - 1

Physics

1. **(B, C)** We know that, $|F| = \frac{dv}{dr} = \frac{d}{dr}\left[\frac{kr^2}{2}\right] = kr$

$\because$ Potential energy, V(r) = $Kr^2/2$ given

For $r = R$, $F = kR$

Also $F = \frac{mv^2}{R}$ (circular motion)

$\therefore \quad \frac{mv^2}{R} = kR \quad \therefore \ v = \sqrt{\frac{k}{m}} \times R$

Angular momentum $L = mvR = m\left(\sqrt{\frac{k}{m}}R\right)R = \sqrt{km}R^2$

2. **(A, C)** Given $\vec{F} = t\hat{i} + \hat{j} \quad \therefore \ \frac{md\vec{v}}{dt} = t\,\hat{i} + \hat{j}$

$\therefore d\vec{v} = t\,dt\,\hat{i} + dt\,\hat{j} \quad [\because \ m = 1]$

$\therefore \int_0^v d\vec{v} = \int_0^t t\,dt\,\hat{i} + \int_0^t dt\,\hat{j}$

$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$

At $t = 1s$, $\vec{v} = \frac{1}{2}\hat{i} + \hat{j} = \frac{1}{2}(\hat{i} + 2\hat{j})\text{ms}^{-1}$

Further $\frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j}$

$\therefore \quad d\vec{r} = \frac{t^2}{2}dt\,\hat{i} + t\,dt\,\hat{j}$

$\therefore \quad \int_0^r d\vec{r} = \int_0^t \frac{t^2}{2}dt\,\hat{i} + \int_0^t t\,dt\,\hat{j}$

$\Rightarrow \quad \vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$

At $t = 1$, $\vec{r} = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j} \quad \therefore \ |\vec{r}| = \sqrt{\frac{1}{36} + \frac{1}{4}} = \sqrt{\frac{10}{36}}$

$\vec{\tau} = \vec{r} \times \vec{F} = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right) \times (\hat{i} + \hat{j})$ (At $t = 1$s)

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{6} & \frac{1}{2} & 0 \\ 1 & 1 & 0 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}\left(\frac{1}{6} - \frac{1}{2}\right) = -\frac{1}{3}\hat{k}$$

$\therefore \quad |\vec{\tau}| = \frac{1}{3}$ Nm

3. **(A, C)** We know that $h = \frac{2\sigma\cos\theta}{r\rho g_{eff}}$

As 'r' increases, h decreases

[all other parameter remaining constant]

Also $h \propto \sigma$

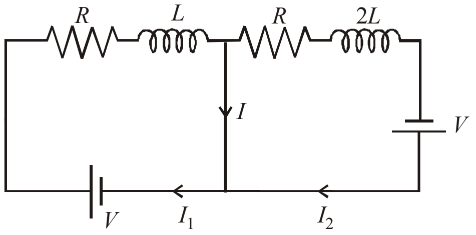
Further if lift is going up with an acceleration 'a' then $g_{\text{eff}} = g + a$. As g_{eff} increases, 'h' decreases.

Also h is not proportional to 'θ' but $h \propto \cos\theta$

4. **(B, D)** Here $I + I_2 = I_1 \quad \therefore \ I = I_1 - I_2$

$\therefore \quad I = \frac{V}{R}\left[1 - e^{\frac{-Rt}{2L}}\right] - \frac{V}{R}\left[1 - e^{\frac{-Rt}{L}}\right]$

$\Rightarrow I = \frac{V}{R}\left[e^{\frac{-Rt}{L}} - e^{\frac{-Rt}{2L}}\right]$



For I to be maximum, $\frac{dI}{dt} = 0$

$\therefore \quad \frac{V}{R}\left[\frac{-R}{L}e^{\frac{-Rt}{L}} - \left(\frac{-R}{2L}\right)e^{\frac{-Rt}{2L}}\right] = 0$

$\therefore \quad e^{\frac{-Rt}{2L}} = \frac{1}{2} \Rightarrow \left(\frac{R}{2L}\right)t = \ln 2$

$\therefore \quad t = \frac{2L}{R}\ln 2$

This is the time when I is maximum

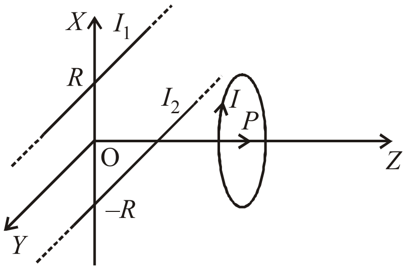
Further $I_{\max} = \frac{V}{R}\left[e^{\frac{-R}{L}\left(\frac{2L}{R}\ell n2\right)} - e^{\frac{-R}{2L}\left(\frac{2L}{R}\ell n2\right)}\right]$

$\Rightarrow I_{\max} = \frac{V}{R}\left[\frac{1}{4} - \frac{1}{2}\right]$

$\therefore \quad I_{\max} = \frac{V}{4R}$

5. **(A, B, D)** If $I_1 = I_2$, then the magnetic fields due to I_1 and I_2 at origin 'O' will cancel out each other. But the magnetic field at 'O' due to the circular loop will be present. Therefore 'A' is correct.

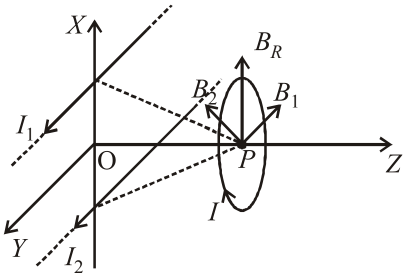
If $I_1 > 0$ and $I_2 < 0$, then the magnetic field due to both current will be in $+Z$ direction and add-up. The magnetic field due to current I will be in $-Z$ direction and if its magnitude is equal to the combined magnitudes of I_1 and I_2, then $\vec{B}$ can be zero at the origin. Therefore option 'B' is correct.



If $I_1 < 0$ and $I_2 > 0$ then their net magnetic field at origin will be in $-Z$ direction and the magnetic field due to I at origin will also be in $-Z$ direction. Therefore $\vec{B}$ at origin cannot be zero. Therefore 'C' is incorrect.

If $I_1 = I_2$ then the resultant of the magnetic field B_R at P (the centre of the circular loop) is along $+X$ direction. Therefore the magnetic field at P is only due to the current I which is in $-Z$ direction and is equal to

$$\vec{B} = \frac{\mu_0 I}{2R}(-\hat{k}).$$



Therefore option 'D' is correct.

6. **(B, C, D)** In process I, volume is changing. Therefore it is not isochoric. Therefore 'A' is incorrect.

In process II, $q = \Delta U + W$. $\Delta U = 0$ as temperature is constant. Therefore $q = W$. Here $W = P(V_f - V_i)$ is positive therefore q is positive i.e., gas absorbs heat. Therefore 'B' is correct.

For process IV, $q = \Delta U + W$. Here $\Delta U = 0$ and W is negative (volume decreases). Therefore q is negative i.e., gas releases heat. 'C' is correct.

For an isobaric process, $V \propto T$ i.e., we will get a straight inclined line in T-V graph. Therefore I and II are NOT isobaric. 'D' is correct.

7. **(2.00)**

$$|\vec{A} + \vec{B}| = \sqrt{3}\,|\vec{A} - \vec{B}|$$

$$\therefore \quad |a\,\hat{i} + a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j}| = \sqrt{3}\,|a\,\hat{i} - a\cos\omega t\,\hat{i} - a\sin\omega t\,\hat{j}|$$

$$\Rightarrow \quad |(1+\cos\omega t)\hat{i} + \sin\omega t\,\hat{j}| = \sqrt{3}\,|(1-\cos\omega t)\hat{i} - \sin\omega t\,\hat{j}|$$

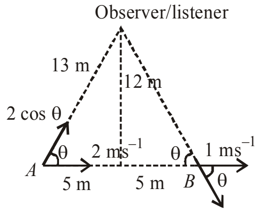
$$\sqrt{2 + 2\cos\omega t} = \sqrt{3}\,\sqrt{2 - 2\cos\omega t}$$

$$\therefore \quad 1 + \cos\omega t = 3(1 - \cos\omega t)$$

$$\Rightarrow \quad 4\cos\omega t = 2 \qquad \therefore\ \cos\omega t = \frac{1}{2} \qquad or,\ \omega t = \frac{\pi}{3}$$

$$\therefore \quad \frac{\pi}{6} \times \tau = \frac{\pi}{3} \qquad \therefore\ \tau = 2.00 \text{ seconds}$$

8. **(5.00)**



$$\nu_A = \nu\left[\frac{v}{v - 2\cos\theta}\right]$$

$$\nu_B = \nu\left[\frac{v}{v + 1\cos\theta}\right]$$

$$\therefore \quad \text{Beat frequency} = \nu\left[\frac{v}{v - 2\cos\theta}\right] - \nu\left[\frac{v}{v + \cos\theta}\right]$$

$$= \nu\, v\left[\frac{1}{v - 2\cos\theta} - \frac{1}{v + \cos\theta}\right]$$

$$= 1430 \times 330\left[\frac{1}{330 - 2\times\frac{5}{13}} - \frac{1}{330 + \frac{5}{13}}\right]$$

$$= 1430 \times 330 \times 13\left[\frac{1}{330\times 13 - 10} - \frac{1}{330 \times 13 + 5}\right]$$

$$= 1430 \times 330 \times 13\left[\frac{1}{4280} - \frac{1}{4295}\right] \approx 5 \text{ Hz}$$

9. **(0.75)**

The time taken to reach the ground is given by

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1 + \frac{I_C}{MR^2}\right)}$$

For ring $\quad t_1 = \dfrac{1}{\sin 60^\circ}\sqrt{\dfrac{2h}{g}\left(1 + \dfrac{MR^2}{MR^2}\right)} = \dfrac{4}{\sqrt{3}}\sqrt{\dfrac{h}{g}}$

For disc $\quad t_2 = \dfrac{1}{\sin 60^\circ}\sqrt{\dfrac{2h}{g}\left(1 + \dfrac{\frac{1}{2}MR^2}{MR^2}\right)} = \dfrac{2}{\sqrt{3}}\sqrt{\dfrac{3h}{g}}$

Given $\quad t_1 - t_2 = \dfrac{2 - \sqrt{3}}{\sqrt{10}}$

$$\therefore \quad \frac{4}{\sqrt{3}}\sqrt{\frac{h}{g}} - \frac{2}{\sqrt{3}}\sqrt{\frac{3h}{g}} = \frac{2 - \sqrt{3}}{\sqrt{10}}$$

$$2\sqrt{\frac{h}{10}} - \sqrt{\frac{3h}{10}} = \frac{(2 - \sqrt{3})}{\sqrt{10}}\left(\frac{\sqrt{3}}{2}\right)$$

$$2\sqrt{h}-\sqrt{3h}=\sqrt{3}-\frac{3}{2}$$

$$\sqrt{h}\,(2-1.732)=1.732-1.5 \qquad \therefore\ \sqrt{h}=\frac{0.232}{0.268}$$

$$\therefore h \approx 0.75 \text{ m}$$

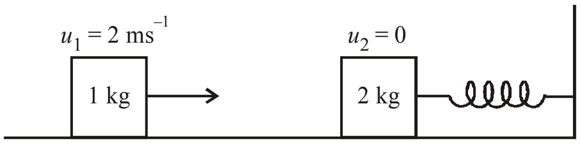
10. **(2.09)**

$$v_1=\frac{(m_1-m_2)u_1}{m_1+m_2}+\frac{2m_2u_2}{m_1+m_2}=\frac{(1-2)2}{1+2}=\frac{-2}{3}\text{ ms}^{-1}$$

$$v_2=\frac{(m_2-m_1)u_2}{m_1+m_2}+\frac{2m_1u_1}{m_1+m_2}=\frac{2\times1\times2}{1+2}=\frac{4}{3}\text{ ms}^{-1}$$

The time period of mass 2 kg after attaining velocity is

$$T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{2}{2}}=2\pi$$



Therefore the time taken to return the original position by 2 kg mass is π sec.

∴ Distance between the two blocks

$$=\frac{2}{3}\times\pi=\frac{2}{3}\times\frac{22}{7}=2.09 \text{ m}$$

11. **(1.50)**

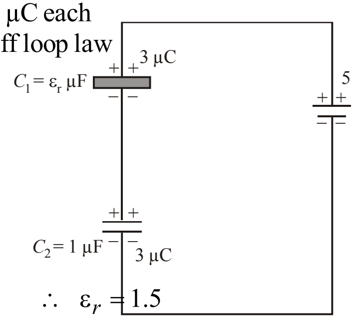
Initially

The charge on C_3 is $q_3 = C_3V = 1 \times 8\ \mu C = 8\ \mu C$

Finally

As the charge on C_3 is found to be 5 μC therefore charges on C_1 and C_2 are 3 μC each

Applying Kirchhoff loop law



$$\frac{5}{1}-\frac{3}{\varepsilon_r}-\frac{3}{1}=0$$

$$\therefore 5 = 3\left[1+\frac{1}{\varepsilon_r}\right]$$

$$\therefore\ \frac{1}{\varepsilon_r}=\frac{5}{3}-1=\frac{2}{3} \qquad \therefore\ \varepsilon_r=1.5$$

12. **(2.00)**

$$\text{Average speed along X-axis}=\frac{D_1+D_2}{t_1+t_2}=\frac{2(R_1+R_2)}{t_1+t_2}$$

$$=2\left[\frac{\dfrac{mv_0}{qB_1}+\dfrac{mv_0}{q(4B_1)}}{\dfrac{\pi m}{qB_1}+\dfrac{\pi m}{q(4B_1)}}\right]$$

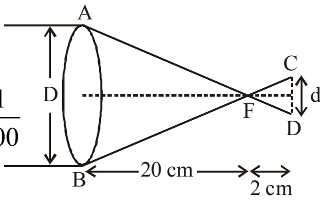
But $V_0 = \pi$

∴ Average speed along X-axis = 2.00

13. **(130.00)**

ΔAFB and ΔCFD are similar

$$\frac{d}{D}=\frac{2}{20}=\frac{1}{10}$$



$$\therefore\ \text{Ratio of area}=\frac{d^2}{D^2}=\frac{1}{100}$$

As there is no energy loss

∴ Average intensity of light at a distance 22 cm

$$=\frac{1.3\times\pi D^2/4}{\pi d^2/4}=1.3\times100=130.00\,\text{kWm}^{-2}$$

14. **(4.00)**

The intermediate temperature is given by the formula

$$T=\frac{\dfrac{k_1A_1T_1}{l_1}+\dfrac{k_2A_2T_2}{l_2}}{\dfrac{k_1A_1}{l_1}+\dfrac{k_2A_2}{l_2}}$$

Here, $T = 200$ k, $T_1 = 300$ k, $T_2 = 100$ k

$l_1 = l_2$ and $A_1 = \pi r^2$, $A_2 = 4\pi r^2$

$$\therefore\ 200=\frac{k_1\pi r^2\times300+k_2\pi(4r^2)100}{k_1\pi r^2+k_2\pi(4r^2)}$$

$$\therefore\ 200=\frac{300k_1+400k_2}{k_1+4k_2}$$

$$\therefore\ 200\,k_1+800\,k_2=300\,k_1+400\,k_2$$

$$\Rightarrow\ 400\,k_2=100\,k_1 \quad \therefore\ \frac{k_1}{k_2}=4.00$$

15. **(C)** We know that, $C=\dfrac{E}{B}$ where C = speed of light

$\therefore\ E = CB = LT^{-1}B$

16. **(D)** We know that

$$C=\frac{1}{\sqrt{\mu_0\varepsilon_0}} \quad \therefore\ C^2=\frac{1}{\mu_0\varepsilon_0}$$

$$\therefore\ \mu_0=\varepsilon_0^{-1}L^{-2}T^2$$

17. **(B)** $r=\dfrac{1-a}{1+a} \quad \therefore\ \dfrac{dr}{da}=\dfrac{(1+a)(-1)-(1-a)}{(1+a)^2}=\dfrac{-2}{(1+a)^2}$

$$\therefore |\Delta r|=\frac{2\Delta a}{(1+a)^2}$$

18. **(C)** We know that, $N=N_0e^{-\lambda t}$

Taking *log* on both sides $\log_e N = \log_e N_0 - \lambda t$

differentiating with respect to 'λ' we get

$$\frac{1}{N}\frac{dN}{d\lambda}=0-t \qquad \therefore\ |d\lambda|=\frac{dN}{tN}=\frac{40}{1\times2000}=0.02$$

[∵ N = 3000 – 1000 = 2000]

Chemistry

19. (B,C)

(A) $NH_4NO_3 \xrightarrow[\text{below 300°C}]{} N_2O + 2H_2O$

$N_2O \xrightarrow[\text{above 600°C}]{} N_2 + O_2$

(B) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$

(C) $Ba(N_3)_2 \xrightarrow[180°C]{\Delta} Ba + 3N_2$

(D) $Mg_3N_2 \xrightarrow[\text{above 700°C}]{} 3Mg + N_2$

Hence only $(NH_4)_2Cr_2O_7$ and $Ba(N_3)_2$ can provide N_2 gas on heating below 300°C

20. (B, C)

(A) $[(Fe(CO_5)]$ & $[Ni(CO)_4]$ complexes have 18-electrons in their valence shell.

(B) Due to strong ligand field, carbonyl complexes are predominantly low spin complexes.

(C) As electron density increases on metals (with lowering oxidation state on metals), the extent of synergic bonding increases. Hence M-C bond strength increases

(D) While positive charge on metals increases and the extent of synergic bond decreases and hence C–O bond becomes stronger.

21. (A, B, C)

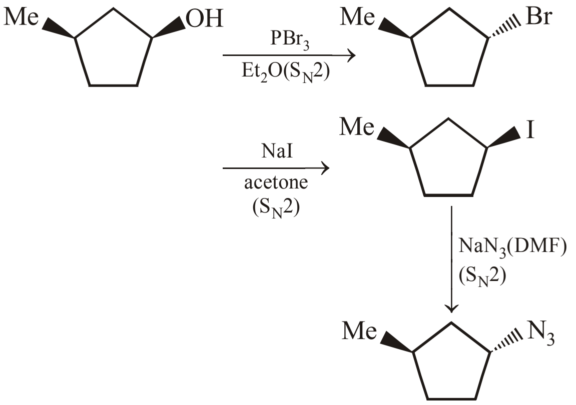
(A) Basic character of oxides increase on moving down the group therefore Bi_2O_5 is more basic than N_2O_5.

(B) Covalent nature depends on electronegativity difference between bonded atoms. In NF_3, N and F are non-metals but in BiF_3, Bi is metal while F is non metal therefore NF_3 is more covalent than BiF_3.

(C) In PH_3 hydrogen bonding is absent but in NH_3 hydrogen bonding is present, therefore PH_3 boils at lower temperature than NH_3.

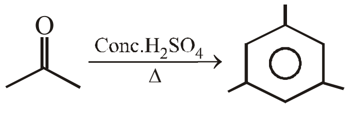
(D) Due to small size in N–N single bond l.p. – l.p. repulsion is more than P–P single bond therefore N–N single bond is weaker than the P–P single bond.

22. (B)

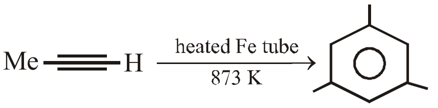


23. (A, B, D)

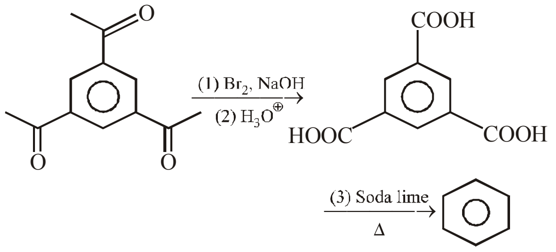
(A)



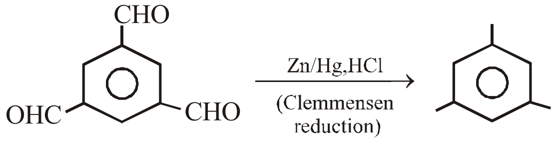
(B)



(C)



(D)



24. (B, C)

A– C ⇒ isochoric process

A– B ⇒ isothermal process

B– C ⇒ isobaric process

$\Rightarrow q_{AC} = \Delta U_{AC} = nC_{V,m}(T_2 - T_1) = \Delta U_{BC}$

$\Rightarrow W_{AB} = -nRT_1 \ln\left(\frac{V_2}{V_1}\right)$

$\Rightarrow W_{BC} = -P_2(V_1 - V_2) = P_2(V_2 - V_1)$

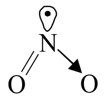
$\Rightarrow q_{BC} = \Delta H_{BC} = nC_{P,m}(T_2 - T_1) = \Delta H_{AC}$

$\Rightarrow \Delta H_{CA} = nC_{P,m}(T_1 - T_2)$

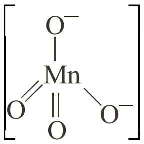
$\Rightarrow \Delta U_{CA} = nC_{V,m}(T_1 - T_2)$

$\Delta H_{CA} < \Delta U_{CA}$ since both are negative $(T_1 < T_2)$

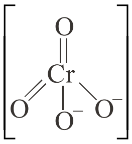
25. (1)

- H-atom = $1s^1$ Paramagnetic
- NO_2 =  odd electron species Paramagnetic
- O_2^- (superoxide) = One unpaired electron in π* M.O. Paramagnetic
- S_2 (in vapour phase) = same as O_2, two unpaired e⁻s are present in π* M.O. Paramagnetic
- $Mn_3O_4 = 2\overset{+2}{Mn}O . \overset{+4}{Mn}O_2$ Paramagnetic
- $(NH_4)_2[FeCl_4] = Fe^{2+} = 3d^6 4s^0$ Paramagnetic
- $(NH_4)_2[NiCl_4] = Ni = 3d^8 4s^2$

  $Ni^{2+} = 3d^8 4s^0$ Paramagnetic

- $K_2MnO_4 = 2K^+$  $, Mn^{6+} = [Ar]\,3d^1$

Paramagnetic

- $K_2CrO_4 = 2K^+$  $, Cr^{6+} = [Ar]\,3d^0$

Diamagnetic

26. **(2992)**

$$\underset{\substack{1584\,g\\=12\,mole}}{(NH_4)_2SO_4} + Ca(OH)_2 \rightarrow \underset{\substack{gypsum(M=172)\\12\,mole}}{CaSO_4.2H_2O} + \underset{24\,mole}{2NH_3}$$

$$\underset{952\,g=4\,mole}{NiCl_2.6H_2O} + \underset{24\,mole}{6NH_3} \rightarrow \underset{\substack{(M=232)\\4\,mole}}{[Ni(NH_3)_6]Cl_2} + 6H_2O$$

Total mass = 12 × 172 + 4 × 232 = 2992 g

27. **(3)** As per given information, cations form fcc lattice and anions occupy all the octahedral voids.

So	M^+	X^-	Formula MX
	4 ions	4 ions	
After step I	4 ions	1 ion	
After step II	1 ion	4 ions	
After step III	0 ion	4 ions	
After step IV	1 ion	3 ions	

$$\text{So ratio of } \frac{\text{No.of anions}}{\text{No.of cations}} = \frac{3}{1}$$

28. **(10)**

$$Mg(s) \longrightarrow Mg^{2+}(aq) + 2e^-$$

$$Cu^{2+}(aq) + 2e^- \longrightarrow Cu(s)$$

$$Mg(s) + Cu^{2+}(aq) \longrightarrow Mg^{2+}(aq) + Cu(s)$$

$$E_{cell} = E^\circ_{cell} - \frac{RT}{nF}\ln x$$

$$E = 2.67 = 2.7 - \frac{RT}{nF}\ln\frac{x}{1}$$

$$0.03 = \frac{300}{2 \times 11500}\ln x$$

$$2.3 = \ln x$$

$$x = 10$$

29. **(2.22)** $P_1 = 5$ bar $\quad P_2 = 1$ bar

$V_1 = 1\,m^3 \quad V_2 = 3\,m^3$

$T_1 = 400\,K \quad T_2 = 300\,K$

$$n_1 = \frac{5}{400R} \quad n_2 = \frac{3}{300R}$$

Let volume be $(V + x)$ $\quad V = (3 - x)$

$$\frac{P_A}{T_A} = \frac{P_B}{T_B}$$

$$\Rightarrow \frac{n_{b_1} \times R}{V_{b_1}} = \frac{n_{b_2} \times R}{V_{b_2}}$$

$$\Rightarrow \frac{5}{400(4 \times x)} = \frac{3}{300R\,(3 - x)}$$

$$\Rightarrow 5(3 - x) = 4 + 4x$$

$$\Rightarrow x = \frac{11}{9}$$

$$V = 1 + x = 1 + \frac{11}{9} = \left(\frac{20}{9}\right) = 2.22$$

30. **(19)** $P_T = p^\circ_A X_A + p^\circ_B X_B$

$$45 = 20(0.5) + p^o_B\,(0.5)$$

$$p^o_B = 70$$

$$22.5 = 20\,X_A + 70(1 - X_A)$$

$$50X_A = 47.5$$

$$X_A = \frac{4.75}{5} = 0.95$$

$$X_B = 0.05$$

$$\frac{X_A}{X_B} = 19$$

31. **(4.47)** $S = \sqrt{K_{sp}\left(\frac{[H^+]}{K_a} + 1\right)} = \sqrt{20 \times 10^{-10}\left(\frac{10^{-3}}{10^{-8}} + 1\right)}$

$$\simeq \sqrt{2 \times 10^{-5}} = 4.47 \times 10^{-3}\,M$$

32. **(0.05)** From graph

For solvent 'X' $\quad \Delta T_{b(x)} = 362 - 360 = 2$

$$\Delta T_{b(x)} = m_{NaCl} \times K_{b(x)} \qquad ...(1)$$

For solvent 'Y' $\quad \Delta T_{b(y)} = 368 - 367 = 1$

$$\Delta T_{b(y)} = m_{NaCl} \times K_{b(y)} \qquad ...(2)$$

Dividing equation (1) by (2)

$$\Rightarrow \frac{K_{b(x)}}{K_{b(y)}} = 2$$

For solute S

Given solute S dimerizes in solvent. Hence,

$$\underset{(1-\alpha)}{\overset{}{\underset{1}{2(S)}}} \rightarrow \underset{\alpha/2}{S_2}$$

$$i = (1 - \alpha/2)$$

$$\Delta T_{b(x)(s)} = \left(1 - \frac{\alpha_1}{2}\right) K_{b(x)}$$

$$\Delta T_{b(y)(s)} = \left(1 - \frac{\alpha_2}{2}\right) K_{b(y)}$$

Given $\Delta T_{b(x)(s)} = 3\Delta T_{b(y)(s)}$

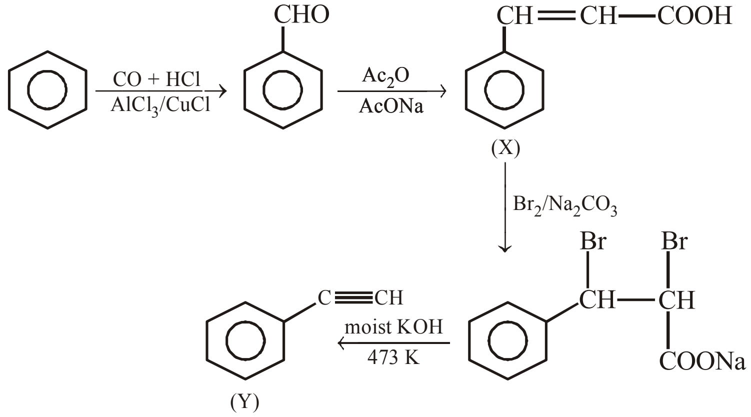
$$\left(1 - \frac{\alpha_1}{2}\right) K_{b(x)} = 3 \times \left(1 - \frac{\alpha_2}{2}\right) \times K_{(b)(y)}$$

$$2\left(1 - \frac{\alpha_1}{2}\right) = 3\left(1 - \frac{\alpha_2}{2}\right)$$

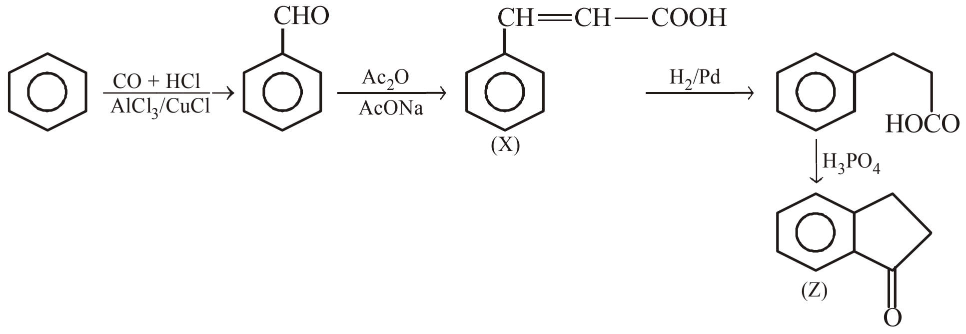
$$\alpha_2 = 0.7$$

so $\alpha_1 = 0.05$

33. (C)

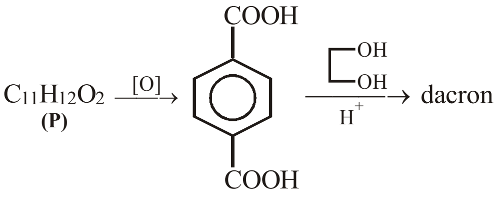


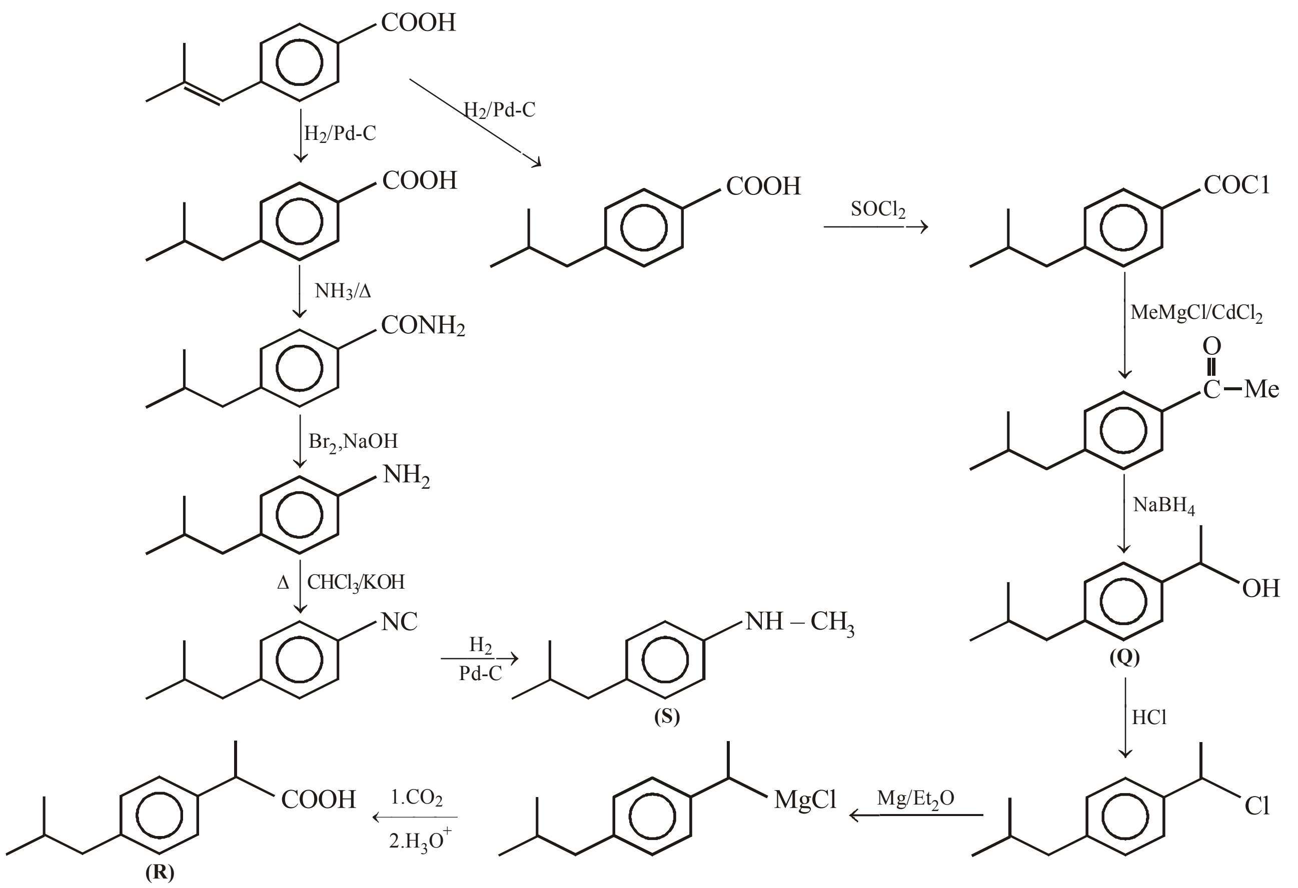
34. (A)



35. (A) 36. (B)

Sol. 35 & 36





Mathematics

37. (A, B, D)

(A) $\arg(-1-i) = \frac{-3\pi}{4}$

$\therefore$ (A) is false

(B) $f(t) = \arg(-1+it) = \begin{cases} \pi - \tan^{-1}(t), t \geq 0 \\ -\pi + \tan^{-1}(t), t < 0 \end{cases}$

$\lim\limits_{t \to 0^-} f(t) = -\pi$ and $\lim\limits_{t \to 0^+} f(t) = \pi$

LHL $\neq$ RHL $\Rightarrow$ f is discontinuous at $t = 0$

$\therefore$ (B) is false.

(C) $\arg\left(\frac{z_1}{z_2}\right) - \arg z_1 + \arg z_2$

$= 2n\pi + \arg z_1 - \arg z_2 - \arg z_1 + \arg z_2$

$= 2n\pi$, multiple of 2π

$\therefore$ (C) is true.

(D) $\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$

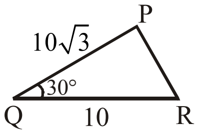
$$\Rightarrow \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)} = k, \qquad k \in R$$

$$\Rightarrow \left(\frac{z-z_1}{z-z_3}\right) = k\left(\frac{z_2-z_1}{z_2-z_3}\right)$$

$\Rightarrow z, z_1, z_2, z_3$ are concyclic. i.e. z lies on a circle.

$\therefore$ (D) is false.

38. (B, C, D)



(A) $\cos 30° = \frac{PQ^2 + QR^2 - PR^2}{2PQ \cdot QR}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{(10\sqrt{3})^2 + 10^2 - PR^2}{2 \times 10\sqrt{3} \times 10}$

$\Rightarrow PR^2 = 100$ or $PR = 10$

$\therefore \angle P = \angle Q = 30°$

$\therefore$ (A) is false.

(B) Area of $\Delta PQR = \frac{1}{2} PQ \times QR \times \sin 30°$

$= \frac{1}{2} \times 10\sqrt{3} \times 10 \times \frac{1}{2} = 25\sqrt{3}$

Also $\angle R = 180° - 30° - 30° = 120°$

$\therefore$ (B) is true.

(C) $r = \frac{\Delta}{s} = \frac{25\sqrt{3}}{\left(\frac{10\sqrt{3}+10+10}{2}\right)} = \frac{25\sqrt{3}}{10+5\sqrt{3}} = \frac{5\sqrt{3}}{2+\sqrt{3}}$

$= 5\sqrt{3}(2-\sqrt{3}) = 10\sqrt{3}-15$

∴ (C) is true.

(D) $R = \frac{abc}{4\Delta} = \frac{10\sqrt{3}\times10\times10}{4\times25\sqrt{3}} = 10$

∴ Area of circumcircle $= \pi R^2 = 100\pi$

∴ (D) is true.

39. (C,D)

(A) Direction ratios of line of intersection of two planes will be given by $\vec{n}_1 \times \vec{n}_2$.

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} + 3\hat{k}$$

∴ dr's of line of intersection of P_1 and P_2 are 1, – 1, 1

∴ (A) is false.

(B) Given line can be written as

$$\frac{x-\frac{4}{3}}{3} = \frac{y-\frac{1}{3}}{-3} = \frac{z}{3}$$

Clearly this line is parallel to line of intersection of P_1 and P_2

∴ (B) is false.

(C) If θ is the angle between P_1 and P_2 then

$$\cos\theta = \left|\frac{2\times1+1\times2+(-1)\times1}{\sqrt{6}\sqrt{6}}\right| = \frac{3}{6} = \frac{1}{2}$$

∴ $\theta = 60°$

Hence (C) is true.

(D) Equation of plane P_3:

$1(x-4) - 1(y-2) + 1(z+2) = 0$

or $x - y + z = 0$

Distance of (2, 1, 1) from $P_3 = \frac{2-1+1}{\sqrt{1+1+1}} = \frac{2}{\sqrt{3}}$

∴ (D) is true.

40. (A, B, D)

(A) f(x) being twice differentiable, it is continuous but can't be constant throughout the domain.

∴ We can find $x \in (r, s)$ such that f(x) is one one.

Hence (A) is true.

(B) By Lagrange's Mean Value theorem for f(x) in [–4, 0], there exists

$x_0 \in (-4, 0)$ such that

$$f'(x_0) = \frac{f(0)-f(-4)}{0-(-4)}$$

$\Rightarrow |f'(x_0)| = \left|\frac{f(0)-f(-4)}{4}\right|$

$\because -2 \le f(x) \le 2$

$\therefore -4 \le f(0) - f(-4) \le 4$

$\Rightarrow |f'(x_0)| \le 1$

∴ (B) is true.

(C) If we consider $f(x) = \sin(\sqrt{85}x)$ then f (x) satisfies the given condition $[f(0)]^2 + [f'(0)]^2 = 1$

But $\lim\limits_{x\to\infty} (\sin\sqrt{85}x)$ does not exist

∴ (C) is false.

(D) Let us consider $g(x) = [f(x)]^2 + [f'(x)]^2$

By Lagrange's Mean Value theorem

$|f'(x)| \le 1$

Also $|f(x_1)| \le 2$ as $f(x) \in [-2, 2]$

$\therefore g(x_1) \le 5$, for same $x_1 \in (-4, 0)$

Similarly $g(x_2) \le 5$, for same $x_2 \in (0, 4)$

Also $g(0) = 85$

Hence g(x) has maxima in (x_1, x_2) say at α such that

$g'(\alpha) = 0$ and $g(\alpha) \ge 85$

$g'(\alpha) = 0 \Rightarrow 2f(\alpha)f'(\alpha) + 2f'(\alpha)f''(\alpha) = 0$

$\Rightarrow 2f'(\alpha)[f(\alpha) + f''(\alpha)] = 0$

If $f'(\alpha) = 0 \Rightarrow g(\alpha) = [f(\alpha)]^2$

and $[f(\alpha)]^2 \le 4$

$\therefore g(\alpha) \ge 85$ (is not possible.)

Hence $f(\alpha) + f''(\alpha) = 0$ for $\alpha \in (x_1, x_2) \in (-4, 4)$

∴ (D) is true.

41. (B, C)

Given $f'(x) = e^{(f(x)-g(x))} \cdot g'(x)$

$\Rightarrow e^{-f(x)} f'(x) = e^{-g(x)} g'(x)$

Integrating both sides, we get

$-e^{-f(x)} = -e^{-g(x)} + c$

$\Rightarrow -e^{-f(x)} + e^{-g(x)} = c$

$\Rightarrow -e^{-f(1)} + e^{-g(1)} = -e^{-f(2)} + e^{-g(2)}$

But given that $f(1) = g(2) = 1$

$\therefore\ -e^{-1}+e^{-g(1)}=-e^{-f(2)}+e^{-1}$

$\Rightarrow\ e^{-f(2)}+e^{-g(1)}=\frac{2}{e}$

$\Rightarrow\ e^{-f(2)}<\frac{2}{e}$ and $e^{-g(1)}<\frac{2}{e}$

$\Rightarrow\ -f(2)<\ln 2-1$ and $-g(1)<\ln 2-1$

$\Rightarrow\ f(2)>1-\ln 2$ and $g(1)>1-\ln 2$

$\therefore$ (B) and (C) are True.

42. **(B, C)**

$$f(x)=1-2x+\int_0^x e^{x-t}f(t)\,dt$$

$$\Rightarrow\quad f(x)=1-2x+e^x\int_0^x e^{-t}f(t)dt$$

$$\Rightarrow\quad f'(x)=-2+e^x\int_0^x e^{-t}f(t)\,dt+e^x[e^{-x}f(x)]$$

$$\Rightarrow\quad f'(x)=-2+[f(x)-1+2x]+f(x)$$

$$\Rightarrow\quad f'(x)-2f(x)=2x-3$$

Its a linear differential equation.

$$IF=e^{\int -2dx}=e^{-2x}$$

Solution: $f(x)\times e^{-2x}=\int e^{-2x}(2x-3)\,dx$

$$f(x)\times e^{-2x}=\frac{e^{-2x}}{-2}(2x-3)-\int\frac{e^{-2x}}{-2}\times 2\,dx$$

$$e^{-2x}f(x)=\frac{e^{-2x}}{-2}(2x-3)+\frac{e^{-2x}}{-2}+c$$

$$f(x)=-x+\frac{3}{2}+\frac{1}{-2}+ce^{2x}$$

$$f(x)=-x+1+ce^{2x}$$

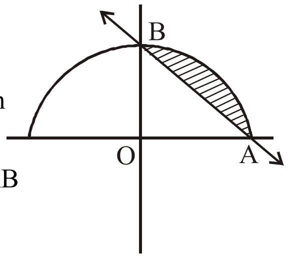
From definition of function, $f(0)=1$

$\therefore\ 1=1+c\quad\Rightarrow\quad c=0$

$\therefore\ f(x)=1-x$

Clearly curve $y=1-x$, does not pass through (1, 2) but it passes through (2, – 1)

$\therefore$ (A) is false and (B) is true



Also the area of the region $1-x\le y\le\sqrt{1-x^2}$, is shown in the figure, is given by

= Area of quadrant – Area ΔOAB

$$=\frac{1}{4}\times\pi\times 1^2-\frac{1}{2}\times 1\times 1=\frac{\pi-2}{4}$$

$\therefore$ (C) is true and (D) is false.

43. **(8)** $$((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}}\times(\sqrt{7})^{\frac{1}{\log_4 7}}$$

$$=(\log_2 9)^{2\times\log_{(\log_2 9)}2}\times 7^{\frac{1}{2}\times\log_7 4}$$

$$=(\log_2 9)^{\log_{(\log_2 9)}4}\times 7^{\log_7 2}$$

$=4\times 2=8.$

44. **(625)** The last 2 digits, in 5-digit number divisible by 4, can be 12, 24, 32, 44 or 52.

Also each of the first three digits can be any of $\{1,2,3,4,5\}$

Hence 5 options for each of the first three digits and total 5 options for last 2-digits

$\therefore$ Required number of 5 digit numbers are

$=5\times 5\times 5\times 5=625$

45. **(3748)**

The given sequences upto 2018 terms are

1, 6, 11, 16, ..., 10086

and 9, 16, 23, ..., 14128

The common terms are

16, 15, 86, ... upto n terms, where $T_n\le 10086$

$\Rightarrow\quad 16+(n-1)\,35\le 10086$

$\Rightarrow\quad 35n-19\le 10086$

$\Rightarrow\quad n\le\frac{10105}{35}=288.7$

$\therefore\quad n=288$

$\therefore\quad n(X\cup Y)=n(X)+n(Y)-n(X\cap Y)$

$=2018+2018-288=3748$

46. **(2)** $$\sin^{-1}\left(\frac{x^2}{1-x}-x\cdot\frac{\frac{x}{2}}{1-\frac{x}{2}}\right)=\sin^{-1}\left(\frac{-\frac{x}{2}}{1+\frac{x}{2}}-\frac{-x}{1+x}\right)$$

$$\Rightarrow\ \frac{x^2}{1-x}-\frac{x^2}{2-x}=\frac{-x}{2+x}+\frac{x}{1+x}$$

$$\Rightarrow\ \frac{x^2}{1-x}-\frac{x}{1+x}+\frac{x}{2+x}-\frac{x^2}{2-x}=0$$

$$\Rightarrow\ \frac{x(x+x^2-1+x)}{1-x^2}+\frac{x(2-x-2x-x^2)}{4-x^2}=0$$

$$\Rightarrow\ \frac{x(x^2+2x-1)}{1-x^2}+\frac{x(2-3x-x^2)}{4-x^2}=0$$

$$\Rightarrow\ x[(x^2+2x-1)(4-x^2)+(1-x^2)(2-3x-x^2)]=0$$

$\Rightarrow x[x^3+2x^2+5x-2]=0$

$\Rightarrow x = 0 \text{ or } x^3+2x^2+5x-2=0=p(x)\text{ (say)}$

We observe $p(0) < 0$ and $p\left(\frac{1}{2}\right) > 0$

$\therefore$ One root of $p(x) = 0$ lies in $\left(0, \frac{1}{2}\right)$.

Thus two solutions lie between $-\frac{1}{2}$ and $\frac{1}{2}$.

47. **(1)** $y_n = \left(\frac{n+1}{n}.\frac{n+2}{n}.\frac{n+3}{n}.....\frac{n+n}{n}\right)^{1/n}$

$\Rightarrow \log y_n = \frac{1}{n}\sum_{r=0}^{n} \log\left(1+\frac{r}{n}\right)$

$\Rightarrow \left(\lim_{n\to\infty} y_n\right) = \lim_{n\to\infty} \frac{1}{n}\sum_{r=0}^{n}\log\left(1+\frac{r}{n}\right)$

$\Rightarrow \log L = \int_0^1 \log(1+x)dx = [x\log(1+x)]_0^1 - \int_0^1 \frac{x}{1+x}dx$

$= \log 2 - [x - \log|1+x|]_0^1$

$= \log 2 - 1 + \log 2 = 2\log 2 - 1$

$= \log 4 - \log e = \log\left(\frac{4}{e}\right)$

$\therefore \quad L = \frac{4}{e} \Rightarrow [L] = \left[\frac{4}{e}\right] = 1$

48. **(3)** Given $|\vec{a}| = |\vec{b}| = 1, \quad \vec{a}.\vec{b} = 0, \quad |\vec{c}| = 2$

$\vec{c}$ makes angle α with both $\vec{a}$ and $\vec{b}$

Also, $\vec{c} = x\vec{a} + y\vec{b} + \vec{a}\times\vec{b}$

$\vec{c}.\vec{a} = 2\cos\alpha \Rightarrow x = 2\cos\alpha$

$\vec{c}.\vec{b} = 2\cos\alpha \Rightarrow y = 2\cos\alpha$

$|\vec{c}|^2 = \vec{c}.\vec{c} = \left|(2\cos\alpha)\vec{a} + (2\cos\alpha)\vec{b} + \vec{a}\times\vec{b}\right|^2$

$\Rightarrow (2)^2 = 4\cos^2\alpha + 4\cos^2\alpha + |\vec{a}\times\vec{b}|^2$

$\Rightarrow 4 = 8\cos^2\alpha + 1 \quad \left(\because |\vec{a}\times\vec{b}| = 1\times 1\times\sin 90° = 1\right)$

$\Rightarrow 8\cos^2\alpha = 3$

49. **(0.5)** Given that the equation

$\sqrt{3}\, a\cos x + 2b\sin x = c$

has two roots α and β, such that $\alpha+\beta = \frac{\pi}{3}$

$\therefore \quad \sqrt{3}\, a\cos\alpha + 2b\sin\alpha = c \qquad ...(1)$

and $\sqrt{3}\, a\cos\beta + 2b\sin\beta = c \qquad ...(2)$

subtracting equation (2) from (1) we get

$\sqrt{3}\, a(\cos\alpha - \cos\beta) + 2b(\sin\alpha - \sin\beta) = 0$

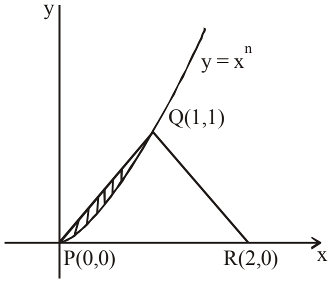
$\Rightarrow -\sqrt{3}a2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} + 2b.2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} = 0$

$\Rightarrow -2\sqrt{3}a\ \sin\frac{\pi}{6} + 4b\cos\frac{\pi}{6} = 0$

$\Rightarrow -2\sqrt{3}a\times\frac{1}{2} + 4b\frac{\sqrt{3}}{2} = 0$

$\Rightarrow \frac{b}{a} = \frac{1}{2} = 0.5$

50. **(4)**



Shaded area $= \frac{30}{100}\times Ar(\Delta PQR)$

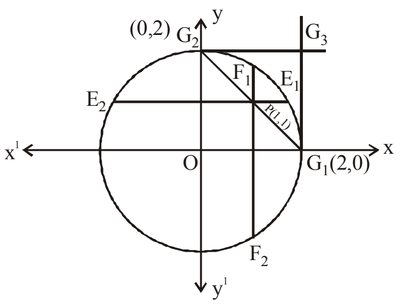
$\Rightarrow \int_0^1 (x - x^n)dx = \frac{3}{10}\times\frac{1}{2}\times 2\times 1$

$\Rightarrow \left(\frac{x^2}{2} - \frac{x^{n+1}}{n+1}\right)_0^1 = \frac{3}{10}$

$\Rightarrow \frac{1}{2} - \frac{1}{n+1} = \frac{3}{10}$

$\Rightarrow \frac{1}{n+1} = \frac{1}{2} - \frac{3}{10} = \frac{1}{5} \quad \Rightarrow \quad n = 4$

51. **(A)**



Equation of $E_1 E_2 : y = 1$

Equation of $F_1 F_2 : x = 1$

Equation of $G_1 G_2 : x + y = 2$

By symmetry, tangents at E_1 and E_2 will meet on y-axis and tangents at F_1 and F_2 will meet on x-axis

$E_1 \equiv (\sqrt{3}, 1)$ & $F_1 \equiv (1, \sqrt{3})$

Equation of tangent at $E_1 : \sqrt{3}x + y = 4$

Equation of tangent at $F_1 : x + \sqrt{3}y = 4$

$\therefore$ Points $E_3(0, 4)$ and $F_3(4, 0)$

Tangents at G_1 and G_2 are $x = 2$ and $y = 2$ intersecting each other at $G_3(2, 2)$.

Clearly E_3, F_3 and G_3 lie on the curve $x + y = 4$.

52 **(D)** Let point P be $(2 \cos \theta, 2 \sin \theta)$

Tangent at P : $x \cos \theta + y \sin \theta = 2$

$\therefore$ $M\left(\frac{2}{\cos\theta}, 0\right)$· and $N\left(0, \frac{2}{\sin\theta}\right)$

Mid point of MN $= \left(\frac{1}{\cos\theta}, \frac{1}{\sin\theta}\right)$

For locus of mid point (x, y) of MN,

$$x = \frac{1}{\cos\theta}, \quad y = \frac{1}{\sin\theta}$$

$$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = 1$$

$$\Rightarrow x^2 + y^2 = x^2y^2$$

53. **(A)** No. of dearrangements for 4 students

$$= 4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$$

$= 12 - 4 + 1 = 9$

Total no. of arrangements of seating of 5 students $= 5! = 120$

$\therefore$ required probability $= \frac{9}{120} = \frac{3}{40}$

54. **(C)** Total cases = 5! = 120

Favourable cases :

$$\left.\begin{array}{l}1,\ 3,\ 5,\ 2,\ 4\\ 1,\ 4,\ 2,\ 5,\ 3\end{array}\right\}2$$

$$\left.\begin{array}{l}2,\ 4,\ 1,\ 3,\ 5\\ 2,\ 5,\ 3,\ 1,\ 4\\ 2,\ 4,\ 1,\ 5,\ 3\end{array}\right\}3$$

$$\left.\begin{array}{l}3,\ 1,\ 4,\ 2,\ 5\\ 3,\ 5,\ 2,\ 4,\ 1\\ 3,\ 1,\ 5,\ 2,\ 4\end{array}\right\}3$$

$$\left.\begin{array}{l}4,\ 2,\ 5,\ 1,\ 3\\ 4,\ 2,\ 5,\ 3,\ 1\\ 4,\ 1,\ 3,\ 5,\ 2\end{array}\right\}3$$

$$\left.\begin{array}{l}5,\ 2,\ 4,\ 1,\ 3\\ 5,\ 3,\ 1,\ 4,\ 2\end{array}\right\}2$$

$\therefore$ favourable cases = 14

$\therefore$ required probability $= \frac{14}{120} = \frac{7}{60}$

Paper - 2

Physics

1. **(A, B, D)** $\frac{dk}{dt}=\gamma t$ and $k=\frac{1}{2}mV^2$ $\quad\therefore\ \frac{d}{dt}\left(\frac{1}{2}mV^2\right)=\gamma t$

$$\Rightarrow\ \frac{m}{2}\times 2V\frac{dV}{dt}=\gamma t \qquad \therefore\ mV\frac{dV}{dt}=\gamma t$$

$$\therefore\ m\int_0^V VdV=\gamma\int_0^t tdt \qquad \Rightarrow\ \frac{mV^2}{2}=\frac{\gamma t^2}{2}$$

$\therefore\ V=\sqrt{\frac{\gamma}{m}}\times t.$ i.e., V ∝ t

As V is proportional to 't', distance cannot be proportional to 't'.

Now $F=ma=m\frac{dV}{dt}=m\frac{d}{dt}\left[\sqrt{\frac{\gamma}{m}}\times t\right]=m\sqrt{\frac{\gamma}{m}}=\sqrt{\gamma m}$ =constant

2. **(A, C, D)** We know that $|F|=\eta A\frac{u_0}{h}$

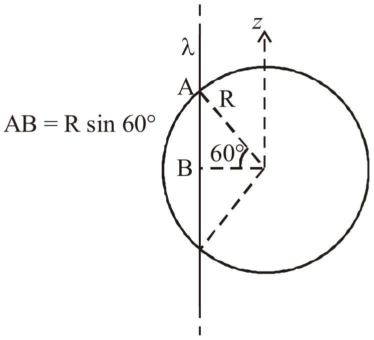
where $\frac{u_0}{h}$ = velocity gradient

Also $\frac{|F|}{A}=\eta\frac{u_0}{h}$

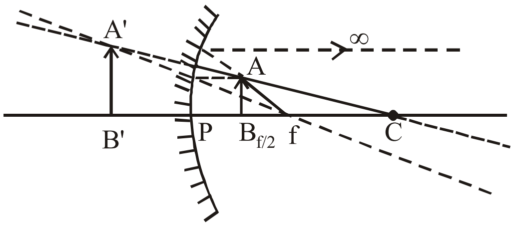
3. **(A, B)** According to Gauss's Law,

Electric flux, $\phi=\frac{1}{\epsilon_0}q_{in}=\frac{1}{\epsilon_0}[\lambda\times 2R\sin 60°]=\frac{\sqrt{3}\lambda R}{\epsilon_0}$

Further electric field is perpendicular to the wire therefore its z-component will be zero.



4. **(D)** The image of AB will be A'B' as AB lies between pole and focus. Further as the object is moved towards the focus the image also moves away.



The object distance decreases from $\frac{f}{2}$ to f. Therefore the final result is (D).

5. **(A, C)** No. of α-particles $=\frac{232-212}{4}=\frac{20}{4}=5$

$$\therefore\ {}^{232}_{90}\text{Th}\longrightarrow {}^{212}_{82}\text{Pb}+5\,{}^{4}_{2}\text{He}+2\,{}^{0}_{-1}\beta$$

6. **(A, B, C)** Given $(2n+1)\frac{\lambda}{4}=50.7+e$

and $(2n+3)\frac{\lambda}{4}=83.9+e$

If $n=1$, $\frac{3\lambda/4}{5\lambda/4}=\frac{50.7+e}{83.9+e}$ $\quad\therefore 3\times 83.9+3e=5\times 50.7+5e$

$\therefore\quad 2e=1.8 \qquad\Rightarrow e=0.9$ cm

$\therefore\quad \frac{3\lambda}{4}=50.7+0.9=51.6 \quad\therefore \lambda=66.4$ cm

Also $V=v\lambda=500\times 0.664\ \text{ms}^{-1}=332.0\ \text{ms}^{-1}$

7. **(6.30)** Impulse = Change in linear momentum

$\therefore\quad J=mV_0$ or $V_0=\frac{J}{m}=\frac{1}{0.4}=2.5\ \text{ms}^{-1}$

Also $V=v_0e^{-t/\tau}$ $\quad\therefore\ \frac{ds}{dt}=v_0e^{-t/\tau}$ $\quad\Rightarrow ds=v_0e^{-t/\tau}dt$

$$\therefore\quad s=v_0\int_0^\tau e^{-t/\tau}dt=v_0\tau(1-e^{-1})=2.5\times 4\times 0.63=6.30\ \text{m}$$

8. **(30.00)** $H=\frac{u^2\sin^2\theta}{2g}$ $\quad\Rightarrow 120=\frac{u^2\left(\frac{1}{2}\right)}{2g}$

$\therefore\quad u^2=480$ g

$\therefore\quad \text{K.E}_{\text{initial}}=\frac{1}{2}mu^2=240$ mg

$$\text{K.E}_{\text{final}} = \frac{1}{2}(240\,\text{mg}) = 120\,\text{mg}$$

$$\therefore \quad \frac{1}{2}mv^2 = 120\,\text{mg} \qquad \therefore\ v^2 = 240\text{ g}$$

$$\therefore \quad H' = \frac{v^2\sin^2\theta}{2g} = \frac{240g\times\left(\frac{1}{4}\right)}{2g} = 30\,\text{m}$$

9. **(2.00)** Given $E = \sin 10^3 t\ \hat{i}$

F = ma

$$\therefore \quad qE = m\frac{dv}{dt} \qquad \therefore\ dv = \frac{qEdt}{m} = \frac{q\sin 1000t\ \hat{i}}{m}dt$$

$$\therefore \int_0^v dv = \frac{q}{m}\int_0^{\pi/\omega}\sin 1000t\ dt \quad \left[\text{max. speed is at } \frac{T}{2} = \frac{2\pi}{\omega\times 2}\right]$$

$$\therefore \quad V = -\frac{q}{m}\left[\frac{\cos 1000t}{1000}\right]_0^{\pi/\omega} = -\frac{1}{10^{-3}}\times\frac{[\cos 1000t]_0^{\pi/\omega}}{1000}$$

$$\therefore \quad V = -\left[\cos 1000\times\frac{\pi}{1000} - \cos 0\right] = -[-1-1] = 2\text{ ms}^{-1}$$

10. **(5.56)** We know that $C\theta = NBA\,I_g$

$$\therefore \quad I_g = \frac{C\theta}{NBA} = \frac{10^{-4}\times 0.2}{50\times 2\times 10^{-4}\times 0.02} = 0.1\text{A}$$

Further for a galvanometer

$$I_g \times G = (I - I_g)\,S$$

$$\therefore \quad S = \frac{I_gG}{I - I_g} = \frac{0.1\times 50}{1-0.1} = \frac{50}{9}\ \Omega$$

11. **(3.00)** We know that $\Delta l = \dfrac{Fl}{AY}$

$$= \frac{1.2\times 10\times 1}{\pi\left(\frac{5\times 10^{-4}}{2}\right)^2\times 2\times 10^{11}} \simeq 0.3\,\text{mm}$$

The third marking of vernier scale will coincide with the main scale because least count is 0.1 mm.

12. **(900.00)** For an adiabatic process $TV^{\gamma-1} = T_2\,(8V)^{\gamma-1}$

where $\gamma = \dfrac{5}{3}\ \therefore\ T_2 = \dfrac{T}{4}$

Further $\Delta V = nC_v\Delta T = n\left(\dfrac{f}{2}R\right)\Delta T = \dfrac{nfR}{2}\left(\dfrac{-3T}{4}\right)$

$$\therefore \quad \Delta V = -\frac{1\times 3\times 8}{2}\times\frac{3}{4}\times 100 = -900\text{J}$$

13. **(24.00)** Number of electrons emitted per second

$$= \frac{200\ W}{6.25\times 1.6\times 10^{-19}J}$$

Force = Rate of change of linear momentum = $N\sqrt{2mk}$

$$= \frac{200}{6.25\times 1.6\times 10^{-19}}\times\sqrt{2\times 9\times 10^{-31}\times 1.6\times 10^{-19}\times 500}$$

[$\because$ K = eV: e = 1.6 × 10^{-19} = V = 500]

= 24.00

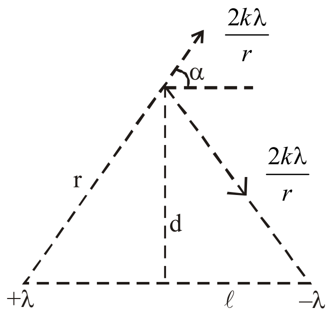
14. **(3.00)** $\Delta E_{2-1} = 74.8 + \Delta E_{3-2}$

$$13.6z^2\left[1-\frac{1}{4}\right] = 74.8 + 13.6z^2\left[\frac{1}{4}-\frac{1}{9}\right]$$

$\therefore$ z = 3

15. **(B)** For a point charge $E = \dfrac{kQ}{d^2}$ and for a dipole $E = \dfrac{kp}{d^3}$

Further for an infinite long line charge $E = \dfrac{2k\lambda}{d}$ and for infinite plane charge $E = \dfrac{\sigma}{2\in_0}$



Also for two infinite wires carrying uniform linear charge density.

$$E = \frac{2k\lambda}{r}\cos\alpha = \frac{2k\lambda}{\sqrt{d^2+\ell^2}}\times\frac{\ell}{\sqrt{d^2+\ell^2}} = \frac{2k\lambda\ell}{d^2+\ell^2}$$

16. **(B)** $V_o = \sqrt{\dfrac{GM}{R}},\quad \therefore\ \dfrac{V_1}{V_2} = \sqrt{\dfrac{R_2}{R_1}} = \dfrac{2}{1}$

Further $\dfrac{L_1}{L_2} = \dfrac{m_1v_1R_1}{m_1v_2R_2} = \dfrac{2\times 2\times 1}{1\times 1\times 4} = \dfrac{1}{1}$

Also K.E. = $\dfrac{GMm}{R}$. Therefore $\dfrac{k_1}{k_2} = \dfrac{m_1}{m_2}\times\dfrac{R_1}{R_2} = \dfrac{2\times 4}{1\times 1} = \dfrac{8}{1}$

Further $T^2 \propto R^3$ $\qquad \therefore\ \dfrac{T_1}{T_2} = \left(\dfrac{R_1}{R_2}\right)^{3/2} = \dfrac{1}{8}$

17. **(C)** Process 1 is adiabatic therefore $\Delta Q = 0$

Process 2 is isobaric therefore $W = P(V_2 - V_1) = 3P_0(3V_0 - V_0) = 6P_0V_0$

Process 3 is isochoric therefore $W = P(V_2 - V_1) = 0$

Process 4 is isothermal therefore temperature is constant, $\Delta u = 0$

18. **(A)** P → $\vec{v} = \frac{d\vec{r}}{dt} = \alpha\hat{i} + \beta\hat{j}$ which is constant

$\therefore \quad \vec{a} = 0$

Further $\vec{P} = m\vec{v}$ is constant

and K = $\frac{1}{2}mv^2$ is constant

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}\right) = 0 \qquad (\because \vec{a} \text{ is constant})$$

⇒ U = constant
Also E = K + U

$$\therefore \quad \frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F} = 0 \qquad \therefore \vec{L} = \text{constant}$$

$$Q \rightarrow \vec{v} = \frac{d\vec{r}}{dt} = -\alpha\omega(\sin\ \omega t)\hat{\mathrm{i}} + \beta\omega(\cos\ \omega t)\hat{\mathrm{j}}$$

$$\therefore \quad \vec{a} = \frac{d\vec{v}}{dt} = -\omega^2[\alpha\cos\omega t\ \hat{\mathrm{i}} + \beta\sin\ \omega t\,\hat{\mathrm{j}}] = -\omega^2\vec{r}$$

Also $\vec{\tau} = \vec{r} \times \vec{F} = 0$ $\quad$ ($\therefore \vec{r}$ and $\vec{F}$ are parallel)

$$\Delta U = -\int \vec{F}.\vec{dr} = +\int_0^r m\omega^2 r dr = \frac{m\omega^2 r^2}{2} \qquad \therefore U \propto r^2$$

Also r $= \sqrt{\alpha^2\cos^2\omega t + \beta^2\sin^2\omega t}$ $\quad \therefore\ r = f(t)$

As the force is central therefore total energy remains constant.

$$R \rightarrow \vec{v} = \frac{d\vec{r}}{dt} = \alpha[-\omega\sin\omega t\,\hat{i} + \omega\cos\omega t\ \hat{j}]$$

$\therefore \quad$ v = αω i.e., speed is constant

$$\vec{a} = \frac{d\vec{v}}{dt} = -\alpha\omega^2[\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}\,]$$

$$\therefore \quad \vec{a} = -\omega^2\vec{r}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = 0$$

Force is central in nature
U and K are also constant.

$$S \rightarrow \vec{v} = \frac{d\vec{r}}{dt} = \alpha t\,\hat{i} + \beta t\,\hat{j} \qquad \therefore V = f(t)$$

$\vec{a} = \beta\hat{j}$ i.e., constant

$\vec{F} = m\vec{a}$ constant

$$\Delta U = -\int \vec{F}\cdot d\vec{r} = -m\int_0^t \beta\hat{j}\cdot(\alpha\hat{i} + \beta t\hat{j})dt = \frac{-m\beta^2 t^2}{2}$$

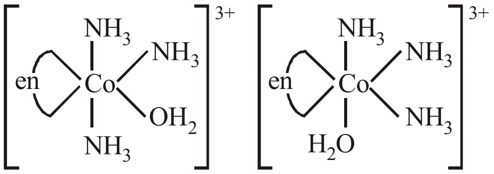
$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(\alpha^2 + \beta^2 t^2)$$

Also E = K + U = $\frac{1}{2}m\alpha^2$ which is constant.

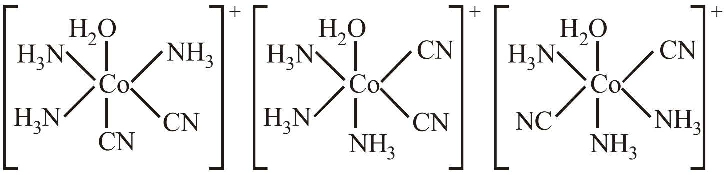
Chemistry

19. **(A, B, D)**

(A) $[Co(en)(NH_3)_3(H_2O)]^{3+}$ has 2 geometrical isomers.



(B) Compound $[Co(CN)_2(NH_3)_3(H_2O)]^+$ will have three geometrical isomers.



(C) $[Co(en)(NH_3)_3(H_2O)]^{3+}$ is diamagnetic

(D) $[Co(en)(NH_3)_4]^{3+}$ has larger gap between e_g and t_{2g} than $[Co(en)(NH_3)_3(H_2O)]^{3+}$. So $[Co(en)(NH_3)_3(H_2O)]^{3+}$ absorbs light at longer wavelength as compared to $[Co(en)(NH_3)_4]^{3+}$.

20. (B, D)

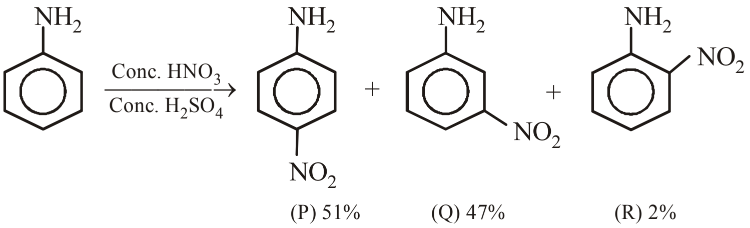
(A) Cu^{2+} shows characteristic green colour in the flame test whereas Mn^{2+} shows the pale colour in flame test.

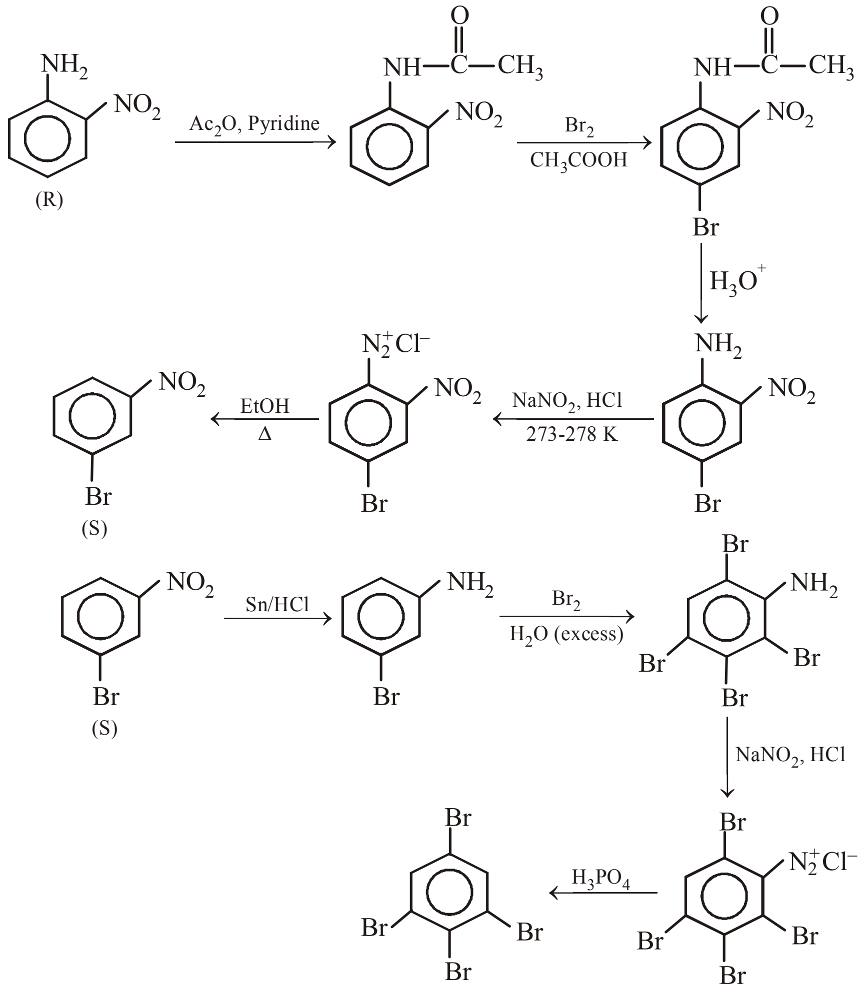
(B) Only Cu^{2+} can give black precipitate of CuS in acidic medium on passing H_2S.

(C) Both Cu^{2+} and Mn^{2+} show the formation of precipitate by passing H_2S in faintly basic medium.

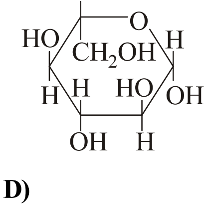
(D) $E^{\circ}_{Cu^{2+}/Cu}$ (+ 0.34V) > $E^{\circ}_{Mn^{2+}/Mn}$ (– 1.18V) as per electrochemical series.

21. (D)





22. (D) Structure of β-L-glucopyranose is



23. (A, D)

$$A(g) \xrightarrow{\text{First order}} 2B(g) + C(g) \quad V = \text{constant}, T = 300K$$

$$\begin{array}{llll} t=0 & P_0 & & \\ t=t_{1/3} & \left(P_0 - \frac{2P_0}{3}\right) = \frac{P_0}{3} & \frac{4P_0}{3} & \frac{2P_0}{3} \\ t=t & P_0 - x & 2x & x \end{array}$$

So, $P_t = P_0 - x + 2x + x = P_0 + 2x$

or $2x = P_t - P_0$

$$t=\frac{1}{k}\ln\frac{P_0}{(P_0-x)}$$

or $$t=\frac{1}{k}\ln\frac{P_0}{P_0-\frac{(P_t-P_0)}{2}}=\frac{1}{k}\ln\frac{2P_0}{2P_0-P_t+P_0}$$

or $$kt=\ln\frac{2P_0}{3P_0-P_t},\ kt=\ln 2P_0-\ln(3P_0-P_t)$$

or $$\ln(3P_0-P_t)=-kt+\ln 2P_0$$

Graph between $\ln(3P_0-P_t)$ vs 't' is a straight line with negative slope.
Since rate constant is a constant quantity and independent of initial concentration.
So graph (A) and (D) are correct.

24. (A, C)

On increasing temperature, concentration of product decreases
Hence reaction is exothermic $\Rightarrow \Delta H°<0$

$$\frac{\ln K_{T_1}}{\ln K_{T_2}}>1\Rightarrow \ln K_{T_1}>\ln K_{T_2}\text{ so, }K_{T_1}>K_{T_2}$$

Also, $$\frac{\ln K_{T_1}}{\ln K_{T_2}}>\frac{T_2}{T_1}$$

or $$T_1\ln K_{T_1}>T_2\ln K_{T_2}\Rightarrow -RT_1\ln K_{T_1}>-RT_2\ln K_{T_2}$$

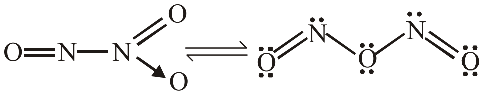
or $$\Delta G^{\circ}_{T_1}<\Delta G^{\circ}_{T_2}\qquad(\because \Delta G=-RT\ln K)$$

or $$\Delta H^{\circ}-T_1\Delta S^{\circ}<\Delta H^{\circ}-T_2\Delta S^{\circ}$$

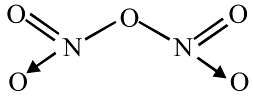
(Also $\Delta G°=\Delta H°-T\Delta S°$: Gibbs Helmholtz equation)

As $\Delta G^{\circ}_{T_1}<\Delta G^{\circ}_{T_2}$, this is possible only when $\Delta S°<0$

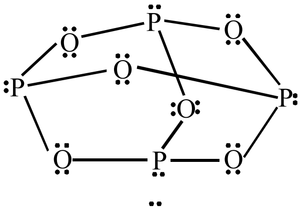
25. (6) N_2O_3=



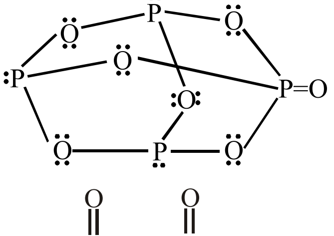
N_2O_5=



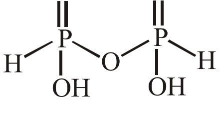
P_4O_6=



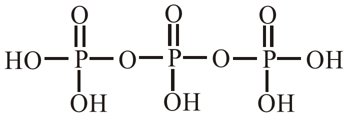
P_4O_7=



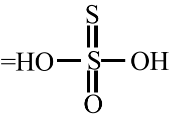
$H_4P_2O_5$=



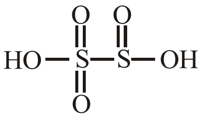
$H_5P_3O_{10}$=



$H_2S_2O_3$=



$H_2S_2O_5$=



26. (6.47)

$$PbS+O_2\longrightarrow Pb+SO_2$$

No. of moles of $O_2=\frac{10^3}{32}$

Moles of Pb formed $=\frac{10^3}{32}$

$\therefore$ Mass of Pb formed $=\frac{10^3}{32}\times 207=6468.75$ g
$=6.46875$ kg
$=6.47$ kg

27. (126)

The balanced equations are

$$2MnCl_2+5K_2S_2O_8+8H_2O\longrightarrow 2KMnO_4+4K_2SO_4+6H_2SO_4+4HCl\quad ...(1)$$

$$2KMnO_4+5H_2C_2O_4+3H_2SO_4\longrightarrow K_2SO_4+2MnSO_4+8H_2O+10CO_2\quad ...(2)$$

Mass of oxalic acid added = 225 mg

Milimoles of oxalic acid added $=\frac{225}{90}=2.5$

From equation (2)
Milimoles of $KMnO_4$ used to react with oxalic acid = 1
(5 m mole $H_2C_2O_4$ = 2 m mole of $KMnO_4$)
and milimoles of $MnCl_2$ required initially = 1
$\therefore$ Mass of $MnCl_2$ required initially = 1 × 126 = 126 mg
(Molar mass of $MnCl_2$ = 126)

28. (7)

29. (495) Moles of D formed = 10 × 0.6 × 0.5 × 0.5 × 1 = 1.5
Mass of D formed = 1.5 × 330 = 495 g

30. (– 14.6)

(i) $2Cu(s)+\frac{1}{2}O_2(g)\longrightarrow Cu_2O(s):\Delta G^{\circ}=-78$ kJ/mol

(ii) $H_2(g)+\frac{1}{2}O_2(g)\longrightarrow H_2O(g),\ \Delta G^{\circ}=-178$ kJ/mol

(i) – (ii) then

$$2Cu(s)+H_2O(g)\longrightarrow Cu_2O(s)+H_2(g)$$

$\Delta G°=-78+178=100$ kJ/mol $=10^5$ J/mol

Now for the above reaction

$$\Delta G=\Delta G^{\circ}+RT\ln\left(\frac{P_{H_2}}{P_{H_2O}}\right)$$

To prevent the above reaction:
$\Delta G\geq 0$

$$\Delta G^\circ + RT\ln\left(\frac{P_{H_2}}{P_{H_2O}}\right) \geq 0$$

$$10^5 + 8 \times 1250 \ln\left(\frac{P_{H_2}}{P_{H_2O}}\right) \geq 0$$

$$10^4(\ln P_{H_2} - \ln P_{H_2O}) \geq -10^5$$

$$\ln P_{H_2} \geq -10 + \ln P_{H_2O}$$

Now,

$$P_{H_2O} = X_{H_2O} \times P_{total} = 0.01 \times 1 = 10^{-2}$$

$$\geq -10 - 2\ln 10$$

$$\geq -10 - 2 \times 2.3 \qquad \text{(Given } \ln 10 = 2.3)$$

$$\ln P_{H_2} \geq -10 - 4.6$$

$$\ln P_{H_2} \geq -14.6$$

$\therefore$ Minimum $\ln P_{H_2} = -14.6$

31. (– 8500)

$$A(g) + B(g) \rightleftharpoons AB(g)$$

$$E_{a_b} = E_{a_f} + 2RT \qquad \& \qquad A_f = 4A_b$$

Now,

Rate constant of forward reaction $k_f = A_f e^{-Ea_f/RT}$

Rate constant of reverse reaction $K_b = A_b e^{-E_{ab}/RT}$

Equilibrium constant

$$K_{eq} = \frac{K_f}{K_b} = \frac{A_f}{A_b} e^{-(E_{af} - E_{ab})/RT}$$

$$K_{eq} = 4e^{2RT/RT} = 4e^2$$

Now, $\Delta G^\circ = -RT \ln K_{eq} = -2500 \ln(4e^2)$

$= -2500 (\ln 4 + \ln e^2)$

$= -2500(1.4 + 2)$

$= -2500 \times 3.4$

$= -8500$ J/mol.

32. (– 11.62)

A (s) | A^{n+} (aq, 2M) || B^{2n+} (aq, 1M) | B (s)

Reactions

Anode $\quad (A \longrightarrow A^{n+} + ne^-) \times 2$

Cathode $\quad B^{2n+} + 2ne^- \longrightarrow B$

Overall reaction :

$$2A + B^{2n+} \longrightarrow 2A^{n+} + B$$

$$E = E^\circ - \frac{RT}{2nF}\ln Q$$

$$0 = E^\circ - \frac{RT}{2nF}\ln\frac{[A^{n+}]^2}{[B^{2n+}]}$$

$$E^\circ = \frac{RT}{2nF}\ln\frac{2^2}{1}$$

$$E^\circ = \frac{RT}{2nF}\ln 4$$

Now,

$$\Delta G^\circ = -2nFE^\circ = -\frac{2nFRT}{2nF}\ln 4 = -RT\ln 4$$

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = 2\Delta G^\circ - T\Delta S^\circ$ (Given $\Delta H^\circ = 2\Delta G^\circ$)

$T\Delta S^\circ = \Delta G^\circ$

$$\Delta S^\circ = \frac{\Delta G^\circ}{T} = \frac{-RT\ln 4}{T} = -R\ln 4$$

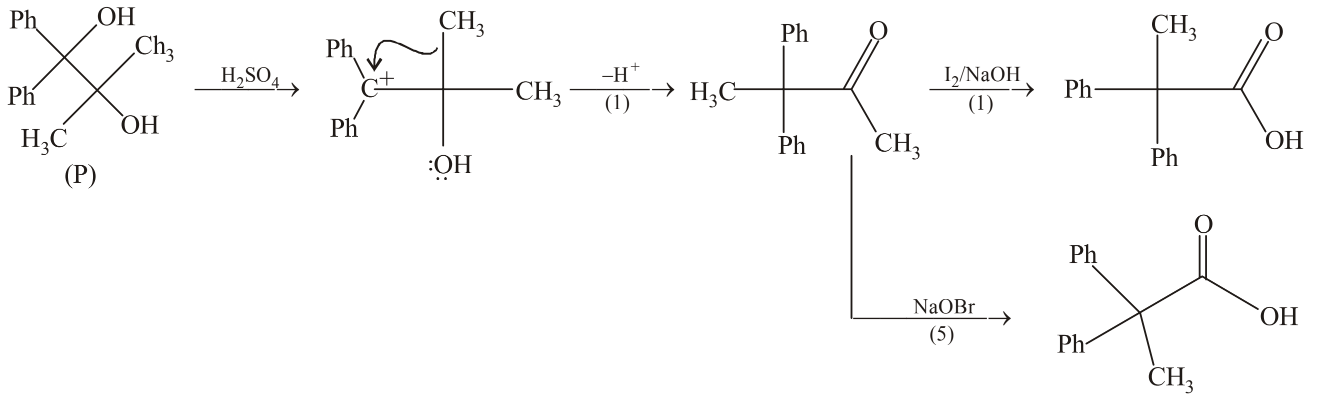
$= -8.3 \times 2 \times 0.7 = -11.62$ $JK^{-1}\,mol^{-1}$

33. (c) P – 6; Q – 4, 5; R – 1; S – 2, 3

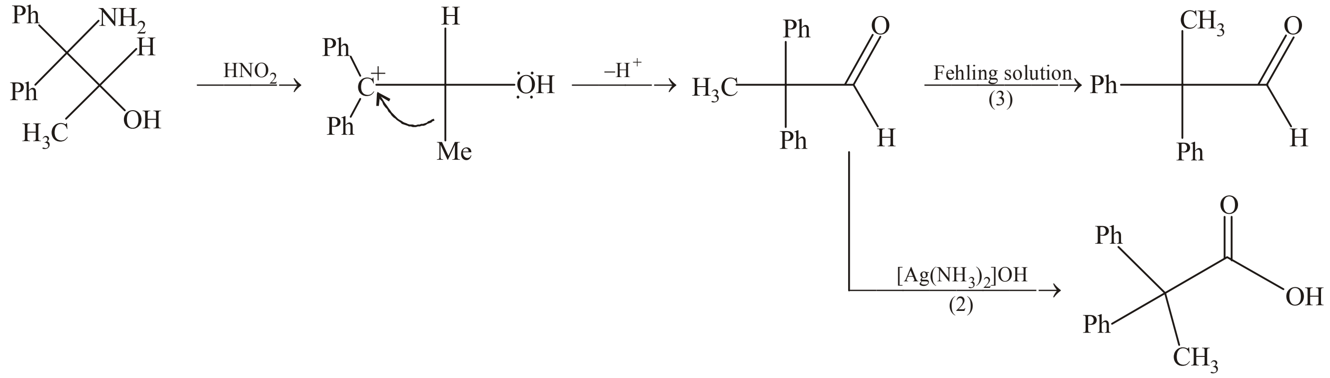
1. $[FeF_6]^{4-}$, $Fe^{2+} = 3d^6$ & F^- is weak field ligand
   $\therefore$ Hybridization is sp^3d^2 (high spin complex)
2. $[Ti(H_2O)_3Cl_3]$, $Ti^{3+} = 3d^1$ (No effect of ligand field strength)
   $\therefore$ Hybridization is d^2sp^3
3. $[Cr(NH_3)_6]^{3+}$, $Cr^{3+} = 3d^3$ (No effect of ligand field strength)
   $\therefore$ Hybridization is d^2sp^3
4. $[FeCl_4]^{2-}$, $3d^6$ & Cl^- is weak field ligand
   $\therefore$ Hybridization is sp^3
5. Ni $(CO)_4$, Ni = $3d^{10}$ & CO is strong field ligand
   $\therefore$ Hybridization is sp^3
6. $[Ni(CN)_4]^{2-}$, $Ni^{2+} = 3d^8$ & CN is strong field ligand
   $\therefore$ Hybridization is dsp^2

34. (D)

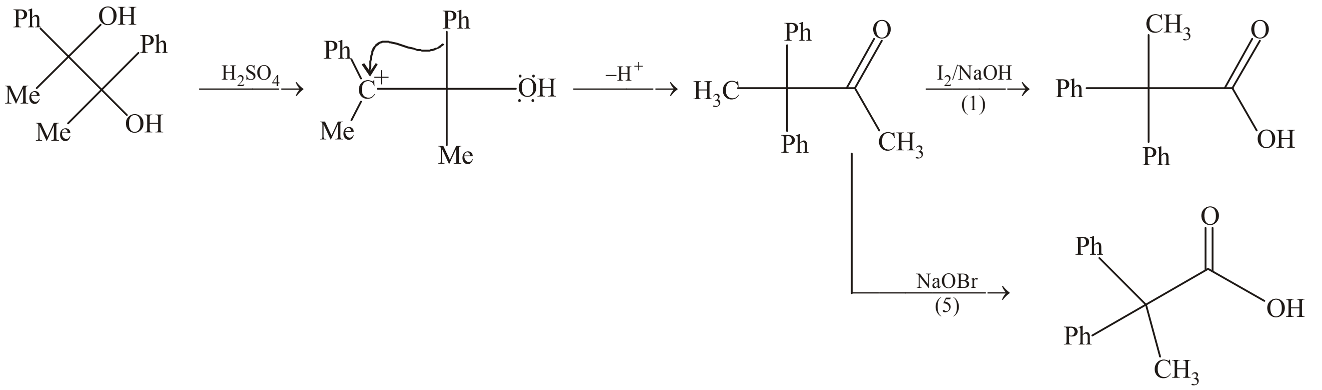
(P)



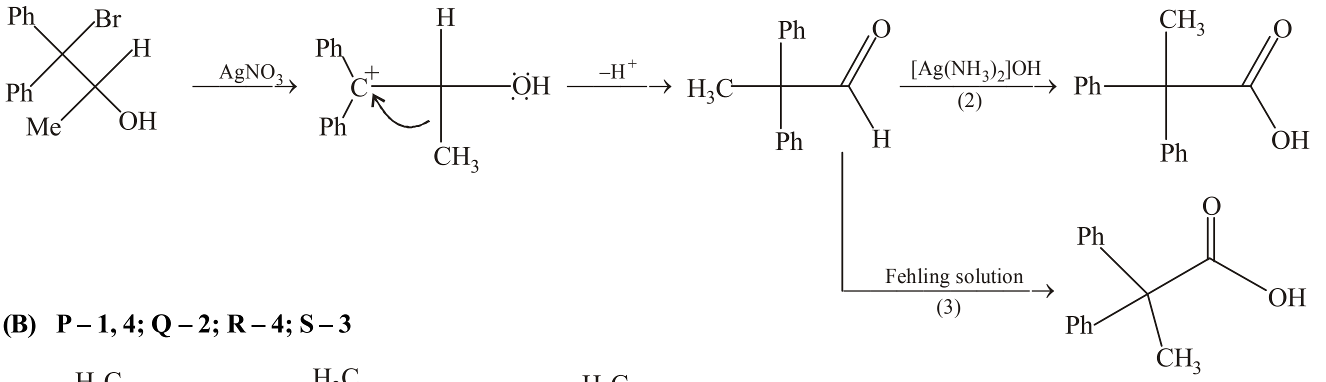
(Q)



(R)

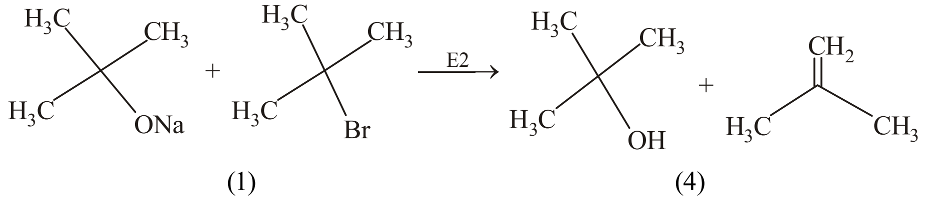


(S)

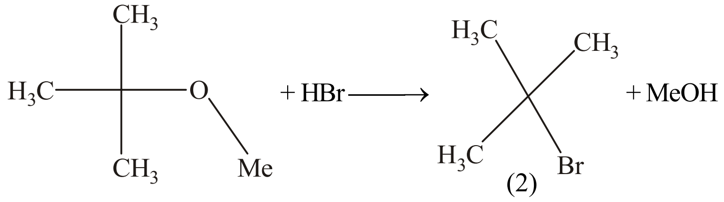


35. **(B)** **P – 1, 4; Q – 2; R – 4; S – 3**

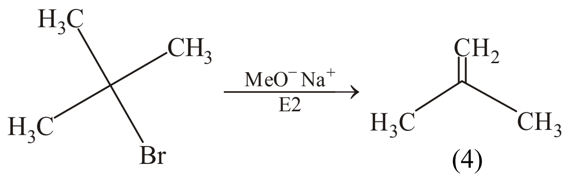
P:



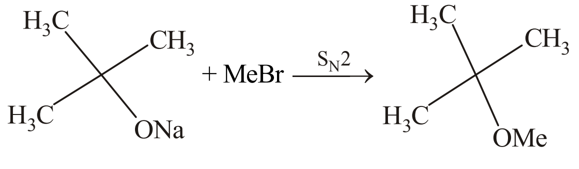
Q :



R :



S :



(3)

36. **(D) P – 1; Q – 5; R – 4; S – 1**

(P)
$$\begin{array}{lll} NaOH & + \quad CH_3COOH & \longrightarrow CH_3COONa + H_2O \\ \text{m mole} & 10\times 0.1 & 20\times 0.1 \\ & =1\text{ m.mol} & =2\text{ m.mol} \end{array}$$

∴ Solution contains 1 m. mol CH_3COOH & 1 m.mol CH_3COONa in 30 mL solution.

It is a Buffer solution. Hence pH does not change with dilution.

(Q)
$$\begin{array}{lll} NaOH & + \quad CH_3COOH & \longrightarrow CH_3COONa + H_2O \\ \text{m mole} & 20\times 0.1 & 20\times 0.1 \\ & =2\text{ m.mol} & =2\text{ m.mol} \end{array}$$

∴ Solution contains 2 m. mol of CH_3COONa in 40 mL solution (salt of weak acid and strong base)

For salts of weak acid and strong base :

$$[H^+]_{\text{initial}} = \sqrt{\frac{K_w K_a}{C}}$$

On dilution upto 80 mL, new conc. will be = $\frac{C}{2}$

$$\therefore [H^+]_{\text{new}} = \sqrt{\frac{K_w K_a}{C/2}} = [H+]_{\text{initial}} \times \sqrt{2}$$

(R)
$$\begin{array}{llll} & HCl & + \quad NH_3 & \longrightarrow NH_4Cl \\ \text{m mole} & 20\times 0.1 & 20\times 0.1 & \\ & =2\text{m.mol} & =2\text{ m.mol} & \end{array}$$

∴ Solution contains 2 m. mol of NH_4Cl in 40 mL solution (salt of strong acid and weak base)

For salts of strong acid and weak base

$$[H^+]_{\text{initial}} = \sqrt{\frac{K_w C}{K_b}}$$

On dilution upto 80 mL, new conc. will be = $\frac{C}{2}$.

$$\therefore [H^+]_{\text{new}} = \sqrt{\frac{K_w C}{K_b 2}} = \frac{[H^+]_{\text{initial}}}{\sqrt{2}}$$

(S) $Ni(OH)_2(s) \longrightarrow Ni^{2+} + 2OH^-$

∵ it is sparingly soluble salt

∴ On dilution $[OH^-]$ conc. in saturated solution of $Ni(OH)_2$ remains constant

$$\therefore [H^+]_{\text{new}} = [H^+]_{\text{initial}}$$

Mathematics

37. **(D)** $f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right)$

$$= \sum_{j=1}^{n} \tan^{-1}\left[\frac{(x+j)-(x+j-1)}{1+(x+j)(x+j-1)}\right]$$

$$= \sum_{j=1}^{n}[\tan^{-1}(x+j) - \tan^{-1}(x+j-1)]$$

$$\Rightarrow f_n(x) = \tan^{-1}(x+n) - \tan^{-1}(x)$$

$$= \tan^{-1}\left(\frac{n}{1+x(n+x)}\right)$$

$$\Rightarrow f_n'(x) = \frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$$

and $f_n(0) = \tan^{-1}(n)$ ∴ $\tan^2(\tan^{-1}n) = n^2$

Here x = 0 is not in the given domain, i.e., $x \in (0, \infty)$.

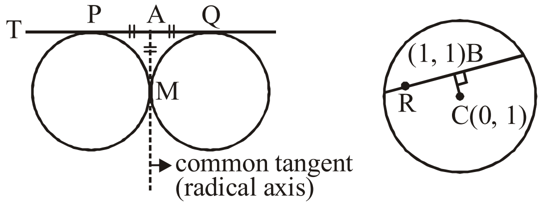
∴ Options (A) & (B) are not correct options.

(C) $\lim_{x\to\infty} \tan(f_n(x)) = \lim_{x\to\infty}\left(\frac{n}{1+x(n+x)}\right) = 0$

(D) $\lim_{x\to\infty} \sec^2(f_n(x)) = \lim_{n\to\infty} 1 + \tan^2(f_n(x))$

$$= 1 + \lim_{x\to\infty} \tan^2(f_n(x)) = 1$$

38. **(B, D)**



Here, AP = AQ = AM

∴ locus of M is a circle having PQ as its diameter and is given as :

$E_1 : (x-2)(x+2) + (y-7)(y+5) = 0$ and $x \neq \pm 2$

and its centre is (0, 1)

Locus of B (midpoint) is a circle having RC as its diameter and is given as :

$E_2 : x(x-1) + (y-1)^2 = 0$

(A) Option (A) is incorrect although it satisfies E_1 otherwise the line T would touch the second circle on two points.

(B) Also (4/5, 7/5) satisfies E_2 but again in this case one end of the chord would be (–2, 7) which is not included in E_1. Therefore $\left(\frac{4}{5}, \frac{7}{5}\right)$ does not lie in E_2.

(C) (1/2, 1) does not satisfy E_2 therefore it does not lie in E_2.

(D) (0, 3/2) does not satisfy E_1, so it does not lie in E_1.

39. **(A, D)** Here Δ= 0 so for at least one solution, we have

$$\Delta_1 = \Delta_2 = \Delta_3 = 0$$

$$\Rightarrow b_1 + 7b_2 = 13b_3 \qquad \text{(i)}$$

(A) $\Delta \neq 0$

∴ The equations have unique solution

∴ Option (A) is correct.

(D) $\Delta \neq 0$

∴ The equations have unique solution

∴ option (D) is correct

(C) $\Delta = 0$

⇒ equations are $x - 2y + 5z = -b_1$

$$x-2y+5z=\frac{b_2}{2}$$

$$x-2y+5z=b_3$$

The planes given in option (c) are parallel so they must be coincident

$$\Rightarrow \quad -b_1=\frac{b_2}{2}=b_3 \qquad \text{...(ii)}$$

$\therefore$ Equation (ii) satisfies equation (i) for all b_1, b_2, b_3, but not every value of (b_1, b_2, b_3) of equation (i) satisfies equation (ii)

$\therefore$ Option (C) is not correct.

$$\text{(B)}\ \Delta=\begin{vmatrix}1&1&3\\5&2&6\\-2&-1&-3\end{vmatrix}=-3\begin{vmatrix}1&1&1\\5&2&2\\2&1&1\end{vmatrix}=0$$

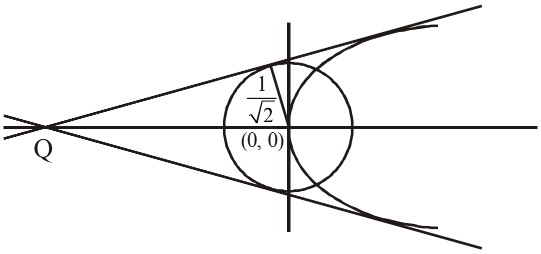
Also $\Delta_1=\begin{vmatrix}-1&1&3\\2&2&6\\1&-1&-3\end{vmatrix}b_3=0$

For infinite solutions, Δ_2 and Δ_3 must be 0

$$\Rightarrow \begin{vmatrix}1&b_1&3\\5&b_2&6\\-2&b_3&-3\end{vmatrix}=0$$

$\Rightarrow b_1+b_2+3b_3=0$ which does not satisfy (i) for all b_1, b_2, b_3 so option (B) is incorrect.

40. **(A, C)**



Let the equation of common tangent is : $y=mx+\frac{1}{m}$

$$\therefore \quad \left|\frac{0+0+\frac{1}{m}}{\sqrt{1+m^2}}\right|=\frac{1}{\sqrt{2}}$$

$\Rightarrow \quad m^4+m^2-2=0 \quad \Rightarrow \quad m=\pm1$

$\therefore$ Equation of common tangents are

$y=x+1$ and $y=-x-1$

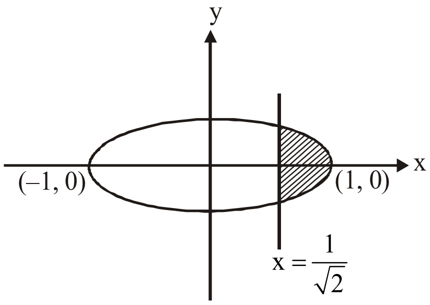
$\therefore \quad Q\equiv(-1, 0)$

$\therefore$ Equation of ellipse is : $\frac{x^2}{1}+\frac{y^2}{1/2}=1$

$$\text{(A)}\quad e=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$$

$$\text{and latus rectum}=\frac{2b^2}{a}=\frac{2\left(\frac{1}{\sqrt{2}}\right)^2}{1}=1$$

(C)



$$\therefore \text{ Required area }=2\cdot\int_{\frac{1}{\sqrt{2}}}^{1}\frac{1}{\sqrt{2}}\cdot\sqrt{1-x^2}\,dx$$

$$=\sqrt{2}\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{1/\sqrt{2}}^{1}$$

$$=\sqrt{2}\left[\frac{\pi}{4}-\left(\frac{1}{4}+\frac{\pi}{8}\right)\right]=\sqrt{2}\left(\frac{\pi}{8}-\frac{1}{4}\right)=\frac{\pi-2}{4\sqrt{2}}$$

41. **(A, C, D)**

$$sz+t\bar{z}+r=0 \qquad \text{...(i)}$$

$$\bar{s}\,\bar{z}+\bar{t}\,z+\bar{r}=0 \qquad \text{...(ii)}$$

Adding (i) and (ii), we get :

$$(t+\bar{s})\bar{z}+(s+\bar{t})z+(r+\bar{r})=0 \qquad (1)$$

Subtracting (ii) from (i), we get:

$$(t-\bar{s})\bar{z}+(s-\bar{t})z+(r-\bar{r})=0 \qquad (2)$$

Equation (1) and (2) represent set of lines.

For equation (1) and (2) to have unique solution, we have:

$$\frac{t+\bar{s}}{t-s}\neq\frac{s+\bar{t}}{s-t}$$

On solving the above equation we get

$|t|\neq|s|$

$\therefore$ Option (A) is correct

For equation (1) and (2) to have infinitely many solutions, we have :

$$\frac{t+\bar{s}}{t-s}=\frac{\bar{t}+s}{s-\bar{t}}=\frac{r+\bar{r}}{r-\bar{r}}\Rightarrow|t|=|s|$$

and $\bar{t}r-\bar{t}\,\bar{r}+sr-s\bar{r}=sr+s\bar{r}-\bar{t}r-\bar{t}\,\bar{r}$

$\Rightarrow \quad 2\bar{t}r=2s\bar{r} \quad \Rightarrow \quad \bar{t}r=s\bar{r}$

$\therefore \quad |\bar{t}||r|=|s||\bar{r}|$

$\Rightarrow \quad |t||r|=|s||r| \quad \Rightarrow \quad |t|=|s|$

$\therefore$ If $|t|=|s|$, lines will be parallel for sure but may not be coincident (i.e., does not have infinitely many solutions).

(C) : Locus of Z is a null set or singleton set or a line, in all threse cases it will intersect given circle at most two points.

(D) : in this case locus of Z is a line so L has infinte elements.

42. **(B, C, D)**

$$\lim_{t\to x}\frac{f(x)\sin t-f(t)\sin x}{t-x}=\sin^2x$$

$$\Rightarrow \quad \lim_{t\to x}\frac{f(x)\cos t-f'(t)\sin x}{1}=\sin^2x$$

(Using L'Hospital's Rule)

$$\Rightarrow \quad f(x)\cos x - f'(x)\sin x = \sin^2 x$$

$$\Rightarrow \quad -\left(\frac{f'(x)\sin x - f(x)\cos x}{\sin^2 x}\right) = 1$$

$$\Rightarrow \quad -d\left(\frac{f(x)}{\sin x}\right) = 1 \quad \Rightarrow \quad \frac{f(x)}{\sin x} = -x + c$$

Now, Put $x = \frac{\pi}{6}$ also it is given that $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$

$$\therefore \quad \frac{\frac{-\pi}{12}}{\frac{1}{2}} = -\frac{\pi}{6} + c \quad \Rightarrow \quad \frac{-\pi}{12} = \frac{-\pi}{12} + c$$

$\Rightarrow \quad c = 0 \Rightarrow f(x) = -x \sin x$

(A) $f\left(\frac{\pi}{4}\right) = \frac{-\pi}{4}\frac{1}{\sqrt{2}}$

(B) $f(x) = -x \sin x$

as $\sin x > x - \frac{x^3}{6} \forall x \in (0, \pi)$

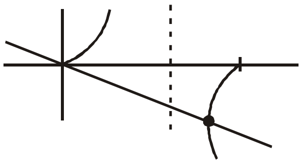
$\therefore -x \sin < -x^2 + \frac{x^4}{6}$

$\therefore \; f(x) < -x^2 + \frac{x^4}{6} \forall x \in (0, \pi)$

(C) $f'(x) = -\sin x - x\cos x$

$f'(x) = 0 \Rightarrow \tan x = -x$

$\Rightarrow$ there exist $\alpha \in (0, \pi)$ for which $f'(\alpha) = 0$



(D) Here, $f''(x) = -2\cos x + x \sin x$

$\therefore \; f''\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$

$\Rightarrow \; f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

43. (2) Let $I = \int_0^{\frac{1}{2}} \frac{(1+\sqrt{3})\,dx}{\left[(1+x)^2\,(1-x)^6\right]^{1/4}}$

$$= \int_0^{\frac{1}{2}} \frac{(1+\sqrt{3})\,dx}{(1+x)^2\left[\frac{(1-x)^6}{(1+x)^6}\right]^{1/4}}$$

Now put $\frac{1-x}{1+x} = t \quad \Rightarrow \quad \frac{-2dx}{(1+x)^2} = dt$

$$\therefore \quad I = \int_1^{1/3} \frac{(1+\sqrt{3})\,dt}{-2t^{6/4}} = \frac{-(1+\sqrt{3})}{2} \times \left|\frac{-2}{\sqrt{t}}\right|_1^{1/3}$$

$$= \left(1+\sqrt{3}\right)\left(\sqrt{3}-1\right) = 2$$

44. (4) Suppose $P = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

So, $\det(P) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$= a_1\,(b_2c_3 - b_3c_2) - a_2\,(b_1c_3 - b_3c_1) + a_3\,(b_1c_2 - b_2c_1)$

Maximum value can be 6 when $a_1 = 1, a_2 = -1, a_3 = 1$ and $b_2c_3 = b_1c_3 = b_1c_2 = 1$ & $b_3c_2 = b_3c_1 = b_2c_1 = -1$.

So, $(b_2c_3)\,(b_3c_1)\,(b_1c_2) = -1$

and $\quad (b_1c_3)\,(b_3c_2)\,(b_2c_1) = 1$.

Therefore $\; b_1b_2b_3c_1c_2c_3$ has two values 1 and -1 which is not possible.

Contradiction also occurs if $a_1 = 1, a_2 = 1, a_3 = 1$ and $b_2c_2 = b_3c_1 = b_1c_2 = 1$ & $b_3c_2 = b_1c_3 = b_1c_2 = -1$.

For maximum value to be 5 one of the terms should be zero but this will make 2 terms zero therefore answer should not be 5.

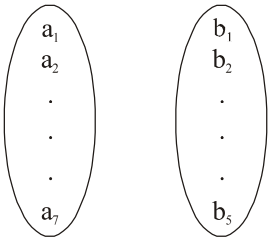
As $\begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{vmatrix} = 4.$

Hence maximum value of the determinant of $P = 4$.

45. (119) Here $n\,(X) = 5$ and $n\,(Y) = 7$

Number of one-one function $= \alpha = {}^7C_5 \times 5!$

and Number of onto function Y to X is given as:



1, 1, 1, 1, 3 1, 1, 1, 2, 2

$$\beta = \frac{7!}{3!\,4!} \times 5! + \frac{7!}{(2!)^3\,3!} \times 5! = ({}^7C_3 + 3 \times {}^7C_3)\,5!$$

$$= 4 \times {}^7C_3 \times 5!$$

$$\Rightarrow \quad \frac{\beta - \alpha}{5!} = 4 \times {}^7C_3 - {}^7C_5 = 4 \times 35 - 21 = 119$$

46. (0.4) $\frac{dy}{dx} = (5y+2)\,(5y-2) = 25\left(y + \frac{2}{5}\right)\left(y - \frac{2}{5}\right)$

$$\Rightarrow \quad \frac{1}{25}\int \frac{dy}{\left(y+\frac{2}{5}\right)\left(y-\frac{2}{5}\right)} = \int dx$$

$$\Rightarrow \quad \frac{1}{25}\int \frac{5}{4}\left[\frac{1}{y-\frac{2}{5}}-\frac{1}{y+\frac{2}{5}}\right]dy = \int dx$$

$$\Rightarrow \quad \frac{1}{25}\times\frac{5}{4}\ell n\left|\frac{y-\frac{2}{5}}{y+\frac{2}{5}}\right| = x + c, \text{ where c is constant of}$$

integration.

$$\Rightarrow \quad \frac{1}{20}\ell n\left|\frac{5y-2}{5y+2}\right| = x + c$$

As, $f(0) = 0 \Rightarrow$ at $x = 0, y = 0 \Rightarrow 0 = 0 + c \Rightarrow c = 0$

Therefore, $\left|\frac{5y-2}{5y+2}\right| = e^{20x}$

$$\Rightarrow \quad \lim_{x\to-\infty}\left|\frac{5\,f(x)-2}{5\,f(x)+2}\right| = \lim_{x\to-\infty} e^{20x} = e^{-\infty} = 0$$

$$\Rightarrow \quad 5\lim_{x\to-\infty} f(x) - 2 = 0 \Rightarrow \lim_{x\to-\infty} f(x) = \frac{2}{5} = 0.4$$

47. **(2)** $f(x+y) = f(x)\,f'(y) + f'(x)\,f(y)$ (1)

After putting $x = y = 0$, we get

$f(0) = 2f'(0)\,f(0) \Rightarrow f'(0) = \frac{1}{2}$ $[\because f(0) = 1]$

Now putting $y = 0$ in equation (1), we get

$f(x) = f(x)\;f'(0) + f'(x)\;f(0)$

$$\Rightarrow \quad f'(x) = \frac{f(x)}{2} \Rightarrow \int \frac{f'(x)}{f(x)}dx = \frac{1}{2}\int dx$$

$[\because f(0) = 1 \text{ and } f'(0) = \frac{1}{2}]$

$$\Rightarrow \quad \log_e f(x) = \frac{x}{2} + \log_e c$$

$\Rightarrow \quad f(x) = ce^{x/2} \Rightarrow f(x) = e^{x/2}$ $(\because f(0) = 1)$

$$\Rightarrow \quad \log_e(f(x)) = \frac{x}{2} \Rightarrow \log_e(f(4)) = 2$$

48. **(8)** Suppose coordinates of P are (a, b, c).

So, coordinates of Q are (0, 0, c) and coordinates of R are (a, b, –c).

Here, PQ is perpendicular to the plane $x + y = 3$.

So, PQ is parallel to the normal of given plane

i.e. $(a\hat{i} + b\hat{j})$ is parallel to $(\hat{i} + \hat{j})$

$\Rightarrow \quad a = b$

As mid-point of PQ lies in the plane $x + y = 3$, so

$$\frac{a}{2} + \frac{b}{2} = 3 \Rightarrow a + b = 6 \Rightarrow a = 3 = b$$

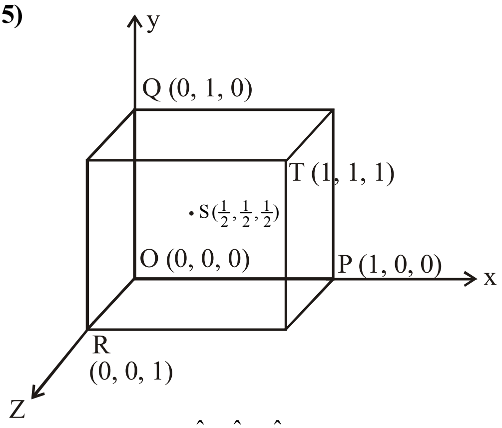
Therefore, distance of P from the x-axis $= \sqrt{b^2 + c^2} = 5$ (given)

$\Rightarrow \quad b^2 + c^2 = 25 \Rightarrow c^2 = 25 - 9 = 16$

$\Rightarrow \quad c = \pm 4$

Hence, $PR = |2c| = 8$

49. **(0.5)**



Here, $\vec{p} = \overrightarrow{SP} = \frac{\hat{i}-\hat{j}-\hat{k}}{2}$

$\vec{q} = \overrightarrow{SQ} = \frac{-\hat{i}+\hat{j}-\hat{k}}{2}$

$\vec{r} = \overrightarrow{SR} = \frac{-\hat{i}-\hat{j}+\hat{k}}{2}$

$\vec{t} = \overrightarrow{ST} = \frac{\hat{i}+\hat{j}+\hat{k}}{2}$

$$\text{So, } \vec{p}\times\vec{q} = \frac{1}{4}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -1\\ -1 & 1 & -1\end{vmatrix} = \frac{2\hat{i}+2\hat{j}}{4} = \frac{\hat{i}+\hat{j}}{2}$$

$$\text{and, } \vec{r}\times\vec{t} = \frac{1}{4}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & -1 & 1\\ 1 & 1 & 1\end{vmatrix} = \frac{-2\hat{i}+2\hat{j}}{4} = \frac{-\hat{i}+\hat{j}}{2}$$

$$\text{Hence, } (\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t}) = \frac{1}{4}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 0\\ -1 & 1 & 0\end{vmatrix} = \frac{\hat{k}}{2}$$

$$\Rightarrow \quad |(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})| = \frac{1}{2} = 0.5$$

50. **(646)**

$$\sum_{r=0}^{n} r({}^{n}C_r)^2 = n\sum_{r=0}^{n} {}^{n}C_r\,{}^{n-1}C_{r-1}$$

$$= n\sum_{r=1}^{n} {}^{n}C_{n-r}\,{}^{n-1}C_{r-1} = n^{2n-1}C_{n-1}$$

So, $X = ({}^{10}C_1)^2 + 2({}^{10}C_2)^2 + 3({}^{10}C_3)^2 + \ldots + 10({}^{10}C_{10})^2$

$$= \sum_{n=0}^{10} r({}^{10}C_r)^2 = 10^{19}C_9$$

$$\text{Hence, } \frac{X}{1430} = \frac{1}{143}\,{}^{19}C_9 = 646$$

51. **(A)** For $E_1, \frac{x}{x-1} > 0$ and $x \neq 1 \Rightarrow x \in (-\infty, 0) \cup (1, \infty)$

For E_2, $-1 \leq \log_e\left(\frac{x}{x-1}\right) \leq 1 \Rightarrow \frac{1}{e} \leq \frac{x}{x-1} \leq e$

$$\Rightarrow \frac{1}{e} \leq 1 + \frac{1}{x-1} \leq e$$

$$\frac{1}{e}-1\le\frac{1}{x-1}\le e-1\Rightarrow(x-1)\in\left(-\infty,\frac{e}{1-e}\right]\cup\left[\frac{1}{e-1},\infty\right)$$

$$\Rightarrow x\in\left(-\infty,\frac{1}{e-1}\right]\cup\left[\frac{e}{e-1},\infty\right)$$

For E_1, $\frac{x}{x-1}\in(0,\infty)-\{1\}$

$$\Rightarrow \log_e\left(\frac{x}{x-1}\right)\in(-\infty,\infty)-\{0\}$$

$$\Rightarrow f(x)\in(-\infty,0)\cup(0,\infty)$$

$$g(x)=\sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$$

52. (C) Here, a committee has to be formed form a group of 6 Boys and 5 girls.

Total number of ways for selecting exactly 3 boys and 2 girls $= {}^6C_3\times{}^5C_2=20\ \times 10=200=\alpha_1$.

Total number of ways for selecting at least 2 members with equal number of boys and girls

$$=({}^6C_1\times{}^5C_1)+({}^6C_2\times{}^5C_2)+({}^6C_3\times{}^5C_3)+({}^6C_4\times{}^5C_4)$$
$$+({}^6C_5\times{}^5C_5)={}^{11}C_5-1=461=\alpha_2.$$

Total number of ways for selecting 5 members having at least 2 girls $={}^{11}C_5-{}^6C_5-{}^6C_4\times{}^5C_1={}^{11}C_5-81=381=\alpha_3$

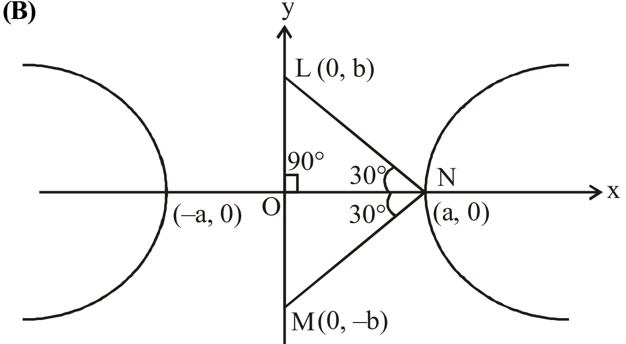
Total number of ways for selecting 4 memebrs having at least 2 girls M_1 and G_1 are not selected together

$= n(M_1$ selected & G_1 not selected$) + n(G_1$ selected & M_1 not selected$) + n(M_1$ and G_1 both not selected$)$

$$=({}^4C_2\times{}^5C_1+{}^4C_3)+({}^4C_1\times{}^5C_2+{}^4C_2\times{}^5C_1+{}^4C_3)$$
$$+({}^4C_4+{}^4C_3\times{}^5C_1+{}^4C_2\times{}^5C_2)$$

$$=34+4+81=189=\alpha_4$$

53. (B)



Area of $\Delta LMN = 4\sqrt{3}$ (given)

$$\Rightarrow \frac{1}{2}\times LM\times ON=4\sqrt{3}$$

$$\Rightarrow \frac{1}{2}(2b)(\sqrt{3}b)=4\sqrt{3}$$

$$\Rightarrow b^2=4\Rightarrow b=2$$

So, length of the conjugate axis of H = 2b = 4

$$\tan 30°=\frac{OL}{ON}=\frac{b}{a}\ \Rightarrow a=\sqrt{3b}\Rightarrow a=2\sqrt{3}$$

$\therefore\ b^2=a^2(e^2-1)$

$\Rightarrow 4=12(e^2-1)$

$$\Rightarrow e^2=1+\frac{1}{3}=\frac{4}{3}$$

$\Rightarrow$ The eccentricity of H= $e=\frac{2}{\sqrt{3}}$ and

The distance between the foci of H = 2ae

$$=2\times2\sqrt{3}\times\frac{2}{\sqrt{3}}=8$$

and length of latus ractum of H $=\frac{2b^2}{a}=\frac{2\times4}{2\sqrt{3}}=\frac{4}{\sqrt{3}}$

54. (D)

(i) $$f'_1(0)=\lim_{h\to0}\left[\frac{\sin\sqrt{1-e^{-h^2}}-0}{h}\right]$$

$$=\lim_{h\to0}\left[\frac{\sin\sqrt{1-e^{-h^2}}}{\sqrt{1-e^{-h^2}}}\times\frac{\sin\sqrt{1-e^{-h^2}}}{h^2}\times\frac{|h|}{h}\right]$$

$$=\lim_{h\to0}\left[1\times1\times\frac{|h|}{h}\right]=\lim_{h\to0}\frac{|h|}{h}\qquad\left[\because\ \lim_{x\to0}\frac{\sin x}{x}=1\right]$$

which does not exit.

So for (P), (2) is correct.

(ii) $$\lim_{x\to0}f_2(x)=\lim_{x\to0}\left[\frac{|\sin x|}{\tan^{-1}x}\right]$$

$$\lim_{x\to0}\left[\frac{|\sin x|}{|x|}\times\frac{x}{\tan^{-1}x}\times\frac{|x|}{x}\right]$$

$$\lim_{x\to0}\left[1\times1\times\frac{|x|}{x}\right]=\lim_{x\to0}\frac{|x|}{x}\qquad\left[\because\ \lim_{x\to\infty}\frac{x}{\tan^{-1}x}=1\right]$$

which does not exist, so for Q, (1) is correct.

(iii) $$\lim_{x\to0}f_3(x)=\lim_{x\to0}[\sin(\log_e(x+2))]$$

$$\text{if } x\to0\Rightarrow(x+2)\to2$$

$$\Rightarrow\ \log_e(x+2)\to\log_e 2<1$$

$$\Rightarrow 0<\lim_{x\to0}\sin(\log_e(x+2)<\sin1$$

$$\Rightarrow\ \lim_{x\to0}[\sin(\log_e(x+2))]=0$$

$$f_3(x)=0\qquad\forall x\in[-1,e^{\pi/2}-2)$$

$$\Rightarrow\quad f'_3(x)=0\quad\forall x\in(-1,e^{\pi/2}-2)$$

$$\Rightarrow\quad f''_3(x)=0\quad\forall x\in(-1,e^{\pi/2}-2)$$

So for (R), (4) is correct.

(iv) $$\lim_{x\to0}f_4(x)=\lim_{x\to0}\left(x^2\sin\frac{1}{x}\right)=\lim_{x\to0}x^2\left(\sin\frac{1}{x}\right)=0$$

$$f'_4(0)=\lim_{x\to0}\frac{h^2\sin\left(\frac{1}{h}\right)-0}{h}=\lim_{x\to0}h\sin\left(\frac{1}{4}\right)=0$$

$$f'_4(x)=-\cos\frac{1}{x}+2x\sin\frac{1}{x},\ x\ne0$$

$$\lim_{x\to0}f'_4(x)=\lim_{x\to0}\left[-\cos\frac{1}{x}+2x\ \sin\frac{1}{x}\right]=-\lim_{x\to0}\cos\frac{1}{x}$$

which does not exist

So for (S), (3) is correct.

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PAPER - 1

Section - I (Maximum Marks: 12)

Directions for (Qs. 1 - 4, 19 - 22 and 37 - 40).

- This section contains FOUR (04) questions.
- Each question has FOUR options. ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :
  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases

Section - II (Maximum Marks: 32)

Directions for (Qs. 5 - 12, 23 - 30 and 41 - 48).

- This section contains EIGHT (08) questions.
- Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is(are)correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : + 4 If only (all) the correct options(s) is(are) chosen;
  Partial Marks : + 3 If all the four options are correct but Only three options are chosen;
  Partial Marks : + 2 If three or more options are correct but ONLY two options are chosen and both of which are correct;
  Partial Marks : + 1 If two or more options are correct but ONLY one option are chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : – 1in all other cases.
  For example, in a question if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get –1 mark.

Section - III (Maximum Marks: 18)

Directions for (Qs. 13 - 18, 31 - 36 and 49 - 54).

- This section contains **SIX (06)** questions. The answer to each question is **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. if the numerical value has more than two decimal places, **truncate/round-off** the value to **Two** decimal places.
- Answer to each question will be evaluated according to the following marking scheme.
  Full Marks : + 3 If ONLY the correct numerical value is entered;
  Zero Marks : 0 In all other cases.

PHYSICS

Section - I

1. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2$ ($a << 1$) is made on the shell without affecting the rest of the shell. Which one of the following statements is correct ?

(a) The magnitude of electric field at a point, located on a line passing through the hole and shell's center, on a distance $2R$ from the centre of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

(b) The ratio of the potential at the centre of the shell to that of the point at $\frac{1}{2}R$ from the centre towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

(c) The magnitude of electric field at the centre of the shell is reduced by $\frac{\alpha V_0}{2R}$

(d) The potential at the centre of the shell is reduced by $2\alpha V_0$

2. Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radical distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common centre with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is
[G is universal gravitational constant]

(a) $\dfrac{3K}{\pi r^2 m^2 G}$ (b) $\dfrac{K}{2\pi r^2 m^2 G}$

(c) $\dfrac{K}{\pi r^2 m^2 G}$ (d) $\dfrac{K}{6\pi r^2 m^2 G}$

3. A current carrying wire heats a metal rod. The wire provides a constant power P to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as $T(t) = T_0\,(1+\beta t^{1/4})$ Where β is a constant with appropriate dimensions while T_0 is a constant with dimensions of temperature. The heat capacity of metal is :

(a) $\dfrac{4P(T(t)-T_0)}{\beta^4 T_0^2}$ (b) $\dfrac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$

(c) $\dfrac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (d) $\dfrac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$

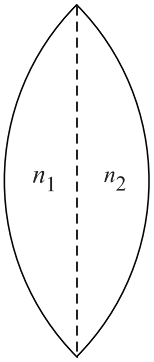
4. In a radioactive sample, ${}^{40}_{19}K$ nuclei either decay into stable ${}^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable ${}^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei are produced by the ${}^{40}_{19}K$ nuclei only. In time $t\times10^9$ years, if the ratio of the sum of stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei to the radioactive ${}^{40}_{19}K$ nuclei is 99, the value of t will be [Given: ln 10 = 2.3]

(a) 9.2 (b) 4.6

(c) 1.15 (d) 2.3

Section - II

5. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surface are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n-1)$ and $1 < n < 2$, the correct statement(s) is/are.



(a) If $\dfrac{\Delta n}{n} < 0$ then $\dfrac{\Delta f}{f} > 0$

(b) The relation between $\dfrac{\Delta f}{f}$ and $\dfrac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surface of the same radius of curvature.

(c) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place).

(d) $\left|\dfrac{\Delta f}{f}\right| < \left|\dfrac{\Delta n}{n}\right|$

6. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct ?
[ε_0 is the permittivity of free space]

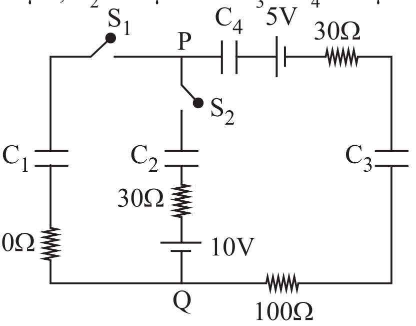
(a) If $h > 2R$ and $r = 3R/5$ then $\phi = Q/5\varepsilon_0$

(b) If $h > 2R$ and $r > R$ then $\phi = Q/\varepsilon_0$

(c) If $h < 8R/5$ and $r = 3R/5$ then $\phi = 0$

(d) If $h > 2R$ and $r = 4R/5$ then $\phi = Q/5\varepsilon_0$

7. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\mu F$, $C_2 = 30\mu F$ and $C_3 = C_4 = 80\mu F$.



Which of the statement(s) is/are correct?

(a) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10V.

(b) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2A (round off to 1st decimal place)

(c) At time $t = 0$, the key S_1 is closed, the instantaneous current in the closed circuit will be 25mA.

(d) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitors C_1 will be 4V.

8. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement(s) is/are correct?

(a) The dimension of force is L^{-3}

(b) The dimension of linear momentum is L^{-1}

(c) The dimension of energy is L^{-2}

(d) The dimension of power is L^{-5}

9. Two identical moving coil galvanometers have 10 Ω resistance and full scale deflection at $2\mu A$ current. One of them is converted into a voltmeter of 100mV full scale reading and the other into an Ammeter of 1mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with R = 1000Ω resistor by using an ideal cell. Which of the following statement(s) is/are correct?

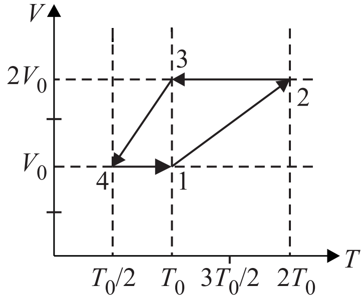
(a) The resistance of the voltmeter will be $100k\,\Omega$

(b) The measured value of R will be $978\,\Omega < R < 982\,\Omega$

(c) If the ideal cell is replaced by a cell having internal resistance of $5\,\Omega$ then the measured value of R will be more than $1000\,\Omega$

(d) The resistance of the Ammeter will be $0.02\,\Omega$ (round off to 2nd decimal place)

10. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V–T) diagram. The correct statement(s) is/are: [R is the gas constant]



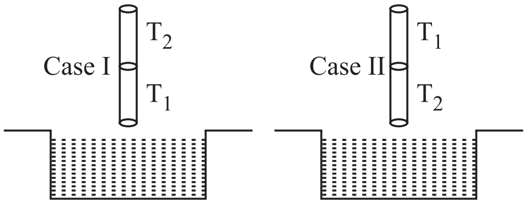
(a) Work done in this thermodynamic cycle $(1\to 2\to 3\to 4\to 1)$ is $|W| = \frac{1}{2}RT_0$

(b) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(c) The ratio of heat transfer during processes $1\to 2$ and $2\to 3$ is $\left|\frac{Q_{1\to 2}}{Q_{2\to 3}}\right| = \frac{5}{3}$

(d) The ratio of heat transfer during processes $1\to 2$ and $3\to 4$ is $\left|\frac{Q_{1\to 2}}{Q_{3\to 4}}\right| = \frac{1}{2}$

11. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0° and 60°, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct? [surface tension of water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2]



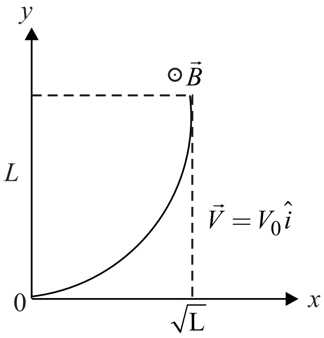
(a) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

(b) For case II, if the capillary joint is 5cm above the water surface, the height of water column raised in the tube will be 3.75cm. (Neglect the weight of the water in the meniscus).

(c) For case I, if the joint is kept at 8cm above the water surface, the height of water column in the tube will be 7.5cm. [Neglect the weight of the water in the meniscus]

(d) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. [Neglect the weight of the water in the meniscus]

12. A conducting wire of parabolic shape, initially $y = x^2$ is moving with velocity $\vec{V} = V_0\hat{i}$ in a non – uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are:



(a) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis.

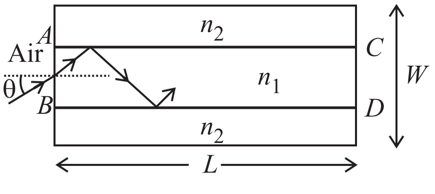
(b) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, $y = x$ initially, of length $\sqrt{2}L$

(c) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$

(d) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$

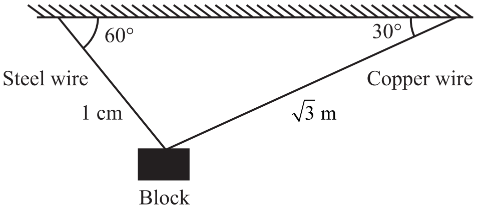
Section - III

13. A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If $L >> W$, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For $L = 9.6$ M, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}s$, where t is _____



14. A block of weight 100N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30^o and 60^o, respectively. If elongation in copper wire is (Δl_c) and elongation in steel wire is (Δl_s), then the ratio $\frac{\Delta l_c}{\Delta l_s}$ is_____

[Young's modulus for copper and steel are 1×10^{11} N/M^2 and 2×10^{11} N/M^2 , respectively.]

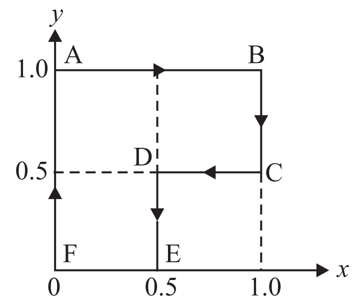


15. A liquid at 30^{o}C is poured very slowly into a Calorimeter that is at temperature of 110^{o}C. The boiling temperature of the liquid is 80^{o}C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50 ^{o}C. The ratio of latent heat of the liquid to its specific heat will be ___ ^{o}C.
[Neglect the heat exchange with surrounding]

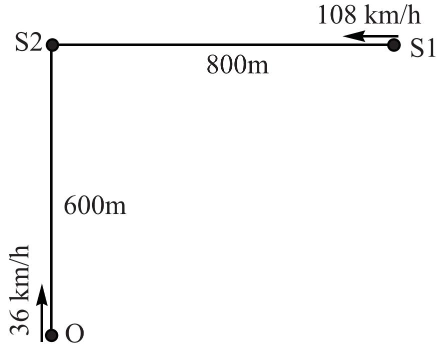
16. A parallel plate capacitor of capacitance *C* has spacing *d* between two plates having area *A*. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the mth layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large N(>10^3), the capacitance C is $\alpha\left(\frac{K \in_0 A}{d \ln 2}\right)$. The value of α will be ___

[$\in_0$ is the permittivity of free space]

17. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})N$, where *x* and *y* are in meter and $\alpha = -1Nm^{-1}$. The work done on the particle by this force $\vec{F}$ will be ___ Joule.



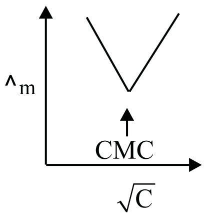
18. A train S1, moving with a uniform velocity of 108 km/h approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36km/h towards S2 as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600m away from S2 and distance between S1 and S2 is 800m, the number of beats heard by O is _____
[Speed of the sound = 330m/s]

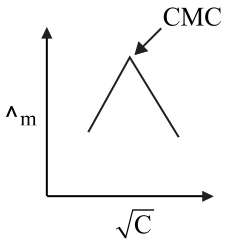


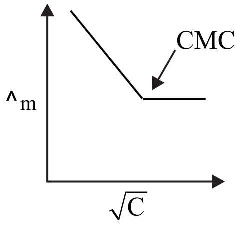
CHEMISTRY

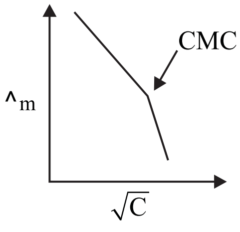
Section - I

19. Molar conductivity $(\wedge_m)$ of aqueous solution of sodium stearate, which behaves as a strong electrolyte is recorded at varying concentration (C) of sodium stearate. Which one of the following plots provides the correct representation of micelle formation in the solution?
(Critical micelle concentration (CMC) is marked with an arrow in the figures)

(a)


(b)


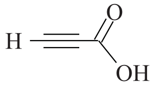
(c)


(d)


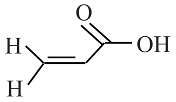
20. Calamine, malachite, magnetite and cryolite, respectively, are
(a) $ZnCO_3$, $CuCO_3.Cu(OH)_2$, Fe_3O_4, Na_3A1F_6
(b) $ZnSO_4$, $Cu(OH)_2$, Fe_3O_4, Na_3AlF_6
(c) $ZnSO_4$, $CuCO_3$, Fe_2O_3, AlF_3
(d) $ZnCO_3$, $CuCO_3$, Fe_2O_3, Na_3AlF_6

21. The green colour produced in the borax bead test of a chromium (III) salt is due to
(a) $Cr_2(B_4O_7)_3$ (b) Cr_2O_3
(c) $Cr(BO_2)_3$ (d) CrB

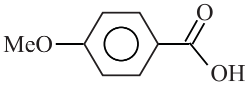
22. The correct order of acid strength of the following carboxylic acid is



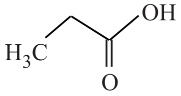
I



II



III



IV

(a) I > II > III > IV (b) II > I > IV > III
(c) I > III > II > IV (d) III > II > I > IV

Section - II

23. Which of the following statement(s) is (are) true?

(a) Oxidation of glucose with bromine water gives glutamic acid.

(b) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose.

(c) The two six-membered cyclic hemiacetal forms of D-(+)- glucose are called anomers.

(d) Monosaccharides cannot be hydrolysed to give polyhydroxy aldehydes and ketones

24. Fusion of MnO_2 with KOH in presence of O_2 produces a salt W. Alkaline solution of W upon electrolytic oxidation yields another salt X. The manganese containing ions present in W and X, respectively are Y and Z. Correct statement(s) is (are)

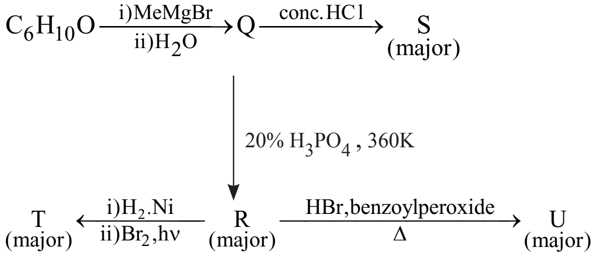
(a) In both Y and Z, π-bonding occurs between *p*-orbitals of oxygen and *d*-orbitals of manganese

(b) In aqueous acidic solution, Y undergoes disproportionation reaction to give Z and MnO_2

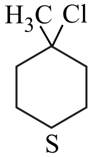
(c) Both Y and Z are coloured and have tetrahedral shape'

(d) Y is diamagnetic in nature while Z is paramagnetic

25. Choose the correct option(s) for the following set of reactions



(a)



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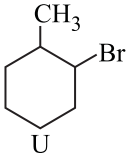

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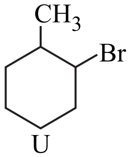
(b)



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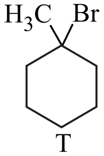

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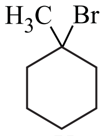
(c)



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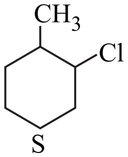
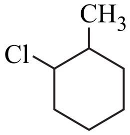


S

(d)



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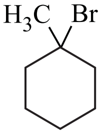


T

26. Choose the reaction(s) from the following options, for which the standard enthalpy of reaction is equal to the standard enthalpy of formation

(a) $\frac{1}{8}S_8(s)+O_2(g)\to SO_2(g)$

(b) $2H_2(g)+O_2(g)\to 2H_2O(l)$

(c) $\frac{3}{2}O_2(g)\to O_3(g)$

(d) $2C(g)+3H_2(g)\to C_2H_6(g)$

27. A tin chloride Q undergoes the following reactions (not balanced)

$Q+Cl^- \to X$
$Q+Me_3N\to Y$
$Q+CuCl_2 \to Z+CuCl$

X is a mono anion having pyramidal geometry. Both Y and Z are neutral compounds. Choose the correct option(s).

(a) The oxidation state of the central atom in Z is +2

(b) The central atom in Z has one lone pair of electrons

(c) The central atom in X is sp^3 hybridized

(d) There is a coordinate bond in Y

28. Each of the following options contains a set of four molecules. Identify the option(s) were all four molecules posses permanent dipole moment at room temperature.

(a) BF_3, O_3, SF_6, XeF_6

(b) NO_2, NH_3, $POCl_3$, CH_3Cl

(c) SO_2, C_6H_5Cl, H_2Se, BrF_5

(d) $BeCl_2$, CO_2, BCl_3, $CHCl_3$

29. Which of the following statement(s) is(are) correct regarding the root mean square speed (U_{rms}) of a molecule in a gas at equilibrium?

(a) ε_{av} at a given temperature does not depend on its molecular mass

(b) U_{rms} is inversely proportional to the square root of its molecular mass

(c) U_{rms} is doubled when its temperature is increased four times

(d) ε_{av} is doubled when its temperature is increased four times

30. In the decay sequence,

$$^{238}_{92}U \xrightarrow{-x_1} {}^{234}_{90}Th \xrightarrow{-x_3} {}^{234}_{91}Pa \xrightarrow{-x_2} {}^{234}Z \xrightarrow{-x_4} {}^{230}_{90}Th$$

x_1, x_2, x_3 and x_4 are particles/radiation emitted by the respective isotopes. The correct option(s) is (are)

(a) Z is an isotope of uranium
(b) x_1 will deflect towards negatively charged plate
(c) x_3 is γ-ray
(d) x_2 is β-ray

Section - III

31. For the following reaction, the equilibrium constant K_c at 298 K is 1.6×10^{17}

$$Fe^{2+}(aq) + S^{2-}(aq) \rightleftharpoons FeS(s)$$

When equal volumes of 0.06 M Fe^{2+} (aq) and 0.2M S^{2-} (aq) solutions are mixed, the equilibrium concentration of Fe^{2+} (aq) is found to be $Y \times 10^{-17}$ M. The value of Y is

32. At 143 K, the reaction of XeF_4 with O_2F_2, produces a xenon compound Y. The total number of lone pair(s) of electron present on the whole molecule of Y is ______

33. Consider the kinetic data given in the following table for the reaction A + B + C $\rightarrow$ product.

Experiment No.	[A] (mol dm^{-3})	[B] (mol dm^{-3})	[C] (mol dm^{-3})	Rate of reaction (mol dm^{-3})
1	0.2	0.1	0.1	6.0×10^{-5}
2	0.2	0.2	0.1	6.0×10^{-5}
3	0.2	0.1	0.2	1.2×10^{-4}
4	0.3	0.1	0.1	9.0×10^{-5}

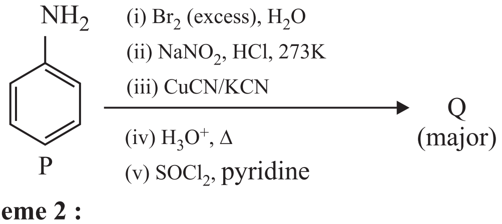
The rate of the reaction for [A] = 0.15 mol^{-3}, [B] = 0.25 mol dm^{-3} and [C] = 0.15 mol dm^{-3} is found to be $Y \times 10^{-5}$ mol dm^{-3} s^{-1}. The value of Y is _____

34. On dissolving 0.5 g of a non-volatile non-ionic solute to 39 g of benzene, its vapour pressure decreases from 650 mm Hg to 640 mm Hg. The depression of freezing point of Benzene (in K) upon addition of the solute is _____
(Given data : Molar mass and the molal freezing point depression constant of benzene are 78 g mol^{-1} and 5.12 K kg mol^{-1}, respectively)

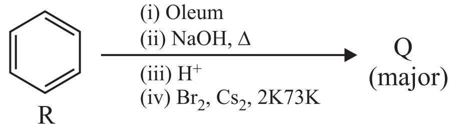
35. Among B_2H_6, $B_3N_3H_6$, N_2O, N_2O_4, $H_2S_2O_3$, $H_2S_2O_8$, the total number of molecules containing covalent bond between two atoms of the same kind is _______

36. **Schemes 1** and **2** describes the conversion of P and Q and R to S, respectively, scheme 3 describes the synthesis of T from Q and S. The total number of Br atoms in a molecule of T is ____

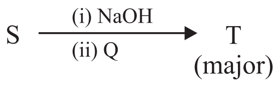
Scheme 1 :



Scheme 2 :



Scheme 3 :



MATHEMATICS

Section - I

37. The area of the region $\left\{(x, y : xy \le 8, 1 \le y \le x^2\right\}$ is

(a) $8\log_e 2 - \frac{14}{3}$
(b) $16\log_e 2 - \frac{14}{3}$
(c) $8\log_e 2 - \frac{7}{3}$
(d) $16\log_e 2 - 6$

38. A line y = mx + 1 intersectrs the circle $(x - 3)^2 + (y + 2)^2 = 25$ at the points P and Q. If the midpoint of the line segment PQ has x-coordinate $-\frac{3}{5}$, then which one of the following options is correct ?

(a) $2 \le m < 4$
(b) $-3 \le m < -1$
(c) $4 \le m < 6$
(d) $6 \le m < 8$

39. Let S be the set of all complex numbers z satisfying $|z - 2 + i| \ge \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0 - 1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in S\right\}$, then the principal argument of $\frac{4 - z_0 - \overline{z}_0}{z_0 - \overline{z}_0 + 2i}$ is

(a) $\frac{\pi}{4}$
(b) $\frac{3\pi}{4}$
(c) $\frac{\pi}{2}$
(d) $-\frac{\pi}{2}$

40. Let $M = \begin{bmatrix} \sin^4\theta & -1 - \sin^2\theta \\ 1 + \cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

Where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2 × 2 identity matrix. If α^* is the minimum of the set $\{\alpha(\theta) : \theta \in [0, 2\pi)\}$ and β^* is the minimum of the set $\{\beta(\theta) : \theta \in [0, 2\pi)\}$. Then the value of $\alpha^* + \beta^*$ is

(a) $-\frac{31}{16}$
(b) $-\frac{17}{16}$
(c) $-\frac{37}{16}$
(d) $-\frac{29}{16}$

Section - II

41. In a non-right angled triangle ΔPQR, let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, RS and PE intersect at O. If $p = \sqrt{3}, q = 1$ and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct ?

(a) Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2}(2 - \sqrt{3})$
(b) Area of $\Delta SOE = \frac{\sqrt{3}}{12}$
(c) Length of OE = $\frac{1}{6}$
(d) Length of RS = $\frac{\sqrt{7}}{2}$

42. Let Γ denote a curve $y = y(x)$ which is in the first quadrant and let the point (1, 0) lie on it. Let the tangent to Γ at a point P intersect the *y-axis* at Y_p. If PY_p has length 1 for each point P on Γ, then which of the following options is/are correct?

(a) $y = -\log_e\left(\dfrac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

(b) $xy' - \sqrt{1-x^2} = 0$

(c) $y = \log_e\left(\dfrac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

(d) $xy' + \sqrt{1-x^2} = 0$

43. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 1$$

$b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}, n \geq 2$

Then which of the following options is/are correct ?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\alpha_n}{10^n} = \frac{10}{89}$

(b) $b_n = \alpha^n + \beta^n$ for all $n \geq 1$

(c) $a_1 + a_2 + a_3 + ... a_n = a_{n+2} - 1$ for all $n \geq 1$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

44. Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and $(\text{adj } M) = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ where a and b are real numbers. Which of the following options is/are correct ?

(a) $a + b = 3$

(b) $\det(\text{adj } M^2) = 81$

(c) $(adjM)^{-1} + adjM^{-1} = -M$

(d) If $M \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

45. Let L_1 and L_2 denote the lines
$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in R$ and $\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in \text{R}$
respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3 ?

(a) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(b) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + \text{t}(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(c) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), \text{t} \in R$

(d) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

46. Define the collections $\{E_1, E_2, E_3, ...\}$ of ellipses and $\{R_1, R_2, R_3, ...\}$ of rectangles as follows :

$E_1 : \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$;

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1 ;

E_n : ellipse $\dfrac{x^2}{a_n^2} + \dfrac{y^2}{b_n^2} = 1$ of largest area inscribed in R_{n-1}, $n > 1$;

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n, $n > 1$.

Then which of the following options is/are correct?

(a) The eccentricities of E_{18} and E_{19} are NOT equal

(b) The length of latus rectum of E_9 is $\dfrac{1}{6}$

(c) $\displaystyle\sum_{n=1}^{N}$ (area of R_n) < 24, for each positive integer N

(d) The distance of a focus from the centre in E_9 is $\dfrac{\sqrt{5}}{32}$

47. There are three bags B_1, B_2, and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities $\dfrac{3}{10}, \dfrac{3}{10}$ and $\dfrac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct ?

(a) Probability that the selected bag is B_3 and the chosen ball is green equals $\dfrac{3}{10}$

(b) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\dfrac{3}{8}$

(c) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\dfrac{5}{13}$

(d) Probability that the chosen ball is green equals $\dfrac{39}{80}$

48. Let $f : R \to R$ given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \leq x < 1; \\ \dfrac{2}{3}x^3 - 4x^2 + 7x - \dfrac{8}{3}, & 1 \leq \text{x} < 3; \\ (x-2)\log_e(x-2) - x + \dfrac{10}{3}, & x \geq 3 \end{cases}$$

Then which of the following options is/are correct ?

(a) f' has a local maximum at $x = 1$

(b) f is increasing on $(-\infty, 0)$

(c) f' is NOT differentiable at $x = 1$

(d) f is onto

Section - III

49. Let S be the sample space of all 3 × 3 matrices with entries from the set {0,1}. Let the events E_1 and E_2 be given by
$E_1 = \{A \in S : det\ A = 0\}$ and
$E_2 = \{A \in \text{S}: \textit{sum of entries of A is 7}\}$.

If a matrix is chosen at random from S, then the conditional probability $P(E_1/E_2)$ equals _____.

50. Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\left\{\left|a+b\omega+c\omega^2\right|^2 : a, b, c \text{ distinct non-zero integers}\right\}$ equals _____.

51. Let the point B be the reflection of the point $A(2,3)$ with respect to the line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centers A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is _____.

52. Let AP(a;d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference $d > 0$. If $AP(1;3) \cap AP(2;5) \cap AP(3;7) = AP(a;d)$ then $a + d$ equals ____.

53. If $I = \dfrac{2}{\pi}\displaystyle\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$, then $27\,I^2$ equals ____.

54. Three lines are given by

$\vec{r} = \lambda\hat{i}, \lambda \in R$

$\vec{r} = \mu\left(\hat{i}+\hat{j}\right), \mu \in \mathrm{R}$ and

$\vec{r} = v\left(\hat{i}+\hat{j}+\hat{k}\right), v \in R$.

Let the lines cut the plane x + y + z = 1 at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $\left(6\Delta\right)^2$ equals ____.

PAPER - 2

Section - I (Maximum Marks: 32)

Directions for (Qs. 1 - 8, 19 - 26 and 37 - 44).

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are)correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : + 4 If only (all) the correct options(s) is(are) chosen;
Partial Marks : + 3 If all the four options are correct but Only three options are chosen;
Partial Marks : + 2 If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks : + 1 If two or more options are correct but ONLY one option are chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : – 1in all other cases.

For example, in a question if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 marks;
choosing ONLY (B) will get +1 marks;
choosing ONLY (D) will get +1 marks;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get –1 mark.

Section - II (Maximum Marks: 18)

Directions for (Qs. 9 - 14, 27 - 32 and 45 - 50).

- This section contains **SIX (06)** questions. The answer to each question is **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. if the numerical value has more than two decimal places, **truncate/round-off** the value to **Two** decimal places.
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : + 3 If ONLY the correct numerical value is entered;
Zero Marks : 0 In all other cases.

Section - III (Maximum Marks: 12)

Directions for (Qs. 15 - 18, 33 - 36 and 51 - 54).

- This section contains **TWO (02)** List Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions.
- For List-Match set has two list: List-I and List-II.
- List-I has four entires (i), (ii), (iii) and (iv) and List-II has Six entires (P), (Q), (R), (S), (T) and (U).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple choice Question.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 if ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases

PHYSICS

Section - I

1. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical ?
[g is the acceleration due to gravity]

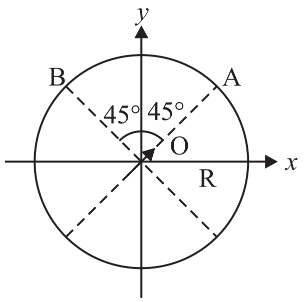
(a) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(b) The radical acceleration of the rod's center of mass will be $\frac{3g}{4}$

(c) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

(d) The angular acceleration of the rod will be $\frac{2g}{L}$

2. An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are :
(ε_0 is permittivity of free space. R >>dipole size)



(a) The magnitude of total electric field on any two points of the circle will be same

(b) Total electric field at point B is $\vec{E}_B = 0$

(c) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0\left(\hat{i}+\hat{j}\right)$

(d) $R = \left(\frac{p_0}{4\pi \epsilon_0 E_0}\right)^{1/3}$

3. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state n= 1 to the state n = 4. Immediately after that the electron jumps to $n = m$ state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are p_a and Δp_e, respectively. If $\lambda_a / \lambda_e = \frac{1}{5}$, which of the option(s) is/are correct?
[Use hc = 1242 eV nm; 1 nm = 10^{-9} m, h and c are Planck's constant and speed of light, respectively]

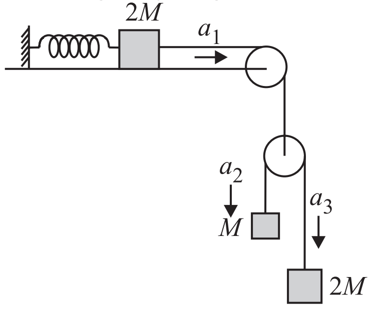
(a) $\Delta p_a / \Delta p_e = \frac{1}{2}$

(b) The ratio of kinetic energy of the electron in the state $n = m$ to the state $n = 1$ is $\frac{1}{4}$

(c) $m = 2$

(d) $\lambda_e = 418$ nm

4. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The acceleration of the blocks are a_1, a_2 and a_3 as shown in the figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct? [g is the acceleration due to gravity. Neglect friction]



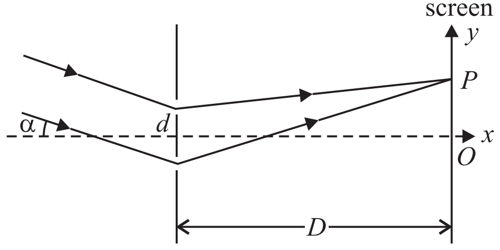
(a) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

(b) $x_0 = \frac{4Mg}{k}$

(c) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(d) $a_2 - a_1 = a_1 - a_3$

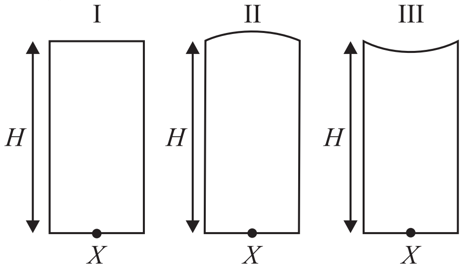
5. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1 m. A parallel beam of light of wavelength 600 nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct ?



(a) For $\alpha = 0$, there will be constructive interference at point P.
(b) Fringe spacing depends on α
(c) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P.
(d) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.

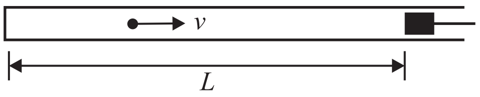
6. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0, and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are, (Given $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant)
(a) The work $|W|$ done during the process is $13RT_0$
(b) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$
(c) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$
(d) Adiabatic constant of the gas mixture is 1.6

7. Three glass cylinders of equal height H = 30 cm and same refractive index $n = 1.5$ are placed on a horizontal surface as shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3 m), If H_1, H_2, and H_3 are the apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are :



(a) $0.8 \text{ cm} < (H_2 - H_1) < 0.9 \text{ cm}$
(b) $H_2 > H_1$
(c) $H_3 > H_1$
(d) $H_2 > H_3$

8. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct ?

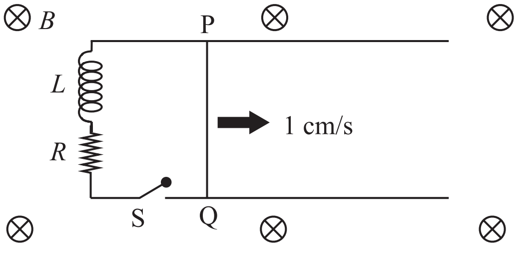


(a) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$
(b) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$
(c) The rate at which the particle strikes the piston is v/L
(d) After each collision with the piston, the particle speed increases by 2 V.

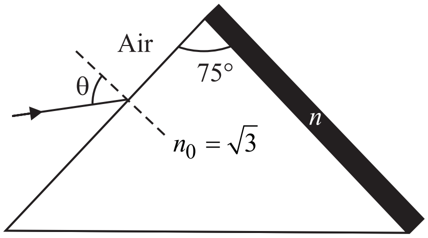
Section - II

9. Suppose a ${}^{226}_{88}Ra$ nucleus at rest and in ground state undergoes α-decay to a ${}^{222}_{86}Rn$ nucleus in its excited state. The kinetic energy of the emitted α particle is found to be 4.44 MeV. ${}^{222}_{86}Rn$ nucleus then goes to its ground state by γ -decay. The energy of the emitted γ photon is ____ keV. [Given : atomic mass of ${}^{226}_{88}Ra$ = 226.005 u, atomic mass of ${}^{222}_{86}Rn$ = 222.000 u, atomic mass of α particle = 4.000 u, 1 u = 931 MeV/c^2, c is speed of the light]

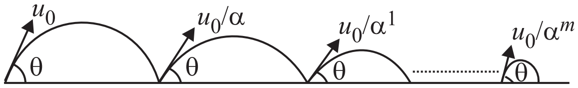
10. A 10 cm long perfectly conducting wire PQ is moving with a velocity 1 cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1 mH and a resistance R = 1 Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1 T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3}$A, where the value of x is _____
[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed. Given : $e^{-1} = 0.37$, where e is base of the natural logarithm]



11. A monochromatic light is incident from air on a refracting surface of a prism of angle 75^0 and refractive index $n_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \leq 60^0$. The value of n^2 is

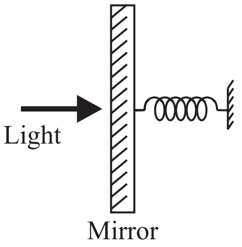


12. A ball is thrown from ground at angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of u_0/a. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is ____



13. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24}m^{-2}$ with h as Planck's constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1μm. If the value of N is $x \times 10^{12}$, then the value of x is ______

[Consider the spring as massless]



14. An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is ___

Section - III

15. *Answer the following by appropriately matching the lists based on the information given in the paragraph.*

A musical instrument is made using four different metal strings 1, 2, 3 and 4 with mass per unit length μ, 2μ ,3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

List-I	List-II
(I) String-1 (μ)	(P) 1
(II) String-2 (2 μ)	(Q) 1/2
(III) String-3 (3 μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4 μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

The length of the strings 1, 2, 3 and 4 are kept fixed at L_0, $\frac{3L_0}{2}$, $\frac{5L_0}{4}$, and $\frac{7L_0}{4}$, respectively. Strings 1, 2, 3, and 4 are vibrated at their 1st, 3rd, 5th, and 14th harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be

(a) I→T, II→Q, III→R, IV→U
(b) I→P, II→Q, III→T, IV→U
(c) I→P, II→Q, III→R, IV→T
(d) I→P, II→R, III→T, IV→U

16. *Answer the following by appropriately matching the lists based on the information given in the paragraph.*

A musical instrument is made using four different metal strings 1, 2, 3 and 4 with mass per unit length μ, 2μ,3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

List-I	List-II
(I) String-1 (μ)	(P) 1
(II) String-2 (2 μ)	(Q) 1/2
(III) String-3 (3 μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4 μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

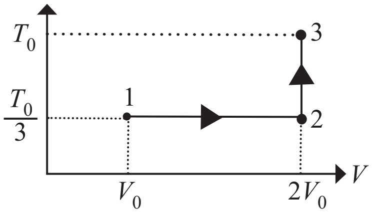
(a) I→ Q,II→ P, III→ R, IV→ T
(b) I→ Q, II→ S, III→ R, IV→ P
(c) I→ P, II→ R, III→ S, IV→ Q
(d) I→ P, II→ Q, III→ T, IV→ S

17. In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by TΔX, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $x = \frac{3}{2}\text{R In}\left(\frac{T}{T_A}\right) + \text{R In}\left(\frac{V}{V_A}\right)$. Here R is gas constant, V is volume of gas. T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I		List-II	
(I)	Work done by the system in process1→2→3	(P)	$\frac{1}{3}RT_0 ln2$
(II)	Change in internal energy in process 1→2→3	(Q)	$\frac{1}{3}RT_0$
(III)	Heat absorbed by the system in process 1→2→3	(R)	RT_0
(IV)	Heat absorbed by the system in process 1→2	(S)	$\frac{4}{3}RT_0$
		(T)	$\frac{1}{3}RT_0(3 + ln2)$
		(U)	$\frac{5}{6}\ RT_0$

If the process on one mole of monatomic ideal gas is as shown in figure in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$,the correct match is,



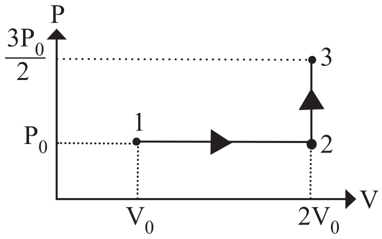
(a) I→P, II→T, III→Q, IV→T
(b) I→S, II→T, III→Q, IV→U
(c) I→P, II→R, III→T, IV→P
(d) I→P, II→R, III→T, IV→S

18. In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by TΔX, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}\text{R 1n}\left(\frac{T}{T_A}\right) + \text{R 1n}\left(\frac{V}{V_A}\right)$. Here R is gas constant, V is volume of gas. T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I		List-II	
(I)	Work done by the system in process1→2→3	(P)	$\frac{1}{3}RT_0 ln2$
(II)	Change in internal energy in process 1→2→3	(Q)	$\frac{1}{3}RT_0$
(III)	Heat absorbed by the system in process 1→2→3	(R)	RT_0
(IV)	Heat absorbed by the system in process 1→2	(S)	$\frac{4}{3}RT_0$
		(T)	$\frac{1}{3}RT_0(3 + ln2)$
		(U)	$\frac{5}{6}\ RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in the figure in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(a) I→Q, II→R, III→S, IV→U
(b) I→S, II→R, III→Q, IV→T
(c) I→Q, II→R, III→P, IV→U
(d) I→Q, II→S, III→R, IV→U

CHEMISTRY

Section - I

19. With reference to *aqua regia*, choose the correct option(s)

(a) Reaction of gold with *aqua regia* produces NO_2 in the absence of air
(b) Reaction of gold with *aqua regia* produces an anion having Au in +3 oxidation state
(c) *Aqua regia* is prepared by mixing conc. HCl and conc. HNO_3 in 3 : 1 (v/v) ratio
(d) The yellow colour of *aqua regia* is due to the presence of NOCl and Cl_2

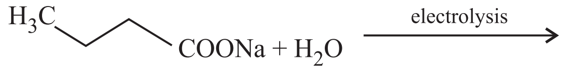
20. The cyanide process of gold extraction involves leaching out gold from its ore with CN^- in the presence of Q in water to form R. Subsequently, R is treated with T to obtain Au and Z. Choose the correct option(s)

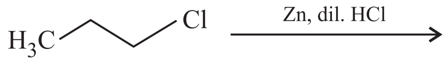
(a) Q is O_2 (b) T is Zn

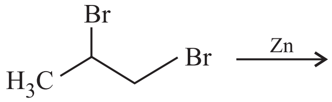
(c) Z is $[Zn(CN)_4]^{2-}$ (d) R is $[Au(CN)_4]^-$

21. Which of the following reactions produce(s) propane as a major product ?

(a) 

(b) 

(c) 

(d) 

22. Choose the correct option(s) from the following.

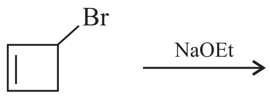
(a) Nylon-6 has amide linkages

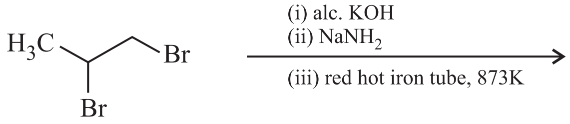
(b) Cellulose has only α-D-glucose units that are joined by glycosidic linkages

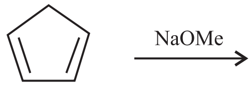
(c) Teflon is prepared by heating tetrafluoroethene in presence of a persulphate catalyst at high pressure

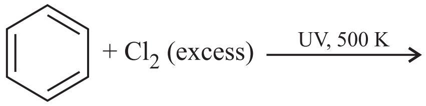
(d) Natural rubber is polyisoprene containing *trans* alkene units

23. Choose the correct option(s) that give(s) an aromatic compound as the major product.

(a) 

(b) 

(c) 

(d) 

24. The ground state energy of hydrogen atom is –13.6 eV. Consider an electronic state ψ of He^+ whose energy, azimuthal quantum number and magnetic quantum number are –3.4 eV, 2 and 0, respectively. Which of the following statement(s) is (are) true from the state ψ?

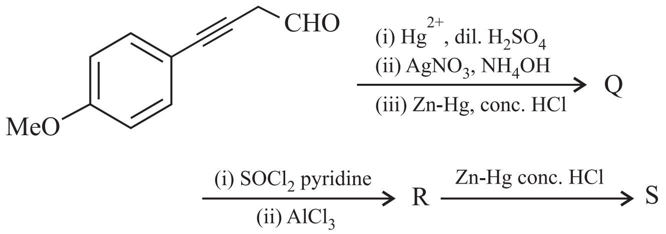
(a) It is a 4*d* state

(b) It has 3 radial nodes

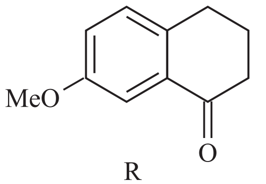
(c) It has 2 angular nodes

(d) The nuclear charge experienced by the electron in this state is less than 2*e*, where *e* is the magnitude of the electronic charge

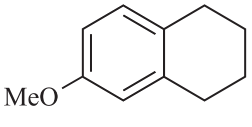
25. Choose the correct option(s) for the following reaction sequence



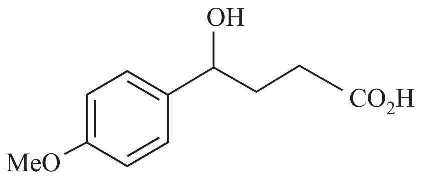
Consider Q, R and S as major products.

(a) 

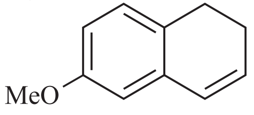
R



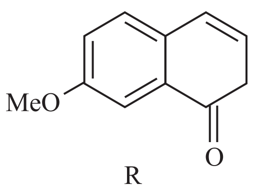
S

(b) 

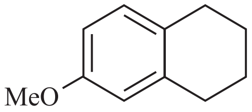
Q



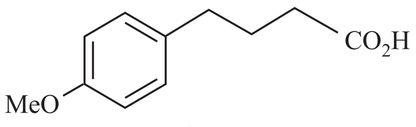
S

(c) 

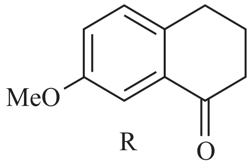
R



S

(d) 

Q



R

26. Consider the following reactions (unbalanced) Zn + hot conc. $H_2SO_4 \rightarrow G + R + X$

Zn + conc. NaOH→ T + Q

$G + H_2S + NH_4OH \rightarrow$ Z (a precipitate) + X + Y

Choose the correct option(s)

(a) The oxidation state of Zn in T is + 1

(b) Bond order of Q is 1 in its ground state

(c) Z is dirty white in colour

(d) R is a V-shaped molecule

Section - II

27. Total number of *cis* N – Mn – Cl bond angles (that is Mn – N and Mn – Cl bonds in *cis* positions) present in a molecule of *cis* –[Mn(en)$_2$Cl$_2$] complex is ___ (en = $NH_2CH_2CH_2NH_2$)

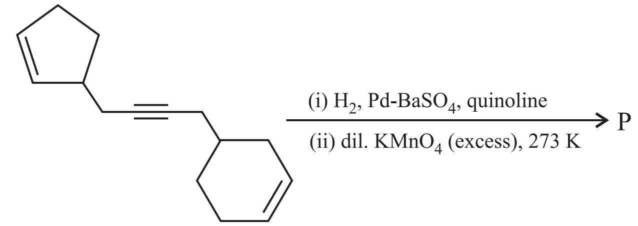
28. The amount of water produced (in g) in the oxidation of 1 mole of rhombic sulphur by conc.HNO_3 to a compound with the highest oxidation state of sulphur is ____ (Given data : Molar mass of water = 18g mol^{-1})

29. The decomposition reaction

$$2N_2O_2(g) \xrightarrow{\Delta} 2N_2O_4(g) + O_2(g)$$

is started in a closed cylinder under isothermal isochoric condition at an initial pressure of 1 atm. After Y×10^{3}s the pressure inside the cylinder is found to be 1.45 atm. If the rate constant of the reaction is 5×10^{-4} s^{-1}, assuming ideal gas behaviour, the value of Y is __

30. Total number of hydroxyl groups present in a molecule of major product P is ___



31. Total number of isomers, considering both structural and stereoisomers of cyclic ethers with the molecular formula C_4H_8O is____

32. The mole fraction of urea in an aqueous urea solution containing 900 g of water is 0.05. If the density of the solution is 1.2 g cm^{-3}, the molarity of urea solution is ___ (Given data: Molar masses of urea and water are 60 g mol^{-1} and 18 g mol^{-1}, respectively)

Section - III

33. Consider the Bohr's model of a one – electron atom where the electron moves around the nucleus. In the following List–I contains some quantities for the nth orbit of the atom and List–II contains options showing how they depend on *n*

List–I	List–II
(I) Radius of the nth orbit	(P) $\propto n^{-2}$
(II) Angular momentum of the electron in the nth orbit	(Q) $\propto n^{-1}$
(III) Kinetic energy of the electron in the nth orbit	(R) $\propto n^{0}$
(IV) Potential energy of the electron in the nth orbit	(S) $\propto n^{1}$
	(T) $\propto n^{2}$
	(U) $\propto n^{1/2}$

Which of the following options has the correct combination considering List–I and List–II?

(a) (II), (R) (b) (II), (Q)

(c) (I), (P) (d) (I), (T)

34. Consider the Bohr's model of a one-electron atom where the electron moves around the nucleus. In the following List–I contains some quantities for the nth orbit of the atom and List–II contains options showing how they depend on *n*.

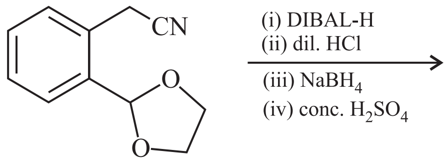
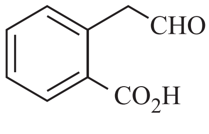
List–I	List–II
(I) Radius of the nth orbit	(P) $\propto n^{-2}$
(II) Angular momentum of the electron in the nth orbit	(Q) $\propto n^{-1}$
(III) Kinetic energy of the electron in the n^{th} orbit	(R) $\propto n^{0}$
(IV) Potential energy of the electron in the nth orbit	(S) $\propto n^{1}$
	(T) $\propto n^{2}$
	(U) $\propto n^{1/2}$

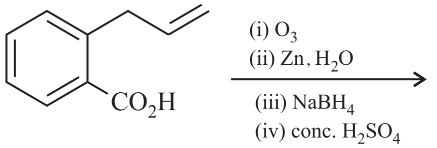
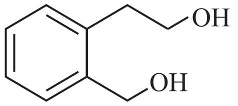
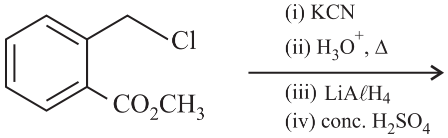
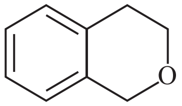
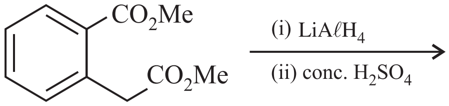
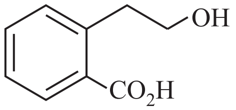
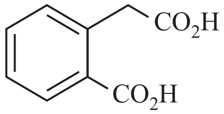
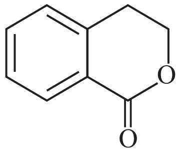
Which of the following options has the correct combination considering List–I and List–II?

(a) (III), (S) (b) (IV), (Q)

(c) (III), (P) (d) (IV), (U)

35. Answer the following by appropriately matching the lists based on the information given in the paragraph. List–I includes starting materials and reagents of selected chemical reactions. List–II gives structures of compounds that may be formed as intermediate products and/ or final products from the reactions of List–I.

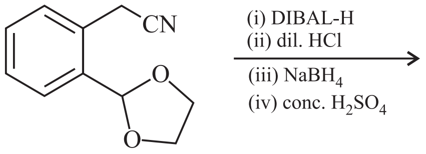
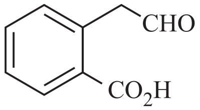
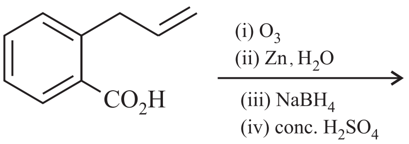
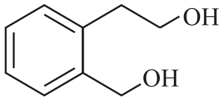
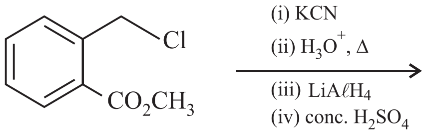
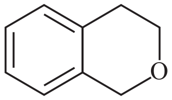
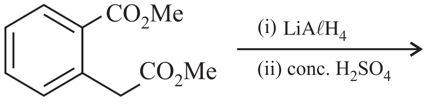
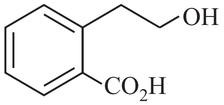
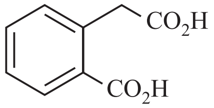
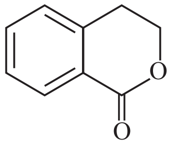
List–I		List–II	
(I)		(P)	

(II)		(Q)	
(III)		(R)	
(IV)		(S)	
		(T)	
		(U)	

Which of the following options has correct combination considering List–I and List–II?

(a) (I), (S), (Q), (R) (b) (II), (P), (S), (U) (c) (I), (Q), (T), (U) (d) (II), (P), (S), (T)

36. List – I includes starting materials and reagents of selected chemical reactions. List – II gives structures of compounds that may be formed as intermediate products and/ or final products from the reactions of List – I

List–I		List–II	
(I)		(P)	
(II)		(Q)	
(III)		(R)	
(IV)		(S)	
		(T)	
		(U)	

Which of the following options has correct combination considering List – I and List –II?

(a) (IV), (Q), (R) (b) (IV), (Q), (U) (c) (III), (S), (R) (d) (III), (T), (U)

MATHEMATICS

Section -I

37. Let $f : R \to R$ be given by $f(x) = (x-1)(x-2)(x-5)$.

Define $F(x) = \int\limits_0^x f(t)dt, x > 0.$

Then which of the following options is/are correct?

(a) F has a local maximum at $x = 2$

(b) F has a local minimum at $x = 1$

(c) F has two local maxima and one local minimum in $(0, \infty)$

(d) $F(x) \neq 0$ for all $x \in (0,5)$

38. Let

$$P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

and $X = \sum\limits_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$

Where P_k^T denotes the transpose of the matrix P_k. Then which of the following options is/are correct?

(a) X is a symmetric matrix

(b) The sum of diagonal entries of X is 18

(c) $X - 30I$ is an invertible matrix

(d) If $X\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $\alpha = 30$

39. For non-negative integers n, let

$$f(n) = \frac{\sum\limits_{k=0}^{n} \sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum\limits_{k=0}^{n} \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0, \pi]$, which of the following options is/are correct?

(a) $\lim\limits_{n\to\infty} f(n) = \frac{1}{2}$

(b) $f(4) = \frac{\sqrt{3}}{2}$

(c) If $\alpha = \tan\left(\cos^{-1} f(6)\right)$, then $\alpha^2 + 2\alpha - 1 = 0$

(d) $\sin(7\cos^{-1} f(5)) = 0$

40. Let $f(x) = \frac{\sin \pi x}{x^2}, x > 0$

Let $x_1 < x_2 < x_3 < \ldots\ldots.. < x_n < \ldots\ldots.$ be all the points of local maximum of f and $y_1 < y_2 < y_3 < \ldots\ldots. < y_n < \ldots..$ be all the points of local minimum of f. Then which of the following options is/are correct?

(a) $x_{n+1} - x_n > 2$ for every n

(b) $x_n \in \left(2n, 2n + \frac{1}{2}\right)$ for every n

(c) $|x_n - y_n| > 1$ for every n

(d) $x_1 < y_1$

41. Three lines

$L_1 : \vec{r} = \lambda\hat{i}, \lambda \in R$

$L_2 : \vec{r} = \hat{k} + \mu\hat{j}, \mu \in R$ and

$L_3 : \vec{r} = \hat{i} + \hat{j} + v\hat{k}, v \in R$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear?

(a) $\hat{k} - \frac{1}{2}\hat{j}$ (b) $\hat{k}$ (c) $\hat{k} + \hat{j}$ (d) $\hat{k} + \frac{1}{2}\hat{j}$

42. For, $a \in R, |a| > 1$, let

$$\lim_{x\to\infty}\left(\frac{1 + \sqrt[3]{2} + \ldots.. + \sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2} + \frac{1}{(an+2)^2} + \ldots. + \frac{1}{(an+n)^2}\right)}\right) = 54$$

Then the possible value(s) of a is/are

(a) -9 (b) 7 (c) -6 (d) 8

43. Let $x \in R$ and let

$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix}$ and $R = PQP^{-1}$

Then which of the following options is/are correct?

(a) $\det R = \det\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$, for all $x \in R$

(b) For $x = 1$, there exists a unit vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ for which $R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(c) There exists a real number x such that $PQ = QP$

(d) For $x = 0$, if $R = \begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$, then $a + b = 5$

44. Let $f : R \to R$ be a function. We say that f has

PROPERTY 1 if $\lim\limits_{h\to 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim\limits_{h\to 0} \frac{f(h) - f(0)}{h^2}$ exists and is finite

Then which of the following options is/are correct?

(a) $f(x) = x^{2/3}$ has **PROPERTY 1**

(b) $f(x) = \sin x$ has **PROPERTY 2**

(c) $f(x) = |x|$ has **PROPERTY 1**

(d) $f(x) = x|x|$ has **PROPERTY 2**

Section - II

45. Suppose

$$\det\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^{n}C_{k}k^{2} \\ \sum_{k=0}^{n} {}^{n}C_{k}k & \sum_{k=0}^{n} {}^{n}C_{k}3^{k} \end{bmatrix} = 0$$

holds for some positive integer n. The $\sum_{k=0}^{n} \frac{{}^{n}C_{k}}{k+1}$ equals_____

46. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\hat{a} + \beta\hat{b},\ \alpha, \beta \in R$. If the projection of $\vec{c}$ on the vector $\left(\vec{a} + \vec{b}\right)$ is $3\sqrt{2}$, then the minimum value of $\left(\vec{c} - \left(\vec{a} \times \vec{b}\right)\right).\vec{c}$ equals_____

47. Let $|X|$ denote the number of elements in a set X. Let $S = \{1, 2, 3, 4, 5, 6\}$ be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A, B) such that $1 \leq |B| < |A|$, equals ____.

48. Five persons A, B, C, D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is ___.

49. The value of

$$\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10} \sec\left(\frac{7\pi}{12} + \frac{k\pi}{2}\right) \sec\left(\frac{7\pi}{12} + \frac{(k+1)\pi}{2}\right)\right)$$

in the interval $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$ equals ____.

50. The value of the integral

$$\int_{0}^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta} + \sqrt{\sin\theta}\right)^{5}} d\theta$$ equals _____.

Section - III

51. Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order.

$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$

$Z = \{x : g(x) = 0\},\ W = \{x : g'(x) = 0\}$

List – I contains the sets X, Y, Z and W. List – II contains some information regarding these sets.

List-I	List-II
(I) X	(P) $\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) not an arithmetic progression
(IV) W	(S) $\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
	(T) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(U) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination?

(a) (IV), (P), (R), (S)
(b) (III), (P), (Q), (U)
(c) (III), (R), (U)
(d) (IV), (Q), (T)

52. Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order :

$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$

$Z = \{x : g(x) = 0\},\ W = \{x : g'(x) = 0\}$

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	(T) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(U) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination?

(a) (I), (Q), (U)
(b) (I), (P), (R)
(c) (II), (R), (S)
(d) (II), (Q), (T)

53. Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x - 3)^2 + (y - 4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x - h)^2 + (y - k)^2 = r^2$ satisfies the following conditions:

(i) Centre of C_3 is collinear with the centres of C_1 and C_2
(ii) C_1 and C_2 both lie inside C_3, and
(iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expressions given in the List – I whose values are given in List – II below

List-I	List-II
(I) $2h + k$	(P) 6
(II) $\dfrac{\text{Length of } ZW}{\text{Length of } XY}$	(Q) $\sqrt{6}$
(III) $\dfrac{\text{Area of triangle } MZN}{\text{Area of triangle } ZMW}$	(R) $\dfrac{5}{4}$
(IV) α	(S) $\dfrac{21}{5}$
	(T) $2\sqrt{6}$
	(U) $\dfrac{10}{3}$

Which of the following is the only CORRECT combination?

(a) (I), (U) (b) (I), (S)

(c) (II), (T) (d) (II), (Q)

54. Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x - 3)^2 + (y - 4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x - h)^2 + (y - k)^2 = r^2$ satisfies the following conditions:

(i) Centre of C_3 is collinear with the centres of C_1 and C_2

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N

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	(T) $2\sqrt{6}$
	(U) $\dfrac{10}{3}$

Which of the following is the only INCORRECT combination?

(a) (IV), (S) (b) (I), (P)

(c) (III), (R) (d) (IV), (U)

SOLUTIONS

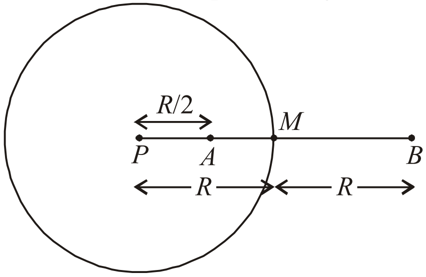
Paper - 1

Physics

1. (b) Let Q be the total charge on the sphere. Then surface charge density is $\frac{Q}{4\pi R^2}$. A hole is now cut of area $\alpha(4\pi R^2)$. The charge on this hole is

$$\frac{q}{\alpha(4\pi R^2)} = \frac{Q}{4\pi R^2} \quad \therefore q = \alpha Q \text{ Also } V_0 = \frac{KQ}{R}.$$

Now we visualise this situation as a complete spherical distribution of positive charge on the surface with a negative charge of the same surface charge density on the hole. This negative charge can be treated as a point charge.



Potential

$$V_P = \frac{KQ}{R} - \frac{K\alpha Q}{R} = \frac{KQ}{R}(1-\alpha)$$
$$= V_0(1-\alpha) = V_0 - \alpha V_0$$

Therefore option (d) is incorrect.

$$V_A = \frac{KQ}{R} - \frac{K\alpha Q}{R/2} = \frac{KQ}{R}(1-2\alpha) \quad \therefore \quad \frac{V_P}{V_A} = \frac{1-\alpha}{1-2\alpha}$$

Therefore option (b) is correct.

Electric field

$$\left(E_B\right)_{\text{initial}} = \frac{KQ_0}{(2R)^2}$$

$$\left(E_B\right)_{\text{final}} = \frac{KQ}{(2R)^2} - \frac{K(\alpha Q)}{R^2} = \frac{KQ}{(2R)^2} - \alpha V_0$$

Option (a) is incorrect.

$$\left(E_P\right)_{\text{initial}} = 0$$

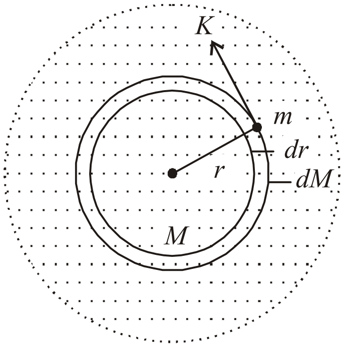
$$\left(E_P\right)_{\text{final}} = \frac{K(\alpha Q)}{R^2} = \frac{\alpha V_0}{R}$$

Option (c) is incorrect.

2. (b) The required centripetal force of particle of mass 'm' to revolve in a circular path is provided by gravitational pull of the mass 'M' present in the sphere of radius 'r'. Therefore

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2r} \Rightarrow K = \frac{GMm}{2r}$$

$$\therefore \quad M = \frac{2Kr}{Gm}$$



Differentiating the above equation w.r.t 'r' we get

$$\frac{dM}{dr} = \frac{2K}{Gm} \text{ or } dM = \frac{2K}{Gm}dr$$

$$\therefore \quad 4\pi r^2 dr\, \rho = \frac{2K}{Gm}dr \quad \therefore \quad \rho = \frac{K}{2\pi r^2 mG}$$

$$\therefore \quad \frac{\rho}{m} = \frac{K}{2\pi r^2 m^2 G}$$

3. (d) $P = \frac{dQ}{dt} = \frac{d}{dt}(mc)T = (mc)\frac{dT}{dt} = (mc)\frac{d}{dt}\left[T_0(1+\beta t^{1/4})\right]$

$P = (mc)T_0 = \frac{\beta t^{-3/4}}{4}$ where (mc) is the heat capacity

$$\therefore \quad (mc) = \frac{4Pt^{3/4}}{T_0\beta} \quad \text{..... (i)}$$

But $\quad t^{1/4} = \frac{T(t)-T_0}{\beta T_0}$

$$\therefore \quad t^{3/4} = \frac{\left[T(t)-T_0\right]^3}{\beta^3 T_0^3} \quad \text{..... (ii)}$$

From (i) & (ii),

$$(mc) = \frac{4P}{T_0\beta}\frac{\left[T(t)-T_0\right]^3}{\beta^3 T_0^3} = \frac{4P\left[T(t)-T_0\right]^3}{\beta^4 T_0^4}$$

4. (a) Here $-\frac{dN}{dt} = \lambda_1 N + \lambda_2 N$

$$\therefore \quad t = \frac{2.303}{\lambda_1+\lambda_2}\log_{10}\frac{N_0}{N} = \frac{2.303}{5\times10^{-10}}\log_{10}\frac{100}{1}$$

$$\therefore \quad t = 9.2 \times 10^9 \text{ year}$$

5. (a, b, c) $\frac{1}{f} = (n-1)\frac{2}{R} \Rightarrow f = \frac{R}{2(n-1)}$

$$\frac{1}{f+\Delta f} = \frac{(n-1)}{R} + \frac{(n+\Delta n-1)}{R} = \frac{2(n-1)+\Delta n}{R}$$

$$\therefore \quad f+\Delta f = \frac{R}{2(n-1)+\Delta n}$$

$$\therefore \quad \frac{f+\Delta f}{f} = \frac{R}{2(n-1)+\Delta n} \times \frac{\left[2(n-1)\right]}{R}$$

$$= \frac{[2(n-1)]}{[2(n-1)+\Delta n]}$$

$$\therefore \quad \frac{\Delta f}{f} = \frac{2n-2-2n+2-\Delta n}{[2(n-1)+\Delta n]}$$

$$= \frac{-\Delta n}{2n-2+\Delta n} = \frac{-\Delta n}{2(n-1)} \quad \text{..... (i)}$$

$$[\because \ \Delta n << (n-1)]$$

From equation (i) if $\frac{\Delta n}{n} < 0$, then $\frac{\Delta f}{f} > 0$. Therefore option (a) is correct.

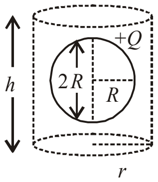
From equation (i) if convex surface are replaced by concave surface of the same radius of curvature then the relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged. Therefore option (b) is also correct.

For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm then from (i)

$$\frac{\Delta f}{20} = -\frac{10^{-3}}{2(1.5-1)} \Rightarrow \Delta f = -0.02 \text{ cm}$$

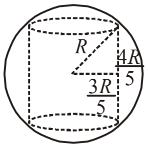
or $\quad |\Delta f| = 0.02$ cm $\therefore$ Option (c) is correct.

6. **(a, b, c)** (b)



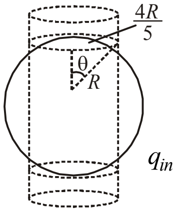
$$\phi = \frac{q_{in}}{\in_0} = \frac{Q}{\in_0}$$

(c)



$$\phi = \frac{q_{in}}{\in_0} = 0$$

(d)



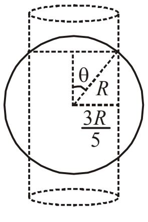
$$\sin\theta = \frac{4R/5}{R} = \frac{4}{5} = 0.8$$

$$\Rightarrow \theta = 53°$$

$$q_{in} = Q[1-\cos\theta] = Q\left[1-\frac{3}{5}\right] = \frac{2Q}{5}$$

$$\therefore \quad \phi = \frac{2Q}{5\in_0}$$

(a)

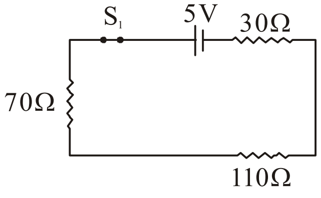


$$\sin\theta = \frac{3R/5}{R} = \frac{3}{5} = 37°$$

$$q_{in} = Q[1-\cos 37°] = Q\left[1-\frac{4}{5}\right] = \frac{Q}{5}$$

$$\therefore \quad \phi = \frac{Q}{5\in_0}$$

7. **(c, d)** S_1 closed and S_2 open ($t = 0$)

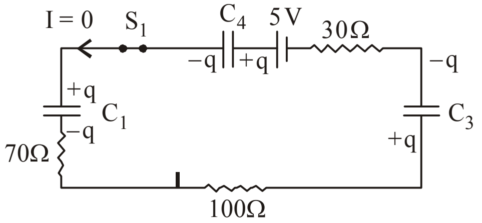


At $t = 0$, capacitors do not have any charge. Therefore

$$I = \frac{5}{70+100+30} = 0.025 \text{ A} = 25 \text{ mA}.$$

Option (c) is correct

when S_1 is closed for a long time the all the capacitors are fully charged. As the capacitors are in series these carry equal charge q. Current in the circuit is now zero. Applying Krichhoff's law



$$5 - \frac{q}{80} - \frac{q}{10} - \frac{q}{80} = 0 \qquad \therefore \ q = 40 \ \mu c$$

Potential difference across C_1 is

$$\frac{q}{C_1} = \frac{40\times10^{-6}}{10\times10^{-6}} = 4\text{V}$$

(d) is the correct option.

$V_P - 4 - 70 \times 25 \times 10^{-3} = V_Q$

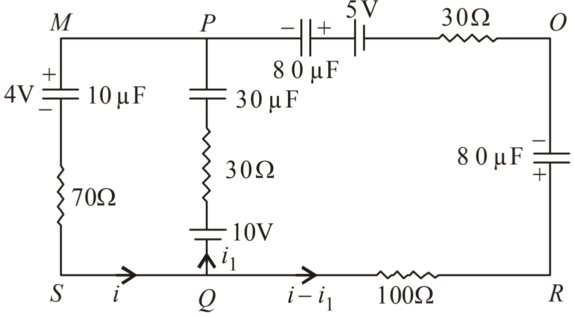
$\therefore \ V_P - V_Q = 4 + 1.75 = 5.75$ V

(a) is an incorrect option.

Now when key S_2 is closed

In loop MPQS $+10 - 30\,i_1 - 4 - 70\,i = 0$

$70\,i + 30\,i_1 = 6$ ..... (i)



In loop QROPQ,

$$+10 - 30\,i_1 + \frac{40}{80} - 5 + (i - i_1) \times 130 + \frac{40}{80} = 0$$

$130\,i - 160\,i_1 = -6$ ..... (ii)

On solving (i) & (ii), we get $i = 0.05$ A $\therefore \ i_1 = 0.077$ A

(b) is an incorrect option.

8. **(a, b, c)** Given $[mvr] = M^0L^0T^0$ and $[m] = M^0L^0T^0$

$ML^2T^{-1} = M^0L^0T^0 \Rightarrow T = L^2$

Given, momentum

$$p = mv \ \frac{mvr}{r} = \frac{M^0L^0T^0}{L} = L^{-1}$$

$$\text{Energy } E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = L^{-2}$$

$$\text{Power } \ P = \frac{E}{t} = \frac{L^{-2}}{T} = \frac{L^{-2}}{L^2} = L^{-4}$$

$$\text{Force } \ F = \frac{E}{x} = \frac{L^{-2}}{L} = L^{-3}$$

9. **(b, d)** $G = 10\,\Omega$; $I_g = 2\times10^{-6}$ A, $V = 100$ mV $= 0.1$V, $I = 10^{-3}$A

Here V = I_g (G + R) where R is the resistance connected in series with galvanometer

$0.1 = 2\times10^{-6}R_v$

$\therefore R_v = 5\times10^4\,\Omega$ = Resistance of voltmeter

Also $I_g G = (I - I_g)s$

$2\times10^{-6}\times10 = (10^{-3} - 2\times10^{-6})s \therefore S = 2\times10^{-2}\,\Omega$

$$R_A = \frac{GS}{G+S} = \frac{10\times0.02}{10+0.02} \approx 0.02\Omega$$

option (d) is correct

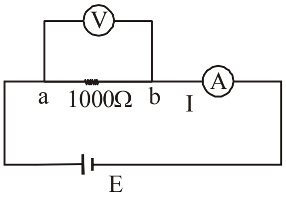
$$I = \frac{E}{\frac{50,000\times1000}{51000}+0.02} = \frac{E}{980.41}$$

$$V_{ab} = \frac{E}{980.41}\times\frac{50,000\times1000}{51000} = \frac{E}{980.41}\times980.39$$

$$\therefore R_{measured} = \frac{V_{ab}}{I} = \frac{E}{980.41}\times\frac{980.39}{E/980.41} = 980.39\Omega$$

Option (b) is Correct

If the ideal cell is replaced by a cell having internal resistance of 5Ω

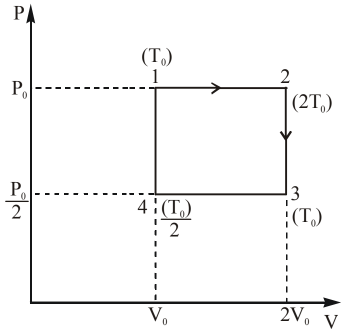


then $I = \frac{E}{985.41}$

and $V_{ab} = \frac{E}{985.41}\times\frac{50,000\times1000}{51000} = \frac{E}{985.41}\times980.39$

$$\therefore R_{measured} = \frac{V_{ab}}{I} = \frac{E\times980.39}{985.41\ \ E}\times985.41 = 980.39$$

10. **(a, c)** Let P_0 be the pressure at 1



$1\to2$ Process is isobaric pressure at '2' is P_0

$2\to3$ Process is isochoric $\frac{P_0}{2T_0} = \frac{P_3}{T_0} \therefore P_3 = \frac{P_0}{2}P_0$

$3\to4$ Process is isobaric

$4\to1$ Process is isochoric

Option (b) is incorrect

The P-V diagram is shown

$|\Delta Q_{1\to2}| = |nC_p\,\Delta T| = |nC_p(2T_0 - T_0)| = |nC_pT_0|$

$|\Delta Q_{2\to3}| = |\Delta U| = |nC_v\,\Delta T| = |nC_v\,T_0|$

$$\therefore \left|\frac{\Delta Q_{1\to2}}{\Delta Q_{2\to3}}\right| = \frac{C_p}{C_v} = \frac{5}{3} \text{ (monoatomic gas)}$$

option (c) is correct

$$|\Delta Q_{3\to4}| = nC_p\frac{T_0}{2} \qquad \therefore \left|\frac{\Delta Q_{1\to2}}{\Delta Q_{3\to4}}\right| = \frac{nC_pT_0}{nC_p\frac{T_0}{2}} = \frac{2}{1}$$

$\therefore$ option (d) is correct.

Work done during cyclic process = area enclosed in the loop $= \frac{P_0}{2}V_0$

For point, 1, $P_0V_0 = nRT_0$ $\qquad \therefore \frac{P_0V_0}{2} = \frac{nRT_0}{2}$

$\therefore$ loop done $= \frac{nRT_0}{1} = \frac{RT_0}{2}$ $\quad [\text{as } n = 1]$

(a) is the correct option.

11. **(a, b, c) For case 1**

$$h_1 = \frac{2T\cos\theta_1}{r\rho g} = \frac{2\times0.75\times\cos0^\circ}{2\times10^{-4}\times1000\times10} = 7.5\,\text{cm}$$

option (c) is correct

For case 2

$$h_2 = \frac{2T\cos\theta_2}{r\rho g} = \frac{2\times0.75\times\cos60^\circ}{2\times10^{-4}\times1000\times10} = 3.75\,\text{cm}$$

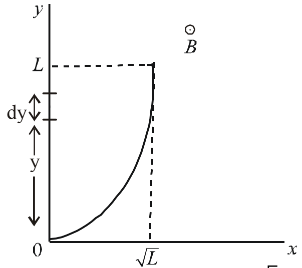
option (b) is correct

The correction in the height of water column raised in the tube, due to weight a in the meniscus will be different in both option (a) is correct

In case I, if the capillary joint is 5 cm above the water surface then the height of water raised in tube 1 will be 5 cm and the shape of liquid meniscus will change to adjust extra liquid.

12. **(a, b, d)** Let us consider the projection of wire on f-axis

We now consider a infinite small length of wire dy at a distance y from the origin on this projection. The induced emf developed across 'dy' is



$$|d\phi| = B(dy)V_0 \qquad |d\phi| = B_0\left[1+\left(\frac{y}{L}\right)^{B}\right]V_0dy$$

Therefore the induced emf across the complete projection

$$|\Delta\phi| = B_0V_0\int_0^L\left[1+\left(\frac{y}{L}\right)^{\beta}\right]V_0dy = B_0V_0L\left[1+\frac{1}{\beta+1}\right]$$

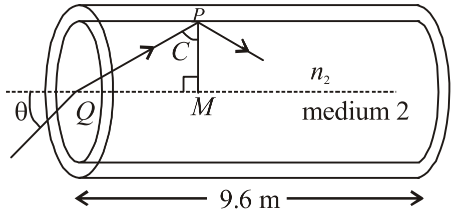
Option (a) is correct

For $\beta = 0, |\Delta\phi| = 2B_0V_0L$. Option (c) is correct

For a straight wire of length $\sqrt{2L}$ placed along $y = x$ then the value of $|\Delta\phi|$ will remain the same as its projection of y-axis is same as that of previous case. Therefore option (a) is also correct

13. **(50)** Let 'C' be the critical angle. Then

$$\sin C = \frac{1.44}{1.50}$$



But sin C $= \frac{QM}{QP}$

$\therefore \frac{QM}{QP} = \frac{1.44}{1.50} \quad \therefore QP = \frac{1.50}{1.44} QM$

If we replace QM by 9.6 then the total path length travelled by light is $\frac{1.50}{1.44} \times 9.6 = 10m$

Now time taken to do so is

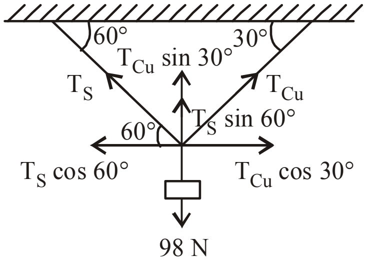
$t = \frac{d}{v_2}$ where v_2 is velocity of light in medium 2

$\therefore t = \frac{10}{3 \times 10^8 / 1.5} = 5 \times 10^8 s = 50 \times 10^{-9} s$ $\left[\because v_2 = \frac{C}{n_2}\right]$

t = 50.00

14. **(2)** Due to equilibrium

$T_s \cos 60° = T_{cu} \cos 30° \Rightarrow T_s = \sqrt{3}\, T_{cu}$ ..... (1)



Now $\frac{\Delta l_{cu}}{\Delta l_s} = \frac{\frac{T_{cu} l_{cu}}{A \times l_{cu}}}{\frac{T_s l_s}{A \times y_s}} = \frac{T_{cu} l_{cu}}{y_{cu}} \times \frac{y_s}{T_s l_s}$

$= \frac{T_{cu} \times \sqrt{3}}{10^{11}} \times \frac{2 \times 10^{11}}{\sqrt{3} T_{cu} \times 1} = \frac{2}{1} = 2.00$

15. **(270)** Let m_c is mass of calorimeter and its specific heat. Capacity Further let C be the specific heat capacity of liquid and L to latent heat of voportion

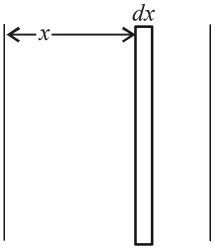
$5 \times C \times 50 + 5 \times L = m_c s_c (110 - 80)$ ... (1)

For the second case $80 \times C \times 20 = m_c s_c (80 - 50)$ ... (2)

on dividing we get $\frac{250C + 5L}{1600c} = \frac{30}{30}$

$\therefore \frac{L}{C} = 270°C = 270.00°C$

16. **(1.00)** $dc = \frac{k\left(1 + \frac{m}{N} \in_0 A\right)}{dx}$



$\therefore \frac{1}{dc} = \frac{dx}{k\left(1 + \frac{m}{N}\right) \in_0 A}$

$\frac{1}{C} = \int dc = \int \frac{dx}{k\left(1 + \frac{m}{N}\right) \in_0 A}$

$= \int \frac{dx}{k\left(1 + \frac{x}{d}\right) \in_0 A} \qquad \because \frac{x}{m} = \frac{D}{N}$

$= \frac{d}{k \in_0 A} \int_0^d \frac{dx}{d + x}$

$\frac{1}{C} = \frac{d}{k \in_0 A} \ln_2 \Rightarrow C = \frac{k \in_0 A}{d \ln^2} \quad \therefore \alpha = 1.00$

17. **(0.75)** We know that

$dW = \vec{F} \cdot d\vec{r} = \left(\alpha y \hat{i} + 2\alpha x \hat{j}\right) \cdot \left(dx \hat{i} + dy \hat{j}\right)$

$\therefore dW = \alpha y dx + 2\alpha x dy$

Work done from A → B dy = 0, as y = 1

$\therefore W_1 = \int_0^1 \alpha y\, dx = \alpha \int_0^1 dx = \alpha$

Work done from B → C dx = 0, as x = 1

$\therefore W_2 = \int_1^{0.5} 2\alpha n\, dy = \int_1^{0.5} 2\alpha dy = 2\alpha(-0.5) = -\alpha$

Work done from C → D dy = 0, as y = 0.5

$\therefore W_3 = \int_1^{0.5} \alpha \times 0.5\, dx = -\frac{\alpha}{4}$

Work done from D to E dx = 0, as x = 0.5

$\therefore W_4 = \int_{0.5}^{0} 2\alpha \times 0.5\, dy = -\frac{\alpha}{2}$

Work done from E to F dy = 0, as y = 0

$\therefore W_5 = \int \alpha \times 0 \times dx = 0$

Work done from F to A dx = 0 as x = 0

$\therefore W_6 = \int 2\alpha \times 0 dx = 0$

Total work $W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6$

$= \alpha - \alpha - \frac{\alpha}{4} - \frac{\alpha}{2} = -\frac{3\alpha}{4} = \frac{-3(-1)}{4} = +0.75J$

18. **(8.13)** Frequency heard by observer due to source S_1 is

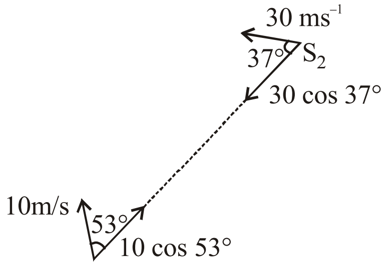
$v_1 = v\left[\frac{v + v_0}{v}\right] = 120\left[\frac{330 + 10}{330}\right] = 120 \times \frac{34}{33} = 123.636$

Frequency heard by observer due to S_2 is

$v_2 = v\left[\frac{v + v_0}{v - v_s}\right] = 120\left[\frac{330 + 10\cos 53°}{330 - 30\cos 37°}\right]$

$\therefore v_2 = 120\left[\frac{330 + 10 \times 0.6}{330 - 30 \times 0.8}\right] = 120\left[\frac{336}{306}\right] = 131.764$

$\therefore$ Beat frequency $= v_2 - v_1 = 8.125$ Hz ≈ 8.13 Hz



Chemistry

19. **(d)** Sodium stearate at low concentration (i.e., below CMC) behaves as normal strong electrolyte, but at higher concetration (i.e. above CMC) exhibits colloidal behaviour due to the formation of micelles. Thus, plot (d) correctly represents relation between $\wedge_m$ and $\sqrt{C}$ for sodium strearate.

20. **(a)** Calamine → $ZnCO_3$

Malachite → $CuCO_3.Cu(OH)_2$

Magnetite → Fe_3O_4

Cryolite → Na_3AlF_6

21. **(c)**

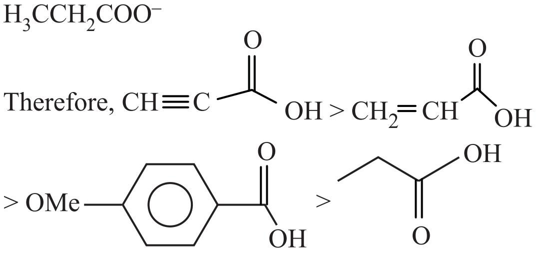
$$Na_2B_4O_7.10H_2O \xrightarrow{\Delta} Na_2B_4O_7 \xrightarrow{\Delta} 2NaBO_2 + B_2O_3$$

$$Cr_2O_3 + 3B_2O_3 \longrightarrow \underset{\text{(green colour)}}{\underset{\text{Chromium meta borate}}{2Cr(BO_2)_3}}$$

22. **(a)** Acidic strength depends upon the stability of anion formed. If stability of anion formed is high, its acidic strength will also be high. Order of stability of formed anion : $HC \equiv COO^- > H_2C = CHCOO^- > C_6H_4(OMe)COO^- >$

$H_3CCH_2COO^-$

Therefore,



23. **(b, c, d)**

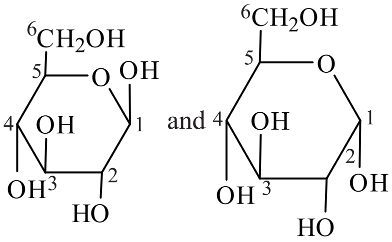
(a) Glucose $\xrightarrow[H_2O]{Br_2}$ Gluconic acid.

Bromine water oxidises only aldehyde group to carboxylic group. It neither oxidises –OH group nor $\rangle C = O$ group.

(b) Sucrose $\xrightarrow{H_2O} \underset{(+)}{\text{Glucose}} + \underset{(-)}{\text{Fructose}}$

Sucrose is dextrorotatory (+ 66.6°) in nature but on hydrolysis it gives dextrorotatory glucose and laevorotatory fructose (–90.4°). Overall solution is laevorotatory since laevorotation is more than dextrorotation.

(c)



are anomers because they differ in configuration only around C_1 position.

(d) Monosaccharides do not undergo further hydrolysis due to absence of glycosidic linkages.

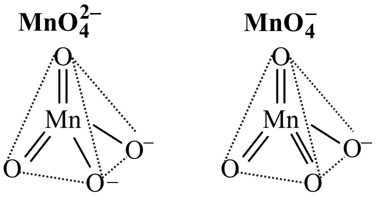
24. **(a, b, c)**

$$2MnO_2 + 4KOH + O_2 \xrightarrow{\Delta} \underset{(W)}{2K_2MnO_4} + 2H_2O$$

$$\underset{(W)}{2K_2MnO_4} + 2H_2O \rightarrow 2KMnO_4 + \underset{(X)}{2KOH\,(g)} + H_2$$

$$K_2MnO_4 \rightarrow 2K^+ + \underset{(Y)}{MnO_4^{2-}} \text{ (mangnate ion)}$$

$$KMnO_4 \rightarrow K^+ + \underset{(Z)}{MnO_4^-} \text{ (permangnate ion)}$$



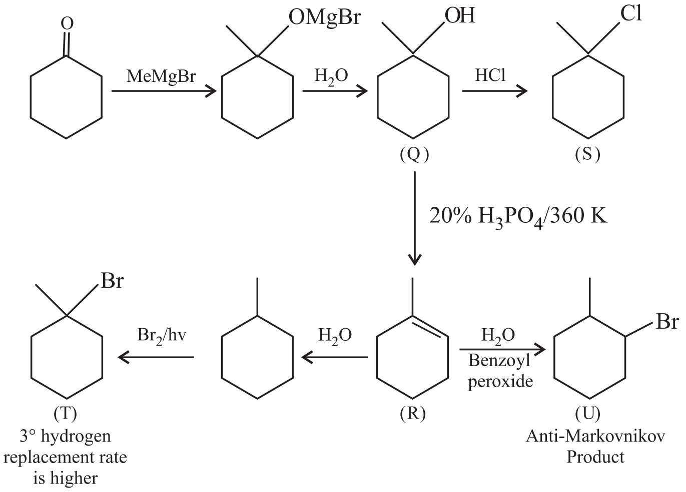
• sp^3, Tetrahedral	• sp^3, Tetrahedral
• Green colour	• Purple colour
• $Mn^{6+}: [Ar]\, 4S^0 3d^1$	• $Mn^{7+}: [Ar]\, 4S^0 3d^0$
• Paramagnatic	• Diamagnetic

Disproportionation of MnO_4^{2-} undergoes in acidic medium but not in base, concerned reaction is as under :

$$3MnO_4^{2-} + 4H^+ \longrightarrow MnO_2 + 2MnO_4^- + 2H_2O$$

a, b, c are correct

25. **(a, b)**



26. (a, c) Enthalpy of formation is the enthalpy change for formation of 1 mole of substance from its element present in the most stable natural form.

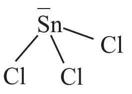
27. (c, d)

$$\underset{(X)}{SnCl_2 + Cl^- \rightarrow SnCl_3^-}$$

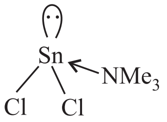
$$SnCl_2 + Me_3N \rightarrow \underset{(Y)}{SnCl_2[NMe_3]}$$

There is a coordinate bond between NMe_3 and $SnCl_2$ due to sharing of lone pairs of NMe_3 with $SnCl_2$.

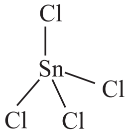
Structure of X, Y, Z are respectively :



$SnCl_3^-$
(X)



$SnCl_2[NMe_3]$
(Y)



$SnCl_4$
(Z)

$$SnCl_2 + 2CuCl_2 \rightarrow \underset{(Z)}{SnCl_4} + 2CuCl$$

28. (b, c) Dipole moment (μ) value of BF_3, SF_6, $BeCl_2$, CO_2, BCl_3 is Zero.

29. (a, b, c) According to kinetic theory of gases, all gases at a given temperature have same average kinetic energy.

$$E_{av} = \frac{3}{2}RT \;; \qquad E_{av} \propto T \text{ (absolute temp)}$$

Root mean square velocity is directly proportional to square root of absolute temperature and inversely proportional to square root of molecular weight of the gas.

$$V_{rms} \propto \sqrt{T} \text{ (absolute temp) : } V_{rms} \propto \frac{1}{\sqrt{M}}$$

30. (a, b, d)

$${}^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}He;\ x_1 = \alpha$$

$${}^{234}_{90}Th \longrightarrow {}^{234}_{91}Pa + {}^{0}_{-1}e;\ x_2 = \beta$$

$${}^{234}_{91}Pa \longrightarrow {}^{234}_{92}Z + {}^{0}_{-1}e;\ x_3 = \beta$$

$${}^{234}_{92}Z \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}He;\ x_4 = \alpha$$

31. (8.93)

$$Fe^{2+} + S^{2-} \rightleftharpoons FeS(K_c = 1.6 \times 10^{17})$$

t = 0 0.03M 0.1 M

At equilibrium (0.03 – x) (0.1 – x)

Since,

$K_c >> 10^3$; $0.03 - x \approx 0$

$\therefore$ x = 0.03 and 0.1 – x = 0.07

$$K_c = \frac{1}{(0.07) \times [Fe^{2+}]} = 1.6 \times 10^{17}$$

$$[Fe^{2+}] = \frac{1}{0.07 \times 1.6} \times 10^{-17} = \frac{250}{28} \times 10^{-17}$$

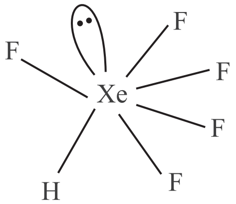
$$= 8.93 \times 10^{-17} \qquad \text{(i)}$$

comparing (i) with given value in question we get

Y = 8.93.

32. (19.00)

$$XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$$



Shape of XeF_6 is distorted octahedral. It contains one lone pair of e⁻ s on central atom 3 lone pair of e⁻ s on each F atom surrounded by Xe.

Total no. of lone pairs: 1 + 18 = 19.00

33. (6.75)

Rate of the reaction = $K[A]^x [B]^y [C]^{-z}$

Comparing experiment 1 with 2 we get that, y = 0

Comparing experiment 1 with 3 we get that z = 1

Comparing experiment 1 with 4 we get that, putting values of x, y, z in rate equation for experiment 1.
x = 1

$6 \times 10^{-5} = K \times 0.2 \times 0.1$

$K = 3 \times 10^{-3}$

Now, for the given concentration of A, B and C rate of reaction will be,

Rate = $3 \times 10^{-3} \times 0.15 \times 1 \times 0.15 = 6.75 \times 10^{-5}$

Therefore value of y = 6.75.

34. (1.03) We know that,

$$\frac{P^0 - P}{P^0} = \frac{w_2 \times m_1}{m_2 \times w_1};\frac{10}{640} = \frac{0.5 \times 78}{m_2 \times 39}$$

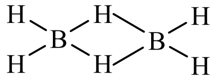
m_2 = 64 g

$$\Delta T_f = \frac{K_f \times w_2 \times 1000}{m_2 \times w_1}$$

$$\Delta T_f = \frac{5.12 \times 0.5 \times 1000}{64 \times 39} = 1.0256 \approx 1.03$$

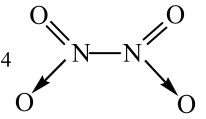
35. (4.00)

B_2H_6

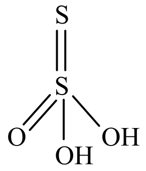


N_2O $\overset{-}{N} = \overset{+}{N} = O \longleftrightarrow N \equiv \overset{+}{N} - \overset{-}{O} \longleftrightarrow \overset{2-}{N} \equiv \overset{+}{N} \equiv \overset{+}{O}$

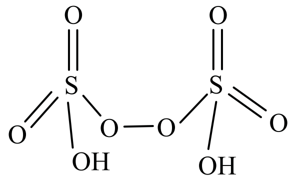
N_2O_4



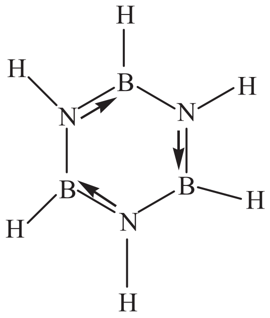
$H_2S_2O_3$



$H_2S_2O_8$



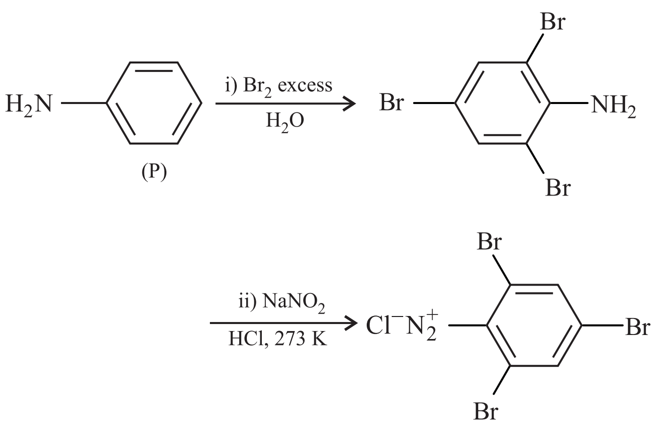
$B_3N_3H_6$

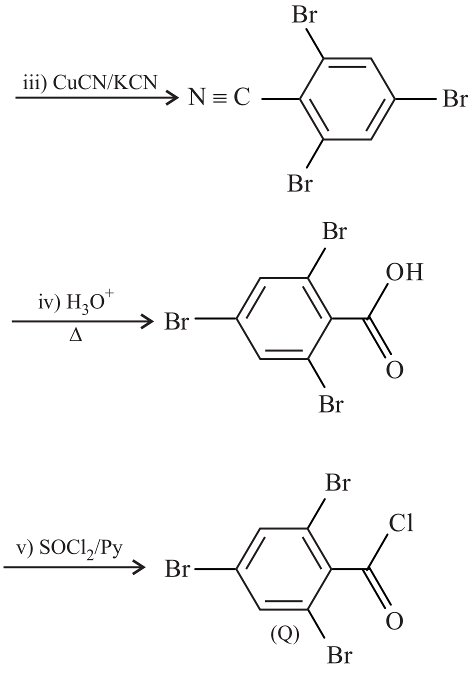


Total no. of molecules containing covalent bond between two atoms of the same kind are 4.

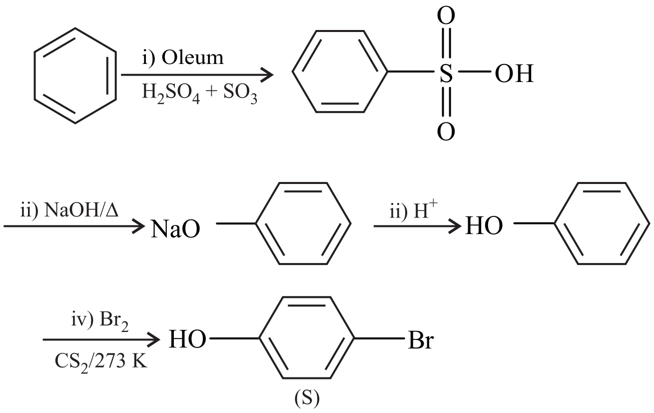
36. (4.00)

Scheme – I

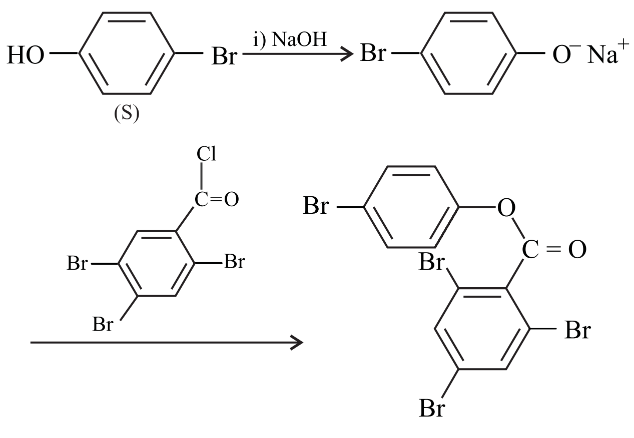




Scheme – II



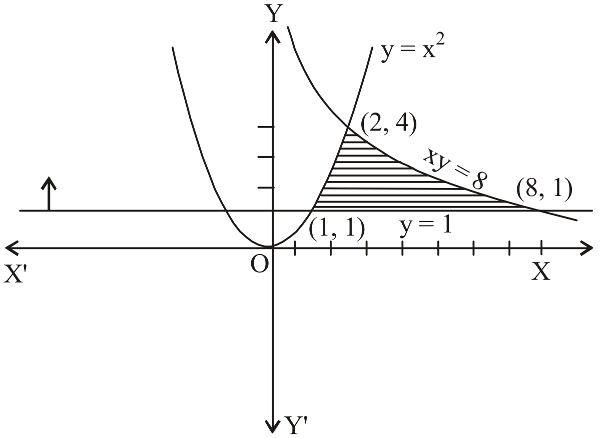
Scheme – III



Mathematics

37. (b) $xy \le 8,\ \ 1 \le y \le x^2$

Intersection points of $xy = 8$ and $y = 1$ is (8, 1); $xy = 8$ and $y = x^2$ is (2, 4) and $y = x^2$ and $y = 1$ is (1, 1)



Required area $= \int_1^2 x^2 dx + \int_2^8 \frac{8}{x} dx - \int_1^8 1 dx$

$$= \left(\frac{x^3}{3}\right)_1^2 + (8 \ln x)_2^8 - (x)_1^8$$

$$= \frac{8}{3} - \frac{1}{3} + 8 \ln 8 - 8 \ln 2 - (8-1)$$

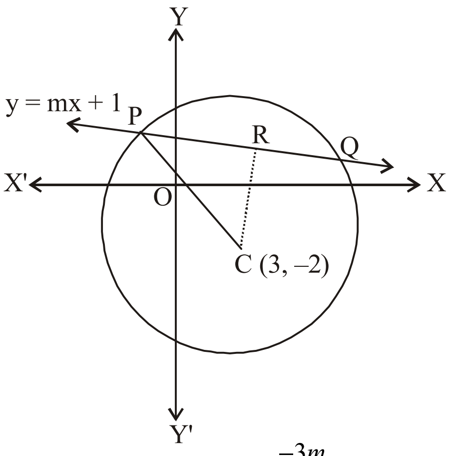
$$= \frac{7}{3} + 24 \ln 2 - 8 \ln 2 - 7 \ \ = 16 \ \ln 2 - \frac{14}{3}$$

∴ correct option is (b)

38. (a) Circle $(x-3)^2 + (y+2)^2 = 25$, with centre $C(3, -2)$ and radius 5 is intersected by a line y = mx + 1 at p & Q such that mid point R of PQ has its

x-coordinate as $-\frac{3}{5}$.

Let $R\left(-\frac{3}{5}, \frac{-3m}{5} + 1\right)$



Then $CR \perp PQ \quad \Rightarrow \quad \dfrac{\dfrac{-3m}{5} + 1 + 2}{-\dfrac{3}{5} - 3} \times m = -1$

$$\Rightarrow \quad \frac{(-3m+15)m}{-18} = -1 \Rightarrow m^2 - 5m + 6 = 0$$

$$\Rightarrow \quad m = 2, 3$$

Alternately :- The x-coordinates of intersection points P and Q of circle and line can be given by solving their equations for x.

Putting $y = mx + 1$ in $(x-3)^2 + (y+2)^2 = 25$, we get $(x-3)^2 + (mx+3)^2 = 25$

$$\Rightarrow \quad (m^2+1)x^2 + 6(m-1)x - 16 = 0$$

$\Rightarrow \quad x_1 + x_2 = \dfrac{-6(m-1)}{m^2+1}$ where x_1 and x_2 are x-coordinates of P and Q respectively. As per question, x-coordinate of mid point of PQ is $\dfrac{-3}{5}$

$$\therefore \quad \frac{x_1 + x_2}{2} = -\frac{3}{5} \Rightarrow \frac{-3(m-1)}{m^2+1} = \frac{-3}{5}$$

$$\Rightarrow \quad m^2 - 5m + 6 = 0 \Rightarrow m = 2, 3$$

39. (d) S : $|z - 2 + i| \ge \sqrt{5}$ represents boundary and outer region of circle with centre (2, –1) and radius $\sqrt{5}$.

$z_0 \in S$, such that $\dfrac{1}{|z_0 - 1|}$ is the maximum.

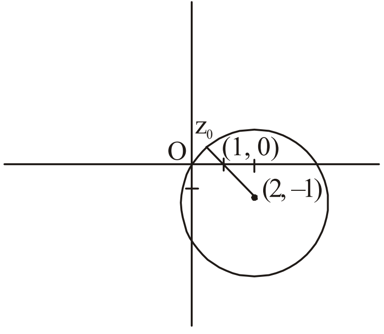
∴ $|z_0 - 1|$ is minimum

$z_0 \in S$ with $|z_0 - 1|$ as minimum will be a point on boundary of circle of region S which lies on radius of this circle, and passes through (1, 0).

∴ z_0, 1, $2 - i$ are collinear, or (x_0, y_0), (1, 0), (2, –1) are collinear.

∴ Using slopes of paralled lines,

$$\frac{y_0}{x_0 - 1} = \frac{-1}{2-1} \quad \Rightarrow \quad y_0 = 1 - x_0$$



Now, $\dfrac{4 - z_0 - \overline{z}_0}{z_0 - \overline{z}_0 + 2i} = \dfrac{4 - (z_0 + \overline{z}_0)}{(z_0 - \overline{z}_0) + 2i}$

$$= \frac{4 - 2x_0}{2iy_0 + 2i} = \frac{4 - 2x_0}{2i - 2x_0 i + 2i} = \frac{2(2 - x_0)}{2(2 - x_0)i} = \frac{1}{i} = -i$$

$$\therefore \quad Arg\left(\frac{4 - z_0 - \overline{z}_0}{z_0 - \overline{z}_0 - 2i}\right) = Arg(-i) = \frac{-\pi}{2}$$

40. **(d)** $M=\begin{bmatrix}\sin^4\theta & -1-\sin^2\theta\\ 1+\cos^2\theta & \cos^4\theta\end{bmatrix}$

$$|M|=\sin^4\theta\cos^4\theta+1+\sin^2\theta+\cos^2\theta$$
$$+\sin^2\theta\cos^2\theta$$
$$=2+\sin^2\theta\cos^2\theta+\sin^4\theta\cos^4\theta$$

$$M^{-1}=\frac{1}{|M|}\begin{bmatrix}\cos^4\theta & 1+\sin^2\theta\\ -1-\cos^2\theta & \sin^4\theta\end{bmatrix}$$

Given that $M=\alpha I+\beta M^{-1}$

$$\Rightarrow\begin{bmatrix}\sin^4\theta & -1-\sin^2\theta\\ 1+\cos^2\theta & \cos^4\theta\end{bmatrix}=\begin{bmatrix}\alpha & 0\\ 0 & a\end{bmatrix}$$
$$+\frac{\beta}{|M|}\begin{bmatrix}\cos^4\theta & 1+\sin^2\theta\\ -1-\cos^2\theta & \sin^4\theta\end{bmatrix}$$

$$\Rightarrow\quad\frac{\beta}{|M|}=-1\text{ and }\alpha+\frac{\beta}{|M|}\cos^4\theta=\sin^4\theta$$

$\Rightarrow\quad \alpha=\sin^4\theta+\cos^4\theta$

$\Rightarrow\quad \beta=-[2+\sin^2\theta\cos^2\theta+\sin^4\theta\cos^4\theta]$

Now, $\alpha=(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta$

$$=1-2\sin^2\theta\cos^2\theta=1-\frac{1}{2}\sin^2 2\theta$$

For α to be minimum $\sin^2 2\theta$ is maximum i.e.1.

$$\therefore\quad \alpha^*=1-\frac{1}{2}=\frac{1}{2}$$

Also, $\beta=-\left[2+\frac{1}{4}\sin^2 2\theta+\frac{1}{16}\sin^4 2\theta\right]$

For β to be minimum, $\sin^2 2\theta$ is maximum i.e.

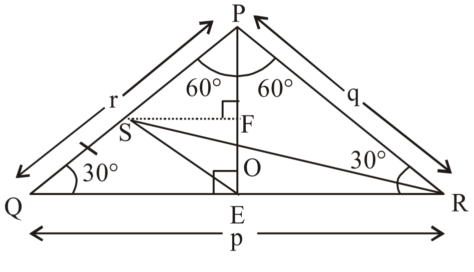
$$\therefore\ \beta^*=-\left[2+\frac{1}{4}+\frac{1}{16}\right]=-\frac{32+4+1}{16}=\frac{-37}{16}$$

$$\therefore\ \alpha^*+\beta^*=\frac{1}{2}-\frac{37}{16}=\frac{-29}{16}$$

41. **(a, c, d)** RS is median and $PE\perp QR$.

$p=\sqrt{3},\ q=1$

radius of circumcircle $(R)=1$



Using sine law in ΔPQR

$$\frac{p}{\sin P}=\frac{q}{\sin Q}=\frac{r}{\sin R}=2\times\text{radius of circumcircle},$$

We get, $\dfrac{\sqrt{3}}{\sin P}=\dfrac{1}{\sin Q}=\dfrac{r}{\sin R}=2\times 1=2$

$$\Rightarrow \sin P=\frac{\sqrt{3}}{2}\text{ and }\sin Q=\frac{1}{2}$$

$\Rightarrow$ $\angle P=60°$ or $120°$ and $\angle Q=30°$ or $150°$

$\because$ Δ cannot have two obtuse angles

$\therefore$ $\angle P=120°$, $\angle Q=150°$ is rejected.

$\because$ sum of three angles can not be $>180°$,

$\therefore$ $\angle P=60°$, $\angle Q=150°$ is rejected.

$\because$ ΔPQR is non right triangle, $\therefore$ $\angle P=60°$ and $\angle Q=30°$ is rejected.

$\therefore$ $\angle P=120°$ and $\angle Q=30°$ is the only option.

$\Rightarrow \angle QPE=\angle RPE=60°$ and $\angle PRQ=30°$

$$\Rightarrow\ \frac{r}{\sin 30°}=2\Rightarrow r=1$$

Also PE is bisector of $\angle QPR$

$$\therefore\ QE:ER=PQ:PR\Rightarrow\ QE=ER=\frac{\sqrt{3}}{2}.$$

Area of ΔPQR

$$=\frac{pqr}{4(\text{radius of circumcircle})}\qquad\left[\because\ \Delta=\frac{abc}{4R}\right]$$

$$=\frac{\sqrt{3}}{4}$$

$\therefore$ In ΔPQR, radius of incircle

$$=\frac{\text{Ar}(\Delta\text{PQR})}{\text{Semi perimeter}}\qquad\left(\because\ r=\frac{\Delta}{s}\right)$$

$$=\frac{\frac{\sqrt{3}}{4}}{\frac{\sqrt{3}+1+1}{2}}=\frac{\sqrt{3}}{2(2+\sqrt{3})}=\frac{\sqrt{3}}{2}(2-\sqrt{3})$$

$\therefore$ option (a) is correct.

$\because$ RS and PE are medians of ΔPQR intersecting at O, O is centriod of ΔPQR.

$$\therefore\ \text{ar}(\Delta\text{OQR})=\frac{1}{3}\text{ar}(\Delta\text{PQR})=\frac{\sqrt{3}}{12}$$

$$\Rightarrow\quad \frac{1}{2}\times\sqrt{3}\times\text{OE}=\frac{\sqrt{3}}{12}\ \Rightarrow\ \text{OE}=\frac{1}{6}$$

$\therefore$ option (c) is correct.

Now $\text{Ar}(\Delta\text{SOE})=\frac{1}{2}\text{OE}\times\text{SF}$

$$=\frac{1}{2}\times\frac{1}{6}\times\frac{r}{2}\sin 60°$$

$$= \frac{1}{12} \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{48}$$

$\therefore$ option (b) is incorrect. Also in ΔRQS, by cosine law

$$\cos 30° = \frac{QS^2 + QR^2 - RS^2}{2QS \times QR}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{\left(\frac{1}{2}\right)^2 + (\sqrt{3})^2 - RS^2}{2 \times \frac{1}{2} \times \sqrt{3}}$$

$$\Rightarrow \frac{3}{2} = \frac{13}{4} - RS^2$$

$$\Rightarrow RS^2 = \frac{13}{4} - \frac{3}{2} = \frac{7}{4} \quad \text{or} \quad RS = \frac{\sqrt{7}}{2}$$

$\therefore$ Option (d) is correct.

42. **(c, d)** Tangent to the curve y = y (x) at point P (x, y) is given by $Y - y = y'(x)(X - x)$

It intersects y-axis at Y_p, putting x = 0

$$Y - y = -xy'(x)$$

$$\Rightarrow Y = y - xy'(x) \qquad \therefore Y_p(0, y - xy'(x))$$

Given $PY_p = 1$

$$\Rightarrow \sqrt{(x-0)^2 + (y - y + xy'(x))^2} = 1$$

$$\Rightarrow x^2 + x^2(y'(x))^2 = 1$$

$$\Rightarrow y'(x) = \pm\frac{\sqrt{1-x^2}}{x}$$

Now y = y (x) less in first quadrant and its tangent passes through (1, 0), therefore it has to be a decreasing function, so derivative should be negative

$$\therefore y'(x) = \frac{-\sqrt{1-x^2}}{x} \qquad \left[\text{or } xy'(x) + \sqrt{1-x^2} = 0\right]$$

$$\Rightarrow \; y(x) = -\int \frac{\sqrt{1-x^2}}{x}\,dx$$

put $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$$y = -\int \frac{\cos\theta}{\sin\theta}\cos\theta\, d\theta = -\int(\operatorname{cosec}\theta - \sin\theta)\, d\theta$$

$$y = +\log|\operatorname{cosec}\theta + \cot\theta| - \cos\theta + c$$

$$y = \log\left|\frac{1+\sqrt{1-x^2}}{x}\right| - \sqrt{1-x^2} + c$$

for x = 1 and y = 0, we get c = 0

$$y = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$$

Hence options (c) and (d) are correct.

43. **(a, b, c)** Given α, β are roots of $x^2 - x - 1 = 0$ with $\alpha > \beta$

$$\therefore \qquad \alpha = \frac{1+\sqrt{5}}{2}, \; \beta = \frac{1-\sqrt{5}}{2}$$

Also, $\alpha + \beta = 1, \quad \alpha\beta = -1, \quad \alpha - \beta = \sqrt{5}$

$\alpha^2 - \alpha - 1 = 0$ and $\beta^2 - \beta - 1 = 0$

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 1; \; b_1 = 1 \text{ and } b_n = a_{n-1} + a_{n+1}, n \geq 2$$

Let us now check the given options, one by one

(a) $$\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \sum_{n=1}^{\infty} \frac{\alpha^n - \beta^n}{\sqrt{5}(10^n)}$$

$$= \frac{1}{\sqrt{5}}\left[\sum_{n=1}^{\infty}\left(\frac{\alpha}{10}\right)^n - \sum_{n=1}^{\infty}\left(\frac{\beta}{10}\right)^n\right]$$

$$= \frac{1}{\sqrt{5}}\left[\frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} - \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}\right] = \frac{1}{\sqrt{5}}\left[\frac{\alpha}{10-\alpha} - \frac{\beta}{10-\beta}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\frac{10\alpha - \alpha\beta - 10\beta + \alpha\beta}{(10-\alpha)(10-\beta)}\right]$$

$$= \frac{1}{5}\left[\frac{10(\alpha-\beta)}{100 - 10(\alpha+\beta) + \alpha\beta}\right] = \frac{1}{5}\left[\frac{10\sqrt{5}}{100-10-1}\right] = \frac{10}{89}$$

$\therefore$ option (a) is correct.

(b) $\quad b_n = a_{n+1} + a_{n-1}$

$$= \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta} + \frac{\alpha^{n-1} - \beta^{n-1}}{\alpha - \beta}$$

$$= \frac{(\alpha^{n+1} + \alpha^{n-1}) - (\beta^{n+1} + \beta^{n-1})}{\alpha - \beta}$$

$$= \frac{\alpha^{n-1}(\alpha^2 + 1) - \beta^{n-1}(\beta^2 + 1)}{\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}}\left[\alpha^{n-1}(\alpha + 2) - \beta^{n-1}(\beta + 2)\right]$$

(using $\alpha^2 = \alpha + 1, \beta^2 = \beta + 1$)

$$= \frac{1}{\sqrt{5}}\left[\alpha^{n-1}\left(\frac{1+\sqrt{5}}{2} + 2\right) - \beta^{n-1}\left(\frac{1-\sqrt{5}}{2} + 2\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\alpha^{n-1}\left(\frac{5+\sqrt{5}}{2}\right) - \beta^{n-1}\left(\frac{5-\sqrt{5}}{2}\right)\right]$$

$$= \frac{\sqrt{5}}{\sqrt{5}}\left[\alpha^{n-1}\left(\frac{\sqrt{5}+1}{2}\right) + \beta^{n-1}\left(\frac{1-\sqrt{5}}{2}\right)\right]$$

$= \alpha^{n-1}\alpha + \beta^{n-1}\beta = \alpha^n + \beta^n$

∴ option(b) is correct

(c) $a_1 + a_2 + a_3 + ... + a_n$

$$= \frac{\alpha^1-\beta^1}{\alpha-\beta} + \frac{\alpha^2-\beta^2}{\alpha-\beta} + \frac{\alpha^3-\beta^3}{\alpha-\beta} + ... + \frac{\alpha^n-\beta^n}{\alpha-\beta}$$

$$= \frac{1}{\sqrt{5}}\,[(\alpha+\alpha^2+\alpha^3+...+\alpha^n)-(\beta+\beta^2+\beta^3+...+\beta^n)]$$

$$= \frac{1}{\sqrt{5}}\left[\frac{\alpha(1-\alpha^n)}{1-\alpha} - \frac{\beta(1-\beta^n)}{1-\beta}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\frac{\alpha(1-\beta)(1-\alpha^n)-\beta(1-\alpha)(1-\beta^n)}{(1-\alpha)(1-\beta)}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\frac{(\alpha-\alpha\beta)(1-\alpha^n)-(\beta-\alpha\beta)(1-\beta^n)}{(1-\alpha)(1-\beta)}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\frac{(\alpha+1)(1-\alpha^n)-(\beta+1)(1-\beta^n)}{1-(\alpha+\beta)+\alpha\beta}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\frac{\alpha^2(1-\alpha^n)-\beta^2(1-\beta^n)}{1-1-1}\right]$$

(using $\alpha^2 - \alpha - 1 = 0, \beta^2 - \beta - 1 = 0$)

$$= \frac{1}{\sqrt{5}}\left[\frac{(\alpha^2-\beta^2)-(\alpha^{n+2}-\beta^{n+2})}{-1}\right]$$

$$= \left[\frac{-(\alpha-\beta)(\alpha+\beta)}{\alpha-\beta} + \frac{(\alpha^{n+2}-\beta^{n+2})}{\alpha-\beta}\right] = -1 + a_{n+2}$$

∴ option (c) is correct.

(d) $$\sum_{n=1}^{\infty}\frac{b_n}{10^n} = \sum_{n=1}^{\infty}\frac{\alpha^n+\beta^n}{10^n}$$

(using $b_n = \alpha^n + \beta^n$ as proved in option (b))

$$= \sum_{n=1}^{\infty}\left(\frac{\alpha}{10}\right)^n + \left(\frac{\beta}{10}\right)^n = \frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}$$

$$= \frac{\alpha}{10-\alpha} + \frac{\beta}{10-\beta} = \frac{\alpha(10-\beta)+\beta(10-\alpha)}{(10-\alpha)(10-\beta)}$$

$$= \frac{10(\alpha+\beta)-2\alpha\beta}{100-10(\alpha+\beta)+\alpha\beta} = \frac{10+2}{100-10-1} = \frac{12}{89}$$

∴ option (d) is incorrect.

44. (a, c, d)

$$M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix};\ \text{Adj}\,M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

Cofactor of a_{11} in M = 2 – 3b = –1 = a_{11} in Adj M

$\Rightarrow b = 1$

Cofactor of a_{31} in M = 3 – 2a = –1 = a_{13} in Adj M

$\Rightarrow a = 2$

∴ $a + b = 2 + 1 = 3 \Rightarrow$ option (a) is correct.

$$|M| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = -(1-9) + 2(1-6) = 8 - 10 = -2$$

$|\text{adj}\ M^2| = |M^2|^2 = |M|^4 = (-2)^4 = 16$

∴ option (b) is incorrect.

Also $(\text{adj}\ M)^{-1} = \text{adj}\ M^{-1}$

∴ $(\text{adj}\ M)^{-1} + \text{adj}\ M^{-1} = 2\text{adj}\ (M^{-1})$

$$= \frac{2\,|M^{-1}|}{|M^{-1}|}\text{adj}(M^{-1}) = 2|M^{-1}| \cdot (M^{-1})^{-1}$$

$$= 2 \times \frac{1}{-2} \times M = -M$$

∴ option (c) is correct.

$$\text{Now,}\ M\begin{bmatrix}\alpha \\ \beta \\ \gamma\end{bmatrix} = \begin{bmatrix}1 \\ 2 \\ 3\end{bmatrix} \Rightarrow \begin{bmatrix}\alpha \\ \beta \\ \gamma\end{bmatrix} = M^{-1}\begin{bmatrix}1 \\ 2 \\ 3\end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix}1 \\ 2 \\ 3\end{bmatrix} \Rightarrow \begin{bmatrix}\alpha \\ \beta \\ \gamma\end{bmatrix} = \frac{1}{-2}\begin{bmatrix}-2 \\ 2 \\ -2\end{bmatrix} = \begin{bmatrix}1 \\ -1 \\ 1\end{bmatrix}$$

∴ $\alpha - \beta + \gamma = 1 - (-1) + 1 = 3$ ∴option (d) is correct.

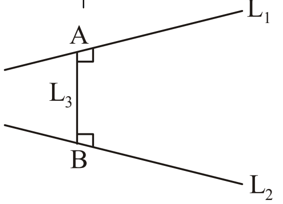
45. (a, b, d) $L_1 : \vec{r} = \hat{i} + \lambda(-i + 2j + 2\hat{k})$

$L_2 : \vec{r} = \mu(2i - j + 2\hat{k})$

L_3 being perpendicular to both L_1 and L_2, is the shortest distance line between L_1 & L_2.

L_3 is in the direction of $(-\hat{i} + 2\hat{j} + 2\hat{k}) \times (2\hat{i} - \hat{j} + 2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{vmatrix} = 6\hat{i} + 6\hat{j} - 3\hat{k}$$



If AB is the line of shortest distance then we can consider $A(1-\lambda, 2\lambda, 2\lambda),\ B(2\mu, -\mu, 2\mu)$.

$\therefore$ dr's of AB $= 2\mu+\lambda-1, -\mu-2\lambda, 2\mu-2\lambda$

$\because$ AB and L_3 are representing the same line

$$\therefore \quad \frac{2\mu+\lambda-1}{6} = \frac{-\mu-2\lambda}{6} = \frac{2\mu-2\lambda}{-3}$$

$$\Rightarrow \quad 3\lambda+3\mu=1 \qquad ...(1)$$

$$6\lambda-3\mu=0 \qquad ...(2)$$

Solving eqn. (1) and (2): $\lambda=\frac{1}{9}, \mu=\frac{2}{9}$

$$\therefore \quad A\left(\frac{8}{9},\frac{2}{9},\frac{2}{9}\right) \text{ and } B\left(\frac{4}{9},\frac{-2}{9},\frac{4}{9}\right)$$

$\therefore$ L_3 can be described by

$$\vec{r} = \frac{2}{9}(4\hat{i}+\hat{j}+\hat{k})+t(2i+2j-\hat{k})$$

or
$$\vec{r} = \frac{2}{9}(2\hat{i}-\hat{j}+2\hat{k})+t(2i+2j-\hat{k})$$

Also mid-point of AB is $\left(\frac{2}{3},0,\frac{1}{3}\right)$

$\therefore$ L_3 can also be given by

$\vec{r} = \frac{1}{3}(2\hat{i}+\hat{k})+t(2\hat{i}+2\hat{j}-\hat{k})$, where $t \in R$

Clearly (0, 0, 0) does not lie on

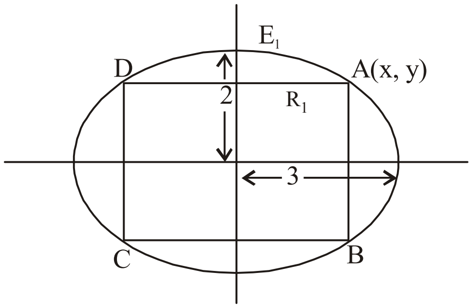
$$\vec{r} = \frac{2}{9}(4\hat{i}+\hat{j}+\hat{k})+t(2\hat{i}+2\hat{j}-\hat{k})$$

$\therefore$ $\vec{r} = t(2\hat{i}+2\hat{j}-\hat{k})$ can not describe the line L_3.

Hence options (a), (b) and (d) are correct, but (c) is incorrect.

46. (b, c) $E_1 : \frac{x^2}{9}+\frac{y^2}{4}=1$

R_1 : rectangle ABCD with largest area.



Area of $R_1 = A = 2x \times 2y$

$$\Rightarrow \quad A = 4x \times \frac{2}{3}\sqrt{9-x^2} = \frac{8}{3}x\sqrt{9-x^2}$$

$$\frac{dA}{dx} = \frac{8}{3}\left[\sqrt{9-x^2}-\frac{x^2}{\sqrt{9-x^2}}\right]$$

$$\Rightarrow \frac{8}{3}\left[\frac{9-2x^2}{\sqrt{9-x^2}}\right]=0 \Rightarrow x=\frac{3}{\sqrt{2}}, y=\frac{2}{3}\sqrt{9-\frac{9}{2}}=\frac{2}{\sqrt{2}}$$

$$\therefore \quad \text{for } E_2 : a=\frac{3}{\sqrt{2}}, b=\frac{2}{\sqrt{2}}$$

Similarly for $E_3 : a=\frac{3}{(\sqrt{2})^2}, b=\frac{2}{(\sqrt{2})^2}$ and so on.

Now eccentricity depends on $\frac{b}{a}$ which is same for all E_n, therefore eccentricity for all the E_n's will remain $\sqrt{1-\frac{4}{9}}=\frac{\sqrt{5}}{3}$

$\therefore$ option (a) is incorrect

for $E_9 : a=\frac{3}{(\sqrt{2})^8}, b=\frac{2}{(\sqrt{2})^8}$

$$\therefore \quad \text{length of latus rectum} = \frac{2b^2}{a} = \frac{2\times\frac{4}{256}}{\frac{3}{16}} = \frac{1}{6}$$

$\therefore$ option (b) is correct.

$$\text{Area of } R_1 = 4\times\frac{3}{\sqrt{2}}\times\frac{2}{\sqrt{2}}=\frac{24}{2}$$

$$\text{Area of } R_2 = 4\times\frac{3}{(\sqrt{2})^2}\times\frac{2}{(\sqrt{2})^2}=\frac{24}{2^2}$$

$$\text{Area of } R_3 = 4\times\frac{3}{(\sqrt{2})^3}\times\frac{2}{(\sqrt{2})^3}=\frac{2}{(2)^3} \text{ and so on}$$

We observe

$$\sum_{n=1}^{\infty}\text{area of } R_n = \frac{24}{2}+\frac{24}{2^2}+\frac{24}{2^3}+....=\frac{12}{1-\frac{1}{2}}=24$$

$\therefore$ $\sum_{n=1}^{N}(\text{area of } R_n) < 24$ for each positive integer N.

$\therefore$ option (c) is correct.

For $E_9 : a=\frac{3}{(\sqrt{2})^8}, b=\frac{2}{(\sqrt{2})^8}, e=\frac{\sqrt{5}}{3}$

$\therefore$ focus $= (ae, 0) = \left(\frac{\sqrt{5}}{16},0\right)$

$\therefore$ distance of focus from centre $= \frac{\sqrt{5}}{16}$

$\therefore$ option (d) is incorrect.

47. (b, d) $B_1\begin{cases}5R\\5G\end{cases} \qquad B_2\begin{cases}3R\\5G\end{cases} \qquad B_3\begin{cases}5R\\3G\end{cases}$

$$P(B_1)=\frac{3}{10}, P(B_2)=\frac{3}{10}, P(B_3)=\frac{4}{10}$$

$$P(G/B_1)=\frac{5}{10}, P(G/B_2)=\frac{5}{8}, P(G/B_3)=\frac{3}{8}$$

$$P(B_3\cap G)=P(B_3)\,P(G/B_3)=\frac{4}{10}\times\frac{3}{8}=\frac{3}{20}$$

$\therefore$ option (a) is incorrect $P(G/B_3)=\frac{3}{8}$

∴ option (b) is correct.

$$P(B_3/G)=\frac{P(G/B_3)P(B_3)}{P(G/B_1)\,P(B_1)+P(G/B_2)P(B_2)+P(G/B_3)P(B_3)}$$

$$=\frac{\frac{3}{8}\times\frac{4}{10}}{\frac{5}{10}\times\frac{3}{10}+\frac{5}{8}\times\frac{3}{10}+\frac{3}{8}\times\frac{4}{10}}=\frac{\frac{12}{80}}{\frac{15}{100}+\frac{15}{80}+\frac{12}{80}}$$

$$=\frac{12}{80}\times\frac{400}{60+75+60}=\frac{60}{195}=\frac{4}{13}$$

∴ Option (c) is incorrect.

$$P(G)=P(G/B_1)P(B_1)+P(G/B_2)P(B_2)+P(G/B_3)P(B_3)$$

$$=\frac{5}{10}\times\frac{3}{10}+\frac{5}{8}\times\frac{3}{10}+\frac{3}{8}\times\frac{4}{10}=\frac{60+75+60}{400}=\frac{195}{400}=\frac{39}{80}$$

∴ option (d) is correct.

48. (a, c, d)

$$f(x)=\begin{cases}(x^5+5x^4+10x^3+10x^2+5x+1)-2x &, x<0\\ x^2-2\times\frac{1}{2}\times x+\frac{1}{4}+\frac{3}{4} &, 0\le x<1\\ \frac{2}{3}x^3-4x^2+7x-\frac{8}{3} &, 1\le x<3\\ (x-2)\log_e(x-2)-x+\frac{10}{3} &, x\ge 3\end{cases}$$

$$=\begin{cases}(x+1)^5-2x &, x<0\\ \left(x-\frac{1}{2}\right)^2+\frac{3}{4} &, 0\le x<1\\ \frac{2}{3}x^3-4x^2+7x-\frac{8}{3} &, 1\le x<3\\ (x-2)\log_e(x-2)-x+\frac{10}{3} &, x\ge 3\end{cases}$$

For $x=0, f(x)=1$. For $x<0$, $f(x)=(x+1)^5-2x$

It decreases to $-\infty$. ∴ $f(x)\in(-\infty, 1]$ for $x\le 0$

For $x=3$, $f(x)=\frac{1}{3}$

For $x\ge 3$, $f(x)$ increases to ∞

∴ $f(x)\in\left[\frac{1}{3},\infty\right)$ for $x\ge 3$

Combining the two $f(x)\in R$ ⇒ f is onto.

∴ option (d) is correct.

$$f'(x)=\begin{cases}5(x+1)^4-2 &, x<0\\ 2x-1 &, 0\le x<1\\ 2(x-2)^2-1 &, 1\le x<3\\ \log_e(x-2) &, x\ge 3\end{cases}$$

$Lf''(1)=2,\ Rf''(1)=-4,$

⇒ f' is not differentiable at x = 1

∴ option (c) is correct. For $x<0$, $f'(x)=5(x+1)^4-1$

$$f'(x)=0 \Rightarrow (x+1)^4=\frac{1}{5} \Rightarrow x=-1\pm\left(\frac{1}{5}\right)^{1/4}$$

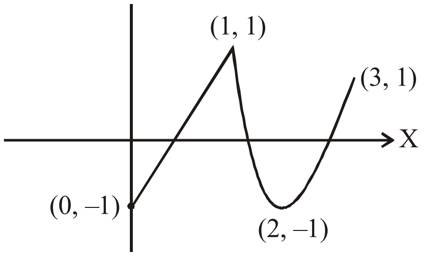
⇒ f is changing its nature at $x=-1-\left(\frac{1}{5}\right)^{1/4}$

∴ f is not increasing on $(-\infty, 0)$

∴ option (b) is incorrect.

$$f'x=\begin{cases}2x-1 &, 0\le x<1\\ 2(x-2)^2-1 &, 1\le x<3\end{cases}$$

From its graph $f'(x)$ has local maxima at $x=1$.



∴ option (a) is correct.

49. (0.50) $n(S)=2^9=512$

E_2 contains those matrices in which sum of entries is 7.

∴ there will be 7 one's and 2 zeroe's.

∴ $n(E_2)={}^9C_2=36$

$E_1\cap E_2$ contains those matrices in which 7 ones, 2 zeroes are there and det is zero.

Det(A) = 0 in cases when two rows/columns are identical for example

$$\begin{vmatrix}0&0&1\\1&1&1\\1&1&1\end{vmatrix} \text{ or } \begin{vmatrix}1&0&0\\1&1&1\\1&1&1\end{vmatrix} \text{ or } \begin{vmatrix}0&1&0\\1&1&1\\1&1&1\end{vmatrix}$$

∴ $n(E_1\cap E_2)={}^3C_1\times{}^3C_1\times 2=18$

$$\therefore P(E_1/E_2)=\frac{P(E_1\cap E_2)}{P(E_2)}=\frac{18/512}{36/512}=\frac{1}{2}=0.50$$

50. (3) a, b, c are distinct non zero integers

Min. value of $|a+b\omega+c\omega^2|^2$ is to be found

$|a+b\omega+c\omega^2|^2$

$$=\left|a+b\left(\frac{-1+i\sqrt{3}}{2}\right)+c\left(\frac{-1-i\sqrt{3}}{2}\right)\right|^2$$

$$=\left|\frac{1}{2}(2a-b-c)+\frac{i\sqrt{3}}{2}(b-c)\right|^2$$

$$=\frac{1}{4}(2a-b-c)^2+\frac{3}{4}(b-c)^2$$

$$=\frac{1}{4}(4a^2+b^2+c^2-4ab+2bc-4ac+3b^2+3c^2-6bc)$$

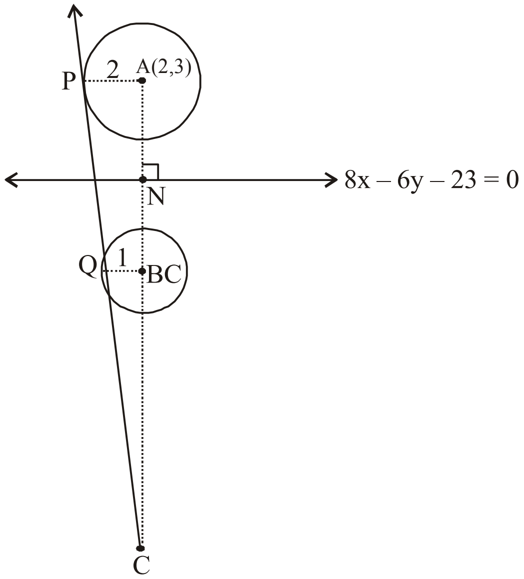
$$=a^2+b^2+c^2-ab-bc-ca$$

$$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

For min. value, let us consider $a=3, \quad b=2, \ c=1$

$\therefore \quad \text{min value} = \frac{1}{2}[1+1+4] = \frac{6}{2} = 3$

51. (10) $\text{AN} = \left|\frac{16-18-23}{\sqrt{64+36}}\right| = \frac{25}{10} = \frac{5}{2} = \text{BN}$



From $\Delta\text{CPA} \sim \Delta\text{CQB}$

$$\frac{CA}{CB} = \frac{PA}{QB} \Rightarrow \frac{CA}{CA-5} = \frac{2}{1}$$

$\Rightarrow \quad CA = 2CA - 10 \quad \Rightarrow CA = 10$

52. (157) $AP(1,3)$: $1, 4, 7, 10, 13$ ....

$AP(2,5)$: $2, 7, 12, 17, 22$ ....

$AP(3,7)$: $3, 10, 17, 24, 31$ ....

For $AP(1,3) \cap AP(2,5) \cap AP(3,7)$

first term will be the minimum common value of a term.

$\therefore$ we need to find that minimum number which

when divided by 7 leaves remainder 3 $(7m+3)$

and when divided by 5 leaves remainder 2 $(5p+2)$

and when divided by 3 leaves remainder 1 $(3q+1)$

By hit and trial 52 is such number $(7 \times 7 + 3)$

$\therefore$ first term 'a' of intersection $AP = 52$

Also common difference 'd' of intersection

$AP = \text{LCM}(7,5,3) = 105 \ \therefore a + d = 52 + 105 = 157$

53. (4) $$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{\left(1+e^{\sin x}\right)(2-\cos 2x)} \qquad \text{......(1)}$$

$$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{\left(1+e^{-\sin x}\right)(2-\cos 2x)}$$

$$\left[\text{using} \int_a^b f(x)\,\text{dx} = \int_a^b f(a+b-x)dx\right]$$

$$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{e^{\sin x}}{\left(e^{\sin x}+1\right)(2-\cos 2x)} \qquad \text{........(2)}$$

Adding (1) and (2):

$$2I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{1+e^{\sin x}}{\left(e^{\sin x}+1\right)(2-\cos 2x)}dx$$

$$= \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{1}{2-\cos 2x}dx$$

$$I = \frac{2}{\pi}\int_0^{\pi/4} \frac{1}{2-\cos 2x}dx = \frac{2}{\pi}\int_0^{\pi/4} \frac{dx}{1+2\sin^2 x}$$

$$= \frac{2}{\pi}\int_0^{\pi/4} \frac{\sec^2 x}{\sec^2 x + 2\tan^2 x}dx = \frac{2}{\pi}\int_0^{\pi/4} \frac{\sec^2 x}{1+3\tan^2 x}dx$$

Put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

At $x = 0, t = 0$; At $x = \frac{\pi}{4}, t = 1$

$$\therefore \ I = \frac{2}{\pi}\int_0^1 \frac{1}{1+3t^2}dt = \frac{2}{\pi}\left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}\,t\right)\right]_0^1$$

$$= \frac{2}{\pi}\left[\frac{1}{\sqrt{3}} \times \frac{\pi}{3}\right] = \frac{2}{3\sqrt{3}} \quad \therefore 27I^2 = 4$$

54. (0.75) Given lines are $\vec{r} = \lambda\hat{i}$ ......(1)

$\vec{r} = \mu(\hat{i}+\hat{j})$ ......(2)

$\vec{r} = v(\hat{i}+\hat{j}+\hat{k})$ ....(3)

These lines cut the plane $x + y + z = 1$ at points

$A(\lambda, 0, 0)$, $B(\mu, \mu, 0)$ and $C(\nu, \nu, \nu)$ respectively

$\because A$ lies on plane

$\Rightarrow \lambda = 1 \Rightarrow A(1, 0, 0) \because B$ lies on plane

$$\Rightarrow \mu + \mu = 1 \Rightarrow \mu = \frac{1}{2} \Rightarrow B\left(\frac{1}{2}, \frac{1}{2}, 0\right)$$

$\because C$ lies on plane

$$\Rightarrow \nu + \nu + \nu = 1 \ \Rightarrow \nu = \frac{1}{3} \Rightarrow C\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

$$\text{Area}(\Delta ABC) = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} \end{vmatrix}$$

$$= \frac{1}{2}\left|\frac{1}{6}\hat{i} + \frac{1}{6}\hat{j} + \frac{1}{6}\hat{k}\right| = \frac{1}{2} \times \frac{1}{6}\sqrt{3} = \frac{\sqrt{3}}{12}$$

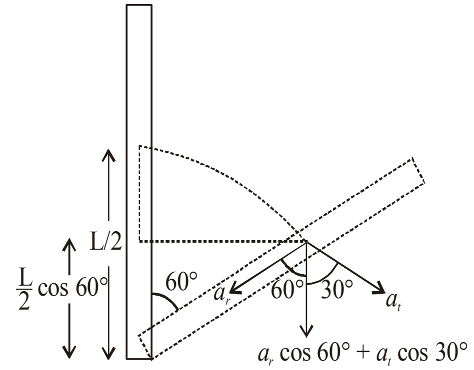
$$\therefore (6\Delta)^2 = 36 \times \frac{3}{144} = \frac{3}{4} = 0.75$$

Paper - 2

Physics

1. **(a, b, c)**

Gain in kinetic energy = Loss in potential energy



$$\frac{1}{2}I\omega^2 = mg\frac{l}{2}(1 - \cos 60°)$$

$$\therefore \quad \frac{ml^2}{3}\omega^2 = mg\frac{l}{2} \quad \therefore \quad \omega = \sqrt{\frac{3g}{2l}}$$

Option (a) is correct

Now, $\tau = I\alpha$

$$\therefore \quad mg \times \frac{l}{2}\sin 60° = \frac{1}{3}ml^2\alpha$$

$$\therefore \quad \alpha = \frac{3\sqrt{3}}{4}\frac{g}{l}$$

∴ (d) is an incorrect option

Further $a_t = \frac{l}{2}\alpha = \frac{3\sqrt{3}g}{8}$

Also $a_r = \omega^2\frac{l}{2} = \frac{3g}{2l}\times\frac{l}{2} = \frac{3g}{4}$

So (b) is a correct option.

For vertical motion of C.M

$mg - N = m(a_r\cos 60° + a_t\cos 30°)$

$$\therefore \quad mg - N = m\left[\frac{3g}{4}\times\frac{1}{2} + \frac{3\sqrt{3}g}{8}\times\frac{\sqrt{3}}{2}\right]$$

$$\therefore \quad N = \frac{4/Mg}{16}$$

Option (c) is correct

2. **(b, d)** It is given at all points on the circle are at the same potential. We further know that electric field is perpendicular to such a line that is the direction of electric field is either radial or the magnitude of electric field should be zero at points on the circle. Now considering point A, the electric field due to dipole should be $\frac{2Kp}{R^3}$ (directed from O to A) as point A lies on the axial line of electric dipole. The external electric field E_0 should also be in the direction of O to A. Now considering point B which is a point on the equitorial line of the electric dipole.

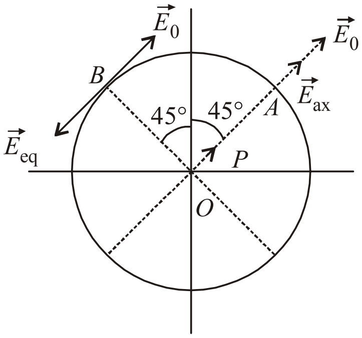
The electric field here due to dipole is $\frac{Kp}{R^3}$ in a direction opposite to the dipole. The external electric field should cancel out this field. Therefore (b) is the correct option.

Further $\vec{E}_0 = \frac{-K\vec{p}}{R^3}$ ...(i)

The electric field at A is

$$\vec{E}_A = \frac{2K\vec{p}}{R^3} + \vec{E}_0 = -2\vec{E}_0 + \vec{E}_0 = -\vec{E}_0$$

Option (c) is wrong.



Further from (i) $E_0 = \frac{1}{4\pi\epsilon_0}\frac{p_0}{\sqrt{2}(R^3)}\times\sqrt{2}$

$$\therefore \quad R^3 = \frac{p_0}{4\pi\epsilon_0 E_0} \quad \therefore \quad R = \left[\frac{p_0}{4\pi\epsilon_0 E_0}\right]^{1/3}$$

Option (d) is correct.

3. **(b, c)** Here, $E_4 - E_1 = \frac{hc}{\lambda_a} = \Delta p_a \times C$ ...(i)

$E_4 - E_m = \frac{hC}{\lambda_e} = \Delta p_e \times C$ ...(ii)

$$\therefore \quad \frac{\Delta p_a}{\Delta p_e} = \frac{\lambda_e}{\lambda_a} = 5$$

∴ (a) is an incorrect option.

$$\therefore \quad \frac{\Delta p_a}{\Delta p_e} = \frac{-0.85-(-13.6)}{-0.85-(-E_m)} = 5$$

⇒ $12.75 = 5(E_m - 0.85) \Rightarrow E_m = 3.4\text{eV}$

⇒ $m = 2$

⇒ (c) is a correct option

$$\frac{K\cdot E_2}{K\cdot E_1} = \frac{V_2^2}{V_1^2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$\Rightarrow$ (b) is the correct option.

From (ii), $E_4 - E_2 = \dfrac{hc}{\lambda_e}$

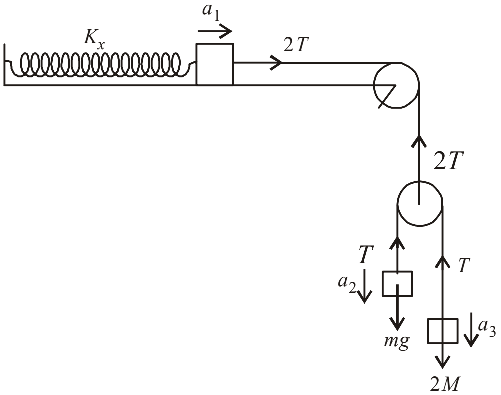
$\therefore \quad \lambda_e = \dfrac{1242}{-0.85-(-3.4)} = 487 \text{ nm}$

$\Rightarrow$ Option (d) is incorrect.

4. (d) According to constraint relation

$$a_1 = \frac{a_2 + a_3}{2} \Rightarrow a_1 - a_3 = a_2 - a_1$$

$\Rightarrow$ Option (d) is correct



Let 'x' be the extension of the spring at a certain instant. Then

$2Mg - T = 2Ma_3$

$Mg - T = Ma_2$

On solving we get,

$$a_1 = \frac{4g}{7} - \frac{2kx}{14M} \qquad \text{...(i)}$$

$$\therefore \quad \omega^2 = \frac{3k}{14M} \quad \therefore \quad \omega = \sqrt{\frac{3k}{14M}}$$

$$\text{and} \quad T = \frac{4Mg}{7} + \frac{3kx}{7} \qquad \text{...(ii)}$$

For $a_1 = 0$ (Maximum extension of spring) we have from (i)

$$\frac{4g}{7} - \frac{3kx}{14M} = 0$$

$$\therefore \quad 4g = \frac{3kx}{2M} \quad \therefore \quad x = \frac{8Mg}{3k}$$

$$\therefore \quad x_0 = 2x = \frac{16Mg}{3k}$$

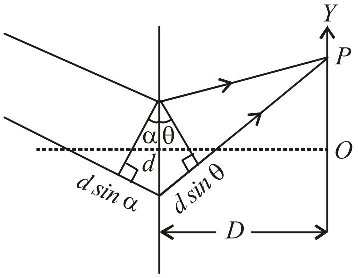
$$\text{For} \quad x = \frac{x_0}{4} = \frac{1}{4}\left(\frac{16Mg}{3k}\right) = \frac{4Mg}{3k}$$

$$\text{From (i) } a_1 = \frac{4g}{7} - \frac{3k}{14M} \times \frac{4Mg}{3x} = \frac{2g}{7}$$

At $x = \dfrac{x_0}{2}$ the particle is at the mean position and its

$$\text{velocity} = A\omega = \frac{x_0}{2}\sqrt{\frac{3k}{14M}} = \frac{8Mg}{3k}\sqrt{\frac{3k}{14M}}$$

5. (c)



Path difference $= d \sin \alpha + d \sin \theta = d\alpha + \dfrac{yd}{D}$

[when α and θ are small]

For $\alpha = 0$, path difference $= \dfrac{yd}{D}$

$= 11 \times 0.3 \times 10^{-3} = 33 \times 10^{-4}$ mm

$$\text{Now } \frac{\text{path difference}}{\lambda} = \frac{33\times10^{-4}}{600\times10^{-6}} = \frac{11}{2}$$

$=$ odd multiple of $\dfrac{1}{2}$

This implies destructive interference at P.

$\therefore$ Option (a) is wrong.

Fringe width $\beta = \dfrac{\lambda D}{d}$ is independent of path difference

$\therefore$ Option (b) is wrong.

For $\alpha = \dfrac{0.36}{\pi}$ degree (at point P)

$$\text{Path difference} = d\left[\alpha + \frac{y}{D}\right]$$

$$= 0.3 \times 10^{-3}\left[\frac{0.36}{180} + \frac{11\times10^{-3}}{1}\right] \text{m} = 3900 \text{ nm}$$

Now path difference $= \dfrac{3900}{600} = \dfrac{13}{2}$ is an odd multiple of $\dfrac{1}{2}$

This implies destructive intereference at P.

Option (c) is correct.

For $\alpha = \dfrac{0.36}{\pi}$ degree (at point O)

$$\text{Path difference} = d\alpha = 0.3 \times 10^{-3} \times \frac{0.36}{180}$$

$= 600 \times 10^{-9}$m $= 600$ nm

$$\text{Now } \frac{\text{path difference}}{\lambda} = 1$$

This implies constructive interference at O. Option (d) is incorrect.

6. **(a, b, d)** $\gamma_m = \frac{n_1C_{p1} + n_2C_{p2}}{n_1C_{v1} + n_2C_{v2}} = \frac{5\times\frac{5R}{2}+1\times\frac{7R}{2}}{5\times\frac{3R}{2}+1\times\frac{5R}{2}} = 1.6$

Option (d) is correct.

For an adiabatic process

$$P = P_0\left(\frac{V_0}{V}\right)^{1.6} = P_0\left(4\right)^{1.6}$$

$$= P_0(2^2)^{1.6} = P_0\ 2^{3.2} = 9.2\ P_0$$

Option (b) is correct.

$$W = \frac{P_2V_2 - P_1V_1}{1-\gamma} = \frac{9.2P_0 \times (V_0/4) - P_0V_0}{1-1.6} = \frac{-13P_0V_0}{6}$$

But $P_0V_0 = 6RT_0$ (as n = 5 + 1 = 6)

$$\therefore\ W = \frac{-13\left(6RT_0\right)}{6} = -13RT_0 \quad \therefore\ |W| = 13\ RT_0$$

Option (a) is correct.

$$\overline{K.E} = n(C_v)_T$$

where $T = \frac{PVT_0}{P_0V_0} = \frac{9.2P_0\left(V_0/4\right)T_0}{P_0V_0} = 2.3T_0$

$$= 6\times\left[\frac{5\times\frac{3R}{2}+1\times\frac{5R}{2}}{5+1}\right]\times 2.3T_0 = 23T_0$$

Option (c) is incorrect.

7. **(b, d)** **Case - I**

$$H_1 = \frac{H}{1.5} = \frac{30}{1.5} = 20\text{cm}$$

Case - II

$$\frac{-n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$\therefore\ \frac{-1.5}{-30} + \frac{1}{V} = \frac{1-1.5}{-300}$$

$$\therefore\ \frac{1}{V} = \frac{1}{600} - \frac{1}{20}$$

$$\therefore\ V = -20.68\text{ cm}$$

$$\therefore\ H_2 = 20.68\text{ cm}$$

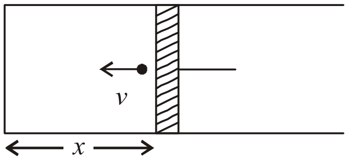
Case - III

$$\therefore\ \frac{-1.5}{-30} + \frac{1}{V} = \frac{1-1.5}{300}$$

$$\therefore\ \frac{1}{V} = -\frac{1}{600} - \frac{1}{20} \Rightarrow V = -19.35\text{cm}$$

$$\therefore\ H_3 = 19.35\text{ cm}$$

8. **(a,d)** When the small particle moving with velocity v_0 undergoes an elastic collision with the heavy movable that piston moving with velocity V, it acquires a new velocity v_0 + 2v. Therefore the increase in velocity after every collision is 2V. Option (d) is correct.



Time period of collision when the piston is at a distance 'x' from the closed end is

$$T = \frac{\text{distance}}{\text{speed}} = \frac{2x}{v}$$

Where v is the speed of the particle at that time.

$\therefore$ rate at which the particle strikes the piston = $\frac{v}{2x}$

when x = L, the frequency = $\frac{v}{2L}$

Option (c) is incorrect.

The rate of change of speed of the particle

$$= \frac{dv}{dt} = (\text{frequency}) \times 2v$$

$$\therefore\quad dv = \frac{v}{2x}2vdt \ \therefore\quad \frac{dv}{v} = \frac{Vdt}{x} = \frac{-dx}{x}$$

Where dx is the distance travelled by the piston in time dt. The minus sign indicates decrease in 'x' with time.

Therefore option (b) is incorrect.

$$\therefore\quad \int_{V_0}^{V}\frac{dv}{v} = -\int_{L_0}^{x}\frac{dx}{x}$$

$$\therefore\quad \ell n\frac{v}{v_0} = -\ \ell n\frac{x}{L_0} \quad \text{or} \quad |v| = \frac{v_0L_0}{x}$$

when $x = \frac{L_0}{2}$ we have $|V| = \frac{V_0L_0}{L_0/2} = 2V_0$

$$\therefore\ K.E_{L_{0/2}} = \frac{1}{2}m(2V_0)^2$$

when x = L_0 we have |V| = v_0

$$\therefore\ K.E_{L_0} = \frac{1}{2}mV_0^2 \qquad \therefore\ \frac{K.E_{L_{0/2}}}{K.E_{L_0}} = 4$$

$\therefore$ Option (a) is correct.

9. (135.00) $^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$

By momentum conservation

$$\sqrt{2m_\alpha K.E_\alpha} = \sqrt{2m_{Rn} K.E_{Rn}}$$

$$\therefore\ K.E_{Rn} = \frac{4}{222} \times 4.44 = 0.08 \text{ Mev}$$

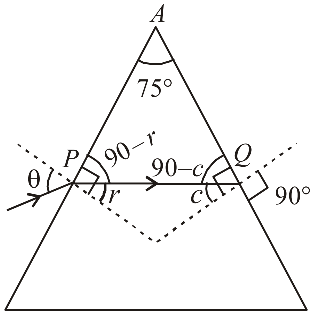
Energy of γ-photon = $[\Delta m \times 931] - (K.E_\alpha + K.E_{Rn})$
= [226.005 – (222 + 4)] × 931 – (4.44 + 0.08) MeV
= 135 KeV

10. (0.63) Here $i = \frac{E}{R}(1 - e^{-\frac{Rt}{L}}) = \frac{Blv}{R}[1 - e^{-\frac{Rt}{L}}]$

$$= \frac{1 \times 0.1 \times 10^{-2}}{1}\left[1 - e^{\frac{-1\times10^{-3}}{10^{-3}}}\right]$$

$= 10^{-3}\,[1 - e^{-1}] = 10^{-3}\,[1 - 0.37] = 0.63 \times 10^{-3}$

11. (1.50) In ΔAPQ
90 × r + 90 – C + 75 = 180
$\therefore$ r + C = 75 `....(i)
Applying snell's law at *Q* we have



$\sqrt{3}$ sin C = n sin 90°

$\sqrt{3}$ sin C = n ....(ii)

Applying snell's law at P we have

1 × sinθ = $\sqrt{3}$ sin r = $\sqrt{3}$ sin (75 – C) From (i)

For θ = 60°

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin(75 - C) \qquad \therefore C = 45°$$

From (ii) $n = \sqrt{3}\sin 45° = \frac{\sqrt{3}}{\sqrt{2}}$

$\therefore n^2 = 1.50$

12. (4.00) $V_1 = \frac{R}{T} = \frac{u_0^2 \sin 2\theta \times g}{g \times 2u_0 \sin\theta} = u_0\cos\theta$ (i)

Now

$$\Sigma R = \frac{u_0^2}{g}(\sin 2\theta)\left[1 + \frac{1}{\alpha^2} + \frac{1}{\alpha^4} + ...\right]$$

$$= \frac{2u_0^2}{g}\sin\theta\cos\theta\left[\frac{1}{1 - \frac{1}{\alpha^2}}\right]$$

$$\Sigma T = \frac{2u_0\sin\theta}{g}\left[1 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + ...\right] = \frac{2u_0\sin\theta}{g}\left[\frac{1}{1 - \frac{1}{\alpha}}\right]$$

$$\therefore 0.8V_1 = \frac{\Sigma R}{\Sigma T}$$

$$\therefore 0.8(u_0\cos\theta) = (u_0\cos\theta)\frac{\left[\frac{\alpha^2}{\alpha^2 - 1}\right]}{\left[\frac{\alpha}{\alpha - 1}\right]}$$

$$\therefore 0.8 = \frac{\alpha^2}{\alpha^2 - 1} \times \frac{\alpha - 1}{\alpha} = \frac{\alpha}{\alpha + 1} \quad \therefore 0.8\alpha + 0.8 = \alpha$$

$\therefore \alpha = 4.00$

13. (1) Change in momentum of photon = change in momentum of mirror

$$2\left[N\left(\frac{h}{\lambda}\right)\right] = MV_{max}$$

$$\therefore\ 2\frac{Nh}{\lambda} = M(A\Omega) \qquad [\because V_{max} = Aw]$$

$$N = \left(\frac{M\Omega}{h}\right)\frac{A\lambda}{2} = \frac{10^{24}}{4\pi} \times \frac{10^{-6} \times 8\pi \times 10^{-6}}{2}$$

$= 1 \times 10^{12}$

14. (1.39%)

u ± Δu = (75 – 45) ± (0.25 + 0.25) = (30 ± 0.5) cm

v ± Δv = (135 – 75) ± (0.25 + 0.25) = (60 ± 0.5) cm

We know that

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore\ +\frac{dv}{v^2} + \frac{du}{u^2} = \frac{df}{f^2} \qquad ....(i)$$

Now $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{60} - \frac{1}{-30} = \frac{1}{f}$

$\therefore\ \frac{1}{f} = \frac{1}{20} \quad \therefore\ f = 20\text{cm}$

Substituting the values in (1)

$$\frac{0.5}{(60)^2} + \frac{0.5}{(30)^2} = \frac{df}{(20)^2} \quad \therefore\ df = 0.277$$

$$\therefore\ \frac{df}{f} \times 100 = \frac{0.277}{20} \times 100 = 1.388\% = 1.39\%$$

15. **(b)** We know that $\nu = \frac{p}{2\ell}\sqrt{\frac{T}{m}} \Rightarrow T = \frac{\nu^2 \ell^2 m}{p^2}$

Case 1 $T_0 = \frac{f_0^2 4L_0^2 \mu}{\ell^2}$

Case 2 $T_2 = \frac{f_0^2 4\left(\frac{3}{2}\right)^2 L_0^2 (2\mu)}{(3)^2} = \frac{T_0}{2}$

Case 3 $T_3 = \frac{f_0^2 4\left(\frac{5}{2}\right)^2 L_0^2 (3\mu)}{5^2} = \frac{3}{16}T_0$

Case 4 $T_4 = \frac{f_0^2 4\left(\frac{7}{4}\right)^2 L_0^2 (4\mu)}{(14)^2} = \frac{T_0}{16}$

∴ Correct option is I-P, II-Q, III-T, IV-U

16. **(c)** We know that $\nu = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ for first mode of vibration

For 'ν' to be maximum, 'ℓ' should be minimum.

Case (i) $f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

Case (ii) $f_2 = \frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu}} = \frac{f_0}{\sqrt{2}}$

Case (iii) $f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{\sqrt{3}}$

Case (iv) $f_4 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{2}$

The correct option is I - P, II - R, III - S, IV - Q

17. **(c)** **I.** $W_{1\to2\to3} = W_{1\to2} + W_{2-3}$

$1R\frac{T_0}{3}\ell n\frac{2V_0}{V_0} + \text{zero}$

[∵ n = 1, and $W_{2\to3}$ there is no change in volume]

∴ I – P

II. $\Delta U_{1\to2\to3} = \Delta U_{1\to2} + \Delta U_{2\to3}$

$= O + nC_V\Delta T = n\frac{f}{2}R\Delta T$

[There is no change in temperature from 1 → 2]

$= 1\times\frac{3}{2}R\left(T_0 - \frac{T_0}{3}\right) = RT_0$

[For monoatomic gas f = 3]

∴ II – R

III. $Q_{1\to2\to3} = Q_{1\to2} + Q_{2\to3} = W_{1\to3} + \Delta U_{2\to3}$

$= \frac{RT_0}{3}\ell n_2 + RT_0 = \frac{RT_0}{3}[\ell n_2 + 3]$

∴ III – T

IV. $Q_1\rightarrow_2 = W_{1\to2}$ [∵ $\Delta U_{1\to2} = 0$]

$= \frac{RT_0}{3}\ell n_2$

IV – P

18. **(a)** **I.** $W_{1\to2\to3} = W_{1\to2} + W_{2\to3}$

[$W_{2\to3} = 0$ as there is no change in volume]

$= P_0 \times V_0 + O$

$= P_0V_0 = \frac{RT_0}{3}$ [∵ $P_0V_0 = \frac{1}{3}RT_0$ given]

∴ I → Q

II. $\Delta U_{1\to2\to3} = \Delta U_{1\to2} + \Delta U_{2\to3}$

$= nC_v\Delta T_{1\to2} + nC_v\Delta T_{2\to3}$

$= 1\times\frac{3R}{2}(T_f - T_i)_{1\to2} + 1\times\frac{3R}{2}[T_f - T_i]_{2\to3}$

$= \frac{3}{2}[2P_0V_0 - P_0V_0] + \frac{3}{2}\left[\frac{3P_0}{2}\times2V_0 - P_0\times2V_0\right]$

$= 3P_0V_0 = RT_0$

II → R

III. $Q_{1\to2\to3} = Q_{1\to2} + Q_{2\to3}$

$= nC_p\Delta T_{1\to2} + nC_v\Delta T_{2\to3}$

$= \frac{5}{2}P_0V_0 + 1\times\frac{3}{2}\left[\frac{3P_0}{2}\times2V_0 - P_2(2V_0)\right]$

$= \frac{8}{2}P_0V_0 = \frac{8}{2}\times\frac{RT_0}{3} = \frac{4}{3}RT_0$

III – S

IV. $Q_{1\to2} = nC_p\Delta T_{1\to2} = nCp(T_f - T_i)$

$= 1\times\frac{5}{2}R\left[\frac{P_0(2V_0)}{R} - \frac{P_0V_0}{R}\right]$

[∵ PV = nRT]

$= \frac{5}{2}P_0V_0 = \frac{5}{2}\left(\frac{RT_0}{3}\right) = \frac{5RT_0}{6}$

∴ IV – U

Chemistry

19. (b, c, d) Aqua regia is a mixture of conc. HCl and conc. HNO_3 in 3:1 ratio.

When gold dissolves in aqua regia, the species formed is $AuCl_4^-$ in which gold is in +3 oxidation state.

In the absence of air the reaction between gold and aquaregia

$$Au + HNO_3 + 4HCl \rightarrow H^+ + \left[AuCl_4\right]^- + 2H_2O + NO$$

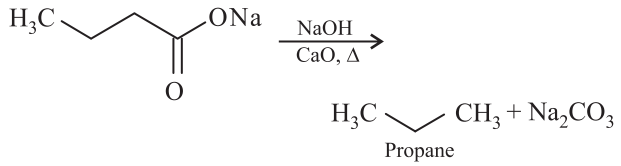
NO_2 will not be produced. Yellow colour is due to its decomposition into NOCl and Cl_2.

20. (a, b, c) Gold extraction:

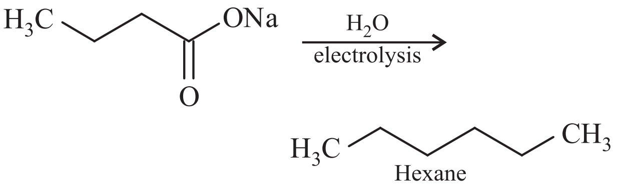
$$4Au + 8NaCN + 2H_2O + \underset{(Q)}{O_2} \rightarrow 4Na\left[\underset{(R)}{Au(CN)_2}\right] + 4NaOH$$

$$4Na\left[\underset{(R)}{Au(CN)_2}\right] + \underset{(T)}{2Zn} \rightarrow 2Na_2\left[\underset{(Z)}{Zn(CN)_4}\right] + 4Au$$

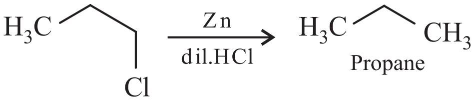
21. (a, c)



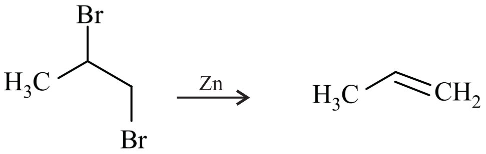
This process of elimination of carbon dioxide from a carboxylic acid is known as decarboxylation.



This is Kolbe's electrolytic method.



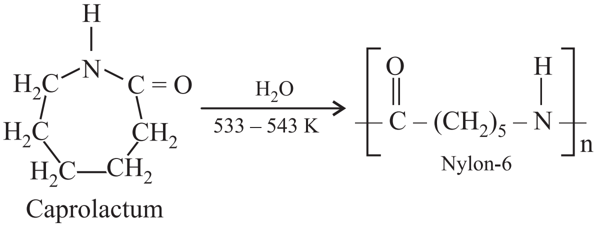
Alkyl halides (except fluorides) on reduction with zinc and dilute hydrochloric acid give alkanes.



This is a dehalogenation reaction of vicinal dihalide. In this reaction, an alkene is produced on treatment of dihalide with Zn metal by loosing a molecule of ZnX_2.

22. (a, c)

Nylon-6 is obtained by heating caprolactum with water at high temperature. It has amide linkages.



Cellulose has only β-D-glucose units that are joined by glycosidic linkages between C–1 of one glucose unit and C–4 of the next glucose unit

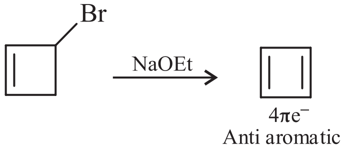
Teflon is prepared by heating tetrafluoromethane in presence of a persulphate catalyst at high pressure.

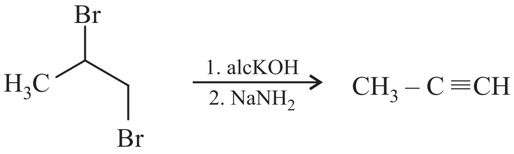
$$CF_2 = CF_2 = \xrightarrow[\text{High pressure}]{\text{Catalyst}} \left[CF_2 - CF_2 \right]_n$$

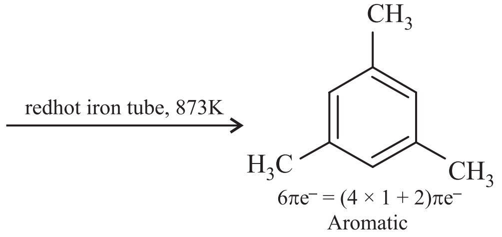
Tetrafluoroe thene Teflon

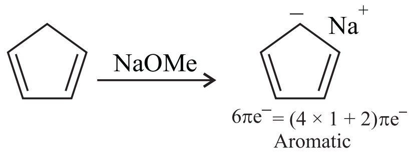
Natural rubber is a linear polymer of isoprene (2-methyl-1, 3-butadine) containing cis alkene units. It is also called cis - 1, 4 - polyisoprene.

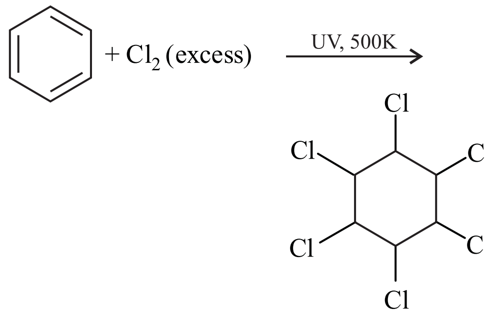
23. (b, c)











Non aromatic

24. **(a, c)** Given, azimuthal quantum no. (l) = 2 (d-subshell)

Magnetic quantum no.(m) = 0(dz_2)

$$E = -13.6\frac{z^2}{n^2} = -13.6 \times \frac{2^2}{n^2} = -3.4$$

$$13.6 \times \frac{2^2}{n^2} = 3.4$$

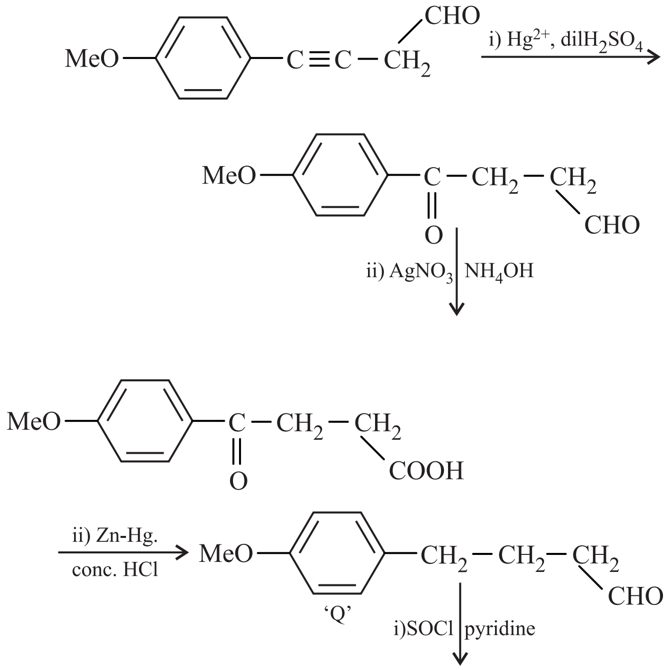
$n^2 = 4^2 \Rightarrow n = 4$

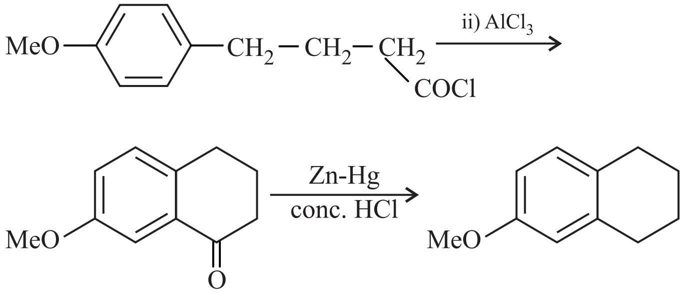
Radial node = n – l – 1 = 4 – 2 – 1 = 1

Angular node = l = 2

Wave function corresponds to $\psi_{4,2,0}$. It represents $4d_{z^2}$ -orbital which has only one radial nodes and two angular nodes. It experiences nuclear charge of 2e units.

25. **(a, d)**





26. **(b, c, d)**

$$\mathrm{Zn} + \underset{\text{Hot \& Conc.}}{\mathrm{H_2SO_4}} \rightarrow \mathrm{G + R + X}$$

$$\mathrm{Zn} + \mathrm{H_2SO_4} \rightarrow \underset{(G)}{\mathrm{ZnSO_4}} + \underset{(R)}{\mathrm{SO_2}} + \underset{(X)}{\mathrm{2H_2O}}$$

$$\mathrm{Zn} + \mathrm{Conc.\,NaOH} \rightarrow \underset{(T)}{\mathrm{Na_2\,ZnO_2}} + \underset{(Q)}{\mathrm{H_2}}$$

$$\underset{(G)}{\mathrm{ZnSO_4}} + \mathrm{H_2S} + \mathrm{NH_4OH} \rightarrow \underset{(Z)}{\mathrm{ZnS}} + \underset{(Y)}{\mathrm{(NH_4)_2SO_4}} + \underset{(X)}{\mathrm{H_2O}}$$

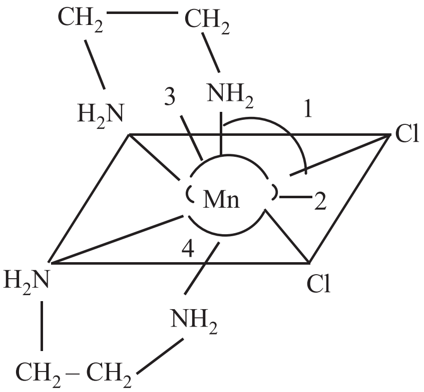
(a) Oxidation state of Zn in Na_2ZnO_2 is +2

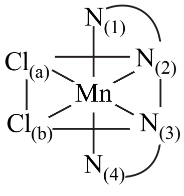
(b) Bond order of Q is one for H_2.

(c) ZnS is white in colour

(d) SO_2 is angular in shape

27. **(6)**





cis Bond angles are:

$Cl_{(a)} - Mn - N_{(1)}$

$Cl_{(a)} - Mn - N_{(2)}$

$Cl_{(a)} - Mn - N_{(4)}$

$Cl_{(b)} - Mn - N_{(1)}$

$Cl_{(b)} - Mn - N_{(3)}$

$Cl_{(b)} - Mn - N_{(4)}$

28. **(288)** Concentrated nitric acid oxidises sulphur to sulphuric acid (+6 oxidation state).

$$\underset{\text{Rhombic sulphur}}{S_8} + 48HNO_3 \rightarrow 8H_2SO_4 + 48NO_2 + 16H_2O$$

1 mole S_8 produces 16 × 18 g of H_2O i.e. 288 g of H_2O.

29. (2.30) $2N_2O_5(g) \xrightarrow{\Delta} 2N_2O_4(g) + O_2(g)$

Given rate constant of the reaction = 5 × $10^{-4}$$sec^{-1}$

$$\frac{1}{2}\frac{dP_{N_2O_5}}{dt} = K_{overall}P_{N_2O_5}$$

$$\Rightarrow \frac{dP_{N_2O_5}}{dt} = 2K_{overall}P_{N_2O_5}$$

$$2N_2O_5(g) \rightarrow 2N_2O_4(g) + O_2(g)$$

$$1 + \frac{x}{2} = 1.45$$

$x = 0.90$ atm

For a first order reaction,

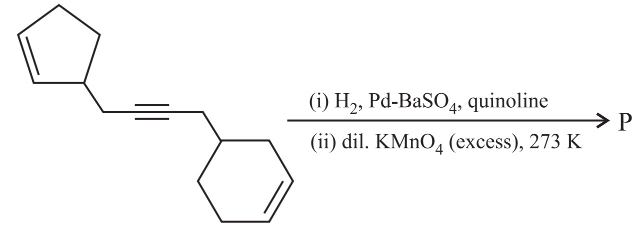
$$t = \frac{2.303}{K}\log\frac{[P]_o}{[P]}$$

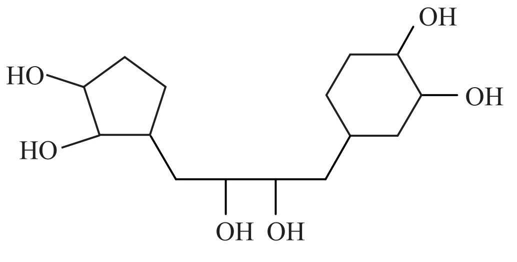
$$t = \frac{2.303}{2K}\log\frac{[P]_o}{[P]}$$

$$y \times 10^3 = \frac{2.303}{2 \times 5 \times 10^{-4}}\log\frac{1}{0.1}$$

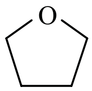
$$y = \frac{23.03}{10} = 2.30$$

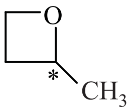
30. (6)

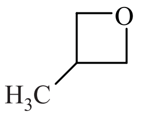


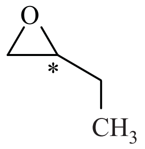


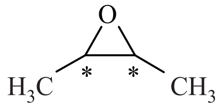
31. (10)

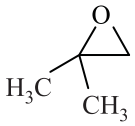












32. (2.98)

$X_{urea} = 0.05$

Mass of water = 900 g

Number of moles of water = $n_{H_2O} = \frac{900}{18} = 50$

Let moles of urea = n_U

$$\frac{n_U}{n_U + 50} = 0.05$$

$$n_U = \frac{50}{19} = 2.63$$

Mass of urea = $2.63 \times 60 = 157.8$

Given density (d) of solution = 1.2 g/mL.

$$V_{solution} = \frac{\text{mass of solution}}{\text{density (g/cc)}} = \frac{900 + 157.8}{1.2} = 881.5 \text{ mL}$$

$$M = \frac{n_V}{V_{mL}} \times 1000 = \frac{2.63}{881.5} \times 1000 = 2.98$$

33. (d) $r \propto \frac{n^2}{Z}$ or $r = 0.529 \times \frac{n^2}{Z}$

$$|L| \propto n \ or \ mvr = \frac{nh}{2\pi}$$

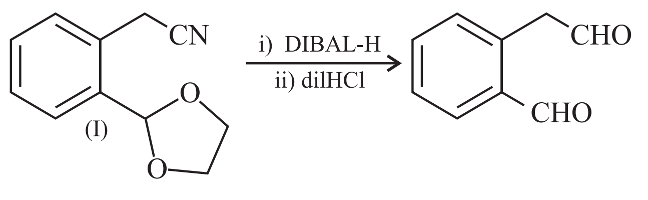
$$KE \propto \frac{Z^2}{n^2} \quad or \quad KE = +13.6\frac{Z^2}{n^2}$$

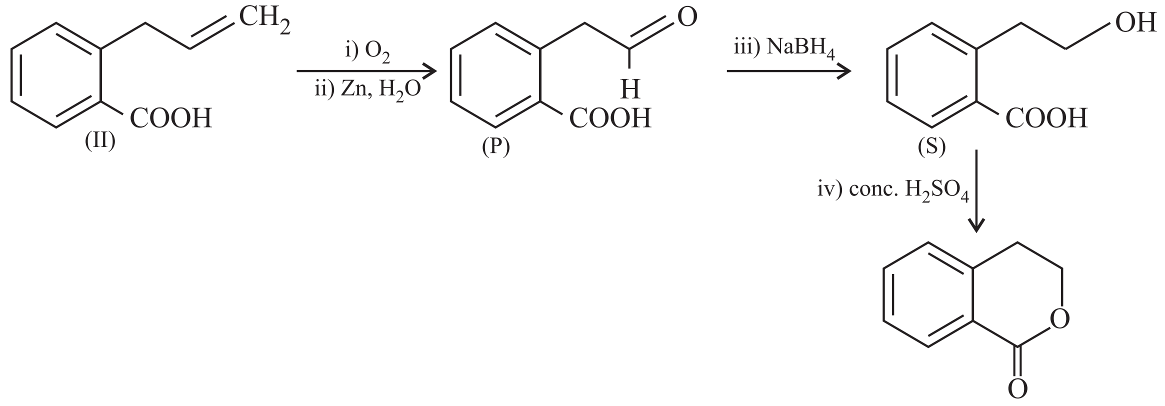
$$PE \propto \frac{-Z^2}{n^2} \quad or \quad PE = -2 \times 13.6 \times \frac{Z^2}{n^2}$$

34. (c) $r\alpha\frac{n^2}{Z}$; $|L|\alpha\ n$

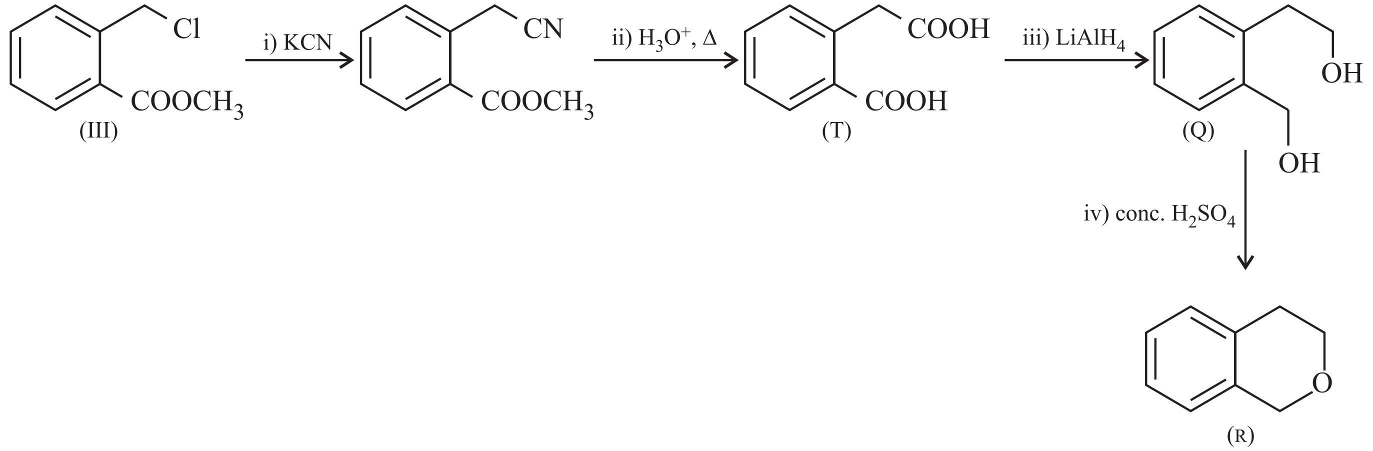
$$KE\ \alpha\ \frac{Z^2}{n^2};\ PE\ \alpha\ \frac{-Z^2}{n^2}$$

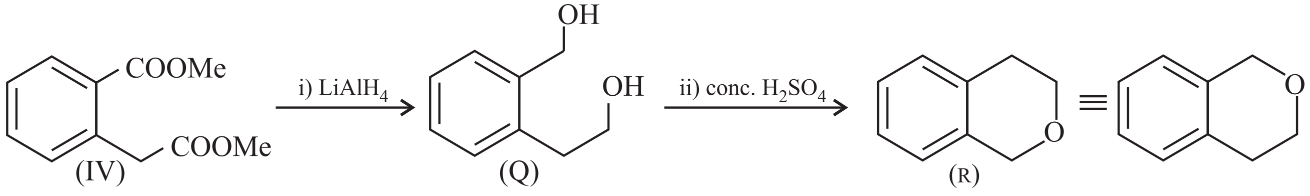
35. (b)


CN
(I)
O
O
i) DIBAL-H
ii) dilHCl
CHO
CHO
NaBH$_4$
OH
(Q)
OH
iii) conc. H$_2$SO$_4$
O
(R)


CH$_2$
COOH
(II)
i) O$_2$
ii) Zn, H$_2$O
O
H
COOH
(P)
iii) NaBH$_4$
OH
COOH
(S)
iv) conc. H$_2$SO$_4$
O
O

36. (a)


Cl
COOCH$_3$
(III)
i) KCN
CN
COOCH$_3$
ii) H$_3$O$^+$, Δ
COOH
COOH
(T)
iii) LiAlH$_4$
OH
(Q)
OH
iv) conc. H$_2$SO$_4$
O
(R)


COOMe
(IV)
COOMe
i) LiAlH$_4$
OH
OH
(Q)
ii) conc. H$_2$SO$_4$
O
(R)
≡
O

Mathematics

37. **(a, b, d)** $F(x) = (x-1)(x-2)(x-5)$

$$F(x) = \int_0^x f(t)\,dt, x > 0$$

$$F'(x) = f(x) = (x-1)(x-2)(x-5)$$

$$F'(x) = 0 \Rightarrow x = 1, 2, 5$$

$$F''(x) = (x-2)(x-5) + (x-1)(x-5) + (x-1)(x-2)$$

$F''(1) = +$ ve, $F''(2) = -$ve, $F''(5) = +$ve

$\therefore$ F(x) has local minima at x = 1 and x = 5 and local maxima at x = 2.

Also

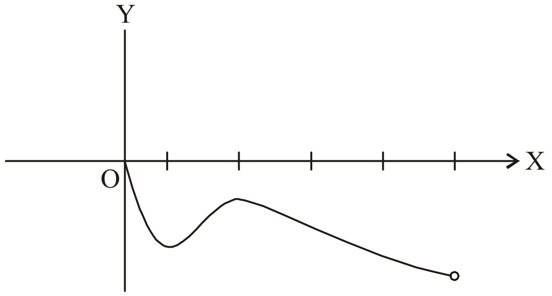
$$F(x) = \int_0^x (t-1)(t-2)(t-5)dt = \int_0^x (t^3 - 8t^2 + 17t - 10)dt$$

$$= \frac{x^4}{4} - \frac{8x^3}{3} + \frac{17x^2}{2} - 10x$$

$$= \frac{x}{12}(3x^3 - 32x^2 + 102x - 120)$$

F(1), F(2), F(5) < 0

$\therefore$ approximate graph of F(x) for $x \in (0, 5)$ is



$\therefore$ $F(x) < 0$, for all $-x \in (0,5)$

Hence options (a), (b) and (d) are correct but (c) is incorrect.

38. **(a,b, d)** We observe that $P_1' = P_1, P_2' = P_2, P_3' = P_3$,

$P_4' = P_5, P_5' = P_4, P_6' = P_6$,

Also $P_k P_k' = I$ for $k = 1$ to 6.

Now

$$X = \sum_{k=1}^{6} P_k A P_k' \quad \text{where } A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} \text{ and } A' = A$$

$$X' = \left(\sum_{k=1}^{6} P_k A P_k'\right) = \sum_{k=1}^{6} (P_k A P_k')'$$

$$= \sum_{k=1}^{6} (P'_k)' A' P_x' = \sum_{k=1}^{6} P_k A P_k' = X$$

$\therefore$ X is a symmetric matrix.

Option (a) is correct.

Sum of diagonals entries of X = Trace X

$$= \text{Trace} \sum_{k=1}^{6} (P_k A P_k') = \sum_{k=1}^{6} \text{Trace } (P_k A P_k')$$

$$= \sum_{k=1}^{6} \text{Trace}(A P_k P_k')(\text{using Trace AB} = \text{Trace BA})$$

$$= \sum_{k=1}^{6} \text{Trace (AI)} = \sum_{k=1}^{6} \text{Trace} A = 6 \times \text{Trace} A$$

$= 6 \times (2 + 0 + 1) = 18$

$\therefore$ option (b) is correct

$$X\begin{bmatrix}1\\1\\1\end{bmatrix} = \sum_{k=1}^{6} P_k A \left(P_k' \begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \sum_{k=1}^{6} P_k A \begin{bmatrix}1\\1\\1\end{bmatrix} = \sum_{k=1}^{6} P_k \begin{bmatrix}6\\3\\6\end{bmatrix}$$

$$= \begin{bmatrix}2 & 2 & 2\\2 & 2 & 2\\2 & 2 & 2\end{bmatrix}\begin{bmatrix}6\\3\\6\end{bmatrix} = \begin{bmatrix}30\\30\\30\end{bmatrix} = 30\begin{bmatrix}1\\1\\1\end{bmatrix} = \alpha\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$\therefore \alpha = 30$

Hence option (d) is correct.

$$\text{Also } X\begin{bmatrix}1\\1\\1\end{bmatrix} = 30\begin{bmatrix}1\\1\\1\end{bmatrix} \quad \Rightarrow (X - 30\,I)\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$\Rightarrow X - 30\,I = 0 \Rightarrow |X - 30\,I| = 0$

$\Rightarrow X - 30\,I$ is not invertible

$\therefore$ option (c) is incorrcet.

39. **(b, c, d)** $$f(n) = \frac{\sum_{k=0}^{n} 2\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n} 2\sin^2\left(\frac{k+1}{n+2}\right)\pi}$$

where n is non negative integer

$$= \frac{\sum_{k=0}^{n}\left[\cos\left(\frac{\pi}{n+2}\right) - \cos\frac{(2k+3)\pi}{n+2}\right]}{\sum_{k=0}^{n}\left[1 - \cos\frac{2(k+1)\pi}{n+2}\right]}$$

$$= \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right) - \left[\cos\frac{3\pi}{n+2} + \cos\frac{5\pi}{n+2} + \ldots\ldots\ldots + \cos\frac{(2n+3)\pi}{n+2}\right]}{n+1 - \left[\cos\frac{2\pi}{n+2} + \cos\frac{4\pi}{n+2} + \ldots\ldots\ldots + \cos\frac{2(n+1)\pi}{n+2}\right]}$$

$$= \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right) - \frac{\frac{\sin(n+1)\pi}{n+2}}{\sin\left(\frac{\pi}{n+2}\right)}.\cos\frac{(2n+6)\pi}{2(n+2)}}{n+1-\frac{\frac{\sin(n+1)\pi}{n+2}}{\sin\left(\frac{\pi}{n+2}\right)}.\cos\frac{(2n+4)\pi}{2(n+2)}}$$

$$= \frac{(n+1)\cos\frac{\pi}{n+2}+\cos\frac{\pi}{n+2}}{n+1+1} = \frac{(n+2)\cos\left(\frac{\pi}{n+2}\right)}{n+2}$$

$$\therefore f(n) = \cos\left(\frac{\pi}{n+2}\right)$$

$$\lim_{n\to\infty} f(n) = \lim_{n\to\infty} \cos\left(\frac{\pi}{n+2}\right) = 1$$

$\therefore$ option (a) is incorrect.

$$f(4) = \cos\left(\frac{\pi}{4+2}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$\therefore$ option (b) is correct

It $\alpha = \tan\left(\cos^{-1} f(6)\right)$

$$= \tan\left(\cos^{-1}\left(\cos\frac{\pi}{8}\right)\right) = \tan\frac{\pi}{8}$$

$$\tan\frac{\pi}{4} = 1 \Rightarrow \frac{2\tan\frac{\pi}{8}}{1-\tan^2\pi/8} = 1$$

$$\Rightarrow \frac{2\alpha}{1-\alpha^2} = 1 \text{ or } \alpha^2 + 2\alpha - 1 = 0$$

$\therefore$ option (c) is correct

$$\sin\left(7\cos^{-1} f(5)\right) = \sin\left(7\cos^{-1}\left(\cos\frac{\pi}{7}\right)\right) = \sin\left(7\times\frac{\pi}{7}\right)$$

$= \sin\pi = 0$

$\therefore$ option (d) is correct.

40. (a, b, c) $f(x) = \frac{\sin\pi x}{x^2}, x > 0$

For points of local max/min, f' (x) = 0

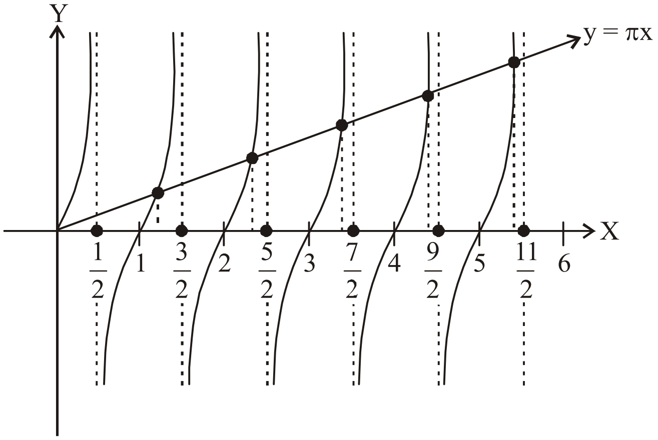
$$\Rightarrow \frac{\pi x^2\cos\pi x - 2x\sin\pi x}{x^4} = 0$$

$$\Rightarrow \frac{x\pi\cos\pi x - 2\sin\pi x}{x^3} = 0$$

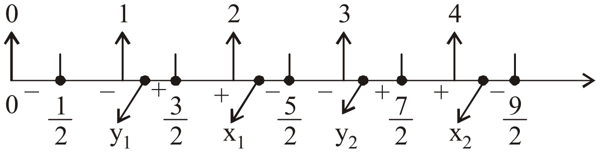
$$\Rightarrow \frac{\cos\pi x\left(\pi x - 2\tan\pi x\right)}{x^3} = 0$$

$$\Rightarrow \cos\pi x = 0 \Rightarrow x = \frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2},\ldots$$

and πx– 2tanπx = 0 which can be solved by drawing the graphs of y = πx and y = 2 tanπx, as follows



Plotting the stationary points on number line and finding the sign of f'(x) in different intervals we observe



i.e. $x_{n+1} - x_n > 2$ for every n

$x_n \in \left(2n, 2n+\frac{1}{2}\right)$ for every n

$|x_n - y_n| > 1$ for every n

$x_1 > y_1$

Hence options a, b, c are correct, but option d is incorrect.

41. (a, d) We can choose

$P(\lambda, 0, 0)$ on L_1

$Q(0, \mu, 1)$ on L_2

$R(1, 1, v)$ on L_3

If P, Q, R are collinear, $\overrightarrow{PQ} \parallel \overrightarrow{PR}$

$$\Rightarrow \frac{\lambda}{\lambda-1} = \frac{+\mu}{+1} = \frac{+1}{+v} \Rightarrow \mu = \frac{\lambda}{\lambda-1}, v = \frac{\lambda-1}{\lambda}$$

Clearly $\lambda \neq 0, 1$

$$\theta\left(0, \frac{\lambda}{\lambda-1}, 1\right)$$

(a) For $Q = \hat{k} - \frac{1}{2}\hat{j}$

$$\frac{\lambda}{\lambda - 1} = -\frac{1}{2} \Rightarrow 3\lambda = +1 \text{, which is possible.}$$

(b) For $Q = \hat{k}$

$$\frac{\lambda}{\lambda - 1} = 0 \Rightarrow \lambda = 0 \text{, not possible}$$

(c) For $Q = \hat{k} + \hat{j}$

$$\frac{\lambda}{\lambda - 1} = 1 \Rightarrow \lambda = \lambda - 1, \text{ not possible}$$

(d) For $Q = \hat{k} + \frac{1}{2}\hat{j}$

$$\frac{\lambda}{\lambda - 1} = \frac{1}{2} \Rightarrow 2\lambda = \lambda - 1 \Rightarrow \lambda = -1,$$

which is possible

Hence options (a) and (d) are correct and options (b) and (c) are incorrect.

42. (a, d)

$$\lim_{n\to\infty} \frac{\sqrt[3]{1} + \sqrt[3]{2} + \ldots + \sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2} + \frac{1}{(an+2)^2} + \ldots + \frac{1}{(an+n)^2}\right)} = 54$$

$$\Rightarrow \frac{\lim\limits_{n\to\infty} \frac{1}{n}\sum\limits_{r=1}^{n}\left(\frac{r}{n}\right)^{1/3}}{\lim\limits_{n\to\infty} \frac{1}{n}\sum\limits_{r=1}^{n}\frac{1}{\left(a + \frac{r}{n}\right)^2}} = 54$$

$$\Rightarrow \frac{\int_0^1 x^{1/3}dx}{\int_0^1 \frac{1}{(a+x)^2}dx} = 54$$

$$\Rightarrow \frac{\left(\frac{3x^{4/3}}{4}\right)_0^1}{\left(\frac{-1}{a+x}\right)_0^1} = 54 \Rightarrow \frac{\frac{3}{4}}{\frac{1}{a} - \frac{1}{a+1}} = 54$$

$$\Rightarrow \frac{3}{4} \times \frac{a(a+1)}{1} = 54 \quad \Rightarrow a^2 + a - 72 = 0$$

$\Rightarrow (a+9)(a-8) = 0 \quad \Rightarrow a = 8 \text{ or } -9$

$\therefore$ options (a) and (d) are correct.

43. (a, d) $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}; Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix}$

$R = PQP^{-1}$

$\Rightarrow \det R = \det(PQP^{-1}) = |P|\,|Q|\,|P^{-1}| = |Q| = 4(12 - x^2)$

$$\text{Also, } \det\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8 = 4(10 - x^2) + 8 = 4(12 - x^2)$$

$$\therefore \det R = \det\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8 \ \forall x \in R$$

$\therefore$ option (a) is correct

For x = 1, $\det R = 4(12 - 1) \neq 0$

$$\therefore R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ will have only trivial solution}$$

i.e., $\alpha = 0\ \beta = 0, \gamma = 0$

$\therefore$ option (b) is incorrect

For PQ = QP

a_{11} in PQ = a_{11} in QP

$\Rightarrow 2 + x = 2 \Rightarrow x = 0$

a_{12} in PQ = 2x + 4 and a_{12} in QP = 2 + 2x

$\Rightarrow 2x + 4 = 2 + 2x \Rightarrow 4 = 2$ not possible

$\therefore$ PQ = QP is not possible for any real x.

$\therefore$ option (c) is incorrect.

For x = 0,

$$R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}\begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & -1/3 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 0 & 8 & 12 \\ 0 & 0 & 18 \end{bmatrix}\begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & -1/3 \\ 0 & 0 & 1/3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2/3 \\ 0 & 4 & 4/3 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\text{Now } R\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 & 2/3 \\ 0 & 4 & 4/3 \\ 0 & 0 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = \begin{bmatrix} 6 \\ 6a \\ 6b \end{bmatrix}$$

$$\Rightarrow 2 + a + \frac{2}{3}b = 6 \Rightarrow 3a + 2b = 12$$

$$4a + \frac{4}{3}b = 6a \Rightarrow 3a - 2b = 0$$

Solving a = 2, b = 3 ⇒ a + b = 5

$\therefore$ option (d) is correct

44. **(a,c)** Let's check each option for the given properties.

(a) f(x) = $x^{2/3}$ for PROPERTY 1 $\lim\limits_{h\to 0}\frac{f(h)-f(0)}{\sqrt{|h|}}$ exists and is finite.

Then $\lim\limits_{h\to 0}\frac{h^{2/3}-0}{\sqrt{|h|}}=\lim\limits_{h\to 0}\frac{|h|^{2/3}}{|h|^{1/2}}=\lim\limits_{h\to 0}|h|^{1/6}=0$

∴ option (a) is correct.

(b) f(x) = sin x for PROPERTY 2

$$\lim_{h\to 0}\frac{\sin h-\sin 0}{h^2}=\lim_{h\to 0}\frac{\sin h}{h}\times\frac{1}{h}$$

when does not exist.

∴ (b) is incorrect option.

(c) f(x) = |x| for PROPERTY 1

$$\lim_{h\to 0}\frac{|h|-0}{\sqrt{|h|}}=\lim_{h\to 0}\sqrt{|h|}=0$$

∴ option (c) is correct

(d) f(x) = x |x| for PROPERTY 2

$$\lim_{h\to 0}\frac{h|h|-0}{h^2}=\lim_{h\to 0}\frac{|h|}{h}$$

LHL = –1 and RHL = 1

∴ $\lim\limits_{h\to 0}\frac{|h|}{h}$ does not exist

∴ option (d) is incorrect.

45. **(6.20)** Here $\sum\limits_{k=0}^{n}k=\frac{n(n+1)}{2}$

$$\sum_{k=0}^{n}{}^{n}C_k k^2=\sum_{k=1}^{n}\frac{n}{k}.{}^{n-1}C_{k-1}.k^2=\sum_{k=1}^{n}n.{}^{n-1}C_{k-1}.k$$

$$=n\sum_{k=1}^{n}{}^{n-1}C_{k-1}(k-1+1)$$

$$=n\left[\sum_{k=2}^{n}\frac{n-1}{k-1}{}^{n-2}C_{k-2}(k-1)+\sum_{k=1}^{n}{}^{n-1}C_{k-1}\right]$$

$$=n(n-1)2^{n-2}+n\times 2^{n-1}$$

$$\sum_{k=0}^{n}{}^{n}C_k\times k=\sum_{k=1}^{n}\frac{n}{k}{}^{n-1}C_{k-1}\times k=n\times 2^{n-1}$$

$$\sum_{k=0}^{n}{}^{n}C_k 3^k=4^n$$

$$\therefore\ \det\begin{bmatrix}\sum\limits_{k=0}^{n}k & \sum\limits_{k=0}^{n}{}^{n}C_k k^2\\ \sum\limits_{k=0}^{n}{}^{n}C_k k & \sum\limits_{k=0}^{n}{}^{n}C_k 3^k\end{bmatrix}=0$$

$$\Rightarrow\begin{vmatrix}\frac{n(n+1)}{2} & n(n-1)2^{n-2}+n\times 2^{n-1}\\ n\times 2^{n-1} & 4^n\end{vmatrix}=0$$

$$\Rightarrow n(n+1)\times 2^{2n-1}-n^2\left[(n-1)2^{2n-3}+2^{2n-2}\right]=0$$

$$\Rightarrow 2^{2n-3}\times n\left[4(n+1)-n[n-1+2]\right]=0$$

$$\Rightarrow 2^{2n-3}\times n\left[4n+4-n^2-n\right]=0$$

$$\Rightarrow n^2-3n-4=0$$

$$\Rightarrow n^2-4n+n-4=0\Rightarrow n=4$$

$$\therefore\sum_{k=0}^{n}\frac{{}^{n}C_k}{k+1}=\sum_{k=0}^{4}\frac{{}^{4}C_k}{k+1}={}^{4}C_0+\frac{{}^{4}C_1}{2}+\frac{{}^{4}C_2}{3}+\frac{{}^{4}C_3}{4}+\frac{{}^{4}C_4}{5}$$

$$=1+2+2+1+\frac{1}{5}=6.20$$

46. **(18)** $\vec{a}=2\hat{i}+\hat{j}-\hat{k},\ \vec{b}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{c}=\alpha\vec{a}+\beta\vec{b}=(2\alpha+\beta)\hat{i}+(\alpha+2\beta)\hat{j}+(-\alpha+\beta)\hat{k}$

$\vec{a}+\vec{b}=3\hat{i}+3\hat{j}$

Projection of $\vec{c}$ on $\vec{a}+\vec{b}=3\sqrt{2}$

$$\Rightarrow\frac{\vec{c}.(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|}=3\sqrt{2}$$

$$\Rightarrow\frac{3(2\alpha+\beta)+3(\alpha+2\beta)}{3\sqrt{2}}=3\sqrt{2}$$

$\Rightarrow\ \alpha+\beta=2$ ....(1)

Using eq^n (1) $\vec{c}=(\alpha+2)\hat{i}+(4-\alpha)\hat{j}+(2-2\alpha)\hat{k}$

Now $\vec{c}$ is in the plane of $\vec{a}$ and $\vec{b}$ $(\because\vec{c}=\alpha\vec{a}+\beta\vec{b})$

$\therefore\ (\vec{a}\times\vec{b}).\vec{c}=0$

Hence $\left(\vec{c}-(\vec{a}\times\vec{b})\right).\vec{c}=\vec{c}.\vec{c}$

$$=(\alpha+2)^2+(4-\alpha)^2+(2-2\alpha)^2=6\left(\alpha^2-2\alpha+4\right)$$

$$=6\left((\alpha-1)^2+3\right)$$

which has minimum value of 18 when $\alpha=1$

47. **(422)** Let $n(A) = a, n(B) = b, n(A \cap B) = c$

Then as per question $1 \le b < a$

Also given that A and B are independent events

$\therefore \quad P(A \cap B) = P(A)\,P(B)$

$$\Rightarrow \quad \frac{n(A \cap B)}{n(S)} = \frac{n(A)}{n(S)} \times \frac{n(B)}{n(S)}$$

$$\Rightarrow \quad \frac{c}{6} = \frac{a}{6} \times \frac{b}{6}$$

$\Rightarrow \quad ab = 6c$

If $a = 6$ then $b = c = 5, 4, 3, 2, 1$ $\quad (\because b < a)$

There is only one way to select all 6 elements of set A. Number of ways of selecting 5, 4, 3, 2 or 1 elements in B and $A \cap B$ are

$^6C_5 + {}^6C_4 + {}^6C_3 + {}^6C_2 + {}^6C_1 = 2^6 - 2 = 62$

If $a = 5$ then $b = \frac{6c}{5}$, which is not possible because if $c = 5$ then $b = 6$, while $b < a$.

If $a = 4$ then $b = \frac{6c}{4} = \frac{3c}{2}$

$\Rightarrow \quad c = 2$ and $b = 3$

2 elements in $A \cap B$ can be selected in $^6C_2$ ways

2 additional elements in A can be selected in $^4C_2$ ways

1 additional element in B can be selected in $^2C_1$ way

$\therefore$ No. of ways for $a = 4, b = 3, c = 2$ are

$^6C_1 \times {}^4C_1 \times {}^2C_1 = 15 \times 6 \times 2 = 180$

If $\quad a = 3$ then $b = 2c \;\Rightarrow\; c = 1, b = 2$

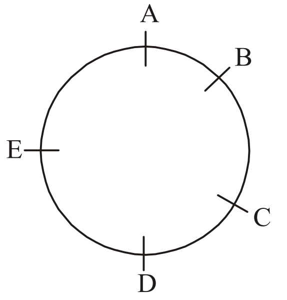
which can be done in $^6C_1 \times {}^5C_1 + {}^4C_2 = 6 \times 5 \times 6 = 180$ ways.

If $\quad a = 2$ then $b = 3c$ which is not possible

$\therefore$ Total number of required ways

$= 62 + 180 + 180 = 422.$

48. **(30)** 5 persons A, B, C, D and E are seated in circular arrangement.



Let A be given red hat, then there will be two cases.

Case I : B and E have same coloured hat blue/green.

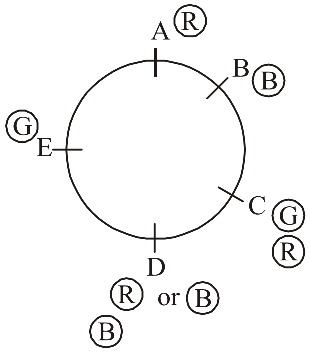
Say B and E have blue hat.

Then C and D can have either red and green or green and red i.e. 2 ways.

Similarly if B & E have green hat, there will be 2 ways for C & D.

Hence there are 2 + 2 = 4 ways.

Case II : B and E have different coloured hats blue and green or green or blue.



Let B has blue and E has green.

If C has green then D can have red or blue.

If C has red then D can have only blue.

$\therefore$ three ways.

Similarly 3 ways will be there when B has green and E has blue.

Hence there are 3 + 3 = 6 ways.

Combining the two cases, there will be 4 + 6 = 10 ways.

When similar discussion is repeated with A as blue and green hat, we get 10 ways for each.

Hence in all, there will be 10 + 10 + 10 = 30 ways.

49. **(0)** $\sec^{-1}\left[\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right]$

$$= \sec^{-1}\left[\frac{1}{2}\sum_{k=0}^{10}\frac{1}{2\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}+\frac{\pi}{2}\right)}\right]$$

$$= \sec^{-1}\left[\frac{1}{2}\sum_{k=0}^{10}\frac{-1}{\sin\left(\frac{7\pi}{6}+k\pi\right)}\right]$$

For k = even, $\quad \sin\left(\frac{7\pi}{6}+k\pi\right) = \sin\frac{7\pi}{6} = \frac{-1}{2}$

For k = odd, $\quad \sin\left(\frac{7\pi}{6}+k\pi\right) = -\sin\frac{7\pi}{6} = \frac{1}{2}$

So in summation all pairs from 0 to 9 will be cancelled and term for k = 10 will be left.

∴ we get

$$\sec^{-1}\left[\frac{1}{2}\left(\frac{-1}{-1/2}\right)\right]=\sec^{-1}(1)=0$$

50. (0.50) $I=\int_0^{\pi/2}\frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5}d\theta$ ...(1)

$$I=\int_0^{\pi/2}\frac{3\sqrt{\sin\theta}}{\left(\sqrt{\sin\theta}+\sqrt{\cos\theta}\right)^5}d\theta \quad ...(2)$$

Adding two values of I, we get:

$$\frac{2}{3}I=\int_0^{\pi/2}\frac{1}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^4}d\theta$$

$$\frac{2}{3}I=\int_0^{\pi/2}\frac{\sec^2\theta}{\left(1+\sqrt{\tan\theta}\right)^4}d\theta$$

(put $\tan\theta=t^2\Rightarrow\sec^2\theta\,d\theta=2t\,dt$)

$$\Rightarrow \quad \frac{2I}{3}=\int_0^{\infty}\frac{2+dt}{(1+t)^4}$$

$$\Rightarrow \quad I=3\int_0^{\infty}\frac{t+1-1}{(t+1)^4}dt=3\int_0^{\infty}\left(\frac{1}{(t+1)^3}-\frac{1}{(t+1)^4}\right)dt$$

$$=3\left[-\frac{1}{2(t+1)^2}+\frac{1}{3(t+1)^3}\right]_0^{\infty}=3\left[\frac{1}{2}-\frac{1}{3}\right]=\frac{1}{6}$$

$$\Rightarrow \quad I=\frac{1}{2}=0.50$$

For Q 51 and 52.

$f(x)=0\Rightarrow\sin(\pi\cos x)=0$

$\Rightarrow \pi\cos x == n\pi$

$\Rightarrow \cos x = n$

$\Rightarrow \cos x = -1, 0, 1$

$$\Rightarrow x=\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi,\frac{5\pi}{2},3\pi,\frac{7\pi}{2},4\pi........$$

$$\therefore X=\left\{\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi,\frac{5\pi}{2},3\pi,\frac{7\pi}{2},4\pi,......\right\}$$

∴ (I) – P, Q

$f'(x)=0$

$$\Rightarrow \cos(\pi\cos x)(-\pi\sin x)=0$$

$$\Rightarrow \cos(\pi\cos x)=0,\ \sin x=0$$

$$\Rightarrow \pi\cos x=(2n-1)\frac{\pi}{2},\ \ x=n\pi$$

$$\Rightarrow \cos x=(2n-1)\frac{1}{2},\ \ x=\pi,2\pi,3\pi,.......$$

$$\Rightarrow \cos x=\frac{-1}{2},\frac{1}{2}.$$

$$\Rightarrow x=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\frac{8\pi}{3},\frac{10\pi}{3},\frac{11\pi}{3},\frac{13\pi}{3},.....$$

$$\therefore Y=\left\{\frac{\pi}{3},\frac{2\pi}{3},\pi,\frac{4\pi}{3},\frac{5\pi}{3},2\pi,...........\right\}.$$

∴ (II) – Q, T.

$g(x)=0\Rightarrow\cos(2\pi\sin x)=0$

$$\Rightarrow 2\pi\sin x=(2n-1)\frac{\pi}{2}\ \Rightarrow\ \sin x=\frac{2n-1}{4}$$

$$\Rightarrow \sin x=\frac{1}{4},\frac{-1}{4},\frac{3}{4},\frac{-3}{4}$$

$$\therefore Z=\left\{-\sin^{-1}\frac{3}{4},-\sin^{-1}\frac{1}{4},\sin^{-1}\frac{1}{4},\sin^{-1}\frac{3}{4}\right\}.$$

(III) – R.

$g'(x)=0\Rightarrow-\sin(2\pi\sin x).2\pi\cos x=0$

$$\Rightarrow \sin(2\pi\sin x)=0,\ \cos x=0$$

$$\Rightarrow 2\pi\sin x=n\pi,\ \ x=(2n-1)\frac{\pi}{2}.$$

$$\Rightarrow \sin x=\frac{n}{2},$$

$$x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2},........$$

$$\Rightarrow \sin x=-1,\frac{-1}{2},0,\frac{1}{2},1.$$

$$\Rightarrow x=\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6},\pi,\frac{3\pi}{2},\frac{11\pi}{6},2\pi,\frac{13\pi}{6},.........$$

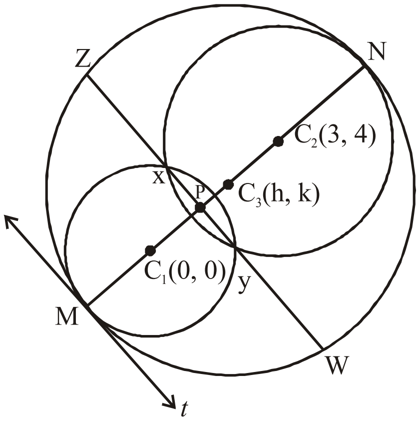
$$\therefore W=\left\{\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6},\pi,\frac{3\pi}{2},\frac{11\pi}{6},2\pi,\frac{13\pi}{6},......\right\}.$$

(IV) – P, R, S.

51. option (a) is correct.

52. option (d) is correct.

For Q 53and 54



$C_1 : x^2 + y^2 = 9$

$C_2 : (x-3)^2 + (y-4)^2 = 16$

$C_3 : (x-h)^2 + (y-k)^2 = r^2$

Centres of $C_1(0, 0)$, $C_2(3, 4)$, $C_3(h, k)$

radii of $C_1 : 3$, $C_2 : 4$, $C_3 : r$

Eq^n of $C_1C_2 : y = \frac{4}{3}x$

C_1, C_2, C_3 are collinear $\Rightarrow C_3\left(h, \frac{4}{3}h\right)$

$MN = MC_1 + C_1C_2 + C_2N = 3 + 5 + 4 = 12 \Rightarrow r = 6$

$\therefore \quad C_1C_3 = 6 - 3 = 3$

$\Rightarrow \quad h^2 + \frac{16}{9}h^2 = 9 \Rightarrow h^2 = \frac{81}{25}$

$\Rightarrow h = \frac{9}{5}$ taking h +ve, as lies between C_1 & C_2

$\therefore \quad C_3\left(\frac{9}{5}, \frac{12}{5}\right)$

$\therefore \quad 2h + k = \frac{18}{5} + \frac{12}{5} = \frac{30}{5} = 6$

$\therefore \quad$ (I) – (P)

XY is common chord of C_1 and C_2

$\therefore \quad$ Eq^n of XY : $S_1 - S_2 = 0 \Rightarrow 6x + 8y - 18 = 0$

or $\quad 3x + 4y - 9 = 0$

Length of $\perp$ from C_1 to $XY = \frac{9}{5} = C_1P$

Also $C_1X = 3 \;\therefore\; PX = \sqrt{9 - \frac{81}{25}} = \sqrt{\frac{225-81}{25}} = \frac{12}{5}$

$\therefore \quad XY = 2PX = \frac{24}{5}$

ZW is chord of C_3.

$C_3P = MC_3 - MP = 6 - \left(3 + \frac{9}{5}\right) = 6 - \frac{24}{5} = \frac{6}{5}$

$\therefore \quad ZP = \sqrt{6^2 - \left(\frac{6}{5}\right)^2} = \frac{6\sqrt{24}}{5} = \frac{12\sqrt{6}}{5}$

$\therefore \quad ZW = \frac{24\sqrt{6}}{5}$

Hence, $\frac{\text{Length of ZW}}{\text{Length of XY}} = \frac{24\sqrt{6}/5}{24/5} = \sqrt{6}$

$\therefore \quad$ (II) – (Q)

Area of $\Delta MZN = \frac{1}{2} MN \times ZP = \frac{1}{2} \times 12 \times \frac{12\sqrt{6}}{5}$

$= \frac{72\sqrt{6}}{5}$

Area of $\Delta ZMW = \frac{1}{2} \times ZW \times MP$

$= \frac{1}{2} \times \frac{24\sqrt{6}}{5} \times \frac{24}{5} = \frac{288\sqrt{6}}{25}$

$\therefore \quad \frac{\text{Area of } \Delta MZN}{\text{Area of } \Delta ZMW} = \frac{72\sqrt{6}}{5} \times \frac{25}{288\sqrt{6}} = \frac{5}{4}$

$\therefore \quad$ (III) – (R)

Now common tangent of C_1 and C_3 is $S_1 - S_3 = 0$

$\Rightarrow \quad 2hx + 2ky - h^2 - k^2 = 9 - r^2$

or $\quad \frac{18}{5}x + \frac{24}{5}y - \frac{81}{25} - \frac{144}{25} = 9 - 36$

$\Rightarrow \quad 3x + 4y + 15 = 0$

It is tangent to $x^2 = 8\alpha y$

Putting value of y from common tangent in parabola, we get

$x^2 = -8\alpha\left(\frac{3x+15}{4}\right)$

$\Rightarrow \quad x^2 + 6\alpha x + 30\alpha = 0$

It should have equal roots

$\therefore \quad 36\alpha^2 - 4 \times 30\alpha = 0 \Rightarrow \alpha = \frac{10}{3}$

$\therefore \quad$ (IV) – (U)

Thus (II) – (Q) is the only correct combination and (IV) – (S) is the only incorrect combination.

53. option (d) is correct

54. option (a) is incorrect