AIRPORT AUTHORITY OF INDIA
Solved Paper-2018

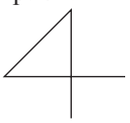
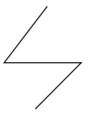
GENERAL KNOWLEDGE

1. Substances like phosphorus burns in air at room temperature. This type of combustion in which a material suddenly bursts into flames, without application of any apparent cause, is called.
   (a) Explosive combustion
   (b) Spontaneous combustion
   (c) Running combustion
   (d) Rapid combustion
2. Which of the following nations won the Cricket World Cup 2015?
   (a) India (b) England
   (c) Australia (d) West Indies
3. The Elephanta caves are dedicated to which deity?
   (a) Shiva (b) Tirthankar Mahavir
   (c) Vishnu (d) Buddha
4. In which of the following states is the Simlipal bio-reserve located?
   (a) Himachal Pradesh (b) Punjab
   (c) Uttarakhand (d) Odisha
5. Name a book maintained internationally by an organization which keeps a record of all the endangered animals and plants. India also maintains it for plants and animals found in India.
   (a) Primitive Books (b) Vintage Book
   (c) Extinct Book (d) Red Data Book
6. As of August 2018. who among the following is the Chairman of SEBI?
   (a) Sanjeev Kaushik (b) U. K. Sinha
   (c) C. B. Bhave (d) Ajay Tyagi
7. ————is/was the Speaker of the _____ Lok Sabha.
   (a) Somnath Chatterjee; 12th
   (b) Somnath Chatterjee; 16th
   (c) Sumitra Mahajan; 14th
   (d) Sumitra Mahajan; 16th
8. Which of the following Articles of the Constitution of India deals with establishment and constitution of Supreme Court?
   (a) Article 117 (b) Article 114
   (c) Article 124 (d) Article 106
9. Which of the following styles of painting belongs to Maharashtra?
   (a) Miniature (b) Madhubani
   (c) Kalam (d) Warli
10. What was the rank of India in world Press Freedom Index 2018?
   (a) 38 (b) 138
   (c) 101 (d) 1

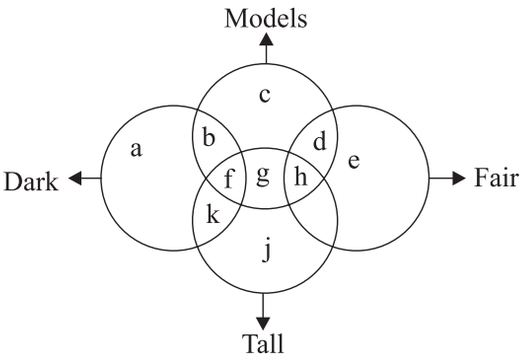
REASONING ABILITY

11. Ravi's father Mahesh has a brother Mukesh who has a daughter Chavi. Vinayak is Chavi's brother. How is vinayak related to Mahesh's mother.
   (a) Nephew (b) Son
   (c) Grandson (d) Grand nephew
12. Select the option that is related to the third number in the same way as the second number is related to the first number.
   169 : 14 : : 64 : ?
   (a) 9 (b) 8 (c) 7 (d) 13
13. Select the option that will correctly come in the place of question mark to complete the below number series.
   6, 11, 19, 30, ?, 61, 81
   (a) 43 (b) 42 (c) 40 (d) 44
14. In the following number sequence, how many such 8s are there, each of which is exactly divisible by its immediate preceding number but not divisible by its immediately succeeding number?
   2 8 7 5 3 4 8 6 2 8 4 5 8 4 3 2 8 9
   (a) 1 (b) 3 (c) 2 (d) 4
15. Select the option that is related to the third term in the same way as the second term is related to the first term.
   KJPO : PQKL :: LINO : ?
   (a) ONIL (b) ORML (c) YVZI (d) ORLM
16. Select the word that CANNOT be formed by using the letters of the given word.
   COURAGEOUS
   (a) SOURCE (b) AROUSE
   (c) COURSES (d) USAGE
17. Among given word pairs, three bear a certain common relationship. Choose the pair in which the words are differently related.
   (a) Colt : Horse (b) Piglet : Pig
   (c) Bitch : Dog (d) Cub : Bear

18. In a certain code, 'SURPRISE' is coded as T5SQS3T2. How will GRACE be coded in that language?
(a) HSBD2 (b) HSID2
(c) H2SID (d) H5BD2

19. Ajay's residence is 35 metre away from that of Sachin's towards South-west direction. Tarun's house is 35 metre away from Sachin's house towards West. Uday's house is 35 metre away from Ajay's house towards East. Which of the given options resembles the shape of the above description?
(a)  (b) 
(c)  (d) 

20. In the given Venn diagram, which region represents Tall and Dark Models?



(a) b (b) f (c) k (d) g

21. Which one set of letters, from the options below, when sequentially placed (left to right) at the gaps in the given letter series, shall complete it:
s_ut_vwu_wx_vw_y_?
(a) tuvyxz (b) tuvwxy
(c) tuvxyz (d) tuvwxz

22. 'Chaotic' is related to 'Jumbled' in the same way 'Orderly' is related to:
(a) Confused (b) Erratic
(c) Chaos (d) Tabulated

23. If the given interchanges are made in signs and numbers, which one of the four equations would be correct?
(a) 3 + 6 ÷ 6 = 10 (b) 6 ÷ 3 + 3 = 6
(c) 6 + 3 ÷ 4 = 2.5 (d) 3 + 6 ÷ 2 = 4

24. Select the number-pair that is different from the other three.
(a) 15 – 841 (b) 6 – 72
(c) 12 – 576 (d) 9 – 243

25. Select the term that is different from the other three options.
(a) KLQR (b) CDYZ
(c) GIUV (d) FGVW

MATHEMATICS (QUANTITATIVE APTITUDE)

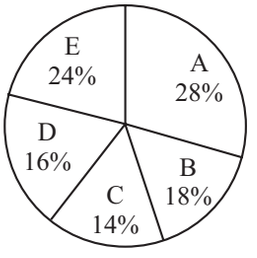
26. The number of students in sections A and B are 60 and 70 respectively. The average score in mathematics of students in A is 70 and that of students in A and B together is 63. What is the average score of students of section B in mathematics?
(a) 57 (b) 56.5 (c) 50 (d) 57.5

27. Two trains start at the same time, one from A to B and the other from B to A. After passing each other on their way, they take 50 min and 3 h 20 min respectively to reach their respective destinations. If the speed of the train starting from A is 72 km/h, then what is the speed (in km/h) of the other train?
(a) 36 (b) $33\frac{1}{3}$
(c) 54 (d) $66\frac{2}{3}$

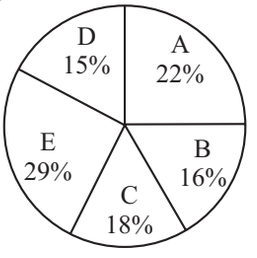
28. A person sold an article for ₹1,500. Had he offered a discount of 10% on the selling price, he would have earned a profit of 8%. If he sells the article now at a profit of 12%, then the selling price will be;
(a) ₹ 1,460 (b) ₹ 1,380
(c) ₹ 1,372 (d) ₹ 1,400

29. The compound interest accrued on a sum of ₹ 4400 at the end of 2 years is ₹ 1,119.36. What would be the simple interest on the same at the same rate for double the time?
(a) ₹ 2,211 (b) ₹ 2,121
(c) ₹ 2,122 (d) ₹ 2,112

30. Study the following pie chart and answer the question.
Percentage break-up of number of children in five villages



Total number of children = 5,400
Break-up of children attending school from these villages (A, B, C, D & E)



Total number of children attending School = 3,200
What is the ratio of the total number of children in village C and D to the total number of children attending schools in A and C?

(a) 81 : 71 (b) 91 : 71
(c) 91 : 64 (d) 81 : 64

31. The value of $\frac{5\frac{3}{4}-\frac{3}{7}of\,15\frac{3}{4}+2\frac{2}{35}\div\frac{1}{4}of\,5\frac{19}{25}}{\left(\frac{1}{2}+\frac{1}{4}\right)\times\frac{4}{7}\times\left(8\times\frac{2}{3}\div\frac{4}{5}\right)}$ lies between:

(a) 0.3 and 0.4 (b) 0.1 and 0.2
(c) 0.2 and 0.3 (d) 0.4 and 0.5

32. The value of $\frac{4.1\times26.21-12.3\times4.93}{(26.21)^2-9\times(4.93)^2}$ is:

(a) 0.2 (b) 0.02
(c) 0.01 (d) 0.1

33. 10 persons begin to work together on a job, but after some days, 4 persons leave. As a result, the job, which could have been completed in 40 days, was completed in 50 days. How many days after commencement of the work did the four persons leave?

(a) 25 (b) 24
(c) 30 (d) 20

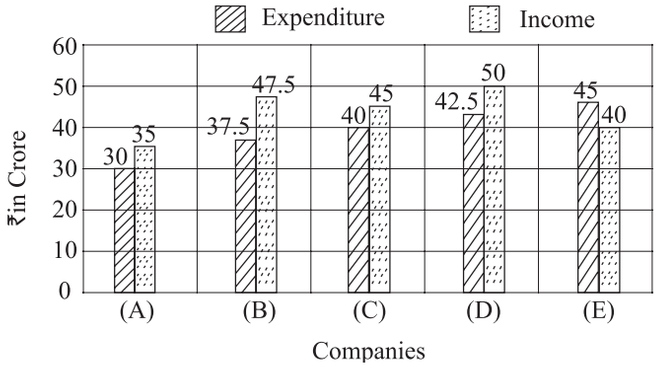
34. The ratio of the mean proportion between 1.6 and 32.4 and the third proportional between 0.8 and 1.2 is:

(a) 3 : 2 (b) 4 : 1
(c) 2 : 1 (d) 5 : 2

35. Study the following bar graph which shows the account of income and expenditure (in crore rupees) of 5 companies in the year 2016, and answer the question.

$$\text{Profit} = \frac{\text{Income - Expenditure}}{\text{Expenditure}}\times100$$

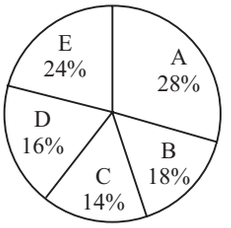
Account of Income and Expenditure



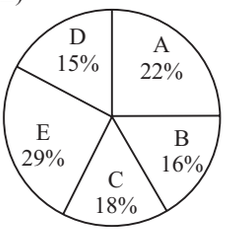
What is the approximate percentage of profit earned by all the companies together in 2016?

(a) 11.5 (b) 12.4
(c) 10.8 (d) 12.2

36. Study the following pie chart and answer the question.
Percentage break-up of number of children in five villages



Total number of Children = 5,400
Break-up of children attending school from these villages (A, B, C, D & E)



Total number of children attending School = 3,200
The total number of children NOT attending school in villages A, E and D is approximately what percentage of the total number of children in A, B and C?

(a) 50 (b) 46 (c) 45 (d) 48

37. Sudha bought an article for ₹ 6,200 and sold it at a loss 2.5%. With this amount, she bought another article and sold it at a gain of 4%. What was her gain/loss?

(a) 1.5% loss (b) 1.4% gain
(c) 5% loss (d) 15% gain

38. A field is in the shape of a trapezium, whose parallel sides measure 250 m and 110 m and non-parallel sides measure 150 m and 130 m. What is the area (in hectares) of the field?

(a) 2.16 (b) 2.12 (c) 2.54 (d) 3.24

39. A sum of ₹ 4,200 is divided among A, B, C and D such that the share of B is equal to $\frac{2}{3}$ of the share of C and the share of C is equal to $\frac{9}{13}$ of the share of D. If the ratio of the share of A and B is 8 : 9, then the difference between the shares of B and D will be:

(a) ₹ 860 (b) ₹ 882 (c) ₹ 840 (d) ₹ 924

40. The salary of A is 50% more than the salary of B. If A got a 50% rise in his salary and B got a 25% rise in his salary, then the percentage increase in their combined salaries will be:

(a) $33\frac{1}{3}$ (b) 40

(c) $35\frac{1}{2}$ (d) 75

ENGLISH

41. Select the most appropriate option to fill in the blank.
You must reach the theatre a little early, _____ you will not get a seat.
(a) otherwise (b) therefore
(c) moreover (d) besides

42. Select the most appropriate option to fill in the blank.
He _____ his breakfast when his friends arrived and asked him to come with them.
(a) is just finishing (b) just finishes
(c) has just finished (d) had just finished

43. Select the most appropriate antonym of the given word.
APPARENT
(a) Hamper (b) Clear
(c) Obvious (d) Obscure

44. Select the most appropriate option to fill in the blank.
The ____ church on the riverside looked very beautiful from the boat.
(a) old stone red (b) stone red old
(c) old red stone (d) red stone old

45. Select the option that is NOT an antonym of a word by way of adding the prefix 'in-'.
(a) increment (b) incurable
(c) inconsistent (d) incomparable

46. Select the correct passive form of the given sentence.
The rules forbid the people to cross the railway line.
(a) The people are forbidden by rules to cross the railway line.
(b) The people are being forbidden by rules to cross the railway line.
(c) The poeple have forbidden by rules to cross the railway line.
(d) The railway line are forbidden to cross the people.

47. Select the most appropriate option to fill in the blank.
Sachin is _____ in having a good time than to study.
(a) interested (b) as interested
(c) most interested (d) more interested

48. Select the most appropriate indirect form of the given sentence.
She said, "I can't come to the party on Saturday."
(a) She said that I can't come to the party on Saturday.
(b) She said that she can't come to the party on Saturday.
(c) She said that she wouldn't come to the party on Saturday.
(d) She said that she couldn't come to the party on Saturday.

49. Select the most appropriate option to fill in the blank.
I looked for my suitcase at the station but _____ had disappeared.
(a) its (b) it
(c) he (d) she

50. Select the correct active form of the given sentence.
He will surely be elected by the people in the next elections.
(a) People would surely be electing him in the next election.
(b) He will surely elect the people in the next election.
(c) People will surely be electing him in the next election.
(d) People will surely elect him in the next election.

51. In the following sentence, four words or phrases have been underlined. One of them is incorrect. Choose the INCORRECT word or phrase from the given options.
<u>There are not</u> a great deal of <u>difference between</u> the two schools <u>but I think</u> mine is <u>slightly</u> better.
(a) difference between (b) but I think
(c) slightly (d) There are not

52. Select the most appropriate option to fill in the blank.
Although she couldn't speak Spanish, she managed to make _____ understood in Spain.
(a) oneself (b) itself
(c) herself (d) himself

53. Select the most appropriate synonym of the given word.
Reproach
(a) Blame (b) Appreciate
(c) Confuse (d) Approach

54. Select the correctly spelt word.
(a) associassion (b) asociation
(c) associasion (d) association

55. Select the most appropriate option to fill in the blank.
_____ the A.C. was on, the room was not cool.
(a) In spite of (b) Although
(c) Whether (d) In case

56. Select the most appropriate synonym of the given word.
GRACEFUL
(a) Natural (b) Decorated
(c) Elegant (d) Artificial

57. Select the most appropriate option to fill in the blank.
She _____ her daughter to school before she goes to work.
(a) takes (b) taking
(c) has taken (d) took

58. In the following sentence, four words or phrases have been underlined. One of them in incorrect. Choose the INCORRECT word or phrase from the given options:
As heavy rain <u>continued to lash</u> Kerala <u>for</u> the third consecutive day, the State Government <u>has been sounding</u> red alert <u>in eight districts</u>.
(a) has been sounding
(b) continued to lash
(c) in eight districts
(d) for

59. Select the most appropriate direct form the given sentence. He said that he was not feeling well and that he wanted to go to bed.
(a) He said, "He was not feeling well. He wanted to go to bed."
(b) He said, "I am not feeling well. I want to go to bed."
(c) He said, "I am not feeling well and that I want to go to bed."
(d) He said, "I was not feeling well. I wanted to go to bed."

60. Select the wrongly spelt word.
(a) morphine (b) moralise
(c) mortuary (d) mortguage

PHYSICS

61. Which of the following materials is used for the generation of ultrasonic waves by using magnetostriction effect?
(a) Paramagnetic material
(b) Ferromagnetic material
(c) Diamagnetic materials
(d) Both paramagnetic and diamagnetic materials

62. Which of the following particles in motion, having the same kinetic energy, has the shortest wavelength?
(a) A neutron (b) An alpha-particle
(c) A proton (d) An electron

63. A position is an anti-particle of an electron, having the same mass but opposite charge to electron. Then, the minimum energy released in annihilation of matter of a pair of an electron and a positron is nearly :
(a) 2.01 MeV (b) 1.20 MeV
(c) 1.02 MeV (d) 0.51 MeV

64. Ultrasonic pulse echo technique, a non-destructive ultrasonic testing, is employed for any possible flaw detection in a metallic bar of thickness 30 cm. If the arrival times of the ultrasonic pulse are 45 μs and 90 μs respectively, the distance of the flaw from one end of the steel bar at which the ultrasonic pulse initially enters the steel bar, will be :
(a) 14 cm (b) 15 cm
(c) 18 cm (d) 16 cm

65. In an experimental arrangement of a Fresnel's biprism, monochromatic light of wavelength λ is used to produce interference fringe pattern. On introducing a thin, transparent glass sheet of refractive index 1.50 and thickness of 7 microns in the path of one of the interfering beams, the central fringe of the pattern is shifted to the position of 6^{th} bright fringe. Then, the wavelength λ of the monochromatic light used is nearly :
(a) 555 nm (b) 650 nm
(c) 618 nm (d) 581 nm

66. Which of the following pairs of phenomena illustrate the particle nature of the electromagnetic radiation?
(a) Compton effect and Photoelectric effect
(b) Compton effect and Pauli's principle
(c) Bragg's diffraction and photoelectric effect
(d) Bragg's diffraction and Compton effect

67. The frequency of ultrasonic waves is :
(a) Less than 20 Hz
(b) More than 20 KHz
(c) Equal to 20 Hz
(d) Greater than 20 Hz and less than 20 KHz

68. A suger solution in a tube of length 2.0 dm produces optical rotation of 12°. Then, the suger solution is diluted to one half of its initial concentration. If the dilute solution is contained in another tube of length 3.0 dm, the optical rotation produced by it will be :
(a) 9° (b) 7° (c) 10° (d) 11°

69. Which of the following equations correctly represents the momentum p of a photon of Energy E?
(a) E/c (b) E^2c
(c) Ec (d) Ec^2

70. The group velocity of matter waves associated with a moving particle is :
(a) the same as phase velocity
(b) less than the particle velocity
(c) equal to the particle velocity
(d) more than the particle velocity

71. The speed of a fast moving electron, having total energy of 2 MeV, is nearly :
(a) 0.96 C (b) 0.99 C
(c) 0.98 C (d) 0.97 C

72. Which of the following materials behaves as a dielectric?
(a) Tungsten (b) Copper
(c) Germanium (d) Mica

73. The ratio of the amplitudes of the electric field and magnetic field strengths has the same dimensions as that of :
(a) Permittivity (b) Inductance
(c) Capacitance (d) Impedance

74. For a non-relativistic particle, the ratio of phase velocity to group velocity of de Broglie waves is :
(a) 1 : 2 (b) 2 : 3
(c) 3 : 2 (d) 2 : 1

75. Which of the following phenomena establishes the transverse nature of light waves?
(a) Polarisation (b) Diffraction
(c) Photoelectric effect (d) Compton effect

76. Ultrasonic waves, propagating through a medium, can be detected by :
(a) Quinck's tube (b) Light meter
(c) Envelope detector (d) Piezoelectric detector

77. Maximum angle of diffraction in a plane transmission diffraction grating is :
(a) 90° (b) 45°
(c) 180° (d) 135°

78. Which of the following is the energy quantum of radiation?
(a) Phantom (b) Phonon
(c) Positron (d) Photon

79. The refractive indices of quartz crystal for right handed and left handed circularly polarized light of wavelength 762.9 nm are 1.5391 and 1.5392 respectively. The angle of rotation produced by the crystal plate of thickness 0.5 mm is :
(a) 25.5° (b) 11.8°
(c) 13.8° (d) 18.1°

80. The wavelength produced by a He-Ne laser corresponds to the transition in :
(a) both Ne and He atoms
(b) Ne atoms only
(c) He atoms only
(d) most favourable to He atoms than Ne atoms

81. In order to investigate the internal atomic structure of crystals, we make use of :
(a) Orange light (b) X-rays
(c) Ultraviolet radiation (d) Infrared radiation

82. A ray of light is incident on a transparent medium at an angle of 60°. The reflected ray of light is founded to be completely polarised. Then, the refractive index of the transparent medium is nearly :
(a) 1.73 (b) 1.62
(c) 1.52 (d) 1.33

83. A metre stick moves along its length with a certain speed. The apparent length of the moving metre stick as measured by a stationary observer on the ground is found to be 98 cm. Then, the velocity v of the metre stick in terms of the speed of light in vacuum c will be :
(a) 0.17 c (b) 0.19 c
(c) 0.98 c (d) 0.14 c

84. Which of the following materials exhibits piezoelectric effect?
(a) Copper (b) Aluminium
(c) Iron (d) Quartz

85. The experimental evidence that the electron exhibits wave-like characteristics was first provided by :
(a) Huygens (b) De Broglie
(c) Davisson and Germer (d) Dirac

86. Newton's rings can be obtained in :
(a) both transmitted and reflected light system
(b) transmitted light system only
(c) any light system with narrow source of light
(d) reflected light system only

87. The curie law, $\chi = C/T$, expressing susceptibility χ varying with absolute temperature T, is obeyed by :
(a) Both ferromagnetic and paramagnetic materials
(b) Ferromagnetic materials only
(c) Paramagnetic materials only
(d) Diamagnetic materials only

88. Which of the following materials is most suitable for marking electromagnetics and core of transformer?
(a) Copper (b) Aluminum
(c) Iron (d) Cobalt

89. Which of the following optical phenomena CANNOT convert unpolarised light to plane polarised light?
(a) Double refraction of light (b) Scattering of light
(c) Diffraction of light (d) Reflection of light

90. The Poynting vector of electromagnetic waves has the same dimensions as that of :
(a) electric field
(b) electric current density
(c) electromagnetic power density
(d) electric charge density

MATHEMATICS

91. Consider the below data:

x	:	0	1	2
$f(x)$	:	4	3	12

The value of $\int_0^2 f(x)\,dx$ by Trapezoidal rule will be :
(a) 11 (b) 12
(c) 15 (d) 9

92. For what value of λ, do the simultaneous equations $2x + 3y = 1$, $4x + 6y = \lambda$ have infinite solutions?
(a) $\lambda = 0$ (b) $\lambda = 1$
(c) $\lambda \neq 2$ (d) $\lambda = 2$

93. Consider two differential equations :
I : $e^y dx + (xe^y + 2y)dy = 0$
II : $xdy + 2y^2\,dx = 0$
Which of the following statements is correct?
(a) Both I and II are exact differential equations.
(b) I is not exact but II is an exact differential equation.
(c) Neither I nor II are exact differential equations.
(d) I is exact but II is not an exact differential equation.

94. A partial differential equation derived from the equation $z = ae^{by} \sin bx$ will be :
(a) $\frac{\partial z}{\partial y} = 2y\left(\frac{\partial z}{\partial x}\right)^2$ (b) $\left(1+\frac{\partial z}{\partial y}\right)\frac{\partial z}{\partial x} = z$
(c) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ (d) $2z = x\frac{\partial z}{\partial x} + y\frac{\partial x}{\partial y}$

95. The value of $\int \frac{e^x}{e^{2x-4}} dx$ will be _____, where C is an arbitrary constant.

(a) $\frac{1}{2}\log\left|\frac{e^x+1}{e^x-1}\right|+C$ (b) $\frac{1}{3}\log\left|\frac{2e^x-1}{2e^x+1}\right|+C$

(c) $\frac{1}{4}\log\left|\frac{e^x-2}{e^x+2}\right|+C$ (d) $\frac{1}{2}\log\left|\frac{e^{2x}+2}{e^{2x}-2}\right|+C$

96. General Solution of partial differential equation.

$x\left(y^2+z\right)\frac{\partial z}{\partial x}-y\left(x^2+z\right)\frac{\partial z}{\partial y}=\left(x^2-y^2\right)z$ will be :

(a) $\varphi\,(xyz,\ x^2+y^2-2z)=0$, where φ is an arbitrary function.

(b) $\varphi\,(x+y+z,\ x^2+y^2)=0$, where φ is an arbitrary function.

(c) $\varphi\,(x^2+y^2+z^2,\ x+y+z)=0$, where φ is an arbitrary function.

(d) $\varphi\,(x+y-z,\ xy+yz+zx)=0$, where φ is an arbitrary function.

97. A biased six-faced dice when thrown, is thrice as likely to show an odd number than an even number. If it is thrown twice, then the probability that the sum of two numbers thrown is odd will be :

(a) 3/8 (b) 1/4 (c) 1/8 (d) 1/2

98. Let p be a prime number. Then $\sqrt{p}$ is :

(a) a rational number

(b) an integer

(c) a prime number

(d) not a rational number

99. The value of $\int_0^1 \frac{\sin^{-1}x}{x}dx$ is :

(a) $\pi \log 2$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}\log 2$

100. The standard ordered basis of $\mathbb{R}^3$ is $\{e_1, e_2, e_3\}$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that $T(e_1) = Te_1 - 5e_3$, $T(e_2) = -2e_2 + 9e_3$, $T(e_3) = e_1 + e_2 + e_3$. The standard matrix of T is :

(a) $\begin{pmatrix} 7 & 0 & 1 \\ 0 & -2 & 1 \\ -5 & 9 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 7 & -2 & 1 \\ -5 & 9 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 7 & 0 & -5 \\ 0 & -2 & 9 \\ 1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 7 & -5 & 0 \\ -2 & 9 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

101. The value of $\int \frac{1}{(x+1)\sqrt{x^2-1}}dx$ will be _____, where C is an arbitrary constant.

(a) $\sqrt{\frac{x^2+1}{x^2-1}}+C$ (b) $\sqrt{\frac{x-1}{x+1}}+C$

(c) $\log\frac{x+1}{x-1}+C$ (d) $\log\frac{x^2-1}{x^2+1}+C$

102. The value of $\oint_c \frac{1}{\bar{z}^2+4}d\bar{z}$ where C is $|\bar{z} - 2i| = 1$ will be:

(a) 0 (b) 1/5 (c) $\pi/2$ (d) $\pi/3$

103. The value of $\lim_{x\to 0}\left(x\sin\frac{1}{x}\right)$ is :

(a) α (b) –1

(c) 0 (d) 1

104. The value of the integral $\int_0^{1+i}\left(x-y+ix^2\right)d\bar{z}$ along the straight line from $\bar{z} = 0$ to $\bar{z} = 1 + i$ will be :

(a) $\frac{1}{3}(i+1)$ (b) $\frac{2}{3}(1+i)$

(c) $\frac{1}{3}(i-1)$ (d) $\frac{2}{3}(i-1)$

105. The general solution of $\frac{dx}{y^2}=\frac{dy}{x^2}=\frac{dz}{x^2y^2z^2}$ will be :

(a) $x+3y=C_1, y^2+z^3=C_2$, where C_1 and C_2 are arbitrary constants.

(b) $x^3 + y^2 = C_1$, $x^2 + 3z^2 = C_2$, where C_1 and C_2 are arbitrary constants.

(c) $x^3 - y^3 = C_1$, $x^3 + 3z^{-1} = C_2$, where C_1 and C_2 are arbitrary constants.

(d) $x^2 + 2y^2 = C_1$, $x^3 - 3z = C_2$, where C_1 and C_2 are arbitrary constants.

106. Let A be a non-singular diagonalisable matrix of order 3 with eigenvalue $\lambda_1, \lambda_2, \lambda_3$. A^{-1} is diagonalisable if :

(a) $\lambda_1 = 2, \lambda_2 = 0, \lambda_3 = -1$

(b) $\lambda_1 = 0, \lambda_2 = 3, \lambda_3 = -2$

(c) $\lambda_1 = -1, \lambda_2 = 2, \lambda_3 = -3$

(d) $\lambda_1 = -3, \lambda_2 = 1, \lambda_3 = 0$

107. A complete solution of partial differential equation

$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}-z=\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}$ will be :

(a) $Z = ax + by - ab$, where a, b are arbitrary constants.

(b) $Z = x^2 + y^2 - 2ab$, where a, b are arbitrary constants.

(c) $Z = ax^2 + by^2 + abxy$, where a, b are arbitrary constants.

(d) $Z = ax^2 - by + ab$, where a, b are arbitrary constants.

108. The solution of differential equation $u_{n+3} - 4u_{n+2} + u_{n+1} + 1 + 6u_n = 0$ will be :
(a) $u_n = C_1 (1)^n + C_2(2)^n + C_3 (-3)^n$, where C_1, C_2, C_3 are constants.
(b) $u_n = C_1 (-1)^n + C_2(-2)^n + C_3 (-3)^n$, where C_1, C_2, C_3 are constants.
(c) $u_n = C_1 (1)^n + C_2(-2)^n + C_3 (3)^n$, where C_1, C_2, C_3 are constants.
(d) $u_n = C_1 (-1)^n + C_2(2)^n + C_3 (3)^n$, where C_1, C_2, C_3 are constants.

109. The value of $\int_0^{2\pi}\int_0^{\pi/4}\int_0^1 r^2 \sin\theta \, dr \, d\theta \, d\varphi$ will be :
(a) $\frac{\sqrt{2}\pi}{3}\left(\sqrt{2}+\sqrt{3}\right)$ (b) $\frac{2\pi}{3}\left(\sqrt{3}-1\right)$
(c) $\frac{2\pi}{3}\left(\sqrt{3}-\sqrt{2}\right)$ (d) $\frac{\sqrt{2}\pi}{3}\left(\sqrt{2}-1\right)$

110. The curve represented by $\mathbb{Z}\overline{\mathbb{Z}} + (1 + i)\mathbb{Z} + (1 - i)\overline{\mathbb{Z}} = 0$ will be :
(a) a circle with centre at (–1, 1) and radius as $\sqrt{2}$.
(b) an ellipse with semi-major axis as 2 and semi-minor axis as 1
(c) a straight line with x- intercept as –2
(d) a parabola with vertex at (–1, 0).

111. The solution e^x, e^{-x} and e^{2x} of $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$ will be :
(a) linearly dependent for $x \in$ [–2, 2] and linearly independent elsewhere.
(b) linearly dependent for $x \in$ [–1, 1] and linearly dependent elsewhere.
(c) linearly independent on every real interval.
(d) linearly dependent for all real x.

112. If 3, 5 are the eigenvalues of a square matrix A of order 2, then the eigenvalues of the matrix A^2 will be :
(a) 27, 125 (b) 9, 15
(c) 3, 5 (d) 9, 25

113. The solution of differential equation
$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ will be :
(a) $y = C_1 + C_2 x \log x + x^2 \log x$, where C_1 and C_2 are arbitrary constants.
(b) $y = (C_1 + C_2 \log x)x + \log x + 2$, where C_1 and C_2 are arbitrary constants.
(c) $y = C_1x + C_2 \log x + 3$, where C_1 and C_2 are arbitrary constants.
(d) $y = (C_1x + C_2 \log x)\, x + \log (2x) + 2$, where C_1 and C_2 are arbitrary constants.

114. For a complex variable $z = x + iy$, which of the following statements is true ?
(a) Both sin $h\,\mathbb{Z}$ and cos $h\,\mathbb{Z}$ are entire functions.
(b) Neither sin $h\,\mathbb{Z}$ nor cos $h\,\mathbb{Z}$ are entire functions.
(c) sin $h\,\mathbb{Z}$ is entire but cos $h\,\mathbb{Z}$ is not an entire function.
(d) sin $h\,\mathbb{Z}$ is not entire but cos $h\,\mathbb{Z}$ is an entire function.

115. If $f(0) = 3, f(1) = 5, f(3) = 21$, then the unique polynomial of degree 2 or less using Newton divided difference interpolation will be :
(a) $2x^2 + 2x + 1$ (b) $2x^2 - 3x + 1$
(c) $2x^2 + 3$ (d) $x^2 + 3x - 2$

116. The 2nd approximation to a root of the equation $x^2 - x - 1 = 0$ in the interval (1, 2) by Bisection method will be :
(a) 1.75 (b) 1.35
(c) 1.25 (d) 1.5

117. Which of the following statements is FALSE?
(a) The series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is convergent.
(b) A Cauchy sequence of real numbers need not be bounded.
(c) A sequence of real numbers is convergent if and only if it is a Cauchy sequence.
(d) If $\{x_n\}$ is a convergent sequence of real numbers, then $\{x_n\}$ is a Cauchy sequence.

118. The length of the curve $y = x^{3/2}$ over the interval [0, 1] will be :
(a) $\frac{1}{27}\left[(13)^{3/2} - 8\right]$ units
(b) $\frac{1}{16}\left[(11)^{5/2} - 3\right]$ units
(c) $\frac{57}{5}$ units
(d) $\frac{1}{9}\left[(15)^{1/2} + 4\right]$ units

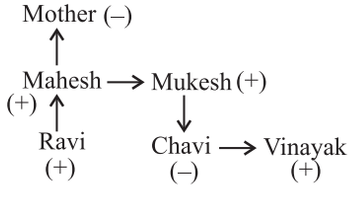
119. Consider two subsets of $\mathbb{R}^3$ given as,
$S_1 = \{[2, 3, 1], [1, 0, 5], [0, 1, 0], [0, 0, 1]\}$ and
$S_2 = \{[1, 0, 0], [0, 1, 1], [0, 0, 0]\}$
Which of the following statements is true?
(a) S_1 is linearly dependent but S_2 is linearly independent.
(b) Both S_1 and S_2 are linearly independent.
(c) S_1 is linearly independent but S_2 is linearly dependent.
(d) Both S_1 and S_2 are linearly dependent.

120. The p-series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges for :
(a) $0 < p \le 1$ (b) $p \in$ [2, 3, 5, 7, 11,....]
(c) $p > 1$ (d) $p \in$ [2, 4, 6,.....]

ANSWER KEY																			
1	(b)	13	(d)	25	(c)	37	(b)	49	(b)	61	(b)	73	(a)	85	(c)	97	(a)	109	(d)
2	(c)	14	(b)	26	(a)	38	(a)	50	(d)	62	(b)	74	(d)	86	(a)	98	(d)	110	(a)
3	(a)	15	(b)	27	(a)	39	(b)	51	(d)	63	(c)	75	(a)	87	(c)	99	(d)	111	(c)
4	(d)	16	(c)	28	(d)	40	(b)	52	(c)	64	(b)	76	(d)	88	(c)	100	(a)	112	(a)
5	(d)	17	(c)	29	(d)	41	(a)	53	(a)	65	(d)	77	(a)	89	(c)	101	(b)	113	(a)
6	(d)	18	(b)	30	(d)	42	(d)	54	(d)	66	(a)	78	(d)	90	(c)	102	(c)	114	(a)
7	(d)	19	(d)	31	(b)	43	(d)	55	(b)	67	(b)	79	(b)	91	(a)	103	(c)	115	(c)
8	(c)	20	(b)	32	(d)	44	(c)	56	(c)	68	(a)	80	(b)	92	(d)	104	(c)	116	(a)
9	(d)	21	(a)	33	(a)	45	(a)	57	(a)	69	(a)	81	(b)	93	(d)	105	(*)	117	(b)
10	(b)	22	(d)	34	(b)	46	(a)	58	(a)	70	(c)	82	(a)	94	(c)	106	(c)	118	(a)
11	(c)	23	(d)	35	(a)	47	(d)	59	(b)	71	(d)	83	(b)	95	(c)	107	(a)	119	(d)
12	(a)	24	(a)	36	(d)	48	(d)	60	(d)	72	(d)	84	(d)	96	(a)	108	(d)	120	(a)

Solutions

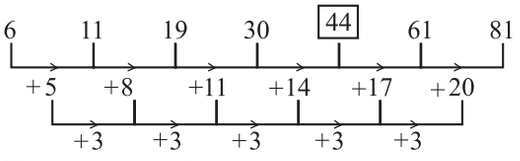
1. (b) 2. (c) 3. (a) 4. (d) 5 (d) 6. (d)
7. (d) 8. (c) 9. (d) 10. (b)
11. (c) Mother (–)



So, Vinayak is Grandson of Mahesh's mother.

12. (a) 169 : 14 :: 64 : ?
$(13)^2$ 14 $(8)^2$ 9

13. (d)



14. (b) There are only 3 8s
[287, 486, 289]

15. (b) Second term is related to first term
= 27 – 16 = 11
= 27 – 17 = 10
= 27 – 11 = 16
= 27 – 12 = 15] KJPO

in the same way fourth term is related to third term
= 27 – 15 = 12
27 – 18 = 9
27 – 13 = 14
27 – 12 = 15] ORML

So, option (b) is correct.

16. (c) Word COURSES can not formed, because there is only one 's' in the given word.

17. (c) All the three are related with each other as, young male and adult male except. Option 'c'.
in option c, they are female and male.

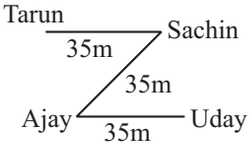
18. (b) As,

S	U	R	P	R	I	S	E
↓+1	|	↓+1	↓+1	↓+1	|	↓+1	|
T	5	S	Q	S	3	T	2

Similarly,

G	R	A	C	E
↓+1	↓+1	|	↓+1	|
H	S	I	D	2

Here vowel are coded as its position in English alphabet.

19. (d) Tarun


20. (b) "f" represents Tall and Dark Models.

21. (a) S T U T U V W U V W X Y V W X Y Z

22. (d) As chaotic and Jumbled are a state of confusion in the same way orderly and Tabulated are the state of proper arrangement.

23. (d) Given,
$3 + 6 \div 2 = 4$
After interchanging
$6 \div 3 + 2 = 4$
$2 + 2 = 4$
$4 = 4$
So, option 'd' is correct.

24. (a) In all the number- pairs , Second number is divisible by first number, except option ' (a).

25. (c) K L (+1) Q R (+1)
C D (+1) Y Z (+1)
G I (+1) U V (+1)
F G (+1) V W (+1)

26. (a) Number of students in section A = 60
Average of students in A = 70
$\therefore$ Total score = $60 \times 70 \Rightarrow 4200$
Number of students in section A and B together = 60 + 70 $\Rightarrow$ 130
Average of students in section A and B together = 63
$\therefore$ Total score = $130 \times 63 \Rightarrow 8190$
$\therefore$ Total score of section B = $8190 - 4200 \Rightarrow 3990$
$\therefore$ Average score of section B = $\frac{3990}{70} = 57$

27. (a) Let speed of other train is x Km/h.
$\therefore \quad \frac{S_1}{S_2} = \sqrt{\frac{T_2}{T_1}}$

$\frac{72}{x} = \sqrt{\frac{10}{\frac{3}{5/6}}}$ $\left\{\because 3\text{h } 20\text{min} = 3\frac{1}{3} \Rightarrow \frac{10}{3}h \right\}$ $\left\{50\,\text{min} = \frac{50}{60} \Rightarrow \frac{5}{6}h.\right\}$

$\frac{72}{x} = \sqrt{\frac{10}{3} \times \frac{6}{5}} = \sqrt{4}$

$x = 72/2 = 36$ Km/h.

28. (d) SP of the article = ₹ 1500.
After discount = $1500 \times \frac{90}{100} = ₹1350$.
ATQ,
CP = $\frac{1350}{108} \times 100 = ₹1250$.
$\therefore$ SP. after 12% profit = $\frac{1250 \times 112}{100} = ₹1400$

29. (d) Principal amount = ₹ 4400
Time = 2 Years.
Let Rate is r%
ATQ,
$4400 \times \left(\frac{100+r}{100}\right) \times \left(\frac{100+r}{100}\right)$
= 4400 + (1119.36)

$(100 + r)\,(100 + r) = \frac{5519.36}{44} \times 100$
$(100 + r)^2 = \frac{551936}{44}$
$(100 + r)^2 = 12544$
Taking square root on both sides
$100 + r = 112$.
$\boxed{r = 12\%}$
$\therefore$ SI for four years = $\frac{4400 \times 12 \times 4}{100} = ₹\,2112$

30. (d) Total number of children in village C and D
= (14 + 16)% of 5400 = $5400 \times \frac{30}{100} = 1620$
Total no. of children attending Schools in A and C
= (22 + 18)% of 3200 = $3200 \times \frac{40}{100} = 1280$
Required Ratio = $\frac{1620}{1280} = 81 : 64$

31. (b) $= \frac{\frac{23}{4} - \frac{3}{7} \times \frac{63}{4} + \frac{72}{35} \div \frac{1}{4} \times \frac{144}{25}}{\frac{3}{4} \times \frac{4}{7} \times \frac{20}{3}}$

$= \frac{\frac{23}{4} - \frac{27}{4} + \frac{72}{35} \div \frac{36}{25}}{\frac{20}{7}} = \frac{\frac{23}{4} - \frac{27}{4} + \frac{10}{7}}{\frac{20}{7}}$

$$= \frac{\frac{-4}{4}+\frac{10}{7}}{\frac{20}{7}} = \frac{\frac{-7+10}{7}}{\frac{20}{7}} = \frac{3}{20} = 0.15$$

So, it lies between 0.1 and 0.2.

32. (d) $\frac{107.461-60.639}{686.9641-9\times24.3049}$

$$= \frac{46.83}{686.96-218.73}$$

$$= \frac{46.83}{468.3} = 0.1$$

33. (a) Let, after x days, four persons leaves.

Total work = 10 persons × 40 days = 400 units

Remaining persons = (10 – 4) = 6 Persons.

ATQ,

$10 \times 40 = 10 \times x + 6 \times (50 - x)$

$400 = 10x + 300 - 6x$

$4x = 100$

$\boxed{x = 25}$

34. (b) Mean Proportion of 1.6 and 32.4

$= \sqrt{1.6 \times 32.4} = \sqrt{51.84}$

= 7.2

Third Proportional of 0.8 and 1.2 = $\frac{1.2\times1.2}{0.8} \Rightarrow 1.8$

Required Ratio = $\frac{7.2}{1.8} = \frac{4}{1} \Rightarrow 4:1$

35. (a) Total income by all the companies = (35 + 47.5 + 45 + 50 + 40) = 217.5 Crore.

Total expenditure by all the companies = (30 + 37.5 + 40 + 42.5 + 45) = 195 Crore

Profit % = $\frac{217.5-195}{195}\times100$

$= \frac{22.5}{195}\times100$

= 11.538

= 11.5%

36. (d) Total no. of children in village A, E and D = (28 + 24 + 16)% of 5400

$= 5400 \times \frac{68}{100} = 3672$

Total no. of children attending school in village A, E and D

= (22 + 29 + 15)% of 3200

$= 3200 \times \frac{66}{100} = 2112$

Total no. of children not attending school in village A, E and D = (3672 – 2112) = 1560

Total no. of children in village A, B and C

= (28 + 18 + 14)% of 5400.

$= 5400 \times \frac{60}{100} = 3240$

Required Percentage = $\frac{1560}{3240}\times100$

= 48.148%

$\Rightarrow$ 48%

37. (b) According to first conditions: SP.

$= 6200 \times \frac{97.5}{100} = ₹\,6045$

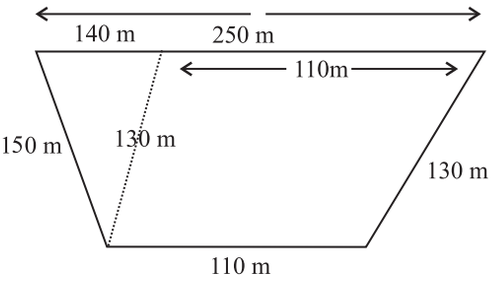
According to second condition SP.

$= 6045 \times \frac{104}{100} = ₹\,6286.8$

Gain % = $\left(\frac{6286.8-6200}{6200}\right)\times100$

Gain % = $\frac{86.8}{6200}\times100 = 1.4\%$

38. (a)



Area of trapezium

$= \left[\frac{\text{Sum of Parallel sides}}{\text{difference between two Parallel sides}}\right]$

$\times \sqrt{\left[S(S-a)(S-c)(S-d)\right]}$

$\boxed{\text{Where, } a = 140,\ b = 150,\ c = 130}$

Difference between two Parallel sides (k) = 250 – 110

$\Rightarrow$ 140 m

Semi Perimeter (S) = (140 + 150 + 130)/2

= 420/2 = 210 m

Area of trapezium

$= \left[(250+110/140)\right]\times\sqrt{\left[210(210-140)(210-150)(210-130)\right]}$

$[360/140]\times\sqrt{210\times70\times60\times80}$

$\Rightarrow$ [360/140] × 8400

$\Rightarrow$ 21600 m^2

$\Rightarrow$ 2.16 $hectare^2$

39. (b) $B = \frac{2}{3}C$

$\Rightarrow \frac{B}{C} = \frac{2}{3}$ ....(1)

$C = \frac{9}{13}D$

$\Rightarrow \frac{C}{D} = \frac{9}{13}$ ....(2)

To make the ratio equal of 'c' in both the eq^n, multiply by. 3 in numerator and denominator of eq^n (1)

$\frac{B}{C} = \frac{6}{9}$ and $\frac{C}{D} = \frac{9}{13}$

$\therefore$ B : C : D = 6 : 9 : 13 ....(3)

From the question A : B = 8 : 9

$\frac{A}{B} = \frac{8}{9}$ ....(4)

To make the ratio equal of B, multiply by 3 in eq^n (3) and multiply by 2 in eq^n (4) – in numerator and denominator B : C : D = 18 : 27 : 39.

$\frac{A}{B} = \frac{16}{18}$

$\therefore$ A : B : C : D = 16 : 18 : 27 : 39

Required difference = $\frac{4200}{100} \times 21 = ₹\ 882$

40. (b) Let salary of B is ₹ 100.

$\therefore$ Salary of A = ₹ 150

ATQ,

A's Salary after 25% rise

$= \frac{150 \times 150}{100} = ₹\ 225$

B's Salary after 50% rise

$= 100 \times \frac{125}{100} = ₹\ 125$

% increase in their Combined salaries

$= \frac{(225+125)-(150+100)}{(150+100)} \times 100$

$= \frac{350-250}{250} \times 100 = 40\%$

41. (a) The correct word to fill here will be '**otherwise**'. It is used for saying that if one thing does not happen, something else (usually bad) will happen. 'Otherwise' can also be used to suggest that something is true because the situation will be different if it was not true.

42. (d) 'Had just finished' will fit in the blank to make the sentence grammatically correct and contextually meaningful. Here the sentence is in past perfect tense. The past perfect tense is used to emphasize that an action was completed before another took place. So, (d) will be the correct alternative.

43. (d) Apparent : Clearly visible or understood; obvious.
Obscure : Not clearly expressed or easily understood.
Hence, '**Obscure**' is the most appropriate antonym of the given word.

44. (c) Here, **'old red stone'** will be the appropriate option to fill in the blank. Here, both 'old' and 'red' are adjectives qualifying the noun 'church'. Hence, (c) is the right answer choice.

45. (a) The antonym of 'increment' is 'decrement' or reduction'.
Incurable is the antonym of curable.
Inconsistent is the antonym of consistent.
Incomparable is the antonym of comparable.
Hence, (a) is the most viable answer choice.

46. (a) The correct passive voice of the given sentence is,
"The people are forbidden by rules to cross the railway line'.
To change into passive voice from active voice, Subject (Rules) and Object (the people) of the sentence are interchanged. And 'by' is used before the object of the final sentence. In other words, in passive voice, the subject will be 'the people' and the object will be 'rules'. Hence, (a) is the right passive form of the given sentence.

47. (d) 'More interested' will be the most appropriate option to fill in the blank. Here, use of 'than' in the later part of the sentence indicates comparative degree of adjective.
Interested-more interested-most interested.
Hence, (d) is the right answer choice.

48. (d) The correct indirect form of the given sentence will be,
"She said that she couldn't come to the party on Saturday'.
To change in to indirect form from direct speech, 'that' is used instead of 'comma'. Further, since this sentence is present indefinite tense, it will be changed into past indefinite tense i.e. "can't" will be changed into 'couldn't. 'She' will be changed into second person.
Hence, (d) is the right answer choice.

49. (b) Here, 'it' will be used in the given blank. Note that, the subject of the sentence is referring to 'suitcase' which is a non-living thing and 'it' is used for indicating a non-living thing. It is used for a thing previously mentioned or easily identified. The pronoun 'IT' also serves as a placeholder subject in sentences with no identifiable actor, such as "It rained last night".

50. (d) The correct active form of the given sentence will be, **People will surely elect him in the next election.**
To change into active from passive voice, the subject (He) and the object (people) are interchanged. Here, 'him' will be changed into 'he'.
The forms **he**, **she** and **they** are used when a pronoun is the subject of a sentence. The forms **him, her** and **them** are used when a pronoun is the object of a sentence **'by'** will be removed. Hence, (d) is the right answer choice.

51. (d) 'There is not' will be correct instead of 'there are not' because 'deal' is singular in number. Hence, **option (d)** will be right answer choice.

52. (c) The most appropriate option to fill in the blank will be 'herself'.
The final sentence will be,
'although she couldn't speak Spanish, she managed to make herself understood in Spain.
Herself is a third person singular reflexive pronoun. **Herself** is used when the object of a verb or preposition refers to the same person as the subject of the verb. You use **herself** to refer to a woman, girl, or female animal. E.g. She let **herself** out of the room.

53. (a) Reproach : Express to (someone) one's disapproval of or disappointment in their actions.
Blame : Feel or declare that (someone or something) is responsible for a fault or wrong.
Appreciate : Recognize the full worth of.
Confuse: Make (someone) bewildered or perplexed.
Approach : Come near or nearer to (someone or something) in distance or time. Speak to (someone) for the first time about a proposal or request.
Hence, **option (a)** is the right answer choice.

54. (d) The correctly spelt word among the given options is **'Association'**.
Association **: (often in names) a group of people organized for a joint purpose.**
Hence, (d) is the most viable answer choice.

55. (b) **'Although'** will be the most appropriate option to fill in the blank.
You use although to introduce a subordinate clause which contains a statement which makes the main clause of the sentence seem surprising or unexpected.
Hence, (b) is the right answer choice.

56. (c) Graceful : Having or showing grace or elegance.
Elegant : Graceful and stylish in appearance or manner.
Artificial : Made or produced by human beings rather than occurring naturally, especially as a copy of something natural.
Decorated : Made (something) look more attractive by adding extra items or images to it.
Natural : Existing in or derived from nature; not made or caused by humankind.
Hence, (c) is the right answer choice.

57. (a) The correct verb here will be '**takes**' as the sentence is in simple present tense. The final sentence will be,
She ***takes*** **her daughter to school before she goes to work.'**
Hence, (a) is the right answer choice.

58. (a) In the given sentence, the incorrect phrase is as given in (a). **'Sounded'** should be used instead of **'has been sounding'**. Hence, (a) is the right answer choice.

59. (b) The correct direct form of the given sentence will be, He **said, "I am not feeling well, I want to go to bed."**
The tense of indirect speech is in past tense and it will be converted into present tense.

60. (d) The wrongly spelt word among the given options is **'Mortguage'**. The correct spelling is '**Mortgage**'.
Mortgage : A legal agreement by which a bank, building society, etc. lends money at interest in exchange for taking title of the debtor's property, with the condition that the conveyance of title becomes void upon the payment of the debt.

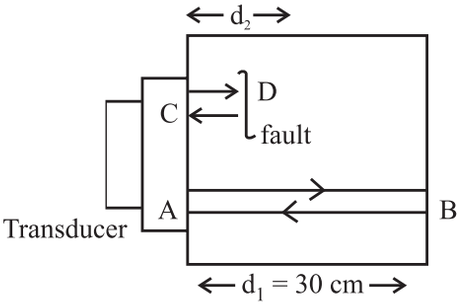
61. (b) Ferromagnetic materials i.e, iron, cobalt, nickel, etc. is used for the generation of ultrasonic waves by using magnetostriction effect.

62. (b) $\lambda = \frac{h}{\sqrt{2mKE}} \Rightarrow \lambda\alpha\frac{1}{\sqrt{m}}$

As mass is greatest for α-particle hence it has the shortest wavelength among the particles.

63. (c) Rest mass energy of each of positron and electron
$Eo = m_o c^2 = (9.1 \times 10^{-31}\ kg) \times (3 \times 10^8\ m/s)^2 = 8.2 \times 10^{-14} J$
$= 0.5\ 1\ MeV$
Hence for pair production, the minimum energy released $= 2 \times 0.51 = 1.02$ MeV.

64. (b) Let time taken by pulse 1 from A to B and back to A be t_1 and time taken by pulse 2 to travel from C to D and back to C be t_2
Let AB = d_1 = 30cm and CD = d_2
For pulse 1



$2d_1 = vt_1$

$\Rightarrow v = \frac{2d_1}{t_1}$ ...(i)

here v is the velocity of pulse

For pulse 2

$2d_2 = vt_2 \Rightarrow d_2 = \frac{vt_2}{2}$

Using equation (i)

$d_2 = \left(\frac{2d_1}{t_1}\right)\frac{t_2}{2}$

$= \frac{d_1 t_2}{t_1} = \frac{0.3 \times 45}{90} = 15$ cm

65. (d) In the absence of transparent glass sheet,

$y = \frac{nD\lambda}{d}$ .....(i)

When transparent glass sheet is introduced, shift in fringe,

$y = \frac{D(\mu - 1)t}{d}$ ....(ii)

According to question

$\frac{nD\lambda}{d} = \frac{D(\mu - 1)t}{d}$

$n\lambda = (\mu - 1)t$

$\Rightarrow \lambda = \frac{(\mu - 1)t}{n}$

Here $n = 6$, $\mu = 1.5$

and $t = 7$ μm

So,

$\lambda = \frac{(1.5 - 1) \times 7 \times 10^{-6}}{6}$

$= \frac{0.5 \times 7 \times 10^{-6}}{6}$

$= 581 \times 10^{-9}$m $= 581$ nm.

$\therefore$ The wavelength of monochromatic light used is 581 nm.

66. (a) Electromagnetic radiation has the dual nature. It exhibits wave properties as well as particle properties. The phenomena like interference and diffraction illustrate the wave nature. The phenomena photoelectric effect, compton effect illustrate particle nature.

67. (b) Ultrasonic waves is sound waves with frequencies higher than the upper audible limit of human hearing. Which is more than 20 KHz.

68. (a) Concentration of sugar solution, $C_1 = \frac{x}{\pi r^2 l_1}$

$\therefore C_1 \propto \frac{1}{l_1}$

After dilution, $C_2 \propto \frac{1}{l_2}$

Optical rotation, $\theta = SlC$

$\therefore \frac{\theta_2}{\theta_1} = \frac{Sl_2C_2}{Sl_1C_1} = \frac{3}{2} \times \frac{1}{2}$

$\Rightarrow \theta_2 = \frac{3}{4} \times \cancel{12}^{3} = 9°$

69. (a) Einstein explained the energy and momentum of a photon are related by the equation

$E = pc$

where, c speed of light

$\therefore p = \frac{E}{c}$

70. (c) From Planck Einstein relation

$E = h\nu$

Momentum, $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

From the De-broglie relation

Wavelength of electron

$\lambda = \frac{h}{p}$

De-broglie proposed that electron has properties of both waves and particle and above relation holds for both wave and particle, which means velocity of particle must be equal to group velocity of waves.

71. (d) We know that

Total energy = K.E + Rest mass energy of electron

2 Mev = K.E + 0.511 MeV

$\Rightarrow$ K.E = 1.5 MeV

$\frac{1}{2}\frac{m_0}{\sqrt{1 - \frac{V^2}{C^2}}} \times V^2 = 1.5 \times 1.6 \times 10^{-13} J$

$\Rightarrow V^2 = \frac{3 \times 1.6 \times 10^{-13}}{9.1 \times 10^{-31}}\sqrt{1 - \frac{V^2}{C^2}}$

$\Rightarrow V^4 = \frac{25 \times 10^{-26}}{81 \times 10^{-62}}\left(1 - \frac{V^2}{C^2}\right) \Rightarrow V = 0.97C$

72. (d) Dielectrics are insulating (non-conducting) materials which transmits electric effect without conducting.

73. (None) the ratio of the amplitude of electric field (E) and magnetic field (B) strength i.e., $\frac{E}{B} = C$
Dimensions [M° LT^{-2}]
Permittivity [M^{-1} L^{-3} T^4 A^2];
Capacitance [M^{-1} L^{-2} T^4 A^2] ; Impedance [ML^{2}T^{-3}A^{-2}];
Inductance [ML^{2}T^{-2}A^{-2}]

74. (d) $\because$ Group velocity, $V_g = \frac{\text{phase velocity } V_P}{2}$

$\therefore \frac{V_P}{V_g} = 2$

75. (a) Polarisation establishes the transverse nature of light waves as there are two directions for light waves to oscillate.

76. (d) Quink's tube is used to measure velocity of sound in air light meter measure the amount of light in space. Envelop detector is used to demodulate a previously modulated signal.

77. (a) Since diffraction condition is given by
$d \sin\theta = n\lambda$

$\sin\theta = \frac{n\lambda}{d}$

sin θ is maximum when θ = 90°.
Here θ is the angle of diffraction.

78. (d) According to Einstein's quantum theory light propagates in the form of bundles (packets or quanta of energy, each bundle is called a photon and possessing energy $E = h\nu$.

79. (b) Given,
Refractive index of quartz crystal for right handed circularly polarized light, $\mu_R = 1.5391$
Refractive index for left handed vibration, $\mu_L = 1.5392$
Angle of rotation, $= \frac{\pi}{\lambda}(\mu_L - \mu_R)t$

$= \frac{3.14(1.5392 - 1.5391)}{762.9 \times 10^{-9}} \times 0.5 \times 10^{-9} = 11.8°$

80. (b) In transition in He-Ne laser, an electric discharge in gas pumps the helium atom to higher energy states. These He atoms collide with the ground state neon atoms and excite to higher states and produce a photon beam.

81. (b) X-rays have smaller wavelength and they have higher energy. With their higher energy, they can penetrate matter more easily. Hence, they are used to investigate the internal atomic structure of crystals.

82. (a) According to Brewster's law
$\tan(i_p) = \mu$
Here i_p is palarising angle
$\mu = \tan(60°)$

$= \sqrt{3}$

= 1.73

83. (b) Contraction of length of metre stick when it is moving at relativistic speed v is given by

$L' = \frac{L_0}{\gamma}$

Here L' is the length observed by an observer in motion relative to the object.
L_0 is the length of metre stick in rest frame
and $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

There $L' = L_0\sqrt{1 - \frac{v^2}{c^2}}$

$\frac{L'}{L_0} = \sqrt{1 - \frac{v^2}{c^2}}$

Given, L' = 98 cm = 0.98 m, Lo = 1 m
Putting these value

$\frac{0.98}{1} = \sqrt{1 - \frac{v^2}{c^2}}$

or, $(0.98)^2 = 1 - \frac{v^2}{c^2}$

$\frac{v^2}{c^2} = 1 - (0.98)^2$

$\frac{v}{c} = \sqrt{0.0396} = 0.19$

$\Rightarrow v = 0.19\ c$

84. (d) Piezoelectric crystal are those which produce electricity in response to applied mechanical stress and the effect is called piezoelectric effect. Quartz is an example of piezoelectric crystal. Copper, Aluminium and iron produce electricity only when they are connected to external source of electricity.

85. (c) The wave nature of electrons was first experimentally verified by C.J. Davisson and L.H. Germer in 1927.

86. (a) Newton's rings can be obtained in both transmitted and reflected light systems Newton's rings are concentric circles of bright and dark fringe formed by interference of light due to reflected and transmitted wave.

87. (c) For ferromagnetic materials magnetic susceptibility χ is inversely proportional to temperature T with relation.

$\chi = \frac{C}{T}$

88. (c) Iron is most suitable for making electromagnets and core of transformer because out of given materials, iron has less rententivity and coercivity.

89. (c) Unpolarised light can be converted to plane polarised light by double refraction of light, scattering of light and reflection of light but unpolarised light cannot be converted into plane polarised light by diffraction of light.

90. (c) Poynting vector represents the direction of energy flux whose SI unit is watt per square meter (W/m^2) which is the same as the unit of electromagnetic power density. So, Poynting vector and power density both has dimension of MT^{-3}.

91. (a) Here $a = 0$, $b = 2$ and $n = 2$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{2} = 1$$

By trapezoidal rule:

$$\int_0^2 f(x)dx = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + f(x_2)]$$

$$= \frac{1}{2} \times 1\,[4 + 2(3) + 12]$$

$$= \frac{1}{2} \times [22] = 11$$

92. (d) Equations $2x + 3y = 1$ and $4x + 6y = \lambda$ have infinite solutions.

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \Rightarrow \quad \frac{2}{4} = \frac{3}{6} = \frac{1}{\lambda}$$

$$\Rightarrow \quad \lambda = 2$$

93. (d) I : $P = e^y$, $Q = xe^y + 2y$

$$\therefore \quad \frac{\partial Q}{\partial x} = e^y + 0 = e^y \text{ and } \frac{\partial p}{\partial y} = e^y$$

$$\because \quad \frac{\partial Q}{\partial x} = \frac{\partial p}{\partial y}$$ So, it is exact.

II : $P = 2y^2$, $Q = x$

$$\therefore \quad \frac{\partial Q}{\partial x} = 1, \quad \frac{\partial p}{\partial y} = 4y$$

$$\because \quad \frac{\partial Q}{\partial x} \neq \frac{\partial p}{\partial y}$$ So, it is not exact.

94. (c) $\because Z = ae^{by} \sin bx$

$$\frac{\partial z}{\partial x} = ae^{by} \cos bx.b$$

$$\frac{\partial^2 z}{\partial x^2} = -ae^{by} \sin bx.b^2 \qquad ...(i)$$

$$\frac{\partial z}{\partial y} = ae^{by} b. \sin bx$$

$$\Rightarrow \quad \frac{\partial^2 z}{\partial y^2} = ae^{by} . b^2 \sin bx \qquad ...(ii)$$

Adding (i) and (ii), we get

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$

95. (c) $I = \int \frac{e^x}{e^{2x} - 4} dx = \int \frac{e^x}{(e^x)^2 - (2)^2} dx$

Let $e^x = t \quad \Rightarrow \quad e^x dx = dt$

$$I = \int \frac{1}{t^2 - 2^2} dt = \frac{1}{4} \log \left| \frac{t-2}{t+2} \right| + C$$

$$= \frac{1}{4} \log \left| \frac{e^x - 2}{e^x + 2} \right| + C$$

96. (a) The characteristic equations and the compatibility condition are

$$\frac{dx}{x(y^2 + z)} = \frac{dy}{-y(x^2 + z)} = \frac{dz}{(x^2 - y^2)z}$$

Case 1. $\frac{y\,dx}{xy^3 + xyz} = \frac{x\,dy}{-x^3 y - xyz} = \frac{dz}{(x^2 - y^2)z}$

$$\Rightarrow \frac{y\,dx + x\,dy}{xy\,(y^2 - x^2)} = \frac{dz}{(x^2 - y^2)z}$$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

On integration we get

$\Rightarrow xyz = c_1$ …(i)

Case 2. $\frac{x\,dx}{x^2\left(y^2 + z\right)} = \frac{y\,dy}{-y^2\left(x^2 + z\right)} = \frac{dz}{\left(x^2 - y^2\right)z}$

$$\Rightarrow \frac{x\,dx + y\,dy}{z\left(x^2 - y^2\right)} = \frac{dz}{\left(x^2 - y^2\right)z}$$

$\Rightarrow \quad x\,dx + y\,dy = dz$

$\Rightarrow \quad x^2 + y^2 - 2z = cz$

$\therefore \quad \phi\,(xyz, x^2 + y^2 - 2z) = 0$. Where ϕ in an arbitrary function.

97. (a) Let P = Probability of even number

$\therefore$ 3P = Probability of odd number

$\therefore$ 3P + P + 3P + P + 3P + P = 1

$\Rightarrow 12\,P = 1 \Rightarrow P = \frac{1}{12}$

$\Rightarrow P\,(even) = \frac{1}{12}, \quad P\,(odd) = \frac{1}{4}$

Sum of two numbers in twice thrown a dice will be odd if one number is odd and other is even.

$\therefore$ P (Sum is odd) = $2.3c_1.3c_1 \times \frac{1}{12} \times \frac{1}{4} = \frac{3}{8}$

98. (d) Since P is prime number then it is not perfact square. So, $\sqrt{p}$ is irrational number.

99. (d) $I = \int_0^1 \frac{\sin^{-1} x}{x} dx$

Substitute $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$x \to 0$ then $\theta \to 0$

$x \to 1$ then $\theta \to \frac{\pi}{2}$

$\therefore \quad I = \int_0^{\pi/2} \frac{\theta}{\sin\theta} \cos\theta\, d\theta$

$= \int_0^{\pi/2} \theta \cdot \cot\theta\, d\theta$

$\because \quad \int \theta \cdot \cot\theta\, d\theta = \theta \cdot \int \cot\theta\, d\theta - \int \left[\frac{d\theta}{d\theta} \cdot \int \cot\theta\, d\theta\right] d\theta$

$= \theta \log|\sin\theta| - \int \log|\sin\theta|\, d\theta$

$\therefore \quad I = [\theta \log|\sin\theta|]_0^{\pi/2} - \int_0^{\pi/2} \log|\sin\theta|\, d\theta \quad \ldots(1)$

Let $I_1 = \int_0^{\pi/2} \log|\sin\theta|\, d\theta = \int_0^{\pi/2} \log\left|\sin\left(\frac{\pi}{2} - 0\right)\right| d\theta$

$I_1 = \int_0^{\pi/2} \log|\cos\theta|\, d\theta$

$\therefore \quad 2I_1 = \int_0^{\pi/2} \log|\sin\theta|\, d\theta + \int_0^{\pi/2} \log|\cos\theta|\, d\theta$

$2I_1 = \int_0^{\pi/2} \log|\sin\theta \cdot \cos\theta|\, d\theta$

$[\because \log m + \log n = \log(mn)]$

$2I_1 = \int_0^{\pi/2} \log\left|\frac{2\sin\theta \cdot \cos\theta}{2}\right| d\theta$

$2I_1 = \int_0^{\pi/2} \log\left|\frac{\sin 2\theta}{2}\right| d\theta \quad [\because \sin 2\theta = 2\sin\theta \cdot \cos\theta]$

$= \int_0^{\pi/2} \log|\sin 2\theta|\, d\theta - \int_0^{\pi/2} \log 2\, d\theta$

$= 2\int_0^{\pi/4} \log|\sin 2\theta|\, d\theta - \int_0^{\pi/2} \log 2\, d\theta$

$= 2\int_0^{\pi/2} \log|\sin u| \frac{du}{2} - \log 2\left(\frac{\pi}{2} - 0\right)$

(Put $u = 2\theta \Rightarrow du = 2d\theta$)

$= \int_0^{\pi/2} \log|\sin\theta| d\theta - \frac{\pi}{2}\log 2$

$2I_1 = I_1 - \frac{\pi}{2}\log 2$

$2I_1 - I_1 = -\frac{\pi}{2}\log 2$

$I_1 = -\frac{\pi}{2}\log 2$

From eqn. (1)

$I = [\theta \log|\sin\theta|]_0^{\pi/2} - I_1$

$= \frac{\pi}{2}\log(1) - 0 - \left(-\frac{\pi}{2}\log 2\right)$

$= \frac{\pi}{2} \times 0 + \frac{\pi}{2}\log 2$

$= \frac{\pi}{2}\log 2$

100. (a) The standard matrix of T is

$$[T(e_1)T(e_2)T(e_3)] = \begin{bmatrix} 7 & 0 & 1 \\ 0 & -2 & 1 \\ -5 & 9 & 1 \end{bmatrix}$$

101. (b) $\int \frac{1}{(x+1)\sqrt{x^2-1}} dx = \int \frac{1}{(x+1)(x+1)^{\frac{1}{2}}(x-1)^{\frac{1}{2}}} dx$

$= \int \frac{1}{(x+1)^{3/2}(x-1)^{1/2}} dx$

Substitute $(x-1)^{1/2} = u \Rightarrow x = u^2 + 1 \Rightarrow dx = 2u\, du$

$\therefore \quad \frac{1}{(x+1)\sqrt{x^2-1}} dx = \int \frac{1}{(x+1)^{3/2}(x-1)^{1/2}} dx$

$= \int \frac{2u\, du}{(u^2+2)^{3/2} u} = 2\int \frac{du}{(u^2+2)^{3/2}}$

$= 2\int \frac{dv}{2\sec v}, u = 2^{1/2}\tan v$

$= \int \cos v\, dv$

$= \sin v + c$

$= \sin\left(\tan^{-1}\left(\frac{u}{2^{1/2}}\right) + C\right)$

$= \sin\left[\tan^{-1}\frac{(x-1)^{1/2}}{\sqrt{2}}\right] + C$

$= \sin\left[\tan^{-1}\sqrt{\frac{x-1}{2}}\right] + C$

$$= \sin\left[\sin^{-1}\sqrt{\frac{x-1}{x+1}}\right] + C$$

$$= \sqrt{\frac{x-1}{x+1}} + C$$

102. (c) $\oint_c \frac{1}{z^2+4}dz = \frac{1}{2}\left[\tan^{-1}\frac{z}{2}\right]_c$

$\because |z - 2i| = 1$ Shows a circle of centre (0, 2) and radius 1.

$0 \le \tan^{-1} z \le 2\pi \Rightarrow 0 \le \tan^{-1}\frac{z}{2} \le \pi$

$$\therefore \oint_c \frac{1}{z^2+4}dz = \frac{1}{2}\times\pi = \frac{\pi}{2}$$

103. (c) $\lim_{x\to 0}\left(x\sin\frac{1}{x}\right)$

$= \lim_{x\to 0} x \cdot \lim_{x\to 0}\sin\frac{1}{x}$

$= 0 \times$ some finite value

$= 0$

104. (c) Let $z = x + iy$

$\Rightarrow \quad dz = dx + i\,dy$

Since, integration is along the line

$z = 0$ to $z = 1 + i$

That is from (0, 0) to (1, 1)

Then, the equation of straight line passing through (0, 0) to (1, 1)

$$y - 0 = \frac{1-0}{1-0}[x-0]$$

$\Rightarrow \quad y = x \Rightarrow dy = dx$

$$\therefore \quad I = \int_0^1 \left(x - y + ix^2\right)dz = \int_0^1 \left(x - x + ix^2\right)(dx + idy)$$

$$= \int_0^1 \left(ix^2\right)(dx + idx) \qquad [\because dy = dx]$$

$$= (1+i)\int_0^1 ix^2 dx = i(1+i)\left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{i(1+i)}{3} = \left(\frac{i-1}{3}\right)$$

105. (*) $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$

From ratio (1) and (2)

$$\frac{dx}{y^2} = \frac{dy}{x^2} \Rightarrow \int x^2 dx = \int y^2 dy$$

$$\Rightarrow \frac{x^3}{3} = \frac{y^3}{3} + c \Rightarrow x^2 - y^3 = c_1,\ c_1 = 3c$$

From ratio (1) and (3)

$$\frac{dx}{y^2} = \frac{dz}{x^2y^2z^2} \Rightarrow \frac{dx}{1} = \frac{dz}{x^2z^2}$$

$$\Rightarrow \quad x^2dx = \frac{dz}{z^2} \Rightarrow \int x^2dx = \int z^{-2}dz$$

$$\Rightarrow \quad \frac{x^3}{3} = -z^{-1} + k \Rightarrow x^3 = -3z^{-1} + 3k$$

$\Rightarrow \quad x^3 + 3z^{-1} = c_2,\ c_2 = 3k$

$\therefore \quad x^3 - y^3 = c_1,\ x^3 + 3z^{-1} = c_2$

where c_1 and c_2 are arbitrary constant.

106. (c) Since, matrix A is non-singular.

$\therefore \quad |A| \ne 0 \Rightarrow$ All eigenvalues of A are non-zero

$\Rightarrow \quad$ All eigenvalue of A^{-1} are non-zero.

$\Rightarrow \quad \lambda_i \ne 0\ \forall\ i = 1, 2, 3$

107. (a) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} - z = \frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y}$

$$z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} - \frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y}$$

This is the Clairaut's equation

so, $\frac{\partial z}{\partial x} = a, \frac{\partial z}{\partial y} = b$

$\therefore \quad z = ax + by - ab$

108. (d) Let $U_n = D$

$\therefore D^4 - 4D^3 + D^2 + 6D = 0$

$\Rightarrow D(D^3 - 4D^2 + D + 6) = 0$

$\Rightarrow D(D^3 + D^2 - 5D^2 - 5D + 6D + 6) = 0$

$\Rightarrow D[D^2(D+1) - 5D(D+1) + 6(D+1)] = 0$

$\Rightarrow D(D+1)[D^2 - 5D + 6] = 0$

$\Rightarrow D(D+1)[D^2 - 3D - 2D + 6] = 0$

$\Rightarrow D(D+1)(D-3)(D-2) = 0$

$\therefore D = 0, D = -1, D = 3, D = 2.$

$\therefore Un = c_1(-1)^n + c_2(2)^n + c_3(3)^n$. Where c_1, c_2, c_3 are constants.

109. (d) $\int_0^{2\pi}\int_0^{\pi/4}\int_0^1 r^2 \sin\theta\, dr\, d\theta\, d\varphi$

$$= \int_0^{2\pi}\int_0^{\pi/4}\left[\int_0^1 r^2 dr\right]\sin\theta\, d\theta\, d\varphi$$

$$= \int_0^{2\pi}\int_0^{\pi/4}\left[\frac{r^3}{3}\right]_0^1 \sin\theta\, d\theta\, d\varphi$$

$$= \frac{1}{3}\int_0^{2\pi}\left[\int_0^{\pi/4}\sin\theta\, d\theta\right]d\varphi$$

$$= \frac{1}{3}\int_0^{2\pi}\left[-\cos\theta\right]_0^{\pi/4} d\varphi = \frac{1}{3}\int_0^{2\pi}\left[\frac{-1}{\sqrt{2}} + 1\right]d\varphi$$

$$= \frac{\sqrt{2}-1}{3\sqrt{2}}\int_0^{2\pi} d\varphi = \frac{\sqrt{2}-1}{3\sqrt{2}}[\varphi]_0^{2\pi}$$

$$= \frac{2\pi\left(\sqrt{2}-1\right)}{3\sqrt{2}} = \frac{\sqrt{2}\left(\sqrt{2}-1\right)\pi}{3}$$

110. (a) $\because Z\,\bar{Z} + (1+i)Z + (1-i)\bar{Z} = 0$

Let $Z = x + iy \Rightarrow \bar{Z} = x - iy$

$\Rightarrow$ $(x + iy)(x - iy) + (1 + i)(x + iy) + (1 - i)(x - iy) = 0$

$\Rightarrow$ $x^2 + y^2 + x + iy + ix - y + x - iy - ix - y = 0$

$\Rightarrow$ $x^2 + 2x + y^2 - 2y = 0$

$\Rightarrow$ $(x + 1)^2 + (y - 1)^2 = \left(\sqrt{2}\right)^2$

This represent equation of circle with centre (–1, 1) and radius $\sqrt{2}$.

111. (c) $\because \frac{d^3y}{dx^3} - \frac{2d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$

The characteristic equation:

$D^3 - 2D^2 - D + 2 = 0$

$\Rightarrow$ $D^2(D - 2) - 1(D - 2) = 0$

$\Rightarrow$ $(D - 2)(D^2 - 1) = 0 \Rightarrow D = -1, 1, 2.$

$\therefore$ The general solutions are

$y = c_1e^{-x} + c_2e^{x} + c_3e^{2x}$, where $c_1, c_2, c_3 \neq 0$

It will be zero when $c_1 = c_2 = c_3 = 0$

So, e^{-x}, e^{x} and e^{2x} are linearly independent on every real interval.

112. (a) We know that if λ_i are eigenvalue of square matrix A of order 2 then eigenvalues of A^n are $(\lambda_i)^n$.

Given 3, 5 are eigenvalues of a square matrix A of order 2.

$\therefore$ eigenvalues of the matrix A^3

$= (3)^3$ and $(5)^3$ i.e 27 and 125.

113. (a) $\because \frac{x^2d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ ....(1)

Substitute $x = e^z$ or $z = \log x$

Then $\frac{x\,dy}{dx} = Dy$ and $x^2\frac{d^2y}{dx^2} = D(D-1)y$

$\therefore$ Equation (1) becomes

$D(D - 1)y - Dy + y = z$

$\Rightarrow (D^2 - 2D + 1)\,y = z$ ....(2)

Auxiliary equation is

$\lambda^2 - 2\lambda + 1 = 0$

i.e $(\lambda - 1)^2 = 0$

$\Rightarrow \lambda = 1, 1$

C.F $= (c_1 + c_2 z)\,e^x$

$= (c_1 + c_2 \log x)x$

For particular integral

$$\text{P.I} = \frac{1}{D^2 - 2D + 1}z = \left(1 + 2D + 3D^2\right)z$$

$= z + 2 = \log x + 2$

y = C. F. + P.I.

$= (c_1 + c_2 \log x)\,x + \log x + 2.$

114. (a)

115. (c) Suppose $f(x)$ Since, $f(0) = 3, f(1) = 5, f(3) = 21$

Then the data points are (0, 3), (1, 5), (3, 21)

Suppose $f(x)$ be any quadratic polynomial and (x_0, y_0), (x_1, y_1), (x_2, y_2), be some points then by Newton's divided difference polynomial method.

$f(x) = a + b(x - x_0) + c(x - x_0)(x - x_1)$

$\because x_0 = 0, x_1 = 1$

$\therefore f(x) = a + bx + cx(x - 1)$ ...(1)

Since, $f(0) = 3$

$\therefore f(0) = a + 0 + 0$

$3 = a$

$\Rightarrow a = 3$

Put $a = 3$ in equation (1)

$f(x) = 3 + bx + c\,x(x - 1)$ ...(2)

$\because f(1) = 5$

$\therefore$ $3 + b + 0 = 5$

$\Rightarrow b = 5 - 3 = 2$

Put b = 2 in eqn. (2)

$f(x) = 3 + 2x + cx(x - 1)$ (3)

$\because f(3) = 21$

$\Rightarrow c = 2$

Put c = 2 in eqn. (3), then

$f(x) = 3 + 2x + 2x(x - 1) = 3 + 2x^2$

116. (a) Let $f(x) = x^2 - x - 1$ and x_1 be the mid-point of the interval (1, 2). Then, 1st approximation

$$x_1 = \frac{1+2}{2} = \frac{3}{2}$$

$$f(x_1) = f\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{3}{2} - 1$$

$$= \frac{9-6-4}{4} = -\frac{1}{4}$$

$$\Rightarrow f(x_1) = -\frac{1}{4} < 0$$

$\therefore$ Root lies between $\left(\frac{3}{2}, 2\right)$

Now, 2nd approximation

$$x_2 = \frac{\frac{3}{2}+2}{2} = \frac{7}{4} = 1.75$$

117. (b) Suppose (x_n) be a Cauchy sequence.

Then $\forall \in > O \exists N$ & t $n > N$ and $m > N$

$\Rightarrow$ $|x_n - x_m| < \in$

$\Rightarrow$ $x_m - \in < x_n < x_m + \in$

$\Rightarrow$ (x_n) is a bounded sequence

$\Rightarrow$ Cauchy sequence is a bounded sequence.

118. (a) $y = x^{3/2}$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$$

Length of the curve over the internal [0, 1] is

$$L = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^1 \sqrt{1+\left(\frac{3}{2}x^{\frac{1}{2}}\right)^2}\,dx$$

$$= \int_0^1 \sqrt{1+\frac{9}{4}x}\,dx$$

Substitute $1 + \frac{9}{4}x = u \Rightarrow dx = \frac{4}{9}du$

$x \to 0$ then $u \to 1$ and $x \to 1$ then $u \to \frac{13}{4}$

$$\therefore \quad L = \int_1^{\frac{13}{4}} \sqrt{u}\left(\frac{4}{9}\right)du = \frac{4}{9}\int_1^{\frac{13}{4}} \sqrt{u}\,du$$

$$= \frac{4}{9}\left[\frac{4^{3/2}}{\frac{3}{2}}\right]_1^{13/4} = \frac{8}{27}\left[u^{3/2}\right]_1^{13/4}$$

$$= \frac{1}{27}\left[(13)^{3/2} - 8\right] \text{ units}$$

119. (d) Find the rank of matrix corresponding to the vectors of S_1

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 1 & 5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} (C_2 \to C_2 - 5C_3)$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} (C_1 \to C_1 - 2C_2 - 3C_2 - C_3)$$

Rank (A) = 3 less than number of vectors in S_1

$\Rightarrow$ S_1 contains linearly dependent vector

$\Rightarrow$ S_1 is linearly dependent

Since, S_2 contains 0 vector and the set containing 0 vector is always linearly dependent.

So, S_1 and S_2 both are linearly dependent.

120. (a) Since, the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ and $\int_1^{\infty} \frac{1}{x^p}dx$ be have alike

Thus $\int_1^{\infty} \frac{1}{x^p}dx = \left[\frac{x^{-p+1}}{-p+1}\right]_1^{\infty}$

For $0 < p \le 1$ integral $\int_1^{\infty} \frac{1}{x^p}dx$ diverges

$\Rightarrow$ $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges

For $p > 1$ integral $\int_1^{\infty} \frac{1}{x^p}dx$ converges

$\Rightarrow$ $\int_{n=1}^{\infty} \frac{1}{n^p}$ converges