NDA SOLVED PAPER 2023-I

MATHEMATICS

1. If ω is a non-real cube root of 1, then what is the value of $\left|\frac{1-\omega}{\omega+\omega^2}\right|$?

(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 1 (d) $\frac{4}{\sqrt{3}}$

2. What is the number of 6 digit numbers that can be formed only by using 0, 1, 2, 3, 4 and 5 (each once); and divisible by 6?

(a) 96 (b) 120 (c) 192 (d) 312

3. What is the binary number equivalent to decimal number 1011?

(a) 1011 (b) 111011
(c) 11111001 (d) 111110011

4. Let A be a matrix of order 3 × 3 and |A| = 4. If $|2\ \text{adj}(3A)|=2^{\alpha}3^{\beta}$, then what is the value of (α + β)?

(a) 12 (b) 13 (c) 17 (d) 24

5. If α and β are the distinct roots of equation $x^2-x+1=0$, then what is the value of $\left|\frac{\alpha^{100}+\beta^{100}}{\alpha^{100}-\beta^{100}}\right|$?

(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 1 (d) $\frac{1}{\sqrt{3}}$

6. Let *A* and *B* be symmetric matrices of same order, then which one of the following is correct regarding (*AB* –*BA*)?

1. Its diagonal entries are equal but non-zero
2. The sum of its non-diagonal entries is zero

Select the correct answer using the code given below:

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

7. Consider the following statements in respect of square matrices *A*, *B*, *C* each of same order *n*:

1. $AB = AC \Rightarrow B = C$ if *A* is non-singular
2. If $BX = CX$ for every column matrix X having n rows then $B = C$

Which of the statement given above is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

8. The system of linear equations $x + 2y + z = 4$, $2x + 4y + 2z = 8$ and $3x + 6y + 3z = 10$ has

(a) a unique solution (b) infinite many solutions
(c) no solution (d) exactly three solutions

9. Let $AX = B$ be a system of 3 liner equations with 3-unknowns. Let X_1 and X_2 be its two distinct solutions. If the combination aX_1+bX_2 is a solution of $AX = B$; where a, b, are real numbers, then which one of the following is correct?

(a) $a = b$ (b) $a + b = 1$
(c) $a + b = 0$ (d) $a - b = 1$

10. What is the sum of the roots of the equation $\begin{vmatrix} & x-a & x-b \\ 0 & 0 & x \quad c \\ x+b & x+c & \end{vmatrix}=0$

(a) a + b + c (b) a – b + c
(c) a + b – c (d) $a - b - c$

11. If $2-i\sqrt{3}$ where $i=\sqrt{-1}$ is a root of the equation $x^2+ ax + b = 0$, then what is the value of $(a + b)$?

(a) –11 (b) –3 (c) 0 (d) 3

12. If $\frac{1}{1}\ \frac{\sqrt{3}}{\sqrt{3}}$ where $i=\sqrt{-1}$, then what is the argument of z?

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{5\pi}{6}$

13. If *a*, *b*, *c* are in AP, then what is $\begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+3 \end{vmatrix}$ equal to?

(a) –1 (b) 0 (c) 1 (d) 2

14. If $\log_x a$, a^x and $\log_b x$ are in G.P, then what is *x* equal to?

(a) $\log_a(\log_b a)$
(b) $\log_b(\log_a a)$
(c) $\frac{\log_a(\log_b a)}{2}$
(d) $\log_b(\log_a\ )$

15. If $2^{\frac{1}{c}}, 2^{\frac{b}{ac}}, 2^{\frac{1}{a}}$ are in GP, then which one of the following is correct?
(a) *a, b, c* are in AP (b) *a, b, c* are in GP
(c) *a, b, c* are in HP (d) *ab*, *bc*, *ca* are in AP

16. The first and the second terms of an AP are $\frac{5}{2}$ and $\frac{23}{12}$ respectively. If n^{th} terms is the largest negative term, what is the value of *n*?
(a) 5 (b) 6
(c) 7 (d) *n* cannot be determined

17. For how many integral values of *k*, the equation $x^2 - 4x + k = 0$, where *k* is an integer has real roots and both off them lie in the interval (0, 5)?
(a) 3 (b) 4 (c) 5 (d) 6

18. In an AP, the first term is *x* and the sum of the first *n* terms is zero. What is the sum of next *m* terms?
(a) $\frac{mx(m+n)}{n-1}$ (b) $\frac{mx(m+n)}{1-n}$
(c) $\frac{nx(m+n)}{m-1}$ (d) $\frac{nx(m+n)}{1-m}$

19. Consider the following statements:
1. (25)! + 1 is divisible by 26
2. (6)! + 1 is divisible by 7
Which of the above statement is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

20. If *z* is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then what is $|z|$ equal to?
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 1 (d) 2

21. How many real numbers satisfy the equation $|x-4|+|x-7|=15$?
(a) Only one (b) Only two
(c) Only three (d) Infinitely many

22. A mapping $f: A \to B$ defined as $f(x)=\frac{2x+3}{3x+5}, x \in A.$. If *f* is to be onto, then what are *A* and *B* equal to?
(a) $A = R \setminus \left\{-\frac{5}{3}\right\}$ and $B = R \setminus \left\{-\frac{2}{3}\right\}$
(b) A = R and B = $B = R \setminus \left\{-\frac{5}{3}\right\}$
(c) $A = R \setminus \left\{-\frac{3}{2}\right\}$ and B = R\{0}
(d) $A = R \setminus \left\{-\frac{5}{3}\right\}$ and $B = R \setminus \left\{\frac{2}{3}\right\}$

23. α and β are distinct real roots of the quadratic equation $x^2 + ax + b = 0$. Which of the following statements is/are sufficient to find α?
1. $\alpha+\beta=0, \alpha^2+\beta^2=2$ 2. $\alpha\beta^2=-1, a=0$
Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

24. If the sixth term in the binomial expansion of $\left(x^{-\frac{8}{3}}+x^2\log_{10}x\right)^8$ is 5600, then what is the value of *x*?
(a) 6 (b) 8 (c) 9 (d) 10

25. How many terms are there in the expansion of $(3x-y)^4(x+3y)^4$?
(a) 9 (b) 12 (c) 15 (d) 17

26. p, q, r and s are in AP such that p + s = 8 and qr = 15. What is the difference between largest and smallest numbers?
(a) 6 (b) 5 (c) 4 (d) 3

27. Consider the following statements for a fixed natural number *n*:
1. C(*n*, *r*) is greatest if $n = 2r$
2. C(*n*, *r*) is greatest if $n = 2r - 1$ and $n = 2r + 1$
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

28. *m* parallel lines cut *n* parallel lines giving rise to 60 parallelograms. What is the value of $(m + n)$?
(a) 6 (b) 7 (c) 8 (d) 9

29. Let *x* be the number of permutations of the word 'PERMUTATIONS' and y be the number of permutations of the word 'COMBINATIONS'. Which one of the following is correct?
(a) $x = y$ (b) $y = 2x$ (c) $x = 4y$ (d) $y = 4x$

30. 5- digit numbers are formed using the digits 0, 1, 2, 4, 5 without repetition. What is the percentage of numbers which are greater than 50,000?
(a) 20% (b) 25% (c) $\frac{100}{3}\%$ (d) $\frac{110}{3}\%$

Consider the following for the next two (02) items that follow:
Let $\sin\beta$ be the GM of sin α and cos α; tan γ be the AM of sin α and cos α

31. What is cos 2β equal to?
(a) $(\cos\alpha-\sin\alpha)^2$ (b) $(\cos\alpha+\sin\alpha)^2$
(c) $(\cos\alpha-\sin\alpha)^3$ (d) $\frac{(\cos\alpha-\sin\alpha)^2}{2}$

32. What is the value of of sec 2γ?
(a) $\frac{3-\sin 2\alpha}{5+2\sin 2\alpha}$ (b) $\frac{5+\sin 2\alpha}{3-\sin 2\alpha}$
(c) $\frac{3-2\sin 2\alpha}{4+\sin 2\alpha}$ (d) $\frac{3-\sin 2\alpha}{4+3\sin 2\alpha}$

Consider the following for the next two (02) items that follow:

A flagstaff 20 m long standing on a pillar 10 m high subtends an angle $\tan^{-1}(0.5)$ at a point P on the ground. Let θ be the angle subtended by the pillar at this point P.

33. If x is the distance of P from bottom of the pillar, then consider the following statements:

1. x can take two values which are in the ratio 1 : 3
2. x can be equal to height of the flagstaff

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

34. What is a possible value of tan θ?

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Consider the following for the next two (02) items that follow:

The perimeter of a triangle ABC is 6 times the AM of sine of angle of the triangle. Further $BC = \sqrt{3}$ and $CA = 1$.

35. What is the perimeter of the triangle?

(a) $\sqrt{3}+1$ (b) $\sqrt{3}+2$ (c) $\sqrt{3}+3$ (d) $2\sqrt{3}+1$

36. Consider the following statements:

1. ABC is right angled triangle
2. The angles of the triangle are in AP

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Consider the following for the next two (02) items that follow:

Let $x = \frac{\sin^2 A + \sin A + 1}{\sin A}$ where $0 < A \le \frac{\pi}{2}$

37. What is the minimum value of x?

(a) 1 (b) 2 (c) 3 (d) 4

38. At what value of A does x attain the minimum value?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Consider the following for the next two (02) items that follow:

In the triangle ABC, $a^2 + b^2 + c^2 = ac + \sqrt{3}bc$

39. What is the nature of the triangle?

(a) Equilateral
(b) Isosceles
(c) Right angled triangle
(d) Scalene but not right angled

40. If c = 8, what is the area of the triangle?

(a) $4\sqrt{3}$ (b) $6\sqrt{3}$
(c) $8\sqrt{3}$ (d) $12\sqrt{3}$

Consider the following for the next two (02) items that follow:

Consider the function

$f(x) = |x-2| + |3-x| + |4-x|$, where $x \in R$.

41. At what value of x does the function attain minimum value?

(a) 2 (b) 3 (c) 4 (d) 0

42. What is the minimum value of the function?

(a) 2 (b) 3 (c) 4 (d) 0

Consider the following for the next two (02) items that follow:

Consider the sum

S = 0! + 1! + 2! + 3! + 4! + … + 100!

43. If the sum S is divided by 8, what is the remainder?

(a) 0 (b) 1
(c) 2 (d) cannot be determined

44. If the sum S is divided by 60, what is the remainder?

(a) 1 (b) 3 (c) 17 (d) 34

Consider the following for the next two (02) items that follow:

In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the in-circle of the triangle touches the sides PQ, QR and RP at N, L and M respectively such that the lengths PN, QL and RM are n, $n+2$, $n+4$ respectively where n is an integer.

45. What is the value of n?

(a) 4 (b) 6 (c) 8 (d) 10

46. What is the length of the smallest side?

(a) 12 (b) 14 (c) 16 (d) 18

Consider the following for the next two (02) items that follow:

Given that

$\sin x + \cos x + \tan x + \cot x + \sec x + \text{cosec} x = 7$

47. The given equation can be reduced to

(a) $\sin^2 2x - 44\sin 2x + 36 = 0$
(b) $\sin^2 2x + 44\sin 2x - 36 = 0$
(c) $\sin^2 2x - 22\sin 2x + 18 = 0$
(d) $\sin^2 2x - 22\sin 2x + 18 = 0$

48. If $\sin 2x = a - b\sqrt{c}$, where a and b are natural numbers and c is prime number, then what is the value of a – b + 2c ?

(a) 0 (b) 14 (c) 21 (d) 28

Consider the following for the next two (02) items that follow:

A quadratic equation is given by

$(3+2\sqrt{2})x^2 - (4+2\sqrt{3})x + (8+4\sqrt{3}) = 0$

49. What is the HM of the roots of the equation?

(a) 2 (b) 4
(c) $2\sqrt{2}$ (d) $2\sqrt{3}$

50. What is the GM of the root of the equation?

(a) $\sqrt{2}(\sqrt{6}-\sqrt{3}+\sqrt{2}-1)$ (b) $\sqrt{2}(\sqrt{6}+\sqrt{3}-\sqrt{2}-1)$

(c) $(\sqrt{6}-\sqrt{3}+\sqrt{2}-1)$ (d) $(\sqrt{6}+\sqrt{3}+\sqrt{2}-1)$

Consider the following for the next two (02) items that follow:

Let $\Delta(a,b,c,\alpha) = \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$

51. If $\Delta(a, b, c, \alpha) = 0$ for every $\alpha > 0$, then which one of the following is correct ?
(a) a, b, c are in AP (b) a, b, c are in GP
(c) a, $2b$, c are in AP (d) a, $2b$, c are in GP

52. If $\Delta(7,4,2,\alpha) = 0$, then α is a root of which one of the following equation?
(a) $7x^2 + 4x + 2 = 0$ (b) $7x^2 - 4x + 2 = 0$
(c) $7x^2 + 8x + 2 = 0$ (d) $7x^2 - 8x + 2 = 0$

Consider the following for the next two (02) items that follow:
Given that $m(\theta) = \cot^2\theta + n^2\tan^2\theta + 2n$, where n is a fixed positive real number.

53. What is the least value of m (θ)?
(a) n (b) $2n$ (c) $3n$ (d) $4n$

54. Under what condition does m attain the least value?
(a) $n = \tan^2\theta$ (b) $n = \cot^2\theta$
(c) $n = \sin^2\theta$ (d) $n = \cos^2\theta$

Consider the following for the next two (02) items that follow:
A quadrilateral is formed by the lines x = 0, y = 0. x + y = 1 and $6x + y = 3$.

55. What is the equation of diagonal through origin?
(a) $3x + y = 0$ (b) $2x + 3y = 0$
(c) $3x - 2y = 0$ (d) $3x + 2y = 0$

56. What is the equation of other diagonal?
(a) $x + 2y - 1 = 0$ (b) $x - 2y - 1 = 0$
(c) $2x + y + 1 = 0$ (d) $2x + y - 1 = 0$

Consider the following for the next two (02) items that follow:
P(x, y) is any point on the ellipse $x^2 + 4y^2 = 1$. Let E, F be the foci of the ellipse.

57. What is $PE + PF$ equal to?
(a) 1 (b) 2 (c) 3 (d) 4

58. Consider the following points:
1. $\left(\frac{\sqrt{3}}{2}, 0\right)$ 2. $\left(\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$
3. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{4}\right)$

Which of the following points lie on latus rectum of ellipse?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

Consider the following for the next two (02) items that follow:
The line y = x partitions the circle $(x-a)^2 + y^2 = a^2$ in two segments.

59. What is the area of minor segment?
(a) $\frac{(\pi-2)a^2}{4}$ (b) $\frac{(\pi-1)a^2}{4}$
(c) $\frac{(\pi-2)a^2}{2}$ (d) $\frac{(\pi-1)a^2}{2}$

60. What is the area of major segment?
(a) $\frac{(3\pi-2)a^2}{4}$ (b) $\frac{(3\pi+2)a^2}{4}$
(c) $\frac{(3\pi-2)a^2}{2}$ (d) $\frac{(3\pi+2)a^2}{2}$

Consider the following for the next two (02) items that follow:
Let A(1, –1, 2) and B(2, 1, –1) be the end points of the diameter of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$.

61. What is $u + v + w$ equal to?
(a) –2 (b) –1 (c) 1 (d) 2

62. If $P(x, y, z)$ is any point on the sphere, then what is $PA^2 + PB^2$ equal to?
(a) 15 (b) 14 (c) 13 (d) 6.5

Consider the following for the next two (02) items that follow:
Consider two lines whose direction ratios are (2, –1, 2) and (k, 3, 5). They are inclined at an angle $\frac{\pi}{4}$.

63. What is the value of k?
(a) 4 (b) 2 (c) 1 (d) –1

64. What are the direction ratios of a line which is perpendicular to both the lines?
(a) (1, 2, 10) (b) (–1, –2, 10)
(c) (11, 12, –10) (d) (11, 2, –10)

Consider the following for the next two (02) items that follow:
Let $\vec{a} = 3\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{j} - \hat{k}$. Let $\vec{b}$ be such that $\vec{a}\cdot\vec{b} = 27$ and $\vec{a}\times\vec{b} = \overrightarrow{9c}$

65. What is $\vec{b}$ equal to?
(a) $3\hat{i} + 4\hat{j} + 2\hat{k}$ (b) $5\hat{i} + 2\hat{j} + 2\hat{k}$
(c) $5\hat{i} - 2\hat{j} + 6\hat{k}$ (d) $3\hat{i} + 3\hat{j} + 4\hat{k}$

66. What is the angle between $(\vec{a} + \vec{b})$ and $\vec{c}$?
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

Consider the following for the next two (02) items that follow:
Let a vector $\vec{a} = 4\hat{i} - 8\hat{j} + \hat{k}$ make angles α, β, γ with the positive directions of x, y, z axes respectively.

67. What is cos α equal to?
(a) $\frac{1}{3}$ (b) $\frac{4}{9}$
(c) $\frac{5}{9}$ (d) $\frac{2}{3}$

68. What is $\cos 2\beta + \cos 2\gamma$ equal to?

(a) $-\frac{32}{81}$ (b) $-\frac{16}{81}$

(c) $\frac{16}{81}$ (d) $\frac{32}{81}$

Consider the following for the next two (02) items that follow:

The position vectors of two points A and B are $\hat{i} - \hat{j}$ and $\hat{j} + \hat{k}$ respectively.

69. Consider the following points:

1. (–1, –3, 1)
2. (–1, 3, 2)
3. (–2, 5, 3)

Which of the above points lie on the line joining *A* and *B*?

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

70. What is the magnitude of $\overrightarrow{AB}$?

(a) 2 (b) 3

(c) $\sqrt{6}$ (d) $\sqrt{3}$

Consider the following for the next three (03) items that follow:

Let $f(x) = Pe^x + Qe^{2x} + Re^{3x}$, where *P*, *Q*, *R* are real numbers. Further $f(0) = 6$, $f'(\ln 3) = 282$ and $\int_0^{\ln 2} f(x)dx = 11$

71. What is the value of Q?

(a) 1 (b) 2

(c) 3 (d) 4

72. What is the value of R?

(a) 1 (b) 2

(c) 3 (d) 4

73. What is $f'(0)$ equal to?

(a) 18 (b) 16

(c) 15 (d) 14

Consider the following for the next two (02) items that follow:

Suppose E is the differential equation representing family of curves $y^2 = 2cx + 2c\sqrt{c}$ where c is a positive parameter.

74. What is the order of the differential equation?

(a) 1 (b) 2

(c) 3 (d) 4

75. What is the degree of the differential equation?

(a) 2 (b) 3

(c) 4 (d) Degree does not exist

Consider the following for the next three (03) items that follow:

Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

76. What is $f(0)$ equal to?

(a) –1 (b) 0

(c) 1 (d) 2

77. What is $\lim_{x\to 0} \frac{f(x)}{x}$ equal to?

(a) – 1 (b) 0

(c) 1 (d) 2

78. What is $\lim_{x\to 0} \frac{f(x)}{x^2}$ equal to?

(a) –1 (b) 0

(c) 1 (d) 2

Consider the following for the next two (02) items that follow:

Let $f(x) = \sin[\pi^2]x + \cos[-\pi^2]x$ where [.] is a greatest integer function.

79. What is $f\left(\frac{\pi}{2}\right)$ equal to?

(a) –1 (b) 0

(c) 1 (d) 2

80. What is $f\left(\frac{\pi}{4}\right)$ equal to?

(a) $-\frac{1}{\sqrt{2}}$ (b) –1 (c) 1 (d) $\frac{1}{\sqrt{2}}$

Consider the following for the next three (03) items that follow:

Let $I_1 = \int_0^{\pi} \frac{x}{1+\cos^2 x}dx$ and $I_2 = \int_0^{\pi} \frac{1}{1+\sin^2 x}dx$

81. What is the value of $\frac{I_1 + I_2}{I_1 - I_2}$?

(a) 1 (b) π (c) π^2 (d) $\frac{\pi+1}{\pi-1}$

82. What is the value of $8I_1^2$?

(a) π (b) π^2 (c) π^3 (d) π^4

83. What is the value of I_2?

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2\sqrt{2}}$

(c) $\frac{3\pi}{2\sqrt{2}}$ (d) $\frac{\pi}{4\sqrt{2}}$

Consider the following for the next two (02) items that follow:

Let $l = \int_a^b \frac{|x|}{x} dx, \quad a < b$

84. What is l equal to when $a < 0 < b$?
(a) $a + b$ (b) $a - b$
(c) $b - a$ (d) $\frac{(a+b)}{2}$

85. What is l equal to when a < b < 0?
(a) $a + b$ (b) $a - b$
(c) $b - a$ (d) $\frac{(a+b)}{2}$

Consider the following for the next three (03) items that follow:

Let $f(x) = |\ln x|, x \neq 1$

86. What is the derivative of $f(x)$ at $x = 0.5$?
(a) -2 (b) -1
(c) 1 (d) 2

87. What is the derivative of $f(x)$ at $x = 2$?
(a) $-\frac{1}{2}$ (b) -1 (c) $\frac{1}{2}$ (d) 2

88. What is the derivative of $fof(x)$, where 1 < x < 2?
(a) $\frac{1}{\ln x}$ (b) $\frac{1}{x \ln x}$ (c) $-\frac{1}{\ln x}$ (d) $-\frac{1}{x \ln x}$

Consider the following for the next two (02) items that follow:

Let $f(x) = \begin{cases} x+6, & x \leq 1 \\ px+q, & 1 < x < 2 \\ 5x, & x \geq 2 \end{cases}$

and $f(x)$ is continuous.

89. What is the value of p?
(a) 2 (b) 3 (c) 4 (d) 5

90. What is the value of q?
(a) 2 (b) 3 (c) 4 (d) 5

91. Consider the following statements:
1. $f(x) = \ln x$ is increasing in $(0, \infty)$
2. $g(x) = e^x + e^{\frac{1}{x}}$ is decreasing in $(0, \infty)$

Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

92. What is the derivative of $\sin^2 x$ with respect to $\cos^2 x$?
(a) -1 (b) 1
(c) $\sin 2x$ (d) $\cos 2x$

93. For what value of m with $m < 0$, is the area bounded by the lines $y = x$, $y = mx$ and $x = 2$ equal to 3?
(a) $-\frac{1}{2}$ (b) -1 (c) $-\frac{3}{2}$ (d) -2

94. What is the derivative of cosec(x°)?
(a) – cosec(x°) cot (x°) (b) $-\frac{\pi}{180}\text{cosec}(x^\circ)\cot(x^\circ)$
(c) $\frac{\pi}{180}\text{cosec}(x^\circ)\cot(x^\circ)$ (d) $-\frac{\pi}{180}\text{cosec}(x)\cot(x)$

95. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = 0$ is
(a) $y = 2x$ (b) $y = 2x + 4$
(c) $y = x^2 - 1$ (d) $y = \frac{(x^2-2)}{2}$

96. If $f(x) = x^2 + 2$ and $g(x) = 2x - 3$, then what is $(fg)(1)$ equal to?
(a) 3 (b) 1 (c) –2 (d) –3

97. What is the range of the function $f(x) = x + |x|$ if the domain is the set of real numbers?
(a) $(0, \infty)$ (b) $[0, \infty)$
(c) $(-\infty, \infty)$ (d) $[1, \infty)$

98. If $f(x) = x(4x^2 - 3)$, then what is $f(\sin\theta)$ equal to?
(a) $-\sin 3\theta$ (b) $-\cos 3\theta$
(c) $\sin 3\theta$ (d) $-\sin 4\theta$

99. What is $\lim_{x \to 5} \frac{5-x}{|x-5|}$ equal to?
(a) –1 (b) 0
(c) 1 (d) Limit does not exist

100. What is $\lim_{x \to 1} \frac{x^9 - 1}{x^3 - 1}$ equal to?
(a) –1 (b) –3
(c) 3 (d) Limit does not exist

101. The mean and variance of five observations are 14 and 13.2 respectively. Three of the five observations are 11, 16 and 20. What are the other two observations?
(a) 8 and 15 (b) 9 and 14
(c) 10 and 13 (d) 11 and 12

102. Let A and B be two independent events such that $P(\bar{A}) = 0.7$, $P(\bar{B}) = k$, $P(A \cup B) = 0.8$

What is the value of k?
(a) $\frac{5}{7}$ (b) – (c) $\frac{2}{7}$ (d) $\frac{1}{7}$

103. A biased coin with the probability of getting head equal t $\frac{1}{4}$ is tossed five times. What is the probability of getting tail in all the first four tosses followed by head?
(a) $\frac{81}{512}$ (b) $\frac{81}{1024}$
(c) $\frac{81}{256}$ (d) $\frac{27}{1024}$

104. A coin is biased so that heads comes up thrice as likely as tails. In four independent tossess of the coin, what is probability of getting exactly three heads?

(a) $\frac{81}{256}$ (b) $\frac{27}{64}$

(c) $\frac{27}{256}$ (d) $\frac{9}{256}$

105. Let X and Y be two random variables such that X + Y = 100. If X follows Binomial distribution with parameters n = 100 and $p = \frac{4}{5}$, what is the variance of Y?

(a) 1 (b) $\frac{1}{2}$

(c) 16 (d) $\frac{1}{16}$

106. If two lines of regression are $x + 4y + 1 = 0$ and $4x + 9y + 7 = 0$, then what is the value of x when $y = -3$?

(a) –13 (b) –5

(c) 5 (d) 7

107. The central angles p, q, r and s (in degrees) of four sectors in a Pie Chart satisfy the relation $9p = 3q = 2r = 6s$. What is the value of $4p - q$?

(a) 12 (b) 24

(c) 30 (d) 36

108. The observations 4, 1, 4, 3, 6, 2, 1, 3, 4, 5, 1, 6 are outputs of 12 dices thrown simultaneously. If m and M are means of lowest 8 observations and highest 4 observations respectively, then what is $(2m + M)$ equal to?

(a) 10 (b) 12

(c) 17 (d) 21

109. The bivariate data set contains only two points (–1, 1) and (3, 2) What will be the line of regression of y on x?

(a) $x - 4y + 5 = 0$ (b) $3x + 2y - 1 = 0$

(c) $x + 4y + 1 = 0$ (d) $5x - 4y + 1 = 0$

110. A die is thrown 10 times and obtained the following outputs:
1, 2, 1, 1, 2, 1, 4, 6, 5, 4
What will be the mode of data so obtained?

(a) 6 (b) 4

(c) 2 (d) 1

111. Consider the following frequency distribution:

x	1	2	3		5
f	4	6	9		7

What is the value of median of the distribution?

(a) 1 (b) 2

(c) 3 (d) 3.5

112. For data –1, 1, 4, 3, 8, 12, 17, 19, 9, 11; if M is the median of first 5 observations and N is the median of last five observations then what is the value of $4M - N$?

(a) 7 (b) 4

(c) 1 (d) 0

113. Let P, Q, R represent mean, median and mode. If for some distribution $5P = 4Q = \frac{R}{2}$, then what is $\frac{P+Q}{2P+0.7R}$ equal to?

(a) $\frac{1}{12}$ (b) $\frac{1}{7}$

(c) $\frac{2}{9}$ (d) $\frac{1}{4}$

114. If G is the geometric mean of numbers $1, 2, 2^2, 2^3, \ldots, 2^{n-1}$, then what is the value of $1 + 2 \log_2 G$?

(a) 1 (b) 4

(c) $n - 1$ (d) n

115. If H is the harmonic mean of numbers $1, 2. 2^2, 2^3, \ldots, 2^{n-1}$, then what is n/H equal to?

(a) $2 - \frac{1}{2^{n+1}}$ (b) $2 - \frac{1}{2^{n-1}}$

(c) $2 + \frac{1}{2^{n-1}}$ (d) $2 - —$

116. Let P be the median, Q be the mean and R be the mode of observations $x_1, x_2, x_3, \ldots x_n$. Let $S = \sum_{i=1}^{n} (2x_i - a)^2$. S takes minimum value, when a is equal to

(a) P (b) $\frac{Q}{2}$ (c) $2Q$ (d) R

117. One bag contains 3 white and 2 black balls, another bag contains 2 white and 3 black balls. Two balls are drawn from the first bag and put it into the second bag and then a ball is drawn from the second bag. What is the probability that it is white?

(a) $\frac{6}{7}$ (b) $\frac{33}{70}$ (c) $\frac{3}{10}$ (d) $\frac{1}{70}$

118. Three dice are thrown. What is the probability that each face shows only multiples of 3?

(a) $\frac{1}{9}$ (b) $\frac{1}{18}$ (c) $\frac{}{27}$ (d) $\frac{1}{3}$

119. What is the probability that the month of December has 5 Sundays ?

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{3}{7}$ (d) $\frac{2}{7}$

120. A natural number n is chosen from the first 50 natural numbers. What is the probability that $n + \frac{50}{n} < 50$?

(a) $\frac{23}{25}$ (b) $\frac{47}{50}$

(c) $\frac{24}{25}$ (d) $\frac{49}{50}$

GENERAL ABILITY

PART - A: ENGLISH

Directions : Each item in this section has a sentence with three underlined parts labelled (a), (b) and (c). Read each sentence to find out whether there is any error in any underlined part. Indicate your response in the Answer Sheet against the corresponding letter *i.e.,* (a) or (b) or (c). If you find no error, your response should be indicated as (d).

1. Over long periods of time, /(a) layers of sediments builds up /(b) to a height of a few kilometers. /(c) No Error /(d).

2. When a gas is /(a) cooled down it turns into a liquid /(b) from a process called condensation. /(c) No Error /(d).

3. If you want /(a) to know the news /(b) you can read a newspaper. /(c) No Error /(d).

4. Columbus made his /(a) first voyage from Europe to America /(b) on 1492. /(c) No Error /(d).

5. Whenever the sky is /(a) clear, you can see /(b) the stars in the night. /(c) No Error /(d).

6. I'm not working tomorrow, /(a) so I don't had to /(b)get up early /(c) No Error /(d).

7. She didn't /(a) tell anybody /(b) about her plans. /(c) No Error /(d).

8. She wouldn't have /(a) has an accident, /(b) if she had driven carefully. /(c) No Error /(d).

9. I will watch film /(a) if I finish the work /(b) in time. /(c) No Error /(d).

10. In 1989, the government /(a) did an about-face and Iran restored /(b) it's family planning program. /(c) No Error /(d).

Directions : Each item in this section consists of a sentence with an underlined word followed by four options, (a), (b), (c) and (d). Select the option that is ***nearest in meaning*** to the underlined word and mark your response in your Answer Sheet accordingly.

11. During the pandemic the indigent people had to suffer a lot.
(a) very poor (b) opulent
(c) solvent (d) prosperous

12. She had no idea what made him angry in one minute and jovial the next.
(a) aggrieved (b) melancholic
(c) doleful (d) mirthful

13. It is sheer lunacy to drive a car in this frosty weather.
(a) prudence (b) normalcy
(c) insanity (d) sanity

14. Operating on a child with cancer needs meticulous planning and teamwork.
(a) strong (b) long (c) playful (d) scrupulous

15. The thrill of over-speeding the vehicle can be exhilarating, but it is important not to take the consequences lightly.
(a) humdrum
(b) dreary
(c) exciting
(d) agitating

16. The redemption will now depend on his new strategy of inclusiveness.
(a) retrieval (b) forfeiture
(c) corporation (d) desecration

17. Only three candidates are now in contention for the title.
(a) involved in dispute
(b) in agreement with each other
(c) chance of winning
(d) amiable to each other

18. All my fishing paraphernalia is in the car.
(a) boxes (b) accessories
(c) fuel (d) food

19. The public watched in astonishment as he took a sudden jump the bridge.
(a) anticipation (b) hurriedly
(c) wonderment (d) calmness

20. Drinking inordinate amount of liquor is not good for health.
(a) temperate (b) exorbitant
(c) moderate (d) regular

Directions : Each of the following items in this section consists of a sentence, the parts of which have been jumbled. These parts have been labelled P, Q, R and S. Given below each sentence are four sequences namely (a), (b), (c) and (d). You are required to re-arrange the jumbled parts of the sentence correctly and mark your response accordingly.

21. because I feel /(P) you remember it better /(Q) when you write something, /(R) I make at least three drafts of a song /(S)
The correct sequence should be :
(a) SPRQ (b) RQPS
(c) QRSP (d) PRSQ

22. for hours in the shop /(P) a gunman who held /(Q) a hostage /(R) demanded ransom of ₹10 lakh /(S)
The correct sequence should be :
(a) SPRQ (b) RQPS
(c) QRPS (d) PRSQ

23. living in Russia /(P) country immediately /(Q) Ukraine has urged its citizens /(R) to leave the /(S)
The correct sequence should be :
(a) SPRQ (b) RPSQ
(c) RSQP (d) PRSQ

24. ripping roofs off houses and raising fears of /(P) coast of Madagascar in the early hours, /(Q) cyclone Emnati crashed into the southeastern /(R) flooding and food shortages in the region /(S)
The correct sequence should be :
(a) SPRQ (b) RPSQ
(c) RQPS (d) PRSQ

25. ecology is protected and aquatic life thrives /(P) we need to utilize the resources of water /(Q) for different purposes while /(R) ensuring that its natural /(S)

The correct sequence should be :
(a) SPRQ (b) RQPS
(c) QRSP (d) PRSQ

26. very difficult /(P) the poor visibility /(Q) made the movement of traffice /(R) due to fog /(S)
The correct sequence should be :
(a) SPRQ (b) RQPS
(c) QRPS (d) QSRP

27. avenues for the patients to indict /(P) but rarely is it the other way round /(Q) there are multiple legal /(R) the doctors (S)
The correct sequence should be :
(a) SPRQ (b) RPSQ
(c) QRPS (d) QSRP

28. flight, and each person /(P) her own journey /(Q) life is a solo /(R) makes his or /(S)
The correct sequence should be :
(a) SPRQ (b) RPQS
(c) RPSQ (d) PQRS

29. made it evolve /(P) technologies and equipment has /(Q) into a rare centre of excellence /(R) adoption of the latest /(S)
The correct sequence should be :
(a) PQRS (b) RPQS
(c) SQPR (d) QSRP

30. please mark the same /(P) be made in response to this notice /(Q) if you have any complaint to /(R) to the concerned authority /(S)
The correct sequence should be :
(a) PQRS (b) RPQS
(c) SQPR (d) RQPS

Directions : Each item in this section consists of a sentence with an underlined word followed by four options, (a), (b), (c) and (d). Select the option that is ***opposite in meaning*** to the underlined word and mark your response in your Answer Sheet accordingly.

31. The parental support can fortify a child's learning.
(a) weaken (b) encourage
(c) brace (d) strengthen

32. Now we can have another dazzling thought, as an outcome of my brilliant research!
(a) splendid (b) murky
(c) dazing (d) fulgent

33. The group held some clandestine meetings to resolve the matter.
(a) secret (b) covert
(c) sneaky (d) public

34. In the winters, the countryside view is bleak and the house is drafty.
(a) cheerless
(b) verdant
(c) desolate
(d) bare

35. Amid commotion the leader and his supporters remained passive.
(a) quietude (b) uproar
(c) tempest (d) stir

36. He had the audacity to blame him.
(a) boldness (b) vigour
(c) temerity (d) politeness

37. They are making plans for the abatement of the nuisance within a specified time.
(a) subsidence (b) ebbing
(c) accumulation (d) mitigation

38. There is exponential growth in the development of the scientific researches in the recent few decades.
(a) aggresive (b) rampant
(c) meager (d) augmented

39. He was known for his sagacity.
(a) prudence (b) wisdom
(c) ignorance (d) sapience

40. The river Ganges culminates in the Bay of Bengal.
(a) concludes (b) flows
(c) originates (d) merges

Directions : Given below are some idioms/phrases followed by four alternative meanings for each. Choose the most appropriate answer from among the options (a), (b), (c) or (d).

41. The lion's share
(a) A portion of something
(b) The largest and the best part of something
(c) An important decision
(d) An aggressive statement

42. Spill the beans
(a) Reveal a secret
(b) Forced to leave belongings
(c) Share the news
(d) Take offence at someone's rude behaviour

43. Cook someone's goose
(a) To assist others (b) Hypnotize other's mind
(c) To spoil other's plans (d) Inviting misfortune

44. Disappear into thin air
(a) Lost forever (b) Become forgetful
(c) Become uncaring (d) Disappear suddenly

45. Put the screws on
(a) Forcing someone to do what you want
(b) Never letting go of things
(c) Keep the past alive
(d) Using someone's trick for own benefit

46. Sit on the fence
(a) Avoid meeting someone
(b) Waiting patiently
(c) Avoid taking sides
(d) Hiding from someone

47. Be as sharp as a tack
(a) A talkative person (b) A shrewd person
(c) A clumsy person (d) A clerver person

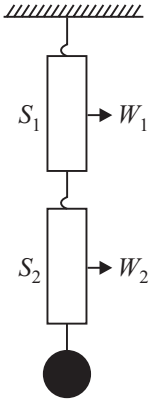
48. From pillar to post
(a) Upside down
(b) Keep moving from one place to another
(c) Constructing a huge building
(d) Jumping from the top of the mountain

49. Heath Robinson
(a) Very complicated system or machine for doing a simple task
(b) Treating everyone as enemy, when they are not
(c) Sleeping all day, doing nothing
(d) Very lazy person, slow in action

50. As thick as thieves
(a) Dumb person (b) Good for nothing
(c) Someone who uses everyone for personal gain
(d) Very close friends who are looked upon with suspicion

PART- B: GENERAL KNOWLEDGE

51. Two identical spring balances S_1 and S_2 are connected one after the other and are held vertically as shown in the figure. A mass of 10 kg is hanging from S_2. If the readings on S_1 and S_2 are W_1 and W_2 respectively, then:

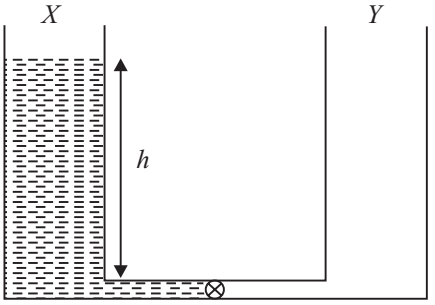


(a) W_1 = 5 kg and W_2 = 10 kg
(b) W_1 = 10 kg and W_2 = 5 kg
(c) W_1 = 5 kg and W_2 = 5 kg
(d) W_1 = 10 kg and W_2 = 10 kg

52. A stone is thrown horizontally from the top of a 20 m high building with a speed of 12 m/s. It hits the ground at a distance R from the building. Taking g = 10 m/s^2 and neglecting air resistance will give :
(a) R = 12 m (b) R = 18 m
(c) R = 24 m (d) R = 30 m

53. A sphere of volume V is made of a material with lower density than water. While on Earth, it floats on water with its volume f_1V ($f_1 < 1$) submerged. On the other hand, on a spaceship accelerating with acceleration $a < g$ (g is the acceleration due to gravity on Earth) in outer space, its submerged volume in water is f_2V. Then :
(a) $f_2 = f_1$ (b) $f_2 = \left(1 - \frac{a}{g}\right) f_1$
(c) $f_2 > f_1$ (d) $f_2 = \frac{a}{g} f_1$

54. Two identical containers X and Y are connected at the bottom by a thin tube of negligible volume. The tube has a valve in it, as shown in the figure. Initially container X has a liquid filled up to height h in it and container Y is empty. When the valve is opened, both containers have equal amount of liquid in equilibrium. If the initial (before the valve is opened) potential energy of the liquid is P_1 and the final potential energy is P_2 then :



(a) $P_1 = P_2$ (b) $P_1 = 4P_2$
(c) $P_1 = 2P_2$ (d) $P_1 = 8P_2$

55. A particle is moving in a circle of radius R with a constant speed v. Its average acceleration over the time when it moves over half the circle is :
(a) $\frac{v^2}{R}$ (b) $\frac{\pi v^2}{2R}$
(c) $\frac{2v^2}{\pi R}$ (d) 0

56. Two forces of 5.0 N each are acting on a point mass. If the angle between the forces is 60°, then the net force acting on the point mass has magnitude close to :
(a) 8.6 N (b) 4.3 N
(c) 5.0 N (d) 6.7 N

57. Which one of the following is ***not*** an igneous rock?
(a) Granite (b) Slate
(c) Basalt (d) Gabbro

58. Which of the following statements is/are correct?
1. Hypocenter is the point on the surface of the Earth, nearest to the focus.
2. Velocity of earthquake waves is higher in denser materials.
3. P waves move faster and are the first to arrive at the surface of the Earth.

Select the correct answer using the code given below :
(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 3 only

59. In terms of geological time scale, the quaternary period consists of two epochs. They are :
(a) Pleistocene and Pliocene
(b) Holocene and Pleistocene
(c) Pleistocene and Miocene
(d) Holocene and Eocene

60. Which one of the following is the correct sequence of arrangement of the given planets is descending order of their density (in gm/cm^3)?
(a) Earth > Jupiter > Venus > Saturn
(b) Jupiter > Earth > Saturn > Venus
(c) Earth > Venus > Jupiter > Saturn
(d) Earth > Venus > Saturn > Jupiter

61. Which one of the following is ***not*** a cold current?
(a) Western Australian Current
(b) Eastern Australian Current
(c) Benguela Current
(d) Peru Current

62. The process of Podsolization is predominantly found in:
(a) Equatorial forest (b) Monsoon forest
(c) Taiga forest (d) Mediterranean forest

63. Joint Military exercise 'Keen Sword 23' was conducted between :
(a) India and Japan (b) India and USA
(c) USA and Japan (d) Japan and Taiwan

64. The Battle of Rezang La, an epic battle in hostile conditions, was fought by the Indian Army in :
(a) 1948 (b) 1956 (c) 1962 (d) 1972

65. Consider the following statements about 'Exercise Sea Vigil-22' :
1. Its aim is to assess India's preparedness in the domain of Maritime security and coastal defence.
2. Naval forces of USA and Japan also took part in the exercise.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

66. Consider the following statements :
1. England is the only country that won the ICC T20 World Cup twice.
2. Virat Kohli is the only player to be adjudged as the Player of the Series in the ICC T20 World Cup twice.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

67. Who among the following is ***not*** a recipient of Nobel Prize in Chemistry in 2022?
(a) Carolyn R. Bertozzi (b) Benjamin List
(c) Morten Meldal (d) K. Barry Sharpless

68. Consider the following statements regarding cell wall composition :
1. Bacterial cell wall is made of peptidoglycan.
2. Fungal cell wall is made of cellulose.
3. Animals lack cell wall and have extracellular matrix made up of sugar and proteins.
Select the correct answer using the code given below :
(a) 2 only (b) 1 and 2 only
(c) 1 and 3 only (d) 1, 2 and 3

69. Which one of the following structures is ***not*** present in a prokaryotic cell?
(a) Cell wall (b) Ribosomes
(c) Nucleus (d) Plasma membrane

70. In a plant cell, which one of the following contains their own DNA?
(a) Nucleus and Endoplasmic Reticulum
(b) Ribosome and Golgi apparatus
(c) Mitochondria and Chloroplast
(d) Chloroplast and Vacuoles

71. One advantage of sexual reproduction over asexual reproduction is that it helps species to survive over long evolutionary time. This is because sexual reproduction produces :
(a) more offspring in each reproductive cycle.
(b) robust and healthy offspring.
(c) genetically similar offspring.
(d) more variation in offspring.

72. Which one of the following structures or components is ***not*** always present in living cells?
(a) Cell wall (b) Plasma membrane
(c) Cytoplasm (d) Genetic material

73. Browning of the chopped apple can be minimized by :
(a) using table sugar. (b) preserving in a container.
(c) using lemon juice. (d) using milk of magnesia.

74. Which of the hydrocarbons are arranged as per the increasing order of their boiling points?
(a) Methane, Butane, Propane, Heptane
(b) Propane, Butane, Pentane, Octane
(c) Propane, Butane, Heptane, Methane
(d) Octane, Ethane, Methane, Propane

75. Which one of the following apparatus is sued for separating benzene and water mixture?
(a) Round bottom flask (b) Conical flask
(c) Separating funnel (d) Dean and Stark apparatus

76. An iron nail dipped in copper sulphate solution turns brown. This is due to which one of the following types of reactions?
(a) Addition reaction (b) Decomposition reaction
(c) Substitution reaction (d) Displacement reaction

77. Among the following, which is ***not*** the correct method for keeping the curd?
(a) Keeping in stainless steel vessel
(b) Keeping in copper vessel
(c) Keeping in plastic vessel
(d) Keeping in glass vessel

78. Toothpaste prevents tooth decay by :
(a) neutralizing the excess acidity.
(b) means of emulsification.
(c) the action of fluoride.
(d) making a coat of calcium over the teeth surface.

79. Freedom fighter Kanaklata Barua was martyred in :
(a) Sepoy Mutiny
(b) Quit India Movement
(c) Non-Cooperation Movement
(d) Peasant Uprising of 1893 - 1894

80. Which one among the following statements about the Mansabdari system is correct?
(a) All army troopers were allotted mansabs.
(b) Mansabs were usually assigned on the basis of ancestry.
(c) Position and salary of mansabdars were indicated by a numerical designation called zat.
(d) Mansabdars were never paid in cash.

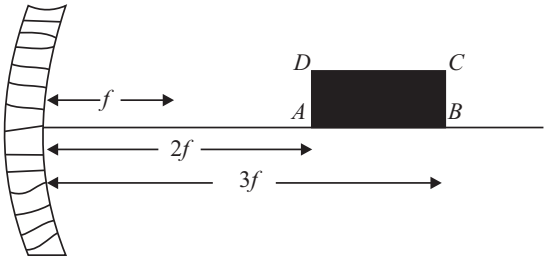
81. Which Governor General of Bengal underwent impeachment proceedings in the British Parliament?
(a) Robert Clive (b) Henry Vansittart
(c) Warren Hastings (d) Lord Cornwallis

82. Who among the following composed the 'Prayag Prashasti' of Samudragupta?
(a) Harishena (b) Chand Bardai
(c) Vishakhadatta (d) Kalidasa

83. Which one of the following kingdoms was founded by the two brothers Harihar and Bukka?
(a) Bahmani (b) Vijayanagara
(c) Malwa (d) Maratha

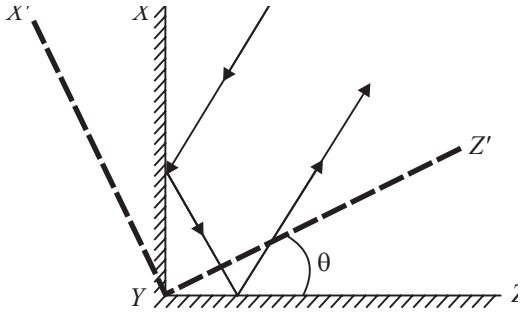
84. At which one of the following places did the Danes establish their settlement in India?
(a) Chinsura (b) Karaikal
(c) Mahe (d) Tranquebar

85. A rectangle *ABCD* is kept in front of a concave mirror of focal length *f* with its corners *A* and *B* being, respectively, at distances 2*f* and 3*f* from the mirror with *AB* along the principal axis as shown in the figure. It forms an image *A'B'C'D'* in front of the mirror. What is the ratio of *B'C'* to *A'D'*?



(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

86. Shown in the figure are two plane mirrors *XY* and *YZ* $(XY \perp YZ)$ joined at their edge. Also shown is a light ray falling on one of the mirrors and reflected back parallel to its original path as a result of this arrangement. The two mirrors are now rotated by an angle θ to their new position *X'YZ'*, as shown. As a result the new reflected ray is at an angle from the original reflected ray. Then:



(a) $\alpha = 0$ (b) $\alpha = \theta$ (c) $\alpha = 2\theta$ (d) $\alpha = 4\theta$

87. A railway wagon (open at the top) of mass M_1 is moving with speed v_1 along a straight track. As a result of rain, after some time it gets partially filled with water so that the mass of the wagon becomes M_2 and speed becomes v_2. Taking the rain to be falling vertically and water stationary inside the wagon, the relation between the two speeds v_1 and v_2 is :
(a) $v_1 = v_2$ (b) $\frac{1}{2}M_1v_1^2 < \frac{1}{2}M_2v_2^2$
(c) $M_1v_1 = M_2v_2$ (d) $M_1v_1 < M_2v_2$

88. Which one of the following statements is ***not*** true for a flute, a musical instrument?
(a) Momentum of waves on the blowing jet determines the loudness of the produced note.
(b) Arrival time of the waves on the blowing jet determines the pitch of the produced note.
(c) Sound comes from a vibrating column of air inside the flute.
(d) Sound comes from a vibrating column of air inside as well as outside the flute.

89. A positive charge is moving towards south in a space where magnetic field is pointing in the north direction. The moving charge will experience :
(a) a deflecting force towards north direction.
(b) a deflecting force towards east direction.
(c) a deflecting force towards west direction.
(d) no deflecting force.

90. Which one of the following is ***not*** a main greenhouse gas?
(a) Water vapour (b) Oxygen
(c) Carbon dioxide (d) Methane

91. Which one of the following statements about *X*-rays is ***not*** true?
(a) They have wavelengths of about 1 Å.
(b) These can be generated by bombarding a metal target by high energy electrons.
(c) Due to their wavelengths being shorter, these can be used for radar systems.
(d) These are also used for the treatment of certain forms of cancer.

92. Which one among the following is the northernmost geographical location?
(a) New Delhi (b) Kathmandu
(c) Thimphu (d) Dhaka

93. Consider the following statements :
1. Distance between the longitudes becomes zero on North Pole and South Pole.
2. Distance between the longitudes is maximum on the Equator.
3. Number of longitudes is more than number of latitudes.

Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) 1 and 3 only (d) 1, 2 and 3

94. Which one among the following states is the leading producer of Manganese in India?
(a) Madhya Pradesh (b) Jharkhand
(c) Rajasthan (d) Karnataka

95. Mica is mainly used in :
(a) food and beverage industry.
(b) iron and steel industry.
(c) aluminium industry.
(d) electrical and electronic industries.

96. According to Koppen's climatic classifications, the Great Northern Plains of India have which one of the following climates?
(a) Aw climate (b) Cwg climate
(c) Amw climate (d) Dfc climate

97. Which of the following statements about Lachit Borphukan is/are correct?
1. He was a General of the Ahom Force.
2. He is known for his leadership in the Battle of Saraighat.
3. Lachit Borphukan Gold Medal is given to the best cadet at the National Defence Academy.

Select the correct answer using the code given below :
(a) 1, 2 and 3 (b) 2 and 3 only
(c) 1 only (d) 2 only

98. The 4th edition of joint military exercise 'DUSTLIK' between the Indian Army and the Uzbekistan Army was held in :
(a) Ranikhet (b) Gangtok
(c) Pithoragarh (d) Leh

99. Which of the following is India's first privately developed rocket launched by ISRO recently?
(a) Skyroot (b) Prarambh
(c) Bazoomq (d) Vikram-5

100. Which one among the following villages was recently declared as India's first 24 × 7 solar-powered village?
(a) Mawlynnong, Meghalaya
(b) Modhera, Gujarat
(c) Ziro village, Arunachal Pradesh
(d) Malana, Himachal Pradesh

101. Tableaux of which one of the following States was adjudged the best in the Republic Day Parade, 2023?
(a) Uttarakhand (b) Punjab
(c) Gujarat (d) Uttar Pradesh

102. Organisms capable of using CO_2 as principal carbon source are called :
(a) Autotrophs
(b) Heterotrophs
(c) Parasites
(d) Decomposers

103. When yeast cells are O_2 starved, fermentation serves as the source of energy. This results in the production of :
(a) ATP + CO_2 + Ethanol
(b) ATP + O_2 + Pyruvate
(c) ATP + CO_2 + Lactic acid
(d) ATP + O_2 + Acetaldehyde

104. During a laboratory experiment, a student immerses epidermal leaf peel in a hypertonic solution. After some time, the student examined the cells under a microscope and observed that :
(a) the cells swelled.
(b) the cells were plasmolysed.
(c) the cells built up turgor pressure.
(d) the cells size was unaffected.

105. Which one of the following is ***not*** a characteristic feature of fungi?
(a) Cell wall is made of chitin
(b) Filamentous mycelium is present
(c) Can carry out photosynthesis
(d) Asexual spores are produced

106. Which one of the following statements about bryophytes is ***not*** correct?
(a) The plant body is a gametophyte.
(b) They are also called the amphibians of plant kingdom.
(c) The plant body is attached to the substratum by rhizoids.
(d) Specialized water-conducting tissues are present.

107. What is the specific purpose of using potassium hydroxide during the saponification process?
(a) To obtain soaps which are hard on the skin
(b) To obtain soaps which are soft on the skin
(c) To obtain natural fragrance
(d) To make the saponification very economical

108. Which one of the following is the correct arrangement of metals in the decreasing order of their reactivity?
(a) Iron, Sodium, Silver, Copper
(b) Silver, Copper, Iron, Sodium
(c) Sodium, Copper, Silver, Iron
(d) Sodium, Iron, Copper, Silver

109. Which among the following statements is ***not*** correct with respect to allotropQes of Carbon?
(a) Graphite is a good conductor of electricity.
(b) Diamond is the hardest substance known.
(c) Fullerene is one of the allotropes of carbon.
(d) Next to diamond, graphite is the second hardest known substance.

110. Which among the following is the popular method for manufacture of ammonia?
(a) Ostwald's process
(b) Haber-Bosch process
(c) Electric furnace process
(d) Electrolysis process

111. For manufacturing of glass, which among the following is used as a source of silica?
(a) Fine clay soil (b) Wood powder
(c) Coconut shell (d) Sand

112. Which among the following is the correct arrangement of halogens in the increasing order of their oxidizing nature?
(a) F, Cl, Br, I (b) Cl, Br, F, I
(c) Br, I, Cl, F (d) I, Br, Cl, F

113. Copper sulphate crystals available in the market are blue coloured crystals. By careful heating, they turn to white colour. Which one of the following is responsible for the blue colour?
(a) Oxygen (b) Nitrogen
(c) Water (d) Hydrogen

114. Equal volume of all gases, when measured at the same temperature and pressure, contain an equal number of particles. Who proposed the above law?
(a) Charles (b) Boyle
(c) Avagadro (d) Lussac

115. Airbags work on the principle of a chemical reaction triggered by the impact producing a gaseous product that causes a sudden volume change. Which one among the following chemical conversions is responsible for this?
(a) Sodium azide into nitrogen gas
(b) Solid carbon dioxide into gaseous carbon dioxide
(c) Carbon dioxide into carbon monoxide
(d) Sudden conversion of gaseous carbon dioxide into carbon monoxide

116. Which one among the following districts of Eastern India held a referendum in 1947 to decide whether or not to join Pakistan?
(a) Sylhet (b) Goalpara
(c) Cachar (d) Jalpaiguri

117. Which one of the following statements about Rabatak inscription is ***not*** correct?
(a) It throws important light on Kushana genealogy.
(b) It refers to Kanishka as 'a king of kings and a son of God'.
(c) The 23-line inscription is written in Gandhari language.
(d) It mentions names of States which were part of Kanishka's empire.

118. Who among the following was the author of the famous Sanskrit work *'Mrichchhakatika'*?
(a) Kalidasa (b) Bhasa
(c) Valmiki (d) Shudraka

119. Yashovarman was ruler of which one of the following kingdoms?
(a) Kannauj (b) Mewar (c) Marwar (d) Kalinga

120. Which one among the following rulers established Pataliputra as the capital of the Magadhan Empire?
(a) Bimbisara (b) Bindusara
(c) Ajatashatru (d) Ashoka

121. Which one of the following astronomers proved that the Earth and other planets revolve around the Sun?
(a) Copernicus (b) Kepler
(c) Galileo (d) Newton

122. Sand falls vertically on a conveyor belt at a rate of 0.1 kg/s. In order to keep the belt moving at a uniform speed of 2 m/s, the force required to be applied on the belt is :
(a) 0 N (b) 0.2 N (c) 1.0 N (d) 2.0 N

123. The power required to lift a mass of 8.0 kg up a vertical distance of 4 m in 2 s is (taking acceleration due to gravity as 10 m/s^2) :
(a) 80 W (b) 160 W
(c) 320 W (d) 640 W

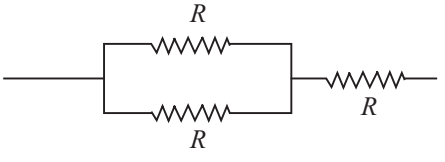
124. One block of 2.0 kg mass is placed on top of another block of 3.0 kg mass. The coefficient of static friction between the two blocks is 0.2. The bottom block is pulled with a horizontal force F such that both the blocks move together without slipping. Taking acceleration due to gravity as 10 m/s^2, the maximum value of the frictional force is :
(a) 50 N (b) 30 N (c) 4 N (d) 10 N

125. A mass is attached to a spring that hangs vertically. The extension produced in the spring is 6 cm on Earth. The acceleration due to gravity on the surface of the Moon is one-sixth of its value on the surface of the Earth. The extension of the spring on the Moon would be :
(a) 6 cm (b) 1 cm (c) 0 cm (d) 36 cm

126. Which one of the following is an example of Second Class Lever?
(a) A pair of scissors (b) A bottle opener
(c) A cricket bat (d) A bow and arrow

127. In an electric circuit, a wire of resistance 10 Ω is used. If this wire is stretched to a length double of its original value, the current in the circuit would become :
(a) half of its original value.
(b) double of its original value.
(c) one-fourth of its original value.
(d) four times of its original value.

128. What is the total resistance in the following circuit element?



(a) $R/2$ (b) $3R$
(c) $3R/2$ (d) $2R/3$

129. Which of the following scheme(s) is/are included under *Bharatmala Pariyojana*?
1. Develop the road connectivity to border areas
2. Development of coastal roads
3. Improvement in the efficiency of National Corridors

Select the correct answer using the code given below :
(a) 1 only (b) 3 only
(c) 2 and 3 only (d) 1, 2 and 3

130. Deendayal Port was earlier known as :
(a) Paradip Port (b) Tuticorin Port
(c) Kandla Port (d) Visakhapatnam Port

131. The Counter Insurgency and Jungle Warfare School of Indian Army is situated at :
(a) Dehradun (b) Vairengte
(c) Gulmarg (d) Mhow

132. In the soil-forming regime, which one of the following occurs in a region where evapotranspiration exceeds precipitation significantly?
(a) Calcification (b) Laterization
(c) Podosolization (d) Gleization

133. Which one of the following is found in the innermost part of the Earth?
(a) Conrad discontinuity
(b) Moho discontinuity
(c) Guttenberg discontinuity
(d) Lehman discontinuity

134. Which of the following tree species is/are found on Himalayas?
1. Oak
2. Rhododendron
3. Rosewood

Select the correct answer using the code given below :
(a) 1 only (b) 2 and 3
(c) 1 and 3 (d) 1 and 2

135. Which one of the following is the lowermost /innermost intrusive igneous rock?
(a) Laccolith (b) Batholith
(c) Lopolith (d) Phacolith

136. If it is 12 noon in New Delhi, what will be the time in London, UK?
(a) 6:30 A.M. (b) 6:30 P.M.
(c) 5:30 A.M. (d) 5:30 P.M.

137. Which of the following positions of Sun, Earth and Moon is/are suitable for Spring Tide?
1. SYZYGY Conjunction
2. SYZYGY Opposition
3. Quadrature

Select the correct answer using the code given below :
(a) 1 only (b) 2 only
(c) 1 and 2 (d) 1 and 3

138. Which Renaissance artist painted 'The Last Supper'?
(a) Michelangelo (b) Donatello
(c) Botticelli (d) Leonardo da Vinci

139. Which one of the following European explorer conquered Mexico?
(a) Vasco da Gama (b) Bartholomew Diaz
(c) Magellan (d) Hernán Cortés

140. Near which one of the following cities in India have large statues of Kushana rulers been discovered?
(a) Karnal (b) Ropar
(c) Hisar (d) Mathura

141. The Treaty of Yandabo was concluded as part of which one of the following wars?
(a) First Anglo-Burmese War
(b) Second Anglo-Burmese War
(c) Anglo-Kuki War
(d) Anglo-Maratha War

142. Plan allocation in agriculture and irrigation as percentage of total plan outlay was highest in :
(a) Seventh Five-Year Plan
(b) Third Five-Year Plan
(c) First Five-Year Plan
(d) Second Five-Year Plan

143. The UN COP-27 Summit relates to :
(a) Russia-Ukraine war
(b) Terrorism and counter-terrorism
(c) Climate change
(d) Interpol

144. Which one of the following tribes from India's North-East had the earliest known association with cultivation and production of tea?
(a) Khasis (b) Garos
(c) Singphos (d) Jayantias

145. By which one of the following amendments, was Article 51A, relating to the Fundamental Duties, inserted into the Constitution of India?
(a) The Constitution (42nd Amendment) Act
(b) The Constitution (44th Amendment) Act
(c) The Constitution (85th Amendment) Act
(d) The Constitution (92nd Amendment) Act

146. Which one of the following statements is true?
(a) The force of gravity of the Earth on the Moon is greater than the force of gravity of the Moon on the Earth.
(b) The force of gravity of the Moon on the Earth is greater than the force of gravity of the Earth on the Moon.
(c) The force of gravity of the Earth on the Moon and of the Moon on the Earth are equal in magnitude and are in the same direction.
(d) The force of gravity of the Earth on the Moon and of the Moon on the Earth are equal in magnitude but are in opposite directions.

147. An electric bulb is rated as 220 V and 80 W. When it is operated on 110 V, the power rating would be :
(a) 80 W (b) 60 W (c) 40 W (d) 20 W

148. In the dispersion of white light by a common glass prism, which one among the following is correct?
(a) Red light deviates the most because red light has highest speed in prism
(b) Blue light deviates the most because blue light has highest speed in prism
(c) Red light deviates the most because red light has lowest speed in prism
(d) Blue light deviates the most because blue light has lowest speed in prism

149. Which one among the following is true for the speed of sound in a given medium?
(a) Speed of sound remains same at all frequencies
(b) Speed of sound is faster at higher frequencies
(c) Speed of sound is slower at higher frequencies
(d) Speed of sound is slower at higher wavelengths

150. Which one of the following telescopes contains only mirrors?
(a) Galilean telescope (b) Keplerian telescope
(c) Newtonian telescope (d) Schmidt telescope

HINTS & SOLUTIONS

MATHEMATICS

1. (a) $\left|\frac{1-w}{w+w^2}\right| = \left|\frac{1-w}{-1}\right| = |w-1|$ [$\because$ 1 + w + w^2 = 0]

$= \left|\frac{-1+\sqrt{3}i}{2} - 1\right|$

$= \left|\frac{-1+\sqrt{3}i-2}{2}\right|$

$= \left|\frac{-3}{2} + \frac{\sqrt{3}}{2}i\right|$

$= \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{9}{4}+\frac{3}{4}} = \sqrt{3}.$

2. (d) Sum of digits = 0 + 1 + 2 + 3 + 4 + 5 = 15

All numbers are divisible by 3.

To make it divisible by 6, it must be divisible by 2 i.e. even number at unit place.

Case 1 : When 0 is at unit place

					0
5	4	3	2	1	

Number of numbers = 51 = 120

Case 2 : 2, 4 at unit place

					2 / 4
4	4	3	2	1	2

Number of numbers = 41 × 4 × 2 = 192

Total numbers = 120 + 192 = 312.

3. (Bonus) $(1011)_{10} = (1111110011)_2$

2	1011	
2	505	1
2	252	1
2	126	0
2	63	0
2	31	1
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

$\therefore$ $(1011)_{10} = (1111110011)_2$.

4. (b) $|\,2 \text{ adj } (3A)\,|$

$= 2^3 \cdot |\,\text{adj } (3A)\,|$ [$\because$ order of matrix = 3]

$= 2^3 \cdot |\,3A\,|^{3-1} = 2^3 \cdot |\,3A\,|^2$

$= 2^3 (3^3 \cdot |\,A\,|)^2 = 2^3 \cdot 3^6 \cdot |\,A\,|^2$

$= 2^3 \cdot 3^6\, 4^2 = 2^3 \cdot 3^6 \cdot 2^4 = 2^7 \cdot 3^6$

$\therefore$ $\alpha = 7, \beta = 6$

$\alpha + \beta = 13.$

5. (d) $x^2 - x + 1 = 0$

We know that roots of $x^2 + x + 1 = 0$ are ω and ω^2.

$\therefore$ Roots of $x^2 - x + 1 = 0$ are $-\omega$ and $-\omega^2$

where ω, ω^2 are complex cube roots of unity

$\therefore$ $\left|\frac{\alpha^{100}+\beta^{100}}{\alpha^{100}-\beta^{100}}\right| = \left|\frac{\omega^{100}+(\omega^2)^{100}}{\omega^{100}-(\omega^2)^{100}}\right|$

$$= \left|\frac{\omega+\omega^2}{\omega-\omega^2}\right| = \left|\frac{-1}{\omega-\omega^2}\right|$$

$$= \left|\frac{-1}{\frac{-1+\sqrt{3}i}{2} - \frac{(-1-\sqrt{3}i)}{2}}\right|$$

$$= \left|\frac{-1}{\sqrt{3}i}\right| = \left|\frac{i}{\sqrt{3}}\right| = \frac{1}{\sqrt{3}}.$$

6. (b) For symmetric matrices,

AB and BA have identical diagonal elements.

Hence $AB - BA$: Diagonal entries are equal and zero.

Sum of all non diagonal elements = 0

$\therefore$ Only 2 is correct.

7. (a) $AB = AC$

If A is non singular hence A^{-1} exist.

$A^{-1}AB = A^{-1}AC$

$\therefore \quad B = C$

but $BX = CX \Rightarrow B = C$

e.g. $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}0\\ 0\end{bmatrix} = \begin{bmatrix}2 & 3\\ 7 & 5\end{bmatrix}\begin{bmatrix}0\\ 0\end{bmatrix}$.

8. (b) $x + 2y + z = 4$

$2x + 4y + 2z = 8$

$3x + 6y + 3z = 10$

$$D = \begin{vmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 3 & 6 & 3\end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix}4 & 2 & 1\\ 8 & 4 & 2\\ 10 & 6 & 3\end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix}1 & 4 & 1\\ 2 & 8 & 2\\ 3 & 10 & 3\end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix}1 & 2 & 4\\ 2 & 4 & 8\\ 3 & 6 & 10\end{vmatrix} = 0$$

$\because D = D_1 = D_2 = D_3 = 0$

$\therefore$ System has infinite solutions.

9. (b) $AX = B$

$A(aX_1 + bX_2) = B$

$aAX_1 + bAX_2 = B$

$aB + bB = B$

$B(a + b) = B$

$\therefore \quad a + b = 1.$

10. (b) $\begin{vmatrix}0 & x-a & x-b\\ 0 & 0 & x-c\\ x+b & x+c & 1\end{vmatrix} = 0$

Expand along C_1,

$(x + b)(x - a)(x - c) = 0$

Clearly roots are $a, -b$ and c

Sum of roots $= a - b + c$.

11. (d) If $2-\sqrt{3}i$ is one root.

$\Rightarrow \quad 2+\sqrt{3}i$ is other root

$\therefore \quad -a \quad$ = Sum of roots

$= 2-\sqrt{3}i+2+\sqrt{3}i = 4$

$a = -4$

and b = product of roots

$b = (2-\sqrt{3}i)(2+\sqrt{3}i) = 4+3 = 7$

$a + b = -4 + 7 = 3.$

12. (b) $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} \times \frac{1+\sqrt{3}i}{1+\sqrt{3}i}$

$= \frac{(1+\sqrt{3}i)^2}{1+3}$

$z = \frac{1-3+2\sqrt{3}i}{4}$

$z = \frac{-1}{2} + \frac{\sqrt{3}}{2}i = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$\arg(z) = \frac{2\pi}{3}.$

13. (b) $\begin{vmatrix}x+1 & x+2 & x+3\\ x+2 & x+3 & x+4\\ x+a & x+b & x+3\end{vmatrix}$

Here a_{33} element is wrong it must be $x + c$

$\because$ a, b, c are in AP

$2b = a + c$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$

$\begin{vmatrix}x+1 & 1 & 1\\ x+2 & 1 & 1\\ x+a & b-a & c-b\end{vmatrix}$

$\because \quad b - a = c - b$

$\therefore \quad C_2$ and C_3 are identical.

$\therefore \quad$ Value of determinant = 0.

14. (c) $\log_x a,\ a^x,\ \log_b x$ are in G.P.

$$(a^x)^2 = \log_x a \times \log_b x$$

$$(a^x)^2 = \frac{\log a}{\log x} \times \frac{\log x}{\log b}$$

$$a^{2x} = \frac{\log a}{\log b} = \log_b a$$

Taking log with base a

$$\log_a a^{2x} = \log_a (\log_b a)$$

$$2x = \log_a (\log_b a)$$

$$x = \frac{\log_a (\log_b a)}{2}$$

15. (a) $2^{\frac{1}{c}},\ 2^{\frac{b}{ac}},\ 2^{\frac{1}{a}}$ are in GP

$$\left(2^{\frac{b}{ac}}\right)^2 = 2^{\frac{1}{c}} \cdot 2^{\frac{1}{a}}$$

$$2^{\frac{2b}{ac}} = 2^{\frac{1}{c}+\frac{1}{a}}$$

$\therefore \quad \frac{2b}{ac} = \frac{1}{c} + \frac{1}{a} = \frac{a+c}{ac}$

$\therefore \quad 2b = a + c$

$\therefore \quad a, b, c$ are in AP.

16. (b) $a = \frac{5}{2}$

$$a + d = \frac{23}{12}$$

$\therefore \quad d = \frac{23}{12} - \frac{5}{2} = \frac{-7}{12}$

The largest negative term will be the first negative term as it is decreasing AP.

i.e. $T_n < 0$

$a + (n-1)\,d < 0$

$$\frac{5}{2} + (n-1)\left(-\frac{7}{12}\right) < 0$$

$30 - 7\,(n-1) < 0$

$30 - 7n + 7 < 0$

$7n > 37$

$$n > \frac{37}{7}$$

For integer value, $n = 6$.

17. (b) $x^2 - 4x + K = 0$

For real roots, $D \geq 0$

$\Rightarrow \quad (-4)^2 - 4(1)(K) \geq 0$

$16 - 4K \geq 0$

$4 - K \geq 0$

$K \geq 4$

Both roots lie in (0, 5).

$f(0) > 0$ and $f(5) > 0$

$K > 0$ and $(5)^2 - 4(5) + K > 0$

$K > 0$ and $K > -5$ i.e. $K > 0$

$\therefore \quad K \in (0, 4]$

$K = 1, 2, 3, 4$ (integers)

Number of integral values = 4.

18. (b) $a = x$

$S_n = 0$

$$\frac{n}{2}[2a + (n-1)d] = 0$$

$2a + (n-1)d = 0$

$$d = \frac{-2a}{n-1} = \frac{-2x}{n-1}$$

Sum of next n terms = Sum of $m + n$ terms

$$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$$

$$= \frac{m+n}{2}\left[2x - \frac{2x(m+n-1)}{n-1}\right]$$

$$= (m+n)x\left[1 - \frac{m+n-1}{n-1}\right]$$

$$= (m+n)x\left[\frac{n-1-m-n+1}{n-1}\right]$$

$$= \frac{-m(m+n)x}{n-1} = \frac{m(m+n)x}{1-n}$$

19. (b) 1. $25! + 1$

$\because$ for any $n > 2$, $n!$ is even (divisible by 2)

$\therefore$ $25! + 1$ is odd hence can't be divided by 2.

$\therefore$ $25! + 1$ also can not be divided by 26 as $26 = 13 \times 2$

2. $6! + 1 = 720 + 1 = 721$.

Clearly 721 is divisible by 7.

$\therefore$ 2 only is correct.

20. (c) Let $z = x + iy$

$$\frac{z-1}{z+1} = \frac{(x-1)+iy}{(x+1)+iy}$$

$$= \frac{[(x-1)+iy]\times[(x+1)-iy]}{[(x+1)+iy]\times[(x+1)-iy]}$$

$$= \frac{(x^2-1)+y^2+i[y(x+1)-y(x-1)]}{(x+1)^2+y^2}$$

$$= \frac{(x^2+y^2-1)}{(x+1)^2+y^2} + i\frac{(2y)}{(x+1)^2+y^2}$$

$\because$ It is purely imaginary,

$$\text{Re}\left(\frac{z-1}{z+1}\right) = 0$$

$$\frac{x^2+y^2-1}{(x+1)^2+y^2} = 0$$

$\therefore$ $x^2 + y^2 - 1 = 0$

$x^2 + y^2 = 1$

$|z|^2 = 1$

$|z| = 1.$

21. (b) $|x - 4| + |x - 7| = 15$

Case 1 : $x < 4$

$4 - x + 7 - x = 15$

$11 - 2x = 15$

$2x = -4$

$x = -2$

One solution.

Case 2 : $4 \le x < 7$

$x - 4 + 7 - x = 15$

No solution

Case 3 : $x \ge 7$

$x - 4 + x - 7 = 15$

$2x = 26$

$x = 13$

$\therefore$ Total 2 solutions; $x = -2, 13$.

22. (d) $f(x) = \frac{2x+3}{3x+5}$

Clearly for domain,

$$3x + 5 \neq 0 \Rightarrow x \neq -\frac{5}{3}$$

$\therefore$ Domain $= A = R - \left\{-\frac{5}{3}\right\}$

$\because$ for onto function, Range = Co-domain

B = Range of f

$$y = \frac{2x+3}{3x+5}$$

$3xy + 5y = 2x + 3$

$x(3y - 2) = 3 - 5y$

$$x = \frac{3-5y}{3y-2}$$

Clearly, $3y - 2 \neq 0 \Rightarrow y \neq \frac{2}{3}$

$\therefore$ $B = R - \left\{\frac{2}{3}\right\}$

23. (c) $x^2 + ax + b = 0$

(i) $\alpha + \beta = 0 \Rightarrow \beta = -\alpha$

$\alpha^2 + \beta^2 = 2$

$(\alpha + \beta)^2 - 2\alpha\beta = 2$

$0 - 2\alpha\beta = 2$

$\alpha\beta = -1 \Rightarrow \alpha \cdot (-\alpha) = -1 \Rightarrow \alpha^2 = 1$

(ii) $\alpha\beta^2 = -1$

$a = 0 \Rightarrow \alpha + \beta = 0 \Rightarrow \beta = -\alpha$

$\alpha(-\alpha)^2 = -1 \Rightarrow \alpha^3 = -1 \Rightarrow \alpha = -1$

$\therefore$ Both 1 and 2 are sufficient.

24. (d) $(x^{-8/3} + x^2 \log_{10} x)^8$

$T_6 = {}^8C_5 (x^{-8/3})^3 \cdot (x^2 \log_{10} x)^5$

$= 56 \cdot x^{-8} \cdot x^{10} \cdot \log_{10} x$

$= 56 \cdot x^2 \cdot \log_{10} x = 5600$

$\therefore$ $x^2 \log_{10} x = 100 = (10)^2 \cdot \log_{10} 10$

$\therefore$ $x = 10.$

25. (c) $(3x - y)^4 (x + 3y)^4$

$= [(3x - y)(x + 3y)]^4$

$= (3x^2 + 8xy - 3y^2)^4$

Number of terms $= {}^{4+3-1}C_{3-1} = {}^6C_2 = 15.$

26. (a) p, q, r, s in AP

$p + s = 8$

$\therefore$ $q + r = 8$ and $qr = 15$

By observation, q and r can take values 3 or 5.

$\therefore$ The possible AP can be 1, 3, 5, 7 or 7, 5, 3, 1.

$\therefore$ Difference = 7 – 1 = 6.

27. (d) $C(n, r)$ is greatest

when n is even then $n = 2r$

when n is odd,

For maximum value, $n = 2r + 1$ and $2r - 1$.

28. (d) Number of parallelogram $= {}^{m}C_2 \times {}^{n}C_2$

$${}^{m}C_2 \times {}^{n}C_2 = 60$$

$$\frac{m(m-1)}{2} \times \frac{n(n-1)}{2} = 60$$

$$m(m-1) \times n(n-1) = 240$$

By observation,

$m\,(m-1) = 20 = 5 \times 4$

$n\,(n-1) = 12 = 4 \times 3$

$m = 5, n = 4$

$m + n = 9$

29. PERMUTATIONS $\quad x = \frac{12!}{2!}$ [$\because$ T Comes 2 times]

COMBINATIONS $\quad y = \frac{12!}{2!\,2!\,2!} = \frac{x}{4}$

[$\because$ O, N, I Comes 2 times]

$\therefore \quad x = 4y.$

30. (b) Total numbers $= 4 \times 4 \times 3 \times 2 \times 1 = 96$

Numbers greater than 50,000 :

5				
1	4	3	2	1

$= 4 \times 3 \times 2 \times 1 = 24$

Percentage of Numbers $= \frac{24}{96} \times 100 = 25\%.$

31. (b) $(\sin\beta)^2 = \sin\alpha \cdot \cos\alpha$ ....(i)

$$\tan\gamma = \frac{\sin\alpha + \cos\alpha}{2}$$

$\cos 2\beta = 1 - 2\sin^2\beta = 1 - 2\sin\alpha\cos\alpha$ [$\because$ By (i)]

$= \sin^2\alpha + \cos^2\alpha - 2\sin\alpha\cos\alpha$

$= (\cos\alpha + \sin\alpha)^2.$

32. (b) $\sec 2\gamma = \frac{1}{\cos 2\gamma} = \frac{1+\tan^2\gamma}{1-\tan^2\gamma}$

A stan $\gamma = \frac{\text{Sin}\alpha + \text{Cos}\alpha}{2}$

$$= \frac{1 + \left(\frac{\sin\alpha + \cos\alpha}{2}\right)^2}{1 - \left(\frac{\sin\alpha + \cos\alpha}{2}\right)^2}$$

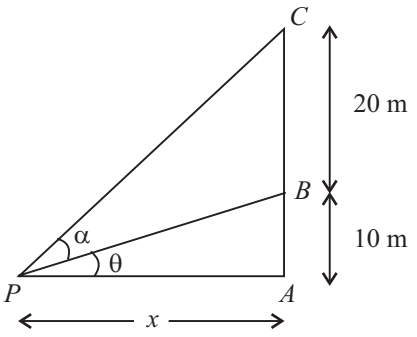
$$= \frac{4 + (\sin\alpha + \cos\alpha)^2}{4 - (\sin\alpha + \cos\alpha)^2}$$

$$= \frac{4 + (1 + \sin 2\alpha)}{4 - (1 + \sin 2\alpha)} = \frac{5 + \sin 2\alpha}{3 - \sin 2\alpha}.$$

33. (a) $\angle BPC = \tan^{-1}(0.5) = \alpha$

$\therefore \quad \tan\alpha = \frac{1}{2}$

(a) Let $PA = x$



$$\tan\theta = \frac{10}{x}$$

$$\tan(\theta + \alpha) = \frac{30}{x}$$

$$\frac{\tan\theta + \tan\alpha}{1 - \tan\theta \cdot \tan\alpha} = \frac{30}{x}$$

$$\Rightarrow \frac{\frac{10}{x} + \frac{1}{2}}{1 - \frac{10}{2x}} = \frac{30}{x}$$

$$\Rightarrow \frac{20 + x}{2x - 10} = \frac{30}{x}$$

$$\Rightarrow 20x + x^2 = 60x - 300$$

$$\Rightarrow x^2 - 40x + 300 = 0$$

$\Rightarrow (x - 30)\,(x - 10) = 0$

$x = 10, 30$

$\therefore \quad x$ can take 2 values which are in ratio 1 : 3

but $x \neq 20$ (height of flagstaff)

$\therefore \quad$ Statement 1 is correct.

34. (c) $\tan\theta = \frac{10}{x}$

$$\tan\theta = \frac{10}{10} \text{ or } \frac{10}{30}$$

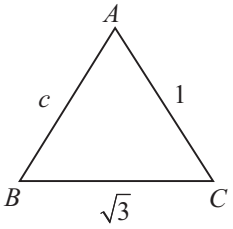
$$\tan\theta = 1 \text{ or } \frac{1}{3}$$

$$\therefore \tan\theta = \frac{1}{3}$$

35. (c) As P = 6 (Am of Sine of angle)

$$\Rightarrow \quad \sqrt{3}+1+c=\frac{6(\sin A+\sin B+\sin C)}{3}$$

$$\sqrt{3}+1+c=2(\sin A+\sin B+\sin C)$$



We know that,

$a = K\sin A,\ b = K\sin B,\ c = K\sin C$

$$a+b+c=K(\sin A+\sin B+\sin C)$$

where $K = 2R$

$\therefore$ $2R = 2 \Rightarrow R = 1$

also, $\sin A+\sin B+\sin C=\frac{\sqrt{3}}{2}+\frac{1}{2}+\frac{c}{2}$ by observations.

For sin A and sin B take values of A & B can be 60° and 30°

i.e. $A + B = 90°$

$\therefore$ $C = 90°$

$\therefore$ Triangle is right angled.

$\therefore$ $BC^2 + AC^2 = AB^2$

$$(\sqrt{3})^2+(1)^2=AB^2$$

$\therefore$ $AB = 2$

$\therefore$ Perimeter $= 2+1+\sqrt{3}=3+\sqrt{3}$

36. (c) Clearly triangle ABC is right angled where angles are 30°, 60° and 90° which are also in AP.

$\therefore$ 1 and 2 both are correct.

37. (c) $x=\frac{\sin^2 A+\sin A+1}{\sin A}=\sin A+\frac{1}{\sin A}+1$

Using AM ≥ GM

$$\frac{\sin\ \ \overline{\sin}}{2}\geq\sqrt{\sin\ \ \cdot\overline{\sin}}$$

$$\sin A+\frac{1}{\sin A}\geq 2$$

Minimum value of $\sin A+\frac{1}{\sin A}=2$

$x_{\text{minimum}} = 2+1=3.$

38. (d) For minimum value,

$$\sin A+\frac{1}{\sin A}=2$$

$$\sin^2 A-2\sin A+1=0$$

$$(\sin A-1)^2=0$$

$$\sin A = 1$$

$\therefore$ $A=\frac{\pi}{2}$

39. (c) $a^2+b^2+c^2=ac+\sqrt{3}bc$

$$(a^2+c^2-ac)+(b^2-\sqrt{3}bc)=0$$

$$\left[a^2+\left(\frac{c}{2}\right)^2-2\cdot a\left(\frac{c}{2}\right)\right]+\left[b^2+\left(\frac{\sqrt{3}c}{2}\right)^2-2\cdot b\cdot\frac{\sqrt{3}c}{2}\right]=0$$

$$\left(a-\frac{c}{2}\right)^2+\left(b-\frac{\sqrt{3}c}{2}\right)^2=0$$

It is only possible if

$$a-\frac{c}{2}=0 \text{ and } b-\frac{\sqrt{3}c}{2}=0$$

$$a=\frac{c}{2} \text{ and } b=\frac{\sqrt{3}c}{2}$$

$$a:b:c=\frac{c}{2}:\sqrt{3}\frac{c}{2}:c$$

$$=\frac{1}{2}:\frac{\sqrt{3}}{2}:1$$

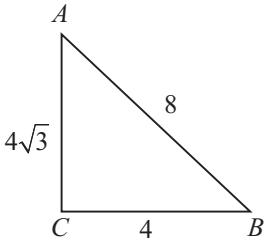
= sin 30° : sin 60° : sin 90°

$\therefore$ Angles of triangle are 30°, 60° and 90° i.e. triangle is right angled.

40. (c) $c = 8$

$\therefore$ $a = 4$

$$b=\frac{\sqrt{3}}{2}\times 8=4\sqrt{3}$$



$\therefore$ Area of $\Delta=\left(\frac{1}{2}\times 4\sqrt{3}\times 4\right)=8\sqrt{3}.$

41. (b) $f(x) = |x-2| + |3-x| + |4-x|$

$f(2) = 0 + 1 + 2 = 3$

$f(3) = 1 + 0 + 1 = 2$

$f(4) = 2 + 1 + 0 = 3$

$f(0) = 2 + 3 + 4 = 9$

$\therefore$ $x = 3$ is correct.

42. (b) Minimum value is at $x = 3$

$f(3) = 2$

43. (c) $S = 0! + 1! + 2! + ... + 100!$

Clearly all factorial from 4 to 100 have 4×2 i.e. 8 in them hence all of these are completely divisible by 8.

$\therefore$ Remainder will be from $0! + 1! + 2! + 3!$

$= 1 + 1 + 2 + 6 = 10.$

After dividing 10 from 8, remainder is 2

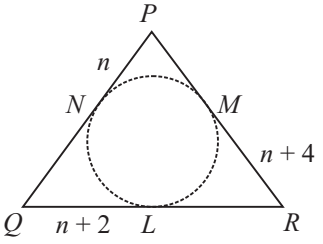
44. (d) $\because$ $5! = 120$

$\therefore$ All factorials of 5 or above are divisible by 60.

$\therefore$ Remainder will be from

$0! + 1! + 2! + 3! + 4! = 34$

45. (c) $\cos P = \frac{1}{3}$



$\because$ $PM = PN = n$

$QN = QL = n + 2$

$RM = RL = n + 4$

$PQ = 2n + 2$

$QR = 2n + 6$

$PR = 2n + 4$

Now, $\cos P = \frac{PQ^2 + PR^2 - QR^2}{2(PQ)(PR)}$

$\Rightarrow \frac{1}{3} = \frac{(2n+2)^2 + (2n+4)^2 - (2n+6)^2}{2(2n+2)(2n+4)}$

$\frac{1}{3} = \frac{(n+1)^2 + (n+2)^2 - (n+3)^2}{2(n+1)(n+2)}$

$2(n+1)(n+2) = 3[(n+1)^2 + (n+2)^2 - (n+3)^2]$

$\Rightarrow 2n^2 + 6n + 4 = 3n^2 - 12$

$\Rightarrow n^2 - 6n - 16 = 0$

$\Rightarrow (n-8)(n+2) = 0$

$\because$ $n \in Z$

$\therefore$ $n = 8$

46. (d) $\because$ Smallest side is $PQ = 2n + 2$

$\therefore$ $PQ = 2(8) + 2 = 18$

47. (a) $\sin x + \cos x + \tan x + \cot x + \sec x + \operatorname{cosec} x = 7$

$$(\sin x + \cos x) + \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) + \left(\frac{1}{\sin x} + \frac{1}{\cos x}\right) = 7$$

$$(\sin x + \cos x) + \frac{1}{\sin x \cos x} + \frac{\sin x + \cos x}{\sin x \cos x} = 7$$

$$(\sin x + \cos x) \times \left(1 + \frac{1}{\sin x \cos x}\right) + \frac{1}{\sin x \cos x} = 7$$

$$\frac{(\sin x + \cos x)(\sin x \cos x + 1) + 1}{\sin x \cos x} = 7$$

$$(\sin x + \cos x)(\sin x \cos x + 1) + 1 = 7 \sin x \cos x$$

$$(\sin x + \cos x)(\sin x \cos x + 1) = 7 \sin x \cos x - 1$$

$$(\sin x + \cos x)\left(\frac{1}{2} \cdot \sin 2x + 1\right) = \frac{7}{2} \sin 2x - 1$$

Squaring both sides

$$(1 + \sin 2x)\left(\frac{1}{4}\sin^2 2x + 1 + \sin 2x\right)$$

$$= \frac{49}{4}\sin^2 2x + 1 - 7 \sin 2x$$

$$(1 + \sin 2x)(\sin^2 2x + 4 \sin 2x + 4)$$

$$= 49 \sin^2 2x - 28 \sin 2x$$

$$\Rightarrow \sin^2 2x + 4 \sin 2x + 4 + \sin^3 2x + 4 \sin^2 2x + 4 \sin 2x$$

$$= 49 \sin^2 2x - 28 \sin 2x + 4$$

$$\Rightarrow \sin^3 2x - 44 \sin^2 2x + 36 \sin 2x = 0$$

$$\Rightarrow \sin 2x(\sin^2 2x - 44 \sin 2x + 36) = 0$$

$$\Rightarrow \sin^2 2x - 44 \sin 2x + 36 = 0.$$

48. (d) $\sin 2x = a - b\sqrt{c}$

$\sin^2 2x - 44 \sin 2x + 36 = 0$

$$\sin 2x = \frac{44 \pm \sqrt{(44)^2 - 4(36)}}{2}$$

$$= \frac{44 \pm 4\sqrt{11^2 - 9}}{2} = 22 \pm 2\sqrt{112} = 22 \pm 8\sqrt{7}$$

$\because$ $\sin 2x \le 1$

$\therefore \quad \sin 2x = 22 - 8\sqrt{7} = a - b\sqrt{c}$

$a = 22, b = 8, c = 7$

$a - b + 2c = 22 - 8 + 14 = 28$

49. (b) $(3+2\sqrt{2})x^2 - (4+2\sqrt{3})x + (8+4\sqrt{3}) = 0$

If roots are α, β

$$\text{HM} = \frac{2\alpha\beta}{\alpha+\beta} = \frac{2\left(\frac{8+4\sqrt{3}}{3+2\sqrt{2}}\right)}{\frac{4+2\sqrt{3}}{(3+2\sqrt{2})}}$$

$$= \frac{2(8+4\sqrt{3})}{4+2\sqrt{3}} = 4\frac{(4+2\sqrt{3})}{4+2\sqrt{3}} = 4.$$

50. (a) $\text{GM} = \sqrt{\alpha\beta} = \sqrt{\frac{8+4\sqrt{3}}{3+2\sqrt{2}}}$

$$= \sqrt{\frac{(8+4\sqrt{3})(3-2\sqrt{2})}{(3+2\sqrt{2})(3-2\sqrt{2})}}$$

$$= \sqrt{4(2+\sqrt{3})(3-2\sqrt{2})}$$

$$= \sqrt{4(6-4\sqrt{2}+3\sqrt{3}-2\sqrt{6})}$$

$$= \sqrt{2(12-8\sqrt{2}+6\sqrt{3}-4\sqrt{6})}$$

$$= \sqrt{2[(\sqrt{6}-\sqrt{3})^2 + (\sqrt{2}-1)^2 + 2(\sqrt{6}-\sqrt{3})(\sqrt{2}-1)}$$

$$= \sqrt{2(\sqrt{6}-\sqrt{3}+\sqrt{2}-1)^2} = \sqrt{2}(\sqrt{6}-\sqrt{3}+\sqrt{2}-1).$$

51. (b) $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$

$R_3 \to R_3 - \alpha R_1 - R_2$

$$\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ 0 & 0 & -a\alpha^2 - b\alpha - b\alpha - c \end{vmatrix} = 0$$

Expanding along R_3,

$-(a\alpha^2 + 2b\alpha + c)(b^2 - ac) = 0$

$\alpha \neq 0$

$b^2 - ac = 0$

$b^2 = ac$

a, b, c are in GP

52. (c) $\Delta(7, 4, 2, \alpha) = 0$

$a = 7, b = 4, c = 2$

$a\alpha^2 + 2b\alpha + c = 0$

$7\alpha^2 + 8\alpha + 2 = 0$

$\therefore \quad \alpha$ is a root.

$\Rightarrow 7x^2 + 8x + 2 = 0$

53. (d) $m(\theta) = \cot^2\theta + n^2\tan^2\theta + 2n$

$\text{AM} \geq \text{GM}$

$$\frac{\cot^2\theta + n^2\tan^2\theta}{2} \geq \sqrt{\cot^2\theta \cdot n^2\tan^2\theta}$$

$$\cot^2\theta + n^2\tan^2\theta \geq 2n$$

For minimum value,

$$\cot^2\theta + n^2\tan^2\theta = 2n$$

$\therefore \quad m(\theta)_{\min} = 2n + 2n = 4n.$

54. (b) For minimum value,

$$\cot^2\theta + n^2\tan^2\theta - 2n = 0$$

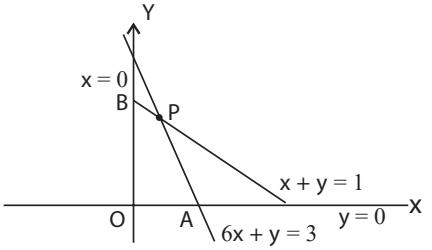
$$(\cot\theta)^2 + (n\tan\theta)^2 - 2(\cot\theta \cdot n\tan\theta) = 0$$

$$(\cot\theta - n\tan\theta)^2 = 0$$

$$\cot\theta = n\tan\theta$$

$\therefore \quad \cot^2\theta = n$

55. (c)



$\therefore$ The quadrilateral is $OAPB$

P is point of intersection of $x + y = 1$ and $6x + y = 3$

$\therefore \quad P\left(\frac{2}{5}, \frac{3}{5}\right)$

$\therefore$ Equation of diagonal through O,

$$y = \frac{3/5}{2/5}(x)$$

$2y = 3x$

$3x - 2y = 0$

56. (d) $6x + y = 3$

$$\frac{2x}{1} + \frac{y}{3} = 1$$

$$\frac{x}{\left(\frac{1}{2}\right)}+\frac{y}{3}=1$$

$\therefore \quad A\left(\frac{1}{2}, 0\right)$ and $x + y = 1$

$\therefore \quad B\ (0, 1).$

Equation of diagonal AB,

$$y-0=\frac{1-0}{0-\frac{1}{2}}\left(x-\frac{1}{2}\right)$$

$$y=-2\left(x-\frac{1}{2}\right)$$

$y = -2x + 1$

$2x + y - 1 = 0.$

57. (b) $x^2 + 4y^2 = 1$

$$\frac{x^2}{1}+\frac{y^2}{\left(\frac{1}{4}\right)}=1$$

$$a^2=1,\ b^2=\frac{1}{4}$$

$$a=1,\ b=\frac{1}{2}$$

$\because$ E and F are foci

$\therefore \quad PE + PF = 2a =$ major axis $= 2(1) = 2.$

58. (d) $b^2 = a^2\ (1 - e^2)$

$$\frac{1}{4}=1-e^2 \Rightarrow e=\frac{\sqrt{3}}{2}$$

$$\text{focii } (\pm ae,\ 0)=\left(\pm\frac{\sqrt{3}}{2}, 0\right)$$

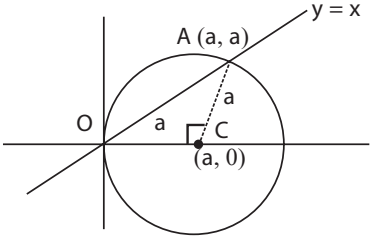
Equation of latus rectum, $x=\pm\frac{\sqrt{3}}{2}$

$\therefore$ All 3 points lie on latus rectum

59. $(x - a)^2 + y^2 = a^2$

Centre : $(a, 0)$

Radius $= a$



Clearly sector $OCAO$ is one quarter of circle

$\therefore \quad \text{Area } (\Delta OCA) = \frac{1}{2}\times a\times a=\frac{a^2}{2}$

Area of minor segment

$$=\frac{1}{4}(\text{Area of circle})-\text{Area}\,(\Delta OCA)$$

$$=\frac{1}{4}\cdot\pi a^2-\frac{1}{2}a^2=\frac{(\pi-2)a^2}{4}.$$

60. (b) Area of major segment

= Area of circle – Area of minor segment

$$=\pi a^2-\frac{(\pi-2)a^2}{4}$$

$$=a^2\left[\pi-\frac{\pi-2}{4}\right]$$

$$=a^2\,\frac{(3\pi+2)}{4}$$

61. $A(1, –1, 2), B(2, 1, –1)$

Centre of sphere $= (–u, –v, –w)$

Centre is mid point of AB

$$=\left(\frac{1+2}{2}, \frac{-1+1}{2}, \frac{2-1}{2}\right)$$

$$=\left(\frac{3}{2}, 0, \frac{1}{2}\right)$$

$$\therefore \quad u=\frac{-3}{2},\ v=0,\ w=\frac{-1}{2}$$

$$u+v+w=\frac{-3}{2}+0-\frac{1}{2}=-2$$

62. (b) $\because \quad \angle APB = 90°$

$\therefore \quad PA^2 + PB^2 = AB^2$

$= (2 – 1)^2 + (1 + 1)^2 + (–1 – 2)^2$

$= 1 + 4 + 9 = 14.$

63. (a) $A\ (2, –1, 2)\ B\ (k, 3, 5)$

$$\cos\theta=\frac{2k-3+10}{\sqrt{2^2+(-1)^2+2^2}\cdot\sqrt{k^2+3^2+5^2}}$$

$$\Rightarrow \cos\frac{\pi}{4}=\frac{2k+7}{3\cdot\sqrt{34+k^2}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow 3\sqrt{34+k^2}=\sqrt{2}(2k+7)$$

$$\Rightarrow 9(34+k^2)=2(2k+7)^2$$

$$\Rightarrow 306+9k^2=8k^2+98+56k$$

$\Rightarrow\ k^2 - 56k + 208 = 0$

$\Rightarrow\ (k-52)(k-4) = 0$

$k = 4, 52$

$\therefore\ k = 4$

64. (d) Line perpendicular to both lines will be obtained by cross product of both

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 4 & 3 & 5 \end{vmatrix}$$

$$= -11\hat{i} - 2\hat{j} + 10\hat{k}$$

Direction Ratio : (–11, –2, 10) or (11, 2, –10).

65. (b) $\vec{a} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$\vec{c} = \hat{j} - \hat{k}$

$\vec{a} \cdot \vec{b} = 27$

$\vec{a} \times \vec{b} = 9\vec{c}$

Let $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a} \cdot \vec{b} = 3x + 3y + 3z = 27$

$\Rightarrow x + y + z = 0$

Also, $\vec{a} \times \vec{b} = 9\vec{c}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 3 \\ x & y & z \end{vmatrix} = 9(\hat{j} - \hat{k})$$

$\Rightarrow (3z - 3y)\hat{i} + (3x - 3z)\hat{j} + (3y - 3x)\hat{k} = 9\hat{j} - 9\hat{k}$

$\therefore\ 3z - 3y = 0 \Rightarrow y = z$ ...(1)

$3x - 3z = 9 \Rightarrow x - z = 3$ ...(2)

$3y - 3x = -9 \Rightarrow y - x = -3$ ...(3)

Clearly $y = z$ is satisfied in

$\therefore\ \vec{b} = 5\hat{i} + 2\hat{j} + 2\hat{k}.$

66. (a) $\vec{a} + \vec{b} = 8\hat{i} + 5\hat{j} + 5\hat{k}$

$\vec{c} = \hat{j} - \hat{k}$

$\Rightarrow (\vec{a} + \vec{b}) \cdot \vec{c} = |\vec{a} + \vec{b}| \cdot |\vec{c}| \cdot \cos\theta$

$\Rightarrow 5 - 5 = |\vec{a} + \vec{b}| \cdot |\vec{c}| \cos\theta$

$\Rightarrow \cos\theta = 0$

$\therefore\ \theta = \frac{\pi}{2}$

67. (b) $\vec{a} = 4\hat{i} - 8\hat{j} + \hat{k}$

for $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

If angle made with X axis = α

$$\cos\alpha = l = \frac{a_1}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

$$\Rightarrow \cos\alpha = \frac{4}{\sqrt{4^2 + (-8)^2 + (1)^2}}$$

$$\Rightarrow \cos\alpha = \frac{4}{9}$$

68. (a) $\cos\beta = \frac{a_2}{\sqrt{a_1^2 + a_2^2 + a_3^2}} = \frac{-8}{9}$

$$\cos\gamma = \frac{a_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}} = \frac{1}{9}$$

$\cos 2\beta + \cos 2\gamma = 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$$= 2\left(\frac{-8}{9}\right)^2 + 2\left(\frac{1}{9}\right)^2 - 2$$

$$= \frac{128}{81} + \frac{2}{81} - 2 = \frac{-32}{81}.$$

69. (b) $A(\hat{i} - \hat{j})$ and $B(\hat{j} + \hat{k})$

Equation of $\overrightarrow{AB} = (\hat{j} + \hat{k}) - (\hat{i} - \hat{j}) = -\hat{i} + 2\hat{j} + \hat{k}$

$\therefore$ Direction ratio of line parallel to AB = (–1, 2, 1).

$\therefore$ Equation of line passing through A and pass to AB,

$$\frac{x-1}{-1} = \frac{y+1}{2} = \frac{z-0}{1}$$

Clearly (–1, 3, 2) and (–2, 5, 3) is satisfying *i.e.* only 2 and 3.

70. (c) $\overrightarrow{AB} = -\hat{i} + 2\hat{j} + \hat{k}$

$\therefore\ |\overrightarrow{AB}| = \sqrt{(-1)^2 + (2)^2 + (1)^2} = \sqrt{6}$

71. (b) $f(x) = Pe^x + Qe^{2x} + Re^{3x}$

$f(0) = P + Q + R = 6$ ...(1)

$f'(x) = Pe^x + 2Qe^{2x} + 3Re^{3x}$

$f'(\ln 3) = Pe^{\ln 3} + 2Qe^{2\ln 3} + 3Re^{3\ln 3}$

$\Rightarrow 3P + 18Q + 81R = 282$

$\Rightarrow P + 6Q + 27R = 94$ ...(2)

$$\int f(x)\,dx = Pe^x + \frac{Q}{2}e^{2x} + \frac{R}{3}e^{3x}$$

$$\Rightarrow \int_0^{\ln 2} f(x)\,dx = \left(Pe^{\ln 2} + \frac{Q}{2}e^{2\ln 2} + \frac{R}{3}e^{3\ln 2} - P - \frac{Q}{2} - \frac{R}{3}\right) = 11$$

$\Rightarrow\ 2P+2Q+\frac{8R}{3}-P-\frac{Q}{2}-\frac{R}{3}=11$

$\Rightarrow\ P+\frac{3Q}{2}+\frac{7R}{3}=11$ ...(3)

(2) – (1) and (3) – (1),

$5Q + 26R = 88$ ...(4)

and, $\frac{Q}{2}+\frac{4R}{3}=5$

$3Q + 8R = 30$ ...(5)

From (5), $R=\frac{30-3Q}{8}$

From (4), $5Q+26\left(\frac{30-3Q}{8}\right)=88$

$\Rightarrow 20Q + 390 – 39Q = 352$

$\Rightarrow 19Q = 38$

$\Rightarrow Q = 2$

72. (c) $\because\ Q = 2$

$\therefore\ R=\frac{30-3Q}{8}=\frac{30-6}{8}$

$\therefore\ R = 3$

73. (d) $\because\ P + Q + R = 6$

$P + 2 + 3 = 6$

$\Rightarrow P = 1$

$\therefore\ f(x)=e^x+2e^{2x}+3e^{3x}$

$f'(x)=e^x+2\cdot 2e^{2x}+3\cdot 3e^{3x}$

$f(0) = 1 + 4 + 9 = 14.$

74. (a) $y^2=2cx+2c\sqrt{c}$

Differentiate wrt x,

$2y\frac{dy}{dx}=2c$

$\Rightarrow\ c=y\frac{dy}{dx}$

$\Rightarrow\ y^2=2x\cdot y\frac{dy}{dx}+2\left(y\frac{dy}{dx}\right)^{3/2}$

$\Rightarrow\ y^2-2xy\frac{dy}{dx}=2\cdot\left(y\frac{dy}{dx}\right)^{3/2}$

Squaring both sides,

$y^4+4x^2y^2\left(\frac{dy}{dx}\right)^2-4xy^3\frac{dy}{dx}=4y^3\left(\frac{dy}{dx}\right)^3$

Clearly order = 1.

75. (b) As highest power of $\frac{dy}{dx}=3$

$\therefore$ Degree = 3

76. (b) $f(x)=\begin{vmatrix}\cos x & x & 1\\ 2\sin x & x^2 & 2x\\ \tan x & x & 1\end{vmatrix}$

$f(0)=\begin{vmatrix}\cos 0 & 0 & 1\\ 2\sin 0 & 0 & 0\\ \tan 0 & 0 & 1\end{vmatrix}=0$

77. (b) Solving $f(x)$

$f(x)=(x^2-2x^2)\cos x-x(2\sin x-2x\tan x)$

$+1(2x\sin x-x^2\tan x)$

$=-x^2\cos x-2x\sin x+2x^2\tan x+2x\sin x-x^2\tan x$

$f(x)=x^2\tan x-x^2\cos x$

$=x^2(\tan x-\cos x)$

$\therefore\ \lim_{x\to 0}\frac{f(x)}{x}$

$=\lim_{x\to 0}\frac{x^2(\tan x-\cos x)}{x}$

$=\lim_{x\to 0}x(\tan x-\cos x)$

$= 0.$

78. (a) $\lim_{x\to 0}\frac{f(x)}{x^2}$

$=\lim_{x\to 0}\frac{x^2(\tan x-\cos x)}{x^2}$

$=\lim_{x\to 0}\tan x-\cos x$

$= 0 – 1 = –1.$

79. (b) $f(x)=\sin[\pi^2]x+\cos[-\pi^2]x$

$\because\ \pi^2=(3\cdot 14)^2$

$[\pi^2]=9$

$[-\pi^2]=-10$

$f(x)=\sin 9x+\cos(-10x)$

$=\sin 9x+\cos 10x$

$f\left(\frac{\pi}{2}\right)=\sin\frac{9\pi}{2}+\cos\frac{10\cdot\pi}{2}$

$$= \sin\left(4\pi + \frac{\pi}{2}\right) + \cos 5\pi$$

$$= \sin\frac{\pi}{2} + \cos\pi$$

$= 1 - 1 = 0.$

80. (d) $f\left(\frac{\pi}{4}\right) = \sin\frac{9\pi}{4} + \cos\frac{10\pi}{4}$

$$= \sin\left(2\pi + \frac{\pi}{4}\right) + \cos\left(2\pi + \frac{\pi}{2}\right)$$

$$= \sin\frac{\pi}{4} + \cos\frac{\pi}{2} = \frac{1}{\sqrt{2}}.$$

81. (Bonus) $I_1 = \int_0^{\pi} \frac{x}{1+\cos^2 x}dx$ ...(1)

$$I_1 = \int_0^{\pi} \frac{(\pi - x)}{1+\cos^2(\pi - x)}dx$$ ...(2)

(1) + (2),

$$2I_1 = \int_0^{\pi} \frac{\pi}{1+\cos^2 x}dx$$

$$2I_1 = \int_0^{\pi} \frac{\pi\sec^2 x}{1+\sec^2 x}dx$$

$$I_1 = \frac{\pi}{2}\int_0^{\pi} \frac{\sec^2 x}{2+\tan^2 x}dx$$

$\left[\because f(\pi - x) = f(n)\right]$

$$I_1 = \frac{\pi}{2}\cdot 2\int_0^{\pi/2} \frac{\sec^2 x\,dx}{2+\tan^2 x}$$

Let $\tan x = t$, $\sec^2 x\,dx = dt$

$$I_1 = \pi\int_0^{\infty} \frac{dt}{t^2 + (\sqrt{2})^2}$$

$$I_1 = \frac{\pi}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)\Bigg|_0^{\infty}$$

$$= \frac{\pi}{\sqrt{2}}\left(\frac{\pi}{2} - 0\right) = \frac{\pi^2}{2\sqrt{2}}$$

Similarly,

$$I_2 = \int_0^{\pi} \frac{1}{1+\sin^2 x}dx$$

$$= 2\int_0^{\pi/2} \frac{dx}{1+\sin^2 x}$$

$$= 2\int_0^{\pi/2} \frac{dx}{2\sin^2 x + \cos^2 x}$$

$$= 2\int_0^{\pi/2} \frac{\sec^2 x\,dx}{2\tan^2 x + 1}$$

$$= \int_0^{\pi/2} \frac{\sec^2 x\,dx}{\tan^2 x + \left(\frac{1}{\sqrt{2}}\right)^2}$$

Let $\tan x = t$, $\sec^2 x\,dx = dt$

$$\int_0^{\infty} \frac{dt}{t^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{2}\tan^{-1}\sqrt{2}t\,\Big|_0^{\infty}$$

$$= \sqrt{2}\left(\frac{\pi}{2} - 0\right) = \frac{2\pi}{2\sqrt{2}}$$

$$\frac{I_1 + I_2}{I_1 - I_2} = \frac{\frac{\pi^2}{2\sqrt{2}} + \frac{2\pi}{2\sqrt{2}}}{\frac{\pi^2}{2\sqrt{2}} - \frac{2\pi}{2\sqrt{2}}} = \frac{\pi^2 + 2\pi}{\pi^2 - 2\pi}$$

$$\Rightarrow \frac{I_1 + I_2}{I_1 - I_2} = \frac{\pi + 2}{\pi - 2}.$$

82. (d) $\because \quad I_1 = \frac{\pi^2}{2\sqrt{2}}$

$$\therefore \quad I_1^2 = \frac{\pi^4}{8} \Rightarrow 8I_1^2 = \pi^4$$

83. (a) $I_2 = \frac{2\pi}{2\sqrt{2}} = \frac{\pi}{\sqrt{2}}.$

84. (a) $I = \int_a^b \frac{|x|}{x}dx, \qquad a < D < b$

$$\int_a^b \frac{|x|}{x}dx = \int_a^0 \frac{|x|}{x}dx + \int_0^b \frac{|x|}{x}dx$$

$$= \int_a^0 \frac{-x}{x}dx + \int_0^b \frac{x}{x}dx$$

$$= \int_a^0 -1\, dx + \int_0^b 1\, dx$$

$$= -[x]_a^0 + [x]_0^b = -(0-a) + (b-0) = a + b.$$

85. (b) $a < b < 0$

$$\int_a^b \frac{|x|}{x} dx = \int_a^b \frac{-x}{x} dx$$

$$= \int_a^b -1\, dx = [-x]_a^b$$

$= -b - (-a) = a - b.$

86. (a) $f(x) = |\ln x|$

$$f'(x) = \begin{matrix} \ln x, & x > 1 \\ -\ln x, & x < 1 \end{matrix}$$

for $x = 0.5$

$f(x) = -\ln x$

$$f'(x) = -\frac{1}{x}$$

$$f'(0.5) = -\frac{1}{0.5} = -2.$$

87. (c) At $x = 2$,

$f(x) = \ln x$

$$f'(x) = \frac{1}{x},$$

$$f'(2) = \frac{1}{2}$$

88. (d) $y = fof(x) = |\ln|\ln x|| = \ln|\ln x|$ $\quad 1 < x < 2$

$$y' = \frac{-1}{\ln x} \times \frac{1}{x} \qquad \ln 1 < \ln x < \ln 2$$

$$y' = \frac{-1}{x \ln x} \qquad 0 < \ln x < 1$$

89. (b) $f(x) = \begin{cases} x + 6, & x \le 1 \\ px + q, & 1 < x < 2 \\ 5x, & x \ge 2 \end{cases}$

$\because$ $f(x)$ is continuous

$\therefore$ LHL = RHL = $f(1)$ at $x = 1$

$1 + 6 = p + q$

$p + q = 7$ ...(1)

LHL = RHL = $f(2)$ at $x = 2$

$p(2) + q = 5$ (2)

$2p + q = 10$ ...(2)

(2) – (1), $p = 3$

90. (c) $\because$ $p = 3$ and $p + q = 7$

$q = 7 - 3 = 4.$

91. (a) $f(x) = \ln x$

$$f'(x) = \frac{1}{x}$$

for $x \in (0, \infty)$

$f'(x) > 0$

$\therefore$ $f(x)$ is increasing in $(0, \infty)$.

$$g(x) = e^x + e^{1/x}$$

$$g'(x) = e^x - e^{1/x}$$

when $x < 1$, $g'(x) < 0$

at $x = 1$, $g'(x) = 0$

at $x > 1$, $g'(x) > 0$

$\therefore$ in $(0, \infty)$. $g(x)$ is neither increasing nor decreasing.

Only 1 is correct.

92. (a) $$\frac{d(\sin^2 x)}{d(\cos^2 x)} = \frac{\frac{d}{dx}\sin^2 x}{\frac{d}{dx}\cos^2 x}$$

$$\frac{d}{dx}\sin^2 x = 2\sin x \cos x$$

$$\frac{d}{dx}\cos^2 x = -2\cos x \sin x$$

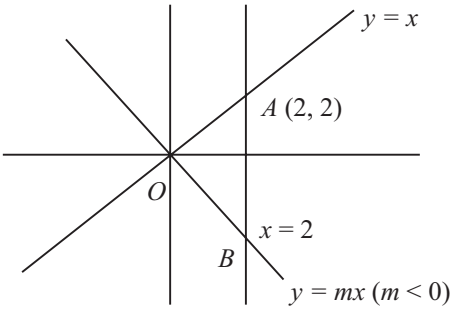
$$\therefore \quad \frac{d(\sin^2 x)}{d(\cos^2 x)} = \frac{2\sin x \cos x}{-2\sin x \cos x} = -1.$$

93. (c) O (0, 0)

A = (2, 2)

B (2, $2m$)

Ar (ΔOAB) = 3



$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 2 & 2 & 1 \\ 2 & 2m & 1 \end{vmatrix} = 3$$

$\Rightarrow$ $|4m - 4| = 6$

$4m - 4 = \pm 6$

$4m = 10, -2$

$\therefore \quad m = \frac{10}{4}, \frac{-2}{4}$

$\because \quad m < 0$

$m = -\frac{1}{2}.$

94. (b) $\frac{d}{dx}\text{cosec}\,(x°) = \frac{d}{dx}\text{cosec}\left(\frac{x\pi}{180}\right)$

$= \frac{-\pi}{180}\text{cosec}\left(\frac{\pi x}{180}\right)\cdot\cot\left(\frac{\pi x}{180}\right)$

$= \frac{-\pi}{180}\text{cosec}\,(x°)\cdot\cot\,(x°)$

95. (d) $\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}\left[\frac{dy}{dx} - x\right] = 0$

$\frac{dy}{dx} = 0 \text{ or } \frac{dy}{dx} = x$

$dy = x\,dx$

$\int dy = \int x\,dx$

$y = \frac{x^2}{2} + C$

$y = \frac{x^2 + 2C}{2}$

Comparing with options, $y = \frac{x^2 - 2}{2}$

96. (d) $f(x) = x^2 + 2$

$g(x) = 2x - 3$

$(fg)\,(1) = f(1) \times g(1) = (1 + 2)\,(2 - 3) = -3.$

97. (b) $f(x) = x + |\,x\,|$

when $x < 0$

$f(x) = x - x = 0$

when $x \geq 0$

$f(x) = x + x = 2x$

$\therefore \quad f(x) \in (0, \infty).$

98. (a) $f(x) = x\,(4x^2 - 3)$

$f(\sin\theta) = \sin\theta(4\sin^2\theta - 3)$

$= 4\sin^3\theta - 3\sin\theta$

$= -(3\sin\theta - 4\sin^3\theta) = -\sin 3\theta$

$= -\sin 3\theta$

$\therefore \quad f(\sin\theta) = -\sin 3\theta.$

99. (d) $\lim_{x\to 5}\frac{5-x}{|\,x-5\,|}$

LHL : $\lim_{x\to 5^-}\frac{5-x}{-(x-5)} = \lim_{x\to 5^-}\frac{5-x}{5-x} = 1$

RHL : $\lim_{x\to 5^+}\frac{5-x}{(x-5)} = \lim_{x\to 5^+}(-1) = -1$

$\because$ LHL $\neq$ RHL

$\therefore$ Limit does not exist.

100. (c) $\lim_{x\to 1}\frac{x^9 - 1}{x^3 - 1}$

$\lim_{x\to 1}\frac{(x^3)^3 - 1}{x^3 - 1} = \lim_{x\to 1}\frac{(x^3 - 1)(x^6 + x^3 + 1)}{x^3 - 1}$

$= \lim_{x\to 1} x^6 + x^3 + 1 = 3.$

101. $\bar{x} = 14$

Var $(x) = 13.2$

Observations = 11, 16, 20, a, b

$\bar{x} = \frac{11 + 16 + 20 + a + b}{5} = 14$

$a + b = 23 \qquad ...(1)$

$\text{var}(X) = \frac{\Sigma x_i^2}{n} = (\bar{x})^2$

$13.2 = \frac{11^2 + 16^2 + 20^2 + a^2 + b^2}{5} - (14)^2$

$66 = 777 - 980 + a^2 + b^2$

By (1) & (2). we get

$a^2 + b^2 = 269 \qquad ...(2)$

$\therefore \quad a = 13, b = 10$

102. $P(\bar{A}) = 0.7 \Rightarrow P(A) = 0.3$

$P(\bar{B}) = k \Rightarrow P(B) = 1 - k$

$P(A \cup B) = 0.8$

For independent events

$P(A \cap B) = P(A)\cdot P(B)$

$P(A \cup B) = P(A) + P(B) - P(A)\cdot P(B)$

$\Rightarrow 0.8 = 0.3 + 1 - k - 0.3\,(1 - k)$

$\Rightarrow 0.5 = 1 - k - 0.3 + 0.3k$

$\Rightarrow 0.5 = 0.7 - 0.7k$

$\Rightarrow 0.7\,k = 0.2$

$\Rightarrow k = \frac{2}{7}.$

103.(b) $P(\text{Head}) = \frac{1}{4}$

$\therefore \quad P(\text{Tail}) = \frac{3}{4}$

P (4 Tail and 1 Head)

$= P(T) \cdot P(T) \cdot P(T) \cdot P(T) \cdot P(H)$

$= (P(T))^4 \cdot P(H)$

$= \left(\frac{3}{4}\right)^4 \cdot \left(\frac{1}{4}\right) = \frac{81}{1024}.$

104.(b) $\because$ Heads comes up thrice as likely as tails

$\Rightarrow P(H) = 3P(T)$

and $P(H) + P(T) = 1$

$\therefore \quad P(H) = \frac{3}{4}$

$P(T) = \frac{1}{4}$

P (3 head and 1 tail) $= \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{27}{256}.$

105.(c) $\text{var}(X) = npq = 100 \times \frac{4}{5} \times \frac{1}{5} = 16$

$\text{var}(Y) = \text{var}(100 - X) = \text{var}(X) = 16$

106. (c) $x + 4y + 1 = 0$ ...(1)

$4x + 9y + 7 = 0$ ...(2)

Putting $y = -3$ in (1) and (2)

From (1), $x = 11$ which is not in option

From (2), $x = 5$

107. (d) $p + q + r + s = 360°$ ...(1)

$9p = 3q = 2r = 6s = k$ (let)

$p = \frac{k}{9}, q = \frac{k}{3}, r = \frac{k}{2}, s = \frac{k}{6}$

From (1),

$\frac{k}{9} + \frac{k}{3} + \frac{k}{2} + \frac{k}{6} = 360°$

$\Rightarrow \frac{2k + 6k + 9k + 3k}{18} = 360°$

$\Rightarrow 20k = 360° \times 18 = 324°.$

$\therefore \quad 4p - q = \frac{4k}{9} - \frac{k}{3} = \frac{k}{9} = \frac{324}{9}$

$\therefore \quad 4p - q = 36°.$

108.(a) Observations in ascending order

1, 1, 1, 2, 3, 3, 4, 4, 4, 5, 6, 6

m = mean of 1, 1, 1, 2, 3, 3, 4, 4

$= \frac{1+1+1+2+3+3+4+4}{8} = \frac{19}{8}$

M = Mean of highest 4 observations (4, 5, 6, 6)

$M = \frac{4+5+6+6}{4} = \frac{21}{4}$

$2m + M = 2\left(\frac{19}{8}\right) + \frac{21}{4}$

$\therefore \quad 2m + M = 10.$

109. (a) For line of regression of y on x, the line will be one which satisfy both points.

Clearly,

$x - 4y + 5 = 0$ satisfy $(-1, 1)$ and $(3, 2)$ both.

Hence line is $x - 4y + 5 = 0$.

110. (d) Mode = Data observations with maximum frequency

$\therefore$ Mode = 1.

111. (c)

x	f	CF
1	4	4
2	6	10
3	9	19
5	7	26

$\therefore \quad N = 26$

$\frac{N}{2} = 13$

$\because$ 13 is less than 19

$\therefore$ Median = 3.

112. (d) First 5 observations : –1, 1, 4, 3, 8

i.e. –1, 1, 3, 4, 8

$\therefore$ Median $(M) = 3$

Last 5 observations : 12, 17, 19, 9, 11

i.e. 9, 11, 12, 17, 19

Median $(N) = 12$

$\therefore \quad 4M - N = (3 \times 4) - 12 = 0.$

113. (d) $5P = 4Q = \frac{R}{2} = k$

$P = \frac{k}{5}, Q = \frac{k}{4}, R = 2k$

$\therefore \quad \frac{P+Q}{2P + 0.7R} = \frac{\frac{k}{5} + \frac{k}{4}}{\frac{2k}{5} + 0.7(2k)} = \frac{k\left(\frac{1}{5} + \frac{1}{4}\right)}{k\left(\frac{2}{5} + \frac{7}{5}\right)}$

$= \frac{\frac{9}{20}}{\frac{9}{5}} = \frac{1}{4}.$

114. (d) 1, 2, 2^2, 2^3, ... 2^{n-1}

Number of terms = n

$\therefore$ GM $= (1\times 2\times 2^2\times 2^3\times .....\times 2^{n-1})^{1/n}$

$= (2^{1+2+3+....+(n-1)})^{1/n}$

$= \left[2^{\frac{n(n-1)}{2}}\right]^{1/n}$

$G = 2^{\frac{n-1}{2}}$

$\therefore \quad 1+2\log_2 G = 1+2\log_2 2^{\frac{n-1}{2}}$

$= 1+2\frac{(n-1)}{2}\log_2 2$

$= 1 + n - 1 = n$

115. (b) $\because \quad \frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+.....+\frac{1}{a_n}} = H$

$\frac{n}{H} = \frac{1}{a_1}+\frac{1}{a_2}+.....+\frac{1}{a_n}$

$= 1+\frac{1}{2}+\frac{1}{2^2}+.....+\frac{1}{2^{n-1}}$

$= \frac{1\left(\left(\frac{1}{2}\right)^n-1\right)}{\left(\frac{1}{2}-1\right)} = 2\left(1-\frac{1}{2^n}\right)$

$\therefore \quad \frac{n}{H} = 2-\frac{1}{2^{n-1}}$

116. (c) $\Sigma(x_i-a)^2$ is minimum for mean = a

$\therefore$ For $\Sigma(2x_i-a)^2 = \Sigma 4\left(x_i-\frac{a}{2}\right)^2$

$= \frac{a}{2}$ must be mean

$= \frac{a}{2} = Q$

$\Rightarrow a = 2Q.$

117. (Bonus) $\boxed{\begin{matrix}2B\\3W\end{matrix}}_{I} \quad \boxed{\begin{matrix}3B\\2W\end{matrix}}_{II}$

When 2 balls are drawn from I, it can be done as both black, both white or 1 black 1 white.

Now in such cases, new number of balls will be

	bag 1	bag 2	Required Probability
$2B$	$3W$	$5B, 2W$	$\frac{^2C_2}{^5C_2}\times\frac{2}{7}$
$1B+1W$	$1B, 2W$	$4B, 3W$	$\frac{^2C_1\times{}^3C_1}{^5C_2}\times\frac{3}{7}$
$2W$	$2B, 1W$	$3B, 4W$	$\frac{^3C_2}{^5C_2}\times\frac{4}{7}$

Total Probability

$= \left(\frac{^2C_2}{^5C_2}\times\frac{2}{7}\right)+\left(\frac{^2C_1\times{}^3C_1}{^5C_2}\times\frac{3}{7}\right)+\left(\frac{^3C_2}{^5C_2}\times\frac{4}{7}\right)$

$= \frac{32}{70} = \frac{16}{35}.$

118. (c) On throw of 3 dice,

Number of possible outcomes = $6^3 = 216$

Favourable outcomes

= {(3, 3, 3), (3, 3, 6), (3, 6, 3), (6, 3, 3), (3, 6, 6)
(6, 3, 6), (6, 6, 3), (6, 6, 6)}

Probability $= \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}}$

$= \frac{8}{216} = \frac{1}{27}.$

119. (c) Number of days in month of December = 31

Out of which there will be 4 full weeks by 28 days.

Remaining days = 3

$\therefore$ P (having 5 Sunday) $= \frac{3}{7}.$

120. (b) $n+\frac{50}{n} < 50$

Clearly for n = 1, 49, 50 this relation is not true.

$\therefore$ Number of favourable values of n = 47

$\therefore$ Probability $= \frac{47}{50}.$

GENERAL ABILITY

PART- A: ENGLISH

1. (b) As per Subject-Verb concord, a singular subject takes the singular form of verb and a plural subject is conjugated with the plural form of verb. In the sentence, 'layers' is plural in 'layers of sediments', thus 'build' (without the ending 's') will be used with it. The correct sentence is 'Over long periods of time, layers of sediments **build** up to a height of few kilometres.'

2. (b) We do not use the indefinite articles 'a' or 'an' with uncountable nouns. Thus, 'a liquid' is incorrect. The correct sentence will be 'When a gas is cooled down it turns into liquid from a process called condensation.'.

3. (d) There is no error in the sentence

4. (c) We use the preposition 'in' with years and months. Hence, the right sentence is 'Columbus made his first voyage from Europe to America in 1492.'

5. **(c)** In this sentence, the preposition that should be used is 'at'. Therefore, the correct sentence is 'Whenever the sky is clear, you can see the stars at night.'

6. **(b)** The verb is not conjugated with 'do not or don't'. With it, we use the infinitive form of verb. Thus, the correct form is 'I'm not working tomorrow, so I don't have to get up early.'

7. (b) 'Anybody' is not completely incorrect. However, 'anyone' is more appropriate. We use 'anyone' when we do not have any specific person in mind to refer whereas 'anybody' is used when we intend to target a group or a member of a particular group.

8. (b) Here, Conditional Type 3 is not correctly used. To create a Type 3 Conditional sentence, we use the following structure-

If + past perfect + would have + past participle

Hence, the correct sentence will be 'She wouldn't have had an accident, if she had driven carefully.'

9. **(c)** We will use the preposition 'on' instead of 'in' to indicate that the action need to end at a certain time and not later than that. 'I will watch film if I finish the work on time', should be the right sentence.

10. (c) 'It's' is a contraction of 'it is' or 'it has'. However, in the given sentence we are supposed to show possession for which we use 'Its'. The correct sentence is 'In 1989, the government did an about-face and Iran restored its family planning programme.'

11. (a) Indigent means 'needy' or 'very poor'. Thus, the correct response is option (a).
Other words on the list have the following meaning-
Opulent- luxurious and wealthy

Solvent- a substance that dissolves other substances

Prosperous- wealthy and successful

12. (d) Jovial means 'happy' or 'mirthful'. Other words have the following meaning- Aggrieved- feeling resentment at having been treated unfairly

Melancholic- sad

Doleful- mournful

13. (c) 'Lunacy' means 'madness' or 'insanity'. Other words and their meanings are - Prudence- marked by good judgement or wisdom Normalcy- state of being normal Sanity- ability to think and behave in a normal and rational manner.

14. (d) 'Meticulous' or 'scrupulous' means 'extreme attention to details'.

15. (c) The synonym of 'exhilarating' is 'exciting'. Thus, option (c) is the correct response.

'Humdrum' means 'monotonous', 'dreary' means 'depressingly bleak' and 'agitating' means 'make someone nervous or troubled'.

16. (a) 'Redemption' means 'retrieval', 'recovery' or 'act of saving'. Other words have the following meaning- Forfeiture- giving up of something as a penalty

Corporation- large company that acts as a single entity

Desecration- act of violation

17. (a) 'Contention' means 'heated argument' or 'dispute'. Thus, option (a) is the correct response.

18. (b) 'Paraphernalia' refers to the equipments or accessories that are needed for a particular purpose.

19. (c) 'Astonishment' means 'amazement' or 'wonderment'. Other words on the list do not have the same meaning.

20. (b) 'Inordinate' like 'exorbitant' means 'unreasonably high or excessive'. Thus, option (b) is the right answer. Note that the words 'temperate' and 'moderate' are synonymous.

21. (a) The right order- I make at least three drafts of a song because I feel when you right something, you remember it better.

22. (c) The right order- A gunman who held a hostage for hours in the shop demanded ransom of ₹ 10 lakh.

23. (b) The right order- Ukraine has urged its citizens living in Russia to leave the country immediately.

24. (c) The right order- Cyclone Emnati crashed into the south-eastern coast of Madagascar in the early hours, ripping roofs off houses and raising fears of flooding and food shortages in the region.

25. (c) The right order- We need to utilize the resources of water for different purposes while ensuring that its natural ecology is protected and aquatic life thrives.

26. (d) The right order- The poor visibility due to fog made the movement of traffic very difficult.

27. (b) The right order- There are multiple legal avenues for the patients to indict the doctors but rarely is it the other way round.

28. (c) The right order- Life is a solo flight and each person makes his or her own journey.

29. (c) The right order- Adoption of the latest technologies and equipment has made it evolve into a rare centre of excellence.

30. (d) The right order- If you have any complaint to be made in response to this notice please mark the same to the concerned authority.

31. (a) 'Fortify' means 'strengthen' or 'encourage'. Its opposite will be 'weaken'.

32. (b) The antonym of 'dazzling' is 'murky'. The former means 'bright' or 'fulgent' whereas the latter means 'dark' or 'gloomy'.

33. (d) 'Clandestine' means 'hidden'. The opposite of the word will be 'public'. Other words on the list are synonymous to 'clandestine'.

34. (b) 'Bleak' refers to something that is gloomy and dull. 'Verdant' means 'lush or bright green'. The words are opposite in their meaning. Hence, the two are antonyms.

35. (a) 'Quietude' is the antonym of 'commotion', which means 'noise' or 'confusion'.

36. (d) 'Audacity', like 'temerity' means 'willingness or boldness in doing something '. The apt antonym is 'politeness'.

37. (c) 'Abatement' refers to the act of removing or ending something. On the other hand, 'accumulation' means 'to gather' or 'bring together something'. Thus, they are antonyms. Note, the other words on the list of options imply reduction or removal, therefore they cannot be opposites.

38. (c) 'Exponential', which means 'to grow rapidly' is the antonym of the word 'meagre' which means 'lack' or 'inadequacy'.

39. (c) 'Sagacity', like 'prudence' and 'sapience' means 'wisdom'. Its opposite is 'ignorance'.

40. (c) 'Culminates' means 'to end'. The correct antonym of the word from the given list is 'originates', which means 'to start' or 'to begin'.

41. (b) The idiom 'the lion's share' means the largest and best part of something.

42. (a) The idiom 'to spill the beans' means 'to reveal a secret or confidential information'.

43. (c) 'To cook someone's goose' means 'to spoil someone's plans.'

44. (d) The idiom 'to disappear into thin air' means 'to disappear completely and suddenly'.

45. (a) The idiom 'to put the screws on' means 'to intimidate or pressurize someone'. Thus, option (a) is the correct response.

46. (c) 'To sit on the fence' means 'to avoid taking a particular side'.

47. (d) The idiom 'to be as sharp as a tack' means 'to be witty or clever'.

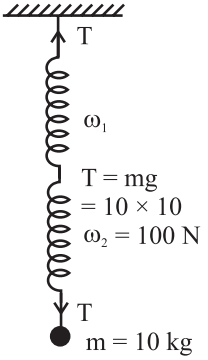
48. (b) 'From pillar to post' means 'to move fruitlessly from one place to another'.

49. (a) 'Heath Robinson' is used to denote a machine which is meant to do a simple task but is unnecessarily complicated.

50. (d) The phrase 'as thick as thieves', is used to refer to people who are very close friends but their friendship is looked upon with suspicion.

PART- B: GENERAL KNOWLEDGE

51. (d)



In both end tension will be same.

W_1 = 10 kg and W_2 = 10 kg

So, the reading on S_1 and S_2 are W1 = 10 kg and W2 = 10 kg respectively.

52. (c) Height of building, h = 20 m

Speed of stone, v = 12 m/s

Time required to reach the ground

$$t = \sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2\times 20}{10}}$$

= 2s

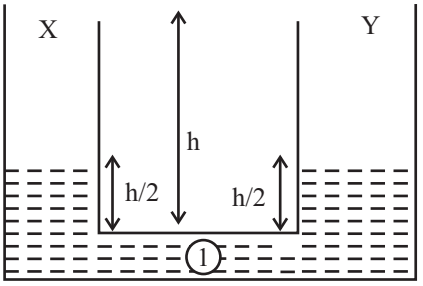
then $R = vt$

$= 12 \times 2$

= 24 m

53. (c)

54. (c) Consider the mass of all the liquid is m, then mass of containers x and y is $\frac{m}{2}$. After opening the value, the liquid will be divided equally in container x and as shown in below.



Container Y is same placed before and after condition. So gravitational potential energy due to container y does not changed.

Initial potential energy of the liquid $p_1 = \frac{mgh}{2}$

Final potential energy of the liquid $p_2 = \frac{m}{2}g\frac{h}{2} = \frac{mgh}{4}$

$$\frac{p_1}{p_2} = \frac{mgh}{2} \times \frac{4}{mgh}$$

$$\therefore \quad p_1 = 2p_2$$

55. (c) Change in momentum = Final momentum – Initial momentum

$$= mv - (-mv)$$
$$= 2mv$$

Now, time taken $= \frac{\text{distance}}{\text{speed}} = \frac{\pi R}{v}$

$$= \frac{2mv}{\frac{\pi R}{v}}$$

$$= \frac{2mv^2}{\pi R}$$

The average acceleration will be

$$a = \frac{F}{m}$$

$$= \frac{2mv^2}{\pi Rm}$$

$$a = \frac{2v^2}{\pi R}$$

56. (a) The net force acting on the point mass has magnitude close to

$$F_{net} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60°}$$

$(\because F_1 = F_2 = 5N)$

$$= \sqrt{5^2 + 5^2 + 2 \times 5 \times 5 \times \frac{1}{2}}$$

= 8.6 N

57.(b) Igneous rocks form when hot, molten rock crystallizes and solidifies. The melt originates deep within the Earth near active plate boundaries or hot spots, then rises toward the surface.

Example of igneous rocks includes andesite, basalt, dacite, obsidian, pumice, rhyolite, scoria, and tuff.

Slate is a type of metamorphic rock which is formed when shale gets metamorphosed.

58.(b) The subsurface location (focus) at which the energy of an earthquake is released is known as hypocentre. Earthquakes generally occur at depths less than about 30 km.

P waves are the fastest body waves and arrive at stations before the S waves, or secondary waves. Their velocity in the crust varies between 5.0 and 7.0 km/s. Seismic waves move more slowly through a liquid than a solid. Molten areas within the Earth slow down P waves and stop S waves because their shearing motion cannot be transmitted through a liquid.

59. (b) The Quaternary Period is divided into two epochs, Holocene and Pleistocene. Holocene is the youngest epoch of Quaternary period while Pleistocene is the oldest.

60. (c) Earth > Venus > Jupiter> Saturn

Planet	Average Density (gm/cm3)
Mercury	5.4
Venus	5.2
Earth	5.5
Mars	3.9
Jupiter	1.3
Saturn	0.7
Uranus	1.3
Neptune	1.6

61. (b) The East Australian Current is a large-scale flow of water that runs south along the east coast of Australia. It is a warm, southward, western boundary current.

62. (a) Podzolization usually occurs under forest or heath vegetation and is common in cool and humid climates as these climates inhibit the activity of soil microbes in the topsoil.

63. (c) Keen Sword 23 is a joint/bilateral exercise involving U.S. military and Japan Self-Defense Force (JSDF) personnel designed to enhance Japan-U.S. bilateral relations.

It is a joint, bilateral field training exercise designed to enhance U.S.-Japan combat readiness and interoperability while demonstrating the U.S. resolve to support the security interests of allies and partners in the Indo-Pacific region.

64. (c) Rezang La is a mountain pass on the Line of Actual Control between Indian-administered Ladakh and the Spanggur Lake basin that is illegally occupied by

China. On 18 November 1962, the Charlie Company of the 13 Kumaon Battalion, Kumaon Regiment, fought a Chinese attack at Rezang La Pass in Ladakh, India. Of the 120 men and officers of this company, 114 died. Yet they managed to inflict relatively higher fatalities on the enemy side. Indian troops staved off waves after waves of Chinese attack and put up enough of a fight to ensure that Chushul wouldn't fall into Chinese hands.

65. (a) Exercise Sea Vigil-22 the two-day Coastal Defence exercise was conducted from 15-16 Nov 22. The conceptual and geographical expanse of Sea Vigil included contingencies ranging from Peace to War-time across the entire coastline and EEZ of the country. In addition, mitigation measures, on shore, in case of any breach in Coastal Security mechanism were also validated.

The exercise involved participation of more than 17 Government agencies from nine Coastal States and four Union Territories that are involved in the Coastal Defence Mechanism and Coastal Security construct.

66. (c) England emerged as winners of the ICC Men's T20 World Cup twice during 2010 and 2022. The maximum number of the "Man of the Tournament" won by Virat Kohli first in 2014then in 2016.

67. (b) Barry Sharpless,Morten Meldal and Carolyn Bertozzi, were awarded the Nobel Prize in Chemistry 2022.

Benjamin List and David MacMillan were awarded the Nobel Prize in Chemistry 2021 for their development of a precise new tool for molecular construction known as organocatalysis.

68. (c) The cell wall is present in all plant cells, fungi, bacteria, algae, and some archaea. In bacteria the primary component of the cell wall is peptidoglycan.

The characteristic structure of fungal cell wall is composed mainly of glucans, chitin and glycoproteins.

Animal cells are typical of the eukaryotic cell, enclosed by a plasma membrane and containing a membrane-bound nucleus and organelles. The extracellular matrix of animal cells. The extracellular matrix of animal cells is made up of proteins and carbohydrates and holds cells together to form a tissue and allow tissues to communicate with each other.

69. (c) A typical prokaryotic cell lacks a prominent nuclei. In Prokaryotic cells, the genetic material DNA simply exists in a region of the cell called the nucleoid. The genetic material exists as a free-floating chromosome that is usually circular and is not enclosed in a nuclear membrane of typical nucleus.

70. (c) Both chloroplasts and mitochondria contain their own genetic systems, and replicate by division. However, chloroplasts are larger and more complex than mitochondria, and they perform several critical tasks in addition to the generation of ATP.

71. (d) Sexual reproduction involves two parents, whereas asexual reproduction involves only one parent. The offspring resulting from sexual reproduction therefore have characteristics from both the parents and therefore, will be different from each parent. This will bring variations in the population. Variations are important for the survival of species over time.

72. (a) Cell Wall is the characteristics of plant cell only and not the all types of cell including animal cells. Plasma membrane, cytoplasm, genetic material these all are essential cell component.

The Plasma Membrane is a double membranous cell organelle, composed of the phospholipid bilayer and is present both in prokaryotic and eukaryotic cells.

Cytoplasm is the fluid inside a cell but outside the cell's nucleus. Most chemical reactions in a cell take place in the cytoplasm.

The part of the cell which carries the genetic information that can be inherited is called genetic material. It is necessary for the reproduction and propagation of genes from one generation to another. DNA/ RNA are types of genetic material inside a cell.

73. (c) Browning of the chopped apple can be minimized by adding lemon juice or similar types of antioxidants. "Enzymatic browning of apple occurs when the sliced pieces are exposed to oxygen. This triggers an enzyme known as polyphenol oxidase (PPO) which oxidizes polyphenols in the apple's flesh and turn it into brown.

74. (b) Boiling points of alkenes increases with their molecular masses due to stranger intermolecular forces.

The order of their masses is:–

Methane < Ethane < Propane < Butane < Pentane < Hexane < Heptane < Octane

Thus, the boiling points increases in the same order.

Therefore, (b) propane < butane < pentane < octane <, is the correct option

75. (c) Benzene and water form an immiscible mixture and can be separated by a separating funnel wherein benzene forms the upper layer and can be separated. Dean and Stark apparatus is used for mixtures that are separated by distillation.

76. (d) $Fe(S) + CUSO4(aq) \longrightarrow FeSO4(aq.)$ (blue) $+$ $CU(S)$ (brown)

The brown colour is due to Cu deposits and this is a displacement reaction.

77. (b) Curd contains acids that might react with the metallic copper so it should not be kept in a copper vessel. Other materials are not reactive.

78. (a) Toothpastes are basic in nature and they react with and neutralize the acids produced in our teeth by bacterial activities. This way they prevent tooth decay.

79. (b) Kanaklata Baruah was an eminent freedom fighter of Assam. She played a pivotal role in 1942 Quit India

Movement by sacrificing her life for the cause of the nation.

On September 20, 1942 the revolutionary group Mrityu Bahini members proceeded towards the police station under the leadership of Kanaklata Baruah to unfurl the national flag at the local police station.

She was shot during the process by the British Police but kept on holding the flag till the time her companion Mukanda Kakati took it from her. Kanaklata Baruah succumbed to her injury and attained martyrdom.

80. (c) The term mansabdar refers to an individual who holds a mansab, meaning a position or rank. It was a grading system used by the Mughals to fix rank, salary and military responsibilities. Rank and salary were determined by a numerical value called zat. The higher the zat, the more prestigious was the noble's position in court and the larger his salary.

81. (c) The impeachment of Warren Hastings, the first governor-general of Bengal Presidency, was attempted between 1787 and 1795 in the Parliament of Great Britain. Hastings was accused of misconduct during his time in Calcutta, particularly relating to mismanagement and personal corruption.

82. (a) Prayag Prasashti is an inscription found at present day Prayagraj. It is written in sanskrit. As the name suggests it is a Prashasti or eulogy written by Harisena who was the court poet of Samudragupta. In this Prasashti he mentioned the achievements of Samudragupta.

83. (b) Brothers Harihar and Bukkaroy founded Vijaynagar, an independent Hindu kingdom. They were the sons of Sangama, a Hoysala ruler's court chieftain. The Vijyanagar kingdom, located on the southern bank of the river Tungabhadra, was perhaps South India's most famous empire.

84. (d) The Danish East India Company established their first factory at Tranquebar in Tamil Nadu in 1620.

85. (c)

86. (d) $\delta = \delta_1 + \delta_2$
$= 180 - 25 + 180 - 2i$
$= 360 - 4i$
Hence, $\delta = 4\theta$

87. (c) Applying the conservation of momentum
initial momentum = final momentum
$m_1v_1 = m_2v_2$

88. (c) In flute, sound is produced only inside due to vibration of the air column.

89. (d) We have
Charge move with speed $\vec{v} = v_0(-\hat{j})$,
Magnetic field, $\vec{B} = B_0\hat{j}$
From $F = q\left(\vec{v} \times \vec{B}\right)$
$= q\left(v_0 - \hat{j} \times B_0\hat{j}\right)$
$= 0 \qquad \left(\because \hat{j} \times \hat{j} = 0\right)$
The moving charge will experience no deflecting force

90.(b) Oxygen is not an example of greenhouse gases. The main greenhouse gases whose concentrations are rising are carbon dioxide (CO2), Methane (CH4), Nitrous Oxide, Hydrochlorofluorocarbons (HCFCs), Hydrofluorocarbons (HFCs) and Ozone in the lower atmosphere.

91. (a) X-rays have very small wavelengths, between 0.03 and 3 nanometers. The generation of X-rays occurs when electrons are accelerated under a potential difference and turned into electromagnetic radiation.

92. (a) Among the locations of New Delhi, Thimphu, Kathmandu and Dhaka, Capital of India New Delhi is located geographically northern relative to other three.

93. (d) Lines of latitude start at 0 degrees at the equator and end at 90 degrees at the North and South Poles (for a total to 180 degrees of latitude). Therefore, the higher the value of degree of latitude, the closer it is to the North or South Pole.

The distance between longitudes reaches zero at the north poles and south as the lines of meridian converge at that point.

The distance between the longitudes is the maximum at the equator.

The total number of latitudes is 180 and the total number of longitudes is 360.

94. (a) Madhya Pradesh is the leading producing State of manganese ore in India. The state accounts for 33% of total production in last few years followed by Maharashtra, Odisha, Karnataka and other states.

95. (d) Mica is useful in a wide range of industries, including mining, plastics, electronics, automotive, paint, and much more.

96. (b) According to Koppen's climatic classifications of India, Great Northern plains are represented by Cwg climate. This type of climate is found in most parts of the Northern Ganga Plains. In this type of climate, the summer temperature rises to 40°C which falls to 27°C in winter.

97. (a) Lachit Borphukan is regarded as the brave General of the Royal Army of the Ahom Kingdom of Assam who defeated the Mughals and successfully halted the ever-expanding ambitions of the Mughals under Aurangzeb.

Lachit Borphukan inspired the Assamese soldiers in the Battle of Saraighat fought in 1671, and delivered a crushing and humiliating defeat on the Mughals. The heroic fight of Lachit Borphukan and his army remains one of the most inspiring military feats of resistance in the history of our country.

To keep his valour and inspire younger generation, the Lachit Barphukan Gold Medal is given to the best

cadet at the National Defence Academy (NDA) since 1999.

98. (c) The 4th edition of joint military exercise 'DUSTLIK' between the Indian Army and Uzbekistan Army commenced took place in Foreign Training Node, Pithoragarh (Uttarakhand). 45 Soldiers each from Uzbekistan and Indian Army participated in this 14 day exercise.

DUSTLIK is aimed at promoting positive relations between both the armies.

99. (d) Vikram-S became India's first privately-developed rocket. The rocket was developed by Hyderabad-based startup Skyroot Aerospace. The rocket was launched under Mission 'Prarambh' and carried three payloads.

100. (b) Modhera in Gujarat state became India's first 24×7 solar-powered village. This village in the Mehsana district of Gujarat is India's first village to be powered by solar all day, every day. It has cost $9.7 million and been paid for by the paid for by the central government and the government of Gujarat state.

101. (a) The tableau of Uttrakhand was given away the best tableau award among the 17 states/union territories tableaux displayed at the Republic Day 2023. It was based on the theme 'Manaskhand'.

102. (a) Autotrophs obtain carbon from inorganic sources like carbon dioxide (CO_2). These organisms use CO_2 as primary source of carbon and are capable of producing their own food using light, water, carbon dioxide, or other chemicals.

103. (a) During fermentation, yeast cells convert cereal-derived sugars into ethanol and CO_2. This anaerobic reaction can be written as,

Carbohydrate------Yeast--------→Ethanol+ CO_2 + ATP

At the same time, hundreds of secondary metabolites that influence the aroma and taste of beer are produced.

104. (b) When plant cells are exposed to hyperosmotic stress they undergo Plasmolysis.

Plasmolysis is defined as the process of contraction or shrinkage of the protoplasm of a plant cell and is caused due to the loss of water in the cell.

105. (c)

- Fungi are eukaryotic organisms means they have true nucleus which are enclosed in membranes.
- Fungi are heterotrophic organisms. They obtains its food and energy from organic substances, plant and animal matters.
- They are non-vascular organisms. They do not have vascular system. Xylem and Phloem are absent.
- Fungi have cell walls (plants also have cell walls, but animals have no cell walls).
- he structure of cell wall is similar to plants but chemically the fungi cell wall are composed of chitin
- There is no embryonic stage for fungi.

106. (*) Bryophytes include the various mosses and liverworts that are found commonly growing in moist shaded areas in the hills.

Bryophytes are also called amphibians of the plant kingdom because these plants can live in soil but are dependent on water for sexual reproduction. They usually occur in damp, humid and shaded localities. They play an important role in plant succession on bare rocks/soil.

The plant body of bryophytes is more differentiated than that of algae. It is thallus-like and prostrate or erect, and attached to the substratum by unicellular or multicellular rhizoids.

The main plant body of the bryophyte is haploid. It produces gametes, hence is called a gametophyte. The sex organs in bryophytes are multicellular. The male sex organ is called antheridium. They produce biflagellate antherozoids. The female sex organ called archegonium is flask-shaped and produces a single egg. Both bryophytes and tracheophytes have developed conducting tissues consisting of **cells** specialized for the distribution of food and water.

107. (b) The process of converting fats, oil, and lipid into soap using an aqueous alkali is called saponification. Vegetable oil and animal fats are triesters or triglycerides that can be saponified in one or two steps. During this process, the triglyceride reacts with an aqueous hydroxide ion to form a mixture of glycerol and fatty acid derivative. The sodium or potassium salts of long-chained fatty acids are a significant component of soap

The alkali used in this process is sodium hydroxide (NaOH) or lye for hard soap and potassium hydroxide (KOH) for soft soap.

108. (d) The arrangement of metals as per decreasing order of their reactivity is-

Sodium > Iron > Copper > Silver

109. (d) Graphite is a good conductor of electricity because of the presence of free electrons in its. The hardness of minerals is compared using the Mohs Hardness Scale, a relative scale numbered 1 (softest) to 10 (hardest).

- Graphite is very soft and has a hardness of 1 to 2 on this scale.
- Diamonds are the hardest known natural substance and have a hardness of 10. No other naturally occurring substance has a hardness of 10.

110. (b) Ammonia is used as raw material for the production of various nitrogen compounds, including nitric acid, and a variety of fertilisers and polymers. Also, ammonia is used as refrigerant and neutraliser for NOx emission from fuel combustion.

The Haber-Bosch process takes nitrogen gas from the atmosphere and combines it to form ammonia gas with molecular hydrogen gas.

111. (d) Glass is made from natural and abundant raw materials (sand, soda ash and limestone) that are melted at very high temperature to form a new material: glass. At high temperature glass is structurally similar to liquids, however at ambient temperature it behaves like solids.

The source of silica during the process is generally sand.

112. (d) The increasing order of Oxidizing nature of halogens is, Iodine<Bromine<Chlorine<Fluorine

113. (c) The most common form of copper sulphate is its pentahydrate, given by the chemical formula $CuSO_4.5H_2O$. It is a blue coloured chemical. When it is heated, the blue colour disappears and it turns white. Heating removes the water of crystallization which is responsible for blue colour of the crystal.

$CuSO_4.5H_2O(Blue) \xrightarrow{Heat} CuSO_4$ (White)+ $5H_2O$

After it cools, if a few drops of water are put into the test tube, the copper sulphate turns blue again as water of crystallization make the copper sulphate hydrated again.

114. (c) According to Avogadro's law, under the same conditions of temperature and pressure, equal volumes of different gases contain an equal number of molecules.

115. (a) The chemical at the heart of the air bag reaction is called sodium azide (NaN3). High impact during crashes trip sensors in cars that send an electric signal to an ignitor. The heat generated causes sodium azide to decompose into sodium metal and nitrogen gas, which inflates the car's air bags.

116. (a) The 1947 Sylhet referendum was held in the Sylhet District of the Assam Province of British India to decide whether the district would remain in Undivided Undivided Assam and therefore within the post-independence Dominion of India, or leave Assam for East Bengal and consequently join the newly-created Dominion of Pakistan.

117. (c) The Rabatak Inscription is a stone inscribed with text written in the Bactrian language and Greek script, found in 1993 at Rabatak, near Surkh Kotal in Afghanistan. The inscription relates to the rule of the Kushan emperor Kanishka, and gives remarkable clues on the genealogy of the Kushan dynasty. It dates to the 2nd century CE.

The inscription includes 23 lines in Bactrian Language and a Greco- Bactrian script. The Bactrian Language is one of the Eastern Middle Iranian Languages. It was spoken by the people of northern part of Afghanistan.

118. (d) Mricchakatika is a ten-act Sanskrit drama attributed to Śūdraka, an ancient playwright who is possibly from the 5th century.

Rhizoids are unicellular or multicellular filamentous outgrowths that extend from the thallus of bryophytes like liverworts and mosses. Rhizoids are functionally similar to roots, though they are structurally different.

119. (a) Yashovarman was a medieval Indian ruler of Kannauj who founded the Varman dynasty of Kannauj.

120. (c) Ajatshatru s credited to move his capital from Rajagriha to Pataliputra. Some historians also mentioned his Son Udayin who finally moved the capital to Patna and this status was maintained during the reign of the Mauryas and the later dynasties.

121. (a) Nicolaus Copernicus detailed his radical theory of the Universe in which the Earth, along with the other planets, rotated around the Sun.

122. (b) According to question

Speed, $\nu = 2m/s$

$$\frac{dm}{dt} = 0.1 kg/s$$

$$\text{Force} = \frac{d}{dt}(m\nu)$$

$$= \nu\frac{dm}{dt}$$

$$= 2 \times 0.1$$

$$= 0.2\ N$$

123. (b) Mass of lift, m = 8kg

vertical distance, h = 4m

time, t = 2s

work, W = mgh

$$= 8 \times 10 \times 4$$

$$= 320J$$

$$\text{Power} = \frac{\text{work}}{\text{time}}$$

$$= \frac{320}{2} = 160W$$

124. (d) Coefficient of static friction, $\mu = 0.2$

Maximum frictional force, $f_{max} = \mu (m_1 + m_2)g$

$$= 0.2 \times (2 + 3) \times 10$$

$$= 10N$$

125 (b) Given, $g'_m = \frac{g_e}{6}$

For earth

$F = mg = kx$

$$k = \frac{mg}{x} \qquad \text{(i)}$$

For moon

$F' = mg' = kx'$

$$\frac{mg}{6} = kx'$$

$$x' = \frac{mg}{6k}$$

Using equation (i), we have

$$x' = \frac{mg \times x}{6 \times mg} = \frac{x}{6}$$

As extension in the spring, $x = 6cm$

The extension of the spring on the moon

$$x' = \frac{6}{6} = 1\,\text{cm}$$

126. (b) A bottle opener

127. (c) Resistance of a wire, R = 10Ω
Consider initial length of wire is *l*.
After wire is stretched, length of wire $l' = 2l$
Initial Volume (V_1) = Final Volume (V_2)
$Al = A'l$
$Al = A'2l$
$A' = \frac{A}{2}$
We have
$R' = \frac{\rho l'}{A'} \qquad \left(\because R = \frac{\rho l}{A}\right)$
$R' = \frac{\rho 2l}{A/2}$
$= 4\frac{\rho l}{A} = 4R$
$I \propto \frac{1}{R} \qquad (\because V = IR)$
Then $I' = \frac{1}{4}I$
Hence, the current in the circuit would become one-fourth of its original value.

128. (c) R′ R
$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R}$
$\frac{1}{R'} = \frac{2}{R}$
$R' = \frac{R'}{2}$
the total resistance in the circuit element
$R_{eq} = R' + R$
$= \frac{R}{2} + R = \frac{3R}{2}$

129. (d) The Bharatmala Pariyojana envisages development of about 26,000 km length of Economic Corridors, which along with Golden Quadrilateral (GQ) and North-South and East-West (NS-EW) Corridors are expected to carry majority of the Freight Traffic on roads.
The Bharatmala Project Components are,
- Economic Corridor
- Feeder Route or Inter Corridor
- National Corridor Efficiency Improvement
- Border Road and International Connectivity
- Port Connectivity and Coastal
- Green Field Expressway
- Balance NHDP Works

130. (c) The old name of Deendayal Port Authority is Kandla port. This seaport is located in the Kutch district of Gujarat state in Western India, near the city of Gandhidham.

131. (b) The Counter-Insurgency and Jungle Warfare School (CIJWS) of Indian Army is located in Vairengte, Mizoram. It is a training and research establishment of the Indian Army.
CIJWS is one of the premier counter-insurgency training institutions in the world. The school is specialised in unconventional warfare, especially counter-insurgency and guerrilla warfare.

132. (a) Calcification occurs when evapotranspiration exceeds precipitation causing the upward movement of dissolved alkaline salts from the groundwater. At the same time, the movement of rain water causes a downward movement of the salts. The net result is the deposition of the translocated cations in the B horizon. Calcification is common in the prairie grasslands.

133. (d) Transition zone between Outer core and Inner core is known as **Lehman Discontinuity**.
Other discontinuities/transition Zones inside the earth are,
- Conrad Discontinuity: Transition zone between upper and lower Crust.
- Mohorovicic Discontinuity: Transition zone between the Crust and Mantle.
- Repiti Discontinuity: Transition zone between Outer mantle and Inner mantle.
- Gutenberg Discontinuity: Transition zone between Μαντλε ανδ Χορε.

134. (d) Indian Rosewood mainly grows on the banks of the river below 900 meter elevation. Rhododendrons are native chiefly in the north temperate zone, especially in the moist acidic soil of the Himalayas and into Southeast Asia to the mountains of New Guinea.
Oaks form the backbone of the temperate moist forest biome occurring between 1,500 m and 2,700 m in the Himalayas.

135. (b) Batholith is a type of igneous rock that forms when magma rises into the earth's crust, but does not erupt onto the surface. Batholiths are large areas of intrusive (usually granitic) rock typically associated with subduction zones or hot spots.

136. (a) The time in London is 5 hours and 30 minutes behind the time in India. Indian Standard Time (IST) is GMT+5:30, while the time in UK is GMT. Hence when it is 12 noon in the New Delhi, time in London will be 12-5.30 =6:30 am

137. (a) spring tide, tide of maximal range, near the time of new and full moon when the Sun and Moon are in syzygy—i.e., aligned with the Earth. Conjunction is

the time during new moon when the Sun and Moon lie on the same side of the Earth.

138. (d) The 'last supper' was Created in 1495–1498 by the Italian High Renaissance artist Leonardo da Vinci, The Last Supper is a testament to the talent of the artist. The painting represents the scene of the Last Supper of Jesus with the Twelve Apostles, as it is told in the Gospel of John

139. (d) Between 1519 and 1521 Hernán Cortés and a small band of men brought down the Aztec empire in Mexico. He was the Spanish conquistador responsible for conquering the Aztec Empire and building Mexico City which secured Spain's position in the New World.

140. (d) From a place known as Mat near Mathura almost life-size statues of Kushana kings were recovered. One of the idols includes a monumental sculpture of King Kanishka I characterized by its frontality and martial stance holding sword and a mace firmly.

141. (a) The Treaty of Yandabo was signed on February 24, 1826. The treaty marked the end of the First Anglo-Burmese War, which had been raging for two years. The treaty was signed between the East India Company and the King of Ava. This treaty had significant consequences, including the cession of Assam, Manipur, Arakan, and Tenasserim to the British.

142. (c) Agriculture and irrigation including power was accorded the highest priority in first five year plan. During the plan period (1951-56) the maximum priority was given to this sector.

143. (c) The 2022 United Nations Climate Change Conference or 27th Conference of the Parties of the UNFCCC (COP27) brought together countries from around the world to increase ambition and implement existing goals and strengthen commitments towards

144. (c) The Singpho tribe, which lives in the areas between Assam and Arunachal Pradesh, had been involved in cultivation and production of Tea since generations. The people of this tribe were growing tea and steeping its leaves to procure a herbal drink for hundreds of years based as part of their traditional knowledge.

145. (a) Article 51A was not a part of the Constitution of India 1950. It was inserted by the Constitution (Forty-second Amendment) Act, 1976. Article 51A deals with Fundamental Duties of every citizens of India. The Fundamental Duties are non-enforceable and non-justiciable. There is no provision in the constitution for direct enforcement of these duties

146. (a) The greater an object's mass, the more gravitational force it exerts. Earth weighs 81 times more than the moon. The moon's density is 3.34 grams per cubic centimeter that is about 60% of Earth's density.

This is the reason Earth has a greater gravitational pull than the moon because the Earth is more massive. The moon's gravitational pull also affects Earth but it is less as compared to Earth's gravitational pull, though; the rising and falling of the tides are an effect of the moon's gravity.

147. (c) As, $P = IV$

Here, $v_1 = 220v$, $p_1 = 80W$ and $v_2 = 110v$

$$\frac{p_2}{p_1} = \frac{v_2}{v_1}$$

$$p_2 = \frac{p_1 v_2}{v_1}$$

$$= \frac{80 \times 110}{220}$$

$$p_2 = 40W$$

148.(d) In the dispersion of white light by a prism, blue light deviates the most because light has lowest speed and least wavelength in prism.

149.(a) Speed of sound remain same at all frequencies.

150. (c) A reflecting telescope (also called a reflector) is a telescope that uses a single or a combination of curved mirrors that reflect light and form an image.

The Newtonian telescope is a type of reflecting telescope invented by Sir Isaac Newton, using a concave primary mirror and a flat diagonal secondary mirror.