TS/EAMCET Solved Paper 2019

Held on May 3

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 marks.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. Match the functions of List-I with their nature in List-II and choose the correct option.

List-I	List-II
(A) $f : R \to R$ defined by $f(x) = \cos(112x - 37)$	(i) Injection but not surjection
(B) $f : A \to B$ defined by $f(x) = x\,|x|$ when $A = [-2, 2]$ and $B = [-4, 4]$	(ii) Surjection but not injection
(C) $f : R \to R$ defined by $f(x) = (x-2)(x-3)(x-5)$	(iii) Bijection
(D) $f : N \to N$ defined by $f(n) = n + 1$	(iv) Neither injection nor surjection
	(v) Composite function

Then the correct match is

	A	B	C	D
(a)	(i)	(ii)	(iii)	(iv)
(b)	(iv)	(i)	(ii)	(iii)
(c)	(iv)	(iii)	(ii)	(i)
(d)	(iv)	(iii)	(ii)	(i)

2. If $[x]$ denotes the greatest integer function, then the domain of the function $f(x) = \sqrt{\dfrac{x-[x]}{\log(x^2-x)}}$, is

(a) $(1, \infty)$ (b) $(1, \infty) - Z$

(c) $R - \left[\dfrac{1-\sqrt{5}}{2}, \dfrac{1+\sqrt{5}}{2}\right]$ (d) $\left[\dfrac{1-\sqrt{5}}{2}, \dfrac{\sqrt{5}+1}{2}\right]$

3. **Assertion :** If $|x| < 1$, then

$$\sum_{n=0}^{\infty} (-1)^n x^{n+1} = \frac{x}{x+1}$$

Reason : If $|x| < 1$, then $(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$

Which one of the following is true?

(a) (A) and (R) are true, (R) is a correct explanation of (A)

(b) (A) and (R) are true but (R) is not a correct explanation of (A)

(c) (A) is true, but (R) is false

(d) (A) is false but (R) is true

4. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 1 & 6 \end{bmatrix}$, then $(\text{Adj}(\text{Adj } A))^{-1} =$

(a) $\dfrac{1}{6}\begin{bmatrix} 8 & -6 & 3 \\ 5 & 1 & -2 \\ -5 & 3 & 1 \end{bmatrix}$ (b) $\dfrac{1}{6}\begin{bmatrix} 13 & -9 & 1 \\ 4 & 0 & -2 \\ -5 & 3 & 1 \end{bmatrix}$

(c) $\dfrac{1}{36}\begin{bmatrix} 13 & -9 & 1 \\ 4 & 0 & -2 \\ -5 & 3 & 1 \end{bmatrix}$ (d) $\dfrac{1}{12}\begin{bmatrix} 4 & -3 & 2 \\ 3 & 4 & 2 \\ -5 & 2 & 1 \end{bmatrix}$

5. If $\begin{vmatrix} x^2+3x & x+1 & x-3 \\ x-1 & 2-x & x+4 \\ x-3 & x-3 & 3x \end{vmatrix} = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$, then $(a_1 + a_3) + 2(a_0 + a_2 + a_4) =$

(a) -1 (b) 0 (c) 1 (d) -29

6. Let $AX = D$ be a system of three linear non-homogeneous equations. If $|A| = 0$ and rank (A) = rank $([AD]) = \alpha$, then

(a) $AX = D$ will have infinite number of solutions when $\alpha = 3$
(b) $AX = D$ will have unique solution when $\alpha < 3$
(c) $AX = D$ will have infinite number of solutions when $\alpha < 3$
(d) $AX = D$ will have no solution when $\alpha < 3$

7. If $x + iy = (1 + i)^6 - (1 - i)^6$, then which one of the following is true?

(a) $x + y = 16$ (b) $x + y = -16$
(c) $x + y = -8$ (d) $x + y = 8$

8. $i^2 + i^3 + \ldots + i^{4000} =$

(a) 1 (b) 0
(c) i (d) $-i$

9. If 1, ω and ω^2 are the cube roots of unity, then $(a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega) =$

(a) $a^3 + b^3 + c^3$ (b) $a^3 + b^3 + c^3 - 3abc$
(c) $(a + b + c)^3 - 3abc$ (d) $a^3 + b^3 + c^3 + 3abc$

10. If $a_k = \cos \alpha_k + i \sin \alpha_k$, $k = 1, 2, 3$ and a_1, a_2, a_3 are the roots of the equation $x^3 + bx + c = 0$, then the real part of $b =$

(a) -1 (b) 1 (c) 0 (d) $\frac{2}{3}$

11. The set of all values of 'a' for which the expression $\frac{ax^2 - 2x + 3}{2x - 3x^2 + a}$ assumes all real values for real values of x, is

(a) $[2, 3]$ (b) $R - (2, 3)$
(c) ϕ (d) $[1, 5]$

12. If both the roots of the equation $x^2 - 4ax + 1 - 3a + 4a^2 = 0$ exceed 1, then a lies in the interval

(a) $\left(-\infty, \frac{7 - \sqrt{17}}{8}\right)$ (b) $\left(\frac{7 + \sqrt{17}}{8}, \infty\right)$
(c) $\left(\frac{7 - \sqrt{17}}{8}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{7 + \sqrt{17}}{8}\right)$

13. If the cubic equation $x^3 - ax^2 + ax - 1 = 0$ is identical with the cubic equation whose roots are the squares of the roots of the given cubic equation, then the non-zero real value of 'a' is

(a) $\frac{1}{2}$ (b) 2 (c) 3 (d) $\frac{7}{2}$

14. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha) =$

(a) $p - qr$ (b) $r - pq$ (c) $q - rp$ (d) $r + pq$

15. Let x denote the number of ways of arranging m boys and m girls in a row so that no two boys sit together. If y and z give the number of ways of arranging m boys and m girls in a row and around a circular table respectively so that boys and girls sit alternately, then $x : y : z =$

(a) $m + 1 : m : m - 1$ (b) $3 : 2 : 1$
(c) $m - 1 : m : 2$ (d) $(m + 1)m : 2m : 1$

16. The number of even numbers greater than 1000000 that can be formed using all the digits 1, 2, 0, 2, 4, 2 and 4 is

(a) 120 (b) 240 (c) 310 (d) 480

17. The greatest integer less than or equal to $(\sqrt{3} + 2)^5$ is

(a) 721 (b) 722 (c) 723 (d) 724

18. The sixth term in the expansions of $\left(3 - \sqrt{\frac{17}{4} + 3\sqrt{2}}\right)^{10}$ is a

(a) positive rational number
(b) negative rational number
(c) positive irrational number
(d) negative irrational number

19. Let $\frac{1}{(x^2 - 3)^2} = \frac{A_1}{x - \sqrt{3}} + \frac{A_2}{(x - \sqrt{3})^2} + \frac{A_3}{x + \sqrt{3}} + \frac{A_4}{(x + \sqrt{3})^2}$.

Then, consider the following statements.

(i) All the A_i's are not distinct.
(ii) There exists a pair, A_p and A_q such that $A_p^2 = A_q^2$ $(p \neq q)$
(iii) $\sum_{i=1}^{4} A_i = \frac{1}{6}$
(iv) $\sum_{i=1}^{4} A_i = 1$

Which one of the following is true?

(a) Only statement (iii) is false
(b) Both the statements (ii) and (iv) are false
(c) Only statement (iv) is false
(d) Both the statements (i) and (iii) are false

20. The period of $\cos(x + 8x + 27x + \ldots + n^3x)$ is

(a) $\frac{2\pi}{n}$ (b) $\frac{2\pi}{n^2(n+1)^2}$
(c) $\frac{8\pi}{n^2(n+1)^2}$ (d) $\frac{8\pi}{n^3(n+1)^2}$

21. $\sin^2(3°) + \sin^2(6°) + \sin^2(9°) + \ldots + \sin^2(84°) + \sin^2(87°) + \sin^2(90°) =$

(a) $\frac{31}{2}$ (b) $\frac{39}{2}$ (c) $\frac{59}{2}$ (d) 36

22. $\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} - \cos\frac{6\pi}{7} =$

(a) 0 (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) 1

23. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution, is

(a) 4 (b) 6 (c) 8 (d) 10

24. $\sec\left(\tan^{-1}\frac{y}{2}\right) =$

(a) $\sqrt{\frac{4+y^2}{2}}$ (b) $\sqrt{\frac{4-y^2}{2}}$

(c) $\frac{\sqrt{4+y^2}}{2}$ (d) $\frac{\sqrt{4-y^2}}{2}$

25. The number of roots of the equation

$\sqrt{2} + e^{\cos h^{-1} x} - e^{\sin h^{-1} x} = 0$, is

(a) 0 (b) 1 (c) 2 (d) 3

26. A wire of length 44 cm is bent into an arc of a circle of radius 12 cm. The angle (in degrees) subtended by the arc at the centre of the circle is

(a) $\left(\frac{11}{3}\right)^\circ$ (b) $\left(\frac{660}{\pi}\right)^\circ$ (c) 150° (d) $\left(\frac{5}{3}\right)^\circ$

27. In any triangle ABC, if $a : b : c = 2 : 3 : 4$, then $R : r =$

(a) 8 : 3 (b) 16 : 9 (c) 5 : 16 (d) 16 : 5

28. Corresponding to a triangle ABC, match the items given in List-I with the items given in List-II.

	List-I		**List-II**
(A)	$rr_2 = r_1r_3$	(i)	$\angle A = 90°$
(B)	$r_1 + r_2 = r_3 - r$	(ii)	$b^2 = c^2 + a^2$
(C)	$r_1 = r + 2R$	(iii)	$\angle C = 90°$
		(iv)	$\angle B = 120°$

Then the correct match is

	A	B	C
(a)	(ii)	(iii)	(i)
(b)	(ii)	(i)	(iii)
(c)	(i)	(iv)	(iii)
(d)	(iii)	(i)	(iv)

29. Let the position vectors of two points A and B be $\mathbf{a} + \mathbf{b} + \mathbf{c}$ and $\mathbf{a} - 2\mathbf{b} + 3\mathbf{c}$, respectively. If the points P and Q divide AB in the ratio 1 : 3 internally and externally respectively, then $3\,|\boldsymbol{AB}| =$

(a) $4\,|\boldsymbol{PQ}|$ (b) $3\,|\boldsymbol{PQ}|$ (c) $\frac{1}{2}|\boldsymbol{PQ}|$ (d) $2\,|\boldsymbol{PQ}|$

30. If **a, b** and **c** are three non-collinear points and $k\mathbf{a} + 2\mathbf{b} + 3\mathbf{c}$ is a point in the plane of **a, b c**, then $k =$

(a) 4 (b) 5 (c) − 5 (d) − 4

31. If the vector $\mathbf{a} = 3\hat{j} + 4\hat{k}$ is the sum of two vectors $\mathbf{a}_1$ and $\mathbf{a}_2$, vector $\mathbf{a}_1$ is parallel to $\mathbf{b} = \hat{i} + \hat{j}$ and vector $\mathbf{a}_2$ is perpendicular to **b**, then $\mathbf{a}_1 =$

(a) $\frac{1}{2}(\hat{i} + \hat{j})$ (b) $-(\hat{i}\quad\hat{j})$

(c) $\frac{2}{3}(\hat{i} + \hat{j})$ (d) $\frac{3}{2}(\hat{i} + \hat{j})$

32. The angle between the line of intersection of the two planes $\mathbf{r}\cdot(2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$, $\mathbf{r}\cdot(3\hat{i} + 3\hat{j} - 5\hat{k}) = 3$ and the line $\boldsymbol{r} = 3\hat{i} + 2\hat{j} + \hat{k} + t(5\hat{i} + 5\hat{j} - 7\hat{k})$ is

(a) $\cos^{-1}\left(\frac{-1}{\sqrt{28}}\right)$ (b) $\cos^{-1}\left(\frac{41}{\sqrt{17}\sqrt{99}}\right)$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

33. Let $\mathbf{x} = \hat{i} + \hat{j}$ and $\mathbf{y} = 3\hat{i} - 2\hat{k}$. Then, the vector $\boldsymbol{r}$ of magnitude $\sqrt{21}$ satisfying $\boldsymbol{r} \times \boldsymbol{x} = \boldsymbol{y} \times \boldsymbol{x}$ and $\boldsymbol{r} \times \boldsymbol{y} = \boldsymbol{x} \times \boldsymbol{y}$ is

(a) $-\hat{i} + 4\hat{j} - 2\hat{k}$ (b) $-\hat{i} - 4\hat{j} - 2\hat{k}$

(c) $4\hat{i} + \hat{j} - 2\hat{k}$ (d) $4\hat{i} - \hat{j} - 2\hat{k}$

34. The acute angle between $\mathbf{r} = (-\hat{i} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\mathbf{r}\cdot(10\hat{i} + 2\hat{j} - 11\hat{k}) = 3$, is

(a) $\sin^{-1}\left(\frac{8}{21}\right)$ (b) $\cos^{-1}\left(\frac{8}{21}\right)$

(c) $\sin^{-1}\left(\frac{5}{21}\right)$ (d) $\cos^{-1}\left(\frac{5}{21}\right)$

35. In a data with 15 number of observations $x_1, x_2, x_3, \ldots, x_{15}$, $\sum_{i=1}^{15} x_i^2 = 3600$ and $\sum_{i=1}^{15} x_i = 175$. If the value of one observation 20 was found wrong and was replaced by its correct value 40, then the corrected variance of that data is

(a) 151 (b) 149 (c) 145 (d) 144

36. If the coefficient of variation and variance of a frequency distribution are 7.2 and 3.24 respectively, then its mean is

(a) 45 (b) 25 (c) 20 (d) 16

37. If five dice are thrown simultaneously, then the probability that atleast three of them show the same numbered face is

(a) $\frac{16}{6^4}$ (b) $\frac{452}{6^5}$ (c) $\frac{226}{6^4}$ (d) $\frac{123}{6^5}$

38. If two unbiased dice are rolled simultaneously until a sum of the number appeared on these dice is either 7 or 11, then the probability that 7 comes before 11, is

(a) $\frac{3}{8}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{2}{9}$

39. A box contains 10 mangoes out of which 4 are spoiled. 2 mangoes are taken together at random. If one of them is found to be good, then the probability that the other is also good, is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{8}{15}$ (d) $\frac{5}{13}$

40. Two dice are rolled. If a random variable X is defined as the absolute difference of the two numbers that appear on them, then the mean of X is

(a) 0 (b) $\frac{13}{18}$ (c) $\frac{19}{9}$ (d) $\frac{35}{18}$

41. If getting a head on a coin when it is tossed is considered as success, then the probability of having more number of failures when ten fair coins are tossed simultaneously, is

(a) $\frac{105}{2^8}$ (b) $\frac{73}{2^7}$ (c) $\frac{193}{2^9}$ (d) $\frac{638}{2^{10}}$

42. The set of all points that forms a triangle of area 15 sq. units with the points (1, – 2) and (– 5, 3) lies on

(a) $5x + 6y + 23 = 0$
(b) $(5x + 6y - 23)(5x + 6y + 37) = 0$
(c) $25x^2 + 36y^2 + 24x - 30y - 227 = 0$
(d) $5x + 6y - 37 = 0$

43. Suppose the new axes X, Y are generated by rotating the coordinate axes x, y about the origin through an angle of 30° in the anti-clockwise direction. Then, the transformed equation of $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ with respect to new axes X, Y is

(a) $X^2 - Y^2 = a^2$
(b) $X^2 + Y^2 = 2a^2$
(c) $X^2 + 2\sqrt{3}XY - Y^2 = 2a^2$
(d) $X^2 - Y^2 = 2a^2$

44. A line L makes intercepts a and b on the coordinate axes. The axes are rotated through an angle θ in the positive direction, keeping the origin fixed. If the line L makes intercepts p and q on the new coordinate axes, then $\frac{1}{a^2} + \frac{1}{b^2} =$

(a) $\frac{1}{p^2q^2}$ (b) $\frac{1}{p^2} - \frac{1}{q^2}$ (c) $\frac{1}{p^2} + \frac{1}{q^2}$ (d) $\frac{pq}{p^2 + q^2}$

45. If m_1, m_2 $(m_1 > m_2)$ are the slopes of the lines which make an angle of 30° with the line joining the points (1, 2) and (3, 4), then $\frac{m_1}{m_2} =$

(a) $2 + \sqrt{3}$ (b) $2 - \sqrt{3}$ (c) $7 + 4\sqrt{3}$ (d) $7 - 4\sqrt{3}$

46. If $A(-2, 1)$, $B(0, -2)$, $C(1, 2)$ are the vertices of a triangle ABC, then the perpendicular distance from its circum centre to the side BC is

(a) $\frac{7\sqrt{13}}{22}$ (b) $\frac{3\sqrt{17}}{22}$ (c) $\frac{5\sqrt{10}}{11}$ (d) $\frac{\sqrt{2026}}{22}$

47. If one of the lines $ax^2 + 2hxy + by^2 = 0$ bisects the angle between the positive coordinate axes, then

(a) $a + b = 2h$ (b) $a - b = 2|h|$
(c) $(a + b)^2 = 4h^2$ (d) $(a - b)^2 = 4h^2$

48. The equation of the pair of perpendicular lines passing through origin and forming an isosceles triangle with the line $2x + 3y = 6$, is

(a) $5x^2 - 24xy - 5y^2 = 0$ (b) $4x^2 - 12xy - 4y^2 = 0$
(c) $6x^2 - 5xy - 6y^2 = 0$ (d) $9x^2 + 5xy - 9y^2 = 0$

49. If one of the diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the bigger circle is

(a) 6 (b) 4 (c) 2 (d) 3

50. The line $3x - y + k = 0$ touches the circle $x^2 + y^2 + 4x - 6y + 3 = 0$. If $k_1, k_2 (k_1 < k_2)$ are the two values of k, then the equation of the chord of contact of the point (k_1, k_2) with respect to the given circle is

(a) $19x + y - 18 = 0$ (b) $x + 19y - 3 = 0$
(c) $x + 16y - 56 = 0$ (d) $20x + 18y - 7 = 0$

51. If the line $ax + by = 1$ is a tangent to the circle $S_r \equiv x^2 + y^2 - r^2 = 0$, then which one of the following is true?

(a) (a, b) lies on the circle $S_1 = 0$
(b) (a, b) lies inside the circle $S_{1/2} = 0$
(c) (a, b) lies outside the circle $S_2 = 0$
(d) (a, b) lies on the circle $S_3 = 0$

52. Each of the two orthogonal circles C_1 and C_2 passes through both the points (2, 0) and (– 2, 0). If $y = mx + c$ is a common tangent to these circles, then

(a) $c^2 = 4(1 + 2m^2)$ (b) $c^2 = 2(1 + 2m^2)$

(c) $c^2 = 1 + m^2$ (d) $c^2m^2 = 4(1 + m^2)$

53. The equation of the circle whose diameter is the common chord of the circles $x^2 + y^2 - 3x + y - 10 = 0$ and $x^2 + y^2 - x + 2y - 20 = 0$ is

(a) $x^2 + y^2 - 3x + 6y + 15 = 0$

(b) $x^2 + y^2 - 6x + 4y + 10 = 0$

(c) $x^2 + y^2 - 9x + 2y + 20 = 0$

(d) $x^2 + y^2 - 9x - 2y + 20 = 0$

54. Study the following statements.

(I) The vertex of the parabola $x = ly^2 + my + n$ is $\left(n - \frac{m^2}{4l}, -\frac{m}{2l}\right)$

(II) The focus of the parabola $y = lx^2 + mx + n$ is $\left(n + \frac{1 - m^2}{4l}, -\frac{m}{2l}\right)$

(III) The pole of the line $lx + my + n = 0$ with respect to the parabola $x^2 = 4ay$ is $\left(-\frac{2al}{m}, \frac{n}{m}\right)$

Then, the correct option among the following is:

(a) All the three statements are true

(b) Statement I & II are true but III is false

(c) Statement I & III are true but II is false

(d) Statement II & III are true but I is false

55. Let P represent the point (3, 6) on the parabola $y^2 = 12x$. For the parabola $y^2 = 12x$, if l_1 is the length of the normal chord drawn at P and l_2 is the length of the focal chord drawn through P, then $\frac{l_1}{l_2} =$

(a) $2\sqrt{2}$ (b) 3 (c) $4\sqrt{2}$ (d) 5

56. A tangent is drawn at $(3\sqrt{3}\cos\theta, \sin\theta)$ $\left(\quad < \theta < - \right)$ to the ellipse $\frac{x^2}{27} + \frac{y^2}{1} = 1$. The value of θ for which the sum of the intercepts on the coordinate axes made by this tangent attains the minimum, is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{2\pi}{4}$

57. A line perpendicular to the X-axis cuts the circle $x^2 + y^2 = 9$ at A and the ellipse $4x^2 + 9y^2 = 36$ at B such that A and B lie in the same quadrant. If θ is the greatest acute angle between the tangents drawn to the curves at A and B, then $\tan\theta =$

(a) $\frac{1}{12}$ (b) $\frac{1}{2\sqrt{6}}$ (c) $\frac{5}{24}$ (d) $\frac{5}{4\sqrt{6}}$

58. If e_1 is the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and e_2 is the eccentricity of a hyperbola passing through the foci of the given ellipse and $e_1e_2 = 1$, then the equation of such a hyperbola among the following is

(a) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (b) $\frac{y^2}{9} - \frac{x^2}{16} = 1$

(c) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (d) $\frac{x^2}{25} - \frac{y^2}{9} = 1$

59. If (1, 0, 3), (2, 1, 5), (–2, 3, 6) are the mid-points of the sides of a triangle, then the centroid of the triangle is

(a) $\left(\frac{1}{3}, \frac{4}{3}, -\frac{14}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{4}{3}, \frac{14}{3}\right)$

(c) $\left(\frac{1}{3}, -\frac{4}{3}, \frac{14}{3}\right)$ (d) $\left(-\frac{1}{3}, \frac{4}{3}, \frac{14}{3}\right)$

60. If a plane P passes through the points (1, 0, 0), (0, 1, 0) and makes an angle $\frac{\pi}{4}$ with the plane $x + y = 3$, then the direction ratios of a normal to that plane P is

(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$ (c) 1, 1, 2 (d) $\sqrt{2}, 1, 1$

61. A variable plane is at a distance of 6 units from the origin. If it meets the coordinate axes in A, B and C, then the equation of the locus of the centroid of the ΔABC is

(a) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{4}$ (b) $x^2 + y^2 + z^2 = 4$

(c) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 1$ (d) $\frac{1}{x^2} + \frac{1}{y^2} - \frac{1}{z^2} = \frac{1}{4}$

62. $\lim_{x \to \infty} \left(\frac{2x^2 + 3x + 4}{x^2 - 3x + 5}\right)^{\frac{3|x|+1}{2|x|-1}} =$

(a) $\frac{3}{2}$ (b) $2\sqrt{2}$ (c) 3 (d) $\sqrt{2}$

63. If 'a' is the point of discontinuity of the function

$$f(x) = \begin{cases} \cos 2x, & \text{for } -\infty < x < 0 \\ e^{3x}, & \text{for } 0 \le x < 3 \\ x^2 - 4x + 3, & \text{for } 3 \le x \le 6 \\ \frac{\log(15x - 89)}{x - 6}, & \text{for } x > 6 \end{cases}$$

Then, $\lim_{x \to a} \frac{x^2 - 9}{x^3 - 5x^2 + 9x - 9} =$

(a) 1 (b) 0 (c) 6 (d) 3

64. If $y = (x + 1)(x^2 + 1)(x^4 + 1)(x^8 + 1)$, then $\lim_{x \to -1} \frac{dy}{dx} =$

(a) 0 (b) 2 (c) – 4 (d) 8

65. $f(x)$ is a twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If $h(x) = (f(x))^2 + (g(x))^2$ and $h(1) = 2$, then $h(2) =$

(a) 0 (b) 1 (c) 2 (d) 4

66. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \dots \infty}}}}$, then $\frac{dy}{dx} =$

(a) $\frac{y^3 - x}{2y^2 - 2xy + 1}$ (b) $\frac{x + y^3}{2y^2 - x}$

(c) $\frac{y + x}{y^2 - 2x}$ (d) $\frac{y^2 - x}{2y^3 - 2xy - 1}$

67. The angle between the tungents of the curves $xy = 1$ and $x^2 + 8y = 0$, is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{6}{7}$ (d) $\frac{3}{7}$

68. The slope of the tangent at (1, 2) to the curve $x = t^2 - 7t + 7$ and $y = t^2 - 4t - 10$, is

(a) $\frac{8}{5}$ (b) $\frac{5}{8}$ (c) $-\frac{8}{5}$ (d) $-\frac{5}{8}$

69. Consider the function $f(x) = 2x^3 - 3x^2 - x + 1$ and the intervals $I_1 = [-1, 0], I_2 = [0, 1], I_3 = [1, 2], I_4 = [-2, -1]$. Then,

(a) $f(x) = 0$ has a root in the intervals I_1 and I_4 only
(b) $f(x) = 0$ has a root in the intervals I_1 and I_2 only
(c) $f(x) = 0$ has a root in every interval except in I_4
(d) $f(x) = 0$ has a root in all the four given intervals

70. If $f(x) = \int_x^{x+1} e^{-t^2} dt$, then the interval in which $f(x)$ is decreasing is

(a) $\left(-\frac{1}{2}, \infty\right)$ (b) $(-\infty, 2)$

(c) $(-\infty, 0)$ (d) $(-2, 2)$

71. If $\int \frac{\cos x + x}{1 + \sin x} dx = f(x) + \int \frac{3\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}} dx + c$, then $f(x) =$

(a) $\frac{-2x}{1 + \tan\frac{x}{2}}$ (b) $\frac{-x\cos\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$

(c) $\frac{2x}{1 + \tan\frac{x}{2}}$ (d) $\frac{x\cos\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$

72. $\int \frac{\sqrt{\cos 2x}}{\sin x} dx =$

(a) $\frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2} + \sqrt{1 - \tan^2 x}}{\sqrt{2} - \sqrt{1 - \tan^2 x}}\right| - \frac{1}{2}\log\left|\frac{1 - \sqrt{1 - \tan^2 x}}{1 + \sqrt{1 - \tan^2 x}}\right| + c$

(b) $\frac{1}{\sqrt{2}} \log\left|\frac{\sqrt{2} + \sqrt{1 - \tan^2 x}}{\sqrt{2} - \sqrt{1 - \tan^2 x}}\right| - \frac{1}{2}\log\left|\frac{1 + \sqrt{1 - \tan^2 x}}{1 - \sqrt{1 - \tan^2 x}}\right| + c$

(c) $\frac{1}{4\sqrt{2}} \log\left|\frac{\sqrt{2} - \sqrt{1 - \tan^2 x}}{\sqrt{2} + \sqrt{1 - \tan^2 x}}\right| + \frac{1}{2}\log\left|\frac{1 - \sqrt{1 - \tan^2 x}}{1 + \sqrt{1 - \tan^2 x}}\right| + c$

(d) $\frac{1}{4\sqrt{2}} \log\left|\frac{2 - \sqrt{1 - \tan^2 x}}{2 + \sqrt{1 - \tan^2 x}}\right| + \frac{1}{2\sqrt{2}}\log\left|\frac{1 - \sqrt{1 - \tan^2 x}}{1 + \sqrt{1 - \tan^2 x}}\right| + c$

73. If $\int \frac{(2x + 3)}{x(x + 1)(x + 2)(x + 3) + 1} dx = -\frac{1}{px^2 + qx + r} + c$, then $\frac{3p - q}{r} =$

(a) 0 (b) 1 (c) 2 (d) – 1

74. $\int (\log x)^2 dx =$

(a) $x \log x - 2x \log x + c$
(b) $x \log x + 2x \log x + c$
(c) $x(\log x)^2 - 2x(\log x - 1) + c$
(d) $x(\log x)^2 + 2x(\log x - 1) + c$

75. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{k}{n^2 + k^2} =$

(a) $\frac{1}{2}\log 2$ (b) $2\log 2$

(c) $\frac{1}{3}\log 2$ (d) $3\log 2$

76. $\int_0^{10} (5-\sqrt{10x-x^2})\,dx =$

(a) $50-25\pi$ (b) $(100-25\pi)$

(c) $\frac{1}{2}(100-25\pi)$ (d) $\frac{1}{4}(100-25\pi)$

77. Area of the region (in sq. units) bounded by the curves $y=\sqrt{x}$, $x=\sqrt{y}$ and the lines $x=1$, $x=4$, is

(a) $\frac{8}{3}$ (b) $\frac{49}{3}$ (c) $\frac{16}{3}$ (d) $\frac{14}{3}$

78. The differential equation representing the family of circles of constant radius r is

(a) $r^2y'' = [1+(y')^2]^2$ (b) $r^2(y'')^2 = [1+(y')^2]^2$

(c) $r^2(y'')^2 = [1+(y')^2]^3$ (d) $(y'')^2 = r^2[1+(y')^2]^2$

79. The solution of the differential equation $(2x-3y+5)\,dx + (9y-6x-7)\,dy = 0$, is

(a) $3x-3y+8\log|6x-9y-1| = c$

(b) $3x-9y+8\log|6x-9y-1| = c$

(c) $3x-9y+8\log|2x-3y-1| = c$

(d) $3x-9y+4\log|2x-3y-1| = c$

80. The solution of the differential equation $\sqrt{1-y^2}\,dx + x\,dy - \sin^{-1}y\,dy = 0$, is

(a) $x = \sin^{-1}y - 1 + ce^{-\sin^{-1}y}$

(b) $y = x\sqrt{1-y^2} + \sin^{-1}y + c$

(c) $x = 1 + \sin^{-1}y + ce^{\sin^{-1}y}$

(d) $y = \sin^{-1}y - 1 + x\sqrt{1-y^2} + c$

PHYSICS

81. Match the following fundamental forces of nature with their relative strength.

	List-I		List-II
(A)	Strong nuclear force	(i)	10^{-2}
(B)	Weak nuclear force	(ii)	1
(C)	Electromagnetic force	(iii)	10^{-39}
(D)	Gravitational force	(iv)	10^{-13}

The correct match is

	A	B	C	D
(a)	(ii)	(iv)	(i)	(iii)
(b)	(iii)	(ii)	(iv)	(i)
(c)	(ii)	(iii)	(iv)	(i)
(d)	(iv)	(ii)	(i)	(iii)

82 Identify the incorrect statement among the following.

(a) A true length of 5.678 km has been measured in two experiments as 5.5 km and 5.51 km, respectively. The second measurement has more precision.

(b) Length of 1 m and 0.5 m have been both measured with the same absolute error of 0.01 m. Both the measurement are equally accurate.

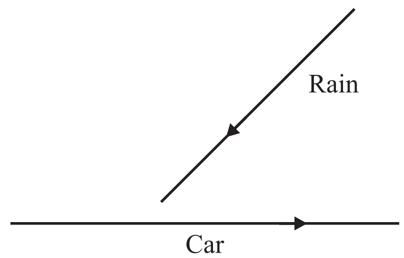
(c) The numbers of significant digits in 1.6 and 0.60 are both two.

(d) The number 2.445 can be rounded to two decimal place as 2.45.

83. Ball-1 is dropped from the top of a building from rest. At the same moment, ball-2 is thrown upward towards ball-1 with a speed 14 m/s from a point 21 m below the top of building. How far will the ball-1 have dropped when it passes ball-2. (Assume acceleration due to gravity, $g = 10\text{ m/s}^2$.)

(a) $\frac{45}{4}$ m (b) $\frac{52}{6}$ m (c) $\frac{37}{2}$ m (d) $\frac{25}{2}$ m

84. Rain is falling at an angle of 30° from the vertical due to the wind with a speed of 40 m/s. A car is travelling horizontally in the direction opposite to the wind, at a speed of 40 m/s. At what angle from the vertical will it experience the rain falling from?



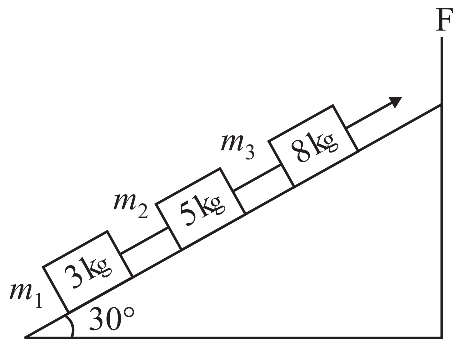
(a) 30° (b) 60° (c) 90° (d) 120°

85. Two touching blocks 1 and 2 are placed on an inclined plane forming an angle 60° with the horizontal. The masses are m_1 & m_2 and the coefficient of friction between the inclined plane and the two blocks are 1.5 μ and 1.0 μ, respectively. The force of reaction between the blocks during the motion is (g = acceleration due to gravity)

(a) $(m_2-m_1)\,\mu g$ (b) $(m_2+m_1)\,\mu g$

(c) $\frac{1}{2}\frac{m_1m_2}{m_1+m_2}\mu g$ (d) $\frac{1}{4}\frac{m_1m_2}{m_1+m_2}\mu g$

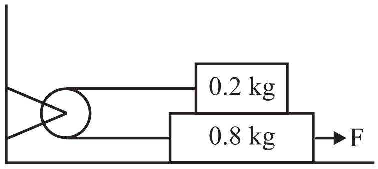
86. Three blocks are connected by massless strings on a frictionless inclined plane of 30° as shown in the figure. A force of 104 N is applied upward along the incline to mass m_3 causing an upward motion of the blocks. What is the acceleration of the blocks? (Assume, acceleration due to gravity, $g = 10\text{ m/s}^2$)



(a) 6.0 m/s^2 (b) 4.5 m/s^2
(c) 3.0 m/s^2 (d) 1.5 m/s^2

87. Consider a system of two masses and a pulley shown in the figure. The coefficient of friction between the two blocks and also between block and table is 0.1. Find the force F, that must be given to the 0.8 kg block such that it attains accelerations of 5 m/s^2.

(Assume, acceleration due to gravity, $g = 10$ m/s^2.)



(a) 6.4 N (b) 7.1 N (c) 6.0 N (d) 7.8 N

88. A box of mass 3 kg moves on a horizontal frictionless table and collides with another box of mass 3 kg initially at rest on the edge of the table at height 1 m. The speed of the moving box just before the collision is 4 m/s. The two boxes stick together and fall from the table. The kinetic energy just before the boxes strike the floor is (Assume, acceleration due to gravity, $g = 10$ m/s^2)

(a) 40 J (b) 80 J (c) 96 J (d) 72 J

89. A ball of mass 2 kg is thrown from a tall building with velocity, $\mathbf{v} = (20\text{ m/s})\,\hat{\boldsymbol{i}} + (24\text{ m/s})\,\hat{\boldsymbol{j}}$ at time $t = 0$ s.

Change in the potential energy of the ball after, $t = 8$ s is (The ball is assumed to be in air during its motion between 0 s and 8 s, $\hat{\boldsymbol{i}}$ is along the horizontal and $\hat{\boldsymbol{j}}$ is along the vertical direction. (Take $g = 10$ m/s^2)

(a) – 2.56 kJ (b) 0.52 kJ
(c) 1.76 kJ (d) – 2.44 kJ

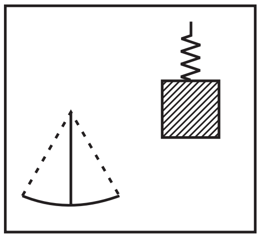
90. The balls A, B and C of masses 50 g, 100 g and 150 g, respectively are placed at the vertices of an equilateral triangle. The length of each side is 1 m. If A is placed at (0, 0) and B is placed at (1, 0) m, find the coordinates (x, y) for the centre of mass of this system of the balls

(a) $\left(\frac{7}{12}, \sqrt{\frac{3}{4}}\right)$ m (b) $\left(\frac{5}{18}, \sqrt{\frac{1}{4}}\right)$ m
(c) $\left(\frac{7}{12}, \sqrt{\frac{3}{2}}\right)$ m (d) $\left(\frac{5}{18}, \sqrt{\frac{3}{4}}\right)$ m

91. Three bodies, a ring, a solid disc and a solid sphere roll down the same inclined plane without slipping. The radii of the bodies are identical and they start from rest. If V_S, V_R and V_D are the speeds of the sphere, ring and disc, respectively when they reach the bottom, then the correct option is

(a) $V_S > V_R > V_D$ (b) $V_D > V_S > V_R$
(c) $V_R > V_D > V_S$ (d) $V_S > V_D > V_R$

92. A vertical spring mass system has the same time period as simple pendulum undergoing small oscillations. Now, both of them are put in an elevator going downwards with an acceleration 5 m/s^2. The ratio of time period of the spring mass system to the time period of the pendulum is (Assume, acceleration due to gravity, $g = 10$ m/s^2)



(a) $\sqrt{\frac{3}{2}}$

(b) $\sqrt{\frac{2}{3}}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\sqrt{2}$

93. Consider a spherical planet which is rotating about its axis such that the speed of a point on its equator is v and the effective acceleration due to gravity on the equator is $\frac{1}{3}$ of its value at the poles. What is the escape velocity for a particle at the pole of this planet.

(a) $3v$ (b) $2v$
(c) $\sqrt{3}v$ (d) $\sqrt{2}v$

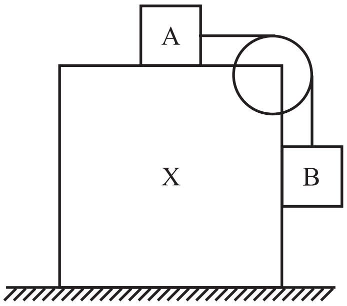
94. Consider a system of blocks X, A and B as shown in the figure. The blocks A and B have equal mass and are connected by a massless string through a massless pully. The coefficient of friction between block A and X or B and X is 0.5. If block X moves on the horizontal frictionless surface what should be its minimum acceleration such that blocks A and B remain stationary. (g = acceleration due to gravity.)

(a) $\frac{g}{3}$

(b) $3g$

(c) $\frac{g}{4}$

(d) $\frac{3g}{4}$



95. How much pressure (in atm) is needed to compress a sample of water by 0.4%?

(Assume, Bulk modulus of water $\approx 2.0 \times 10^9$ Pa)

(a) 60 atm (b) 70 atm (c) 80 atm (d) 90 atm

96. The tension in a massless cable connected to an iron ball of 100 kg when it is submerged in sea water is

($\rho_{iron} = 8 \times 10^3$ kg/m^3 and $\rho_{sea\ water} = 1000$ kg/m^3, $g = 10$ m/s^2)

(a) 950 N (b) 846 N (c) 875 N (d) 933 N

97. The area of a circular copper coin increases by 0.4% when its temperature is raised by 100°C. The coefficient of linear expansion of the coin is:

(a) 1×10^{-5}/°C (b) 2×10^{-5}/°C

(c) 3×10^{-5}/°C (d) 4×10^{-5}/°C

98. A 210 W heater is used to heat 100 g water. The time required to raise the temperature of this water from 25°C to 100°C is (specific heat capacity of water = 4200 J/Kg-°C)

(a) 100 s (b) 125 s (c) 150 s (d) 200 s

99. One mole of nitrogen gas being initially at a temperature of $T_0 = 300$ K is adiabatically compressed to increase its pressure 10 times. The final gas temperature after compression is (Assume, nitrogen gas molecules as rigid diatomic and $100^{1/7} = 1.9$)

(a) 120 K (b) 750 K (c) 650 K (d) 570 K

100. Two gases A and B are contained in two separate, but otherwise identical containers. Gas A consists of monatomic molecules, each with atomic mass of 4 u whereas u as B consists of rigid diatomic molecules, each with atomic mass of 40 u. If gas A is kept at 27°C, at what temperature should gas B be kept so that both have the same rms speed?

(a) 27°C (b) 54°C (c) 270°C (d) 62°C

101. Standing waves are produced in a string 16 m long. If there are 9 nodes between the two fixed ends of the string and the speed of the wave is 32 m/s. What is the frequency of the wave?

(a) 5 Hz (b) 10 Hz (c) 30 Hz (d) 20 Hz

102. A highway truck has two horns A and B. When sounded together, the driver records 50 beats in 10 seconds. With horn B blowing and the truck moving towards a wall at a speed of 10 m/s, the driver noticed a beat frequency of 5 Hz with the echo. When frequency of A is decreased the beat frequency with two horns sounded together increases. Calculate the frequency of horn A.

(Speed of sound in air = 330 m/s)

(a) 75 Hz (b) 85 Hz (c) 90 Hz (d) 95 Hz

103. When light of an unknown polarisation is examined with a polaroid, it is found to exhibit maximum intensity I_0 along y-axis and minimum intensity $\frac{2I_0}{3}$ along x-axis. The intensity transmitted through a polaroid with pass axis at 45° to y-axis (in x-y plane) is

(a) $\frac{5I_0}{8}$ (b) $\frac{I_0}{2}$ (c) $\frac{5I_0}{6}$ (d) $\frac{I_0}{4}$

104. In a Young's double slit experiment, m^{th} order and n^{th} order of bright fringes are formed at point P on a distant screen, if monochromatic source of wavelength 400 nm and 600 nm are used respectively. The minimum value of m and n are respectively.

(a) 4, 6 (b) 3, 2 (c) 2, 3 (d) 4, 2

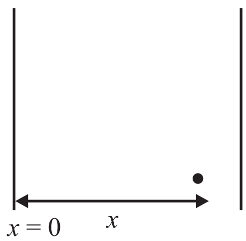
105. Two small conducting balls of identical mass 20 g and identical charge 10^{-10} C hang from non-conducting threads of length, $L = 300$ cm. If the equilibrium separation of balls is x and $x << L$ then the magnitude of x is

(Assume, $4\pi\epsilon_0 = \frac{1}{9 \times 10^9}$ F/m and $g = 10$ m/s^2)

(a) $\frac{2}{5^{1/3}}$ mm (b) $\frac{3}{10^{1/3}}$ mm

(c) $\frac{3^{1/3}}{10}$ mm (d) $\frac{3^{2/3}}{5}$ mm

106. The space between the two large parallel plates is filled with a material of uniform charge density ρ. Assume that one of the plate is kept at $x = 0$. The potential at any point x between these plates is given by (A and B are constants).



(a) $-\frac{\rho x^3}{2\epsilon_0}$ (b) $-\left(\frac{\rho x^2}{2\epsilon_0} + Ax\right)$

(c) $-\left(\frac{\rho x^2}{2\epsilon_0} + Ax + B\right)$ (d) $-\left(\frac{\rho x^3}{4\epsilon_0} + Ax^2 + Bx\right)$

107. Identify the correct statement among the following:

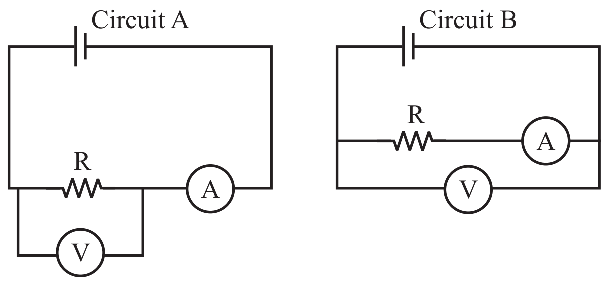
(a) Resistivity of metals decreases with temperature because more electrons are available for conduction.

(b) Resistivity of metals increases with temperature because number of electrons decreases.

(c) Resistivity of metals increases with temperature because number of collisions between electrons increases

(d) Resistivity of metals decreases with temperature because superconductivity sets in.

108. For the circuit A and B as shown in the figure, identify the correct option.



(a) Circuit A is for accurate measurement of high resistance and B is for low resistance.

(b) Circuit A is for accurate measurement of low resistance and B is for high resistance.

(c) Both circuits can accurately measured high resistance only.

(d) Both circuits can accurately measured low resistance only.

109. Two infinitely long straight wires A and B, each carrying current I are placed on x and y-axis, respectively. The current in wires A and B flow along $-\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$ directions respectively. The force on a charged particle having charge q, moving from position, $\boldsymbol{r} = d(\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}})$ with velocity $\mathbf{v} = v\hat{\boldsymbol{i}}$ is

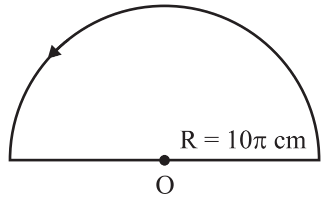
(a) $\frac{\mu_0 I q v}{2\pi d}\hat{\boldsymbol{j}}$ (b) $\frac{\mu_0 I q v}{\pi d}\hat{\boldsymbol{j}}$

(c) $\frac{\mu_0 I q v}{\sqrt{2}\pi d}\hat{\boldsymbol{k}}$ (d) 0

110. A long straight wire carrying current 16 A is bent at 90° such that half of the wire lies along the positive x-axis and other half lies along the positive y-axis. What is the magnitude of the magnetic field at the point, $\boldsymbol{r} = (-2\hat{\boldsymbol{i}} + 0\hat{\boldsymbol{j}})$ mm? (Assume, $\frac{\mu_0}{4\pi} = 10^{-7}\ \text{Hm}^{-1}$)

(a) 1.2 mT (b) 0.8 mT (c) 3.2 mT (d) 1.6 mT

111. The magnitude of the force vector acting on a unit length of a thin wire carrying a current I = 8 A at a point O, if the wire is bent as shown in the figure with a radius, R = 10π cm is



(a) 64 μN/m (b) 32 μN/m

(c) 20 μN/m (d) 100 μN/m

112. A 10 Ω coil of 180 turns and diameter 4 cm is placed in a uniform magnetic field so that the magnetic flux is maximum through the coil's cross-sectional area. When the field is suddenly removed a charge of 360 μC flows through a 618 Ω galvanometer connected to the coil, find the magnetic field.

(a) 12 T (b) 6 T (c) 1 T (d) 8 T

113. An inductor coil is connected to a capacitor and an AC source of rms voltage 8 V in series. The rms current in the circuit is 16 A and is in phase with emf. If this inductor coil is connected to 6 V DC battery, the magnitude of steady current is

(a) 8 A (b) 10 A (c) 12 A (d) 16 A

114. An electromagnetic wave of frequency 3.0 MHz passes from vacuum into a non-magnetic medium with permittivity, $\epsilon = 16\epsilon_0$. Where ϵ_0 is the free space permittivity. The charge in wavelength is

(a) – 75 m (b) + 75 m (c) – 50 m (d) + 50 m

115. A particle of charge q, mass m and energy E has de-Broglie wavelength λ. For a particle of charge $2q$, mass $2m$ and energy 2E, the de-Broglie wavelength is

(a) $\frac{\lambda}{4}$ (b) 2λ (c) 8λ (d) $\frac{\lambda}{2}$

116. The collision of an electron with kinetic energy 5.5 eV and a hydrogen atom in its ground state can be described as

(a) completely inelastic

(b) may be completely inelastic

(c) may be partially elastic

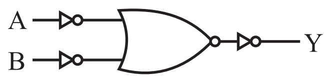
(d) elastic

117. An alloy is composed of two radioactive materials A and B having equal weight. The half life of A and B are 10 yrs and 20 yrs respectively. After time t, the alloy was found to consist of $\left(\frac{1}{e}\right)$ kg of A and 1 kg of B. If the atomic weight of A and B are same, then the value of t is (Assume, ln 2 = 0.7)

(a) $\left(\frac{200}{7}\right)$ yrs (b) $\left(\frac{10}{7}\right)$ yrs
(c) 7 yrs (d) 70 yrs

118. When a zener diode is used as a regulator with zener voltage of 10 V, nearly five times the load current passes through the zener diode. What should be the series resistance for the zener diode. If load resistance is 2 kΩ and the unregulated voltage supplied is 16 V.
(a) 500 Ω (b) 100 Ω (c) 200 Ω (d) 800 Ω

119. The logic circuit below has the truth table, same as that of



(a) NOR gate (b) NAND gate
(c) AND gate (d) OR gate

120. A message signal is used to modulate a carrier frequency. If the peak voltages of message signal and carrier signal are increased by 0.1% and 0.3% respectively, then the percentage change in modulation index is
(a) 0.4 (b) 0.0 (c) – 0.4 (d) – 0.2

CHEMISTRY

121. From the following energy levels of hydrogen atom, the values of E_∞ and E_3 in J are, respectively
________ E_∞ = ...
________ E_3 = ...
________ $E_2 = -0.545 \times 10^{-18}$ J
________ $E_1 = -2.18 \times 10^{-18}$ J
(a) $1, -0.242 \times 10^{-18}$ (b) $\infty, -0.726 \times 10^{-18}$
(c) $0, -0.242 \times 10^{-18}$ (d) $0, -0.321 \times 10^{-18}$

122. Match the following:

	List-I		List-II
(A)	Nodes	(i)	Three dimensional shape of the orbital
(B)	Subsidiary quantum number	(ii)	Significant only for motion of microscopic objects
(C)	White light	(iii)	$\lvert \psi \rvert^2$ is zero
(D)	Heisenberg uncertainty principle	(iv)	Spin state of electron
		(v)	Continuous spectrum

	A	B	C	D
(a)	(v)	(iv)	(ii)	(i)
(b)	(iii)	(iv)	(v)	(ii)
(c)	(iv)	(iii)	(ii)	(i)
(d)	(iii)	(i)	(v)	(ii)

123. In which group of the periodic table the element with Z = 120 be placed?
(a) 2 (b) 1 (c) 14 (d) 15

124. Common oxidation state of *f*-block elements is III. The other stable oxidation states of ^{63}Eu and ^{65}Tb are respectively
(a) II, IV (b) IV, II (c) II, V (d) V, II

125. How many of the following species are diamagnetic?
$He_2^+, H_2, H_2^+, H_2^-$, He
(a) 1 (b) 2 (c) 3 (d) 0

126. In which of the following hydrogen bonding is strongest?
(a) O—H·····N (b) O—H·····O
(c) O—H·····F (d) F—H·····F

127. What is the approximate most probable velocity of oxygen? If the kinetic energy of one mole of oxygen is 3741.3 J.
(a) $\sqrt{43851}$ J kg^{-1} (b) $\sqrt{48321}$ J kg^{-1}
(c) $\sqrt{155887}$ J kg^{-1} (d) $\sqrt{3950}$ J kg^{-1}

128. What is the correction term in the pressure for real gas in comparison to the ideal gas?
(a) $\frac{n^2}{V^2}$ (b) $\frac{aV^2}{n^2}$ (c) $\frac{an^2}{V^2}$ (d) $\frac{an^2}{V} - nb$

129. In a 1 L vessel, 10 moles of methane and 50 moles of O_2 are present. The number of moles of O_2, water and CO_2 present in the vessel are respectively after the vessel was heated to burn methane completely.
(a) 30, 20, 20 (b) 30, 20, 10
(c) 20, 30, 10 (d) 20, 10, 30

130. Identify the oxidation states of Mn, when MnO_4^{2-} ion undergoes disproportionation reaction under acidic medium.
(a) + 2, + 7 (b) + 2, + 5
(c) + 4, + 4 (d) + 7, + 4

131. Find the heat required to make water of 30°C from 10 g of ice at 0.0°C. (Enthalpy of fusion of ice = 33355 J g^{-1}, C_P of water = 4.18 Jg^{-1} K^{-1})
(a) 4.0 kJ (b) 5.0 kJ (c) 3.59 kJ (d) 4.59 kJ

132. For the reaction,
$0.5C(s) + 0.5CO_2(g) \rightleftharpoons CO(g)$
the equilibrium pressure is 12 atm. If CO_2 conversion is 50%, the value of K_p, in atm is
(a) 4 (b) 1 (c) 0.5 (d) 2

133. What is the solubility product (K_{sp}) of calcium phosphate in pure water? [S = molar solubility]

(a) $108\ S^5$ (b) $72\ S^3$ (c) $6\ S^5$ (d) $121\ S^2$

134. What is the approximate volume (in mL) of 10 vol H_2O_2 solution that will react completely with 1 L of 0.02 M $KMnO_4$ solution in acidic medium?

(a) 56.05 (b) 113.5 (c) 90.8 (d) 75.75

135. Which of the following products are formed on hydrolysis of NaO_2?

(A) NaOH (B) H_2O_2 (C) O_2 (D) H_2O

(a) A, D (b) A, C, D (c) A, B, D (d) A, B, C

136. The oxidation state (n) and coordination number (CN) of Al and number of valence electrons around Al(N) in Al_2Cl_6 are respectively

(a) 3, 3, 6 (b) 3, 4, 8
(c) 4, 4, 8 (d) 3, 4, 6

137. ΔH_f° values (in kJ mol^{-1}) for graphite, diamond and C_{60} are respectively

(a) 0; 1.9; 38.1 (b) 1.8; 1.9; 38.1
(c) 0; 0; 21.4 (d) 1.8; 1.9; 2.0

138. What is the formula of the product formed when F^- reacts with enamel on teeth?

(a) $CaSO_4$, CaF_2 (b) $3Ca_3(PO_4)_2$, PF_5
(c) $Ca_2SO_4 \cdot CaF_2$ (d) $3[Ca_3(PO_4)_2 \cdot CaF_2]$

139. Which one of the following methods can be used to find out the percentage composition of halogen present in an organic compound?

(a) Kjeldahl method (b) Duma's method
(c) Lassaigne's method (d) Carius method

140. Which of the following reaction produce alkane as the product?

I. $R—CO\overset{\ominus}{O}\overset{\oplus}{Na} + NaOH \xrightarrow[\Delta]{CaO}$

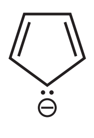
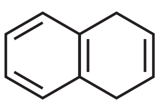
II. $CH_3\overset{\overset{O}{\|}}{C}CH_3 \xrightarrow[\text{Conc. HCl}]{\text{Zn-Hg}}$

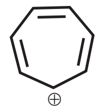
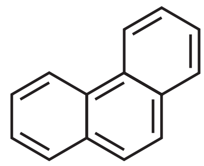
III. $CH_3C{\equiv}CCH_3 \xrightarrow{LiAlH_4}$

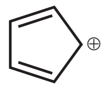
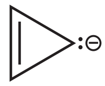
IV. $CH_3—\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}—Cl \xrightarrow{NaBH_4}$

(a) I, II, III (b) I, III, IV (c) I, II, IV (d) II, III, IV

141. Which of the following are not aromatic?

(A)  (B) 

(C)  (D) 

(E)  (F) 

(a) A, C, E (b) B, E, F
(c) B, C, F (d) C, D, E

142. How many of the following compounds show ferrimagnetism?

Fe_3O_4; $MgFe_2O_4$; $NiFe_2O_4$; MnO; CrO_2

(a) 1 (b) 2 (c) 3 (d) 4

143. The molality (in mol kg^{-1}) of 1 mole of solute in 50g of solvent is

(a) 10 (b) 20 (c) 30 (d) 40

144. The freezing point of solution containing 10 mL of non-volatile and non-electrolyte liquid "A" in 500 g of water is – 0.413°C. If K_f of water is 1.86 K kg mol^{-1} and the molecular weight of A = 60 g mol^{-1}, what is the density of the solution in g mL^{-1}?

(Assume $\Delta_{mix}V = 0$)

(a) 1.13 (b) 1.3 (c) 0.90 (d) 0.993

145. A current of 19296 C is passed through an aqueous solution of copper sulphate using copper electrodes. What is the mass (in g) of copper deposited at the cathode? (Molar mass of Cu = 63.5 g mol^{-1})

(a) 3.17 (b) 1.58 (c) 6.35 (d) 0.79

146. For a zero-order reaction, the correct expression for rate constant (k) at half-life time ($t_{1/2}$) is (R_0 = initial concentration of reactant)

(a) $k = \frac{2.303}{t_{1/2}} \log \frac{[R_0]}{\frac{[R_0]}{2}}$ (b) $k = \frac{2.303}{t} \log \frac{[R_0]}{[R_0]}$

(c) $k = \frac{[R_0] - \frac{1}{2}[R_0]}{t_{1/2}}$ (d) $k = \frac{2.303}{(t_2 - t_1)} \log [R_0]$

147. Which one of the following is used as an eye lotion?

(a) Milk of magnesia (b) Silver sol
(c) Colloidal antimony (d) Chromium salt sol

148. Identify A and B respectively in the following reactions:

$4Au(s) + 8CN^-(aq) + 2H_2O(aq)$

$\xrightarrow{O_2(g)} A(aq) + 4OH^-(aq) \xrightarrow{Zn(s)} B + 2Au(s)$

(a) $[Au(CN)_2]^-$; $[Zn(CN)_4]^{2-}$
(b) $Au(CN)_4$; $[Zn(CN)_4]^{2-}$
(c) HCN; $[Au(CN)_4]^{2-}$
(d) AuCN; [HCN]

149. **Statement:** Sulphur vapour is paramagnetic.

Statement : Reaction of dil. HCl with finely divided iron forms $FeCl_3$ and H_2 gas.

(a) Statement I is correct, but Statement II is wrong.
(b) Both the statements are correct.
(c) Statement I is wrong, but Statement II is correct.
(d) Both the statements are wrong.

150. The reason for the noble gases to have low boiling and low melting point is

(a) atoms of the noble gases have weak covalent interaction
(b) atoms of the noble gases have weak dipole interaction
(c) atoms of the noble gases have weak van der Waal's interaction
(d) atoms of the noble gases have weak dispersion forces

151. Match the following:

	List-I		**List-II**
(A)	Co^{2+}	(i)	Yellow
(B)	Fe^{2+}	(ii)	Dark-green
(C)	Ni^{2+}	(iii)	Blue
(D)	Cu^{2+}	(iv)	Pale-green
		(v)	Pink

	A	**B**	**C**	**D**
(a)	(v)	(iv)	(ii)	(iii)
(b)	(i)	(ii)	(iii)	(iv)
(c)	(v)	(i)	(iv)	(ii)
(d)	(i)	(v)	(iv)	(ii)

152. Which one of the following complex has the highest magnitude of crystal field splitting energy (Δ_0)?

(a) $[Co(H_2O)_6]^{3+}$ (b) $[Co(NH_3)_6]^{3+}$
(c) $[Co(C_2O_4)_3]^{3-}$ (d) $[CoF_6]^{3-}$

153. A polymer contains 50 molecules with molecular mass 5000, 100 molecules with molecular mass 10,000 and 50 molecules with molecular mass 15,000. Calculate number average molecular mass?

(a) 5,000 (b) 75,000 (c) 10,000 (d) 20,000

154. Which of the following are reducing sugars?

(A) Sucrose, (B) Maltose, (C) Lactose, (D) Fructose

(a) A, B, C (b) A, B, D (c) A, C, D (d) B, C, D

155. Identify opiates from the following:

(A) Codein, (B) Tymine, (C) Epinephrine, (D) Morphine, (E) Thiamine and (F) Heroin

(a) A, D, F (b) C, D, E (c) B, E, F (d) A, B, C

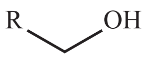
156. Match the following:

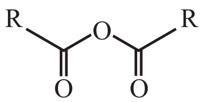
	List-I		List-II Mirror Product
(A)	$CH_3—CHBr—CH_2Br \xrightarrow{KOH/C_2H_5OH}$	(i)	1°-alkyl bromide
(B)	$CH_3—CH_2—CH=CH_2 \xrightarrow[(C_6H_5CO)_2O_2,\Delta]{HBr}$	(ii)	2°-alkyl bromide
(C)	$CH_3CH_2CH_3 \xrightarrow{Br_2, h\nu}$	(iii)	Allyl bromide
(D)	$CH_3—CH=CH_2 \xrightarrow[\Delta]{NBS}$	(iv)	Alkenyl bromide

	A	B	C	D
(a)	(i)	(iv)	(ii)	(iii)
(b)	(iv)	(iii)	(i)	(ii)
(c)	(ii)	(iii)	(i)	(iv)
(d)	(iv)	(i)	(ii)	(iii)

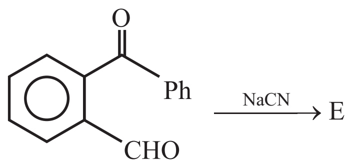
157. Find the suitable product for the following reaction.

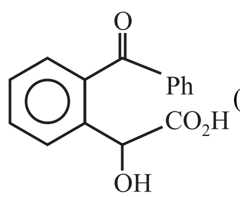
$$R—CO_2H \xrightarrow[(ii)\ H_2O/H^{\oplus}]{(i)\ B_2H_6}$$

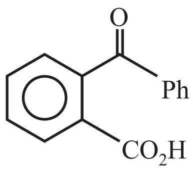
(a) R—CHO (b) 

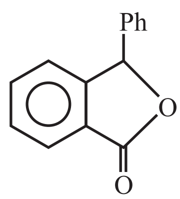
(c) $R—CO_2R$ (d) 

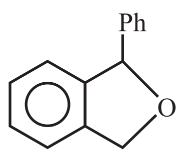
158. What is the product E in the following reaction?



(a) 

(b) 

(c) 

(d) 

159. The decreasing order of acidic strength for following acids is:

(A) CH_3COOH (B) $CH_3CHClCH_2COOH$

(C) $ClCH_2COOH$ (D) $Cl_2CHCOOH$

(a) B > C > A > D (b) D > C > B > A

(c) D > B > C > A (d) C > D > B > A

160. Identify X, Y and Z respectively in the following reaction sequence:

$$\text{Acetic acid} \xrightarrow[\text{(iii) } C_6H_5SO_2Cl,\ \text{Pyridine/343K}]{\text{(i) } NH_3,\ \text{(ii) } \Delta} X + Y + Z$$

(a) $C_6H_5SO_3H$, CH_3NC, HCl

(b) $CH_3CONH{-}\overset{\overset{\displaystyle O}{\|}}{\underset{\underset{\displaystyle O}{\|}}{S}}{-}C_6H_5$, H_2O, HCl

(c) $C_6H_5SO_3H$, CH_3CN, HCl

(d) $C_6H_5SO_2Cl$, CH_3NC, H_2O

ANSWER KEY

1	(d)	**2**	(c)	**3**	(a)	**4**	(c)	**5**	(a)	**6**	(c)	**7**	(b)	**8**	(d)	**9**	(b)	**10**	(c)
11	(c)	**12**	(b)	**13**	(c)	**14**	(b)	**15**	(d)	**16**	(c)	**17**	(c)	**18**	(d)	**19**	(c)	**20**	(c)
21	(a)	**22**	(d)	**23**	(c)	**24**	(c)	**25**	(b)	**26**	(b)	**27**	(d)	**28**	(a)	**29**	(a)	**30**	(c)
31	(d)	**32**	(c)	**33**	(c)	**34**	(a)	**35**	(a)	**36**	(b)	**37**	(Bonus)	**38**	(b)	**39**	(d)	**40**	(d)
41	(c)	**42**	(b)	**43**	(d)	**44**	(c)	**45**	(c)	**46**	(b)	**47**	(c)	**48**	(a)	**49**	(d)	**50**	(c)
51	(a)	**52**	(a)	**53**	(d)	**54**	(c)	**55**	(a)	**56**	(a)	**57**	(b)	**58**	(b)	**59**	(b)	**60**	(b)
61	(a)	**62**	(b)	**63**	(a)	**64**	(d)	**65**	(c)	**66**	(d)	**67**	(c)	**68**	(a)	**69**	(c)	**70**	(a)
71	(a)	**72**	(b)	**73**	(a)	**74**	(c)	**75**	(a)	**76**	(c)	**77**	(b)	**78**	(c)	**79**	(b)	**80**	(a)
81	(a)	**82**	(b,d)	**83**	(a)	**84**	(b)	**85**	(d)	**86**	(d)	**87**	(a)	**88**	(d)	**89**	(a)	**90**	(None)
91	(d)	**92**	(c)	**93**	(c)	**94**	(a)	**95**	(c)	**96**	(c)	**97**	(b)	**98**	(c)	**99**	(d)	**100**	(c)
101	(b)	**102**	(a)	**103**	(c)	**104**	(b)	**105**	(b)	**106**	(c)	**107**	(c)	**108**	(b)	**109**	(b)	**110**	(b)
111	(a)	**112**	(c)	**113**	(c)	**114**	(a)	**115**	(d)	**116**	(d)	**117**	(a)	**118**	(c)	**119**	(b)	**120**	(d)
121	(c)	**122**	(d)	**123**	(a)	**124**	(a)	**125**	(b)	**126**	(d)	**127**	(c)	**128**	(c)	**129**	(b)	**130**	(d)
131	(d)	**132**	(a)	**133**	(a)	**134**	(a)	**135**	(d)	**136**	(b)	**137**	(a)	**138**	(d)	**139**	(d)	**140**	(c)
141	(b)	**142**	(c)	**143**	(b)	**144**	(d)	**145**	(c)	**146**	(c)	**147**	(b)	**148**	(a)	**149**	(a)	**150**	(c, d)
151	(a)	**152**	(b)	**153**	(c)	**154**	(d)	**155**	(a)	**156**	(d)	**157**	(b)	**158**	(c)	**159**	(b)	**160**	(b)

Hints & Solutions

MATHEMATICS

1. **(d)** (A) We have $f: R \to R$, $f(x) = \cos(112x - 37)$. Cosine is not one-one function and range lies between –1 to 1. So, it is not onto on R. (A) → (iv)

(B) $f(x) = x|x|$ for $f: A \to B$.

Here, A = [–2, 2] and B = [–4, 4]

For every $x \in A$ there exist a unique value between [–4, 4] and range is equal to codomain. So, it is one-one & onto both. Hence bijective (B) → (iii).

(C) $f(x) = (x-3)(x-2)(x-5)$

It gives zero at x = 3, 2, 5 and range R to every real number. So, (C) → (iii).

(D) $f(x) = x + 1$, $f: N \to N$.

For x = 1, 2, 3, ..., so, range of f(x) would be $[2, \infty)$

So, (D) → (i).

2. **(c)** We have given the function,

$$f(x) = \sqrt{\frac{x-[x]}{\log(x^2-x)}}$$

It is defined

$\log(x^2 - x) > 0$ $\quad [\because x - [x] = [x] \geq 0, \forall x \in R]$

$\Rightarrow x^2 - x > e^0 \Rightarrow x^2 - x > 1 \Rightarrow x^2 - x - 1 > 0$

$\because x^2 - x - 1 = 0 \quad \Rightarrow x = \dfrac{1 \pm \sqrt{5}}{2}$

$\therefore x \in \left(-\infty, \dfrac{1-\sqrt{5}}{2}\right) \cup \left(\dfrac{1+\sqrt{5}}{2}, \infty\right)$

$\therefore$ Domain of $f(x) = R - \left[\dfrac{1-\sqrt{5}}{2}, \dfrac{1+\sqrt{5}}{2}\right]$

3. **(a)** **Assertion:** We have,

$\dfrac{x}{x+1} = x(1+x)^{-1} = x(1 - x + x^2 - x^3 + x^4 - \dots)$

$= x - x^2 + x^3 - x^4 + x^5 - \dots$

$= \sum_{n=0}^{\infty} (-1)^n x^{n+1}$

$\therefore$ Assertion is true.

Reason: $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \dots$

Hence, (A) and (R) are true, (R) is correct explanation of (A).

4. **(c)** We have given that,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 1 & 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 1 & 6 \end{vmatrix}$$

$|A| = 1(18 - 5) - 2(6 - 10) + 3(1 - 6)$

$= 13 + 8 - 15 = 6$

$$\text{Adj } A = \begin{bmatrix} 13 & -9 & 1 \\ 4 & 0 & -2 \\ -5 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{6}\begin{bmatrix} 13 & -9 & 1 \\ 4 & 0 & -2 \\ -5 & 3 & 1 \end{bmatrix}$$

Now, Adj (Adj A) = $|A|\, A = 6A$

$$(\text{Adj}(\text{Adj } A))^{-1} = (6A)^{-1} = \frac{1}{36}\begin{bmatrix} 13 & -9 & 1 \\ 4 & 0 & -2 \\ -5 & 3 & 1 \end{bmatrix}$$

5. **(a)** We have $\begin{vmatrix} x^2+3x & x+1 & x-3 \\ x-1 & 2-x & x+x \\ x-3 & x-3 & 3x \end{vmatrix}$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\Rightarrow \begin{vmatrix} x^2+2x+1 & 2x-1 & -7 \\ 2 & 5-2x & 4-2x \\ x-3 & x-3 & 3x \end{vmatrix}$$

$C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

$$\Rightarrow \begin{vmatrix} x^2+2 & 2x+6 & -7 \\ 2x-3 & 1 & 4-2x \\ 0 & -x-3 & 3x \end{vmatrix}$$

Expand w.r.t. R_1

$\Rightarrow (x^2 + 2)[3x - (-x - 3)(4 - 2x)]$
$\quad - (2x + 6)[(2x - 3)3x - 0] - 7[(2x - 3)(-x - 3) - 0].$

$\Rightarrow (x^2 + 2)[3x + (4x - 2x + 12 - 6x)] - 12x^3 + 18x^2 - 36x^2 + 54x - 7[(2x - 3)(-x - 3) - 0]$

$\Rightarrow 4x^4 - 7x^3 + 6x^2 + 64x - 39$

Here, $a_4 = 4$, $a_3 = -7$, $a_2 = 6$, $a_1 = 64$, $a_0 = 39$

Now, $(a_1 + a_3) + 2(a_4 + a_2 + 90)$

$\Rightarrow (64 - 7) + 2(4 + 6 - 39)$

$\Rightarrow 57 - 58 = -1$

6. **(c)** Given,

$AX = D$ be a system of three linear non-homogeneous equation.

Here, $|A| = 0 \Rightarrow$ No unique solution.

But rank (A) = rank $(AD) = \alpha$

$\therefore$ If $\alpha < 3$, then equation has infinite number of solutions.

7. **(b)** It is given that,

$x + iy = (1 + i)^6 - (1 - i)^6$

$1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$(1 + i)^6 = (\sqrt{2})^6\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^6$

$= 8\left(\cos\frac{6\pi}{4} + i\sin\frac{6\pi}{4}\right)$

Similarly, $(1 - i)^6 = 8\left(\cos\frac{6\pi}{4} - i\sin\frac{6\pi}{4}\right)$

$\therefore\ x + iy = 8\left[\cos\frac{6\pi}{4} + i\sin\frac{6\pi}{4} - \cos\frac{6\pi}{4} + i\sin\frac{6\pi}{4}\right]$

$x + iy = 8\left[2i\sin\frac{3\pi}{2}\right] = -16i \qquad \left(\because\ \sin\frac{3\pi}{2} = -1\right)$

$\therefore\ x = 0, y = -16$

Hence, $x + y = 0 - 16 = -16$

8. **(d)** $\because\ i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \forall\ n \in$ Integer

So, $i^2 + i^3 + i^4 + \ldots + i^{4000} = i - i + i^2 + i^3 + \ldots + i^{4000}$

$= -i + [i + i^2 + i^3 + \ldots + i^{4000}] = -i$

9. **(b)** Here 1, ω and ω^2 are the cube roots of unity.

$(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega) = a^2 + b^2 + c^2 - ab - bc - ca$

and $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

So, $(a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

$= a^3 + b^3 + c^3 - 3abc$

10. **(c)** Given that $a_k = \cos\alpha_k + i\sin\alpha_k$

$\therefore\ a_1 = \cos\alpha_1 + i\sin\alpha_2$

$a_2 = \cos\alpha_2 + i\sin\alpha_2$

$a_3 = \cos\alpha_b + i\sin\alpha_3$

Also, given that a_1, a_2, a_3 are rots of $x^3 + bx + c = 0$

We have at least one root of cubic equation as real.

$\therefore\ \sin\alpha_1 = 0 \Rightarrow \cos\alpha_1 = 1$

$\therefore\ \alpha_1 = 0$

And other roots are congugate pair.

$\therefore\ a_3 = \cos\alpha_2 - i\sin\alpha_2$

Now, $a_1 + a_2 + a_3 = 0$

$1 + \cos\alpha_2 + i\sin\alpha_2 + \cos\alpha_2 - i\sin\alpha_2 = 0$

$\Rightarrow \cos\alpha_2 = -\frac{1}{2} \Rightarrow \sin\alpha_2 = \frac{-\sqrt{3}}{2}$

$a_2 = \frac{-1}{2} - \frac{\sqrt{3}}{2}i$

$a_3 = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$

Now, $a_1a_2 + a_2a_3 + a_3a_1 = b$

$\Rightarrow \frac{-1}{2} - \frac{\sqrt{3}}{2}i + \frac{1}{4} + \frac{3}{4} - \frac{1}{2} + \frac{\sqrt{3}}{2}i = b$

$\Rightarrow -1 + 1 = b \Rightarrow b = 0$

11. **(c)** Let $y = \frac{ax^2 - 2x + 3}{2x - 3x^2 + a}$

$2xy - 3x^2y + ay = ax^2 - 2x + 3$

$ax^2 + 3x^2y - 2x - 2xy - ay + 3 = 0$

$x^2(a + 3y) - 2x(y + 1) + 3 - ay = 0$

As $x \in R$, $D \geq 0$ for real values,

$\therefore\ 4(y + 1)^2 - 4(a + 3y)(3 - ay) \geq 0$

$(y^2 + 2y + 1) - [3a - a^2y + 9y - 3ay^2] \geq 0$

$y^2(3a + 1) + (a^2 - 7)y + 1 - 3a \geq 0$

As $y \in R$, $D \leq 0$

$\therefore\ (a^2 - 7)^2 + 4(3a + 1)(3a - 1) \leq 0$

$a^4 - 14a^2 + 49 + (12a + 4)(3a - 1) \leq 0$

$a^4 + 22a^2 + 45 \leq 0 \qquad [\because\ a^4 + 22a^2 + 45 > 0, \forall\ a \in R]$

$a \in \phi$

12. **(b)** We have $x^2 - 4ax + 1 - 3a + 4a^2 = 0$

When both roots greater than 1 then $D > 0$

$D = b^2 - 4ac$

$= 16a^2 - 4 \times 1 \times (4a^2 - 3a + 1) = -12a - 4$

$= 12a - 4 > 0$

$a > \frac{1}{3}$.

So, $a \in \left(\frac{1}{3}, \infty\right)$

For $x = 1$, $f(1) = 1 - 4a \times 1 + 1 - 3a + 4a^2$

$4a^2 - 7a + 2 = 0$

$a = \frac{7 \pm \sqrt{49 - 32}}{8} = \frac{7 \pm \sqrt{17}}{8}$

Required interval $a \in \left(\frac{7 \pm \sqrt{17}}{8}, \infty\right)$

13. **(c)** Let α, β, γ are roots of equation

$x^3 - ax^2 + ax - 1 = 0 \qquad \ldots(i)$

$\therefore\ \alpha + \beta + \gamma = a$

$\alpha\beta + \beta\gamma + \gamma\alpha = a$

$\alpha\beta\gamma = -1$

Cubic equation whose roots are $\alpha^2, \beta^2, \gamma^2$ is

$x^2 - (\alpha^2 + \beta^2 + \gamma^2)x^2 + (\alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2)x$

$- \alpha^2\beta^2\gamma^2 = 0 \qquad \ldots(ii)$

Eqs. (i) and (ii) are identical.

$\therefore\ \frac{a}{\alpha^2 + \beta^2 + \gamma^2} = \frac{a}{\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2} = \frac{1}{\alpha^2\beta^2\gamma^2}$

$a = \alpha^2 + \beta^2 + \gamma^2 \qquad [\alpha\beta\gamma = -1]$

$a = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$a = a^2 - 2a \ \Rightarrow\ a^2 = 3a$

$\Rightarrow\ a = 3 \qquad [\because\ a$ is non-zero real$]$

14. **(b)** We have $x^3 + px^2 + qx + r = 0$

$\alpha + \beta + \gamma = -p$

$\alpha\beta\gamma = -r$

$\alpha\beta + \beta\gamma + \gamma\alpha = q$

So, $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha) = (-p-\gamma)(-p-\alpha)(-p-\beta)$
$= (p^2 + p\alpha + \gamma p + \gamma\alpha)(-p-\beta)$
$= -p^3 - p^2\beta - p^2\alpha - p\alpha\beta - p^2\gamma - p\gamma\beta - p\alpha\gamma - \alpha\beta\gamma$
$= -p^3 - p^2(\alpha+\beta+\gamma) - p(\alpha\beta+\beta\gamma+\alpha\gamma) - \alpha\beta\gamma$
$= -p^3 + p^3 - p \times q + r$
$= r - pq$

15. **(d)** x denotes the number of ways of arranging m boys and m girls in a row such that no two boys sit together, $x = (m+1)!\, m!$

y denotes the number of ways of arranging m boys and m girls in a row such that boys and girls sit alternately, $y = m! \times m! \times 2!$

z denotes the number of ways of arranging m boys and m girls in a circular table such that boys and girls sit alternately, $z = (m-1)!\, m!$

$\therefore\ x : y : z = (m+1)!\, m! : m!m! \times 2 : (m-1)!\, m!$
$= (m+1)\, m : 2m : 1$

16. **(c)** Total numbers greater than 1000000 that can be formed using all the digits 1, 2, 0, 2, 4, 2 and 4,

$= \dfrac{7!}{3!\,2!} - \dfrac{6!}{3!\,2!} = 360$

The odd numbers greater than 1000000 that can be formed by using all the digits 1, 2, 0, 2, 4, 2 and 4,

$= \dfrac{6!}{3!\,2!} - \dfrac{5!}{3!\,2!} = 50$

$\therefore$ Total number of even numbers = 360 – 50 = 310

17. **(c)** Let $(\sqrt{3}+2)^5 = (2+\sqrt{3})^5 = I + F$

[$\because$ I is an integer, F is fraction]

$\because\ 2-\sqrt{3} < 1 \Rightarrow (2-\sqrt{3})^5 < 1$

Let $(2-\sqrt{3})^5 = F'$

$\therefore\ I + F + F' = (2+\sqrt{3})^5 + (2-\sqrt{3})^5$
$= 2(2^5 + {}^5C_2 2^3(3) + {}^5C_4 2 \cdot 3^2)$
$I + F + F' = 2(32 + 10 \times 24 + 5 \times 18)$
$I + F + F' = 2(32 + 240 + 90)$
$I + F + F' = 724$
$I = 724 - (F + F')$
$I = 724 - 1 = 723$ [$\because F + F' = 1$]

18. **(d)** In the expansion of

$$\left(3 - \sqrt{\frac{17}{4} + 3\sqrt{2}}\right)^{10}$$

$\Rightarrow T_6 = T_{5+1} = -\,{}^{10}C_5 (3)^5 \left(\dfrac{17}{4} + 3\sqrt{2}\right)^{5/2}$

$\therefore\ T_6$ = negative irrational number.

19. **(c)** We have

$$\frac{1}{(x^2-3)^2} = \frac{A_1}{x-\sqrt{3}} + \frac{A_2}{(x-\sqrt{3})^2} + \frac{A_3}{(x+\sqrt{3})} + \frac{A_4}{(x+\sqrt{3})^2}$$

$1 = A_1(x-\sqrt{3})(x^2 + 2\sqrt{3x} + 3) + A_2(x^2 + 2\sqrt{3x} + 3)$
$+ A_3(x+\sqrt{3})(x^2 - 2\sqrt{3x} + 3) + A_4(x^2 - 2\sqrt{3x} + 3)$

$A_1 + A_3 = 0$ ... (i)
$\sqrt{3}A_1 + A_2 - \sqrt{3}A_3 + A_4 = 0$ ... (ii)
$-3A_1 + 2\sqrt{3}A_2 - 3A_3 - 2\sqrt{3}A_4 = 0$ ... (iii)
$-3\sqrt{3}A_1 + 3A_2 + 3\sqrt{3}A_3 + 3A_4 = 1$ ... (iv)

So, $A_1 = -A_3$

Now, satisfy $A_1 = -A_3$ in eqs (ii), (iii) and (iv)

$2\sqrt{3}A_1 + A_2 + A_4 = 0$

$2\sqrt{3}A_2 = 2\sqrt{3}A_4$

$A_2 = A_4$

$-6\sqrt{3}A_1 + 3A_2 + 3A_4 = 1$

Now, $2\sqrt{3}A_1 + 2A_2 = 0$

$\Rightarrow -\sqrt{3}A_1 = A_2$ ... (a)

$-6\sqrt{3}A_1 + 6A_2 = 1$

$-6\sqrt{3}A_1 - 6\sqrt{3}A_2 = 1 \Rightarrow A_1 = \dfrac{-1}{12\sqrt{3}}$

Similarly, $A_2 = \dfrac{1}{12} = A_4, A_3 = \dfrac{1}{12\sqrt{3}}$

So, all A_1, A_2, A_3, A_4 are not distinct, $A_1^2 = A_3^2$. Which is correct.

Take III point $\sum_{i=1}^{4} A_i = A_1 + A_2 + A_3 + A_4$

$= \dfrac{1}{12\sqrt{3}} + \dfrac{1}{12} + \dfrac{1}{12\sqrt{3}} + \dfrac{1}{12} \Rightarrow \sum_{i=1}^{4} A_i = \dfrac{1}{6}$

So, $\sum_{i=1}^{4} A_i \neq 1$, them statement (iv) is false.

20. **(c)** Let $f(x) = \cos(x + 8x + 27x + \ldots + n^3x)$

$= \cos\{(1^3 + 2^3 + 3^3 + \ldots + n^3)x\} = \cos\left(\left\{\dfrac{n(n+1)}{2}\right\}^2 x\right)$

Period of $f(x) = \dfrac{2\pi \times 4}{n^2(n+1)^2} = \dfrac{8\pi}{n^2(n+1)^2}$

21. **(a)** Given,

$\sin^2(3°) + \sin^2(6°) + \sin^2(9°) + \ldots + \sin^2(84°) + \sin^2(87°) + \sin^2(90°)$

$= \sin^2(3°) + \sin^2(87°) + \sin^2(6°) + \sin^2(84°) + \ldots + \sin^2(90°)$

($\because \sin^2\theta + \sin^2(90-\theta) = 1$)

$= 1 + 1 + 1 + \ldots + 15$ times $+ \sin^2 45$

$= 1 + 1 + 1 + \ldots + 15$ times $+ \left(\dfrac{1}{\sqrt{2}}\right)^2 = 15 + \dfrac{1}{2} = \dfrac{31}{2}$

22. **(d)**

23. **(c)** Let $f(x) = 7\cos x + 5\sin x = 2k + 1$

Maximum and minimum value of $f(x) = 7\cos x + 5\sin x$ is

$\sqrt{49+25}, -\sqrt{49+25} \Rightarrow \sqrt{74}, -\sqrt{74}$

$\therefore\ -\sqrt{74} \le 2k + 1 \le \sqrt{74}$

$\because$ k is an integer value.

$\Rightarrow -8 \le 2k + 1 \le 8 \quad \Rightarrow -9 \le 2k \le 7$

$\Rightarrow -\dfrac{9}{2} \le k \le \dfrac{7}{2}\ \therefore\ k = \{-4, -3, -2, -1, 0, 1, 2, 3\}$

Total number of values of $k = 8$.

24. **(c)** Assume, $\tan^{-1}\dfrac{y}{2} = x \Rightarrow \dfrac{y}{2} = \tan x$

Squaring on both the sides, we get

$\Rightarrow \dfrac{y^2}{4} = \tan^2 x \Rightarrow \dfrac{y^2}{4} = \sec^2 x - 1$

$\Rightarrow \sec^2 x = 1 + \dfrac{y^2}{4} = \dfrac{4+y^2}{4}$

$\Rightarrow \sec x = \sqrt{\dfrac{4+y^2}{4}}$

$\Rightarrow \sec\left(\tan^{-1}\dfrac{y}{2}\right) = \dfrac{\sqrt{4+y^2}}{2}$

25. **(b)** We have $\sqrt{2} + e^{\cosh^{-1}x} - e^{\sinh^{-1}x} = 0$

$\cosh^{-1}x = \log\left(x+\sqrt{x^2-1}\right), \sinh^{-1}x = \log\left(x+\sqrt{x^2+1}\right)$

$\Rightarrow \sqrt{2} + e^{\log(x+\sqrt{x^2-1})} - e^{\log(x+\sqrt{x^2+1})} = 0$

$\Rightarrow \sqrt{2} + x + \sqrt{x^2-1} - x - \sqrt{x^2+1} = 0$

$\Rightarrow \left(\sqrt{2}+\sqrt{x^2-1}\right)^2 = \left(\sqrt{x^2+1}\right)^2$

$\Rightarrow 2 + x^2 - 1 + 2\sqrt{2(x^2-1)} = x^2 + 1$

$\Rightarrow 2\sqrt{2(x^2-1)} = 0$

$\Rightarrow x^2 - 1 = 0, \Rightarrow x = \pm 1 \Rightarrow x = 1$

26. **(b)** We have, length of an arc = 44 cm and radius = 12 cm

We know that, $\theta = \dfrac{l}{r} \Rightarrow \theta = \dfrac{44}{12} = \dfrac{11}{3}$ radian

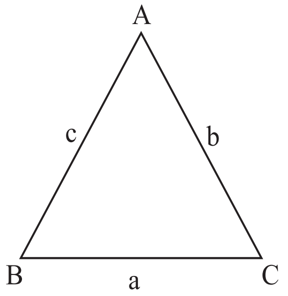
$\theta(\text{in degree}) = \dfrac{11}{3} \times \dfrac{180}{\pi} = \left(\dfrac{660}{\pi}\right)^\circ$

27. **(d)** $R = \dfrac{abc}{4\Delta}$ and $r = \dfrac{\Delta}{5}$

We have a : b : c = 2 : 3 : 4

a = 2k, b = 3k, c = 4k.

$S = \dfrac{2k+3k+4k}{2} = \dfrac{9k}{2}$



Now, $\dfrac{R}{r} = \dfrac{2k\times 3k \times 4k}{4\left(\dfrac{9k}{2}-2k\right)\left(\dfrac{9k}{2}-3k\right)\left(\dfrac{9k}{2}-4k\right)}$

$\Rightarrow \dfrac{R}{r} = \dfrac{2k\times 3k\times 4k}{4\times\dfrac{5k}{2}\times\dfrac{3k}{2}\times\dfrac{1}{2}k} = \dfrac{16}{5} \quad \Rightarrow R : r = 16 : 5$

28. **(a)**

(A) In ΔABC, we have

$rr_2 = r_1r_3$

$\therefore\ \dfrac{\Delta}{s} = \dfrac{\Delta}{s-b} = \dfrac{\Delta}{s-a}.\dfrac{\Delta}{s-c} = \dfrac{(s-a)(s-c)}{s(s-b)} = 1$

$\Rightarrow \tan^2\dfrac{B}{2} = 1 \Rightarrow \tan\dfrac{B}{2} = \tan 45^\circ \Rightarrow B = 90^\circ$

$\therefore b^2 = a^2 + c^2 \therefore A \to II$

(B) We have, $r_1 + r_2 = r_3 - r$

$\dfrac{\Delta}{s-a} + \dfrac{\Delta}{s-b} = \dfrac{\Delta}{s-c} - \dfrac{\Delta}{s}$

$\Rightarrow \dfrac{s-b+s-a}{(s-a)(s-b)} = \dfrac{s-s+c}{s(s-c)} \Rightarrow \dfrac{25-(a+b)}{(s-a)(s-b)} = \dfrac{c}{s(s-c)}$

$\Rightarrow \tan^2\dfrac{c}{z} = 1 \Rightarrow \angle c = 90^\circ \therefore B \to III$

(C) $r_1 = r + 2R$

$\dfrac{\Delta}{s-a} = \dfrac{\Delta}{s} + \dfrac{a}{\sin A} \Rightarrow \sin A = \dfrac{s(s-a)}{\Delta}$

$\Rightarrow 2\sin\dfrac{A}{2}\cos\dfrac{A}{2} = \sqrt{\dfrac{s(s-a)}{(s-b)(s-c)}} = \cot\dfrac{A}{2}$

$\Rightarrow \sin^2\dfrac{A}{2} = \dfrac{1}{2} \Rightarrow \sin^2\dfrac{A}{2} = \dfrac{1}{\sqrt{2}} \Rightarrow \angle A = 90^\circ$

$\therefore C \to I$

29. **(a)** Given that $\overrightarrow{OA} = \vec{a} + \vec{b} + \vec{c}$ and $\overrightarrow{OB} = \vec{a} - 2\vec{b} + 3\vec{c}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= -3\vec{b} + 2\vec{c}$, $|AB| = \sqrt{a+x} = \sqrt{13}$

$\overrightarrow{OP} = \dfrac{m(\overrightarrow{OB}) + n(\overrightarrow{OA})}{m+n}$

$\overrightarrow{OP} = \dfrac{1(\vec{a}-2\vec{b}+3\vec{c}) + 3(\vec{a}+\vec{b}+\vec{c})}{4} = \dfrac{4\vec{a}+\vec{b}+6\vec{c}}{4}$

$\overrightarrow{OQ} = \dfrac{1(\vec{a}-2\vec{b}+3\vec{c}) - 3(\vec{a}+\vec{b}+\vec{c})}{-2} = \dfrac{-2\vec{a}-5\vec{b}}{-2}$

$\overrightarrow{OQ} = \dfrac{4\vec{a}+10\vec{b}}{4}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \dfrac{4\vec{a}+10\vec{b}}{4} - \dfrac{4\vec{a}+\vec{b}+6\vec{c}}{4} = \dfrac{9\vec{b}-6\vec{c}}{4}$

$$|\overrightarrow{PQ}| = \sqrt{\left(\frac{9}{4}\right)^2 + \left(-\frac{6}{4}\right)^2} = \sqrt{\frac{81}{16} + \frac{36}{16}} = \frac{\sqrt{117}}{4}$$

$$|\overrightarrow{PQ}| = \frac{3\sqrt{13}}{4} = \frac{3}{4}|\overrightarrow{AB}|$$

$$\Rightarrow 3|\overrightarrow{AB}| = 4|\overrightarrow{PQ}|$$

30. **(c)** It is given that, $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-collinear point and $k\vec{a} + 2\vec{b} + 3\vec{c}$ is a point in the plane $\vec{a}, \vec{b}, \vec{c}$.

$$\therefore\ k\vec{a} + 2\vec{b} + 3\vec{c} = 0 \Rightarrow k + 2 + 3 = 0 \quad \Rightarrow k = -5$$

31. **(d)** Given that,

$\vec{a} = 3\hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$

Here, vector $\vec{a}$ is the sum of two vectors $\vec{a}_1$ and $\vec{a}_1$.

$$\vec{a} = \vec{a}_1 + \vec{a}_2$$

Vector $\vec{a}_1$ is parallel to vector $\vec{b}$.

$$\therefore\ \vec{a}_1 = \lambda\vec{b} = \lambda(\hat{i} + \hat{j})$$

Vector $\vec{a}_1$ is perpendicular to vector $\vec{b}$.

$$\vec{a}_2 \cdot \vec{b} = 0$$

$$(\vec{a} - \vec{a}_1) \cdot \vec{b} = 0$$

$$(3\hat{j} + 4\hat{k} - \lambda(\hat{i} + \hat{j})) \cdot (\hat{i} + \hat{j}) = 0$$

$$(-\lambda\hat{i} + (3 - \lambda)\hat{j} + 4\hat{k}) \cdot (\hat{i} + \hat{j}) = 0$$

$$\Rightarrow -\lambda + 3 - \lambda = 0 \Rightarrow \lambda = \frac{3}{2}\ \therefore\ \vec{a}_1 = \frac{3}{2}(\hat{i} + \hat{j})$$

32. **(c)** We have, $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$

$\vec{r} \cdot (3\hat{i} + 3\hat{j} - 5\hat{k}) = 3$

Direction ratio of line of intersection of two planes is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -3 \\ 3 & 3 & -5 \end{vmatrix} = -\hat{i} + \hat{j}$$

$$n_1 \times n_2 = (2\hat{i} + 2\hat{j} - 3\hat{k}) \times (3\hat{i} + 3\hat{j} - 5\hat{k})$$

Direction ratio of line

$$\vec{r} = 3\hat{i} + 2\hat{j} + \hat{k} + t(5\hat{i} + 5\hat{j} - 7\hat{k})$$

is $5\hat{i} + 5\hat{j} - 7\hat{k}$

Angle between these lines.

$$\therefore\ \theta = \cos^{-1}\frac{(-\hat{i} + \hat{j}) \cdot (5\hat{i} + 5\hat{j} - 7\hat{k})}{|-\hat{i} + \hat{j}||5\hat{i} + 5\hat{j}|} = \cos^{-1} 0$$

$$\theta = 90° = \frac{\pi}{2}$$

33. **(c)**

34. **(a)** We have,

Line $\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

and plane $\vec{r} \cdot (10\hat{i} + 2\hat{j} - 11\hat{k}) = 3$

Let angle between line and plane is θ, then

$$\sin\theta = \frac{(2\hat{i} + 3\hat{j} + 6\hat{k}) \cdot (10\hat{i} + 2\hat{j} - 11\hat{k})}{|2\hat{i} + 3\hat{j} + 6\hat{k}||10\hat{i} + 2\hat{j} - 11\hat{k}|}$$

$$\sin\theta = \frac{|20 + 6 - 66|}{\sqrt{4 + 9 + 36}\sqrt{100 + 4 + 121}}$$

$$\sin\theta = \frac{40}{7 \times 15} \quad \Rightarrow \sin\theta = \frac{8}{21} \Rightarrow \theta = \sin^{-1}\left(\frac{8}{21}\right)$$

35. **(a)** In a data, 15 observations are $x_1, x_2, x_3, \ldots, x_{15}$

Given, $\sum_{i=1}^{15} x_i^2 = 3600, \sum_{i=1}^{15} x_i = 175$

When the observation 20 is replaced by the observation 40 then

$$\sum_{i=1}^{15} x_i = 175 - 20 + 40 = 195$$

$$\sum_{i=1}^{15} x_i^2 = 3600 - (20)^2 + (40)^2 = 4800$$

$$\therefore\ \text{Corrected variance} = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$= \frac{4800}{15} - \left(\frac{195}{15}\right)^2 = 320 - 169 = 151$$

36. **(b)** Let mean $= \bar{x}$

Given, coefficient of variation, C.V. = 7.2

Variance, $\sigma^2 = 3.24$

$$\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100$$

$$7.2 = \frac{1.8}{\bar{x}} \times 100 \quad \Rightarrow \bar{x} = \frac{1.8}{7.2} \times 100 \quad \Rightarrow \bar{x} = 25$$

37. **(Bonus)** Five dice are thrown simultaneously $\Rightarrow n = 5$.

p = probability of show same numbered face

$$\therefore\ p = \frac{1}{6}, q = 1 - \frac{1}{6} = \frac{5}{6}$$

Probability that at least three of them show the same numbered face.

$$= 6(P(X \geq 3)) = 6[P(X = 3) + P(X = 4) + P(X = 5)]$$

$$= 6\left[{}^5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 + {}^5C_4\left(\frac{1}{6}\right)^4\frac{5}{6} + {}^5C_5\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^0\right]$$

$$= \frac{6}{6^5}(10 \times 25 + 25 + 1) = \frac{276}{6^4}$$

38. **(b)** Let A is the events sum appeared on two unbiased dice is 7 and B is the event sum appeared on two unbiased dice is either 7 or 11.

$$\therefore\ P(A) = \frac{6}{36}, P(B) = \frac{6}{36} + \frac{2}{36} = \frac{2}{9}$$

$\therefore$ Probability that 7 comes before 11,

$= P(A) + P(\bar{B}A) + P(\bar{B}\bar{B}A) + P(\bar{B}\bar{B}\bar{B}A) + \dots$

$= \frac{1}{6} + \frac{7}{9}\times\frac{1}{6} + \left(\frac{7}{9}\right)^2 \times \frac{1}{6} + \dots = \frac{1}{6}\left[1 + \frac{7}{9} + \left(\frac{7}{9}\right)^2 + \dots\right]$

$1 \cdot \frac{7}{9} \cdot \left(\frac{7}{9}\right)^2 + \dots$ follows G.P., so $S_\infty = \frac{a}{1-r}$

$= \frac{1}{6}\left(\frac{1}{1-\frac{7}{9}}\right) = \frac{1}{6}\times\frac{9}{2} = \frac{3}{4}$

39. **(d)** Let A is the event that the first mango is good and B is the event that second is good.

If one of them is good. Then probability that the other is also good,

$= P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)}$

$P(A) = \frac{^6C_2}{^{10}C_2} + \frac{^6C_1 \times {}^4C_1}{^{10}C_2}$ and $P(A\cap B) = \frac{^6C_2}{^{10}C_2}$

Now, $P\left(\frac{B}{A}\right) = \frac{}{^6C_2 + {}^6C_1 \times {}^4C_1}$

$= \frac{15}{15+24} = \frac{15}{39} = \frac{5}{13}$

40. **(d)** Let X = Absolute difference of two numbers

X	$P(X)$	P_iX_i
0	6/36	0
1	10/36	10/36
2	8/36	16/36
3	6/36	18/36
4	4/36	16/36
5	2/36	10/36

Mean $= \sum P_iX_i = \frac{0+10+16+18+16+10}{36}$

Mean $= \frac{70}{36} = \frac{35}{18}$

41. **(c)** Number of coins tossed, $n = 10$

Probability of getting head, $p = \frac{1}{2}, q = \frac{1}{2}$

$\therefore$ Required probability $= P(X \geq 6)$

$= P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= {}^{10}C_6\left(\frac{1}{2}\right)^{10} + {}^{10}C_7\left(\frac{1}{2}\right)^{10} + {}^{10}C_8\left(\frac{1}{2}\right)^{10} + {}^{10}C_9\left(\frac{1}{2}\right)^{10} + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$

$= \frac{1}{2^{10}}({}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + 1)$

$= \frac{1}{2^{10}}(210+120+45+10+1) = \frac{386}{2^{10}} = \frac{193}{2^9}$

42. **(b)** Two points $A(1, -2)$, $B(-5, 3)$ are given, then and area of triangle = 15 sq. unit

Let a point $C(x, y)$ which completes the triangle.

$$\Delta = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & -2 & 1 \\ -5 & 3 & 1 \end{vmatrix}$$

$15 = \frac{1}{2}\,|\,x(-2-3) - y(1+5) + 1(3-10)\,|$

$30 = |-5x - 6y - 7|$

$5x + 6y + 7 = \pm 30$

$5x + 6y + 37 = 0$ or $5x + 6y - 23 = 0$

Hence, point lies on

$(5x + 6y + 37)(5x + 6y - 23) = 0$

43. **(d)** Given rotated angled for axes is 30º.

Then, $x = x'\cos\theta - y'\sin\theta = x'\cos30^\circ - y'\sin30^\circ$,

$x = \frac{\sqrt{3}}{2}x' - \frac{y'}{2}$

$y = x'\sin\theta + y'\cos\theta$

$y = x'\left(\frac{1}{2}\right) + y'\left(\frac{\sqrt{3}}{2}\right)$

Satisfy in equation $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$

$\Rightarrow \frac{\sqrt{3}}{4}x'^2 + \frac{y'^2}{4} - \frac{\sqrt{3}}{2}x'y' + 2\sqrt{3}\left(\frac{\sqrt{3}}{4}x'^2 + \frac{3}{4}x'y' - \frac{x'y'}{2} - \frac{\sqrt{3}}{4}y'^2\right) - \left(\frac{x'^2}{4} + \frac{3}{4}y'^2 + \frac{\sqrt{3}}{2}x'y'\right) = 2a^2$

After solving the above equation, we get

$2x'^2 - 2y'^2 = 2a^2$

$x'^2 - y'^2 = 2a^2$

As $(x', y') \rightarrow (x, y)$

$x^2 - y^2 = 2a^2$

44. **(c)** Required equation of line L is $\frac{x}{a} + \frac{y}{b} = 1$

Replace x by $(x\cos\alpha - y\sin\alpha)$ and y by $(x\sin\alpha + y\cos\alpha)$.

$\Rightarrow \frac{1}{b}(x\sin\alpha + y\cos\alpha) + \frac{1}{a}(x\cos\alpha - y\sin\alpha) = 1$

Here, p and q are intercepts then the required coordinates are (p, 0) and (0, q).

$\frac{1}{p} = \frac{1}{a}\cos\alpha + \frac{1}{b}\sin\alpha$ ... (i)

$\frac{1}{q} = -\frac{1}{a}\sin\alpha + \frac{1}{b}\cos\alpha$ ... (ii)

After solving (i) & (ii), we get

$\frac{1}{p^2} + \frac{1}{q^2} = \frac{1}{a^2} + \frac{1}{b^2}$

45. **(c)** Let two points be $A(1, 2)$ and $B(3, 4)$. Then slope of $AB = \frac{4-2}{3-1} = 1$

Let the slope of line $l_1 = m_1$

$\therefore\ \tan 30° = \left|\dfrac{m_1 - 1}{1 + m_1}\right| \Rightarrow \pm\dfrac{1}{\sqrt{3}} = \dfrac{m_1 - 1}{1 + m_1}$

$m_1 + 1 = \pm\sqrt{3}(m_1 - 1)$

$m_1 + 1 = \sqrt{3}(m_1 - 1)$

$m_1(\sqrt{3} - 1) = \sqrt{3} + 1$

$m_1 = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = 2 + \sqrt{3}$

Similarly, we get the slope of line l_2,

$m_2 = 2 - \sqrt{3}\ \therefore\ \dfrac{m_1}{m_2} = \dfrac{2+\sqrt{3}}{2-\sqrt{3}} = 7 + 4\sqrt{3}$

46. **(b)** Here, OA = OB = OC

$OA^2 = OB^2$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

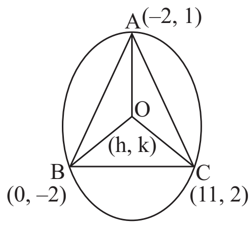
$(h + 2)^2 + (k - 1)^2$

$= (h - 0)^2 + (k + 2)^2$

$h^2 + 4 + 4h + k^2 + 1 - 2k$

$= h^2 + k^2 + 4 + 4k$

$4h + 1 - 2k - 4k = 0$



$4h - 6k + 1 = 0$ ... (i)

Similarly, $OB^2 = OC^2$, then $2h + 8k - 1 = 0$ ... (ii)

After solving (i) & (ii)

$O(h, k) \rightarrow \left(\dfrac{-1}{22}, \dfrac{3}{22}\right)$

Equation of line BC $\Rightarrow (y - y_1) = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$\Rightarrow (y + z) = \dfrac{2+z}{1-0}(x) \Rightarrow 4x - y - z = 0$

Distance of O(h, k) from the line $4x - y - z = 0$

$d = \left|\dfrac{ah + bk + c}{\sqrt{a^2 + b^2}}\right| \Rightarrow d = \left|\dfrac{4\left(\dfrac{-1}{22}\right) + (-1)\left(\dfrac{3}{22}\right) - 2}{\sqrt{17}}\right|$

$d = \dfrac{3\sqrt{17}}{22}$

47. **(c)** Given, $ax^2 + 2hxy + by^2 = 0$ …(i)

Equation of the triangle bisector of the line between positive coordinate axes is $y = x$.

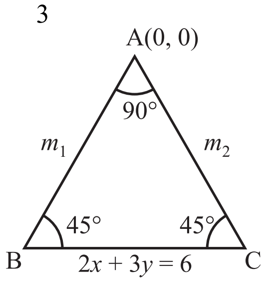
Substituting $y = x$ in eq. (i), we get

$ax^2 + 2hx^2 + bx^2 = 0;\quad a + 2h + b = 0$

$a + b = -2h \quad \therefore\ (a + b)^2 = 4h^2$

48. **(a)** Let the equation of line BC is $2x + 3y = 6$.

Slope of $BC = \dfrac{-2}{3}$



Let the slope of AB and AC are m_1 and m_2 respectively.

Angle between the lines AB and BC,

$\tan 45° = \left|\dfrac{m_1 + \dfrac{2}{3}}{1 - \dfrac{2}{3}m_1}\right| \Rightarrow 1 - \dfrac{2}{3}m_1 = m_1 + \dfrac{2}{3}$

$\Rightarrow m_1\left(1 + \dfrac{2}{3}\right) = 1 - \dfrac{2}{3} \Rightarrow m_1 = \dfrac{1}{5}$

$\because$ Line AB and AC are perpendicular to each other,

$\therefore\ m_2 = -5$

Equation of line AB

$\Rightarrow y = \dfrac{1}{5}x \Rightarrow 5y - x = 0$

Equation of line $AC \Rightarrow y = -5x \Rightarrow y + 5x = 0$

Combining both the equations,

$(5y - x)(y + 5x) = 0$

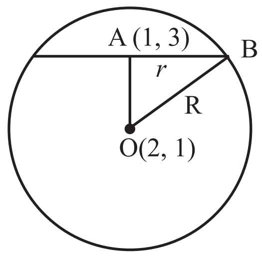
$\Rightarrow 5y^2 + 24xy - 5x^2 = 0 \Rightarrow 5x^2 - 24xy - 5y^2 = 0$

49. **(d)** Let the radius of smaller and bigger circles are r and R.

Equation of the circle,

$x^2 + y^2 - 2x - 6y + 6 = 0$

Centre (1, 3) and radius, $r = \sqrt{1 + 9 - 6} = 2$



$OA = \sqrt{(2-1)^2 + (1-3)^2}$

$\Rightarrow OA = \sqrt{1+4} = \sqrt{5}$

$\because AB = r = 2$ and $R^2 = r^2 + OA^2 = (2)^2 + (\sqrt{5})^2$

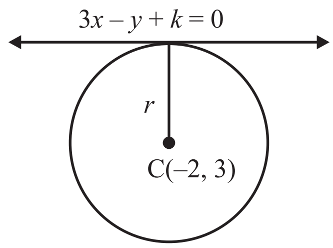
$\Rightarrow R^2 = 4 + 5 \quad \Rightarrow R = 3$

50. **(c)** Line $3x - y + k = 0$ touches the circle

$x^2 + y^2 + 4x - 6y + 3 = 0$ having

Centre (– 2, 3) and radius,

$r = \sqrt{4 + 9 - 3} = \sqrt{10}$



Distance between centre and line

$\because\ r = \left|\dfrac{-6 - 3 + k}{\sqrt{10}}\right|$

$10 = |-9 + k|$

$\Rightarrow \pm 10 = k - 9$

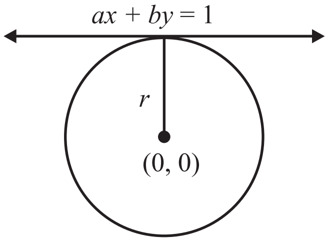
$\because k = -1, 19 \Rightarrow k_1 = -1, k_2 = 19$

Equation of chord of contact of the point $(k_1, k_2) = (-1, 19)$ to the circle

$-x + 19y + 2(x-1) - 3(y+19) + 3 = 0$

$\Rightarrow x + 16y - 56 = 0$

51. **(a)** Given that, $ax + by = 1$ is tangent of circle $x^2 + y^2 - r^2 = 0$



Radius of circle = Distance between the centre and line

$\therefore r = \dfrac{1}{\sqrt{a^2 + b^2}} \Rightarrow r^2 = \dfrac{1}{a^2 + b^2}$

$\Rightarrow a^2 + b^2 = \dfrac{1}{r^2}$

$r = 1$, (a, b) lies on the circle

$\therefore (a, b)$ lies on the circle when $S_1 = 0$

52. **(a)** Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Since, this circle is passes through (2, 0) and (– 2, 0), we get

$\therefore 4 + 4g + c = 0 \quad \text{...(i)}$

$4 - 4g + c = 0 \quad \text{...(ii)}$

On solving eqs. (i) and (ii), we get

$g = 0, c = -4$

$\therefore$ Equation of circle $x^2 + y^2 + 2fy - 4 = 0$

Now, $y = mx + c$ is tangent of circle

$\therefore \left|\dfrac{-f + c}{\sqrt{1 + m^2}}\right| = \sqrt{f^2 + 4} \Rightarrow (c - f)^2 = (1 + m^2)(f^2 + 4)$

$c^2 - 2cf + f^2 = f^2 + 4 + m^2f^2 + 4m^2$

$\Rightarrow m^2f^2 + 2cf + 4(1 + m^2) - c^2 = 0$

Let f_1 and f_2 are roots of equation,

$\therefore f_1 f_2 = \dfrac{4(1 + m^2) - c^2}{m^2}$

Now, equation of this circle are

$x^2 + y^2 + 2f_1y - 4 = 0$ and $x^2 + y^2 + 2f_2y - 4 = 0$

Since, both the circles are orthogonals.

$\therefore 2f_1 f_2 = c_1 + c_2$

$2f_1 f_2 = -8 \Rightarrow f_1 f_2 = -4$

$\dfrac{4(1 + m^2) - c^2}{m^2} = -4$

$4 + 4m^2 - c^2 = -4m^2 \Rightarrow c^2 = 4(1 + 2m^2)$

53. **(d)** We have $S \equiv x^2 + y^2 - 3x + y - 10 = 0$ and

$s' = x^2 + y^2 + x + 2y - 20 = 0$

Common chord $s - s' = 0$

$x^2 + y^2 - 3x + y - 10 - x^2 - y^2 + x - 2y + 20 = 0$

$-2x - y + 10 = 0$

$2x + y - 10 = 0$

Equation of circle passing through s and s' is

$s + \lambda L = 0$

$x^2 + y^2 - 3x + y - 10 + \lambda(2x + y - 10) = 0$

$x^2 + y^2 + x(2\lambda - 3) + y(1 + \lambda) - 10(1 + \lambda) = 0$ ... (i)

Centre of required circle is $\left(\dfrac{-(2\lambda - 3)}{2}, \dfrac{-1 + \lambda}{2}\right)$.

Satisfy the coordinate of centre in the equation

$za + y - 10 = 0$

$\Rightarrow z\left(\dfrac{-2\lambda + 3}{2}\right) + \left(\dfrac{1 + \lambda}{2}\right) - 10 = 0$

$\Rightarrow -4\lambda + 6 + 1 + \lambda - 20 = 0$

$\Rightarrow -5\lambda - 15 = 0$

$\lambda = \dfrac{-15}{5} = -3$

From (i),

$x^2 + y^2 + x(2\lambda - 3) + y(1 + \lambda) - 10(1 + \lambda) = 0$

$x^2 + y^2 + x(-6 - 3) + y(-2) - 10(-2) = 0$

$x^2 + y^2 - 9x - 2y + 20 = 0$

So, option (d) is correct.

54. **(c)** **I.** $x = ly^2 + my + n$

$\Rightarrow y^2 + \dfrac{m}{1}y = \dfrac{x}{1} - \dfrac{n}{1}$

$\Rightarrow y^2 + \dfrac{m}{1}y + \dfrac{m^2}{4l^2} - \dfrac{x}{1} - \dfrac{x}{1} + \dfrac{m^2}{4l^2}$

$\Rightarrow \left(y + \dfrac{m}{2l}\right)^2 = \dfrac{1}{l}\left[x - \left(x - \dfrac{m^2}{4l}\right)\right]$

Vertex $= \left(n - \dfrac{m^2}{4l}, \dfrac{-m}{2l}\right)$

Statement I is true.

II. We have,

$y = lx^2 + mx + n = \dfrac{y}{l} - \dfrac{n}{l} = x^2 + \dfrac{mx}{l}$

$\dfrac{y}{l} - \dfrac{n}{l} + \dfrac{m^2}{nl^2} = \left(x + \dfrac{m}{2l}\right)^2$

$\dfrac{1}{l}\left[y - \left(n - \dfrac{m^2}{4l}\right)\right] = \left(x + \dfrac{m}{2l}\right)^2$

Focus $= \left(\dfrac{-m}{2l}, n - \dfrac{m^2}{4l} + \dfrac{1}{4l}\right)$

Option (c) is true.

55. **(a)** As we know, length of the normal chord of parabola. $y^2 = 4ax$ at $(at^2, 2at)$ = $8a\sqrt{t^2+1}$ and length of focal

chord = $a\left(t+\frac{1}{t}\right)^2$

Point P (3, 6) lies on the parabola $y^2 = 12x$

$\therefore$ Here $a = 3, t = 1$

Now, $l_1 = 8\times 3\sqrt{1+1} = 24\sqrt{2}$

$l_2 = 3(1+1)^2 = 12$

$\frac{l_1}{l_2} = \frac{24\sqrt{2}}{12} = 2\sqrt{2}$

56. **(a)** Ellipse, $\frac{x^2}{27}+\frac{y^2}{1} = 1$

Equation of tangent at $(3\sqrt{3}\cos\theta, \sin\theta)$ on the ellipse,

$$\frac{3\sqrt{3}x\cos\theta}{27}+\frac{y\sin\theta}{1} = 1$$

$$\frac{x}{3\sqrt{3}}\cos\theta + y\sin\theta = 1$$

Sum of intercepts on the coordinate axes made by tangent

i.e., $L = 3\sqrt{3}\sec\theta + \text{cosec}\,\theta$

$\because \quad \frac{dL}{d\theta} = 3\sqrt{3}\sec\theta\tan\theta - \text{cosec}\,\theta\cot\theta$

For maxima or minima $\frac{dL}{d\theta} = 0$

$3\sqrt{3}\sec\theta\tan\theta - \text{cosec}\,\theta\cot\theta = 0$

$$\frac{3\sqrt{3}\sin\theta}{\cos^2\theta} = \frac{\cos\theta}{\sin^2\theta}$$

$$\tan^3\theta = \frac{1}{3\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$$

Minimum at $\theta = \frac{\pi}{6}$

57. **(b)** A line cuts the circle $x^2 + y^2 = 9$ at A $(3\cos\alpha, 3\sin\alpha)$

Equation of tangent at A is $x\cos\alpha + y\sin\alpha = 3$

Slope = $-\cot\alpha$.

Similarly, cut the elipse $4x^2 + 9y^2 = 36$ at B is $(3\cos\alpha, 2\sin\alpha)$.

Equation of tangent at B is

$2\cos\alpha + 35\sin\alpha = 6$

Slope = $\frac{-2}{3}\cot\alpha$

Angle between tangents is θ, then

$$\tan\theta = \frac{\left(-\frac{2}{3}+1\right)\cot\alpha}{1+\frac{2}{3}\cot^2\alpha} = \frac{\cot\alpha}{3+2\cot^2\alpha}$$

Let $\tan\theta = z$

$$\Rightarrow z = \frac{\cot\alpha}{3+2\cot^2\alpha}$$

$$\frac{dz}{d\alpha} = \left[\frac{(3+2\cot^2\alpha)(\text{cosec}^2\alpha) - \cot\alpha(4\cot\alpha\,\text{cosec}^2\alpha)}{-(3+2\cot^2\alpha)}\right]$$

For maxima or minima $\frac{dz}{d\alpha} = 0$

$\therefore 3\,\text{cosec}^2\alpha + 2\cot^2\alpha\,\text{cosec}^2\alpha - 4\cot^2\alpha\,\text{cosec}^2\alpha = 0$

$\Rightarrow \cos^2\alpha = \frac{3}{2} \Rightarrow \cos\alpha = \sqrt{\frac{3}{2}}$

$$\tan\theta = \frac{\sqrt{\frac{3}{2}}}{3+2\times\frac{3}{2}} = \sqrt{\frac{3}{2}}\times\frac{1}{6} = \frac{1}{2\sqrt{6}}$$

58. **(b)** Equation of ellipse, $\frac{x^2}{16}+\frac{y^2}{25} = 1$

Foci of the ellipse are (0, ± 3)

Given $e_1e_2 = 1$

Eccentricity of ellipse

$\Rightarrow e_1 = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$

Let equation of hyperbola $\frac{y^2}{b^2}-\frac{x^2}{a^2} = 1$ and it passes through (0, ± 3).

$\therefore b^2 = 9$

Eccentricity of hyperbola,

$e_2 = \sqrt{1+\frac{a^2}{b^2}}$

$e_2^2 = 1+\frac{a^2}{b^2}$

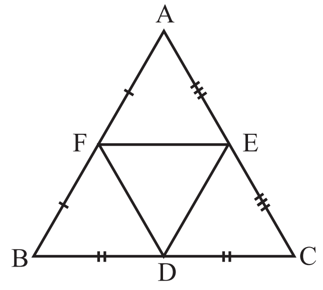
$\frac{1}{e_1^2} = 1+\frac{a^2}{9} \Rightarrow \frac{25}{9} = 1+\frac{a^2}{9}$ $\quad [\because e_1e_2 = 1]$

$a^2 = 16$

Hence, the equation of hyperbola is $\frac{y^2}{9}-\frac{x^2}{16} = 1$.

59. **(b)** Given that, (1, 0, 3), (2, 1, 5), (– 2, 3, 6) are the midpoint of the sides of triangle.

As we know that, centroid of ΔABC is also the centroid of ΔDEF.



$\therefore$ Centroid, $G = \left(\frac{1+2-2}{3}, \frac{0+1+3}{3}, \frac{3+5+6}{3}\right)$

$= \left(\frac{1}{3}, \frac{4}{3}, \frac{14}{3}\right)$

60. **(b)** Let the direction ratio of normal to the plane P is (a, b, c).

$\because$ Equation of another given plane $x + y = 3$ having direction ratio of normal = (1, 1, 0)

Angle between two planes,

$$\therefore \cos\frac{\pi}{4} = \frac{(a,b,c)\cdot(1,1,0)}{\sqrt{a^2+b^2+c^2}\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{a+b}{\sqrt{a^2+b^2+c^2}\sqrt{2}}$$

$\Rightarrow (a+b)^2 = a^2+b^2+c^2 \Rightarrow 2ab = c^2$

Option (b) satisfies the above equations

$\therefore a = 1, b = 1, c = \sqrt{2}$

61. **(a)** Let the equation of plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$$\therefore 6 = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{36}$$

Centroid of plane is $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$.

Let $x = \frac{a}{3} \Rightarrow a = 3x$

$b = 3y, c = 3z$

From (i),

$$\frac{1}{9x^2}+\frac{1}{9y^2}+\frac{1}{9z^2}=\frac{1}{36} \Rightarrow \frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{4}$$

62. **(b)** Let $L = \lim_{x\to\infty}\left(\frac{2x^2+3x+4}{x^2-3x+5}\right)^{\frac{3|x|+1}{2|x|-1}}$

$$= \lim_{x\to\infty}\left[\frac{x^2\left(2+\frac{3}{x}+\frac{4}{x^2}\right)}{x^2\left(1-\frac{3}{x}+\frac{5}{x^2}\right)}\right]^{\frac{|x|\left(3+\frac{1}{|x|}\right)}{|x|\left(2-\frac{1}{|x|}\right)}}$$

$$= \lim_{x\to\infty}\left(\frac{2+\frac{3}{x}+\frac{4}{x^2}}{1-\frac{3}{x}+\frac{5}{x^2}}\right)^{\frac{3+\frac{1}{|x|}}{2-\frac{1}{|x|}}} = \left(\frac{2}{1}\right)^{\frac{3}{2}} = 2\sqrt{2}$$

63. **(a)** Given that,

$$f(x) = \begin{cases} \cos 2x, & -\infty < x < 0 \\ e^{3x}, & 0 \le x < 3 \\ x^2-4x+3, & 3 \le x \le 6 \\ \frac{\log(15x-89)}{x-6}, & x > 6 \end{cases}$$

At $x = 3$,

$\because$ LHL $= \lim_{x\to 3^-} e^{3x} = e^9$ and

RHL $= \lim_{x\to 3^+} x^2-4x+3 = 9-12+3 = 0$

Clearly, $\lim_{x\to 3^-} f(x) \ne \lim_{x\to 3^+} f(x)$

$\therefore f(x)$ is discontinuous at $x = 3$.

Now, at $a = 3$

$$\lim_{x\to 3}\frac{x^2-9}{x^3-5x^2+9x-9} = \lim_{x\to 3}\frac{(x-3)(x+3)}{(x-3)(x^2-2x+3)}$$

$$= \lim_{x\to 3}\frac{x+3}{x^2-2x+3} = \frac{3+3}{(3)^2-2(3)+3} = \frac{6}{6} = 1$$

64. **(d)** We have $y = (x+1)(x^2+1)(x^4+1)(x^8+1)$

$y = (x^6+x^2+x^4+1)(x^9+x+x^8+1)$

Differentiate w.r.t. x

$$\frac{dy}{dx} = (6x^5+4x^3+2x)(x^9+x+x^8+1)(x^6+x^2+x^4+1)(9x^8+8x^7+1)$$

Take limit $x \to -1$

$$\lim_{x\to -1}\frac{dy}{dx} = (-6-4-2)(-1+1-1+1)+(1+1+1+1)(9-8+1)$$

$$\lim_{x\to -1}\frac{dy}{dx} = 0+4(2) = 8$$

65. **(c)** Given that,

$h(x) = (f(x))^2 + (g(x))^2$

Differentiating w.r.t. x,

$h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$

$h'(x) = 2f(x)g(x) + 2g(x)f''(x)$ $\quad[\because f'(x) = g(x)$ and $f''(x) = g'(x)]$

$h'(x) = 2f(x)g(x) - 2g(x)f(x)$ $\quad[\because f''(x) = -f'(x)]$

$h'(x) = 0$ represents $h(x)$ = Constant function

Here $h(1) = 2 \quad \therefore h(2) = 2$.

66. **(d)** Given

$$y = \sqrt{x+\sqrt{y+\sqrt{x+\sqrt{y+\ldots\infty}}}}$$

$$y = \sqrt{x+\sqrt{y+y}} \Rightarrow y^2 = x+\sqrt{2y}$$

$$y^2 - x = \sqrt{2y} \Rightarrow (y^2-x)^2 = 2y$$

$y^4 - 2xy^2 + x^2 = 2y$

$y^4 - 2xy^2 - 2y + x^2 = 0$

On differentiating w.r.t. to x, we get

$$4y^3\frac{dy}{dx} - 2y^2 - 4yx\frac{dy}{dx} - \frac{2dy}{dx} + 2x = 0$$

$$\frac{dy}{dx}(4y^3-4xy-2) = -2x+2y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2-x}{2y^3-2xy-1}$$

67. **(c)** We have $xy = 1$ and $x^2 + 8y = 0$

Slope of the curve $xy = 1$

$$x\frac{dy}{dx}+y = 0 \Rightarrow m_1 = \frac{dy}{dx} = \frac{-y}{x} \quad \ldots (i)$$

Slope of the curve $2x + \frac{8dy}{dx} = 0$

$m_2 = \frac{dy}{dx} = \frac{-2x}{8} = \frac{-x}{4}$ ... (ii)

From the curves, $x^2 + 8y = 0$

$\Rightarrow x^2 + \frac{8}{x} = 0$

$\Rightarrow x^3 + 8 = 0 \Rightarrow (x^3) + (2)^3 = 0$

$(x + 2)(x^2 + 4 + 2x) = 0$

$x + 2 = 0 \Rightarrow x = -2$

From curve $xy = 1 \Rightarrow y = \frac{-1}{2}$

$(x, y) \to \left(-2, \frac{-1}{2}\right)$

$m_1 = \frac{-\left(\frac{-1}{2}\right)}{(-2)} = \frac{-1}{4}$

$m_2 = \frac{2}{4} = \frac{1}{2}$

$\tan\theta = \left|\frac{\frac{1}{2} + \frac{1}{4}}{1 - \frac{1}{8}}\right| = \frac{6}{7}$

68. **(a)** Given that,

$x = t^2 - 7t + 7, y = t^2 - 4t - 10$

Put $x = 1$

$1 = t^2 - 7t + 7 \Rightarrow t^2 - 7t + 6 = 0$

$\Rightarrow (t - 6)(t - 1) = 0 \Rightarrow t = 1, 6$

Put $y = 2$

$2 = t^2 - 4t - 10 \Rightarrow t^2 - 4t - 12 = 0$

$(t - 6)(t + 2) = 0 \Rightarrow t = -2, 6$

Hence, $t = 6$ satisfies both equations.

Now, $\frac{dx}{dt} = 2t - 7$ and $\frac{dy}{dt} = 2t - 4$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t-4}{2t-7}$

$\left(\frac{dy}{dx}\right)_{t=6} = \frac{2(6)-4}{2(6)-7}$

$\frac{dy}{dx} = \frac{8}{5}$

69. **(c)** Given $f(x) = 2x^3 - 3x^2 - x + 1$

Let $g(x) = \int f(x)\,dx = \frac{x^4}{2} - x^3 - \frac{x^2}{2} + x$

$g(-1) = \frac{1}{2} + 1 - \frac{1}{2} - 1 = 0$ and $g(0) = 0$

$\therefore I_1 = [-1, 0], f(x) = 0$ has roots

Similarly, $g(0) = g(1) = 0$

$I_2 = [0, 1]$ interval, $f(x)$ has roots.

$g(2) = \frac{16}{2} - 8 - \frac{4}{2} + 2 = 8 - 8 - 2 + 2 = 0$

$g(1) = g(2) = 0$

$I_3 = [1, 2], f(x)$ has again roots.

But, $g(-2) \neq 0$

$\therefore f(x) = 0$ has roots in every interval except $I_4 = [-2, -1]$.

70. **(a)** Given, $f(x) = \int_x^{x+1} e^{-t^2}\,dt$

Apply Lebnitz rule,

$\frac{d}{dx}\int_{g(x)}^{h(x)} l(t)\,dt = l(h(x))\,h'(x) - l(g(x))\,g'(x)$

$\Rightarrow f'(x) = e^{-(x+1)^2} - e^{-x^2}$

$f'(x) = \frac{1}{e^{(x+1)^2}} - \frac{1}{e^{x^2}}$

Since, $f(x)$ is decreasing function.

$\therefore f'(x) < 0$

$\therefore \frac{1}{e^{(x+1)^2}} - \frac{1}{e^{x^2}} < 0 \Rightarrow \frac{1}{e^{(x+1)^2}} < \frac{1}{e^{x^2}}$

$\Rightarrow e^{x^2} < e^{(x+1)^2}$

$x^2 < (x + 1)^2 \Rightarrow x^2 < x^2 + 2x + 1$

$2x + 1 > 0 \Rightarrow x > -\frac{1}{2} \quad \therefore x \in \left(-\frac{1}{2}, \infty\right)$

71. **(a)** We have

$$\int \frac{\cos x + x}{1 + \sin x}dx = f(x) + \int \frac{3\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}dx + C$$

Let $I = \int \frac{\cos x}{1 + \sin x}dx + \int \frac{x}{1 + \sin x}dx$

$$I = \int \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}dx + \int x\sec^2 dx - \int x\sec x\tan x\,dx$$

$$I = \int \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}dx + \left[\int x\tan x - x\sec x + \int \frac{(1 - \sin x).dx}{\cos x}\right] + C$$

$$I = x\tan x - x\sec x + \int \frac{\left(-\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\cos\frac{x}{2} + \sin\frac{x}{2}}dx + \frac{3\cos\frac{x}{2} - \sin\frac{x}{2}.dx}{\cos\frac{x}{2} + \sin\frac{x}{2}} + C$$

$$\Rightarrow I = x\tan x - x\sec x - x + \int \frac{3\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}.dx + C$$

$$\Rightarrow I=\left(\frac{\sin x-1-\cos x}{\cos x}\right)+\int\frac{3\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}.dx+C$$

$$\Rightarrow I=\frac{-2x\cos\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}+\int\frac{3\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}.dx+C$$

$$\Rightarrow I=\frac{-2x}{1+\tan\frac{x}{2}}+\int\frac{3\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}.dx+C$$

72. **(b)** $I=\int\frac{\sqrt{\cos 2x}}{\sin x}.dx \Rightarrow I=\int\frac{\sqrt{(2\cos^2 x-1)}}{\sin x}.dx$

$\Rightarrow I=\int\sqrt{\cot^2 x-1}.dx$

Let cotx = secθ

$$dx=\frac{\sec\theta}{1+\cos^2\theta}.d\theta$$

$$I=-\int\frac{\tan\theta\sin\theta}{\cos^2\theta+1}.d\theta=-\int\frac{\sin^2\theta}{\cos\theta(\cos^2\theta+1)}.d\theta$$

Let $\sin\theta=t\Rightarrow d\theta=\frac{1}{\sqrt{1-t^2}}dt$

$$I=-\int\frac{t^2}{(1-t^2)(2-t^2)}.dt\ =-\int\frac{t^2}{(t^2-1)(t^2-2)}.dt$$

$$=-\int\left(\frac{2}{t^2-2}-\frac{1}{t^2-1}\right).dt$$

$$=-\int\left(\frac{2}{t^2-2}\right).dt+\int\left(\frac{1}{t^2-1}\right).dt$$

$$=-\left(2.\frac{1}{2\sqrt2}\log\left|\frac{t-\sqrt2}{t+\sqrt2}\right|\right)+\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+c$$

$$=-\left(\frac{1}{\sqrt2}\log\left|\frac{\sin\theta-\sqrt2}{\sin\theta+\sqrt2}\right|\right)+\frac{1}{2}\log\left|\frac{\sin\theta-1}{\sin\theta+1}\right|+c$$

$$=\frac{1}{\sqrt2}\log\left|\frac{\sqrt2+\sqrt{1-\tan^2 x}}{\sqrt2-\sqrt{1-\tan^2 x}}\right|-\frac{1}{2}\log\left|\frac{1+\sqrt{1-\tan^2 x}}{1-\sqrt{1-\tan^2 x}}\right|+c$$

73. **(a)** Given that,

$$I=\int\frac{2x+3}{x(x+1)(x+2)(x+3)+1}dx$$

$$=-\frac{1}{px^2+qx+r}+c$$

$$I=\int\frac{2x+3}{(x^2+3x)(x^2+3x+2)+1}dx$$

Let $x^2+3x=t\quad\Rightarrow (2x+3)\,dx=dt$

$$\therefore\ I=\int\frac{dt}{t(t+2)+1}=\int\frac{dt}{t^2+2t+1}$$

$$I=\int\frac{dt}{(t+1)^2}=\frac{-1}{t+1}+c$$

$$I=\frac{-1}{x^2+3x+1}+c$$

$\therefore\ p=1,\ q=3,\ r=1$

Now, $\frac{3p-q}{r}=\frac{3-3}{1}=0$

74. **(c)** Let $I=\int(\log x)^2\,dx$

$$I=(\log x)^2\int dx-\int\left[\frac{d(\log x)^2}{dx}\int dx\cdot dx\right]+c$$

$$I=x(\log x)^2-\int 2\log x\,dx+c$$

$$I=x(\log x)^2-2(x\log x-x)+c$$

$$I=x(\log x)^2-2x\log x+2x+c$$

$$I=x(\log x)^2-2x(\log x-1)+c$$

75. **(a)** Given that,

$$I=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2+k^2}=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{k/n}{1+\left(\frac{k}{n}\right)^2}$$

Let $\frac{k}{n}=x\ \Rightarrow\ I=\int_0^1\frac{x}{1+x^2}dx=\frac{1}{2}\int_0^1\frac{2x}{1+x^2}dx$

$$=\frac{1}{2}[\log(1+x^2)]_0^1=\frac{1}{2}\log 2$$

76. **(c)** Let $I=\int_0^{10}(5-\sqrt{10x-x^2})\,dx$

$$I=\int_0^{10}(5-\sqrt{(5)^2-(x-5)^2}\,dx$$

$$I=\left[5x-\frac{x-5}{2}\sqrt{10x-x^2}-\frac{25}{2}\sin^{-1}\frac{x-5}{5}\right]_0^{10}$$

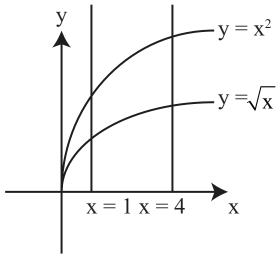
$$I=\left(50-0-\frac{25}{2}\sin^{-1}1\right)-\left(0-0-\frac{25}{2}\sin^{-1}(-1)\right)$$

$$I=50-\frac{25}{2}\times\frac{\pi}{2}-\frac{25}{2}\times\frac{\pi}{2}\ =50-\frac{25\pi}{4}-\frac{25\pi}{4}$$

$$I=50-\frac{25\pi}{2}\quad\Rightarrow I=\frac{1}{2}(100-25\pi)$$

77. **(b)** We have curve $y=\sqrt{x}$, $x=\sqrt{y}$

Required are a $=\int_1^4(x^2-\sqrt{x})\,dx$



$$= \frac{x^3}{3}\Big|_1^4 - \frac{2}{3}x^{\frac{3}{2}}\Big|_1^4$$

$$= \frac{1}{3}\left[(4)^3 - (1)^3\right] - \frac{2}{3}\times\left((2)^{2\times\frac{3}{2}} - 1\right)$$

$$= \frac{1}{3}(64-1) - \frac{2}{3}\times 7 = \frac{63}{3} - \frac{14}{3} = 21 - \frac{14}{3} = \frac{49}{3}$$

78. **(c)** Equation of family of circle of constant radius r is,

$(x - a)^2 + (y - b)^2 = r^2$

Let $x - a = r\cos\theta$, $y - b = r\sin\theta$

$$\frac{dx}{d\theta} = -r\sin\theta,\ \frac{dy}{d\theta} = r\cos\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\cot\theta$$

$$\frac{d^2y}{dx^2} = \text{cosec}^2\theta\frac{d\theta}{dx} = \frac{-\text{cosec}^3\theta}{r}$$

$$\frac{d^2y}{dx^2} = \frac{-(1+\cot^2\theta)^{3/2}}{r} \qquad (\because \text{cosec}^2\theta = 1 + \cot^2\theta)$$

$ry'' = -(1 + (y')^2)^{3/2}$

Squaring on both sides, we get

$r^2(y'')^2 = (1 + (y')^2)^3$

79. **(b)** We have $(2x - 3y + 5)\,dx + (9y - 6x - 7)\,dy = 0$

$$\frac{dy}{dx} = \frac{-(2x-3y+5)}{(9y-6x-7)} = \frac{(2x-3y+5)}{[3(2x-3y)+7]}$$

Let $2x - 3y = t$

$$\Rightarrow 2 - \frac{3dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{3}\left(2 - \frac{dt}{dx}\right) = \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{3}\left(2 - \frac{dt}{dx}\right) = \frac{(t+5)}{(3t+7)}$$

$$\Rightarrow \frac{2}{3}(3t+7) - \frac{1}{3}(3t+7)\frac{dt}{dx} = t+5$$

$$\Rightarrow t + \frac{14}{3} - 5 = \left(t + \frac{7}{3}\right)\frac{dt}{dx}$$

$$\Rightarrow \int dx = \int\frac{\left(t+\frac{7}{3}\right)}{\left(t-\frac{1}{3}\right)}dt$$

On integration, we get

$\Rightarrow (3x - 9y) + 8\log|6x - 9y - 1| = c$

80. **(a)** Given that

$$\sqrt{1-y^2}\,dx + x\,dy - \sin^{-1}y\,dy = 0$$

$$\sqrt{1-y^2}\,dx + x\,dy = \sin^{-1}y\,dy$$

$$\frac{dx}{dy} + \frac{x}{\sqrt{1-y^2}} = \frac{\sin^{-1}y}{\sqrt{1-y^2}}$$

Here, $P(y) = \dfrac{1}{\sqrt{1-y^2}}$, $Q(y) = \dfrac{\sin^{-1}y}{\sqrt{1-y^2}}$

Integrating factor, IF $= e^{\int\frac{dy}{\sqrt{1-y^2}}dx} = e^{\sin^{-1}y}$

Solution of given differential equation,

$$xe^{\sin^{-1}y} = \int e^{\sin^{-1}y}\frac{\sin^{-1}y}{\sqrt{1-y^2}}dy$$

$$xe^{\sin^{-1}y} = e^{\sin^{-1}y}(\sin^{-1}y - 1) + c$$

$$x = \sin^{-1}y - 1 + ce^{-\sin^{-1}y}$$

PHYSICS

81. **(a)** The four fundamental forces of nature are:

(i) **Strong nuclear force** the strong nuclear force is very strong but very short ranged. Its strength is of the order of 1. It act between nuclear particle like quarks.

(ii) **Weak nuclear force** its strength is low and of the order of 10^{-13} and short ranged. This force is responsible for radioactive decay.

(iii) **Electromagnetic force** act between the electrically charged particles. Its strength is of the order of 10^{-2}. But its range is infinite.

(iv) **Gravitational force** is the force of attraction acting between two pieces of matter of the universe. Its strength is weak of the order of 10^{-39}. But it's range is infinite.

82. **(b, d)** In statement (b), the percentage error in case of 1 m length is:

$$\frac{\Delta l_1}{l_1}\times 100 = \frac{0.01}{1}\times 100 = 1\%$$

while in case of 0.5 m length, it is

$$\frac{\Delta l_2}{l_2}\times 100 = \frac{0.01}{0.5}\times 100 = 2\%$$

As percentage error in case of 0.5 m length is greater. So, accuracy is less as compared to that of 1 m length. Hence this statements is incorrect.

In statement (d), according to the result of rounding off, if the number to be rounded off is 5, then the preceding digit remains unchanged if it is even or increased by 1, if it is odd. So, the correct result is 2.44. Hence, this is also an incorrect statement.

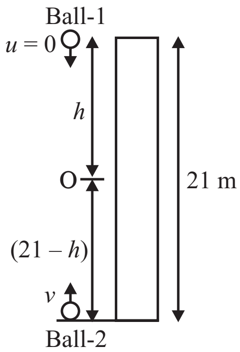
83. (a) For downward motion of ball-1

Using $h = ut + \frac{1}{2}gt^2$

$\Rightarrow h = 0 \times t + \frac{1}{2} \times 10t^2$

$[\because u = 0]$

$\Rightarrow h = 5t^2$ ...(i)



For upward motion of ball-2,

$h = ut - \frac{1}{2}gt^2$

Given, speed of ball-2,

$v = 14$ m/s, acceleration due to gravity, $g = 10$ m/s^2

Substituting these values, we get

$21 - h = 14t - \frac{1}{2} \times 10t^2$

$\Rightarrow 21 - h = 14t - 5t^2$ ...(ii)

Adding Eq. (i) and (ii), we get

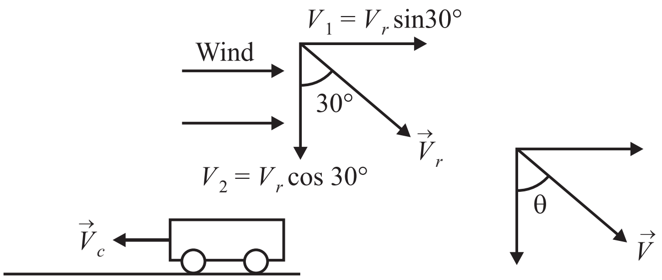
$21 = 14t \Rightarrow t = \frac{3}{2}$ s $= 1.5$ s

$\therefore$ From Eq. (i), we get

$h = 5 \times (1.5)^2 = 11.25$ m $= \frac{45}{4}$ m

Therefore, the ball-1 will have dropped $\frac{45}{4}$ m when it passes ball-2.

84. (b) $\vec{V}_{r,c} = \vec{V}_r - \vec{V}_c = V_1\hat{i} - V_2\hat{j} + V_c\hat{i} = (V_1 + V_c)\hat{i} - V_2\hat{j}$



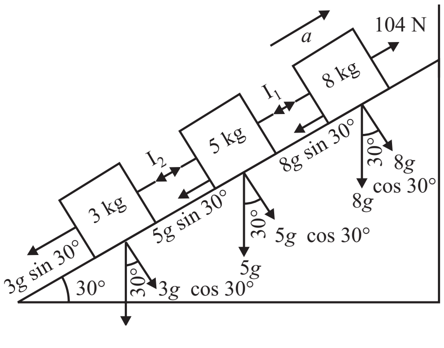
$\tan\theta = \frac{V_1 + V_c}{4} = \frac{V_r \sin 30 + 40}{V_r \cos 30}$

$= \frac{20 + 40}{20\sqrt{3}} = \sqrt{3}$ $[\because V_r = 40$ m/s$]$

So, $\theta = \tan^{-1}(\sqrt{3}) = 60°$

85. (d)

86. (d) The given situation is shown in the figure below,



For 8 kg block.

$104 - T_1 - 8g \sin 30° = 8a$

$\Rightarrow 104 - T_1 - 8 \times 10 \times \frac{1}{2} = 8a$ $[\because g = 10$ m/s$^2]$

$64 - T_1 = 8a$ ...(i)

For 5 kg block

$T_1 - T_2 - 5g \sin 30° = 5a$

$T_1 - T_2 - 25 = 5a$ ...(ii)

for 3 kg block

$T_2 - 3g \sin 30° = 3a$

$T_2 - 15 = 3a$ ...(iii)

Adding Eqs. (i), (ii) and (iii), we get

$24 = 16a \Rightarrow a = 1.5$ m/s^2

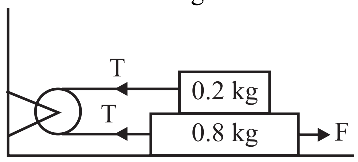
Hence, acceleration a of block is 1.5 m/s^2.

87. (a) Given,

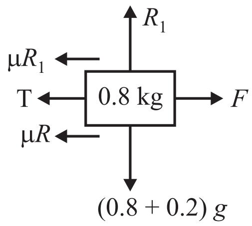
Acceleration of block, $a = 5$ m/s^2

Coefficient of friction between two blocks and table, $\mu = 0.1$

Let tension in the string be T.



Force acting on the 0.8 kg block are shown in the figure.



$\therefore$ Equation of motion for 0.8 kg block,

$F - T - \mu R - \mu R_1 = 0.8a$

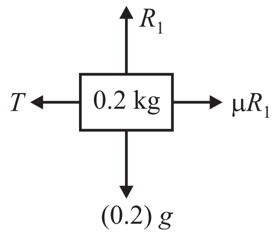
putting the given values, we get

$F - T - 0.1(0.8 + 0.2)g - 0.1 \times 0.2g = 0.8a$

$\Rightarrow F - T - 0.1 \times 10 - 0.1 \times 0.2 \times 10 = 0.8 \times 5$

$\Rightarrow F - T = 5.2$ ...(i)

Free body diagram for 0.2 kg body.



For 0.2 kg block, friction force due to 0.8 kg block is in opposite direction of tension force.

So, $T - \mu R_1 = 0.2a$

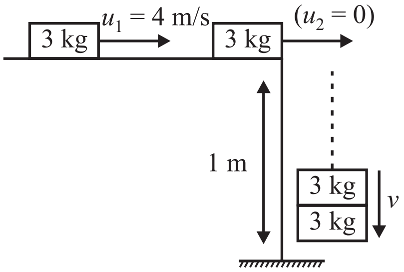
$T - 0.1 \times 0.2g = 0.2 \times 5$, $T - 0.02 \times 10 = 1.0$, $T = 1.2$ ...(ii)

From Eqs. (i) and (ii), we get

$F - 1.2 = 5.2$

$F = 6.4$ N

88. **(d)** The given situation can be shown as below.



By law of conservation of momentum, total momentum before collision = total momentum after collision.

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$m_1 = m_2 = 3$ kg

$3 \times 4 + 3 \times 0 = (3 + 3)\, v$

$\Rightarrow v = \frac{12}{6} = 2\ \text{m/s}^2$

Thus, the two bodies move with the velocity of 2 m/s.

Applying law of conservation of energy.

$KE_1 + PE_1 = KE_2 + PE_2$

$\frac{1}{2}(m_1 + m_2)v^2 + (m_1 + m_2)gh = KE_2$

$\Rightarrow KE_2 = \frac{1}{2} \times 6 \times 4 + 6 \times 10 \times 1 = 12 + 60 = 72\text{J}$

Hence, the kinetic energy just before the boxes strike the floor is 72 J.

89. **(a)** $\mathbf{v} = \mathbf{u} - gt$

$\mathbf{v} = (20\hat{\mathbf{i}} + 24\hat{\mathbf{j}}) - (10\hat{\mathbf{j}})$

$= 20\hat{\mathbf{i}} + 24\hat{\mathbf{j}} - 80\hat{\mathbf{j}} = 20\hat{\mathbf{i}} + 56\hat{\mathbf{j}}$ …(i)

From the law of conservation of energy, change in potential energy of the ball = change in kinetic energy of the ball

$\Rightarrow \Delta PE = \frac{1}{2}m\mathbf{u}^2 - \frac{1}{2}m\mathbf{v}^2$

$= \frac{1}{2}m(\mathbf{u}^2 - \mathbf{v}^2) = \frac{2}{2}(\mathbf{u}\cdot\mathbf{u} - \mathbf{v}\cdot\mathbf{v})$

$= [(20\hat{\mathbf{i}} + 24\hat{\mathbf{j}})\cdot(20\hat{\mathbf{i}} + 24\hat{\mathbf{j}})]$

$- [(20\hat{\mathbf{i}} - 56\hat{\mathbf{j}})\cdot(20\hat{\mathbf{i}} - 56\hat{\mathbf{j}})]$

$= (20)^2 + (24)^2 - (20)^2 - (56)^2$

$= 576 - 3136 = -2560 = -2.56$ kJ

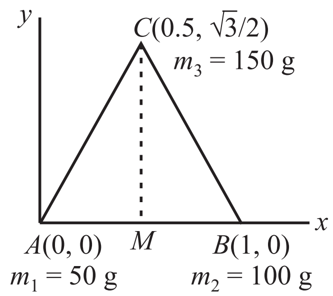
Hence, change in potential energy of the ball after $t = 85$ s is -2.56 kJ.

90. **(None)** Given,

mass of ball A, $m_1 = 50$ g

mass of ball B, $m_2 = 100$ g

mass of ball C, $m_3 = 150$ g



From the triangle ACB,

$\because\ y_3 = \sqrt{AC^2 - AM^2} = \sqrt{1 - (0.5)^2}$

$= \sqrt{0.75} = \frac{\sqrt{3}}{2}$ $[\because AM = x_3 = 0.5]$

Coordinates of centre of mass

$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$= \frac{50 \times 0 + 100 \times 1 + 150 \times 0.5}{50 + 100 + 150} = \frac{175}{300} = \frac{7}{12}$

and $y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$

$= \frac{50 \times 0 + 100 \times 0 + 150 \times \frac{\sqrt{3}}{2}}{50 + 100 + 150} = \frac{75\sqrt{3}}{300} = \frac{\sqrt{3}}{4}$

The coordinates of centre of the mass of this system of balls is $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$. So, no option given is correct.

91. **(d)** Velocity of body rolling down on inclined plane is given by

$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$, where v = velocity of body when it reaches the ground.

where, h = height of inclined plane, R = radius of body, and k = radius of gyration

Find a ring, $k^2 = R^2$

$\therefore$ Speed of the ring, $v_R = \sqrt{\frac{2gh}{1 + \frac{R^2}{R^2}}} = \sqrt{gh}$ …(i)

For a solid sphere, $k^2 = \frac{2R^2}{5}$

$\therefore$ Speed of the solid sphere, $v_s = \sqrt{\frac{2gh}{1 + \frac{2}{5}}}$

$\Rightarrow \sqrt{\frac{10gh}{7}} = 1.19\sqrt{gh}$ …(ii)

For a solid disc, $k^2 = \frac{R^2}{2}$

$\therefore$ Speed of the solid disc, $v_D = \sqrt{\dfrac{2gh}{1+\dfrac{1}{2}}}$

$\Rightarrow \sqrt{\dfrac{4gh}{3}} = 1.15\sqrt{gh}$ ...(iii)

From Eqs. (i), (ii) and (iii), it is clear that $V_S > V_D > V_R$.

92. **(c)** For spring mass system.

Time period $T_1 = 2\pi\sqrt{\dfrac{m}{k}}$

Here, m = mass of body

and k = force constant of the spring

For simple pendulum,

$T_2 = 2\pi\sqrt{\dfrac{l}{g}}$

According to the question

Time period, $T_1 = T_2$

$\therefore \quad 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{l}{g}} \Rightarrow \sqrt{\dfrac{m}{k}} = \sqrt{\dfrac{l}{10}}$ ...(i)

[$\because g = 10\ \text{m/s}^2$]

As time period of spring mass system is independent of g, no effect occurs on the time period of spring mass system, when put in an elevator going downwards with an acceleration 5 m/s^2.

i.e., $T'_1 = T_1 = 2\pi\sqrt{\dfrac{m}{k}}$

Acceleration of the pendulum when elevator is accelerating downwards with a m/s^2.

$T'_2 = 2\pi\sqrt{\dfrac{l}{g-a}} = 2\pi\sqrt{\dfrac{l}{10-5}} = 2\pi\sqrt{\dfrac{l}{5}} = 10-5$

$\therefore \quad \dfrac{T'_1}{T'_2} = \dfrac{2\pi\sqrt{\dfrac{m}{k}}}{2\pi\sqrt{\dfrac{l}{5}}} = \dfrac{\sqrt{\dfrac{l}{10}}}{\sqrt{\dfrac{l}{5}}}$ [From Eq. (i)]

$\therefore \quad \dfrac{T'_1}{T'_2} = \dfrac{1}{\sqrt{2}}$

93. **(c)** Escape velocity of a particle from the surface of a planet is given by

$v_e = \sqrt{2gR}$

where, g = acceleration due to gravity,

and R = radius of the planet

Given, acceleration due to gravity at equator,

$g_E = \dfrac{1}{3}g_P \Rightarrow g_P = 3g_E$ ...(i)

Escape velocity of a particle at equator,

$\therefore \; v_E = \sqrt{2g_E R}$ ...(ii)

At poles,

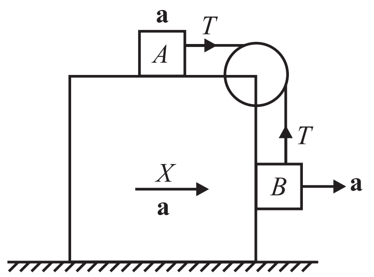
$v_P = \sqrt{2g_P R}$

$= \sqrt{2\times 3g_E R} = \sqrt{3}\sqrt{2g_E R}$ [From Eq. (i)]

$= \sqrt{3}v_E = \sqrt{3}v$ [From Eq. (ii)] [$\because v_E = V$]

Hence, $\sqrt{3}v$ is the escape velocity for a particle at the pole of this planet.

94. **(a)** Block X moves with acceleration a such that block A and B remains stationary. Block B is stationary only when, Tension force on block B is equal to friction force on B.



i.e., $T = \mu R$

$T = \mu ma$ ...(i) [$\because R = ma$]

Block A is stationary only when friction force on block A is equal to the sum of tension force on block A and applied force on block A.

$\mu R = T + ma$

$\mu mg = \mu ma + ma$ [From Eq. (i)]

$\mu g = \mu a + a$

$\mu g = a(\mu + 1)$

$a = \dfrac{\mu g}{\mu+1} = \dfrac{0.5g}{0.5+1} = \dfrac{0.5g}{1.5} = \dfrac{g}{3} \quad \therefore \; a = \dfrac{g}{3}$

Thus, $g/3$ should be its minimum acceleration such that blocks A and B remains stationary.

95. **(c)** Given, Bulk modulus of water, $B = 2\times 10^9$ Pa

Bulk modulus, $B = \dfrac{-pV}{\Delta V} \Rightarrow p = \dfrac{-\Delta V}{V}B$

Given, $\dfrac{\Delta V}{V} = -0.4\% = \dfrac{0.4}{100}$

$\therefore$ Pressure required, $p = B\left(-\dfrac{\Delta V}{V}\right)$

$= \dfrac{0.4}{100}\times 2\times 10^9\ \text{Pa} = \dfrac{0.4}{100}\times\dfrac{2\times 10^9}{101325}$ atm

= 79 atm ≈ 80 atm

96. **(c)**

97. **(b)** Given, Change in area,

$\dfrac{\Delta A}{A} = \dfrac{0.4}{100} = 0.004$

Increase in temperature, $\Delta T = 100°\text{C}$

Coefficient of areal expansion, $\beta = \left(\dfrac{\Delta A}{A}\right)\cdot\dfrac{1}{\Delta T}$

$= \dfrac{0.004}{100} = 4\times10^{-5}/°C$

$\therefore$ Coefficient of linear expansion, of the coin is

$\alpha = \dfrac{\beta}{2} = \dfrac{4\times10^{-5}}{2}/°C = 2\times10^{-5}/°C$

98. (c) Given, specific heat of water,

$C_{water} = 4200$ J/kg°C $= 4.2 \times 10^3$ J/kg°C

Change in temperature of water,

$\Delta T = T_2 - T_1 = 100 - 25 = 75°C$

Mass of water, $m = 100$ g $= 10^{-1}$ kg

Power of heater, $\rho = \dfrac{Q}{t}$

$\Rightarrow Q = \rho \times t = 210t$ ...(i)

Heat, $Q = mc\Delta T$

$\therefore 210t = mc\Delta T$ [Using Eqs. (i)]

$\Rightarrow 210t = 10^{-1} \times 4.2 \times 10^3 \times 75$

$\Rightarrow t = \dfrac{10^{-1}\times4.2\times10^3\times75}{210} = 150s$

99. (d) Given, initial temperature of 1 mole of N_2 gas

$T_0 = 300$ K

initial pressure of gas, $p_1 = p$

and final pressure of gas, $p_2 = 10\,p$

For an adiabatic process, $P^{1-\gamma}T^{\gamma}$ = Constant

$\therefore p_1^{1-\gamma}\, T_1^{\gamma} = p_2^{1-\gamma} \cdot T_2^{\gamma}$

$\Rightarrow \left(\dfrac{p_1}{p_2}\right)^{1-\gamma} = \left(\dfrac{T_2}{T_1}\right)^{\gamma} \Rightarrow \left(\dfrac{p}{10p}\right)^{1-\gamma} = \left(\dfrac{T_2}{300}\right)^{\gamma}$

$\Rightarrow 10^{\gamma-1} = \dfrac{T_2^{\gamma}}{300^{\gamma}} \Rightarrow T_2^{\gamma} = 10^{\gamma-1} \times 300^{\gamma}$

$\Rightarrow T_2 = 10^{\gamma} \times 300 = 10^{1\frac{1}{\gamma}} \times 300$

$= 10^{1-\frac{1}{7/5}} \times 300$ $\left[\because \text{For diatomic, } \gamma = \dfrac{7}{5}\right]$

$= 10^{2/7} \times 300 = (100)^{-7} \times 300$

$= 1.9 \times 300$ $[\because 100^{1/7} = 1.9]$

$= 570$ K

Therefore, the final gas temperature after compression is 570 K.

100. (c) For monatomic gas A,

Atomic mass of molecules of gas, $m_A = 4u$

Temperature of gas, $T_A = 27°C$

For diatomic gas B,

Atomic mass of molecules of gas,

$m_B = 2 \times 20u = 40u$

Temperature of gas $= T_B$

RMS speed of gas molecules, $v_{rms} = \sqrt{\dfrac{3RT}{m}}$

We have given, $(v_{rms})_A = (v_{rms})_B$

$\therefore \sqrt{\dfrac{3RT_A}{m_A}} = \sqrt{\dfrac{3RT_B}{m_B}} \Rightarrow \dfrac{T_A}{m_A} = \dfrac{T_B}{m_B}$

$\Rightarrow \dfrac{27}{4u} = \dfrac{T_B}{40u} \Rightarrow T_B = 270°C$

101. (b) As there are 9 nodes in between fix ends.

So, $L = (g+1)\dfrac{\lambda}{2} \Rightarrow L = 5\lambda$ as $f = \dfrac{v}{\lambda}$

$f = \dfrac{v}{L/5} = \dfrac{32}{16} \times 5 = 10$ Hz

102. (a) Here, let frequencies of horn A and B are n_A and n_B, respectively. So beat frequency $= n_A - n_B = \dfrac{50}{10}$

$n_A - n_B = 5$ beats

When the horn B is blown while moving towards the wall the apparent frequency.

$n'_B = n_B \dfrac{v + v_0}{v - v_s}$

Since, both the observer and source are moving with truck.

So, $v_s = v_0 = 10$ m/s

$n'_B = n_B\left(\dfrac{330+10}{330-10}\right)$ $n'_B = n_B\, 1.0625$

$= n'_B - n_B = 0.0625 n_B$

As given $n'_B - n_B = 5$

$0.0625\, n_B = 5 \Rightarrow n_B = 80$ Hz

$\because$ Given that if frequency of A is decreased then beat frequency is increases.

So, $n_B > n_A$

$\therefore n_A = n_B - 5 = 80 - 5 \Rightarrow n_A = 75$ Hz

103. (c) The intensity transmitted, with pass axis 45° to y-axis (i.e., 45° to x-axis also) is given by Brewster's law as shown below

$I = I_x \cos^2 45° + I_y \cos^2 45°$

$= I_0 \cos^2 45° + \dfrac{2I_0}{3}\cos^2 45°$

$= I_0 \cdot \dfrac{1}{2} + \dfrac{2I_0}{3}\cdot\dfrac{1}{2} = \dfrac{I_0}{2} + \dfrac{I_0}{3} = \dfrac{5I_0}{6}$

104. (b) $x_m = x_n$

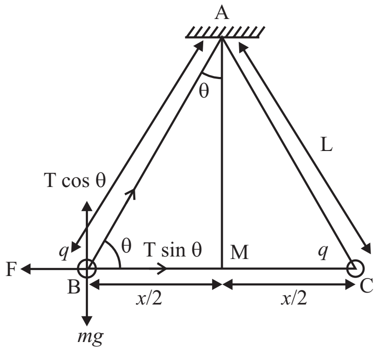
$\dfrac{m\lambda_m D}{d} = \dfrac{n\lambda_n D}{d}$ $\left[\because x_n = \dfrac{n\lambda_n D}{d}\right]$

where, x_n is position of n^{th} maxima

$\Rightarrow \dfrac{m}{n} = \dfrac{\lambda_n}{\lambda_m} = \dfrac{600\times10^{-9}}{400\times10^{-9}}$ $\therefore \dfrac{m}{n} = \dfrac{3}{2}$

So, least integral values of m and n satisfying above requirement are $m = 3$ and $n = 2$.

105. (b)



At point B,

$T\cos\theta = mg$ …(i)

$T\sin\theta = F$

$$T\sin\theta = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{x^2} \quad \text{…(ii)}$$

From Eqn. (i) and (ii), we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{x^2}}{mg}$$

$$\therefore\ mg\tan\theta = 9\times10^9\cdot\frac{q^2}{x^2} \quad \text{…(iii)}$$

From ΔABM,

$$\tan\theta = \frac{\frac{x}{2}}{\sqrt{L^2-\left(\frac{x}{2}\right)^2}} = \frac{x}{\sqrt{4L^2-x^2}} = \frac{x}{2L}\ (\because L >> x)$$

$\therefore$ From Eq. (iii), we get

$$mg\cdot\frac{x}{2L} = \frac{9\times10^9\times q^2}{x^2}$$

Putting the given values, we get

$$2\times10^{-2}\times10\times\frac{x}{2\times3} = \frac{9\times10^9\times10^{-20}}{x^2}$$

$$\Rightarrow x^3 = \frac{27}{10^{10}}\text{ m} \Rightarrow x = \frac{3}{10^3\times10^{1/3}} = \frac{3}{10^{1/3}}\text{ mm}$$

106. (c) Poisson's equation as given below

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}, \text{ where } V = \text{potential} \quad \text{…(i)}$$

Hence, from Eq. (i), we get

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon_0} \qquad \left[\because \frac{d^2V}{dy^2} = \frac{d^2V}{dz^2} = 0\right]$$

Integrate on both the sides, we get

$$\Rightarrow \int\frac{d^2V}{dx^2}dx = -\int\frac{\rho}{\varepsilon_0}dx \Rightarrow \frac{dV}{dx} = \frac{-\rho x}{\varepsilon_0} - A$$

Integrate on both sides again, we get

$$\Rightarrow \int\frac{dV}{dx}dx = \int\frac{-\rho x}{\varepsilon_0}dx - \int A\,dx$$

$$V = \frac{-\rho x^2}{2\varepsilon_0} - Ax - B \ \therefore\ V = -\left(\frac{+\rho x^2}{2\varepsilon_0} + Ax + B\right]$$

107. (c) The resistivity of a metal is given by

$$\rho = \frac{m}{ne^2\tau} \Rightarrow \rho \propto \frac{1}{\tau} \text{ and } T \propto \frac{1}{\tau}$$

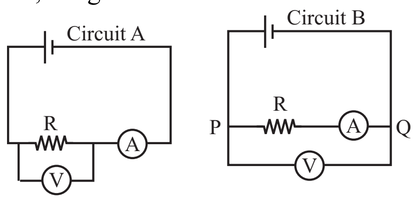
[Because as temperature increases collision frequency increases]

So, on τ decreases, resistivity '*s*' increases.

108. (b) Circuits A represents an ammeter, which is used to measured the current it is a low resistive device, 80 that it does not affect the circuit. It is connected in series with resistor.

Similarly, *V* represents a voltmeter, which is used to measure the potential drop. It is usually a high resistive device, so that the negligible current passes through it. Thus, does not affect current passing through the resistor. It is connected in parallel with to the resistor. This implies that, these devices are used measure. Current and voltage drop across a resistor, without actually affecting any of these quantities because of their addition in the circuit.

So, the given circuits A and B as shown below.

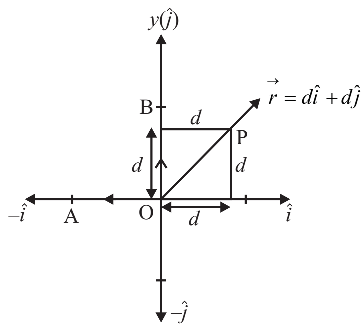


We can conclude that the circuit A has been correctly connected. Therefore, it can be used to measure *R* accurately, which can be calculated with the help of ammeter and voltmeter reading as,

$$R = \frac{\text{voltage (V)}}{\text{current (I)}} \quad \text{[From Ohm's law]}$$

However, when the arrangement of ammeter and voltmeter is as per given in circuit B. The circuit then is usually used for measuring higher resistance as compare to circuit A.

109. (b)



The magnetic field due to the wire along $-\hat{i}$ direction at P is $\vec{\mathbf{B}}_1 = \frac{\mu_0}{2\pi}\cdot\frac{I}{d}(-\hat{k})$

Similarly, magnetic field due to wire along $\hat{j}$ direction at P is $\vec{\mathbf{B}}_2 = \frac{\mu_0}{2\pi}\cdot\frac{I}{d}(-\hat{k})$

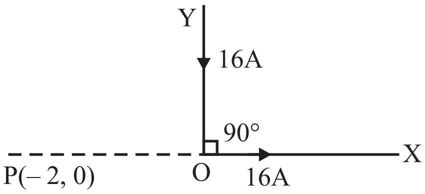
$\therefore$ Resultant magnetic field at point P due to both the wires,

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = 2\cdot\frac{\mu_0}{2\pi}\cdot\frac{I}{d}(-\hat{k})\,;\quad \vec{\mathbf{B}} = \frac{\mu_0}{\pi}\cdot\frac{I}{d}(-\hat{k})$$

$\therefore$ Force on the charged particles,

$$\vec{\mathbf{F}} = q(\vec{v}\times\vec{\mathbf{B}}) = q\left[v\hat{i}\times\frac{\mu_0}{\pi}\frac{I}{d}\cdot(-\hat{k})\right] = \frac{\mu_0 Iqv}{\pi d}\cdot\hat{j}$$

110. (b) A long straight wire is bent at 90° as shown in the figure.



Magnitude of the magnetic field at point P due to current carrying wire along x-axis is zero because point P lies in the direction of conducting wire along x-axis.

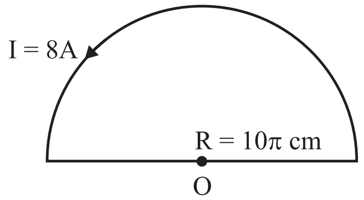
$\therefore$ Magnitude of the magnetic field due to current carrying wire along y-axis at point P is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}$$

Putting the given values in above relation, we get

$$= \frac{\mu_0}{4\pi}\times\frac{16}{2\times10^{-3}} = 10^{-7}\times8\times10^{3} = 0.8\times10^{-3}\,\text{T} = 0.8\,\text{mT}$$

111. (a) According to the question,



$\because$ Magnetic field by semi-circular current wire at centre O.

$$B = \frac{\mu_0 I}{4R} \Rightarrow B = \frac{4\pi\times10^{-7}\times8}{4\times10\pi\times10^{-2}} \Rightarrow B = 8\times10^{-6}\,\text{T}$$

$\therefore$ Magnitude of the force on thin wire per unit length.

$f = IB$, where $f = \frac{F}{L}$

Putting the given values,

$= 8\times8\times10^{-6} = 64\times10^{-6}$ N/m $= 64\ \mu$N/m

112. (c) When magnetic field is suddenly removed, then some charge flows through the galvanometer.

As $q = I\cdot\Delta t$

$$\Rightarrow q = \frac{E}{R_{eq}}\Delta t = \frac{\Delta\phi}{\Delta t}\cdot\frac{\Delta t}{R_{eq}} \qquad \left[\because E = N\frac{\Delta\phi}{\Delta t}\right]$$

So, for N turns, $q = \frac{N}{R_{eq}}\Delta\phi$

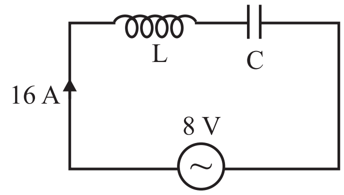
$$360\times10^{-6} = \frac{180}{(618+10)}\cdot(BA-0)$$

$$2\times10^{-6}\times628 = B\cdot\pi\times(2\times10^{-2})^2$$

$$\Rightarrow B = \frac{2\times10^{-6}\times628}{314\times4\times10^{-4}} = 1\,\text{T}$$

Hence, the magnetic field is 1 T.

113. (c)



$\therefore$ Impedance of the circuit,

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{8}{16}$$

$Z = 0.5\ \Omega$

When the inductor coil is connected to a 6 V DC battery then $V^0_{\text{rms}} = 6\text{ V} = V_{DC}$

$\therefore$ Magnitude of steady current flowing through inductor

$$I^0 = \frac{V^0_{\text{rms}}}{Z} = \frac{6}{0.5} = 12\text{ A}$$

114. (a) Wavelength of EM wave,

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^6}$$

$\lambda = 100$ m

Velocity of electromagnetic (EM) into non-magnetic material.

$$v = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times10^8}{\sqrt{16}} = \frac{3}{4}\times10^8\text{ m/s}$$

$$\left[\because v = \frac{c}{\sqrt{\mu_r\varepsilon_r}}\text{ and }\mu_r = 1\right]$$

$\therefore$ Wavelength, $\lambda' = \frac{v}{f} = \frac{\frac{3}{4}\times10^8}{3\times10^6} = 25$ m

$\therefore$ Change in wavelength $= \lambda' - \lambda$

$= 25 - 100 = -75$ m

115. (d) As $\lambda = \frac{h}{\sqrt{2mE}}$ $\quad\therefore\quad \lambda\propto\frac{1}{\sqrt{mE}}$ …(i)

$$\frac{\lambda_i}{\lambda_f} = \frac{\sqrt{2m \cdot 2E}}{\sqrt{mE}}$$

$$\frac{\lambda_i}{\lambda_f} = 2 \quad \Rightarrow \lambda_f = \frac{\lambda_i}{2}$$

116. (d) The energy required to excite a hydrogen atom from ground state ($n = 1$) to first excited state ($n = 2$) is

$\Delta E = E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV

When an electron collides with hydrogen atom in the ground state with energy of 5.5 eV which is less than 10.2 eV, then it will not be able to excite hydrogen atom into first excited state. Therefore, electron will not give any energy to the hydrogen atom. Hence, the total kinetic energy of electron remains conserved. So, collision is elastic.

117. (a) $(N_0)_A = (N_0)_B$ ...(i)

Half life of element A, $(t_{1/2})_A = 10$ year

and half life of element B, $(t_{1/2})_B = 20$ year

After time t, remaining the amount of element A,

$$N_A = \frac{1}{e} \text{ kg}$$

and remaining amount of element B,

$N_B = 1$ kg

For A,

$$N_A = (N_0)_A \left(\frac{1}{2}\right)^{\frac{t}{(t_{1/2})_A}}$$

$$\frac{1}{e} = (N_0)_A \left(\frac{1}{2}\right)^{\frac{t}{10}} \quad \text{...(ii)}$$

For B,

$$N_B = (N_0)_B \left(\frac{1}{2}\right)^{\frac{t}{20}}$$

$$1 = (N_0)_B \left(\frac{1}{2}\right)^{\frac{t}{20}} \quad \text{...(iii)}$$

Dividing Eqs. (ii) and (iii),

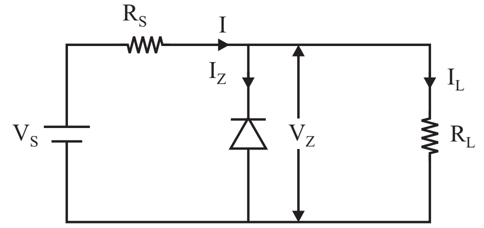
$$\frac{1}{e} = \frac{\left(\frac{1}{2}\right)^{\frac{t}{10}}}{\left(\frac{1}{2}\right)^{\frac{t}{20}}} = \left(\frac{1}{2}\right)^{\frac{t}{20}}$$

Taking log on the both sides, we get

$$\log \frac{1}{e} = \frac{t}{20} \log \frac{1}{2}$$

$$t = \frac{20}{\log 2} = \frac{20}{0.7} = \frac{200}{7} \text{ years} \qquad [\because \log 2 = 0.7]$$

118. (c)



The current through load resistance is

$$I_L = \frac{\text{voltage } (V_Z)}{\text{resistance } (R_L)} = \frac{10}{2 \times 10^3} = 5 \times 10^{-3} \text{ A} \quad \text{...(i)}$$

The current through in the resistance in series

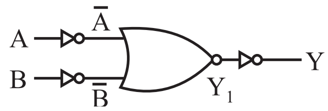
$I = I_Z + I_L = 5I_L + I_L = 6I_L = 6 \times 5 \times 10^{-3} = 3 \times 10^{-2}$ A

Also, $I = \dfrac{V_S - V_Z}{R_S} \Rightarrow R_S = \dfrac{V_S - V_Z}{I}$ ...(ii)

Substituting the values of Eqs. (ii), we get

$$R_S = \frac{16 - 10}{3 \times 10^{-2}} = \frac{6}{3 \times 10^{-2}} = 200\ \Omega$$

119. (b)



$Y_1 = \overline{\overline{A} + \overline{B}}$

$Y = \overline{Y_1} = \overline{A} + \overline{B} = \overline{A \cdot B}$

As $Y = \overline{A \cdot B}$, so the logic circuit, is NAND gate.

120. (d) We know that, the modulation index is given by

$$\mu = \frac{A_m}{A_c} \quad \text{...(i)}$$

where, A_m = maximum amplitude of message signal and A_c = maximum amplitude of carrier-signal.

Given, A_m and A_c are increased by 0.1% and 0.3%, then the modulation index becomes.

$$\mu_f = \frac{A_m + 0.001A_m}{A_c + 0.003A_c} = \frac{1001A_m}{1003A_c} = \frac{1001}{1003}\mu$$

[From Eq. (i)]

$$\Rightarrow \frac{\mu_f}{\mu} = \frac{1001}{1003}$$

The percentage change in modulation index,

$$= \frac{\mu_f - \mu}{\mu} \times 100 = \frac{1001 - 1003}{1003} \times 100$$

$$= -\frac{2}{1003} \times 100 \simeq -0.2\%$$

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121. (c) $E_n = -2.18 \times 10^{-18} \cdot \frac{Z^2}{n^2}$

For hydrogen, $Z = 1$

Therefore, $E_3 = -2.18 \times 10^{-18} \times \frac{1}{3^2}$

$= -\frac{2.18 \times 10^{-18}}{9} = -0.242 \times 10^{-18}$

and $E_\infty = -2.18 \times 10^{-18} \times \frac{1}{\infty}$

$E_\infty = 0$

Hence, $E_\infty = 0$ and $E_3 = -0.242 \times 10^{-18}$ J

122. (d) A– (III), B – (I), C – (V), D – (II)

(A) **For nodes:** $|\psi^2|$ is zero because $|\psi^2|$ represents the region where probability of electron finding is zero.

(B) **Subsidiary quantum Number:** It is a three dimensional shape of the orbital. It is also called azimuthal quantum number (l). The values of (l) gives three dimensional shapes of orbitals.

(C) **White light:** When white light is passed through a prism which causes dispersion of light and gives continuous spectrum. When energy waves are so close to each other they appear as continuous spectrum.

(D) **Heisenberg's uncertainty principle:** Significant only for motion of microscopic objects, because, according to this principle, it is impossible to determine simultaneously, the exact position and exact momentum (or velocity) of an electron.

123. (a) IUPAC name of element with $Z = 120$ is unbinilium. It is a hypothetical element.

Electronic configuration: 2, 8, 18, 32, 32, 18, 8, 2.

Thus, it is expected to belong to group 2 of s-block, in periodic table as it has two electrons in its outermost subshell.

124. (a) Common oxidation state of f-block elements is (+)3, but Eu and Tb can also show (+)2 and (+)4 oxidation states respectively.

125. (b) According to molecular orbital theory, species with all paired electrons in their electronic configuration are diamagnetic in nature.

$He_2^+ : \sigma 1s^2\ \sigma^* 1s^1$

$H_2 : \sigma 1s^2$

$H_2^+ : \sigma 1s^1$

$H_2^- : \sigma 1s^2\ \sigma^* 1s^1$

$He : \sigma 1s^2\ \sigma^* 1s^2$

Hence, among the given species, only H_2 and He which are diamagnetic and thus, option (b) is the correct answer.

126. (d) More be the electronegativity difference between H-atom and the atom bonded with H-atom, more stronger is the H-bonding.

Since, electronegativity difference between H-atom and F-atom is maximum. Therefore, option (d) is the correct answer.

127. (c) $\because$ Kinetic energy (KE) $= \frac{3}{2}RT$

$\therefore RT = \frac{2}{3}\text{KE}$ ...(i)

Given, KE = 3741.3 J

Most probable velocity $(\mu_{mp}) = \sqrt{\frac{2RT}{M}}$ ...(ii)

Substituting the value of RT from Eq. (i), we have,

$\mu_{mp} = \sqrt{\frac{2 \times 2(\text{KE})}{3 \times \text{M}}} = \sqrt{\frac{2 \times 2 \times 3741.3}{3 \times 32} \times 1000}$

$= \sqrt{155.88 \times 1000} = \sqrt{155887}$ J kg^{-1}

Hence, option (c) is the correct answer.

128. (c) According to vander Waal's equation, the correction term of pressure is $\frac{an^2}{V^2}$ which represents the intermolecular interaction that causes the non-ideal behaviour.

129. (b) The combustion reaction for methane is as follows:

	CH_4	+ $2O_2$ ⟶	CO_2	+ $2H_2O$
Initial moles	10	50	0	0
Final moles	0	(50 – 20)= 30	10	20

[$\because$ According to balanced equation, ratio of moles of O_2, H_2O and CO_2 are 3 : 2 : 1 respectively]

Hence, final moles of O_2 = 30 moles, H_2O = 20 moles

CO_2 = 10 moles

130. (d) The disproportionation reaction for MnO_4^{2-} occurs as follows:

$$3\overset{+6}{Mn}O_4^{2-} + 4H^+ \longrightarrow \overset{+4}{Mn}O_2 + 2\overset{+7}{Mn}O_4^- + 2H_2O$$

Hence, MnO_4^{2-} is oxidised to MnO_4^- and reduced to MnO_2. Thus, oxidation state of Mn when MnO_4^{2-} undergoes disproportionation reaction under acidic medium are + 7 and + 4.

$\therefore$ Hence, option (d) is the correct answer.

131. (d) Given,

Initial temperature = 0°C, Final temperature = 30°C

Thus, ΔT = 30°C

Mass of ice (m) = 10 g

Enthalpy of fusion (L_f) = 333.5 J g^{-1}

C_p of water = 4.18 J g^{-1} K^{-1}

$\because$ Heat required $(Q) = mL + mC_p\Delta T$

= 10 × 333.5 + 10 × 4.18 × 30

= 3335.0 + 1254.00 = 4589 J = 4.59 kJ

Hence, option (d) is the correct answer.

132. (a) For the reaction,

$0.5C(s) + 0.5CO_2(g) \rightleftharpoons CO(g)$

If 50% conversion of $CO_2(g)$ takes place.

We have concentration of $CO_2(g) = [0.25]$

$\because$ K_P is calculated only for gaseous species, thus

$$K_p = \frac{[\text{Product}]_{(g)}}{[\text{Reactant}]_{(g)}} = \frac{[1]}{[0.25]} = 4$$

Hence, option (a) is the correct answer.

133. (a) $Ca_2(PO_4)_3 \rightarrow \underset{2s}{2Ca^{2+}} + \underset{3s}{3PO_4^{3-}}$

$K_{sp} = [Ca^{2+}]^2 [PO_4^{3-}]^3 = [2s]^2[3s]^3 = 108\, s^5$

In $Ca_2(PO_4)_3$

$K_{sp} = 108\, s^5$ and option (a) is the correct answer.

134. (a) The balanced equation between reaction of $KMnO_4$ and H_2O_2 in acidic medium is as follows:

$$2KMnO_4 + 5H_2O_2 + 3H_2SO_4 \longrightarrow 5O_2 + 2MnSO_4 + K_2SO_4 + 8H_2O$$

$$\text{Normality of } H_2O_2 = \frac{\text{Volume strength}}{5.6} = \frac{10}{5.6}$$

According to law of equivalence

$N_1V_1\ (KMnO_4) = N_2V_2\ (H_2O_2)$

$$0.02 \times 5 \times 1000 \text{ mL} = \frac{10}{5.6} \times V_2$$

$$V_2 = \frac{0.02 \times 5 \times 1000 \times 5.6}{10} = 56 \text{ mL}$$

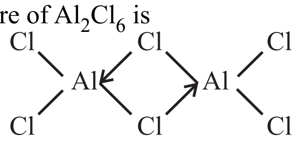
Hence, option (a) is correct.

135. (d) On hydrolysis of NaO_2, (a superoxide of sodium), it gives O_2, H_2O_2 and NaOH as products, i.e.

$2NaO_2 + 2H_2O \rightarrow 2NaOH + O_2 + H_2O_2$

Hence, option (d) is the correct answer.

136. (b) Structure of Al_2Cl_6 is



Therefore,

(i) Oxidation state (x) of 'Al' in Al_2Cl_6:

$= 2x - 6 = 0$ or $x = (+)3$

(ii) Coordination number of Al in Al_2Cl_6 is four (4), it is bonded to 4 chlorine atoms in Al_2Cl_6.

(iii) Number of valence electrons around Al in $Al_2Cl_6 = 2 \times 4 = 8$

Hence, option (b) is the correct answer.

137. (a) $\because$ Graphite is the most stable state of carbon and its ΔH_f° is considered as zero ($\because$ has more van der Waals' force) also ΔH_f° for $C_{60} > \Delta H_f^\circ$ for diamond. ΔH_f° values of graphite, diamond and C_{60} are 0, 1.9 and 38.1 kJ mol^{-1} respectively.

138. (d) $\underset{\text{Teeth enamel}}{Ca_{10}(PO_4)_6(OH)_2} + 2F^- \rightarrow \underset{\text{Fluoro apatite}}{Ca_{10}(PO_4)_6F_2} + 2OH^-$

$Ca_{10}(PO_4)_6F_2$ can be rewritten as:

$3[Ca_3(PO_4)_2]CaF_2$ or $3[Ca_3(PO_4)_2 \cdot CaF_2]$

Hence, option (d) is the correct answer.

139. (d) (a) **Kjeldahl method** can be used for estimation of nitrogen in an organic sample.

(b) **Dumas method** can be used for estimation of nitrogen in an organic sample.

(c) **Lassaigne method** can be used for estimation of N, S, P and halogens in an organic sample.

(d) **Carius method** can be used to find out the percentage composition of halogen present in an organic compound using Carius tube.

Hence, option (d) is the correct answer.

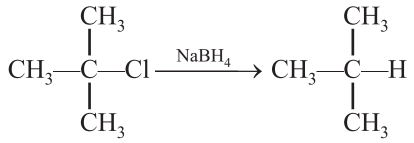
140. (c) (I) $R{-}CO\overset{-}{O}\overset{+}{N}a + NaOH \xrightarrow[\Delta \text{ Decarboxylation}]{NaOH + CaO} \underset{\text{(Alkane)}}{(R{-}H)}$

(II) $CH_3{-}\underset{\underset{O}{\|}}{C}{-}CH_3 \xrightarrow[\text{conc. HCl, Clemmensen reduction}]{\text{Zn-Hg}} \text{Alkane}$

The above reaction is called Clemmensen-reduction. It converts aldehydes and ketones to corresponding alkane.

(III) $CH_3C \equiv CCH_3 \xrightarrow{LiAlH_4} \text{Alkene}$

This reaction is used for reduction of alkynes which gives corresponding alkenes.

(IV) 

Hence, option (c) is the correct answer.

141. (b) Conditions for species to be an aromatic are:

(i) The aromatic structures follows Huckel's rule, i.e.,species have $(4n + 2)$ π-electrons in its structure. where, n = integer 0, 1, 2, 3, … so on.

(ii) Should be planar ring structure.

(A) **Structure A:** It follows Huckel's rule and has planar structure thus, is an aromatic ($\because n = 1, \pi e^- = 6$).

(B) **Structure B:** It does not follow Huckel's rule ($\because$ has 8π electrons with $n = 2$).

(C) **Structure C:** It follows Huckel's rule and has planar structure. Thus, is an aromatic ($\because n = 1, \pi e^- = 6$).

(D) **Structure D:** It follows Huckel's rule and has planar structure as it has $n = 3$ and 14π electrons.

(E) **Structure E:** It does not follow Huckel's rule, thus is not an aromatic compound ($n = 1, \pi e^- = 4$).

(F) **Structure F:** It also does not follow Huckel's rule. Thus, is not an aromatic ($n = 0, \pi e^- = 4$).

142. (c) In ferrimagnetic substances, the magnetic moments of the domains are alligned in parallel and antiparallel directions in unequal number. The net magnetic moment is small. Thus, these are weakly attracted by magnetic field than ferromagnetic substances and they can also lose ferrimagnetism on heating and become paramagnetic. e.g. Fe_3O_4 and ferrites like $ZnFe_2O_4$, $MgFe_2O_4$ and $NiFe_2O_4$ etc.

$\uparrow \uparrow \downarrow \uparrow \uparrow \downarrow$
Ferrimagnetism

CrO_2 is ferromagnetic, while MnO is anti-ferromagnetic.

143. (b) Given,
Moles of solute (n) = 1 mol
Mass of solvent (w_A) = 50 g

$\because$ Molality (m) = $\dfrac{n \times 1000}{w_A(\text{in g})}$

$\therefore\ m = \dfrac{1000}{50} = 20 \text{ mol kg}^{-1}$

Hence, option (b) is the correct answer.

144. (d) Given,
Volume of solute = 10 mL
$\Delta T_f = 0 - (-0.413) = 0.413°C$
w (solvent) = 500 g
K_f (water) = 1.86 K kg mol^{-1} (= w_A)
Molecular weight of (A) or M_B = 60 g
Let mass of solute (A) = w_B
$\Delta T_f = K_f m$

$\therefore\ \Delta T_f = K_f \times \dfrac{w_B}{M_B} \times \dfrac{1000}{w_A}$

$0.413 = 1.86 \times \dfrac{w_B}{60} \times \dfrac{1000}{500} \quad \therefore\ w_B = \dfrac{0.413 \times 60 \times 500}{1.86 \times 1000}$

Mass of solute = 6.66 g
Also,

$\because$ Density (d) of solution = $\dfrac{\text{Total mass}}{\text{Total volume}}$

$= \dfrac{6.66 + 500}{500 + 10} = \dfrac{506.66}{510} = 0.993\,(\text{g mL}^{-1})$

Density (d) of solution = 0.993 g mL^{-1}
Hence, option (d) is the correct answer.

145. (c) Given,
Charge used = 19296 C
Molar mass of Cu = 63.5
$\because$ 2 × 96500 C of charge give = 63.5 g of Cu

Thus, 19296 C of charge give = $\dfrac{63.5 \times 19296}{2 \times 96500}$
= 6.35 g of copper (Cu).
Hence, option (c) is the correct answer.

146. (c) For zero-order reaction, integrated rate equation is given as:

$k = \dfrac{[R_0] - [R]}{t}$

k = rate constant
where, $[R_0]$ = initial concentration and [R] = concentration at time (t)

When, $t = t_{1/2}$, then $[R] = \dfrac{[R_0]}{2}$

Thus, $k = \dfrac{[R_0] - \frac{1}{2}[R_0]}{t_{1/2}}$

Hence, option (c) is the correct answer.

147. (b) Silver sols can be used as an eye-lotion because it can heal eye infections.
Hence, option (b) is the correct answer.

148. (a) $4Au + 8CN^-(aq) + 2H_2O(aq)$

$\xrightarrow{O_2(g)} \underset{(A)}{4[Au(CN)_2]^-} + 2H_2O$

$\xrightarrow{Zn(s)} \underset{(B)}{2[Zn(CN)_4]^{2-}} + 2Au$

Hence, option (a) is the correct answer.

149. (a) **Statement (I)** is correct statement as sulphur vapour has formula S_2, i.e. similar to O_2 and O_2 has two unpaired electrons in its anti-bonding molecular orbitals.

Statement (II) Reaction of HCl (dil.) with finely divided iron forms $FeCl_2$ and H_2 gas.

$Fe + 2HCl\,(\text{dil.}) \rightarrow FeCl_2 + H_2(g)$

The $H_2(g)$ produced will prevent the formation of Fe^{3+} (i.e. $FeCl_3$).

150. (c,d) Noble gases have low boiling point and low melting point because of
(i) weak van der Waal's interactions and,
(ii) weak dispersion forces.
Hence, option (c) and (d) both are the correct answer.

151. (a) (A) $Co^{2+} \longrightarrow$ Pink colour
(B) $Fe^{2+} \longrightarrow$ Pale green colour
(C) $Ni^{2+} \longrightarrow$ Dark green colour
(D) $Cu^{2+} \longrightarrow$ Blue colour

152. (b) The crystal field splitting, i.e., Δ_0, depends upon the field produced by the ligand and charge on the metal ion.
Ligands have been arranged in a series in the order of increasing field strength. This series is known as spectrochemical series.
e.g., $I^\ominus < Br^\ominus < SCN^\ominus < Cl^\ominus < F^\ominus < OH^\ominus < C_2O_4^{2-} < H_2O < NCS^\ominus < EDTA^{4-} < NH_3 < en < CN^\ominus < CO$.
$\because$ Among the given options, ligand (NH_3) has highest magnetic field therefore, it has maximum value for Δ_0. Hence option (b) is the correct answer.

153. (c) $\because$ Number average molecular mass ($\bar{M}_n$)

$= \dfrac{N_1M_1 + N_2M_2 + N_3M_3}{N_1 + N_2 + N_3}$

where, N_1, N_2 and N_3 are number of molecules and M_1, M_2 and M_3 are respectively their molecular masses.

Given, $N_1 = 50$ and $M_1 = 5000$

$N_2 = 100$ and $M_2 = 10{,}000$

$N_3 = 50$ and $M_3 = 15{,}000$

$$\text{Thus, } \bar{M}_n = \frac{N_1M_1 + N_2M_2 + N_3M_3}{N_1 + N_2 + N_3}$$

$$= \frac{(50 \times 5000) + (100 \times 10{,}000) + (50 \times 15{,}000)}{50 + 100 + 50}$$

$$= \frac{(25{,}0000) + (1{,}000{,}000) + (75{,}00{,}00)}{200}$$

$\bar{M}_n = 10{,}000$

Hence, option (c) is the correct answer.

154. (d) The species which contain free —CHO group are reducing sugars.

Sucrose is a non-reducing sugar, while maltose, lactose and fructose are reducing sugars.

Hence, option (d) is the correct answer.

155. (a) (A) Codeine, (D) morphine and (F) heroin are opiates or narcotics. The most common effect of medicinal opiates is pain relief.

(B) Thymine is a nucleoside present in DNA.

(C) Epinephrine is used under extreme stress. It affects on hormones which release stress producing chemicals.

(E) Thiamine is a vitamin.

156. (d) (A) $CH_3—CHBr—CH_2Br \xrightarrow{KOH/C_2H_5OH}$ Alkenyl bromide (IV)

∵ KOH(alc.) will remove one H and one Br to give alkenyl bromide.

(B) $CH_3—CH_2—CH{=}CH_2 \xrightarrow[(C_6H_5CO)_2O_2\cdot\Delta]{HBr}$ (I)

gives 1°alkyl bromide due to presence of peroxide and HBr. (*Anti*-markownikoff's addition)

(C) $CH_3—CH_2—CH_3 \xrightarrow{Br\ ,\ h\nu}$ (II), gives 2° alkyl bromide because 2° carbocation is more stable.

(D) $CH_3—CH{=}CH_2 \xrightarrow{NBS}$ (III) gives allyl bromide due to presence of NBS.

Hence, option (d) is the correct answer.

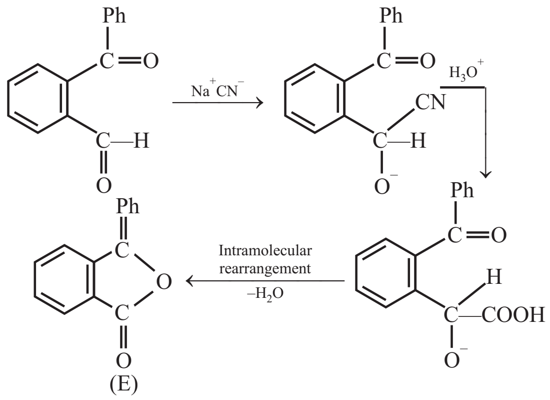
157. (b) $R—COOH \xrightarrow[(ii)\ H_2O/H^+]{(i)\ B_2H_6}$ Product

In the above reaction, when R—COOH is reduced by B_2H_6, followed by acidic hydrolysis;

—C(=O)—OH group changes to $—CH_2—OH$ and therefore $R—CH_2—OH$ is the main product.

158. (c) The given reaction proceed in a similar way as that of benzoin condensation.

The complete reaction is given as below:



Hence, option (c) is the correct answer.

159. (b) Acidic strength increases with presence of EWG (i.e., groups show –I effect) and decreases with the presence of EDG (i.e., groups show +I or hyperconjugation effect). More be the number and more close be the EWG to the —COOH group, more is the acidic strength.

(A) $\overset{2}{C}H_3—\overset{1}{C}OOH$, has only one EDG, i.e. (by hyperconjugation) one CH_3 group is bonded with —COOH group.

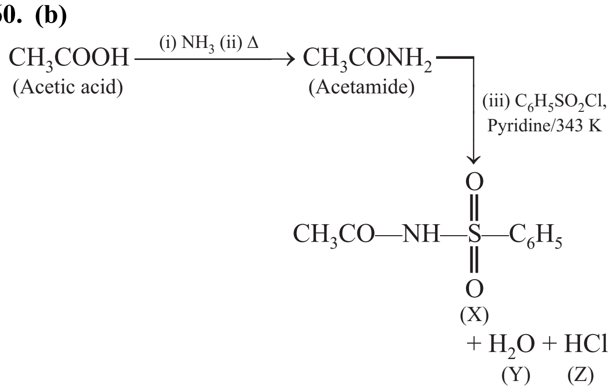
(B) $CH_3—\overset{3}{C}H(Cl)—\overset{2}{C}H_2—\overset{1}{C}OOH$ has one EWG (i.e., Cl) at C-3 position and CH_3, i.e., one EDG (by hyperconjugation) at C_3-position.

(C) $Cl—\overset{2}{C}H\ —\overset{1}{C}OOH$, has one EWG (i.e., Cl-atoms) at C_2-position.

(D) $Cl_2—\overset{2}{C}H—\overset{1}{C}OOH$, has two EWG's (i.e., two Cl-atom) at C_2-position.

Thus, order of acidic strength is (D) > (C) > (B) > (A) and option (b) is the correct answer.

160. (b)



Hence, answer (b) is the correct option.