TS/EAMCET Solved Paper 2020

Held on September 9

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. Match the items of List-I with those of the items of List-II

	List-I		**List-II**
A.	Range of $\sec^{-1}[1+\cos^2 x]$, [.] denote greatest integer function	I.	odd function
B.	Domain of $f(x)$, where $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}$	II.	$\left\{0,\frac{1}{2}\right\}$
C.	$f(x+y)=f(x)+f(y); f(1)=5$	III.	$\{\sec^{-1}5, \sec^{-1}4\}$
D.	$\sin^{-1} x-\cos^{-1} x+\sin^{-1}(1-x)=0 \Rightarrow x \in$	IV.	$R-(-2, 2)$
		V.	$\{\sec^{-1} 1, \sec^{-1}2\}$

	A	B	C	D
(a)	V	IV	I	II
(b)	III	IV	II	I
(c)	V	II	III	IV
(d)	III	II	I	IV

2. The domain of the function

$$f(x)=\sec^{-1}(3x-4)+\tanh^{-1}\left(\frac{x+3}{5}\right) \text{ is}$$

(a) $(-8,1)\cup\left(\frac{5}{3},2\right)$ (b) $\left(1,\frac{5}{3}\right)$

(c) $[-8,1]\cup\left[\frac{5}{3},2\right]$ (d) $(-8,1]\cup\left[\frac{5}{3},2\right)$

3. Let $m=(9n^2+54n+80)(9n^2+45n+54)(9n^2+36n+35)$. The greatest positive integer which divides m, for all positive integers n, is

(a) 720 (b) 724 (c) 696 (d) 842

4. If A is a 3 × 3 matrix and the matrix obtained by replacing the elements of A with their corresponding cofactors is $\begin{bmatrix}1 & -2 & 1\\ 4 & -5 & -2\\ -2 & 4 & 1\end{bmatrix}$ then a possible value of the determinant of A is

(a) 4 (b) 3 (c) 2 (d) 1

5. If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$, then $(A^2)^{-1}=$

(a) A^2 (b) $2A$

(c) A^3 (d) $\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ -2 & 2 & -1\end{bmatrix}$

6. The equations $x+y+z=3$, $x+2y+2z=6$ and $x+ay+3z=b$ have

(a) No solution when $a \neq 3$, b is any value

(b) Infinite number of solutions when $b \neq 9$

(c) Unique solution when $a \neq 3$, b is any value

(d) Unique solution when $a = 3$ and $b \neq 9$

7. Let $z \in \boldsymbol{C}$ and $i=\sqrt{-1}$, if $a, b, c \in (0, 1)$ be such that $a^2+b^2+c^2=1$ and $b+ic=(1+a)z$, then $\frac{1+iz}{1-iz}=$

(a) $\frac{a+ib}{1+c}$ (b) $\frac{a-ib}{1+c}$ (c) $\frac{a-ib}{1-c}$ (d) $\frac{a+ib}{1-c}$

8. If A = $\left\{z = x + iy / \text{ real part of } \frac{\bar{z}-1}{z-i} = 2\right\}$, then the locus of the point $P(x, y)$ in the cartesian plane is

(a) a pair of lines passing through (–1, –1)

(b) a circle of radius $\frac{1}{\sqrt{2}}$ and the centre $\left(\frac{-1}{2}, \frac{3}{2}\right)$

(c) a pair of lines passing through (–1, –2)

(d) a circle of radius $\frac{1}{2}$

9. If ω is a complex cube root of unity, then $(1 - \omega + \omega^2)^6 + (1 - \omega^2 + \omega)^6 =$

(a) 0 (b) 6 (c) 64 (d) 128

10. If e^{ix} is a solution of the equation

$z^n + p_1 z^{n-1} + p_2 z^{n-2} + ... + p_n = 0$, where p_i are real (i = 1, 2, 3, ..., n), then

$p_n \sin nx + p_{n-1} \sin(n-1)x + ... + p_1 \sin x + 1 =$

(a) $\cos(n + 1)x$ (b) $\sin(n(n + 1))x$

(c) 1 (d) 0

11. **Assertion:** $3x^2 - 16x + 4 > -16$ is satisfied for some values of real x in $\left(0, \frac{10}{3}\right)$.

Reason: $ax^2 + bx + c$ and a will have the same sign for some values of $x \in \boldsymbol{R}$ when $b^2 - 4ac > 0$.

(a) (A) is true, (R) is true and (R) is the correct explanation for (A)

(b) (A) is true, (R) is true but (R) is not the correct explanation for (A)

(c) (A) is true, but (R) is false

(d) (A) is false, but (R) is true.

12. If the roots of the quadratic equation $ax^2 + bx + c = 0$ are imaginary, then for all real values of x, the minimum value of the expression $3a^2x^2 + 6abx + 2b^2$ is

(a) $< 4ab$ (b) $> 4ac$ (c) $> -4ac$ (d) $< -4ab$

13. Let α, β, γ be the roots of the equation $x^3 + px + q = 0$ and $f(x) = 3p^2x^2 + p^2x + 3q$. Then $\Sigma\alpha^2\beta + \Sigma\alpha^4 =$

(a) $f(1)$ (b) $f(-1)$ (c) $f(0)$ (d) $f(2)$

14. If α, β, γ are the roots of the equation $x^3 + ax^2 - bx + c = 0$, then $\Sigma\beta^2(\gamma + \alpha) =$

(a) $\frac{a^2 + b - c}{3ab}$ (b) $ac + b^3$

(c) $\frac{bc + a^2}{3ab}$ (d) $ab + 3c$

15. If the number of all possible permutations of the letters of the word MATHEMATICS is which the repeated letters are not together is 982(X), then X =

(a) 5040 (b) 14400 (c) 21600 (d) 86400

16. There are 10 red and 5 yellow roses of different sizes. If x is the number of garlands that can be formed with all these flowers so that no two yellow roses come together and y is the number of garlands formed with all these flowers so that all the red roses coming together, then $\frac{2(x-y)}{10!} =$

(a) $\frac{9!}{5!} - 5!$ (b) $(11)^2.(4!)$

(c) $10! - 6!$ (d) $6! \times (5! - 2)$

17. If $y = \frac{3}{4} + \frac{3 \cdot 5}{4 \cdot 8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + ...\infty$, then

(a) $y^2 - 2y + 5 = 0$ (b) $y^2 + 2y - 7 = 0$

(c) $y^2 - 3y + 4 = 0$ (d) $y^2 + 4y - 6 = 0$

18. If the coefficient of x^{13} in the expansion of $\frac{(1+x)^2}{(1-2x)^3}$ is $A \times 2^{10}$, then A =

(a) 862 (b) 1304 (c) 1360 (d) 1724

19. If the partial fraction decomposition of

$$\frac{x^2 + 1}{x^3 + 3x^2 + 3x + 2} \text{ is } \frac{A}{x+2} + \frac{B}{x^2 + x + 1} + \frac{C}{(x+2)(x^2 + x + 1)},$$ then $A - B + C =$

(a) 0 (b) 2 (c) 3 (d) 4

20. If $\sqrt{x} + \frac{1}{\sqrt{x}} = 2\cos\theta$, then $x^6 + x^{-6} =$

(a) 2 cos6θ (b) 2 cos12θ

(c) 2 cos3θ (d) 2 sin3θ

21. If A = sinθ |sinθ|, B = cos θ |cos θ| and $\frac{99\pi}{2} \le \theta \le \frac{100\pi}{2}$, then

(a) A + B = 1 (b) A + B = –1

(c) B – A = 1 (d) B – A = –1

22. If $\frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)}+\frac{\cos(\theta_3-\theta_4)}{\cos(\theta_3+\theta_4)}=0$, then $\cot\theta_1.\cot\theta_2.$ $\cot\theta_3.\cot\theta_4=$

(a) 1 (b) –1 (c) 2 (d) 1/2

23. The equation $\sin^4 x-(K+3)\sin^2 x-K-4=0$ has a solution if

(a) $K>4$

(b) $-4\le K\le -3$

(c) K is any positive integer

(d) $K=0$

24. Let Z denote the set of integers. Then match the items in list-I, with those of the items in list-II.

List-I	**List-II**
A. $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)+\sin^{-1}\frac{1}{3}$	I. $n\pi\pm(-1)^n\frac{\pi}{6}$
B. $\sin^{-1}\left(\frac{(-1)^n}{2}\right)=x, n\in\boldsymbol{Z}$	II. $n\pi\pm 1, n\in\boldsymbol{Z}$
C. $\tan^{-1}\left(\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right)$	III. $\frac{3}{2}$
D. $\sin^{-1}\lvert\sin x\rvert=\sqrt{\sin^{-1}\lvert\sin x\rvert}$ $\Rightarrow x\in$	IV. $\frac{3\pi}{8}$
	V. $\frac{\pi}{2}$

The correct match is

	A	B	C	D
(a)	V	I	III	II
(b)	IV	II	V	I
(c)	V	I	IV	II
(d)	IV	II	V	III

25. $\sinh^{-1}(-2)+\operatorname{cosech}^{-1}(-2)+\coth^{-1}(-2)=$

(a) $\log\left(\frac{7-3\sqrt{5}}{2\sqrt{3}}\right)$ (b) $\log\left(\frac{3-\sqrt{5}}{2\sqrt{3}}\right)$

(c) $\log\left(\frac{7+3\sqrt{5}}{2\sqrt{3}}\right)$ (d) $\log\left(\frac{3+\sqrt{5}}{2\sqrt{3}}\right)$

26. In ΔABC, if $a:b:c=4:5:6$, then the ratio of the circumradius to its inradius is

(a) 16 : 7 (b) 7 : 16 (c) 4 : 5 (d)5 : 4

27. In a ΔABC, if $b=10$, $a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=15$, and the area of the triangle is $15\sqrt{3}$ sq. units, then $\cot\frac{B}{2}=$

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{5}{\sqrt{3}}$

28. If d_1, d_2, d_3 are the diameters of three ex-circles of a ΔABC, then $d_1d_2+d_2d_3+d_3d_1=$

(a) $(a+b+c)^2$ (b) $ab+bc+ca$

(c) $4\Delta^2$ (d) $2s^2$

29. A(a), B(b), C(c), D(d) are four concyclic points, such that $xa+yb+zc+td=0$, $x+y+z+t=0$ where x, y, z, t are constants not all zero. If the chords AB and CD intersect at P, then

(a) $\lvert xy\rvert\lvert a+c\rvert^2=\lvert zt\rvert\lvert b+d\rvert^2$

(b) $\lvert xy\rvert\lvert a-b\rvert^2=\lvert zt\rvert\lvert c-d\rvert^2$

(c) $\lvert xt\rvert\lvert a-c\rvert^2=\lvert yz\rvert\lvert b-c\rvert^2$

(d) $\lvert xz\rvert\lvert b+d\rvert^2=\lvert yt\rvert\lvert a+c\rvert^2$

30. If a and b are two non collinear vectors, then $\frac{a\times(b\times a)}{\lvert a\rvert^2}$ represent

(a) a vector perpendicular to the plane of *a, b*

(b) projection of *b* along a vector perpendicular to the vector *a*.

(c) projection of a along the vector perpendicular to *b*.

(d) a vector on the plane of a, b whose magnitude is equal to $\lvert a\rvert+\lvert b\rvert$.

31. If A (1, 2, 3), B (3, 7, –2), C (6, 7, 7) and D (–1, 0, –1) are points in a plane, then the vector equation of the line passing through the centroids of ΔABD and ΔACD is

(a) $r=2\hat{i}+3\hat{j}+3\hat{k}+t(\hat{i}+3\hat{j})$

(b) $r=(1+t)\hat{i}+3\hat{j}+3t\hat{k}$

(c) $r=\hat{i}+\hat{j}+\hat{k}+t(2\hat{i}-\hat{j})$

(d) $r=2\hat{i}-\hat{j}+t(\hat{j}+4\hat{k})$

32. If $a=\hat{i}+(\tan\theta)\hat{j}+\left(\frac{3}{\sqrt{\sin\frac{\theta}{2}}}\right)\hat{k}$ and

$b=\tan\theta(\hat{j}-\hat{i})-\left(2\sqrt{\sin\frac{\theta}{2}}\right)\hat{k}$ are orthogonal vectors and

$c = (\sin 2\theta)\ \hat{i} - 2\hat{j} + 2\hat{k}$ makes an obtuse angle with X-axis, then $\theta =$

(a) $(2n+1)\pi + \tan^{-1} 2, n \in Z$

(b) $n\pi - \tan^{-1} 2, n \in Z$

(c) $(2n+1)\pi - \tan^{-1} 3, n \in Z$

(d) $(2n+1)\pi + \tan^{-1} 3, n \in Z$

33. If *a, b* and *c* be three non-coplanar vectors and *p*, *q* and *r* be defined by $p = \frac{b \times c}{a \cdot (b \times c)}, q = \frac{c \times a}{b \cdot (c \times a)}, r = \frac{a \times b}{c \cdot (a \times b)}$ such that $\alpha = (a+b) \cdot p + (b+c) \cdot q + (c+a) \cdot r$ and $\beta = \frac{(a+b) \cdot (b+c) \times (a+b+c)}{b \cdot (a \times c)}$, then $\alpha + \beta =$

(a) 2 (b) 3 (c) 4 (d) 0

34. If $a + lb + l^2c = 0$ and $a \times b + b \times c + c \times a = 3\ (b \times c)$, then the minimum value of such *l* is

(a) 1 (b) –2 (c) $\frac{-9}{4}$ (d) 0

35. The mean deviation about the mean for the following data is

Class Interval	0-4	4-8	8-12	12-16
Frequency	4	3	2	1

(a) 6 (b) 3.6 (c) 3.2 (d) 10

36. The means of two groups of observations A and B are $\overline{x}, \overline{y}$ respectively and their standard deviations are respectively 2 and 3. In order that the group A is to be more consistent than the group $B \frac{\overline{y}}{\overline{x}} >$.

(a) $\frac{3}{2}$ (b) $\frac{5}{1}$ (c) $\frac{2}{3}$ (d) $\frac{6}{5}$

37. When two dice are rolled, let *x* be the probability of getting a sum of the numbers appear on the dice is at most 7. Let *y* be the probability of getting a sum 7 at least once when a pair of dice are rolled *n* times. In order to have $y > x$, the minimum *n* is

(a) 3 (b) 6 (c) 5 (d) 4

38. Two numbers are selected at random from the set {1, 2, 3, ..., 13}. If the sum of the selected numbers is even, the probability that both the numbers are odd is

(a) $\frac{2}{13}$ (b) $\frac{1}{2}$ (c) $\frac{7}{12}$ (d) $\frac{5}{26}$

39. Let X be the discrete random variable representing the number (x) appeared on the face of a biased die when it is rolled. The probability distribution of X is

X = x	1	2	3	4	5	6
P(X = x)	0.1	0.15	0.3	0.25	k	k

Then variance of X is

(a) 1.64 (b) 1.93 (c) 2.16 (d) 2.28

40. 2*n* unbiased coins are tossed. The probability of having the number of heads is not equal to the number of tails, is

(a) $\frac{2n!}{(n!)^2}\left(\frac{1}{2}\right)^{2n}$ (b) $1 - \frac{2n!}{(n!)^2}$

(c) $\frac{2n!}{(n!)^2}$ (d) $1 - \frac{2n!}{(n!)^2} \cdot \frac{1}{4^n}$

41. Let *p*(*x*) represent the probability mass function of a poisson distribution. If its mean $\lambda = 3.725$, then value of *x* at which *p*(*x*) is maximum is

(a) 2 (b) 3 (c) 4 (d) 5

42. Let A = (0, 4) and B = (2 cosθ, 2sinθ), for some $0 < \theta < \frac{\pi}{2}$. Let P divide the line segment AB in the ratio 2 : 3 internally. The locus of P is

(a) Circle (b) Ellipse

(c) Parabola (d) Hyperbola

43. The transformed equation of $3x^2 - 4xy = r^2$, when the coordinate axes are rotated through an angle $\tan^{-1}(2)$ is

(a) $X^2 - 4Y^2 = r^2$ (b) $2XY + r^2 = 0$

(c) $4Y^2 - X^2 = r^2$ (d) $XY = r^2$

44. If $x \cos\theta + y \sin\theta = p$ is the normal form and $y = mx + c$ is the slope-intercept form of the line $x + 2y + 1 = 0$, then $\tan^{-1}(\tan\theta + m + c) =$

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) $\frac{\pi}{4}$

45. If the family of straight lines $ax + by + c = 0$, where $2a + 3b = 4c$ is concurrent at the point *P*(*l, m*), then the foot of the perpendicular drawn from P to the line $x + y + 1 = 0$ is

(a) $\left(\frac{-3}{8}, \frac{-5}{8}\right)$ (b) $\left(\frac{-2}{5}, \frac{-3}{5}\right)$

(c) (3, –4) (d) (–5, 4)

46. $O(0,0), A(-3,-1)$ and $B(-1,-3)$ are the vertices of a ΔOAB. *P* is a point on the perpendicular *AD* drawn from *A* on *OB* such that $\frac{AP}{PD} = \frac{3}{4}$. Then the equation of the line *L* parallel to *OB* and passing through *P*, is

(a) $3x - y + 3 = 0$ (b) $21x - 7y + 32 = 0$

(c) $15x - 5y + 32 = 0$ (d) $3x - y + 35 = 0$

47. A straight line passing through the point (1, 0) and not parallel to X-axis intersects the curve $2x^2 + 5y^2 - 7x = 0$ at two points A and B. The angle subtended by the line segment AB at the origin is

(a) 30° (b) 45° (c) 60° (d) 90°

48. If (p, q) is the centroid of the triangle formed by the lines $8x^2 - 14xy + 5y^2 = 0$, $x - 2y + 3 = 0$, then

(a) $p + q = -1$ (b) $q = 2p$

(c) $p = 2q$ (d) $p = q$

49. The area (in sq. units) of the triangle formed by the two tangents drawn from the external point O (0, 0) to the circle $x^2 + y^2 - 2gx - 2hy + h^2 = 0$ and their chord of contact is

(a) $\dfrac{gh}{h^3 + g^2}$ (b) $\dfrac{gh}{h^2 + g^3}$

(c) $\dfrac{hg^3}{h^2 + g^2}$ (d) $\dfrac{gh^3}{h^2 + g^2}$

50. If the circle $x^2 + y^2 - 6x + 2y = 28$ cuts off a chord of length λ units of the line $2x - 5y + 18 = 0$, then the value of λ is

(a) 3 (b) 6 (c) 12 (d) 9

51. Consider the circles

$S_1 : x^2 + y^2 + 2x + 8y - 23 = 0$ and

$S_2 : x^2 + y^2 - 4x + 10y + 19 = 0$

If the polars of the centre of a circle with respect to the another circle are L_1 and L_2, then L_1, L_2 are

(a) Parallel and separated by a distance of $4\sqrt{10}$ units

(b) Perpendicular and intersect at (1, 3)

(c) Perpendicular and intersect at (1, –5)

(d) Parallel and separated by a distance of $2\sqrt{10}$ units.

52. If the circle $S \equiv x^2 + y^2 - 4 = 0$ intersects another circle $S' = 0$ of radius $\dfrac{5\sqrt{2}}{2}$ in such a manner that the common chord is of maximum length with slope equal to $\dfrac{1}{4}$, then the centre of $S' = 0$ is

(a) (–1, 4) or (1, – 4)

(b) $\left(+\dfrac{\sqrt{2}}{2}, 2\sqrt{2}\right)$ or $\left(\dfrac{\sqrt{2}}{2}, -2\sqrt{2}\right)$

(c) $\left(-2\sqrt{2}, \dfrac{\sqrt{2}}{2}\right)$ or $\left(2\sqrt{2}, \dfrac{-\sqrt{2}}{2}\right)$

(d) (4, –1) or (– 4, 1)

53. If the angle between the circles $x^2 + y^2 - 4x - 6y + k = 0$ and $x^2 + y^2 + 8x - 4y + 11 = 0$ is $\dfrac{\pi}{3}$, then the value of k is

(a) –3 (b) 36 (c) 3 (d) 2

54. Let $A(1, 2), B(4, -4), C(2, 2\sqrt{2})$ be points on the parabola $y^2 = 4x$. If α and β respectively represent the area of ΔABC and the area of the triangle formed by the tangents at A, B, C to the above parabola, then αβ =

(a) 6 (b) $3\sqrt{2}$ (c) 9 (d) $6\sqrt{2}$

55. If $x - 2y + k = 0$ is a tangent to the parabola $y^2 - 4x - 4y + 8 = 0$, then the slope of the tangent drawn at (l, k) on the given parabola is

(a) $\dfrac{-5}{2}$ (b) 2 (c) –2 (d) $\dfrac{2}{5}$

56. The equation of the ellipse in the standard form whose length of the latus rectum is 4 and whose distance between the foci is $4\sqrt{2}$, is

(a) $\dfrac{x^2}{2} + \dfrac{y^2}{3} = 1$ (b) $2x^2 + y^2 = 8$

(c) $x^2 + 2y^2 = 16$ (d) $x^2 + 5y^2 = 25$

57. The value of b^2 in order that the foci of the hyperbola $\dfrac{x^2}{144} - \dfrac{y^2}{81} = \dfrac{1}{25}$ and the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{b^2} = 1$ coincide is

(a) 1 (b) 5 (c) 7 (d) 9

58. If the pole of the line $3x - 16y + 48 = 0$ with respect to the hyperbola $9x^2 - 16y^2 = 144$ is (α, β), then α – β =

(a) 0 (b) –3 (c) 2 (d) –7

59. A (2, 3, – 4), B (–3, 3, –2), C(–1, 4, 2) and D(3, 5, 1) are the vertices of a tetrahedron. If E, F, G are the centroids of its faces containing the point A, then the centroid of the triangle EFG is

(a) $\left(\dfrac{1}{9}, \dfrac{15}{9}, \dfrac{-3}{9}\right)$ (b) $\left(\dfrac{1}{4}, \dfrac{15}{4}, \dfrac{-3}{4}\right)$

(c) $\left(\dfrac{4}{9}, \dfrac{11}{3}, \dfrac{-10}{9}\right)$ (d) $\left(\dfrac{-1}{9}, \dfrac{12}{9}, \dfrac{1}{9}\right)$

60. If l, m, n are the direction cosines of a line which makes angles α, β and γ with the coordinate axes X, Y, Z respectively, then $lm + mn + nl$ takes the maximum value when

(a) α, β, γ are in Arithmetic progression

(b) α = β = γ

(c) any two of α, β, γ are the same

(d) one of α, β, γ is zero and the remaining two are non-zero and unequal

61. The combined equation for a pair of planes is $S \equiv 2x^2 - 6y^2 - 12z^2 + 18yz + 2zx + xy = 0$. If one of the planes is parallel to $x + 2y - 2z = 5$, then the acute angle between the planes $S = 0$ is

(a) $\cos^{-1}\left(\frac{16}{21}\right)$ (b) $\frac{\pi}{2}$

(c) $\frac{2\pi}{3}$ (d) $\sin^{-1}\left(\frac{7}{15}\right)$

62. $f : R^+ \to R$ be an increasing function such that $f(x) > 0$ for all x. If $\lim_{x\to\infty} \frac{f(9x)}{f(3x)} = 1$, then $\lim_{x\to\infty} \frac{f(6x)}{f(3x)} =$

(a) 1 (b) 2 (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

63. Let $[x]$ denotes the greatest integer less than or equal to x. Then $f(x) = \frac{1 + \sin([\cos x])}{\cos([\sin x])}$ is

(a) continuous on $\left(0, \frac{\pi}{2}\right)$

(b) continuous on $(0, \pi)$

(c) discontinuous on $\left(\pi, \frac{3\pi}{2}\right)$

(d) continuous on $(\pi, 2\pi)$

64. For $x \neq -1, y \neq -1$, if $x = \frac{1 - \sqrt[3]{y}}{1 + \sqrt[3]{y}}$, then $\frac{dx}{dy} =$

(a) $\frac{-6(1-x)^2}{(1+x)^4}$ (b) $\frac{-(1+x)^4}{6(1-x)^2}$

(c) $\frac{4(1-x)^4}{(1+x)^6}$ (d) $\frac{-6(1+x)^2}{(1-x)^4}$

65. **Assertion:** For x < 0, $\frac{d^2}{dx^2}(\log|x|) = \frac{1}{|x|^2}$

Reason: For $x < 0$. $|x| = -x$

The correct option among the following is

(a) (A) is true, (R) is true and (R) is the correct explanation to (A).

(b) (A) is true, (R) is true but (R) is not a correct explanation to (A).

(c) (A) is true, (R) is false.

(d) (A) is false, (R) is true.

66. If $ax^2 + 2hxy + by^2 = 0$, then $\frac{d^2y}{dx^2} =$

(a) $\frac{h^2 - ab}{(hx + by)^3}$ (b) $\frac{2(h^2 - ab)}{(hx + by)^3}$

(c) $\frac{(hx + by)^3}{h^2 - ab}$ (d) 0

67. The number of points on the parabola $y^2 = x$ at which the slope of the normal drawn at the point is equal to the x-coordinate of that point is

(a) ∞ (b) 1 (c) 2 (d) 0

68. Let α, β be two roots of the quadratic equation $x^2 + ax - b = 0$, $b \neq 0$. If the straight line $x \cos\theta + y \sin\theta = C$ touches the curve $\left(\frac{x}{\alpha}\right)^n + \left(\frac{y}{\beta}\right)^n = 2$ at the point (α, β), then $\left(\frac{a}{b}\right)^2 + \frac{2}{b} =$

(a) $\frac{1}{2C^2}$ (b) $\frac{4}{C^2}$ (c) $\frac{2}{C^2}$ (d) $\frac{1}{C^2}$

69. The number of admissible values of C obtained when the Lagrange's mean value theorem is applied for the function $f(x) = x$ on [2, 5] is

(a) 0 (b) only one

(c) infinite (d) finitely many

70. The function $f(x) = x^3 - 4x^2 + 4x + 3$ defined on $[-1, 3]$ has

(a) minimum value – 6 at $x = -1$

(b) minimum value 6 at $x = 3$

(c) minimum value 3 at $x = 2$

(d) minimum value 9 at $x = 3$

71. $\int \frac{\tan^{-1} x}{x^3} dx =$

(a) $\frac{-(x^2+1)}{2x} \tan^{-1} x - \frac{1}{2x} + C$

(b) $\frac{-(x^2+1)}{2x^2+1} \tan^{-1} x - \frac{1}{2x^2} + C$

(c) $\frac{-1}{2x} - \left(\frac{1}{2} + \frac{1}{2x^2}\right) \tan^{-1} x + C$

(d) $\frac{1}{2x} + \frac{1}{2x^2} \tan^{-1} x + C$

72. If $\int \frac{1 - (\cot x)^{2019}}{\tan x + (\cot x)^{2020}} dx = \frac{1}{n} \ln|(f(x))^n$ $+ (g(x))^n| + c$, then the value of $n[(f(x))^4 + (g(x))^4]_{x=\frac{\pi}{3}} =$

(a) $\frac{10105}{16}$ (b) $\frac{10012}{15}$ (c) $\frac{20210}{9}$ (d) $\frac{10105}{8}$

73. If $x \neq 1$ and

$$\int \frac{(x^3 + x^2 - x - 1)}{(x^5 + x^4 + 3x^3 + 3x^2 + x + 1)\tan^{-1}\left(\frac{x^2+1}{x}\right)} dx$$

$= A \log (f(x)) + C$, then $A - \tan (f(2)) =$

(a) $\frac{-3}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{7}{2}$ (d) -2

74. If $I_n = \int x^n \sin x \, dx$ and

$I_6 - 360 I_2 = f(x) \cos x + g(x) \sin x$, then $f(l) + g(l) =$

(a) –85 (b) 0 (c) –53 (d) 75

75. $\int_0^{\pi/2} x^3 \sin x dx =$

(a) $\frac{3\pi^2}{4} - 3\pi + 6$ (b) $\frac{3\pi^2}{4} + 3\pi - 6$

(c) $\frac{3\pi^2}{4} + 6$ (d) $\frac{3\pi^2}{4} - 6$

76. $\lim_{n \to \infty} \prod_{r=1}^{n} \left(1 + \frac{r^2}{n^2}\right)^{2r/n^2}$:

(a) $\log\left(\frac{4}{e}\right)$ (b) $\log\left(\frac{2}{e}\right)$

(c) $\frac{2}{e}$ (d) $\frac{4}{e}$

77. The larger of the two areas into which the circle $x^2 + y^2 = 16a^2$ is divided by the parabola $y^2 = 6ax$ is

(a) $\frac{4a^2}{3}(8\pi - \sqrt{3})$ (b) $\frac{4a^2}{3}(4\pi - \sqrt{3})$

(c) $\frac{2a^2}{3}(4\pi + \sqrt{3})$ (d) $\frac{4a^2}{3}(4\pi + \sqrt{3})$

78. The differential equation for which $\sqrt{1 + y^2} = Cxe^{\tan^{-1} x}$ is the general solution of

(a) $xy(1 + x^2)dy - e^{\tan^{-1} x}(1 + x + x^2)\, dx = 0$

(b) $xy(1 + y^2)dy - (1 + x^2)(1 + y + y^2)dx = 0$

(c) $(1 + y^2)\tan^{-1} x \frac{dy}{dx} = \frac{1 + x^2}{xy}$

(d) $xy(1 + x^2)dy - (1 + y^2)(1 + x + x^2)dx = 0$

79. The general solution of the differential equation $\frac{dy}{dx} = \frac{x + y - 3}{x + y - 7}$ is

(a) $(x + y - 5)^2 = Ce^{y+x}$

(b) $(x + y - 5)^2 = Ce^{y-x}$

(c) $2\log(x + y - 5) = 3x + 2y + C$

(d) $\log (x + y - 3) = 3 (x + y - 2)^2 + C$

80. If $y = f(x)$ is the solution of the differential equation $x\frac{dy}{dx} = x^2 + 3y, x > 0, y(2) = 4$, then $f(4) =$

(a) 48 (b) 260 (c) 80 (d) 36

PHYSICS

81. The scientific principle that forms the basis of the tokamak technology is

(a) controlled nuclear fission

(b) motion of charged particles in electromagnetic fields

(c) magnetic confinement of plasma

(d) superconductivity

82. If E and E_0 represent the energies, t and t_0 represent the times, then which of the following is dimensionally correct relation?

(a) $E = E_0 e^{-t}$ (b) $E = E_0 t_0 e^{-t/t_0}$

(c) $E = E_0 t_0 e^{-t^2}$ (d) $E = E_0 e^{-t/t_0}$

83. A ball is dropped from a bridge that is 45 m above the water. It falls directly into a boat which is moving with constant velocity. The boat is 12 m away from the point of impact when the ball is dropped. The speed of the boat is (take, $g = 10$ m/s^2)

(a) 2 m/s (b) 3 m/s (c) 4 m/s (d) 5 m/s

84. A ball is dropped from a height H from rest. The ball travels $\frac{H}{2}$ in last 1.0 s. The total time taken by the ball to hit the ground is

(a) 3.85 s (b) 3.41 s (c) 2.55 s (d) 4.65 s

85. A planet is moving in a circular orbit. It completes 2 revolutions in 360 days. What is its angular frequency?

(a) 1.5×10^{-2} rad/day (b) 2.5×10^{-2} rad/day

(c) 3.5×10^{-2} rad/day (d) 4.5×10^{-2} rad/day

86. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces

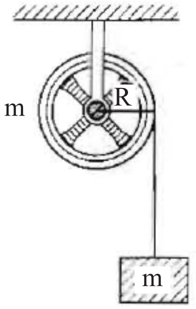
(a) cannot be predicted

(b) always are equal to each other

(c) are equal to each other in magnitude

(d) are not equal to each other in magnitude

87. A mass m is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, then with what acceleration will the mass release?
(Assume, g = acceleration due to gravity)



(a) $\frac{2g}{3}$ (b) $\frac{g}{2}$
(c) $\frac{5g}{6}$ (d) g

88. A particle of mass m_1 collides with a particle of mass m_2 at rest. After the elastic collision, the two particles moves at an angle of 90° with respect to each other. The ratio $\frac{m_2}{m_1}$ is

(a) 1.0 (b) 1.5 (c) 2.0 (d) 2.5

89. A block of mass 100 g moving at a speed of 2 m/s compresses a spring through a distance 2 cm before its speed is halved. Find the spring constant of the spring.

(a) 1250 N/m (b) 750 N/m
(c) 1000 N/m (d) 1500 N/m

90. A bullet of mass m moving horizontally with speed v_0 hits a wooden block of mass M that is suspended from a massless string. The bullet gets lodged into the block and comes into halt. If the block-bullet combine swings to a maximum height h, then how much of the initial kinetic energy of the bullet is lost in the collision?

(a) $\frac{1}{2}mv_0^2\left(\frac{M}{m+M}\right)$ (b) $\frac{1}{2}mv_0^2\left(\frac{M+m}{M}\right)$
(c) $\frac{1}{2}mv_0^2\left(\frac{M^2}{(m+M)^2}\right)$ (d) $\frac{1}{2}mv_0^2\left(\frac{(M+m)^2}{M^2}\right)$

91. A ball of mass 100 g is dropped at a time $t = 0$. A second ball of mass 200 g is dropped from the same point at $t = 0.2$ s. The distance between the centre of mass of two balls and the release point at $t = 0.4$ s is
(Assume, $g = 10$ m/s^2)

(a) 0.4 m (b) 0.5 m (c) 0.6 m (d) 0.8 m

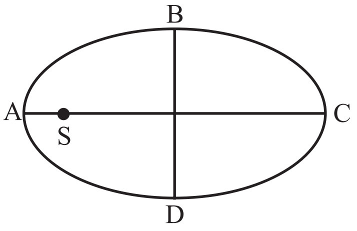
92. A ball of mass M moving with a speed of 2 m/s hits another ball of mass 1 kg moving in the same direction with a speed of 1 m/s. If the kinetic energy of centre of mass is $\frac{4}{3}$ J, then the magnitude of M is

(a) 1 kg (b) 0.25 kg (c) 0.50 kg (d) 2 kg

93. A simple harmonic oscillator of frequency 1 Hz has a phase of 1 rad. By how much should the origin be shifted in time, so as to make the phase of the oscillator vanish?
[Time in seconds (s)].

(a) $-\frac{1}{\pi}s$ (b) $-\frac{1}{2\pi}s$ (c) $-\frac{\pi}{2}s$ (d) $-\pi s$

94. A planet is revolving around the sun in which of the following is correct statement?



(a) The time taken in travelling DAB is less than that for BCD.
(b) The time taken in travelling DAB is greater than that for BCD.
(c) The time taken in travelling CDA is less than that for ABC.
(d) The time taken in travelling CDA is greater than that for ABC.

95. Which of the following statement is incorrect?

(a) The bulk modulus for solids is much larger than for liquids.
(b) Gases are least compressible.
(c) The incompressibility of the solids is due to the tight coupling between neighbouring atoms.
(d) The reciprocal of the bulk modulus is called compressibility.

96. The speed of the water in a river is v near the surface. If the coefficient of viscosity of water is η and the depth of the river is H, then the shearing stress between the horizontal layers of water is

(a) $\eta\frac{H}{v}$ (b) $\eta\frac{v}{H}$ (c) $\frac{v}{\eta H}$ (d) $\eta v H$

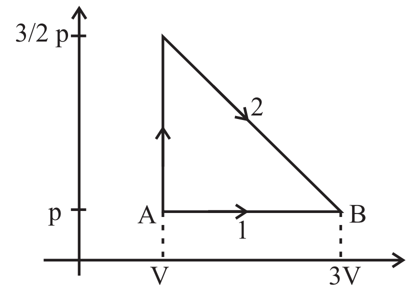
97. A steel rod at 25°C is observed to be 1 m long when measured by another metal scale which is correct at 0°C. The exact length of steel rod at 0°C is ($\alpha_{\text{steel}} = 12 \times 10^{-6}$/°C and $\alpha_{\text{metal}} = 20 \times 10^{-6}$/°C)

(a) 1.00002 m (b) 1.0002 m
(c) 0.998 m (d) 0.9998 m

98. Latent heat of vaporisation of water is $22{\cdot}6 \times 10^5$ J/kg. The amount of heat needed to convert 100 kg of water at 100°C into vapour at 100°C is

(a) 11.3×10^5J (b) 11.3×10^6J
(c) 22.6×10^6J (d) 22.6×10^7J

99. The p-V diagram shown below indicates two paths along which a sample of gas can be taken from state A to state B. The energy equal to 5pV in the form of heat is required to be transferred, if the Path-I is chosen. How much energy in the form of heat should be transferred, if Path-2 is chosen?



(a) $\frac{11}{2}pV$ (b) $6\,pV$ (c) $\frac{9}{2}pV$ (d) $7\,pV$

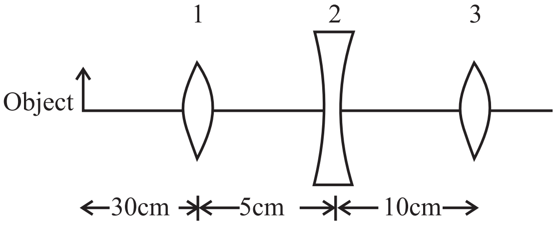
100. Two vessels separately contain two ideal gases A and B at the same temperature. The pressure of gas A is three times the pressure of gas B. Under these conditions, the density of gas A is found to be two times the density of B. The ratio of molecular weights of gas A and B, i.e. $\frac{M_A}{M_B}$ is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

101. Two coherent plane waves of identical frequency and intensity (I) interfere at a point, where they differ in phase by 60°. What is the resulting intensity?

(a) l (b) 2l (c) 3l (d) 4l

102. The position of final image formed by the given lens combination from the third lens will be at a distance of ($f_1 = +10$ cm, $f_2 = -10$ cm, and $f_3 = +30$ cm)



(a) 15 cm (b) infinity (c) 45 cm (d) 30 cm

103. An object is placed at a distance of 40 cm in front of a concave mirror of focal length 20 cm. The image produced is

(a) real, inverted and smaller in size
(b) real, inverted and of same size
(c) real and erect
(d) virtual and inverted

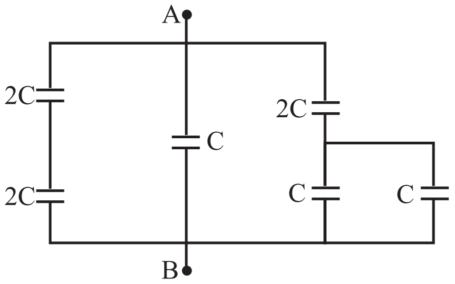
104. Two beams of monochromatic light with intensities 64 mW and 4 mW interfere constructively to produce an intensity of 100 mW. If one of the beams is shifted by an angle θ, the intensity is reduced to 84 mW. The magnitude of θ is.

(a) 30° (b) 60°
(c) 45° (d) $\cos^{-1}\left(\frac{1}{3}\right)$

105. A particle of mass 2×10^{-6} kg with a charge 5×10^{-6} C is hanging in air above a similarly charged conducting surface. The charge density of the surface is (assume, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$ and $g = 10\ m/s^2$)

(a) $35.4 \times 10^{-12}\ C/m^2$ (b) $23.6 \times 10^{-12}\ C/m^2$
(c) $53.1 \times 10^{-12}\ C/m^2$ (d) $17.7 \times 10^{-12}\ C/m^2$

106. Find the equivalent capacitance between point A and B.

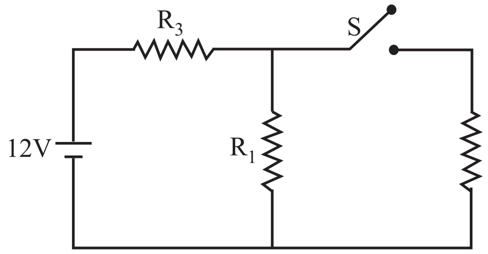


(a) 4C (b) 3C (c) 2C (d) 1C

107. A circular wire has current density $J = \left(2 \times 10^{10} \frac{A}{m^2}\right) r^2$, where r is the radial distance, out of the wire radius is 2 mm. The end-to-end potential applied to the wire is 50 V. How much energy (in joule) is converted to thermal energy in 100 s?

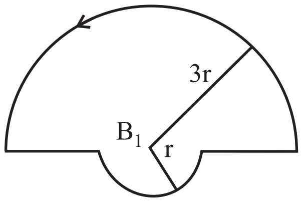
(a) 1200 π (b) 800 π
(c) 3200 π (d) 600 π

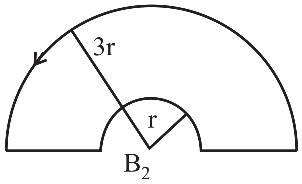
108. The resistance in following circuit are $R_1 = R_2 = R_3 = 6.0\Omega$. The emf of the battery is 12 V. When switch S is closed, the potential across resistance R_1 is changed by an amount.

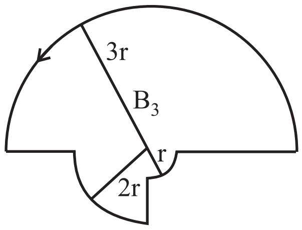


(a) –2V (b) +2V (c) –4V (d) +4V

109. Figure below shows three circuits consisting of concentric circular arcs and straight radial lines. The centre of the circle is shown by the dot. Same current flows through each of the circuits. If B_1, B_2, B_3 are the magnitudes of the magnetic field at the centre. Which of the following is true?







(a) $B_1 > B_2 > B_3$ (b) $B_1 > B_3 > B_2$

(c) $B_3 > B_1 > B_2$ (d) $B_3 > B_2 > B_1$

110. A charged particle moves through a magnetic field perpendicular to its direction. Then,

(a) kinetic energy changes but the momentum is constant

(b) the momentum changes but the kinetic energy is constant

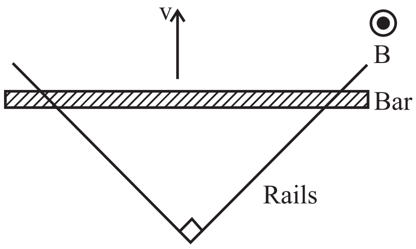
(c) both momentum and kinetic energy of the particles are not constant

(d) both momentum and kinetic energy of the particles are constant

111. The length *l* of a magnet is large compared to its width and breadth. The time-period of its oscillation in a vibration magnetometer is 2s. The magnet is cut into three euqal parts of length – each. If these parts are placed on each other with their like poles together, then the time-period of this combination is

(a) $2\sqrt{3}$ s (b) $\frac{2}{3}$s (c) 2s (d) $\frac{2}{\sqrt{3}}$s

112. Two straight conducting rails form a right angle as shown below. A conducting bar in contact with the rails starts at the vertex at time t = 0 and moves with constant velocity of v = 5 m/s along them. A magnetic field with B = 0.1 T is directed out of the page. The absolute value of the emf around the triangle at the time t = 4 s will be



(a) 10 V (b) 15 V (c) 20 V (d) 30 V

113. A current of 6 A is flowing at 220V in the primary coil of a transformer. If the voltage produced in the secondary coil is 1100 V and 40% of power is lost, then the current in the secondary coil will be

(a) 0.28 A (b) 0.36 A (c) 0.48 A (d) 0.42 A

114. The phase difference between the following two waves y_2 and y_1 is

$$y_1 = a\sin(\omega t - kx);\ y_2 = b\cos\left(\omega t - kx + \frac{\pi}{3}\right)$$

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) π

115. A beam of photons with an energy of 10.5 eV strike a metal plate. The photoelectrons are emitted with maximum velocity of 1.6×10^6 m/s. The work-function of the metal is (assume, mass of electron = 9×10^{-31} kg and charge of electron = 1.6×10^{-19} C)

(a) 3.0 eV (b) 3.1 eV (c) 3.3 eV (d) 3.5 eV

116. In the hydrogen atom spectrum, let E_1 and E_2 are energies for the transition $n = 2 \rightarrow n = 1$ and $n = 3 \rightarrow n = 2$, respectively. The ratio E_2/E_1 is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{2}{9}$ (d) $\frac{5}{27}$

117. What is the binding energy of ${}^{29}_{14}\text{Si}$, whose atomic mass is 28.976495 *u*?

Mass of proton = 1.007276 *u*
Mass of neutron = 1.008664 *u*
(Neglect the electron mass)
(Assume, 1 u = 931.5 MeV)

(a) 237.84 MeV (b) 421.72 MeV

(c) 387.21 MeV (d) 116.35 MeV

118. Which of the following statements is incorrect?

(a) The resistance of intrinsic semiconductors decrease with increase of temperature.

(b) Doping pure Si with trivalent impurities give *p*-type semiconductors.

(c) The majority carriers in *n*-type semiconductors are holes.

(d) A *p-n* junction can act as a semiconductor device.

119. A piece of copper and another of germanium are cooled from room temperature to 77°K. The resistance of

(a) copper increases and germanium decreases

(b) both decreases

(c) both increases

(d) copper decreases and germanium increases

120. A carrier signal of frequency v_1 and peak voltage of V_1 is modulated by a message signal of frequency v_2 and peak voltage of V_2. Let m be the modulation index and v_+, v_- be side bands produced. Which of the correct statement?

(a) $m = \frac{V_1}{V_2}$ (b) $v_1 = \frac{v_+ + v_-}{2}$

(c) $v_2 = \frac{v_+ + v_-}{2}$ (d) $m > \frac{V_2}{V_1}$

CHEMISTRY

121. Two series of spectral lines of atomic hydrogen which do not belong to infrared spectral region are

(a) Lyman and Paschen (b) Balmer and Brackett

(c) Pfund and Lyman (d) Lyman and Balmer

122. Which of the activities can be compared to the concept of quantisation?

(a) A car is travelling on the road

(b) An apple is falling form the tree

(c) A person can stand on any step of a stair case

(d) Throwing a playing disc

123. What will be the IUPAC symbol and name for the element with atomic number 123?

(a) Unt and unniltrium (b) Ubq and unbiquadium

(c) Ubt and unbitrium (d) Unb and unnilbium

124. Which of the following statements is incorrect?

(a) The enthalpy of atomisation decreases down a group in s-block elements

(b) The enthalpy of atomisation decreases down a group in p-block elements

(c) The enthalpy of atomisation decreases down a group in d-block elements

(d) The enthalpy of atomisation increases down a group in d-block elements

125. Find out the correct order of ionic character in the following molecules.

(i) SO_2 (ii) K_2O

(iii) N_2 (iv) LiF

(a) (iv) > (ii) > (iii) > (i) (b) (iv) > (ii) > (i) > (iii)

(c) (ii) > (iv) > (i) > (iii) (d) (ii) > (iv) > (iii) > (i)

126. Find out the bond order in He_2, He^+_2, O_2 and O^+_2, respectively

(a) 0, 0.5, 2 and 3 (b) 0.5, 0, 3 and 2

(c) 0, 0.5, 2 and 1 (d) 0, 0.5, 2 and 2.5

127. One mole of an ideal gas occupies 12 L at 297° C. What is the pressure of the gas?

(a) 207 kPa (b) 395 kPa

(c) 395 Pa (d) 207 Pa

128. 4.4 grams of a gas at 0°C and 0.82 atm pressure occupies a volume of 2.73 L. The gas can be

(a) O_2 (b) CO (c) NO_2 (d) CO_2

129. How many moles of ammonia are produced by 5 moles of hydrogen?

(a) 2.3 (b) 8.3 (c) 10.3 (d) 3.3

130. C_2H_4 can react with H_2 in presence of a catalyst to form C_2H_6 as per the following reaction,

$C_2H_4(g) + H_2(g) \xrightarrow{\text{Catalyst}} C_2H_6(g)$. The amount of C_2H_4 in grams required to produce 50 grams of C_2H_6 is

(a) 36.44 g (b) 22.18 g (c) 46.67 g (d) 57.11 g

131. For which of the following systems, the difference between ΔH and ΔU is not significant?

(i) Solids

(ii) gases

(iii) Mixture of gases and liquids

(iv) Liquids

(a) (i) and (iv) (b) (i), (iii) and (iv)

(c) (ii) and (iv) (d) (ii) and (iii)

132. At 25°C, the ionisation constant for anilinium hydroxide is 5.00×10^{-10}. The hydrolysis constant of anilinium chloride is

(a) 2.00×10^{-5} (b) 4.00×10^{-3}

(c) 1.50×10^{-6} (d) 2.50×10^{-4}

133. For the given equilibrium reaction, $2A(g) \rightleftharpoons 2B(g) + C(g)$ the equilibrium constant (K_C) at 1000 K is 4×10^{-4}. Calculate K_p for the reaction at 800 K temperature

(a) 0.044 (b) 0.026 (c) 0.33 (d) 1

134. Hydrogen peroxide is stored away from light because it can get decomposed when exposed to light as

(a) H_2O and O_2 (b) H_2 and O_2

(c) H_2O and H_2 (d) H_2O and O_3

135. Which of the following reactions represents "slaking of lime"?

(a) $CaCO_3 \rightleftharpoons CaO + CO_2$

(b) $CaO + CO_2 \longrightarrow CaCO_3$

(c) $CaO + H_2O \longrightarrow Ca(OH)_2$

(d) $CaO + SiO_2 \longrightarrow CaSiO_3$

136. H_3BO_3 is

(a) monobasic and weak Lewis acid

(b) monobasic and weak Bronsted acid

(c) monobasic and strong Lewis acid

(d) tribasic and weak Bronsted acid

137. In CO_2 molecule, the hybridisations of carbon and oxygen atoms are respectively.

(a) Carbon Oxygen
sp sp^2

(b) Carbon Oxygen
sp^2 sp^2

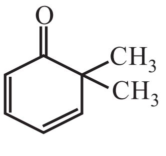
(c) Carbon Oxygen
sp sp

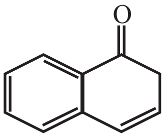
(d) Carbon Oxygen
sp^2 sp^3

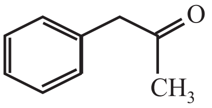
138. The consequence of global warming may be the following.

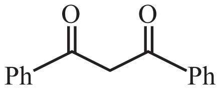
(a) Decrease in average temperature of the earth
(b) Melting of Himalayan glaciers
(c) Eutrophication
(d) Increased biochemical oxygen demand

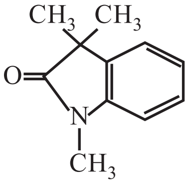
139. Which of the following carbonyl compounds will exhibit enolisation?

(i) 

(ii) 

(iii) 

(iv) 

(v) 

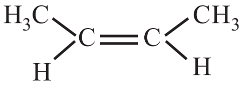
(a) (i), (ii) and (iii) (b) (ii), (iii) and (iv)
(c) (iii), (iv) and (v) (d) (i), (iii) and (v)

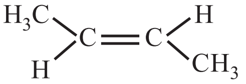
140. The major product formed in the following reaction is

$$H_3C—C\equiv C—CH_3 + Na \xrightarrow{\text{Liquid } NH_3}$$

(a) $H_3C—CH_2—C\equiv C^-Na^+$

(b) $H_3C—C\equiv C—CH_2^-Na^-$

(c) 

(d) 

141. Which one of the following reactions gives the product with less number of carbon atoms than the starting material?

(a) $R—X \xrightarrow{\text{Na, Ether}}$

(b) $RCO_2Na \xrightarrow[\Delta]{\text{NaOH,CaO}}$

(c) $RCO_2K \xrightarrow{\text{Electrolysis}}$

(d) $R—X \xrightarrow{\text{Zn,HCl}}$

142. $LiCoO_2$ crystallises in a rhomobohedral structure. Consider a situation where 50% lithium (Li) is extracted from the lattice. To keep the crystal electrically neutral, change in average oxidation state of Co is

(a) 16.66% decrease (b) 16.66% increase
(c) 50% increase (d) 50% decrease

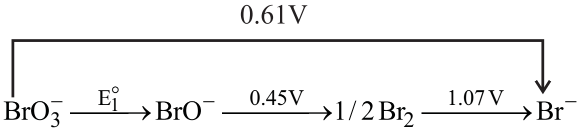
143. The freezing point depression of 0.001 M of $A_xB_y[Fe(CN)_6]$ is 5.58×10^{-3} K. If the oxidation state of Fe is +2 and K_f = 1.86 kg mol^{-1}, then the total number of possibilities for different types of A and B cations are

(a) 1 (b) 2 (c) 3 (d) 4

144. When 2.44 grams of benzoic acid (C_6H_5COOH) dissolved in 25 grams of benzene, it shows depression of freezing point equal to 2.2 K. Molal depression constant of benzene is 5.0 K kg mol^{-1} . What is the percentage association of acid, if it forms dimer in solution?

(a) 50% (b) 77% (c) 95% (d) 90%

145.



$$BrO_3^- \xrightarrow{E_1^\circ} BrO^- \xrightarrow{0.45V} 1/2\,Br_2 \xrightarrow{1.07\,V} Br^-$$

The value of $E°_1$ is

(a) 0.76 V (b) 0.535 V (c) 0.428 V (d) 1.12 V

146. The rate of chemical reaction doubles with every 10°C rise in temperature. If the reaction is carried out in the vicinity of 22°C, the activation energy of the reaction is (given R = 8.3 J K^{-1} mol^{-1}, ln 2 = 0.69 and ln 3 = 1.1)

(a) 1.69 kJ mol^{-1} (b) 0.169 kJ mol^{-1}
(c) 49.8 kJ mol^{-1} (d) 498 J mol^{-1}

147. In an adsorption experiment, a graph between $\log\left(\frac{x}{m}\right)$ vs log p was found to be linear with a slope of 45°. The intercept on the $\log\left(\frac{x}{m}\right)$ axis was found to be 0.3. The amount of the gas adsorbed per gram of charcoal under a pressure of 1 atm is

(a) 1 (b) 2 (c) 3 (d) 4

148. Ellingham diagram is drawn between

(a) change in potential of pH
(b) change in free energy and oxidation state
(c) change in free energy and temperature
(d) change in free energy and polarisability

149. Brown ring test is used to detect the presence of

(a) NO_3^- (b) Cl^- (c) I^- (d) Br^-

150. Oxidation state of S in $H_2S_2O_8$ is

(a) 8 (b) 6 (c) 4 (d) 7

151. Which of the following sets correctly represents the increase in the paramagnetic property of the ions?

(a) $Cu^{2+} < V^{2+} < Cr^{2+} < Mn^{2+}$
(b) $Cu^{2+} < Cr^{2+} < V^{2+} < Mn^{2+}$
(c) $Mn^{2+} < V^{2+} < Cr^{2+} < Cu^{2+}$
(d) $Mn^{2+} < Cu^{2+} < Cr^{2+} < V^{2+}$

152. The coordination complex $[Co(OH_2)_6]^{2+}$ has one unpaired electron, which of the following statements are true?

(i) The complex is octahedral.
(ii) The complex is an outer orbital complex.
(iii) The complex is diamagnetic.

(a) (i) and (iii) only
(b) (i), (ii) and (iii)
(c) (i) and (ii) only
(d) (ii) and (iii) only

153. Which one of the following is made by using step growth polymerisation?

(a) Nylon 6, 6
(b) Teflon
(c) Rubber
(d) Neoprene

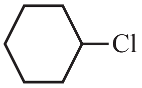
154. Insulin and glucagon, which are responsible to maintain the blood glucose level, fall under

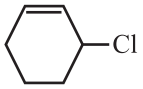
(a) antibodies
(b) harmones
(c) enzymes
(d) transport agents

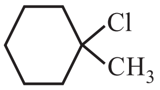
155. Which one of the following statements is "not correct" about cationic detergents?

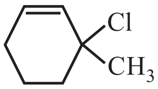
(a) Cationic detergents are quaternary ammonium salts
(b) Cationic part contains a long hydrocarbon chain
(c) Cationic detergents exhibit germicidal properties
(d) Cationic detergents are cheap and are widely used

156. Which one of the following will be most reactive for S_N1 reaction?

(a)


(b)


(c)


(d)


157. Which of the following alcohols gives white turbidity almost immediately with the Lucas reagent at room temperature?

(i) *n*-butanol
(ii) *tert*-butanol
(iii) Benzyl alcohol
(iv) Allylic alcohol

(a) (i), (ii) and (iii)
(b) (i), (iii) and (iv)
(c) (ii), (iii) and (iv)
(d) (i), (ii) and (iv)

158. Which one among the following reaction products gives iodoform test?

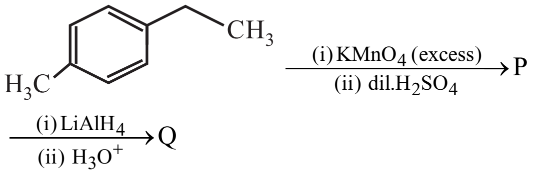
(a) $CH_3CH_2CH_2C \equiv N \xrightarrow[\text{(ii)}\,H_3O^+]{\text{(i)}\,C_2H_5MgBr}$

(b) $(C_6H_5)_2Cd \xrightarrow{CH_3COCl}$

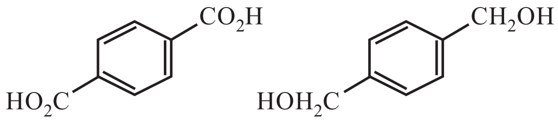
(c) $CH_2 = CHCH_2CH(OH)C_6H_5 \xrightarrow{PCC}$

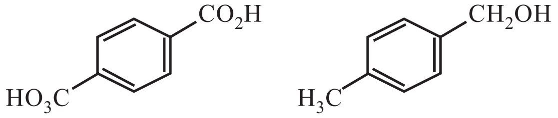
(d) $C_6H_5CH_2CN \xrightarrow[\text{(ii)}\,H_3O^+]{\text{(i)}\,C_2H_5MgBr}$

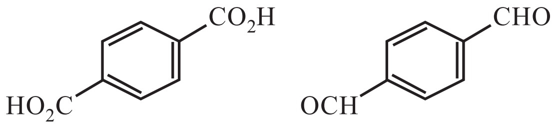
159. The major products P and Q in the following reaction sequence are

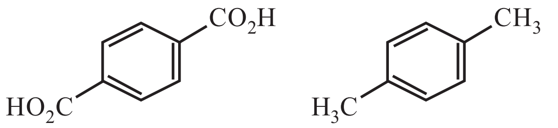


P Q

(a)


(b)


(c)


(d)


160. Which one of the following compounds undergoes Hofmann degradation reaction?

(a) CH_3CN
(b) $CH_3CONHCH_3$
(c) CH_3CONH_2
(d) CH_3NC

ANSWER KEY

1	(a)	**2**	(d)	**3**	(a)	**4**	(b)	**5**	(a)	**6**	(c)	**7**	(a)	**8**	(Bonus)	**9**	(d)	**10**	(c)
11	(a)	**12**	(c)	**13**	(b)	**14**	(d)	**15**	(a)	**16**	(a)	**17**	(b)	**18**	(Bonus)	**19**	(d)	**20**	(b)
21	(c)	**22**	(b)	**23**	(b)	**24**	(c)	**25**	(a)	**26**	(a)	**27**	(d)	**28**	(a)	**29**	(b)	**30**	(b)
31	(b)	**32**	(b)	**33**	(a)	**34**	(c)	**35**	(c)	**36**	(c)	**37**	(c)	**38**	(c)	**39**	(b)	**40**	(d)
41	(b)	**42**	(a)	**43**	(c)	**44**	(d)	**45**	(a)	**46**	(b)	**47**	(d)	**48**	(d)	**49**	(d)	**50**	(b)
51	(a)	**52**	(b)	**53**	(a)	**54**	(c)	**55**	(d)	**56**	(c)	**57**	(c)	**58**	(c)	**59**	(c)	**60**	(b)
61	(a)	**62**	(a)	**63**	(a)	**64**	(b)	**65**	(d)	**66**	(d)	**67**	(c)	**68**	(b)	**69**	(c)	**70**	(a)
71	(c)	**72**	(d)	**73**	(a)	**74**	(a)	**75**	(d)	**76**	(d)	**77**	(a)	**78**	(d)	**79**	(b)	**80**	(a)
81	(c)	**82**	(d)	**83**	(c)	**84**	(b)	**85**	(c)	**86**	(c)	**87**	(b)	**88**	(a)	**89**	(b)	**90**	(a)
91	(a)	**92**	(c)	**93**	(b)	**94**	(a)	**95**	(b)	**96**	(b)	**97**	(b)	**98**	(d)	**99**	(a)	**100**	(a)
101	(c)	**102**	(d)	**103**	(b)	**104**	(b)	**105**	(a)	**106**	(b)	**107**	(b)	**108**	(a)	**109**	(b)	**110**	(b)
111	(b)	**112**	(c)	**113**	(c)	**114**	(b)	**115**	(c)	**116**	(d)	**117**	(a)	**118**	(c)	**119**	(d)	**120**	(b)
121	(d)	**122**	(c)	**123**	(c)	**124**	(c)	**125**	(b)	**126**	(d)	**127**	(b)	**128**	(d)	**129**	(d)	**130**	(c)
131	(a)	**132**	(a)	**133**	(b)	**134**	(a)	**135**	(c)	**136**	(a)	**137**	(a)	**138**	(b)	**139**	(b)	**140**	(d)
141	(b)	**142**	(b)	**143**	(c)	**144**	(d)	**145**	(b)	**146**	(c)	**147**	(b)	**148**	(c)	**149**	(a)	**150**	(b)
151	(a)	**152**	(c)	**153**	(a)	**154**	(b)	**155**	(d)	**156**	(d)	**157**	(c)	**158**	(b)	**159**	(a)	**160**	(c)

Hints & Solutions

MATHEMATICS

1. **(a)** (A) We know that

Since, $0 \le \cos^2 x \le 1$

$\Rightarrow \quad 1 \le 1 + \cos^2 x \le 2$

$\therefore \quad [1 + \cos^2 x]$ [1, 2]

So, range of $\sec^{-1} [1 + \cos^2 x] = \{\sec^{-1} 1, \sec^{-1} 2\}$

(A) → (V)

(B) We have,

$$f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$$

Put y = $x+\frac{1}{x}$

Now, AM ≥ GM

Case I: When $x > 0, x+\frac{1}{x} \ge 2\sqrt{x\cdot\frac{1}{x}}$

$\Rightarrow \quad x+\frac{1}{x} \ge 2$

Case II : When $x < 0$, $\dfrac{-|x|+\frac{1}{-|x|}}{2} \ge \sqrt{(-|x|)\times\frac{1}{-|x|}}$

$\Rightarrow \quad -\left(|x|+\frac{1}{|x|}\right) \ge 2 \Rightarrow |x|+\frac{1}{|x|} \le -2$

$\therefore \quad x+\frac{1}{x} \in \mathbf{R}$

R – (–2, 2)

So, domain of f(x) is R – (–2, 2)

(B) → (IV)

(C) We have

$f(x + y) = f(x) + f(y)$

Put $x = y = 0$, we get

$f(0) = f(0) + f(0)$

$\Rightarrow \quad f(0) = 2f(0)$

And $2f(0) - f(0) = 0 \Rightarrow f(0) = 0$

Now, put $y = -x$, we get

$f(x - x) = f(x) + f(-x)$

$\Rightarrow \quad f(0) = f(x) + f(-x)$

$\Rightarrow \quad f(x) + f(-x) = 0 \qquad [\because f(0) = 0]$

$\Rightarrow \quad f(-x) = -f(x)$

So $f(x)$ is a odd function.

(C) → I

(D) Given

$\sin^{-1} x - \cos^{-1} x + \sin^{-1} (1 - x) = 0$

or $\sin^{-1}(1 - x) = \cos^{-1} x - \sin^{-1} x$

or $\sin^{-1}(1 - x) = \frac{\pi}{2} - 2\sin^{-1} x$

or $1 - x = \sin\left(\frac{\pi}{2} - 2\sin^{-1} x\right)$

or $1 - x = \cos (2 \sin^{-1}x)$

We know that

$2 \sin^{-1} (x) = \sin^{-1}(2x\sqrt{1-x^2})$

or $1 - x = \cos(\sin^{-1} 2x\sqrt{1-x^2})$

or $1 - x = \cos \cos^{-1} (1 - 2x^2)$

or $1 - x = 1 - 2x^2$

or $2x^2 - x = 0 \Rightarrow x (2x - 1) = 0$

$x = 0, 1/2 \Rightarrow x \in \left\{0, \frac{1}{2}\right\}$

(D) → II

2. **(d)** We have,

$f(x) = \sec^{-1} (3x - 4) + \tan h^{-1} \left(\frac{x+3}{5}\right)$

$\because \tan h^{-1} = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

$$= \sec^{-1}(3x-4) + \frac{1}{2}\log\left(\frac{1+\frac{x+3}{5}}{1-\frac{x+3}{5}}\right)$$

$$= \sec^{-1}(3x-4) + \frac{1}{2}\log\left(\frac{8+x}{2-x}\right)$$

Now, for $\sec^{-1} (3x - 4)$

$3x - 4 \le -1 \cup 3x - 4 \ge 1$

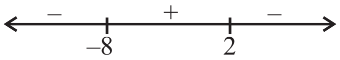
$\Rightarrow \quad 3x \le 3 \cup 3x \ge 5$

$\Rightarrow \quad x \le 1 \cup x \ge \frac{5}{3}$

$\therefore \quad x \in (-\infty, 1] \cup x \in \left[\frac{5}{3}, \infty\right)$

Again, for $\ln \left(\frac{8+x}{2-x}\right)$

$\frac{8+x}{2-x} > 0$ or $\frac{x+8}{x-2} < 0$



$x \in (-8, 2)$

$\therefore \quad$ Domain of $f(x) = (-8, 1] \cup \left[\frac{5}{3}, 2\right)$

3. **(a)** Given,

$m = (9n^2 + 54n + 80) (9n^2 + 45n + 54) (9n^2 + 36n + 35)$

$= [(9n^2 + 30n + 24n + 80)] [(9n^2 + 27n + 18n + 54)]$
$[(9n^2 + 15n + 21n + 35)]$

$= [3n (3n + 10) + 8 (3n + 10)] [9n (n + 3) + 18 (n + 3)]$
$[3n (3n + 5) + 7 (3n + 5)]$

$= (3n+10)(3n+8)(n+3)(9n+18)(3n+5)(3n+7)$

$= (3n+5)(3n+6)(3n+7)(3n+8)(3n+9)(3n+10)$

Now, we know that

$n(n+1)(n+2)\ldots(n+r-1)$ is divisible by r!

$\therefore$ m is divisible by 6! i.e., 720.

4. **(b)** We have, according to question,

$$(\text{adj}\,(A))^T = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -5 & -2 \\ -2 & 4 & 1 \end{bmatrix}$$

$$\Rightarrow \text{adj}\,(A) = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -5 & -2 \\ -2 & 4 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\therefore |\text{adj}\,A| = \begin{vmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{vmatrix}$$

$= 1(-5+8) - 4(-2-4) - 2(+4+5) = 9$

Now, we know that

$|\text{adj}\,A| = |A|^{n-1}$

$\Rightarrow |A|^{3-1} = 9 \Rightarrow |A|^2 = 9 \Rightarrow |A| = \pm 3$

5. **(a)** Given,

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

then $A^2 = A.A$

$$\text{Hence, } A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

Now, $|A^2| = 3(3-0) + 4(0) + 4(0-2) = 1$

Now, cofactors of A^2 to obtain adj (A^2)

$C_{11} = 3, C_{12} = 0, C_{13} = -2, C_{21} = -4, C_{22} = -1,$

$C_{23} = 2, C_{31} = -4, C_{32} = 0, C_{33} = -3$

$$\therefore \text{adj}\,(A^2) = \begin{bmatrix} 3 & 0 & -2 \\ -4 & -1 & 2 \\ -4 & 0 & -3 \end{bmatrix}^T = \begin{bmatrix} 3 & -4 & -4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$$\therefore (A^2)^{-1} = \frac{1}{|A^2|}\text{adj}(A^2) = \frac{1}{1}\begin{bmatrix} 3 & -4 & -4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} = A^2$$

$\therefore (A^2)^{-1} = A^2$

6. **(c)** We have, $x + y + z = 3$

$x + 2y + 2z = 6$

$x + ay + 3z = b$

Above equations can be written as AX = B.

$$\text{Where, A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & a & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 3 \\ 6 \\ b \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & a & 3 \end{vmatrix} = 1[6-2a] - 1[3-2] + 1[a-2]$$

$= 6 - 2a - 1 + a - 2 = 3 - a$

For unique solution

$|A| \neq 0 \Rightarrow 3 - a \neq 0 \Rightarrow a \neq 3$

$\therefore$ Given system have unique solution when $a \neq 3$, b can have any value.

7. **(a)** Given C is a complex number

$$= \frac{1+iz}{1-iz} = \frac{1+a+ib-c}{1+a-ib+c} = \frac{(1+a-c)+ib}{(1+a+c)-ib}$$

$$= \frac{(1+a-c)+ib}{(1+a+c)-ib} \times \frac{(1+a+c)+ib}{(1+a+c)+ib} \qquad \left\{\text{Given, } z = \frac{b+ic}{1+a}\right\}$$

$$\text{Here } iz = \frac{ib-c}{1+a} \Rightarrow -iz = \frac{-ib+c}{1+a}$$

$$= \frac{(1+a-c)(1+a+c)+(1+a-c)bi + (1+a+c)bi - b^2}{(1+a+c)^2+b^2}$$

$$= \frac{(1+a+c+a+a^2+ac-c-ac-c^2-b^2) + ib(1+a-c+1+a+c)}{(1+a+c)^2+b^2}$$

$$= \frac{1+2a+a^2-(1-a^2)+2(1+a)ib}{1+a^2+c^2+2a+2ac+2c+b^2}$$

$$= \frac{2a(1+a)+2(1+a)ib}{2+2a+2ac+2c} = \frac{2(1+a)(a+ib)}{2(1+a)+2c(1+a)}$$

$$= \frac{2(1+a)(a+ib)}{2(1+a)(1+c)} = \frac{a+ib}{1+c}$$

8. **(Bonus)** If $z = x + iy$, then $\bar{z} = x - iy$,

$$\text{As } \frac{\bar{z}-1}{z-i} = 2 \Rightarrow \left|\frac{\bar{z}-1}{z-i}\right| = 2$$

$$\text{or, } \left|\frac{\bar{z}-1}{z-i}\right| = 2$$

$\Rightarrow (x-1)^2 + y^2 = 4[x^2 + (y-1)^2]$

$\Rightarrow 3x^2 + 3y^2 + 2x - 8y + 3 = 0$

$$\Rightarrow x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + 1 = 0$$

So, the locus of point $P(x, y)$ in the cartesian plane is a circle of radius $\frac{\sqrt{10}}{3}i$ and the centre $\left(-\frac{1}{3}, \frac{4}{3}\right)$

No option is correct.

9. **(d)** We have, $(1-\omega+\omega^2)^6+(1-\omega^2+\omega)^6$

$=(-\omega-\omega)^6+(-\omega^2-\omega^2)^6$ $[\because 1+\omega+\omega^2=0]$

$=(-2\omega)^6+(-2\omega^2)^6$

$(-2)^6[\omega^6+\omega^{12}]=2^6.2=2^7=128$ $[\because \omega^3=1]$

10. **(c)** We have

$z^n+p_1z^{n-1}+p_2z^{n-2}+\ldots+p_n=0$

Given equation is polynomial in z

Hence dividing equation (1) by z^n, we get

$\Rightarrow 1+p_1z^{-1}+p_2z^{-2}+\ldots+p_nz^{-n}=0$

Now, $z=e^{ix}$ satisfied the above equation

$\therefore 1+p_1e^{-ix}+p_2e^{-2ix}+\ldots+p_ne^{-inx}=0$

Since, $e^{i\theta}=\cos 3\theta+i\sin\theta$

$\Rightarrow (1+p_1\cos x+p_2\cos x+\ldots+p_n\cos nx)$
$-i(p_1\sin x+p_2\sin 2x+\ldots+p_n\sin nx)=0$

$\therefore p_1\sin x+p_2\sin 2x+\ldots+p_n\sin nx=0$

$\Rightarrow p_n\sin nx+p_{n-1}\sin(n-1)x+\ldots+p_2\sin 2x$
$+p_1\sin x+1=1$

11. **(a)** The quadratic expression ax^2+bx+c and a will have the same sign for some value of $x\in R$ when $b^2-4ac>0$

And the quadratic inequality

$3x^2-16x+4>-16$

$\Rightarrow 3x^2-16x+20>0$ for some value of $x\in$ R as

or $x^2-\frac{16}{3}x+\frac{20}{3}>0$ ...(i)

Here roots are $\alpha,\beta=\frac{\frac{16}{3}\pm\frac{4}{3}}{2}=\frac{16\pm4}{6}=\frac{10}{3},2$

Here $x\in(-\infty,2)\cup\left(\frac{10}{3},\infty\right)$

$\Rightarrow$ Expression (1) is satisfied for some values of $x\in(-\infty,2)$

i.e. for $x=\frac{1}{2},\frac{1}{3},\cdots,1$

$\Rightarrow$ Assertion (A) if true

12. **(c)** We have,

Equation $ax^2+bx+c=0$ has imaginary roots

$\therefore b^2-4ac<0\Rightarrow b^2<4ac$

Now, $f(x)=3a^2x^2+6abx+2b^2$

$$=3a^2\left(x^2+\frac{2b}{a}x\right)+2b^2$$

$$=3a^2\left[\left(x+\frac{b}{a}\right)^2-\frac{b^2}{a^2}\right]+2b^2$$

$$=3a^2\left(x+\frac{b}{a}\right)^2-3b^2+2b^2=3a^2\left(x+\frac{b}{a}\right)^2-b^2$$

$\therefore$ Minimum value of $f(x)$ is $-b^2>-4ac$ $[\because b^2<4ac]$

13. **(b)** We have, $x^3+px+q=0$

Since, α,β,γ are the roots of the given equation

$\alpha+\beta+\gamma=-\frac{b}{a}=0\Rightarrow\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}=p$

$\alpha\beta\gamma=\frac{-d}{a}=-q$

Let $S_1=\alpha+\beta+\gamma=\Sigma\alpha$

$S_2=\alpha\beta+\beta\gamma+\gamma\alpha=\Sigma\alpha\beta$ and $S_3=\alpha\beta\gamma$

Now, $\Sigma\alpha^2\beta=S_1S_2-3S_3=(0)(p)-3(-q)=3q$

Again, $x^3+px+q=0$

$\Rightarrow x^3=-[px+q]\Rightarrow x^4=-[px^2+qx]$

$\therefore \Sigma\alpha^4=-[p\Sigma\alpha^2+q\Sigma\alpha]$

$=-[p((\Sigma\alpha)^2-2(\Sigma\alpha\beta))+q\Sigma\alpha]$

$=-[p(S_1^2-2S_2)+qS_1]$

$=-[p(0-2p)+q\times0]=2p^2$

$\therefore \Sigma\alpha^2\beta+\Sigma\alpha^4=3q+2p^2$

Now, $f(x)=3p^2x^2+p^2x+3q$

$\therefore f(1)=3p^2+p^2+3q=4p^2+3q$

$f(-1)=3p^2-p^2+3q=2p^2+3q$

$f(0)=3q$

$f(2)=12p^2+2p^2+3q=14p^2+3q$

$\therefore \Sigma\alpha^2\beta+\Sigma\alpha^4=f(-1)$

14. **(d)** We have, $x^3+ax^2-bx+c=0$

$\therefore$ Let $\Sigma\alpha=-\frac{(a)}{1}=-a$

$\Sigma\alpha\beta=\frac{(-b)}{1}=-b$

$\alpha\beta\gamma=-\frac{(c)}{1}=-c$

$$\begin{bmatrix}\text{Let } \Sigma\alpha=S_1\text{(say)}\\ \Sigma\alpha\beta=S_2\text{(say)}\\ \alpha\beta\gamma=S_3\text{ (say)}\end{bmatrix}$$

Now, $\Sigma\beta^2(\gamma+\alpha)=S_1S_2-3S_3$

$=(-a)(-b)-3(-c)=ab+3c$

15. **(a)** Total number of words that can be formed with the letters of the word 'MATHEMATICS'

$=\frac{11!}{2!2!2!}=4989600$

Number of words in which all the repeated letters are together $=8!\times\frac{2!}{2!}\times\frac{2!}{2!}\times\frac{2!}{2!}=8!=40320$

$\therefore$ Required words $=4989600-40320=4949280$

It is given that

$982(X)=4949280$

$\Rightarrow X=\frac{4949280}{982}=5040$

16. **(a)** We have, 10 Red and 5 Yellow roses of different sizes.

Now, x = number of garlands that can be formed with all these flowers so that no two yellow roses come together

Here, 5 yellow roses can be arrange in 10 places

$$=\frac{1}{2}(10-1)!\times{}^{10}P_5=\frac{1}{2}\times9!\times\frac{10!}{5!}=x=\frac{10!}{2}\left[\frac{9!}{5!}\right]$$

$$\Rightarrow\frac{2x}{10!}=\frac{9!}{5!}$$

and y = number of garlands formed with all these flowers so that all the red roses coming together

$$Y=\frac{1}{2}(6-1)!\times10!=\frac{1}{2}\times5!\times10!$$

$$\Rightarrow \quad \frac{2y}{10!}=5! = \frac{2(x-y)}{10!}=\frac{9!}{5!}-5!$$

17. **(b)** We have,

$$y=\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+...\infty$$

$$y+1=1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+...\infty$$

$$=1+\left(\frac{-3}{2}\right)\left(\frac{-1}{2}\right)+\frac{\left(\frac{-3}{2}\right)\left(\frac{-3}{2}-1\right)\left(\frac{-1}{2}\right)^2}{2!}$$

$$+\frac{\left(\frac{-3}{2}\right)\left(\frac{-3}{2}-1\right)\left(\frac{-3}{2}-2\right)\left(\frac{-1}{2}\right)^3}{3!}+...$$

$$=\left(1-\frac{1}{2}\right)^{-3/2}=\left(\frac{1}{2}\right)^{-3/2}=2^{3/2}=2\sqrt{2}$$

$$y+1=2\sqrt{2}$$

$\Rightarrow \quad (y+1)^2 = 8 \Rightarrow y^2+2y-7=0$

18. **(Bonus)** Given expression is $\dfrac{(1+x)^2}{(1-2x)^3}$

$= (1+x)^2 (1-2x)^{-3}$
$= (1+2x+x^2)(1-2x)^{-3}$

Now, the coefficient of x^{13} in the expansion of $\dfrac{(1+x)^2}{(1-2x)^3}$
$=$ Coefficient of x^{11} in $(1-2x)^{-3}$
$+2$ {coefficient of x^{12} in $(1-2x)^{-3}$}
$+$ Coefficient of x^{13} in $(1-2x)^{-3}$

$$= {}^{3+11-1}C_{11}(-2)^{11}+2\{{}^{3+12-1}C_{12}(-2)^{12}\}$$
$$+{}^{3+13-1}C_{13}(-2)^{13}$$
$$= {}^{15}C_{13}(-2)^{13}+{}^{14}C_{12}(2)^{13}+{}^{13}C_{11}(-2)^{11}$$

$= 2^{10}[-112-156] = (-268)\times 2^{10} = A\times 2^{10}$ (given)
Hence A = – 268

19. **(d)** We have,

$$\frac{x^2+\mathrm{I}}{x^3+3x^2+3x+2}=\frac{A}{x+2}+\frac{B}{x^2+x+1}$$
$$+\frac{C}{(x+2)(x^2+x+1)}$$

Multiplying by x^3+3x^2+3x+2 both side
Hence, $x^2+1 = A(x^2+x+1)+B(x+2)+\mathrm{C}$
$\Rightarrow \quad x^2+1 = Ax^2+Ax+A+Bx+2B+\mathrm{C}$
$\Rightarrow \quad x^2+1 = Ax^2+(A+B)x+A+2B+\mathrm{C}$
On comparing, we get A = 1
$A+B=0$
$A+2B+C=1$
$\therefore A=1, \mathrm{B}=-1, C=2$
$\therefore A-B+C=1-(-1)+2=4$

20. **(b)** We have, $\sqrt{x}+\dfrac{1}{\sqrt{x}}=2\cos\theta$

On squaring both side

$$\Rightarrow \quad \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=4\cos^2\theta \Rightarrow x+\frac{1}{x}=4\cos^2\theta-2$$

$$\Rightarrow \quad x+\frac{1}{x}=2(2\cos^2\theta-1) \Rightarrow x+\frac{1}{x}=2\cos 2\theta$$

Again, $\left(x+\dfrac{1}{x}\right)^2=4\cos^2 2\theta \Rightarrow x^2+\dfrac{1}{x^2}=2\cos 4\theta$

On cubing both side $\Rightarrow \left(x^2+\dfrac{1}{x^2}\right)^3=8\cos^3 4\theta$

$$x^6+\frac{1}{x^6}+3\left(x^2+\frac{1}{x^2}\right)=8\left[\frac{\cos 12\theta+3\cos 4\theta}{4}\right]$$

$$\Rightarrow \quad x^6+\frac{1}{x^6}+6\cos 4\theta=2\cos 12\theta+6\cos 4\theta$$

$\Rightarrow \quad x^6+x^{-6}=2\cos 12\theta$

21. **(c)** We have, $\dfrac{99\pi}{2}\le\theta\le\dfrac{100\pi}{2}\left[\text{or}, 49\pi+\dfrac{\pi}{2}\le\theta\le 50\pi\right]$

Hence $\theta \in$ IV quadrant
where $|\sin\theta| = -\sin\theta$ and $|\cos\theta| = \cos\theta$
$\therefore \quad A = \sin|\sin\theta| = \sin(-\sin\theta) = -\sin^2\theta$
and $B = \cos\theta \ |\cos\theta| = \cos\theta \ (\cos\theta) = \cos^2\theta$
$\therefore \quad B-A = \cos^2\theta-(-\sin^2\theta)$
$= \cos^2\theta+\sin^2\theta = 1$

22. **(b)** We have,

$$\frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)}+\frac{\cos(\theta_3-\theta_4)}{\cos(\theta_3+\theta_4)}=0$$

$$\Rightarrow \quad \frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)}=-\frac{\cos(\theta_3-\theta_4)}{\cos(\theta_3+\theta_4)}$$

By using componendo and dividendo

$$\Rightarrow \quad \frac{\cos(\theta_1+\theta_2)-\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)+\cos(\theta_1-\theta_2)}$$

$$=\frac{-\cos(\theta_3-\theta_4)-\cos(\theta_3+\theta_4)}{-\cos(\theta_3-\theta_4)+\cos(\theta_3+\theta_4)}$$

$$\Rightarrow \quad \frac{-2\sin\theta_1\sin\theta_2}{2\cos\theta_1\cos\theta_2}=\frac{-2\cos\theta_3\cos\theta_4}{-2\sin\theta_3\sin\theta_4}$$

$$\Rightarrow \quad -\tan\theta_1\tan\theta_2=\cot\theta_3\cot\theta_4$$

$$\Rightarrow \quad \cot\theta_1\cot\theta_2\cot\theta_3\cot\theta_4=-1$$

23. **(b)** We have

$\sin^4 x-(K+3)\sin^2 x-K-4=0$

Let $\sin^2 x = y, y \in [0, 1]$

$$\text{Hence } y=\frac{(K+3)\pm\sqrt{(K+3)^2+4(K+4)}}{2}$$

$$=\frac{(K+3)\pm\sqrt{(K+5)^2}}{2}$$

$$=\frac{K+3\pm(K+5)}{2}=\frac{2K+8}{2},-1$$

$= K + 4, -1 \Rightarrow y = K + 4$ $\{\because y \in [0, 1]\}$

or $0 \le \sin^2 x \le 1$

$\Rightarrow$ $0 \le K + 4 \le 1 \Rightarrow -4 \le \text{K} \le -3$

24. **(c)** A. We have,

$$\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) + \sin^{-1}\left(\frac{1}{3}\right)$$

We know,

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$$

Hence, $\sin^{-1}\left[\frac{2\sqrt{2}}{3}\sqrt{1-\frac{1}{9}} + \frac{1}{3}\sqrt{1-\frac{8}{9}}\right]$

$$= \sin^{-1}\left[\frac{2\sqrt{2}}{3} \times \frac{2\sqrt{2}}{3} + \frac{1}{3} \times \frac{1}{3}\right]$$

$$= \sin^{-1}\left[\frac{8}{9} + \frac{1}{9}\right] = \sin^{-1} 1 = \frac{\pi}{2}$$

$\therefore$ (A) → (V)

B. We have $\sin^{-1}\left(\frac{(-1)^n}{2}\right) = x \;\because D_{f(x)} \equiv [-1, 1]$

$\Rightarrow$ $\sin x = \frac{(-1)^n}{2}$

$\Rightarrow$ $\sin x = \frac{-1}{2}$ or $\frac{1}{2}, x \in \boldsymbol{Z}$

Now, $\sin x = \frac{1}{2}$

$\Rightarrow$ $x = n\pi \pm (-1)^n \frac{\pi}{6}$ and $\sin x = -\frac{1}{2}$

$\Rightarrow$ $x = n\pi \pm (-1)^n \left(\frac{-\pi}{6}\right)$

$\therefore$ $x = n\pi \pm (-1)^n \frac{\pi}{6}$

(B) → (I)

C. We have,

$$\tan^{-1}\left(\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right) = \tan^{-1}(\sqrt{2}+1) = \frac{3\pi}{8}$$

(C) → (IV)

D. Given,

$$\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$$

$\Rightarrow$ $y = \sqrt{y}$, where $y = \sin^{-1}|\sin x|$

$\Rightarrow$ $y^2 - y = 0$

$\Rightarrow$ $y(y-1) = 0 \Rightarrow y = 0, 1$

Now, $\sin^{-1}|\sin x|$ is periodic with period π In $(0, \pi)$,

$$\sin^{-1}|\sin x| = \begin{cases} 1, & x = 1 \\ \pi - 1, & \text{otherwise} \end{cases}$$

$\therefore$ General solution, $x = n\pi \pm 1, n \in \boldsymbol{Z}$

(D) → (II).

25. **(a)** We have,

$\sin \text{h}^{-1}(-2) + \operatorname{cosec} h^{-1}(-2) + \cot h^{-1}(-2)$

$$= \log\left(-2 + \sqrt{(-2)^2 + 1}\right) + \log\left(\sqrt{1 + \frac{1}{(-2)^2}} + \frac{1}{(-2)}\right)$$

$$+ \frac{1}{2}\left[\log\left[1 + \frac{1}{(-2)}\right] - \log\left(1 - \frac{1}{(-2)}\right)\right]$$

$$= \log(\sqrt{5} - 2) + \log\left(\frac{\sqrt{5}}{2} - \frac{1}{2}\right) + \frac{1}{2}\left[\log\frac{1}{2} - \log\frac{3}{2}\right]$$

$$= \log(\sqrt{5} - 2) + \log\frac{(\sqrt{5}-1)}{2} + \frac{1}{2}\log\frac{1}{3}$$

$$= \log\left[(\sqrt{5} - 2) \times \frac{(\sqrt{5}-1)}{2} \times \frac{1}{\sqrt{3}}\right]$$

$$= \log\left[\frac{5 - \sqrt{5} - 2\sqrt{5} + 2}{2\sqrt{3}}\right] = \log\left[\frac{7 - 3\sqrt{5}}{2\sqrt{3}}\right]$$

26. **(a)** We have, $a : b : c = 4 : 5 : 6$

$\Rightarrow$ $a = 4\text{K}, b = 5\text{K}, c = 6\text{K}$

We know, $R = \frac{abc}{4\Delta}$ and $r = \frac{\Delta}{s}$

Then, $\frac{R}{r} = \frac{\left(\frac{abc}{4\Delta}\right)}{\left(\frac{\Delta}{s}\right)} = \frac{abc\,s}{4\Delta^2}$

$\because \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

Hence $\Delta^2 = s(s-a)(s-b)(s-c)$

and $s = \frac{a+b+c}{2} = \frac{4K + 5K + 6K}{2} = \frac{15K}{2}$

$\therefore$ $\frac{R}{r} = \frac{\left[abc\frac{(a+b+c)}{2}\right]}{[4s(s-a)(s-b)(s-c)]}$

$$= \frac{abc}{8(s-a)(s-b)(s-c)}$$

$$= \frac{4K \times 5K \times 6K}{4\left(\frac{15K}{2} - 4K\right)\left(\frac{15K}{2} - 5K\right)\left(\frac{15K}{2} - 6K\right)}$$

$$= \frac{120K^3}{4 \times \frac{7K}{2} \times \frac{5K}{2} \times \frac{3K}{2}} = \frac{16}{7}$$

Required ratio = 16 : 7

27. **(d)** Given

$a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2} = 15$ $\left[\because \cos A = 2\cos^2\frac{A}{2} - 1\right]$

$\Rightarrow$ $a\left[\frac{1+\cos C}{2}\right] + c\left[\frac{1+\cos A}{2}\right] = 15$

$$\left[\because \cos C = \frac{b^2+a^2-c^2}{2ab}\right]$$

$$\Rightarrow \quad \frac{a}{2}\left(1+\frac{a^2+b^2-c^2}{2ab}\right)+\frac{c}{2}\left(1+\frac{b^2+c^2-a^2}{2bc}\right)=15$$

$$\Rightarrow \quad \frac{a}{2}\left[\frac{(a+b)^2-c^2}{2ab}\right]+\frac{c}{2}\left[\frac{(b+c)^2-a^2}{2bc}\right]=15$$

$$\Rightarrow \quad \frac{1}{4b}[(a+b)^2-c^2]+\frac{1}{4b}[(b+c)^2-a^2)]=15$$

$$\Rightarrow \quad \frac{1}{4b}[a^2+b^2+2ab-c^2+b^2+c^2+2bc-a^2]=15$$

$$\Rightarrow \quad \frac{1}{4b}[2b^2+2ab+2bc]=15=\frac{2b}{4b}[b+a+c]=15$$

$$\Rightarrow \quad a+b+c=30$$

Now, $\cot\frac{B}{2}=\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$

$$=\sqrt{\frac{s^2(s-b)^2}{s(s-a)(s-b)(s-c)}} \quad =\frac{s(s-b)}{\Delta}=\frac{15(15-10)}{15\sqrt{3}}$$

$$\left[\because s=\frac{a+b+c}{2}=\frac{30}{2}=15, b=10, \Delta=15\sqrt{3}\right]$$

$$=\frac{5}{\sqrt{3}}$$

28. **(a)** Given d_1, d_2, d_3 are diameter

Let $d_1 = 2r_1, d_2 = 2r_2, d_3 = 2r_3$

Now, $d_1d_2 + d_2d_3 + d_3d_1 = (2r_1)(2r_2) + (2r_2)(2r_3) + (2r_3)(2r_1)$

$= 4[r_1r_2 + r_2r_3 + r_3r_1]$ [By formula]

$$=4\left[\frac{\Delta}{s-a}\times\frac{\Delta}{s-b}+\frac{\Delta}{s-b}\times\frac{\Delta}{s-c}+\frac{\Delta}{s-c}\times\frac{\Delta}{s-a}\right]$$

$$=4\left[\frac{\Delta^2}{(s-a)(s-b)}+\frac{\Delta^2}{(s-b)(s-c)}+\frac{\Delta^2}{(s-c)(s-a)}\right]$$

$$=4\Delta^2\left[\frac{s-c+s-a+s-b}{(s-a)(s-b)(s-c)}\right]$$

$$=4\Delta^2\left[\frac{3s-(a+b+c)}{(s-a)(s-b)(s-c)}\right]$$

$$=4\Delta^2\left[\frac{3/2(a+b+c)-(a+b+c)}{(s-a)(s-b)(s-c)}\right]$$

$$=\frac{4\Delta^2}{2}\left[\frac{a+b+c}{(s-a)(s-b)(s-c)}\right]$$

$$=\frac{2s(s-a)(s-b)(s-c)(a+b+c)}{(s-a)(s-b)(s-c)}$$

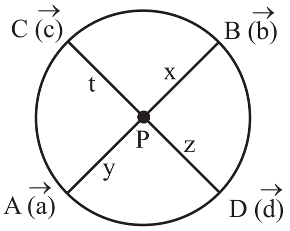
$= (a + b + c)(a + b + c) = (a + b + c)^2$

29. **(b)** Given, $A(\vec{a}), B(\vec{b}), C(\vec{c}), D(\vec{d})$ are concyclic points and $x\vec{a}+y\vec{b}+z\vec{c}+t\vec{d}=0$

$x + y + z + t = 0 \Rightarrow x + y = -(z + t)$

$$\frac{x\vec{a}+y\vec{b}}{x+y}=-\frac{(z\vec{c}+t\vec{d})}{x+y}$$

Here *x, y, z, t* not all zero at a time



$$\frac{x\vec{a}+y\vec{b}}{x+y}=\frac{(z\vec{c}+t\vec{d})}{z+t}$$

$$P=\frac{x\vec{a}+y\vec{b}}{x+y} \text{ or } P=\frac{z\vec{c}+t\vec{d}}{z+t}$$

$$AP=\frac{x\vec{a}+y\vec{b}}{x+y}-\vec{a}=x(\vec{a}-\vec{b})$$

$$BP=\frac{x\vec{a}+y\vec{b}}{x+y}-\vec{b}=y(\vec{a}-\vec{b})$$

$$CP=\frac{z\vec{c}+t\vec{d}}{z+t}-\vec{c}=t(\vec{d}-\vec{c})$$

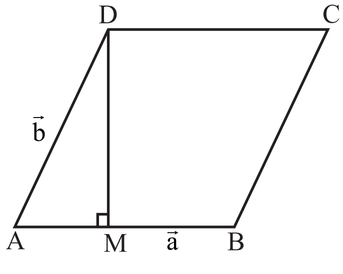
$$DP=\frac{z\vec{c}+t\vec{d}}{z+t}-\vec{d}=z(\vec{d}-\vec{c})$$

$\therefore$ AP × BP = CP × DP [By similarity]

$|x(\vec{a}-\vec{b})||y(\vec{a}-\vec{b})|=|t(\vec{d}-\vec{c})||z(\vec{d}-\vec{c})|$

$|xy||\vec{a}-\vec{b}|^2=|zt||\vec{c}-\vec{d}|^2$

30. **(b)**



Clearly,

AM = projection of $\vec{b}$ on $\vec{a}=\frac{\vec{a}.\vec{b}}{|\vec{a}|}$

$$\therefore \quad AM=\left\{\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right\}\vec{a}$$

In Δ AMD, we have

AD = AM + MD $\Rightarrow$ DM = AM – AD

$\Rightarrow \quad DM = \dfrac{(\vec{a}.\vec{b})\vec{a}}{|\vec{a}|^2} - \vec{b}$

Also, $DM = \dfrac{1}{|\vec{a}|^2}\{(\vec{a}.\vec{b})\vec{a} - |\vec{a}|^2\,\vec{b}\}$

$= \dfrac{1}{|\vec{a}|^2}\{(\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b}\} = \dfrac{\vec{a}\times(\vec{a}\times\vec{b})}{|\vec{a}|^2}$

31. **(b)** Given

$A(1, 2, 3)$, $B(3, 7, -2)$, $C(6, 7, 7)$, $D(-1, 0, -1)$

Let M be the centroid of ΔABD, then

$$M = \left(\frac{1+3-1}{3}, \frac{2+7+0}{3}, \frac{3-2-1}{3}\right) = (1, 3, 0)$$

and N be the centroid of ΔACD, then

$$N = \left(\frac{1+6-1}{3}, \frac{2+7+0}{3}, \frac{3+7-1}{3}\right) = (2, 3, 3)$$

$\therefore$ Equation of line passing through M (1, 3, 0) and N (2, 3, 3) is

$r = \hat{i} + 3\hat{j} + 0\hat{k} + t[(2-1)\hat{i} + (3-3)\hat{j} + (3-0)\hat{k})]$

$= \hat{i} + 3\hat{j} + t(\hat{i} + 3\hat{k}) = (1+t)\hat{i} + 3\hat{j} + 3t\hat{k}$

32. **(b)** Given, $\vec{a}$ and $\vec{b}$ are orthogonal vectors

$\therefore \quad \vec{a}.\vec{b} = 0$

$\Rightarrow \quad -\tan\theta + \tan^2\theta + \dfrac{3}{\sqrt{\sin\frac{\theta}{2}}}\left(-2\sqrt{\sin\frac{\theta}{2}}\right) = 0$

$\Rightarrow \quad -\tan\theta + \tan^2\theta - 6 = 0 \Rightarrow \tan^2\theta - \tan\theta - 6 = 0$

$\Rightarrow \quad (\tan\theta - 3)(\tan\theta + 2) = 0 \Rightarrow \tan\theta = 3, -2$

Now, c makes an obtuse angle with X-axis

$\therefore \quad \sin 2\theta < 0 \qquad [\because \sin\theta < 1]$

Since, $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$\Rightarrow \quad \dfrac{2\tan\theta}{1+\tan^2\theta} < 0 \Rightarrow \tan\theta < 0$

$\therefore \quad \tan\theta = -2 \Rightarrow \tan\theta = \tan(\tan^{-1}(-2))$

$\Rightarrow \quad \theta = n\pi + \tan^{-1}(-2) = n\pi - \tan^{-1}2$

33. **(a)** We have,

$(\vec{a}+\vec{b}).\vec{p} = (\vec{a}+\vec{b}).\dfrac{\vec{b}\times\vec{c}}{\vec{a}.(\vec{b}\times\vec{c})}$

$= \dfrac{\vec{a}.(\vec{b}\times\vec{c}) + \vec{b}.(\vec{b}\times\vec{c})}{\vec{a}.(\vec{b}\times\vec{c})} = \dfrac{[\vec{a}\,\vec{b}\,\vec{c}] + [\vec{b}\,\vec{b}\,\vec{c}]}{[\vec{a}\,\vec{b}\,\vec{c}]}$

$= \dfrac{[\vec{a}.\vec{b}\,\vec{c}]}{[\vec{a}\,\vec{b}\,\vec{c}]} = 1 \qquad [\because [\vec{b}\,\vec{b}\,\vec{c}] = 0]$

Similarly, $(\vec{b}+\vec{c}).\vec{q} = (\vec{c}+\vec{a}).\vec{r} = 1$

$\therefore \quad \alpha = 1 + 1 + 1 = 3$

Now, $(\vec{a}+\vec{b}).(\vec{b}+\vec{c})\times(\vec{a}+\vec{b}+\vec{c})$

$= [\vec{a}+\vec{b}\;\vec{b}+\vec{c}\;\vec{a}+\vec{b}+\vec{c}]$

$$= \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} [\vec{a}\,\vec{b}\,\vec{c}]$$

$= [1(1-1) - 1(0-1)][\vec{a}\,\vec{b}\,\vec{c}] = [\vec{a}\,\vec{b}\,\vec{c}]$

and $\vec{b}.(\vec{a}\times\vec{c}) = [\vec{b}\,\vec{a}\,\vec{c}] = -[\vec{a}\,\vec{b}\,\vec{c}]$

$\therefore \quad \beta = \dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{-[\vec{a}\,\vec{b}\,\vec{c}]} = -1$

$\therefore \quad \alpha + \beta = 3 - 1 = 2$

34. **(c)** It is given that, $\vec{a} + l\vec{b} + l^2\vec{c} = 0$

$\therefore \quad \vec{a}\times\vec{b} = l^2(\vec{b}\times\vec{c})$ and $\vec{c}\times\vec{a} = l(\vec{b}\times\vec{c})$

and $\vec{a}\times\vec{b} = l(\vec{c}\times\vec{a})$ and it is also given that,

$\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = 3(\vec{b}\times\vec{c})$

$\Rightarrow \quad \vec{a}\times\vec{b} + \vec{c}\times\vec{a} = 2(\vec{b}\times\vec{c})$

$\Rightarrow \quad l^2(\vec{b}\times\vec{c}) + l(\vec{b}\times\vec{c}) = 2(\vec{b}\times\vec{c})$

$\Rightarrow \quad l^2 + l - 2 = 0$

Here minimum value of $l = \dfrac{-D}{4a} = -\dfrac{9}{4}$

35. **(c)**

Cl	f_i	x_i	f_ix_i	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
0-4	4	2	8	4	16
4-8	3	6	18	0	0
8-12	2	10	20	4	8
12-16	1	14	14	8	8
Total	$\Sigma f_i = 10$		$\Sigma f_ix_i = 60$		$\Sigma f_i\lvert x_i - \bar{x}\rvert = 32$

Now, $\bar{x} = \dfrac{\Sigma f_ix_i}{\Sigma f_i} = \dfrac{60}{10} = 6$

$\therefore \quad$ Mean Deviation $= \dfrac{\Sigma f_i|x_i - \bar{x}|}{\Sigma f_i} = \dfrac{32}{10} = 3.2$

36. **(c)** Given,

$CV_A = \dfrac{\sigma_x}{\bar{x}}\times 100 = \dfrac{2}{x}\times 100$

and $CV_B = \dfrac{\sigma_y}{\bar{y}}\times 100 = \dfrac{3}{y}\times 100$

Now, A is more consistent than B

$\therefore \quad CV_A < CV_B$

$\Rightarrow \quad \dfrac{2}{x}\times 100 < \dfrac{3}{y}\times 100 \Rightarrow \dfrac{2}{x} < \dfrac{3}{y} \Rightarrow \dfrac{\bar{x}}{\bar{y}} > \dfrac{2}{3}$

Note : There must be $\dfrac{\bar{x}}{\bar{y}}$ in place of $\dfrac{\bar{y}}{\bar{x}}$ in question

37. **(c)** Here S = 36 and sum = 2/3/4/5/6/7

$$=\frac{1}{36}+\frac{2}{36}+\frac{3}{36}+\frac{4}{36}+\frac{5}{36}+\frac{6}{36}=\frac{21}{36}$$

And y = P (getting sum = 7 atleast once when pair of dice rolled n times)

= 1 – P (getting sum = 7 zero times)

$$=1-{}^{n}C_0\left(\frac{6}{36}\right)^0\left(\frac{30}{36}\right)^n=1-\left(\frac{30}{36}\right)^n$$

Now $y > x$

$$1-\left(\frac{5}{6}\right)^n>\frac{7}{12}\Rightarrow 1-\frac{7}{12}>\left(\frac{5}{6}\right)^n$$

Let $\frac{5}{6}=a$

$$(a)^n<\left(\frac{a}{2}\right)\Rightarrow (a)^n<\frac{a}{6}\times\frac{1}{2}\Rightarrow (a)^{n-1}<\frac{1}{2}$$

∴ Minimum value of n is 5

$$\Rightarrow \frac{5\times5\times5\times5}{6\times6\times6\times6}<\frac{1}{2}$$

38. **(c)** Let E be the event that sum of two numbers is even and F be the event that both the numbers are odd.

Since sum of two even numbers is even and sum of two odd numbers is even

Hence, $n(E)={}^6C_2+{}^7C_2=\frac{6\times5}{2\times1}+\frac{7\times6}{2\times1}$

= 15 + 21 = 36

$n(F)={}^7C_2=\frac{7\times6}{2\times1}=21$

and $n(E\cap F)={}^7C_2=\frac{7\times6}{2\times1}=21$

Required probability = P(F/E)

$$=\frac{P(F\cap \text{E})}{P(E)}=\frac{n(F\cap \text{E})}{n(E)}=\frac{21}{36}=\frac{7}{12}$$

39. **(b)** We know that,

$$\sum_{r=0}^{m}P(X)=1$$

$\Rightarrow 0.1 + 0.15 + 0.3 + 0.25 + k + k = 1$

$\Rightarrow 0.80 + 2k = 1 \Rightarrow 2k = 0.2 \Rightarrow k = 0.1$

Now, mean, $\bar{X}=\Sigma XP(X)$

$= 1 \times 0.1 + 2 \times 0.15 + 3 \times 0.3 + 4 \times 0.25 + 5k + 6k$

= 3.4 [K = 0.1]

and $\Sigma X^2P(X) = 1 \times 0.1 + 4 \times 0.15 + 9 \times 0.3 + 16 \times 0.25 + 25k + 36k = 13.5$

∴ Variance $\sigma^2 = \Sigma X^2P(X) – (\Sigma XP(X))^2$

$= 13.5 – (3.4)^2 = 13.5 – 11.56$

= 1.94 or 1.93

40. **(d)** Required probability = 1 – P (Number of head and tail are equal)

$$=1-{}^{2n}C_n\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^n$$

$$=1-\frac{2n!}{n!n!}\left(\frac{1}{4}\right)^n=1-\frac{2n!}{(n!)^2}.\frac{1}{4^n}$$

41. **(b)** In Poisson distribution

$$p(x)=\frac{e^{-\lambda}\lambda^r}{r!}\Rightarrow p(2)=\frac{e^{-\lambda}(3.725)^2}{2!}\qquad [\because \lambda = 3.725]$$

$$p(2)=e^{-\lambda}\left(\frac{13.875}{2}\right)=e^{-\lambda}(6.937)$$

$$p(3)=\frac{e^{-\lambda}(3.725)^3}{3!}=e^{-\lambda}(8.614)$$

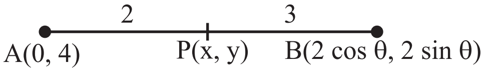
$$p(4)=\frac{e^{-\lambda}(3.725)^4}{4!}=e^{-\lambda}(8.022)$$

$$p(5)=\frac{e^{-\lambda}(3.725)^5}{5!}=e^{-\lambda}(5.976)$$

Clearly, $p(3)$ has maximum value

∴ Value of $x = 3$

42. **(a)**



Let P be (x, y). Then

$$(x,y)=\left(\frac{4\cos\theta+0}{5},\frac{4\sin\theta+12}{5}\right)$$

$$\therefore\ x=\frac{4\cos\theta}{5}\text{ and }y=\frac{4\sin\theta+12}{5}$$

$$\Rightarrow\quad x=\frac{4}{5}\cos\theta\text{ and }y-\frac{12}{5}=\frac{4}{5}\sin\theta$$

On eliminating θ, we get

$$\therefore\ x^2+\left(y-\frac{12}{5}\right)^2=\frac{16}{25}\cos^2\theta+\frac{16}{25}\sin^2\theta$$

$$\Rightarrow\quad x^2+\left(y-\frac{12}{5}\right)^2=\frac{16}{25}\qquad [\because\ \cos^2\theta+\sin^2\theta=1]$$

which is equation of a circle

43. **(c)** Let $\tan^{-1}(2) = \theta$

Then, $\tan\theta = 2$

$$\cos\theta=\frac{1}{\sqrt5},\sin\theta=\frac{2}{\sqrt5};\ \therefore\ x=X\cos\theta-Y\sin\theta=\frac{X-2Y}{\sqrt5}$$

$$\text{and } y=X\sin\theta+Y\cos\theta=\frac{2X+Y}{\sqrt5}$$

The equation $3x^2 – 4xy = r^2$ reduces to

$$3\left(\frac{X-2Y}{\sqrt5}\right)^2-4\left(\frac{X-2Y}{\sqrt5}\right)\left(\frac{2X+Y}{\sqrt5}\right)=r^2$$

$$\Rightarrow 3\ (X^2-4XY+4Y^2)-4\ (2X^2-2Y^2-3\times4)=5r^2$$

$\Rightarrow 20Y^2 - 5X^2 = 5r^2$

$\Rightarrow 4Y^2 - X^2 = r^2$

44. **(d)** Given, $x + 2y + 1 = 0$

$x + 2y + 1 = 0$

$$\Rightarrow \quad 2y = -x - 1 \Rightarrow y = \frac{-1}{2}x - \frac{1}{2}$$

$$\therefore \quad m = \frac{-1}{2}, c = \frac{-1}{2}$$

$$\Rightarrow \quad x + 2y = -1 \Rightarrow -x - 2y = 1$$

$$\Rightarrow \quad \frac{-1}{\sqrt{5}}x - \frac{2}{\sqrt{5}}y = \frac{1}{\sqrt{5}}$$

$$\therefore \quad P = \frac{1}{\sqrt{5}}, \cos\theta = \frac{-1}{\sqrt{5}}, \sin\theta = \frac{-2}{\sqrt{5}}$$

$$\therefore \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = 2$$

Now $\tan^{-1}(\tan\theta + m + c) = \tan^{-1}\left(2 - \frac{1}{2} - \frac{1}{2}\right)$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

45. **(a)** Given, $ax + by + c = 0$ and $2a + 3b = 4c$

Since line are concurrent.

Hence

Putting $c = \frac{2a+3b}{4}$ in $ax + by + c = 0$

We get $ax + by + \frac{2a+3b}{4} = 0$

$$\Rightarrow \quad 4ax + 4by + 2a + 3b = 0$$

$$\Rightarrow \quad (4x+2) + \frac{b}{a}(4y+3) = 0$$

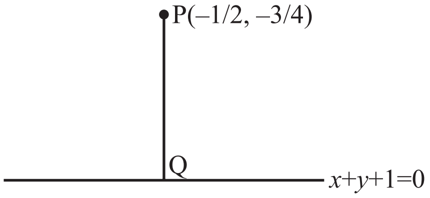
By family of lines $L_1 + \lambda L_2 = 0$.

So, set of lines are

$4x + 2 = 0$ and $4y + 3 = 0$

These two lines intersect at $\left(\frac{-1}{2}, \frac{-3}{4}\right)$

$$\therefore \ P(l, m) = \left(\frac{-1}{2}, \frac{-3}{4}\right)$$



Equation of line passing through $\left(\frac{-1}{2}, \frac{-3}{4}\right)$ and perpendicular to $x + y + 1 = 0$ is

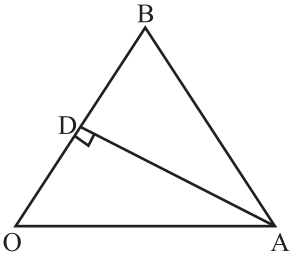
$$y + \frac{3}{4} = 1\left(x + \frac{1}{2}\right)$$

$$\Rightarrow \quad 4y = 4x - 1 \Rightarrow 4x - 4y - 1 = 0$$

On solving $x + y + 1 = 0$ and $4x - 4y - 1 = 0$

we get $Q\left(-\frac{3}{8}, -\frac{5}{8}\right)$

46. **(b)** Given, ΔOAB with coordinates O (0, 0) A (–3, –1) and B (–1, –3)



Here, slope of $OB = \frac{-3-0}{-1-0} = 3$

$$\therefore \quad \text{Slope of } AD = \frac{-1}{\text{Slope of } OB} = \frac{-1}{3}$$

[$\because$ AD is $\perp$ on BO]

Now, equation of OB is given by

$y - 0 = 3(x - 0) \Rightarrow y = 3x$ ...(i)

and equation of AD is given by

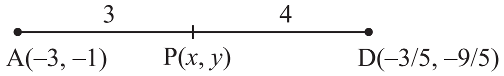
$$y + 1 = \frac{-1}{3}(x + 3)$$

$x + 3y = -6$ ...(ii)

On solving eqs. (i) and (ii), we get

$$D\left(\frac{-3}{5}, \frac{-9}{5}\right)$$

Now, P divides AD in the ratio 3 : 4



$$\therefore \quad P(x, y) = \left(\frac{\frac{-9}{5} - 12}{7}, \frac{\frac{-27}{5} - 4}{7}\right) = \left(\frac{-69}{35}, \frac{-47}{35}\right)$$

$\therefore$ Equation of line passing through $P\left(\frac{-69}{35}, \frac{-47}{35}\right)$ and parallel to OB is

$$y + \frac{47}{35} = 3\left(x + \frac{69}{35}\right)$$

$$\Rightarrow \quad 35y + 47 = 3\,(35x + 69)$$

$$\Rightarrow \quad 21x - 7y + 32 = 0$$

47. **(d)** Let the equation of line through the point (1, 0) having slope m is $y = mx - m$.

$m = \tan\theta$ where $\theta \in (0, \pi)$

Since curve satisfy the origin (0, 0) then making homogenization.

Now, the combined equation of pair of straight lines joining the point of intersections of given curve $2x^2 + 5y^2 - 7x = 0$ and line $y = mx - m$ to the origin is

$$2x^2+5y^2-7x\left(\frac{mx-y}{m}\right)=0$$

⇒ $2mx^2 + 5my^2 - 7x\,(mx - y) = 0$

⇒ $-5mx^2 + 5my^2 + 7xy = 0$ ...(i)

Since, the Eq. (i) represents the perpendicular lines, because sum of coefficients of x^2 terms and y^2 terms is zero.

48. **(d)** We have, $8x^2 - 14xy + 5y^2 = 0$

On breaking it into two linear equations

$\Rightarrow 8x^2 - 4xy - 10xy + 5y^2 = 0$

$\Rightarrow (2x - y)(4x - 5y) = 0$

∴ Sides of triangles are

$2x - y = 0,\ 4x - 5y = 0,\ x - 2y + 3 = 0$

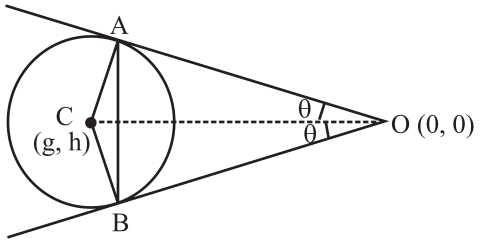
On solving above equations, we get point of intersections as (0, 0), (1, 2) and (5, 4)

Since (p, q) is the centroid of the triangle

$$\therefore\ (p,q)=\left(\frac{0+1+5}{3},\frac{0+2+4}{3}\right)$$

$\Rightarrow (p, q) = (2, 2) \Rightarrow p = q = 2$

49. **(d)**



Given, equation of circle is

$x^2 + y^2 - 2gx - 2hy + h^2 = 0$

Hence

∴ Radius of circle, AC = $\sqrt{g^2+h^2-h^2}=g$

Now length of tangent,

$$OA=\sqrt{0+0-0-0+h^2}=h$$

Now, in ΔOAC

$$\tan\theta=\frac{AC}{OA}=\frac{g}{h}$$

$$\therefore\ \sin 2\theta=\frac{2\tan\theta}{1+\tan^2\theta}=\frac{2\times\frac{g}{h}}{1+\frac{g^2}{h^2}}=\frac{2gh}{h^2+g^2}$$

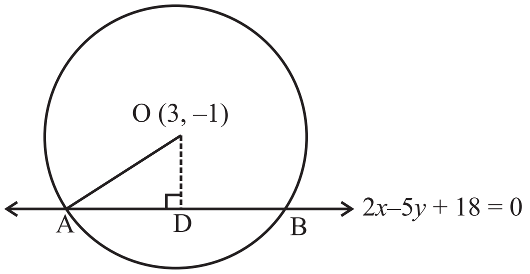
Now Area of ΔOAB $=\frac{1}{2}OA\times OB\times\sin 2\theta$

$$=\frac{1}{2}\times h\times h\times\frac{2gh}{h^2+g^2}=\frac{gh^3}{h^2+g^2}$$

50. **(b)** Given, equation of circle is

$x^2 + y^2 - 6x + 2y - 28 = 0$

∴ Centre of circle, 0 = (3, –1)



and radius of circle, $OA=\sqrt{9+1+28}=\sqrt{38}$

$$\text{Now, } OD=\left|\frac{3\times 2-(-1)\times 5+18}{\sqrt{2^2+(-5)^2}}\right|$$

$$=\left|\frac{6+5+18}{\sqrt{4+25}}\right|=\frac{29}{\sqrt{29}}=\sqrt{29}$$

Now, in ΔOAD

$$AD=\sqrt{OA^2-OD^2}=\sqrt{38-29}=\sqrt{9}=3$$

∴ λ = AB = 2AD = 2 × 3 = 6

51. **(a)** Given equation of two circles

$S_1 : x^2 + y^2 + 2x + 8y - 23 = 0$

$S_2 : x^2 + y^2 - 4x + 10y + 19 = 0$

Hence, centres of both the circles are O_1 (–1, –4) and O_2 (2, –5)

∴ Pole of O_1 (–1, –4) with respect to S_2 is

$-x - 4y - 2(x - 1) + 5(y - 4) + 19 = 0$

$\Rightarrow -x - 4y - 2x + 2 + 5y - 20 + 19 = 0$

$\Rightarrow 3x - y - 1 = 0$ ...(i)

and Pole of O_2 (2, –5) with respect to S_1 is

$2x - 5y + (x + 2) + 4(y - 5) - 23 = 0$

$3x - y - 41 = 0$ ...(ii)

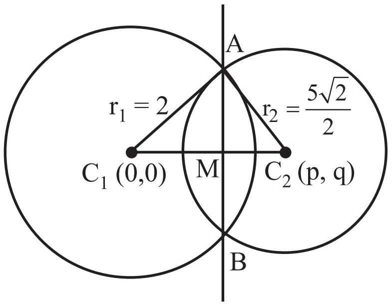
Hence, both the equations shows parallel lines

Now distance between them

$$=\left|\frac{-41+1}{\sqrt{9+1}}\right|=\left|\frac{-40}{\sqrt{10}}\right|=4\sqrt{10}\text{ units}$$

52. **(b)** Given $S_1 : x^2+y^2-4=0 \Rightarrow x^2+y^2=4$

$$S_2:(x-p)^2+(y-q)^2=\frac{25}{2}$$



Let $r_1 = 2$ $r_2=\frac{5\sqrt{2}}{2}$;

Let $O_2 : (p, q)$

So, equation of common chord will be

$S_1 - S_2 = 0$

$$\Rightarrow \quad 2px + 2qy - p^2 - q^2 = -\frac{17}{2}$$

Now, slope of common chord $= \frac{1}{4}$

$$\therefore \quad \frac{-2p}{2q} = \frac{1}{4} \Rightarrow q = -4p \qquad \text{...(i)}$$

Now, for maximum length of chord $C_1M = 0$

So, that AB become diameter of S_1

$$\therefore \quad \left| \frac{-p^2 - q^2 + \frac{17}{2}}{\sqrt{4p^2 + 4q^2}} \right| = 0$$

$$\Rightarrow p^2 + q^2 = \frac{17}{2} \qquad \text{...(ii)}$$

On solving eqs. (i) and (ii), we get

$$(p, q) = \left(\frac{\sqrt{2}}{2}, \mp 2\sqrt{2} \right)$$

53. **(a)** Given, equation of circles

$S_1 : x^2 + y^2 - 4x - 6y + k = 0$

and $S_2 : x^2 + y^2 + 8x - 4y + 11 = 0$

Let O_1, O_2 are centres of circles and r_1, r_2 are radius of circles S_1 and S_2 respectively.

$\therefore$ $O_1 = (2, 3)$, $O_2 = (-4, 2)$

$$r_1 = \sqrt{4 + 9 - K} = \sqrt{13 - x}, r_2 = \sqrt{16 + 4 - 11} = 3$$

We know angle between circles is given by

$$\cos\theta = \frac{r_1^2 + r_2^2 - O_1O_2^2}{2r_1 r_2} \qquad \left[\because \cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2} \right]$$

$$\cos\frac{\pi}{3} = \frac{13 - k + 9 - (\sqrt{(2+4)^2 + (3-2)^2})^2}{2 \times \sqrt{13 - k} \times 3}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{22 - k - 37}{6\sqrt{13 - k}}$$

$\Rightarrow$ $3\sqrt{13 - k} = -15 - k \Rightarrow 9\,(13 - k) = (15 + k)^2$

$\Rightarrow$ $117 - 9k = 225 + k^2 + 30k$

$\Rightarrow$ $k^2 + 39k + 108 = 0$

$\Rightarrow$ $(k + 3)\,(k + 36) = 0$

$\Rightarrow$ $k = -3, -36$

$\therefore$ $k = -3$

54. **(c)** Given, $y^2 = 4x$

Now tangents at A, B, C

$$\Rightarrow \quad 2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

At A (1, 2)

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{2}{2} = 1$$

$\therefore$ Tangent is given by

$y - 2 = 1(x - 1)$

$y = x + 1$ ...(i)

At B (4, –4)

$$\left.\left(\frac{dy}{dx}\right)\right|_{(4,-4)} = \frac{2}{-4} = -\frac{1}{2}$$

$\therefore$ Tangent is given by $y + 4 = \frac{-1}{2}(x - 4)$

$x + 2y + 4 = 0$ ...(ii)

At $C(2, 2\sqrt{2})$

$$\left.\frac{dy}{dx}\right|_{(2,2\sqrt{2})} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\therefore$ Tangent is given by

$\Rightarrow$ $\sqrt{2}y - 4 = x - 2$

$\Rightarrow$ $x - \sqrt{2y} + 2 = 0$ ...(iii)

Let P, Q, R the vertices of triangle formed by tangents (i), (ii), (iii). On solving eqs. (i), (ii) and (iii), we get

$P(-2, -1), Q(-2\sqrt{2}, \sqrt{2} - 2)$ and $R(\sqrt{2}, \sqrt{2} + 1)$

$$\text{Now, ar } (\Delta ABC) = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 4 & -4 & 1 \\ 2 & 2\sqrt{2} & 1 \end{vmatrix}$$

Using $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 0 \\ 1 & (2\sqrt{2} \quad 2) & 0 \end{vmatrix}$$

$1(3(2\sqrt{2} - 2) + 6 \;= 3\sqrt{2}$ sq unit

$$\text{and } ar(\Delta PQR) = \frac{1}{2}\begin{vmatrix} -2 & -1 & 1 \\ -2\sqrt{2} & \sqrt{2} - 2 & 1 \\ \sqrt{2} & \sqrt{2} + 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[-2[\sqrt{2} - 2 - \sqrt{2} - 1] + 1[-2\sqrt{2} - \sqrt{2}]$$
$$+1[-4 - 2\sqrt{2} - 2 + 2\sqrt{2}]$$

$$= \frac{1}{2}[6 - 3\sqrt{2} - 6] = \frac{-3}{2}\sqrt{2} \quad = \frac{3}{2}\sqrt{2} \text{ sq unit}$$

$$\therefore \; a = 3\sqrt{2} \text{ and } \beta = \frac{3}{2}\sqrt{2}$$

$$\therefore \; \alpha\beta = 3\sqrt{2} \times \frac{3}{2}\sqrt{2} = 9$$

55. **(d)** Given, $x - 2y + k = 0$ is tangent to the parabola $y^2 - 4x - 4y + 8 = 0$, then

To find point of tangency

Now put $x = 2y - k$ in equation of parabola, we get

$\therefore$ $y^2 - 4\,(2y - k) - 4y + 8 = 0$

$\Rightarrow$ $y^2 - 12y + 4k + 8 = 0$

Since line is tangent to parabola

∴ $D = 0$ or $b^2 = 4ac$

$(-12)^2 - 4\,(4k + 8) = 0$

⇒ $16k = 112 \Rightarrow k = 7$

Now, we have

⇒ $y^2 - 4x - 4y + 8 = 0$

Slope of tangent at (l, K)

$$\therefore \quad 2\,\frac{dy}{dx} - 4 - \frac{dy}{dx} = 0$$

$$\Rightarrow \quad 2\frac{dy}{dx}(y-2) = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y-2}$$

∴ Slope of tangent at (l, 7) = $\dfrac{2}{k-2} = \dfrac{2}{5}$ $\quad [\because k = 7]$

56. **(c)** Let the standard equation of ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$$

Given LR = 4

$$\Rightarrow \quad \frac{2b^2}{a} = 4 \Rightarrow b^2 = 2a \qquad \text{...(i)}$$

Given distance between foci $= 4\sqrt{2}$

⇒ $2ae = 4\sqrt{2} \Rightarrow ae = 2\sqrt{2}$

⇒ $a^2e^2 = 8$

$$\Rightarrow \quad a^2\left(1 - \frac{b^2}{a^2}\right) = 8 \qquad \left[\because e = \sqrt{1 - \frac{b^2}{a^2}}\right.$$

⇒ $a^2 - 2a = 8$ [From Eq. (i)]

⇒ $(a - 4)(a + 2) = 0 \Rightarrow a = 4, -2$

⇒ $a = 4$

∴ $b^2 = 2a = 2 \times 4 = 8$

∴ Equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{8} = 1 \Rightarrow x^2 + 2y^2 = 16$$

57. **(c)** Given equation of hyperbola is

$$\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$$

$$\Rightarrow \quad \frac{x^2}{\left(\frac{144}{25}\right)} - \frac{y^2}{\left(\frac{81}{25}\right)} = 1 \Rightarrow \frac{x^2}{\left(\frac{12}{5}\right)^2} - \frac{y^2}{\left(\frac{9}{5}\right)^2} = 1$$

Standard equation of hyperbola is $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

where $b^2 = a^2\,(e^2 - 1)$

∴ Foci = (± ae, 0)

$$= \left(\pm\frac{12}{5}\sqrt{1 + \left(\frac{9/5}{12/5}\right)^2}, 0\right) \qquad \left[\because a = \frac{12}{5} \text{ and } e = \frac{5}{4}\right]$$

$$= \left(\frac{12}{5} \times \frac{5}{4}, 0\right) = (\pm 3, 0)$$

Equation of ellipse is

$$\frac{x^2}{16} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{4^2} + \frac{y^2}{b^2} = 1$$

∴ Foci = (± ae, 0)

$$= \left(\pm 4\sqrt{1 - \frac{\sqrt{b^2}}{4^2}}, 0\right) = \left(\pm\frac{4\sqrt{16 - b^2}}{4}, 0\right)$$

$$= (\pm\sqrt{16 - b^2}, 0)$$

Since both have same foci

$$\therefore \quad \sqrt{16 - b^2} = 3 \Rightarrow 16 - b^2 = 9 \Rightarrow b^2 = 7$$

58. **(c)** Given, equation of hyperbola is

$9x^2 - 16y^2 = 144$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1 \Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$$

Since, the pole of the line $lx + my + n = 0$ to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is given as $\left(-\dfrac{a^2 l}{n}, \dfrac{b^2 m}{n}\right)$

Hence pole of $3x - 16y + 48 = 0$ with respect to the hyperbola $\dfrac{x^2}{4^2} - \dfrac{y^2}{3^2} = 1$ will be $\left(\dfrac{-(4)^2 \times 3}{48}, \dfrac{(3)^2(-16)}{48}\right)$

∴ $(\alpha, \beta) = (-1, -3)$

∴ $\alpha = -1, \beta = -3$

∴ $\alpha - \beta = -1 - (-3) = 2$

59. **(c)** Given vertices of tetrahedron are

$A\,(2, 3, -4), B\,(-3, 3, -2), C\,(-1, 4, 2), D\,(3, 5, 1)$

Now, E = centroid of ΔABC

$$= \left(\frac{2-3-1}{3}, \frac{3+3+4}{3}, \frac{-4-2+2}{3}\right) = \left(\frac{-2}{3}, \frac{10}{3}, \frac{-4}{3}\right)$$

F = centroid of ΔABD

$$\left(\frac{2-3+3}{3}, \frac{3+3+5}{3}, \frac{-4-2+1}{3}\right) = \left(\frac{2}{3}, \frac{11}{3}, \frac{-5}{3}\right)$$

G = centroid of ΔACD

$$= \left(\frac{2-1+3}{3}, \frac{3+4+5}{3}, \frac{-4+2+1}{3}\right) = \left(\frac{4}{3}, 4, \frac{-1}{3}\right)$$

Now centroid of ΔEFG Let ‘O’

$$\therefore\ O = \left(\frac{\frac{-2}{3} + \frac{2}{3} + \frac{4}{3}}{3}, \frac{\frac{10}{3} + \frac{11}{3} + 4}{3}, \frac{\frac{-4}{3}\frac{-5}{3}\frac{-1}{3}}{3}\right)$$

$$= \left(\frac{4}{9}, \frac{11}{3}, \frac{-10}{9}\right)$$

60. **(b)** It is a conceptual question solved by using Cauchy’s inequality

$$(lm + mn + nl)^2 \leq (l^2 + m^2 + n^2)^2$$

∵ sum of square of direction cosines is zero

⇒ $(lm+mn+nl)^2 \le 1$ $\{\because l^2+m^2+n^2=0\}$

⇒ $-1 \le (lm+mn+nl) \le 1$

Therefore, the maximum value of $lm + mn + nl = 1$ and it is possible only when $l = m = n$, as $l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$, so $\alpha = \beta = \gamma$

61. **(a)** Given, combined equation of planes is
$S \equiv 2x^2 - 6y^2 - 12z^2 + 18yz + 2zx + xy = 0$
Let the planes are $a_1x + b_1y + c_1z + d_1 = 0$
and $a_2x + b_2y + c_2z + d_2 = 0$
∴ $a_1a_2 = 2$ $b_1b_2 = -6$, $c_1c_2 = -12$
Now, one plane say $a_1x + b_1y + c_1z + d_1$ is parallel to
$x + 2y - 2z = 5$
∴ $a_1 = 1, b_1 = 2, c_1 = -2$
∴ $a_2 = 2, b_2 = -3, c_2 = 6$
Let θ be acute angle between the planes

$$\therefore \quad \cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

$$= \left|\frac{2-6-12}{\sqrt{1+4+4}\sqrt{4+9+36}}\right| = \frac{16}{21}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{16}{21}\right) \qquad [\because f(x) > 0 \text{ and } x \in \mathbf{R}^+]$$

62. **(a)** Since $f(x) > 0$ and $x \in \mathbf{R}^+$
Hence $3x < 6x < 9x$ $[\because x \in \mathbf{R}^+]$
⇒ $f(3x) < f(6x) < f(9x)$
[as $f(x)$ is a increasing function]

$$\text{Now, } \frac{f(3x)}{f(3x)} < \frac{f(6x)}{f(3x)} < \frac{f(9x)}{f(3x)} \qquad [\because f(x) > 0 \forall x]$$

$$\Rightarrow \lim_{x\to\infty}\frac{f(3x)}{f(3x)} < \lim_{x\to\infty}\frac{f(6x)}{f(3x)} < \lim_{x\to\infty}\frac{f(9x)}{f(3x)}$$

$$\Rightarrow 1 < \lim_{x\to\infty}\frac{f(6x)}{f(3x)} < 1$$

$$\therefore \lim_{x\to\infty}\frac{f(6x)}{f(3x)} = 1$$

63. **(a)** Given $f(x) = \dfrac{1+\sin([\cos x])}{\cos([\sin x])}$

Let $f(x)$ is to be defined in various intervals
Hence,

$$f(x) = \begin{cases} 1, & 0 < x < \frac{\pi}{2} \\ 1-\sin 1, & \frac{\pi}{2} < x < \pi \\ \frac{1-\sin 1}{\cos 1}, & \pi < x < \frac{3\pi}{2} \\ \frac{1}{\cos 1}, & \frac{3\pi}{2} < x < 2\pi \end{cases}$$

From above definition of $f(x)$, we have $f(x)$ is continuous on $\left(0, \frac{\pi}{2}\right)$.

64. **(b)** Given,

$$\Rightarrow x + x\sqrt[3]{y} = 1 - \sqrt[3]{y}x = \frac{1-\sqrt[3]{y}}{1+\sqrt[3]{y}}$$

$$= \sqrt[3]{y}(x+1) = 1-x$$

$$\Rightarrow \sqrt[3]{y} = \frac{1-x}{1+x} \Rightarrow y = \frac{(1-x)^3}{(1+x)^3}$$

Now $\dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{3(1-x)^2(-1)(1+x)^3 - (1-x)^3(1+x)^3}{(1+x)^6}$$

$$= \frac{-3(1-x)^2(1+x)^2[(1+x)+1-x]}{(1+x)^6}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-6(1-x)^2}{(1+x)^4}$$

$$\therefore \frac{dx}{dy} = \frac{-(1+x)^4}{6(1-x)^2}$$

65. **(d)** Since derivative of log $|x|$ is

$$\text{Hence, } \log|x| = \begin{cases} \log(-x), & x < 0 \\ \log x, & x \ge 0 \end{cases}$$

$$\text{Now } \frac{d}{dx}\log|x| = \begin{cases} \frac{-1}{x}(-1), & x < 0 \\ \frac{1}{x}, & x \ge 0 \end{cases} = \begin{cases} \frac{1}{x}, & x < 0 \\ \frac{1}{x}, & x \ge 0 \end{cases}$$

$$\text{and } \frac{d^2}{dx^2}(\log|x|) = \begin{cases} \frac{-1}{x^2}, & x < 0 \\ \frac{-1}{x^2}, & x \ge 0 \end{cases}$$

$$\frac{d^2}{dx^2}f(x) = \frac{-1}{x^2} = \frac{-1}{|x|^2}$$

So, A is false.
We know that, $|x| = -x$, when $x < 0$
So, R is true

66. **(d)** Given,
$ax^2 + 2hxy + by^2 = 0$ ...(i)
Hence differentiating equation (i)
$2ax + 2hy + 2hxy' + 2byy' = 0$

$$\Rightarrow y' = \frac{-(ax+hy)}{(hx+by)}$$

Now,

$$\frac{d^2y}{dx^2} = -\left[\frac{(a+hy')(hx+by)-(ax+hy)(h+by')}{(hx+by)^2}\right]$$

$$= -\left[\frac{[a(hx+by)-h(ax+hy)]+[h(hx+by) - b(ax+hy)y'}{(hx+by)^2}\right]$$

$$= -\left[\frac{(ahx+aby-ahx-h^2y)+[h^2x+hby - abx-bhy]y'}{(hx+by)^2}\right]$$

$$= -\left[\frac{(ab-h^2)y+(h^2-ab)x\,y'}{(hx+by)^2}\right]$$

$$= -\frac{(h^2-ab)}{(hx+by)^2}\left[-y+x\left(-\left(\frac{ax+hy}{hx+by}\right)\right)\right]$$

$$= -\frac{(h^2-ab)}{(hx+by)^3}[-hxy-by^2-ax^2-hxy]$$

$$= \frac{h^2-ab}{(hx+by)^3}[ax^2+2hxy+by^2]=0 \qquad \text{[From Eq. (i)]}$$

67. **(c)** Given, $y^2 = x$

Hence $y' = \dfrac{1}{2y}$

Let the point be (x_1, y_1).

$\therefore\ y_1^2 = x_1$ ...(i)

$\therefore$ Slope of normal at $(x_1, y_1) = \dfrac{-1}{(y')_{(x_1,y_1)}}$

$$= \frac{-1}{\left(\frac{1}{2y_1}\right)} = -2y_1$$

It is given that, $-2y_1 = x_1$ ...(ii)

From Eqs. (i) and (ii), we have

$y_1^2 = -2y_1 \Rightarrow y_1^2 + 2y_1 = 0$

$\Rightarrow\ y_1(y_2 + 2) = 0 \Rightarrow y_1 = 0, -2$

$\therefore\ x_1 = 0, 4$

Required points are (0, 0), (4,–2)

68. **(b)** Given, $x^2 + ax - b = 0$ has roots α and β

Where $\alpha + \beta = -a$ and $\alpha\beta = -b$

Given, equation of curve is

$$\frac{x^n}{\alpha^n} + \frac{y^n}{\beta^n} = 2$$

Now slope of tangent at $(\alpha, \beta) = \left.\dfrac{dy}{dx}\right|_{(\alpha,\beta)}$

$$\therefore\ \frac{n}{\alpha^n}x^{n-1} + \frac{n}{\beta^n}y^{n-1}\frac{dy}{dx} = 0$$

$$\Rightarrow\ \frac{dy}{dx} = -\frac{\beta^n}{\alpha^n}\frac{x^{n-1}}{y^{n-1}} = \frac{-\beta}{\alpha}$$

Hence, equation of tangent at (α, β)

$$y - \beta = \frac{-\beta}{\alpha}(x-\alpha)$$

$\Rightarrow\ \alpha y - \alpha\beta = -\beta x + \alpha\beta$

$\Rightarrow\ \beta x + \alpha y = 2\alpha\beta$

But equation that touches the curve is

$x \cos\theta + y \sin\theta = C$

$\therefore$ Hence $\dfrac{\cos\theta}{\beta} = \dfrac{\sin\theta}{\alpha} = \dfrac{C}{2\alpha\beta}$

$\Rightarrow\ \cos\theta = \dfrac{C}{2\alpha}$ and $\sin\theta = \dfrac{C}{2\beta}$

$\therefore\ \cos^2\theta + \sin^2\theta = \dfrac{C^2}{4\alpha^2} + \dfrac{C^2}{4v\beta^2}$

$\Rightarrow\ \dfrac{C^2}{4}\left[\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}\right] = 1$

$\Rightarrow\ \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = \dfrac{4}{C^2} \Rightarrow \dfrac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \dfrac{4}{C^2}$

$\Rightarrow\ \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \dfrac{4}{C^2} \Rightarrow \dfrac{a^2+2b}{b^2} = \dfrac{4}{C^2}$

$\Rightarrow\ \left(\dfrac{a}{b}\right)^2 + \dfrac{2}{b} = \dfrac{4}{C^2}$

69. **(c)** According to Lagrange's mean value theorem function is continuous and differentiate in (a, b)

then, $f'(C) = \dfrac{f(b)-f(a)}{b-a}$

Now, $f(x) = x \Rightarrow f'(x) = 1$

Also, $f(b) = f(5) = 5$

$f(a) = f(2) = 2$

$\therefore\ \dfrac{f(b)-f(a)}{b-a} = \dfrac{5-2}{5-2} = 1$

$\therefore\ f'(C) = \dfrac{f(b)-f(a)}{b-a}$ is always true

For infinite value of $C \in (2, 5)$

70. **(a)** Given,

$f(x) = x^3 - 4x^2 + 4x + 3$

$\therefore\ f'(x) = 3x^2 - 8x + 4$

Now, $f'(x) = 0 \Rightarrow 3x^2 - 8x + 4 = 0$

$\Rightarrow\ x = 2, \dfrac{2}{3}$

Now, $f(-1) = -1 -4 -4 + 3 = -6$

$$f\left(\frac{2}{3}\right) = \frac{8}{27} - 4\times\frac{4}{9} + 4\times\frac{2}{3} + 3$$

$$= \frac{8-48+72+81}{27} = \frac{113}{27}$$

$f(2) = 8 - 14 + 8 + 3 = 5$

$f(3) = 27 - 36 + 12 + 3 = 6$

$\therefore f(x)$ has minimum value at $x = -1$

71. **(c)** Let $I = \int \frac{\tan^{-1}x}{x^3}dx = \int \underset{\text{I}}{\tan^{-1}x}.\underset{\text{II}}{\frac{1}{x^3}}dx$

Using by parts

$$= \tan^{-1}x\left[\frac{-1}{2x^2}\right] - \int \frac{-1}{2x^2}.\frac{1}{1+x^2}dx$$

$$= -\frac{1}{2x^2}\tan^{-1}x + \frac{1}{2}\int \frac{1}{x^2(1+x^2)}dx$$

$$= \frac{-1}{2x^2}\tan^{-1}x + \frac{1}{2}\int\left(\frac{1}{x^2} - \frac{1}{1+x^2}\right)dx$$

$$= -\frac{1}{2x^2}\tan^{-1}x + \frac{1}{2}\left[\frac{-1}{x} - \tan^{-1}x\right] + C$$

$$= \frac{-1}{2x} - \left(\frac{1}{2x^2} + \frac{1}{2}\right)\tan^{-1}x + C$$

72. **(d)** Let $I = \int \frac{1-(\cot x)^{2019}}{\tan x + (\cot x)^{2020}}dx$

$$= \int \frac{1 - \frac{(\cos x)^{2019}}{(\sin x)^{2019}}}{\frac{\sin x}{\cos x} + \frac{(\cos x)^{2020}}{(\sin x)^{2020}}}dx$$

$$= \int \frac{\frac{(\sin x)^{2019} - (\cos x)^{2019}}{(\sin x)^{2019}}}{\frac{\sin^{2021}x + \cos^{2021}x}{\cos x(\sin x)^{2020}}}$$

$$= \int \frac{\sin^{2019}x - \cos^{2019}x}{\sin^{2021}x + \cos^{2021}x}.\sin x\cos x\,dx$$

Let $\sin^{2021}x + \cos^{2021}x = \text{t}$

$\Rightarrow$ $[2021 \sin^{2020}x.\cos x + 2021 \cos^{2020}.(-\sin x)]dx = \text{dt}$

$\Rightarrow$ $\sin x \cos x\,[\sin^{2019}x - \cos^{2019}x]\ dx = \frac{1}{2021}dt$

$\therefore$ $I = \frac{1}{2021}\int\frac{1}{t}\text{dt} = \frac{1}{2021}\log t + C$

$$= \frac{1}{2021}\log[\sin^{2021}x + \cos^{2021}x] + C$$

$\therefore$ $f(x) = \sin x$, $g(x) = \cos x$, $n = 2021$

$\therefore$ $n[(f(x)^4) + (g(x)^4)]_{x=\frac{\pi}{3}} = 2021\left[\sin^4\frac{\pi}{3} + \cos^4\frac{\pi}{3}\right]$

$$= 2021\left[\left(\frac{\sqrt{3}}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right]$$

$$= 2021\left[\frac{9}{16} + \frac{1}{16}\right] = \frac{2021\times 10}{16} = \frac{10105}{8}$$

73. **(a)** Let

$$I = \int \frac{x^3 + x^2 - x - 1}{(x^5 + x^4 + 3x^3 + 3x^2 + x + 1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

as $x \neq 1$

$$= \int \frac{x(x^2-1) + x^2 - 1}{[x(x^4+3x^2+1) + (x^4+3x^2+1)]\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$= \int \frac{(x+1)(x^2-1)}{(x^4+3x^2+1)(x+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$= \int \frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

Let $\tan^{-1}\left(\frac{x^2+1}{x}\right) = t$ $\left[\because \frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}\right]$

$$\Rightarrow \frac{1}{1+\left(\frac{x^2+1}{x}\right)^2}.\frac{2x(x)-(x^2+1)}{x^2}dx = dt$$

$$\Rightarrow \frac{x^2-1}{x^4+3x^2+1}dx = dt \Rightarrow I = \int\frac{1}{t}dt = \log t + C$$

$$= \log\tan^{-1}\left(\frac{x^2+1}{x}\right) + C$$

$\therefore$ $A = 1, f(x) = \tan^{-1}\left(\frac{x^2+1}{x}\right)$

$\therefore$ $A - \tan f(2) = 1 - \tan\tan^{-1}\frac{5}{2} = 1 - \frac{5}{2} = -\frac{3}{2}$

74. **(a)** Given, $I_n = \int x^n \sin x\,dx$

Hence, $I_6 = \int x^6 \sin x\,dx$ [Using by parts]

$$= x^6(-\cos x) - \int -\cos x.(6x^5)dx$$

$$= -x^6\cos x + 6\int x^5\cos x\,dx$$

$$= -x^6\cos x + 6[x^5\sin x - \int \sin x.(5x^4)dx]$$

$$= -x^6\cos x + 6x^5\sin x - 30\int x^4\sin x\,dx$$

$= -\,x^6\cos x + 6x^5 \sin x - 30$

$[x^4(-\cos x) - \int -\cos x(4x^3)dx]$

$= -\ x^6\cos x + 6x^5 \sin x + 30\,x^4\cos x\ -120\int x^3\cos x\,dx$

$= -\ x^6\cos x + 6x^5 \sin x + 30x^4 \cos x - 120$

$$[x^3 \sin x - \int \sin x (3x^2)\, dx]$$

$$= - x^6 \cos x + 6x^5 \sin x + 30x^4 \cos x - 120x^3 \sin x + 360\int x^2 \sin x\, dx$$

$$= - x^6 \cos x + 6x^5 \sin x + 30x^4 \cos x - 120\, x^3 \sin x + 360 I_2$$

$\therefore\quad I_6 - 360\, I_2 = (-x^6 + 30x^4) \cos x + (6x^5 - 120x^3) \sin x$

$\therefore\quad f(x) = -x^6 + 30x^4$ and $g(x) = 6x^5 - 120x^3$

$\therefore\quad f(1) + g(1) = -1 + 30 + 6 - 120 = -85$

75. **(d)** Let $I = \int x^3 \underset{\text{II}}{\sin x}\, dx$

$$= x^3(-\cos x) - \int (-\cos x) 3x^2 dx$$

$$= -x^3 \cos x + 3\int \underset{\text{I}}{x^2} \underset{\text{II}}{\cos x}\, dx$$

$$= -x^3 \cos x + 3[x^2 \sin x - \int \sin x . 2x\, dx]$$

$$= -x^3 \cos x + 3x^2 \sin x - 6\int \underset{\text{I}}{x} \underset{\text{II}}{\sin x}\, dx$$

$$= -x^3 \cos x + 3x^2 \sin x - 6[x(-\cos x) - \int -\cos x . 1 dx$$

$$= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x$$

$$\therefore \int_0^{\frac{\pi}{2}} x^3 \sin x\, dx = [-x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x]_0^{\pi/2}$$

$$= 3 \times \frac{\pi^2}{4} - 6 = \frac{3\pi^2}{4} - 6$$

76. **(d)** Given, $\lim_{n\to\infty} \prod_{r=1}^{n} \left(1 + \frac{r^2}{n^2}\right)^{2r/n^2}$

Taking log both sides

$$\text{Log } l = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{2r}{n^2} \log\left(1 + \frac{r^2}{n^2}\right)$$

$$\Rightarrow \quad \log l = 2 \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{r}{n} \log\left(1 + \left(\frac{r}{n}\right)^2\right)$$

$$\Rightarrow \quad \log l = 2\int_0^1 x \log(1 + x^2) dx$$

$$= 2\left[\left[\log(1 + x^2) . \frac{x^2}{2}\right]_0^1 - \int_0^1 \frac{x^2}{2} . \frac{1}{1 + x^2} . 2x\, dx\right]$$

$$= 2\left[\frac{1}{2} \log 2\right] - 2\int_0^1 \frac{x^3}{1 + x^2} dx$$

$$= \log 2 - 2\int_0^1 \left(x - \frac{x}{1 + x^2}\right) dx$$

$$= \log 2 - 2\left[\frac{x^2}{2} - \frac{1}{2} \log(1 + x^2)\right]_0^1$$

$$= \log 2 - 2\left[\frac{1}{2} - \frac{1}{2} \log 2\right]$$

$$\log 2 - 1 + \log 2 = \log 4 - \log e = \log\left(\frac{4}{e}\right)$$

$$\therefore \quad l = \frac{4}{e}$$

77. **(a)** Given, $x^2 + y^2 = 16a^2$ and $y^2 = 6ax$

On solving both equations, simultaneously

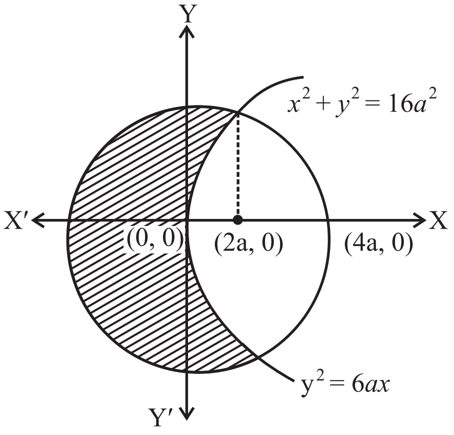
$x^2 + 6ax = 16a^2$

$\Rightarrow\quad x^2 + 6ax - 16a^2 = 0$

$\Rightarrow\quad (x + 8a)(x - 2a) = 0$

$\Rightarrow\quad x = -8a, 2a$

$\Rightarrow\quad x = 2a \qquad [x \neq -8a$, as $y^2 = 6ax]$



$\therefore\quad$ Required area = Area of circle − Area of unshaded region

$$= \pi(4a)^2 - 2\left[\int_0^{2a} \sqrt{6ax} dx + \int_{2a}^{4a} \sqrt{16a^2 - x^2}\, dx\right]$$

$$= 16a^2\pi - 2\left[\sqrt{6a} . \left[\frac{x^{3/2}}{3/2}\right]_0^{2a} + \left[\frac{x}{2}\sqrt{16a^2 - x^2} + \frac{16a^2}{2} \sin^{-1} \frac{x}{4a}\right]_{2a}^{4a}\right]$$

$$= 16a^2\pi - 2\left[\frac{2}{3}\sqrt{6a}\,(2\sqrt{2}a)^{3/2} + \frac{16a^2}{2} . \frac{\pi}{2} - \frac{2a}{2} \times 2\sqrt{3a} - \frac{16a^2}{2^2} \times \frac{\pi}{6}\right]$$

$$= 16a^2\pi - \frac{16\sqrt{3}}{3} a^2 - 8a^2\pi + 4\sqrt{3}a^2 + \frac{8}{3} a^2\pi$$

$$= \frac{32\pi a^2}{3} - \frac{4a^2\sqrt{3}}{3} = \frac{4a^2}{3}(8\pi - \sqrt{3}) \text{ sq unit}$$

78. **(d)** We have, $\sqrt{1 + y^2} = Cx\, e^{\tan^{-1} x}$ ...(i)

$$\Rightarrow \quad \frac{1}{2\sqrt{1 + y^2}} . 2y\, y' = Ce^{\tan^{-1} x} + \frac{c\, x\, e^{\tan^{-1} x}}{1 + x^2}$$

$$\Rightarrow \quad \frac{y}{\sqrt{1+y^2}}\, y' = C\, e^{\tan^{-1}x}\left[1+\frac{x}{1+x^2}\right]$$

$$\Rightarrow \quad \frac{y}{\sqrt{1+y^2}}\, y' = \frac{\sqrt{1+y^2}}{x\, e^{\tan^{-1}x}} . e^{\tan^{-1}x}\left[\frac{1+x^2+x}{1+x^2}\right]$$

[From Eq. (i)]

$$\Rightarrow \quad \frac{y}{\sqrt{1+y^2}}\, y' = \frac{\sqrt{1+y^2}\,(1+x+x^2)}{x(1+x^2)}$$

⇒ $xy\,(1+x^2)\,dy = (1+y^2)\,(1+x+x^2)$

⇒ $xy\,(1+x^2)\,dy = (1+y^2)\,(1+x+x^2)\,dx$

⇒ $xy\,(1+x^2)\,dy - (1+y^2)\,(1+x+x^2)\,dx = 0$

which is required differential equation

79. **(b)** Given

$$\frac{dy}{dx} = \frac{x+y-3}{x+y-7} \Rightarrow \frac{dy}{dx} = \frac{x+y-5+2}{x+y-5-2}$$

Let $x + y - 5 = V$

$$\Rightarrow \quad 1+\frac{dy}{dx} = \frac{dV}{dx} \Rightarrow \frac{dy}{dx} = \frac{dV}{dx} - 1$$

$$\therefore \quad \frac{dV}{dx} - 1 = \frac{V+2}{V-2} \Rightarrow \frac{dV}{dx} = \frac{V+2}{V-2} + 1$$

$$\Rightarrow \quad \frac{dV}{dx} = \frac{2V}{V-2} \Rightarrow \int \frac{V-2}{V}\, dV = 2\int dx$$

$$\Rightarrow \quad \int \left(1-\frac{2}{V}\right) dV = 2\int dx$$

⇒ $V - 2\log V = 2x + K$

⇒ $x + y - 5 - 2\log(x + y - 5) = 2x + K$

⇒ $y - x - 5 - K = \log(x + y - 5)^2$

⇒ $(x + y - 5)^2 = e^{y-x-5-k}$

⇒ $(x + y - 5)^2 = e^{y-x} . e^{-5-k}$

⇒ $(x + y - 5)^2 = Ce^{y-x}$, where $C = e^{-5-k}$

Which is required solution.

80. **(a)** We have, $x\frac{dy}{dx} = x^2 + 3y$

Equation is of the form of $\frac{dy}{dx} + P(y)$

$$\Rightarrow \quad x\frac{dy}{dx} - 3y = x^2 \Rightarrow \frac{dy}{dx} - \left(\frac{3}{x}\right) y = x$$

$$\therefore \quad \text{IF} = e^{\int \frac{-3}{x} dx} = e^{-3\log x} = \frac{1}{x^3}$$

∴ Solution is given by

$$y.\frac{1}{x^3} = \int x.\frac{1}{x^3}\, dx \Rightarrow \frac{y}{x^3} = \frac{-1}{x} + C$$

$y = -x^2 + Cx^3$

Now, it is given $y(2) = 4$

∴ $4 = -4 + 8C \Rightarrow C = 1$

$y = x^3 - x^2$

∴ $y(4) = 64 - 16 = 48$

PHYSICS

81. **(c)** Tokamak is a device which confine plasma using magnetic field.

82. **(d)** Since exponent should be dimensionless

$\left[\frac{t}{t_0}\right]$ dimensionless

83. **(c)** In given situation,

Time of fall of ball = $\sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 45}{10}} = 3s$

In 3s boat covers 12 m

∴ Speed = $\frac{\text{distance}}{\text{time}} = \frac{12}{3} = 4$ m/s

84. **(b)** Let t be the total time to cover H.

$$H = \frac{1}{2}gt^2 \qquad \text{...(i)}$$

In $(t - 1)$, ball will fall by $\frac{H}{2}$ height

$$\text{So, } \frac{H}{2} = \frac{1}{2}g(t-1)^2 \qquad \text{...(ii)}$$

Now, dividing eq.(i) by eq. (ii),

$$\frac{H}{\frac{H}{2}} = \frac{\frac{1}{2}gt^2}{\frac{1}{2}g(t-1)^2} \Rightarrow \sqrt{2} = \frac{t}{t-1}$$

$$\sqrt{2}t - \sqrt{2} = t$$

$$\therefore \quad t = \frac{\sqrt{2}}{\sqrt{2}-1} = \frac{\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= 2+\sqrt{2} = 2+1.41 = 3.41 \text{ s}$$

85. **(c)** Angle covered in 360 days = 2 × 2π rad

Angular frequency = $\frac{\text{angle covered}}{\text{time taken}} = \frac{4\pi}{360}$

$= 3.5 \times 10^{-2}$ rad/day

86. **(c)** Let's denote vectors by $\vec{a}\ \&\ \vec{b}$

⇒ $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0$

⇒ $a^2 - b^2 = 0 \Rightarrow a^2 = b^2$

So, magnitudes of vectors are equal.

87. **(b)** Let T be tension in string.

$$mg - T = ma \quad \text{...(i)}$$

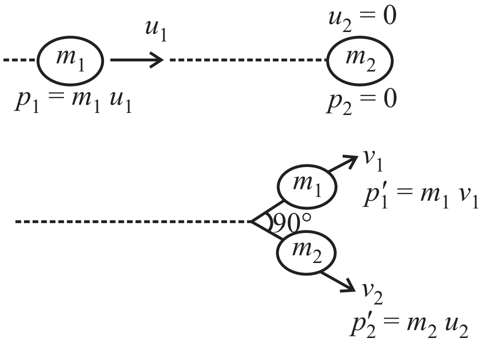
$$TR = I\alpha = MR^2\alpha \quad \text{...(ii)}$$

$$a = \alpha R \quad \text{...(iii)}$$

From (i) (ii) & (iii)

$$a = \frac{g}{2}$$

88. **(a)** According to given situation,



Momentum conservation gives,

$$p_1^{'2} + p_2^{'2} = p_1^2 \quad \text{...(i)}$$

KE conservation gives,

$$\frac{p_1^{'2}}{m_1} + \frac{p_2^{'2}}{m_2} = \frac{p_1^2}{m_1} \qquad \left[\therefore k = \frac{p^2}{2m}\right] \quad \text{...(ii)}$$

Put value of p_1 from (i) into (ii), we have

$$\frac{p_1^{'2}}{m_1} + \frac{p_2^{'2}}{m_2} = \frac{1}{m_1}(p_1^{'} + p_2^{'})^2$$

$$\frac{p_1^{'2}}{m_1} + \frac{p_2^{'2}}{m_2} = \frac{p_1^{'2}}{m_1} + \frac{p_2^{'2}}{m_1} + \frac{2p_1^{'}p_2^{'}}{m_1} \quad \text{...(iii)}$$

As angle between $\overline{p_1^{'}}$ & $\overline{p_2^{'}}$ is 90°

$$\Rightarrow \quad \overline{p_1^{'}} \cdot \overline{p_2^{'}} = 0$$

$$p_1^{'}.p_2^{'}\cos\theta = 0 \Rightarrow p_1^{'}p_2^{'} = 0$$

Putting value of $p_1^{'}p_2^{'}$ eqn. (iii)

$$\Rightarrow \quad \frac{p_2^{'2}}{m_2} = \frac{p_2^{'2}}{m_1} \text{ or } \frac{m_2}{m_1} = 1$$

89. **(b)** PE stored in spring

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}kx^2 \quad \text{…(i)}$$

Here, $m = 100\ g = 0.1$ kg

$$u = 2ms^{-1}, v = \frac{u}{2} = \frac{4}{2} = 1\,ms^{-1}, x = 2\text{cm}$$

$$\frac{1}{2}\times 0.1\times(2)^2 - \frac{1}{2}\times 0.1\times(1)^2 = \frac{1}{2}\ \ (2\times 10\ \ ^2)^2$$

$$0.4 - 0.1 = 4 \times 10^{-4}\,k$$

$$\therefore\ k = \frac{0.3}{4\times 10^{-4}} = 750 \text{ N/m}$$

90. **(a)** According to conservation of linear momentum

$$mv_0 = (m + M)\ v$$

[v is common velocity of bullet & block)

$$v = \frac{mv_0}{(m+M)}$$

Loss of KE = final KE – initial KE

$$= \frac{1}{2}(m+M)v^2 - \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}(m+M)\frac{m^2v_0^2}{(m+M)^2} - \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}mv_0^2\left(\frac{m}{m+M} - 1\right) = \frac{1}{2}mv_0^2\left(\frac{M}{m+M}\right)$$

91. **(a)** Height fallen by first ball in 0.4 s

$$x_1 = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times(0.4)^2 = 0.8\text{m}$$

Height fallen by second ball in 0.2 s

$$x_2 = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times(0.2)^2 = 0.2\text{m}$$

Position of centre of mass is given by

$$y_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{100\times 0.8 + 200\times 0.2}{100 + 200} = 0.4\,\text{m}$$

92. **(c)** Without external force velocity of centre of mass will not change before & after collission

$$v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \Rightarrow v_{CM} = \frac{2M + 1}{M + 1}$$

Kinetic energy of centre of mass $= \frac{4}{3}J.$

$= \frac{1}{2}$ (Total mass) vCM2

$$\Rightarrow \quad \frac{4}{3} = \frac{1}{2}(M+1).v_{CM}^2$$

$$\Rightarrow \quad \frac{4}{3} = \frac{1}{2}(M+1)\left(\frac{2M+1}{M+1}\right)^2 \Rightarrow \frac{8}{3} = \frac{(2M+1)^2}{M+1}$$

$\Rightarrow$ 8(M + 1) = 3 (2M + I)^2

$\Rightarrow$ 12M^2 + 4M – 5 = 0

$\Rightarrow$ $12M^2 + 10M - 6M - 5 = 0$

$\Rightarrow$ $2M(6M + 5) - 1(6M + 5) = 0$

$\Rightarrow$ $(2M - 1)(6M + 5) = 0$

$\Rightarrow$ $M = \frac{1}{2}$ or $M = \frac{-5}{6}$

Ignoring negative value

$M = \frac{1}{2} = 0.5$ kg

93. **(b)** Let equation of SHM be = A sin $(\omega t + \phi)$

Given, $\omega = 2\pi\nu = 2\pi \times 1 = 2\pi$ & $\phi = 1$

For zero phase

$\omega t + \phi = 0$

$2\pi t + 1 = 0$

$\therefore\ t = \frac{-1}{2\pi}$

94. **(a)** Planet is nearer to seen in DAB than in BCD. Hence speed in DAB is greator compared to BCD.
So time to cover DAB is lesser than that to cover BCD.

95. **(b)** Gases are highly compressible as interatomic distance between their atoms is large.

96. **(b)** Viscous force $= \eta A\left(\frac{dv}{dx}\right)$

between top layer & river bed

$$\frac{dv}{dx} = \frac{v}{H}$$

So, shearing stress $= \frac{\text{Viscous force}}{\text{Area}} = \eta\frac{\text{dv}}{\text{dx}} = \eta.\frac{v}{H}$

97. **(b)** As both steel rod and metal scale expands at 25°C,

True value of length of rod at 25°C

= Scale reading × $(1 + \alpha(\theta' - \theta))$ ...(i)

Where, α = coefficient of linear expansion

$(= 20 \times 10^{-6}/°C)$

θ' = temperature at which observation is taken

(= 25°C),

and θ = temperature at which metre scale reads correctly (= 0°C)

Subtituting values in (i), we get

Length of rod at 25°C

$= 1 \times [1 + 20 \times 10^{-6}(25 - 0)]$

$= 1 + 5 \times 10^{-4} = 1.0005$ m

Now, if l_2 = length of steel rod at 25°C and

l_1 = length of steel rod at 0°C, then we have

$l_2 = l_1(l + \alpha\,\Delta\,\theta)$

Substituting values, we have

$1.0005 = l_1[1 + 12 \times 10^{-6} \times (25 - 0)]$

$\Rightarrow$ $l_1 = \frac{1.0005}{1.0003} = 1.000199 \approx 1.0002\text{m}$

98. **(d)** Heat required to convert water at 100°C into steam at 100°C due to phase change

Q = mL

Given, m = mass = 100 kg and L = latent heat

$= 226 \times 10^5$ J/kg

$\therefore$ $Q = 100 \times 22.6 \times 10^5 = 22.6 \times 10^7$J

99. **(a)** Work done in path 1

= Area under AB

$= p\Delta V = p(3V - V) = 2pV$

From first law of thermodynamics,

$Q_1 = \Delta U + W$

Given, $Q_1 = 5pV$

$\Rightarrow$ $5pV = \Delta U + 2pV$

$\Rightarrow$ $\Delta U = 3PV$

ΔU will be same for path 1 & path 2

Consider path 2

Also, work done for process 2 is

W_2 = area under process 2

$$= \frac{1}{2} \times 2V \times \frac{1}{2}p + p \times 2V = \frac{5}{2}pV$$

Again, using first law of thermodynamics, heat required in process 2 is

$$Q_2 = \Delta U + W_2 = 3pV + \frac{5}{2}pV = \frac{11}{2}pV$$

100. **(a)** Pressure of A is thrice of that of B

$\Rightarrow$ $P_A = 3P_B$

Density of A is 2 times that of B

$\Rightarrow$ $P_A = 2P_B$

from gas equation

$p_A V_A = n_A RT_A$ and $p_B V_B = n_B RT_B$ but given $T_A = T_B$

$\Rightarrow$ $\frac{p_A V_A}{n_A} = \frac{p_B V_B}{n_B}$

$\Rightarrow$ $\frac{p_A V_A}{m_A / M_A} = \frac{p_B V_B}{m_B / M_B}$ $\left[\because n = \frac{m}{M}\right]$

$\Rightarrow$ $\frac{p_A M_A}{\rho_A} = \frac{p_B M_B}{\rho_B}$ $\left[\because \frac{m}{V} = \rho = \text{density}\right]$

Putting values, we get

$\frac{3p.M_A}{2\rho} = \frac{p.M_B}{\rho}$

$\Rightarrow$ $\frac{M_A}{M_B} = \frac{2}{3}$

101. (c) Resultant intensity is given by

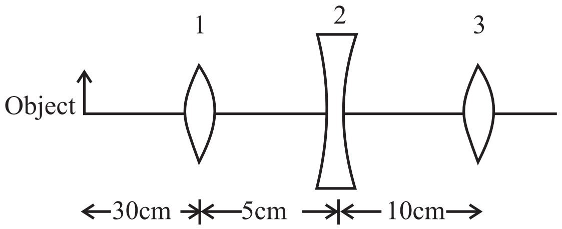
$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$$

$I_1 = I_2 = I$

and $\phi = 60°$

$$\Rightarrow \quad I = I + I + 2\sqrt{I}\sqrt{I}\cos 60° = 3I$$

102. (d) Given, lens combination and position of object is as shown below:



Consider first lens, first

$u = -30$ cm, $f = 10$ cm

By lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{15} \text{ or } v = 15 \text{ cm}$$

Now, image of first lens will behave as object for second lens.

Consider second lens,

$u = +10$ cm, $f = -10$ cm

Again, using lens formula, $\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-10} + \frac{1}{10}$

$$\Rightarrow \quad \frac{1}{v} = 0 \text{ or } v = \infty$$

So light rays emerge parallel after refraction from the second lens.

So parallel light rays passes through third lens, image is formed at focus. So image as formed 30 cm from 3rd lens.

103. (b) Image will be formed at the centre of curvature which will be real inverted and of the same size as that of object.

104. (b) When one of wave is shifted by angle θ, intensity will be

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\theta$$

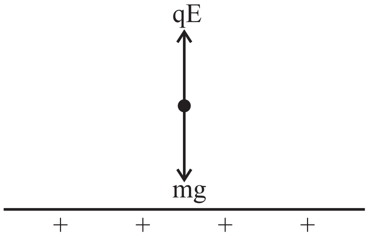
Given, $I = 84$ m W, $I_1 = 64$ mW and $I_2 = 4$mW

Putting values of I, I_1 & I_2 we get

$$84 = 64 + 4 + 2\sqrt{64}\sqrt{4}\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

105. (a)



Here, $mg = qE \Rightarrow mg = q\frac{\sigma}{\varepsilon_0}$

$$\left(\because E = \frac{\sigma}{\varepsilon_0}, \text{ near a conducting surface}\right)$$

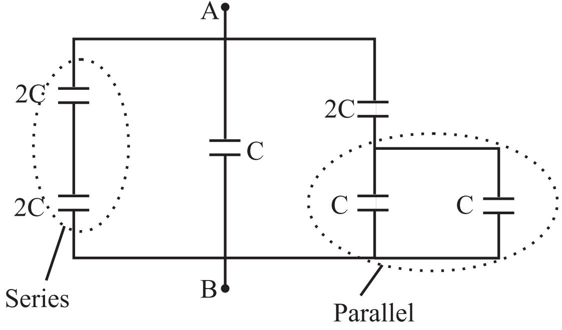
$$\Rightarrow \sigma = \frac{\varepsilon_0 mg}{q}$$

Given $m = 2 \times 10^{-6}$ kg, $q = 5 \times 10^{-6}$ C

$g = 10$ m/s^2 and $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2

$$\Rightarrow \sigma = \frac{8.85 \times 10^{-12} \times 2 \times 10^{-6} \times 10}{5 \times 10^{-6}} = 35.4 \times 10^{-12}\,\text{C/m}^2$$

106. (b)

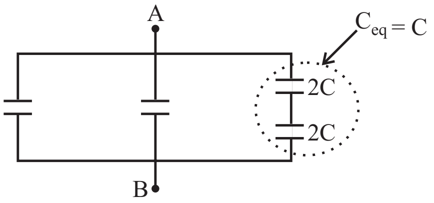


$$\frac{1}{C_{eq}} = \frac{1}{2C} + \frac{1}{2C}$$

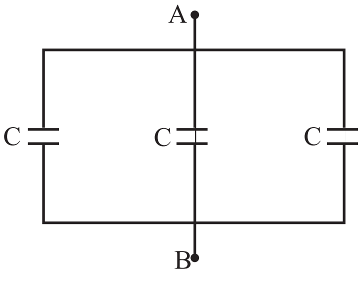
$\therefore \quad C_{eq} = C$

$\therefore \quad C_{eq} = C + C = 2C$

Now, the circuit is drawn as



Circuit's equivalent is

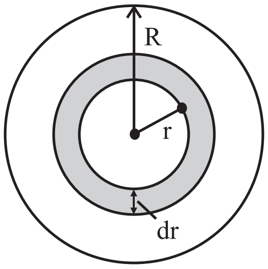


3 capacitors are in parallel.

$\therefore C_{\text{equivalent}} = C + C + C = 3C$

107. (b) Heat produced through dr width in time t

Heat = $\int_0^R V.t.j.2\pi r\,dr$



Given V = 50 V, t = 100 s

$j = 2 \times 10^{10} r^2$, $R = 2 \times 10^{-3}$ m

Total heat flowing through wire

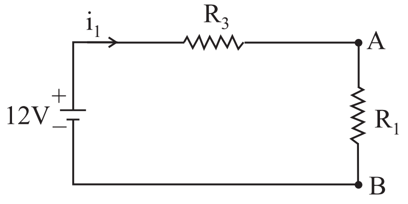
$$= \int_0^R 50 \times 100 \times 2 \times 10^{10} r^2 \times 2\pi r \times dr$$

$$= \int_0^R 2\pi \times 10^{14} \cdot r^3 \cdot dr = 2\pi \times 10^{14} \times \left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{\pi}{2} \times 10^{14} \times (2 \times 10^{-3})^4$$

$= 8 \times \pi \times 10^{14} \times 10^{-12} = 800\pi$ J

108. (a) When switch S is open



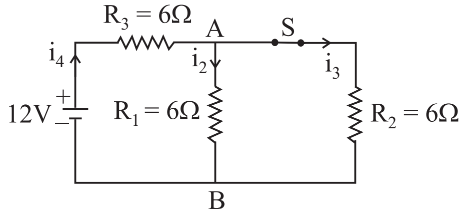
Net resistance = 6 + 6 = 12Ω

Current $i_1 = \frac{12V}{12\Omega} = 1A$

Potential across R_1 is

$V_{AB} = i_1 \times R_1 = 1 \times 6 = 6V$

After switch is closed



Total resistance is here

$$= 6 + (6 \parallel 6) = 6 + \frac{6 \times 6}{6+6} = 6 + 3 = 9\Omega$$

Current passing through cell $i_4 = \frac{V}{R_{\text{Total}}} = \frac{12}{9} = \frac{4}{3}$A

$\therefore \quad i_2 = i_3 = i_4 / 2 = \frac{4}{3 \times 2} = \frac{2}{3}$A

So, potential across R_1

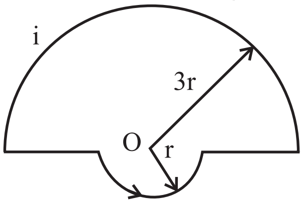
$V'_{AB} = i_2 R_1 = \frac{2}{3} \times 6 = 4V$

So, change in potential of R_1 is

$= V'_{AB} - V_{AB} = 4 - 6 = -2V$

109. (b) Same current i is flowing each of the circuit

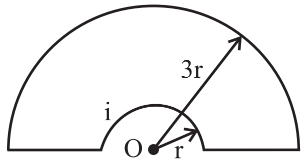
(i)



Magnetic field at O is $B_1 = B_{3r} + B_r$

$$= \frac{\mu_0 I}{4(3r)} + \frac{\mu_0 I}{4(r)} = \frac{4\mu_0 I}{12r} = \frac{\mu_0 I}{3r}$$

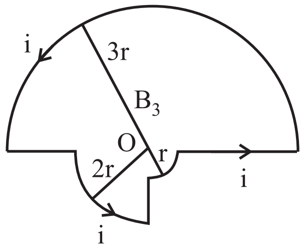
(ii)



$B_2 = B_r - B_r$

$$\frac{\mu_o I}{4(3r)} - \frac{\mu_o I}{4(r)} = \mu_o I/6r$$

(iii)



$= B_{3r} + B_{2r} + B_r$

$$= \frac{\mu_0 I}{4(3r)} + \frac{\mu_0 I}{8(2r)} + \frac{\mu_0 I}{8r} = \frac{13}{48}\frac{\mu_0 I}{r}$$

So, $B_1 > B_3 > B_2$

110. (b) Magnetic field does not do any work on charge particle. So kinetic energy remains constant but momentum changes as direction.

111. (b) Time period in magnetometer

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

where, I = moment of inertia = $\frac{ml^2}{12}$

and M = magnetic dipole moment of magnet.

Before cutting $T = 2S = 2\pi\sqrt{\frac{ml^2}{12 \times MB}}$

When magnet is cut into 3 parts, mass of each part will be m/3 each part length is l/3

So, moment of inertia of each part will be

$$I' = \frac{m'l'^2}{12} = \frac{\frac{m}{3}\times\frac{l^2}{9}}{12}$$

Moment of inertia of 3 parts together

$$= 3\times\frac{m}{3}\times\frac{l^2}{9}\times\frac{l}{12} = \frac{ml^2}{9\times 12}$$

Also, magnetic dipole moment of each part $\frac{m}{3}$

So dipole moment of 3 parts together

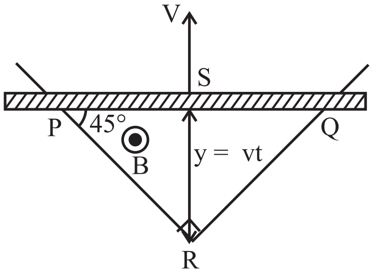
$$= 3\times\frac{M}{3} = M$$

Hence, time period of oscillation of combination

$$T' = 2\pi\sqrt{\frac{I'}{M_1B}} = 2\pi\sqrt{\frac{ml^2}{9\times 12MB}} = \frac{2\pi}{3}\sqrt{\frac{ml^2}{12MB}}$$

$$= \frac{T}{3} = \frac{2}{3}s$$

112. (c)



In triangle $y = v \times t$

at $t = 4$s,

$y = 5 \times 4 = 20$ m

Now in ΔQRS,

$$\frac{SR}{SQ} = \tan 45° \Rightarrow SR = SQ \qquad (\because \tan 45° = 1)$$

So length PQ of rod is 2 × SR = 2y = 40 m

$$\text{Area of } \Delta\text{PQR} = \frac{1}{2}\times\text{base}\times\text{height}$$

$$\Delta\text{PQR} = \frac{1}{2}\times 2y = y^2$$

So, magnitude of emf induced in triangular loop is

$$E = \frac{d\phi}{dt} = B\left(\frac{dA}{dt}\right) = B\times\frac{dy^2}{dt} = B2y\times\frac{dy}{dt} \leq 2ByV$$

= 2 × 0.1 × 20 × 5 = 20 V

113. (c) Power given = VI

= 220 × 6J

As 40% power is lost so power in secondary coil = 60% of input power

$$\text{Output power} = \frac{60}{100}\times \text{ input power}$$

$$\Rightarrow \frac{60}{100}\times 220\times 6 = 1100\times I_s$$

$$\Rightarrow I_s = \frac{6\times 22\times 6}{1100} = 0.72\text{A}$$

Answer does not matches with any option, however if we put 40% as output power option c matches.

114. (b) Given, waves are $y_1 = a\sin(\omega t - kx)$

$$y_2 = b\cos\left(\omega t - kx + \frac{\pi}{3}\right)$$

$$\Rightarrow y_2 = b\sin\left(\frac{\pi}{2} + \omega t - kx + \frac{\pi}{3}\right)$$

[Using $\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$]

$$y_2 = b\sin\left(\omega t - kx + \frac{5\pi}{6}\right)$$

So, phase difference of y_1 and y_2 is $\frac{5\pi}{6}$ rad.

115. (c) Maximum kinetic energy of photoelectrons is

$$= \frac{1}{2}mv_{\max}^2 = \frac{1}{2}\times 9.1\times 10^{-31}\times(1.6\times 10^6)^2 J$$

$$= \frac{9.1\times 10^{-31}\times(1.6\times 10^6)^2}{2\times 1.6\times 10^{-19}}\text{eV} = 7.2\text{ eV}$$

So, work function = photon energy – maximum kinetic energy of electron

= 10.5 – 7.2 = 3.3 eV

116. (d) Energy value of n th state for H-electron is

$$E_n = \frac{-13.6}{n^2}$$

Energy radiated in transition $n = 3$ to $n = 2$ is

$$E_2 = \frac{-13.6}{9} - \frac{-13.6}{4} = \frac{5}{36}\times 13.6$$

Also, energy radiated in transition $n = 2$ to $n = 1$ is

$$E_1 = \frac{-13.6}{4} - \frac{-13.6}{1} = \frac{13.6}{n^2}$$

$$\text{So } \frac{E_2}{E_1} = \frac{5}{27}$$

117. (a) Given, mass of $^{29}_{14}\text{Si} = 28.976495u$

Now, mass of 14 protons

= 14 × 1.007276 = 14.101864 u

Also, mass of 29 – 14 = 15 neutrons

= 15 × 1.008664 = 15.12996

Total mass of constituents particles

= 14.101864 + 15.12996 = 29.231824

So, mass defect = mass of constituents – mass of atom

= 29.231824 – 28.976495 = 0.255329 u

Binding energy = mass defect × c^2

= mass defect × c^2 × 931.5 Mev/c^2

= 0.255329 × 931.5 = 237.84 MeV

118. (c) Majority carriers in n-type semiconductors are electrons

119. (d) Germanium is a semiconductor and copper is a metal. On cooling semiconductors resistance increases whereas conductors resistance decreases. Semiconductors resistance increases as no. of electrons & holes concentration decreases due to decrease in temperature.

120. (b) Modulation index

$$m = \frac{\text{peak value message signal}}{\text{peak value of carrier signal}} = \frac{V_2}{V_1}$$

Upper side band (USB)

$v_+ = v_1 + v_2$ ...(i)

And lower side band (LSB)

$v_- = v_1 - v_2$ ...(ii)

Adding eqs. (i) and (ii);

$$\Rightarrow \quad v_1 = \frac{v_+ + v_-}{2} \text{ and } v_2 = \frac{v_+ - v_-}{2}$$

CHEMISTRY

121. (d) Series of spectral lines

Series	n_1	n_2	Spectral Region
Lyman	1	2,3...	Ultraviolet
Balmer	2	3,4...	Visible
Paschen	3	4,5...	Infrared
Brackett	4	5, 6...	Infrared
P-fund	5	6,7...	Infrared

122. (c) Incidence (C) can be divided into small steps in which all the steps are equal. Hence we can say that option (c) can be comparable to the concept of quantisation.

123. (c) IUPAC name for atomic number 123 is unbitrium and symol is ubt.

Digit	Name	appr
1	Un	u
2	bi	b
3	tri	t

124. (c) Energy of atomisation depends upon the number of unpaired electrons. The enthalpy of atomisation of transition elements increases as the number of unpaired electrons increases down the group.

125. (b) Ionic character increases with increasing difference in electronegativety of bonded atoms in a molecule. Hence, correct order of ionic character will be

$$\underbrace{LiF > K_2O > SO_2}_{\substack{\text{Polar} \\ \text{(Ionic bond)}}} > \underbrace{N_2}_{\substack{\text{Non-polar} \\ \text{(Covalent bond)}}}$$

126. (d) Bond order $= \frac{1}{2}[N_B - N_A]$

$$= \frac{\text{bonding electrons} - \text{antibonding electrons}}{2}$$

Molecules	Bond order
He_2	$\frac{2-2}{2} = 0$
He_2^+	$\frac{2-1}{2} = 0.5$
O_2	$\frac{10-6}{2} = 2$
O_2^+	$\frac{10-5}{2} = 2.5$

127. (b) Given,

Volume of gas = 12L

mole of gas = 1

$R = 0.0821 \text{ L-atm mol}^{-1} K^{-1}$

T = 297°C + 273 = 570 K

From gas equation, pV = nRT

$$p = \frac{nRT}{V} = \frac{1 \times 0.0821 \times 570}{12}$$

= 38.99 atm (1 atm = 101325 Pa)

= 395066.18 Pa = 395 kPa

128. (d) From gas equation,

$pV = nRT$

$$pV = \frac{W \times R \times T}{M}$$

$$M_{CO_2} = \frac{4.4 \times 0.0821 \text{ atm.L mol}^{-1} K^{-1} \times 273 \text{ K}}{0.82 \text{ atm} \times 2.73 \text{ L}} = 44 \text{ g mol}^{-1}$$

where M is the molecular weight of gas

129. (d) $N_2 + 3H_2 \longrightarrow 2NH_3$

3 mol H_2 gives 2 mol NH_3

$\therefore$ 5 mol H_2 gives $= \frac{2}{3} \times 5 = 3.3 \text{mol}$

130. (c) $\underset{28 \text{ g}}{C_2H_4(g)} + H_2(g) \xrightarrow{\text{Catalyst}} \underset{30\text{g}}{C_2H_6(g)}$

28 = molar mass of C_2H_4

30 = molar mass of C_2H_6

$\because$ 28 g C_2H_4 gives 30 g C_2H_6

$\therefore$ 50 g C_2H_6 is obtained by $= \frac{28 \times 50}{30} = 46.67 \text{g } C_2H_4$

131. (a) Since for solids and liquids, volume changes are insignificate.

$\therefore \Delta V = 0$

$\therefore \Delta H = \Delta U + P\Delta V \Rightarrow \Delta H = \Delta U$

Therefore, the difference between ΔH and ΔU is almost negligible for solids and liquids.

132. (a) Given, K_b of $NH_4OH = 5.00 \times 10^{-10}$

$K_w = 10^{-14}$ at 298 K

$$K_h = \frac{K_w}{K_b} = \frac{10^{-14}}{5 \times 10^{-10}} = 2 \times 10^{-5}$$

133. (b) Given, $K_C = 4 \times 10^{-4}$

T = 1000 K

$K_p = K_C.(RT)^{\Delta n_g}$

Δn_g = moles of product (n_p) – moles of reactant (n_R)

= 3 – 2 = 1

$K_p = 4 \times 10^{-4} (0.0821 \times 800)^1$

$K_p = 0.026$

134. (a) Hydrogen peroxide in presence of light can easily break down, or decompose, into water and oxygen

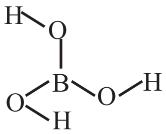
$$2H_2O_2 \longrightarrow 2H_2O + O_2$$

135. (c) Slaking is the addition of water to calcium oxide powder (lime). The resulting product is calcium hydroxide. (Milk of lime)

$$CaO + H_2O \longrightarrow Ca(OH)_2$$

This reaction is exothermic, so the mixture heats up.

136. (a) Boric acid (H_3BO_3) or orthoboric acid is a weak, monobasic Lewis acid of boron.



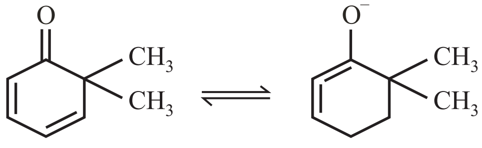
137. (a) In CO_2 molecule, carbon atom has two double bonds with oxygen. The molecule can be represented as

$$\underset{sp^2}{O} = \underset{sp}{C} = \underset{sp^2}{O}$$

138. (b) Global warming stresses ecosystems through temperature rises and rise in temperature is responsible for melting of Himalayan glaciers.

139. (b) Organic esters ketons and aldehydes with an α-hydrogen, often form enols. This phenomenon is known as enolisation.

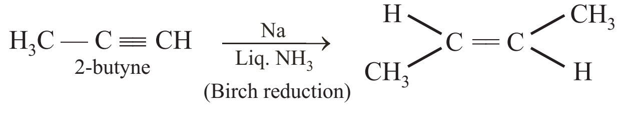
In case of compound (i), it will have α-hydrogen atom. However and product is not stable due to ring strain and electron repulsion. Hence the equilibrium will tends to more toward keto product



(less stable)

Compound (V) do not have α-hydrogen atom therefore it will not show enolisation.

140. (d)



141. (b) (a) $2RX + 2Na \xrightarrow[\text{ether}]{\text{Dry}} \underset{\text{(same no. of carbon)}}{R — R} + 2Na^+X^-$

(b) $RCO_2Na \xrightarrow{\text{NaOH+CaO}} \underset{\text{Less number of carbon}}{RH} + Na_2CO_3$

(c) $2R — COOK \xrightarrow{\text{Electrolysis}} R — R + 2CO_2$

(d) $R — X + H_2 \xrightarrow{\text{Zn.HCl}} \underset{\text{Samenumber of carbon}}{R — H} + HCl$

142. (b) $LiCoO_2$ crystallises in a rhomobohedral structure. Let at initial 100 atoms are present of both Li and Co Li carries (+1) charge and Co carries (+3) charge. So the total charge will be 400.

100 Li + 100 Co = 400

After 50% Li is extracted then cobalt have to increase its oxidation state to balance the total charge.

50 Li + 100 Co* = 400

100 Co* = 350

Co* = 3.5

So, after extraction of 50% Li atom new oxidation state of Co to 3.5.

Change in oxidation state = final oxidation state – initial oxidation state

= 3.5 – 3

Change in average oxidation state of Co

= 0.5 % of change in average oxidation state of Co

$$= \frac{0.5}{3} \times 100 = 16.66\% \qquad \text{(increases)}$$

143. (c) Given, $m = 0.001$

$\Delta_y = 5.58 \times 10^{-3}$ K

$K_f = 1.86$ kg mol^{-1}

$\Delta T_f = i.K_f.m$

$$i = \frac{\Delta T_f}{K_f m} = \frac{5.58 \times 10^{-3}\,K}{1.86 \times 0.001} = 3$$

∵ $i = 3$

∴ The total number of possibilities for different types of *A* and *B* cations is 3.

144. (d) Given,

$$2C_6H_5COOH \rightarrow (C_6H_5COOH)_2$$

Initial moles	1	0
Final moles	1–α	α/2

Total number of moles at eqm. = 1 – α + α/2

= 1 – α/2

$$\therefore i = \frac{\text{Total moles at equilibrium}}{\text{Initial moles}} = \frac{1 - \frac{\alpha}{2}}{1}$$

$\because \Delta T_f = iK_f m$

$$2.2 = \left(1 - \frac{\alpha}{2}\right) \times 5.0\text{k kg mol}^{-1} \times \frac{2.44}{122} \times \frac{1000}{25}$$

$$1 - \frac{\alpha}{2} = \frac{2.2 \times 122 \times 25}{2.44 \times 1000 \times 5}$$

$$\Rightarrow \quad 1 - \frac{\alpha}{2} = 0.55$$

α = 0.90

% degree of association = 90%

145. (b) Given

$$BrO^- \xrightarrow{+1e^-} \frac{1}{2}Br_2 \qquad n = 1, E_2^\circ = -0.45V$$

$$\frac{1}{2}Br \xrightarrow{+1e^-} Br \qquad n = 1, E_3^\circ = -1.07V$$

$$BrO_3^- \xrightarrow{+6e^-} Br^- \qquad n = 6, E_4^\circ = -0.61V$$

$$BrO_3^- \xrightarrow{+4} BrO^- \qquad n = 4, E_1^\circ = ?$$

$$\begin{tikzcd} BrO_3^- \arrow[r, "E_1^\circ"'] \arrow[rrr, bend right, "0.61V"', "E_4^\circ" below] & BrO^- \arrow[r, "0.45V", "E_2^\circ"'] & \frac{1}{2}Br \arrow[r, "1.07V", "E_3^\circ"'] & Br^- \end{tikzcd}$$

$$\Delta G_4^\circ = \Delta G_1^\circ + \Delta G_2^\circ + \Delta G_3^\circ$$

$$(-n_4FE_4^\circ) = (-n_1FE_1^\circ) + (-n_2FE_2^\circ) + (-n_3FE_3^\circ)$$

$$E_1^\circ = \frac{n_4E_4^\circ - n_2E_2^\circ - n_3E_3^\circ}{n_1}$$

$$E_1^\circ = \frac{(6\times 0.61) - (1\times 0.45) - (1\times 1.07)}{4}$$

$$= \frac{3.66 - 0.45 - 1.07}{4}$$

$$E_1^\circ = 0.53V$$

146. (c) Given, $T_1 = 22°\,C + 273 = 295\ K$
$T_2 = 32°C + 273 = 305K$

$$\log\frac{K_1}{K_2} = \frac{E_a}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\log\frac{2}{1} = \frac{E_a}{2.303\times 8.3}\left(\frac{10}{295\times 305}\right)$$

$$0.3010 = \frac{E_a}{2.303\times 8.3}\left(\frac{10}{295\times 305}\right)$$

$$\frac{0.3010\times 2.303\times 8.3\times 295\times 305}{10} = E_a$$

$E_a = 51.76$ kJ mol^{-1} (nearest value to 49.8 kJ mol^{-1})

147. (b) We know, $\frac{x}{m} = Kp^{1/n}$

or
$$\log\frac{x}{m} = \log K + \frac{1}{n}\log p$$

Slope = $\frac{1}{n}$ of intercept = log K

$$\frac{1}{n} = \tan\theta = \tan 45° = 1, \text{i.e.} n = 1$$

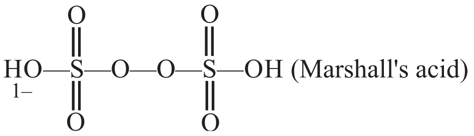
and log K = 0.3 = antilog (0.3) = 2

At p = 1 atm, $\frac{x}{m} = Kp^{1/n} = 2\times(1)^{1/1} = 2$

148. (c) Ellingham diagram is a plot between change in free energy and temperature.

149. (a) Brown ring test is used to detect the presence of nitrate ion (NO_3^-).
It can be performed by adding iron sulphate to a solution of a nitrate, then slowly adding concentrated sulphuric acid at the bottom of aqueous solution. A brown ring ($[Fe(H_2O)_5(NO)]^{2+}$) will appear at the junction of the two layers, indicating the presence of the nitrate ion.

150. (b)



$$\underset{1-}{HO}-\overset{\overset{O}{\|}}{\underset{\underset{O}{\|}}{S}}-O-O-\overset{\overset{O}{\|}}{\underset{\underset{O}{\|}}{S}}-OH \text{ (Marshall's acid)}$$

2x + 4 (–1) + 4 (–2) = 0 ⇒ 2x – 4 – 8 = 0
2x = 12 ⇒ x = +6

151. (a) Paramagnetic nature ∝ number of unpaired electron

Metal ion	d-electrons	Number of unpaired e$^-$
Cu^{2+}	⇅ ⇅ ⇅ ↑ ↑	2
V^{2+}	↑ ↑ ↑ ☐ ☐	3
Cr^{2+}	↑ ↑ ↑ ↑ ☐	4
Mn^{2+}	↑ ↑ ↑ ↑ ↑	5

Hence, increasing order of paramagnetic nature is,
$Cu^{2+} < V^{2+} < Cr^{2+} < Mn^{2+}$

152. (c) Coordination complex = $[Co(H_2O)_6]^{2+}$

- The complex is octahedral.
- H_2O is weak field ligand, so it will for m outer orbital complexes.
- Complex is paramagnetic in nature due to presence of unpaired electrons.

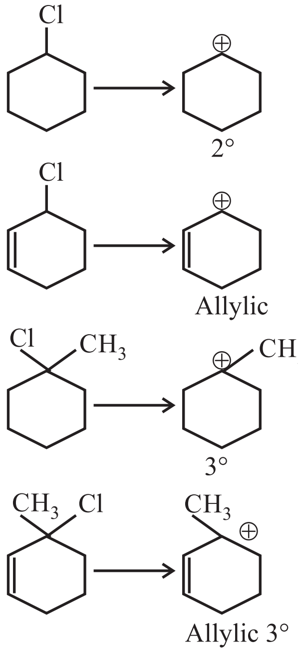
153. (a) Step-growth polymerisation refers to a type of polymerisation mechanism in which bi-functional or multifunctional monomers react stepwise to form long chain polymers.
Nylon-6,6 is type of polyamide or nylon. It is formed by step-growth polymerisation by polycondensation of hexamethylene diamine and adipic acid.

154. (b) The pancreas secretes insulin and glucagon. Both hormones play a vital role in regulating blood sugar levels. If the level of one hormone is higher or lower than the ideal range, blood sugar levels may become imbalanced.

155. (d) A type of detergent in which the active part of the molecule is a positive ion (cation) is known as cationic detergents are usually quaternary ammonium salts and often also have bactericidal properties. It contains a long-chain cations that is responsible for their surface-active properties. It is not cheap and not used widely.

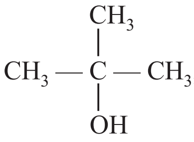
156. (d) In S_N1 nucleophilic reaction, the first and the slow step is the formation of a carbocation.

Reactivity of alkyl halides towards S_N1 follows order allylic 3° > 2° allylic > 3° > 2°

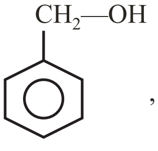


157. (c) Lucas reagent is used to differentiate between primary, secondary and tertiary alcohols. Since benzylic, allylic and tertiary carbocations are very stable they react immediately with Lucas reagent and gives turbidity due to the low solubility of the organic chloride in the aqueous mixture.

(i) $\underset{\text{n-butanol}}{CH_3-CH_2-CH_2-CH_2-OH}$, 1° alcohol does not give reaction immediately with Lucas reagent

(ii) 

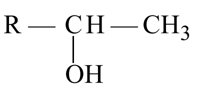
$CH_3-\overset{CH_3}{\underset{OH}{C}}-CH_3$, 3° alcohol-white turbidity

(iii) 

, 1° alcohol benzylic cation-white turbidity

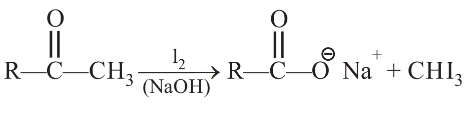
(iv) $CH_3 = CH - CH_2 - OH$, 1° alcohol allylic cation-white turbidity.

158. (b) **Iodoform test** is used to check the presence of carbonyl compounds with the structure of R—CO—CH_3 or alcohols with the structure



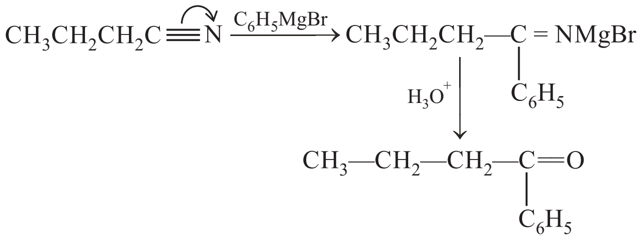
$R-\underset{OH}{CH}-CH_3$ in a given unknown substance.

Reaction:



$$R-\overset{O}{\overset{\|}{C}}-CH_3 \xrightarrow[(NaOH)]{I_2} R-\overset{O}{\overset{\|}{C}}-\overset{\ominus}{O}\,\overset{+}{Na} + CHI_3$$

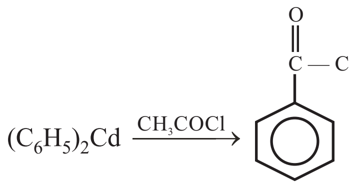
(a)



$$CH_3CH_2CH_2C\equiv N \xrightarrow{C_6H_5MgBr} CH_3CH_2CH_2-\underset{C_6H_5}{C}=NMgBr \xrightarrow{H_3O^+} CH_3-CH_2-CH_2-\underset{C_6H_5}{C}=O$$

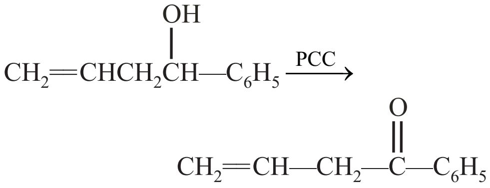
Does not give iodoform test

(b)



Give iodoform test

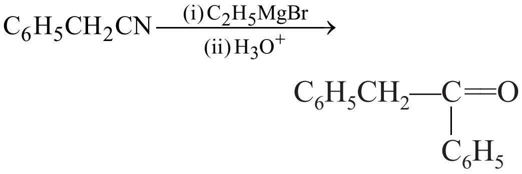
(c)



$$CH_2=CHCH_2\overset{OH}{CH}-C_6H_5 \xrightarrow{PCC} CH_2=CH-CH_2-\overset{O}{\overset{\|}{C}}-C_6H_5$$

Does not give iodoform test

(d)

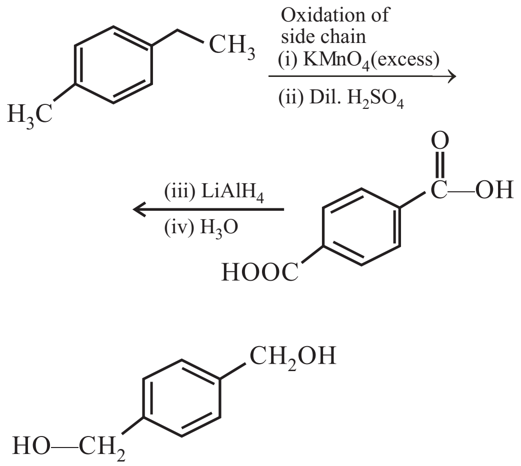


$$C_6H_5CH_2CN \xrightarrow[(ii)\,H_3O^+]{(i)\,C_2H_5MgBr} C_6H_5CH_2-\underset{C_6H_5}{C}=O$$

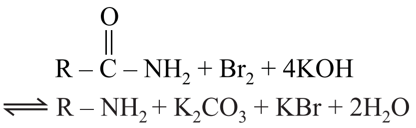
Doesnot give iodoform test

Hence, correct option is (b).

159. (a)



160. (c) Hofmann degradation reaction:



$$R-\overset{O}{\overset{\|}{C}}-NH_2 + Br_2 + 4KOH \rightleftharpoons R-NH_2 + K_2CO_3 + KBr + 2H_2O$$