TS/EAMCET Solved Paper 2021
Held on August 5

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. A and B are subsets of R. Every element x of A is mapped to an element of B by the rule

$$y(x) = \begin{cases} \dfrac{5x}{(x-3)(x+3)}, & \text{if } x \neq -1 \\ -1 \quad , & \text{if } x = -1 \end{cases} \quad \text{then A is equal to}$$

(a) R / {– 3, + 3, – 0} (b) R/ {– 3, + 3}
(c) R / {– 3, 3, 0, – 1} (d) R

2. The domain and range of $y(x) = \cos x - 3$ are respectively
(a) R and [–1,1] (b) R and [–4, –2]
(c) R / {0} and [0, 1] (d) $R / \left\{(2n+1)\frac{\pi}{2}\right\}$ and [–1,1]

3. If $f(1) = 0$ and $f(n + 1) - f(n) = 5n$, $\forall\, n \in N$, then $f(n)$ is equal to
(a) $\frac{5}{2}\left(n^2+n\right)$ (b) $\frac{5}{2}\left(n^2-n\right)$
(c) $\frac{5}{3}\left(3n^2-n\right)$ (d) $\frac{5}{4}\left(4n^2-1\right)(n-1)$

4. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then adj (adj A) is equal to
(a) A (b) $36A$ (c) $6A$ (d) $\frac{1}{6}A$

5. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then A^5 is equal to
(a) A (b) Identity matrix
(c) Null matrix (d) A^{-1}

6. Let A and B be two 3 × 3 non-singular matrices, such that det $(ATBA) = 27$ and $\det(AB^{-1}) = 8$. Then det $(BTA^{-1}B)$ is equal to
(a) $\frac{3}{32}$ (b) $\frac{1}{16}$ (c) 1 (d) 16

7. If $z_1 = 1 - 2i$, $z_2 = 1 + i$ and $z_3 = 3 + 4i$, then $\left|\left(\frac{1}{z_1}+\frac{2}{z_2}\right)\frac{z_3}{z_2}\right|$ is equal to
(a) $\frac{\sqrt{7}}{2}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\sqrt{\frac{45}{2}}$ (d) $\frac{\sqrt{15}}{2}$

8. If $2 + 2\sqrt{3}\, i = k(\cos\theta + i\sin\theta)$, $(k > 0)$, then $\frac{1}{\sqrt{3}}[\cos 6\theta + i \sin 6\theta]$ is equal to
(a) 1 (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

9. If $x = \frac{4}{5}+\frac{3}{5}$ i and y = $\frac{\sqrt{3}}{\sqrt{8}}-\frac{\sqrt{5}}{\sqrt{8}}i$, then $\left(x^2+\frac{1}{x^2}\right)\left(y^2-\frac{1}{y^2}\right)$ is equal to
(a) $\frac{-7\sqrt{3}}{5\sqrt{5}}i$ (b) $\frac{7}{125}i$ (c) $\frac{7\sqrt{3}}{5\sqrt{5}}i$ (d) $\frac{\sqrt{15}}{\sqrt{8}}i$

10. If $x^2 - 5x - 14 > 0$ and x lie outside $[\alpha, \beta]$, then $\frac{\alpha}{\beta}$ is equal to
(a) $\frac{-2}{7}$ (b) $\frac{-7}{2}$ (c) $\frac{2}{7}$ (d) $\frac{7}{2}$

11. For $x \in R / \{-6\}$, the value of $\frac{(x+2)(x+5)}{(x+6)}$ does not lie in the interval
(a) [–9, –1] (b) [– 5, – 2]
(c) (–5, 2) (d) (–9, –1)

12. If $x = 2 + 2^{2/3} + 2^{1/3}$, then $x^3 - 6x^2 + 6x$ is equal to
(a) 3 (b) 2 (c) 1 (d) 0

13. The roots of the cubic equation $3x^3 + 4x^2 - 5x - 2 = 0$ are diminished by h and a cubic equation with these diminished roots is formed. If the transformed equation does not contain x^2 term, then the roots of the transformed equation are

(a) $\frac{-7}{3}, \frac{2}{3}, \frac{5}{3}$ (b) $\frac{7}{3}, \frac{-2}{3}, \frac{-5}{3}$

(c) $\frac{13}{9}, \frac{-14}{9}, \frac{1}{9}$ (d) $\frac{-13}{9}, \frac{14}{9}, \frac{-1}{9}$

14. If $0 < r < s < n$ and $npr = nps$, then $r + s$ is equal to

(a) 2n – 2 (b) 2n – 1 (c) 2 (d) 1

15. Three and four digit numbers are formed from the digits 1, 3, 5, 6, 8. If e_1 is number of three digit even numbers with no digit repeated and e_2 is number of four digit even numbers with no digit repeated. Also, O_1, represent the number of three digit odd numbers in which no digit is repeated and O_2 represent the number of four digit odd numbers in which no digit is repeated. Then

(a) $e_1 = O_1, e_2 = O_2$

(b) $e_1 + e_2 + O_1 + O_2 = 5p_3 + 5^3$

(c) $\frac{e_1 + e_2}{2} = \frac{O_1 + O_2}{3} = 6^2$

(d) $\frac{e_1 + e_2}{O_1 + O_2} = \frac{3}{2}$

16. If $f(n)$ is the coefficient of xn in the expansion of $(1 + x)(1 - x)n$, then $f(2021)$ is equal to

(a) –2019 (b) 2020 (c) 2021 (d) – 2022

17. If p and q are respectively the coefficient of x^{-3} and x^{-5} in the expansion of $\left(x^{1/3} + \frac{1}{2x^{1/3}}\right)$, $x > 0$, then $\frac{5p}{4q}$ equal to

(a) 102 (b) 408 (c) 182 (d) 468

18. For $n, p \in N - \{1\}$, the coefficient of x^3 in $\frac{(1-x)^{-1/p}}{(1-x)^n}$ is equal to

(a) $\frac{(np+1)(np+p+1)(np+2p+1)}{p^3 \times 3!}$

(b) $\frac{(np+1)(np+p)(np+2p)}{3!\,p^3}$

(c) $\frac{(np+p)(np+2p)(np+3p)}{3!\,p^3}$

(d) $\frac{(np+1)(np+2)(np+3)}{3!\,p^3}$

19. If $\frac{1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$ and $\frac{x}{(x-1)(x-2)(x-3)} = \frac{P}{x-1} + \frac{Q}{x-2} + \frac{R}{x-3}$, then A + 2B + 3C is equal to

(a) P + Q + R (b) P + 2Q + 3R

(c) 3P + 2Q + R (d) AP + BQ + CR

20. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$ is equal to

(a) – 1 (b) 1 (c) 0 (d) 12

21. $\left(\frac{\sin 35°}{\cos 55°}\right)^2 + \left(\frac{\cos 55°}{\sin 35°}\right)^2 - 2\cos 30°$ is equal to

(a) $2 + \sqrt{3}$ (b) $2 - \sqrt{3}$ (c) $2\sqrt{3}$ (d) $3\sqrt{2}$

22. If $\cos\theta = \frac{-3}{5}$ and $\pi < \theta < 3\pi/2$, then $\tan\left(\frac{\theta}{2}\right)$ is equal to

(a) 2 (b) – 2 (c) 1 (d) – 1

23. $\frac{1 - \tan^2 15°}{1 + \tan^2 15°}$ is equal to

(a) 1 (b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) 2

24. If $6\cos 2\theta + 2\cos^2\left(\frac{\theta}{2}\right) + 2\sin^2\theta = 0$, $-\pi < \theta < \pi$, then θ is equal to

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{3}, \cos^{-1}\left(\frac{3}{5}\right)$

(c) $\cos^{-1}\left(\frac{3}{5}\right)$ (d) $\pm\frac{\pi}{3}, \pm\left(\pi - \cos^{-1}\frac{3}{5}\right)$

25. $\sin\left(\tan^{-1}\frac{4}{5} + \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{9} - \tan^{-1}\frac{1}{7}\right)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

26. Let $k > 0$ and $t = \sec h^{-1}\left(\frac{1}{2}\right) - \operatorname{cosec} h^{-1}\left(\frac{3}{k}\right)$. If $3e t = 2 + \sqrt{3}$, then k is equal to

(a) 2 (b) 4

(c) $3\sqrt{3}$ (d) $3\sqrt{2}$

27. In a ΔABC, if $4a = b + c$, then $\tan\frac{B}{2}\tan\frac{C}{2}$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{3}{5}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

28. If 4 times the area of a ΔABC is $c^2 - (a - b)^2$, then sin C

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) 1

29. In a ΔABC, if $\cot\frac{A}{2}\cot\frac{B}{2} = k$, then all the possible values of k lies in

(a) (0,1] (b) $[1, \infty)$ (c) $(1, \infty)$ (d) (0, 1)

30. In a Δ ABC, $b^2 \sin 2C + c^2 \sin 2B$ is equal to

(a) 0 (b) 4Δ (c) 2Δ (d) Δ

31. The position vectors of the points P and Q are respectively $-2\hat{i} - 3\hat{j} + \hat{k}$ and $3\hat{i} + 3\hat{j} + 2\hat{k}$. The ratio in which the point having position vector $\frac{-9}{2}\hat{i} - 6\hat{j} + \frac{1}{2}\hat{k}$ divides the line segment joining P and Q is

(a) – 3 : 2 (b) 1 : 2 (c) 2 : 1 (d) – 1 : 3

32. Consider the vectors $\boldsymbol{a} = 3\hat{i}+5\hat{j}+2\hat{k}$, $\boldsymbol{b} = 2\hat{i}-3\hat{j}-5\hat{k}$ and $\boldsymbol{c} = -5\hat{i}-2\hat{j}+3\hat{k}$. If l, m and n are length of projections of $\boldsymbol{a}$ on $\boldsymbol{b}$, $\boldsymbol{b}$ on $\boldsymbol{c}$ and $\boldsymbol{c}$ on a respectively, then
(a) $l+m-n=0$ (b) $l=m=n$
(c) $l-m+n=0$ (d) $m+n-l=0$

33. Let $(x, y) \in (R \times R)$ and $\boldsymbol{a} = x\hat{i}+2\hat{j}-\hat{k}$, $\boldsymbol{b} = 6\hat{i}-y\hat{j}+2\hat{k}$ be two vectors. If $|\boldsymbol{a} \times \boldsymbol{b}|^2 + |\boldsymbol{a}.\boldsymbol{b}|^2 = f(x)g(y)$, then $f(x) + g(y) - 46 = 0$ represents
(a) a pair of lines (b) an ellipse
(c) a hyperbola (d) a circle

34. If $\boldsymbol{a} = 2\hat{i}+2\hat{j}+\hat{k}$, $|\boldsymbol{b}| = 6$ and the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is $\frac{\pi}{6}$, then the area of the triangle (in square units) with $\boldsymbol{a}$ and $\boldsymbol{b}$ as two of its sides is
(a) $\frac{3\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{5}{4}$ (d) $\frac{9}{2}$

35. If $\boldsymbol{r}$ is a vector perpendicular to both the vectors $2\hat{i}+3\hat{j}-4\hat{k}$ and $3\hat{i}-\hat{j}+\hat{k}$ satisfy $\boldsymbol{r}.(3\hat{i}-3\hat{j}+4\hat{k}) = 5$, then $|\boldsymbol{r}|$ is equal to
(a) $\sqrt{366}$ (b) $\sqrt{222}$ (c) $\sqrt{318}$ (d) $\sqrt{246}$

36. The perpendicular distance from the origin to the plane containing the points having position vectors $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}+3\hat{j}-4\hat{k}$ and $3\hat{i}-4\hat{j}+5\hat{k}$, is
(a) $\frac{10}{\sqrt{60}}$ (b) $\frac{12}{\sqrt{30}}$ (c) $\frac{15}{\sqrt{127}}$ (d) $\frac{25}{\sqrt{57}}$

37. Let O be the mean deviation of the first five odd natural numbers about their mean and P be the mean deviation of the first five prime numbers about their mean. Then P – O is equal to
(a) 0.3 (b) 0.32 (c) 0.23 (d) 0.2

38. If a proper divisor of the integer 2520 is selected at random, then the probability that it is an odd number is
(a) $\frac{11}{46}$ (b) $\frac{12}{46}$ (c) $\frac{11}{48}$ (d) $\frac{1}{4}$

39. Each of the two boxes P and Q contain 100 chits numbered from 1 to 100. If one chit is drawn at random from each box, then the probability that the number on the chits drawn from P is square of the number on the chits drawn from Q, is
(a) 0.1% (b) 10% (c) 1% (d) 0.01%

40. A problem in algebra is given to two students A and B whose chances of solving it are 2/5 and 3/4 respectively. The probability that the problem is solved, if both of them try independently, is
(a) $\frac{17}{20}$ (b) $\frac{1}{2}$ (c) $\frac{3}{20}$ (d) $\frac{13}{20}$

41. The probability distribution of a random variable X is given below

X = x	0	1	2	3	4	5	6	7
P(x)	0.01	0.10	0.26	0.33	0.18	0.06	K	0.04

Then, $P(X \geq 3) - P(X < 6)$ is equal to
(a) 0.24 (b) – 0.27 (c) 0.57 (d) – 0.31

42. If $P(X = x) = 5rx$, $x = 1, 2, 3$ ... is the probability density function of a discrete random variable X, then r is equal to
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

43. If a point $P(x, y)$ moves such that the sum of the squares of its coordinates is equal to their product, then the locus of P excluding origin
(a) $\frac{1}{x^2}+\frac{1}{y^2}=1$ (b) $\frac{1}{x}+\frac{1}{y}=1$
(c) $\frac{x}{y}+\frac{y}{x}=1$ (d) $x^2 + y^2 - xy = 1$

44. When the origin is shifted to (–1, 2) by the translation of axes, the transformed equation of $x^2 + y^2 + 2x - 4y + 1 = 0$ is
(a) $X^2 + Y^2 = 4$ (b) $X^2 + Y^2 = 16$
(c) $X^2 + 2X + Y^2 = 4$ (d) $X^2 - 2X + Y^2 = 16$

45. If the portion of a straight line intercepted between the coordinate axes is divided by the point (2, 3) in the ratio 2 : 3, then the product of the intercepts made by this line on the axes is
(a) 25 (b) $\frac{29}{6}$ (c) 50 (d) $\frac{31}{3}$

46. The slope of a line L is 2. If m_1 and m_2 are slopes of two lines which are inclined at an angle of $\frac{\pi}{6}$ with L, then $m_1 + m_2$ is equal to
(a) –11 (b) 16 (c) 11 (d) – 16

47. In a ΔABC, $2x + 3y + 1 = 0$ and $x + 2y - 12 = 0$ are the perpendicular bisectors of its sides AB and AC respectively and if A is (3, 2), then the slope of the side BC is
(a) 1 (b) $\frac{1}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{2}$

48. If the equation of a line having a slope $m(m \in Z)$, passing through (1, 1) and making an angle of $\tan^{-1}\left(\frac{5}{7}\right)$ with the line $x + y - 3 = 0$ is $ax + y + c = 0$, then ac is equal to
(a) – 7 (b) – 42 (c) – 21 (d) 12

49. From the point (3, – 4) perpendicular lines L_1 and L_2 are drawn on each of the lines $S \equiv 2x^2 + 3xy - 2y^2 - 7x + y + 3 = 0$. The area of the quadrilateral formed by the pair of lines S = 0, L_1 and L_2 is (in square units)
(a) $\frac{64}{5}$ (b) $\frac{72}{5}$ (c) 25 (d) 35

50. The line $\frac{x}{3}+\frac{y}{2}=1$ and a pair of lines both passing through origin forms an isosceles triangle. If this pair of lines are perpendicular, then the equation of the pair of straight lines is
(a) $5(x^2 - y^2) + 24xy = 0$ (b) $5(x^2 - y^2) - 24xy = 0$
(c) $5(x^2 - y^2) + 12xy = 0$ (d) $5(x^2 - y^2) - 12xy = 0$

51. If $(2, a)$ does not lie outside the circles $x^2 + y^2 = 13$ and $x^2 + y^2 + x - 2y = 14$, then a lies in
(a) $(-\infty, -3) \cup (4, \infty)$ (b) $[-3, 4]$
(c) $(-\infty, -1) \cup (3, \infty)$ (d) $[-2, 3]$

52. The locus of the centre of the circles passing through the origin and cutting off a chord of length 2 units on the line $x = 1$ is
(a) a straight line (b) a circle
(c) a parabola (d) an ellipse

53. The number of common tangents that can be drawn to the circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 2x - 6y + 6 = 0$ is
(a) 4 (b) 0 (c) 2 (d) 1

54. The equation of the circle passing through the points of intersection of two circles $x^2 + y^2 + 2x + 3y + 1 = 0$, $x^2 + y^2 + 4x + 3y + 2 = 0$ and the point $(-1, 1)$ is
(a) $x^2 + y^2 + 10x + 3y + 5 = 0$
(b) $x^2 + y^2 + 10x - 3y + 11 = 0$
(c) $x^2 + y^2 + 20x - 3y + 21 = 0$
(d) $x^2 + y^2 + 20x + 3y + 15 = 0$

55. If the circle $x^2 + y^2 + 2kx + 4y - 4 = 0$ has its centre in 4th quadrant and touches the circle $x^2 + y^2 + 6x - 2y + 6 = 0$, then k is equal to
(a) -5 (b) $\frac{-15}{7}$ (c) $\frac{-23}{5}$ (d) -1

56. If a parabola having horizontal axis and passes through the points $(-2, 1)$, $(1, 2)$ and $(-1, 3)$, then the y-coordinate of the focus of that parabola is
(a) $\frac{37}{40}$ (b) $\frac{21}{10}$ (c) $\frac{41}{40}$ (d) $\frac{-41}{40}$

57. If $ax^2 + 2hxy + by^2 - 82x + 98y + 144 = 0$ is the equation of a parabola with focus $(2, -3)$ and directrix $3x - 2y + 5 = 0$, then $ax^2 + 2hxy + by^2 = 0$ represents
(a) two lines making an angle $\frac{\pi}{3}$ at origin
(b) a conic with eccentricity $\frac{a}{b}$
(c) two perpendicular lines
(d) two coincident lines

58. An ellipse has its major axis along Y-axis and minor axis along X-axis. If its length of latus rectum is 2/3 times of its minor axis, then the eccentricity of the ellipse is
(a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{\sqrt{5}}{3}$ (d) $\frac{\sqrt{2}}{5}$

59. If the length and breadth of a rectangle of maximum area that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are $8\sqrt{2}$ and $4\sqrt{2}$ respectively, then the eccentricity of that ellipse is
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

60. If $\frac{x^2}{\alpha+3}+\frac{y^2}{2-\alpha}=1$ represents a hyperbola, then α lies in
(a) $(-3, 2)$ (b) $(-3, \infty)$
(c) $(-\infty, -2)$ (d) $(-\infty, -3) \cup (2, \infty)$

61. The four points A$(2, -1, 3)$, B$(4, -2, 1)$, C$(4, 5, -7)$ and D$(2, 6, -5)$ forms a
(a) Square (b) Parallelogram
(c) Rectangle (d) Rhombus

62. Suppose L_1 and L_2 are two lines having the direction ratios $1, -2, -2$ and $0, 2, 1$ respectively. If the direction cosines of a line perpendicular to both L_1 and L_2 are l, m, n then $|l| + |m| + |n|$ is equal to
(a) 3 (b) $\frac{5}{3}$ (c) $\sqrt{3}$ (d) $\frac{7}{3}$

63. The Cartesian equation of a plane parallel to the plane $\vec{r}\cdot(2\hat{i}+3\hat{j}-4\hat{k})=1$ and at a distance of 2 units from it is
(a) $2x + 3y - 4z = 3$
(b) $2x + 3y - 4z = 1 \pm 2\sqrt{29}$
(c) $2x + 3y - 4z = -1 \pm 2\sqrt{29}$
(d) $2x + 3y - 4z = -3$

64. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$, where $[x]$ denotes the greatest integer less than or equal to x, then $\lim\limits_{x\to 0^-} f(x)$
(a) exist and equal to 1.
(b) exist and equal to sin 1.
(c) exist and equal to $-$ sin 1.
(d) Does not exist.

65. $\lim\limits_{x\to-\infty} \frac{3|x|^3 - x^2 + 2|x| - 5}{-5|x|^3 + 3x^2 - 2|x| + 7}$ is equal to
(a) $\frac{3}{5}$ (b) $\frac{-5}{7}$ (c) $\frac{5}{7}$ (d) $\frac{-3}{5}$

66. Let $f(x) = \begin{cases} \frac{\tan(2p-7)x + \tan 3x}{x} & , x < 0 \\ p - q & , x = 0 \\ q\left(\frac{\sqrt{x^2+x} - \sqrt{x}}{x^{\frac{3}{2}}}\right), & , x > 0 \end{cases}$

If $f(x)$ is continuous at $x = 0$, then $\frac{q}{p}$ is equal to
(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

67. If $xexy = y + \sin^2 x$, then $\frac{dy}{dx}$ at $x = 0$ is
(a) 0 (b) -1 (c) -2 (d) 1

68. If $f(x) = x^2 \sin\frac{1}{x}$, when $x \neq 0$ and $f(0) = 0$, then $\lim\limits_{x\to 0} f'(x)$
(a) Does not exist (b) 0
(c) ∞ (d) 1

69. The derivative of $(\log x)^{\sin x}$ with respect to $\cos x$ at $x = \frac{\pi}{2}$ is
(a) $\frac{-4}{\pi}$ (b) $\frac{-\pi}{2}$ (c) $\frac{-2}{\pi}$ (d) $\frac{-\pi}{4}$

70. Let $f(x)$ be differentiable function for all $x \in$ R and $f(x+y) = f(x) + f(y) - 3xy$. If $\lim_{h \to 0} \frac{f(h)}{h} = 7$, then f′(x) is equal to

(a) $-3x + 7$ (b) $3x - 7$
(c) $3x + 7$ (d) $-7 - 3x$

71. The equation of the tangent to the curve $xy^5 + 2x^2y - x^3 + y + 1 = 0$ at $x = 0$ is

(a) $3x + 4y + 4 = 0$ (b) $y = x - 1$
(c) $5x + 7y + 7 = 0$ (d) $x + y + 1 = 0$

72. A particle moves in a straight line such that its displacement S (in m) at a time t (in s) is given by $S(t) = t^3 - 4t^2 + 7t$. The instantaneous velocity v at $t = 4$ is

(a) 21 m/s (b) 23 m/s (c) 20 m/s (d) 19 m/s

73. If the two curves $x = y^2$ and $xy = K$ cut each other at right angles, then a possible value of K is

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

74. The Rolle's theorem is not applicable to

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 \le x \le 2 \end{cases} \text{ on } [0, 2] \text{ because}$$

(a) $f(x)$ is not defined everywhere
(b) $f(x)$ is not continuous
(c) $f(0) \neq f(2)$
(d) $f(x)$ is not differentiable

75. $\int \frac{x e^{\frac{x^2}{x^2-2}}}{x^4 - 4x^2 + 4} dx$ is equal to

(a) $\left(\frac{-1}{4}\right) e^{\frac{x^2}{x^2-2}} + C$ (b) $\frac{1}{4} e^{\frac{x^2}{x^2-2}} + C$

(c) $\frac{1}{x^2 - 2} e^{\frac{x^2}{x^2-2}} + C$ (d) $\frac{-1}{\left(x^2-2\right)^4} e^{\frac{x^2}{x^2-2}} + C$

76. If $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx = \frac{1}{2} f(x) + C$, then $f(1) - f(0)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{18}$ (c) $\frac{1}{27}$ (d) $\frac{1}{54}$

77. $\int \frac{\sqrt{\cot x}}{\sin 2x} dx$ is equal to

(a) $\sqrt{\cot x} + C$ (b) $-\sqrt{\cot x} + C$
(c) $\sqrt{\tan x} + C$ (d) $-\sqrt{\tan x} + C$

78. $\lim_{n \to \infty} \frac{1}{n} \log\left(\frac{(2n)!}{n^n \cdot n!}\right) = \int_1^2 f(x)\,dx$, then f(x) is equal to

(a) $\log(1 + x)$ (b) $\log\left(\frac{1}{x}\right)$

(c) $\log x$ (d) $\log\left(\frac{x+1}{x}\right)$

79. $\int_{-\pi/2}^{\pi/2} \sin(x - [x])dx$ is equal to

(a) 3(1 – cos1) + sin2 – sin1
(b) cos2 – sin2
(c) 3(1 – cos1) + cos2 – sin1
(d) 0

80. The solution of $\frac{dy}{dx} = \sqrt{1 - y^2}$, $y(0) = 1$, is

(a) $\sin^{-1} y = x - \sin^{-1}(1)$ (b) $\sin^{-1} y = x + \sin^{-1}(1)$
(c) $\cos^{-1} y = x + \cos^{-1}(1)$ (d) $\sin^{-1} y + x = \sin^{-1}(1)$

PHYSICS

81. The physics behind fusion test reactor is

(a) Newton's law of motion
(b) Trapping and cooling of atoms by laser beams and magnetic fields
(c) Magnetic confinement of plasma
(d) Motion of charged particles in electromagnetic fields

82. Consider an expression $QV = kpTL^{\alpha}$, where V, p, T, L are volume, pressure, time and length respectively. The quantity $[Q]$ has dimension $ML^{-1}T^{-1}$ and k is dimensionless constant. The value of integer α is

(a) 2 (b) –2 (c) 3 (d) –1

83. Two cars A and B initially at rest are moving in same direction with accelerations a_1 and a_2, respectively. After a certain time, they achieve velocities v_1 and v_2 and separated by a distance of 50 m. If $(a_1 - a_2) = 4$ ms^{-2}, then the quantity $(v_1 - v_2)$ will be

(a) 24 ms^{-1} (b) 20 ms^{-1}
(c) 40 ms^{-1} (d) 12 ms^{-1}

84. A rocket lifts off from ground and accelerate upwards at 1 ms^{-2} 20 s after, lift off a piece breaks off from the bottom of rocket. After breaking off, how much time it takes approximately to reach the ground?
(Take g = 10 ms^{-2})

(a) 6.3 s (b) 4.5 s (c) 10.5 s (d) 8.5 s

85. Two bodies were thrown simultaneously from the origin, one straight up and the other, at angle 60° to the vertical. The initial velocity of each body is equal to 10 ms^{-1}. Neglecting the air resistance, the distance between the two bodies after $t = 2$ s is (Take, $g = 10$ m / s^2)

(a) 20 m (b) $20\sqrt{2}$ m (c) $5\sqrt{3}$ m (d) 30 m

86. A particle leaves the origin with initial velocity $\boldsymbol{v} = (3\hat{\boldsymbol{i}})$ m/s and a constant acceleration $\boldsymbol{a} = (-1\hat{\boldsymbol{i}} - 0.5\hat{\boldsymbol{j}})$ m/s^2.

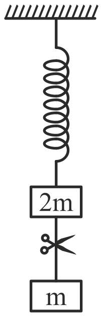
(a) $\frac{9}{2}(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}})m$ (b) $\frac{9}{2}\left(\hat{\boldsymbol{i}} - \frac{\hat{\boldsymbol{j}}}{2}\right)m$

(c) $\frac{9}{2}\left(-\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}}\right)m$ (d) $\frac{9}{2}\left(\frac{\hat{\boldsymbol{i}}}{2} - \hat{\boldsymbol{j}}\right)m$

87. A ball of mass 0.2 kg moving with a speed of 20 m/s is brought to rest in 0.1 s. The average force applied to the ball is

(a) 20 N (b) 30 N (c) 40 N (d) 60 N

88. A system as shown in figure is in equilibrium and is at rest. The spring and string are massless, now the string is cut. The acceleration of the masses 2m and m just after the string is cut, will be



(a) $\frac{g}{2}$ upwards, g downwards
(b) g upwards, $\frac{g}{2}$ downwards
(c) g upwards, 2g downwards
(d) 2g upwards, g downwards

89. A constant force of 5 Naccelerates a stationary particle of mass 500 g through a displacement of 5 m. The average power delivered is
(a) 6.25 W (b) 25 W (c) 62.5 W (d) 50 W

90. Which of the following is a incorrect statement?
(a) Work done by conservative force is equal to negative change in potential energy.
(b) Total energy of system is always conserved.
(c) Work done by non-conservative force in a closed path is equal to zero.
(d) In stable equilibrium, the potential energy is minimum.

91. A small block of mass 200 g is placed on a horizontal slab at a height of 2 m above the floor. The block is pressed against a horizontal spring fixed at one end to compress the spring through 10.0 cm. Upon releasing, the block moves horizontally till it leaves the spring. Calculate the horizontal distance covered by the block after leaving the slab and just before hitting the ground.
The spring constant is 50 N/m. (Take, g = 10 m/s^2)
(a) 0.99 m (b) 0.55 m (c) 0.44 m (d) 0.33 m

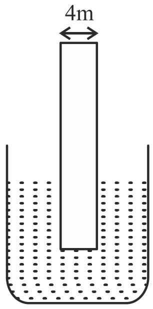
92. Three bodies a ring, a solid cylinder and a solid sphere roll down an inclined plane without slipping. They start from rest. Which of the bodies reaches bottom of plane with minimum velocity?
(a) Ring (b) Solid cylinder
(c) Solid sphere (d) Both ring and solid sphere

93. The displacement of a particle in simple harmonic motion (SHM) is given by $y = \sqrt{3\pi}\sin\left(\frac{100}{\pi}t + \frac{\pi}{4}\right)$. What will be the displacement of the particle from the mean position, when its kinetic energy is eight times that of its potential energy?
(a) $\sqrt{\frac{\pi}{3}}$ (b) $\sqrt{\frac{3\pi}{2}}$ (c) $\sqrt{\pi}$ (d) $\sqrt{3\pi}$

94. Two stars of equal masses M are orbiting in a circle of radius R, their orbital time period is proportional to
(a) $R^{3/2}$ (b) R (c) R^2 (d) $R^{1/2}$

95. Two springs of force constants k_1 and k_2 are loaded with weights w_1 and w_2 respectively. Assume that, length of each string is increased by same amount. If $k_1 = 2k_2$, then the ratio w_2/w_1 is
(a) 1 (b) 0.5 (c) 0.25 (d) 4

96. A metal sheet 4 cm on a side and of negligible thickness is attached to a balance and inserted into container fluid. The balance to which metal sheet is attached read 0.50 N and the contact angle is found to be zero.



A small amount of oil is then spread over the metal sheet. The contact angle now becomes 180° and the balance now reads 0.49 N. The surface tension of the fluid is
(a) 6.25×10^{-2} N/m (b) 1.25×10^{-1} N/m
(c) 4.25×10^{-2} N/m (d) 0.1 N/m

97. Match the column I and with column II.

	Column **I**		**Column II**
A.	Stoke's law	I.	Pressure and energy
B.	Turbulence	II.	Hydraulic lift
C.	Bernoulli's principle	III.	Viscous drag
D.	Pascal's law	IV.	Reynold's number

	A	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	III	IV	I	II	(b)	I	II	III	IV
(c)	II	I	IV	III	(d)	III	IV	II	I

98. An ideal gas undergoes an adiabatic process. If the pressure of the gas is reduced by 0.1%, then the volume is changed by $\left(\text{Given}, \gamma = \frac{C_p}{C_v} = 5/3\right)$
(a) 0.1% (b) 0.05% (c) 0.06% (d) – 0.05%

99. A body cools from 70°C to 40°C in 5 min. Calculate the time it takes to cool from 60°C to 40°C. The temperature of the surrounding is 20°C.
(a) 3.77 min (b) 3.56 min
(c) 3.68 min (d) 3.89 min

100. The Carnot heat engine have an efficiency of 50%. The temperature of sink is maintained at 500 K. To increase the efficiency upto 80%, the increment in the source temperature is
(a) 1500 K (b) 2500 K (c) 500 K (d) 2000 K

101. Consider an ideal gas in a closed container at 300 K. The container is then heated, so that the average velocity of a particles of the gas increases by a factor of 4. What would be the final temperature?
(a) 4500°C (b) 4527°C
(c) 4617°C (d) 4600°C

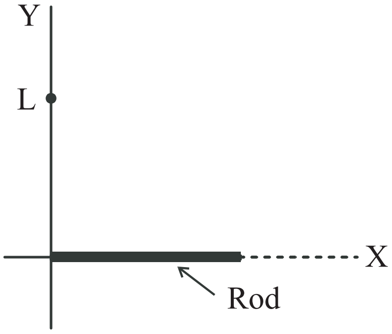
102. The mean free path for a gas at temperature 300 K and pressure 600 torr is 10^{-7} m. The mean free path of the gas at a temperature 400 K and pressure 200 torr will be
(a) 2.5×10^{-8} m (b) 4.4×10^{-8} m
(c) 3.3×10^{-8} m (d) 4×10^{-7} m

103. A second wave of frequency 200 Hz is travelling in air. The speed of sound in the air is 340 ms^{-1}. What is the phase difference (in rad) at a given instant between two points separated by a distance of 85 cm along the direction of propagation?
(a) π (b) 2π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

104. An object is placed infront of a spherical concave mirror between the focal point and the radius of curvature. Its image is
(a) inverted, real, further than radius of curvature from mirror
(b) inverted, virtual, closer than focal point to mirror
(c) upright, real, further than radius of curvature from mirror
(d) inverted, real, closer than radius of curvature to mirror

105. The amplitude of the wave resulting from the superposition of three waves given by
$x_1 = A\cos\omega t$, $x_2 = 2A\sin\omega t$ and
$x_3 = \sqrt{2}A\cos\left(\omega t + \frac{\pi}{4}\right)$ is
(a) $\sqrt{7}A$ (b) $\sqrt{5}A$ (c) $\left(3+\sqrt{2}\right)A$ (d) $\sqrt{2}A$

106. A semi-infinite non-conducting rod lies along X-axis with its left end at the origin. The rod has uniform linear charge density λ. The magnitude of electric field $|E|$ at a point on the Y-axis at the distance L from the origin, will be



(a) $\frac{\lambda}{4\pi\varepsilon_0 L}$ (b) $\frac{\lambda}{2\pi\varepsilon_0 L}$ (c) $\frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 L}$ (d) $\frac{\sqrt{2}\lambda}{\pi\varepsilon_0 L}$

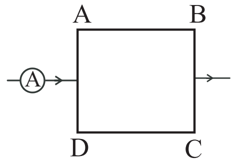
107. Two capacitors $C_1 = 2\mu F$ and $C_2 = 8\ \mu F$ are connected in series across a 300 V source, then
(a) the charge on each capacitor is 4.8×10^{-4}C
(b) the potential difference across C_1 is 60 V
(c) the potential difference across C_2 is 240 V
(d) the energy stored in the system is 52×10^{-2} J

108. A cylindrical metallic wire is stretched to increase its length. If the resistance of the wire is increased by 4%, then the percentage increase in its length is
(a) 4% (b) 8% (c) 1% (d) 2%

109. Two long parallel wires are separated by a distance of 2.5 cm. The force per unit length that each wire exerts on the other is 4×10^{-5} N/m and the wires repel each other. The current in one wire is 0.5 A. What is the current in the second wire? (Take $\mu_0 = 4\pi \times 10^{-7}$ SI unit)
(a) 12 A (b) 8 A (c) 6 A (d) 10 A

110. What is the magnetic moment of orbiting electron in simple hydrogen atom?
(Assume, e = charge of electron, m_e = mass of electron and L = orbital angular momentum of electron)
(a) $\mu = \left(\frac{e}{m_e}\right)L$ (b) $\mu = \left(\frac{e}{2m_e}\right)L$
(c) $\mu = \left(\frac{2e}{m_e}\right)L$ (d) $\mu = \left(\frac{e}{4m_e}\right)L$

111. *ABCD* is a rectangular loop made of uniform wire. If $AD = BC = 2$ cm, what is the magnetic force per unit length acting on wire DC due to wire AB, if ammeter reads 20 A? (The length of AB and DC are large in comparison with other two sides)



(a) 10^{-1} Nm^{-1} (b) 10^{-2} Nm^{-1}
(c) 10^{-3} Nm^{-1} (d) 10^{-4} Nm^{-1}

112. A coil of resistance 50 Ω is connected across a 5.0 V battery. If the current in the coil is found to be 50 mA after time t = 0.1 s battery is connected, then the inductance of the coil is
(a) $\frac{5}{\ln(2)}$ (b) 10 ln (2) (c) $5e^4$ (d) $\frac{10}{e^4}$

113. How much current is drawn by the primary coil of a transformer, which steps down 220 V to 55 V to operate a device with an impedance of 275 Ω ?
(a) 0.05 A (b) 0.02 A (c) 0.2 A (d) 0.15 A

114. Blue light travelling in vacuum has a wavelength of 450 nm. It enters a medium whose refractive index is 1.5. What is its frequency in the medium?
(Speed of light in vacuum = 3×10^8 m/s)
(a) 6.67×10^{14} Hz (b) 10^{15} Hz
(c) 4.45×10^{14} Hz (d) 10^{14} Hz

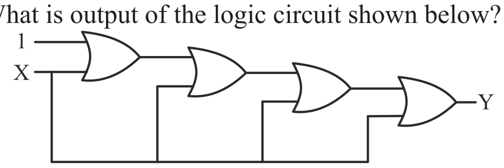
115. The de-Broglie wavelength of an electron having kinetic energy 100 eV is
(Take, $h = 4.14 \times 10^{-15}$ eV-s, mass of electron $= \frac{0.5\times10^6}{c^2}$ eV, 1 pm = 10^{-12} m)
(a) 150.1 pm (b) 124.2 pm
(c) 115.5 pm (d) 120.8 pm

116. If the light from Balmer series of hydrogen is used to eject photoelectrons from a metal then the maximum work-function of the metal can be
(a) 1.89 eV (b) 3.4 eV (c) 3.8 eV (d) 5.1 eV

117. The mass number and the volume of a nucleus is M and V, respectively. If the mass number is increased to 2M, then the volume is changed to
(a) 4V (b) $\frac{V}{2}$ (c) 2V (d) 8V

118. The length of germanium rod is 0.928 cm and its area of cross-section is 1 mm^2. If for germanium.
$\mathbf{n_1} = 2.5 \times 10^{19}$ m^{-3}, $\mu_n = 0.15$ m^2 V^{1}s^{-1}, $\mu_e = 0.35$ m^2 V^{-1} s^{-1} then resistivity is
(a) 50 Ω-cm (b) 25 Ω-cm
(c) 50 Ω-mm (d) 100 Ω-m

119. What is output of the logic circuit shown below?



(a) Y = 0 (b) Y = 1 (c) Y = X (d) Y = $\overline{X}$

120. The height of transmitting antenna, if TV programs have to cover the population in area of radius 64 km is
(Take, radius of earth = 6.4×10^6 m)
(a) 160 m (b) 200 m (c) 240 m (d) 320 m

CHEMISTRY

121. A microscope using appropriate photons is engaged to track an electron in an atom within distance of 0.001 nm. What will be the uncertainty involved in measuring its velocity?
(a) 5.79×10^7 m/s (b) 5.79×10^6 m/s
(c) 4.79×10^7 m/s (d) 3.7×10^6 m/s

122. An orbital with one angular node shows three maxima in its radial probability distribution curve, the orbital is
(a) 3*s* (b) 4*p* (c) 5*d* (d) 3*p*

123. Starting from the 1st, the successive ionisation potentials of an element are respectively 5.98, 18.8, 28.4, 120.1, 154 eV. The element is
(a) B (b) Al (c) P (d) Mg

124. The orbital with 4 radial and 1 angular nodes is
(a) $5p_y$ (b) $6p_z$ (c) $4d_{xy}$ (d) $5d_{yz}$

125. The number of H_2O molecules participating in hydrogen bonding in $CuSO_4 . 5H_2O$ is/are
(a) 4 (b) 2 (c) 1 (d) 0

126. One Debye is equal to how many coulomb metre?
(a) 3.33×10^{-30} (b) 2.22×10^{-20}
(c) 1.11×10^{-10} (d) 4.44×10^{-24}

127. The dipole-dipole interaction energy between stationary polar molecules and rotating polar molecules, respectively is proportional to [r is the distance between the polar molecules]
(a) $r^3; \frac{1}{r^2}$ (b) $\frac{1}{r^3}; \frac{1}{r^6}$ (c) $\frac{1}{r^2}; r^2$ (d) $\frac{1}{r^2}; \frac{1}{r^4}$

128. Which gas has a density of 1.24 g/L at 0°C and 1 atm pressure?
(a) F (b) CH_4 (c) CO (d) CO_2

129. From the given reactions, identify the disproportionation reaction.
(i) $Cl_2(g) + 2KI\,(aq) \longrightarrow 2KCl\,(aq) + I_2(s)$
(ii) $Cl_2(g) + 2OH^-(aq) \longrightarrow ClO^-(aq) + Cl^-(aq) + H_2O(l)$
(iii) $Mg(s) + 2HCl\,(aq) \longrightarrow MgCl_2\,(aq) + H_2(g)$
(iv) $2H_2O_2\,(aq) \longrightarrow 2H_2O(l) + O_2(g)$
(a) (i) and (iv) (b) (ii) and (iv)
(c) (ii) and (iii) (d) (i) and (ii)

130. $KMnO_4$ oxidises oxalic acid in acidic medium. The number of CO_2 molecules produced per mole of $KMnO_4$ is
(a) 5 (b) 4 (c) 3 (d) 1.5

131. Which of the following is not an intensive property?
(a) Entropy (b) Melting point
(c) Specific gravity (d) Refractive index

132. The equilibrium constant (K_p) for the formation of ammonia from its constituent elements at 27°C is 1.2×10^{-4} and at 127°C is 0.60×10^{-4}. Calculate the mean heat of formation of ammonia per mole in this temperature range.
(a) – 82.64 cal (b) – 826.4 cal
(c) – 1652.8 cal (d) – 165.2 cal

133. For a reaction, $2A \rightleftharpoons B + C$, K_c is 2×10^{-3}. At a given time, the reaction mixture has $[A] = [B] = [C] = 3 \times 10^{-4}$ M. Which of the following options is correct?
(a) The system is at equilibrium
(b) The reaction proceeds to the left
(c) The reaction proceeds to the right
(d) The reaction is complete

134. Match the following columns.

Column I (Reaction)	Column II (Main product)
A. $SO_3 \xrightarrow{\text{Heavy water}}$	I. C_2D_2
B. $CaC_2 \xrightarrow{\text{Heavy water (excess)}}$	II. CD_4
C. $Al_4C_3 \xrightarrow{\text{Heavy water (large excess)}}$	III. D_2SO_3
	IV. C_2D_4
	V. D_2SO_4

A B C A B C
(a) III I IV (b) III IV II
(c) V I II (d) V I IV

135. Powdered beryllium burns in air to give
(a) BeO; Be_2N_2 (b) Be_2O_3; Be_3N_2
(c) BeO; Be_3N_2 (d) BeO; Be_2N

136. Match the following columns.

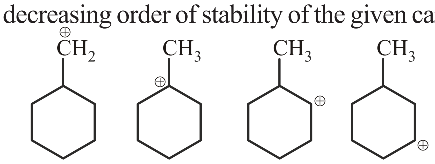
Column I (Reaction)	Column II (Main product)
A. $B_2H_6 + 2CO \rightarrow$	I. B_2O_3
B. $B_2H_6 + 3O_2 \rightarrow$	II. $2BH_3.CO$
C. $B_2H_6 + 6H_2O \rightarrow$	III. $2H_3BO_3$
	IV. $2BH_2(CO)$
	V. $2HBO_2$

A B C A B C
(a) IV I III (b) II III V
(c) IV III I (d) II I III

137. During the process of fermentation, the number of moles of CO_2 liberated from one mole of glucose is
(a) 2 (b) 3 (c) 4 (d) 1

138. Which of the given statements are correct, when carboxyheamoglobin reaches to 3-4% in blood?
I. Leads to headache.
II. Results in cardiovascular problem.
III. Increases the body temperature.
IV. Leads to diarrhoea.
(a) I and II (b) I and III
(c) III and IV (d) II and III

139. The decreasing order of stability of the given carbocations is



(A) (B) (C) (D)

(a) (D) > (C) > (A) > (B) (b) (D) > (A) > (C) > (B)
(c) (B) > (D) > (C) > (A) (d) (B) > (C) > (D) > (A)

140. The reaction,

$CH_2 = CH_2 + H_2 \xrightarrow[300°C]{Ni} CH_3 — CH_3$ is

(a) Wurtz reaction
(b) Kolbe reaction
(c) Sabatier-Senderens reaction
(d) Dow's reaction

141. Benzene reacts with *n*-propyl chloride in the presence of anhydrous $AlCl_3$ to give predominantly

(a) *n*-propyl benzene
(b) isopropyl benzene
(c) 3-propyl-1-chloro benzene
(d) 1-chloro-3-*n*-propyl benzene

142. The fraction of the total volume occupied by the atoms in a simple cube is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

143. Match the following columns.

	Column I		Column II
A.	Ebullioscopic constant	I.	Depression of freezing point
B.	Cryoscopic constant	II.	Total pressure is the sum of partial pressure of the components
C.	Henry's law	III.	Elevation of boiling point
D.	Dalton's law	IV.	Solubility of a gas in liquid

	A	B	C	D		A	B	C	D
(a)	III	I	II	IV	(b)	I	III	II	IV
(c)	III	I	IV	II	(d)	I	III	IV	II

144. On mixing urea, the boiling point of H_2O changed to 100.5°C. Calculate the freezing point of the solution, if K_f of water is 1.87 K kg mol^{-1} and K_b of water is 0.52 K kg mol^{-1}.

(a) –1°C (b) –0.5°C (c) –1.8°C (d) 0°C

145. When the same quantity of electricity is passed through the aqueous solutions of the given electrolytes for the same amount of time, which metal will be deposited in maximum amount on the cathode?

(a) $ZnSO_4$ (b) $FeCl_3$ (c) $AgNO_3$ (d) $NiCl_2$

146. Calculate the activation energy of a reaction, whose rate constant doubles on raising the temperature from 300 K to 600 K.

(a) 3.45 kJ/mol (b) 6.90 kJ/mol
(c) 9.68 kJ/mol (d) 19.6 kJ/mol

147. The most effective coagulating agent among the options for Sb_2S_3 sol is

(a) Na_2SO_4 (b) $Al_2(SO_4)_3$ (c) $CaCl_2$ (d) NH_4Cl

148. The correct order of reducing ability of the following hydrides is

(a) $BiH_3 > SbH_3 > PH_3 > NH_3$
(b) $NH_3 > PH_3 > SbH_3 > BiH_3$
(c) $SbH_3 > BiH_3 > PH_3 > NH_3$
(d) $PH_3 > BiH_3 > SbH_3 > NH_3$

149. Sulphur dioxide reacts with chlorine in the presence of charcoal to give

(a) H_2SO_3 (b) $SOCl_2$ (c) SO_2Cl_2 (d) H_2SO_4

150. **Assertion:** The bond dissociation energy increases from F_2 to Cl_2 and then decreases to I_2.

Reason: The low bond energy of fluorine is due to the repulsion between the lone pairs of electrons in two fluorine atoms.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

151. The magnetic moment of the high spin complex is 5.92 BM. What is the electronic configuration?

(a) $t_{2g}^3 e_g^1$ (b) $t_{2g}^4 e_g^2$ (c) $t_{2g}^3 e_g^2$ (d) $t_{2g}^5 e_g^0$

152. According to Valence Bond Theory, the number of unpaired electrons present in $[MnCl_6]^{3-}$, $[Fe(CN)_6]^{3-}$ and $[Co(C_2O_4)_3]^{3-}$, respectively, are

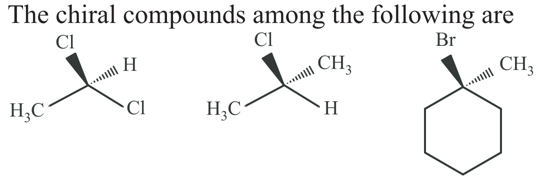
(a) 0 : 5 : 0 (b) 4 : 3 : 2 (c) 4 : 1 : 0 (d) 5 : 4 : 3

153. **Assertion:** In aqueous solution α-amino acids exists as internal salt called Zwitter ion.

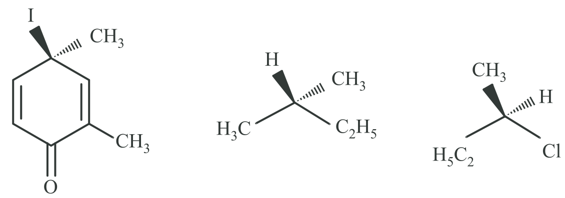
Reason: Proline is a natural amino acid having a secondary amino group.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

154. The chiral compounds among the following are



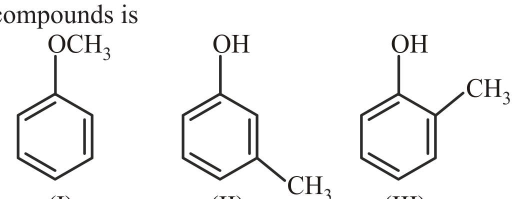
(I) (II) (III)



(IV) (V) (VI)

(a) I and III (b) II and IV
(c) IV and V (d) IV and VI

155. The correct order of boiling points of following compounds is



(I) (II) (III)

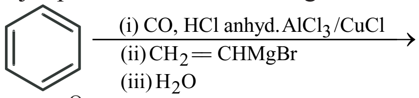
(a) I > II > III (b) II > I > III
(c) III > II > I (d) II > III > I

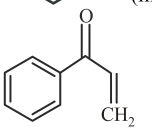
156. Match the following columns.

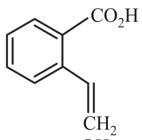
Column I (Compound/reagent)	**Column II (Other name/ chemical/ name of the process)**
A. Methanol	I. Lucas reagent
B. $ZnCl_2$/Conc.HCl	II. Baeyer's reagent
C. Rectified spirit	III. Wood spirit
D Dil. $KMnO_4$	IV. 95% C_2H_5OH
	V. 75% C_2H_5OH
	VI. 90% C_3H_7OH

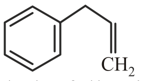
	A	B	C	D		A	B	C	D
(a)	IV	I	III	II	(b)	III	I	VI	II
(c)	III	I	IV	II	(d)	III	II	V	I

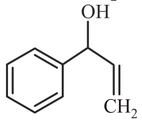
157. The major product of the following reaction sequence is



(a) 

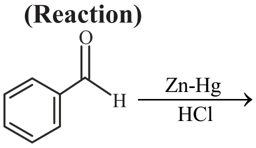
(b) 

(c) 

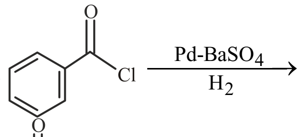
(d) 

158. Match the following columns.

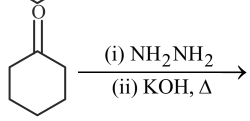
Column I (Reaction) **Column II (Product)**

A. 

I. Cyclohexanol

B. 

II. Benzyl alcohol

C. 

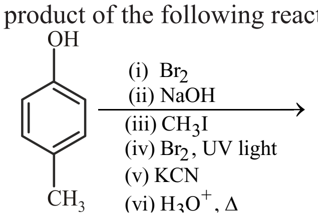
III. Toluene

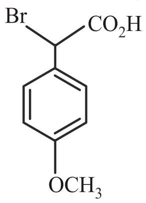
IV. Benzaldehyde

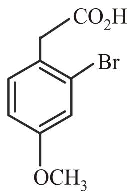
V. Cyclohexane

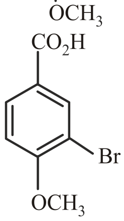
	A	B	C		A	B	C
(a)	III	IV	V	(b)	III	II	V
(c)	III	IV	I	(d)	II	IV	I

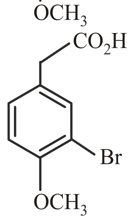
159. The major product of the following reaction sequence is



(a) 

(b) 

(c) 

(d) 

160. The test that distinguishes primary amines from other amines is
(a) Iodoform test (b) Victor Meyer test
(c) Lucas test (d) Carbylamine test

ANSWER KEY

1	(b)	**2**	(b)	**3**	(b)	**4**	(c)	**5**	(d)	**6**	(a)	**7**	(c)	**8**	(b)	**9**	(a)	**10**	(a)
11	(d)	**12**	(b)	**13**	(c)	**14**	(b)	**15**	(c)	**16**	(b)	**17**	(b)	**18**	(a)	**19**	(a)	**20**	(a)
21	(b)	**22**	(b)	**23**	(c)	**24**	(d)	**25**	(d)	**26**	(b)	**27**	(b)	**28**	(d)	**29**	(c)	**30**	(b)
31	(d)	**32**	(b)	**33**	(d)	**34**	(d)	**35**	(c)	**36**	(b)	**37**	(b)	**38**	(a)	**39**	(a)	**40**	(a)
41	(d)	**42**	(a)	**43**	(c)	**44**	(a)	**45**	(a)	**46**	(d)	**47**	(c)	**48**	(b)	**49**	(b)	**50**	(b)
51	(d)	**52**	(c)	**53**	(a)	**54**	(a)	**55**	(d)	**56**	(b)	**57**	(d)	**58**	(c)	**59**	(b)	**60**	(d)
61	(b)	**62**	(b)	**63**	(b)	**64**	(b)	**65**	(d)	**66**	(a)	**67**	(d)	**68**	(a)	**69**	(c)	**70**	(a)
71	(b)	**72**	(b)	**73**	(d)	**74**	(d)	**75**	(a)	**76**	(b)	**77**	(b)	**78**	(c)	**79**	(a)	**80**	(b)
81	(c)	**82**	(c)	**83**	(b)	**84**	(d)	**85**	(a)	**86**	(a)	**87**	(c)	**88**	(a)	**89**	(b)	**90**	(c)
91	(a)	**92**	(a)	**93**	(a)	**94**	(a)	**95**	(b)	**96**	(a)	**97**	(a)	**98**	(c)	**99**	(d)	**100**	(a)
101	(b)	**102**	(d)	**103**	(a)	**104**	(a)	**105**	(b)	**106**	(c)	**107**	(a)	**108**	(d)	**109**	(d)	**110**	(b)
111	(c)	**112**	(a)	**113**	(a)	**114**	(a)	**115**	(b)	**116**	(b)	**117**	(c)	**118**	(a)	**119**	(b)	**120**	(d)
121	(a)	**122**	(b)	**123**	(b)	**124**	(b)	**125**	(c)	**126**	(a)	**127**	(b)	**128**	(c)	**129**	(b)	**130**	(a)
131	(a)	**132**	(b)	**133**	(b)	**134**	(c)	**135**	(c)	**136**	(d)	**137**	(a)	**138**	(a)	**139**	(c)	**140**	(c)
141	(b)	**142**	(c)	**143**	(c)	**144**	(c)	**145**	(c)	**146**	(a)	**147**	(b)	**148**	(a)	**149**	(c)	**150**	(a)
151	(c)	**152**	(c)	**153**	(b)	**154**	(d)	**155**	(d)	**156**	(c)	**157**	(d)	**158**	(a)	**159**	(d)	**160**	(d)

Hints & Solutions

MATHEMATICS

1. **(b)** Given that, $y(x)=\begin{cases}\dfrac{5x}{(x-3)(x+3)}, & \text{if } x\neq 1\\ -1, & \text{if } x=-1\end{cases}$

and A is the set of domain and its denominator must be non-zero. So, $(x-3)(x+3)\neq 0$

$\therefore x\neq -3, 3$. Hence, $x\in R-\{-3,3\}$

2. **(b)** Given that $y(x)=\cos x-3$

As, we know that, Domain of $\cos x$ is R. By subtracting 3 from $\cos x$, there will not be any change in the domain. So, domain of $y(x)$ will also be R. Now, to find out the range

$-1\le\cos x\le 1 \Rightarrow -1-3\le\cos x-3\le 1-3$

$\Rightarrow -4\le\cos x-3\le -2 \Rightarrow$ Range $\in[-4,-2]$.

3. **(b)** Given that $f(1)=0$

$f(n+1)-f(n)=5n$

$f(2)-f(1)=5\Rightarrow f(2)=5$

$f(3)-f(2)=10\Rightarrow f(3)=15$

$f(4)-f(3)=15\Rightarrow f(4)=30$

$f(5)-f(4)=20\Rightarrow f(5)=50$

So, 0, 5, 15, 30, 50

Taking difference of consecutive terms,

5, 10, 15, 20... $\Rightarrow$ Forms an A.P.

$Sn=T_1+T_2+T_3+...+Tn$

$Sn=T_1+T_2+...+Tn_{-1}+Tn$

On subtracting, we get

$0=T_1+(T_2-T_1)+(T_3-T_2)+...+(Tn-Tn_{-1})-Tn$

$Tn=T_1+5+10+...+5(n-1)$

$Tn=0+5[1+2+...+(n-1)]$

$T_n=\dfrac{5(n-1)(n-1+1)}{2}=\dfrac{5}{2}n(n-1)=f(n)$

4. **(c)** According to the property of determinant,

adj (adj$A$) $=|A|n^{-2}A$

$|A|=\begin{vmatrix}1&0&0\\0&2&0\\0&0&3\end{vmatrix}=6$

$\therefore$ adj (adj A) $=6^{3-2}\cdot A=6A$ ...(i)

5. **(d)** $A=\begin{pmatrix}1&-1&1\\2&-1&0\\1&0&0\end{pmatrix}$

$A^2=\begin{pmatrix}1&-1&1\\2&-1&0\\1&0&0\end{pmatrix}\begin{pmatrix}1&-1&1\\2&-1&0\\1&0&0\end{pmatrix}=\begin{pmatrix}0&0&1\\0&-1&2\\1&-1&1\end{pmatrix}$

and $A^3=A\cdot A^2=\begin{pmatrix}1&-1&1\\2&-1&0\\1&0&0\end{pmatrix}\begin{pmatrix}0&0&1\\0&-1&2\\1&-1&1\end{pmatrix}$

$A^3=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=I$

$\therefore A^5=A^2\cdot A^3=A^2\cdot I=A^2(AA^{-1})$

$=A^3\cdot A^{-1}=I\cdot A^{-1}$ $\therefore$ $A^5=A^{-1}$

6. **(a)** Given, $|A|\neq 0$ and $|B|\neq 0$

As we know that,

$|AB|=|A|\,|B|\Rightarrow|A^T|=|A|$

$\therefore |A^TBA|=|A^T|\,|B|\,|A|=27\Rightarrow|A|\,|B|\,|A|=27$

$|A|^2|B|=27$ ...(i)

Now, $|AB^{-1}|=|A|\,|B^{-1}|=8 \Rightarrow \dfrac{|A|}{|B|}=8 \Rightarrow |A|=8|B|$

Now, using eq. (i) $|A|^2\dfrac{|A|}{8}=27$

$\Rightarrow|A|^3=27\times 8\Rightarrow|A|=6;|B|=\dfrac{6}{8}=\dfrac{3}{4}$

$|B^TA^{-1}B|=|B|\dfrac{1}{|A|}|B|=\dfrac{3}{4}\times\dfrac{1}{6}\times\dfrac{3}{4}=\dfrac{3}{32}$

7. **(c)** $z_1=1-2i$, $z_2=1+i$, $z_3=3+4i$

To calculate $\left|\left(\dfrac{1}{z_1}+\dfrac{2}{z_2}\right)\dfrac{z_3}{z_2}\right|$,

$\dfrac{1}{z_1}=\dfrac{1}{1-2i}\times\dfrac{1+2i}{1+2i}=\dfrac{1+2i}{5}$

$\dfrac{2}{z_2}=\dfrac{2}{1+i}\times\dfrac{1-i}{1-i}=1-i$

$\dfrac{z_3}{z_2}=(3+4i)\left(\dfrac{1-i}{2}\right)=\dfrac{7+i}{2}$

$\left|\left(\dfrac{1}{z_1}+\dfrac{2}{z_2}\right)\dfrac{z_3}{z_2}\right|=\left|\left(\dfrac{1+2i}{5}+1-i\right)\left(\dfrac{7+i}{2}\right)\right|$

$\Rightarrow\left|\left(\dfrac{6-3i}{5}\right)\left(\dfrac{7+i}{2}\right)\right|=\left|\left(\dfrac{45-15i}{5\cdot 2}\right)\right|=\left|3\left(\dfrac{3-i}{2}\right)\right|$

$\Rightarrow\dfrac{3}{2}\sqrt{9+1}=\dfrac{3}{2}\sqrt{10}=\sqrt{\dfrac{9}{4}\cdot 10}=\sqrt{\dfrac{45}{2}}$

8. **(b)** Given, $2+2\sqrt{3}i=k(\cos\theta+i\sin\theta)$

$4\left(\dfrac{1}{2}+\dfrac{\sqrt{3}i}{2}\right)=k(\cos\theta+i\sin\theta)$

$4\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)=k(\cos\theta+i\sin\theta)$

$\therefore\theta=\dfrac{\pi}{3},k=4\Rightarrow\dfrac{1}{\sqrt{3}}(\cos 6\theta+i\sin 6\theta)$

$=\dfrac{1}{\sqrt{3}}\left[\cos\left(\dfrac{6\pi}{3}\right)+i\sin\left(\dfrac{6\pi}{3}\right)\right]=\dfrac{1}{\sqrt{3}}$

9. **(a)** $x=\frac{4}{5}+\frac{3i}{5}, y=\frac{\sqrt{3}}{\sqrt{8}}-\frac{\sqrt{5}}{\sqrt{8}}i$

$|x|=1=|y| \Rightarrow x\bar{x}=1=y\bar{y}$

$\bar{x}=\frac{1}{x}$ and $\bar{y}=\frac{1}{y}$

$$\left(x^2+\frac{1}{x^2}\right)\left(y^2-\frac{1}{y^2}\right)=\left[\left(x+\frac{1}{x}\right)^2-2\right]\left(y^2-\frac{1}{y^2}\right)$$

$$=[(x+\bar{x})^2-2x\bar{x}](y+\bar{y})(y-\bar{y})$$

$$=\left[\left(\frac{8}{5}\right)^2-2\right]\left(\frac{2\sqrt{3}}{\sqrt{8}}\right)\left(\frac{-2\sqrt{5}}{\sqrt{8}}\right)i$$

$$=\frac{14}{25}\times\left(\frac{-\sqrt{15}}{2}i\right)=\frac{-7\sqrt{3}}{5\sqrt{5}}i$$

10. **(a)** Given, $x^2-5x-14>0 \Rightarrow (x+2)(x-7)>0$

$\Rightarrow x<-2$ or $x>7 \Rightarrow x$ lies outside $[\alpha, \beta]$

$\Rightarrow \alpha=-2$ or $\beta=7$ $\qquad \left\{\therefore \frac{\alpha}{\beta}=\frac{-2}{7}\right\}$

11. **(d)** Let $y=\frac{(x+2)(x+5)}{(x+6)}$

$x^2+7x+10=y\ (x+6)$

$x^2+(7-y)x+10-6y=0$

$\because\ x \in R$

$D=b^2-4ac \Rightarrow (7-y)^2-4(10-6y)>0$

$y^2-14y+49-40+24y>0$

$\Rightarrow y^2+10y+9>0 \Rightarrow (y+9)(y+1)>0$

$\Rightarrow y<-9$ or $y>-1 \Rightarrow$ So, $y \notin (-9,-1)$

12. **(b)** Given, $x=2+2^{\frac{1}{3}}+2^{\frac{2}{3}}$

Taking cube on both the sides,

$$(x-2)^3=\left(2^{\frac{1}{3}}\right)^3+\left(2^{\frac{2}{3}}\right)^3+3\cdot 2^{\frac{1}{3}}\cdot 2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2^{\frac{2}{3}}\right)$$

$\Rightarrow (x-2)^3=2+4+6(x-2)$

$\Rightarrow x^3-8-6x^2+12x=6+6x-12 \Rightarrow x^3-6x^2+6x=2$

13. **(c)** Let, $f(x)=3x^3+4x^2-5x-2$

$=(x+2)(x-1)(3x+1)$. Now, $f(x)=0$ we get,

roots $=-2,1,-\frac{1}{3}$

If the coefficient of x^2 in new cubic equation is zero, then, it means that sum of roots is zero.

$$(-2-h)+(1-h)+\left(-\frac{1}{3}-h\right)=0$$

$$-\frac{4}{3}-3h=0 \Rightarrow h=-\frac{4}{9}$$

So, new roots $=-2+\frac{4}{9},1+\frac{4}{9},-\frac{1}{3}+\frac{4}{9}=-\frac{14}{9},\frac{13}{9},\frac{1}{9}$

14. **(b)** Given, $0<r<s<n$ and $n\text{p}_\text{r}=n\text{p}_\text{s}$

$\Rightarrow \frac{n!}{(n-r)!}=\frac{n!}{(n-s)!} \Rightarrow (n-r)!=(n-s)!$

As, $0<r<s<n \Rightarrow (n-r)>(n-s)$

So, the only possibility to hold this inequality is

$n-r=1$ and $n-s=0$

$r=n-1$ and $s=n \therefore r+s=n+(n-1)=2n-1$

15. **(c)** According to the question,

$e_1=\boxed{}\ \boxed{}\ \boxed{6 \text{ or } 8}$

3 choice 4 choice 2 choice $=3\times4\times2=24$

$e_2=\boxed{}\times\boxed{}\times\boxed{}\times\boxed{6 \text{ or } 8}$

2 choice 3 choice 4 choice 2 choice $=2\times3\times4\times2=48$

$O_1=\boxed{}\ \boxed{}\ \boxed{1/3/5}$

3 choice 4 choice 3 choice $=3\times4\times3=36$

$O_2=\boxed{}\ \boxed{}\ \boxed{}\ \boxed{1/3/5}$

2 choice 3 choice 4 choice 3 choice $=2\times3\times3\times4=72$

$$\frac{e_1+e_2}{2}=\frac{24+48}{2}=36 \text{ and } \frac{O_1+O_2}{3}=\frac{36+72}{3}=36$$

16. **(b)** In the expansion of

$(1+x)(1-x)^n$

$=(1+x)(C_0-C_1x+C_2x^2...+(-1)^{n-1}C_{n-1}x^{n-1}+(-1)^nC_nx^n)$

Coefficient of xn, $f(n)=[(-1)^n C_n+(-1)^{n-1}C_{n-1}]$

$=[(-1)^n+(-1)^{n-1}n]$

$f(2021)=[(-1)^{1021}+(-1)^{2021}(-1)2021]=(-1+2021)=2020$

17. **(b)** In the expansion of, $\left(x^{1/3}+\frac{1}{2x^{1/3}}\right)^{21}$

General Term, $T_{r+1}={}^{21}C_r\cdot x^{r/3}\cdot\frac{1}{(2x^{1/3})^{21-r}}$

$$\frac{{}^{21}C_r}{2^{21-r}}\cdot\frac{x^{r/3}}{x^{7-\frac{r}{3}}}=\left(\frac{{}^{21}C_r}{2^{21-r}}\right)x^{\left(\frac{2r}{3}-7\right)}$$

When, $\frac{2r}{3}-7=-3 \Rightarrow r=6$ and $\frac{2r}{3}-7=-5 \Rightarrow r=3$

$\therefore p=\frac{{}^{21}C_6}{2^{15}}$ and $q=\frac{{}^{21}C_3}{2^{18}}$

Now, $\frac{5p}{4q}=\left(\frac{5}{4}\right)\left(\frac{{}^{21}C_6}{{}^{21}C_3}\times\frac{2^{18}}{2^{15}}\right)=408$

18. **(a)** $(1-x)^{-n}(1-x)^{-\frac{1}{p}}=(1-x)^{-n-\frac{1}{p}}$

Let $y=-n-\frac{1}{p}=\frac{-pn-1}{p}$

$\therefore (1-x)y=1-yx+\frac{y(y-1)}{2}x^2-\frac{y(y-1)(y-2)}{6}x^3+...$

Now, Coefficient of

$$x^3=\frac{-1}{6}\left(\frac{-np-1}{p}\right)\left(\frac{-np-1}{p}-1\right)\left(\frac{-np-1}{p}-2\right)$$

$$=\frac{(np+1)(np+p+1)(np+2p+1)}{6p^3}$$

19. **(a)** Given,

$$\frac{1}{(x-1)(x-2)(x-3)}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$$

$1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

$x = 2 \Rightarrow 1 = -B \Rightarrow x = 3 \Rightarrow 1 = 2C \Rightarrow x = 1 \Rightarrow 1 = 2A$

Thus, $A=\frac{1}{2}, B=-1$ and $C=\frac{1}{2}$

Given, $\frac{x}{(x-1)(x-2)(x-3)}=\frac{P}{x-1}+\frac{Q}{x-2}+\frac{R}{x-3}$

$x = P(x-2)(x-3) + Q(x-1)(x-3) + R(x-1)(x-2)$

$x = 2 \Rightarrow 2 = -Q \Rightarrow x = 3 \Rightarrow 3 = 2R$

$x = 1 \Rightarrow 1 = 2P$

Thus, $P=\frac{1}{2}, Q=-2$ and $R=\frac{3}{2}$

$$A+2B+3C=\frac{1}{2}+(-2)+\frac{3}{2}=0$$

and $P+Q+R=\frac{1}{2}-2+\frac{3}{2}-0$

20. **(a)** $2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)$ ...(i)

$\sin^6\theta+\cos^6\theta$

$=(\sin^2\theta+\cos^2\theta)(\sin^4\theta+\cos^4\theta-\sin^2\theta\cos^2\theta)$

$\sin^6\theta+\cos^6\theta=\sin^4\theta+\cos^4\theta-\sin^2\theta\cos^2\theta$

Substituting in Eq. (i), $=2\sin^4\theta+2\cos^4\theta$

$-2\sin^2\theta+\cos^2\theta-3\sin^4\theta-3\cos^4\theta$

$=-\sin^4\theta-\cos^4\theta-2\sin^2\theta\cos^2\theta$

$=-(\sin^2\theta+\cos^2\theta)2=-1$

21. **(b)** As we know that, $\sin(90°-\theta)=\cos\theta$

If $\theta=55° \Rightarrow \sin 35°=\cos 55°$

If $\theta=35° \Rightarrow \sin 55°=\cos 35°$

$$\left(\frac{\sin 35°}{\cos 55°}\right)^2+\left(\frac{\cos 55°}{\sin 35°}\right)^2-2\cos 30°$$

$$=1^2+1^2-2\cdot\frac{\sqrt{3}}{2}=2-\sqrt{3}$$

22. **(b)** $\cos\theta=-\frac{3}{5}$, $\sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{9}{25}}$

$\because \sin\theta=-\frac{4}{5}$ $\quad\because \pi<\theta<\frac{3\pi}{2}$

$$\tan\frac{\theta}{2}=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}=\frac{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$=\frac{\sin\theta}{1+\cos\theta}=\frac{\left(-\frac{4}{5}\right)}{1+\left(-\frac{3}{5}\right)}=\frac{-\frac{4}{5}}{\frac{2}{5}}=-2$$

23. **(c)** $$\frac{1-\tan^2 15°}{1+\tan^2 15°}=\frac{1-\frac{\sin^2 15°}{\cos^2 15°}}{1+\frac{\sin^2 15°}{\cos^2 15°}}$$

$$\frac{\cos^2 15°-\sin^2 15°}{\cos^2 15°+\sin^2 15°}=\cos(15°+15°)=\frac{\sqrt{3}}{2}$$

24. **(d)** $6\cos 2\theta+2\cos^2\frac{\theta}{2}+2\sin^2\theta=0$

$\Rightarrow 6(2\cos^2\theta-1)+1+\cos\theta+2(1-\cos^2\theta)$

$\Rightarrow 10\cos^2\theta+\cos\theta-3=0$

$\Rightarrow (2\cos\theta-1)(5\cos\theta+3)=0$

$\Rightarrow \cos\theta=\frac{1}{2}$ or $\cos\theta=-\frac{3}{5} \Rightarrow$ As, $-\pi<\theta<\pi$

So, $\theta=\pm\frac{\pi}{3}$ or $\pm\left(\pi-\cos^{-1}\frac{3}{5}\right)$

25. **(d)** $\sin\left(\tan^{-1}\frac{4}{5}+\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{9}-\tan^{-1}\frac{1}{7}\right)$

$$=\sin\left[\left(\tan^{-1}\frac{4}{5}+\tan^{-1}\frac{1}{9}\right)+\left(\tan^{-1}\frac{4}{3}-\tan^{-1}\frac{1}{7}\right)\right]$$

$$=\sin\left[\tan^{-1}\left(\frac{\frac{4}{5}+\frac{1}{9}}{1-\frac{4}{5}\frac{1}{9}}\right)+\tan^{-1}\left(\frac{\frac{4}{3}-\frac{1}{7}}{1+\frac{4}{3}\cdot\frac{1}{7}}\right)\right]$$

$$=\sin\left(\tan^{-1}\frac{41}{41}+\tan^{-1}\frac{25}{25}\right)=\sin[\tan^{-1}(1)+\tan^{-1}(1)]$$

$$=\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right)=\sin\frac{\pi}{2}=1$$

26. **(b)** $t=\operatorname{sech}^{-1}\left(\frac{1}{2}\right)-\operatorname{cosech}^{-1}\left(\frac{3}{k}\right)$

$$\operatorname{sech}^{-1}(x)=\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$$

$$\therefore \operatorname{sech}^{-1}\left(\frac{1}{2}\right)=\ln\left(\frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)=\ln(\sqrt{3}+2)$$

$$\operatorname{cosech}^{-1}(x)=\ln\left(\frac{1+\sqrt{1+x^2}}{x}\right)$$

$$\therefore \operatorname{cosech}^{-1}\left(\frac{3}{k}\right)=\ln\left(\frac{k+\sqrt{k^2+9}}{3}\right)$$

Now $t=\ln(\sqrt{3}+2)-\ln\left(\frac{k+\sqrt{k^2+9}}{3}\right)$

$$t=\ln\left[\frac{(\sqrt{3}+2)3}{k+\sqrt{k^2+9}}\right]$$

$$3(\sqrt{3}+2)=\left(k+\sqrt{k^2+9}\right)e^t$$

$$3(\sqrt{3}+2)=\left(k+\sqrt{k^2+9}\right)\left(\frac{2+\sqrt{3}}{3}\right)$$

If k = 4, then

$$3(\sqrt{3}+2)=(4+5)\left(\frac{2+\sqrt{3}}{3}\right)$$

$k=4$ satifies the equation.

27. **(b)** In ΔABC, $4a=b+c$

$$\Rightarrow \tan\frac{B}{2}\tan\frac{C}{2}=\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}\cdot\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$=\frac{(s-a)}{s}$$

As we know, $s=\dfrac{a+b+c}{2}=\dfrac{a+4a}{2}=\dfrac{5a}{2}$

Now, $\tan\dfrac{B}{2}\tan\dfrac{C}{2}=\dfrac{s-a}{s}=\dfrac{\frac{5a}{2}-a}{\frac{5a}{2}}=\dfrac{3}{5}$

28. **(d)** Given, x Area of $\Delta ABC = c^2-(a-b)^2$

$$4\cdot\frac{1}{2}ab\sin C=c^2-a^2-b^2+2ab$$

$$\sin C=1-\frac{a^2+b^2-c^2}{2ab}$$

$\Rightarrow \sin C = 1 - \cos C \Rightarrow \sin C + \cos C = 1$

$$\Rightarrow \sqrt{2}\left(\sin C\cdot\frac{1}{\sqrt{2}}+\cos C\cdot\frac{1}{\sqrt{2}}\right)=1$$

$$\Rightarrow \sqrt{2}\sin\left(C+\frac{\pi}{4}\right)=1$$

$$C+\frac{\pi}{4}=\frac{3\pi}{4} \qquad \therefore\ C=\frac{\pi}{2}$$

29. **(c)** In $\Delta ABC, \cot\dfrac{A}{2}\cot\dfrac{B}{2}=k$

$$\left[\frac{s(s-a)}{(s-b)(s-c)}\frac{s(s-b)}{(s-a)(s-c)}\right]^{\frac{1}{2}}=k$$

$$\frac{s}{s-c}=k\ \Rightarrow\ \frac{\frac{a+b+c}{2}}{\frac{a+b+c}{2}-c}=k$$

$$\frac{a+b+c}{a+b-c}=k\ \Rightarrow\ \frac{a+b-c+2c}{a+b-c}=k\ \Rightarrow\ 1+\frac{2c}{a+b-c}=k$$

Now, in any $\Delta, a+b>c$

So, $\dfrac{2c}{a+b-c}$ is always greater than 0. $\therefore k\in(1,\infty)$

30. **(b)** In ΔABC, $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}=2R=\dfrac{abc}{2\Delta}$

$b^2\sin 2C+c^2\sin 2B$

$=2(b^2\sin C\cos C+c^2\sin B\cos B)$

$$=2\left[b^2\cdot\frac{2\Delta}{ab}\cdot\frac{a^2+b^2-c^2}{2ab}+c^2\frac{2\Delta}{ac}\cdot\frac{a^2+c^2-b^2}{2ac}\right]$$

$$=2\Delta\left[\frac{a^2+b^2-c^2}{a^2}+\frac{a^2+c^2-b^2}{a^2}\right]=2\Delta\times\frac{2a^2}{a^2}=4\Delta$$

31. **(d)** $P(-2,-3,1)$ and $Q(3,3,2)$

Let position of point R divides the position of point P and Q in the ratio of $\lambda:1$

$$R\left(\frac{3\lambda-2}{\lambda+1},\frac{3\lambda-3}{\lambda+1},\frac{2\lambda+1}{\lambda+1}\right)=\left(\frac{-9}{2},-6,\frac{1}{2}\right)$$

comparing the z-coordinate,

$$4\lambda+2=\lambda+1\ \Rightarrow\ \lambda=-\frac{1}{3}$$

$\therefore$ Required ratio $\lambda:1=-\dfrac{1}{3}:1$ or $-1:3$

32. **(b)** Given, $l=a\cdot\hat{b}, m=b\cdot\hat{c}$ and $n=c\cdot\hat{a}$

$$l=(3,5,2)\frac{(2,-3,-5)}{\sqrt{4+9+25}}=\frac{6-15-10}{\sqrt{38}}=\frac{-19}{\sqrt{38}}$$

$$m=(2,-3,-5)\frac{(-5,-2,3)}{\sqrt{25+4+9}}=\frac{-10+6-15}{\sqrt{38}}=\frac{-19}{\sqrt{38}}$$

$$n=(-5,2,3)\frac{(3,5,2)}{\sqrt{9+25+4}}=\frac{-15-10+6}{\sqrt{38}}=-\frac{19}{\sqrt{38}}$$

Hence, $l=m=n$

33. **(d)** Given, $\vec{a}=(x,2,-1)$ $\vec{b}=(6,-y,2)$

$|\vec{a}|^2=x^2+4+1$ and $|\vec{b}|^2=y^2+36+4$

If $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=f(x)g(y)$

$=|\vec{a}|^2|\vec{b}|^2\sin^2\theta+|\vec{a}|^2|\vec{b}|^2\cos^2\theta$

$$\left(\because \cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|},\sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}\right)$$

$=|\vec{a}|^2|\vec{b}|^2(\sin^2\theta+\cos^2\theta)=|\vec{a}|^2|\vec{b}|^2$

$=(x^2+4+1)(y^2+36+4)$

$=(x^2+5)(y^2+40)=f(x)\,g(y)$

$\therefore f(x)=x^2+5$ and $g(y)=y^2+40$

$f(x)+g(y)-46=0\Rightarrow x^2+5+y^2+40-46=0$

$x^2+y^2-1=0\Rightarrow x^2+y^2=1$

This represents the equation of circle.

34. **(d)** $\vec{a}=2\hat{i}+2\hat{j}+\hat{k}, |\vec{a}|=\sqrt{4+4+2}$

$|\vec{a}|=3, |\vec{b}|=6, C=\dfrac{\pi}{6}$

Area of triangle $=\dfrac{1}{2}|\vec{a}||\vec{b}|\sin C$

$$=\frac{1}{2}(3)\times6\times\frac{1}{2}=\frac{9}{2}$$

35. **(c)** Let $\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + \hat{k}$

Vector r is perpendicular to vectors $\vec{a}$ and $\vec{b}$

$\therefore\ \vec{r} = \vec{a} \times \vec{b}$

$$\vec{r} = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -4 \\ 3 & -1 & 1 \end{vmatrix} = \lambda(-\hat{i} - 14\hat{j} - 11\hat{k})$$

It is given that, $\vec{r}\cdot(3\hat{i} - 3\hat{j} + 4\hat{k}) = 5$

$\Rightarrow \lambda(-\hat{i} - 14\hat{j} - 11\hat{k})(3\hat{i} - 3\hat{j} + 4\hat{k}) = 5$

$\Rightarrow \lambda(-3 + 42 - 44) = 5 \Rightarrow \lambda = -1$

$|\vec{r}| = |\lambda|\sqrt{1 + 196 + 121} = \sqrt{318}$

36. **(b)** Let, $P(1, 2, 3)$, $Q(2, 3, -4)$, $R(3, -4, 5)$ and $T \equiv (x, y, z)$

$PQ = (1, 1, -7)$; $QR = (1, -7, 9)$; $PT = (x - 1, y - 2, z - 3)$

All the points are lying in the same plane, So, PQ, QR and PT are coplanar.

$$\therefore \quad \begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -7 \\ 1 & -7 & 9 \end{vmatrix} = 0$$

$-40(x - 1) - 16(y - 2) - 8(z - 3) = 0$

$40x + 16y + 8z = 96$

This is the equation of plane.

Now, Distance from origin (0, 0, 0)

$$D = \frac{|0 + 0 + 0 - 96|}{\sqrt{40^2 + 16^2 + 8^2}} = \frac{96}{8\sqrt{30}} = \frac{12}{\sqrt{30}}$$

37. **(b)** First 5 odd natural numbers = 1, 3, 5, 7, 9

Mean $= \dfrac{1+3+5+7+9}{5} = 5$

Deviation = |1 – 5|, |3 – 5|, |5 – 5|, |7 – 5|, |9 – 5|

= 4, 2, 0, 2, 4

$\therefore$ O = Mean deviation $= \dfrac{4+2+0+2+4}{5} = 2.4$

First 5 prime numbers = 2, 3, 5, 7, 11

Mean $= \dfrac{2+3+5+7+11}{5} = 56$

Deviation = |2 – 5.6|, |3 – 5.6|, |5 – 5.6|, |7 – 5.6|, |11–5.6| 3.6, 2.6, 0.6, 1.4, 5.4

P = Mean deviation

$= \dfrac{3.6+2.6+0.6+1.4+5.4}{5} = 2.72$

P – O = 2.72 – 2.4 = 0.32

38. **(a)** Given integer, $2520 = 2^3.3^2.5^1.7$

Number of divisors = (3 + 1) (2 + 1) (1 + 1) (1+ 1)

= 4 × 3 × 2 × 2 = 48

excluding 1 and 2520

Number of proper divisors = 48 – 2 = 46

Number of odd divisors = (2 + 1) (1 + 1) (1+1) = 12

Number of odd proper divisors = 12 – 1 = 11

$\therefore$ 2520 is even so it would not be taken as consideration)

$\therefore$ Probability $= \dfrac{11}{46}$

39. **(a)** Box P and Q each containing 100 chits.

Perfect squares from 1 to 100 are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Probability of picking one chit which is perfect square from set $P = \dfrac{1}{10}$

Probability of picking one chit form set $Q = \dfrac{1}{10}$

$\therefore$ Required probability $= \dfrac{10}{100} \times \dfrac{1}{100} = \dfrac{1}{1000}$ or 0.1%

40. **(a)** Probability of A solving the problem, $P(A) = \dfrac{2}{5}$

Probability of B solving, $P(B) = \dfrac{3}{4}$

Probability of neither solving $= \left(1 - \dfrac{2}{5}\right)\cdot\left(1 - \dfrac{3}{4}\right) = \dfrac{3}{20}$

$\therefore$ Probability that the problem will be solved

$= 1 - \dfrac{3}{20} = \dfrac{17}{20}$

41. **(d)** $\displaystyle\sum_{r=0}^{7} P(X = x) = 1$

0.01 + 0.1 + 0.26 + 0.33 + 0.18 + 0.06 + k + 0.04 = 1

k = 0.02

$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7)$

= 0.33 + 0.18 + 0.06 + k + 0.04 = 0.63

$P(X < 6) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$

= 0.01 + 0.1 + 0.26 + 0.33 + 0.18 + 0.06 = 0.94

So, $P(x \geq 3) - P(X < 6) = 0.63 - 0.94 = -0.31$

42. **(a)** $P(X = x) = 5r^x$, where x = 1, 2, 3,...

r^x Follows infinite G.P. series so its sum will be $\dfrac{a}{1-r}$.

$\Sigma P(X = r) = \dfrac{5r}{1-r} = 1 \Rightarrow 5r = 1 - r \Rightarrow r = \dfrac{1}{6}$

43. **(c)** According to the question,

$x^2 + y^2 = xy \Rightarrow \dfrac{x^2}{xy} + \dfrac{y^2}{xy} = 1 \Rightarrow \dfrac{x}{y} + \dfrac{y}{x} = 1$

44. **(a)** After translation of origin by (h, k),

$X = x - h$, $Y = y - k$

$\therefore$ $X = x - 1$, $Y = y - 2$

So, $x^2 + y^2 + 2x - 4y + 1 = 0$

$x^2 + 2x + 1 + y^2 - 4y + 4 = 4$

$\Rightarrow (x + 1)^2 + (y - 2)^2 = 4 \Rightarrow X^2 + Y^2 = 4$

45. **(a)** Let the points on the coordinates axes are $(a, 0)$ and $(0, b)$. Here Point (2, 3) divided the line formed by them in the ratio of 2 : 3

2 : 3

(a, 0) ——————— (0, b)

(2·3)

$2 = \dfrac{2\times 0 + 3\times a}{5} \Rightarrow 10 = 3a$; $3 = \dfrac{2\times b + 3\times 0}{5} \Rightarrow 15 = 2b$

$ab = \dfrac{10}{3} \times \dfrac{15}{2} = 25$

46. **(d)** Angle between two lines is $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ and $\theta = \frac{\pi}{6}$,

So, $\tan\left(\frac{\pi}{6}\right) = \left|\frac{m-2}{1+2m}\right| \Rightarrow |1+2m| = \sqrt{3}\,|m-2|$

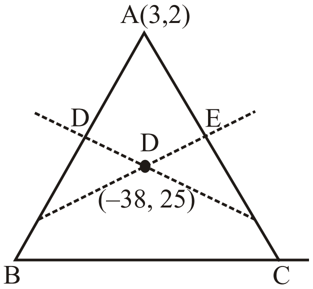
$2m_1 + 1 = \sqrt{3}(m_1 - 2)$ or $2m_2 + 1 = \sqrt{3}(2 - m_2)$

$\therefore\ m_1 = \frac{-2\sqrt{3}-1}{2-\sqrt{3}} m_2 = \frac{2\sqrt{3}-1}{2+\sqrt{3}}$

$m_1 + m_2 = \frac{-2\sqrt{3}-1}{2-\sqrt{3}} + \frac{2\sqrt{3}-1}{2+\sqrt{3}} = -16$

$= \frac{-4\sqrt{3}-2-6-\sqrt{3}+4\sqrt{3}-2-6+\sqrt{3}}{4-3}$

47. **(c)**



$2x + 3y + 1 = 0$ is perpendicular bisector of AB

So, $m_{AB} = \frac{3}{2}$; $AB : y - 2 = \frac{3}{2}(x-3)$

Equation of line $AB \Rightarrow 3x - 2y - 5 = 0$

$x + 2y - 12 = 0$ is perpendicular bisector of AC

So, $mAC = 2$; $AC : (y - 2) = 2(x - 3)$

Equation of line $AC \Rightarrow 2x - y - 4 = 0$

On solving $3x - 2y - 5 = 0$ and $2x + 3y + 1 = 0$

We get coordinates of $D \equiv (1, -1)$

So, and $D = \frac{A+B}{2} \Rightarrow A + B = 2D$

$B + (3, 2) = (2, -2)\ \therefore\ B \equiv (-1, -4)$

Similarly, on solving

$x + 2y - 12 = 0$ and $2x - y - 4 = 0$ we get

Coordinates of $E \equiv (4, 4)$ and $E = \frac{A+C}{2} \Rightarrow A + C = 2E$

$(3, 2) + C = (8, 8) \Rightarrow C \equiv (5, 6)$

So, $B \equiv (-1, -4)$ and $C \equiv (5, 6)$

$\therefore\ m_{BC} = \frac{6+4}{5+1} = \frac{10}{6} = \frac{5}{3}$

48. **(b)** Line 'L' passes through (1, 1) and makes an angle of $\tan^{-1}\frac{5}{7}$ with $x + y - 3 = 0$

$\therefore\ \tan\theta = \left|\frac{m+1}{1-m}\right| = \frac{5}{7}$

$7m + 7 = 5 - 5m \Rightarrow m \neq -\frac{1}{6}$ $\quad (\because m \in Z)$

$\therefore m = -\frac{1}{6}$ is not acceptable or $7m + 7 = 5m - 5 \Rightarrow m = -6$

Equation of line $m \in z \Rightarrow m = -6$ is acceptable.

$y - 1 = -6(x - 1) \Rightarrow y = -6x + 7 \Rightarrow -6x - y + 7 = 0$

On comparing with $ax + by + c = 0$, we get

$\therefore\ a = -6, c = 7 \Rightarrow ac = -42$

49. **(b)** $S = 2x^2 + 3xy - 2y^2 - 7x + y + 3 = 0$

$= (x + 2y - 3)(2x - y - 1) = 0$

Equation of line L_1 drawn from (3, –4) on $x + 2y - 3 = 0$

$L_1 : (y + 4) = 2(x - 3) \Rightarrow 2x - y - 10 = 0$

Equation of line L_2 drawn from (3, –4) on $(2x - y - 1 = 0)$

$L_2 : (y+4) = -\frac{1}{2}(x-3) \Rightarrow x + 2y + 5 = 0$

Distance between parallel lines $x + 2y + 5 = 0$ and $x + 2y - 3 = 0$ is

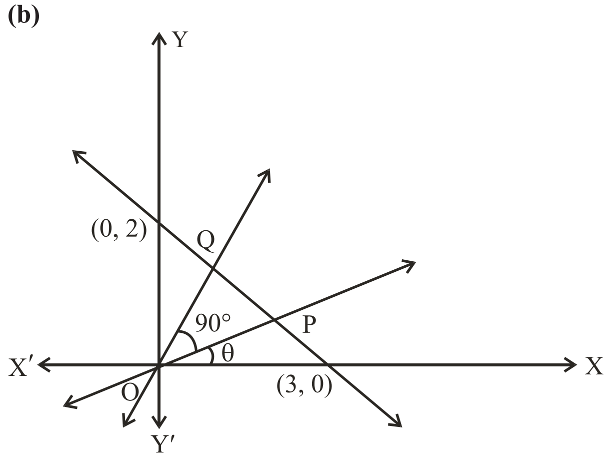
$D_1 = \frac{|5+3|}{\sqrt{1+4}} = \frac{8}{\sqrt{5}}$

Distance between parallel lines $2x - y - 1 = 0$ and $2x - y - 10 = 0$ is

$D_2 = \frac{|10-1|}{\sqrt{1+4}} = \frac{9}{\sqrt{5}}$

$\therefore$ Area $= D_1 \times D_2 = \frac{8}{\sqrt{5}} \times \frac{9}{\sqrt{5}} = \frac{72}{5}$ sq. unit

50. **(b)**



Polar coordinates of

$P(r\cos\theta, r\sin\theta)$ and $Q(-r\sin\theta, r\cos\theta)$

Equation of line $OP \Rightarrow y = x\tan\theta$

Equation of line $OQ \Rightarrow y = -x\cot\theta$

Let $P(x, y) \Rightarrow Q = (-x, y)$

$\because \frac{x}{3} + \frac{y}{2} = 1 \Rightarrow 2x + 3y = 6$

$\Rightarrow r(2\cos\theta + 3\sin\theta) = 6$ ...(i)

and $r(-2\sin\theta + 3\cos\theta) = 6$ ...(ii)

$2\cos\theta + 3\sin\theta = -2\sin\theta + 3\cos\theta$

$5\sin\theta = \cos\theta \Rightarrow \tan\theta = \frac{1}{5} \Rightarrow y = \frac{x}{5} \Rightarrow 5y - x = 0$

and $y = -5x \Rightarrow 5x + y = 0$ So, $(5y - x)(5x + y) = 0$

$5(x^2 - y^2) - 24xy = 0$

This is the equation of pair of lines.

51. **(d)** Point (2, a) does not lie outside the circle $x^2 + y^2 = 13$

$4 + a^2 - 13 \not> 0 \Rightarrow a^2 \not> 9 \Rightarrow a^2 \le 9$

$\therefore -3 \le a \le 3$ and

Similarly $4 + a^2 + 2 - 2a - 14 \not> 0$ ...(i)

$a^2 - 2a - 8 \le 0 \Rightarrow (a-1)^2 \le 9 \Rightarrow -3 \le a - 1 < 3$

$\Rightarrow -2 \le a \le 4$ ...(ii)

From, Eqs. (i) and (ii), we get

$a \in [-2, 3]$

52. **(c)** General equation of circle, $x^2 + y^2 + 2gx + 2fy + c = 0$

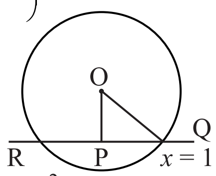
As, it passes through the origin $\Rightarrow c = 0$

centre = $(-g, -f)$

radius (OQ) = $(g+1)^2 + 1^2 \left(\sqrt{g^2+f^2}\right)^2$

OP = $|g + 1|$, PQ = 1

(Given RQ = 2 units)



$\therefore (g+1)^2 + 1^2 \left(\sqrt{g^2+f^2}\right)^2$

$g^2 + 2g + 2 = g^2 + f^2 \Rightarrow f^2 = 2(g + 1) \Rightarrow y^2 = 2(n + 1)$

$\therefore$ Its represents parabola.

53. **(a)** $c_1 : x^2 + y^2 = 1$; $c_1(0, 0)$ and $r_1 = 1$

$c_2 : x^2 + y^2 - 2x - 6y + 6 = 0$; $c_1c_2 = \sqrt{1^2 + 3^2} = \sqrt{10} = 312$

and $r_1 + r_2 = 1 + 2 = 3$

$\therefore c_1c_2 > r_1 + r_2 \Rightarrow$ So, there will be 4 tangents.

54. **(a)** Equation of the circle passing through the intersection of two circles, $S_1 + \lambda S_2 = 0$

$(x^2 + y^2 + 2x + 3y + 1) + \lambda(x^2 + y^2 + 4x + 3y + 2) = 0$

This circle passes through (–1, 1)

$(1 + 1 - 2 + 3 + 1) + \lambda(1 + 1 - 4 + 3 + 2) = 0$

$4 + 3\lambda = 0 \Rightarrow \lambda = -\frac{4}{3}$

$$x^2\left(1 - \frac{4}{3}\right) + y^2\left(1 - \frac{4}{3}\right) + x\left(2 - \frac{16}{3}\right) + y\left(3 - \frac{4}{3}\cdot 3\right) + 1 - \frac{8}{3} = 0$$

$x^2 + y^2 + 10x + 3y + 5 = 0$

55. **(d)** Circle $x^2 + y^2 + 2kx + 4y - 4 = 0$ has centre c $(-k_1, -2)$.

Since it lies in fourth quadrant $\Rightarrow -k > 0 \Rightarrow k < 0$

Centre of circle $x^2 + y^2 + 6x - 2y + 6 = 0$ is (–3, 1)

and radius $= \sqrt{9 + 1 - 6} = 2$

$\because r_1 + r_2 = c_1c_2 \Rightarrow \sqrt{k^2 + 4 + 4} + 2 = \sqrt{(k-3)^2 + (-2-1)^2}$

$\sqrt{k^2 + 8} + 2 = \sqrt{k^2 - 6k + 18}$

$k = -1$ satisfies the equation.

56. **(b)** Let equation of parabola $\Rightarrow y^2 + Ay + Bx + C = 0$

Passes through (–2, 1) (1, 2) and (–1, 3)

At (–2, 1) $\Rightarrow 1 + A - 2B + C = 0$ ...(i)

At (1, 2) $\Rightarrow 4 + 2A + B + C = 0$ ...(ii)

At (–1, 3) $\Rightarrow 9 + 3A - B + C = 0$ ...(iii)

$3 + A + 3B = 0 \Rightarrow 5 + A - 2B = 0 \Rightarrow 2 - 5B = 0$

Solving the above three equations, we get

$B = \frac{2}{5} A = -\frac{21}{5}$ and $C = 4 \Rightarrow y^2 - \frac{21}{5}y + \frac{2x}{5} + 4 = 0$

$\left(y - \frac{21}{10}\right)^2 + \left(\frac{2}{5}\right)x + (4) = 0$

This is the required equation of parabola.

As, y coordinate of focus will be same as y coordinate of vertex. So, it is $\frac{21}{10}$.

57. **(d)** Focus (2, –3), Directrix 3x – 2y + 5 = 0

Equation of parabola,

$(x-2)^2 + (y+3)^2 = \frac{(3x - 2y + 5)^2}{3^2 + 2^2}$

$\therefore 13x^2 - 52x + 52 + 13y^2 + 78y + 117$

$= 9x^2 + 4y^2 + 25 - 12xy - 20y + 30x$

$\therefore 4x^2 + 12xy + 9y^2 - 82x + 98y + 144 = 0$

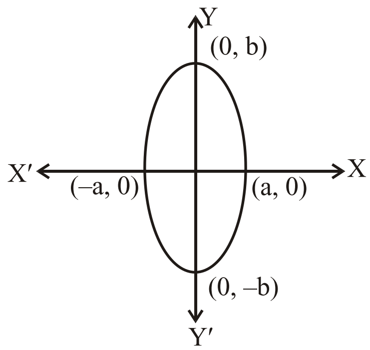
$\therefore 4x^2 + 12xy + 9y^2 = (2x + 3y)^2$

Hence, they represent two coincident lines.

58. **(c)** Latus rectum of ellipse $= \frac{2}{3}$ (Minor axis)

XX' = Minor axis = $2a$

YY' = Major axis = $2b$



$\frac{2a^2}{b} = \frac{2}{3}(2a) \Rightarrow \frac{a}{b} = \frac{2}{3} \Rightarrow a = \frac{2}{3}b$

$a^2 = b^2(1 - e^2) \Rightarrow \frac{4b^2}{9} = b^2(1 - e^2) \quad \therefore e = \frac{\sqrt{5}}{3}$

59. **(b)** Given length $= 8\sqrt{2}$ and Breadth $= 4\sqrt{2}$

$a\sqrt{2} = 8\sqrt{2} \Rightarrow a = 8 \Rightarrow b\sqrt{2} = 4\sqrt{2} \Rightarrow b = 4$

$\Rightarrow b^2 = a^2(1 - e^2) \Rightarrow 16 = 64(1 - e^2)$

$1 - e^2 = \frac{1}{4} \quad \therefore e = \frac{\sqrt{3}}{2}$

60. **(d)** Given $\frac{x^2}{\alpha + 3} + \frac{y^2}{2 - \alpha} = 1$

General equation of hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore \alpha + 3 > 0$ and $2 - \alpha < 0$

$(\alpha + 3)(\alpha - 2) > 0 \Rightarrow \alpha \in (-\infty, -3) \cup (2, \infty)$

61. **(b)** $AB=\sqrt{(4-2)^2+(-2+1)^2+(3-1)^2}=3$

$BC=\sqrt{0+49+64}=\sqrt{113}$; $CD=\sqrt{4+1+4}=\sqrt{9}=3$

$AD=\sqrt{0+49+64}=\sqrt{113}$; $AB=CD=3$

$BC=AD=\sqrt{113}$; $AC=\sqrt{4+36+100}=\sqrt{140}$

$BD=\sqrt{4+64+36}=\sqrt{104}$; $AC\neq BD$

Its parallel sides are equal. But its diagonals are unequal. So, there points form a parallelogram.

62. **(b)** $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & -2\\ 0 & 2 & 1\end{vmatrix}=2\hat{i}-\hat{j}+2\hat{k}$

unit vector $=\frac{2}{3}\hat{i}+-\frac{1}{3}\hat{j}+\frac{2}{3}\hat{k}$

$|l|=\left|\frac{2}{3}\right|\Rightarrow|l|=\frac{2}{3}$; $|m|=\left|-\frac{1}{3}\right|\Rightarrow|m|=\frac{1}{3}$

$|n|=\left|\frac{2}{3}\right|\Rightarrow|n|=\frac{2}{3}$; $\therefore |l|+|m|+|n|=\frac{5}{3}$

63. **(b)** Given equation of plane,

$\vec{r}\cdot(2\hat{i}+3\hat{j}-4\hat{k})=1$ ...(i)

$\therefore$ Equation of plane parallel to the plane in Eq. (i),

$\vec{r}\cdot(2\hat{i}+3\hat{j}-4\hat{k})=P$

$\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}=2\Rightarrow\frac{|P-1|}{\sqrt{4+9+16}}=2$

$P-1=\pm2\sqrt{29}\Rightarrow P=1\pm2\sqrt{29}$

Hence, equation of required plane

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(2\hat{i}+3\hat{j}-4\hat{k})=1\pm2\sqrt{29}$

$2x+3y-4z=1\pm2\sqrt{29}$

64. **(b)** $f(x)=\begin{cases}\frac{\sin[x]}{[x]}, & \text{if } [x]\neq 0\\ 0, & \text{if } [x]=0\end{cases}$

$\Rightarrow \lim_{x\to0^-}f(x)=\lim_{x\to0^-}\frac{\sin(-1)}{(-1)}=\sin 1$

65. **(d)** $\lim_{x\to-\infty}\frac{3|x|^3-x^2+2|x|-5}{-5|x|^3+3x^2-2|x|+7}$

$x\to-\infty\Rightarrow|x|=-x$

$\lim_{x\to-\infty}\frac{-3x^3-x^2-2x-5}{5x^3+3x^2+2x+7}$

$=\lim_{x\to-\infty}\frac{x^3\left[-3-\frac{1}{x}-\frac{2}{x^2}-\frac{5}{x^3}\right]}{x^3\left[5+\frac{3}{x}+\frac{2}{x^2}+\frac{7}{x^3}\right]}=-\frac{3}{5}$

66. **(a)** Since $f(x)$ is continuous at $x=0$

$\therefore \lim_{x\to0^-}f(x)=f(0)=\lim_{x\to0^+}f(x)$

$\Rightarrow \lim_{x\to0^-}f(x)=f(0)$

$\lim_{x\to0^-}\frac{\tan(2p-7)x+\tan 3x}{x}=p-q$

$\lim_{x\to0^-}\left[\frac{(2p-7)\tan(2p-7)x}{(2p-7)x}+\frac{3\tan 3x}{3x}\right]=p-q$

$2p-7+3=p-q\Rightarrow 2p-p+q=-3+7$

$p+q=4\Rightarrow \lim_{x\to0^+}f(x)=f(0)$

$\lim_{x\to0^+}q\left(\frac{\sqrt{x^2+x}-\sqrt{x}}{x\sqrt{x}}\right)=p-q$

$\lim_{x\to0^+}\frac{\sqrt{x}(\sqrt{x+1}-1)}{x\sqrt{x}}=\frac{p}{q}-1\Rightarrow\lim_{x\to0^+}\frac{\sqrt{x+1}-1}{x}=\frac{p}{q}-1$

Applying L' Hospital Rule,

$\lim_{x\to0^+}\frac{\frac{1}{2\sqrt{x+1}}-0}{1}=\frac{p}{q}-1$; $\therefore\frac{p}{q}-1=\frac{1}{2}$

$\frac{p}{q}=\frac{1}{2}+1=\frac{3}{2}$ Hence, $\frac{q}{p}=\frac{2}{3}$

67. **(d)** $xcxy=y+\sin^2x$

Differentiating with respect to x,

$xe^{xy}\frac{d(xy)}{dx}+e^{xy}\cdot1=\frac{dy}{dx}+2\sin x\cos x$

$xe^{xy}\left(x\frac{dy}{dx}+y(1)\right)+e^{xy}=\frac{dy}{dx}+2\sin x\cos x$

At $x=0$, $0+1=\frac{dy}{dx}+0\Rightarrow\frac{dy}{dx}=1$

68. **(a)** $f(x)=x^2\sin\left(\frac{1}{x}\right)$

Differentiating with respect to x,

$f'(x)=x^2\cos\left(\frac{1}{x}\right)\left(\frac{-1}{x^2}\right)+2x\sin\left(\frac{1}{x}\right)$

$\lim_{x\to0}f'(x)=\lim_{x\to0}\left[-\cos\left(\frac{1}{x}\right)+2x\sin\left(\frac{1}{x}\right)\right]$

Range of cosine function is [–1, 1]. So it will lie between – 1 and 1, $\therefore \lim_{x\to0}f'(x)=[-1,1]+0$

So, limit does not exist.

69. **(c)** $y=(\log x)^{\sin x}$

$\log y=\sin x\cdot\log(\log x)$

Differentiating with respect to x,

$\frac{1}{y}\frac{dy}{dx}=\sin x\cdot\left(\frac{1}{\log x}\right)\left(\frac{1}{x}\right)+\log(\log x)\cos x$

At $x=\frac{\pi}{2}, \left[\frac{1}{\log\left(\frac{\pi}{2}\right)}\right] y' = \frac{2}{\pi}\left[\frac{1}{\log\left(\frac{\pi}{2}\right)}\right] = \frac{2}{\pi}$

Let $t = \cos x$

$dt/dx = -\sin x$

At $x=\frac{\pi}{2}\cdot\frac{dt}{dx}=-1$

$$\therefore \frac{dy}{dt}=\frac{\frac{dy}{dx}}{\frac{dt}{dx}}=\frac{\left(\frac{2}{\pi}\right)}{-1}=\frac{-2}{\pi}$$

70. (a) According to the definition of differentiation,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x)+f(h)-3xh-f(x)}{h} = \lim_{h\to 0}\frac{f(h)-3xh}{h}$$

$$=\lim_{h\to 0}\frac{f(h)}{h}-\lim_{h\to 0}\frac{3xh}{h}=7-3x$$

$$\therefore f'(x)=-3x+7$$

71. (b) At $x = 0 \Rightarrow y + 1 = 0, y = -1 \Rightarrow P(0, -1)$

$xy^5 + 2x^2y - x^3 + y + 1 = 0$

On differentiating w.r.t. x,

$$5xy^4\cdot y' + y^5 + 2x^2y' + 4xy - 3x^2 + y' = 0$$

At $P(0,-1), 0-1+0+0-0+y'=0$

$y'=1$

Equation of tangent at (0, –1)

$(y + 1) = 1(x - 0) \Rightarrow y = x - 1$

72. (b) Equation of displacement S at a time t is,

$S(t) = t^3 - 4t^2 + 7t$

$$V=\frac{dS(t)}{dt}=3t^2-8t+7$$

At $t = 4$ $v = 3\times 16 - 8\times 4 + 7 = 23$ m/s

73. (d) We given that two curves are

$x = y^2$ .... (i) & $x.y = k$ ... (ii)

Now from eqn (i)

$$1=2y\frac{dy}{dx}\Rightarrow \boxed{m_1=\frac{1}{2y}}$$

Now from eqn (ii)

$$x.\frac{dy}{dx}+y.1=0\Rightarrow x.m_2+y=0$$

$$\Rightarrow x.m_2=-y \Rightarrow \boxed{m_2=\frac{-y}{x}}$$

Now given that $m_1.m_2=-1$

$$\Rightarrow\left(\frac{1}{2y}\right)\left(-\frac{y}{x}\right)=-1\Rightarrow -\frac{1}{2x}=-1$$

$$\Rightarrow 2x = 1 \Rightarrow \boxed{x=\frac{1}{2}}$$

Now from eqn (i)

$$\frac{1}{2}=y^2\Rightarrow \boxed{y=\pm\frac{1}{\sqrt{2}}}$$

$$\therefore k=x.y=\frac{1}{2}.\frac{1}{\sqrt{2}}=\frac{1}{2\sqrt{2}}\Rightarrow \boxed{k=\frac{1}{2\sqrt{2}}}$$

74. (d) $f(x)=\begin{cases} x, & 0\le x\le 1 \\ 2-x, & 1\le x\le 2\end{cases}$

To check the continuity of $f(x)$,

LHL $=\lim_{x\to 1^-} f(x)=1$

RHL $=\lim_{x\to 1^+} f(x)=1$

and $f(1) = 1$

To check the differentiability of $f(x)$,

$f(x)=x\Rightarrow f'(x)=1$

LHD $=\lim_{x\to 1^-} f'(x)=1$

$f(x)=2-x\Rightarrow f'(x)=-1$

RHD $=\lim_{x\to 1^+} f'(x)=-1$

$\therefore f(x)$, is not differentiable.

75. (a) $I=\int\frac{xe^{\frac{x^2}{x^2-2}}}{x^4-4x^2+4}dx=\int\frac{xe^{\frac{x^2}{x^2-2}}}{(x^2-2)^2}dx$ ...(i)

Let $t=\frac{x^2}{x^2-2}$ ...(ii)

$$\therefore\frac{dt}{dx}=\frac{(x^2-2)\cdot 2x-x^2(2x)}{(x^2-2)^2}=\frac{-4x}{(x^2-2)^2}$$

$\Rightarrow\frac{x}{(x^2-2)^2}dx=\frac{dt}{-4}$ ...(iii)

Using Equations (ii) and Eq. (iii) in Eq. (i),

$$I=\int e^t\left(\frac{dt}{-4}\right)=\left(-\frac{1}{4}\right)e^t+C=\left(-\frac{1}{4}\right)e^{\frac{x^2}{x^2-2}}+C$$

76. (b) $I=\int\frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$

$I=\int\frac{2x^{12}+5x^9}{x^{15}\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx=\int\frac{\left(\frac{2}{x^3}\right)+\left(\frac{5}{x^6}\right)}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx$ …(i)

Let $1+\frac{1}{x^2}+\frac{1}{x^5}=t$ ...(ii)

$\left(\frac{-2}{x^3}+\frac{(-5)}{x^6}\right)dx=dt$ ...(iii)

Using Eqs. (ii) and (iii) in Eq. (i)

$$I=\int-\frac{dt}{t^3}=-\left(\frac{t^{-2}}{-2}\right)+C=\frac{1}{2t^2}+C$$

$$=\left(\frac{1}{2}\right)\left(\frac{x^5}{x^5+x^3+1}\right)^2+C=\frac{1}{2}f(x)+c$$

$$\therefore f(x)=\frac{1}{2}\left(\frac{x^5}{x^5+x^3+1}\right)^2$$

$f(1)=\frac{1}{18}$ and $f(0)=0$ Thus, $f(1)-f(0)=\frac{1}{18}$

77. **(b)** $I=\int\frac{\sqrt{\cot x}}{\sin 2x}dx=\int\frac{\sqrt{\cot x}}{2\sin x\cos x}dx$

$$=\int\frac{\sqrt{\cot x}}{\cot x}\frac{dx}{2\sin^2 x}=\frac{1}{2}\int(\cot x)^{-\frac{1}{2}}\operatorname{cosec}^2 x\,dx$$

Let cot x = t ⇒ – cosec^2 x dx = dt

Now, $I=\frac{1}{2}\int t^{-1/2}(-dt)=\frac{1}{2}\left(\frac{-t^{1/2}}{1/2}\right)+C$

$$=-t^{1/2}+C=-\sqrt{t}+C=-\sqrt{\cot x}+C$$

78. **(c)** Given, $\lim_{n\to\infty}\frac{1}{n}\log\left[\frac{(2n)!}{n^n n!}\right]=\int_1^2 f(x)dx$

$$\frac{(2n)!}{n^n}n!=\frac{2n\times(2n-1)\times(2n-2)\times...\times(n+1)\times n!}{n\times n\times n\times...\times n\times n!}$$

$$=2\times\left(2-\frac{1}{n}\right)\times\left(2-\frac{2}{n}\right)\times\left(2-\frac{3}{n}\right)\times...\times\left(1+\frac{1}{n}\right)$$

$$=\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\times...\left(1+\frac{n}{n}\right)$$

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)\left[\log\left(1+\frac{1}{n}\right)+\log\left(1+\frac{2}{n}\right)+...\log\left(1+\frac{n}{n}\right)\right]$$

$$=\lim_{n\to\infty}\left(\frac{1}{n}\right)\sum_{r=1}^{n}\log\left(1+\frac{r}{n}\right)$$

Lower limit $=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)=1$

Upper limit $=\lim_{n\to\infty}\left(1+\frac{n}{n}\right)=2=\int_1^2\log(x)dx$

$\therefore f(x)=\log x$

79. **(a)** $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin(x-[x])dx$

$$=\int_{-\frac{\pi}{2}}^{-1}\sin(x+2)dx+\int_{-1}^{0}\sin(x+1)dx+\int_{0}^{1}\sin(x)dx+\int_{1}^{\frac{\pi}{2}}\sin(x-1)dx$$

$$=\cos\left(2-\frac{\pi}{2}\right)-\cos(1)+\cos 0-\cos 1+\cos 0-\cos 1+\cos 0-\cos\left(\frac{\pi}{2}-1\right)$$

= 3(1 – cos1) + sin 2 – sin 1

80. **(b)** Given, $\frac{dy}{dx}=\sqrt{1-y^2}$ and $y(0)=1$

Integrating both the sides, we get

$$\int\frac{dy}{\sqrt{1-y^2}}=\int dx$$

$\sin^{-1}y=x+C$

$\because y(0)=1$

$\Rightarrow \sin^{-1}(1)=0+C\Rightarrow C=\sin^{-1}(1)$

$\sin^{-1}y=x+\sin^{-1}(1)$.

PHYSICS

81. **(c)** The physics behind fusion test reactor is magnetic confinement of plasma in which particle and energy of hot plasma are held together by strong magnetic field, that confines the movement of deuterium-tritium plasma and prevents them from coming in contact with reactor wall.

82. **(c)** From question $QV=kpTL^{\alpha}$

Here, dimensional formula of

$[Q]=[ML^{-1}T^{-1}]$, $[k]=[M^0L^0T^0]$

$[V]=[M^0L^3T^0]$, $[p]=[ML^{-1}T^{-2}]$

$[T]=[M^0L^0T^{+1}]$, $[l]=[M^0L^1T^0]^{\alpha}$

Therefore $[Q][V]=[K][P][T][L]^{\alpha}$

$$\Rightarrow[ML^{-1}T^{-1}][M^0L^3T^0]=[M^0L^0T^0][ML^{-1}T^{-2}][M^0L^0T^1][M^0L^1T^0]^{\alpha}$$

$$\Rightarrow[ML^2T^{-1}]=[M^{0+1+0+0}L^{0-1+0+\alpha}T^{0-2+1+0}]$$

$\therefore -1+\alpha=2$ or, $\alpha=3$

83. **(b)** Using $v^2-u^2=2as$

$\therefore (v_1-v_2)^2-0=2(a_1-a_2)^5$ [$\because$ Cars are at rest initially]

$\Rightarrow(v_1-v_2)^2=2\times4\times50=400$

$\therefore v_1-v_2=20\text{ ms}^{-1}$

84. **(d)** Distance covered by rocket in $t=20$ s,

$$\because s=ut+\frac{1}{2}at^2$$

$$\Rightarrow s=0\times20+\frac{1}{2}\times1\times(20)^2=200\,m$$

Now using v = u + at

where, v is speed of rocket after time 20s.

v = 0 + 1 × 20 = 20 ms^{-1}

Now, after 20 s the piece of rocket breaks off, the time taken by the piece to reach on ground be t'. Speed of rocket after 20 s (v) = speed of broken part of rocket (u')

i.e. u' = v

$u'=v_{a'g}$

$\because s' = -u't + \frac{1}{2}a'(t')^2$ $[\because s' = s = 200\,m]$ $a = 1\text{ms}^{-2}$
$u = 0$

$\Rightarrow\ 200 = -20t + \frac{1}{2} \times 10 \times t^2$

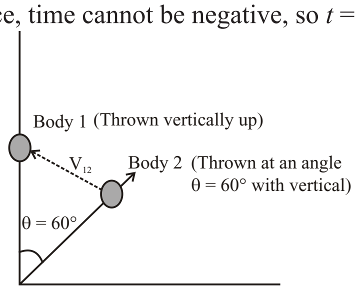
$\Rightarrow 40 = -4t + t^2 \Rightarrow t^2 - 4t - 40 = 0$

Solving we get

$t = 8.5$s or $t = -4.63$s

Since, time cannot be negative, so $t = 8.5$s

85. **(a)**



Using triangle law, relative speed

$$v_{12} = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$$

$$= \sqrt{10^2 + 10^2 - 2(10)^2 \times \cos 60°}$$

$$= \sqrt{200 - 200 \times \frac{1}{2}} = \sqrt{100} = 10\,\text{m/s and}$$

$$v_{12} = \frac{s_{12}}{t} \Rightarrow s_{12} = v_{12}t = 10 \times 2 = 20\,m$$

86. **(a)** From $s = ut + \frac{1}{2}at^2$

$$s_x = u_xt + \frac{1}{2}a_xt^2 \quad \text{...(i)}$$

and $s_y = u_yt + \frac{1}{2}a_yt^2$

Here t be the time taken by the particle to reach maximum height.

$$\therefore s_x = 3t - \frac{1}{2}(1)t^2 \quad \therefore \frac{ds_x}{dt} = 3 - t = v_x$$

Here, v_x is final speed of particle along X-axis.

$$\therefore \frac{ds_x}{dt} = 0 \Rightarrow 3 - t = 0 \quad [\because v_{\max} = 0]$$

or $t = 3s$

$$\therefore s_x = 3 \times 3 - \frac{1}{2}(1) \times 3^3 = 9 - \frac{9}{2} = \frac{9}{2}m$$

$$s_x = \frac{9}{2}\hat{i}m \text{ and } s_y = u_yt - \frac{1}{2}a_yt^2$$

$$\Rightarrow s_y = 0 \times 3 - \frac{1}{2} \times \frac{1}{2} \times 3^2 = \frac{-9}{4}m$$

$$\therefore s_y = -\frac{9}{4}(\hat{j})$$

Since particle reaches to maximum height and returns

$$\therefore s_y = -\frac{9}{4} \times 2\hat{j} = -\frac{9}{2}\hat{j}$$

$$\therefore\ s = s_x\hat{i} + s_y\hat{j} = \frac{9}{2}\hat{i} - \frac{9}{2}\hat{j} = \frac{9}{2}(\hat{i} - \hat{j})$$

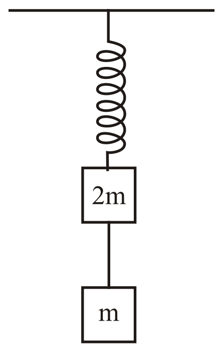
87. **(c)** From Newton's 2nd law of motion,

$$F = ma = m\left(\frac{v-u}{t}\right)$$

$$\therefore\ F = \frac{2}{10}\left(\frac{0-20}{\frac{1}{10}}\right) = \frac{2}{10}(-200) = -40N$$

Here, –ve sign shows the direction of force is opposite to the direction of velocity.

88. **(a)** When string between m and $2m$ is cut, mass m undergoes free fall.



$\therefore$ Acceleration of 2m be $a_2 = g$ and equal force is experienced by $2m$

$\therefore\ mg = 2ma_1$

where, a_1 = acceleration of mass $2m$, when string is cut

$$\therefore a_1 = \frac{g}{2}$$

89. **(b)** From work-energy theorem,

Work done, $W = \frac{1}{2}m(v^2 - u^2)$ ...(ii)

and by using third equation of motion

$v^2 - u^2 = 2as \Rightarrow v^2 = 2$as

$$\Rightarrow v = \sqrt{2as} \quad [\because u = 0]$$

$$\therefore\ v = \sqrt{2 \times 10 \times 5} = \sqrt{100} = 10ms^{-1}$$

$$\left[\because a = \frac{F}{m} = \frac{5}{0.5} = 10ms^{-2}\right]$$

$$\therefore\ W = \frac{1}{2} \times 0.5(10^2 - 0^2) = \frac{1}{4}(100) = 25J$$

$$P = \frac{W}{t} = \frac{w}{\left(v - \frac{u}{a}\right)} \quad [\because v = u + at]$$

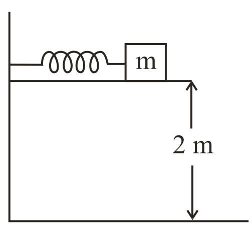
$$= \frac{25}{\frac{10-0}{10}} = \frac{25}{1} = 25W$$

90. **(c)** In case of non-conservative force, work done is path dependent and independent of initial and final positions and for closed path, displacement $s \neq 0$

Work (W) = force (F) × displacement(s) cos θ

$\therefore$ W ≠ 0J

91. **(a)**



Using law of conservation of energy,

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$$

where, Δx = horizontal displacement and v = velocity of block in horizontal direction.

$$\because v^2 = \frac{k(\Delta x)^2}{m} \Rightarrow v = \sqrt{\frac{k(\Delta x)^2}{m}} = \Delta x\sqrt{\frac{k}{m}}$$

$$= \frac{10}{100}\sqrt{\frac{50}{0.2}} = \frac{1}{10}\sqrt{\frac{500}{2}} = \sqrt{\frac{5}{2}} = \sqrt{2.5} = 1.58ms^{-1}$$

$$\because h = ut + \frac{1}{2}gt^2$$

$$\because h = 0\times t + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 2}{10}}$$

$$\therefore\ t = \sqrt{\frac{4}{10}} = \sqrt{0.4}s$$

$\therefore$ Distance travelled along X-axis.

$$x = vt = 1.58\times\sqrt{0.4} = 0.99m$$

92. **(a)** In case of rolling without slipping, velocity of body at bottom

$$v = \sqrt{\frac{2mgh}{R^2 + m}}$$

So, $\because\ I_{\text{ring}} = MR^2$

$$I_{\text{solid cylinder}} = \frac{MR^2}{2} = 0.5MR^2$$

$$I_{\text{solid sphere}} = \frac{2}{5}MR^2 = 0.4MR^2$$

Therefore, $I_{\text{ring}} > I_{\text{solid cylinder}} > I_{\text{solid sphere}}$

$\therefore$ v_{ring} is least.

93. **(a)** Let displacement be x from mean position :

Then KE is given by $KE = \frac{1}{2}k(A^2 - x^2)$

And potential energy (PE) is $\frac{1}{2}kx^2$

As kinetic energy (KE) = 8 × potential energy (PE)

$$\Rightarrow \frac{1}{2}k(A^2 - x^2) = 8\times\frac{1}{2}kx^2$$

Here, k is propagation constant, A is amplitude and x is wave displacement.

Therefore, $A^2 - x^2 = 8x^2$

$$\Rightarrow A^2 = 9x^2 \quad \Rightarrow X = \frac{A}{3}$$

Here $A = \sqrt{3\pi}$

$$\therefore x = \frac{\sqrt{3\pi}}{3} = \sqrt{\frac{\pi}{3}}$$

94. **(a)** Let M be the mass of two stars and R be its orbital radius. From Kepler's second law of planetary motion,

$$T^2 \propto R^3 \ \Rightarrow T \propto R^{3/2}$$

95. **(b)** Here we have two spring having spring constant k_1 and k_2 and their weight is W_1 and W_2. Let Δx_1 and Δx_2 be the change in length of string.

Then, $\Delta x_1 = \Delta x_2$ and $k_1 = 2k_2$

Here, Weight, $w = k\Delta x$

$$\therefore \frac{w_2}{w_1} = \frac{k_2}{k_1}\frac{\Delta x_2}{\Delta x_1} \Rightarrow \frac{w_2}{w_1} = \frac{k_2}{2k_2}\frac{\Delta x}{\Delta x} \Rightarrow \frac{w_2}{w_1} = \frac{1}{2} = 0.5$$

96. **(a)** Let θ be the contact angle and F be the force

So, θ_1 when θ = 0°, F = 0.5 N

when θ = 180°, F = 0.49 N

So, this system, we have l = length of sheet

Surface tension, S = ΔF/4l

Here, $\Delta F = F_1 - F_2$

= 0.50 – 0.49 = 0.01 N

$$\therefore S = \frac{\Delta F}{4l}$$

$$= \frac{0.01}{4\times 4\times 10^{-2}} = \frac{1}{16} = 0.0625\,N/m = 6.25 \times 10^{-2}\text{ N/m}$$

97. **(a)** According to Stoke's law for spherical body

Viscous force, $F = 6\pi\eta Rv$

$\therefore$Stoke's law is related with viscous drag.

Fluid is turbulent or not is determined by Reynold's number

C. Bernouli's principle

According to this principle,

$$p + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

where, p is pressure energy per unit volume, ρgh is potential energy per unit volume and $\frac{1}{2}\rho v^2$ is kinetic energy per unit volume.

D. Pascal's law

In a fluid if a pressure is changed at anypoint, same change in pressure appears at other point in fluid.

i.e. $\Delta p_1 = \Delta p_2$

which is used in hydraulic lift.

Hence, $A \to III, B \to IV, C \to I$ and $D \to II$ are correct.

98. **(c)** Let initial pressure be p_i and final pressure be p_f.

then, $p_f = p - 0.1\%$ of $p = p - \frac{0.1}{100}p$

pV^γ = constant

$$\Rightarrow p_iV_i^\gamma = p_fV_f^\gamma$$

$$\Rightarrow pV^\gamma = \left(p - \frac{0.1}{100}p\right)V_f^\gamma \Rightarrow V^\gamma = \left(\frac{99.9}{100}\right)V_f^\gamma$$

$$\Rightarrow V_f^\gamma = \left(\frac{100}{99.9}\right)V^\gamma \Rightarrow V_f = \left(\frac{100}{99.9}\right)^{\frac{1}{\gamma}}V$$

$\therefore$ Change in volumetric %

$$\frac{\Delta V}{V}\times 100 = \frac{V_f - V_i}{V_i}\times 100 = \frac{\left(\frac{100}{99.9}\right)^{\frac{1}{\gamma}}V - V}{V}\times 100$$

$$=\left[\left(\frac{100}{99.9}\right)^{\frac{3}{5}}-1\right]\times 100 = 0.06\% \quad \left[\because \gamma=\frac{5}{3}\right]$$

99. **(d)** Given, temperature of surrounding, $T_s = 20°C$.
We know by Newton's law of looking

$$\frac{T_i - T_f}{t} = K\left(\frac{T_i+T_f}{2} - T_s\right) \quad ...(i)$$

According to first case,
By using Eq. (i), we get

$$\frac{T_1-T_2}{t_1} = K\left(\frac{T_1+T_2}{2}-T_s\right)$$

$$\frac{70-40}{5} = K\left(\frac{70+40}{2}-20\right) [\because T_1 = 70°C T_2 = 40°C]$$

$t = 15$ sec

$$\frac{30}{5} = K\left(\frac{110}{2}-20\right) \Rightarrow 6 = K(35)$$

$$K = \frac{6}{35}$$

According to second case, again by using Eq. (i), we get

$$\frac{T_3-T_4}{T_2} = K\left(\frac{T_3+T_4}{2}-T_s\right) \Rightarrow \frac{60-40}{t_2} = \frac{6}{35}\left(\frac{60+40}{2}-20\right)$$

$[\because T_3 = 60°C, T_4 = 40°C]$

$$\Rightarrow \frac{60-40}{t_2} = \frac{6}{35}\times 30 \Rightarrow \frac{20}{t_2} = \frac{6}{7}\times 6$$

$$\Rightarrow t_2 = \frac{20\times 7}{36} = \frac{140}{36} = 3.89 \text{ min}$$

100. **(a)** Here,
Initial efficiency $\eta_1 = 50\%$
Temperature of sink, $T_2 = 500$ K
Final efficiency, $\eta_2 = 80\%$
T_{1_i} and T_{1f} be the initial and final temperature of source.
As we know that,

$$\eta - 1 - \frac{T_2}{T_1} \Rightarrow \eta_1 = \frac{50}{100} = 1 - \frac{500}{T_{1_i}}$$

$$\Rightarrow \frac{500}{T_{1_i}} = 1-\frac{1}{2} \Rightarrow \frac{500}{T_{1_i}} = \frac{1}{2} \Rightarrow T_{1_i} = 1000K$$

Similarly, $\eta_2 = 1 - \frac{T_2}{T_{1_f}} \Rightarrow \frac{80}{100} = 1 - \frac{500}{T_{1_f}}$

$$\Rightarrow \frac{500}{T_{1_f}} = 1 - \frac{80}{100} = \frac{20}{100} = \frac{1}{5} \Rightarrow T_{1_f} = 2500K$$

$\therefore$ Change in temperature,

$$\Delta T = T_{1_f - T_{1_i}} = 2500 - 1000 = 1500K$$

101. **(b)** Given,
Initial temperature of container, $T_i = 300$K
Let v_i and v_f be the initial and final velocity of particle.
where, $v_f = 4v_i$
and final temperature of container be T_f
As we know that,

Average velocity, $v_{av} = \sqrt{\frac{8RT}{\pi m}}$

Here R is universal gas constant, T is temperature and m be molar mass.
On squaring both sides, we get

$$v_{av}^2 = \frac{8RT}{\pi m} \Rightarrow V_{av}^2 \propto T$$

$$\therefore \left(\frac{V_F}{V_i}\right)^2 = \left(\frac{T_F}{T_i}\right) \Rightarrow \left(\frac{4V_i}{V_i}\right)^2 = \left(\frac{T_1}{T_i}\right)$$

$\Rightarrow 16 \times T_i = T_f \Rightarrow T_f = 16 \times 300 = 4800$ K
$\therefore T_f = (4800 - 273)°C = 4527°C$

102. **(d)** Given, we know that mean free path is given by,

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 \cdot p} \Rightarrow \lambda \propto \frac{T}{p} \therefore \frac{\lambda_1}{\lambda_2} = \frac{T_1}{T_2}\times\frac{p_2}{p_1}$$

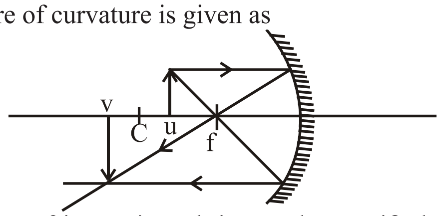
$$\Rightarrow \frac{10^{-7}}{\lambda_2} = \frac{300}{400}\times\frac{200}{600} \Rightarrow \lambda_2 = 10^{-7}\times\frac{4\times 6}{3\times 2} = 4\times 10^{-7} m$$

103. **(a)** We know that

$\Delta\phi = \frac{\omega}{v}\Delta x = \frac{2\pi f}{v}\Delta x$ Here, $f = 200$ Hz
$V = 340$ m/s
$\Delta x = 0.85$ m

$$= \frac{2\pi\times 200}{340}\times\frac{85}{100} = 3.14 \text{ rad} = \pi \text{ rad}$$

104. **(a)** Ray diagram when object is in between focus and centre of curvature is given as



Nature of image is real, inverted, magnified and behind centre of curvature.

105. **(b)** We have equation of SHM as

$x_1 = A\cos\omega t, x_2 = 2A\sin\omega t$ and $x_3 = \sqrt{2}A\cos\left(\omega t + \frac{\pi}{4}\right)$

By superposition principal

$x = x_1 + x_2 + x_3$

$$= A\cos\omega t + 2A\sin\omega t + \sqrt{2A}\cos\left(\omega t + \frac{\pi}{4}\right)$$

$$= \sqrt{5}A\left(\frac{1}{\sqrt{5}}\cos\omega t + \frac{2}{\sqrt{5}}\sin\omega t\right) + \sqrt{2}A\cos\left(\omega t + \frac{\pi}{4}\right)$$

$$= \sqrt{5}A(\cos\phi\cos\omega t + \sin\phi\sin\omega t) + \sqrt{2}A\cos\left(\omega t + \frac{\pi}{4}\right)$$

$$x = \sqrt{5}A\cos(\omega t - \phi) + \sqrt{2}A\cos\left(\omega t + \frac{\pi}{4}\right) \quad ...(i)$$

Phase difference between above two waves,

$$\Delta\phi = \left(\omega t + \frac{\pi}{4}\right) - (\omega t - \phi) = \Delta\phi = \frac{\pi}{4} + \phi$$

$$\Rightarrow \cos\Delta\phi = \cos\left(\frac{\pi}{4} + \phi\right) = \cos\frac{\pi}{4}\cos\phi - \sin\frac{\pi}{4}\sin\phi$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}} \cdot \frac{2}{\sqrt{5}} = \frac{l}{\sqrt{10}} - \frac{2}{\sqrt{10}} = \frac{-1}{\sqrt{10}}$$

$$\cos\Delta\phi = -\frac{1}{\sqrt{10}} \qquad \text{...(ii)}$$

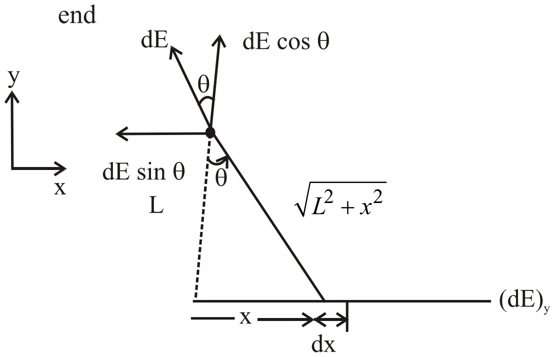
From Eq. (i), amplitude of resultant wave,

$$A = \sqrt{(\sqrt{5}A)^2 + (\sqrt{2}A)^2 + 2\sqrt{5}A \times \sqrt{2}A'\cos\phi'}$$

$$= \sqrt{5A^2 + 2A^2 + 2\sqrt{10}A^2 \times \left(\frac{-1}{\sqrt{10}}\right)} \qquad \text{[From Eq. (ii)]}$$

$$= \sqrt{5A^2} = \sqrt{5}A$$

106. (c) Let us suppose a small section 'dl' at x distance from its end



$$dq = \lambda dx$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{L^2+x^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{L^2+x^2}$$

$$(dE)_y = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx.L}{(L^2+x^2)^{3/2}} \quad [\because dE_y = dE\cos\theta]$$

Let $x = L\tan\theta$

$dx = L\sec^2\theta d\theta$

$$\therefore dE_y = \frac{1}{4\pi\epsilon_0}\frac{\lambda L \cdot L\sec^2\theta d\theta}{(L^2 + L^2\tan^2\theta)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda L^2\sec^2\theta d\theta}{(L^2\sec^2\theta)^{3/2}}$$

$$E_y = \frac{\lambda}{4\pi\epsilon_0 L}\int_0^{\pi/2}\cos\theta d\theta$$

$$= \frac{\lambda}{4\pi\epsilon_0 L}[\sin\theta]_0^{\pi/2} = \frac{\lambda}{4\pi\epsilon_0 L}$$

Similarly, $dE_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{\lambda dx}{(L^2+x^2)^{3/2}}x$

Let $L^2 + x^2 = t^2$

$x\,dx = t\,dt$

$x \to 0 \Rightarrow t = L$

So, $x \to \infty \Rightarrow t \to \infty$

$$\text{So } dE_x = \frac{1\lambda}{4\pi\epsilon_0}\frac{tdt}{t^3} = \frac{\lambda}{4\pi\epsilon_0}\frac{dt}{t^2}$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0}\left[-\frac{1}{t}\right]_L^\infty = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{t}\right]_\infty^L = \frac{\lambda}{4\pi\epsilon_0 L}$$

$$\therefore E_{net} = \sqrt{E_x^2 + E_y^2} = \frac{\lambda}{4\pi\epsilon_0}\sqrt{\frac{2}{L^2}} = \frac{\lambda}{2\sqrt{2}\pi L\epsilon_0}$$

107. (a) Given, $C_1 = 2\mu F = 2 \times 10^{-6}$ F

$C_2 = 8\mu F = 8 \times 10^{-6}$ F

V = 300 V

For series equivalent capacitance,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \frac{1}{C_{eq}} = \frac{C_1 + C_2}{C_2 \times C_2}$$

$$\Rightarrow C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{2\times10^{-6}\times8\times10^{-6}}{2\times10^{-6}+8\times10^{-6}}$$

$$= \frac{16\times10^{-12}}{10\times10^{-6}} = 1.6\times10^{-6}F$$

But $C_{eq} = \dfrac{q}{V} \Rightarrow q = C_{eq}V$

$q = 1.6\times10^{-6} \times 300 = 4.8 \times 10^{-4}$C

108. (d) As volume = constant

$\Rightarrow$ Area (A) × Length(l) = constant

$$\Rightarrow \frac{dA}{A} + \frac{dl}{l} = 0$$

$$\frac{dA}{A} = -\frac{dl}{l} \qquad \text{...(i)}$$

As $R = \dfrac{pl}{A}$

$$\Rightarrow \frac{dR}{R}\times100 = \left(\frac{dp}{p} + \frac{dl}{l} - \frac{dA}{A}\right)\times100$$

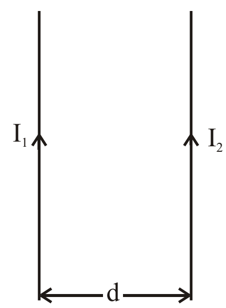
$$\Rightarrow \frac{dR}{R}\times100 = \left(0 + \frac{dl}{l} + \frac{dl}{l}\right)\times100 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow 4 = 2\frac{dl}{l} \Rightarrow \frac{dl}{l} = 2\%$$

109. (d) Given, Force per unit length $\dfrac{F}{l} = 4\times10^{-5}$ N/m

$I_1 = 0.5$A and the need to determine I_2

As we know that,



$$f = \frac{\mu_0}{2\pi}\frac{I_1I_2}{d}, \text{ where } f = \frac{F}{L}$$

$$\therefore I_2 = (f)\times\frac{d\times2\pi}{\mu_0I_1}$$

$$= 4\times10^{-5}\times\frac{2.5\times10^{-2}\times2\pi}{4\pi\times10^{-7}\times0.5}$$

$$= 2\times5\times10^{-7+7} = 10A$$

110. (b) We know that magnetic moment is given by
μ = current (I) × Area (A)

$$\Rightarrow \mu = \frac{q}{T} A \quad ...(i)$$

Substituting the value of T and A in Eq. (i), we get

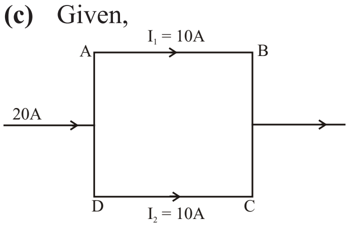
$$\mu = \frac{qv}{2\pi r} \cdot \pi r^2 = \frac{qvr}{2} \quad ...(ii)$$

and $L = m_e vr \Rightarrow vr = \frac{L}{m_e}$

Substituting in Eq. (ii), we get

$$\therefore \mu = \left(\frac{q}{2m_e}\right) L = \left(\frac{e}{2m_e}\right) L \quad [\because q = e]$$

111. (c) Given,



$AD = BC = d = 2$ cm $= 2 \times 10^{-2}$m

As resistance of both *AB* and *CD* is same and current in main branch is 20A, then current in separate branches, is $I_1 = I_2 = 10$A. We will assume *AB* and CD an infinite wire

As we know that,

$$f = \frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$$

where, (f) is force per unit length.

$$\therefore f = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{10 \times 10}{2 \times 10^{-2}}$$

$$\Rightarrow f = 2 \times 10^{-7} \times \frac{10^4}{2} = 10^{-3} Nm^{-1}$$

112. (a) As we know that,

$$I = I_0 \left(1 - e^{-\frac{t}{\tau}}\right)^7 \text{ where, } I_0 = \text{peak current}, \quad ...(i)$$

I = current in coil and τ = time constant $= \frac{L}{R}$

Since, by using Ohm's law, $V = I_0 R$

$$\Rightarrow I_0 = \frac{V}{R} = \frac{5}{50}$$

Put in Eq. (i), we get

$$\Rightarrow 50 \times 10^{-3} = \frac{5}{50}\left(1 - e^{-\frac{0.1}{\tau}}\right) \quad [\because I = 50m\text{A}, t = 0.1\text{sec.}]$$

$$\Rightarrow 5 \times 10^{-3+2} = 1 - e^{-0.1/\tau}$$

$$\Rightarrow e^{-0.1/\tau} = 1 - 0.5 = 0.5$$

Taking log both sides, we get

$$\Rightarrow -\frac{0.1}{\tau} \log_e e = \ln\frac{1}{2} \Rightarrow -\frac{0.1}{\tau} = \ln 1 - \ln 2 \Rightarrow \frac{0.1}{\tau} = \ln 2$$

$$\Rightarrow \tau = \frac{0.1}{\ln 2} = \frac{L}{R} \Rightarrow L = \frac{0.1R}{\ln 2} = \frac{0.1 \times 50}{\ln 2} = \frac{5}{\ln 2}$$

113. (a) Transformation ratio, $k = \frac{V_p}{V_s} = \frac{I_s}{I_p}$ ...(i)

as, $I_s = \frac{V_s}{R_s} = \frac{55}{275}$

From Eq. (i), we get

$$I_p = I_s \left(\frac{V_s}{V_p}\right) = \frac{55}{275}\left(\frac{55}{220}\right) = 0.05A$$

114. (a) Let λ_m is the wavelength of light in medium, whereas v_m is the speed of light in medium.
We will find refractive index of medium with respect to air,

$$\mu_{ma} = \frac{\mu_m}{\mu_a} \quad \left[\because \mu \propto \frac{1}{\lambda}\right]$$

$$\Rightarrow \frac{\mu_m}{\mu_a} = \frac{\lambda_a}{\lambda_m} \therefore \lambda_m = \frac{\lambda_a \mu_a}{\mu_m}$$

$$= 450 \times 10^{-9} \times \frac{1}{1.5} = \frac{4500}{15} \times 10^{-9} = 3 \times 10^{-7}\text{m}$$

Since, $\frac{\mu_m}{\mu_a} = \frac{c}{v_m}$

$$\Rightarrow v_m = c\frac{\mu_a}{\mu_m} \Rightarrow v_m = 3 \times 10^8 \times \frac{1}{1.5} \Rightarrow v_m = 2 \times 10^8 ms^{-1}$$

$$f_m = \frac{v_m}{\lambda} = \frac{2 \times 10^8}{3 \times 10^{-7}} = 6.67 \times 10^{14}\,\text{Hz}$$

115. (b) de-Broglie wavelength of the particle is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e E}}$$

where, λ is the wavelength and p is the momentum of the particle.

$$\therefore \lambda = \frac{4.14 \times 10^{-15}}{\sqrt{2 \times \frac{0.5 \times 10^6}{c^2} \times 100}} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{\sqrt{2 \times 0.5 \times 10^8}}$$

$= 4.14 \times 10^{-15+8-4} \times 3 = 1242 \times 10^{-11}$
$= 124.2 \times 10^{-12} = 124.2$ pm

116. (b) $\Delta E_n = +13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$

In Balmer series, $n_f = 2$
ΔE_n will be maximum when n_i is ∞

So, $(\Delta E)_{\max} = 13.6\left(\frac{1}{\infty^2} - \frac{1}{2^2}\right) = -3.4eV$

So this can be maximum energy of photon. So metal can have work function of maximum 3.4eV.

117. (c) Radius of nuclei is given by

$R = R_0 A^{1/3} \Rightarrow R \propto A^{1/3}$ ...(i)

where, R_0 is radius of nucleus and A is mass number.

Since, volume $V = \frac{4}{3}\pi R^3$ ...(ii)

$\Rightarrow R \propto (V)^{1/3}$

Now, from Eqs. (i) and (ii), we get

$R \propto A^{1/3} \propto V^{1/3} \Rightarrow A \propto V$

$\Rightarrow \frac{A_1}{A_2} = \frac{V_1}{V_2} \Rightarrow \frac{M_1}{M_2} = \frac{V_1}{V_2}$

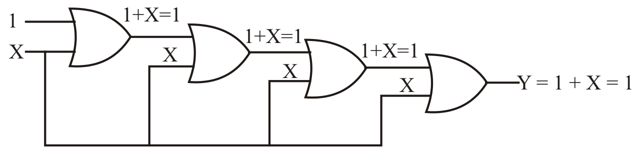
Suppose $M_1 = M$ and $M_2 = 2M$

$\Rightarrow \frac{M}{2M} = \frac{V}{V_2} \Rightarrow V_2 = 2V$

118. (a) We have,
Intrinsic charge carrier, $n_i = 2.5\times10^{19}\text{m}^{-3}$,
Mobility of hole, $\mu_p = 0.15\ \text{m}^2\text{V}^{-1}\ \text{s}^{-1}$
and mobility of electron, $\mu_n = 0.35\text{m}^2\text{V}^{-1}\text{s}^{-1}$
As we know that, conductivity is given by
$\sigma = n_i e(\mu_n + \mu_p)$
$= 2.5 \times 10^{19} \times 1.6 \times 10^{-19}(0.15 + 0.35)$
$= 2.5 \times 1.6 \times 0.50 = 4 \times 0.5 = 2\ \Omega^{-1}\text{m}^{-1}$

Resistivity, $\rho = \frac{1}{\sigma} = \frac{1}{2} = 0.5\ \Omega - m = 50\ \Omega - cm$

119. (b) We can see the lower input of all the OR gate is same. So lower input all OR gate is X.



According to given diagram, there are 4 OR gates and in case of OR gate
$1 + X = 1$
$\therefore Y = 1$

120. (d) If R_e is radius of earth and R is radius of area of population

Then, height of antenna, $H = \frac{R^2}{2R_e}$

$\because [R = 64\,\text{km}, R_e = 6400\text{km}]$

$\therefore H = \frac{64\times64}{2\times6400} = \frac{64}{200} = 0.32\text{km} = 320\text{m}$

CHEMISTRY

121. (a) $\Delta x \cdot \Delta P \geq \frac{h}{4\pi} \Rightarrow \Delta x \cdot m \cdot \Delta v \geq \frac{h}{4\pi}$

$m = 9.11\times10^{-31}$ kg; $h = 6.626 \times 10^{-34}$ Js
$\Delta x = 0.001\text{nm} = 10^{-12}$ m

$\Delta v \geq \frac{h}{4\pi\times m\times\Delta x} \Rightarrow \Delta v \geq \frac{6.626\times10^{-34}}{4\times3.14\times9.11\times10^{-31}\times10^{-12}}$

$\Delta v \geq \frac{6.626\times10^{-34}}{114.4216\times10^{-43}} \Rightarrow \Delta v \geq 5.79\times10^7$ m/s

122. (b) $4p$ orbital has one angular node and three maxima in its radial probability distribution curve.
Number of angular node
$= l =$ azimuthal quantum number $= 1$
(for p-orbital, $l = 1$)
$\because$ For 3 maxima, the no. of radial nodes will be 2.
$\therefore n - l - 1 = 2 \Rightarrow n - 1 - 1 = 2$
$\Rightarrow n = 4$, Thus, the orbital is $4p$.

123. (b) The IE values are 5.98, 18.8, 28.4, 120.1...
There is sudden jump from 3rd IE and 4th IE. So the element has 3 valence e^-, i.e. Al.

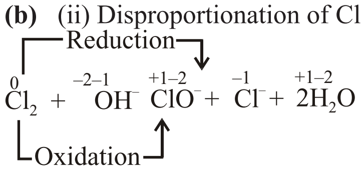
124. (b) Number of radial nodes $= n - l - 1$
Number of angular nodes $= l$
For $6p_z$ orbital $n = 6$, $l = 1$
Angular node $= l = 1$
Radial nodes $= n - l - 1$
$= 6 - 1 - 1 = 4$
Hence, orbital with 4 radial and 1 angular node is $6p_z$.

125. (c) Cu is coordinated with 4 water molecules and two more oxygen atoms from sulphate ion.
Fifth water molecule is hydrogen-bonded with other water molecules.
Hence, 4 water molecules are coordinated and the fifth one is the only hydrogen bonded.

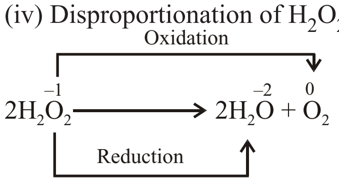
126. (a) 1 Debye $= 10^{-10}$ esu $\times 10^{-8}$ C-m
$= 10^{-10} \times 3.335 \times 10^{-10}\text{C} \times 10^{-8} \times 10^{-2}\text{m}$
$= 3.335 \times 10^{-30}$ C-m.

127. (b) 128. (c)

129. (b) (ii) Disproportionation of Cl



(iv) Disproportionation of H_2O_2.



130. (a) $2KMnO_4 + 5H_2C_2O_4 + 3H_2SO_4 \rightarrow 2MnSO_4 + 10CO_2 + K_2SO_4 + 8H_2O$
According to balanced chemical equation, 2 moles of $KMnO_4$ produced 10 moles of CO_2, hence 1 mole of $KMnO_4$ produced 5 moles of CO_2.

131. (a) Entropy, $S = Q/T$ and $Q = mCT$
Thus, we can see that entropy (S) depends on mass (m) and hence, is an extensive property.

132. (b) $\log\frac{K_2}{K_1} = \frac{-\Delta H}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

Substituting the values in above equation

$\log\frac{0.6\times10^{-4}}{1.2\times10^{-4}} = \frac{-\Delta H}{2.303\times2}\left[\frac{1}{300} - \frac{1}{400}\right]$

$\Rightarrow \log 0.5 = -\frac{\Delta H}{4.606}\left[\frac{100}{120000}\right] \Rightarrow \Delta H = -1663.69$

Mean heat of formation

$= \dfrac{\Delta H}{2} = \dfrac{-1663.69}{2} \approx -826.43\,\text{cal}$

133. (b) $Q_c = \dfrac{[B][C]}{[A]^2}$

As, [A] = [B] = [C]

$Q_c = \dfrac{[A][A]}{[A]^2} = 1 \Rightarrow \because Q_c > K_c$

$\therefore$ The reaction will proceed in the reverse direction.

134. (c) (A) $\underset{\text{Sulphur trioxide}}{SO_3} + \underset{\text{Heavy water}}{D_2O} \rightarrow \underset{\text{Deutero sulphuric acid}}{D_2SO_4}$

(B) $\underset{\text{Calcium carbide}}{CaC_2} + 2D_2O \rightarrow \underset{\text{Calcium deuteroxide}}{Ca(OD)_2} + \underset{\text{Deutero acetylene}}{C_2D_2}$

(C) $\underset{\text{Aluminium carbide}}{Al_4C_3} + \underset{\text{Heavy water}}{12\,D_2O} \rightarrow \underset{\text{Aluminium deuteroxide}}{4\,Al(OD)_3} + \underset{\text{Deutero methane}}{3CD_4}$

135. (c) $2\,Be + O_2 \rightarrow \underset{\text{(beryllium oxide)}}{2\,BeO}$

$3\,Be + N_2 \rightarrow \underset{\text{(beryllium nitride)}}{Be_3N_2}$

136. (d) (A) $\underset{\text{Diborane}}{B_2H_6} + \underset{\text{Carbonmonoxide}}{2\,CO} \rightarrow \underset{\text{Borane carbonmonoxide}}{2\,BH_3 \cdot CO}$

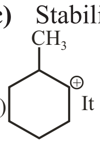
(B) $\underset{\text{Diborane}}{B_2H_6} + \underset{\text{Oxygen}}{3O_2} \rightarrow \underset{\text{Borane trioxide}}{B_2O_3} + 3H_2O$

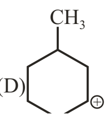
(C) $\underset{\text{Diborane}}{B_2H_6} + \underset{\text{Water}}{6H_2O} \rightarrow \underset{\text{Orthoboric acid}}{2\,H_3BO_3} + \underset{\text{Hydrogen}}{6\,H_2}$

137. (a) $C_6H_{12}O_6 \xrightarrow{\text{Yeast}} 2C_2H_5OH + 2\,CO_2$

138. (a)

139. (c) Stability of carbocation decreases as 3° > 2° > 1°.

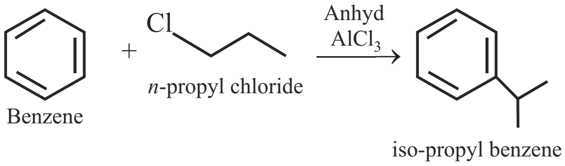
(c)  It is 2° carbocation and it has 3α-H

(D)  It is also 2° carbocation but it has 4α-H. It is more stable than (C) due to more hyperconjugation

So, the order is B > D > C > A.

140. (c)

141. (b) This is Friedel-Craft's alkylation reaction in which *n*-propyl carbocation rearranges to form isopropyl benzene.



142. (c) In a simple cube,

Number of atoms present = 1/8 × 8 = 1

Hence, volume occupied $= 1 \times \dfrac{4}{3}\pi r^3$

So, fraction of total volume occupied $= \dfrac{\frac{4}{3}\pi r^3}{a^3}$

$= \dfrac{\frac{4}{3} \times \pi \times \left(\frac{a}{2}\right)^3}{a^3} = \dfrac{4}{3} \times \pi \times \dfrac{1}{8} = \dfrac{\pi}{6}$

143. (c)

144. (c) $T_b = 100.5°C$; $K_b = 0.52$ K kg mol^{-1}

$K_f = 1.87$ K kg mol^{-1}

$\Delta T_b = 100.5 - 100 = 0.5°C$

$\Delta T_b = K_b \times$ molality $\Rightarrow$ Molality $= \Delta T/K_b = 0.5/0.52$

$\Delta T_f = K_f \times$ molality $\Rightarrow = 1.87 \times 0.5/0.52 = 1.80$

$\therefore$ Freezing point of solution = 0 – 1.80 = –1.80°C

145. (c) $\dfrac{\text{Mass of metal}_1\text{ deposited}}{\text{Mass of metal}_2\text{ deposited}} = \dfrac{\text{Eq. wt. of metal}_1}{\text{Eq. wt. of metal}_2}$

The equivalent weight of silver is highest among the given options.

146. (a) $\log\left(\dfrac{k_2}{k_1}\right) = \dfrac{E_a}{2.303R}\left[\dfrac{1}{T_1} - \dfrac{1}{T_2}\right]$

or, $\log\left(\dfrac{2k}{k}\right) = \dfrac{E_a}{2.303 \times 8.3} \times \dfrac{[600-300]}{300 \times 600}$

$E_a \approx 3.45\,\text{kJ/mol}$

147. (b) Sb_2S_3 sol is negatively charged sol. thus it will be coagulated by positively charged ions.

A^{3+} ion has maximum charge among the given electrolytes, so it is the most effective coagulating agent.

148. (a) The reducing character of the hydrides of group V elements depends upon the bond energy of hydrides. With decrease in bond energy, the reducing character of hydrides increases as we move down the group. The order of reducing abilities of V group hydrides is

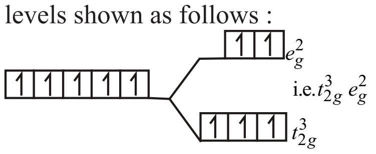
$NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$

149. (c) $SO_2 + Cl_2 \rightarrow \underset{\text{Sulphuryl chloride}}{SO_2Cl_2}$

150. (a) The bond dissociation enthalpy of F_2 is lower than Cl_2 and Br_2. It is due to the presence of lone pair of electron on fluorine atom which creates greater repulsion due to small size of fluorine. Hence, the bond dissociation enthalpy order is :

$I_2 < F_2 < Br_2 < Cl_2$

151. (c) The magnetic moment of 5.92 BM corresponds to the presence of five unpaired electrons in the *d*-orbitals. 5 unpaired electrons will go into t_{2g} and e_g energy levels shown as follows :



152. (c) $[\overset{x}{Mn}Cl_6]^{3-}$

$3d^4$

(Cl is a weak ligand)

$x + 6(-1) = -3 \Rightarrow x = +3$

Mn = [Ar] $3d^5 4s^2$,

Mn^{3+} = [Ar] $3d^4$

There are 4 unpaired electrons.

CN is strong field ligand.

$[\overset{x}{F_e}(CN)_6]^{3-}$

$3d^5$

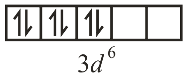
$x + 6(-1) = -3 \Rightarrow x = +3$

$Fe = [Ar]\, 3d^6 4s^2 \Rightarrow Fe^{3+} = [Ar]\, 3d^5$

1 unpaired electron in Fe^{3+} C_2O_4 is strong field ligand.

$[Co(C_2O_4)_3]^{3-}$

$x + 3(-2) = -3 \Rightarrow x = +3$



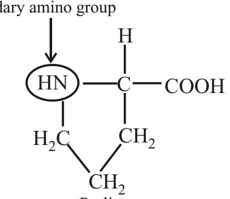
$Co = [Ar]\, 3d^7 4s^2 \Rightarrow Co^{3+} = [Ar]\, 3d^6$

Hence, 0 unpaired electron.

153. **(b)** In aqueous solutions, amino acids mostly exist as $H_3N^+ - CHR - COO^-$ (Zwitter ion)

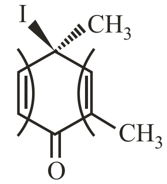
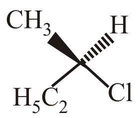
Proline is a natural amino acid which has a secondary amino group alpha to the carboxyl group.

Secondary amino group



Proline

154. **(d)**

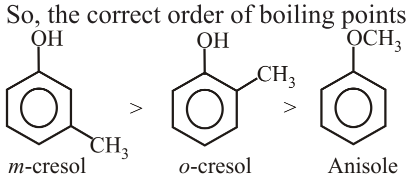
(iv)  , (vi) 

All groups are different (no symmetry), so they are chiral.

155. **(d)** Alcohols have more boiling point compared to ethers due to hydrogen bonding.

Due to effective intermolecular H-bonding in *m*-isomer, it has more boiling point.

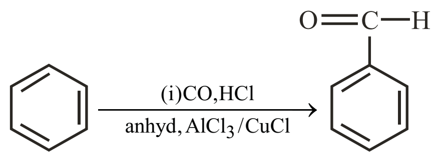
So, the correct order of boiling points is



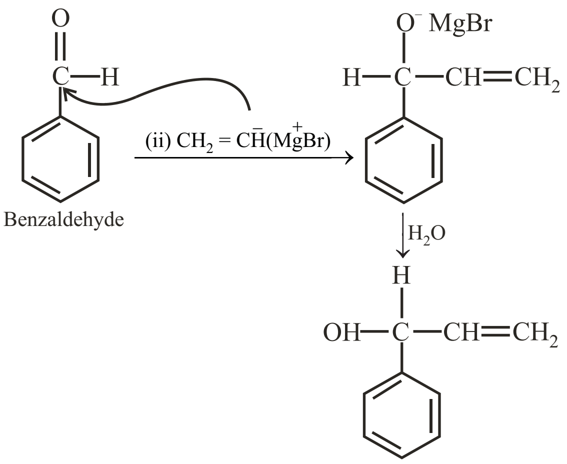
m-cresol > *o*-cresol > Anisole

156. **(c)**

157. **(d)**

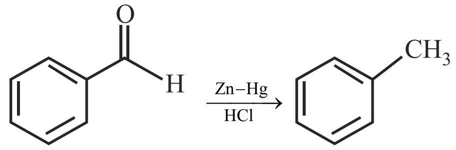


Benzaldehyde



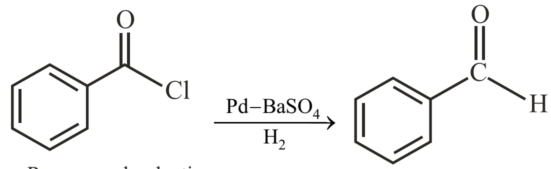
Benzaldehyde

158. **(a)** A.



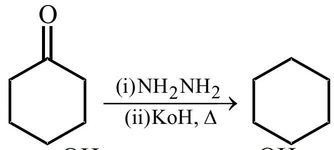
Clemmensen reduction

B.

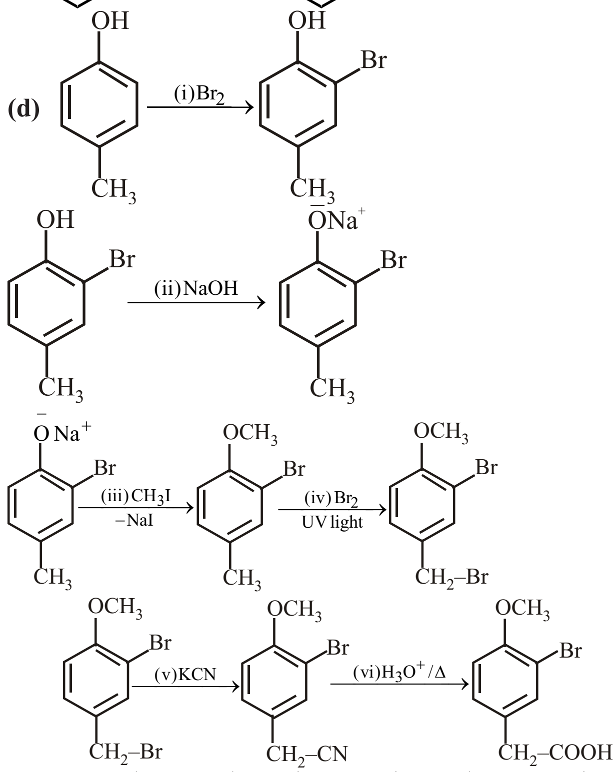


Rosenmund reduction

C.



159. **(d)**



160. **(d)** Secondary and tertiary amines do not give carbylamine reaction test.