AP/EAPCET Solved Paper 2019
Held on April 20

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. $f(x)=\frac{x}{e^x-1}+\frac{x}{2}+2\cos^3\frac{x}{2}$ on $R-\{0\}$ is
(a) one one function (b) bijection
(c) algebraic function (d) even function

2. Consider the following lists.

	List-I		**List-II**
(A)	$f(x)=\frac{\lvert x+2\rvert}{x+2}, x\neq -2$	1.	$\left[\frac{1}{3},1\right]$
(B)	$g(x)=\lvert[x]\rvert, x\in R$	2.	Z
(C)	$h(x)=\lvert x-[x]\rvert, x\in R$	3.	W
(D)	$f(x)=\frac{1}{2-\sin 3x}, x\in R$	4.	$[0, 1)$
		5.	$\{-1, 1\}$

	A	B	C	D
(a)	5	3	2	1
(b)	3	2	4	1
(c)	5	3	4	1
(d)	1	2	3	4

3. **Assertion (A) :** $(1)+(1+2+4)+(4+6+9)+(9+12+16)+\ldots+(81+90+100)=1000$

Reason (R) : $\sum_{r=1}^{n}\left(r^3-[r-1]^3\right)=n^3$ for any natural number n
(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true but (R) is not correct explanation of (A)
(c) (A) is true but (R) is false
(d) (A) is false but (R) is true

4. If $A=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}, P=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $X=APA^T$, then $A^TX^{50}A=$
(a) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}2 & 1\\ 0 & -1\end{bmatrix}$
(c) $\begin{bmatrix}25 & 1\\ 1 & -25\end{bmatrix}$ (d) $\begin{bmatrix}1 & 50\\ 0 & 1\end{bmatrix}$

5. If $[x]$ is the greatest integer less than or equal to x and $|x|$ is the modulus of x, then the system of three equations $2x+3|y|+5[z]=0$, $x+|y|-2[z]=4$, $x+|y|+[z]=1$ has
(a) a unique solution (b) finitely many solutions
(c) infinitely many solutions (d) no solution

6. Investigate the values of λ and μ for the system $x+2y+3z=6$, $x+3y+5z=9$, $2x+5y+\lambda z=\mu$ and match the values in List-I with the items in List-II.

List-I	**List-II**
(A) $\lambda=8, \mu\neq 15$	1. Infinitely many solutions
(B) $\lambda\neq 8, \mu\in R$	2. No solution
(C) $\lambda=8, \mu=15$	3. Unique solution

	A	B	C
(a)	1	2	3
(b)	2	3	1
(c)	3	1	2
(d)	3	2	1

7. If $z=x+iy$, $x, y\in R$, $(x, y)\neq(0, -4)$ and $\arg\left(\frac{2z-3}{z+4i}\right)=\frac{\pi}{4}$, then the locus of z is
(a) $2x^2+2y^2+5x+5y-12=0$
(b) $2x^2-3xy+y^2+5x+y-12=0$
(c) $2x^2+3xy+y^2+5x+y+12=0$
(d) $2x^2+2y^2-11x+7y-12=0$

8. If $z=x+iy$, $x, y\in R$ and the imaginary part of $\frac{\bar{z}-1}{\bar{z}-i}$ is 1, then the locus of z is
(a) $x+y+1=0$
(b) $x+y+1=0, (x, y)\neq(0, -1)$
(c) $x^2+y^2-x+3y+2=0$
(d) $x^2+y^2-x+3y+2=0, (x, y)\neq(0, -1)$

9. If ω represents a complex cube root of unity, then $\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+\ldots+\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)=$

(a) $\dfrac{n\left(n^2+1\right)}{3}$ (b) $\dfrac{n\left(n^2+2\right)}{3}$

(c) $\dfrac{n\left(n^2-2\right)}{3}$ (d) $\dfrac{n^2\left(n-1\right)}{6}$

10. If ω is a complex cube root of unity, then

$\sum_{r=1}^{9} r\left(r+1-\omega\right)\left(r+1-\omega^2\right)=$

(a) 5025 (b) 4020 (c) 2016 (d) 3015

11. If α and β are the roots of $x^2 + 7x + 3 = 0$ and $\dfrac{2\alpha}{3-4\alpha}, \dfrac{2\beta}{3-4\beta}$ are the roots of $ax^2 + bx + c = 0$ and GCD of a, b, c is 1, then $a + b + c =$

(a) 11 (b) 0 (c) 243 (d) 81

12. If α, β are the roots of $x^2 + bx + c = 0$, γ, δ are the roots of $x^2 + b_1x + c_1 = 0$ and $\gamma < \alpha < \delta < \beta$, then $(c - c_1)^2 <$

(a) $(b_1 - b)(bc_1 - b_1c)$ (b) 1

(c) $(b - b_1)^2$ (d) $(c - c_1)(b_1c - b_1c_1)$

13. Let a, b and c be the sides of a scalane triangle. If λ is a real number such that the roots of the equation $x^2 + 2(a + b + c)\,x + 3\lambda\,(ab + bc + ca) = 0$ are real, then the interval in which λ lies is

(a) $\left(-\infty, \dfrac{4}{3}\right)$ (b) $\left(\dfrac{5}{3}, \infty\right)$ (c) $\left(\dfrac{1}{3}, \dfrac{5}{3}\right)$ (d) $\left(\dfrac{4}{3}, \infty\right)$

14. The polynomial equation of degree 4 having real coefficients with three of its roots as $2 \pm \sqrt{3}$ and $1 + 2i$, is

(a) $x^4 - 6x^3 - 14x^2 + 22x + 5 = 0$

(b) $x^4 - 6x^3 - 19x + 22x - 5 = 0$

(c) $x^4 - 6x^3 + 19x - 22x + 5 = 0$

(d) $x^4 - 6x^3 + 14x^2 - 22x + 5 = 0$

15. All the letters of the word ANIMAL are permuted in all possible ways and the permutations thus formed are arranged in dictionary order. If the rank of the word ANIMAL is x, then the permutation with rank x, among the permutation obtained by permuting the letters of the word PERSON and arranging the permutations thus formed in dictionary order is

(a) ENOPRS (b) NOSPRE

(c) NOEPRS (d) ESORNP

16. A student is allowed to choose atmost n books from a collection of $2n + 1$ books. If the total number of ways in which he can select atleast one book is 255, then the value of n is

(a) 4 (b) 5 (c) 6 (d) 7

17. The sum of all the coefficient in the binomial expansion of $(1 + 2x)^n$ is 6561. Let $R = (1 + 2x)^n = I + F$ where $I \in N$ and $0 < F < 1$. If $x = \dfrac{1}{\sqrt{2}}$, then $1 - \dfrac{F}{1+\left(\sqrt{2}-1\right)^4} =$

(a) $\left(3\sqrt{2}-4\right)$ (b) $4\left(3\sqrt{2}+4\right)$

(c) $\left(\sqrt{2}-1\right)^4$ (d) 1

18. If $\dfrac{\left(1-px\right)^{-1}}{\left(1-qx\right)} = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots\ldots$, then $a_n =$

(a) $\dfrac{p^{n+1}-q^{n+1}}{q-p}$ (b) $\dfrac{p^{n+1}-q^{n+1}}{p-q}$

(c) $\dfrac{p^n-q^n}{q-p}$ (d) $\dfrac{p^n-q^n}{p-q}$

19. If $\dfrac{3}{\left(x-1\right)\left(x^2+x+1\right)} = \dfrac{1}{x-1} - \dfrac{x+2}{x^2+x+1} = f_1(x) - f_2(x)$

$\dfrac{x+1}{\left(x-1\right)^2\left(x^2+x+1\right)} = Af_1\left(x\right) + \left(B + \dfrac{D}{x-1}\right)$

$f_2\left(x\right) + \dfrac{C}{\left(x-1\right)^2}\; A + B + C + D =$

(a) 1 (b) $\dfrac{-1}{3}$ (c) 0 (d) $\dfrac{1}{3}$

20. Let M and m respectively denote the maximum and the minimum values of $\left[f\left(\theta\right)\right]^2$, where $f\left(\theta\right) = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$. Then $M - m =$

(a) $a^2 + b^2$ (b) $(a - b)^2$ (c) $a^2 b^2$ (d) $(a + b)^2$

21. If $\cos A = \dfrac{-60}{61}$ and $\tan B = -\dfrac{7}{24}$ and neither A nor B is the second quadrant, then the angle $A + \dfrac{B}{2}$ lies in the quadrant

(a) 1 (b) 2 (c) 3 (d) 4

22. $\cos^2 5° - \cos^2 15° - \sin^2 15° + \sin^2 35° + \cos 15° \sin 15° - \cos 5° \sin 35° =$

(a) 0 (b) 1 (c) $\dfrac{3}{2}$ (d) 2

23. If $\cos\theta \neq 0$ and $\sec\theta - 1 = \left(\sqrt{2}-1\right)\tan\theta$, then $\theta =$

(a) $n\pi + \dfrac{\pi}{8}, n \in Z$ (b) $2n\pi + \dfrac{\pi}{4}$ or $2n\pi$, $n \in Z$

(c) $2n\pi + \dfrac{\pi}{8}$, $n \in Z$ (d) $2n\pi - \dfrac{\pi}{4}$, or $2n\pi$, $n \in Z$

24. $\cot\left[\sum_{n=3}^{32} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right] =$

(a) $\dfrac{10}{3}$ (b) $\dfrac{8}{3}$ (c) $\dfrac{14}{3}$ (d) $\dfrac{16}{3}$

25. If $\sin x \cos h\, y = \cos\theta$, $\cos x \sin h\, y = \sin\theta$ and $4 \tan x = 3$. Then, $\sin h^2 y =$

(a) $\dfrac{4}{5}$ (b) $\dfrac{9}{16}$ (c) $\dfrac{9}{25}$ (d) $\dfrac{16}{25}$

26. In triangle ABC, if $\frac{b+c}{9}=\frac{c+a}{10}=\frac{a+b}{11}$, then $\frac{\cos A+\cos B}{\cos C}=$

(a) $\frac{9}{10}$ (b) $\frac{10}{11}$ (c) $\frac{11}{12}$ (d) $\frac{12}{13}$

27. In a ΔABC, with usual notation, match the items in List-I with the items in List-II and choose the correct option,

List-I	List-II
(A) $r_1 r_2\sqrt{\left(\frac{4R-r_1-r_2}{r_1+r_2}\right)}$	1. b
(B) $\frac{r_2(r_3+r_1)}{\sqrt{r_1r_2+r_2r_3+r_3r_1}}$	2. a^2, b^2, c^2 are in AP
(C) $\frac{a}{c}=\frac{\sin(A-B)}{\sin(B-C)}$	3. Δ
(D) $bc\cos^2\frac{A}{2}$	4. $R\,r_1 r_2 r_3$
	5. $s(s-a)$

	A	B	C	D
(a)	4	3	1	5
(b)	5	4	3	2
(c)	3	1	2	5
(d)	4	5	2	1

28. If a, b and c are the sides of ΔABC for which $r_1 = 8$, $r_2 = 12$ and $r_3 = 24$, then the ordered triad $(a, b, c) =$

(a) (12, 20, 16) (b) (12, 16, 20)
(c) (16, 12, 20) (d) (20, 16, 12)

29. If $4\hat{\mathbf{i}}+7\hat{\mathbf{j}}+8\hat{\mathbf{k}}, 2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}}, 2\hat{\mathbf{i}}+5\hat{\mathbf{j}}+7\hat{\mathbf{k}}$ are respectively the position vectors of the vertices A, B, C of ΔABC, then the position vector of the point where the bisector of angle A meet BC is

(a) $2\hat{\mathbf{i}}+\frac{13}{3}\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ (b) $2\hat{\mathbf{i}}-\frac{13}{3}\hat{\mathbf{j}}+6\hat{\mathbf{k}}$

(c) $2\hat{\mathbf{i}}+13\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ (d) $2\hat{\mathbf{i}}+\frac{13}{3}\hat{\mathbf{j}}+6\hat{\mathbf{k}}$

30. The equation of the plane passing through the point $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and perpendicular to the line of intersection of the planes $\mathbf{r}.(3\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}})=1$ and $\mathbf{r}.(\hat{\mathbf{i}}+4\hat{\mathbf{j}}-2\hat{\mathbf{k}})=2$, is

(a) $\mathbf{r}.(-2\hat{\mathbf{i}}-5\hat{\mathbf{j}}+\hat{\mathbf{k}})=0$ (b) $\mathbf{r}.(\hat{\mathbf{i}}+7\hat{\mathbf{j}}+4\hat{\mathbf{k}})=0$

(c) $\mathbf{r}.(2\hat{\mathbf{i}}-7\hat{\mathbf{j}}-13\hat{\mathbf{k}})=1$ (d) $\mathbf{r}.(-2\hat{\mathbf{i}}+7\hat{\mathbf{j}}+13\hat{\mathbf{k}})=0$

31. If the position vectors of the vertices A, B and C of ΔABC are $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-5\hat{\mathbf{k}}, -2\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ respectively, then $\angle B =$

(a) $\cos^{-1}\left(\frac{7}{3\sqrt{10}}\right)$ (b) $\cos^{-1}\left(\frac{8}{\sqrt{105}}\right)$

(c) $\cos^{-1}\left(\frac{1}{\sqrt{42}}\right)$ (d) $\cos^{-1}\left(-\frac{7}{3\sqrt{10}}\right)$

32. If the position vectors of the vertices of a ΔABC are $\mathbf{OA}=3\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$, $\mathbf{OB}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $\mathbf{OC}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}}$ then the length of the altitude of ΔABC drawn from A is

(a) $\sqrt{\frac{3}{2}}$ (b) $\frac{3}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{3}{2}$

33. A new tetrahedron is formed by joining the centroids of the faces of a given tetrahedron $OABC$. Then the ratio of the volume of the new tetrahedron to that of the given tetrahedron is

(a) $\frac{3}{25}$ (b) $\frac{1}{27}$ (c) $\frac{5}{62}$ (d) $\frac{1}{162}$

34. Let $\mathbf{A}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $\mathbf{B}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$. If C is a vector such that $\mathbf{A}\cdot\mathbf{C}=|\mathbf{C}|$, $|\mathbf{C}-\mathbf{A}|=2\sqrt{2}$ and the angle between A × B and C is 30°, then the value of $|(\mathbf{A}\times\mathbf{B})\times\mathbf{C}|$ is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) 2

35. If $a_0, a_1, ..., a_{11}$ are in an arithmetic progression with common difference d, then their mean deviation from their arithmetic mean is

(a) $\frac{30}{11}|d|$ (b) 2 |d| (c) 3 |d| (d) 12 |d|

36. The variance of the following continuous frequency distribution is

Class Interval	0-10	10-20	20-30	30-40
Frequency	2	3	4	1

(a) 201 (b) 62 (c) 19 (d) 84

37. If two sections of strengths 30 and 45 are formed from 75 students who are admitted in a school, then the probability that two particular students are always together in the same section is

(a) $\frac{66}{185}$ (b) $\frac{19}{37}$ (c) $\frac{29}{185}$ (d) $\frac{18}{37}$

38. A bag contains $2n$ coins out of which $n-1$ are unfair with heads on both sides and the remaining are fair. One coin is picked from the bag at random and tossed. If the probability that head falls in the toss is $\frac{41}{56}$, then the number of unfair coins in the bag is

(a) 18 (b) 15 (c) 13 (d) 14

39. Bag A contains 6 green and 8 red balls and bag B contains 9 green and 5 red balls. A card is drawn at random from a well shuffled pack of 52 playing cards. If it is a spade, two balls are drawn at random from bag A, otherwise two balls are drawn at random from bag B. If the two balls drawn are found to be of the same colour then the probability that they are drawn from bag A is

(a) $\frac{43}{181}$ (b) $\frac{1}{4}$ (c) $\frac{48}{131}$ (d) $\frac{43}{138}$

40. A random variable X has the probability distribution

$X=x_i$	1	2	3	4	5	6
$P(X=x_i)$	0.2	0.3	0.12	0.1	0.2	0.08

If $A = \{x_i / x_i$ is a prime number$\}$,
$B = \{x_i / x_i < 4\}$ are two events, then
$P(A \cup B) =$

(a) 0.31 (b) 0.62 (c) 0.82 (d) 0.41

41. In a poisson distribution with unit mean, $\sum_{x=0}^{\infty}|x-\bar{x}|P(X=x) =$ ($\bar{x}$ is the mean of the distribution)

(a) e (b) $\frac{1}{e}$ (c) $\frac{2}{e}$ (d) $\frac{2}{3e}$

42. Two straight rods of lengths $2a$ and $2b$ move along the coordinate axes in such a way that their extremities are always concyclic. Then the locus of the centres of such circles is

(a) $2(x^2 + y^2) = a^2 + b^2$ (b) $2(x^2 - y^2) = a^2 + b^2$
(c) $x^2 + y^2 = a^2 + b^2$ (d) $x^2 - y^2 = a^2 - b^2$

43. When the coordinate axes are rotated about the origin in the positive direction through an angle $\frac{\pi}{4}$, if the equation $25x^2 + 9y^2 = 225$ is transformed to $\alpha x^2 + \beta xy + \gamma y^2 = \delta$, then $\left(\alpha+\beta+\gamma-\sqrt{\delta}\right)^2 =$

(a) 3 (b) 9 (c) 4 (d) 16

44. The equation of the line through the point of intersection of the lines $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$ and making equal non-zero intercepts on the coordinate axes is

(a) $2x + 2y = 3$ (b) $23x + 23y = 6$
(c) $23x + 23y = 11$ (d) $2x + 2y = 7$

45. The line through $P(a, 2)$, where $a \neq 0$, making an angle 45° with the positive direction of the X-axis meets the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$ at A & D and the coordinate axes at B and C. If PA, PB, PC and PD are in a geometric progression, then $2a =$

(a) 13 (b) 7 (c) 1 (d) –13

46. The equation of the perpendicular bisectors of the sides AB and AC of a ΔABC are $x - y + 5 = 0$ and $x + 2y + 5 = 0$, respectively. If A is $(1, -2)$, then the equation of the straight line BC is

(a) $14x + 23y - 40 = 0$ (b) $12x + 17y - 28 = 0$
(c) $14x - 29y - 30 = 0$ (d) $7x - 12y + 15 = 0$

47. If each line of a pair of lines passing through origin is at a perpendicular distance of 4 units from the point (3, 4), then the equation of the pair of lines is

(a) $7x^2 + 24xy = 0$ (b) $7y^2 + 24xy = 0$
(c) $7y^2 - 24xy = 0$ (d) $7x^2 - 24xy = 0$

48. Variable straight lines $y = mx + c$ make intercepts on the curve $y^2 - 4ax = 0$ which subtend a right angle at the origin. Then the point of concurrence of these lines $y = mx + c$ is

(a) $(4a, 0)$ (b) $(2a, 0)$ (c) $(-4a, 0)$ (d) $(-2a, 0)$

49. The abscissae of two points P, Q are the roots of the equation $2x^2 + 4x - 7 = 0$ and their ordinates are the roots of the equation $3x^2 - 12x - 1 = 0$. Then the centre of the circle with PQ as a diameter is

(a) (–1, 2) (b) (–2, 6) (c) (1, –2) (d) (2, –6)

50. If the angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α, then the equation of the locus of P is

(a) $x^2 + y^2 + 4x - 6y + 4 = 0$
(b) $x^2 + y^2 + 4x - 6y - 9 = 0$
(c) $x^2 + y^2 - 4x + 6y - 4 = 0$
(d) $x^2 + y^2 + 4x - 6y + 9 = 0$

51. The equation of the circle whose radius is 3 and which touches internally the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ at the point (–1, –1) is

(a) $5x^2 + 5y^2 + 9x - 6y - 7 = 0$
(b) $5x^2 + 5y^2 - 8x - 14y - 32 = 0$
(c) $5x^2 + 5y^2 - 6x + 8y - 8 = 0$
(d) $5x^2 + 5y^2 + 6x - 8y - 12 = 0$

52. Suppose that the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ has its centre on $2x + 3y - 7 = 0$ and cuts the circles $x^2 + y^2 - 4x - 6y + 11 = 0$ and $x^2 + y^2 - 10x - 4y + 21 = 0$ orthogonally. Then $5g - 10f + 3c =$

(a) 0 (b) 1 (c) 3 (d) 9

53. If the radical axis of the circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $2x^2 + 2y^2 + 3x + 8y + 2c = 0$ touches the circle $x^2 + y^2 + 2x + 2y + 1 = 0$, then $(4g - 3)(f - 2) =$

(a) 0 (b) –1 (c) 1 (d) 2

54. The parabola $x^2 = 4ay$ makes an intercept of length $\sqrt{40}$ units on the line $y = 1 + 2x$, then a value of $4a$ is

(a) 2 (b) –2 (c) –1 (d) 4

55. The locus of the points of intersection of perpendicular normals to the parabola $y^2 = 4ax$ is

(a) $y^2 - 2ax + a^2 = 0$ (b) $y^2 + ax + 2a^2 = 0$
(c) $y^2 - ax + 2a^2 = 0$ (d) $y^2 - ax + 3a^2 = 0$

56. P is a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with foci F_1 and F_2. If A is the area of the triangle PF_1F_2, then the maximum value of A is

(a) $\frac{e}{ab}$ (b) $\frac{ae}{b}$ (c) aeb (d) $\frac{ab}{e}$

57. If the line joining the points $A(\alpha)$ and B(β) on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ is a focal chord, then one possible value of $\cot\frac{\alpha}{2}\cdot\cot\frac{\beta}{2}$ is

(a) –3 (b) 3 (c) –9 (d) 9

58. The equation of a tangent to the hyperbola $16x^2 - 25y^2 - 96x + 100y - 356 = 0$ which makes an angle 45° with its transverse axis is

(a) $x - y + 2 = 0$ (b) $x - y + 4 = 0$
(c) $x + y + 2 = 0$ (d) $x + y + 4 = 0$

59. If $P(0, 7, 10)$, Q(–1, 6, 6) and $R(-4, 9, 6)$ are three points in the space, then PQR is

(a) right angled isosceles triangle
(b) equilateral triangle
(c) isosceles but not right angled triangle
(d) scalene triangle

60. $A(2, 3, 5)$, $B(\alpha, 3, 3)$ and $C(7, 5, \beta)$ are the vertices of a triangle. If the median through A is equally inclined with the co-ordinate axes, then $\cos^{-1}\left(\frac{\alpha}{\beta}\right) =$

(a) $\cos^{-1}\left(\frac{-1}{9}\right)$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\cos^{-1}\left(\frac{2}{5}\right)$

61. The plane $3x + 4y + 6z + 7 = 0$ is rotated about the line $\mathbf{r} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) + t(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}})$ until the plane passes through origin. The equation of the plane in the new position is

(a) $x + y + z = 0$ (b) $6x + 3y - 4z = 0$

(c) $4x - 5y - 2z = 0$ (d) $x + 2y + 4z = 0$

62. If $\lim_{x\to\infty}\left\{\frac{x^3+1}{x^2+1} - (\alpha x + \beta)\right\}$ exists and equal to 2, then the ordered pair (α, β) of real numbers is

(a) (1, –1) (b) (–2, 1) (c) (–1, 1) (d) (1, –2)

63. For $k > 0$, $\sum_{x=0}^{\infty} \frac{k^x}{x!} \lim_{n\to\infty} \frac{n!}{(n-x)!}\left(1-\frac{k}{n}\right)^{n-x}\left(\frac{1}{n}\right)^x =$

(a) 0 (b) k (c) x (d) 1

64. Let $f: R \to R$ be the function defined by

$$f(x) = \begin{cases} 5, & \text{if } x \le 1 \\ a + bx & \text{if } 1 < x < 3 \\ b + 5x, & \text{if } 3 \le x < 5 \\ 30, & \text{if } x \ge 5 \end{cases}$$ then f is

(a) continuous if $a = 5$ and $b = 5$

(b) continuous if $a = 0$ and $b = 5$

(c) continuous if $a = -5$ and $b = 10$

(d) not continuous for any values of a and b

65. Let $[x]$ denote the greatest integer less than or equal to x. Then the number of points where the function $y = [x] + |1 - x|$, $-1 \le x \le 3$ is not differentiable is

(a) 1 (b) 2 (c) 3 (d) 4

66. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$, then $y^2\frac{dy}{dx} =$

(a) $\sqrt{\frac{1-y^6}{1-x^6}}$ (b) $x\sqrt{\frac{1-y^6}{1-x^6}}$

(c) $x^2\sqrt{\frac{1-y^6}{1-x^6}}$ (d) $\frac{1}{x^2}\sqrt{\frac{1-y^6}{1-x^6}}$

67. If $y = f(x)$ is twice differentiable function such that at a point P, $\frac{dy}{dx} = 4, \frac{d^2y}{dx^2} = -3$ then $\left(\frac{d^2x}{dy^2}\right)_P =$

(a) $\frac{64}{3}$ (b) $\frac{16}{3}$ (c) $\frac{3}{16}$ (d) $\frac{3}{64}$

68. The time T of oscillation of a simple pendulum of length L is governed by $T = 2\pi\sqrt{\frac{L}{g}}$, where g is constant. The percentage by which the length be changed in order to correct an error of loss equal to 2 minutes of time per day is

(a) $-\frac{5}{18}$ (b) $-\frac{2}{9}$ (c) $\frac{1}{6}$ (d) $\frac{1}{9}$

69. Let A, G, H and S respectively denote the arithmetic mean, geometric mean, harmonic mean and the sum of the numbers $a_1, a_2, a_3, ..., a_n$. Then the value of x at which the function $f(x) = \sum_{k=1}^{n}(x - a_k)^2$ has minimum is

(a) S (b) H (c) G (d) A

70. For $m > 1, n > 1$, the value of c for which the Rolle's theorem is applicable for the function $f(x) = x^{2m-1}(a-x)^{2n}$ in $(0, a)$ is

(a) $\frac{2am-1}{m+2n-1}$ (b) $\frac{a(m-n+1)}{2m+2n}$

(c) $\frac{a(2m-1)}{2m+2n-1}$ (d) $\frac{a(2m+1)}{m+n-1}$

71. If the function $f: [-1, 1] \to R$ defined by

$$f(x) = \begin{cases} 2^x + 1, & \text{for } x \in [-1, 0) \\ 1, & \text{for } x = 0 \\ 2^x - 1, & \text{for } x \in (0, 1] \end{cases}$$

then, in $[-1, 1]$, $f(x)$ has

(a) a maximum

(b) a minimum

(c) both maximum and minimum

(d) neither maximum nor minimum

72. $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}dx =$

(a) $2\tan^{-1}\left(\sqrt{\frac{1+x+x^2}{x}}\right) + c$

(b) $\tan^{-1}\left(\sqrt{\frac{1+x+x^2}{x}}\right) + c$

(c) $\tan^{-1}\left(\sqrt{\frac{x}{1+x+x^2}}\right) + c$

(d) $\tan^{-1}\left(\sqrt{\frac{1+x^2}{x}}\right) + c$

73. If $I(x) = \int x^2(\log x)^2 dx$ and $I(1) = 0$, then $I(x)$

(a) $\frac{x^3}{18}\left[8(\log x)^2 - 3\log x\right] + \frac{7}{18}$

(b) $\frac{x^3}{27}\left[9(\log x)^2 + 6\log x\right] - \frac{2}{27}$

(c) $\frac{x^3}{27}\left[9(\log x)^2 - 6\log x + 2\right] - \frac{2}{27}$

(d) $\frac{x^3}{27}\left[9(\log x)^2 - 6\log x - 2\right] + \frac{2}{27}$

74. $\int \frac{x^5 dx}{(x^2+x+1)(x^6+1)(x^4-x^3+x-1)} =$

(a) $\log_e\left|\frac{x^6-1}{x^6+1}\right|+c$ (b) $\frac{1}{12}\log_e\left|\frac{x^6-1}{x^6+1}\right|+c$

(c) $\frac{1}{12}\log_e\left|\frac{x^4+1}{x^4-1}\right|+c$ (d) $\log_e\left|\frac{x^8+4}{x^6-1}\right|+c$

75. $\int \frac{dx}{x+\sqrt{x-1}} =$

(a) $\log_e\left|x+\sqrt{x-1}\right|-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)+c$

(b) $\frac{1}{\sqrt{3}}\log_e\left|x+\sqrt{x-1}\right|-\tan^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)+c$

(c) $\frac{2}{\sqrt{3}}\log_e\left|x+\sqrt{x-1}\right|-\tan^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)+c$

(d) $\log_e\left|x+\sqrt{x-1}\right|-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)+c$

76. $\int_{\log_e 2}^{x} \frac{dt}{\sqrt{e^t-1}} = \frac{\pi}{6} \Rightarrow x =$

(a) $2\cdot\log_e 2$ (b) $3\cdot\log_e 2$ (c) $4\cdot\log_e 2$ (d) $8\cdot\log_e 2$

77. $\int_0^1 \frac{\log_e(1+x)}{1+x^2}dx =$

(a) $\frac{\pi}{4}\log_e 2$ (b) $\frac{\pi}{6}\log_e 6$

(c) $\frac{\pi}{2}\log_e 8$ (d) $\frac{\pi}{8}\log_e 2$

78. If the area of the circle $x^2 + y^2 = 2$ is divided into two parts by the parabola $y = x^2$, then the area (in sq units) of the larger part is

(a) $\frac{3\pi}{2}-\frac{1}{3}$ (b) $6\pi-\frac{4}{3}$ (c) $\frac{4\pi}{3}-\frac{2}{3}$ (d) $4\pi-\frac{1}{4}$

79. If c is a parameter, then the differential equation of the family of curves $x^2 = c(y + c)^2$ is

(a) $x\left(\frac{dy}{dx}\right)^3+y\left(\frac{dy}{dx}\right)^2-1=0$

(b) $x\left(\frac{dy}{dx}\right)^3-y\left(\frac{dy}{dx}\right)^2+1=0$

(c) $x\left(\frac{dy}{dx}\right)^3+y\left(\frac{dy}{dx}\right)^2+1=0$

(d) $x\left(\frac{dy}{dx}\right)^3-y\left(\frac{dy}{dx}\right)^2-1=0$

80. If $f(x)$, $f'(x)\, f''(x)$, are positive functions and $f(0) = 1$, $f'(0) = 2$, then the solution of the differential equation $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$ is

(a) e^2x (b) $2\sin x + 1$

(c) $\sin^2x + 2x + 1$ (d) e^4x

PHYSICS

81. If the charge of electron e, mass of electron m, speed of light in vacuum c and Planck's constant h are taken as fundamental quantities, then the permeability of vacuum μ_0 can be expressed as

(a) $\frac{h}{mc^2}$ (b) $\frac{hc}{me^2}$ (c) $\frac{h}{ce^2}$ (d) $\frac{mc^2}{he^2}$

82. The velocity of an object moving in a straight line path is given as a function of time by $v = 6t - 3t^2$, where v is in ms^{-1}, t is in s. The average velocity of the object between, $t = 0$ and $t = 2$ s is

(a) 0 (b) 3 ms^{-1} (c) 2 ms^{-1} (d) 4 ms^{-1}

83. A gun and a target are at the same horizontal level separated by a distance of 600 m. The bullet is fired from the gun with a velocity of 500 ms^{-1}. In order to hit the target, the gun should be aimed to a height h above the target. The value of h is

(Acceleration due to gravity, $g = 10\ ms^{-2}$)

(a) 2.4 m (b) 3.6 m (c) 7.2 m (d) 10.8 m

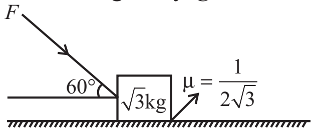
84. A projectile is thrown in the upward direction making an angle of 60° with the horizontal with a velocity of 140 ms^{-1}. Then the time after which its velocity makes an angle 45° with the horizontal is

(Acceleration due to gravity, $g = 10\ ms^{-2}$)

(a) 0.5124 s (b) 51.24 s (c) 5.124 s (d) 512.4 s

85. The maximum value of the applied force F such that the block as shown in the arrangement does not move is

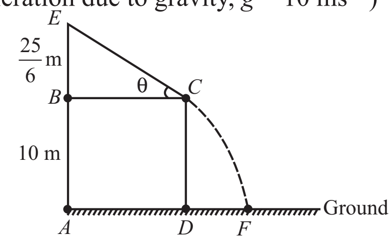
(Acceleration due to gravity, $g = 10\ ms^{-2}$)



(a) 20 N (b) 15 N (c) 25 N (d) 10 N

86. A rough inclined plane BCE of height $\left(\frac{25}{6}\right)$ m is kept on a rectangular wooden block $ABCD$ of height 10 m, as shown in the figure. A small block is allowed to slide down from the top E of the inclined plane. The coefficient of kinetic friction between the block and the inclined plane is $\frac{1}{8}$ and the angle of inclination of the inclined plane is $\sin^{-1}$ (0.6). If the small block finally reaches the ground at a point F, then DF will be

(Acceleration due to gravity, $g = 10\ ms^{-2}$)



(a) $\frac{5}{3}$m (b) $\frac{10}{3}$m (c) $\frac{13}{3}$m (d) $\frac{20}{3}$m

87. Two particles P and Q each of mass 3m lie at rest on the X-axis at points $(-a, 0)$ and $(+a, 0)$, respectively. A third particle R of mass 2 m initially at the origin moves towards the particle Q. If all the collisions of the system of 3 particles are elastic and head on, the total number of collisions in the system is
(a) 2 (b) 3 (c) 4 (d) 5

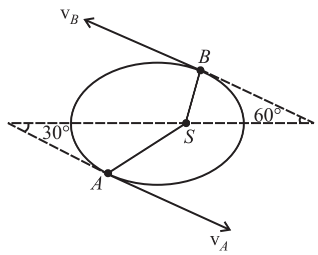
88. A motor engine pumps 1800 L of water per minute from a well of depth 30 m and allows to pass through a pipe of cross-sectional area 30 cm^2. Then the power of the engine is (Acceleration due to gravity, $g = 10\ ms^{-2}$)
(a) 20.5 kW (b) 15.5 kW
(c) 10.5 kW (d) 9.5 kW

89. A solid sphere of 100 kg and radius 10 m moving in a space becomes a circular disc of radius 20 m in one hour. Then the rate of change of moment of inertia in the process is
(a) $\frac{40}{9}$ kg $m^2\ s^{-1}$ (b) $\frac{10}{9}$ kg $m^2\ s^{-1}$
(c) $\frac{50}{9}$ kg $m^2\ s^{-1}$ (d) $\frac{25}{9}$ kg $m^2\ s^{-1}$

90. A semicircular plate of mass m has radius r and centre c. The centre of mass of the plate is at a distance x from its centre c. Its moment of inertia about an axis passing through its centre of mass and perpendicular to its plane is
(a) $\frac{mr^2}{2}$ (b) $\frac{mr^2}{4}$
(c) $\frac{mr^2}{2} + mx^2$ (d) $\frac{mr^2}{2} - mx^2$

91. Two bodies of masses m_1 and m_2 initially at rest at infinite distance apart move towards each other under gravitational force of attraction. Their relative velocity of approach when they are separated by a distance r is (G = universal gravitational constant.)
(a) $\left[\frac{2G(m_1 - m_2)}{r}\right]^{\frac{1}{2}}$ (b) $\left[\frac{2G(m_1 + m_2)}{r}\right]^{\frac{1}{2}}$
(c) $\left[\frac{r}{2G(m_1 m_2)}\right]^{\frac{1}{2}}$ (d) $\left[\frac{r}{2G} m_1 m_2\right]^{\frac{1}{2}}$

92. A planet is revolving around the Sun as shown in the figure. The radius vectors joining the Sun and the planet at points A and B are 90×10^6 km and 60×10^6 km, respectively. The ratio of velocities of the planet at the points A and B when its velocities make angle 30° and 60° with major-axis of the orbit is



(a) $\frac{3}{2\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

93. A solid copper cube of 7 cm edge is subjected to a hydraulic pressure of 8000 kPa. The volume contraction of the copper cube is
(Bulk modulus of copper = 140 GPa)
(a) 196×10^{-3} cm^3 (b) 19.6×10^{-6} cm^3
(c) 19.6×10^{-3} cm^3 (d) 196×10^3 cm^3

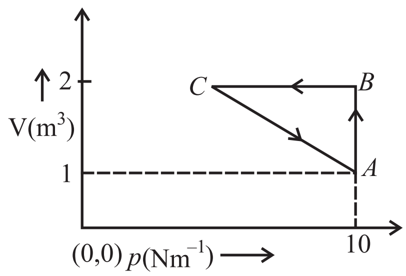
94. A long cylindrical glass vessel has a pinhole of diameter 0.2 mm at its bottom. The depth to which the vessel can be lowered vertically in a deep water bath without the water entering into the vessel is
(surface tension of water, $T = 0.07\ Nm^{-1}$, acceleration due to gravity, $g = 10\ ms^{-2}$)
(a) 14 cm (b) 7 cm (c) 21 cm (d) 28 cm

95. The focal length of a spherical mirror made of steel is 150 cm. If the temperature of the mirror increases by 200 K, its focal length become (coefficient of linear expansion of steel $\alpha = 12 \times 10^{-6}\ °C^{-1}$.)
(a) 186.3 cm (b) 153.6 cm
(c) 150.036 cm (d) 150.36 cm

96. A metal rod of length 10 cm and area of cross-section $2.8 \times 10^{-4}\ m^2$ is covered with a non-conducting substance/ One end of it is maintained at 80°C, while the other end is put in ice at 0°C. It is found that 20 gm of ice melts in 5 min. The thermal conductivity of the metal in $Js^{-1}\ m^{-1}\ K^{-1}$ is (Latent heat of ice is 80 cal g^{-1}.)
(a) 70 (b) 80 (c) 90 (d) 100

97. A gas expands with temperature according to the relation, $V = kT^{2/3}$, where k is a constant. Work done when the temperature changes by 60 K is (R = universal gas constant.)
(a) 10 R (b) 20 R (c) 50 R (d) 40 R

98. An ideal gas is taken through the cycle
$A \to B \to C \to A$
as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J. The magnitude of work done during the process C → A is



(a) 5 J (b) 10 J (c) 15 J (d) 20 J

99. The average translational kinetic energy of a molecule in a gas becomes equal to 0.69 eV at temperature about,
[Boltzmann's constant = $1.38 \times 10^{-23}\ J\ K^{-1}$]
(a) 3370°C (b) 3388°C (c) 5333°C (d) 5060°C

100. An earthquake generates both transverse S and longitudinal P waves in the earth with speeds 4.5 km s^{-1} and 8.0 km s^{-1}, respectively. A seismograph records that the first P-wave arrives 3.5 minutes earlier than the first S-wave. From the seismograph, the epicentre of the earthquake is located at a distance.
(a) 1080 km (b) 2468 km
(c) 2160 km (d) 4320 km

101. An observer moves towards a stationary source of sound with a speed $\frac{1}{5}^{\text{th}}$ of the speed of sound. The wavelength and frequency of the waves emitted by the source are λ and f respectively. The apparent frequency and wavelength heard by the observer are respectively.

(a) $1.2 f, \lambda$ (b) $f, 1.2\lambda$
(c) $0.8 f, 0.8\lambda$ (d) $1.2 f, 1.2\lambda$

102. An object is placed 0.1 m infront of a convex lens of focal length 20 cm made of a material of refractive index 1.5. The surface of the lens away from the object is silvered. If the radius of curvature of the silvered surface is 22 cm, then the distance of the final image from the silvered surface is

(a) 10 cm (b) 11 cm (c) 12 cm (d) 13 cm

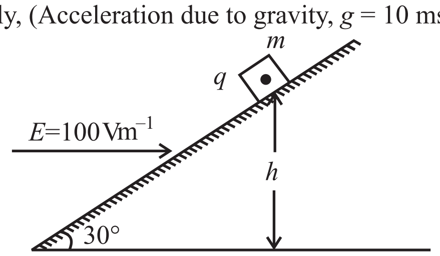
103. In a Young's double slit experiment, if the slit separation is twice the wavelength of light used, then the maximum number of interference maxima is

(a) 0 (b) 3 (c) 5 (d) 7

104. Three charges of each magnitude 100 μC are placed at the corners *A, B* and *C* of an equilateral triangle of side 4 m. If the charges at points *A* and *C* are positive and the charge at point *B* is negative, then the magnitude of total force acting on the charge at *C* and angle made by it with *AC* are

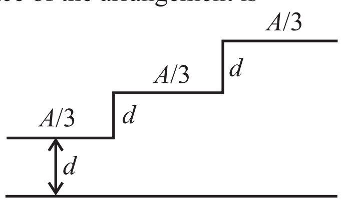
(a) 5.625 N, 60° (b) 0.5625 N, 60°
(c) 5.625 N, 30° (d) 0.5625 N, 30°

105. An inclined plane making an angle 30° with the horizontal is placed in a uniform horizontal electric field of 100 Vm^{-1} as shown in the figure. A small block of mass 1 kg and charge, 0.01C is allowed to slide down from rest from a height, h = 1 m. If the coefficient of friction is 0.2, then the acceleration of the block is nearly, (Acceleration due to gravity, $g = 10\ ms^{-2}$)



(a) $1.3\ ms^{-2}$ (b) $2.3\ ms^{-2}$
(c) $3.3\ ms^{-2}$ (d) $4.3\ ms^{-2}$

106. A capacitor is made of a flat plate of area *A* and a second plate of stair-like structure as shown in the figure. The area of each stair is $\frac{A}{3}$ and the height is *d*. The capacitance of the arrangement is



(a) $\frac{\varepsilon_0 A}{3d}$ (b) $\frac{6\varepsilon_0 A}{11d}$ (c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{11\varepsilon_0 A}{18d}$

107. The radius of a soap bubble is r and the surface tension of the soap solution is S. The electric potential to which the soap bubble be raised by charging it so that the pressure inside the bubble becomes equal to the pressure outside the bubble is
(ε_0 = permittivity of the free space

(a) $\sqrt{\frac{Sr}{8\varepsilon_0}}$ (b) $\sqrt{\frac{Sr}{4\varepsilon_0}}$ (c) $\sqrt{\frac{4Sr}{\varepsilon_0}}$ (d) $\sqrt{\frac{8Sr}{\varepsilon_0}}$

108. The ratio of heats generated through shunt and galvanometer is 7 : 5 when they are connected to make an ammeter. If the resistance of the galvanometer is 112 Ω then the resistance of the shunt is

(a) 80 Ω (b) 8 Ω (c) 15.6 Ω (d) 1.56 Ω

109. When an inductor of inductance $\frac{6}{\pi}$H, a capacitor of capacitance $\frac{50}{\pi}\mu$F and resistor of resistance R are connected in series with an AC supply of rms voltage 220 V and frequency 50 Hz, the rms current through the circuit is 440 mA. Match the inductive reactance, X_L the capacitive reactance, X_C the resistance R and the impedance Z of the circuit given in List-I with the corresponding values given in List-II.

	List-I		List-II
(A)	X_L	(i)	200 Ω
(B)	X_C	(ii)	300 Ω
(C)	R	(iii)	500 Ω
(D)	Z	(iv)	600 Ω

	A	B	C	D
(a)	(iv)	(ii)	(i)	(iii)
(b)	(iv)	(iii)	(i)	(ii)
(c)	(iv)	(i)	(ii)	(iii)
(d)	(i)	(iv)	(iii)	(ii)

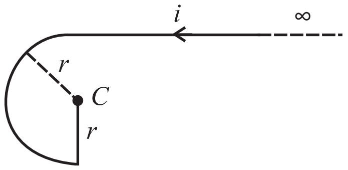
110. **Assertion (A) :** When proton and a neutron enter into a transverse magnetic field with equal speeds, then they trace a circular paths of equal radii.
Reason (R) : In a transverse magnetic field the period of revolution of a charged particle in a circular path is directly proportional to the mass of the particle.

(a) Both (A) and (R) are correct and (R) is the correct explanation of (A).
(b) Both (A) and (R) are correct but (R) is not the correct explanation of (A).
(c) (A) is correct but (R) is not correct.
(d) (A) is not correct but (R) is correct.

111. If only $\frac{1}{51}^{\text{th}}$ of the main current is to be passed through a galvanometer then the shunt required is R_1 and if only $\frac{1^{\text{th}}}{11}$ of the main voltage is to be developed across the galvanometer, then the resistance required R_2. Then $\frac{R_2}{R_1}$

(a) $\frac{1}{500}$ (b) $\frac{50}{9}$ (c) $\frac{500}{3}$ (d) 500

112. The magnetic field at the centre C of the arrangement shown in figure is



(a) $\frac{\mu_0 i}{2\pi r}(1+\pi)$ (b) $\frac{\mu_0 i}{4\pi r}(1+\pi)$

(c) $\frac{\mu_0 i}{\pi r}(1+\pi)$ (d) $\frac{\mu_0 i}{r}(1+\pi)$

113. To measure a magnetic field between the magnetic poles of a loud speaker, a small coil having 30 turns and 2.5 cm^2 area is placed perpendicular to the field and removed immediately. If the total charge flown through the coil is 7.5×10^{-3}C and the total resistance of wire and galvanometer is 0.3 Ω, then the magnitude of the magnetic field is

(a) 0.03T (b) 0.3T (c) 3T (d) 3×10^2T

114. In an oscillating *LC* circuit, the maximum charge on the capacitor is *Q*. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is

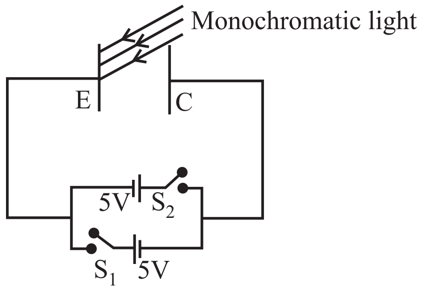
(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$ (c) Q (d) $\frac{Q}{\sqrt{2}}$

115. An electromagnetic wave of frequency 1×10^{14} Hz is propagating along z-axis. The amplitude of electric field is 4 Vm^{-1}, then energy density of the electric field will be (Permittivity of free space = 8.8×10^{-12} $C^2 N^{-1} m^{-2}$)

(a) 35.2×10^{-13} Jm^{-3} (b) 70.4×10^{-13} Jm^{-3}

(c) 70.4×10^{-12} Jm^{-3} (c) 35.2×10^{-12} Jm^{-3}

116. In a photoelectric experiment, a monochromatic light is incident on the emitter plate E, as shown in the figure. When switch S_1 is closed and switch S_2 is open, the photoelectrons strike the collector plate *C* with a maximum kinetic energy of 1 eV. If switch S_1 is open and switch S_2 is closed and the frequency of the incident light is doubled the photoelectrons strike the collector plate with a maximum kinetic energy of 20 eV. The threshold wavelength of the emitter plate is



(a) 5233.3 Å (b) 4133.3 Å

(c) 4166.7 Å (d) 5336.7 Å

117. In a system, a particle *A* of mass *m* and charge –2q is moving in the nearest orbit around a very heavy particle *B* having charge +*q*. Assuming Bohr's model of the atom to be applicable to this system, the orbital angular velocity of the particle *A* is

(a) $\frac{2\pi m^2 q^2}{\varepsilon_0 h^4}$ (b) $\frac{3\pi m^3 q^2}{\varepsilon_0^3 h^2}$

(c) $\frac{2\pi m q^4}{\varepsilon_0^2 h^3}$ (d) $\frac{5\pi m^2 q^3}{\varepsilon_0^3 h^2}$

118. In a nuclear reactor the activity of a radioactive substance is 2000/s. If the mean life of the product is 50 minutes, then in the steady power generation, the number of radio nuclides is

(a) 12 × 105 (b) 60 × 105

(c) 90 × 105 (d) 15 × 105

119. In a *p*-type semiconductor the donor level is at 50 meV above the valence band. To produce one electron, the maximum wavelength of light photon required is (Planck's constant, $h = 6.6 \times 10^{-34}$ Js and speed of light in vacuum, $c = 3 \times 10^8$ ms^{-1})

(a) 0.0248 mm (b) 0.248 mm

(c) 2.48 mm (d) 24.8 mm

120. A signal of frequency 10 kHz and peak voltage 10V is used to amplitude modulate a carrier of frequencies 1 MHz and peak voltage 20 V. The side-band frequenciesin kHz are

(a) 1010, 990 (b) 910, 1090

(c) 10, 11 (d) 1.01, 0.99

CHEMISTRY

121. The wavelength of a microscopic particle of mass 9.1×10^{-31} kg is 182 nm, its kinetic energy in J is ($h = 6.625 \times 10^{-34}$ J s)

(a) 7.28×10^{-23} (b) 7.28×10^{-24}

(c) 3.64×10^{-23} (d) 3.64×10^{-24}

122. The energy of an electron in an orbit of hydrogen like ion with an orbit radius of 52.9 pm in J is (ground state energy of electron in hydrogen atom is $= -2.18 \times 10^{-18}$ J)

(a) -4.36×10^{-18} (b) -1.09×10^{-17}

(c) -8.72×10^{-18} (d) -6.54×10^{-18}

123. In which of the following the electron gain enthalpy of elements is correctly arranged ?

(a) S > Se > Te > O (b) F > Cl > Br > I

(c) Na > Li > K > Rb (d) O > S > Se > Te

124. Which of the following orders are correct against the property given ?

(i)	Dipole moment	$NF_3 > NH_3 > BF_3$
(ii)	Covalent bond length	C—O > N—O > O—H
(ii)	Bond order	$C_2 > B_2 > He_2$

(a) i, ii only(b) ii, iii only (c) i, iii only (d)i, ii, iii

125. The molecule on having diamagnetic nature and a bond order of 1.0 is

(a) He_2^+ (b) Li_2^+ (c) B_2 (d) C_2

126. If the kinetic energy of O_2 gas is 4.0 kJ mol^{-1}, its RMS speed in cm s^{-1} is

(a) 5.0×10^2 (b) 5.0×10^3

(c) 5.0×10^4 (d) 5.0×10^{-4}

127. The volume of 0.02 M acidified permanganate solution required for complete reaction of 60 mL, of 0.01 M I^- ion solution to from I_2 in mL is
(a) 60 (b) 20 (c) 40 (d) 6

128. 6g of graphite is burnt in a bomb calorimeter at 25°C and 1 atm pressure. The temperature of water increased from 25°C to 31°C. If ΔH of this reaction is –248 kJ mol^{-1}, find out C_V (in kJ K^{-1}) of bomb calorimeter.
(a) 20.667 (b) 41.33 (c) 1488 (d) 0.145

129. In which of the following, the solubility of AgCl will be minimum?
(a) 0.1 M KNO3 (b) 0.1 M KCl
(c) 0.2 M KNO3 (d) Water

130. The number of species of the following that can act both as Bronsted acids and bases is
$HCl, ClO_4^-, {}^-OH, H^+, H_2O, HSO_4^-, SO_4^{2-}, H_2SO_4, Cl^-$
(a) 4 (b) 3 (c) 1 (d) 2

131. Which one of the following properties has same value for H_2 and D_2?
(a) Density
(b) Enthalpy of bond dissociation
(c) Bond length
(d) Melting point

132. Identify the correct statements from the following :
I. Tendency to form halide hydrates gradually increases from Be to Ba down the group.
II. Tendency to form stable super oxides increases from Li to Cs down the group.
III. Low solubility of LiF is due to its high lattice energy.
IV. Solubility of carbonates of group-2 elements increases down the group.
(a) I, II (b) III, IV (c) II, III (d) I, III

133. Which of the following metals exist in liquid state during summer season ?
(a) Ga (b) Al (c) Pb (d) Sn

134. Formic acid is heated with conc. H_2SO_4 at 100°C to form *A* and *B*. When Fe_2O_3 is heated strongly with *B*, the products formed are *C* and *D*. *C* can also be obtained by reacting $CaCO_3$ with dil.HCl. What is *D*?
(a) Fe (b) CO (c) CO_2 (d) Fe_3O_4

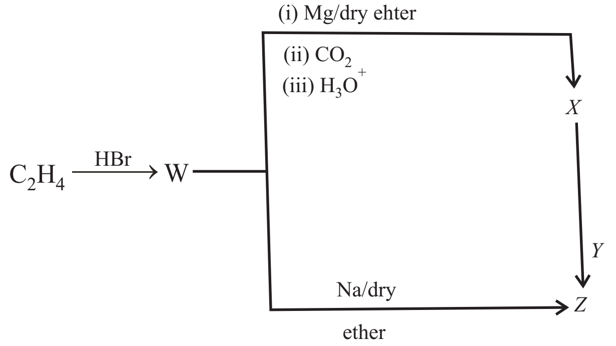
135. The non-biodegradable waste formed in fertilizer industries is
(a) fly ash (b) carbon monoxide
(c) gypsum (d) lead

136. Which one of the following has maximum number of hybrid orbitals?
(a) C_6H_6 (b) $(CH_3)_4C$
(c) $(CH_3)_2C = O$ (d) $CH_3 — CH = CH—CN$

137. In Dumas method one gram of carbon compound gives 50 mL of N_2 at 300 K and 740 mm Hg pressure. If the aqueous tension at 300 K is 15 mm Hg, what is the percentage of nitrogen in it?
(a) 5.42 (b) 10.84 (c) 21.68 (d) 2.71

138. Sodium acetate was electrolysed by Kolbe's method to form two gases *A* and *B* at anode. *C* and *D* are formed, when *B* is heated with regulated supply of O_2 or air in the presence of $(CH_3COO)_2Mn$. *C* reacts with NaOH to form a salt. *A* and *D* are respectively,
(a) CO_2, CH_3COOH (b) CO_2, H_2O
(c) C_2H_6, H_2O (d) CO_2, H_2O_2

139. What are *Y* and *Z* in the following reaction sequence ?



	Y	*Z*
(a)	NaOH/CaO	$CH_3(CH_2)_2CH_3$
(b)	NaOH/electrolysis	$H_3C—CH_3$
(c)	NaOH/CaO	$H_3C—CH_3$
(d)	NaOH/electrolysis	$CH_3(CH_2)_2CH_3$

140. At *T*(K), copper (atomic mass = 63.5 u) has fcc unit cell structure with edge length of *x* Å. What is the approximate density of Cu in g cm^{-3} at that temperature? ($N_A = 6.0 \times 10^{23} mol^{-1}$)
(a) $\frac{42.3}{x^3}$ (b) $\frac{4.23}{x^3}$ (c) $\frac{423}{x^3}$ (d) $\frac{212}{x^3}$

141. The number of moles of solute present in the solutions of I, II and III is respectively
I. 500 mL of 0.2 M NaOH
II. 200 mL of 0.1 N H_2SO_4
III. 6 g of urea in 1 kg of water
(a) 0.1, 0.01, 0.1 (b) 0.1, 0.02, 0.1
(c) 0.2, 0.01, 0.1 (d) 0.1, 0.01, 0.2

142. 6 g of a mixture of naphthalene ($C_{10}H_8$) and anthracene ($C_{14}H_{10}$) is dissolved in 300 gram of benzene. If the depression in freezing point is 0.70 K, the composition of naphthalene and anthracene in the mixture respectively in g are (molal depression constant of benzene is 5.1 K kg mol^{-1})
(a) 2.60, 3.40 (b) 3.40, 2.60
(c) 2.90, 3.10 (d) 3.10, 2.90

143. Under which of the following conditions *E* value of the cell, for the cell reaction given is maximum ?
$$Zn(s) + Cu^{2+}(aq, c_1) \rightleftharpoons Cu(s) + Zn^{2+}(aq, c_2)$$
$$\left(\frac{2.303\,RT}{F} \text{ at } 298 \text{ K} = 0.059 \text{ V}, \; E^\circ_{Zn^{2+}/Zn} = -0.76\text{V}, E^\circ_{Cu^{2+}/Cu} = +0.34 \text{ V}\right)$$
(a) $C_1 = 0.1\,M, C_2 = 0.01\,M$ (b) $C_1 = 0.01\,M, C_2 = 0.1\,M$
(c) $C_1 = 0.1\,M, C_2 = 0.2\,M$ (d) $C_1 = 0.2\,M, C_2 = 0.1\,M$

144. In the first order thermal decomposition of $C_2H_5I(g) \rightarrow C_2H_4(g) + HI(g)$, the reactant in the beginning exerts a pressure of 2 bar in a closed vessel at 600 K. If the partial perssure of the reactant is 0.1 bar after 1000 minutes at the same temperature the rate constant in min^{-1} is (log 2 = 0.3010)
(a) 6.0×10^{-4} (b) 6.0×10^{-3}
(c) 3.0×10^{-3} (d) 3.0×10^{-4}

145. Identify the correct statements from the following :
I. Sulphur sol is an example of a multimolecular colloid.
II. Tyndall effect is observed when the diameter of the dispersed particles is not much smaller than the wavelength of the light used.
III. The process of removing a dissolved substance from a colloidal solution by means of diffusion through a suitable membrane is called peptisation.
IV. Eosin, gelatin are examples of negatively charged sols.
(a) I, II, III (b) I, II, IV (c) I, III, IV (d) II, III, IV

146. Which of the following are carbonate ores ?
I Magnetite II. Kaolinite
III. Siderite IV. Calamine
(a) I, II, III (b) II, III, IV
(c) I, II only (d) III, IV only

147. Which of the following statements is not correct ?
(a) From SO_2 to TeO_2 reducing power decreases
(b) The order of boiling points of hydrides of 16th group elements is $H_2S < H_2Se < H_2Te < H_2O$
(c) Rhombic sulphur has S_8 molecules while monoclinic sulphur has S_6 molecules.
(d) The bond angle in ozone molecule is 117°

148. Noble metals, like gold and platinum are soluble in which of the following mixtures?
(a) 1 : 1 mixture of conc. HNO_3 and conc. H_2SO_4
(b) 1 : 3 mixture of conc. HCl and conc. HNO_3
(c) 1 : 3 mixture of conc. HNO_3 and conc. HCl
(d) 1 : 3 mixture of conc. H_2SO_4 and conc. HCl

149. Identify the set of acidic oxides.
(a) Na_2O, CaO, BaO
(b) ZnO, PbO, BeO
(c) CO, NO, N_2O
(d) Mn_2O_7, CrO_3, V_2O_5

150. The wavelengths of light absorbed by the complexes $[Ni(H_2O)_6]^{2+}$, $[Ni(en)_3]^{2+}$, $[Ni(H_2O)_4\,en]^{2+}$ are λ_1, λ_2, λ_3 respectively. The correct order of wavelengths is
(a) $\lambda1 > \lambda2 > \lambda3$ (b) $\lambda3 > \lambda2 > \lambda1$
(c) $\lambda1 > \lambda3 > \lambda2$ (d) $\lambda2 > \lambda3 > \lambda1$

151. $KMnO_4$ oxidises $S_2O_3^{2-}$ to SO_4^{2-} in medium *x* and NO_2^- to NO_3^- in medium *y*, *x* and *y* are respectively
(a) acidic, basic (b) acidic, acidic
(c) acidic, neutral (d) neutral, acidic

152. Match the following :

	List I		List II
(A)	Teflon	I.	$SnCl_2$
(B)	Anionic polymerisation	II.	C_2F_4
(C)	Cationic polymerisation	III.	Bakelite
(D)	Thermosetting polymer	IV.	Polystyrene
		V.	RLi

The correct answer is

	A	B	C	D
(a)	II	I	V	III
(b)	II	V	I	IV
(c)	II	V	I	III
(d)	V	II	I	IV

153. Identify the correct set of monosaccharides present in sucrose (*X*), lactose (*Y*) and maltose (*Z*).

	X	*Y*	*Z*
(a)	glucose, fructose	galactose, glucose	glucose, fructose
(b)	glucose, fructose	galactose, glucose	glucose, glucose
(c)	glucose, glucose	galactose, glucose	glucose, glucose
(d)	galactose, glucose	glucose, fructose	glucose, glucose

154. Which of the following are broad spectrum antibiotics ?

Penicillin G	Chloram-phenicol	Ofloxacin	Ampicillin
(I)	(II)	(III)	(IV)

(a) I, II only (b) I, II, III
(c) II, III, IV (d) I, III only

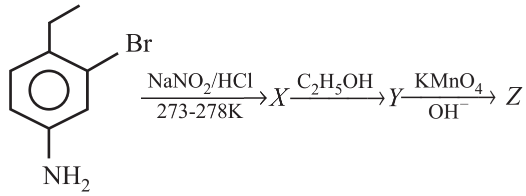
155. Arrange the following organic halides in correct order of reactivity towards S_N2 displacement.

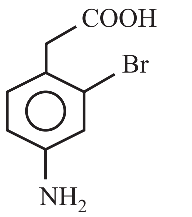
$(CH_3)_2C(Br)CH_2CH_3$, (*P*) $BrCH_2(CH_2)_3CH_3$, (*Q*) $CH_3CH(Br)(CH_2)_2CH_3$ (*R*)

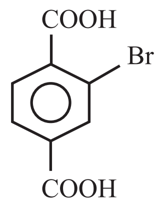
(a) P > Q > R (b) R > P > Q
(c) P > R > Q (d) Q > R > P

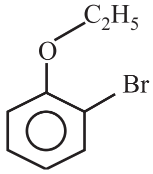
156. The bond angle between C—O and O—H bonds in alcohols is close to
(a) 109° (b) 120° (c) 180° (d) 90°

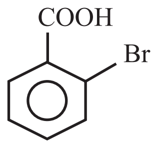
157. Identify *Z* in the following sequence of reactions.



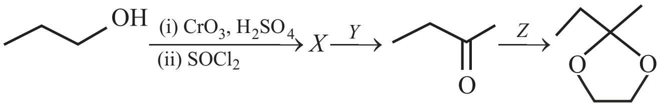
(a)


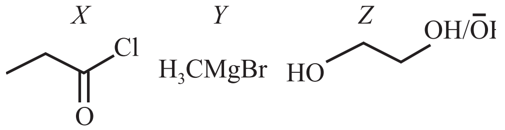
(b)


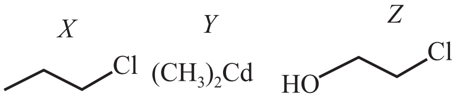
(c)


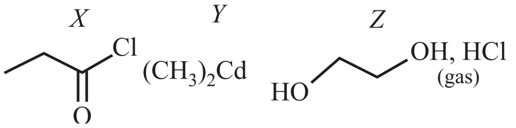
(d)


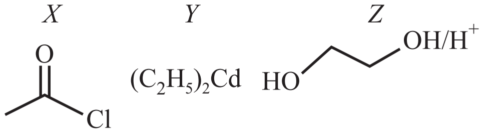
158. Identify *X, Y, Z* in the following reaction sequence



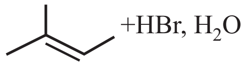
(a) 

(b) 

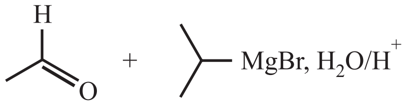
(c) 

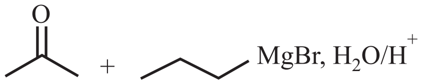
(d) 

159. 2-methyl-2-butene on hydration gave an alcohol *X*. Isomer of *X* could be prepared from which of the following ?

(a) 

(b) 

(c) 

(d) 

160. Acetic acid on heating with NH_3 forms *A*. When *A* reacts with $LiAlH_4$ followed by hydrolysis gives *B*. When *B* is heated with chloroform in KOH medium gives *C*. What are *B* and *C* respectively?

(a) CH_3CONH_2, CH_3CH_2NC
(b) CH_3CH_2NH2, CH_3CH_2NC
(c) $CH_3CH_2NH_2$, CH_3COOH
(d) $CH_3CH_2CH_2NH_2$, CH_3CH_2NC

ANSWER KEY

1	(d)	17	(c)	33	(b)	49	(a)	65	(d)	81	(c)	97	(d)	113	(b)	129	(b)	145	(b)
2	(c)	18	(b)	34	(b)	50	(d)	66	(c)	82	(c)	98	(a)	114	(d)	130	(d)	146	(d)
3	(a)	19	(c)	35	(c)	51	(b)	67	(d)	83	(c)	99	(d)	115	(c)	131	(c)	147	(c)
4	(d)	20	(b)	36	(d)	52	(d)	68	(a)	84	(c)	100	(c)	116	(b)	132	(c)	148	(c)
5	(c)	21	(a)	37	(b)	53	(a)	69	(d)	85	(a)	101	(a)	117	(c)	133	(a)	149	(d)
6	(b)	22	(a)	38	(c)	54	(b)	70	(c)	86	(d)	102	(b)	118	(b)	134	(a)	150	(c)
7	(a)	23	(b)	39	(a)	55	(d)	71	(d)	87	(a)	103	(c)	119	(d)	135	(c)	151	(d)
8	(d)	24	(a)	40	(c)	56	(c)	72	(a)	88	(c)	104	(a)	120	(a)	136	(b)	152	(c)
9	(b)	25	(d)	41	(c)	57	(c)	73	(c)	89	(a)	105	(b)	121	(b)	137	(a)	153	(b)
10	(d)	26	(c)	42	(d)	58	(a)	74	(b)	90	(d)	106	(d)	122	(c)	138	(b)	154	(c)
11	(d)	27	(c)	43	(b)	59	(a)	75	(d)	91	(b)	107	(d)	123	(a)	139	(d)	155	(d)
12	(a)	28	(b)	44	(c)	60	(a)	76	(a)	92	(b)	108	(a)	124	(b)	140	(c)	156	(a)
13	(a)	29	(d)	45	(a)	61	(a)	77	(d)	93	(c)	109	(c)	125	(N)	141	(a)	157	(d)
14	(d)	30	(c)	46	(a)	62	(d)	78	(a)	94	(a)	110	(d)	126	(c)	142	(b)	158	(c)
15	(d)	31	(b)	47	(b)	63	(d)	79	(d)	95	(d)	111	(d)	127	(d)	143	(a)	159	(c)
16	(a)	32	(b)	48	(a)	64	(d)	80	(a)	96	(d)	112	(b)	128	(a)	144	(c)	160	(b)

Hints & Solutions

MATHEMATICS

1. **(d)** Given that function

$f(x)=\frac{x}{e^x-1}+\frac{x}{2}+2\cos^3\frac{x}{2}$ define on $R-\{0\}$.

$\because\quad f(-x)=\frac{-x}{e^{-x}-1}-\frac{x}{2}+2\cos^3\left(-\frac{x}{2}\right)$

$=\frac{xe^x}{e^x-1}-\frac{x}{2}+2\cos^3\frac{x}{2}$

$=\frac{x}{e^x-1}+\frac{x}{2}+2\cos^3\frac{x}{2}=f(x)$

$\because\quad f(-x)=f(x),\ \forall\ x\in R-\{0\}$

$\therefore\quad f(x)$ is an even function.

2. **(c)** (A) Since, $f(x)=\frac{|x+2|}{x+2},\ x\neq-2$

$$=\begin{cases}\frac{x+2}{x+2}, & x>-2\\ -\frac{x+2}{x+2}, & x<-2\end{cases}=\begin{cases}1, & x>-2\\ -1, & x<-2\end{cases}$$

So, range of $f(x)$ is $\{-1, 1\}$.

(B) Since, $g(x)=|[x]|,\ x\in R$

We have, $[x]\in I\Rightarrow|[x]|\in W$

So, range of $g(x)$ is W.

(C) Since, $h(x)=|x-[x]|,\ x\in R\Rightarrow|\{x\}|\in[0,1)$

$[\because\{x\}=x-[x]$ and $\{x\}\in[0,1)]$

So, range of $h(x)$ is $[0, 1)$.

(D) Since, $f(x)=\frac{1}{2-\sin 3x},\ x\in R$

We have, $-1\le\sin 3x\le1,\ \forall\ x\in R$

$\Rightarrow\quad -1\le-\sin 3x\le1$

$\Rightarrow\quad 2-1\le2-\sin 3x\le2+1$

$\Rightarrow\quad \frac{1}{3}\le\frac{1}{2-\sin 3x}\le\frac{1}{1}$

So, range of $f(x)$ is $\left[\frac{1}{3}, 1\right]$.

3. **(a)** Since, $1+(1+2+4)+(4+6+9)+(9+12+16)+\ldots+(81+90+100)$

$=1+(1^2+(2\times1)+2^2)+(2^2+(2\times3)+3^2)+(3^2+(3\times4)+4^2)+\ldots+(9^2+(9\times10)+10^2)$

$$=\sum_{k=1}^{10}[(k-1)^2+k(k-1)+k^2]$$

$$=\sum_{k=1}^{\alpha}[k^3-(k-1)^3]$$

$=(1^3-0^3)+(2^3-1^3)+(3^3-2^3)+\ldots+(10^3-9^3)$

$=10^3-0^3=1000$

So, both (A) and (R) are true and (R) is the correct explanation of (A).

4. **(d)** Since $AA^T=I$, therefore matrix A is orthogonal matrix.

Now, $A^TX^{50}A=A^TX^{49}(APA^T)A$

$=A^TX^{49}AP(A^TA)=A^TX^{49}AP$

$=A^TX^{48}(APA^T)AP=A^TX^{48}AP^2$........

$=A^TAP^{50}=1P^{50}=P^{50}$

$\because\quad P=\begin{bmatrix}1&1\\0&1\end{bmatrix}\Rightarrow P^2=\begin{bmatrix}1&2\\0&1\end{bmatrix}$

$\Rightarrow\quad P^3=\begin{bmatrix}1&3\\0&1\end{bmatrix}$.....

$\Rightarrow\quad P^{50}=\begin{bmatrix}1&50\\0&1\end{bmatrix}$

So, $A^TX^{50}A=P^{50}=\begin{bmatrix}1&50\\0&1\end{bmatrix}$

5. **(c)** Given that system of equations are

$2x+3|y|+5[z]=0$

$x+|y|-2[z]=4$

and $x+|y|+[z]=1$

According to Cramer's rule,

$x=\frac{\Delta_x}{\Delta},\ |y|=\frac{\Delta_y}{\Delta}$ and $[z]=\frac{\Delta_z}{\Delta}$

Now, $\Delta=\begin{vmatrix}2&3&5\\1&1&-2\\1&1&1\end{vmatrix}=-3.$

$\Delta_x=\begin{vmatrix}0&3&5\\4&1&-2\\1&1&1\end{vmatrix}=-18+15=-3.$

$\Delta_y=\begin{vmatrix}2&0&5\\1&4&-2\\1&1&1\end{vmatrix}=-3$

and $\Delta_z=\begin{vmatrix}2&3&0\\1&1&4\\1&1&1\end{vmatrix}=3$

Now, $x=\frac{-3}{-3}=1,\ |y|=\frac{-3}{-3}=1$ and $[z]=\frac{-3}{3}=-1$

$\therefore\quad x=1,\ |y|=1\Rightarrow y=\pm1$ and $[z]=-1$

$\Rightarrow\quad z\in[-1, 0)$

Hence, the given system of equations has infinitely many solution.

6. **(b)** Given that system of linear equations are
$x + 2y + 3z = 6$
$x + 3y + 5z = 9$
and $2x + 5y + \lambda z = \mu$
According to Cramer's rule,

now, $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & \lambda \end{vmatrix} = \lambda - 8$

$\Delta_x = \begin{vmatrix} 6 & 2 & 3 \\ 9 & 3 & 5 \\ \mu & 5 & \lambda \end{vmatrix} = \mu - 15$

$\Delta_y = \begin{vmatrix} 1 & 6 & 3 \\ 1 & 9 & 5 \\ 2 & \mu & \lambda \end{vmatrix} = 3\lambda - 2\mu + 6$

and $\Delta_z = \begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 9 \\ 2 & 5 & \mu \end{vmatrix} = \mu - 15$

1. Let system of equations has infinite number of solutions.
$\therefore \quad \Delta = 0 \Rightarrow \lambda = 8$ and $\Delta_x = 0 \Rightarrow \mu = 15$.
2. Let system of equations has no solutions.
$\therefore \quad \Delta = 0 \Rightarrow \lambda = 8$ and $\Delta_x \neq 0 \Rightarrow \mu \neq 15$
3. Let system of equations has unique solution.
$\therefore \quad \Delta \neq 0 \Rightarrow \lambda \neq 8$ and $\Delta_x \neq 0 \Rightarrow \mu \neq 15$

7. **(a)** Let $z = x + iy,\ x, y \in R,\ (x, y) \neq (0, -4)$

$$\frac{2z-3}{z+4i} = \frac{(2x-3)+2iy}{x+i(y+4)} \times \frac{x-i(y+4)}{x-i(y+4)}$$

$$= \frac{(2x^2 - 3x + 2y^2 + 8y) + i(12 + 3y - 8x)}{x^2 + (y+4)^2}$$

So, $\arg\left(\frac{2z-3}{z+4i}\right) = \tan^{-1}\left(\frac{12+3y-8x}{2x^2-3x+2y^2+8y}\right) = \frac{\pi}{4}$ (given)

$$\Rightarrow \quad \frac{12+3y-8x}{2x^2-3x+2y^2+8y} = 1$$

$$\Rightarrow \quad 2x^2 + 2y^2 + 5x + 5y - 12 = 0$$

8. **(d)** Let $z = x + iy$

Now, $\frac{\bar{z}-1}{\bar{z}-i} = \frac{(x-1)-iy}{x-i(y+1)} \times \frac{x+i(y+1)}{x+i(y+1)}$

$$= \frac{[x(x-1) + y(y+1)] + i[(y+1)(x-1) - xy]}{x^2 + (y+1)^2}$$

$\therefore \quad \text{Im}\left(\frac{\bar{z}-1}{\bar{z}-i}\right) = \frac{xy - y + x - 1 - xy}{x^2 + (y+1)^2} = 1$ (given)

$$\Rightarrow \quad x^2 + y^2 - x + 3y + 2 = 0, (x, y) \neq (0, -1)$$

9. **(b)** Given that ω is a complex cube root of unity
$\therefore \quad 1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$ ...(i)

$\because \quad \left(k + \frac{1}{\omega}\right)\left(k + \frac{1}{\omega^2}\right) = k^2 + k\left(\frac{1}{\omega} + \frac{1}{\omega^2}\right) + \frac{1}{\omega^3}$

$= k^2 + \left(\frac{\omega^2 + \omega}{\omega^3}\right)k + \frac{1}{\omega^3} = k^2 - k + 1$ [From Eq. (i)]

Now, $\sum_{k=1}^{n}\left(k + \frac{1}{\omega}\right)\left(k + \frac{1}{\omega^2}\right)$

$= \sum_{k=1}^{n}(k^2 - k + 1) = \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + n$

$= \frac{n(n+1)}{6}[2n + 1 - 3] + n = \frac{n(n+1)}{6}(2n-2) + n$

$= \frac{n}{3}(n^2 - 1) + n = \frac{n}{3}[n^2 - 1 + 3] = \frac{n(n^2+2)}{3}$

10. **(d)** Given that ω is a complex cube root of unity, then
$1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$ ...(i)

$\because \quad r(r + 1 - \omega)(r + 1 - \omega^2)$

$= r[(r+1)^2 - (\omega + \omega^2)(r+1) + \omega^3]$

$= r[(r+1)^2 + (r+1) + 1]$ [from Eq. (i)]

$= r^3 + 3r^2 + 3r$

Now,

$$\sum_{r=1}^{9} r(r+1-\omega)(r+1-\omega^2) = \sum_{r=1}^{9}(r^3 + 3r^2 + 3r)$$

$$= \left(\frac{9(10)}{2}\right)^2 + 3\frac{9(10)(19)}{6} + 3\frac{9 \times 10}{2}$$

$$= (45)^2 + (45 \times 19) + (3 \times 45) = 3015.$$

11. **(d)** Let $\frac{2\alpha}{3 - 4\alpha} = y \Rightarrow 2\alpha = 3y - 4\alpha y$

$\Rightarrow \quad \alpha(2 + 4y) = 3y \Rightarrow \alpha = \frac{3y}{2+4y}$

Given that α is root of quadratic equation
$x^2 + 7x + 3 = 0$,

$\therefore \quad \left(\frac{3y}{2+4y}\right)^2 + 7\left(\frac{3y}{2+4y}\right) + 3 = 0$

$\Rightarrow \quad 9y^2 + 84y^2 + 42y + 48y^2 + 48y + 12 = 0$

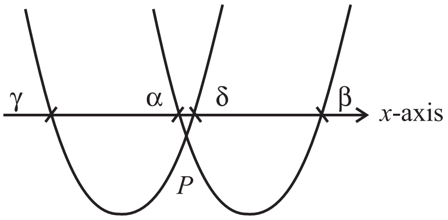
$\Rightarrow \quad 141y^2 + 90y + 12 = 0$

$\Rightarrow \quad 47y^2 + 30y + 4 = 0$ ...(i)

$\because \quad y = \frac{2\alpha}{3 - 4\alpha}$ is root of quadratic equation

So, compare equation (i) with $ax^2 + bx + c = 0$.
$\therefore$ $a = 47$, $b = 30$ and $c = 4$ and GCD of 47, 30, 4 is 1.
$\therefore \quad a + b + c = 47 + 30 + 4 = 81.$

12. **(a)** Since it is given that roots are in order, $\gamma < \alpha < \delta < \beta$ and, $x^2 + b_1x + c_1 = 0,\ x^2 + bx + c = 0$



For x-coordinate of point P, on subtracting given quadratic equations, we get

$$x^2 + b_1x + c_1 = 0$$
$$x^2 + bx + c = 0$$
$$(b_1 - b)x + (c_1 - c) = 0$$
$$\Rightarrow \quad x = \left(\frac{c - c_1}{b_1 - b}\right)$$

Now, with respect to quadratic expression
$f(x) = x^2 + bx + c$

$$f\left(x = \frac{c - c_1}{b_1 - b}\right) < 0$$
$$\Rightarrow \quad \left(\frac{c - c_1}{b_1 - b}\right)^2 + b\left(\frac{c - c_1}{b_1 - b}\right) + c < 0$$
$$\Rightarrow \quad (c - c_1)^2 < b(c_1 - c)(b_1 - b) - c(b_1 - b)^2$$
$$\Rightarrow \quad (c - c_1)^2 < (b_1 - b)(bc_1 - cb_1)$$

Hence, option (a) is correct.

13. **(a)** Given that roots of the quadratic equation
$x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$
are real, so $D \geq 0$

$$\Rightarrow \quad 4(a + b + c)^2 - 4 \times 3\lambda(ab + bc + ca) \geq 0$$
$$\Rightarrow \quad \lambda \leq \frac{(a + b + c)^2}{3(ab + bc + ca)}$$

Now, for scalane triangle.

$\because$ For $\Delta ABC\ |b - c| < a,\ |c - a| < b$ and $|a - b| < c$

$$\Rightarrow \quad (b - c)^2 + (c - a)^2 + (a - b)^2 < a^2 + b^2 + c^2$$
$$\Rightarrow \quad a^2 + b^2 + c^2 + 2(ab + bc + ca) < 4(ab + bc + ca)$$
$$\Rightarrow \quad \frac{(a + b + c)^2}{3(ab + bc + ca)} < \frac{4}{3}$$
$$\therefore \quad \lambda < \frac{4}{3}$$

14. **(d)** Since it is given that, the polynomial equation of degree 4 having real three coefficients of its roots as $2 \pm \sqrt{3}$ and $1 + 2i$, hence the remaining root is $1 - 2i$. Now, the quadratic equation whose roots as $2 \pm \sqrt{3}$ is $x^2 - 4x + 1 = 0$, and the quadratic equation whose roots as $1 \pm 2i$, is
$x^2 - 2x + 5 = 0$
So, the required polynomial equation is

$$(x^2 - 4x + 1)(x^2 - 2x + 5) = 0$$
$$\Rightarrow \quad x^4 - 6x^3 + 14x^2 - 22x + 5 = 0$$

15. **(d)** On arranging the letters of word ANIMAL in alphabetic order, we get AAILMN, now to find the rank of word ANIMAL, x (given)
AA .... 4!, AI .... 4!
AL .... 4!, AM .... 4!
ANA .... 3!, ANIA .... 2!
ANIL .... 2!, ANIMAL 1
$107 = x$
Similarly for word PERSON on arranging the letters in alphabetic order, we get ENOPRS, now to find the word having 107 ways,
EN .... 4!, EO .... 4!,
EP .... 4!, ER .... 4!,
ESN .... 3!, ESON .... 2!
ESOP .... 2!, ESORNP 1
107 same as x.
So, ESORNP is required word.

16. **(a)** Since number of ways of taken book from $(2n + 1)$ books can be given as

$${}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots + {}^{2n+1}C_n = x \quad \text{(Let)}$$
$$\because \quad {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots\ldots + {}^{2n+1}C_n + \ldots. + {}^{2n+1}C_{2n+1} = 2^{2n+1}$$
$$\Rightarrow \quad 2x + {}^{2n+1}C_0 + {}^{2n+1}C_{2n+1} = 2^{2n+1} \quad [\because {}^nC_{n-r} = {}^nC_r]$$
$$\Rightarrow \quad 2x + 2 = 2^{2n+1} \quad [\because {}^nC_0 = {}^nC_n = 1]$$
$$\Rightarrow \quad x = 2^{2n} - 1 = 255 \quad \text{(given)}$$
$$\Rightarrow \quad 2^{2n} = 256 = 2^8 \Rightarrow n = 4$$

17. **(c)** Since it is given that sum of all the coefficients in the binomial expansion of $(1 + 2x)^n$ is
$6561 = (1 + 2)^n$ on putting $x = 1$

$$\Rightarrow \quad 3^n = 6561 \Rightarrow n = 8$$

Now, at $x = \frac{1}{\sqrt{2}}$, then $R = (1 + 2x)^n = I + F$

$$\Rightarrow \quad R = (\sqrt{2} + 1)^8 = I + F,$$

where $I \in N$ and $0 < F < 1$

$$\therefore \quad (\sqrt{2} - 1)^8 = F', \text{ where } 0 < F' < 1$$
$$\therefore \quad (\sqrt{2} + 1)^8 + (\sqrt{2} - 1)^8 = I + (F + F')$$
$$\Rightarrow \quad 2[(\sqrt{2})^8 + {}^8C_2(\sqrt{2})^6 + {}^8C_4(\sqrt{2})^4 + {}^8C_6(\sqrt{2})^2 + {}^8C_8]$$
$$= I + (F + F')$$

For even integer $= I + (F + F')$

$$\Rightarrow \quad F + F' \in \text{Integer}$$
$$\because \quad 0 < F < 1 \text{ and } 0 < F' < 1 \Rightarrow 0 < F + F' < 2$$

So, $F + F' = 1$

$\Rightarrow \quad F = 1 - F' = 1 - (\sqrt{2}-1)^8$

So, $1 - \dfrac{F}{1+(\sqrt{2}-1)^4} = 1 - \dfrac{1-(\sqrt{2}-1)^8}{1+(\sqrt{2}-1)^4}$

$= 1 - [1-(\sqrt{2}-1)^4] = (\sqrt{2}-1)^4$

18. **(b)** Since we are given that

$$\frac{(1-px)^{-1}}{(1-qx)} = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$$

We know

$(1-px)^{-1} = 1 + px + p^2x^2 + p^3x^3 + \ldots + p^nx^n + \ldots$

and $(1-qx)^{-1} = 1 + qx + q^2x^2 + q^3x^3 + \ldots + q^nx^n + \ldots$

hence, coefficient of x^n in the expansion of

$(1-px)^{-1}(1-qx)^{-1}$

$= p^n + p^{n-1}q + p^{n-2}q^2 + p^{n-3}q^3 + \ldots + q^n$

$$a_n = \frac{p^n\left(1-\left(\frac{q}{p}\right)^{n+1}\right)}{1-\frac{q}{p}} = \frac{p^n(p^{n+1}-q^{n+1})p}{(p-q)p^{n+1}}$$

So, $a_n = \dfrac{p^{n+1}-q^{n+1}}{p-q}$

19. **(c)** Since, we are given that

$$\frac{3}{(x-1)(x^2+x+1)} = \frac{1}{x-1} - \frac{x+2}{x^2+x+1}$$

$= f_1(x) - f_2(x)$

and $\dfrac{x+1}{(x-1)^2(x^2+x+1)} = Af_1(x) + \left(B + \dfrac{D}{x-1}\right)$

$f_2(x) + \dfrac{C}{(x-1)^2}$

$\Rightarrow \quad f_1(x) = \dfrac{1}{x-1}$ and $f_2(x) = \dfrac{x+2}{x^2+x+1}$

Therefore, $\dfrac{x+1}{(x-1)^2(x^2+x+1)}$

$= \dfrac{A}{x-1} + \dfrac{(Bx-B+D)(x+2)}{(x^2+x+1)(x-1)} + \dfrac{C}{(x-1)^2}$

$\Rightarrow \quad x+1 = A(x-1)(x^2+x+1) + B(x-1)$

$(x+2)(x-1) + D(x+2)(x-1) + C(x^2+x+1)$

On comparing we get

coefficient of $x^3 = 0 \Rightarrow A + B = 0$

coefficient of $x^2 = 0 \Rightarrow D + C = 0$

Hence, $A + B + C + D = 0$

20. **(b)** Since, it is given that

$f(\theta) = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

So,

$[f(\theta)]^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$

$+2\sqrt{(a^2\cos^2\theta + b^2\sin^2\theta)(a^2\sin^2\theta + b^2\cos^2\theta)}$

$\because [f(\theta)]^2$ will be maximum, if $\sin^2\theta = \cos^2\theta = \dfrac{1}{2}$ and will be minimum, if either $\sin^2\theta = 0$ or $\cos^2\theta = 0$.

$\therefore \quad M = (a^2+b^2) + 2\sqrt{\left(\dfrac{a^2}{2}+\dfrac{b^2}{2}\right)^2} = 2(a^2+b^2)$

and $m = (a^2+b^2) + 2\sqrt{a^2b^2} = a^2 + b^2 + 2ab$

$\therefore \quad M - m = 2(a^2+b^2) - [a^2+b^2+2ab]$

$= a^2 + b^2 - 2ab = (a-b)^2$

Hence, option (b) is correct.

21. **(a)** Given, $\cos A = \dfrac{-60}{61}, \tan B = \dfrac{-7}{24}$ where A and B do not lie in second quadrant so A and B will be in third and fourth quadrant respectively. Thus, $\dfrac{B}{2}$ will be in second quadrant.

Now, we know $\tan B = \dfrac{2\tan\frac{B}{2}}{1-\tan^2\frac{B}{2}} = -\dfrac{7}{24}$ (given)

$\Rightarrow \quad 7\tan^2\dfrac{B}{2} - 48\tan\dfrac{B}{2} - 7 = 0$

$\Rightarrow \quad \left(7\tan\dfrac{B}{2}+1\right)\left(\tan\dfrac{B}{2}-7\right) = 0$

$\Rightarrow \quad \tan\dfrac{B}{2} = -\dfrac{1}{7}$ $\left[\because \dfrac{B}{2} \in \text{IInd quadrant}\right]$

Now, $\tan\left(A+\dfrac{B}{2}\right) = \dfrac{\tan A + \tan\frac{B}{2}}{1-\tan A\tan\frac{B}{2}} = \dfrac{\frac{11}{60}-\frac{1}{7}}{1+\frac{11}{420}}$

$\left[\because \cos A = -\dfrac{60}{61} \text{ and } A \in \text{IIIrd Quadrant} \quad \therefore \tan A = \dfrac{11}{60}\right]$

$= \dfrac{77-60}{420+11} = \dfrac{17}{431}$

$\because$ A $\in$ IIIrd quadrant and $\dfrac{B}{2} \in \left(\dfrac{3\pi}{4}, \pi\right)$ and $\tan\left(A+\dfrac{B}{2}\right)$ is positive.

$\therefore \quad \left(\quad + — \right) \in$ Ist quadrant.

22. **(a)** Given, $\cos^2 5° - \cos^2 15° - \sin^2 15° + \sin^2 35°$
$+ \cos 15° \sin 15° - \cos 5° \sin 35°$
After rearranging the terms
$= \cos 5°(\cos 5° - \sin 35°) - \cos 15°(\cos 15° - \sin 15°)$
$+ \sin^2 35° - \sin^2 15°$
$= \cos 5°(\cos 5° - \cos 55°) - \cos 15°(\cos 15° - \cos 75°)$
$+ \sin 50° \sin 20°$
$= -\dfrac{\cos 15°}{\sqrt{2}} + \cos 5° \sin 25° + \sin 50° \sin 20°$
$= -\dfrac{\cos 15°}{\sqrt{2}} + \dfrac{1}{2}[\cos 70° + \cos 60° + \cos 30° - \cos 70°]$
$= -\dfrac{\cos 15°}{\sqrt{2}} + \dfrac{1}{2}[2\cos 45° \cos 15°]$
$= -\dfrac{\cos 15°}{\sqrt{2}} + \dfrac{\cos 15°}{\sqrt{2}} = 0$

23. **(b)** It is given that
$\cos\theta \neq 0$ and $\sec\theta - 1 = (\sqrt{2} - 1)\tan\theta$
$\Rightarrow \dfrac{1 - \cos\theta}{\cos\theta} = (\sqrt{2} - 1)\dfrac{\sin\theta}{\cos\theta}$
$\Rightarrow \sin^2\dfrac{\theta}{2} = (\sqrt{2} - 1)\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$
$\Rightarrow$ Either $\sin\dfrac{\theta}{2} = 0$ or $\tan\dfrac{\theta}{2} = \sqrt{2} - 1$
$\Rightarrow$ Either $\dfrac{\theta}{2} = n\pi,\ n \in Z$
or $\theta = 2n\pi + \dfrac{\pi}{4}$ or $2n\pi,\ n \in Z$.

24. **(a)** Since, given expression is sum of even natural numbers so,
$\because \displaystyle\sum_{k=1}^{n} 2k = n(n+1)$
$\therefore \cot\left(\displaystyle\sum_{n=3}^{32} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right)$
$= \cot\left(\displaystyle\sum_{n=3}^{32} \cot^{-1}(1 + n(n+1))\right)$
$\cot\left(\displaystyle\sum^{32} \tan \left(\dfrac{(n+1) - n}{1 + (n+1)n}\right)\right)$
$\because \displaystyle\sum_{n=3}^{32} \tan^{-1}\left(\dfrac{(n+1) - n}{1 + (n+1)n}\right) = \sum_{n=3}^{32}[\tan^{-1}(n+1) - \tan^{-1} n]$
$= (\tan^{-1} 4 - \tan^{-1} 3 + (\tan^{-1} 5 - \tan^{-1} 4) + ...$
$+(\tan^{-1} 33 - \tan^{-1} 32)$
$= \tan^{-1} 33 - \tan^{-1} 3 = \tan^{-1}\left[\dfrac{33 - 3}{1 + 99}\right] = \tan^{-1}\left(\dfrac{3}{10}\right)$

$\therefore \cot\left(\displaystyle\sum_{n=3}^{32} \tan^{-1}\dfrac{(n+1) - n}{1 + (n+1)n}\right) = \cot\left[\tan^{-1}\left(\dfrac{3}{10}\right)\right]$
$= \cot\left[\cot^{-1}\left(\dfrac{10}{3}\right)\right] = \dfrac{10}{3}$
$\left\{\because \tan^{-1}(x) = \cot^{-1}\left(\dfrac{1}{x}\right) \text{if } x \in \text{I}^{\text{st}} \text{ quadrant}\right\}$

25. **(d)** Here, it is given $\tan x = \dfrac{3}{4}$
$\Rightarrow \sin^2 x = \dfrac{9}{25}$ and $\cos^2 x = \dfrac{16}{25}$
and $\sin x \cosh y = \cos\theta,\ \cos x \sinh y = \sin\theta$
$\because \cos^2\theta + \sin^2\theta = 1$
$\Rightarrow \dfrac{9}{25}(1 + \sinh^2 y) + \dfrac{16}{25}\sinh^2 y = 1$
$\Rightarrow 9 + 9\sin\text{h}^2 y + 16\sin\text{h}^2 y = 25$
$\Rightarrow \sin\text{h}^2 y = \dfrac{16}{25}$

26. **(c)** Given, in ΔABC, $\dfrac{b+c}{9} = \dfrac{c+a}{10} = \dfrac{a+b}{11}$
Let $\dfrac{b+c}{9} = \dfrac{c+a}{10} = \dfrac{a+b}{11} = k$
$\Rightarrow$ $b + c = 9k$, $c + a = 10k$ and $a + b = 11k$ and $a + b + c = 15k$
$\therefore$ $a = 6k$, $b = 5k$ and $c = 4k$
$\because \dfrac{\cos A + \cos B}{\cos C} = \dfrac{\dfrac{b^2 + c^2 - a^2}{2bc} + \dfrac{a^2 + c^2 - b^2}{2ac}}{\dfrac{a^2 + b^2 - c^2}{2ab}}$
$= \dfrac{\dfrac{25 + 16 - 36}{40} + \dfrac{36 + 16 - 25}{48}}{\dfrac{36 + 25 - 16}{60}} = \dfrac{11}{12}.$
Hence, option (c) is correct.

27. **(c)** Since, given information for ΔABC
(A) $r_1 r_2\sqrt{\dfrac{4R - r_1 - r_2}{r_1 + r_2}} = \left[\dfrac{\Delta^2}{(s-a)(s-b)}\right.$
$= \sqrt{\dfrac{4R - 4R\cos\dfrac{C}{2}\left(\sin\dfrac{A}{2}\cos\dfrac{B}{2} + \sin\dfrac{B}{2}\cos\dfrac{A}{2}\right)}{4R\cos\dfrac{C}{2}\left(\sin\dfrac{A}{2}\cos\dfrac{B}{2} + \sin\dfrac{B}{2}\cos\dfrac{A}{2}\right)}}\ \Bigg]$
$= \dfrac{\Delta^2}{(s-a)(s-b)}\sqrt{\dfrac{4R\left(1 - \cos^2\dfrac{C}{2}\right)}{4R\cos^2\dfrac{C}{2}}}$

$$= \frac{\Delta^2}{(s-a)(s-b)} \tan\frac{C}{2}$$

$$= \frac{\Delta^2}{(s-a)(s-b)} \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{\Delta^2}{\Delta} = \Delta$$

(B) $$\frac{r_2(r_3+r_1)}{\sqrt{r_1r_2+r_2r_3+r_3r_1}} = \frac{\frac{\Delta}{s-b}\left(\frac{\Delta}{s-c}+\frac{\Delta}{s-a}\right)}{\sqrt{\frac{(s-a)+(s-b)+(s-c)}{(s-a)(s-b)(s-c)}}}$$

$$= \frac{\frac{\Delta}{(s-b)} \times \frac{2S-a-c}{(s-a)(s-c)}}{\sqrt{\frac{3S-a-b-c}{(s-a)(s-b)(s-c)}}} = \frac{\Delta(b)}{\sqrt{s(s-a)(s-b)(s-c)}} = b$$

(C) Given, $\frac{a}{c} = \frac{\sin(A-B)}{\sin(B-C)} \Rightarrow \frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$

$\Rightarrow \sin A \sin(B-C) = \sin(A-B)\sin C$

$\Rightarrow$ sin A(sin B cos C – sin C cos B)

$\Rightarrow 2\sin A\cos B\sin C = \sin A\sin B\cos C + \sin B\cos A\sin C$

$$\Rightarrow 2\frac{a}{2R} \times \frac{a^2+c^2-b^2}{2ac} \times \frac{c}{2R}$$

$$= \left(\frac{a}{2R} \times \frac{b}{2R} \times \frac{a^2+b^2-c^2}{2ab}\right) + \left(\frac{b}{2R} \times \frac{b^2+c^2-a^2}{2bc} \times \frac{c}{2R}\right)$$

$\Rightarrow 2(a^2+c^2-b^2) = a^2+b^2-c^2+b^2+c^2-a^2$

$\Rightarrow 2a^2+2c^2-2b^2 = 2b^2 \Rightarrow 2b^2 = a^2+c^2$

Hence, a^2, b^2, c^2 are in AP.

(D) $bc\cos^2\frac{A}{2} = bc \times \frac{s(s-a)}{bc} = s(s-a)$

28. **(b)** Given, a, b and c are sides of ΔABC where $r_1 = 8$, $r_2 = 12$ and $r_3 = 24$ then we have,

$$r_1 = 8 = \frac{\Delta}{s-a} \quad ...(i)$$

$$r_2 = 12 = \frac{\Delta}{s-b} \quad ...(ii)$$

and $r_3 = 24 = \frac{\Delta}{s-c}$ ...(iii)

From Eqs. (i) and (ii), we get

$\frac{s-b}{s-a} = \frac{2}{3} \Rightarrow 3s-3b = 2s-2a$

$\Rightarrow 5a + c = 5b$ ...(iv)

From Eqs. (ii) and (iii), we get

$\frac{s-c}{s-b} = \frac{1}{2}$

$\Rightarrow 2s - 2c = s - b$

$\Rightarrow a + 3b = 3c$ ...(v)

and from Eqs. (i) and (iii), we get

$\frac{s-c}{s-a} = \frac{1}{3}$

$\Rightarrow 3s - 3c = s - a$

$\Rightarrow 2a + b = 2c$

On solving Eqs. (iv), (v) and (vi), we get

$(a, b, c) = (12, 16, 20)$

Hence, option (b) is correct.

29. **(d)** Given, $\overline{OA} = 4\hat{i}+7\hat{j}+8\hat{k}$, $\overline{OB} = 2\hat{i}+3\hat{j}+4\hat{k}$ and $\overline{OC} = 2\hat{i}+5\hat{j}+7\hat{k}$

Since we know that bisector of angle A divides the BC in ratio $c : b$

where c is length of side $AB = \sqrt{4+16+16} = 6$

and b is length of side $AC = \sqrt{4+4+1} = 3$

$\therefore$ Position vector of the point where the bisector of angle A meet $\overline{BC}$ is

$$\frac{6(2\hat{i}+5\hat{j}+7\hat{k})+3(2\hat{i}+3\hat{j}+4\hat{k})}{6+3}$$

$$= \frac{18\hat{i}+39\hat{j}+54\hat{k}}{9} = 2\hat{i}+\frac{13}{3}\hat{j}+6\hat{k}$$

30. **(c)** Given equation of plane passes through the point $(\hat{i}+2\hat{j}-\hat{k})$.

Hence (1, 2, –1) having direction ratios to normal are a, b, c is

$a(x-1) + b(y-2) + c(z+1) = 0$ ...(i)

$\because$ Plane (i) is perpendicular to line of intersection of planes $\mathbf{r}\cdot(3\hat{i}-\hat{j}+\hat{k}) = 1$ and $\mathbf{r}\cdot(\hat{i}+4\hat{j}-2\hat{k}) = 2$

$\therefore$ $3a - b + c = 0$ and $a + 4b - 2c = 0$

$\Rightarrow \frac{a}{2-4} = \frac{-b}{-6-1} = \frac{c}{12+1}$

$\Rightarrow \frac{a}{2} = \frac{b}{-7} = \frac{c}{-13}$

So, equation of required plane is

$2(x-1) - 7(y-2) - 13(z+1) = 0$

$\Rightarrow 2x - 7y - 13z = 1$

or $\mathbf{r}\cdot(2\hat{i}-7\hat{j}-13\hat{k}) = 1$

31. **(b)** Given, p.v. of the vertices A, B and C are $\hat{i}+2\hat{j}-5\hat{k}$, $-2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$.

$\because$ $\overrightarrow{BA} = 3\hat{i}-6\hat{k}$ and $\overrightarrow{BC} = 4\hat{i}-\hat{j}-2\hat{k}$

$\therefore$ $\cos(\angle B) = \frac{|\overrightarrow{BA}\cdot\overrightarrow{BC}|}{|\overrightarrow{BA}|.|\overrightarrow{BC}|}$

$$= \frac{12+12}{\sqrt{9+36}\sqrt{16+1+4}} = \frac{24}{\sqrt{45\times 21}} = \frac{8}{\sqrt{105}}$$

$$\Rightarrow \angle B = \cos^{-1}\left(\frac{8}{\sqrt{105}}\right)$$

32. **(b)** Here, length of altitude of ΔABC drawn from A is

$$d=\frac{(\text{Area of }\Delta ABC)}{\frac{1}{2}|\overrightarrow{BC}|}=\frac{\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|}{\frac{1}{2}|\overrightarrow{BC}|}$$

Now, $\overrightarrow{AB}=-2\hat{i}+\hat{j}+\hat{k}$, $\overrightarrow{AC}=-\hat{i}+2\hat{j}-\hat{k}$

and $\overrightarrow{BC}=\hat{i}+\hat{j}-2\hat{k}$

So, $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 1 & 1\\ -1 & 2 & -1\end{vmatrix}$

$=\hat{i}(-1\cdot 2)-\hat{j}(2+1)+\hat{k}(-4+1)=-3\hat{i}-3\hat{j}-3\hat{k}$

$\therefore \quad |\overrightarrow{AB}\times\overrightarrow{AC}|=3\sqrt{3}$ and $|\overrightarrow{BC}|=\sqrt{6}$

$\therefore \quad d=\frac{3}{\sqrt{2}}$

33. **(b)** Let the position vectors of the vertices of tetrahedron $OABC$ are $OA=a$, $OB=b$ and $\overrightarrow{OC}=c$

Since, volume of tetrahedron $OABC=\frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$

Now, position vectors of vertices of new tetrahedron are $\frac{\vec{a}+\vec{b}}{3},\frac{\vec{b}+\vec{c}}{3},\frac{\vec{c}+\vec{a}}{3}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$, so, coterminous edge vectors of the new tetrahedron are $\frac{\vec{a}}{3},\frac{\vec{b}}{3}$ and $\frac{\vec{c}}{3}$.

$\therefore$ Volume of new tetrahedron is

$$\frac{1}{6}\left[\frac{\vec{a}}{3}\ \frac{\vec{b}}{3}\ \frac{\vec{c}}{3}\right]=\frac{1}{6\times 27}[\vec{a}\ \vec{b}\ \vec{c}]$$

So, the required ratio $=\frac{1}{27}$

34. **(b)** Given, $\vec{A}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{B}=\hat{i}+\hat{j}$, so

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 1 & 1 & 0\end{vmatrix}=\hat{i}(2)-\hat{j}(2)+\hat{k}(2-1)$$

$=2\hat{i}-2\hat{j}+\hat{k}$

Now $|(\vec{A}\times\vec{B})\times\vec{C}|=|\vec{A}\times\vec{B}||\vec{C}|\sin 30°$

[as angle between $\vec{A}\times\vec{B}$ and $\vec{C}=30°$ (given)]

$=\sqrt{4+4+1}\,|\vec{C}|\left(\frac{1}{2}\right)=\frac{3}{2}|\vec{C}|$ ...(i)

$|\vec{C}-\vec{A}|=2\sqrt{2}$ (Given)

$\Rightarrow \quad |\vec{C}|^2+|\vec{A}|^2-2\vec{C}\cdot\vec{A}=8$

$\Rightarrow \quad |\vec{C}|^2+9-2|\vec{C}|=8$ [as $\vec{A}\cdot\vec{C}=|\vec{C}|$]

$\Rightarrow \quad (|\vec{C}|-1)^2=0\Rightarrow|\vec{C}|=1$

So, $|(\vec{A}\times\vec{B})\times\vec{C}|=\frac{3}{2}$ [from Eq. (i)]

35. **(c)** Here, mean of given data is

$$\bar{x}=\frac{a_0+a_1+a_2+\ldots+a_{11}}{12}=\frac{a_0+a_{11}}{2}$$

$$=\frac{a_1+a_{10}}{2}=\ldots.$$

Now, deviations from their mean are

$$|\bar{x}-a_0|=\frac{|a_{11}-a_0|}{2}=\frac{11|d|}{2}$$

$$|\bar{x}-a_1|=\frac{|a_{10}-a_1|}{2}=\frac{9|d|}{2}, \text{ .... and so on}$$

So, sum of deviations

$$=2\left[\frac{11|d|}{2}+\frac{9|d|}{2}+\frac{7|d|}{2}+\frac{5|d|}{2}+\frac{3|d|}{2}+\frac{|d|}{2}\right]$$

$=36|d|$

$\therefore$ Mean deviation from their arithmetic mean is

$$\frac{36|d|}{12}=3|d|.$$

36. **(d)** Let table of frequency distribution.

Class Interval	Frequency (f_i)	x_i	$x_i f_i$	$(\bar{x}-x_i)^2$	$f_1(\bar{x}-x_i)^2$
0-10	2	5	10	196	392
10-20	3	15	45	16	48
20-30	4	25	100	36	144
30-40	1	35	35	256	256
	$N=\Sigma f_i=10$		$\Sigma x_i f_i=190$		$\Sigma f_i(\bar{x}-x_i)^2=840$

$\because \quad \bar{x}=\frac{\Sigma x_i f_i}{N}=\frac{190}{10}=19$

$\therefore \quad$ Variance $(\sigma)^2=\frac{1}{N}\Sigma f_i(\bar{x}-x_i)^2=\frac{1}{10}(840)=84$

37. **(b)** Since, two particular students are always together so according to question, the required probability

$$=\frac{{}^{73}C_{28}+{}^{73}C_{43}}{{}^{75}C_{30}}$$

$$=\frac{\frac{1}{45\times 44}+\frac{1}{30\times 29}}{\frac{75\times 74}{(30\times 29)(45\times 44)}}=\frac{(30\times 29)+(44\times 45)}{(75\times 74)}$$

$$=\frac{29+(22\times 3)}{5\times 37}=\frac{29+66}{5\times 37}=\frac{95}{5\times 37}=\frac{19}{37}.$$

38. **(c)** Since, bag contains $2n$ coins out of which $n-1$ are unfair with heads on both sides.

Now, according to question

$$\frac{(n-1)\times 1+(n+1)\frac{1}{2}}{2n}=\frac{41}{56}$$

$\Rightarrow \quad \dfrac{2n-2+n+1}{4n} = \dfrac{41}{56}$

$\Rightarrow \quad 42n - 14 = 41n$

$\Rightarrow \quad n = 14$

Hence, number of unfair coins in the bag is $(n-1) = 13$.

39. **(a)** Since Bag A has 6 Green and 8 Red balls while B has 9 Green and 5 Red and by using other informations we deduce to.

Required probability

$$= \frac{\frac{1}{4}\left(\frac{{}^6C_2 + {}^8C_2}{{}^{14}C_2}\right)}{\frac{1}{4}\left(\frac{{}^6C_2 + {}^8C_2}{{}^{14}C_2}\right) + \frac{3}{4}\left(\frac{{}^9C_2 + {}^5C_2}{{}^{14}C_2}\right)}$$

$$= \frac{(6\times 5)+(8\times 7)}{[(6\times 5)+(8\times 7)] + 3[(9\times 8)+(5\times 4)]} = \frac{43}{181}$$

40. **(c)** For a random variable x, the given probability distribution is,

$x = x_i$	1	2	3	4	5	6
$P(X = x_i)$	0.2	0.3	0.12	0.1	0.2	0.08

and two events $A = \{x_i \mid x_i \text{ is a prime number}\}$
$= \{2, 3, 5\}$

and $B = \{x_i \mid x_i < 4\} = \{1, 2, 3\}$

So,

$P(A \cup B) = P(X=1) + P(X=2) + P(X=3) + P(X=5)$
$= 0.2 + 0.3 + 0.12 + 0.2 = 0.82$

Hence, option (c) is correct.

41. **(c)** Since the unit mean is $\bar{x} = 1$

Hence $\displaystyle\sum_{x=0}^{\infty} |x - \bar{x}|\, P(X = x) = \sum_{x=0}^{\infty} |x-1| \frac{e^{-1}}{x!}$

$$= \frac{1}{e}\left[\frac{1}{0!} + \sum_{x=1}^{\infty}\frac{x-1}{x!}\right] = \frac{1}{e}\left[1 + \sum_{x=1}^{\infty}\frac{1}{(x-1)!} - \sum_{x=1}^{\infty}\frac{1}{x!}\right]$$

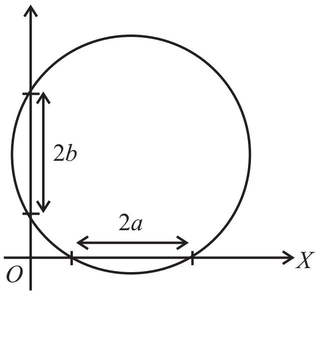
$$= \frac{1}{e}[1 + e - (e-1)] = \frac{2}{e}.$$

42. **(d)** Let the equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

Hence x-intercept and y-intercept are given as,

$\because \quad 2\sqrt{g^2 - c} = 2a$ and $2\sqrt{f^2 - c} = 2b$



then $g^2 - a^2 = 0$ and $f^2 - b^2 = 0$

so, $g^2 - a^2 = f^2 - b^2$

$\Rightarrow \quad g^2 - f^2 = a^2 - b^2$

Hence locus of the centre $(-g, -f)$, we get

$x^2 - y^2 = a^2 - b^2$

43. **(b)** Since coordinate axes are rotates about the origin through angle $\dfrac{\pi}{4}$

Hence, the new coordinates are (X, Y) have relation with older coordinates (x, y) as

$(x, y) = [(X\cos\theta - Y\sin\theta)\cdot(Y\cos\theta + X\sin\theta)]$

$\Rightarrow \quad \left(\left(\dfrac{X}{\sqrt{2}} - \dfrac{Y}{\sqrt{2}}\right), \left(\dfrac{Y}{\sqrt{2}} + \dfrac{X}{\sqrt{2}}\right)\right) \qquad \left\{\because \theta = \dfrac{\pi}{4}\right\}$

Now equation $25x^2 + 9y^2 = 225$ becomes

$$25\left(\frac{X-Y}{\sqrt{2}}\right)^2 + 9\left(\frac{X+Y}{\sqrt{2}}\right)^2 = 225$$

$\Rightarrow \quad 34X^2 + 34Y^2 - 32XY = 450$

$\Rightarrow \quad 17X^2 + 17Y^2 - 16XY = 225$

On comparing, we get

$\alpha = \gamma = 17, \beta = -16$ and $\delta = 225$

$\therefore \quad (\alpha + \beta + \gamma - \sqrt{\delta})^2 = (34 - 16 - 15)^2 = 3^2 = 9$

44. **(c)** Since we know that equation of line through intersection of lines L_1 and L_2 is given as $L_1 + \lambda L_2 = 0$

$\Rightarrow \quad (3x - 4y + 1) + \lambda(5x + y - 1) = 0$

$\Rightarrow \quad (3 + 5\lambda)x + (\lambda - 4)y = (\lambda - 1)$

$$\Rightarrow \quad \frac{x}{\frac{\lambda - 1}{3 + 5\lambda}} + \frac{y}{\frac{\lambda-1}{\lambda - 4}} = 1$$

Since inten. cells are equal

$\dfrac{\lambda - 1}{3 + 5\lambda} = \dfrac{\lambda - 1}{\lambda - 4}$ and $\lambda \neq 1$

$\Rightarrow \quad 4\lambda = -7 \Rightarrow \lambda = -\dfrac{7}{4}$

So, equation of required line is

$$\left(3 - \frac{35}{4}\right)x + \left(-\frac{7}{4} - 4\right)y = \left(-\frac{7}{4} - 1\right)$$

$\Rightarrow \quad -\dfrac{23}{4}x - \dfrac{23}{4}y = -\dfrac{11}{4}$

$\Rightarrow \quad 23x + 23y = 11$

45. **(a)** Given that a line passes through $P\ (a, 2)$, $a \neq 0$ making an angle 45° with positive direction of the X-axis is :

hence $\dfrac{x-a}{\frac{1}{\sqrt{2}}} = \dfrac{y-2}{\frac{1}{\sqrt{2}}} = r \Rightarrow x = a + \dfrac{r}{\sqrt{2}}$ and $y = 2 + \dfrac{r}{\sqrt{2}}$

Hence for point B, $r = -2\sqrt{2}$ (as it is on X-axis)

and $B(a-2, 0)$ for point C, $r = -a\sqrt{2}$ (as it is on Y-axis)

so, $C\ (0, 2-a)$

$\therefore \quad PB = \sqrt{4+4} = 2\sqrt{2}$

$PC = \sqrt{a^2 + a^2} = a\sqrt{2}$

For points A and D

$$\frac{\left(a+\frac{r}{\sqrt{2}}\right)^2}{9} + \frac{\left(2+\frac{r}{\sqrt{2}}\right)^2}{4} = 1$$

$$\Rightarrow \quad 4\left(a+\frac{r}{\sqrt{2}}\right)^2 + 9\left(2+\frac{r}{\sqrt{2}}\right)^2 = 36$$

$$\Rightarrow \quad 13\left(\frac{r^2}{2}\right) + \left(\frac{8a}{\sqrt{2}} + \frac{36}{\sqrt{2}}\right)r + 4a^2 = 0,$$

let having roots $r_1 = PA$ and $r_2 = PD$, so $r_1 r_2 = \frac{4a^2}{\frac{13}{2}}$

$\Rightarrow (PA)(PD) = \frac{8a^2}{13} = (PA)(PD)$

Therefore PA, PB, PC and PD are in GP. So,

$$(PA)(PD) = (PB)(PC) \Rightarrow \frac{8a^2}{13} = 4a \Rightarrow 2a = 13$$

46. **(a)** Image of A in $x - y + 5 = 0$ is

$$\Rightarrow \quad \frac{x-1}{1} = \frac{x+2}{-1} = -2\frac{1+2+5}{2} = -8$$

$\Rightarrow \quad x = -7, x = 6$

so, $B(-7, 6)$.

Image of A in $x + 2y$, is

$$\Rightarrow \quad \frac{x-1}{1} = \frac{y+2}{2} = -2\frac{1-4}{5} = \frac{6}{5}$$

$$\Rightarrow \quad x = \frac{11}{5} \text{ and } y = \frac{2}{5}$$

So, $C\left(\frac{11}{5}, \frac{2}{5}\right)$.

Hence equation of line BC is

$$y - 6 = \frac{28}{-46}(x+7)$$

$$\Rightarrow \quad y - 6 = \frac{-14}{23}(x+7)$$

$\Rightarrow \quad 14x + 23y = 138 - 98$

$\Rightarrow \quad 14x + 23y - 40 = 0$

Hence, option (a) is correct.

47. **(b)** Since equation of line passing through origin is $y - mx = 0$.

Hence $4 = \frac{|4-3m|}{\sqrt{1+m^2}}$ where m is slope of line.

or $\quad 16 + 16m^2 = 16 + 9m^2 - 24m$

$$\Rightarrow \quad 7m^2 + 24m = 0 \Rightarrow m = 0 \text{ or } m = -\frac{24}{7}$$

so combined equation of required lines

$$y\left(y + \frac{24}{7}x\right) = 0 \Rightarrow 7y^2 + 24xy = 0$$

Therefore, option (b) is correct.

48. **(a)** Let homogenisation of the curve $y^2 - 4ax = 0$.

For line $y = mx + C$ where $C = y - mx$

Getting combined equation of straight lines which subtend 90° at centre is

$$y^2 - 4ax\left(\frac{y-mx}{c}\right) = 0$$

$\Rightarrow \quad c + 4am = 0 \quad$ ...(i)

$y = m(x - 4a)$, represent family of line passes through $(4a, 0)$. Hence, option (a) is correct.

49. **(a)** Given abscissae of points P, Q are roots of x of the equation $2x^2 + 4x - 7 = 0$ and their ordinates are the roots of the equation $3x^2 - 12x - 1 = 0$.

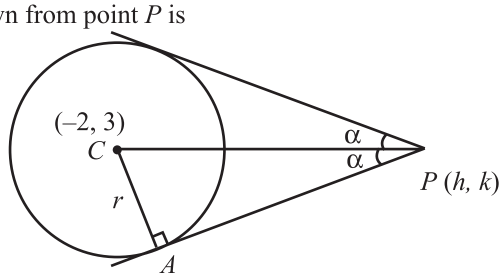
Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, then

$x_1 + x_2 = -2$ and $y_1 + y_2 = 4$

Now, centre of the circle with PQ as a diameter is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (-1, 2).$$

50. **(d)** Let a circle $C : (-2, 3)$ with centre and tangents drawn from point P is



$$\because \quad \tan\alpha = \frac{AC}{PA}$$

$$\Rightarrow \quad \tan\alpha = \frac{\sqrt{4+9-9\sin^2\alpha - 13\cos^2\alpha}}{\sqrt{h^2+k^2+4h-6k+9\sin^2\alpha+13\cos^2\alpha}}$$

$$= \frac{\sqrt{13\sin^2\alpha - 9\sin^2\alpha}}{\sqrt{h^2+k^2+4h-6k+9+4\cos^2\alpha}}$$

$$= \sqrt{\frac{4\sin^2\alpha}{h^2+k^2+4h-6k+9+4\cos^2\alpha}}$$

$$\Rightarrow \quad \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{4\sin^2\alpha}{h^2+k^2+4h-6k+9+4\cos^2\alpha}$$

$$\Rightarrow \quad h^2+k^2+4h-6k+9+4\cos^2\alpha = 4\cos^2\alpha$$

$$\Rightarrow \quad h^2+k^2+4h-6k+9 = 0$$

On taking locus of point $P(h, k)$, we get

$x^2 + y^2 + 4x - 6y + 9 = 0$

51. **(b)** Since, given equation of circle is,

$x^2 + y^2 - 4x - 6y - 12 = 0$ with centre $C_1(2, 3)$ and radius $r_1 = \sqrt{4+9+12} = 5$.

Let $C_2(h, k)$ and $r = 3$ (given) for required circle touches the given circle at $A(-1, -1)$.

The point $A(-1, -1)$ divides the line joining the centres $C_1(2, 3)$ and $C_2(h, k)$ externally in 5 : 3 so

$\Rightarrow \quad (-1, -1) = \left(\frac{5h-6}{2}, \frac{5k-9}{2}\right)$

$\Rightarrow \quad h = \frac{4}{5}$ and $k = \frac{7}{5}$

so equation of required circle is

$\left(x - \frac{4}{5}\right)^2 + \left(y - \frac{7}{5}\right)^2 = (3)^2$

$\Rightarrow \quad x^2 + y^2 - \frac{8x}{5} - \frac{14y}{5} + \frac{65}{25} = 9$

$\Rightarrow \quad 5x^2 + 5y^2 - 8x - 14y - 32 = 0$

52. **(d)** Since given equation of the circle is,

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

With centre $(-g, -f)$ lies on the line

$2x + 3y - 7 = 0$

will satisfy it

$\Rightarrow \quad 2g + 3f + 7 = 0$ ...(ii)

Since circle (i) cuts the given circles

$x^2 + y^2 - 4x - 6y + 11 = 0$

and $x^2 + y^2 - 10x - 4y + 21 = 0$

orthogonally

so, $2g(-2) + 2f(-3) = c + 11$

$\Rightarrow \quad 4g + 6f + c + 11 = 0$ ...(iii)

and $2g(-5) + 2f(-2) = c + 21$

$\Rightarrow \quad 10g + 4f + c + 21 = 0$ ...(iv)

From Eqs. (iii) and (iv), we get

$6g - 2f + 10 = 0$ ...(v)

From Eq. (ii) and (v), we get

$11f + 11 = 0 \Rightarrow f = -1$

so, $g = -2 \Rightarrow c = 3$

$\therefore \quad 5g - 10f + 3c = -10 + 10 + 9 = 9.$

53. **(a)** Given radical axis of the circles

$x^2 + y^2 + 2gx + 2fy + c = 0$

and $2x^2 + 2y^2 + 3x + 8y + 2c = 0$

is $(4g - 3)x + (4f - 8)y = 0$ ...(i)

Since, the radical axis (i) touches the circle

$x^2 + y^2 + 2x + 2y + 1 = 0$, so

$$\sqrt{1+1-1} = \frac{|-(4g-3)-(4f-8)|}{\sqrt{(4g-3)^2+(4f-8)^2}}$$

$\Rightarrow \quad (4g-3)^2 + (4f-8)^2 + 2(4g-3)(4f-8)$

$= (4g-3)^2 + (4f-8)^2$

$\Rightarrow \quad 8(4g-3)(f-2) = 0$

$\Rightarrow \quad (4g-3)(f-2) = 0$

54. **(b)** Here length of the chord intercepted by the line $y = mx + c$ by the given parabola $x^2 = 4ay$ is given as

$4\sqrt{a(1+m^2)(c+am^2)}$

So, $\sqrt{40} = 4\sqrt{a(1+4)(1+a(4))}$

$\Rightarrow \quad 40 = 16(5a(1+4a))$

or $1 = 2a(1 + 4a)$

$\Rightarrow \quad 8a^2 + 2a - 1 = 0$

or $8a^2 + 4a - 2a - 1 = 0$

$\Rightarrow \quad a = \frac{1}{4}$ or $-\frac{1}{2}$

Hence, option (b) is correct.

55. **(d)** Here, equation of normal to parabola $y^2 = 4ax$, having slope 'm' is $y = mx - 2am - am^3$

It passes through (h, k) so $k = mh - 2am - am^3$ is cubic equation in 'm'. Let having roots m_1, m_2 and m_3.

So, $m_1m_2m_3 = -\frac{k}{a}$

and $m_1m_2 = -1$ due to perpendicular normals.

So $m_3 = \frac{k}{a}$

$k = \frac{k}{a}h - 2a\frac{k}{a} - a\left(\frac{k}{a}\right)^3 \Rightarrow 1 = \frac{h}{a} - 2 - \frac{k^2}{a^2}$

$\Rightarrow \quad \frac{k^2}{a^2} = \frac{h}{a} - 3 \Rightarrow k^2 = a(h - 3a)$

On taking locus (h, k), we get

$y^2 - ax + 3a^2 = 0$

56. **(c)** Let a parametric point $P(a\cos\theta, b\sin\theta)$ on the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

Hence area of $\Delta PF_1F_2 = \frac{1}{2}(2ae)b|\sin\theta|$

$= aeb|\sin\theta| = A$

For maximum value of A, $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

so $A_{max} = aeb$.

57. **(c)** Given ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$, so equation of chord joining the points $A(\alpha)$ and $B(\beta)$ on the ellipse is

$\frac{x}{5}\cos\frac{\alpha+\beta}{2} + \frac{y}{3}\sin\frac{\alpha+\beta}{2} = -\cos\frac{\alpha-\beta}{2}$ ...(i)

Hence, obtained chord is the focal chord so, it will pass through focus (4, 0).

Thus, $\frac{4}{5}\cos\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$

$\Rightarrow \quad 4\left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2} - \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)$

$= 5\left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2} + \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)$

$\Rightarrow \quad 4\left(\cot\frac{\alpha}{2}\cot\frac{\beta}{2} - 1\right) = 5\left(\cot\frac{\alpha}{2}\cot\frac{\beta}{2} + 1\right)$

$\Rightarrow \quad \cot\frac{\alpha}{2}\cot\frac{\beta}{2} = -9$

58. **(a)** Let's deduce the given equation of hyperbola $16x^2 - 25y^2 - 96x + 100y - 356 = 0$ in to

$$\frac{(x-3)^2}{25} - \frac{(y-2)^2}{16} = 1$$

Now equation of tangent to the hyperbola with slope '1' is

$$y - 2 = 1(x-3) + \sqrt{25(1) - 16}$$

$\Rightarrow$ $y - 2 = x - 3 + 3$

or $x - y - 4 = 0$

$\Rightarrow$ $x - y + 2 = 0$

59. **(a)** Given points $P(0, 7, 10)$, $Q(-1, 6, 6)$ and $R(-4, 9, 6)$ Here, we think acc. to options and to find side lengths.

$\therefore$ $PQ = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

$QR = \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$

and $PR = \sqrt{16+4+16} = \sqrt{36} = 6$

$\because$ $PQ^2 + QR^2 = 18 + 18 = 36 = PR^2$ and $PQ = QR$

$\therefore$ PQR is a right angled isosceles triangle.

60. **(a)** Given, points $A(2, 3, 5)$, $B(\alpha, 3, 3)$ and $C(7, 5, \beta)$ are Vertices of a triangle ABC,

$\therefore$ Mid-point of BC is $D\left(\frac{\alpha+7}{2}, 4, \frac{3+\beta}{2}\right)$

$\because$ Direction ratios of line joining points $A(2, 3, 5)$ and $D\left(\frac{\alpha+7}{2}, 4, \frac{3+\beta}{2}\right)$ is $\left(\frac{\alpha+3}{2}, 1, \frac{\beta-7}{2}\right)$.

Since it is given that the line segment AD is equally inclined with the co-ordinate axes, so

$$\frac{\alpha+3}{2} = 1 = \frac{\beta-7}{2}$$

$\Rightarrow$ $\alpha = -1$ and $\beta = 9$

$\therefore$ $\cos^{-1}\left(\frac{\alpha}{\beta}\right) = \cos^{-1}\left(-\frac{1}{9}\right)$

61. **(a)** Equation of plane passes through the origin and containing the line

$\vec{\mathbf{r}} = (\hat{i} + 2\hat{j} - 3\hat{k}) + t(2\hat{i} - 3\hat{j} + \hat{k})$ is

$$(\vec{\mathbf{r}} - 0)\cdot[(\hat{i} + 2\hat{j} - 3\hat{k}) \times (2\hat{i} - 3\hat{j} + \hat{k})] = 0$$

$\Rightarrow$ $\vec{\mathbf{r}} \cdot [\hat{i}(2-9) - \hat{j}(1+6) + \hat{k}(-3-4)] = 0$

$\Rightarrow$ $\vec{\mathbf{r}} \cdot (-7\hat{i} - 7\hat{j} - 7\hat{k}) = 0$

$\Rightarrow$ $\vec{\mathbf{r}} \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$ or $x + y + z = 0$

62. **(d)** Since it is given that,

$\lim\limits_{x\to\infty}\left\{\frac{x^3+1}{x^2+1} - (\alpha x + \beta)\right\}$ exists and equals to 2.

$$\lim_{x\to\infty}\left\{\frac{x^3+1}{x^2+1} - (\alpha x + \beta)\right\} = 2$$

or, $\lim\limits_{x\to\infty} \frac{x^3 + 1 - \alpha x^3 - \beta x^2 - \alpha x - \beta}{x^2 + 1} = 2$

For the existance of limit, coefficient of $x^3 = 0$

$\therefore$ $\alpha = 1$

$\therefore$ $\lim\limits_{x\to\infty} \frac{-\beta x^2 - x - \beta + 1}{x^2 + 1} = 2$

$\Rightarrow$ $\lim\limits_{x\to\infty} \frac{-\beta - \frac{1}{x} - \frac{\beta}{x^2} + \frac{1}{x^2}}{1 + \frac{1}{x^2}} = 2$

$\Rightarrow$ $-\beta = 2 \Rightarrow \beta = -2$

$\therefore$ $(\alpha, \beta) = (1, -2)$

63. **(d)** Since for $k > 0$, given

$$\sum_{x=0}^{\infty} \frac{k^x}{x!} \lim_{n\to\infty} \frac{n!}{(n-x)!}\left(1 - \frac{k}{n}\right)^{n-x}\left(\frac{1}{n}\right)^x$$

$$= \lim_{n\to\infty} \sum_{x=0}^{n} \frac{n!}{x!(n-x)!}\left(1 - \frac{k}{n}\right)^{n-x}\left(\frac{k}{n}\right)^x$$

$$= \lim_{n\to\infty} \sum_{x=0}^{n} {}^nC_x \left(1 - \frac{k}{n}\right)^{n-x}\left(\frac{k}{n}\right)^x$$

$$= \lim_{n\to\infty}\left(1 - \frac{k}{n} + \frac{k}{n}\right)^n = \lim_{n\to\infty} 1^n = 1.$$

64. **(d)** Since given function $f : R \to R$ in such a case

$$f(x) = \begin{cases} 5, & \text{if } x \le 1 \\ a + bx, & \text{if } 1 < x < 3 \\ b + 5x, & \text{if } 3 \le x < 5 \\ 30, & \text{if } x \ge 5 \end{cases}$$

Here we check continuity at all the given points,

$x = 1$, $a + b = 5$

At $x = 3$, $a + 3b = b + 15$

$\Rightarrow$ $a + 2b = 15$

At $x = 5$, $b + 25 = 30 \Rightarrow b = 5$

From above equations, $a = 5$

Hence given function is continuous at $x = 1$, $x = 3$ and $x = 5$.

But $a = 5, b = 5 \not\Rightarrow a + b = 5$

So, $f : R \to R$ is not continuous for any values of a and b.

Hence, option (d) is correct.

65. **(d)** The given function $y = [x] + |1 - x|$ have point of discontinuity at $x = 0$, 1, 2 and 3 for $-1 \le x \le 3$. So function $y = [x] + |1 - x|$ is not differentiable at 4 points.

Hence, option (d) is correct.

66. **(c)** Since the given equation is

$$\sqrt{1 - x^6} + \sqrt{1 - y^6} = a(x^3 - y^3)$$

This can be written as

$$\frac{\sqrt{1 - x^6} + \sqrt{1 - y^6}}{x^3 - y^3} = a$$

Now differentiating both sides w.r.t., x, we get

$$\frac{\left[(x^3-y^3)\left(\frac{-6x^5}{2\sqrt{1-x^6}}-\frac{6y^5}{2\sqrt{1-y^6}}\frac{dy}{dx}\right)-\sqrt{1-x^6}+\sqrt{(1-y^6)}\left(3x^2-3y^2\frac{dy}{dx}\right)\right]}{(x^3-y^3)^2}=0$$

$$\Rightarrow \left(y^2(\sqrt{1-x^6}+\sqrt{1-y^6})-\frac{y^5(x^3-y^3)}{\sqrt{1-y^6}}\right)\frac{dy}{dx}$$

$$=x^2(\sqrt{1-x^6}+\sqrt{1-y^6})+\frac{x^5}{\sqrt{1-x^6}}(x^3-y^3)$$

$$\Rightarrow y^2\frac{dy}{dx}\left[\frac{\sqrt{1-x^6}+\sqrt{1-y^6}+(1-y^6)-y^3x^3+y^6}{\sqrt{1-y^6}}\right]$$

$$\left[\frac{(1-x^6)+\sqrt{1-y^6}\sqrt{1-x^6}+x^6-x^3y^3}{\sqrt{}}\right]$$

$$\Rightarrow y^2\frac{dy}{dx}\left[\frac{\sqrt{1-x^6}\sqrt{1-y^6}+1-x^3y^3}{\sqrt{1-y^6}}\right]$$

$$=x^2\left[\frac{1+\sqrt{1-y^6}\sqrt{1-x^6}-x^3y^3}{\sqrt{1-x^6}}\right]$$

$$\Rightarrow y^2\frac{dy}{dx}=x^2\sqrt{\frac{1-y^6}{1-x^6}}$$

67. **(d)** Given, $f(x)$ is a twice differentiable function at point P, where $\frac{dy}{dx}=4,\ \frac{d^2y}{dx^2}=-3$

Hence, $\frac{d^2x}{dy^2}=\frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{dx}{dy}\frac{d}{dx}\left(\frac{1}{\left(\frac{dy}{dx}\right)}\right)$

$$=\left(\frac{dx}{dy}\right)\left(\frac{-\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^2}\right)=-\frac{\left(\frac{d^2y}{dx^2}\right)}{\left(\frac{dy}{dx}\right)^3}$$

$$\therefore \left(\frac{d^2x}{dy^2}\right)_p=-\frac{(-3)}{(4)^3}=\frac{3}{64}.$$

68. **(a)** Since from the given formula T is directly proportional to and error is 2 minutes less per day.
Hence, for simple pendulum of length L, the time of oscillation is given as

$$T=2\pi\sqrt{\frac{L}{g}}\Rightarrow\frac{\Delta T}{T}\times100=\frac{1}{2}\frac{\Delta L}{L}$$

$$\Rightarrow \frac{\Delta L}{L}\%=-\frac{2\times2}{24\times60}\times100 \quad (\because \Delta T=2\text{ min})$$

$$=-\frac{5}{18}$$

$\therefore \frac{\Delta L}{L}=-\frac{5}{18}$ percentage.

69. **(d)** Given $A\Rightarrow$ A.M., $G\Rightarrow$ G.M., $H\Rightarrow$ H.M. of S
$\Rightarrow$ Sum of the numbers
The function $f(x)$ is

$$=\sum_{k=1}^{n}(x-a_k)^2=\sum_{k=1}^{n}(x^2-2xa_k+a_k^2)$$

$$=nx^2-2x(a_1+a_2+a_3+...a_n)+(a_1^2+a_2^2+...+a_n^2)$$

$\because$ The quadratic expression ax^2+bx+c has its minimum value at $x=-\frac{b}{2a}$.

$\therefore$ $f(x)$ has it's minimum value at

$$x=-\frac{-2(a_1+a_2+a_3+...+a_n)}{2n}$$

$$=\frac{a_1+a_2+a_3+...+a_n}{n}\Rightarrow x=A$$

70. **(c)** Given at $x=c,\ m>1,\ n>1$
Since applicability of Rolle's theorem for the function
$f(x)=x^{2m-1}(a-x)^{2n}$ in $(0, a)$. So,

$$f'(x)=(2m-1)x^{2m-2}(a-x)^{2n}-2n(a-x)^{2n-1}x^{2m-1}$$

$$f'(c)=0$$

$$\Rightarrow (2m-1)c^{2m-2}=2nc^{2m-1}(a-c)^{2n-1}$$

$$\Rightarrow \frac{(2m-1)}{c}=\frac{2n}{a-c}$$

$$\Rightarrow c=\frac{a(2m-1)}{2n+2m-1}$$

71. **(d)** Since the given function is defined from $f:[-1,1]\to R$

and given as, $f(x)=\begin{cases}2^x+1, & \text{for } x\in[-1,0)\\ 1, & \text{for } x=0\\ 2^x-1, & \text{for } x\in(0,1]\end{cases}$

According to definition,
the given function $f(x)$ in $x\in(-1,0)$ strictly increasing and $f(0^-)\to2$

but $f(0)=1$ and in interval $x\in(0,1)$, again it is strictly increasing, but $f(0^+)=0$.
So, function has neither maximum nor minimum.

72. **(a)** Given $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}dx$

Multiplying by x in numerator and denominator

$$= \int \frac{x^2-1}{(x+1)^2 x\sqrt{1+\frac{1}{x}+x}}dx$$

$$= \int \frac{1-\frac{1}{x^2}}{\left(1+\frac{1}{x}+x\right)\sqrt{1+\frac{1}{x}+x}}dx = \int \frac{x^2-1}{\left(1+1+\frac{1}{x}+x\right)x^2\sqrt{1+\frac{1}{x}+x}}dx$$

Let $1+\frac{1}{x}+x = t^2$

$$\Rightarrow \left(1-\frac{1}{x^2}\right)dx = 2t\,dt = \int \frac{2t\,dt}{(1+t^2)t} = 2\int \frac{dt}{1+t^2}$$

$$= 2\tan^{-1}(t)+c = 2\tan^{-1}\left(\sqrt{\frac{1+x+x^2}{x}}\right)+C$$

73. **(c)** Given integral

$I(x) = \int x^2(\log x)^2\,dx$ and $I(1) = 0$

$$\Rightarrow I(x) = \int x^2(\log x)^2 dx = \frac{x^3}{3}(\log x)^2 - \int \frac{x^3}{3}\frac{2\log x}{x}dx$$

[using integration by parts]

$$= \frac{x^3}{3}(\log x)^2 - \frac{2}{3}\left[\frac{x^3}{3}(\log x) - \int \frac{x^3}{3}\left(\frac{1}{x}\right)dx\right]$$

$$= \frac{x^3}{3}(\log x)^2 - \frac{2}{3}\left[\frac{x^3}{3}(\log x) - \frac{1}{3}\frac{x^3}{3}\right]+C$$

$$= \frac{x^3}{27}[9(\log x)^2 - 6(\log x)+2]+C$$

$\because \quad I(1) = 0$

$\Rightarrow C = \frac{-2}{27}$

Hence $I(x) = \frac{x^3}{27}[9(\log x)^2 - 6(\log x)+2] - \frac{2}{27}$

74. **(b)** Let integral

$$I = \int \frac{x^5\,dx}{(x^2+x+1)(x^6+1)(x^4-x^3+x-1)}$$

Here $(x^2+x+1)(x^4-x^3+x-1) = (x-1)(x^3+1)$

$(x^2+x+1) = (x^3-1)(x^3+1) = x^6-1$

Hence $I = \int \frac{x^5\,dx}{(x^6+1)(x^6-1)}$

Let $x^6 = t \Rightarrow 6x^5\,dx = dt$

$$I = \frac{1}{6}\int \frac{dt}{(t+1)(t-1)} = \frac{1}{12}\log_e\left|\frac{t-1}{t+1}\right|+c$$

$$= \frac{1}{12}\log_e\left|\frac{x^6-1}{x^6+1}\right|+c$$

75. **(d)** Let integral, $I = \int \frac{dx}{x+\sqrt{x-1}}$

and, put $x-1 = t^2 \Rightarrow dx = 2t\,dt$

then $I = \int \frac{2t}{(t^2+1)+t}dt = \int \frac{(2t+1)-1}{t^2+t+1}dt$

$$= \int \frac{2t+1}{t^2+t+1}dt - \int \frac{dt}{t^2+t+1}$$

$$= \log_e|t^2+t+1| - \int \frac{dt}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$= \log_e|t^2+t+1| - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+c$$

Since, $t = \sqrt{x-1}$,

So, $I = \log_e|x+\sqrt{x-1}| - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)+c$

76. **(a)** Let integral

$$I = \int_{\log_e 2}^{x} \frac{dt}{\sqrt{e^t-1}} = \frac{\pi}{6} \qquad \text{(given)}$$

$$\Rightarrow \int_{\log_e 2}^{x} \frac{e^{-\frac{1}{2}}}{\sqrt{1-(e^{-t/2})^2}}dt = \frac{\pi}{6}$$

Let $e^{-t/2} = u$, at $t = \log_e 2$, $u = \frac{1}{\sqrt{2}}$ and at $t = x$,

$u = e^{-x/2}$ and $e^{-t/2}dt = -2du$

So, $I = \int_{\frac{1}{\sqrt{2}}}^{e^{-x/2}} \frac{-2du}{\sqrt{1-u^2}} = \frac{\pi}{6}$

$$\Rightarrow [-2\sin^{-1}u]_{\frac{1}{\sqrt{2}}}^{e^{-\frac{x}{2}}} = \frac{\pi}{6} \Rightarrow \sin^{-1}(e^{-x/2}) - \frac{\pi}{4} = -\frac{\pi}{12}$$

$$\Rightarrow \sin^{-1}(e^{-x/2}) = \frac{\pi}{6} \Rightarrow e^{-x/2} = \sin\frac{\pi}{6} - \frac{1}{2}$$

$$\Rightarrow -\frac{x}{2} = \log_e\left(\frac{1}{2}\right) = -\log_e(2) \Rightarrow x = 2\cdot\log_e(2)$$

77. **(d)** Let integral

$$I=\int_0^1 \frac{\log_e(1+x)}{1+x^2}dx$$

Put $x=\tan\theta$, so at $x=0$, $\theta=0$ and

at $x=1$, $\theta=\frac{\pi}{4}$

and $dx=\sec^2\theta\, d\theta$

$$\text{Now, } I=\int_0^{\frac{\pi}{4}}\frac{\log_e(1+\tan\theta)}{\sec^2\theta}\sec^2\theta\, d\theta$$

$$=\int_0^{\frac{\pi}{4}}\log_e(1+\tan\theta)\,d\theta \qquad \text{...(i)}$$

On, applying property,

$$\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx,$$

we get

$$I=\int_0^{\frac{\pi}{4}}\log_e\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$$

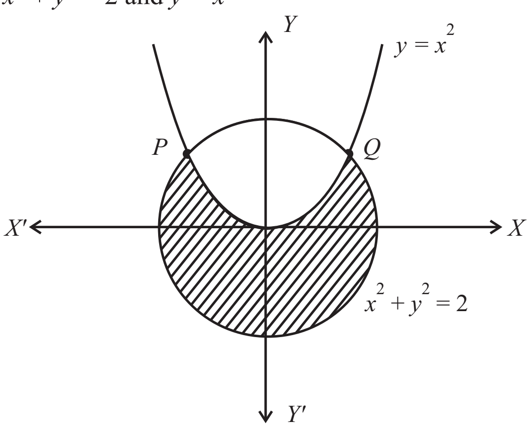
$$=\int_0^{\frac{\pi}{4}}\log_e\left(\frac{2}{1+\tan\theta}\right)d\theta$$

$$=\int_0^{\frac{\pi}{4}}\log_e(2)\,dx-\int_0^{\frac{\pi}{4}}\log_e(1+\tan\theta)\,d\theta$$

$$\Rightarrow\quad I=\frac{\pi}{4}\log_e(2)-I \qquad \text{[from Eq. (i)]}$$

$$\Rightarrow\quad 2I=\frac{\pi}{4}\log_e(2)\Rightarrow I=\frac{\pi}{8}\log_e(2)$$

78. **(a)** Given equations of circle and parabola are $x^2+y^2=2$ and $y=x^2$



On solving the given equation, we get

$y^2+y-2=0 \Rightarrow y^2+2y-y-2=0$

$\Rightarrow\quad y(y+2)-1(y+2)=0$

$\Rightarrow\quad y=1 \qquad [\because\ y>0]$

Hence the required area

$$=\pi+2\int_0^1(\sqrt{(2-y^2)}-\sqrt{y})\,dy$$

$$=\pi+2\left[\frac{y}{2}\sqrt{2-y^2}+\frac{2}{2}\sin^{-1}\frac{y}{\sqrt{2}}-\frac{2}{3}y^{3/2}\right]_0^1$$

$$=\pi+2\left[\frac{1}{2}+\frac{\pi}{4}-\frac{2}{3}\right]=\frac{3\pi}{2}+1-\frac{4}{3}$$

$$=\frac{3\pi}{2}-\frac{1}{3}\text{ sq. units}$$

79. **(d)** Given, c is a parameter and equation of the family of curves is

$x^2=c(y+c)^2$

or, $\quad x=\sqrt{c}(y+c) \qquad$ ...(i)

On differentiating both sides

$$1=\sqrt{c}\frac{dy}{dx}$$

$$\Rightarrow\quad \sqrt{c}=\frac{dx}{dy} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii) on eliminating 'c', we get

$$x=\left(\frac{dy}{dx}\right)\left[y+\left(\frac{dx}{dy}\right)^2\right]$$

$$\Rightarrow\quad x\left(\frac{dy}{dx}\right)^3=y\left(\frac{dy}{dx}\right)^2+1$$

$$\Rightarrow\quad x\left(\frac{dy}{dx}\right)^3-y\left(\frac{dy}{dx}\right)^2-1=0$$

80. **(a)** Given $f(x)$, $f'(x)$ are positive functions and $f(0)=1$, $f'(0)=2$

$$\text{Since, }\begin{vmatrix} f(x) & f'(x)\\ f'(x) & f''(x)\end{vmatrix}=0 \qquad \text{(given)}$$

$$\Rightarrow\quad f(x)\,f''(x)-(f'(x))^2=0$$

$$\Rightarrow\quad f(x)\,f''(x)=(f'(x))^2$$

$$\Rightarrow\quad \frac{f''(x)}{f'(x)}=\frac{f'(x)}{f(x)}$$

On, integrating both sides, we get

$$\int\frac{f''(x)}{f'(x)}dx=\int\frac{f'(x)}{f(x)}dx$$

$$\Rightarrow\quad \log_e(f'(x))=\log_e(f(x))+c$$

$\because\quad f(0)=1$ and $f'(0)=2$

$\therefore\quad c=\log_e 2$

$$\log_e(f'(x))=\log_e(f(x))+\log_e 2$$

$$\Rightarrow\quad f'(x)=2f(x)$$

$$\Rightarrow\quad \frac{f'(x)}{f(x)}=2$$

$$\Rightarrow\quad \int\frac{f'(x)}{f(x)}dx=\int 2\,dx\Rightarrow \log_e f(x)=2x+c$$

$\because\quad f(0)=1$, so $c=0$

$\because\quad \log_e f(x)=2x\Rightarrow f(x)=e^{2x}$

PHYSICS

81. **(c)** Suppose μ_0 is related with fundamental quantities as shown

$\mu_0 \propto e^a m^b c^c h^d \Rightarrow [\mu_0] = [e]^a [m]^b [c]^c [h]^d$

$[\mu_0] = [\text{M L T}^{-2}\ \text{A}^{-2}]$,

$[e] = [\text{A T}]$,

$[m] = [\text{M}]$,

$[c] = [\text{L T}^{-1}]$,

and $[h] = [\text{M L}^2\ \text{T}^{-1}]$

Using dimension of above physical quantities in Eq.

$[\text{M L T}^{-2}\ \text{A}^{-2}] = [\text{A T}]^a [\text{M}]^b [\text{L T}^{-1}]^c [\text{M L}^2\ \text{T}^{-1}]^d$

$[\text{M L T}^{-2}\ \text{A}^{-2}] = [\text{M}]^{b+d} [\text{L}]^{c+2d} [\text{T}]^{a-c-d} [\text{A}]^a$

Comparing dimensions of M, L, T and A on the both sides, we get

$b + d = 1$ ...(i)

$c + 2d = 1$ ...(ii)

$a - c - d = -2$ ...(iii)

$a = -2$ ...(iv)

After solving above equations we get

$a = -2, b = 0, c = -1, d = 1$

Putting values of *a, b, c, d* in eqn., we get

$$\mu_0 = \frac{h}{ce^2}$$

82. **(c)** Given, velocity, $v = 6t - 3t^2$

Velocity (v)

$$v = \frac{dx}{dt} \Rightarrow dx = v\,dt$$

Total displacement of the particle.

$$\int dx = \int v\,dt$$

$$\Rightarrow \Delta x = \int_0^2 v\,dt = \int_0^2 (6t - 3t^2)\,dt$$

$$= \left[\frac{6t^2}{2}\right]_0^2 - \left[\frac{3t^3}{3}\right]_0^2$$

$= [3(2)^2 - 3(0)^2] - [(2)^3 - (0)^2] = 4$ m.

Average velocity,

$$v_{\text{avg}} = \frac{\text{Total displacement}}{\text{Time interval}} = \frac{4}{2} = 2 \text{ m/s}$$

Hence, option (c) is correct.

83. **(c)** Given,

Distance between gun and target = 600 m

Velocity of bullet = 500 ms^{-1}

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{600}{500} = 1.2 \text{ s}$$

Vertical displacement equals height.

$$h = 0 \times (1.2) + \frac{1}{2} \times (-10) \times (1.2)^2$$

$$h = \frac{1}{2} \times (-10)(1.2 \times 1.2), \text{h} = -7.2 \text{ m}$$

84. **(c)** Given, angle of projection, $\theta = 60°$ and velocity, $u = 140\ \text{ms}^{-1}$ = initial velocity.

Horizontal component, $u_x = u \cos 60° = 140 \cos 60°$

and Vertical component, $u_y = \text{u} \sin 60° = 140 \sin 60°$

Suppose after time t, the inclination of particle with horizontal be 45° and at time t velocity along $x = v_x$ and along $y = v_y$.

$$\tan 45° = \frac{v_y}{v_x} = 1 \Rightarrow v_x = v_y$$

The horizontal component of velocity does not change

$\Rightarrow u \cos 60° = 140 \cos 60° = v \cos 45° = v_y = v_x$

Using 1st eqn. of motion in vertical direction

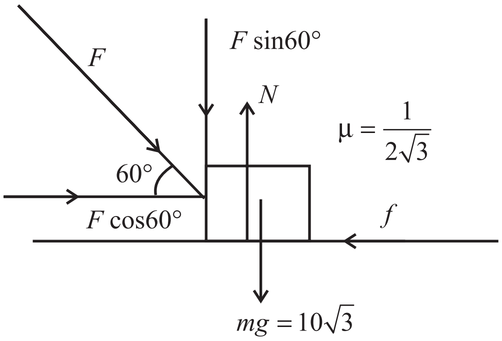
$v_y = u_y - gt$

$140 \cos 60° = 140 \sin 60° - 10t$ $[\because v_y = v_x]$

$$\Rightarrow 140 \times \frac{1}{2} = 140 \times \frac{\sqrt{3}}{2} - 10t$$

$\Rightarrow 7 \times (0.7320) = 5.124$ s.

85. **(a)** Free body diagram



$N = F \sin 60° + mg$

Force of friction, $f = \mu N$ [Limiting]

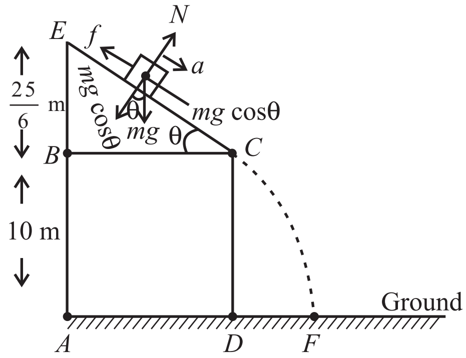
$$f = \mu(mg + F \sin 60°)$$

$$F \cos 60° = \frac{1}{2\sqrt{3}}(10\sqrt{3} + F \sin 60°)$$

$$\Rightarrow F \times \frac{1}{2} = \frac{1}{2\sqrt{3}}\left(10\sqrt{3} + F \times \frac{\sqrt{3}}{2}\right)$$

$\Rightarrow F = 20$ N.

86. **(d)** According to question, a small block in slide down from top E of inclined plane as shown in figure,



From newton's 2nd law

$\Rightarrow \quad mg\sin\theta - f = ma$ ...(1)

Due to slipping, $f = \mu_k N$

or $\quad f = \mu_k(mg\cos\theta)$ (From figure)

Putting value of f in eq^n. (1)

$\Rightarrow \quad mg\sin\theta - \mu_k mg\cos\theta = ma$

$\Rightarrow \quad a = 10\sin\theta - 1/8 \times 10\cos\theta$ ...(2)

$\because$ Given, angle of the inclined plane, $\theta = \sin^{-1}(0.6)$

or $\quad \sin\theta = 0.6$ and $\mu_k = \frac{1}{8}$

We know $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-(0.6)^2}$

$\cos\theta = 0.8$

Putting values in eqn. (2)

$a = 10(0.6) - \frac{1}{8} \times 10(0.8)$ or $a = 5$ ms^{-2}

Let v be velocity at point C then from third equation of the motion

$v^2 = u^2 + 2as$

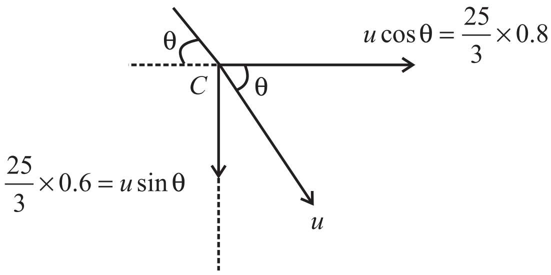
or $\quad v = \sqrt{2as}$ $\quad [u = 0]$

From $\triangle CBE$, $CE = \frac{EB}{\sin\theta} \Rightarrow s = \frac{25/6}{0.6} = \frac{25}{6 \times 0.6}$

$s = \frac{125}{18}$ m $= CE$

$\Rightarrow \quad V = \sqrt{2 \times 5 \times \frac{125}{18}} = \frac{25}{3}$ ms^{-1}

Resolving velocity at point C



Using equation of motion in vertical direction

$\Rightarrow \quad \Delta y = h = ut + \frac{1}{2}gt^2$

$\Rightarrow \quad 10 = u\sin\theta t + \frac{1}{2}gt^2$

$\Rightarrow \quad 10 = \frac{25}{3} \times 0.6t + \frac{1}{2} \times 10 \times t^2$ $\quad$ [as $\sin\theta = 0.6$]

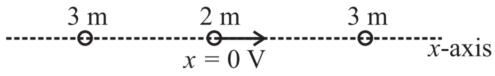
$\Rightarrow \quad \frac{25}{3} \times \frac{6}{10}t + \frac{1}{2} \times 10 \times t^2 = 10$

$\Rightarrow \quad 5t + 5t^2 = 10 \Rightarrow t^2 + t - 2 = 0$ or $t = 1$ sec.

Now, again from second Eqs. of motion in x-direction,

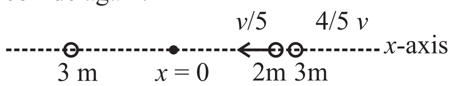
$\Rightarrow \quad DF = v\cos\theta \cdot t + 0$ or $DF = \frac{25}{3} \times 0.8 = \frac{20}{3}$ m

87. **(a)** According to question, three particles are situated as shown below,



Initially

After Ist collision velocity of balls will be as shown. 2m will slow down to $\frac{4}{5}v$ will collide with 3 m on left. After 2nd collision 2 m will further slow down and will not collide again.



After collision

So, collision of R and Q

Total no. of collision = 2.

88. **(c)** Given,

Depth of a well, $d = 30$ m

Water quantity per minute = 1800 litre

Cross-section area of pipe, $A = 30$ cm^2

$= 30 \times 10^{-4}$ m^2

Area × Velocity = Volume of water per second

$\Rightarrow$ velocity of water a jet $= \frac{1800 \times 10^{-3}}{(30 \times 10^{-4}) \times 60} = 10$ m/s

and work done by engine

= Change in P.E. + Change in K.E.

$= mgd + \frac{1}{2}mv^2$

$= 1800 \times 10 \times 30 + \frac{1}{2} \times 1800 \times (10)^2$

= 630000 J

Power of engine, $P = \frac{W}{t} = \frac{630000}{60}$

= 10500 W = 10.5 kW

89. **(a)** Given, mass of solid sphere or disc $M = 100$ kg

Radius of solid sphere, $R = 10$ m

Radius of circular disc, $r = 20$ m
and time = 1 hour = 60 minute = 60 × 60 sec
Moment of inertia of the solid sphere,

$$I_s = \frac{2}{5}MR^2 = \frac{2}{5}\times 100\times(10)^2 = 4000 \text{ kg-m}^2$$

Similarly,
Moment of inertia of the disc,

$$I_c = \frac{1}{2}Mr^2$$

$$= \frac{1}{2}\times 100\times(20)^2 = 20,000 \text{ kg-m}^2$$

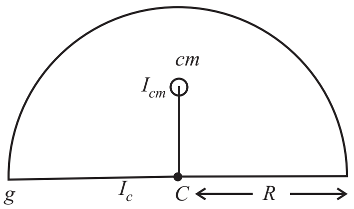
Rate of change of moment of inertia

$$= \frac{I_c - I_s}{t} = \frac{20000-4000}{60\times 60} = \frac{16000}{60\times 60} = \frac{160}{36}$$

$$= \frac{40}{9} \text{ kg-m}^2\text{s}^{-1}$$

90. **(d)** Given,
Mass (plate) = m
Radius (plate) = R
Using parallel axis theorem,

$$I_c = I_{cm} + mx^2$$



$$\frac{mR^2}{2} = I_{cm} + mx^2$$

Using $I_c = \frac{mR^2}{2}$

$$\Rightarrow \quad I_{cm} = \frac{mR^2}{2} - mx^2$$

91. **(b)** Loss in potential energy = gain in KE

Loss in PE $= \frac{Gm_1m_2}{r^2} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ ...(i)

If v_1 and v_2 are their respective velocities when they are a distance r apart then, from law of conservation of momentum.

$m_1 v_1 = m_2 v_2$ ...(ii)

From Eq^n. (i) and Eq^n. (ii),

$$v_1 = \sqrt{\frac{2Gm_2^2}{(m_1+m_2)r}}, \; v_2 = \sqrt{\frac{2Gm_1^2}{(m_1+m_2)r}}$$

Therefore, their relative velocity of approach is

$$v_1 + v_2 = \sqrt{\frac{2Gm_2}{r}} + \sqrt{\frac{2Gm_1}{r}} = \sqrt{\frac{2G}{r}(m_1+m_2)}$$

$$= \left(\frac{2G(m_1\times m_2)}{r}\right)^{\frac{1}{2}}$$

92. **(b)** As there is no external torque on planet, angular momentum (J) of a planet is constant.

$$\Rightarrow \quad mu_A r_A \sin\theta_A = mu_B r_B \sin\theta_B$$

$$r_A = 90\times 10^6 \text{ km}, \; r_B = 60\times 10^6 \text{ km}$$

$$\theta_A = 30°, \; \theta_B = 60°$$

Putting values of given quantities we get $\frac{u_A}{u_B} = \frac{2}{\sqrt{3}}$

Option (b) is correct.

93. **(c)** Volume of solid copper cube $= l^3 = 7^3 \times 10^{-6}$ m
Increase in pressure, p = 8000 kPa = 8000 × 10^3 Pa
Bulk modulus of copper, β = 140 Gpa = 140 × 10^9 Pa

Bulk modulus $B = \frac{p}{\left(\frac{\Delta V}{V}\right)}$

$$\Rightarrow \quad \Delta V = \frac{pV}{B} = \frac{8000\times 10^3\times(l)^3}{140\times 10^9} = 19.6\times 10^{-3} \text{ cm}^3$$

94. **(a)** Given, diameter of a pinhole, d = 0.2 mm
So, radius of pinhole,

$$r = \frac{0.2}{2} = 0.1 \text{ mm} = 0.1\times 10^{-3} \text{ m}$$

Pressure due to depth must be equal to excess pressure

$$\Rightarrow \quad h\rho g = \frac{2S}{r} \Rightarrow h = \frac{2T}{\rho g r} = \frac{2\times 0.007}{10^3\times 10\times 0.1\times 10^{-3}}$$

= 0.14 m = 14 cm.

95. **(d)** Given, focal length of spherical mirror, f = 150 cm
coefficient of linear expansion of steel,
$\alpha = 12\times 10^{-6}$° C^{-1}
As temperature changes by dt Radius R changes by dR we can define coefficient of linear exapnsion,

$$\alpha = \frac{dR/R}{dt} \Rightarrow \frac{dR}{R} = \alpha T$$

As $\frac{dR}{R} = \frac{df}{f}$ [As $R = 2f$]

$$\Rightarrow \quad \frac{df}{f} = \alpha dT \Rightarrow df = f\alpha dT$$

$$f' - f = f\alpha\Delta T$$

where, f = focal length of the mirror at temperature t and f' = final focal length of mirror at $t + dt$

$$f' = f + f\alpha dT$$

$$\Rightarrow \quad f' = f(1+\alpha dT)$$

$$\Rightarrow \quad f' = f(1+\alpha\Delta T)$$ (for large change in temperature)

$$\Rightarrow \quad f' = 150(1+12\times 10^{-6}\times 200)$$ [As $\Delta T = 200$]

f' = 150.36 cm.

96. **(d)** Given,

Length of rod, $l = 10$ cm = 0.1 m

Area of cross-section of rod, $A = 2.8 \times 10^{-4}$ m^2

Temperature difference = $\Delta T = 80 - 0 = 80°C$

Quantity of melted ice, $m = 20$ gm

Total time, $t = 5$ min = 300 sec.

and latent heat of ice, $s = 80$ cal g^{-1}

Heat flow per second $= \frac{mL \times 4.184}{t}$ J/s

$$= \frac{20 \times 80 \times 4.184}{300} = 22.314 \text{ J/s} \quad ...(1)$$

Heat current, $\frac{\Delta Q}{\Delta t} = \frac{kA \cdot \Delta T}{l}$

$$= \frac{k(2.8 \times 10^{-4}) \times 80}{0.1} \quad ...(2)$$

Equating (1) and (2),

$$\Rightarrow \quad 22314 = \frac{k(2.8 \times 10^{-4}) \times 80}{0.1}$$

$$\therefore \quad k = 99.61 \approx 100 \text{ Js}^{-1}\text{m}^{-1}\text{K}^{-1}$$

97. **(d)** Given, $V = kT^{2/3}$

Work done in changing volume by

$$dV = dW = P\,dV = \frac{RT}{V}dV \quad \left[\text{As } P = \frac{RT}{V}\right]$$

$$= \frac{RT}{kT^{2/3}}dV \quad \left[Put\ V = kT^{2/3}\right]$$

Taking derivative of $V = kT^{2/3}$ on the both sides, we get

$$dV = K\frac{2}{3}T^{-1/3}dT$$

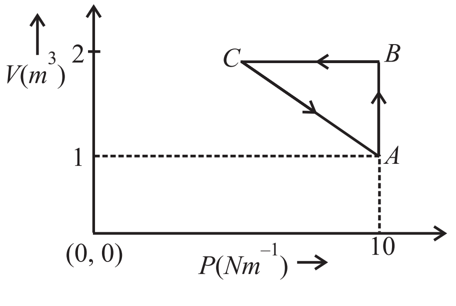
Putting value of dV in expression for dW

$$dW = \frac{2}{3}RdT$$

$$\Rightarrow \quad W\int dW = \frac{2}{3}R\int_{T_1}^{T_2} dT = \frac{2}{3}R(T_2 - T_1)$$

$$= \frac{2}{3}R(60-0) = \frac{2}{3} \times R \times 60 = 40\,R$$

98. **(a)** An ideal gas is taken through the cycle $A \to B \to C \to A$ as shown in figure,



$$W_{AB} = P \times \Delta V = 10 \times (2-1) = 10 \text{ J}$$

$$W_{BC} = P\,dV.W_{BC} = P \times (0) = 0 \text{ J as } dV = 0$$

Applying first law of thermodynamics

$Q = \Delta U + W$

Here, $\Delta U = 0$ as cyclic process

$Q = W$

$\Rightarrow \quad 5 = W_{AB} + W_{BC} + W_{CA}$

$\Rightarrow \quad 5 = 10 + 0 + W_{CA}$

$|W_{CA}| = 5$ J.

Option (a) is correct.

99. **(d)** Given, Energy 0.69 $eV = 0.69 \times 1.6 \times 10^{-19}$ V

Also average translational kinetic energy $= \frac{3}{2}kT$

$$= \frac{3}{2} \times 1.38 \times 10^{-23}T = 0.69 \times 1.6 \times 10^{-19}$$

$\Rightarrow \quad T = 5333$ K = (5333 – 273)°C = 5060°C.

100. **(c)** Let the distance of epicenter of earthquake from point of observation be d.

speed of S-wave, $v_S = 4.5$ km^{-1}s

speed of P-wave, $v_P = 8$ km^{-1}s

then, $d = V_P t_P = V_S t_S$

or, $\quad 8t_P = 4.5t_S$

$$t_P = \frac{4.5}{8}t_S \quad ...(i)$$

The first P-wave arrives 3.5 min earlier than the first S-wave.

Hence, $t_S - t_P = 3.5 \times 60$

$t_S - t_P = 210 \quad ...(ii)$

From Eq. (1), we get

$$t_S - \frac{4.5}{8}T_s = 210$$

$$\Rightarrow \quad \frac{8t_S - 4.5t_S}{8} = 210$$

$$\Rightarrow \quad 3.5t_S = 210 \times 8$$

$$\Rightarrow \quad t_S = \frac{210 \times 8}{3.5} = 480 \text{ s}$$

Now, $d = v_S t_S = 4.5 \times 480 = 2160$ km.

101. **(a)** Apparent frequency is given by,

$$\nu' = \left(\frac{v + v_0}{v - v_s}\right)\nu_0$$

Put $v_s = 0 \Rightarrow \nu' = \left(\frac{v + v_0}{v}\right)\nu_0$

Putting $v_0 = \frac{v}{5}$

$$\nu' = \left(\frac{v + \frac{v}{5}}{v}\right)\nu_0 = 1.2\nu_0$$

Apparent wavelength does not change by speed of observer.

102. (b) Given,

Focal length of convex lens, $f = 20$ cm

Object distance, $u = -0.1$ m $= -10$ cm

Radius of curvature of silvered surface, $R = 22$ cm

Focal length of the lens is 20 cm.

So, the power of lens, $P_1 = \frac{1}{20}D$

and focal length of concave mirror,

$$f = \frac{R}{2} = -11 \text{ cm} \quad [\text{As } R = 2f] \qquad ...(i)$$

Therefore, the power of the mirror,

$$P_2 = -\frac{1}{f_m} \Rightarrow P_m = \frac{1}{11}D \qquad ...(ii)$$

Total power P = Power of lens + Power of mirror

Light first enters lens then reflected back by mirror finally it passes through lens again be for forming image.

$$P = P_1 + P_2 + P_1 \qquad ...(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$P = \frac{1}{20} + \frac{1}{11} + \frac{1}{20}$$

$$\Rightarrow \quad P = \frac{21}{110}$$

The focal length of equivalent mirror for whole image formation,

$$f = -\frac{110}{21} \text{ cm}$$

Using mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{-21}{110} = \frac{1}{v} - \frac{1}{10}$$

$$\Rightarrow \quad v = -11 \text{ cm}$$

Hence, the distance of final image from the silvered surface is 11 cm.

103. (c) Condition for maxima is, $d\sin\theta = n\lambda$

$\Rightarrow \quad 2\lambda\sin\theta = n\lambda \qquad [\text{As } d = 2\lambda]$

$\Rightarrow \quad 2\sin\theta = n$

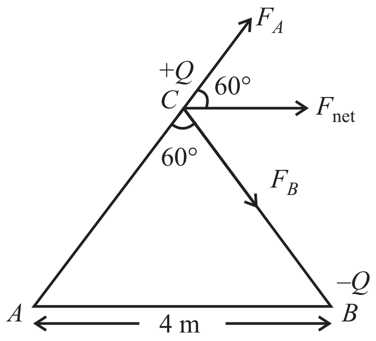
Maximum value of n is when $\sin\theta = 1$

$n_{max} = 2$

Possible values of n are –2, –1, 0, 1, 2. So, 5 maxima.

104. (a) $Q = 100\ \mu C$

F_A is force on charge at C due to A and F_B is force on charge at C dec. to B.



$$|F_A| = |F_B| = \frac{KQ^2}{r^2}$$

$$|F_{net}| = |F_A| = |F_B|$$

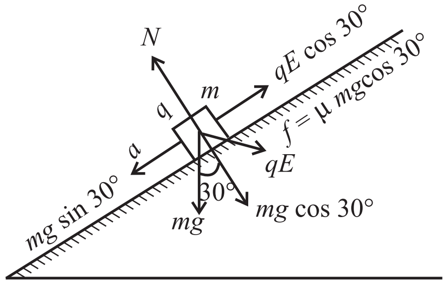
As angle between F_A and F_B is 120°.

Putting the given values in F_{net}, we get

$$= \frac{9\times10^9 \times (100\times10^{-6})^2}{(4)^2}$$

$$F_{net} = 5.625\text{N}, 60°$$

105. (b) Electric field = 100 v/m



Writing forces along inclined plane and $F = ma$

From fig.

$$mg\sin30° - \mu mg\cos30° - qE\cos30° = ma$$

Given,

$\mu = 0.2$, $m = 1$ kg, $q = 0.01C$ and $h = 1$ m and $E = 100$ V/m

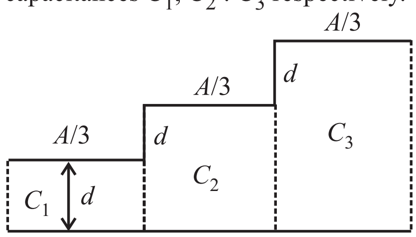
Putting these values, we get in above eq^n

$$10\times\frac{1}{2} - 0.2\times10\times\frac{\sqrt{3}}{2} - 0.01\times100\times\frac{\sqrt{3}}{2} = a$$

$$\Rightarrow \quad a = 2.3 \text{ m/s}^2$$

Option (b) is correct.

106. (d) 3 capacitors have been arranged like this with capacitances C_1, C_2 : C_3 respectively.



For parallel plate capacitors capacitance $= \dfrac{\varepsilon_0 A}{d}$

Capacitance of capacitor 1, $C_1 = \dfrac{\varepsilon_0 A}{3d}$

Capacitance of capacitor 2, $C_2 = \dfrac{\varepsilon_0 A}{3(2d)} = \dfrac{\varepsilon_0 A}{6d}$

Capacitance of capacitor 3, $C_3 = \dfrac{\varepsilon_0 A}{3(3d)} = \dfrac{\varepsilon_0 A}{9d}$

Net cpacitance $C_{net} = C_1 + C_2 + C_3$

$$C_{eq} = \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{6d} + \frac{\varepsilon_0 A}{9d}$$

$$\Rightarrow \quad C_{eq} = \frac{(6\varepsilon_0 + 3\varepsilon_0 + 2\varepsilon_0)A}{18d} = \frac{11\varepsilon_0 A}{18d}$$

Hence, the capacitance of the arrangement is $\dfrac{11\varepsilon_0 A}{18d}$.

107. (d) $\because$ Pressure difference due to surface tension

$$p_i - p_0 = \frac{4S}{r}$$

Electrostatic pressure due to change on the soap bubble, $= \dfrac{\sigma^2}{2\varepsilon_0}$

Here, σ = surface charge density and ε_0 = permittivity of the free space

Electric potential on spherical shell, $V = \dfrac{kq}{r}$

$$\Rightarrow \quad V = \frac{k(\sigma A)}{r} \qquad [\text{as } q = \sigma \cdot A]$$

Putting, $A = 4\pi r^2$ and $k = \dfrac{1}{4\pi\varepsilon_0}$

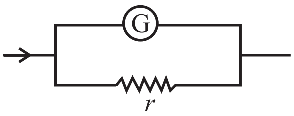
$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{\sigma 4\pi r^2}{r}\right) = \frac{\sigma r}{\varepsilon_0} \Rightarrow \sigma = \frac{\varepsilon_0 V}{r}$$

Pressure difference due to surface tension must cancel pressure difference due to charge.

$$\Rightarrow \quad \frac{4S}{r} = \frac{\sigma^2}{2\varepsilon_0} \Rightarrow \frac{4S}{r} = \frac{\left(\frac{\varepsilon_0 V}{r}\right)^2}{2\varepsilon_0} \Rightarrow V = \sqrt{\frac{8Sr}{\varepsilon_0}}$$

So, the potential of the bubble is $\dfrac{\sqrt{8Sr}}{\varepsilon_0}$.

108. (a) Resistance of galvanometer, $G = 112\ \Omega$



$$\frac{\text{Heat through shunt}}{\text{Heat through galvanometer}} = \frac{7}{5}$$

$$\Rightarrow \quad \frac{\frac{V^2}{r}}{\frac{V^2}{G}} = \frac{7}{5} \quad \text{Putting } G = 112\ \Omega$$

$$\Rightarrow \quad r = \frac{5}{7} \times 112 = 80\ \Omega$$

$\Rightarrow$ Shunt resistance = 80 Ω.

109. (c) Given, $V_{rms} = 220$ V, supply frequency, $f = 50$ Hz rms current, $I_{rms} = 440$ mA

Capacitance reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$$X_C = \frac{\pi \times 10^6}{2\pi \times 50 \times 50} \qquad \left[\text{As } C = \frac{50}{\pi}\mu F\right]$$

$$\Rightarrow \quad X_C = \frac{10^6}{2500 \times 2} \Rightarrow X_C = 200\ \Omega$$

Inductive reactance, $X_L = \omega L$

$X_L = 2\pi f \times L \qquad (\text{As } \omega = 2\pi f)$

$$= 2\pi \times 50 \times \frac{6}{\pi} = 600\ \Omega \qquad \left[\text{As } L = \frac{6}{\pi}H\right]$$

$\therefore \quad X_L = 600\ \Omega$

Now, impedance of LCR circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Putting the given values, we get

$$500 = \sqrt{R^2 + (600 - 200)^2}$$

Squaring both sides

$250000 = R^2 + (400)^2$

$R^2 = 250000 - 160000\ R^2 = 90000$

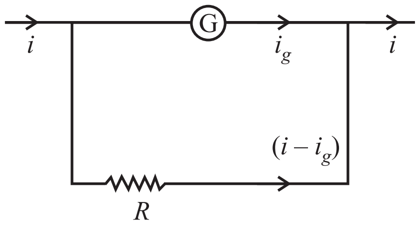
$\Rightarrow \quad R = 300\ \Omega$

Impedance, $Z = \dfrac{V_{rms}}{I_{rms}} = \dfrac{220}{440} \times 10^3 \Rightarrow Z = 500\Omega$.

110. (d) Neutron is neutral so it cannot move in circular path in magnetic field. Only charged particle can do so. Option (d) is correct.

111. (d) According to the question,

Case ***a***



$v_G = v_{R_1}$

$\Rightarrow \quad i_G G = (i - i_G)R_1$

$$\Rightarrow \frac{i}{51}G = \left(i - \frac{i}{51}\right)R_1$$

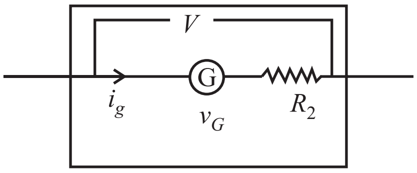
$$\Rightarrow R_1 = \frac{G}{50} \quad ...(i)$$

$$\Rightarrow v_G = \frac{v}{11}$$

$$\Rightarrow v_{R_2} = \frac{10}{11}v$$

$$\Rightarrow \frac{v_G}{v_{R_2}} = \frac{1}{10}$$

Case ***b***



$$v_G = i_g G \Rightarrow v_{R_2} = i_g R_2$$

$$\Rightarrow \frac{v_G}{v_{R_2}} = \frac{i_g G}{i_g R_2} = \frac{G}{R_2} \Rightarrow \frac{1}{10} = \frac{G}{R_2}$$

$$\Rightarrow R_2 = 10G \quad ...(ii)$$

Now, from Eqn. (i) and (ii), we get

$$\frac{R_2}{R_1} = \frac{10G}{\frac{G}{50}} = 500 \qquad \therefore \frac{R_2}{R_1} = 500$$

112. **(b)** Magnetic field at C = magnetic field at C due to straight wire + Magnetic field due to semi circular ring

$$= \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4r} = \frac{\mu_0 i}{4\pi r}(1+\pi)$$

113. **(b)** Given,

Number of turns in the coil, $N = 30$ turns

Area of the coil, $A = 2.5\ \text{cm}^2 = 2.5 \times 10^{-4}\ \text{m}^2$

Total charge flowing through the coil,

$Q_{\text{Net}} = 7.5 \times 10^{-3}$ C

and total resistance of wire and galvanometer,

$R = 0.3\Omega$

We know that,

$$\text{Charge flown} = \frac{\text{Charge in flux}}{\text{Resistance}}$$

$$Q_{\text{net}} = \frac{\Delta\phi}{R}$$

Magnetic flux, $\phi = NBA$

$$\text{Charge flown} = \frac{BNA}{R} \quad [\text{As } \Delta\phi = NBA]$$

Putting the given values, we get

$$7.5 \times 10^{-3} = \frac{B \times 30 \times (2.5 \times 10^{-4})}{0.3}$$

$$7.5 \times 10^{-3} = \frac{B \times 7.5 \times 10^{-3}}{0.3}$$

$B = 0.3$ T

Hence, the magnitude of the magnetic field, $B = 0.3$ T.

114. **(d)** Maximum energy stored on capacitor $= \frac{1}{2}\frac{Q^2}{C}$

When energy is distributed equally between Electric field and magnetic field, energy in capacitor $= \frac{1}{2} \times \frac{Q^2}{2C}$

$$\Rightarrow \frac{1}{2} \times \frac{1}{2}\frac{Q^2}{C} = \frac{q^2}{2C} \quad [q \text{ is charge at that time}]$$

$$\Rightarrow q^2 = \frac{2Q^2}{4} \Rightarrow q = \frac{Q}{\sqrt{2}}.$$

115. **(c)** Given,

Amplitude of the electric field, $E_M = 4\ \text{Vm}^{-1}$

Energy density (energy/volume) is given by

$$= \frac{1}{2}\varepsilon_0 E_m^2 \quad (\text{where, } \varepsilon_0 = 8.8 \times 10^{-2}\ \text{C}^2\text{N}^{-1}\text{m}^{-2})$$

Putting the given values, we get energy density

$$= \frac{1}{2} \times 8.8 \times 10^{-12} \times (4)^2\ \text{J/m}^3$$

$\therefore \quad \mu = 70.4 \times 10^{-12}\ \text{Jm}^{-3}$.

116. **(b)** Say threshold wavelength of emitter plate = λ

Energy of photon in first case is E.

Case I :

When switch S_1 is closed and switch S_2 is open,

So, $E = h\nu_0 + (5 + 1)$ eV ...(1)

Case II :

When switch S_1 is open and switch S_2 is closed and frequency of incident light is doubled.

Then, $2E = hc/\lambda + (20 - 5)$ eV ...(2)

From Eqs. (1) and (2), we get

$$\Rightarrow 2\left(\frac{hc}{\lambda} + 6\text{eV}\right) = \frac{hc}{\lambda} + 15\text{eV}$$

$$\Rightarrow 2\frac{hc}{\lambda} + 12\text{eV} = \frac{hc}{\lambda} + 15\text{eV}$$

$$\Rightarrow \frac{hc}{\lambda} = 3\text{eV}$$

$$\Rightarrow \frac{c}{\lambda} = \frac{3 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 7.25 \times 10^{14}\ \text{Hz}$$

$$\Rightarrow \lambda = \frac{3 \times 10^8}{7.25 \times 10^{14}} = 4133.3\text{Å}$$

Hence, the threshold wavelength of the emitter plate is 4133.3Å.

117. **(c)** Cocilombatration = Centripetal force

$$\frac{1}{4\pi\varepsilon_0}\frac{2qq}{r^2} = mr\omega^2 \quad ...(i)$$

From quantization,

$$mvr = \frac{nh}{2\pi} \Rightarrow mr^2\omega = \frac{nh}{2\pi} \quad ...(ii) \text{ [as } v = \omega r]$$

Solving Eqn. (i) and (ii),

$$\omega = \frac{2\pi mq^4}{\varepsilon_0^2 h^3}$$

Hence, the orbital angular velocity of the particle A is $\frac{2\pi mq^4}{\varepsilon_0^2 h^3}$. (Take $n = 1$ for nearest orbit)

118. **(b)** Given,

Nuclear reactor the activity of a radioactive substance,

$$\frac{dN}{dt} = 2000/s$$

and mean-life of the products = 50 min.

= 50 × 60 sec = 3000s

Also, mean life $= \frac{1}{\lambda} \Rightarrow \quad 3000 = \frac{1}{\lambda}$

$$\Rightarrow \lambda = \frac{1}{3000} \text{ s}^{-1}$$

∴ The rate of decay, $\frac{dN}{dt} = \lambda N$

$$\Rightarrow 2000 = \frac{1}{3000} \times N$$

$$\Rightarrow N = 60 \times 10^5$$

Option (b) is correct.

119. **(d)** p-type semiconductor donor energy level,

$E = 50$ meV $= 50 \times 10^{-3}$ eV

According to the Planck's quantum theory,

$$\therefore \quad E = h\nu \text{ or } E = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{1242nm}{E(eV)}$$

$$\Rightarrow \lambda = \frac{1242 \text{ nm}}{50 \times 10^{-3}}$$

$$\Rightarrow \lambda = 24.8 \ \mu\text{m}.$$

So, the maximum wavelength of light photon required is 24.8 μ m.

120. **(a)** Given, frequency of signal, $f_s = 10$ kHz

Carrier frequency, $f_c = 1$ mHz

Side bank frequency are given by,

$$f_c \pm f_s = 1 \times 10^6 \pm 10 \times 10^3 \text{ Hz}$$

$$= (1000 \pm 10) \text{ kHz}$$

∴ 1010 kHz and 990 kHz.

CHEMISTRY

121. **(b)** $\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m \times \lambda}$

$$\Rightarrow v = \frac{6.626 \times 10^{-34} \text{ Js}}{9.1 \times 10^{-31} \text{ kg} \times 182 \times 10^{-9} \text{ m}}$$

$= 4 \times 10^3$ m/s

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (4 \times 10^3)^2$$

$= 7.28 \times 10^{-24}$ J.

122. **(c)** Given,

Orbit radius = 52.9 pm

Ground state energy of electron in hydrogen atom

$= -2.18 \times 10^{-18}$ J

$$r_n = r_0 \times \frac{n^2}{Z}$$

$$52.9 \text{ pm} = 52.9 \text{ pm} \times \frac{n^2}{Z} \Rightarrow Z = n^2$$

Now, $E_n = E_0 \times \frac{Z^2}{n^2}$

$$\Rightarrow E_n = -2.18 \times 10^{-18} \times \frac{n^4}{n^2}$$

$$\Rightarrow E_n = -2.18 \times 10^{-18} \times n^2$$

When $n = 1$, $E_1 = -2.18 \times 10^{-18}$ J

Which does not match with any option.

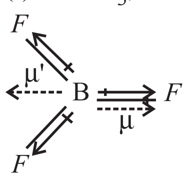
When $n = 2$, $E_2 = 4 \times (-2.18 \times 10^{-18})$

$= -8.72 \times 10^{-18}$ J

123. **(a)** On moving down the group electron gain enthalpy decreases. Due to e^-, e^- repulsions in relatively small $2p$-subshell of O atom, its e^- gain enthalpy is lowest.

124. **(b)**

(I) In BF_3, electronegativity of F-atom is greater than B.



(II) Covalent bond length increases as

C — O > N — O > O — H

Due to increase in size of atom.

(III) Bond order $= \frac{N_b - N_a}{2}$

$C_2 = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^2 \pi 2p_y^2$

$\text{B.O.} = \dfrac{8-4}{2} = 2$

$B_2 = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^1 \pi 2p_y^1$

$\text{B.O.} = \dfrac{6-4}{2} = 1$

$He_2 = \sigma 1s^2 < \sigma^* 1s^2$

$\text{B.O.} = \dfrac{2-2}{2} = 0$

Hence, Bond order :
$C_2 > B_2 > He_2$.

125. (N) $He_2^+ : \sigma 1s^2 < \sigma^* 1s^1$

$BO = \dfrac{2-1}{2} = 0.5$

$Li_2^+ : \sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^1$

$BO = \dfrac{3-2}{2} = 0.5$

$B_2 : \sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^2 < \sigma^* 2s^2 < \pi 2p_x^1 = \pi 2p_y^1$

$BO = \dfrac{6-4}{2} = 1$

Due to 2 unpaired e^-, it is paramagnetic.

$C_2 : \sigma 1s^2 < {}^*\sigma 1s^2 < \sigma 2s^2 < {}^*\sigma 2s^2 < \pi 2p_x^2 = \pi 2p_y^2$

$BO = \dfrac{8-4}{2} = 2$

126. (c) $KE = \dfrac{3RT}{2}$ and $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

$v_{\text{rms}} = \sqrt{\dfrac{2KE}{M}}$

$= \sqrt{\dfrac{2 \times 4 \times 10^3 \text{ J}}{32 \times 10^{-3} \text{ kg mol}^{-1}}}$

$= 5 \times 10^2$ m/s $= 5 \times 10^4$ cm/s.

127. (d) $2K\overset{+7}{Mn}O_4 + 10I^- \longrightarrow 5\overset{0}{I_2} + 2Mn^{+2}$

n-factor of $KMnO_4 = 7 - 2 = 5$

I^- convert into I_2 then, n-factor $= 1 - 0 = 1$

Number of equivalents of $KMnO_4$
= Number of equivalents of I^-

n-factor × moles of $KMnO_4$ = n-factor × moles of I^-

$\Rightarrow V \times 0.02 \times 5 = 60 \times 0.01 \times 1$

$\Rightarrow V = 6$ mL.

128. (a) In bomb calorimeter,

$\Delta H = \Delta U = q$

Given, $\Delta H = -248$ kJ/mol

For, 12 g of graphite energy released = 248 kJ/mol.

For, 6 g of graphite energy released $= \dfrac{248 \times 6}{12}$

From, $q = C_v \Delta T$

$\Rightarrow \dfrac{248 \times 6}{12} = C_v \times (31 - 25)$

$\Rightarrow C_v = \dfrac{248}{2 \times 6} = 20.6$ kJ/K

129. (b) Solubility of AgCl in 0.1 M KCl solution will be minimum because of common ion effect of Cl^-.

130. (d) $H_2O + H_2O \longrightarrow H_3O^+ + OH^-$ (Acid, Base) (Bronsted acid as well as base)

$HSO_4 \xrightarrow{H^+} H_2SO_4$ (acting as acid)

$HSO_4 \xrightarrow{-H^+} SO_4^{2-}$ (acting as base)

131. (c) Bond lengths of H_2 and D_2 are same because of same size.

132. (c)

(II) On moving down in the group the size of the atom increases. The bigger atom can stabilise more the bigger superoxide ion effectively.

(III) Lattice energy of LiF is more than the hydration energy. Hence, LiF has very low solubility.

133. (a) Gallium (Ga) exist in liquid phase at room temperature due to presence of Ga_2 dimers in solid phase.

134. (a) $HCOOH \xrightarrow{\text{Conc. } H_2SO_4} \underset{(B)}{CO} + \underset{(A)}{H_2O}$

$\underset{(B)}{3CO} + Fe_2O_3 \longrightarrow \underset{(D)}{2Fe} + \underset{(C)}{3CO_2}$

$CaCO_3 + 2HCl \text{ (dil.)} \longrightarrow \underset{(C)}{CO_2} + H_2O + CaCl_2$

135. (c)

136. (b)

(a) $\mathbf{C_6H_6}$ All six C-atoms show sp^2-hybridisation, *i.e.*, 3-hybrid orbitals by each C-atom.

Total number of hybrid orbitals = 6 × 3 = 18

(b) $\mathbf{(CH_3)_4 C}$ All five C-atoms show sp^3 - hybridisation, *i.e.,* 4-orbitals by each C-atom.

Total number of hybrid orbitals = 5 × 4 = 20.

(c) $\underset{sp^3}{H_3C} - \underset{sp^2}{\overset{|}{C}}(CH_3)\,(sp^3) = \underset{sp^2}{O}$

$$\overset{sp^3}{H_3C} - \overset{sp^2}{C}(\underset{sp^3}{CH_3}) = \underset{sp^2}{O}$$

Thus, total number of hybrid orbitals

= 4 + 4 + 3 + 3 = 14.

(d) $$\underset{sp^3}{\overset{4}{CH_3}} - \underset{sp^2}{\overset{3}{CH}} = \underset{sp^2}{\overset{2}{CH}} - \underset{sp}{\overset{1}{C}} \equiv \underset{sp}{N}$$

- C-1 show *sp*-hybridisation (*i.e.* 2-hybrid-orbitals)
- C-2 and 3 show sp^2-hybridisation (*i.e.* 3-hybrid orbital by each C-atom)
- C-4 show sp^3-hybridisation (*i.e.* 4-hybrid orbitals)
- N-atom show *sp*-hybridisation (*i.e.* 2-hybrid orbitals)

Thus, total number of hybrid orbitals

= 2 + 2 + 3 + 3 + 4 = 14.

137. (a) $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$

$\Rightarrow \dfrac{725 \times 50}{300} = \dfrac{760 \times V_2}{273}$

$\Rightarrow V_2 = \dfrac{725 \times 50 \times 273}{300 \times 760} = 43.4$ mL

$\Rightarrow$ 22400 mL of $N_2 \equiv$ 28 g of N_2 at STP

$\Rightarrow$ 43.4 mL of $N_2 \equiv \dfrac{28 \times 43.4}{22400} = 0.054$ g of N_2

% mass of $N_2 = \dfrac{0.054 \text{ g}}{1 \text{ g}} \times 100 = 5.4\%$.

138. (b) Kolbe electrolysis of $CH_3COO^-Na^+$.

At anode,

$$CH_3-\underset{\underset{O}{\|}}{C}-O^- \longrightarrow CH_3-\underset{\underset{O}{\|}}{C}-O^\bullet + e^-$$

$$CH_3-\underset{\underset{O}{\|}}{C}-O^\bullet \longrightarrow \underset{(A)}{CO_2} + {}^\bullet CH_3$$

$$^\bullet CH_3 + {}^\bullet CH_3 \longrightarrow \underset{(B)}{H_3C-CH_3}$$

$$\underset{(B)}{CH_3-CH_3} + 5O_2 \xrightarrow{(CH_3COO)_2Mn} \underset{(C)}{2CO} + \underset{(D)}{3H_2O}$$

$$\underset{(C)}{CO} + NaOH \rightleftharpoons \underset{\text{(Salt)}}{HCOONa}$$

Hence, A is CO_2 and D is H_2O.

139. (d) $H_2C = CH_2 + HBr \longrightarrow \underset{(W)}{H_3C-CH_2-Br}$

$$\xrightarrow{\text{(i) Mg/dry ether}} H_3C-CH_2-MgBr$$

$$\xrightarrow{\text{(ii) } O=C=O} H_3C-CH_2-\underset{\underset{O}{\|}}{C}-OMgBr$$

$$\xrightarrow{\text{(iii) } H_3O^+} \underset{(X)}{CH_3-CH_2-COOH} + Mg\begin{matrix}OH\\Br\end{matrix}$$

$$\xrightarrow[\text{electrolysis}]{\text{NaOH}} \underset{(Z)}{CH_3-CH_2-CH_2-CH_3}$$

$$\underset{(W)}{H_3C-CH_2-Br} \xrightarrow[\text{dry/ether}]{\text{Na}} \underset{(Z)}{CH_3-CH_2-CH_2-CH_3}$$

140. (c) For fcc unit cell, $Z = 4$

Edge length = x Å

Atomic mass, $M = 63.5$ g mol^{-1}

$\because$ Density, $d = \dfrac{Z \times M}{x^3 \times N}$

$= \dfrac{4 \times 63.5}{(x \times 10^{-8})^3 \times 6.023 \times 10^{23}}$

$d = \dfrac{423}{x^3}$

141. (a) I. Moles of solute NaOH

= molarity × volume of solution (in L)

$= \dfrac{0.2 \times 500}{1000} = 0.1$

II. Molarity of $H_2SO_4 = \dfrac{0.1}{2} = 0.05$ M

Moles of $H_2SO_4 = \dfrac{0.05 \times 200}{1000} = 0.01$

III. Moles of area $= \dfrac{\text{Mass of urea}}{\text{Molecular mass of urea}}$

$= \dfrac{6}{60} = 0.1$

142. (b) $\Delta T_f = K_f \times m$

$\Rightarrow 0.70 = 5.1 \times m$

$\Rightarrow m = \dfrac{0.70}{5.1} = \dfrac{\text{Moles of solute}}{\text{Total mass of solvent (kg)}}$

Lets, assume x g napthalene is present.

$$0.137 = \frac{\left(\frac{x}{m_{C_{10}H_8}} + \frac{6-x}{m_{C_{14}H_{10}}}\right)\times 1000}{300}$$

$\Rightarrow 0.041 = \dfrac{x}{128} + \dfrac{6-x}{178} = 3.4$ g

Mass of anthracene = 6 – x = 6 – 3.4 = 2.6 g

143. (a) $E^\circ_{cell} = E^\circ_C - E^\circ_A = 0.34 - (-0.76)V = 1.1V$

$\Rightarrow E_{cell} = E^\circ_{Cell} - \dfrac{2.303RT}{nF}\log\left(\dfrac{Zn^{2+}}{Cu^{2+}}\right)$

$\Rightarrow E_{cell} = 1.1 - \dfrac{0.059}{n}\log\dfrac{C_2}{C_1} \quad \left(\begin{matrix} Zn^{2+} = C_2 \\ Cu^{2+} = C_1 \end{matrix}\right)$

E_{Cell} value will be maximum when, $\log\dfrac{C_2}{C_1}$ will be minimum.

From the given options $\log\dfrac{0.01}{0.1} = \log 10^{-1}$ will give the maximum value of E_{Cell}.

144. (c) $C_2H_5I\,(g) \longrightarrow C_2H_4\,(g) + HI\,(g)$

	$C_2H_5I\,(g)$	$C_2H_4\,(g)$	$HI\,(g)$
At $t = 0$	2 bar	0	0
t = 1000 min.	0.1	(2 – 0.1) = 1.9	(2 – 0.1) = 1.9

Since, the reaction is first order,

$k = \dfrac{2.303}{t}\log\dfrac{p_0}{p_0 - x}$

$\Rightarrow k = \dfrac{2.303}{1000}\log\dfrac{2}{0.1}$

$\Rightarrow k = \dfrac{2303}{1000}\log 20 = \dfrac{2.996}{1000}$

$\Rightarrow k = 3 \times 10^{-3}\ \text{min}^{-1}$.

145. (b)

146. (d) (I) Magnetite → Fe_3O_4

(II) Kaolinite → $Al\,(OH)_4, Si_2O_3$

(III) Siderite → $FeCO_3$

(IV) Calamine → $ZnCO_3$

(III), (IV) both are carbonate ores.

147. (c) S_8 molecule is monoclinic as well as rhombic.

148. (c) Nobel metal like gold and platinum are soluble in aqua-regia which is a mixture of nitric acid and hydrochloric acid in molar ratio of 1 : 3.

149. (d) Mn_2O_7, CrO_3, V_2O_5 are acidic.

Due to higher oxidation state of central metal, oxides.

150. (c) The wavelength of the light absorbed by the complexes $\propto \dfrac{1}{\text{strength of ligands}}$.

Strength of ligand : en > H_2O.

Hence, order of wavelength :

$[Ni(H_2O)_6]^{2+} > [(Ni(H_2O)_4 en)]^{2+} > [Ni(en)_3]^{2+}$

$(\lambda_1) \quad (\lambda_3) \quad (\lambda_2)$

151. (d) In neutral or faintly alkaline solution,

$8\,MnO_4^- + 3\,S_2O_3^{-2} + H_2O \longrightarrow 8\,MnO_2 + 6\,SO_4^{-2} + 2\,OH^-$

In acidic medium,

$2\,MnO_4^- + 5\,NO_2^- + 6\,H^+ \longrightarrow 2\,Mn^{+2} + 5\,NO_3^- + 3\,H_2O$

152. (c)

153. (b)

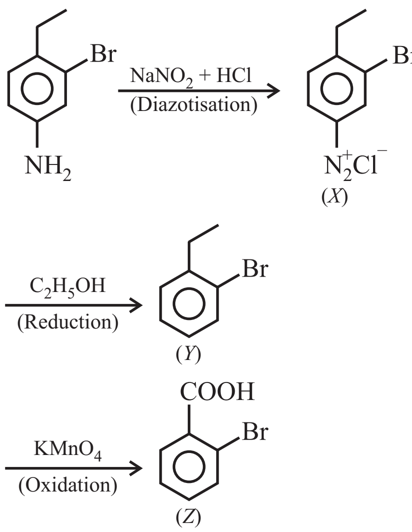
154. (c)

155. (d) Rate of S_N2 reaction $\propto \dfrac{1}{\text{steric hindrance}}$

∴ Rate is 1° > 2° > 3° alkyl halide.

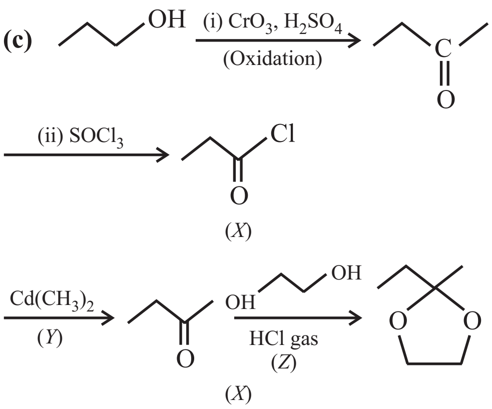
156. (a) The bond angle between C – O and O – H bonds in alcohols is close to 109°. (i.e. 109°28')

157. (d)



Thus, option (d) is correct.

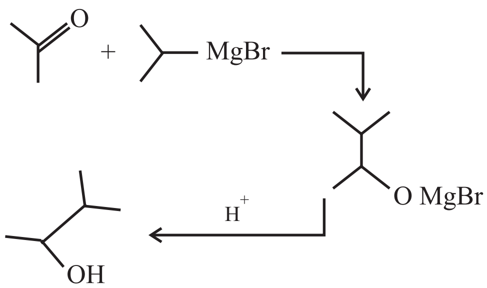
158. (c)



Hence, option (c) is correct.

159. (c) $CH_3-\underset{\underset{CH_3}{|}}{C}=CH-CH_3 \xrightarrow[H^+]{H_2O} CH_3-\underset{\underset{CH_3}{|}}{\overset{\overset{OH}{|}}{C}}-CH_2-CH_3$ (X)

(c)



(isomer of *X*)

160. (b) $\underset{\text{(Acetic acid)}}{CH_3COOH} \xrightarrow{NH_3} CH_3CO\overset{-}{O}\overset{+}{N}H_4$

$\xrightarrow[-H_2O]{\Delta} \underset{(A)}{CH_3CONH_2}$

$\xrightarrow{LiAlH_4} \underset{(B)}{CH_3-CH_2-NH_2}$

$\xrightarrow{CHCl_3,\ KOH} \underset{(C)}{CH_3-CH_2-N\equiv C}$