AP/EAPCET Solved Paper 2019

Held on April 21

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. Let $D = R - \{0, 1\}$ and $f : D \to D$, $g : D \to D$ and h: $D \to D$ be three functions defined by

$f(x) = \frac{1}{x}; g(x) = 1 - x$ and $h(x) = \frac{1}{1-x}$. If $j : D \to D$ is such that $(gojof)(x) = f(x)$ for all $x \in D$, then which one of the following is $j(x)$?

(a) $(fog)(x)$ (b) $f(x)$ (c) $g(x)$ (d) $(goh)(x)$

2. The maximum value of the function

$f(x) = \tan\left(x + \frac{2\pi}{3}\right)$

$- \tan\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right)$ in $\left[-\frac{5\pi}{12}, \frac{-\pi}{3}\right]$ is

(a) $\frac{11\sqrt{2}}{6}$ (b) $\frac{11\sqrt{3}}{6}$ (c) 3 (d) 1

3. $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + ... + (1^2 + 2^2 + ... + n^2) =$

(a) $\frac{n(n+1)(n+2)}{12}$ (b) $\frac{n(n+1)(2n+1)}{6}$

(c) $\frac{n(n+1)^2(n+2)}{12}$ (d) $\frac{n(n+1)(n+2)(n+3)}{12}$

4. If $\begin{vmatrix} a+b+2c & a & b \\ c & 2a+b+c & b \\ c & a & a+2b+c \end{vmatrix} = 2,$

then $a^3 + b^3 + c^3 - 3abc =$

(a) $1 - 3ab - 3bc - 3ca$ (b) 0

(c) $1 - 2ab - 2bc - 2ca$ (d) 1

5. If k is one of the roots of the equation

$x^2 - 25x + 24 = 0$ such that $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & k \end{bmatrix}$ is a non-singular matrix, then $A^{-1} =$

(a) $-\frac{1}{46}\begin{bmatrix} 90 & -94 & 8 \\ -138 & 46 & 0 \\ 2 & 2 & -8 \end{bmatrix}$

(b) $-\frac{1}{92}\begin{bmatrix} 45 & -47 & 4 \\ -69 & 23 & 0 \\ 1 & 1 & -4 \end{bmatrix}$

(c) $-\frac{1}{46}\begin{bmatrix} 45 & -47 & 4 \\ -69 & 23 & 0 \\ 1 & 1 & -4 \end{bmatrix}$

(d) $-\frac{1}{92}\begin{bmatrix} 90 & -94 & 8 \\ -138 & 46 & 0 \\ 2 & 2 & -8 \end{bmatrix}$

6. A value of b for which the rank of the matrix

$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ is 3, is

(a) –2 (b) –4 (c) –6 (d) 3

7. If $z = x - iy$ and $z^{\frac{1}{3}} = a + ib$, then $\frac{\left(\frac{x}{a} + \frac{y}{b}\right)}{a^2 + b^2} =$

(a) –2 (b) –1 (c) 1 (d) 2

8. The equation whose solutions are the non-zero solutions of the equation $\bar{z} = iz^2$, is

(a) $z^3 + i = 0$ (b) $z^3 + z + 1 = 0$

(c) $z^3 - i = 0$ (d) $z^3 + iz + 1 = 0$

9. If $x, y \in R$ and $x^2 + y + 4i$ and $-3 + x^2 yi$ are conjugates to each other, then $(|x| + |y|)^2 =$

(a) 17 (b) 16

(c) 25 (d) 9

10. $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)=$

(a) –1 (b) 0 (c) i (d) $-i$

11. If α satisfies the equation $\sqrt{\frac{x}{2x+1}}+\sqrt{\frac{2x+1}{x}}=2$, then the roots of the equation $\alpha^2x^2+4\alpha x+3=0$ are

(a) l, 3 (b) –1, 1 (c) 2, –3 (d) 3, 4

12. Let $f(x)=x^2+2x+2$, $g(x)=-x^2+2x-1$ and a, b be the extreme values of $f(x)$, $g(x)$ respectively. If c is the extreme value of $\frac{f}{g}(x)$ (for $x\neq 1$), then $a+2b+5c+4=$

(a) 2 (b) 1 (c) 4 (d) 3

13. The set of all real numbers satisfying the inequation $x^2-|x+2|+x>0$ is

(a) $\left[-2,-\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$ (b) $(-\infty,-2)\cup(2,\infty)$

(c) $\left(-\infty,-\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$ (d) $(-\infty,-2)\cup\left(\sqrt{2},\infty\right)$

14. If α, β, γ are the roots of $x^3-6x^2+11x-6=0$, then the equation having the roots $\alpha^2+\beta^2$, $\beta^2+\gamma^2$ and $\gamma^2+\alpha^2$ is

(a) $x^3-28x^2+245x-650=0$

(b) $x^3-28x^2+245x+650=0$

(c) $x^3+28x^2-245x-650=0$

(d) $x^3+28x^2+245x-650=0$

15. If the number of elements in the sets G and A are 3 and 4 respectively, then match the items of List I with those of List II

	List I		List II
(A)	The number of non-bijective functions from G × G to G	(I)	24
(B)	The number of bijective functions from A to A	(II)	0
(C)	The number of functions from G to G × A	(III)	1728
(D)	The number of surjective functions from A to A × A	(IV)	12
		(V)	19683

The correct match is

	A	B	C	D		A	B	C	D
(a)	V	I	III	II	(b)	V	III	IV	II
(c)	III	IV	V	II	(d)	I	II	III	IV

16. There are 20 straight lines in a plane such that no two of them are parallel and no three of them are concurrent. If their points of intersection are joined, then the number of new line segments formed are

(a) 3420 (b) 14535 (c) 2907 (d) 17955

17. Let $a_0, a_1, a_2, \ldots, an \in R$ be in an arithmetic progression and let $C_0, C_1, C_2, \ldots, Cn$ be the binomial coefficients Then $\sum_{k=0}^{n}a_k.C_k=$

(a) $\frac{1}{2}(a_0+a_n)$ (b) $(a_0+an).2n^{-1}$

(c) (a_0+an) (d) 0

18. If $x=\frac{3}{10}+\frac{3.7}{10.15}+\frac{3.7.9}{10.15.20}+\ldots.$, then 5x + 8 =

(a) $\frac{5\sqrt{5}}{3\sqrt{3}}$ (b) $\frac{5\sqrt{5}}{\sqrt{3}}$ (c) $\frac{3\sqrt{3}}{\sqrt{5}}$ (d) $\frac{25\sqrt{5}}{3\sqrt{3}}$

19. If $\frac{x^4}{(x-1)(x-2)(x-3)}=x+k+\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$, then $k+A-B+C=$

(a) 104 (b) 52 (c) 63 (d) $\frac{127}{2}$

20. cosec 48° + cosec 96° + cosec 192° + cosec 384° =

(a) –2 (b) –1 (c) 0 (d) $\frac{\sqrt{3}}{2}$

21. $\sin^4\frac{\pi}{8}+\sin^4\frac{2\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{4\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{6\pi}{8}+\sin^4\frac{7\pi}{8}=$

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) $\frac{7}{2}$

22. If $x:y:z=\tan\left(\frac{\pi}{15}+\alpha\right):\tan\left(\frac{\pi}{15}+\beta\right):\tan\left(\frac{\pi}{15}+\gamma\right)$, then $\frac{z+x}{z-x}\sin^2(\gamma-\alpha)+\frac{x+y}{x-y}\sin^2(\alpha-\beta)+\frac{y+z}{y-z}\sin^2(\beta-\gamma)=$

(a) $\sin^2\theta$ (b) $\cos^2\theta$ (c) 0 (d) 1

23. Let $[x]$ denote the largest integer $\leq x$. If the number of solutions of $\sin x\sqrt{4\cos^2 x}=\frac{2+x-[x]}{1-x+[x]}$ is k, then for $x\in\left[\frac{\pi}{4},\frac{\pi}{3}\right]$, the value of $k^{\tan^2 x}$

(a) is equal to 1 (b) lies in between 2^1 and 2^3

(c) is equal to zero (d) lies in between $\frac{1}{2^3}$ and $\frac{1}{2}$

24. If α and β are the least and the greatest values of $f(x)=(\sin^{-1}x)^2+(\cos^{-1}x)^2$ for all $x\in R$ respectively, then $8(\alpha+\beta)=$

(a) π^2 (b) $11\pi^2$ (c) $9\pi^2$ (d) $25\pi^2$

25. If $x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$, then log sec x =

(a) $2\operatorname{cosech}^{-1}\left(\cot^2\frac{x}{2}-1\right)$

(b) $2\operatorname{cosech}^{-1}\left(\cot^2\frac{x}{2}+1\right)$

(c) $2\coth^{-1}\left(\operatorname{cosec}^2\frac{x}{2}-1\right)$

(d) $2\coth^{-1}\left(\operatorname{cosec}^2\frac{x}{2}+1\right)$

26. The area (in square units) of ΔABC if $\angle A = 75°$, $\angle B = 45°$ and $a = 2\left(\sqrt{3}+1\right)$ is

(a) 6 (b) $2\sqrt{3}$ (c) $6-2\sqrt{3}$ (d) $6+2\sqrt{3}$

27. In a ΔABC, if $3a = b + c$, then $\cot\frac{B}{2}\cot\frac{C}{2} =$

(a) 1 (b) 2 (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

28. In a ΔABC, if $\frac{2r_2r_3}{r_2-r_1} = r_3 - r_1$, then $\frac{r_1(r_2+r_3)}{\sqrt{r_1r_2+r_2r_3+r_3r_1}} =$

(a) $\frac{a^2+b^2+c^2}{\Delta^2}$ (b) $b-c$

(c) $\frac{1}{2R}$ (d) $2R$

29. If $3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}}, 2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-4\hat{\mathbf{k}}, -\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\lambda\hat{\mathbf{k}}$ are respectively the position vectors of four coplanar points P, Q, R and S, then $\lambda =$

(a) $\frac{46}{17}$ (b) $-\frac{46}{17}$ (c) $\frac{146}{17}$ (d) $-\frac{146}{17}$

30. If $\mathbf{OA} = 2\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\mathbf{OB} = 2\hat{\mathbf{i}}+4\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ and the length of the internal bisector of $\angle BOA$ of triangle AOB is k, then $9k^2 =$

(a) $\sqrt{225}$ (b) 136 (c) 712 (d) 20

31. If $a + xb + yc = 0$, $a \times b + b \times c + c \times a = 6(b \times c)$, then the locus of the point (x, y) is

(a) $x^2 + y^2 = 1$ (b) $x + y - 5 = 0$

(c) $2x + 6y = 5$ (d) $x + y + 6 = 0$

32. Let $A = (\alpha, 1, 2\alpha)$, $B = (3, 1, 2)$ and $\mathbf{C} = 4\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}}$. If $\mathbf{AB}\times\mathbf{C} = 6\hat{\mathbf{i}}+9\hat{\mathbf{j}}-5\hat{\mathbf{k}}$, then $\alpha^2 + \alpha + 5 =$

(a) 11 (b) 7 (c) 9 (d) 5

33. The shortest distance between the skew lines

$r = \left(6\hat{i}+2\hat{j}+2\hat{k}\right)+t\left(\hat{i}-2\hat{j}+2\hat{k}\right)$ and

$r = \left(-4\hat{i}-\hat{k}\right)+s\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$ is

(a) 9 (b) $\frac{40}{7}$ (c) 108 (d) 120

34. If a makes an acute angle with b, $r \,.\, a = 0$ and $r \times b = c \times b$, then $r =$

(a) $a \times c - b$ (d) $c \times a$

(c) $c = \left(\frac{c\cdot a}{b\cdot a}\right)b$ (d) $c + \left(\frac{c\cdot a}{b\cdot a}\right)b$

35. For a data consisting of 15 observations x_i, $i = 1, 2, 3, ..., 15$ the following results are obtained : $\sum_{i=1}^{15} x_i = 170; \sum_{i=1}^{15} x_i^2 = 2830$. If one of the observation namely 20 was found wrong and was replaced by its correct value 30, then the corrected variance is

(a) 80 (b) 78 (c) 76 (d) 75

36. The mean deviation about the mean for the following data is

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	4	6	16	28	16	6	4

(a) 35 (b) 10 (c) 15 (d) 12

37. A and B each select one number at random from the distinct numbers $1, 2, 3, ..., n$ and the probability that the number selected by A is less than the number selected by B is $\frac{1009}{2019}$. Now, the probability that the number selected by B is the number immediately next to the number selected by A is

(a) $\frac{2018}{2019}$ (b) $\frac{2018}{(2019)^2}$ (c) $\frac{2000}{(2019)}$ (d) $\frac{2000}{(2019)^2}$

38. There are 3 bags A, B and C. Bag A contains 2 white and 3 black balls, bag B contains 4 white and 2 black balls and bag C contains 3 white and 2 black balls. If a ball is drawn at random from a randomly chosen bag, then the probability that the ball drawn is black, is

(a) $\frac{2}{3}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{1}{9}$

39. The following table shows the probability of selecting the boxes A, B and C and the number of balls of different colours contained in them.

Box	Number of balls			Probability
	White	Green	Red	
A	1	2	3	$\frac{1}{2}$
B	2	3	1	$\frac{1}{3}$
C	3	1	2	$\frac{1}{6}$

(a) $\frac{1}{13}$ (b) $\frac{6}{13}$ (c) $\frac{5}{13}$ (d) $\frac{7}{13}$

40. If a random variable X has the probability distribution given by $P(X = 0) = 3C^3$, $P(X = 2) = 5C - 10C^2$ and $P(X = 4) = 4C - 1$, then the variance of that distribution is

(a) $\frac{68}{9}$ (b) $\frac{22}{9}$ (c) $\frac{612}{81}$ (d) $\frac{128}{81}$

41. A box contains 30 toys of same size in which 10 toys are white and all the remaining toys are blue. A toy is drawn at random from the box and it is replaced in the box after noting down its colour. If 5 toys are drawn in this way, then the probability of getting atmost 2 white toys is

(a) $\left(\frac{6}{9}\right)^2$ (b) $\left(\frac{8}{9}\right)^2$ (c) $\left(\frac{7}{9}\right)^2$ (d) $\left(\frac{2}{3}\right)^5$

42. For any value of θ, if the straight lines $x\sin\theta + (1 - \cos\theta)y = a\sin\theta$ and $x\sin\theta - (1 + \cos\theta)y + a\sin\theta = 0$ intersect at $P(\theta)$, then the locus of $P(\theta)$ is a

(a) straight line (b) circle
(c) parabola (d) hyperbola

43. A line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle θ keeping the origin fixed, this line L has the intercepts p and q. Then

(a) $a^2 + b^2 = p^2 + q^2$ (b) $a^2 + p^2 = b^2 + q^2$

(c) $\frac{1}{a^2}+\frac{1}{p^2}=\frac{1}{b^2}+\frac{1}{q^2}$ (d) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

44. If O is the origin and A & B are points on the line $3x - 4y + 25 = 0$ such that **OA** = **OB** = 13, then the area of ΔOAB (in sq units) is

(a) 30 (b) 120 (c) 60 (d) 65

45. If $P(\alpha, \beta)$ be a point on the line $3x + y = 0$ such that the point P and the point $Q(1, 1)$ lie on either side of the line $3x = 4y + 8$, then

(a) $\alpha > \frac{8}{15}, \beta < \frac{-8}{5}$ (b) $\alpha < \frac{8}{15}, \beta < \frac{-8}{5}$

(c) $\alpha > \frac{8}{15}, \beta > \frac{-8}{5}$ (d) $\alpha < \frac{8}{15}, \beta > \frac{-8}{5}$

46. Two vertices of a triangle are (5, –1) and (–2, 3). If the origin is the orthocentre of this triangle, then the coordinates of the third vertex of that triangle are

(a) (4, 7) (b) $\left(-2, \frac{-7}{2}\right)$

(c) (–4, –7) (d) (–2, 3)

47. The distance from the origin to the orthocentre of the triangle formed by the lines $x + y - 1 = 0$ and $6x^2 - 13xy + 5y^2 = 0$ is

(a) $\frac{11\sqrt{2}}{2}$ (b) 13 (c) 11 (d) $\frac{11\sqrt{2}}{24}$

48. The combined equation of two lines L and L_1 is $2x^2 + axy + 3y^2 = 0$ and the combined equation of two lines L and L_2 is $2x^2 + bxy - 3y^2 = 0$. If L_1 and L_2 are perpendicular, then $a^2 + b^2 =$

(a) 26 (b) 29 (c) 13 (d) 85

49. The power of the point $B(-1, 1)$ with respect to the circle $S \equiv x^2 + y^2 - 2x - 4y + 3 = 0$ is p. If the length of the tangent drawn from B to the circles $S = 0$ is t, then the point (2, 3) with respect to the circle $S' = 0$ having centre at (p, t^2) and passing through the origin.

(a) lies inside the circle $S' = 0$
(b) lies outside the circle $S' = 0$
(c) lies on the circle $S' = 0$
(d) is the centre of the circle $S' = 0$

50. If tangents are drawn to the circle $x^2 + y^2 = 12$ at the points where it intersects the circle $x^2 + y^2 - 5x + 3y - 2 = 0$, then the coordinates of the point of intersection of those tangents are

(a) $\left(-6, \frac{18}{5}\right)$ (b) $\left(6, \frac{18}{5}\right)$

(c) $\left(-6, \frac{-18}{5}\right)$ (d) $\left(6, \frac{-18}{5}\right)$

51. If the point of intersection of the pair of the transverse common tangents and that of the pair of direct common tangents drawn to the circle $x^2 + y^2 - 14x + 6y + 33 = 0$ and $x^2 + y^2 + 30x - 2y + 1 = 0$ are T and D respectively, then the centre of the circle having TD as diameter is

(a) $\left(\frac{39}{2}, \frac{-7}{4}\right)$ (b) $\left(\frac{39}{4}, \frac{7}{2}\right)$

(c) $\left(\frac{39}{4}, \frac{-7}{2}\right)$ (d) $\left(\frac{39}{2}, \frac{-7}{2}\right)$

52. If the circles $x^2 + y^2 + 2\lambda x + 2 = 0$ and $x^2 + y^2 + 4y + 2 = 0$ touch each other, then $\lambda =$

(a) ±1 (b) ±2 (c) ±3 (d) ±4

53. The equation of the circle whose diameter is the common chord of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$ is

(a) $2x^2 + 2y^2 + x + 3y + 2 = 0$
(b) $2x^2 + 2y^2 + 2x + 6y + 1 = 0$
(c) $2x^2 + 2y^2 + 4x - 3y - 1 = 0$
(d) $x^2 + y^2 + 2x + 6y - 2 = 0$

54. If the focus of a parabola divides a focal chord of the parabola into segments of lengths 5, 3 units, then the length of the latusrectum of the parabola is

(a) $\frac{15}{4}$ (b) 20 (c) $\frac{25}{2}$ (d) $\frac{15}{2}$

55. The angle between the tangents drawn to the parabola $y^2 = 4x$ from the point (1, 4) is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{5}$ (d) $\frac{\pi}{6}$

56. If the tangent drawn to the parabola $y^2 = 4x$ at $(t^2, 2t)$ is the normal to the ellipse $4x^2 + 5y^2 = 20$ at $\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$, then

(a) $5t^4 + 4t^2 = 1$ (b) $\frac{5}{t^4}+\frac{100}{t^2}=1$

(c) $t = \sin\theta$ (d) $\cos\theta = t + 1$

57. If the tangents drawn from a point P to the ellipse $4x^2 + 9y^2 - 24x + 36y = 0$ are perpendicular, then the locus of P is

(a) $x^2 + y^2 - 6x + 4y + 13 = 0$

(b) $x^2 + y^2 - 6x + 4y - 13 = 0$

(c) $x^2 + y^2 = 26$

(d) $x^2 + y^2 + 6x - 4y - 13 = 0$

58. The locus of the mid-points of the chords of the circle $x^2 + y^2 = 16$ which are the tangents to the hyperbola $9x^2 - 16y^2 = 144$ is

(a) $3x^2 - 4y^2 = 16(x^2 + y^2)$ (b) $4x^2 - 3y^2 = 9(x^2 + y^2)$

(c) $16x^2 - 9y^2 = (x^2 + y^2)^2$ (d) $16x^2 - 9y^2 = 4(x^2 + y^2)$

59. $A(3, 2, -1), B(4, 1, 1), C(6, 2, 5)$ are three points. If D, E, F are three points which divide BC, CA, AB respectively in the same ratio 2 : 1, then the centroid of ΔDEF is

(a) $\left(\frac{13}{3},\frac{5}{3},\frac{5}{3}\right)$ (b) $(13, 5, 5)$

(c) $(4, 2, 1)$ (d) $\left(\frac{11}{3},\frac{4}{3},\frac{1}{3}\right)$

60. If $A = (1, 8, 4)$, $B = (2, -3, 1)$, then the direction cosines of a normal to the plane AOB is

(a) $\frac{2}{\sqrt{78}},\frac{5}{\sqrt{78}},\frac{-7}{\sqrt{78}}$ (b) $\frac{2\sqrt{10}}{9},\frac{7\sqrt{10}}{90},\frac{-19\sqrt{10}}{90}$

(c) $\frac{4}{\sqrt{218}},\frac{9}{\sqrt{218}},\frac{-11}{\sqrt{218}}$ (d) $\frac{2}{11},\frac{6}{11},\frac{-9}{11}$

61. If the two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ have a point in common, then $k =$

(a) $\frac{2}{9}$ (b) $-\frac{2}{9}$ (c) $\frac{9}{2}$ (d) 0

62. $\lim_{x\to 0}\frac{x^2(\tan 2x-2\tan x)^2}{(1-\cos 2x)^4}=$

(a) 4 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

63. $\lim_{x\to\infty}\left(\frac{6x^2-\cos 3x}{x^2+5}-\frac{5x^3+3}{\sqrt{x^6+2}}\right)=$

(a) 11 (b) 0 (c) –1 (d) 1

64. The number of discontinuities in R for the function $f(x)=\frac{x-1}{x^3+6x^2+11x+6}$ is

(a) 3 (b) 2 (c) 1 (d) 0

65. $\frac{d}{dx}\left(\log\left(\sqrt{x+\sqrt{x^2+a^2}}\right)\right)=$

(a) $\sqrt{x^2+a^2}$ (b) $\frac{1}{\sqrt{x^2+a^2}}$

(c) $\frac{1}{2\sqrt{x^2+a^2}}$ (d) $\frac{1}{2\left(x+\sqrt{x^2+a^2}\right)}$

66. If $f(x)=\cot^{-1}\left(\frac{x^x-x^{-x}}{2}\right)$, then $f'(1) =$

(a) –log 2 (b) log 2 (c) 1 (d) –1

67. If $a \neq 0$, $x = a(t + \sin t)$ and $y = a(1 - \cos t)$, then $\frac{d^2y}{dx^2}$ at $t=\frac{2\pi}{3}$ is

(a) $\frac{4}{a}$ (b) $\frac{1}{4a}$ (c) $4a$ (d) $\frac{a}{4}$

68. The number of tangent to the curve $y^2(x - a) = x^2(x + a)$ $(a > 0)$ that are parallel to the X-axis is

(a) infinitely many (b) 0

(c) 1 (d) 2

69. If $f(x) = (2k + 1)x - 3 - ke^{-x} + 2e^x$ is monotonically increasing for all $x \in R$, then the least value of k is

(a) 1 (b) 0 (c) $-\frac{1}{2}$ (d) –1

70. If the function $f(x) = ax^3 + bx^2 + 11x - 6$ satisfies the conditions of Rolle's theorem in [1, 3] and $f'\left(2+\frac{1}{\sqrt{3}}\right) = 0$ then $a + b =$

(a) –5 (b) –3 (c) 4 (d) 7

71. For $a > 0$, if the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ attains its maximum value at p and minimum value at q such that $p^2 = q$, then $a =$

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 4

72. If $\int \cos x\cdot\cos 2x\cdot\cos 5x\,dx$ $= A\sin 2x + B\sin 4x + C\sin 6x + D\sin 8x + k$ (where k is the arbitrary constant of integration), then $\frac{1}{B}+\frac{1}{C}=$

(a) $\frac{1}{A}-\frac{1}{D}$ (b) $\frac{1}{A}+\frac{1}{D}$ (c) 1 (d) 0

73. If $\int e^x\left(\frac{x+2}{x+4}\right)^2 dx = f(x)+c$ arbitrary constant, then $f(x)=$

(a) $\frac{xe^x}{x+4}$ (b) $\frac{e^x}{x+4}$

(c) $\frac{xe^x}{(x+4)^2}$ (d) $\frac{e^x}{(x+4)^2}$

74. $\int \frac{dx}{\sin x + \sin 2x} =$

(a) $\frac{1}{2}\log_e|1+\cos x| + \frac{1}{6}\log_e|1-\cos x| - \frac{2}{3}\log_e|1+2\cos x| + c$

(b) $\frac{1}{2}\log_e|1+\cos x| - \frac{2}{3}\log_e|1-\cos x| + \frac{1}{2}\log_e|1+2\cos x| + c$

(c) $\frac{1}{2}\log_e|1+\sin x| - \frac{1}{3}\log_e|1-\sin x| - \frac{1}{3}\log_e|1+\cos x| + c$

(d) $\frac{1}{3}\log_e|1-\sin x| + \frac{1}{2}\log_e|1+\cos x| - \frac{2}{3}\log_e|1-2\cos x| + c$

75. If $I_n = \int \frac{\sin nx}{\sin x}dx$ for $n = 1, 2, 3, \ldots$, then $I_6 =$

(a) $\frac{3}{5}\sin 3x + \frac{8}{5}\sin^5 x - \sin x + c$

(b) $\frac{2}{5}\sin 5x - \frac{5}{3}\sin^3 x - 2\sin x + c$

(c) $\frac{2}{3}\sin 5x - \frac{8}{3}\sin^5 x + 4\sin x + c$

(d) $\frac{2}{5}\sin 5x - \frac{8}{5}\sin^3 x + 4\sin x + c$

76. If $\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\ldots\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}} = k,$

then $\log k =$

(a) $\log 4 + \frac{\pi}{2} - 1$ (b) $\log 2 + \frac{\pi}{2} + 1$

(c) $\log 2 + \frac{\pi}{2} - 2$ (d) $\log 2 + \frac{\pi}{2} - 1$

77. $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x \cos x\, dx}{\sin^4 x + \cos^4 x} =$

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

78. The curve $y = ax^2 + bx$ passes through the point (1, 2) and lies above the X-axis for $0 \le x \le 8$. If the area enclosed by this curve, the X-aixs and the line $x = 6$ is 108 square units, then $2b - a =$

(a) 2 (b) 0 (c) 1 (d) –1

79. The differential equation of all parabolas whose axes are parallel to Y-axis is

(a) $\frac{d^3y}{dx^3} = 0$ (b) $\frac{d^2y}{dx^2} = 0$

(c) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (d) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

80. The solution of the equation $\frac{dy}{dx} + 2y\tan x = \sin x$ satisfying $y = 0$ when $x = \frac{\pi}{3}$, is

(a) $y = 2\sin^2 x + \cos x - 2$ (b) $y = 2\sin^2 x - \cos x - 2$

(c) $y = 2\cos^2 x - \sin x + 2$ (d) $y = 2\cos x - \sin^2 x - 1$

PHYSICS

81. Two intervals of time are measured as $\Delta t_1 = (2.00 \pm 0.02)$ s and $\Delta t_2 = (4.00 \pm 0.02)$s The value of $\sqrt{(\Delta t_1)(\Delta t_2)}$ with correct significant figures and error is

(a) (2.828 ± 0.01)s (b) (2.83 ± 0.01)s

(c) (2.828 ± 0.0075)s (d) (2.83 ± 0.0075)s

82. The speed of a particle changes from $\sqrt{5}$ ms^{-1} to $2\sqrt{5}$ ms^{-1} in a time t. If the magnitude of change in its velocity is 5 ms^{-1}, the angle between the initial and final velocities of the particle is

(a) 30° (b) 45° (c) 60° (d) 90°

83. If the maximum height and range of a projectile are 3 m and 4 m respectively, then the velocity of the projectile is (Take, $g = 10$ ms^{-2})

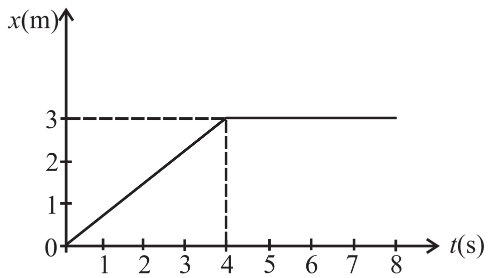
(a) $20\sqrt{\frac{6}{5}}$ ms^{-1} (b) $10\sqrt{\frac{3}{2}}$ ms^{-1}

(c) $10\sqrt{\frac{2}{3}}$ ms^{-1} (d) $20\sqrt{\frac{5}{6}}$ ms^{-1}

84. A body is projected at an angle other than 90° with the horizontal with same velocity. If the time of ascent of the body is 1s, then the maximum height it can reach is (Take; $g = 10$ ms^{-2})

(a) 5 m (b) 10 m (c) 2.5 m (d) 75 m

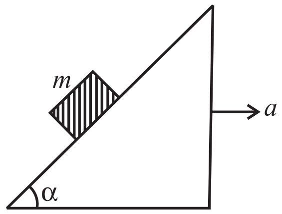
85. The position-time (x-t) graph of a moving body of mass 2 kg is shown in the figure. The impulse on the body at $t = 4$ s is



(a) 1.5 kg-ms^{-1} (b) – 1.5 kg-ms^{-1}

(c) 1 kg-ms^{-1} (d) 2 kg-ms^{-1}

86. A block of mass m is lying on a rough inclined plane having an inclination $\alpha = \tan^{-1}\left(\frac{1}{5}\right)$. The inclined plane is moving horizontally with a constant acceleration of $a = 2$ ms^{-2} as shown in the figure. The minimum value of coefficient of friction, so that the block remains stationary with respect to the inclined plane is (Take, $g = 10$ ms^{-2})



(a) $\frac{2}{9}$ (b) $\frac{5}{12}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

87. Potential energy of a body of mass 1 kg free to move along X-axis is given by $U(x)=\left(\frac{x^2}{2}-x\right)$J. If the total mechanical energy of the body is 2J, then the maximum speed of the body is (Assume only conservative force acts on the body)

(a) $\sqrt{5}\ ms^{-1}$ (b) $5\ ms^{-1}$
(c) $3.5\ ms^{-1}$ (d) $\sqrt{8}\ ms^{-1}$

88. A cylindrical well of radius 2.5 m has water upto a height of 14 m from the bottom. If the water level is at a depth of 6 m from the top of the well, then the time taken (in minutes) to empty the well using a motor of 10 HP is approximately, (Take, $g = 10\ ms^{-2}$)

(a) 30 (b) 80 (c) 98 (d) 90

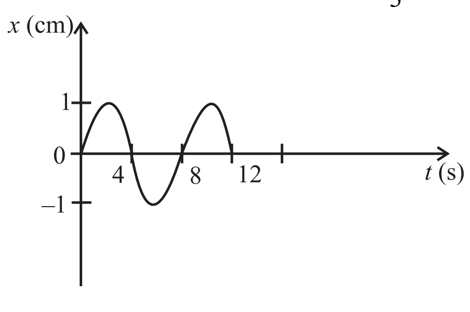
89. A flywheel of mass 1 kg and radius vector $(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ m is at rest. When a force $(3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}})$ N acts on it tangentially, it can rotate freely. Then, its angular velocity after 4.5 s is

(a) $\frac{2}{9}\sqrt{261}$ rad s^{-1} (b) $\frac{3}{2}\sqrt{261}$ rad s^{-1}
(c) $\sqrt{261}$ rad s^{-1} (d) $\frac{5}{9}\sqrt{261}$ rad s^{-1}

90. Three identical spheres each of diameter $2\sqrt{3}$ m are kept on a horizontal surface such that each sphere touches the other two spheres. If one of the sphere is removed, then the shift in the position of the centre of mass of the system is

(a) 12 m (b) 1 m (c) 2 m (d) $\frac{3}{2}$m

91. For a particle executing simple harmonic motion, the displacement-time (x-t) graph is as shown in the figure. The acceleration of the particle at $t=\frac{4}{3}$s is



(a) $-\frac{\sqrt{3}}{32}\pi^2 cm\ s^{-2}$ (b) $\frac{32}{\sqrt{3}}\pi^2\ cm\ s^{-2}$
(c) $+\frac{\sqrt{3}}{32}\pi\ cm\ s^{-2}$ (d) $+\frac{32}{\sqrt{3}}\pi\ cm\ s^{-2}$

92. Two masses 90 kg and 160 kg are separated by a distance of 5m. The magnitude of intensity of the gravitational field at a point which is at a distance 3 m from the 90 kg mass and 4 m from the 160 kg mass is
(Universal gravitational constant, $G = 6.67 \times 10^{-11}$ N-m^2 kg^{-2})

(a) 94.3×10^{-10} N kg^{-1} (b) 9.43×10^{-10} N kg^{-1}
(c) 9.43×10^{-12} N kg^{-1} (d) 94.3×10^{-12} N kg^{-1}

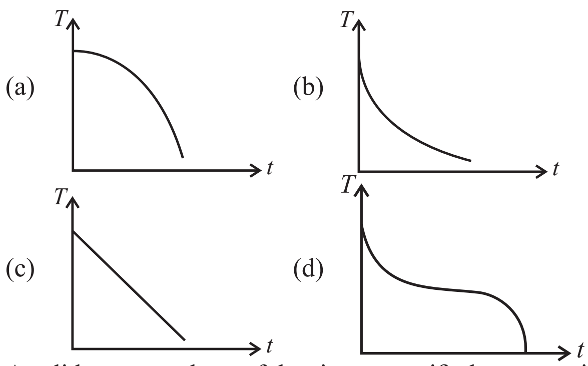
93. The following four wires are made of the same material. If same tension is applied to each, the wire having largest extension is

(a) length 0.5 m, diameter 0.5 mm.
(b) length 1 m, diameter 1 mm.
(c) length 2 m, diameter 2 mm.
(d) length 3 m, diameter 3 mm.

94. A liquid drop of density ρ is floating half immersed in a liquid of surface tension S and density $\frac{\rho}{2}$. If the surface tension S of the liquid is numerically equal to 10 times of acceleration due to gravity, then the diameter of the drop is :

(a) $\sqrt{\frac{20}{\rho}}$ (b) $\sqrt{\frac{80}{\rho}}$ (c) $\sqrt{\frac{60}{\rho}}$ (d) $\sqrt{\frac{40}{\rho}}$

95. A block of metal is heated to a temperature much higher than the room temperature and placed in an evacuated cavity. The curve which correctly represents the rate of cooling (T is temperature of the block and t is the time.)



96. A solid copper sphere of density ρ, specific heat capacity C and radius r is initially at 200K. It is suspended inside a chamber whose walls are at 0 K. The time required (in μs) for the temperature of the sphere to drop to 100 K is (σ is Stefan's constant and all the quantities are in SI units.)

(a) $48\frac{r\rho C}{\sigma}$ (b) $\frac{1}{48}\frac{r\rho C}{\sigma}$ (c) $\frac{27}{7}\frac{r\rho C}{\sigma}$ (d) $\frac{7}{27}\frac{r\rho C}{\sigma}$

97. Match the temperatures of the source and sink (T_1 and T_2 respectively) of a Carnot heat engine given in List-I with the corresponding efficiencies given in List-II.

List-I		List-II	
(A)	$T_1 = 500$ K, $T_2 = 300$ K	(i)	0.2
(B)	$T_1 = 500$ K, $T_2 = 350$ K	(ii)	0.3
(C)	$T_1 = 800$ K, $T_2 = 400$ K	(iii)	0.4
(D)	$T_1 = 450$ K, $T_2 = 360$ K	(iv)	0.5

The correct match is

	A	B	C	D		A	B	C	D
(a)	(iii)	(iv)	(ii)	(i)	(b)	(iv)	(iii)	(ii)	(i)
(c)	(iii)	(i)	(iv)	(ii)	(d)	(iii)	(ii)	(iv)	(i)

98. A hammer of mass 200 kg strikes a steel block of mass 200 g with a velocity 8 ms^{-1}. If 23% of the energy is utilized to heat the steel block, the rise in temperature of the block is (specific heat capacity of steel = 460 J kg^{-1} K^{-1})

(a) 8 K (b) 16 K (c) 12 K (d) 24 K

99. At a temperature of 314 K and a pressure of 100 kPa, the speed of sound in a gas is 1380 ms^{-1}. The radius of each gas molecule is 0.5Å. The frequency of sound at which the wavelength of sound wave in the gas becomes equal to the mean free path of the gas molecules is (Boltzmann constant = 1.38×10^{-23} JK^{-1}.)

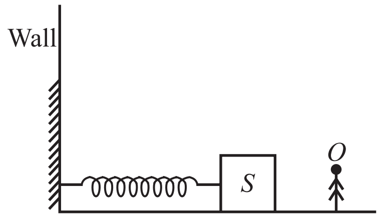
(a) 1000 MHz (b) $1000\sqrt{2}$ MHz

(c) $\frac{1000}{\sqrt{2}}$ MHz (d) 500 MHz

100. At a temperature of 27°C, two identical organ pipes produce notes of frequency 140 Hz. If the temperature of one pipe is raised to 57.75°C, then the number of beats produced per second is

(a) 7 (b) 5 (c) 3 (d) 9

101. A source of sound *S* in the form of a block kept on a smooth horizontal surface is connected to a spring, as shown in the figure. If the spring oscillates with an amplitude of 50 cm along horizontal between the wall and the observer *O*, the maximum frequency heard by the observer is 12.5% more than the minimum frequency heard by him. If the mass of the source of sound is 100 g, the force constant of the spring is (Speed of sound in air is 340 ms^{-1})



(a) 40 Nm^{-1} (b) 80 Nm^{-1}

(c) 160 Nm^{-1} (d) 320 Nm^{-1}

102. A girl of height 150 cm with her eye level at 140 cm stands in front of plane mirror of height 75 cm fixed to a wall. The lower edge of the mirror is at a height of 85 cm above her feet level. The height of her image the girl can see in the mirror is

(a) 130 cm (b) 140 cm (c) 120 cm (d) 150 cm

103. Unpolarised light from air incidents on the surface of a transparent medium of refractive index 1.414 such that the reflected light is completely polarised. Match the angles given in List-I with the corresponding values given in List-II.

List-I	List-II	
(A) Angle of reflection	(i)	$2\sin^{-1}\left(\sqrt{\frac{2}{3}}\right)$
(B) Angle of refraction	(ii)	$\sin^{-1}\left(\sqrt{\frac{2}{3}}\right) - \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(C) Angle between incident and completely polarised light	(iii)	$\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(D) Angle of deviation of the incident ray	(iv)	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

	A	B	C	D		A	B	C	D
(a)	(ii)	(iii)	(i)	(iv)	(b)	(ii)	(iii)	(iv)	(i)
(c)	(iv)	(i)	(iii)	(ii)	(d)	(iv)	(iii)	(i)	(ii)

104. The electric field intensity at a point on the axis of an electric dipole in air is 4 NC^{-1}. Then the electric field intensity at a point on the equatorial line which is at a distance equal to twice the distance on the axial line and if the dipole is in a medium of dielectric constant 4 is

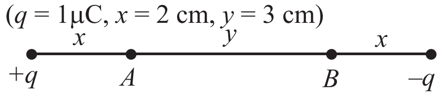
(a) 1NC^{-1} (b) $\frac{1}{8}$$NC^{-1}$

(c) 16 NC^{-1} (d) $\frac{1}{16}$$NC^{-1}$

105. Two small spheres of each charge *q*, mass *m* and material density *d* are suspended from a fixed point with the help of inextensible light thread. When the spheres are in air, the angle between the threads is 90°. When the spheres are suspended in a liquid of density $\frac{2}{3}d$, the angle between the threads is 60°. The value of dielectric constant of the liquid is

(a) $6\sqrt{3}$ (b) $2\sqrt{5}$ (c) $5\sqrt{3}$ (d) $7\sqrt{2}$

106. The potential difference $(V_A - V_B)$ in the arrangement shown in the figure is
($q = 1\mu$C, $x = 2$ cm, $y = 3$ cm)



(a) 5.4×10^5 V (b) 2.7×10^5 V

(c) 5.4×10^2 V (d) 2.7×10^2 V

107. In a parallel plate capacitor the separation between plates in 3*x*. This separation is filled by two layers of dielectrics, in which one layer has thickness *x* and dielectric constant 3*k*, the other layer is of thickness 2x and dielectric constant 5*k*, If the plates of the capacitor are connected to a battery, then the ratio of potential difference across the dielectric layers is

(a) $\frac{1}{2}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{6}$

108. **Assertion (A) :** When a wire of aluminium and another wire of silicon are heated from room temperature to 80°C, then conductivity of aluminium increases and that of silicon decreases.

Reason (R) : Aluminium has positive temperature coefficient of resistivity and silicon has negative temperature coefficient of resistivity.

(a) Both (A) and (R) are correct and (R) is the correct explanation of (A).
(b) Both (A) and (R) are correct but (R) is not the correct explanation of (A).
(c) (A) is correct but (R) is not correct.
(d) (A) is not correct but (R) is correct.

109. The walls of a closed cubical box of edge 60 cm are made of material of thickness 1 mm and thermal conductivity, 4×10^{-4} cal $s^{-1}cm^{-1}°C^{-1}$. The interior of the box is maintained 1000°C above the outside temperature by a heater placed inside the box and connected across 400 V DC supply. The resistance of the heater is

(a) 4.41Ω (b) 44.1Ω (c) 0.441Ω (d) 441Ω

110. A galvanometer of resistance G Ω, is shunted by a resistance S Ω. To keep the main current in the circuit unchanged, the resistance to be connected in series with the galvanometer is

(a) $\frac{G^2}{S+G}$ (b) $\frac{S}{S+G}$ (c) $\frac{S^2}{S+G}$ (d) $\frac{SG}{S+G}$

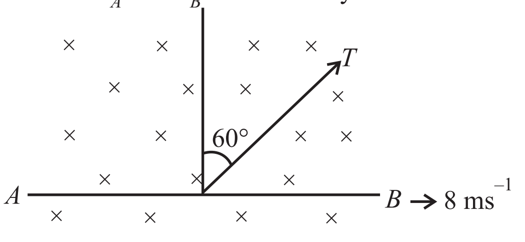
111. A proton and an α-particle are simultaneously projected in opposite direction into a region of uniform magnetic field of 2 mT perpendicular to the direction of the field. After some time it is found that the velocity of proton has changed in direction by 90°. Then at this time, the angle between the velocity vectors of proton and α-particle is

(a) 60° (b) 90° (c) 45° (d) 180°

112. A bar magnet placed in a uniform magnetic field making an angle θ with the field experiences a torque. If the angle made by the magnet with the field is doubled, the torque experienced by the magnet increases by 41.4%. The initial angle made by the magnet with the magnetic field is

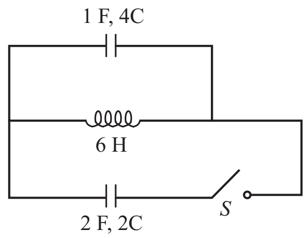
(a) 60° (b) 30° (c) 90° (d) 45°

113. A metal rod AB of length 50 cm is moving at a velocity 8 ms^{-1} in a magnetic field of 2T. If the field is at 60° with the plane of motion as shown in the figure, then the potentials V_A and V_B are related by



(a) $VA - VB = 8$ V (b) $VA - VB = 4$ V
(c) $VB - VA = 8$ V (d) $VB - VA = 4$V

114. In the given electrical circuit, if the switch S is closed then the maximum energy stored in the inductors is :



(a) 3 J (b) 9 J (c) 12 J (d) 6 J

115. Which of the following is/are the property/properties of a monochromatic electromagnetic wave propagating in the free space?

1. Electric and magnetic fields will have a phase difference $\frac{\pi}{2}$.
2. The energy of the wave is distributed equally between electric and magnetic fields.
3. The pressure exerted by the wave is the product of its speed and energy density.
4. The speed of the wave is equal to the ratio of the magnetic field to the electric field.

(a) 1 and 3 (b) Only 2 (c) 2 and 3 (d) Only 4

116. The maximum kinetic energy of a photoelectron liberation from the surface of lithium with work function 2.35 eV by electromagnetic radiation whose electric component varies with time as :

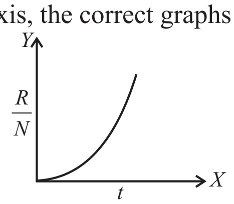
$E = a[1 + \cos(2\pi f_1 t)] \cos 2\pi f_2 t$ (where a is a constant) is ($f_1 = 3.6 \times 10^{15}$ Hz, and $f_2 = 1.2 \times 10^{15}$ Hz and Planck's constant $h = 6.6 \times 10^{-34}$ Js)

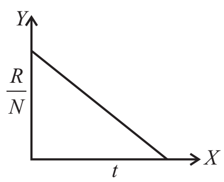
(a) 2.64 eV (b) 7.55 eV
(c) 12.52 eV (d) 17.45 eV

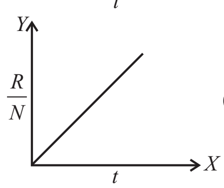
117. Magnetic moment due to the motion of the electron in nth energy state of hydrogen atom is proportional to.....

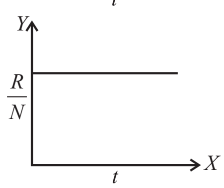
(a) n^{-2} (b) n (c) n^2 (d) n^3

118. The rate of disintegration of a radioactive sample is R and the number of atoms present at any time t is N. When $\frac{R}{N}$ is taken along Y-axis and t is taken along X-axis, the correct graphs is

(a) 

(b) 

(c) 

(d) 

119. For an LED to emit light in visible region of the electromagnetic spectrum, it can have energy band gap in the range of,

(Plank's constant, $h = 6.6 \times 10^{-34}$ Js and speed of light, $e = 3 \times 10^8$ ms^{-1} in vacuum)

(a) 0.1 eV to 0.4 eV (b) 0.9 eV to 1.6 eV
(c) 1.7 eV to 3.1 eV (d) 0.5 eV to 0.8 eV

120. A transmitting antenna of height 20 m and the receiving antenna of height *h* are separated by a distance of 40 km for satisfactory communication in line of sight (Los) mode. Then the value of *h* is
(Give, radius of earth is 6400 km.)
(a) 40 m (b) 45 m (c) 30 m (d) 25 m

CHEMISTRY

121. The energies of an electron in first orbit of He^+ and in third orbit of Li^{2+} in J are respectively
(a) $-8.72 \times 10^{-18}, -2.18 \times 10^{-18}$
(b) $-8.72 \times 10^{-18}, -1.96 \times 10^{-17}$
(c) $-1.96 \times 10^{-17}, -2.18 \times 10^{-18}$
(d) $-8.72 \times 10^{-17}, -1.96 \times 10^{-17}$

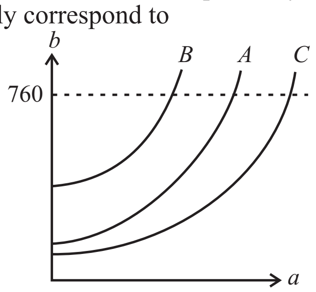
122. How many orbitals is/are possible with $n = 3$, $l = 1$ and $m_l = -1$ value?
(a) 2 (b) 3 (c) 5 (d) 1

123. If four elements with atomic numbers $Z-2$, $Z-1$, Z and $Z+1$ are forming isoelectronic ions, the atomic number of the ion having largest size is
(a) $Z-2$ (b) $Z-1$ (c) Z (d) $Z+1$

124. Identify the molecule in which the arrangement of electron pairs around the central atom is octahedral and shape is not octahedral.
(a) SF_6 (b) XeF_6 (c) BrF_5 (d) XeO_2F_4

125. The wave functions of 1*s*- orbitals of two hydrogen atoms are ψ_A and ψ_B.ψ_A and ψ_B are linearly combined to form two molecular orbitals (σ and σ^*). Which of the following statements are correct?
I. σ^* is equal to $(\psi_A - \psi_B)$.
II. In σ-orbital, one nodal plane is present in between two nuclei.
III. The energy of σ-orbital is lower than the energy of σ^*–orbital.
(a) I, II, III (b) I, II only
(c) II, III only (d) I, III only

126. The variation of vapour pressure (*b*) as a function of temperature (*a*) is studied for $C_2H_5OC_2H_5$, CCl_4 and H_2O at 760 mm Hg and is shown in the figure below. The boiling temperatures of $C_2H_5OC_2H_5$, CCl_4 and H_2O are 308, 350 and 373 K respectively. Curves *A, B, C* respectively correspond to



(a) $H_2O, C_2H_5OC_2H_5, CCl_4$ (b) $C_2H_5OC_2H_5, CCl_4, H_2O$
(c) $CCl_4, C_2H_5OC_2H_5, H_2O$ (d) $CCl_4, H_2O, C_2H_5OC_2H_5$

127. 30.0 mL of the given HCl solution requires 20.0 mL of 0.1 M sodium carbonate solution for complete neutralisation. What is the volume of this HCl solution required to neutralise 30.0 mL of 0.2 M NaOH solution?
(a) 25 mL (b) 50 mL (c) 90 mL (d) 45 mL

128. The heat required to rise the temperature of 54 g of aluminium from 40°C to 60°C in J is (molar heat capacity of aluminium in this temperature range is 24 J mol^{-1} K^{-1}; atomic weight of Al is 27)
(a) 480 (b) 800 (c) 960 (d) 1280

129. The equilibrium constant at 850 K for the reaction $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$ is 0.5625. The equilibrium concentration of NO(*g*) is 3.0×10^{-3} M. If the equilibrium concentration of $N_2(g)$ and $O_2(g)$ are equal, the concentrations of $N_2(g)$ in *M* is
(a) 4.0×10^{-3} (b) 4.0×10^{-2}
(c) 1.6×10^{-3} (d) 3.0×10^{-3}

130. The solubility product of a sparingly soluble salt A_2B is 3.2×10^{-11}. Its solubility in mol L^{-1} is
(a) 4×10^{-4} (b) 2×10^{-4}
(c) 6×10^{-4} (d) 3×10^{-4}

131. What is the volume (in mL) of 20 vol H_2O_2 required to completely react with 500 mL of 0.02 M acidified $KMnO_4$ solution?
(a) 14.0 (b) 7.0 (c) 28.0 (d) 42.0

132. KO_2, reacts with water to form *A, B* and *C*. *B* forms *C* when it reacts with iodine in basic medium. What are *B* and *C* respectively?
(a) KOH, H_2O_2 (b) K_2O_2, H_2O_2
(c) KOH, O_2 (d) H_2O_2, O_2

133. Identify the correct statements from the following :
I. Ga_2O_3 is an amphoteric oxide.
II. The dimer of aluminium chloride has three Al—Cl—Al bridge bonds.
III. Boron is very hard refractory solid of high melting temperature.
(a) I, II only (b) I, III only
(c) II, III only (d) I, II, III

134. Which one of the following methods is used to prepare carbon monoxide on commercial scale?
(a) Dehydration of formic acid with conc. H_2SO_4
(b) Direct oxidation of C in limited supply of oxygen
(c) Passing steam over hot coke
(d) Heating lime stone

135. Match the following :

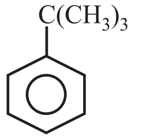
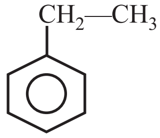
	List-I		List-II
(A)	Insecticide	(I)	COD
(B)	$K_2Cr_2O_7/50\%H_2SO_4$	(II)	PAN
(C)	Bleaching of clothes and paper	(III)	Na_3AsO_3
(D)	Eye irritant	(IV)	BOD
		(V)	H_2O_2

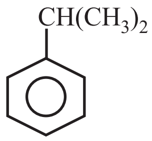
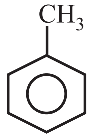
Codes

	A	B	C	D		A	B	C	D
(a)	III	IV	V	II	(b)	III	I	V	II
(c)	III	I	II	V	(d)	V	I	III	II

136. For which of the following, Kjeldahl's method is not used for the estimation of nitrogen?
I. Aniline II. Azobenzene
III. Nitrobenzene IV. Pyridine
(a) II, III, IV (b) II, III only
(c) III, IV only (d) I, III, IV

137. Identify the compound, which has maximum number of no bond resonance structures.

(a)  (b) 

(c)  (d) 

138. Which of the following statements are correct?

I. In nitrating mixture nitric acid participates as an acid.

II. σ complex is the intermediate substance in electrophilic substitution of benzene.

III. Benzene on Friedel-Crafts alkylation with *n*-propyl chloride gives isopropyl benzene.

(a) II, III only (b) I, II only
(c) I, III only (d) I, II, III

139. Which one of the following functional groups is not *meta* directing?

(a) —COOH (b) $—NO_2$
(c) —CHO (d) $—OCH_3$

140. If the radius of an atom of an element which forms a body centred cubic unit cell is 173.2 pm, the volume of unit cell in cm^3 is

(a) 3.12×10^{-23} (b) 6.4×10^{-24}
(c) 3.2×10^{-24} (d) 2.13×10^{-23}

141. A solution is prepared by dissolving 10 g of a non-volatile solute (molar mass, '*M*' g mol^{-1}) in 360 g of water. What is the molar mass in g mol^{-1} of solute if the relative lowering of vapour pressure of solution is 5×10^{-3}?

(a) 199 (b) 99.5 (c) 299 (d) 149.5

142. *x* g of $MgSO_4$(i = 1.8) in 2.5 L of solution has an osmotic pressure of 2.463 atm at 27°C. What is the value of *x* in g?

(a) 33.2 (b) 6.6 (c) 3.3 (d) 16.6

143. The electrode potential for

$M^{2+}(aq) + e^- \longrightarrow M^+(aq)$

$M^+(aq) + e^- \longrightarrow M(s)$

are + 0.15 V and +0.50 V respectively. The value of $E°_{M^{2+}/M}$ will be

(a) 0.150 V (b) 0.300 V (c) 0.325 V (d) 0.650 V

144. The half-life period of a first order reaction at 300 K and 400 K are 50 s and 10 s respectively. The activation energy of the reaction in kJ mol^{-1} is (log 5 = 0.70)

(a) 4.0 (b) 8.0 (c) 16.10 (d) 20.10

145. Which one of the following statements is correct for adsorption of solutes on solids in solutions?

(a) The extent of adsorption increases with an increase in temperature.

(b) The extent of adsorption decreases with an increase of surface area of the adsorbate.

(c) The extent of adsorption decreases with an increase in temperature.

(d) The extent of adsorption does not depend on the amount of the solute in solution.

146. Identify the metal which is not common to German silver and brass.

(a) Cu (b) Zn (c) Fe (d) Ni

147. Which of the following reactions are correct with respect to the formation of products?

I. $2NaOH + SO_2 \longrightarrow Na_2SO_4 + H_2$

II. $2XeF_4 + 3O_2F_2 \xrightarrow{143\text{ K}} 2XeO_3 + 7F_2$

III. $PCl_5 + 4H_2O \longrightarrow H_3PO_4 + 5HCl$

IV. $2NaNO_2 + 2HCl \longrightarrow 2NaCl + NO + NO_2 + H_2O$

(a) II, IV (b) III, IV (c) I, III (d) II, III

148. Chlorine oxidises sulphur dioxide in the presence of water to give an oxyacid *A*. Chlorine also oxidises iodine in the presence of water to give an oxyacid *B*. The oxidation states of *S* and I in *A* and *B* are respectively

(a) + 4, + 5 (b) + 6, + 3 (c) + 6, + 5 (d) + 4, + 7

149. White phosphorus is heated with concentrated NaOH in CO_2 atmosphere to form a gas *A* and compound *B*. When *A* is bubbled into aqueous $CuSO_4$ solution copper phosphide and *C* are formed, *B* and *C* are respectively

(a) PH_3, H_2SO_4 (b) NaH_2PO_2, H_2SO_4
(c) $NaHPO_2$, CuS (d) NaH_2PO_2, Cu_2S

150. Which of the following set of elements do not possess *f*-electrons?

(a) La, U, Lr (b) La, Th, Lr
(c) La, Ac, Th (d) Ce, Ac, Th

151. The Δ_0 of a coordination complex of a metal ion ($3d^1$) is 1000 kJ mol^{-1}. If the energy of t_{2g} orbitals is – 400 kJ mol^{-1}, the energy (in kJ mol^{-1}) of e_g orbitals is

(a) – 600 (b) 600 (c) 1000 (d) 400

152. How many of the following polymers given, come under the category of condensation polymers? Bakelite, Teflon, Nylon-6, Dacron, Polyisoprene, Melamine, Neoprene

(a) 4 (b) 3 (c) 5 (d) 6

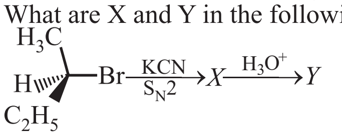
153. Identify the correct set from the following :

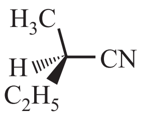
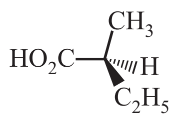
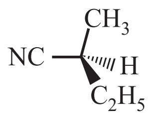
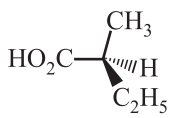
	Vitamin	Source	Deficiency disease
(a)	B_6	Milk	Convulsions
(b)	K	Leaf vegetables	Anaemia
(c)	C	Fish	Scurvy
(d)	D	Citrus fruits	Ricket

154. Which one of the following contains —As = As— in its structure?

(a) Ranitidine (b) Saccharin
(c) Salvarsan (d) Seldane

155. What are X and Y in the following reaction sequence?



	X	*Y*
(a)		
(b)		

(c) 

(d) 

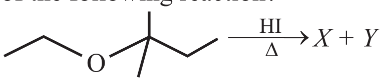
156. Which of the following sets is in the correct order regarding the property mentioned against them?

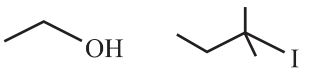
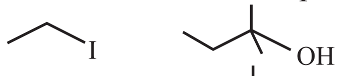
	Sets	Property
I.	$NCCH_2COOH > FCH_2COOH > H_3CCH_2COOH$	Acidity
II.	$CH_3CH_2CHO > PhCOCH_3 > PhCHO$	Reactivity
III.	$H_3COCH_2CH_3 < H_3CCH_2CHO < H_3CCH_2CH_2OH$	Boiling points

(a) I, II only (b) I, III only

(c) II, III only (d) I, II. III

157. Identify the products (*X*, *Y*) and reaction mechanism (*Z*) of the following reaction?



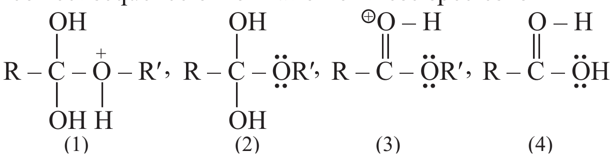
	X	Y	Z
(a)			S_N1
(b)			S_N2
(c)			S_N1
(d)			S_N2

158. What are the products formed when an aldehyde (*R*CHO) is reacted with Tollen's reagent?

(a) Ag, H_2O, RCH_2OH, NH_3 (b) Ag, H_2O, $RCOO^-$, H_2

(c) Ag, H_2O, $RCOO^-$, NH_3 (d) Ag_2, H_2O, $RCOO^-$, NH_3

159. The following species are involved in the formation of an ester from a carboxylic acid in the presence of acid. the correct sequence of formation of these species is



(a) 2 1 4 3 (b) 4 1 2 3 (c) 1 4 3 2 (d) 2 3 1 4

160. Identify the reagents (*X, Y, Z*) used in the conversion of 3-methylaniline to 3-nitrotoluene.

	X	Y	Z
(a)	$NaNO_2$, HCl 273 K	HBF_4	$NaNO_2$, Cu, Δ
(b)	$NaNO_3$, HCl 273 K	HF	$NaNO_3$, Cu, Δ
(c)	$NaNO_2$, HCl	C_2H_5OH	$NaNO_3$, Δ
(d)	$NaNO_3$, HCl	NaOH	$C_6H_5NO_2$

ANSWER KEY

1	(b)	17	(b)	33	(a)	49	(a)	65	(c)	81	(b)	97	(d)	113	(b)	129	(a)	145	(c)
2	(b)	18	(d)	34	(c)	50	(d)	66	(d)	82	(d)	98	(b)	114	(a)	130	(b)	146	(c)
3	(c)	19	(c)	35	(b)	51	(c)	67	(a)	83	(c)	99	(b)	115	(b)	131	(a)	147	(b)
4	(a)	20	(c)	36	(b)	52	(b)	68	(b)	84	(a)	100	(a)	116	(d)	132	(d)	148	(c)
5	(b)	21	(c)	37	(b)	53	(b)	69	(b)	85	(b)	101	(c)	117	(b)	133	(b)	149	(b)
6	(c)	22	(c)	38	(b)	54	(d)	70	(a)	86	(b)	102	(c)	118	(d)	134	(c)	150	(c)
7	(a)	23	(c)	39	(b)	55	(b)	71	(c)	87	(a)	103	(d)	119	(c)	135	(b)	151	(b)
8	(a)	24	(b)	40	(d)	56	(a)	72	(b)	88	(b)	104	(d)	120	(b)	136	(a)	152	(a)
9	(c)	25	(c)	41	(b)	57	(b)	73	(a)	89	(c)	105	(a)	121	(a)	137	(d)	153	(a)
10	(c)	26	(d)	42	(b)	58	(c)	74	(a)	90	(b)	106	(a)	122	(d)	138	(a)	154	(c)
11	(a)	27	(b)	43	(d)	59	(a)	75	(d)	91	(a)	107	(d)	123	(a)	139	(d)	155	(b)
12	(c)	28	(d)	44	(c)	60	(b)	76	(c)	92	(b)	108	(d)	124	(c)	140	(N)	156	(b)
13	(c)	29	(d)	45	(a)	61	(c)	77	(d)	93	(a)	109	(c)	125	(d)	141	(b)	157	(a)
14	(a)	30	(b)	46	(c)	62	(d)	78	(b)	94	(b)	110	(a)	126	(c)	142	(d)	158	(c)
15	(a)	31	(b)	47	(d)	63	(a)	79	(a)	95	(b)	111	(c)	127	(d)	143	(e)	159	(b)
16	(b)	32	(b)	48	(a)	64	(a)	80	(a)	96	(b)	112	(d)	128	(c)	144	(c)	160	(a)

Hints & Solutions

MATHEMATICS

1. **(b)** Since we have,

$(goj\ of)(x) = f(x)$ for all $x \in D$

$$\Rightarrow g(jof)(x)) = \frac{1}{x}$$

$$\Rightarrow 1-(jof(x) = \frac{1}{x} \text{ (given)}$$

$$\Rightarrow 1-j(f(x)) = \frac{1}{x}$$

$$\Rightarrow 1-j\left(\frac{1}{x}\right) = \frac{1}{x} \quad \Rightarrow j\left(\frac{1}{x}\right) = 1-\frac{1}{x}$$

$\Rightarrow j(x) = 1 - x = g(x)$

2. **(b)** Given,

$$f(x) = \tan\left(x+\frac{2\pi}{3}\right) - \tan\left(x+\frac{\pi}{6}\right) + \cos\left(x+\frac{\pi}{6}\right)$$

$$\left\{\therefore \tan\left(x+\frac{2\pi}{3}\right) - \tan\left(x+\frac{\pi}{6}\right) = \frac{\sin\frac{\pi}{2}}{\cos\left(x+\frac{2\pi}{3}\right)\cos\left(x+\frac{\pi}{6}\right)}\right\}$$

$$f(x) = \frac{1\times 2}{2\cos\left(x+\frac{2\pi}{3}\right)\cos\left(x+\frac{\pi}{6}\right)} + \cos(x+30°)$$

$$= \frac{2}{\cos(150°+2x)+\cos 90°} + \cos(x+30°)$$

$$= \cos(x+30°) - \frac{2}{\cos(2x-30°)}$$

for maximum differentiating both side

$$f'(x) = -\sin(x+30°) - \frac{4\sin(2x-30°)}{\cos^2(2x-30°)}$$

Since, it lie $-75° < x < -60°$

then $(30° + x) \in [-45°, -30°]$

and $(2x - 30°) \in [-180°, -150°]$

$\therefore f'(x) > 0$ for all $-75° < x < -60°$

So, f_{max} at $x = -60°$

$f(-60°) = \tan(60°) - \tan(-30°) + \cos(-30°)$

$$= \sqrt{3} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \frac{11\sqrt{3}}{6}$$

3. **(c)** Given $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots + (1^2 + 2^2 + \ldots + n^2)$

$$= \sum_{i=1}^{n}(1^2+2^2+3^2+\ldots+i^2) = \sum_{i=1}^{n}\sum_{j=1}^{i} j^2$$

$$= \sum_{i=1}^{n}\frac{i(i+1)(2i+1)}{6}\left\{\because \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}\right\}$$

$$= \sum_{i=1}^{n}\frac{2i^3+3i^2+i}{6} = \frac{1}{3}\sum_{i=1}^{n} i^3 + \frac{1}{2}.\sum_{i=1}^{n} i^2 + \frac{1}{6}\sum_{i=1}^{n} i$$

$$= \frac{1}{3}\left[\frac{n(n+1)}{2}\right]^2 + \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{1}{6}\left[\frac{n(n+1)}{2}\right]$$

$$\left\{\therefore \Sigma n^3 = \left[\frac{n(n+1)}{2}\right]^2,\quad \Sigma n = \frac{n(n+1)}{2}\right\}$$

$$= \frac{1}{3}.\frac{n^2(n+1)^2}{4} + \frac{1}{2}.\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}.\frac{n(n+1)}{6}$$

$$= \frac{n(n+1)}{12}[n^2+3n+2] = \frac{n(n+1)^2(n+2)}{12}$$

4. **(a)** Given

$$\begin{vmatrix} a+b+2c & a & b \\ c & 2a+b+c & b \\ c & a & a+2b+c \end{vmatrix} = 2$$

$$\Rightarrow 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix} - 2$$

[Applying $C_1 \to C_1 + C_2 + C_3$ and taking $2(a+b+c)$ common from C_1]

$$\Rightarrow 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+a & b \\ 1 & a & c+a+b \end{vmatrix} = 2$$

[Applying $R_1 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$\Rightarrow 2(a+b+c)^3 = 2$ [expanding along C_1]

$\Rightarrow (a+b+c)^3 = 1 \Rightarrow a+b+c = 1$

Now, $a^3 + b^3 + c^3 - 3abc$

$= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= 1[a^2+b^2+c^2-ab-bc-ca]$

$= 1 - 3ab - 3bc - 3ca$

5. **(b)** Given $x^2 - 25x + 24 = 0$ ...(i)

$(x-1)(x-24) = 0 \Rightarrow x = 1, 24$

$\because$ k is one of the root of the Eq. (i),

$\therefore$ $k = 1, 24$

If $k = 1$,

$$\therefore A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

Since $C_1 \equiv C_2$

So $|A| = 0$

$k = 1$, not possible, because given matrix A is singular.

Now, $k = 24$,

$$\therefore A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 24 \end{bmatrix}$$

$|A| = 1 (48 - 3) - 2 (72 - 3) + 1 (3 - 2)$

$= 45 - 138 + 1 = -92 \neq 0$

$$\text{adj } A = \begin{bmatrix} 45 & -69 & 1 \\ -47 & 23 & 1 \\ 4 & 0 & -4 \end{bmatrix}^1 = \begin{bmatrix} 45 & -47 & 4 \\ -69 & 23 & 0 \\ 1 & 1 & -4 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}.\text{adj} A$$

$$= -\frac{1}{92}\begin{bmatrix} 45 & -47 & 4 \\ -69 & 23 & 0 \\ 1 & 1 & -4 \end{bmatrix}$$

6. **(c)** Given,

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$$

For rank to be 3, there must exist 3 non zero row.

Now, applying $R_2 \to R_2 - 4R_1$; $R_3 \to R_3 - 2R_1$

and Applying $R_4 \to R_4 - 9R_1$

$$= \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ b-2 & 0 & 4 & 2 \\ 0 & 0 & b+9 & 3 \end{bmatrix}$$

Again, applying $R_4 \to R_4 - 3R_2$

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ b-2 & 0 & 4 & 2 \\ 0 & 0 & b+6 & 0 \end{bmatrix}$$

for rank = 3,

Last row must have all elements 0.

$\therefore b + 6 = 0 \Rightarrow b = -6$

7. **(a)** Given,

$z = x - iy$ and $z^{\frac{1}{3}} = a + ib$

Cubing $(z^{1/3})^3 = (a + ib)^3$ $\quad [\because i^2 = -1]$

$\Rightarrow z = (a^3 - 3ab^2) - i (b^3 - 3a^2b)$

$\Rightarrow x - iy = (a^3 - 3ab^2) - i (b^3 - 3a^2b)$ $\quad [\because z = x - iy]$

Here, $x = a^3 - 3ab^2$ and $y = b^3 - 3a^2b$

$$\text{Now, } \frac{\left(\frac{x}{a} + \frac{y}{b}\right)}{a^2 + b^2} = \frac{\left(\frac{a^3 - 3ab^2}{a} + \frac{b^3 - 3a^2b}{b}\right)}{a^2 + b^2}$$

$$= \frac{a^2 - 3b^2 + b^2 - 3a^2}{a^2 + b^2} = \frac{-2(a^2 + b^2)}{a^2 + b^2} = -2$$

8. **(a)** Given $\overline{z} = iz^2$

$\Rightarrow z = -i\overline{z}^2 \Rightarrow z = -i[i z^2]^2$

$\Rightarrow z = -i\, i^2 z^4 \Rightarrow z = iz^4$

$\Rightarrow z^4 = \frac{1}{i} z$

$\Rightarrow z^4 + iz = 0 \Rightarrow z (z^3 + i) = 0$

$\Rightarrow z^3 + i = 0$ $\quad (\because z \neq 0)$

9. **(c)** Given $x^2 + y + 4i$ and $-3 + x^2 yi$ are conjugate.

Therefore, $x^2 + y + 4i = -3 - x^2yi$

$\Rightarrow (x^2 + y) + 4i = (-3) - x^2yi$

On comparing both sides, we get

$\Rightarrow x^2 + y = -3$ ...(i)

and $4 = -x^2y$

$\Rightarrow y = \frac{4}{-x^2} = -\frac{4}{x^2}$ ..(ii)

On putting the value of y in eq. (i), we get

$\therefore x^2 - \frac{4}{x^2} = -3 \Rightarrow x^4 - 4 = -3x^2$

$\Rightarrow x^4 + 3x^2 - 4 = 0$

$\Rightarrow (x^2 + 4) (x^2 - 1) = 0$ $\quad (\because x^2 + 4 > 0)$

$x^2 - 1 = 0$

$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

Put the value of $x = \pm 1$ in eq. (ii), we get

$y = \frac{-4}{(1)^2} = -4$

$\therefore x = \pm 1, y = -4$

Hence, $(|x| + |y|)^2$

$= |x|^2 + |y|^2 + 2 |x| |y|$

$= (1)^2 + |(-4)|^2 + 2 |1| |-4|$

$= 1 + 16 + 8 = 25$

10. **(c)** Given $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$

$$=-i\sum_{k=1}^{6}\left[\cos\left(\frac{2\pi k}{7}\right)-\frac{1}{i}\sin\left(\frac{2\pi k}{7}\right)\right]$$

$$=-i\sum_{k=1}^{6}\left[\cos\left(\frac{2\pi k}{7}\right)+i\sin\left(\frac{2\pi k}{7}\right)\right]$$

$$=-i\left[\sum_{k=1}^{6}e^{\frac{i2\pi k}{7}}\right] \quad ...(i)\ \{\because e^{i\theta}=\cos\theta+i\sin\theta\}$$

Now,

$$\left[1+e^{\frac{i2\pi}{7}}+e^{\frac{i4\pi}{7}}+e^{\frac{i6\pi}{7}}+e^{\frac{i8\pi}{7}}+e^{\frac{i10\pi}{7}}+e^{\frac{i12\pi}{7}}=0\right]$$

$$\left\{\because z^7=1 \text{ then, roots } 1, e^{\frac{i2\pi}{7}}, e^{\frac{i4\pi}{7}} ..., e^{\frac{i12\pi}{7}}\right\}$$

$$=1+\sum_{k=1}^{6}e^{i(2\pi k/7)}=0\sum_{k=1}^{6}e^{i(2\pi k/7)}=-1 \quad ...(ii)$$

From eqs. (i) and (ii), we get

11. **(a)** Given equation is $\sqrt{\frac{x}{2x+1}}+\sqrt{\frac{2x+1}{x}}=2$ ...(i)

Let $\sqrt{\frac{x}{2x+1}}=t$

Then, $t+\frac{1}{t}=2$

$\Rightarrow\ t^2-2t+1=0\Rightarrow t=1$

$\Rightarrow \sqrt{\frac{x}{2x+1}}=1 \Rightarrow \frac{x}{2x+1}=1$

$\Rightarrow\ x=-1$

$\because$ α satisfies the eq. (i), therefore $\alpha=-1$

Now, $\alpha^2x^2+4ax+3=0$

Put the value of α

$(-1)^2x^2+4(-1)x+3=0$

$x^2-4x+3=0\Rightarrow(x-1)(x-3)=0$

$x=1, 3$

Hence, the roots of the equation

$\alpha^2x^2+4ax+3=0$ are 1, 3.

12. **(c)** Given, $f(x)=x^2+2x+2$

$=x^2+2x+1+1=(x+1)^2+1$

Hence, $f(x)\in[1,\infty)$

a. d $g(x)=(x^2-2x+1)=-(x-1)^2$

$\Rightarrow g(x)\in(-\infty,0]$

So, $\frac{f}{g}(x)=\frac{x^2+2x+2}{-x^2+2x-1}=y$ (let)

$\Rightarrow x^2+2x+2=-yx^2+2xy-y$

$\Rightarrow x^2(1+y)(2-2y)x+2+y=0$

$\because D\geq 0$

$\therefore (2-2y)^2-4(2+y)(1+y)\geq 0$

$1+y^2-2y-2-3y-y^2\geq 0$

$-5y-1\geq 0$

$\Rightarrow y\leq-\frac{1}{5}\Rightarrow\frac{f}{g}(x)\leq-\frac{1}{5}$

So, $\frac{f}{g}(x)\in\left(-\infty,-\frac{1}{5}\right]$

So, $a=1$, $b=0$ and $c=-\frac{1}{5}$

Hence, $a+2b+5c+4$

$=1+0+5\left(-\frac{1}{5}\right)+4=1-1+4=4$

13. **(c)** Given $x^2-|x+2|+x>0$

Case I: When $x+2\geq 0$

$x^2-(x+2)+x>0$

$\Rightarrow x^2-2>0$

$\Rightarrow(x-\sqrt{2})(x+\sqrt{2})=0$

outside of the roots

$x\in[-2,-\sqrt{2}]\cup[\sqrt{2},\infty]$

Case II: When $x+2<0$

$x^2+(x+2)+x>0$

$\Rightarrow x^2+2x+2>0$

$\Rightarrow(x+1)^2+1>0\Rightarrow x<-2$

Hence, the solution set is

$x\in(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$

14. **(a)** Given cubic equation is

$x^3-6x^2+11x-6=0$ ...(i)

$\Rightarrow(x-1)(x-2)(x-3)=0$

$\Rightarrow x=1, 2, 3\equiv\alpha, \beta, \gamma$

Therefore, $\alpha^2+\beta^2=(1)^2+(2)^2=5=a$ (say)

$\beta^2+\gamma^2=(2)^2+(3)^2=13=b$ (say)

and $\gamma^2+\alpha^2=(3)^2+1=10=c$ (say)

Equation of the having the roots *a, b, c*

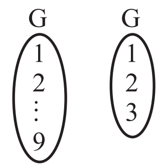
$x^3-(a+b+c)x^2+(ab+bc+ca)x-abc=0$

$\Rightarrow x^3-(5+13+10)x^2+(5\times13+13\times10+10\times5)x-5\times13\times10=0$

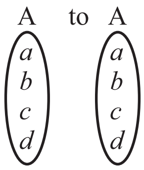
$\Rightarrow x^3-28x^2+245x-650=0$

15. **(a)** Given that, number of elements in the sets G and A are 3 and 4 respectively.

(A) The number of non bijective function from G × G to G $=3^9-19683$



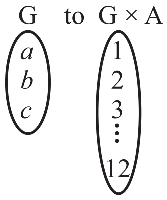
(B) A to A



Here, a has 4 options, b has 3, c has 2 and d has 1

So, number of bijective functions from A to A is 41 = 24

(C) G to G × A



So, number of function from G to G × A is

$= 12^3 = 1728$

(D) A to A × A

The number of surjective from G to G × A

i.e. 4 to 16 is 0

So, A → V, B → I, C → III, D → II.

16. **(b)** Since, we know that if a plane have n, straight lines in which no two are parallel and no three passes through the same point and their point of intersection are jointed then number of new lines are

$$= \frac{1}{8} n(n-1)(n-2)(n-3)$$

$$= \frac{1}{8} \times 20 \times (20-1) \times (20-2) \times (20-3) = 14535$$

17. **(b)** Here, $\sum_{k=0}^{n} a_k . C_k = a_0 C_0 + a_1 C_1 + a_2 C_2 + ... + a_n C_n$

Since a_0, a_1, a_2 are in A.P.

$= a_0 C_0 + (a_0 + d) C_1 + (a_0 + 2d) C_2 + ... + (a_0 + nd) C_n$

$= a_0 (C_0 + C_1 + ... + C_n) + d(C_1 + 2C_2 + 3C_3 + ... + {}^nC_n$

$= a_0 . 2^n + d(n.2^{n-1}) = 2^{n-1} [2a_0 + nd] = (a_0 + a_n) 2^{n-1}$

18. **(d)** Given,

$$x = \frac{3}{10} + \frac{3.7}{10.15} + \frac{3.7.9}{10.15.20} + ...$$

Hence $\frac{5}{5}(x) = \frac{3.5}{5.10} + \frac{3.5.7}{5.10.15} + \frac{3.5.7.9}{5.10.15.20} + ...$

$$\therefore T_{r+1} = \frac{3.5.7...(2r+1)}{5.1.2.3....r}$$

$$= \left(\frac{2}{5}\right)^r \frac{\frac{3}{2} \cdot \frac{5}{2} ... \left(r + \frac{1}{2}\right)}{r!}$$

$$= \frac{\left(-\frac{3}{2}\right)\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}-2\right)...\left(-\frac{3}{2}-r+1\right)\left(-\frac{2}{5}\right)^r}{r!}$$

Comparing with general term of $(1 + x)^n$, $n \in R$

$$\therefore \frac{n(n-1)(n-2)...(n-r+1)x^r}{r!}$$

$$\Rightarrow n = -\frac{2}{2} x = -\frac{2}{5}$$

$$\therefore x + \frac{8}{5} = \left(1 - \frac{2}{5}\right)^{-3/2} = \left(\frac{3}{5}\right)^{-3/2} = \left(\frac{5}{3}\right)^{3/2} = \frac{5\sqrt{5}}{3\sqrt{3}}$$

$$\Rightarrow \frac{5x+8}{5} = \frac{5\sqrt{5}}{3\sqrt{3}} \Rightarrow 5x + 8 = \frac{25\sqrt{5}}{3\sqrt{3}}$$

19. **(c)** Given, $\frac{x^4}{(x-1)(x-2)(x-3)}$

$$= x + k + \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$\Rightarrow x^4 = x(x-1)(x-2)(x-3) + k(x-1)$
$(x-2)(x-3) + A(x-2)(x-3) + B(x-3)$
$(x-1) + C(x-1)(x-2)$

at $x = 1$, $A = \frac{1}{2}$

at $x = 2$, $B = \frac{16}{-1} = -16$

at $x = 3$, $C = \frac{81}{2}$

at $x = 0$,

$$\Rightarrow 0 = k(-1)(-2)(-3) + \frac{1}{3}(-2)(-3) - 16$$
$$(-3)(-1) + \frac{81}{2}(-1)(-2)$$

$\Rightarrow 6k = 3 - 48 + 81 = 36$

$\Rightarrow k = 6$

Now, $k + A - B + C = 6 + \frac{1}{2} + 16 + \frac{81}{2}$

$= 6 + 16 + 41 = 63$

20. **(c)** Given cosec 48° + cosec 96° + cosec 192° + cosec 384°

= cosec (90° – 42°) + cosec 96° + cosec (270° – 78°) + cosec (360° + 24°)

= sec42° + cosec 96° – sec 78° + cosec 24°

$$\Rightarrow = \frac{1}{\cos 42°} + \frac{1}{\sin 96°} - \frac{1}{\cos 78°} + \frac{1}{\sin 24°}$$

$$= \frac{1}{\cos 42°} - \frac{1}{\cos 78°} + \frac{1}{\sin 96°} + \frac{1}{\sin 24°}$$

$$= \frac{(\cos 78° - \cos 42°)}{\cos 42° \times \cos 78°} + \frac{(\sin 24° + \sin 96°)}{\sin 96° \times \sin 24°}$$

$$= \frac{-2\sin 60°\times \sin 18°}{\cos(60°-18°)\cos(60°+18°)} + \frac{2\sin 60°\times \cos 36°}{\sin(60°+36°)\sin(60°-36°)}$$

$$= \frac{-2\times\sqrt{3}/2\times\left(\frac{\sqrt{5}-1}{4}\right)}{\left(\frac{1}{2}\right)^2-\left(\frac{\sqrt{5}-1}{4}\right)^2} + \frac{2\times\sqrt{3}/2\times\left(\frac{\sqrt{5}+1}{4}\right)}{(\sqrt{3}/2)^2-\left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)^2}$$

$$= -2\sqrt{3}+2\sqrt{3} = 0$$

21. **(c)** Given $\sin^4\frac{\pi}{8}+\sin^4\frac{2\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{4\pi}{8}$

$$+\sin^4\frac{5\pi}{8}+\sin^4\frac{6\pi}{8}+\sin^4\frac{7\pi}{8}$$

$$= \left[\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}\right] + \left[\sin^4\frac{2\pi}{8}+\sin^4\frac{4\pi}{8}+\sin^4\frac{6\pi}{8}\right]$$

$$= \left[\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\cos^4\left(\frac{\pi}{2}-\frac{5\pi}{8}\right)\right. \left.+\cos^4\left(\frac{\pi}{2}-\frac{7\pi}{8}\right)\right]+\left[\sin^4\frac{\pi}{4}+\sin^4\frac{\pi}{2}+\sin^4\frac{3\pi}{4}\right]$$

$$= \left[\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\cos^4\left(-\frac{\pi}{8}\right)+\cos^4\left(\frac{-3\pi}{8}\right)\right] + \left[\sin^4\frac{\pi}{4}+\sin^4\frac{\pi}{2}+\sin^4\left(\frac{\pi}{2}+\frac{\pi}{4}\right)\right]$$

$$= \left[\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}\right] + \left[\sin^4\frac{\pi}{4}+\sin^4\frac{\pi}{2}+\cos^4\frac{\pi}{4}\right]$$

$$= \left[\left(\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}\right)+\left(\sin^4\frac{3\pi}{8}+\cos^4\frac{3\pi}{8}\right)\right] + \left[\left(\frac{1}{\sqrt{2}}\right)^4+(1)^4+\left(\frac{1}{\sqrt{2}}\right)^4\right]$$

$$= \left[2-\frac{1}{2}4\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}-\frac{1}{2}4\sin^2\frac{3\pi}{8}\cos^2\frac{3\pi}{8}\right]+\left[\frac{1}{2}+1\right]$$

$$= \left[2-\frac{1}{2}\left(\sin^2\frac{\pi}{4}\right)-\frac{1}{2}\left(\sin^2\frac{3\pi}{4}\right)\right]+\frac{2}{2}$$

$$= 2-\frac{1}{2}\left(\frac{1}{2}\right)-\frac{1}{2}\left(\frac{1}{2}\right)+\frac{3}{2}$$

22. **(c)** Given

$$x : y : z = \tan\left(\frac{\pi}{15}+\alpha\right) : \tan\left(\frac{\pi}{15}+\beta\right) : \tan\left(\frac{\pi}{15}+\gamma\right)$$

Let $\dfrac{x}{\tan\left(\frac{\pi}{15}+\alpha\right)} = k \Rightarrow x = k\tan\left(\frac{\pi}{15}+\alpha\right)$

Similarly $y = k\tan\left(\frac{\pi}{15}+\beta\right)$

$z = k\tan\left(\frac{\pi}{15}+\gamma\right)$

Now, $\dfrac{z+x}{z-x}\sin^2(\gamma-\alpha)$

$$= \frac{\tan(12°+\gamma)+\tan(12°+\alpha)}{\tan(12°+\gamma)-\tan(12°+\alpha)}.\sin^2(\gamma-\alpha)$$

$$= \frac{\sin\{24°+(\gamma+\alpha)\}}{\sin(\gamma-\alpha)}\sin^2(\gamma-\alpha)$$

$= [\sin 24°\cos(\gamma+\alpha) + \cos 24°\sin(\gamma+\alpha)]\times\sin(\gamma-\alpha)$

$= \sin 24° [\cos(\gamma+\alpha)\sin(\gamma-\alpha)] + \cos 24° [\sin(\gamma+\alpha)\sin(\gamma-\alpha)]$

$$= \frac{\sin 24°}{2}(\sin^2\gamma-\sin^2\alpha)-\frac{\cos 24°}{2}(\cos^2\gamma-\cos^2\alpha) \quad \text{...(i)}$$

Similarly,

$$\frac{x+y}{x-y}\sin^2(\alpha-\beta) = \frac{\sin 24°}{2}(\sin^2\alpha-\sin^2(\beta)) - \frac{\cos 24°}{2}(\cos^2\alpha-\cos^2(\beta)) \quad \text{...(ii)}$$

and $$\frac{y+z}{y-z}\sin^2(\beta-\gamma) = \frac{\sin 24°}{2}(\sin^2\beta-\sin^2\gamma) - \frac{\cos 24°}{2}(\cos^2\beta-\cos^2\gamma) \quad \text{...(iii)}$$

By adding eqs. (i), (ii) and (iii), we get

$$\frac{z+x}{z-x}\sin^2(\gamma-\alpha)+\frac{x+y}{x-y}\sin^2(\alpha-\beta) + \frac{y+z}{y-z}\sin^2(\beta-\gamma) = 0$$

23. **(c)** Given, $\sin x\sqrt{4\cos^2 x} = \dfrac{2+x-[x]}{1-x+[x]}$

$\Rightarrow \sin x\sqrt{4\cos^2 x} = \dfrac{2+\{x\}}{1-\{x\}}$ $[\because x-[x]=\{x\}]$

$\Rightarrow 2\sin x\cos x = \dfrac{2+\{x\}}{1-\{x\}} = y$ (let)

$\therefore |\cos x| = \cos x, \forall x \in \left[\frac{\pi}{4},\frac{\pi}{3}\right]$ (given)

$\because y=\frac{2+\{x\}}{1-\{x\}}>0$

Since $\{x\} \in [0, 1]$

So, $2\sin x\cos x=\frac{2+\{x\}}{1-\{x\}}$

$\because$ Maximum value of 2 sin x cos x = sin 2x is '1'.

and minimum value of $\frac{2+\{x\}}{1-\{x\}}$ (at $\{x\}$ = 0) is '2'.

$\therefore$ For the given equation number of solution $k = 0$

$\therefore \forall x\in\left[\frac{\pi}{4},\frac{\pi}{3}\right], k^{\tan^2 x}=0$

24. **(b)** Given, α, β are the least and greatest values of f(x) and, $f(x) = (\sin^{-1} x)^2 + (\cos^{-1} x)^2$ Let $\sin^{-1} x = a$ and $\cos^{-1} x = b$

Then, $f(x) = a^2 + b^2$

$= (a + b)^2 - 2ab$

Put the value of a and b

$f(x) = (\sin^{-1} x + \cos^{-1} x)^2 - 2\sin^{-1} x \cos^{-1} x$

$=\frac{\pi^2}{4}-2\sin^{-1}x\cos^{-1}x\left[\because \sin^{-1}x+\cos^{-1}x=\pi/2\right]$

$=\frac{\pi^2}{4}-2\sin^{-1}x\left(\frac{\pi}{2}-\sin^{-1}x\right)$

$=\frac{\pi^2}{4}-\pi\sin^{-1}x+2(\sin^{-1}x)^2$

For minimum and maximum value,

$f'(x)=0-\pi.\frac{1}{\sqrt{1-x^2}}+4\sin^{-1}x\frac{1}{\sqrt{1-x^2}}=0$

$x=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

Therefore, $f''\left(\frac{1}{\sqrt{2}}\right)=+\text{ve}$

$f(x)_{\min}$ = when $x=\frac{1}{\sqrt{2}}$

$\therefore f(x)_{\min}=\frac{\pi^2}{4}-2\sin^{-1}x\cos^{-1}x$

$\alpha=\frac{\pi^2}{8}$

$f(x)_{\max}=\frac{\pi^2}{4}-2\sin^{-1}x\cos^{-1}x$

We can see that, $f(x)$ is maximum, when $x = -1$

$f(x)_{\max}=\frac{\pi^2}{4}-2\left(-\frac{\pi}{2}\right)(\pi)$

$\beta=\frac{\pi^2}{4}+\pi^2=\frac{5\pi^2}{4}\Rightarrow\beta=\frac{5\pi^2}{4}$

Hence, $8(\alpha+\beta)=8\left[\frac{\pi^2}{8}+\frac{5\pi^2}{4}\right]=11\pi^2$

25. **(c)** For $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

log sec $x = y$ (let)

$\Rightarrow$ sec $= e^y \Rightarrow \cos x = e^{-y}$

$\because \frac{\sin hy}{\cos hy}-\frac{e^y-e^{-y}}{e^y+e^{-y}}$

using CD rule

$\Rightarrow\frac{\cos hy-\sin hy}{\cos hy+\sin hy}=\frac{e^{-y}}{e^y}=\cos^2 x$

$\Rightarrow\left(\frac{\cos h\frac{y}{2}-\sin h\frac{y}{2}}{\cos h\frac{y}{2}+\sin h\frac{y}{2}}\right)=\cos^2 x$

Taking square root both side

$\Rightarrow\frac{\cos h\frac{y}{2}-\sin h\frac{y}{2}}{\cos h\frac{y}{2}+\sin h\frac{y}{2}}=\cos x$

$\Rightarrow\frac{2\cos h\frac{y}{2}}{2\sin h\frac{y}{2}}=\frac{1+\cos x}{1-\cos x}=\cot^2\frac{x}{2}$

$\Rightarrow \cot h\frac{y}{1}=\cot^2\frac{x}{2}=\text{cosec}^2\frac{x}{2}-1$

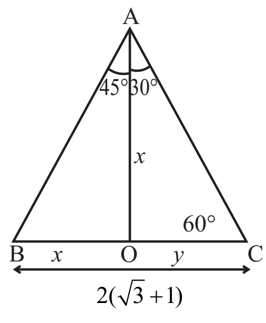
$\Rightarrow\frac{y}{2}=\cot h^{-1}\left(\text{cosec}^2\frac{x}{2}-1\right)$

$= y=2\cot h^{-1}\left(\text{cosec}^2\frac{x}{2}-1\right)$

26. **(d)** Given, $\angle A = 75°$, $\angle B = 45°$ and $a=2(\sqrt{3}+1)$

In ΔAOC, $\tan 60°=\frac{x}{y}\Rightarrow\sqrt{3}=\frac{x}{y}$

$\Rightarrow x=\sqrt{3}y$



Now, $x+y=2(\sqrt{3}+1)$

$\Rightarrow \sqrt{3}y+y=2(\sqrt{3}+1)$

$\Rightarrow y(\sqrt{3}+1)=2(\sqrt{3}+1)$

$\Rightarrow y=2 \Rightarrow x=2\sqrt{3}$

Now, area of ΔABC = area of ΔAOB + area of ΔAOC

$=\frac{1}{2}\times x^2+\frac{1}{2}\times x\times y=\frac{1}{2}x[x+y]$

$=\frac{1}{2}\times 2\sqrt{3}\times 2(\sqrt{3}+1)$

$=2\sqrt{3}(\sqrt{3}+1) \; =6+2\sqrt{3}$ sq units

27. **(b)** Given, $3a=b+c$ ...(i)

Hence, $s=\frac{a+b+c}{2}$

$s=\frac{a+3a}{2}=\frac{4a}{2}=2a$

Now, $\cot\frac{B}{2}\cot\frac{C}{2}$

$=\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}.\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

Put the value of s, we get

$=\sqrt{\frac{2a(2a-b)}{(2a-a)(2a-c)}}.\sqrt{\frac{2a(2a-c)}{(2a-a)(2a-b)}}$

$\sqrt{\frac{(2a(2a-b)}{a(2a-c)}\times\frac{2a(2a-c)}{a(2a-b)}}=\sqrt{4}=2$

28. **(d)** Given in a ΔABC $\frac{2r_2r_3}{r_2-r_1}=r_3-r_1$

$\Rightarrow 2r_2r_3=(r_2-r_1)(r_3-r_1)$

$\Rightarrow 2.\frac{\Delta}{(s-b)}.\frac{\Delta}{(s-c)}=\left(\frac{\Delta}{s-b}-\frac{\Delta}{s-a}\right)\left(\frac{\Delta}{s-c}-\frac{\Delta}{s-a}\right)$

$\Rightarrow \frac{2}{(s-b)(s-c)}=\frac{(b-a)}{(s-b)(s-a)}\times\frac{(c-a)}{(s-c)(s-a)}$

$\Rightarrow \frac{2\Delta^2}{(s-b)(s-c)}=\Delta^2\left\{\frac{s-a-s+b}{(s-b)(s-a)}\right\}\left\{\frac{s-a-s+c}{(s-c)(s-a)}\right\}$

$\Rightarrow 2(s-a)^2=(b-a)(c-a)$

$\Rightarrow \frac{2(b+c-a)^2}{4}=(b-a)(c-a)$

$\Rightarrow b^2+c^2+a^2+2abc-2ca-2ab=2$

$[bc-ba-ac+a^2]$

$\Rightarrow b^2+c^2+a^2+2bc-2ca-2ab$

$=2bc-2ba-2ac+2a^2$

$\Rightarrow b^2+c^2+a^2=2a^2 \Rightarrow a^2=b^2+c^2$

Now from $=\frac{\frac{\Delta}{s-a}\times\Delta\left\{\frac{1}{s-b}+\frac{1}{s-c}\right\}}{s}$

$[\because r_1r_2+r_2r_3+r_3r_1=s^2]$

$=\frac{\Delta^2(2s-b-c)}{s(s-a)(s-a)(s-c)}$

$=\frac{\Delta^2(a+b+c-b-c)}{\Delta^2}=a=2R$

29. **(d)** Given,

$\vec{P}=3\hat{i}-2\hat{j}-\hat{k}$

$\vec{Q}=2\hat{i}+3\hat{j}-4\hat{k}$

$\vec{R}=-\hat{i}+\hat{j}+2\hat{k}$

and $\vec{S}=4\hat{i}+5\hat{j}+\lambda\hat{k}$

Here, $\overline{PS}=(4-3)\hat{i}+(5+2)\hat{j}+(\lambda+1)\hat{k}$

$=\hat{i}+7\hat{j}+(\lambda+1)\hat{k}$

$\overrightarrow{PQ}=-\hat{i}+5\hat{j}-3\hat{k}$

and $\overrightarrow{PR}=-4\hat{i}+3\hat{j}+3\hat{k}$

Now, PQ × PR = $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 5 & -3\\ -4 & 3 & 3\end{vmatrix}$

$=\hat{i}(15+9)-\hat{j}(-3-12)+\hat{k}(-3+20)$

$=24\hat{i}+15\hat{j}+17\hat{k}$

Since, P, Q, R and S are coplanar,

∴ PS . (PQ × PR) = 0

$\rightarrow[\hat{i}+7\hat{j}+(\lambda+1)]\hat{k}.[24\hat{i}+15\hat{j}+17\hat{k}]=0$

$\Rightarrow 24+105+17(\lambda+1)=0 \Rightarrow 129+17\lambda+17=0$

$\Rightarrow 17\lambda=-146 \Rightarrow \lambda=-\frac{146}{17}$

30. **(b)** Given vectors

$\overrightarrow{OA}=2\hat{i}+\hat{j}+\hat{k} \Rightarrow |OA|=3$

and $\overrightarrow{OB}=2\hat{i}+4\hat{j}+4\hat{k} \Rightarrow |OB|=6$

∵ The angle bisector of ∠BOA intersect the side AB at point P in the ratio 3 : 6 = 1 : 2, so

$\overrightarrow{OP}=\frac{2(\overrightarrow{OA})+1(\overrightarrow{OB})}{3}$

$=\frac{6\hat{i}+8\hat{j}+6\hat{k}}{3}=2\hat{i}+\frac{8}{3}\hat{j}+2\hat{k}$

$\therefore |\overrightarrow{OP}| = \sqrt{2^2 + \left(\frac{8}{3}\right)^2 + 2^2}$

$\sqrt{\frac{136}{9}} = k$

$\therefore 9k^2 = 9\left(\frac{136}{9}\right) = 136$

31. **(b)** Given $\vec{a} + x\vec{b} + y\vec{c} = 0$

As we know that, if $\vec{a} + \vec{b} + \vec{c} = 0$

Then, $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq 0$

Now, $\vec{a} + x\vec{b} + y\vec{c} = 0$

Let $x(\vec{a} \times \vec{b}) = xy(\vec{b} \times \vec{c}) = y(\vec{c} \times \vec{a}) = \vec{p}$

$\Rightarrow \vec{a} \times \vec{b} = \frac{\vec{p}}{x}, \vec{b} \times \vec{c} = \frac{\vec{p}}{xy}$ and $\vec{c} \times \vec{a} = \frac{\vec{p}}{y}$

$\therefore \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \frac{\vec{p}}{x} + \frac{\vec{p}}{xy} + \frac{\vec{p}}{y}$

$= \vec{p}\left(\frac{x+y+1}{xy}\right) = \left(\frac{x+y+1}{xy}\right) \times xy \times (\vec{b} \times \vec{c})$

$= (x+y+1)(\vec{b} \times \vec{c})$

Compare with $6(\vec{b} \times \vec{c})$

$x + y + 1 = 6$

$\therefore x + y = 5$

$\Rightarrow x + y - 5 = 0$

Hence, option (b) satisfies it.

32. **(b)** Given, points $A = (\alpha, 1, 2\alpha)$, $B = (3, 1, 2)$ and $\vec{C} = 4\hat{i} - \hat{j} + 3\hat{k}$

Now, $\overrightarrow{AB} = (3 - \alpha)\hat{i} + (2 - 2\alpha)\hat{k}$

$\therefore \overrightarrow{AB} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3-\alpha & 0 & 2-2\alpha \\ 4 & -1 & 3 \end{vmatrix}$

$= \hat{i}(2 - 2\alpha) - \hat{j}(9 - 3\alpha - 8 + 8\alpha) + \hat{k}(-3 + \alpha)$

$= (2 - 2\alpha)\hat{i} + (5\alpha - 1)\hat{j}(\alpha - 3)\hat{k}$

Comparing it with $6\hat{i} + 9\hat{j} - 5\hat{k}$, then

$2 - 2\alpha = = 6$, $5\alpha - 1 = 9$ and $\alpha - 3 = -5$

$\Rightarrow \alpha = -2$

Hence $\alpha^2 + \alpha + 5 = (-2)^2 - 2 + 5 = 7$

33. **(a)** Given,

$\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + t(\hat{i} - 2\hat{j} + 2\hat{k})$

and $\vec{r} = (-4\hat{i} - \hat{k}) + s(3\hat{i} - 2\hat{j} - 2\hat{k})$

Here $\vec{a} = 6\hat{i} + 2\hat{j} + 2\hat{k}, b = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{c} = -4\hat{i} - \hat{k}, d = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$\therefore$ Shortest distance $= \frac{|(\vec{a} - \vec{c}).(\vec{b} \times \vec{d})|}{|\vec{b} \times \vec{d}|}$ ...(i)

Now, $\vec{b} \times \vec{d} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$

$= \hat{i}(4 + 4) - \hat{j}(-2 - 6) + \hat{k}(-2 + 6) = 8\hat{i} + 8\hat{j} + 4\hat{k}$

By Eq. (i),

$= \left|\frac{(10\hat{i} + 2\hat{j} + 3\hat{k}).(8\hat{i} + 8\hat{j} + 4\hat{k})}{\sqrt{64 + 64 + 16}}\right|$

$= \left|\frac{80 + 16 + 12}{\sqrt{144}}\right| = \frac{108}{12} = 9$

34. **(c)** Given $\vec{r}.\vec{a} = 0$

and $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$

So, $\vec{a} \times (\vec{r} \times \vec{b}) = \vec{a} \times (\vec{c} \times \vec{b})$

$\Rightarrow \vec{r}(\vec{a}.\vec{b}) - \vec{b}(\vec{a}.\vec{r}) = \vec{c}(\vec{a}.\vec{b}) - \vec{b}(\vec{a}.\vec{c})$

$\Rightarrow \vec{r} = \vec{c} - \vec{b}\left(\frac{\vec{c}.\vec{a}}{\vec{a}.\vec{b}}\right)$

35. **(b)** Given $\Sigma x = 170$ and $\Sigma x^2 = 2830$

Increase in $\Sigma x = 10$

and increase in $\Sigma x^2 = (30)^2 - (20)^2$

$= 900 - 400 = 500$

$\Sigma x' = 170 + 10 = 180$

and $\Sigma x'^2 = 2830 + 500 = 3330$

We know that, variance $= \frac{\Sigma x^2}{n} = \left(\frac{\Sigma x}{n}\right)^2$

$= \frac{3330}{15} - \left(\frac{180}{15}\right)^2$ [Here, $n = 15$]

$= 222 - (12)^2 = 222 - 144 = 78$

36. **(b)**

Class Interval	f_i	x_i	$f_i x_i$	$\|x_i - 35\|$	$f_i\|x_i - 35\|$
0-10	4	5	20	30	120
10-20	6	15	90	20	120
20-30	16	25	400	10	160
30-40	28	35	980	0	0
40-50	16	45	720	10	160
50-60	6	55	330	20	120
60-70	4	65	260	30	120
Total	80		2800		800

$\because \bar{X} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2800}{80} = 35$

Mean deviation $= \frac{1}{N}\sum_{i=1}^{n} f_i \mid x_i - \bar{x} \mid$

$= \frac{1}{80}\sum_{i=1}^{n} f_i \mid x_i - 35 \mid$ [Here, N = 80]

$= \frac{1}{80} \times 800 = 10$

37. **(b)** Since, A and B each select one number randomly from the distinct numbers 1, 2, 3, ..., n, then the probablity that the number selected by A is less than the number selected by B is

$\frac{{}^nC_2}{n \times n} = \frac{n(n-1)}{2(n \times n)} = \frac{1009}{2019}$ (given)

$\Rightarrow \frac{n-1}{2n} = \frac{1009}{2019}$

$\Rightarrow$ n = 2019

Now, number of ways selecting numbers from the distinct numbers 1, 2, 3, ..., 2019, by B is the number immediately next ot the number selected by A is 2018, because there are 2018 pairs of consecutive numbers.

So, required probability = $\frac{2018}{2018 \times 2019} = \frac{2018}{(2019)^2}$

38. **(b)** Given, there are three bags A, B and C and bag A has 2 white, 3 black balls, B has 4 white and 2 black balls and bag C has 3 white and 2 black balls.

Let event of drawing black ball from bags, A, B and C = $\frac{1}{3}$

$\therefore$ Required probability

= Probability of black ball from bag A
+ Probability of black ball from bag B
+ Probability of black ball from bag C

$= \frac{1}{3}\times\frac{3}{5}+\frac{1}{3}\times\frac{2}{6}+\frac{1}{3}\times\frac{2}{5} = \frac{3}{15}+\frac{2}{18}+\frac{2}{15} = \frac{4}{9}$

39. **(b)** Given, probability of selecting box A, B and C

$\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{6}$ and

Probability of green ball P(G)

$= \frac{1}{2}\times\frac{2}{6}+\frac{1}{3}\times\frac{3}{6}+\frac{1}{6}\times\frac{1}{6} = \frac{1}{6}+\frac{1}{6}+\frac{1}{36} = \frac{13}{36}$

Let probability of drawn ball is green comes from bag C is $P\left(\frac{C}{G}\right)$, then

$P\left(\frac{C}{G}\right) = \frac{P\left(\frac{C}{G}\right)\times P(C)}{P(G)} = \frac{\frac{1}{6}\times\frac{1}{6}}{\frac{13}{36}} = \frac{1}{13}$

40. **(d)** Given,

$P(X = 0) = 3C^3$

$P(X = 2) = 5C - 10C^2$

and $P(X = 4) = 4C - 1$

Since

$\Sigma P(X) = 1$

$\Rightarrow 3C^3 + (5C - 10C^2) + (4C - 1) = 1$

or $3C^3 - 10C^2 + 9C - 2 = 0$

$\Rightarrow (C - 1)(3C - 1)(C - 2) = 0$

$\Rightarrow C = 1, \frac{1}{3}, 2$

$\therefore C = \frac{1}{3}$

Now,

X	0	2	4
P(X)	$\frac{1}{9}$	$\frac{5}{9}$	$\frac{1}{3}$

Hence, variance $\Sigma X^2{}_P - (\Sigma X_P)^2$

$= \left(0^2 \times \frac{1}{9} + 4 \times \frac{5}{9} + 16 \times \frac{1}{3}\right) - \left(\frac{10}{9} + \frac{4}{3}\right)^2$

$= \left(\frac{20}{9} + \frac{16}{3}\right) - \left(\frac{66}{27}\right)^2 = \frac{60+144}{27} - \frac{484}{81}$

$= \frac{204}{27} - \frac{484}{81} = \frac{128}{81}$

41. **(b)** Required probability

= No white ball + one white ball + two white ball

$= \left(\frac{2}{3}\right)^5 + {}^5C_1\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^1 + {}^5C_2\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2$

$= \left(\frac{1}{3}\right)^5 [2^5 + 5.2^4 + 10.2^3]$

$= \left(\frac{1}{3}\right)^5 [32 + 80 + 80] = \frac{192}{3^5} = \frac{64 \times 3}{3^5}$

$= \frac{64}{3^4} = \left(\frac{8}{9}\right)^2$

42. **(b)** Given, equations of straight lines are

$x \sin\theta + (1 - \cos\theta) y = a \sin\theta$ ...(i)

and $x \sin\theta - (1 + \cos\theta) y = -a \sin\theta$ ...(ii)

On solving equation (i) and (ii)

$\Rightarrow y = a \sin\theta$

Putting the value of y in eq. (i), we get

$x \sin\theta + (1 - \cos\theta) a \sin\theta = a \sin\theta$

$\Rightarrow \sin\theta [x + (1 - \cos\theta)a] = a \sin\theta$

$\Rightarrow x + a - a \cos\theta = a$
$\Rightarrow x - a\cos\theta = 0 \Rightarrow x = a\cos\theta$
Now, $x^2 + y^2 = (a\cos\theta)^2 + (a\sin\theta)^2$
$[\because \sin^2\theta + \cos^2\theta = 1]$
$\Rightarrow x^2 + y^2 = a^2$, whose represent a circle.

43. **(d)** Given equation of line L : $\frac{x}{a} + \frac{y}{b} = 1$ ...(i)

rotate the axis with respect to angle α

$$\left.\begin{array}{l} x = x\cos\alpha - y\sin\alpha \\ y = x\sin\alpha + y\cos\alpha \end{array}\right\} \quad ...(ii)$$

Put the values of eq. (ii) in eq. (i), we get

$$\frac{x\cos\alpha - y\sin\alpha}{a} + \frac{x\sin\alpha + y\cos\alpha}{b} =$$

$$x\left(\frac{\cos\alpha}{a} + \frac{\sin\alpha}{b}\right) + y\left(\frac{\cos\alpha}{b} - \frac{\sin\alpha}{\alpha}\right) = 1$$

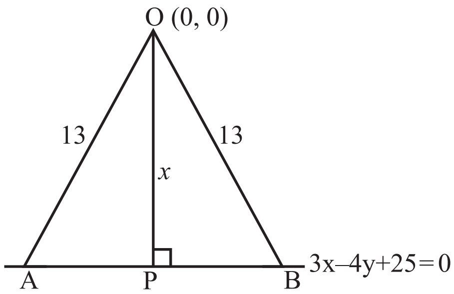
rotated axis $L' : \frac{x}{p} + \frac{y}{p} = 1$

$$\therefore \frac{\cos\alpha}{a} + \frac{\sin\alpha}{b} = \frac{1}{p} \text{ and } \frac{\cos\alpha}{b} - \frac{\sin\alpha}{a} = \frac{1}{q}$$

On squaring and adding we get

$$\Rightarrow \frac{1}{p^2} + \frac{1}{q^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

44. **(c)** Given O is origin and A, B are points on the line



$$\text{Required distance OP} = \left|\frac{0+0+25}{\sqrt{3^2+4^2}}\right| = \left|\frac{25}{5}\right| = 5$$

So, AP = PB = 12 [By Pythagoras theorem in ΔAOP]

Area of ΔOAB = $\frac{1}{2} \times 24 \times 5$ = 12 × 5 = 60

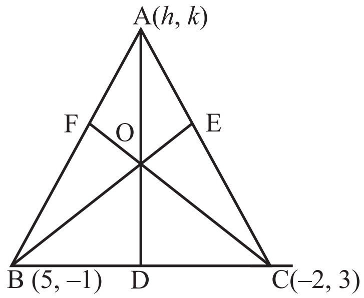
45. **(a)** Line L : $3x = 4y + 8$
$= 3x - 4y - 8$
$L_{(1,1)} = 3 - 4 - 8 = -9 < 0$
Since, α, β lie either side
Hence, $L_{(\alpha, \beta)} > 0$
$3x - 4y - 8 > 0$
$\Rightarrow 3x - 4(-3x) - 8 > 0 \quad [\because y = -3x]$

$$15x - 8 > 0 \Rightarrow x > \frac{8}{15} \Rightarrow \alpha > \frac{8}{15}$$

$3x - 4y - 8 > 0$
$\Rightarrow -5y - 8 > 0 \, [\because 3x = -y]$
$\Rightarrow 5y + 8 < 0$

$$y < -\frac{8}{5} \Rightarrow \beta < \frac{-8}{5}$$

46. **(c)** Since AB and BC are perpendicular
Slope of BC × slope of AD = –1



$$\Rightarrow \frac{3+1}{-2-5} \times \frac{k}{h} = -1$$

$$\Rightarrow \frac{4k}{-7h} = -1 \Rightarrow 4k - 7h = 0 \quad ...(i)$$

Again, slope of AB × slope of CF = –1

$$\Rightarrow M_{AB} \times M_{CF} = -1 \left\{\because M_{CF} = \frac{3-0}{-2-0} = \frac{-3}{2}\right\}$$

$$\Rightarrow M_{AB} \times \frac{-3}{2} = -1 \Rightarrow M_{AB} = \frac{2}{3}$$

Now, equation of AB $y + 1 = \frac{2}{3}(x - 5)$
$\Rightarrow 2x - 3y = 13$
A (h, k) lie on line AB
$2h - 3k = 13$ ...(ii)
From eqs. (i) and (ii), we get
$h = -4, k = -7$
Hence, third vertex of the triangle = (–4, –7)

47. **(d)** Given lines are $x + y - 1 = 0$
and $6x^2 - 13xy + 5y^2 = 0$
Now reduced into linear equations
We have
$6x^2 - 10xy - 3xy + 5y^2 = 0$
$\Rightarrow 2x - y = 0$ or $3x - 5y = 0$
Let orthocentre be (h, k)
Slope of OP × slope of AB = –1

$$\frac{k}{h} \times -1 = -1 \quad \Rightarrow h = k \quad ...(i)$$

Now, slope of OB × slope of AD = –1

$$\Rightarrow 2\times\left(\frac{\frac{3}{8}-k}{\frac{5}{8}-h}\right)=-1\Rightarrow\frac{3}{4}-2h=h-\frac{5}{8}$$

$$\Rightarrow\frac{3}{4}+\frac{5}{8}=3h\Rightarrow h=\frac{11}{24}$$

$$\therefore k=\frac{11}{24}$$

Now, OP = $\sqrt{h^2+k^2}$

$$=\sqrt{\left(\frac{11}{24}\right)^2+\left(\frac{11}{24}\right)^2}=\frac{11}{24}\sqrt{2}$$

48. **(a)** Let L $\Rightarrow y = mx$, $L_1 \Rightarrow y = m_1x$, $L_2 \Rightarrow y = \frac{1}{m_1}x$

Now, $(y - mx)(y - mx) = 0$
$y^2 - ymx - mxy + m\,mx^2 = 0$
$\Rightarrow mm_1x^2 - (m_1 + m)xy + y^2 = 0$
Given that, $2x^2 + axy + 3y^2 = 0$

or $\frac{2}{3}x^2+\frac{a}{3}xy+y^2=0$

On comparing, $mm_1=\frac{2}{3}, -(m_1+m)=\frac{a}{3}$ ...(i)

Now, $(y-mx)\left(y+\frac{x}{m_1}\right)=0$

$$\Rightarrow y^2+\frac{xy}{m_1}-mxy-\frac{mx^2}{m_1}=0$$

$$\Rightarrow -\frac{m}{m_1}x^2+\left(\frac{1}{m_1}-m\right)xy+y^2=0$$

which is $2x^2 + bxy - 3y^2 = 0$

or $-\frac{2}{3}x^3-\frac{b}{3}xy+y^2=0$

On comparing,

$$\frac{-m}{m_1}=\frac{-2}{3}, -\frac{b}{3}=\frac{1}{m_1}-m$$

So, $m=\frac{2m_1}{3}, -\frac{b}{3}=\frac{1-m_1m}{m_1}$ ...(ii)

By solving eqs. (i) and (ii), we get

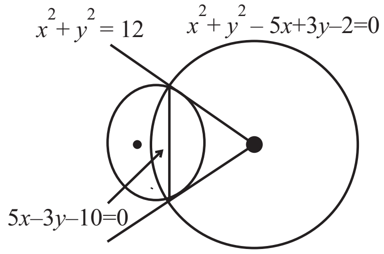
$m=\frac{2}{3}, m_1=1$ and $a = -5$, $b = -1$

$\therefore a^2 + b^2 = 25 + 1 = 26$

49. **(a)** Given circle
$S \equiv x^2 + y^2 - 2x - 4y + 3 = 0$
$\therefore p = (-1)^2 + (1)^2 - 2(-1) - 4(1) + 3$

$\because t=\sqrt{p}\Rightarrow t=\sqrt{3}$
Now, circle whose centre is (p, t^2), i.e. (3, 3)
$(x - 3)^2 + (y - 3)^2 = 18$
Since, this circle passes through (0, 0)
So, circle S′ will be
$(x - 3)^2 + (y - 3)^2 = 18$
Now, point (2, 3) w.r.t. to circle
$(x - 3)^2 + (y - 3)^2 = 18$ is
$\Rightarrow -17 < 0$
So, point (2, 3) lies inside the circle S′ = 0

50. **(d)** Let (p, q) is the point of intersection of the tangents. Then, the chord of contact of tangents is the common chord of the circles $x^2 + y^2 = 12$ and $x^2 + y^2 - 5x + 3y - 2 = 0$
$x^2 + y^2 = 12$
$x^2 + y^2 - 5x + 3y - 2 = 0$
Here equation of the common chord is
$5x - 3y - 10 = 0$ ...(i)



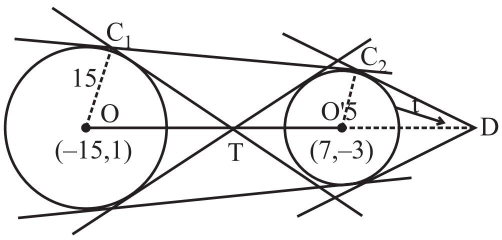
Also, the equation of the chord of contact is
$px + qy - 12 = 0$ ...(ii)
Eqs. (i) and (ii) represents the same line.
Therefore,

$$\frac{p}{5}=\frac{q}{-3}=\frac{-12}{-10}$$

$$\Rightarrow p=6, q=\frac{-18}{5}$$

Hence, the required point is $\left(6,-\frac{18}{5}\right)$

51. **(c)**



Given $S_1 : x^2 + y^2 + 30x - 2y + 1 = 0$
$\therefore$ Centre $(C_1) = (-15, 1)$
and radius $r_1=\sqrt{225+1-1}=15$
and $S_2 : x^2 + y^2 - 14x + 6y + 33 = 0$
$\therefore$ Centre $(C_2) = (7, -3)$
and radius $=\sqrt{49+9-33}=5$

Since, point T divides C_1C_2 in ration 3 : 1 Internally

$$\Rightarrow T = \left(\frac{21-15}{4}, \frac{-9+1}{4}\right) = \left(\frac{3}{2}, -2\right)$$

Also, point D divides C_1C_2 in ratio 3 : 1 Externally.

$$\Rightarrow D = \left(\frac{21+15}{2}, \frac{-9-1}{2}\right) = (18, -5)$$

Now, centre of circle with TD as diameters of mid-point of TD

$$= \left(\frac{18+3/2}{2}, \frac{-2-5}{2}\right) = \left(\frac{39}{4}, \frac{-7}{2}\right)$$

52. (b) Two circles will touch each other, if $C_1C_2 = r_1 + r_2$

Now, the centres of the two circles are $C_1(-\lambda, 0)$ and $C_2(0, -2)$ and their radii are $\sqrt{\lambda^2 - 2}$ and $\sqrt{2}$. So, the

$$\Rightarrow \sqrt{\lambda^2+4} = \sqrt{\lambda^2-2} + \sqrt{2}$$

$$\Rightarrow 4 = 2\sqrt{2}\sqrt{\lambda^2-2} \Rightarrow \sqrt{2} = \sqrt{\lambda^2-2}$$

$$\Rightarrow 2 = \lambda^2 - 2 \Rightarrow \lambda^2 = 4 \Rightarrow \lambda = \pm 2$$

53. (b) Since equation of the common chord is $S_1 - S_2 = 0$
Hence $x^2 + y^2 + 2x + 3y + 1 = 0$ and
$x^2 + y^2 + 4x + 3y + 2 = 0$ is given by
$S_1 - S_2 \equiv 2x + 1 = 0$
The equation of a circle passing through the intersection of the given circles is
$(x^2 + y^2 + 2x + 3y + 1) + \lambda(x^2 + y^2 + 4x + 3y + 2) = 0$
$\Rightarrow x^2(1 + \lambda) + y^2(1 + \lambda) + (1 + 2\lambda)$
$2x + 3y(1 + \lambda) + 1 + 2\lambda = 0$

$$\Rightarrow x^2 + y^2 + \left(\frac{1+2\lambda}{1+\lambda}\right)2x + 3y + \frac{1+2\lambda}{\lambda+1} = 0 \quad \text{...(i)}$$

Since, $2x + 1 = 0$ is a diameter of this circle.

Therefore, its centre $\left(-\frac{2\lambda+1}{\lambda+1}, -\frac{3}{2}\right)$ lies on it

$$\Rightarrow -2\left(\frac{2\lambda+1}{\lambda+1}\right) + 1 = 0$$

$$\Rightarrow -4\lambda - 2 + \lambda + 1 \Rightarrow \lambda = -\frac{1}{3}$$

On putting $\lambda = -\frac{1}{3}$ in eq. (i), we get

$$\Rightarrow x^2 + y^2 + \left(\frac{\frac{1}{3}}{\frac{2}{3}}\right)2x + 3y + \frac{\frac{1}{3}}{\frac{2}{3}} = 0$$

$$\Rightarrow 2x^2 + 2y^2 + 2x + 6y + 1 = 0$$

54. (d) We know that, semi latus rectum is the harmonic mean of segments.

$\therefore$ Length of latus rectum $= 2 \times \frac{2\times5\times3}{5+3}$

55. (b) We know that Equation of a tangent to a parabola $y^2 = 4x$ is

$$y = mx + \frac{1}{m}$$

It passes through point (1, 4) (given)

Hence $4 = m + \frac{1}{m}$ or $m^2 - 4m + 1$

$\Rightarrow m = 2 \pm \sqrt{3}$

$\therefore$ or $m_1 = 2 + \sqrt{3}$ and $m_2 = 2 - \sqrt{3}$

Hence, angle between two slopes is

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = \left|\frac{2\sqrt{3}}{1+1}\right|$$

$$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$$

56. (a) Given, tangent to parabola $y^2 = 4x$ at $(t^2, 2t)$ is
$y.\, 2t = 2(x + t^2) \Rightarrow yt = x + t^2$

Using $y.y_1 = 4\left(\frac{x+x_1}{2}\right)$ ...(i)

Normal to the ellipse $4x^2 + 5y^2 = 20$

or $\frac{x^2}{5} + \frac{y^2}{4} = 1$ at $(\sqrt{5}\cos\theta, 2\sin\theta)$

$\Rightarrow$ Slope of normal $= \frac{5y}{4x} = \frac{\sqrt{5}}{2}\tan\theta$

$\therefore$ Equation of line is

$$y - 2\sin\theta = \frac{\sqrt{5}}{2}\tan\theta(x - \sqrt{5}\cos\theta)$$

$$\Rightarrow y = \frac{\sqrt{5}}{2}\tan\theta \times x - \frac{\sin\theta}{2} \quad \text{...(ii)}$$

On comparing eqs. (i) and (ii), we get

$$t = -\frac{\sin\theta}{2}$$

$\Rightarrow \sin\theta = -2t$ ...(iii)

$$\Rightarrow \tan\theta = \frac{2}{\sqrt{5}t}$$

$$\Rightarrow \sin\theta = \frac{2}{\sqrt{4+5t^2}} \quad \text{...(iv)}$$

From eqs. (iii) and (iv), we get

$$-2t = \frac{2}{\sqrt{4+5t^2}}$$

On squaring both sides, we get
$t^2(4 + 5t^2) = 1 \Rightarrow 5t^4 + 4t^2 = 1$

57. **(b)** Since equation of ellipse is given, as

$4x^2 + 9y^2 - 24x + 36y = 0$

$\Rightarrow 4(x-3)^2 + 9(y+2)^2 = 72$

$$\Rightarrow \frac{(x-3)^2}{18} + \frac{(y+2)^2}{8} = 1$$

Since tangents are perpendicular.

So required locus is director circle. With equation $(x-k)^2 + (y-k)^2 = a^2 + b^2$.

$\therefore (x-3)^2 + (y+2)^2 = 26$

$\Rightarrow x^2 + y^2 - 6x + 4y = 13$

$\Rightarrow x^2 + y^2 - 6x + 4y - 13 = 0$

58. **(c)** Since equation of chord with mid-point (h, k) to the circle $x^2 + y^2 = 16$ is $T = S_1$ (given chord is tangent)

$hx + ky - 16 = h^2 + k^2 - 16$

$$\text{or } y = -\frac{h}{k}x + \left(\frac{h^2+k^2}{k}\right)$$

For hyperbola $9x^2 - 16y^2 = 144$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

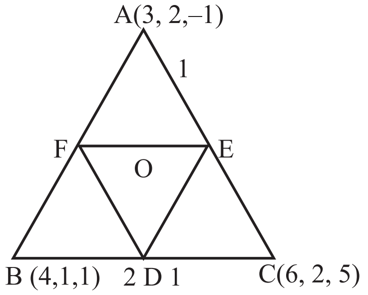
Hence we know that, condition of tangency of a line is $a^2m^2 - b^2 = c^2$ where $l \equiv y = mx + c$

$$\text{So, } 16\left(-\frac{h}{k}\right)^2 - 9 = \left(\frac{h^2+k^2}{k}\right)^2$$

$\Rightarrow 16h^2 - 9k^2 = (h^2 + k^2)^2$

$\therefore$ or $16x^2 - 9y^2 = (x^2 + y^2)^2$

59. **(a)** Given A (3, 2, –1), B (4, 1, 1), C (6, 2, 5) are three poitns while D, E and F are points on BC, CA and AB



From figure

$$\text{Here, D} = \left(\frac{2\times6+4\times1}{3}, \frac{2\times2+1\times1}{3}, \frac{2\times5+1\times1}{3}\right)$$

$$= \left(\frac{16}{3}, \frac{5}{3}, \frac{11}{3}\right)$$

$$\text{Similarly, } E = \left(\frac{12}{3}, \frac{6}{3}, \frac{3}{3}\right) \Rightarrow F = \left(\frac{11}{3}, \frac{4}{3}, \frac{1}{3}\right)$$

Let centroid of ΔDEF is O (x, y, z).

$$(x, y, z) \equiv \left(\frac{13}{3}, \frac{5}{3}, \frac{5}{3}\right)$$

Hence, coordinates are $\left(\frac{13}{3}, \frac{5}{3}, \frac{5}{3}\right)$

60. **(b)** Given, A = (1, 8, 4) and B = (2, –3, 1)

$\therefore \overrightarrow{OA} = \hat{i} + 8\hat{j} + 4\hat{k}$ and $\overrightarrow{OB} = 2\hat{i} - 3\hat{j} + \hat{k}$

$$\text{Now, } \hat{n} = \frac{\overrightarrow{OA}\times\overrightarrow{OB}}{|\overrightarrow{OA}\times\overrightarrow{OB}|}$$

$$\text{Here, } \overrightarrow{OA}\times\overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 8 & 4 \\ 2 & -3 & 1 \end{vmatrix} = 20\hat{i} + 7\hat{j} - 19\hat{k}$$

$$\text{and } |\overrightarrow{OA}\times\overrightarrow{OB}| = \sqrt{(20)^2 + 7^2 + (-19)^2} = 9\sqrt{10}$$

$$\therefore \hat{n} = \frac{20\hat{i} + 7\hat{j} - 19\hat{k}}{9\sqrt{10}}$$

$$\therefore \text{Direction cosines are } \frac{20}{9\sqrt{10}}, \frac{7}{9\sqrt{10}}, \frac{-19}{9\sqrt{10}}$$

$$\text{i.e. } \frac{2\sqrt{10}}{9}, \frac{7\sqrt{10}}{90}, \frac{-19\sqrt{10}}{90}$$

61. **(c)** Given lines sare

$$L_1 \equiv \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda \text{ (say)}$$

$$L_2 \equiv \frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = \mu \text{ (say)}$$

Hence from L_1 and L_2

$2\lambda + 1 = \mu + 3$

$3\lambda - 1 = 2\mu + k$

$4\lambda + 1 = \mu$

On solving we get $\lambda = -\frac{3}{2}, \mu = -5$

Hence, $k = \frac{9}{2}$

Option (c) is correct.

62. **(d)** Given, $\lim\limits_{x\to\infty} \frac{x^2(\tan 2x - 2\tan x)^2}{(1-\cos 2x)^4}$

$$= \lim_{x\to\infty} \frac{x^2\left[\left(2x + \frac{(2x)^3}{3} + \frac{2}{15}(2x)^5 + \ldots\right) - 2\left(x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots\right)\right]^2}{\left[1 - \left(1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \ldots\right)\right]^4}$$

$$= \lim_{x\to\infty} \frac{4x^8\left[\left(\frac{4}{3}-\frac{1}{3}\right)+\frac{2}{15}(16x^2-x^2)+...\right]^2}{16x^8\left[1+\frac{x^2}{3}+...\right]^4}$$

$$= \frac{4}{16} = \frac{1}{4}$$

63. **(a)** Given $\lim_{x\to\infty}\left(\frac{6x^2-\cos 3x}{x^2+5} - \frac{5x^3+3}{\sqrt{x^6+2}}\right)$

$$= \lim_{x\to\infty}\left(\frac{6-\frac{\cos 3x}{x^2}}{1+\frac{5}{x^2}} - \frac{5+\frac{3}{x^3}}{\left(\frac{|x|^3}{x^3}\right)\sqrt{1+\frac{2}{x^6}}}\right)$$

$$\left[\because \lim_{x\to\infty}\frac{\cos 3x}{x^2} = 0 \text{ and } \lim_{x\to\infty}\frac{|x|^3}{x^3} = -1\right]$$

$= 6 + 5 = 11$

64. **(a)** Given,

$$f(x) = \frac{x-1}{x^3+6x^2+11x+6}$$

$$\Rightarrow f(x) = \frac{x-1}{(x+1)(x+2)(x+3)}$$

$\Rightarrow (x + 1)(x + 2)(x + 3) = 0$
Here $x = -1, -2, -3$ are points of discontinuity
Hence, number of discontinuous in R is 3.

65. **(c)** $\frac{d}{dx}\left(\log\left(\sqrt{x+\sqrt{x^2+a^2}}\right)\right)$

$$= \frac{1}{\sqrt{x+\sqrt{x^2+a^2}}} \times \left(\frac{1}{2\sqrt{x+\sqrt{x^2+a^2}}}\right) \times \left(1+\frac{1}{2}.\frac{2x}{\sqrt{x^2+a^2}}\right)$$

$$= \frac{(\sqrt{x^2+a^2}+x)}{2\sqrt{x^2+a^2}(x+\sqrt{x^2+a^2})} = \frac{1}{2\sqrt{x^2+a^2}}$$

66. **(d)** Given, $f(x) = \cot^{-1}\left(\frac{x^2-x^{-x}}{2}\right)$

Let $y = \cot^{-1}\left(\frac{x^{2x}-1}{2x^x}\right)$

Put $x^x = \tan\theta$

$$\Rightarrow y = \cot^{-1}\left(\frac{\tan^2\theta - 1}{2\tan\theta}\right)$$

$y = \pi - \cot^{-1}(\cot 2\theta) \Rightarrow y = \pi - 2\theta$
$y = \pi - 2\tan^{-1}(x^x)$

$$\therefore \frac{dy}{dx} = -\frac{2}{1+x^{2x}}.x^x(1+\log x)$$

So, $\frac{dy}{dx}$ at $x = 1$ $= \frac{-2}{1+(1)^2}.(1+\log 1)$

$$= -\frac{2}{2}.1 = -1$$

67. **(a)** Let
$x = a(t + \sin t)$ ...(i)
and $y = a(1 - \cos t)$ ...(ii)
Now differentiate eq. (i) and (ii) w.r.t. t,

We get $\frac{dx}{dt} = a(1+\cos t)$

and $\frac{dy}{dx} = a\sin t$

Since, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\sin t}{a(1+\cos t)} = \frac{\sin t}{(1+\cos t)}$

Now, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\times\frac{dt}{dx}$

$$= \frac{d}{dt}\left(\frac{\sin t}{1+\cos t}\right)\times\frac{1}{a(1+\cos t)}$$

$$= \frac{(1+\cos t)\cos t - \sin t(-\sin t)}{(1+\cos t)^2}\times\frac{1}{a(1+\cos t)}$$

$$\frac{d^2y}{dx^2} = \frac{1}{a(1+\cos t)^2}$$

At $t = \frac{2\pi}{3}$

$$\frac{d^2y}{dx^2} = \frac{1}{a\left(\frac{1}{2}\right)^2} = \frac{4}{a}$$

68. **(b)** Given, curve is
$y^2(x - a) = x^2(x + a)$
Taking log both side
$\log\{y^2(x - a\} = \log\{x^2(x + a)\}$
$\Rightarrow 2\log y + \log(x - a) = 2\log x + \log(x + a)$
Differentiating both sides w.r.t. x, we get

$$\Rightarrow \frac{2}{y}.\frac{dy}{dx} + \frac{1}{x-a} = \frac{2}{x} + \frac{1}{x+a}$$

$$\Rightarrow \frac{2}{y}.\frac{dy}{dx} = \frac{2}{x} + \frac{1}{x-a} + \frac{1}{x-a}$$

$$= \frac{2(x^2-a^2)+x^2-ax-x^2-ax}{x(x-a)(x+a)}$$

$$\Rightarrow \frac{2}{y}\frac{dy}{dx} = \frac{2x^2 - 2a^2 - 2ax}{x(x-a)(x+a)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 - a^2 - ax}{x(x-a)(x+a)}.x.\sqrt{\frac{x+a}{x-a}}$$

At $\frac{dy}{dx} = 0$

$\therefore x^2 - a^2 - ax = 0$
Here, $p = a^2 + 4a^2 > 0$
So, it has two real roots.

69. **(b)** Given, function
$f(x) = (2k+1)\,x - 3 - ke^{-x} + 2e^x$
is monotonically increasing for all x $\in$ R.
$\Rightarrow f'(x) \geq 0$
$(2k+1) + ke^{-x} + 2e^x \geq 0$
$\Rightarrow e^{-x}(2k+1)\,e^x + k + 2e^{2x} \geq 0$
or $(2k+1)e^x + k + 2e^{2x} \geq 0$
$\Rightarrow 2^{ex}(e^x + k) + 1\,(e^x + k) \geq 0$
$\Rightarrow (2e^x + 1)\,(e^x + k) \geq 0$
$\Rightarrow (e^x + k) \geq 0$ or $k \geq 0$
Hence, least value of k is zero.

70. **(a)** Given, $f(x) = ax^3 + bx^2 + 11x - 6$ satisfies the Rolle's theorem in [1, 3]. So, $f(1) = 0$ and $f(3) = 0$
$\Rightarrow a + b + 5 = 0$ ...(i)
and $27a + 9b + 27 = 0$
$\Rightarrow 9a + 3b + 9 = 0$
$\Rightarrow 3a + b + 3 = 0$ ...(ii)
From Eq. (i) we get $a + b = -5$

71. **(c)** Given,
$f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$
It attains max. value of p and min value of q.
So $f'(p) = 0$ and $f'(a) = 0$
Now, $f'(x) = 6x^2 - 18ax + 12a^2$ has roots p and q

$$\therefore p + q = \frac{18a}{6} = 3a \text{ and } pq = \frac{12a^2}{6} = 2a^2$$

On solving both equations, we get
$p = a$ and $q = 3a$
Given $p^2 = q$
$\therefore \quad a^2 = 2a \Rightarrow a^2 - 2a = 0 \Rightarrow a = 2$ $(\because a \equiv 0)$

72. **(b)** Let $I = \int \cos x.\cos 2x.\cos 5x\,dx$

$$= \frac{1}{2}\int 2\cos x \cos 5x \cos 2x\,dx$$

$$= \frac{1}{2}\int (\cos 6x + \cos 4x)\cos 2x\,dx$$

$$= \frac{1}{4}\int (2\cos 6x \cos 2x + 2\cos 2x \cos 4x)\,dx$$

$$= \frac{1}{4}\int (\cos 8x + \cos 4x + \cos 6x + \cos 2x)\,dx$$

$$= \frac{1}{4}\left[\frac{\sin 8x}{8} + \frac{\sin 4x}{4} + \frac{\sin 6x}{6} + \frac{\sin 2x}{2}\right] + C$$

$$= \frac{\sin 2x}{8} + \frac{\sin 4x}{16} + \frac{\sin 6x}{24} + \frac{\sin 8x}{32} + C$$

Hence,
On comparing, we get

$$A = \frac{1}{8}, B = \frac{1}{16}, C = \frac{1}{24} \text{ and } D = \frac{1}{32}$$

$$\therefore \frac{1}{B} + \frac{1}{C} = 16 + 24 = 40$$

$$\text{Now, } \frac{1}{A} + \frac{1}{D} = 8 + 32 = 40$$

$$\therefore \frac{1}{B} + \frac{1}{C} = \frac{1}{A} + \frac{1}{D}$$

73. **(a)** Given, $\int e^x\left(\frac{x+2}{x+4}\right)^2 dx = f(x) + c$

$$\text{Now, } \int e^x\left(\frac{x+2}{x+4}\right)^2 dx = \int e^x\left(\frac{x^2+4+4x}{(x+4)^2}\right)dx$$

$$= \int e^x\left(\frac{x}{x+4} + \frac{4}{(x+4)^2}\right)dx$$

$$= \int e^x\{g(x) + g'(x)\}dx$$

$$= e^x g(x) + c = e^x\left(\frac{x}{x+4}\right) + c \qquad \therefore f(x) = \frac{xe^x}{x+4}$$

74. **(a)** Given, $\int \frac{dx}{\sin x + \sin 2x}$

$$\Rightarrow \int \frac{dx}{\sin x(1 + 2\cos x)} \Rightarrow = \int \frac{\sin x dx}{\sin^2 x(1 + 2\cos x)}$$

$$= \int \frac{-\sin x\,dx}{(\cos^2 x - 1)(1 + 2\cos x)}$$

Let $\cos x = t$

$$= \int \frac{dt}{(t^2-1)(1+2t)}$$

By partial fraction,

$$\frac{1}{(t-1)(t+1)(2t+1)} = \frac{A}{(t-1)} + \frac{B}{(t+1)} = \frac{C}{(2t+1)}$$

$\Rightarrow 1 = A(t+1)\,(2t+1) + B(t-1)\,(2t+1) + C(t^2 - 1)$
At $t = -1$

At $t = -\frac{1}{2} \Rightarrow C = -\frac{4}{3}$

At $t = 0$,

$$\therefore A = \frac{1}{6}, B = \frac{1}{2} \text{ and } C = -\frac{4}{3}$$

Now, $\int \frac{dt}{(t^2-1)(1+2t)} = \int \frac{1/6}{(t-1)} dt + \frac{1}{2}\int \frac{dt}{t+1}$

$-\frac{4}{6}\int \frac{dt}{\left(t+\frac{1}{2}\right)}$

$= \frac{\ln(t-1)}{6} + \frac{1}{2}\ln(t+1) - \frac{4}{6}\ln\left(t+\frac{1}{2}\right) + C$

$= \frac{\ln(\cos x - 1)}{6} + \frac{1}{2}\ln(\cos x + 1) = \frac{2}{3}\ln\left(\cos x + \frac{1}{2}\right) + C$

$= \frac{1}{2}\ln|(1+\cos x)| + \frac{1}{6}\ln|(1-\cos x|$

$\frac{-2}{3}\ln|(1+2\cos x)| + C$

75. **(d)** Let, $I_n = \int \frac{\sin nx}{\sin x} dx$ ...(i)

Then $I_{n-2} = \int \frac{\sin(n-2)x}{\sin x} dx$ ...(ii)

Subtracting eq. (ii) from eq. (i), we get

$I_n - I_{n-2} = \int \frac{\{\sin nx - \sin(n-2)x\}}{\sin x} dx$

$= \int \frac{2\cos(n-1)x \sin x}{\sin x} dx = \int 2\cos(n-1)x\, dx$

$= \frac{2\sin(n-1)x}{(n-1)}$

Similarly

$\therefore I_6 - I_4 = \frac{2\sin 5x}{5}$ and $I_4 - I_2 = \frac{2\sin 3x}{3}$

Now, $I_2 = \int \frac{\sin 2x}{\sin x} dx$

$= \int \frac{2\sin x \cos x}{\sin x} dx = 2\int \cos x\, dx$

$= 2\sin x + c$

and $I_6 = I_4 + 2\frac{\sin 5x}{5}$

$= 2\frac{\sin 5x}{5} + 2\frac{\sin 3x}{3} + 2\sin x + c$

$= \frac{2\sin 5x}{5} + \frac{2}{3}(3\sin x - 4\sin^3 x) + 2\sin x + c$

$I_6 = \frac{2}{5}\sin 5x - \frac{8}{3}\sin^3 x + 4\sin x + c$

76. **(c)** Given,

$k = \lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^n}{n^2}\right)...\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$

Taking log both side

$\log k = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{n^2}\right) = \int_0^1 \log(1+x^2)\,dx$

$= [\log(1+x^2).x]_0^1 - \int_0^1 \frac{2x}{1+x^2}.x\,dx$

$= \log 2 - 2\int_0^1 \frac{x^2}{1+x^2}dx = \log 2 - 2\int_0^1\left(1-\frac{1}{1+x^2}\right)dx$

$= \log 2 - 2\,[x - \tan^{-1}x]^0{}_1$

$= \log 2 - 2\left[1-\frac{\pi}{4}\right] \Rightarrow \log 2 + \frac{\pi}{2} - 2$

77. **(d)** Let $I = \int_0^{\pi/2}\frac{\sin^3 x\cos x\,dx}{\sin^4 x + \cos^4 x}$

By property

$= \int_0^{\pi/2}\frac{\sin^3\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}$

$I = \int_0^{\pi/2}\frac{\cos^3 x\sin x}{\cos^4 x + \sin^4 x}dx$

Adding Eqs. (i) and (ii), we get

$2I = \int_0^{\pi/2}\frac{(\sin^3 x\cos x + \cos^3 x\sin x)}{\cos^4 x + \sin^4 x}dx$

$= \int_0^{\pi/2}\frac{\sin x\cos x(\sin^2 x + \cos^2 x)}{\cos^4 x + \sin^4 x}dx$

$= \int_0^{\pi/2}\frac{\sin x\cos x}{\sin^4 x + \cos^4 x}dx$

$= \int_0^{\pi/2}\frac{\tan x\sec^2 x}{(\tan^4 x + 1)}dx$

Let $\tan^2 x = t$

Then, $\tan x\sec^2 x\,dx = \frac{dt}{2}$

$\therefore 2I = \frac{1}{2}\int_0^\infty \frac{dt}{t^2+1} = \frac{1}{2}\left[\tan^{-1}t\right]_0^\infty$

$\Rightarrow 2I = \frac{1}{2}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right] = \frac{1}{2}\times\frac{\pi}{2}$

$\Rightarrow 2I = \frac{\pi}{4} \Rightarrow I = \frac{\pi}{8}$

78. **(b)** Since, Given curve $y = ax^2 + bx$ passes through point (1, 2).

Hence, $2 = a(1)^2 + b(1)$

$\Rightarrow a + b = 2$ ...(i)

Given, area under curve $= \int_0^6 (ax^2 + bx)dx = 108$

$$\Rightarrow \left[\frac{ax^3}{3}+\frac{bx^2}{2}\right]_0^6 = 108$$

$\Rightarrow 72a + 18b = 108$

$\Rightarrow 4a + b = 6$ ...(ii)

By solving eqs. (i) and (ii), we get

$a = \frac{4}{3}$ and $b = \frac{2}{3}$

$\therefore 2b - a = 2\times\frac{2}{3} - \frac{4}{3}$

79. **(a)** Let the equation of all parabolas whose area are parallel to y-axis is

$y = Ax^2 + Bx + C$

Differentiate w.r.t. x, we get

$$\frac{dy}{dx} = 2Ax + B$$

Again, differentiating

$$\frac{d^2y}{dx^2} = 2A$$

Again, differentiating

$$\frac{d^3y}{dx^3} = 0$$

This is required equation.

80. **(a)** Given, differential equation is

$$\frac{dy}{dx} + 2y\tan x = \sin x$$

Here, P = 2 tan x and Q = sin x

$\therefore$ I.F. $= e^{\int 2\tan x\, dx}$

$= e^{2\ln|\sec x|} = e^{\ln \sec^2 x} = \sec^2 x$

Now, required solution is,

$$y \times \text{I.F} = \int \text{Q.IF}\, dx + c$$

$$\Rightarrow y\times\sec^2 x = \int \sin x.\sec^2 x\, dx + c$$

$$\Rightarrow y\sec^2 x = \int \sec x \tan x + c$$

$\Rightarrow y \sec^2 x = \sec x + c$ ...(i)

Given, $y = 0$ when $x = \frac{\pi}{3}$

$\Rightarrow 0 = 2 + c$

$\Rightarrow c = -2$

from (i)

$$\Rightarrow y = \frac{\sec x}{\sec^2 x} - \frac{2}{\sec^2 x}$$

$\Rightarrow y = \cos x - 2(1 - \sin^2 x)$

$\Rightarrow y = 2\sin^2 x + \cos x - 2$

PHYSICS

81. **(b)** Given, $\Delta t_1 = (2.00 \pm 0.0)$s $\Delta t_2 = (4.00 \pm 0.02)$ s

Product,

$$T = \sqrt{(\Delta t_1)(\Delta t_2)} = \sqrt{(2.00)(4.00)} = 2.828427\text{s}$$

Maximum fractional error in T

As per rule of significant figure, both T and Δt should have 3 significant figure.

$$\pm\frac{\Delta T}{T} = \pm\left(\frac{1}{2}\frac{\Delta t_1}{t_1} + \frac{1}{2}\frac{\Delta t_2}{t_2}\right)$$

T = 2.83s

Now, $\Delta T = \pm\frac{1}{2}\left(\frac{0.02}{2.00} + \frac{0.02}{4.00}\right)\times 2.8284$

$\Delta T = 0.02121$ s $= 0.02$

$\therefore$ T = (T + ΔT) = 2.83 ± 0.02) s

So, most matched answer is (b).

82. **(d)** Given initial velocity, $v_i = \sqrt{5}$ ms^{-1}

Final velocity, $v_f = 2\sqrt{5}$ ms^{-1} and

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$$\cos\theta = \frac{\Delta v^2 - v_i^2 - v_f^2}{2v_i v_f}$$

$$\Rightarrow \cos\theta = \frac{(5)^2 - (\sqrt{5})^2 - (2\sqrt{5})^2}{2(\sqrt{5})(\sqrt{5})}$$

$$\Rightarrow \cos\theta = \frac{25-5-20}{10} = \frac{0}{10} = 0 \Rightarrow \theta = 90°$$

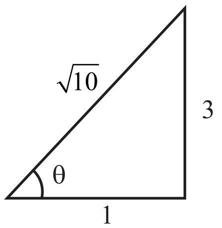
Hence, the correct option is (d).

83. **(c)** Given Maximum height, $H = \frac{u^2\sin^2\theta}{2g} = 3$ ...(i)

Maximum range, $R = \frac{u^2 \sin 2\theta}{g} = 4$

$$\therefore \frac{H}{R} = \frac{\sin^2\theta}{2\sin 2\theta} = \frac{3}{4} \Rightarrow 3\sin 2\theta = 2\sin^2\theta$$

$\Rightarrow \tan\theta = 3$ $(\because 2\sin 2\theta = 2\sin\theta\cos\theta)$



$\therefore \sin\theta = \frac{3}{\sqrt{10}}$ ...(ii)

Using Eqs. (i) and (ii), we get

$3 \times 2 \times 10 = u^2 \sin^2\theta$

$$\Rightarrow 3 \times 2 \times 10 = u^2 \times \left(\frac{3}{\sqrt{10}}\right)^2$$

$$\Rightarrow u^2 = \left(\frac{\sqrt{10}}{3}\right)^2 \times 3 \times 2 \times 10 \Rightarrow u^2 = \frac{100 \times 2}{3}$$

$$\Rightarrow u = 10\sqrt{\frac{2}{3}}$$

Hence, the correct option is (c).

84. **(a)** Given As the body is projected at an angle other 90°, hence it is a projectile motion Time of flight

$T = 2t = 2 \times 1 = 2S$

Using $h = ut + \frac{1}{2}gt^2$

$$\Rightarrow 0 = u \times 2 - \frac{10}{2} \times 2^2$$

$\Rightarrow u = 10$ m/s

Maximum height reached,

$$h = ut - \frac{10}{2}t^2$$

$\Rightarrow h = 10 \times 1 - 5 \times 1$

$\Rightarrow h = 5$ m

Hence, the correct option is (a).

85. **(b)** Given, mass of the body, $m = 2$ kg

time $t = 4$s

Impulse = change in momentum

impulse = $m\,(v_f - v_i)$

From the graph,

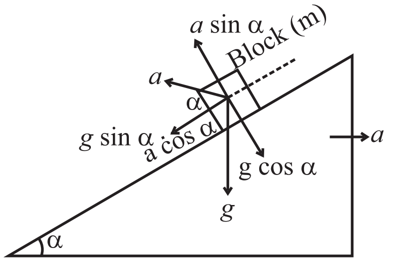
$$\text{Final velocity } v_f = \frac{3-3}{8-4} = 0$$

$$v_i = \frac{3-0}{4-0} = \frac{3}{4}$$

$$\therefore \text{Impulse } = 2 \times \left(0 - \frac{3}{4}\right) = -1.5 \text{ kg.ms}^{-1}$$

Hence, the correct option is (b).

86. **(b)** The block-plane system is shown in the figure,



So, from the above free body diagram (FBD),

Acceleration of body sliding up over inclined plane.

$a = g\sin\alpha + a\cos\alpha$ ...(i)

Acceleration of body sliding down over a rough inclined plane

$a_2 = \mu(g\cos\alpha - a\sin\alpha)$...(ii)

From block to be stationary,

$a_1 = a_2$

$$\Rightarrow \mu = \frac{g\sin\alpha + a\cos\alpha}{g\cos\alpha - a\sin\alpha} \quad ...(iii)$$

Dividing above equation by sin α and putting the values,

$$\mu = \frac{10 + 2\frac{1}{\tan\alpha}}{10\frac{1}{\tan\alpha} - 2} = \frac{10 + 2 \times 5}{10 \times 5 - 2}$$

$$\Rightarrow \mu = \frac{20}{48} = \frac{5}{12}$$

Hence, the correct option is (b)

87. **(a)** Given, mass of the body, $m = 1$ kg,

Mechanical energy $E_{mech} = 2J$

For speed ot be maximum, kinetic energy should be maximum and potential energy should be minimum.

$$\therefore U_{min} = \frac{dU(x)}{dx} = \frac{d}{dx}\left[\frac{x^2}{2} - x\right] = 0$$

$\Rightarrow x - 1 = 0 \Rightarrow x = 1$

$$\text{Hence, } U_{min} = \frac{(1)^2}{2} - 1 = -\frac{1}{2} < 0$$

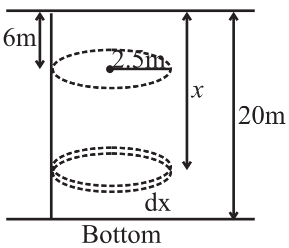
$$\text{Kinetic energy, KE} = \frac{1}{2}mv^2 = \frac{v^2}{2} \; (\because m = 1 \text{ kg})$$

$$\Rightarrow 2 = -\frac{1}{2} + \frac{v^2}{2} \Rightarrow v^2 = 5$$

$\Rightarrow v = \sqrt{5}$m/s

Hence, the correct option is (a).

88. **(b)** A cylindrical well of radius 2.5 m is shown in the figure.



Consider a small volume element dx at a distance x from the surface, then mass of the element,

$dm = \rho dV = \rho\pi f^2 dx$

So, the potential energy of the mass dm

$dU = dmg\,x$

$\Rightarrow dU = g\rho\pi r^2\,xdx$

Integrating on the both sides, we get

$U = g\rho\pi r^2 \int_6^{20} x\,dx$

$\Rightarrow U = 10\times10^3 \times 3.14\times(2.5)^2\left[\frac{20^2}{2}-\frac{6^2}{2}\right]$

$= 10^4 \times 3.14 \times (2.5)^2 \times 182$

$\Rightarrow U = 3571 \times 10^6$ J

Now, the time taken by pump of power 10 HP

$\text{time} = \frac{\text{work done}}{\text{power}} = \frac{35.71\times10^6}{10\times746\times60} \approx 80$ min

[$\because$ 1 HP = 746W]

Hence, the correct option is (b).

89. **(c)** Given, mass of flywheel, M = 1 kg,

Radius vectors $R = (2\hat{i} + \hat{j} + 2\hat{k})\,m$

Force applied,

$F = (3\hat{i} + 2\hat{j} - 4\hat{k})$N

and time, t = 4.5 s

Magnitude of radius

(R) = $\sqrt{(2)^2 + (1)^2 + (2)^2} = \sqrt{9} = 3$ m

Similarly, F = $\sqrt{(3)^2 + (2)^2 + (-4)^2} = \sqrt{29}$N

Torque on the flywheel,

$\tau = I\alpha = F.R = \frac{MR^2}{2}\alpha$

$\Rightarrow \alpha = \frac{2F}{MR} = \frac{2\sqrt{29}}{1\times3} = \frac{2}{3}\sqrt{29}$

Now, using

$\omega = \omega_0 + \alpha t$

$\Rightarrow \omega = 0 + \frac{2}{3}\sqrt{29}\times4.5 \quad (\because \omega_0 = 0)$

$\Rightarrow \omega = \sqrt{261}\ \text{rad}\,s^{-1}$

Hence, the correct option is (c).

90. **(b)** The coordinates of centre of mass of the spheres is given by

$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

As the spheres are identical,

$\therefore m_1 = m_1 = m_3 = m$ (say)

$\Rightarrow x_{CM} = \frac{m}{3m}(0 + 2\sqrt{3} + \sqrt{3}) = \frac{3\sqrt{3}}{3} = \sqrt{3}$

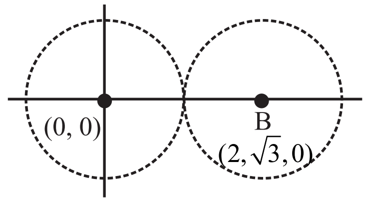
Similarly, $y_{CM} = \frac{m}{3m}(y_1 + y_2 + y_3) = \frac{(0+0+3)}{3} = i$

So, the centre of mass, = $(\sqrt{3}, 1)$

If one sphere is removed (say C), then

$x'_{CM} = \frac{0 + 2\sqrt{2}}{2} = \sqrt{3}$

$y'_{CM} = 0$



So, $C'_{CM} = (\sqrt{3}, 0)$

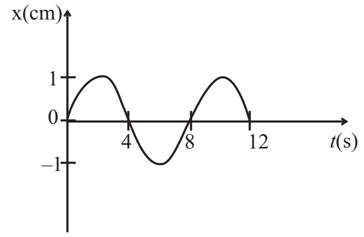
Hence, the centre of mass shifted by 1 m in $-y$ direction. The correct option is (b).

91. **(a)** From the graph, Amplitude, a = 1

Time period, T = 8s

The graph represnts a sine wave.

$\therefore$ Displacement, $x = 1\sin\omega t$



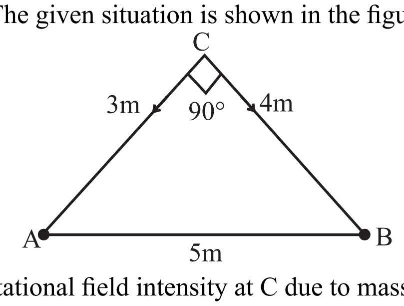
and, $\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$ rad/s

Acceleration in SHM, $\alpha = \frac{d^2x}{dy^2} - \omega^2 a\sin\omega t$

$\alpha t = \frac{4}{3}s\left(\frac{\pi}{4}\right)^2 \times1\times\sin\left(\frac{\pi}{4}\times\frac{4}{3}\right)$

$\Rightarrow \alpha = \frac{\pi^2}{16}\times\frac{\sqrt{3}}{2} = -\frac{\pi^2\sqrt{3}}{32}$ cm s^{-2}

92. **(b)** The given situation is shown in the figure,



Gravitational field intensity at C due to mass A. will be

$E_A = \frac{GM_A}{r_{CA}^2} = G\frac{90}{(3^2)} = 10G$ along CA

Gravitational field intensity at C due to mass B will be

$E_B = G\times\frac{160}{4^2} = 10G$ along CB

In ΔABC, $(AB)^2 = (AC)^2 + (BC)^2$

$\therefore (5)^2 = (3)^2 + (4)^2$

$\therefore \Delta ABC$ is a right angle triangle.

Resultant gravitational intensity at C will be

$$E = \sqrt{E_A^2 + E_B^2} = \sqrt{(10G)^2 + (10G)^2} = \sqrt{2} \times 10G$$

$$\Rightarrow E = \sqrt{2} \times 10 \times 6.67 \times 10^{-11}$$

$= 9.43 \times 10^{-10}\ Nkg^{-1}$

Hence, the correct option is (b).

93. (a) Given,

Young's modules, $Y = \frac{F/A}{\Delta^2}$

$$\Rightarrow \Delta L = \frac{FL}{YA} = \frac{4F}{Y\pi} \times \frac{L}{D^2} \quad \left(\because A = \frac{\pi D^2}{4}\right)$$

$$\Rightarrow \Delta L \propto \frac{L}{D^2}$$

(a) For length, L = 0.5 m, Diameter D = 0.5 mm = 0.5×10^{-3} m

$$\Delta L = \frac{4F}{Y\pi} \times \frac{0.5}{(0.5 \times 10^{-3})^2} = 2 \times 10^6 \times \frac{4F}{Y\pi}$$

(b) For L = 1 m, D = 1 mm

$$\Delta L = \frac{4F}{Y\pi} \times 1.0 \times 10^6$$

(c) For L = 2m, D = 2 mm

$$\Delta L = \frac{4F}{Y4\pi} \times 0.5 \times 10^6$$

(d) For L = 3m, D = 3 mm

$$\Delta L = \frac{4F}{Y\pi} \times 0.33 \times 10^6$$

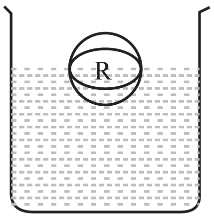
In option (a) we get highest value of ΔL.

94. (b) Given surface, tension of liquid, S = log

Force acting on the drop due to surface tension,

$F_s = S \times \pi D = 10g\ (\pi D)$ ...(i)

$W = F_s + F_b$



$$F_s = W = F_b = \rho Vg - \frac{\rho}{2}\frac{V}{2}g = \frac{3}{4}\rho Vg \quad ...(ii)$$

$$10g\rho D = \frac{3}{4}\rho g \frac{\pi D^3}{6} \left(\because V = \frac{4}{3}\pi r^3\right)$$

$$D = \sqrt{\frac{80}{\rho}} m$$

Hence, the correct option is (b).

95. (b) From the Newton's law of cooling is given by

$$-\frac{dT}{dt} = K(T - T_0)$$

Here, K = proportionality constant

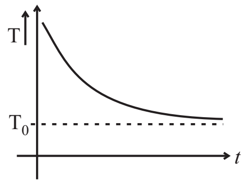
$$\int K \frac{dT}{(T - T_0)} = \int dt$$

$$\Rightarrow \frac{T - T_0}{T_1 - T_0} e^{-Kt}$$

$$\text{In}\left(\frac{T - T_0}{T_1 - T_0}\right) = -Kt$$

$T = T_0 + (T_i - T_0)\ e^{-Kt}$

Hence, the graph as shown below, shows temperature of a body (done) varies exponentially with time from T to T_0 ($T_0 < T$).



Thus, the correct option is (b)

96. (b) Heat lost form the sphere = Heat lost from radiation

$$\therefore mS\frac{\Delta T}{\Delta t}$$

$= 6Ae\ (T^4 - T_0^4)$

$$\Rightarrow \frac{\Delta T}{\Delta t} = \frac{\sigma Ae(T^4 - T_0^4)}{ms}$$

where, σ = Stefan's constant, A = area of sphere, and e = emittivity = 1 and S = secific heat capacity.

$$\therefore = \frac{ms\Delta T}{\sigma A(T^4 - T_4^4)} = \frac{(\rho V)C(200K - 100K)}{\sigma(A)(200^4 - 0^4)}$$

($\because$ T = 200K)

$$= \frac{\rho\frac{4}{3}\rho r^3 C \times 100}{\sigma 4\pi r^2 \times (200)^4} = \frac{1}{48}\frac{r\rho C}{\sigma} \times 10^{-6} s = \frac{1}{48}\frac{\rho r C}{\sigma}\mu s$$

Hence, the correct option is (b)

97. (d) Efficiency of a carnot engine (n) is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Here T_1 = temperature of source and T_2 = temperature of sin k

(a) For $T_1 = 500$ K, $T_2 = 300$ K

$$\eta = 1 - \frac{300}{500} = 0.4$$

(b) For $T_1 = 500$ K, $T_2 = 350$ K

$$\eta = 1 - \frac{350}{500} = 0.3$$

(c) For $T_1 = 800$ K, $T_2 = 400$ K

$$\eta = 1 - \frac{400}{800} = 0.5$$

(d) For $T_1 = 450$ K, $T_2 = 360$ K

$$\eta = 1 - \frac{360}{450} = 0.2$$

Hence, the correct code is matches to option (d).

98. **(b)** Given, mass of hammer, $m = 200$ kg,
steel block of the mass = 200 g = 0.2 kg an specific heat capacity of steel, $s = 460$ J kg^{-1} K^{-1}
velocity of hammer, $v = 8$ ms^{-1}
As we know that,

Kinetic energy, KE = $\frac{1}{2}mv^2 = \frac{1}{2} \times 200 \times 8^2 = 6400$J

Hence, the 23 % of this energy is converted to heat.

23% of 6400 = $\frac{23}{100} \times 6400 = 1472$ J

The rise in temperature of steel,

$$\Delta T = \frac{Q}{ms} = \frac{1472}{460 \times 0.2} = 16\text{K}$$

Hence, the rise in temperature is 16 K.

99. **(b)** Given
Temperature of gas T = 314 K,
Presspure of gas $p = 100$ kPa, = 1.0×10^5 Pa,
Sound, $v = 1380$ ms^{-1}
Diameter of gas molecule, $d = 10^{-10}$ m or radius

$$r = \frac{1}{2} \times 10^{-10} m = \frac{1}{2}\text{Å}$$

Mean free path (r) is given by

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 p}$$

$\because$ frequency, $f = \frac{v}{\lambda}$

$$\therefore f = \frac{\sqrt{2}\pi d^2 p \times v}{kT}$$

Substituting the respective values, we get

$$= \frac{\sqrt{2} \times 3.14 \times 10^{-20} \times 10^5 \times 1380}{1.38 \times 10^{-23} \times 314}$$

$$v = \sqrt{2} \times 10^9 \text{ Hz} \Rightarrow v = 1000\sqrt{2} \text{ MHz}$$

Hence, the correct option is (b)

100. **(a)** Here, $n_0 = 140$ Hz, $T_0 = 27°C = 300$ K and $T_2 = 57.75°C = 330.75$K

Speed of sound in gas, $v_s = \sqrt{\frac{y RT}{M}}$

$$\Rightarrow v_s \propto \sqrt{T}$$

$$\therefore \frac{v_1}{v_0} = \sqrt{\frac{T}{T_0}}$$

$$\Rightarrow v_1 = v_0\sqrt{\frac{330.75}{300}} = 1.05 v_0$$

Here v_0 = speed of sound at the room temperature
Hence, the beats produced in oragan pipe.
Number of beats, $n = n_1 - n_0 = 140 \times 1.05 - 140 = 7$

$$\left(\because n_1 = \frac{v_1}{4L} \text{ and } n_0 = \frac{v_0}{4L}\right)$$

So, the correct option is (a).

101. **(c)** Given, amplitude of spring, $A = 50 \times 10^{-2}$ m,
We have given, $f_{max} = 1.125 f_{min}$
Mass of sound source $m = 100$ g = 0.0 kg
Speed of sound, $v = 340$ ms^{-1}
Apparent frequency heard by observer for moving source,

$$f = \frac{f_0 v}{v - v_s}$$

For maximum value of f, $v_s = v_{s\,max}$
For minimum value of f, $v_s = -v_{s\,max}$

$$f_{max} = \frac{f_0 v}{v - v_{s\,max}} \quad \text{...(i)}$$

$$f_{min} = \frac{f_0 v}{v + v_{s\,max}} \quad \text{...(ii)}$$

By dividing Eqs. (i) by (ii), we get

$$\frac{1.125 f_{min}}{f_{min}} = \frac{v + v_{s\,max}}{v - v_{s\,max}}$$

$\Rightarrow 1.125 (v - v_{s\,max}) = (v + v_{s\,max})$
$\Rightarrow v\,(0.125) = 2.125\, v_{s\,max}$

$$\Rightarrow \frac{340 \times 0.125}{2125} = v_{s\,max} \Rightarrow v_{s\,max} = 20\,\text{ms}^{-1}$$

Velocity in spring mass system,

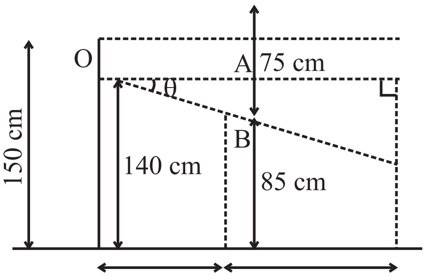
$$v_{s\,max} = A\omega = A\sqrt{\frac{k}{m}}$$

$$\Rightarrow k = \frac{v_{s\,max}^2 m}{A^2}$$

$\Rightarrow k = \dfrac{20\times 20\times 0.10}{0.50\times 0.50} = 160\,\text{N}\,m^{-1}$

Hence, the correct option is (c).

102. (c) The given situation is shown in figure.



From the property of similar triangle in OAB and OC

$\tan\theta = \dfrac{AB}{OA} = \dfrac{CD}{OC}$

Here AB = 75 + 140 – 75 + 85 = 55 cm

OA = x and OC = 2

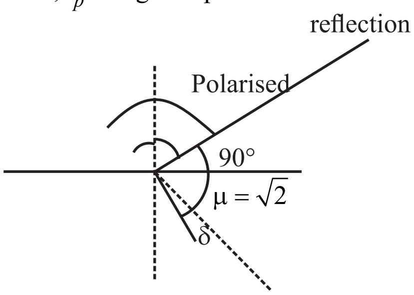
$\therefore \tan\theta = \dfrac{55}{x} = \dfrac{x}{2x} \Rightarrow x = 55\times 2 = 110$ cm

So, the maximum height seen by the girl,

H=110 + 10 = 120 cm

Since, the height above her eyes, has not effected the mirror height, because it is just in front of the mirror. Hence, the correct option is (c).

103. (d) Given, i_p = angle of polarisation



$i_p = \tan^{-1}\mu$

$\Rightarrow i_p = \tan^{-1}\sqrt{2} = \sin^{-1}\sqrt{\dfrac{2}{3}}$

By Snell's law,

$\sin i_p = \mu \sin r$

$\Rightarrow \sin r = \dfrac{\sin i_p}{\mu} \Rightarrow s\sin r = \dfrac{1}{\sqrt{2}}\dfrac{\sqrt{2}}{\sqrt{3}}$

$\Rightarrow r = \sin^{-1}\dfrac{1}{\sqrt{3}}$

Angle of deviation, $\delta = i_p - r$

$\delta = \sin^{-1}\sqrt{\dfrac{2}{3}} - \sin^{-1}\dfrac{1}{\sqrt{3}}$

Angle of reflection,

$\theta = 90° - r$

$\Rightarrow r = 90° - \theta$

$\Rightarrow \sin^{-1}\dfrac{1}{\sqrt{3}} = 90° - \theta \Rightarrow \dfrac{1}{\sqrt{3}} = \sin(90° - \theta)$

$\Rightarrow \cos\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\dfrac{1}{\sqrt{3}}$

Angle between incident and completely polarised

$\theta_1 = i_p + \theta$

$\theta_1 = \sin^{-1}\sqrt{\dfrac{2}{3}} + \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = 2\sin^{-1}\sqrt{\dfrac{2}{3}}$

Hence, the correct option is (d).

104. (d) As, electric field intensity of the axis,

$E_{axis} = \dfrac{2kpr}{(r^2 - a^2)^2}$

For $r >> a$

$E_{axis} = \dfrac{2kp}{r^3} = 4 \quad (\because \text{given}, E_{axis} = 4)$

$\Rightarrow \dfrac{kP}{r^3} = 2$ ...(i)

Electric field intensity on equatorial line,

$E_{eq} = \dfrac{k'p}{\left(r_1^2 + a^2\right)^{3/2}}$

For $r >> a$

$E_{eq} = \dfrac{k'P}{r_1^3}$

Here,

$r_1 = 2r$ and $k' = \dfrac{k}{4}$

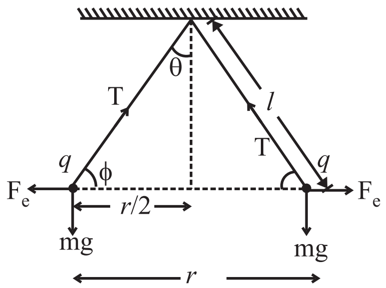
$\therefore E_{eq} = \dfrac{kP}{4\times 8r^3}$

Using (i), we get

$E_{eq} = \dfrac{2}{4\times 8} = \dfrac{1}{16}\,\text{NC}^{-1}$

Hence, the correct option is (d).

105. (a)



$F_e = \dfrac{kq^2}{r^2}$

$r = 2l\cos\theta$

$\tan\theta = \dfrac{F_e}{mg} \simeq \theta$

So, for $\theta = 45°$ and $\phi = 45°$

$$r = \frac{2l}{v^2} = \sqrt{2}l \Rightarrow F_e = \frac{Kq^2}{2l}$$

$$\tan 45° = \frac{kq^2}{2l\,mg} \Rightarrow mg = \frac{kq^2}{2l} \qquad (\because \tan 45° = 1)$$

For, $\theta = 30°$ and $\phi = 60°$

$$r = 2l \times \frac{2}{2} = l$$

$$\text{So, } F_e^{¢} = \frac{kq^2}{l^2} \Rightarrow \tan 30° = \frac{k'q^2}{l^2 - m'g}$$

As, the sphere is suspended in a liquid of density $\frac{2}{3}d$, then the observed weight of the body.

$$m' = V\left(d - \frac{2d}{3}\right) = \frac{m}{3}$$

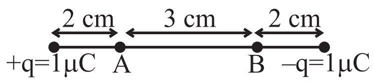
$$\frac{1}{\sqrt{3}} = \frac{3k'q^2}{l^2 mg} \Rightarrow mg = \frac{3\sqrt{3}k'q^2}{l^2} \qquad ...(ii)$$

So, from eq. (i) and (ii), we get

$$3\sqrt{3}k' = \frac{k}{2} \Rightarrow k' = \frac{1}{4\pi\varepsilon_0\varepsilon_r}, K = \frac{1}{4\pi\varepsilon_0}$$

$$\Rightarrow \varepsilon_r = 6\sqrt{3}$$

106. (a) The given situation is shown in the figure below



Potential at a point A = potential due to charge $+q$ + potential due to charge $-q$

$$V_A = \frac{1}{4\pi\varepsilon_0 x} + \frac{1(-q)}{4\pi\varepsilon_0(x+y)}$$

$$\frac{9\times10^5\times10^{-6}}{5\times10^{-2}} - \frac{9\times10^9\times10^{-6}}{5\times10^{-2}}$$

$$= 9\times10^5\left[\frac{1}{2} - \frac{1}{5}\right]$$

Similarly, potential at a point B,

$$V_B = \frac{1}{4\pi\varepsilon_0(x+y)} + \frac{1(-q)}{4\pi\varepsilon_0 x}$$

$$= 9\times10^5\left[\frac{1}{5} - \frac{1}{2}\right]$$

$$V_A - V_B \; 9\times10^5\left[\frac{1}{2} - \frac{1}{5} - \frac{1}{5} + \frac{1}{2}\right]$$

$\Rightarrow V_A - V_B = 5.4 \times 10^5 V$

Hence, the correct option is (a).

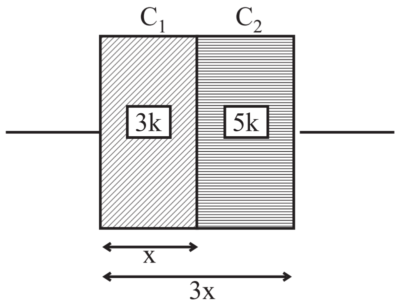
107. (d) We know that capacitance of a parallel plate capacitor is given by

$$C = \varepsilon_r \frac{\varepsilon_0 A}{d}$$

Here ε_r = dielectric constant.

A = area of parallel plate

d = distance between the plate.



$$\text{Now, } C_1 = 3k\frac{\varepsilon_0 A}{x}$$

$$C_2 = 5k\frac{\varepsilon_0 A}{2x}$$

In series combination of capacitor, stored charge in each capacitor is same i.e. $\theta_1 = \theta_2 = \theta$

$$\text{So, } V_1 = \frac{Q}{C_1} \text{ and } V_2 \frac{Q}{C_2}$$

$$\Rightarrow V_1 = \frac{\theta x}{3k\varepsilon_0 A} \text{ and } V_2 = \frac{\theta 2x}{5k\varepsilon_0 A}$$

$$\text{The ratio } \frac{V_1}{V_2} = \frac{\frac{3k\varepsilon_0 A}{\theta 2x}}{5k\varepsilon_0 A} = \frac{5}{3\times2} = \frac{5}{6}$$

Hence, option (d) is correct.

108. (d) With rise in temperature, the collision of electrons with fixed lattice ions/atoms increases. So relaxation time decreases. So, the conductivity of metal decreases.

All metals like aluminimum have positive temperature coefficient of resistivity and semiconductor like silicon has negative temperature coefficient of resistivity. Hence, assertion (A) is incorrect but reason (R) is correct.

109. (c) Given side of cube, a = 60 cm.

$\therefore$ area of cube = 6 (area of one sid eof the cube) = 6a thickness $d = 0.1$ cm

Temperature difference, $(T_1 - T_0) = 1000°C$

Rate of neat transfer through conduction wall is given by

$$\frac{dQ}{dt} = kA\frac{(T_1 - T_0)}{d}$$

$$\therefore \text{ Power, } P = \frac{d\theta}{dt} = \frac{kA \times 4.184 \times (T_1 - T_0)}{d}$$

$$= \frac{k6a^2 \times 4.184 \times 10^3}{d}$$

$$\Rightarrow P = \frac{4\times10^{-4} \times 6 \times (60)^2 \times 10^3 \times 4.184}{0.1} \text{J=361.49W}$$

Given, voltage of DC supply, V = 400 V

$$\therefore \text{ Power, } P = \frac{V^2}{R} = 361.49$$

$$\Rightarrow R = \frac{V^2}{P} = \frac{400 \times 400}{361.49 \times 10^3} = 0.4426\Omega$$

Hence, the correct option is (c).

110. **(a)** Given,

Let resistance to be connected in series be R. so the net resistance of circuit after connected R,

$$R_0 = R + \frac{G \times S}{G + S}$$

$$\therefore G = R + \frac{G \times S}{G + S}$$

$$\Rightarrow R = G - \frac{GS}{G+S} \quad \Rightarrow R = \frac{G^2}{G+2}$$

Hence, the correct option is (a).

111. **(c)** Given, magnetic field, B = 2 *m*T

Time-period of revolution of proton in the magnetic field (T_p) is given by

$$T_P = \frac{2\pi m_p}{eB} \quad \text{...(i)}$$

Time-period of revolution of α-particle in the magnetic field (T_τ) is given by

$$T_P = \frac{2\pi m_\propto}{eB} \Rightarrow T_\propto = \frac{2\pi(4m_p)}{(2e)B}$$

Dividing Eq. (i) and (ii), we get

$$\frac{T_p}{T_a} = \frac{2\pi m_p}{eB} = \frac{1}{2}$$

$$\Rightarrow T_\alpha = 2T_B$$

112. **(d)** Torque in a magnetic field is given by

$\tau = MB\sin\theta$

if $\theta = \theta_1$ then $\tau_1 = MB \sin\tau_1$ ....(i)

if $\theta = \theta_2$ then $\tau_2 = MB \sin\theta_2 = MB \sin 2\theta_1$

($\because$ Given, $\theta_2 = 2\theta_1$)

Given $\tau_2 = 141.4\%$ of τ_1

$= 1.414\ \tau_2 = \sqrt{2}\tau_1$

$$\Rightarrow T_2 = \sqrt{2}\tau_1 = MB\sin 2\theta_1 \quad \text{...(ii)}$$

Dividing eqs. (i) by (ii), we get

$$\frac{1}{\sqrt{2}} = \frac{\sin\theta_1}{\sin 2\theta_1}$$

$$\Rightarrow \sin 2\theta_1 = \sqrt{2}\sin\theta_1 \quad \text{...(iii)}$$

As we know that $\sin 2\theta = 2 \sin\theta \cos\theta$

Hence, $2 \sin\theta_1 \cos\theta_1 = \sqrt{2}\sin\theta_1$

$$2\cos\theta = \sqrt{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$$

113. **(b)** Given, velocity, V = 8 m/s Length of metal rod, l = 50 cm

Emf included in the metal rod,

$\Rightarrow V_A - V_B = l. (v \times B)$

$V_A - V_B = \text{iv B} \sin(90° - 60°)$

$\Rightarrow V_A - V_B = 50 \times 10^{-2} \times 8 \times 2 \times \sin 30°$

$$= 8 \times \frac{1}{2} = 4V$$

Hence, the correct option is (b).

114. **(a)** Let U_0 be the initial energy of capacitors.

Energy in IF capacitor.

$$u_1 = \frac{1}{2}CV^2 = \frac{1}{2}C\left(\frac{Q}{C}\right)^2$$

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} \times \frac{4^2}{1} = 8J$$

$$\text{Similarly, } u_2 = \frac{1}{2}\frac{Q^2}{C} = \frac{2^2}{2\times2} = IJ$$

So, the total energy $u_0 = u_1 + u_2 = 8 + 1 = 9J$

When, switch's is closed then the common potential of Capacitors

$$V_{common} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{1\times4 + 2\times1}{1+2} = \frac{6}{3} = 2V$$

When switch is closed,

$$E_1 = \frac{1}{2}C_1V_{common}^2 = \frac{1}{2} \times 1 \times 4 = 2J$$

and $E_2 = \frac{1}{2}C_2V_{common}^2 = \frac{1}{2}\times 2\times 4 = 4J$

Now, from conservation of energy,
Energy stored in the inductor, $E_L \approx u_0 - (E_1 + E_2)$
$= 9 - (2 + 4) = 3J$

Hence, the inductor has 3 J of energy.

115. (b) The energy of electromagnetic wave is equally divided between electric and magnetic field.

The equation consisting of frequency f_2, $(f_1 + f_2)$ and $(f_1 - f_2)$

The maximum frequency is $(f_1 + f_2)$.

Electric and magnetic field will have zero phase difference.

The pressure exerted by electromagnetic wave is the ratio of energy density to the speed of light.

$$C = \frac{E}{B}$$

Hence, the correct option is (b).

116. (d) Here, work function, $W_0 = 2.35$ eV and the electric compnent of electromagnetic radiation

$E = a\ [1 + \cos(1gf)]\cos(2\pi fl)]$

$E = [a\cos(2\pi ft) + a\cos(2\pi f_1 d)\cos(2\pi f_2 t)$

$$\because \cos A\cos B = \frac{1}{2}[\cos(A+B) - \cos(A-B)]$$

$$\Rightarrow E = a\cos(2\pi f_2 t) + \frac{a}{2}\cos 2\pi(f_1 + f_2)t - \frac{a}{2}\cos 2\pi(f_1 - f_2)t$$

So, the maximum kinetic energy of photoelectron, we take photon of maximum frequency, Hence,

$$E_{max} = \frac{hv_{max}}{e} = \frac{6.6\times 10^{-14}\times(3.610^{15} + 1.2\times 10^{15})}{1.6\times 10^{-19}}$$

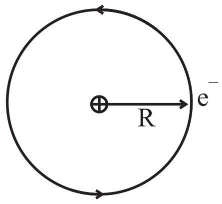
$= 19.8$ eV

Hence, the maximum kinetic energy,

$KE_{max} = E_{max} - W_0 = 19.8 - 2.35 = 17.45$ eV

Hence, the correct option is (d).

117. (b) Consider an electron of charge q is revolving in a circle of radius R as shown in figure.



$$\text{Current, } i = \frac{\text{Charge}}{\text{Time}} = \frac{q}{T} = \frac{q\omega}{2\pi} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

As, magnetic moment,

$$M = iA = \frac{q\omega}{2\pi}\pi R^2$$
$(\because A = \pi R^2)$

$$\Rightarrow M = \frac{qv}{2R}R^2 = \frac{eVR}{2} \qquad \left(\because \omega = \frac{v}{R}\right)$$

Angular momentum, $L = mvr$

$$\therefore \text{Magnetic moment, } M = \frac{qL}{2m}$$

$$\text{From Bohr's model, Angular momentum, } L = \frac{nh}{2\pi}$$

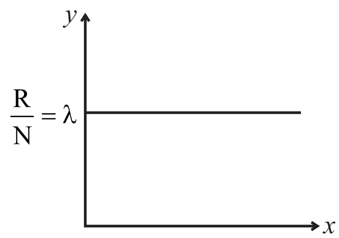
$$\therefore M = \frac{enh}{22\pi} \Rightarrow M \propto n$$

$\because$ e, h, m are constant of electron,

Hence, the correct option is (b).

118. (d) From the law of radioctive decay,

$$\text{Rate of disintegration, } R = \frac{dN}{dt} = -\lambda N$$



$\Rightarrow R = -\lambda N$

$$\Rightarrow \frac{R}{N} = -\lambda = \text{constant}$$

So, the correct graph is in option (d)

119. (c) We know that Frequency of visible rays range from 4×10^{14} Hz to 7×10^{14} Hz.

Maximum frequency, $V = 4\times 10^{14}$ Hz

Maximum energy of photon in visible light,

$$E_{max} = hv = \frac{6.6\times 10^{-34}\times 7\times 10^{14}}{1.6\times 10^{-19}}$$

$= 2.88$ eV ≈ 3eV

Minimum frequency of visible light

$V' = 7\times 10^{14}$ Hz

Minimum energy of photon is

$$E_{min} = hv' = \frac{6.6\times 10^{-34}\times 4\times 10^{14}}{1.6\times 10^{-19}}$$

$= 1.65$ eV ≈ 1.7 eV

So, the correct option matches between the energy 1.7 eV to 3 eV is (c),

120. (b) Given, height of transmitting antenna

h_r = 20 m, distance of LOS, d_m = 40 km

As maximum distance of line of sight mode

$$d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

$$\Rightarrow 40 = \sqrt{2 \times 6.4 \times 10^3 + 20 \times 10^{-3}} + \sqrt{2 \times 6.4 \times 10^3 \times h_R \times 10^{-3}}$$

$$\Rightarrow 40 = 16 + \sqrt{2 \times 6.4 \times 10^3 \times 10^{-3} \times h_R}$$

$\Rightarrow 2 \times 6.4 \times h_T = 24 \times 24 \Rightarrow h_T = 45$ m

Hence, the correct option is (b)

CHEMISTRY

121. (a) Energy of electron in *n*th orbitals is given by

$$E_n = -2.18 \times 10^{-18} . \frac{Z^2}{n^2} J$$

For helium ion (He^+)

Z = 2, n = 1

$$\therefore E_n = -2.18 \times 10^{-18} \times \frac{2^2}{1^2}$$

$$= -8.72 \times 10^{-18} J$$

For lithium ion (Li^{2+})

Z = 3, n = 3

$$\therefore E_n = -2.18 \times 10^{-18} \times \frac{3^2}{3^2} = -2.18 \times 10^{-18} J$$

122. (d) Given, $n = 3$, l = 1 and $m_1 = -1$

Means an orbital is present in p-subshell of 3rd shell having orientation value = –1. (i.e. p_x or p_y).

Thus, only one orbital is possible for the given set of values of n, l and m_1.

123. (a) For an isoelectronic species, the species with more electrons with respect protons in nucleus has larger size.

If 'Z' represents number of protons in a nucleus.

then, Z – 2 has least number of protons, whereas Z + 1 has maximum number of protons.

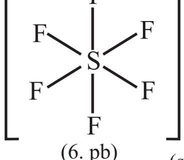
Hence, order of size for isoelectronic species will e

Z – 2 > Z – 1 > Z > Z + 1

Element with atomic number Z – 2 is of largest size and option (a) is the correct answer.

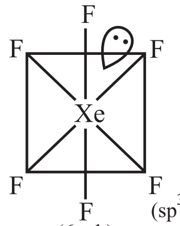
124. (c) The structure of given species are :

(a) SF_6



(6. pb) Octahedral (sp^3d^2 hyb.)

(b) XeF_6

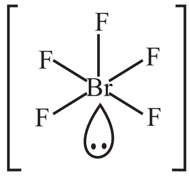


(sp^3d^2hyb.)

(6. pb)

Distorted octahedral

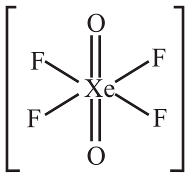
(c) BrF_5



(5 bp, 1*l*p) (sp^3d^2hyb.)

Square pyramidal

(d) XeO_2F_4



Octahedral (sp^3d^2hyb.)

125. (d) According to molecular orbital theory:

I. σ-orbital are formed by the substraction of two wave functions say ψ_A and ψ_B and therefore $\sigma^* = \psi_A - \psi_B$.

II. σ-orbital are formed when two orbitals are in same phase Hence, they do not have nodal plane in between two nuclei.

III. Combination of two sigma (σ) atomic orbitals give two types of molecular orbitals one is bonding molecular orbital σ and another one is antibonding molecular orbital (σ*) Energy of σ is lower than that of σ* orbital.

126. (c) Boiling point of a liquid is that temperature at which its vapour pressure becomes equal to the atmospheric pressure. Now, weaker is the attractive force between the molecues of given species, more is its volatile nature and smaller is its boiling point.

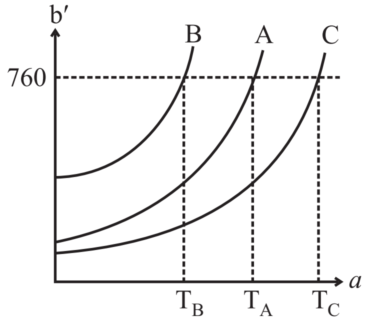
$$\text{Volatile nature} \propto \text{vapour pressure of } \frac{1}{B.Pt}$$

Force of attraction; $C_2H_5OC_2H_5 < CCl_4 < H_2O$

order of B.Pt ; $C_2H_5OC_2H_5 < CCl_4 < H_2O$

308 k 350k 373k

Now, drawing perpendicular on x-axis, i.e. on (a) from points B, A and C, we have



Hence, (A) = CCl_4, (B) = $C_2H_5OC_2H_5$ (C) = H_2O

127. (d) Applying

Volume of HCl solution (V_1) = 30 mL

Volume of sodium carbonate solution (Na_2CO_3) (V_2) = 20 mL

Concentration of Na_2CO_3 solution (M_2) = 0.1 M

Volume of NaOH solution (V_3) = 30.0 mL

Concentration of NaOH solution (M_3) = 0.2 M

N_1V_1 (Na_2CO_3)

$M_1 \times n_f \times v_1$ (HCl) = $M_2 \times n_f \times v_2$ (Na_2CO_3)

N = M × nf_i n_f of HCl = 1 n_f of Na_2CO_3 = 2

$M_1 \times 30 = 20 \times 2 \times 0.1$

$M_1 = 0.133$

Now, volue of HCl solution required to neutralise 30 mL of 0.2 M NaOH can be calculated by

Applying $M_1V_1 = M_3V_3$

$0.133 \times V_1 = 0.2 \times 30$

$V_1 = 45.1$ mL

128. (c) Moles of (Al), $(n) = \frac{w}{M} = \frac{54}{27} = 2$ mol

Heat required (Q) = nCΔT

= 2 × 24 × 20 = 960 J

Hence, option (c) is the correct answer.

129. (a) $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

$$K_C = \frac{[NO]^2}{[N_2][O_2]}$$

But, $[N_2] = [O_2]$ (Given)

$$\therefore K_C = \frac{[NO]^2}{[N_2]^2}$$

$$0.5625 = \frac{[3 \times 10^{-3}]^2}{[N_2]^2}$$

$$[N_2]^2 = \frac{[3 \times 10^{-3}]^2}{0.5625}$$

$[N_2]^2 = 16 \times 10^{-6}$

$[N_2] = 4 \times 10^{-3}$

130. (b) $A_2B \rightarrow 2A^+ + B^{2-}$

2S S

$sp = [A^+]^2 [B^{2-}] = (2S)^2 (S) = 4S^3$

Given, K_{sp} for $A_2B = 3.2 \times 10^{-11}$

$\therefore 3.2 \times 10^{-11} = 4S^3$

$$s = \left(\frac{3.2}{4} \times 10^{-11}\right)^{1/3} = 8 \times 10^{-12}$$

Thus $s = 2 \times 10^{-4}$

131. (a) Given,

Volume strength of H_2O_2 = 20 vol

$\because$ Normality of $H_2O_2 = \frac{\text{Volume strength}}{5.6}$

$= \frac{20}{5.6} = 3.57N$

$\because$ For $KMnO_4$, reaction is in acidic medium.

$Mn^{7+} + 5e^- \longrightarrow Mn^{2+}$

$\therefore$ Thus, valence factors is 5,

Normality for $KMnO_4$ = Molarity × n_f

Now, applying normality equation,

$N_1V_1(H_2O_2) = N_2V_2(KMnO_4)$

$3.57 \times V_1 = 0.02 \times 5 \times 500$

$$V_1 = \frac{50}{3.57} = 14.0 \text{ mL}$$

132. (d) $\underset{(B)}{2KO_2} + 2H_2O \longrightarrow \underset{(A)}{2KOH} + \underset{(B)}{H_2O_2} + \underset{(C)}{O_2}$

$\underset{(B)}{H_2O_2} + I_2 + 2KOH \xrightarrow{\text{Basic medium}} 2KI + 2H_2O + \underset{(C)}{O_2}$

KOH can not reduce I_2,

133. (b) Ga_2O_3 is an amphoteric oxide

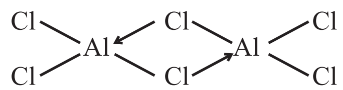
Reaction with an acid

$Ga_2O_3 + 6HCl \rightarrow 2GaCl_3 + 3H_2O$

Reaction with a base :

$Ga_2O_3 + 2NaOH \rightarrow 2\ NaGaO_2 + H_2O$

(II) The dimer of aluminium chloride has three Al-Cl-Al bridge bond is not the correct statement as structure of Al_2Cl_6 is as follows:



(i.e. has only two Al—Cl—Al bridge bonds)

(III) Boron is very hard refractory solid of high melting temperature as its melting point is 2349 K.

134. (c)

(a) HCOOH + H_2SO_4 (conc.) → CO + H_2O + H_2SO_4

But this process is not used on commercial scale.

(b) Direct oxidation of carbon in limited supply of oxygen can give CO but the process is not used on commericial scale.

$2C + O_2$ (limited supply) $\rightarrow$ 2CO

(c) When stream (H_2O) is passed over coke (C), it gives $CO + H_2$, as :

$$\text{Coke(C)} + \underset{\text{(stream)}}{H_2O} \xrightarrow[\text{Ni}]{\Delta} CO(g) + H_2(g)$$

The above process is used to produce (CO) on commercial scale.

(d) Lime stone is $CaCO_3$, on heating it gives oxygen (O_2) and CaO as :

$$CaCO_3 \xrightarrow{\Delta} CaO + O_2$$

135. (b) A-(III), B-(i), C-(V), D-(II)

A. Insecticide	III. Na_3AsO_3, as it is used in killing insects or pests
B. $K_2Cr_2O_7$/ 50% H_2SO_4	I. COD, as $K_2CR_2O_7$/ 50% H_2SO_4 is used to find "Chemical Oxygen Demand"(COD) value.
C. Bleaching of clothes and paper	V. H_2O_2, as H_2O_2 is a bleaching agent and can be used to bleach clothes/ paper.
D. Eye irritant	II. PAN (Peroxy acyl nitrates) can produce irritation in eyes.

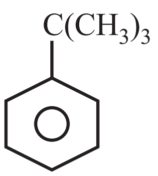
136. (a) The groups in which nitrogen contain double bond or present in heterocyclic ring can not be estimated by Kjeldahl's method.

Nitrogen in aniline does not contain double bond, it estimated by Kjeldahl's method.

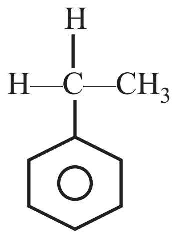
nitrogen can not be estimated by Kjeldahl's method.

137. (d) No bond resonance is a special case of resonance in which delocalization of sigma electron or orbital containg lone pair of electron with adjacent p-orbital. It is also called hyperconjugation.

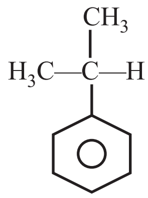
More be the H-atoms associated (bonded) with C-atom, which is bonded with the benzene ring, more will be the number of 'no bond resonance'



C-attached with the ring has no H-atom

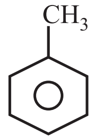


C-attached with the ring has 2 H-atoms.



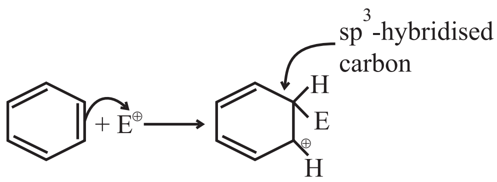
C-attached with the ring has one H-atom.

the ring has 3H-atoms



138. (a)

(I) In nitrating mixture ($HNO_3 + H_2SO_4$) nitric acid (HNO_3) with H_2SO_4) behaves as a base.

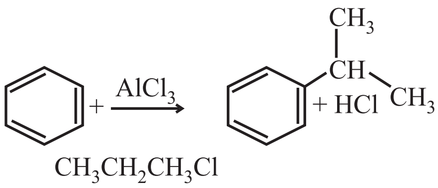


Sigma complex
(arenium ion)

(II) Benzene on Friedel-Craft's alkylation with n-propyl chloride gives isopropyl benzene due to formation carbocation more stable 2°

1° carbocation.

The reaction occurs as follows



n-propyl chloride

139. (d) OCH_3 due to +R effect increase electron density at

O and P position.

140. (N) Given,

Radius of an atom (bcc)

= 173.2 pm, = 1.732×10^{-8} cm

$\because$ For bcc structure

$\sqrt{3}.a = 4r$ where, a = edge-length
r = radius of atom.
and a^3 = V (volume of cubic unit cell).

$$\therefore = a\frac{4}{\sqrt{3}}.r$$

$$a = \frac{4}{1.73} \times 1.732 \times 10^{-8}\,\text{cm}$$

$a = 4 \times 10^{-8}$ cm
Therefore,
Volume (V) = $(a^3) = [10^{-8}]^3 \times (4)^3$
$64 \times 10^{-24}\ \text{cm}^3$
$6.4 \times 10^{-23}\ \text{cm}^3$

141. **(b)** Given,
Mass of solute (w_B) = 10 g
Molar mass of solute (M_B) = M_B
Mass of solvent (w_A) = 360g
Molar mass of water (M_A) = 18 g mol^{-1}

$$\frac{\Delta p}{p^\circ} = \chi_B = \frac{n_B}{n_A + n_B} = 5 \times 10^{-3}$$

$$\frac{\Delta p}{p^\circ} = \frac{\frac{w_B}{M_B}}{\frac{w_A}{M_A} + \frac{w_B}{M_B}}$$

$$5 \times 10^{-3} = \frac{\frac{10}{M_B}}{\frac{360}{18} + \frac{10}{M_B}}$$

$$5 \times 10^{-3} = \frac{10}{20M_B + 10}$$

$(20\ M_B + 10)\ 5 \times 10^{-3} = 10$

$$M_B = \frac{1990}{20} = 99.5\ \text{g mol}^{-1}$$

142. **(d)** Given,
Mass of $MgSO_4$ (W) = x g
Volume of solution (V) = 2.5 L
vant Hoff factor (i) = 1.8
Osmotic pressure (π) = 2.463 atm
Temperature (T) = 27 + 273 = 300 K
Molar mass of $MgSO_4$ (M) = 120 g mol^{-1}

$$\pi = iCRT = \frac{i \times W \times RT}{M \times V(\text{in} L)}$$

$$2.463 = \frac{1.8 \times x \times 0.0821 \times 300}{120 \times 2.5}$$

($\because$ R = 0.0821 L - atm K^{-1} mol^{-1})

x = 16.68g

143. **(e)** Given,
Free energy, [$\Delta G = -\,nFE^\circ$]
where n = number of electron invoved
E° = standard electrode potential
F = Faraday's constant

(iii) $M^{2+}_{(aq)} + 2e^- \longrightarrow M_{(s)} \Delta C^\circ_{13} = n_3FE^\circ_3$

For reaction (iii)

$$\Delta G^\circ_3 = \Delta G^\circ_1 + \Delta G^\circ_2$$

$$-n_3FE^\circ_3 = -\left(n_1FE^\circ + n_2FE^\circ_2\right)$$

$$-2E^\circ_3 = -(nE^\circ_1 + nE^\circ_2)$$

$$2E^\circ_3 = E^\circ_1 + E^\circ_2$$

$$2E^\circ_3 = 0.15 + 0.5$$

$$\therefore E^\circ_3 = \frac{0.65}{2} = 0.325\,\text{V}$$

144. **(c)** Rate constant $k = \frac{0.693}{t_{1/2}}$

$$\therefore k_1\ (\text{at 300 K}) = \frac{0.693}{50} = 0.014\,s^{-1}$$

$$\text{and}\ k_2(\text{at 400K})\frac{0.693}{10} = 0.07\,s^{-1}$$

According to Arrhenius theory

$$\log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{T_2 - T_1}{T_1.T_2}\right]$$

Thus,

$$\log\frac{0.07}{0.014} = \frac{E_a}{2.303 \times 8.314}\left[\frac{400 - 300}{400 \times 300}\right]$$

$$\log 5 = \frac{E_a}{2.303 \times 8.314}\left[\frac{100}{400 \times 300}\right]$$

$$\text{or,}\ 0.70 = \frac{E_a}{19.15 \times 1200}$$

or, E_a = 16086 J = 16.08 kJ mol^{-1}
E_a = 16.10 kJ mol^{-1}

145. **(c)**
(a) An increse in temperature reduces the interaction between the absorbate molecules and adsorbent molecules.
(b) The extent of adsorption increases with an increase of surface area of the adsorbate.
(c) The extent of adsorption decreases with an increased in temperature.

$$\because \text{Adsorption} \propto \frac{1}{\text{temperature}}$$

(d) The extent of adsorption depends on amount of solute in solution

$$\frac{x}{m} \propto \text{conc.}(C)^{1/n}$$

146. (c) Composition of German silver is zinc (35%)
Copper 50%
Nickel (15%)
Composition of brass
Zinc (30%) and copper (–70%) and some quantity of nickel (Ni).
Therefore, iron (Fe) is not common to German silver and brass.

147. (b) For the given reactions :
(I) $2NaOH + SO_2 \rightarrow Na_2SO_3 + H_2O$

(II) $XeF_4 + O_2F_2 \xrightarrow{143\,K} XeF_6 + O_2$

(III) $PCl_5 + H_2O \rightarrow POCl_3 + 2HCl$
$POCl_3 + 3H_2O \rightarrow H_3PO_4 + 3\ HCl$
Overall reaction,

$$PCl_5 + 4H_2O \longrightarrow H_3PO_4 + 5HCl$$

(IV) $2NaNO_2 + 2HCl \longrightarrow 2\,NaCl + NO_2 + H_2O$
also the correct reaction as HCl can decomposes salts like $NaNO_2$ to NaCl, NO_2 and H_2O.

148. (c) (i) When chlorine oxidises sulphur dioxide in presence of water, it gives H_2SO_4 as oxyacid (A).
The reaction occurs as follows :

$$Cl_2 + SO_2 + 2H_2O \longrightarrow H_2SO_4 + 2HCl$$

The oxidation number of sulphur in H_2SO_4
$= 2 + x + (4 \times -2) = 0 \Rightarrow x = +6$
(ii) When chlorine (Cl_2) oxidises iodine (I_2) in presence of water, it gives HIO_3 as oxyacid (B), the reaction occurs as follows :

$$5Cl_2 + I_2 + 6H_2O \longrightarrow 2HIO_3 + 10HCl$$

The oxidation state of (1) in HIO_3 is
$= 1 + y + 3 \times (-2) = 0$
$\Rightarrow y = +5$

149. (b) (i)

$$\underset{\text{White phosphorus}}{P_4} + 3NaOH + 3H_2O \xrightarrow[\Delta]{CO_2} \underset{(A)}{PH_3(g)} + \underset{\substack{\text{(Sodiumhypophosphite)} \\ (B)}}{3NaH_2PO_2}$$

(ii) When (A), i.e. $PH_3(g)$ is bubbled into aqueous $CuSO_4$ solution, copper phosphide (Cu_3P_2) and H_2SO_4(C) is formed. The reaction occurs as follows :

$$2PH_3 + 3CuSO_4 \longrightarrow \underset{\text{(Copper phosphide)}}{Cu_3P_2} + \underset{(C)}{3H_2SO_4}$$

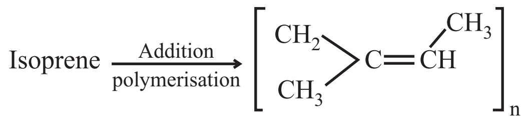
150. (c) Electronic configurations of
(i) Lanthanum : (Z = 57) $[Xe]_{54}\,5d^1\,6s^2$
(ii) Uranium : (Z = 92) $[Rn]_{86}\,5f^3\,6d^1\,7s^2$
(iii) Lawrencium : (Z = 103) $[Rn]_{86}\,5f^{14}\,6d^1\,7s^2$
(iv) Thorium : (Z = 90) $[Rn]_{54}\,6d^2\,7s^2$
(v) Actinium : (Z = 89) $[Rn]_{86}\,6d^1\,7s^2$
(vi) Cerium: (Z = 58) $[Xe]_{54}\,4f^1\,5d^1\,6s^2$
Hence, elements La, Ac, and Th do not possess f-electrons.

151. (b) Given, Δ_0 for ($3d^2$) = 1000 kJ mol^{-1}
Energy of (t_{2g}) E(t_{2g}) = –400 kJ mol^{-1}
$\because \Delta_0$ = Energy of e_g – Energy of t_{2g}
$1000 = E_{(eg)} - (-400)$
$E_{(eg)} = 1000 - 400$
$E_{(eg)} = 600$ kJ/mol

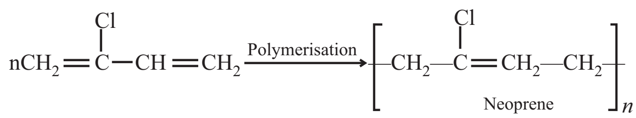
152. (a) It is prepared by heating tetrafluoroethylene at high pressure in the presence of peroxides or ammonium peroxo sulphate [$(NH_4)_2S_2O_8$] catalyst.
It is chemically and resistant to attack.

$$\underset{\text{Tetrafluoroethene}}{nCF_2 = CF_2} \xrightarrow[\text{High pressure}]{n(NH_4)_2S_2O_8} \underset{\text{Teflon}}{\left(CF_2 - CF_2 \right)_n}$$

It is an addition polymer.
Polyisoprene :



It also an addition polymer
Neoprene : it is also called polychloroprene and formed by the free radial polymerisation of chloroprene and has superior resistance to vegetable and mineral oils.



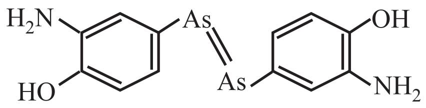
It is an addition polymer.
Bakelite, Nylon-6, Dacron and Melamine are condensation polymers

153. (a)

	Vitamin	Source	Deficiency disease
(a)	B_6	Milk	Convulsion
(b)	K	Green leafy vegetables	Difficulty in blooc clotting
(c)	C	Citrus fruits	Scurby

(d) D Sunlight egg yolk Ricket

154. **(c)** Salvarsan

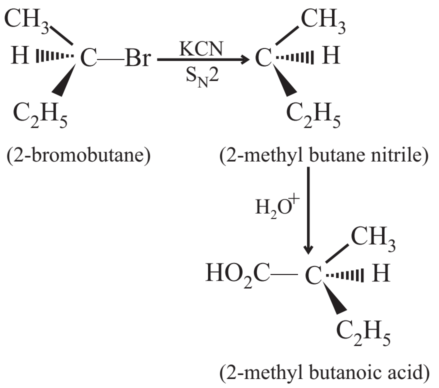


Structure of salvarsan contain

—As = As — in its structure.

155. **(b)** 2 bromobutane (2° haloalkane) reacts with KCN via S_N2 mechanism due to less steric hinderance and we get 2-methyl butane nitrile (X) with inversion in structure which

On hydrolysis gives methyl butanoic acid (Y) having inverted structure as that of haloalkane.



156. **(b)** (–) I group, if are present along with —COOH group is in acid, they increases the acidic property while (+) I groups of groups show hyperconjugation will decreases the acidic property.

N CH_2COOH > FCH_2COOH> H_3C CH_2COOH + I effect

(II) Structures which show more resonance structures or hyperconjugation among aldehydes are less reactive.

Ketones are less reactive than aldehyole's due to steric hindrance.

Hence, correct order of reactivity is

$CH_3CH_2.CHO$ > PhCHO > $PhCOCH_3$

(III) Boiling point of alcohols (due to ability to form H-bond) are higher than that of aldehydes which are more than that of ketones. (due to less steric-hinderance and more surface area in aldehyde group, having same number of C-atoms)

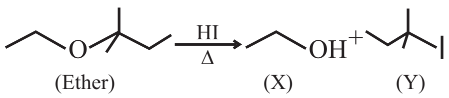
Hence order of b.pt is $H_3COCH_2CH_3$ < H_3C CH_2CHO < $H_3CCH_3CH_2OH$

Hence option (b) is the correct answer.

157. **(a)**

(i) Reaction of ether with HI give an alcohol and alkyl iodide (R—I), in which —I is bonded with more substituted part of ether.

Thus, the given reaction occurs as follows:



158. **(c)** $R-CHO+2[Ag(NH_3)_2]^+ + 3OH^- \longrightarrow$ (Tollen's reagent)

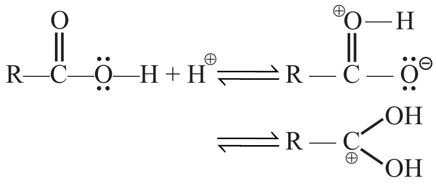
$RCOO^- + 2Ag + 4NH_3 + 2H_2O$

159. **(b)** $R-COOH + HOR' \xrightarrow{\text{Conc.} H_2SO_4}$

$R—COOR' + H_2O$

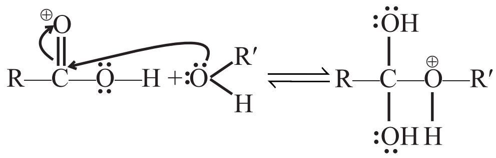
Mechanism of esterification

Step-I



(4)

Step-II

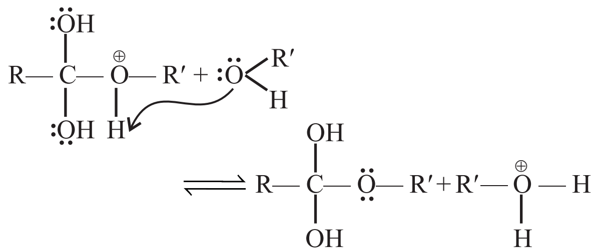


(Oxonium ion)

(1)

Formation of ester (esterification) involves reaction between carboxylic acid and alcohol in the presence of acid.

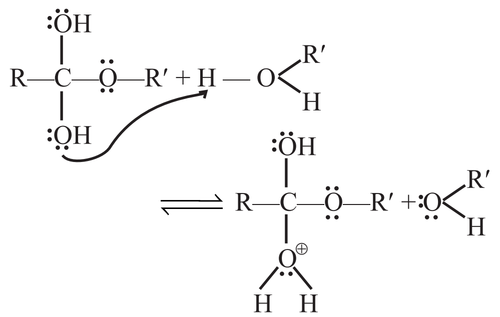
Step-III



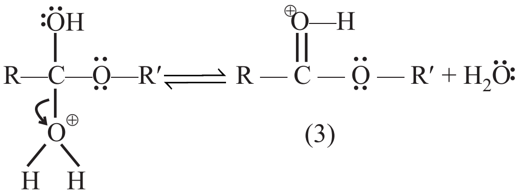
Tetahedral intermediate

(2)

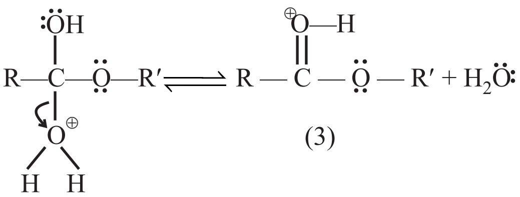
Step-IV



Step-V



Step-VI



160. (a)