AP/EAPCET Solved Paper 2020
Held on September 18

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. If sec(x) = cos h(θ), then find $\tan h^2\left(\frac{\theta}{2}\right)$

(a) $\sec^2\left(\frac{x}{2}\right)$ (b) $\tan^2\left(\frac{x}{2}\right)$

(c) $\tan h^2\left(\frac{x}{2}\right)$ (d) $\sec h^2\left(\frac{x}{2}\right)$

2. The equation of the tangent to the parabola $y^2 = 12x$, which makes an angle 30° with the positive direction of X-axis is given by $x-\sqrt{3}y+9=0$, then its points of contact is

(a) $\left(-9,-6\sqrt{3}\right)$ (b) $\left(9,-6\sqrt{3}\right)$

(c) $\left(-9,6\sqrt{3}\right)$ (d) $\left(9,6\sqrt{3}\right)$

3. The solution of $\frac{d^2y}{dx^2}=0$ represents

(a) straight lines (b) a circle

(c) a parabola (d) a point

4. There are two dice A and B. Die A has 4 red and 2 white faces and B has 2 red and 4 white faces. A coin is tossed once, if it shows head, die A is rolled, if it shows tail, die B is rolled, if the probability that die A is used is $\left(\frac{32}{33}\right)$ when it is given that red turns up every time in first n throws, then n =

(a) 5 (b) 6 (c) 4 (d) 3

5. $\int_0^{\pi/4}\frac{dx}{\cos^3(x)\cdot\sqrt{2\sin(2x)}}=$

(a) $\frac{6}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{8}{5}$

6. $\lim_{n\to\infty}\frac{n!}{(n+1)!-n!}=$

(a) 1 (b) – 1 (c) 2 (d) 0

7. $\int_{-1}^{2}|x|\,dx=$

(a) 1 (b) 2 (c) $\frac{5}{2}$ (d) $\frac{3}{2}$

8. Which of the following is false?

1. If (a, b, c) are direction ratios of a line, then $a^2+b^2+c^2\neq 1$.
2. The direction cosines of a line can be its direction ratios but not vice-versa.
3. If (l, m, n) is one set of direction cosines, then ($-l$, $-m$, $-n$) is also a valid set.
4. If (l_1, m_1, n_1) and (l_2, m_2, n_2) are direction cosines of perpendicular lines, then $l_1l_2+m_1m_2+n_1n_2=1$.

(a) 1 (b) 2 (c) 3 (d) 4

9. If the sum of the roots of the quadratic equations is 1 and sum of the squares of the roots is 13, then find that equation.

(a) $x^2+x-6=0$ (b) $x^2-x+6=0$

(c) $x^2-x-6=0$ (d) $x^2+x+6=0$

10. If a diagonal of a square is along the line $8x-15y=0$ and one of its vertices is (1, 2), then the equations of the sides of the square passing through this vertex are

(a) $23x-7y+9=0$, $7x+23y+53=0$

(b) $23x-7y-9=0$, $7x+23y-53=0$

(c) $23x+7y-9=0$, $7x+23y-53=0$

(d) $23x+7y-9=0$, $7x+23y+53=0$

11. In triangle ABC, $\frac{\tan A}{2}=\frac{\tan B}{3}=\frac{\tan C}{4}$, then the value of $\sec^2A+\sec^2B+\sec^2C=$

(a) $\frac{101}{8}$ (b) $\frac{111}{8}$ (c) $\frac{121}{8}$ (d) $\frac{91}{8}$

12. If $\frac{x^2+x+1}{x^2+2x+1}=A+\frac{B}{x+1}+\frac{C}{(x+1)^2}$, then $A-B$ is equal to

(a) $4C$ (b) $4C+1$ (c) $3C$ (d) $2C$

13. Which of the following is false?

1. If A is a skew symmetric matrix of order 5×5, then the rank of A is less than 5.
2. If P is a non-zero column matrix and Q is a non-zero row matrix, then the rank of PQ is 1
3. Rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$ is 2
4. If the lines $a_r x + b_r y + c_r = 0$ $(r = 1, 2, 3)$ are distinct and intersect at a point, then rank of $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is 3

(a) 1 (b) 2 (c) 3 (d) 4

14. Choose the correct option regarding the following statements

1. $C_0 + C_2 + C_4 + ... + C_n = 2^{n-1}$, if n is even
2. $C_1 + C_3 + C_5 + ... + C_{n-1} = 2^{n-1}$, if n is even

(a) 1 is true, 2 is false (b) 1 is false, 2 is true
(c) Both 1 and 2 are false (d) Both 1 and 2 are true

15. The number of different words that can be formed from the letters of the word "INTERMEDIATE" such that two vowels never come together, is

(a) $\frac{6!}{2!} \times \frac{7!}{2!3!}$ (b) $\frac{5!}{2!} \times \frac{6!}{3!}$
(c) $6! \times \frac{7!}{2!3!}$ (d) $\frac{6!}{2!} \times \frac{6!}{2!3!}$

16. A straight line is drawn through the point $A(1, 2)$ such that its point of intersection with the straight line $x + y = 4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point 'A'. Find the angle which the line makes with the positive direction of X-axis.

(a) $\theta = 15°$ and $75°$ (b) $\theta = 75°$ and $45°$
(c) $\theta = 45°$ and $60°$ (d) $\theta = 60°$ and $30°$

17. In an apartment there are 30 kids. If each kid plays table tennis with other kid, then the total number of matches played by them

(a) ${}^{30}C_2$ (b) ${}^{30}P_2$ (c) ${}^{30}C_2 - 1$ (d) ${}^{30}P_2 - 1$

18. The curve $3y^2 = 2ax^2 + 6b$ passes through the point $P(3, -1)$ and the gradient of the curve at P is "–1", then the values of a and b are

(a) $a = \frac{1}{2}, b = -1$ (b) $a = \frac{-1}{2}, b = 1$
(c) $a = \frac{1}{2}, b = 1$ (d) $a = \frac{-1}{2}, b = -1$

19. $\int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx =$

(a) $x[\log(\log x) + \log x] + c$ (b) $\frac{x}{\log(\log x)} + c$
(c) $x \log(\log x) + c$ (d) $x\left[\log(\log x) - \frac{1}{\log x}\right] + c$

20. Choose the correct option about the matrices given below

$$A = \begin{bmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} & 0 \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{3} & \sin\frac{\pi}{3} \\ 0 & -\sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{bmatrix}$$

$$C = \begin{bmatrix} \cos\frac{\pi}{6} & 0 & \sin\frac{\pi}{6} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{6} & \cos\frac{\pi}{6} & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} & 0 \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) $A^{2020} = I$ (b) $B^{2020} = I$
(c) $D^{2019} = I$ (d) $B^{2022} = I$

21. In a ΔABC, $\angle C = 60°$ and $\angle A = 75°$. If D is a point on AC such that the area of ΔBAD is $\sqrt{3}$ times the area of ΔBCD, then the measure of $\angle ABD$ is

(a) 30° (b) 45° (c) 60° (d) 90°

22. The value of x in $\left(0, \frac{\pi}{2}\right)$ satisfying the equation $(\sin x)(\cos x) = \frac{1}{4}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{12}$

23. The lines $ax^2 + 2hxy + by^2 = 0$ are at right angles if

(a) a + b = 0 (b) $a + b = 1$
(c) $h^2 - ab = 0$ (d) $a = b$

24. In an ellipse, two vertices are (5, 0) and (0, –4). Then the equation of the ellipse is

(a) $\frac{x^2}{16} + \frac{y^2}{25} = 1$ (b) $\frac{x^2}{25} + \frac{y^2}{16} = 1$
(c) $\frac{x^2}{5} + \frac{y^2}{4} =$ (d) $x^2 + y^2 = 41$

25. Find the value of 'k', if it is given that

$$\int_0^{b-c} f(x+c)\,dx = k\int_c^b f(x)\,dx$$

(a) 1 (b) 2 (c) 0 (d) – 2

26. Let u and v be two non-zero vectors. Then the magnitude of the cross product $u \times v$ is always

(a) $< |u|\,|v|$ (b) $= |u|\,|v|$ (c) $> |u|\,|v|$ (d) $= 0$

27. The differential equation of the family of all straight lines passing through the origin is

(a) $x = y\dfrac{dy}{dx}$ (b) $\dfrac{dy}{dx} = 0$

(c) $y = x\dfrac{dy}{dx}$ (d) $\dfrac{d^2y}{dx^2} = \dfrac{y}{x}$

28. Let the complex numbers α and $\left(\dfrac{1}{\bar{\alpha}}\right)$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha| =$

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{\sqrt{7}}$ (d) $\dfrac{1}{3}$

29. $\int_0^{\pi/2} e^{\sin x} \cdot \cos x \, dx =$

(a) $1 - e$ (b) $1 + e$ (c) $e - 1$ (d) e

30. Suppose X has the following probability mass function $P(X = 0) = 0.2$, $P(X = 1) = 0.5$, $P(X = 2) = 0.3$. What $E[X^2] = ?$

(a) 2.89 (b) 1.70 (c) 1.10 (d) 1.21

31. $\int \sqrt{x-1}\left(x\sqrt{x+1}\right)^{-1} dx =$

(a) $\ln\left|x + \sqrt{x^2 - 1}\right| - \sec^{-1}(x) + c$

(b) $\ln\left|x - \sqrt{x^2 - 1}\right| - \tan^{-1}(x) + c$

(c) $\ln\left|x + \sqrt{x^2 - 1}\right| + \sec^{-1}(x) + c$

(d) $\ln\left|x + \sqrt{x^2 - 1}\right| - \tan^{-1}(x) + c$

32. Let ABC be a triangle. Let $u = \mathbf{AB}$ and $v = \mathbf{AC}$. If D is a middle point of BC, then $\mathbf{AD} =$

(a) $\dfrac{u-v}{2}$ (b) $\dfrac{v-u}{2}$ (c) $\dfrac{u+v}{2}$ (d) $u + v$

33. $a^n + b^n$ is divisible by ...., if n is any odd positive integer.

(a) $a - b$ (b) $a^2 - b^2$ (c) $a^2 + b^2$ (d) $a + b$

34. The equation of the smallest circle passing through the intersection of the line $x + y = 1$ and the circle $x^2 + y^2 = 9$ is

(a) $x^2 + y^2 - 9 - (x + y + 1) = 0$

(b) $x^2 + y^2 - 9 - (x + y - 1) = 0$

(c) $x^2 + y^2 - 9 - x + y - 1 = 0$

(d) $x^2 + y^2 - 9 + x + y - 1 = 0$

35. If $A = \begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a+ib & -c-id \\ -c+id & a-ib \end{bmatrix}$. Find $(a^2 + b^2 + c^2 + d^2)$.

(a) 1 (b) -1 (c) i (d) $-i$

36. $\lim_{x \to 0} (1 + 3x)^{2/x} =$

(a) 6 (b) e^6 (c) e^{-6} (d) $e^{\frac{1}{6}}$

37. Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be three vectors such that $\mathbf{u} + \mathbf{v} + \mathbf{w} = 0$, $|\mathbf{u}| = 3$, $|\mathbf{v}| = 5$ and $|\mathbf{w}| = 7$. Then the angle between $\mathbf{u}$ and $\mathbf{v}$ is

(a) 60° (b) 70° (c) 80° (d) 90°

38. A circle is drawn touching the X-axis, with its centre at the point of reflection of (m, n) on the line $y - x = 0$. Then the equation of the circle is

(a) $x^2 + y^2 - 2mx - 2ny + m^2 = 0$

(b) $x^2 + y^2 - 2mx + 2ny + m^2 = 0$

(c) $x^2 + y^2 + 2nx - 2my - n^2 = 0$

(d) $x^2 + y^2 - 2nx - 2my + n^2 = 0$

39. Number of roots common to the equations $x^3 + x^2 - 2x - 2 = 0$ and $x^3 - x^2 - 2x + 2 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 0

40. In a ΔABC, $b : c = \sqrt{3} : \sqrt{2}$ and the angles A, B, C are in AP, then $\angle A =$

(a) 45° (b) 65° (c) 55° (d) 75°

41. In a game, a person wins 5 rupees for getting a number greater than 4 and loses 1 rupee otherwise, when a fair die is thrown thrice. A man participated in the game, but decided to quit as and when he gets a number greater than 4. Then the expected value (mean value) of the amount he wins/loses is

(a) $\dfrac{9}{19}$ (b) $\dfrac{8}{19}$ (c) $\dfrac{19}{9}$ (d) $\dfrac{19}{8}$

42. If α and β are non-real roots of $x^3 - x^2 - x - 2 = 0$, then $\alpha^{2020} + \beta^{2020} + \alpha^{2020} \cdot \beta^{2020} =$

(a) 1 (b) 2020 (c) $1 + \alpha + \beta$ (d) -1

43. If $y = \sqrt{2x + \cos^2\left(2x + \dfrac{\pi}{4}\right)}$, then $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{4}$.

(a) $\dfrac{2\sqrt{2}}{\sqrt{\pi+1}}$ (b) $2\sqrt{\pi+1}$

(c) $\dfrac{2}{\sqrt{\pi+1}}$ (d) $\dfrac{\sqrt{2}}{\sqrt{\pi+1}}$

44. If α is the angle between two vector $\mathbf{p} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{q} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, then $\sin(\alpha) =$

(a) $\sqrt{\dfrac{145}{156}}$ (b) $\sqrt{\dfrac{135}{156}}$ (c) $\sqrt{\dfrac{155}{156}}$ (d) $\sqrt{\dfrac{165}{156}}$

45. A polygon has 54 diagonals. The number of sides of this polygon is ....

(a) 12 (b) 15 (c) 16 (d) 9

46. Equation of the perpendicular bisector of the line joining the points whose position vectors are $\mathbf{a}$ and $\mathbf{b}$ respectively is

(a) $(2r - \mathbf{a} - \mathbf{b})\cdot(\mathbf{a} - \mathbf{b}) = 0$

(b) $(2r - \mathbf{a} - \mathbf{b})\cdot(\mathbf{a} + \mathbf{b}) = 0$

(c) $(2r + \mathbf{a} + \mathbf{b})\cdot(\mathbf{a} - \mathbf{b}) = 0$

(d) $(2r - \mathbf{a} + \mathbf{b})\cdot(\mathbf{a} + \mathbf{b}) = 0$

47. The angle between the straight lines $x^2 + 4xy + y^2 = 0$ is ....

(a) 30° (b) 45° (c) 60° (d) 90°

48. If $f(x) = x^4 - x^3 + 7x^2 + 14$, then what is the value of $f'(5)$?

(a) 594 (b) 549 (c) 954 (d) 495

49. The radical centre of the circles
$x^2 + y^2 - 4x - 6y + 5 = 0$,
$x^2 + y^2 - 2x - 4y - 1 = 0$ and
$x^2 + y^2 - 6x - 2y = 0$ is equal to

(a) $\left(\frac{33}{4}, \frac{20}{3}\right)$ (b) $\left(\frac{33}{4}, \frac{10}{3}\right)$

(c) $\left(\frac{33}{4}, \frac{-20}{3}\right)$ (d) $\left(\frac{33}{4}, \frac{-10}{3}\right)$

50. To which point the origin is to be shifted in order to eliminate first powers of x and y (x^1 and y^1 terms) from the equation $4x^2 + 9y^2 - 8x + 36y + 4 = 0$?

(a) (1, 2) (b) (– 1, 2) (c) (1, – 2) (d) (– 1, – 3)

51. The combined equation of the lines passing through the origin and having slopes $\frac{2}{3}$ and $\frac{-2}{3}$ is

(a) $2x^2 - 9y^2 = 0$ (b) $4x^2 - xy - 9y^2 = 0$

(c) $4x^2 - 9y^2 = 0$ (d) $4x^2 + xy - 9y^2 = 0$

52. Let PQ and RS be tangents at the extremities of a diameter PR of a circle of radius r such that PS and RQ intersect at a point X on the circumference of the circle, then $2r$ equals

(a) $\sqrt{PQ \cdot RS}$ (b) $\frac{PQ + RS}{2}$

(c) $\frac{2PQ \cdot RS}{PQ + RS}$ (d) $\sqrt{\frac{(PQ)^2 + (RS)^2}{2}}$

53. If the line joining the points $(k, 3, 4)$, $(4, 7, 8)$ is parallel to the line joining the points $(-1, -2, 1)$, $(1, 2, l)$, then $k + 1 =$

(a) 2 (b) 5 (c) 7 (d) – 3

54. $\frac{1 - \cos(2x) + \sin(x)}{\sin(2x) + \cos(x)} =$

(a) $\sin(x)$ (b) $\cos(x)$ (c) $\tan(x)$ (d) cosec (x)

55. Find $\sum_{t=1}^{39} f(t)$ if $f : \mathbf{R} \to \mathbf{R}$ is defined as $f(x + y) = f(x) + f(y)$, $x, y \in \mathbf{R}$ and $f(1) = 7$

(a) 5187 (b) 5460 (c) 5740 (d) 5407

56. The value of 'k' for which the function $f(x) = k(x + \sin x) + k$ is increasing, is equal to

(a) $k < 0$ (b) $k > 0$

(c) $k = 0$ (d) Data Insufficient

57. A number is selected at random from the set {1, 2, 3, 4, ..., 1000}, then the probability of getting a number which is a perfect cube or a natural having odd number of divisors is

(a) $\frac{481}{500}$ (b) $\frac{483}{500}$ (c) $\frac{479}{500}$ (d) $\frac{477}{500}$

58. A homogeneous equation of second degree in x and y represents which of the following ?

(a) Two lines

(b) A pair of straight lines through the origin

(c) Only one line through origin

(d) A circle whose centre isn't the origin

59. If $4 \cos x + 3 \sin x = 5$, then find the value of $\tan x =$

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{-3}{4}$ (d) $\frac{-4}{3}$

60. Find $\alpha^4 + \beta^4$ if α, β are the roots of equation $x^2 + x + 1 = 0$.

(a) $\frac{1}{\alpha\beta}$ (b) $\frac{2}{\alpha\beta}$ (c) $\alpha\beta$ (d) $-\alpha\beta$

61. Given that lines $L_1 : y = m_a x$, $L_2 : y = m_b x$ and $L_3 : y = m_c x$ make equal intercepts on the line $x + y = 1$, then

(a) $2(1 + m_a)(1 + m_c) = (1 + m_b)(1 + m_c)$

(b) $2(1 + m_a)(1 + m_c) = (1 + m_b)(2 + m_a + m_c)$

(c) $(1 + m_a)(1 + m_b) = (2 + m_c)(1 + m_a + m_c)$

(d) $(1 + m_a)(1 + m_b) = (1 + m_b)(2 + m_a + m_c)$

62. Find the value of 'k', if

$$\begin{vmatrix} k-2 & 2k-3 & 3k-4 \\ k-4 & 2k-9 & 3k-16 \\ k-8 & 2k-27 & 3k-64 \end{vmatrix} = 0$$

(a) 1 (b) 2 (c) 3 (d) 4

63. The lengths of the sides of the rectangle of greatest area that can be inscribed in the ellipse $x^2 + 4y^2 = 64$ are

(a) $6\sqrt{2}, 4\sqrt{2}$ (b) $8\sqrt{2}, 4\sqrt{2}$

(c) $8\sqrt{2}, 8\sqrt{2}$ (d) $16\sqrt{2}, 4\sqrt{2}$

64. Equation of the line passing through the intersection of the plane $x + 2y + 3z = 4$ and the line $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-1}{-1}$ and parallel to the vector $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}) \times (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$ is

(a) $\frac{x-5}{3} = \frac{y-1}{2} = \frac{z+1}{-7}$ (b) $\frac{x-5}{-3} = \frac{y-1}{-2} = \frac{z-1}{7}$

(c) $\frac{x-5}{-3} = \frac{y-1}{-2} = \frac{z+1}{-7}$ (d) $\frac{x-5}{-3} = \frac{y-1}{2} = \frac{z+1}{7}$

65. The area of an equilateral triangle inscribed in the circle $x^2 + y^2 - 6x + 2y - 28 = 0$ is sq. units

(a) $\frac{27\sqrt{3}}{2}$ (b) $\frac{37\sqrt{3}}{2}$ (c) $\frac{31\sqrt{3}}{2}$ (d) $\frac{57\sqrt{3}}{2}$

66. If $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{b} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, then find the angle between the vectors $2\mathbf{a} + \mathbf{b}$ and $\mathbf{a} + 2\mathbf{b}$.

(a) $\cos^{-1}\left(\frac{36}{\sqrt{42 \times 35}}\right)$ (b) $\cos^{-1}\left(\frac{72}{\sqrt{24 \times 32}}\right)$

(c) $\cos^{-1}\left(\frac{52}{\sqrt{74 \times 65}}\right)$ (d) $\cos^{-1}\left(\frac{24}{\sqrt{18 \times 32}}\right)$

67. For A, B and C, if $A + B + C = 0$, then $\sin(2A) + \sin(2B) + \sin(2C)$ is equal to
(a) $4 \sin(A) \cdot \sin(B) \cdot \sin(C)$
(b) $2 \sin(A) \cdot \sin(B) \cdot \sin(C)$
(c) $-4 \sin(A) \cdot \sin(B) \cdot \sin(C)$
(d) $-2 \sin(A) \cdot \sin(B) \cdot \sin(C)$

68. If $f(x)=\sqrt{x+2\sqrt{2x-4}}+\sqrt{x-2\sqrt{2x-4}}$, then the value of $10 \times f'(102) =$
(a) 1 (b) 2 (c) 102 (d) -1

69. If $(a, 8)$ is a point on the join of $(2, 5)$ and $(4, -1)$ then
(a) $a=\frac{8}{3}$ (b) $a=\frac{3}{8}$ (c) $a=1$ (d) $a=-1$

70. $\int \frac{\cos 7x-\cos 8x}{1+2\cos 5x}\,dx =$
(a) $\sin 2x-\frac{1}{3}\sin 3x+c$ (b) $\frac{1}{2}\sin 2x-\frac{1}{3}\sin 3x+c$
(c) $\frac{1}{2}\sin 2x-\sin 3x+c$ (d) $\frac{1}{3}\sin 2x-\frac{1}{2}\sin 3x+c$

71. In the expansion of $\left(\sqrt[5]{3}+\sqrt[3]{2}\right)^{15}$
(a) Number of rational terms is 3
(b) Sum of all rational terms is 58
(c) Sum of all rational terms is greater than the sum of all irrational terms
(d) Sum of all irrational terms is greater than the sum of all rational terms

72. Evaluate $\int \sin\left(\sqrt{k}\right) dk$ on $(0, \infty)$
(a) $2\left[\cos\left(\sqrt{k}\right)-\sqrt{k}\sin\left(\sqrt{k}\right)\right]+c$
(b) $2\left[\cos\left(\sqrt{k}\right)+\sqrt{k}\sin\left(\sqrt{k}\right)\right]+c$
(c) $2\left[\sqrt{k}\cos\left(\sqrt{k}\right)-\sqrt{k}\sin\left(\sqrt{k}\right)\right]+c$
(d) $2\left[\sin\left(\sqrt{k}\right)-\sqrt{k}\cos\left(\sqrt{k}\right)\right]+c$

73. The binomial distribution whose mean is 9 and whose standard deviation is $\frac{3}{2}$ equal to
(a) $\left(\frac{1}{4}+\frac{3}{4}\right)^{12}$ (b) $\left(\frac{3}{4}+\frac{1}{4}\right)^{12}$
(c) $\left(\frac{1}{2}+\frac{3}{2}\right)^{12}$ (d) $\left(\frac{3}{2}+\frac{1}{2}\right)^{12}$

74. The equation of the normal to the curve $x = a\cosh(t)$, $y = b\sinh(t)$ at any point t is
(a) $ax + by = a^2 + b^2$
(b) $ax\,\text{sech}(t) + by\,\text{cosech}(t) = a^2 + b^2$
(c) $ax\,\text{sech}(t) - by\,\text{cosech}(t) = a^2 - b^2$
(d) $\frac{ax}{\sinh(t)}+\frac{by}{\cosh(t)}=a^2+b^2$

75. Find the function $g(t)$ if $f(t) = 3t - 2$ and $(gof)^{-1}(t) = t - 2$.
(a) $g(t)=\frac{(t-8)}{3}$ (b) $g(t)=\frac{(t+8)}{3}$
(c) $g(t)=\frac{(8-t)}{3}$ (d) $g(t) = 3t - 8$

76. $\frac{d}{dx}\left(e^{\log_e \sqrt{1+\tan^2 x}}\right)=$
(a) $\sec^2(x) \cdot \tan(x)$ (b) $\sec(x) \cdot \tan^2(x)$
(c) $\sec(x) \cdot \tan(x)$ (d) $\tan^2(x)$

77. The equation of the tangent to the parabola $y^2 = 16x$, which is perpendicular to the line $3x - 4y + 5 = 0$ is given by
(a) $4x - 3y + 9 = 0$ (b) $4x + 3y - 9 = 0$
(c) $4x + 3y + 9 = 0$ (d) $4x - 3y - 9 = 0$

78. Find the equation of the normal to the curve $y=\frac{(x-7)}{(x-2)(x-3)}$ at the point where it cuts the X-axis.
(a) $20x + y + 140 = 0$
(b) $x - 20y - 140 = 0$
(c) $x + 20y + 140 = 0$
(d) $20x + y - 140 = 0$

79. The solution of the differential equation $\cos(x + y)dy = dx$ given that $y(0) = 0$ is
(a) $y=\tan\left(\frac{x+y}{2}\right)$ (b) $y=\sin\left(\frac{x+y}{2}\right)$
(c) $y=\tan\left(\frac{y}{2}\right)$ (d) $y=\tan\left(\frac{x}{2}\right)$

80. Which among the following equations has roots which are negative of the roots of the equation $x^3 - x^2 + x - 4 = 0$?
(a) $x^3 - x^2 + x - 4 = 0$ (b) $x^3 + x^2 + x + 4 = 0$
(c) $x^3 - x^2 + x - 4 = 0$ (d) $x^3 - x^2 - x - 4 = 0$

PHYSICS

81. A transformer works on the principle of
(a) self-induction
(b) electrical inertia
(c) magnetic effect of electric current
(d) mutual induction

82. A proton moving with a velocity 2.5×10^7 m/s, enters a magnetic field of intensity 2.5 T making an angle 30° with the magnetic field. The force on the proton is
(a) 3×10^{-12} N (b) 5×10^{-12} N
(c) 6×10^{-12} N (d) 9×10^{-12} N

83. A person of mass $M = 90$ kg standing on a smooth horizontal plane of ice throws a body of mass $m = 10$ kg horizontally on the same surface. If the distance between the person and body after 10 s is 10 m, then the KE of the person (in J is)
(a) 0.55 J (b) 4.5 J (c) 0.90 J (d) 0

84. A horizontal overhead powerline at a height of 5 m from the ground and carries a current of 150 A from East to West. The magnetic field directly below it on ground is
(a) 6×10^{-6} T, towards South
(b) 6×10^{-6} T, towards West
(c) 7×10^{-6} T, towards East
(d) 8×10^{-7} T, towards North

85. The tension in the spring is

(a) zero (b) 10 N (c) 2.5 N (d) 5 N

86. The door of an operating refrigerator is kept open. As a result, the temperature of the room will
(a) remain unchanged
(b) increases
(c) decreases
(d) depends on the contents inside the refrigerator

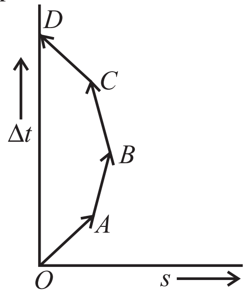
87. If 60% of the kinetic energy of water falling from 210 m high water fall is converted into heat. The raise in temperature of water at the bottom of the falls is nearly (specific heat of water = 4.2×10^3 J kg^{-1}K^{-1})
(a) 0.6°C (b) 0.3°C (c) 1.2 K (d) 2.4 K

88. An AM wave has 1800 W of total power content for 100% modulation the carrier should have power content equal to
(a) 1000 W (b) 1200 W
(c) 1500 W (d) 1600 W

89. For a perfect black body, the absorption coefficient is
(a) $a = 1$ (b) $a < 1$ (c) $a > 1$ (d) $a = 0$

90. In a balanced meter bridge, the segment of wire opposite to a known resistance of 70 Ω is 70 cm. The unknown resistance is
(a) 30 Ω (b) 60 Ω (c) 90 Ω (d) 15 Ω

91. Which of the following options is correct for the object having a straight-line motion represented by the following graph?



(a) The object moves with constantly increasing velocity from O to A and then it moves with constant velocity.
(b) Velocity of the object increases uniformly.
(c) Average velocity is zero.
(d) The graph shown is impossible.

92. If a force F is applied on a body and it moves with a velocity v, the power will be
(a) $F \cdot v$ (b) $\frac{F}{v}$ (c) $\frac{F}{v^2}$ (d) $F \cdot v^2$

93. A ballet dancer suddenly folds her outstretched arms. Her angular velocity
(a) increases (b) decreases
(c) remains the same (d) may increase or decrease

94. Resistance of a tungsten wire at 150°C is 133Ω. Its temperature coefficient of resistance is 0.0045°C^{-1}. The resistance of this wire at 500°C is
(a) 180 Ω (b) 225 Ω (c) 258 Ω (d) 317 Ω

95. At high altitude, a body explodes which is at rest into two equal fragments with one fragment receiving horizontal velocity of 10ms^{-1}. Time taken by the two radius vectors connecting point of explosion to fragments to make 90° is
(a) 10 s (b) 4 s (c) 2 s (d) 1 s

96. A mass m is in rest on an inclined plane of mass M which is further resting on a smooth horizontal plane. Now, if the mass m starts moving under gravity, the position of centre of mass of system will
(a) remain unchanged
(b) change along the horizontal direction
(c) move up in vertical direction
(d) move down in the vertical direction and changes along the horizontal

97. The energy stored in a 50 mH inductor carrying a current of 4 A is
(a) 0.4 J (b) 4.0 J (c) 0.8 J (d) 0.04 J

98. Find the apparent weight of a body of mass 0.1 kg falling with an acceleration of 10 ms^{-2}.
(g = 10 ms^{-2})
(a) 1 kg-wt (b) 2 kg-wt
(c) 0 (d) 0.5 kg-wt

99. The displacement of a body when it covers a distance of C/4 (where, C is circumference along the circumference of the circle of radius r with a uniform speed u is)
(a) r (b) $r\sqrt{2}$ (c) $2r$ (d) $\frac{r}{2}$

100. Two waves of frequency f and amplitude a superimpose with each other. The total intensity is directly proportional to
(a) a (b) $2a$ (c) $2a^2$ (d) $4a^2$

101. A hydrogen like atom has one electron revolving round a stationary nucleus. If the energy required to excite the electron from the 2nd orbital to 3rd orbit is 47.2 eV, find the atomic number of the given atom.
(a) 3 (b) 4 (c) 5 (d) 6

102. On getting reflected at a surface, the intensity of sound is found to be decreased by 20%. If A be the amplitude of the incident sound waves, then the amplitude of reflected sound waves is
(a) $\frac{4}{5}A$ (b) $\frac{2}{\sqrt{5}}A$ (c) $\frac{\sqrt{2}}{5}A$ (d) $\frac{1}{\sqrt{5}}A$

103. A light wave has a frequency of 4×10^{14} Hz and a wavelength of 5×10^{-7} m in a medium. The refractive index of the medium is
(a) 1.5 (b) 1.33 (c) 1.0 (d) 0.66

104. A particle is moving an X-axis has potential energy $U = 2 - 20x + 5x^2$ J along X-axis. The particle is released at $x = -3$. The maximum value of x will be (x is in metre and U is in joules)

(a) 5m (b) 3m (c) 7m (d) 8m

105. The average kinetic energy of H_2 molecules at 300 K is E. At the same temperature the average kinetic energy of O_2 molecule is

(a) E (b) $\frac{E}{4}$ (c) $\frac{E}{16}$ (d) 16 E

106. The escape velocity for a planet whose radius is 1.7×10^6 m and acceleration due to gravity is 1.7 ms^{-2} is

(a) 1.7 kms^{-1} (b) 2.89 kms^{-1}
(c) $1.7\sqrt{2}$ kms^{-1} (d) 3.4 kms^{-1}

107. At what height from surface of earth the value of acceleration due to gravity will fall to half that on the surface of the earth?

(a) 2625 m (b) 2625 km
(c) 2526 m (d) 2526 km

108. Choose the physical quantity pair with the same dimensions.

(a) Angular momentum and work
(b) Work and torque
(c) Potential energy and linear momentum
(d) Kinetic energy and velocity

109. Practically ozone layer absorbs radiations of wavelength

(a) less than 3×10^{-7} m
(b) greater than 3×10^{-7} m
(c) equal to 3×10^{-7} m
(d) All of the above

110. Simultaneously, from the top of a tower, when ball-1 is thrown horizontally and ball-2 is just dropped, in the absence of air resistance which among the following options is correct?

(a) Ball-1 reaches the ground first.
(b) Ball-2 reaches the ground first.
(c) Both will reach the ground simultaneously.
(d) Either ball-1 or ball-2 reach the ground first depending on which ever is heavier.

111. In a photodiode, the value of the emf produced by a monochromatic light beam is directly proportional to

(a) the barrier potential at p-n junction
(b) the intensity of light falling on the photodiode
(c) the frequency of light falling on the photodiode
(d) the voltage applied at the p-n junction

112. The coefficient of mutual inductance between the primary and the secondary coil of a transformer is 0.2 H. When the current in the primary changes by 5 As^{-1}, then the induced emf in the secondary will be

(a) 5 V (b) 1 V (c) 25 V (d) 10 V

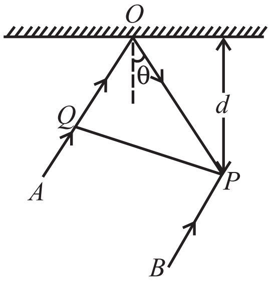
113. A magnet of magnetic moment M is rotated through 360° in a magnetic field H, the work done will be

(a) MH (b) $2MH$ (c) $2\pi MH$ (d) 0

114. Three charges $4q$, Q and q are placed at positions 0, $\frac{l}{2}$ and l respectively along a straight line. If the resultant force on q is zero, then Q is equal to

(a) $-q$ (b) $-2q$ (c) $-\frac{q}{2}$ (d) $4qt$

115. PQ represents a wavefront and AO and BP, the corresponding two rays. Find the condition on O for constructive interference at P between ray BP and reflected ray OP.



(a) $\cos\theta = \frac{3\lambda}{2d}$ (b) $\cos\theta = \frac{\lambda}{4d}$
(c) $\sec\theta - \cos\theta = \frac{\lambda}{d}$ (d) $\sec\theta - \cos\theta = \frac{4\lambda}{d}$

116. The radius of the bore of a capillary tube is r and the angle of contact of the liquid is θ. When the tube is dipped in the liquid, the radius of curvature of the meniscus of liquid rising in the tube is

(a) $r\sin\theta$ (b) $\frac{r}{\sin\theta}$ (c) $r\cos\theta$ (d) $\frac{r}{\cos\theta}$

117. There is no change in internal energy of an ideal gas when it undergoes

(a) isothermal expansion (b) adiabatic expansion
(c) free expansion (d) isobaric expansion

118. When two bodies collide elastically, then

(a) kinetic energy of the system along is conserved
(b) only momentum is conserved
(c) both kinetic energy and momentum is conserved
(d) Neither kinetic energy nor momentum is conserved

119. Which of the following parameters does not characterise the thermodynamic state of matter?

(a) Temperature (b) Pressure
(c) Work (d) Volume

120. The retarding potential necessary to stop the emission of photoelectrons, when a target material of work function 1.24 eV is irradiated with light of wavelength 4.36×10^{-7}m is

(a) 4.08 eV (b) 2.84 eV
(c) 1.60 eV (d) 0.36 eV

CHEMISTRY

121. Calculate the coordination number of Na^+ in NaCl crystal, given radius of Na^+ and Cl^- are 95 pm and 181 pm respectively.

(a) 8 (b) 4 (c) 6 (d) 12

122. The yellow colour of chromate ion and orange colour of dichromate ion is due to
(a) *d-d* transition only
(b) charge transfer only
(c) both *d-d* transition and charge transfer
(d) *s-d* transitions only

123. Gas '*X*' is obtained on heating $KClO_3$ with catalyst MnO_2. This gas (in excess) on reaction with white phosphorus forms an acidic oxide '*Y*'. '*Y*' on dissolving in water forms a compound '*Z*'. Identify *X, Y* and *Z*.
(a) O_2, P_2O_5, H_3PO_4 (b) O_3, P_2O_5, H_3PO_4
(c) O_2, P_2O_3, H_3PO_3 (d) O_2, P_2O_5, H_3PO_3

124. The 'dry ice' is ....
(a) dry CO_2 gas (b) solid CO_2
(c) dry SO_2 gas (d) solid NH_3

125. Which among the following is used in detergent
(a) Sodium acetate (b) Sodium stearate
(c) Calcium stearate (d) Sodium lauryl sulphate

126. Which of the following complex ion shows geometrical isomerism?
(a) $[Cr(H_2O)_4Cl_2]^+$ (b) $[Pt(NH_3)_3Cl]^+$
(c) $[Co(NH_3)_6]^{3+}$ (d) $[Co(CN)_5(NC)]^{3-}$

127. Which of the following molecules can be represented as AB_2E_2, where *A*-central atom, *B*-bond pairs of electrons, *E*-lone pairs of electrons?
(a) SO_2 (b) H_2O_2 (c) H_2O (d) XeF_2

128. Which among the following is used as food preservative?
(a) $C_6H_5CH_2ONa$ (b) $C_6H_5COOONa$
(c) C_6H_5COONa (d) $C_6H_5CH{=}CHCOONa$

129. 0.12 g of an organic compound '*X*' containing phosphorus on reacting with magnesia mixture gave 0.22 g of magnesium pyrophosphate ($Mg_2P_2O_7$). The percentage of phosphorus in the compound '*X*' is ....
(a) 45.30% (b) 28.70% (c) 64.25% (d) 51.20%

130. Which of the following statements is not correct?
1. Reduction of alumina to give aluminium by magnesium is thermodynamically feasible.
2. The point of intersection of AlO_3 and MgO curves in Ellingham diagram is below 1665 K.
3. Use of magnesium as reducing agent in metallurgy of aluminium is economical.
4. Ellingham diagram represents the graphical plot of Gibbs energy *vs* temperature for the formation of the oxides of common metals and reducing agents.

(a) 1 (b) 2 (c) 3 (d) 4

131. Number of different bonds present in P_4O_{10} is ....
(a) 8 P — O bonds and 4 P = O bonds
(b) 12 P — O bonds and 3 P = O bonds
(c) 12 P — O bonds and 4 P = O bonds
(d) 8 P — O bonds and 3 P = O bonds

132. The order of boiling points of the following compounds is
1. $(CH_3)_3\ddot{N}$ 2. $CH_3CH_2CH_2\ddot{N}H_2$
3. $CH_3CH_2\ddot{N}HCH_3$

(a) $1 > 3 > 2$ (b) $3 > 1 > 2$
(c) $2 > 1 > 3$ (d) $2 > 3 > 1$

133. The rate of a reaction doubles, when the temperature is changed from 300 K to 310 K. Activation energy of the change is ....
($R = 8.314\ JK^{-1}\ mol^{-1}$, log 2 = 0.301)
(a) 53.6 kJ mol^{-1} (b) 48.6 kJ mol^{-1}
(c) 58.5 kJ mol^{-1} (d) 60.5 kJ mol^{-1}

134. Which of the following statements is correct about photoelectric effect?
1. The number of electrons ejected from metal surface is inversely proportional to intensity of light.
2. Below threshold frequency, photoelectric effect can be observed.
3. At higher frequency than threshold frequency, the ejected electrons have certain kinetic energy.
4. At higher frequency than threshold frequency, the electron is still on the metal surface.

(a) 1 (b) 2 (c) 3 (d) 4

135. The oxidation number of an element in a compound is evaluated on the basis of certain rules. Which of the following rules is not correct?
1. The oxidation number of hydrogen is always +1.
2. The algebraic sum of all the oxidation numbers of all elements in a compound is zero.
3. An element in the free or the uncombined state bears oxidation number zero.
4. In all its compounds, the oxidation number of fluorine is – 1.

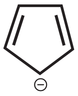
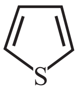
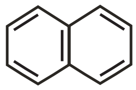
(a) 1 (b) 2 (c) 3 (d) 4

136. Calculate the amount of CO_2 gas produced, when 32 g moles of CH_4 is burned with sufficient amount of oxygen. (Given, atomic weights of C = 12, O = 16, H = 1)
(a) 132 g (b) 44 g (c) 88 g (d) 176 g

137. **Assertion:** Standard boiling point of a liquid is slightly higher than the normal boiling point.
Reason: 1 bar pressure is slightly less than 1 atm pressure.
(a) Assertion and Reason are correct statements and Reason is the correct explanation for Assertion.
(b) Assertion and Reason are correct statement and Reason is not the explanation for Assertion.
(c) Assertion is correct, Reason is incorrect.
(d) Assertion is incorrect, Reason is correct.

138. Which transition metal oxide among the following has electrical conductivity similar to that of copper?
(a) MnO (b) FeO (c) RgO_3 (d) TiO_2

139. The non-aromatic compound among the following is

(a) 
(b) 
(c) 
(d) 

140. Which of the following statements is correct about "shielding effect"?

1. It is effective, when the orbitals in the inner shells are completely filled.'
2. Inner shells do not show any effect.
3. With increase of shielding down the group, ionisation energy increases.
4. With increasing nuclear charge, the shielding effect increases.

(a) 1 (b) 2 (c) 3 (d) 4

141. Which of the following compounds are formed, when boron trichloride is treated with water?

(a) $H_3BO_3 + HCl$ (b) $B_2H_6 + HCl$
(c) $B_2O_3 + HCl$ (d) $HBO_3 + HCl$

142. "No two electrons in an atom can have the same set of all four quantum numbers". This principle of called

(a) Zeeman effect (b) Pauli's exclusion principle
(c) Stark effect (d) Heisenberg principle

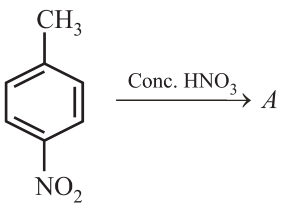
143. Intermolecular forces in nylon-6, 6 are

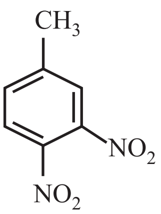
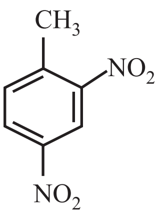
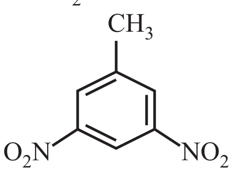
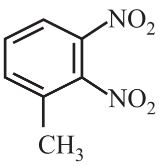
(a) dipole-dipole interactions
(b) hydrogen bonding
(c) van der Waal's forces
(d) ionic bonds

144. The main constituent of vinegar is ....

(a) formic acid (b) oxalic acid
(c) nitric acid (d) acetic acid

145. What will be the product *A* in the reaction given below?



(a) 
(b) 
(c) 
(d) 

146. Oxidation number of Cr in $[Cr(CO)_6]$ is

(a) + 6 (b) – 6 (c) + 3 (d) 0

147. Which of the following is not a semi-synthetic polymer?

(a) *Cis*-polyisoprene (b) Cellulose nitrate
(c) Cellulose acetate (d) Vulcanised rubber

148. The types of hybridisation on the five carbon atoms from left to right in pent-1-en-4-yne are

(a) sp^2, sp^2, sp^3, sp, sp (b) sp, sp, sp^3, sp^2, sp^2
(c) sp^2, sp^3, sp, sp, sp^3 (d) sp^2, sp, sp^3, sp, sp^2

149. Which of the products indicates the presence of sulphur atom in an organic compound, in qualitative elemental analysis?

(a) Fe_2SO_4 (b) ZnS (c) MgS (d) PbS

150. The uncertainty in the position of an electron moving with velocity of 3×10^4 cm/s is (given mass of electron $= 9.1 \times 10^{-28}$ g, uncertainty in velocity = 0.02%)

(a) 1.8×10^{-3} cm (b) 9×10^{-3} cm
(c) 3.8×10^{-2} cm (d) 1.8×10^{-4} cm

151. When SbF_5 reacts with XeF_4 to form an adduct, the shapes of cation and anion in the adduct respectively are

(a) square planar, trigonal bipyramidal
(b) T-shaped, octahedral
(c) square pyramidal, octahedral
(d) plane triangular, trigonal bipyramidal

152. Which of the following is not a characteristic of chemisorption?

(a) Adsorption is highly specific
(b) Heat of adsorption is around 400 kJ mol^{-1}
(c) The process is irreversible
(d) It forms multimolecular layer

153. A 200 W, 100 V bulb is connected in series with an electrolytic cell. If an aqueous solution of an Sn-salt is electrolysed for 5 hrs, 11.1 g of Sn gets deposited. The chemical formula of the compound is .... (Given atomics weight of Sn is 118.7 g mol^{-1}).

(a) SnO (b) $SnCl_2$ (c) $SnCl_4$ (d) SnO_2

154. An example for hydrophobic sol among the following is ....

(a) gum solution
(b) arsenic sulphide solution
(c) starch solution
(d) protein solution

155. Strongest conjugate base among the following is

(a) Cl^- (b) F^- (c) Br^- (d) I^-

156. Match the items of List-I with those of List-II and choose the correct option given below.

	List-I		List-II
A.	Calcination	1.	Galena
B.	Roasting	2.	Magnetite
C.	Magnetic separation	3.	Pyrometallurgy
D.	Carbon reduction of metal oxide	4.	Malachite

Codes

	A	B	C	D		A	B	C	D
(a)	4	1	3	2	(b)	4	1	2	3
(c)	2	4	3	2	(d)	3	2	1	4

157. For a reversible reaction, if the concentration of the reactants is reduced to half, the equilibrium constant will be ....

(a) doubled (b) halved
(c) reduced to one-fourth (d) remains same

158. The temperature at which 4 moles of a gas occupy 5 dm^3 volume at 3.32 bar pressure is

(a) 50 K (b) 50°C (c) 27°C (d) 100 K

159. Which of the following statements is incorrect?

1. The property of liquid drops to have minimum surface area is called surface tension.
2. Surface tension of liquids decreases with rise in temperature.
3. The SI unit of surface tension in Nm^{-1}.
4. Magnitude of surface tension is less, when there is strong attractive force between the molecules.

(a) 1 (b) 2 (c) 3 (d) 4

160. What is the hybridisation of Be in BeF_2 molecule?

(a) dsp^2 (b) sp^2d (c) sp (d) sp^3

ANSWER KEY

1	(b)	**17**	(a)	**33**	(d)	**49**	(Bonus)	**65**	(d)	**81**	(d)	**97**	(a)	**113**	(d)	**129**	(d)	**145**	(b)
2	(d)	**18**	(a)	**34**	(b)	**50**	(c)	**66**	(Bonus)	**82**	(b)	**98**	(c)	**114**	(a)	**130**	(c)	**146**	(d)
3	(a)	**19**	(d)	**35**	(a)	**51**	(c)	**67**	(c)	**83**	(a)	**99**	(b)	**115**	(b)	**131**	(c)	**147**	(a)
4	(a)	**20**	(d)	**36**	(b)	**52**	(a)	**68**	(a)	**84**	(a)	**100**	(d)	**116**	(d)	**132**	(d)	**148**	(a)
5	(a)	**21**	(a)	**37**	(a)	**53**	(c)	**69**	(c)	**85**	(b)	**101**	(c)	**117**	(c)	**133**	(a)	**149**	(d)
6	(d)	**22**	(d)	**38**	(d)	**54**	(c)	**70**	(b)	**86**	(b)	**102**	(b)	**118**	(c)	**134**	(c)	**150**	(b)
7	(c)	**23**	(a)	**39**	(b)	**55**	(b)	**71**	(d)	**87**	(b)	**103**	(a)	**119**	(c)	**135**	(a)	**151**	(b)
8	(d)	**24**	(b)	**40**	(d)	**56**	(b)	**72**	(d)	**88**	(b)	**104**	(c)	**120**	(c)	**136**	(c)	**152**	(d)
9	(c)	**25**	(a)	**41**	(c)	**57**	(Bonus)	**73**	(b)	**89**	(a)	**105**	(a)	**121**	(c)	**137**	(d)	**153**	(c)
10	(b)	**26**	(a)	**42**	(c)	**58**	(b)	**74**	(b)	**90**	(a)	**106**	(c)	**122**	(b)	**138**	(c)	**154**	(b)
11	(b)	**27**	(c)	**43**	(a)	**59**	(a)	**75**	(b)	**91**	(c)	**107**	(b)	**123**	(a)	**139**	(b)	**155**	(b)
12	(d)	**28**	(c)	**44**	(c)	**60**	(d)	**76**	(c)	**92**	(a)	**108**	(b)	**124**	(b)	**140**	(a)	**156**	(b)
13	(d)	**29**	(c)	**45**	(a)	**61**	(b)	**77**	(c)	**93**	(a)	**109**	(a)	**125**	(d)	**141**	(a)	**157**	(d)
14	(d)	**30**	(b)	**46**	(a)	**62**	(d)	**78**	(d)	**94**	(c)	**110**	(c)	**126**	(a)	**142**	(b)	**158**	(a)
15	(a)	**31**	(a)	**47**	(c)	**63**	(b)	**79**	(a)	**95**	(c)	**111**	(b)	**127**	(c)	**143**	(b)	**159**	(d)
16	(a)	**32**	(c)	**48**	(d)	**64**	(c)	**80**	(b)	**96**	(c)	**112**	(b)	**128**	(c)	**144**	(d)	**160**	(c)

Hints & Solutions

MATHEMATICS

1. **(b)** Given, $\sec x = \cos h\,(\theta)$

$$\tan h^2\left(\frac{\theta}{2}\right)=\frac{\cos h(\theta)-1}{\cos h(\theta)+1}=\frac{\sec x-1}{\sec x+1}$$

$$\tan h^2\left(\frac{\theta}{2}\right)=\frac{1-\cos x}{1+\cos x}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}=\tan^2\frac{x}{2}$$

2. **(d)** Let the point of tangency

give, parabola $y^2 = 12 \times (x_1, y_1)$

Equation of tangent

at point (x_1, y_1) is $yy_1 = 6\,(x + (x_1))$

Comparing the above equation of tangent with the given tangent $x-\sqrt{3y}+9=0,$ we get,

$$\frac{6}{1}=\frac{-y_1}{-\sqrt{3}}=\frac{6x_1}{9}\Rightarrow(x_1,y_1)=(9,6\sqrt{3})$$

Therefore, point of contact is $(9,6\sqrt{3})$

3. **(a)** Given differential equation $\frac{d^2y}{dx^2}=0$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)=0\ \Rightarrow\ d\left(\frac{dy}{dx}\right)=0$$

Integrating both the sides, we get

$$\Rightarrow\frac{dy}{dx}=a\ \Rightarrow dy=adx$$

Again integrating, we get $y = ax + b$.

It represents a equation of straight line.

4. **(a)** Let events, E_1 : die A is used when head is appeared

E_2 : die B is used when tail is appeared

R : red face appears on the die.

$$P\,(E_1/R)=\frac{32}{33}$$

$$P(E_1)=\frac{1}{2},P(E_2)=\frac{1}{2},$$

$$p(R\,|\,E_1)=\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\ldots n\text{ times}=\left(\frac{2}{3}\right)^n$$

Similarly, $P(R\,|\,E_2)=\left(\frac{1}{3}\right)^n$

According to the Baye's theorem,

$$P(E_1\,|\,R)=\frac{P(E_1)P(R\,|\,E_1)}{P(E_1)P(R\,|\,E_1)+P(E_2)P(R\,|\,E_2)}$$

$$\Rightarrow\frac{32}{33}=\frac{\frac{1}{2}\times\left(\frac{2}{3}\right)^n}{\frac{1}{2}\times\left(\frac{2}{3}\right)^n+\frac{1}{2}\times\left(\frac{1}{3}\right)^n}\Rightarrow\frac{32}{33}=\frac{2^n}{1+2^n}\Rightarrow n=5$$

5. **(a)** Let $I=\int_0^{\pi/4}\frac{dx}{\cos^3 x\sqrt{2\sin 2x}}=\int_0^{\pi/4}\frac{dx}{\cos^3 x\sqrt{4\sin x\cos x}}$

$$=\int_0^{\pi/4}\frac{dx}{2\cos^4 x\sqrt{\tan x}}$$

$$=\int_0^{\pi/4}\frac{\sec^4 x}{2\sqrt{\tan x}}dx=\frac{1}{2}\int_0^{\pi/4}\frac{\sec^2 x(1+\tan^2 x)}{\sqrt{\tan x}}dx$$

Put $\tan x = t^2$, $\sec^2 x\,dx = 2t\,dt$

Limits : $x=0\Rightarrow t=0$ and $x=\frac{\pi}{4}\Rightarrow t=1$

So, $I=\int_0^1\frac{(1+t^4)2t\,dt}{2\sqrt{t^2}}=\int_0^1(1+t^4)\,dt$

$$=\left[t+\frac{t^5}{5}\right]_0^1=1+\frac{1}{5}=\frac{6}{5}$$

6. **(d)** $$\lim_{n\to\infty}\frac{n!}{(n+1)!-n!}=\lim_{n\to\infty}\frac{n!}{(n+1)n!-n!}$$

$$=\lim_{n\to\infty}\frac{1}{(n+1)-1}=\lim_{n\to\infty}\frac{1}{n}=0$$

7. **(c)** Let $\text{I}=\int_{-1}^{2}|\,x\,|dx$

$-1<x<0\Rightarrow|x|=-x$

$0<x<2\Rightarrow|x|=x$

$$=\int_{-1}^{0}(-x)dx+\int_0^2(x)\ dx$$

$$=\left[\frac{-x^2}{2}\right]_{-1}^0+\left[\frac{x^2}{2}\right]_0^2=\left(0+\frac{1}{2}\right)+\left(\frac{4}{2}-0\right)=\frac{5}{2}$$

8. **(d)** If the direction cosines of two lines are l_1, m_1, n_1 and l_2, m_2, n_2.

$l_1l_2 + m_1m_2 + n_1n_2 = 0.$

and, these two lines are perpendicular.

9. **(c)** Let the roots of the quadratic equation are α and β then according to the given data,

$\alpha + \beta = 1$ and $\alpha^2 + \beta^2 = 13$

$$\alpha\beta = \frac{1}{2}[(\alpha+\beta)^2 - (\alpha^2+\beta^2)] = \frac{1}{2}[1-13] = -6$$

Hence, equation of required quadratic is

$x^2 - (\alpha + \beta)\, x + \alpha\beta = 0 \Rightarrow x^2 - x - 6 = 0$

10. **(b)** Let L_1 be the line of diagonal & given as $8x - 15y = 0$

$\therefore L_1 : 8x - 5y = 0$

Equation of line passes through point (1, 2) and inclined with an angle 45° with respect to line $8x - 15y = 0$, because sides of a square inclined with diagonal at 45°, is

$$y - y_1 = \frac{m_1 \pm \tan\theta}{1 \mp m_1 \tan\theta}(x - x_1)$$

$$y - 2 = \frac{8/15 \pm 1}{1 \mp \left(\frac{8}{15} \times 1\right)}(x-1)$$

$$y - 2 = \frac{23}{7}(x-1) \text{ and } y - 2 = \frac{-7}{23}(x-1)$$

$\therefore 23x - 7y - 9 = 0$ and $7x + 23y - 53 = 0$

11. **(b)** In a ΔABC, Given that

$$\frac{\tan A}{2} = \frac{\tan B}{3} = \frac{\tan C}{4} = k \text{ (say)}$$

$\therefore \tan A = 2k$, $\tan B = 3k$ and $\tan C = 4k$

Since in ΔABC, we know that

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$$9k = 24k^3 \Rightarrow k^2 = \frac{3}{8}$$

Now, $\sec^2 A + \sec^2 B + \sec^2 C$

$= 1 + \tan^2 A + 1 + \tan^2 B + 1 + \tan^2 C$

$= 3 + 4k^2 + 9k^2 + 16k^2$

$= 3 + 29k^2$

$= 3 + \tan^2 A + \tan^2 B + \tan^2 C$

$$= 3 + \frac{3}{8}(29) = \frac{24+87}{8} = \frac{111}{8}$$

12. **(d)** We have,

$$\frac{x^2+x+1}{x^2+2x+1} = A + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$\frac{x^2+x+1}{(x+1)^2} = A + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$x^2 + x + 1 = A(x + 1)^2 + B(x + 1) + C$

$\Rightarrow x^2 + x + 1 = A(x^2 + 2x + 1) + B(x + 1) + C$

$\Rightarrow x^2 + x + 1 = Ax^2 + (2A + B)\, x + (A + B + C)$

On comparing the coefficient of all the terms.

We get $A = 1$, $2A + B = 1$ and $A + B + C = 1$

$\therefore B = -1$, and $C = 1$

$\therefore A - B = 2 = 2C$.

13. **(d)** 1. The skew-symmetric matrix of order 5 × 5, has rank less than 5 because determinant of an odd ordered skew symmetric matrix is zero.

2. If P is non-zero column matrix and Q is a non-zero row matrix, then PQ is a matrix of order 1 × 1, so rank of matrix $PQ = 1$

3. Since, the

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{vmatrix} = 1(21-24) - 2(14-20) + 3(12-15)$$

$= -3 + 12 - 9 = 0$ and there is no cofactor of any elements is zero, so rank = 2.

4. If the lines $a_r x + b_r y + c_r = 0$ $(r = 1, 2, 3)$ are distinct and intersect at a point, then matrix

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \Rightarrow |A| = 0 \text{, so rank of } A \neq 3.$$

14. **(d)** According to the binomial theorem.

$(1 + x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + ... + C_n x^n$

Put $x = 1$ and $x = -1$ respectively, we get

$2^n = C_0 + C_1 + C_2 + C_3 + ... + C_n$

and $0 = C_0 - C_1 + C_2 - C_3 + ... + (-1)^n C_n$

If n is even, then

$C_0 + C_1 + C_2 + C_3 + ... + C_n = 2^n$ ...(i)

and $C_0 - C_1 + C_2 - C_3 + ... + C_n = 0$ ...(ii)

By adding and subtracting eqs. (i) and (ii), we get

$C_0 + C_2 + C_4 + ... + C_n = 2^{n-1}$

and $C_1 + C_3 + C_5 + ... + C_{n-1} = 2^{n-1}$

15. **(a)** Given word "INTERMEDIATE"vowels are I, E, E, I, A, E and consonants are N, T, R, M, D, T

Now number of ways to arrange consonants first is $\frac{6!}{2!}$

Now number of ways to arrange six vowels in the seven favourable positions are $\frac{{}^7P_6}{3!2!} = \frac{7!}{3!2!}$

Therefore, total number of arrangements $= \frac{6!}{2!} \times \frac{7!}{2!3!}$

16. **(a)** Let θ be the angle which the line makes with positive direction of x-axis and this line passes through point $A(1, 2)$, so equation of line is

$$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = \pm\frac{\sqrt{6}}{3}$$

$\therefore$ General point on the line is

$$P\left(1\pm\frac{\sqrt{6}}{3}\cos\theta, 2\pm\frac{\sqrt{6}}{3}\sin\theta\right)$$

Let this point P is on the straight line $x + y = 4$, so

$$1\pm\frac{\sqrt{6}}{3}\cos\theta+2\pm\frac{\sqrt{6}}{3}\sin\theta=4$$

$$3\pm\frac{\sqrt{6}}{3}(\sin\theta+\cos\theta)=4$$

$$\Rightarrow \pm\frac{\sqrt{6}}{3}(\sin\theta+\cos\theta)=1$$

On squaring both sides, we get

$$\frac{6}{9}\left(\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta\right)=1$$

$$1+\sin 2\theta=\frac{3}{2}$$

$$\sin 2\theta=\frac{1}{2}\Rightarrow 2\theta=30°\text{and } 150°\Rightarrow\theta=15°\text{ and } 75°$$

17. **(a)** Total number of kids $= 30$

The number of required matches played is same as the number ways to choose two kids among 30 kids.

So number of matches = $^{30}C_2$

18. **(a)** Given curve,

$3y^2 = 2ax^2 + 6b$ ...(i)

This curve passes through point P(3, –1),

$3(-1)^2 = 2a(3)^2 + 66$

$3 = 18a + 6b$

$6a + 2b = 1$ ...(ii)

On differentiating the eq. (i), we get

$$6y\frac{dy}{dx}=4ax$$

$\Rightarrow 6\times(-1)\times(-1) = 4a(3)$ $\quad\left(\because \frac{dy}{dx}=-1\right)$

$\Rightarrow a = 1/2$

Put the value of 'a' in eq. (ii), we get $b = -1$

19. **(d)** Let, $I=\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$

Put, $\log x = tx = e^t \Rightarrow dx = e^t\,dt$

$$=\int e^t\left[\left(\log t+\frac{1}{t}\right)-\left(\frac{1}{t}-\frac{1}{t^2}\right)\right]dt$$

$$=\int e^t\left(\log t+\frac{1}{t}\right)dt-\int e^t\left(\frac{1}{t}-\frac{1}{t^2}\right)dt$$

We know that,

$$\int e^x[f(x)+f'(x)]dx=\int e^x f(x)+C,$$

So, $I=e^t\log t-\frac{e^t}{t}+C$

Now, put $t = \log x$

$$=x\log(\log x)-\frac{x}{\log x}+C$$

$$I=x\left[\log(\log x)-\frac{1}{\log x}\right]+C$$

20. **(d)** Given matrix

$$A=\begin{bmatrix}\cos\frac{\pi}{4} & \sin\frac{\pi}{4} & 0\\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$A^2 = A.A$

$$\Rightarrow A^2=\begin{bmatrix}\cos\frac{\pi}{2} & \sin\frac{\pi}{2} & 0\\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0\\ 0 & 0 & 1\end{bmatrix}=0$$

$$\therefore A^2=\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\therefore A^4 = A^2.A^2$

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}=D^2\Rightarrow A^8=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=0$$

4 = 1

$\therefore A^{2020} = (A^{505})^4 = ((A^8)^{63}.A)^4 = (I^{63}.A)^4 = A^4 \neq 1$

Similarly, $D^4 = I$, so $D^{2019} \neq I$

$$\text{Now, } B=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\frac{\pi}{3} & \sin\frac{\pi}{3}\\ 0 & -\sin\frac{\pi}{3} & \cos\frac{\pi}{3}\end{bmatrix}$$

$B^2 = B.B$

$$\Rightarrow B^4=B^2.B^2=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\frac{4\pi}{3} & \sin\frac{4\pi}{3}\\ 0 & -\sin\frac{4\pi}{3} & \cos\frac{4\pi}{3}\end{bmatrix}\neq 1$$

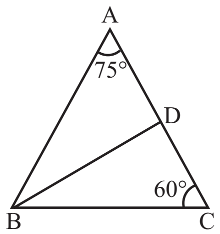
$$\Rightarrow B^6 = B^2.B^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{6\pi}{3} & \sin\frac{6\pi}{3} \\ 0 & -\sin\frac{6\pi}{3} & \cos\frac{6\pi}{3} \end{bmatrix} = I$$

$$\therefore B^{2020} \neq I \text{ but } B^{2022} = (B^6)^{337} = I^{337} = I$$

21. **(a)** In a ΔABC, $\angle A = 75°$ and $\angle C = 60°$

and $\Delta BAD = \sqrt{3}(\Delta BCD) \Rightarrow (AD) = \sqrt{3}(CD)$

$$\Rightarrow \frac{AD}{CD} = \sqrt{3}$$



As $\angle B = 45°$, Now let $\angle ABD = \alpha$, then

$\angle DBC = 45° - \alpha$

Now, according to sine law

In ΔABD,

$$\frac{\sin\alpha}{AD} = \frac{\sin 75°}{BD}$$

and in ΔDBC, $\frac{\sin(45° - \alpha)}{CD} = \frac{\sin 60°}{BD}$

$$\therefore \frac{(CD)\sin\alpha}{(AD)\sin(45° - \alpha)} = \frac{\sin 75°}{\sin 60°} = \frac{\sin(30° + 45°)}{\sin 60°}$$

$$\Rightarrow \frac{1}{\sqrt{3}} \times \frac{\sin\alpha}{\sin(45° - \alpha)} = \frac{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow \frac{\sin\alpha}{\sin(45° - \alpha)} = \frac{\sqrt{3}+1}{\sqrt{2}} \Rightarrow \frac{\sin(45° - \alpha)}{\sin\alpha} = \frac{\sqrt{3}-1}{\sqrt{2}}$$

$$\frac{\sin 45\cos\alpha - \cos 45\sin\alpha}{\sin\alpha} = \frac{\sqrt{3}-1}{\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cot\alpha - \frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{\sqrt{2}} \quad \Rightarrow \cot\alpha = \sqrt{3} \Rightarrow \alpha = 30°$$

22. **(d)** Given that the equation $\sin x\cos x = \frac{1}{4}$

$$2\sin x\cos x = \frac{1}{2}$$

$$\Rightarrow \sin 2x = \frac{1}{2} \Rightarrow 2x = \frac{\pi}{6}, \frac{5\pi}{6}$$

or $x = \frac{\pi}{12}, \frac{5\pi}{12} \in \left(0, \frac{\pi}{2}\right)$

23. **(a)** Let θ be the angle between lines represented by equation $ax^2 + 2h\,xy + by^2 = 0$, is

$$\theta = \tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{|a+b|}\right)$$

If lines are perpendicular then $\theta = 90°$

$$\tan 90° = \frac{1}{0} = \frac{2\sqrt{h^2 - ab}}{|a+b|}$$

24. **(b)** Let the equation of an ellipse,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Since, points (5, 0) and (0, –4) are the vertices of ellipse 50

At (5, 0), $\frac{25}{a^2} = 1 \Rightarrow a^2 = 25$

At (0, –4), $\frac{16}{b^2} = 1 \Rightarrow b^2 = 16$

$\therefore$ The required equation of ellipse,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

25. **(a)** Given that,

$$\int_0^{b-c} f(x+c)\,dx = K\int_c^b f(x)\,dx$$

Let $x + c = t \Rightarrow dx = dt$

Limits : $x = 0 \Rightarrow t = c$

$x = b - c \Rightarrow t = b$

$$\therefore \int_c^b f(t)\,dt = K\int_c^b f(x)\,dx$$

Therefore, $K = 1$

26. **(a)** For given non-zero vectors u and v, let θ be the angle between them, then cross product of the vectors u and v can be given by

$|u \times v| = |u|\,|v|\,|\sin\theta|$

As we know, $-1 \leq \sin\theta \leq 1$

So $|\sin\theta| \leq 1$

Thus, $|u \times v| < |u|\,|v|$

27. **(c)** We know that, equation of family of lines passing through origin,

$$y = mx \Rightarrow \frac{y}{x} = m$$

Differentiating w.r.t. x, we get

$$\frac{x\frac{dy}{dx} - y}{x^2} = 0 \quad \Rightarrow \quad y = x\frac{dy}{dx}$$

This is the required differential equation.

28. **(c)** Given, $z_0 = x_0 + iy_0$

Complex number α lies on the circle

$(x - x_0)^2 + (y - y_0)^2 = r^2$

$\therefore |\alpha - z_0|^2 = r^2$

$\Rightarrow |\alpha|^2 + |z_0|^2 - (\alpha\bar{z}_0 + \bar{\alpha}z_0) = r^2$ ...(i)

Also, $\frac{1}{\alpha}$ lies on the circle $(x - x_0)^2 + (y - y_0)^2 = 4r^2$

$$\therefore \left|\frac{1}{\alpha} - z_0\right|^2 = 4r^2$$

$$\Rightarrow \frac{1}{|\alpha|^2} + |z_0|^2 - \left(\frac{\alpha\bar{z}_0}{|\alpha|^2} + \frac{\bar{\alpha}z_0}{|\alpha|^2}\right) = 4r^2$$

$$\Rightarrow 1 + |z_0|^2|\alpha^2| - (\alpha\bar{z}_0 + \bar{\alpha}z_0) = 4r^2|\alpha|^2 \quad ...(ii)$$

By subtracting Eqs. (i) and (ii), we get

$$1 - |\alpha|^2 - |z_0|^2(1 - |\alpha|^2) = r^2(4|\alpha|^2 - 1)$$

$$\Rightarrow (|\alpha|^2 - 1)(|z_0|^2 - 1) = r^2(4|\alpha|^2 - 1)$$

Given $2|z_0|^2 = r^2 + 2 \Rightarrow |z_0|^2 = \frac{r^2}{2} + 1$

$$(|\alpha|^2 - 1)\frac{r^2}{2} = r^2(4|\alpha|^2 - 1)$$

$$|\alpha|^2 - 1 = 8|\alpha|^2 - 2$$

$$7|\alpha|^2 = 1 \Rightarrow |\alpha| = \frac{1}{\sqrt{7}}$$

29. **(c)** Let, $I = \int_0^{\pi/2} e^{\sin x} \cos x\, dx$

Put $\sin x = t$,

$\cos x\, dx = dt$

Limits :

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

So, $I = \int_0^1 e^t dt = [e^t]_0^1 = e^1 - 1 = e - 1$

30. **(b)** Given probability distribution is

X = x	0	1	2
P(X = x)	0.2	0.5	0.3

where x = mass probability function

We know that,

$$E(X^2) = \frac{\Sigma p_i x_i^2}{\Sigma p_i} = \frac{((0.2)\times 0^2) + (0.5\times 1^2) + (0.3\times 2^2)}{0.2 + 0.5 + 0.3}$$

$$= \frac{0.5 + 1.2}{1} = 1.7$$

31. **(a)** Let, $I = \int \sqrt{x-1}(x\sqrt{x+1})^{-1}dx = \int \frac{1}{x}\cdot\sqrt{\frac{x-1}{x+1}}dx$

Put $\frac{x-1}{x+1} = t^2 \Rightarrow x = \frac{1+t^2}{1-t^2}$

So, $dx \frac{(1-t^2)(2t) - (1+t^2)(-2t)}{(1-t^2)^2} dt$

$$\Rightarrow dx = \frac{4t}{(1-t^2)^2} dt$$

Now, $I = \int \left(\frac{1-t^2}{1+t^2}\right) t \frac{4t}{(1-t^2)^2} dt = \int \frac{4t^2}{(1+t^2)(1-t^2)} dt$

$$= 2\int \frac{2t^2}{(1+t^2)(1-t^2)} dt = 2\int \frac{t^2 + 1 + t^2 - 1}{(1+t^2)(1-t^2)} dt$$

$$= 2\int \left(\frac{1}{1-t^2} - \frac{1}{1+t^2}\right) dt$$

$$= 2\left[-\frac{1}{2}\log\left(\frac{1-t}{1+t}\right) - \tan^{-1}(t)\right] + c$$

Put $t^2 = \frac{x-1}{x+1}$

$$= -\log_e \left(\frac{1 - \sqrt{\frac{x-1}{x+1}}}{1 + \sqrt{\frac{x-1}{x+1}}}\right) - 2\tan^{-1}\sqrt{\frac{x-1}{x+1}} + c$$

$$= -\log_e \left(\frac{x + 1 + x - 1 - 2\sqrt{x^2 - 1}}{(x+1) - (x-1)}\right) - \tan^{-1}\sqrt{x^2 - 1} + c$$

$$= -\log_e(x - \sqrt{x^2 - 1}) - \sec^{-1} x + c$$

$$= \log_e(x + \sqrt{x^2 - 1}) - \sec^{-1} x + c$$

32. **(c)** In ΔABC **AB** = **u** and **AC** = **v** and D is mid-point of **BC** then by using mid-point formula, then

$$\mathbf{AD} = \frac{\mathbf{AB} + \mathbf{AC}}{2} = \frac{\mathbf{u} + \mathbf{v}}{2}$$

33. **(d)** Let $y = a^n + b^n$

Here, n is an odd positive integer. So,

Put $n = 1$, $y = a + b$

Put $n = 3$, $y = a^3 + b^3$

$= (a + b)(a^2 - ab + b^2)$

Thus, It can be observed that if n is any odd positive integer, $a^n + b^n$ is divisible by $(a + b)$.

34. **(b)** Equation of the family of circles passes through the intersection of circles $x^2 + y^2 = 9$ and line $x + y = 1$ is

$(x^2 + y^2 - 9) + \lambda(x + y - 1) = 0$

$x^2 + y^2 + \lambda x + \lambda y - (\lambda + 9) = 0$

Its centre is $\left(-\frac{\lambda}{2}, -\frac{\lambda}{2}\right)$

For the smallest circle, the line $x + y - 1 = 0$ must be the diameter of circle, so This line will pass through the centre

$\therefore -\frac{\lambda}{2}-\frac{\lambda}{2}=1 \Rightarrow \lambda=-1$

So, equation of the required circle is

$x^2 + y^2 - x - y - 8 = 0$

or, $(x^2 + y^2 - 9) - (x + y - 1) = 0$

35. **(a)** Given matrix

$A=\begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix} \Rightarrow |A| = a^2 + b^2 + c^2 + d^2$

and we know that, $A^{-1}=\frac{1}{|A|}$ adj A

$=\frac{1}{a^2+b^2+c^2+d^2}\begin{bmatrix} a-ib & -c-id \\ c-id & a+ib \end{bmatrix}$

$=\begin{bmatrix} a+ib & -c-id \\ -c+id & a-ib \end{bmatrix}$

Comparing the respective terms, we get

$\therefore b = 0$ and $c = d = 0$

and $a^2 + b^2 + c^2 + d^2 = 1$

36. **(b)** Let $l=\lim_{x\to 0}(1+3x)^{\frac{2}{x}}=\lim_{x\to 0}\left((1+3x)^{\frac{1}{3x}}\right)^6$

Since, $\lim_{x\to 0}(1+ax)^{\frac{1}{ax}}=e,(a\neq 0)$

$\therefore \ell = e^6$

37. **(a)** Given that, u, v and w be three vectors such that $|u| = 3$, $|v| = 5$ and $|w| = 7$

and $u + v + w = 0 \Rightarrow u + v = -w$

$\Rightarrow |u|^2 + |v|^2 + 2u.v = |w|^2$

$\Rightarrow 9 + 25 + 2 \times 3 \times 5 \cos\theta = 49,$

$\Rightarrow \cos\theta=\frac{1}{2}\Rightarrow \theta=60°$

38. **(d)** The point of reflection of point (m, n) on the line $y - x = 0$ will be (n, m), so the equation of circle having centre (n, m) and let radius r is

$(x - n)^2 + (y - m)^2 = r^2$ ...(i)

Given that, this circle touches the X-axis, so $r = m$

Now, equation of required circle becomes

$(x - n)^2 + (y - m)^2 = m^2$

$x^2 - 2nx + n^2 + y^2 - 2my + m^2 = m^2$

or $x^2 + y^2 - 2nx - 2my + n^2 = 0$

39. **(b)** Let α is the common root of the equation

$x^3 + x^2 - 2x - 2 = 0$ and $x^3 - x^2 - 2x + 2 = 0$

So, α will satisfy both the equations

Now, $\alpha^3 + \alpha^2 - 2\alpha - 2 = 0$ ...(i)

and $\alpha^3 - \alpha^2 - 2\alpha + 2 = 0$ ...(ii)

On subtractiing Eq. (ii) from Eq. (i), we get

$2\alpha^2 - 2 = 0 \Rightarrow \alpha = \pm 1.$

So, there are 2 common roots.

40. **(d)** In a ΔABC, angles A, B, C are in

$\therefore 2B = A + C$ and $A + B + C = 160°$

$\therefore B = 60°$

Also given that,

$\frac{b}{c}=\frac{\sqrt{3}}{\sqrt{2}}\Rightarrow\frac{\sin B}{\sin C}=\frac{\sqrt{3}}{\sqrt{2}}$

$\frac{\frac{\sqrt{3}}{2}}{\sin C}=\frac{\sqrt{3}}{\sqrt{2}}\Rightarrow \sin C=\frac{1}{\sqrt{2}}\Rightarrow C=45°$

So, $A = 180° - B - C = 180° - 105° = 75°$

41. **(c)** In a game, A fair die is thrown.

The probability to win a throw is $\frac{2}{6}$ and loss a throw is $\frac{4}{6}$.

Now, following cases are possible

(i) W (ii) L, W

(iii) L, L, W (iv) L, L, L

So, the expected value (mean value) of the amount he wins/loses is

$$E(x)=5\left(\frac{2}{6}\right)+4\left(\frac{4}{6}\times\frac{2}{6}\right)+3\left(\frac{4}{6}\times\frac{4}{6}\times\frac{2}{6}\right)+(-3)\left(\frac{4}{6}\times\frac{4}{6}\times\frac{4}{6}\right)$$

$=\frac{10}{6}+\frac{32}{36}+\frac{96}{216}-\frac{192}{216}$

$=\frac{360+192+96-192}{216}=\frac{456}{216}=\frac{19}{9}$

42. **(c)** Given equation, $x^3 - x^2 - x - 2 = 0$

$(x - 2)(x^2 + x + 1) = 0$

$x=2,\ \frac{-1\pm\sqrt{3}i}{2}$

$\therefore \alpha$ and β are $\frac{-1\pm\sqrt{3}i}{2}$

Non-real complex roots of this equation

So, let $\alpha = \omega$ and $\beta = \omega^2$, where $\omega^3 = 1$ and $\omega^2 + \omega + 1 = 0.$

$\therefore \alpha^{2020} + \beta^{2020} + \alpha^{2020}\ \beta^{2020}$

$= \omega^{2020} + \omega^{4040} + \omega^{2020}\ \omega^{4040}$

$= (\omega^3)^{673}\ \omega + (\omega^3)^{1346}\ \omega^2 + (\omega^3)^{673}\ \omega\ (\omega^3)^{1346}\ \omega^2$

$= \omega + \omega^2 + \omega^3 = 1 + \omega + \omega^2 = 1 + \alpha + \beta$

43. **(a)** We have, $y-\sqrt{2x+\cos^2\left(2x+\frac{\pi}{4}\right)}$

On squaring both the sides, we get

$$y^2 = 2x + \cos^2\left(2x + \frac{\pi}{4}\right)$$

On differentiating w.r.t. x we get

$$2y\frac{dy}{dx} = 2 + 2\cos\left(2x+\frac{\pi}{4}\right) \times \left[-\sin\left(2x+\frac{\pi}{4}\right)\right]\times 2$$

$$= \frac{1-\cos 4x}{\sqrt{2x+\cos^2\left(2x+\frac{\pi}{4}\right)}}$$

$$2y\frac{dy}{dx} = 2 - 2\sin\left(4x+\frac{\pi}{2}\right)$$

$$= 2 - 2\cos 4x \Rightarrow \frac{dy}{dx} = \frac{1-\cos 4x}{y}$$

$$\Rightarrow \left(\frac{dy}{dx}\right) x = \frac{\pi}{4} = \frac{1-(-1)}{\sqrt{\frac{\pi}{2}+\frac{1}{2}}} = \frac{2\sqrt{2}}{\sqrt{\pi+1}}$$

44. **(c)** Angle between vectors p and q.

$$\sin\alpha = \frac{|p\times q|}{|p||q|} = \frac{\sqrt{(4-1)^2+(-3-2)^2+(-3-8)^2}}{\sqrt{9+16+1}\sqrt{4+1+1}}$$

$$= \frac{\sqrt{9+25+121}}{\sqrt{26}\sqrt{6}} = \sqrt{\frac{155}{156}}$$

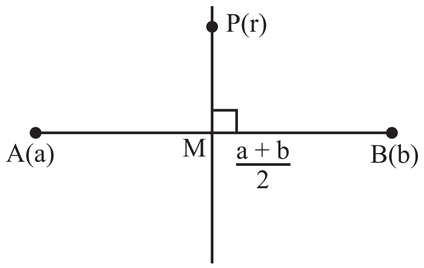
45. **(a)** Let the number of sides of the polygon is n, then number of diagonals,

$$\frac{n(n-3)}{2} = 54$$

$n^2 - 3n - 108 = 0 \Rightarrow (n-12)(n+9) = 0$

$\therefore n = 12$

46. **(a)** The mid-point of line joining points a and b is $M\left(\frac{a+b}{2}\right)$ and the direction ratio vector of line joining of given points a and b is $(a-b)$



Let a variable point P(r) on the perpendicular bisector of AB, so MP is perpendicular to AB

MP. AB = 0

$$\left(r - \frac{a+b}{2}\right)(a-b) = 0$$

$(2r-a-b)\,.\,(a-b) = 0$

47. **(c)** Let Q be the angle between the straight lines $x^2 + 4xy + y^2 = 0$

Then, $\tan\theta = \frac{2\sqrt{h^2-ab}}{(a+b)} \Rightarrow \tan\theta = \frac{2\sqrt{(2)^2-1}}{1+1}$

$= \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = 60°$

48. **(d)** Given that, $f(x) = x^4 - x^3 + 7x^2 + 14$

Differentiating w.r.t. x we get

$f'(x) = 4x^3 - 3x^2 + 14x$

At $x = 5$,

$\therefore f'(5) = 500 - 75 + 70 = 495$

49. **(Bonus)** Let, $S_1 : x^2 + y^2 - 4x - 6y + 5 = 0$

$S_2 : x^2 + y^2 - 2x - 4y - 1 = 0$

$S_3 : x^2 + y^2 - 6x - 2y = 0$

$\therefore$ Radical axis of circles S_1 and S_2 is $S_1 - S_2$,

$2x + 2y - 6 = 0 \Rightarrow x + y = 3$ ...(i)

Similarly, the radical axis of circles S_2 and S_3 is $S_2 - S_3$,

$-4x + 2y + 1 = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get the radical centre as

$y = \frac{11}{6}$ and $x = \frac{7}{6}$

Here, no option is correct.

50. **(c)** Given equation,

$4x^2 + 9y^2 - 8x + 36y + 4 = 0$

$4x^2 - 8x + 4 + 9y^2 + 36y + 36 = 36$

$\Rightarrow 4(x-1)^2 + 9(y+2)^2 = 36$

$$\Rightarrow \frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 1$$

On shifting the origin to (1, –2), the equation becomes $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and we can observe that it is free from x and y terms.

51. **(c)** The equations of lines passes through origin having slopes $\frac{2}{3}$ and $-\frac{2}{3}$ are respectively.

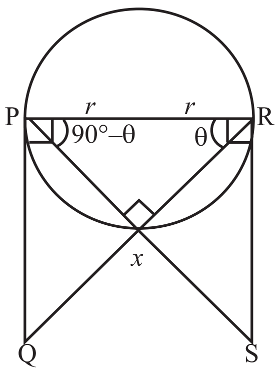
$2x + 3y = 0$ and $2x - 3y = 0$,

Thus, the combined equation

$(2x - 3y)(2x + 3y) = 0$

$4x^2 - 9y^2 = 0$

52. **(a)** Given that PQ and RS be tangents at the extremities of a diameter PR of a circle of radius r such that PS and RQ intersect at a point x on the circumference of a circle



From the diagram, in ΔPQR

$$\tan\theta = \frac{PQ}{PR} \Rightarrow PR = PQ\cot\theta$$

and in ΔPRS,

$$\tan(90° - \theta) = \frac{RS}{PR} \Rightarrow PR = RS\tan\theta$$

From Eqs (i) and (ii), we have
PQ cot θ = RS tan θ

$$\tan\theta = \sqrt{\frac{PQ}{RS}}$$

From Eqs. (ii) and (iii), we have

$$PR = RS\sqrt{\frac{PQ}{RS}} = \sqrt{PQ.RS}$$

Since PR = $2r$, then
$16 + 9\tan^2 x + 24\tan x = 25(\tan x + 1)$

$$2r = \sqrt{PQ.RS}$$

53. **(c)** The direction ratios of line joining points (k, 3, 4) and (4, 7, 8) is (k – 4, – 4, – 4) and similarly the direction ratios of line joining points (–1, –2, 1) and (1, 2, ℓ) is (–2, –4, 1 – ℓ)
Here, given that both line segments are parallel, so

$$\frac{k-4}{-2} = \frac{-4}{-4} = \frac{-4}{1-\ell} = \frac{k-4}{-2} = 1 \text{ and } \frac{-4}{1-\ell} = 1$$

$k = 2$ and $\ell = 5$
$k + \ell = 7$

54. **(c)** $$\frac{1-\cos 2x + \sin x}{\sin 2x + \cos x} = \frac{2\sin^2 x + \sin x}{2\sin x\cos x + \cos x}$$

($\because \cos 2x = 1 - 2\sin^2 x, \sin 2x = 2\sin x\cos x$)

$$\frac{\sin x(2\sin x + 1)}{\cos x(2\sin x + 1)} = \tan x.$$

55. **(b)** Given that, $f(x + y) = f(x) + f(y)$
and $f(1) = 7$
$\therefore f(x) = 7x$

Now, $\sum_{i=1}^{39} f(t) = 7[1 + 2 + 3 + ... + 39]$

$$= \frac{7 \times 39(39+1)}{2} = 5460$$

56. **(b)** We have $f(x) = k(x + \sin x) + k$
On differentiating w.r.t. x, we get $f'(x) = k(1 + \cos x)$
Since, $f(x)$ is an increasing function, so
$f'(x) \geq 0$
$k(1 + \cos x) \geq 0$
$k > 0$ $\{\because 1 + \cos x \geq 0$ and $\cos x \in [-1, 1]\}$

57. **(Bonus)** The perfect cube numbers in given set {1, 2, 3, 4, ..., 100} are 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000 and the natural numbers having odd number of divisions are perfect square numbers and the perfect square numbers in given set are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961.
Hence, required probability

$$= \frac{10+31-3}{1000} = \frac{38}{1000} = \frac{19}{500}$$

58. **(b)** A homogeneous equation of second degree in terms of x and y can be given as $ax^2 + 2hxy + by^2 = 0$, represents a pair of straight lines through origin, provided that $h^2 \geq ab$.

59. **(a)** Given that, $4\cos x + 3\sin x = 5$
Dividing by $\cos x$. We get
$\Rightarrow 4 + 3\tan x = 5\sec x$
On squaring both sides, we get
$16 + 9\tan^2 x + 24\tan x = 25\sec^2 x$
$\Rightarrow 16\tan^2 x - 24\tan x + 9 = 0$

$$\Rightarrow (4\tan x - 3)^2 = 0 \Rightarrow \tan x = \frac{3}{4}$$

60. **(d)** The roots of the equation $x^2 + x + 1 = 0$ are α and β which are non-real roots of unity.
Thus, $\alpha^3 = \beta^3 = 1$ and $\alpha + \beta + 1 = 0$
$\therefore \alpha^4 + \beta^4 = \alpha^3 + \beta^3\beta = \alpha + \beta$
$\alpha^4 + \beta^4 = -1 = -\alpha\beta$
Since product of roots $\alpha\beta = 1$

61. **(b)** $L_1 : m_a x$, $L_2 : y = m_b x$ and $L_3 : y = m_c x$
the point of intersecting of given lines with line $x + y = 1$ are

$$A\left(\frac{1}{1+m_a}, \frac{ma}{1+m_a}\right), B\left(\frac{1}{1+m_b}, \frac{m_b}{1+m_b}\right) \text{ and}$$

$$C\left(\frac{1}{1+m_c}, \frac{m_c}{1+m_c}\right) \text{ respectively.}$$

According to the given data,

$AB = BC \Rightarrow AB^2 = BC^2$

$$\Rightarrow \left(\frac{1}{1+m_a} - \frac{1}{1+m_b}\right)^2 + \left(\frac{m_a}{1+m_a} - \frac{m_b}{1+m_b}\right)^2$$

$$= \left(\frac{1}{1+m_b} - \frac{1}{1+m_c}\right)^2 + \left(\frac{m_b}{1+m_b} - \frac{m_c}{1+m_c}\right)^2$$

$$\Rightarrow \frac{(m_b - m_a)^2}{(1+m_a)^2(1+m_b)^2} + \frac{(m_a - m_b)^2}{(1+m_a)^2(1+m_b)^2}$$

$$\Rightarrow 2\frac{(m_a - m_b)^2}{(1+m_a)^2(1+m_b)^2} = 2\frac{(m_b - m_c)^2}{(1+m_b)^2(1+m_c)^2}$$

$$\frac{m_a - m_b}{1+m_a} = \frac{m_b - m_c}{1+m_c}$$

$$\Rightarrow (m_a - m_b)(1+m_c) = (m_b - m_c)(1+m_a)$$

$$\Rightarrow [(1+m_a) - (1+m_b](1+m_c)$$

$$= [(1+m_b) - (1+m_c)](1+m_a)$$

$$\Rightarrow 2(1+m_a)(1+m_c) = (1+m_b)(1+m_c+1+m_a)$$

$$\Rightarrow 2(1+m_a)(1+m_c) = (1+m_b)(2+m_a+m_c)$$

62. **(d)** We have,

$$\begin{vmatrix} k-2 & 2k-3 & 3k-4 \\ k-4 & 2k-9 & 3k-16 \\ k-8 & 2k-27 & 3k-64 \end{vmatrix} = 0$$

On applying the elementary transformation $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\Rightarrow \begin{vmatrix} k-2 & 2k-3 & 3k-4 \\ -2 & -6 & -12 \\ -6 & -24 & -60 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} k-2 & 2k-3 & 3k-4 \\ 1 & 3 & 6 \\ 1 & 4 & 10 \end{vmatrix} = 0$$

$\Rightarrow (k-2)[30-24] - (2k-3)[10-6] + (3k-4)$
$[4-3] = 0$

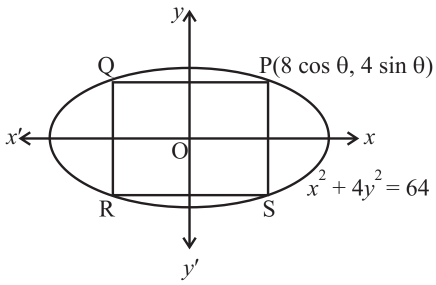
$\Rightarrow 6k - 12 - 8k + 12 + 3k - 4 = 0 \Rightarrow k = 4$

63. **(b)** Equation of the given ellipse,

$x^2 + 4y^2 = 64$

$$\Rightarrow \frac{x^2}{64} + \frac{y^2}{16} = 1$$

Let P (8 cosθ, 4 sin θ) be the vertex of the rectangle



Area of rectangle, A = 4 (8 cos θ) (4 sin θ)
= 64 sin 2θ
Since, rectangle has the greatest area
sin 2θ = 1

So A = 64 sq. units and θ = will be $\frac{\pi}{4}$.

Therefore point P (8 cosθ, 4 sin θ) $\Rightarrow P(4\sqrt{2}, 2\sqrt{2})$

Hence, length of the sides are $8\sqrt{2}$ and $4\sqrt{2}$

64. **(c)** Given equation of the line,

$$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-1}{-1} = r \text{ (say) ...(i)}$$

Let the point $P(2r+1, r-1, 1-r)$
Lies on the above line and the point P is the intersection of line in Eq. (i) and the plane $x + 2y + 3z = 4$, also, then
$2r + 1 + 2r - 2 + 3 - 3r = 4 \Rightarrow r = 2$
Thus, point P (5, 1, –1)

$$\text{Now, } (2\hat{i} - 3\hat{j}) \times (\hat{i} + 2\hat{j} - \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 0 \\ 1 & 2 & -1 \end{vmatrix}$$

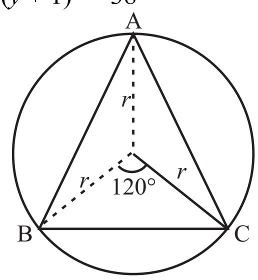
$$= \hat{i}(3) - \hat{j}(-2) + \hat{k}(4+3) = 3\hat{i} + 2\hat{j} + 7\hat{k}$$

$\therefore$ Equation of required line which is parallel to $3\hat{i} + 2\hat{j} + 7\hat{k}$

$$\frac{x-5}{3} = \frac{y-1}{2} = \frac{z+1}{7} \text{ or } \frac{x-5}{-3} = \frac{y-1}{-2} = \frac{z+1}{-7}$$

65. **(d)** Equation of the given circle,

$x^2 + y^2 - 6x + 2y - 28 = 0$
$x^2 - 6x + 9 + y^2 + 2y + 1 - 10 - 28 = 0$
$\Rightarrow (x-3)^2 + (y+1)^2 = 38$



Now, area of equilateral triangle
ΔABC = 3 × Area of ΔGBC

$$= 3 \times \frac{1}{2} r^2 \sin 120° = \frac{3}{2} \times 38 \times \frac{\sqrt{3}}{2} = \frac{57\sqrt{3}}{2}$$

66. **(Bonus)** Given vectors $a = 2\hat{i} + \hat{j} - 3\hat{k}$ and $b = 3\hat{i} - \hat{j} + 2\hat{k}$
.
Let C= $2a + b = 2(2\hat{i} + \hat{j} - 3\hat{k}) + 3\hat{i} - \hat{j} + 2\hat{k} = 7\hat{i} + \hat{j} - 4\hat{k}$
and $d = a + 2b = 2\hat{i} + \hat{j} - 3\hat{k} + 2(3\hat{i} - \hat{j} + 2\hat{k}) = 8\hat{i} - \hat{j} + \hat{k}$
Let θ be the angle between the vectors c and d. Then

$= \cos^{-1}\left(\frac{c.d}{|c||d|}\right) \theta = \cos^{-1}\frac{(7\hat{i} + \hat{j} - 4\hat{k})}{\sqrt{49+1+16}} \cdot \frac{(8\hat{i} - \hat{j} + \hat{k})}{\sqrt{64+1+1}}$

$= \cos^{-1}\frac{56-1-4}{\sqrt{66}\sqrt{66}} = \cos^{-1}\left(\frac{51}{66}\right)$

No option is correct.

67. **(c)** Given that, for A, B, and C, A + B + C = 0
sin 2A + sin 2B + sin 2C
= 2 sin (A + B) cos (A – B) + 2 sin C cos C
= – 2 sin C cos (A – B) + 2 sin C cos (A + B)
= – 2 sin C [cos (A – B) – cos (A + B)]
= – 2 sin c [cosA cos B + sin A sin B – cos A cos B + sin A sin B]
= – 2 sin C [2 sin A sin B]
= – 4 sin A sin B sin C.

68. **(a)** We have, $f(x) = \sqrt{x + 2\sqrt{2x-4}} + \sqrt{x - 2\sqrt{2x-4}}$

$= \sqrt{(x-2) + 2 + 2\sqrt{2}\sqrt{x-2}} + \sqrt{(x-2) + 2 - 2\sqrt{2}\sqrt{x-2}}$

$\sqrt{(\sqrt{x-2} + \sqrt{2})^2} + \sqrt{(\sqrt{x-2} - \sqrt{2})^2}$

$f(x) = \sqrt{x-2} + \sqrt{2} + \sqrt{x-2} - \sqrt{2} = 2\sqrt{x-2}$

On differentiating w.r.t x, we get

$f(x) = \frac{1}{\sqrt{x-2}}$

At $x = 102$, $f(102) = \frac{1}{\sqrt{102-2}} = \frac{1}{\sqrt{100}}$

So, $10 \times f'(102) = 10 \times \frac{1}{\sqrt{100}} = 1$

69. **(c)** Given that point A(, A B) is on the line segment of B (2, 5) and C (4, –1). So points A, B and C are collinear.
So, slope of line AB = slope of line BC

$\frac{3}{a-2} = \frac{6}{-2} \Rightarrow a - 2 = -1 \Rightarrow a = 1$

70. **(b)** Let, $I = \int \frac{\cos 7x - \cos 8x}{1 + 2\cos 5x} dx$

$= \int \frac{\cos 7x - \cos 8x}{1 + 2 - 4\sin^2\left(\frac{5x}{2}\right)} dx$

($\because \cos 2x = 1 - 2\sin^2 x$ and $\cos\alpha - \cos\beta$

$= -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$= \int \frac{2\sin\frac{15x}{2}\sin\frac{x}{2}}{3 - 4\sin^2\left(\frac{5x}{2}\right)} dx = \int \frac{2\sin\frac{15x}{2}\sin\frac{x}{2}\sin\frac{5x}{2}}{3\sin\frac{5x}{2} - 4\sin^3\frac{5x}{2}} dx$

$= \frac{2\sin\frac{15x}{2}\sin\frac{x}{2}\sin\frac{5x}{2}}{\sin\frac{15x}{2}} dx \quad (\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta)$

$= \int 2\sin\frac{x}{2}\sin\frac{5x}{2} dx = \int \cos 2x - \cos 3x\, dx$

$= \frac{1}{2}\sin 2x - \frac{1}{3}\sin 3x + c$

71. **(d)** In the expansion of $(\sqrt[5]{3} + \sqrt[3]{2})^{15}$

General term, $T_{r+1} = {}^nc_r a^{n-1} b^r = {}^{15}C_r 3^{\frac{15-r}{5}} \frac{r}{2^3}$

$= {}^{15}C_r 3^{3-r/5} 2^{r/3}$

For rational terms, r must be a multiple of 15, so the possible values of r = 0 and 15 $(\because 0 \le r \le 15)$
$\therefore$ Sum of rational terms = ${}^{15}C_0\, 3^3 + {}^{15}C_{15}\, 2^5$
= 27 + 32 = 59.
$\therefore$ The sum of all irrational terms is greater than the sum of all rational terms.

72. **(d)** For $K \in (0, \infty) I = \int \sin(\sqrt{k}) dk$
Put $k = t^2 \Rightarrow dk = 2t\, dt$

$\therefore I = 2\int t \sin t\, dt$

By using integration by parts,

$= -2t\cos t - 2\int 1.(-\cos t)\, dt$

$= -2t \cos t + 2 \sin t + c$

$I = 2[\sin(\sqrt{k}) - \sqrt{k}\cos(\sqrt{k})] + c$

73. **(b)** For binomial distribution we know that mean = $np = 9$
and standard deviation $= \sqrt{npq} = \frac{3}{2}$

$\therefore q = \frac{1}{4}$ and $p = \frac{3}{4}$ and $n = 12$

Hence, binomial distribution $x(p + q)^n$

$= \left(\frac{3}{4} + \frac{1}{4}\right)^{12}$

74. **(b)** Given that, $x = a \cos h(t)$ and $y = b \sin h(t)$
This curves represents, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and it is a hyperbola.

Thus, equation of normal at a point 't' is

$$\frac{x-a\cos\text{h}t}{\dfrac{a\cos\text{h}t}{a^2}}=-\frac{y-b\sin\text{h}t}{\dfrac{b\sin\text{h}t}{b^2}}$$

$\Rightarrow ax\sec ht + by\,\text{cosec}\,ht = a^2 + b^2$

75. **(b)** Given that, $(go\,f)^{-1}(t) = t - 2$

$(go\,f)(t) = t + 2$

$\Rightarrow g(f(t)) = t + 2$

As $f(t) = 3t - 2$

$g(3t - 2) = (t + 2)$

Now replace t by $\frac{t}{3}$, we get

$$g\left(3\times\frac{t}{3}.2\right)=g(t-2)=\left(\frac{t}{3}+2\right)$$

Again replace t by $t + 2$, we get

$$g(t+2-2)=g(t)=\frac{t+2}{3}+2=\frac{t+8}{3}$$

76. **(c)** $\frac{d}{dx}(e^{\log_e\sqrt{1+\tan^2 x}}\frac{d}{dx}(\sqrt{1+\tan^2 x})$

$$=\frac{d}{dx}(\sqrt{\sec^2 x})=\frac{d}{dx}(\sec x)=\sec x\tan x$$

77. **(c)** Given parabola $y^2 = 16x$

Here, the tangent is perpendicular to the line $3x - 4y + 5 = 0$

So, slope of tangent is $\left(-\frac{4}{3}\right)$

Therefore equation of tangent is

$$y=-\frac{4}{3}x+\frac{4}{-4/3}\Rightarrow y=-\frac{4}{3}x-3$$

$\Rightarrow 4x + 3y + 9 = 0$

78. **(d)** Given curve, $y=\frac{(x-7)}{(x-2)(x-3)}$, cuts the X-axis at point P (7, 0).

Now, $\frac{dy}{dx}\frac{(x-2)(x-3)-(x-7)[(x-2)+(x-3)]}{(x-2)^2(x-3)^2}$

$$\therefore\left(\frac{dy}{dx}\right)_{x=7}=\frac{5\times4}{5^2\times4^2}=\frac{1}{20}$$

So, the slope of normal $=-\frac{1}{\left(\frac{dy}{dx}\right)_{x=7}}=-20$

Therefore equation of normal,

$y - 0 = -20\,(x - 7) \Rightarrow 20x + y - 140 = 0$

79. **(a)** Given differential equation,

$$\cos(x+y)dy=dx\Rightarrow\frac{dy}{dx}=\frac{1}{\cos(x+y)}=\sec(x+y)$$

Let $x+y=t\Rightarrow1+\frac{dy}{dx}=\frac{dt}{dx}\Rightarrow\frac{dy}{dx}=\frac{dt}{dx}-1$

$$\frac{dt}{dx}-1=\sec(t)\Rightarrow\frac{dt}{1+\sec t}=dx$$

Integrating both sides, we get

$$\int\frac{dt}{1+\sec t}=\int dx$$

$$\int\frac{\cos t}{1+\cos t}dt=\int dx$$

$$\int\left(1-\frac{1}{2\cos^2\frac{t}{2}}\right)dt=\int dx$$

$$\int\left[1-\frac{1}{2}\sec^2\left(\frac{t}{2}\right)\right]dt=\int dx$$

$$t-\tan\left(\frac{t}{2}\right)=x+c$$

Put $t = x + y$,

$$x+y-\tan\left(\frac{x+y}{2}\right)=x+c\;\Rightarrow\;y=\tan\left(\frac{x+y}{2}\right)+c$$

At $x = 0$,

$\because f(0) = 0 \Rightarrow c = 0$

The solution of differential equation is,

$$\therefore y=\tan\left(\frac{x+y}{2}\right)$$

80. **(b)** Let α is the root of the given equation,

$x^3 - x^2 + x - 4 = 0$

Now, we have given that root is the negative of the root of given equation so, put $(-\alpha) = x$, so we get the required equation,

$(-x)^3 - (-x)^2 + (-x) - 4 = 0$

$x^3 + x^2 + x + 4 = 0$

PHYSICS

81. **(d)** Transformer works on the principle of mutual induction

AC generator works on the principle of magnetic effect of electric current.

82. **(b)** Magnetic force on proton

$F = Bqv\sin\theta$

$= 2.5 \times 1.6 \times 10^{-19} \times 25 \times 10^7 \sin 30°$

$$=6.25\times1.6\times10^{-12}\times\frac{1}{2}=5\times10^{-12}\,\text{N}$$

83. **(a)** As per conservation of linear momentum i.e., total linear momentum is conserved.

Initial total moment = Final total moment

$0 + 0 = M.v' + mv$

where, v' is velocity of person.

$$v' = -\frac{mv}{M} = -\frac{10\times\frac{10}{10}}{90} = -\frac{1}{9}\text{ m/s}$$

Since body travels 10 m in 10 s

$\therefore$ Kinetic energy of person $= \frac{1}{2}Mv'^2$

$$= \frac{1}{2}\times 90\times\left(-\frac{1}{9}\right)^2 = 0.55\text{ J}$$

84. **(a)** Magnetic field $B = \frac{\mu_0}{2\pi}.\frac{I}{r}$

$$= 2\times 10^{-7}\times\frac{150}{5} \qquad [\because \mu_0 = 4\pi\times 10^{-7}\,TmA^{-1}]$$

$= 6\times 10^{-6}$ T

And as per right hand palm rule the magnetic field is directed towards south.

85. **(b)** 5N  5N

Since, two equal forces of 5N in opposite direction

So total tension of

$T_{total} = 5 + (-5) = 5 + 5 = 10$ N.

86. **(b)** If a refrigerator's door is kept open, then temperature of the room increases because refrigerator exhaust more heat into the room.

87. **(b)** According to question,

Heat produced = 60% of KE = 60% of PE

$$\therefore mc\Delta T = \frac{60}{100}\times mgh$$

or, $\Delta T = \frac{0.6gh}{c} = \frac{0.6\times 10\times 210}{4.2\times 10^3} = 0.3°\text{C}$

88. **(b)** Using $P_t = P_c\left(1+\frac{m^2}{2}\right)$

And for 100% modulation, depth of modulation $m = 1$

$$\therefore P_t = P_c\left(1+\frac{1}{2}\right) \Rightarrow P_t = \frac{3}{2}P_c$$

Given, $P_t = 1800$W

$$\therefore P_c = \frac{2}{3}P_t = \frac{2}{3}\times 1800 = 1200\,W$$

89. **(a)** Absorption coefficient

$$a = \frac{\text{amount of absorb radiation } (Q_a)}{\text{amount of incident radiation} (Q_i)}$$

And for a perfectly black body $Q_a = Q_i$

$\therefore$ a = 1

90. **(a)** In balanced meter bridge unknown resistance,

$$R = S\left(\frac{100-l}{l}\right) = \frac{70(100-70)}{70} = 30\Omega$$

91. **(c)** As per given displacement time graph, initial and final position of the object is same, so total displacement = 0

Therefore average velocity of object

$$= \frac{\text{Total displacement}}{\text{Time interval}} = \frac{0}{\Delta t} = 0$$

92. **(a)** Power is rate of doing work

$$P = \frac{dW}{dt} = \frac{d}{dt}[F.s] \qquad [\because \text{W = F.s}]$$

$$= F.\frac{ds}{dt} = \text{F.v} \qquad \left[\therefore v = \frac{ds}{dt}\right]$$

93. **(a)** From law of conservation of angular momentum, Iω = constant

When a ballet dancer suddenly folds her outstretched arms, then her moment of inertia decreases, hence her angular velocity will increases.

94. **(c)** Since $R_t = R_0(1+\alpha t)$

$R_0(1+\alpha t) = 133 \qquad [\because R_{150} = 133\Omega]$

$\Rightarrow R_0(1+150\alpha) = 133$

Similarly, resistance of wire at 500°C

$R_{500} = R_0(1+500\alpha)$

$R_0(1+500\alpha) = R_{500}$

$$\therefore \frac{R_0(1+150\alpha)}{R_0(1+500\alpha)} = \frac{133}{R_{500}}$$

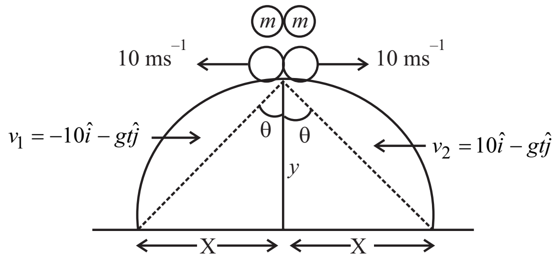
$$\Rightarrow R_{500} = \frac{133(1+500\alpha)}{1+150\alpha} = \frac{133(1+500\times 0.0045)}{1+150\times 0.0045}$$

$[\because \alpha = 0.0045°\ C^{-1}]$

$$= \frac{43225}{1.675} = 258.06\Omega \simeq 258\Omega$$

95. **(c)** As per question,

i.e., $\theta + \theta = 90° \Rightarrow \theta = 45°$ angle between two radius vector is 90°.



According to question,

According to conservation of linear momentum,

$m_1v_1 = m_2v_2 \Rightarrow mv_1 = mv_2 \Rightarrow v_1 = v_2$

$v_1 \cdot v_2 = 0$

If 't' be the time, taken by the two radius vectors connecting point of explosion to fragments

$[\because \vec{v}_1 + \vec{v}_2, \perp$ to each other]

$$(-10\hat{i} - gt\hat{j}).10\hat{i} - gt\hat{j} = 0 \Rightarrow -100 + g^2t^2 = 0$$

$$\Rightarrow t^2 = \frac{100}{g^2} = \frac{100}{10^2} = 1\text{s} \qquad \therefore t = 1\text{ sec}$$

96. **(c)** As there is zero net force on the system in horizontal direction because mass m starts moving under gravity but there is a net force in vertical direction so centre of mass of the system changes in vertical direction

97. **(a)** Energy stored in the inductor

$$E=\frac{1}{2}LI^2=\frac{1}{2}\times 5\times 10^{-2}\times 4^2=0.4\text{J}$$

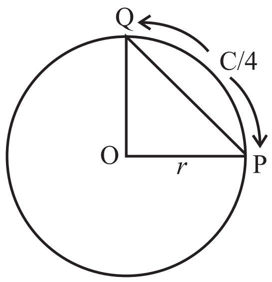
98. **(c)** When a body is falling freely i.e., under the gravity only then $a = q$

$\therefore$ Apparent weight = $m(g-a) = m \times 0 = 0$

99. **(b)** When body covers $\frac{C}{4}$ distance i.e., P to Q then

Displacement, $AB=\sqrt{OA^2+OB^2}$

$=\sqrt{r^2+r^2}=\sqrt{2r^2}=r\sqrt{2}$



100. **(d)** Resultant amplitude, when two waves of same frequency f and same amplitude a superimpose,

A = a + a = 2a

And intensity I of resultant wave

i.e., $I \propto A^2 \Rightarrow I \propto (2a)^2 \quad \therefore \quad I \propto 4a^2$

101. **(c)** Energy of nth orbit

$$E_n=\frac{-RhCz^2}{n^2};RhC=13.6\,\text{eV}$$

For hydrogen atom, $z = 1$ and $n = 1$,

$$\therefore E_1=\frac{-RhC.1^2}{1^2}=-RhC=-13.6eV$$

$n = 2$

And for $E_2=\frac{-RhCz^2}{2^2}=\frac{-13.6z^2}{4} \quad \therefore E_2=\frac{-13.6z^2}{4}$

Similarly, for $n = 3$,

$$E_3=\frac{-13.6}{9}z^2$$

Given, $\Delta E = E_3 - E_2 = 47.2$

$$\frac{-13.6}{9}z^2-\left(\frac{-13.6z^2}{4}\right)=47.2$$

or, $13.6\times\frac{5}{36}z^2=47.2$

$\Rightarrow \quad z^2 = 24.98 \approx 25$

$\Rightarrow \quad z = 5$

102. **(b)** From the relation between intensity and amplitude of the sound wave.

i.e., $I \propto A^2$

$$\left(\frac{I_{\text{incident}}}{I_{\text{reflected}}}\right)=\left(\frac{A_{\text{incident}}}{A_{\text{reflected}}}\right)^2$$

$I_{\text{incident}} = I$

$I_{\text{reflected}} = I - 20\%$ of $I = I-\frac{1}{5}=\frac{4}{5}I$

$A_{\text{incident}} = A$

therefore,

$$\therefore \frac{I}{\frac{4I}{5}}=\frac{A^2}{(A_{\text{reflected}})^2}$$

$$\therefore A_{\text{reflected}}=\sqrt{\frac{4}{5}A^2}=\frac{2}{\sqrt{5}}A$$

103. **(a)** Refractive index of medium

$$\mu=\frac{\lambda_{\text{air}}}{\lambda_{\text{medium}}}=\frac{V_{\text{air}}}{\frac{f_{\text{air}}}{\lambda\text{medium}}}=\frac{3\times 10^8}{4\times 10^{14}\times 5\times 10^{-7}}$$

$$=\frac{7.5\times 10^{-7}}{5\times 10^{-7}}=1.5$$

104. **(c)** Travel maximum distance if whole of potential energy is converted into kinetic energy (K) and is totally exhausted

i.e., K = U = 0

$\Rightarrow 2 - 20x + 5x^2 = 0$

$5x^2 - 20x + 2 = 0$

$$\therefore x=\frac{-(-20)\pm\sqrt{(-20)^2-4\times 5\times 2}}{2\times 5}$$

$$=\frac{20\pm\sqrt{360}}{10}=\frac{20\pm 18.97}{10}$$

$x = 2 \pm 1.9$

For maximum value of x,

$x = 2 + 1.9 = 3.9$

$\therefore$ Total distance = $|-3| + 39 = 6.9$ m ≈ 7 m

105. **(a)** Average kinetic energy

$$E=\frac{3}{2}K_BT \qquad \therefore E \propto T$$

Therefore, at same temperature average kinetic energy of O_2 molecule will be same as E.

106. (c) Escape velocity on the surface of planet

$$v_e = \sqrt{2gR} = \sqrt{2\times1.7\times1.7\times10^6}$$

$$= 1.7\sqrt{2}\times10^3\,\text{ms}^{-1} = 1.7\sqrt{2}\ \text{km}\,s^{-1}$$

107. (b) Acceleration due to gravity 'g' varies with height h as

$$g_h = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} \Rightarrow \frac{g}{2} = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} \qquad \left[\because g_h = \frac{g}{2}\right]$$

$$\Rightarrow \left(1+\frac{h}{R_e}\right)^2 = 2 \Rightarrow 1+\frac{h}{R_e} = \sqrt{2}$$

$$\therefore h = (\sqrt{2}-1)R_e = (1.414-1)6400 = 2625\text{ km}$$

108. (b) Work, W = F.d cos θ
& Torque, T = F.d
Hence the dimensions of work and torque
= $[MLT^{-2}]$ [L] $[ML^2T^{-2}]$ are same.

109. (a) Practically ozone layer absorbs u-v radiations. Wavelength of ultraviolet radiation become less than 3×10^{-7} m

110. (c) Both balls ball-1 drops from the top of a tower horizontally and ball-2 is dropped vertically downward from the tower at the same time will reach at the same time on the ground as both balls will move under the effect of same value of gravitational acceleration g.

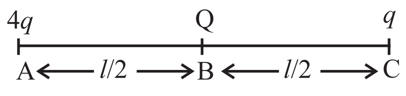
111. (b) In a photodiode, the value of the emf produced by monochromatic light beam is directly proportional to the intensity of light falling on the photodiode.

112. (b) Induced emf in the secondary coil,

$$e = M\frac{dl}{dt} = 0.2\times5 = 1V$$

113. (d) Using W = MB ($\cos\theta_1 - \cos\theta_2$)
Here, $\theta_1 = 0°$ and $\theta_2 = 360°$
∴ W = MB (cos0° – cos 360°) = MB (1 – 1) = 0

114. (a)

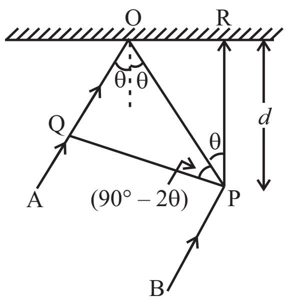


Resultant force on charge q is zero

$$\therefore K.\frac{4q.q}{l^2} + \frac{KQq}{(1/2)^2} = 0$$

$$\Rightarrow \frac{4q}{l^2} + \frac{4Q}{l^2} = 0 \Rightarrow q + Q = 0 \therefore Q = -q$$

115. (b) Point P and point Q are at same phase as they are in the same phase



From figure, ΔPQR,

$$\cos\theta = \frac{PR}{OP} = \frac{d}{OP} \therefore OP = \frac{d}{\cos\theta}$$

In ΔQOP,

$$\sin(90° - 2\theta) = \frac{OQ}{OP} \Rightarrow \cos 2\theta = \frac{OQ}{OP} \therefore OQ = OP\cos 2\theta$$

∴ Path difference,
Δ = OP + OQ = OP + OP cos 2θ
= OP (1 + cos 2θ) [∵ 1 + cos 2θ = 2 cos²θ]

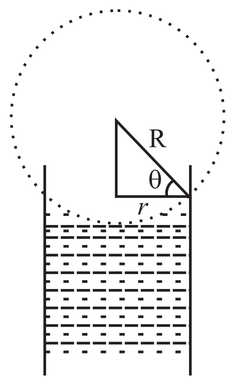
$$= \frac{d}{\cos\theta}.2\cos^2\theta = 2d\cos\theta$$

$$\text{or, } 2d\cos\theta = \frac{\lambda}{2}\left[\because \Delta = \frac{\lambda}{2}\right]$$

$$\therefore \cos\theta = \frac{\lambda}{4d}$$

116. (d) From figure, $\frac{r}{R} = \cos\theta$
So radius of curvature of
meniscus whose angle of contact

$$\Rightarrow R = \frac{r}{\cos\theta}$$



117. (c) There is no change in internal energy of an ideal gas when it undergoes free expansion as in free expansion the ideal gas is allowed to expand in vacuum.

118. (c) In elastic collision, both kinetic energy and momentum are conserved. Total energy is also conserved.

119. (c) Work is not a state variable it depends on the way it was achieved.

120. **(c)** From Einstein's photoelectric equation,

$$eV_0 = \frac{hc}{\lambda} - \phi_0$$

$$\Rightarrow eV_0 = \frac{6.63\times10^{-34}\times3\times10^8}{4.36\times10^{-7}} - 1.24\times1.6\times10^{-19}$$

$$V_0 = \frac{6.63\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times4.36\times10^{-7}} - 1.24$$

$\therefore V_0 = 2.85 - 1.24 = 1.60V$

CHEMISTRY

121. **(c)** Radius ratio, $\frac{R_{Na^+}}{R_{Cl^-}} = \frac{95}{181} = 0.525,$ which is, in the range of 0.414 – 0.732. This corresponds to coordination number of 6.

122. **(b)** Cr is present in +6 oxidation state in both of chromate (CrO_4^{2-}) and dichromate $(Cr_2O_7^{2-})$ ions i.e. d^0-configuration. So, d-d transition is absent in them. The yellow colour of CrO_4^{2-} and orange colour of $Cr_2O_7^{2-}$ is due to charge transfer of oxygen ligand get transfered into vacant *d*-orbital of metal Cr (VI).

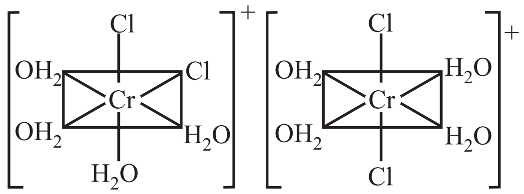
123. **(a)** $KClO_3 \xrightarrow[-KCl]{MnO_2/\Delta} \underset{(X)}{O_2} \xrightarrow{P_4(\text{white})} \underset{(Y)}{P_2O_5}$

$\xrightarrow{H_2O} \underset{[Z]}{H_3PO_4}$

124. **(b)** Dry ice is solid CO_2.

125. **(d)** Sodium stearate is used in soap and sodium lauryl sulphate $CH_3(CH_2)_{10}CH_2-OSO_3^{\ominus}Na^{\oplus}$ is an anionic detergent.

126. **(a)**



Cis *Trans*

127. **(c)** In H_2O, the central atom O forms two sigma bonds and it has two lone pair of e^-

128. **(c)**

129. **(d)** $P\% = \frac{62}{222}\times\frac{m_1}{m}\times100 = \frac{62}{222}\times\frac{0.22}{0.12}\times100 = 51.20\%$

130. **(c)** Though, Mg can reduce Al_2O_3, The process is uneconomical.

131. **(c)**

132. **(d)** Due to H- bonding in liquid state, the Boiling point order is :

$\underset{(2)}{CH_3CH_2CH_2-NH_2} >$

$\underset{(3)}{CH_3CH_2-NH-CH_3} > \underset{(1)}{CH_3-\underset{\underset{CH_3}{|}}{N}-CH_3}$

133. **(a)** From Arrhenius equation,

$$\log\frac{k_{310}}{k_{300}} = \frac{E_a}{2.303R}\left(\frac{1}{300}-\frac{1}{310}\right)$$

$$\Rightarrow \log 2 = \frac{E_a}{2.303\times8.314\times10^{-3}}\left(\frac{10}{300\times310}\right)$$

$E_a = 53.6\ kJ\ mol^{-1}$

134. **(c)** Kinetic energy of photoelectrons

$KE = h(v - v_0)$

When $v > v_0$ (threshold frequency) $KE > 0$

135. **(a)** In alkali and alkaline earth metal hydride oxidation state of hydrogen atom is –1.

136. **(c)** $\underset{\substack{16g\\32g}}{CH_4} + 2O_2 \longrightarrow \underset{\substack{44g\\88g}}{CO_2} + 2H_2O$

137. **(d)** Standard boiling point (b.p. at 1 bar pressure) of a liquid is slightly lower than the normal boiling point (b.p. at 1 atm pressure). It is because 1 bar pressure is sightly lower than 1 atm pressure.

138. **(c)**

139. **(b)** Due to 4n e^-, cyclopentadiene is non-aromatic.

140. **(a)** Due to more e^- in inner shells, shielding effect is more effective.

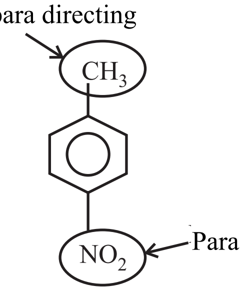
141. **(a)** $\underset{\text{Boron trichloride}}{BCl_3} + 3H_2O \longrightarrow \underset{\text{Boric acid}}{H_3BO_3} + 3HCl$

142. **(b)**

143. **(b)** Nylon – 6, 6 has strong intermolecular H-bonding.

144. **(d)** Vinegar contains 5-8% acetic acid by volume.

145. **(b)** Ortho, para directing



So this position is activated by both groups directing group.

146. (d) Oxidation number of the metal in metal carbonyl compounds is zero.

147. (a) *cis*-polyisoprene is natural rubber.

148. (a) $\overset{1}{\underset{\underset{sp^2}{\uparrow}}{CH_2}} = \overset{2}{\underset{\underset{sp^2}{\uparrow}}{CH}} - \overset{3}{\underset{\underset{sp^3}{\uparrow}}{CH_2}} — \overset{4}{\underset{\underset{sp}{\uparrow}}{C}} \equiv \overset{5}{\underset{\underset{sp}{\uparrow}}{CH}}$

149. (d) Lassaigne test

$$Na_2S \xrightarrow{(CH_3COO)_2Pb} \underset{\text{Black}}{PbS\downarrow} + 2CH_3COONa$$

150. (b) $\Delta x \times \Delta P \geq \frac{h}{4\pi}$

$$\Delta x \times m\Delta v \geq \frac{h}{4\pi} \Rightarrow \Delta x \geq \frac{h}{4\pi \times m \times \Delta v}$$

$$\geq \frac{6.626 \times 10^{-27}}{4 \times 3.14 \times 9.1 \times 10^{-28} \times 3 \times 10^4 \times 0.02 \times 10^{-2}}$$

$$\geq 9.66 \times 10^{-3}\ \text{cm} \approx 9 \times 10^{-3}\ \text{cm}$$

151. (b) $xeF_4 + SbF_5 \longrightarrow \underset{\text{Cation}}{[:\ddot{X}eF_3]^{\oplus}}\ \underset{\text{Anion}}{[SbF_6]^{\ominus}}$

Hybridisation	sp^3d (Xe)	sp^3d^2 (Sb)
Geometry	Trigonal bipyramidal	octahedral
Shape	T-shaped	octahedral

152. (d) In chemisorption, a single layer of adsorbate gets formed on the surface of absorbent.

153. (c) $\therefore w = \frac{Eit}{F} = \frac{M \times i \times t}{n \times F} i = \frac{P}{V}$

$$\Rightarrow n = \frac{M \times i \times t}{w \times F} = \frac{118.7 \times 2 \times (5 \times 3600)}{11.1 \times 96500} = 3.989$$

SnO and SnO_2 are non-electrolytes (aqueous).

Hence, the compound is $SnCl_4$

154. (b)

155. (b) HF is weakest acid, thus, F^- is the strongest conjugate base.

156. (b)

157. (d) The value of equilibrium constant does not depend on initial concentrations of reactants and products.

158. (a) pV = nRT

$$\Rightarrow T = \frac{pV}{nR} = \frac{3.32 \times 5}{4 \times 0.082}$$

= 50.60 K ≃ 50 K

159. (d) Stronger intermolecular attractive force between molecules will give higher magnitude of surface tension.

160. (c) F—Be—F

No. of lp + bp of e^- with Be = 2

Hence, hybridisation = sp