AP/EAPCET Solved Paper 2021
Held on August 19

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. Let $f: R \to R$ and $g: R \to R$ be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$ determine $(gof)(x)$ is equal to

(a) $2x^2 - 3$ (b) $4x^2 + 4x - 1$
(c) $4x^2 + 4x + 1$ (d) $2x^2 - 4$

2. Given, the function $f(x) = \dfrac{a^x + a^{-x}}{2}$ $(a > 2)$, then $f(x + y) + f(x - y)$ is equal to

(a) $f(x) - f(y)$ (b) $f(y)$
(c) $2f(x)f(y)$ (d) $f(x)f(y)$

3. If f is a function defined on (0, 1) by $f(x)$ = min $\{x - [x], -x - [x]\}$, then $(fofofof)(x)$ is equal to $\longrightarrow$ ([·] greatest integer function)

(a) x (b) $-x$ (c) $4x$ (d) $2x$

4. $n \in N$ then, the statement $8n + 16 \leq 2^n$ is true for

(a) $n = 2$ (b) $n = 3$ (c) $n = 6$ (d) $n = 5$

5. The equation whose roots are the values of the equation $\begin{vmatrix} 1 & -3 & 1 \\ 1 & 6 & 4 \\ 1 & 3x & x^2 \end{vmatrix} = 0$ is

(a) $x^2 + x + 2 = 0$ (b) $x^2 + x - 2 = 0$
(c) $x^2 + 2x + 2 = 0$ (d) $x^2 - x - 2 = 0$

6. Let a and b be non-zero real numbers such that $ab = 5/2$ and given $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ and AA^T $20I$(l is unit matrix), then the equation whose roots are a and b is

(a) $x^2 \mp 10x + 5 = 0$ (b) $2x^2 \pm 10x + 5 = 0$
(c) $x^2 - 5x + \dfrac{5}{2} = 0$ (d) $x^2 - 25x + \dfrac{5}{2} = 0$

7. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ and $B = A^{-1}$, then the value of α is

(a) 2 (b) 0 (c) 5 (d) 4

8. The rank of the matrix $\begin{bmatrix} 4 & 2 & (1-x) \\ 5 & k & 1 \\ 6 & 3 & (1+x) \end{bmatrix}$ is 1, then,

(a) $k = \dfrac{5}{2}, x = \dfrac{1}{5}$ (b) $k = \dfrac{5}{2}, x \neq \dfrac{1}{5}$
(c) $k = \dfrac{1}{5}, x = \dfrac{5}{2}$ (d) $k \neq \dfrac{5}{2}, x = \dfrac{1}{5}$

9. If $a_1, a_2, \ldots a_9$ are in GP, then $\begin{vmatrix} \log a_1 & \log a_2 & \log a_3 \\ \log a_4 & \log a_5 & \log a_6 \\ \log a_7 & \log a_8 & \log a_9 \end{vmatrix}$ is equal to

(a) $\log(a_1, a_2, \ldots a_n)$ (b) 1
(c) $(\log a_9)^9$ (d) 0

10. $(\sin\theta - i\cos\theta)^3$ is equal to

(a) $i^3 (\cos 3\theta + i \sin 3\theta)$ (b) $\cos 3\theta + i \sin 3\theta$
(c) $\sin 3\theta - i \cos 3\theta$ (d) $(-i)^3 (\cos 3\theta + i \sin 3\theta)$

11. Real part of $(\cos 4 + i\sin 4 + 1)^{2020}$ is

(a) $2^{2020}\cos^{2020}2\cos 2020$ (b) $2^{2020}\cos^{2020}2\cos 4040$
(c) $2^{1020}\cos^{2020}2\cos 4040$ (d) $2^{2020}\cos^{2020}1\cos 2020$

12. If $(x^2 + 5x + 5)^{x+5} = 1$, then the number of integers satisfying this equation is

(a) 2 (b) 3 (c) 4 (d) 5

13. Let $f(x) = x^3 + ax^2 + bx + c$ be polynomial with integer coefficients. If the roots of $f(x)$ are integer and are in Arithmetic Progression, then a cannot take the value

(a) –642 (b) 1214 (c) 1323 (d) 1626

14. The sum of the roots of the equation $e^{4t} - 10e^{3t} + 29e^{2t} - 22e^{t} + 4 = 0$ is
(a) $\log_e 10$ (b) $2\log_e 2$ (c) $\log_2 29$ (d) $2\log_{10} 2$

15. If a person has 3 coins of different denominations, the number of different sums can be formed is
(a) 3 (b) 7 (c) 8 (d) 3!

16. There are 7 identical white balls and 3 identical black balls. The number of distinguishable arrangements in a row of all the balls, so that no two black balls are adjacent is
(a) 120 (b) $89\cdot(8!)$ (c) 56 (d) 42×5^4

17. The number of ways of distributing eight identical rings to three different girls so that every girl gets at least one rings is
(a) 21 (b) 120 (c) ${}^8P_3$ (d) ${}^8P_3 - 6$

18. If $\dfrac{x^4}{(x-1)(x-2)} = f(x) + \dfrac{A}{x-1} + \dfrac{B}{x-2}$, then
(a) $f(x) = x^2 - 3x + 7$ (b) $f(x) = x^2 + 3x + 7$
(c) $A + B = 17$ (d) $A - B = -18$

19. $\tan 2\alpha\cdot \tan(30° - \alpha) + \tan 2\alpha \cdot \tan(60° - \alpha) + \tan(60° - \alpha)\cdot\tan(30° - \alpha)$ is equal to
(a) $\tan 3\alpha$ (b) $\tan^2 2\alpha - \tan^2 60°$
(c) 1 (d) 0

20. If $\sin\alpha - \cos\alpha = m$ and $\sin 2\alpha = n - m^2$, where $-\sqrt{2} \le m \le \sqrt{2}$, then n is equal to
(a) 0 (b) 1 (c) 2 (d) -2

21. The value of x satisfying the equation $3 \operatorname{cosec} x = 4\sin x$ are
(a) $\dfrac{\pi}{6}, \dfrac{\pi}{3}$ (b) $\pm\dfrac{\pi}{6}$ (c) $\pm\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{3}, \dfrac{\pi}{4}$

22. If $\tan^{-1}\left[\dfrac{1}{1+1.2}\right] + \tan^{-1}\left[\dfrac{1}{1+2.3}\right] + \ldots + \tan^{-1}\left[\dfrac{1}{1+n(n+1)}\right] = \tan^{-1}[x]$, then x is equal to
(a) $\dfrac{1}{n+1}$ (b) $\dfrac{n}{n+1}$ (c) $\dfrac{1}{n+2}$ (d) $\dfrac{n}{n+2}$

23. If $\sin h\, u = \tan\theta$, then $\cosh u$ is equal to
(a) $-\sec\theta$ (b) $\sec\theta$ (c) $\sin\theta$ (d) $\cot\theta$

24. In a ΔABC, if $a = 3$, $b = 4$ and $\sin A = \dfrac{3}{4}$, then $\angle CBA$ is equal to
(a) 60° (b) 75° (c) 90° (d) 45°

25. In ΔABC, $A = 75°$ and $B = 45°$, then the value of $b + c\sqrt{2}$ is equal to
(a) a (b) $3a$ (c) $2a$ (d) $4a$

26. In ΔABC, suppose the radius of the circle opposite to an $\angle A$ is denoted by r_1, similarly $r_2 \leftrightarrow \angle B$ and $r_3 \leftrightarrow \angle C$. If r is the radius of inscribed circle, then, what is the value of $\dfrac{ab - r_1 r_2}{r_3}$ is equal to
(a) $r_1 r_2 r_3$ (b) r (c) $r_1 r_2 \dfrac{r_3}{2}$ (d) $\dfrac{r}{2}$

27. A vector makes equal angles α with X and Y-axis, and 90° with Z-axis. Then, α is equal to
(a) 60° or 120° (b) 30° and 150°
(c) 45° and 135° (d) 90°

28. Angle made by the position vector of the point $(5, -4, -3)$ with the positive direction of X-axis is
(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{6}$ (c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{3}$

29. If D, E and F are respectively mid-points of AB, AC and BC in ΔABC, then BE + AF is equal to
(a) DC (b) $\dfrac{3}{2}$BF (c) $\dfrac{1}{2}$BF (d) $\dfrac{1}{2}$DC

30. If the volume of the parallelopiped formed by the vectors $\hat{\mathbf{i}} + a\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{j}} + a\hat{\mathbf{k}}$ and $a\hat{\mathbf{i}} + \hat{\mathbf{k}}$ becomes minimum, then a is equal to
(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{\sqrt{3}}$ (c) $\dfrac{2}{\sqrt{3}}$ (d) $\dfrac{2}{3}$

31. If $\mathbf{a} = \dfrac{3}{2}\hat{\mathbf{k}}$ and $\mathbf{b} = \dfrac{2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}}{2}$, then angle between a + b and a – b is
(a) 45° (b) 90° (c) 30° (d) 60°

32. Let $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{c} = 7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$, then the area of parallelogram having diagonals **a** + **b** and **b** + **c** is
(a) $4\sqrt{6}$ sq units (b) $2\sqrt{6}$ sq units
(c) $\sqrt{6}$ sq units (d) $6\sqrt{6}$ sq units

33. If $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{c} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, then the value of $\begin{vmatrix} \mathbf{a}\cdot\mathbf{a} & \mathbf{a}\cdot\mathbf{b} & \mathbf{a}\cdot\mathbf{c} \\ \mathbf{b}\cdot\mathbf{a} & \mathbf{b}\cdot\mathbf{b} & \mathbf{b}\cdot\mathbf{c} \\ \mathbf{c}\cdot\mathbf{a} & \mathbf{c}\cdot\mathbf{b} & \mathbf{c}\cdot\mathbf{c} \end{vmatrix}$ is equal to
(a) 2020 (b) 2025 (c) 2030 (d) 1849

34. If a and b are two vectors such that $|a| = 2$, $|b| = 3$ and $a + tb$ and $a - tb$ are perpendicular, where t is a positive scalar, then
(a) $t = \pm\dfrac{2}{3}$ (b) $t = \dfrac{4}{9}$ (c) $t = \dfrac{2}{3}$ (d) $t = \dfrac{2}{9}$

35. The variance of the variates 112, 116, 120, 125 and 132 about their AM is
(a) 58.8 (b) 60 (c) 48.8 (d) 61.8

36. Which of the following set of data has least standard deviation?
(a) 10, 20, 30, 40 (b) 2, 4, 6, 8
(c) 3, 6, 9, 12 (d) 1, 2, 3, 4

37. 12 balls are distributed among 3 boxes, then the probability that the first box will contain 3 balls is
(a) $\dfrac{{}^{12}C_3 \times 2^9}{3^{12}}$ (b) $\dfrac{{}^{12}C_3 \times 2^9}{3^{10}}$
(c) $\dfrac{{}^{12}C_3}{3^{12}}$ (d) $\dfrac{{}^{12}C_3}{3^{10}}$

38. If the letters of the word REGULATIONS be arranged in such a way that relative positions of the letters of the word GULATIONS remain the same, then the probability that there are exactly 4 letters between R and E is

(a) $\frac{3}{55}$ (b) $\frac{6}{55}$ (c) $\frac{9}{55}$ (d) $\frac{7}{55}$

39. A random variable X has the probability distribution

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E = \{X$ is a prime number$\}$ and $F = \{X < 4\}$, then $P(E \cup F)$ is

(a) 0.77 (b) 0.87 (c) 0.35 (d) 0.50

40. A die is tossed thrice. If event of getting an even number is a success, then the probability of getting at least 2 successes is

(a) $\frac{7}{8}$ (b) $\frac{1}{4}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

41. If the axes are rotated through an angle 45°, the coordinates of the point $\left(2\sqrt{2}, -3\sqrt{2}\right)$ in the new system are

(a) $\left(3\sqrt{3}, -5\right)$ (b) $(-1, -5)$

(c) $\left(5\sqrt{3}, -7\right)$ (d) $\left(7, -\sqrt{3}\right)$

42. The sum of the squares of the intercepts made the line $5x - 2y = 10$ on the coordinate axes equals

(a) 29 (b) 25 (c) 4 (d) 100

43. For three consecutive odd integers $a \cdot b$ and c, if the variable line $ax + by + c = 0$ always passes through the point (α, β), the value of $\alpha^2 + \beta^2$ equals

(a) 9 (b) 4 (c) 5 (d) 3

44. The line which is parallel to X-axis and crosses the curve $y = \sqrt{x}$ at an angle of 45° is

(a) $y = \frac{1}{4}$ (b) $y = \frac{1}{2}$ (c) $y = 1$ (d) $y = 4$

45. If $2x + 3y + 4 = 0$ is the perpendicular bisector of the line segment joining the points $A(1, 2)$ and $B(\alpha, \beta)$, then the value of $13\alpha + 13\beta$ equals

(a) – 81 (b) – 99 (c) 99 (d) 81

46. The equation of the pair of straight lines perpendicular to the pair $2x^2 + 3xy + 2y^2 + 10x + 5y = 0$ and passing though the origin is

(a) $2x^2 + 5xy + 2y^2 = 0$ (b) $2x^2 - 3xy + 2y^2 = 0$

(c) $2x^2 + 3xy + y^2 = 0$ (d) $2x^2 - 5xy + 2y^2 = 0$

47. If the centroid of the triangle formed by the lines $2y^2 + 5xy - 3x^2 = 0$ and $x + y = k$ is $\left(\frac{1}{18}, \frac{11}{18}\right)$, then the value of k equals

(a) –1 (b) 0 (c) 1 (d) 2

48. If m_1 and m_2, $(m_1 > m_2)$ are the slopes of the lines represented by $5x^2 - 8xy + 3y^2 = 0$, then $m_1 : m_2$ equals

(a) 5 : 1 (b) 2 : 1 (c) 5 : 3 (d) 3 : 2

49. If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is the square of the other then, $\left|\frac{a+b}{h} + \frac{8h^2}{ab}\right|$ is equal to

(a) 3 (b) 2 (c) 6 (d) 4

50. Find the equations of the tangents drawn to the circle $x^2 + y^2 = 50$ at the points where the line $x + 7 = 0$ meets it.

(a) $7x + y + 50 = 0$ and $7x - y + 50 = 0$

(b) $x + y = 0$ and $x - y = 0$

(c) $x + 7y + 5 = 0$ and $y - 7x + 5 = 0$

(d) $x + 7y + 50 = 0$ and $x - 7y + 50 = 0$

51. If the chord of contact of tangents from a point on the circle $x^2 + y^2 = r_1^2$ to the circle $x^2 + y^2 = r_2^2$ touches the circle $x^2 + y^2 = r_3^2$, then r_1, r_2 and r_3 are in

(a) AP (b) HP (c) GP (d) AGP

52. Find the equation of the circle passing through (1, –2) and touching the X-axis at (3, 0).

(a) $x^2 + y^2 + 6x - 4y - 9 = 0$

(b) $x^2 + y^2 - 6x - 4y + 9 = 0$

(c) $x^2 + y^2 - 6x - 4y - 9 = 0$

(d) $x^2 + y^2 - 6x + 4y + 9 = 0$

53. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations represent L_1

(a) $x + y = 0$ and $x + 7y = 0$

(b) $x - y = 0$ and $x + 7y = 0$

(c) $x - 7y = 0$ and $x + y = 0$

(d) $x - 7y = 0$ and $x - y = 0$

54. The radius of the circle whose centre lies at (1, 2) while cutting the circle $x^2 + y^2 + 4x + 16y - 30 = 0$ orthogonally, is .... units.

(a) $\sqrt{41}$ (b) $\sqrt{31}$ (c) $\sqrt{21}$ (d) $\sqrt{11}$

55. The point which has the same power with respect to each of the circles $x^2 + y^2 - 8x + 40 = 0$, $x^2 + y^2 - 5x + 16 = 0$ and $x^2 + y^2 - 8x + 16y + 160 = 0$ is

(a) $\left(-8, \frac{-15}{2}\right)$ (b) $\left(8, \frac{-15}{2}\right)$

(c) $\left(8, \frac{15}{2}\right)$ (d) $\left(-8, \frac{15}{2}\right)$

56. If one end of focal chord of the parabola $y^2 = 8x$ is $\left(\frac{1}{2}, 2\right)$, then the length of the focal chord is .... units.

(a) $\frac{625}{4}$ (b) $\frac{5}{\sqrt{2}}$ (c) $\frac{25}{2}$ (d) 25

57. If a point $P(x, y)$ moves along the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and if C is the centre of the ellipse, then the sum of maximum and minimum values of CP is

(a) 25 (b) 9 (c) 4 (d) 5

58. The asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, with any tangent to the hyperbola form a triangle whose area is $a^2\tan(\alpha)$. Then, its eccentricity equals

(a) $\sec(\alpha)$ (b) $\text{cosec}(\alpha)$ (c) $\sec^2(\alpha)$ (d) $\text{cosec}^2(\alpha)$

59. The ratio in which the YZ-plane divides the line joining (2, 4, 5) and (3, 5, –4) is

(a) 2 : 3 internally (b) 3 : 2 internally
(c) 3 : 2 externally (d) 2 : 3 externally

60. The direction cosines of a line which makes equal angles with the coordinate axes are

(a) $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ (b) $\left(\frac{-1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{-1}{\sqrt{3}}\right)$

(c) $\left(\frac{\pm1}{\sqrt{3}},\frac{\pm1}{\sqrt{3}},\frac{\pm1}{\sqrt{3}}\right)$ (d) $\left(\frac{12}{15},\frac{5}{13},0\right)$

61. Let O be the origin and P be a point which is at a distance of 3 units from the origin. If the direction ratios of $\overline{OP}$ are (1, –2, –2), then the coordinates of P are

(a) (1, –2, –2) (b) (3, –6, –6)

(c) $\left(\frac{1}{3},\frac{-2}{3},\frac{-2}{3}\right)$ (d) $\left(\frac{1}{9},\frac{-2}{9},\frac{-2}{9}\right)$

62. $\lim_{z\to1}\frac{z^{(1/3)}-1}{z^{(1/6)}-1}$ is equal to

(a) –1 (b) 1 (c) 2 (d) –2

63. $f(x)=\begin{cases}\dfrac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}, & x\neq 0\\ K\log 2\log 3, & x=0\end{cases}$

Find the value of k for which the function f is continuous.

(a) $\sqrt{2}$ (b) 24 (c) $18\sqrt{3}$ (d) $24\sqrt{2}$

64. If the function $f(x)$, defined below is continuous in the interval $[0, \pi]$, then

$$f(x)=\begin{cases}x+a\sqrt{2}(\sin x), & 0\le x<\frac{\pi}{4}\\ 2x(\cot x)+b, & \frac{\pi}{4}\le x\le\frac{\pi}{2}\\ a(\cos 2x)-b(\sin x), & \frac{\pi}{2}<x\le\pi\end{cases}$$

(a) $a=\frac{\pi}{6},b=\frac{\pi}{12}$ (b) $a=\frac{-\pi}{6},b=\frac{\pi}{12}$

(c) $a=\frac{-\pi}{6},b=\frac{-\pi}{12}$ (d) $a=\frac{\pi}{6},b=\frac{-\pi}{12}$

65. If $y=x+\frac{1}{x}$, then which among the following holds?

(a) $x^2y'+xy=0$ (b) $x^2y'+xy+2=0$
(c) $x^2y'-xy+2=0$ (d) $x^2y'+xy-2=0$

66. If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, where $x^2\le 1$. Then, find $\frac{dy}{dx}$ is equal to

(a) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}(x^2)$ (b) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}(x^2)$

(c) $\frac{-x}{\sqrt{1-x^4}}$ (d) $\frac{-2x}{\sqrt{1-x^4}}$

67. If $3\sin xy+4\cos xy=5$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{3\sin xy+4\cos xy}{3\cos xy-4\sin xy}$ (b) $\frac{3\cos xy+4\sin xy}{4\cos xy-3\sin xy}$

(c) $\frac{-y}{x}$ (d) $\frac{x}{y}$

68. $f(x)=\sqrt{x^2+1}:g(x)=\frac{x+1}{x^2+1}:h(x)=2x-3$, then the value of $f'[h'(g'(x))]$ is equal to

(a) $\sqrt{5}$ (b) $\frac{2}{\sqrt{5}}$ (c) $\frac{\sqrt{5}}{2}$ (d) $\frac{1}{\sqrt{5}}$

69. If the error committed in measuring the radius of a circle is 0.05%, then the corresponding error in calculating its area would be

(a) 0.05% (b) 0.0025% (c) 0.25% (d) 0.1%

70. The stationary points of the curve $y=8x^2-x^4-4$ are

(a) (0, – 4), (2, 12), (–2, 12)
(b) (0, 4), (–2, 12), (1, 2)
(c) (0, – 4), (–1, 2), (2, 12)
(d) (0, 4), (–1, 2), (1, 2)

71. Which statement among the following is true?

(i) the function $f(x)=x|x|$ is strictly increasing on $R-\{0\}$.
(ii) the function $f(x)=\log_{(1/4)}x$ is strictly increasing on $(0,\infty)$.
(iii) a one-one function is always an increasing function.
(iv) $f(x)=x^{1/3}$ is strictly decreasing on R

(a) (i) (b) (ii) (c) (iii) (d) (iv)

72. For which value (s) of $af(x)=-x^3+4ax^2+2x-5$ is decreasing for every x?

(a) (1, 2) (b) (3, 4)
(c) R (d) no value of a

73. The distance between the origin and the normal to the curve $y=e^{2x}+x^2$ drawn at $x=0$ is .... units

(a) 2 (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{1}{2}$

74. If $\int\frac{dx}{x\left(\sqrt{x^4-1}\right)}=\frac{1}{k}\sec^{-1}\left(x^k\right)$, then the value of k is equal to

(a) 1 (b) 2 (c) 3 (d) 4

75. $\int \frac{e^x(x+3)}{(x+5)^3}dx$ is equal to

(a) $\frac{e^x}{(x+5)^2}+C$ (b) $e^x(x+5)^2+C$

(c) $e^x(x+3)^2+C$ (d) $\frac{e^x}{(x+3)^2}+C$

76. If $\int \frac{(x-1)^2}{(x^2+1)^2}dx = \tan^{-1}(x)+g(x)+k,$ then $g(x)$ is equal to

(a) $\tan^{-1}\left(\frac{x}{2}\right)$ (b) $\frac{1}{x^2+1}$

(c) $\frac{1}{2(x^2+1)}$ (d) $\frac{2}{x^2+1}$

77. If $\int \frac{1-(\cot x)^{2021}}{\tan x+(\cot x)^{2022}}dx =$

$\frac{1}{A}\log|(\sin x)^{2023}+(\cos x)^{2023}|+c$, then A is equal to

(a) 2020 (b) 2021 (c) 2022 (d) 2023

78. $\int_2^4 \{|x-2|+|x-3|\}\,dx$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

79. $\int_{-1/2}^{1/2}\left\{[x]+\log\left(\frac{1+x}{1-x}\right)\right\}dx$ is equal to

(a) 2log(1/2) (b) 0

(c) $\frac{-1}{2}$ (d) 1

80. The solution of the differential equation $\frac{d^2y}{dx^2}+y=0$ is

(a) $y=3\sin x+4\cos x$ (b) $y=x^2$

(c) $y=x+2$ (d) $y=\log x$

PHYSICS

81. An electric generator is based on

(a) Faraday's laws of electromagnetic induction
(b) Motion of charged particles in an electromagnetic field
(c) Fission of uranium by slow neutrons
(d) Newton's laws of motion

82. Which of the following decreases, in motion on a straight line, with constant retardation?

(a) Speed (b) Acceleration

(c) Displacement (d) Distance

83. When a ball is thrown with a velocity of 50 ms^{-1} at an angle 30° with the horizontal, it remains in the air for .... s. (Take, g = 10 ms^{-2})

(a) 5 (b) 2.5 (c) 1.25 (d) 0.625

84. One of the rectangular components of a force of 40 N is $20\sqrt{3}$ N. What is the other rectangular component?

(a) 10 N (b) 20 N (c) 30 N (d) 25 N

85. An object dropped in a stationary lift takes time t_1 to reach the floor. It takes time t_2 when lift is moving up with constant acceleration. Then,

(a) $t_2 > t_1$ (b) $t_1 > t_2$ (c) $t_1 \approx t_2$ (d) $t_1 = t_2$

86. When a body is placed on a rough plane (coefficient of friction = μ) inclined at an angle θ to the horizontal, its acceleration is .... (acceleration due to gravity = g)

(a) $g(\sin\theta - \mu\cos\theta)$ (b) $g(\sin\theta - \cos\theta)$

(c) $g\mu(\sin\theta - \cos\theta)$ (d) $g(\mu\sin\theta - \cos\theta)$

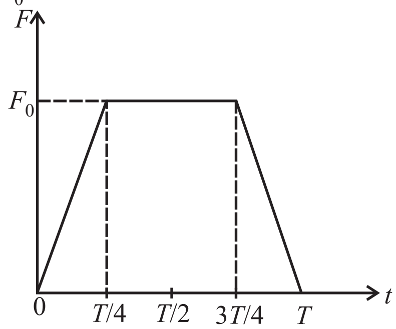
87. A metal ball of mass 2 kg moving with a velocity of 36 km/h has a head on collision with a stationary ball of mass 3 kg. After the collision, if both balls move together, then the loss in kinetic energy due to collision is

(a) 40 J (b) 60 J (c) 100 J (d) 140 J

88. A body of mass 8 kg, under the action of a force, is displaced according to the equation, $s=\frac{t^2}{4}$ m, where t is the time. Find the work done by the force in the first 4 s.

(a) 9 J (b) 16 J (c) 6 J (d) 3 J

89. A particle of mass m, moving with a velocity v makes an elastic collision in one dimension with a stationary particle of mass m. During the collision, they remain in contact with each other for an extremely small time T. Their force of contact, with time is shown in the figure. Then, F_0



(a) $\frac{2mv}{T}$ (b) $\frac{4mv}{3T}$ (c) $\frac{mv}{T}$ (d) $\frac{3mv}{4T}$

90. Which of the following type of wheels of same mass and radius will have largest moment of inertia?

(a) Ring (b) Angular disc

(c) Solid disc (d) Cylindrical disc

91. The sum of moments of all the particles in a system about its centre of mass is always

(a) minimum (b) zero

(c) maximum (d) infinite

92. **Assertion :** Two identical trains move in opposite senses in equatorial plane with same speeds relative to the Earth's surface. They have equal magnitude of normal reaction.

Reason : The trains have different centripetal accelerations due to different speeds.

(a) Both A and R are true and R is a correct explanation for A.
(b) Both A and R are true but R is not a correct explanation for A.
(c) A is true, R is false.
(d) A is false, R is true.

93. A spring is stretched by 0.40 m when a mass of 0.6 kg is suspended from it. The period of oscillations of the spring loaded by 255 g and put to oscillations is close to ($g = 10$ ms^{-2})
(a) 11 s (b) 48.6 s (c) 0.82 s (d) 4.86 s

94. A heavy brass sphere is hung from a spring and it executes vertical vibrations with period T. The sphere is now immersed in a non-viscous liquid with a density (1/10)th that of brass. When set into vertical vibrations with the sphere remaining inside liquid all the time, the time period will be
(a) $\left(\sqrt{\frac{9}{10}}\right)T$ (b) $\left(\sqrt{\frac{10}{9}}\right)T$
(c) $\left(\frac{9}{10}\right)T$ (d) Unchanged

95. A particle is kept on the surface of a uniform sphere of mass 1000 kg and radius 1 m. The work done per unit mass against the gravitational force between them is
[$G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}]
(a) 3.35×10^{-10} Jkg^{-1} (b) -3.35×10^{-10} Jkg^{-1}
(c) 6.67×10^{-8} Jkg^{-1} (d) -3.35×10^{-8} Jkg^{-1}

96. The acceleration due ot gravity at a height (1/20)th of the radius of Earth above the Earth's surface is 9 ms^{-2}. Its value at an equal depth below the surface of earth is
(a) 9 ms^{-2} (b) 9.25 ms^{-2} (c) 9.5 ms^{-2} (d) 9.8 ms^{-2}

97. The Young's modulus of a rubber string of length 12 cm and density 1.5 kgm^{-3} is 5×10^8 Nm^{-2}. When this string is suspended vertically, the increase in its length due to its own weight is (Take, $g = 10$ ms^{-2})
(a) 2.16×10^{-10} m (b) 9.6×10^{-11} m
(c) 9.6×10^{-3} m (d) 2.16×10^{-3} m

98. The lower end of a capillary tube is dipped into water and it is observed that the water in capillary tube rises by 7.5 cm. Find the radius of the capillary tube used, if surface tension of water is 7.5×10^{-2} Nm^{-1}. Angle of contact between water and glass is 0° and acceleration due to gravity is 10 ms^{-2}.
(a) 0.2 cm (b) 0.1 cm (c) 0.4 mm (d) 0.2 mm

99. An ideal liquid flows through a horizontal tube of variable diameter. The pressure is lowest where the
(a) velocity is highest (b) velocity is lowest
(c) diameter is largest (d) velocity is intermediate

100. In a steady state, the temperature at the end A and end B of a 20 cm long rod AB are 100°C and 0°C. The temperature of a point 9 cm from A is
(a) 55°C (b) 45°C (c) 65°C (d) 50°C

101. If two rods of length L and $2L$, having coefficients of linear expansion α and 2α respectively are connected end-to-end, then find the average coefficient of linear expansion of the composite rod.
(a) $\frac{3\alpha}{2}$ (b) $\frac{5\alpha}{2}$ (c) $\frac{5\alpha}{4}$ (d) $\frac{5\alpha}{3}$

102. A system is taken from state-A to state-B along two different paths. The heat absorbed and work done by the system along these two paths are Q_1, Q_2 and W_1, W_2 respectively, then
(a) $Q_1 = Q_2$ (b) $W_1 = W_2$
(c) $Q_1 - W_1 = Q_2 - W_2$ (d) $Q_1 + W_1 = Q_2 + W_2$

103. A gas ($\gamma = 1.5$) is suddenly compressed to (1/4)th its initial volume. Then, find the ratio of its final to initial pressure.
(a) 1 : 16 (b) 1 : 8 (c) 1 : 4 (d) 8 : 1

104. A cylinder has a piston at temperature of 30°C. There is all round clearance of 0.08 mm between the piston and cylinder wall if internal diameter of the cylinder is 15 cm. What is the temperature at which piston will fit into the cylinder exactly?
($\alpha_p = 1.6 \times 10^{-5}$/°C and $\alpha_c = 1.2 \times 10^{-5}$/°C)
(a) 298° C (b) 273° C (c) 305° C (d) 268° C

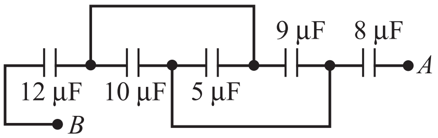
105. A balloon contains 1500 m^3 of He at 27° C and 4 atmospheric pressure, the volume of He at –3° C temperature and 2 atmospheric pressure will be
(a) 1500 m^3 (b) 1700 m^3
(c) 1900 m^3 (d) 2700 m^3

106. The sources of sound A and B produce a wave of 350 Hz in same phase. A particle P is vibrating under an influence of these two waves. If the amplitudes at P produced by the two waves is 0.3 mm and 0.4 mm, the resultant amplitude of the point P will be, when $AP - BP = 25$ cm and the velocity of sound is 350 ms^{-1}
(a) 0.7 mm (b) 0.1 mm (c) 0.2 mm (d) 0.5 mm

107. In a diffraction pattern due to a single slit of width a, the first minimum is observed at an angle 30° when light of wavelength 500 nm is incident on the slit. The first secondary maximum is observed at an angle of
(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$
(c) $\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\sin^{-1}\left(\frac{2}{3}\right)$

108. Which statement(s) among the following are incorrect?
(i) A negative test charge experiences a force opposite to the direction of the field.
(ii) The tangent drawn to a line of force represents the direction of electric field.
(iii) The electric field lines never intersect.
(iv) The electric field lines form a closed loop.
(a) Only (i)
(b) Both (ii) and (iii)
(c) Only (iii)
(d) Only (iv)

109. In the given circuit, if the potential difference between A and B is 80 V, then the equivalent capacitance between A and B and the charge on $10\mu F$ capacitor respectively, are



(a) $4\mu F$ and $133\mu C$ (b) $164\mu F$ and $150\mu C$
(c) $15\mu F$ and $200\ \mu C$ (d) $4\mu F$ and $50\ \mu C$

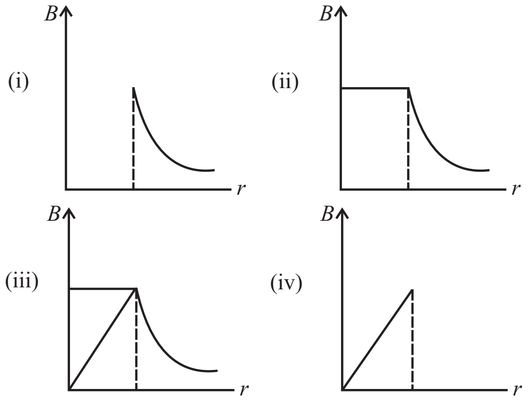
110. A cell of emf 1.8 V gives a current of 17 A when directly connected to an ammeter of resistance 0.06 Ω. Internal resistance of the cell is

(a) 0.046 Ω (b) 0.066 Ω (c) 0.10 Ω (d) 10 Ω

111. In which of the following case no force exerted by a magnetic field on a charge?

(a) Moving with constant velocity
(b) Moving in a circle
(c) At rest
(d) Moving along a curved path

112. A long thin hollow metallic cylinder of radius R has a current i ampere. The magnetic induction B away from the axis at a distance r from the axis varies as shown is



(a) (i) (b) (ii) (c) (iii) (d) (iv)

113. The plane of a dip circle is set in the geographic meridian and the apparent dip is δ_1. It is then set in a vertical plane perpendicular to the geographic meridian. The apparent dip angle is δ_2. The declination θ at the place is

(a) $\tan^{-1}(\tan\delta_1 \tan\delta_2)$ (b) $\tan^{-1}(\tan\delta_1 + \tan\delta_2)$
(c) $\tan^{-1}\left(\dfrac{\tan\delta_1}{\tan\delta_2}\right)$ (d) $\tan^{-1}(\tan\delta_1 - \tan\delta_2)$

114. **Assertion :** It is more difficult to move a magnet into a coil with more loops.

Reason : This is because emf induced in each current loop resists the motion of the magnet.

(a) Both A and R are true and R is a correct explanation for A.
(b) Both A and R are true but R is not a correct explanation for A.
(c) A is true and R is false.
(d) A is false, R is true.

115. Two inductors A and B when connected in parallel are equivalent to a single inductor of inductance 1.5 H and when connected in series are equivalent to a single inductor of inductance 8H. Find the difference in the inductances of A and B.

(a) 3 H (b) 7.5 H (c) 2 H (d) 4 H

116. A resonant frequency of a current is f. If the capacitance is made four times the initial value, then the resonant frequency will become

(a) $\dfrac{f}{2}$ (b) $2f$ (c) f (d) $\dfrac{f}{4}$

117. The law which states that a variation in an electric field causes magnetic field, is

(a) Faraday's law (b) Bio-Savart law
(c) Modified Ampere's law (d) Lenz's law

118. Radiation of wavelength 300 nm and intensity 100 W-m^{-2} falls on the surface of a photosensitive material. If 2% of the incident photons produced photoelectron, the number of photoelectrons emitted from an area of 2 cm^2 of the surface is nearly

(a) 15×10^{11} (b) 6.04×10^{14}
(c) 1.5×10^{12} (d) 60.4×10^{15}

119. Potential energy between a proton and an electron is given by $U = \dfrac{ke^2}{3R^3}$, then radius of Bohr's orbit can be given by

(a) $\dfrac{Ke^2m}{h^2}$ (b) $\dfrac{6\pi^3Ke^2m}{n^3h^2}$
(c) $\dfrac{2\pi}{n}\dfrac{Ke^2m}{h^2}$ (d) $\dfrac{4\pi^2Ke^2m}{n^3h^2}$

120. A transistor is connected in common emitter configuration. The collector supply is 8 V and the voltage drop across a resistance of 800 Ω in the collector circuit is 0.5 V. If the current gain factor α is 0.96, then the base current is

(a) 2.6×10^{-5} A (b) 3.6×10^{-5} A
(c) 5.6×10^{-5} A (d) 6.6×10^{-5} A

CHEMISTRY

121. If two particles A and B are moving with the same velocity, but wavelength of A is found to be double than that of B. Which of the following statement is correct?

(a) Both A and B have same mass.
(b) Mass of A is half that of B.
(c) Mass of B is half that of A.
(d) Mass of B is one-fourth that of A.

122. The spectrum of helium is expected to be similar to that of

(a) Li^+ (b) H (c) Na (d) He^+

123. On the basis of Bohr's model, the radius of the 3rd orbit is

(a) equal to the radius of 1st orbit
(b) 3 times the radius of 1st orbit
(c) 5 times the radius of 1st orbit
(d) 9 times the radius of 1st orbit

124. To which group of the periodic table does an element having electronic configuration [Ar]$3d^5 4s^2$ belong?
(a) Second (b) Fourth (c) Seventh (d) Third

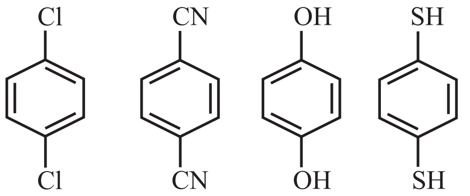
125. Given that ionisation potential and electron gain enthalpy of chlorine are 13eV and 4eV respectively. The electronegativity of chlorine on Mulliken scale, approximately equals to
(a) 8.5 eV (b) 6.0 eV (c) 3.0 eV (d) 1.5 eV

126. Which of the following represents the correct order of increasing electron gain enthalpy with negative sign for the elements?
1. Nitrogen (N) 2. Phosphorus (P)
3. Chlorine (Cl) 4. Fluorine (F)
(a) $P < N < F < Cl$ (b) $N < P < F < Cl$
(c) $Cl < F < P < N$ (d) $F < Cl < N < P$

127. Which of the following will have maximum dipole moment?
(a) NF_3 (b) NCl_3 (c) NBr_3 (d) NH_3

128. In which of the following molecules/ions, the central atom is sp^2 hybridised?
BF_3, NO_2^-, NH_2^- and H_2O
(a) NH_2^- and H_2O (b) NO_2^- and H_2O
(c) BF_3 and NO_2^- (d) NO_2^- and NH_2^-

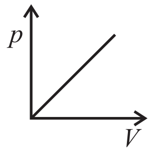
129. For which molecules among the following, the resultant dipole moment $(\mu) \neq 0$?

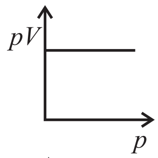


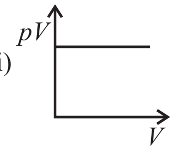
(i) (ii) (iii) (iv)

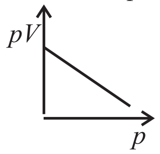
(a) (iii) and (iv) (b) (i) and (ii)
(c) (ii) and (iii) (d) Only (iv)

130. Which of the following graphs correctly represents Boyle's Law?

(i)


(ii)


(iii)


(iv)


(a) (i), (ii) and (iii) (b) (ii) and (iii)
(c) (iii) and (iv) (d) (ii) and (iv)

131. The density of an ideal gas can be given by ...., where *p, V, M, T* and *R* respectively denote pressure, volume, molar-mass, temperature and universal gas constant.
(a) $\frac{pM}{RT}$ (b) $\frac{pV}{RT}$ (c) $\frac{RT}{pM}$ (d) $\frac{RT}{pV}$

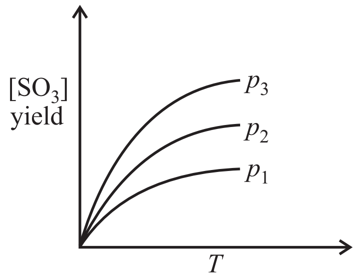
132. When 20 g of $CaCO_3$ is treated with 20 g of HCl, the mass of CO_2 formed would be
(a) 10 g (b) 8.8 g (c) 22.2 g (d) 20 g

133. Which among the following species acts as a self-indicator?
(a) H_2O_2 (b) I^- (c) $Cr_2O_7^{2-}$ (d) MnO_4^-

134. If a chemical reaction is known to be non-spontaneous at 298 K but spontaneous at 350 K, then which among the following conditions is true for the reaction?
(a) $\Delta G = -ve, \Delta H = -ve, \Delta S = +ve$
(b) $\Delta G = +ve, \Delta H = +ve, \Delta S = +ve$
(c) $\Delta G = -ve, \Delta H = +ve, \Delta S = +ve$
(d) $\Delta G = +ve, \Delta H = +ve, \Delta S = -ve$

135. Standard entropies of X_2, Y_2 and XY_3 are 60, 40 and 50 JK^{-1} mol^{-1} respectively. At what temperature, the following reaction will be at equilibrium?
[given : $\Delta H° = -30$ kJ]
$\frac{1}{2}X_2 \quad \frac{3}{2}Y_2 \rightleftharpoons XY_3$
(a) 500 K (b) 750 K (c) 1000 K (d) 1250 K

136. For the reaction $SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$, the percentage yield of product at different pressure is shown in the figure. Then, which among the following is true?



(a) Pressure has no effect (b) $p_1 < p_2 < p_3$
(c) $p_1 > p_2 > p_3$ (d) $p_1 = p_2 = p_3 \neq 0$

137. Which among the following denotes the correct relationship between K_p and K_c for the reaction, $2A(g) \rightleftharpoons B(g) + C(g)$
(a) $K_p > K_c$ (b) $K_c > K_p$
(c) $K_c = (K_p)^2$ (d) $K_p = K_c$

138. Which metal oxide among the following gives H_2O_2 on treatment with dilute acid?
(a) BaO_2 (b) RbO_2 (c) MnO_2 (d) Al_2O_3

139. **Assertion :** K, Rb and Cs form superoxides.
Reason : The stability of superoxides increases from K to Cs due to decrease in lattice energy.
(a) Both A and R are true and R is a correct explanation of A.
(b) Both A and R are true but R is not a correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

140. When borax is dissolved in water, it gives an alkaline solution. The alkaline solution consists the following products

(a) NaOH and BH_3 (b) NaOH and H_3BO_3
(c) $NaHCO_3$ and H_3BO_3 (d) Na_2CO_3 and H_3BO_3

141. Identify (*P*) and (*Q*) in the following reaction.

$$CH_3Cl + Si \xrightarrow[570\,K]{Cu-powder} (P) \xrightarrow{H_2O} (Q)$$

$\longrightarrow$ Straight chain polymer

(a) $P : (CH_3)_3SiCl$, $Q : (CH_3)_3SiOH$
(b) $P : (CH_3)_2SiCl_2$, $Q : (CH_3)_2Si(OH)_2$
(c) $P : (CH_3)_2SiCl_2$, $Q : (CH_3)_2Si(OH)Cl$
(d) $P : (CH_3)_2SiCl_2$, $Q : (CH_3)_2SiO$

142. Green chemistry refers to reactions which

(a) reduce the use and production of hazardous chemicals
(b) study of the extremely show reactions
(c) are related to soil erosion
(d) study of green leaves

143. **Assertion :** Sodium acetate on Kolbe's electrolysis gives ethane.

Reason : Methyl free radical is formed at cathode.

(a) Both A and R are true and R is a correct explanation of A.
(b) Both A and R are true but R is not a correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

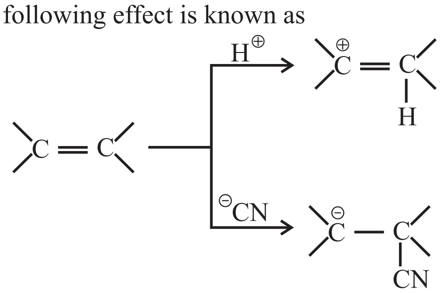
144. When difference in boiling points of two liquids is too small, then the separation is carried out by

(a) steam distillation (b) simple distillation
(c) fractional distillation (d) vacuum distillation

145. In Lassaigne's test for halogens, it is necessary to remove *X* and *Y* from the sodium fusion extract, if nitrogen and sulphur are present. This is done by boiling the extract with *Z*. Identify *X, Y* and *Z*.

(a) $X = NaNO_3$, $Y = Na_2SO_4$, $Z =$ conc. HNO_3
(b) $X = NaNH_2$, $Y = NaSH$, $Z =$ conc. HCl
(c) $X = Na_2NO_2$, $Y = NaSO_3H$, $Z =$ conc. H_2SO_4
(d) $X = NaCN$, $Y = Na_2S$, $Z =$ conc. HNO_3

146. The following effect is known as



(a) inductive effect (b) electromeric effect
(c) resonance effect (d) hyperconjugation

147. Which of the following will form an ideal solution?

(a) C_2H_5OH and H_2O
(b) HNO_3 and H_2O
(c) $CHCl_3$ and CH_3COCH_3
(d) C_6H_6 and $C_6H_5CH_3$

148. The molal elevation constant is the ratio of elevation in boiling point to

(a) molarity
(b) molality
(c) mole fraction of solute
(d) mole fraction of solvent

149. When a current of 10 A is passes through molten $AlCl_3$ for 1.608 minutes. The mass of Al deposited will be [Atomic mass of Al = 27 g]

(a) 0.09 g (b) 0.81 g (c) 1.35 g (d) 0.27 g

150. The molar conductivities $\left(\lambda^{\circ}_{m}\right)$ at infinite dilution of KBr, HBr and KNH_2 are 120.5, 420.6 and 90.48 S cm^2 mol^{-1} respectively. Find the value of λ°_{m} for NH_3.

(a) 511.0 S cm^2 mol^{-1} (b) 390.5 S cm^2 mol^{-1}
(c) 256.2 S cm^2 mol^{-1} (d) 240.9 S cm^2 mol^{-1}

151. If the rate constant for a first order reaction is 2.303×10^{-3} s^{-1}. Find the time required to reduce 4 g of the reactant to 0.2 g.

(a) 1.30 hours (b) 21.60 hours
(c) 0.36 hours (d) 2.60 hours

152. A plot of $\log(x / m)$ *versus* $\log(p)$ for adsorption of a gas on a solid gives a straight line with a slope of

(a) $-\log k$ (b) $\log(1/n)$
(c) $\frac{1}{n}$ (d) antilog$(1/n)$

153. Match the following compounds with their corresponding physical properties.

	Column I		Column II
A.	IBr	1.	Orange solid
B.	ClF_3	2.	Yellow-green liquid
C.	BrF_3	3.	Black solid
D.	ICl_3	4.	Colourless gas

	A	B	C	D		A	B	C	D
(a)	2	1	4	3	(b)	1	3	2	4
(c)	3	4	2	1	(d)	4	2	1	3

154. What is coordination number of the metal in $[Co(en)_2Cl_2]^{2+}$?

(a) 3 (b) 4 (c) 5 (d) 6

155. A compound *A* is used in paints instead of salts of lead. Compound *A* is obtained when a white compound *B* is strongly heated. Compound *B* is insoluble in water but dissolves in NaOH solution forming a solution of compound *C*. The compound *A* on heating with coke gives a volatile metal *D* and a gas *E* which burns with a blue flame. Identify the possible species *D* and *C* can be respectively?

(a) $D = Hg$, $C = Hg(OH)_2$
(b) $D = Cd$, $C = Na_2(CdO_2)$
(c) $D = Zn$, $C = Na_2ZnO_2$
(d) $D = Zn$, $C = Zn(OH)_2$

156. Identify the product of the following reaction.

$$(C_6H_{10}O_5)_n + nH_2O \xrightarrow[\substack{393\text{ K} \\ 2\text{–}3\text{ atm}}]{H^+} ?$$

(a) Fructose (b) Glucose
(c) Lactose (d) Maltose

157. The number of optical isomers possible for 2-bromo-3-chloro butane are

(a) 8 (b) 10 (c) 4 (d) 2

158. During the action of enzyme 'zymase' glucose is converted into ...., with the liberation of carbon dioxide gas.

(a) phenol (b) ethanol
(c) methanol (d) isopropyl alcohol

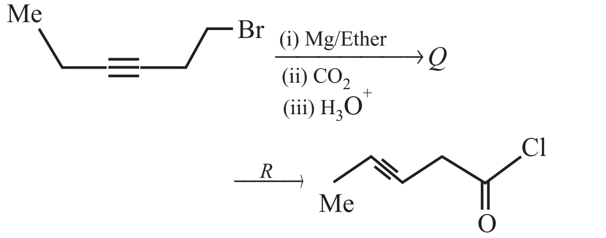
159. The total number of products formed in the following reaction sequence is

$$CH_3COCl \xrightarrow{(i)\,(CH_3)_2Cd} (P)$$

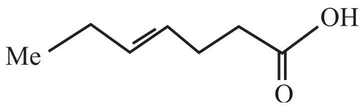
$$(P) + CH_3CHO \xrightarrow{(ii)\ NaOH\,(aq),\,\Delta} ?$$

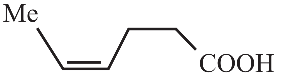
(a) 2 (b) 4 (c) 1 (d)3

160. In the following reaction sequence, identify product 'Q' and reagent 'R'.



(a)  and $SOCl_2$

(b)  and SO_2Cl_2

(c)  and $SOCl_2$

(d)  and $CH_3SO_2Cl_2$

ANSWER KEY

1	(b)	**17**	(a)	**33**	(b)	**49**	(c)	**65**	(c)	**81**	(a)	**97**	(a)	**113**	(c)	**129**	(a)	**145**	(d)
2	(c)	**18**	(b)	**34**	(c)	**50**	(a)	**66**	(c)	**82**	(a)	**98**	(d)	**114**	(a)	**130**	(b)	**146**	(b)
3	(a)	**19**	(c)	**35**	(c)	**51**	(c)	**67**	(c)	**83**	(a)	**99**	(a)	**115**	(d)	**131**	(a)	**147**	(d)
4	(c)	**20**	(b)	**36**	(d)	**52**	(d)	**68**	(b)	**84**	(b)	**100**	(a)	**116**	(a)	**132**	(b)	**148**	(b)
5	(d)	**21**	(c)	**37**	(a)	**53**	(b)	**69**	(d)	**85**	(b)	**101**	(d)	**117**	(c)	**133**	(d)	**149**	(a)
6	(b)	**22**	(d)	**38**	(b)	**54**	(d)	**70**	(a)	**86**	(a)	**102**	(c)	**118**	(b)	**134**	(c)	**150**	(b)
7	(c)	**23**	(b)	**39**	(a)	**55**	(b)	**71**	(a)	**87**	(b)	**103**	(d)	**119**	(*)	**135**	(b)	**151**	(c)
8	(a)	**24**	(c)	**40**	(d)	**56**	(c)	**72**	(d)	**88**	(b)	**104**	(c)	**120**	(a)	**136**	(b)	**152**	(c)
9	(d)	**25**	(c)	**41**	(b)	**57**	(b)	**73**	(c)	**89**	(b)	**105**	(d)	**121**	(b)	**137**	(d)	**153**	(c)
10	(d)	**26**	(b)	**42**	(a)	**58**	(a)	**74**	(b)	**90**	(a)	**106**	(d)	**122**	(a)	**138**	(a)	**154**	(d)
11	(b)	**27**	(c)	**43**	(c)	**59**	(d)	**75**	(a)	**91**	(b)	**107**	(b)	**123**	(d)	**139**	(c)	**155**	(c)
12	(b)	**28**	(c)	**44**	(b)	**60**	(c)	**76**	(b)	**92**	(d)	**108**	(d)	**124**	(c)	**140**	(b)	**156**	(b)
13	(b)	**29**	(a)	**45**	(a)	**61**	(a)	**77**	(d)	**93**	(c)	**109**	(a)	**125**	(a)	**141**	(b)	**157**	(c)
14	(b)	**30**	(b)	**46**	(b)	**62**	(c)	**78**	(c)	**94**	(d)	**110**	(a)	**126**	(b)	**142**	(a)	**158**	(b)
15	(b)	**31**	(b)	**47**	(c)	**63**	(d)	**79**	(c)	**95**	(c)	**111**	(c)	**127**	(d)	**143**	(c)	**159**	(b)
16	(c)	**32**	(a)	**48**	(c)	**64**	(d)	**80**	(a)	**96**	(c)	**112**	(a)	**128**	(c)	**144**	(c)	**160**	(a)

Hints & Solutions

MATHEMATICS

1. **(b)** Given, $f(x) = 2x + 1$, $g(x) = x^2 - 2$
Since, $gof(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 - 2$
$= 4x^2 + 4x + 1 - 2 = 4x^2 + 4x - 1$

2. **(c)** Given, $f(x) = \dfrac{a^x + a^{-x}}{2}$
Then, $f(x + y) + f(x - y)$
put $x + y \to x$ and $x - y \to x$
$= \dfrac{a^{x+y} + a^{-(x+y)}}{2} + \dfrac{a^{x-y} + a^{-(x-y)}}{2}$
$= \dfrac{(a^x + a^{-x})(a^y + a^{-y})}{2}$
$= 2.\dfrac{(a^x + a^{-x})}{2}.\dfrac{(a^y + a^{-y})}{2} = 2f(x)f(y)$

3. **(a)** We know that
$\{x\} = x - [x]$ and $f : (0, 1) \to (0, 1)$
Defined by $f(x) = \min \{x - [x], -x - [x]\}$ which is an identity function, as $x \in (0, 1)$ and $\{x\} \in (0, 1)$
Where, $\{.\}$ is fractional part function.
$\Rightarrow (fofofof)(x) = x$

4. **(c)** Given the statement $8n + 16 \le 2^n$, $n \in N$
$\Rightarrow 8(n + 2) \le 2^n \Rightarrow n + 2 \le 2^{n-3}$
Clearly, for n = 6
$6 + 2 \le 2^{6-3}$ or $8 \le 8$ is true.

5. **(d)** Since if we put $x = -1$
We can see R_1 and R_3 became identical so A = 0
Let $A = \begin{vmatrix} 1 & -3 & 1 \\ 1 & 6 & 4 \\ 1 & 3x & x^2 \end{vmatrix} = 0$
Hence $x = -1$ is a root of the given equation
$\Rightarrow$ from option, $x^2 - x - 2 = 0$

6. **(b)** Given $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ and, $ab = \dfrac{5}{2}$
Then, $A^T = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$
$\because AA^T = 20I$
$\Rightarrow \begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ -b & a \end{bmatrix} = 20\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$\therefore a^2 + b^2 = 20 \Rightarrow (a + b)^2 - 2ab = 20$
$\Rightarrow (a+b)^2 = 20 + 2 \times \dfrac{5}{2} \quad \left[\because ab = \dfrac{5}{2}\right]$
or $a + b = \pm 5$
Equation whose roots are a and b is given by
$x^2 + (a + b)x + ab = 0$
or $x^2 \pm 5x + \dfrac{5}{2} = 0$ or $2x^2 \pm 10x + 5 = 0$

7. **(c)** Since $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ Hence, $|A| = 10$ ...(i)
and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$
$\therefore |10B| = 4(2\alpha) - 2(-15 - \alpha) + 2(10) = 10\alpha + 50$
$\Rightarrow 10^3 |B| = 10\alpha + 50 \quad [\because |kA| = k^n |A|]$ ...(ii)
$\because B = A^{-1}$
$\Rightarrow |B| = |A^{-1}| = |A|^{-1} \Rightarrow |B| = \dfrac{1}{|A|} = \dfrac{1}{10}$
From Eq. (ii), we get
$1000.\dfrac{1}{10} = 10\alpha + 50$ or $\alpha = \dfrac{50}{10} = 5$

8. **(a)** Given $A = \begin{bmatrix} 4 & 2 & 1-x \\ 5 & k & 1 \\ 6 & 3 & 1+x \end{bmatrix}$
and, rank of matrix A is 1.
$\Rightarrow$ All the minors of order 2 are zero.
$\therefore M_{33} = 0 \Rightarrow \begin{vmatrix} 4 & 2 \\ 5 & k \end{vmatrix} = 0$
or $4k - 10 = 0$ or $k = \dfrac{5}{2}$
and $M_{11} = \begin{vmatrix} k & 1 \\ 3 & 1+x \end{vmatrix} = 0$
or $\begin{vmatrix} \frac{5}{2} & 1 \\ 3 & 1+x \end{vmatrix} = 0$
or $\dfrac{5}{2}x = \dfrac{1}{2}$ or $x = \dfrac{1}{5}$

9. **(d)** Given $a_1, a_2, ..., a_9$ are in GP.
Then, $\dfrac{a_2}{a_1} = \dfrac{a_3}{a_2} = \dfrac{a_9}{a_8} = \lambda$ (say) [common ratio]

Now, $\begin{vmatrix} \log a_1 & \log a_2 & \log a_3 \\ \log a_4 & \log a_5 & \log a_6 \\ \log a_7 & \log a_8 & \log a_9 \end{vmatrix}$

$C_3 \to C_3 - C_2, C_2 \to C_2 - C_1$

$$\begin{vmatrix} \log a_1 & \log \frac{a_2}{a_1} & \log \frac{a_3}{a_2} \\ \log a_4 & \log \frac{a_5}{a_4} & \log \frac{a_6}{a_5} \\ \log a_7 & \log \frac{a_8}{a_7} & \log \frac{a_9}{a_8} \end{vmatrix}$$

or $\begin{vmatrix} \log a_1 & \log \lambda & \log \lambda \\ \log a_4 & \log \lambda & \log \lambda \\ \log a_7 & \log \lambda & \log \lambda \end{vmatrix} = 0$

[$\because$ By property of determinant C_2 and C_3 are identical]

10. **(d)** Given $(\sin\theta - i\cos\theta)^3 = (-i)^3 (\cos\theta + i\sin\theta)^3$
$= (-i)^3 (\cos 3\theta + i \sin 3\theta)$ [by De-Moivre theorem]

11. **(b)** Let the real part of $(\cos 4 + i \sin 4 + 1)^{2020}$
$= (2\cos^2 2 + i\, 2 \sin 2 \cos 2)^{2020}$
$= 2^{2020} \cos^{2020}(2) (\cos 2 + i \sin 2)^{2020}$
$= 2^{2020} \cos^{2020}(2) (\cos 4040 + i \sin 4040)$
$= 2^{2020} \cos^{2020} 2. \cos 4040$ $[z + \bar{z} = 2\text{Re}(z)]$

12. **(b)** Given equation $(x^2 + 5x + 5)^{x+5} = 1$
If $x + 5 = 0 \Rightarrow x = -5$
and $x^2 + 5x + 5 = 1$ or $x^2 + 5x + 4 = 0$
$\Rightarrow x = -1, -4$
$x = -1, -4$ and -5 are the three integers satisfying given equation.

13. **(b)** Given $f(x) = x^3 + ax^2 + bx + c$
Let roots of $f(x)$ are α, β, γ
since roots are in AP. $\Rightarrow 2\beta = \alpha + \gamma$
Now, sum of roots

$\alpha + \beta + \gamma = -a \Rightarrow \beta = -\frac{a}{3}$

It is given that, roots are integers.
$\therefore$ β is an integer when a is multiple of 3.
Option (a), (c) and (d) are multiple of 3.
$\Rightarrow a \neq 1214$

14. **(b)** Given $e^{4t} - 10e^{3t} + 29e^{2t} - 22e^{t} + 4 = 0$ ...(i)
Let $e^t = x$
then, $x^4 - 10x^3 + 29x^2 - 22x + 4 = 0$
Here, roots are x_1, x_2, x_3, x_4
Hence product of roots $x_1.x_2.x_3.x_4 = 4$

$\Rightarrow e^{t_1}.e^{t_2}.e^{t_3}.e^{t_4} = 4 \Rightarrow e^{t_1+t_2+t_3+t_4} = 4$

$\Rightarrow t_1 + t_2 + t_3 + t_4 = \log_e 4 = \log_e 2^2 = 2\log_e 2$
Sum of roots $= 2 \log_e 2$

15. **(b)** Given total number of coins = 3
The number of different sums $= {}^3C_1 + {}^3C_2 + {}^3C_3$
$= 3 + 3 + 1 = 7$

16. **(c)** Given number of white balls = 7
and number of black balls = 3
Here, B – B – B – B – B – B – B – B
are the places in row where black balls can be placed.

So, number of arrangements $= {}^8C_3 = \frac{8\times7\times6}{1\times2\times3} = 56$

17. **(a)** Since, each girl is to given at least one ring so, 3 rings to give 3 girls in one way since rings are identical, so, balls can be arranged in ${}^{n-1}C_{r-1}$ ways. Then, number of ways is ${}^7C_2 = 21$.

18. **(b)** Given $\frac{x^4}{(x-1)(x-2)} = f(x) + \frac{A}{x-1} + \frac{B}{x-2}$

$\frac{x^4}{(x-1)(x-2)} = \frac{x^4}{x^2 - 3x + 2}$ is an improper fraction.

$$\therefore \frac{x^4}{(x-1)(x-2)} = x^2 + 3x + 7 + \frac{15x - 14}{(x-1)(x-2)}$$
$$= x^2 + 3x + 7 + \frac{A}{x-1} + \frac{B}{x-2}$$

On comparing with given equation,

$\frac{x^4}{(x-1)(x-2)} = f(x) + \frac{A}{x-1} + \frac{B}{x-2}$, we have

$f(x) = x^2 + 3x + 7$

19. **(c)** Let, $A = 2\alpha$, $B = 30° - \alpha$, $C = 60° - \alpha$
then $A + B + C = 2\alpha + 30° - \alpha + 60° - \alpha = 90°$
$\because \tan(A + B + C)$

$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

If $A + B + C = 90°$, then
$1 - \tan A \tan B - \tan B \tan C - \tan C \tan A = 0$
or $\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$
$\therefore \tan 2\alpha \tan(30° - \alpha) + \tan 2\alpha.\ \tan(60° - \alpha)$
$+ \tan(30° - \alpha)\tan(60° - \alpha) = 1$

20. **(b)** Given $\sin\alpha - \cos\alpha = m$ and $\sin 2\alpha = n - m^2$
$\therefore (\sin\alpha - \cos\alpha)^2 = m^2$
$\sin^2\alpha + \cos^2\alpha - 2\sin\alpha\cos\alpha = m^2$
or $1 - \sin 2\alpha = m^2$
or $\sin 2\alpha = 1 - m^2$
But $\sin 2\alpha = n - m^2$ (given)
$\Rightarrow n = 1$

21. **(c)** Given $3 \operatorname{cosec} x = 4 \sin x$
or $\sin^2 x = 3/4$

or $\sin^2 x = \sin^2 \frac{\pi}{3} \Rightarrow x = n\pi \pm \frac{\pi}{3}, n \in Z$

or $x = \pm\frac{\pi}{3}$

22. **(d)** Given $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \ldots$

$$+ \tan^{-1}\left(\frac{}{1+n(n+1)}\right) = \tan^{-1}(\ )$$

$$\Rightarrow \tan^{-1}\left(\frac{2-1}{1+1.2}\right)+\tan^{-1}\left(\frac{3-2}{1+2.3}\right)+...$$

$$+\tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)=\tan^{-1}(x)$$

$\Rightarrow \tan^{-1}(2) - \tan^{-1}(1) + \tan^{-1}(3) - \tan^{-1}(2) + ...$
$+ \tan^{-1}(n + 1) - \tan^{-1}(n) = \tan^{-1}(x)$

$\Rightarrow \tan^{-1}(n + 1) - \tan^{-1}(1) = \tan^{-1}(x)$

$$\left\{\because \tan^{-1}(A)-\tan^{-1}(B)=\tan^{-1}\left(\frac{A-B}{1+A.B}\right)\right\}$$

$$\Rightarrow \tan^{-1}\left(\frac{n}{n+2}\right)=\tan^{-1}(x)\Rightarrow x=\frac{n}{n+2}$$

23. **(b)** Since, given sinh u = tanθ

$\Rightarrow \tan\theta = \sinh u$

$\Rightarrow \tan\theta=\sqrt{\cosh^2 u-1}$ $\quad [\cosh^2 x - \sinh^2 x = 1]$

$\Rightarrow \cosh u=\sqrt{1+\tan^2\theta}\Rightarrow \cosh u=\sec\theta$

24. **(c)** Given in ΔABC, a = 3, b = 4, sin A = 3/4

$\therefore \cos A=\sqrt{1-\sin^2 A}=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$

Since, $\cos A=\frac{b^2+c^2-a^2}{2bc}$ $\Rightarrow \frac{\sqrt{7}}{4}=\frac{4^2+c^2-3^2}{2.4.c}$

$\Rightarrow c^2-2\sqrt{7}c+7=0$ $\Rightarrow (c-\sqrt{7})^2=0$

$\Rightarrow c=\sqrt{7}$

So, $\cos B=\frac{c^2+a^2-b^2}{2ac}$

$\Rightarrow \cos B=\frac{7+9-16}{2.3.\sqrt{7}}=0\Rightarrow B=90°$ $\therefore \angle CBA = 90°$

25. **(c)** Since A = 75°, B = 45°, then C = 60°

$\because \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$ where R is circum radius

$\Rightarrow$ a = 2R sin A, b = 2R sin B, c = 2R sin C

Then, $b+c\sqrt{2}=2R(\sin B+\sqrt{2}\sin C)$

$=2R(\sin 45°+\sqrt{2}\sin 60°)$

$=2R\left(\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{2}}\right)=2R\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)$

= 2. (2R sin 75°) = 2 (2R sin A) = 2a

26. **(b)** In ΔABC,

Sides opposite to ∠A, and ∠B can be given as

$a=2R\sin A=4R\sin\frac{A}{2}\cos\frac{A}{2}$

$b=2R\sin B=4R\sin\frac{B}{2}\cos\frac{B}{2}$

$r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$r_1=4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

$r_2=4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

$r_3=4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Now, $ab-r_1r_2=16R^2\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{A}{2}\cos\frac{B}{2}\left(1-\cos^2\frac{C}{2}\right)$

$=16R^2\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{A}{2}\cos\frac{B}{2}\sin^2\frac{C}{2}$

$\therefore \frac{ab-r_1r_2}{r_3}=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=r$

27. **(c)** Here cosα = cosβ and cosγ = 0 $\quad (\because \gamma = 90°)$

Hence, $\cos^2\alpha + \cos^2\alpha + \cos^2\gamma = 1$

$\Rightarrow 2\cos^2\alpha = 1$ $\Rightarrow \cos\alpha=\pm\frac{1}{\sqrt{2}}$

$\Rightarrow$ α = 45° or 135°

28. **(c)** P.V. of pt A is $5\hat{i}-4\hat{j}-3\hat{k}$

and P.V. of x-axis i.e. $(\hat{i}+0\hat{j}+0\hat{k})$

Hence, angle,

$$\cos\theta=\frac{5.1+(-4).0+(-3).0}{\sqrt{5^2+(-4)^2+(-3)^2}\sqrt{1^2+0^2+0^2}}$$

$=\frac{5}{\sqrt{50}}=\frac{1}{\sqrt{2}}$ $\therefore \theta=\frac{\pi}{4}$

29. **(a)** Let P.V. of A, B, C is represented as $\vec{a},\vec{b},\vec{c}$ respectively.

Mid-points D, E, F is $\frac{\vec{a}+\vec{b}}{2},\frac{\vec{a}+\vec{c}}{2},\frac{\vec{b}+\vec{c}}{2}$ respectively.

Then, $\overrightarrow{BE}=\frac{\vec{a}-2\vec{b}+\vec{c}}{2},\overrightarrow{AF}=\frac{\vec{b}-2\vec{a}+\vec{c}}{2}$

$\overrightarrow{CD}=\frac{\vec{a}+\vec{b}-2\vec{c}}{2}$

$\overrightarrow{BE}+\overrightarrow{AF}=\frac{2\vec{c}-\vec{a}-\vec{b}}{2}=\frac{-(\vec{a}+\vec{b}-2\vec{c})}{2}$

$=-\overrightarrow{CD}=\overrightarrow{DC}$

30. **(b)** Let $\vec{a}=\hat{i}+a\hat{j}+\hat{k}$, $\vec{b}=\hat{j}+a\hat{k}\Rightarrow \vec{a}=a\,\hat{i}+\hat{k}$

Volume of parallelopiped = $[\vec{a}\,\vec{b}\,\vec{c}]$

So, $[\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix}1 & a & 1\\ 0 & 1 & a\\ a & 0 & 1\end{vmatrix}$

$\Rightarrow$ V = $a^3 - a + 1$

Differentiating V with respect to a

$$\frac{dV}{da} = 3a^2 - 1$$

For stationary points $\frac{dV}{da} = 0$

$\Rightarrow 3a^2 - 1 = 0$ or $a = \pm\frac{1}{\sqrt{3}}$

At, $a = \frac{1}{\sqrt{3}}$,

$\because \frac{d^2V}{da^2} = 6a$, $\Rightarrow \frac{d^2V}{da^2} = 6.\frac{1}{\sqrt{3}} > 0$

$\therefore$ Volume is minimum at $a = \frac{1}{\sqrt{3}}$

31. **(b)** Given $\vec{a} = \frac{3}{2}\hat{k}, \vec{b} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{2}$

$$\vec{a} + \vec{b} = \frac{3}{2}\hat{k} + \frac{2\hat{i} + 2\hat{j} - \hat{k}}{2} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{a} - \vec{b} = \frac{3}{2}\hat{k} - \frac{2\hat{i} + 2\hat{j} - \hat{k}}{2} = -\hat{i} - \hat{j} + 2\hat{k}$$

Let θ be the angle between (a + b) and (a – b) then

$$= \frac{1(-1) + 1(-1) + 1(2)}{\sqrt{1^2 + 1^2 + 1^2}\sqrt{(-1)^2 + (-1)^2 + 2^2}} = 0$$

$\therefore \theta = \frac{\pi}{2}$ or 90°

32. **(a)** Given $\overline{a} = \hat{i} + \hat{j} + \hat{k}$, $\overline{b} = \hat{i} + 3\hat{j} + 5\hat{k}$

$\overline{c} = 7\hat{i} + 9\hat{j} + 11\hat{k}$

Then, $\overline{a} + \overline{b} = 2\hat{i} + 4\hat{j} + 6\hat{k}$

and $\overline{b} + \overline{c} = 8\hat{i} + 12\hat{j} + 16\hat{k}$

Area of parallelogram $= \frac{1}{2}|(\overline{a} + \overline{b}) \times (\overline{b} + \overline{c})|$

$$= \frac{1}{2}|2(\hat{i} + 2\hat{j} + 3\hat{k}) \times 4(2\hat{i} + 3\hat{j} + 4\hat{k})|$$

$$= 4\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 4|-\hat{i} + 2\hat{j} - \hat{k}| = 4\sqrt{6} \text{ sq. units.}$$

33. **(b)** Given $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = \hat{i} + 3\hat{j} - \hat{k}, \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$

$\vec{a}.\vec{a} = 4 + 1 + 9 = 14$

$\vec{a}.\vec{b} = \vec{b}.\vec{a} = 2 + 3 - 3 = 2$

$\vec{a}.\vec{c} = \vec{c}.\vec{a} = 6 - 1 - 6 = -1$

$\vec{b}.\vec{b} = 1 + 9 + 1 = 11$

$\vec{b}.\vec{c} = \vec{c}.\vec{b} = 3 - 3 + 2 = 2$

$\vec{c}.\vec{c} = 9 + 1 + 4 = 14$

Now, $\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = \begin{vmatrix} 14 & 2 & -1 \\ 2 & 11 & 2 \\ -1 & 2 & 14 \end{vmatrix} = 2025$

34. **(c)** Given $|\vec{a}| = 2, |\vec{b}| = 3$

$\because (\vec{a} + t\vec{b}) \perp (\vec{a} - t\vec{b})$

$\Rightarrow (\vec{a} + t\vec{b}).(\vec{a} - t\vec{b}) = 0$

$\Rightarrow \vec{a}.\vec{a} - t(\vec{a}.\vec{b}) + t(\vec{b}.\vec{a}) - t^2(\vec{b}.\vec{b}) = 0$

$\Rightarrow |\vec{a}|^2 - t^2|\vec{b}|^2 = 0$

$\Rightarrow t^2 = \frac{|\vec{a}|^2}{|\vec{b}|^2} \quad \Rightarrow t = \frac{|\vec{a}|}{|\vec{b}|} = \frac{2}{3}$

35. **(c)** Given variates x = 112, 116, 120, 125, 132

x	$(x-\overline{x})^2$
112	81
116	25
120	1
125	16
132	121

$\Sigma x = 605; \ \Sigma(x - \overline{x})^2 = 244$

Mean $= \overline{x} = \frac{605}{5} = 121$

Variance $= \frac{\Sigma(x - \overline{x})^2}{n} = \frac{244}{5} = 48.8$

36. **(d)** Clearly, difference of values from arithmetic mean is least in option (d).

$\Rightarrow$ Option (d) has least standard deviation.

37. **(a)** Since total distribution = 3^{12}

Number of distribution when first box contains three balls = ${}^{12}C_3 \times 2^9$

Required probability = $\frac{{}^{12}C_3 \times 2^9}{3^{12}}$

38. **(b)** Since number of letters in the word REGULATIONS = 11. So, number of possible arrangements = 11!

We are required 4 letter between R and E.

So we can select 4 letters out of 9 letters

Favourable number of arrangements

= ${}^9C_4 \times 4! \times 2! \times 6!$

Required probability = $\frac{{}^9C_4 \times 4! \times 2!6!}{11!} = \frac{6}{55}$

39. **(a)** Since events E = {X is a prime number}
So, P(E) = P(X = 2) + P (X = 3) + P (X = 5) + P (X = 7)
= 0.23 + 0.12 + 0.20 + 0.07 = 0.62
Given F = $\{x < 4\}$
P(F) = P (X = 1) + P (X = 2) + P (X = 3)
= 0.15 + 0.23 + 0.12 = 0.50
P (E ∩ F) = P (X = 2) + P (X = 3)
= 0.23 + 0.12 = 0.35
P (E ∪ F) = P(E) + P(F) – P (E ∩ F)
= 0.62 + 0.50 – 0.35 = 0.77

40. **(d)** Since sample spaces of a dice S = {1, 2, 3, 4, 5, 6}
Let E = getting even number ⇒ E = {2, 4, 6}
Hence, P(E) $= \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

If dice is thrown three times
Let X = success, then

X	$x = 0$	$x = 1$	$x = 2$	$x = 3$
P(X = x)	${}^3C_0\left(\frac{1}{2}\right)^3$	${}^3C_1\left(\frac{1}{2}\right)^3$	${}^3C_2\left(\frac{1}{2}\right)^3$	${}^3C_3\left(\frac{1}{2}\right)^3$

Probability of getting at least 2 successes
= P (X ≥ 2) = P(X = 2) + P (X = 3)
$$= {}^3C_2\left(\frac{1}{2}\right)^3 + {}^3C_3\left(\frac{1}{2}\right)^3 = \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$$

41. **(b)** Given, coordinate $(x, y) = (2\sqrt{2}, -3\sqrt{2})$
Let new coordinates be (x', y').
Then, the new coordinate is given as
$x' = x\cos\theta + y\sin\theta$ and $y' = y\cos\theta - x\sin\theta$
Since, $\theta = 45°$
$$\therefore x' = 2\sqrt{2}\cos 45° + (-3\sqrt{2})\sin 45°$$
$$= 2\sqrt{2}.\frac{1}{\sqrt{2}} - 3\sqrt{2}.\frac{1}{\sqrt{2}} = 2 - 3 = -1$$
and $y' = (-3\sqrt{2})\cos 45° - 2\sqrt{2}\sin 45° = -2 - 3 = -5$
New coordinate is (–1, –5).

42. **(a)** Given line, $5x - 2y = 10$
$$\Rightarrow \frac{x}{2} + \frac{y}{-5} = 1$$
Sum of the squares of the intercepts
$= 2^2 + (-5)^2 = 29$

43. **(c)** Since, a, b and c are three consecutive odd integers.
So, a, b and c will be in A.P.
$\Rightarrow 2b = a + c \Rightarrow a - 2b + c = 0$
which is similar to the line $ax + by + c = 0$ passing through (1, –2).
$\therefore (\alpha, \beta) = (1, -2)$
$\Rightarrow \alpha^2 + \beta^2 = 1^2 + (-2)^2 = 5$

44. **(b)** Let the equation of line parallel to X-axis is $y = k$
Then, point of intersection of the line and the curve $y = \sqrt{x}$ is (k^2, k)
Now, slope of the curve is $y' = \frac{1}{2\sqrt{x}}$
or, $\left(\frac{dy}{dx}\right)_{(k^2, k)} = \tan 45° = 1$
$$\Rightarrow \frac{1}{2\sqrt{k^2}} = 1 \Rightarrow k = \frac{1}{2} \quad \therefore \text{ Line is } y = \frac{1}{2}$$

45. **(a)** Since line $2x + 3y + 4 = 0$ is perpendicular bisector of line through A (1, 2) and B (α, β)
Hence, distance of point A from the line will be equal to the distance of B from line.
$$\therefore \frac{|2+6+4|}{\sqrt{13}} = \frac{|2\alpha + 3\beta + 4|}{\sqrt{13}}$$
$\Rightarrow 2\alpha + 3\beta + 4 = \pm 12$
$\Rightarrow 2\alpha + 3\beta = 8$ ...(i)
$2\alpha + 3\beta = -16$ ...(ii)
Also, the line joining A and B is perpendicular to $2x + 3y + 4 = 0$
⇒ Product of slopes = –1
$$\therefore \frac{\beta - 2}{\alpha - 1} \times \left(\frac{-2}{3}\right) = -1$$
$\Rightarrow 2\beta - 3\alpha = 1$ ...(iii)
Solving eqs. (ii) and (iii), we have
$$\alpha = \frac{-35}{13}, \beta = \frac{-46}{13}$$
$\therefore 13\alpha + 13\beta = -35 - 46 = -81$

46. **(b)** Since the equation of the pair of lines perpendicular to the pair $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ and passes through origin is given by $bx^2 - 2hxy + ay^2 = 0$
$$\Rightarrow a = 2, b = 2, h = \frac{3}{2}$$
So, required pair of lines $2x^2 - 3xy + 2y^2 = 0$

47. **(c)** Given lines,
$x + y = k$ ...(i)
and $2y^2 + 5xy - 3x^2 = 0$
or $(y + 3x)(2y - x) = 0$
$\Rightarrow y + 3x = 0$ ...(ii)
$2y - x = 0$ ...(iii)
On solving eqs. (i), (ii) and (iii), we get vertices of triangle, which are (0, 0), $\left(\frac{2k}{3}, \frac{k}{3}\right)$ and $\left(\frac{-k}{2}, \frac{3k}{2}\right)$
$$\therefore \text{Centroid} = \left(\frac{k}{18}, \frac{11k}{18}\right)$$
Comparing with given centroid $\left(\frac{1}{18}, \frac{11}{18}\right)$, we have k = 1

48. **(c)** Given homogeneous equation is $5x^2 - 8xy + 3y^2 = 0$
It can be broken as
$\Rightarrow (5x - 3y)(x - y) = 0$
$\Rightarrow 5x - 3y = 0$ or $x - y = 0$
Slopes $m_1 = \frac{5}{3}, m_2 = 1 \Rightarrow m_1 : m_2 = \frac{5}{3} : 1 = 5 : 3$

49. **(c)** Let m be the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$
Hence, the other line has slope m^2
Now, $m + m^2 = \frac{-2h}{b}$ and $m.m^2 = \frac{a}{b}$
$\because (m + m^2)^3 = m^3 + (m^2)^3 + 3m.m^2(m + m^2)$
$\Rightarrow \frac{-8h^3}{b^3} = \frac{a}{b} + \frac{a^2}{b^2} - \frac{6ah}{b^2}$
$\Rightarrow \frac{a}{b} + \frac{a^2}{b^2} - \frac{6ah}{b^2} + \frac{8h^3}{b^3} = 0$
or $ab(a + b) + 8h^3 = 6abh$
or $\frac{a+b}{h} + \frac{8h^2}{ab} = 6$
or $\left|\frac{a+b}{h} + \frac{8h^2}{ab}\right| = 6$

50. **(a)** Given equation of circle $x^2 + y^2 = 50$ ...(i)
and a line $x + 7 = 0$ ...(ii)
On solving both the equations simultaneously
We get (–7, –1) and (–7, 1) as pt. of inter-section
$\therefore$ (–7, 1) and (–7, –1) are points of contact of eqs. (i) and (ii),
for circle $x^2 + y^2 = r^2$
Since, equation of tangent at (x_1, y_1) is
$xx_1 + yy_1 = r^2$
$\therefore$ Equation of tangent of at (–7, 1) and (–7, –1) are
$-7x + y = 50$ or $7x - y + 50 = 0$
and $-7x - y = 50$ or $7x + y + 50 = 0$

51. **(c)** Let be any point on the circle $x^2 + y^2 = r_1^2$
$\Rightarrow h_1^2 + k_1^2 = r_1^2$ ...(i)
Equation of chord of contact of tangents from (h, k) to the circle $x^2 + y^2 = r_2^2$ is
$hx + ky = r_2^2$ ...(ii)
$\because$ Eq. (ii) touches the circle, $x^2 + y^2 = r_3^2$
The length of perpendicular from centre (0, 0) on Eq. (ii) is radius = r_3
$\Rightarrow \frac{r_2^2}{\sqrt{h^2 + k^2}} = r_3 \Rightarrow \frac{r_2^2}{\sqrt{r_1^2}} = r_3$
$\Rightarrow r_2^2 = r_1 r_3 \Rightarrow r_1, r_2, r_3$ are in G.P.

52. **(d)** Since circle touches at (3, 0) hence its centre is $(3, k)$
So, equation of circle is $(x - 3)^2 + (y - k)^2 = r^2$
it passes through (1, –2)
and $4 + k^2 + 4k + 4 = k^2$
$\Rightarrow 8 + 4k = 0 \Rightarrow k = -2$
$\therefore$ Required equation of circle
$(x - 3)^2 + (y + 2)^2 = 2^2$
or $x^2 + y^2 - 6x + 4y + 9 = 0$

53. **(b)** Since equation of line passes through origin
Let $L_1 : y = mx$ given, intercepts made by L_1 and L_2 are equal
$\Rightarrow L_1$ and L_2 are at the same distance from centre of circle $x^2 + y^2 - x + 3y = 0$
Now, centre $\left(\frac{1}{2}, \frac{-3}{2}\right)$ [$\because$ centre = (–g, –f)]
Distance of L_1 from $\left(\frac{1}{2}, -\frac{3}{2}\right) = \frac{\left|m.\frac{1}{2} + \frac{3}{2}\right|}{\sqrt{m^2 + 1}}$
Distance of L_2 from $\left(\frac{1}{2}, -\frac{3}{2}\right) = \frac{\left|\frac{1}{2} - \frac{3}{2} - 1\right|}{\sqrt{1+1}}$
$\because L_2 : x + y - 1 = 0$ [$\because y = mx \Rightarrow mx - y = 0$]
$\Rightarrow \frac{\left|\frac{m+3}{2}\right|}{\sqrt{m^2+1}} = \frac{2}{\sqrt{2}}$
Squaring both sides, we get
$\frac{(m+3)^2}{4(m^2+1)} = \frac{4}{2} \Rightarrow 7m^2 - 6m - 1 = 0$
$\Rightarrow m = \frac{6 \pm \sqrt{36 + 28}}{14} = \frac{6 \pm 8}{14} = 1, \frac{-1}{7}$
$\therefore L_1$ is $y = x$ and $y = \frac{-1}{7}x$
or $x - y = 0$ or $x + 7y = 0$

54. **(d)** Given, circle
$x^2 + y^2 + 4x + 16y - 30 = 0$ ...(i)
Where, $g_1 = 2, f_1 = 8, c_1 = -30$
Now equation of circle whose centre at (1, 2) is
$x^2 + y^2 - 2x - 4y + c = 0$ ...(ii)
Here, $g_2 = -1, f_2 = -2, c_2 = c$
Since, circles cut each other orthogonally
So, $2(g_1g_2 + f_1f_2) = c_1 + c_2$
$\Rightarrow 2(-2, -16) = -30 + c$ or $c = -6$
From eqs. (ii), we get
$x^2 + y^2 - 2x - 4y - 6 = 0$
Radius $r = \sqrt{(-1)^2 + (-2)^2 - (-6)} = \sqrt{11}$

55. **(b)** Let the circles as S_1, S_2 and S_3

Hence, $S_1 : x^2 + y^2 - 8x + 40 = 0$

$S_2 : x^2 + y^2 - 5x + 16 = 0$

$S_3 : x^2 + y^2 - 8x + 16y + 160 = 0$

To obtain the point which has same power is

$S_2 - S_1 = 0$ and $S_3 - S_1 = 0$

$\Rightarrow 3x - 24 = 0$ and $16y + 120 = 0$

$\Rightarrow x = 8$ and $y = \frac{-15}{2} \Rightarrow 2at = 2$

$\therefore$ Required point is $\left(8, \frac{-15}{2}\right)$.

56. **(c)** Here given the parabola, $y^2 = 8x$

$\Rightarrow a = 2$ and given one end of focal chord $\left(\frac{1}{2}, 2\right)$

Let parametric form of ends of focal chord = $(at_1^2, 2at)$

$\Rightarrow 2at = 2$

$\therefore 2.2.t = 2 \Rightarrow t = \frac{1}{2}$

Hence, length of focal chord = $a\left(t + \frac{1}{t}\right)^2$

$$= 2\left(\frac{1}{2} + 2\right)^2 = 2.\left(\frac{5}{2}\right)^2 = \frac{25}{2}$$

57. **(b)** Given equation of ellipse, $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Centre $C = (0, 0)$

If $P(x, y)$ be any point on the ellipse, then maximum value of CP = length of the semi-major axis = 5 minimum value of CP = length of semi-minor axis = 4

Their sum = 5 + 4 = 9

58. **(a)** Since, we know that any tangent to hyperbola forms a triangle with asymptotes which has constant area = ab

Given $ab = a^2 \tan\alpha \Rightarrow \frac{b}{a} = \tan\alpha$

Eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \tan^2\alpha} = \sec\alpha$

59. **(d)** Let the line joining the points A (2, 4, 5) and B (3, 5, –4) is divided by YZ-plane at (0, y, z) in the ratio 1 : λ . Then, by section formula $\frac{2\lambda + 3}{\lambda + 1} = 0$

$\Rightarrow \frac{1}{\lambda} = -\frac{2}{3}$ or 2 : 3 (externally)

60. **(c)** Since the line makes equal angles with the coordinate axes, then $\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

or $3\cos^2\alpha = 1$ or $\cos\alpha = \pm\frac{1}{\sqrt{3}}$

Direction cosines are $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$

61. **(a)** Let $P(x, y, z)$ and $O(0, 0, 0)$

DR's of $OP = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$

But it is given that DR's of $OP = (1, -2, -2)$

$\because x = 1, y = -2, z = -2$

$\therefore$ Coordinate of $P = (1, -2, -2)$

62. **(c)** Given $\lim_{z \to 1} \frac{z^{1/3} - 1}{z^{1/6} - 1}$ $\left[\frac{0}{0}\text{ form}\right]$

$$= \lim_{z \to 1} \frac{\frac{1}{3} z^{-2/3}}{\frac{1}{6} z^{-5/6}} = \frac{6}{3} \frac{(1)^{5/6}}{(1)^{2/3}} = \frac{6}{3} = 2$$

63. **(d)** Given f is continuous at $x = 0$

$\therefore \lim_{x \to 0} f(x) = f(0)$

$$\Rightarrow \lim_{x \to 0} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}} = k \log 2 \log 3$$

$\Rightarrow K \log 2. \log 3$

$$= \lim_{x \to 0} \frac{(9^x - 1)(8^x - 1)(\sqrt{2} + \sqrt{1 + \cos x})}{2 - (1 + \cos x)}$$

$$= \lim_{x \to 0}\left[\frac{(9^x - 1)}{x}.\left(\frac{8^x - 1}{x}\right).\frac{x^2}{1 - \cos x}.(\sqrt{2} + \sqrt{1 + \cos x})\right]$$

$$= \log 9. \log 8. 2(\sqrt{2} + \sqrt{2}) \quad \left\{\because \lim_{x \to 0} \frac{a^x - 1}{x} = \log a\right\}$$

$\Rightarrow k \log 3. \log 2 = 24\sqrt{2} \log 3. \log 2 \Rightarrow k = 24\sqrt{2}$

64. **(d)** $\because$ Given function $f(x)$ is continuous in $[0, \pi]$

$\Rightarrow f$ is continuous at $x = \frac{\pi}{4}, \frac{\pi}{2}$

at $x = \frac{\pi}{4}$

$$\Rightarrow \lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{x \to \frac{\pi}{4}^+} f(x) = 2\left(\frac{\pi}{4}\right)\cot\frac{\pi}{4} + b$$

$$\Rightarrow \lim_{x \to \frac{\pi}{4}^-} (x + a\sqrt{2}\sin x) = \lim_{x \to \frac{\pi}{4}^+} (2x\cot x + b) = \frac{\pi}{2} + b$$

$$\Rightarrow \frac{\pi}{4} + a\sqrt{2}\sin\frac{\pi}{4} = 2.\frac{\pi}{4}\cot\frac{\pi}{4} + b$$

$\Rightarrow a - b = \frac{\pi}{4}$ ...(i) at $x=\frac{\pi}{2}$

$$\Rightarrow \lim_{x\to\frac{\pi^-}{2}} f(x)=\lim_{x\to\frac{\pi^+}{2}} f(x) \Rightarrow \lim_{x\to\frac{\pi^-}{2}}(2x\cot x+b)$$

$$=\lim_{x\to\frac{\pi^+}{2}}(a\cos 2x-b\sin x)$$

$$\Rightarrow 2.\frac{\pi}{2}\cot\frac{\pi}{2}+b=a\cos\pi-b\sin\frac{\pi}{2}$$

$\Rightarrow$ a = – 2b ...(ii)

On solving eqs. (i) and (ii), we get

$$a=\frac{\pi}{6}, b=-\frac{\pi}{12}$$

65. **(c)** Given $y=x+\frac{1}{x}$ ...(i)

On differentiating we get, $y'=1-\frac{1}{x^2}$

or $x^2y'=x^2-1$ ...(ii)

$\Rightarrow x^2=xy-1$ $\left\{y'=1-\frac{1}{x^2}\right\}$

$\therefore$ From eq. (ii), we have

$x^2y'=xy-1-1$ or $x^2y'-xy+2=0$

66. **(c)** Given $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

Putting $x^2=\cos 2\theta$ (where $x^2\le 1$)

$$\Rightarrow \theta=\frac{1}{2}\cos^{-1}(x^2)$$

$$\therefore y=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$$

$$=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)\Rightarrow y=\frac{\pi}{4}+\theta$$

$$\Rightarrow y=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}(x^2)$$

$$\therefore \frac{dy}{dx}=\frac{1}{2}.\left(\frac{-2x}{\sqrt{1-x^4}}\right)=\frac{-x}{\sqrt{1-x^4}}$$

67. **(c)** Given 3 sin xy + 4 cos xy = 5

Differentiating with respect to x, we get

$$3\cos xy.\left(x\frac{dy}{dx}+y\right)-4\sin xy\left(x\frac{dy}{dx}+y\right)=0$$

$$\Rightarrow x\frac{dy}{dx}+y=0 \text{ or } \frac{dy}{dx}=\frac{-y}{x}$$

68. **(b)** Given $f(x)=\sqrt{x^2+1}$, $g(x)=\frac{x+1}{x^2+1}$

and $h(x)=2x-3$

$$\Rightarrow f'(x)=\frac{2x}{2\sqrt{x^2+1}}=\frac{x}{\sqrt{x^2+1}}$$

and $h'(x)=2$ So, $h'(g'(x))=2$

$$\therefore f'(h'(g'(x)))=f'(2)=\frac{2}{\sqrt{2^2+1}}=\frac{2}{\sqrt{5}}$$

69. **(d)** Let radius of circle is r and error is Δr

Since area of circle

$$A=\pi r^2 \Rightarrow \frac{dA}{dr}=2\pi r$$

We know $\left(\frac{dA}{dr}\right)=\Delta r$

$\Rightarrow$ $(2\pi p)$ 0.05% $\Rightarrow$ 2. (0.05) = 0.1

$\therefore$ Error in calculating area = 0.1%

70. **(a)** Given curve is $y=8x^2-x^4-4$

Differentiating w.r.t. x

$$\frac{dy}{dx}=16x-4x^3=4x(2-x)(2+x)$$

For stationary points $\frac{dy}{dx}=0$

or $4x(2-x)(2+x)=0$

$\Rightarrow x=0, x=2, x=-2$

Putting in the given curve we get y = 0, 2, –2

Hence, Stationary points are

(0, –4), (2, 12), (–2, 12).

71. **(a)** Given (i) $f(x)=x|x|=\begin{cases}x(-x), & x<0\\ x(x), & x\ge 0\end{cases}$

$$f(x)=\begin{cases}-x^2, & x<0\\ x^2, & x\ge 0\end{cases}$$

$$f'(x)=\begin{cases}-2x, & x<0\\ 2x, & x\ge 0\end{cases}$$

$$f'(x)>0, \forall x\in R-\{0\}$$

$\Rightarrow f(x)$ is strictly increasing on R – {0}.

72. **(d)** $f(x)=-x^3+4ax^2+2x-5$

$f'(x)=-3x^2+8ax+2$

For decreasing $f'(x)<0$

Discriminant of $f'(x)=(8a)^2-4(-3)(2)$

$=64a^2+24>0, \forall a\in$ R

Hence, $f(x)$ is decreasing for no value of a.

73. **(c)** Given curve, $y=e^{2x}+x^2$

At $x=0$, $y=e^0+0=1$

$$\Rightarrow \frac{dy}{dx}=2e^{2x}+2x$$

Let, find slope at (0, 1)

and $\frac{dy}{dx} = 2.e^0 + 0 = 2$

$y - 1 = \frac{-1}{2}(x - 0) \Rightarrow x + 2y - 2 = 0$

Distance between normal to the curve and origin is

$\frac{|1.0 + 2.0 - 2|}{\sqrt{1^2 + 2^2}} = \frac{2}{\sqrt{5}}$

74. **(b)** Given $\int \frac{dx}{x\sqrt{x^4 - 1}}$

$\Rightarrow \frac{1}{2}\int \frac{2x\,dx}{x^2\sqrt{x^4 - 1}} = \frac{1}{k}\sec^{-1}(x^k)$

Let $x^2 = t \Rightarrow 2x\,dx = dt$

$= \frac{1}{2}\int \frac{dt}{t\sqrt{t^2 - 1}} = \frac{1}{2}\sec^{-1} t$

$\Rightarrow \frac{1}{k}\sec^{-1}(x^k) = \frac{1}{2}\sec^{-1}(x^2) \Rightarrow k = 2$

75. **(a)** Let $I = \int \frac{e^x(x+3)}{(x+5)^3}dx = \int e^x\left[\frac{1}{(x+5)^2} - \frac{2}{(x+5)^3}\right]dx$

Since $\int [f(x) + f'(x)]e^x dx = e^x f(x) + C$

$\Rightarrow I = \frac{e^x}{(x+5)^2} + C$

76. **(b)** Given $\int \frac{(x-1)^2}{(x^2+1)^2}dx = \tan^{-1}(x) + g(x) + k$

$= \int\left(\frac{1}{x^2+1} - \frac{2x}{(x^2+1)^2}\right)dx = \tan^{-1}(x) - \int \frac{2x}{(x^2+1)^2}dx$

Let $x^2 + 1 = t \Rightarrow 2x\,dx = dt$

$= \tan^{-1}(x) - \int t^{-2}dt = \tan^{-1}(x) + \frac{1}{t} + k$

$\Rightarrow \int \frac{(x-1)^2}{(x+1)^2}dx = \tan^{-1}(x) + \frac{1}{x^2+1} + k$

Comparing with $\int \frac{(x-1)^2}{(x^2+1)^2}dx = \tan^{-1}(x) + g(x) + k$

we have $g(x) = \frac{1}{x^2+1}$

77. **(d)** On solving the given equation we get

$\frac{1}{A}\log|(\sin x)^{2023} + (\cos x)^{2023}| + C$

$= \int \frac{1 - (\cot x)^{2021}}{\tan x + (\cot x)^{2022}}dx = \int \frac{1 - \frac{(\cos x)^{2021}}{(\sin x)^{2021}}}{\frac{\sin x}{\cos x} + \frac{(\cos x)^{2022}}{(\sin x)^{2022}}}dx$

$= \int \frac{\cos x(\sin x)^{2022} - \sin x(\cos x)^{2022}}{(\sin x)^{2023} + (\cos x)^{2023}}dx$

$\int \frac{f'(x)}{f(x)}dx = \log|f(x)| + C$

$\Rightarrow \frac{1}{A}\log|(\sin x)^{2023} + (\cos x)^{2023}| + C$

$= \frac{1}{2023}\log|(\sin x)^{2023} + (\cos x)^{2023}| + C$

$\Rightarrow A = 2023$

78. **(c)** Let $I = \int_2^4 (|x-2| + |x-3|)\,dx$

$= \int_2^4 (x-2)\,dx - \int_2^3 (x-3)\,dx + \int_3^4 (x-3)\,dx$

$= \left(\frac{x^2}{2} - 2x\right)_2^4 - \left(\frac{x^2}{2} - 3x\right)_2^3 + \left(\frac{x^2}{2} - 3x\right)_3^4$

$= \left[\left(\frac{16}{2} - 8\right) - \left(\frac{4}{2} - 4\right)\right] - \left[\left(\frac{9}{2} - 9\right) - \left(\frac{4}{2} - 6\right)\right]$

$+ \left[\left(\frac{16}{2} - 12\right) - \left(\frac{9}{2} - 9\right)\right]$

$= [2 + 9 - 8] = 3$

79. **(c)** Let $I = \int_{-1/2}^{1/2} \left\{[x] + \log\left(\frac{1+x}{1-x}\right)\right\}dx$

By property

$\because \int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & f(-x) = f(x) \\ 0, & f(-x) = -f(x) \end{cases}$

$= \int_{-1/2}^{1/2}[x]dx + \int_{-1/2}^{1/2} \log\left(\frac{1+x}{1-x}\right)dx$

Let $f(x) = \log\left(\frac{1+x}{1-x}\right)$

$= \int_{-1/2}^{1/2} [x]dx + 0 = \int_{-1/2}^{0} [x]dx + \int_0^{1/2} [x]dx$

$= \int_{-1/2}^{0} (-1)\,dx + \int_0^{1/2} 0\,dx$

$= -(x)_{-1/2}^{0} + 0 = -\left(0 + \frac{1}{2}\right) = -\frac{1}{2}$

80. **(a)** Here the differential equation $\frac{d^2y}{dx^2}+y=0$

A solution is function of x which satisfies given differential equation.

Consider option (a) $y = 3\sin x + 4\cos x$

$$\frac{dy}{dx}=3\cos x-4\sin x$$

$$\frac{d^2y}{dx^2}=-3\sin x-4\cos x=-y$$

$$\Rightarrow \frac{d^2y}{dx^2}+y=0$$

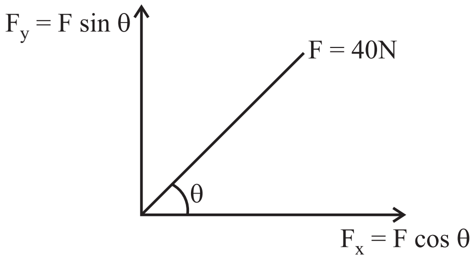
PHYSICS

81. **(a)** According to Faraday's law of EMI, when we move a conductor in magnetic field region, there will be induced current in the conductor and the same phenomenon is used in case of electric generator.

82. **(a)** In retardation, the speed reduces continuously and as the body is moving in a straight line, Therefore, displacement will be equal to distance covered by body.

83. **(a)** Time of flight (T) = 2usinθ/g $=\frac{2\times 50\times \sin 30°}{10}=5\text{s}$

84. **(b)**



Clearly, $40\cos\theta=20\sqrt{3}$

$$\cos\theta=\frac{\sqrt{3}}{2}\Rightarrow \theta=30$$

and $F_y = 40\sin 30 = 20$ N

85. **(b)** **Case 1 :** $s = 0.t_1 + 1/2gt_1^2 \Rightarrow s=\frac{1}{2}gt_1^2$ ...(A)

[$\because a = g$]

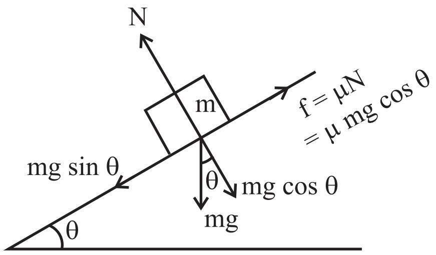
If a_{net} is net acceleration of ball, then in case 2 when lift is going up with acceleration a

$$a_{net}=(g+a)\Rightarrow s=\frac{1}{2}(g+a)t_2^2 \quad ...(B)$$

From eqs. (A) and (B), we get

$t_1 > t_2$

86. **(a)**



Force along the plane, $mg\sin\theta - f = ma$

$\Rightarrow mg\sin\theta - \mu N = ma$ ...(i)

Since, $N = mg\cos\theta$ ...(ii)

From eqs. (i) and (ii), we get

$mg\sin\theta - \mu mg\cos\theta = ma$

$a = g(\sin\theta - \mu\cos\theta)$

87. **(b)** After collision, the two balls will stick together and their combined velocity be v_0

Now, by using law of conservation of momentum,

$m_1u_1 + m_2u_2 = (m_1 + m_2)v_0$

$$\Rightarrow v=\frac{2\times 10+3\times 0}{5}=4\text{ms}^{-1}$$

$$=\left(\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2\right)-\frac{1}{2}(m_1+m_2)V_0^2$$

$$=\left(\frac{1}{2}\times 2\times 10^2+\frac{1}{2}\times 3\times 0^2\right)-\frac{1}{2}(2+3)4^2$$

= 100 – 40 = 60J

88. **(b)** Given, mass of body, $m = 8$ kg

Equation of displacement, $s = t^2/4$

$$V=\frac{ds}{dt}=\frac{t}{2}$$

By work energy theorem

W = ΔK

$$K_f - K_i=\frac{1}{2}mv_f^2-0 = \frac{1}{2}\times 8\times\left(\frac{4}{2}\right)^2 = 16\text{ J}$$

89. **(b)** $\int_{p_1}^{p_2} dp=\int_0^T Fdt$ = Area under graph

$$=\frac{1}{2}\times\frac{T}{4}\times F_0+\frac{2T}{4}\times F_0+\frac{1}{2}\times\frac{T}{4}\times F_0 = \frac{3F_0T}{4}$$

So, $p_2-p_1=\frac{3F_0T}{4}\Rightarrow mv-0=\frac{3F_0T}{4}\Rightarrow F_0=\frac{4mv}{3T}$

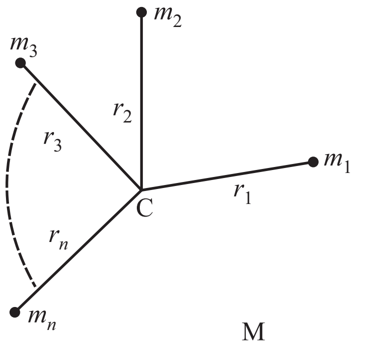
90. **(a)** Moment of inertia (I) for following bodies are

(i) Ring = MR^2 (ii) Angular disc $=\frac{M}{2}(R^2-r^2)$

(iii) Solid disc = $MR^2/2$ (iv) Cylindrical disc = $MR^2/2$

Clearly, MoI of ring is maximum

91. **(b)** as $r = \frac{m_1r_1 + m_2r_2 + m_3r_3 + ... + m_nr_n}{M}$ ...(i)



If CoM lies at origin then, $\vec{r} = 0$

$\therefore$ From eq. (i), we get

$m_1r_1 + m_2r_2 + m_3r_3 + ... m_nr_n = 0$ ...(ii)

Net moment of all particles in the system about centre of mass C.

$\tau_c = m_1gr_1 + m_2gr_2 + m_3gr_3 + ... m_ngr_n$

$= g\,[m_1r_1 + m_2r_2 + m_3r_3 + ... m_nr_n]$

$= g \times 0 = 0$ [from Eq. (ii)]

Hence, sum of moments of all the particles in a system about its COM is always zero.

92. **(d)** As we know that,

Earth rotates from West to East

$\therefore$ The train going towards East will have high speed with respect to earth from the train going towards West

$$N \propto V^2$$

and speeds of different trains are different. Hence, magnitude of normal reaction is different.

$\therefore$ Assertions will be false and reason will be true because their relative speeds are different.

But actual speeds of both the trains are same.

93. **(c)** Since, force F = $mg = k\Delta x$

$$k = \frac{mg}{\Delta x}$$

$$\Rightarrow k = \frac{m_1g}{\Delta x_1} = \frac{0.6 \times 10}{0.4} = 15\,\text{N/m}$$

and time period,

$$T_2 = 2\pi\sqrt{\frac{m_2}{k}}\quad T_2 = 2\pi\sqrt{\frac{0.255}{15}}$$

$$= 2\pi\sqrt{0.017} = 0.82\text{s}$$

94. **(d)** As $T = 2\pi\sqrt{\frac{m}{k}}$.

So T depends on m & k which remain unchanged

So, time period will remain same.

95. **(c)** Work done per unit mass = $\frac{W}{M_1}$

$$\therefore \frac{W}{M_1} = \frac{GM_2}{r} \qquad \left[\because W = V = \frac{GM_1M_2}{r}\right]$$

$$= \frac{6.67 \times 10^{-11} \times 1000}{1} = 6.67 \times 10^{-8}\ \text{J/kg}$$

96. **(c)** $a_n = \frac{GM}{(R+h)^2} = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$$\frac{a_n}{n} = \frac{R}{20} = \frac{g}{\left(1+\frac{1}{20}\right)^2} = \frac{20^2 g}{21^2} \Rightarrow g = \frac{20^2}{21^2}g$$

$$\Rightarrow g = \frac{21^2}{20^2} \times 9\,\text{m/s}^2$$

$$a_{d=\frac{R}{20}} = g\left(1 - \frac{d}{R}\right)$$

$$= \frac{21^2}{20^2} \times 9\left(1 - \frac{1}{20}\right) = \frac{21^2}{20^2} \times 9 \times \frac{19}{20} = 9.426$$

≈ 9.5 m/s^2

97. **(a)** Here, $\Delta l = \frac{mgl}{2AY} \Rightarrow Y = \frac{mgl}{A\Delta l} \times \frac{1}{2}$

$$\therefore\ Y = \frac{\rho(A.l).g.l}{2A.\Delta l} \qquad [\because M = \rho Al]$$

where, A is area of cross-section

$$\therefore\ Y = \frac{\rho l^2 g}{2\Delta l} \Rightarrow \Delta l = \frac{\rho l^2 g}{2Y}$$

$$= \frac{1.5 \times (12 \times 10^{-2})^2 \times 10}{2 \times 5 \times 10^8} = 2.16 \times 10^{-10}\ \text{m}$$

98. **(d)** As, H = $\frac{2T\cos\theta}{\rho gR}$

$$\Rightarrow R = \frac{2T\cos\theta}{\rho gH} = \frac{2 \times 7.5 \times 10^{-2} \times \cos 0°}{1000 \times 10 \times 7.5 \times 10^{-2}}$$

$= 2 \times 10^{-4}$ m = 0.2 mm

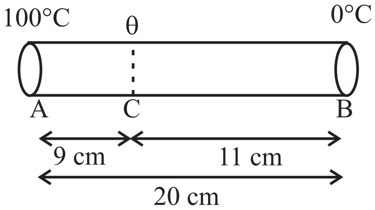
99. **(a)** As h = constant

$$P + \frac{1}{2}\rho v^2$$

$\Rightarrow$ PW $\Rightarrow$ VM

So, where pressure is least, velocity is maximum

100. (a) As the flow is steady,



$\therefore \frac{KA(100-\theta)}{9} = \frac{KA(\theta-0)}{11}$

$\Rightarrow 1100 - 11\theta = 9\theta \Rightarrow \theta = 55°C$

101. (d) As, $\Delta\ell = \ell_0\alpha\Delta T$

let final heat coefficient be α''

$\therefore$ (L + 2L) $\alpha''\Delta T = L\alpha\Delta T + 2L \cdot 2\alpha \cdot \Delta T$

$\Rightarrow 3\alpha'' = \alpha + 4\alpha = 5\alpha$

$\Rightarrow \alpha'' = 5\alpha/3$

102. (c) By using 1st law of thermodynamics,

$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = \Delta Q - \Delta W$

In both path ΔU is same

So, $\Delta Q_1 - \Delta W_1 = \Delta Q_2 - \Delta W_2$

or, $Q_1 - W_1 = Q_2 - W_2$

103. (d) Since, pV^{γ} = constant

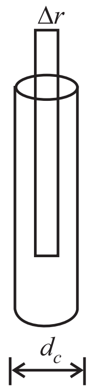
$\therefore p_1 V_1^{y} = p_2 V_2^{\gamma}$

$\Rightarrow p_1/p_2 = (V_2/V_1)^y = \left(\frac{1}{4}\right)^{3/2}$ $[\because V_1 = yV_2]$

$\Rightarrow p_1/p_2 = \frac{1}{8}$ Hence, $p_2 : p_1 = 8 : 1$

104. (c) Radius of piston, $r_p = (7.5 - 0.008) \times 10^{-2}$ m

$= 7.492 \times 10^{-2}$ m



Let, Δr_c and Δr_p is increase in radius of cylinder and piston respectively, for fully filled piston in cylinder.

New radius will be $r'_c = r_c + \Delta r_c$

$= r_c + r_c\alpha_c\Delta T = r_c(1 + \alpha_c\Delta T)$

$= 7.5 \times 10^{-2}[1 + 1.2 \times 10^{-5}(T - 303)]$

Similarly, new radius of piston,

$r'_p = 7.492 \times 10^{-2}[I + 1.6 \times 10^{-5}(T - 303)]$

Now, for fully fitted piston, $r_c = r_p$

$7.5 \times 10^{-2}[1 + 1.2 \times 10^{-5}(T - 303)]$

$= 7.492 \times 10^{-2}[1 + 1.6 \times 10^{-5}(T - 303)]$

$\Rightarrow 1.0011 + 1.2013 \times 10^{-5}(T - 303)$

$= 1 + 1.6 \times 10^{-5}(T - 303)$

$\Rightarrow 1.1 \times 10^{-3} = (T - 303) \times 10^{-5}(1.6 - 1.2013)$

$= (T - 303) \times 10^{-5} \times 0.3987$

$\Rightarrow T - 303 = 275$

$\Rightarrow T = 578\text{ K} = 305°C$

105. (d) For an ideal gas, we have

$\frac{pV}{T} = \text{constant} \Rightarrow \frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

$\Rightarrow V_2 = \frac{p_1V_1T_2}{T_1p_2}$ $\Rightarrow V_2 = \frac{4\times1500\times270}{300\times2} = 2700\,\text{m}^3$

106. (d) We have

$\lambda = \frac{(\text{speed})v}{(\text{frequency})f} = \frac{350}{350} = 1\text{ m} = 100\text{ cm}$

$\therefore \Delta\phi = \frac{2\pi}{100}\times25 = \frac{\pi}{2}$ $\left[\because \Delta\phi = \frac{2\pi}{\lambda}\Delta x\right]$

As, resultant amplitude,

$a_{net} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

$\Rightarrow a_{net} = \sqrt{0.3^2 + 0.4^2 + 2\times0.3\times0.4\cos\frac{\pi}{2}}$

$= \sqrt{\frac{9}{100}+\frac{16}{100}+0} = \sqrt{\frac{25}{100}} = 0.5$ mm

107. (b) Let first secondary maximum occur at angle θ_2.

$\because n\lambda = a\sin\theta_1$

$\Rightarrow a = \frac{n\lambda}{\sin 30°} = 2\lambda$ $[\because n = 1]$

and for maxima and $\theta_1 = 30°$

$(2n+1)\frac{\lambda}{2} = a\sin\theta_2 \Rightarrow (2+1)\frac{\lambda}{2} = a\sin\theta_2$

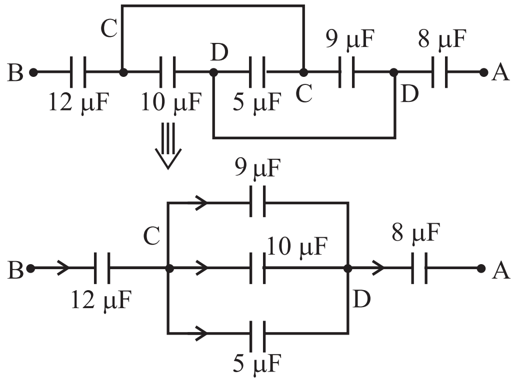
$\Rightarrow \sin\theta_2 = \frac{3\lambda}{2a}$ ...(i)

Substituting the value of $a = 2\lambda$ in Eq. (i), we get

$\therefore \sin\theta_2 = \frac{3\lambda}{2\times2\lambda} = \frac{3}{4}$ $\Rightarrow \theta_2 = \sin^{-1}\left(\frac{3}{4}\right)$

108. (d) Statement (d) is incorrect because electric field lines cannot form closed loop.

109. (a) According to given circuit diagram.



Parallel equivalent capacitance,

$C'_{eq} = C_1 + C_2 + C_2$

$\therefore C'_{eq} = 9 + 10 + 5 = 24\ \mu F$

and series equivalent capacitance,

$$\frac{1}{C_{eq}} = \frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{12} + \frac{1}{24} + \frac{1}{8} \Rightarrow \frac{1}{C_{eq}} = \frac{2+1+3}{24}$$

$$\Rightarrow C_{eq} = \frac{24}{6}\mu F = 4\mu F$$

Since, $C = \frac{Q}{V}$ $\therefore$ Q = CV

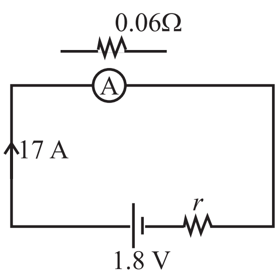
$= 4 \times 10^{-6} \times 80 = 320\ \mu C$

Now, 9V + 10V + 5V = 320

$\Rightarrow V = \frac{40}{3}$ volt. So, $q_{10\mu F} = 10 \times \frac{40}{3} = 133.3\mu C$

110. (a) Clearly, 1.8 = 17 (0.06 + r)

$$\Rightarrow \frac{1.8}{17} = 0.06 + r$$



$$\Rightarrow r = \frac{1.8}{17} - 0.06 = 0.046\ \Omega$$

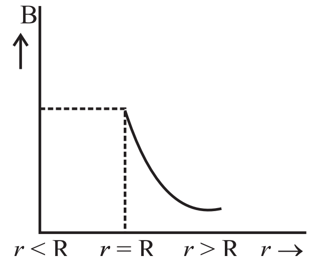
111. (c) $F = Bqv \sin\theta$, clearly if $V = 0 \Rightarrow F = 0$

112. (a) Here, $B(r < R) = 0$

$$B(r = R) = \frac{1}{2\pi}\frac{\mu_0 I}{R}$$

and $B(r > R) = \frac{1}{2\pi}\frac{\mu_0 I}{r}$

So, these variations are correctly shown as below.



113. (c) Here, first apparent dip = δ_1

Second apparent dip = δ_2

Declination = θ

Then, $\tan\theta = \frac{\tan\delta_1}{\tan\delta_2} \Rightarrow \theta = \tan^{-1}\left(\frac{\tan\delta_1}{\tan\delta_2}\right)$

114. (a) As, we know that,

emf induced, $\varepsilon = -\frac{Nd\phi}{dt}$

and, A is area of cross-section.

$\therefore\ \varepsilon = -\frac{NBA\cos\theta}{t} \Rightarrow \varepsilon \propto N$

$\therefore$ If n increases, emf also increases and this back emf resist motion of magnet.

Therefore, option (a) is correct.

115. (d) L_{eq_1} (series) $= L_1 + L_2$

$\therefore\ L_{eq_1} = L_1 + L_2 = 8$ ...(i)

and $\frac{1}{L_{eq2}}$(parallel) $= \frac{1}{L_1} + \frac{1}{L_2} \Rightarrow L_{eq_2} = \frac{L_1 L_2}{L_2 + L_1}$

$$\Rightarrow \frac{3}{2} = \frac{L_1(8 - L_1)}{8}$$

$\Rightarrow L_1^2 - 8L_1 + 12 = 0$

$\Rightarrow L_1(L_1 - 6) - 2(L_1 - 6) = 0$

$\therefore L_1 = 2H$ or $6\ H$

and $L_2 = (8 - 2)$ or $(8 - 6)$

= 6H or 2H

$\therefore$ Difference in inductance is

$(L_1 - L_2)$ or $(L_2 - L_1) = 6 - 2 = 4H$

116. (a) $f_r \propto \frac{1}{\sqrt{C}} \Rightarrow f_{r_2} = f_{r_1}\sqrt{\frac{C_1}{C_2}}$

$\Rightarrow f_{r_2} = \frac{f_{r_1}}{2}$ $[\because C_2 = yC_1]$

117. **(c)** According to modified Ampere's law or Maxwell-Ampere's law,

$\vec{\nabla}\times\vec{B}=\mu_0(\vec{J}+\vec{J}_d)$

Where $\vec{J}_d=\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}$

118. **(b)** Since $I=\dfrac{E}{At}$

So, energy fall = IA

per sec = $100 \times 2 \times 10^{-4}$

= 2×10^{-2} J/s

Energy of 1 photon = $\dfrac{12400}{3000}eV$

= 4.13 eV = 6.61×10^{-19} J

Number of photon falling per sec $=\dfrac{2\times10^{-2}}{6.61\times10^{-19}}$

= 3.02×10^{16}

Number of photoelecton emitted

= $0.02 \times 3.02 \times 10^{16} = 6.04 \times 10^{14}$

119. **(*)** Force, $F=\dfrac{-dU}{dR}$

$$\therefore F=-\frac{Ke^2}{3}\frac{d}{dR}R^{-3}\Rightarrow\frac{mv^2}{R}=-\frac{Ke^2}{3}.(-3)R^{-4}$$

$$\therefore\frac{mv^2}{R}=\frac{Ke^2}{R^4}$$

$$\Rightarrow R^3=\frac{Ke^2}{mv^2} \qquad ...(i)$$

By Bohr's quantization rule

$$L=mvR=\frac{nh}{2\pi}\Rightarrow v=\frac{nh}{2\pi mR}$$

Substituting the value in Eq. (i), we get

$$R^3=\frac{Ke^2}{m.\dfrac{n^2h^2}{4\pi^2m^2R^2}}\Rightarrow R^3=\frac{Ke^2.4\pi^2m^2R^2}{mn^2h^2}$$

$$\therefore R=\frac{4\pi^2.Ke^2.m}{n^2h^2}$$

120. **(a)**

CHEMISTRY

121. **(b)** $\lambda_A=2\times\lambda_B;\ 2\times\dfrac{h}{m_Bv_B}=\dfrac{h}{m_Av_A}$

Given, $\upsilon_A=\upsilon_B;\ \dfrac{2}{m_B}=\dfrac{1}{m_A}\Rightarrow m_A=\dfrac{m_B}{2}$

122. **(a)** Number of electrons in Li^+ = Number of electrons in helium = 2.

123. **(d)** For first orbit (n = 1), $r_1=\left(\dfrac{1}{z}\right)\times0.529$Å

For third orbit (n = 3), $r_3=\left(\dfrac{9}{z}\right)\times0.529$Å

Hence, $r_3 = 9r_1$

124. **(c)** For electronic configuration = [Ar] $3d^54s^2$

Group = number of valence electrons = 5 + 2 = 7

125. **(a)** $EN=\dfrac{IE+EA}{2}$ ...(i)

$EN=\dfrac{13+4}{2}=\dfrac{17}{2}=8.5$ eV

126. **(b)** Due to small size of F atom, it has lower EGE than that of Cl atom

Order of increasing electron gain enthalpy

N < P < F < Cl

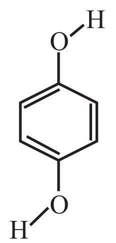
127. **(d)** In NH_3, the high dipole moment is due to same direction of dipole moments due to lp and bp of electrons.

Dipole moment order :

NH_3	>	NBr_3	>	NCl_3	>	NF_3
(1.4 D)		(0.8 D)		(0.6 D)		(0.24 D)

128. **(c)** The central atoms in BF_3 and NO_2^- are sp^2 hybridised.

129. **(a)** The O–H and S–H bond dipoles in (iii) and (iv) molecules do not cancel each other as they exist in different conformations.



130. **(b)** pV = constant at cons. T and n.

131. **(a)** $pV = nRT$ $V = n\ RT/p$

Density, $D = M/V$ (for 1 mole)

$$\Rightarrow D=\frac{M}{RT/p}\Rightarrow D=\frac{pM}{RT}$$

132. **(b)**

$$\underset{\substack{\text{Calcium}\\\text{carbonate}\\(100g)}}{CaCO_3(s)}+\underset{\substack{\text{Hydrochloric}\\\text{acid}\\(73g)}}{2HCl}\rightarrow\underset{\substack{\text{Calcium}\\\text{chloride}}}{CaCl_2(aq)}+H_2O(l)+\underset{(44g)}{CO_2(g)}$$

As amount of $CaCO_3$ and HCl are same, $CaCO_3$ will be the limiting reagent.

$\therefore$ 100g of $CaCO_3$ gives = 44g of CO_2 gas

$\therefore$ 20g of $CaCO_3$ will give = $\dfrac{44}{100}\times$ 20g of CO_2 = 8.8g of CO_2

133. (d) Pink colour of MnO_4^- converts to colourless MnO_2 after reduction.

134. (c) $\Delta G = \Delta H - T\Delta S$

If ΔH and ΔS are +ve then, ΔG will be positive till $\Delta H > T\Delta S$.

When the temperature is increased further, then

$\Delta H < T\Delta S$ and ΔG becomes –ve. Which is the essential condition for spontaneous reaction.

135. (b) $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \rightleftharpoons XY_3; \Delta H = -30kJ$

$X_2 + 3Y_2 \rightleftharpoons 2XY_3; \Delta H = -60kJ$

$\Delta S_r = 2 \times 50 - 3 \times 40 - 1 \times 60 = -80\ JK^{-1}\ mol^{-1}$

$\Delta G = \Delta H - T\Delta S$

When, $\Delta G = 0$

$0 = \Delta H - T\Delta S \Rightarrow \Delta H = T\Delta S$

$1000 \times (-60) = T \times (-80) \Rightarrow T = 750\ K.$

136. (b) $SO_2 + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

When pressure is increased forward reaction is favoured because product side has lower no. of moles.

From p_1 to p_3, yield of SO_3 is increasing because higher pressure favours forward reaction.

137. (d) $2(A)g \rightleftharpoons B(g) + C(g)$

$\Delta n = 2 - 2 = 0$

$K_p = K_C (RT)^0 \Rightarrow K_P = K_C$

138. (a) Peroxides such as BaO_2 on treatment with dilute H_2SO_4 give H_2O_2.

$$\underset{\text{Barium peroxide}}{BaO_2} + H_2SO_4 \longrightarrow \underset{\text{Barium sulphate}}{BaSO_4} + \underset{\text{Hydrogen peroxide}}{H_2O_2}$$

139. (c) K, Rb and Cs form superoxides when they are burned in the air. As we move down the group, the size of an atom from K to Cs increases.

Bigger cation can stabilize bigger O_2^- more easily

Hence, stability of superoxide increases.

140. (b) $\underset{\text{Borax}}{Na_2B_4O_7} + 7H_2O \longrightarrow \underset{\text{Sodium hydroxide}}{2NaOH} + \underset{\text{Boric acid}}{4H_3BO_3}$

141. (b) $\underset{\text{methyl chloride}}{2CH_3Cl} + Si \xrightarrow[570K]{Cu-powder} \underset{\text{Dimethyl silanes}}{(CH_3)_2} - \underset{\text{dichloro (P)}}{SiCl_2}$

$$\xrightarrow[-2HCl]{2H_2O} \underset{(Q)}{(CH_3)_2Si(OH)_2} \longrightarrow -O\left(\begin{matrix} CH_3 \\ | \\ Si - O \\ | \\ CH_3 \end{matrix}\right)\begin{matrix} CH_3 \\ | \\ Si - \\ | \\ CH_3 \end{matrix}$$

142. (a)

143. (c) Kolbe's electrolysis

$2CH_3COONa + 2H_2O \rightarrow CH_3 - CH_3 + 2CO_2 + 2NaOH + H_2$

Reaction at anode :

$2CH_3CO\dot{O} \longrightarrow 2\dot{C}H_3 + 2CO_2$

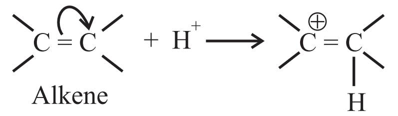
$2\dot{C}H_3 \longrightarrow C_2H_6$

$\dot{C}H_3$ radical is produced at anode.

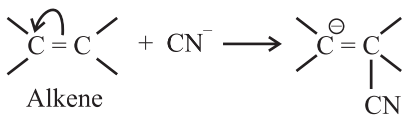
144. (c) Fractional distillation is used for the separation of a mixture of miscible liquids for which the difference in boiling points is less than 25°C.

145. (d) HNO_3 to remove NaCN(X) and Na_2S (Y) so that they do not interfere with the test for halogen.

146. (b) Positive electromeric effect (+E)



Negative electromeric effect (–E)



147. (d) Benzene and toluene have very similar molecular structure. Thus, they form an ideal solution.

148. (b) $\Delta T_b = K_b \times m$

$\Rightarrow K_b = \frac{\Delta T_b}{m}$

149. (a) Time, t = 1.608 min or 96.5 s

Formula, $W_{Al} = \frac{z \times i \times t}{96500}$

$\Rightarrow W_{Al} = \frac{27}{3} \times \frac{10 \times 96.5}{96500}$

$\Rightarrow W_{Al} = 0.09$ g

150. (b) $\lambda^\circ_m (NH_3) = \lambda^\circ (H^+) + \lambda^\circ (NH^-_2)$

$\Rightarrow \lambda^\circ_m (NH_3) = \lambda^\circ (NH^-_2) + \lambda^\circ (H^+) + \lambda^\circ (KBr) - \lambda^\circ (KBr)$

$= \lambda^\circ (KNH_2) + \lambda^\circ (HBr) - \lambda^\circ (KBr)$

$= 90.48 + 420.6 - 120.5$

$= 390.5\ S\ cm^2\ mol^{-1}$

151. (c) For first order reaction;

$t = \frac{2.303}{k}\log\left(\frac{a}{a-x}\right)$

$t = \frac{2.303}{2303 \times 10^{-3}}\log\left(\frac{4}{0.2}\right)$

$\Rightarrow t = 10^3 \times \log 20$

$\Rightarrow t = 1301\ s = 0.36\ hr$

152. (c) $\Rightarrow \log \frac{x}{m} = \log k + \frac{1}{n} \log p$

Slope (m) = 1/n

153. (c)

154. (d) Co binds with two bidentate ligands en and two monodentate ligand.

$\therefore$ Its coordination number = 4 + 2 = 6

155. (c) $\underset{(B)}{Zn(OH)_2} \xrightarrow{\Delta} \underset{(A)}{ZnO} + H_2O$

$\underset{(B)}{Zn(OH)_2} + 2NaOH \longrightarrow Na_2[Zn(OH)_4]$

$\xrightarrow[H_2O]{} \underset{(C)}{Na_2ZnO_2}$

$\underset{\text{Zinc oxide}}{ZnO} + C \longrightarrow \underset{(D)}{Zn} + \underset{\substack{(E)\\ \text{Carbon}\\ \text{monoxide}}}{CO}$

156. (b) $\underset{\text{Starch}}{(C_6H_{10}O_5)_n} + nH_2O \xrightarrow[343K, 2-3 atom]{H^+} \underset{\text{Glucose}}{nC_6H_{12}O_6}$

157. (c)

$$H_3C - \overset{\displaystyle H}{\underset{\displaystyle Br}{\overset{*}{C}}} - \overset{\displaystyle H}{\underset{\displaystyle Cl}{\overset{*}{C}}} - CH_3$$

Number of optical isomers = 2^n

S = 2^2 = 4

158. (b) $\underset{\text{Glucose}}{C_6H_{12}O_6} + \text{Zymase} \rightarrow \underset{\text{Ethanol}}{2C_2H_5OH} + \underset{\substack{\text{Carbon}\\ \text{dioxide}}}{2CO_2}$

159. (b) When CH_3COCl react with $(CH_3)_2$ Cd, it will form acetone as a product (P). On further step, acetone (P) react with acetaldehyde to give aldol condensation reaction and formed

α, β–unsaturated alkene product.

$$\underset{\text{Methyl chloride}}{CH_3 - \overset{O}{\overset{\|}{C}} - Cl} \xrightarrow{(i) (CH_3)_2Cd}$$

$$\underset{\text{Acetone (P)}}{CH_3 - \overset{O}{\overset{\|}{C}} - CH_3} + CH_3CdCl$$

$$\underset{(P)}{CH_3 - \overset{O}{\overset{\|}{C}} - CH_3} + \underset{\text{Acetaldehyde}}{CH_3 - \overset{O}{\overset{\|}{C}} - H}$$

$$\Delta \downarrow NaOH$$

$$\underset{\substack{\text{(cis / trans)}\\ \text{(2 products)}}}{CH_3 - \overset{CH_3}{\overset{|}{C}} = \overset{H}{\overset{|}{C}} - \overset{O}{\overset{\|}{C}} - H} \quad +$$

$$\underset{\substack{(Cis/trans)\\ \text{(2 products)}}}{CH_3 - \overset{O}{\overset{\|}{C}} - CH = CH - CH_3}$$

$$1 + CH_3 - CH = CH - CHO + \frac{H_3C}{H_3C} \!> C = CH - \overset{O}{\overset{\|}{C}} - CH_3$$

Hence, total number of products = 4 (without considering isomers).

160. (a)