AP/EAPCET Solved Paper 2021
Held on August 20

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. $f(x) = \sin x + \cos x \cdot g(x) = x^2 - 1$, then $g(f(x))$ is invertible if

(a) $\frac{-\pi}{4} \le x \le \frac{\pi}{4}$ (b) $\frac{-\pi}{2} \le x \le 0$

(c) $\frac{-\pi}{2} \le x \le \pi$ (d) $0 \le x \le \frac{\pi}{2}$

2. If $f: Z \to Z$ is defined by $f(x) = x^9 - 11x^8 - 2x^7 + 22x^6 + x^4 - 12x^3 + 11x^2 + x - 3$, $\forall x \in Z$, then $f(11)$ is equal to

(a) 7 (b) 8 (c) 6 (d) 9

3. Let $f(x) = x^3$ and $g(x) = 3^x$, then the quadratic equation whose roots are solutions of the equation $(fog)(x) = (gof)(x)$ (for $x \neq 0$) is

(a) $x^2 - 6x + 3 = 0$ (b) $x^2 - 6x + 9 = 0$

(c) $x^2 - x + 3 = 0$ (d) $x^2 - 3 = 0$

4. The trace of the matrix $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$ is

(a) 17 (b) 25 (c) 3 (d) 12

5. If A, B and C are the angles of a triangle, then the system of equations

$-x + y\cos C + z\cos B = 0$,

$x\cos C - y + z\cos A = 0$ and

$x\cos B + y\cos A - z = 0$

(a) Only zero solution

(b) A non-zero solution for all ΔABC

(c) Only zero solution but for certain values of A, B and C

(d) A non-zero solution if ΔABC is an equilateral triangle and not for all triangles.

6. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

(a) $a = 1, b = 1$

(b) $a = \sin 2\theta$ and $b = \cos 2\theta$

(c) $a = \cos 2\theta$ and $b = \sin 2\theta$

(d) $a = 0$ and $b = 0$

7. If $z_1 = 2 + 3i$ and $z_2 = 3 + 2i$, where $i = \sqrt{-1}$, then $\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}\begin{bmatrix} \bar{z}_1 & -z_2 \\ \bar{z}_2 & z_1 \end{bmatrix}$ is equal to

(a) $13I$ (b) I

(c) $26I$ (d) zero matrix

8. What is the value of $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$?

(a) $a^3 + b^3 + c^3 + 3abc$

(b) $a^3 + b^3 + c^3 - 3abc$

(c) $a^3 + b^3 + c^3 - 6abc$

(d) $a^3 + b^3 + c^3 + 6abc$

9. The radius of the circle represented by $(1 + i)(1 + 3i)(1 + 7i) = x + iy$ is $\left(i = \sqrt{-1}\right)$.

(a) 1000 (b) $10\sqrt{10}$ (c) 10000 (d) 100

10. If 1, α_1, α_2, α_3 and α_4 are the roots of $z^5 - 1 = 0$ and ω is a cube roots of unity, then $(\omega - 1)(\omega - \alpha_1)(\omega - \alpha_2)(\omega - \alpha_3)(\omega - \alpha_4) + \omega$ is equal to

(a) 0 (b) -1 (c) -2 (d) 1

11. If $a > 0$ and $z = x + iy$, then $\log_{\cos^2\theta}|z - a| > \log_{\cos^2\theta}|z - ai|, (\theta \in R)$ implies

(a) $x > y$ (b) $x < y$

(c) $x + y = \cos\theta$ (d) $x + y < 0$

12. If one root of the equation $ix^2 - 2(i + 1)x + (2 - i) = 0$ is $(2 - i)$, then the other root is

(a) $-i$ (b) $2 + i$ (c) i (d) $2 - i$

13. If α and β are the roots of the quadratic equation $x^2 + x + 1 = 0$, then the equation whose roots are α^{2021}, β^{2021} is given by

(a) $x^2 - x + 1 = 0$ (b) $x^2 + x - 1 = 0$

(c) $x^2 - x - 1 = 0$ (d) $x^2 + x + 1 = 0$

14. If 2, 1 and 1 are roots of the equation $x^3 - 4x^2 + 5x - 2 = 0$, then the roots of

$$\left(x+\frac{1}{3}\right)^3 - 4\left(x+\frac{1}{3}\right)^2 + 5\left(x+\frac{1}{3}\right) - 2 = 0$$

(a) $\frac{7}{3}, \frac{4}{3}, \frac{4}{3}$ (b) $\frac{5}{3}, \frac{2}{3}, \frac{2}{3}$

(c) $\frac{-5}{3}, \frac{-2}{3}, \frac{-2}{3}$ (d) $\frac{-7}{3}, \frac{-4}{3}, \frac{-4}{3}$

15. If $f(x) = 2x^3 + mx^2 - 13x + n$ and 2, 3 are the roots of the equation $f(x) = 0$, then the values of m and n are

(a) $-5, -30$ (b) $-5, 30$

(c) $5, 30$ (d) $5, -30$

16. The value of ${}^6P_4 + 4 \cdot {}^6P_3$ is

(a) 5040 (b) 2520 (c) 840 (d) 720

17. The number of ways in which 3 boys and 2 girls can sit on a bench so that no two boys are adjacent is

(a) 6 (b) 10 (c) 12 (d) 32

18. In how many ways can 5 balls be placed in 4 tins if any number of balls can be placed in any tin?

(a) ${}^5P_4$ (b) ${}^5C_4$ (c) 4^5 (d) 5^4

19. Given, $\frac{3x-2}{(x+1)^2(x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}$, then $4A + 2B + 4C$ is equal to

(a) 5 (b) -5 (c) -3 (d) 3

20. What is the value of $\cos\left(22\frac{1}{2}\right)^\circ$?

(a) $\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$ (b) $\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$

(c) $\sqrt{2}-1$ (d) $\sqrt{2}+1$

21. If $\cos\theta = -\sqrt{\frac{3}{2}}$ and $\sin\alpha = \frac{-3}{5}$, where 'θ' does not lie in the third quadrant, then the value of $\frac{2\tan\alpha + \sqrt{3}\tan\theta}{\cot^2\theta + \cos\alpha}$ is equal to

(a) $\frac{7}{22}$ (b) $\frac{5}{22}$ (c) $\frac{9}{22}$ (d) $\frac{22}{5}$

22. If $\tan\beta = \frac{\tan\alpha + \tan\gamma}{1 + \tan\alpha\tan\gamma}$, then $\frac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha \sin 2\gamma}$ is equal to

(a) $\sin 2\beta$ (b) $\cos 2\beta$ (c) $\tan 2\beta$ (d) $\sec 2\beta$

23. If $\sin\left(\frac{\pi}{4}\cos\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$, then θ is equal to

(a) $2n\pi + \frac{\pi}{4}$ (b) $2n\pi \pm \frac{\pi}{4}$

(c) $2n\pi - \frac{\pi}{4}$ (d) $n\pi + \frac{\pi}{4}$

24. If $x = \sin(2\tan^{-1} 2)$, $y = \cos(2\tan^{-1} 3)$ and $z = \sec(3\tan^{-1} 4)$, then

(a) $x < y < z$ (b) $y < z < x$

(c) $z < x < y$ (d) $z < y < x$

25. In ΔABC, medians AD and BE are drawn. If $AD = 4$, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of ΔABC is

(a) $\frac{8}{3}$ sq units (b) $\frac{16}{3}$ sq units

(c) $\frac{32}{3\sqrt{2}}$ sq units (d) $\frac{64}{3}$ sq units

26. In a ΔABC, $2\Delta^2 = \frac{a^2b^2c^2}{a^2+b^2+c^2}$, then the triangle is

(a) equilateral (b) isosceles

(c) right angled (d) acute angled triangle

27. The sides of a triangle inscribed in a given circle subtend angles α, β, γ at the centre. The minimum value of the AM of $\cos\left(\alpha+\frac{\pi}{2}\right), \cos\left(\beta+\frac{\pi}{2}\right)$ and $\cos\left(\gamma+\frac{\pi}{2}\right)$, is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{-\sqrt{3}}{2}$ (c) $\frac{-2}{\sqrt{3}}$ (d) $\sqrt{2}$

28. The position vectors of the points A and B with respect to O are $2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. The length of the internal bisector of $\angle BOA$ of ΔAOB is (take proportionality constant is 2)

(a) $\frac{\sqrt{136}}{9}$ (b) $\frac{\sqrt{136}}{3}$ (c) $\frac{20}{3}$ (d) $\frac{25}{3}$

29. Let $\mathbf{u} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{v} = -3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and $\mathbf{w} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. Then which of the following statement is true?

(a) **u** is perpendicular to **v** but not **w**

(b) **v** is perpendicular to **w** but not **u**

(c) **w** is perpendicular to **u** but not **v**

(d) **u** is perpendicular to both **v** and **w**

30. If the lines, $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$ are coplanar, then $\sin^{-1}(\sin\lambda) + \cos^{-1}(\cos\lambda)$ is equal to

(a) $8 - 2\pi$ (b) $6 - \pi$ (c) $3\pi - 8$ (d) $4\pi - 8$

31. If a = (1, 1, 0) and b = (1, 1, 1), then unit vector in the plane of a & b and perpendicular to a is

(a) (0, 1, 0) (b) (1, – 1, 0)

(c) **k** (d) (1, 0, 1)

32. The line passing through (1, 1, –1) and parallel to the vector $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ meets the line $\frac{x-3}{-1} = \frac{y+2}{5} = \frac{z-2}{-4}$ at A and the plane $2x - y + 2z + 7 = 0$ at B. Then AB is equal to

(a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) $3\sqrt{6}$ (d) $4\sqrt{6}$

33. Let $\mathbf{a} = \hat{\mathbf{i}}$ and $\mathbf{b} = \hat{\mathbf{j}}$, the point of intersection of the lines r × a = b × a and r × b = a × b is

(a) $\mathbf{r} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ (b) $\mathbf{r} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$

(c) $\mathbf{r} = \hat{\mathbf{k}}$ (d) $\mathbf{r} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}}$

34. The mean deviation from the mean of the set of observation –1, 0, 4 is
(a) 3 (b) 1 (c) – 2 (d) 2

35. Let an angle of a triangle is 60°. If the variance of the angles of the triangle is 1014°, then the other two angles are
(a) 23° and 97° (b) 22° and 98°
(c) 21° and 99° (d) 20° and 100°

36. One card is selected at random from 27 cards numbered from 1 to 27. What is the probability that the number on the card is even or divisible by 5.
(a) $\frac{15}{27}$ (b) $\frac{16}{27}$ (c) $\frac{17}{27}$ (d) $\frac{18}{27}$

37. Nine balls one drawn simultaneously from a bag containing 5 white and 7 black balls. The probability of drawing 3 white and 6 black balls is
(a) $\frac{{}^7C_3}{{}^{12}C_9}$ (b) $\frac{7}{22}$ (c) $\frac{3}{22}$ (d) $\frac{7}{11}$

38. The probabilities that A and B speak truth are $\frac{4}{5}$ and $\frac{3}{4}$ respectively. The probability that they contradict each other when asked to speak on a fact is
(a) $\frac{1}{5}$ (b) $\frac{3}{20}$ (c) $\frac{4}{20}$ (d) $\frac{7}{20}$

39. The mean and variance of a binomial variable X are 2 and 1 respectively. The probability that X takes values greater than 1 is
(a) $\frac{5}{16}$ (b) $\frac{8}{16}$ (c) $\frac{11}{16}$ (d) $\frac{1}{16}$

40. For the random variable X with probability distribution is given by the table

$X = x$	0	1	2	3
$P(X = x)$	K	$K+\frac{1}{7}$	$2K$	$\frac{2}{5}$

The mean of X is
(a) $\frac{31}{35}$ (b) $\frac{57}{35}$ (c) $\frac{63}{35}$ (d) $\frac{67}{35}$

41. The locus of a point, which is at a distance of 4 units from (3, –2) in xy-plane is
(a) $x^2 + y^2 + 6x - 4y + 16 = 0$
(b) $x^2 + y^2 - 6x - 4y + 3 = 0$
(c) $x^2 + y^2 - 6x + 4y - 16 = 0$
(d) $x^2 + y^2 - 6x + 4y - 3 = 0$

42. When the axes are rotated through an angle 45°, the new coordinates of a point P are (1, –1). The coordinates of P in the original system are
(a) $(\sqrt{2}, \sqrt{2})$ (b) $(\sqrt{2}, 0)$
(c) $(0, \sqrt{2})$ (d) $(-\sqrt{2}, 0)$

43. Find the equation of a straight line passing through (–5, 6) and cutting off equal intercepts on the coordinate axes.
(a) $6x - 5y = 30$ (b) $x - y = -11$
(c) $x + y = 11$ (d) $x + y = 1$

44. Line has slope m and y-intercept 4. The distance between the origin and the line is equal to
(a) $\frac{4}{\sqrt{1-m^2}}$ (b) $\frac{4}{\sqrt{m^2-1}}$
(c) $\frac{4}{\sqrt{m^2+1}}$ (d) $\frac{4m}{\sqrt{m^2+1}}$

45. The equation of the base of an equilateral triangle is $x + y = 2$ and one vertex is (2, – 1), then the length of the side of the triangle is
(a) $\sqrt{3/2}/\sqrt{2/3}$ (b) $\sqrt{2}$
(c) $\sqrt{2/3}$ (d) $\sqrt{3/2}$

46. The equation of a straight line which passes through the point ($a\cos^3\theta$, $a\sin^3\theta$) and perpendicular to ($x\sec\theta + y\operatorname{cosec}\theta) = a$ is
(a) $\frac{x}{a} + \frac{y}{b} = a\cos\theta$
(b) $x\cos\theta - y\sin\theta = a\cos 2\theta$
(c) $x\cos\theta + y\sin\theta = a\cos 2\theta$
(d) $x\cos\theta + y\sin\theta - a\cos 2\theta = 1$

47. The acute angle between lines $6x^2 + 11xy - 10y^2 = 0$ is
(a) $\tan^{-1}\left(\frac{\sqrt{361}}{2}\right)$ (b) $\tan^{-1}\left(\frac{\sqrt{361}}{4}\right)$
(c) $\tan^{-1}\left(\frac{361}{2}\right)$ (d) $\tan^{-1}\left(\frac{361}{4}\right)$

48. If the lines, joining the origin to the points of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and the line $x + 2y = k$, are at right angles, then k^2 equals
(a) 4 (b) 3 (c) 2 (d) 1

49. The equation of bisector of the angle between the lines represented by $3x^2 - 5xy + 4y^2 = 0$ is
(a) $9x^2 + 6y^2 - 2x = 0$ (b) $5(x^2 - y^2) = 2xy$
(c) $3x^2 + 2xy - y^2 = 0$ (d) $5x^2 + xy + 4y^2 = 0$

50. If the bisectors of the pair of lines $x^2 - 2mxy - y^2 = 0$ is represented by $x^2 - 2nxy - y^2 = 0$, then
(a) $mn + 1 = 0$ (b) $mn - 1 = 0$
(c) $m + n = 0$ (d) $m - n = 0$

51. Find the equation of the circle which passes through origin and cuts off the intercepts – 2 and 3 over the X and Y-axes respectively.
(a) $x^2 + y^2 - 2x + 8y = 0$
(b) $2(x^2 + y^2) + 2x - 3y = 0$
(c) $x^2 + y^2 - 2x - 8y = 0$
(d) $x^2 + y^2 + 2x - 3y = 0$

52. The angle between the pair of tangents drawn from (1, 1) to the circle $x^2 + y^2 + 4x + 4y - 1 = 0$ is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

53. If the circle $x^2 + y^2 - 4x - 8y - 5 = 0$ intersects the line $3x - 4y - m = 0$ in two distinct points, then the number of integral values of 'm' is
(a) 52 (b) 51 (c) 50 (d) 49

54. Let **C** be the circle centre (0, 0) and radius 3 units. The equation of the locus of the mid-points of the chords of the circle **c** that subtends an angle of $\frac{2\pi}{3}$ at its centre is

(a) $x^2+y^2=\frac{1}{4}$ (b) $x^2+y^2=\frac{27}{4}$

(c) $x^2+y^2=\frac{9}{4}$ (d) $x^2+y^2=\frac{5}{4}$

55. The length of the common chord of the circles $x^2 + y^2 + 3x + 5y + 4 = 0$ and $x^2 + y^2 + 5x + 3y + 4 = 0$ is ______ units.

(a) 3 (b) 2 (c) 6 (d) 4

56. Find the equation of the circle which passes through the point (1, 2) and the points of intersection of the circles $x^2 + y^2 - 8x - 6y + 21 = 0$ and $x^2 + y^2 - 2x - 15 = 0$

(a) $x^2 + y^2 - 18x - 12y + 27 = 0$

(b) $2(x^2 + y^2) - 18x - 12y + 27 = 0$

(c) $3(x^2 + y^2) - 18x - 12y + 27 = 0$

(d) $4(x^2 + y^2) - 18x - 12y + 27 = 0$

57. The coordinates of the focus of the parabola described parametrically by $x = 5t^2 + 2$ and $y = 10t + 4$ (where t is a parameter) are

(a) (7, 4) (b) (3, 4) (c) (3, –4) (d) (–7, 4)

58. If $\tan\theta_1$, $\tan\theta_2=\frac{-a^2}{b^2}$, then the chord joining 2 points θ_1 and θ_2 one the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ will subtend a right angle at

(a) focus (b) centre

(c) end of major axis (d) end of minor axis

59. If one focus of a hyperbola is (3, 0), the equation of its directrix is $4x - 3y - 3 = 0$ and its eccentricity $e = 5/4$, then the coordinates of its vertex is

(a) $\left(\frac{3}{5},\frac{11}{5}\right)$ (b) $\left(\frac{11}{5},\frac{3}{5}\right)$

(c) $\left(\frac{7}{5},\frac{4}{5}\right)$ (d) $\left(\frac{4}{5},\frac{7}{5}\right)$

60. If the vertices of the triangles are (1, 2, 3), (2, 3, 1,), (3, 1, 2) and if H, G, S and I respectively denote its orthocenter, centroid, circumcenter and incenter, then $H + G + S + I$ is equal to

(a) (2, 2, 2) (b) (4, 4, 4) (c) (6, 6, 6) (d) (8, 8, 8)

61. $A(2, 3, 4)$, $B(4, 5, 7)$, $C(2, -6, 3)$ and $D(4, -4, k)$ are four points. If the line AB is parallel to CD, then k is equal to

(a) 2 (b) 4 (c) 5 (d) 6

62. If the direction cosines of two lines are $\left(\frac{2}{3},\frac{2}{3},\frac{1}{3}\right)$ and $\left(\frac{5}{13},\frac{12}{13},0\right)$, then identify the direction ratios of a line which is bisecting one of the angle between them.

(a) (40, 60, 13) (b) (41, 60, 10)

(c) (41, 62, 13) (d) (1, 2, 3)

63. $\lim\limits_{n\to\infty}\frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$ is equal to

(a) 0 (b) 4 (c) 2 (d) ∞

64. If the function $f(x)$, defined below, is continuous on the interval [0, 8], then

$$f(x)=\begin{cases}x^2+ax+b & , \ 0\le x<2\\ 3x+2 & , \ 2\le x\le 4\\ 2ax+5b & , \ 4<x\le 8\end{cases}$$

(a) $a = 3, b = -2$ (b) $a = -3, b = 2$

(c) $a = -3, b = -2$ (d) $a = 3, b = 2$

65. If $f(x)$, defined below, is continuous at $x = 4$, then

$$f(x)=\begin{cases}\frac{x-4}{|x-4|}+a & , \ x<4\\ a+b & , \ x=4\\ \frac{x-4}{|x-4|}+b & , \ x>4\end{cases}$$

(a) $a = 0$ and $b = 0$

(b) $a = 1$ and $b = 1$

(c) $a = -1$ and $b = 1$

(d) $a = 1$ and $b = -1$

66. If $f(x) = 2x^2 + 3x - 5$, then the value of $f'(0) + 3f'(-1)$ is equal to

(a) 1 (b) 0 (c) 3 (d) 2

67. If $y=\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)...\left(1+\frac{n}{x}\right)$ and $x \neq 0$. When $x=-1$, $\frac{dy}{dx}$ is equal to

(a) $n!$ (b) $(n-1)!$ (c) $(-1)^n (n-1)!$ (d) $(-1)^n n!$

68. $\frac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)\right\}$ is equal to

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{-1}{2}$ (d) -1

69. If $y=\tan^{-1}\left\{\frac{ax-b}{bx+a}\right\}$, then y' is equal to

(a) $\frac{1}{1+x^2}+\frac{a^2}{a^2+b^2}$ (b) $\frac{1}{1+x^2}$

(c) $\frac{1}{1+\left(\frac{ax-b}{bx+a}\right)^2}$ (d) $\frac{bx+a}{1+(ax-b)^2}$

70. If $y = 4x - 6$ is a tangent to the curve $y^2 = ax^4 + b$ at (3, 6), then the values of a and b are

(a) $a=\frac{4}{9}$ and $b=\frac{-4}{9}$ (b) $a=0$ and $b=\frac{4}{9}$

(c) $a=\frac{-4}{9}$ and $b=\frac{-4}{9}$ (d) $a=\frac{4}{9}$ and $b=0$

71. Find the positive value of a for which the equality $2\alpha + \beta = 8$ holds, where α and β are the points of maximum and minimum, respectively, of the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$.

(a) 0 (b) 2 (c) 1 (d) $\frac{1}{4}$

72. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area,

(a) $2.16\ \pi\ cm^2$ (b) $21.6\ \pi\ cm^2$
(c) $216\ \pi\ cm^2$ (d) $0.216\ \pi\ cm^2$

73. The diameter and altitude of a right circular cone, at a certain instant, were found to be 10 cm and 20 cm respectively. If its diameter is increasing at a rate of 2 cm/s, then at what rate must its altitude change, in order to keep its volume constant?

(a) 4 cm/s (b) 6 cm/s (c) – 4cm/s (d) – 8 cm/s

74. $\int \frac{\sin\alpha}{\sqrt{1+\cos\alpha}}\, d\alpha$ is equal to

(a) $-2\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+C$ (b) $2\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+C$

(c) $\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+C$ (d) $-\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+C$

75. If $\int \frac{\cos 4x+1}{\cot x-\tan x} = k\cos 4x + C$, then k is equal to

(a) $\frac{-1}{2}$ (b) $\frac{-1}{8}$

(c) $\frac{-1}{3}$ (d) $\frac{-1}{5}$

76. If $\int\left[\cos(x)\cdot\frac{d}{dx}(\text{cosec}(x))\right]dx = f(x)+g(x)+c,$ then $f(x)\cdot g(x)$ is equal to

(a) $x\cot(x)$ (b) $x\tan(x)$
(c) $x\cos(x)$ (d) 1

77. If $\int\frac{(2x+1)^6}{(3x+2)^8}dx = P\left(\frac{2x+1}{3x+2}\right)^Q + R,$ then $\frac{P}{Q}$ is equal to

(a) $\frac{1}{7^2}$ (b) $\frac{1}{7}$ (c) 7^2 (d) 7

78. If $\int_0^a \frac{dx}{4+x^2} = \frac{\pi}{8}$, then the value of a is equal to

(a) 1 (b) 2 (c) 3 (d) 4

79. $\int_1^2 \frac{x^3-1}{x^2}dx$ is equal to

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) 1 (d) – 1

80. The solution of the differential equation $2x\left(\frac{dy}{dx}\right) - y = 4$ represents a family of

(a) ellipse (b) parabola (c) straight line (d) circle

PHYSICS

81. The displacements of a particle starting from rest at $t = 0$ is given by $s = 9t^2 - 2t^3$. The time in seconds at which the particle will attain zero velocity is

(a) 8 s (b) 6 s (c) 4 s (d) 3 s

82. The range of a projectile is 100 m. Its kinetic energy will be maximum after covering a distance of

(a) 25 m (b) 50 m (c) 75 m (d) 100 m

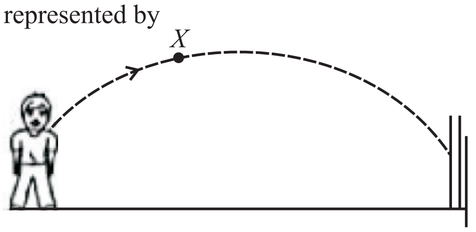
83. Two cars A and B are moving with a velocity of 30 km/h in the same direction. They are separated by 10 km. The speed of another car C moving in the opposite direction, if it meets these two cars at an interval of eight minutes is

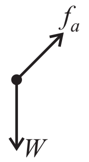
(a) 45 km/h (b) 40 km/h
(c) 15 km/h (d) 30 km/h

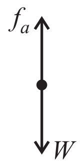
84. A book is lying on a table. What is the angle between the normal reaction acting on the book on the table and the weight of the book?

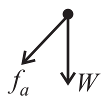
(a) 0° (b) 45° (c) 90° (d) 180°

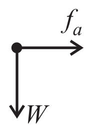
85. A boy throws a cricket ball from the boundary to the wicket keeper. If the frictional force due to air (f_a) cannot be ignored, the forces acting on the ball at the position X are represented by



(a) 

(b) 

(c) 

(d) 

86. When a force F = $17 - 2x + 6x^2$ N acts on a body of mass 2 kg and displaces it from $x = 0$ m to $x = 8$ m, the work done is

(a) 1096 J (b) 270 J (c) 35 J (d) 135 J

87. A rifle bullet loses $\left(\frac{1}{25}\right)$th of its velocity in passing through a plank. The least number of such planks required just to stop the bullet is

(a) 5 (b) 10 (c) 11 (d) 13

88. A uniform chain has a mass m and length l. It is held on a frictionless table with one-sixth of its length hanging over the edge. The work done in just pulling the hanging part back on the table is

(a) $\frac{mgl}{72}$ (b) $\frac{mgl}{36}$ (c) $\frac{mgl}{12}$ (d) $\frac{mgl}{6}$

89. A sphere and a hollow cylinder without slipping, roll down two separate inclined planes A and B, respectively. They cover same distance in a given duration. If the angle of inclination of plane A is 30°, then the angle of inclination of plane B must be (approximately)

(a) 60° (b) 53° (c) 45° (d) 37°

90. Four spheres each of diameter $2a$ and mass m are placed in a way that their centres lie on the four corners of a square of side b. Moment of inertia of the system about an axis along one of the sides of the square is

(a) $\frac{8}{5}ma^2$ (b) $\frac{4}{5}ma^2+5mb^2$

(c) $\frac{4}{5}ma^2+2mb^2$ (d) $\frac{8}{5}ma^2+2mb^2$

91. If an energy of 684 J is needed to increase the speed of a flywheel from 180 rpm to 360 rpm, then find its moment of inertia.

(a) 0.7 kg/m^2 (b) 1.28 kg/m^2

(c) 2.75 kg/m^2 (d) 7.28 kg/m^2

92. A particle executing simple harmonic motion along a straight line with an amplitude A, attains maximum potential energy when its displacement from mean position equals

(a) 0 (b) $\pm\frac{A}{\sqrt{2}}$ (c) $\pm A$ (d) $\pm\frac{A}{2}$

93. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out the time period of oscillation would

(a) remain unchanged

(b) increase towards a saturation value

(c) first increase and then decrease to the original value

(d) first decrease and then increase to the original value

94. The gravitational potential energy is maximum at

(a) infinity

(b) the earth's surface

(c) the centre of the earth

(d) twice the radius of the earth

95. A geostationary satellite is taken to a new orbit, such that its distance from centre of the earth is doubled. Then, find the time period of this satellite in the new orbit.

(a) 24 h (b) 4.8 h (c) $48\sqrt{2}$ h (d) $24\sqrt{2}$ h

96. A body of mass 10 kg is attached to a wire of 0.3 m length. The breaking stress is 4.8×10^7 Nm^{-2}. The area of cross-section from the wire is 10^{-6} m^2. The maximum angular velocity with which it can be rotated in a horizontal circle is

(a) 4 rad s^{-1} (b) 8 rad s^{-1}

(c) 16 rad s^{-1} (d) 32 rad s^{-1}

97. A glass flask weighting 390 g, having internal volume 500 cc just floats when half of it is filled with water. Specific gravity of the glass is

(a) 2.8 (b) 1.8 (c) 1.0 (d) 2.5

98. Water does not wet an oily glass because

(a) cohesive force of oil is greater than adhesive force between oil and glass

(b) cohesive force of oil is greater than cohesive force of water

(c) oil repels water

(d) cohesive force of water is greater than adhesive force between water and oil molecules

99. Boiling water is changing into steam. The specific heat of boiling water is

(a) zero (b) one (c) infinity (d) less than one

100. If the volume of a block of metal changes by 0.12% when heated through 20°C, then find its coefficient of linear expansion.

(a) 4×10^{-5} °C^{-1} (b) 4×10^{-4} °C^{-1}

(c) 2×10^{-5} °C^{-1} (d) 2×10^{-4} °C^{-1}

101. Isothermal process is the graph between

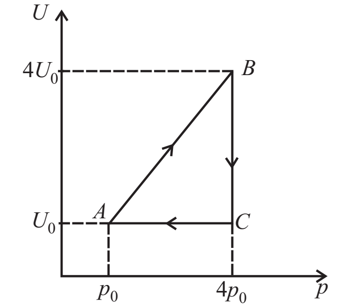
(a) pressure and temperature

(b) pressure and volume

(c) volume and temperature

(d) pV and temperature

102. For a monoatomic ideal gas is following the cyclic process $ABCA$ shown in the U *versus* p plot, identify the incorrect option.



(a) Molar heat capacity of the process AB is $\frac{R}{2}$.

(b) Heat is rejected by the system in path BC.

(c) Molar heat capacity for the process BC is $\frac{2R}{3}$.

(d) Work done by the system in the process CA is $\frac{2U_0}{3}\ln 4$.

103. The pressure of a gas is proportional to

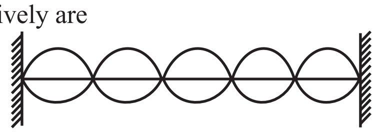
(a) the sum of kinetic and potential energies

(b) potential energy

(c) kinetic energy

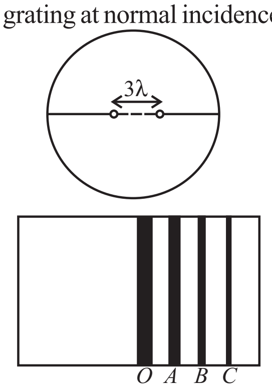
(d) None of the above

104. A string fixed at both ends vibrate in 5 loops as shown in the figure. The total number of nodes and anti-nodes respectively are



(a) 6 and 5 (b) 6 and 10 (c) 2 and 5 (d) 10 and 6

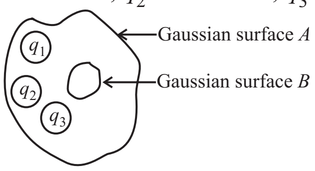
105. The position of the direct image obtained at O, when a monochromatic beam of light is passed through a plane transmission grating at normal incidence is shown in figure.



The diffracted images A, B and C correspond to the first, second and third order diffraction. When the source is replaced by another source of shorter wavelength,

(a) all the four will shift in the direction C to O
(b) all the four will shift in the direction O to C
(c) the images C, B and A will shift towards O
(d) the images C, B and A will shift away from O

106. What is the electric flux for Gaussian surface A that encloses the charged particles in free space?
[Given, $q_1 = -14$ nC, $q_2 = 78.85$ nC, $q_3 = -56$ nC]



(a) 10^3 N-m^2 C^{-1} (b) 10^3 C-N^{-1} m^{-2}
(c) 632×10^3 N-m^2 C^{-1} (d) 632×10^3 C-N^{-1} m^{-2}

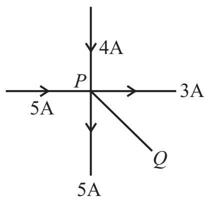
107. Two charges 8 μC each are placed at the corners A and B of an equilateral triangle of side 0.2 m in air. The electric potential at the third corner C is

(a) 7.2×10^5 V (b) 1.8×10^5 V
(c) 3.6×10^5 V (d) 3.6×10^4 V

108. A 60, μF parallel plate capacitor whose plates are separated by 6 mm is charged to 250 V, and then the charging source is removed. When a slab of dielectric constant 5 and thickness 3 mm is placed between the plates, find the change in the potential difference across the capacitor.

(a) 250 V (b) 100 V (c) 150 V (d) 75 V

109. Five current carrying conductors meet at a point P. What is the magnitude and direction of the current in the fifth conductor?

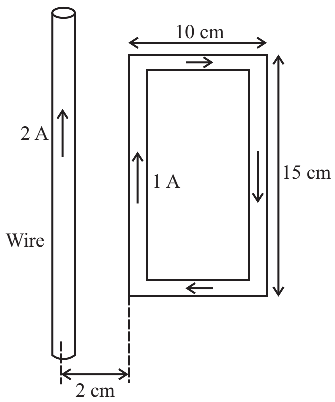


(a) 1A from Q to P (b) 1A from P to Q
(c) 3A from P to Q (d) 2A from Q to P

110. A wire of length L metre carrying a current I ampere is bent in the form of a circle. Magnitude of its magnetic moment is

(a) $\dfrac{L^2I^2}{4\pi}$ (b) $\dfrac{L^2I}{4\pi}$ (c) $\dfrac{LI}{4\pi}$ (d) $\dfrac{LI^2}{4\pi}$

111. What is the net force on the square coil?



(a) 25×10^{-7} N moving towards wire
(b) 25×10^{-7} N moving away from wire
(c) 35×10^{-7} N moving towards wire
(d) 35×10^{-7} N moving away from wire

112. A paramagnetic sample showing a net magnetisation of 0.8 Am^{-1}, when placed in an external magnetic field of strength 0.8 T, at a temperature 5K. If the temperature is raised to 20 K, then the magnetisation becomes

(a) 0.8 A m^{-1} (b) 0.2 A m^{-1}
(c) 0.1 A m^{-1} (d) 0.4 A m^{-1}

113. The induced emf cannot be produced by

(a) moving a magnet near a circuit
(b) moving a circuit near a magnet
(c) changing the current in one circuit placed near the other
(d) maintaining large but constant current in a circuit

114. **Assertion (A) :** When plane of coil is perpendicular to magnetic field, magnetic flux linked with the coil is minimum, but induced emf is zero.

Reason (R) : $\phi = nAB\cos\theta$ and $e = \dfrac{d\phi}{dt}$

(a) Both A and R are true and R is the correct explanation for A.
(b) Both A and R are true but R is not the correct explanation for A.
(c) A is true, R is false.
(d) A is false, R is true.

115. A 20 V AC is applied to a circuit consisting of a resistor and a coil with negligible resistance. If the voltage across the resistor is 12 V, the voltage across the coil is

(a) 16 V (b) 10 V (c) 8 V (d) 6 V

116. The electric and the magnetic fields associated with an electromagnetic wave propagating along the z-axis, can be represented by

(a) $\left[\mathbf{E} = E_0\hat{\mathbf{i}}\ \ \mathbf{B} = B_0\hat{\mathbf{j}}\right]$

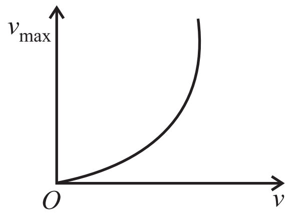
(b) $\left[\mathbf{E} = E_0\hat{\mathbf{k}}, \mathbf{B} = B_0\hat{\mathbf{i}}\right]$

(c) $\left[\mathbf{E} = E_0\hat{\mathbf{j}}, \mathbf{B} = B_0\hat{\mathbf{i}}\right]$

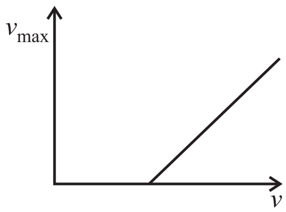
(d) $\left[\mathbf{E} = E_0\hat{\mathbf{j}}, \mathbf{B} = B_0\hat{\mathbf{k}}\right]$

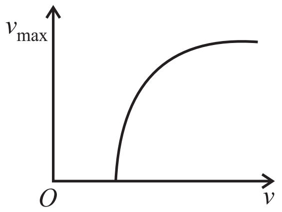
117. The graph between the maximum speed (v_{max}) of a photoelectron and frequency (v) of the incident radiation, in photoelectric effect is correctly represented by

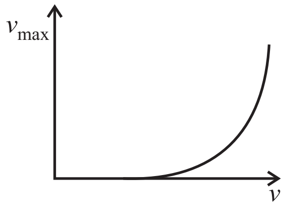
(a)



(b)



(c) 

(d) 

118. The angular momentum of the orbital electron is integral multiple of

(a) h (b) $2\pi h$ (c) $\frac{h}{2\pi}$ (d) $3\pi h$

119. Which of the following values is the correct order of nuclear density?

(a) 5×10^5 kg m^{-3} (b) 9×10^{10} kg m^{-3}
(c) 3×10^{21} kg m^{-3} (d) 2×10^{17} kg m^{-3}

120. The truth table given below corresponds to logic gate.

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(a) NAND (b) OR (c) AND (d) XOR

CHEMISTRY

121. The number of protons, neutrons and electrons in $^{13}_{6}C$ respectively are

(a) 6, 7, 6 (b) 13, 6, 6 (c) 6, 7, 13 (d) 6, 6, 13

122. The masses of an electron, a proton and a neutron respectively will be in the ratio

(a) 1 : 1836.15 : 1838.68 (b) 1 : 1856.15 : 1858.68
(c) 1 : 1834.15 : 1836.68 (d) 1 : 1846.15 : 1848.68

123. Match the following species with the correct number of electrons present in them.

	Species		Number of electrons
A.	Be^{2+}	(i)	0
B.	H^+	(ii)	10
C.	Na^+	(iii)	2
D.	Mg^+	(iv)	11
		(v)	4

Codes

	A	B	C	D		A	B	C	D
(a)	(iv)	(iii)	(ii)	(i)	(b)	(i)	(ii)	(iii)	(iv)
(c)	(v)	(iv)	(i)	(iii)	(d)	(iii)	(i)	(ii)	(iv)

124. The correct order of electronegativity of carbon in various hybridisation states is

(a) $sp < sp^2 < sp^3$ (b) $sp > sp^2 > sp^3$
(c) $sp^2 > sp < sp^3$ (d) $sp = sp^2 < sp^3$

125. Which of the following is not arranged in the correct sequence?

(a) MO, M_2O_3, MO_2, M_2O_5 (Decreasing basic nature)
(b) Sc, V, Cr, Mn (Increasing number of oxidation states)
(c) d^5, d^3, d^1, d^4 (Increasing magnetic moment)
(d) Mn^{2+}, Fe^{2+}, Cr^{2+}, Co^{2+} (Decreasing stability)

126. Which of the following statement is incorrect?

(a) Tl^{3+} salts are oxidising agents.
(b) Ga^+ salts are reducing agents.
(c) Pb^{4+} salts are better oxidising agents.
(d) As^{+5} salts are better oxidising agents.

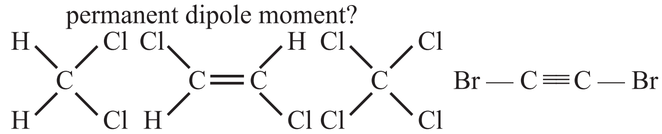
127. Bond order is an inverse measure of

(a) bond length
(b) bond angle
(c) bond dissociation energy
(d) stability

128. Which of the following molecule has the maximum dipole moment?

(a) NH_3 (b) CS_2 (c) C_2H_6 (d) NCl_3

129. Which compound among the following will have a permanent dipole moment?



(a) Only (i) (b) Only (ii) (c) Only (iii) (d) Only (iv)

130. Which among the following statements is/are incorrect regarding real gases?

(i) Their compressibility factor is never equal to unity ($Z \neq 1$).
(ii) The deviations from ideal behaviour are less at low pressure and high temperatures.
(iii) Intermolecular forces among gas molecules are equal to zero.
(iv) They obey van der Waal's equation, $pV = nRT$

(a) (i), (ii) and (iv) (b) (ii) and (iv)
(c) Only (ii) (d) (iii) and (iv)

131. Which among the following species does not shown disproportionation reaction?

(a) ClO^- (b) ClO_2^- (c) ClO_3^- (d) ClO_4^-

132. An alloy of metals X and Y weighs 12 g and contains atoms X and Y in the ratio of 2 : 5. The percentage of metal X in the alloy is 20 by mass. If the atomic mass of X is 40 amu what is the atomic mass of metal Y?

(a) 64 amu (b) 32 amu (c) 60 amu (d) 50 amu

133. For the reaction, $H_2O(l) \longrightarrow H_2O(g)$ at $T = 100°C$ and $p = 1$ atm, choose the correct option.

(a) $\Delta S_{system} > 0$ and $\Delta S_{surrounding} > 0$
(b) $\Delta S_{system} > 0$ and $\Delta S_{surrounding} < 0$
(c) $\Delta S_{system} < 0$ and $\Delta S_{surrounding} > 0$
(d) $\Delta S_{system} < 0$ and $\Delta S_{surrounding} < 0$

134. At 60°C, dinitrogen tetroxide is 50% dissociated. Find it's standard free energy change at this temperature and one atmosphere. [Given log1.33 = 0.1239]

(a) – 650 J mol^{-1} (b) – 830 J mol^{-1}
(c) – 790 J mol^{-1} (d) – 875 J mol^{-1}

135. The solubility of AgBr(*s*), having solubility product 5×10^{-10} in 0.2 M NaBr solution equals
(a) 5×10^{-10} M (b) 25×10^{-10} M
(c) 0.5 M (d) 0.002 M

136. Le-Chateliers' principle is not applicable to
(a) $H_2(g)+I_2(g) \rightleftharpoons 2HI(g)$
(b) $Fe(s)+S(s) \rightleftharpoons FeS(s)$
(c) $N_2(g)+3H_2(g) \rightleftharpoons 2NH_3(g)$
(d) $N_2(g)+O_2(g) \rightleftharpoons 2NO(g)$

137. Which of the following does not form double salts?
(a) Li_2SO_4 (b) Na_2SO_4 (c) K_2SO_4 (d) Rb_2SO_4

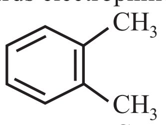
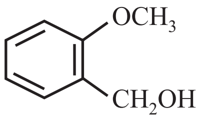
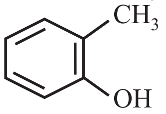
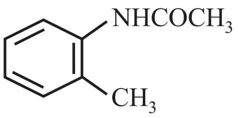
138. AlF_3 is soluble in HF only in the presence of KF due to formation of
(a) AlH_3 (b) $[AlH_6]^{3-}$ (c) $[AlF_6]^{3-}$ (d) $K[AlF_3H]$

139. What would be the product of following reaction?
$$SiCl_4 \xrightarrow{\text{Excess of } H_2O} ?\,(\text{Major product})$$
(a) $SiCl_3(OH)$ (b) $Si(OH)_4$
(c) $SiCl_2(OH)_2$ (d) $SiCl_4$ (no reaction)

140. Which among the following is not a greenhouse gas?
(a) Nitrous oxide (b) Water vapour
(c) Sulphur dioxide (d) Methane

141. An organic compound of molecular formula $C_6H_6Br_2$ has six carbon atoms in a ring system, two non-conjugate double bonds and two bromo groups at 1, 4-positions. Then the compound is
(a) aromatic but non-homo-cyclic
(b) aromatic and hetero-cyclic
(c) homo-cyclic but not aromatic
(d) neither homo-cyclic nor hetero-cyclic

142. Using Kjeldahl's method over 1 g of a soil sample, the ammonia evolved could neutralise 25 mL of 1 MH_2SO_4. Then, the percentage of nitrogen present in the sample is
(a) 100% (b) 60% (c) 70% (d) 25%

143. Which compound among the following is most reactive towards electrophilic reagents?
(a) 
(b) 
(c) 
(d) 

144. Which of the following is not explained by hyperconjugation?
(a) Stability order of carbanions
(b) Stability order of free radicals
(c) Stability order of carbocations
(d) Stability of alkenes

145. In the face centred unit cell, the lattice points are present at
(a) only the corners of the unit cell
(b) the corners and the centre of the unit cell
(c) the corners and the face centres of the unit cell
(d) only the face centres of the unit cell

146. If the K_H values for Ar(*g*), CO_2(*g*), HCHO(*g*) and CH_4(*g*) respectively are 40.39, 1.67, 1.83×10^{-5} and 0.413, then identify the correct increasing order of their solubilities.
(a) HCHO < CH_4 < CO_2 < Ar
(b) HCHO < CO_2 < CH_4 < Ar
(c) Ar < CO_2 < HCHO < CH_4
(d) Ar < CO_2 < CH_4 < HCHO

147. If 500 mL of $CaCl_2$ solution contains 3.01×10^{22} chloride ions, molarity of the solution will be
(a) 0.05 M (b) 0.01 M (c) 0.1 M (d) 0.02 M

148. Which statement among the following is incorrect?
(a) Unit of rate of disappearance is M s^{-1}.
(b) Unit of rate of reaction is M s^{-1}.
(c) Unit of rate constant *k* depends upon order of reaction.
(d) Unit of rate constant *k* for a first order reaction is M s^{-1}.

149. For zero order reaction, a plot of $t_{1/2}$ *versus* $[A]_0$ will be
(a) a straight line passing through the origin and slope = *k*
(b) a horizontal line (parallel to *x*-axis)
(c) a straight line with slope –*k*
(d) a straight line passing through origin and slope $= \dfrac{1}{2k}$

150. If hydrogen electrods dipped in two solutions of pH = 3 and pH = 6 are connected by a salt bridge, the emf of the resulting cell is
(a) 0.177 V (b) 0.3 V (c) 0.052 V (d) 0.104 V

151. In an adsorption experiment, a graph between log(*x*/*m*) *versus* log *p* was found to be linear with a slope of 45°. The intercept on log(*x*/*m*) axis was found to be 0.3010. The amount of gas adsorbed per gram of charcoal under a pressure of 0.5 atm is
(a) 0.5 g (b) 1.0 g (c) 1.5 g (d) 0.75 g

152. The correct order of sulphur-oxygen bond in SO_3, $S_2O_3^{2-}$ and SO_4^{2-} is
(a) $SO_4^{2-} < S_2O_3^{2-} < SO_3$ (b) $SO_4^{2-} < SO_3 < S_2O_3^{2-}$
(c) $S_2O_3^{2-} < SO_4^{2-} < SO_3$ (d) $S_2O_3^{2-} < SO_3 < SO_4^{2-}$

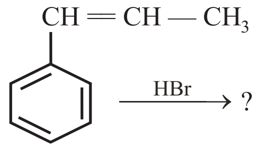
153. Potassium cyanide is made alkaline with NaOH and boiled with thiosulphate ions. The solutions is cooled and acidified with HCl and this solution with iron (III) chloride produces
(a) prussian blue colour solution
(b) blood red colour solution
(c) dark brown colour solution
(d) green colour solution

154. What among the following is coloured?
(a) CuCl (b) $ScCl_3$ (c) $CuCl_2$ (d) $TiCl_4$

155. Which of the following complexes formed by nickel is tetrahedral and paramagnetic?
(a) $[Ni(CN)_4]^{2-}$ (b) $[Ni(CO)_4]$
(c) $[Ni(Cl)_4]^{2-}$ (d) $[Ni(NH_3)_6]^{2+}$

156. Vitamin-B_1 is
(a) riboflavin (b) cobalamine
(c) thiamine (d) pyridoxine

157. Identify the product of the following reaction.



(a) $C_6H_5—CH_2—CH—CH_3$
      Br

(b) $C_6H_5—CH—CH_2—CH_3$
      Br

(c) $C_6H_5—CH_2—CH_2—CH_2—Br$

(d) $C_6H_4—CH_2—CH_2—CH_3$
      Br

158. The correct order of acidic strength among the followings is

(a) $FCH_2CO_2H > C_6H_5CO_2H > CH_3CH_2CHClCO_2H > CH_3CO_2H$

(b) $C_6H_5CO_2H > CH_3CO_2H > CH_3CH_2CHClCO_2H > FCH_2CO_2H$

(c) $CH_3CH_2CHClCO_2H > FCH_2CO_2H > CH_3CO_2H > C_6H_5CO_2H$

(d) $FCH_2CO_2H > CH_3CH_2CHClCO_2H > C_6H_5CO_2H > CH_3CO_2H$

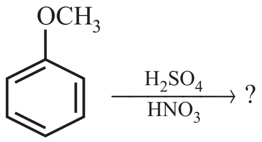
159. Identify (Z) in the following reaction.

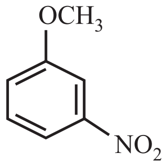
$$CH_3COOH \xrightarrow{LiAlH_4} (X) \xrightarrow[573\text{ K}]{Cu} (Y) \xrightarrow{\text{dil. NaOH}} (Z)$$

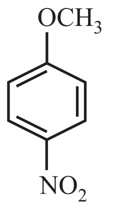
(a) Aldol (b) Ketol

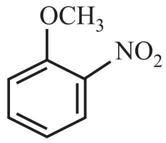
(c) Acetol (d) Butanol

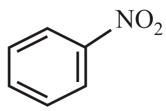
160. Identify the major product of the following reaction.



(a)


(b)


(c)


(d)


ANSWER KEY

1	(a)	**17**	(c)	**33**	(a)	**49**	(b)	**65**	(d)	**81**	(d)	**97**	(a)	**113**	(d)	**129**	(a)	**145**	(c)
2	(b)	**18**	(c)	**34**	(d)	**50**	(a)	**66**	(b)	**82**	(d)	**98**	(d)	**114**	(d)	**130**	(d)	**146**	(d)
3	(d)	**19**	(b)	**35**	(c)	**51**	(d)	**67**	(c)	**83**	(a)	**99**	(c)	**115**	(a)	**131**	(d)	**147**	(a)
4	(a)	**20**	(b)	**36**	(b)	**52**	(a)	**68**	(c)	**84**	(d)	**100**	(c)	**116**	(a)	**132**	(a)	**148**	(d)
5	(b)	**21**	(Bonus)	**37**	(b)	**53**	(d)	**69**	(b)	**85**	(c)	**101**	(b)	**117**	(c)	**133**	(b)	**149**	(d)
6	(c)	**22**	(a)	**38**	(d)	**54**	(c)	**70**	(d)	**86**	(a)	**102**	(a, c)	**118**	(c)	**134**	(c)	**150**	(a)
7	(c)	**23**	(d)	**39**	(c)	**55**	(d)	**71**	(b)	**87**	(d)	**103**	(c)	**119**	(d)	**135**	(b)	**151**	(b)
8	(b)	**24**	(d)	**40**	(d)	**56**	(d)	**72**	(a)	**88**	(a)	**104**	(a)	**120**	(b)	**136**	(b)	**152**	(c)
9	(b)	**25**	(c)	**41**	(d)	**57**	(a)	**73**	(d)	**89**	(c)	**105**	(c)	**121**	(a)	**137**	(a)	**153**	(b)
10	(c)	**26**	(c)	**42**	(b)	**58**	(b)	**74**	(a)	**90**	(d)	**106**	(a)	**122**	(a)	**138**	(c)	**154**	(c)
11	(a)	**27**	(b)	**43**	(d)	**59**	(b)	**75**	(b)	**91**	(b)	**107**	(a)	**123**	(d)	**139**	(b)	**155**	(c)
12	(a)	**28**	(b)	**44**	(c)	**60**	(d)	**76**	(a)	**92**	(c)	**108**	(b)	**124**	(b)	**140**	(c)	**156**	(c)
13	(d)	**29**	(a)	**45**	(c)	**61**	(d)	**77**	(a)	**93**	(c)	**109**	(b)	**125**	(c)	**141**	(c)	**157**	(b)
14	(b)	**30**	(c)	**46**	(b)	**62**	(c)	**78**	(b)	**94**	(a)	**110**	(b)	**126**	(d)	**142**	(c)	**158**	(d)
15	(b)	**31**	(c)	**47**	(b)	**63**	(b)	**79**	(c)	**95**	(c)	**111**	(a)	**127**	(a)	**143**	(c)	**159**	(a)
16	(c)	**32**	(b)	**48**	(d)	**64**	(a)	**80**	(b)	**96**	(a)	**112**	(b)	**128**	(a)	**144**	(a)	**160**	(b)

Hints & Solutions

MATHEMATICS

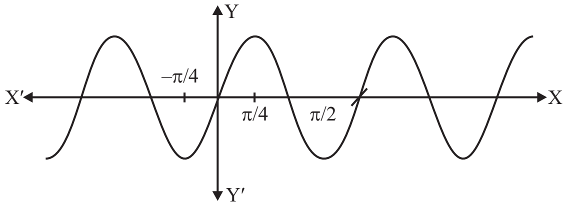
1. **(a)** Given that, $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$

$g[f(x)] = (\sin x + \cos x)^2 - 1$

$= \sin^2 x + \cos^2 x + 2 \sin x \cos x - 1$

$= 1 + \sin 2x - 1 = \sin 2x$

$(\because \sin^2 x + \cos^2 x = 1, \sin x = 2 \sin x \cos x)$



Among the given options, $\sin 2x$ is monotonous (here strictly increasing) in $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$.

So, $g(f(x))$ is invertible in $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$.

2. **(b)** $f(x) = x^9 - 11x^8 - 2x^7 + 22x^6 + x^4 - 12x^3 + 11x^2 + x - 3$

$= x^8(x - 11) - 2x^6(x - 11) + x^3(x - 11) - x^2(x - 11) + (x - 11) + 8$

$= (x - 11)[x^8 - 2x^6 + x^3 - x^2 + 1] + 8$

At $x = 11$, $f(11) = 0 + 8 = 8$

3. **(d)** Given, $f(x) = x^3$ and $g(x) = 3^x$

Here, $fog(x) = gof(x) \Rightarrow f[g(x)] = g[f(x)]$

$(3^x)^3 = 3^{x^3} \Rightarrow 3x = x^3$

$\Rightarrow x(x^2 - 3) = 0$

As, $x \ne 0$, So $x^2 - 3 = 0$

4. **(a)** We have, $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$

Trace of A = Sum of diagonal elements of $A = 1 + 7 + 9 = 17$

5. **(b)** Given system of equations,

$-x + y \cos C + z \cos B = 0$

$x \cos C - y + z \cos A = 0$

$x \cos B + y \cos A - Z = 0$

$$\Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$$

$\Rightarrow -1(1 - \cos^2 A) + \cos C(\cos A \cos B + \cos C + \cos B(\cos A \cos C + \cos B)]$

$= -1(\sin^2 A) + \cos A \cos B \cos C + \cos^2 C + \cos A \cos B \cos C + \cos^2 B$

$= -\sin^2 A + \cos^2 B + \cos^2 C + 2 \cos A \cos B \cos C$

Clearly, we can observe that $\Delta > 0$.

Hence, system of equations has a non-zero solution.

6. **(c)** Given,

$$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$

$$A^{-1} = \frac{\text{adj}(A)}{|A|} = \left(\frac{1}{\sec^2\theta}\right)\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

So, $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1}$

$$= \frac{1}{\sec^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$= \frac{1}{\sec^2\theta}\begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & -\tan^2\theta+1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

Comparing the above matrix with $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, we get

$\therefore \; a = \cos 2\theta, b = \sin 2\theta$

7. **(c)** We have given, $z_1 = 2 + 3i$, $z_2 = 3 + 2i$

$|z_1| = 13, |z_2| = 13$

$$\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}\begin{bmatrix} \bar{z}_1 & -z_2 \\ \bar{z}_2 & z_1 \end{bmatrix} = \begin{bmatrix} z_1\bar{z}_1 + z_2\bar{z}_2 & -z_1z_2 + z_1z_2 \\ -\bar{z}_2\bar{z}_1 + \bar{z}_1\bar{z}_2 & z_2\bar{z}_2 + z_1\bar{z}_1 \end{bmatrix}$$

$$= \begin{bmatrix} |z_1|^2 + |z_2|^2 & 0 \\ 0 & |z_1|^2 + |z_2|^2 \end{bmatrix}$$

$$= \begin{bmatrix} 13+13 & 0 \\ 0 & 13+13 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 0 \\ 0 & 26 \end{bmatrix} = 26I$$

8. **(b)** Given $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$ $[R_1 \to R_1 + R_3]$

$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$

$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ a+c & b+a & b+c \\ b+c & c+a & a+b \end{vmatrix}$

$[R_2 \to R_2 + R_3]$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ a+c & b-c & b-a \\ b+c & a-b & a-c \end{vmatrix}$

$= (a+b+c)[(b-c)(a-c)-(b-a)(a-b)]$

$= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= a^3+b^3+c^3-3abc$

9. **(b)** We have,

$(1+i)(1+3i)(1+7i) = x+iy$

$(1+3i+i+3i^2)(1+7i) = x+iy$

$-2-14i+4i+28i^2 = x+iy$

$-30-10i = x+iy$

$|x+iy| = \sqrt{30^2+10^2} = \sqrt{1000} = 10\sqrt{10}$

10. **(c)** We have given that

$1, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the roots of $z^5 - 1 = 0$

$\therefore$ We can write that

$(z-1)(z-\alpha_1)(z-\alpha_2)(z-\alpha_3)(x-\alpha_4) = z^5 - 1$

Since, ω is the cube root of unity.

$(\omega-1)(\omega-\alpha_1)(\omega-\alpha_2)(\omega-\alpha_3)(\omega-\alpha_4) = \omega^5 - 1$

$= (\omega^5-1)+\omega = \omega^2-1+\omega$ $(\because \omega^2+\omega+1=0)$

$= -1-1 = -2$

11. **(a)** We have,

$a > 0, z = x+iy$

$\log_{\cos^2\theta}|z-a| > \log_{\cos^2\theta}|z-ai|$

We know that,

$0 < \cos^2\theta < 1$

So, $|z-a| < |z-ai|$

$(x-a)^2+y^2 < x^2+(y-a)^2$

$-2ax < -2ay \Rightarrow x > y$

12. **(a)** Given equation,

$ix^2 - 2(i+1)x + (2-i) = 0$

Let the other roots be k

$[x-(2-i)][x-k] = 0$

$x^2-(2-i+k)x-(k(2-i)) = 0$

{comparing with Eq. (i)}

$2i+1+ki = 2i+2$

$ki = 1$

$k = \frac{1}{i} = -i$

13. **(d)** Given quadratic equation,

$x^2+x+1 = 0$

$\therefore$ Roots are $\alpha = \omega$ and $\beta = \omega^2$

where, $\omega = \frac{-1+\sqrt{3}i}{2}$ and $\omega^3 = 1$

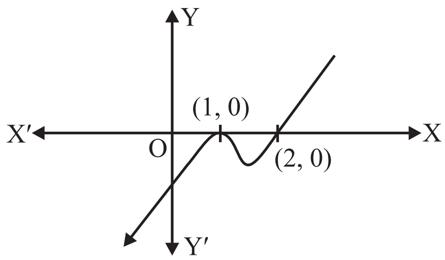
Now, $\alpha^{2021} = \omega^{2021} = \omega^{2019}\omega^2 = (\omega^3)^{673}\omega^2 = \omega^2$

and $\beta^{2021} = \omega^{4042} = \omega^{4041}\omega = (\omega^3)^{1347}\omega = \omega$

Hence, equation whose roots are α^{2021} and β^{2021} will be,

$(x-\omega)(x-\omega^2) = x^2+x+1$

14. **(b)**



This is the graph of $f(x) = x^3-4x^2+5x-2$

Also given,

$$f\left(x+\frac{1}{3}\right) = \left(x+\frac{1}{3}\right)^3 - 4\left(x+\frac{1}{3}\right)^2 + 5\left(x+\frac{1}{3}\right) - 2 = 0$$

So, the graph of $f\left(x+\frac{1}{3}\right)$ will shift towards left by 1/3 units,

So the new roots will also shift by 1/3 units towards left.

$\therefore$ New roots $= \left(1-\frac{1}{3}, 1-\frac{1}{3}, 2-\frac{1}{3}\right) = \left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}\right)$

15. **(b)** Given equation, $f(x) = 2x^3+mx^2-13x+n$

Let α, β, γ are the roots of $f(x) = 0$ where $\alpha = 2, \beta = 3$ and let $\gamma = k$

As we know,

$\alpha+\beta+\gamma = -\frac{m}{2} = 2+3+k$ …(i)

$\alpha\beta+\beta\gamma+\gamma\alpha = -\frac{13}{2} = 6+5k$ …(ii)

$\alpha\beta\gamma = -\frac{n}{2} = 6k$ …(iii)

From Eq. (ii), $-13 = 12+10k \Rightarrow k = \frac{-5}{2}$

$n = -12k = -12\left(\frac{-5}{2}\right) = 30$

$m = -10 - 2k = -10 - 2\left(\frac{-5}{2}\right)$

16. (c) We have given

${}^6P_4 + 4 \cdot {}^6P_3$

$= \left(\frac{6!}{2!} + 4 \cdot \frac{6!}{3!}\right)$

$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} + \frac{4 \times 6 \times 5 \times 4 \times 3!}{3!}$

$= 360 + 480 = 840 \qquad \left[\because \; {}^nP_r = \frac{n!}{(n-r)!}\right]$

17. (c) Let two girls are G_1 and G_2 and three boys are B_1, B_2, B_3.

$B_1\, G_1\, B_2\, G_2\, B_3$

According to the given situation,

G_1 and G_2 girls can be arranged in 2! ways.

B_1, B_2 and B_3 boys can be arranged in 3! ways.

$\therefore$ Required number of arrangements = 2! × 3! = 12

18. (c) We have,

Number of balls = 5

Number of tins = 4

So, each ball can be placed in 4 ways.

Thus, total number of ways = 4 × 4 × 4 × 4 × 4 = 4^5

19. (b) Given that,

$$\frac{3x-2}{(x+1)^2(x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+3)}$$

$3x - 2 = A(x+1)(x+3) + B(x+3) + C(x+1)^2$

Put $x = -1$, $3(-1) - 2 = B(-1+3)$

$-5 = 2B$

$B = \frac{-5}{2}$

Put $x = -3$, $3(-3) - 2 = C(-3+1)^2$

$\Rightarrow C = \frac{-11}{4}$

Put $x = 0$,

$-2 = 3A + 3B + C$

$\Rightarrow -2 = 3A - \frac{15}{2} - \frac{11}{4}$

$\frac{33}{4} = 3A \quad \Rightarrow A = \frac{11}{4}$

Therefore,

$4A + 2B + 4C = 4\left(\frac{11}{4}\right) + 2\left(\frac{-5}{2}\right) + 4\left(\frac{-11}{4}\right)$

$= 11 - 5 - 11 = -5$

20. (b) $\cos\left(22\frac{1}{2}\right)^\circ = \cos\left(\frac{\pi}{8}\right) = \sqrt{\frac{1 + \cos\frac{\pi}{4}}{2}}$

$\cos\left(22\frac{1}{2}\right)^\circ = \sqrt{\frac{1 + \frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$

21. (Bonus) Given, $\cos\theta = -\sqrt{\frac{3}{2}}$, $\sin\alpha = \frac{-3}{5}$

$\cos\theta = -1.2247$

But as we know,

$-1 \le \cos\theta \le 1$

It means the given value of $\cos\theta$ is incorrect.

Hence, it is not the correct question.

22. (a) Given, $\tan\beta = \frac{\tan\alpha + \tan\gamma}{1 + \tan\alpha\tan\gamma}$

$$\tan\beta = \frac{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\gamma}{\cos\gamma}}{1 + \frac{\sin\alpha}{\cos\alpha}\frac{\sin\gamma}{\cos\gamma}}$$

$$\tan\beta = \frac{\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)} = \frac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha \sin 2\gamma}$$

$$= \frac{2\sin(\alpha+\gamma)\cos(\alpha-\gamma)}{1 + (2\sin\alpha\cos\gamma)(2\sin\gamma\cos\alpha)}$$

$\left(\because \sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right),\right.$

$\left.\sin 2x = 2\sin x\cos x\right)$

$$= \frac{2\sin(\alpha+\gamma)\cos(\alpha-\gamma)}{1 + [\sin(\alpha+\gamma) + \sin(\alpha-\gamma)][\sin(\alpha+\gamma) - \sin(\alpha-\gamma)]}$$

$$= \frac{2\sin(\alpha+\gamma)\cos(\alpha-\gamma)}{1 + \sin^2(\alpha+\gamma) - \sin^2(\alpha-\gamma)}$$

$$= \frac{2\sin(\alpha+\gamma)\cos(\alpha-\gamma)}{\sin^2(\alpha+\gamma) + \cos^2(\alpha-\gamma)}$$

Dividing by $\cos(\alpha-\gamma)$ in numerator and denominator, we get

$$= \frac{2\tan\beta}{1+\tan^2\beta} = \sin 2\beta$$

23. (d) Given, $\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$

Let $A = \frac{\pi}{4}\cot\theta$, $B = \frac{\pi}{4}\tan\theta$

$\therefore \sin A = \cos B$

To satisfy $\sin A = \cos B$, $A + B$ must be $\frac{\pi}{2}$.

$A + B = \frac{\pi}{2}$, $\frac{\pi}{4}\cot\theta + \frac{\pi}{4}\tan\theta = \frac{\pi}{2}$

$\frac{\pi}{4}(\tan\theta + \cot\theta) = \frac{\pi}{2} \Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0 \Rightarrow (\tan\theta - 1)^2 = 0$

$\Rightarrow \tan\theta - 1 = 0$

$\tan\theta = 1$

$\therefore \theta = n\pi + \frac{\pi}{4}$

24. **(d)** Given that, $x = \sin(2\tan^{-1} 2)$

Let $\tan^{-1} 2 = \alpha$

$\tan\alpha = 2$

$\therefore x = \sin(2\alpha) = \frac{2\tan\alpha}{1+\tan^2\alpha}$

$= \frac{2\times 2}{1+(2)^2} = \frac{4}{5}$

$\Rightarrow y = \cos(2\tan^{-1} 3)$

Let $\tan^{-1} 3 = \beta$, $\tan\beta = 3$

$\therefore y = \cos(2\beta) = \frac{1-\tan^2\beta}{1+\tan^2\beta} = \frac{1-9}{1+9} = \frac{-4}{5}$

$\Rightarrow z = \sec(3\tan^{-1} 4)$

Let $\tan^{-1} 4 = \gamma$

$\tan\gamma = 4$

$\sec^2\gamma = 1 + \tan^2\gamma = 1 + 16 = 17$

$\therefore \cos\gamma = \frac{1}{\sqrt{17}}$

$\therefore z = \sec(3\gamma) \Rightarrow z = \frac{1}{\cos 3\gamma}$

$z = \frac{1}{4\cos^3\gamma - 3\cos\gamma}$

$z = \frac{1}{4\left(\frac{1}{17\sqrt{17}}\right) - 3\left(\frac{1}{\sqrt{17}}\right)} \Rightarrow z = \frac{-17\sqrt{17}}{47}$

Thus, $x = \sin(2\tan^{-1} 2) = \frac{4}{5} = 0.8$

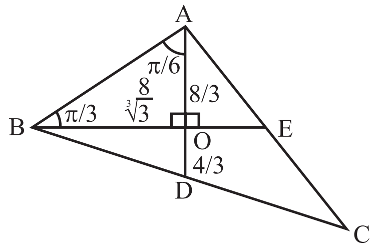
$y = \cos(2\tan^{-1} 3) = \frac{-4}{5} = -0.8$

$z = \sec(3\tan^{-1} 4) = \frac{-17\sqrt{17}}{47} = -1.49$

$\therefore z < y < x$

25. **(c)** Given in ΔABC,

$AD = 4$; $\angle DAB = \frac{\pi}{6}$; $\angle ABE = \frac{\pi}{3}$



Since point O divides AD in the ratio of 2 : 1.

$\therefore AO = \frac{8}{3}$ and $OD = \frac{4}{3}$

Now, area of $\Delta ABC = 2 \times$ Area of ΔABE

$= 2\times\left[\frac{3}{2}(\text{Area of } \Delta AOB)\right]$

$= 3 \times$ Area of ΔAOB

$= 3\times\frac{1}{2}\times BO\times AO$

$= 3\cdot\frac{1}{2}\cdot\frac{8}{3\sqrt{3}}\cdot\frac{8}{3}$

$\left\{\because \tan\frac{\pi}{6} = \frac{BO}{AO} \Rightarrow BO = \frac{1}{\sqrt{3}}\times\frac{8}{3}\right\}$

$\therefore$ Area of $\Delta ABC = \frac{32}{3\sqrt{3}}$ sq. units

26. **(c)** In ΔABC,

Given that

$2\Delta^2 = \frac{a^2b^2c^2}{a^2+b^2+c^2}$

$2\Delta^2(a^2+b^2+c^2) = a^2b^2c^2$

$a^2+b^2+c^2 = \left(\frac{abc}{\Delta}\right)^2\cdot\frac{1}{2} = 8R^2$

[$\because$ $abc/\Delta = 4R$ and $a = 2R\sin A$, $b = 2R\sin B$, $c = 2R\sin C$]

$4R^2\sin^2 A + 4R^2\sin^2 B + 4R^2\sin^2 C = 8R^2$

$4R^2(\sin^2 A + \sin^2 B + \sin^2 C) = 8R^2$

$\sin^2 A + \sin^2 B + \sin^2 C = 2$

$\frac{1-\cos 2A}{2} + \frac{1-\cos 2B}{2} + \frac{1-\cos 2C}{2} = 2$

$\left(\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$

$(\cos 2A + \cos 2B + \cos 2C) = -1$

$-1 - 4\cos A\cos B\cos C = -1$

$\cos A\cos B\cos C = 0$

So, any one among A, B and C has to be $\frac{\pi}{2}$.

Hence, ΔABC is a right angled triangle.

27. **(b)** Given that, sides of a triangle inscribed in a circle subtend angles of α, β, γ at the centre.

$$\cos\left(\alpha+\frac{\pi}{2}\right)=-\sin\alpha,\ \cos\left(\beta+\frac{\pi}{2}\right)=-\sin\beta$$

$$\cos\left(\gamma+\frac{\pi}{2}\right)=-\sin\gamma$$

As we know,

$AM \geq GM$

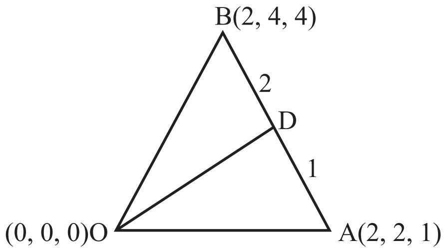
$$\frac{-(\sin\alpha+\sin\beta+\sin\gamma)}{3} \geq (-\sin\alpha\sin\beta\sin\gamma)^{1/3}$$

Equating holds, when $\alpha=\beta=\gamma$

$\alpha+\beta+\gamma=360°$, $\alpha=\beta=\gamma=120°$

$$\Rightarrow AM=\frac{1}{3}\left[\left(-\frac{\sqrt{3}}{2}\right)+\left(-\frac{\sqrt{3}}{2}\right)+\left(-\frac{\sqrt{3}}{2}\right)\right]=-\frac{\sqrt{3}}{2}$$

28. **(b)** Given position vectors of A and B are $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$



$$OA=\sqrt{2^2+2^2+1^2}=3$$

$$OB=\sqrt{2^2+4^2+4^2}=6$$

By angle bisector theorem,

$$\frac{OA}{OB}=\frac{AD}{BD}=\frac{3}{6}$$

Coordinates of D,

$$=\frac{2(2,2,1)+1(2,4,4)}{2+1}=\frac{1}{3}(6,8,6)=\left(2,\frac{8}{3},2\right)$$

$$\Rightarrow OD=\sqrt{2^2+\left(\frac{8}{3}\right)^2+2^2}=\frac{\sqrt{136}}{3}$$

29. **(a)** Given, $\mathbf{u}=2\hat{i}+3\hat{j}+\hat{k}$, $\mathbf{v}=-3\hat{i}+2\hat{j}$,

$\mathbf{w}=\hat{i}-\hat{j}+4\hat{k}$

$\mathbf{u}\cdot\mathbf{v}=-6+6+0=0$

$\mathbf{u}\cdot\mathbf{w}=2-3+4=3\neq 0$

u is perpendicular to **v** but **u** is not perpendicular to **w**.

30. **(c)** Given lines are

$$L_1:\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{6}$$

$\therefore\ x_1=3, y_1=2, z_1=1$

$a_1=2, b_1=3, c_1=\lambda$

$$L_2:\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-2}{3}$$

$x_2=2, y_2=3, z_2=2$

$a_2=3, b_2=2, c_2=3$

Since we know that two lines are coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 2 & 3 & \lambda \\ 3 & 2 & 3 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 2 & 5 & \lambda+2 \\ 3 & 5 & 6 \end{vmatrix}=0$$

$30-5\lambda-10=0, \lambda=4$

Now, $\sin^{-1}(\sin\lambda)+\cos^{-1}(\cos\lambda)$

$=\sin^{-1}(\sin 4)+\cos^{-1}(\cos 4)$

$=(\pi-4)+(2\pi-4)=(3\pi-8)$

31. **(c)** $a=(1,1,0), b=(1,1,1)$

$\therefore$ Unit vector in the plane of a and b will be $a+\lambda b$.

Given that $a+\lambda b$ is perpendicular to a

$0=(a+\lambda b)\cdot a$

$a\cdot a+\lambda a\cdot b=0$

$(1,1,0)\cdot(1,1,1)+\lambda(1,1,0)\cdot(1,1,1)=0$

$(1+1+0)+\lambda(1+1+0)=0$

$\lambda=-1$

$\therefore\ a+\lambda b=(0,0,1)=k$

32. **(b)** Let L, be the line passing through $(1, 1, -1)$ and parallel to the vector $\hat{i}+2\hat{j}-\hat{k}$ is

$$L_1:\frac{x-1}{1}=\frac{y-1}{2}=\frac{z+1}{-1}=\lambda$$

$$L_2:\frac{x-3}{-1}=\frac{y+2}{5}=\frac{z-2}{-4}=\mu$$

The line L_1 meets L_2 at point A. So

$(\lambda+1, 2\lambda+1, -\lambda-1)$

To calculate the intersection point of these lines,

$=(-\mu+3, 5\mu-2, -4\mu+2)$

$\lambda+\mu=2$ …(i)

and $2\lambda+1=5\mu-2$

Substitute the value of λ from Eq. (i),

$4-2\mu+1=5\mu-2$

$\therefore\ \mu=1$

Then $\lambda=1$

$A:(2,3,-2)$

Plane $P: 2x-y+2z+7=0$

Plane P meets with the line L_1 at B. So,

$2(\lambda+1)-(2\lambda+1)+(-2\lambda-2)+7=0$

$-2\lambda + 6 = 0$

$\Rightarrow \lambda = 3$

$B : (4, 7, -4)$

$\therefore$ Point $AB^2 = 2^2 + 4^2 + 2^2 = 24 = 2\sqrt{6}$

33. **(a)** Given, $\mathbf{a} = \mathbf{i}$, $\mathbf{b} = \mathbf{j}$

$\therefore (\mathbf{r} \times \mathbf{a}) = (\mathbf{b} \times \mathbf{a})$

$\Rightarrow (\mathbf{r} \times \mathbf{a}) - (\mathbf{b} \times \mathbf{a}) = 0, (\mathbf{r} - \mathbf{b}) \times \mathbf{a} = 0$

$\Rightarrow \mathbf{r} - \mathbf{b} = \lambda$

$\mathbf{r} = \mathbf{b} + \lambda\mathbf{a}$

$\mathbf{r} = \lambda\hat{i} + \hat{j}$

$\mathbf{r} \times \mathbf{b} = \mathbf{a} \times \mathbf{b}$

$L_2 : (\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$

$\mathbf{r} = \mathbf{a} + \mu\mathbf{b}$

$\gamma = \mathbf{i} + \mu\mathbf{j}$

From Eqs. (i) and (ii), we get

$\lambda\mathbf{i}, \mathbf{i} - \mu\mathbf{j} + \mathbf{j} = 0$

$\mathbf{i}(\lambda - 1) + \mathbf{j}(1 - \mu) = 0$

$\Rightarrow \lambda = 1, \mu = 1$

Hence, $\mathbf{r} = \mathbf{i} + \mathbf{j}$

34. **(d)** Observations are $-1, 0, 4$.

Mean, $\bar{x} = \dfrac{-1+0+4}{3} = 1$

Deviation $= |-1-1|, |0-1|, |4-1| = 2, 1, 3$

Mean deviation, $\text{M.D.}(\bar{x}) = \dfrac{2+1+3}{3} = 2.$

35. **(c)** Let the angles of triangle are α, β and γ.

Given $\alpha = 60°$ and variance $= 1014$

Mean, $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{\alpha+\beta+\gamma}{3}$

(As we know, sum of angles of a triangle is always 180°)

$\therefore \bar{x} = \dfrac{180}{3} = 60$

Now, variance

$$V = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$$

$$1014 = \frac{\alpha^2 + \beta^2 + \gamma^2}{3} - (60)^2$$

$$\frac{(60)^2 + \beta^2 + \gamma^2}{3} = 1014 + 3600$$

$\beta^2 + \gamma^2 = 4614 \times 3 - 3600$

$\beta^2 + \gamma^2 = 10242$ ...(i)

and mean, $\bar{x} = 60 = \dfrac{\alpha+\beta+\gamma}{3}$

$60 + \beta + \gamma = 60 \times 3$

$\beta + \gamma = 120$ ...(ii)

Solving Eqs. (i) and (ii), we get

$\beta = 21$ and $\gamma = 99$

36. **(b)** One card is selected at random from 27 cards numbered from 1 to 27

$\Rightarrow n = 27$

Even or divisible by 5

Possible numbers which are

$A = (2, 4, 5, 6, 8, 10, 12, 14, 15, 16, 18, 20, 22, 24, 25, 26)$

$r = 16$

$\therefore$ Probability $= \dfrac{\mathbf{r}}{\mathbf{n}} = \dfrac{16}{27}$

37. **(b)** White balls = 5

Black balls = 7

Total balls = 12

Probability of drawing 3 white and 6 black balls

$$= \frac{{}^5C_3 \times {}^7C_6}{{}^{12}C_9}$$

$$= \frac{7 \times 10}{{}^{12}C_9} = \frac{70}{12 \cdot 11 \cdot 10} = \frac{7}{22}$$

38. **(d)** Probability of speaking truth by A and B.

$P(A) = \dfrac{4}{5}$

$P(B) = \dfrac{3}{4}$

They will contradict each other in two case.

Case I: $A \to$ True, $B \to$ False

Case II: $A \to$ False, $B \to$ True

Probability $= P(A)\,P(B') + P(B)\,P(A')$

$$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{7}{20}$$

39. **(c)** For a binomial distribution,

Mean $= np = 2$

Variance $= npq = 1$

$\Rightarrow q = \dfrac{1}{2}$ and $p = \dfrac{1}{2}$

$\Rightarrow np = 2\, n\left(\dfrac{1}{2}\right) = 2$

$\Rightarrow n = 4$

Probability that x is greater than 1

$P(x > 1) = P(x = 2) + P(x = 3) + P(x = 4)$

$$= ({}^4C_2 + {}^4C_3 + {}^4C_4)\frac{1}{2^4} = \frac{11}{16}$$

40. **(d)** $\Sigma P(x_i) = 1$

$$K + K + \frac{1}{7} + 2K + \frac{2}{5} = 1$$

$$\Rightarrow \; 4K = \frac{3}{5} - \frac{1}{7} \Rightarrow \; 4K = \frac{16}{35}$$

$$\Rightarrow K = \frac{4}{35}$$

Mean of X,

$$\mu = \frac{\Sigma P_i X_i}{\Sigma P_i} = \Sigma P_i X_i = K + \frac{1}{7} + 4K + \frac{6}{5}$$

$$= 5K + \frac{47}{35} = 5\left(\frac{4}{35}\right) + \frac{47}{35} = \frac{67}{35}$$

41. **(d)** Let the point be (h, k) is at the distance 4 units from the point $(3, -2)$. So,

$$(h-3)^2 + (k+2)^2 = 4^2$$

$$h^2 + 9 - 6h + k^2 + 4 + 4k = 16$$

Now, replacing, h and k by x and y

$x^2 + y^2 - 6x + 4y - 3 = 0$

42. **(b)** If initial coordinates of any point is (x, y) and after rotation through an angle of 45° new coordinates are (X, Y). Then

$x = X \cos 45° - Y \sin 45°$

$y = X \sin 45° + Y \cos 45°$

Here, $X = 1$, $Y = -1$

$$x = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} \quad \Rightarrow \quad y = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0 .$$

So, point in original system is $(\sqrt{2}, 0)$.

43. **(d)** Equation of line in the intercept form is $\frac{x}{a} + \frac{y}{b} = 1$

Here intercepts are equal and passes through (5, 6), then

$$\frac{-5}{a} + \frac{6}{a} = 1$$

$$\frac{1}{a} = 1 \quad \Rightarrow \quad a = 1$$

From Eq. (i) we get $\;x + y = 1$

44. **(c)** Equation of the line in slope intercept form,

$$y = mx + c$$

Here, y intercept $= c = 4$

$\therefore \;\; y = mx \times 4$

Distance from origin,

$$D = \frac{|m.0 - 0 + 4|}{\sqrt{1+m^2}} = \frac{4}{\sqrt{1+m^2}} .$$

45. **(c)** For an equilateral triangle, equation of its base $x + y = 2$ and vertex : $(2, -1)$

vertex $(2, -1)$ does not lie on $x + y = 2$

because $2 - 1 = 1 \neq 2$

$\therefore$ Length of side of triangle = Distance between vertex and base

$\therefore$ Length of side

$$= \frac{2}{\sqrt{3}}\left(\frac{|2-1-2|}{\sqrt{1^2+1^2}}\right) = \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{3}} .$$

46. **(b)** Let the slope of required line $= m$

As it is given that required line is perpendicular to the line $x \sec\theta + y \operatorname{cosec}\theta = a$

$$\therefore \quad m\left(-\frac{\sec\theta}{\operatorname{cosec}\theta}\right) = -1$$

$$\Rightarrow \quad m = \frac{\operatorname{cosec}\theta}{\sec\theta} = \cot\theta$$

Equation of the required line

$(y - a\sin^3\theta) = \cot\theta\,(x - a\cos^3\theta)$

$y = x\cot\theta - a\cot\theta\cos^3\theta + a\sin^3\theta$

$y\sin\theta = x\cos\theta - a(\cos^4\theta - \sin^4\theta)$

$y\sin\theta = x\cos\theta - a(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$\Rightarrow \quad x\cos\theta - y\sin\theta = a\cos 2\theta.$

47. **(b)** Given, $6x^2 + 11xy - 10y^2 = 0$

$6x^2 + 15xy - 4xy - 10y^2 = 0$

$(3x - 2y)(2x + 5y) = 0$

$$y = \left(\frac{3}{2}\right)x \quad \text{or} \quad y = \left(\frac{-2}{5}\right)x$$

Angle between these lines,

$$\tan\theta = \left|\frac{M_1 - M_2}{1 + M_1M_2}\right| = \left|\frac{\frac{3}{2} + \frac{2}{5}}{1 - \frac{3}{2}\cdot\frac{2}{5}}\right| = \left|\frac{\frac{19}{10}}{\frac{2}{5}}\right| = \frac{19}{4}$$

$$\therefore \;\; \theta = \tan^{-1}\left(\frac{\sqrt{361}}{4}\right)$$

48. **(d)** Given curve,

C : $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$

and line, L : $x + 2y = k$ ...(i)

$$\frac{x+2y}{k} = 1 \qquad \text{...(ii)}$$

Using Eq. (ii) in Eq. (i), we get

$(2x^2 - 2xy + 3y^2) + (2x - y) . 1 - 1^2 = 0$

$(2x^2 - 2xy + 3y^2) + (2x - y)$

$$\cdot \frac{(x+2y)}{k} - \left(\frac{x+2y}{k}\right)^2 = 0$$

$k^2(2x^2 - 2xy + 3y^2) + k(2x - y)(x + 2y) - (x + 2y)^2 = 0$

$x^2(2k^2 + 2k - 1) + xy(-2k^2 + 3k - 4) + y^2(3k^2 - 2k - 4) = 0$

Given that these lines are at right angles,

$\therefore$ Coefficient of x^2 + Coefficient of $y^2 = 0$

$(2k^2 + 2k - 1) + (3k^2 - 2k - 4) = 0$

$5k^2 - 5 = 0 \Rightarrow k = \pm 1 \quad \Rightarrow k^2 = 1.$

49. **(b)** Given, $3x^2 - 5xy + 4y^2 = 0$

$\therefore \quad a = 3, 2h = -5$ and $b = 4$

Equation of bisector of the angle between the pair of lines,

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

$$\frac{x^2 - y^2}{3 - 4} = \frac{xy}{-\frac{5}{2}}$$

$\Rightarrow 5x^2 - 5y^2 = 2xy$

$\Rightarrow 5x^2 - 2xy - 5y^2 = 0$

50. **(a)** Bisectors of $ax^2 + 2hxy + by^2 = 0$ is

$hx^2 - (a - b)xy - hy^2 = 0$

Here, $x^2 - 2mxy - y^2 = 0$ and

$a = 1, b = -1, h = -m$

$\therefore$ Equation of bisector $-mx^2 - 2xy + my^2 = 0$

Now comparing this bisector with the given bisector

$x^2 - 2nxy - y^2 = 0$

$$\frac{-m}{1} = \frac{-2}{-2n} = \frac{m}{-1}$$

$$\frac{-m}{1} = \frac{-2}{-2n}$$

$1 + mn = 0$

51. **(d)** For a given circle,

x-Intercept $= 2\sqrt{g^2 - c} = 2$

y-Intercept $= 2\sqrt{f^2 - c} = 3$ and $c = 0$

$g = \pm 1, f = \pm \frac{3}{2}$

$x^2 + y^2 \pm 2x \pm 3y = 0$

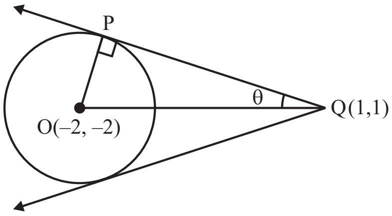
It also passes through $(-2, 0)$ and $(0, 3)$.

So, its centre must be in second quadrant.

The coordinate of centre is negative. So, $x^2 + y^2 + 2x - 3y = 0$.

52. **(a)** Given circle, $x^2 + y^2 + 4x + 4y - 1 = 0$

$x^2 + 4x + 4 + y^2 + 4y + 4 + 1 - 1 = 9$



$\text{OQ} = \sqrt{(1+2)^2 + (1+2)^2} = 3\sqrt{2}$

$$\sin\theta = \frac{\text{OP}}{\text{OQ}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\theta = 45° \quad \Rightarrow \quad 2\theta = 90°$

53. **(d)** Given circle, $x^2 + y^2 - 4x - 8y - 5 = 0$

Centre = (2, 4)

Radius $= 2\sqrt{4 + 16 + 5} = 10$

Line : $3x - 4y - m = 0$

Distance of L from Centre < Radius

$$\frac{|3.2 - 4.4 - m|}{\sqrt{3^2 + 4^2}} < 10$$

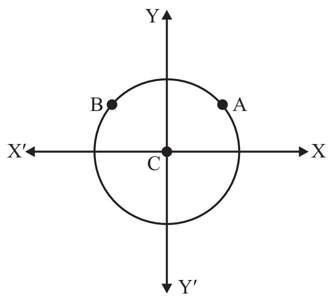
$\Rightarrow |-10 - m| < 50$

$\Rightarrow -50 < -10 - m < 50$

$\Rightarrow -60 < m < 40$

Only (d) satisfies the above inequality.

54. **(c)** For a given circle, C(0, 0), $r = 3$ units



Let $A = (3\cos\theta, 3\sin\theta)$

Then, $B = \left(3\cos\left(\theta + \frac{2\pi}{3}\right), 3\sin\left(\theta + \frac{2\pi}{3}\right)\right)$

Let (h, k) be the required point

$2h = 3\cos\theta + 3\cos\left(\theta + \frac{2\pi}{3}\right)$

$= 3\cos\theta + 3\left(\cos\theta\cos\frac{2\pi}{3} - \sin\theta\sin\frac{2\pi}{3}\right)$

$= \frac{3}{2}(\cos\theta - \sqrt{3}\sin\theta)$

$2k = 3\sin\theta + 3\sin\left(\theta + \frac{2\pi}{3}\right)$

$$= 3\sin\theta + 3\left(\sin\theta\cos\frac{2\pi}{3} + \cos\theta\sin\frac{2\pi}{3}\right)$$

$$= \frac{3}{2}(\sin\theta + \sqrt{3}\cos\theta)$$

Therefore, $\cos\theta - \sqrt{3}\sin\theta = \dfrac{4h}{3}$ ...(i)

and $\sqrt{3}\cos\theta + \sin\theta = \dfrac{4k}{3}$ ...(ii)

Now, From eq. (i) $\cos\theta - \sqrt{3}\sin\theta = \dfrac{4h}{3}$

$\sqrt{3}\cos\theta - 3\sin\theta = \dfrac{4h}{\sqrt{3}}$ ...(iii)

Substract Eq. (iii), from (ii),

$$4\sin\theta = \frac{4}{3}(k - \sqrt{3}h)$$

$$\sin\theta = \left(\frac{k - \sqrt{3}h}{3}\right)$$

Similarly,

$$\cos\theta = \left(\frac{3k + \sqrt{3}h}{3\sqrt{3}}\right) = \left(\frac{\sqrt{3}k + h}{3}\right)$$

We know that $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{k - \sqrt{3}h}{3}\right) + \left(\frac{\sqrt{3}k + h}{3}\right)^2 = 1$$

$$4(k^2 + h^2) = 9 \Rightarrow x^2 + y^2 = \frac{9}{4}.$$

55. **(d)** Given circles, $C_1: x^2 + y^2 + 3x + 5y + 4 = 0$ ...(i)

$C_2: x^2 + y^2 + 5x + 3y + 4 = 0$ ...(ii)

On solving these two circles, we get $x = y$

Substituting $x = y$ in Eq. (i), we get

$2x^2 + 8x + 4 = 0$

$x^2 + 4x + 2 = 0 \Rightarrow (x + 2)^2 = 2$

$x + 2 = \pm\sqrt{2} \Rightarrow x = -2 \pm \sqrt{2}$

$x = y \Rightarrow y = -2 \pm \sqrt{2}$

$$\text{Length} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{2}(x_1 - x_2)$$

$$= \sqrt{2}[(-2 + \sqrt{2}) - (-2 - \sqrt{2})] = 4.$$

56. **(d)** Equation of the circle passing through the intersection of the circles,

$(x^2 + y^2 - 8x - 6y + 21) + \lambda(x^2 + y^2 - 2x - 15) = 0$

It passes through the point (1, 2). So,

$(1 + 4 - 8 - 12 + 21) + \lambda(1 + 4 - 2 - 15) = 0$

$6 + (-12\lambda) = 0 \Rightarrow \lambda = 1/2$

Hence the equation becomes

$3(x^2 + y^2) - 18x - 12y + 27 = 0$

57. **(a)** $x = 5t^2 + 2$ ...(i)

$y = 10t + 4 \Rightarrow t = \dfrac{y - 4}{10}$

From Eq. (i), $(x - 2) = 5\left(\dfrac{y - 4}{10}\right)$

$(y - 4)^2 = 20(x - 2)$

Comparing it with $(y - h)^2 = 4a(x - k)$

$\therefore\ 4a = 20 \Rightarrow a = 5$

Focus $= (a + k, 0 + h) = (5 + 2, 0 + 4) = (7, 4)$.

58. **(b)** Let two points be $A(a\cos\theta_1\ b\sin\theta_1)$ and $B(a\cos\theta_2, b\sin\theta_2)$

$$m_{OA} = \frac{b\sin\theta_1}{a\cos\theta_1} = \frac{b}{a}\tan\theta_1$$

$$m_{OB} = \frac{b}{a}\tan\theta_2$$

$$m_{OA} \times m_{OB} = -1 \Rightarrow \frac{b^2}{a^2}\tan\theta_1\tan\theta_2 = -1.$$

59. **(b)** Distance from focus to directrix is $a\left(e - \dfrac{1}{e}\right)$

Given, $e = \dfrac{5}{4}$ and directrix, $4x - 3y - 3 = 0$

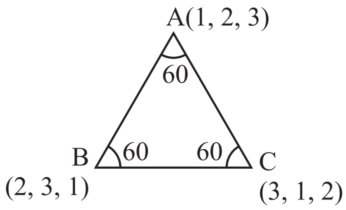
$$a\left(\frac{5}{4} - \frac{4}{5}\right) = \frac{4.3 - 3}{|5|}$$

$$\Rightarrow a\left(\frac{25 - 16}{20}\right) = \frac{12 - 3}{5} \Rightarrow a = 4$$

$$\text{Vertex} = \left[3 \pm \left(\frac{-4}{5}\right), 0 \pm \left(\frac{3}{5}\right)\right]$$

$$= \left(3 - \frac{4}{5}, \frac{3}{5}\right) = \left(\frac{11}{5}, \frac{3}{5}\right)$$

60. **(d)** Let $A(1, 2, 3)$ $B(2, 3, 1)$ and $C(3, 1, 2)$ be the vertices of ΔABC.



$$\text{Centroid} = \left(\frac{1 + 2 + 3}{3}, \frac{2 + 3 + 1}{3}, \frac{3 + 1 + 2}{3}\right)$$

$= (2, 2, 2)$

$\because$ Triangle is equilateral

Orthocentre, creamcentre and incentre will be same as centroid.

$\therefore$ H + G + S + I = (2, 2, 2) + (2, 2, 2) + (2, 2, 2) + (2, 2, 2) = (8, 8, 8)

61. **(d)** We have given four points

$A(2, 3, 4)$, $B(4, 5, 7)$, $C(2, -6, 3)$ and $D(4, -4, k)$

$AB = (4, 5, 7) - (2, 3, 4) = (2, 2, 3)$

$CD = (4, -4, k) - (2, -6, 3) = (2, 2, k-3)$

Here, AB is parallel to CD.

$\therefore$ $AB = \lambda CD$

$(2, 2, 3) = \lambda(2, 2, k-3)$

Comparing x and z component to calculate the value of k,

$\therefore$ $2\lambda = 2 \Rightarrow \lambda = 1$

$\lambda(k-3) = 3 \Rightarrow k = 6$

62. **(c)** Let A be a vector which bisects the direction cosines of two lines,

$$A = \frac{1}{2}\left[\left(\frac{2}{3} \pm \frac{5}{13}\right), \left(\frac{2}{3} \pm \frac{12}{13}\right), \left(\frac{1}{3} \pm 0\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{25 \pm 15}{39}, \frac{26 \pm 36}{39}, \frac{1}{3}\right)\right]$$

$$A = \frac{1}{2}\left(\frac{41}{39}, \frac{62}{39}, \frac{1}{3}\right) \quad \text{or} \quad \frac{1}{2}\left(\frac{11}{39}, -\frac{10}{39}, \frac{1}{3}\right)$$

$\therefore$ According to the given options.

Direction ratio of a line can be considered as (41, 62, 13)

63. **(b)** Let $l = \lim\limits_{n \to \infty} \dfrac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$

$$l = \lim_{n \to \infty} \frac{n^3\left(2 + \frac{1}{n}\right)^2}{n^3\left(1 + \frac{2}{n}\right)\left(1 + \frac{3}{n} - \frac{1}{n^2}\right)}$$

$$l = \frac{(2)^2}{(1)(1)} = 4$$

64. **(a)** Given function,

$$f(x) = \begin{cases} x^2 + ax + b, & 0 \le x < 2 \\ 3x + 2, & 2 \le x \le 4 \\ 2ax + 5b, & 4 < x \le 8 \end{cases}$$

Here, $f(x)$ is continuous on the interval [0, 8]. So it will also be continuous on 2 and 4.

At $x = 2$,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\lim_{x \to 2^-} (x^2 + \text{ax} + \text{b}) = \lim_{x \to 2^+} (3x + 2)$$

$4 + 2a + b = 3(2) + 2$

$\therefore$ $2a + b = 4$ ...(i)

At $x = 4$,

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$$

$$\lim_{x \to 4^-} (3x + 2) = \lim_{x \to 4^+} 2ax + 5b$$

$3(4) + 2 = 2a(4) + 5b$

$\therefore$ $8a + 5b = 14$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = 3$ and $b = -2$.

65. **(d)** Given function,

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & x < 4 \\ a + b, & x = 4 \\ \frac{x-4}{|x-4|} + b, & x > 4 \end{cases}$$

Here, $f(x)$ is continuous at $x = 4$.

$$\lim_{x \to 4^-} f(x) = f(4) = \lim_{x \to 4^+} f(x)$$

$$\lim_{x \to 4^-} \frac{x-4}{|x-4|} + a = f(4) = \lim_{x \to 4^+} \frac{x-4}{|x-4|} + b$$

$$\lim_{x \to 4^-} \frac{x-4}{-(x-4)} + a = a + b = \lim_{x \to 4^+} \frac{x-4}{x-4} + b$$

$-1 + a = a + b = 1 + b$

$\therefore$ $-1 + a = a + b$ and $a + b = 1 + b$

$b = -1$, $a = 1$

66. **(b)** Given, $f(x) = 2x^2 + 3x - 5$

$f'(x) = 4x + 3$

$f'(0) = 3, f'(-1) = -1$

Hence, $f'(0) + 3f'(1) = 3 - 3 = 0$.

67. **(c)** Given, $y = \left(1 + \frac{1}{x}\right)\left(1 + \frac{2}{x}\right)\left(1 + \frac{3}{x}\right) + \ldots + \left(1 + \frac{n}{x}\right)$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \left(-\frac{1}{x^2}\right)\left(1 + \frac{2}{x}\right)\ldots\left(1 + \frac{n}{x}\right)$$

$$+\left(1 + \frac{1}{x}\right)\left(-\frac{2}{x^2}\right)\ldots\left(1 + \frac{n}{x}\right) + \ldots$$

At $x = -1$

$$\frac{dy}{dx} = -1(-1)(-2)(-n+1) + 0 = (-1)^n(n-1)!.$$

68. **(c)** Let $x = \cos^2\theta$

Now, $\dfrac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\dfrac{1+x}{1-x}}\right)\right\}$

$= \frac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}\right)\right\}$

$= \frac{d}{dx}\sin^2\theta$

$= \frac{d}{dx}\left(\frac{1-\cos 2\theta}{2}\right) = \frac{d}{dx}\left(\frac{1-x}{2}\right)$

$= \frac{d}{dx}\left(\frac{1}{2}-\frac{x}{2}\right) = -\frac{1}{2}$

69. **(b)** Given, $y = \tan^{-1}\left\{\frac{ax-b}{bx+a}\right\}$

Differentiating w.r.t. x, we get

$y' = \left[\frac{1}{1+\left(\frac{ax-b}{bx+a}\right)^2}\right]\frac{(bx+a)a-(ax-b)b}{(bx+a)^2}$

$= \left[\frac{1}{(a^2+b^2)(x^2+1)}\right](a^2+b^2) = \frac{1}{x^2+1}$.

70. **(d)** Given curve $y^2 = ax^4 + b \Rightarrow 2y\frac{dy}{dx} = 4ax^3$

At point (3, 6), $(dy/dx)_{(3,6)} = (2a)(27/6) = 9a$ ...(i)

Slope of $y = 41 - 6 \Rightarrow m = 4$

From eq. (i),

$9a = 4 \Rightarrow a = \frac{4}{9}$

$\Rightarrow$ Now, $y^2 = ax^4 + b$ At point (3, 6)

$36 = \frac{4}{9}(81) + b \Rightarrow b = 0.$

71. **(b)** Given equality, $2\alpha + \beta = 8$

Here, α and β are the values at which maximum and minimum occurs.

Now, $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$f'(x) = 6x^2 - 18ax + 12a^2$

$f'(x) = 0 \Rightarrow 6x^2 - 18ax + 12a^2 = 0$

$(x-a)(x-2a) = 0 \Rightarrow x = a, x = 2a$

$f''(x) = 12x - 18a$ at $x = a$

$f''(a) = -6a < 0$, as $a > 0$

$x = a$ is point of maxima at $x = 2a$

$f''(2a) = 6a > 0$ $x = 2a$ is point of minima

$\therefore \alpha = a, \beta = 2a$

$\because 2\alpha + \beta = 8$

$2a + 2a = 8 \Rightarrow a = 2.$

72. **(a)** Radius of sphere, $r = 9$ cm

Error in radius, $dr = 0.03$ cm

Surface area of sphere $\Rightarrow A = 4\pi r^2$

$dA = 4\pi \times 2r\,dr$

$= 4\pi \times 2(9) \times 0.03$

$dA = 2.16\pi$ cm^2

73. **(d)** Given diameter, $x = 10$ cm

Altitude, $h = 20$ cm

$\frac{dx}{dt} = 2$ cm/s, $\frac{dh}{dt} = ?$

Volume of right circular cone.

$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}\times\frac{x^2}{4}\times h \Rightarrow V = \frac{\pi}{12}x^2 h$

As volume remains constant,

$0 = \frac{\pi}{12}\left[x^2\frac{dh}{dt} + h\times 2x\frac{dx}{dt}\right]$

$x\frac{dh}{dt} + 2h\frac{dx}{dt} = 0$

$\frac{dh}{dt} = \frac{-2\times 20}{10}\times 2$

$\frac{dh}{dt} = -8$ cm/s

74. **(a)** Let $I = \int \frac{\sin\alpha\, d\alpha}{\sqrt{1+\cos\alpha}} = \int \frac{\sin\alpha\sqrt{1-\cos\alpha}}{\sqrt{1-\cos^2\alpha}}\,d\alpha$

$= \int \frac{\sin\alpha\sqrt{1-\cos\alpha}}{\sin\alpha}\,d\alpha$

$= \int\sqrt{1-\cos\alpha}\,d\alpha$

$= \int\sqrt{2\sin^2\frac{\alpha}{2}}\,d\alpha$

$= \sqrt{2}\int\sin\frac{\alpha}{2}\,d\alpha$

$= -2\sqrt{2}\cos\frac{\alpha}{2}$.

75. **(b)** Let $I = \int\frac{1+\cos 4x}{\cot x - \tan x}\,dx = \frac{2\cos^2 2x}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}\,dx$

$= \equiv \frac{2\cos^2 2x}{\frac{\cos 2x}{\cos x \sin x}}\,dx$

$= \int(2\cos 2x\cos x\sin x)\frac{1}{2} = \int 2\cos 2x\sin 2x\,dx$

$= \frac{1}{2}\int\sin 4x\,dx = \frac{-1}{8}\cos 4x + C$

Comparing it with $k\cos 4x + C$, we get $\Rightarrow k = -\frac{1}{8}$.

76. **(a)** $u + I = \int \left[\cos x \dfrac{d}{dx} \operatorname{cosec} x\right] dx \;\; I = \int -\cos x \operatorname{cosec} x$

$\cot x \, dx$

$= -\int \cot^2 x \, dx$

$= \int (1 - \operatorname{cosec}^2 x) \, dx = x + \cot x + c$

Comparing it with $f(x) + g(x) + c$, we get

$\therefore \; f(x) = x$ and $g(x) = \cot x$

Hence, $f(x) \, . \, g(x) = x \cot x$

77. **(a)** Let $I = \int \dfrac{(2x+1)^6}{(3x+2)^8} dx$

$$I = \int \left(\frac{2x+1}{3x+2}\right)^6 \left(\frac{1}{3x+2}\right)^2 dx$$

Take $\dfrac{2x+1}{3x+2} = t$

$$\Rightarrow \left(\frac{1}{3x+2}\right)^2 dx = dt$$

$$I = \int t^6 \, dt = \frac{t^7}{7} + C$$

Comparing it with $P\left(\dfrac{2x+1}{3x+2}\right)^Q$, we get

$$\Rightarrow \; P = \frac{1}{7}, Q = 7.$$

$P/Q = 1/49 = 1/7^2$.

78. **(b)** Let $I = \int_0^a \dfrac{dx}{4 + x^2} = \dfrac{\pi}{8}$

$$\Rightarrow \left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_0^a = \frac{\pi}{8}$$

$$\tan^{-1}\left(\frac{a}{2}\right) = \frac{\pi}{4}.$$

79. **(c)** Let $I = \int_1^2 \dfrac{x^3 - 1}{x^2} dx = \int_1^2 \left(x - \dfrac{1}{x^2}\right) dx = \left[\dfrac{x^2}{2} + \dfrac{1}{x}\right]$

$$= \left[\left(\frac{4}{2} + \frac{1}{2}\right) - \left(\frac{1}{2} + 1\right)\right] = \frac{5}{2} - \frac{3}{2} = 1.$$

80. **(b)** Given differential equation is $2x\dfrac{dy}{dx} - y = 4$

$$\frac{dy}{dx} - \left(\frac{y}{2x}\right) = \left(\frac{4}{2x}\right)$$

It represents the linear differential equation.

$$\text{IF} = e^{\int \frac{-1}{2x} dx} = e^{\frac{\log x}{-2}} = \frac{1}{\sqrt{x}}$$

Solution of D.E., $y \, (\text{IF}) = \int Q(x) \, (\text{IF}) \, dx + c$

$$\left(\frac{1}{\sqrt{x}} y\right) = \frac{2}{x\sqrt{x}} dx \;\; \Rightarrow \;\; \frac{y}{\sqrt{x}} = 2\left(\frac{x^{-\frac{3}{2}+1}}{\frac{-3}{2}+1}\right) + C$$

$$\frac{y}{\sqrt{x}} = 2\left(\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}\right) + C$$

$$y = C\sqrt{x} - 4 \;\; \Rightarrow \;\; (y+4)^2 = C^2 x$$

This equation represents a parabola.

PHYSICS

81. **(d)** Displacement of particle, $s = 9t^2 - 2t^3$

Velocity, $v = \dfrac{ds}{dt} = \dfrac{d}{dt}(9t^2 - 2t^3)$

$\Rightarrow \; v = 18t - 6t^2$

When, $v = 0 \;\; \Rightarrow \; 18t - 6t^2 = 0 \; \Rightarrow \; t = 3$ s

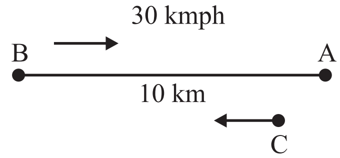
82. **(d)** If initial and final velocities is u and v.

Here always $v < u$

So, kinetic energy will be again maximum, when it will be again on ground at 100 m from point of projection.

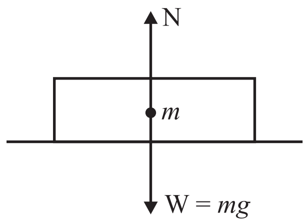
83. **(a)** $V_{CB} = V_{CA} = 30 + V$

So, $(30 + V)\dfrac{8}{60} = 10$



$\Rightarrow 240 + 8V = 600 \; \Rightarrow \; V = 45$ km/hr

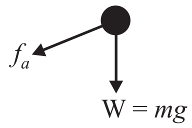
84. **(d)** We have, FBD of book as shown



Clearly, angle between, W and $N = 180°$

85. **(c)** We know that, weight W is always perpendicular towards earth and friction force f_a is always in opposite direction of net force.

$\therefore$ Free body diagram of ball will be



86. **(a)** Work, $\int dW = \int \mathbf{F} \cdot \mathbf{dx}$

$\Rightarrow W = \int (17 - 2x + 6x^2)\, dx$

$\Rightarrow W = 17x - \frac{2x^2}{2} + \frac{6x^3}{3} \Big|_0^8 = 1096 \text{ J}$

87. **(d)** Let $V_i = u$

Then, $V_f = u - \frac{u}{25} = \frac{24u}{25}$

$\therefore\ a = \frac{v^2 - u^2}{2s} = \frac{\left(\frac{24}{25}u\right)^2 - u^2}{2s} = -\frac{49u^2}{625 \times 2s}$

Suppose it take N planks to make final velocity zero.

Then, $0^2 - u^2 = 2\left(\frac{49u^2}{625 \times 2s}\right)(Ns)$

$\Rightarrow N = \frac{625}{49} = 12.7 \approx 13$

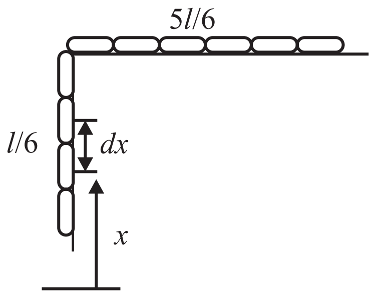
88. **(a)** Here, $w_{\text{ext}} = \Delta U$

Here,

$dU = dmgx = \frac{m}{l}\, dxgx$

$\Delta U = \int_0^{l/6} \frac{mg}{l}\, x\, dx = \frac{mg}{2l}\left(\frac{l}{6}\right)^2 = \frac{mgl}{72}$

So, $w_{\text{ext}} = \frac{mgl}{72}$



89. **(c)** For a rolling body on inclined plane,

$a = \frac{mg \sin\theta}{m + \frac{I}{R^2}} = \frac{g \sin\theta}{1 + \frac{I}{mR}}$

$\because$ Distance covered by body, $s = \frac{1}{2}at^2$

$\because$ $S_{\text{sphere}} = S_{\text{cylinder}}$

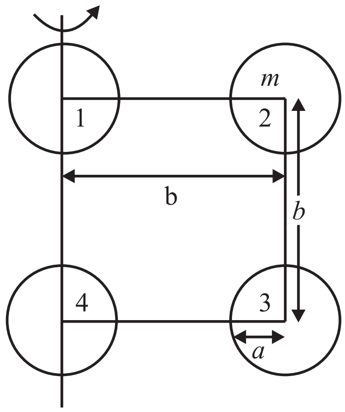
$\frac{1}{2}a_{\text{sphere}} \times t^2 = \frac{1}{2} \times a_{\text{cylinder}} \times t^2$

$\Rightarrow a_{\text{sphere}} = a_{\text{cylinder}}$

$\Rightarrow \frac{g \sin 30^\circ}{1 + \frac{\frac{2}{5}mR^2}{mR^2}} = \frac{g \sin\theta}{1 + \frac{mR^2}{mR^2}} \Rightarrow \frac{\frac{1}{2}}{\frac{7}{5}} = \frac{\sin\theta}{2}$

$\Rightarrow \sin\theta = \frac{5}{7} \Rightarrow \theta = \sin^{-1}\left(\frac{5}{7}\right) = 45.6^\circ \approx 45^\circ$

90. **(d)**



So, net moment of inertia on sphere

$I = (I_1 + I_4) + (I_2 + I_3)$

$\Rightarrow I = 2 \times \left(\frac{2}{5}ma^2 + mb^2\right) + 2 \times \left(\frac{2}{5}ma^2\right)$

$= \frac{8ma^2}{5} + 2mb^2$

91. **(b)** Initial angular frequency, $\omega_i = 180$ rpm $= 6\pi$ rad/sec

Final angular frequency, $\omega_f = 360$ rpm $= 12\pi$ rad/sec

According to question,

$\Delta E = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$

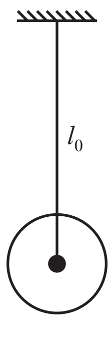
$\Rightarrow 684 = \frac{1}{2}I[(12\pi)^2 - (6\pi)^2]$

$\Rightarrow I = \frac{684 \times 2}{(144\pi^2 - 36\pi^2)} = 1.28 \text{ kg-m}^2$

92. **(c)** When displacement is maximum, body comes at rest. So entire mechanical energy at this instant is potential energy.

So, at this moment potential energy is maximum.

93. **(c)** When sphere is fully filled with water, centre of mass lies at centre.



Case I: For, water level below centre of sphere

$l_{\text{eff}} > l_0$

Case II: When sphere is completely empty, $l_{\text{eff}} = l_0$, because COM come back to original position.

In case I, time period increase.

In case II, time period decrease.

94. **(a)** Gravitational potential energy, $(U) = -\frac{GM_e m}{r}$

If $r = \infty$, $U = 0$ which is maximum.

95. **(c)** By using Kepler's law of planetary motion,

$T^2 \propto R^3$

$$\Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

$$\Rightarrow \left(\frac{24}{T_2}\right)^2 = \left(\frac{R}{2R}\right)^3$$

$$\Rightarrow \frac{24\times 24}{T_2^2} = \frac{1}{8}$$

$$\Rightarrow T_2 = 24\times 2\sqrt{2} = 48\sqrt{2}\text{ h}$$

96. **(a)** Let angular velocity be ω_0.

$\because$ Stress, $\sigma = \dfrac{\text{Force (F)}}{\text{A}}$

$$\Rightarrow F = \sigma A = m\omega_0^2 l$$

$$\Rightarrow \omega_0 = \sqrt{\frac{\sigma A}{ml}}$$

$$\Rightarrow \omega_0 = \sqrt{\frac{4.8\times 10^7\times 10^{-6}}{10\times 0.3}}$$

$$= \sqrt{16} = 4\text{ rad s}^{-1}$$

97. **(a)** Let specific gravity of glass = ρ

Let V = Volume of water displaced

$m = dV$

$$\Rightarrow \left(390 + \frac{500}{2}\right) = 1\times V$$

$\Rightarrow (390 + 250) = V$

$\Rightarrow V = 640$ cc

$\therefore$ Volume of glass, $V_g = V - V_{\text{in}}$

$= 640 - 500 = 140$ cc

$$\therefore \rho = \frac{390}{140} = 2.78\text{ gcc}^{-1} = 2.8\text{ gcc}^{-1}$$

98. **(d)** Cohesive force is defined as the force of attraction between intermolecular particles. Since, cohesive force between the molecules of water is greater than adhesive force between water and oil molecules. So this is reason water cannot wet oily glass.

99. **(c)** For boiling of water to turn into steam, $\Delta T = 0$

Since, $\Delta Q = nC_p\Delta T$

$$C_p = \frac{\Delta Q}{\Delta T} = \frac{\Delta Q}{0} = \infty \qquad [\because \Delta T = 0]$$

100. **(c)** Volume change, $\dfrac{\Delta V}{V} = \dfrac{0.12}{100} = 12\times 10^{-4}$

$\Delta T = 20°\text{C}$

Since, $\dfrac{\Delta V}{V} = 3\alpha\Delta T \qquad [\because \gamma = 3\alpha]$

$$\therefore \alpha = \frac{\Delta V}{V}\times\frac{1}{3\Delta T}$$

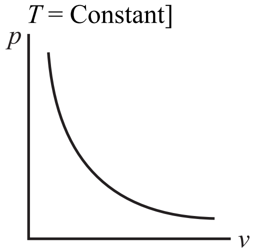
$$\Rightarrow \alpha = \frac{12\times 10^{-4}}{3\times 20} = 2\times 10^{-5}\ °\text{C}^{-1}$$

101. **(b)** As $\dfrac{pV}{T}$ = constant

$\Rightarrow pV$ = constant $\qquad [\because T = \text{Constant}]$

$\Rightarrow p \propto \dfrac{1}{V}$

$\therefore$ p-V graph is given as:



102. **(a, c)**

According to given graph,

For path AB,

$U \propto P$

$\Rightarrow T \propto P$

$\Rightarrow V$ = conts.

$$\therefore C = C_V = \frac{f}{2}R = \frac{3}{2}R$$

Option (a) is incorrect.

For path BC,

Pressure is constant

$$\text{So, } C = C_p = \left(\frac{f}{2}+1\right)R = \frac{5}{2}R$$

Option **(c)** is incorrect

As, $\Delta Q = nC\Delta T$

Here ΔT is –ve. So heat is rejected.

For path CA (isothermal)

$$W = nRT\ \ln\frac{p_2}{p_1} = nRT\ \ln\frac{4p_0}{p_0} = nRT\ \ln 4$$

and $U_0 = \dfrac{3}{2}nRT$

$$\therefore W = \frac{2}{3}U_0\ (\ln 4)$$

103. **(c)** As we know that,

Kinetic energy, per unit volume, $E = \dfrac{1}{2}\rho V^2 \qquad \ldots\text{(i)}$

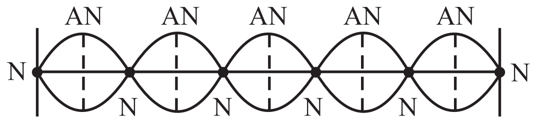
and pressure per unit volume, $p = \dfrac{1}{3}\rho V^2 \qquad \ldots\text{(ii)}$

From Eqs. (i) and (ii), we get

$$\therefore p = \frac{1}{3}2E \Rightarrow p = \frac{2}{3}E$$

Hence, $p \propto E$.

104. (a)



Clearly, number of node, $N = 6$

and number of anti-node, $A = 5$

105. (c) As $d \sin\theta = n\lambda$ for minima

$\sin\theta = \dfrac{n\lambda}{d}$. So, if $\lambda\downarrow\downarrow \Rightarrow \theta\downarrow\downarrow$

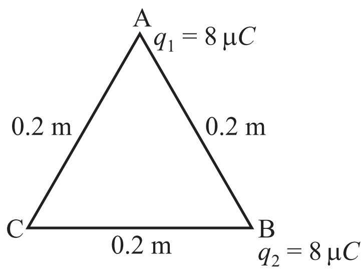
$\Rightarrow$ so image will shift towards.

106. (a) Flux, $\phi = \dfrac{Q_{\text{enclosed}}}{\varepsilon_0}$

$$\Rightarrow \phi_{\text{net}} = \frac{(-14 + 78.85 - 56)\times 10^{-9}}{8.854\times 10^{-12}}$$

$= 10^3$ N-m^2 C^{-1}

107. (a)



$\therefore$ Potential at third point,

$$V_C = \frac{kq_1}{r} + \frac{kq_2}{r} = \frac{2kq_1}{r}$$

$$\Rightarrow V_3 = \frac{9\times 10^9 \times 8\times 10^{-6}}{0.2}$$

$= 7.2\times 10^5$ V

108. (b) We have,

$q_0 = C_0 V_0$

$= 60\times 10^{-6}\times 250 = 15\times 10^{-3}$ C

and $C_{\text{new}} = \dfrac{AE_0}{d - \left(t - \dfrac{t}{k}\right)}$

$$\text{New capacitance, } C = \frac{\varepsilon_0 A}{\left[6 - 3\left(1 - \frac{1}{5}\right)\right]\times 10^{-3}} \quad \ldots\text{(i)}$$

and $\varepsilon_0 A = C_0 d$

$\Rightarrow \varepsilon_0 A = 60\times 10^{-6}\times 6\times 10^{-3} \Rightarrow \varepsilon_0 A = 360\times 10^{-9}$

Put this value in Eq. (i), we get

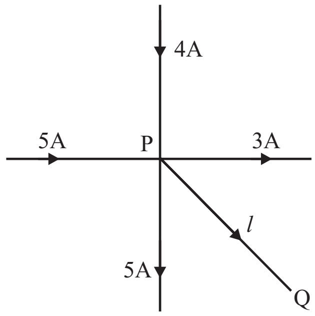
$$C = \frac{360\times 10^{-9}}{3.6\times 10^{-3}} = 10^{-4}\text{ F}$$

Final potential stored in capacitor

$$= \frac{q_0}{C'} = \frac{15\times 10^{-3}}{10^{-4}} = 150\text{ V}$$

Hence, voltage difference = 250 – 150 = 100 V

109. (b)



Input current at $P = 9$ A

Output current at $P = 8$ A

So current in 5$^{\text{th}}$ conductor is 1 A from P to Q.

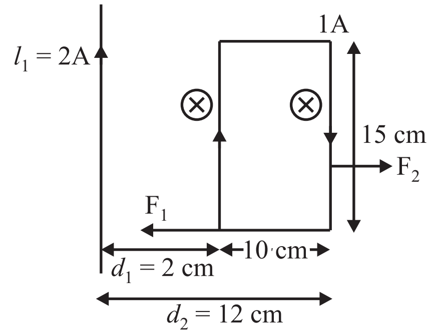
110. (b) We have,

$2\pi r = L$

$\Rightarrow r = \dfrac{L}{2\pi}$

$$m = IA = I\,x\,\pi\left(\frac{L}{2\pi}\right)^2 = \frac{L^2}{4\pi}$$

111. (a) According to given diagram,



As we know that, $\dfrac{F}{l} = \dfrac{\mu_0}{2\pi}\cdot\dfrac{I_1 I_2}{d}$

$$\therefore \text{ Net force, } F_{\text{net}} = \left(\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_1} - \frac{\mu_0}{2\pi}\frac{I_1\cdot I_2}{d_2}\right)l$$

$$= \frac{\mu_0}{2\pi}\left(\frac{2\times 1}{2\times 10^{-2}} - \frac{2\times 1}{12\times 10^{-2}}\right)15\times 10^{-2}$$

$= 25\times 10^{-7}$ N, towards wire

112. (b) Since, $M = \dfrac{CB}{T}$, where M = magnetisation

where, C is Curie's temperature.

$\Rightarrow m \propto \dfrac{1}{T} \quad \therefore \dfrac{m_1}{m_2} = \dfrac{T_2}{T_1}$

$\Rightarrow m_2 = \frac{m_1 T_1}{T_2}$

$= \frac{0.8 \times 5}{20} = 0.2 \text{ Am}^{-1}$

113. (d) emf cannot be produced by maintaining large but constant current in a circuit because change in flux is important for production of induced EMF.

114. (d) If $\theta = 0°$

Then, $\phi = BA \cos 0° = BA$

$\Rightarrow \phi_{max} = BA$

So, when plane of coil is perpendicular to $\vec{B}$, flux is maximum.

Therefore, A is false and R is true.

115. (a) $V = \sqrt{V_R^2 + V_L^2}$

$\Rightarrow V^2 = {V_R}^2 + {V_L}^2$

$\Rightarrow {V_L}^2 = V^2 - {V_R}^2 = (20)^2 - (12)^2$

$= 400 - 144 = 256 \text{ V} \Rightarrow V_L = 16 \text{ V}$

116. (a) As we know that, electric field **E**, magnetic field **B** and propagation **k** all are mutually perpendicular to each other. Direction of $\vec{k}$ *is* ||r to $\vec{E} \times \vec{B}$.

So, $\mathbf{E} = E_0\hat{\mathbf{i}}$ and $\mathbf{B} = B_0\hat{\mathbf{j}}$ is the best choice.

117. (c) Given, maximum speed of photoelectron = v_{max}

According to Einstein's photoelectric equation,

$\frac{1}{2}mv_{max}^2 = hv - hv_0$

$\Rightarrow hv = hv_0 + \frac{1}{2}mv_{max}^2$

$\Rightarrow \frac{2h}{m}V - \frac{2hV_0}{m} = V_{max}^2$

So, graph between v_{max} and v is a parabola as shown in option **(c)**.

118. (c) As we know that, angular momentum of e^- in H-atom is $L = nh/2\pi$.

$\therefore$ L is integral multiple of $h/2\pi$.

119. (d) As we know that, nuclear density of any atom $= 2 \times 10^{17}$ kg/m^3

$\therefore$ Option (d) is correct.

120. (b)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Here, $A + B = x$.

So gate is OR gate.

CHEMISTRY

121. (a) Atomic number of carbon = 6

Atomic mass of carbon = 13

Atomic number = no. of protons

= no. of electrons = 6

Atomic mass = no. of protons + no. of neutrons

$13 = 6 + n$

$\Rightarrow n = 7$

122. (a) Mass of proton (p) = 1.6726×10^{-27} kg

Mass of neutron (n) = 1.6749×10^{-27} kg

Mass of electron (e) = 9.1×10^{-31} kg

Mass of proton is 1836.16 times heavier than electron.

Mass of neutron is 1838.68 times heavier than electron.

Ratio of mass of electron, proton and neutron will be

1 : 1836.15 : 1838.68.

123. (d) A – (iii), B – (i), C – (ii), D – (iv)

Number of electron = Atomic no. – no. of +ve charge + no. of –ve charge

(i) Be^{2+}

$\Rightarrow$ Atomic number = 4

$\therefore$ Number of electrons = 2

(ii) H^+

$\Rightarrow$ Atomic number = 1

$\Rightarrow$ Number of electrons = 0

(iii) Na^+

$\Rightarrow$ Atomic number = 11

$\Rightarrow$ Number of electrons = 10

(iv) Mg^+

$\Rightarrow$ Atomic number = 12

$\Rightarrow$ Number of electrons = 11

124. (b) $sp > sp^2 > sp^3$

50% > 33% > 25%

125. (c) n = no.umber of unpaired electron

Configuration	No. of unpaired	Magnetic moment
d^5	5	5.90
d^3	3	3.82
d^1	1	1.92
d^4	4	4.90

So, the correct order of magnetic moment is

$$d^5 > d^4 > d^3 > d^1$$

126. (d) **(a)** Tl in +1 oxidation state is more stable than +3 oxidation state because of inert pair effect therefore Tl^{3+} salts are oxidising agents.

(b) $\therefore$ Ga^+ is a reducing agent because it is more stable in +3 O.S.

(c) Pb^{2+} is more stable than

Pb^{4+} hence Pb^{4+} is an oxidising agent.

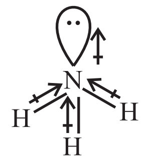
(d) $As^{5+} + 2e^- \rightarrow As^{3+}$ (Less stable)

Thus, As cannot be reduced easily hence, it is not a good oxidising agent.

An oxidising agent is defined as substance whose oxidation number decreases while reducing agent defined as substance whose oxidation number increases.

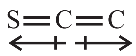
127. (a) Bond order $\propto \dfrac{1}{\text{Bond length}}$

128. (a)



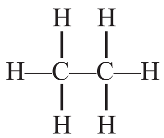
$\mu \neq 0$

(Resultant dipole moment and dipole due to l_0 are in same direction)



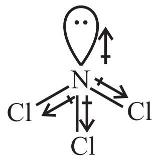
$\mu = 0$

(Symmetric)



$\mu = 0$

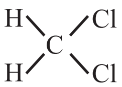
(Symmetric)



$\mu \neq 0$

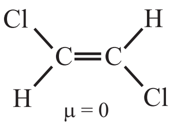
(lone pair decreases the resultant N–Cl bond moments)

129. (a) (i)



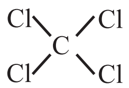
$\mu \neq 0$

(ii)



$\mu = 0$

(iii)



$\mu = 0$

(iv) $Br — C \equiv C — Br$

$\mu = 0$

As dipole moment is vector quantity. Due to electron density in opposite direction, net dipole moment cancel out each other.

130. (d)

(i) For real gases compressibility factor (Z) cannot be unity, while compressibility factor for an ideal gas is one.

(ii) A gas can behave like an ideal gas at very high temperature and low pressure. Hence, show very less deviation.

(iii) Intermolecular forces among gas molecules are weak as compare to liquid and solid but, these forces cannot be zero.

(iv) Ideal gas follows van der Waal's equation

$pV = nRT$

Gases which do not follow this equation are real gases.

131. (d) In disproportionation reaction a species simultaneously undergo reduction and oxidation to form two different products.

In ClO_4^-, chlorine is present in its highest oxidation state +7 and cannot further undergo oxidation.

132. (a) Mass of sample of alloy = 12 g

% of metal X in sample = 20

$\therefore$ If x is the mass of metal X in sample

$$\frac{x}{12} \times 100 = 20$$

$x = 2.4$ g

If y is the mass of metal Y in sample, then

$y = 12 - 2.4 = 9.6$ g

Number of atoms of X = $\dfrac{6.022 \times 10^{23} \times 24}{40}$

$= 3.61 \times 10^{22}$

Atoms of X and atom of $y = \dfrac{2}{5}$

Number of atoms of $y = \dfrac{3.61 \times 10^{22} \times 5}{2}$

$= 9.025 \times 10^{22}$ atoms.

9.025×10^{22} atoms of Y are present in 9.6 g.

6.022×10^{23} atoms of Y are present in

$$\frac{9.6}{9.025 \times 10^{22}} \times 6.022 \times 10^{23} = 64 \text{ g}$$

Hence, atomic mass of Y = 64 amu.

133. (b) $H_2O(l) \rightarrow H_2O(g)$ at T = 100°C

$\Rightarrow p = 1$ atm

The process represents the boiling of liquid which is an endothermic process, entropy increases during this change i.e., $\Delta S = +ve$,

$\therefore$ If $\Delta S_{system} > 0$ then $\Delta S_{surrounding} > 0$.

134. (c) $N_2O_4(g) \rightleftharpoons 2NO_2(g)$

	$N_2O_4(g)$	$2NO_2(g)$
Initial moles	1 mol	0 mol
At equilibrium	(1 – 0.5) mol	2 × 0.5 mol

Total number of moles = 0.5 + 1 = 1.5 mol

According to law of chemical equilibrium

$$K_p = \frac{p^2_{NO_2}}{p_{N_2O_4}} = \frac{\left(\frac{1}{1.5} \times 1\right)}{\left(\frac{0.5}{1.5} \times 1\right)} = 1.33 \text{ atm}$$

We know,

$\Delta G^\circ = -2.303RT \log K_p$

$\Delta G^\circ = -2.303 \times 0.082\ JK^{-1}\ mol^{-1} \times 333\ K \times \log 1.33$

$= -7.79$ atm L

atm L = 101.32500 joules = – 789.2 J mol^{-1}

= – 790 J mol^{-1}

135. (b) Given, $[K_{sp}]$ of AgBr = 5×10^{-10}

[NaBr] = 0.2 M; $[Na^+] = [Br^-] = 0.2$ M

Now, $[K_{sp}] = [Ag^+][Br^-]$

$S(0.2) = 5^5 \times 10^{-10}$ $\quad (S << 0.2)$

$$S = \frac{5 \times 10^{-10}}{0.2} = 25 \times 10^{-10}\ M$$

136. (b) Le-Chatelier principle is not applicable to pure solids and liquids because they experience negligible change in concentration during chemical equilibrium.

137. (a) Li_2SO_4 is the only alkali metal sulphate that does not form double salt due to small size.

138. (c) AlF_3 is insoluble in anhydrous HF because F^- ion is not available for H-bonding but it is soluble in HF only in presence of KF due to formation of $K_3[AlF_6]$.

$$\underset{\text{Aluminium trifluoride}}{AlF_3(s)} \xrightarrow{HF} AlF_3 + 3KF$$

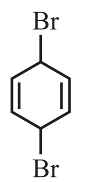
$$\xrightarrow[KF]{HF} \underset{\substack{\text{(Soluble)} \\ \text{Potassium hexafluorido} \\ \text{aluminate (III)}}}{K_3[AlF_6]}$$

139. (b) $$\underset{\text{Silicon tetrachloride}}{SiCl_4} + 4H_2O \longrightarrow \underset{\textit{ortho}\text{-silicic acid}}{Si(HO)_4} + 4HCl$$

140. (c) Water vapour, CO_2, CH_4, nitrous oxide (N_2O), O_3 and CFC are the greenhouse gas. Which helps in keeping the earth surface warm.

SO_2 (sulphur dioxide) is not a greenhouse gas.

141. (c) The compound contains ring system, with two non-conjugate double bond and two bromo group at 1, 4-position.



Two double bonds means 4π-electrons are present in the compound. Therefore, it is anti-aromatic as aromatic compound must have $(4n + 2)$ π-electron.

Bromo group is attached to ring system as substituent because it consists of carbon atom within the ring.

142. (c) 25 mL of 1M H_2SO_4 corresponds to 25 milli mol.

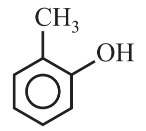
It will neutralise 50 milli mol of NH_3.

weight of N = $14 \times 50 \times 10^{-3}$ g = 0.70 g

Thus 1 g of sample of organic compound contains 0.70 g of nitrogen.

% of nitrogen in organic sample is $\frac{0.70}{1} \times 100 = 70\%$.

143. (c) The most reactive towards electrophilic reagent is *o*-cresol.



The phenolic (—OH) group increases the electron density on benzene ring through resonance. Hence, the reactivity towards electrophilic reagent increases.

Note: The decreasing order of activating influence of substituents towards electrophilic reagent is

NR_2 > —NHR > —NH_2 > —OH > —OR > —NHCOR > —Ph > —R.

144. (a) Hyperconjugation is the delocalisation of σ electron with adjacent *p*-orbital.

It helps in explaining stability order of free radicals, carbocations, alkanes but cannot explain stability of carbanion due to absence of vacant π or *p*-orbitals.

145. (c) In the face centred unit cell, (fcc) the lattice points are present at the corners and face centres of unit cell.

146. (d) Higher the value of K_H, lower fcc is the solubility of the gas in the liquid at given pressure.

i.e., solubility $\propto \frac{1}{K_H}$

So, the correct order is

Ar < CO_2 < CH_4 < HCHO

147. (a) $CaCl_2 \rightarrow Ca^{2+} + 2Cl^-$

Cl^- in solution = 3.01×10^{22} ions

No. of $CaCl_2$ present in solution = $\frac{3.01 \times 10^{22}}{2}$

= 1.505×10^{22} $CaCl_2$

$$\text{Molarity of solution} = \frac{\text{No. of moles of } CaCl_2}{\text{Vol. of solution (in litre)}}$$

$$= \frac{1.51 \times 10^{22}}{6.022 \times 10^{23}} \times \frac{1000}{500} = 0.05$$

148. (d) (d) For 1st order

$$\frac{-d[A]}{dt} = k[A]$$

$Ms^{-1} = kM^2$

$k = s^{-1}$

(a) Rate of reaction

$$\frac{d[\text{change in conc. of substance}]}{dt} = M\,s^{-1}$$

(b) Rate of disappearance

$A \rightarrow B$

$$\frac{-d[A]}{dt} = M\,s^{-1}$$

So, k depends on order of reaction and unit of k for 1st order reaction is s^{-1}.

149. (d) For zero order reaction,

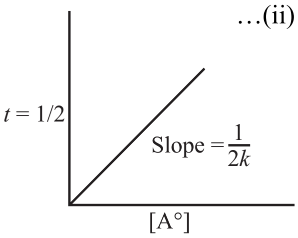
$[A] = [A_0] - kt$ ...(i)

When $t = t_{1/2}$, $[A] = \dfrac{[A_0]}{2}$ ...(ii)

From eqs. (i) and (ii)

$$\frac{[A_0]}{2} = [A_0] - kt_{1/2}$$

$$\Rightarrow t_{1/2} = \frac{[A_0]}{2k}$$



Plot of $t_{1/2}$ *versus* $[A_0]$ will be straight line.

150. (a) pH of solution 1 = 3

$\therefore$ $[H^+]_1 = 10^{-3}$ M

pH of solution 2 = 6

$\therefore$ $[H^+]_2 = 10^{-6}$ M

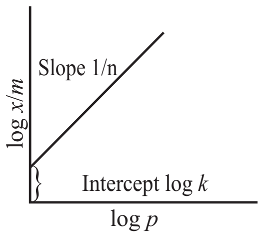
$$\text{Now, } E_{cell} = E^\circ_{cell} - \frac{0.059}{1}\log\frac{10^{-6}}{10^{-3}} = 0 + 0.059 \times 3$$

$E_{cell} = 0.177$ V

151. (b) $\dfrac{x}{m} = kp^{1/n}$

[adsorption isotherm]

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log p$$



Given slope i.e., $1/n = \tan\theta$

$= \tan 45° = 1$ i.e., $n = 1$

$\log k = 0.3010$

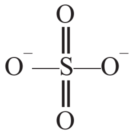
or k = antilog (0.3010) = 2

At $p = 0.5$ atm,

$x/m = kp^{1/n}$

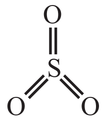
$\Rightarrow x/m = 2 \times (0.5)^1 = 1.0$

152. (c) (i) SO_4^{2-} :



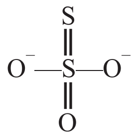
Average bond order = 6/4 = 1.5

(ii) SO_3 :



Average bond order = 6/3 = 2

(iii) $S_2O_3^{2-}$:



Average bond order = 4/3 = 1.33

Correct sequence of bond order is

$SO_3 > SO_4^{2-} > S_2O_3^{2-}$

153. (b) $KCN \xrightarrow[S_2O_3^{2-}]{NaOH} \underset{\text{Potassium thiocyanate}}{KSCN}$

$FeCl_3 + 3KSCN \longrightarrow \underset{\text{Blood red colour}}{Fe(SCN)_3} + 3KCl$

Reaction of $FeCl_3$ with KSCN gives a blood red solution.

154. (c) In all other compounds

$CuCl_2 \rightarrow [Cu^{2+}] \Rightarrow [Ar]\,3d^9$

$CuCl \rightarrow [Cu^+] \Rightarrow 3d^{10}\,4s^0$

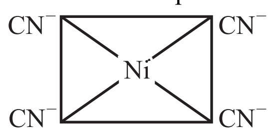
$ScCl_3 \rightarrow [Sc^{3+}] \Rightarrow 3p^6\,4s^2$

$TiCl_4 \rightarrow [Ti^{4+}] \Rightarrow 3p^6\,4s^2$

$CuCl_2$ contains one unpaired electron.

Hence, it is coloured compound.

155. (c) (a)



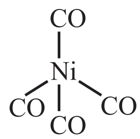
Ni^{2+} : $[Ar]3d^8$

dsp^2 hybridisation

Shape : Square planar

(CN^- causes pairing of electron hence, diamagnetic)

(b)



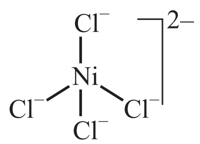
Ni (in zero O.S.) $[Ar]3d^8$

sp^3 hybridisation 45°

Shape : Tetrahedral

(Diamagnetic)

(c)

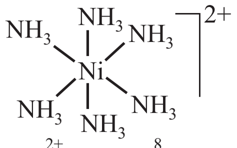


Ni^{2+} : $[Ar]3d^8$

sp^3 hybridisation

Shape : Tetrahedral

(Cl^- weak field ligant cause no pairing of electron hence, paramagnetic)

(d) 

Ni^{2+} : $[Ar]3d^8$

sp^3d^2 hybridisation

Shape : Octahedral paramagnetic

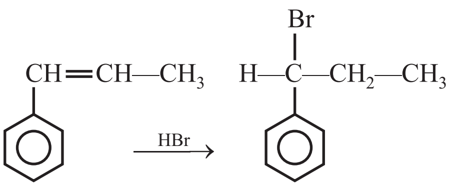
156. **(c)** Thiamine – Vitamin B_1

Riboflavin – Vitamin B_2

Cobalamine – Vitamin B_{12}

Pyridoxine – Vitamin B_6

157. **(b)**



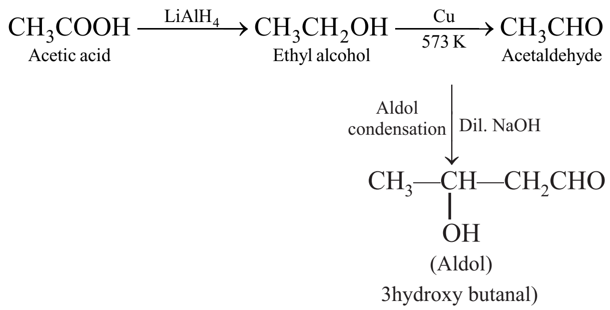
According to Markownikov rule, in addition reaction of unsymmetrical alkenes, electron rich component of reagent add to carbon atom with fewer hydrogen atoms bonded to it.

158. **(d)** Carboxylic acids are the most acidic compounds among the organic compounds. Due to presence of electron withdrawing group i.e. chlorine and fluorine, FCH_2COOH and $CH_3CH_2CHClCOOH$ are more acidic, than C_6H_5COOH and CH_3COOH. Fluorine is more electron withdrawing group than chlorine.

Therefore, $\underset{\text{Fluoro ethanoic acid}}{FCH_2COOH} > \underset{\text{2-chlorobutanoic acid}}{CH_3CH_2CHClCOOH}$

$> \underset{\text{Benzoic acid}}{C_6H_5COOH} > \underset{\text{Acetic acid}}{CH_3COOH}$

159. **(a)** $LiAlH_4$ (Lithium aluminium hydride) reduces acetic acid to ethyl alcohol which further oxidises in presence of Cu to ethanal and undergoes aldol condensation.



160. **(b)** Methoxy group is activating and *ortho-para* directing group. Hence with mixture of conc. HNO_3 and conc. H_2SO_4 anisole gives *ortho* and *para*-nitro anisole.