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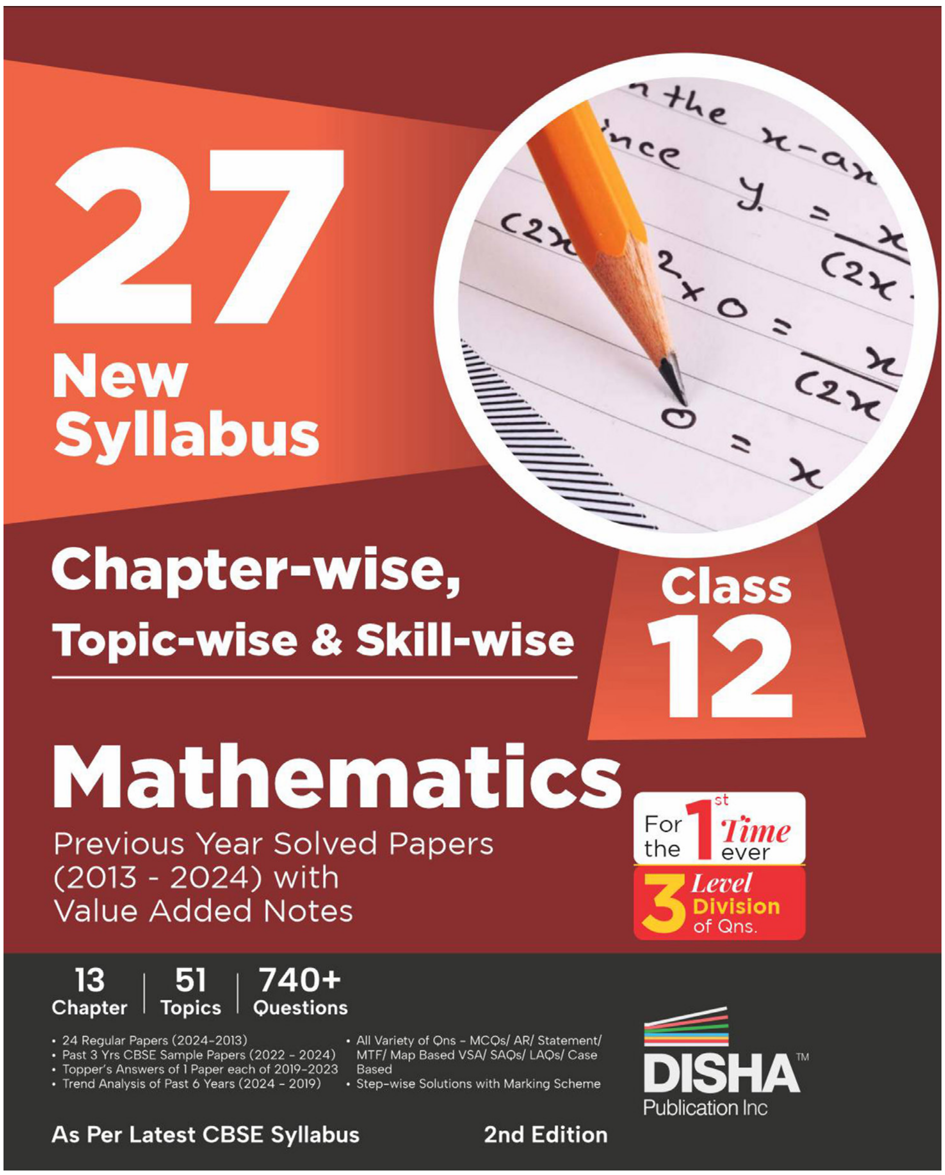
All India 2024 Solved Paper

This sample is taken from the **“27 New Syllabus Chapter-wise, Topic-wise & Skill-wise CBSE Class 12 Mathematics Previous Year Solved Papers (2013 - 2024) with Value Added Notes 2nd Edition”**



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Chapterwise Topicwise CBSE 2024
All India and Delhi Solved Paper

Chapter 1 : Relations and Functions

Topic-1: *Types of Relations*

3 ***Assertion Reason/ Two Statement Type Questions (1 Mark)***

Assertion (A) and Reason (R) based questions carrying 1 marks each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below:

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

1. **Assertion (A) :** The relation $R = \{(x, y) : (x + y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation.

Reason (R) : The number '2n' is composite for all natural numbers n. **[All India 2024, K]**

6 ***Long Answer Questions (5 Marks)***

2. Check whether the relation S in the set of real numbers R defined by $S = \{(a, b) :$ where $a - b + \sqrt{2}$ is an irrational number$\}$ is reglexive, symmetric or transitive. **[All India 2024, K]**

3. A relation R is defined on N × N (where N is the set of natural numbers) as: **[Delhi 2024, A]**

$(a, b)\ R\ (c, d) \Leftrightarrow a - c = b - d$

Show that R is an equivalence relation.

Topic-2: *Types of Functions*

1 ***Multiple Choice Questions (1 Mark)***

4. Let $f : \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, R_+ is the set of all non-negative real numbers. Then, f is: **[Delhi 2024, Ap]**

(a) one-one
(b) onto
(c) bijective
(d) neither one-one more onto

6 ***Long Answer Questions (5 Marks)***

5. Let $A = R - \{5\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$, defined by $f(x) = \dfrac{x-3}{x-5}$. Show that f is one - one and onto. **[All India 2024, Ap]**

6. Show that a function $f : R \to$ defined by $f(x) = \dfrac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function. **[Delhi 2024, K]**

Chapter 2 : Inverse Trigonometric Functions

Topic-1: *Definition, Range, Domain and Principal Value Branch*

4 ***Very Short Answer Questions (2 Marks)***

7. Find the principal value of

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right).$$

[All India 2024, U]

8. Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range. **[Delhi 2024, K]**

Topic-2: Simplest Form, Graph Inverse Trigonometric Functions.

4 Very Short Answer Questions (2 Marks)

9. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form. **[All India 2024, K]**

10. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$. **[Delhi 2024, U]**

Chapter 3 : Matrices

Topic-1: Matrix, Types of Matrices

1 Multiple Choice Questions (1 Mark)

11. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of a + 2 b + 3c + 4d is: **[All India 2024, K]**
(a) 0 (b) 5
(c) 10 (d) 25

12. If the sum of all the elements of a 3 × 3 scalar matrix is 9, then the product of all its elements is: **[Delhi 2024, K]**
(a) 0 (b) 9
(c) 27 (d) 729

13. If $A = [a_{ij}]$ be a 3 × 3 matrix, where $a_{ij} = i - 3j$, then which of the following is false? **[Delhi 2024, K]**
(a) $a_{11} < 0$ (b) $a_{12} + a_{21} = -6$
(c) $a_{13} > a_{31}$ (d) $a_{31} = 0$

Topic-2: Operations on Matrices

1 Multiple Choice Questions (1 Mark)

14. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $A^2 + 7I = kA$, then the value of k is : **[All India 2024, U]**
(a) 1 (b) 2
(c) 5 (d) 7

15. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \frac{1}{3}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & \lambda \end{bmatrix}$. If AB = I, then the value of λ is: **[All India 2024, U]**
(a) $\frac{-9}{4}$ (b) –2
(c) $\frac{-3}{2}$ (d) 0

16. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \ldots$ is: **[All India 2024, Ap]**
(a) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

17. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is: **[Delhi 2024, Ap]**
(a) 1 (b) 2
(c) 0 (d) –2

Topic-4: Symmetric and Skew Symmetric Matrices

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(d) Assertion (A) is false, but Reason (R) is true.

18. **Assertion (A):** For any symmetric matrix A, B' AB is a skew- symmetric matrix.
Reason (R): A square matrix P is skew-symmetric if P' = –P. **[Delhi 2024, K]**

Chapter 4 : Determinants

Topic-4: Adjoint and Inverse of a Matrix

1 Multiple Choice Questions (1 Mark)

19. Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is:

[All India 2024, U]

(a) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(c) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (d) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

20. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is:

[Delhi 2024, U]

(a) 0 (b) 1
(c) 2 (d) d

6 Long Answer Questions (5 Marks)

21. (b) If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that

$A' A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$. **[Delhi 2024, K]**

Topic-5: Solutions of System of Equations

6 Long Answer Questions (5 Marks)

22. Find A^{-1}, if $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{bmatrix}$. Hence, solve the following system of equations: **[All India 2024, Ap]**

$x + 2y + z = 5$
$2x + 3y = 1$
$x - y + z = 8$

23. Solve the following system of equations, using matrices: **[Delhi 2024, Ap]**

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

where $x, y, z \neq 0$

Chapter 5 : Continuity and Differentiability

Topic-1: Continuity

1 Multiple Choice Questions (1 Mark)

24. The number of points of discontinuity of f(x)

$= \begin{cases} |x| & \text{if } x \le -3 \\ -2x & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \ge 3 \end{cases}$ is **[Delhi 2024, K]**

(a) 0 (b) 1
(c) 2 (d) infinite

Topic-2: Differentiability

1 Multiple Choice Questions (1 Mark)

25. The function $f(x) = |x| + |x - 2|$ is
(a) continuous, but not differentiable at $x = 0$ and $x = 2$.
(b) differentiable but not continuous at $x = 0$ and $x = 2$.
(c) continuous but not differentiable at $x = 0$ only.
(d) neither continuous nor differentiable at $x = 0$ and $x = 2$. **[All India 2024, A]**

4 Very Short Answer Questions (2 Marks)

26. If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ and $x = \frac{\pi}{3}$.

[Delhi 2024, Ap]

5 Short Answer Question (3 Marks)

27. Show that:

$\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$ **[Delhi 2024, K]**

Topic-3: Derivatives of Implicit and Inverse Trigonometric Functions

1 Multiple Choice Questions (1 Mark)

28. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is: **[Delhi 2024, K]**

(a) $\frac{x}{1+x^4}$ (b) $\frac{2x}{1+x^4}$

(c) $-\frac{2x}{1+x^4}$ (d) $\frac{1}{1+x^4}$

4 *Very Short Answer Questions (2 Marks)*

29. If $y = \cos^3 (\sec^2 2t)$, find $\frac{dy}{dt}$. **[All India 2024, K]**

30. If $y = \text{cosec}(\cot^{-1} x)$, then prove that

$\sqrt{1+x^2}\frac{dy}{dx} - x = 0$ **[Delhi 2024, Ap]**

Topic-5: Logarithmic Differentiation

4 *Very Short Answer Questions (2 Marks)*

31. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

[All India 2024, A]

5 *Short Answer Question (3 Marks)*

32. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1}\sqrt{x}$ is given.

[All India 2024, Ap]

Topic-6: Derivatives of Functions in Parametric Forms

1 *Multiple Choice Questions (1 Mark)*

33. Derivative of x^2 with respect to x^3, is: **[All India 2024, K]**

(a) $\frac{2}{3x}$ (b) $\frac{3x}{2}$

(c) $\frac{2x}{3}$ (d) $6x^5$

5 *Short Answer Question (3 Marks)*

34. (a) If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that

$\frac{dy}{dx} = -\frac{y \log x}{x \log y}$. **[Delhi 2024, U]**

Chapter 6 : Application of Derivatives

Topic-1: Rate of Change of Quantities

4 *Very Short Answer Questions (2 Marks)*

35. The volume of a cube is increasing at the rate of $6cm^{3/}s$. How fast is the surface area of cube increasing, when the length of an edge is 8 cm? **[All India 2024, U]**

Topic-2: Increasing and Decreasing Functions

1 *Multiple Choice Questions (1 Mark)*

36. The function $f(x) = kx - \sin x$ is strictly increasing for

[All India 2024, Ap]

(a) $k > 1$ (b) $k < 1$

(c) $k > -1$ (d) $k < -1$

37. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is: **[Delhi 2024, U]**

(a) strictly decreasing on R

(b) strictly increasing on R

(c) neither strictly increasing nor strictly decreasing on R

(d) strictly decreasing on $(-\infty, 0)$

4 *Very Short Answer Questions (2 Marks)*

38. Show that the function f given by $f(x) = \sin x + \cos x$, is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

[All India 2024, Ap]

39. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain. **[Delhi 2024, Ap]**

Topic-3: Maxima and Minima

4 *Very Short Answer Questions (2 Marks)*

40. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ respectively, find the value of $(M - m)$. **[Delhi 2024, Ap]**

7 *Case Based Questions (4 Marks)*

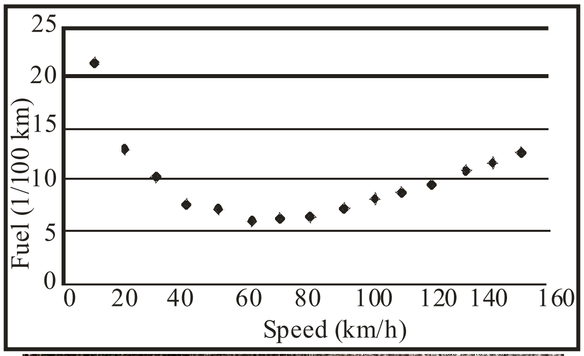
41. A store has been selling calculators at ₹ 350 each. A market survey indicated that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$. **[All India 2024, Ap]**

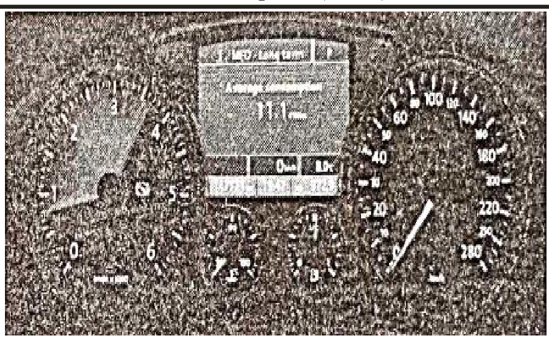
Based on the above information, answer the following questions:

(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also, verify the result. **(2)**

(ii) What rebate in price of calculator should the store give to maximise the revenue? **(2)**

42. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/hr. **[Delhi 2024, Ap]**





The relation between fuel consumption F(l/100 km) and speed V (km/h) under some constraints is given as

$$F = \frac{V^2}{500} - \frac{V}{4} + 14.$$

On the basis of the above information, answer the following questions:

(i) Find F, when V = 40 km/h. **(1)**

(ii) Find $\frac{dF}{dV}$. **(1)**

(iii) (a) Find the speed V for which fuel consumption F is minimum. **(2)**

OR

(b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$. **(2)**

Chapter 7 : Integrals

Topic-2: Integration by substitution

4 *Very Short Answer Questions (2 Marks)*

43. Find:

$$\int \frac{e^{4x} - 1}{e^{4x} + 1} dx$$ **[Delhi 2024, U]**

5 *Short Answer Question (3 Marks)*

44. (b) Find:

$$\int \frac{1}{x\left[(\log x)^2 - 3\log x - 4\right]} dx$$ **[Delhi 2024, U]**

45. Find:

$$\int x^2 . \sin^{-1}\left(x^{3/2}\right) dx$$ **[Delhi 2024, U]**

Topic-5: Integration by Partial Fractions

4 *Very Short Answer Questions (1 Mark)*

46. Find : $\int \frac{2x}{(x^2+1)(x^2-4)} dx.$ **[All India 2024, K]**

Topic-6: Integration by Parts

5 *Short Answer Question (3 Marks)*

47. Find : $\int \sec^3 \theta d\theta$ **[All India 2024, U]**

48. (b) Find : $\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1+x^2}}\right] dx$

[All India 2024, Ap]

Topic-8: Evaluation of Definite Integrals by Substitution

1 Multiple Choice Questions (1 Mark)

49. The value of $\int_0^{\pi} \tan^2\left(\frac{\theta}{3}\right) d\theta$ is: **[All India 2024, K]**

(a) $\pi + \sqrt{3}$ (b) $3\sqrt{3} - \pi$

(c) $\sqrt{3} - \pi$ (d) $\pi - \sqrt{3}$

Topic-9: Some Properties of Definite Integrals

1 Multiple Choice Questions (1 Mark)

50. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x + \cos x} dx$ is equal to: **[Delhi 2024, Ap]**

(a) π (b) Zero (0)

(c) $\int_0^{\pi/2} \frac{2\sin x}{1 + \sin x + \cos x} dx$ (d) $\frac{\pi^2}{4}$

5 Short Answer Question (3 Marks)

51. Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x dx}{1 + \cos 2x + \sin 2x}$ **[All India 2024, Ap]**

52. Evaluate: **[Delhi 2024, K]**

$\int_{-2}^{2} \sqrt{\frac{2 - x}{2 + x}} dx$

Chapter 8 : Application of Integrals

Topic-1: Area under Simple Curves

6 Long Answer Questions (5 Marks)

53. (a) Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X - axis and the ordinates $x = -2$ and $x = 2$, usig integration. **[All India 2024, A]**

54. Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X - axis. **[All India 2024, Ap]**

55. If A_1 denotes the area of region bounded by $y^2 = 4x$. $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$. **[Delhi 2024, K]**

Chapter 9 : Differential Equations

Topic-1: Order and Degree of a differential equation

1 Multiple Choice Questions (1 Mark)

56. The degree of the differential equation $(y'')^2 + (y')^3 = x (y')$ is: **[Delhi 2024, K]**

(a) 1 (b) 2

(c) 3 (d) not defined

Topic-3: Differential equations with variables separable

5 Short Answer Question (3 Marks)

57. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = y \cos 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$. **[All India 2024, Ap]**

Topic-4: Homogeneous differential equations

1 Multiple Choice Questions (1 Mark)

58. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if F(x, y) is: **[Delhi 2024, U]**

(a) $\cos x - \sin\left(\frac{y}{x}\right)$ (b) $\frac{y}{x}$

(c) $\frac{x^2 + y^2}{xy}$ (d) $\cos^2\left(\frac{x}{y}\right)$

5 *Short Answer Question (3 Marks)*

59. Find the particular solution of the differential equation $\left(xe^{\frac{y}{x}}+y\right)dx = x\,dy$, given that y = 1 when x = 1.

[All India 2024, U]

60. Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2\frac{dy}{dx} = 0$; y = 2, when x = 1. **[Delhi 2024, Ap]**

Topic-5: Linear differential equations

1 *Multiple Choice Questions (1 Mark)*

61. The integrating factor of the differential equation $\frac{dy}{dx}+\frac{2}{x}y = 0, x \neq 0$ is: **[All India 2024, Ap]**

(a) $\frac{2}{x}$ (b) x^2

(c) $e^{\frac{2}{x}}$ (d) $e^{\log(2x)}$

62. The integrating factor of the differential equation $(x+2y^2)\frac{dy}{dx} = y\,(y>0)$ is: **[All India 2024, Ap]**

(a) $\frac{1}{x}$ (b) x

(c) y (d) $\frac{1}{y}$

5 *Short Answer Question (3 Marks)*

63. Find the general solution of the differential equation: $y\,dx = (x+2y^2)\,dy$ **[Delhi 2024, K]**

Chapter 10 : Vectors Algebra

Topic-3: Scalar (or dot) product of two vectors

1 *Multiple Choice Questions (1 Mark)*

64. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is : **[All India 2024, U]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{5\pi}{6}$ (d) $\frac{11\pi}{6}$

65. The vectors $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}, \vec{b} = \hat{i}-3\hat{j}-5\hat{k}$ and $\vec{c} = -3\hat{i}+4\hat{j}+4\hat{k}$ represents the sides of **[All India 2024, U]**

(a) an equilaterlal triangle
(b) an obtuse - angled triangle
(c) an isosceles triangle
(d) a right - angled triangle

66. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true? **[Delhi 2024, K]**

(a) $\vec{a}.\vec{b} \geq |\vec{a}||\vec{b}|$ (b) $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|$

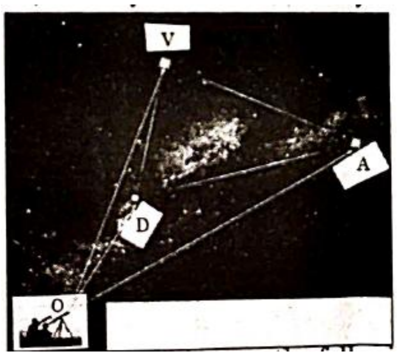
(c) $\vec{a}.\vec{b} \leq |\vec{a}||\vec{b}|$ (d) $\vec{a}.\vec{b} < |\vec{a}||\vec{b}|$

5 *Short Answer Question (3 Marks)*

67. The position vectors of vertices of Δ ABC are $A(2\hat{i}-\hat{j}+\hat{k})$, $B(\hat{i}-3\hat{j}-5\hat{k})$ and $C(3\hat{i}-4\hat{j}-4\hat{k})$. Find all the angles of ΔABC. **[Delhi 2024, K]**

7 *Case Based Questions (4 Marks)*

68. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i}+3\hat{j}+4\hat{k}, 7\hat{i}+5\hat{j}+8\hat{k}$ and $-3\hat{i}+7\hat{j}+11\hat{k}$ respectively. **[All India 2024, Ap]**



Based on the above information, answer the following questions:

(i) How far is the star V from star A? **(1)**

(ii) Find a unit vector in the direction of $\overrightarrow{DA}$. **(1)**

(iii) Find the measure of $\angle VDA$. **(2)**

OR

(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$? **(2)**

Topic-4: *Vector (or cross) product of two vectors*

1 *Multiple Choice Questions (1 Mark)*

69. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2$ is : **[All India 2024, K]**

(a) a^2 (b) $2a^2$

(c) $3a^2$ (d) 0

70. The unit vector perpendicular to both vector $\hat{i}+\hat{k}$ and $\hat{i}-\hat{k}$ is: **[Delhi 2024, Ap]**

(a) $2\hat{i}$ (b) $\hat{j}$

(c) $\dfrac{\hat{i}-\hat{k}}{\sqrt{2}}$ (d) $\dfrac{\hat{i}+\hat{k}}{\sqrt{2}}$

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(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

71. **Assertion (A):** For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$.

Reason (R): For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a}\times\vec{b} = \vec{b}\times\vec{a}$. **[Delhi 2024, K]**

Chapter 11 : Three Dimensional Geometry

Topic-1: *Direction Cosines and Direction Ratios of a Line*

1 *Multiple Choice Questions (1 Mark)*

72. The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by: **[Delhi 2024, K]**

(a) (1, 0, 0) (b) (2, 0, 0)

(c) $(\sqrt{5}, 0, 0)$ (d) (0, 0, 0)

73. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is: **[Delhi 2024, Ap]**

(a) 90° (b) 120°

(c) 60° (d) 0°

74. Direction ratios of a vector parallel to line $\dfrac{x-1}{2} = -y = \dfrac{2z+1}{6}$ are: **[Delhi 2024, U]**

(a) 2, –1, 6 (b) 2, 1, 6

(c) 2, 1, 3 (d) 2, –1, 3

Topic-2: *Equation of a Line in Space*

1 *Multiple Choice Questions (1 Mark)*

75. The Cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{i}-\hat{j}$ and parallel to the line $\vec{r} = \hat{i}+\hat{k}+\mu(2\hat{i}-\hat{j})$, is **[All India 2024, U]**

(a) $\dfrac{x-2}{1} = \dfrac{y+1}{0} = \dfrac{z}{1}$ (b) $\dfrac{x-1}{2} = \dfrac{y+1}{-1} = \dfrac{z}{0}$

(c) $\dfrac{x+1}{2} = \dfrac{y+1}{-1} = \dfrac{z}{0}$ (d) $\dfrac{x-1}{2} = \dfrac{y}{-1} = \dfrac{z-1}{0}$

6 *Long Answer Questions (5 Marks)*

76. Find the equation of the line which bisects the line segment joining points A(2, 3, 4) and B(4, 5, 8) and is perpendicular to the lines $\dfrac{x-8}{3} = \dfrac{y+19}{-16} = \dfrac{z-10}{7}$ and $\dfrac{x-15}{3} = \dfrac{y-29}{8} = \dfrac{z-5}{-5}$. **[Delhi 2024, K]**

Topic-3: Angle between Two Lines

1 Multiple Choice Questions (1 Mark)

77. The lines $\frac{1-x}{2}=\frac{y-1}{3}=\frac{z}{1}$ and $\frac{2x-3}{2p}=\frac{y}{-1}=\frac{z-4}{7}$ are perpendicular to each other for p equal to:

[All India 2024, Ap]

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 2 (d) 3

6 Long Answer Questions (5 Marks)

78. If the line $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7). **[All India 2024, K]**

Topic-4: Shortest Distance between Two Lines

6 Long Answer Questions (5 Marks)

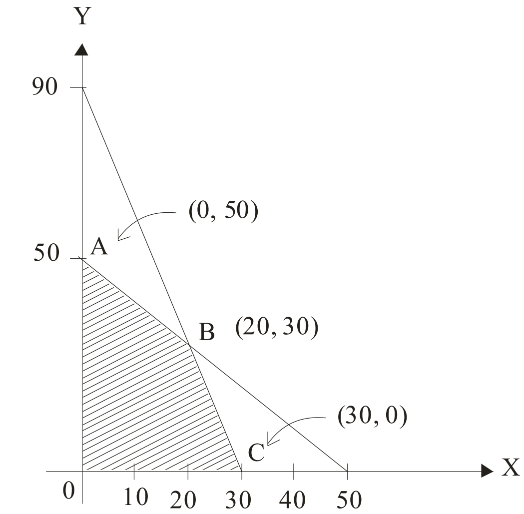
79. Find the distance between the line $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, –5). **[All India 2024, Ap]**

Chapter 12 : Linear Programming

Topic-1: Linear Programming Problem and its Mathematical Formulation

1 Multiple Choice Questions (1 Mark)

80. The maximum value of Z = 4x + y for a L. P. P. whose feasible region is given below is: **[All India 2024, K]**



(a) 50 (b) 110

(c) 120 (d) 170

81. The common region determined by all the constraints of a linear programming problem is called: **[Delhi 2024, K]**

(a) an unbounded region (b) an optimal region

(c) a bounded region (d) a feasible region

3 Assertion Reason/ Two Statement Type Questions (1 Mark)

Assertion (A) and Reason (R) based questions carrying 1 marks each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below:

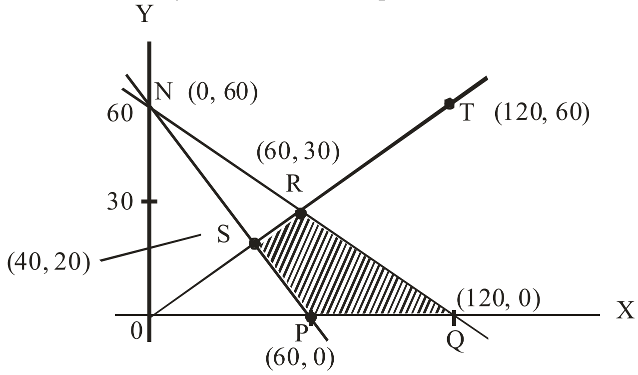
(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

82. Assertion (A) : The corner points of the bounded feasible region of a L. P. P. are shown below. The maximum value of Z = x + 2y occurs at infinite points.

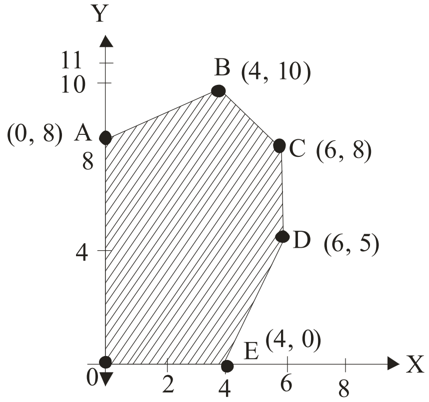


Reason (R) : The optimal solution of a LPP having bounded feasible region must occur at corner points.

[All India 2024, K]

5 *Short Answer Question (3 Marks)*

83. The corner points of the feasible region determined by the system of linear constraints are as shown in the following figure: **[All India 2024, U]**



(i) If $Z = 3x - 4y$ be the objective function, then find the maximum value of Z.

(ii) If $Z = px + qy$ where $p, q > 0$ be the objective function. Find the condition on p and q so that maximum value of Z occurs at B(4, 10) and C(6, 8).

7 *Case Based Questions (4 Marks)*

84. The month of September is celebrated as the Rashitrya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

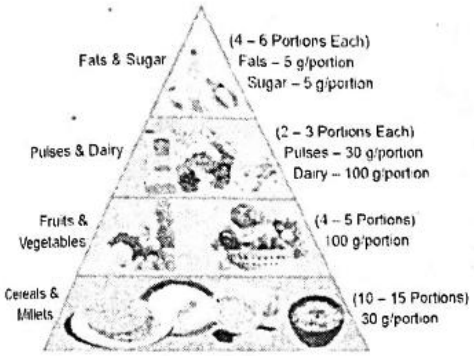


Figure-1

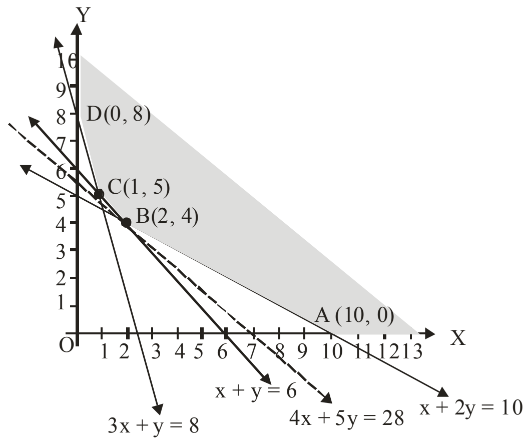


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X(x kg) and food Y(y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions:

(i) Identify and write all the constraints which determine the given feasible region in Figure-2. **(2)**

(ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region. **(2) [Delhi 2024, Ap]**

Chapter 13 : Probability

Topic-1: Conditional Probability

1 *Multiple Choice Questions (1 Mark)*

85. Let E be an event of a sample space S of an experiment, then $P(S|E) =$ **[Delhi 2024, K]**

(a) $P(S \cap E)$ (b) P(E)

(c) 1 (d) 0

Topic-2: Multiplication Theorem on Probability and Independent Events

7 Case Based Questions (4 Marks)

86. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.

Based on the above information, answer the following questions: **[All India 2024, K]**

(i) What is the probability that at least one of them is selected? **(1)**

(ii) Find $P(G \mid \overline{H})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected. **(1)**

(iii) Find the probability that exactly one of them is selected. **(2)**

OR

(iii) Find the probability that exactly two of them are selected. **(2)**

Topic-3: Bayes' Theorem

5 Short Answer Question (3 Marks)

87. A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King. **[All India 2024, K]**

Topic-4: Random Variable and its Probability Distributions, Mean of Random Variable

1 Multiple Choice Questions (1 Mark)

88. The probability distribution of a random variable X is:

X	0	1	2	3	4
P(X)	0.1	k	2k	k	0.1

where k is some unknown constant.
The probability that the random variable X takes the value 2 is: **[All India 2024, AP]**

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{4}{5}$ (d) 1

4 Very Short Answer Questions (1 Mark)

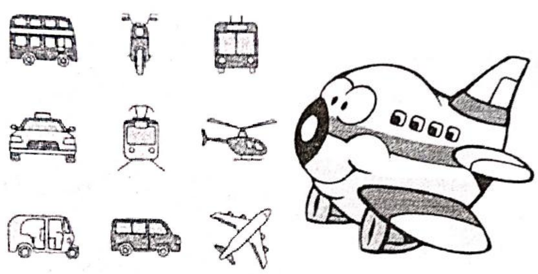
89. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X. **[Delhi 2024, U]**

5 Short Answer Question (3 Marks)

90. A biased die is twice as likely to show an even number as an odd number. If such a dies is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution. **[All India 2024, K]**

7 Case Based Questions (4 Marks)

91. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals. **[Delhi 2024, Ap]**



Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event the passengers survive after the journey.

On the basis of the above information, answer the following questions:

(i) Find the probability that the airplane will not crash. **(1)**

(ii) Find $P(A|E_1) + P(A|E_2)$. **(1)**

(iii) (a) Find P(A). **(2)**

OR

(b) Find $P(E_2|A)$. **(2)**

Solutions

Chapter 1 : Relations and Functions

Topic-1: Types of Relations

1. **(c)** $(a, a) \in R$ implies $a + a$ is a prime number
But $a + a = 2a$ is not always a prime number.
Hence, R is not reflexive.
Let $n = 1$
$2n = 2$ which is a prime number.
2n is not composite for all natural numbers n. **[1 mark]**

2. **(b)** Consider $2\sqrt{2}\ R\sqrt{2}$ as $2\sqrt{2} - \sqrt{2} + \sqrt{2} = 2\sqrt{2}$ (irrational)
$\sqrt{2}R3\sqrt{2}$ as $\sqrt{2} - 3\sqrt{2} + \sqrt{2} = -\sqrt{2}$ (irrational)
But $2\sqrt{2} \not R\ 3\sqrt{2}$
$2\sqrt{2} - 3\sqrt{2} + \sqrt{2} = 0$ (rational number)
Hence, R is not transitive. **[2 marks]**
$(a, a) \in R$ as $a - a + \sqrt{2} = \sqrt{2}$
which is irrational number $\forall\ a \in R$.
Hence, R is reflexive. **[1 mark]**
Consider $(2\sqrt{2}, \sqrt{2})$
$2\sqrt{2} - \sqrt{2} + \sqrt{2} = 2\sqrt{2}$ (irrational number)
$(\sqrt{2}, 2\sqrt{2}) \notin R$
as $\sqrt{2} - 2\sqrt{2} + \sqrt{2} = 0$ (not irrational)
Hence, R is not symmetric. **[2 marks]**

3. Reflexive
$(a, b)\ R\ (a, b) \Rightarrow a - a = b - b = 0$
Which is true for all $(a, b) \in N \times N$. **[1 Mark]**
Symmetric
$(a, b)R(c, d) \Rightarrow a - c = b - d \Rightarrow c - a = d - b$
$\Rightarrow$ $(c, d)\ R\ (a, b)\ \forall\ (a, b) \in N \times N$ and $(c, d) \in N \times N$
Hence R is symmetric. **[2 Marks]**
Transitive
Let $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$
$\Rightarrow$ $a - c = b - d$ ...(i)
$\Rightarrow$ $c - e = d - f$ ...(ii)
Adding (i) & (ii)
$a - e = b - f \Rightarrow (a, b)\ R\ (e, f)$
Hence R is transitive.
R is an equivalence Relation. **[2 Marks]**

Topic-2: Types of Functions

4. **(c)** To show function is one-one, let $f(x_1) = f(x_2)$ where $x_1, x_2 \in (0, \infty)$

$\Rightarrow$ $9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow$ $9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow$ $(x_1 - x_2)[9(x_1 + x_2) + 6] = 0$

$\Rightarrow$ $x_1 = x_2$ $[\because 9(x_1 + x_2) + 6 > 0]$

Hence function f(x) is one-one
Given, $f(x) = 9x^2 + 6x - 5 = (3x + 1)^2 - 6$

$\Rightarrow$ $y = (3x+1)^2 - 6 \Rightarrow x = \dfrac{\sqrt{y+6} - 1}{3}$

Clearly, $\forall y \in [-5, \infty)$, There exist pre-image $x \in [0, \infty)$.
Hence f(x) is onto. Therefore we can say that f(x) is bijective. **[1 Mark]**

5. Let $f(x_1) = f(x_2)$

$\dfrac{x_1 - 3}{x_1 - 5} = \dfrac{x_2 - 3}{x_2 - 5}$

$x_1x_2 - 5x_1 - 3x_2 + 15 = x_1x_2 - 3x_1 - 5x_2 + 15$

$2x_1 = 2x_2 \Rightarrow x_1 = x_2$

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Hence f(x) is one one.
[2½ marks]

Let $y = \dfrac{x-3}{x-5}$

$xy - 5y = x - 3$
$x(y - 1) = 5y - 3$

$x = \dfrac{5y - 3}{y - 1}$

x is defined $\forall y \in R - \{1\}$
Range of $f = R - \{1\}$
Range = codomain
$\Rightarrow$ f is onto **[2½ marks]**

6. Let $f(x) = \dfrac{1}{2} = \dfrac{2x}{1 + x^2}$

$1 + x^2 = 4x \Rightarrow x^2 - 4x + 1 = 0$

$\Rightarrow$ $x = \dfrac{4 \pm \sqrt{16 - 4}}{2} - \dfrac{4 \pm 2\sqrt{3}}{2}$

$= 2 \pm \sqrt{3}$

For $f(x) = \dfrac{1}{2}$ there are two values of x belonging to domain. Hence f is not one one. **[2 Marks]**

$$y = \frac{2x}{1+x^2}$$

$$y + yx^2 = 2x \Rightarrow yx^2 - 2x + y = 0$$

for x to be real

$4 - 4y^2 \geq 0$

$y^2 \leq 1 \Rightarrow y \in [-1,1]$ **[2 Marks]**

Range of f is [–1, 1]. For $f(x) = 2 \in R$ there is no preimage. f is not onto.

For f to be onto A should be [–1, 1] **[1 Mark]**

Chapter 2 : Inverse Trigonometric Functions

Topic-1: *Definition, Range, Domain and Principal Value Branch*

7. $\tan^{-1}1 = \frac{\pi}{4}, \cos^{-1}\left(\frac{-1}{2}\right) = \cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

$\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{-\pi}{4}$ **[1 mark]**

$\tan^{-1}1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{4} = \frac{2\pi}{3}$

[1 mark]

8. $-1 \leq x^2 - 4 \leq 1 \Rightarrow 3 \leq x^2 \leq 5$

$x \in \left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right]$ **[1 Mark]**

$x^2 - 4 \in [-1, 1]$. Range of $\sin^{-1}(x^2 - 4)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

[1 Mark]

Topic-2: *Simplest Form, Graph of Inverse Trigonometric Functions*

9. $\frac{\cos x}{1 - \sin x}$

$$= \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2} = \frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}$$ **[1 mark]**

$$= \frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right) = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$$

$$= \frac{\pi}{4} + \frac{x}{2}$$ **[1 mark]**

10. $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}$

$\cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3}$

$\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right] = \tan^{-1}(-1) = \frac{-\pi}{4}$ **[1 Mark]**

$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right)$

$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{6} - \frac{\pi}{4} = \frac{-\pi}{12}$ **[1 Mark]**

Chapter 3 : Matrices

Topic-1: *Matrix, Types of Matrices*

11. **(d)** Given that $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is scalar matrix

$\therefore$ a = d = 5 and b = c = 0

Now, a + 2b + 3c + 4d = 5 + 0 + 0 + 20 = 25 **[1 mark]**

12. **(a)** Let a 3 × 3 scalar matrix be $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$

Now, Product of all its element $= 0.k^3 = 0$ **[1 Mark]**

13. **(c)** Putting values of i, j in $a_{ij} = i - 3j$

we get $a_{11} = -2 < 0$

$a_{12} + a_{21} = -5 - 1 = -6$

$a_{31} = 0$, $a_{13} = -8$

$\therefore$ $a_{13} < a_{31} \Rightarrow$ Option (c) is false statement. **[1 Mark]**

Topic-2: *Operations on Matrices*

14. **(c)** $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$A^2 + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

$A^2 + 7I = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = 5A.$

$\therefore$ k = 5.

15. **(b)** $A=\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}, B=\frac{1}{3}\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & \lambda\end{bmatrix}$

Since, $AB=I \Rightarrow \begin{bmatrix}1 & 0 & 4+2\lambda\\0 & 1 & -6-3\lambda\\0 & 0 & 9+4\lambda\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

Comparing both sides, we get

$4+2\lambda=0 \Rightarrow \lambda=-2$

$-6-3\lambda=0 \Rightarrow \lambda=-2$

and $9+4\lambda=1 \Rightarrow \lambda=-2$.

16. **(a)** $A^2=\begin{bmatrix}2 & 1\\-4 & -2\end{bmatrix}\begin{bmatrix}2 & 1\\-4 & -2\end{bmatrix}$

$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$A^2=A^3=A^4=0$

$I-A+A^2-A^3$ ....$=I-A$

$=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}2 & 1\\-4 & -2\end{bmatrix}$

$=\begin{bmatrix}-1 & -1\\4 & 3\end{bmatrix}$ **[1 mark]**

17. **(b)** $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\\sin x & \cos x & 0\\0 & 0 & 1\end{bmatrix}$

$\Rightarrow [F(x)]^2=\begin{bmatrix}\cos x & -\sin x & 0\\\sin x & \cos x & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos x & -\sin x & 0\\\sin x & \cos x & 0\\0 & 0 & 1\end{bmatrix}$

$\Rightarrow [F(x)]^2=\begin{bmatrix}\cos^2 x-\sin^2 x & -2\sin x\cos x & 0\\2\sin x\cos x & \cos^2 x-\sin^2 x & 0\\0 & 0 & 1\end{bmatrix}$

$=\begin{bmatrix}\cos 2x & -\sin 2x & 0\\\sin 2x & \cos 2x & 0\\0 & 0 & 1\end{bmatrix}$

$\Rightarrow$ $[F(x)]^2=F(2x)$ ...(i)

Also, Given that $[F(x)]^2=F(kx)$ ...(ii)

From equation (i) and (ii) : $k=2$ **[1 Mark]**

Topic-4: *Transpose of a Matrix*

18. **(d)** For symmetric matrix A

$A'=A$

$(B'AB)'=B'A'(B')'$

$=B'AB$

Hence B'AB is symmetric matrix.

For skew symmetric matrix P

$P'=-P$

Assertion is false but reason is true. **[1 Mark]**

Chapter 4 : Determinants

Topic-4: *Adjoint and Inverse of a Matrix*

19. **(b)** Given, $A^{-1}=\frac{1}{7}\begin{bmatrix}2 & 1\\-3 & 2\end{bmatrix}$

Now, $A=(A^{-1})^{-1}$

$|A^{-1}|=\frac{1}{7}$

$\text{adj}(A^{-1})=\begin{bmatrix}2/7 & -1/7\\3/7 & 2/7\end{bmatrix}$

$(A^{-1})^{-1}=\frac{\text{adj}(A^{-1})}{|A^{-1}|}=\begin{bmatrix}2 & -1\\3 & 2\end{bmatrix}$ **[1 mark]**

20. **(d)** given $\begin{vmatrix}-a & b & c\\a & -b & c\\a & b & -c\end{vmatrix}=kabc$

On expending the given matrices, we get

$\Rightarrow$ $4abc=kabc \Rightarrow k=4$ **[1 Mark]**

21. $A=\begin{bmatrix}1 & \cot x\\-\cot x & 1\end{bmatrix}$

$|A|=\text{cosec}^2 x$

$\text{adj}(A)=\begin{bmatrix}1 & -\cot x\\\cot x & 1\end{bmatrix}$ **[2 Marks]**

$A^{-1}=\frac{1}{\text{cosec}^2 x}\begin{bmatrix}1 & -\cot x\\\cot x & 1\end{bmatrix}$

$=\begin{bmatrix}\sin^2 x & -\cos x\sin x\\\cos x\sin x & \sin^2 x\end{bmatrix}$

$A'=\begin{bmatrix}1 & -\cot x\\\cot x & 1\end{bmatrix}$

$$A'A^{-1}=\begin{bmatrix}1 & -\cot x\\ \cot x & 1\end{bmatrix}\begin{bmatrix}\sin^2 x & -\cos x\sin x\\ \cos x\sin x & \sin^2 x\end{bmatrix}$$

$$=\begin{bmatrix}\sin^2 x-\cos^2 x & -2\sin x\cos x\\ 2\cos x\sin x & \sin^2 x-\cos^2 x\end{bmatrix}$$

$$=\begin{bmatrix}-\cos 2x & -\sin 2x\\ \sin 2x & -\cos 2x\end{bmatrix}$$ **[3 Marks]**

Topic-5: Solutions of System of Equations

22. $A=\begin{bmatrix}1 & 2 & 1\\ 2 & 3 & -1\\ 1 & 0 & 1\end{bmatrix}$

$|A|=3-2(2+1)+(0-3)=3-6-3=-6$ **[1 mark]**

$$\text{adj}(A)=\begin{bmatrix}3 & -2 & -5\\ -3 & 0 & 3\\ -3 & 2 & -1\end{bmatrix}$$

$$A^{-1}=\frac{\text{adj}(A)}{|A|}=\frac{-1}{6}\begin{bmatrix}3 & -2 & -5\\ -3 & 0 & 3\\ -3 & 2 & -1\end{bmatrix}$$ **[2 marks]**

Matrix form of system of equations

$$\begin{bmatrix}1 & 2 & 1\\ 2 & 3 & 0\\ 1 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}5\\ 1\\ 8\end{bmatrix}$$

$A^T X=B$

$X=(A^T)^{-1}B=(A^{-1})^T B$

$$X=\frac{-1}{6}\begin{bmatrix}3 & -3 & -3\\ -2 & 0 & 2\\ -5 & 3 & -1\end{bmatrix}\begin{bmatrix}5\\ 1\\ 8\end{bmatrix}$$

$$=\frac{-1}{6}\begin{bmatrix}15-3-24\\ -10+0+16\\ -25+3-8\end{bmatrix}$$

$$=\frac{-1}{6}\begin{bmatrix}-12\\ 6\\ -30\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ -1\\ 5\end{bmatrix}$$

$x=2, y=-1, z=5$ **[2 marks]**

23. (a) Let $\frac{1}{x}=a, \frac{1}{y}=b, \frac{1}{z}=c$ **[1 Mark]**

$2a+3b+10c=4$

$4a-6b+5c=1$

$6a+9b-20c=2$

$$\text{Let } A=\begin{bmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{bmatrix}$$

$$B=\begin{bmatrix}4\\ 1\\ 2\end{bmatrix},\quad X=\begin{bmatrix}a\\ b\\ c\end{bmatrix}$$

$AX=B\Rightarrow X=A^{-1}B$

$$\text{Adj }(A)=\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$$ **[2 Marks]**

$|A|=2(120-45)-3\times(-110)+10\times 72$

$=150+330+720=1200$

$$A^{-1}=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$$

$$X=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}\begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$$

$$=\frac{1}{1200}\begin{bmatrix}600\\ 400\\ 240\end{bmatrix}\Rightarrow a=\frac{1}{2}, b=\frac{1}{3}, c=\frac{1}{5}$$

$x=2, y=3, z=5$ **[2 Marks]**

Chapter 5 : Continuity and Differentiability

Topic-1: Continuity

24. (b) $\because \forall x \leq -3$ we have $|x| = -x$

Hence f(x) is, $f(x) = \begin{cases} -x+3 & \text{if} \quad x \leq -3 \\ -2x & \text{if} \quad -3 < x < 3 \\ 6x+2 & \text{if} \quad x \geq 3 \end{cases}$

$\because \lim_{h \to 0} f(-3-h) = \lim_{h \to 0} f(-3+h) = f(-3) = 6$

$\Rightarrow$ f(x) is continuous at $x = -3$

Now, since $\lim_{h \to 0} f(3+h) = 20$ and $\lim_{h \to 0} f(3-h) = 6$

$\Rightarrow$ at $x = 3$, RHL $\neq$ LHL

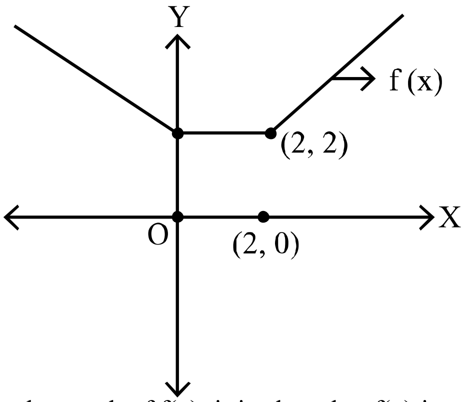
$\Rightarrow$ f(x) is discontinuous at $x = 3$

Hence, the number of points of discontinuity = 1 **[1 Mark]**

Topic-2: Differentiability

25. Given function is,

$f(x) = |x| + |x-2|$



from the graph of f(x), it is clear that f(x) is continuous everywhere but not differentiable at $x = 0$ and $x = 2$.

26. $f(x) = |\tan 2x|$

$\tan\left(2 \times \frac{\pi}{3}\right) = -\sqrt{3}. < 0$

$f(x) = -\tan 2x$ at $x = \frac{\pi}{3}$

$f'(x) = -2\sec^2 2x$ **[1 Mark]**

$f'\left(\frac{\pi}{3}\right) = \frac{-2}{\cos^2 2x}$

$= \frac{-2}{\cos^2\left(\frac{2\pi}{3}\right)} = \frac{-2}{(-1/2)^2}$

$= -8$ **[1 Mark]**

27. for $x > 0$, $|x| = x$

$\frac{d}{dx}|x| = \frac{d}{dx}(x) = 1 = \frac{x}{x} = \frac{x}{|x|}$ **[1½ Marks]**

for $x < 0$, $|x| = -x$

$\frac{d}{dx}(|x|) = \frac{d}{dx}(-x) = -1 = \frac{x}{-x} = \frac{x}{|x|}$ **[1½ Marks]**

Topic-3: Derivatives of Implicit and Inverse Trigonometric Functions

28. (b) $\frac{d(\tan^{-1}x^2)}{dx} = \frac{d(\tan^{-1}x^2)}{dx^2} \cdot \frac{d(x^2)}{dx}$

$= \frac{1}{1+(x^2)^2} \cdot (2x) = \frac{2x}{1+x^4}$ **[1 Mark]**

29. $y = \cos^3(\sec^2 2t)$

$\frac{dy}{dt} = 3\cos^2(\sec^2 2t) \times (-\sin(\sec^2 2t)) \times$

$2\sec(2t) \times \sec 2t \tan 2t \times 2$

$\frac{dy}{dt} = -12\cos^2(\sec^2 2t)\sin(\sec^2 2t) \times \sec^2 2t \tan 2t$

[2 marks]

30. Let $\cot^{-1}x = \theta$

$\cot\theta = x$

$\text{cosec}\theta = \sqrt{1+x^2}$

$\Rightarrow \text{cosec}(\cot^{-1}x) = \sqrt{1+x^2} = y$ **[1 Mark]**

$\frac{dy}{dx} = \frac{1}{2\sqrt{1+x^2}} \times 2x = \frac{x}{\sqrt{1+x^2}}$

$\sqrt{1+x^2}\frac{dy}{dx} - x = 0$ **[1 Mark]**

Topic-5: Logarithmic Differentiation

31. $x^y = c^{x-y}$

Take (log) on both sides

$y \ln x = x - y \Rightarrow \frac{y}{x} + (\ln x)\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$$(1+\ln x)\frac{dy}{dx}=1-\frac{y}{x}$$

$$y\ln x = x - y \Rightarrow \frac{y}{x}(1+\ln x)=1$$ **[1 mark]**

$$(1+\ln x)\frac{dy}{dx}=1-\frac{1}{(1+\ln x)}=\frac{\ln x}{(1+\ln x)}$$

$$\frac{dy}{dx}=\frac{\ln x}{(1+\ln x)^2}$$ **[1 mark]**

32. Let $u=(\cos x)^x$

$\log u = x \log \cos x$

$$\frac{1}{u}\frac{dy}{dx}=\log(\cos x)-x\tan x$$ **[1 mark]**

$$\frac{dy}{dx}=(\cos x)^x\left[\log(\cos x)-x\tan x\right]$$

$$y=(\cos x)^x+\cos^{-1}\sqrt{x}$$

$$\frac{dy}{dx}=(\cos x)^x\left[\log(\cos x)-x\tan x\right]+\frac{-1}{\sqrt{1-x}}\times\frac{1}{2\sqrt{x}}$$

$$=(\cos x)^x\left[\log(\cos x)-x\tan x\right]-\frac{1}{2\sqrt{x-x^2}}$$ **[2 marks]**

Topic-6: *Derivatives of Functions in Parametric Forms*

33. **(a)** Derivative of x^2 with respect to x^3 is,

$$\frac{d(x^2)}{d(x^3)}=\frac{\frac{d(x^2)}{dx}}{\frac{d(x^3)}{dx}}=\frac{2x}{3x^2}=\frac{2}{3x}$$

34. **(a)** $x=e^{\cos 3t} \Rightarrow \ln x=\cos 3t$

$y=e^{\sin 3t} \Rightarrow \ln y=\sin 3t$

$(\ln x)^2+(\ln y)^2=\cos^2 3t+\sin^2 3t=1$

$(\ln x)^2+(\ln y)^2=1$ **[1 Mark]**

differentiating both sides wrt. x

$$\frac{2(l\,x)}{x}+\frac{2\ln y}{y}\cdot\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-y\ln x}{x\ln y}$$ **[2 Marks]**

Chapter 6 : Application of Derivatives

Topic-1: *Rate of Change of Quantities*

35. $\frac{dV}{dt}=6\text{ cm}^3/\text{sec} \Rightarrow V=a^3 \Rightarrow 3a^2\frac{da}{dt}=6$

$$3\times 8^2\frac{da}{dt}=6\Rightarrow\frac{da}{dt}=\frac{2}{8^2}$$ **[1 mark]**

$S=6a^2$

$$\frac{dS}{dt}=12a\frac{da}{dt}=12\times 8\times\frac{2}{8^2}=3\text{cm}^2/\text{s}$$ **[1 mark]**

Topic-2: *Increasing and Decreasing Functions*

36. **(a)** $f(x)=kx-\sin x$

$f'(x)=k-\cos x$

for strictly increasing

$k-\cos x>0\Rightarrow \cos x<k$

$\because \quad \cos x\le 1<k$

$\therefore \quad k>1$ **[1 mark]**

37. **(b)** Given, $f(x)=x^3-3x^2+12x-18$ **[1 Mark]**

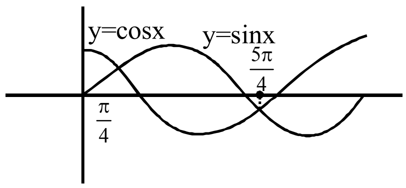
$\Rightarrow \quad f'(x)=3x^2-6x+12=3(x-1)^2+9$

$\because \quad \forall x\in R,\ f'(x)>0$

$\Rightarrow \quad f(x)$ is strictly increasing on R **[1 Mark]**

38. $f(x)=\sin x+\cos x$

$f'(x)=\cos x-\sin x$



$\forall x\in\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$

$\cos x<\sin x$

$\cos x-\sin x<0$

$$f'(x)<0\ \forall x\in\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$$

Hence f(x) is strictly decreasing in the internal $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$

[1 mark]

Note : If function f(x) is strictly decreasing in (a, b) then $f'(x)<0 \quad \forall x\in(a,b)$

39. $f(x)=e^x-e^{-x}+x-\tan^{-1}x$

$$f'(x)=e^x+e^{-x}+1-\frac{1}{1+x^2}$$

$$e^x+e^{-x}\ge 2\ \forall x\in R\ \&\ \frac{1}{1+x^2}\le 1\ \forall x\in R$$

$f'(x)\ge 2+1-1=2>0$

Hence, f(x) is strictly increasing in its domain. **[2 Marks]**

Topic-3: Maxima and Minima

40. $f(x) = x + \frac{1}{x}$

$f'(x) = 1 - \frac{1}{x^2}$

For maxima or minima

$f'(x) = 1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1$ **[1 Mark]**

$f''(x) = \frac{2}{x^3}$

$f''(1) = 2 > 0 \Rightarrow x = 1$ is point of minima.

$f''(-1) = -2 < 0 \Rightarrow x = -1$ is point of maxima

$M = f(-1) = -2$

$m = f(1) = 2$

$M - m = -2 - 2 = -4$

41. $R(x) = x \times P(x)$

$= 450x - \frac{x^2}{2}$

$\frac{dR}{dx} = 450 - x$

$\frac{d^2R}{dx^2} = -1 < 0$

For maximum revenue

$\frac{dR}{dx} = 0 \Rightarrow x = 450$

$\left.\frac{d^2R}{dx^2}\right|_{x=450} < 0$

At x = 450 maximum revenue is obtained 450 units should be sold to obtain maximum revenue. **[1 mark]**

(ii) $P(450) = 450 - \frac{450}{2} = 225$

Rebate = 350 – 225

₹ 125 for maximum revenue. **[2 marks]**

42. $F = \frac{V^2}{500} - \frac{V}{4} + 14$

(i) $F(40) = \frac{1600}{500} - \frac{40}{4} + 14$

$= 3.2 - 10 + 14 = 7.2$ **[1 Mark]**

(ii) $\frac{dF}{dV} = \frac{V}{250} - \frac{1}{4}$ **[1 Mark]**

(iii) For minimum F

$\frac{dF}{dV} = 0$ & $\frac{d^2F}{dV^2} > 0$

$\Rightarrow \frac{V}{250} - \frac{1}{4} = 0$

$\Rightarrow V = 62.5$

$\frac{d^2F}{dV^2}$ at $V = 62.5$ is $\frac{1}{250} > 0$

V = 62.5 for minimum fuel consumption. **[2 Marks]**

(b) $\frac{V}{250} - \frac{1}{4} = 0.01$

$\frac{V}{250} = 0.25 - 0.01 = 0.24$

$V = 250 \times 0.24 = 60$ km/hrs **[1 Mark]**

$F = \frac{3600}{500} - \frac{60}{4} + 14$

$= 7.2 - 15 + 14 = 21.2 - 15 = 6.2(\ell/100 \text{ km})$ **[1 Mark]**

Chapter 7 : Integrals

Topic-2: Integration by substitution

43. $I = \int \frac{e^{4x} - 1}{e^{4x} + 1} dx$ **[1 Mark]**

$= \int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx$

Let $u = e^{2x} + e^{-2x}$ **[1 Mark]**

$du = 2(e^{2x} - e^{-2x})dx$

$I = \int \frac{du}{2u} = \frac{1}{2}\ell n\ u + C$

$= \frac{1}{2}\ln(e^{2x} + e^{-2x}) + C$

44. $\log x = t \Rightarrow \frac{1}{x}dx = dt$ **[1 Mark]**

$I = \int \frac{dt}{t^2 - 3t - 4} = \int \frac{dt}{(t-4)(t+1)}$

$= \frac{1}{5}\int\left(\frac{1}{t-4} - \frac{1}{t+1}\right)dt = \frac{1}{5}\ln\left(\frac{t-4}{t+1}\right) + C$

$= \frac{1}{5}\ln\left(\frac{\ln x - 4}{\ln x + 1}\right) + C$ **[2 Marks]**

45. Let $X^{3/2} = t$

$$dt = \frac{3}{2}\sqrt{x}\ dx$$ **[1 Mark]**

$$I = \int x^{3/2}.\sin^{-1} x^{3/2}\sqrt{x}\ dx$$

$$= \int t \sin^{-1} t \times \frac{2}{3} dt$$

$$= \frac{2}{3}\left[\frac{t^2}{2}\sin^{-1} t - \int \frac{1}{2\sqrt{1-t^2}} t^2 dt\right]$$ **[1 Mark]**

$$= \frac{1}{3}\left[t^2 \sin^{-1} t + \int \frac{1-t^2-1}{\sqrt{1-t^2}} dt\right]$$

$$= \frac{1}{3}\left[t^2 \sin^{-1} t - \sin^{-1} t + \int \sqrt{1-t^2}\, dt\right]$$

$$= \frac{1}{3}\left[t^2 \sin^{-1} t - \sin^{-1} t + \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1} t\right] + C$$

$$= \frac{1}{3}\left[t^2 \sin^{-1} t - \frac{1}{2}\sin^{-1} t + \frac{t}{2}\sqrt{1-t^2}\right] + C$$ **[1 Mark]**

Transpose of a Matrix

46. $$\frac{2x}{(x^2+1)(x^2-4)} = \frac{2x}{5}\left[\frac{1}{x^2-4} - \frac{1}{x^2+1}\right]$$

$$\int \frac{2x}{(x^2+1)(x^2-4)}\, dx = \frac{1}{5}\left[\int \frac{2x dx}{x^2-4} - \int \frac{2x dx}{x^2+1}\right]$$ **[1 mark]**

$$= \frac{1}{5}\left[\ln(x^2-4) - \ln(x^2+1)\right] + c$$

$$= \frac{1}{5}\ln\left(\frac{x^2-4}{x^2+1}\right) + c$$ **[1 mark]**

47. $$I = \int \sec^3 \theta\, d\theta$$

$$I = \int \sec\theta \cdot \sec^2 \theta\, d\theta$$ **[1 mark]**

$$= \sec\theta \tan\theta - \int \sec\theta \tan\theta \cdot \tan\theta d\theta$$

$$= \sec\theta \tan\theta - \int \sec\theta(\sec^2\theta - 1)\, d\theta$$

$$= \sec\theta \tan\theta + \int \sec\theta d\theta - \int \sec^3 \theta d\theta$$

$$2I = \sec\theta \tan\theta + \log|\sec\theta + \tan\theta|$$

$$I = \frac{1}{2}[\sec\theta \tan\theta + \log|\sec\theta + \tan\theta|]$$ **[2 marks]**

48. Let $f(x) = \dfrac{x}{\sqrt{1+x^2}}$

$$f'(x) = \frac{1}{\sqrt{1+x^2}} - \frac{1 \times 2x \times x}{2(1+x^2)^{3/2}}$$ **[1 mark]**

$$= \frac{1}{\sqrt{1+x^2}} - \frac{x^2}{(1+x^2)^{3/2}}$$

$$= \frac{1}{(1+x^2)^{3/2}}$$

$$I = \int e^x \left(f(x) + f'(x)\right) dx$$

$$= e^x f(x) + C = \frac{xe^x}{\sqrt{1+x^2}} + C$$ **[2 marks]**

49. **(b)** $$\int_0^{\pi} \tan^2\left(\frac{\theta}{3}\right) d\theta = \int_0^{\pi}\left(\sec^2\left(\frac{\theta}{3}\right) - 1\right) d\theta$$

$$= \left[3\tan\left(\frac{\theta}{3}\right) - \theta\right]_0^{\pi} = \left(3\tan\frac{\pi}{3} - \pi\right) - 0$$

$$= 3\sqrt{3} - \pi$$ **[1 mark]**

Topic-9: Some Properties of Definite Integrals

50. **(b)** $$I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x + \cos x} dx \quad \text{...(i)}$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sin(\pi/2 - x) - \cos(\pi/2 - x)}{1 + \sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$\left\{\because \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx\right\}$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x + \cos x}\, dx \quad \text{...(ii)}$$

Eq (i) + (ii),

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \frac{0}{1 + \sin x + \cos x} dx = 0$$

$$\Rightarrow \quad I = 0$$

51. $I=\int_0^{\frac{\pi}{4}}\frac{x\,dx}{1+\cos 2x+\sin 2x}$

$I=\int_0^{\frac{\pi}{4}}\frac{\left(\frac{\pi}{4}-x\right)dx}{1+\cos\left[2\left(\frac{\pi}{4}-x\right)\right]+\sin\left[2\left(\frac{\pi}{4}-x\right)\right]}$ **[1 mark]**

$=\int_0^{\frac{\pi}{4}}\frac{\left(\frac{\pi}{4}-x\right)dx}{1+\sin 2x+\cos 2x}$

$2I=\int_0^{\frac{\pi}{4}}\frac{\frac{\pi}{4}dx}{2\cos^2 x+2\sin x\cos x}$ **[1 mark]**

$I=\frac{\pi}{16}\int_0^{\frac{\pi}{4}}\frac{\sec^2 x\,dx}{1+\tan x}$

$=\frac{\pi}{16}\left[\ln(1+\tan x)\right]_0^{\pi/4}$

$=\frac{\pi}{16}(\ln 2-\ell n_1)$

$I=\frac{\pi\ln 2}{16}$ **[1 mark]**

$=$

52. $I=\int_{-2}^{2}\sqrt{\frac{2-x}{2+x}}\,dx$

$I=\int_{-2}^{2}\frac{\sqrt{2-x}}{\sqrt{2+x}}\times\frac{\sqrt{2-x}}{\sqrt{2-x}}dx$ **[1 Mark]**

$=\int_{-2}^{2}\frac{2-x}{\sqrt{4-x^2}}dx=\int_{-2}^{2}\frac{2}{\sqrt{4-x^2}}-\int_{-2}^{2}\frac{xdx}{\sqrt{4-x^2}}$

$=\left[2\sin^{-1}\left(\frac{x}{2}\right)+\sqrt{4-x^2}\right]_{-2}^{2}$

$=(2\sin^{-1}(1)+0)-(2\sin^{-1}(-1)+0)$

$=\pi-(-\pi)=2\pi$ **[2 Marks]**

Chapter 8 : Application of Integrals

Topic-1: Area under Simple Curves

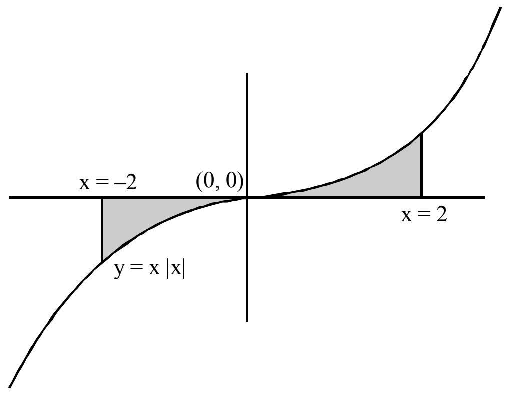
6 *Long Answer Questions (4 or 5 Marks)*

53. $y=x^2 \quad \forall x>0$ **[1 mark]**

$=-x^2 \quad \forall x<0$

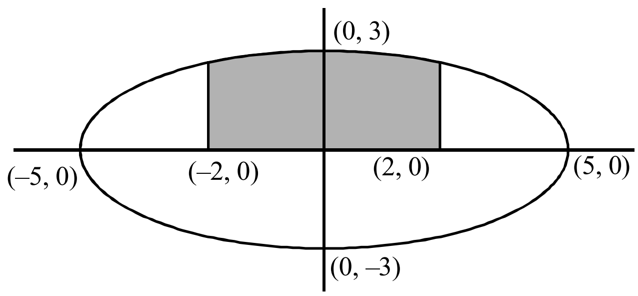
Required Area

$=2\times\int_0^2 x^2dx$ **[2 marks]**



$=2\left[\frac{x^3}{3}\right]_0^2=\frac{16}{3}$ **[2 marks]**

54. $y=\frac{\sqrt{225-9x^2}}{5}$



Required Area

$=\int_{-2}^{2}\sqrt{\frac{225-9x^2}{5}}\,dx$

$= \frac{2}{5}\int_0^2 \sqrt{(225-9x^2)}\ dx$

$= \frac{2}{5}\times 3\int_0^2 \sqrt{25-x^2}\,dx$

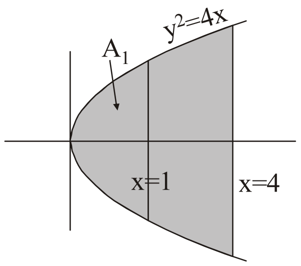
$= \frac{6}{5}\left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_0^2$

$= \frac{3}{5}\left[\left(2\sqrt{21} + 25\sin^{-1}\frac{2}{5}\right) - 0\right]$

$= \frac{6}{5}\sqrt{21} + 15\sin^{-1}\left(\frac{2}{5}\right)$ **[3 marks]**

55. $y^2 = 4x$

$y = 2\sqrt{x}$



$A_1 = \int_0^1 2\sqrt{x}\ dx = \frac{4}{3}\left[x^{3/2}\right]_0^1 = \frac{4}{3}$ **[2 Marks]**

$A_2 = \int_0^4 \left[2\sqrt{x} - (-2\sqrt{x})\right]dx$ **[2 Marks]**

$A_2 = 4\times\frac{2}{3}\times\left[x^{3/2}\right]_0^4 = \frac{8}{3}\times 8 = \frac{64}{3}$

$\frac{A_1}{A_2} = \frac{4/3}{64/3} = \frac{1}{16} \Rightarrow A_1 : A_2 = 1:16$ **[1 Mark]**

Chapter 9 : Differential Equations

Topic-1: *Order and Degree of a differential equation*

56. **(d)** given $(y'')^2 + (y')^3 = x\sin(y')$

Clearly the given differential equation is not in the polynomial form of (y').

Hence its degree is not defind. **[1 Mark]**

Topic-3: *Differential equations with variables separable*

57. **(a)** $\frac{dy}{dx} = y\cos 2x$

$\Rightarrow \int \frac{dy}{y} = \int \cos 2x\,dx$

$\ln y = \frac{1}{2}\sin 2x + C$ **[1 mark]**

$y\left(\frac{\pi}{4}\right) = 2$

$\ln 2 = \frac{1}{2}\sin\left(2\times\frac{\pi}{4}\right) + C$

$C = \ln 2 - \frac{1}{2}$ **[1 mark]**

Particular solution is

$\ln y = \frac{1}{2}\sin 2x + \ln\ 2 - \frac{1}{2}$ **[1 mark]**

Topic-4: *Homogeneous differential equations*

58. **(a)** If $F(x, y) = \cos(x) - \sin\left(\frac{y}{x}\right)$ then **[1 Mark]**

$\frac{dy}{dx} = F(x, y)$ will not be Homogeneous.

59. **(b)** $\left(x\,e^{\frac{y}{x}} + y\right)dx = x\,dy$

$\frac{dy}{dx} = e^{y/x} + \frac{y}{x}$

Let $y = vx$ **[1 mark]**

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = e^v + V$

$\int e^{-v}dv = \int \frac{dx}{x}$

$-e^{-v} = \ln x + C$

$\ln x + e^{-v} + C = 0$

$y(1) = 1$

$\ell n + e^{-1} + C = 0 \Rightarrow e = -e^{-1}$

Particular solution is

$\ln x + C^{\frac{-y}{x}} = e^{-1}$ **[1 mark]**

Note : If the given differential equation is homogeneous then put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$

60. $\frac{dy}{dx} = \frac{y^2 + 2xy}{2x^2}$

$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ **[1 Mark]**

$v + x\frac{dv}{dx} = \frac{v^2x^2 + 2x \times vx}{2x^2}$

$x\frac{dv}{dx} = \frac{v^2 + 2v}{2} - v = \frac{v^2}{2}$

$\int \frac{dv}{v^2} = \int \frac{dx}{2x} \Rightarrow \frac{-1}{v} = \frac{1}{2}\ln x + C'$

$\Rightarrow \quad \frac{-2x}{y} = \ln x + C'$

$\ln x + \frac{2x}{y} = C$ **[1 Mark]**

$y(1) = 2$

$\ln 1 + \frac{2 \times 1}{2} = C \Rightarrow C = 1$

Particular solution is

$\ln x + \frac{2x}{y} = 1$ **[1 Mark]**

Topic-5: Linear differential equations

1 *Multiple Choice Questions (1 Mark)*

61. **(b)** $\frac{dy}{dx} + \frac{2}{x}y = 0 \quad x \neq 0$

Here, $P = \frac{2}{x}, \ Q = 0$

$I.F. = e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$ **[1 mark]**

62. **(d)** $(x + 2y^2)\frac{dy}{dx} = y \Rightarrow \frac{dx}{dy} = \frac{x}{y} + 2y$

$\frac{dx}{dy} - \frac{x}{y} = 2y \Rightarrow I.F. = e^{\int \frac{-1}{y}dy} = e^{-1ny} = \frac{1}{y}$

$I.F. = \frac{1}{y}$

63. $ydx = (x + 2y^2)dy$

$\frac{dx}{dy} - \frac{x}{y} = zy$ **[1 Mark]**

$IF = \int_{e} -\frac{1}{y}dy, e^{-lx\,y}$

$\frac{x}{y} = \int \frac{1}{y} \times 2y\,dy + c$

$\frac{x}{y} = 2y + c$ **[2 Marks]**

Chapter 10 : Vector Algebra

Topic-3: Scalar (or dot) product of two vectors

64. **(c)** Given : $|\vec{a}| = 1, |\vec{b}| = 2$

and $\vec{a} \cdot \vec{b} = \sqrt{3} \Rightarrow 2\vec{a} \cdot (-\vec{b}) = -2\sqrt{3}$

$\Rightarrow \quad |2\vec{a}| \cdot |-\vec{b}| \cdot \cos\theta = -2\sqrt{3}$

$\Rightarrow \quad 2 \times 1 \times 2\cos\theta = -2\sqrt{3}$

$\Rightarrow \quad \cos\theta = --\frac{\sqrt{3}}{2}$

$\Rightarrow \quad \theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= \cos^{-1}\left(\cos\frac{5\pi}{6}\right)$

$\Rightarrow \quad \theta = \frac{5\pi}{6}$

65. **(d)** Given : $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}, \vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$

$|\vec{a}| = \sqrt{4+1+1} = \sqrt{6}$

$|\vec{b}| = \sqrt{1+9+25} = \sqrt{35}$

$|\vec{c}| = \sqrt{9+16+16} = \sqrt{41}$

$\vec{a}.\vec{b} = 2 + 3 - 5 = 0$

$\therefore$ The given vectors represent a right angled triangle.

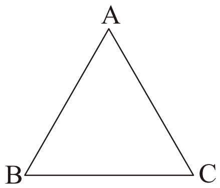
66. **(a)** We know that $\cos\theta \geq 1$ **[1 Mark]**

$\Rightarrow \quad |\vec{a}||\vec{b}|\cos\theta \geq |\vec{a}||\vec{b}|$

$\Rightarrow \quad \vec{a}.\vec{b} \geq |\vec{a}||\vec{b}|$

67. $\overrightarrow{AB} = (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$

$= -\hat{i} - 2\hat{j} - 6\hat{k} \Rightarrow |\overrightarrow{AB}| = \sqrt{41}$



$\overrightarrow{AC} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$

$= \hat{i} - 3\hat{j} - 5\hat{k} \Rightarrow |\overrightarrow{AC}| = \sqrt{35}$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = -1 + 6 + 30 = 35$

$\cos A = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}| \cdot |\overrightarrow{AC}|} = \dfrac{35}{\sqrt{35}.\sqrt{41}} = \sqrt{\dfrac{35}{41}}$ **[1 Mark]**

$A = \cos^{-1}\sqrt{\dfrac{35}{41}}$

$\overrightarrow{BC} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$

$= 2\hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{BA} = (2\hat{i} - \hat{j} + \hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$

$= \hat{i} + 2\hat{j} + 6\hat{k}$

$\overrightarrow{BC} \cdot \overrightarrow{BA} = 2 - 2 + 6 = 6$

$\cos B = \dfrac{\overrightarrow{BC}.\overrightarrow{BA}}{(\overrightarrow{BC}) \cdot (\overrightarrow{BA})} = \dfrac{6}{\sqrt{6}.\sqrt{41}} = \sqrt{\dfrac{6}{41}}$

$B = \cos^{-1}\sqrt{\dfrac{6}{41}}$ **[1 Mark]**

$\overrightarrow{CA} = (2\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k})$

$= -\hat{i} + 3\hat{j} + 5\hat{k}$

$\overrightarrow{CB} = (\hat{i} - 3\hat{j} - 5\hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k})$

$= -2\hat{i} + \hat{j} - \hat{k}$

$\cos C = \dfrac{\overrightarrow{CA}.\overrightarrow{CB}}{|\overrightarrow{CA}| \cdot |\overrightarrow{CB}|} = \dfrac{2 + 3 - 5}{\sqrt{35}.\sqrt{6}} = 0$

$C = \cos^{-1} 0 = 90^\circ$ **[1 Mark]**

68. (i) $\overrightarrow{AV} = (-3\hat{i} + 7\hat{j} + 11\hat{k}) - (7\hat{i} + 5\hat{j} + 8\hat{k})$

$= -10\hat{i} + 2\hat{j} + 3\hat{k}$

Distance between A & V is

$= \sqrt{10^2 + 2^2 + 3^2} = \sqrt{113}$ **[1 mark]**

(ii) $\overrightarrow{DA} = (7\hat{i} + 5\hat{j} + 8\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k}) = 5\hat{i} + 2\hat{j} + 4\hat{k}$

$\widehat{DA} = \dfrac{(5\hat{i} + 2\hat{j} + 4\hat{k})}{\sqrt{25 + 16 + 4}} = \dfrac{5\hat{i} + 2\hat{j} + 4\hat{k}}{3\sqrt{5}}$ **[1 mark]**

(iii) $\overrightarrow{DV} = (-3\hat{i} + 7\hat{j} + 11\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k})$

$= -5\hat{i} + 4\hat{j} + 7\hat{k}$

$\overrightarrow{DA} = 5\hat{i} + 2\hat{j} + 4\hat{k}$ **[1 mark]**

$\cos\theta = \dfrac{\overrightarrow{DV}.\overrightarrow{DA}}{|\overrightarrow{DV}| \times |\overrightarrow{DA}|}$

$= \dfrac{-25 + 8 + 28}{3\sqrt{5} \times \sqrt{25 + 16 + 49}}$

$= \dfrac{11}{3\sqrt{5} \times \sqrt{90}} = \dfrac{11}{45\sqrt{2}}$

$\angle VDA = \cos^{-1}\left(\dfrac{11}{45\sqrt{2}}\right)$ **[1 mark]**

OR

Projection of $\overrightarrow{DV}$ on $\overrightarrow{DA} = \dfrac{(-5\hat{i} + 4\hat{j} + 7\hat{k}).(5\hat{i} + 2\hat{j} + 4\hat{k})}{3\sqrt{5}}$

$= \dfrac{-25 + 8 + 28}{3\sqrt{5}} = \dfrac{11}{3\sqrt{5}}$ **[2 marks]**

Note : projection of $\vec{a}$ on $\vec{b}$ is given by $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

Topic-4: *Vector (or cross) product of two vectors*

69. **(b)** $|\vec{a} \times \hat{i}|^2 = |(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times \hat{i}|^2$

$= |-a_2\hat{k} + a_3\hat{j}|^2 = a_2^2 + a_3^2$

Similarly, $|\vec{a} \times \hat{j}|^2 = a_1^2 + a_3^2$

and $|\vec{a} \times \hat{k}|^2 = a_1^2 + a_2^2$

Now, $|\vec{a}\times\hat{i}|^2+|\vec{a}\times\hat{j}|^2+|\vec{a}\times\hat{k}|$

$= a_2^2+a_3^2+a_1^2+a_3^2+a_1^2+a_2^2 = 2\left(a_1^2+a_2^2+a_3^2\right)$

$= 2|\vec{a}|^2 = 2a^2$

70. **(b)** given $\vec{a}=\hat{i}+\hat{k},\ \vec{b}=\hat{i}-\hat{k}$

Hence the unit vector perpendicular to $\vec{a}$ and $\vec{b}$ is

$\hat{n}=\dfrac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

$\because\quad \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & 1\\ 1 & 0 & -1\end{vmatrix}=2\hat{j}$

$|\vec{a}\times\vec{b}|=|2\hat{j}|=2.$

$\therefore\quad \hat{n}=\dfrac{2\hat{j}}{2}=\hat{j}$ **[1 Mark]**

71. **(c)** $\vec{a}.\vec{b}=\vec{b}.\vec{a}$

Assertion is true.

$\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$

Reason is false **[1 Mark]**

Chapter 11 : Dimensional Geometry

Topic-1: Direction Cosines and Direction Ratios of a Line

72. **(d)** Let the coordinates of foot of perpendicular on x-axis be (a, 0, 0)

$\therefore$ direction ratios of the line joining points (a, 0, 0) and (0, 1, 2) will be a, –1, –2 since this line is perpendicular to x-axis hence $a\,.\,1+(-1)(0)+(-2)(0)=0 \Rightarrow a=0$

$\therefore$ Required coordinate is (0, 0, 0) **[1 Mark]**

73. **(a)** $\alpha=30°, \beta=120°$

We know that,

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow\quad \cos^2 30°+\cos^2 120+\cos^2\gamma=1$

$\Rightarrow\quad \left(\dfrac{\sqrt{3}}{2}\right)^2+\sin^2 30°+\cos^2\gamma=1$

$\Rightarrow\quad \dfrac{3}{4}+\dfrac{1}{4}+\cos^2\gamma=1$

$\Rightarrow\quad \cos^2\gamma=1$

$\Rightarrow\quad \cos^2\gamma=0$

$\Rightarrow\quad \cos\gamma=0 \qquad \Rightarrow\quad \gamma=90°$ **[1 Mark]**

74. **(d)** The given equation of line is:

$\dfrac{x-1}{2}=-y=\dfrac{2z+1}{6}\Rightarrow\dfrac{x-1}{2}=\dfrac{y}{-1}=\dfrac{z+\frac{1}{2}}{3}$

$\therefore$ Direction ratios of the given line is (2, –1, 3)

So, the direction ratios of a vector parallel to given line 2, [–1], 3. **[1 Mark]**

Topic-2: Equation of a Line in Space

75. **(b)** Here $\vec{a}=\hat{i}-\hat{j}\Rightarrow$ Point (1, –1, 0)

and $\vec{b}=2\hat{i}-\hat{j}\Rightarrow \ell=2, m=-1, n=0$

$\therefore$ Equation of straight line

$\dfrac{x-1}{2}=\dfrac{y+1}{-1}=\dfrac{z}{0}$ **[1 mark]**

76. Vector perpendicular to the two lines is

$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -16 & 7\\ 3 & 8 & -5\end{vmatrix}=24\hat{i}+36\hat{j}+72\hat{k}$ **[2 Marks]**

DR's of required lines are

$<24, 36, 72>$ or $<2, 3, 6>$

Mid Point of AB is (3, 4, 6)

Equation of the required line are

$\dfrac{x-3}{2}=\dfrac{y-4}{3}=\dfrac{z-6}{6}$ **[3 Marks]**

Topic-3: Angle between Two Lines

77. **(c)** $\dfrac{1-x}{2}=\dfrac{y-1}{3}=\dfrac{z}{1}\Rightarrow\dfrac{x-1}{-2}=\dfrac{y-1}{3}=\dfrac{z}{1}$

Here, $l_1=-2,\ m_1=3, n_1=1$

$\dfrac{2x-3}{2p}=\dfrac{y}{-1}=\dfrac{z-4}{7}\Rightarrow\dfrac{x-3/2}{p}=\dfrac{y}{-1}=\dfrac{z-4}{7}$

Here, $l_2=p, m_2=-1, n_2=7$

Since, both are perpendicular.

$\therefore\quad -2.p-3+7=0\Rightarrow p=2$ **[1 mark]**

Note : If two lines are perpendicular then $l_1l_2+m_1m_2+n_1n_2=0$

78. The lines are perpendicular

$-3\times 3k+2k\times 1-2\times 7=0$

$-7k=14\Rightarrow k=-2$

DR's of $\ell_1=(-3, -4, 2)$

DR's of $\ell_2 = (-6, 1, -7)$ **[2 marks]**

Vector perpendicular to ℓ_1 & ℓ_2 is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -4 & 2 \\ -6 & 1 & -7 \end{vmatrix}$

$= 26\hat{i} - 33\hat{j} - 27\hat{k}$ **[2 marks]**

Equation of required line is

$\vec{r} = (3\hat{i} - 4\hat{j} + 7\hat{k}) + \lambda(26\hat{i} - 33\hat{j} - 27\hat{k})$ **[1 mark]**

Note : A vector perpendicular to two vector $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b}$.

Topic-4: Shortest Distance between Two Lines

79. $\ell : \frac{x}{2} = \frac{y-3}{2} = \frac{z-1}{1} = \lambda$

Distance between the two parallel lines is equal to distance of Point (4, 0, –5) from the line ℓ.

A Point on ℓ is $(2\lambda, 2\lambda + 3, \lambda + 1)$ **[1 mark]**

$< 2\lambda - 4, 2\lambda + 3, \lambda + 6 >$ are the directional ratios of line perpendicular to ℓ.

$2(2\lambda - 4) + 2(2\lambda + 3) + \lambda + 6 = 0 \Rightarrow 9\lambda + 4 = 0$

$\lambda = -\frac{4}{9}$

Foot of perpendicular from (4, 0, –5) on line is

$\left(-\frac{8}{9} - 4, \frac{-8}{9} + 3, 6 - \frac{4}{9}\right)$

$= \left(-\frac{44}{9}, \frac{19}{9}, \frac{50}{9}\right)$ **[1 mark]**

Required distance $= \sqrt{\left(4 + \frac{44}{9}\right)^2 + \left(\frac{-19}{9}\right)^2 + \left(-5 - \frac{50}{9}\right)^2}$

$= \sqrt{\left(\frac{80}{9}\right)^2 + \left(\frac{19}{9}\right)^2 + \left(\frac{95}{9}\right)^2} \approx 14$ **[1 mark]**

Chapter 12 : Linear Programming

Topic-1: Linear Programming Problem and its Mathematical Formulation

80. (c)

Corner Points	Value of $z = 4x + y$
A (0, 50)	$z = 50$
B (20, 30)	$z = 80 + 30 = 110$
C (30, 0)	$z = 120$ (Maximum)
0 (0, 0)	$z = 0$

[1 mark]

81. (d) a feasible region **[1 Mark]**

82. (b)

Corner Point	$Z = x + 2y$
(40, 20)	80
(60, 30)	120 (maximum value)
(120, 0)	120 (maximum value)
(60, 0)	60

Since maximum value occurs at two adjacent corner points, maximum value occurs at all point on line segment joining those two points.

Assertion is true.

Reason is also true but is not correct explanation of Assertion. **[1 mark]**

83.

Corner Point	$Z = 3x - 4y$
(0, 8)	–32
(4, 10)	–28
(6, 8)	–14
(6, 5)	–2
(4, 0)	$12 \rightarrow$ Maximum
(0, 0)	0

$Z_{max} = 12$ **[1 mark]**

(ii) The value of Z at B is same as the value of Z at C.

$4p + 10q = 6p + 8q$

$2p = 2q$

$p = q$ **[2 marks]**

84. (i) The constraints are

$x + 2y > 10$

$x + y > 6$

$3x + y > 8$ **[2 Marks]**

(ii) $Z = 16x + 20y$

Corner Points	**Value of Z**
(0, 8)	160
(1, 5)	116
(2, 4)	112 -minimum
(10, 0)	160

for minimum cost $x = 2, y = 4$

Minimum cost is ₹ 112.

Chapter 13 : Probability

Topic-1: Conditional Probability

85. **(c)** $P(S|E) = \frac{P(S \cap E)}{P(E)}$

$= \frac{P(E)}{P(E)} \{\because E \in S\}$

$= 1$ **[1 Mark]**

Topic-2: Multiplication Theorem on Probability and Independent Events

86. **(i)** Probability that none of

then is selected $= \frac{4}{5} \times \frac{2}{3} \times \frac{3}{4} = \frac{2}{5}$

Probability that at least one is selected is

$1 - \frac{2}{5} = \frac{3}{5}$ **[1 mark]**

(ii) $P\left(\frac{G}{\overline{H}}\right) = \frac{P(G \cap \overline{H})}{P(\overline{H})} = \frac{P(G).P(\overline{H})}{P(\overline{H})} = P(G)$

$= \frac{1}{3} = P(G)$ [events are a independent] **[1 mark]**

(iii) Let Probability of Alia's selection be P(A).

Required probability $= P(A).P(\overline{G}).P(\overline{H}) +$

$P(\overline{A}).P(G).P(\overline{H}) + P(\overline{A}).P(\overline{G}).P(H)$

$= \frac{1}{4} \times \frac{2}{3} \times \frac{4}{5} + \frac{3}{4} \times \frac{1}{3} \times \frac{4}{5} + \frac{3}{4} \times \frac{2}{3} \times \frac{1}{5}$

$= \frac{(8+12+6)}{4 \times 3 \times 5} = \frac{26}{4 \times 3 \times 5} = \frac{13}{30}$ **[2 marks]**

OR

(iii) Probability that exactly two are selected is

$P(A).P(G).P(\overline{H}) + P(\overline{A}).P(G).P(H) + P(A).P(\overline{G}).P(H)$

$= \frac{1}{4} \times \frac{1}{3} \times \frac{4}{5} + \frac{3}{4} \times \frac{1}{3} \times \frac{1}{5} + \frac{1}{4} \times \frac{2}{3} \times \frac{1}{5}$

$= \frac{9}{4 \times 3 \times 5} = \frac{3}{20}$ **[2 marks]**

Topic-3: Bayes' Theorem

87. Let E_1 be the event that lost card is king.

$P(E_1) = \frac{4}{52} = \frac{1}{13}$ **[1 mark]**

$P(\overline{E}_1) = \frac{12}{13}$.

Let A be the event that a king is drawn.

$P\left(\frac{A}{E_1}\right) = \frac{3}{51}$ & $P\left(\frac{A}{\overline{E}_1}\right) = \frac{4}{51}$ **[1 mark]**

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(\overline{E}_1) \cdot P\left(\frac{A}{\overline{E}_1}\right)}$$

$$= \frac{\frac{1}{13} \times \frac{3}{51}}{\frac{1}{13} \times \frac{3}{51} + \frac{12}{13} \times \frac{4}{51}} = \frac{3}{3+48}$$

$= \frac{3}{51} = \frac{1}{17}$ **[1 mark]**

Topic-4: Random Variable and its Probability Distributions , Mean of Random Variable

88. **(b)** $\because \Sigma P(X) = 1$

$\therefore \quad 0.1 + K + 2K + K + 0.1 = 1 \Rightarrow K = 0.2$

Now, $P(2) = 2K = 2 \times 0.2 = 0.4 = \frac{2}{5}$ **[1 mark]**

89. Total number of out comes = 6 × 6 = 36

Let x denote the absolute difference of numbers appearing.

For x = 0 out comes are

(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

$P(X = 0) = \frac{6}{36} = \frac{1}{6}$

For x = 1 Possible out comes are

(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 5) (5, 4), (5, 6), (6, 5)

$P(x = 1) = \frac{10}{36} = \frac{5}{18}$

For (x = 2) Possible out comes

(1, 3) (3, 1), (2, 4), (4, 2), (5, 3), (3, 5), (4, 6), (6, 4)

$$P(X=2)=\frac{8}{36}=\frac{2}{9}$$ **[1 Mark]**

for (X = 3) possible out comes are
(1, 4), (4, 1), (2, 5), (5, 2), (3, 6), (6, 3)

$$P(X=3)=\frac{6}{36}=\frac{1}{6}$$ **[1 Mark]**

For X = 4 possible out comes are
(1, 5), (5, 1), (2, 6), (6, 2)

$$P(X=4)=\frac{4}{36}=\frac{1}{9}$$

For X = 5 out comes are
(1, 6), (6, 1)

$$P(X=5)=\frac{2}{36}=\frac{1}{18}$$

X	0	1	2	3	4	5
P(X)	1/6	5/18	2/9	1/6	1/9	1/18

[1 Mark]

90. 4

Outcome	1	2	3	4	5	6
Probability	P	2P	P	2P	P	2P

$$9P=1 \Rightarrow P=\frac{1}{9}$$ **[1 mark]**

Probability of number appearing is 6, is, $2P=\frac{2}{9}$

Probability of not getting six $=\frac{7}{9}$

$$P(X=0)=\frac{7}{9}\times\frac{7}{9}=\frac{49}{81}$$

$$P(X=1)=\frac{2}{9}\times\frac{7}{9}\times 2=\frac{28}{81}$$

$$P(X=2)=\frac{2}{9}\times\frac{2}{9}=\frac{4}{81}$$

X	0	1	2
P(X)	$\frac{49}{81}$	$\frac{28}{81}$	$\frac{4}{81}$

[2 marks]

Note : If x is a random variable then $\sum P(xi)=1$

91. $P(E_1)=10^{-7}$
$P(E_2)=1-10^{-7}$

(i) Probability that the airplane will not crash
$=P(E_2)=0.9999999$ **[1 Mark]**

(ii) $P(A/E_1)=0.95;\ P(A/E_2)=1$ **[1 Mark]**
$P(A/E_1)+P(A/E_2)=1.95$

(iii) (a) $$P(A)=P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)$$

$$=10^{-7}\times 0.95+\left(1-10^{-7}\right)\times 1$$
$$=9.5\times 10^{-8}+1-10^{-7}$$
$$=1-0.05\times 10^{-7}=1-5\times 10^{-9}$$ **[2 Marks]**

(b) $$P(E_2/A)=\frac{P(A/E_2)\times P(E_2)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)}$$

$$=\frac{(1-10^{-7})\times 1}{10^{-7}\times 0.95+(1-10^{-7})\times 1}$$

$$=\frac{1-10^{-7}}{1-0.05\times 10^{-7}}=\frac{1-10^{-7}}{1-5\times 10^{-9}}\simeq 1.$$ **[2 Marks]**