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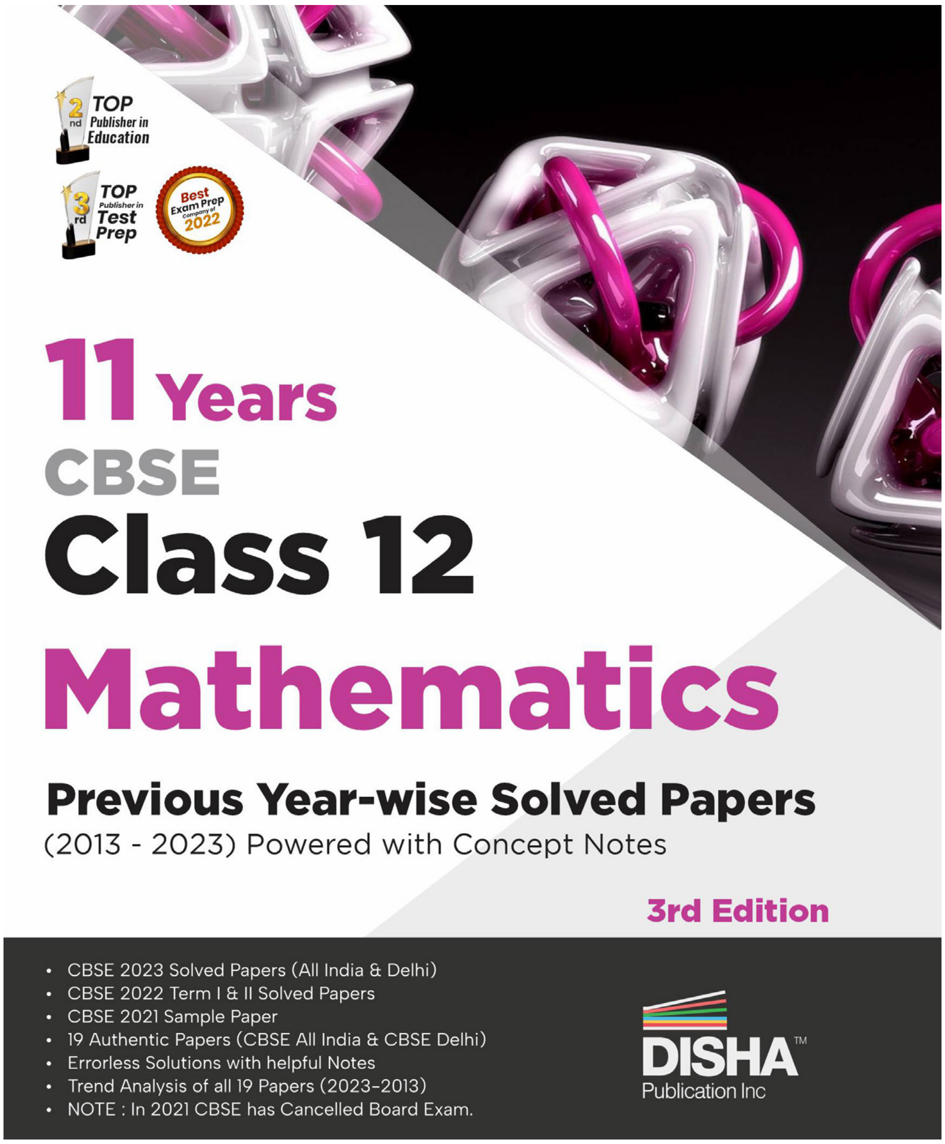
All India 2023 Solved Paper

This sample is taken from the **"11 Years CBSE Class 12 Mathematics Previous Year-wise Solved Papers (2013 - 2023) powered with Concept Notes 3rd Edition | Previous Year Questions PYQs"**



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All India 2023

CBSE Board Solved Paper

Time Allowed : 3 Hours ***Maximum Marks : 80***

General Instructions:

(i) This question paper contains **38** questions. **All** questions are compulsory.
(ii) Question paper is divided into **Five** Sections - Sections **A, B, C, D** and **E**.
(iii) In Section **A** - Question Number **1** to **18** are Multiple Choise Questions (MCQ) type and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.
(iv) In Section **B** - Question Number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.
(v) In Section **C** - Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
(vi) In Section **D** - Question Number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.
(vii) In Section **E** - Question Number **36** to **38** are case study based questions carrying **4** marks each where **2 VSA** type questions are of **1** mark each and **1 SA** type question is of **2** marks. Internal choice is provided in **2** marks question in each case study.
(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section - **B**, **3** questions in Section - **C**, **2** questions in Section - **D** and **2** questions in Section - **E**.
(ix) Use of calculators is **NOT** allowed.

SECTION - A

Select the correct option out of the four given options:

1. If A is a 3×4 matrix and B is a matrix such that A'B and AB' are both defined, then the order of the matrix B is:

(a) 3×4 (b) 3×3 (c) 4×4 (d) 4×3

2. If the area of a triangle with vertices (2, – 6), (5, 4) and (k, 4) is 35 sq. units, then k is:

(a) 12 (b) –2 (c) –12, –2 (d) 12, –2

3. If $f(x) = 2|x| + 3|\sin x| + 6$, then the right hand derivative of f(x) at x = 0 is:

(a) 6 (b) 5 (c) 3 (d) 2

4. If $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$, then :

(a) x = 1, y = 2 (b) x = 2, y = 1

(c) x = 1, y = –1 (d) x = 3, y = 2

5. If a matrix A = [1 2 3], then the matrix AA' (where A' is the transpose of A) is:

(a) 14 (b) $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix}$ (d) [14]

6. The product $\begin{bmatrix}a & b\\-b & a\end{bmatrix}\begin{bmatrix}a & -b\\b & a\end{bmatrix}$ is equal to:

(a) $\begin{bmatrix}a^2+b^2 & 0\\0 & a^2+b^2\end{bmatrix}$ (b) $\begin{bmatrix}(a+b)^2 & 0\\(a+b)^2 & 0\end{bmatrix}$

(c) $\begin{bmatrix}a^2+b^2 & 0\\a^2+b^2 & 0\end{bmatrix}$ (d) $\begin{bmatrix}a & 0\\0 & b\end{bmatrix}$

7. Distance of the point (p, q, r) from y-axis is :

(a) q (b) |q| (c) |q| + |r| (d) $\sqrt{p^2 + r^2}$

8. The solution set of the inequation $3x + 5y < 7$ is:

(a) whole xy-plane except the points lying on the line $3x + 5y = 7$.

(b) whole xy-plane along with the points lying on the line $3x + 5y = 7$.

(c) open half plane containing the origin except the points of line $3x + 5y = 7$.

(d) open half plane not containing the origin.

9. If $\int_0^a 3x^2 dx = 8$, then the value of 'a' is:

(a) 2 (b) 4 (c) 8 (d) 10

10. The sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ is:

(a) $\sqrt{\frac{5}{21}}$ (b) $\frac{5}{\sqrt{21}}$ (c) $\sqrt{\frac{3}{21}}$ (d) $\frac{4}{\sqrt{21}}$

11. The order and degree (if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x\sin\left(\frac{dy}{dx}\right)$ respectively are:

(a) 2, 2 (b) 1, 3
(c) 2, 3 (d) 2, degree not defined

12. $\int e^{5\log x} dx$ is equal to:

(a) $\frac{x^5}{5} + C$ (b) $\frac{x^6}{6} + C$
(c) $5x^4 + C$ (d) $6x^5 + C$

13. A unit vector along the vetor $4\hat{i} - 3\hat{k}$ is:

(a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$ (b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$
(c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$ (d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

14. Which of the following points satisfies both the inequations $2x + y \le 10$ and $x + 2y \ge 8$?

(a) (–2, 4) (b) (3, 2) (c) (–5, 6) (d) (4, 2)

15. If $y = \sin^2(x^3)$, then $\frac{dy}{dx}$ is equal to:

(a) $2\sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$
(c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2(x^3)$

16. The point (x, y, 0) on the xy-plane divides the line segment joining the points (1, 2, 3) and (3, 2, 1) in the ratio:

(a) 1 : 2 internally (b) 2 : 1 internally
(c) 3 : 1 internally (d) 3 : 1 externally

17. The events E and F independent. If P(E) = 0.3 and $P(E \cup F) = 0.5$, then P(E/F) – P(F/E):

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{35}$ (d) $\frac{1}{70}$

18. The integrating factor for solving the differential equation $x\frac{dy}{dx} - y = 2x^2$ is:

(a) e^{-y} (b) e^{-x} (c) x (d) $\frac{1}{x}$

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

19. **Assertion (A):** The lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are perpendicular, when $\vec{b_1}.\vec{b_2} = 0$.

Reason (R): The angle θ between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by $\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$.

20. **Assertion (A):** All trigonometric functions have their inverses over their respective domains.

Reason (R): The inverse of $\tan^{-1}x$ exists for some $x \in R$.

SECTION - B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. If $xy = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$

22. (a) Find the domain of $y = \sin^{-1}(x^2 - 4)$.

OR

(b) Evaluate:

$$\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$$

23. If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.

24. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.

25. (a) Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines.

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}; \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}.$$

OR

(b) The equations of a line are 5x -3 = 15y + 7 = 3 - 10z. Write the direction cosines of the line and find the coordinates of a point through which it passes.

SECTION - C

The section comprises of Short Answer (SA) type questions of 3 marks each.

26. Find $\int \frac{2}{(1-x)(1+x^2)}dx$.

27. (a) Evaluate $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}dx$.

OR

(b) Evaluate: $\int_{1}^{3}\{|(x-1)|+|(x-2)|\}dx$

28. Solve the following linear programming problem graphically:

Maximise $z = 5x + 3y$

subject to the constraints

$3x + 5y \le 15$,

$5x + 2y \le 10$,

$x, y \ge 0$.

29. From a lot of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.

30. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{x+y}{x}, y(1) = 0$$

OR

(b) Find the general solution of the differfntial equation

$$e^x \tan y dx + (1-e^x)\sec^2 y\, dy = 0$$

31. (a) Evaluate: $\int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

OR

(b) Evaluate $\int_{-2}^{2}\frac{x^2}{1+5^x}dx$

SECTION - D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. (a) Find the image of the point (2, – 1, 5) in the line

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$$

OR

(b) Vertices B and C of ΔABC lie on the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z}{4}$. Find the area of ΔABC given that point A has coordinates (1, – 1, 2) and the line segment BC has length of 5 units.

33. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse, A^{-1}, solve the sysyem of linear equations

x-y+ 2z -1; 2y -3z =1; 3x -2y +4z =3

34. Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

35. (a) If N denotes the set of all natural numbers and R is the relation on N × N defined by (a,b,) R (c,d), if ad(b+c) = bc(a + d). Show that R is an equivalence relation.

OR

(b) Let $f : R - \left\{-\frac{4}{3}\right\} \to R$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is one -one function. Also, check whether f is an onto function or not.

SECTION - E

This section comprises of 3 Case Study/Passage-Bassed questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub – parts (I) and (II) of marks 2 each.

Case Study-I

36. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let: E_1 : represent the events when many workers were not present for the job;

E_2 : represent the events when all workers were present; and

E : represent completing the condtruction work on time.

Based on the above information, answer the following questions:

(i) What is the probability that all the workers are present for the job ?

(ii) What is the probability that construction will be completed on time?

(iii) (a) What is the probability that many workers are not present given that the costruction work is completed on time ?

OR

(IV) (b) What is the probability that all workers were present given that the construction job was completed on time?

Case Study - II

37. Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) :

$$Lf'(a) = \lim_{h \to 0} \frac{f(a-h)-f(a)}{-h}$$

- Right Hand Derivative (R.H.D) :

$$Rf'(a) = \lim_{h \to 0} \frac{f(a+b)-f(a)}{h}$$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$

answer the following questions:

(i) What is R.H.D. of f(x) at x = 1

(ii) What is L.H.D. of f(x) at x = 1

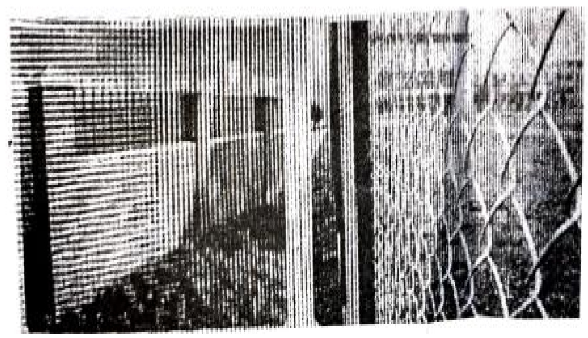
(iii) (a) Check if the function f(x) is differentiable at x =1

OR

(iii) (b) Find the f'(2) and f'(-1)

Case Study-III

38. Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 meters of fencing wire.



Based on the above information, answer the following questions;

(i) Let 'x' meters denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x) the area of the garden.

(ii) Determine the maximum value of A(x)

Solutions

1. **(a)** Order of matrix A is 3×4

Let order of B is $m \times n$ them B′ is $n \times m$ and A′ is 4×3

$\Rightarrow$ Number of columns of A′ must equal number of rows of B, because A′ B is defined, so m = 3

Also (B A′) is defined that is 2 why n = 4 **(1 mark)**

2. **(d)** Area of triangle having vertices (2,– 6), (5, 4),

(k, 4) is $A = \frac{1}{2}\begin{bmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{bmatrix} = 35$

$\Rightarrow A = \frac{1}{2}\left|\left[2(4-4)+6(5-k)+20-4k\right]\right| = 35$

$\Rightarrow |30-6k+20-4k| = 70$

$\Rightarrow 50-10k = \pm 70$

$\Rightarrow 50-10k = -70; 50-10k = 70$

$\Rightarrow k = \frac{-20}{10} = -2, k = \frac{120}{10} = 12$ **(1 mark)**

3. **(b)** Let x = 0 + h, where $h \to 0$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{2|h|+3|\sin h|+6-6}{h}$

for h > 0

$\Rightarrow f'(x) = \lim_{h\to 0}\left(\frac{2h}{h}+\frac{3\sin h}{h}\right) = 5$ **(1 mark)**

4. **(b)** Given, $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$

We can write given equation as,

$\Rightarrow x+2y=4$ (1)

and, $2x+5y=9$ (2)

By multiplying 2 in equation (1)

$$\begin{array}{l} 2x+4y=8 \\ 2x+5y=9 \\ -\ \ -\ \ \ \ - \\ \hline -y=-1 \Rightarrow y=1 \end{array}$$

Put y=1 in equation (i)

$\Rightarrow x+2=4 \quad \Rightarrow x=2$ **(1 mark)**

5. **(a)** Given, $A = [1 \ \ 2 \ \ 3]$

Then, $A' = \begin{bmatrix}1\\2\\3\end{bmatrix}$

$\Rightarrow AA' = [1 \ \ 2 \ \ 3]\begin{bmatrix}1\\2\\3\end{bmatrix}$

$\Rightarrow AA' = [1+4+9] = 14$ **(1 mark)**

6. **(a)** Product of given matrix define as.

$\Rightarrow \begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$ **(1 mark)**

7. **(d)** Let point on y-axis is (0, q, 0) that is nearest from (p. q, r)

Distance (d) = $\sqrt{p^2+(q-q)^2+r^2} = \sqrt{p^2+r^2}$ **(1 mark)**

Note

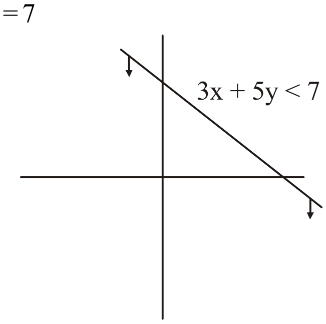
If A ≡ (x_1, y_1, z_1) and B ≡ (x_2, y_2, z_2)

then $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

8. **(c)** Given inequation, $3x+5y<7$

solution defines in open half plane containing the origin except the points of line

$3x+5y=7$



(1 mark)

9. **(a)** Given, $\int_0^a 3x^2 dx = 8$

By integrating the equation,

$\Rightarrow 3\left[\frac{x^3}{3}\right]_0^a = 8$

$\Rightarrow a^3 = 8 \Rightarrow a = 2$ **(1 mark)**

10. **(a)** Let Angle between $\vec{a}$ and $\vec{b}$ is θ

Then, $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{(3\hat{i}+\hat{j}+2\hat{k})(\hat{i}+\hat{j}+2\hat{k})}{\sqrt{3^2+1^2+2^2}\sqrt{1^2+1^2+2^2}}$

$\Rightarrow \cos\theta = \frac{3+1+4}{\sqrt{14}\sqrt{6}} = \frac{8}{\sqrt{7}\sqrt{2}\sqrt{3}\sqrt{2}} = \frac{4}{\sqrt{21}}$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{16}{21}} = \sqrt{\frac{5}{21}}$ **(1 mark)**

11. **(d)** Given differential equation is not in polynomial form Order = 2, but degree is not defined **(1 mark)**

12. **(b)** Given, $\int e^{5\log x}dx$

where, $e^{5\log x} = e^{\log x^5} = x^5$ $\left[\because e^{\log_e^x} = x\right]$

Now, $\int e^{5\log x}dx = \int x^5 dx$

By integrating the given equation.

$\Rightarrow \frac{x^6}{6} + C$ **(1 mark)**

13. **(b)** Given, $4\hat{i} - 3\hat{k}$

Unit vector $= \frac{4\hat{i}-3\hat{k}}{\sqrt{4^2+3^2}}$

$\Rightarrow \frac{4\hat{i}}{5} - \frac{3\hat{k}}{5} = \frac{1}{5}(4\hat{i}-3\hat{k})$ **(1 mark)**

14. **(d)** At (–2, 4)

$\Rightarrow 2x + y \le 10 \Rightarrow -4+4 \le 10 \Rightarrow 0 \le 10$

$\Rightarrow x + 2y \ge 8 \Rightarrow -2+8 \ge 8 \Rightarrow 6 \not\ge 8$

At (3, 2)

$\Rightarrow 2x + y \le 10 \Rightarrow 6+2 \le 10 \Rightarrow 8 \le 10$

$\Rightarrow x + 2y \ge 10 \Rightarrow 3+4 \ge 10 \Rightarrow 7 \not\ge 10$

At (–5, 6)

$\Rightarrow 2x + y \le 10 \Rightarrow -10+6 \le 10 \Rightarrow -4 \le 10$

$\Rightarrow x + 2y \ge 8 \Rightarrow -5+12 \ge 8 \Rightarrow 7 \not\ge 8$

At (4, 2)

$\Rightarrow 2x + y \le 10 \Rightarrow 8+2 \le 10$

$\Rightarrow 10 \le 10 \Rightarrow x + 2y \ge 8$

$\Rightarrow 4+4 \ge 8 \Rightarrow 8 \ge 8$ **(1 mark)**

15. **(c)** Given, $y = \sin^2(x^3)$

By Chain Rule differentiation the above equation

$\Rightarrow \frac{dy}{dx} = 2\sin(x^3)\frac{d}{dx}\left(\sin(x^3)\right)$

$\Rightarrow \frac{dy}{dx} = 2\sin(x^3)\frac{d}{dx}(x^3)$

$\Rightarrow \frac{dy}{dx} = 2\sin(x^3)\ \cos(x^3)\cdot(3x^2)$

$= 6x^2 \sin x^3 . \cos x^3$ **(1 mark)**

16. **(c)** Let (x, y, 0) divides the point in k : 1

Then, $x = \frac{3k-1}{k+1}, y = \frac{2k-2}{k+1}, z = \frac{k-3}{k+1}$

In (x, y, 0), z = 0

$\Rightarrow \frac{k-3}{k+1} = 0 \Rightarrow k = 3$

So, (x, y, 0) divides the line segment in ratio 3 : 1 internally. **(1 mark)**

17. **(d)** The events E and F are independent

so, $P(E\cap F) = P(E)\cdot P(F) = (0.3)(P(F))$

We know, $P(E\cup F) = P(E) + P(F) - P(E\cap F)$

$\Rightarrow 0.5 = 0.3 + P(F) - (0.3)(P(F))$

$\Rightarrow (0.7)P(F) = 0.2 \Rightarrow P(F) = \frac{2}{7}$

Now $P\left(\frac{E}{F}\right)$=P(E) and $P\left(\frac{F}{E}\right)$= P(E) because E & F are independent.

$\Rightarrow P(E) - P(F) = 0.3 - \frac{2}{7} = \frac{1}{70}$ **(1 mark)**

> ***Note***
>
> If E_1 & E_2 are independent events then
> $P(E_1 \cap E_2) = P(E_1)\cdot P(E_2)$

18. **(d)** Given, $x\frac{dy}{dx} - y = 2x^2$

or, $\frac{dy}{dx} - \frac{y}{x} = 2x$

Integrating factor = $e^{-\int \frac{1}{x}dx}$

$\Rightarrow e^{-(\log x)} = e^{\log\frac{1}{x}} = \frac{1}{x}$ **(1 mark)**

19. **(a)** Given Asseration is true and Reason is correct explanation of Asseration.

$\Rightarrow \cos\theta = \frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$

If $\vec{b}_1$ and $\vec{b}_2$ are perpendicular then

$\Rightarrow \cos 90^o = \frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|} = 0 \Rightarrow \vec{b}_1\cdot\vec{b}_2 = 0$ **(1 mark)**

20. **(d)** Assertion is not true for all trigonometric function. Domain of sin x is R but $\sin^{-1}x$ is not define on R

$\Rightarrow$ Reason is true $\tan^{-1}x$ inverse exist for some $x \in R$

(1 mark)

21. Given, $xy = e^{x-y} = e^x \cdot e^{-y}$

Differentiate above function w.r.t. x

$$\Rightarrow x\frac{dy}{dx} + y = e^x \frac{d}{dx}e^{-y} + e^{-y}\frac{d}{dx}e^x$$

$$\Rightarrow x\frac{dy}{dx} + y = -e^x e^{-y}\frac{dy}{dx} + e^x e^{-y}$$

$$\Rightarrow x\frac{dy}{dx} + y = -xy\frac{dy}{dx} + xy$$ **(1 mark)**

$$\Rightarrow (x + xy)\frac{dy}{dx} = -y + xy$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$$ **(1 mark)**

22. **(a)** Domain of $\sin^{-1}x$ is $[-1, 1]$

So, domain of $\sin^{-1}(x^2 - 4)$ is **(1 mark)**

$$\Rightarrow -1 \le x^2 - 4 \le 1$$

$$\Rightarrow 3 \le x^2 \le 5$$

$$\Rightarrow \sqrt{3} \le |x| \le \sqrt{5}$$ **(1 mark)**

OR

(b) $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$

$$\Rightarrow \cos^{-1}\left[\cos\left(-2\pi - \frac{\pi}{3}\right)\right]$$ **(1 mark)**

$$= \cos^{-1}\cos\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$$ **(1 mark)**

23. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = P\hat{i} + \hat{j} - 2\hat{k}$

Then projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$\Rightarrow \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{(\hat{i}+\hat{j}+\hat{k})(P\hat{i}+\hat{j}-2\hat{k})}{\sqrt{P^2+1^2+2^2}} = \frac{1}{3}$$ **(1 mark)**

$$\Rightarrow \frac{P+1-2}{\sqrt{P^2+5}} = \frac{1}{3}$$

$\Rightarrow P = 2$ **(1 mark)**

Note

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then $\vec{a}\cdot\vec{b} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})\cdot(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

$= a_1b_1 + a_2b_2 + a_3b_3$

24. Given, $y^2 = 8x$

Derivative of given equation, w.r.t. x

$$\Rightarrow 2y\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{8}{2y} = 1$$

$\Rightarrow 2y = 8 \Rightarrow y = 4$ **(1 mark)**

or derivative w.r.t. y is

$$2y = 8\frac{dx}{dy} \Rightarrow 2y = 8 \Rightarrow y = 4$$

At $y = 4$, $16 = 8x \Rightarrow x = 2$ **(1 mark)**

25. **(a)** Let the required line Parallel to the vector $\vec{b}$

$\vec{b}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

The position vector of (2, 1, 3) & parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r}(2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$ ...(i)

Given line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ ...(ii)

$\Rightarrow \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$ ...(iii)

are $\perp$ to each other

$\therefore b_1 + 2b_2 + 3b_3 = 0$ ...(iv)

(1 mark)

lines (i) and (iii) are $\perp$ to each other

$\therefore -3b_1 + 2b_2 + 5b_3 = 0$ ...(v)

Then

$$\frac{b_1}{2(5)-3(2)} = \frac{b_2}{5-3(-3)} = \frac{b_3}{2-2(-3)}$$

$$\Rightarrow \frac{b_1}{2} = \frac{-b_2}{7} = \frac{b_3}{4}$$

Direction ratios of $\vec{b}$ are 2, –7, 4

Then required equation is

$\vec{r}(2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$ **(1 mark)**

OR

(b) Given, $5x - 3 = 15y + 7 = 3 - 10z$

$$\Rightarrow \frac{x - \frac{3}{5}}{\frac{1}{5}} = \frac{y - \left(-\frac{7}{15}\right)}{\frac{1}{15}} = \frac{z - \frac{3}{10}}{-\frac{1}{10}}$$

Compare to $\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3}$

Then, $b_1 = \frac{1}{5}, b_2 = \frac{1}{15}, b_3 = \frac{-1}{10}$ **(1 mark)**

The direction cosines are the components of unit vector

$$\hat{b} = \frac{b_1\hat{i} + b_2\hat{j} + b_3\hat{k}}{\sqrt{b_1^2 + b_2^2 + b_3^2}} = \frac{\frac{1}{5}\hat{i} + \frac{1}{15}\hat{j} - \frac{1}{10}\hat{k}}{\frac{7}{30}}$$

$\Rightarrow \quad \frac{6}{7}\hat{i}+\frac{2}{7}\hat{j}-\frac{3}{7}\hat{k}$

Direction cosines are $\frac{6}{7},\frac{2}{7},-\frac{3}{7}$ **(1 mark)**

26. Let $I=\int\frac{2}{(1-x)(1+x^2)}dx$

$\Rightarrow \frac{-2}{(x-1)(1+x^2)}=\frac{A}{(x-1)}+\frac{Bx+C}{1+x^2}$

$\Rightarrow -2=A(1+x^2)+(Bx+C)(x-1)$ **(1 mark)**

On comparing, we get

$A+B=0$ ...(i)

$-B+C=0$ ...(ii)

$A-C=-2$ ...(iii)

On solving equations (i), (ii) and (iii), we get

$A=-1, B=1, C=1$

So, $\int\frac{-2dx}{(x-1)(1+x^2)}=\int\frac{-1}{x-1}dx+\int\frac{x+1}{x^2+1}dx$ **(1 mark)**

$\Rightarrow -\log|x-1|+\frac{1}{2}\int\frac{2xdx}{x^2+1}+\int\frac{dx}{x^2+1}$

$I=-\log|x-1|+\frac{1}{2}\log(x^2+1)+\tan^{-1}x+C$ **(1 mark)**

$\left[\because \int\frac{f'(x)}{f(x)}dx=\log f(x)\right]$

> ***Note***
>
> $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$ *can be written as*
>
> $\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+C}$

27. (a) Let $I=\int_{1/3}^{1}\frac{(x-x^3)1/3}{x^4}dx$

$\Rightarrow \int_{1/3}^{1}\frac{x\left(\frac{1}{x^2}-1\right)^{1/3}}{x^4}dx=\int_{1/3}^{1}\frac{\left(\frac{1}{x^2}-1\right)^{1/3}}{x^3}dx$ **(1 mark)**

Let $\left(\frac{1}{x^2}-1\right)=t$, then, $-\frac{2}{x^3}dx=dt$

$\Rightarrow \frac{1}{2}\int -t^{1/3}dt=\frac{-3t^{4/3}}{4}\times\frac{1}{2}$

$\Rightarrow I=\frac{-3}{8}\left[\left(\frac{1}{x^2}-1\right)^{4/3}\right]_{1/3}^{1}$ **(1 mark)**

or $I=\frac{-3}{8}\left[0-(8)^{4/3}\right]=\frac{3}{8}\left(2^3\right)^{4/3}$

$\Rightarrow I=\frac{3}{8}\times 16=6$ **(1 mark)**

OR

(b) $I=\int_1^3\{|x-1|+|x-2|\}dx$

In given limit $|x-1|=(x-1)$

And, $|x-2|=\begin{cases}-(x-2), & x<2\\ x-2, & x>2\end{cases}$ **(1 mark)**

$\Rightarrow I=\int_1^2[(x-1)-(x-2)]dx+\int_2^3[(x-1)+(x-2)]dx$

$\Rightarrow I=\int_1^2 dx+\int_2^3(2x-3)dx$ **(1 mark)**

$\Rightarrow I=[x]_1^2+\left[x^2-3x\right]_2^3$

$\Rightarrow I=(2-1)+[0-(4-6)]$

$\Rightarrow I=1+2=3$ **(1 mark)**

28. Let $3x+5y\le 15$

$5x+2y\le 10$

By solving both equation we get (0, 3), (2, 0)

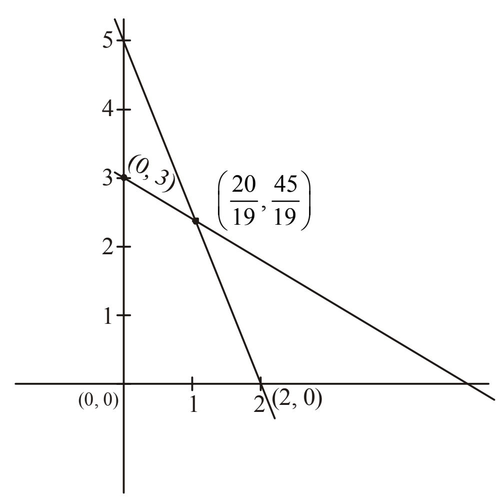
$\left(\frac{20}{19},\frac{45}{19}\right)$ **(1 mark)**

Corner points are

(0, 3), (2, 0), $\left(\frac{20}{19},\frac{45}{19}\right)$ **(1 mark)**

and value of z are $9,\frac{235}{19},10$

Maximum value is $\frac{235}{19}=12.36$



(1 mark)

29. Non-defective bulbs = 30 – 6 = 24

Let X be the random variable that denotes the number of defective bulbs.

P(X = 0) = P(2 non-defective and 0 defective)

$$= {}^2C_0 \cdot \frac{24}{30} \cdot \frac{24}{30} = \frac{16}{25}$$ **(1 mark)**

$$P(X=1) = {}^2C_1 \cdot \frac{24}{30} \cdot \frac{6}{30} = \frac{8}{25}$$ **(1 mark)**

$$P(X=2) = {}^2C_2 \cdot \frac{6}{30} \cdot \frac{6}{30} = \frac{1}{25}$$ **(1 mark)**

Required Probability distribution is,

X	0	1	2
P(x)	$\frac{16}{25}$	$\frac{8}{25}$	$\frac{1}{25}$

30. (a) Given, $\frac{dy}{dx} = \frac{x+y}{x}$, $y(1) = 0$ (1)

Let y = vx **(1 mark)**

then, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{x+vx}{x}$

$\Rightarrow \quad v + x\frac{dv}{dx} = 1 + v \Rightarrow \frac{dv}{dx} = \frac{1}{x}$ **(1 mark)**

$\Rightarrow \quad \int dv = \int \frac{dx}{x}$

$\Rightarrow \quad v = \log x + c$

$\Rightarrow \quad \frac{y}{x} = \log x + c$

At x = 1, y = 0

$\Rightarrow \quad 0 + c \quad \Rightarrow c = 0$

Then, y = x log x **(1 mark)**

OR

(b) Given, $e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$

or $\frac{e^x}{1-e^x}dx + \frac{\sec^2 y}{\tan y}dy = 0$ **(1 mark)**

$\Rightarrow \quad \int \frac{\sec^2 y}{\tan y} dy = \int \frac{e^x dx}{(e^x - 1)}$ **(1 mark)**

$\Rightarrow \quad \log|\tan y| = \log|e^x - 1| + c$ **(1 mark)**

$$\left[\because \int \frac{f^1(x)}{f(x)} dy = \log f(x)\right]$$

Note

For homogeneous function, power of x & y (should be) same in given function. Then put y = vx

31. (a) Let $I = \int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

$$\Rightarrow \quad I = \int_{\pi/4}^{\pi/2} e^{2x}\left[\frac{1}{1-\cos 2x} - \frac{\sin 2x}{1-\cos 2x}\right]dx$$

$$\Rightarrow \quad I = \int_{\pi/4}^{\pi/2} e^{2x}\left[\frac{1}{2\sin^2 x} - \frac{2\sin x\cos x}{2\sin^2 x}\right]dx$$

$$\Rightarrow \quad I = \int_{\pi/4}^{\pi/2} e^{2x}\left[\frac{\operatorname{cosec}^2 x}{2} - \cot x\right]dx$$ **(1 mark)**

Put 2x = t

$$\Rightarrow \quad x = t/2 \Rightarrow dx = \frac{dt}{2}$$

$$\therefore \quad I = -\int e^t\left(\cot\frac{t}{2} - \frac{1}{2}\operatorname{cosec}^2\frac{t}{2}\right)\frac{dt}{2}$$

$$\left[\because \int e^t\left(f(t) + f^1(t)\right)dt = e^t f(t) + c\right]$$

$$\Rightarrow \quad I = -\frac{1}{2}\left(e^t \cot\frac{t}{2}\right)$$ **(1 mark)**

$$\Rightarrow \quad I = \left[-\frac{1}{2}e^{2x}\cot x\right]_{\pi/4}^{\pi/2}$$

$$\Rightarrow \quad I = -\left[0 - \frac{1}{2}e^{\pi/2}\right] = \frac{e^{\pi/2}}{2}$$ **(1 mark)**

OR

(b) Let $I = \int_{-2}^{2} \frac{x^2 dx}{1+5^x}$ ...(i)

$$\left[\because \int_a^b f(x)dx = f(a+b-x)dx\right]$$

$$\Rightarrow \quad I = \int_{-2}^{2} \frac{(-x)^2}{1+5^{-x}}dx$$ ...(ii)

(1½ marks)

Add equations (i) and (ii)

$$\Rightarrow \quad 2I = \int_{-2}^{2}\left[\frac{5^x x^2}{1+5^x} + \frac{x^2}{1+5^x}\right]dx = \int_{-2}^{2} x^2 dx$$

$$I = \frac{1}{2}\left[\frac{x^3}{3}\right]_{-2}^{2} = \frac{1}{2} \times \frac{16}{3} = 8/3$$ **(1½ marks)**

32. Let N is foot of perpendicular drawn from the point P(2, –1, 5)

Any point on line is N (11 + 10t, –2–4t, –8–11t)

Now, direction ratio of NP is

$\langle 9+10t, -1-4t, -13-11t \rangle$ **(2 marks)**

Direction ratio of line is $\langle 10, -4, -11 \rangle$

$\Rightarrow 10(9+10t)+4(1+4t)+11(13+11t)=0$

$\Rightarrow 237t+237=0$

$\Rightarrow t=-1$

Now, $(1, 2, 3)=\left(\frac{2+x}{2}, \frac{-1+y}{2}, \frac{5+z}{2}\right)$

$\Rightarrow \frac{x+2}{2}=1 \Rightarrow x=0$

$\Rightarrow \frac{y-1}{2}=2 \Rightarrow y=5$ **(2 marks)**

$\Rightarrow \frac{z+5}{2}=3 \Rightarrow z=1$

So image is (0, 5, 1) **(1 mark)**

OR

(b) BC lie on $\frac{x+2}{2}=\frac{y-1}{1}=\frac{z}{4}$

Coordinates of D can be expressed as,

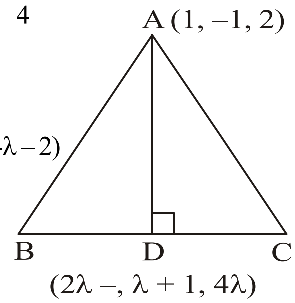
$(2\lambda-2, \lambda+1, 4\lambda)$

Dr's of AD are $(2\lambda-3, \lambda+2, 4\lambda-2)$

AD is perpendicular to BC

So, $2(2\lambda-3)+(\lambda+2)$

$+4(4\lambda-2)=0$



$\Rightarrow \lambda=\frac{12}{21} \Rightarrow z=1$

(3 marks)

$|AD|=\sqrt{(2\lambda-3)^2+(\lambda+2)^2+(4\lambda-2)^2}$

$=\sqrt{\left(\frac{39}{21}\right)^2+\left(\frac{54}{21}\right)^2+\left(\frac{6}{21}\right)^2}=\sqrt{\frac{4473}{441}}$

$\text{Area}=\frac{1}{2}\times 5\times\sqrt{\frac{4473}{441}}=\sqrt{\frac{1775}{28}}$ **(2 marks)**

33. Given, $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$

$\Rightarrow |A|=1(8-6)+(0+9)-2(0-6)=-1$ **(1 mark)**

On finding adjoint of A

$\Rightarrow F_{11}=\begin{vmatrix}2 & -3\\ -2 & 4\end{vmatrix}=2, F_{12}=\begin{vmatrix}0 & -3\\ 3 & 4\end{vmatrix}=-9$

$\Rightarrow F_{13}=\begin{vmatrix}0 & 2\\ 3 & -2\end{vmatrix}=-6, F_{21}=\begin{vmatrix}-1 & 2\\ -2 & 4\end{vmatrix}=0$

$\Rightarrow F_{22}=\begin{vmatrix}1 & 2\\ 3 & 4\end{vmatrix}=-2, F_{23}=-\begin{vmatrix}1 & -1\\ 3 & -2\end{vmatrix}=1$

$\Rightarrow F_{31}=\begin{vmatrix}-1 & 2\\ 2 & -3\end{vmatrix}=-1, F_{32}=-\begin{vmatrix}1 & 2\\ 0 & -3\end{vmatrix}=3$

$\Rightarrow F_{33}=\begin{vmatrix}1 & -1\\ 0 & 2\end{vmatrix}=2$

Matrix formed by adjoint of A is

$\Rightarrow \text{adj } A=B^T=\begin{bmatrix}2 & 0 & -1\\ -9 & -2 & -3\\ -6 & -1 & 2\end{bmatrix}$

Then, $A^{-1}=\begin{bmatrix}2 & -9 & -6\\ 0 & -2 & -1\\ -1 & 3 & 2\end{bmatrix}$ **(2 marks)**

$\Rightarrow =\frac{1}{-1}\begin{bmatrix}2 & 0 & -1\\ -9 & -2 & 3\\ -6 & -1 & 2\end{bmatrix}$

Now, AX = B

Then $X=A^{-1}B$

$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{(-1)}\begin{bmatrix}2 & 0 & -1\\ -9 & -2 & 3\\ -6 & -1 & 2\end{bmatrix}\begin{bmatrix}1\\ 1\\ 3\end{bmatrix}=\frac{1}{(-1)}\begin{bmatrix}-1\\ -2\\ -1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}$

$x=1, y=2, z=1$ **(2 marks)**

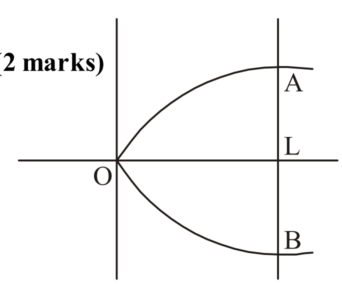
34. Given, $y^2=4ax$

Then the equation of latus rectum is x = a

Required area = 2(area of AOL) **(1 mark)**

$=2\int_0^a y\,dx=2\int_0^a 2\sqrt{a}\sqrt{x}\,dx$

$=4\sqrt{a}\int_0^a \sqrt{x}\,dx$ **(2 marks)**



$=4\sqrt{a}\left[\frac{2x^{3/2}}{3}\right]_0^a$

$=4\sqrt{a}\times\frac{2}{3}\left[a^{3/2}-0\right]$

$= \frac{8}{3}a^2$ sq units. **(2 marks)**

Note

ar. AOL = ar. BOL
because both are symmetric figure

35. **(a)** Let R be defined on N × N

Reflexivity: Sum and product of natural numbers obeys commutative property

$(a, b) R (c, d) \Leftrightarrow ad (b + c) = bc(a + d)$

Hence, R is reflexive. **(1½ marks)**

Symmetry: Let (a, b) R(c, d)

$\Rightarrow$ $ad(b + c) = bc(a + d)$

$\Rightarrow$ $da(c + b) = cb(d + a)$

$\Rightarrow$ $(c, d) = R(a, b)$ **(1½ marks)**

So, R is symmetric.

Transitivity: Let $(a, b), (c, d), (e, f) \in N \times N$

(a, b) R(c, d) and (c, d) R (e, f)

$ad(b + c) = bc(a + d)$ and $cf(d + e) = de (c + f)$

$$\Rightarrow \frac{ab}{a-b} = \frac{cd}{c-d}, \frac{cd}{c-d} = \frac{ef}{e-f}$$

$$\Rightarrow \frac{ab}{a-b} = \frac{ef}{e-f} \Rightarrow (a, b) R(e, f)$$

Hence, R is transitive.

OR

(b) Given, $f(x) = \frac{4x}{3x+4} \Rightarrow f(x) = f(y)$

$$\Rightarrow \frac{4x}{3x+4} = \frac{4y}{3y+4} \Rightarrow 12xy + 16x = 12xy + 16y$$

$\Rightarrow$ $16x = 16y \Rightarrow x = y$

$\therefore$ f is one-one $\Rightarrow f(x) = \frac{4x}{3x+4}$ **(2½ marks)**

$$\Rightarrow 4x = 3xy + 4y \Rightarrow x = \frac{4y}{4-3y}$$

So $y \in R - \left\{\frac{4}{3}\right\}$

So, every element in $R-\left\{\frac{4}{3}\right\}$ has pre-image in $R-\left\{-\frac{4}{3}\right\}$

Hence, f is onto. **(2½ marks)**

Note

If f(x) = f(y) $\Rightarrow$ x = y for one-one function & for onto $x = f^{-1}(y)$ exists i.e. Range = codomain.

36. Given that

E_1 = Represent the event when many workers where not present

E_2 = Represent the event when all workers where present

E = Represent completing the construction work on time.

$\therefore$ $P(E_1) = 0.65,\ P(E/E_1) = 0.35$

and $P(E/E_2) = 0.80$

(i) P(All workers are present for the job)

$= P(E_2) = 1 - 0.65 = 0.35$ **(1 mark)**

(ii) P(Construction will be completed on time)

$= P(E) = P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)$

$= 0.65 \times 0.35 + 0.35 \times 0.80$

$$= \frac{13}{20} \times \frac{7}{20} + \frac{7}{20} \times \frac{16}{20} = \frac{91}{400} + \frac{112}{400}$$

$$= \frac{203}{400} = 0.51$$ **(1 mark)**

(iii) (a) P(Many workers are not present given that construction work is completed on time)

$$= P(E_1/E) = \frac{P(E/E_1) \cdot P(E_1)}{P(E/E_1)P(E_1) + P(E/E2) \cdot P(E_2)}$$ **(1 mark)**

$$= \frac{\frac{13}{20} \times \frac{7}{20}}{\frac{203}{400}} = \frac{\frac{91}{400}}{\frac{203}{400}}$$

$$= \frac{91}{203} = 0.45$$ **(1 mark)**

OR

(b) P(all workers were present given that the construction job was completed on time)

$$= P(E_2/E) = \frac{P(E/E_2) \cdot P(E_2)}{P(E/E_1)P(E_1) + P(E/E_2) \cdot P(E_2)}$$

$$= \frac{112}{203} = 0.55$$ **(2 marks)**

37. (i) Given, $f(x) = \begin{cases} |x-3|, & x \ge 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

RHD of f(x) at x = 1 is define as $\lim_{x \to 1^+} f(1^+)$

$$|x-3| = \begin{cases} -(x-3), & x < 3 \\ (x-3), & x > 3 \end{cases}$$

$$\Rightarrow \lim_{x \to 1^+} |x-3| = \lim_{x \to 1^+} (3-x) = 3 - 1 = 2$$ **(1 mark)**

(ii) LHD of f(x) at x = 1 is define as $\lim_{x \to 1^-} f(1^-)$

At $x \to 1^-, f(x) = \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}$

$$\Rightarrow \lim_{x \to 1^-} f(1^-) = \frac{1}{4} - \frac{3}{2} + \frac{13}{4} = \frac{1}{4} - \frac{3}{4} + \frac{13}{4}$$

$$\Rightarrow \frac{11}{4}$$ **(1 mark)**

(iii) (a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

LHS

At $x \to 1 + h$, where $h \to 0$

$$f'(x) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \frac{-(1+h-3)-2}{h}$$

$$= \frac{-h+2-2}{h} = -1$$ **(1 mark)**

RHS

At $x \to 1 - h$, where $h \to 0$

$$f'(x) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{h}$$

$$= \frac{\frac{(1-h)^2}{4} - \frac{6(1-h)}{4} + \frac{13}{4} - \left(\frac{1}{4} - \frac{6}{4} + \frac{13}{4}\right)}{h}$$

$$\Rightarrow \frac{h^2 - 2h + 6h}{4h} = \frac{h-2+6}{4} = 1$$ **(1 mark)**

So given function is not differentiable at x = 1 RHS ≠ LHS

OR

(b) At $x = 2$, $f(x) = -(x-3)$

Then, $f'(x) = -1$

so, $f'(2) = -1$

And, at $x = -1$, $f(x) = \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}$

$$\Rightarrow f'(x) = \frac{2x}{4} - \frac{3}{2}$$

$$\Rightarrow f'(x) = -\frac{1}{2} - \frac{3}{2} = -2$$ **(2 marks)**

38. (i) Three sides are wire fencing

So $2x + y = 200$ ...(i)

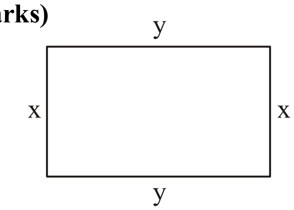
Area of rectangular garden is,

$A = xy$

Put value of y from equation (i)

$A(x) = x(200 - 2x)$

$A(x) = 200x - 2x^2$ **(2 marks)**



(ii) For maximum area

$A'(x) = 200 - 4x = 0$

$\Rightarrow 4x = 200 \Rightarrow x = 50$

Put value of x in equation (i)

$\Rightarrow 100 + y = 200 \Rightarrow y = 100$

So area $= xy = 100 \times 50 = 5000$ **(2 marks)**

Note

To find maximum or minimum area put $\frac{dy}{dx} = 0$ *for finding the values of x.*