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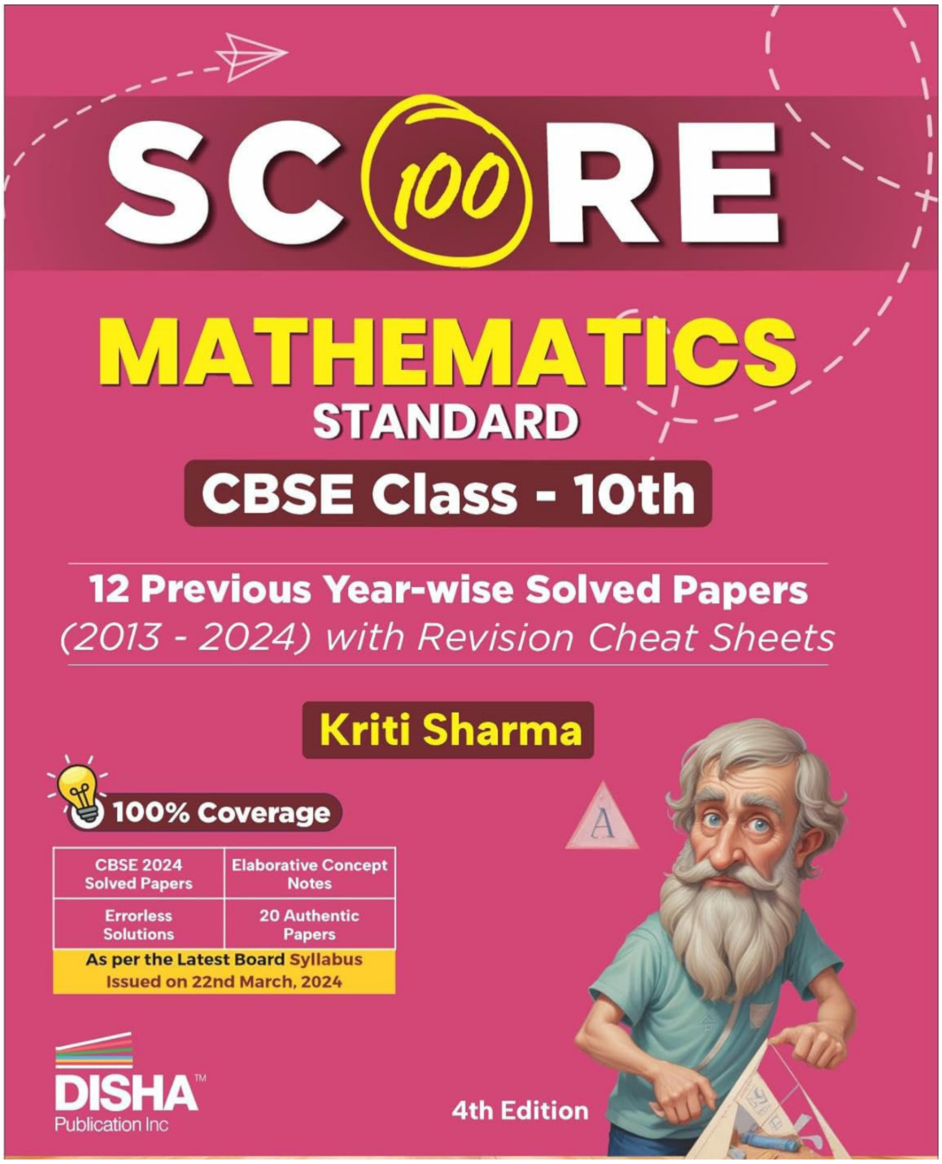
All India 2023 Solved Paper

This sample is taken from the **"Score 100 Mathematics (Standard) CBSE Class 10th 12 Previous Year-wise Solved Papers (2013 - 2024) with Revision Cheat Sheets 4th Edition"**



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All India 2023

CBSE Board Solved Paper

Time Allowed : 3 Hours *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them :

(i) This question paper contains **38** questions. **All** questions are **compulsory**.
(ii) This question paper is divided into **five Sections** - **A, B, C, D** and **E.**
(iii) In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
(iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
(v) In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
(vi) In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.
(vii) In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each. Internal choice is provided in **2** marks questions in each case-study.
(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section B, **2** questions in Section C, **2** questions in **Section D** and **3** questions in **Section E**.
(ix) Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
(x) Use of calculators is **not** allowed

SECTION - A

This section comprises multiple choice questions (MCQs) of **1 mark**

1. $\left(\frac{1-\tan^2 30°}{1+\tan^2 30°}\right)$ is equal to :

(a) sin 60° (b) cos 60°
(c) tan 60° (d) cos 30°

2. In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$. Which of the following makes the two triangles similar?

(a) ∠A = ∠D (b) ∠B = ∠D
(c) ∠B = ∠E (d) ∠A = ∠F

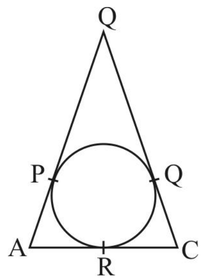
3. The 13th term from the end of the A.P. : 20, 13, 6, –1, ..., –148:

(a) 57 (b) –57
(c) 64 (d) –64

4. Two dice are rolled together. What is the probability of getting a sum greater than 10?

(a) $\frac{1}{9}$ (b) $\frac{1}{6}$
(c) $\frac{1}{12}$ (d) $\frac{5}{18}$

5. In the given figure, AB = BC = 10 cm. If AC = 7 cm, then the length of BP is



(a) 3·5 cm (b) 7 cm
(c) 6·5 cm (d) 5 cm

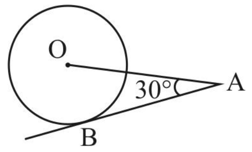
6. Water in a river which is 3 m deep and 40 m wide is flowing at the rate of 2 km/h. How much water will fall into the sea in 2 minutes?

(a) 800 m^3 (b) 4000 m^3
(c) 8000 m^3 (d) 2000 m^3

7. If the mean and the median of a data are 12 and 15 respectively, then its mode is:

(a) 13·5 (b) 21
(c) 6 (d) 14

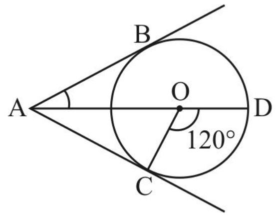
8. In the given figure, AB is a tangent to the circle centered at O. If OA = 6 cm and $\angle OAB = 30°$, then the redius of the circle is:



(a) 3 cm (b) $3\sqrt{3}$ cm

(c) 2 cm (d) $\sqrt{3}$ cm

9. In the given figure, AC and AB are tangents to a circle centered at O. If $\angle COD = 120°$, then $\angle BAO$ is equal to:



(a) 30° (b) 60° (c) 45° (d) 90°

10. Which of the following numbers cannot be the probability of happening of an event:

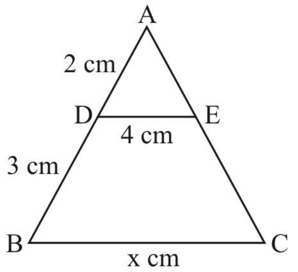
(a) 0 (b) $\frac{7}{0\cdot01}$ (c) 0.07 (d) $\frac{0\cdot07}{3}$

11. If every term of the statistical data consisting of n terms is decreased by 2, then the mean of the data:

(a) decreases by 2 (b) remains unchanged

(c) decreases by 2n (d) decreases by 1

12. In the given figure, DE ∥ BC. The value of x is :



(a) 6 (b) 12.5 (c) 8 (d) 10

13. A quadratic equation whose roots are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ is:

(a) $x^2 - 4x + 1 = 0$ (b) $x^2 + 4x + 1 = 0$

(c) $4x^2 - 3 = 0$ (d) $x^2 - 1 = 0$

14. If $\tan\theta = \frac{5}{12}$, then the value of $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$ is :

(a) $-\frac{17}{7}$ (b) $\frac{17}{7}$

(c) $\frac{17}{13}$ (d) $-\frac{7}{13}$

15. If end points of a diameter of a circle are (–5, 4) and (1, 0), then the radius of the circle is:

(a) $2\sqrt{13}$ units (b) $\sqrt{13}$ units

(c) $4\sqrt{2}$ units (d) $2\sqrt{2}$ units

16. The number of polynomials having zeroes –1 and 2 is:

(a) exactly 2 (b) only 1

(c) at most 2 (d) infinite

17. The pair of equations $ax + 2y = 9$ and $3x + by = 18$ represent parallel lines, where a, b are integers, if:

(a) a = b (b) 3a = 2b

(c) 2a = 3b (d) ab = 6

18. The common differences of the A.P. whose n^{th} term is given by $a_n = 5n - 7$ is :

(a) –7 (b) 7

(c) 5 (d) –2

Directions (Q. 19-20): Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A)

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The number, 5^n cannot end with the digit 0, where n is a natural number.

Reason (R) : Prime factorisation of 5 has only two factors, 1 and 5.

20. **Assertion (A) :** If the points A(4, 3) and B(x, 5) lie on a circle with centre O(2, 3), then the value of x is 2

Reason (R) : Centre of a circle is the mid-point of each chord of the circle.

SECTION - B

This section comprises very short answer (VSA) type questions of **2 marks**.

21. Using prine factorisation, find HCF and LCM of 96 and 120.

22. Find the ratio in which line y = x divides the line segment joining the points (6, –3) and (1, 6).

23. (a) If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then prove that $a^2 + b^2 = m^2 + n^2$.

OR

(b) Prove that:

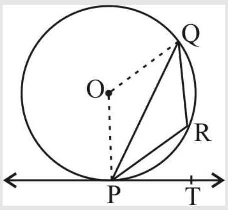
$$\sqrt{\frac{\sec A-1}{\sec A+1}}+\sqrt{\frac{\sec A+1}{\sec A-1}} = 2\text{cosec } A$$

24. (a) The line segment joining the points A(4, –5) and B(4, 5) is divided by the point P such that AP : AB = 2 : 5. Find the coordinates of P.

OR

(b) Point (x, y) is equidistant from points A(5, 1) and B(1, 5). Prove that x = y

***25.** In the given figure, PQ is a chord of the circle centered at O. PT is a tangent to the circle at P. If ∠QPT = 55°, then find ∠PRQ.



SECTION - C

This section comprises short answer (SA) type questions of **3 marks.**

26. Find the mean of the following distribution

Classes	0–15	15–30	30–45	45–60	60–75	75–90
Frequency	17	20	18	21	15	9

27. A 2-digit number is seven times the sum of its digits. The number formed by reversing the digits is 18 less than the given number. Find the given number.

28. Prove that:

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\text{cosec}\,\theta$$

29. (a) Prove that $\sqrt{3}$ is an irrational number.

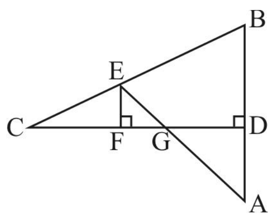
OR

(b) The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change together next?

30. In an A.P., the sum of the first n terms is given by $S_n = 6n - n^2$. Find the 30th term.

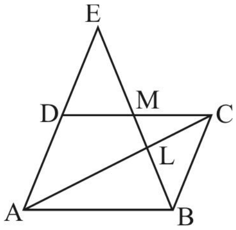
31. (a) In the given figure, CD is the perpendicular bisector of AB. EF is perpendicular to CD. AE intersects CD at G.

Prove that $\frac{CF}{CD}=\frac{FG}{DG}$.



OR

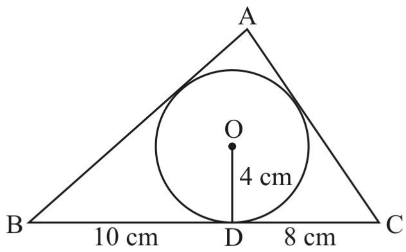
(b) In the given figure, ABCD is a parallelogram. BE bisects CD at M and intersects AC at L. Prove that EL = 2BL.



SECTION - D

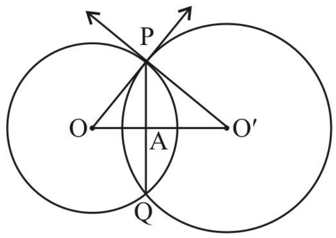
This section comprises long answer (LA) type questions of **5 marks**.

32. (a) A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 10 cm and 8 cm respectively. Find the lengths of the sides of AB and AC, if it is given that area ΔABC = 90 cm^2.



OR

(b) Two circles with centres O and O′ of radii 6 cm and 8 cm, respectively intersect at two points P and Q such that OP and O′P are tangents to the two circles. Find the length of the common chord PQ.



33. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze. Also, find the increase in grazing area if length of rope is increased to 10 m. (Use π = 3.14)

34. The angle of elevation of the top of a vertical tower from a point P on the ground is 60°. From another point Q, 10 m vertically above the first point P, its angle of elevation is 30°. Find :

(a) The height of the tower.

(b) The distance of the point P from the foot of the tower.

(c) The distance of the point P from the top of the tower.

35. (a) A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the journey, what was its first average speed?

OR

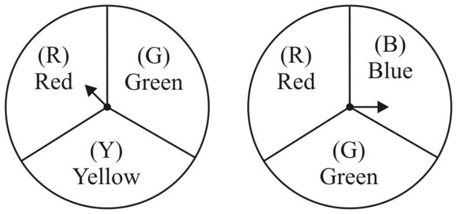
Two pipes together can fill a tank in $\frac{15}{8}$ hours. The pipe with larger diameter takes 2 hours less than the pipe with smaller diameter to fill thetank separately. Find the time in which each pipe can fill the tank separately.

SECTION - E

This section comprises **3** case study based questions of **4 marks** each.

Case Study-1

36. A middle school decided to run the following spinner game as a fund-raiser on Christmas Carnival.



I II

Making Purple : Spin each spinner once. Blue and red make purple. So, if one spinner shows Red (R) and another Blue (B), then you 'win'. One such outcome is written as 'RB'.

Based on the above, answer the following questions:

(i) List all possible outcomes of the game.

(ii) Find the probability of 'Making Purple'.

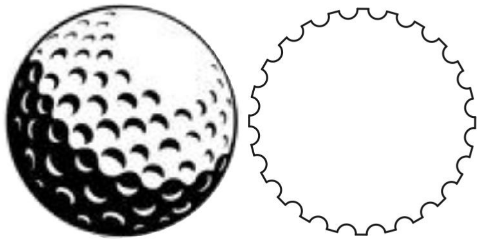
(iii) (a) For each win, a participant gets ₹10, but if he/she loses, he/she has to pay ₹5 to the school.
If 99 participants played, calculate how much fund could the school have collected.

OR

(iii) (b) If the same amount of ₹ 5 has been decided for winning or losing the game, then how much fund had been collected by school? (Number of participants =99)

Case Study-2

37. A golf ball is spherical with about 300 – 500 dimples that help increase its velocity while in play. Golf balls are traditionally white but available in colours also. In the given figure, a golf ball has diameter 4.2 cm and the surface has 315 dimples (hemi-spherical) of radius 2 mm.



Based on the above, answer the following questions:

(i) Find the surface area of one such dimple.

(ii) Find the volume of the material dug out to make one dimple.

(iii) (a) Find the total surface area exposed to the surroundings.

OR

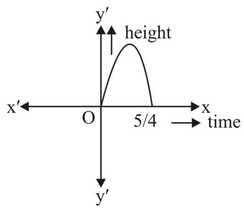
(iii) (b) Find the volume of the golf ball.

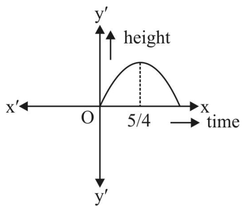
38. In a pool at an aquarium, a dolphin jumps out of the water travelling at 20 cm per second. Its height above water level after t seconds is given by $h = 20t - 16t^2$.

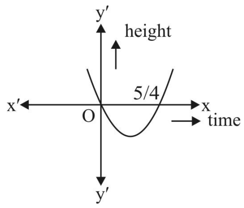
Based on the above, answer the following questions.

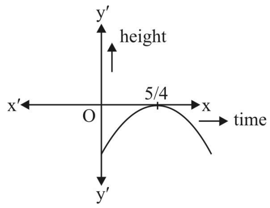
(i) Find zeroes of polynomial $p(t) = 20t - 16t^2$.

(ii) Which of the following types of graph represents p(t)?

(a) 

(b) 

(c) 

(d) 

(iii) (a) What would be the value of h at $t = \frac{3}{2}$? Interpret the result.

OR

(iii) (b) How much distance has the dolphin covered before hitting the water level again?

Solutions

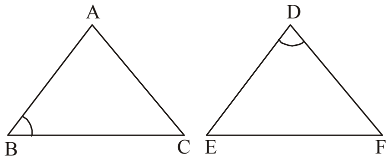
SECTION - A

1. (b) $\left(\dfrac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ}\right)$

$\Rightarrow \left(\dfrac{1-\left(\dfrac{1}{\sqrt{3}}\right)^2}{1+\left(\dfrac{1}{\sqrt{3}}\right)^2}\right) = \dfrac{\left(1-\dfrac{1}{3}\right)}{\left(1+\dfrac{1}{3}\right)} = \dfrac{2}{4}$

$\Rightarrow \dfrac{1}{2} = \sin 30^\circ$ or $\cos 60^\circ$ **(1 Mark)**

2. (b) Given, $\dfrac{AB}{DE} = \dfrac{BC}{FD}$



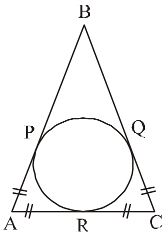
In AB and BC, $\angle B$ is common
And In DE and FD, $\angle D$ is common
For similar triangle $\angle B = \angle D$ **(1 Mark)**

3. (d) There is common difference $d = -7$
Then series from end is, $-148, -141, -134, ...20$
Where, $a = -148$, $d = 7$; 13^{th} term $= a + 12d$
$= -148 + 12 \times 7 = -148 + 84 = -64$

4. (c) Getting a sum greater than $10 = (5, 6), (6, 5), (6, 6)$
$\Rightarrow$ Total cases $= 6 \times 6 = 36$

Probability $= \dfrac{\text{Number of event}}{\text{Total cases}} = \dfrac{3}{36} = \dfrac{1}{12}$ **(1 Mark)**

5. (c) $AP = AR$ and $CR = CQ$



$\Rightarrow AR + CR = 7$
or $AP + CQ = 7$
$\Rightarrow (AB - BP) + (BC - BQ) = 7$
$\Rightarrow (10 - BP) + (10 - BQ) = 7$
$\Rightarrow 20 - 7 = 2x \quad [BP = BQ = x]$
$\Rightarrow x = \dfrac{13}{2} = 6.5$ **(1 Mark)**

6. (c) Area of river $= 40 \times 3 = 120\ m^2$

Rate of flowing $= \dfrac{2000}{60} = \dfrac{200}{6} = \dfrac{100}{3}$

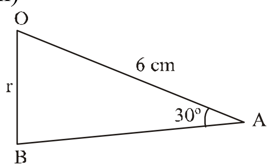
Rate of flowing in 2 min $\dfrac{100}{3} \times 2 = \dfrac{200}{3}$

Water fall into the sea $= \dfrac{200}{3} \times 120$

$\Rightarrow 200 \times 40 = 8000\ m^3$ **(1 Mark)**

7. (b) Mode $= 3(\text{median}) - 2(\text{mean})$
$\Rightarrow$ Mode $= 3(15) - 2(12)$
$= 45 - 24 = 21$

8. (a) For ΔOAB



$\sin 30^\circ = \dfrac{r}{6}$

$\Rightarrow r = 6 \sin 30^\circ = 6 \times \dfrac{1}{2} = 3$ cm

9. (a) Given ΔOAC
$\angle DOA = 120^\circ + \angle AOC = 180^\circ$
$\Rightarrow \angle AOC = 60^\circ \Rightarrow 60^\circ + 90^\circ + \angle OAC = 180^\circ$
$\Rightarrow \angle OAC = 180^\circ - 150^\circ = 30^\circ$
$\Rightarrow \angle BAO = 30^\circ$ **(1 Mark)**

10. (b) Probability always should be ≤ 1

$\dfrac{7}{0.01} = 700 > 1$ So, $\dfrac{7}{0.01}$ is not possible. **(1 Mark)**

Note

If E be any event then $0 \le P(E) \le 1$

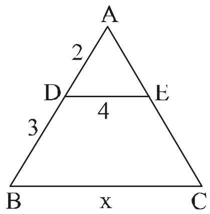
11. (a) Let $\dfrac{a+b+c...+n \text{ terms}}{n} = Z$

And, $\dfrac{(a-2)+(b-2)+...+n \text{ terms}}{n}$

$\Rightarrow \dfrac{(a+b+...+n \text{ terms})-(2+2...n \text{ terms})}{n}$

$\Rightarrow \dfrac{(a+b+...n \text{ terms})}{n} - \dfrac{2n}{n} = Z - 2$ **(1 Mark)**

12. (d) Given, $DE \parallel BC$
$\Delta ADE \sim \Delta ABC$



$\Rightarrow \dfrac{2}{4} = \dfrac{5}{x}$

$\Rightarrow x = \dfrac{20}{2} = 10$ **(1 Mark)**

13. (a) Given, $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are roots

The equation is, $\left(x-(2+\sqrt{3})\right)\left(x-(2-\sqrt{3})\right) = 0$

$\Rightarrow (x-2-\sqrt{3})(x-2+\sqrt{3}) = 0$
$\Rightarrow (x-2)^2 - 3 = x^2 + 4 - 4x - 3 = 0 \Rightarrow x^2 - 4x + 1 = 0$ **(1 Mark)**

14. (a) Given, $\tan\theta = \dfrac{5}{12}$

$\Rightarrow \dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{\cos\theta\left(\dfrac{\sin\theta}{\cos\theta}+1\right)}{\cos\theta\left(\dfrac{\sin\theta}{\cos\theta}-1\right)}$

$\Rightarrow \left(\dfrac{\tan\theta+1}{\tan\theta-1}\right) = \dfrac{\left(\dfrac{5}{12}+1\right)}{\left(\dfrac{5}{12}-1\right)} = -\dfrac{17}{7}$ **(1 Mark)**

15. (b) Given points are $(-5, 4)$ and $(1, 0)$

Diameter $= \sqrt{(-5-1)^2 + (4-0)^2}$

$= \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Then radius $= \dfrac{\text{diameter}}{2} = \dfrac{2\sqrt{13}}{2} = \sqrt{13}$ units

16. **(d)** Polynomial define in factor form $K(x+1)(x-2)=0$ **(1 Mark)**

$\Rightarrow$ $K(x^2-2x+x-2)=0 \Rightarrow K(x^2-x-2)=0$

So infinite many Polynomial having zeroes –1, and 2

> ***Note***
>
> *If α, β are the roots/zeros of the Polynomial then the Polynomial is k(x – a)(x– b) where k ∈ R*

17. **(d)** Given here, $ax+2y=9$ ...(i)

$3x+by=18$ ...(ii)

For parallel, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$. So, to make cofficient equal, multiply(i) eq. by (ii).

or $2ax+4y=18$, $3x+by=18$

Given pair are parallel if

$2a=3, b=4 \Rightarrow 2ab=12 \Rightarrow ab=6$ **(1 Mark)**

18. **(c)** Given, $a_n=5n-7$; $a_1=-2$, $a_2=3$

Common difference $(d)=a_2-a_1=3-(-2)=5$ **(1 Mark)**

19. **(d)** The number 5^n end with multiple of 5 for all $n \in N$

$\Rightarrow$ R : Prime factorisation of 5 are 1, 5 **(1 Mark)**

20. **(c)** A : Lenght of OA = Length of OB

$\Rightarrow$ $\sqrt{(4-2)^2+(3-3)^2}=\sqrt{(x-2)^2+(5-3)^2}$

$\Rightarrow$ $4=x^2+4-4x+4 \Rightarrow (x-2)^2=0 \Rightarrow x=2$

R : Centre of circle is the mid point of each diameter of the circle. **(1 Mark)**

SECTION - B

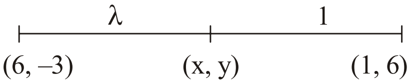
21. **(24)** $96=2^2\times 3\times 2^3=2^5\times 3$ **(1 Mark)**

$120=2^2\times 3\times 5\times 2=2^3\times 3\times 5$ **(1 Mark)**

HCF $=2^3\times 3=24$; LCM $=2^5\times 3\times 5=480$

22. Let the ratio is λ : 1

$x=\frac{\lambda+6}{\lambda+1}, y=\frac{6\lambda-6}{\lambda+1}$



for $y=x$ **(1 Mark)**

$\Rightarrow$ $\frac{\lambda+6}{\lambda+1}=\frac{6\lambda-6}{\lambda+1} \Rightarrow 5\lambda=9 \Rightarrow \lambda=\frac{9}{5}$

So the ratio is 9 : 5 **(1 Mark)**

23. **(a)** $m^2=a^2\cos^2\theta+b^2\sin^2\theta+2ab\cos\theta\sin\theta$

$n^2=a^2\sin^2\theta+b^2\cos^2\theta-2ab\cos\theta\sin\theta$ **(1 Mark)**

$m^2+n^2=(a^2+b^2)\cos^2\theta+(a^2+b^2)\sin^2\theta$

$\Rightarrow$ $m^2+n^2=(a^2+b^2)(\cos^2\theta+\sin^2\theta)$

$\Rightarrow$ $m^2+n^2=a^2+b^2$ $[\because \cos^2\theta+\sin^2\theta=1]$ **(1 Mark)**

OR

(b) $\frac{\sqrt{\sec A-1}}{\sqrt{\sec A+1}}+\frac{\sqrt{\sec A+1}}{\sqrt{\sec A-1}}=\frac{\sec A-1+\sec A+1}{\sqrt{\sec^2 A-1}}$ **(1 Mark)**

$=\frac{2\sec A}{\sqrt{\tan^2 A}}=\frac{2\sec A}{\tan A}$ $[\because \sec^2 A-1=\tan^2 A]$

$=2\left(\frac{1}{\cos A}\right)\left(\frac{\cos A}{\sin A}\right)=2\operatorname{cosec} A$ **(1 Mark)**

24. **(a)** Let coordinates of P is (a, b)

By section formula **(1 Mark)**

$a=\frac{2\times 4+3\times 4}{5}=\frac{20}{5}=4 \Rightarrow b=\frac{2\times 5-3\times 5}{5}=\frac{-5}{5}=-1$

coordinates of P is (4, –1) **(1 Mark)**

OR

(b) Length of AP = Length of BP

$\Rightarrow$ $\sqrt{(5-x)^2+(1-y)^2}=\sqrt{(1-x)^2+(5-y)^2}$

$\Rightarrow$ $\sqrt{x^2+25-10x+y^2+1-2y}$

$=\sqrt{x^2+1-2x+y^2-10y+25}$ **(1 Mark)**

$\Rightarrow$ $x^2+25-10x+y^2+1-2y=x^2+1-2x+y^2-10y+25$

$\Rightarrow$ $-10x-2y=-2x-10y$

$\Rightarrow$ $8x=8y \Rightarrow y=x$ **(1 Mark)**

25.** ***Not in Syllabus

SECTION - C

26.

Classes	Frequency	x_i	f_ix_i
0 – 15	17	7.5	127.5
15 – 30	20	22.5	450
30 – 45	18	37.5	675
45 – 60	21	52.5	1102.5
60 – 75	15	67.5	1012.5
75 – 90	9	82.5	742.5
	$\Sigma f=100$	$\Sigma x_i=270$	$\Sigma f_ix_i=4110$

(2 Marks)

Now, Mean $=\frac{\sum f_ix_i}{\sum f_i}=\frac{4110}{100}=41.1$ **(1 Mark)**

27. $10x+y=7(x+y)=7x+7y$

$\Rightarrow$ $3x=6y \Rightarrow x=2y$ **(1 Mark)**

Number formed by reversing the digit yx

$10y+x=10x+y-18$

put $x=2y$

$\Rightarrow$ $10y+2y=10\times 2y+y-18$ **(1 Mark)**

$\Rightarrow$ $21y=18+12y \Rightarrow y=2 \therefore x=2y=2\times 2=4$

The number is 42 **(1 Mark)**

28. $\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$ **(1 Mark)**

$\Rightarrow$ $\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$\Rightarrow$ $\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$[\because a^3-b^3=(a-b)(a^2+b^2+ab)]$

$=\frac{(\sin\theta-\cos\theta)(1+\cos\theta\sin\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$ $[\because \sin^2\theta+\cos^2\theta=1]$

$\Rightarrow \quad \dfrac{1+\cos\theta\sin\theta}{\sin\theta\cos\theta} = 1 + \dfrac{1}{\sin\theta\cos\theta}$ **(1 Mark)**

$= \quad 1 + \sec\theta \operatorname{cosec}\theta$ **(1 Mark)**

29. **(a)** Let $\sqrt{3}$ is rational no. So $\sqrt{3}$ can be written as

$\sqrt{3} = \dfrac{p}{q}, q \neq 0$, HCF (P, q) = 1 **(1 Mark)**

i.e. $p \neq q$ are co-prime to each other

Squaring both sides

$3 = \dfrac{p^2}{q^2} \Rightarrow p^2 = 3q^2$

$\Rightarrow$ 3 is a factor of p^2 ...(i)

$\Rightarrow$ 3 is a factor of p ...(ii) **(1 Mark)**

So p = 3 m from (i), where m is any integer. $p^2 = 9m^2$

$3q^2 = 9m^2$

$q^2 = 3m^2$

$\Rightarrow$ 3 is factor of $q^2 \Rightarrow$ 3 is a factor of q

HCF (p,q) ≠ 1 contradicts our & supposition. So $\sqrt{3}$ is irrational. **(1 Mark)**

OR

(b) Take the LCM of given time

$48 = 2^4 \times 3;\ 72 = 2^3 \times 3^2;\ 108 = 2^2 \times 3^3$

Then, LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 432$ **(2 Marks)**

After 432 seconds, they will change simultaneously.

$\Rightarrow$ 432 seconds = 7 min 12 sec

Time = 7 : 07 : 12 am **(1 Mark)**

30. Given $S_n = 6n - n^2$

We know, $T_n = S_n - S_{n-1}$ **(1 Mark)**

$\Rightarrow \quad T_{30} = S_{30} - S_{29} = 6(30) - (30)^2 - (6(29) - (29)^2)$

$= \quad 180 - 900 - 174 + 841$

$= \quad -53$ **(2 Marks)**

> **Note**
>
> *If S_n be the sum of n terms of an AP then its nth term is $t_n = S_{n-1} - S_{n-1}$*

31. **(a)** In ΔEFC and ΔBDC

∠EFC = ∠BDC = 90° ∠ECF = ∠BCD

by similarity ΔEFC~ΔBDC **(1 Mark)**

$\Rightarrow \quad \dfrac{EF}{BD} = \dfrac{CF}{DC}$

In ΔEGF and ΔDGF

∠EGF = ∠DGA, ∠EFG = ∠GDA **(1 Mark)**

by similarity ΔEGF~ΔDGA $\Rightarrow \dfrac{EF}{AD} = \dfrac{FG}{DG}$

And AD = BD so, $\dfrac{FC}{DC} = \dfrac{FG}{DG}$ **(1 Mark)**

OR

(b) ΔBMC = ΔEMD (ASA criterion)

BC = ED (Parts of congruent triangle) (i)

BC = AD (Opposite side of parallelogram) (ii)

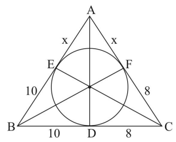
2BC = DE + AD ⇒ 2BC = AE (adding i & ii) **(2 Marks)**

$\dfrac{BC}{AE} = \dfrac{1}{2}$ (iii)

ΔBCL~ΔEAL (AA corollary)

$\dfrac{BC}{EA} = \dfrac{BL}{EL} \Rightarrow \dfrac{1}{2} = \dfrac{BL}{EL} \Rightarrow EL = 2BL$ **(1 Mark)**

SECTION - D



32. **(a)** BD = BE = 10; DC = CF = 8

And, AE = AF = x

In ΔABC **(1 Mark)**

Area of ΔABC $= \dfrac{1}{2}\left[\begin{array}{r}\text{Area of }\Delta AOE + \text{Area of }\Delta OBD \\ + \text{Area of }\Delta ODC\end{array}\right]$ **(2 Marks)**

Area $= \dfrac{1}{2}\left[\dfrac{1}{2}(4x) + \dfrac{1}{2}(10\times4) + \dfrac{1}{2}(8\times4)\right] = 90$

$\Rightarrow \quad 4x + 40 + 32 = 90 \Rightarrow 4x = 18 \Rightarrow x = \dfrac{18}{4} = 4.5$

Then, AB = 10 + 4.5 = 14.5

And AC = 8 + 4.5 = 12.5 **(2 Marks)**

OR

(b) By ΔPOO′

$(OO')^2 = 64 + 36 = 100$

$\Rightarrow OO' = 10$ **(1 Mark)**

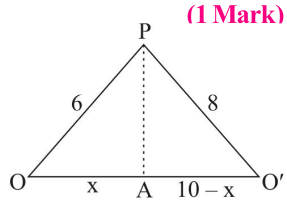
By ΔPOA, and ΔPO′A

$36 = x^2 + (PA)^2$

$\Rightarrow (PA)^2 = 36 - x^2$

$64 = (10 - x)^2 + (PA)^2$

$\Rightarrow (PA)^2 = 64 - (10 - x)^2$



$\Rightarrow \quad 36 = x^2 = 64 - (10 - x)^2 \quad \Rightarrow x = \dfrac{18}{5}$ **(2 Marks)**

Then (PA) $= \sqrt{36 - \left(\dfrac{18}{5}\right)^2} = \sqrt{\dfrac{900 - 324}{25}} = \dfrac{24}{25}$

Then PQ = 2(PA) $= 2 \times \dfrac{24}{25} = \dfrac{48}{5}$ **(2 Marks)**

33. Side of square = 15 m

Length of rope = 5 m

Area of quadrant of circle $= \dfrac{\pi r^2}{4} = \dfrac{\pi(5)^2}{4}$

$= 19.625\ m^2$ **(2 Marks)**

If length is increased to 10 m, then r = 10 m

Area of quadrant $= \dfrac{\pi \times (10)^2}{4} = 78.5\ m^2$

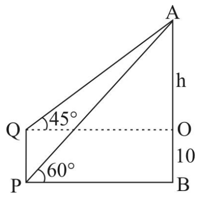
∴ Increase in area = 78.5 – 19.625 = 58.875 m^2 **(3 Marks)**

> **Note**
>
> *Area of Sector* $= \dfrac{\theta}{360°} \times \pi r^2$; *For Q adrant θ = 90°*

34. In ΔAQO

$\tan 45° = \frac{h}{OQ}$

In $\Delta APB \Rightarrow \tan 60° = \frac{h+10}{PB}$



where $PB = OQ \Rightarrow \frac{h}{\tan 45°} = \frac{h+10}{\tan 60°}$

$\Rightarrow h = \frac{h+10}{\sqrt{3}}$

$\Rightarrow \sqrt{3}h - h = 10 \Rightarrow h = \frac{10}{\sqrt{3}-1}$ **(2 Marks)**

(a) height of tower $= h + 10 = \frac{10}{\sqrt{3}-1} + 10 = \frac{10\sqrt{3}}{\sqrt{3}-1}$ **(1 Mark)**

(b) distance of $PB = h = \frac{10}{\sqrt{3}-1}$ **(1 Mark)**

(c) distance of $PA = \sqrt{(h+10)^2 + h^2} = \sqrt{h^2+100+20h+h^2}$

$\Rightarrow \sqrt{\frac{300}{(\sqrt{3}-1)^2} + \frac{100}{(\sqrt{3}-1)^2}} = \frac{20}{(\sqrt{3}-1)}$ **(1 Mark)**

35. **(a)** D = 54 km

Let average speed be x.

Distance = 63 km

The average speed to cover distance = x + 6

Time taken = 3 hr **(3 Marks)**

So, $\frac{54}{x} + \frac{63}{x+6} = 3 \Rightarrow 54x + 324 + 63x = 3x^2 + 18x$

$\Rightarrow x^2 - 33x - 108 = 0 \Rightarrow (x-36)(x+3) = 0$

Then, x = 36 km/hr **(2 Marks)**

(b) Time taken by larger dia pipe = x – 2
where x is time to fill smaller dia tap tank filled in one hour

by both pipes $= \frac{8}{15}$ **(2 Marks)**

$\Rightarrow \frac{1}{x} + \frac{1}{x-2} = \frac{8}{15} \Rightarrow 3x - 30 = 8x^2 - 16x$

$\Rightarrow 8x^2 - 46x + 30 = 0 \Rightarrow (x-5)\left(x - \frac{3}{4}\right) = 0$

Then, x = 5 and, x – 2 = 5 – 2 = 3

Hence two pipe take 5 hours and 3 hours respectively. **(3 Marks)**

Note

$\frac{\text{Distance}}{\textit{Speed}} = \textit{time}$

SECTION - E

36. (i) The total possible outcomes of a player spinning the game is 9.

(ii) The probability of making purple is $\frac{1}{9}$.

For making purple we need R in spinner 1 and B in spinner 2. So favourable outcome is 1

And the total outcomes is 9 **(2 Marks)**

(iii) (i) $P(W) = \frac{1}{9}, P(L) = 1 - \frac{1}{9} = \frac{8}{9}$

Total fund collected $= \left(99 \times \frac{1}{9}\right)(-10) + \left(99 \times \frac{8}{9}\right)5$

$= -110 + 11 \times 40 = 330$ **(2 Marks)**

OR

(ii) Fund collected by school.

$= 99 \times \frac{1}{9} \times (-5) + 99 \times \frac{8}{9} \times 5$

$= 11(-5) + (11 \times 40) = 385$

37. (i) Surface area of half circle $= 2\pi r^2$

$\Rightarrow 2\pi\left(\frac{2}{10}\right)^2 = \frac{8\pi}{100}\ cm^2$ **(1 Mark)**

(ii) Volume of half sphere $= \frac{2}{3}\pi r^3$

One half sphere volume $= \frac{2}{3}\pi\left(\frac{2}{10}\right)^3 = \frac{16\pi}{3000}\ cm^3$ **(1 Mark)**

(iii) (a) Total surface area exposed to surrounding = surface area of ball – surface area of 315 dimples

$= 4\pi r^2 - 315 \times \frac{8\pi}{100}$

$= 70.56\pi - 252\pi = 45.36\pi\ cm^2$ **(2 Marks)**

OR

(b) Volume of ball $= \frac{4}{3}\pi r^3$ – (volume of dimples)

$= \frac{4}{3}\pi r^3 - 315 \times \frac{16\pi}{3000}$

$= \frac{4}{3}\pi(2.1)^3 - 315 \times \frac{16\pi}{3000}$ **(2 Marks)**

$= 38.78 - 5.28 = 33.5$

38. (i) P(t) = 0

$\Rightarrow 20t - 16t^2 = 0 \Rightarrow t(20 - 16t) = 0 \Rightarrow t = \frac{20}{16} = \frac{5}{4}, t = 0$

$\Rightarrow t = 0, \frac{5}{4}$ **(2 Marks)**

(ii) Let P(t) = y, t = x

$\Rightarrow y = 20x - 16x^2 \Rightarrow y = -(16x^2 - 20x)$

That is similar to parabola $y = -(x-a)^2$

At, $t = 0, \frac{5}{4} \Rightarrow P(t) = 0$

Both condition are similar to option (a) **(2 Marks)**

Note

Equation of parbola can be in the form of

$y^2 = \pm 4ax,\ x^2 = \pm 4ay$