REVISION CHEAT SHEETS

1. REAL NUMBERS

(i) Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, a part from the order in which the prime factors occur.

(ii) If p is a prime and p divides a^2, then p divides a, where a is a positive integer.

2. POLYNOMIALS

Relationship between Zero(ES) and coefficient of a polynomial

(i) Zero of a linear polynomial $ax + b$, is $x = -\dfrac{b}{a}$

(ii) If quadratic polynomial $ax^2 + bx + c = k(x - \alpha)(x - \beta)$, where k is any real constant; then α and β are zeroes of quadratic polynomial $ax^2 + bx + c$ where $a, b, c \in R$ and $a \neq 0$.

$$\alpha + \beta = -\frac{b}{a}, \text{ i.e., sum of zeroes} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and $\alpha \cdot \beta = \dfrac{c}{a}$

i.e., product of zeroes $= \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

(iii) If cubic polynomial $ax^3 + bx^2 + cx + d = k(x - \alpha)(x - \beta)(x - \gamma)$ where k is any real constant, then α, β and γ are zeroes of cubic polynomial $ax^3 + bx^2 + cx + d$ where $a, b, c, d \in R$ and $a \neq 0$

$$\alpha + \beta + \gamma = -\frac{b}{a}, \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \quad \text{and } \alpha\beta\gamma = -\frac{d}{a}$$

3. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

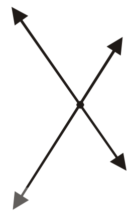
Consistent, Dependent and Inconsistent System of Equations

System of a pair of linear equations in two variables :

$$a_1 x + b_1 y + c_1 = 0$$
$$a_2 x + b_2 y + c_2 = 0$$

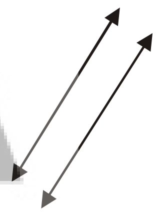
The given system of a pair of linear equations in two variables has either one solution, infinite solutions or no solution.

(i) If $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$, then the system has one (or unique) solution, and the system is called consistent. In this case, a pair of straight lines represented by the system intersect each other at only one point.



(ii) If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$, then the given system has infinite solution and the system is called dependent. In this case, a pair of lines represented by the system coincides with each other.
So the intersect each other at infinite number of points.

(iii) If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$, then the given system has no solution and hence the system is called inconsistent. In this case, a pair of lines represented by the system are parallel to each other.
So they do not intersect each other at any point.



4. QUADRATIC EQUATIONS

Solutions of Quadratic Equations

Method I : Solution of a Quadratic Equation by Factorisation

Quadratic equation : $ax^2 + bx + c = 0$

By spliting the middle term 'bx' of L.H.S, factorise the L.H.S $(ax^2 + bx + c)$ in to linear factors. Then after equating each factor to zero, we find the values of the variable 'x' of the quadratic equation $ax^2 + bx + c = 0$.

These value of x are the solutions/roots of the given quadratic equation.

Method II : Solution of a Quadratic Equation by Using the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{...(i)}$$

We can use this equation (i) as a formula to find the solutions/roots of the quadratic equation $ax^2 + bx + c = 0$.

Nature of Roots : For the quadratic equation: $ax^2 + bx + c = 0$ $(a \neq 0)$, value of $(b^2 - 4ac)$ is called discriminant of the quadratic equation. The value of $(b^2 - 4ac)$ is denoted by D.

$\therefore \quad D = b^2 - 4ac$

The discriminant plays an important role in finding the nature of the roots of the quadratic equation.

(i) If $D = 0$, then roots are real and equal.

(ii) If $D > 0$, then roots are real and unequal.

(iii) If $D < 0$, then roots are not real.

5. ARITHMETIC PROGRESSIONS

The sequence $\langle x_1, x_2, x_3, \ldots\ldots\ldots, x_n, \ldots\ldots \rangle$ is called an arithmetic progression (A.P.), if

$x_2 - x_1 = x_3 - x_2 = \ldots\ldots = x_n - x_{n-1} = \ldots\ldots$

In general, $x_{n+1} - x_n$ = Constant (denoted by d), here n is a natural number.

The constant difference d is called the *common difference* of the A.P. First term x_1 of the A.P. is denoted by 'a'. Hence the standard form of

A.P. is $a, a+d, a+2d, \ldots\ldots$

Formula for General Term of an A.P.

The n^{th} term of the A.P., is given by $a_n = a + (n-1)d, n \in N$

Here a_n is the n^{th} term of the A.P.

Formula for Sum of First n Terms of an A.P.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + \ell)$$

where ℓ = Last term up to which the sum of the A.P. is to find.

6. TRIANGLES

Criteria for similarity of triangles

(i) AAA Similarity Criterion or (Equi - angular criterion)

The corresponding angles of two triangles are equal, then their corresponding sides are proportional and hence the triangles are similar.

(ii) SSS Similarity Criterion : In this criterion, the corresponding sides of two triangles are proportional, then their corresponding angles are equal. Hence the triangles are said to be similar.

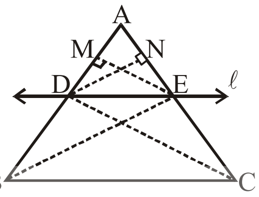
(iii) SAS Similarity Criterion : In this case, if one angle of a triangle is equal to one angle of the other and the sides including these angles are proportional.

Basic Proportionality Theorem or Thale's Theorem

Statement : "If a line is drawn parallel to one side of a triangle, it divides the other two sides in the same ratio."

In ΔABC, a line parallel to BC intersects AB at D and AC at E.

then $\frac{AD}{DB} = \frac{AE}{EC}$



Converse of B.P. Theorem

Statement : "If a line divides any two sides of a triangle in the same ratio, the line parallel to the third side".

In ΔABC, a line intersecting AB in D and AC in E. such that $\frac{AD}{DB} = \frac{AE}{EC}$ then DE || BC.

7. COORDINATE GEOMETRY

Distance Formula : The distance between two points whose co-ordinates are P (x_1, y_1) and Q (x_2, y_2) is given by the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance From Origin

$$\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

Section Formula : The coordinates of the point p (x, y) which divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) internally in the ratio $m_1 : m_2$ are

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \quad \text{and} \quad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\overset{m_1}{} \qquad \overset{m_2}{}$$
$$A(x_1, y_1) \quad P(x, y) \quad B(x_2, y_2)$$

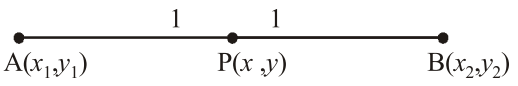
Note : If the ratio in which P (x, y) divides AB is K : 1, then the coordinates of the point P will be

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$$

Coordinates of Mid-Point : The mid-point of a line segment divides the line segment in the ratio 1 : 1

$\therefore$ The coordinates of the mid-point P of the join of the points A (x_1, y_1) and B (x_2, y_2) is

$$\left(\frac{1.x_1+1.x_2}{1+1}, \frac{1.y_1+1.y_2}{1+1}\right) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



REMARKS

(I) Four points will form :

(i) a **parallelogram** if its opposite sides are equal, but diagonals are unequal.

(ii) a **rectangle** if opposite sides are equal and two diagonals are also equal.

(iii) a **rhombus** if all the four sides are equal, but diagonals unequal,

(iv) a **square** if all sides are equal and diagonals are also equal.

(II) Three points will form:

(i) an equilateral triangle if all the three sides are equal.

(ii) an isosceles triangle if any two sides are equal.

(iii) a right angled triangle if sum of square of any two sides is equal to square of the third side.

(iv) a triangle if sum of any two sides (distances) is greater than the third side (distance).

(III) Three points A, B and C are collinear or lie on a line if one of the following holds

(i) AB + BC = AC (ii) AC + CB = AB

(iii) CA + AB = CB.

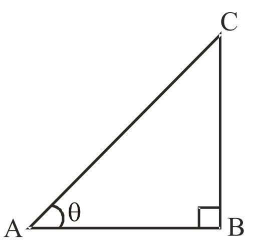
8. INTRODUCTION TO TRIGONOMETRY

Trigonometrical Ratios : (T - Ratios)

For right ΔABC, $B = 90°$, $A = \theta$

(i) $\frac{BC}{AC} = \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

(ii) $\frac{AB}{AC} = \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$



(iii) $\frac{BC}{AB} = \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

(iv) $\frac{AC}{BC} = \text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

(v) $\frac{AC}{AB} = \sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Base}}$

(vi) $\frac{AB}{BC} = \cot\theta = \frac{1}{\tan\theta} = \frac{\text{Base}}{\text{Perpendicular}}$

These six ratios are called trigonometric-ratios for the angle θ. We can write

$$\tan\theta = \frac{BC}{AB} = \frac{BC/AC}{AB/AC} = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Relations

(i) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ **(ii)** $\sec\theta = \frac{1}{\cos\theta}$ **(iii)** $\cot\theta = \frac{1}{\tan\theta}$

Trigonometric Ratios of some specific angles

The table for all T-ratios of some angles is given below :

θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric Identities

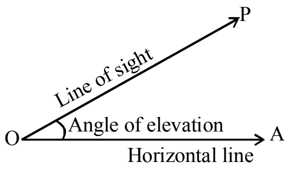
Identity I : $\sin^2\theta + \cos^2\theta = 1$
Identity II : $\sec^2\theta = 1 + \tan^2\theta$
Identity III : $\text{cosec}^2\theta = 1 + \cot^2\theta$

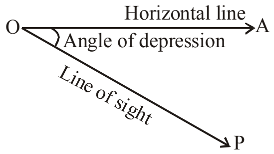
These are concerned with any right angled triangle with any acute angle θ.

9. APPLICATION OF TRIGONOMETRY

ANGLE OF ELEVATION AND ANGLE OF DEPRESSION

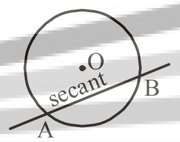
Let an observer at the point O is observing an object at the point P. The line OP is called the LINE OF SIGHT of the point P. Let OA be the horizontal line in the vertical plane passing through OP. If object P be above the horizontal line OA, then the acute angle AOP, between the line of sight and the horizontal line is known as ANGLE OF ELEVATION of object P. If the object P is below the horizontal line OA, then the acute angle AOP, between the line of sight and horizontal line is known as ANGLE OF DEPRESSION of object P.





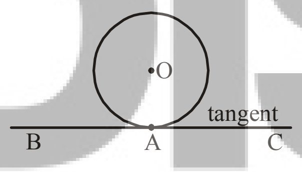
10. CIRCLES

Secant of a circle : A line which intersects a circle in two distinct points is called a secant of the circle.



Note : A line can meet a circle at most in two distinct points.

Tangent to a circle : A line which meets a circle exactly at one point is called a tangent to the circle. In adjoining figure, the line BAC is a tangent to the circle with centre O.



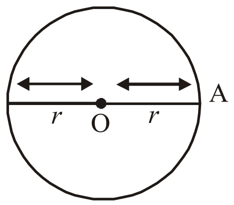
Theorems Related to Tangent to a circle

Theorem 1 : The tangent at any point of a circle is perpendicular to the radius through the point of contact. We have given a circle with centre O and a tangent XY to the circle at a point P then OP is perpendicular to XY.

Theorem 2 : The lengths of tangents drawn from an external point to a circle are equal. We have a circle with centre O, a point P lying outside the circle and two tangents PQ, PR on the circle from P then PQ = PR.

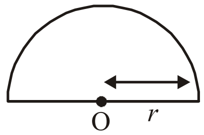
11. AREA RELATED TO CIRCLES

Area of a circle : Area of a circle = πr^2 where 'r' is the radius of the circle.



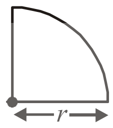
Area of a Semi-circle :

Area of a semi-circle = $\frac{1}{2}\pi r^2$

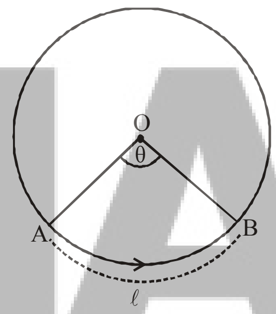


Area of a Quadrant :

Area of a quadrant = $\frac{\pi r^2}{4}$



Perimeter and Area of Sector of a Circle : Two radii OA and OB enclose a portion of the circular region which makes a central angle θ. The region is called a sector of the circle. In fig., AOB is the sector with central angle θ. Let 'ℓ' be the length of arc AB. Then,



$$\ell = 2\pi r.\frac{\theta}{360} = \frac{\pi r\theta}{180}$$

$$\text{Perimeter of the sector} = OA + OB + AB = 2r + \frac{\pi r\theta}{180}$$

Also, area of sector is given by $\frac{\text{Area of sector } AOB}{\text{Area of the circle}} = \frac{\theta}{360}$

$$\text{Area of sector } AOB = \pi r^2 \frac{\theta}{360}$$

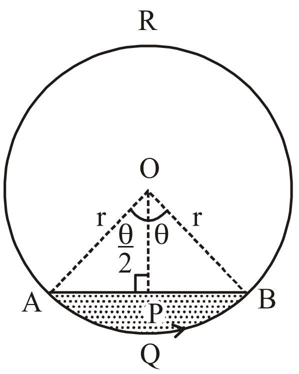
Area of a Segment

In right ΔOPA,

$$AP = r\sin\left(\frac{\theta}{2}\right),\ AB = 2AP = 2r\sin\left(\frac{\theta}{2}\right),\ OP = r\cos\left(\frac{\theta}{2}\right)$$

Area of minor segment $AQBPA$ = (Area of sector OAQBO) – (Area of ΔOAB)

$$= \frac{\pi r^2\theta}{360} - \frac{1}{2}\times AB\times OP$$



$$= \frac{\pi r^2\theta}{360} - r^2 \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

Area of major segment *APBRA* = (Area of the circle) – (Area of minor segment *AQBPA*)

$$= \pi r^2 - \frac{\pi r^2\theta}{360} + r^2 \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right).$$

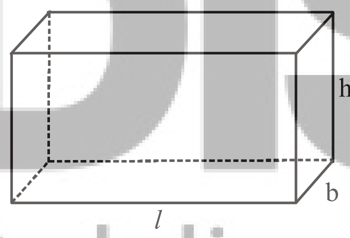
12. SURFACE AREAS AND VOLUMES

Surface Areas and Volumes of Solids

(i) CUBOID

If 'l' be the length, 'b' be the breadth and 'h' be the height (or depth) of a cuboid, then

Volume = length × breadth × height = $l \times b \times h$



Total surface area = $2\,(lb + bh + hl)$

Lateral surface area = $2\,(bh + hl)$

Diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

(ii) CUBE

In a cube, all the three dimensions i.e., its length, breadth and height are equal.

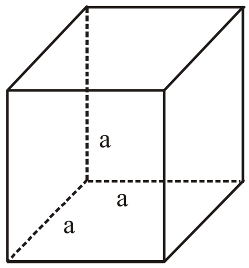
If 'a' be the edge of a cube, then

Volume = a^3

Total surface area = $6a^2$

Lateral surface area = $4a^2$

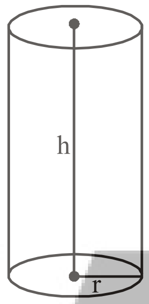
Diagonal of a cube = $\sqrt{3}$ × edge = $\sqrt{3}\,a$



(iii) CYLINDER

(a) Right circular cylinder : For a right circular cylinder of base radius r and height h, we have

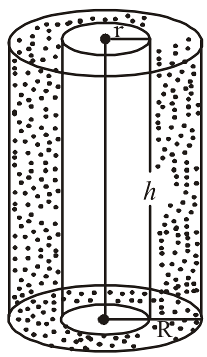
- Area of each end = Area of base = πr^2
- Area of curved surface = $2\pi rh$
- Total surface area = $2\pi r\,(h + r)$
- Volume of the cylinder = $\pi r^2 h$



(b) Right circular hollow cylinder : A cylinder from which a smaller cylinder of the same height and of the same axis is cut out is called a hollow cylinder.

If 'r' & 'R' be the internal & external radii respectively of a hollow cylinder of height 'h', then

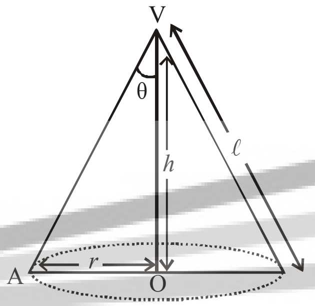
- Volume of the hollow cylinder = $(\pi R^2 - \pi r^2)h$
  $= \pi\,(R^2 - r^2)h$
- Area of the curved surface = $2\pi Rh + 2\pi rh$
  $= 2\pi(R + r)\,h$
- Total surface area = area of curved surface + 2 (area of a base)
  $= 2\pi\,(R + r)h + 2\pi\,(R^2 - r^2)$
  $= 2\pi\,(R + r)\,(h + R - r)$



(c) Right circular cone : It is a solid generated by the revolution of a right angled triangle about one of its sides containing the right angle as axis.

For a right circular cone of height 'h', slant height ℓ and radius of base 'r' we have

- $\ell^2 = h^2 + r^2$
- Area of base $= \pi r^2$
- Curved surface area $= \pi r\ell$



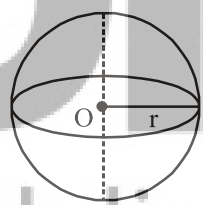
- Total surface area = Curved surface area + Area of the base $= \pi r\ell + \pi r^2$

Volume $= \frac{1}{3}\pi r^2 h$

(iv) SPHERE

(a) Sphere

For a sphere of radius 'r', we have

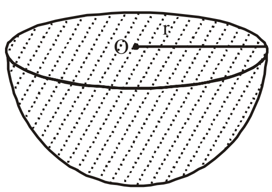


Surface area $= 4\pi r^2$

Volume $= \frac{4}{3}\pi r^3$

(b) Hemisphere

For a hemisphere of radius r, we have



Curved surface area $= 2\pi r^2$

Total surface area $= 3\pi r^2$

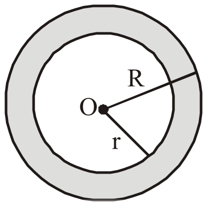
Volume $= \frac{2}{3}\pi r^3$

(c) Hollow Sphere/Spherical Shell

From a sphere, a smaller sphere having the same centre of the sphere, is cut off, then hollow sphere is obtained.

External surface area $= 4\pi R^2$

Internal surface area $= 4\pi r^2$

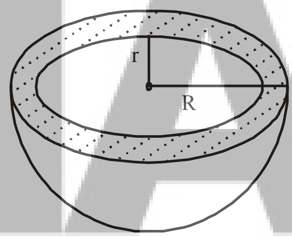


Volume $= \frac{4}{3}\pi\left(R^3 - r^3\right)$

(d) Hemispherical Shell

If a spherical shell is cut into two halves by a plane passing through the centre of spherical shell, then each of the two halves is called a hemispherical shell.

Internal curved surface area $= 2\pi r^2$ sq. units



External curved surface area $= 2\pi R^2$ sq. units

Total surface area = Internal surface area + Ext. surface area + Area of ring

$= 2\pi r^2 + 2\pi R^2 + \pi(R^2 - r^2) = \pi r^2 + 3\pi R^2 = \pi(r^2 + 3R^2)$ units

Volume of the material used to form hemispherical shell $= \frac{2}{3}\pi\left(R^3 - r^3\right)$ cubic units

13. STATISTICS

Mean of Grouped data

In frequency distribution with frequencies for the values of $x_1, x_2, x_3 \ldots\ldots\ldots x_n$ are $f_1, f_2, f_3 \ldots\ldots\ldots\ldots f_n$, respectively. Then,

$$\text{Mean } (\bar{x}) = \frac{f_1x_1 + f_2x_2 + \ldots\ldots\ldots\ldots + f_nx_n}{f_1 + f_2 + \ldots\ldots\ldots\ldots + f_n} = \sum_{i=1}^{n}\frac{f_ix_i}{\Sigma f_i}$$

Short cut method (Assumed mean Method)

As per this method : $\bar{x} = A + \frac{1}{N}\sum_{i=1}^{n} f_i d_i$ where A is assumed mean and $d_i = x_i - A$, $N = \Sigma f_i$

Note :

1. This method is used when large calculation is involved in frequency data calculations and it is tedious to calculate mean value etc. by conventional method.
2. Generally the middle most value is considered as assumed mean.

Step Deviation Method

If the deviation **di's** are divisible by any common number **C**, then

$u_i = \frac{x_i - A}{C}$ and we use formula $\bar{x} = A + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)C$

where 'C' is differences between successive x_is.or we can say C = class size.

Note :

1. The step-deviation method will be convenient to apply if all the d_is have a common factor.
2. The mean obtained by all the three methods is the same.

Mode

Mode is that value of observations having maximum frequency.

Mode of grouped data

Mode = $\ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ where,

ℓ = lower limit of the modal class*
h = size of the class interval
f_1 = frequency of the modal class
f_0 = frequency of the class preceeding the modal class
f_2 = frequency of the class succeeding the modal class.

Note : Modal class is the class having maximum frequency.

Median

Median is the value of the middle-most observation in the data.

Median of ungrouped data

We first put the data values in the ascending order, then the median is the $\left(\frac{n+1}{2}\right)$th observation if n is odd, and if n is even, then the median is $\frac{1}{2}\left[\frac{n}{2}\text{th} + \left(\frac{n}{2}+1\right)\text{th}\right]$ observation.

Median of Grouped Data

Step I : Make cumulative frequency table.

Step II : Choose the median class. Median class is the class whose cumulative frequency is greater than and nearest to $\frac{n}{2}$ where n is the sum of all frequencies.

Step III : Use this formula

$$\text{Median} = \ell + \left[\frac{\frac{n}{2} - c.f}{f}\right] \times h$$

where, ℓ = lower limit of median class
n = sum of all frequencies
c.f = cumulative frequency of class preceding the median class
f = frequency of the median class
h = class size

Note : The relationship between three measures of central tendency.

3 Median = Mode + 2 Mean

14. PROBABILITY

EXPERIMENT : An operation which can produce some well defined outcomes, is known as an experiment.

TRIAL : Performing of an experiment is called Trial. *For example* : Tossing a coin, throwing a dice.

EVENT : The outcomes of an experiment are called events. *For example :* Getting a head or tail tossing a coin is an Event.

SAMPLE SPACE : The set of all possible out comes in a trial is called sample space.

For instance :

(i) If a fair coin is tossed, there are two possible outcomes, namely head (H) & Tail (T).
$\therefore$ Sample space S = {H, T}

(ii) In unbaised die is thrown; S = {1, 2, 3, 4, 5, 6}

(iii) When two coins are tossed ; S = {HH, HT, TH, TT}

Probability

Mathematically, Probability of an event E, is defined as,

$$P(E)=\frac{n(E)}{n(S)}=\frac{\text{No.of outcomes of favourable cases to E}}{\text{Total No. of possible outcomes}}$$

The probability of an event E is a number between 0 and 1 inclusive i.e., $0 \le P(E) \le 1$

(i) If $P(E) = 0$, then the event cannot possibly occur. An event that cannot occur has 0 probability; Such an event is called impossible event.

(ii) If $P(E) = 1$, then the event is certain to occur. An event that is certain to occur has probability equal to one and is called a sure event.

Complementary Event

Let $\bar{E}$ denote the event 'E does not occur'. Then

$$P(\bar{E})=\frac{n(\bar{E})}{n(S)}=\frac{n(S)-n(E)}{n(S)}=1-\frac{n(E)}{n(S)}$$

$$\Rightarrow P(\bar{E})=1-P(E) \Rightarrow P(E)+P(\bar{E})=1$$

i.e. P(E) + P(not E) = 1

Thus P (not E) = 1– P (E), this event is said to be a complementary event.