# MATHEMATICS CLASS-XII
# REVISION CHEAT SHEET

## RELATIONS AND FUNCTIONS

- A relation R from a set A to a set B is a subset of the cartesian product $A \times B$ obtained by describing a relationship between the first element x and the second element y of the ordered pairs in $A \times B$.
- **Function :** A function f from a set A to a set B is a specific type of relation for which every element x of set A has one and only one image y in set B. We write $f : A \to B$, where $f(x) = y$.
- A function $f : X \to Y$ is one-one (or injective) if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \ \forall \ x_1, x_2 \in X$.
- A function $f : X \to Y$ is onto (or surjective) if given any $y \in Y, \exists \ x \in X$ such that Range = codomain.
- **Many-One Function :**
  A function $f : A \to B$ is called many- one, if two or more different elements of A have the same f- image in B.
- **Into function :**
  A function $f : A \to B$ is into if there exist at least one element in B which is not the f - image of any element in A.
- **Many One -Onto function :**
  A function $f : A \to R$ is said to be many one- onto if f is onto but not one-one.
- **Many One -Into function :**
  A function is said to be many one-into if it is neither one-one nor onto.
- A function $f : X \to Y$ is invertible if and only if f is one-one and onto.

## INVERSE TRIGONOMETRIC FUNCTIONS

- **Properties of inverse trigonometric function**

$$\tan^{-1} x + \tan^{-1} y = \begin{cases} \tan^{-1}\left(\dfrac{x+y}{1-xy}\right), & \text{if } xy < 1 \\ \pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right), & \text{if } x > 0, y > 0 \text{ and } xy > 1 \\ -\pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right), & \text{if } x < 0, y < 0 \text{ and } xy > 1 \end{cases}$$

$$\tan^{-1} x - \tan^{-1} y = \begin{cases} \tan^{-1}\left(\dfrac{x-y}{1+xy}\right), & \text{if } xy > -1 \\ \pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right), & \text{if } x > 0, y < 0 \text{ and } xy < -1 \\ -\pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right), & \text{if } x < 0, y > 0 \text{ and } xy < -1 \end{cases}$$

$$\sin^{-1} x + \sin^{-1} y =$$

$$\begin{cases} \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}, & \text{if } -1 \le x, \ y \le 1 \text{ and } x^2 + y^2 \le 1 \text{ or if } xy < 0 \text{ and } x^2 + y^2 > 1 \\ \pi - \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}, & \text{if } 0 < x, \ y \le 1 \text{ and } x^2 + y^2 > 1 \\ -\pi - \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}, & \text{if } -1 \le x, \ y < 0 \text{ and } x^2 + y^2 > 1 \end{cases}$$

$$\cos^{-1} x + \cos^{-1} y =$$

$$\begin{cases} \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}, & \text{if } -1 \le x, y \le 1 \text{ and } x + y \ge 0 \\ 2\pi - \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}, & \text{if } -1 \le x, y \le 1 \text{ and } x + y \le 0 \end{cases}$$

$$2\sin^{-1} x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } -\dfrac{1}{\sqrt{2}} \le x \le \dfrac{1}{\sqrt{2}} \\ \pi - \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } \dfrac{1}{\sqrt{2}} \le x \le 1 \\ -\pi - \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } -1 \le x \le -\dfrac{1}{\sqrt{2}} \end{cases}$$

$$2\tan^{-1} x = \begin{cases} \tan^{-1}\left(\dfrac{2x}{1-x^2}\right), & \text{if } -1 < x < 1 \\ \pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right), & \text{if } x > 1 \\ -\pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right), & \text{if } x < -1 \end{cases}$$

## THREE DIMENSIONAL GEOMETRY

- **Conditions of Parallelism and Perpendicularity of Two Lines:**
  **Case-I :** When dc's of two lines AB and CD, say $\ell_1, m_1, n_1$ and $\ell_2, m_2, n_2$ are known.

$$AB \,||\, CD \Leftrightarrow \frac{\ell_1}{\ell_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

$$AB \perp CD \Leftrightarrow \ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$$

**Case-II :** When dr's of two lines AB and CD, say $a_1, b_1, c_1$ and $a_2, b_2, c_2$ are known

$$AB || CD \Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$AB \perp CD \Leftrightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$$

- If $\ell_1, m_1, n_1$ and $\ell_2, m_2, n_2$ are the direction cosines of two lines; and θ is the acute angle between the two lines; then $\cos\theta = |\ell_1\ell_2 + m_1m_2 + n_1n_2|$.
- Equation of a line through a point $(x_1, y_1, z_1)$ and having direction cosines $\ell$, m, n is $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$
- Shortest distance between $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is $\left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

## DIFFERENTIAL CALCULUS

- **Existence of Limit :**

$\lim_{x\to a} f(x)$ exists $\Rightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = \ell$

Where $\ell$ is called the limit of the function

- (i) If $f(x) \le g(x)$ for every x in the deleted nbd of a, then $\lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$

(ii) If $f(x) \le g(x) \le h(x)$ for every x in the deleted nbd of a and $\lim_{x\to a} f(x) = \ell = \lim_{x\to a} h(x)$ then $\lim_{x\to a} g(x) = \ell$

(iii) $\lim_{x\to a} fog(x) = f\left(\lim_{x\to a} g(x)\right) = f(m)$ where $\lim_{x\to a} g(x) = m$

(iv) If $\lim_{x\to a} f(x) = +\infty$ or $-\infty$, then $\lim_{x\to a} \frac{1}{f(x)} = 0$

### CONTINUITY AND DIFFERENTIABILITY OF FUNCTIONS

- A function f(x) is said to be continuous at a point x = a if $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$
- **Discontinuous Functions :**

**(a) Removable Discontinuity:** A function f is said to have removable discontinuity at x = a if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ but their common value is not equal to f (a).

**(b) Discontinuity of the first kind:** A function f is said to have a discontinuity of the first kind at x = a if $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ both exist but are not equal.

**(c) Discontinuity of second kind:** A function f is said to have a discontinuity of the second kind at x = a if neither $\lim_{x\to a^-} f(x)$ nor $\lim_{x\to a^+} f(x)$ exists.

Similarly, if $\lim_{x\to a^+} f(x)$ does not exist, then f is said to have discontinuity of the second kind from the right at x = a.

- **For a function f :**

Differentiability ⇒ Continuity;

Continuity ⇏ derivability

Not derivibaility ⇏ discontinuous ;

But discontinuity ⇒ Non derivability

- **Differentiation of infinite series:**

**(i)** If $y = \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x) + ........\infty}}}$

$$\Rightarrow \quad y = \sqrt{f(x) + y} \Rightarrow y^2 = f(x) + y$$

$$2y\frac{dy}{dx} = f'(x) + \frac{dy}{dx} \qquad \therefore \frac{dy}{dx} = \frac{f'(x)}{2y-1}$$

**(ii)** If $y = f(x)^{f(x)^{f(x)^{....\infty}}}$ then $y = f(x)^y$.

$\therefore \quad \log y = y \log [f(x)]$

$$\frac{1}{y}\frac{dy}{dx} = \frac{y.f'(x)}{f(x)} + \log f(x).\left(\frac{dy}{dx}\right)$$

$$\therefore \quad \frac{dy}{dx} = \frac{y^2 f'(x)}{f(x)[1 - y\log f(x)]}$$

**(iii)** If $y = f(x) + \cfrac{1}{f(x) + \cfrac{1}{f(x) + ....}}$ then $\frac{dy}{dx} = \frac{y f'(x)}{2y - f(x)}$

## DIFFERENTIATION AND APPLICATION

- **Interpretation of the Derivative :** If y = f(x) then, $\frac{dy}{dx} = f'(x)$ is rate of change of y with respect to x.
- **Increasing/Decreasing :**

(i) If $f'(x) > 0$ for all x in an interval I then f(x) is increasing on the interval I.

(ii) If $f'(x) < 0$ for all x in an interval I then f(x) is decreasing on the interval I.

(iii) If $f'(x) = 0$ for all x in an interval I then f(x) is constant on the interval I.

- **Test of Local Maxima and Minima –**

**First Derivative Test** – Let f be a differentiable function defined on an open interval I and $c \in I$ be any point. f has a local maxima or a local minima at x = c, $f'(c) = 0$.

Put $\frac{dy}{dx} = 0$ and solve this equation for x. Let $c_1, c_2........c_n$ be the roots of this.

If $\frac{dy}{dx}$ changes sign from +ve to –ve as x increases through $c_1$ then the function attains a local max at $x = c_1$

If $\frac{dy}{dx}$ changes its sign from –ve to +ve as x increases through $c_1$ then the function attains a local minimum at $x = c_1$

If $\frac{dy}{dx}$ does not changes sign as x increases through $c_1$ then $x = c_1$ is neither a point of local max$^m$ nor a point of local min$^m$. In this case x is a point of inflexion.

- **Rate of change of variable :**

The value of $\frac{dy}{dx}$ at $x = x_0$ i.e. $\left(\frac{dy}{dx}\right)_{x=x_0}$ represents the rate of change of y with respect to x at $x = x_0$

If $x = \phi(t)$ and $y = \psi(t)$, then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, provided that $\frac{dx}{dt} \neq 0$

Thus, the rate of change of y with respect to x can be calculated by using the rate of change of y and that of x each with respect to t.

## INTEGRAL CALCULUS AND APPLICATIONS

- **Two standard forms of integral :**

$\int e^x [f(x) + f'(x)\ dx = e^x f(x) + c$

$\Rightarrow \int e^x [f(x) + f'(x)]\ dx = \int e^x f(x) dx + \int e^x f'(x) dx$

$= e^x f(x) - \int e^x f'(x) dx + \int e^x f'(x)$

(on integrating by parts) $= e^x f(x) + c$

- Table shows the partial fractions corresponding to different type of rational functions :

| S. No. | Form of rational function | Form of partial fraction |
|---|---|---|
| 1. | $\frac{px+q}{(x-a)(x-b)}$ | $\frac{A}{(x-a)} + \frac{B}{(x-b)}$ |
| 2. | $\frac{px^2+qx+r}{(x-a)^2(x-b)}$ | $\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$ |
| 3. | $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$ | $\frac{A}{(x-a)} + \frac{Bx+C}{x^2+bx+c}$ |

- Area between curves :

$$y = f(x) \Rightarrow A = \int_a^b [\text{upper function}] - [\text{lower function}]\, dx$$

and $$x = f(y) \Rightarrow A = \int_c^d [\text{right function}] - [\text{left function}]\, dy$$

If the curves intersect then the area of each portion must be found individually.

- **Symmetrical area :** If the curve is symmetrical about a coordinate axis (or a line or origin), then we find the area of one symmetrical portion and multiply it by the number of symmetrical portion to get the required area.

## PROBABILITY

- **Probability of an event:** For a finite sample space with equally likely outcomes Probability of an event is

$P(A) = \frac{n(A)}{n(S)}$, where n (A) = number of elements of an event A, n (S) = Total number of sample space.

- **Theorem of total probability :** Let $\{E_1, E_2, ..., E_n\}$ be a partition of a sample spaces S and suppose that each of $E_1, E_2, ..., E_n$ has nonzero probability. Let A be any event associated with S, then

$P(A) = P(E_1) P(A | E_1) + P(E_2) P(A | E_2) + ...$
$+ P(E_n) P(A | E_n)$

- **Bayes' theorem:** If $E_1, E_2, ..., E_n$ are events which constitute a partition of sample space S, i.e. $E_1, E_2, ..., E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup ... \cup E_n = S$ and A be any event with nonzero probability, then

$$P(E_i | A) = \frac{P(E_i) P(A | E_i)}{\sum_{j=1}^{n} P(E_j) P(A | E_j)}$$

- Let X be a random variable whose possible values $x_1, x_2, x_3, ..., x_n$ occur with probabilities $p_1, p_2, p_3, ... p_n$ respectively.

The mean of X, denoted by $\mu$, is the number $\sum_{i=1}^{n} x_i p_i$

The mean of a random variable X is also called the expectation of X, denoted by E (X).

## MATRICES

- **Properties of Transpose**

(i) $(A^T)^T = A$

(ii) $(A \pm B)^T = A^T \pm B^T$

(iii) $(AB)^T = B^T A^T$  (iv) $(kA)^T = k(A)^T$

(v) $I^T = I$ (vi) $tr(A) = tr(A)^T$

(vii) $(A_1A_2A_3.....A_{n-1}A_n)^T = A_n^T A_{n-1}^T.....A_3^T A_2^T A_1^T$

- **Symmetric Matrix :** A square matrix $A = [a_{ij}]$ is called symmetric matrix if

$a_{ij} = a_{ji}$ for all i, j or $A^T = A$

- **Skew-Symmetric Matrix :** A square matrix $A = [a_{ij}]$ is called skew-symmetric matrix if

$a_{ij} = -a_{ji}$ for all i, j or $A^T = -A$

Also every square matrix A can be uniquely expressed as a sum of a symmetric and skew-symmetric matrix.

## DETERMINANTS

- **Differentiation of a determinants :** If $A = \begin{vmatrix} f(x) & g(x) \\ h(x) & \ell(x) \end{vmatrix}$ then

  $\frac{dA}{dx} = \begin{vmatrix} f'(x) & g'(x) \\ h(x) & \ell(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) \\ h'(x) & \ell'(x) \end{vmatrix}$ is a differentiation of Matrix A

- **Properties of adjoint matrix :** If A, B are square matrices of order n and $I_n$ is corresponding unit matrix, then
  (i) $A(\text{adj. } A) = |A| I_n = (\text{adj } A) A$
  (ii) $|\text{adj } A| = |A|^{n-1}$
  (Thus A (adj A) is always a scalar matrix)
  (iii) $\text{adj}(\text{adj } A) = |A|^{n-2} A$
  (iv) $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$
  (v) $\text{adj}(A^T) = (\text{adj } A)^T$
  (vi) $\text{adj}(AB) = (\text{adj } B)(\text{adj } A)$
  (vii) $\text{adj}(A^m) = (\text{adj } A)^m, m \in N$
  (viii) $\text{adj}(kA) = k^{n-1}(\text{adj. } A), k \in R$
  (ix) $\text{adj}(I_n) = I_n$

- **Properties of Inverse Matrix :** Let A and B are two invertible matrices of the same order, then
  (i) $(A^T)^{-1} = (A^{-1})^T$
  (ii) $(AB)^{-1} = B^{-1}A^{-1}$
  (iii) $(A^k)^{-1} = (A^{-1})^k, k \in N$
  (iv) $\text{adj}(A^{-1}) = (\text{adj } A)^{-1}$
  (v) $(A^{-1})^{-1} = A$
  (vi) $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$
  (vii) If $A = \text{diag}(a_1, a_2, \ldots, a_n)$, then
  $A^{-1} = \text{diag}(a_1^{-1}, a_2^{-1}, \ldots\ldots\ldots a_n^{-1})$
  (viii) A is symmetric matrix $\Rightarrow A^{-1}$ is symmetric matrix.

## VECTOR ALGEBRA

- Vector perpendicular to both $\vec{a}$ and $\vec{b}$ is equal to $\vec{a} \times \vec{b}$.

## DIFFERENTIAL EQUATIONS

- **Methods of solving a first order first degree differential equation :**

**(a) Differential equation of the form** $\frac{dy}{dx} = f(x)$

$\frac{dy}{dx} = f(x) \Rightarrow dy = f(x)\,dx$

Integrating both sides we obtain

$\int dy = \int f(x)\,dx + c$ or $y = \int f(x)\,dx + c$

**(b) Differential equation of the form** $\frac{dy}{dx} = f(x)\,g(y)$

$\frac{dy}{dx} = f(x)\,g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)\,dx + c$

**(c) Differential equation of the form of**

$\frac{dy}{dx} = f(ax + by + c)$ :

To solve this type of differential equations, we put

$ax + by + c = v$ and $\frac{dy}{dx} = \frac{1}{b}\left(\frac{dv}{dx} - a\right)$

$\therefore \frac{dv}{a + b f(v)} = dx$

So solution is by integrating $\int \frac{dv}{a + b f(v)} = \int dx$

**(d) Differential Equation of homogeneous type :**

An equation in x and y is said to be homogeneous if it can be put in the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ where f (x ,y) and g (x ,y) are both homogeneous functions of the same degree in each term.

So to solve the homogeneous differential equation $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$, substitute $y = vx$ and so $\frac{dy}{dx} = v + x\frac{dV}{dx}$

Thus $v + x\frac{dv}{dx} = f(v) \Rightarrow \frac{dx}{x} = \frac{dv}{f(v) - v}$

Therefore solution is $\int \frac{dx}{x} = \int \frac{dv}{f(v) - v} + c$

- **Linear differential equations :**

$\frac{dy}{dx} + Py = Q$ ....... (1)

Where P and Q are either constants or functions of x.

Multiplying both sides of (1) by $e^{\int P\,dx}$, we get

$e^{\int P\,dx}\left(\frac{dy}{dx} + Py\right) = Q\,e^{\int P\,dx}$

On integrating both sides with respect to x we get

$y\,e^{\int P\,dx} = \int Q\,e^{\int P\,dx} + c$

which is the required solution, where c is the constant and $e^{\int P\,dx}$ is called the integration factor.