AP/EAMCET Solved Paper 2015

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. If $f : R \to R$, $g : R \to R$ are defined by $f(x) = 5x - 3$, $g(x) = x^2 + 3$, then $gof^{-1}(3)$ is equal to

(a) $\frac{25}{3}$ (b) $\frac{111}{25}$ (c) $\frac{9}{25}$ (d) $\frac{25}{111}$

2. If $A = \left\{x \in R / \frac{\pi}{4} \le x \le \frac{\pi}{3}\right\}$ and $f(x) = \sin x - x$, then $f(A)$ is equal to

(a) $\left[\frac{\sqrt{3}}{2} - \frac{\pi}{3}, \frac{1}{\sqrt{2}} - \frac{\pi}{4}\right]$ (b) $\left[\frac{-1}{\sqrt{2}} - \frac{\pi}{4}, \frac{\sqrt{3}}{2} - \frac{\pi}{3}\right]$

(c) $\left[-\frac{\pi}{3}, -\frac{\pi}{4}\right]$ (d) $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$

3. The value of the sum $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \ldots$upto n terms is equal to

(a) $\frac{1}{6}n^2\left(2n^2+1\right)$

(b) $\frac{1}{6}\left(n^2-1\right)\left(2n-1\right)\left(2n+3\right)$

(c) $\frac{1}{8}\left(n^2+1\right)\left(n^2+5\right)$

(d) $\frac{1}{4}n\left(n+1\right)\left(n+2\right)\left(n+3\right)$

4. The value of the determinant

$$\begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix}$$ is

(a) abc (b) $a + b + c$

(c) 0 (d) $ab + bc + ca$

5. If A is a square matrix of order 3, then $|\text{adj (adj } A^2)|$ is equal to

(a) $|A|^2$ (b) $|A|^4$ (c) $|A|^8$ (d) $|A|^{16}$

6. The system $2x + 3y + z = 5$, $3x + y + 5z = 7$ and $x + 4y - 2z = 3$ has

(a) unique solution (b) finite number of solution

(c) infinite solutions (d) no solution

7. $\sum_{k=1}^{6}\left[\sin\frac{2k\pi}{7} - i\cos\frac{2k\pi}{7}\right]$ is equal to

(a) -1 (b) 0 (c) $-i$ (d) i

8. If ω is a complex cube root of unity, then

$\omega^{\left(\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\ldots\infty\right)} + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\ldots\infty\right)}$ is equal to

(a) 1 (b) -1 (c) ω (d) i

9. The common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$, $z^{2014} + z^{2015} + 1 = 0$ are

(a) ω , ω^2 (b) $1, \omega, \omega^2$

(c) $-1, \omega$, ω^2 (d) $-\omega, -\omega^2$

10. If a, b, c are distinct and the roots of $(b - c) x^2 + (c - a) x + (a - b) = 0$ are equal, then a, b and c are in

(a) arithmetic progression

(b) geometric progression

(c) harmonic progression

(d) arithmetico-geometric progression

11. If the roots of $x^3 - kx^2 + 14x - 8 = 0$ are in geometric progression, then k is equal to

(a) -3 (b) 7 (c) 4 (d) 0

12. If the harmonic mean of the roots $\sqrt{2}x^2 - bx + \left(8 - 2\sqrt{d}\right) = 0$ is 4, then the value of b is
(a) 2 (b) 3 (c) $4-\sqrt{5}$ (d) $4+\sqrt{5}$

13. For real value of x, the range of $\dfrac{x^2+2x+1}{x^2+2x-1}$ is
(a) $(-\infty, 0) \cup (1, \infty)$ (b) $\left[\dfrac{1}{2}, 2\right]$
(c) $\left(-\infty, \dfrac{-2}{9}\right] \cup (1, \infty)$ (d) $(-\infty, -6] \cup (-2, \infty)$

14. T_m denotes the number of triangles that can be formed with the vertices of a regular polygon of m sides. If $T_{m+1} - T_m = 15$, then m is equal to
(a) 3 (b) 6 (c) 9 (d) 12

15. If $|x| < 1$, then the coefficient of x^5 in the expansion of $\dfrac{3x}{(x-2)(x+1)}$ is
(a) $\dfrac{33}{32}$ (b) $\dfrac{-33}{32}$ (c) $\dfrac{31}{32}$ (d) $\dfrac{-31}{32}$

16. If the coefficients of x^9, x^{10} and x^{11} in the expansion of $(1+x)^n$ are in arithmetic progression, then $n^2 - 41n$ is equal to
(a) 399 (b) 298 (c) –398 (d) 198

17. If $\sin\theta + \cos\theta = p$ and $\tan\theta + \cot\theta = q$, then $q(p^2 - 1)$ is equal to
(a) $\dfrac{1}{2}$ (b) 2 (c) 1 (d) 3

18. $\tan\dfrac{\pi}{5} + 2\tan\dfrac{2\pi}{5} + 4\cot\dfrac{4\pi}{5}$ is equal to
(a) $\cot\dfrac{\pi}{5}$ (b) $\cot\dfrac{2\pi}{5}$ (c) $\cot\dfrac{3\pi}{5}$ (d) $\cot\dfrac{4\pi}{5}$

19. If $\sin A + \sin B + \sin C = 0$ and
$\cos A + \cos B + \cos C = 0$, then
$\cos(A+B) + \cos(B+C) + \cos(C+A)$ is equal to
(a) $\cos(A+B+C)$ (b) 2
(c) 1 (d) 0

20. If $\tan\theta \cdot \tan(120^\circ - \theta)\tan(120^\circ + \theta) = \dfrac{1}{\sqrt{3}}$, then θ is equal to
(a) $\dfrac{n\pi}{3} + \dfrac{\pi}{18}, n \in \mathbf{Z}$ (b) $\dfrac{n\pi}{3} + \dfrac{\pi}{12}, n \in \mathbf{Z}$
(c) $\dfrac{n\pi}{12} + \dfrac{\pi}{12}, n \in \mathbf{Z}$ (d) $\dfrac{n\pi}{3} + \dfrac{\pi}{6}, n \in \mathbf{Z}$

21. If $\sin^{-1}\left(x - \dfrac{x^2}{2} + \dfrac{x^3}{4} - \ldots\infty\right) + \cos^{-1}\left(x^2 - \dfrac{x^4}{2} + \dfrac{x^6}{4} - \ldots\infty\right) = \dfrac{\pi}{2}$ and $0 < x < \sqrt{2}$, then x is equal to
(a) $\dfrac{1}{2}$ (b) 1
(c) $-\dfrac{1}{2}$ (d) –1

22. If $2\sinh^{-1}\left(\dfrac{a}{\sqrt{1-a^2}}\right) = \log\left(\dfrac{1+x}{1-x}\right)$, then x is equal to
(a) a (b) $\dfrac{1}{a}$ (c) $\sqrt{1-a^2}$ (d) $\dfrac{1}{\sqrt{1-a^2}}$

23. If in a ΔABC, $r_1 = 2r_2 = 3r_3$, then the perimeter of the triangle is equal to
(a) $3a$ (b) $3b$
(c) $3c$ (d) $3(a+b+c)$

24. If M_1, M_2, M_3 and M_4, are respectively the magnitudes of the vectors $\mathbf{a}_1 = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{a}_2 = -3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, $\mathbf{a}_3 = -\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{a}_4 = -\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$, then the correct order of M_1, M_2, M_3 and M_4 is
(a) $M_3 < M_1 < M_4 < M_2$ (b) $M_3 < M_1 < M_2 < M_4$
(c) $M_3 < M_4 < M_1 < M_2$ (d) $M_3 < M_4 < M_2 < M_1$

25. If $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ are unit vectors such that $\hat{\mathbf{a}} + \hat{\mathbf{b}} + \hat{\mathbf{c}} = \mathbf{0}$, then the $\hat{\mathbf{a}} \cdot \hat{\mathbf{b}} + \hat{\mathbf{b}} \cdot \hat{\mathbf{c}} + \hat{\mathbf{c}} \cdot \hat{\mathbf{a}}$ is equal to
(a) $\dfrac{3}{2}$ (b) $-\dfrac{3}{2}$ (c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{2}$

26. If $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{c} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$, then the vector r satisfying r × b = c × b and r . a = 0 is
(a) $\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ (b) $\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$
(c) $\hat{\mathbf{i}} - 8\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ (d) $-\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

27. If **a**, **b** and **c** are three vectors such that $|\mathbf{a}| = 1$, $|\mathbf{b}| = 2$, $|\mathbf{c}| = 3$ and $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0$, then $|[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]|$ is equal to
(a) 2 (b) 3 (c) 4 (d) 6

28. If $[\mathbf{a} \times \mathbf{b}\ \ \mathbf{b} \times \mathbf{c}\ \ \mathbf{c} \times \mathbf{a}] = \lambda\,[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2$, then λ is equal to
(a) 0 (b) 1 (c) 2 (d) 3

29. The cartesian equation of the plane passing through the point (3, –2, –1) and parallel to the vectors $\mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\mathbf{c} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ is
(a) $2x - 17y - 8z + 63 = 0$
(b) $3x + 17y + 8z - 36 = 0$
(c) $2x + 17y + 8z + 36 = 0$
(d) $3x - 16y + 8z - 63 = 0$

30. The arithmetic mean of the observations 10, 8, 5, a, b is 6 and their variance is 6.8, then ab is equal to
(a) 6 (b) 4 (c) 3 (d) 12

31. If the median of the data 6, 7, $x - 2$, x, 18 and 21 written in ascending order is 16, then the variance of that data is
(a) $30\dfrac{1}{5}$ (b) $31\dfrac{1}{3}$ (c) $32\dfrac{1}{2}$ (d) $33\dfrac{1}{3}$

32. Two persons A and B are throwing an unbiased six faced dice alternatively, with the condition that the person who throws 3 first wins the game. If A starts the game, then probabilities of A and B to win the same are, respectively

(a) $\frac{6}{11}, \frac{5}{11}$ (b) $\frac{5}{11}, \frac{6}{11}$ (c) $\frac{8}{11}, \frac{3}{11}$ (d) $\frac{3}{11}, \frac{8}{11}$

33. The letters of the word 'QUESTION' are arranged in a row at random. The probability that there are exactly two letters between Q and S is

(a) $\frac{1}{14}$ (b) $\frac{5}{7}$ (c) $\frac{1}{7}$ (d) $\frac{5}{28}$

34. If $\frac{1+3P}{3}, \frac{1-2P}{2}$ are probabilities of two mutually exclusive events, then P lies in the interval.

(a) $\left[-\frac{1}{3}, \frac{1}{2}\right]$ (b) $\left(\frac{-1}{2}, \frac{1}{2}\right)$

(c) $\left[-\frac{1}{3}, \frac{2}{3}\right]$ (d) $\left(\frac{-1}{3}, \frac{2}{3}\right)$

35. The probability that an event does not happen in one trial is 0.8 The probability that the event happens atmost once in three trials is

(a) 0.896 (b) 0.791 (c) 0.642 (d) 0.592

36. The probability distribution of a random variable is given below

$X = x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Then, $P(0 < x < 5)$ is equal to

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{8}{10}$ (d) $\frac{7}{10}$

37. If the equation to the locus of points equidistant from the points (–2, 3), (6, –5) is $ax + by + c = 0$, where $a > 0$, then the ascending order of a, b, c is

(a) a, b, c (b) c, b, a (c) b, c, a (d) a, c, b

38. The point (2, 3) is first reflected in the straight line $y = x$ and then translated through a distance of 2 units along the positive direction X-axis. The coordinates of the transformed point are

(a) (5, 4) (b) (2, 3)
(c) (5 , 2) (d) (4, 5)

39. If the straight line $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$ form a triangle with origin as orthocentre, then (a, b) is equal to

(a) (6, 4) (b) (–3, 3)
(c) (–8, 8) (d) (0, 7)

40. The point on the line $4x - y - 2 = 0$ which is equidistant from the points (–5, 6) and (3, 2) is

(a) (2, 6) (b) (4, 14)
(c) (1, 2) (d) (3, 10)

41. If the lines $x + 2ay + a = 0$, $x + 3by + b = 0$, $x + 4cy + c = 0$ are concurrent, then a, b and c are in

(a) arithmetic progression
(b) geometric progression
(c) harmonic progression
(d) arithmetico-geometric progression

42. If the slope of one of the lines represented by $ax^2 - 6xy + y^2 = 0$ is the square of the other, then the value of a is

(a) –27 or 8 (b) –3 or 2
(c) –64 or 27 (d) –4 or 3

43. The sum of the minimum and maximum distance of the point (4, –3) to the circle $x^2 + y^2 + 4x - 10y - 7 = 0$, is

(a) 10 (b) 12
(c) 16 (d) 20

44. The locus of centres of the circles, which cut the circles $x^2 + y^2 + 4x - 6y + 9$ and $x^2 + y^2 - 5x + 4y + 2 = 0$ orthogonally, is

(a) $3x + 4y - 5 = 0$ (b) $9x - 10y + 7 = 0$
(c) $9x + 10y - 7 = 0$ (d) $9x - 10y + 11 = 0$

45. If $x - y + 1 = 0$ meets the circle $x^2 + y^2 + y - 1 = 0$ at A and B, then the equation of the circle with AB as diameter is

(a) $2(x^2 + y^2) + 3x - y + 1 = 0$
(b) $2(x^2 + y^2) + 3x - y + 2 = 0$
(c) $2(x^2 + y^2) + 3x - y + 3 = 0$
(d) $x^2 + y^2 + 3x - y + 4 = 0$

46. An equilateral triangle is inscribed in the parabola $y^2 = 8x$, with one of its vertices is the vertex of the parabola. Then, length of the side of that triangle is

(a) $24\sqrt{3}$ units (b) $16\sqrt{3}$ units
(c) $8\sqrt{3}$ units (d) $4\sqrt{3}$ units

47. The point (3, 4) is the focus and $2x - 3y + 5 = 0$ is the directrix of a parabola. Its latusrectum is

(a) $\frac{2}{\sqrt{13}}$ (b) $\frac{4}{\sqrt{13}}$ (c) $\frac{1}{\sqrt{13}}$ (d) $\frac{3}{\sqrt{13}}$

48. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having its centre at (0,3) is

(a) 6 (b) 4 (c) 3 (d) 2

49. The values that m can take, so that the straight line $y = 4x + m$ touches the curve $x^2 + 4y^2 = 4$ is

(a) $\pm\sqrt{45}$ (b) $\pm\sqrt{60}$ (c) $\pm\sqrt{65}$ (d) $\pm\sqrt{72}$

50. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then, the value of b^2 is

(a) 5 (b) 7 (c) 9 (d) 1

51. If (2, – 1, 2) and (K, 3, 5) are the triads of direction ratios of two lines and the angle between them is 45°, then the value of K is

(a) 2 (b) 3 (c) 4 (d) 6

52. The length of perpendicular from the origin to the plane which makes intercepts $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively on the coordinate axes is

(a) $\frac{1}{5\sqrt{2}}$ (b) $\frac{1}{10}$ (c) $5\sqrt{2}$ (d) 5

53. Match the following columns.

	Column I	Column II
(A)	The centroid of the triangle formed by (2, 3, –1), (5, 6, 3),(2, –3 ,1) is	(p) (2, 2, 2)
(B)	The circumcentre of the triangle formed by (1, 2, 3), (2, 3, 1), (3, 1, 2) is	(q) (3, 1, 4)
(C)	The orthocentre of the triangle formed by (2, 1, 5), (3, 2, 3), (4, 0, 4) is	(r) (1, 1, 0)
(D)	The incentre of the triangle formed by (0, 0, 0), (3, 0, 0), (4, 0, 4) is	(s) (3, 2, 1)
(E)	The incentre of the triangle formed by (0, 0, 0), (3, 0, 0), (4, 0, 4) is	(t) (0, 0, 0)

	A	B	C	D		A	B	C	D
(a)	s	p	q	r	(b)	p	q	r	s
(c)	s	r	q	r	(d)	s	p	t	r

54. If $g(x) = \frac{x}{[x]}$ for x > 2, then $\lim_{x \to 2} \frac{g(x) - g(2)}{x - 2}$ is equal to

(a) –1 (b) 0 (c) $\frac{1}{2}$ (d) 1

55. $\lim_{\to -} \left(\frac{-\pi}{\cos} \right)$ is equal to

(a) 0 (b) $\frac{1}{2}$ (c) –2 (d) 5

56. If f is defined by $f(x) = \begin{cases} x \text{ for } 0 \le x < 1 \\ 2 - x \text{ for } x \ge 1 \end{cases}$, then at $x = 1$, $f(x)$ is

(a) continuous and differentiable
(b) continuous but not differentiable
(c) discontinuous but differentiable
(d) neither continuous nor differentiable

57. If $x^2 + y^2 = t + \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{x}{y}$ (b) $-\frac{y}{x}$ (c) $\frac{x^2}{y^2}$ (d) $\frac{y^2}{x^2}$

58. If $x = at^2$ and $y = 2at$, then $\frac{d^2y}{dx^2}$ at $t = \frac{1}{2}$ is

(a) $-\frac{2}{a}$ (b) $\frac{4}{a}$ (c) $\frac{8}{a}$ (d) $\frac{-4}{a}$

59. The volume of sphere is increasing at the rate of 1200 cu cm/s. The rate of increase in its surface area when the radius is 10 cm is

(a) 120 sq cm/s (b) 240 sq cm/s
(c) 200 sq cm/s (d) 100 sq cm/s

60. The slope of the tangent to the curve $y = \int_0^x \frac{1}{1+t^3} dt$ at the point, where $x = 1$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

61. If $x^2 + y^2 = 25$, then $\log_5 [\max(3x + 4y)]$ is

(a) 2 (b) 3
(c) 4 (d) 5

62. $\int \frac{dx}{(x-1)\sqrt{x^2-1}}$ is equal to

(a) $-\sqrt{\frac{x-1}{x+1}} + C$ (b) $\sqrt{\frac{x-1}{x^2+1}} + C$

(c) $-\sqrt{\frac{x+1}{x-1}} + C$ (d) $\sqrt{\frac{x^2+1}{x-1}} + C$

63. $\int e^x \frac{x^2+1}{(x+1)^2} dx$ is equal to

(a) $\frac{e^x}{x+1} + C$ (b) $\frac{-e^x}{x+1} + C$

(c) $e^x \left(\frac{x-1}{x+1} \right) + C$ (d) $e^x \left(\frac{x+1}{x-1} \right) + C$

64. $\int \frac{x+1}{x(1+xe^x)} dx$ is equal to

(a) $\log \left| \frac{1+xe^x}{xe^x} \right| + C$ (b) $\log \left| \frac{xe^x}{1+xe^x} \right| + C$

(c) $\log |xe^x(1 + xe^x)| + C$ (d) $\log |1 + xe^x| + C$

65. $\int \frac{f(x)g'(x) - f'(x)g(x)}{f(x)g(x)} \times [\log g(x) - \log f(x)] \, dx$ is equal to

(a) $\log \left[\frac{g(x)}{f(x)} \right] + C$ (b) $\frac{1}{2} \left[\log \frac{g(x)}{f(x)} \right]^2 + C$

(c) $\frac{g(x)}{f(x)} \log \left[\frac{g(x)}{f(x)} \right] + C$ (d) $\log \left[\frac{g(x)}{f(x)} \right] - \frac{g(x)}{f(x)} + C$

66. $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x} dx$ is equal to

(a) $\frac{1}{2} \log 3$ (b) $\log 2$ (c) $\log 3$ (d) $\frac{1}{4} \log 3$

67. $\int_{-1}^{1}\frac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{\sqrt{1+x+x^2}+\sqrt{1-x+x^2}}dx$ is equal to

(a) $\frac{3\pi}{2}$ (b) $\frac{\pi}{2}$ (c) 0 (d) –1

68. The area of the region described by $\{(x, y) / x^2+y^2 \le 1$ and $y^2 \le 1-x\}$ is

(a) $\frac{\pi}{2}-\frac{2}{3}$ (b) $\frac{\pi}{2}+\frac{2}{3}$ (c) $\frac{\pi}{2}+\frac{4}{3}$ (d) $\frac{\pi}{2}-\frac{4}{3}$

69. The solution of $\frac{dy}{dx}+\frac{1}{x}=\frac{e^y}{x^2}$ is

(a) $2x=(1+Cx^2)e^y$ (b) $x=(1+Cx^2)e^y$
(c) $2x^2=(1+Cx^2)e^{-y}$ (d) $x^2=(1+Cx^2)e^{-y}$

70. The differential equation $\frac{dy}{dx}=\frac{1}{ax+by+c}$, where a,b,c are all non-zero real numbers, is

(a) linear in y
(b) linear in x
(c) linear in both x and y
(d) homogeneous equation

71. $\left(\frac{1+\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}}{1+\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}}\right)^8$ is equal to

(a) 1 (b) – 1 (c) 2 (d) $\frac{1}{2}$

72. The number of four-digit numbers formed by using the digits 0, 2, 4, 5 and which are not divisible by 5, is

(a) 10 (b) 8 (c) 6 (d) 4

73. If $x=\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.5}+....\infty$, then $3x^2+6x$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

74. In a ΔABC, $(a+b+c)(b+c-a)=\lambda bc$, then

(a) $\lambda < -6$ (b) $\lambda > 6$ (c) $0 < \lambda < 4$ (d) $\lambda > 4$

75. In a ΔABC, $\frac{a}{\tan A}+\frac{b}{\tan B}+\frac{c}{\tan C}$ is equal to

(a) $2r$ (b) $r+2R$ (c) $2r+R$ (d) $2(r+R)$

76. The angle between the straight lines represented by $(x^2+y^2)\sin^2\alpha=(x\cos\alpha-y\sin\alpha)^2$ is

(a) $\frac{\alpha}{2}$ (b) α (c) 2α (d) $\frac{\pi}{2}$

77. The equation of the circle passing through (2, 0) and (0,4) and having the minimum radius, is

(a) $x^2+y^2=20$ (b) $x^2+y^2-2x-4y=0$
(c) $x^2+y^2=4$ (d) $x^2+y^2=16$

78. If $x^2+y^2-4x-2y+5=0$ and $x^2+y^2-6x-4y-3=0$ are members of a coaxial system of circles, then the centre of a point circle in the system is

(a) (– 5, – 6) (b) (5, 6)
(c) (3, 5) (d) (– 8, – 13)

79. Let D be the domain of a twice differentiable function f. For all $x \in D$, $f''(x)+f(x)=0$ and $f(x)=\int g(x)dx+$ constant. If $h(x)=\{f(x)\}^2+\{g(x)\}^2$ and $h(0)=5$, then $h(2015)-h(2014)$ is equal to

(a) 5 (b) 3 (c) 0 (d) 1

80. If f is defined in [1, 3] by $f(x)=x^3+bx^2+ax$, such that $f(1)-f(3)=0$ and $f'(c)=0$, where $c=2+\frac{1}{\sqrt{3}}$, then (a, b) is equal to

(a) (–6, 11) (b) $\left(2-\frac{1}{\sqrt{3}}, 2+\frac{1}{\sqrt{3}}\right)$
(c) (11, –6) (d) (6, 11)

PHYSICS

81. The pressure on a circular plate is measured by measuring the force on the plate and the radius of the plate. If the errors in measurement of the force and the radius are 5% and 3% respectively, the percentage of error in the measurement of pressure is

(a) 8 (b) 14 (c) 11 (d) 12

82. A body is projected vertically from the surface of the earth of radius R with a velocity equal to half of the escape velocity. The maximum height reached by the body is

(a) $\frac{R}{2}$ (b) $\frac{R}{3}$ (c) $\frac{R}{4}$ (d) $\frac{R}{5}$

83. A particle aimed at a target, projected with an angle 15° with the horizontal is short of the target by 10 m. If projected with an angle of 45° is away from the target by 15 m, then the angle of projection to hit the target is

(a) $\frac{1}{2}\sin^{-1}\left(\frac{1}{10}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{10}\right)$

(c) $\frac{1}{2}\sin^{-1}\left(\frac{9}{10}\right)$ (d) $\frac{1}{2}\sin^{-1}\left(\frac{7}{10}\right)$

84. A man running at a speed of 5 km/h, find that the rain falls vertically. When he stops running, he finds that the rain is falling at an angle of 60° with the horizontal. The velocity of rain with respect to running man is

(a) $\frac{5}{\sqrt{3}}$ km/h (b) $\frac{5\sqrt{3}}{2}$ km/h

(c) $\frac{4\sqrt{3}}{2}$ km/h (d) $5\sqrt{3}$ km/h

85. A horizontal force just sufficient to move a body of mass 4 kg lying on a rough horizontal surface, is applied on it. Coefficients of static and kinetic frictions are 0.8 and 0.6 respectively. If the force continues to act even after the body has started moving, the acceleration of the body is (take, $g = 10\ ms^{-2}$).

(a) $6\ ms^{-2}$ (b) $8\ ms^{-2}$ (c) $2\ ms^{-2}$ (d) $4\ ms^{-2}$

86. A force $(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}})$N acts on a body, which is initially at rest. At the end of 20 s the velocity of the body is $(4\hat{\mathbf{i}}+2\hat{\mathbf{j}}-2\hat{\mathbf{k}})\text{ms}^{-1}$, then the mass of the body is

(a) 8 kg (b) 10 kg
(c) 5 kg (d) 4.5 kg

87. A man of weight 50 kg carries an object to a height of 20 m in a time of 10 s. The power used by the man in this process is 2000 W, then find the weight of the object carried by the man (take, $g = 10\ ms^{-2}$)

(a) 100 kg (b) 25 kg
(c) 50 kg (d) 10 kg

88. A ball P moving with a speed of $v\ ms^{-1}$ collides directly with another identical ball Q moving with a speed $10\ ms^{-1}$ in the opposite direction. P comes to rest after the collision. If the coefficient of restitution is 0.6, the value of v is

(a) $30\ ms^{-1}$ (b) $40\ ms^{-1}$
(c) $50\ ms^{-1}$ (d) $60\ ms^{-1}$

89. A particle of mass $m = 5$ units is moving with uniform speed $v = 3\sqrt{2}$ units in the XY-plane along the line $Y = X + 4$. The magnitude of the angular momentum about origin is

(a) zero (b) 60 units (c) 7.5 units (d) 40 units

90. The kinetic energy of a circular disc rotating with a speed of 60 r.p.m. about an axis passing through a point on its circumference and perpendicular to its plane is (mass of circular disc = 5 kg, radius of disc = 1m) approximately.

(a) 170 J (b) 160 J
(c) 150 J (d) 140 J

91. The amplitude of a simple pendulum is 10 cm. When the pendulum is at a displacement of 4 cm from the mean position, the ratio of kinetic and potential energies at that point is

(a) 5.25 (b) 2.5 (c) 4.5 (d) 7.5

92. A satellite revolving around a planet has orbital velocity 10 km/s. The additional velocity required for the satellite to escape from the gravitational field of the planet is

(a) 14.14 km/s (b) 11.2 km/s
(c) 4.14 km/s (d) 41.4 km/s

93. The length of a metal wire is l_1 when the tension in it is F_1 and l_2 when the tension is F_2. Then, original length of the wire is

(a) $\dfrac{l_1F_1+l_2F_2}{F_1+F_2}$ (b) $\dfrac{l_2-l_1}{F_2-F_1}$

(c) $\dfrac{l_1F_2-l_2F_1}{F_2-F_1}$ (d) $\dfrac{l_1F_1-l_2F_2}{F_2-F_1}$

94. The average depth of Indian ocean is about 3000 m. The value of fractional compression $\left(\dfrac{\Delta V}{V}\right)$ of water at the bottom of the ocean is (given that the bulk modulus of water is $2.2 \times 10^9\ Nm^{-2}$, $g = 9.8$, $\rho_{H_2O} = 1000\ kg.m^{-3}$)

(a) 3.4×10^{-2} (b) 1.34×10^{-2}
(c) 4.13×10^{-2} (d) 13.4×10^{-2}

95. The ratio energies of emitted radiation by a black body at 600 K and 933 K when the surrounding temperature is 300 K

(a) $\dfrac{5}{16}$ (b) $\dfrac{7}{16}$ (c) $\dfrac{3}{16}$ (d) $\dfrac{9}{16}$

96. The specific heat of helium at constant volume is 12.6 $Jmol^{-1}\ K^{-1}$. The specific heat of helium at constant pressure in $Jmol^{-1}\ K^{-1}$ is about (Assume the temperature of the gas is moderate, universal gas constant, $R = 8.314\ Jmol^{-1}\ K^{-1}$)

(a) 12.6 (b) 16.8 (c) 18.9 (d) 21

97. A gas does 4.5 J of external work during adiabatic expansion. If its temperature falls by 2K, then its internal energy will be

(a) increased by 4.5 J
(b) decreased by 4.5 J
(c) decreased by 2.25 J
(d) increased by 9.0 J

98. The relation between efficiency η of a heat engine and the coefficient of performance α of a refrigerator is

(a) $\eta = \dfrac{1}{1-\alpha}$ (b) $\eta = \dfrac{1}{1+\alpha}$

(c) $\eta = 1 + \alpha$ (d) $\eta = 1 - \alpha$

99. A flask contains argon and chlorine in the ratio of 2 : 1 by mass. The temperature of the mixture is 27° C. The ratio of average kinetic energies of two gases per molecule is

(a) 1 : 1 (b) 2 : 1 (c) 3 : 1 (d) 6 : 1

100. A transverse wave is represented by the equation $y = 2\sin(30t - 40x)$ and the measurements of distances are in meters, then the velocity of propagation is

(a) $15\ ms^{-1}$ (b) $0.75\ ms^{-1}$
(c) $3.75\ ms^{-1}$ (d) $300\ ms^{-1}$

101. Two closed pipes have the same fundamental frequency. One is filled with oxygen and the other with hydrogen at the same temperature. Ratio of their lengths respectively is

(a) 1 : 4 (b) 4 : 1 (c) 1 : 2 (d) 2 : 1

102. An image is formed at a distance of 100 cm from the glass surface when light from point source in air falls on a spherical glass surface with refractive index 1.5. The distance of the light source from the glass surface is 100 cm. The radius of curvature is

(a) 20 cm (b) 40 cm (c) 30 cm (d) 50 cm

103. Two coherent sources of intensity ratio 9 : 4 produce interference. The intensity ratio of maxima and minima of the interference pattern is

(a) 13 : 6 (b) 5 : 1 (c) 25 : 1 (d) 3 : 2

104. The energy of a parallel plate capacitor when connected to a battery is E. With the battery still in connection, if the plates of the capacitor are separated, so that the distance between them is twice the original distance, then the electrostatic energy becomes

(a) $2E$ (b) $\frac{E}{4}$ (c) $\frac{E}{2}$ (d) $4E$

105. Two point charges +8μC and +12μC repel each other with a force of 48 N. When an additional charge of –10μC is given to each of these charges (the distance between the charges is unaltered) then the new force is

(a) repulsive force of 24 N
(b) attractive force of 24 N
(c) repulsive force of 12 N
(d) attractive force of 2 N

106. If the dielectric constant of a substance is $K = \frac{4}{3}$, , then the electric susceptibility ψ_e is

(a) $\frac{\varepsilon_0}{3}$ (b) $3\varepsilon_0$

(c) $\frac{4}{3}\varepsilon_0$ (d) $\frac{3}{4}\varepsilon_0$

107. In a region of uniform electric field of intensity E, an electron of mass m_e is released from rest. The distance travelled by the electron in a time t is

(a) $\frac{2m_e t^2}{e}$ (b) $\frac{eEt^2}{2m_e}$ (c) $\frac{m_e g t^2}{eE}$ (d) $\frac{2Et^2}{em_e}$

108. A constant potential difference is applied between the ends of the wire. If the length of the wire is elongated 4 times, then the drift velocity of electrons will be

(a) increases 4 times (b) decreases 4 times
(c) increases 2 times (d) decreases 2 times

109. In a meter bridge, the gaps are enclosed by resistances of 2Ω and 3Ω. The value of shunt to be added to 3Ω resistor to shift the balancing point by 22.5 cm is

(a) 1 Ω (b) 2 Ω (c) 2.5 Ω (d) 5 Ω

110. Two long straight parallel conductors 10 cm apart, carry equal currents of magnitude 3A in the same direction. Then, the magnetic induction at a point midway between them is

(a) 2×10^{-5} T (b) 3×10^{-5} T
(c) zero (d) 4×10^{-5} T

111. In a crossed field, the magnetic field induction is 2.0 T and electric field intensity is 20×10^3 V/m. At which velocity the electron will travel in a straight line without the effect of electric and magnetic fields?

(a) $\frac{20}{16} \times 10^3 \text{ ms}^{-1}$ (b) $10 \times 10^3 \text{ ms}^{-1}$
(c) $20 \times 10^3 \text{ ms}^{-1}$ (d) $40 \times 10^3 \text{ ms}^{-1}$

112. A material of 0.25 cm^2 cross-sectional area is placed in a magnetic field of strength (H) 1000 Am^{-1}. Then, the magnetic flux produced is
(Susceptibility of material is 313)
(Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1})

(a) 8.33×10^{-8} Wb (b) 1.84×10^{-6} Wb
(c) 9.87×10^{-6} Wb (d) 3.16×10^{-6} Wb

113. The magnitude of the induced emf in a coil of inductance 30 mH in which the current changes from 6 A to 2 A in 2 s is

(a) 0.06 V (b) 0.6 V (c) 1.06 V (d) 6 V

114. In an AC circuit V and I are given below, then find the power dissipated in the circuit

$V = 50\sin(50t)\ v\ I = 50\sin\left(50t + \frac{\pi}{3}\right)\text{mA}$

(a) 0.625 W (b) 1.25 W
(c) 2.50 W (d) 5.0 W

115. Light with an energy flux of 9 Wcm^{-2} falls on a non-reflecting surface at normal incidence. If the surface has an area of 20 cm^2. The total momentum delivered for complete absorption in one hour is

(a) 2.16×10^{-4} kgms^{-1} (b) 1.16×10^{-3} kgms^{-1}
(c) 2.16×10^{-3} kgms^{-1} (d) 3.16×10^{-4} kgms^{-1}

116. The ratio of the de-Broglie wavelengths for the electron and proton moving with the same velocity is (m_p-mass of proton, m_e-mass of electron)

(a) $m_p : m_e$ (b) $m_p^2 : m_e^2$

(c) $m_e : m_p$ (d) $m_e^2 : m_p^2$

117. The ratio of longest wavelength lines in the Balmer and Paschen series of hydrogen spectrum is

(a) $\frac{5}{36}$ (b) $\frac{7}{20}$ (c) $\frac{7}{144}$ (d) $\frac{5}{27}$

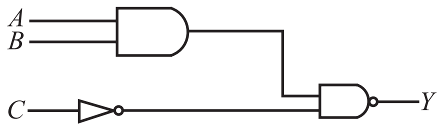
118. In the following nuclear reaction x stands for

$n \rightarrow p + e^- + x$

(a) α-particle (b) positron

(c) nutrino (d) antinutrino

119. In the following circuit, the output Y becomes zero for the input combinations.



(a) $A = 1, B = 0, C = 0$

(b) $A = 0, B = 1, C = 1$

(c) $A = 0, B = 0, C = 0$

(d) $A = 1, B = 1, C = 0$

120. The maximum amplitude of an amplitude modulated wave is 16 V, while the minimum amplitude is 4 V. The modulation index is

(a) 0.4 (b) 0.5

(c) 0.6 (d) 4

CHEMISTRY

121. Which of the following sets of quantum numbers is correct for an electron in 3*d*-orbital?

(a) $n = 3, l = 2, m = -3, s = +\frac{1}{2}$

(b) $n = 3, l = 3, m = +3, s = -\frac{1}{2}$

(c) $n = 3, l = 2, m = -2, s = +\frac{1}{2}$

(d) $n = 3, l = 2, m = -3, s = -\frac{1}{2}$

122. If the kinetic energy of a particle is reduced to half, de-Broglie wavelength becomes

(a) 2 times (b) $\frac{1}{\sqrt{2}}$ times

(c) 4 times (d) $\sqrt{2}$ times

123. Identify the most acidic oxide among the following oxides based on their reaction.

(a) SO_3 (b) P_4O_{10}

(c) Cl_2O_7 (d) N_2O_5

124. Match the following.

List I	List II
(A) Rubidium	(1) Germanium
(B) Platinum	(2) Radioactive chalcogen
(C) Eka-silicon	(3) s-block element
(D) Polonium	(4) Atomic number 78

	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	3	4	1	2
(c)	2	1	4	3	(d)	4	3	1	2

125. Which of the following does not have triple bond between the atoms?

(a) N_2 (b) CO (c) NO (d) C_2^{2-}

126. In which one of the following pairs the two species have identical shape, but differ in hybridisation?

(a) I_3^-, $BeCl_2$ (b) NH_3, BF_3

(c) XeF_2, I_3^- (d) NH_4^+, SF_4

127. On the top of a mountain, water boils at

(a) high temperature (b) same temperature

(c) high pressure (d) low temperature

128. Which one of the following is the wrong statement about the liquid?

(a) It has intermolecular force of attraction

(b) Evaporation of liquids increase with the decrease of surface area

(c) It resembles a gas near the critical temperature

(d) It is an intermediate state between gaseous and solid state

129. A carbon compound contains 12.8% of carbon, 2.1% of hydrogen and 85.1% of bromine. The molecular weight of the compound is 187.9. Calculate the molecular formula of the compound.

(Atomic weight: H = 1.008, C = 12.0, Br = 79.9)

(a) CH_3Br (b) CH_2Br_2I

(c) $C_2H_4Br_2$ (d) $C_2H_3Br_3$

130. 3.011×10^{22} atoms of an element weighs 1.15 g. The atomic mass of the element is

(a) 23 (b) 10

(c) 16 (d) 35.5

131. Which one of the following is applicable for an adiabatic expansion of an ideal gas?

(a) $\Delta E = 0$ (b) $\Delta W = \Delta E$

(c) $\Delta W = -\Delta E$ (d) $\Delta W = 0$

132. On increasing temperature, the equilibrium constant of exothermic and endothermic reactions, respectively
(a) increases and decreases
(b) decreases and increases
(c) increases and increases
(d) decreases and decreases

133. What is the pH of the NaOH solution when 0.04 g of it dissolved in water and made to 100 mL solution?
(a) 2 (b) 1
(c) 13 (d) 12

134. Which of the following methods is used for the removal of temporary hardness of water?
(a) Treatment with washing soda
(b) Calgon method
(c) Ion-exchange method
(d) Clark's method

135. **Assertion (A) :** Alkali metals are soft and have low melting and boiling points.
Reason (R) : This is because interatomic bonds are weak.
(a) Both (A) and (R) are not true
(b) (A) is true but (R) is not the correct explanation of (A)
(c) (A) is true but (R) is false
(d) Both (A) and (R) are true and (R) is the correct explanation of (A)

136. Identify the correct statement.
(a) Lead forms compounds in +2 oxidation state due to inert pair effect
(b) All halogens show only negative oxidation state
(c) Catenation property increases from boron to oxygen
(d) Oxidation state of oxygen is –1 in ozonides

137. **Assertion (A) :** Noble gases have very low boiling points.
Reason (R) : All noble gases have general electronic configuration of $ns^2\, np^6$ (except He).
(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) (A) is false but (R) is true
(c) (A) is true but (R) is false
(d) Both (A) and (R) are true and (R) is the not the correct explanation of (A)

138. Which of the following statements are correct?
A. Ocean is sink for CO_2.
B. Greenhouse effect causes lowering of temperature of earth's surface.
C. To control CO emission by automobiles, usually catalytic convertor are fitted into exhaust pipes.
D. H_2SO_4, herbicides and insecticides form mist.
(a) C and D (b) A and B
(c) B and D (d) A and D

139. The bond angle of bond in methoxy methane is
(a) 111.7° (b) 109°
(c) 108.9° (d) 180°

140. Which of the following compounds has zero dipole moment?
(a) 1, 4-dichlorobenzene
(b) 1, 2-dichlorobenzene
(c) 1, 3-dichlorobenzene
(d) 1-chloro-2-methyl benzene

141. Which of the following reagent is used to find out carbon-carbon multiple bonds?
(a) Grignard reagent
(b) Baeyer's reagent
(c) Sandmeyer's reagent
(d) Gattermann reagent

142. Pure silicon doped with phosphorus is
(a) amorphous
(b) *p*-type semiconductor
(c) *n*-type semiconductor
(d) insulator

143. 18 g of glucose is dissolved in 90 g of water. The relative lowering of vapour pressure of the solution is equal to
(a) 6 (b) 0.2 (c) 5.1 (d) 0.02

144. A gas '*X*' is dissolved in water at 2 bar pressure. Its mole fraction is 0.02 in solution. The mole fraction of water when the pressure of gas is doubled at the same temperature is
(a) 0.04 (b) 0.98 (c) 0.96 (d) 0.02

145. Calculate $\Delta G°$ for the following cell reaction.

$$Zn(s) + Ag_2O(s) + H_2O(l) \longrightarrow Zn^{2+}(aq) + 2Ag(s) + 2^{-}OH(aq)$$

$$E^\circ_{Ag^+/Ag} = +0.80\text{V and } E^\circ_{Zn^{2+}/Zn} = -0.76\text{ V}$$

(a) –305 kJ/mol (b) –301 kJ/mol
(c) 305 kJ/mol (d) 301 kJ/mol

146. The time required for a first order reaction to complete 90% is '*t*'. What is the time required to complete 99% of the same reaction?
(a) 2*t* (b) 3*t* (c) *t* (d) 4*t*

147. Which of the following is the most effective in causing coagulation of ferric hydroxide solution?
(a) KCl (b) KNO_3
(c) K_2SO_4 (d) $K_3[Fe(CN)_6]$

148. Which of the following process does not involve heating?
(a) Calcination (b) Smelting
(c) Roasting (d) Levigation

149. Which one of the following is correct with respect to basic character?
(a) $P(CH_3)_3 > PH_3$ (b) $PH_3 > P(CH_3)_3$
(c) $PH_3 > NH_3$ (d) $PH_3 = NH_3$

150. When $AgNO_3$ solution is added in excess to 1M solution of $CoCl_3 \cdot X NH_3$, one mole of AgCl is formed. What is the value of '*X*'?
(a) 1 (b) 4 (c) 3 (d) 2

151. In which of the following coordination compounds, the central metal ion is in zero oxidation state?
(a) $[Fe(H_2O)_6]Cl_3$ (b) $K_4[Fe(CN)_6]$
(c) $Fe(CO)_5$ (d) $[Fe(H_2O)_6]Cl_2$

152. The percentage of lanthanides and iron, respectively, in misch metal are
(a) 50, 50 (b) 75, 25 (c) 90, 10 (d) 95, 5

153. Sea divers use a mixture of
(a) O_2, N_2 (b) O_2, H_2 (c) O_2, He (d) N_2, H_2

154. The polymer obtained with methylene bridges by condensation polymerisation
(a) PVC (b) buna-S
(c) polyacrylonitrile (d) bakelite

155. The amino acid containing indole part is
(a) tryptophan (b) tyrosine
(c) proline (d) methionine

156. The drug used as post operative analgesic in medicine is
(a) L-dopa (b) amoxycillin
(c) sulphapyridine (d) morphine

157. $C_2H_5OH + 4I_2 + 3Na_2CO_3 \longrightarrow X + HCOONa + 5NaI + 3CO_2 + 2H_2O$
In the above reaction, '*X*' is
(a) diiodomethane (b) triiodomethane
(c) iodomethane (d) tetraiodomethane

158. Phenol on oxidation in air gives
(a) quinone (b) catechol
(c) resorcinol (d) *o*-cresol

159. Identify the reagents *A* and *B* respectively in the following reactions.

$$CH_3COOH \xrightarrow{A} CH_3COCl \xrightarrow{B} CH_3CHO$$

(a) $SOCl_2$, H_2/Pd-$BaSO_4$
(b) H_2/Pd -$BaSO_4$, $SOCl_2$
(c) $SOCl_2$, H_2O_2
(d) $SOCl_2$, O_5O_4

160. Predict respectively '*X*' and 'Y' in the following reactions.

$$Ar - NH_2 \xrightarrow{x} Ar - \overset{+}{N} \equiv NCl^- \xrightarrow{y} Ar - Cl$$

(a) $NaNO_3$ and Cl_2
(b) $NaNO_3$ - HCl and HCl
(c) $NaNO_2$ - HCl and Cu /HCl
(d) $NaNO_2$ - HCl and $NaNH_2$

Hints & Solutions

MATHEMATICS

1. **(b)** Given, $f(x) = 5x - 3$ and $g(x) = x^2 + 3$

Let, $y = f(x)$, $\therefore$ $y = 5x - 3$

$y + 3 = 5x \Rightarrow x = \frac{y+3}{5}$

$\therefore$ $f^{-1}(y) = \frac{y+3}{5} \Rightarrow f^{-1}(x) = \frac{x+3}{5}$

Now, $g(x) = x^2 + 3$;

So, $gof^{-1}(3) = g[f^{-1}(3)]$

$= g\left(\frac{3+3}{5}\right) = g\left(\frac{6}{5}\right) = \frac{(6)^2}{(5)^2} + 3 = \frac{36}{25} + 3 = \frac{111}{25}$

2. **(a)** Given, $f(x) = \sin x - x$

Here, $\frac{\pi}{4} \le x \le \frac{\pi}{3}$

$f\left(\frac{\pi}{4}\right) \ge f(x) \ge f\left(\frac{\pi}{3}\right)$

$\left(\because f(x) \text{ is decreasing function in } x \in \left[\frac{\pi}{4}, \frac{\pi}{3}\right]\right)$

$\sin\frac{\pi}{4} - \frac{\pi}{4} \ge f(x) \ge \sin\frac{\pi}{3} - \frac{\pi}{3}$

$\frac{1}{\sqrt{2}} - \frac{\pi}{4} \ge f(x) \ge \frac{\sqrt{3}}{2} - \frac{\pi}{3}$

$\therefore$ $f(A) \in \left[\frac{\sqrt{3}}{2} - \frac{\pi}{3} \cdot \frac{1}{\sqrt{2}} - \frac{\pi}{4}\right]$

3. **(d)** Let given series be

$S = 1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + \dots n$ terms

Now, general term

$T_n = \{1 + (n-1) \cdot 1\}\{2 + (n-1) \cdot 1\}\{3 + (n-1) \cdot 1\}$

$= (1 + n - 1)(2 + n - 1)(3 + n - 1)$

$= n(n+1)(n+2)$

$= n(n^2 + 3n + 2)$

$= n^3 + 3n^2 + 2n$

Now,

Sum of the series S $= \Sigma T_n = \Sigma(n^3 + 3n^2 + 2n)$

$= \Sigma n^3 + 3\Sigma n^2 + 2\Sigma n$

$= \frac{n^2(n+1)^2}{4} + \frac{3n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + (2n+1) + 2\right]$

$= \frac{n(n+1)}{2}\left[\frac{n^2 + n + 4n + 2 + 4}{2}\right]$

$= \frac{n(n+1)(n^2 + 5n + 6)}{4}$

$\therefore$ S$= \frac{n(n+1)(n+2)(n+3)}{4}$

4. **(c)** Let D $= \begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix}$

$= \begin{vmatrix} b(b-a) & b - c & c(b-a) \\ a(b-a) & a - b & b(b-a) \\ c(b-a) & c - a & a(b-a) \end{vmatrix}$

$= (b-a)(b-a)\begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$

$= (b-a)^2\begin{vmatrix} b & 0 & c \\ a & 0 & b \\ c & 0 & a \end{vmatrix}$ [using $C_2 \to C_2 - (C_1 - C_3)$]

$= 0$

5. **(c)** Given matrix A is of order 3.

We know that,

$|\operatorname{adj}(\operatorname{adj} A)| = |A|^{(n-1)^2}$

Similarly,

$|\operatorname{adj}(\operatorname{adj} A^2)| = |A^2|^{(3-1)^2}$

$= |A^2|^{2^2}$ $[\because n = 3]$

$= |A^2|^4 = |A|^8$

6. **(d)** Given, $2x + 3y + z = 5$,

$3x + y + 5z = 7$,

$x + 4y - 2z = 3$

The given system can be written as $AX = B$, where

$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 5 \\ 1 & 4 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 1 & 5 \\ 1 & 4 & -2 \end{vmatrix}$

$= 2\,(-2-20) - 3\,(-6-5) + 1\,(12-1)$

$= 2\,(-22) - 3\,(-11) + 1\,(11)$

$= -44 + 33 + 11 = 0$

$|A| = 0$

As $|A| = 0$, this system can have no solution (or) infinitely many solutions. So we check (adj A) B.

$$\text{adj } A = \begin{bmatrix} -22 & 10 & 14 \\ 11 & -5 & -7 \\ 11 & -5 & -7 \end{bmatrix}$$

$$(\text{adj } A)\,B = \begin{bmatrix} -22 & 10 & 14 \\ 11 & -5 & -7 \\ 11 & -5 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -110+70+42 \\ 55-35-21 \\ 55-35-21 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \neq 0$$

Hence, there exist no solution

7. **(d)** We have, $\sum_{k=1}^{6}\left[\sin\frac{2k\pi}{7} - i\cos\frac{2\pi k}{7}\right]$

$$= \sum_{k=1}^{6}\left[(-i)\left(\cos\frac{2k\pi}{7} + i\sin\frac{2\pi k}{7}\right)\right]$$

$$= (-i)\sum_{k=1}^{6}\left(\cos\frac{2k\pi}{7} + i\sin\frac{2\pi k}{7}\right) = (-i)\sum_{k=1}^{6}\alpha^k$$

Let $\alpha = \cos\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}$

$= (-i)\,(\alpha + \alpha^2 + \alpha^3 + \ldots + \alpha^5)$

Here, $\alpha + \alpha^2 + \alpha^3 + \ldots + \alpha^6$ follows G.P. so its sum.

$$S = \frac{\alpha\left(1-\alpha^6\right)}{1-\alpha} = \frac{\alpha - \alpha^7}{1-\alpha} = \frac{\alpha - 1}{1-\alpha} = i$$

$$\left[\because\ \alpha^7 = \cos 2\pi + i\sin 2\pi = 1\right]$$

Thus, $\alpha = -i\,(-1)$

$\alpha = i$

8. **(b)** We have, $\omega^{\left(\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\ldots.\infty\right)} + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\ldots.\infty\right)} = ?$

Here, $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \ldots.\infty$

Follows infinite G.P. series so its sum,

$$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{2}{3}}$$

$$= \frac{1}{3}\times\frac{3}{1} = 1 \quad \therefore \quad \omega^{\left(\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\ldots\infty\right)} = \omega$$

Now, $\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \ldots.. + \infty$

also follows infinite G.P. series so its sum

$$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{3}{4}} = \frac{1}{2}\times\frac{4}{1} = 2$$

$$\therefore \quad \omega^{\left(\frac{1}{2}+\frac{3}{8}+\ldots+\infty\right)} = \omega^2$$

Hence, $\omega^{\left(\frac{1}{3}+\frac{2}{9}+\ldots+\infty\right)} + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\ldots+\infty\right)} = \omega + \omega^2 = -1$

$$\left[\because\ \omega^2 + \omega + 1 = 0 \Rightarrow \omega^2 + \omega = -1\right]$$

9. **(a)** Given equation,

$z^3 + 2z^2 + 2z + 1 = 0$

$(z+1)\,(z^2 + z + 1) = 0$

Its roots are -1, ω and ω^2.

Let $f(z) = z^{2014} + z^{2015} + 1 = 0$

Put $z = -1$, ω and ω^2 respectively, we get

$f(-1) = (-1)^{2014} + (-1)^{2015} + 1 = 0 = 1 \neq 0$

Therefore, -1 is not a root of the equation $f(z) = 0$

Again, $f(\omega) = (\omega)^{2014} + (\omega)^{2015} + 1$

$= (\omega^3)^{671} \cdot \omega + (\omega^3)^{671} \cdot \omega^2 + 1$

$= \omega + \omega^2 + 1$

$= \omega^2 + \omega + 1$

$= 0$

Therefore, ω is a root of the equation $f(z) = 0$

Similarly,

$f(\omega^3) = (\omega^2)^{2014} + (\omega^3)^{2015} + 1$

$= (\omega^3)^{1342} \cdot \omega^2 + (\omega^3)^{1343} \cdot \omega + 1$

$= \omega^2 + \omega + 1$

$= 0$

Hence ω and ω^2 are the common roots of $z^{2014} + z^{2015} + 1 = 0$

10. **(a)** Given that the roots of equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal, so

$D = 0$

$(c-a)^2 - 4\,(a-b)\,(b-c) = 0$

$c^2 + a^2 - 2ca - 4ab + 4ac + 4b^2 = 0$

$c^2 + a^2 + 2ac + 4b^2 - 4b\,(c+a) = 0$

$(c+a)^2 + (2b)^2 - 2 \cdot 2b\,(c+a) = 0$

$[(c+a) - (2b)]^2 = 0$

$c + a - 2b = 0$

$2b = a + c$

Hence, we can conclude that a, b and c are in AP.

11. **(b)** Given equation, $x^3 - kx^2 + 14x - 8 = 0$

Roots of the above equation is in G.P. So let $\frac{a}{r}$, a and ar are the roots of equation.

Then, product of roots

$\frac{a}{r}\cdot a\cdot ar=\frac{D}{A} \Rightarrow \frac{a}{r}\cdot a\cdot ar=8 \Rightarrow a^3=8$

$\therefore \quad a=2$

Therefore, at $a = 2$ is the root of equation.

$\therefore \quad x^3 - kx^2 + 14x - 8 = 0$

$(2)^3 - k(2)^2 + 14(2) - 8 = 0$

$8 - 4k + 28 - 8 = 0 \Rightarrow k = 7$

12. **(c)** Given equation

$\sqrt{2}x^2 - bx + \left(8-2\sqrt{5}\right) = 0$

Let the roots of the given equation are α and β.

Sum of roots, $\alpha+\beta=\frac{-b}{a}=-\left(\frac{-b}{\sqrt{2}}\right)=\frac{b}{\sqrt{2}}$

and product of roots, $\alpha\beta=\frac{c}{a}=\frac{8-2\sqrt{5}}{\sqrt{2}}$

Now, as given in the question

$$\frac{2\alpha\beta}{\alpha+\beta}=4 \Rightarrow \frac{2\left(\frac{8-2\sqrt{5}}{\sqrt{2}}\right)}{\frac{b}{\sqrt{2}}}=4$$

$$\Rightarrow \quad 2\left(\frac{8-2\sqrt{5}}{\sqrt{2}}\right)=b \Rightarrow b=4-\sqrt{5}$$

13. **(Bonus)**

Let $y=\frac{x^2+2x+1}{x^2+2x-1}$

$y(x^2 + 2x - 1) = x^2 + 2x + 1$

$yx^2 + 2xy - y = x^2 + 2x + 1$

$yx^2 - x^2 + 2xy - 2x - y - 1 = 0$

$(y-1)x^2 + 2(y-1)x - y - 1 = 0$

For real values of x, $b^2 - 4ac \geq 0$

$[2(y-1)]^2 + 4(y-1)(y+1) \geq 0$

$4(y-1)^2 + 4(y-1)(y+1) \geq 0$

$4(y-1)(y-1+y+1) \geq 0$

$4(y-1)(2y) \geq 0$

$8y(y-1) \geq 0$

At $y = 0$, $x^2 + 2x + 1 = 0$

$(x+1)^2 = 0 \Rightarrow x = -1 \in R$

At $y = 1$, $x^2 + 2x + 1 = x^2 + 2x - 1$

$1 \neq -1 \Rightarrow y \neq 1$

$y \in (-\infty, 0] \cup (1, \infty)$

14. **(b)** Given, T_m = Number of triangles formed with the vertices of a polygon of m sides.

Also, $T_{m+1} - T_m = 15$

$\Rightarrow \quad {}^{m+1}C_3 - {}^{m}C_3 = 15$ ... (i)

As we know,

${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$

$\therefore \quad {}^{m}C_2 + {}^{m}C_3 = {}^{m+1}C_{2+1}$

${}^{m}C_2 + {}^{m}C_3 = 15 + {}^{m}C_3$ [from eq. (i)]

$\therefore \quad {}^{m}C_2 = 15$

$\frac{m!}{2!(m-2)!}=15$

$m(m-1) = 30$

$m^2 - m - 30 = 0$

$(m-6)(m+5) = 0$

$m = 6 - 5$

$\therefore \quad m = 6$ $[\because m \neq -5]$

15. **(b)** Given, $\frac{3x}{(x-2)(x+1)}$ can be written as,

$\frac{3x}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}$ ... (i)

$3x = A(x+1) + B(x-2)$

At $x = 2$,

$\therefore \quad A=\frac{3(2)}{2+1} \Rightarrow A=\frac{3(2)}{3}=A=2$

At $x=-1$, $B=\frac{3(-1)}{-1-2}$

$B=\frac{-3}{-3} \Rightarrow B=1$

Substitute the values of A and B in Eq. (i), we get

$\frac{3x}{(x-2)(x+1)}=\frac{2}{x-2}+\frac{1}{x+1}$

$$=\frac{2}{-2\left(1-\frac{x}{2}\right)}+\frac{1}{(x+1)}=-\left(1-\frac{x}{2}\right)^{-1}+(1+x)^{-1}$$

$$=-\left[1+\frac{x}{2}+\left(\frac{x}{2}\right)^2+\ldots+\left(\frac{x}{2}\right)^5+\ldots\right]$$

$$+\left(1-x+x^2-x^3+x^4-x^5+\ldots\right)$$

$\therefore$ Coefficient of $x^5=-\left(\frac{1}{2}\right)^5-1=\frac{-1}{32}-1=-\frac{33}{32}$

16. **(c)** Given that coefficient of x^9, x^{10} and x^{11} in the expansion of $(1+n)^n$ are in A.P.

It means ${}^{n}C_9 \cdot {}^{n}C_{10} \cdot {}^{n}C_{11}$ are in A.P.

$\therefore \quad 2\,{}^{n}C_{10} = {}^{n}C_9 + {}^{n}C_{11}$

$2=\frac{{}^{n}C_9}{{}^{n}C_{10}}+\frac{{}^{n}C_{11}}{{}^{n}C_{10}}$

$$\Rightarrow \quad 2=\frac{n!}{9!(n-9)!}\times\frac{10!(n-10)!}{n!}$$

$$+\frac{n!}{11!(n-11)!}\times\frac{10!(n-10)!}{n!}$$

$\Rightarrow \quad 2 = \frac{10}{n-9} + \frac{n-10}{11}$

$2 = \frac{110 + (n-10)(n-9)}{11(n-9)}$

$2 = \frac{110 + n^2 - 9n - 10n + 90}{11n - 99}$

$22n - 198 = 200 + n^2 - 19n$

$\therefore \quad n^2 - 41n = -398$

17. **(b)** We have given,

$\sin\theta + \cos\theta = p$ ... (i)

and $\tan\theta + \cot\theta = q$ ... (ii)

From Eq. (i)

$(\sin\theta + \cos\theta)^2 = p^2$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = p^2$

[$\because \sin^2\theta + \cos^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$]

$1 + \sin 2\theta = p^2$

$\sin 2\theta = p^2 - 1$ ... (iii)

From Eq. (ii), $\tan\theta + \cot\theta = q$

$\tan\theta + \frac{1}{\tan\theta} = q \Rightarrow \frac{\tan^2\theta + 1}{\tan\theta} = q$

$\frac{\tan^2\theta + 1}{2\tan\theta} = \frac{q}{2} \quad \left[\because \operatorname{cosec} 2\theta = \frac{\tan^2\theta + 1}{2\tan\theta}\right]$

$\therefore \quad \operatorname{cosec} 2\theta = \frac{q}{2}$

$\sin 2\theta = \frac{2}{q}$

$p^2 - 1 = \frac{2}{q}$ [from eq. (iii)]

$q(p^2 - 1) = 2$

18. **(a)** Given, $\tan\frac{\pi}{5} + 2\tan\frac{2\pi}{5} + 4\cot\frac{4\pi}{5}$

$= \tan\frac{\pi}{5} + 2\left[2\cot\frac{4\pi}{5} + \tan\frac{2\pi}{5}\right]$

We know that,

$2\cot 2A + \tan A = \cot A$ ... (i)

$= \tan\frac{\pi}{5} + 2\cot\frac{2\pi}{5}$ [from Eq. (i)]

$= \cot\frac{\pi}{5}$

19. **(d)** Here, we have the pattern of data in $\cos x$ and $\sin x$, so let's use the method of polar form of complex number.

Polar form of complex number,

$z = \cos\theta + i\sin\theta$

then $\bar{z} = \cos\theta - i\sin\theta$

and $z\bar{z} = 1$

$\Rightarrow \quad \bar{z} = \frac{1}{z}$

Let $z_1 = \cos A + i\sin A$

$z_2 = \cos B + i\sin B$

and $z_3 = \cos C + i\sin C$

Then, $\quad \bar{z}_1 = \cos A - i\sin A$

$\bar{z}_2 = \cos B - i\sin B$

$z_3 = \cos C - \sin C$

Now, to find $\cos(A+B) + \cos(B+C) + \cos(C+A)$

$\bar{z}_1 + \bar{z}_2 + \bar{z}_3 = (\cos A - i\sin A)$

$+ (\cos B - i\sin B) + (\cos C - i\sin C)$

$= (\cos A + \cos B + \cos C) - i(\sin A + \sin B + \sin C) = 0$

$\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0 \quad \left[\because \bar{z} = \frac{1}{z}\right]$

$z_1z_2 + z_2z_3 + z_3z_1 = 0$

$\Sigma(\cos A + i\sin A)(\cos B + i\sin B) = 0$

$\Sigma(\cos A\cos B - \sin A\sin B)$

$+ i(\sin A\cos B + \cos A\sin B) = 0$

$\Sigma\cos(A+B) + i\sin(A+B) = 0$

$\Sigma\cos(A+B) = 0$

Comparing the real part, we get

$\cos(A+B) + \cos(B+C) + \cos(C+A) = 0$

20. **(a)** We have given,

$\tan\theta \cdot \tan(120° - \theta)\tan(120° + \theta) = \frac{1}{\sqrt{3}}$

Since, we know

$\tan\theta\tan(120° - \theta)\tan(120° + \theta) = \tan 3\theta$

$\therefore \quad \tan 3\theta = \frac{1}{\sqrt{3}} \Rightarrow \tan 3\theta = \tan\frac{\pi}{6}$

$3\theta = n\pi + \frac{\pi}{6}$

$\theta = \frac{n\pi}{3} + \frac{\pi}{18}, \; n \in Z$

21. **(b)** Given that, $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^2}{4} - \ldots\infty\right)$

$\cos^{-1}\left(x - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\infty\right) = \frac{\pi}{2}$

Here, $\quad x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots\infty$

Forms G.P. so its sum can be given as $\frac{x}{1 - \frac{x}{2}}$

Similarly, for series

$x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\infty \Rightarrow \text{sum} = \frac{x^2}{1 - \frac{x^2}{2}}$

$$\sin^{-1}\left[\frac{x}{1-\frac{x}{2}}\right]+\cos^{-1}\left[\frac{x^2}{1-\frac{x^2}{2}}\right]=\frac{\pi}{2}$$

$$\sin^{-1}\left[\frac{2x}{2-x}\right]+\cos^{-1}\left[\frac{2x^2}{2-x^2}\right]=\frac{\pi}{2}$$

$$\left[\because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\right]$$

$$\frac{2x}{2-x}=\frac{2x^2}{2-x^2}$$

$$\frac{1}{2-x}=\frac{x}{2-x^2}$$

$(2-x^2)=x(2-x)$

$2-x^2=2x-x^2$

$2x=2 \Rightarrow x=1$

22. **(a)** We have given,

$$2\sinh^{-1}\left(\frac{a}{\sqrt{1-a^2}}\right)=\log\left(\frac{1+x}{1-x}\right) \quad \text{... (i)}$$

As we know,

$$\sinh^{-1}x=\log x+\sqrt{1+x^2}$$

So, $2\sinh^{-1}x=2\log\left(x+\sqrt{1+x^2}\right)$

$$=\log\left(x+\sqrt{1+x^2}\right)^2$$

$$=\log\left[x^2+1+x^2+2x\sqrt{1+x^2}\right]$$

$$2\sinh^{-1}x=\log\left[2x^2+1+2x\sqrt{1+x^2}\right]$$

Now, put $x=\frac{a}{\sqrt{1-a^2}}$

$$2\sinh^{-1}\left(\frac{a}{\sqrt{1-a^2}}\right)=\log\left[\frac{2a^2}{1-a^2}+1+2\times\frac{a}{\sqrt{1-a^2}}\times\sqrt{1+\frac{a^2}{1-a^2}}\right]$$

$$=\log\left[\frac{2a^2}{1-a^2}+1+\frac{2a}{\sqrt{1-a^2}}\times\frac{1}{\sqrt{1-a^2}}\right]$$

$$=\log\left[\frac{2a^2}{1-a^2}+1+\frac{2a}{1-a^2}\right]$$

$$=\log\left[\frac{2a^2+1-a^2+2a}{1-a^2}\right]$$

$$=\log\left[\frac{a^2+2a+1}{1-a^2}\right]$$

Now, by using Eq. (i),

$$\log\left[\frac{a^2+2a+1}{1-a^2}\right]=\log\left[\frac{1+x}{1-x}\right]$$

$$\frac{a^2+2a+1}{1-a^2}=\frac{1+x}{1-x}$$

[by using componendo and dividendo rule]

$$\frac{a^2+2a+1+1-a^2}{a^2+2a+1-1+a^2}$$

$$=\frac{1+x+1-x}{1+x-1+x} \Rightarrow \frac{2+2a}{2a^2+2a}=\frac{1}{x}$$

$$\Rightarrow \frac{2(1+a)}{2a(1+a)}=\frac{1}{x} \Rightarrow \frac{1}{a}=\frac{1}{x} \Rightarrow x=a$$

23. **(b)** Given that, $r_1=2r_2=3r_3$

$$\frac{\Delta}{s-a}=\frac{2\Delta}{s-b}=\frac{3\Delta}{s-c}=\frac{1}{K} \quad \text{(say)}$$

Then, $s-a=\Delta K$ ... (i)

$s-b=2\Delta K$ ... (ii)

and $s-c=3\Delta K$ ... (iii)

Now, adding Eqs. (i), (ii) and (iii), we get

$(s-a)+(s-b)+(s-c)=\Delta K+2\Delta K+3\Delta K$

$3s-(a+b+c)=6\,\Delta K$

Since, we know $a+b+c=2s$

$$3s-2s=6\times\left(\frac{s-b}{2}\right) \quad \text{[Using Eq. (ii)]}$$

$3s-2s=3(s-b)$

$s=3s-3b$

$2s=3b$

24. **(a)** Given that, $a_1=2\hat{i}-\hat{j}+\hat{k}$

$a_2=3\hat{i}-4\hat{j}-4\hat{k}$

$a_3=-\hat{i}+\hat{j}-\hat{k}$

$a_4=-\hat{i}+3\hat{j}+\hat{k}$

As M_1, M_2, M_3 and M_4 are the magnitudes of the vectors a_1, a_2, a_3 and a_4 respectively.

$\therefore \quad M_1=|a_1|$

$=\sqrt{(2)^2+(-1)^2+(1)^2}$

$=\sqrt{4+1+1}=\sqrt{6}$

$M_2=|a_2|$

$=\sqrt{(3)^2+(-4)^2+(-4)^2}$

$=\sqrt{9+16+16}=\sqrt{41}$

$M_3=|a_3|$

$=\sqrt{(-1)^2+(1)^2+(-1)^2}$

$=\sqrt{1+1+1}=\sqrt{3}$

$M_4=|a_4|$

$= \sqrt{(-1)^2+(3)^2+(-1)^2}$

$= \sqrt{1+9+1} = \sqrt{11}$

Hence, the correct order of magnitudes

$M_3 < M_1 < M_4 < M_2$

25. **(b)** Given that $\hat{a}, \hat{b}, \hat{c}$ are unit vectors and $\hat{a}+\hat{b}+\hat{c}=0$

$\therefore \quad |\hat{a}| = |\hat{b}| = |\hat{c}| = 1$

Here, $\hat{a}+\hat{b}+\hat{c}=\hat{0}$

Then, $(\hat{a}+\hat{b}+\hat{c})^2 = (\hat{0})^2$

$\Rightarrow \quad (\hat{a}+\hat{b}+\hat{c})\cdot(\hat{a}+\hat{b}+\hat{c}) = 0$

$\Rightarrow \quad \hat{a}\cdot\hat{a}+\hat{a}\cdot\hat{b}+\hat{a}\cdot\hat{c}+\hat{b}\cdot\hat{a}+\hat{b}\cdot\hat{b}$

$\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}+\hat{c}\cdot\hat{b}+\hat{c}\cdot\hat{c}=0$

$\Rightarrow \quad |\hat{a}|^2+\hat{a}\cdot\hat{b}+\hat{c}\cdot\hat{a}+\hat{a}\cdot\hat{b}+|\hat{b}|^2+\hat{b}\cdot\hat{c}$

$+\hat{c}\cdot\hat{a}+\hat{b}\cdot\hat{c}+|\hat{c}|^2=0$

$\Rightarrow \quad (1)^2+2\hat{a}\cdot\hat{b}+2\hat{b}\cdot\hat{c}+2\hat{c}\cdot\hat{a}+(1)^2+(1)^2=0$

$\Rightarrow \quad 2\left[\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}\right]+3=0$

$2\left[\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}\right]=-3$

$\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}=-\frac{3}{2}$

26. **(d)** Three vectors a, b and c are given as,

$a = 2\hat{i}+\hat{k}, b=\hat{i}+\hat{j}+\hat{k}$ and $c = 4\hat{i}-3\hat{j}+7\hat{k}$

Given condition, $r \times b = c \times b$

$r \times b - c \times b = 0$

$(r - c) \times b = 0$

It means, $(r - c) \parallel b$

So, $r - c = \lambda b$

$r = c + \lambda b$ ...(i)

Also given, $r \cdot a = 0$

$(c + \lambda b) \cdot a = 0$ [using Eq. (i)]

$(4\hat{i}-3\hat{j}+7\hat{k}+\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k})\cdot(2\hat{i}+\hat{k})=0$

$\left[(4+\lambda)\cdot\hat{i}+(-3+\lambda)\hat{j}+(7+\lambda)\hat{k}\right]\cdot(2\hat{i}+\hat{k})=0$

$(4+\lambda)\cdot 2+(7+\lambda)\cdot 1=0$

$8+2\lambda+7+\lambda=0$

$3\lambda=-15$

$\therefore \quad \lambda=-5$

Put the value of λ in Eq. (i), we get

$r = 4\hat{i}-3\hat{j}+7\hat{k}-5(\hat{i}+\hat{j}+\hat{k})$

$= 4\hat{i}-3\hat{j}+7\hat{k}-5\hat{i}-5\hat{j}-5\hat{k} = -\hat{i}-8\hat{j}+2\hat{k}$

27. **(d)** For three vectors a, b and c

$|a|=1, |b|=2, |c|=3$

and $a\cdot b = b\cdot c = c\cdot a = 0$

$$\because \quad |[a\ b\ c]|^2 = \begin{vmatrix} a\cdot a & a\cdot b & a\cdot c \\ b\cdot a & b\cdot b & b\cdot c \\ c\cdot a & c\cdot b & c\cdot c \end{vmatrix}$$

$$= \begin{vmatrix} |a|^2 & 0 & 0 \\ 0 & |b|^2 & 0 \\ 0 & 0 & |c|^2 \end{vmatrix} \qquad [\because\ a\cdot a = |a|]$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 9 & 9 \end{vmatrix}$$

$= 1\,(36 - 0)$

$\Rightarrow \quad |[a\ b\ c]|^2 = 36$

$\therefore \quad |[a\ b\ c]| = 6$

28. **(b)** Given,

$[a \times b\ \ b \times c\ \ c \times a] = \lambda [a\ b\ c]^2$

As we know, the properties of cross product of three vectors, we have

$[a \times b\ \ b \times c\ \ c \times a] = [a\ b\ c]^2$

$\therefore \quad \lambda = 1$

29. **(c)** Vector b and c are given as,

$b = \hat{i}-2\hat{j}+4\hat{k}$

and $c = 3\hat{i}+2\hat{j}-5\hat{k}$

Cartesian equation of plane passing through the point (3, –2, –1)

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-3 & y+2 & z+1 \\ 1 & -2 & 4 \\ 3 & 2 & -5 \end{vmatrix} = 0$$

$(x-3)(10-8)-(y+2)(-5-12)+(z+1)(2+6)=0$

$(x-3)(2)-(y+2)(-17)+(z+1)\,8=0$

$2x-6+17y+34+8z+8=0$

$2x+17y+8z+36=0$

30. **(d)** Given observations are 10, 8, 5, a, b.

Arithmetic Mean,

$$\text{A.M.} = \frac{10+8+5+a+b}{5} = 6$$

$23 + a + b = 30$

$a + b = 7$ ... (i)

Here,

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
10	4	16
8	2	4
5	–1	1
a	$a - 6$	$(a - 6)^2$
b	$b - 6$	$(b - 6)^2$

$\therefore \quad \Sigma(x_i - \bar{x})^2 = a^2 + b^2 + 93 - 12(a + b)$

Variance,

$$\frac{1}{n}\Sigma(x_i - \bar{x})^2 = 6.8$$

$$\frac{a^2 + b^2 + 93 - 12(a+b)}{5} = 6.8$$

$a^2 + b^2 + 93 - 12 \times 7 = 6.8 \times 5$

$a^2 + b^2 + 93 - 84 = 34$ $\quad [\because \; a + b = 7]$

$a^2 + b^2 = 25$ $\quad$ ... (ii)

$[\because \; (a + b)^2 = a^2 + b^2 + 2ab]$

$(7)^2 = 25 + 2ab$

[using Eqs. (i) and (ii)]

$49 = 25 + 2ab$

$ab = 12$

31. (b) Since, there are six observations and are written in ascending order then

$$\text{Median} = \left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \frac{\text{observation}}{2}$$

$$\frac{x - 2 + x}{2} = 16$$

$2x - 2 = 32$

$2x = 34 \;\Rightarrow\; x = 17$

So the observations are 6, 7, 15, 17, 18 and 21.

$$\therefore \quad \text{Mean} = \frac{6 + 7 + 15 + 17 + 18 + 21}{6}$$

$$\bar{x} = \frac{84}{6} = 14$$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
6	–8	64
7	–7	49
15	1	1
17	3	9
18	4	16
21	7	49
		$\Sigma(x_i - \bar{x})^2 = 188$

$$\therefore \quad \text{Variance} = \frac{1}{n}\Sigma(x_i - x)^2 = \frac{188}{6} = \frac{94}{3} = 31\frac{1}{3}$$

32. (c) Here, probability of success,

$p = \frac{1}{6}$ and probability

of failure. $q = \frac{5}{6}$

Let A starts the game, then

$$A = \frac{1}{6} + \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2 + \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4 + ...$$

$$P(A) = \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{\frac{1}{6}}{1 - \frac{25}{36}}$$

$$= \frac{\frac{1}{6}}{\frac{11}{36}} = \frac{6}{11}$$

Total probability, $P(A) + P(B) = 1$

$\therefore \quad P(B) = 1 - P(A)$

$$= 1 - \frac{6}{11} = \frac{5}{11}$$

33. (d) The given word is 'QUESTION', let E be the event that there are exactly two letters between Q and S.

Here, number of sample space,

$n(S) = 8!$

number of possible events,

$n(E) = {}^6P_2 \times 2! \times 5!$

$\therefore$ Required probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^6P_2 \times 2! \times 5!}{8!} = \frac{6! \times 2! \times 5!}{4! \times 8!}$$

$$= \frac{6! \times 2! \times 5! \times 4!}{4! \times 7 \times 8 \times 6!} = \frac{2 \times 5}{7 \times 8} = \frac{5}{28}$$

34. (a) Given,

$$0 \le \frac{1 + 3P}{3} \le 1 \text{ and } 0 \le \frac{1 - 2P}{3} \le 1$$

$\Rightarrow \quad 0 \le 1 + 3P \le 3$ and $0 \le 1 - 2P \le 2$

$\Rightarrow \quad -1 \le 3P \le 2$ and $-1 \le -2P \le 1$

$\Rightarrow \quad -\frac{1}{3} \le P \le \frac{2}{3}$ and $-\frac{1}{2} \le P \le \frac{1}{2}$

$\therefore \quad P \in \left[-\frac{1}{3}, \frac{1}{2}\right]$

35. (a) Given, $p = 0.2$, $q = 0.8$, $n = 3$

From binomial distribution,

$P(X = r) = {}^nC_2 \, P^r q^{n-r}$

Probability that event happens at most once,

$P(X \le 1) = P(X = 0) + P(X = 1)$

$= {}^3C_0 (0.2)^0 (0.8)^{3-0} + {}^3C_1 (0.2)^1 (0.8)^{3-1}$

$= \frac{3!}{0!3!} \times 1 \times \left(\frac{8}{10}\right)^3 + \frac{3!}{1!2!}\left(\frac{2}{10}\right)\left(\frac{8}{10}\right)^2$

$= \frac{64}{125} + \frac{48}{125} = \frac{112}{115} = 0.896$

36. **(c)** As we know, $\sum_{i=1}^{8} P(x_i) = 1$

$0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$9K + 10K^2 = 1$

$10K^2 + 9K - 1 = 0$

$10K^2 + 10K - K - 1 = 0$

$10K(K + 1) - 1(K + 1) = 0$

$(K + 1)(10K - 1) = 0$

$\therefore \quad K = -1, \frac{1}{10}$

As the probability cannot be negative. So K must be greater than 0.

$\therefore \quad K = \frac{1}{10}$

$P(0 < x < 5) = P(X = 1) + P(X = 2)$

$+ P(X = 3) + P(X = 4)$

$= K + 2K + 2K + 3K = 8K$

$= \frac{8}{10}$

37. **(b)** Let $P(x, y)$ be the required point whose locus is given by $ax + by + c = 0$

Also given that P is equidistant from $A(-2, 3)$ and $B(6, -5)$. Then,

$PA = PB$ or $PA^2 = PB^2$

$(x + 2)^2 + (y - 3)^2 = (x - 6)^2 + (y + 5)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9$

$= x^2 - 12x + 36 + y^2 + 10y + 25$

$16x - 16y - 48 = 0$

$x - y - 3 = 0$

On comparing with $ax + by + c = 0$, we get

$a = 1, b = -1, c = -3$

Hence, the ascending order is c, b, a.

38. **(c)** Let $P(2, 3)$ be the given point and Q be the reflection of point $P(2, 3)$ about the line $y = x$. Then, the coordinates of Q are $(3, 2)$

Now, the point Q is translated through a distance of 2 units along the positive direction of X-axis.

Let the new position of Q be R.

Then, the coordinates of R are

$R = (3 + 2, 2)$

$R = (5, 2)$

39. **(c)** Here, point A is the intersection of line AB and AC so equation of line passing through A.

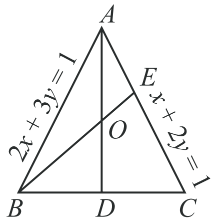
$(x + 2y - 1) + \lambda(2x + 3y - 1) = 0$ ... (i)

This line passes through the orthocentre (0,0), then

$-1 - \lambda = 0$

$\lambda = -1$

On substituting $\lambda = -1$ in Eq. (*i*), we get $x + y = 0$ as the equation of AD. Since $AD \perp BC$, therefore



$-1 \times -\frac{a}{b} = -1$

$\Rightarrow \quad a + b = 0$ ... (ii)

Similarly, by applying the condition that BE is perpendicular to CA, we get

$a + 2b = 8$ ... (iii)

Now, solving Eqs. (ii) and (iii), we get $a = -8, b = 8$

40. **(b)** Let the required point be P (x_1, y_1) and it is on the line $4x - y - 2 = 0$. Then,

$4x_1 - y_1 - 2 = 0$ ... (i)

Also given, point P is equidistant from $A(-5, 6)$ and $B(3, 2)$.

$\therefore \quad PA^2 = PB^2$

$(x_1 + 5)^2 + (y_1 - 6)^2 = (x_1 - 3)^2 + (y_1 - 2)^2$

$x_1^2 + 10x_1 + 25 + y_1^2 - 12y_1 + 36$

$= x_1^2 - 6x_1 + 9 + y_1^2 - 4y_1 + 4$

$= 16x_1 - 8y_1 + 48 = 0$

$= 2x_1 - y_1 + 6 = 0$ ...(ii)

On solving Eq. (i) in Eq. (ii), we get

$\therefore \quad x_1 = 4$

and $y_1 = 14$

41. **(c)** Let these three lines be L_1, L_2 and L_3

$L_1 = x + 2ay + a = 0$

$L_2 = x + 3by + b = 0$

$L_3 = x + 2cy + c = 0$

If L_1, L_2 and L_3 are concurrent, then

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$,

$$\begin{vmatrix} 0 & 2a-3b & a-b \\ 0 & 3b-4c & b-c \\ 1 & 4c & c \end{vmatrix} = 0$$

$\Rightarrow$ $1[(2a-3b)(b-c)-(3b-4c)(a-b)] = 0$

$\Rightarrow$ $(2a-3b)(b-c) = (3b-4c)(a-b)$

$\Rightarrow$ $2ab - 2ca - 3b^2 + 3bc = 3ab - 3b^2 - 4ca + 4bc$

$\Rightarrow$ $ab + bc = 2ca$

$\Rightarrow$ $\frac{1}{a}+\frac{1}{c}=\frac{2}{b}$

Hence, a, b and c are in Harmonic progression.

42. **(a)** Given pair of lines, $ax^2 - 6xy + y^2 = 0$

Let the slope of one line be m, then slope of another line will be m^2.

We know that

$\Rightarrow$ $m + m^2 = \frac{-(-6)}{1} = 6$ ... (i)

$\Rightarrow$ $m \cdot m^2 = m^3 = \frac{a}{1} = a$ ... (ii)

On cubing Eq. (i) both sides, we get

$(m + m^2)^3 = (6)^3$

$\Rightarrow$ $m^3 + m^6 + 3m^3(m + m^2) = 216$

$\Rightarrow$ $m^3 + m^6 + 18m^3 = 216$ $[\because m^3 = a]$

$\Rightarrow$ $a + a^2 + 18a = 216$

$\Rightarrow$ $a^2 + 19a - 216 = 0$

$a^2 + 27a - 8a - 216 = 0$

$a(a + 27) - 8(a + 27) = 0$

$(a + 27)(a - 8) = 0$

$\therefore$ $a = -27, 8$

43. **(d)** Let the given point be $P(4, -3)$

and the given circle is

$x^2 + y^2 + 4x - 10y - 7 = 0$

Centre of circle = C $(-2, 5)$

Radius = $\sqrt{(-2)^2+(5)^2+7}$

$= \sqrt{4+25+7} = \sqrt{36} = 6$

Maximum distance, $a = CP + r$

Minimum distance, $b = CP - r$

Sum of the maximum and minimum distance,

$a + b = CP + r + CP - r = 2CP$

and $CP = \sqrt{(-2-4)^2+(5+3)^2}$

$= \sqrt{36+64} = \sqrt{100} = 10$

Thus, $a + b = 2CP = 2 \times 10 = 20$

44. **(b)** Let the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

Cuts the circle

$x^2 + y^2 + 4x - 6y + 9 = 0$ and

$x^2 + y^2 - 5x + 4y + 2 = 0$

orthogonally.

For circle

$x^2 + y^2 + 4x - 6y + 9 = 0$

$2(g_1 g_2 + f_1 f_2) = c_1 + c_2$

$2[(g)(-2) + (f)(3)] = c + 9$

$-4g + 6f = c + 9$ ... (ii)

For circle,

$x^2 + y^2 - 5x + 4y + 2 = 0$

$2\left[(g)\left(\frac{5}{2}\right)+(f)(-2)\right] = c + 2$

$5g - 4f = c + 2$ ... (iii)

On subtracting Eq. (iii) from Eq. (ii), we get

$-9g + 10f = 7$

$9g - 10f = -7$

Replace g and f by x and y to get the locus of centre,

$9x - 10y = -7$

$\Rightarrow$ $9x - 10y + 7 = 0$

45. **(a)** Given that circle, $S = x^2 + y^2 + y - 1 = 0$

Line, L : $x - y + 1 = 0$

Equation of the circle passing through the intersection of line and circle is given by

$S + \lambda L = 0$

$(x^2 + y^2 + y - 1) + \lambda(x - y + 1) = 0$

$x^2 + y^2 + \lambda x + (1 - \lambda)y + \lambda - 1 = 0$

Centre of above circle = $\left(\frac{-\lambda}{2}, \frac{\lambda-1}{2}\right)$

Since, centre lies on $x - y + 1 = 0$

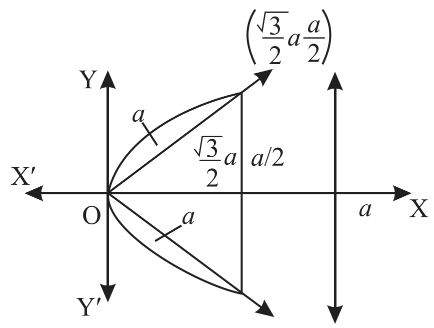
$\therefore$ $\frac{-\lambda}{2} - \left(\frac{\lambda-1}{2}\right) + 1 = 0 \Rightarrow \lambda = \frac{3}{2}$

Now, the required equation of circle

$x^2 + y^2 + y - 1 + \frac{3}{2}(x - y + 1) = 0$

$\therefore$ $2(x^2 + y^2) + 3x - y + 1 = 0$

46. **(b)** Let a be the length of the side of an equilateral triangle.



Then, from above figure, we can say that the point $\left(\frac{\sqrt{3}}{2}a, \frac{a}{2}\right)$ will lie on parabola $y^2 = 8x$.

So, $\left(\frac{a}{2}\right)^2 = 8\left(\frac{\sqrt{3}}{2}a\right) \Rightarrow a^2 = 16\sqrt{3}a \Rightarrow a = 16\sqrt{3}$ units

47. **(a)** Given that focus of parabola is (3, 4) and equation of directrix is $2x - 3y + 5 = 0$

As we know, Length of the latusrectum

= 2 × Length of perpendicular from focus on directrix

$$= \left|\frac{2\times 3 - 3\times 4 + 5}{\sqrt{2^2 + (-3)^2}}\right| = \frac{2}{\sqrt{13}}$$

48. **(b)** Equation of ellipse, $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$a^2 = 16 \Rightarrow a = 4,\ b^2 = 9 \Rightarrow b = 3$

Here, $a > b$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

∴ Focus = $(\pm ae, 0) = \pm\sqrt{7},\ 0$

Since, the circle passes through P$\left(\pm\sqrt{7},\ 0\right)$ and centre at C (0, 3).

∴ Radius = $rcp = \sqrt{(0-7)^2 + (3-0)^2}$

$= \sqrt{7+9} = \sqrt{16} = 4$

49. **(c)** As the straight line, $y = mx + 4$ touches the circle $x^2 + 4y^2 = 4$,

Put $y = 4x + m$ in $x^3 + 4y^2 = 4$, we get

$x^2 + 4(4x + m)^2 = 4$

$\Rightarrow x^2 + 4(16x^2 + 8mx + m^2) = 4$

$\Rightarrow x^2 + 64x^2 + 32mx + 4(m^2 - 1) = 0$

$\Rightarrow 65x^2 + 32mx + 4(m^2 - 1) = 0$

Since, line is tangent to the given curve

$\Rightarrow D = 0$

$\therefore (32m)^2 - 4(65)[4(m^2 - 1)] = 0$

$\Rightarrow m = \pm\sqrt{65}$

50. **(b)** Equation of ellipse, $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

Eccentricity,

$\therefore e = \sqrt{1 - \frac{b^2}{16}} = \frac{\sqrt{16 - b^2}}{4}$

So, the focus will be $\left(\pm\sqrt{16 - b^2}, 0\right)$

Also, the equation of hyperbola,

$$\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25} \Rightarrow \frac{x^2}{\left(\frac{144}{25}\right)} - \frac{y^2}{\left(\frac{81}{25}\right)} = 1$$

$$\therefore e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{\frac{81}{25}}{\frac{144}{25}}} = \pm\frac{15}{12}$$

So, the focus will be

$\left(\pm\frac{12}{5}\times\frac{15}{12}, 0\right)$ i.e., $(\pm 3, 0)$

On comparing the focus of ellipse with hyperbola, we get

$\sqrt{16 - b^2} = 3$

$16 - b^2 = 9$

$b^2 = 7$

51. **(c)** Given direction ratios are (2, –1, 2) and (K, 3, 5)

Let θ be the angle between two lines, then

$$\cos\theta = \frac{(2,-1,2)(K,3,5)}{\sqrt{4+1+4}\,\sqrt{K^2+9+25}}$$

$$\cos 45° = \frac{2K - 3 + 10}{\sqrt{9}\,\sqrt{K^2 + 34}}$$

$$\frac{1}{\sqrt{2}} = \frac{2K + 7}{3\times\sqrt{K^2 + 9 + 25}}$$

$3\sqrt{K^2 + 34} = \sqrt{2}(2K + 7)$

$9(K^2 + 34) = 2(2K + 7)^2$

$9K^2 + 306 = 8K^2 + 56K + 98$

$K^2 - 56K + 208 = 0 \Rightarrow K = 52, 4$

∴ According to the given options, $K = 4$

52. **(a)** Here, intercepts made with the coordinate axes X, Y and Z are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively.

Intercept form of a plane is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

We have, $a = \frac{1}{3}, b = \frac{1}{4}, c = \frac{1}{5}$

∴ Equation of plane is

$3x + 4y + 5z = 1$

$\Rightarrow 3x + 4y + 5z - 1 = 0$

Distance from origin

$$= \left|\frac{3\times 0 + 4\times 0 + 5\times 0 - 1}{\sqrt{3^2 + 4^2 + 5^2}}\right| = \frac{1}{5\sqrt{2}}$$

53. **(a)** (A) Given coordinates of triangle are

$P(2, 3, -1), Q(5, 6, 3), R(2, -3, 1)$

$$\therefore \text{Centroid, } G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

$$= \left(\frac{2+5+2}{3}, \frac{3+6-3}{3}, \frac{-1+3+1}{3}\right)$$

$= \left(\frac{9}{3}, \frac{6}{3}, \frac{3}{3}\right) = (3, 2, 1)$

(B) Given coordinates of triangle are P (1, 2, 3), Q (2, 3, 1) and R (3, 1, 2)

Since, ΔPQR is an equilateral triangle.

$\therefore$ Circumcentre of the triangle,

$$C = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

$$= \left(\frac{1+2+3}{3}, \frac{2+3+1}{3}, \frac{3+1+2}{3}\right) = (2, 2, 2)$$

(C) Given coordinates of triangle are P (2, 1, 5), Q (3, 2, 3) and R (4, 0, 4).

Since, ΔPQR is an equilateral triangle.

$\therefore$ Orthocentre of the triangle,

$$O = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

$$= \left(\frac{2+3+4}{3}, \frac{1+2+0}{3}, \frac{5+3+4}{3}\right) = (3,1,4)$$

(D) Given coordinates of triangle are P (0, 0, 0), Q (3, 0, 0) and R (0, 4, 0).

Since, ΔPQR is a right angled triangle.

Here, $a = 5, b = 4, c = 3$

$\therefore$ Incentre of the triangle,

$$I = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}, \frac{az_1+bz_2+cz_3}{a+b+c}\right)$$

$$= \left(\frac{5\times0+4\times3+3\times0}{3+4+5}, \frac{5\times0+4\times0+3\times4}{3+4+5}, \frac{5\times0+4\times0+0\times3}{3+4+5}\right)$$

$$= \left(\frac{12}{12}, \frac{12}{12}, 0\right) = (1,1,0)$$

54. **(c)** Given, $g(x) = \frac{x}{[x]}$

When, $x > 2$, then $[x] = 2 \Rightarrow g(x) = \frac{x}{2}$

Now, $\lim\limits_{x\to2} \frac{g(x)-g(2)}{x-2} = \lim\limits_{x\to2} \frac{\frac{x}{2}-1}{x-2}$

$= \lim\limits_{x\to2} \frac{x-2}{2(x-2)} = \lim\limits_{x\to2}\left(\frac{1}{2}\right) = \frac{1}{2}$

55. **(c)** Let $\ell = \lim\limits_{x\to2} \frac{\pi}{2}\left(\frac{2x-\pi}{\cos x}\right)$ $\left[\text{form } \frac{0}{0}\right]$

Use L' Hospital rule $= \lim\limits_{x\to\pi/2} \frac{2}{-\sin x} = \frac{2}{-1} = -2$

56. **(b)** Give function,

$$f(x) = \begin{cases} x & \text{for } 0 \le x < 1 \\ 2-x & \text{for } x \ge 1 \end{cases}$$

At $x = 1$

$\Rightarrow f(x) = 2 - x$

$\Rightarrow f(1) = 2 - 1 = 1$

LHL $= \lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^-} x = 1$

RHL $= \lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+} (2-x) = 2-1 = 1$

LHL = RHL = $f(1)$

$\therefore$ $f(x)$ is continuous at $x = 1$

$\Rightarrow$ To check the differentiability of $f(x)$,

$$f'(x) = \begin{cases} 1 & \text{for } 0 \le x < 1 \\ -1 & \text{for } x \ge 1 \end{cases}$$

LHD $= \lim\limits_{x\to1^-} f'(x) = \lim\limits_{x\to1^-} 1 = 1$

RHD $= \lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+} (-1) = -1$

LHD $\ne$ RHD

$\therefore$ $f(x)$ is not differentiable at $x = 1$

57. **(b)** We have,

$$x^2 + y^2 = t + \frac{1}{t} \quad \text{.. (i)}$$

and $$x^4 + y^4 = t^2 + \frac{1}{t^2} \quad \text{... (ii)}$$

From Eq. (i), squaring both the sides

$$\left(x^2+y^2\right)^2 = \left(t+\frac{1}{t}\right)^2$$

$$x^4 + y^4 + 2x^2y^2 = t^2 + \frac{1}{t^2} + 2$$

$x^4 + y^4 + 2x^2y^2 = x^4 + y^4 + 2$ [using Eq. (ii)]

$2x^2y^2 = 2$

Differentiating w.r.t. x, we get

$$2xy^2 + 2x^2y\frac{dy}{dx} = 0$$

$$\therefore \quad \frac{dy}{dx} = \frac{-2xy^2}{2x^2y} = -\frac{y}{x}$$

58. **(d)** We have, $x = at^2$

$\Rightarrow \frac{dx}{dt} = 2at \Rightarrow \frac{dt}{dx} = \frac{1}{2at}$

and $y = 2at \Rightarrow \frac{dy}{dt} = 2a$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2a}{2at} = \dfrac{1}{t}$

Differentiating w.r.t x, we get

Now, $\dfrac{d^2y}{dx^2} = \dfrac{-1}{t^2} \cdot \dfrac{dt}{dx} = \dfrac{-1}{t^2} \cdot \dfrac{1}{(2at)} \quad \left[\because \dfrac{dt}{dx} = \dfrac{1}{2at}\right]$

$= \dfrac{-1}{2at^3}$

At $t = \dfrac{1}{2} \left(\dfrac{d^2y}{dx^2}\right)_{t=\frac{1}{2}} = \dfrac{-1}{2a \times \frac{1}{8}} = \dfrac{-4}{a}$

59. **(b)** Let V be the volume, S be the surface area and r be the radius of the sphere.

It is given that, $\dfrac{dV}{dt} = 1200$ cu cm/s

and $r = 10$ cm

Now, Volume of sphere $= V = \dfrac{4}{3}\pi r^3$

$\therefore \quad \dfrac{dV}{dt} = \dfrac{4}{3}\pi 3r^2 \dfrac{dr}{dt}$

$1200 = 4\pi(10)^2 \dfrac{dr}{dt}$

$\dfrac{dr}{dt} = \dfrac{3}{\pi}$

and Surface Area of sphere,

$S = 4\pi r^2 = \dfrac{dS}{dt} = 4\pi \cdot 2r \dfrac{dr}{dt}$

$\Rightarrow \quad \dfrac{dS}{dt} = 4\pi \cdot 2(10)\dfrac{dr}{dt}$

$\Rightarrow \quad \dfrac{dS}{dt} = 4\pi \cdot 2 \times 10\left(\dfrac{3}{\pi}\right)$

$\therefore \quad \dfrac{dS}{dt} = 240$ sq cm/s

60. **(c)** We have, $y = \int_0^x \dfrac{1}{1+t^3} dt$

Differentiating w.r.t. x, we get $\dfrac{dy}{dx} = \dfrac{1}{1+x^3}$

At $x = 1$,

$\therefore \quad \left(\dfrac{dy}{dx}\right)_{x=1} = \dfrac{1}{1+1} = \dfrac{1}{2}$

61. **(a)** Given $\quad x^2 + y^2 = 25$

$\Rightarrow \quad y = \sqrt{25 - x^2}$

Let $z = 3x + 4y$

$\Rightarrow \quad z = 3x + 4\sqrt{25 - x^2}$

Differentiating w.r.t. x, we get

$\dfrac{dz}{dx} = 3 + \dfrac{4}{2\sqrt{25 - x^2}}(-2x)$

For maxima put $\dfrac{dz}{dx} = 0$

$3 = \dfrac{4x}{\sqrt{25 - x^2}}$

$3\sqrt{25 - x^2} = 4x$

$9\,(25 - x^2) = 16x^2$

$9 \times 25 - 9x^2 = 16x^2$

$9 \times 25 = (16 + 9)x^2$

$x^2 = 9$

$\therefore \quad x = 3$

$\Rightarrow \quad y = \sqrt{25 - x^2} \Rightarrow y = 4$

Now, $\quad z = 3x + 4y$

$= 3\,(3) + 4\,(4) = 25$

So, $\log_5 [\max (3x + 4y)] = \log_5 [\max (z)]$

$= \log_5 [(25)] = \log_5 (5)^2$

$= 2 \log_5 5 = 2$

62. **(c)** Let $I = \int \dfrac{dx}{(x-1)\sqrt{x^2 - 1}}$

Put $x - 1 = \dfrac{1}{t} \Rightarrow x = 1 + \dfrac{1}{t}$

Then, $\quad dx = \dfrac{-1}{t^2} dt$

$\therefore \quad I = \int \dfrac{-\frac{1}{t^2} dt}{\frac{1}{t}\sqrt{\left(1+\frac{1}{t}\right)^2 - 1}} = -\int \dfrac{dt}{t\sqrt{1 + \frac{1}{t^2} + \frac{2}{t} - 1}}$

$= -\int \dfrac{dt}{t\sqrt{\frac{1+2t}{t^2}}} = -\int \dfrac{dt}{\sqrt{1+2t}}$

$= -\int (1+2t)^{-\frac{1}{2}} dt$

$= -\dfrac{(1+2t)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \times \dfrac{1}{2}$

$= -\sqrt{1+2t} + C$

$= -\sqrt{1 + 2\left(\dfrac{1}{x-1}\right)} + C = -\sqrt{\dfrac{x-1+2}{x-1}} + C$

Thus, $I = -\sqrt{\dfrac{x+1}{x-1}} + C$

63. **(c)** Let $I = \int e^x \dfrac{x^2+1}{(x+1)^2} dx = \int e^x \left(\dfrac{x^2 - 1 + 2}{(x+1)^2}\right) dx$

$= \int e^x \left(\dfrac{x^2 - 1}{(x+1)^2} + \dfrac{2}{(x+1)^2}\right) dx$

$= \int e^x \left(\dfrac{x-1}{x+1} + \dfrac{2}{(x+1)^2}\right) dx$

Put $f(x)=\dfrac{x-1}{x+1}$

$\Rightarrow \quad f'(x)=\dfrac{(x+1)\dfrac{d}{dx}(x-1)-(x-1)\dfrac{d}{dx}(x+1)}{(x+1)^2}$

$=\dfrac{(1)(x+1)-(x-1)(1)}{(x+1)^2}$

$=\dfrac{2}{(x+1)^2}$

As we know, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C$

$\therefore \quad I=e^x\left(\dfrac{x-1}{x+1}\right)+C$

64. **(b)** Let $I=\int\dfrac{x+1}{x\left(1+xe^x\right)}dx$

Put $xe^x=t$

$\Rightarrow \quad (xe^x+e^x)\,dx=dt$

$\Rightarrow \quad e^x(x+1)\,dx=dt$

$\Rightarrow \quad (x+1)dx=\dfrac{dt}{e^x}$

$\Rightarrow \quad I=\int\dfrac{dt}{xe^x\left(1+xe^x\right)}=\int\dfrac{1}{t(1+t)}dt=\int\dfrac{t+1-t}{t(t+1)}dt$

$\therefore \quad I=\int\left(\dfrac{1}{t}-\dfrac{1}{t+1}\right)dt$

$=\log t-\log(t+1)+C$

$=\log\left|\dfrac{t}{t+1}\right|+C$

Now, Substitute the value of t, we get

$=\log\left|\dfrac{xe^x}{xe^x+1}\right|+C$

65. **(b)** Let $I=\int\dfrac{f(x)g'(x)-f'(x)g(x)}{f(x)g(x)}$

$\left[\log g(x)-\log f(x)\right]dx$

$=\int\log\left[\dfrac{g(x)}{f(x)}\right]\cdot d\left\{\log\dfrac{g(x)}{f(x)}\right\}$

Put $t=\log\left[\dfrac{g(x)}{f(x)}\right]$

$dt=d\left(\log\left[\dfrac{g(x)}{f(x)}\right]\right)$

Now, $I=\int t\,dt=\dfrac{t^2}{2}+C$

$=\dfrac{1}{2}\left\{\log\dfrac{g(x)}{f(x)}\right\}^2+C$

66. **(d)** Let $I=\int_0^{\pi/4}\dfrac{\sin x+\cos x}{3+\sin 2x}dx$

Put $\sin x-\cos x=t$

$(\sin x+\cos x)\,dx=dt$

Limits:

$x=0$

$\Rightarrow \quad t=\sin\theta-\cos\theta=-1$

$x=\dfrac{\pi}{4}\Rightarrow t=\sin\dfrac{\pi}{4}-\cos\dfrac{\pi}{4}=0$

$\sin 2x=1-1+\sin 2x$

$=1-(1-\sin 2x)$

$=1-(\sin^2 x+\cos^2 x-2\sin x\cos x)$

$=1-(\sin x-\cos x)^2$

$=1-t^2$

Now, $I=\int_{-1}^{0}\dfrac{dt}{3+1-t^2}=\int_{-1}^{0}\dfrac{dt}{(2)^2-(t)^2}$

$=\dfrac{1}{4}\left[\log\left(\dfrac{2+t}{2-t}\right)\right]_{-1}^{0}$

$=-\dfrac{1}{4}\log\dfrac{1}{3}=\dfrac{1}{4}\log 3$

67. **(c)** $I=\int_{-1}^{1}\dfrac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{\sqrt{1+x+x^2}+\sqrt{1-x+x^2}}dx$

Let $f(x)=\dfrac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{\sqrt{1+x+x^2}+\sqrt{1-x+x^2}}$

Replacing x by $-x$, we get

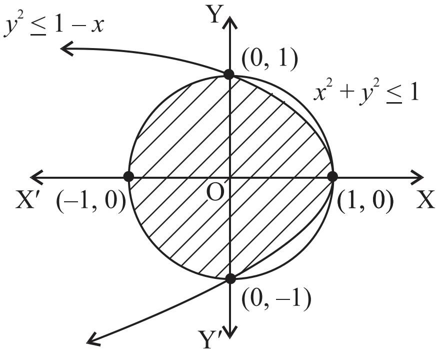
$f(-x)=\dfrac{\sqrt{1-x+x^2}-\sqrt{1+x+x^2}}{\sqrt{1-x+x^2}+\sqrt{1+x+x^2}}$

$=\dfrac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{\sqrt{1+x+x^2}+\sqrt{1-x+x^2}}=-f(x)$

So, $f(x)$ is an odd function.

$\therefore \quad \int_{-1}^{1}f(x)dx=0$

68. **(c)** Region: $\left\{(x,y)\mid x^2+y^2\le 1 \text{ and } y^2\le 1-x\right\}$



Required area

$$A = 2\left[\int_{-1}^{0}\sqrt{1-x}\,dx + \int_{0}^{1}\sqrt{1-x}\,dx\right]$$

$$= 2\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{-1}^{0} + 2\left[\frac{-2}{3}(1-x)^{\frac{3}{2}}\right]_{0}^{1}$$

$$= \frac{\pi}{2} + \frac{4}{3}$$

69. **(a)** Given differential equation,

$$\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$$

Dividing the eqn. by e^y, we get

$$\Rightarrow \quad e^{-y} = \frac{dy}{dx} + \frac{1}{x}e^{-y} = \frac{1}{x^2} \quad \text{... (i)}$$

Let $e^{-y} = v$

$$\Rightarrow \quad e^{-y}\frac{dy}{dx} = \frac{dv}{dx}$$

From eqn. (i), we get

$$\therefore \quad \frac{dv}{dx} - \frac{1}{x}v = \frac{-1}{x^2}$$

Integrating Factor,

$$\text{IF} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = e^{\log\frac{1}{x}} = \frac{1}{x}$$

Now, the required solution is

$$v(\text{IF}) = \int -\frac{1}{x^2}(\text{IF})\,dx + C_1$$

$$v\cdot\left(\frac{1}{x}\right) = \int \frac{-1}{x^2}\cdot\frac{1}{x}\,dx + C_1$$

$$\frac{v}{x} = -\int x^{-3}dx + C_1$$

$$\frac{v}{x} = \frac{1}{2x^2} + C_1 \Rightarrow \frac{v}{x} = \frac{1+2x^2C_1}{2x^2}$$

$$2xv = 1 + Cx^2 \quad [\because 2C_1 = C]$$

$$2xe^{-y} = 1 + Cx^2 \quad [\because v = e^{-y}]$$

$$2x = \left(1 + Cx^2\right)e^y$$

70. **(b)** Given differential equation,

$$\frac{dy}{dx} = \frac{1}{ax+by+c}$$

$$\frac{dy}{dx} = ax+by+c$$

$$\frac{dx}{dy} - ax = by + c$$

Hence, this equation is a linear differential equation in x.

71. **(b)** $$\left(\frac{1+\cos\frac{\pi}{8} - i\sin\frac{\pi}{8}}{1+\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}}\right)^8$$

$$= \left(\frac{2\cos^2\frac{\pi}{16} - 2i\sin\frac{\pi}{16}\cos\frac{\pi}{16}}{2\cos^2\frac{\pi}{16} + 2i\sin\frac{\pi}{16}\cos\frac{\pi}{16}}\right)^8$$

$$= \left(\frac{2\cos\frac{\pi}{16}\left(\cos\frac{\pi}{16} - i\sin\frac{\pi}{16}\right)}{2\cos\frac{\pi}{16}\left(\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right)}\right)^8$$

$$= \left(\frac{\cos\frac{\pi}{16} - i\sin\frac{\pi}{16}}{\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}}\right)^8$$

$$= \left(\frac{e^{-\frac{\pi}{16}i}}{e^{\frac{\pi}{16}i}}\right)^8$$

$$= \left(e^{-\frac{2\pi}{16}i}\right)^8 \quad \text{[Using eular's theorem]}$$

$$= \left(e^{-8\times\frac{2\pi}{16}i}\right) \quad \text{[Using De-Moivre's theorem]}$$

$= e^{-\pi i}$

$= \cos\pi - i\sin\pi$

$= -1$

72. **(b)** Given digits are 0, 2, 4, 5

Total number of four-digits numbers,

3	3	2	1

$3 \times 3 \times 2 \times 1 = 18$

Total number of four-digit numbers that are divisible by 5 = Number which ends with 0 and 5.

Number ends with 0.

3	2	1	1

$3 \times 2 \times 1 \times 1 = 6$

Number ends with 5.

2	2	1	1

$2 \times 2 \times 1 \times 1 = 4$

$\therefore$ Total number of four digits numbers that are not divisible by 5 = 18 – (6 + 4) = 8

73. **(b)** Given that,

$$x=\frac{1}{5}+\frac{1\cdot3}{5\cdot10}+\frac{1\cdot3\cdot5}{5\cdot10\cdot5}+\cdots$$

$$=\frac{1}{5}+\frac{1\cdot3}{2\times1}\left(\frac{1}{5}\right)^2+\frac{1\cdot3\cdot5}{3\times2\times1}\left(\frac{1}{5}\right)^3+\cdots$$

$$=\frac{1}{5}+\frac{1\cdot3}{2!}\left(\frac{1}{5}\right)^2+\frac{1\cdot3\cdot5}{3!}\left(\frac{1}{5}\right)^3+\cdots$$

On adding 1 both the sides, we get

$$1+x=1+\frac{1}{5}+\frac{1\cdot3}{2!}\left(\frac{1}{5}\right)^2+\frac{1\cdot3\cdot5}{3!}\left(\frac{1}{5}\right)^3+\cdots$$

Now,

According to the binomial theorem for any index,

$$\left(1-\frac{2}{5}\right)^{-1/2}=1+\frac{1}{5}+\frac{1\cdot3}{2!}\left(\frac{1}{5}\right)^2+\frac{1\cdot3\cdot5}{3!}\left(\frac{1}{5}\right)^3+\cdots$$

$$1+x=\left(1-\frac{2}{5}\right)^{-1/2}\Rightarrow 1+x=\left(\frac{3}{5}\right)^{-1/2}$$

$$1+x=\left(\frac{5}{3}\right)^{1/2}$$

$$(1+x)^2=\frac{5}{3}$$

$$1+2x+x^2=\frac{5}{3}$$

$$3+6x+3x^2=5$$

$$3x^2+6x=2$$

74. **(c)** **75.** **(d)** **76.** **(c)** **77.** **(b)**
78. **(a)** **79.** **(c)** **80.** **(c)**

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81. **(c)** We have

$$p=\frac{F}{A}=\frac{F}{\pi R^2}$$

So, $\frac{\Delta p}{p}\times100=\frac{\Delta F}{F}\times100+2\frac{\Delta R}{R}\times100$

$$=5+2(3)=5+6=11$$

$\Rightarrow\quad \frac{\Delta p}{p}\times100=11\%$

82. **(b)** $K_f+P_f=K_i+P_i$

$$O-\frac{GMm}{R+h}=\frac{1}{2}m\left(\frac{V_e}{2}\right)^2-\frac{GMm}{R}$$

$$-\frac{GMm}{1}\left[-\frac{1}{R}+\frac{1}{R+h}\right]=\frac{1}{8}mV_e^{\,2}$$

$$-GMm\left[\frac{-h}{R(R+h)}\right]=\frac{1}{8}mV_e^{\,2}$$

$$-GM\left[-\frac{}{R(R\quad h)}\right]=-\times\left(\sqrt{2R.\frac{GM}{}}\right)$$

$$\frac{h}{R^2+Rh}=\frac{1}{4R}\Rightarrow 4Rh=R^2+Rh$$

$\Rightarrow\quad R^2=3Rh$

$\Rightarrow\quad h=R/3$

83. **(d)** As Range $\propto\sin 2\theta$

So, $\frac{R-10}{R+15}=\frac{\sin2\theta_1}{\sin2\theta_2}=\frac{\sin30°}{\sin90°}=\frac{1}{2}$

$\Rightarrow\quad 2R-20=R+15\Rightarrow R=35$

For the range to be maximum $\sin 2\theta=\sin 90°=1$

So, $R_{\max}=\frac{u^2}{g}$

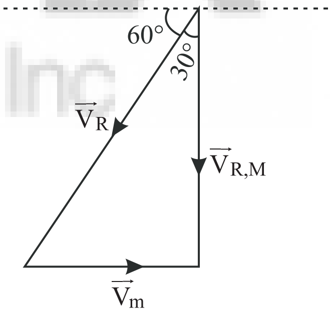
As $R_1=\frac{u^2\sin2\theta}{g}\Rightarrow 50=\frac{u^2}{g}\Rightarrow u^2=50g$

and $35=\frac{50g\times\sin2\theta}{g}[\because u=50g]$

$\therefore\quad \sin2\theta=\frac{35}{50}=\frac{7}{10}\quad\Rightarrow 2\theta=\sin^{-1}\left[\frac{7}{10}\right]$

$\Rightarrow\quad \theta=\frac{1}{2}\sin^{-1}\left[\frac{7}{10}\right]$

84. **(d)** Consider the situational diagram as shown below

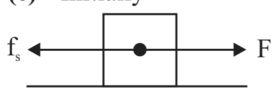


In ΔOAP, $\tan 30°=\frac{v_m}{v_{rm}}$

$$\frac{1}{\sqrt{3}}=\frac{\text{Velocity of man}}{\text{Velocity of rain with respect to man}}$$

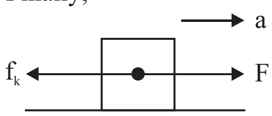
$\Rightarrow\quad v_{r\,m}=\frac{5}{1/\sqrt{3}}=5\sqrt{3}\text{ km/h}$

85. **(c)** Initially



F = $f_s=\mu_s N$
$=0.4\times4\times10=32$ N

Finally,



$F - f_k = ma$

$32 - \mu_k\, mg = \ ma$

$\Rightarrow \ 32 - 0.6 \times 40 = 4a$

$\Rightarrow \ 8 = 4a$

$\Rightarrow \ a = 2\ m/s^2$

86. **(b)** $|F| = F = \sqrt{4+1+1} = \sqrt{6}$

$v = \sqrt{16+4+4} = \sqrt{24}$

Now, $F = \dfrac{dp}{dt} = m\left(\dfrac{v-u}{t}\right)$

where, m = mass of body

$\Rightarrow \ \sqrt{6} = m\left(\dfrac{\sqrt{24}-0}{20}\right) \Rightarrow m = \dfrac{\sqrt{6}\times 20}{\sqrt{24}} = 10\text{kg}$

87. **(c)** P = w/t

w= p × t

$\Rightarrow \ (m + M)\, gh = 2000 \times 10$

$\Rightarrow \ (m + 50)\, 200 = 2000 \times 10$

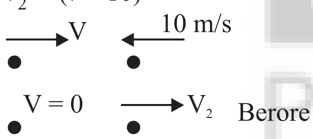
$\Rightarrow \ m + 50 = 100$

$\Rightarrow \ m = 50$ kg

88. **(b)** From conservation of moment,

$m(v - 10) = mv_2$

$v_2 = (v - 10)$



The coefficient of restitution,

$$e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

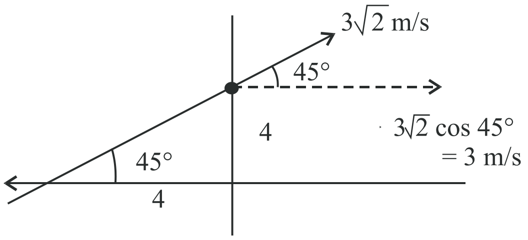
$$= \frac{v_2 - v_1}{u_1 + u_2} = \frac{(v-10)-0}{(v+10)}$$

$\Rightarrow \ 0.6 = \dfrac{v-10}{v+10}$

$\Rightarrow \ 0.6\, v + 6 = v - 10$

$\Rightarrow \ 0.4\, v = \ 16 \Rightarrow v = 40\ \text{ms}^{-1}$

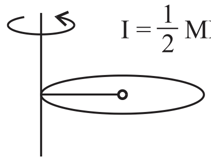
89. **(b)**



$L = m\, V\perp r$

$= 5 \times 3 \times 4 = 60$ units

90. **(c)** $KE = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\times\dfrac{3}{2}mr^2 4\pi^2 f^2$



$I = \dfrac{1}{2}MR^2 + MR^2 = \dfrac{3}{2}MR^2$

$\Rightarrow \ KE = 3\, mr^2 \times \pi^2 f^2 = 3 \times 5 \times 1 \times 10 \times 1$

KE = 150 J

91. **(a)** $K.E = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\,\omega^2\left(A^2 - x^2\right)$

$P.E = \dfrac{1}{2}kx^2 = \dfrac{1}{2}m\,\omega^2\, x^2$

Therefore, the ratio

$$\frac{KE}{PE} = \frac{\frac{1}{2}m\omega^2\left(A^2-x^2\right)}{\frac{1}{2}m\omega^2 x^2} \Rightarrow \frac{KE}{PE} = \frac{A^2-x^2}{x^2}$$

$\Rightarrow \ \dfrac{10^2-4^2}{4^2} = \dfrac{84}{16}$

$\Rightarrow \ \dfrac{KE}{PE} = \dfrac{21}{4} = 5.25$

92. **(c)** The relation between two velocities is given by the relation

$v_e = \left(\sqrt{2}-1\right)v_0$

$= (1.414 - 1)\, v_0$

$= 0.414 \times 10 = 4.14$ km/s

93. **(c)** We have, $F_1 \propto \quad (l_1 - l)$,

where ℓ = original length

Similarly, $F_2 \propto \quad (l_2 - l)$,

The ratio, $\dfrac{F_1}{F_2} = \dfrac{l_1 - l}{l_2 - l}$

Solving, we get

$\Rightarrow \ l = \dfrac{F_2 l_1 - F_1 l_2}{F_2 - F_1}$

94. **(b)** We have,

$$\frac{\Delta V}{V} = \frac{h\rho g}{B}\left[\because B = \frac{\Delta P}{\Delta V/V} \Rightarrow \frac{\Delta V}{V} = \frac{\Delta P}{B} = \frac{h\rho g}{B}\right]$$

$\Rightarrow \ \dfrac{\Delta V}{V} = \dfrac{3\times 10^3 \times 10^3 \times 9.8}{2.2\times 10^9} = 1.34\times 10^{-2}$

95. **(c)** $\dfrac{E_1}{E_2} = \dfrac{T_1^4 - T_S^4}{T_2^4 - T_S^4}\ \left[\because E = \sigma A T^4\right]$

$\Rightarrow \ \dfrac{E_1}{E_2} = \dfrac{(600)^4 - (300)^4}{(900)^4 - (300)^4} = \dfrac{3}{16}$

96. **(d)** As $C_V = 12.6\ \text{J mol}^{-1}\ \text{K}^{-1}$ is given in SI unit

So, $R = 8314\ \text{J mol}^{-1}\ \text{K}^{-1}$

Then, $C_p = C_V + R = 12.6 + 8.314 = 21$

97. **(b)** From first law of thermodynamics,

$dQ = dU + dW$

$\Rightarrow \quad dU = -dW$ [In adiabatic process $dQ = 0$]

$dU = -4.5$ J

As, the temperature falls so internal energy will decrease by 4.5 J

98. **(b)** $\eta = \dfrac{1}{1+\alpha}$. This is the required relation.

99. **(a)** The average kinetic energy of molecules is given by

$KE_{av} = \dfrac{3}{2}KT$. So it depends only on temperature

$KE_1 : KE_2 = 1 : 1$

100. **(b)** We have $y = \ 2\sin(30t - 40x)$

The standard transverse wave is

$y = A\sin(\omega t - kx)$

So, $\omega = 30$ and $k = 40$

On comparing, we get

$v = \dfrac{\omega}{k} = \dfrac{30}{40} = 0.75\,\text{m/s}$

101. **(a)** The fundamental frequency,

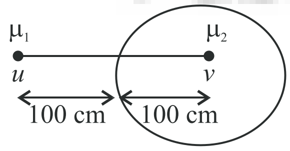
$f = \dfrac{v}{4l} = \dfrac{1}{4e}\sqrt{\dfrac{\gamma RT}{M_0}}$

$\ell \propto \dfrac{1}{\sqrt{M}} \Rightarrow \dfrac{\ell_1}{\ell_2} = \sqrt{\dfrac{M_1}{M_2}} = \sqrt{\dfrac{2}{32}} = \dfrac{1}{4}$

So, $\ell_1 : \ell_2 = 1 : 4$

102. **(a)** The refraction formula is

$\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$



$\mu_2 = 1.5$, μ_1 (air) $= 1$, $u = -100$ cm, $v = 100$ cm

$\Rightarrow \quad \dfrac{1.5}{100} + \dfrac{1}{100} = \dfrac{1.5-1}{R} \Rightarrow \dfrac{25}{100} = \dfrac{0.5}{R} \Rightarrow R = 20\text{ cm}$

103. **(c)** $\dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\dfrac{\sqrt{9}+\sqrt{4}}{\sqrt{9}-\sqrt{4}}\right)^2$

$= \left(\dfrac{3+2}{3-2}\right)^2 = 25$

104. **(c)** $E = \dfrac{1}{2}CV^2 \Rightarrow C \propto \dfrac{1}{\text{d}}$

$\Rightarrow \quad \dfrac{C_1}{C_2} = \dfrac{d_1}{d_2} = \dfrac{2d}{d} = 2 \left[\because C = \dfrac{A\in_0}{d}\right]$

Now, $\dfrac{E_1}{E_2} = \dfrac{C_1}{C_2} = \dfrac{C}{C/2} = \dfrac{2}{1}$

$\Rightarrow \quad \dfrac{E}{E_2} = \dfrac{2}{1} \Rightarrow E_2 = \dfrac{E}{2}$

105. **(d)** When an amount of charge = 10μC is added to both of the charges then, these becomes

$q_1 = -2\mu C$ and $q_2 = +2\mu C$

$F \propto |q_1 q_2|$

$\Rightarrow F_1 \propto |q_1 q_2|$ and $F_2 \propto \left|q_1^1\, q_2^1\right|$

and $\dfrac{F_1}{F_2} = \dfrac{q_1 q_2}{q_1\, q_2} \quad \Rightarrow \quad \dfrac{48}{F_2} = \dfrac{8\times 12}{2\times 2}$

$\Rightarrow \quad F_2 = 2\text{N}$

106. **(a)** The dielectric constant permittivity and susceptibility are relative as,

$K = 1 + \dfrac{\chi_e}{\varepsilon_0} \quad \Rightarrow \quad \dfrac{4}{3} = 1 + \dfrac{\chi_e}{\varepsilon_0}$

$\Rightarrow \quad \chi_e = \varepsilon_0\left(\dfrac{4}{3} - 1\right)$

$\Rightarrow \quad \chi_e = \dfrac{\varepsilon_0}{3}$

The electric susceptibility, $\chi_e = \dfrac{\varepsilon_0}{3}$

107. **(b)** $a = \dfrac{F}{m_e} = \dfrac{eE}{m_e}$

As, $S = \dfrac{1}{2}at^2 = \dfrac{1}{2}\dfrac{eE}{m_e}t^2$

108. **(b)** $v_d \propto \dfrac{1}{l}\ \dfrac{v_{d_1}}{v_{d_2}} = \dfrac{I_1}{I_2} = \dfrac{4I}{I} \Rightarrow v_{d_2} = \dfrac{v_{d_1}}{4}$

So drift velocity will decrease by 4 times

109. **(b)** (Initial part)

$\dfrac{x}{100} - x = \dfrac{2}{3} \Rightarrow x = 40\text{ cm}$

If there is shifting by 22.5 cm. Then,

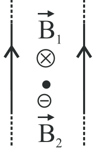
to obtain the balance point in meter bridge

$\dfrac{2(3+x)}{3x} = \dfrac{62.5}{37.5} \Rightarrow (6+2x)37.5 = 62.5\times 3x$

$\Rightarrow \quad 225 + 75x = 187.5x$

$187.5x - 75x = 225 \Rightarrow 112.5x = 225 \Rightarrow x = 2$

110. **(c)**



At the mid point of two conductors, the magnetic induction. $B_1 = B_2 = B$ and $\vec{B}_1$ is anti||r to $\vec{B}_2$.

Therefore, the net magnetic induction due to two wire at the mid points is

$B_N = B_1 - B_2 = 0$

111. **(b)** $eVB = eE$

where, e = electronic charge and

v = velocity of electron

$\Rightarrow \quad v = \frac{E}{B}v = \frac{20\times10^3}{2.0} = 10\times10^3\,\text{ms}^{-1}$

112. **(c)** Magnetic flux $\phi = BA = \mu HA$

$= \mu_0 (1 + \chi_m) HA \ \left[\because \mu = \mu_0(1+\chi_m)\right]$

$= 4\pi \times 10^{-3} \times (1 + 313) \times 1000 \times 0.25 \times 10^{-4}$

$= 9.87 \times 10^{-6}$ Wb

113. **(a)** $e = -L\frac{dl}{dt} \Rightarrow e = -30\times10^{-3}\times\left(\frac{2-6}{2}\right)$

$= +30\times10^{-3}\times\frac{4}{2} = +0.06\,V$

114. **(a)** $P = E_{rms}V_{rms}\cos\theta = \frac{E_0V_0}{2}\cos\phi$

So, $P = \frac{50\times50\times10^{-3}\times1}{2\times2} = 0.625$ W

115. **(c)** We have total momentum as $\Delta p = \left(\frac{IA}{c}\right)t$ for non reflecting surface,

$\Delta p = \left(\frac{9\times10^4\times20\times10^{-4}}{3\times10^8}\times3600\right)$

$= 216 \times 10^{-3}$ kgms^{-1}

116. **(a)** As, we know that de-Broglie wave lengths

$\lambda = \frac{h}{mv} \Rightarrow \lambda \propto \frac{1}{m}$ So, $\frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e}$

117. **(b)** $\frac{1}{\lambda_{BL}} = R\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = \frac{5}{36}R \Rightarrow \lambda_B = \frac{36}{5R}$ ... (A)

$\frac{1}{\lambda_{PL}} = R\left[\frac{1}{3^2}-\frac{1}{4^2}\right] = \frac{7}{144}R \Rightarrow \lambda_P = \frac{144}{7R}$ ... (B)

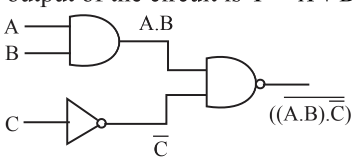
On dividing Eq. (B) by Eq. (A), we get

$\frac{\lambda_{BL}}{\lambda_{PL}} = \frac{7}{20}$

118. **(d)** Here $\frac{n}{p}\downarrow\downarrow$.

So it is β^- decay. So, x stands for antinutrino.

119. **(d)** The output of the circuit is $Y = \overline{A} + \overline{B} + C$



$Y = \overline{\text{A}\,.\,\text{B}} + \overline{\overline{\text{C}}} = \overline{\text{A}} + \overline{\text{B}} + \text{C}$

If the output is zero,

i.e. $Y = 0$ then $A = 1$

$B = 1$ and $C = 0$

120. **(c)** We have, $m = \frac{A_m}{A_c} = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$

Here, $E_{max} = 16$ V

and $E_{min} = 4$ V

$\Rightarrow \quad m = \frac{16-4}{16+4}$

$= \frac{12}{20} = \frac{3}{5} = 0.6$

CHEMISTRY

121. **(c)** For 3d-orbitals; $n = 3$, $\ell = 2$. m can take values from $-\ell$ to $+\ell$, i.e., –2 to +2. s can take values $+\frac{1}{2}$ or $-\frac{1}{2}$

122. **(d)** $\lambda \propto \frac{1}{\sqrt{\text{K.E.}}} \quad \Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{(\text{KE})_2}{(\text{KE})_1}}$

$\text{KE}_2 = \frac{(\text{KE})_1}{2}$

Thus, $\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{1}{2}} \Rightarrow \lambda_2 = \sqrt{2}\lambda_1$

123. **(c)** As we move from left to right within a period, the acidic character increases. As we move down a group, the acidic character decreases. Order of acidic strength:

$SO_3 < P_4O_{10} < N_2O_5 < Cl_2O_7$

124. **(b)**

125. **(c)** Nitric oxide (NO) has 11 electrons in its valence shells and its bond order is 2.5. Hence, it does not contain triple bond between the atoms.

126. **(a)** Central I in I_3^- undergoes sp^3d hybridisation giving a linear shape with three lone pairs at equatorial positions. Beryllium chloride has linear structure with sp hybridisation of Be atom.

127. **(d)** As we go higher in altitude, the atmospheric pressure decreases. We know that, P $\propto$ T. This results in decreasing the boiling point at higher altitude.

128. **(b)** When surface area decreases, less liquid will be exposed to air and therefore, less amount of water can evaporate in a given time period.

129. **(c)**

Element	% of element	Atomic weight	Relative number	Mole Ratio
Carbon	12.8	12	$\frac{12.8}{12} = 1.06$	$\frac{1.06}{1.06} = 1$
Hydrogen	2.1	1	$\frac{2.1}{1} = 2.1$	$\frac{2.1}{1.06} = 2$
Bromine	85.1	80	$\frac{85.1}{80} = 1.06$	$\frac{1.06}{1.06} = 1$

Empirical formula = CH_2Br

Empirical formula mass = 12 + 2 + 80 = 94

Factor, $n = \dfrac{\text{Molecular mass}}{\text{Empirical formula mass}}$

$= \dfrac{187.9}{94} = 2$

Molecular formula = $(CH_2Br)_2 = C_2H_4Br_2$

130. **(a)** From Avogadro's law,

Avogadro's number = Atomic mass of the element

$\because$ 3.011×10^{22} atoms of an element weigh = 1.15 g

$\therefore$ 6.023×10^{23} atoms weigh

$= \dfrac{1.15 \times 6.023 \times 10^{23}}{3.011 \times 10^{22}} = 23$

131. **(b)** $\Delta E = q + W$

For an adiabatic reaction,

$\Delta q = 0$

So, $\Delta E = \Delta W$

132. **(b)** For exothermic reaction, k_f decreases and k_b increases with increase of temperature so that K_{eq} decreases. For endothermic reaction, k_f increases and k_b decreases with increase of temperature so that K_{eq} increases.

133. **(d)** Molecular mass of NaOH = 23 + 16 + 1 = 40

$$\text{Molarity} = \frac{\text{Mass of solute (in g)}}{\text{Molecular mass of the solute} \times \text{Volume of solution (in L)}}$$

$= \dfrac{0.04 \times 1000}{40 \times 100} = 10^{-2}$

$OH^- = 10^{-2}$ mol/L

So, pOH = $-\log[OH^-]$

pOH + pH = 14 or pH = 14 – 2 = 12

134. **(d)**

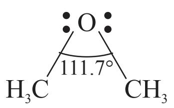
135. **(d)** Melting and boiling points of alkali metals are very low due to weak metallic bond.

136. **(a)** Due to inert pair effect, +2 O.S. is more stable than +4 O.S. for lead.

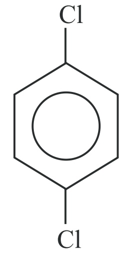
137. **(d)** Boiling points of noble gases are low due to only acting weak van der Waal's forces of attraction between the atoms.

138. **(d)** Ocean absorb CO_2 via photochemical and biological process. (d) is also correct.

139. **(a)** Due to repulsion between two $-CH_3$ groups, the bond angle is more than the tetrahedral angle of 109°28′.



140. **(a)** $\mu = 0$



1, 4-dichlorobenzene

141. **(b)** With cold dilute alkaline $KMNO_4$ (Baeyer's reagent), alkenes give addition products and the purple colour of $KMNO_4$ disappears.

142. **(c)** Doping of silicon with group 15 elements such as phosphorus give rise to *n*-type semiconductor.

143. **(d)** From Rault's law for non-volatile solute, we know that

$$\frac{p^\circ - p_s}{p^\circ} = \frac{n_2}{n_1 + n_2}$$

For dilute solution, relative lowering of vapour pressure,

$n_2 = \dfrac{18}{180} = 0.1$

$n_1 = \dfrac{90}{18} = 5$

$\dfrac{p^\circ - p_s}{p^\circ} = \dfrac{0.1}{5} = 0.02$

144. **(c)** From Henry's law, if we increase the pressure then the mole fraction of gas in the water will increase by the same multiple. Hence, mole fraction of gas = 0.04. Mole fraction of water = 0.96

145. **(b)** $E^\circ_{cell} = E^\circ_C - E^\circ_A = 0.80 - (-0.76) = 1.56\text{V}$

From the equation $\Delta G^\circ = -nFE^\circ$

$= -2 \times 96500 \times 1.56$ ($n = 2$)

$= -301080$ J/mol

$= -301$ kJ/mol

146. **(a)** For the first order reaction,

$t = \dfrac{2303}{k} \log \dfrac{a}{a - x}$

Now putting the values

$t_{90\%} = t = \dfrac{2303}{k} \log\left(\dfrac{100}{100 - 90}\right)$

$t_{99\%} = \dfrac{2303}{k} \log\left(\dfrac{100}{100 - 99}\right)$

$$\frac{t}{t_{99\%}} = \frac{\dfrac{2303}{k}\log\left(\dfrac{100}{10}\right)}{\dfrac{2303}{k}\log\left(\dfrac{100}{1}\right)} = \frac{1}{2}$$

$t_{99\%} = 2t$

147. **(d)** Ferric hydroxide solution, is a negatively charged sol.

$$KCl \rightleftharpoons K^+ + Cl^-$$

$$KNO_3 \rightleftharpoons K^+ + NO_3^-$$

$$K_2SO_4 \rightleftharpoons 2K^+ + NO_4^{2-}$$

$$K_3\left[Fe(CN)_6\right] \rightleftharpoons 3K^- + \left[Fe(CN)_6\right]^{3-}$$

Hence, $K_3[Fe(CN)_6]$ will be most effective in causing coagulation of ferric hydroxide solution.

148. **(d)** Levigation or gravity separation is the process of separating the light impure particles from the heavier ore particles by washing in current of water.

149. **(a)** Due to +I effect of 3 methyl groups, the correct basic order is: $P(CH_3)_3 > PH_3$

150. **(b)** It means that there is only one counter ion, i.e., the possible complex is $[Co(NH_3)_4Cl_2]$ Cl. Hence, X is 4.

151. **(c)** Let oxidation state of Fe = x

For $Fe(CO)_5$, $x + (5 \times 0) = 0$

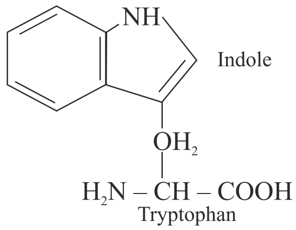
$\Rightarrow$ $x = 0$

152. **(d)**

153. **(c)** In place of nitrogen, helium is mixed with oxygen because helium is not soluble in blood even under high pressure.

154. **(d)**

155. **(a)**



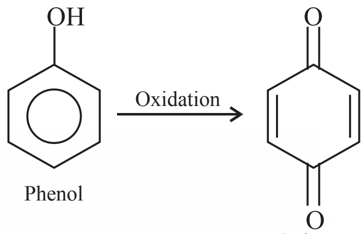
Tryptophan

156. **(d)**

157. **(b)** $C_2H_6OH + 4\,I_2 + 3\,Na_2CO_3 \longrightarrow$

$$\underset{\text{Triiodomethane}}{CHI_2} + \underset{\text{Sodium formate}}{HCOONa} + 5\,NaI + 3\,CO_2 + 2\,H_2O$$

158. **(a)**



Quinone

159. **(a)** $\underset{\text{Acetic acid}}{CH_3COOH} \xrightarrow{SOCl_2} \underset{\text{Acetyl chloride}}{CH_3COCl}$

$$\xrightarrow{H_2/Pd-BaSO_4} \underset{\text{Acetaldehyde}}{CH_3CHO}$$

160. **(c)** The sequence of reaction is

$$Ar_2 - NH_2 \xrightarrow[(X)]{NaNO_2 + HCl} Ar - \overset{+}{N} \equiv NCl^-$$

$$\xrightarrow[(Y)]{Cu/HCl} Ar - Cl + N_2$$