AP/EAMCET Solved Paper 2016

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. The domain of the function $f(x)=\sqrt{\log_{0.5} x!}$ is

(a) $\{0, 1, 2, 3, \ldots\}$ (b) $\{1, 2, 3, \ldots\}$
(c) $(0, \infty)$ (d) $\{0, 1\}$

2. If $f(x) = |x-1| + |x-2| + |x-3|$, $2 < x < 3$, then f is

(a) an onto function but not one-one
(b) one-one function but not onto
(c) a bijection
(d) neither one-one nor onto

3. The greatest positive integer which divides $(n + 16)(n + 17)(n + 18)(n + 19)$, for all positive integers n, is

(a) 6 (b) 24 (c) 28 (d) 20

4. If a, b, c are distinct positive real numbers, then the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

(a) < 0 (b) > 0 (c) 0 (d) ≥ 0

5. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in geometric progression with the same common ratio, then the points (x_1, y_1), (x_2, y_2), (x_3, y_3) are

(a) vertices of an equilateral triangle
(b) vertices of a right angled triangle
(c) vertices of a right angled isosceles triangle
(d) collinear

6. The equations $x - y + 2z = 4$
$3x + y + 4z = 6$
$x + y + z = 1$ have

(a) unique solution (b) infinitely many solutions
(c) no solution (d) two solutions

7. The locus of the point representing the complex number z for which $|z + 3|^2 - |z - 3|^2 = 15$ is

(a) a circle (b) a parabola
(c) a straight line (d) an ellipse

8. $\dfrac{(1+i)^{2016}}{(1-i)^{2014}}$ is equal to

(a) $-2i$ (b) $2i$ (c) 2 (d) -2

9. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is

(a) 3 (b) 4 (c) 8 (d) 2

10. If $1, z_1, z_2, \ldots, z_{n-1}$ are the nth roots of unity, then $(1 - z_1)(1 - z_2) \ldots (1 - z_{n-1})$ is equal to

(a) 0 (b) $n - 1$ (c) n (d) 1

11. If $12^{4+2x^2} = \left(24\sqrt{3}\right)^{3x^2-2}$, then x is equal to

(a) $\pm\sqrt{\dfrac{13}{12}}$ (b) $\pm\sqrt{\dfrac{14}{5}}$ (c) $\pm\sqrt{\dfrac{12}{13}}$ (d) $\pm\sqrt{\dfrac{5}{14}}$

12. The product and sum of the roots of the equation $|x^2| - 5|x| - 24 = 0$ are respectively

(a) $-64, 0$ (b) $-24, 5$ (c) $5, -24$ (d) $0, 72$

13. The number of real roots of the equation $x^5 + 3x^3 + 4x + 30 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 5

14. If the coefficients of the equation whose roots are k times the roots of the equation, $x^3+\dfrac{1}{4}x^2-\dfrac{1}{16}x+\dfrac{1}{144}=0$ are integers, then a possible value of k is

(a) 3 (b) 12 (c) 9 (d) 4

15. The sum of all 4-digit numbers that can be formed using the digits 2, 3, 4, 5, 6 without repetition, is

(a) 533820 (b) 532280 (c) 533280 (d) 532380

16. If a set A has 5 elements, then the number of ways of selecting two subsets P and Q from A such that P and Q are mutually disjoint, is

(a) 64 (b) 128 (c) 243 (d) 729

17. The coefficient of x^4 in the expansion of $(1 - x + x^2 - x^3)^4$ is

(a) 31 (b) 30 (c) 25 (d) -14

18. If the middle term in the expansion of $(1 + x)^{2n}$ is the greatest term, then x lies in the interval

(a) $\left(\dfrac{n}{n+1}, \dfrac{n+1}{n}\right)$ (b) $\left(\dfrac{n+1}{n}, \dfrac{n}{n+1}\right)$
(c) $(n - 2, n)$ (d) $(n - 1, n)$

19. To find the coefficient of x^4 in the expansion of $\frac{3x}{(x-2)(x-1)}$, the interval in which the expansion isvalid, is

(a) $-2 < x < \infty$ (b) $-\frac{1}{2} < x < \frac{1}{2}$

(c) $-1 < x < 1$ (d) $-\infty < x < \infty$

20. If (1 + tan α) (1 + tan 4α) = 2, $\alpha \in \left(0, \frac{\pi}{16}\right)$, then α is equal to

(a) $\frac{\pi}{20}$ (b) $\frac{\pi}{30}$ (c) $\frac{\pi}{40}$ (d) $\frac{\pi}{60}$

21. If $\cos\theta = \frac{\cos\alpha - \cos\beta}{1 - \cos\alpha\cos\beta}$, then one of the values of $\tan\frac{\theta}{2}$ is

(a) $\cot\frac{\beta}{2}\tan\frac{\alpha}{2}$ (b) $\tan\alpha\tan\frac{\beta}{2}$

(c) $\tan\frac{\beta}{2}\cot\frac{\alpha}{2}$ (d) $\tan^2\frac{\alpha}{2}\tan^2\frac{\beta}{2}$

22. The value of the expression

$$\frac{1+\sin 2\alpha}{\cos(2\alpha - 2\pi)\tan\left(\alpha - \frac{3\pi}{4}\right)}$$

$$-\frac{1}{4}\sin 2\alpha\left[\cot\frac{\alpha}{2} + \cot\left(\frac{3\pi}{2} + \frac{\alpha}{2}\right)\right]$$ is

(a) 0 (b) 1 (c) $\sin^2\frac{\alpha}{2}$ (d) $\sin^2\alpha$

23. If $\frac{1}{6}$ sinθ, cosθ and tanθ are in geometric progression, then the solution set of θ is

(a) $2n\pi \pm \left(\frac{\pi}{6}\right)$ (b) $2n\pi \pm \left(\frac{\pi}{3}\right)$

(c) $n\pi + 1(-1)^n\left(\frac{\pi}{3}\right)$ (d) $n\pi + \left(\frac{\pi}{3}\right)$

24. If $x = \sin(2\tan^{-1}2)$ and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$,then

(a) $x > y$ (b) $x = y$ (c) $x = 0 = y$ (d) $x < y$

25. If cos h(x) $= \frac{5}{4}$, then cos h(3x) is equal to

(a) $\frac{61}{16}$ (b) $\frac{63}{16}$ (c) $\frac{65}{16}$ (d) $\frac{61}{63}$

26. In ΔABC if $x = \tan\left(\frac{B-C}{2}\right)\tan\frac{A}{2}$,

$y = \tan\left(\frac{C-A}{2}\right)\tan\frac{B}{2}$, and $z = \tan\left(\frac{A-B}{2}\right)\tan\frac{C}{2}$, then $(x + y + z)$ is equal to

(a) xyz (b) $-xyz$ (c) $2xyz$ (d) $\frac{1}{2}xyz$

27. In ΔABC, if the sides a, b, c are in geometric progression and the largest angle exceeds the smallest angle by 60°, then cos B is equal to

(a) $\frac{\sqrt{13}+1}{4}$ (b) $\frac{1-\sqrt{13}}{4}$

(c) 1 (d) $\frac{\sqrt{13}-1}{4}$

28. In a ΔABC, if ∠A = 90°, then $\cos^{-1}\left(\frac{R}{r_2 + r_3}\right)$ is equal to

(a) 90° (b) 30° (c) 60° (d) 45°

29. The cartesian equation of the plane whose vector equation is $\gamma = (1+\lambda-\mu)\hat{\mathbf{i}} + (2-\lambda)\hat{\mathbf{j}} + (3-2\lambda+2\mu)\hat{\mathbf{k}}$ where λ, μ are scalars, is

(a) $2x + y = 5$ (b) $2x - y = 5$

(c) $2x - z = 5$ (d) $2x + z = 5$

30. For three vectors **p, q** and **r**, if **r** = 3**p** + 4**q** and 2**r** = **p** – 3**q**, then

(a) |**r**| < 2|**q**| and **r, q** have the same direction

(b) |**r**| > 2|**q**| and **r, q** have opposite directions

(c) |**r**| < 2|**q**| and **r, q** have opposite directions

(d) |**r**| > 2|**q**| and **r, q** have the same direction

31. If $\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$, $\mathbf{b} = m\hat{\mathbf{i}} + n\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$ and **a** × **b** = **0**, then (m, n) is equal to

(a) $\left(\frac{-24}{5}, \frac{-36}{5}\right)$ (b) $\left(\frac{-24}{5}, \frac{36}{5}\right)$

(c) $\left(\frac{24}{5}, \frac{-36}{5}\right)$ (d) $\left(\frac{24}{5}, \frac{36}{5}\right)$

32. If |**a**|= 3, |**b**|= 4 and the angle between **a** and **b** is 120°, then |4**a** + 3**b**| is equal to

(a) 25 (b) 7 (c) 13 (d) 12

33. If **a, b, c** are non-zero vectors such that (**a** × **b**) × **c** $= \frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$, $\mathbf{c} \perp \mathbf{a}$ and θ is the angle between the vectors **b** and **c**, then sin θ is equal to

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{1}{3}$ (c) $\frac{\sqrt{2}}{3}$ (d) $\frac{2}{3}$

34. If a(α × β) + b(β × γ) + c(γ × α) = 0 and atleast one of the scalars a, b, c is non-zero, then the vectors α, β, γ are

(a) parallel

(b) non coplanar

(c) coplanar

(d) mutually perpendicular

35. If the mean of 10 observations is 50 and the sum of the squares of the deviations of the observations from the mean is 250, then the coefficient of variation of those observations is

(a) 25 (b) 50 (c) 10 (d) 5

36. The variance of the first 50 even natural numbers is

(a) $\frac{833}{4}$ (b) 833 (c) 437 (d) $\frac{437}{4}$

37. 3 out of 6 vertices of a regular hexagon are chosen at a time at random. The probability that the triangle formed with these three vertices is an equilateral triangle, is

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{20}$

38. A speaks truth in 75% of the cases and B in 80% of the cases. Then, the probability that their statements about an incident do not match, is

(a) $\frac{7}{20}$ (b) $\frac{3}{20}$ (c) $\frac{2}{7}$ (d) $\frac{5}{7}$

39. If the mean and variance of a binomial distribution are 4 and 2 respectively, then the probability of 2 successes of that binomial variate X, is

(a) $\frac{1}{2}$ (b) $\frac{219}{256}$ (c) $\frac{37}{256}$ (d) $\frac{7}{64}$

40. In a city, 10 accidents take place in a span of 50 days. Assuming that the number of accidents follow the Poisson distribution,the probability that three or more accidents occur in a day, is

(a) $\sum_{k=3}^{\infty}\frac{e^{-\lambda}\lambda^k}{k!}, \lambda = 0.2$ (b) $\sum_{k=3}^{\infty}\frac{e^{\lambda}\lambda^k}{k!}, \lambda = 0.2$

(c) $1-\sum_{k=0}^{3}\frac{e^{-\lambda}\lambda^k}{k!}, \lambda = 0.2$ (d) $\sum_{k=0}^{3}\frac{e^{-\lambda}\lambda^k}{k!}, \lambda = 0.2$

41. Equation of the locus of the centroid of the triangle whose vertices are $(a \cos k, a \sin k)$, $(b \sin k, -b \cos k)$ and $(1, 0)$, where k is a parameter, is

(a) $(1 - 3x)^2 + 9y^2 = a^2 + b^2$

(b) $(3x - 1)^2 + 9y^2 = 2a^2 + 2b^2$

(c) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$

(d) $(3x + 1)^2 + (3y)^2 = 3a^2 + 3b^2$

42. If the coordinate axes are rotated through an angle $\frac{\pi}{6}$ about the origin, then the transformed equation of $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$ is

(a) $\sqrt{3}y^2 + xy = 0$ (b) $x^2 - y^2 = 0$

(c) $\sqrt{3}y^2 - xy = 0$ (d) $\sqrt{3}y^2 - 2xy = 0$

43. If the lines $x + 3y - 9 = 0$, $4x + by - 2 = 0$ and $2x - y - 4 = 0$ are concurrent, then the equation of the line passing through the point $(b, 0)$ and concurrent with the given lines, is

(a) $2x + y + 10 = 0$ (b) $4x - 7y + 20 = 0$

(c) $x - y + 5 = 0$ (d) $x - 4y + 5 = 0$

44. The mid-point of the line segment joining the centroid and the orthocentre of the triangle whose vertices are (a, b), (a, c) and (d, c), is

(a) $\left(\frac{5a+d}{6}, \frac{b+5c}{6}\right)$ (b) $\left(\frac{a+5d}{6}, \frac{5b+c}{6}\right)$

(c) (a, c) (d) $(0, 0)$

45. The distance from the origin to the image of $(1, 1)$ with respect to the line $x + y + 5 = 0$ is

(a) $7\sqrt{2}$ (b) $3\sqrt{2}$ (c) $6\sqrt{2}$ (d) $4\sqrt{2}$

46. The equation of the pair of lines joining the origin to the points of intersection of $x^2 + y^2 = 9$ and $x + y = 3$, is

(a) $x^2 + (3 - y)^2 = 9$ (b) $(3 + y)^2 + y^2 = 9$

(c) $x^2 - y^2 = 9$ (d) $xy = 0$

47. The orthocentre of the triangle formed by the lines $x + y = 1$ and $2y^2 - xy - 6x^2 = 0$ is

(a) $\left(\frac{4}{3}, \frac{4}{3}\right)$ (b) $\left(\frac{2}{3}, \frac{2}{3}\right)$

(c) $\left(\frac{2}{3}, \frac{-2}{3}\right)$ (d) $\left(\frac{4}{3}, \frac{-4}{3}\right)$

48. Let L be the line joining the origin to the point of intersection of the lines represented by $2x^2 - 3xy - 2y^2 + 10x + 5y = 0$. If L is perpendicular to the line $kx + y + 3 = 0$, then k is equal to

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) -1 (d) $\frac{1}{3}$

49. A circle $S = 0$ with radius $\sqrt{2}$ touches the line $x + y - 2 = 0$ at $(1, 1)$. Then, the length of the tangent drawn from the point $(1,2)$ to $S = 0$ is

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

50. The normal drawn at $P(-1, 2)$ on the circle $x^2 + y^2 - 2x - 2y - 3 = 0$ meets the circle at another point Q. Then, the coordinates of Q are

(a) $(3, 0)$ (b) $(-3, 0)$ (c) $(2, 0)$ (d) $(-2, 0)$

51. If the lines $kx + 2y - 4 = 0$ and $5x - 2y - 4 = 0$ are conjugate with respect to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$, then k is equal to

(a) 0 (b) 1 (c) 2 (d) 3

52. The angle between the tangents drawn from the origin to the circle $x^2 + y^2 + 4x - 6y + 4 = 0$ is

(a) $\tan^{-1}\left(\frac{5}{13}\right)$ (b) $\tan^{-1}\left(\frac{5}{12}\right)$

(c) $\tan^{-1}\left(\frac{-12}{2}\right)$ (d) $\tan^{-1}\left(\frac{13}{2}\right)$

53. If the angle between the circles $x^2 + y^2 - 2x - 4y + c = 0$ and $x^2 + y^2 - 4x - 2y + 4 = 0$ is 60°, then c is equal to

(a) $\frac{3 \pm \sqrt{5}}{2}$ (b) $\frac{6 \pm \sqrt{5}}{2}$

(c) $\frac{9 \pm \sqrt{5}}{2}$ (d) $\frac{7 \pm \sqrt{5}}{2}$

54. A circle S cuts three circles
$x^2 + y^2 - 4x - 2y + 4 = 0$,
$x^2 + y^2 - 2x - 4y + 1 = 0$
and $x^2 + y^2 + 4x + 2y + 1 = 0$ orthogonally.
Then, the radius of S is

(a) $\sqrt{\frac{29}{8}}$ (b) $\sqrt{\frac{28}{11}}$ (c) $\sqrt{\frac{29}{7}}$ (d) $\sqrt{\frac{29}{5}}$

55. The distance between the vertex and the focus of the parabola $x^2 - 2x + 3y - 2 = 0$ is

(a) $\frac{4}{5}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{5}{6}$

56. If (x_1, y_1) and (x_2, y_2) are the end points of a focal chord of the parabola $y^2 = 5x$, then $4x_1x_2 + y_1y_2$ is equal to

(a) 25 (b) 5 (c) 0 (d) $\frac{5}{4}$

57. The distance between the focii of the ellipse $x = 3\cos\theta$, $y = 4\sin\theta$ is

(a) $2\sqrt{7}$ (b) $7\sqrt{2}$ (c) $\sqrt{7}$ (d) $3\sqrt{7}$

58. The equations of the latusrectum of the ellipse $9x^2 + 25y^2 - 36x + 50y - 164 = 0$ are

(a) $x - 4 = 0, x + 2 = 0$ (b) $x - 6 = 0, x + 2 = 0$

(c) $x + 6 = 0, x - 2 = 0$ (d) $x + 4 = 0, x + 5 = 0$

59. The values of m for which the line $y = mx + 2$ becomes a tangent to the hyperbola $4x^2 - 9y^2 = 36$ is

(a) $\pm\frac{2}{3}$ (b) $\pm\frac{2\sqrt{2}}{3}$ (c) $\pm\frac{8}{9}$ (d) $\pm\frac{4\sqrt{2}}{3}$

60. The harmonic conjugate of (2, 3, 4) with respect to the points (3, –2, 2), (6, –17, –4) is

(a) $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$ (b) $\left(\frac{18}{5}, -5, \frac{4}{5}\right)$

(c) $\left(\frac{-18}{5}, \frac{5}{4}, \frac{4}{5}\right)$ (d) $\left(\frac{18}{5}, -5, \frac{-4}{5}\right)$

61. If a line makes angles α, β, γ and δ with the four diagonals of a cube, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta$ is

(a) $\frac{4}{3}$ (b) $\frac{8}{3}$ (c) $\frac{7}{3}$ (d) $\frac{5}{3}$

62. If the plane $56x + 4y + 9z = 2016$ meets the coordinate axes in A, B, C, then the centroid of the ΔABC is

(a) (12, 168, 224) (b) (12, 168, 112)

(b) $\left(12, 168, \frac{224}{3}\right)$ (d) $\left(12, -168, \frac{224}{3}\right)$

63. The value (s) of x for which the function

$$f(x) = \begin{cases} 1 - x, & x < 1 \\ (1-x)(2-x), & 1 \le x \le 2 \\ 3 - x, & x > 2 \end{cases}$$

fails to be continuous is(are)

(a) 1 (b) 2

(c) 3 (d) all real numbers

64. $\lim\limits_{x\to 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$ is equal to

(a) $(\log_e 2)\log_e 3$ (b) $\log_e 5$

(c) $\log_e 6$ (d) 0

65. Define $f(x) = \begin{cases} x^2 + bx + c, & x < 1 \\ x, & x \ge 1 \end{cases}$. If $f(x)$ is differentiable at $x = 1$, then $(b - c)$ is equal to

(a) –2 (b) 0 (c) 1 (d) 2

66. If $x = a$ is a root of multiplicity two of a polynomial equation $f(x) = 0$, then

(a) $f'(a) = f''(a) = 0$ (b) $f''(a) = f(a) = 0$

(c) $f'(a) \ne 0 \ne f''(a)$ (d) $f(a) = f'(a) = 0; f''(a) \ne 0$

67. If $y = \log_2(\log_2 x)$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{\log_e 2}{x\log_e x}$ (b) $\frac{1}{\log_e(2x)^x}$

(c) $\frac{1}{(x\log_e x)\log_e 2}$ (d) $\frac{1}{x(\log_2 x)^2}$

68. The angle of intersection between the curves $y^2 + x^2 = a^2\sqrt{2}$ and $x^2 - y^2 = a^2$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

69. If $A > 0$, $B > 0$ and $A + B = \frac{\pi}{3}$, then the maximum value of $\tan A \tan B$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\sqrt{3}$

70. The equation of the common tangent drawn to the curves $y^2 = 8x$ and $xy = -1$ is

(a) $y = 2x + 1$ (b) $2y = x + 6$ (c) $y = x + 2$ (d) $3y = 8x + 2$

71. Suppose $f(x) = x(x + 3)(x - 2)$, $x \in [-1, 4]$. Then, a value of c in $(-1, 4)$ satisfying $f'(c) = 10$ is

(a) 2 (b) $\frac{5}{2}$ (c) 3 (d) $\frac{7}{2}$

72. If $\int x^3 e^{5x} dx = \frac{e^{5x}}{5^4}\left[f(x)\right] + C$, then $f(x)$ is equal to

(a) $\frac{x^3}{5} - \frac{3x^2}{5^2} + \frac{6x}{5^3} - \frac{6}{5^4}$ (b) $5x^3 - 5^2x^2 + 5^3x - 6$

(c) $5^2x^3 - 15x^2 + 30x - 6$ (d) $5^3x^3 - 75x^2 + 30x - 6$

73. $\int \frac{x}{\left(x^2 + 2x + 2\right)^2} dx$ is equal to

(a) $\frac{x^2 + 2}{x^2 + 2x + 2} - \frac{1}{2}\tan^{-1}(x + 1) + C$

(b) $\frac{x^2 + 2}{2\left(x^2 + 2x + 2\right)} - \frac{1}{2}\tan^{-1}(x - 1) + C$

(c) $\frac{x^2 - 2}{4\left(x^2 + 2x + 2\right)} - \frac{1}{2}\tan^{-1}(x + 1) + C$

(d) $\frac{2(x - 1)}{\left(x^2 + 2x + 2\right)} + \frac{1}{2}\tan^{-1}(x + 1) + C$

74. If $\int \log\left(a^2 + x^2\right) dx = h(x) + C$, then h$(x)$ is equal to

(a) $x\log\left(a^2 + x^2\right) + 2\tan^{-1}\left(\frac{x}{a}\right)$

(b) $x^2\log\left(a^2 + x^2\right) + x + a\tan^{-1}\left(\frac{x}{a}\right)$

(c) $x\log\left(a^2 + x^2\right) - 2x + 2a\tan^{-1}\left(\frac{x}{a}\right)$

(d) $x^2\log\left(a^2 + x^2\right) + 2x - a^2\tan^{-1}\left(\frac{x}{a}\right)$

75. For $x > 0$, if $\int (\log x)^5 dx$ is equal to $x[A(\log x)^5 + B(\log x)^4 + C(\log x)^3 + D(\log x)^2 + E(\log x) + F]$ + constant, then $A + B + C + D + E + F$ is equal to
(a) –44 (b) –42 (c) –40 (d) –36

76. The area included between the parabola $y = \frac{x^2}{4a}$ and the curve $y = \frac{8a^3}{(x^2 + 4a^2)}$ is

(a) $a^2\left(2\pi + \frac{2}{3}\right)$ (b) $a^2\left(2\pi - \frac{8}{3}\right)$

(c) $a^2\left(\pi + \frac{4}{3}\right)$ (d) $a^2\left(2\pi - \frac{4}{3}\right)$

77. By the definition of the definite integral, the value of
$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2-1}} + \frac{1}{\sqrt{n^2-2^2}} + \ldots + \frac{1}{\sqrt{n^2-(n-1)^2}}\right)$$
is equal to

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

78. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\frac{x + \frac{\pi}{4}}{2 - \cos 2x}\right)dx$ is equal to

(a) $\frac{8\pi\sqrt{3}}{5}$ (b) $\frac{2\pi\sqrt{3}}{9}$ (c) $\frac{4\pi^2\sqrt{3}}{9}$ (d) $\frac{\pi^2}{6\sqrt{3}}$

79. The solution of the differential equation
$(1 + y^2) + \left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$, is

(a) $xe^{\tan^{-1} y} = \tan^{-1} y + C$
(b) $xe^{2\tan^{-1} y} = e^{-\tan^{-1} y} + C$
(c) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + C$
(d) $x^2 e^{\tan^{-1} y} = 4e^{2\tan^{-1} y} + C$
(C is an arbitrary constant)

80. The solution of the differential equation
$(2x - 4y + 3)\frac{dy}{dx} + (x - 2y + 1) = 0$ is

(a) $\log[(2x - 4y) + 3] = x - 2y + C$
(b) $\log[2(2x - 4y) + 3] = 2(x - 2y) + C$
(c) $\log[2(x - 2y) + 5] = 2(x + y) + C$
(d) $\log[4(x - 2y) + 5] = 4(x + 2y) + C$
(C is an arbitrary constant)

PHYSICS

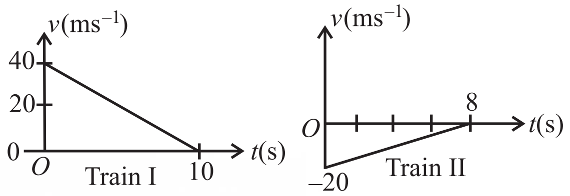
81. Match the List-I with List-II.

List-I	List-II
A. Boltzmann constant	I. $[ML^0T^0]$
B. Coefficient of viscosity	II. $[ML^{-1}T^{-1}]$
C. Water equivalent	III. $[MLT^{-3}K^{-1}]$
D. Coefficient of thermal conductivity	IV. $[ML^2T^{-2}K^{-1}]$

The correct match in the following is

	A	B	C	D		A	B	C	D
(a)	III	I	II	IV	(b)	III	II	I	IV
(c)	IV	II	I	III	(d)	IV	I	II	III

82. Two trains, which are moving along different tracks in opposite directions are put on the same track by mistake. On noticing the mistake, when the trains are 300 m apart the drivers start slowing down the trains. The graphs given below show decrease in their velocities as function of time. The separation between the trains when both have stopped is



(a) 120 m (b) 20 m (c) 60 m (d) 280 m

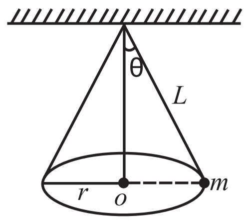
83. A point object moves along an arc of a circle of radius R. Its velocity depends upon the distance covered s as $v = K\sqrt{s}$, where K is a constant. If θ is the angle between the total acceleration and tangential acceleration, then

(a) $\tan\theta = \sqrt{\frac{s}{R}}$ (b) $\tan\theta = \sqrt{\frac{s}{2R}}$

(c) $\tan\theta = \frac{s}{2R}$ (d) $\tan\theta = \frac{2s}{R}$

84. A body projected from the ground reaches a point X in its path after 3 seconds and from there it reaches the ground after further 6 seconds. The vertical distance of the point X from the ground is (acceleration due to gravity = 10 ms^{-2})
(a) 30 m (b) 60 m (c) 80 m (d) 90 m

85. A particle of mass m is suspended from a ceiling through a string of length L. If the particle moves in a horizontal circle of radius r as shown in the figure, then the speed of the particle is



(a) $r\sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$ (b) $g\sqrt{\frac{r}{\sqrt{L^2 - r^2}}}$

(c) $r\sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$ (d) $g\sqrt{\frac{r}{\sqrt{L^2 - r^2}}}$

86. A particle is placed at rest inside a hollow hemisphere of radius R. The coefficient of friction between the particle and the hemisphere is $\mu = \frac{1}{\sqrt{3}}$. The maximum height upto which the particle can remain stationary is

(a) $\frac{R}{2}$ (b) $\left(1-\frac{\sqrt{3}}{2}\right)R$

(c) $\frac{\sqrt{3}}{2}R$ (d) $\frac{3R}{8}$

87. A 1 kg ball moving with a speed of 6 ms^{-1} collides head-on with a 0.5 kg ball moving in the opposite direction with a speed of 9 ms^{-1}. If the coefficient of restitution is $\frac{1}{3}$, then the energy lost in the collision is

(a) 303.4 J (b) 66.7 J (c) 33.3 J (d) 67.8 J

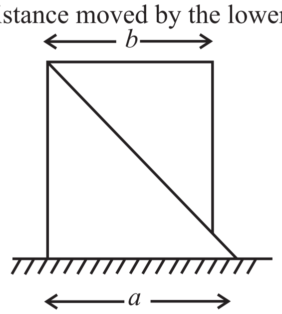
88. A ball is thrown vertically down from a height of 40 m from the ground with an initial velocity v. The ball hits the ground, loses $\frac{1}{3}$rd of its total mechanical energy and rebounds back to the same height. If the acceleration due to gravity is 10 ms^{-2}, then the value of v is

(a) 5 ms^{-1} (b) 10 ms^{-1} (c) 15 ms^{-1} (d) 20 ms^{-1}

89. Three identical uniform thin metal rods form the three sides of an equilateral triangle. If the moment of inertia of the system of these three rods about an axis passing through the centroid of the triangle and perpendicular to the plane of the triangle is n times. The moment of inertia of one rod separately about an axis passing through the centre of the rod and perpendicular to its length, the value of n is

(a) 3 (b) 6 (c) 9 (d) 12

90. Two smooth and similar right angled prisms are arranged on a smooth horizontal plane as shown in the figure. The lower prism has a mass 3 times the upper prism. The prisms are held in an initial position as shown and are then released. As the upper prism touches the horizontal plane, the distance moved by the lower prism is



(a) $a-b$ (b) $\frac{a-b}{3}$ (c) $\frac{b-a}{2}$ (d) $\frac{a-b}{4}$

91. A particle is executing simple harmonic motion with an amplitude of 2 m. The difference in the magnitudes of its maximum acceleration and maximum velocity is 4. The time-period of its oscillation and its velocity when it is 1 m away from the mean position are respectively

(a) $2s, 2\sqrt{3}\ ms^{-1}$ (b) $\frac{7}{22}s, 4\sqrt{3}\ ms^{-1}$

(c) $\frac{22}{7}s, 2\sqrt{3}\ ms^{-1}$ (d) $\frac{44}{7}s, 4\sqrt{3}\ ms^{-1}$

92. Two bodies of masses m and $9m$ are placed at a distance r. The gravitational potential at a point on the line joining them, where gravitational field is zero, is (G is universal gravitational constant)

(a) $\frac{-14Gm}{r}$ (b) $\frac{-16Gm}{r}$

(c) $\frac{-12Gm}{r}$ (d) $\frac{-8Gm}{r}$

93. When a load of 80 N is suspended from a string, its length is 101 mm. If a load of 100 N is suspended, its length is 102 mm. If a load of 160 N is suspended from it, then length of the string is (Assume the area of cross-section unchanged)

(a) 15.5 cm (b) 13.5 cm (c) 16.5 cm (d) 10.5 cm

94. A sphere of material of relative density 8 has a concentric spherical cavity and just sinks in water. If the radius of the sphere is 2 cm, then the volume of the cavity is

(a) $\frac{76}{3}cm^3$ (b) $\frac{79}{3}cm^3$ (c) $\frac{82}{3}cm^3$ (d) $\frac{88}{3}cm^3$

95. A hunter fired a metallic bullet of mass m kg from a gun towards an obstacle and it just melts when it is stopped by the obstacle. The initial temperature of the bullet is 300 K. If $\frac{1}{4}$th of heat is absorbed by the obstacle, then the minimum velocity of the bullet is
(Melting point of bullet = 600 K,
Specific heat of bullet = 0.03 cal $g^{-1}\ °C^{-1}$,
Latent heat of fusion of bullet = 6 cal g^{-1})

(a) 410 ms^{-1} (b) 260 ms^{-1}

(c) 460 ms^{-1} (d) 310 ms^{-1}

96. M kg of water at t °C is divided into two parts so that one part of mass m kg when converted into ice at 0°C would release enough heat to vapourise the other part, then $\frac{m}{M}$ is equal to (Specific heat of water = 1 cal $g^{-1}\ °C^{-1}$,
Latent heat of fusion of ice = 80 cal g^{-1},
Latent heat of steam = 540 cal g^{-1})

(a) $640-t$ (b) $\frac{720-t}{640}$ (c) $\frac{640+t}{720}$ (d) $\frac{640-t}{720}$

97. A diatomic gas (γ = 1.4) does 300 J work when it is expanded isobarically. The heat given to the gas in this process is

(a) 1050 J (b) 950 J (c) 600 J (d) 550 J

98. When the absolute temperature of the source of a Carnot heat engine is increased by 25%, its efficiency increases by 80%. The new efficiency of the engine is

(a) 12% (b) 24% (c) 48% (d) 36%

99. A cylinder of fixed capacity 67.2 litres contains helium gas at STP. The amount of heat needed to rise the temperature of the gas in the cylinder by 20°C is
(R = 8.31 J $mol^{-1}K^{-1}$)

(a) 748 J (b) 374 J (c) 1000 J (d) 500 J

100. For a certain organ pipe, three successive resonance frequencies are observed at 425, 595 and 765 Hz, respectively. The length of the pipe is (speed of sound in air = 340 ms^{-1})
(a) 0.5 m (b) 1 m (c) 1.5 m (d) 2 m

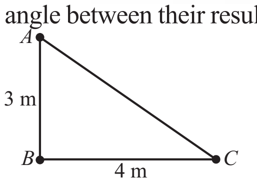
101. A student holds a tuning fork oscillating at 170 Hz. He walks towards a wall at a constant speed of 2 ms^{-1}. The beat frequency observed by the student between the tuning fork and its echo is (Velocity of sound = 340 ms^{-1})
(a) 2.5 Hz (b) 3 Hz (c) 1 Hz (d) 2 Hz

102. An infinitely long rod lies along the axis of a concave mirror of focal length f. The nearer end of the rod is at a distance u, $(u > f)$ from the mirror. Its image will have a length
(a) $\frac{uf}{u+f}$ (b) $\frac{uf}{u-f}$ (c) $\frac{f^2}{u+f}$ (d) $\frac{f^2}{u-f}$

103. In Young's double slit experiment, red light of wavelength 6000 Å is used and the nth bright fringe is obtained at a point P on the screen. Keeping the same setting, the source of light is replaced by green light of wavelength 5000 Å and now $(n + 1)$th bright fringe is obtained at the point P on the screen. The value of n is
(a) 4 (b) 5 (c) 6 (d) 3

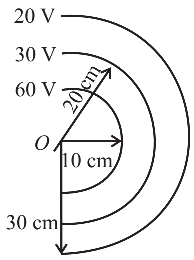
104. Two charges each of charge + 10 μC are kept on Y-axis at $y = -a$ and $y = +a$, respectively. Another point charge – 20 μC is placed at the origin and given a small displacement x $(x << a)$ along X-axis. The force acting on the point charge is
$\left(x \text{ and } a \text{ are in metres}, \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{ N} - \text{m}^2\text{C}^{-2}\right)$
(a) $\frac{3.6x}{a^2}$N (b) $\frac{2.4x^2}{a}$N (c) $\frac{3.6x}{a^3}$N (d) $\frac{4.8x}{a^2}$N

105. Three identical charges, each 2 μC lie at the vertices of a right angled triangle as shown in the figure. Forces on the charge at B due to the charges at A and C respectively are F_1 and F_2. The angle between their resultant force and F_2 is



(a) $\tan^{-1}\left(\frac{9}{16}\right)$ (b) $\tan^{-1}\left(\frac{9}{7}\right)$
(c) $\tan^{-1}\left(\frac{16}{9}\right)$ (d) $\tan^{-1}\left(\frac{7}{9}\right)$

106. The figure shows equipotential surfaces concentric at O. The magnitude of electric field at a distance r metres from O is



(a) $\frac{9}{r^2}$Vm^{-1} (b) $\frac{16}{r^2}$Vm^{-1}
(c) $\frac{2}{r^2}$Vm^{-1} (d) $\frac{6}{r^2}$Vm^{-1}

107. A region contains a uniform electric field $\mathbf{E} = \left(10\hat{\mathbf{i}} + 30\hat{\mathbf{j}}\right)\text{Vm}^{-1}$. A and B are two points in the field at (1, 2, 0) m and (2, 1, 3) m, respectively. The work done when a charge of 0.8 C moves from A to B in a parabolic path is
(a) 8 J (b) 80 J (c) 40 J (d) 16 J

108. When a long straight uniform rod is connected across an ideal cell, the drift velocity of electrons in it is v. If a uniform hole is made along the axis of the rod and the same battery is used, then the drift velocity of electrons becomes
(a) v (b) $> v$ (c) $< v$ (d) zero

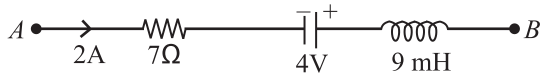
109. In a meter bridge experiment, when a nichrome wire is in the right gap, the balancing length is 60 cm. When the nichrome wire is uniformly stretched to increase its length by 20% and again connected in the right gap, the new balancing length is nearly
(a) 61 cm (b) 31 cm (c) 51 cm (d) 41 cm

110. A loop of flexible conducting wire lies in a magnetic field of 2.0 T with its plane perpendicular to the field. The length of the wire is 1 m. When a current of 1.1 A is passed through the loop, it opens into a circle, then the tension developed in the wire is
(a) 0.15 N (b) 0.25 N (c) 0.35 N (d) 0.45 N

111. A charge q is spread uniformly over an isolated ring of radius R. The ring is rotated about its natural axis with an angular velocity ω. Magnetic dipole moment of the ring is
(a) $\frac{q\omega R^2}{2}$ (b) $\frac{q\omega R}{2}$ (c) $q\omega R^2$ (d) $\frac{q\omega}{2R}$

112. A magnetic dipole of moment 2.5 Am^2 is free to rotate about a vertical axis passing through its centre. It is released from East-West direction. Its kinetic energy at the moment, it takes North-South position is
$(B_H = 3 \times 10^{-5}$ T$)$
(a) 50 μJ (b) 100 μJ (c) 175 μJ (d) 75 μJ

113. A branch of a circuit is shown in the figure. If current is decreasing at the rate of 10^3 As^{-1}, then the potential difference between A and B is



(a) 1 V (b) 5 V (c) 10 V (d) 2 V

114. The natural frequency of an L-C circuit is 125 kHz. When the capacitor is totally filled with a dielectric material, the natural frequency decreases by 25 kHz. Dielectric constant of the material is nearly
(a) 3.33 (b) 2.12 (c) 1.56 (d) 1.91

115. Choose the correct sequence of the radiation sources in increasing order of the wavelength of electromagnetic waves produced by them.
(a) X-ray tube, Magnetron valve, Radioactive source, Sodium lamp
(b) Radioactive source, X-ray tube, Sodium lamp, Magnetron valve
(c) X-ray tube, Magnetron valve, Sodium lamp, Radioactive source
(d) Magnetron valve, Sodium lamp, X-ray tube, Radioactive source

116. A photo sensitive metallic surface emits electrons when X-rays of wavelength λ fall on it. The de-Broglie wavelength of the emitted electrons is (Neglect the work function of the surface, m is mass of the electron, h is Planck's constant, c is the velocity of light)
(a) $\sqrt{\frac{2mc}{h\lambda}}$ (b) $\sqrt{\frac{h\lambda}{2mc}}$ (c) $\sqrt{\frac{mc}{h\lambda}}$ (d) $\sqrt{\frac{h\lambda}{mc}}$

117. An electron in a hydrogen atom undergoes a transition from a higher energy level to a lower energy level. The incorrect statement of the following is
(a) Kinetic energy of the electron increases
(b) Velocity of the electron increases
(c) Angular momentum of the electron remains constant
(d) Wavelength of de-Broglie wave associated with the motion of electron decreases

118. The radius of germanium (Ge) nuclide is measured to be twice the radius of ${}_{4}^{9}Be$. The number of nucleons in Ge will be
(a) 72 (b) 73 (c) 74 (d) 75

119. For a common-emitter transistor amplifier, the current gain is 60. If the emitter current is 6.6 mA, then its base current is
(a) 6.492 mA (b) 0.108 mA
(c) 4.208 mA (d) 0.343 mA

120. If a transmitting antenna of height 105m is placed on a hill, then its coverage area is
(a) 4224 km^2 (b) 3264 km^2
(c) 6400 km^2 (d) 4864 km^2

CHEMISTRY

121. The product of uncertainty in velocity and uncertainty in position of a micro particle of mass 'm' is not less than in which of the following?
(a) $h \times \frac{3\pi}{m}$ (b) $\frac{h}{3\pi} \times m$ (c) $\frac{h}{4\pi} \times \frac{1}{m}$ (d) $\frac{h}{4\pi} \times m$

122. An element has [Ar] $3d^1$ configuration in its +2 oxidation state. Its position in the periodic table is
(a) period-3, group-3 (b) period-3, group-7
(c) period-4, group-3 (d) period-3, group-9

123. In which of the following molecules, all bond lengths are not equal?
(a) SF_6 (b) PCl_5 (c) BCl_3 (d) CCl_4

124. In which of the following molecules, maximum number of lone pairs is present on the central atom?
(a) NH_3 (b) H_2O (c) ClF_3 (d) XeF_2

125. Which one of the following is the kinetic energy of a gaseous mixture containing 3 g of hydrogen and 80 g of oxygen at temperature $T(K)$?
(a) $3RT$ (b) $6RT$ (c) $4RT$ (d) $8RT$

126. A, B, C and D are four different gases with critical temperatures 304.1, 154.3, 405.5 and 126.0 K respectively. While cooling the gas, which gets liquefied first?
(a) B (b) A (c) D (d) C

127. 40 mL of xM $KMnO_4$ solution is required to react completely with 200 mL of 0.02 M oxalic acid solution in acidic medium. The value of x is
(a) 0.04 (b) 0.01 (c) 0.03 (d) 0.02

128. Given that,
$C(s) + O_2(g) \longrightarrow CO_2(g); \Delta H^\circ = -x$ kJ mol^{-1}
$2CO(g) + O_2(g) \longrightarrow 2CO_2(g); \Delta H^\circ = -y$ kJ mol^{-1}
The enthalpy of formation of CO will be
(a) $\frac{y-2x}{3}$ (b) $\frac{y-2x}{2}$ (c) $\frac{2x-y}{2}$ (d) $\frac{x-y}{2}$

129. At 400 K, in a 1.0 L vessel, N_2O_4 is allowed to attain equilibrium, $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ At equilibrium, the total pressure is 600 mm Hg, when 20% of N_2O_4 is dissociated. The value of K_p for the reaction is
(a) 50 (b) 100 (c) 150 (d) 200

130. In which of the following salts, only cationic hydrolysis is involved?
(a CH_3COONH_4 (b) CH_3COONa
(c) NH_4Cl (d) Na_2SO_4

131. Calgon is
(a) Na_2HPO_4 (b) Na_3PO_4
(c) $Na_6P_6O_{18}$ (d) NaH_2PO_4

132. Consider the following statements.
I. Cs^+ ion is more highly hydrated than other alkali metal ions.
II. Among the alkali metals, only lithium forms a stable nitride by direct combination with nitrogen.
III. Among the alkali metals Li, Na, K, Rb, the metal, Rb has the highest melting point.
IV. Among the alkali metals Li, Na, K, Rb, only Li forms peroxide when heated with oxygen.
Select the correct statement is
(a) I (b) II (c) III (d) IV

133. **Assertion :** $AlCl_3$ exists as a dimer through halogen bridged bonds.
Reason : $AlCl_3$ gets stability by accepting electrons from the bridged halogen.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(c) (A) is true, but (R) is false
(d) (A) is false, but (R) is true

134. Which of the following causes ''blue baby syndrome''?
(a) High concentration of lead in drinking water
(b) High concentration of sulphates in drinking water
(c) High concentration of nitrates in drinking water
(d) High concentration of copper in drinking water

135. Which of the following belongs to the homologous series of $C_5H_8O_2N$?
(a) $C_6H_{10}O_3N$ (b) $C_6H_8O_2N_2$
(c) $C_6H_{10}O_2N_2$ (d) $C_6H_{10}O_2N$

136. In Dumas method, 0.3 g of an organic compound gave 45 mL of nitrogen at STP. The percentage of nitrogen is
(a) 16.9 (b) 18.7 (c) 23.2 (d) 29.6

137. The IUPAC name of
$(CH_3)_2CH - CH = CH—CH = CH - \underset{\displaystyle C_2H_5}{\underset{|}{CH}} - CH_3$ is
(a) 2, 7-dimethyl-3, 5-nonadiene
(b) 2, 7-dimethyl-2-ethylheptadiene
(c) 2-methyl-7-ethyl-3, 5-octadiene
(d) 1, 1-dimethyl-6-ethyl-2, 4-heptadiene

138. Match the following.

	List-I (Magnetic property)		**List-II (Substance)**
(A)	Ferromagnetic	(1)	O_2
(B)	Antiferromagnetic	(2)	CrO_2
(C)	Ferrimagnetic	(3)	MnO
(D)	Paramagnetic	(4)	Fe_3O_4
		(5)	C_6H_6

	A	B	C	D		A	B	C	D
(a)	3	2	4	1	(b)	2	3	4	1
(c)	1	3	5	4	(d)	4	2	3	5

139. The vapour pressure of pure benzene and toluene are 160 and 60 mm Hg respectively. The mole fraction of benzene in vapour phase in contact with equimolar solution of benzene and toluene is
(a) 0.073 (b) 0.027 (c) 0.27 (d) 0.73

140. 6 g of a non-volatile, non-electrolyte X dissolved in 100 g of water freezes at –0.93°C. The molar mass of X in g mol^{-1} is (K_f of H_2O = 1.86 K kg mol^{-1})
(a) 60 (b) 140 (c) 180 (d) 120

141. The products obtained at the cathode and anode respectively during the electrolysis of aqueous K_2SO_4 solution using platinum electrodes are
(a) O_2, H_2 (b) H_2, O_2 (c) H_2, SO_2 (d) K, SO_2

142. The slope of the graph drawn between ln k and $\frac{1}{T}$ as per Arrhenius equation gives the value (R = gas constant, E_α = Activation energy)
(a) $\frac{R}{E_a}$ (b) $\frac{E_a}{R}$ (c) $\frac{-E_a}{R}$ (d) $\frac{-R}{E_a}$

143. Which of the following statements is not correct in respect of chemisorption?
(a) Highly specific adsorption
(b) Irreversible adsorption
(c) Multilayered adsorption
(d) High enthalpy of adsorption

144. Which of the following is a carbonate ore?
(a) Cuprite (b) Siderite (c) Zincite (d) Bauxite

145. Which one of the following statements is not correct?
(a) O_3 is used as a germicide
(b) In O_3, O—O bond length is identical with that of molecular oxygen
(c) O_3 is an oxidising agent
(d) The shape of O_3 molecule is angular

146. Which of the following reactions does not take place?
(a) $F_2 + 2Cl^- \longrightarrow 2F^- + Cl_2$
(b) $Br_2 + 2I^- \longrightarrow 2Br^- + I_2$
(c) $Cl_2 + 2Br^- \longrightarrow 2Cl^- + Br_2$
(d) $Cl_2 + 2F^- \longrightarrow 2Cl^- + F_2$

147. Which of the following statements regarding sulphur is not correct?
(a) At about 1000 K, it mainly consists of S_2 molecules
(b) The oxidation state of sulphur is never less than + 4 in its compounds
(c) S_2 molecule is paramagnetic
(d) Rhombic sulphur is readily soluble in CS_2

148. Which of the following reactions does not involve liberation of oxygen?
(a) $XeF_4 + H_2O \longrightarrow$ (b) $XeF_4 + O_2F_2 \longrightarrow$
(c) $XeF_2 + H_2O \longrightarrow$ (d) $XeF_6 + H_2O \longrightarrow$

149. Select the correct IUPAC name of
$[Co\ (NH_3)_5(CO_3)]\ Cl$.
(a) Penta ammoniacarbonate cobalt (III) chloride
(b) Pentaamminecarbonatecobalt chloride
(c) Pentaamminecarbonatocobalt (III) chloride
(d) Cobalt (III) pentaamminecarbonate chloride

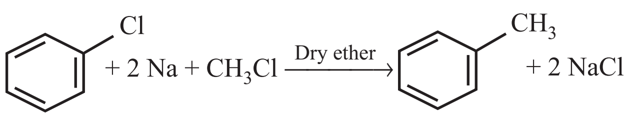
150. Which of the following characteristics of the transition metals is associated with their catalytic activity?
(a) Colour of hydrated ions
(b) Diamagnetic behaviour
(c) Paramagnetic behaviour
(d) Variable oxidation states

151. Observe the following polymers.
I. PHBV II. Nylon-2-nylon-6
III. Glyptal IV. Bakelite
Biodegradable polymer(s) from the above is/are
(a) III (b) I and II (c) IV (d) III and IV

152. Observe the following statements.
I. Sucrose has glycosidic linkage.
II. Cellulose is present in both plants and animals.
III. Lactose contains D-galactose and D-glucose units.
Which of the following statements are correct?
(a) (I), (II) and (III) (b) (I) and (II)
(c) (II) and (III) (d) (I) and (III)

153. Identify the antioxidant used in foods.
(a) Aspartame
(b) Sodium benzoate
(c) *ortho-sulphobenzimide*
(d) Butylated hydroxy toluene

154.



This reaction is known as
(a) Wurtz-Fittig reaction (b) Wurtz reaction
(c) Fittig reaction (d) Friedel-Crafts reaction

155. What is Z in the following sequence of reaction?
2-methyl-2-bromopropane

$$\xrightarrow[\text{Dry ether}]{\text{Mg}} X \xrightarrow{H_2O} Z$$

(a) Propane (b) 2-methyl propene
(c) 2-methyl propane (d) 2-methyl butane

156. In which of the following reactions, the product is not correct?

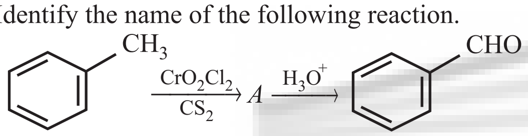
(a) $CH_3CHO \xrightarrow{LiAlH_4} CH_3CH_2OH$

(b) $CH_3COCH_3 \xrightarrow[HCl]{Zn-Hg} CH_3-\underset{\underset{OH}{|}}{OH}-CH_3$

(c) $CH_3CH_2CHO \xrightarrow[\text{(ii) KOH, Ethylene glycol}/\Delta]{\text{(i) } H_2N-NH_2} CH_3CH_2CH_3$

(d) $CH_3CH_2CHO \xrightarrow{KMnO_4} CH_3CH_2COOH$

157. Identify the name of the following reaction.



(a) Gattermann Koch reaction
(b) Gattermann reaction
(c) Stephen reaction
(d) Etard reaction

158. What is C in the following sequence of reaction?

$$CH_3OH \xrightarrow{PCl_3} A \xrightarrow{KCN} B \xrightarrow{\text{Hydrolysis}} C$$

(a) CH_3CH_2OH (b) CH_3CHO
(c) CH_3COOH (d) $HOCH_2-CH_2OH$

159. The order of basic strength of the following in aqueous solution is

I. $C_6H_5NH_2$ II. $(CH_3)_3N$
III. NH_3 IV. CH_3NH_2
V. $(CH_3)_2NH$

(a) IV > I > V > III > II (b) II > V > IV > III > I
(c) V > IV > II > III > I (d) IV > III > V > II > I

160. Yellow dye can be prepared by a coupling reaction of benzene diazonium chloride in acidic medium with X. Identify X from the following.

(a) Aniline (b) Phenol
(c) Cumene (d) Benzene

Hints & Solutions

MATHEMATICS

1. **(b)** We have, $f(x) = \sqrt{\log_{0.5} x!}$

So, $f(x)$ will be defined when,

$\log_{0.5} x! \geq 0$

$x! \leq (0.5)^0$

$x! \leq 1 \quad \therefore x \in \{0, 1\}$

Hence, the domain of the function is $\{0, 1\}$.

2. **(c)** We have,

$f(x) = |x - 1| + |x - 2| + |x - 3|$

$$\Rightarrow \quad f(x) = \begin{cases} 6 - 3x, & x < 1 \\ 4 - x, & 1 < x < 2 \\ x, & 2 < x < 3 \\ 3x - 6, & x > 3 \end{cases}$$

For $2 < x < 3$, then $f(x) = x$

So, $f(x)$ is one-one and onto function.

It implies that $f(x)$ is a bijection.

3. **(b)** Given, $(n + 16)(n + 17)(n + 18)(n + 19)$

As we can observe that these numbers are th product of four consecutive natural numbers.

This number gets divided by $4! = 24$.

4. **(c)** Let $A = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ $\quad [C_1 \to C_1 \to C_2 \to C_3]$

$$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$[R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1]$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$$

$= (a + b + c)\,[(c - b)(b - c) - (a - c)(a - b)]$

$= (a + b + c)\,[bc - b^2 - c^2 + bc - a^2 + ab + ac - abc]$

$= -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= -\frac{1}{2}(a + b + c)\,[(a - b)^2 + (b - c)^2 + (c - a)^2]$

Since it is given that a, b, c are distinct positive numbers. So, the value of determinant A is less than 0.

5. **(d)** It is given that x_1, x_2, x_3 and y_1, y_2, y_3 are in GP with the same common ratio.

Let r be the common ratio.

$\therefore \quad x_1 = x_1,\ x_2 = xr$ and $x_3 = xr^2$

Similarly, $y_1 = y,\ y_2 = yr$ and $y_3 = yr^2$

$$\therefore \quad \text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_1 & 1 \\ x_3 & y_1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} x & y_1 & 1 \\ xr & yr & 1 \\ xr^2 & yr^2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix} = \frac{1}{2} \times 0 = 0$$

$[\because C_1 \text{ and } C_2 \text{ are identical}]$

$\therefore$ The given points do not form any triangle. They are collinear.

6. **(b)** Given equations, $x - y + 2z = 4$

$3x + y + 4z = 6$

$x + y + z = 1$

$$\text{Let } \Delta = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 1(1 - 4) + 1(3 - 4) + 2(3 - 1)$

$= -3 - 1 + 4 = 0$

$$\text{and } \Delta_1 = \begin{vmatrix} 4 & -1 & 2 \\ 6 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 4(1 - 4) + 1(6 - 4) + 2(6 - 1)$

$= -12 + 2 + 10 = 0$

Now, $\Delta = 0$ and $\Delta_1 = 0$

$\therefore$ These equations have infinitely many solutions.

7. **(c)** Let the comlex number, $z = x + iy$

Now, $|z + 3|^2 - |z - 3|^2 = 15$

$\therefore \quad |x + iy + 3|^2 - |x + iy - 3|^2 = 15$

$(x + 3)^2 + y^2 - (x - 3)^2 - y^2 = 15$

$x^2 + 6x + 9 - x^2 + 6x - 9 = 15$

$12x = 15 \quad \Rightarrow 4x = 5$

$\therefore$ It represents a straight line.

8. **(a)** We have,

$$\frac{(1+i)^{2016}}{(1-i)^{2014}} = \frac{\left[(1+i)^2\right]^{1008}}{\left[(1-i)^2\right]^{1007}} = \frac{\left(1+2i+i^2\right)^{1008}}{\left(1-2i+i^2\right)^{1007}}$$

$$= \frac{(2i)^{1008}}{(-2i)^{1007}} = \frac{(2i)^{1008}}{(-1)^{1007}(2i)^{1007}}$$

$= -(2i)^{1008-1007} = -2i$

9. **(d)** We have,

$|z_1| = 1, |z_2| = 2, |z_3| = 3$ and

$|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$

Since, we know $|z|^2 = z z$

Now, $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$

$$\left| |z_3|^2 z_1z_2 + |z_2|^2 z_1z_2 + |z_1|^2 z_2z_3 \right| = 12$$

$$\left| z_3\bar{z}_3z_1z_2 + z_2\bar{z}_2z_1z_2 + z_1\bar{z}_1z_2z_3 \right| = 12$$

$$\left| z_1z_2z_3\left(\bar{z}_3 + \bar{z}_2 + \bar{z}_1\right) \right| = 12$$

$$\left| z_1z_2z_3 \right|\left|\bar{z}_1 + \bar{z}_2 + \bar{z}_3\right| = 12$$

$$|z_1||z_2||z_3|\left|\bar{z}_1 + \bar{z}_2 + \bar{z}_2\right| = 12$$

$$1 \times 2 \times 3\left|\bar{z}_1 + \bar{z}_2 + \bar{z}_3\right| = 12$$

$$\left|\bar{z}_1 + \bar{z}_2 + \bar{z}_3\right| = 2$$

$$\left|z_1 + z_2 + z_3\right| = 2$$

10. **(c)** We have given that, $1, z_1, z_2, z_3, \ldots\ldots z_{n-1}$ are the nth root of unity.

$\therefore \quad z^n - 1 = (z-1)(z-z_1)(z-z_2), \ldots\ldots, (z-z_{n-1})$

$(z-1)(z^{n-1} + z^{n-2} + \ldots\ldots + z^2 + z + 1)$

$= (z-1)(z-z_1) + \ldots\ldots (z-z_{n-1})$

$(z^{n-1} + z^{n-2} + \ldots\ldots + z^2 + z + 1) = (z-z_1)$

$(z-z_1) = (z-z_{n-1})$

Put $z = 1$, we get

$(1 + 1 + \ldots. + n$ times$)$

$= (1-z_1)(1-z_2) + \ldots. (1-z_{n-1})$

$n = (1-z_1)(1-z_2) \ldots (1-z_{n-1})$

11. **(b)** We have, $(12)^{4-2x^2} = \left(24\sqrt{3}\right)^{3x^2-2}$

$$(12)^{4+2x^2} = (12)^{\frac{3}{2}\left(3x^2-2\right)}$$

$$4 + 2x^2 = \frac{3}{2}\left(3x^2 - 2\right)$$

$8 + 4x^2 = 9x^2 - 6$

$9x^2 - 4x^2 = 8 + 6$

$\Rightarrow \quad 5x^2 = 14 \quad \Rightarrow x^2 = \frac{14}{5} \qquad \therefore \quad x = \pm\sqrt{\frac{14}{5}}$

12. **(a)** We have, $|x|^2 - 5|x| - 24 = 0$

$\Rightarrow \quad (|x| - 8)(|x| + 3) = 0$

$\Rightarrow \quad |x| - 8$ or $|x| = -3$ is rejected.

$\Rightarrow \quad |x| = 8$

$\Rightarrow \quad x = \pm 8$

$\therefore \quad$ The roots of this equation. $ax = 8$ and -8.

Hence, product of roots $8 \times (-8) = -64$

and sum of roots $= +8 - 8 = 0$

13. **(a)** Let $f(x) = x^5 + 3x^3 + 4x + 30$

$\Rightarrow \quad f'(x) = 5x^4 + 9x^2 + 4$

As $f'(x)$ consist of the terms which has even powers of x. $f'(x) > 0$ for all $x \in$ R

Hence, the $f(x) = 0$ has only one real root.

14. **(b)** Given equation,

$$x^3 + \frac{1}{4}x^2 - \frac{1}{16}x + \frac{1}{144} = 0 \qquad \ldots. \text{(i)}$$

The equation whose roots are k times the roots of the eq. (i), is

$$\left(\frac{x}{k}\right)^3 + \frac{1}{4}\left(\frac{x}{k}\right)^3 - \frac{1}{16}\left(\frac{x}{k}\right) + \frac{1}{144} = 0$$

$$\Rightarrow x^3 + \frac{k}{4}x^2 - \frac{k^2}{16}x + \frac{k^3}{144} = 0$$

Also given that, coefficients of the above equation are integer.

$k = 12$

15. **(c)** Given digits are 2, 3, 4, 5, 6

Sum of all 4-digit numbers formed by the using 2, 3, 4, 5, 6 without repetition

$= 41\,(2 + 3 + 4 + 5 + 6)\,(10^3 + 10^2 + 10^1 + 10^0)$

$= 24(20)\,(1000 + 100 + 10 + 1)$

$= (480)\,(1111) = 533280$

16. **(c)** Given set A has 5 elements, there are two subsets P and Q which are mutually disjoint. So, the required number of ways of selecting two subsets will be $3^5 = 243$.

17. **(a)** In the expansion $(1 - x + x^2 - x^3)^4$

$= (1 + x^2 - x - x^3)^4 = [(1 + x^2) - x(1 + x^2)^4]$

$= [(1 + x^2)(1 - x)^4] = (1 + x^2)^4 (1 - x)^4$

$= (1 + 4x^2 + 1 + 6x^4 + 4x^6 + x^8)$

$(1 - 4x^2 + 6x^2 - 4x^3 + x^4)$

$\therefore$ Coefficient of $x^4 = 1 + 4 \times 6 + 6$

$= 1 + 24 + 6 = 31$

18. **(a)** In the expansion of $(1 + x)^{2n}$, middle term is ${}^{2n}C_n\, x^n$. Since, middle term is the greatest term.

$\therefore \quad {}^{2n}C_n\, x^n > {}^{2n}C_{n-1}\, x^{n-1}$

and ${}^{2n}C_r . x^n > {}^{2n}C_{n+1}\, x^{n+1}$

$\Rightarrow \quad x > \dfrac{{}^{2n}C_{n-1}}{{}^{2n}C_n}$ and $x < \dfrac{{}^{2n}C_n}{{}^{2n}C_{n+1}}$

Hence, $x \in \left(\dfrac{{}^{2n}C_n}{{}^{2n}C_{n+1}}, \dfrac{{}^{2n}C_n}{{}^{2n}C_{n+1}}\right)$

$$x \in \left(\begin{array}{c}\dfrac{(2n)!}{(n-1)!(2n-n+1)} \times \dfrac{n!(2n-n)!}{(2n)!}, \\ \dfrac{(2n)!}{n!(2n-n)!} \times \dfrac{(n-1)!(2n-n+1)}{(2n)!}\end{array}\right)$$

$$x \in \left(\frac{n(n-1)!n!}{n(n-1)!(n+1)n!}, \frac{(n+1)n!(n-1)!}{n(n-1)!n!}\right)$$

$\therefore \quad x \in \left(\dfrac{n}{n+1}, \dfrac{n+1}{n}\right)$

19. **(c)** Given, $\dfrac{3x}{(x-2)(x-1)}$ can be written as $\dfrac{6}{x-2} - \dfrac{3}{x-1}$

$\therefore \quad \dfrac{3x}{(x-2)(x-1)} = \dfrac{6}{x-2} - \dfrac{3}{x-1}$

$= 6(x-2)^{-1} - 3(x-1)^{-1}$

$= -3\left(1 - \dfrac{x}{2}\right) + 3(x-1)^{-1}$

It is valid iff $\left|\dfrac{x}{2}\right| < 1$ and $|x| < 1$

$\Rightarrow \quad |x| < 2$ and $|x| < 1$

$\Rightarrow \quad x \in (-2, 2)$ and $x \in (-1, 1) \quad \therefore \quad x \in (-1, 1)$

Hence, $-1 < x < 1$

20. **(a)** Given that $(1 + \tan\alpha)(1 + \tan 4\alpha) = 2,\ \alpha \in \left(0, \dfrac{\pi}{16}\right)$

$\Rightarrow \quad 1 + \tan\alpha + \tan 4\alpha + \tan\alpha \tan 4\alpha = 2$

$\Rightarrow \quad \tan\alpha + \tan 4\alpha = 1 - \tan\alpha \tan 4\alpha$

$\Rightarrow \quad \dfrac{\tan\alpha + \tan 4\alpha}{1 - \tan\alpha + \tan 4\alpha} = 1$

$\Rightarrow \quad \tan(\alpha + 4\alpha) = 1$

$\Rightarrow \quad \tan 5\alpha = \tan\dfrac{\pi}{4}$

$\Rightarrow \quad 5\alpha = \dfrac{\pi}{4} \quad \Rightarrow \tan\alpha = \dfrac{\pi}{20}$

21. **(a)** We have, $\cos\theta = \dfrac{\cos\alpha - \cos\beta}{1 - \cos\alpha\cos\beta}$

Appling componendo and dividendo, we get

$$\frac{\cos\theta + 1}{\cos\theta - 1} = \frac{\cos\alpha - \cos\beta + 1 - \cos\alpha\cos\beta}{\cos\alpha - \cos\beta - 1 + \cos\alpha\cos\beta}$$

$$\frac{\cos\theta + 1}{\cos\theta - 1} = \frac{(\cos\alpha + 1)(1 - \cos\beta)}{(\cos\alpha - 1)(\cos\beta + 1)}$$

$$\frac{1 + \cos\theta}{1 - \cos\theta} = \frac{(1 + \cos\alpha)(1 - \cos\beta)}{(1 - \cos\alpha)(1 + \cos\beta)}$$

$$\frac{2\cos^2\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \frac{2\cos^2\frac{\alpha}{2}}{2\sin^2\frac{\alpha}{2}} \cdot \frac{2\sin^2\frac{\beta}{2}}{2\cos^2\frac{\beta}{2}}$$

$$\cot^2\frac{\theta}{2} = \cot^2\frac{\alpha}{2}\tan^2\frac{\beta}{2}$$

$$\tan^2\frac{\theta}{2} = \tan^2\frac{\alpha}{2}\cot^2\frac{\beta}{2}$$

$$\tan\frac{\theta}{2} = \tan\frac{\alpha}{2}\cot\frac{\beta}{2}$$

22. **(d)** Given expression,

$$\frac{1 + \sin 2\alpha}{\cos(2\alpha - 2\pi)\tan\left(\alpha - \frac{3\pi}{4}\right)} - \frac{1}{4}\sin 2\alpha\left[\cot\frac{\alpha}{2} + \cot\left(\frac{3\pi}{2} + \frac{\alpha}{2}\right)\right]$$

$$\Rightarrow \quad \frac{(\cos\alpha + \sin\alpha)^2}{\cos 2\alpha\left(\dfrac{\tan\alpha - \tan\frac{3\pi}{4}}{1 + \tan\alpha\tan\frac{3\pi}{4}}\right)} - \frac{1}{4}2\sin\alpha\cos\alpha\left(\cot\frac{\alpha}{2} - \tan\frac{\alpha}{2}\right)$$

$$= \frac{(\cos\alpha + \sin\alpha)^2}{\cos^2\alpha - \sin^2\alpha\left(\dfrac{\tan\alpha + 1}{1 - \tan\alpha}\right)} - \frac{1}{4}2\sin\alpha\cos\alpha\left(\frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} - \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\right)$$

$$= \frac{(\cos\alpha + \sin\alpha)\times(\cos\alpha - \sin\alpha)}{(\cos\alpha - \sin\alpha)\times(\sin\alpha + \cos\alpha)} - \frac{1}{2}\sin\alpha\cos\alpha\left(\frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\right)$$

$$= 1 - \frac{\sin\alpha\cos^2\alpha}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} = 1 - \frac{\sin\alpha\cos^2\alpha}{\sin\alpha}$$

$= 1 - \cos^2\alpha = \sin^2\alpha$

23. **(b)** Given that, terms $\dfrac{1}{6}\sin\theta$, $\cos\theta$ and $\tan\theta$ are in GP.

$\therefore \quad \cos^2\theta = \dfrac{1}{6}\sin\theta \times \tan\theta$

$\Rightarrow \quad \cos^2\theta = \dfrac{\sin^2\theta}{6\cos\theta}$

$\Rightarrow \quad 6\cos^3\theta - \sin^2\theta = 0$

$\Rightarrow \quad 6\cos^3\theta - 1 + \cos^2\theta = 0$

$\Rightarrow \quad 6\cos^3\theta + 1 + \cos^2\theta = 0$

$\Rightarrow \quad 6\cos^3\theta + \cos^2\theta - 1 = 0$

$\Rightarrow \quad 6\cos^3\theta + 4\cos^2\theta - 3\cos^2\theta + 2\cos\theta - 2\cos\theta - 1 = 0$

$\Rightarrow \quad 6\cos^3\theta - 3\cos^2\theta + 4\cos^2\theta - 2\cos\theta + 2\cos\theta - 1 = 0$

$\Rightarrow \quad 3\cos^2\theta\,(2\cos\theta - 1) + 2\cos\theta\,(2\cos\theta - 1) + 1(2\cos\theta - 1) = 0$

$\Rightarrow \quad (2\cos\theta - 1)(3\cos^2\theta + 2\cos\theta + 1) = 0$

For $3\cos^2\theta + 2\cos\theta + 1 = 0$ value of $\cos\theta$ will be imaginary so only $2\cos\theta - 1 = 0$ will be considered to find $\cos\theta$.

$\Rightarrow \quad 2\cos\theta - 1 = 0 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \cos\theta = \cos\frac{\pi}{3}$

$\therefore \quad \theta = 2n\pi \pm \frac{\pi}{3}$

24. **(a)** We have, $x = \sin(2\tan^{-1} 2)$ and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$

$\Rightarrow \quad x = \sin(2\tan^{-1} 2)$

Let $\tan^{-1} 2 - \alpha \quad \Rightarrow \tan\alpha = 2$

$\therefore \quad x = \sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha} = \frac{2(2)}{1+(2)^2} = \frac{4}{5}$

Now, $\quad y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$

Let $\tan^{-1}\frac{4}{3} = \beta$

$\tan\beta = \frac{4}{3}, \cos\beta = \frac{1}{\sqrt{1+\tan^2\beta}} = \frac{3}{5}$

Then, $y = \sin\left(\frac{\beta}{2}\right)$

$= \sqrt{\frac{1-\cos\beta}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{1}{5}}$

$\therefore \quad x > y$

25. **(c)** Given, $\cos h(x) = \frac{5}{4}$

Now, $\cos h(3x) = 4\cos h^3(x) - 3\cos h(x)$

$= 4\left(\frac{5}{4}\right)^3 - 3\times\frac{5}{4} = \frac{125}{16} - \frac{15}{4}$

$= \frac{125-60}{16} = \frac{65}{16}$

26. **(b)** In ΔABC, given

$x = \tan\left(\frac{B-C}{2}\right)\tan\frac{A}{2}$

$y = \tan\left(\frac{C-A}{2}\right)\tan\frac{B}{2}$

and $y = \tan\left(\frac{A-B}{2}\right)\tan\frac{C}{2}$

Since, $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\tan\frac{A}{2}$

$\Rightarrow \quad x = \frac{b-c}{b+c}$

Similarly, $y = \frac{c-a}{c+c}$ and $z = \frac{a-b}{a+b}$

Now, by componendo and dividendo

$\frac{1+x}{1-x} = \frac{b+c+b-c}{b+c-b+c} = \frac{b}{c}$

Similarly, $\frac{1+y}{1-y} = \frac{c}{a}$ and $\frac{1+z}{1-z} = \frac{a}{b}$

$\therefore \quad \left(\frac{1+x}{1-x}\right)\left(\frac{1+y}{1-y}\right)\left(\frac{1+z}{1-z}\right) = \frac{b}{c}\times\frac{c}{a}\times\frac{a}{b} = 1$

$\Rightarrow \quad (1+x)(1+y)(1+z)$

$= (1-x)(1-y)(1-z)$

$\Rightarrow \quad 1 + x + y + z + xy + yz + zx + xyz$

$= 1 - (x+y+z) + xy + yz + zx - xyz$

$\Rightarrow \quad x + y + z = -xyz$

27. **(d)** In ΔABC, sides a, b and c are in GP.

$\therefore \quad b^2 = ac \quad$ ....(i)

Given, largest angle exceeds the smallest angle by 60°.

$C - A = 60^\circ \quad$ ....(ii)

$\cos(C-A) = \cos 60^\circ$

$\cos C\cos A + \sin C\sin A = \frac{1}{2}$

We know, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$

$\Rightarrow \quad 2\cos C\cos A + 2\sin C\sin A = 1$

$\Rightarrow \quad \cos(C+A) + \cos(C-A) + 2k^2 ac = 1$

$\Rightarrow \quad \cos(\pi + B) + \cos 60^\circ + 2k^2b^2 = 1 \quad [\therefore ac = b^2]$

$\Rightarrow \quad -\cos B + \frac{1}{2} + 2\sin^2 B = 1 \quad [\therefore bk = \sin B]$

$\Rightarrow \quad -\cos B + 1 + 4\sin^2 B = 2$

$\Rightarrow \quad -2\cos B + 1 + 4(1-\cos^2 B) = 2$

$\Rightarrow \quad 4\cos^2 B + 2\cos B - 3 = 0$

$\therefore \quad \cos B = \frac{-2\pm\sqrt{4+48}}{2\times 4} = \frac{-2\pm 2\sqrt{13}}{8}$

$= \frac{\sqrt{13}-1}{4}$ or $\frac{-\sqrt{13}-1}{4}$

28. **(c)** Given, $\angle A = 90^\circ$

In ΔABC, we know that

$r_2 + r_3 = 4R\cos^2\frac{A}{2}$

$\Rightarrow \quad r_2 + r_3 = 4R\cos^2 45^\circ$

$\Rightarrow \quad r_2 + r_3 = 4R\times\frac{1}{2}$

$\Rightarrow \quad r_2 + r_3 = 2R$

$\therefore \quad \cos^{-1}\left(\frac{R}{r_2+r_3}\right) = \cos^{-1}\left(\frac{R}{2R}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$

29. **(d)** $\gamma = (1+\lambda-\mu)\hat{i} + (2-\lambda)\hat{j} + (3-2\lambda+2\mu)\hat{k}$

$= \hat{i} + \lambda\hat{i} - \mu\hat{i} + 2\hat{j} - \lambda\hat{j} + 3\hat{k} - 2\lambda\hat{k} + 2\mu\hat{k}$

$= (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} - 2\hat{k}) + \mu(-\hat{i} + 2\hat{k})$

Equation of plane is given by

$\{r - (\hat{i} + 2\hat{j} + 3\hat{k})\} + [(\hat{i} - \hat{j} - 2\hat{k})\times(-\hat{i} + 2\hat{k})] = 0$

$\Rightarrow \quad \{r-(\hat{i}+2\hat{j}+3\hat{k})\}.(-2\hat{i}-\hat{k})=0$

$\Rightarrow \quad r(2\hat{i}+\hat{k})-5=0$

$\Rightarrow \quad r(2\hat{i}+\hat{k})=5$

Now, equation of plane in cartesian form

$(x\hat{i}+y\hat{j}+z\hat{k}).(2\hat{j}+\hat{k})=5$

$\therefore \quad 2x+z=5$

30. **(b)** For three vectors p, q and r,

$r=3p+4q \quad$ ... (i)

$2r=p-3q \quad$ ... (ii)

From eqs. (i) and (ii), we get

$r=3(2r+3q)+4q$

$r=6r+9q+4q$

$\Rightarrow \quad 5r=-13q \Rightarrow \quad r=\frac{13}{5}q \quad \Rightarrow \quad |r|=\frac{13}{5}|q|$

$\therefore \quad r$ and q have opposite directions

Hence, $|r|>2|q|$

31. **(a)** We have, $a=2\hat{i}+3\hat{j}-5\hat{k}$

$b=m\hat{i}+n\hat{j}+12\hat{k}$

Since, $a\times b=0 \qquad \Rightarrow a \parallel b$

$\therefore \quad \frac{2}{m}=\frac{3}{n}=\frac{-5}{12}$

$\Rightarrow m=-\frac{24}{5}$ and $n=-\frac{36}{5}$

Thus, $(m,n)=\left(-\frac{24}{5},-\frac{36}{5}\right)$

32. **(d)** We have, $|a|=3$, $|b|=4$

$\therefore \quad |4a+3b|^2=(4a+3b)(4a+3b)$

$= \quad 16|a|^2+9|b|^2+24a.b$

$=16|a|^2+9|b|^2+24|a|.|b|\cos\theta$

$=16(3)^2+9(4)^2+24\times 3\times 4\times\cos 120°$

$=16\times 9+9\times 16+24\times 12\times\left(\frac{-1}{2}\right)=144$

So, $|4a+3b|=\sqrt{144} \quad =12$

33. **(a)** Here, a, b and c are non-zero vectors $C\perp a \Rightarrow c.a=0$ and θ is the angle between the vectors b and c.

$(a\times b)\times c=\frac{1}{3}|b|\,|c|\,a$

$(c.a)\,b-(c.b)\,a=\frac{1}{3}|b|\,|c|\,a$

$\Rightarrow \quad 0-(|c|\,|b|\cos\theta)\,a=\frac{1}{3}|b|\,|c|\,a$

$\Rightarrow \quad \cos\theta=-\frac{1}{3} \qquad \Rightarrow \cos^2\theta=-\frac{1}{9}$

$\Rightarrow \quad 1-\sin^2\theta=\frac{1}{9} \quad \Rightarrow \sin^2\theta=1-\frac{1}{9}=\frac{8}{9}$

$\therefore \quad \sin\theta=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$

34. **(c)** Given that,

$a(\alpha\times\beta)+b(\beta\times\gamma)+c(\gamma\times\alpha)=0$

On taking dot product of α with the given cross product, we get

$a\alpha.(\alpha\times\beta)+b\alpha.(\beta\times\gamma)+c\alpha.(\gamma\times\alpha)=0$

$a[\alpha\ \alpha\ \beta]+b(\alpha\ \beta\ \gamma)+c(\alpha\ \gamma\ \alpha)=0$

$\therefore \quad [x\ x\ y]=[x\ y\ x]=[y\ x\ x]=0$

$\Rightarrow \quad b[\alpha\ \alpha\ \beta]=0$

Since, $\quad b\neq 0$

$\therefore \quad [\alpha\ \alpha\ \beta]=0$

Hence, α, β, γ are coplanar.

35. **(c)** Given, number of observation, $n=10$,

Mean, $\bar{x}=50$ and $\sum|xi-\bar{x}|^2=250$

Variance, $\sigma^2=\sum_{i-1}^{n}\frac{|x_i-\bar{x}|^2}{n}$

$\Rightarrow \sigma^2=\sum_{i-1}^{10}\frac{\left|x_i-\bar{x}^2\right|}{10}$

$=\frac{250}{10}=25 \quad \Rightarrow \sigma=5$

$\therefore$ Cofficient of variation,

C.V. $=\frac{\sigma}{x}\times 100=\frac{5}{50}\times 100=10$

36. **(b)** Given, 50 even natural numbers are 2, 4, 6, ... 100

Mean, $\bar{x}=\frac{\sum xi}{n}=\frac{2+4+6+\ ....+98+100}{50}$

$\bar{x}=\frac{2(1+2+3+\ ....+49+50)}{50}$

$\bar{x}=\frac{2\times 50\times 51}{2\times 50}=51$

Variance, $\sigma^2=\frac{\sum x_1^2}{n}-(\bar{x})^2$

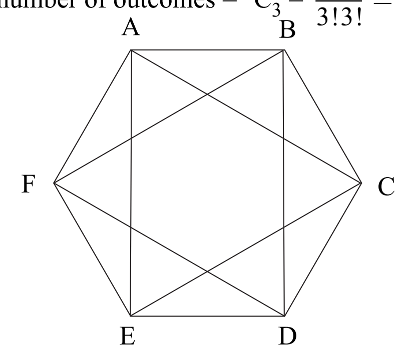
$=\frac{(2^2+4^2+6^2+....+98^2+100^2)}{50}-(51)^2$

$=\frac{2^2(1^2+2^2+3^2+....+49^2+50^2)-(51)^2\times 50}{50}$

$=\frac{4(50)(51)(101)-(51)\times(51)\times 50\times 6}{6\times 50}=833$

37. **(c)** Give that, 3 out of 6 vertices of hexagon are chosen.

Total number of outcomes $={}^5C_3=\frac{6!}{3!3!}=20$



Only two equilateral triangles are possible of regular hexagon, i.e. ΔAEC and ΔBFD.

$\therefore$ Probability $= \dfrac{2}{20} = \dfrac{1}{10}$

38. **(a)** Let Event A: A speaks the truth

Event B: B speaks the truth

$P(A) = 75\% = \dfrac{75}{100} = \dfrac{3}{4}$

and $P(B) = 80\% = \dfrac{80}{100} = \dfrac{4}{5}$

$\therefore$ Required probability

$$P\left(A\overline{B}\right) + P\left(\overline{A}B\right) = P(A) \times P\left(\overline{B}\right) + P\left(\overline{A}\right) \times P(B)$$

$$= P(A) \times [1 - P(B)] + P(A) \times P(B)$$

$$= \frac{3}{4} \times \left(1 - \frac{4}{5}\right) + \left(1 - \frac{3}{4}\right) \times \frac{4}{5}$$

$$= \frac{3}{4} \times \frac{1}{5} + \frac{1}{4} \times \frac{4}{5} = \frac{7}{20}$$

39. **(d)** For a binomial distribution, Mean, $\mu = n\rho = 4$ ... (i)

Variance, $\sigma^2 = npq = 2$ ... (ii)

On solving eqs. (i) and (ii), we get

$n = 8$, $p = \dfrac{1}{2}$, $q = \dfrac{1}{2}$

Probability of getting 2 successes,

$$p\,(X = 2) = {}^{8}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^6 = \frac{{}^{8}C_2}{2^8} = \frac{28}{256} = \frac{7}{64}$$

40. **(a)** For poisson distribution,

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

Where λ = mean of distribution = $n\rho$

k = probability of success

Here, 10 accidents take place in 50 days.

So, $p = \dfrac{10}{50} = \dfrac{1}{5}$ and n = 1

$\therefore \lambda = 1 \times \dfrac{1}{5} = 0.2$

Probability that three or more accidents occur in a day,

$P(x \geq 3) = P(x = 3) = P(x = 4) + ...$

$$= \sum_{k=3}^{\lambda} \frac{e^{-\lambda}\lambda^k}{k!}, \ \lambda = 0.2$$

41. **(a)** Let A, B ad C the vertices of triangle

$A = (a \cos k, a \sin k)$

$B = (b \sin k - b \cos k)$

$C = (1, 0)$

Let $G\,(x, y)$ be the centroid,

$\therefore \quad x = \dfrac{a\cos k + b\sin k + 1}{3}$

$\Rightarrow \quad 3x - 1 = a \cos k + b \sin k$ ....(i)

and $y = \dfrac{a\sin k - b\cos k + 0}{3}$

and $3y = a \sin k - b \cos k$ ....(ii)

On squaring and then adding Eqs. (i) and (ii) we get

$(3x - 1)^2 + 3y^2 = a^2(\sin^2 k + \cos^2 k) + b^2(\sin^2 k + \cos^2 k)$

$\therefore \quad (3x - 1)^2 + 9y^2 = a^2 + b^2$

$\therefore \quad (1 - 3x)^2 + 9y^2 = a^2 + b^2$

42. **(c)** Given equation, $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$

Coordinate axes are rotated through an angle of $\dfrac{\pi}{6}$ about the origin

Now, axis is $x \cos \dfrac{\pi}{6} \quad y \cos \dfrac{\pi}{6}$

and $x \cos \dfrac{\pi}{6} + y \cos \dfrac{\pi}{6}$

Thus, equation becomes $\sqrt{3}\left(x\cos\dfrac{\pi}{6} - y\cos\dfrac{\pi}{6}\right)^2$

$$-4\left(x\cos\frac{\pi}{6} - y\cos\frac{\pi}{6}\right)$$

$$\left(x\cos\frac{\pi}{6} + y\cos\frac{\pi}{6}\right) + \sqrt{3}\left(x\cos\frac{\pi}{6} + y\cos\frac{\pi}{6}\right)^2 = 0$$

$$\Rightarrow \sqrt{3}\left(\frac{\sqrt{3}x}{2} - \frac{1}{2}y\right)^2 - 4\left(\frac{\sqrt{3}x - y}{2}\right)$$

$$\left(\frac{x + \sqrt{3}y}{2}\right) + \sqrt{3}\left(\frac{x + \sqrt{3}y}{2}\right)^2 = 0$$

$$\Rightarrow \sqrt{3}\left(3x^2 - 2\sqrt{3}xy + y^2\right) - 4\left(\sqrt{3}x^2 + 3xy - xy - \sqrt{3}y^2\right)$$

$$+\sqrt{3}\left(x^2 + 2\sqrt{3}xy + 3y^2\right) = 0$$

$$\Rightarrow \quad 3\sqrt{3}x^2 - 6xy + \sqrt{3}y^2 - 4\sqrt{3}x^2 - 12xy$$

$$+4xy + 4\sqrt{3}y^2 + \sqrt{3}x^2 + 6xy + 3\sqrt{3}y^2 = 0$$

$$\Rightarrow \quad 8\sqrt{3}y^2 - 8xy = 0$$

$$\therefore \quad \sqrt{3}y^2 - xy = 0$$

43. **(d)** Given that, $x + 3y - 9 = 0$, $\quad 4x + by - 2 = 0$

$2x - y - 4 = 0$ are concurrent.

$$\therefore \quad \begin{vmatrix} 1 & 3 & -9 \\ 4 & b & -2 \\ 2 & -1 & -4 \end{vmatrix} = 0$$

$1\,(-4b - 2) - 3\,(-16 + 4) - 9\,(-4 - 2b) = 0$

$-4b - 2 + 36 + 36 + 18b = 0$

$14b = -70 \quad \Rightarrow b = -5$

From equations $x + 3y - 9 = 0$ and $2x - y - 4 = 0$

$\Rightarrow$ $x = 3$ and $y = 2$

$\therefore$ Concurrency point is (3, 2)

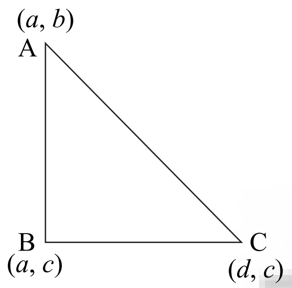
Equation of line passing through (–5, 0) and (3, 2) is

$$y - 0 = \frac{2-0}{3+5}(x+5)$$

$$y = \frac{1}{4}(x+5) \quad \Rightarrow 4y = x + 5$$

$\Rightarrow$ $x - 4y + 5 = 0$

44. **(a)** The vertices of the triangle are given as A(a, b), B(a, c) and C (d,c)



Centroid of ΔABC = $\left(\frac{a+a+d}{3}, \frac{b+c+c}{3}\right)$

$\equiv \left(\frac{2a+d}{3}, \frac{2c+b}{3}\right)$

Since, x-coordinate of point A and B are same. Also y-coordinate of point B and C are same so it forms a right angle triangle.

$\therefore$ Orthocentre of ΔABC is (a, c)

Mid-point of centroid and orthocentre is

$$= \left(\frac{\frac{2a+d}{3}+a}{2}, \frac{\frac{2c+b}{3}+c}{2}\right) = \left(\frac{5a+d}{6}, \frac{5c+d}{6}\right)$$

45. **(c)** As we know that the image of (1, 1) with respect to line $x + y + 5 = 0$ is

$$\frac{x-1}{1} = \frac{y-1}{1} = \frac{2(1+1+5)}{1+1}$$

$\Rightarrow$ $x - 1 = -7, y - 1 = -7$

$\Rightarrow$ $x = -6, y = -6$

$\therefore$ Image of point (1, 1) is (–6, –6)

Now, distance from origin

This is the required transformed equation.

$$D = \sqrt{(0+6)^2 + (0+6)^2}$$

$$D = \sqrt{72} = 6\sqrt{2}$$

46. **(d)** Given, $x^2 + y^2 = 9$ and $x + y = 3$

Equation of pair of lines.

$$x^2 + y^2 = 9\left(\frac{x+y}{3}\right)^2$$

$\Rightarrow$ $x^2 + y^2 = (x + y)^2$

$\Rightarrow$ $x^2 + y^2 = x^2 + y^2 + 2xy$

$\therefore$ $xy = 0$

47. **(a)** Given that, $x + y = 1$

and $2y^2 - xy - 6x^2 = 0$

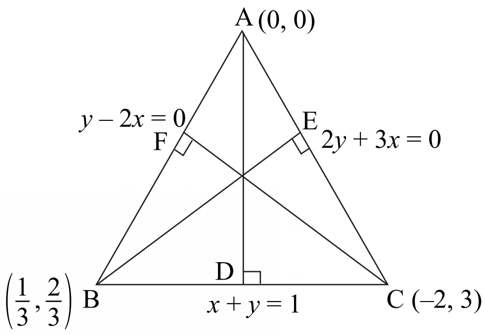
$\Rightarrow$ $2y^2 - 4xy + 3xy - 6x^2 = 0$

$\Rightarrow$ $(2y - 3x) + (y - 2x) = 0$

$\Rightarrow$ $2y + 3x$ and $y - 2x = 0$

$\therefore$ Equation of sides are

$x + y = 1$, $2y + 3x = 0$ and $y - 2x = 0$



Solving these equations simultaneously, we get the coordinate of the points A(0, 0), B $\left(\frac{1}{3}, \frac{2}{3}\right)$ and C(–2, 3)

Equation of altitude AD,

$x - y = 0$ ....(i)

Equation of altitude CF,

$x + 2y = \lambda$ ....(ii)

Since, this passes through (–2, 3)

$\therefore$ $-2 + 6 = \lambda$ $\Rightarrow \lambda = 4$

So, equation of altitude CF

$x + 2y = 4$

On solving eqs. (i) and (ii), we get

$x = \frac{4}{3}, y = \frac{4}{3}$

$\therefore$ Orthocentre of the ΔABC is $\left(\frac{4}{3}, \frac{4}{3}\right)$

48. **(b)** Given that,

$2x^2 - 3xy - 2y^2 + 10x + 5y = 0$

$(2x + y)(x - 2y + 5) = 0$

$2x + y = 0$ and $x - 2y + 5 = 0$

Now, equation of line passing through origin is

$2x^2 + y = 0$ $\Rightarrow m_1 = -2$

Since, this line is perpendicular to the line

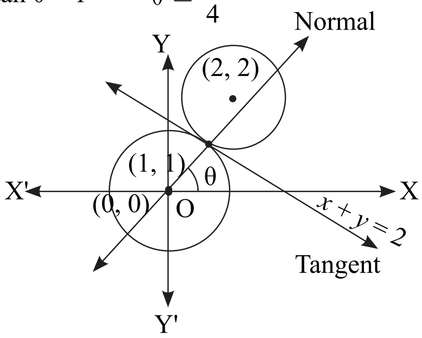
$kx + y + 3 = 0$ $\Rightarrow m_2 = -k$

$\therefore$ $m_1 \times m_2 = -1$

$\therefore$ $(-2) + (-k) = -1$ $\Rightarrow k = -\frac{1}{2}$

49. **(c)** Equation of line at (1, 1) is $x + y - 2 = 0$
Slope of this line is –1.
So, slope of line perpendicular to this line is 1.

$\therefore \quad \tan\theta = 1 \qquad \theta = \frac{\pi}{4}$



Let, centre of circle (h, k)
i.e. $x = h \pm r\cos\theta$ and $y = k \pm r\sin\theta$
As, it passes through (1, 1).

$\therefore \quad h = 1 \pm \sqrt{2}\cos\frac{\pi}{4}$

$k = 1 \pm \sqrt{2}\sin\frac{\pi}{4}$

$\Rightarrow \quad h = 1 \pm \frac{\sqrt{2}}{\sqrt{2}}$ and $k = 1 \pm \frac{\sqrt{2}}{\sqrt{2}}$

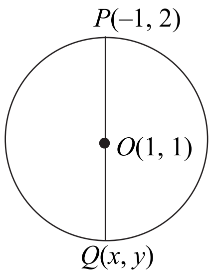
$\Rightarrow \quad h = 2, 0$ and $k = 2, 0$

$\therefore$ Centres are (2, 2) or (0, 0)
Hence, equation of circle
$x^2 + y^2 = 2$
or $(x - 2)^2 + (y - 2)^2 = 2$

$\therefore$ Length of tangent from (1, 2)

$= \sqrt{1^2 + 2^2 - 2} = \sqrt{3}$

50. **(a)** Equation of circle,
$x^2 + y^2 + 2x - 2y - 3 = 0$



Its centre is (1, 1)

Given that,

$\therefore$ PQ being normal to the circle represent the diameter of circle and O is the mid-point of PQ.

$\therefore \quad \frac{x-1}{2} = 1, \frac{y+2}{2} = 1$

$x = 3, y = 0$

So, coordinate of Q = (3, 0)

51. **(b)** The lines $kx + 2y - 4 = 0$ and $5x - 2y - 4 = 0$ are conjugate with respect to the circle

$x^2 + y^2 - 2x - 2y + 1 = 0$

Then, necessary condition

$r^2(a_1a_2 + b_1b_2) = (a_1g + b_1f - c_1)$

$(a_2g + b_2f - c_2) \qquad$ ... (i)

From the above equation,

$a_1 = k, \quad b_1 = 2 \quad c_1 = -4$

$a_2 = 5, \quad b_2 = -2 \quad c_2 = -4$

$g = -1, \quad f = -1 \quad c = 1$

$r^2 = g^2 + f^2 - c = 1 + 1 - 1 = 1$

Now, by using equation (i)

$1(5k - 1) = (-k - 2 + 1)(-5 + 2 + 4)$

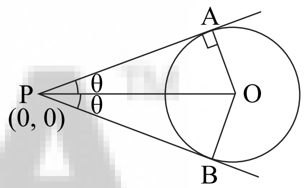
$5k - 4 = -k + 2$

$6k = 6$

$\Rightarrow \quad k = 1$

52. **(c)** We have,

$x^2 + y^2 + 4x - 6y + 4 = 0$



Centre ≡ $(-g, -f) \equiv (-2, 3)$

Radius of circle, OA = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{(2)^2 + (-3)^2 - 4}$

$= \sqrt{4 + 9 - 4} = 3$

Length of tangent

PA = PB

$= \sqrt{(0)^2 + (0)^2 + 4(0) - 6(0) + 4}$

$= \sqrt{4} = 2$

In ΔOAP, $\tan\theta = \frac{OA}{PA} = \frac{3}{2}$

Now, the angle between the tangent drawn from origin

$2\theta = \tan^{-1}\left(\frac{2\tan\theta}{1 - \tan^2\theta}\right)$

$2\theta = \tan^{-1}\left(\frac{2 \times \frac{3}{2}}{1 - \frac{9}{4}}\right)$

$\Rightarrow \quad 2\theta = \tan^{-1}\left(\frac{-12}{5}\right)$

53. **(d)** We have,

C_1: $x^2 + y^2 - 2x - 4y + c = 0$ ....(i)

C_2: $x^2 + y^2 - 4x - 2y + 4 = 0$ ....(ii)

So, centre of circle $C_1 = (1, 2)$, $r_1 = \sqrt{5-c}$

and centre of circle $C_2 = (2, 1)$, $r_2 = 1$

Now, angle between the two circles,

$$\cos\theta = \frac{(C_1C_2)^2 - (r_1^2 + r_2^2)}{2r_1r_2}$$

$$\cos\theta = \frac{\left[(2-1)^2 + (1-2)^2\right] - (5-c+1)}{2\sqrt{5-c}(1)}$$

Given, $\theta = 60°$

$$\cos 60° = \frac{1+1-5+c-1}{2\sqrt{5-c}}$$

$$\frac{1}{2} = \frac{c-4}{2\sqrt{5-c}}$$

$$\sqrt{5-c} = c-4$$

On squaring both sides, we get

$5 - c = c^2 - 8c + 16$

$c^2 - 5c + 11 = 0$

$$c = \frac{7 \pm \sqrt{49-44}}{2}$$

$$c = \frac{7 \pm \sqrt{5}}{2}$$

54. **(a)** Let the circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

$x^2 + y^2 - 4x - 2y + 4 = 0$

$x^2 + y^2 - 2x - 4y + 1 = 0$

and $x^2 + y^2 + 4x + 2y + 1 = 0$, respectively.

$\therefore$ $2g(2) + 2f(1) = c + 4$

$\Rightarrow$ $4g + 2f = c + 4$ ... (i)

$2g(1) + 2f(2) = c + 1$

$\Rightarrow$ $2g + 4f = c + 1$ ... (ii)

and $2g(-2) + 2f(-1) = 1 + c$

$\Rightarrow$ $-4g - 2f = 1 + c$ ... (iii)

On solving eqs. (i), (ii) and (iii), we get

$$g = \frac{3}{4}, f = -\frac{3}{4}, c = -\frac{10}{4}$$

$\therefore$ Equation of circle,

$$S = x^2 + y^2 + \frac{3}{2}x - \frac{3}{2}y - \frac{10}{4} = 0$$

$$\therefore \quad \text{Radius} = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{-3}{4}\right)^2 + \frac{10}{4}}$$

$$= \sqrt{\frac{9}{16} + \frac{9}{16} + \frac{10}{4}} = \sqrt{\frac{58}{16}} = \sqrt{\frac{29}{8}}$$

55. **(b)** Given equation of parabola

$x^2 - 2x + 3y - 2 = 0$

$\Rightarrow$ $x^2 - 2x + 1 = -3y + 2 + 1$

$= -3y + 3$

$\Rightarrow$ $(x-1)^2 = -3(y-1)$

Distance between the vertex and focus of parabola

$= \frac{1}{4} \times$ Length of latusrectum

$= \frac{1}{4} \times 4|a| = \left|\frac{-3}{4}\right| = \frac{3}{4}$

56. **(c)** Given parabola $y^2 = 5x$

Let (x_1, y_1) and (x_2, y_2) be the end points of a focal chord.

So, $x_1 = \frac{5}{4}t_1^2$ and $y_1 = \frac{5}{4}t_1$

Since, $t_1t_2 = -1$ as t_1 and t_2 are the end points of a focal chord

$\therefore$ $x_2 = \frac{5}{4}\left(\frac{1}{t_1^2}\right)$ and $y_2 = \frac{5}{2}\left(\frac{-1}{t_1}\right)$

Now, $x_1x_2 = \frac{25}{16}$

and $y_1y_2 = -\frac{25}{4}$

$\therefore$ $4x_1x_2 + y_1y_2 = 4\left(\frac{25}{16}\right) - \frac{25}{4}$

$\frac{25}{4} - \frac{25}{4} = 0$

57. **(a)** Given that, $x = 3\cos\theta$

$\Rightarrow$ $\frac{x}{3} = \cos\theta$ ....(i)

and $y = 4\sin\theta$

$\Rightarrow$ $\frac{y}{4} = \sin\theta$ ....(ii)

On squaring and adding eqs. (i) and (ii) we get

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = \cos^2\theta + \sin^2\theta$$

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

$\therefore$ Distance between their foci,

$f_1f_2 = 2\sqrt{b^2 - a^2} = 2\sqrt{16-9} = 2\sqrt{7}$

58. **(b)** We have ,

$\Rightarrow$ $9x^2 + 25y^2 - 36x + 50y - 164 = 0$

$\Rightarrow$ $9x^2 - 36x + 36 + + 25y^2 + 50y - 25 = 164 + 36 + 25$

$\Rightarrow$ $9(x^2 - 4x + 4) - 25(y^2 + 2y - 1) = 225$

$\Rightarrow$ $9(x-2)^2 - 25(y+1)^2 = 225$

$\Rightarrow \quad \frac{(x-2)^2}{25}+\frac{(y+1)^2}{9}=1$

Eccentricity e $=\sqrt{1-\left(\frac{b}{a}\right)^2}$

$=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

Equation of the latusrectum,
$x-2=\pm ae$

$\Rightarrow \quad x-2=\pm 5\times\frac{4}{5}=\pm 4$

$\Rightarrow \quad x=\pm 4+2$
$\Rightarrow \quad x=6$ and $x=-2$
$\therefore \quad x-6=0,\ x+2=0$

59. **(b)** We have, line, $y=mx+2$ ... (i)
Hyperbola,
$4x^2-9y^2=36$ ....(ii)
On solving eqs. (i) and (ii), we get
$x^2(4-9m^2)-36mx-72=0$
Since, this line is a tangent of hyperbola
$\therefore \quad D=b^2-4ac=0$
$\therefore \quad (36m)^2+4\times 72(4-9m^2)=0$
$\Rightarrow \quad 36\times 36m^2+4\times 36\times 2(4-9m^2)=0$
$\Rightarrow \quad 9m^2+8-18m^2=0$
$\Rightarrow \quad 9m^2=8$
$\Rightarrow \quad m^2=\frac{8}{9}$
$\therefore \quad m^2=\pm\sqrt{\frac{8}{9}}=\pm\frac{2\sqrt{2}}{3}$

60. **(b)** Let we points P (2, 3, 4), A (3, –2, 2) and B (6, –17, –4).
and P divides AB in the ratio $k:1$ then

$(2, 3, 4)=\left(\frac{6k+3}{k+1},\frac{-17k-2}{k+1},\frac{-4k+2}{k+1}\right)$

$2=\frac{6k+3}{k+1}$
$\Rightarrow \quad 2k+2=6k+3$

$\Rightarrow \quad -4k=1 \quad \Rightarrow \quad k=-\frac{1}{4}$

Harmonic conjugate Q divides in the ratio $-k:1$, So Ratio will be $\frac{1}{4}:1$

$\therefore$ Coordinates of Q

$=\left(\frac{\frac{1}{4}(6)+3}{\frac{1}{4}+1},\frac{\frac{1}{4}(-17)-2}{\frac{1}{4}+1},\frac{\frac{1}{4}(-4)+2}{\frac{1}{4}+1}\right)$

$=\left(\frac{6+12}{5},\frac{-17-8}{5},\frac{-4+8}{5}\right)$

$=\left(\frac{18}{5},-5\frac{4}{5}\right)$

61. **(b)** Given that, line makes angles α, β, γ, δ with the four diagonals of a cube, then we know that

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$
Since, $\cos^2\theta=1-\sin^2\theta$

$1-\sin^2\alpha+1-\sin^2\beta+1-\sin^2\gamma+1-\sin^2\delta=\frac{4}{3}$

$\therefore \quad \sin^2\alpha+\sin^2\beta+\sin^2\gamma+\sin^2\delta=4-\frac{4}{3}=\frac{8}{3}$

62. **(c)** Given that equation of plane,

$56x+4y+9z=2016$

$\frac{x}{\frac{2016}{56}}+\frac{y}{\frac{2016}{4}}+\frac{z}{\frac{2016}{9}}=1$

Also given, this plane meets the coordinate axes at points A, B and C.

Coordinate of A $=\left(\frac{2016}{56},0,0\right)$

Coordinate of B $=\left(0,\frac{2016}{4},0\right)$

Coordinate of C $=\left(0,0,\frac{2016}{9}\right)$

Now, centroid of ΔABC

$G=\left(\frac{2016}{56\times 3},\frac{2016}{4\times 3},\frac{2016}{9\times 3}\right)=\left(12,168,\frac{224}{3}\right)$

63. **(b)** Given function,

$$f(x)=\begin{cases}1-x, & x<1\\ (1-x)(2-x), & 1\le x\le 2\\ 3-x & x>2\end{cases}$$

For $f(x)$ to be continuous at $x=1$.
$\lim_{x\to 1^+}f(x)=\lim_{x\to 1^+}f(x)=f(1)$
$\Rightarrow \quad \lim_{x\to 1^+}(1-x)=\lim_{x\to 1^+}(1-x)(2-x)$
$(1-1)=(1-1)(2-1)=0$
and $f(1)=(1-1)(2-1)=0$
$\therefore$ $f(x)$ is continuous at $x=1$.
Now, at $x=2$, $\lim_{x\to 2^-}f(x)=\lim_{x\to 2^-}f(x)=f(2)$
$\Rightarrow \lim_{x\to 2^-}(1-x)(2-x)=\lim_{x\to 2^+}(3-x)$
$\Rightarrow \quad (1-2)(2-2)=(3-2)$
$\Rightarrow \quad 0\neq 1$
$\therefore$ $f(x)$ is discontinuous at $x=2$.

64. **(a)** Let,

$l=\lim_{x\to 0}\frac{6^x-3^x-2^x+1}{x^2}=\lim_{x\to 0}\frac{3^x.2^x-3^x-2^x+1}{x^2}$

$$= \lim_{x\to 0}\frac{3^x\left(2^x-1\right)-1\left(2^x-1\right)}{x^2}$$

$$= \lim_{x\to 0}\frac{\left(2^x-1\right)\left(3^x-1\right)}{x\times x}$$

$$= \lim_{x\to 0}\frac{2^x-1}{x}\times\lim_{x\to 0}\frac{3^x-1}{x}$$

$= (\log_a 2)\ (\log_e 3)$

65. **(a)** Given function,

$$f(x)=\begin{cases}x^2+bx+c, & x<1\\ x, & x\ge 1\end{cases}$$

$$f'(x)=\begin{cases}2x+b, & x<1\\ 1, & x\ge 1\end{cases}$$

Since, $f(x)$ is differentiable at $x = 1$.

$\lim_{x\to 1^+} f'(x) = \lim_{x\to 1^+} f'(x)$

$\lim_{x\to 1^+} (2x+b) = \lim_{x\to 1^+} 1$

$2 + b = 1 \Rightarrow b = -1$

As form is differentiable at $n = 1$ So, it will be continuous at $x = 1$ also.

$\Rightarrow\ \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} f'(x) = f(1)$

$\Rightarrow\ \lim_{x\to 1^+} x^2+bx+c = \lim_{x\to 1^+} x = 1$

$\Rightarrow\ 1 + b + c = 1$

$\Rightarrow\ 1 - 1 + c = 1 \quad \Rightarrow c = 1$

Hence, $b - c = -1 - 1 = -2$

66. **(d)** Given that, $x = a$ is a root of multiplicity two of a polynomial equation $f(x) = 0$.

Let $f(x) = (x-a)\,g(x)$

On differentiating w.r.t. x, we get

$\Rightarrow\ f'(x) = 2(x-a)\,g(x) + (x-a)^2\,g'(x)$

Again, differentiating w.r.t x, we get

Now, $f''(x) = 2g(x) + 2(x-a)g'(x) + 2(x-a)g'(x) + (x-a)^2 g''(x)$

$= 2g(x) + 4(x-a)g'(x) + (x-a)^2g''(x)$

At $x = a$,

$\Rightarrow\ f'(a) = 2(a-a)\,g(a) + (a-a)^2\,g'(a) = 0$

$\Rightarrow\ f''(a) = 2g(a) + 4(a-a)g'(a) + (a-a)^2\,g''(a)$

$= 2g(a)$

Hence, $f(a) = f'(a) = 0, f''(a) \ne 0$

67. **(c)** We have, $y = \log_2(\log_2 x)$

Since, $\log_a^b = \dfrac{\log b}{\log a}$

$$\therefore\quad y = \log_2\left(\frac{\log x}{\log 2}\right)$$

$$\therefore\quad y = \frac{\log\dfrac{\log x}{\log 2}}{\log 2}$$

$$\Rightarrow\quad y = \frac{\log(\log x)-\log(\log 2)}{\log 2}$$

On differentiating w.r.f x, we get

$$\therefore\quad \frac{dy}{dx} = \frac{1}{\log 2}\left[\frac{1}{\log_e x}\times\frac{1}{x}-0\right] = \frac{1}{\log 2.\log_e x.x}$$

$$\frac{dy}{dx} = \frac{1}{(x\log_e x)\log_e 2}$$

68. **(b)** Given curves, $y^2 + x^2 = a^2\sqrt{2}$ ....(i)

and $x^2 - y^2 = a^2$ ....(ii)

On solving eqs. (i) and (ii), we get the point of intersection

$$x = a\sqrt{\frac{\sqrt{2}+1}{2}},\ y = a\sqrt{\frac{\sqrt{2}-1}{2}}$$

Now, for curve -1: $y^2 + x^2 = a^2\sqrt{2}$

$$2y\left(\frac{dy}{dx}\right)_1 + 2x = 0 \quad\Rightarrow \left(\frac{dy}{dx}\right)_1 = -\frac{x}{y} = m_1$$

$$m_1 = \frac{-a\sqrt{\dfrac{\sqrt{2}+1}{2}}}{a\sqrt{\dfrac{\sqrt{2}-1}{2}}} = -\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}$$

and curve -2: $x^2 - y^2 = a^2$

On differentiating

$$\left(\frac{dy}{dx}\right)_2 = \frac{x}{y}$$

$$\therefore\quad m_2 = -\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}$$

Then, $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

$$= \frac{-\sqrt{\dfrac{\sqrt{2}+1}{\sqrt{2}-1}}-\sqrt{\dfrac{\sqrt{2}+1}{\sqrt{2}-1}}}{1+\left(-\sqrt{\dfrac{\sqrt{2}+1}{\sqrt{2}-1}}\right)\left(\sqrt{\dfrac{\sqrt{2}+1}{\sqrt{2}-1}}\right)}$$

$$= \frac{-2\sqrt{\dfrac{\sqrt{2}+1}{\sqrt{2}-1}}}{1-\dfrac{\sqrt{2}+1}{\sqrt{2}-1}}$$

$$\tan\theta = \frac{-2\sqrt{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)}}{\sqrt{2}-1-\sqrt{2}-1} = \frac{-2\sqrt{2-1}}{-2} = 1$$

$$\therefore\quad \theta = \frac{\pi}{4}$$

69. **(b)** Given, $A + B = \frac{\pi}{3}$

Let $y = \tan A \tan B$

$y = \tan A \tan\left(\frac{\pi}{3} - A\right)$

Differentiating w.r.t A, we get

$\Rightarrow \quad \frac{dy}{dA} = \sec^2 A \tan\left(\frac{\pi}{3} - A\right) - \sec^2\left(\frac{\pi}{3} - A\right)\tan A$

For maxima or minima, $\frac{dy}{dA} = 0$

$\therefore \quad \sec^2 A \tan\left(\frac{\pi}{3} - A\right) - \sec^2\left(\frac{\pi}{3} - A\right)\tan A = 0$

$\Rightarrow \quad \sec^2 A \tan\left(\frac{\pi}{3} - A\right) = \tan A \sec^2\left(\frac{\pi}{3} - A\right)$

$\Rightarrow \quad \left(1 + \tan^2 A\right)\tan\left(\frac{\pi}{3} - A\right)$

$= \tan A\left[1 + \tan^2 A\left(\frac{\pi}{3} - A\right)\right]$

$\Rightarrow \quad \tan\left(\frac{\pi}{3} - A\right) + \tan^2 A \tan\left(\frac{\pi}{3} - A\right)$

$= \tan A + \tan A \tan^2\left(\frac{\pi}{3} - A\right)$

$\Rightarrow \quad \left[\tan\left(\frac{\pi}{3} - A\right) - \tan A\right]$

$\left[1 - \tan A \tan\left(\frac{\pi}{3} - A\right)\right] = 0$

$\Rightarrow \quad \tan\left(\frac{\pi}{3} - A\right) = \tan A$

$\Rightarrow \quad \frac{\neq}{2} - A = A \quad \Rightarrow \quad 2A = \frac{\neq}{2}$

$\Rightarrow \quad A = \frac{\pi}{6}$

$\therefore \quad A = B = \frac{\pi}{6}$

Maximum value of $\tan A \tan B = \tan\frac{\neq}{2}\ \tan\frac{\pi}{6}$

$= \left(\tan\frac{\pi}{6}\right)^2$

$= \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$

70. **(c)** Let $P\left(t, \frac{-1}{t}\right)$ be a point on $xy = -1$

Equation of tangent of P

$= \frac{1}{2}\left[x\left(\frac{-1}{t}\right) + xy\right] = -1 \quad -x + t^2y = -2t$

$y = \frac{x}{t^2} + \frac{2}{t}$

This is also at tangent of $y^2 = 8x$

i.e. $c = \frac{a}{m}$

$\frac{2}{t} = \frac{2}{\frac{1}{t^2}}$

$\Rightarrow \quad t^3 = 1$

$\Rightarrow \quad t = 1$

Hence, the equation of common tangent

$y = \frac{x}{1} + \frac{2}{1}$

$y = x + 2$

71. **(a)** We have,

$f(x) = x(x + 3)(x - 2), x \in [-1, 4]$

$= x(x^2 + x - 6)$

$\Rightarrow \quad f(x) = x^3 + x^2 - 6x$

Differentiating w.r.f x, we get

$\Rightarrow \quad f'(x) = 3x^3 + 2x - 6$

Given $f(c) = 10$

$\Rightarrow \ 3c^2 + 2c - 6 = 10$

$\Rightarrow \ 3c^2 + 2c - 16 = 0$

$\Rightarrow \ 3c^2 + 8c - 6c - 16 = 0$

$\Rightarrow \ (3c + 8)(c - 2) = 0$

$\therefore c = -\frac{8}{3}$ or $c = 2$

$c = 2 \in [-1, 4]$

72. **(d)** We have given that,

$\int x^3 e^{5x} dx = \frac{e^{5x}}{5^4}\left[\int(x)\right] + c$

By using the method of intergration by parts, we get

$\int x^3 e^{5x} dx = x^3 \int e^{5x} dx - \int\left\{\frac{dx^3}{dx}\int e^{5x} dx\right\} dx$

$= \frac{x^3 e^{5x}}{5} - \int \frac{3x^2 e^{5x}}{5} dx + C$

By using the method of integration by parts, we get

$= \frac{x^3 e^{5x}}{5} - \frac{3}{5}x^2 \int e^{5x} dx$

$+ \frac{3}{5}\int\left\{\frac{dx^2}{dx}\int e^{5x} dx\right\} dx + C$

$= \frac{x^3 e^{5x}}{5} - \frac{3x^3 e^{5x}}{25} + \frac{6}{25}\int x e^{5x} dx$

$= \frac{x^3 e^{5x}}{5} - \frac{3x^3 e^{5x}}{25} + \frac{6x}{25}\int e^{5x} dx$

$-\frac{6}{25}\int\left\{\frac{dx}{dx}\int e^{5x} dx\right\} dx$

$= \frac{x^3 e^{5x}}{5} - \frac{3x^3 e^{5x}}{25} + \frac{6x e^{5x}}{125} - \frac{6}{125}\int e^{5x} dx$

$= \frac{x^3e^{5x}}{5} - \frac{3x^3e^{5x}}{25} + \frac{6xe^{5x}}{125} - \frac{6}{125} + C$

$= \frac{e^{5x}}{625}[125x^3 - 75x^2 + 30x - 6] + C$

Comparing the above equation with

$\frac{e^{5x}}{5^4}\left[f(x)\right] + C,$ we get

$f(x) = 5^3x^3 - 75x^2 + 30x - 6$

73. **(c)** Let, $I = \int \frac{x}{\left(x^2+2x+2\right)^2}dx$

$I = \int \frac{x}{\left[(x+1)^2+1\right]^2}dx$

Put $x + 1 = \tan\theta$

$\Rightarrow \quad dx = \sec^2\theta \, d\theta$

$I = \int \frac{(\tan\theta - 1)\sec^2\theta}{\left(\tan^2\theta+1\right)^2}d\theta$

$= \int \frac{(\tan\theta - 1)\sec^2\theta}{\sec^4\theta}d\theta$

$= \int \left(\frac{\tan\theta}{\sec^2\theta} - \frac{1}{\sec^2\theta}\right)d\theta$

$= \int \left(\sin\theta\cos\theta - \cos^2\theta\right)d\theta$

$= \frac{1}{2}\int \left(2\sin\theta\cos\theta - 2\cos^2\theta\right)d\theta$

$= \frac{1}{2}\int (\sin 2\theta - 1 - \cos 2\theta)d\theta$

$= \frac{1}{2}\int \left(\frac{-\cos 2\theta}{2} - \theta - \frac{\sin 2\theta}{2}\right) + C$

$= -\frac{1}{4}[\cos 2\theta + \sin 2\theta] - \frac{1}{2}\theta + C$

$= -\frac{1}{4}\left[\frac{1-\tan^2\theta}{1-\tan^2\theta} + \frac{2\tan\theta}{1+\tan^2\theta}\right] - \frac{1}{2}\tan^{-1}(x+1) + C$

$= -\frac{1}{4}\left[\frac{1-\tan^2\theta + 2\tan\theta}{1+\tan^2\theta}\right] - \frac{1}{2}\tan^{-1}(x+1) + C$

$= -\frac{1}{4}\left[\frac{1-(x+1)^2 + 2(x+1)}{1+(x+1)^2}\right] - \frac{1}{2}\tan^{-1}(x+1) + C$

$\therefore I = -\frac{1}{4}\left[\frac{x^2-2}{x^2+2x+2}\right] - \frac{1}{2}\tan^{-1}(x+1) + C$

74. **(c)** Let $I = \int \log\left(a^2+x^2\right)dx$

By using method of integration by parts, we get

$= \log\left(a^2+x^2\right)\int dx - \int\left\{\frac{d\left(\log\left(a^2+x^2\right)\right)}{dx}\int dx\right\}dx + C$

$= \log\left(a^2+x^2\right)\int \frac{2x}{x^2+a^2}.x\,dx + C$

$= x\log\left(a^2+x^2\right) - 2\int \frac{x^2}{x^2+a^2}dx + C$

$= x\log\left(a^2+x^2\right) - 2\int \frac{x^2+a^2}{x^2+a^2}dx + 2\int \frac{a^2}{x^2+a^2}dx$

$= x\log\left(a^2+x^2\right) - \frac{2a^2}{a}\tan^{-1}\frac{x}{a} + C$

$I = x\log\left(a^2+x^2\right) - 2x + 2a\tan^{-1}\frac{x}{a} + C$

Comparing the above equation with $\int\log(x^2 + a^2)dx = h(x) + C$, we get

$h(x) = x\log\left(a^2+x^2\right) - 2x + 2a\tan^{-1}\left(\frac{x}{a}\right)$

75. **(a)** Given that,

$\int(\log x^2)dx = x[A(\log x)^5 + B(\log x)^4]$
$+ C(\log x)^3 + D(\log x)^2 + E(\log x) + F] + C$

Let $I = \int(\log x)^5 dx$

Put $\log x = t \quad \Rightarrow x = e^t \Rightarrow dx = e^t dt$

$I = \int t^5 e^5 dt$

$I = e^t[t^5 - 5t^4 + 20t^3 - 60t^2 + 120t - 120] + C$

Now, put E = $\log x$

$I = x[(\log x)^5 - 5(\log x)^4 + 20(\log x)^3 - 60(\log x)^2 + 120(\log x) - 120] + C$

$\therefore \quad A + B + C + D + E + F$

$= 1 - 5 + 20 - 60 + 120 - 120 = -44$

76. **(d)** Given curves,

$y = \frac{x^2}{4a}$ ... (i)

$y = \frac{8a^3}{x^2+4a^2}$ ... (ii)

For point of intersection equation (i) and (ii),

$\frac{x^2}{4a} = \frac{8a^3}{x^2+4a^2}$

$x^2(x^2 + 4a^2) = 4a(8a^3)$

$x^4 + 4a^2x^2 - 32a^2 = 0$

$(x^2 - 4a^2)(x^2 + 8a^2) = 0$

$x^2 - 4a^2 = 0$ or $x^2 + 8a^2 = 0$

$x^2 = 4a^2 \quad [x^2 = -8a^2$ is not possible$]$

$x = \pm 2a \quad \Rightarrow x = \pm 2a$

So, we take limits from 0 to $2a$.

Now, Area enclosed by the two curves

$A = 2 \times \int_0^{2a}\left[\frac{8a^3}{\left(x^2+4a^2\right)} - \frac{x^2}{4a}\right]dx$

$= 2 \times \left[\int_0^{2a}\frac{8a^3}{\left(x^2+4a^2\right)}dx - \int_0^{2a}\frac{x^2}{4a}dx\right]$

$$= 2\times\left[8a^3 \times \int_0^{2a} \frac{1}{x^2+\left(2a^2\right)}dx - \frac{1}{4a}\int_0^{2a} x^2 dx\right]$$

$$= 2\times\left\{8a^3\left[\frac{1}{2a}\tan^{-1}\left(\frac{x}{2a}\right)\right]_0^{2a} - \frac{1}{4a}\left[\frac{x^3}{3}\right]_0^{2a}\right\}$$

$$= 2\times\left\{\frac{8a^3}{2a}\left[\tan^{-1}\left(\frac{2a}{2a}\right)-\tan^{-1}\left(\frac{0}{2a}\right)\right] - \frac{1}{4a}\left[\frac{(2a)^3}{3}-\frac{(0)^3}{3}\right]\right\}$$

$$= 2\times\left\{4a^2\left[\tan^{-1}(1)-\tan^{-1}(0)\right]-\frac{1}{4a}\left(\frac{8a^3}{3}-0\right)\right\}$$

$$= 2\times\left\{4a^2\left(\frac{\pi}{4}-0\right)-\frac{1}{4a}\left(\frac{8a^3}{3}\right)\right\}$$

$$= 2\times\left\{4a^2\times\frac{\pi}{4}-\frac{1}{4a}\times\frac{8a^3}{3}\right\}$$

$$= 2\left\{a^2\left(\pi-\frac{2}{3}\right)\right\}$$

$\therefore \quad A = a^2\left(2\pi-\frac{4}{3}\right)$ sq. units

77. **(b)** Given that,

$$\lim_{n\to\infty}\left\{\frac{1}{\sqrt{n^2-1^2}}+\frac{1}{\sqrt{n^2-2^2}}+\ldots+\frac{1}{\sqrt{n^2-(n-1)^2}}\right\}$$

$$\lim_{n\to\infty}\left\{\frac{1}{n\sqrt{1-\left(\frac{1}{n}\right)^2}}+\frac{1}{n\sqrt{1-\left(\frac{2}{n}\right)^2}}+\ldots+\frac{1}{n\sqrt{1-\left(\frac{n-1}{n}\right)^2}}\right\}$$

$$\lim_{n\to\infty}\left\{\frac{1}{\sqrt{1-\left(\frac{1}{n}\right)^2}}+\frac{1}{\sqrt{1-\left(\frac{2}{n}\right)^2}}+\ldots+\frac{1}{\sqrt{1-\left(\frac{n-1}{n}\right)^2}}\right\}$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n-1}\frac{1}{n\sqrt{1-\left(\frac{1}{n}\right)^2}} = \int_0^1\frac{dx}{\sqrt{1-x^2}} = \left[\sin^{-1}x\right]_0^1$$

$$= \sin^{-1}1-\sin^{-1}0 = \frac{\pi}{2}$$

78. **(d)** Given, $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos^2 x}dx$

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos^2 x}dx + \frac{\pi}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{2-\cos^2 x}dx$$

Let, $I_1 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x}{2-\cos^2 x}dx$

$$f(x) = \frac{x}{2-\cos^2 x}$$

$$f(-x) = \frac{-x}{2-\cos 2(-x)} = \frac{-x}{2-\cos 2x} = -f(x)$$

$f(x)$ is an odd function.

Thus, $I_1 = 0$

Let $I_1 = \frac{\pi}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{2-\cos 2x}dx$

$$g(x) = \frac{\pi}{4(2-\cos 2x)}$$

$$g(-x) = \frac{\pi}{4(2-\cos 2(-x))} = \frac{\pi}{4(2-\cos 2x)} = g(x)$$

$\therefore \quad g(x)$ is an even function

Thus, $I_2 = 2\times\frac{\pi}{4}\int_0^{\frac{\pi}{4}}\frac{dx}{2-\cos 2x}$

Now, $I = I_2 = \frac{\pi}{4}\int_0^{\frac{\pi}{4}}\frac{dx}{2-\frac{1-\tan^2 x}{1+\tan^2 x}}$

$$I = \frac{\pi}{4}\int_0^{\frac{\pi}{4}}\frac{1+\tan^2 x}{2+\tan^2 x-1+\tan^2 x}dx$$

$$I = \frac{\pi}{4}\int_0^{\frac{\pi}{4}}\frac{\sec^2 x}{1+3\tan^2 x}dx$$

Put $\tan x = t \quad \Rightarrow \sec^2 x\, dx = dt$

Limits : $x = 0 \quad \Rightarrow t = 0$

$x = \frac{\pi}{4} \quad \Rightarrow t = 1$

$$\therefore \quad I = \frac{\pi}{4}\int_0^1\frac{dt}{1+3t^2}$$

$$I = \frac{\pi}{2}\times\frac{1}{\sqrt{3}}\left[\tan^{-1}\sqrt{3}t\right]_0^1$$

$$I = \frac{\pi}{2\sqrt{3}}\left[\tan^{-1}\left(\sqrt{3}\right)-\tan^{-1}(0)\right]$$

$$I = \frac{\pi}{2\sqrt{3}}\left(\frac{\pi}{3}\right) = \frac{\pi^2}{6\sqrt{3}}$$

79. **(c)** Given differential equation,

$$\left(1+y^2\right)\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$$

$$\frac{dy}{dx}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$$

$$P=\frac{x}{1+y^2},\ Q=\frac{e^{\tan^{-1}}y}{1+y^2}$$

Now, integration factor,

$$\text{IF}=e^{\int pdy}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$$

The solution of differential equation,

$$x.IF=\int Q.IFdy+C$$

$$\Rightarrow xe^{\tan^{-1}y}=\int\frac{e^{2\tan^{-1}y}}{1+y^2}dy+C$$

$$\Rightarrow xe^{\tan^{-1}y}=\frac{e^{2\tan^{-1}y}}{1+y^2}+C$$

$$\Rightarrow 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+C$$

80. **(d)** Given differential equation,

$$\left(2x-4y+3\right)\frac{dy}{dx}+\left(x-2y+1\right)=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{\left(x-2y+1\right)}{2\left(x-2y+3\right)}\quad \text{... (i)}$$

Let, $x-2y=v \quad \Rightarrow 1-2\frac{dy}{dx}=\frac{dy}{dx}$

$$\therefore\ \frac{1}{2}\left(1-\frac{dy}{dx}\right)=\frac{dy}{dx}$$

Now, substitute $v=x-2y$ and $\frac{dy}{dx}$ in eq. (i) integration both the sides, we get

$$\therefore\ \frac{1}{2}\left(1-\frac{dv}{dx}\right)=-\left(\frac{v+1}{2v+3}\right)$$

$$\Rightarrow \frac{dv}{dx}=1+\frac{2v+2}{2v+3}$$

$$\Rightarrow \frac{2v+2}{4v+5}dv=dx$$

$$\frac{1}{2}\int\frac{4v+6}{4v+5}dv=\int dx$$

$$\Rightarrow \frac{1}{2}\int\left(\frac{4v+6}{4v+5}+\frac{1}{4v+5}\right)dv=\int dx$$

$$\Rightarrow \frac{1}{2}v+\frac{1}{2\times 4}\log\left[4v+5\right]=x+C$$

$$\Rightarrow 4v+\log\left[4v+5\right]=8x+C$$

$$\Rightarrow 4(x-2y)+\log\left[4(x-2y)+5\right]=8x+C$$

$$\Rightarrow \log\left[4(x-2y)+5\right]=4(x+2y)+C$$

PHYSICS

81. **(c)** (A) The value of Boltzmann constant = 1.38×10^{-23} kg $m^2s^{-2}k^{-1}$

So dimensions of Boltzmann constant = $[ML^2T^{-2}K^{-1}]$

(B) From stokes' law, $F=6\pi\eta vr$

Coefficient of viscosity $\eta=\frac{F}{6\pi vr}$

Coefficient of viscosity = $[ML^{-1}T^{-1}]$

(C) Water equivalent (W) is the mass of water which would absorb or evolve the same amount of heat as in done by the body in rising or falling through the same range of temperature.

Water equivalent = $[ML^0T^0]$

(D) Since $\frac{Q}{t}=\frac{KA\left(\theta_1-\theta_2\right)}{l}$

$\therefore$ Coefficient of thermal conductivity

$$K=\frac{Q.l}{A\left(\theta_1-\theta_2\right)t}\ \text{so}$$

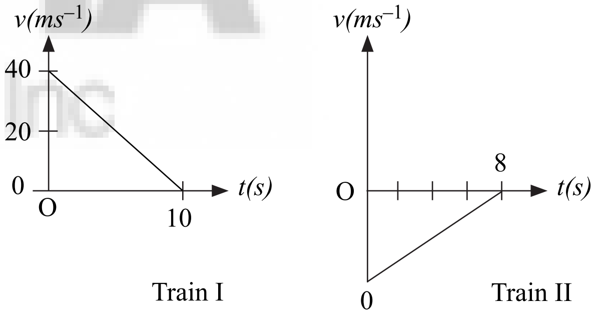
Coefficient of thermal conductivity = $[MLT^{-3}K^{-1}]$

82. **(b)** Given initial distance between trains = 300 m.

From given graph and as per question two trains are moving in opposite direction. So the separation between the trains when both have stopped

$$=300-\left(\frac{1}{2}\times10\times40+\frac{1}{2}\times9\times20\right)$$

$= 300 - 200 + 80 = 20$ m



Train I Train II

Given θ is the angle between the total acceleration and tangential acceleration.

83. **(d)** $\because\quad \tan\theta=\frac{a_{\text{radial}}}{a_{\text{tangential}}}$

$$=\frac{\frac{V^2}{R}}{a_{\text{tangential}}}$$

Givn $V=K\sqrt{s}$

$$\Rightarrow \tan\theta=\frac{1}{a_{\text{tangential}}}\left[\frac{1}{R}\times K^2s\right]$$

$$\Rightarrow \tan\theta=\frac{1}{a_{\text{tangential}}}\left[\frac{K^2s}{R}\right]$$

Now, $a_{\text{tangential}} = \frac{dv}{dt} = \frac{d}{dt} = \left[K\sqrt{s}\right]$

or $a_{\text{tangential}} = K \times \frac{1}{2\sqrt{s}} \times \frac{ds}{dt}$

or $a_{\text{tangential}} = \frac{K}{2\sqrt{s}} \times V$

$= \frac{K}{2\sqrt{s}} \times K\sqrt{s} = \frac{K^2}{2}$

$\therefore \tan\theta = \frac{\frac{K^2 s}{R}}{\frac{K^2}{2}} \quad \Rightarrow \tan\theta = \frac{2s}{R}$

84. **(d)** Total time of flight (t) = $\frac{2u}{g}$

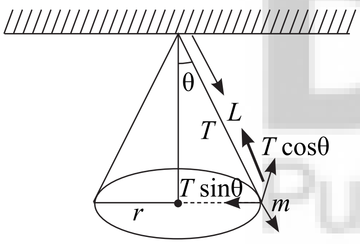
or, $9 = \frac{2u}{g}$ or, $u = \frac{9g}{2}$

or, $u = \frac{9 \times 10}{2}$ or, $u = 45\ m/s$

Since, in covering the vertical distance, g becomes – (ve) using $h = ut - \frac{1}{2}gt^2$

$= 45 \times (3) - \frac{1}{2} \times 10 \times (3)^2 = 135 - 45 = 90\ m$

85. **(a)** From the diagram,



$\frac{T\sin\theta}{T\cos\theta} = \frac{F_{\text{Centripetal}}}{\text{weight}} = \frac{mv^2}{r} \times \frac{1}{mg}$

$\Rightarrow \tan\theta = \frac{v^2}{rg}$

Also, $\tan\theta = \frac{t}{\sqrt{L^2 - r^2}}$

$\therefore \frac{v^2}{rg} = \frac{t}{\sqrt{L^2 - r^2}}$

or, $v = r\sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$

86. **(b)** Using $h_{\max} = \left(1 - \frac{1}{\sqrt{\mu^2 + 1}}\right)R$

$= \left(1 - \frac{1}{\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}}\right) \times R \quad \left(\because \mu = \frac{1}{\sqrt{3}}\right)$

$= \left(1 - \frac{1}{\sqrt{\frac{1}{3} + 1}}\right) \times R = \left(1 - \frac{1}{\sqrt{\frac{1+3}{3}}}\right) \times R$

or, $h_{\max} = \left(1 - \frac{\sqrt{3}}{2}\right)R$

87. **(c)** Using $\Delta\text{KE} = \frac{m_1 m_2}{2(m_1 + m_2)}\left(1 - e^2\right)\left(u_1 + u_2\right)^2$

Given: $m_1 = 1$ kg

$m_2 = \frac{1}{2}$ kg;

$u_1 = 6\ ms^{-1}$; $u_2 = 9\ ms^{-1}$ and $e = \frac{1}{3}$

$\therefore \Delta\text{KE} = \frac{1 \times \frac{1}{2}}{2\left(1 + \frac{1}{2}\right)}\left[1 - \left(\frac{1}{3}\right)^2\right](6 + 9)^2$

$= \frac{1}{2} \times \frac{2}{6} \times \frac{8}{9} \times (15)^2$

$= \frac{1}{6} \times \frac{8}{9} \times 225$

= 33.33 J

88. **(d)** As per question, the ball hits the ground loses $\frac{1}{3}$ rd of its total mechanical energy and rebounds back to the same height.

i.e., $\frac{2}{3}\left(\frac{1}{2}mv^2 + mgh\right) = mgh$

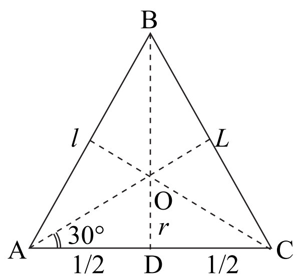
$\Rightarrow \frac{2}{3}\left(\frac{1}{2}v^2 + gh\right) = gh$

$\Rightarrow \frac{v^2}{3} = gh - \frac{2gh}{3}$

$\Rightarrow \frac{v^2}{3} = \frac{gh}{3} \Rightarrow v = \sqrt{gh}$

$\therefore v = \sqrt{400} = 20\ m/s$

89. **(b)** Let the length of the rod be l.



The moment of inertia of one rod separately about an axis passing through the centre of the rod and perpendicular to its length, $I = I_{Ac} + mr^2$

$= \frac{1}{12}ml^2 + m\left(\frac{l}{2\sqrt{3}}\right)^2$

$= \frac{ml^2}{12} + \frac{ml^2}{12} = \frac{2ml^2}{12}$

Moment of inertia of the system of 3rods

$= 3 \times \frac{2ml^2}{12} = 6 \times \frac{ml^2}{12} = 6l$

According to the question $\frac{6ml^2}{12} = l = n.\left(\frac{ml^2}{12}\right)$

$n = 6$

90. **(d)** If lower prism moves through a horizontal distance k since horizontal position of centre of mass of the two prims system remains unchanged

$3mk = m[(a - b) - k]$

$\Rightarrow 3k = (a - b)$

$\Rightarrow 4k = a - b$

or $k = \frac{a-b}{4}$

91. **(c)** As per question,

$|A\omega^2| - |A\omega| = 4 \quad [\therefore A = 2m]$

$2\omega^2 - 2\omega - 4 = 0$

$\Rightarrow \omega = 2 \text{ rad/s} \Rightarrow \frac{2\pi}{T} = 2$

$\Rightarrow T = \pi = \frac{22}{7} s$

Velocity, $v = \omega\sqrt{A^2 - y^2} = 2\sqrt{(2)^2 - (1)^2}$

$= 2\sqrt{4-1} = 2\sqrt{3} \text{ ms}^{-1}$

92. **(b)** Rom the body of mass m, the distance of the point where gravitational field is zero

$x = \frac{d}{\sqrt{\frac{m_2}{m_1}} + 1} \Rightarrow x = \frac{r}{\sqrt{\frac{9m}{m}} + 1} \quad \therefore \quad x = \frac{1}{4}$

Again $(r - x) = r - \frac{r}{4} = \frac{3r}{4}$

$\therefore$ Potential, $v = v_1 + v_2$

$= \frac{Gm_2}{X} - \frac{Gm_1}{r-x} = -\frac{Gm}{r/4} - \frac{G(9m)}{3r/4}$

$= -\frac{4Gm}{r} - \frac{12Gm}{r} = -\frac{16Gm}{r}$

93. **(d)** Natural length $(l) = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}$

$= \frac{101 \times 100 - 102 \times 80}{(100 - 80)}$

$= \frac{101 \times 100 - 102 \times 80}{20}$

$= \frac{101 \times 100}{20} - \frac{102 \times 80}{20}$

$= 505 - 408 = 97$ cm

Again $\frac{T_1}{T_2} = \frac{4}{l_3 - 97}$

$\frac{80}{160} = \frac{4}{l_3 - 97}$ or, $l_3 - 97 = 8$

$\therefore \quad l_3 = 105$ mm or 10.5 cm

94. **(d)** Let r be the radius of cavity

$\therefore$ Volume of cavity $= \frac{4}{3}\pi r^3$

Now, $\frac{4}{3}\pi(R^3 - r^3)dm\ g = \frac{4}{3}\pi R^3 d_w g$

$\Rightarrow 1 - \frac{R^3}{r^3} = \frac{1}{8} \Rightarrow \frac{R^3}{r^3} = 1 - \frac{1}{8}$

$\Rightarrow \frac{R^3}{r^3} = \frac{7}{8}$

$\Rightarrow r^3 = \frac{8}{7}R^3 = \frac{7}{8}(2)^3 = 7$

There Volume of cavity $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7 = \frac{88}{3}$

95. **(a)** As per question $\frac{1}{4}$th of heat is absorbed by the obstacle so, $\frac{3}{4}\left(\frac{1}{2}mv^2\right) = ms\Delta\theta + mL$

$\frac{3}{4}v^2 = 0.03 \times 4200 \times 300 + 6 \times 4200$

$\Rightarrow \quad v^2 = \frac{8}{3}[0.01 \times 4200 \times 300 + 2 \times 4200]$

$\Rightarrow \quad v^2 = 40 \times 4200$

$\Rightarrow \quad v = \sqrt{168000} = 410$ m/s

96. **(c)** Specific heat of water = 1 cal $g^{-1\circ} C^{-1}$

Latent heat of fusion of ice = 80 cal g^{-1}

Latent heat of steam = 540 cal g^{-1}

$m \times 80 + m \times 1 \times t = (M - m) \times 1 \times (100 - t) + 540\,(M - m)$

$\Rightarrow m \times 80 + mt = (M - m) \times 100 - (M - m)t + 540\,M - m \times 540$

$\Rightarrow 80m + mt = M \times 100 - m \times 100 - Mt + mt + 540\,M - 540m$

$\Rightarrow 80m + 100m + 540\,m = 640M - Mt$

or, $720\,m = M\,(640 - t)$

$\therefore \frac{m}{M} = \frac{640 - t}{720}$

97. **(a)** For isobarically expansion, $\frac{dW}{dQ} = 1 - \frac{1}{\gamma}$

$\frac{300}{dQ} = 1 - \frac{1}{14}$ [$\because$ For diatomic gas $\gamma = 1.4$ & dw = 300J]

$\Rightarrow \frac{300}{dQ} = \frac{14-1}{14} = \frac{0.4}{14}$

$\therefore \quad dQ = \frac{300 \times 14}{0.4} = 1050\text{J}$

98. **(a)** According to the question, initially

$\eta = \frac{100 - T_2}{100}$

$= \eta + \eta \times \frac{80}{100} = 1.8\eta = \frac{125 - T_2}{125}$

$\frac{\eta}{\eta'} = \frac{100 - T_2}{100} \times \frac{125}{125 - T_2}$

$\Rightarrow \frac{1}{18} = \frac{100 - T_2}{100} \times \frac{125}{125 - T_2}$

or, $\frac{5}{9} = \frac{100 - T_2}{100} \times \frac{5}{(125 - T_2)}$

or, $9(100 - T_2) = 500 - 4T_2$

or, $900 - 9T_2 = 500 - 4T_2$

or, $400 = 5T_2$

or, $T_2 = \frac{400}{5} = 80$

The new efficiency,

$\eta'' = 1 - \frac{T_2}{T_1} = 1 - \frac{80}{125}$

$\frac{125 - 80}{125} = \frac{45}{125} = \frac{45}{125} \times 100 = 36\%$

99. **(a)** Amount of heat needed

$dQ = \eta C_v dT\, \eta\left(\frac{3}{2}R\right)dt \quad \left[\because Q = \frac{3}{2}R\right]$

$= \frac{67.2}{22.4} \times \frac{3}{2} \times R \times dT$

$= 3 \times \frac{831 \times 3}{2} \times 20 = 784\text{J}$

100. **(b)** For closed organ pipe only odd harmonies are possible.

$\therefore$ Fundamental frequency

Fundamental frequency for a closed organ pipe

$v_0 = \frac{v_0}{4l} \Rightarrow 85 = \frac{340}{4 \times l}$

$\left[\because v_0 = \frac{595 - 425}{2} = \frac{765 - 595}{2} = 85\,\text{Hz}\right]$

$l = \frac{340}{4 \times 85} = 1\,m$

101. **(d)** From Doppler's effect, apparent frequency,

$n' = \left(\frac{v + v_0}{v - v_0}\right)n$

or, $n' = \left(\frac{340 + 2}{340 - 2}\right) \times 170$

or, $n' = \frac{342}{338} \times 170$

or $n' = 172.01 = 172$ Hz

Therefore, beat frequency observed

$(172 - 170) = 2\text{Hz}$

102. **(d)** Using mirror formula,

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

or, $\frac{-1}{f} = \frac{-1}{u} + \frac{1}{v}$ [$\therefore$ u & f negative for concave mirror]

$\Rightarrow \frac{1}{v} = \frac{1}{u} - \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{f - u}{uf}$

$v = \frac{uf}{u - f}$

Length of image,

$L = |v| - |f| = v = \frac{uf}{u - f} - f$

$L = \frac{uf - f(u - f)}{u - f}$

$= \frac{uf - uf + f^2}{u - f} = \frac{f^2}{u - f}$

103. **(b)** Fringe width, $\beta_G = n\frac{\lambda D}{d}$

And for n number of fringes,

$\beta_G = n\frac{\lambda D}{d}$ or, $\beta_R = n\frac{\lambda D}{d}$

For $(n + 1)$ th number of fringes,

$\beta_G = (n + 1)\frac{\lambda D}{d}$

$\because \beta_R = \beta_G$

or, $n\frac{\lambda_R D}{d} = (n + 1)\frac{\lambda_R D}{d}$

$n.6000 \times 10^{-10}\frac{D}{d} = (n + 1)5000 \times 10^{-10}\frac{D}{d}$ or,

$\Rightarrow \quad 6n = (n + 1)5$

$\Rightarrow \quad 6n = 5n + 5$ or, $6n - 5n = 5$

$\therefore \quad n = 5$

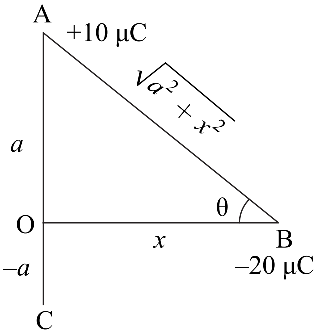
104. **(c)** If force acting on each charge + 10μC be F and x be the displaced position

$\therefore$ Net force $F_{net} = F\cos\theta + F\cos\theta$

$= 2F\cos\theta$

From coulomb's law,

$$F = k\frac{q_1 q_2}{r^2}$$



$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$F = 9\times10^9 \times \frac{10\times10^{-6}\times20\times10^{-6}}{\left(\sqrt{a^2+x^2}\right)^2}$$

$$= 9\times10^9 \times \frac{200\times10^{-12}}{\left(a^2+x^2\right)}$$

$$F_{net} = 2F.\cos\theta = 2\times10\times10^9 \times \frac{200\times10^{-12}}{\left(a^2+x^2\right)}\cos\theta$$

$$\therefore \quad F_{net} = 2\times9\times\frac{2\times10^{-1}}{\left(a^2+x^2\right)}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

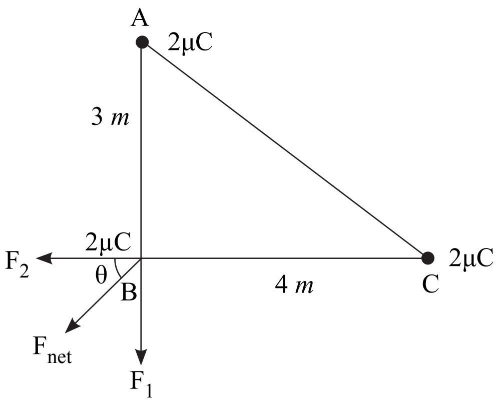
$$\left(\because \cos\theta = \frac{x}{\sqrt{a^2+x^2}}\right)$$

$$= 36\times10^{-1}\frac{x}{\left(a^2+x^2\right)^{3/2}}$$

Given, $x << a$ so, neglecting x^2

$$\therefore F_{net} = 36\frac{x}{\left(a^2\right)^{3/2}} \approx \frac{36x}{a^3}\text{ N}$$

105. **(c)** From figure the net force F_{net} due to F_1 and F_2 makes an angle θ with force F_2



$$\tan\theta = \frac{F_1}{F_2}$$

Also, $F_1 = k.\dfrac{q_1 q_2}{(3)^2}$

$\because \quad q_1 = q_2 = q_3 = 2\mu C$

$\Rightarrow \quad F_2 = k.\dfrac{q_1 q_3}{(4)^3}$

$$\therefore \quad \tan\theta = k.\frac{q_1 q_2 / (3)^2}{q_1 q_3 / (4)^2} = \frac{(4)^2}{(3)^2} = \frac{16}{9}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{16}{9}\right)$$

106. **(d)** Potential $v = \dfrac{kq}{r}$

For 30 V equipotential surface,

$30 = \dfrac{kq}{t}$, here $r = 20$ cm $= 20 \times 10^{-2}$ m

$$30 = \frac{kq}{20\times10^{-2}}$$

$\therefore \quad$ kq = 60 × 10 × 10^{-2} = 6

Therefore, electric field at distance r from charge q.

$$E = \frac{kq}{r^2} = \frac{6}{r^2}\text{ V}m^{-1}$$

107. **(d)** Using $E = \dfrac{dV}{dr}$ or, dV = E dr

Displacement, $dr = r_B - r_A$

$= (2\hat{i} + \hat{j} + 3\hat{k}) - (\hat{i} + 2\hat{j})$

$= (10\,\hat{i} + 30\,\hat{j})\,(\hat{i} - \hat{j} + 3\,\hat{k})$

$= 10\,\hat{i}\,.\,\hat{i} - 30\,\hat{i}\,.\,\hat{j} + 0$

$= 10 - 30 = -20$

Work done, W = q × |dV| = 0.8 (20) = 16J

108. **(a)** The drift velocity,

$$v_a = \frac{1}{neA} = \frac{V}{neA}\left(\because l = \frac{V}{R}\right)$$

$$\therefore\ v_d = \frac{V}{\rho\dfrac{l}{A}neA}\quad\left(\because R = \rho\frac{l}{A}\right)$$

$$\therefore\ v_d = \frac{V}{\rho lne}$$

As drift velocity is independent of cross-sectional area (A) so no change in drift velocity.

109. **(c)** Since $R \propto l$

$$\therefore\ \frac{R_L}{R_R} = \frac{60}{100-60} = \frac{60}{40} = \frac{6}{4} = \frac{3}{2} \qquad \text{....(i)}$$

$$\Rightarrow \quad \frac{R_L}{144R_R} = \frac{l}{100-l} \qquad \text{....(ii)}$$

From eqs. (i) and (ii)

$$\frac{144R_R}{R_R} = \frac{3/2}{l/100-l} = \frac{3}{2}\times\frac{100-l}{l}$$

$$\Rightarrow \frac{100-l}{l} = \frac{2}{3}\times 144 = 2 \times 0.48 = 0.96$$

$\therefore$ $100 - i = 0.96l$

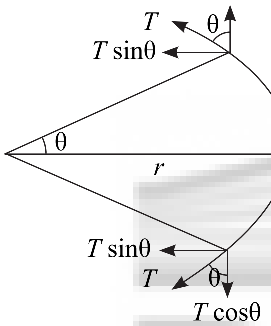
or, $1.96l = 100$

or, $l = \frac{100}{196} = 51.02$ cm = 51 cm

110. (c) Let the tension developed is T.

Let radius of loop be r.

For smaller element



$$BI\,(dl) = 2T\sin\left(\frac{d\theta}{2}\right)$$

$$\Rightarrow BI\,(r d\theta) = 2T\left(\frac{d\theta}{2}\right)$$

θ is small $\sin \simeq \theta$

$$\therefore \quad T = irB = \frac{ilB}{2\pi} = \frac{1.1\times1\times2.0}{2\times3.14} = 0.35\text{ N}$$

111. (a) Magnetic dipole moment

$M = niA$, $n = 1$, $M = iA$

The current in the ring, $i = q/t = q \times f$

Where, f = frequency of charge

$i = q\frac{\omega}{2\pi}$ ($\because \omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi}$)

$\therefore \quad M = q.\frac{\omega}{2\pi}.\pi R^2 = \frac{1}{2}q\omega R^2$

112. (d) When magnetic dipole is released from E-W a torque acts on it. So, in the displacement from E-W to N-S work is done by the torque.

KE = work done & $W = \int_{\theta_1}^{\theta_2} \tau.d\theta$

$= MB(\cos\theta_1 - \cos\theta_2)$

$\therefore$ KE = MBcos 0°

$= 2.5 \times 3 \times 10^{-5}$

$= 7.5 \times 10^{-5}\text{J} = 75 \times 10^{-6}\ \mu\text{J}$

113. (a) Voltage of source, V = 4V

$V_L = L.\frac{dl}{dt} = 9 \times 10^{-3} \times 10^3\text{V} = 9\text{V}$

$V_{AB} - V_R + V + V_L = 0$

or, $V_{AB} - 14 + 4 + 9 = 0$

$\Rightarrow \quad V_{AB} - 1 = 0$

$\therefore \quad V_{AB} = 1\text{V}$

114. (c) The natural frequency, $f_N = \frac{1}{2\pi\sqrt{KLC}}$ = Fe

When capacitor is totally filled with dielectric material of dielectric constant K then capacitance C' = KC

$$f_{C'} = \frac{1}{2\pi\sqrt{KLC}}$$

$$\frac{f_C}{f_{C'}} = \frac{1/2\pi\sqrt{LC}}{1/2\pi\sqrt{KLC}} = \sqrt{K}$$

$$\Rightarrow \frac{125\times10^3}{100\times10^3} = \sqrt{K} \Rightarrow \frac{S}{4} = \sqrt{K}$$

[f_c = 125 kHz – 25kHz = 100 kHz = 100 × 10^3 Hz]

$$\therefore \quad K = \left(\frac{5}{4}\right)^2 = \frac{25}{16} = 1.562$$

115. (b) The correct sequence of the radiation source in increasing order of the wavelength is Radioactive source ($\sim10^{-12}$ m) → X-ray tube ($\sim10^{-10}$ m) → Sodium lamp ($\sim10^{-9}$ m) → Magnetron value ($\sim10^{-3}$ m).

116. (b) de-Broglie wavelength

$$\lambda_0 = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}$$

And $E_K = hv = \frac{hc}{\lambda}$

$$\therefore \quad \lambda_v = \frac{h}{\sqrt{2m\times\frac{hc}{\lambda}}} = \sqrt{\frac{\lambda h}{2mc}}$$

117. (c) As the radius of the orbit will change due to transition from a higher energy level to a lower energy level.

Therefore angular momentum L = mvr will not remain constant.

118. (a) Using radius of a nucleus, $R \propto A^{1/3}$

Where, A = number of nucleons

$$\therefore \quad \frac{R_{Ge}}{R_{Be}} = \left(\frac{A_{Ge}}{A_{Be}}\right)^{1/3}$$

$$2 = \left(\frac{A_{Ge}}{A_{Be}}\right)^{1/3} \text{ or, } 2^3 = \frac{A_{Ge}}{A_{Be}} \quad (\therefore R_{Ge} = 2R_{Be})$$

$$2^3 = \frac{A_{Ge}}{9} \quad [\therefore A_{Be} = 9]$$

$\therefore \quad A_{Ge} = 2^3 \times 9 = 8 \times 9 = 72$

119. (b) Using $l_E = l_B + l_C$

$\Rightarrow \quad l_B = l_E - l_C$

$= 6.6 \times 10^{-3} - 60 l_B$

$\Rightarrow \quad l_B + 60 l_B = 6.6 \times 10^{-3}$

or, $\quad 61 l_B = 6.6 \times 10^{-3}$

$\therefore \quad l_B = \dfrac{6.6 \times 10^{-3}}{61} \simeq 0.108 mA$

$= 0.108 \times 10^{-3}$

$= 0.108\ mA$

120. (a) Coverage area A = πd^2 and d = $\sqrt{2Rh}$

$= \pi\left(\sqrt{2Rh}\right)^2 = \pi \times 2Rh$

Here h = 105 m, $R = 6.4 \times 10^6$ m

$\therefore \quad A = 3.14 \times 2 \times 6.4 \times 10^6 \times 105\ m^2$

$= 314 \times 2 \times 64 \times 10^6 \times 105 \times 10^3 m^2$

$= 4220.6\ km^2 = 4224\ km^2$

CHEMISTRY

121. (c) According to uncertainty principle,

$\Delta P . \Delta x \geq \dfrac{h}{4\pi}$

$\Rightarrow \quad m.\Delta v \times \Delta x \geq \dfrac{h}{4\pi}$

$\Rightarrow \Delta v \times \Delta x \geq \dfrac{h}{4\pi \times m}$

122. (c) Electronic configuration of the element in +2 oxidation state is [Ar] $3d^1$. Its electronic configuration in neutral state:

[Ar] $3d^1 4s^2$

Period = Max.principal quantum no. n = 4

Group = Total number of valence shell electrons = 3

123. (b) In PCl_5 bond length of axial bond is greater than the length of equatorial bond.

124. (d)

125. (b) Number of moles of hydrogen

$= \dfrac{3g}{2g\ mol^{-1}} = 1.5$

Number of moles of oxygen

$= \dfrac{80g}{32g\ mol^{-1}} = 2.5 = 2.5$

Total number of moles in gaseous mixture of hydrogen and oxygen,

$n = 1.5 + 2.5 = 4$

$KE = \dfrac{3}{2} nRT = \dfrac{3}{2} \times 4 \times RT = 6RT$

126. (d) C has highest critical temperature, i.e., 405.5K. Henc, it gets liquefied first.

127. (a) $2\ KMnO_4 + 3\ H_2SO_4 + 5(COOH)_2 \rightarrow K_2SO_4 + 2\ MnSO_4 + 8\ H_2O + 10\ CO_2$

$\therefore \quad \dfrac{M_1 V_1}{n_1} = \dfrac{M_2 V_2}{n_2}$

$\Rightarrow \quad \dfrac{x \times 40\,mL}{2} = \dfrac{0.02 \times 200\,mL}{5}$

$\Rightarrow \quad x = 0.04\ M$

128. (b) $C(s) + O_2(g) \rightarrow CO_2(g)$

$\Delta H^\circ = -x$ kJ mol^{-1} .... (i)

$2\ CO(g) + O_2(g) \rightarrow 2\ CO_2(g)$,

$\Delta H^\circ = -y$ kJ mol^{-1} ... (ii)

by applying eq. (i) – $\dfrac{1}{2}$ × Eq. (ii)

$C(s) + \dfrac{1}{2} O_2 g \rightarrow CO(g)$

$\therefore \quad \Delta f H(CO) = -x - \left(-\dfrac{y}{2}\right)$

$= -x + \dfrac{y}{2} = \dfrac{y - 2x}{2}$

129. (b)

	$N_2O_4 \rightleftharpoons$	$2NO_2$
Initial moles	1	0
At equilibrium	(1 – 0.2)	2 × 0.2= 0.8

Total moles = 0.4 + 0.8 = 12

$$K_P = \frac{\left(P_{NO_2}\right)^2}{\left(P_{N_2O_4}\right)} \Rightarrow K_P = \frac{\left(\frac{0.4}{1.2} \times 600\right)^2}{\left(\frac{0.8}{1.2} \times 600\right)} = 100$$

130. (c) Dissolution $NH_4Cl \rightarrow NH_4^+ + Cl^-$

Cl^- will form Cl^- (aq) and will not undergo hydrolysis.

Hydrolysis $NH_4^- + H_2O \rightleftharpoons NH_3 + H_3O^+$

131. (c)

132. (b) Li is the only alkali metal which reacts directly with N_2 to give nitride (Li_3N)

$6Li + N_2 \rightarrow 2Li_3N$

133. (a) $AlCl_3$ exists as a dimer through halogen bridged bonds. In dimer Al_2Cl_6 aluminium completes its octet through coordinates bond by chloine atom.

134. (c)

135. (d) The general formula for this series is C_nH_{2n-2}. Hence, $C_6H_{10}O_2N$ belongs to the given homologous series.

136. (b) In Dumes method,

% of nitrogen = $\dfrac{28V \times 100}{22400 \times W} = \dfrac{28 \times 45 \times 100}{22400 \times 0.3} = 18.75$

137. (a) **138. (b)**

139. (d) For equimolar solution,

$\chi_b = \chi_t = 0.5$

$p_b = \chi_b \times p_b^0 = 0.5 \times 160$

= 80 mm

$p_b = \chi_t \times p_b^0 = 0.5 \times 60$

= 30 mm

$p_{total} = 80 + 30 = 110\ mm$

Mole fraction of benzene in vapour phase

$= \frac{p_b}{p_{total}} = \frac{80}{110} = 0.727 = 0.73$

140. (d) $\Delta T_f = K_f m$

$0.93 = \frac{1.86 \times 6 \times 1000}{M \times 100} \Rightarrow M \approx 120$

141. (b) $K_2SO_4 \rightarrow 2K^{-1} + SO_4^{2-}$

Due to platinum electrodes, self ionization of water will take place.

$H_2O \rightarrow 2H^+ + OH^-$

At cathode: $2H^+ + 2e- \rightarrow H_2$

At anode: $OH \rightarrow 2H_2O + O_2 + 4e$

Due to lower electrode potentials, H_2 gas will generate at cathode and O_2 gas will generate at anode.

142. (d) Arrhenius equation,

$k = Ae^{-E_a/RT}$

On taking log on both sides, we get

$ln\ k = ln\ A - \frac{E_a}{RT}$

A graph between in k and 1/T is a straight line with $-\frac{E_a}{R}$ slope.

143. (c)

144. (b) Siderite is $FeCO_3$

145. (b) In oxygen, the bond is pure double bond but in ozone, it is partial single and double bond.

146. (d) Oxidise cannot oxidise F^- to F_2.

147. (a) At high temperature about 1000k, sulphur consists of mixture of forms S_2, S_4, S_6, S_8, etc.

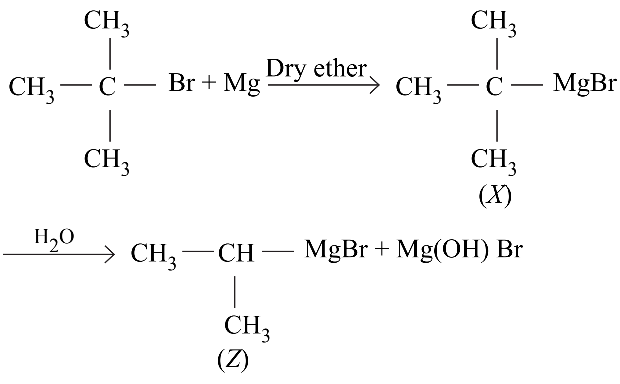
148. (d) $XeF_6 + 3H_2O \rightarrow XeO_3 + 6HF$

149. (c)

150. (d) Catalytic activity of transition elements is due to their variable oxidation states and to form complexes.

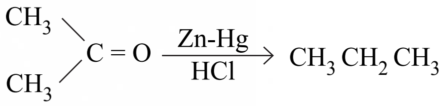
151. (b) **152. (d)** **153. (b)** **154. (a)**

155. (c)



2-methyl propane

156. (b) Clemmensen reduction:



157. (d)

158. (c)

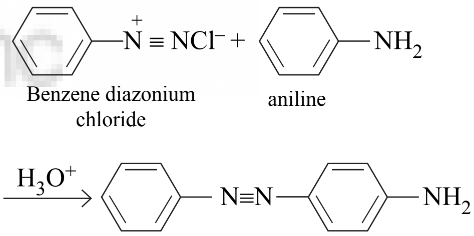
$CH_3OH \xrightarrow{PCl_3} \underset{(A)}{CH_3Cl} \xrightarrow{KCN} \underset{(B)}{CH_3CN}$

$\xrightarrow{H_3O^+} \underset{(C)}{CH_3COOH}$

Hence, C is CH_3COOH.

159. (c) $(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N > NH_3 > C_6H_5NH_2$

160. (a)



yellow dye