AP/EAMCET Solved Paper 2017

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. In ΔABC, if $b\cos\theta = c - a$, (where θ is an acute angle), then $(c - a)\tan\theta =$

(a) $2\sqrt{ca}\cos\frac{B}{2}$ (b) $2\sqrt{ca}\sin\frac{B}{2}$

(c) $2ca\cos\frac{B}{2}$ (d) $2ca\sin\frac{B}{2}$

2. Given below is the distribution of a random variable X

X = x	1	2	3	4
P(X = x)	λ	2λ	3λ	4λ

If $\alpha = P(X < 3)$ and $\beta = P(X > 2)$, then $\alpha : \beta =$

(a) 2 : 5 (b) 3 : 4 (c) 4 : 5 (d) 3 : 7

3. If $f : \mathbf{R} \to \mathbf{R}$ is defined by

$$f(x) = \begin{cases} x-1, & \text{for } x \le 1 \\ 2-x^2, & \text{for } 1 < x \le 3 \\ x-10, & \text{for } 3 < x < 5 \\ 2x, & \text{for } x \ge 5 \end{cases}$$

then the set of points of discontinuity of f is

(a) $\mathbf{R} - \{1, 5\}$ (b) $\{1, 3, 5\}$

(c) $\{1, 5\}$ (d) $\mathbf{R} - \{1, 3, 5\}$

4. If the pair of lines $x^2 - 16\,pxy - y^2 = 0$ and $x^2 - 16qxy - y^2 = 0$ are such that each pair bisects the angle between the other pair, then pq =

(a) $\frac{-1}{64}$ (b) $\frac{1}{64}$ (c) $\frac{-1}{8}$ (d) $\frac{1}{8}$

5. If a non-zero vector **a** is parallel to the line of intersection of the plane determined by the vectors $\hat{\mathbf{j}} - \hat{\mathbf{k}}, 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and the plane determined by the vectors $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}, \hat{\mathbf{i}} - 3\hat{\mathbf{j}}$, then the angle between the vectors **a** and $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ is

(a) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (b) $\cos^{-1}\left(\pm\frac{2}{\sqrt{3}}\right)$

(c) $\tan^{-1}\sqrt{3}$ (d) $\cos^{-1}\left(\pm\frac{1}{\sqrt{3}}\right)$

6. If three numbers are drawn at random successively without replacement from a set $S = \{1, 2, ...10\}$, then the probability that the minimum of the chosen numbers is 3 or their maximum is 7.

(a) $\frac{11}{40}$ (b) $\frac{5}{40}$ (c) $\frac{3}{40}$ (d) $\frac{1}{40}$

7. For $x^2 - 4 \ne 0$, the value of

$$\frac{d}{dx}\left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}\right] \text{ at } x = 3 \text{ is}$$

(a) $\frac{8}{5}$ (b) 2 (c) 1 (d) $\frac{8e^3}{5}$

8. If $y = \frac{\sin h^{-1} x}{\sqrt{1+x^2}}$, then $(1 + x^2)y_2 + 3xy_1 + y =$

(a) 2 (b) 1 (c) –1 (d) 0

9. The area (in sq. units) enclosed between the curves $y = x^2$ and $y = |x|$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) 1

10. $\int \frac{5x^2 + 3}{x^2(x^2 - 2)}dx =$

(a) $\frac{13}{2\sqrt{2}}\log\left|\frac{\sqrt{2}-x}{\sqrt{2}+x}\right| + \frac{3}{2x} + C$

(b) $\frac{13}{4\sqrt{2}}\log\left|\frac{x+\sqrt{2}}{x-\sqrt{2}}\right| + \frac{3}{2x} + C$

(c) $\frac{13}{4\sqrt{2}}\log\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + \frac{3}{2x} + C$

(d) $\frac{5}{3\sqrt{2}}\log\left|\frac{x+\sqrt{2}}{x-\sqrt{2}}\right| + \frac{3}{5}x + C$

11. If $y=\tan^{-1}\left\{\frac{x}{1+\sqrt{1-x^2}}\right\}$

$+\sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$, then $\frac{dy}{dx}=$

(a) $\frac{1-2x}{2\sqrt{1-x^2}}$ (b) $\frac{1-2x}{x\sqrt{1-x^2}}$

(c) $\frac{2x+1}{x\sqrt{1-x}}$ (d) $\frac{2-x}{2\sqrt{1-x^2}}$

12. The equation of the plane through (4, 4, 0) and perpendicular to the planes $2x + y + 2z + 3 = 0$ and $3x + 3y + 2z - 8 = 0$ is

(a) 4x + 3y + 3z = 28 (b) 4x – 2 y – 3z = 8

(c) 4x + 2 y + 3z = 24 (d) 4x + 2 y – 3z = 24

13. $\lim_{n\to\infty}\left[\frac{1^k+2^k+3^k+...+n^k}{n^{k+1}}\right]=$

(a) $\frac{1}{k}$ (b) $\frac{2}{k+1}$ (c) $\frac{1}{k+1}$ (d) $\frac{2}{k}$

14. The coefficient of x^5 in the expansio of $(1+x)^{21}+(1+x)^{22}+...+(1+x)^{30}$ is

(a) ${}^{31}C_6-{}^{21}C_6$ (b) ${}^{51}C_5$

(c) ${}^{9}C_5$ (d) ${}^{30}C_5+{}^{20}C_5$

15. Let $A = \{-4, -2, -1, 0, 3, 5\}$ and $f: A \to \mathbf{R}$ be defined by

$$f(x)=\begin{cases}3x-1 & \text{for} \quad x>3\\ x^2+1 & \text{for} \quad -3\le x\le 3\\ 2x-3 & \text{for} \quad x<-3\end{cases}$$

Then the range of f is

(a) {– 11, 5, 2, 1,10, 14} (b) {– 11, – 7, 2, 1, 8, 14}

(c) {– 11, 5, 2, 1, 8, 14} (d) {– 11, – 7, – 5, 1, 10, 14}

16. The incentre of the triangle formed by the straight lines $y=\sqrt{3}x, y=-\sqrt{3}x$ and $y=3$ is

(a) (0, 2) (b) (1, 2) (c) (2, 0) (d) (2, 1)

17. The solution of the equation

$\left(x-4y^3\right)\frac{dy}{dx}-y=0,\left(y>0\right)$ is

(a) $x=y^3+cy$ (b) $x+2y^3=cy$

(c) $y=x^3+cx$ (d) $y+2x^3=cx$

18. If a circle with radius 2.5 units passes through the points (2, 3) and (5, 7), then its centre is

(a) (1.5, 2) (b) (7, 10) (c) (3, 4) (d) (3.5, 5)

19. The circumcentre of the triangle formed by the points (1, 2, 3), (3, – 1, 5) , (4, 0, – 3) is

(a) (1, 1, 1) (b) (2, 2, 2) (c) (3, 3, 3) (d) $\left(\frac{7}{2},\frac{-1}{2},1\right)$

20. A bag P contains 5 white marbles and 3 black marbles. Four marbles are drawn at random from P and are put in an empty bag Q. If a marble drawn at random from Q is found to be black then the probability that all the three black marbles in P are transfered to the bag Q is.

(a) $\frac{1}{7}$ (b) $\frac{6}{7}$ (c) $\frac{1}{8}$ (d) $\frac{7}{8}$

21. $\operatorname{sech}^{-1}\left(\frac{1}{\sqrt{2}}\right)+\operatorname{cosech}^{-1}(-1)=$

(a) 0 (b) $\sqrt{2}+1$ (c) $\sqrt{2}$ (d) $\sqrt{2}-1$

22. If the points having the position vectors $3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}}, 2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-4\hat{\mathbf{k}}, -\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\lambda\hat{\mathbf{k}}$ are coplanar, then λ =

(a) $\frac{46}{17}$ (b) 8 (c) – 8 (d) $\frac{146}{17}$

23. The lines $y=2x+\sqrt{76}$ and $2y+x=8$ touch the ellipse $\frac{x^2}{16}+\frac{y^2}{12}=1$. If the point of intersection of these two lines lie on a circle. whose centre coincides with the centre of that ellipse, then the equation of that circle is

(a) $x^2+y^2=28$ (b) $x^2+y^2=16$

(b) $x^2+y^2=12$ (d) $x^2+y^2=\left(4+\sqrt{8}\right)^2$

24. The equation of the pair of lines through the point (2, 1) and perpendicular to the pair of lines $4xy + 2x + 6y + 3 = 0$ is

(a) xy – x – 2 y + 2 = 0 (b) xy + x – 2 y – 2 = 0

(c) xy + x + 2 y – 6 = 0 (d) xy – x + 2 y – 2 = 0

25. The harmonic mean of two numbers is $-\frac{8}{5}$ and their geometric mean is 2. The quadratic equation whose roots are twice those numbers is

(a) $x^2+5x+4=0$ (b) $x^2+10x+16=0$

(c) $x^2-10x+16=0$ (d) $x^2-5x+4=0$

26. If z is a complex number with $|z| \ge 5$. Then the least value of $\left|z+\frac{2}{z}\right|$ is

(a) $\frac{24}{5}$ (b) $\frac{26}{5}$ (c) $\frac{23}{5}$ (d) $\frac{29}{5}$

27. ΔABC is formed by A(1, 8, 4), B (0, –11, 4) and C (2, – 3,1). If D is the foot of the perpendicular from A to BC. Then the coordinates of D are

(a) (– 4, 5, 2) (b) (4, 5, – 2)

(c) (4, – 5, 2) (d) (4, – 5, – 2)

28. For the function $f(x) = (x - 1)(x - 2)$ defined on $\left[0,\frac{1}{2}\right]$, the value of c satisfying Lagrange's mean value theorem is

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{1}{7}$ (d) $\frac{1}{4}$

29. A container is the shape of an inverted cone. Its height is 6 m and radius is 4 m at the top. If it is filled with water at the rate of $3m^3$/min then the rate of change of height of water (in mt/min) when the water level is 3 m, is

(a) $\frac{3}{4\pi}$ (b) $\frac{2}{9\pi}$ (c) 16π (d) 2π

30. If the roots of the equation $x^3 - 7x^2 + 14x - 8 = 0$ are in geometric progression, then the difference between the largest and the smallest roots is

(a) 4 (b) 2 (c) $\frac{1}{2}$ (d) 3

31. If the mean and variance of a binomial variate X are $\frac{4}{3}, \frac{8}{9}$ respectively, then $P(X = 2) =$

(a) $\frac{4}{27}$ (b) $\frac{16}{81}$ (c) $\frac{8}{27}$ (d) $\frac{8}{81}$

32. If α is a non-real root of $x^7 = 1$, then $\alpha(1 + \alpha)(1 + \alpha^2 + \alpha^4) =$

(a) 1 (b) 2 (c) -1 (d) -2

33. If $\cot\left(\cos^{-1} x\right) = \sec\left\{\tan^{-1}\left(\frac{a}{\sqrt{b^2 - a^2}}\right)\right\}$: $b > a$, then x =

(a) $\frac{b}{\sqrt{2b^2 - a^2}}$ (b) $\frac{\sqrt{b^2 - a^2}}{ab}$

(c) $\frac{a}{\sqrt{2b^2 - a^2}}$ (d) $\frac{\sqrt{b^2 - a^2}}{a}$

34. In ΔABC, L, M, N are points on BC, CA, AB respectively, dividing them in the ratio 1 : 2, 2 : 3 , 3 : 5. If the point K divides AB in the ratio 5 : 3, then $\frac{\left|\overline{AL} + \overline{BM} + \overline{CN}\right|}{\left|\overline{CK}\right|} =$

(a) $\frac{5}{8}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{15}$

35. The point to which the origin is to be shifted to remove the first degree terms from the equation $2x^2 + 4xy - 6y^2 + 2x + 8y + 1 = 0$ is

(a) $\left(\frac{7}{8}, \frac{3}{8}\right)$ (b) $\left(\frac{-7}{8}, \frac{-3}{8}\right)$ (c) $\left(\frac{-7}{8}, \frac{3}{8}\right)$ (d) $\left(\frac{7}{8}, \frac{-3}{8}\right)$

36. If α, β, γ are the lengths of the tangents from the vertices of a triangle to its incircle. Then

(a) $\alpha + \beta + \gamma = \frac{1}{r^2}(\alpha\beta\gamma)$ (b) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = r(\alpha\beta\gamma)$

(c) $\alpha + \beta + \gamma = \frac{1}{r}(\alpha\beta\gamma)$ (d) $\alpha^2 + \beta^2 + \gamma^2 = \frac{2}{r}(\alpha\beta\gamma)$

37. If $\int_0^{10} f(x)\,dx = 5$, then $\sum_{k=1}^{10} \int_0^1 f(k - 1 + x)\,dx =$

(a) 50 (b) 10 (c) 5 (d) 20

38. The angle between the tangents drawn from the point (1, 2) to the ellipse $3x^2 + 2y^2 = 5$ is

(a) $\tan^{-1}\left(\frac{12\sqrt{5}}{5}\right)$ (b) $\tan^{-1}\left(\frac{12\sqrt{5}}{13}\right)$

(c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

39. If $lx + my = 1$ is a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $a^2m^2 - b^2l^2 =$

(a) $\frac{m^2}{l^2}\left(a^2 + b^2\right)^2$ (b) $(l^2 + m^2)(a^2 + b^2)^2$

(c) $\frac{l^2}{m^2}\left(a^2 + b^2\right)^2$ (d) $l^2m^2(a^2 + b^2)^2$

40. $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x}\,dx =$

(a) $\frac{-1}{e}\log\left|x^e + e^x\right| + C$ (b) $-e\log|x^e + e^x| + C$

(c) $\frac{1}{e}\log\left|x^e + e^x\right| + C$ (d) $e\log|x^e + e^x| + C$

41. If $\Delta = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix}$, then Δ lies in the interval

(a) [2, 4] (b) (2, 4) (c) [1, 4] (d) [– 1, 1]

42. The equation of the circle whose diameter is the common chord of the circles $x^2 + y^2 + 2x + 2y + 1 = 0$ and $x^2 + y^2 + 4x + 6y + 4 = 0$ is

(a) $10x^2 + 10y^2 + 14x + 8y + 1 = 0$

(b) $3x^2 + 3y^2 - 3x + 6y - 8 = 0$

(c) $2x^2 + 2y^2 - 2x + 4y + 1 = 0$

(d) $x^2 + y^2 - x + 2y + 4 = 0$

43. $\frac{x-1}{3x+4} < \frac{x-3}{3x-2}$ holds, for all x in the interval

(a) $\left(\frac{-4}{3}, \frac{2}{3}\right)$ (b) $\left(-\infty, \frac{-5}{4}\right)$

(c) $\left(-\infty\right)$ (d) $\left(-\infty, \frac{-5}{4}\right) \cup \left(\frac{3}{4}, -\infty\right)$

44. There are 10 intermediate stations on a railway line between two particular stations. The number of ways that a train can be made to stop at 3 of these intermediate stations so that no two of these halting stations are consecutive is

(a) 56 (b) 20 (c) 126 (d) 120

45. The figure formed by the pairs of lines $6x^2 + 13xy + 6y^2 = 0$ and $6x^2 + 13xy + 6y^2 + 10x + 10y + 4 = 0$, is a

(a) Square (b) Parallelogram

(c) Rhombus (d) Rectangle

46. If the point of intersection of the tangents drawn at the points where the line $5x + y + 1 = 0$ cuts the circle $x^2 + y^2 - 2x - 6y - 8 = 0$ is $(\underline{a, b})$, then $5a + b =$

(a) 3 (b) – 44 (c) – 1 (d) 4

47. If a, b and c are non-zero vectors such that a and b are not perpendicular to each other, then the vector **r** which is perpendicular to **a** and satisfying r × b = c × b is

(a) $\dfrac{(\mathbf{a}\times\mathbf{b})\times\mathbf{c}}{\mathbf{c}\,.\,\mathbf{a}}$ (b) $\dfrac{\mathbf{b}\times(\mathbf{a}\times\mathbf{c})}{\mathbf{b}\,.\,\mathbf{c}}$

(c) $\dfrac{(\mathbf{b}\times\mathbf{c})\times\mathbf{a}}{\mathbf{a}\,.\,\mathbf{b}}$ (d) $\dfrac{(\mathbf{c}\times\mathbf{b})\times\mathbf{a}}{\mathbf{a}\,.\,\mathbf{c}}$

48. The tangents to the parabola $y^2 = 4ax$ from an external point P make angles θ_1 and θ_2 with the axis of the parabola. Such that $\tan\theta_1 + \tan\theta_2 = b$, where b is constant. Then P lies on

(a) $y = x + b$ (b) $y + x = b$

(c) $y = \dfrac{x}{b}$ (d) $y = bx$

49. The points on the straight line $3x - 4y + 1 = 0$ which are at a distance of 5 units from the point (3, 2) are

(a) $\left(-2,-\dfrac{7}{4}\right),\left(-3,\dfrac{-5}{2}\right)$ (b) $\left(4,\dfrac{11}{4}\right),(-1,-1)$

(c) $\left(1,\dfrac{1}{2}\right),\left(2,\dfrac{5}{4}\right)$ (d) (7, 5), (–1, –1)

50. $\displaystyle\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x} =$

(a) $\dfrac{-1}{4}$ (b) $\dfrac{1}{2}$ (c) 1 (d) 2

51. An angle between the curves $x^2 = 3y$ and $x^2 + y^2 = 4$ is

(a) $\tan^{-1}\dfrac{5}{\sqrt{3}}$ (b) $\tan^{-1}\sqrt{\dfrac{5}{3}}$

(c) $\tan^{-1}\dfrac{2}{\sqrt{3}}$ (d) $\dfrac{\pi}{3}$

52. If a, b, c are non-zero real numbers and if the equations $(a - 1)\,x = y + z$, $(b - 1)\,y = z + x$, $(c - 1)\,z = x + y$ have a non-trivial solution, then $ab + bc + ca =$

(a) $a^2b^2c^2$ (b) 0 (c) abc (d) $a + b + c$

53. The number of different ways of preparing a garland using 6 distinct white roses and 5 distinct red roses such that no two red roses come together is

(a) 21600 (b) 43200 (c) 86400 (d) 151200

54. If a cylindrical vessel of given volume V with no lid on the top is to be made from a sheet of metal, then the radius (r) and height (h) of the vessel so that the metal sheet used is minimum, is

(a) $r = \sqrt[3]{\dfrac{\pi}{V}}, h = \sqrt[3]{\dfrac{\pi}{V}}$ (b) $r = \sqrt{\pi V}, h = \sqrt{\pi V}$

(c) $r = \sqrt[3]{\dfrac{V}{\pi}}, h = \sqrt[3]{\dfrac{V}{\pi}}$ (d) $r = \sqrt{\dfrac{V}{\pi}}, h = \sqrt{\dfrac{V}{\pi}}$

55. $\displaystyle\int\frac{x+\sin x}{1+\cos x}dx =$

(a) $x\tan\dfrac{x}{2} + C$ (b) $x\sin\dfrac{x}{2} + \cos\dfrac{x}{2} + C$

(c) $x\tan\dfrac{x}{2} + \sec\dfrac{x}{2} + C$ (d) $x\sec\dfrac{x}{2} + \tan\dfrac{x}{2} + C$

56. If $I_n = \displaystyle\int\frac{\sin nx}{\cos x}dx$, then $I_n =$

(a) $\dfrac{-2}{n-1}\cos(n-1)x - I_{n-2}$

(b) $\dfrac{2}{n-1}\cos(n-1)x + I_{n-2}$

(c) $\dfrac{-2}{n+1}\sin(n+1)x - I_{n-2}$

(d) $\dfrac{-2}{n+1}\cos(n-1)x - I_{n-2}$

57. If $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + px + q = 0$, then the value of $\sin^2(\alpha + \beta) + p\cos(\alpha + \beta)\sin(\alpha + \beta) + q\cos^2(\alpha + \beta)$ is

(a) $p + q$ (b) p (c) q (d) $\dfrac{p}{p+q}$

58. If ω is a complex cube root of unity, then for any $n > 1$, $\displaystyle\sum_{r=1}^{n-1} r(r+1-\omega)(r+1-\omega^2) =$

(a) $\dfrac{n^2(n+1)^2}{4}$ (b) $\dfrac{n(n+1)(2n+1)}{6}$

(c) $\dfrac{n(n-1)}{4}(n^2+3n+4)$ (d) $\dfrac{n(n+1)(2n+1)}{4}$

59. Let N be the set of all natural numbers, Z be the set of all integers and $\sigma : N \to Z$ be defined by

$$\sigma(n) = \begin{cases} \dfrac{n}{2}, & \text{if } n \text{ is even} \\ -\dfrac{n-1}{2}, & \text{if } n \text{ is odd} \end{cases}$$. Then,

(a) σ is onto but not one-one
(b) σ is one-one but not onto
(c) σ is neither one-one nor onto
(d) σ is one-one and onto

60. $^{37}C_4 + \displaystyle\sum_{r=1}^{5}{}^{(42-r)}C_r =$

(a) $^{41}C_4$ (b) $^{39}C_4$ (c) $^{38}C_4$ (d) $^{42}C_4$

61. The variance of the following data is

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

(a) 33.4 (b) 34.3 (c) 43.4 (d) 44.3

62. The differential equation corresponding to the family of circles in the plane touching the Y-axis at the origin, is

(a) $\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$ (b) $\frac{dy}{dx}=\frac{2xy}{x^2+y^2}$

(c) $\frac{dy}{dx}=\frac{x^2-y^2}{2xy}$ (d) $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$

63. p, x_1, x_2 ..., x_n and q, y_1, y_2,..., y_n are two arithmetic progressions with common differences a and b respectively. If α and β are the arithmetic means of $x_1, x_2, ... x_n$, and $y_1, y_2, ..., y_n$ respectively. Then the locus of $P(\alpha, \beta)$ is

(a) $a(x-p)=b(y-q)$ (b) $b(x-p)=a(y-q)$

(c) $\alpha(x-p)=\beta(y-q)$ (d) $p(x-\alpha)=q(y-\beta)$

64. If $\alpha \neq 0$ and the mean deviation of the observations $\{k\alpha\}$, for $k = 1, 2, \ldots\ldots, 50$ about its median is 50, then $|\alpha|=$

(a) 4 (b) 3 (c) 2 (d) 5

65. If $2kx + 3y - 1 = 0$, $2x + y + 5 = 0$ are conjugate lines with respect to the circle $x^2 + y^2 - 2x - 4y - 4 = 0$, then $k =$

(a) 3 (b) 4 (c) 1 (d) 2

66. If $\frac{5x^2+2}{x^3+x}=\frac{A_1}{x}+\frac{A_2x+A_3}{x^2+1}$, then $(A_1, A_2, A_3) =$

(a) (0, 2, 3) (b) (3, 0, 2) (c) (2, 3, 0) (d) (2, 0, 3)

67. The sum of first n terms of $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots$ is

(a) $\frac{3n}{2(3n+2)}$ (b) $\frac{3n}{3n+2}$

(c) $\frac{n}{2(3n+2)}$ (d) $\frac{n}{3n+2}$

68. The equations of the parabola whose axis is parallel to the X-axis and which passes through the points (– 2, 1),(1, 2)(–1, 3) is

(a) $18y^2 - 12x - 21y - 21 = 0$

(b) $5y^2 + 2x - 21y + 20 = 0$

(c) $15y^2 + 12x - 11y + 20 = 0$

(d) $25y^2 - 2x - 65y + 36 = 0$

69. In ΔABC, if θ is any angle, then $b\cos(C+\theta) + c\cos(B-\theta) =$

(a) $a\cot\theta$ (b) $a\cos\theta$ (c) $a\tan\theta$ (d) $a\sin\theta$

70. The number of solutions of the trigonometric equation $1 + \cos x.\cos 5x = \sin^2 x$ in $[0, 2\pi]$ is

(a) 8 (b) 12 (c) 10 (d) 6

71. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, then the value of $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)$ is

(a) $(r-p)^2+(r-q)^2$ (b) $(1+p)^2+(1+q)^2$

(c) $(r+p)^2+(q+1)^2$ (d) $(r-p)^2+(q-1)^2$

72. If a system of three linear equations in three unknowns, which is in the matrix equation form of $AX = D$, is inconsistent, then $\frac{\text{rank of } A}{\text{rank of } AD}$ is

(a) less than one (b) greater than or equal to one

(c) one (d) greater than one

73. The angle between the two circles, each passing through the centre of the other is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) π

74. If $\log_{\frac{1}{\sqrt{3}}}\left\{\frac{|z|^2-|z|+1}{2+|z|}\right\}>-2$, then z lies inside

(a) a triangle (b) an ellipse

(c) a circle (d) a square

75. A circle having centre at the origin passes through the three vertices of an equilateral triangle the length of its median being 9 units. Then the equation of that circle is

(a) $x^2 + y^2 = 9$ (b) $x^2 + y^2 = 18$

(c) $x^2 + y^2 = 36$ (d) $x^2 + y^2 = 81$

76. $1 + \cos10° + \cos20° + \cos30° =$

(a) 4 sin10° sin20° sin30° (b) 4 cos5° cos10° cos15°

(c) 4 cos10° cos20° cos30° (d) 4 sin5°sin10°sin15°

77. The set of all values of a such that both the points (1, 2) and (3, 4) lie on the same side of the line $3x - 5y + a = 0$

(a) $\{x \in \text{IR} : x > 11\}$

(b) $\{x \in \text{IR} : x > 11\} \cup \{x \in \text{IR} : x < 7\}$

(c) $\{x \in \text{IR} : x < 7\}$

(d) ϕ

78. If $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+\ldots$ to infinite terms, then $9x^2 + 24x =$

(a) 31 (b) 11 (c) 41 (d) 21

79. The triad (x, y, z) of real number such that

$\left(3\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}\right)=\left(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)x+\left(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}\right)y+\left(-2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}\right)z$ is

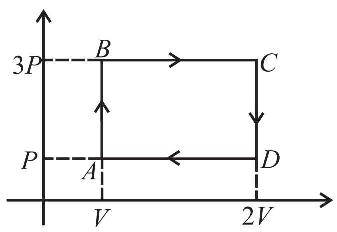
(a) (–2, 5, 3) (b) (2, –5, 3) (c) (2, 5, 3) (d) (2, 5, –3)

80. If the volume of the tetrahedron formed by the coterminous edges **a, b** and **c** is 4, then the volume of the parallelopiped formed by the coterminous edges **a × b, b × c** and **c × a** is

(a) 576 (b) 48 (c) 16 (d) 144

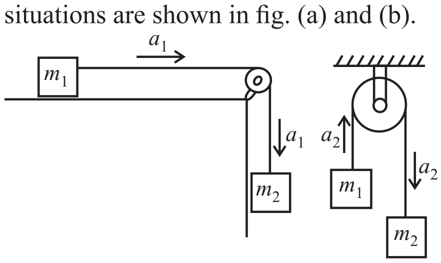
PHYSICS

81. A monoatomic ideal gas goes through a cyclic process as shown in the figure. The efficiency of this process is



(a) 78% (b) 42% (c) 62% (d) 21%

82. Two situations are shown in fig. (a) and (b).



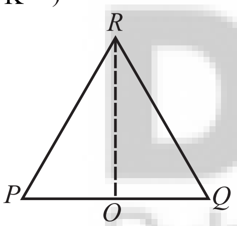
(a) (b)

In each case, m_1 = 3 kg and m_2 = 4 kg. If a_1, a_2 are the respective accelerations of the blocks in these situations, then the values of a_1 and a_2 are respectively [g = 10 ms^{-2}]

(a) $\frac{20}{7}\text{ms}^{-2}, \frac{10}{7}\text{ms}^{-2}$ (b) $\frac{10}{7}\text{ms}^{-2}, \frac{25}{7}\text{ms}^{-2}$

(c) $\frac{40}{7}\text{ms}^{-2}, \frac{10}{7}\text{ms}^{-2}$ (d) $\frac{30}{7}\text{ms}^{-2}, \frac{5}{7}\text{ms}^{-2}$

83. Three uniform thin aluminium rods each of length 2 m form an equilateral triangle PQR as shown in the figure. The mid point of the rod PQ is at the origin of the coordinate system. If the temperature of the system of rods increases by 50°C, the increase in y-coordinate of the centre of mass of the system of the rods is .... mm. (Coefficient of volume expansion of aluminium $= 12\sqrt{3}\times10^{-6}\text{K}^{-1}$)



(a) 0.05 (b) 0.8 (c) 0.1 (d) 0.2

84. An infinitely long thin straight wire has uniform linear charge density of $\frac{1}{3}$Cm^{-1}. Then the magnitude of the force acting on a charge 3 μC situated at a point of 18 cm away from the wire is

$$\left(\frac{1}{4\pi\varepsilon_0}=9\times10^9\,\text{Nm}^2\text{C}^{-2}\right)$$

(a) 2×10^5 N (b) 10^5 N

(c) $\frac{1}{3}\times10^6$ N (d) 3×10^{11} N

85. An electrostatic paint sprayer has a metal sphere of diameter 18 cm and at a potential of 25 kV. The charge on the metal sphere is

(a) 0.25 μC (b) 2.5 μC (c) 0.5 μC (d) 25 μC

86. A body is projected from the top of a tower with a velocity $\vec{u}=3\hat{i}+4\hat{j}+5\hat{k}$ ms^{-1}, where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along east, north and vertically upwards respectively. If the height of the tower is 30 m, horizontal range of the body on the ground is (g = 10 ms^{-2})

(a) 15 m (b) 25 m (c) 9 m (d) 12 m

87. Two long parallel conducting wires carrying currents are separated by a distance 'x'. Work done per unit length in changing the distance between the wires is proportional to

(a) $\frac{1}{\log_e x}$ (b) $\frac{1}{x}$ (c) $\log_e x$ (d) x

88. A Zener diode voltage regulator operated in the range 120V – 180V produces a constant supply of 110 V and 250 mA to the load. If the maximum current is equally shared between the load and the Zener diode, then the values of load resistance (R_L) and series resistance (R_S) are respectively

(a) R_L = 280 Ω, R_S = 70 Ω
(b) R_L = 440 Ω, R_S = 140 Ω
(c) R_L = 70 Ω, R_S = 280 Ω
(d) R_L = 440 Ω, R_S = 1400 Ω

89. Equation of a projectile is given by $y = Px - Qx^2$, where P and Q are constants. The ratio of maximum height to the range of the projectile is

(a) $\frac{Q^2}{2P}$ (b) $\frac{P^2}{Q}$ (c) $4P$ (d) $\frac{P}{4}$

90. The transverse displacement of a string of a linear density 0.01 kg m^{-1}, clamped at its ends is given by $Y_{(x,t)}=0.03\sin\left(\frac{2\pi x}{3}\right)\cos(60\pi t)$, where x and y are in metres and time t is in seconds. Tension in the string is

(a) 9 N (b) 36 N (c) 162 N (d) 81 N

91. A girl of mass 50 kg swinging on a cradle. If she moves with a velocity of 2 ms^{-1} upwards in a direction making an angle 60° with the vertical, then the power generated is (g = 9.8 ms^{-2})

(a) 245 W (b) $490\sqrt{2}$W
(c) $490\sqrt{3}$W (d) 980 W

92. Two point sources S_1 and S_2 are 24 cm apart. Where should a convex lens of focal length 9 cm be placed in between them, so that the images of both sources are formed at the same place?

(a) 8 cm (b) 12 cm (c) 6 cm (d) 10 cm

93. **Assertion (A) :** It is more difficult to push a magnet into a coil with more number of turns.

Reason (R) : The emf induced in a coil opposes the motion of a magnet when it is moved towards the coil.

(a) A is false, R is true
(b) Both A and R are true. R is correct explanation of A
(c) A is true, R is false
(d) Both A and R are true. R is not correct explanation of A

94. A wall is made of equally thick layers P and Q of different materials. Thermal conductivity of Q is half of that of the P. In the steady state, if the temperature difference across the wall is 24°C, then the temperature difference across the layer 'P' is ....

(a) 12°C (b) 16°C (c) 4°C (d) 8°C

95. In the determination of the internal resistance of a cell with a potentiometer, the error in the measurement of the balancing length is ± 1 mm. When the cell alone is connected in the circuit, the balancing length is obtained at 60 cm and when the cell is shunted with a resistance of 10 Ω ± 2%, the balancing length is obtained at 50 cm. The error in the determination of the internal resistance of the cell is ....

(a) 2.4% (b) 4.2% (c) 1.8% (d) 5.6 %

96. The half life of a stream of radioactive particles moving along a straight path with a constant kinetic energy of 4 eV is 1 minute. The percentage of particles which decay before travelling a distance of 3.6 km is (Mass of the radioactive particles = 3.2×10^{-21} kg and charge of the electron = 1.6×10^{-19}C).

(a) 87.5 (b) 175 (c) 37.5 (d) 75

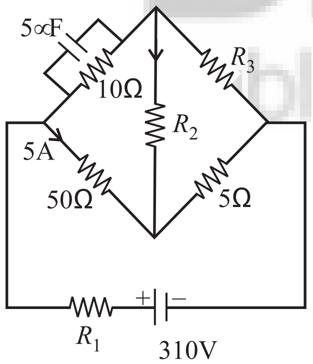
97. The process of recovering the modulating signal from the modulated carrier wave is called

(a) amplification (b) detection
(c) rectification (d) noise

98. Two bodies of masses 4m and 9m are separated by a distance 'r'. The gravitational potential at a point on this line joining them where the gravitational field becomes zero is

(a) $\dfrac{-25Gm}{r}$ (b) $\dfrac{-4Gm}{r}$
(c) $\dfrac{-9Gm}{r}$ (d) $\dfrac{-13Gm}{r}$

99. If the charge on the capacitor is 1 mC in the given circuit,



then $\dfrac{R_1R_2}{R_3} =$ .... Ω .

(a) 6 (b) 0.4 (c) 0.6 (d) 10

100. The amplitude of electric field in an electromagnetic wave is 60 Vm^{-1}. Then the amplitude of magnetic field is

(a) 2×10^{-7} T (b) 2×10^{7} T
(c) 6×10^{7} T (d) 6×10^{-7} T

101. A bird is tossing (flying to and fro) between two cars moving towards each other on a straight road. One car has speed of 54 kmh^{-1} while the other has the speed of 36 kmh^{-1}. The bird starts moving from first car towards the other and is moving with the speed of 36 kmh^{-1} where the two cars were separated by 36 km. The total distance covered by the bird before the cars meet each other is

(a) 14400 m (b) 1440 m
(c) 244 m (d) 24400 m

102. If the average translational kinetic energy of a molecule in a gas is equal to the kinetic energy of an electron accelerating from rest through 10 V, then the temperature of the gas molecule is

(Boltzmann constant = 1.38×10^{-23} JK^{-1})

(a) 7.73×10^3 K (b) 730 K
(c) 73.7 K (d) 77.3×10^3 K

103. A closed organ pipe of length 'L' and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. If the frequencies of their first overtones are same, then the length of the open organ pipe is

(a) $\dfrac{4L}{3}\sqrt{\dfrac{\rho_2}{\rho_1}}$ (b) $\dfrac{4L}{3}\sqrt{\dfrac{\rho_1}{\rho_2}}$

(c) $\dfrac{4L}{3}$ (d) $\dfrac{L}{3}$

104. One mole of a gas expands such that its volume 'V' changes with absolute temperature 'T' in accordance with the relation $V = KT^2$ where 'K' is a constant. If the temperature of the gas changes by 60°C, then work done by the gas is (R is universal gas constant).

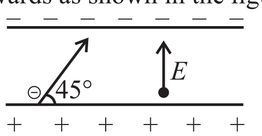
(a) $KR \ln 60$ (b) $R \ln 60$ (c) $40KR$ (d) $120\,R$

105. When a coil is connected to AC supply of frequency 50 Hz, a current of 4 A flows in it and it consumes 240 W power. If the potential difference across the coil is 100 V, then the inductance value of the coil is

(a) L = (5π)H (b) $L = \dfrac{\pi}{5}$ H

(c) $L = \dfrac{1}{5\pi}$ H (d) $L = \dfrac{1}{25\pi}$ H

106. The uniform electric field intensity between the two plates of a parallel plate capacitor is 1 × 103 Vm^{-1} acting vertically upwards as shown in the figure.



The plates are sufficiently long and have separation 2 cm. A particle of negative charge 1 μC and mass 2 g is projected at an angle 45° with the electric field from the lower plate with a velocity 'u'. The maximum velocity acquired by the particle, if it is not hit the upper plate is

(a) 2 ms^{-1} (b) 1 ms^{-1} (c) 0.1ms^{-1} (d) 0.2 ms^{-1}

107. Both an electron and a photon have same de-Broglie wavelength of 1.2 Å. The ratio of their energies is nearly

(a) 1 : 100 (b) 1 : 10 (c) 1 : 1000 (d) 1 : 1

108. When the terminals of a cell are connected by a wire of resistance 4 Ω, the potential difference across the cell is 1.6 V. If a wire of the same resistance is connected in parallel with the first, the potential difference becomes 1.33 V. The emf and internal resistance of the cell are respectively

(a) 1 V, 1 Ω (b) 2 V, 1 Ω (c) 1 V, 2 Ω (d) 2 V, 2 Ω

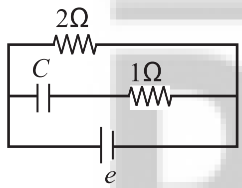
109. Two concentric coils of 20 turns each are placed in same plane. Their radii are 30 cm and 60 cm and carry 0.4 A and 0.6 A currents respectively in opposite directions. The magnetic induction at the centre in tesla is ....

(a) $\frac{8}{3}\mu_0$ (b) $\frac{2}{3}\mu_0$ (c) $\frac{5}{3}\mu_0$ (d) $\frac{10}{3}\mu_0$

110. Time period of a simple pendulum of length 'L' is T_1. Time period of a uniform rod of same length 'L' suspended from one end and oscillating in a vertical plane is T_2. Amplitude of oscillation is small in both the cases. Then $\frac{T_1}{T_2}$ is

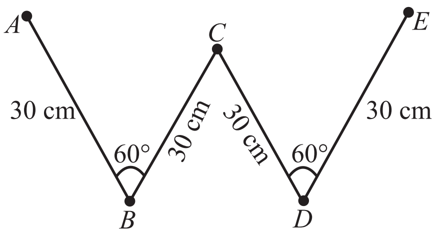
(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\sqrt{\frac{4}{3}}$ (d) 1

111. In steady state, a capacitor of capacitance 2 μ F is charged to 4 μC, as shown in figure. If the internal resistance of the cell is 0.5 Ω, then the emf of the cell is



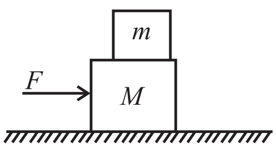
(a) 4 V (b) 5 V (c) 2.5 V (d) 2 V

112. A uniform thin rod of 120 cm length and 1600 g mass is bent as shown in the figure. The moment of inertia of the bent rod about an axis passing through the point 'O' and perpendicular to the plane of the paper is .... kg-m^2.



(a) 0.084 (b) 0.360 (c) 0.018 (d) 0.120

113. Two blocks of masses 'M' and 'm' are placed on one another on a smooth horizontal surface as shown in the figure.



The force 'F' is acting on the mass 'M' horizontally during time interval 't'. Assuming no relative sliding between the blocks, the work done by friction on the blocks is ....

(a) $\frac{Ft}{2(M+m)}$ (b) $\frac{M+m}{mt^2}$

(c) $\frac{mF^2t^2}{2(M+m)^2}$ (d) $\frac{F^2t^2}{(M+m)}$

114. A solid sphere is projected up along an inclined plane of inclination 30° with the horizontal with a speed of 4 ms^{-1}. If it rolls without slipping, the maximum distance traversed by it is .... (g = 10 ms^{-2})

(a) 2.24 m (b) 112 m (c) 1.12 m (d) 22.4 m

115. Fully filled open water tank has two holes on either sides of its walls. One is a square hole of side x cm at a depth of 2 m from the top, and the other hole is equilateral triangle of side 4 cm at a depth of 6 m from the top. If the rate of flow of water is same from both the holes, then 'x' is

(a) 1.73 cm (b) 12 cm (c) 6.92 cm (d) 3.46 cm

116. The radius of orbit of an electron and the speed of electron in the ground state of hydrogen atom are 5.5 × 10^{-11} m and 4 × 10^{6} ms^{-1} respectively. Then, the orbital period of this electron in the first excited state will be ....

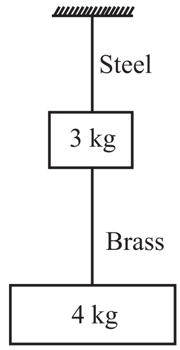
(a) 6.908 10^{-16} s (b) 9.608 × 10^{-16} s

(c) 7.806 × 10^{-16} s (d) 8.9068 × 10^{-16} s

117. Two slits separated by 0.5 mm are illuminated by light of wavelength 500 nm. The screen is at a distance of 120 cm from the slits. The phase difference between the interfering waves at a point 3 mm on the screen from the central bright fringe is ....

(a) 5π (b) π (c) 3π (d) 7π

118. The ratios of lengths, areas of cross-section and Young's modulii of steel to that of brass wires shown in the figure are a, b and c respectively. The ratio of increase in the lengths of brass to that of steel wires is [Assume that the masses of steel and brass wires are negligible]



(a) $\frac{4a}{7bc}$ (b) $\frac{7bc}{4a}$ (c) $\frac{4bc}{7a}$ (d) $\frac{7a}{4bc}$

119. A person of 60 kg mass is in a lift which is coming down such that the man exerts a force of 150 N on the floor of the lift. Then the acceleration of the lift is (g = 10 ms^{-2})

(a) 7.5 ms^{-2} (b) 40.0 ms^{-2}

(c) 22.5 ms^{-2} (d) 15.0 ms^{-2}

120. Match the following

List-I	List-II
(A) High retentivity	(i) Telephone diaphram
(B) High resistivity	(ii) Diamagnet
(C) Low coercivity	(iii) To decrease eddy current losses
(D) Negative susceptibility	(iv) Permanent magnet

The correct answer is

(a) A-(i), B-(iv), C-(iii), D-(ii)
(b) A-(iv), B-(iii), C-(i), D-(ii)
(c) A-(i), B-(ii), C-(iii), D-(iv)
(d) A-(iv), B-(ii), C-(i), D-(iii)

CHEMISTRY

121. (i) $H_3PO_4(aq) \rightleftharpoons H^+(aq) + H_2PO_4^-(aq)$

(ii) $H_2PO_4^-(aq) \rightleftharpoons H^+(aq) + HPO_4^{2-}(aq)$

(iii) $HPO_4^{2-}(aq) \rightleftharpoons H^+(aq) + PO_4^{3-}(aq)$

The equilibrium constants for the above reactions at a certain temperature are K_1, K_2 and K_3 respectively. The equilibrium constant for the reaction $H_3PO_4(aq) \rightleftharpoons 3H^+(aq) + PO_4^{3-}(aq)$ in terms of K_1, K_2 and K_3 is

(a) $K_1 + K_2 + K_3$ (b) $\frac{K_1}{K_2 + K_3}$
(c) $\frac{K_3}{K_1 K_2}$ (d) $K_1 K_2 K_3$

122. Which among the following are having diamagnetic property?

(i) B_2 (ii) N_2 (iii) O_2 (iv) C_2
(a) ii, iii (b) i, iv (c) ii, iv (d) i, ii

123. Which one of the following statements is not correct?

(a) The hydration enthalpies of alkali metal ions decrease down the group.
(b) Lithium halides are some what covalent in nature.
(c) Alkali metals react with water liberating oxygen gas.
(d) KO_2 is paramagnetic.

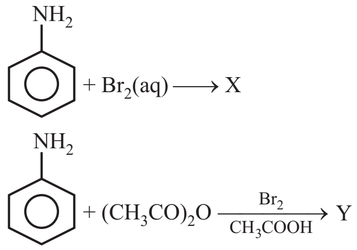
124. Which one of the following is more reactive towards S_N2 reaction ?

(a) $(CH_3)_3CX$ (b) $(CH_3)_2CHX$
(c) CH_3CH_2X (d) CH_3X

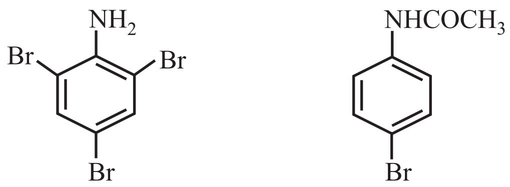
125. Identify, from the following, the diamagnetic, tetrahedral complex

(a) $[Ni(Cl)_4]^{2-}$ (b) $[Co(C_2O_4)_3]^{3-}$
(c) $[Ni(CN)_4]^{2-}$ (d) $[Ni(CO)_4]$

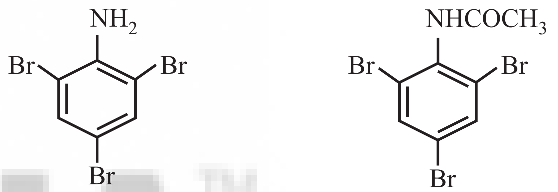
126. What are X and Y in the following reactions ?



X *Y*

(a) 

(b) 

(c) 

(d) 

127. Which of the following forms holes in the ozone layer?

(a) CO (b) SO_2 (c) CO_2 (d) CF_2Cl_2

128. Which one of the following is not used as an initiator in ionic polymerisation?

(a) $NaNH_2$ (b) $SnCl_2$
(c) $AlCl_3$ (d) $(C_6H_5CO)_2O_2$

129. Identify the statement which is not correct?

(a) Dehydrobromination of 2-bromopentane gives pent-1-ene as the major product.
(b) Freon 12 is manufactured by Swarts reaction.
(c) $CHCl_3$ is stored in closed, dark coloured bottles.
(d) Chronic exposure to $CHCl_3$ causes liver damage.

130. To prepare $XeF_6 \cdot Xe$ and F_2 are mixed at 573 K and 60-70 bar in the ratio of

(a) 20 : 1 (b) 1: 5 (c) 5 : 1 (d) 1: 20

131. Which one of the following solutions of compounds show highest osmotic pressure? (AB, AB_2 and A_2B_3 are ionic compounds)

(a) 5.0 M urea $i = 1.0$ and temperature is 67°C
(b) 1.5 M A_2B_3 type $i = 4.1$ and temperature is 27°C
(c) 3.0 M AB type $i = 1.6$ and temperature is 27°C
(d) 2.5 M AB_2 type $i = 2.5$ and temperature is 57°C

132. In which of the following reactions, hydrogen is liberated?
(i) $Al(s) + HCl(aq) \longrightarrow$ (ii) $Al(s) + NaOH(aq) \longrightarrow$
(iii) $NaBH_4 + I_2 \longrightarrow$
(a) i, ii (b) ii, iii (c) i, iii (d) i, ii, iii

133. 31g of ethylene glycol ($C_2H_6O_2$) is dissolved in 600 g of water. The freezing point depression of the solution is (K_f for water is 1.86 K kg mol^{-1})
(a) 0.77 K (b) 1.55 K (c) 4.65 K (d) 3.10 K

134. The equilibrium constant (K_C) for the following equilibrium

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$$

at 563 K is 100. At equilibrium, the number of moles of SO_3 in the 10 litre flask is twice the number of moles of SO_2, then the number of moles of oxygen is
(a) 0.4 (b) 0.3 (c) 0.2 (d) 0.1

135. The energy and radius of electron present in second orbit of He^+ respectively are
(a) -1.09×10^{-18} J, 105.8 pm
(b) -8.72×10^{-18} J, 211.6 pm
(c) -4.36×10^{-18} J, 52.9 pm
(d) -2.18×10^{-18} J, 105.8 pm

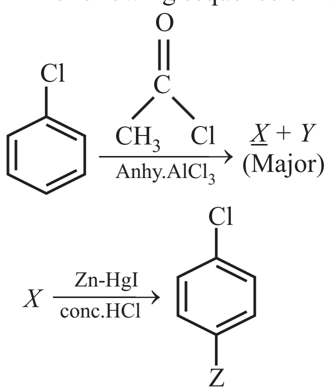
136. What are X and Y in the following reactions?

(i) $MnO_4^- + I^- \xrightarrow{H^+} X$ (ii) $MnO_4^- + I^- \xrightarrow{H_2O} Y$

(a) I_2, IO_4^- (b) I_2, IO_3^-
(c) IO_3^-, IO_3^- (d) IO_3^-, I_2

137. Assertion (A) : Na^+ and Mg^{2+} ions are isoelectronic but the ionic radius of Na^+ is greater than that of Mg^{2+}.
Reason (R) : The effective nuclear charge of Na^+ ion is less than that of Mg^{2+} ion.
(a) Both (A) and (R) are correct but (R) is not the correct explanation of (A).
(b) Both (A) and (R) are correct and (R) is correct explanation of (A).
(c) (A) is not correct but (R) is correct.
(d) (A) is correct but (R) is not correct.

138. What is $\underline{Z}$ in the following sequence of reactions?



(a) $-\underset{\underset{O}{\|}}{C}-CH_3$ (b) $-\underset{\underset{O}{\|}}{C}-Cl$
(c) $-CH_3$ (d) $-CH_2-CH_3$

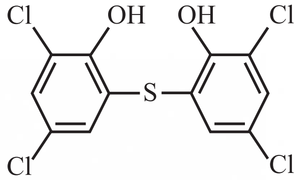
139. What are X and Y in the following reactions?

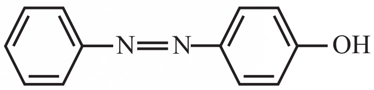
$$CH_2O \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } X} CH_3(CH_2)_2CH_2OH$$

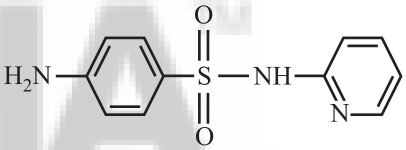
$$Y \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } C_2H_5MgBr} CH_3CH_2C(CH_3)_2OH$$

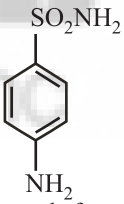
	X	***Y***
(a)	$CH_3-\underset{\underset{CH_3}{\vert}}{CH}-MgBr$	$C_2H_5COCH_3$
(b)	$CH_3CH_2CH_2MgBr$	$CH_3-CO-CH_3$
(c)	CH_3-CH_2-MgBr	CH_3CH_2CHO
(d)	$(CH_3)_3CMgBr$	$CH_3-CO-CH_3$

140. Which of the following structure represents the compound, generally added to soaps to impart antiseptic properties?

(a) 

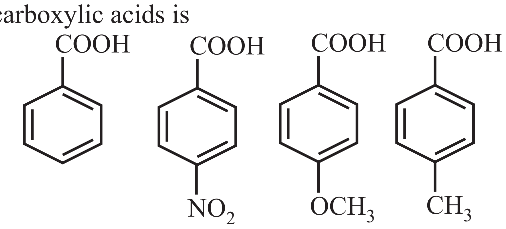
(b) 

(c) 

(d) 

141. The work functions of Ag, Mg, K and Na respectively in eV are 4.3, 3.7, 2.25, 2.30. When an electromagnetic radiation of wavelength of 300 nm is allowed to fall on these metal surface, the number of metals from which the electrons are ejected is
(**1eV** = 1.6022×10^{-19}J)
(a) 4 (b) 3 (c) 2 (d) 5

142. The increasing order of acidity of the following carboxylic acids is



I II III IV

(a) III < IV < II < I (b) II < III < IV < I
(c) I < II < IV < III (d) III < IV < I < II

143. Colloidal solution of gold is in different colours like red, purple, blue and golden because of

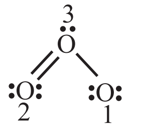
(a) variable oxidation states of gold.

(b) size difference in the particles of gold.

(c) presence of impurities.

(d) difference in the concentration of gold particles.

144. The Lewis structure for O_3 molecule is given below. The correct formal charges on oxygen atoms labelled 1, 2, 3 are respectively.



(a) –1, 0, + 1

(b) + 1, 0, – 1

(c) + 1, – 1, 0

(d) 0, + 1, – 1

145. Identify the statement which is not correct from the following.

(i) Carbohydrates are stored as glycogen in animals.

(ii) In glycylalanine, $\overset{\displaystyle O}{\overset{\|}{—C—}}$ of peptide bond belongs to alanine.

(iii) Base - sugar- phosphate unit is known as nucleoside

(iv) Obesity is due to hypothyroidism.

The correct answer is

(a) i, iv (b) ii, iii, iv

(c) i, iii, iv (d) ii, iii

146. The enthalpy of formation (ΔH_f) of methanol, formaldehyde and water are –239, –116 and –286 kJ mol^{-1} respectively. The enthalpy change for the oxidation of methanol to formaldehyde and water in kJ is

(a) – 136 (b) – 173

(c) 163 (d) – 163

147. Copper matte contains

(a) CuO, FeS (b) Cu_2S, FeS

(c) CuO, Cu_2S (d) Cu_2S, FeO

148. At 27°C in a 10 L flask 4.0 g of an ideal gaseous mixture containing. He (molar mass 4.0 g mol^{-1}) and Ne (molar mass 20g mol^{-1}) has a pres sure of 1.23 atm. What is the mass % of neon ? (R = 0.082 L atm $K^{-1}mol^{-1}$)

(a) 25.2 (b) 62.5

(c) 84.2 (d) 74.2

149. S + Conc. $H_2SO_4 \longrightarrow X + Y$

Here X is a gas and Y is a liquid and both are triatomic molecules. The number of electron lone pairs present on the central atoms of X and Y are respectively.

(a) 2, 1 (b) 1, 0

(c) 1, 2 (d) 2, 2

150. Identify the correct statements from the following

(i) Electromeric effect is a permanent effect.

(ii) Hyper conjugation is a temporary effect.

(iii) Fractional distillation is used to separate two liquids from a mixture if the difference in their boiling points is less.

(iv) Different compounds are adsorbed on an adsorbent different extents.

(a) ii, iii, iv (b) i, ii, iii

(c) ii, iv (d) iii, iv

151. Which of the following is used in the estimation of carbon monoxide?

(a) I_2O_4 (b) BrO_3

(c) Cl_2O_7 (d) I_2O_5

152. Which one of the following statements is not correct?

(a) In CO_2 molecule, carbon hybridisation is *sp*.

(b) Fullerenes are made by heating graphite in an electric arc in the presence of argon gas.

(c) Both $[SiF_6]^{2-}$ and $[SiCl_6]^{2-}$ ions are known.

(d) In CO molecule, there are one 'sigma' (σ) and two "pi" (π) bonds.

153. The drug, which was designed to prevent the interaction of histamine with the receptors present in the stomach wall is

(a) prontosil (b) cimetidine

(c) aspartame (d) equanil

154. Identify the correct statements for a ring system to exhibit aromaticity

(i) It must not be planar.

(ii) It must possess $(4n + 2)$ π-electrons.

(iii) It must be planar.

(iv) It must possess $4n$ π-electrons.

The correct answer is

(a) ii, iv (b) i, ii

(c) i, iv (d) ii, iii

155. Two oxides of a X contain 50% and 40% of non-metal respectively. If the formula of the first oxide is XO_2. Then the formula of second oxide is

(a) X_2O_3 (b) X_2O_5

(c) XO_3 (d) X_2O

156. An element has a body centered cubic structure with a unit cell edge length of 400 pm. Atomic mass of an element is 24 g mol^{-1}. What is the density of the element?

($N_A = 6 \times 10^{23}$ mol^{-1})

(a) 2.50 g cm^{-3}

(b) 1.80 g cm^{-3}

(c) 3.60 g cm^{-3}

(d) 1.25 g cm^{-3}

157. 20% of a first order reaction was found to be completed at 10:00 a.m at 11.30 a.m. on the same day, 20% of the reaction was found to be remaining. The half life period in minutes of the reaction is

(a) 90 (b) 45

(c) 60 (d) 30

158. The gaseous products formed at cathode (X) and anode (Y), when an aqueous solution of sodium acetate is electrolysed are

	X	Y
(a)	CO_2	C_2H_6, H_2
(b)	H_2, CO_2	C_2H_6
(c)	H_2	C_2H_6, CO_2
(d)	C_2H_6, H_2	CO_2

159. How many millilitres of 20 volume H_2O_2 solution is needed to react completely with 500 mL of acidified 1 N $KMnO_4$ solution?

(a) 224 (b) 280 (c) 140 (d) 56

160. Same amount of electricity is passed through aqueous solutions of $AgNO_3$ and $CuSO_4$. The number of Ag and Cu atoms deposited are x and y respectively. The correct relationship between x and y is

(a) $x < y$ (b) $x = 2y$ (c) $x = y$ (d) $y = 2x$

Hints & Solutions

MATHEMATICS

1. **(b)** We have, $b\cos\theta = c-a \Rightarrow \cos\theta = \dfrac{c-a}{b}$

$$\therefore \quad \sin\theta = \frac{\sqrt{b^2-(c-a)^2}}{b}$$

and $\tan\theta = \dfrac{\sqrt{b^2-(c-a)^2}}{(c-a)}$

Now, $(c-a)\tan\theta = \sqrt{b^2-(c-a)^2}$

$= \sqrt{b^2-c^2-a^2+2ac} = \sqrt{-\left(c^2+a^2-b^2\right)+2ac}$

$= \sqrt{2ac-2ac\cos B}$ [Using cosine rule]

$= \sqrt{2ac}\sqrt{1-\cos B}$

$= \sqrt{2}\sqrt{ac}\sqrt{2\sin^2\dfrac{B}{2}} = 2\sqrt{ac}\sin\dfrac{B}{2}$

2. **(d)** For a distribution of random variable x,

$\alpha = P(X^6 < 3) = P(X^6 = 1) + P(X^6 = 2) = \lambda + 2\lambda = 3\lambda$

and $\beta = P(X^6 > 2) = P(X^6 = 3) + P(X^6 = 4)$

$= 3\lambda + 4\lambda = 7\lambda$

$\therefore \quad \alpha : \beta = 3 : 7$

3. **(c)** Given function

$$f(x) = \begin{cases} x-1, & \text{for } x \le 1 \\ 2-x^2, & \text{for } 1 < x \le 3 \\ x-10, & \text{for } 3 < x < 5 \\ 2x, & \text{for } x \ge 5 \end{cases}$$

So, $f(x)$ will be continuous in the intervals $(-\infty, 1)$, $(1, 3)$, $(3, 5)$ and $(5, \infty)$

Now, let us check the continuity at $x = 1$, 3 and 5

Here,

(*i*) At $x = 1$,

$\lim\limits_{x\to1^-} f(x) = 1-1 = 0$ and $\lim\limits_{x\to1^+} f(x) = 2-1 = 1$

therefore F is not continuous at $x = 1$

(*ii*) At $x = 3$,

$\lim\limits_{x\to3^-} f(x) = 2-9 = -7$ and

$f(3) = -7$ and $\lim\limits_{x\to3^+} f(x) = 3-10 = -7$

therefore f is continuous at $x = 3$

(*iii*) At $x = 5$, $\lim\limits_{x\to5^-} f(x) = 5-10 = -5$ and,

$\lim\limits_{x\to5^+} f(x) = 2\times5 = 10$

therefore f is not continuous at $x = 5$

Hence, points of discontinuity of f are $\{1, 5\}$

4. **(a)** We have, equations of pair of straight lines

$$x^2 - 16pxy - y^2 = 0 \qquad \text{...}(i)$$

and $$x^2 - 16qxy - y^2 = 0 \qquad \text{...}(ii)$$

Now, equations of bisectors of these line are

$$-8px^2 - 2xy + 8py^2 = 0$$

i.e. $$4px^2 + xy - 4py^2 = 0 \qquad \text{...}(iii)$$

and $$-8qx^2 - 2xy + 8qy^2 = 0$$

i.e. $$4qx^2 + xy - 4qy^2 = 0 \qquad \text{...}(iv)$$

According to the given condition in the data, Eqs. (*i*) and (*iv*), and Eqs. (*ii*) and (*iii*) must be coincident. So,

$$\frac{1}{4q} = \frac{-16p}{1} = \frac{-1}{-4q}$$

$$1 = -64pq$$

$$\therefore \quad pq = \frac{-1}{64}$$

5. **(d)** Normal of the plane P_1 determined by the vectors $\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$,

$$n_1 = \left[\left(\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)\times\left(3\hat{\mathbf{j}}-2\hat{\mathbf{k}}\right)\right] = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 1 & -1 \\ 0 & 3 & -2 \end{vmatrix} = \hat{\mathbf{i}}$$

Normal of the plane P_2 determined by the vectors $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$,

$$n_2 = \left[\left(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}\right)\times\left(\hat{\mathbf{i}}-3\hat{\mathbf{j}}\right)\right]$$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & 0 \\ 1 & -3 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0) - \hat{\mathbf{j}}(0) + \hat{\mathbf{k}}(-6-3) = -9\hat{\mathbf{k}}$$

Since, a is parallel to the line of intersection of planes P_1 and P_2.

$$a = \pm(n_1 \times n_2) = \pm\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 0 \\ 0 & 0 & -9 \end{vmatrix} = \pm 9\hat{\mathbf{j}}$$

Now, angle θ between $9\hat{\mathbf{j}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ is

$$\cos\theta = \pm\left(\frac{9}{9\sqrt{3}}\right) = \pm\left(\frac{1}{\sqrt{3}}\right)$$

$$\therefore \quad \theta = \cos\left(\pm\frac{}{\sqrt{}}\right)$$

6. **(a)** Given, set $S = \{1, 2, ..., 10\}$

Here, three numbers are drawn at random from the given set. So, total possible outcomes, $n = {}^{10}C_3 = 120$

Let A be the event that minimum of chosen number is 3.

$\therefore \quad n(A) = \{4, 5, 6, 7, 8, 9, 10\} = {}^7C_2 = 21$

B be the event that maximum at chosen number is 7.

$\therefore \quad n(B) = \{1, 2, 3, 4, 5, 6\} = {}^6C_2 = 15$

So, $n(A \cap B) = \{4, 5, 6\} = {}^3C_1 = 3$

Hence, required probability

$$P = \frac{n(A)+n(B)-n(A\cap B)}{n} = \frac{21+15-3}{120}$$

$$P = \frac{11}{40}$$

7. **(a)** Let $y = \log\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$

$$= \log e^x + \log\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}} = x\log e + \frac{3}{4}\log\left(\frac{x-2}{x+2}\right)$$

$$= x + \frac{3}{4}\log\left(\frac{x-2}{x+2}\right)$$

Now, differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 1 + \frac{3}{4}\frac{d}{dx}\log\left(\frac{x-2}{x+2}\right)$$

$$= 1 + \frac{3}{4}\left[\frac{x+2}{x-2}\cdot\left\{\frac{(x+2)\cdot 1-(x-2)\cdot 1}{(x+2)^2}\right\}\right]$$

$$= 1 + \frac{3}{4}\times\frac{4}{x^2-4} \quad = 1 + \frac{3}{x^2-4}$$

At $x = 3$,

$$\left(\frac{dy}{dx}\right)_{x=3} = 1 + \frac{3}{5} = \frac{8}{5}$$

8. **(d)** Given, $y = \frac{\sinh^{-1}x}{\sqrt{1+x^2}}$

$$\sqrt{1+x^2}\,y = \sinh^{-1}x$$

On differentiating w.r.t. x, we get

$$\sqrt{1+x^2}\,y_1 + y\frac{1}{2\sqrt{1+x^2}}\cdot 2x = \frac{1}{\sqrt{1+x^2}}$$

$$(1+x^2)y_1 + xy = 1$$

Again, on differentiating w.r.t. x, we get

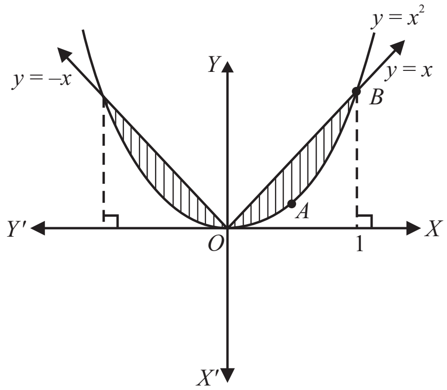
$$(1+x^2)y_2 + y_1\cdot 2x + xy_1 + y = 0$$

$$(1+x^2)y_2 + 3xy_1 + y = 0$$

9. **(c)** Given curves are

$$y = x^2 \qquad ...(i)$$

and $y = |x| = \begin{cases} x, & \text{if } x \ge 0 \\ -x, & \text{if } x < 0 \end{cases}$ ... (ii)



On solving Eqs. (i) and (ii), we get

$x = 0, x = 1$

Thus, coordinates of point B are (1, 1)

$\therefore$ Total area = 2 (Area of region $OABO$)

$$= 2\int_0^1 (x - x^2)\,dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= 2\times\frac{1}{6} = \frac{1}{3} \text{ sq unit}$$

10. **(c)** Let, $I = \int \frac{5x^2+3}{x^2(x^2-2)}dx$

Put $x^2 = y$. Then,

$$I = \int \frac{5y+3}{y(y-2)}dx \qquad ...(i)$$

Now, $\frac{5y+3}{y(y-2)} = \frac{A}{y} + \frac{B}{y-2}$ ...(ii)

$$5y+3 = (y-2)A + yB$$

$$5y+3 = y(A+B) - 2A$$

On comparing the coefficients, we get

$A + B = 5$ and $3 = -2A$

$A = \frac{-3}{2}$ and $B = \frac{13}{2}$

Thus, $\frac{5y+3}{y(y-2)} = \frac{5x^2+3}{x^2(x^2-2)} = -\frac{3}{2}\frac{1}{x^2} + \frac{13}{2}\frac{1}{x^2-2}$

Now, $I = -\frac{3}{2}\int\frac{dx}{x^2} + \frac{13}{2}\int\frac{dx}{x^2-2}$

$$= -\frac{3}{2}\left(-\frac{1}{x}\right) + \frac{13}{2}\times\frac{1}{2\sqrt{2}}\log\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + C$$

$$= \frac{3}{2x} + \frac{\sqrt{13}}{4\sqrt{2}}\log\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + C$$

11. **(a)** Given that,

$$y=\tan^{-1}\left\{\frac{x}{1+\sqrt{1-x^2}}\right\}+\sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$$

Let $x=\cos 2\theta$

$$y=\tan^{-1}\left\{\frac{\cos 2\theta}{1+\sqrt{\sin^2 2\theta}}\right\}+\sin\left\{2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right\}$$

$$\Rightarrow \quad y=\tan^{-1}\left\{\frac{\cos 2\theta}{1+\sin 2\theta}\right\}$$

$$+\sin\left\{2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right\}$$

$$\Rightarrow \quad y=\tan^{-1}\left\{\frac{\cos^2\theta-\sin^2\theta}{(\cos\theta+\sin\theta)^2}\right\}$$

$$+\sin\left\{2\tan^{-1}(\tan\theta)\right\}$$

$$\Rightarrow \quad y=\tan^{-1}\left\{\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right\}+\sin 2\theta$$

$$y=\tan^{-1}\left\{\frac{\cos\theta\left(1-\frac{\sin\theta}{\cos\theta}\right)}{\cos\theta\left(1+\frac{\sin\theta}{\cos\theta}\right)}\right\}+\sin 2\theta$$

$$\Rightarrow \quad y=\tan^{-1}\left\{\frac{1-\tan\theta}{1+\tan\theta}\right\}+\sin 2\theta$$

$$\Rightarrow \quad y=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)+\sin 2\theta$$

$$\Rightarrow \quad y=\frac{\pi}{4}-\theta+\sqrt{1-\cos^2 2\theta}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{2}.\frac{1}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{1-x^2}}(\ \ 2x)$$

Now, on differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{2}.\frac{1}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$\therefore \quad \frac{dy}{dx}=\frac{1-2x}{2\sqrt{1-x^2}}$$

12. **(b)** Let DR's of normal to the plane is < a, b, c >. Then, equation plane passing through (4, 4, 0)

$(r-a) . n=0$

$a(x-4)+b(y-4)+c\,(z)=0$

Since this plane is $\perp$ to the given planes. So, we get

$2a+b+2c=0$ and $3a+3b+2c=0$

By using cross-multiplication method,

$$\frac{a}{2-6}=\frac{-b}{4-6}=\frac{c}{6-3}$$

$$\Rightarrow \quad \frac{a}{-4}=\frac{b}{2}=\frac{c}{3}$$

So, the required equation of plane

$-4(x-4)+2(y-4)+3(z)=0$

$-4x+16+2y-8+3z=0$

$4x-2y-3z=8$

13. **(c)** We have,

$$\lim_{n\to\infty}\left[\frac{1^k+2^k+3^k+....+n^k}{n^{k+1}}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\frac{1^k+2^k+......+n^k}{n^k}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\left(\frac{1}{n}\right)^k+\left(\frac{2}{n}\right)^k+\left(\frac{3}{n}\right)^k+....+\left(\frac{n-1}{n}\right)^k+1\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\left(\frac{r}{n}\right)^k+\lim_{n\to\infty}\frac{1}{n}$$

$$=\int_0^1 x^k dx+0 \quad =\left[\frac{x^{k+1}}{k+1}\right]_0^1$$

$$\therefore \quad \lim_{n\to\infty}\left[\frac{1^k+2^k+3^k+....+n^k}{n^{k+1}}\right]=\frac{1}{k+1}$$

14. **(a)** As we know, coefficient of x^r in the binomial expansion of $(1+x)^n$ is given by ${}^nC_r$.

So, coefficient of x^5 in the binomial expansion of

$(1+x)^{21}+(1+x)^{22}+....+(1+x)^{30}$

$={}^{21}C_5+{}^{22}C_5+....+{}^{30}C_5$

$=\left({}^{21}C_6+{}^{21}C_5+{}^{22}C_5+....+{}^{30}C_5\right)-{}^{21}C_6$

$=\left({}^{22}C_6+{}^{22}C_5+....+{}^{30}C_5\right)-{}^{21}C_6$

$$\left[\because {}^nC_r+{}^nC_{r-1}={}^{n+1}C_r\right]$$

$=\left({}^{23}C_6+{}^{23}C_6+....+{}^{30}C_5\right)-{}^{21}C_6$

....

....

....

$=\left({}^{30}C_6+{}^{30}C_5\right)-{}^{21}C_6={}^{31}C_6-{}^{21}C_6$

15. **(a)** Here, function is defined for $f:A\to R$

$$f(x)=\begin{cases}3x-1 & \text{for} \quad x>3\\ x^2+1 & \text{for} \quad -3\le x\le 3\\ 2x-3 & \text{for} \quad x<-3\end{cases}$$

and $A=\{-4,-2,-1,0,3,5\}$

$f(-4)=2(-4)-3=-11$

$f(-2)=(-2)^2+1=4+1=5$

$f(-1) = (-1)^2 + 1 = 1 + 1 = 2$

$f(0) = 0^2 + 1 = 1$

$f(3) = 3^2 + 1 = 10$

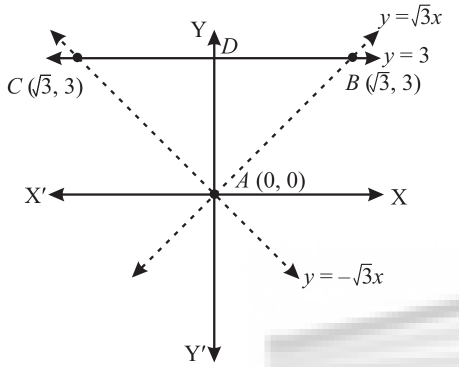
$f(5) = 3(5) - 1 = 14$

Hence, range of f is $\{-11, 5, 2, 1, 10, 14\}$.

16. **(a)** The triangle formed by the lines

$y = \sqrt{3}x, y = -\sqrt{3}x$ and $y = 3$

is shown in the figure.



We can observe that the triangle ABC is an isosceles triangle. Therefore, the incentre lie on the median to the base. Since D is mid-point of BC. So, OD is median to the base BC.

Thus, incentre of ΔABC lie on Y-axis.

17. **(b)** Here, the differential equation

$$\left(x - 4y^3\right)\frac{dy}{dx} - y = 0, \ (y > 0)$$

$$\Rightarrow \quad \left(x - 4y^3\right)\frac{dy}{dx} = y \quad \Rightarrow \quad \frac{dx}{dy} = \frac{x - 4y^3}{y}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{x}{y} - 4y^2 \quad \Rightarrow \quad \frac{dx}{dy} - \frac{x}{y} = -4y^2$$

$$\therefore \quad P = -\frac{1}{y} \cdot Q = -4y^2$$

Integrating factor,

$$\text{I.F.} = e^{\int P\,dy} = e^{\int -\frac{1}{y}dy} = e^{-\log y} = e^{\log y^{-1}} = \frac{1}{y}$$

Now, the solution is given by

$$x \cdot (\text{I.F.}) = \int (\text{I.F.}) \cdot Q\,dy + C$$

$$\frac{x}{y} = \int \frac{1}{y} \times \left(-4y^2\right)dy + C$$

$$\frac{x}{y} = \int -4y\,dy + C$$

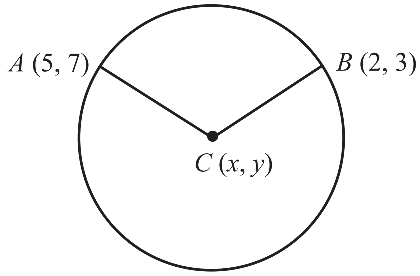
$$\frac{x}{y} = -4\frac{y^2}{2} + C$$

$$x = -2y^3 + Cy$$

$$x + 2y^3 = Cy$$

$\therefore$ This is the required solution.

18. **(d)** Let centre of the circle be $c\,(x, y)$.



Here, CA and CB both represents the radius of the circle. So, it will be equal.

Then, $\quad CA = CB$

$\Rightarrow \quad CA^2 = CB^2$

$\Rightarrow \quad (5 - x)^2 + (7 - y)^2 = (2 - x)^2 + (3 - y)^2$

$\Rightarrow \quad 25 + x^2 - 10x + 49 + y^2 - 14y$

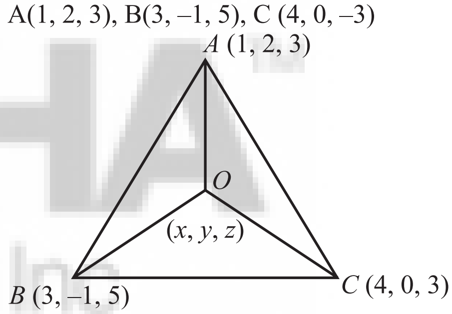
$\qquad = 4 + x^2 - 4x + 9 + y^2 - 6y$

$\Rightarrow \quad 74 - 13 = 6x + 8y$

$\Rightarrow \quad 6x + 8y = 61$

This equation is satisfied by the coordinates given in option (d).

19. **(d)** Let A, B, C be the vertices of the triangle given as, A(1, 2, 3), B(3, –1, 5), C (4, 0, –3)



Now, let $O\,(x, y, z)$ be the circumcentre of ΔABC.

$\therefore \quad OA = OB = OC$

$OA = OB \Rightarrow OA^2 = OB^2$

$\Rightarrow \quad (x - 1)^2 + (y - 2)^2 + (z - 3)^2$

$\qquad = (x - 3)^2 + (y + 1)^2 + (z - 5)^2$

$x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9$

$\qquad = x^2 - 6x + 9 + y^2 + 2y + 1 + z^2 - 10z + 25$

$\Rightarrow \quad 4x - 6y + 4z - 21 = 0 \qquad ...(i)$

Similarly, $OB = OC$

$(x - 3)^2 + (y + 1)^2 + (z - 5)^2$

$\qquad = (x - 4)^2 + (y - 0)^2 + (z + 3)^2$

$x^2 - 6x + 9 + y^2 + 2y + 1 + z^2 - 10z + 25$

$\qquad = x^2 - 8x + 16 + y^2 + z^2 + 6z + 9$

$\Rightarrow \quad x + y - 8z + 5 = 0 \qquad ...(ii)$

Similarly, $OA = OC$

$$(x-1)^2+(y-2)^2+(z-3)^2$$
$$=(x-4)^2+(y-0)^2+(z+3)^2$$
$$x^2-2x+1+y^2-4y+4+z^2-6z+9$$
$$=x^2-8x+16+y^2+z^2+6z+9$$
$$\Rightarrow \quad 6x-4y-12z-11=0 \quad ...(iii)$$

On solving Eqs. (*i*), (*ii*) and (*iii*), we get

$$x=\frac{7}{2},\ y=-\frac{1}{2},\ z=1$$

$\therefore$ Circumcentre of ΔABC is $\left(\frac{7}{2},\frac{-1}{2},1\right)$

20. **(a)** Here, W = White, B = Black. Given that bag P contains $5W$ marbles and $3B$ marbles. 4 marbles are drawn from the bag P and A black marble is drawn from bag Q.

Let Event E_1: 1W and 3B marbles are transferred

Event E_2: 2W and 2B marbles are transferred

Event E_3: 3W and 1B marbles are transferred

Event E_4: 4W and 0B marbles are transferred

and Event A: a black marble is drawn from bag Q.

Then, $P(E_1)=\frac{{}^5C_1\times{}^3C_3}{{}^8C_4}=\frac{5}{{}^8C_4}$

$$P(E_2)=\frac{{}^5C_2\times{}^3C_2}{{}^8C_4}=\frac{30}{{}^8C_4}$$
$$P(E_3)=\frac{{}^5C_3\times{}^3C_1}{{}^8C_4}=\frac{30}{{}^8C_4}$$
$$P(E_4)=\frac{{}^5C_4\times{}^3C_0}{{}^8C_4}=\frac{5}{{}^8C_4}$$
$$P(A/E_1)=\frac{3}{4}$$
$$P(A/E_2)=\frac{2}{4}$$
$$P(A/E_3)=\frac{1}{4}$$
$$P(A/E_4)=0$$

By using Bayes' theorem,

$$P(E_1/A)=\frac{P(E_1)\cdot P(A/E_1)}{(P(E_1)\cdot P(A/E_1)+P(E_2)\cdot P(A/E_2)+P(E_3)P(A/E_3)+P(E_4).P(A/E_4)}$$
$$=\frac{\frac{5}{{}^8C_4}\cdot\frac{3}{4}}{\frac{5}{{}^8C_4}\cdot\frac{3}{4}+\frac{30}{{}^8C_4}\times\frac{2}{4}+\frac{30}{{}^8C_4}\times\frac{1}{4}+0}$$
$$=\frac{15}{15+60+30}=\frac{15}{105}=\frac{1}{7}$$

21. **(a)** We have,

$$\sec h^{-1}\left(\frac{1}{\sqrt{2}}\right)+\operatorname{cosec} h^{-1}(-1)$$
$$=\log_e\left(\frac{1+\sqrt{1-1/2}}{\frac{1}{\sqrt{2}}}\right)+\log_e\left(\frac{1-\sqrt{1+1}}{-1}\right)$$
$$=\log_e\left(\frac{\frac{\sqrt{2}+1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right)+\log_e\left(\sqrt{2}-1\right)$$
$$=\log_e\left(\sqrt{2}+1\right)+\log_e\left(\sqrt{2}-1\right)$$
$$=\log_e\left[\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)\right]$$
$$\left[\because \log m+\log n=\log(m\times n)\right]$$
$$=\log_e\left(\left(\sqrt{2}\right)^2-1\right)=\log_e(2-1)=\log_e 1$$
$$\therefore \quad \sec h^{-1}\left(\frac{1}{\sqrt{2}}\right)+\operatorname{cosec} h^{-1}(-1)=0$$

22. **(a)** Let A, B, C and D are the position vectors of four points given as

$A\left(3\hat{i}-2\hat{j}-\hat{k}\right), B\left(2\hat{i}+3\hat{j}-4\hat{k}\right), C\left(-\hat{i}+\hat{j}+2\hat{k}\right)$

and $D\left(4\hat{i}+5\hat{j}+\lambda\hat{k}\right)$

Here, $\quad AB=-\hat{i}+5\hat{j}-3\hat{k}$

$\quad BC=-3\hat{i}-2\hat{j}+6\hat{k}$

and $\quad CD=5\hat{i}+4\hat{j}+(\lambda-2)\hat{k}$

As given that, these points are coplanar, therefore $[AB\ \ BC\ \ CD]=0$

$$\begin{vmatrix}-1 & 5 & -3\\ -3 & -2 & 6\\ 5 & 4 & \lambda-2\end{vmatrix}=0$$
$$\Rightarrow \quad -1(-2\lambda+4-24)-5(-3(\lambda-2)-30)$$
$$-3(-12+10)=0$$
$$\Rightarrow \quad 2\lambda+20+15(\lambda-2)+150+36-30=0$$
$$\Rightarrow \quad 17\lambda+20-30+150+6=0$$
$$\Rightarrow \quad 17\lambda+146=0 \Rightarrow \lambda=-\frac{146}{17}$$

23. **(a)** Given, equation of tangents to the ellipse are,

$$y=2x+\sqrt{76} \text{ and } 2y+x=8 \Rightarrow y=-\frac{1}{2}x+4$$
$$\Rightarrow \quad m_1=2 \text{ and } m_2=--$$

$\because$ Here, $m_1\times m_2=-1$

It means that the given tangents are perpendicular to each other.

Thus, required circle will be the director circle to ellipse.

Hence, required equation of circle is

$$x^2 + y^2 = a^2 + b^2 = 16 + 12$$

$$\Rightarrow \quad x^2 + y^2 = 28$$

24. **(a)** Given pair of lines,

$$4xy + 2x + 6y + 3 = 0$$

$$\Rightarrow \quad 2x(2y+1) + 3(2y+1) = 0$$

$$\Rightarrow \quad (2x+3)(2y+1) = 0$$

So, equations of lines are $x = \frac{-3}{2}$ and $y = -\frac{1}{2}$

Equations of line passing through (2, 1) and perpendicular to the pair of lines $4xy + 2x + 6y + 3 = 0$ are

$$y - 1 = 0 \text{ and } x - 2 = 0$$

$\therefore$ Combining the equation of lines are

$$(y-1)(x-2) = 0 \Rightarrow xy - x - 2y + 2 = 0$$

25. **(b)** Let two numbers be a and b

$$\text{G.M.} = \sqrt{ab} = 2$$

$\therefore \quad ab = 4 \quad ...(i)$

Now, $\quad \text{H.M.} = \frac{2ab}{a+b} = -\frac{8}{5}$

$\frac{2 \times 4}{a+b} = -\frac{8}{5}$ (from Eq. (i))

$\therefore \quad a + b = -5$

Here, roots are twice of these numbers. So,

$(2a)(2b) = 4ab = 4 \times 4 = 16$

$2a + 2b = 2(a + b) = 2 \times -5 = -10$

$\therefore$ Quadratic Equation $x^2 + 10x + 16 = 0$

26. **(c)** We have given, $|z| \geq 5$

Now, $\left| z + \frac{2}{z} \right| \geq |z| - \left| \frac{2}{z} \right| = \left| |z| + \frac{2}{z} \right|$

$= \left| |z| + 2\left(\frac{-1}{|z|} \right) \right| \geq \left| 5 - \frac{2}{5} \right|$

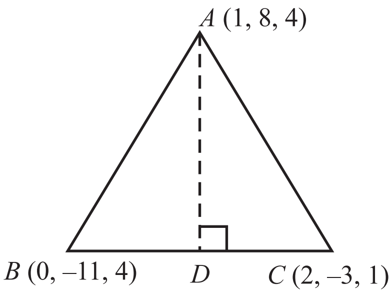
$\therefore \quad \left| z + \frac{2}{z} \right| \geq \frac{23}{5}$

Thus, the least value of $\left| \quad \frac{2}{} \right|$ is $\frac{23}{}$

27. **(b)** The vertices of ΔABC are given as A (1, 8, 4), B (0, –11, 4) and C (2, –3, 1).

Equation of line BC,

$$\frac{x}{2} = \frac{y+11}{8} = \frac{z-4}{-3} = \lambda \text{ (say)}$$



As point D is on line BC, so coordinates of D are $(2\lambda, 8\lambda - 11, -3\lambda + 4)$

Since, $AD \perp BC$

$\therefore \quad AD \cdot BC = 0$

$(2\lambda - 1, 8\lambda - 19, -3\lambda) \cdot (2, 8, -3) = 0$

$\therefore \quad 2(2\lambda - 1) + 8(8\lambda - 19) + (-3)(-3\lambda) = 0$

$4\lambda - 2 + 64\lambda - 152 + 9\lambda = 0$

$77\lambda = 154$

$\lambda = 2$

Hence, the coordinates of D are (4, 5, –2)

28. **(d)** Given function, $f(x) = (x-1)(x-2)$

$= x^2 - 3x + 2$ and $x \in \left[0, \frac{1}{2}\right]$

By using Lagrange's mean value theorem,

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f\left(\frac{1}{2}\right) - f(0)}{\frac{1}{2} - 0}$$

$$\Rightarrow \quad 2 \quad -3 = \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) - (-1)(-2)}{-}$$

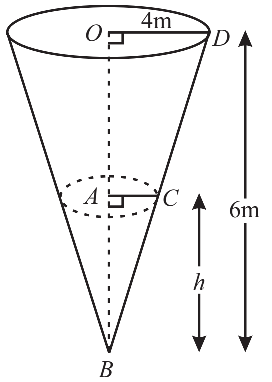
$$\Rightarrow \quad 2c - 3 = 2\left(\frac{3}{4} - 2\right)$$

$$\Rightarrow \quad 2c - 3 = -\frac{5}{2}$$

$$\Rightarrow \quad 4c - 6 = -5$$

$$\Rightarrow \quad c = \frac{1}{4} \in \left(0, \frac{1}{2}\right)$$

29. **(a)** Let V be the volume, r be the radius and h be the height of an inverted cone at any time t. Then



Given, $\frac{dv}{dt} = 3\,\text{m}^3/\text{min}$

We know that,

$$V = \frac{1}{3}\pi r^2 h \qquad \text{...}(i)$$

From similarity of triangle

$$\frac{BO}{BA} = \frac{OD}{AC}$$

$$\Rightarrow \quad \frac{6}{h} \quad \frac{4}{r} \Rightarrow r = \frac{2}{3}h$$

From Eq. (i),

$$V = \frac{1}{3}\pi\left(\frac{2}{3}h\right)^2 h = \frac{4}{27}\pi h^3$$

On differentiating w.r.t. t, we get

$$\Rightarrow \quad \frac{dv}{dt} = \frac{4}{27}\pi 3h^2 \frac{dh}{dt}$$

$$\Rightarrow \quad 3 = \frac{4}{27}\pi\, 3h^2 \frac{dh}{dt}$$

$$\Rightarrow \quad \frac{dh}{dt} = \frac{3\times 27}{4\pi\, 3h^2}$$

$$\Rightarrow \quad \left(\frac{dh}{dt}\right)_{h=3} = \frac{3\times 27}{4\pi\, 27} = \frac{3}{4\pi}\,\text{mt/min}$$

30. **(d)** Given equation, $x^3 - 7x^2 + 14x - 8 = 0$

Let the roots of this equation are $\frac{a}{r}, a, ar$.

Then, we get

Sum of roots = 7

$$\frac{a}{r} + a + ar = 7 \qquad \text{...}(i)$$

Product of consecutive roots = 14

$$\frac{a}{r}\cdot a + a\cdot ar + ar\cdot\frac{a}{r} = 14 \qquad \text{...}(ii)$$

Product of roots = 8

$$\frac{a}{r}\cdot a\cdot ar = 8 \qquad \text{...}(iii)$$

From Eq. (iii), we get $a = 2$

Now, from Eq. (i), we get

$$\frac{2}{r} + 2r = 5$$

$$\Rightarrow \quad 2 + 2r^2 = 5r$$

$$\Rightarrow \quad 2r^2 - 5r + 2 = 0$$

$$\Rightarrow \quad 2r^2 - 4r - r + 2 = 0$$

$$\Rightarrow \quad 2r(r-2) - 1(r-2) = 0$$

$$\Rightarrow \quad (r-2)(2r-1) = 0$$

$$\Rightarrow \quad r = 2 \text{ or } r = \frac{1}{2}$$

Thus, the roots are 1, 2, 4.

Hence, difference of largest and smallest number $= 4 - 1 = 3$

31. **(c)** For a binomial variate x,

$$\text{Mean, } np = \frac{4}{3} \text{ and} \qquad \text{...}(i)$$

$$\text{Variance, } npq = \frac{8}{9} \qquad \text{...}(ii)$$

From Eqs. (i) and (ii), we have

$$q = \frac{2}{3}, p = \frac{1}{3} \text{ and } n = 4$$

$$\text{Now, } P(X = 2) = {}^4C_2\; p^2 q^2$$

$$= 6\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 = \frac{24}{81} = \frac{8}{27}$$

32. **(c)** We have given that α is non-real root of $x^7 = 1$

$\therefore\ \alpha^7 = 1$ and $\alpha \neq 1$

Now, $\alpha(1 + \alpha)(1 + \alpha^2 + \alpha^4)$

$= \alpha(1 + \alpha^2 + \alpha^4 + \alpha + \alpha^3 + \alpha^5)$

$= \alpha + \alpha^3 + \alpha^5 + \alpha^2 + \alpha^4 + \alpha^6$

$= \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6$

$$= \frac{\alpha(1-\alpha^6)}{1-\alpha} \quad [\because\ \alpha \neq 1]$$

$$= \frac{\alpha - \alpha^7}{1-\alpha} = \frac{\alpha - 1}{1-\alpha} \quad [\because\ \alpha^7 = 1]$$

$$\therefore \quad \alpha(1+\alpha)(1+\alpha^2+\alpha^4) = -1$$

33. **(a)** Given that,

$$\cot(\cos^{-1}x) = \sec\left\{\tan^{-1}\left(\frac{a}{\sqrt{b^2-a^2}}\right)\right\}$$

$$\text{Since, } \cos^{-1}x = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\text{and } \tan^{-1}x = \sec^{-1}\left(\sqrt{1+x^2}\right)$$

$$\Rightarrow \quad \cot\left(\cot^{-1}\frac{x}{\sqrt{1-x^2}}\right) = \sec$$

$$\left\{\sec^{-1}\sqrt{1+\left(\frac{a}{\sqrt{b^2-a^2}}\right)^2}\right\}$$

$$\Rightarrow \quad \frac{x}{\sqrt{1-x^2}} = \sqrt{\frac{b^2-a^2+a^2}{b^2-a^2}}$$

$$\Rightarrow \quad \frac{x}{\sqrt{1-x^2}} = \frac{b}{\sqrt{b^2-a^2}}$$

On squaring both the sides, we get

$$\frac{x^2}{1-x^2}=\frac{b^2}{b^2-a^2}$$

$\Rightarrow \quad x^2b^2-x^2a^2=b^2-b^2x^2$

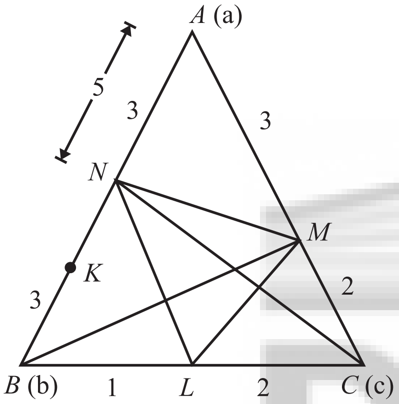
$x^2b^2+b^2x^2-x^2a^2=b^2$

$2x^2b^2-x^2a^2=b^2$

$\Rightarrow \quad x^2\left(2b^2-a^2\right)=b^2$

$\Rightarrow \quad x=\dfrac{b}{\sqrt{2b^2-a^2}}$

34. (d) Given that, in ΔABC, L, M, N are points on BC, CA and AB respectively, dividing in the ratio 1 : 2, 2 : 3 and 3 : 5. Also, point K divides AB in the ratio 5 : 3.



Now, according to the section formula,

$L=\dfrac{2\mathbf{b}+\mathbf{c}}{3},\ M=\dfrac{2\mathbf{a}+3\mathbf{c}}{5}$

$N=\dfrac{3\mathbf{b}+5\mathbf{a}}{8},\ K=\dfrac{5\mathbf{b}+3\mathbf{a}}{8}$

$\overrightarrow{AL}=\dfrac{2\mathbf{b}+\mathbf{c}}{3}-\mathbf{a}=\dfrac{2\mathbf{b}+\mathbf{c}-3\mathbf{a}}{3}$

$\overrightarrow{BM}=\dfrac{2\mathbf{a}+3\mathbf{c}}{5}-\mathbf{b}=\dfrac{2\mathbf{b}+3\mathbf{c}-5\mathbf{b}}{5}$

$\overrightarrow{CN}=\dfrac{3\mathbf{b}+5\mathbf{a}}{8}-\mathbf{c}=\dfrac{3\mathbf{b}+5\mathbf{a}-8\mathbf{c}}{8}$

$\overrightarrow{CK}=\dfrac{5\mathbf{b}+3\mathbf{a}}{8}-\mathbf{c}=\dfrac{5\mathbf{b}+3\mathbf{a}-8\mathbf{c}}{8}$

$\because \quad \left|\dfrac{\overrightarrow{AL}+\overrightarrow{BM}+\overrightarrow{CN}}{\overrightarrow{CK}}\right|=$

$$\left|\frac{\frac{2\mathbf{b}+\mathbf{c}-3\mathbf{a}}{3}+\frac{2\mathbf{a}+3\mathbf{c}-5\mathbf{b}}{5}+\frac{3\mathbf{b}+5\mathbf{a}-8\mathbf{c}}{8}}{\frac{5\mathbf{b}+3\mathbf{a}-8\mathbf{c}}{8}}\right|$$

$$=\left|\frac{(5\mathbf{b}+3\mathbf{a}-8\mathbf{c})}{15(5\mathbf{b}+3\mathbf{a}-8\mathbf{c})}\right|=\frac{1}{15}$$

35. (c) $2x^2+4xy-6y^2+2x+8y+1=0$

is in the form of

$ax^2+2hxy+by^2+2gx+2fy+c=0$, we get

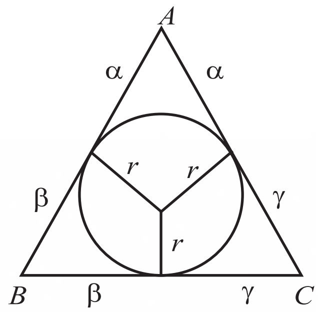
$a=2,\ h=2,\ b=-6,\ g=1,\ f=4,\ c=1$

Required point

$$=\left(\frac{bg-fh}{h^2-ab},\frac{af-gh}{h^2-ab}\right)=\left(\frac{-6-8}{4+12},\frac{8-2}{4+12}\right)$$

$$=\left(\frac{-14}{16},\frac{6}{16}\right)=\left(\frac{-7}{8},\frac{3}{8}\right)$$

36. (a) It is given that α, β, γ are the length of tangents from the vertices of a triangle to its incircle.



Semi-perimeter of ΔABC,

$\Rightarrow \quad S=\alpha+\beta+\gamma$

Area of $\Delta ABC=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{(\alpha+\beta+\gamma)(\alpha)(\beta)(\gamma)}$

As we know,

$r=\Delta/s$

$\because \quad r=\dfrac{\sqrt{(\alpha+\beta+\gamma)(\alpha\beta\gamma)}}{\alpha+\beta+\gamma}$

$\Rightarrow \quad r^2=\dfrac{(\alpha+\beta+\gamma)(\alpha\beta\gamma)}{(\alpha+\beta+\gamma)^2}$

$\Rightarrow \quad$ Hence, $\alpha+\beta+\gamma=\dfrac{\alpha\beta\gamma}{r^2}$

37. (c) Given, $\displaystyle\int_0^{10} f(x)\,dx=5$ ...(*i*)

Let $I=\displaystyle\int_0^1 f(k-1+x)dx$

Put $k-1+x=t \Rightarrow dx=dt$

Limits:

$x=0 \Rightarrow t=k-1$

$x=1 \Rightarrow t=k$

$\therefore \quad I = \int_{k-1}^{k} f(t)\,dt$

It can also be written as,

$I = \int_{k-1}^{k} f(x)\,dx$

Now, $\sum_{k=1}^{10} \int_0^1 f(k-1+x)\,dx$

$= \sum_{k=1}^{10} \int_{k-1}^{k} f(x)\,dx$

$= \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + ... + \int_9^{10} f(x)\,dx$

$= \int_0^{10} f(x)\,dx = 5$ (from Eq. (*i*))

38. **(a)** Given Ellipse,

$3x^2 + 2y^2 - 5 = 0$

Then, equations of pairs of tangents drawn to the ellipse S from the point (1, 2).

$SS_1 = T^2$

$$\left(3x^2 + 2y^2 - 5\right)\left(3(1)^2 - 2(2)^2 - 5\right) = \left(3x(1) + 2y(2) - 5\right)^2$$

$$\left(3x^2 + 2y^2 - 5\right)(6) = (3x + 4y - 5)^2$$

$$18x^2 + 12y^2 - 30 = 9x^2 + 16y^2 + 25 + 24xy - 40y - 30$$

$$9x^2 - 4y^2 - 24xy + 30x + 40y - 55 = 0$$

Now, the angle θ between these lines is given by

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\tan\theta = \frac{2\sqrt{144+36}}{5} = \frac{2\sqrt{180}}{5} = \frac{12\sqrt{5}}{5}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{12\sqrt{5}}{5}\right)$$

39. **(d)** Given equation of normal, $lx + my = 1$

$y = \frac{-l}{m} + \frac{1}{m}$

Which is of the form $y = mx + c$

Since, the lines $y = mx + c$ will be normal to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \text{ if } c^2 = \frac{m^2\left(a^2+b^2\right)^2}{a^2 - b^2m^2}$$

$$\therefore \quad \left(\frac{1}{m}\right)^2 = \frac{\left(-\frac{l}{m}\right)^2\left(a^2+b^2\right)^2}{a^2 - b^2\left(-\frac{l}{m}\right)^2}$$

$$\frac{1}{m^2} = \frac{\frac{l^2}{m^2}\left(a^2+b^2\right)^2}{\frac{m^2a^2 - l^2b^2}{m^2}}$$

$$\frac{1}{m^2} = \frac{l^2\left(a^2+b^2\right)^2}{m^2a^2 - l^2b^2}$$

$$a^2m^2 - b^2l^2 = l^2m^2\left(a^2+b^2\right)^2$$

40. **(c)** Let $I = \int \frac{x^{e-1} + e^{x-1}}{x^e + e^x}\,dx$ ...(*i*)

Now, put $x^e + e^x = t$ ...(*ii*)

$\Rightarrow \quad \left(ex^{e-1} + e^x\right)dx = dt$

$\Rightarrow \quad e\left(x^{e-1} + \frac{e^x}{e}\right)dx = dt$

$\Rightarrow \quad e\left(x^{e-1} + e^{x-1}\right)dx = dt$

$\Rightarrow \quad \left(x^{e-1} + e^{x-1}\right)dx = \frac{dt}{e}$ ...(*iii*)

Now, putting values from Eqs. (*ii*) and (*iii*) in Eq. (*i*)

$\therefore \quad I = \frac{1}{e}\int \frac{dt}{t} = \frac{1}{e}\log|t| + C$

$= \frac{1}{e}\log\left|x^e + e^x\right| + C$

41. **(a)** Given that,

$$\Delta = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ 0 & 0 & 2 \end{vmatrix} \quad [\text{Applying } R_3 \to R_3 + R_1]$$

Now, on expanding along R_3, we get

$\Delta = 2\left(1 + \cos^2\theta\right)$

As we know,

$0 \le \cos^2\theta \le 1$

$1 \le 1 + \cos^2\theta \le 2$

$\Rightarrow \quad 2 \le 2\left(1 + \cos^2\theta\right) \le 4$

Thus, Δ lies in the interval [2, 4].

42. **(a)** Given, $S_1 \equiv x^2 + y^2 + 2x + 2y + 1 = 0$

and $S_2 \equiv x^2 + y^2 + 4x + 6y + 4 = 0$

Now, the equation of common chord of these two circles.

$S_1 - S_2 = 0$

$$\left(x^2+y^2+2x+2y+1\right)-$$
$$\left(x^2+y^2+4x+6y+4\right)=0$$

$\Rightarrow \quad -2x-4y-3=0$ i.e. $2x+4y+3=0$

The equation of required circle,

$$S_1+\lambda\left(S_2-S_1\right)=$$
$$\left(x^2+y^2+2x+2y+1\right)+\lambda\left(2x+4y+3\right)=0$$
$$x^2+y^2+2x\left(1+\lambda\right)+2y\left(2\lambda+1\right)+3\lambda+1=0$$

Diameter of the above circle is the common chord of circles S_1 and S_2.

So, $2\left(-\left(1+\lambda\right)\right)+4\left(-\left(2\lambda+1\right)\right)+3=0$

$$-2-2\lambda-8\lambda-4+3=0$$
$$10\lambda=-3 \Rightarrow \lambda=\frac{-3}{10}$$

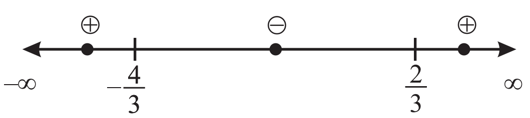
Hence, the required circle

$$x^2+y^2+2x\left(1-\frac{3}{10}\right)+2y\left(-\frac{6}{10}+1\right)-\frac{9}{10}+1=0$$
$$x^2+y^2+\frac{14x}{10}+\frac{8y}{10}+\frac{1}{10}=0$$
$$10x^2+10y^2+14x+8y+1=0$$

43. **(a)** Given that, $\dfrac{x-1}{3x+4}-\dfrac{x-3}{3x-2}<0$

$$\frac{(x-1)(3x-2)-(x-3)(3x+4)}{(3x+4)(3x-2)}<0$$
$$\frac{\left(3x^2-2x-3x+2\right)-\left(3x^2+4x-9x-12\right)}{(3x+4)(3x-2)}<0$$
$$\frac{-5x+2+5x+12}{(3x+4)(3x-2)}<0$$
$$\frac{14}{(3x+4)(3x-2)}<0$$

Thus, $(3x+4)(3x-2)<0$



$\therefore$ x lies in the interval $\left(-\dfrac{4}{3},\dfrac{2}{3}\right)$.

44. **(a)** Given that, there are 10 intermediate stations between two particular stations.

Here, the train stops at only 3 stations and no two of them should be consecutive.

Therefore, there are 8 stations to select these 3 stations.

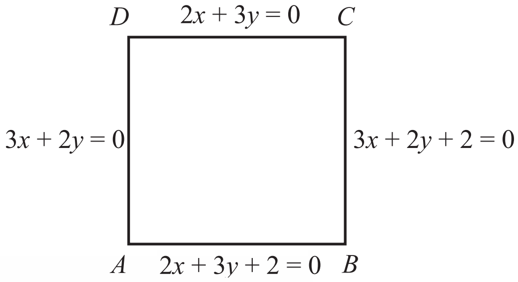
Hence, number of ways $={}^8C_3=56$

45. **(c)** Equation of pair of lines,

$$6x^2+13xy+6y^2=0$$
$$6x^2+9xy+4xy+6y^2=0$$
$$3x\left(2x+3y\right)+2y\left(2x+3y\right)=0$$
$$\left(3x+2y\right)\left(2x+3y\right)=0$$

$\therefore \quad 3x+2y=0$ and $2x+3y=0$

Similarly, for another equation of pairs of lines $6x^2+13xy+6y^2+10x+10y+4=0$ are $3x+2y+2=0$ and $2x+3y+2=0$



Here, AB is parallel to CD and BC is parallel to AD.

$\therefore \quad ABCD$ is a parallelogram.

But also $AB=BC=CD=AD=\dfrac{2}{\sqrt{13}}$

Hence, $ABCD$ is a rhombus.

46. **(b)** Given circle,

$$x^2+y^2-2x-6y-8=0$$

So, chord of contact of two tangents drawn from the point $P(a, b)$ to the given circle is

$$xa+yb-\left(x+a\right)-3\left(y+b\right)-8=0$$

$\Rightarrow \quad \left(a-1\right)x+\left(b-3\right)y-\left(a+3b+8\right)=0$

As, this line coincides with $5x+y+1=0$

$\therefore \quad \dfrac{a-1}{5}=\dfrac{b-3}{1}=\dfrac{-\left(a+3b+8\right)}{1}$

$\Rightarrow \quad a-1=-5\left(a+3b+8\right)$ and $b-3=-a-3b-8$

$\Rightarrow \quad 6a+15b=-39$ and $a+4b=-5$

$\Rightarrow \quad 2a+5b=-13$ ...(*i*)

and $\quad a+4b=-5$ ...(*ii*)

On solving Eqs. (*i*) and (*ii*), we get

$a=-9$ and $b=1$

Hence, $5a+b=-45+1=-44$

47. **(c)** Given that, a, b and c are non-zero vectors such that $\mathbf{a}$ & $\mathbf{b}$ are not perpendicular.

Also given, $\mathbf{r}\times\mathbf{b}=\mathbf{c}\times\mathbf{b}$

$\Rightarrow \quad \mathbf{a}\times\left(\mathbf{r}\times\mathbf{b}\right)=\mathbf{a}\times\left(\mathbf{c}\times\mathbf{b}\right)$

Since, $\mathbf{r}\perp\mathbf{a}$, therefore $\mathbf{a}\cdot\mathbf{r}=0$

$\Rightarrow \quad \left(\mathbf{a}\cdot\mathbf{b}\right)\mathbf{r}-\left(\mathbf{a}\cdot\mathbf{r}\right)\mathbf{b}=\mathbf{a}\times\left(\mathbf{c}\times\mathbf{b}\right)$

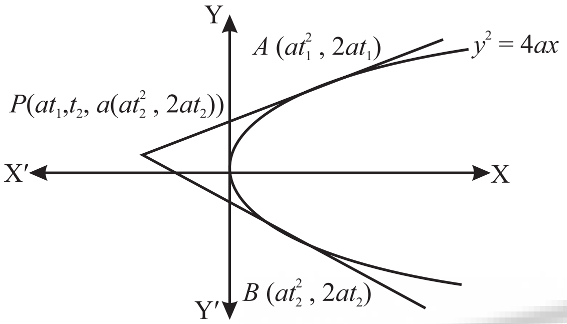
$\Rightarrow \quad \left(\mathbf{a}\cdot\mathbf{b}\right)\mathbf{r}=\mathbf{a}\times\left(\mathbf{c}\times\mathbf{b}\right)$

$\Rightarrow \quad = \dfrac{\mathbf{a}\times(\mathbf{c}\times\mathbf{b})}{\mathbf{a}\cdot\mathbf{b}} = -\dfrac{[(\mathbf{c}\times\mathbf{b})\times\mathbf{a}]}{\mathbf{a}\cdot\mathbf{b}}$

$= -\dfrac{[-(\mathbf{b}\times\mathbf{c})\times\mathbf{a}]}{\mathbf{a}\cdot\mathbf{b}} = \dfrac{(\mathbf{b}\times\mathbf{c})\times\mathbf{a}}{\mathbf{a}\cdot\mathbf{b}}$

48. **(d)** Here, P is intersecting point of tangents at $A\left(at_1^2, 2at_1\right)$ and $B\left(at_2^2, 2at_2\right)$

$P\left(at_1t_2 \;\; a\left(t_1 \quad t_2\right)\right)$



Slope of line PA

$= \tan\theta_1 = \dfrac{2at_1 - at_1 - at_2}{at_1^2 - at_1t_2} = \dfrac{a(t_1 - t_2)}{at_1(t_1 - t_2)} = \dfrac{1}{t_1}$

Similarly,

Slope of line PB $= \tan\theta_2 = \dfrac{2at_2 - at_1 - at_2}{at_2^2 - at_1t_2}$

$= \dfrac{a(t_2 - t_1)}{at_2(t_2 - t_1)} = \dfrac{1}{t_2}$

According to the question,

$\tan\theta_1 + \tan\theta_2 = b$

$\therefore \quad \dfrac{1}{t_1} + \dfrac{1}{t_2} = b$

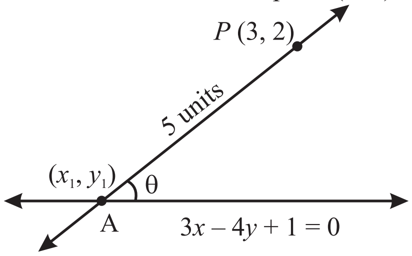
$t_2 + t_1 = bt_1t_2$

$\dfrac{y}{a} = \dfrac{bx}{a} \qquad \left[\because\ x = at_1t_2, y = a(t_1 + t_2)\right]$

$y = bx$

$\therefore$ P lies on the line $y = bx$

49. **(d)** Let the point A (x, y) on the line $3x - 4y + 1$ is at distance 5 units from the point (3, 2).



Equation of line PA

$\dfrac{x_1 - 3}{\cos\theta} = \dfrac{y_1 - 2}{\sin\theta} = \pm 5$

$\Rightarrow \quad x_1 = 3 \pm 5\cos\theta \qquad ...(i)$

$y_1 = 2 \pm 5\sin\theta \qquad ...(ii)$

Since, (x_1, y_1) lies on the line $3x - 4y - 1 = 0$

$\therefore \quad 3(3 \pm 5\cos\theta) - 4(2 \pm 5\sin\theta) - 1 = 0$

$\Rightarrow \quad 9 \pm 15\cos\theta - 8 + 20\sin\theta - 1 = 0$

$\Rightarrow \quad \pm 15\cos\theta - 20\sin\theta = 0$

$\Rightarrow \quad 3\cos\theta = \pm 4\sin\theta \Rightarrow \tan\theta = \pm 3/4$

$\cos\theta = \pm 4/5 \Rightarrow \sin\theta = \pm 3/5$

From Eqs. (i) and (ii), we get

$x_1 = 3 \pm \left(\dfrac{4}{5}\right)5 = 7, -1$

$y_1 = 2 \pm 5\left(\dfrac{3}{5}\right) = 5, -1$

$\therefore$ Coordinates are (7, 5) and (–1, –1)

50. **(d)** Let $l = \lim\limits_{x\to 0} \dfrac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$

$= \lim\limits_{x\to 0} \dfrac{2\sin^2 x \cdot (3 + \cos x)}{x \tan 4x}$

$= 2 \cdot \lim\limits_{x\to 0} \dfrac{\sin^2 x}{x^2} \dfrac{1}{4} \lim\limits_{x\to 0} \dfrac{4x}{\tan 4x} \cdot \lim\limits_{x\to 0} (3 + \cos x)$

According to Sandwich theorem,

$\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$ and $\lim\limits_{x\to 0} \dfrac{\tan x}{x} = 1$

$= 2 \cdot 1 \cdot \dfrac{1}{4} \cdot 1 \cdot (3 + 1) = 2$

51. **(a)** Given curves are

$x^2 = 3y \qquad ...(i)$

$x^2 + y^2 = 4 \qquad ...(ii)$

On solving Eqs. (i) and (ii), we get

$\therefore \quad x = \pm\sqrt{3}, y = 1$

Thus, their points of intersection are $(\sqrt{3}, 1)$ and $(-\sqrt{3}, 1)$.

Now, from Eq. (i), $\dfrac{dy}{dx} = \dfrac{2x}{3}$

and from Eq. (ii), $\dfrac{dy}{dx} = -\dfrac{x}{y}$

To calculate angle, any of the point can be taken so we are taking $(\sqrt{3}, 1)$ as the point of intersection.

Now, let m_1 and m_2 be the slope of tangent to the curves at $(\sqrt{3}, 1)$. Then,

$m_1 = \dfrac{2\sqrt{3}}{3}$ and $m_2 = -\sqrt{3}$

Now, the angle between two curves

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{2\sqrt{3}}{3} - \left(-\sqrt{3}\right)}{1 + \frac{2\sqrt{3}}{3} \times \left(-\sqrt{3}\right)}\right|$$

$$= \left|\frac{\left(\frac{5\sqrt{3}}{3}\right)}{(-1)}\right| = \frac{5}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$$

52. **(c)** We have system of equation,

$(a-1)x - y - z = 0$; $x - (b-1)y + z = 0$

and $x + y - (c-1)z = 0$

It is a homogeneous system of equations.

Now, for non-trivial solution.

$$\begin{vmatrix} a-1 & -1 & -1 \\ 1 & -(b-1) & 1 \\ 1 & 1 & -(c-1) \end{vmatrix} = 0$$

$$\begin{vmatrix} a-1 & -1 & -1 \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix} = 0$$

$$(a-1)[(1-b)(1-c)-1] + 1[1-c-1] - [1-(1-b)] = 0$$

$$(a-1)(1-b)(1-c) - a + 1 - c - b = 0$$

$$(a-1)(1-c-b+bc) - a - b - c + 1 = 0$$

$$a - ac - ab + abc - 1 + c + b - bc - a - b - c + 1 = 0$$

$$\therefore \quad ab + bc + ca = abc$$

53. **(b)** Given, White roses = 6

Red roses = 5

$\therefore$ Total number of ways for making garlands such that no two red roses come together is

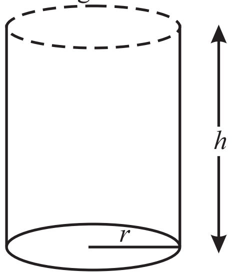
$$= \frac{6! \times 5!}{2} = 43200$$

54. **(c)** We have given,

V = volume of cylindrical vessel

r = radius and

h = height



As we know for cylindrical vessel

$$v = \pi r^2 h$$

$$\therefore \quad h = \frac{v}{\pi r^2} \qquad \ldots(i)$$

Let S be the area of metal sheet used to form a cylindrical vessel. Then,

$$S = 2\pi rh + \pi r^2 = 2\pi r\frac{V}{\pi r^2} + \pi r^2 \quad \text{[using Eq. } (i)\text{]}$$

$$\Rightarrow \quad S(r) = \frac{2V}{r} + \pi r^2$$

For minimum value of $S(r)$, $S'(r) = 0$

$$\Rightarrow \quad \frac{-2V}{r} + 2\pi r = 0 \Rightarrow \frac{2V}{r^2} = 2\pi r$$

$$\Rightarrow \quad r^3 = \frac{V}{\pi} \Rightarrow r = \sqrt[3]{\frac{V}{\pi}}$$

Now, by using the value of r in Eq. (i)

$$V = \pi\left(\frac{V}{\pi}\right)^{\frac{2}{3}} \cdot h$$

$$\Rightarrow \quad h = \frac{V}{\pi} \times \frac{\pi^{2/3}}{v^{2/3}} \Rightarrow h = \frac{V^{1/3}}{\pi^{1/3}} = \sqrt[3]{\frac{V}{\pi}}$$

Hence, the required radius is $r = \sqrt[3]{\frac{V}{\pi}}$ and height is $h = \sqrt[3]{\frac{V}{\pi}}$.

55. **(a)** Let $I = \int \frac{x + \sin x}{1 + \cos x} dx$

$$= \int \frac{x}{1+\cos x} dx + \int \frac{\sin x}{1 + \cos x} dx$$

$$= \int \frac{x}{2\cos^2 \frac{x}{2}} dx + \int \frac{\sin x}{2\cos^2 \frac{x}{2}} dx$$

$$= \frac{1}{2}\int x\sec^2\frac{x}{2} dx + \int \frac{2\sin x/2 \cos x/2}{2\cos^2 x/2} dx$$

$$= \frac{1}{2}\int x\sec^2\frac{x}{2} dx + \int \tan\frac{x}{2} dx$$

$$= \frac{1}{2}\left[x\frac{\tan\frac{x}{2}}{1/2} - \int \frac{\tan x/2}{\frac{1}{2}} dx\right] + \int \tan\frac{x}{2} dx$$

$$= x\tan\frac{x}{2} - \int \tan\frac{x}{2} dx + \int \tan\frac{x}{2} dx \quad = x\tan\frac{x}{2} + C$$

56. **(a)** Given, $I_n = \int \frac{\sin nx}{\cos x} dx \qquad \ldots(\text{i})$

$$= \int \frac{\sin(nx - x + x)}{\cos x} dx$$

$$= \int \frac{\sin[(n-1)x + x]}{\cos x} dx$$

$$\int \frac{\sin(n-1)x\cos x + \cos(n-1)x\sin x}{\cos} dx$$

$= \int \sin(n-1)x\,dx + \int \frac{\cos(n-1)x \sin x}{\cos x} dx$

$= \int \sin(n-1)x\,dx + \frac{1}{2}\int \frac{2\sin x \cos(n-1)x}{\cos x} dx$

Since, $2\sin x \cos y = +\sin(x+y) + \sin(x-y)$

$= -\frac{\cos(n-1)x}{(n-1)} + \frac{1}{2}\int \frac{\sin nx + \sin(2-n)x}{\cos x} dx$

$= \frac{-\cos(n-1)x}{(n-1)}$

$+ \frac{1}{2}\int \frac{\sin nx}{\cos x} dx - \frac{1}{2}\int \frac{\sin(n-2)x}{\cos x} dx$

From Eq. (*i*),

$\therefore I_n = -\frac{\cos(n-1)x}{(n-1)} + \frac{1}{2} I_n - \frac{1}{2} I_{n-2}$

$\left(1-\frac{1}{2}\right) I_n = -\frac{\cos(n-1)x}{(n-1)} - \frac{1}{2} I_{n-2}$

$I_n = \frac{-2}{n-1}\cos(n-1)x - I_{n-2}$

57. **(c)** Given that, tan α and tan β are the roots of the equation $x^2 + px + q = 0$

$\therefore \quad \tan\alpha + \tan\beta = -p$ and $\tan\alpha \cdot \tan\beta = q$

Now, $\quad \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$= \frac{-p}{1-q} = \frac{p}{q-1}$

$\sec(\alpha+\beta) = \sqrt{1+\tan^2(\alpha+\beta)}$

$\sec(\alpha+\beta) = \sqrt{1+\frac{p^2}{(q-1)^2}}$

$\therefore \quad \cos(\alpha+\beta) = \frac{1}{\sqrt{1+\frac{p^2}{(q-1)^2}}}$

$\sin^2(\alpha+\beta) + p\cos(\alpha+\beta)\sin(\alpha+\beta)$

$+ q\cos^2(\alpha+\beta)$

$= \cos^2(\alpha+\beta)\left[\tan^2(\alpha+\beta) + p\tan(\alpha+\beta) + q\right]$

$= \frac{1}{1+\frac{p^2}{(q-1)^2}}\left[\frac{p^2}{(q-1)^2} + \frac{p^2}{q-1} + q\right]$

$= \frac{(q-1)^2}{(q-1)^2+p^2}\left[\frac{p^2 + p^2(q-1) + q(q-1)^2}{(q-1)^2}\right]$

$= \frac{p^2 + p^2 q - p^2 + q(q-1)^2}{p^2 + (q-1)^2}$

$= \frac{q\left\{p^2 + (q-1)^2\right\}}{p^2 + (q-1)^2} = q$

58. **(c)** We have,

$= \sum_{r=1}^{n-1} r(r+1-w)\left(r+1-w^2\right)$

$= \sum_{r=1}^{n-1} r\left[(r+1)^2 - (r+1)w^2 - w(r+1) + w^3\right]$

$= \sum_{r=1}^{n-1} r\left[(r+1)^2 - (r+1)\left(\left(w+w^2\right) + w^3\right)\right]$

$= \sum_{r=1}^{n-1} r\left[(r+1)^2 - (r+1)((-1)+1)\right]$

$\left[\because \; w^3 = 1 \text{ and } 1 + w + w^2 = 0\right]$

$= \sum_{r=1}^{n-1} r\left(r^2 + 3r + 3\right)$

$= \sum_{r=1}^{n-1} r^3 + 3\sum_{r=1}^{n-1} r^2 + 3\sum_{r=1}^{n-1} r$

$= \left(\frac{(n-1)n}{2}\right)^2 + \frac{3(n-1)(n)(2n-1)}{6} + \frac{3(n-1)n}{2}$

$\frac{n^2(n-1)^2}{4} + \frac{n(n-1)(2n-1)}{2} + \frac{3}{2}n(n-1)$

$= \frac{n(n-1)}{4}\left[n(n-1) + 2(2n-1) + 6\right]$

$= \frac{n(n-1)}{4}\left[n^2 - n + 4n - 2 + 6\right]$

$= \frac{n(n-1)}{4}\left[n^2 + 3n + 4\right]$

59. **(d)** Given that,

$$\sigma(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ -\frac{(n-1)}{2} & \text{if } n \text{ is odd} \end{cases}$$

Case-I : If n is even

$\sigma(n) = 1, 2, 3, 4, 5, \ldots$

Case-II: If n is odd

$\sigma(n) = 0, -1, -2, -3, -4, \ldots$

Thus, for every value of n $\sigma(n)$ has an unique image.

$\therefore \quad \sigma(n)$ is one-one function.

So, range of $\sigma(n) = Z$

$\therefore \quad$ Range of $\sigma(n)$ = codomain of $\sigma(n) = Z$.

Hence, $\sigma(n)$ is one-one and onto function.

60. **(d)** Given that,

$$ {}^{37}C_4 + \sum_{r=1}^{5} {}^{(42-r)}C_r $$

$$ = {}^{37}C_4 + {}^{41}C_1 + {}^{40}C_2 + {}^{39}C_3 + {}^{38}C_3 + {}^{37}C_3 $$

$$ = {}^{37}C_3 + {}^{37}C_4 + {}^{38}C_3 + {}^{39}C_3 + {}^{40}C_3 + {}^{41}C_3 $$

As we know,

$$ {}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r $$

$$ = {}^{38}C_4 + {}^{38}C_3 + {}^{39}C_3 + {}^{40}C_3 + {}^{41}C_3 $$

$$ = {}^{39}C_4 + {}^{39}C_3 + {}^{40}C_3 + {}^{41}C_3 $$

$$ = {}^{40}C_4 + {}^{40}C_3 + {}^{41}C_3 $$

$$ = {}^{41}C_4 + {}^{41}C_3 = {}^{42}C_4 $$

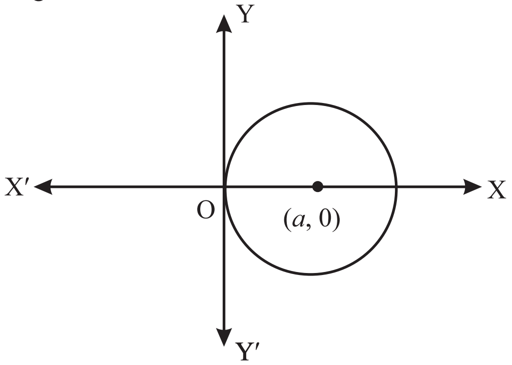
61. **(c)**

x_i	f_i	$d_i = x_i - a$	d_i^2	$f_i d_i$	$f_i d_i^2$
6	2	–12	144	–24	288
10	4	– 8	64	–32	256
14	7	– 4	16	–28	112
18 = a	12	0	0	0	0
24	8	6	36	48	288
28	4	10	100	40	400
30	3	12	144	36	432
	$N = 40$			$\Sigma f_i d_i = 40$	$\sum f_i d_i^2 = 1776$

Now, variance $= \dfrac{\sum f_i d_i^2}{N} - \left(\dfrac{\sum f_i d_i}{N}\right)^2$

$$ = \frac{1}{40} \times 1776 - 1 = \frac{1736}{40} = 43.4 $$

62. **(a)** Here, it is given that circle touches the y-axis at origin.



So, equation of family of circles

$$ (x-a)^2 + y^2 = a^2 $$

$$ \Rightarrow \quad x^2 + a^2 - 2ax + y^2 = a^2 $$

$$ \Rightarrow \quad x^2 + y^2 = 2ax \qquad ...(i) $$

On differentiating w.r.t. x, we get

$$ 2x + 2y\frac{dy}{dx} = 2a \qquad ...(ii) $$

Now, from Eqs. (i) and (ii), we get

$$ x^2 + y^2 = \left(2x + 2y\frac{dy}{dx}\right)x $$

$$ \Rightarrow \quad x^2 + y^2 = 2x^2 + 2xy\frac{dy}{dx} $$

$$ \Rightarrow \quad \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} $$

$\therefore$ This is the required differential equation.

63. **(b)** It is given that $p, x_1, x_2, x_3 .... x_n$ and $q, y_1, y_2, y_3 .... y_n$ are in A.P. whose common difference are a and b respectively.

$\therefore \quad x_1 = p + a,\ x_n = p + na$

$y_1 = q + b,\ y_n = q + nb$

Also, given α is A.M. of $x_1, x_2, x_3 .... x_n$

$$ \therefore \quad \alpha = \frac{x_1 + x_2 + x_3 + ... + x_n}{n} $$

$$ \Rightarrow \quad \alpha = \frac{n}{2}\frac{(x_1 + x_n)}{n} $$

$$ \Rightarrow \quad \alpha = \frac{x_1 + x_n}{2} $$

Similarly, β is A.M. of $y_1, y_2, y_3 ... y_n$. So,

$$ \beta = \frac{y_1 + y_n}{2} $$

Thus, substituting the value of x, x_n, y and y_n, we get

$$ \alpha = \frac{p + a + p + na}{2} = \frac{2p + a(n+1)}{2} \qquad ...(i) $$

$$ \beta = \frac{q + a + q + na}{2} = \frac{2q + b(n+1)}{2} \qquad ...(ii) $$

From Eqs. (i) and (ii) eliminate (n + 1), we get

$$ \frac{2\alpha - 2p}{a} = \frac{2\beta - 2q}{b} $$

$$ \Rightarrow \quad b(\alpha - p) = a(\beta - q) $$

Hence, locus of $p(\alpha, \beta)$ is $b(x-p) = a(y-q)$

64. **(a)** Given observations = { Kα }

Here, K = 1, 2, 3, ..., 50

$\therefore$ { Kα } = {α, 2α, 3α, ..., 50α }

Number of observations = 50 (Even)

Median of given observation $= \dfrac{25\alpha + 26\alpha}{2}$

M = 25.5α

Now, mean deviation about median

$$\text{M,D. (M)} = \frac{\sum_{k=1}^{n}|x_i - m|}{n}$$

$$50 = \frac{\sum_{k=1}^{50}|k\alpha - 25.5\alpha|}{50}$$

$$\Rightarrow \quad 50\times 50 = \sum_{k=1}^{50}\left|\alpha(K-25.5)\right|$$

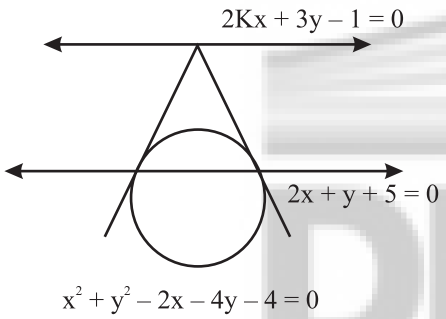
$$\Rightarrow \quad |\alpha|\sum_{k=1}^{50}|k-25.5| = 2500$$

$$\Rightarrow \quad 2|\alpha|[24.5+23.5+..+1.5+0.5] = 2500$$

$$\Rightarrow \quad 2|\alpha|\times\frac{25}{2}\times(24.5+0.5) = 2500$$

$$\Rightarrow \quad |\alpha|\times 25 = 1400 \Rightarrow |\alpha| = 4$$

65. **(c)** Let point (x_1, y_1) lie on the line $2kx+3y-1=0$.



Given circle, $x^2+y^2-2x-4y-4=0$

Equation of chord of contact from (x_1, y_1) to the circle

$$xx_1 + yy_1 - \frac{2(x+x_1)}{2} - \frac{4(y+y_1)}{2} - 4 = 0$$

$$\Rightarrow \quad x(x_1-1)+y(y_1-2)-x_1-2y_1-4=0$$

Now, $2x+y+5=0$ and $2kx+3y+1=0$ are conjugate line

$\therefore \quad 2x+y+5=0$ and $x(x_1-1)+y(y_1-2)$

$-x_1-2y_1-4=0$ are coincide.

$$\therefore \quad \frac{x_1-1}{2} = \frac{y_1-2}{1} = \frac{x_1+2y_1+4}{-5} = \lambda$$

$$\Rightarrow \quad x_1 = 2\lambda+1, y_1 = \lambda+2$$

$x_1+2y_1+4=-5\lambda$

Put x_1, y_1 in $x_1-2y_1-4=-5k$

$2\lambda+1+2\lambda+4+4=-5\lambda \Rightarrow \lambda=-1$

$\therefore \quad x_1=-1, y_1=1$

Now, put $x_1=-1$ and $y_1=1$

$2kx+3y-1=0$, we get

$-2k+3-1=0 \Rightarrow k=1$

66. **(c)** Given that, $\frac{5x^2+2}{x^3+x} = \frac{A_1}{x} + \frac{A_2x+A_3}{x^2+1}$

$$\Rightarrow \quad \frac{5x^2+2}{x(x^2+1)} = \frac{A_1(x^2+1)+(A_2x+A_3)x}{x(x^2+1)}$$

$$\Rightarrow \quad 5x^2+2 = A_1(x^2+1)+A_2x^2+A_3x$$

$$\Rightarrow \quad 5x^2+2 = (A_1+A_2)x^2+A_3x+A_1$$

On comparing the coefficients of x^2, x and constant term, we get

$\therefore \quad A_1=2; A_2=3$ and $A_3=0$

67. **(c)** Let, $S_n = \frac{1}{2\cdot 5}+\frac{1}{5\cdot 8}+\frac{1}{8\cdot 11}+\text{.....upto } n \text{ terms}$

$$= \frac{1}{3}\left[\frac{1}{2\cdot 5}+\frac{1}{5\cdot 8}+\frac{1}{8\cdot 11}+\text{.....upto } n \text{ terms}\right]$$

$$= \frac{1}{3}\left[\frac{5-2}{2\cdot 5}+\frac{8-5}{5\cdot 8}+\frac{11-8}{8\cdot 11}+\text{.....upto } n \text{ terms}\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{8}\right)+\left(\frac{1}{8}-\frac{1}{11}\right) + \left(\frac{1}{3n-1}-\frac{1}{3n+2}\right)\right]$$

$$= \frac{1}{3}\left[\frac{1}{2}-\frac{1}{3n+2}\right] = \frac{1}{3}\left[\frac{3n+2-2}{2(3n+2)}\right]$$

$$\therefore \quad S_n = \frac{n}{2(3n+2)}$$

68. **(b)** Let, general equation of this parabola

$$x = Ay^2+By+C$$

It is given that, it passes through the points (–2, 1), (1, 2) and (–1, 3) so, we get

$$-2 = A+B+C \qquad ...(i)$$

$$1 = 4A+2B+C \qquad ...(ii)$$

and $-1 = 9A+3B+C$ ...(*iii*)

On solving Eqs. (*i*), (*ii*) and (*iii*), we get

$$A = \frac{-5}{2}, B = \frac{21}{2} \text{ and } C = -10$$

Hence, equation of parabola

$$x = -\frac{5}{2}y^2+\frac{21}{2}y-10$$

$$\Rightarrow \quad 2x = -5y^2+21y-20$$

$$\Rightarrow \quad 5y^2+2x-21y+20=0$$

69. **(b)** Given that,

$$b\cos(C+\theta)+c\cos(B-\theta)$$

$$= b(\cos C\cos\theta - \sin C\sin\theta)$$

$$+ c(\cos B\cos\theta + \sin B\sin\theta)$$

$$= b\cos C\cos\theta + c\cos B\cos\theta - b\sin C\sin\theta + c\sin B\sin\theta$$

$$= \cos\theta\left(b\cos C + c\cos B\right) - \sin\theta\left(b\sin C - c\sin B\right)$$

Since by projection formula and by sine rule

$$\frac{b}{\sin B} \quad \frac{c}{\sin C} \Rightarrow b\sin C - c\sin B = 0$$

$$b\cos\left(C+\theta\right) + c\cos\left(B-\theta\right)$$

$$= a\cos\theta - \sin\theta\cdot 0$$

$$= a\cos\theta - \sin\theta\cdot 0 = a\cos\theta$$

70. **(c)** We have,

$$1 + \cos x\cdot\cos 5x = \sin^2 x$$

$$\Rightarrow 1 - \sin^2 x + \cos x\cdot\cos 5x = 0$$

$$\Rightarrow \cos^2 x + \cos x\cdot\cos 5x = 0$$

$$\Rightarrow \cos x\left(\cos x + \cos 5x\right) = 0$$

$$\Rightarrow \cos x\left[2\cos\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right] = 0$$

$$\Rightarrow \cos x\left(2\cos 3x\cos 2x\right) = 0$$

$$\Rightarrow \cos x = 0;\ \cos 3x = 0 \text{ or } \cos 2x = 0$$

$$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \text{ or}$$

$$3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2} \text{ or}$$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \qquad \left[\because x\in\left[0, 2\pi\right]\right]$$

$$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

Thus, there are 10 solutions in $[0, 2\pi]$

71. **(d)** Given that α, β and γ are the roots of the equation

$$x^3 + px^2 + qx + r = 0$$

$$\alpha + \beta + \gamma = -p \qquad \ldots(i)$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = q \qquad \ldots(ii)$$

and $\alpha\beta\gamma = -r \qquad \ldots(iii)$

Now, $\left(1+\alpha^2\right)\left(1+\beta^2\right)\left(1+\gamma^2\right)$

$$= \left(1+\alpha^2\right)\left(1+\beta^2+\gamma^2+\left(\beta\gamma\right)^2\right)$$

$$= 1+\beta^2+\gamma^2+\left(\beta\gamma\right)^2+\alpha^2+\left(\alpha\beta\right)^2+\left(\alpha\gamma\right)^2+\left(\alpha\beta\gamma\right)^2$$

$$= 1+\left(\alpha^2+\beta^2+\gamma^2\right)+\left(\left(\alpha\beta\right)^2+\left(\beta\gamma\right)^2+\left(\gamma\alpha\right)^2\right) + \left(\alpha\beta\gamma\right)^2$$

$$= 1+\left[\left(\alpha+\beta+\gamma\right)^2 - 2\left(\alpha\beta+\beta\gamma+\gamma\alpha\right)\right] + \left[\left(\alpha\beta+\beta\gamma+\gamma\alpha\right)^2 - 2\alpha\beta\gamma\left(\alpha+\beta+\gamma\right)\right]+\left(\alpha\beta\gamma\right)^2$$

From Eq. (*i*), (*ii*) and (*iii*), we get

$$= 1+\left[p^2-2q\right]+\left[q^2-2rp\right]+r^2$$

$$= 1+p^2-2q+q^2-2rp+r^2$$

$$= \left(q-1\right)^2+\left(r-p\right)^2$$

72. **(a)** We have given a system of linear equations with the three unknowns.

Here, equations are in the matrix form $AX = D$ such that the system is inconsistent.

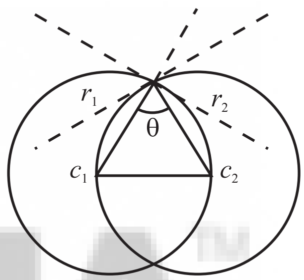
$\therefore$ Rank of Augmented matrix AD > Rank of coefficient matrix A

Rank of AD > Rank of A

$$\therefore \quad \frac{\text{Rank of A}}{\text{Rank of AD}} < 1$$

73. **(a)** Let θ be the angle between two circles. As each circle passing through the centre at each other. Then,

$$c_1c_2 = r_1 = r_2$$



From the figure,

$$\cos\theta = \frac{r_1^2+r_2^2-c_1c_2}{2r_1r_2}$$

$$\cos\theta = \frac{r_1^2+r_2^2-r_1^2}{2r_1r_2}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Hence, angle between two circle is either $\frac{\pi}{3}$ or

$$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

74. **(c)** Given that,

$$\log_{\frac{1}{\sqrt{3}}}\left\{\frac{|z|^2-|z|+1}{2+|z|}\right\} > -2$$

Since, $\log_a b > c, b < a^c$ if $0 < a < 1$

$$\Rightarrow \frac{|z|^2-|z|+1}{2+|z|} < \left(\frac{1}{\sqrt{3}}\right)^{-2}$$

$$\Rightarrow \frac{|z|^2-|z|+1}{2+|z|} < \left(\sqrt{3}\right)^2$$

$\Rightarrow \quad |z|^2 - |z| + 1 < \left(\sqrt{3}\right)^2 (2 + |z|)$

$\Rightarrow \quad |z|^2 - |z| + 1 < 6 + 3|z|$

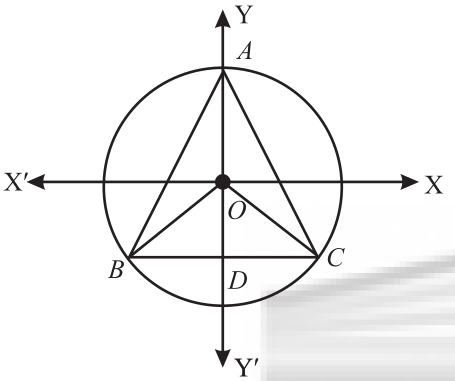
$\Rightarrow \quad |z|^2 - 4|z| + 1 < 6$

$\Rightarrow \quad (|z| - 2)^2 < 9 \ \Rightarrow -3 < |z| - 2 < 3$

$\Rightarrow \quad -1 < |z| < 5 \ \Rightarrow \ 0 < |z| < 5$

Hence, z lies inside the circle.

75. **(c)** We have, length of median of $\Delta ABC = 9$



$\therefore \quad AO = \frac{2}{3} AD \ \Rightarrow \ AO = \frac{2}{3} \times 9 = 6$

O is the circumcentre of ΔABC.

We know that in equilateral triangle circumcentre, incentre, centroid coincide.

$\therefore$ Origin O (0, 0) is the centre and AO is radius of circle.

Hence, equation of circle

$$(x - \sigma)^2 + (y - \sigma)^2 = (6)^2$$

$\Rightarrow \quad x^2 + y^2 = 36$

76. **(b)** We have,

$1 + \cos 10° + \cos 20° + \cos 30°$

$= (1 + \cos 10°) + (\cos 20° + \cos 30°)$

$= 2\cos 5° + 2\cos 25° \cos 5°$

$= 2\cos 5°(\cos 5° + \cos 25°)$

$= 2\cos^2 5°(2\cos 15° \cos 10°)$

$= 4\cos 5° \cos 10° \cos 15°$

77. **(b)** $(a - 7)(a - 11) > 0$

$a < 7$ and $a > 11$

78. **(b)** Given that,

$$x = \frac{1 \cdot 3}{3 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{3 \cdot 6 \cdot 9} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{3 \cdot 6 \cdot 9 \cdot 12} + \ldots \infty \text{ terms}$$

$$\Rightarrow \quad x = \frac{1 \cdot 3}{3^2 (2!)} + \frac{1 \cdot 3 \cdot 5}{3^3 (3!)} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{3^4 (4!)} + \ldots \infty \text{ terms}$$

$$\Rightarrow \quad x = \frac{\frac{1}{2}\left(\frac{1}{2} + 1\right)}{2!}\left(\frac{2}{3}\right)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2} + 1\right)\left(\frac{1}{2} + 2\right)}{3!}\left(\frac{2}{3}\right)^3 + \ldots$$

$$\Rightarrow \quad x = \left[1 + \frac{1}{2}\left(\frac{2}{3}\right) + \frac{\frac{1}{2}\left(\frac{1}{2} + 1\right)}{2!}\left(\frac{2}{3}\right)^2 + \frac{\frac{1}{2}\left(\frac{1}{2} + 1\right)\left(\frac{1}{2} + 1\right)}{3!}\left(\frac{2}{3}\right)^3 \ldots\right] - \left(1 + \frac{1}{3}\right)$$

Now, by using binomial expansion for any index

$\Rightarrow \quad x = \left(1 - \frac{2}{3}\right)^{-1/2} - \frac{4}{3} \ \Rightarrow \ x = \left(\frac{1}{3}\right)^{-1/2} - \frac{4}{3}$

$\Rightarrow \quad x = \sqrt{3} - \frac{4}{3} \ \Rightarrow \ 3x + 4 = 3\sqrt{3}$

On squaring both the sides, we get

$\Rightarrow \quad (3x + 4)^2 = 3\left(\sqrt{3}\right)^2 \ \Rightarrow \ 9x^2 + 24x + 16 = 27$

$\Rightarrow \quad 9x^2 + 24x = 11$

79. **(c)** It is given that,

$$\left(3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) = \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}\right)x + \left(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right)y + \left(-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}\right)z$$

$$= \hat{\mathbf{i}}(2x + y - 2z) + \hat{\mathbf{j}}(3x - 2y + z) + \hat{\mathbf{k}}(-x + 2y - 2z)$$

On comparing both the sides, we get

$2x + y - 2z = 3$

$3x - 2y + z = -1$

$-x + 2y - 2z = 2$

Now, by checking the options for coordinates (x, y, z), option (c) satisfies them.

80. **(a)** As we know, the volume of tetrahedron formed by the coterminous edges **a**, **b** and **c**,

$$V = \frac{1}{6}[\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}]$$

$$4 = \frac{1}{6}[\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}]$$

$\therefore \quad [\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}] = 24 \qquad \ldots(i)$

Now, the volume of parallelopiped formed, by the coterminous edges $\mathbf{a} \times \mathbf{b}$, $\mathbf{b} \times \mathbf{c}$ and $\mathbf{c} \times \mathbf{a}$.

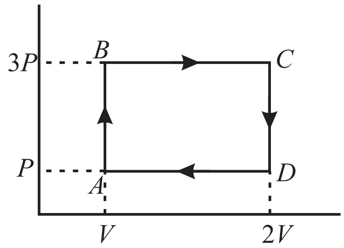
$V = [\mathbf{a} \times \mathbf{b} \quad \mathbf{b} \times \mathbf{c} \quad \mathbf{c} \times \mathbf{a}]$

$V = [\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}]^2$

$V = (24)^2 = 576 \qquad$ (from Eq. (i))

PHYSICS

81. **(*)** (No options match)



Work done in complete cycle

= Area under closed curve = 2 PV

Heat given to the system from A to B $= nC_V\Delta T$

$$= n\frac{3}{2}R\Delta T = \frac{3}{2}\times V\times\Delta P = \frac{3}{2}\times V\times(3P-P) = 3PV$$

Similarly, heat given to the system from B to C

$$= nC_P\Delta T$$

$$= n\left(\frac{5}{2}R\right)\Delta T = \frac{5}{2}(3P)(\Delta V)$$

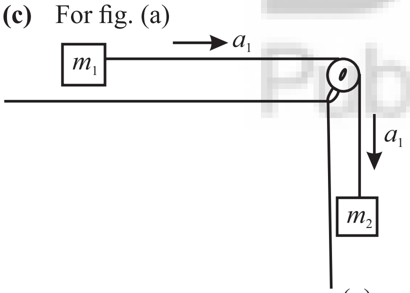
$$= \frac{5}{2}\times(3P)\cdot(2V-V) = \frac{15}{2}PV$$

The heat is released from C to D and D to A.

$$\text{Efficiency } (\eta) = \frac{\text{Work done by gas}}{\text{Heat given to gas}}\times 100$$

$$= \frac{2PV}{3PV+\frac{15}{2}PV}\times 100 = \frac{4}{21}\times 100 = 19.04\%$$

82. **(c)** For fig. (a)



(a)

$m_2g - T = m_2a_1$ ...(i)

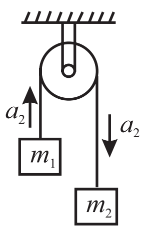
$T = m_1a_1$ ...(ii)

From (i) & (ii), we get

$$a_1 = \frac{m_2g}{m_1+m_2}$$

as $m_1 = 3$ kg, $m_2 = 4$ kg

$$a_1 = \frac{4\times 10}{3+4} = \frac{40}{7}\ \text{m/s}^2$$

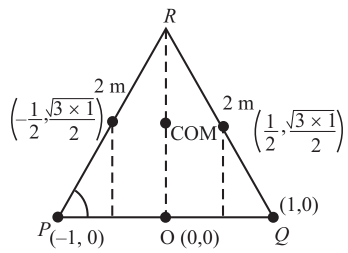


(b)

For fig. (b)

$$a = \left(\frac{m_2-m_1}{(m_1+m_2)}\right)g = \frac{10}{7}\ \text{m/s}^2$$

83. **(d)**



Initially,

$$\mathbf{r}_{\text{COM}} = \frac{m_1\mathbf{r}_1+m_2\mathbf{r}_2+m_3\mathbf{r}_3}{(m_1+m_2+m_3)}$$

$$\Rightarrow\ \mathbf{r}_{\text{COM}} = \frac{1}{3}\left(\frac{\sqrt{3}}{2}\times 1\times 2\right)\hat{\mathbf{j}}\ \Rightarrow\ \mathrm{r}_{\text{COM}} = \frac{1}{\sqrt{3}}\hat{\mathbf{j}}$$

Final length,

$l_f = l_i\,(1+\alpha\Delta T)$

or, $l_f = l_i\left(1+\frac{\gamma}{3}\Delta T\right)$ $[\because\ \gamma = 3\alpha]$

or, $l_f = 2\left(1+\frac{12\sqrt{3}\times 10^{-6}}{3}\times 50\right)$

or, $l_f = 2\left(1+4\times 50\times\sqrt{3}\times 10^{-6}\right)$

or, $l_f = 2\left(1+2\sqrt{3}\times 10^{-4}\right)$

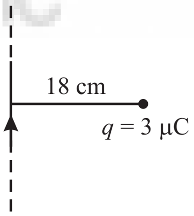
So, $(\mathbf{r}_{\text{COM}})_{\text{final}} = \frac{1}{3}\times\frac{\sqrt{3}}{2}\times\left(1+2\sqrt{3}\times 10^{-4}\right)\times 2\,\hat{\mathbf{j}}$

$$= \left(\frac{1}{\sqrt{3}}+2\times 10^{-4}\right)\hat{\mathbf{j}}$$

So, $\Delta y = (\mathrm{r}_{\text{COM}})_{\text{final}} - (\mathrm{r}_{\text{COM}})_{\text{initial}}$

$= 2\times 10^{-4}$ m = 0.2 mm

84. **(b)**



$$E = \frac{\lambda}{2\pi\varepsilon_0 .\, r} = \frac{2\lambda}{4\pi\varepsilon_0 .\, r} = \frac{9\times 10^9\times 2\times 1}{3\times 18\times 10^{-2}}$$

$$= \frac{1}{3}\times 10^{11}\ \text{N/C}$$

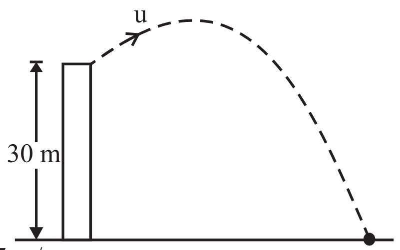
So, $F = qE = 3\times 10^{-6}\times\frac{1}{3}\times 10^{11} = 10^5$ N

85. **(a)** As, $V = \frac{kq}{r}$

So, $q = \frac{V\times r}{k} = \frac{25\times 10^3\times 9\times 10^{-2}}{9\times 10^9}$

$= 25\times 10^{-8}\ C = 0.25\ \mu C$

86. **(a)**



$u = 5$ m/s

($\hat{\mathbf{k}}$ is given vertically upward direction)

$a = -10$ m/s^2

$h = -30$ m

As, $S = ut + \frac{1}{2}at^2 \Rightarrow -30 = 5t - \frac{1}{2} \times 10 \times t^2$

$\Rightarrow \quad t^2 - t - 6 = 0$

$\Rightarrow \quad t = -2\ s$ (Not possible)

or, $t = 32$ s. This is time in which body reaches the ground

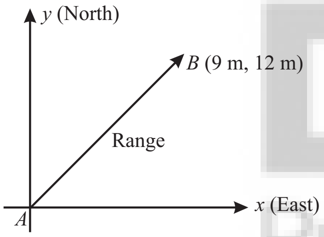
In this time, projectile moving in East and North with speeds 3 m/s and 4 m/s. Distances covered in these directions are;

In east (x - coordinate) $= 3 \times 3 = 9$ m

and in North (y - coordinate) $= 4 \times 3 = 12$ m

So, Projectile land at $(x, y) \equiv$ (9 m, 12 m) mark.

So, horizontal range of body on ground is:



Range $= \sqrt{9^2 + 12^2} = 15$ m

87. **(c)** When conducting wire is displaced by (x) dx distance, then work done per unit length is;

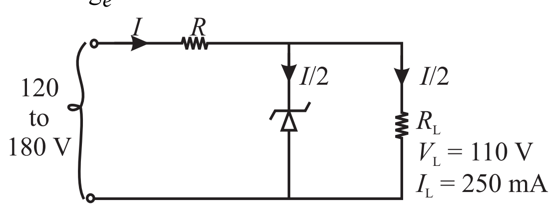
$dW = F/\ell.dx$

or, $dW = \frac{\mu_0 I_1 I_2}{2\pi x} dx \left[\because \frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi x}\right]$

or, $\int dW = \int \frac{\mu_0 I_1 I_2}{2\pi x} dx$ or, $W = \left(\frac{\mu_0 I_1 I_2}{2\pi}\right) \log_e x$

or, $W \propto \log_e x$

88. **(b)**



We have, $R_L = \frac{V_L}{I_L} = \frac{110}{250 \times 10^{-3}} = 440\ \Omega$

As current is equally shared by Zener diode and resistance R. So, $I_Z = 250$ mA

Then, $I = 2 \times 250 = 500$ mA $\quad [\because I = I_L + I_Z]$

$= 0.5$ mA

So, $R_s = \frac{V_s}{I} = \frac{(180 - 110)}{0.5} = \frac{70}{0.5} = 140\ \Omega$

$[\because$ when current is maximum $V = 180\ V]$

89. **(d)** We have, $y = Px - Qx^2$ ...(*i*)

Equation of trajectory is

$$y = x \tan\theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2\theta} \quad ...(ii)$$

After comparing Eqs. (*i*) and (*ii*), we get

$P = \tan\theta$ ...(*iii*)

$$Q = \frac{g}{2u^2 \cos^2\theta} \quad ...(iv)$$

We know that

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin\theta \sin\theta}{2g \times 2\cos\theta} \times 2\cos\theta$$

$$\Rightarrow \quad H = \frac{u^2 \sin 2\theta}{g} \times \frac{\tan\theta}{4}$$

$$\Rightarrow \quad H = \frac{R \tan\theta}{4} \quad \Rightarrow \frac{H}{R} = \frac{\tan\theta}{4}$$

$$\Rightarrow \quad \frac{H}{R} = \frac{P}{4} \quad [\because \tan\theta = p]$$

90. **(d)** Transverse displacement

$y_{(x,t)} = 0.03 \sin\left(\frac{2\pi x}{3}\right) \cos 60\pi t$.

This is equation of standing wave.

The standard equation of standing wave is

$y = a \sin kx \cos \omega t$

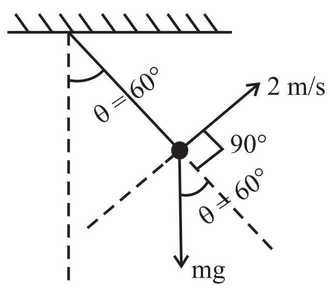
Here, $\omega = 60\pi$, $k = 2\pi/3$

So, $V = \frac{60\pi}{2\pi} \times 3 = 90$

So, $\sqrt{\frac{T}{\mu}} = 90$

$\Rightarrow \quad T = 90^2 \mu \Rightarrow \quad T = 90^2 \times 10 = 81$ N

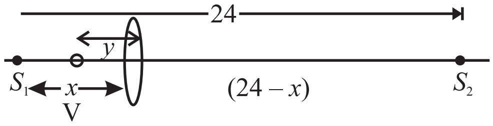
91. **(c)**



$P = \vec{F} \cdot \vec{V} \quad = m\ g\ v \cos 150°$

$= 50 \times 9.8 \times 2 \cos 150° \quad = -490\sqrt{3}\ w$

$P = 490\sqrt{3}\ watt$

92. (b) 

The given condition will be satisfied only if one source (S_1) placed on one side such that $u < f$ (i.e. it lies under the focus. The other source (S_2) is placed on the other side of the lens such that $u > f$ (i.e. it lies beyond the focus).

If S_1 is the object for the lens, then

$$\frac{1}{f} = \frac{1}{-y} - \frac{1}{-x} \quad ...(i)$$

If S_2 is the object for the lens, then

$$\frac{1}{f} = \frac{1}{+y} - \frac{1}{-(24-x)} \quad ...(ii)$$

Adding (*i*) & (*ii*), we get

$$\frac{1}{x} + \frac{1}{(24-x)} = \frac{2}{f} = \frac{2}{7}$$

$\Rightarrow \quad x^2 - 24x + 108 = 0 \Rightarrow x = 18$ cm and $x = 6$ cm

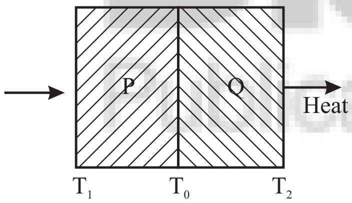
So, the correct answer will be 6 cm.

93. (b) (A) It is more difficult to push a magnet into a coil with large turns because of opposition caused by induced current.

(B) The emf induced in a coil always opposes the motion of a magnet when it is moved towards the coil. It was experimentally verified by lenz law.

So both *A* and *R* are true. *R* is correct explanation of *A*.

94. (d) Here,



We have,

$$K_Q = \frac{K_P}{2} \Rightarrow K_P = 2K_Q$$

$$\frac{2K_Q\, a(T_1 - T_0)}{x} = \frac{K_Q\, a(T_0 - T_2)}{x}$$

$$\left[\because \left[\frac{Q}{t}\right]_P = \left[\frac{Q}{t}\right]_Q\right]$$

$$2(T_1 - T_0) = T_0 - T_2$$

$\Rightarrow \quad 2T_1 - 3T_0 + T_2 = 0 \quad ...(i)$

According to the question,

$T_1 - T_2 = 24°C \quad ...(ii)$

Now, from Eq. (*i*),

$2T_1 - 3T_0 + T_1 - 24 = 0$

$\Rightarrow \quad 3T_1 - 3T_0 = 24 \Rightarrow T_1 - T_0 = 8°C$

95. (b) $r = R\left(\frac{l_1}{l_2} - 1\right)$

This is value of internal resistance here.

$$\frac{\Delta r}{r} \times 100 = \frac{\Delta R}{R} \times 100 + \frac{\Delta\left(\frac{l_1}{l_2}\right)}{\left(\frac{l_1}{l_2}\right)} \times 100$$

$= 4 + 0.2 = 4.2\%$

96. (a) As, $K.E. = \frac{1}{2}mv^2 \quad \Rightarrow V = \sqrt{\frac{2 \times K.E.}{m}}$

$$\Rightarrow \quad V = \sqrt{\frac{2 \times 4 \times 16 \times 10^{-19}}{32 \times 10^{-21}}} = \sqrt{\frac{2 \times 4}{2 \times 10^{-2}}}$$

$\Rightarrow \quad V = 20$ m/s

Time to travel a distance of 3.6 km is

$$t = \frac{D}{v} = \frac{3.6 \times 1000}{20} s = \frac{3.6 \times 1000}{20 \times 60} \text{min}$$

= 3 min = 3 half-time

In three half life times, the number of particles decayed

$$= N_0 - \frac{N_0}{2^3} = \frac{7}{8} N_0$$

$$\% \text{ req.} = \frac{\frac{7}{8} N_o}{N_o} \times 100 = 87.5\%$$

97. (b) After modulation, recovery of signal is done by demodulating it.

98. (a) At the position when the gravitational field is zero. (Distance *x* is measured from 4 m mass)

$$\frac{G(4m)}{x^2} = \frac{G(9m)}{(r-x)^2}$$

$$\frac{4}{9} = \left(\frac{x}{r-x}\right)^2 \Rightarrow \quad \frac{2}{3} = \frac{x}{r-x}$$

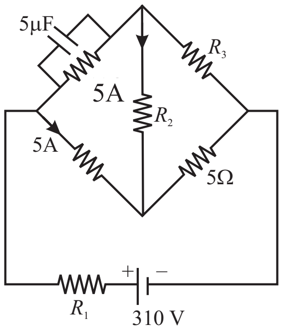
$$2r - 2x = 3x \Rightarrow \quad 2r = 5x \Rightarrow x = \frac{2r}{5}$$

The point *P* is at a distance $\frac{2r}{5}$ from mass 4m and $\left(r - \frac{2}{5}r\right) = \frac{3r}{5}$ from mass m.

$$v = -\frac{G(4m)}{\left(\frac{2}{5}r\right)} - \frac{G(9m)}{\frac{3r}{5}} = \frac{-5Gm}{r}\left[\frac{4}{2} + \frac{9}{3}\right]$$

$$= \frac{-5Gm}{r}[2+3] \qquad = -25\frac{Gm}{r}$$

99. (c)



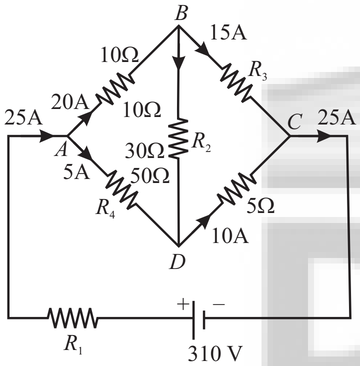
Charge on cap = 1 mC

$$V_{\text{capacitor}} = \frac{Q}{C} = \frac{1\times10^{-3}}{5\times10^{-6}} = \frac{1000}{5} = 200 \text{ V}$$

So, current through R_5

$$I = \frac{V}{R} = \frac{200}{10} = 20 \text{ A}$$

Hence, we have following current distribution.



In loop *BCDB*,

$$-15R_3 + 10\times5 + 5R_2 = 0 \qquad ...(i)$$

In loop *ABCDA*,

$$-200 - 15R_3 + 50 + 250 = 0$$

$$15R_3 = 100 \Rightarrow R_3 = \frac{100}{15}\Omega$$

From Eq. (*i*), we get

$$5R_2 = 50 \Rightarrow R_2 = 10\Omega$$

and from loop *ADCA*,

$$-250 - 50 + 310 - 25R_1 = 0$$

$$25R_1 = 10 \Rightarrow R_1 = \frac{10}{25}\Omega$$

So, $$\frac{R_1\times R_2}{R_3} = \frac{\frac{10}{25}\times10}{\left(\frac{100}{15}\right)} = \frac{15}{25} = \frac{3}{5} = 0.6$$

100. (a) $B_0 = \dfrac{E_0}{c}$ $\quad\left[\because\ c = 3\times10^8 \text{ m/s}\right]$

$$= \frac{60}{3\times10^8} = 20\times10^{-8} = 2\times10^{-7}\text{ T}$$

101. (a) The total time after the two cars which meet

$$= \frac{\text{Total distance}}{\text{Relative velocity of car w.r.t each other}}$$

$$T = \frac{36}{54+36} \Rightarrow T = \frac{36}{90} = 0.4\ h$$

In this duration, the distance travel by bird

= 36 × 0.4 = 14.4 km = 14400 m

102. (d) According to question, the translation KE of a molecule of gas = $\frac{3}{2}kt$

According to the question,

$$\frac{3}{2}kt = eV \Rightarrow \frac{3}{2}kt = 10e$$

$$\Rightarrow T = \frac{20e}{3K_B} = \frac{20\times1.6\times10^{-19}}{3\times1.38\times10^{-23}}$$

$$= 77.3\times10^3\,K$$

103. (b) For closed organ pipe, Ist overtone = 3rd harmonic

$$f = f_2 = 3f_1 = \frac{3\times v}{4L} = \frac{3}{4L}\times\sqrt{\frac{\gamma p}{\rho_1}}$$

For open organ pipe, Ist overtone = 2nd harmonics

$$f = f_2 = 2f_1 = \frac{2v}{2L'} = \frac{v}{L'} = \frac{1}{L'}\times\sqrt{\frac{\gamma p}{\rho_2}}$$

As B is same. So pressure (α β) is also same for both gas

For same sound frequency

$$\frac{3}{4L}\times\sqrt{\frac{\gamma p}{\rho_1}} = \frac{1}{L'}\times\sqrt{\frac{\gamma p}{\rho_2}}$$

$$\therefore\ L' = \frac{4L}{3}\times\sqrt{\frac{\rho_1}{\rho_2}}$$

104. (d) $V = KT^2 \Rightarrow dV = 2KT\,dT$

So, $W = \int P\,dV = R\int\frac{T}{v}dV$

$$= R\int\frac{T}{KT^2}2KT = 2R\int_0^{60}dT = 120R$$

105. (c) $I^2R = 240W,\ I = 4A$

$$\therefore\ R = \frac{240}{16} = 15\Omega$$

As, $V = IZ = I\sqrt{X_L^2 + R^2} \Rightarrow \frac{V}{I} = \sqrt{X_L^2 + R^2}$

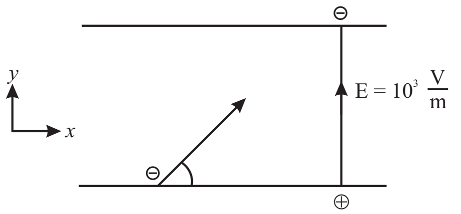
$$\Rightarrow \frac{V^2}{I^2} = X_L^2 + R^2$$

or $$X_L^2 = \frac{100\times100}{4\times4} - 225 = 400$$

$$\therefore\ X_L = 20 = L\omega$$

$$\therefore\ L = \frac{20}{2\pi f} = \frac{20}{2\pi\times50} = \frac{1}{5\pi}H$$

106. **(d)**



We have,

$$h_{\max} = \frac{u^2 \sin^2 \theta}{2\left(\frac{qE}{m}\right)} \quad \left[\text{Here } g \to \frac{qE}{m}\right]$$

$$u^2 = \frac{2hm\, qE}{m \sin^2 45°} \quad u^2 = \frac{2 \times 2 \times 10^{-2} \times 10^{-6} \times 10^3}{2 \times 10^{-3} \times 1/2}$$

$\Rightarrow \quad u^2 = 0.04 \quad \Rightarrow u = 0.2\ m/s$

107. **(a)** The de-Broglie wavelength of electron

$$\lambda_e = \frac{h}{\sqrt{2mK_e}}$$

K_e = Kinetic energy of electron

$$\lambda_e^2 = \frac{h^2}{2m_e K_e} \Rightarrow K_e = \frac{h^2}{\lambda_e^2 \cdot 2m_e}$$

De-Broglie wavelength of photon is λ_p.

Kinetic energy of photon

$$K_P = hv = \frac{hc}{\lambda_P}$$

$$\therefore \quad \frac{K_e}{K_P} = \frac{h^2/\lambda_e^2 \cdot 2m_e}{hc/\lambda_P}$$

As, $\lambda_e = \lambda_p$

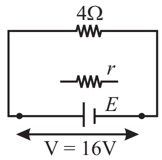
Then, $\dfrac{K_e}{K_P} = \dfrac{h}{c\, \lambda_e \times 2m_e}$

$$= \frac{6.66 \times 10^{-34}}{3 \times 10^8 \times 1.2 \times 10^{-10} \times 2 \times 9.1 \times 10^{-31}} \cong \frac{1}{100}$$

$\therefore \quad K_e : K_P :: 1 : 100$

108. **(b)** $E = V + Ir$

Given, $V = 1.6$ V; $R = 4\ \Omega$

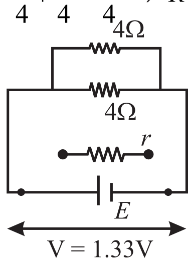


$$I_1 = \frac{1.6}{4} = 0.4\text{A}$$

$E = 1.6 + 0.4\, r \quad ...(i)$

When a resistance of 4 Ω is connected in parallel the resultant resistance

$$\frac{1}{R'} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \Rightarrow R' = 2\Omega$$



$$I_2 = \frac{133}{2} = 0.66\text{ A}$$

So, $E = 1.33 + 0.66r \quad ...(ii)$

From Eqs. (i) and (ii), we get

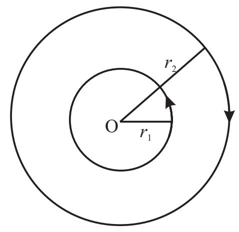
$1.6 + 0.4r = 1.33 + 0.66r$

$$\Rightarrow \quad r = \frac{0.27}{0.26} = 1$$

Substituting of r in Eq. (i), we get

$E = 1.6 + 0.4 \times 1 = 2$ V

109. **(d)**



As $\quad B_{centre} = \dfrac{\mu_0 nI}{2r}$

So, Magnetic field at O,

$$B_O = \frac{\mu_0}{2}\left[\frac{n_1 I_1}{r_1} - \frac{n_2 I_2}{r_2}\right]$$

$n_1 = n_2 = 20$

$$B_O = \frac{\mu_0}{4\pi} \cdot 2\pi \times 20\left[\frac{0.4}{30 \times 10^{-2}} - \frac{0.6}{60 \times 10^{-2}}\right]$$

$$= \frac{\mu_0}{2} \times 20\left[\frac{0.8 - 0.6}{60 \times 10^{-2}}\right] = \frac{10}{3}\mu_0$$

110. **(b)** Length of pendulum = L

Time period of simple pendulum

$$T_1 = 2\pi\sqrt{\frac{L}{g}}$$

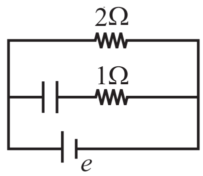
Length of rod = L

Time period of *rod*;

$$T_2 = 2\pi\sqrt{\frac{2L}{3g}} \quad \left[\because T = 2\pi\sqrt{\frac{I}{mg \quad r_{cm}}} \text{ and } r_{cm} = \ell/2\right]$$

$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{L/g}}{2\pi\sqrt{2L/3g}} = \sqrt{\frac{3}{2}}$$

111. **(c)**



$$V_C = \frac{Q}{C} = \frac{4 \times 10^{-6}}{2 \times 10^{-6}} = 2\text{ v}$$

Now, V_C = V (across 2Ω resistor)

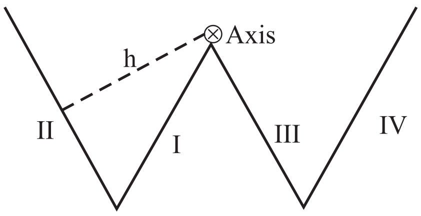
$$\therefore \quad I = \frac{V}{R} = \frac{2}{2} = 1\text{ A}$$

Now, using the relation, $V = E - Ir$

So, $E = V + Ir = 2 + 1 \times 0.5 = 2.5$ V

112. (a) There are four rods each of mass m = 400 g = 0.4 kg and each having a length of ℓ = 30 cm = 0.3 m

So, we can calculate moment of inertia as follows



$$h^2 = l^2 - \left(\frac{l}{2}\right)^2 = \frac{3}{4}l^2$$

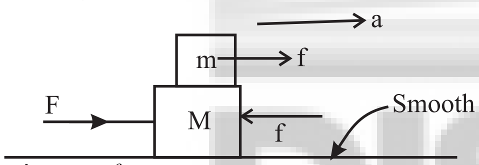
M.I. system = (MI of I) × 2 + (MI of II) × 2

$$= \frac{ml^2}{3} \times 2 + 2\left(\frac{ml^2}{12} + mh^2\right)$$

$$= \frac{2}{3}ml^2 + 2\left(\frac{ml^2}{12} + \frac{3}{4}ml^2\right)$$

$$= \frac{7}{3}ml^2 = \frac{7}{3} \times 0.4 \times 0.3 \times 0.3 = 0.084 \text{ kg-m}^2$$

113. (c)



For 'm' mass $f = ma$ ... (i)

For 'M' mass $F - f = ma$ ... (ii)

From (i) & (ii), we get

$$a = \frac{F}{m+M} \text{ and } f = \frac{mF}{m+M}$$

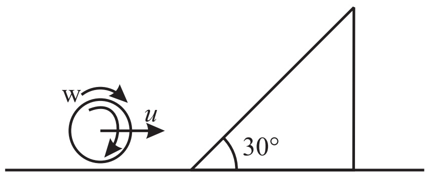
Distances moved in time t (s)

$$= \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{M+m}\right)t^2$$

Work done by friction = $f \times s$

$$= \left(\frac{mF}{M+m}\right) \times \frac{1}{2}\frac{Ft^2}{(M+m)} = \frac{mF^2t^2}{2(M+m)^2}$$

114. (a)



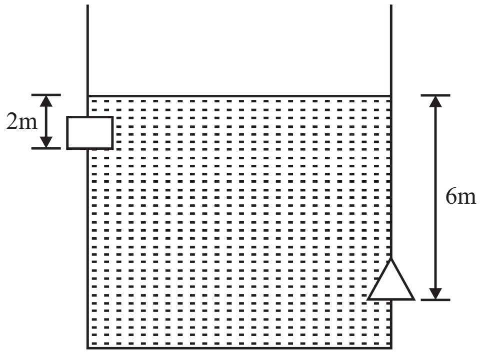
Given, $u = 4 \text{ ms}^{-1}$

$$a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}} = \frac{10 \times \sin 30}{1 + \frac{2}{5}} = \frac{5}{\frac{7}{5}} = \frac{25}{7} \text{ m/s}^2$$

As $v^2 = u^2 + 2aS$

$$\Rightarrow \quad |S| = \frac{u^2}{2a} = \frac{4^2}{2 \times \frac{25}{7}} = \frac{16 \times 7}{50} = \frac{112}{50} = 2.24 \text{ m}$$

115. (d)



Volume flow rate, R_v

R_v = area × velocity

$R_{v_1} = x^2 \times \sqrt{2 \times g \times 2}$ ... square hole ...(i)

$R_{v_2} = \frac{\sqrt{3}}{4} \times 4^2 \times \sqrt{2 \times g \times 6}$ triangle hole ...(ii)

Equating (i) & (ii) and solving, we get

$x = 2\sqrt{3} \Rightarrow x = 3.46$ cm

116. (a) $r_1 = 5.5 \times 10^{-11}$ m

$v_1 = 4 \times 10^6$ m/s

As $r_n \propto n^2$

So, $r_2 = 5.5 \times 10^{-11} \times 2^2 = 22.0 \times 10^{-11}$ m/s

As $v_n \propto \frac{1}{n}$

So, $v_2 = \frac{4 \times 10^6}{2} = 2 \times 10^6$ m/s

The time period of revolution

$$T = \frac{2\pi r_2}{v_2} = \frac{2 \times 3.14 \times 22.0 \times 10^{-11}}{2 \times 10^6}$$

$$= 6.908 \times 10^{-16} \text{ s}$$

117. (a) Given,

d = 0.5×10^{-9} m and $\gamma = 3 \times 10^{-3}$ mm

$\lambda = 500 \times 10^{-9}$ m

$D = 120$ cm $= 120 \times 10^{-2}$ m

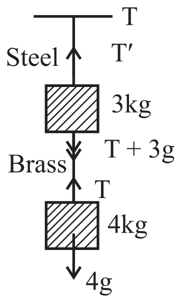
$$\gamma = \frac{\Delta x\, D}{d}$$

$$\Delta x = \frac{\gamma d}{D}$$

$$\Delta x = \frac{3 \times 10^{-3} \times 0.5 \times 10^{-3}}{1.2} = \frac{5}{4} \times 10^{-6}$$

$$\Delta\phi = \frac{2\pi \times 1.25 \times 10^{-6}}{500 \times 10^{-9}} = 5\pi$$

118. (c) Now,



$T' = T + 3g \Rightarrow T = 4g$

So, $T' = 7g$

So, $\frac{F_S}{F_B} = \frac{T'}{T} = \frac{7}{4}$

We know that,

$Y = \frac{F/A}{\Delta L/L} \Rightarrow \Delta L = \frac{FL}{AY}$

$\frac{\Delta L_S}{\Delta L_B} = \left(\frac{F_S}{F_B}\right)\left(\frac{L_S}{L_B}\right)\left(\frac{A_B}{A_S}\right)\left(\frac{Y_B}{Y_S}\right)$

$= \frac{7}{4} \times (a)\left(\frac{1}{b}\right) \times \left(\frac{1}{c}\right)$

or, $\frac{\Delta L_S}{\Delta L_B} = \frac{7a}{4bc}$ or, $\frac{\Delta L_B}{\Delta L_S} = \frac{4bc}{7a}$

119. (a) Here $g_{eff} = g - a$

So, $m(g - a) = 150\ N$

$60(10 - a) = 150$

$a = 7.5\ m/s^2$

120. (b) → We need high retentivity for permanent magnet so that it does not loses its magnetic property even if the external magnetic field is removed.

→ Dimagnetic substance is feebly induced in the direction opposite of magnetic field. So it has negative susceptibility.

(a)	High retentivity	(i)	Permanent magnet
(b)	High resistivity	(ii)	To decrease eddy current losses
(c)	Low coercivity	(iii)	Telephone diaphragm
(d)	Negative susceptibility	(iv)	Dimagnet

CHEMISTRY

121. (d) Required relation

$$H_3PO_4(aq) \rightleftharpoons 3H^+(aq) + PO_4^{3-}(aq)$$

$$K = \frac{[H^+]^3[PO_4^{3-}]}{[H_3PO_4]}$$

For reaction (*i*)

$$K_1 = \frac{\left[H^+\right]^+_{(aq)}\left[H_2PO_4^-\right]_{(aq)}}{H_3PO_{4(aq)}} \quad ...(i)$$

For reaction (*ii*)

$$K_2 = \frac{\left[H^+\right]_{(aq)}\left[HPO_4^{2-}\right]_{(aq)}}{\left[H_2PO_4^-\right]_{(aq)}} \quad ...(ii)$$

For reaction (*iii*)

$$K_3 = \frac{\left[H^+\right]_{(aq)}\left[PO_4^{3-}\right]_{(aq)}}{\left[HPO_4^{2-}\right]_{(aq)}} \quad ...(iii)$$

On multiplying (*i*), (*ii*) and (*iii*)

$K = K_1 \times K_2 \times K_3$

122. (c) Molecular orbital configuration for

(i) $B_2(10) = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi p_x^1 \approx \pi p_y^1$

Two unpaired electrons, paramagnetic.

(ii) $N_2(14)$

$= \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi p_x^2 \approx \pi p_y^2, \sigma 2p_z^2$

No unpaired electrons, diamagnetic.

(iii) O_2 (16)

$= \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 \approx \pi 2$

$\pi^* 2p_x^1, \pi^* 2p_y^1$

Two unpaired electrons, paramagnetic

(iv) C_2 (12)

$= \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^2 \approx \pi 2p_y^2$

No unpaired electron, diamagnetic.

123. (c) $2M(s) + 2H_2O \longrightarrow 2MOH + H_2(g)$

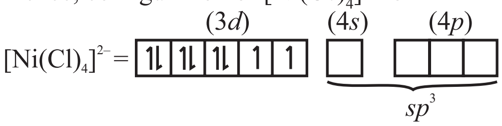
(Where M = Alkali Metal)

124. (d) Bimolecular nucleophilic substitution (S_N2) reaction involves reaction between reactant and nucleophile simultaneously without the formation of carbocation. Less hindered species are more reactive towards S_N2 reaction. As CH_3X is least hindered alkyl halide, it will show more fast reaction towards S_N2 reaction.

125. (d) (a) Ni in $[Ni(Cl)_4]^{2-}$ exist as Ni^{2+} ion.

∵ Cℓ is a weak field ligand (high spin). It will not cause pairing of electrons.

Hence, configuration of $[Ni(C\ell)_4]^{2-}$ is

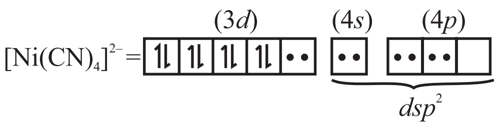


Tetrahedral with two unpaired electrons (i.e., paramagnetic) and has sp^3 hybridisation.

(b) In $\left[Co(C_2O_4)_3\right]$, $(C_2O_4)_3$ is a bidentate ligand thus, give octahedral structure.

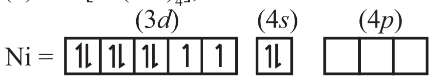
(c) In $\left[Ni(CN)_4\right]^{2-}$, Ni exist as Ni^{2+} ion.

∵ CN^- is a strong field ligand (low spin), causes pairing of electrons of 3*d* orbital.

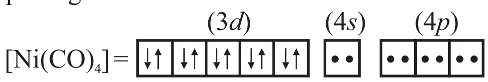


Hence, the structure of $\left[Ni(CN)_4\right]^{2-}$ is square planar and it is diamagnetic.

(d) In $[Ni(CO)_4]$, Ni has zero oxidation state. i.e.,



CO is a strong field ligand causes rearrangement and pairing of electrons of 3*d* and 4*s* orbital.



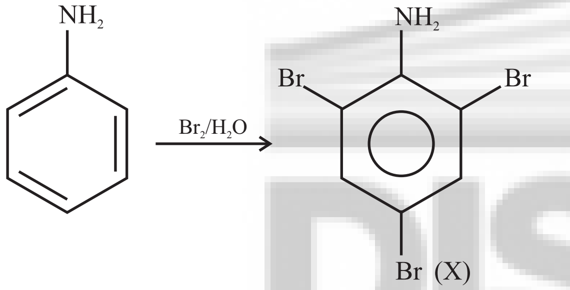
Structure of $Ni(CO)_4$ is tetrahedral with sp^3 hybridisation and it is diamagnetic.

Hence, (d) is the correct answer.

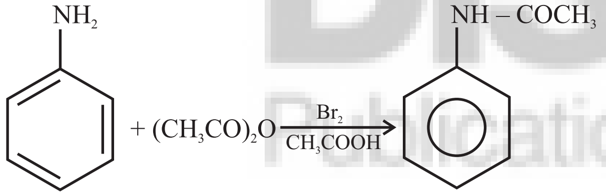
126. **(a)**

(i) Since H_2O is highly polar in nature and aniline is highly reactive towards electrophilic substitution.

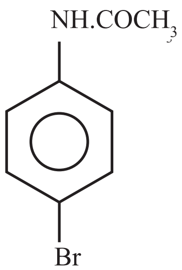
Thus, Br_2 in H_2O gives: 2, 4, 6 tribromo aniline as main product i.e.



(*ii*)



and as a result only mono-substituted bromine at *p*-position is the major product i.e.,



127. **(d)** Chlorofluoro carbons (CFCs) such as CF_2Cl_2, $CFCl_3$, hydrofluoro carbons (HFCs) are responsible for ozone layer depletion or ozone holes.

$$CF_2Cl_2 \xrightarrow{hv} \dot{Cl} + \dot{C}F_2Cl$$

$$\dot{Cl} + O \longrightarrow Cl\dot{O} + O_2$$

$$Cl\dot{O} + O \longrightarrow \dot{Cl} + O_2$$

128. **(d)** In ionic polymerisation ions/ion pairs have ends with ionic center $(C_6H_5CO)_2O_2$ is a covalent compound and will not give any ionic center for chain initiation, thus can not be used as an initiator in ionic polymerisation.

129. **(a)** Elemination of 2-bromopentane follow Saytzeff's rule and give pent-2-ene as major product. Some pent-1-ene also form according to Hofmann rule:

$$CH_3 — CH_2 — CH_2 — \underset{\substack{|\\ Br}}{CH} — CH_3$$

Dehydrobromination →

$CH_3—CH_2—CH{=}CH—CH_3$ (Major) Saytzeff's

$CH_3—CH_2—CH_2—CH{=}CH_2$ (Minor) Hoffman product

130. **(d)** $\underset{(1:20)}{Xe + F_2} \xrightarrow[60–70\ bar]{573\ K} XeF_6$

131. **(d)** $\because$ $\pi = iC\,RT$.

(a) π for 5.0 M urea is

$\pi = 1 \times 5 \times 0.0821 \times 340\ K$

$\pi = 139.57$ atm

(b) π for 1.5 M A_2B_3 type is ($i = 4$)

$\pi = 4.1 \times 1.5 \times 0.0821 \times 300\ K$

$\pi = 151.47$ atm

(c) π for 3.0 M AB type is ($i = 1.6$)

$\pi = 1.6 \times 3 \times 0.0821 \times 300\ K$

$\pi = 118.22$ atm

(d) π for 2.5 M AB_2 type is ($i = 2.5$)

$\pi = 2.5 \times 2.5 \times 0.0821 \times 330\ K$

$\pi = 169.33$ atm

Hence AB_2 highest osmotic pressure at 57°C shows.

132. **(d)**

(i) $2Al(s) + 6HCl(aq) \longrightarrow 2AlCl_3(aq) + 3H_2$

(ii) $2Al(s) + 2NaOH(aq) \longrightarrow$

$$\underset{\text{Sodium tetrahydroxo aluminate (III)}}{2Na\left[Al(OH)_4\right]} + 3H_2$$

(iii) $NaBH_4 + I_2 \longrightarrow BH_3 + NaI + HI$

$$\downarrow (NaBH_4)$$

$$2BH_3 + 2NaI + H_2$$

133. **(b)** Given,

W_B (mass of ethylene glycol) = 31 g

W_A (mass of water) = 600 g

K_f (for water) = 1.86 K kg mol^{-1}

and M_B (for $C_2H_6O_2$) = 62

$$\because \quad \Delta T_f = K_f \cdot \frac{W_B}{M_B} \times \frac{1000}{W_A}$$

$$\therefore \quad \Delta T_f = \frac{1.86 \times 31 \times 1000}{62 \times 600}$$

$$= 1.55K$$

134. (a) Let, number of moles of $SO_2 = x$

moles of $SO_3 = 2x$

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$$

At equilibrium,

$$K_c = \frac{[SO_3]^2}{[SO_2]^2[O_2]}$$

$$100 = \frac{\left[\frac{2x}{10}\right]^2}{\left[\frac{x}{10}\right]^2\left[\frac{n_{O_2}}{10}\right]}$$

$$100 = \frac{40}{n_{O_2}} \quad n_{O_2} = \frac{40}{100} = 0.4$$

Hence, number of moles of oxygen = 0.4

135. (d) For He^+ : $Z = 2$, $n = 2$

$$E_n = -2.18 \times 10^{-18} \cdot \frac{Z^2}{n^2} \cdot J$$

$$\therefore \quad E_n = -2.18 \times 10^{-18} \times \frac{4}{4}$$

$$= -2.18 \times 10^{-18} J$$

$$r_n = 52.9 \times \frac{n^2}{Z} pm = 52.9 \times \frac{4}{2} = 105.8 \text{ pm}$$

136. (b)

(a) $2MnO_4^- + 10I^- + 16H^+ \longrightarrow 2Mn^{2+} + 5I_2 + 8H_2O$

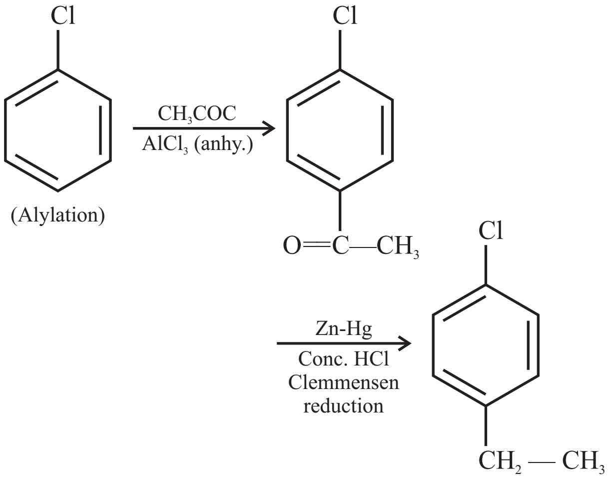
(b) $2MnO_4^- + I^- + H_2O \longrightarrow 2MnO_2 + \underset{Y}{IO_3^-} + \underset{X}{2OH^-}$

Hence, (X) and (Y) are respectively I_2 and IO_3^-.

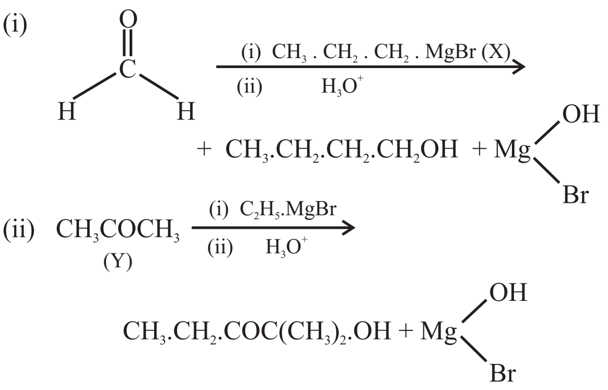
137. (b) Na^+ and Mg^{2+} are isoelectrionic species having $10e^-$ each. But Mg^{2+} has 12 protons while Na^+ has 11 protons. Hence effective nuclear charge is greater in case of Mg^{2+}.

$$\text{ionic radius} \propto \frac{1}{\text{effective nuclear charge}}$$

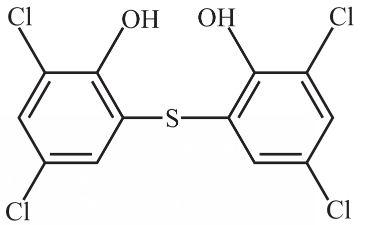
138. (d)



139. (b)



140. (a) Bithionol is added to soap generally to impart antiseptic properties.



Structure of Bithionol

141. (b) Energy of EM wave used

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}}$$

$$E = 0.066 \times 10^{-17} = 6.6 \times 10^{-19} J$$

Metals surface which have work function less than or equal to 6.6×10^{-19} J will eject electrons on radiation with light of 300 nm.

Work function of Ag = 4.3 eV × 1.6022 × 10^{-19} = 6.89 × 10^{-19} J

Mg = 3.7 eV × 1.6022 × 10^{-19} = 5.93 × 10^{-19} J

K = 2.25 eV × 1.6022 × 10^{-19} = 3.60 × 10^{-19} J

Na = 2.30 eV × 1.6022 × 10^{-19} = 3.68 × 10^{-19} J

$\therefore$ Only Mg, K and Na have their work function less than 6.6 × 10^{-19} J hence, they will eject the electrons.

142. (d) (i) NO_2 group increases the acidic nature due to –I effect.

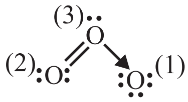
(ii) OCH_3 and CH_3 are electron donating groups (i.e.,+I effect), thus decreases the acidic nature, but OCH_3 will decrease acidic nature more than of CH_3 group

Hence, correct order is

III < IV < I < II

143. (b) Colloidal solution of gold show different colours (like red, purple, blue and golden) due to difference in size gold particles in colloidal solution.

144. (a) $\because$ Formal charge = No. of valency electron in central atom – [No. of lone pair + $\frac{1}{2}$ No. of bond pair of electron]

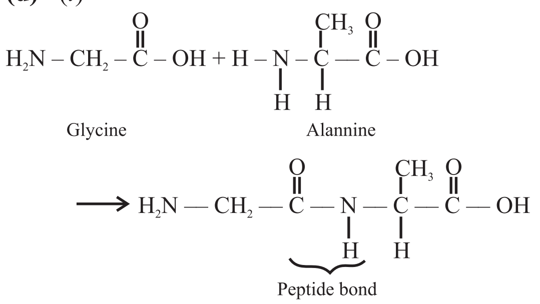


F.C for O labelled as 1 = 6 – [6 + 1] = –1

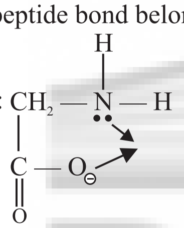
F.C for O labelled as 2 = 6 – [4 + 2] = 0

F.C for O labelled as 3 = 6 – [2 + 3] = +1

145. (d) (*i*)



Hence, $-\overset{\overset{\displaystyle O}{||}}{C}-O$ of peptide bond belongs to glycine

(*ii*) glycylamine is: 

(*iii*) Phosphate group – pentose sugar – Nitrogenous base unit is known as nucleatide.

146. (d) Given ΔH_f for CH_3OH = –239 kJ mol^{-1}

ΔH_f of H.CHO = – 116 kJ mol^{-1}

ΔH_f of H_2O = – 286 kJ mol^{-1}

Required relation:

$$2CH_3OH + O_2 \longrightarrow 2HCHO + 2H_2O$$

$$\Delta H_R = \frac{1}{2} H_{f^\circ}\left[(2x - 116) + (2x - 286)\right]$$

$$-1\left[(2 \times 239)\right]$$

$$\Delta_r H = \Sigma\left(\Delta_f H_{\text{Product}}\right) - \Sigma\left(\Delta_f H_{\text{Reactant}}\right)$$

$$= \left(-116 + (-286)\right) - (-293 + 0)$$

= –163 kJ mol^{-1}

147. (b) $\underset{\substack{\text{Copper pyrites}\\ [\text{ore}]}}{2\text{Cu FeS}_2} + O_2 \longrightarrow \underbrace{Cu_2S + 2FeS}_{\text{Matter}} + SO_2(g)$

148. (b) Given

$T = 27 + 273 = 300K$

$V = 10L$

$W = W_1(He) + W_2(Ne) = 4g$ ...(*i*)

Molar mass (He) = 4

Molar mass (Ne) = 20

P = 123 atm

R = 0.082 L atm K^{-1} mol^{-1}

(*i*) $\because$ $PV = (n_1 + n_2)RT = \left[\frac{W_1}{4} + \frac{W_2}{20}\right]RT$

$\therefore$ $\left[\frac{W_1}{4} + \frac{W_2}{20}\right] = \frac{1.23 \times 10}{0.082 \times 300} = 0.5$

$25W_1 + 5W_2 = 50$

$5W_1 + W_2 = 10$ ...(*ii*)

On solving equation (*ii*) with the help of eq. (*i*)

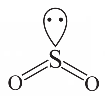
$5W_1 + 4 - W_1 = 10 \Rightarrow \quad W_1 = 1.5$

$W_2 = 2.5$

Mass % of neon = $\frac{2.5 \times 100}{4} = 62.5\%$

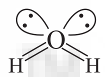
149. (c) $S + 2H_2SO_4(\text{conc.}) \longrightarrow \underset{(X)}{3SO_2} + \underset{(Y)}{2H_2O}$

As SO_2 has one lone pair of electrons on central atom i.e., sulphur.



(X)

And H_2O has two lone pair of electrons on the central atom i.e., oxygen.



(Y)

150. (d) (*i*) Electromeric effect is a temporary effect, it takes place only in the presence of reagent.

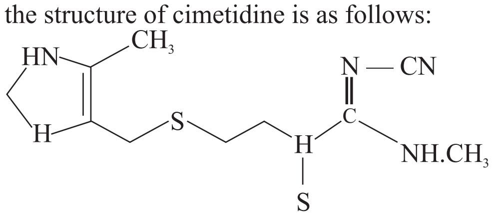
(*ii*) Hyper-conjugation is a permanent effect which allows delocalised sigma electrons to adjacent empty *r* or *p*-orbital.

151. (d) Carbon monoxide is estimated using I_2O_5 as per the following reaction:

$$I_2O_5 + 5CO \longrightarrow \underset{\text{violet}}{I_2} + 5CO_2$$

152. (c) $[SiF_6]^{2-}$ is known to exist but due to large size of chlorine (Cl) atom, $[SiCl_6]^{2-}$ ions can not known.

153. (b) Cimetidine is used to prevent the interaction of histamine with the receptors present in the stomach wall, the structure of cimetidine is as follows:



154. (d) (*i*) For a compound to be aromatic

(*ii*) The ring system must contain (4n + 2)π electrons.

(*iii*) The ring system must be planar.

155. (c) In XO_2, X : O : : 50 : 50

Mass of oxygen = 2 × 16 = 32 g

Hence, mass of X = 32 g

In XO_n, X : O : : 40 : 60

Mass of X, in this case also remains = 32 g

40% ≡ 32 g

$60\% \equiv \frac{32}{40}\times 60 = 48$ g

Mass of oxygen = 48 g

$\Rightarrow \quad n = \frac{48}{16} = 3$

Formula of oxide = XO_3

156. **(d)** Given,

Body centered cubic (bcc) $Z = 2$

Edge length (a) = 400 pm = 400×10^{-10}

M (Atomic mass) = 24 g mol^{-1}

$\because \quad$ Density (d) = $\frac{Z \times M}{a^3 \times N_A}$

$$d = \frac{2\times 24}{\left(400\times 10^{-10}\right)^3 \times 6.022\times 10^{23}}$$

$d = 125\ g\ \text{cm}^{-3}$

157. **(b)** For t = 90 min (from 10 a.m. to 11:30 a.m.)

As 20% was already completed $\therefore$ $a = 80$

$(a - x) = (100 - 80) = 20$

$\because \quad k = \frac{2.303}{t}\log\frac{a}{a-x}$

$k = \frac{2.303}{90}\log\frac{80}{20}$

$k = \frac{2.303}{90}\log 4 = \frac{2.303 \times 0.6020}{90}$

$k = 0.015$ min

$\because \ t_{1/2} = \frac{0.693}{k}$

$\Rightarrow t_{1/2} = \frac{0.693}{0.015} = 46.2 \approx 45\,\text{min}$

158. **(c)** When sodium acetate is electrolysed in aqueous medium, it react as follows

$$2CH_3COONa + 2H_2O \longrightarrow 2CO_2 + 2C_2H_6 + 2H_2$$

At anode (Y): $C_2H_6 + CO_2$

At cathode (X): H_2

159. **(c)** Given,

Normality of $KMnO_4$ (N_1) = 1 N

Volume of $KMnO_4$ (V_1) = 500 mL

Normality of H_2O_2 (N_2) = $\frac{\text{Volume strength}}{5.6\,(\text{eq. mass})} = \frac{20}{5.6}$

$(N_2) = 3.5$

Now, $\quad N_1V_1\,(KMnO_4) = N_2V_2\,(H_2O_2)$

$1 \times 500 = 3.57 \times V_2$

$\therefore \quad V_2 = \frac{500}{3.57} = 140\text{mL}$

160. **(b)** When same amount of electricity is passed through $AgNO_3$ and $CuSO_4$ aqueous solution.

Then 1 mole of Ag^+ will deposit and half mole of Cu^{2+} will deposite.

The relations between Ag^+ and Cu^{2+} ion is as follows

1 mol of Ag^+ = $\frac{1}{2}$ mol of Cu^{2+}

Thus for, $\frac{x \text{ mol of } Ag^+}{y \text{ mol of } Cu^{2+}} = \frac{1\text{ mol}}{1/2\text{ mol}}$

$\therefore \quad x = 2y$