AP/EAMCET Solved Paper 2018

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. If $f : R \to R$ is defined by $f(x) = [2x] - 2[x]$ for $x \in R$, then the range of f is (Here $[x]$ denotes the greatest integer not exceeding x)
(a) Z, the set of all integers
(b) N, the set of all natural numbers
(c) R, the set of all real numbers
(d) $\{0, 1\}$

2. Given that a, b and c are real numbers such that $b^2 = 4ac$ and $a > 0$. The maximal possible set $D \subseteq R$ on which the function $f : D \to R$ given by
$f(x) = \log\{ax^3 + (a+b)x^2 + (b+c)x + c\}$ is defined, is
(a) $R - \left\{-\frac{b}{2a}\right\}$
(b) $R - \left(\left\{-\frac{b}{2a}\right\} \cup (-\infty, -1)\right)$
(c) $R - \left(\left\{-\frac{b}{2a}\right\} \cup (x : x \geq 1)\right)$
(d) $R - \left(\left\{-\frac{b}{2a}\right\} \cup (-\infty, -1]\right)$

3. For any natural number n, $(15 \times 5^{2n}) + (2 \times 2^{3n})$ is divisible by
(a) 7 (b) 11 (c) 13 (d) 17

4. For the matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}, A^{-1} =$
(a) A (b) A^2 (c) A^3 (d) A^4

5. If $A = \begin{bmatrix} k/2 & 0 & 0 \\ 0 & l/3 & 0 \\ 0 & 0 & m/4 \end{bmatrix}$ and
$A^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$, then $k + l + m =$
(a) 1 (b) 9 (c) 14 (d) 29

6. If A and B are the two real values of k for which the system of equations $x + 2y + z = 1$, $x + 3y + 4z = k$. $x + 5y + 10z = k^2$ is consistent, then $A + B =$
(a) 3 (b) 4 (c) 5 (d) 7

7. Let $z = x + iy$ and a point P represent z in the Argand plane. If the real part of $\frac{z-1}{z+i}$ is 1, then a point that lies on the locus of P is
(a) (2016, 2017) (b) (–2016, 2017)
(c) (–2016, –2017) (d) (2016, –2017)

8. If $13e^{i\tan^{-1}\frac{5}{12}} = a + ib$, then the ordered pair $(a, b) =$
(a) (12, 5) (b) (5, 12) (c) (24, 10) (d) (10, 24)

9. If $z_1 = 1 - 2i$; $z_2 = 1 + i$ and $z_3 = 3 + 4i$, then
$\left(\frac{1}{z_1} + \frac{3}{z_2}\right)\frac{z_3}{z_2} =$
(a) $13 - 6i$ (b) $13 - 3i$
(c) $6 - \frac{13}{2}i$ (d) $\frac{13}{2} - 3i$

10. If 1, ω, ω^2 are the cube roots of unity, then
$\frac{1}{1+2\omega} + \frac{1}{2+\omega} - \frac{1}{1+\omega} =$
(a) 1 (b) ω (c) ω^2 (d) 0

11. The number of integral values of x satisfying $5x - 1 < (x+1)^2 < 7x - 3$ is
(a) 0 (b) 1 (c) 2 (d) 3

12. For real number x, if the minimum value of $f(x) = x^2 + 2bx + 2c^2$ is greater than the maximum value of $g(x) = -x^2 - 2cx + b^2$, then
(a) $c^2 > 2b^2$ (b) $c^2 < 2b^2$
(c) $b^2 = 2c^2$ (d) $c^2 = 2b^2$

13. If a, b and c are the roots of $x^3 + qx + r = 0$, then $(a-b)^2 + (b-c)^2 + (c-a)^2 =$
(a) $-6q$ (b) $-4q$ (c) $6q$ (d) $4q$

14. If the sum of two roots of the equation $x^3 - 2px^2 + 3qx - 4r = 0$ is zero, then the value of r is
(a) $\frac{3pq}{2}$ (d) $\frac{3pq}{4}$ (c) pq (d) $2pq$

15. The sum of the four digit even numbers that can be formed with the digits 0, 3, 5, 4 with out repetition is

(a) 14684 (b) 43536 (c) 46526 (d) 52336

16. If x is the number of ways in which six women and six men can be arranged to sit in a row such that no two women are together and if y is the number of ways they are seated around a table in the same manner, then $x : y =$

(a) 12 : 1 (b) 42 : 1 (c) 16 : 1 (d) 6 : 1

17. The number of 5-letter words that can be formed by using the letters of the word SARANAM is

(a) 1120 (b) 6720 (c) 480 (d) 720

18. The number of rational terms in the binomial expansion of $\left(\sqrt[4]{5}+\sqrt[5]{4}\right)^{100}$ is

(a) 50 (b) 5 (c) 6 (d) 51

19. The numerically greatest term in the binomial expansion of $(2a-3b)^{19}$ when $a=\frac{1}{4}$ and $b=\frac{2}{3}$ is

(a) ${}^{19}C_5 \cdot 2^{11}$ (b) ${}^{19}C_3 \cdot \frac{1}{2^{11}}$

(c) ${}^{19}C_4 \cdot \frac{1}{2^{13}}$ (d) ${}^{19}C_3 \cdot 2^{13}$

20. If $\frac{x^2+5x+7}{(x-3)^3}=\frac{A}{(x-3)}+\frac{B}{(x-3)^2}+\frac{C}{(x-3)^3}$, then the equation of the line having slope A and passing through the point (B, C) is

(a) $x+y-20=0$ (b) $x-y+20=0$

(c) $x+y+20=0$ (d) $x-y-20=0$

21. If $\cos\left(x-\frac{\pi}{3}\right), \cos x, \cos\left(x+\frac{\pi}{3}\right)$ are in a harmonic progression, then $\cos x =$

(a) $\frac{3}{2}$ (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{\frac{3}{2}}$

22. $\cos^3 110° + \cos^3 10° + \cos^3 130° =$

(a) $\frac{3}{4}$ (b) $\frac{3}{8}$ (c) $\frac{3\sqrt{3}}{8}$ (d) $\frac{3\sqrt{3}}{4}$

23. If the general solution of $\sin 5x = \cos 2x$ is of the form $a_n \cdot \frac{\pi}{2}$ for $n = 0, \pm 1, \pm 2, \ldots$, then $a_n =$

(a) $\frac{2n}{5+2(-1)^n}$ (b) $\frac{2n+(-1)^n}{5+2(-1)^n}$

(c) $\frac{2n+1}{5+2(-1)^n}$ (d) $\frac{2n-1}{5+2(-1)^n}$

24. Let x, y be real numbers such that $x \neq y$ and $xy \neq 1$. If $ax + b\sec(\tan^{-1} x) = c$ and $ay + b\sec(\tan^{-1} y) = c$, then $\frac{x+y}{1-xy} =$

(a) $\frac{2ab}{a^2-b^2}$ (b) $\frac{2ac}{a^2+c^2}$

(c) $\frac{2ab}{a^2+b^2}$ (d) $\frac{2ac}{a^2-c^2}$

25. $\tan h^{-1}\frac{1}{2}+\cot h^{-1} 3 =$

(a) $\log\sqrt{6}$ (b) $\log 6$ (c) $-\log\sqrt{6}$ (d) $-\log 6$

26. If the median of a ΔABC through A is perpendicular to AC, then $\frac{\tan A}{\tan C} =$

(a) $1+\sqrt{2}$ (b) $-\frac{1}{\sqrt{3}}+1$ (c) -2 (d) $1+\frac{2}{\sqrt{3}}$

27. In ΔABC, $\tan\frac{A}{2}+\tan\frac{B}{2} =$

(a) $\frac{c\cot\frac{C}{2}}{4s}$ (b) $\frac{2c\cot\frac{C}{2}}{a+b+c}$

(c) $\frac{2c\tan\frac{C}{2}}{s}$ (d) $\frac{c\tan\frac{C}{2}}{a+b+c}$

28. In a ΔABC, D, E and F respectively are the points of contact of the incircle with the sides AB, BC and CA such that $AD = \alpha$, $BE = \beta$ and $CF = \gamma$, then $\frac{\alpha\beta\gamma}{\alpha+\beta+\gamma} =$

(a) R^2 (b) $2R$ (c) $2r$ (d) r^2

29. Let **a**, **b** and **c** be three non-coplanar vectors. The vector equation of a line which passes through the point of intersection of two lines, one joining the points **a** + 2**b** – 5**c**, –**a** – 2**b** – 3**c** and the other joining the points – 4**c**, 6**a** – 4**b** + 4**c** is

(a) r = 2a – 4b + 3c + μ(a – 6b + 4c)

(b) r = 3a + 6b – c + μ(a + 2b + c)

(c) r = 2a + 3b – c + μ(a + b – c)

(d) r = –2b + 3c + μ(a – 4b + 3c)

30. In ΔPQR, M is the mid-point of QR and C is the mid-point of PM. If QC when extended meets PR at N, then $\frac{|\overline{QN}|}{|\overline{CN}|} =$

(a) 1 (b) 2 (c) 3 (d) 4

31. If $\mathbf{a}=\hat{\mathbf{i}}-2\hat{\mathbf{j}}-3\hat{\mathbf{k}}, \mathbf{b}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}, \mathbf{c}=\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}}$, then $[(\mathbf{a}\times\mathbf{b})\times(\mathbf{b}\times\mathbf{c})(\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a})(\mathbf{c}\times\mathbf{a})\times(\mathbf{a}\times\mathbf{b})] =$

(a) 160000 (b) –8000 (c) 400 (d) –40

32. If $\mathbf{a}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}, \mathbf{b}=-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}, \mathbf{c}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-2\hat{\mathbf{k}}$, **n** is perpendicular to both **a** and **b** and θ is the angle between **c** and **n** then $\sin\theta =$

(a) $\sqrt{\frac{2}{3}}$ (b) $\frac{\sqrt{2}}{3\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

33. If **a**, **b** and **c** are mutually perpendicular vectors of the same magnitude, then the cosine of the angle between **a** and **a** + **b** + **c** is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

34. It **a**, **b** and **c** are non-coplanar vectors and the four points with position vectors 2**a** + 3**b** – **c**, **a** – 2**b** + 3**c**, 3**a** + 4**b** + 2**c** and k**a** – 6**b** + 6**c** are coplanar, then k =
(a) 0 (b) 1 (c) 2 (d) 3

35. The mean and the standard deviation of a data of 8 items are 25 and 5 respectively. If two items 15 and 25 are added to this data, then the variance of the new data is
(a) 29 (b) 24 (c) 26 (d) 29

36. The mean deviation from the median for the following distribution (corrected to two decimals) is

x_i	6	9	3	12	15	13	21	22
f_i	4	5	3	2	5	4	4	3

(a) 13.42 (b) 5.45 (c) 4.97 (d) 11.25

37. If a die is rolled three times, then the probability of getting a larger number on its face than the previous number each time, is
(a) $\frac{15}{216}$ (b) $\frac{5}{54}$ (c) $\frac{13}{216}$ (d) $\frac{1}{18}$

38. A man is known to speak the truth 2 out of 3 times. If he throws a die and reports that it is six, then the probability that it is actually five, is
(a) $\frac{3}{8}$ (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ (d) $\frac{4}{5}$

39. If the probability function of a random variable X is defined by $P(X=k)=a\left(\frac{k+1}{2^k}\right)$ for k = 0, 1, 2, 3, 4, 5, then the probability that X takes a prime value is
(a) $\frac{13}{20}$ (b) $\frac{23}{60}$ (c) $\frac{11}{20}$ (d) $\frac{19}{60}$

40. If X is a binomial variate with mean 6 and variance 2, then the value of $P(5 \le X \le 7)$ is
(a) $\frac{4762}{6561}$ (b) $\frac{4672}{6561}$ (c) $\frac{5264}{6561}$ (d) $\frac{5462}{6651}$

41. Let A (2, 3), B (3, –6), C (5, –7) be three points. If P is a point satisfying the condition $PA^2 + PB^2 = 2PC^2$, then a point that lies on the locus of P is
(a) (2, –5) (b) (–2, 5)
(c) (13, 10) (d) (–13, –10)

42. If the coordinates of a point P changes to (2, –6) when the coordinate axes are rotated through an angle of 135°, then the coordinates of P in the original system are
(a) (–2, 6) (b) (–6, 2)
(c) $\left(2\sqrt{2}, 4\sqrt{2}\right)$ (d) $\left(\sqrt{2}, -\sqrt{2}\right)$

43. If the portion of a line intercepted between the coordinates axes is divided by the point (2, –1) in the ratio of 3:2, then the equation of that line is
(a) $5x - 2y - 20 = 0$ (b) $2x - y - 5 = 0$
(c) $3x - y - 7 = 0$ (d) $x - 3y - 5 = 0$

44. The equation of the line passing through the point of intersection of the lines $2x + y - 4 = 0$, $x - 3y + 5 = 0$ and lying at a distance of $\sqrt{5}$ units from the origin, is
(a) $x - 2y - 5 = 0$ (b) $x + 2y - 5 = 0$
(c) $x + 2y + 5 = 0$ (d) $x - 2y + 5 = 0$

45. The equation of the line joining the centroid with the orthocentre of the triangle formed by the points (–2, 3), (2, –1), (4, 0) is
(a) $x + y - 2 = 0$ (b) $11x - y - 14 = 0$
(c) $x - 11y + 6 = 0$ (d) $2x - y - 2 = 0$

46. The lines represented by the equations $23x^2 - 48xy + 3y^2 = 0$ and $2x + 3y + 4 = 0$ form
(a) an isosceles triangle (b) a right angled triangle
(c) an equilateral triangle (d) a scalene triangle

47. If the line $x + 2y = k$ intersects the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ at two points A and B and if O is the origin, then the condition for $\angle AOB = 90°$ is
(a) $k^2 + k + 1 = 0$ (b) $k^2 - 2k + 10 = 0$
(c) $2k^2 + 9k - 10 = 0$ (d) $3k^2 + 8k - 1 = 0$

48. If $2x^2 + 3xy - 2y^2 = 0$ represents two sides of a parallelogram and $3x + y + 1 = 0$ is one of its diagonals, then the other diagonal is
(a) $x - 3y + 1 = 0$ (b) $x - 3y + 2 = 0$
(c) $x - 3y = 0$ (d) $3x - y = 0$

49. If the lengths of the tangents drawn from P to the circles $x^2 + y^2 - 2x + 4y - 20 = 0$ and $x^2 + y^2 - 2x - 8y + 1 = 0$ are in the ratio 2 : 1, then the locus P is
(a) $x^2 + y^2 + 2x + 12y + 8 = 0$
(b) $x^2 + y^2 - 2x + 12y + 8 = 0$
(c) $x^2 + y^2 + 2x - 12y + 8 = 0$
(d) $x^2 + y^2 - 2x - 12y + 8 = 0$

50. The equation of a circle touching the coordinate axes and the line $3x - 4y = 12$ is
(a) $x^2 + y^2 + 6x + 6y + 9 = 0$
(b) $x^2 + y^2 + 6x + 6y - 9 = 0$
(c) $x^2 + y^2 - 6x - 6y + 9 = 0$
(d) $x^2 + y^2 - 6x - 6y - 9 = 0$

51. The pole of the straight line $9x + y - 28 = 0$ with respect to the circle $2x^2 + 2y^2 - 3x + 5y - 7 = 0$ is
(a) (3, 1) (b) (3, –1) (c) (–3, 1) (d) (4, –8)

52. The point of intersection of the direct common tangents drawn to the circles $(x + 11)^2 + (y - 2)^2 = 225$ and $(x - 11)^2 + (y + 2)^2 = 25$ is
(a) $\left(\frac{-11}{2}, 1\right)$ (b) (–22, 4)
(c) $\left(\frac{11}{2}, -1\right)$ (d) (22, – 4)

53. In List-I, a pair of circles is given in A, B, C and in List-II, angle between those pair of circles is given. Match the items from List-I to List-II.

	List-I		List-II
A	$(x-2)^2 + y^2 = 2(x-2)^2 + (y-1)^2 = 1$	**I**	**90°**
B	$x^2 + y^2 - 6x - 6y + 9 = 0$ $x^2 + y^2 - 4x + 4y - 9 = 0$	**II**	**135°**
C	$x^2 + y^2 + 4x - 14y + 28 = 0$ $x^2 + y^2 + 4x + 5 = 0$	**III**	**60°**
		IV	**30°**

The correct matching is

	A	B	C		A	B	C
(a)	I	II	III	(b)	II	I	III
(c)	III	I	IV	(d)	IV	III	I

54. If the radical axis of the circles
$x^2 + y^2 + 2gx + 2fy + c = 0$ and
$2x^2 + 2y^2 + 3x + 8y + 2c = 0$ touches the circle
$x^2 + y^2 + 2x + 2y + 1 = 0$, then

(a) $g = \frac{3}{4}$ or $f = 2$ (b) $g \neq \frac{3}{4}, f = 2$

(c) $g = \frac{3}{4}$ or $f \neq 2$ (d) $g = \frac{2}{5}$ or $f = 1$

55. The line $y = 6x + 1$ touches the parabola $y^2 = 24x$. The coordinates of a point P on this line, from which the tangent to $y^2 = 24x$ is perpendicular to the line $y = 6x + 1$, is

(a) (–1, –5) (b) (–2, –11)
(c) (–6, –35) (d) (–7, –41)

56. A point on the parabola whose focus is $S(1, -1)$ and whose vertex is $A(1,1)$ is

(a) $\left(3, \frac{1}{2}\right)$ (b) (1, 2) (c) $\left(2, \frac{1}{2}\right)$ (d) (2, 2)

57. An ellipse having the coordinate axes as its axes and its major axis along Y-axis, passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$. Then its equation is

(a) $3x^2 + 5y^2 - 15 = 0$ (b) $5x^2 + 3y^2 - 32 = 0$
(c) $3x^2 + 5y^2 - 32 = 0$ (d) $5x^2 + 3y^2 - 48 = 0$

58. The product of the perpendicular distances drawn from the points (3, 0) and (–3, 0) to the tangent of the ellipse $\frac{x^2}{36} + \frac{y^2}{27} = 1$ at $\left(3, \frac{9}{2}\right)$ is

(a) 36 (b) 27 (c) 9 (d) 63

59. The equation of the hyperbola whose asymptotes are the lines $3x + 4y - 2 = 0$, $2x + y + 1 = 0$ and which passes through the point (1,1) is

(a) $6x^2 + 11xy + 4y^2 - 30x + 2y + 7 = 0$
(b) $6x^2 + 11xy + 4y^2 - x + 2y - 22 = 0$
(c) $6x^2 + 11xy + 4y^2 - x + 2y + 22 = 0$
(d) $6x^2 + 11xy + 4y^2 - 3x - 7y - 11 = 0$

60. If the orthocentre and the centroid of a triangle are (–3, 5, 2) and (3, 3, 4) respectively, then its circumcentre is

(a) (6, 2, 5) (b) (6, 2, –5)
(c) (6, –2, 5) (d) (6, –2, –5)

61. A plane cuts the coordinate axes X, Y, Z at A, B, C respectively such that the centroid of the ΔABC is (6, 6, 3). Then the equation of that plane is

(a) $x + y + z - 6 = 0$ (b) $x + 2y + z - 18 = 0$
(c) $2x + y + z - 18 = 0$ (d) $x + y + 2z - 18 = 0$

62. If the foot of the perpendicular drawn from the origin to a plane is (1, 2, 3), then a point on that plane is

(a) (3, 2, 1) (b) (7, 2, 1)
(c) (7, 3, –1) (d) (6, –3, 4)

63. If $[x]$ denotes the greatest integer $\leq x$, then

$$\lim_{n\to\infty} \frac{1}{n^3}\left\{\left[1^2 x\right]+\left[2^2 x\right]+\left[3^2 x\right]+\ldots+\left[n^2 x\right]\right\} =$$

(a) $\frac{x}{2}$ (b) $\frac{x}{3}$ (c) $\frac{x}{6}$ (d) 0

64. If a function f defined by

$$f(x) = \begin{cases} \dfrac{1-\sqrt{2}\sin x}{\pi - 4x}, \text{if } x \neq \dfrac{\pi}{4} \\ k, \text{if } x = \dfrac{\pi}{4} \end{cases}$$ is continuous at

$x = \frac{\pi}{4}$, then k =

(a) $\frac{1}{4}$ (b) 1 (c) $\frac{-1}{4}$ (d) 2

65. The derivative of $f(x) = x^{\tan^{-1} x}$ with respect to $g(x) = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$ is

(a) $\frac{1}{2}\sqrt{1-x^2}\, x^{\tan^{-1}x}\left[\frac{\log x}{1+x^2} + \frac{\tan^{-1} x}{x}\right]$

(b) $-\frac{1}{2}\sqrt{1-x^2}\, x^{\tan^{-1}x}\ [\log(\tan^{-1}x) + x(1+x^2)\tan^{-1}x]$

(c) $\dfrac{-2^{\tan^{-1}x}\left[\dfrac{\log x}{1+x^2} + \dfrac{\tan^{-1} x}{x}\right]}{\sqrt{1-x^2}}$

(d) $-\frac{1}{2}\sqrt{1-x^2}\, x^{\tan x^{-1}}\left[\frac{\log x}{1+x^2} + \frac{\tan^{-1} x}{x}\right]$

66. If $x = 3\cos t$ and $y = 4\sin t$, then $\frac{d^2 y}{dx^2}$ at the point $(x_0, y_0) = \left(\frac{3}{2}\sqrt{2}, 2\sqrt{2}\right)$, is

(a) $\frac{4\sqrt{2}}{9}$ (b) $-\frac{4\sqrt{2}}{9}$ (c) $\frac{8\sqrt{2}}{9}$ (d) $-\frac{8\sqrt{2}}{9}$

67. If $y = \frac{2}{\sqrt{a^2 - b^2}}\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right]$, then

$$\left.\frac{d^2 y}{dx^2}\right|_{x=\frac{\pi}{2}} =$$

(a) $\frac{b}{2a^2}$ (b) $\frac{b}{a^2}$ (c) $\frac{2b}{a}$ (d) $\frac{b^2}{2a}$

68. If $f^\circ(x^\circ) = x^3 + ax^2 + bx + 5\sin^2 x$ is an increasing function on R, then

(a) $a^2 - 3b - 15 < 0$ (b) $a^2 - 3b + 15 > 0$
(c) $a^2 - 3b - 15 > 0$ (d) $a^2 + 3b + 15 > 0$

69. The approximate value of cos31° is
(Take 1° = 0.0174)

(a) 0.7521 (b) 0.866 (c) 0.7146 (d) 0.8573

70. If x and y are two positive numbers such that $x + y = 32$, then the minimum value of $x^2 + y^2$ is,

(a) 500 (b) 256 (c) 1024 (d) 512

71. The constant 'c' of Lagrange's mean value theorem for the function $f(x)=\frac{2x+3}{4x-1}$ defined on [1, 2] is

(a) $\frac{1+\sqrt{15}}{3}$ (b) $\frac{1+\sqrt{21}}{4}$

(c) $\frac{5}{3}$ (d) $\frac{3}{2}$

72. $\int\frac{\sin 2x dx}{\sin^4 x+\cos^4 x}=\tan^{-1}(f(x))+c$, then $f\left(\frac{\pi}{3}\right)=$

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{3}$

73. $\int\left(\frac{\log x-1}{1+(\log x)^2}\right)^2 dx=$

(a) $\frac{\log x}{1+(\log x)^2}+c$ (b) $\frac{x}{x^2+1}+c$

(c) $\frac{x}{1+(\log x)^2}+c$ (d) $\frac{-x}{1+(\log x)^2}+c$

74. $\int\frac{dx}{x^3+3x^2+2x}=$

(a) $\log|x|+\log\left|\frac{x+2}{x+1}\right|+c$

(b) $\log|x|-\log|x+1|+\log|x+2|+c$

(c) $\frac{1}{2}\left[\log|x|+\log|x+1|+\log|x+2|\right]+c$

(d) $\frac{1}{2}\log\left(\frac{|x^2+2x|}{(x+1)^2}\right)+c$

75. For $n\geq 2$, If $I_n=\int\sec^n x dx$, then $I_4-\frac{2}{3}I_2=$

(a) $\sec^2 x\tan x+c$ (b) $\frac{1}{3}\sec^2 x\tan x+c$

(c) $\frac{2}{3}\sec^2 x\tan x+c$ (d) $\frac{1}{2}\log|\sec x+\tan x|+c$

76. $\lim_{n\to\infty}\left(\frac{\sqrt{1}+2\sqrt{2}+3\sqrt{3}+...+n\sqrt{n}}{n^{\frac{5}{2}}}\right)=$

(a) 1 (b) $\frac{5}{2}$ (c) 0 (d) $\frac{2}{5}$

77. $\int_0^{\frac{\alpha}{3}}\frac{f(x)}{f(x)+f\left(\frac{\alpha-3x}{3}\right)}dx=$

(a) $\frac{2\alpha}{3}$ (b) $\frac{\alpha}{2}$ (c) $\frac{\alpha}{3}$ (d) $\frac{\alpha}{6}$

78. The area (in sq. units) of the region bounded by the X-axis and the curve $y=1-x-6x^2$ is

(a) $\frac{125}{216}$ (b) $\frac{125}{512}$ (c) $\frac{25}{216}$ (d) $\frac{25}{512}$

79. If m and n are respectively the order and degree of the differential equation of the family of parabolas with focus at the origin and X-axis as its axis, then $mn-m+n=$

(a) 1 (b) 4 (c) 3 (d) 2

80. The general solution of

$$\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$

(a) $ye^{\frac{y}{x}}+x=c$ (b) $ye^{\frac{x}{y}}-x=c$

(c) $ye^{\frac{x}{y}}+y=c$ (d) $ye^{\frac{x}{y}}+x=c$

PHYSICS

81. Two resistance 60.36 Ω and 30.09 Ω are connected in parallel. The equivalent resistance is

(a) 20 ± 0 08 Ω (b) 20 ± 0 06 Ω

(c) 20 ± 0 03 Ω (d) 20 ± 010 Ω

82. **Assertion (A) :** The velocity of a projectile at a point on its trajectory is equal to the slope at that point.

Reason (R) : The velocity vector at a point always along the tangent to the trajectory at that point.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

83. A body is projected from the ground at an angle of $\tan^{-1}\left(\frac{8}{7}\right)$ with the horizontal. The ratio of the maximum height attained by it to its range is

(a) 8 : 7 (b) 4 : 7 (c) 2 : 7 (d) 1 : 7

84. A body is projected with a speed u at an angle θ with the horizontal. The radius of curvature of the trajectory, when it makes an angle $\left(\frac{\theta}{2}\right)$ with the horizontal is (g-acceleration due to gravity)

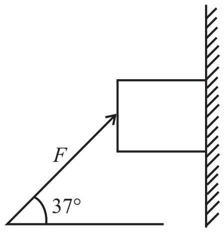
(a) $\frac{u^2\cos^2\theta\sec^3\left(\frac{\theta}{2}\right)}{\sqrt{3}g}$ (b) $\frac{u^2\cos^2\theta\sec^3\left(\frac{\theta}{2}\right)}{2g}$

(c) $\frac{2u^2\cos^3\theta\sec^2\left(\frac{\theta}{2}\right)}{g}$ (d) $\frac{u^2\cos^2\theta\sec^3\left(\frac{\theta}{2}\right)}{g}$

85. Sand is to be piled up on a horizontal ground in the form of a regular cone of a fixed base of radius R. Coefficient of static friction between the sand layers is μ. Maximum volume of the sand can be piled up in the form of cone without slipping on the ground is

(a) $\frac{\mu R^3}{3\pi}$ (b) $\frac{\mu R^3}{3}$ (c) $\frac{\pi R^3}{3\mu}$ (d) $\frac{\mu\pi R^3}{3}$

86. A block of mass 2 kg is being pushed against a wall by a force $F = 90$ N as shown in the figure. If the coefficient of friction is 0.25, then the magnitude of acceleration of the block is (Take, $g = 10$ ms^{-2}) $\left(\sin 37° = \frac{3}{5}\right)$

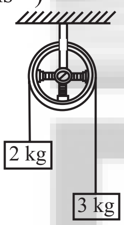


(a) 16 ms^{-2} (b) 8 ms^{-2} (c) 38 ms^{-2} (d) 54 ms^{-2}

87. A body of mass 2 kg thrown vertically from the ground with a velocity of 8 ms^{-1} reaches a maximum height of 3 m. The work done by the air resistance is (acceleration due to gravity = 10 ms^{-2})

(a) 4J (b) 60J (c) 64J (d) 8J

88. The system of two masses 2 kg and 3 kg as shown in the figure is released from rest. The work done on 3 kg block by the force of gravity during first 2 seconds of its motion is ($g = 10$ ms^{-2})

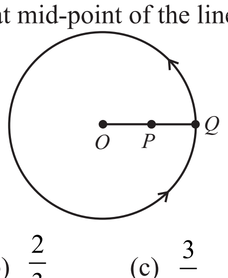


(a) 120 J (b) 80 J (c) 40 J (d) 30 J

89. A rigid metallic sphere is spinning around its own axis in the absence of external torque. If the temperature is raised, its volume increases by 9%. The change in its angular speed is

(a) increases by 9% (b) decreases by 9%
(c) increases by 6% (d) decreases by 6%

90. Two spheres P and Q, each of mass 200 g are attached to a string of length one metre as shown in the figure. The string and the spheres are then whirled in a horizontal circle about O at a constant angular speed. The ratio of the tension in the string between P and Q to that of between P and O is (P is at mid-point of the line joining O and Q)



(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{2}{1}$

91. The potential energy of a simple harmonic oscillator of mass 2 kg at its mean position is 5 J. If its total energy is 9 J and amplitude is 1 cm, then its time period is

(a) $\frac{\pi}{100}$s (b) $\frac{\pi}{50}$s (c) $\frac{\pi}{20}$s (d) $\frac{\pi}{10}$s

92. Three masses m, $2m$ and $3m$ are arranged in two triangular configurations as shown in figure 1 and figure 2. Work done by an external agent in changing, the configuration from figure 1 to figure 2 is

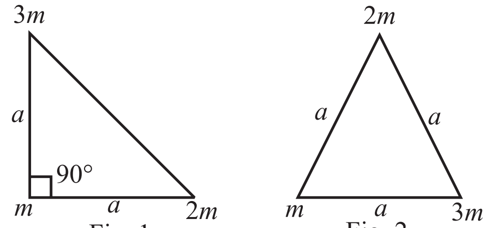
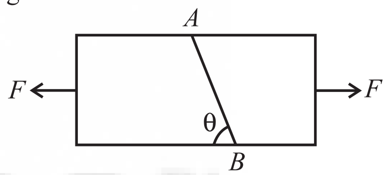


Fig. 1 Fig. 2

(a) $\frac{6Gm^2}{a}\left[2-\frac{6}{\sqrt{2}}\right]$ (b) 0

(c) $\frac{Gm^2}{a}\left[6+\frac{6}{\sqrt{2}}\right]$ (d) $-\frac{Gm^2}{a}\left[6-\frac{6}{\sqrt{2}}\right]$

93. Two equal and opposite forces each F act on a rod of uniform cross-sectional area a as shown in the figure. Shearing stress on the section AB will be



(a) $\frac{F\sin\theta\cos\theta}{a}$ (b) $\frac{F\sin\theta}{a}$

(c) $\frac{F\cos\theta}{a}$ (d) $\frac{F\sin^2\theta}{a}$

94. A body is suspended by a light string. The tensions in the string when the body is in air, when the body is totally immersed in water and when the body is totally immersed in a liquid are respectively 40.2N, 28.4N and 16.6N. The density of the liquid is

(a) 1200 kg -m^{-3} (b) 1600 kg -m^{-3}
(c) 2000 kg -m^{-3} (d) 2400 kg -m^{-3}

95. Steam at 100°C is passed into 1 kg of water contained in a calorimeter at 9°C till the temperature of water and calorimeter is increased to 90°C. The mass of the steam condensed is nearly (water equivalent of calorimeter = 0.1 kg, specific heat of water = 1 calg^{-1} °C^{-1} and latent heat of vaporisation = 540 calg^{-1})

(a) 81g (b) 162 g (c) 243g (d) 486 g

96. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. First and third plates are maintained at absolute temperatures $2T$ and $3T$ respectively. Temperature of the middle plate in steady state is

(a) $\left(\frac{65}{2}\right)^{\frac{1}{4}}$ T (b) $\left(\frac{97}{4}\right)^{\frac{1}{4}}$ T

(c) $\left(\frac{97}{2}\right)^{\frac{1}{4}}$ T (d) $(97)^{\frac{1}{4}}$ T

97. A thermally insulated vessel with nitrogen gas at 27°C is moving with a velocity of 100 ms^{-1}. If the vessel is stopped suddenly, then the percentage change in the pressure of the gas is nearly
(assume entire loss in KE of the gas is given as heat to gas and $R = 8.3$ $Jmol^{-1}$ K^{-1})
(a) 1.1 (b) 0.93 (c) 0.5 (d) 2.25

98. Match the following lists.

	List I		List II
A	Zeroth law of thermodynamics	I	Direction of flow of heat
B	First law of thermodynamics	II	Work done is zero
C	Free expansion of a gas	III	Thermal equilibrium
D	Second law of thermodynamics	IV	Law of conservation of energy

The correct answer is

	A	B	C	D
(a)	II	IV	III	I
(b)	III	IV	II	I
(c)	III	I	II	IV
(d)	I	III	IV	II

99. For a molecule of an ideal gas, the number density is $2\sqrt{2}\times 10^{8}\,cm^{-3}$ and the mean free path is $\frac{10^{-2}}{\pi}$ cm. The diameter of the gas molecule is
(a) 5×10^{-4}cm (b) 0.5×10^{-4}cm
(c) 2.5×10^{-4}cm (d) 4×10^{-4}cm

100. A solid ball is suspended from the ceiling of a motor car through a light string. A transverse pulse travels at the speed 60 cm^{-1} on the string, when the car is at rest. When the car accelerates on a horizontal road, then speed of the pulse is 66 cm^{-1}. The acceleration of the car is nearly ($g = 10$ ms^{-2})
(a) 4.3 ms^{-2} (b) 2.9 ms^{-2} (c) 6.8 ms^{-2} (d) 5.5 ms^{-2}

101. A reflector is moving with 20 ms^{-1} towards a stationary source of sound. If the source is producing sound waves of 160 Hz, then the wavelength of the reflected wave is (speed of sound in air is 340 ms^{-1})
(a) $\frac{17}{8}$m (b) $\frac{17}{11}$m (c) $\frac{17}{9}$m (d) $\frac{17}{16}$m

102. A light ray incidents normally on one surface of an equilateral prism. The angle of deviation of the light ray is (refractive index of the material of the prism = $\sqrt{2}$)
(a) 60° (b) 30° (c) 0° (d) 120°

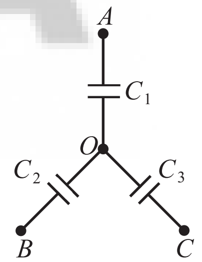
103. Two polaroids are placed in the path of unpolarised light beam of intensity I_0 such that no light is emitted from the second polaroid. If a third polaroid whose polarisation axis makes an angle θ with that of the first polaroid is placed between the polaroids, then intensity of light emerging from the last polaroid is
(a) $\left(\frac{I_0}{8}\right)\sin^2 2\theta$ (b) $\left(\frac{I_0}{4}\right)\sin^2 2\theta$
(c) $\left(\frac{I_0}{2}\right)\cos^2 \theta$ (d) $I_0\cos^2\theta$

104. Two points charges are kept in air with a separation between them. The force between them is F_1, if half of the space between the charges is filled with a dielectric constant 4 and the force between them is F_2, if $\frac{1}{3}$ rd of the space between the charges is filled with dielectric of dielectric constant 9. Then $\frac{F_1}{F_2}$ is
(a) $\frac{27}{64}$ (b) $\frac{16}{81}$ (c) $\frac{81}{64}$ (d) $\frac{100}{81}$

105. A simple pendulum with a bob of mass 40g and charge +2μC makes 20 oscillation in 44 s. A vertical electric field magnitude 4.2×10^4 NC^{-1} pointing downward is applied. The time taken by the pendulum to make 15 oscillation in the electric field is (acceleration due to gravity = 10 ms^{-2})
(a) 30 s (b) 60 s (c) 90 s (d) 15 s

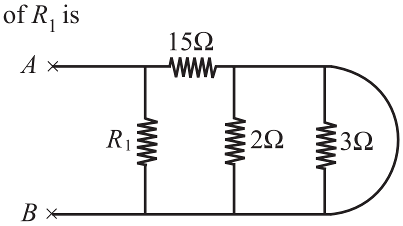
106. A parallel plate capacitor has a capacity 80×10^{-6} F, when air is present between its plates. The space between the plates is filled with a dielectric slab of dielectric constant 20. The capacitor is now connected to a battery of 30V by wires. The dielectric slab is then removed. Then, the charge passing through the wire is
(a) 12×10^{-3}C (b) 25.3×10^{-3}C
(c) 120×10^{-3}C (d) 45.6×10^{-3}C

107. Three uncharged capacitors of capacities C_1, C_2 and C_3 are connected as shown in the figure. *A, B* and *C* are at potentials V_1, V_2 and V_3, respectively, then the potential at *O* is



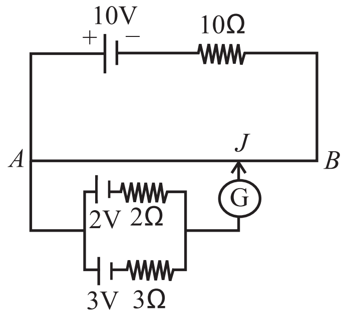
(a) $\frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}$ (b) $\frac{C_1V_1 + C_2V_2 - C_3V_3}{C_1 + C_2 + C_3}$
(c) $\frac{C_1V_1 - C_2V_2 - C_3V_3}{C_1 + C_2 + C_3}$ (d) zero

108. The equivalent resistance between *A* and *B* is 6 Ω. The value of R_1 is



(a) 20 Ω (b) 10 Ω (c) 5 Ω (d) 25 Ω

109. A battery of emf 10 V is connected to a uniform wire AB of 1m length and having a resistance of 10 Ω in series with a 10 Ω resistor as shown in the figure. Two cells of emf 2V and 3V having internal resistance 2 Ω and 3 Ω, respectively are connected as shown in the figure. If the galvanometer shows null deflection at point *J* on the wire, then the distance of point *J* from the point *B* is.



(a) 48 cm (b) 50 cm (c) 52 cm (d) 54 cm

110. Two infinitely long wires carry currents 4A and 3A placed along *X*-axis and *Y*-axis respectively. Magnetic field at a point *P*(0, 0, *d*) *m* will be …… T.

(a) $\frac{4\mu_0}{2\pi d}$ (b) $\frac{3\mu_0}{2\pi d}$ (c) $\frac{7\mu_0}{2\pi d}$ (d) $\frac{5\mu_0}{2\pi d}$

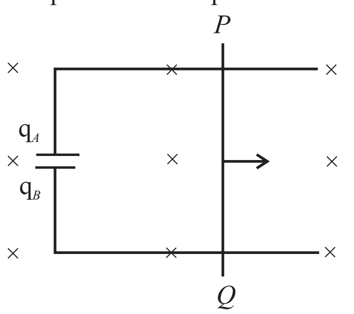
111. Two moving coil galvanometer, *X* and *Y* have coils with resistance 10 Ω and 14 Ω cross-sectional areas 4.8 × 10^{-3}m^2 and 2.4 × 10^{-3}m^2, number of turns 30 and 45 respectively. They are placed in magnetic field of 0.25 T and 0.50 T respectively. Then, the ratio of their current sensitivities and the ratio of their voltage sensitivities are respectively

(a) 2 : 3, 14 : 15 (b) 5 : 7, 2 : 1
(c) 2 :13, 1 : 2 (d) 14 : 15, 2 : 9

112. Two short bar magnets each of magnetic moment of 9Am^2 are placed such that one is at $x = -3$ cm and the other at $y = -3$ cm. If their magnetic moments are directed along positive and negative *X*-directions respectively, then the resultant magnetic field at the origin is

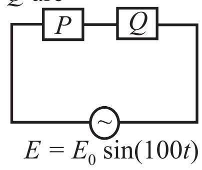
(a) 100T (b) 10T (c) 0.1T (d) 0.001T

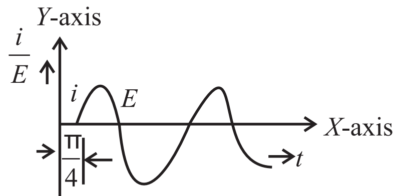
113. A conducting rod *PQ* of length 1 m is moving with a uniform speed 2 ms^{-1} in a uniform magnetic field of 4 T which is directed into the paper. A capacitor of capacity 10 μF is connected as shown in the figure. Then, the charge on the plates of the capacitor are



(a) $q_A = +80$ μC, $q_B = -80$ μC
(b) $q_A = -80$ μC, $q_B = +80$ μC
(c) $q_A = +125$ μC, $q_B = 1.25$ μC
(d) $q_A = -125$ μC, $q_B = +1.25$ μC

114. For the AC circuit shown below, phase difference between emf and current is $\frac{\pi}{4}$ radian as shown in the graph. If the impedance of the circuit is 1414 Ω, then the values of *P* and *Q* are





(a) 1 kΩ, 10 μF (b) 1 kΩ, 1 μF
(c) 1 kΩ, 10 μH (d) 1 kΩ, 1 μH

115. In a plane electromagnetic wave, the electric field oscillates with a frequency 2 × 10^{10} s^{-1} and amplitude 40 Vm^{-1}, then the energy density due to electric field is ($\varepsilon_0 = 8.85 \times 10^{-12}$Fm^{-1})

(a) 1.52 × 10^{-9} Jm^{-3}
(b) 2.54 × 10^{-19} Jm^{-3}
(c) 3.54 × 10^{-9} Jm^{-3}
(d) 4.56 × 10^{-9} Jm^{-3}

116. Photons of frequencies equal to the frequencies of H_β and H_∞ lines of hydrogen incident on a photosensitive plate, whose threshold frequency is equal to the frequency of H_α line of hydrogen. The ratio of the maximum kinetic energies of the emitted electrons is

(a) 7 : 16 (b) 3 : 4 (c) 8 : 27 (d) 5 : 36

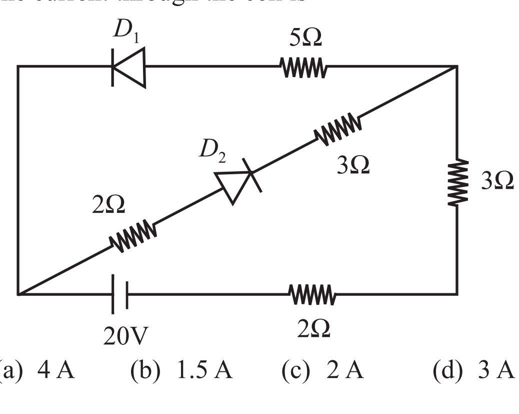
117. Hydrogen atom is in its n^{th} energy state. If de-Broglie wavelength of the electron is λ,then

(a) $\lambda \propto \frac{1}{n^2}$ (b) $\lambda \propto \frac{1}{n}$ (c) $\lambda \propto n^2$ (d) $\lambda \propto n$

118. If 200 MeV of energy is released in the fission of one nucleus of $^{235}_{92}U$, then the number of nuclei that must undergo fission to release an energy of 1000 J is

(a) 3.125 × 10^{13} (b) 6.25 × 10^{13}
(c) 12.5 ×10^{13} (d) 3.125 × 10^{14}

119. If the diodes are ideal in the circuit given below, then the current through the cell is



(a) 4 A (b) 1.5 A (c) 2 A (d) 3 A

120. If a message signal of frequency 10 kHz and peak voltage 12 V is used to modulate a carrier wave of frequency 1 MHz, the modulation index is 0.6. To make the modulation index 0.75, the carrier peak voltage should be
(a) decreased by 25% (b) increased by 25%
(c) decreased by 20% (d) increased by 20%

CHEMISTRY

121. If the radius of electron orbit in the excited state of hydrogen atom is 476.1 pm, the energy of electron in that excited state in J is
(Radius and energy of electron in the first orbit of hydrogen atom are 52.9 pm and -2.18×10^{-18}J respectively)
(a) -2.42×10^{-18} (b) -19.62×10^{-18}
(c) -2.42×10^{-19} (d) -6.05×10^{-19}

122. A light of frequency 1.6×10^{16} Hz when falls on a metal plate emits electrons that have double the kinetic energy compared to the kinetic energy of emitted electrons when frequency of 1.0×10^{16} Hz falls on the same plate. The threshold frequency (v_0) of the metal in Hz is
(a) 1×10^{15} (b) 4×10^{15}
(c) 3×10^{15} (d) 4×10^{13}

123. To which group and period does the element belong if the electronic configuration of an element in its –2 oxidation state is
$1s^2 2s^2 2p^6 3s^2 3p^6$?
(a) period 3, gro0up 16 (b) period 3, group 17
(c) period 4, group 16 (d) period 4, group 17

124. Which set of the following molecules has only one lone pair of electrons on their respective central atoms?
(i) SO_2 (ii) XeF_4 (iii) $PbCl_2$ (iv) SF_4
(v) ClF_3
(a) (i), (iii), (iv) (b) (ii), (iii), (iv)
(c) (i), (ii), (v) (d) (i), (iii), (v)

125. XeF_4 is square planar where as CCl_4 is tetrahedral because
(a) in XeF_4, 'Xe' is sp^2 hybridised and in CCl_4 'C' is sp^3 hybridised
(b) in both XeF_4 and CCl_4 the central atom is sp^3 hybridised
(c) in XeF_4, 'Xe' is sp^3d^2 hybridised but due to the presence of 2 lone pairs of electrons shape is square planar whereas in CCl_4 'C' is sp^3 hybridised
(d) Xe is noble gas, whereas C is a non-metal

126. 16 g each of H_2, He and O_2 are present in a container exerting 10 atm. pressure at *T*(K). The pressure in atm exerted by 16 g each of He and O_2 in the second container of same volume and temperature is
(a) 1.8 (b) 6.4 (c) 3.6 (d) 5.4

127. One litre of 0.15 M Na_2SO_3 aqueous solution is mixed with 500 mL of 0.2 M $K_2Cr_2O_7$ aqueous solution in acid medium. What is the number of moles of $K_2Cr_2O_7$ remaining in the solution after the reaction?
(a) 0.1 (b) 0.0125 (c) 0.025 (d) 0.05

128. From the following data

$$CH_3OH(l) + \frac{3}{2}O_2(g) \longrightarrow CO_2(g) + 2H_2O(l)$$

$\Delta_rH° = -726$ kJ mol^{-1}

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l);$$

$\Delta_rH° = -286$ kJ mol^{-1}

$$C(\text{graphite}) + O_2(g) \longrightarrow CO_2(g);$$

$\Delta_rH° = -393$ kJ mol^{-1}

The standard enthalpy of formation of $CH_3OH(l)$ in kJ mol^{-1} is
(a) –239 (b) 239 (c) 547 (d) –905

129. At 1000 K, the equilibrium constant. K_C for the reaction $2NOCl(g) \rightleftharpoons 2NO(g) + CL_2(g)$ is 4.0×10^{-6} mol L^{-1}. The K_P (in bar) at the same temperature is
(R = 0.083 L bar K^{-1}mol^{-1})
(a) 3.32×10^{-6} (b) 3.32×10^{4}
(c) 3.32×10^{-4} (d) 3.32×10^{-3}

130. If the pK_a of acetic acid and pK_b of dimethylamine are 4.76 and 3.26 respectively, the pH of dimethyl ammonium acetate solution is
(a) 7.75 (b) 6.75 (c) 7.0 (d) 8.5

131. Which of the following statements are correct?
(i) NaH (*s*) reacts violently with water to form NaOH and H_2
(ii) An example for electron rich hydride is NH_3
(iii) Nickel forms saline hydride
(a) (i), (iii) (b) (ii), (iii)
(c) (i), (ii), (iii) (d) (i), (ii)

132. Which of the following nitrates on heating does not give its oxide?
(a) $LiNO_3$ (b) $NaNO_3$ (c) $Ba(NO_3)_2$(d) $Be(NO_3)_2$

133. BF_3 reacts with NaH at 450 K to form NaF and *X*. When *X* reacts with LiH in diethyl ether, *Y* is formed. What is *Y*?
(a) $LiBO_2$ (b) $Li_2B_4O_7$
(c) $LiBH_4$ (d) B_2H_6 .LiH

134. **Assertion (A) :** $[SiF_6]^{2-}$ is formed but $[SiCl_6]^{2-}$ is not
Reason (R) : Electronegativity (EN) of F is higher than EN of Cl
(a) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(b) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(c) (A) is correct but (R) is not correct
(d) (A) is not correct but (R) is correct

135. The environmental friendly chemical now-a-days used for bleaching the paper in the presence of a suitable catalyst is
(a) chlorine (b) sulphur dioxide
(c) hydrogen peroxide (d) bleaching powder

136. The IUPAC name of the following compound is

(a) 5-cyanopentan-2-one
(b) 5-oxohexanenitrile
(c) 4-oxopentanenitrile
(d) 2-oxopentanenitrile

137. Identify the correct statements from the following
(i) Petrol and CNG operated automobiles cause less pollution
(ii) Alkanes having tertiary hydrogen can be oxidised to alcohols by $KMnO_4$
(iii) Methane can be prepared by Kolbe's electrolytic method.
(iv) Alkyl chloride on reduction with zinc and dilute hydrochloric acid gives alkane
(a) (i), (iii), (iv) (b) (i), (ii)
(c) (i), (ii), (iv) (d) (iii), (iv)

138. What are *X* and *Y* in the following reaction?

$$\text{Pent}-2-\text{ene}\xrightarrow[\text{(ii) Zn/H}_2\text{O}]{\text{(i) O}_3}X+Y$$

	X	**Y**
(a)	CH_3CHO	CH_3CH_2CHO
(b)	CH_3CH_2CHO	CH_3CH_2CHO
(c)	CH_3CHO	$(CH_3)_2CO$
(d)	CH_3CHO	CH_3CHO

139. The total number of body centred lattices possible among the 14 bravais lattices is
(a) 2 (b) 1 (c) 4 (d) 3

140. The measured osmotic pressure of a solution prepared by dissolving 17.4 mg of K_2SO_4 in 2L of water at 27°C is 3.735×10^{-3} bar.
The van't Hoff factor is
(R = 0.083 L bar $K^{-1}mol^{-1}$; atomic weights K = 39, S = 32; O = 16)
(a) 2.84 (b) 3.0 (c) 2.0 (d) 2.32

141. Dissolving 120 g of a compound (mol. wt = 60) in 1000 g of water gave a solution of density 1.12 g mL^{-1}. The molarity of solution is
(a) 1.0 M (b) 2.0 M (c) 2.5 M (d) 4.0 M

142. When an aqueous solution of $CuCl_2$ is electrolysed using Pt inert electrodes, the reaction at cathode and anode respectively are

(a) $4H_2O(l)\xrightarrow{+4e^-}2H_2(g)+4\overline{O}H_2(aq);$
$2H_2O(l)\xrightarrow{-4e^-}O_2(g)+4H^+(aq)$

(b) $2Cu^{2+}(aq)\xrightarrow{+4e^-}2Cu(s);$
$2H_2O(l)\xrightarrow{-4e^-}O_2(g)+4H^+(aq)$

(c) $Cu^{2+}(aq)\xrightarrow{+2e^-}Cu(s);$
$2Cl^-(aq)\xrightarrow{-2e^-}Cl_2(g)$

(d) $2H_2O(l)\xrightarrow{+2e^-}H_2(g)+2\overline{O}H(aq);$
$2Cl^-(aq)\xrightarrow{-2e^-}Cl_2(g)$

143. Thermal decomposition of HCOOH is a first order reaction and the rate constant at T(K) is 4.606×10^{-3} s^{-1}. The time required to decompose 90% of initial quantity of HCOOH at T(*K*) in second is
(a) 100 (b) 500 (c) 1000 (d) 50

144. Which one of the following statement is not correct?
(a) A mixture of dinitrogen and dioxygen at room temperature is an example for aerosol
(b) Lyophilic sols are more stable compared to lyophobic sols
(c) Formation of micelles is possible only above Kraft temperature
(d) An example for a soap is sodium stearate and an example for detergent is sodium lauryl sulphate

145. In Ellingham diagram, the plot is drawn between
(a) temperature, ΔH° (b) temperature, ΔG°
(c) pressure, ΔS° (d) temperature, ΔE°

146. Identify the reaction which does not liberate N_2
(a) $NaN_3\xrightarrow[\Delta]{}?$ (b) $(NH_4)_2Cr_2O_7\xrightarrow[\Delta]{}?$
(c) $NH_4Cl+Ca(OH)_2\longrightarrow?$ (d) $Ba(N_3)_2\xrightarrow[\Delta]{}?$

147. Identify the molecules which contains lone pair of electrons on the sulphur atom
(a) H_2SO_5 (b) $H_2S_2O_8$ (c) $H_2S_2O_7$ (d) H_2SO_3

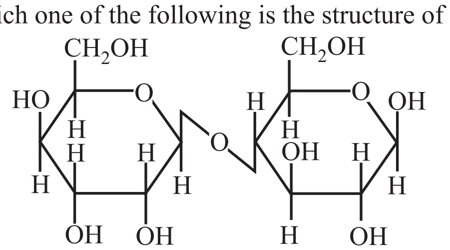
148. Which statement about noble gases is not correct?
(a) 'Xe' forms XeF_6 under suitable conditions
(b) 'Ar' is used in electric bulbs
(c) The number of lone pair of electrons present on Xe in XeF_2 is 3.
(d) 'He' has the highest boiling point among all the noble gases

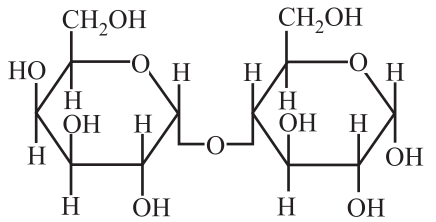
149. Crystal field splitting energies for octahedral (Δ_0) and tetrahedral (Δ_t) geometries caused by the same ligands are related through the expression
(a) $\Delta_0 = \Delta_t$ (b) $4\Delta_0 = 9\Delta_t$ (c) $9\Delta_0 = \Delta_t$ (d) $\Delta_0 = 2\Delta_t$

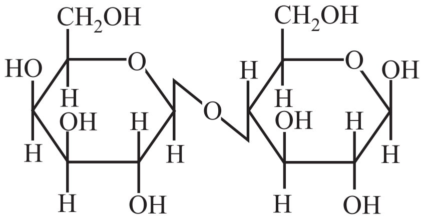
150. In lanthanide series, the element well known to exhibit +4 oxidation state is
(a) Lu (b) Ce (c) Pm (d) Nd

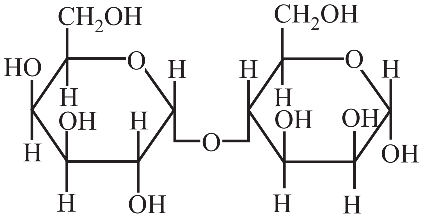
151. In anionic polymerisation, the compound which acts as effective chain initiator is
(a) BF_3 (b) $(CH_3CO)_2O_2$
(c) $SnCl_2$ (d) R - Li

152. Which one of the following is the structure of lactose?

(a)


(b)


(c) 

(d) 

153. Which of the following statements are correct?

(i) Drugs that mimic natural messenger by switching on the receptor are called agonists.

(ii) Shape of the receptor does not change after attachment of chemical messenger.

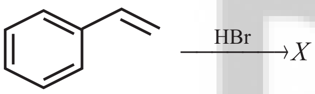
(iii) A cationic detergent is formed when stearic acid reacts with polyethylene glycol.

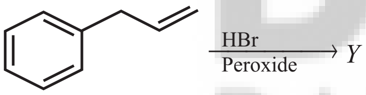
(iv) Seldane is an antihistamine

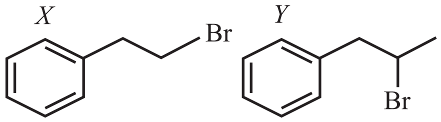
(a) (ii), (iii) (b) (i), (iii), (iv)

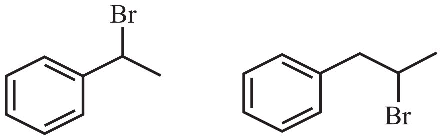
(c) (i), (iv) (d) (i), (ii), (iii)

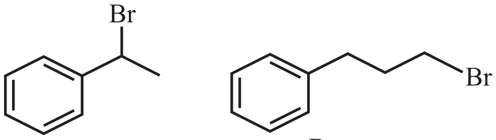
154. Identify the major products X and Y in the following reactions

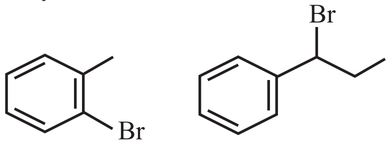
(i) 

(ii) 

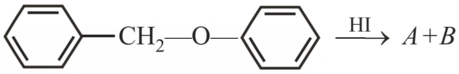
(a) 

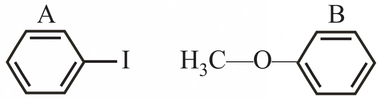
(b) 

(c) 

(d) 

155. Identify A and B is the following reactions



(a) 

(b) 

(c) 

(d) 

156. Identify A, B and C in the following reactions.

Isopropyl chloride $\xrightarrow{\text{NaOH}} A \xrightarrow{\text{Cu/573K}} B \xrightarrow{\text{NaOI}} C + \text{Iodoform}$

	A	B	C
(a)	$CH_3CH_2CH_2OH$	$CH_3CH_2CH_2CHO$	CH_3CH_2COONa
(b)	CH_3CH_2OH	CH_3CHO	$HCOONa$
(c)	$CH_3-CH(OH)-CH_3$	CH_3COCH_3	CH_3COONa
(d)	$CH_3-CH(OH)-CH(OH)-CH_3$	$H_3C-C(=O)-C(=O)-CH_3$	CH_3COONa

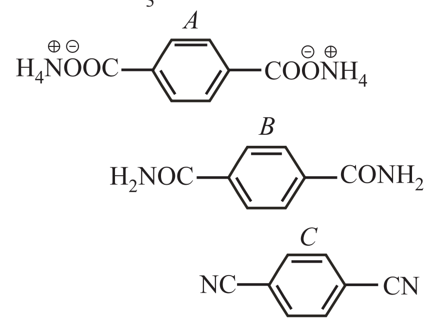
157. Match the following

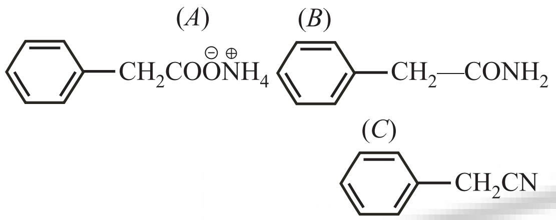
	List-I		List-II
A	Lucas reagent	I	$SnCl_2 + HCl.H_3O^+$
B	Clemmensen reagent	II	$[Ag(NH_3)_2]^+$
C	Tollens' reagent	III	Anhydrous $ZnCl_2$|conc. HCl
D	Stephen reaction	IV	Zn—Hg|conc. HCl
		V	$C_6H_5SO_2Cl$

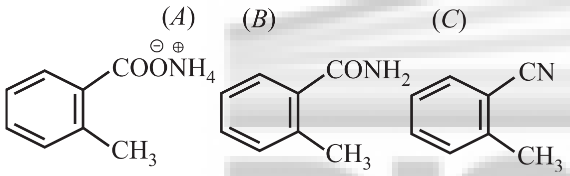
	A	B	C	D
(a)	III	IV	II	I
(b)	III	IV	I	II
(c)	IV	II	III	V
(d)	IV	III	I	V

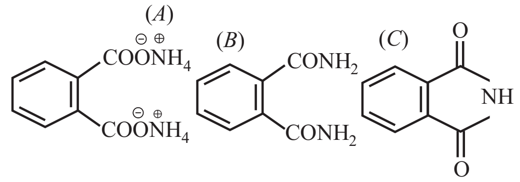
158. What are *A, B* and *C* in the following reactions?

$$\text{Phthalic acid} + NH_3 \longrightarrow A \xrightarrow{\Delta} B \xrightarrow{\text{High temperature}} C$$

(a) 

(b) 

(c) 

(d) 

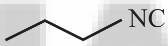
159. What are *A* and *B* in the following reaction sequence?

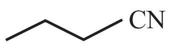
$$\text{Propionitrile} + A \longrightarrow B \xrightarrow{H_3\overset{+}{O}} \text{propiophenone}$$

A	**B**
(a) C_2H_5MgBr	$CH_3CH_2{-}C({=}NMgBr){-}C_2H_5$
(b) C_2H_5MgBr	$CH_3CH_2CH_2{-}C({=}NH){-}C_2H_5$
(c) C_6H_5MgBr	$CH_3CH_2{-}C({=}N.MgBr){-}C_6H_5$
(d) $C_6H_5CH_2MgBr$	$CH_3CH_2CH_2{-}C({=}NH){-}CH_2C_6H_5$

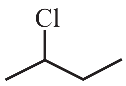
160. $C_2H_5Cl \xrightarrow{KCN} X \xrightarrow{H_2/\text{Catalyst}} Y \xrightarrow[\text{alc.KOH}]{CHCl_3} Z$

What is '*Z*' in the above sequence of reactions?

(a) 

(b) 

(c) 

(d) 

Hints & Solutions

MATHEMATICS

1. **(d)** According to the given data,
$x = [x] + \{x\}$
$[2x] = 2[x] + 2[x]$

$$[2x] = \begin{cases} 2[x] + 0, & 0 < \{x\} < \frac{1}{2} \\ 2[x] + 1, & \frac{1}{2} \le \{x\} < 1 \end{cases}$$

$$\therefore \quad [2x] - 2[x] = \begin{cases} 0, & 0 \le \{x\} < \frac{1}{2} \\ 1, & \frac{1}{2} \le \{x\} < 1 \end{cases}$$

So, the range of f is $\{0, 1\}$

2. **(d)** We have,
$f(x) = \log\{ax^3 + (a + b)x^2 + (b + c)x + c\}$
$= \log\{(ax^2 + bx + c)(x + 1)\}$
For $f(x)$ to be defined
$(ax^2 + bx + c)(x + 1) > 0$
$\Rightarrow \quad x + 1 > 0$
$\{\because \quad a > 0 \text{ and } b^2 = 4ac\}$
and $x \ne -\dfrac{b}{2a}$
So, $\left\{D = x : x \in (-1, \infty) \text{ and } x \ne -\dfrac{b}{2a}\right\}$
or $D = R - \left\{-\dfrac{b}{2a}\right\} \cup (-\infty, -1]$

3. **(d)** Given that, $(15 \times 5^{2n}) + (2 \times 2^{3n})$
Put $n = 1$,
Thus, we have $15 \times 5^2 + 2 \times 2^3$
$= 15 \times 25 + 2 \times 8 = 375 + 16 = 391$
which is divisible by 17
$\therefore \quad (15 \times 5^{2n}) + (2 \times 2^{3n})$ is divisible by 17, $\forall n \in N$.

4. **(c)** Given matrix,

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$$

$= 3(-3 + 4) + 3(2 - 0) + 4(-2 - 0)$
$= 3 + 6 - 8 = 1 \ne 0$

$$\text{adj}(A) = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \text{ adj A} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \quad \text{...(i)}$$

$$A^2 = A \cdot A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$$\therefore \quad A^3 = A^2 \cdot A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $A^{-1} = A^3$

5. **(d)** We have,

$$A = \begin{bmatrix} k/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & m/4 \end{bmatrix} \text{ and } A^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$$

We know that, $AA^{-1} = I$

$$\begin{bmatrix} k/2 & 0 & 0 \\ 0 & l/3 & 0 \\ 0 & 0 & m/4 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} k/4 & 0 & 0 \\ 0 & l/9 & 0 \\ 0 & 0 & m/16 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing the respective terms, we get
$\dfrac{k}{4} = 1, \ \dfrac{l}{9} = 1$ and $\dfrac{m}{16} = 1$
$\Rightarrow \quad k = 4, l = 9, m = 16$
$\therefore \quad k + l + m = 4 + 9 + 16 = 29$

6. **(a)** We have given that,
$x + 2y + z = 1 \qquad x + 3y + 4z = k$
$x + 5y + 10z = k^2$

$$\therefore \quad \Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 1 & 5 & 10 \end{vmatrix}$$

$= 1(30 - 20) - 2(10 - 4) + 1(5 - 3) = 10 - 12 + 2 = 0$
$\Delta = 0$
$\therefore \quad$ Given system of equation is consistent.

Therefore, $\Delta_1 = 0$

$$\Delta_1 = \begin{vmatrix} 1 & 2 & 1 \\ k & 3 & 4 \\ k^2 & 5 & 10 \end{vmatrix} = 0$$

$1(30 - 20) - 2(10k - 4k^2) + (5k - 3k^2) = 0$

$10 - 20k + 8k^2 + 5k - 3k^2 = 0$

$5k^2 - 15k + 10 = 0$

$k^2 - 3k + 2 = 0$

$(k - 2)(k - 1) = 0$

$k = 2, 1$

Hence, the real values of k i.e.

$A = 2$ and $B = 1$

$A + B = 2 + 1 = 3$

7. **(d)** Given $z = x + iy$ be a complex number,

Here $\dfrac{z-1}{z+i}$

$$= \frac{x+iy-1}{x+iy+i} = \frac{(x-1)+iy}{x+(y+1)i}$$

$$= \frac{(x-1)+iy}{x+(y+1)i} \times \frac{x-(y+1)i}{x-(y+1)i}$$

$$= \frac{x(x-1)+ixy-(x-1)(y+1)i+y(y+1)}{x^2+(y+1)^2}$$

$$= \frac{x(x-1)+y(y+1)}{x^2+(y+1)^2} + \frac{[xy-(x-1)(y+1)]i}{x^2+(y+1)^2}$$

Also given, $\text{Re}\left(\dfrac{z-1}{z+i}\right) = 1$

$$\frac{x(x-1)+y(y+1)}{x^2+(y+1)^2} = 1$$

$\therefore\ x(x-1) + y(y+1) = x^2 + (y+1)^2$

$\Rightarrow\ x^2 - x + y^2 + y = x^2 + y^2 + 2y + 1$

$\Rightarrow\ -x + y = 2y + 1$

$\Rightarrow\ x + y + 1 = 0$

$\therefore$ (2016, – 2017) lies on $x + y + 1 = 0$

8. **(a)** Given that,

$$13e^{i\tan^{-1}\frac{5}{12}} = a + ib$$

$$\Rightarrow 13\left[\cos\left(\tan^{-1}\frac{5}{12}\right) + i\sin\left(\tan^{-1}\frac{5}{12}\right)\right] = a + ib$$

$(\because\ e^{i\theta} = \cos\theta + i\sin\theta)$

$$\Rightarrow 13\left[\cos\left(\cos^{-1}\frac{12}{13}\right) + i\sin\left(\sin^{-1}\frac{5}{13}\right)\right] = a + ib$$

$$\left(\because\ \tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}} \text{ and } \tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow 13\left[\frac{12}{13} + i\frac{5}{13}\right] = a + ib$$

$\Rightarrow\ 12 + 5i = a + ib$

On comparing both the sides, we get

$\therefore\ a = 12, b = 5$

$\therefore\ (a, b) = (12, 5)$

9. **(d)** Given that,

$z_1 = 1 - 2i, z_2 = 1 + i, z_3 = 3 + 4i$

Now, $\left(\dfrac{1}{z_1} + \dfrac{3}{z_2}\right)\dfrac{z_3}{z_2} = \left(\dfrac{1}{1-2i} + \dfrac{3}{1+i}\right)\left(\dfrac{3+4i}{1+i}\right)$

$$= \frac{(1+i)+3(1-2i)}{(1-2i)(1+i)} \times \frac{3+4i}{1+i} = \frac{1+i+3-6i}{1+i-2i-2i^2} \times \frac{3+4i}{1+i}$$

$$= \frac{4-5i}{3-i} \times \frac{3+4i}{1+i}$$

$$= \frac{12+16i-15i+20}{3+3i-i+1} = \frac{32+i}{4+2i} = \frac{32+i}{2(2+i)}$$

$$= \frac{(32+i)}{2(2+i)} \times \frac{2-i}{2-i} = \frac{64+2i-32i+1}{2(4+1)}$$

$$= \frac{65-30i}{10} = \frac{13}{2} - 3i$$

10. **(d)** Here, 1, ω and ω^2 are the cube roots of unity

$$\frac{1}{1+2\omega} + \frac{1}{2+\omega} - \frac{1}{1+\omega}$$

$$= \frac{2+\omega+1+2\omega}{(1+2\omega)(2+\omega)} - \frac{1}{(1+\omega)}$$

$$= \frac{3+3\omega}{(1+2\omega)(2+\omega)} - \frac{1}{(1+\omega)}$$

$$= \frac{(3+3\omega)(1+\omega)-(1+2\omega)(2+\omega)}{(1+2\omega)(2+\omega)(1+\omega)}$$

$$= \frac{3+3\omega+3\omega+3\omega^2-(2+\omega+4\omega+2\omega^2)}{(1+2\omega)(2+\omega)(1+\omega)}$$

$$= \frac{3+6\omega+3\omega^2-2-\omega-4\omega-2\omega^2}{(1+2\omega)(2+\omega)(1+\omega)}$$

$$= \frac{1+\omega+\omega^2}{(1+2\omega)(2+\omega)(1+\omega)}$$

According to the property of cube roots of unity,

$\omega^2 + \omega + 1 = 0$

$$\frac{1}{1+2\omega} + \frac{1}{2+\omega} + \frac{1}{1+\omega} = 0$$

11. **(b)** Given, $5x - 1 < (x+1)^2 < 7x - 3$

$\therefore\ 5x - 1 < (x+1)^2$

$\Rightarrow\ 5x - 1 < x^2 + 2x + 1$

$\Rightarrow\ x^2 - 3x + 2 > 0$

$\Rightarrow\ (x-1)(x-2) > 0$

$\Rightarrow\ x \in (-\infty, 1) \cup (2, \infty)$ ...(i)

Similarly, $(x+1)^2 < 7x-3$

$\Rightarrow x^2+2x+1 < 7x-3$

$\Rightarrow x^2-5x+4 < 0$

$\Rightarrow (x-1)(x-4) < 0$

$\Rightarrow x \in (1, 4)$ ...(ii)

From Eqs. (i) and (ii), we get

$\therefore x = 3$

12. (a) We have,

$f(x) = x^2+2bx+2c^2$

$= x^2+2bx+b^2+2c^2-b^2$

$= (x+b)^2+2c^2-b^2$

$\therefore$ Minimum value,

$f(x) = 2c^2-b^2$

Now, $g(x) = -x^2-2cx+b^2$

$= -[x^2+2cx-b^2]$

$= -[x^2+2cx+c^2-b^2-c^2]$

$= -[(x+c)^2-b^2-c^2] = -(x+c)^2+b^2+c^2$

$\therefore$ Maximum value, $g(x) = b^2+c^2$

As given in the question,

Min $[f(x)] >$ Max $[g(x)]$

$2c^2-b^2 > b^2+c^2 \Rightarrow c^2 > 2b^2$

13. (a) Given Equation,

$x^3+qx+r=0$

Since, a, b and c are the roots of equation

$a+b+c=0$

$ab+bc+ca=q$

and $abc=-r$

As we have,

$a+b+c=0$

$(a+b+c)^2=0$

$a^2+b^2+c^2+2ab+2bc+2ca=0$

$a^2+b^2+c^2+2(ab+bc+ca)=0$

$a^2+b^2+c^2=-2q$

Now, $(a-b)^2+(b-c)^2+(c-a)^2$

$= a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ca$

$= 2a^2+2b^2+2c^2-2(ab+bc+ca)$

$= 2(a^2+b^2+c^2)-2(ab+bc+ca)$

$= 2(-2q)-2(q)$ [From Eq. (i)]

$= -4q-2q$

$= -6q$

14. (a) Let the two roots of equation is $a - a$

Now, sum of three roots = $2p$

So, third root will be $2p$,

Product of two consecutive roots = $3q$

$a \times (-a) + a \times (2p) + (-a) \times 2p = 3q$

$\Rightarrow -a^2 = 3q$

As we know, product of roots = $3q$

$a \times (-a) \times 2p = 4r$

$\Rightarrow 3q \times 2p = 4r$

$\Rightarrow r = \frac{3pq \times 2}{4} \quad \Rightarrow r = \frac{3pq}{2}$

15. (b) Given digits are 0, 3, 5, 4. Then four digit numbers which are even and without repetition are 3054, 3504, 5034, 5304, 3450, 3540, 4350, 4530, 5340, 5430

$\therefore$ Required sum

= 3054 + 3504 + 5034 + 5304 + 3450 + 3540 + 4350 + 4530 + 5340 + 5430 = 43536

16. (b) For x number of ways, 6 Boys can be seated in a row in 6_{P_6} ways = 6!

Now, in the 7 gaps 6 girls can be arranged in 7_{P_6} ways.

$\therefore x = 6! \times 7_{P_6} = 6! \times 7!$

For y number of ways. 6 Boys can be seated in a circle in (6 – 1)! ways = 5!

Now, in the 6 gaps 6 girls can be arranged in 6_{P_6} ways.

$\therefore y = 5! \times 6_{P_6} = 5! \times 6!$

$$\frac{x}{y} = \frac{6! \times 7!}{5! \times 6!}$$

$$\frac{x}{y} = \frac{6 \times 5! \times 7 \times 6!}{5! \times 6!}$$

$\therefore x : y = 42 : 1$

17. (c) Given word 'SARANAM' there are 7 letters in this word in which it has 3 A and all other are distinct.

The five letter words may consist of

(*i*) all different letters (using S, A, R, N, M)

$\therefore$ Number of words

= 5! = 120

(*ii*) 2 alike and other different (using 2A and 3 other)

$\therefore$ Number of words

$= {}^4C_3 \times \frac{5!}{2!} = 4 \times \frac{120}{2} = 240$

(*iii*) 3 alike and other different (using 3A and 2 other)

$\therefore$ Number of words

$= {}^4C_2 \times \frac{5!}{3!} = \frac{4 \times 3}{2 \times 1} \times \frac{120}{6} = 120$

$\therefore$ Hence, total words

= 120 + 240 + 120 = 480

18. (c) In the binomial expansion, $(\text{of } 4\sqrt{5}+5\sqrt{4})^{100}$,

General term, $T_{r+1} = {}^{100}C_r\, 5^{\frac{100-r}{4}} 4^{\frac{r}{5}}$

Clearly, T_{r+1} will be an integer if $\frac{100-r}{4}$ and $\frac{r}{5}$ are integers. This is possible when $100 - r$ is a multiple of 4 and r is a multiple of 5

$\Rightarrow 100 - r = 0, 4, 8, 12, \ldots, 96, 100$

and $r = 0, 5, 10, \ldots, 100$

$\Rightarrow r = 0, 4, 8, 12, \ldots, 100$

and $r = 0, 5, 10, 100$

$\Rightarrow r = 0, 20, 40, 60, 80, 100$

Hence, there are 6 rational terms.

19. **(d)** In the expansion of $(2a-3b)^9$, $(2a-3b)^{19} \cdot a^{19}\left(1-\frac{3b}{2a}\right)^{19}$

We know that, the r^{th} term is the greatest term of expansion $(1+x)^n$, then

$$(1+x)^n = \left[\frac{(n+1)|x|}{1+|x|}\right]$$

Here, $n = 19$ $x = -\frac{36}{29}$

$$\therefore \quad r = \left[\frac{(20)\left|\frac{3b}{2a}\right|}{1+\left|\frac{3b}{2a}\right|}\right] = \left[\frac{20}{1+4}\times 4\right]$$

$$\left[\because \; b = \frac{2}{3}, a = \frac{1}{4}\right]$$

$\therefore$ greatest term of $(2a-3b)^{19} = 2^{19}a^{19}\left(1-\frac{3b}{2a}\right)^{19}$ is

$$= {}^{19}C_{16} \times 2^{19}\cdot a^{19}\left(\frac{3b}{2a}\right)^{16}$$

$$= {}^{19}C_3 \times 2^{19} \times \left(\frac{1}{4}\right)^{19} \times (4)^{16} \qquad [\because {}^{19}C_{16} = {}^{19}C_3]$$

$$= {}^{19}C_3 \times 2^{19} \times \frac{1}{2^{38}} \times 2^{32} = {}^{19}C_3 \times 2^{13}$$

20. **(b)** Given that,

$$\frac{x^2+5x+7}{(x-3)^3} = \frac{A}{x-3}+\frac{B}{(x-3)^2}+\frac{C}{(x-3)^3}$$

$\Rightarrow$ $x^2+5x+7 = A(x-3)^2 + B(x-3) + C$ ...(i)

At $x = 3$

$\Rightarrow$ $9+15+7 = C$

$\Rightarrow$ $C = 31$

At $x = 0$

$\Rightarrow$ $7 = 9A - 3B + 31$

$\Rightarrow$ $9A - 3B = -24$

$\Rightarrow$ $3A - B = -8$

At $x = 1$

$\Rightarrow$ $1+5+7 = 4A - 2B + 31$

$\Rightarrow$ $13 = 4A - 2B + 31$

$\Rightarrow$ $4A - 2B = -18$

$\Rightarrow$ $2A - B = -9$

On solving Eqs. (ii) and (iii), we get

$A = 1$

and $B = 11$

$\therefore$ Equation of line having slope A and passing through the points (B, C) is

$y - C = A(x - B)$

$y - 31 = 1(x - 11)$

$y - 31 = x - 11$

$x - y + 20 = 0$

21. **(d)** We have given that,

$\cos\left(x-\frac{\pi}{3}\right)$, $\cos x$, $\cos\left(x+\frac{\pi}{3}\right)$ are in H.P.

$$\therefore \quad \frac{2}{\cos x} = \frac{1}{\cos\left(x-\frac{\pi}{3}\right)} + \frac{1}{\cos\left(x+\frac{\pi}{3}\right)}$$

$$\Rightarrow \cos x = \frac{2\cos\left(x-\frac{\pi}{3}\right)\cos\left(x+\frac{\pi}{3}\right)}{\cos\left(x-\frac{\pi}{3}\right)+\cos\left(x+\frac{\pi}{3}\right)}$$

$$\Rightarrow \cos x = \frac{2\left(\cos^2 x - \sin^2\frac{\pi}{3}\right)}{2\cos x\cos\frac{\pi}{3}}$$

$$\Rightarrow \cos^2 x\cos\frac{\pi}{3} = \cos^2 x - \sin^2\frac{\pi}{3}$$

$$\Rightarrow \cos^2 x\left(1-\cos\frac{\pi}{3}\right) = \left(1-\cos^2\frac{\pi}{3}\right)$$

$$\Rightarrow \cos^2 x\left(1-\cos\frac{\pi}{3}\right) = \left(1-\cos\frac{\pi}{3}\right)\left(1+\cos\frac{\pi}{3}\right)$$

$$\Rightarrow \cos^2 x = 1+\cos\frac{\pi}{3} \Rightarrow \cos^2 a = 1+\frac{1}{2}$$

$$\Rightarrow \cos^2 x = \frac{3}{2}$$

$$\therefore \quad \cos x = \sqrt{\frac{3}{2}}.$$

22. **(c)** We have given, $\cos^3 10 + \cos^3 110 + \cos^3 130°$

As we know,

$$\cos^3 x + \cos^3(120-x) + \cos^3(120+x) = \frac{3}{4}\cos 3x$$

Here, put $x = 10$, we get

$\cos^3 10 + \cos^3(120° - 10°)\cos^3(120° + 10°)$

$$= \left(\frac{3}{4}\right)\cos(3\times 10°)$$

$\cos^3 10 + \cos^3 110° + \cos^3 130°$

$$= \frac{3}{4}\cos 30° = \frac{3}{4}\times\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8}$$

23. **(b)** Given trigonometric equation,

$\sin 5x = \cos 2x$

$$\Rightarrow \sin 5x = \sin\left(\frac{\pi}{2}-2x\right)$$

Comparing with the eqn. $\sin x = \sin y$, we get general solution, $x = n\pi + (-1)^n y$

$$\therefore \quad 5x = n\pi + (-1)^n\left(\frac{\pi}{2}-2x\right)$$

$$5x = n\pi + (-1)^n\frac{\pi}{2} - (-1)^n 2x$$

$$5x + (-1)^n (2x) = \frac{\pi}{2}\{2n + (-1)^n\}$$

$$x(5+(-1)^n 2) = \frac{\pi}{2}(2n+(-1)^n)$$

$$x = \frac{\pi}{2}\left(\frac{2n + (-1)^n}{5 + 2(-1)^n}\right)$$

On comparing with $x = a_n \cdot \frac{\pi}{2}$, we get

$$a_n = \frac{2n + (-1)^n}{5 + 2(-1)^n}$$

24. (d) Given, $ax + b \sec(\tan^{-1}x) = c$ and $ay + b \sec(\tan^{-1}y) = c$

Let $x = \tan\theta$, then we have

$a \tan\theta + b \sec\theta = c$

$$\Rightarrow \frac{a \sin\theta}{\cos\theta} + \frac{b}{\cos\theta} = c$$

$\Rightarrow a \sin\theta + b = c \cos\theta$

$\Rightarrow c \cos\theta - a \sin b = b$

for some α,

$c = r\cos\alpha$, $a = r \sin\alpha$

Then, $\tan\alpha = \frac{a}{c}$

Thus, $\cos\alpha \cos\theta - \sin\alpha \sin\theta = \frac{b}{r}$

$$\Rightarrow \cos(\alpha + \theta) = \frac{b}{r} \Rightarrow \alpha + \theta = \pm \cos^{-1}\frac{b}{r}$$

Similarly, for $ay + b \sec(\tan^{-1} y) = c$

Let $y = \tan\phi$, we get

$$\alpha + \phi = \pm \cos^{-1}\frac{b}{r}$$

Let $\alpha + \theta$ be the positive solution and $\alpha + \phi$ the negative solution, where

$y = \tan\phi$

$\alpha + \phi = -(\alpha + \theta)$

$-2\alpha = \theta + \phi$

$\tan(-2\alpha) = \tan(\theta + \phi)$

$$\frac{-2\tan\alpha}{1 - \tan^2\alpha} = \frac{\tan\theta + \tan\phi}{1 - \tan\theta + \tan\phi}$$

$$\frac{-2a/c}{1 - a^2/c^2} = \frac{x+y}{1-xy} \Rightarrow \frac{2ac}{a^2 - c^2} = \frac{x+y}{1-xy}$$

25. (a) Since, we know

$$\tan h^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$\cot h^{-1}(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$$

Given that, $\tan h^{-1}\left(\frac{1}{2}\right) + \cot h^{-1}(3)$

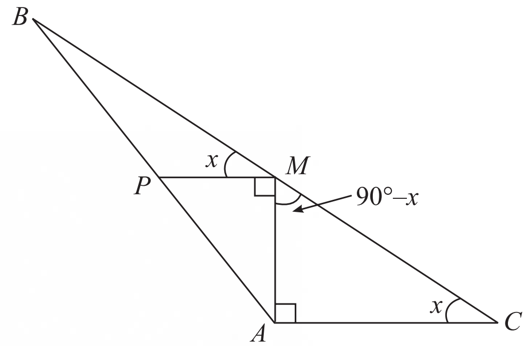
$$= \frac{1}{2}\ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) + \frac{1}{2}\ln\left(\frac{3+1}{3-1}\right)$$

$$= \frac{1}{2}\log\left(\frac{\frac{3}{2}}{\frac{1}{2}}\right) + \frac{1}{2}\log\left(\frac{4}{2}\right) = \frac{1}{2}\log 3 + \frac{1}{2}\log 2$$

$$= \log\sqrt{3} + \log\sqrt{2} = \log(\sqrt{3}\cdot\sqrt{2}) = \log\sqrt{6}$$

26. (c) Since M is mid-point of line

$\therefore BM = MC = \frac{1}{2}BC$



In ΔAMC

$\angle AMC = 90° - x$

$\because$ M lies on line BC

$\therefore$ $\angle BMP + 90° + 90° - x = 180°$

$\Rightarrow$ $\angle BMP = x$

$\therefore$ $PM \parallel AC$ [Alternate angles are equal]

$\therefore$ $PM = \frac{1}{2}AC$ ...(i)

In ΔAPM

$\angle PAM = A - 90°$

$$\therefore \tan(A - 90°) = \frac{PM}{AM}$$

$$\Rightarrow -\cot A = \frac{PM}{AM}$$

$\Rightarrow AM = -PM \tan A$ ...(ii)

In ΔAMC

$$\tan c = \frac{AM}{AC} \Rightarrow AM = AC \tan c$$

From (ii)

$AC \tan c = -PM \tan A$

$$\frac{\tan A}{\tan C} = \frac{-AC}{PM} = -2 \quad \text{[From (i)]}$$

27. (b) In ΔABC, $\tan\frac{A}{2} + \tan\frac{B}{2} = \frac{\Delta}{s(s-a)} + \frac{\Delta}{s(s-b)}$

$$= \frac{\Delta}{s}\left(\frac{1}{s-a} + \frac{1}{s-b}\right) = \frac{\Delta}{s}\left(\frac{s-b+s-a}{(s-b)(s-a)}\right)$$

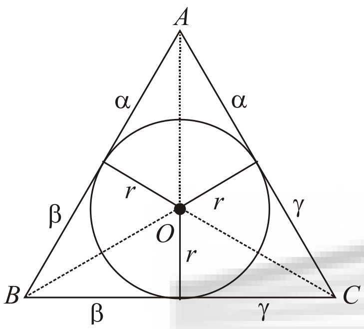
$$= \frac{\Delta}{s}\left(\frac{2s-a-b}{(s-b)(s-a)}\right) = \frac{\Delta}{s}\left(\frac{a+b+c-a-b}{(s-b)(s-a)}\right)$$

$$= \frac{c\Delta}{s(s-a)(s-b)} \qquad (\because 2s = a+b+c)$$

$$= \frac{2c\Delta}{(a+b+c)(s-a)(s-b)}$$

$$= \frac{2c}{(a+b+c)}\sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = \frac{2c\cot\frac{c}{2}}{a+b+c}$$

28. **(d)** Given that, in ΔABC, point D, E and F are the points of contact of the in circle of the sides AB, BC and CA.



From the above figure,
Area of ΔABC = Area of ΔAOB + Area of ΔBOC + Area of ΔCOA

$$\Rightarrow A = \frac{1}{2}cr + \frac{1}{2}ar + \frac{1}{2}br \qquad [\text{where } ar(\Delta ABC) = A]$$

$$\Rightarrow A = \frac{1}{2}r(a+b+c)$$

$$\Rightarrow A = \frac{1}{2}r(2s) \qquad [\text{where } s \text{ is semi perimeter}]$$

$$\Rightarrow A = rs \Rightarrow r = \frac{A}{s} \Rightarrow r^2 = \frac{A^2}{s^2}$$

$$\Rightarrow r^2 = \frac{s(s-a)(s-b)(s-c)}{s^2}$$

$$\Rightarrow r^2 = \frac{\alpha\cdot\beta\cdot\gamma}{\alpha+\beta+\gamma} \qquad [\because 2s = 2\alpha + 2\beta + 2\gamma]$$

29. **(b)** Given, a, b and c be three non-coplanar vectors. Equation of line joining the points,

$\boldsymbol{a} + 2\boldsymbol{b} - 5\boldsymbol{c}$, $-\boldsymbol{a} - 2\boldsymbol{b} - 3\boldsymbol{c}$ is

$l_1 : \boldsymbol{r} = (\boldsymbol{a} + 2\boldsymbol{b} - 5\boldsymbol{c}) + \lambda(2\boldsymbol{a} + 4\boldsymbol{b} - 2\boldsymbol{c})$ ...(i)

Similarly, equation of the line joining the points

$-4\boldsymbol{c}$, $6\boldsymbol{a} - 4\boldsymbol{b} + 4\boldsymbol{c}$ is

$l_2 : \boldsymbol{r} = -4\boldsymbol{c} + t(6\boldsymbol{a} - 4\boldsymbol{b} + 8\boldsymbol{c})$ ...(ii)

Now, for point of intersection of lines l_1 and l_2,

$(2\lambda + 1)\,a + (4\lambda + 2)\,b + (-2\lambda - 5)\,c$

$= (6t)\,a + (-4t)b + (8t + 4)\,c$

On comparing, we get

$2\lambda + 1 = 6t$ ...(iii)

$4\lambda + 2 = -4t$ ...(iv)

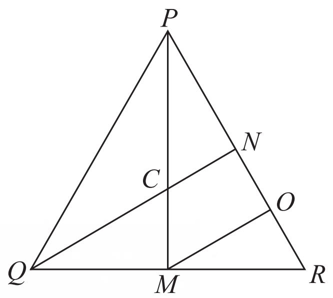
and $-2\lambda - 5 = 8t + 4$ ..(v)

From Eqs. (iii), (iv) and (v)

$\lambda = -\frac{1}{2}$ and $t = 0$

So, the intersection point is $-4c$ and for $\mu = -3$ the line $\boldsymbol{r} = (3\boldsymbol{a} + 6\boldsymbol{b} - \boldsymbol{c}) + \mu(\boldsymbol{a} + 2\boldsymbol{b} + \boldsymbol{c})$ passes through point $-4\boldsymbol{c}$ only.

30. **(d)** Given that,
M is the mid-point of OR and
C is the mid-point of PM.
Draw MO such that MO is parallel to CN.



Since, N is the mid-point of PO

$\therefore\ CN = \frac{1}{2}MO$ ...(i)

M is the mid-point of QR

$\therefore$ MO is parallel to QN

So, $MO = \frac{1}{2}QN$

From Eq. (i),

$\therefore\ CN = \frac{1}{2}\left(\frac{1}{2}QN\right)\ CN = \frac{1}{4}QN$

$$\left|\frac{\mathbf{QN}}{\mathbf{CN}}\right| = 4$$

31. **(a)** Here, vectors **a**, **b** and **c** are given as

$\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\mathbf{c} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

According to the question,

$(\mathbf{a}\times\mathbf{b})\times(\mathbf{b}\times\mathbf{c}) = [\mathbf{a\ b\ c}]\,\mathbf{b} - [\mathbf{a\ b\ b}]\,\mathbf{c} = [\mathbf{a\ b\ c}]\,\mathbf{b}$

$(\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a}) = [\mathbf{b\ c\ a}]\,\mathbf{c} - [\mathbf{b\ c\ c}]\,\mathbf{a} = [\mathbf{a\ b\ c}]\,\mathbf{c}$

$(\mathbf{c}\times\mathbf{a})\times(\mathbf{a}\times\mathbf{b}) = [\mathbf{c\ a\ b}]\,\mathbf{a} - [\mathbf{c\ a\ a}]\,\mathbf{b} = [\mathbf{a\ b\ c}]\,\mathbf{a}$

$$\text{Now, } [\mathbf{a\ b\ c}] = \begin{vmatrix} 1 & -2 & -3 \\ 2 & 1 & -1 \\ 1 & 3 & -2 \end{vmatrix}$$

$= 1(-2 + 3) + 2(-4 + 1) - 3(6 - 1) = 1 - 6 - 15 = -20$

$\therefore\ [(\mathbf{a}\times\mathbf{b})\times(\mathbf{b}\times\mathbf{c})\ (\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a})\ (\mathbf{c}\times\mathbf{a})\times(\mathbf{a}\times\mathbf{b})]$

$= [(-20)\,\mathbf{b}\ (-20)\,\mathbf{c}\ (-20)\,\mathbf{a}]$

$$= \begin{vmatrix} -40 & -20 & 20 \\ -20 & -60 & 40 \\ -20 & 40 & 60 \end{vmatrix}$$

$= -40[-3600 - 1600] + 20[-1200 + 800] + 20[-800 - 1200]$

$= 208000 - 8000 - 40000 = 160000$

32. **(b)** Given that, n is perpendicular to both the vectors a and b.

$\therefore\ \mathbf{n} = \mathbf{a} \times \mathbf{b}$

$$= \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -1 & 2 & 1 \end{vmatrix} = -4i - 4j + 4k$$

Also given , θ be the angle between c and n. Then

$$\sin\theta = \frac{|\mathbf{n} \times \mathbf{c}|}{|\mathbf{n}||\mathbf{c}|}$$

So, $$\mathbf{n} \times \mathbf{c} = \begin{vmatrix} i & j & k \\ -4 & -4 & 4 \\ 1 & 2 & -2 \end{vmatrix} = -4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

$$\therefore\ \sin\theta = \frac{|-4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}|}{|-4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}||\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}|}$$

$$= \frac{\sqrt{(-4)^2 + (-4)^2}}{\sqrt{(-4)^2 + (-4)^2 + (4)^2}\ \sqrt{(1)^2 + (2)^2 + (-2)^2}}$$

$$= \frac{\sqrt{16+16}}{\sqrt{16+16+16}\ \sqrt{1+4+4}} = \frac{4\sqrt{2}}{(4\sqrt{3}) \times 3} = \frac{\sqrt{2}}{3\sqrt{3}}$$

33. **(b)** Given that vectors **a**, **b** and **c** are mutually perpendicular of same magnitude so,

Let $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = k$

Now, $|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = (\mathbf{a} + \mathbf{b} + \mathbf{c}) \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c})$

$= |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a})$

$= k^2 + k^2 + k^2 + 20(0 + 0 + 0)$

$= 3k^2$ $[\because\ \mathbf{a} \perp \mathbf{b}, \mathbf{b} \perp \mathbf{c}, \mathbf{c} \perp \mathbf{a}$, So $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{c} = \mathbf{c}\cdot\mathbf{a} = 0]$

$\therefore\ |\mathbf{a} + \mathbf{b} + \mathbf{c}| = \sqrt{3}k$

Now, let θ be the angle between the vectors **a** and **a** + **b** + **c**

$$\therefore\ \cos\theta = \frac{\mathbf{a}\cdot(\mathbf{a} + \mathbf{b} + \mathbf{c})}{|\mathbf{a}||\mathbf{a} + \mathbf{b} + \mathbf{c}|} = \frac{|\mathbf{a}|^2}{|\mathbf{a}||\mathbf{a} + \mathbf{b} + \mathbf{c}|}$$

$$= \frac{k^2}{k + \sqrt{3}k} = \frac{1}{\sqrt{3}}$$

34. **(b)** Given that vectors **a**, **b** and **c** are non-coplanar vectors.

Let $A(2\mathbf{a} + 3\mathbf{b} - \mathbf{c})$, $B(\mathbf{a} - 2\mathbf{b} + 3\mathbf{c})$, $C(3\mathbf{a} + 4\mathbf{b} - 2\mathbf{c})$ and $D(k\mathbf{a} - 6\mathbf{b} + 6\mathbf{c})$

$\therefore\ \mathbf{AB} = \mathbf{B} - \mathbf{A} = -\mathbf{a} - 5\mathbf{b} + 4\mathbf{c}$

$\mathbf{AC} = \mathbf{C} - \mathbf{A} = \mathbf{a} + \mathbf{b} - \mathbf{c}$

$\mathbf{AD} = \mathbf{D} - \mathbf{A} = (k-2)\,\mathbf{a} - 9\mathbf{b} + 7\mathbf{c}$

Now, As **A**, **B**, **C** and **D** coplanar

$\mathbf{AB} \cdot (\mathbf{AC} \times \mathbf{AD}) = 0$

$$\therefore\ \begin{vmatrix} -1 & -5 & 4 \\ 1 & 1 & -1 \\ k-2 & -9 & 7 \end{vmatrix} = 0$$

$\Rightarrow\ (k-2)(5-4) + 9(1-4) + 7(-1+5) = 0$

$\Rightarrow\ k - 2 - 27 + 28 = 0$

$\Rightarrow\ k - 1 = 0$

$\Rightarrow\ k = 1.$

35. **(a)** Given that, $n = 8$, $\bar{x} = 25$ and $\sigma = 5$

$$\because\ \text{Mean } \bar{x} = \frac{\Sigma x_i}{n}$$

$\Rightarrow\ \Sigma x_i = n\bar{x} = 8 \times 25 = 200$

$\Sigma x_i = 200$

$$\text{Variance, } \sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$$

$$25 = \frac{\Sigma x_i^2}{8} - 625 \Rightarrow \Sigma x_i^2 = 5200$$

As, two new observations 15 and 25 are added, then corrected $\Sigma x_i = 200 + 15 + 25 = 240$

Corrected $\Sigma x_i^2 = 5200 + 225 + 625 = 6050$

$$\therefore\ \text{Corrected variance} = \frac{6050}{10} - \left(\frac{240}{10}\right)^2$$

$= 605 - (24)^2 = 605 - 576 = 29$

36. (*)

x_i	f_i	Cumulative frequency	$d_i = \lvert x_i - 15 \rvert$	$f_i d_i$
6	4	4	9	36
9	5	9	6	30
3	3	12	12	36
12	2	14	3	6
15	5	19	0	0
13	4	23	2	8
21	4	27	6	24
22	3	30	7	21
	$N = \Sigma f_i = 30$			$\Sigma f_i d_i = 161$

The cumulative frequency just greater than $\frac{N}{2}$ is 19 and corresponding value of x is 15.

Therefore, median = 15.

$$\therefore\ \text{Mean deviation} = \frac{\Sigma f_i |x_i - M|}{n} = \frac{\Sigma f_i |x_i - 15|}{30}$$

$= \frac{161}{30}$ 5.40 **(approx)**

37. **(b)** If a die is thrown three times, Total possible outcomes, $n = 6^3 = 216$

For getting a larger number on its face than the previous number each. Then, favourable outcomes $= {}^6C_3 = 20$

$$\text{Now, required probability} = \frac{r}{n} = \frac{20}{216} = \frac{5}{54}$$

38. **(b)** Let

E_1: six occurs on the die

E_2: six does not occurs on the die

A: when man reports that it is six.

$\therefore\ P(E_1) = \frac{1}{6}$,

$P(E_2) = \frac{5}{6}$

and $P\left(\frac{A}{E_1}\right) = \frac{2}{3}, P\left(\frac{A}{E_2}\right) = \frac{1}{3}$

According to Bayes' Theorem,

$$\therefore\quad P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{2}{3}}{\frac{1}{6} \times \frac{2}{3} + \frac{5}{6} \times \frac{1}{3}} = \frac{2}{7}$$

Hence, the probability that it is actually six $= \frac{2}{7}$

The probability that it is not actually six $= 1 - \frac{2}{7}$

$= \frac{5}{7}$

The probability that is actually five $= \frac{1}{5} \times \frac{5}{7} = \frac{1}{7}$

39. **(b)** Given that,

$P(X = k) = a\left(\frac{k+1}{2^k}\right)$

As we know,

$\Sigma P\ (X = k) = 1$

$\Rightarrow\ P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) = 1$

$\Rightarrow\ a + \frac{2a}{2} + \frac{3}{4}a + \frac{4}{8}a + \frac{5}{16}a + \frac{6}{32}a = 1$

$\Rightarrow\ \frac{16a + 16a + 12a + 8a + 5a + 3a}{16} = 1$

$\Rightarrow\ 60a = 16$

$\Rightarrow\ a = \frac{16}{60} \Rightarrow a = \frac{}{15}$

$\therefore$ Required probability

$= P(X = 2) + P(X = 3) + P(X = 4)$

$= \frac{3a}{4} + \frac{1a}{2} + \frac{3a}{16}$

$= \frac{12a + 8a + 3a}{16} = \frac{23a}{16} = \frac{23}{16} \times \frac{4}{15} = \frac{23}{60}$

40. **(b)** For a binomial variable X,

Mean = 6 and variance = – 2

$\therefore\ np = 6$ and $npq = 2$

So $q = \frac{1}{3}$

$p = \frac{2}{3}$

and $np = 6$

$n \times \frac{2}{3} = 6$

$n = 9$

$\therefore\ P(5 \le X \le 7) = P(X = 5) + P(X = 6) + P(X = 7)$

$$= {}^9C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^4 + {}^9C_6\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^3 + {}^9C_7\left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right)^2$$

$$= \frac{1}{3^9}\left[\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times 32 + \frac{9 \times 8 \times 7}{3 \times 2 \times 1} \times 64 + \frac{9 \times 8}{2 \times 1} \times 128\right]$$

$$= \frac{1344 + 1792 + 1536}{6561} = \frac{4672}{6561}$$

41. **(d)** It is given that, points are $A(2, 3)$, $B(3, -6)$, $C(5, -7)$.

Let point P be (x, y), such that satisfying the given condition

$PA^2 + PB^2 = 2PC^2$

$(x - 2)^2 + (y - 3)^2 + (x - 3)^2 + (y + 6)^2 = 2[(x - 5)^2 + (y + 7)^2]$

$\Rightarrow\ x^2 + 4 - 4x + y^2 + 9 - 6y + x^2 + 9 - 6x + y^2 + 36 + 12y = 2[x^2 + 25 - 10x + y^2 + 14y + 49]$

$\Rightarrow\ 2x^2 + 2y^2 - 10x + 6y + 58 = 2x^2 + 2y^2 - 20x + 28y + 148$

$\Rightarrow 10x - 22y = 90 \quad \Rightarrow \quad 5x - 11y = 45$

By checking options, we get to know that point $(-13, -10)$ lies on the locus of P.

42. **(c)** Let (x, y) are old coordinates and (X, Y) are new coordinates, when axes are rotated through an angle of θ, then

$x = X\cos\theta - Y\sin\theta$ and $y = X\sin\theta + Y\cos\theta$

Put $X = 2$, $Y = -6$ and $\theta = 135°$

$\therefore\ x = 2\cos 135° - (-6)\sin 135°$

and $y = 2\sin 135° + (-6)\cos 135°$

$x = 2\left(\frac{-1}{\sqrt{2}}\right) + 6\left(\frac{1}{\sqrt{2}}\right)$ and $y = 2\left(\frac{1}{\sqrt{2}}\right) - 6\left(\frac{-1}{\sqrt{2}}\right)$

$x = \frac{4}{\sqrt{2}}$ and $y = \frac{8}{\sqrt{2}}$

$x = 2\sqrt{2}$ and $y = 4\sqrt{2}$

$\therefore$ Coordinates of point $P(x, y)$ in the original system are $(2\sqrt{2}, 4\sqrt{2})$.

43. **(d)** Let the equation of the line in intercept form,

$\frac{x}{a} + \frac{y}{b} = 1$

The coordinate of the point $(2, -1)$ which divides the line joining $A(a, 0)$ and $B(0, b)$ in the ratio 3 : 2 are

$(2, -1) = \left(\dfrac{3\times 0 + 2\times a}{3+2}, \dfrac{3\times b + 2\times 0}{3+2}\right)$

$(2, -1) = \left(\dfrac{2a}{5}, \dfrac{3b}{5}\right)$

On comparing both the sides, we get

$\therefore \quad \dfrac{2a}{5} = 2$ and $\dfrac{3b}{5} = -1 \quad \Rightarrow \quad a = 5$ and $b = \dfrac{-5}{3}$

Hence, the required equation of line

$\dfrac{x}{5} + \dfrac{y}{\frac{-5}{3}} = 1$

$\Rightarrow \quad \dfrac{x}{5} - \dfrac{3y}{5} = 1$

$\Rightarrow \quad x - 3y = 5$

$\Rightarrow \quad x - 3y - 5 = 0$

44. (b) Let $l_1 : 2x + y - 4 = 0$, $l_2 : x - 3y + 5 = 0$

The equation of a line passing through the intersection of l_1 and l_2,

$(2x + y - 4) + \lambda (x - 3y + 5) = 0$ ...(i)

$x(2 + \lambda) + y(1 - 3\lambda) + 5\lambda - 4 = 0$

This line is at a distance of $\sqrt{5}$ units from the origin.

$\therefore \quad D = \left|\dfrac{5\lambda - 4}{(2+\lambda)^2 + (1-3\lambda)^2}\right| = \sqrt{5}$

$\dfrac{(5\lambda - 4)^2}{4 + \lambda^2 + 4\lambda + 1 + 9\lambda^2 - 6\lambda} = 5$

$\dfrac{(5\lambda - 4)^2}{10\lambda^2 - 2\lambda + 5} = 5$

$25\lambda^2 + 16 - 40\lambda = 50\lambda^2 - 10\lambda + 25$

$25\lambda^2 + 30\lambda + 9 = 0$

$(5\lambda + 3)^2 = 0$

$\therefore \quad \lambda = -\dfrac{3}{5}$

Putting the value of $\lambda = -\dfrac{3}{5}$ in Eq. (i), we get

$(2x + y - 4) - \dfrac{3}{5}(x - 3y + 5) = 0$

$10x + 5y - 20 - 3x + 9y - 15 = 0$

$7x + 14y - 35 = 0$

$x + 2y - 5 = 0$

45. (b) Let AD and BE are altitudes of the triangle.

$\therefore$ Equation of AD is given by

$y - 3 = (\text{Slope of } AD)\,(x + 2)$

$y - 3 = \dfrac{-1}{\text{Slope of BC}}(x + 2)$

$\Rightarrow y - 3 = \dfrac{-1}{\left(\frac{0+1}{4-2}\right)}(x + 2)$

$y - 3 = -2(x + 2)$

$y - 3 = -2x - 4$

$\therefore \quad 2x + y + 1 = 0$ ...(i)

Now, equation of BE is given by

$y + 1 = (\text{Slope of BE})\,(x - 2)$

$y + 1 = \dfrac{-1}{\text{Slope of } AC}(x - 2)$

$\Rightarrow \quad y + 1 = \dfrac{-1}{\left(\frac{0-3}{4+2}\right)}(x - 2)$

$y + 1 = 2(x - 2)$

$y + 1 = 2x - 4$

$\therefore \quad y = 2x - 5$ ...(ii)

Since, orthocentre is the intersecting point of altitudes.

$\therefore$ On solving Eqs. (i) and (ii), we get orthocentre as $(1, -3)$.

Also, centroid of $\Delta ABC = \left(\dfrac{-2+2+4}{3}, \dfrac{3-1+0}{3}\right)$

$= \left(\dfrac{4}{3}, \dfrac{2}{3}\right)$

Equation of line joining centroid $\left(\dfrac{4}{3}, \dfrac{2}{3}\right)$ and orthocentre $(1, -3)$

$y + 3 = \dfrac{-3 - \frac{2}{3}}{1 - \frac{4}{3}}(x - 1)$

$y + 3 = 11(x - 1)$

$y + 3 = 11x - 11$

$11x - y - 14 = 0$

$\therefore$ This is required equation of line.

46. (c) Given that,

$23x^2 - 48xy + 3y^2 = 0$

$\Rightarrow 3y^2 - 48xy + 23x^2 = 0$

Here, $m_1 + m_2 = 16$ ...(i)

and $m_1 m_2 = \dfrac{23}{3}$ ...(ii)

$\therefore \quad m_1 - m_2 = \sqrt{(m_1 + m_2)^2 - 4(m_1 m_2)}$

From Eqs. (i) and (ii) we get

$= \sqrt{(16)^2 - 4\times\dfrac{23}{3}} = \sqrt{256 - \dfrac{92}{3}} = \sqrt{\dfrac{768 - 92}{3}}$

$m_1 - m_2 = \dfrac{26}{\sqrt{3}}$ ...(iii)

On solving Eqs. (i) and (ii), we get

$\therefore \quad m_1 = 8 + \dfrac{13}{\sqrt{3}}$ and $m_2 = 8 - \dfrac{13}{\sqrt{3}}$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\dfrac{\frac{26}{\sqrt{3}}}{1 + \frac{23}{3}}\right| = \sqrt{3}$

$\therefore \quad \theta = 60°$

Angle between line of slope $8 + \dfrac{13}{\sqrt{3}}$ and $-\dfrac{2}{3}$ is

$$\tan\alpha = \frac{8+\frac{13}{\sqrt{3}}+\frac{2}{3}}{1+\left(8+\frac{13}{\sqrt{3}}\right)\left(\frac{2}{3}\right)} = \sqrt{3}$$

$\Rightarrow \alpha = 60°$

Hence, θ and α both are 60° so these lines from a equilateral triangle.

47. (c) Given curve,

$x^2 - xy + y^2 + 3x + 3y - 2 = 0$ ...(i)

and line,

$x + 2y = k \Rightarrow \frac{x+2y}{k} = 1$

Now, By using homogeneous of Eq. (i)

$x^2 - xy + y^2 + 3x(1) + 3y(1) - 2(1)^2 = 0$

$$x^2 - xy + y^2 + 3x\left(\frac{x+2y}{k}\right) + 3y\left(\frac{x+2y}{k}\right) - 2\left(\frac{x+2y}{k}\right)^2 = 0$$

$k^2x^2 - k^2xy + k^2y^2 + 3kx^2 + 6kxy + 3kxy + 6ky^2 - 2x^2 - 8xy - 8y^2 = 0$

$x^2(k^2 + 3k - 2) - (k^2 - 9k + 8)xy + (k^2 + 6k - 8)y^2 = 0$

Since, $\angle AOB = 90°$, it means that

$k^2 + 3k - 2 + k^2 + 6k - 8 = 0$

$\Rightarrow 2k^2 + 9k - 10 = 0$

48. (c) We know that,

If the lines $ax^2 + 2hxy + by^2 = 0$ be two sides of a parallelogram and the line $lx + my = 1$ will be one of its diagonals, then the other diagonal is

$y(bl - hm) = x(am - hl)$ ...(i)

Here, for the given pair of lines, $2x^2 + 3xy - 2y^2 = 0$

$a = 2, b = -2, h = \frac{3}{2}$

$l = -3, m = -1$

Putting all values in Eq. (i), we get

$\therefore\ y\left(6+\frac{3}{2}\right) = x\left(-2+\frac{9}{2}\right) \Rightarrow y\left(\frac{15}{2}\right) = x\left(\frac{5}{2}\right)$

$15y = 5x$

$3y = x \Rightarrow x - 3y = 0$

49. (d) As we know, length of tangent drawn from the point (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

Let point $P(h, k)$ be the point from which the length of tangents are in the ratio of 2 : 1.

$$\therefore\ \frac{\sqrt{h^2+k^2-2h+4k-20}}{\sqrt{h^2+k^2-2h-8k+1}} = \frac{2}{1}$$

$h^2 + k^2 - 2h + 4k - 20 = 4(h^2 + k^2 - 2h - 8k + 1)$

$h^2 + k^2 - 2h + 4k - 20 = 4h^2 + 4k^2 - 8h - 32k + 4$

$3h^2 + 3k^2 - 6h - 36k + 24 = 0$

$h^2 + k^2 - 2h - 12k + 8 = 0$

Replace h and k by x and y,

$x^2 + y^2 - 2x - 12y + 8 = 0.$

$\therefore$ This is the required locus of P.

50. (c) Let the centre of the circle be $c(h_1, k)$ and radius r. Since, circle touches both coordinates axes, then centre will be (h, h) and radius = h

$$\therefore\ \left|\frac{3h-4h-12}{\sqrt{(3)^2+(-4)^2}}\right| = h \Rightarrow \left|\frac{-h-12}{5}\right| = h$$

$\Rightarrow -h - 12 = \pm 5h$

$\Rightarrow -12 = \pm 5h + h \Rightarrow -12 = 6h$ or $-12 = -4h$

$\Rightarrow h = -2$ or $3 \Rightarrow h = 3$ $[\because h > 0]$

Thus, equation of circle

$(x - 3)^2 + (y - 3)^2 = 3^2$

$x^2 - 6x + 9 + y^2 - 6y + 9 = 9$

$x^2 + y^2 - 6x - 6y + 9 = 0$

This is required equation of circle.

51. (b) Given, $L: 9x + y - 28 = 0$

$C: x^2 + y^2 - \frac{3}{2}x + \frac{5}{2}y - \frac{7}{2} = 0$

Let $P(h, k)$ be the pole then, the equation of polar is

$$hx + ky - \frac{3}{4}(x+h) + \frac{5}{4}(y+k) - \frac{7}{2} = 0$$

$$\Rightarrow x\left(h-\frac{3}{4}\right) + y\left(k+\frac{5}{4}\right) - \frac{3}{4}h + \frac{5}{4}k - \frac{7}{2} = 0$$

$\Rightarrow x(4h - 3) + y(4k + 5) - 3h + 5k - 14 = 0$

On comparing this line with $9x + y - 28 = 0$

$4h - 3 = 9$

$4h = 9 + 3$

$4h = 12$

$h = 3$

Similarly,

$4k + 5 = 1$

$4k = -4$

$k = -1$

Hence, the pole of the given line is $(3, -1)$

52. (d) Given circles,

$C_1: (x + 11)^2 + (y - 2)^2 = (15)^2$

$C_2: (x - 11)^2 + (y + 2)^2 = (5)^2$

Centres, $C_1(-11, 2)$ and $C_2(11, -2)$

Radius, $r_1 = 15$ and $r_2 = 5$

The direct common tangents to two circles meet on the line of centres and divide it externally in the ratio of the radii centres of the two circles.

$\therefore$ Point of intersection

$$= \left(\frac{11\times15-(-11)\times5}{15-5}, \frac{-2\times15-2\times5}{15-5}\right)$$

$$= \left(\frac{165+55}{10}, \frac{-30-10}{10}\right) = (22, -4)$$

53. **(b)** We know that, angle between two circles is given by $\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$, where r_1 and r_2 are radius and d is distance between centres.

(A) Given, $r_1 = \sqrt{2}, r_2 = 1, c_1 = (2, 0), c_2 = (2, 1)$

$$\cos\alpha = \frac{(\sqrt{2})^2 + (1)^2 - [\sqrt{(2-2)^2 + (1-0)^2}]^2}{2\times\sqrt{2}\times 1}$$

$$= \frac{2+1-1}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\therefore$ $\alpha = 45°$ or $135°$

(B) Given, $r_1 = 3, r_2 = \sqrt{17}, c_1 = (3, 3), c_2 = (2, -2)$

$$\cos\beta = \frac{(3)^2 + (\sqrt{17})^2 - [\sqrt{(3-2)^2 + (3+2)^2}]^2}{2\times 3\times\sqrt{17}}$$

$$= \frac{9+17-26}{6\sqrt{17}} = 0$$

$\beta = 90°$

(C) Given, $r_1 = 5, r_2 = 3, c_1 = (-2, 7), c_2 = (-2, 0)$

$$\cos\gamma = \frac{(5)^2 + (3)^2 - [\sqrt{(-2+2)^2 + (7-0)^2}]^2}{2\times 5\times 3}$$

$$= \frac{25+9-49}{30} = \frac{-15}{30} = \frac{-1}{2}$$

So, $\gamma = 120°g$ or $60°$.

54. **(a)** Given, $S_1: x^2 + y^2 + 2gx + 2fy + c = 0$

$S_2: 2x^2 + 2y^2 + 3x + 8y + 2c = 0$

$S_3: x^2 + y^2 + 2x + 2y + 1 = 0$

Let point $P(a, b)$, So

$\therefore$ $S_1(a, b) = S_2(a, b)$

$$a^2 + b^2 + 2ga + 2fb + c = a^2 + b^2 + \frac{3}{2}a + 4b + c = 0$$

$$a\left(2g - \frac{3}{2}\right) + b(2f - 4) = 0$$

Now, replace a and b by x and y

So, $x\left(2g - \frac{3}{2}\right) + y(2f - 4) = 0$ is radical axis of given circles

This touches the $x^2 + y^2 + 2x + 2y + 1 = 0$

Its radius = $\sqrt{1^2 + 1^2 - 1} = 1$ and centre = $(-1, -1)$

So, radius of circle = distance between centre and touching point of radical axis.

$$I = \frac{\left|\left(\frac{3}{2} - 2g\right) + (2f - 4)\right|}{\sqrt{\left(\frac{3}{2} - 2g\right)^2 + (2f - 4)^2}}$$

$$\sqrt{\left(\frac{3}{2} - 2g\right)^2 + (2f - 4)^2} = \left|\left(\frac{3}{2} - 2g\right) + (2f - 4)\right|$$

Taking square both sides, we get

$$2\left(\frac{3}{2} - 2g\right)(2f - 4) = 0$$

So, $\frac{3}{2} - 2g = 0$ or $2f - 4 = 0$

$2g = \frac{3}{2}$ or $2f = 4$

$\Rightarrow$ $g = \frac{3}{4}$ or $f = 2$

55. **(c)** Given line, $l: y = 6x + 1$ and parabola: $y^2 = 24x$.

The locus of the point of intersection of perpendicular tangent to a parabola is its directrix.

So, required point will be the point of intersection of $y = 6x + 1$ and directrix $x = -6$.

$\therefore$ $y = 6(-6) + 1 = -35$

Hence, its coordinates are $(-6, -35)$

56. **(a)** The slope of the line joining the focus $S(1, -1)$ and vertex $A(1, 1)$ is

$$m = \frac{-1-1}{1-1} = 0$$

Let $Q(h, k)$ be the point of intersection of the axis AS with the directrix. The $A(1, 1)$ will be the mid-point of OS.

$$(1, 1) = \left(\frac{h+1}{2}, \frac{K-1}{2}\right)$$

$\therefore$ $\frac{h+1}{2} = 1$ and $\frac{k-1}{2} = 1$

$\Rightarrow$ $h = 1$ and $k = 3$ $\Rightarrow$ $Q(1, 3)$

So, the directrix passes through the point (1, 3) and has the gradient 0. So directrix is

$y - 3 = 0$

Let $P(x, y)$ be any point on the parabola and M be the foot of the perpendicular drawn from P on the directrix.

$\therefore$ $PS = PM \Rightarrow PS^2 = PM^2$

$$(x-1)^2 + (y+1)^2 = \left(\frac{y-3}{\sqrt{1}}\right)^2$$

$(x-1)^2 + (y+1)^2 = (y-3)^2$

$(x-1)^2 = 8(1-y)$

By checking option (a),

$$(3-1)^2 = 8\left(1 - \frac{1}{2}\right)$$

$(2)^2 = 8\times\frac{1}{2}$ $\Rightarrow$ $4 = 4$

Hence, point $\left(3, \frac{1}{2}\right)$ lies on the parabola

$(x-1)^2 = 8(1-y)$.

57. **(c)** Let the equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Given that eccentricity, $e = \sqrt{\frac{2}{5}}$

We know,

$b^2 = a^2(1 - e^2)$

$b^2 = a^2\left(1 - \frac{2}{5}\right)$

$a^2 = \frac{5b^2}{3}$

Now, ellipse passes through the point (– 3, 1)

$\frac{(-3)^2}{a^2} + \frac{(1)^2}{b^2} = 1 \quad \Rightarrow \quad 9b^2 + a^2 = a^2 b^2$

Put $a^2 = \frac{5b^2}{3}$, we get

$9b^2 + \frac{5b^2}{3} = \frac{5b^4}{3} \quad \Rightarrow \quad \frac{32b^2}{3} = \frac{5b^4}{3}$

$\therefore \quad b^2 = \frac{32}{5} \quad \Rightarrow \quad a^2 = \frac{5}{3} \times \frac{32}{3} = \frac{32}{3}$

From Eq (i), we get

$\Rightarrow \quad \frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$

$\Rightarrow \quad 3x^2 + 5y^2 = 32$

$\Rightarrow \quad 3x^2 + 5y^2 - 32 = 0$

58. **(b)** Given, ellipse $\frac{x^2}{36} + \frac{y^2}{27} = 1$

Let P $(6\cos\theta,\ 3\sqrt{3}\sin\theta)$ be any point on it

The equation of the tangent,

$\frac{x}{6}\cos\theta + \frac{y}{3\sqrt{3}}\sin\theta = 1$

$\Rightarrow \quad 3\sqrt{3}x\cos\theta + 6y\sin\theta - 18\sqrt{3} = 0$ ...(i)

Let P be the product of the lengths of the perpendiculars from (3, 0) and (– 3, 0) on Eq. (i) is given by

$$P = \left|\frac{3 \times 3\sqrt{3}\cos\theta - 18\sqrt{3}}{\sqrt{27\cos^2\theta + 36\sin^2\theta}}\right| \left|\frac{3 \times 3\sqrt{3}\cos\theta + 18\sqrt{3}}{\sqrt{27\cos^2\theta + 36\sin^2\theta}}\right|$$

$$= \frac{36 \times 27 - 9 \times 27\cos^2\theta}{36\sin^2\theta + 27\cos^2\theta}$$

$$= \frac{9 \times 27\,(4 - \cos^2\theta)}{36(1 - \cos^2\theta) + 27\cos^2\theta}$$

$$= \frac{9 \times 27\,(4 - \cos^2\theta)}{36 - 9\cos^2\theta}$$

$$\therefore \quad P = \frac{9 \times 27(4 - \cos^2\theta)}{9\,(4 - \cos^2\theta)} = 27$$

59. **(b)** Given, asymptotes are

$3x + 4y - 2 = 0$ ...(i)

and $2x + y + 1 = 0$ ...(ii)

From Eqs. (i) and (ii)

$(3x + 4y - 2)(2x + y + 1) = 0$ ...(iii)

As, the equation to the hyperbola will differ from Eq. (iii) only by a constant, so

$(3x + 4y - 2)(2x + y + 1) = \lambda$ ...(iv)

This passes through the point (1, 1), so

$(3 + 4 - 2)(2 + 1 + 1) = \lambda$

$(5)(4) = \lambda \quad \Rightarrow \quad \lambda = 20$

Hence, the equation of the hyperbola will be

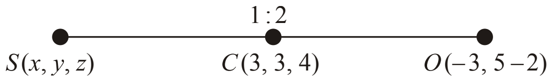
$(3x + 4y - 2)(2x + y + 1) = 20$

$6x^2 + 3xy + 3x + 8xy + 4y^2 + 4y - 4x - 2y - 2 = 20$

$6x^2 + 4y^2 + 11xy - x + 2y - 22 = 0$

$6x^2 + 11xy + 4y^2 - x + 2y - 22 = 0$

60. **(a)** We know that, if O is orthocentre, G is centroid and S is circumcentre, then centroid divides the line segment by circumcentre and orthocentre in the ratio of 1 : 2. Let the coordinates of circumcentre be (x, y, z).



Coordinates of circumference

$= \left(\frac{-3 + 2x}{1 + 2}, \frac{5 + 2y}{1 + 2}, \frac{2 + 2z}{1 + 2}\right)$

$(3, 3, 4) = \left(\frac{-3 + 2x}{3}, \frac{5 + 2y}{3}, \frac{2 + 2z}{3}\right)$

$\frac{-3 + 2x}{3} = 3, \frac{5 + 2y}{3} = 3, \frac{2 + 2z}{3} = 4$

$x = 6, y = 2, z = 5$

$\therefore$ Circumcentre be $S(6, 2, 5)$

61. **(d)** Equation of the plane which cuts the coordinates axes x, y, and z at a, b and c respectively,

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Centroid

$G = \left(\frac{a + 0 + 0}{3}, \frac{0 + b + 0}{3}, \frac{0 + 0 + c}{3}\right)$

$(6, 6, 3) = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

Therefore, $a = 18$, $b = 18$, $c = 9$

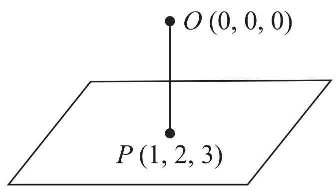
The equation of the plane becomes

$\frac{x}{18} + \frac{y}{18} + \frac{z}{9} = 1 \quad \Rightarrow \quad \frac{x + y + 2z}{18} = 1$

$x + y + 2z = 18$

$\therefore \quad x + y + 2z - 18 = 0$

62. **(b)** Here, foot of the perpendicular P(1, 2, 3) is drawn from the origin $O(0, 0, 0)$



Direction Ratio's of OP = < 1 – 0, 2 – 0, 3 – 0 > = < 1, 2, 3 >

Since, OP is perpendicular to the plane, therefore OP is normal to the plane.

$\therefore$ Equation of plane passing through (1, 2, 3),

$(r - a) \cdot n = 0$

$(x - 1, y - 2, z - 3) \cdot (1, 2, 3) = 0$

$1(x - 1) + 2(y - 2) + 3(z - 3) = 0$

$x + 2y + 3z - 14 = 0$

$\therefore$ According to the given options, (7, 2, 1) lies on the given plane

63. (b) We have,

$$\lim_{n\to\infty} \frac{1}{n^3} \{[1^2 x] + [2^2 x] + [3^2 x] + \cdots + [n^2 x]\}$$

$$= \lim_{n\to\infty} \frac{1}{n^3} \sum_{r=1}^{n} [r^2 x] = \lim_{n\to\infty} \frac{1}{n^3} \sum_{r=1}^{n} (r^2 x - (r^2 x))$$

$$= \lim_{n\to\infty} \frac{1}{n^3} \left(x \sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} \{r^2 n\} \right)$$

$$= \lim_{n\to\infty} \frac{x \times n(n+1)(2n+1)}{n^3 \times 6} - \frac{1}{n^3} \sum_{r=1}^{n} \{r^2 n\}$$

$$= \lim_{n\to\infty} \left(\frac{x}{6} \times \frac{n}{n} \times \frac{n+1}{n} \times \frac{(2n+1)}{n} \right) - \lim_{n\to\infty} \frac{1}{n^3} \sum_{r=1}^{n} \{r^2 n\}$$

$$= \lim_{n\to\infty} \left[\frac{x}{6} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) \right] - \lim_{n\to\infty} \frac{1}{n^2} \sum_{r=1}^{n} \{r^2 n\}$$

$$= \frac{x}{6} \times 1 \times 2 - 0 = \frac{x}{3}$$

64. (a) Given that,

$$f(x) = \begin{cases} \dfrac{1 - \sqrt{2}\sin x}{\pi - 4x}, & x \neq \dfrac{\pi}{4} \\ k, & x = \dfrac{\pi}{4} \end{cases}$$

Since, it is given that $f(x)$ is continuous at $x = \frac{\pi}{4}$

$$\therefore \quad f(\pi/4) = \lim_{\pi \to \frac{\pi}{4}} f(x)$$

$$\Rightarrow \quad k = \lim_{\pi \to \frac{\pi}{4}} \frac{1 - \sqrt{2}\sin x}{\pi - 4x}$$

$$\Rightarrow \quad k = \lim_{\pi \to \frac{\pi}{4}} \frac{-\sqrt{2}\cos x}{-4}$$

[using L' hospital rule]

$$\Rightarrow \quad k = \frac{\sqrt{2}}{4} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad k = \frac{1}{4}.$$

65. (d) Let, $f(x) = x^{\tan^{-1} x}$

Taking log on both the sides,

$\log f(x) = \tan^{-1} x \log x$

Differentiating w.r.t x, we get

$$\therefore \quad \frac{1}{f(x)} \cdot \frac{d}{dx} f(x) = \frac{1}{1 + x^2} \log x + \frac{\tan^{-1} x}{x}$$

$$\Rightarrow \frac{df(x)}{dx} = x^{\tan^{-1} x} \left[\frac{\log x}{1 + x^2} + \frac{\tan^{-1} x}{x} \right]$$

Also, Let $g(x) = \sec^{-1}\left(\dfrac{1}{2x^2 - 1}\right)$

$= \cos^{-1}(2x^2 - 1) = 2\cos^{-1} x$

Differentiating w.r.t x, we get

$$\therefore \quad \frac{d}{dx} g(x) = \frac{-2}{\sqrt{1 - x^2}}$$

$$\therefore \quad \frac{d}{dg(x)} f(x) = \frac{\dfrac{dx}{}}{\dfrac{dg(x)}{dx}}$$

$$= \frac{x^{\tan^{-1} x} \left[\dfrac{\log x}{1 + x^2} + \dfrac{\tan^{-1} x}{x} \right]}{\dfrac{-2}{\sqrt{1 - x^2}}}$$

$$= \frac{-1}{2} \sqrt{1 - x^2} \cdot x^{\tan^{-1} x} \left[\frac{\log x}{1 + x^2} + \frac{\tan^{-1} x}{x} \right]$$

66. (d) Given that,

$x = 3\cos t$ and $y = 4\sin t$

$$\frac{x}{3} = \cos t \Rightarrow \frac{x^2}{9} = \cos^2 t \quad \text{...(i)}$$

$$\frac{y}{4} = \sin t \Rightarrow \frac{y^2}{16} = \sin^2 t \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{16} = \cos^2 t + \sin^2 t$$

$$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{16} = 1$$

Differentiating w.r.t. x, we get

$$\Rightarrow \quad \frac{2x}{9} + \frac{2y}{16} \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-16}{9} \frac{x}{y} \quad \text{...(iii)}$$

Again, differentiating w.r.t. x, we get

$$\Rightarrow \quad \frac{d^2 y}{dx^2} = \frac{-16}{9} \left[\frac{1 \cdot y - x \dfrac{dy}{dx}}{y^2} \right]$$

$$\Rightarrow \quad \frac{d^2 y}{dx^2} = \frac{-16}{9} \left[\frac{y - x - \left(\dfrac{-16}{9} \dfrac{x}{y} \right)}{y^2} \right]$$

[From Eq. (iii)]

$$= \frac{-16}{9}\left[\frac{9y^2+16x^2}{9y^3}\right] = \frac{-16}{9}\times\frac{144}{9y^3}$$

$$\left[\because \frac{x^2}{9}+\frac{y^2}{16}=1\right]$$

$$\left(\frac{d^2y}{dx^2}\right)\left(\frac{3\sqrt{2}}{2}, 2\sqrt{2}\right) = \frac{-16\times 144}{81\times(2\sqrt{2})^3}$$

$$= \frac{-16\times 144}{81\times 16\sqrt{2}} = \frac{-144}{81\times\sqrt{2}} = \frac{-16}{9\sqrt{2}} = \frac{-8\sqrt{2}}{9}$$

67. **(b)** Given that,

$$y = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right]$$

Differentiating w.r.t. x, we get

$$\Rightarrow \frac{dy}{dx} = \left[\frac{2}{\sqrt{a^2-b^2}}\cdot\frac{1}{1+\left(\frac{a-b}{a+b}\right)\tan^2\frac{x}{2}}\cdot\sqrt{\frac{a-b}{a+b}}\cdot\sec^2\frac{x}{2}\cdot\frac{1}{2}\right]$$

$$= \frac{\sec^2 x/2}{a+b}\,\frac{a+b}{(a+b)+(a-b)\tan^2 x/2}$$

$$= \frac{\sec^2 x/2}{a+b+a\tan^2\frac{x}{2}-b\tan^2 x/2}$$

$$= \frac{\sec^2 x/2}{a\left(1+\tan^2\frac{x}{2}\right)+b\left(1-\tan^2\frac{x}{2}\right)}$$

$$= \frac{\sec^2 x/2}{\left(1+\tan^2\frac{x}{2}\right)\left[a+b\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)\right]}$$

$$\frac{dy}{dx} = \frac{1}{a+b\cos x}$$

$$\frac{d^2y}{dx^2} = \frac{b\sin x}{(a+b\cos x)^2}$$

At $x = \pi/2$,

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{x=\frac{\pi}{2}} = \frac{b\sin\pi/2}{\left(a+b\cos\frac{\pi}{2}\right)^2} = \frac{b}{a^2}$$

68. (*) Given function,

$f(x) = x^3 + ax^2 + bx + 5\sin^2 x$

$\Rightarrow$ since, $f(x)$ is an increasing function $f'(x) > 0$

$\Rightarrow 3x^2 + 2ax + b + 10\sin x\cos x > 0$

$\Rightarrow 3x^2 + 2ax + b + 5\sin 2x > 0$

$\Rightarrow 3x^2 + 2ax + b - 5 > 0$

$\because -1 \le \sin 2x \le 1$

$\Rightarrow 3x^2 + 2ax + (b-5) > 0$

Here, since, $f'(x) > 0 \Rightarrow a > 0$ and $D < 0$

$\therefore (2a)^2 - 4\times 3\times(b-5) < 0$

$4a^2 - 12(b-5) < 0$

$a^2 - 3(b-5) < 0$

$a^2 - 3b + 15 < 0.$

69. **(d)** Let $y = \cos x$, $x = 30° = \pi/6$ and $x + \Delta x = 31°$

$$(y)_{x=\frac{\pi}{6}} = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$\Delta x = 1° = 0.0174$ radian.

Consider the function,

$y = f(x) = \cos x$

Differentiating w.r.t. x,

$$\frac{dy}{dx} = -\sin x$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{6}} = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

Let Δy be the change in y due to the change Δx in x.

$$\therefore \Delta y = \frac{dy}{dx}\times\Delta x$$

$$= \left(-\frac{1}{2}\right)\times 0.0174$$

$= (-0.5)\times 0.0174 \approx -0.0087$

$\therefore f(31°) = f(30° + 1°) = y + \Delta y$

$$= \frac{\sqrt{3}}{2} - 0.0087$$

$$= \frac{1.732}{2} - 0.0087$$

$= 0.8660 - 0.0087$

$= 0.8573$

70. **(d)** Given that, x and y are two positive numbers such that $x + y = 32$.

Let, $s = x^2 + y^2$

$= x^2 + (32-x)^2$ $[\because x + y = 32]$

$$\therefore \frac{ds}{dx} = 2x + 2(32-x)(-1)$$

$= 2x - 2(32-x)$

$= 2x - 64 + 2x$

$= 4x - 64$

For maxima or minima,

$$\frac{ds}{dx} = 0$$

$\Rightarrow 4x - 64 = 0$

$\Rightarrow x = 16$

and $y = 32 - x = 32 - 16 = 16$

Again, $\frac{d^2s}{dx} = 4 > 0$

$\therefore$ At $x = 16$, $y = 16$s is minimum

Now, minimum value

$s = 16^2 + 16^2 = 256 + 256 = 512$

71. (b) Given function,

$$f(x) = \frac{2x+3}{4x-1}, x \in [1, 2]$$

Using Lagrange's mean value theorem,

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$f'(c) = \frac{f(2)-f(1)}{2-1}$$

$$\frac{-14}{(4c-1)^2} = \frac{1-\frac{5}{3}}{1}$$

$$\frac{-14}{(4c-1)^2} = -\frac{2}{3}$$

$(4c-1)^2 = 21$

$(4c-1) = \pm\sqrt{21}$

$4c = 1+\sqrt{21}$ or $4c = 1-\sqrt{21}$

$$c = \frac{1+\sqrt{21}}{4} \qquad \left[\because c = \frac{1-\sqrt{21}}{4} \notin [1, 2]\right]$$

72. (c) Let $I = \int \frac{\sin 2x\, dx}{\sin^4 x + \cos^4 x}$

$$I = \int \frac{2\sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

Dividing by $\cos^4 x$ in numerator and denominator, we get

$$I = \int \frac{2\tan x \sec^2 x}{1+(\tan^2 x)^2} dx$$

Put $\tan^2 x = t$

$2\tan x \sec^2 x\, dx = dt$

$$\therefore \quad I = \int \frac{dt}{1+t^2}$$

$I = \tan^{-1} t + C$

$I = \tan^{-1}(\tan^2 x) + C$

By comparing it with $\tan^{-1} f(x) + C$, we get

$f(x) = \tan^2 x$

Now, At $x = \pi/3$

$$\therefore \quad f\left(\frac{\pi}{3}\right) = \left\{\tan\left(\frac{\pi}{3}\right)\right\}^2 = (\sqrt{3})^2 = 3$$

73. (c) Let, $I = \int \left\{\frac{\log x - 1}{1+(\log x)^2}\right\}^2 dx$

Put $\log x = t$, $x = e^t$, $dx = e^t dt$

$$I = \int e^t \frac{(t-1)^2}{(t^2+1)^2} dt,$$

$$= \int e^t \frac{t^2+1-2t}{(t^2+1)^2} dt$$

$$= \int e^t \left\{\frac{1}{t^2+1} + \frac{-2t}{(t^2-1)^2}\right\} dt$$

$$I = \frac{e^t}{t^2+1} + C$$

$$I = \frac{x}{(\log x)^2+1} + C$$

74. (d) Let, $I = \int \frac{dx}{x^3+3x^2+2x}$

$$= \int \frac{dx}{x(x^2+3x+2)}$$

$$= \int \frac{dx}{x(x+1)(x+2)}$$

By using the method of partial fraction,

$$\frac{1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$$

$1 = A(x+1)(x+2) + Bx(x+2) + Cx(x+1)$

At $x = 0 \Rightarrow A = \frac{1}{2}$

At $x = -1 \Rightarrow B = -1$

At $x = -2 \Rightarrow C = \frac{1}{2}$

$$\text{Now, } I = \int \left(\frac{1}{2x} - \frac{1}{x+1} + \frac{1}{2(x+2)}\right) dx$$

$$I = \frac{1}{2}\int \frac{1}{x} dx - \int \frac{1}{x+1} dx + \frac{1}{2}\int \frac{1}{x+2} dx$$

$$= \frac{1}{2}\log x - \log(x+1) + \frac{1}{2}\log(x+2) + C$$

$$= \frac{1}{2}[\log x - 2\log(x+1) + \log(x+2)] + C$$

$$= \frac{1}{2}\log\left|\frac{x(x+2)}{(x+1)^2}\right| + C$$

$$\therefore \quad I = \frac{1}{2}\log\left|\frac{x^2+2x}{(x+1)^2}\right| + C.$$

75. (b) Given that,

$$I_n = \int \sec^n x\, dx$$

Put $n = 2$,

$$I_2 = \int \sec^2 x\, dx = \tan x + c_1 \qquad \text{...(i)}$$

Put $n = 4$,

$$I_4 = \int \sec^4 x\, dx$$

$= \int \sec^2 x \cdot \sec^2 x \, dx$

$= \int (\tan^2 x + 1) \sec^2 x \, dx$

$I_4 = \dfrac{\tan^3 x}{3} + \tan x + c_2 \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$\therefore \quad I_4 - \dfrac{2}{3} I_2 = \dfrac{\tan^3 x}{3} + \tan x + c_2 - \dfrac{2}{3} \tan x - \dfrac{2c_1}{3}$

$= \dfrac{1}{3} \tan^3 x + \dfrac{1}{3} \tan x + c \quad \left[\text{where } c_2 - \dfrac{2}{3} c_1 = c\right]$

$= \dfrac{1}{3} \tan x (\tan^2 x + 1) + c$

$= \dfrac{1}{3} \tan x \sec^2 x + c.$

76. (d) Given that,

$\lim_{x\to\infty} \left[\dfrac{\sqrt{1} + 2\sqrt{2} + 3\sqrt{3} + \cdots + n\sqrt{n}}{n^{5/2}}\right]$

$= \lim_{x\to\infty} \left[\dfrac{1\sqrt{1} + 2\sqrt{2} + 3\sqrt{3} \cdots n\sqrt{n}}{n\sqrt{n}}\right] \dfrac{1}{n}$

$= \lim_{x\to\infty} \left[\dfrac{(1)^{3/2} + (2)^{3/2} + (3)^{3/2} + \cdots + (n)^{3/2}}{(n)^{3/2}}\right] \dfrac{1}{n}$

$= \lim_{x\to\infty} \sum_{r=1}^{n} \left(\dfrac{r}{n}\right)^{3/2} \cdot \dfrac{1}{n}$

$= \int_0^1 x^{3/2} dx = \left[\dfrac{x^{5/2}}{\frac{5}{2}}\right]_0^1 = \dfrac{2}{5}$

77. (d) Let, $I = \int_0^{\alpha/3} \dfrac{f(x)}{f(x) + f\left(\dfrac{\alpha - 3x}{3}\right)} dx \quad$ ...(i)

We know, $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$

$I = \int_0^{\alpha/3} \dfrac{f\left(\dfrac{\alpha}{3} - x\right)}{f\left(\dfrac{\alpha}{3} - x\right) + f\left(\dfrac{\alpha - 3\left(\dfrac{\alpha}{3} - x\right)}{3}\right)} dx$

$\Rightarrow \quad I = \int_0^{\alpha/3} \dfrac{f\left(\dfrac{\alpha - 3x}{3}\right)}{f\left(\dfrac{\alpha - 3x}{3}\right) + f(x)} dx \quad$ ...(ii)

On adding Eqs. (i) and (ii), we get

$I + I = \int_0^{\alpha/3} \dfrac{f(x)}{f(x) + f\left(\dfrac{\alpha - 3x}{3}\right)} dx + \int_0^{\alpha/3} \dfrac{f\left(\dfrac{\alpha - 3x}{3}\right)}{f\left(\dfrac{\alpha - 3x}{3}\right) + f(x)} dx$

$2I = \int_0^{\alpha/3} 1 \, dx$

$2I = [x]_0^{\alpha/3}$

$2I = \dfrac{\alpha}{3}$

$\therefore \quad I = \dfrac{\alpha}{6}$

78. (a) Given curve,

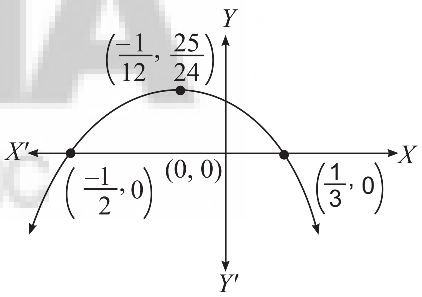
$y = 1 - x - 6x^2$

$y = -[6x^2 + x - 1]$

$y = -6\left[x^2 + \dfrac{x}{6} - \dfrac{1}{6}\right]$

$y = -6\left(x + \dfrac{1}{12}\right)^2 + \dfrac{25}{24}$

$\left(x + \dfrac{1}{12}\right)^2 = -\dfrac{1}{6}\left(y - \dfrac{25}{24}\right)$



$\therefore \quad$ Required area, $A = \int_{-1/2}^{1/3} y \, dx$

$= \int_{-1/2}^{1/3} (1 - x - 6x^2)\, dx = \left[x - \dfrac{x^2}{2} - 2x^3\right]_{-1/2}^{1/3}$

$= \left(\dfrac{1}{3} - \dfrac{1}{18} - \dfrac{2}{27}\right) - \left(-\dfrac{1}{2} - \dfrac{1}{8} + \dfrac{1}{4}\right)$

$= \left(\dfrac{18 - 3 - 4}{54}\right) - \left(\dfrac{-4 - 1 + 2}{8}\right)$

$A = \dfrac{11}{54} + \dfrac{3}{8} = \dfrac{44 + 81}{216} = \dfrac{125}{216}$ sq. unit

79. (c) The equation of the family of parabolas with focus at the origin and X-axis as its axis is given by

$y^2 = 4a(x + a) = 4ax + 4a^2 \quad$ ...(i)

Differentiating w.r.t. x, we get

$\therefore \quad 2y \dfrac{dy}{dx} = 4a$

$$a = \frac{y}{}\left(\frac{dy}{dx}\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$y^2 = 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2$$

$\therefore$ order = m = 1 and degree = n = 2

Hence, $mn - m + n = 1 \times 2 - 1 + 2 = 3$

80. **(d)** Given differential equation,

$$(1+e^{x/y})dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$$

$$(1+e^{x/y})\,dx = -e^{x/y}\left(1-\frac{x}{y}\right)dy$$

$$\frac{dx}{dy} = \frac{-e^{x/y}\left(1-\frac{x}{y}\right)}{(1+e^{x/y})} \quad \text{...(i)}$$

Let $\frac{x}{y} = v \Rightarrow x = vy$

Differentiating w.r.t. y, we get

$$\frac{dx}{dy} = v\frac{d(y)}{dy} + y\frac{dv}{dy} \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$$

Putting value of $\frac{dx}{dy}$ and $v = \frac{x}{y}$ in Eq. (i), we get

$$v + y\frac{dv}{dy} = \frac{-e^v(1-v)}{1+e^v} \Rightarrow y\frac{dv}{dy} = -\frac{e^v + ve^v}{1+e^v} - v$$

$$y\frac{dv}{dy} = \frac{-e^v + ve^v - v - ve^v}{1-e^v}$$

$$y\frac{dv}{dy} = \frac{-[v+e^v]}{1+e^v} \text{ or, } \left[\frac{1+e^v}{v+e^v}\right]dv = -\frac{dy}{y}$$

On integrating both the sides, we get

$$\int\frac{1+e^v}{v+e^v}\,dv = -\int\frac{dy}{y} \quad \text{...(ii)}$$

Put $v + e^v = t$

$(1+e^v)\,dv = dt$

From eq. (ii),

$$\int\frac{dt}{t} = -\int\frac{dy}{y}$$

$\log t = -\log y + \log c$

$\log(v+e^v) = -\log y + \log c$

$\log y(v+e^v) = \log c$

Put value of $v = \frac{x}{y} \Rightarrow \log y\left(\frac{x}{y} + e^{x/y}\right) = \log c$

$$y\left(\frac{x}{y} + e^{x/y}\right) = c \Rightarrow x + ye^{x/y} = c$$

PHYSICS

81. **(a)** We have

$$R_{eq} = \frac{R_1R_2}{R_1+R_2} = \frac{60\times 30}{60+30} = 20\,\Omega$$

and tolerance value is

$$\Rightarrow \Delta R_P = R_{eq}\left\{\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} - \frac{\Delta R_1 + \Delta R_2}{R_1+R_2}\right\}$$

$$= 20\left\{\frac{0.36}{60} + \frac{0.09}{30} - \frac{0.36+0.09}{90}\right\}$$

$= 20\{0.006 + 0.003 - 0.005\} = 0.085\,\Omega$

So, resistance in parallel is $R_p = 20 \pm 0.08$

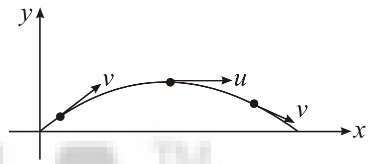
82. **(d)** Trajectory of a projectile is of form

$$y = f(x) = x\tan\theta - \frac{gx^2}{2\pi^2\cos^2\theta} \;\therefore\; \frac{dy}{dx}$$

Slope of $y - x$ curve, which do not gives velocity.

So, Assertion (A) is incorrect.

Also, velocity of a projectile is always along tangent to the trajectory (shown)



Hence, reason (R) is correct.

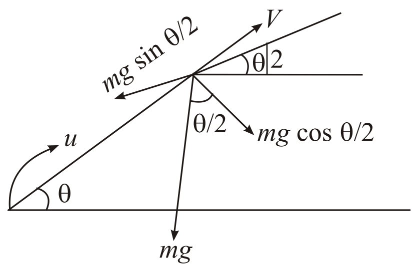
83. **(c)** Required ratio $= \frac{H_{max}}{R} = \frac{\left(\frac{u^2\sin^2\theta}{2g}\right)}{\left(\frac{u^2\sin 2\theta}{g}\right)} = \frac{\tan\theta}{4}$

$$= \frac{8}{7}\times\frac{1}{4} = \frac{2}{7}$$

84. **(d)** Let velocity of projectile is v at an angle $\frac{\theta}{2}$ with horizontal

$$\therefore \quad v\cos\frac{\theta}{2} = u\cos\theta$$

$$\text{or} \quad v = \frac{u\cos\theta}{\cos\frac{\theta}{2}}$$



Now, $\frac{mv^2}{r} = mg\cos\frac{\theta}{2} \Rightarrow r = \frac{v^2}{g\cos\frac{\theta}{2}}$

$$\frac{\frac{u^2\cos^2\theta}{\left(\cos\frac{\theta}{2}\right)^2}}{g\cos\frac{\theta}{2}} \Rightarrow r = \frac{u^2\cos^2\theta\cdot\sec^3\left(\frac{\theta}{2}\right)}{g}$$

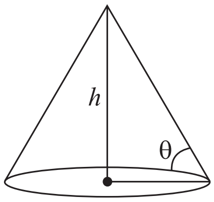
85. (d) $\tan\theta_{max} = \mu$

$\frac{h_{max}}{R} = \mu$

$h_{max} = \mu R$

Now, $V_{max} = \frac{1}{3}\pi R^2\ h_{max}$

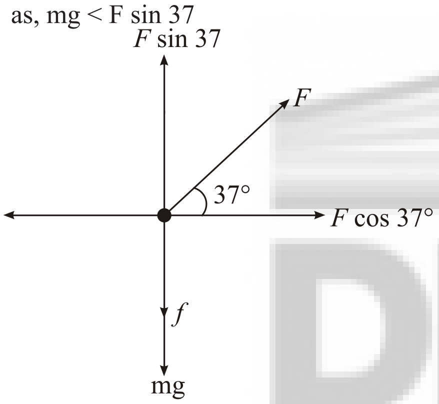
$= \frac{1}{3}\pi R^2 - \mu R = \frac{1}{3}\pi\mu R^3$



86. (b) mg = 20 × 10 = 20N

F sin 37 = 90 × $\frac{3}{5}$ = 54 N

as, mg < F sin 37



So, friction force will act downward

So, F sin 37 – f – mg = m cl

F sin 37 – μ cos 37 – mg = m cl

Putting the value of F_1 μ and m

we get a = 8 m/s^2

87. (a) $E_i = \frac{1}{2}mv^2 = \frac{1}{2}\times 2\times 8^2 = 64$ J

E_f = mgh = 2 × 10 × 3 = 60 J

Work done against air friction

= Loss of energy = 64 – 60 = 4 J

88. (a) $3g - T = 3a$ ...(i)

$T - 2g = 2a$ ...(ii)

From (i) and (ii)

$a = \frac{g}{s} = 2$ m/s^2

Now, $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times 2\,(2)^2$

$\Rightarrow\ s = 4$ m

Work done on block of 3 kg by gravity

$W = Mgs = 3\times 10\times 4 = 120$ J

89. (d) Taken, $v = 1$

$v' = 1.09$

and $\Delta v = 0.09$

$\Rightarrow\ \omega \propto \frac{1}{I}$ $\left[\because\ J = I\omega \Rightarrow I \propto \frac{1}{\omega}\right]$

But $I \propto r^2$ and $\omega \propto v^{1/3}$

$\therefore\ \ \omega \propto \frac{1}{v^{2/3}}$ (or $v^{-2/3}$)

$\Rightarrow\ \omega = kv^{-2/3}$, k = cons

$\therefore$ Change in angular speed

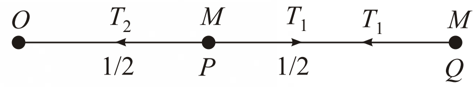
$= \frac{\Delta\omega}{\omega}\times 100 = -\frac{2}{3}\frac{\Delta v}{v}\times 100$

$= -\frac{2}{3}\times 0.09\times 100 = -6\%$

So decrease in angular speed is 6%

90. (b) $T_2 - T_1 = m\omega^2\frac{1}{2}$

$T_1 = m\omega^2 1$



So, $T_2 = m\omega^2\frac{3}{2}$

So, $\frac{T_1}{T_2} = \frac{m\omega^2 1}{m\omega^2\frac{3}{2}} = \frac{2}{3}$

91. (a) Given, total energy = 9J

PE at mean position = 5J, At mean position K · E is maximum

So, maximum KE = 9J – 5J = 4J

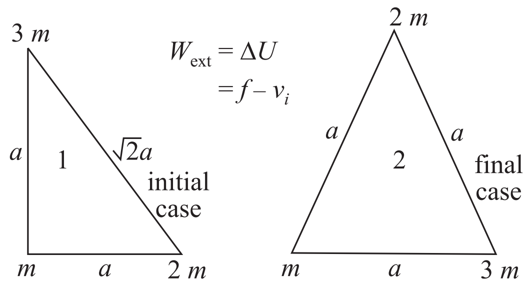
So, $\frac{1}{2}MA^2\omega^2 = 4 \Rightarrow \frac{1}{2}MA^2\times\frac{4\pi^2}{T^2} = 4$

$T^2 = \frac{MA^2\times 4\pi^2}{8}$

Put $A = 10^{-2}$ m

Then, $T = \pi/100$ sec

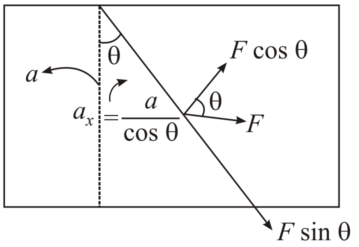
92. (d) Wext = $\Delta U = f - V_i$



$$= -\left[-\frac{Gm^2}{a}\left(2+3+\frac{6}{\sqrt{2}}\right) - \left(-\frac{Gm^2}{a}(2+3+6)\right)\right]$$

$$= -\frac{Gm^2}{a}\left(6-\frac{6}{\sqrt{2}}\right)$$

93. (a)



Shearing stress $= -\frac{F \sin\theta}{a_x} = -\frac{F}{a}\sin\theta\cos\theta$

Magnitude of shearing stress $= \frac{F}{a}\sin\theta\cos\theta$

94. (c) Density of water – Density of air $\propto T_{water} - T_{air}$
Density of liquid – Density of water $\propto T_{liquid} - T_{water}$
Now, from given values,
$T_{water} - T_{air} = T_{liquid} - T_{water}$
Air density = 1 kg/m^3
and water density = 1000 kg/m^3
So, liquid density = 2000 kg/m^3

95. (b) Let mass of the steam condensed in M.
Heat released = Heat gained by water of steam
$\Rightarrow M \times 540 + M \times 1 \times (100 - 90)$
$= 1 \times 1 \times (90 - 9) + 0.1 \times 1 \times (90 - 9)$
$\Rightarrow 540x + 10x = 81 + 8.1$
$\Rightarrow x = \frac{89.1}{550} = 0.162$ kg, $x = 162$g

96. (c) Suppose temp. of middle plate is T_0
$\therefore$ Heat gained by first surface = Heat lost by third surface.
So, $\sigma A\,[(3T)^4 - T_0^4] = \sigma A[T_0^4 - (2T)^4]$
$\Rightarrow 81T^4 - T_0^4 = T_0^4 - 16T^4$
$\Rightarrow T_0^4 = \frac{97}{2}T^2 \quad \Rightarrow \quad T_0 = \left(\frac{97}{2}\right)^{1/4} T$

97. (d) Let there are n moles of Nitrogen gas in the cylinder.
As all of K.E appear in form of heat

So, $n\left(\frac{1}{2}Mv^2\right) = \frac{f}{2}nR\Delta T$ ...(i)

Here, $M = 28\ g = 28 \times 10^{-3}$ kg, $f = 5$

Also, $\Delta p = \frac{nR\Delta T}{V}$

So, $\frac{\Delta p}{p} = \frac{\left(\frac{nR\Delta T}{V}\right)}{\left(\frac{nRT}{V}\right)} = \frac{nR\Delta T}{nRT}$

$\Rightarrow \frac{\Delta p}{p} = \frac{nMv^2}{fnRT} = \frac{Mv^2}{fRT}$ [from (i)]

So, percentage change in pressure is,

$\therefore \frac{\Delta p}{p} \times 100 = \frac{28 \times 10^{-3} \times 100 \times 100}{5 \times 8.3 \times 300} \times 100$

$= 2.25\%$

98. (b) (i) Zeroth law of thermodynamics states about thermal equilibrium of different states in contact.
(ii) First law is based on energy conservation law.
(iii) In free expansion of gases, work done is always zero as no resistance is there.
(iv) Second law of thermodynamics discusses about direction of heat flow.

99. (a) Mean free path,

$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$,

n = number density, d = diameter

$\Rightarrow d^2 = \frac{1}{\sqrt{2}\pi n\lambda} = \frac{1 \times \pi}{\sqrt{2} \times \pi \times 2\sqrt{2} \times 10^8 \times 10^{-2}}$

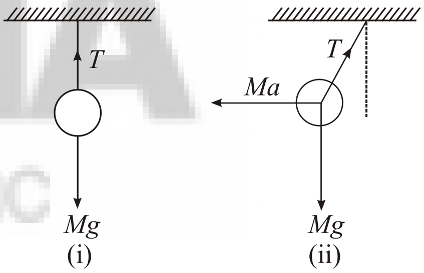
$\Rightarrow d = \frac{1}{2} \times 10^{-3}$ cm $\Rightarrow d = 5 \times 10^{-4}$ cm

100. (c) When car is at rest, tension in string is $T = mg$.

$T = Mg$ amd $V = \sqrt{\frac{T}{\mu}}$ ...(i)

$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{g}{(a^2 + g^2)^{1/2}}}$

$\left(\frac{60}{66}\right) = \sqrt{\frac{g}{(a^2 + g^2)^{1/2}}}$



$\left(\frac{10}{11}\right)^2 = \frac{g}{(a^2 + g^2)^{1/2}}$

$\left(\frac{100}{121}\right)^2 = \frac{g^2}{a^2 + g^2}$

Solving, we get $a = 6.8$ m/s^2

101. (c) Wavelength of the reflected wave is

$\lambda' = \left(\frac{v - v_s}{v + v_s}\right)\lambda = \left(\frac{v - v_s}{v + v_s}\right)\frac{v}{v}$

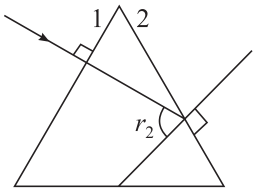
$= \frac{340 - 20}{340 + 20} \times \frac{340}{160} = \frac{120}{360} \times \frac{340}{160}$

$\lambda' = \frac{17}{9}$ m

102. (a) Given, $i = 0°$

So, $\frac{\sin i}{\sin r_1} = \sqrt{2}$, r_1 = angle of refraction at surface 1

$$\frac{\sin 0°}{\sin r_1} = \sqrt{2}$$



$\sin r_1 = 0$ or $r_1 = 0$

Now, $\frac{\sin r_2}{\sin e} = \frac{1}{\mu}$,

r_2 = angle of refraction at surface 2

$\Rightarrow \sin e = \sqrt{2}\sin r_2$

But, $r_1 + r_2 = 60°$ $[\because r_1 + r_2 = A]$

So, $\sin e = \sqrt{2}\ \sin 60°$

$= \sqrt{2} \times \sqrt{3}/2 > 1$

So, light is incident at more than critical angle.

So, it will totally reflect fact

$\therefore$ Deviation angle = $(i + e) - (r_1 + r_2)$

= 0 – 60° = 60° (in magnitude)

103. (a) When unpolarised light passes through first polariser, it becomes plane polarised and its intensity becomes half. Therefore, after first polariser, intensity $I_1 = I_0/2$

After second polariser, intensity $I_2 = \frac{I_0}{2}\cos^2\theta$

$\theta = \theta \quad \theta = \pi/2$

$$I \left| \frac{I_0}{2} \quad I_2 \right| I_3$$

(Malus law)

After third polariser, intensity

$$I_3 = \frac{I_0}{2}\cos^2\theta\cos^2(90° - \theta)$$

(because angle b/w second and third is $(90 - \theta)$)

$$\Rightarrow \quad I_3 = \frac{I_0}{2}\cos^2\theta\sin^2\theta = \frac{I_0}{8}\sin^2 2\theta$$

104. (d) When dielectric of thickness t is introduced in two charges at distance r, the effective force between the charges is given by

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0\,[r - t + t\sqrt{K}]^2}$$

where, K = dielectric constant of medium

In first case, $t = r/2$ and $K = 4$

$$\therefore \quad F_1 = \frac{K\,q_1q_2}{\left[r - r/2 + \frac{r}{2}\sqrt{4}\right]^2} = \frac{Kq_1q_2}{\frac{9}{4}r^2} = \frac{4}{9}\frac{Kq_1q_2}{r^2}$$

In second case, $t = r/3$ and $K = 9$

$\therefore \quad F_2 = Kq_1q_2$

$$\left[r - \frac{r}{3} + \frac{r}{3}\sqrt{3}\right]^2 = \frac{9Kq_1q_2}{25}$$

$$\text{So,} \quad \frac{F_1}{F_2} = \frac{\frac{K\,q_1q_2}{r^2}\times\frac{4}{9}}{\frac{Kq_1q_2}{r_2}\times\frac{9}{25}} = \frac{100}{81}$$

105. (a) Since $a = \frac{q}{m}E$

$$= \frac{2\times10^{-6}}{0.04}\times 4.2\times10^4\ \text{m/s}^2$$

$= 2.1\ \text{m/s}^2$ (downward)

So, effective acceleration on bob,

$a_e = a + g = 12.1\ \text{m/s}^2$

$$\frac{T}{T'} = \sqrt{\frac{g + a_e}{g}} \Rightarrow \frac{T}{T'} = \sqrt{\frac{12.1}{10}} = \frac{11}{10} \quad\Rightarrow\quad T' = \frac{10}{11}T$$

$$\text{Given, } T = \frac{44}{20} \quad\Rightarrow\quad T' = \frac{10}{11}\times\frac{44}{20} = 2s$$

So, time taken in 15 oscillations = 2 × 15 = 30s

106. (d) Charge stored in the presence of air,

$q_{\text{air}} = C_{\text{air}} \times V = 80 \times 30 \times 10^{-6} = 2400\ \mu\text{C}$

Charge stored in presence of dielectric medium,

$q_d = C_{\text{dielectric}} \times V = 1600 \times 30 \times 10^{-6} = 48000\ \mu\text{C}$

$[\because C_{\text{dielectric}} = 20 \times 80 \times 10^{-6} = 1600\ \mu\text{F}]$

When dielectric is removed, the charge passing through wire is,

$q = q_d - q_{\text{air}}$

$\Rightarrow \quad q = (48 - 2.4) \times 10^{-3}C \Rightarrow = q = 45.6 \times 10^{-3}\text{C}$

107. (a) As $Q_i = 0$

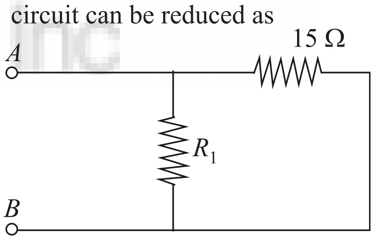
$Q_f = C_1(V_1 - V_0)$, $Q_2 = C_2(V_2 - V_0)$

$Q_3 = C_3(V_3 - V_0)$ and $Q_i = Q_f$

$O = C_1V_1 + C_2V_2 + C_3V_3 - V_0(C_1 + C_2 + C_3)$

$$\Rightarrow V_0 = \frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}$$

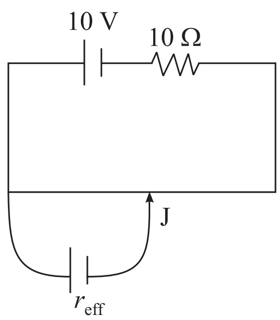
108. (b) Here 2Ω and 3Ω resistances are short circuited, so circuit can be reduced as



$$\text{Req.} = \frac{15R_1}{15 + R_1} = 6$$

$\Rightarrow \quad 15R_1 - 6R_1 = 15 \times 6 \Rightarrow 9R_1 = 90 \Rightarrow R_1 = 10\Omega$

109. (c) $\varepsilon_{\text{eff}} = \frac{\varepsilon_1 t_1 + \varepsilon_2 r_2}{r_1 + r_2} = \frac{2\times2 + 3\times3}{2+3} = \frac{13}{5}V$



$$I_{\text{AB}} = \frac{10}{10 + 10} = 0.5\ A$$

$$V_{\text{AB}} = \frac{5}{1}\times x = 5x$$

But, $V_{\text{AB}} = r_{\text{eff}}$

$$5x = \frac{13}{5}$$

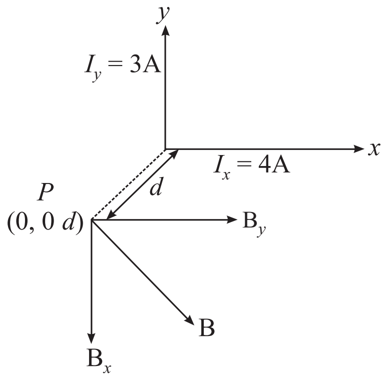
$$x = \frac{13}{25} \quad x = 0.52\ \text{m} = 52\ \text{cm}$$

110. (d) Magnetic field at point P due to I_x

$$\boldsymbol{B}_x = -\frac{\mu_0 \cdot 4}{2\pi d}\hat{\mathbf{j}}$$

magnetic field at point P due to $I_{y'}$.

$$\mathbf{B}_y = \frac{\mu_0 I_y}{2\pi d} \qquad (X = \text{direction})$$



$$\mathbf{B}_y = \frac{\mu_0 \cdot 3}{2\pi d} a\,\hat{\mathbf{i}}$$

∴ Resultant magnetic field,

$$B = \sqrt{B_x^2 + B_y^2}$$

$$\Rightarrow B = \text{——}\sqrt{4^2 \quad 3^2} \Rightarrow B = \frac{5\mu_0}{2\pi d}$$

111. (a) Current sensitivity, $I_S = \frac{N\,BA}{k}$

So, $\frac{I_{S_1}}{I_{S_2}} = \frac{N_1 B_1 A_1 k_2}{N_2 B_2 A_2 k_1}$

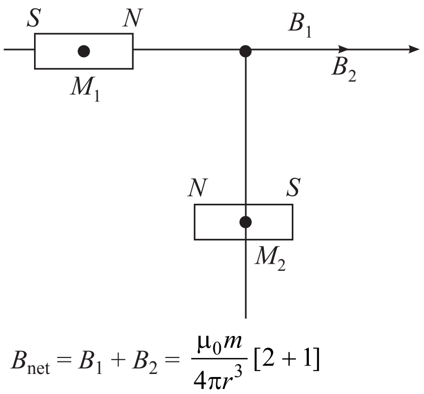
$= \frac{30 \times 0.25 \times 4.8 \times 10^{-3}}{45 \times 0.5 \times 24 \times 10^{-3}} \times \left(\frac{1}{1}\right)$ (Assume $K_1 = K_2$)

$\frac{I_{S_1}}{I_{S_2}} = \frac{2}{3}$

Voltage sensitivity, $V_S = NBA/RK = IS/R$

So, $\frac{V_{S_1}}{V_{S_2}} = \frac{I_{S_1}}{I_{S_2}} \times \frac{R_2}{R_1} = \frac{2}{3} \times \frac{14}{10} = \frac{14}{15}$

112. (c) At origin, both magnetic fields will be in same direction.



$$B_{net} = B_1 + B_2 = \frac{\mu_0 m}{4\pi r^3}[2+1]$$

$$= \frac{3\mu_0 m}{4\pi r^3} = \frac{3 \times 10^{-7} \times 9}{(3 \times 10^{-2})^3} = 0.1\,T$$

113. (a) $V = vBl$ and $q = CV$

So, $a = C(vBl) = 10 \times 10^{-6} \times 2 \times 4 \times 1$

$q = 80\ \mu C$

So, charges on plates are ± 80 μC. By Lenz law direction of induced current will be ACW.

So $q_A = +80\ \mu C$ and $q_B = -80\mu C$.

114. (a) In the shown figure, current is ahead of voltage, so its a RC circuit, so P is a resistor and Q is a capacitor.

As $\cos\phi = \frac{R}{Z}$

$R = Z\cos\phi = 1000\sqrt{2} \times \cos 45 = 1000$ ohm

Now, $Z^2 = X_C^2 + R^2$

$X_C = \sqrt{Z^2 - R^2}$

$= \sqrt{(1000\sqrt{2})^2 - 1000^2} = 1000\ \Omega$

$\therefore \quad \frac{1}{\omega C} = 1000 \Rightarrow C = \frac{1}{100 \times 1000}$ $(\because \omega = 100)$

$\Rightarrow \quad C = 10\ \mu F$

115. (c) Energy density due to electric field is

$U = \frac{1}{2}\varepsilon_0 E^2$

Here, $E = \frac{E_0}{\sqrt{2}} = \frac{40}{\sqrt{2}}$

$\therefore \quad U = \frac{1}{2} \times 8.85 \times 10^{-12} \times \frac{40 \times 40}{2} = 3.54 \times 10^{-9}\ J/m^3$

116. (a) Energy of photons of

H_α line $= \Delta E\,(3 \to 2) = 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$

$= \frac{13.6 \times 5}{36} = 1.89$ eV

Energy of photons of H_β line $= \Delta E\,(4 \to 2)$

$= 13.6\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{12 \times 13.6}{64} = 2.55$ eV

Energy of photons of H_∞ line $= \Delta E\,(\infty \to 2)$

$= \frac{13.6}{4} = 3.4$ eV

Ratio of kinetic energy of emitted photons

$= \frac{255 - 1.89}{3.4 - 1.89} = \frac{0.66}{1.51} \approx \frac{0.7}{1.6} \approx \frac{7}{16}$

177. (d) $\lambda \alpha \frac{1}{V}$ and $V\ \alpha\ \frac{1}{n}$ $\left[\because \lambda = \frac{h}{p}\right]$

So $\lambda\ \alpha\ n$

118. (a) Number of nuclei required

$= \frac{E_{total}}{E_1} = \frac{1000\ J}{200\ MeV} = 31.25 \times 10^{13}$

119. (c) Diode, D_1 = reverse biased (OFF) → will act like open switch

and diode, D_2 = forward biased (ON) → will act like wire

Because diodes are ideal, so voltage drop across D_2 is zero

$R_{eff} = 3 + 2 + 3 + 2 = 10\ \Omega$

$I = V/R = 20/10 = 2A$

120. (a) Modulation index, $M = \frac{(V_n)\text{ signal}}{(V_m)\text{ carrier}}$

Let's (V_m) carrier = V

In first case, $0.6 = \frac{12}{V} \Rightarrow V = 20$ V

In second case, $0.75 = \frac{12}{V'} \Rightarrow V' = \frac{12}{0.75} = 16$ V

Change in peak voltage of carrier wave

$\Delta V = 20 - 16 = 4V$

% change = $\frac{4}{20} \times 100\% = 20\%$ (decrement)

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121. (c) Given, $r_n = 476.1$ pm

$r_1 = 52.9$ pm

$r_n = \frac{n^2 \times r_1}{Z}$; $476.1 = \frac{n^2 \times 52.9}{Z}$

$n = 3$ [Z for hydrogen = 1]

Now, $E_n = -2.18 \times 10^{-18} \cdot \frac{Z^2}{n^2}$

and $E_n = E_3 = -\frac{2.18\, Z^2 \times 10^{-18}\text{ J}}{9} = -2.42 \times 10^{-19}$ J

122. (b) $\frac{hv_1 - hv_0}{hv_2 - hv_0} = \frac{K.E_1}{K.E_2}$

$\frac{h(1.6 \times 10^{16} - v_0)}{h(1.0 \times 10^{16} - v_0)} = \frac{2\text{K.E.}}{\text{K.E.}}$

$1.6 \times 10^{16} - v_0 = 2 \times 1.0 \times 10^{16} - 2v_0$

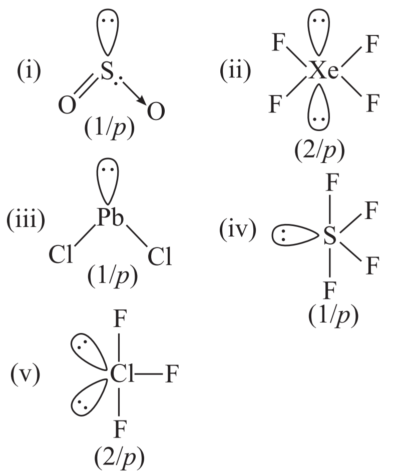
$v_0 = 1.6 \times 10^{16} - 2(1.0 \times 10^{16}) = v_0 = 4 \times 10^{15}$ Hz

123. (a) Electronic configuration = $1s^2, 2s^2, 2p^6, 3s^2, 3p^6$ = 18 electrons

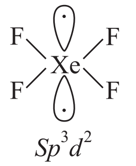
∵ Element is in – 2 oxidation state so, it has 16 electrons.

(Z = 16) refer to group 16 and period 3 i.e. sulphur, Sulphur show – 2 oxidation state.

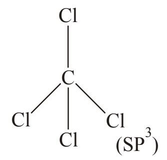
124. (a) SO_2, $PbCl_2$ and SF_4 contain one lone pair.



125. (c) XeF_4 is sp^3d^2 hybridised and due to presence of 2 lone pair of electrons its shape is square planar.



CCl_4 is sp^3 hybridised due to 4 bonds. Its geometry is sigma.



126. (c) Moles of $H_2 = 16/2 = 8$

Moles of He = 16/4 = 4

Moles of $O_2 = 16/32 = 0.5$

Total moles at pressure 10 atm.

$(P_1) = 8 + 4 + 0.5 = 12.5$

Total moles at (P_2) = 4.5 (only He and O_2)

$\frac{P_1V_1}{P_2V_2} = \frac{n_1RT}{n_2RT} \Rightarrow \frac{10}{P_2} = \frac{12.5}{4.5}$

$P_2 = 3.6$

127. (d) When 0.15 M Na_2CO_3 (1L) mixed with 0.2 M $K_2Cr_2O_7$ (5 L) than total molarity (M).

M_1V_1 (Na_2SO_3) + M_2V_2 ($K_2Cr_2O_7$) = M_3V_3 (total)

$M_3 = \frac{0.15 \times 1 + 0.2 \times 0.5}{1.5}$

= 0.15 mol/L

Remaining mole of $K_2Cr_2O_7$

= 0.2 – 0.15 = 0.05 mol/L

128. (a) $CH_3\,OH + 3/2O_2 \rightarrow CO_2 + 2H_2O$,

$\Delta H = -726$ kJ/mol ...(i)

($H_2 + 1/2O_2 \rightarrow H_2O$, $\Delta H = -286$ kJ/mol) ...(ii)

$C + O_2 \rightarrow CO_2$, $\Delta H = -393$ kJ/mol ...(iii)

From Eq. (ii) × 2 + (iii) – Eq. (i)

$\Delta_f H_{(CH_3OH)} = -286 \times 2 - 393 - (-726)$

= – 239 kJ/mol

129. (c) Given, $K_C = 4 \times 10^{-6}$ mol/L

$(\Delta n = n_P - n_R \Rightarrow 3 - 2 = 1)$

$K_p = K_C \times (RT)^{\Delta n_g}$

$= 4 \times 10^{-6}\text{ mol L}^{-1} \times (0.083\text{ L bar k}^{-1}\text{ mol}^{-1} \times 1000\text{ K})^1$

$K_p = 3.32 \times 10^{-4}$ bar

130. (a) pH of the solution of salts of weak acid e.g. acetic acid and weak base e.g. dimethylamine can be calculated as

$\text{pH} = 7 + \frac{1}{2}[\text{p}K_a - \text{p}K_b]$

Here, $pK_a = 4.76$, $pK_b = 3.26$

$= 7 + \frac{1}{2}[4.76 - 3.26]$

$= 7 + \frac{1}{2}[1.50]$

$= 7 + 0.75 = 7.75$

131. (d) (i) $NaH + H_2O \xrightarrow{\Delta} NaOH + H_2$

(ii) Ammonia (NH_3) is electrons rich hydride due to presence of lone pair at N-atom

(i), (ii) both statements are correct.

(iii) Nickle cannot form saline hydride.

132. (b) (a) $4LiNO_3 \xrightarrow[\Delta]{} 2Li_2O + 4NO_2 + O_2$

(b) $2NaNO_3 \xrightarrow[\Delta]{} 2NaNO_2 + O_2$ (Thermal stable)

(c) $2Ba(NO_3)_2 \xrightarrow[\Delta]{} 2BaO + 4NO_2 + O_2$

(d) $2Be(NO_3)_2 \xrightarrow[\Delta]{} 2BeO + 4NO_2 + O_2$

$NaNO_3$ is thermally stable and does not gives its oxide.

133. (c) $2BF_3 + 6NaH \xrightarrow{450\text{ K}} 6NaF + \underset{(X)}{B_2H_6}$

$\underset{\text{Diborane}}{B_2H_6} + 2LiH \xrightarrow{\text{Diethyl ether}} \underset{\substack{(Y)\\ \text{Lithiumborohydride}}}{2LiBH_4}$

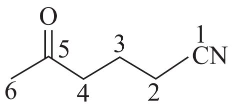
134. (b) Assertion and Reason both are correct but Reason (R) is not the correct explanation of (A).

SiF_6^{2-} is formed but $SiCl_6^{2-}$ is not because Si can accomodate six F^- due to its smaller size but it can not accomodate six Cl^- due to its larger size and steric hindrance.

Electronegativity of F is higher than Cl

135. (c) Hydrogen peroxide is now a days used for bleaching in the presence of catalyst the paper due to ecofriendly nature.

136. (b)



5-Oxohexanenitrile

137. (c) (i) Petrol and CNG operated automobiles causes less pollution.

(ii) Alkane having tertiary hydrogen therefore oxidise in presence of $KMnO_4$ into alcohol.

(iv) Alkyl halide on reduction with zinc and dilute HCl gives alkane.

$R - Cl \xrightarrow[\text{Alkyl Chloride Alkane}]{Zn + HCl} R - H$

138. (a) $CH_3—CH_2—CH_2=CH—CH_3 \longrightarrow$

Pent-2-ene

CH_3CH_2CH

(Y) Propanal

+

CH_3CH

(X) Ethanal

139. (d) 3 body centred lattices possible among the 14 Bravais lattices. Which are in Simple cubic Tetragonal Orthorhombic

140. (b) Given, $\pi = 3.735 \times 10^{-3}$ bar

Mass of $K_2SO_4 = 17.4$ mg

Molar mass (M) $H_2SO_4 = 174$ $i = 3.0$

Volume = 2L

From osmotic pressure of solution.

$\pi = iCRT$

i = van't-Hoff factor

$i = \pi/CRT$

$= \frac{\pi \times M \times V}{W \times R \times T} = \frac{3.735 \times 174 \times 2}{17.4 \times 0.083 \times 300}$

141. (b) Given,

Mass of solute (W) = 120 g

Molar mass of solute (M) = 60

Mass of solvent (w) = 1000 g

$\therefore$ Total mass of solution = 1000 + 120 = 1120 g

$\because$ $d = \frac{\text{Mass}}{\text{Volume }(V)}$

$\therefore$ Total volume (V) = $\frac{1120}{1.12} = 1000$ mL

$\because$ Molarity = $\frac{w}{M} \times \frac{1000}{V}$

$\because$ Molarity = $\frac{120}{60} \times \frac{1000}{1000} = 2.0$

142. (c) When an aqueous solution of $CuCl_2$ is electrolysed using pt inert electrodes. The chloride ion is oxidised to chlorine (Cl_2) at anode and Cu^{2+} ion is reduced to Cu at cathode.

At anode: $2Cl^- \xrightarrow{-2e^-} Cl_2(g)$

At cathode: $Cu^{2+} + 2e^- \longrightarrow Cu$

143. (b) Final quantity $(a - x) = (100 - 90) = 10$

Initial quantity (a) = 100%

$k = 4.606 \times 10^{-3} s^{-1}$

First order reaction

$k = \frac{2.303}{t} \log \frac{a}{(a - x)}$

$k = 4.606 \times 10^{-3}\ s^{-1}$

$t = \frac{2.303}{4.606 \times 10^{-3}} \log \frac{100}{10}$

$t = 500s$

144. (a) A mixture of N_2 and O_2 gases at room temperature form gaseous homogeneous mixture but do not form aerosol.

145. (b) In Ellingham diagram, the plot is drawn between temperature and $\Delta G°$.

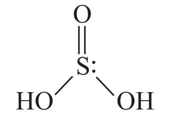
146. (c) (a) $2NaN_3 \xrightarrow{\Delta} 2Na + 3N_2$

(b) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} Cr_2O_3 + N_2 + 4H_2O$

(c) $2NH_4Cl + Ca(OH)_2 \longrightarrow CaCl_2 + 2NH_4OH$

(d) $Ba(N_3)_2 \longrightarrow Ba + 3N_2$

147. (d) Sulphur atom contains lone pair in only in sulphurous acid among the given acids.



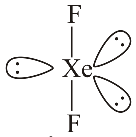
Sulphurous acid

148. (d) (d) Helium has lowest boiling point among all the noble gases. Due to minimum Vander Waal's force of attraction.

(a) $Xe(g) + 3F_2(g) \xrightarrow[60-70\,bar]{573\,k} XeF_6(s)$

(b) Ar used in electric bulbs.

(c) 3 lone pairs are present in XeF_2.



(sp^3d) Linear

149. (b) The relation between octahedral splitting energy (Δ_0) and tetrahedral splitting energy (Δ_t) is

$\Delta_t = \frac{4}{9}\Delta_0 \quad \therefore\ 9\Delta_t = 4\Delta_0$

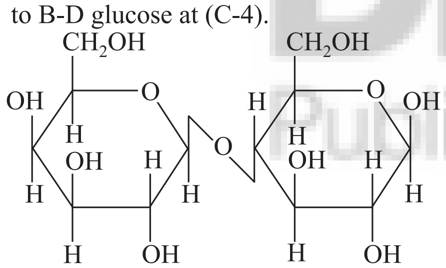
150. (b) Cerium exihibit + 4 oxidation state.

E.C. of Ce (58) = [Xe] $4f^1\ 5d^1\ 6s^2$

Ce^{+4} = [Xe] $4f^0\ 5d^0\ 6s^0$ (stable configuration)

151. (d) Alkyl lithium used as initiators for anionic polymerisation.

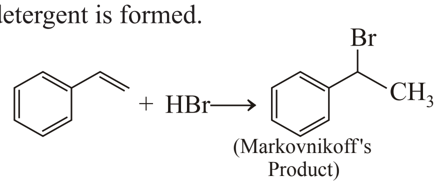
152. (c) Lactose is composed of B-D-galactose (C-1) linked to B-D glucose at (C-4).



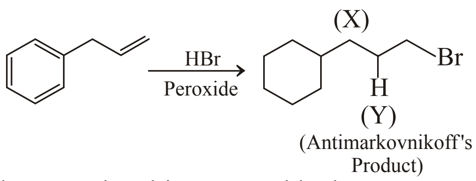
153. (c) (ii) Shape of receptor changes after attachment of chemical messenger.

(ii) When stearic acid reacts with polythene glycol non ionic detergent is formed.

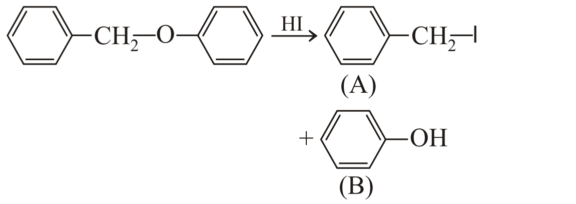
154. (c) (i)



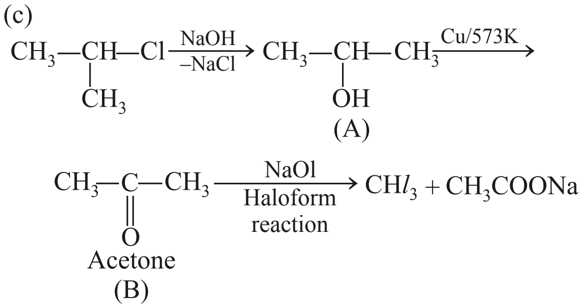
(ii)



155. (b) Aryl oxygen bond is more stable due to resonance. Hence bond will cleaved at benzyl oxygen bond which leads to the formation more stable benzyl carbocation that combine with I^- to form benzyl iodide.



156. (c)



157. (a) A–(iii), B–(iv), C–(ii), D–(i)

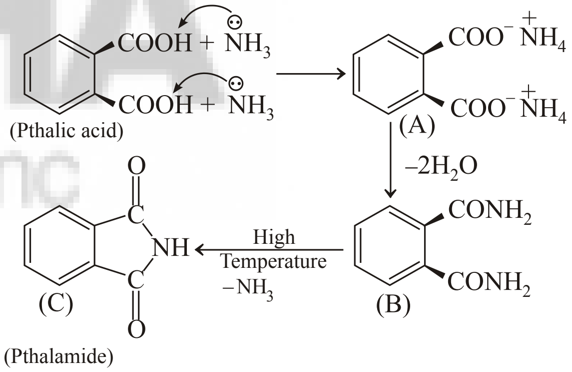
(A) Anhydrous $ZnCl_2$ + conc HCl is known as Lucas reagent and used to distinguish between primary, secondary, tertiary alcohols.

(B) Zn–Hg/HCl is called Clemmensen reagent. Thus this reagent is used in conversion of carbonyl into alkane i.e., Clemmensen reaction.

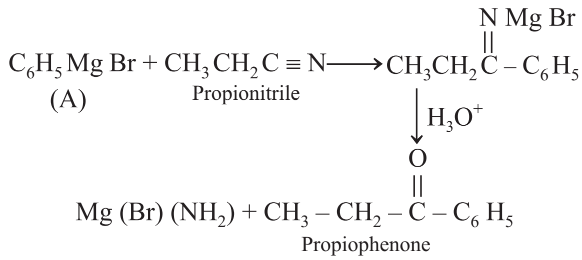
(C) Tollen's reagent $[Ag(NH_3)_2]^+$ used to distinguish aldehyde and ketones as oxidising reagent.

(D) Sephen reagent $SnCl_2$ + HCl used in reduction of nitrogen compounds.

158. (d)



159. (c)



160. (a) $C_2H_5{-}Cl \xrightarrow{KCN} \underset{(X)}{C_2H_5{-}CN} \xrightarrow{H_2/Catalyst}$

$\underset{(Y)}{CH_3{-}CH_2{-}CH_2{-}NH_2} \xrightarrow[\text{ak.KOH, Carbyl amine Test}]{CHCl_3}$

$\underset{(Z)}{CH_3{-}CH_2{-}CH_2{-}NC}$