# TS/EAMCET Solved Paper 2015

## INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

## MATHEMATICS

**1.** An equation of line whose segment between the coordinate axes is divided by the point $\left(\frac{1}{2}, \frac{1}{3}\right)$ in the ratio 2 : 3, is

(a) $6x + 9y = 5$ (b) $9x + 6y = 5$
(c) $4x + 9y = 5$ (d) $9x + 4y = 5$

**2.** The value of $k$ (> 0), such that the angle between the lines $4x - y + 7 = 0$ and $kx - 5y - 9 = 0$ is 45°, is

(a) $\frac{25}{3}$ (b) $\frac{5}{3}$ (c) 3 (d) 5

**3.** The combined equation of the straight lines passing through the point (4, 3) and each line making intercepts on the coordinate axes whose sum is –1, is

(a) $(3x - 2y - 6)(x - 2y + 2) = 0$
(b) $(3x - 2y + 6)(x - 2y + 2) = 0$
(c) $(3x - 2y - 6)(x - 2y - 2) = 0$
(d) $(3x - 2y + 6)(x - 2y - 2) = 0$

**4.** If the origin of a coordinate system is shifted to $(-\sqrt{2}, \sqrt{2})$ and the coordinate system is rotated anti-clockwise through an angle 45°, then the point $P(1, -1)$ in the original system has new coordinates

(a) $(\sqrt{2}, -2\sqrt{2})$ (b) $(0, -2\sqrt{2})$
(c) $(0, -2-\sqrt{2})$ (d) $(0, -2+\sqrt{2})$

**5.** The locus of the point $P$ which is equidistant from $3x + 4y + 5 = 0$ and $9x + 12y + 7 = 0$, is

(a) a hyperbola (b) an ellipse
(c) a parabola (d) a straight line

**6.** The probability of a coin showing head is $p$ and then 100 such coins are tossed. If the probability of 50 coins showing head is same as the probability of 51 coins showing head, then $p$ equals

(a) $\frac{1}{2}$ (b) $\frac{49}{100}$ (c) $\frac{51}{101}$ (d) $\frac{50}{101}$

**7.** Let $X$ be binomial variate with parameters $n = 6$ and $p$. If $4P(X = 4) = P(X = 2)$, then $p$ equals

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/6

**8.** In a certain college, 4% of men and 1% of women are taller than 1.8 m. Also, 60% of students are women. If a student selected at random is found to be taller than 1.8 m, then the probability that the student being a woman is

(a) 3/11 (b) 5/11 (c) 6/11 (d) 8/11

**9.** If $A$ and $B$ are two events such that $P(A/B) = 0.6$, $P(B/A) = 0.3$ and $P(A) = 0.1$, then $P(\overline{A} \cap \overline{B})$ equals

(a) 0.88 (b) 0.12 (c) 0.6 (d) 0.4

**10.** If $A$ and $B$ are two events such that

$P(A \cup B) = \frac{5}{6}$, $P(\overline{A}) = \frac{1}{4}$ and $P(B) = \frac{1}{3}$, then $A$ and $B$ are

(a) mutually exclusive
(b) independent events
(c) exhaustive events
(d) exhaustive and independent events

**11.** Two teams $A$ and $B$ have the same mean and their coefficients of variation are 4, 2, respectively. If $\sigma_A$, $\sigma_B$ are the standard deviations of teams $A$, $B$ respectively, then the relation between them is

(a) $\sigma_A = \sigma_B$ (b) $\sigma_B = 2\sigma_A$
(c) $\sigma_A = 2\sigma_B$ (d) $\sigma_B = 4\sigma_A$

**12.** In a data, if the number $i$ is repeated $i$ times for $i = 1, 2, \ldots, n$, then the mean of the data is

(a) $\frac{2n+1}{6}$ (b) $\frac{2n+1}{4}$ (c) $\frac{2n+1}{3}$ (d) $\frac{2n+1}{2}$

**13.** If $\mathbf{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}$, $\mathbf{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\mathbf{c} = 5\hat{i} - 3\hat{j} - 2\hat{k}$, then the volume of the parallelopiped with coterminous edges **a + b, b + c, c + a**, is

(a) 1 (b) 5 (c) 8 (d) 16

**14.** The shortest distance between the skew-lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$, $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$ is

(a) 6 (b) 7 (c) $3\sqrt{3}$ (d) $\sqrt{35}$

**15.** If the position vectors of the vertices of $\Delta ABC$ are $3\hat{i} + 4\hat{j} - \hat{k}$, $\hat{i} + 3\hat{j} + \hat{k}$ and $5(\hat{i} + \hat{j} + \hat{k})$, respectively. Then, the magnitude of the altitude from $A$ onto the side $BC$ is

(a) $\frac{4}{3}\sqrt{5}$ (b) $\frac{5}{3}\sqrt{5}$ (c) $\frac{7}{3}\sqrt{5}$ (d) $\frac{8}{3}\sqrt{5}$

**16.** $ABCD$ is a parallelogram and $P$ is a point on the segment $AD$ dividing it internally in the ratio 3 : 1. If the line $BP$ meets the diagonal $AC$ in $Q$, then $AQ : QC$ equals

(a) 3 : 4 (b) 4 : 3 (c) 3 : 2 (d) 2 : 3

**17.** If $M$ and $N$ are the mid-points of the sides $BC$ and $CD$ respectively of a parallelogram $ABCD$, then $AM + AN$ equals

(a) $\frac{4}{3}AC$ (b) $\frac{5}{3}AC$ (c) $\frac{3}{2}AC$ (d) $\frac{6}{5}AC$

**18.** $P$ is the point of intersection of the diagonals of the parallelogram $ABCD$. If $S$ is any point in space and $SA + SB + SC + SD = \lambda SP$, then $\lambda$ equals

(a) 2 (b) 4 (c) 6 (d) 8

**19.** In $\Delta ABC$, if $r_1 = 2r_2 = 3r_3$, then $b : c$ equals

(a) 4 : 3 (b) 5 : 4 (c) 2 : 1 (d) 3 : 2

**20.** $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2}$ equals

(a) $\frac{a^2+b^2+c^2}{\Delta}$ (b) $\frac{a^2+b^2+c^2}{\Delta^2}$

(c) $\frac{\Delta^2}{a^2+b^2+c^2}$ (d) $\frac{\Delta}{a^2+b^2+c^2}$

**21.** The angle of $\Delta ABC$ are in an arithmetic progression. If the larger sides $a, b$ satisfy the relation $\frac{\sqrt{3}}{2} < \frac{b}{a} < 1$, then the possible values of the smallest side are

(a) $\frac{a \pm \sqrt{4b^2 - 3a^2}}{2a}$ (b) $\frac{a \pm \sqrt{4b^2 - 3a^2}}{2b}$

(c) $\frac{a \pm \sqrt{4b^2 - 3a^2}}{2c}$ (d) $\frac{a \pm \sqrt{4b^2 - 3a^2}}{2}$

**22.** If cos h $2x$ = 199, then cot h $x$ equals

(a) $\frac{5}{3\sqrt{11}}$ (b) $\frac{5}{6\sqrt{11}}$ (c) $\frac{7}{3\sqrt{11}}$ (d) $\frac{10}{3\sqrt{11}}$

**23.** If $\cos\cot^{-1}\left(\frac{1}{2}\right) = \cot(\cos^{-1} x)$, then the value of $x$ is

(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{-1}{\sqrt{12}}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{-2}{\sqrt{6}}$

**24.** The number of solutions of sec $x$ cos $5x$ + 1 = 0 in the interval $[0, 2\pi]$ is

(a) 5 (b) 8 (c) 10 (d) 12

**25.** In $\Delta ABC$, if $\angle C = \frac{\pi}{3}$, then $\frac{3}{a+b+c} - \frac{1}{a+c}$ equals

(a) $\frac{1}{a+b}$ (b) $\frac{1}{b+c}$ (c) $\frac{1}{2a+b}$ (d) $\frac{1}{b+2c}$

**26.** If $A = \sin^2\theta + \cos^4\theta$, then for all values of $\theta$, $A$ lies in the interval

(a) [1, 2] (b) $\left[\frac{3}{4}, 1\right]$ (c) $\left[\frac{1}{2}, \frac{3}{4}\right]$ (d) $\left[\frac{3}{4}, \frac{19}{16}\right]$

**27.** If $f(x)$ is a real function defined on [– 1, 1], then the function $g(x) = f(5x + 4)$ is defined on the interval

(a) [– 4, 9] (b) [– 1, 9] (c) [– 2, 9] (d) [– 3, 9]

**28.** If $f : N \to R$ is defined by $f(1) = -1$ and $f(n + 1) = 3f(n) + 2$ for $n \geq 1$, then $f$ is

(a) one-one (b) onto

(c) a constant function (d) $f(n) > 0$ for $n > 1$

**29.** The remainder of $n^4 - 2n^3 - n^2 + 2n - 26$ when divided by 24, is

(a) 20 (b) 21 (c) 22 (d) 23

**30.** If $A(x) = \begin{vmatrix} 1 & 2 & 3 \\ x+1 & 2x+1 & 3x+1 \\ x^2+1 & 2x^2+1 & 3x^2+1 \end{vmatrix}$, then $\int A(x)\,dx$ equals

(a) 0 (b) 1 (c) 2 (d) 4

**31.** Let $\begin{vmatrix} x^2+x+1 & x+1 & 2x-3 \\ 3x^2-1 & x+2 & x-1 \\ x^2+5x+1 & 2x+3 & x+4 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$

be an identify in $x$. If $a, b, c, d$ are known, then the value of $e$ is

(a) 29 (b) 24 (c) 16 (d) 9

**32.** The system of equations $4x + y + 2z = 5$, $x - 5y + 3z = 10$, $9x - 3y + 7z = 20$ has

(a) no solution
(b) unique solution
(c) two solutions
(d) infinite number of solutions

**33.** If $1, \omega, \omega^2$ are the cube roots of unity and $\alpha = \omega + 2\omega^2 - 3$, then $\alpha^3 + 12\alpha^2 + 48\alpha + 3$ equals

(a) – 63 (b) – 62 (c) – 61 (d) – 60

**34.** If α, β are the roots of $1 + x + x^2 = 0$, then the value of $\alpha^4 + \beta^4 + \alpha^{-4}\beta^{-4}$ is

(a) 0 (b) 1 (c) –1 (d) 2

**35.** If α, β are the roots of the equation $x^2 - 4x + 8 = 0$, then for any $n \in N$, $\alpha^{2n} + \beta^{2n}$ equals

(a) $2^{2n+1}\cos\frac{n\pi}{2}$ (b) $2^{3n}\cos\frac{n\pi}{2}$

(c) $2^{3n+1}\cos\frac{n\pi}{2}$ (d) $2^{3n}\cos\frac{n\pi}{4}$

**36.** If α, β are non-real cube roots of 2, then $\alpha^6 + \beta^6$ equals

(a) 8 (b) 4 (c) 2 (d) 1

**37.** Let $\alpha \ne \beta$ satisfy $\alpha^2 + 1 = 6\alpha$, $\beta^2 + 1 = 6\beta$. Then, the quadratic equation whose roots are $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$, is

(a) $8x^2 + 8x + 1 = 0$ (b) $8x^2 - 8x - 1 = 0$

(c) $8x^2 - 8x + 1 = 0$ (d) $8x^2 + 8x - 1 = 0$

**38.** The solution set of $|x|^2 - 5|x| + 4 < 0$ is

(a) (– 4, – 1) (b) (1, 4)

(c) $(-4, -1) \cup (1, 4)$ (d) (– 4, 4)

**39.** Let α, β, γ be the roots of $x^3 + x + 10 = 0$ and $\alpha_1 = \frac{\alpha+\beta}{\gamma^2}, \beta_1 = \frac{\beta+\gamma}{\alpha^2}, \gamma_1 = \frac{\gamma+\alpha}{\beta^2}$. Then, the value of $(\alpha_1^3 + \beta_1^3 + \gamma_1^3) - \frac{1}{10}(\alpha_1^2 + \beta_1^2 + \gamma_1^2)$ is

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) $\frac{3}{10}$ (d) $\frac{1}{2}$

**40.** Suppose α, β, γ are the roots of $x^3 + x^2 + x + 2 = 0$. Then, the value of $\left(\frac{\alpha+\beta-2\gamma}{\gamma}\right)\left(\frac{\beta+\gamma-2\alpha}{\alpha}\right)\left(\frac{\gamma+\alpha-2\beta}{\beta}\right)$ is

(a) $-\frac{47}{2}$ (b) $\frac{47}{2}$

(c) – 47 (d) 47

**41.** $\sum_{r=0}^{10} {}^{40-r}C_5$ is equal to

(a) ${}^{41}C_5 - {}^{30}C_5$ (b) ${}^{41}C_6 - {}^{30}C_6$

(c) ${}^{41}C_5 + {}^{30}C_5$ (d) ${}^{41}C_6$

**42.** If the number of diagonals of a regular polygon is 35, then the number of sides of the polygon is

(a) 12 (b) 9 (c) 10 (d) 11

**43.** If $x = 1 + \frac{3}{1!}\times\frac{1}{6} + \frac{3\times7}{2!}\left(\frac{1}{6}\right)^2 + \frac{3\times7\times11}{3!}\left(\frac{1}{6}\right)^3 + ...$, then $x^4$ equals

(a) 81 (b) 54 (c) 27 (d) 8

**44.** If $x$ is so small that $x^2$ and higher powers of $x$ may be neglected, then the approximate value of $\frac{\left(1+\frac{2}{3}x\right)^{-3}(1-15x)^{-1/5}}{(2-3x)^4}$ is

(a) $\frac{1}{8}(1+7x)$ (b) $\frac{1}{16}(1-7x)$

(c) $1 - 7x$ (d) $\frac{1}{16}(1+7x)$

**45.** The coefficient of $x^n$ in the expansion of $\frac{1}{x^2-5x+6}$ for $|x| < 1$ is

(a) $\frac{1}{2^{n-1}} - \frac{1}{3^{n-1}}$ (b) $\frac{1}{2^{n+1}} - \frac{1}{3^{n+2}}$

(c) $\frac{1}{2^{n+1}} - \frac{1}{3^{n+1}}$ (d) $\frac{1}{2^n} - \frac{1}{3^n}$

**46.** In Δ$ABC$, the value of $\angle A$ is obtained from the equation $3\cos A + 2 = 0$. The quadratic equation, whose roots are $\sin A$ and $\tan A$, is

(a) $3x^2 + \sqrt{5}x - 5 = 0$ (b) $6x^2 - \sqrt{5}x - 5 = 0$

(c) $6x^2 + \sqrt{5}x - 5 = 0$ (d) $6x^2 + \sqrt{5}x + 5 = 0$

**47.** Match the differential equations in List-I to their integrating factors in List-II.

| List-I (Differential equation) | List-II (Integrating factor) |
|---|---|
| (P) $(x^3+1)\frac{dy}{dx} + x^2y = 3x^2$ | (1) $x^3$ |
| (Q) $x^2\frac{dy}{dx} + 3xy = x^6$ | (2) $(x^3 + 1)^2$ |
| (R) $(x^3+1)^2\frac{dy}{dx} + 6x^2(x^3+1)y = x^2$ | (3) $(x^2 + 1)^2$ |
| (S) $(x^2+1)\frac{dy}{dx} + 4xy = \ln x$ | (4) $x^2 + 1$ |
| | (5) $(x^3 + 1)^{1/3}$ |
| | (6) $(x^3 + 1)^{1/2}$ |

The correct answer is

| | P | Q | R | S | | P | Q | R | S |
|---|---|---|---|---|---|---|---|---|---|
| (a) | 4 | 1 | 2 | 3 | (b) | 5 | 1 | 2 | 3 |
| (c) | 5 | 2 | 3 | 6 | (d) | 5 | 1 | 3 | 4 |

**48.** The solution of the differential equation $xy' = 2xe^{-y/x} + y$ is

(a) $e^{y/x} + \log|Cx| = 0$

(b) $e^{-y/x} = x + C$

(c) $e^{y/x} = \log|Cx|$

(d) $e^{y/x} = 2\log|Cx|$

**49.** The differential equation of the family of curves $y = ax + \frac{1}{a}$, where $a(\neq 0)$ is an arbitrary constant, has the degree

(a) 4 (b) 3 (c) 1 (d) 2

**50.** The area of the region bounded by the curves $y = 9x^2$ and $y = 5x^2 + 4$ (in sq units) is

(a) 64 (b) $\frac{64}{3}$ (c) $\frac{32}{3}$ (d) $\frac{16}{3}$

**51.** $\int_0^{\pi/2} \frac{16x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ is equal to

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) $\pi^2$ (d) $2\pi^2$

**52.** $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$ is equal to

(a) $\frac{\pi}{2} - 1$ (b) $\frac{\pi}{2} + 1$

(c) $\pi - 1$ (d) $\frac{3\pi}{2}$

**53.** If $\int \frac{x+5}{x^2+4x+5} dx = a \log(x^2+4x+5)$ $+ b \tan^{-1}(x+k) + C$, then $(a, b, k)$ equals

(a) $\left(\frac{1}{2}, 3, 2\right)$ (b) $\left(\frac{1}{2}, 1, 2\right)$

(c) $\left(\frac{1}{2}, 3, 1\right)$ (d) $(1, 3, 2)$

**54.** $\int \sqrt{e^x - 4}\, dx$ equals

(a) $\tan^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right) + \sqrt{e^x-4} + C$

(b) $2\sqrt{e^x-4} - 4\tan^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right) + C$

(c) $2\sqrt{e^x-4} - 4\cot^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right) + C$

(d) $\sqrt{e^x-4} - 4\tan^{-1}(\sqrt{e^x-4}) + C$

**55.** If $\int e^{-x} \tan^{-1}(e^x)\, dx = f(x) - \frac{1}{2}\log(1+e^{2x}) + C$, then $f(x)$ equals

(a) $e^x - e^{-x}\tan^{-1}(e^x)$ (b) $x^2 + e^{-x}\tan^{-1}(e^x)$

(c) $-e^{-x}\tan^{-1}(e^x)$ (d) $x - e^{-x}\tan^{-1}(e^x)$

**56.** $\int \sqrt{\frac{2+x}{2-x}} dx$ is equal to

(a) $2\sin^{-1}\left(\frac{x}{2}\right) + \sqrt{4-x^2} + C$

(b) $\cos^{-1}\left(\frac{x}{2}\right) - \sqrt{4-x^2} + C$

(c) $\sin^{-1}\left(\frac{x}{2}\right) - \sqrt{4-x^2} + C$

(d) $2\sin^{-1}\left(\frac{x}{2}\right) - \sqrt{4-x^2} + C$

**57.** Two particles $P$ and $Q$ located at the points with coordinates $P(t, t^3 - 16t - 3)$, $Q(t+1, t^3 - 6t - 6)$ are moving in a plane. The minimum distance between them in their motion is

(a) 1 (b) 5 (c) 169 (d) 49

**58.** If $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 \le x \le 2 \end{cases}$, then Rolle's theorem is not applicable to $f(x)$ because

(a) $f(x)$ is not defined everywhere on [0, 2]

(b) $f(x)$ is not continuous on [0, 2]

(c) $f(x)$ is not differentiable on (1, 2)

(d) $f(x)$ is not differentiable on (0, 2)

**59.** An equilateral triangle is of side 10 units. In measuring the side, if an error of 0.05 unit is made. Then, the percentage error in the area of the triangle is

(a) 5 (b) 4 (c) 1 (d) 0.5

**60.** If the line $y = -4x + b$ is tangent to the curve $y = \frac{1}{x}$, then $b$ equals

(a) $\pm 4$ (b) $\pm 2$ (c) $\pm 1$ (d) $\pm 8$

**61.** If $x = \frac{1-\sqrt{y}}{1+\sqrt{y}}$, then $(x+1)\frac{d^2y}{dx^2} + \left(\frac{3\sqrt{y}+1}{\sqrt{y}}\right)\frac{dy}{dx}$ equals

(a) $-2y$ (b) 0 (c) $-y$ (d) $y$

**62.** If $x^2 + y^2 = t + \frac{2}{t}$ and $x^4 + y^4 = t^2 + \frac{4}{t^2}$, then $x^3 y \frac{dy}{dx}$ equals

(a) $-1$ (b) $-2$ (c) $\frac{y}{x}$ (d) $xy$

**63.** If $y = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right) + \tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{2}{1+x^2}$ (b) $\frac{4}{1+x^2}$ (c) $\frac{6}{1+x^2}$ (d) $\frac{7}{1+x^2}$

**64.** The value that should be assigned to $f(0)$ so that the function $f(x) = (x+1)^{\cot x}$ is continuous at $x = 0$, is

(a) $e$ (b) 1 (c) 2 (d) $e^{-1}$

**65.** $\lim\limits_{x\to 0}\left[\tan\left(x+\frac{\pi}{4}\right)\right]^{1/x}$ is equal to

(a) $e^2$ (b) $e$ (c) $e^{3/2}$ (d) $e^{-1}$

**66.** A plane meets the coordinate axes at $P, Q, R$ respectively. If the centroid of $\Delta PQR$ is $\left(1, \frac{1}{2}, \frac{1}{3}\right)$, then the equation of the plane is

(a) $2x + 4y + 3z = 5$ (b) $x + 2y + 3z = 3$

(c) $x + 4y + 6z = 5$ (d) $2x - 2y + 6z = 3$

**67.** If the extremities of a diagonal of a square are (1, 2, 3) and (2, – 3, 5), then its side is of length

(a) $\sqrt{6}$ (b) 15 (c) $\sqrt{15}$ (d) 3

**68.** If $A(4, 3, 5)$, $B(0, -2, 2)$ and $C(3, 2, 1)$ are three points. Then, the coordinates of the point in which the bisector of $\angle BAC$ meets the side $BC$, is

(a) $\left(\frac{15}{8}, \frac{4}{8}, \frac{11}{8}\right)$ (b) $\left(\frac{12}{7}, \frac{2}{7}, \frac{10}{7}\right)$

(c) $\left(\frac{9}{5}, \frac{2}{5}, \frac{7}{5}\right)$ (d) $\left(\frac{3}{2}, 0, \frac{3}{2}\right)$

**69.** The product of lengths of perpendicular from any point on the hyperbola $x^2 - y^2 = 16$ to its asymptotes, is

(a) 2 (b) 4 (c) 8 (d) 16

**70.** The centre of the ellipse $\frac{(x+y-3)^2}{9}+\frac{(x-y+1)^2}{16}=1,$ is

(a) (– 1, 2) (b) (1, – 2) (c) (–1, –2) (d) (1, 2)

**71.** For the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1,$ a list of lines given in List-I are to be matched with their equation given in List-II.

| **List-I (Line)** | **List-II (Equation)** |
|---|---|
| (P) Directrix corresponding to the focus (–3, 0) | (1) $y = 4$ |
| (Q) Tangent at the vertex (0, 4) | (2) $3x = 25$ |
| (R) Latusrectum through (3, 0) | (3) $x = 3$ |
| | (4) $y + 4 = 0$ |
| | (5) $x + 3 = 0$ |
| | (6) $3x + 25 = 0$ |

The correct answer is

| | P | Q | R | | P | Q | R |
|---|---|---|---|---|---|---|---|
| (a) | 2 | 1 | 5 | (b) | 6 | 1 | 3 |
| (c) | 2 | 4 | 3 | (d) | 6 | 1 | 5 |

**72.** If $P$ is a point on the parabola $y^2 = 8x$ and $A$ is the point (1, 0), then the locus of the mid-point of the line segment $AP$ is

(a) $y^2 = 4\left(x-\frac{1}{2}\right)$ (b) $y^2 = 2(2x + 1)$

(c) $y^2 = x-\frac{1}{2}$ (d) $y^2 = 2x + 1$

**73.** The equation of the parabola with focus (1, –1) and directrix $x + y + 3 = 0$, is

(a) $x^2+y^2-10x-2y-2xy-5=0$

(b) $x^2+y^2+10x-2y-2xy-5=0$

(c) $x^2+y^2+10x+2y-2xy-5=0$

(d) $x^2+y^2+10x+2y+2xy-5=0$

**74.** The equation of the circle passing through (1, 2) and the points of intersection of the circles $x^2+y^2-8x-6y+21=0$ and $x^2+y^2-2x-15=0,$ is

(a) $x^2+y^2+6x-2y+9=0$

(b) $x^2+y^2-6x-2y+9=0$

(c) $x^2+y^2-6x-4y+9=0$

(d) $x^2+y^2-6x+4y+9=0$

**75.** The length of the common chord of the two circles $(x - a)^2 + y^2 = a^2$ and $x^2 + (y - b)^2 = b^2$, is

(a) $\frac{ab}{\sqrt{a^2+b^2}}$ (b) $\frac{2ab}{\sqrt{a^2+b^2}}$

(c) $\frac{a+b}{\sqrt{a^2+b^2}}$ (d) $\sqrt{a^2+b^2}$

**76.** If (4, 2) and ($k$, –3) are conjugate points with respect to $x^2+y^2-5x+8y+6=0,$ then $k$ equals

(a) $\frac{28}{3}$ (b) $-\frac{28}{3}$

(c) $\frac{3}{28}$ (d) $-\frac{3}{28}$

**77.** The area (in sq units) of the triangle formed by the tangent, normal at $(1, \sqrt{3})$ to the circle $x^2 + y^2 = 4$ and the $X$-axis, is

(a) $4\sqrt{3}$ (b) $\frac{7}{2}\sqrt{3}$

(c) $2\sqrt{3}$ (d) $\frac{1}{2}\sqrt{3}$

**78.** The value of $a$, such that the power of the point (1, 6) with respect to the circle $x^2+y^2+4x-6y-a=0,$ is –16, is

(a) 7 (b) 11 (c) 13 (d) 21

**79.** The line $x + y = k$ meets the pair of straight lines $x^2+y^2-2x-4y+2=0$ at two points $A$ and $B$. If $O$ is the origin and $\angle AOB = 90°$, then the value of $k$ (>1) is

(a) 5 (b) 4 (c) 3 (d) 2

**80.** If two pairs of straight lines with combined equations $xy+4x-3y-12=0$ and $xy-3x+4y-12=0$ form a square. Then, the combined equation of its diagonals is

(a) $x^2-2xy+y^2+x-y=0$

(b) $x^2+2xy+y^2+x+y=0$

(c) $x^2-y^2+x-y=0$

(d) $x^2-y^2+x+y=0$

## PHYSICS

**81.** The moment of inertia of a solid cylinder of mass $M$, length $2R$ and radius $R$ about an axis passing through the centre of mass and perpendicular to the axis of the cylinder is $I_1$ and about an axis passing through one end of the cylinder and perpendicular to the axis of the cylinder is $I_2$, then

(a) $I_2 < I_1$ (b) $I_2 - I_1 = MR^2$

(c) $\dfrac{I_2}{I_1}=\dfrac{19}{12}$ (d) $\dfrac{I_2}{I_1}=\dfrac{7}{6}$

**82.** A body mass of 1 kg, initially at rest explodes and breaks into three parts. The masses of the parts are in the ratio 1 : 1 : 3. The two pieces of equal mass fly off perpendicular to each other with a speed of 30 m/s each. The velocity of the heavier part in m/s is

(a) $10\sqrt{2}$ (b) 6 (c) 3 (d) $6\sqrt{2}$

**83.** A particle of mass 4 kg is executing SHM. Its displacement is given by the equation $y=8,\cos[100t+\pi/4]$ cm. Its maximum kinetic energy is

(a) 128 J (b) 64 J (c) 16 J (d) 32 J

**84.** Infinite number of spheres, each of mass $m$ are placed on the $X$-axis at distances 1, 2, 4, 8, 16, ..... metres from origin. The magnitude of the gravitational field at the origin is

(a) $\dfrac{2}{3}Gm$ (b) $\dfrac{4}{3}Gm$ (c) $Gm$ (d) $6\,Gm$

**85.** When a force $F_1$ is applied on a metallic wire, the length of the wire is $L_1$. If a force $F_2$ is applied on the same wire, the length of the wire is $L_2$. The original length of the wire $L$ is

(a) $\dfrac{L_1F_1+L_2F_2}{F_1+F_2}$ (b) $\dfrac{L_2-L_1}{F_1+F_2}$

(c) $\dfrac{F_1L_2-F_2L_1}{F_1-F_2}$ (d) $\dfrac{F_1L_1-F_2L_2}{F_1-F_2}$

**86.** 1000 spherical drops of water each $10^{-8}$ m in diameter coalesce to form one large spherical drop. The amount of energy liberated in this process (in joule) is (surface tension of water is 0.075 N/m).

(a) $10.75\,\pi\times10^{-15}$ (b) $6.75\,\pi\times10^{-15}$

(c) $8.65\,\pi\times10^{-15}$ (d) $3.88\,\pi\times10^{-15}$

**87.** A thermos flask contains 250 g of coffee at 90 °C. To this 20 g of milk at 5 °C is added. After equilibrium is established, the temperature of the liquid is
(Assume no heat loss to the thermos bottle. Take specific heat of coffee and milk as 1.00 cal/ g °C)

(a) 3.23 °C (b) 3.15 °C (c) 83.7 °C (d) 37.8 °C

**88.** A copper rod of length 75 cm and an iron rod of length 125 cm are joined together end to end. Both are of circular cross-section with diameter 2 cm. The free ends of the copper and iron are maintained at 100 °C and 0 °C respectively. The surfaces of the bars are insulated thermally. The temperature of the copper-iron junction is [Thermal conductivity of copper is 386.4 W/m-K and thermal conductivity of iron is 48.46 W/m-K]

(a) 100 °C (b) 0 °C (c) 93 °C (d) 50 °C

**89.** 1 g of water at 100 °C is completely converted into stream at 100 °C. 1 g of steam occupies a volume of 1650 cc. (Neglect the volume of 1 g of water at 100 °C). At the pressure of $10^5$ N/m$^2$, 1atent heat of steam is 540 cals/g (1 calorie = 4.2 joule). The increase in the internal energy (in joule) is

(a) 2310 (b) 2103 (c) 1650 (d) 2150

**90.** RMS velocity of oxygen molecules at NTP is 0.5 km/s. The RMS velocity for the hydrogen molecule at NTP is

(a) 4 km/s (b) 2 km/s (c) 3 km/s (d) 1 km/s

**91.** A thin wire of length of 99 cm is fixed at both ends as shown in the figure. The wire is kept under a tension and is divided into three segments of lengths $l_1$, $l_2$ and $l_3$ as shown in figure. When the wire is made to vibrate, the segments vibrate respectively with their fundamental frequencies in the ratio 1 : 2 : 3. Then, the lengths $l_1$, $l_2$ and $l_3$ of the segments respectively are (in cm)


(a) 27, 54, 18 (b) 18, 27, 54

(c) 54, 27, 18 (d) 27, 9, 14

**92.** Three thin lenses are combined by placing them in contact with each other to get more magnification in an optical instrument. Each lens has a focal length of 3 cm. If the least distance of distinct vision is taken as 25 cm, the total magnification of the lens combination in normal adjustment is

(a) 9 (b) 26 (c) 300 (d) 3

**93.** A convex lens of glass ($\mu_g = 1.45$) has a focal length $f_a$ in air. The lens is immersed in a liquid of refractive index ($\mu_l = 1.3$). The ratio of the $f_l/f_a$ is

(a) 3.9 (b) 0.23 (c) 0.43 (d) 0.39

**94.** Through a narrow slit of width 2 mm, diffraction pattern is formed on a screen kept at a distance 2 m from the slit. The wavelength of the light used is 6330 Å and falls normal to the slit and screen. Then, the distance between the two minima on either side of the central maximum is

(a) 12.6 mm (b) 1.27 mm

(c) 2.532 mm (d) 25.3 mm

**95.** Charges $Q$ are placed at the ends of a diagonal of a square and charges $q$ are placed at the other two corners. The condition for the net electric force on $Q$ to be zero is

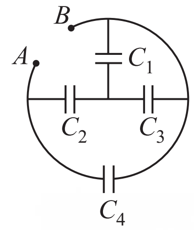
(a) $Q = -2\sqrt{2}q,\ q$ being negative

(b) $Q = -\dfrac{q}{2},\ q$ being negative

(c) $Q = 2\sqrt{2}q,\ q$ being negative

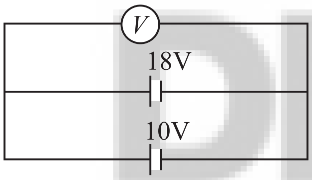
(d) $Q = 2q,\ q$ being negative

**96.** In the arrangement of capacitors shown in the figure, if each capacitor is 9 pF, then the effective capacitance between the points $A$ and $B$ is




(a) 10 pF (b) 15 pF (c) 20 pF (d) 5 pF

**97.** A battery of the emf 18 V and internal resistance of 3Ω and another battery of emf 10 V and internal resistance of 1 Ω are connected as shown in figure. Then, the voltmeter reading is




(a) 10V (b) 12V (c) 16V (d) 8V

**98.** A wire of aluminium and a wire of germanium are cooled to a temperature of 77 K. Then

(a) resistance of each of them decreases

(b) resistance of each of them increases

(c) resistance of aluminium wire increases and that of germanium wire decreases

(d) resistance of aluminium wire decreases and that of germanium wire increases

**99.** A voltmeter of 250 mV range having a resistance of 10Ω is converted into an ammeter of 250 mA range. The value of necessary shunt is (nearly)

(a) 2Ω (b) 0.1Ω (c) 1Ω (d) 10Ω

**100.** A circular loop and a square loop are formed from two wires of same length and cross section. Same current is passed through them. Then, the ratio of their dipole moments is

(a) 4 (b) $\dfrac{2}{\pi}$ (c) 2 (d) $\dfrac{4}{\pi}$

**101.** At a certain place, a magnet makes 30 oscillations per minute. At another place where the magnetic field is doubled, its time period will be

(a) $\sqrt{2}$ s (b) 2 s (c) 4 s (d) 1/2 s

**102.** Match the following :

| Column-I | Column-II |
|---|---|
| (A) Rocket propulsion | (P) Bernoulli's principle in fluid dynamics |
| (B) Aeroplane | (Q) Total internal reflection of light |
| (C) Optical fibres | (R) Newton's laws of motion |
| (D) Fusion test reactor | (S) magnetic confinement of plasma |
| | (T) Photoelectric effect |

The correct answer is

| | A | B | C | D | | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| (a) | R | Q | P | S | (b) | R | P | Q | T |
| (c) | T | P | Q | R | (d) | R | P | Q | S |

**103.** Force $F$ is given by the equation $F = \dfrac{X}{\text{Linear density}}$. Then dimensions of $X$ are

(a) $[M^2L^0T^{-2}]$ (b) $[M^0L^0T^{-1}]$

(c) $[L^2T^{-2}]$ (d) $[M^0L^2T^{-2}]$

**104.** The displacement of a particle moving in a straight line is given by the expression $x = At^3 + Bt^2 + Ct + D$ in metres, where $t$ is in seconds and $A, B, C$ and $D$ are constants. The ratio between the initial acceleration and initial velocity is

(a) $\dfrac{2C}{B}$ (b) $\dfrac{2B}{C}$

(c) $2C$ (d) $\dfrac{C}{2B}$

**105.** $A, B, C$ are points in a vertical line such that $AB = BC$. If a body falls freely from rest at $A$, $t_1$ and $t_2$ are times taken to travel distances $AB$ and $BC$, then ratio $(t_2/t_1)$ is

(a) $\sqrt{2}+1$ (b) $\sqrt{2}-1$

(c) $2\sqrt{2}$ (d) $\dfrac{1}{\sqrt{2}+1}$

**106.** Sum of magnitude of two forces is 25 N. The resultant of these forces is normal to the smaller force and has a magnitude of 10 N. Then the two forces are

(a) 14.5 N, 10.5 N (b) 16 N, 9 N

(c) 13 N, 12 N (d) 20 N, 5 N

**107.** A body of mass $m$ thrown up vertically with velocity $v_1$ reaches a maximum height $h_1$ in $t_1$ seconds. Another body of mass 2 $m$ is projected with a velocity $v_2$ at an angle θ. The second body reaches a maximum height $h_2$ in time $t_2$ seconds. If $t_1 = 2t_2$, then ratio $\left(\dfrac{h_1}{h_2}\right)$ is

(a) 1 : 2 (b) 4 : 1

(c) 1 : 1 (d) 3 : 2

**108.** Hammer of mass $M$ strikes a nail of mass $m$ with a velocity 20 m/s into a fixed wall. The nail penetrates into the wall to a depth of 1 cm. The average resistance of the wall to the penetration of the nail is

(a) $\left(\dfrac{M^2}{M+m}\right)\times 10^3$ (b) $\dfrac{2M^2}{M+m}\times 10^4$

(c) $\dfrac{M+m}{M^2}\times 10^2$ (d) $\dfrac{M^2}{M+m}\times 10^2$

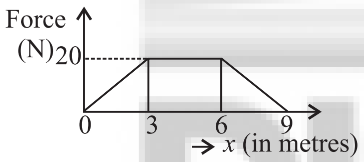
**109.** A body of mass 10 kg is acted upon by a force given by equation $F = (3t^2 - 30)$ newtons. The initial velocity of the body is 10 m/s. The velocity of the body after 5 s is

(a) 4.5 m/s (b) 6 m/s (c) 7.5 m/s (d) 5 m/s

**110.** A ball (initially at rest) is released from the top of tower. The ratio of work done by the force of gravity, in the first, second and third seconds is

(a) 1 : 3 : 5 (b) 1 : 4 : 16 (c) 1 : 9 : 25 (d) 1 : 2 : 3

**111.** A body of mass 2.4 kg is subjected to a force which varies with distance as shown in figure. The body starts from rest at $x = 0$. Its velocity at $x = 9$ m is




(a) $5\sqrt{3}$ m/s (b) $20\sqrt{3}$ m/s

(c) 10 m/s (d) 40 m/s

**112.** A carrier wave of peak voltage 12 V is used to transmit a signal. If the modulation index is 75%, the peak voltage of the modulating signal is

(a) 18 V (b) 22 V (c) 9 V (d) 28 V

**113.** If $n_e$ and $n_h$ are electron and hole concentrations in an extrinsic semiconductor and $n_i$ is electron concentrations in an intrinsic semiconductor, then

(a) $\left(\dfrac{n_e}{n_h}\right) = n_i$ (b) $(n_e + n_h) = n_i$

(c) $(n_e - n_h) = n_i^2$ (d) $n_e n_h = n_i^2$

**114.** In a half wave rectifier, the AC input source of frequency 50 Hz is used. The fundamental frequency of the output is

(a) 50 Hz (b) 150 Hz

(c) 200 Hz (d) 75 Hz

**115.** A radioactive nucleus can decay by two different processes. The half lives of the first and second decay processes are $5 \times 10^3$ and $10^5$ years respectively. Then, the effective half-life of the nucleus is

(a) $105 \times 10^5$ yr (b) 4762 yr

(c) $10^4$ yr (d) 47.6 yr

**116.** The following statements are given about hydrogen atom.

(A) The wavelengths of the spectral lines of Lyman series are greater than the wavelength of the second spectral line of Balmer series.

(B) The orbits correspond to circular standing waves in which the circumference of the orbit equals a whole number of wavelengths.

(a) A is false, B is true (b) A is true, B is false

(c) A is false, B is false (d) A is true, B is true

**117.** If an electron has an energy such that its de-Broglie wavelength is 5500 Å, then the energy value of that electron is ($h = 6.6 \times 10^{-34}$ Js, $m_e = 9.1 \times 10^{-31}$ kg)

(a) $8 \times 10^{-20}$ J (b) $8 \times 10^{-10}$ J

(c) 8 J (d) $8 \times 10^{-25}$ J

**118.** Suppose that the electric flux inside a parallel plate capacitor changes at a rate of $7 \times 10^{14}$ unit/s, then the magnetic induction field density at any point inside the capacitor is

[Area of the plate of the capacitor = 1 m$^2$
Permittivity of free space = $8.8 \times 10^{-12}$ Nm$^2$C$^{-2}$
Permeability of free space = $4\pi \times 10^{-7}$ telsa m/amp]

(a) $7.79 \times 10^{-3}$ T (b) $2 \times 10^{-3}$ T

(c) $8.85 \times 10^{-4}$ T (d) $88.5 \times 10^{-12}$ T

**119.** In a circuit $L$, $C$ and $R$ are connected in series with an alternating voltage source of frequency $f$. When current in the circuit leads the voltage by 45°, the value of $C$ is

(a) $\dfrac{1}{2\pi f(2\pi fL + R)}$ (b) $\dfrac{1}{2\pi f(2\pi fR + L)}$

(c) $\dfrac{2}{2\pi f(R+L)}$ (d) $\dfrac{2}{2\pi f\left(R+\dfrac{1}{L}\right)}$

**120.** A small square loop of wire of side $l$ is placed inside a large square loop of side $L$ ($L > l$). If the loops are coplanar and their centres coincide, the mutual induction of the system is directly proportional to

(a) $l/L$ (b) $l^2/L$

(c) $l/L^2$ (d) $l^2/L^2$

## CHEMISTRY

**121.** When one mole of $A$ and one mole of $B$ were heated in one litre flask at $T$(K), 0.5 mole of $C$ was formed in the equilibrium,

$$A + B \rightleftharpoons C + D$$

the equilibrium constant, $K_C$ is

(a) 0.25 (b) 0.5 (c) 1 (d) 2

**122.** If the solubility of $Ca_3(PO_4)_2$ in water is $x$ mol L$^{-1}$, its solubility product in mol L$^{-1}$ is

(a) $6x^5$ (b) $36x^5$ (c) $64x^5$ (d) $108x^5$

**123.** Which one of the following is not a method to remove permanent hardness of water?
(a) Clark's method
(b) Calgon method
(c) Ion-exchange method
(d) Synthetic resins method

**124.** White metal is an alloy of
(a) Na and Mg (b) Na and Pb
(c) Li and Mg (d) Li and Pb

**125.** Which one of the following elements does not form triiodide on reacting with iodine?
(a) B (b) Tl (c) Al (d) Ga

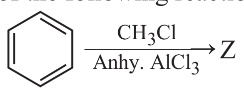
**126.** The buffer system which helps to maintain the pH of blood between 7.26 to 7.42, is
(a) $H_2CO_3 / HCO_3^-$
(b) $NH_4OH / NH_4Cl$
(c) $CH_3COOH / CH_3COO^-$
(d) $CH_3COONH_4$

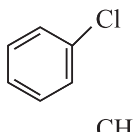
**127.** In municipal sewage, BOD values (in ppm) are
(a) 1-5 (b) 100-4000
(c) 50-90 (d) 20-40

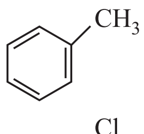
**128.** Two bonds N = O and N — O in $H_3CNO_2$ are of same bond length due to
(a) inductive effect (b) hyperconjugation
(c) electromeric effect (d) resonance effect

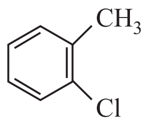
**129.** **Assertion :** Reaction of 1-butene with HBr gives 1-bromobutane as major product.
**Reason :** Addition of hydrogen halides to alkenes proceeds according to Markownikoff's rule.
The correct answer is
(a) A and R are correct and R is the correct explanation of A
(b) A and R are correct but R is not the correct explanation of A
(c) A is correct but R is not correct
(d) A is not correct but R is correct

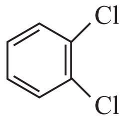
**130.** The product ($Z$) of the following reaction is




(a) 


(b) 


(c) 


(d) 


**131.** An example of covalent solid is
(a) MgO (b) Mg (c) SiC (d) $CaF_2$

**132.** What is the weight (in gram) of $Na_2CO_3$ (molar mass = 106) present in 250 mL of its 0.2 M solution?
(a) 0.53 (b) 5.3 (c) 1.06 (d) 10.6

**133.** An aqueous dilute solution containing non-volatile solute boils at 100.52 °C. What is the molality of solution? ($K_b$ = 0.52 kg mol$^{-1}$K, boiling temperature of water = 100 °C)
(a) 0.1 m (b) 0.01 m (c) 0.001 m (d) 1.0 m

**134.** A lead storage battery is discharged. During the charging of this battery, the reaction that occurs at anode, is
(a) $PbSO_4(s) + 2e^- \longrightarrow Pb(s) + SO_4^{2-}(aq)$
(b) $PbSO_4(s) + 2H_2O(l) \longrightarrow PbO_2(s) + SO_4^{2-}(aq) + 4H^+(aq) + 2e^-$
(c) $PbSO_4(s) \longrightarrow Pb^{2+}(aq) + SO_4^{2-}(aq)$
(d) $PbSO_4(s) + 2H_2O(l) + 2e^- \longrightarrow PbO_2(s) + SO_4^{2-}(aq) + 2H^+(aq)$

**135.** For the reaction,
$$5Br^-(aq) + BrO_3^-(aq) + 6H^+(aq) \longrightarrow 3Br_2(aq) + 3H_2O(l)$$
If, $-\dfrac{\Delta[Br^-]}{\Delta t} = 0.05$ mol L$^{-1}$ min$^{-1}$, $-\dfrac{\Delta[BrO_3^-]}{\Delta t}$ in mol L$^{-1}$ min$^{-1}$ is
(a) 0.005 (b) 0.05 (c) 0.5 (d) 0.01

**136.** Which one of the following is used in the hardening of leather?
(a) Light sensitive silver bromide in gelatin
(b) Sodium lauryl sulphate
(c) Alum
(d) Tannin

**137.** German silver contains which of the following metals?
(a) Cu, Zn (b) Fe, Zn
(c) Zn, Fe, Ni (d) Cu, Zn, Ni

**138.** The key step in the manufacturing of $H_2SO_4$ by contact process is
(a) absorption of $SO_3$ in $H_2SO_4$ to give oleum
(b) dilution of oleum with water
(c) burning of sulphur in air to generate $SO_2$
(d) catalytic oxidation of $SO_2$ with $O_2$ to give $SO_3$

**139.** Ammonia on reaction with chlorine forms an explosive $NCl_3$. What is the mole ratio of $NH_3$ and $Cl_2$ required for this reaction?
(a) 8 : 3 (b) 1 : 1 (c) 1 : 3 (d) 10 : 1

**140.** Which one of the following lanthanide ions does not exhibit paramagnetism?
(a) $Lu^{3+}$ (b) $Ce^{3+}$ (c) $Eu^{3+}$ (d) $Yb^{3+}$

**141.** The increasing order of field strength of ligands is

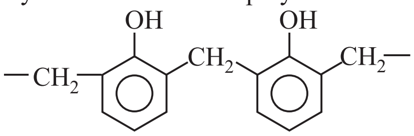
(a) $NH_3 < H_2O < Cl^- < CO < CN^-$

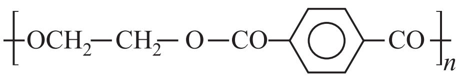
(b) $Cl^- < H_2O < NH_3 < CN^- < CO$

(c) $Cl^- < CO < CN^- < H_2O < NH_3$

(d) $CN^- < CO < NH_3 < Cl^- < H_2O$

**142.** Identify condensation homopolymer from the following :

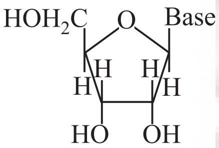
(a) 

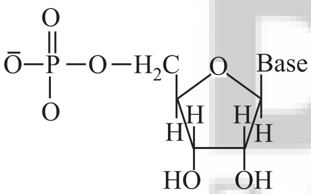
(b) 

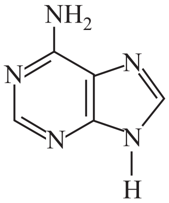
(c) $\left[CO-(CH_2)_5-NH\right]_n$

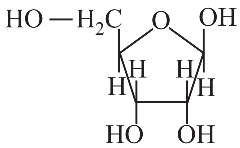
(d) $\left[NH-(CH_2)_6-NH-CO-(CH_2)_4-CO\right]_n$

**143.** Identify the nucleoside from the following :

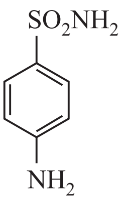
(a) 

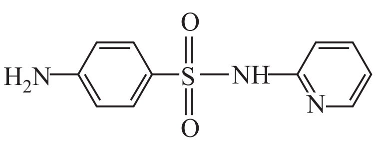
(b) 

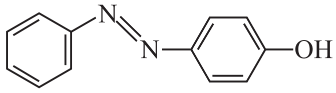
(c) 

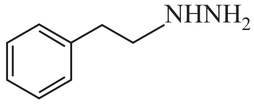
(d) 

**144.** Which one of the following is the correct structure of sulphapyridine?

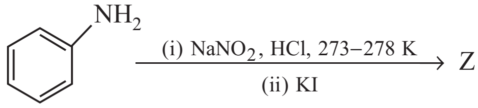
(a) 

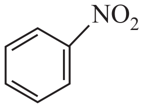
(b) 

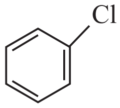
(c) 

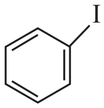
(d) 

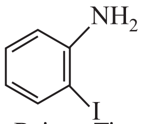
**145.** Identify $Z$ in the following reaction :



(a) 

(b) 

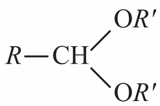
(c) 

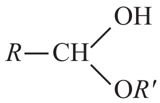
(d) 

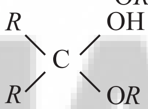
**146.** Which intermediate is formed in the Reimer-Tiemann reaction?

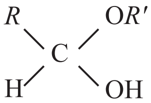
(a) Aldehyde
(b) Carbocation
(c) Carbanion
(d) Substituted benzal chloride

**147.** Which one of the following is an acetal?

(a) 

(b) 

(c) 

(d) 

**148.** $H_3CCH_2CO_2H \xrightarrow[\Delta]{P_2O_5} X \xrightarrow{H_2O} Y \xrightarrow{SOCl_2} Z$

Identify $X$, $Y$ and $Z$.

| | *X* | *Y* | *Z* |
|---|---|---|---|
| (a) | $H_2C{=}CHCO_2H$ | $HOH_2CCHOH$ $CO_2H$ | $HOH_2CCHOH$ $COCl$ |
| (b) | $(H_3CCH_2CO)_2O$ | $H_3CCH_2CO_2H$ | $H_3CCH_2COCl$ |
| (c) | $(H_3CCO)_2O$ | $H_3CCO_2H$ | $ClCH_2COCl$ |
| (d) | $(H_3CCH_2CO)_2O$ | $H_3CCO_2H$ | $H_3CCOCl$ |

**149.** $H_3CCONH_2 + Br_2 + 4NaOH \longrightarrow$

$$Y + Na_2CO_3 + 2NaBr + 2H_2O$$

What is $Y$ in the above reaction?

(a) $H_3CCH_2NH_2$
(b) $H_3CNH_2$
(c) $H_3CCOBr$
(d) $HCONH_2$

**150.** The number of radial nodes present in $3p$ orbital is

(a) 0 (b) 1 (c) 2 (d) 3

**151.** The radiation with maximum frequency is

(a) X-rays
(b) radio waves
(c) UV rays
(d) IR rays

**152.** The equation used to represents the electron gain enthalpy is

(a) $X(g) + e^- \longrightarrow X^-(g)$

(b) $X(s) + e^- \longrightarrow X^-(g)$

(c) $X(g) \longrightarrow X^+(g) + e^-$

(d) $X(s) \longrightarrow X^+(g) + e^-$

**153.** An element in +2 oxidation state has 24 electrons. The atomic number of the element and the number of unpaired electrons in it respectively are

(a) 24 and 4 (b) 26 and 4
(c) 24 and 2 (d) 26 and 5

**154.** Number of bonding electron pairs and number of lone pairs of electrons in $ClF_3$, $SF_4$, $BrF_5$ respectively are

(a) 3, 2; 4, 2; 5, 2 (b) 3, 1; 4, 1; 5, 2
(c) 3, 1; 4, 2; 5, 1 (d) 3, 2; 4, 1; 5, 1

**155.** What is the bond order of $N_2$?

(a) 3 (b) 4 (c) 2 (d) 1

**156.** Match the following

| List-I | List-II |
|---|---|
| (A) Viscosity | P. Critical temperature |
| (B) Ideal gas behaviour | Q. Isobar |
| (C) Liquefaction of gases | R. Compressibility factor |
| (D) Charles law | S. kg $s^{-2}$ |
| | T. $kgm^{-1}\ s^{-1}$ |

The correct answer is

| | A | B | C | D | | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| (a) | S | R | P | Q | (b) | T | R | P | Q |
| (c) | T | R | Q | P | (d) | S | R | Q | P |

**157.** The most probable speed of $O_2$ molecules at $T(K)$ is

(a) $\sqrt{\frac{RT}{4\pi}}$ (b) $\sqrt{\frac{RT}{16\pi}}$ (c) $\sqrt{\frac{RT}{16}}$ (d) $\sqrt{\frac{3RT}{32}}$

**158.** According to significant figure convention, the result obtained by adding 12.11, 18.0 and 1.012 is

(a) 31.12 (b) 31.1 (c) 31 (d) 31.122

**159.** An organic compound having C, H and O has 13.13% H, 52.14% C and 34.73% O. Its molar mass is 46.068 g. What are its empirical and molecular formulae?

(a) $C_2H_6O$, $C_4H_{12}O_2$ (b) $CH_3O$, $C_2H_6O_2$
(c) $C_2H_6O$, $C_2H_6O$ (d) $C_2H_6O_2$, $C_3H_9O_4$

**160.** Which one of the following is not a state function?

(a) Internal energy (b) Work
(c) Entropy (d) Free energy

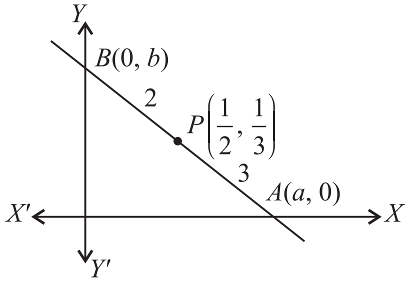
## ANSWER KEY

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | (c) | **2** | (c) | **3** | (a) | **4** | (c) | **5** | (d) | **6** | (c) | **7** | (b) | **8** | (a) | **9** | (a) | **10** | (b) |
| **11** | (c) | **12** | (c) | **13** | (d) | **14** | (d) | **15** | (a) | **16** | (a) | **17** | (c) | **18** | (b) | **19** | (a) | **20** | (b) |
| **21** | (d) | **22** | (d) | **23** | (a) | **24** | (c) | **25** | (b) | **26** | (b) | **27** | (d) | **28** | (c) | **29** | (c) | **30** | (a) |
| **31** | (a) | **32** | (d) | **33** | (d) | **34** | (a) | **35** | (c) | **36** | (a) | **37** | (c) | **38** | (c) | **39** | (c) | **40** | (a) |
| **41** | (b) | **42** | (c) | **43** | (c) | **44** | (d) | **45** | (c) | **46** | (c) | **47** | (b) | **48** | (d) | **49** | (d) | **50** | (d) |
| **51** | (c) | **52** | (a) | **53** | (a) | **54** | (b) | **55** | (d) | **56** | (d) | **57** | (a) | **58** | (d) | **59** | (c) | **60** | (a) |
| **61** | (b) | **62** | (b) | **63** | (d) | **64** | (a) | **65** | (a) | **66** | (b) | **67** | (c) | **68** | (a) | **69** | (c) | **70** | (d) |
| **71** | (b) | **72** | (a) | **73** | (a) | **74** | (c) | **75** | (b) | **76** | (a) | **77** | (c) | **78** | (d) | **79** | (d) | **80** | (c) |
| **81** | (b) | **82** | (a) | **83** | (a) | **84** | (b) | **85** | (c) | **86** | (b) | **87** | (c) | **88** | (c) | **89** | (b) | **90** | (b) |
| **91** | (c) | **92** | (b) | **93** | (a) | **94** | (b) | **95** | (a) | **96** | (b) | **97** | (b) | **98** | (d) | **99** | (c) | **100** | (d) |
| **101** | (a) | **102** | (d) | **103** | (a) | **104** | (d) | **105** | (b) | **106** | (a) | **107** | (b) | **108** | (b) | **109** | (c) | **110** | (a) |
| **111** | (c) | **112** | (c) | **113** | (b) | **114** | (a) | **115** | (b) | **116** | (a) | **117** | (d) | **118** | (*) | **119** | (a) | **120** | (b) |
| **121** | (c) | **122** | (d) | **123** | (a) | **124** | (d) | **125** | (b) | **126** | (a) | **127** | (b) | **128** | (d) | **129** | (d) | **130** | (b) |
| **131** | (c) | **132** | (b) | **133** | (d) | **134** | (b) | **135** | (d) | **136** | (d) | **137** | (d) | **138** | (d) | **139** | (c) | **140** | (a) |
| **141** | (b) | **142** | (c) | **143** | (a) | **144** | (b) | **145** | (c) | **146** | (d) | **147** | (a) | **148** | (b) | **149** | (b) | **150** | (a) |
| **151** | (a) | **152** | (a) | **153** | (b) | **154** | (d) | **155** | (a) | **156** | (b) | **157** | (c) | **158** | (b) | **159** | (c) | **160** | (b) |

# Hints & Solutions

## MATHEMATICS

1. **(c)** Let $A$ and $B$ be the points on the coordinate axes, where the line intersect and $P$ divides the line segment $BA$ in the ratio 2 : 3.




Then, the coordinate of point $P$ will be

$$\frac{2a}{5}=\frac{1}{2} \text{ and } \frac{3b}{5}=\frac{1}{3} \Rightarrow a=\frac{5}{4} \text{ and } b=\frac{5}{9}$$

Hence, equation of line will be,

$$\frac{x}{\frac{5}{4}}+\frac{y}{\frac{5}{9}}=1 \Rightarrow 4x+9y=5.$$

2. **(c)** Here, $m_1 = 4$ and $m_2=\frac{k}{5}\cdot$

Then, angle between two lines whose slopes are $m_1$ and $m_2$, is given by

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\therefore \quad \tan 45^\circ=\left|\frac{4-\frac{k}{5}}{1+4\cdot\frac{k}{5}}\right|$$

$$\Rightarrow \quad 1=\left|\frac{20-k}{5+4k}\right| \qquad \Rightarrow \quad \frac{20-k}{5+4k}=\pm 1$$

$$\Rightarrow \quad \frac{20-k}{5+4k}=1 \text{ or } \frac{20-k}{5+4k}=-1$$

$$\Rightarrow \quad 20-k=5+4k \text{ or } 20-k=-5-4k$$

$$\Rightarrow \quad k=3 \text{ or } -\frac{25}{3}$$

But $k > 0$

$\therefore \quad k = 3$

3. **(a)** Let the intercept of the line on coordinate axes be $a$ and $b$.

Then, $a + b = -1$ (Given)

$\Rightarrow b = -(1 + a)$

$\therefore$ Equation of line with intercepts $a$ and $b$ is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \Rightarrow \frac{x}{a}-\frac{y}{1+a}=1 \qquad \text{...(i)}$$

Since, above line passes through (4, 3).

$$\therefore \quad \frac{4}{a}-\frac{3}{1+a}=1$$

$$\Rightarrow \quad 4+4a-3a=a(1+a) \quad \Rightarrow \quad 4+a=a+a^2$$

$$\Rightarrow \quad a^2=4 \qquad \Rightarrow \qquad a=\pm 2$$

On putting $a = 2$ or $-2$ in Eq. (i), we get

$$\frac{x}{2}-\frac{y}{3}=1 \text{ or } \frac{x}{-2}-\frac{y}{-1}=1$$

$$\Rightarrow \quad 3x-2y-6=0 \text{ or } x-2y+2=0$$

Hence, combined equation of the straight lines is

$(3x - 2y - 6)(x - 2y + 2) = 0$

4. **(c)** Since, the origin is shifted to $(-\sqrt{2}, \sqrt{2})$ and rotates anti-clockwise through an angle $\frac{\pi}{4}$.

Let $(h, k) = (-\sqrt{2}, \sqrt{2})$, $\theta=\frac{\pi}{4}$ and $(x, y) = (1, -1)$.

$$\therefore \quad X=(x-h)\cos\theta+(y-k)\sin\theta$$

$$=(1+\sqrt{2})\frac{1}{\sqrt{2}}+(-1-\sqrt{2})\frac{1}{\sqrt{2}}=0$$

$$\text{and } Y=-(x-h)\sin\theta+(y-k)\cos\theta$$

$$=-(1+\sqrt{2})\frac{1}{\sqrt{2}}+(-1-\sqrt{2})\frac{1}{\sqrt{2}}=-2-\sqrt{2}$$

Hence, the new coordinates are $(0, -2-\sqrt{2})$.

5. **(d)** Since, given lines $3x+4y+5=0$ and $9x+12y+7=0$ are parallel. Hence, locus of the point $P$, which is equidistant from both lines, is a straight line which is an angular bisector.

6. **(c)** It is given that probability of showing head is $p$, then probability of showing tail will be $(1 - p)$.

Probability of showing 50 and 51 heads in lossing a coin 100 times is same.

Hence $P(X = 50) = P(X = 51)$

$$\Rightarrow \quad {}^{100}C_{50}(p)^{50}(1-p)^{50}={}^{100}C_{51}(p)^{51}(1-p)^{49}$$

$$\Rightarrow \quad \frac{100!}{50!\,50!}(1-p)=\frac{100!}{51!\,49!}p \Rightarrow \quad \frac{1-p}{50}=\frac{p}{51}$$

$$\Rightarrow \quad 51-51p=50p \quad \Rightarrow \quad p=\frac{51}{101}$$

7. **(b)** Given, $n = 6$

and also it is given that,

$$4P(X=4)=P(X=2)$$

$$\Rightarrow \quad 4\cdot{}^6C_4\,p^4q^2={}^6C_2p^2q^4$$

$$[\because P(X=r)={}^nC_rp^rq^{n-1}]$$

$\Rightarrow\ 4\times\frac{6\times5}{2\times1}p^2=\frac{6\times5}{2\times1}q^2$

$\Rightarrow\ 2p=q$

$\Rightarrow\ 2p=1-p$ $\quad [\because q=1-p]$

$\Rightarrow\ p=\frac{1}{3}$

**8.** **(a)** Given, students in a college are 60% $W$ and 40% $M$. In which 4% of $M$ and 1% of $W$ are taller than 1.8 m.
Let $E_1$, $E_2$ and $A$ be the events defined as follows:
$E_1$ = Selected students is a woman
$E_2$ = Selected students is a man
$A$ = Student is taller than 1.8 m
We have,

$$P(E_1)=\frac{60}{100},\ P(E_2)=\frac{40}{100}$$

and $P(A/E_1)=\frac{1}{100},\ P(A/E_2)=\frac{4}{100}$

By Baye's theorem, we have

$$P(E_1/A)=\frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)\cdot P(A/E_2)}$$

$$=\frac{\frac{60}{100}\times\frac{1}{100}}{\frac{60}{100}\times\frac{1}{100}+\frac{40}{100}\times\frac{4}{100}}=\frac{3}{11}.$$

**9.** **(a)** Given, $P(A/B)=0.6,\ P(B/A)=0.3,\ P(A)=0.1$

We know that, $P(B/A)=\frac{P(B\cap A)}{P(A)}$

$\Rightarrow\ 0.3=\frac{P(B\cap A)}{0.1}\ \Rightarrow\ P(B\cap A)=0.03$

or $P(A\cap B)=0.03$

Also, $P(A/B)=\frac{P(A\cap B)}{P(B)}$

$\Rightarrow\ 0.6=\frac{0.03}{P(B)}\ \Rightarrow\ P(B)=0.05$

$\therefore\ P(\bar{A}\cap\bar{B})=1-P(A\cup B)$

$=1-[P(A)+P(B)-P(A\cap B)]$

$=1-0.1-0.05+0.03=0.88.$

**10.** **(b)** Given,

$P(A\cup B)=\frac{5}{6},\ P(\bar{A})=\frac{1}{4},\ P(B)=\frac{1}{3}$

$\therefore\ P(A)=1-P(\bar{A})=1-\frac{1}{4}=\frac{3}{4}$

Also, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow\ P(A\cap B)=\frac{1}{4}$ and $P(A)\cdot P(B)=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}$

$[P(A\cap B=P(A)\cdot P(B)]$

Hence, $A$ and $B$ are independent events.

**11.** **(c)** Given, mean and coefficients of variation are $X_A=X_B$ and $CV_A=4,\ CV_B=2$

Let $\sigma_A$ and $\sigma_B$ be the standard deviations of teams $A$ and $B$ respectively.

Then, $CV=\frac{\sigma}{\bar{X}}\times100$ or $\sigma=\frac{CV\times\bar{X}}{100}$

$\therefore\ \sigma_A=\frac{4\bar{X}_A}{100}$ and $\sigma_B=\frac{2\bar{X}_B}{100}$

$\Rightarrow\ \bar{X}_A=25\sigma_A$ and $\bar{X}_B=50\sigma_B$

But $\bar{X}_A=\bar{X}_B$

$\therefore\ 25\sigma_A=50\sigma_B\ \Rightarrow\ \sigma_A=2\sigma_B$

**12.** **(c)** Since, $i$ is repeated $i$ times for $i=1, 2, 3, ..., n$.
Hence, mean of the given data will be

$$\bar{X}=\frac{1\times1+2\times2+3\times3+...+n\times n}{1+2+3+...+n}=\frac{\Sigma n^2}{\Sigma n}$$

$$=\frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}=\frac{2n+1}{3}.$$

**13.** **(d)** Given,
$\vec{a}=2\hat{i}-3\hat{j}+5\hat{k},\ \vec{b}=3\hat{i}-4\hat{j}+5\hat{k}$ and
$\vec{c}=5\hat{i}-3\hat{j}-2\hat{k}$

Now, $\vec{a}+\vec{b}=5\hat{i}-7\hat{j}+10\hat{k}$

and $\vec{b}+\vec{c}=8\hat{i}-7\hat{j}+3\hat{k}$ and $\vec{c}+\vec{a}=7\hat{i}-6\hat{j}+3\hat{k}$

$\therefore$ Volume of the parallelopiped with coterminous edges $\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}=[\vec{a}+\vec{b}\quad \vec{b}+\vec{c}\quad \vec{c}+\vec{a}]$

$$=\begin{vmatrix}5&-7&10\\8&-7&3\\7&-6&3\end{vmatrix}$$

$=5(-21+18)+7(24-21)+10(-48+49)$
$=-15+21+10=16.$

**14.** **(d)** Equations of given lines are :-

$L_1\equiv\frac{x-3}{-1}=\frac{y-4}{2}=\frac{z+2}{1}$

$L_2\equiv\frac{x-1}{1}=\frac{y+7}{3}=\frac{z+2}{2}$

Hence, $(x_1, y_1, z_1)\equiv(3, 4, -2),\ (a_1, b_1, c_1)\equiv(-1, 2, 1)$

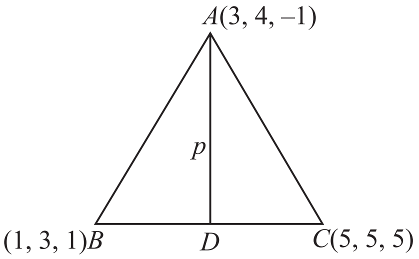
$(x_2, y_2, z_2)\equiv(1, -7, -2),\ (a_2, b_2, c_2)\equiv(1, 3, 2)$

Now,

$$d=\frac{\begin{vmatrix}x_2-x_1&y_2-y_1&z_2-z_1\\a_1&b_1&c_1\\a_2&b_2&c_2\end{vmatrix}}{\sqrt{(b_1c_2-c_1b_2)^2+(a_1c_2-a_2c_1)^2+(a_1b_2-a_2b_1)^2}}$$

$$=\frac{\begin{vmatrix}-2&-11&0\\-1&2&1\\1&3&2\end{vmatrix}}{\sqrt{(1)^2+(-3)^2+(-5)^2}}=\sqrt{35}.$$

**15.** **(a)** Given, position vectors of the point *A, B* and *C* and altitude of a $\Delta ABC$ from *A* on *BC* is *P*,




Now, sides of triangle are

$$AB = -2\hat{i} - \hat{j} + 2\hat{k},\ AC = 2\hat{i} + \hat{j} + 6\hat{k}$$

and $BC = 4\hat{i} + 2\hat{j} + 4\hat{k}$

We know that,

Area of $\Delta ABC = \frac{1}{2}|AB \times AC| = \frac{1}{2}|BC| \times p$ ...(i)

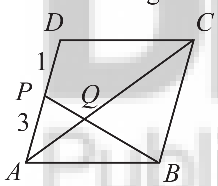
Now, $AB \times AC = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -1 & 2 \\ 2 & 1 & 6 \end{vmatrix} = -8\hat{i} + 16\hat{j} + 0\hat{k}$

From Eq. (i),

$$\frac{1}{2}\sqrt{(-8)^2 + (16)^2 + (0)^2} = \frac{1}{2}\sqrt{4^2 + 2^2 + 4^2} \times p$$

$$= \frac{\sqrt{320}}{6} = \frac{8\sqrt{5}}{6} = \frac{4\sqrt{5}}{3}$$

**16.** **(a)** According to question from fig.




$\Delta AQP \sim \Delta CQB$

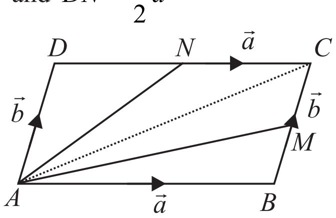
$$\Rightarrow \frac{AQ}{CQ} = \frac{QP}{QB} = \frac{AP}{CB}$$

$\because\ \frac{AP}{AD} = \frac{AP}{CB} = -$ ($\because$ AD = CB)

$\therefore\ \frac{AQ}{CQ} = \frac{3}{4}$

**17.** **(c)** Given, *M* and *N* are the mid-points of *BC* and *CD*.

$\therefore\ BM = \frac{1}{2}\vec{b}$ and $DN = \frac{1}{2}\vec{a}$




Now, in $\Delta ABM$,

$$AM = AB + BM = \vec{a} + \frac{1}{2}\vec{b}$$

and in $\Delta ADN$,

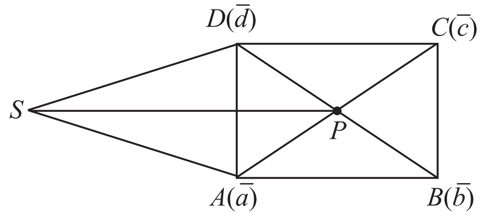
$$AN = AD + DN = \vec{b} + \frac{1}{2}\vec{a}$$

$$\therefore\ \overrightarrow{AM} + \overrightarrow{AN} = \vec{a} + \frac{1}{2}\vec{b} + \vec{b} + \frac{1}{2}\vec{a}$$

$$= \frac{3}{2}\vec{a} + \frac{3}{2}\vec{b} = \frac{3}{2}(\vec{a} + \vec{b})$$

$$\Rightarrow \overrightarrow{AM} + \overrightarrow{AN} = \frac{3}{2}\overrightarrow{AC} \qquad [\because AC = \vec{a} + \vec{b}]$$

**18.** **(b)** Let *S* be the origin and position vectors of *A, B, C* and *D*.




In $\Delta SAP$ are $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$

$$\overline{SA} + \overline{AP} = \overline{SP}$$

$$\bar{a} + \left(\frac{\bar{c} + \bar{a}}{2}\right) - \bar{a} = SP$$

$$\Rightarrow \bar{a} + \bar{c} = 2SP \qquad ...(i)$$

Similarly,

$$\bar{b} + \bar{d} = 2SP \qquad ...(ii)$$

On adding (i) and (ii),

$$\bar{a} + \bar{b} + \bar{c} + \bar{d} = 4SP \Rightarrow \lambda = 4.$$

**19.** **(a)** Given, $r_1 = 2r_2 = 3r_3$

$$\Rightarrow \frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c} = \lambda \qquad \text{[say]}$$

$$\Rightarrow s - a = \frac{\Delta}{\lambda},\ s - b = \frac{2\Delta}{\lambda},\ s - c = \frac{3\Delta}{\lambda}$$

On adding we get, $s = \frac{6\Delta}{\lambda}$

Now,

$$s - a = \frac{\Delta}{\lambda} \Rightarrow \frac{6\Delta}{\lambda} - a = \frac{\Delta}{\lambda} \Rightarrow a = \frac{5\Delta}{\lambda}$$

$$s - b = \frac{2\Delta}{\lambda} \Rightarrow \frac{6\Delta}{\lambda} - b = \frac{2\Delta}{\lambda} \Rightarrow b = \frac{4\Delta}{\lambda}$$

$$s - c = \frac{3\Delta}{\lambda} \Rightarrow \frac{6\Delta}{\lambda} - c = \frac{3\Delta}{\lambda} \Rightarrow c = \frac{3\Delta}{\lambda}$$

$$\therefore\ b : c = \frac{4\Delta}{\lambda} \cdot \frac{3\Delta}{\lambda} \Rightarrow b : c = 4 : 3$$

**20.** **(b)** Since, it is given

$$\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2}$$

$$= \frac{s^2}{\Delta^2} + \frac{(s-a)^2}{\Delta^2} + \frac{(s-b)^2}{\Delta^2} + \frac{(s-c)^2}{\Delta^2}$$

$$= \frac{4s^2 - 2s(a+b+c) + a^2 + b^2 + c^2}{\Delta^2}$$

$$= \frac{4s^2 - 4s^2 + a^2 + b^2 + c^2}{\Delta^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}.$$

**21.** **(d)** Given, *A, B, C* are in an A.P.

Hence $\angle B = \frac{\angle A + \angle C}{2} \Rightarrow \angle B = \frac{\pi - \angle B}{2} \Rightarrow < B = \frac{\pi}{3}$

So $A = 90°$, $B = 60°$ and $C = 30°$

Now, $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \cos 60° = \frac{1}{2} = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow c^2 - ac + a^2 - b^2 = 0$

$\Rightarrow c = \frac{a \pm \sqrt{a^2 - 4(a^2 - b^2)}}{2} = \frac{a \pm \sqrt{4b^2 - 3a^2}}{2}.$

**22.** **(d)** Given, $\cos h\, 2x = 199$

$\Rightarrow \frac{1 + \tan h^2 x}{1 - \tan h^2 x} = 199 \Rightarrow \quad 200 \tan h^2 x = 198$

$\Rightarrow \tan h^2 x = \frac{198}{200} \Rightarrow \tan h^2 x = \frac{99}{100}$

$\Rightarrow \tan h\, x = \frac{3\sqrt{11}}{10} \Rightarrow \cot h\, x = \frac{10}{3\sqrt{11}}$

**23.** **(a)** We have

$\cos\left(\cot^{-1}\left(\frac{1}{2}\right)\right) = \cot(\cos^{-1} x)$

Let $\cot^{-1}\left(\frac{1}{2}\right) = \alpha \Rightarrow \cot\alpha = \frac{1}{2} \Rightarrow \cos^{-1}\frac{1}{\sqrt{5}}$

$\Rightarrow \cos\left(\cos^{-1}\frac{1}{\sqrt{5}}\right) = \cot\left(\cot^{-1}\frac{x}{\sqrt{1-x^2}}\right)$

$\Rightarrow \frac{1}{\sqrt{5}} = \frac{x}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2} = \sqrt{5}x$

On squaring both sides, we get,

$1 - x^2 = 5x^2 \Rightarrow 1 = 6x^2$

$\Rightarrow x = \pm\frac{1}{\sqrt{6}}$

$\therefore \quad x = \frac{1}{\sqrt{6}}$ [neglecting –ve sign]

**24.** **(c)** Given,

$\sec x \cos 5x + 1 = 0$

$\Rightarrow \cos 5x + \cos x = 0 \Rightarrow \quad 2\cos 3x \cos 2x = 0$

$\Rightarrow \cos 3x = 0$ or $\cos 2x = 0$

$\Rightarrow 3x = (2n-1)\frac{\pi}{2}$ or $2x = (2n-1)\frac{\pi}{2}$

$\Rightarrow x = (2n-1)\frac{\pi}{6}$ or $(2n-1)\frac{\pi}{4}$

Hence, total number of solutions is 10 in $[0, 2\pi]$.

**25.** **(b)** Given, $\angle C = \frac{\pi}{3}$

Now, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \cos\frac{\pi}{3} = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow a^2 + b^2 = ab + c^2$

$\Rightarrow a^2 + b^2 + ac + bc = ab + c^2 + ac + bc$

[adding both sides by $ac + bc$]

$\Rightarrow a(c+a) + b(b+c) = (b+c)(c+a)$

$\Rightarrow \frac{a}{b+c} + \frac{b}{c+a} = 1$

$\Rightarrow \frac{a}{b+c} + 1 + \frac{b}{c+a} + 1 = 3$

$\Rightarrow \frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$

$\Rightarrow \frac{3}{a+b+c} - \frac{1}{a+c} = \frac{1}{b+c}$

**26.** **(b)** Given,

$A = \sin^2\theta + \cos^4\theta = 1 - \cos^2\theta + \cos^4\theta$

$= 1 - \cos^2\theta \sin^2\theta = 1 - \frac{1}{4}\sin^2 2\theta$

Since we know, $0 \le \sin^2, 2\theta \le 1$

$\Rightarrow \frac{-1}{4} \le \frac{-1}{4}\sin^2 2\theta \le 0$

$\Rightarrow 1 - \frac{1}{4} \le 1 - \frac{1}{4}\sin^2 2\theta \le 1 \Rightarrow \frac{3}{4} \le A \le 1$

$\therefore \quad A \in \left[\frac{3}{4}, 1\right].$

**27.** **(a)** Since, given that a real function $f(x)$ is defined on $[-1, 1]$.

$\therefore \quad f(5x + 4) = g(x)$ defined for $-1 \le 5x + 4 \le 1$

$\Rightarrow -1 \le x \le \frac{-3}{5} \Rightarrow x \in \left[-1, \frac{-3}{5}\right].$

**28.** **(c)** Given, $f : N \to R$ and $f(1) = -1$

and also given that $f(n+1) = 3f(n) + 2$ for $n \ge 1$

Now, put $n = 1$, we get

$f(2) = 3f(1) + 2$

$= 3(-1) + 2 = -1$

Put $n = 2$, we get

$f(3) = 3f(2) + 2 = 3(-1) + 2 = -1$

So, $f(n) = -1$ for all values of $n$.

Hence, $f(n)$ is a constant function.

**29.** **(c)** Let $f(n) = n^4 - 2n^3 - n^2 + 2n - 26$

Let to check divisibility by 24

$= n^3(n-2) - n(n-2) - 26$

$= (n-2)(n^3 - n) - 26$

$= (n-2)(n-1)n(n+1) - 26 = 24k - 48 + 22$

[$\because$ product of four consecutive natural numbers is divisible by 24]

$= 24[k-2] + 22.$

Hence, remainder is 22.

**30.** **(a)** $A(x) = \begin{vmatrix} 1 & 2 & 3 \\ x+1 & 2x+1 & 3x+1 \\ x^2+1 & 2x^2+1 & 3x^2+1 \end{vmatrix}$

Apply $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$, we get

$$A(x) = \begin{vmatrix} 1 & 1 & 1 \\ x+1 & x & x \\ x^2+1 & x^2 & x^2 \end{vmatrix} = 0$$

[$\because$ $C_2$ and $C_3$ are identical]

$\therefore \int_0^1 A(x)\,dx = \int_0^1 0\,dx = 0.$

**31.** **(a)** Given,

$$\begin{vmatrix} x^2+x+1 & x+1 & 2x-3 \\ 3x^2-1 & x+2 & x-1 \\ x^2+5x+1 & 2x+3 & x+4 \end{vmatrix}$$

$= ax^4 + bx^3 + cx^2 + dx + e$

Hence, on expanding the determinant we get an identity, it will satisfy also for $x = 0$.

Put $x = 0$ on both sides, we get

$$\begin{vmatrix} 1 & 1 & -3 \\ -1 & 2 & -1 \\ 1 & 3 & 4 \end{vmatrix} = e$$

$\Rightarrow e = 1(8+3) - 1(-4+1) - 3(-3-2)$

$= 11 + 3 + 15 = 29.$

**32.** **(d)** Given system of equations written in determinant form

$$D = \begin{vmatrix} 4 & 1 & 2 \\ 1 & -5 & 3 \\ 9 & -3 & 7 \end{vmatrix}$$

$= 4(-35+9) - 1(7-27) + 2(-3+45)$

$= -104 + 20 + 84 = 0.$

$$D_1 = \begin{vmatrix} 5 & 1 & 2 \\ 10 & -5 & 3 \\ 20 & -3 & 7 \end{vmatrix}$$

$= 5(-35+9) - 1(70-60) + 2(-30+100)$

$= -130 - 10 + 140 = 0$

$$D_2 = \begin{vmatrix} 4 & 5 & 2 \\ 1 & 10 & 3 \\ 9 & 20 & 7 \end{vmatrix}$$

$= 4(70-60) - 5(7-27) + 2(20-90)$

$= 40 + 100 - 140 = 0$

and $D_3 = \begin{vmatrix} 4 & 1 & 5 \\ 1 & -5 & 10 \\ 9 & -3 & 20 \end{vmatrix}$

$= 4(-100+30) - 1(20-90) + 5(-3+45)$

$= -280 + 70 + 210 = 0.$

So, the given system of equations has infinite number of solutions.

**33.** **(d)** Given, $\alpha = \omega + 2\omega^2 - 3$

$\Rightarrow \alpha = \omega + \omega^2 + \omega^2 - 3 \Rightarrow \alpha = -1 + \omega^2 - 3$

$\Rightarrow \omega^2 = \alpha + 4$

On cubing both sides

$\Rightarrow (\omega^2)^3 = (\alpha+4)^3 \Rightarrow \omega^6 = \alpha^3 + 12\alpha^2 + 48\alpha + 64$

$\Rightarrow 1 = \alpha^3 + 12\alpha^2 + 48\alpha + 64$

$\Rightarrow \alpha^3 + 12\alpha^2 + 48\alpha + 3 = -60$

**34.** **(a)** Given equation, $1 + x + x^2 = 0$

Hence, roots are $x = \dfrac{-1 \pm \sqrt{1-4}}{2} \Rightarrow x = \dfrac{-1 \pm \sqrt{3}i}{2}$

$\Rightarrow x = \omega, \omega^2$

Given $\alpha, \beta$ are the roots of the equation

$1 + x + x^2 = 0. \Rightarrow \alpha = \omega, \beta = \omega^2$

Now, $\alpha^4 + \beta^4 + \alpha^{-4}\beta^{-4} = \omega^4 + \omega^8 + \omega^{-4}\omega^{-8}$

$= \omega + \omega^2 + \omega^{-12} = -1 + 1 = 0.$

**35.** **(c)** Given, $x^2 - 4x + 8 = 0$

$\therefore x = \dfrac{4 \pm \sqrt{16-32}}{2} = \dfrac{4 \pm 4i}{2} = 2 \pm 2i \Rightarrow x = 2(1 \pm i)$

$\Rightarrow x = 2\sqrt{2}\left(\dfrac{1}{\sqrt{2}} \pm i\dfrac{1}{\sqrt{2}}\right)$

$= 2\sqrt{2}\left(\cos\dfrac{\pi}{4} \pm i\sin\dfrac{\pi}{4}\right) \quad \left(\because \sin\dfrac{\pi}{4} = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}\right)$

Given, $\alpha, \beta$ are the roots of the equation.

$\alpha = 2\sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$

$\beta = 2\sqrt{2}\left(\cos\dfrac{\pi}{4} - i\sin\dfrac{\pi}{4}\right)$

Now, $\alpha^{2n} = (2\sqrt{2})^{2n}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)^{2n}, n \in N$

$= (2\sqrt{2})^{2n}\left(\cos\dfrac{n\pi}{2} + i\sin\dfrac{n\pi}{2}\right)$

Similarly, $\beta^{2n} = (2\sqrt{2})^{2n}\left(\cos\dfrac{n\pi}{2} - i\sin\dfrac{n\pi}{2}\right)$

$\therefore \alpha^{2n} + \beta^{2n} = (2\sqrt{2})^{2n} \cdot 2\cos\dfrac{n\pi}{2}$

$= (2^{3/2})^{2n} \cdot 2\cos\dfrac{n\pi}{2}$

$\Rightarrow \alpha^{2n} + \beta^{2n} = 2^{3n+1}\cos\dfrac{n\pi}{2}$

**36.** **(a)** Given, $\alpha$ and $\beta$ are non-real cube roots of 2.

$\therefore \quad \alpha = 2^{1/3}\omega$ and $\beta = 2^{1/3}\omega^2$

Now, $\alpha^6 + \beta^6 = (2^{1/3}\omega)^6 + (2^{1/3}\omega^2)^6$

$= 2^2\omega^6 + 2^2\omega^{12} = 4(\omega^3)^2 + 4(\omega^3)^4$

$= 4 + 4 = 8 \qquad [\because \omega^3 = 1]$

**37.** **(c)** Given, $\alpha^2 + 1 = 6\alpha$ and $\beta^2 + 1 = 6\beta$ and also $\alpha \neq \beta$

From the above relation $\alpha$, $\beta$ are roots.

Hence, $x^2 - 6x + 1 = 0$

$\Rightarrow \; x = \dfrac{\alpha}{\alpha+1} \Rightarrow \alpha = \dfrac{x}{1-x}$

Hence, required quadratic eqaution whose roots are $\dfrac{\alpha}{\alpha+1}, \dfrac{\beta}{\beta+1}$ is $\left(\dfrac{x}{1-x}\right)^2 - 6\left(\dfrac{x}{1-x}\right) + 1 = 0$

$\Rightarrow \; x^2 - 6x(1-x) + 1(1-x)^2 = 0 \;\Rightarrow\; 8x^2 - 8x + 1 = 0.$

**38.** **(c)** Given, $|x|^2 - 5|x| + 4 < 0$

Let $|x| = y$

$\Rightarrow \; y^2 - 5y + 4 < 0 \;\Rightarrow\; y^2 - 4y - y + 4 < 0$

$\Rightarrow \; (y-4)(y-1) < 0 \;\Rightarrow\; y \in (1, 4) \;\Rightarrow\; 1 < |x| < 4$

$\therefore \quad x \in (-4, -1) \cup (1, 4)$

**39.** **(c)** Given, $\alpha$, $\beta$, $\gamma$ are the roots of the equation $x^3 + x + 10 = 0$.

Hence, $\alpha + \beta + \gamma = 0 \qquad$ ...(i)

and $\alpha_1 = \dfrac{\alpha+\beta}{\gamma^2}, \beta_1 = \dfrac{\beta+\gamma}{\alpha^2}, \gamma_1 = \dfrac{\gamma+\alpha}{\beta^2}$

Now, $\alpha_1 = \dfrac{\alpha+\beta}{\gamma^2} = \dfrac{-\gamma}{\gamma^2} = \dfrac{-1}{\gamma}$

Similarly, $\beta_1 = \dfrac{-1}{\alpha}$ and $\gamma_1 = \dfrac{-1}{\beta}$

$\therefore \; \alpha_1, \beta_1, \gamma_1$ are the roots of equation $f\left(\dfrac{-1}{x}\right) = 0$

Now, $f\left(\dfrac{-1}{x}\right)$ will be $\left(-\dfrac{1}{x}\right)^3 + \left(-\dfrac{1}{x}\right) + 10 = 0$

$\Rightarrow \; 10x^3 - x^2 - 1 = 0 \;\Rightarrow\; 10\alpha_1^3 - \alpha_1^2 - 1 = 0$

$\Rightarrow \; \alpha_1^3 = \dfrac{1}{10}\alpha_1^2 + \dfrac{1}{10}$

$\Rightarrow \; \Sigma\alpha_1^3 = \dfrac{1}{10}\Sigma\alpha_1^2 + \dfrac{1}{10} \;\Rightarrow\; \Sigma\alpha_1^3 - \dfrac{1}{10}\Sigma\alpha_1^2 = \dfrac{3}{10}$

$\therefore \; (\alpha_1^3 + \beta_1^3 + \gamma_1^3) - \dfrac{1}{10}(\alpha_1^2 + \beta_1^2 + \gamma_1^2) = \dfrac{3}{10}.$

**40.** **(a)** Given, $\alpha$, $\beta$, $\gamma$ are the roots of $x^3 + x^2 + x + 2 = 0$.

Then, $\alpha + \beta + \gamma = -1$

Now, $\dfrac{\alpha+\beta-2\gamma}{\gamma} = \dfrac{-1-\gamma-2\gamma}{\gamma} = \dfrac{-1-3\gamma}{\gamma}$

$\therefore \quad x = \dfrac{-1-3\gamma}{\gamma} \Rightarrow \gamma = \dfrac{-1}{x+3}$

Now, the equation whose root is $\dfrac{-1}{x+3}$ is

$\left(\dfrac{-1}{x+3}\right)^3 + \left(\dfrac{-1}{x+3}\right)^2 + \left(\dfrac{-1}{x+3}\right) + 2 = 0$

$\Rightarrow \; -1 + x + 3 - x^2 - 9 - 6x + 2x^3 + 54 + 18x^2 + 54x = 0$

$\Rightarrow \; 2x^3 + 17x^2 + 49x + 47 = 0$

$\therefore \quad \left(\dfrac{\alpha+\beta-2\gamma}{\gamma}\right)\left(\dfrac{\beta+\gamma-2\alpha}{\alpha}\right)\left(\dfrac{\gamma+\alpha-2\beta}{\beta}\right) = \dfrac{-47}{2}$

**41.** **(b)** Given, $\sum_{r=0}^{10} {}^{40-r}C_6$

Let $r = 10, 9, 8, \ldots.\ 0$

$= {}^{30}C_5 + {}^{31}C_5 + {}^{32}C_5 + {}^{33}C_5 + \ldots. + {}^{40}C_5$

$\because \quad {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\Rightarrow \quad {}^nC_{r-1} = {}^{n+1}C_r - {}^nC_r$

$\therefore \quad {}^{31}C_5 = {}^{32}C_6 - {}^{31}C_6$

${}^{32}C_5 = {}^{33}C_6 - {}^{32}C_6$

$\vdots \qquad \vdots \qquad \vdots$

${}^{40}C_5 = {}^{41}C_6 - {}^{30}C_6$

On adding, $\sum_{r=0}^{10} {}^{40-r}C_5 = {}^{41}C_6 - {}^{30}C_6.$

**42.** **(c)** Since the number of diagonals of a regular polygon is given 35.

We know that a regular polygon having $n$ sides is given by $\dfrac{n(n-3)}{2}$.

$\Rightarrow \; \dfrac{n(n-3)}{2} = 35$

$\Rightarrow \; n^2 - 3n - 70 = 0 \;\Rightarrow\; n^2 - 10n + 7n - 70 = 0$

$\Rightarrow \; n(n-10) + 7(n-10) = 0$

$\Rightarrow \; (n-10)(n+7) = 0 \;\Rightarrow\; n = 10, -7 \;\therefore\; n = 10.$

**43.** **(c)** It is given that,

$x = 1 + \dfrac{3}{1!} \times \dfrac{1}{6} + \dfrac{3 \times 7}{2!}\left(\dfrac{1}{6}\right)^2 + \dfrac{3 \times 7 \times 11}{3!}\left(\dfrac{1}{6}\right)^3 + \ldots$

From Binomial expansion of any index $\qquad$ ...(i)

$$\therefore \quad (1-a)^{-p/q} = 1 + \left(\frac{p}{q}\right)\frac{1}{1!}(a) + \frac{\frac{p}{q}\left(\frac{p}{q}+1\right)}{2!}a^2 + \frac{\frac{p}{q}\left(\frac{p}{q}+1\right)\left(\frac{p}{q}+2\right)}{3!}a^3 + \ldots$$

$$= 1 + \frac{p}{1!}\left(\frac{a}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{a}{q}\right)^2$$

$$+ \frac{p(p+q)(p+2q)}{3!}\left(\frac{a}{q}\right)^3 + \ldots \qquad \ldots\text{(ii)}$$

On comparing Eqs. (i) and (ii), we get

$p = 3, p + q = 7, p + 2q = 11$

and $\frac{a}{q} = \frac{1}{6} \Rightarrow q = 4$ and $a = \frac{4}{6} = \frac{2}{3}$

So, let $x = (1-a)^{-p/q} = \left(1 - \frac{2}{3}\right)^{-3/4}$

$$= \left(\frac{1}{3}\right)^{-3/4} = (3)^{3/4}$$

$\therefore \quad x^4 = 3^3 = 27.$

**44.** **(d)** Here, we are given to find the approximate value of

$$\frac{\left(1+\frac{2}{3}x\right)^{-3}(1-15x)^{-1/5}}{(2-3x)^4} = \frac{\left(1+\frac{2}{3}x\right)^{-3}(1-15x)^{-1/5}}{2^4\left(1-\frac{3}{2}x\right)^4}$$

$$= \frac{\left(1+\frac{2}{3}x\right)^{-3}(1-15x)^{-1/5}\left(1-\frac{3}{2}x\right)^{-4}}{16}$$

$$= \frac{1}{16}(1-2x)(1+3x)(1+6x)$$

[$\because$ neglecting higher powers]

$$= \frac{1}{16}(1+x)(1+6x) = \frac{1}{16}(1+7x).$$

**45.** **(c)** Since we have,

$$\frac{1}{x^2-5x+6} = \frac{1}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$$

We get $A = 1$ and $B = -1$

$$\Rightarrow \frac{1}{x-3} - \frac{1}{x-2} = \frac{1}{-3}\left(\frac{1}{1-\frac{x}{3}}\right) - \frac{1}{-2}\left(\frac{1}{1-\frac{x}{2}}\right)$$

$$= \frac{1}{2}\left(1-\frac{x}{2}\right)^{-1} - \frac{1}{3}\left(1-\frac{x}{3}\right)^{-1}$$

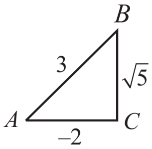
Since $(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots$

$$= \frac{1}{2}\left(\frac{1}{2}\right)^n \times x^n - \frac{1}{3}\left(\frac{1}{3}\right)^n (x)^n$$

$\therefore$ Coefficient of $x^n = \frac{1}{2}\left(\frac{1}{2}\right)^n - \frac{1}{3}\left(\frac{1}{3}\right)^n$

$$= \frac{1}{2^{n+1}} - \frac{1}{3^{n+1}}.$$

**46.** **(c)** Given, $3\cos A + 2 = 0$




$\Rightarrow \quad \cos A = \frac{-2}{3}$

$\therefore \quad \sin A = \frac{\sqrt{5}}{3}$

and $\tan A = \frac{-\sqrt{5}}{2}$

Since, sin$A$ and tan$A$ are the roots of required equation. Hence, equation can be written as

$x^2 - (\sin A + \tan A)x + \sin A \tan A = 0$

$$\Rightarrow x^2 + \frac{\sqrt{5}}{6}x - \frac{5}{6} = 0 \Rightarrow 6x^2 + \sqrt{5}x - 5 = 0$$

**47.** **(b)** (P) $(x^3+1)\frac{dy}{dx} + x^2 y = 3x^2$

$$\Rightarrow \frac{dy}{dx} + \frac{x^2}{x^3+1}y = \frac{3x^2}{x^3+1}$$

It is $\frac{dy}{dx} + P(y) = Q$ type equation

$\therefore$ Integrating factor $= e^{\int \frac{x^2}{x^3+1}dx}$ [Hint : put $x^3 + 1 = t$]

$$= e^{\frac{1}{3}\log(x^2+1)} = (x^3+1)^{1/3}.$$

(Q) $x^2\frac{dy}{dx} + 3xy = x^6$

$$\Rightarrow \frac{dy}{dx} + \frac{3x}{x^2}y = \frac{x^6}{x^2} \Rightarrow \frac{dy}{dx} + \frac{3}{x}y = x^4 \Rightarrow P = \frac{3}{x}$$

$\therefore$ Integrating factor $= e^{\int \frac{3}{x}dx} = e^{3\log x} = x^3.$

(R) $(x^3+1)^2\frac{dy}{dx} + 6x^2(x^3+1)y = x^2$

$$\Rightarrow \frac{dy}{dx} + \frac{6x^2(x^3+1)}{(x^3+1)^2}y = \frac{x^2}{(x^3+1)^2}$$

$$\Rightarrow \frac{dy}{dx} + \frac{6x^2}{x^3+1}y = \frac{x^2}{(x^3+1)^2}$$

$\therefore$ Integrating factor $= e^{\int \frac{5x^2}{x^3+1}dx}$ [same as ($P$)]

$$= e^{2\log(x^3+1)} = (x^3+1)^2$$

(S) $(x^2+1)\frac{dy}{dx} + 4xy = \ln x$

$$\Rightarrow \frac{dy}{dx} + \frac{4x}{x^2+1}y = \frac{\ln x}{x^2+1}$$

$\therefore$ Integrating factor $= e^{\int \frac{4x}{x^2+1}dx}$

$$= e^{2\log(x^2+1)} = (x^2+1)^2.$$

**48.** **(d)** Given differential equation is

$$xy' = 2xe^{-y/x} + y$$

On dividing by $x$, $x \neq 0$

$$\Rightarrow \frac{dy}{dx} = 2e^{-y/x} + \frac{y}{x} \quad \text{...(i)}$$

The above equation is homogeneous differential equation.
Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

On putting these values in Eq. (i), we get

$$v + x\frac{dv}{dx} = 2e^{-v} + v \Rightarrow x\frac{dv}{dx} = 2e^{-v}$$

$$\Rightarrow \int e^v\,dv = \int \frac{2}{x}dx \Rightarrow e^v = 2\log x + \log C$$

$$\Rightarrow e^{y/x} = 2\log x + \log C$$

$$\Rightarrow e^{y/x} = 2\log|Cx|$$

**49.** **(d)** Given family of curves is

$$y = ax + \frac{1}{a} \quad \text{...(i)}$$

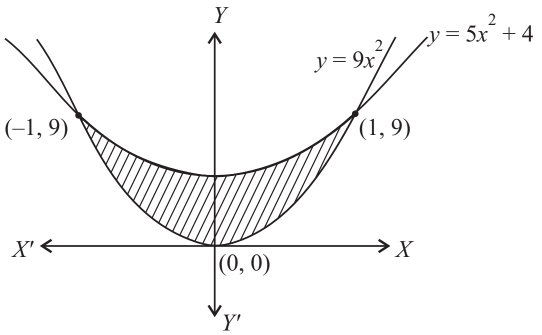
$$\Rightarrow \frac{dy}{dx} = a$$

On putting $a = \frac{dy}{dx}$ in Eq. (i), we get

$$y = x\frac{dy}{dx} + \frac{1}{\left(\frac{dy}{dx}\right)} \Rightarrow y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + 1$$

which is a differential equation of degree 2.

**50.** **(d)** To obtain intersecting point of $y = 9x^2$ and $y = 5x^2 + 4$ is get by solving both the equations simultaneously.
$9x^2 = 5x^2 + 4$
We get (–1, 9) and (1, 9) as point of intersections.




Now, required area $= 2\int_0^1 [(5x^2+4)-(9x^2)]\,dx$

$$= 2\int_0^1 (4-4x^2)\,dx = 2\left[4x - \frac{4x^3}{3}\right]_0^1$$

$$= 2\left(4-\frac{4}{3}\right) - (0-0) = 2\times\frac{8}{3} = \frac{16}{3} \text{ sq. units.}$$

**51.** **(c)** Let $I = \int_0^{\pi/2} \frac{16x\sin x\cos x}{\sin^4 x + \cos^4 x}\,dx$ ...(i)

By property, $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$= \int_0^{\pi/2} \frac{16\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}dx$$

$$= \int_0^{\pi/2} \frac{(8\pi-16x)\sin x\cos x}{\sin^4 x+\cos^4 x}dx \quad \text{...(ii)}$$

On adding Eqs.(i) and (ii), we get

$$2I = 8\pi\int_0^{\pi/2} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$= 4\pi\int_0^{\pi/2} \frac{2\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$\therefore \quad I = 2\pi\int_0^{\pi/2} \frac{2\tan x\sec^2 x}{\tan^4 x+1}dx$$

$$= 2\pi\int_0^{\pi/2} d\{\tan^{-1}(\tan^2 x)\}$$

$$= 2\pi[\tan^{-1}(\tan^2 x)]_0^{\pi/2} = 2\pi\left[\frac{\pi}{2}-0\right] \quad \therefore \ I = \pi^2$$

**52.** **(a)** Let $I = \int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$

$$= \int_0^1 \frac{1-x}{\sqrt{1-x^2}}dx = \int_0^1 \frac{1}{\sqrt{1-x^2}}dx - \int_0^1 \frac{x}{\sqrt{1-x^2}}dx$$

$$= [\sin^{-1} x]_0^1 + [\sqrt{1-x^2}]_0^1$$

$$= (\sin^{-1}1 - \sin^{-1}0) + (\sqrt{1-1} - \sqrt{1-0}) = \frac{\pi}{2} - 1.$$

**53.** **(a)** Given $\int \frac{x+5}{x^2+4x+5}dx = a\log(x^2+4x+5)$

$$+ b\tan^{-1}(x+k) + C$$

Let $I = \int \frac{x+5}{x^2+4x+5}dx$

Put $x + 5 = \lambda(2x+4) + \mu$

On comparing both sides, we get $1 = 2\lambda$ and $5 = 4\lambda + \mu$

$$\Rightarrow \lambda = \frac{1}{2} \text{ and } \mu = 3$$

$$\therefore \quad I = \frac{1}{2}\int \frac{2x+4}{x^2+4x+5}dx + 3\int \frac{dx}{x^2+4x+5}$$

$$= \frac{1}{2}\log(x^2+4x+5) + 3\int \frac{dx}{(x+2)^2+(1)^2}$$

$$= \frac{1}{2}\log(x^2+4x+5) + 3\tan^{-1}(x+2) + C$$

$$\therefore \quad a = \frac{1}{2},\ b = 3,\ k = 2.$$

**54.** **(b)** Given $\int \sqrt{e^x - 4}\, dx$

Let $I = \int \sqrt{e^x - 4}\, dx$

Put $e^x - 4 = t^2 \Rightarrow dx = \frac{2t}{t^2+4} dt$

$$I = 2\int \frac{t^2}{t^2+4} dt = 2\int \frac{t^2+4-4}{t^2+4} dt$$

$$= 2\left[\int 1 - \frac{4}{t^2+(2)^2} dt\right] = 2\left[t - 4\cdot\frac{1}{2}\tan^{-1}\frac{t}{2}\right] + C$$

$$= 2t - 4\tan^{-1}\frac{t}{2} + C$$

$$= 2\sqrt{e^x-4} - 4\tan^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right) + C$$

**55.** **(d)** Given $\int e^{-x}\tan^{-1}(e^x)dx = f(x) - \frac{1}{2}\log(1+e^{2x}) + C$

Let $I = \int e^{-x}\tan^{-1}(e^x)\,dx$

Put $e^x = t$

$\Rightarrow \quad dx = \frac{1}{e^x}dt \Rightarrow dx = \frac{1}{t}dt$

$\therefore \quad I = \int \frac{\tan^{-1} t}{t^2} dt$ [Integration by parts]

$$= -\frac{1}{t}\tan^{-1} t - \int \frac{-1}{t}\cdot\frac{1}{1+t^2} dt$$

$$= \frac{-1}{t}\tan^{-1} t + \int\left(\frac{1}{t} - \frac{t}{1+t^2}\right) dt$$

$$= \frac{-1}{t}\tan^{-1} t + \log t - \frac{1}{2}\log(1+t^2) + C$$

$$= -e^{-x}\tan^{-1}(e^x) + \log(e^x) - \frac{1}{2}\log(1+e^{2x}) + C$$

$\therefore \quad f(x) = -e^{-x}\tan^{-1}(e^x) + x = x - e^{-x}\tan^{-1}(e^x)$

**56.** **(d)** Given $\int \sqrt{\frac{2+x}{2-x}}\, dx$

Let $I = \int \sqrt{\frac{2+x}{2-x}}\, dx = \int \frac{2+x}{\sqrt{4-x^2}} dx$

$$= 2\int \frac{1}{\sqrt{(2)^2 - x^2}} dx + \int \frac{x}{\sqrt{4-x^2}} dx$$

$$= 2\sin^{-1}\frac{x}{2} - \sqrt{4-x^2} + C$$

**57.** **(a)** Given, particles $P:(t,\, t^3 - 16t - 3)$ and $Q:(t+1,\, t^3 - 6t - 6)$ are moving in a plane.

Hence,

$$PQ = \sqrt{(t+1-t)^2 + (t^3 - 6t - 6 - t^3 + 16t + 3)^2}$$

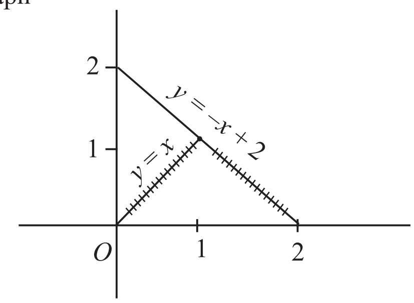
$$= \sqrt{1 + (10t-3)^2}$$

$\therefore \quad PQ^2 = 1 + (10t-3)^2 \geq 1 \qquad \{\because (10t-3)^2 \geq 0\}$

Hence, minimum value of $PQ$ is 1.

**58.** **(d)** We have, $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2-x, & 1 \leq x \leq 2 \end{cases}$

By Graph




We see that a corner point at $x = 1$ on the graph hence option (d) is correct.

**59.** **(c)** It is given that side of an equilateral $\Delta$ is 10 units and error is 0.05.

If $A$ be the area and $x$ be the side then, $A = \frac{\sqrt{3}}{4}x^2$

or, $\frac{dA}{dx} = \frac{\sqrt{3}}{4}(2x) \Rightarrow dA = \frac{\sqrt{3}}{2}x\cdot dx$ or $\Delta A = \frac{\sqrt{3}}{2}x\Delta x$

Now, percentage error in area $= \frac{\Delta A}{A}\times 100$

$$= \frac{\frac{\sqrt{3}}{2}x\Delta x\times 100}{\frac{\sqrt{3}}{4}x^2}$$

$$= \frac{2\times 0.05}{10}\times 100 = 1\%. \qquad [\because \Delta x = 0.05, \text{ given}]$$

**60.** **(a)** Given, equation of the line, $y = -4x + b$

Hence, slope, $m = -4$.

Also, slope of tangent to the curve $y = \frac{1}{x}$ is $\frac{dy}{dx} = \frac{-1}{x^2}$.

Since, given line is tangent to the curve.

$\therefore \quad \frac{-1}{x^2} = -4 \Rightarrow x^2 = \frac{1}{4}$

$\Rightarrow \quad x = \pm\frac{1}{2}$ or $y = \pm 2$

Put these values in $y = -4x + b$, we get $b = \pm 4$.

**61.** **(b)** Given, $x = \frac{1-\sqrt{y}}{1+\sqrt{y}}$

$\Rightarrow \quad x + x\sqrt{y} = 1 - \sqrt{y} \Rightarrow x + (x+1)\sqrt{y} = 1$

On differentiating w.r.t. $x$,

$$1+\frac{(x+1)}{2\sqrt{y}}y'+\sqrt{y}=0$$

$$\Rightarrow\ (x+1)y'+2y=-2\sqrt{y}$$

Again differentiating both sides w.r.t. $x$, we get

$$(x+1)y''+y'+2y'=\frac{-2}{2\sqrt{y}}y'$$

$$\Rightarrow\ (x+1)y''+3y'+\frac{1}{\sqrt{y}}y'=0$$

$$\Rightarrow\ (x+1)y''+\left(\frac{3\sqrt{y}+1}{\sqrt{y}}\right)y'=0$$

**62.** **(b)** Given, $x^2+y^2=t+\frac{2}{t}$ and $x^4+y^4=t^2+\frac{4}{t^2}$

On squaring both sides of, $x^2+y^2=t+\frac{2}{t}$

We get, $x^4+y^4+2x^2y^2=t^2+\frac{4}{t^2}+4$

$$\Rightarrow\ x^4+y^4+2x^2y^2=x^4+y^4+4$$

$$\Rightarrow\ 2x^2y^2=4\Rightarrow x^2y^2=2$$

$$\Rightarrow\ y^2=\frac{2}{x^2}$$

On differentiating, $2y\frac{dy}{dx}=\frac{-4}{x^3}\Rightarrow x^3y\frac{dy}{dx}=-2.$

**63.** **(d)** Given,

$$y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)+\tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)$$

Let $3\tan^{-1}(x)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

and $4\tan^{-1}(x)=\tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)$

$$\Rightarrow\ y=3\tan^{-1}(x)+4\tan^{-1}(x)$$

$$y'=\frac{3}{1+x^2}+\frac{4}{1+x^2}=\frac{7}{1+x^2}$$

**64.** **(a)** Given, $f(x)$ is continuous at $x=0$.

$$\therefore\ f(0)=\lim_{x\to 0}f(x)$$

$$=\lim_{x\to 0}(x+1)^{\cot x}=\lim_{x\to 0}(1+x)^{\cot x}$$

$$\left(\because \lim_{x\to 0}\frac{x}{\tan x}=1\right)$$

$$=e^{\lim_{x\to 0}x\cdot\cot x}=e^{\lim_{x\to 0}\frac{x}{\tan x}}=e^1=e.$$

**65.** **(a)** $\lim_{x\to 0}\left[\tan\left(x+\frac{\pi}{4}\right)\right]^{1/x}$

Since $\lim_{x\to a}\{f(x)\}^{g(x)}=\lim_{x\to a}[1+f(x)-1]^{g(x)}$

$$=e^{\lim_{x\to a}(f(2)-1)g(x)}$$

$$\Rightarrow\ \lim_{x\to 0}\left[1+\tan\left(x+\frac{\pi}{4}\right)-1\right]^{1/x}$$

$$=e^{\lim_{x\to 0}\left[\tan\left(x+\frac{\pi}{4}\right)-1\right]\frac{1}{x}}=e^{\lim_{x\to 0}\left[\frac{\tan x+1}{1-\tan x}-1\right]\frac{1}{x}}$$

$$=e^{\lim_{x\to 0}\frac{\tan x+1-1+\tan x}{x(1-\tan x)}}=e^{\lim_{x\to 0}\frac{2\tan x}{x(1-\tan x)}}=e^2.$$

**66.** **(b)** Let intercept form of the equation of the plane be

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Since, plane meets the coordinate axes at $P$, $O$, $R$ respectively. So, the coordinates of the points $P$, $Q$, $R$ are $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ respectively.

Also, centroid of $\Delta PQR$ is

$$\left(1,\frac{1}{2},\frac{1}{3}\right)\equiv\left(\frac{\Sigma x_1}{3},\frac{\Sigma y_1}{3},\frac{\Sigma z_1}{3}\right)$$

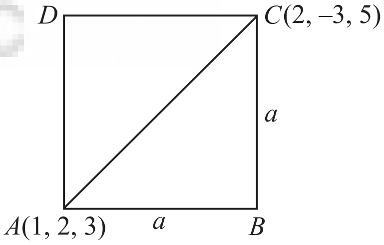
$$\therefore\ \frac{a+0+0}{3}=1\Rightarrow a=3\Rightarrow\ \frac{0+b+0}{3}=\frac{1}{2}\Rightarrow b=\frac{3}{2}$$

and $\frac{0+0+c}{3}=\frac{1}{3}\Rightarrow c=1$

$\therefore$ Equation of plane is $\frac{x}{3}+\frac{y}{3/2}+\frac{z}{1}=1$

$\Rightarrow\ x+2y+3z=3.$

**67.** **(c)** Let the side of the square be a



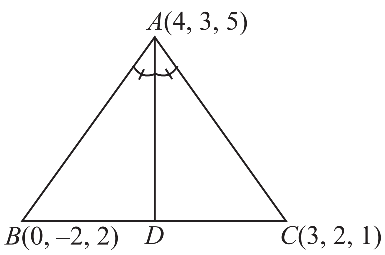

Hence, Diagonal $AC$

$$=\sqrt{(2-1)^2+(-3-2)^2+(5-3)^2}$$

$$=\sqrt{30}=\sqrt{2}\cdot\sqrt{15}=\sqrt{2}\cdot a$$

$$\Rightarrow\ a=\sqrt{15}.$$

**68.** **(a)** Since the angle bisector of $\angle BAC$ meets $BC$ at $D$ which divides $BC$ in the ratio $AB : AC$.



Here $AB \cdot CD = AC \cdot BD$
or $AB : AC = BD : CD$.

Now, $AB = \sqrt{(0-4)^2 + (-2-3)^2 + (2-5)^2} = 5\sqrt{2}$.

$$AC = \sqrt{(3-4)^2 + (2-3)^2 + (1-5)^2} = 3\sqrt{2}.$$

$$\therefore \quad \frac{BD}{CD} = \frac{AB}{AC} = \frac{5\sqrt{2}}{3\sqrt{2}} \Rightarrow \frac{BD}{CD} = \frac{5}{3}$$

Hence, $D$ divides $BC$ in the ratio 5 : 3.
So, coordinates of $D$

$$= \left(\frac{5\times3+3\times0}{5+3}, \frac{5\times2+3(-2)}{5+3}, \frac{5\times1+3\times2}{5+3}\right)$$

$$= \left(\frac{15}{8}, \frac{4}{8}, \frac{11}{8}\right).$$

**69.** **(c)** Since, product of lengths of perpendiculars from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2}$ is $\frac{a^2b^2}{a^2+b^2}$.

Hence, equation of hyperbola, $x^2 - y^2 = 16$

is written as $\frac{x^2}{(4)^2} - \frac{y^2}{(4)^2} = 1$

where $a = 4$ and $b = 4$.

So, $\frac{a^2b^2}{a^2+b^2} = \frac{16\times16}{16+16} = \frac{16\times16}{2\times16} = 8$.

**70.** **(d)** Given, equation of the ellipse

$$\frac{(x+y-3)^2}{9} + \frac{(x-y+1)^2}{16} = 1$$

To determine the centre of ellipse,

$x + y = 3$ ...(i)

$x - y = -1$ ...(ii)

From equations (i) and (ii),

$x = 1$ and $y = 2$

Hence, the centre of the ellipse is (1, 2).

**71.** **(b)** Given equation of the ellipse is

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

Here, $a = 5$ and $b = 4$.

(P) Equation of the directrix corresponding to the focus (–3, 0) is

Since $= \sqrt{1 - \text{——}} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

$$x = \frac{-a}{e} \Rightarrow x = -\frac{5}{3/5}$$

$\Rightarrow \; 3x = -25 \Rightarrow 3x + 25 = 0.$

(Q) Equation of tangent at the vertex (0, 4) is

$$y = b \Rightarrow y = 4$$

(R) Equation of $\angle R$ through (3, 0) is

$x = ae$

$\Rightarrow \; x = 5 \times \frac{3}{5} \Rightarrow x = 3.$

**72.** **(a)** Given $P$ be a point on parabola $y^2 = 8x$ and point $A$ is (1, 0).

Let $P : (2t^2, 4t)$

Now if $(x, y)$ be the mid-point of $P(2t^2, 4t)$ and $A(1, 0)$.

Hence, $x = \frac{2t^2+1}{2}$ ...(i)

and $y = \frac{4t+0}{2} = 2t$ ...(ii)

From equations (i) and (ii) we get,

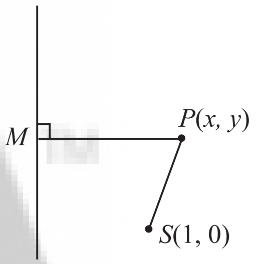
$$x = \frac{y^2+2}{4}$$

$\Rightarrow \; 4x = y^2 + 2 \;\Rightarrow\; y^2 = 4\left(x - \frac{1}{2}\right)$

Hence, the locus of the mid-point of the line segment $AP$ is $y^2 = 4\left(x - \frac{1}{2}\right)$.

**73.** **(a)** According to basic definition of Parabola,

$PS = PM$




$\therefore \quad PS = PM$ or $PS^2 = PM^2$

where, $M$ is the point on directrix.

$$\Rightarrow \; (x-1)^2 + (y+1)^2 = \left(\frac{x+y+3}{\sqrt{2}}\right)^2$$

$$\Rightarrow \; 2(x^2+y^2-2x+2y+2) = (x+y+3)^2$$

$$\Rightarrow \; 2x^2+2y^2-4x+4y+4$$

$$= x^2+y^2+9+2xy+6y+6x$$

$$\Rightarrow \; x^2+y^2-10x-2y-2xy-5=0$$

**74.** **(c)** Basically, we are required a circle passing through three points.

Hence the equation of the circle passing through the points of intersection of the given circles, is

$$S_1 + \lambda S_2 = 0$$

$$\Rightarrow \; (x^2+y^2-8x-6y+21)$$

$$+\lambda(x^2+y^2-2x-15) = 0 \quad \text{...(i)}$$

Now, since this circle passes through point (1, 2), then (1 + 4 – 8 – 12 + 21) + λ(1 + 4 – 2 – 15) = 0

$\Rightarrow \; 6\lambda = 12 \;\Rightarrow\; \lambda = \frac{1}{2}$

On substituting $\lambda = \frac{1}{2}$ in Eq. (i), we get the equation of the required circle as

$$x^2 + y^2 - 8x - 6y + 21 + \frac{x^2}{2} + \frac{y^2}{2} - x - \frac{15}{2} = 0$$

$$\Rightarrow 2x^2 + 2y^2 - 16x - 12y + 42 + x^2 + y^2 - 2x - 15 = 0$$

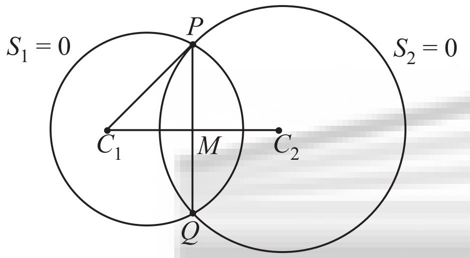
$$\Rightarrow 3x^2 + 3y^2 - 18x - 12y + 27 = 0$$

$$\Rightarrow x^2 + y^2 - 6x - 4y + 9 = 0.$$

**75.** **(b)** Let, $S_1 : x^2 + y^2 - 2ax = 0$

and, $S_2 : x^2 + y^2 - 2by = 0$

Hence, the equation of the common chord $PQ$ of the circles




is $S_1 - S_2 = 0$

$\Rightarrow 2by - 2ax = 0 \Rightarrow ax - by = 0$

The centre of $S_1$ is $(a, 0)$ and radius is $a$.

The length of the perpendicular from $(a, 0)$ to $ax - by = 0$, is

$$C_1M = \left|\frac{a^2}{\sqrt{a^2 + b^2}}\right|$$

Now, $PQ = 2PM = 2\sqrt{C_1P^2 - C_1M^2}$

$$= 2\sqrt{a^2 - \frac{a^4}{a^2 + b^2}} = \frac{2ab}{\sqrt{a^2 + b^2}}.$$

**76.** **(a)** Equation of polar of point (4, 2) w.r.t. circle $x^2 + y^2 - 5x + 8y + 6 = 0$ is $T = 0$.

$\therefore \quad 4x + 2y - \frac{5}{2}(x + 4) + 4(y + 2) + 6 = 0$

$\Rightarrow 3x + 12y + 8 = 0 \qquad$ ...(i)

Since, points (4, 2) and $(k, -3)$ are conjugate points w.r.t. the given circle.

So, $(k, -3)$ satisfied line (i).

$\therefore \quad 3k - 36 + 8 = 0$

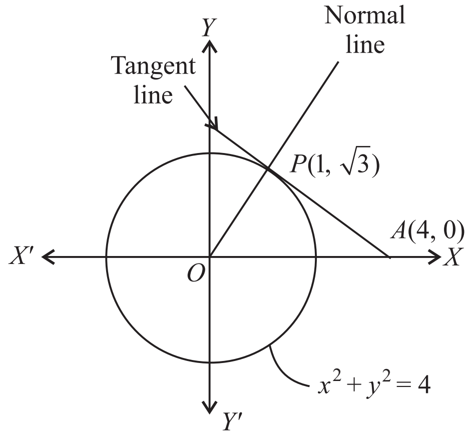
$\Rightarrow k = \frac{28}{3}.$

**77.** **(c)** Let equation of the tangent to the circle $x^2 + y^2 = 4$ at $P(1, \sqrt{3})$ is $xx_1 + yy_1 = a^2$

$\Rightarrow x + \sqrt{3}y = 4$

and normal to the tangent is $y = \sqrt{3}x$

The tangent meets $X$-axis at $A(4, 0)$.




$\therefore$ Area of $\Delta OAP = \frac{1}{2} \times 4 \times \sqrt{3} = 2\sqrt{3}$ sq. units.

**78.** **(d)** Given power of point $P(16)$ w.r.t. the circle is –16.

$\Rightarrow S_1 < 0$

Given equation of the circle is

$$x^2 + y^2 + 4x - 6y - a = 0$$

Hence, $1 + 36 + 4 - 36 - a = -16$

$\Rightarrow 5 - a = -16 \Rightarrow a = 21.$

**79.** **(d)** Given two degree equation,

$$x^2 + y^2 - 2x - 4y + 2 = 0$$

and given line, $L \equiv x + y = k \Rightarrow L = \left(\frac{x + y}{k}\right)$

To make homogenious, the pair of lines

$$x^2 + y^2 - 2x(1) - 4y(1) + 2\left(\frac{x + y}{k}\right)^2 = 0$$

We get

$$x^2 + y^2 - 2x\left(\frac{x + y}{k}\right) - 4y\left(\frac{x + y}{k}\right) + 2\left(\frac{x + y}{k}\right)^2 = 0$$

Since, intersection points of line and pair of lines make an angle 90° at origin $O$.

$\therefore$ Coefficient of $x^2$ + Coefficient of $y^2 = 0$

$$\Rightarrow \left(1 - \frac{2}{k} + \frac{2}{k^2}\right) + \left(1 - \frac{4}{k} + \frac{2}{k^2}\right) = 0$$

$\Rightarrow k^2 - 3k + 2 = 0 \Rightarrow (k - 2)(k - 1) = 0$

$\because \; k > 1$ so, $k = 2$

**80.** **(c)** Since given equation $xy + 4x - 3y - 12 = 0$ can be written as

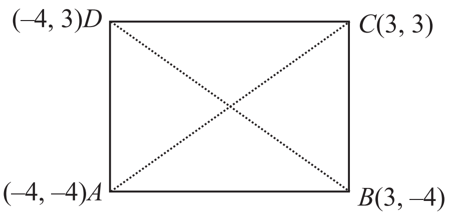
$x(y + 4) - 3(y + 4) = 0$

$\Rightarrow (y + 4)(x - 3) = 0$

Similarly the equation $xy - 3x + 4y - 12 = 0$ can be written as

$x(y - 3) + 4(y - 3) = 0 \Rightarrow (y - 3)(x + 4) = 0$

So, the coordinates of the vertex are $A(-4, -4)$, $B(3, -4)$, $C(3, 3)$ and $D(-4, 3)$.




$\therefore$ Equation of $AC$ : $y+4=\frac{7}{7}(x+4)$

$\Rightarrow$ $y=x \Rightarrow x-y=0$

$\therefore$ Equation $BD$ : $y+4=\frac{7}{7}(x-3)$

$\Rightarrow$ $y+4=-(x-3)$

$\Rightarrow$ $y+4=-x+3 \Rightarrow x+y+1=0$

Hence, combined equation of diagonals is

$(x-y)(x+y+1)=0$

$\Rightarrow$ $x^2+xy+x-xy-y^2-y=0$

$\Rightarrow$ $x^2-y^2+x-y=0$

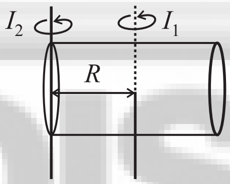
## PHYSICS

**81.** **(b)** According to question,

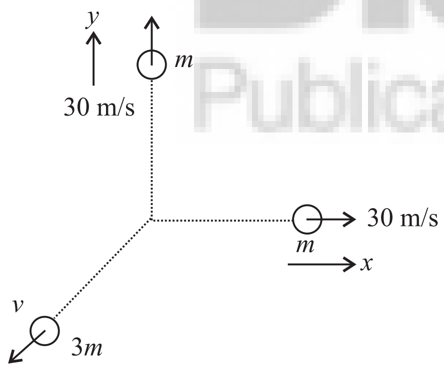
By perpendicular axis theorem

$I_2=I_1+MR^2$

$I_2-I_1=MR^2$




**82.** **(a)** Let $\vec{v}=v_x\hat{i}+v_y\hat{j}$




as $\overrightarrow{p_i}=\overrightarrow{p_f}$

$\Rightarrow$ $0=30m\,\hat{i}+30m\,\hat{j}+3m(v_x\hat{i}+v_y\hat{j})$

$\Rightarrow$ $v_x=-10\hat{i}$ and $v_y=-10\hat{j}$

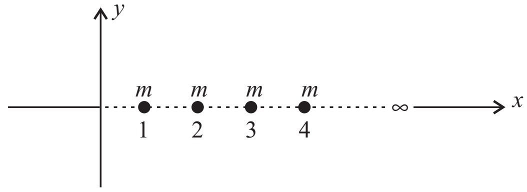
$\therefore$ $v=\sqrt{|v_x|^2+|v_y|^2}=10\sqrt{2}$ m/s

**83.** **(a)** Maximum kinetic energy $=\frac{1}{2}m\omega^2a^2$

Here, $\omega$ = 100 rad/s and $a$ = 8 cm.

$=\frac{1}{2}\times4\times100\times100\times(8\times10^{-2})^2=128$ J.

**84.** **(b)**




$=\frac{Gm}{(1)^2}+\frac{Gm}{(2)^2}+\frac{Gm}{(4)^2}+\frac{Gm}{(8)^2}.....$

$=Gm\left[1+\frac{1}{4}+\frac{1}{16}+.....\right]=Gm\left(\frac{1}{1-\frac{1}{4}}\right)=\frac{4}{3}Gm.$

**85.** **(c)** As, $F\propto\Delta L$

So, $F_1\propto(L_1-L)$

$\Rightarrow$ $F_1=K(L_1-L)$ ...(A)

Similarly, $F_2=K(L_2-L)$ ...(B)

Dividing Eq. (A) by Eq. (B) we get

$\frac{F_1}{F_2}=\frac{L_1-L}{L_2-L} \Rightarrow F_1L_2-F_1L=F_2L_1-F_2L$

$\Rightarrow$ $L=\frac{F_1L_2-F_2L_1}{F_1-F_2}$

**86.** **(b)** As $V_i=V_f \Rightarrow \frac{4}{3}\pi R^3=1000\times\frac{4}{3}\pi r^3 \Rightarrow R=10r$

$\Delta E=\Delta A\times S$

$=(4\pi R^2-1000\,4\pi\,r^2)\times S$

$=4\pi\left(R^2-1000\times\frac{R^2}{100}\right)\times S=-36S\pi R^2.$

$|\Delta E|=36S\pi R^2$

$=36\times0.075\times\pi R^2=6.75\pi\times10^{-15}$ J.

**87.** **(c)** Let after getting equilibrium stage final temperature is $T$.

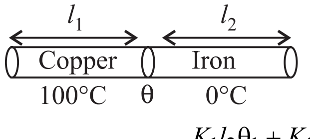
Heat lost by Coffee = Heat gain by Milk

$\Rightarrow$ $(250)(1)(90-T)=20(1)(T-5)$

$\Rightarrow$ $T=83.7°C$.

**88.** **(c)** Let temperature of copper and iron junction is $\theta$.

Then, $\frac{K_1A(\theta_1-\theta)}{l_1}=\frac{K_2A(\theta-\theta_2)}{l_2}$




So, temperature of junction $(\theta)=\frac{K_1l_2\theta_1+K_2l_1\theta_2}{K_1l_2+K_2l_1}$

$=\frac{386.4\times125\times100+48.46\times75\times0}{386.4\times125+48.46\times75}$

$=\frac{48300\times100}{48300+3634.5}=0.93\times100=93°C.$

**89.** **(b)** $dQ = mL$

$= 1 \times 540$ calorie $\quad [\because L_V = 540 \text{ cal/gm}]$

$= 540 \times 4.2 = 2268$ J

Work done in expanding volume

$dW = pdV = 10^5 (V_2 - V_1)$ [$\because$ $P$ = constant

$= 10^5 (1650) \times 10^{-6}$ $\quad$ so, $dW = pdV$]

$= 1650 \times 10^{-1} = 165$ J.

Increase in internal energy

$dU = dQ - dW = 2268 - 165 = 2103$ J.

So, none of the option is correct.

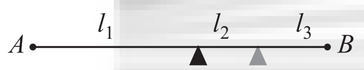
**90.** **(b)** $V_{rms} \propto \dfrac{1}{\sqrt{M}}$ $\quad [\because \text{T = const.}]$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}} \Rightarrow \frac{0.5}{v_2} = \sqrt{\frac{2}{32}} \Rightarrow \frac{0.5}{v_2} = \frac{1}{4}$$

$\Rightarrow v_2 = 2$ km/s.

**91.** **(c)** Ratio of fundamental frequency $(n_1 : n_2 : n_3) = 1 : 2 : 3$

As $f_0 = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$




So, $l_1 : l_2 : l_3 = \dfrac{1}{n_1} : \dfrac{1}{n_2} : \dfrac{1}{n_3}$ $\quad [\because T$ and $\mu$ are constant]

$$= \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2.$$

$\therefore$ $6x + 3x + 2x = 99 \Rightarrow x = 9$

Then, $l_1 = 6x = 54$, $l_2 = 3x = 27$, $l_3 = 2x = 18$

**92.** **(b)** $f_{net} = \left(\dfrac{1}{f}\right) = \dfrac{1}{f_1} + \dfrac{1}{f_2} + \dfrac{1}{f_3}$

$$\Rightarrow \frac{1}{f} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \Rightarrow f = 1 \text{ cm}.$$

Magnification of the lens combination in normal adjustment is

$$M = 1 + \frac{D}{F} = 1 + \frac{25}{1} = 26.$$

**93.** **(a)** Focal length of lens when placed in air

$$f_a = (\mu_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

Focal length of lens when placed in liquid

$$f_{liq} = \left(\frac{\mu_l}{\mu_{liq}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii)

$$\frac{f_{liq}}{f_a} = \frac{(1.45-1)}{\left(\dfrac{1.45-1}{1.3}\right)} = \frac{0.45}{0.15} \times 1.3 = 0.256.$$

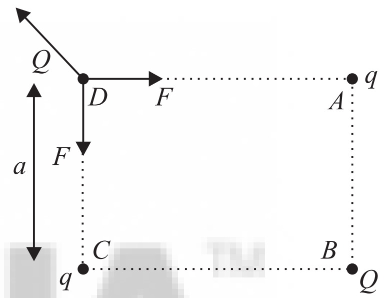
So, none of the option is correct.

**94.** **(b)** Required width $= \dfrac{\lambda D}{a} + \dfrac{\lambda D}{a} = \dfrac{2\lambda D}{a}$

$$= \frac{2 \times 6330 \times 10^{-10} \times 2}{2 \times 10^{-3}}$$

$= 1.27$ mm.

**95.** **(a)**



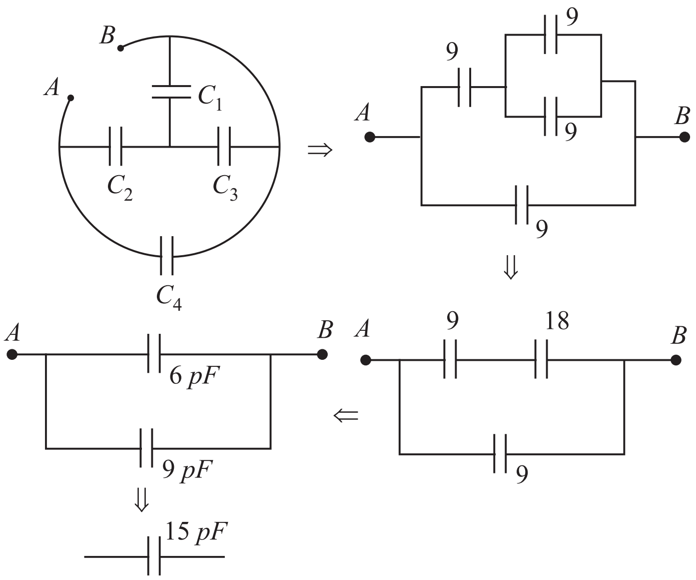

For net electric field on $D$ to be zero

$$K\left[\frac{Q^2}{2a^2} + \frac{\sqrt{2}Qq}{a^2}\right] = 0 \Rightarrow \frac{Q}{2} + \sqrt{2}q = 0$$

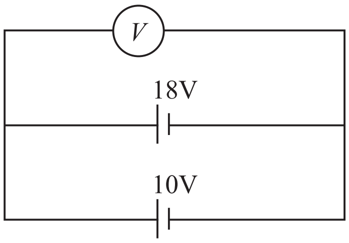
$\Rightarrow Q = -2\sqrt{2}q.$

$\therefore$ $q$ should be negative.

**96.** **(b)** Effective capacitance between the points $A$ and $B$.



**97.** **(b)**




$$E_{net} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{\frac{18}{3} + \frac{10}{1}}{\frac{1}{3} + 1} = 12\,\text{V}$$

$\therefore$ $E_{\text{net}}$ = reading of voltmeter = 12 V.

**98.** **(d)** $R_{\text{cond.}} \propto T$ and $R_{\text{semi cond.}} \propto \frac{1}{T}$

So, resistance of Al decreases and Ge increases.

**99.** **(c)** We have, $V_g = GI_g$

$\Rightarrow$ $250 \times 10^{-3} = 10 \times I_g$

$\Rightarrow$ $I_g = 25 \times 10^{-3}$ A.

Let the shunt required to convert this voltmeter into ammeter be $R_s$.

Then, $(I - I_g)R_s = I_g R_g \;\Rightarrow\; R_s = \frac{I_g R_g}{I - I_g}$

$$= \frac{10 \times (25 \times 10^{-3})}{250 \times 10^{-3} - 25 \times 10^{-3}} = \frac{10}{9} \approx 1\,\Omega.$$

**100.** **(d)** Here, $l = 2\pi r \Rightarrow r = \frac{l}{2\pi}$

and, $l = 4a \Rightarrow a = \frac{l}{4}$

Here, $\frac{M_1}{M_2} = \frac{i\pi r^2}{ia^2} = \pi\left[\frac{\left(\frac{l}{2\pi}\right)^2}{\left(\frac{l}{4}\right)^2}\right] = \pi\frac{16}{4\pi^2} = \frac{4}{\pi}.$

**101.** **(a)** Time period of magnet in magnetic field

$T \propto \frac{1}{\sqrt{B}}$ $\qquad \left[\because T = 2\pi\sqrt{\frac{I}{MB}}\right]$

It magnetic field is increased 2 times, then

$T' \propto \frac{1}{\sqrt{2B}}$

$\Rightarrow T' \propto \frac{T}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}s \qquad \left[\because T = \frac{60}{30} = 2\text{ sec.}\right]$

**102.** **(d)**

(A) Newton's law is used in rocket propulsion.
(B) Bernoulli's principle is used in aeroplane.
(C) Total internal reflection is used by optical fibres.
(D) Magnetic confinement of plasma is used in fusion test reactor by the scientists.

**103.** **(a)** As $F = \frac{X}{m/l}$

$\Rightarrow X = \frac{Fm}{l} \Rightarrow \qquad [X] = \left[\frac{Fm}{l}\right]$

$$= \frac{[MLT^{-2}][M]}{L} = M^2L^0T^{-2}.$$

**104.** **(b)** $v = \frac{dx}{dt} = 3At^2 + 2Bt + C$

$v_i = v(t = 0) = C$

and $a = \frac{d^2x}{dt^2} = 6At + 2B$

$a_i = a(t = 0) = 2B$ So, $\frac{a_i}{v_i} = \frac{2B}{C}$

**105.** **(b)** Let $AB = BC = h$

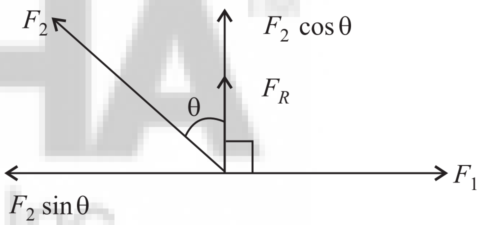
Then, $t_1 = \sqrt{\frac{2h}{g}}$

Time taken to travel from $B$ to $C$,

$t_2 = t_{AC} - t_{AB} = t_{AC} - t_1$

$t_2 = \sqrt{\frac{4h}{g}} - \sqrt{\frac{2h}{g}} \Rightarrow \frac{t_2}{t_1} = \sqrt{2} - 1.$

**106.** **(a)** We have $F_1 + F_2 = 25$ ...(i)




As $F_R \perp F_1$, so $F_2\cos\theta = 10$, $F_2\sin\theta = F_1$

Now, $F_R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos(90 + \theta)$

$\Rightarrow$ $F_R^2 = F_1^2 + F_2^2 + (-2F_1F_2\sin\theta)$

$\Rightarrow$ $100 = F_1^2 + F_2^2 - 2F_1^2$

$\Rightarrow$ $100 = F_2^2 - F_1^2 \Rightarrow 100 = (F_2 - F_1)(F_2 + F_1)$

$\Rightarrow$ $100 = (F_2 - F_1)25$

$\Rightarrow$ $F_2 - F_1 = 4$ ...(ii)

Solving (i) and (ii), we get

$F_1 = 10.5$ N, $F_2 = 14.5$ N.

**107.** **(b)** Maximum vertical height attained by body thrown with velocity $v_1$.

$h_1 = \frac{v_1^2}{2g}$ ...(i)

Another body of mass $2m$ is projected with a velocity $v_2$ at an angle $\theta$.

$\therefore$ Height attained $(h_2) = \frac{v_2^2\sin^2\theta}{2g}$ ...(ii)

Dividing (ii) by (i), we get

$$\frac{h_2}{h_1} = \frac{v_2^2}{v_1^2}\sin^2\theta$$

But $t_1 = 2t_2$

$$\therefore \quad \frac{v_1}{g} = 2\left[\frac{v_2 \sin\theta}{g}\right] \Rightarrow v_1 = 2v_2 \sin\theta$$

$$\therefore \quad \frac{h_1}{h_2} = \left(\frac{2}{1}\right)^2 = \frac{4}{1}.$$

**108. (b)** From law of conservation of energy

$$M \times v + m(0) = (M+m)v' \Rightarrow v' = \frac{M \times 20}{(M+m)}$$

Average resistance of the wall to the penetration of the nail is

$$F = (M+m)\frac{(0^2+v^2)}{2\times 10^{-2}}$$

$$= (M+m)\left(\frac{20}{M \quad m}\right) \times \frac{1}{2\times 10}$$

$$= \frac{2M^2}{(M+m)} \times 10^4.$$

**109. (c)** $F = 3t^2 - 30$

$$a = \frac{1}{m}(3t^2 - 30)$$

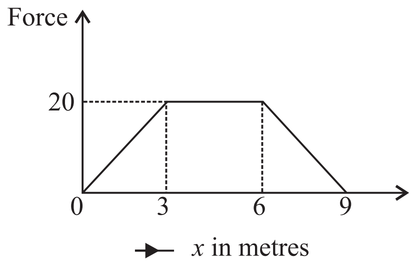
$$\int_{10}^{v} dV = \frac{1}{m}\int_0 (3t^2 - 30)\,dt$$

$$\Rightarrow \quad V - 10 = \frac{1}{10}[t^3 - 30t]_0^5 \Rightarrow V = 7.5 \text{ m/s.}$$

**110. (a)** Here, $W \propto S_n = \frac{1}{2}g(2n-1)$ $n = 1, 2, 3, ....$

$\therefore$ Ratio of work done by the force is 1 : 3 : 5.

**111. (c)** Work done in a moving body is get converted into kinetic energy.




By work-energy theorem, W = $\Delta K$

$$\Rightarrow \quad \int F dx = \frac{1}{2}m(v^2 - 0^2)$$

$$\Rightarrow \quad \frac{1}{2}(9+3)(20) = \frac{1}{2}(2.4)v^2$$

$\Rightarrow$ $v$ = 10 m/s.

**112. (c)** Modulation index $(\mu) = \frac{E_m}{E_c}$

$$\Rightarrow \quad \frac{3}{4} = \frac{E_m}{12} \Rightarrow E_m = 9$$ [no option]

**113. (d)** We know that, $n_e n_h = n_i^2$, so option (d) is correct.

**114. (a)** In half wave rectifier frequency do not change, only signal is C in one half cycle.

$\therefore$ Fundamental frequency of the output is 50 Hz.

**115. (b)** The effective half-life of the nucleus is given,

$\lambda = \lambda_1 + \lambda_2$

$$\Rightarrow \quad \frac{0.693}{T} = \frac{0.693}{T_1} + \frac{0.693}{T_2} \Rightarrow \frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$$

$$\Rightarrow \quad T = \frac{T_1 T_2}{T_1 + T_2}$$

$$= \frac{5\times 10^3 \times 10^5}{5\times 10^3[1+20]} = \frac{5\times 10^5}{105} = 4762 \text{ yr.}$$

**116. (a)** Statement *A* is false as spectral lines of Lyman series has smaller wavelength than second spectral line of Balmer series.

So, statement *A* is false whereas, statement *B* is true.

**117. (d)** de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mK.E.}}$$

$$\Rightarrow \quad K.E. = \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2$$

$$= \frac{1}{2\times 9.1\times 10^{-31}}\left(\frac{6.625\times 10^{-34}}{5.5\times 10^{-7}}\right)^2$$

$= 8 \times 10^{-25}$ J.

**118. (b)** Point where '$B$' to be calculated is not mentioned.

On surface $\int B \cdot dl = \mu_0\left(\varepsilon_0 \frac{d\phi}{dt}\right)$

$$\left[\because \; I = I_d = \text{displacement current } = \varepsilon_0 \frac{d\phi}{dt}\right]$$

$$\Rightarrow \quad B(2\pi r) = \frac{1}{C^2}\frac{d\phi}{dt} \qquad \left[\because \mu_0\varepsilon_0 = \frac{1}{C^2}\right]$$

$= 2 \times 10^{-3}$ $T$

**119. (a)** Current leads the voltage by 45°

$\therefore$ $\phi = 45°$

Since, $\tan\phi = \frac{X_C - X_L}{R} \Rightarrow 1 = \frac{X_C - X_L}{R}$

$$\Rightarrow \quad \frac{1}{\omega C} - \omega L = R \Rightarrow \frac{1}{\omega C} = R + \omega L$$

$$\Rightarrow \quad C = \frac{1}{\omega(R+\omega L)}$$

$$\Rightarrow \ C = \frac{1}{2\pi f[2\pi fL + R]} \qquad [\because \omega = 2\pi f]$$

**120. (b)** $M = \frac{\phi_{SL}}{I_L} = \frac{B_{SL}A_s}{I_L}$

$$B_{SL} = \frac{2\sqrt{2}\ \mu_0 I}{\pi L} \text{ and } M = \frac{\phi}{I} = \frac{2\sqrt{2}\ \mu_0 t^2}{\pi L}$$

$$\phi = B_{SL}A_S = \frac{2\sqrt{2}\ \mu_0 I}{\pi L} l^2 \ \therefore\ M \propto \frac{l^2}{L}$$

## CHEMISTRY

**121. (c)** Given,

| | $A$ | + | $B$ | $\rightleftharpoons$ | $C$ | + | $D$ |
|---|---|---|---|---|---|---|---|
| Initial moles | 1 | | 1 | | 0 | | 0 |
| Moles at equili. | (1–$x$) | | (1–$x$) | | $x$ | | $x$ |
| | (1–0.5) | | (1–0.5) | | 0.5 | | 0.5 |

$$\Rightarrow \ K_C = \frac{[C][D]}{[A][B]} = \frac{0.5\times 0.5}{0.5\times 0.5} = 1$$

**122. (d)** $\underset{x}{Ca_3(PO_4)_2} \rightleftharpoons \underset{3x}{3Ca^{2+}} + \underset{2x}{2PO_4^{3-}}$

$$k_{sp} = [Ca^{2+}]^3[PO_4^{3-}]^2 = [3x]^3[2x]^2 = [27x^3][4x^2]$$

$= 108\,x^5$.

**123. (a)** Clark's method is not used to remove permanent hardness of water. But it is used to remove temporary hardness of water.

$$Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3\downarrow + 2H_2O$$

$$Mg(HCO_3)_2 + 2Ca(OH)_2 \longrightarrow 2CaCO_3\downarrow + Mg(OH)_2\downarrow + 2H_2O$$

**124. (d)** White metal is an alloy of Li and Pb.

**125. (b)** Due ot inert pair effect, Tl in +1 oxidation state is more stable than that of +3 oxidation state.
This is due to the presence of intervening *d*- and *f*- orbitals which do not shield the nucleus effectively. Consequently effective nuclear charge increases, that holds the *ns*-electrons tightly so that they do not participate in bonding (inert pair effect).

**126. (a)** $H_2CO_3$ is a weak dibasic acid. It dissociates in following way

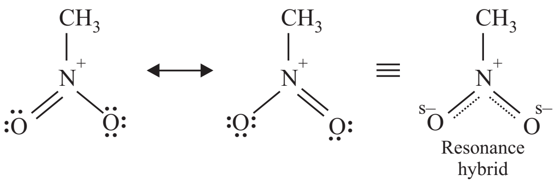
$$H_2CO_3(aq) + H_2O(l) \rightleftharpoons HCO_3^-(aq) + H_3O^+(aq.)$$

$$HCO_3^-(aq) + H_2O(l) \rightleftharpoons CO_3^{2-}(aq) + H_3O^+(aq.)$$

$H_2CO_3/HCO_3^-$ act as buffer system and helps to maintain pH of blood between 7.26 to 7.42.

**127. (b)** The amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water is called biochemical oxygen demand (BOD). More the amount of oxygen required to breakdown the organic material present in water, more will be the BOD values. Clean water have BOD value less than 5 ppm whereas highly polluted water such as municipal sewage has BOD value more than 100 ppm.

**128. (d)** Due to resonance effect, two bond N = O and N — O in $H_3CNO_2$ have same bond length.



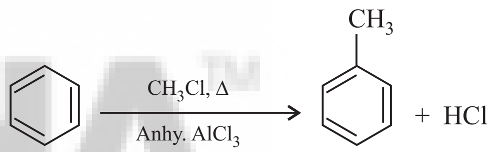

**129. (d)** According to Markownikoff's rule, the addition of reagents such as HCl, HBr, HI, HOCl with unsymmetrical alkenes occurs in such a way that the negative part of the adding molecule goes to that carbon atom of the double bond which carries lesser number of hydrogen atoms.

$$CH_3CH_2CH = CH_2 + HBr \xrightarrow[\text{Absence of peroxides}]{\text{Dark}}$$

$$\underset{\substack{\text{2-bromobutane}\\\text{(major product)}}}{CH_3CH_2 - \underset{|}{\overset{}{C}}H(Br) - CH_3}$$

$$+ \ \underset{\substack{\text{1-bromobutane}\\\text{(minor product)}}}{CH_3CH_3 - CH_2 - CH_2 - Br}$$

**130. (b)**




Benzene Toluene (*Z*)

It is a Friedel-Crafts reaction.

**131. (c)** Silicon carbide (SiC) is a covalent network solid. In this silicon atoms are connected with carbon atoms in a tetrahedral manner.

**132. (b)** Molarity

$$= \frac{\text{Weight of solute}}{\text{Molar mass of solute}} \times \frac{1000}{\text{volume (in mL)}}$$

$$0.2 = \frac{\text{Weight of solute}}{106} \times \frac{1000}{250}$$

$$\therefore \text{ Weight of solute } = \frac{0.2\times 106\times 250}{1000} = 5.3 \text{ g.}$$

**133. (d)** $\Delta T_b = K_b \cdot m$

$T_b - T_b^{\circ} = K_b \cdot m$

where, $K_b$ = molal elevation constant, $m$ = molality

$(100.52 - 100) = (0.52)(m)$

$0.52 = 0.52 \times m \Rightarrow m = 1.0.$

**134. (b)** During recharging, the cell is operated like an electrolytic cell. The electrode reactions are the reverse of those that occur during discharge.

**At cathode:**

$$PbSO_4(s) + 2e^- \longrightarrow Pb(s) + SO_4^{2-}(aq)$$

**At anode:**

$$PbSO_4(s) + 2H_2O(l) \longrightarrow PbO_2(s) + SO_4^{2-}(aq) + 4H^+(aq) + 2e^-$$

**Net reaction:**

$$2PbSO_4(s) + 2H_2O(l) \longrightarrow Pb(s) + PbO_2(s) + 4H^+(aq) + 2SO_4^{2-}(aq)$$

**135. (d)** For the reaction,

$$5Br^-(aq) + BrO_3^-(aq) + 6H^+(aq) \longrightarrow 3Br_2(aq) + 3H_2O(l)$$

$$\text{Rate of reaction} = -\frac{1}{5}\frac{d[Br^-]}{dt} = -\frac{d[BrO_3^-]}{dt}$$

$$= -\frac{1}{6}\frac{d[H^+]}{dt} = +\frac{1}{3}\frac{d[Br_2]}{dt}$$

$$\therefore \quad -\frac{1}{5}\frac{d[Br^-]}{dt} = -\frac{d[BrO_3^-]}{dt}$$

$$\Rightarrow \frac{-d[BrO_3]}{dt} = \frac{0.05}{5} \Rightarrow -\frac{d[BrO_3]}{dt} = 0.01$$

**136. (d)** Animal leather is colloidal in nature and is positively charged. When it is soaked in tannin which is a negatively charged colloid, it results in mutual coagulation and gets harden. Thus, leather get hardened after tanning.

**137. (d)** German silver (nickel silver) is an alloy of copper, zinc and nickel. It is used in cheap jewellery and cutlery.

**138. (d)** The catalytic oxidation of $SO_2$ with $O_2$ to give $SO_3$. It is used as the key step in the manufacturing of sulphuric acid by contact process.

$$2SO_2(g) + O_2(g) \xrightleftharpoons{V_2O_5} 2SO_3(g);$$

$$\Delta_r H^\circ = -196 \text{ kJ mol}^{-1}$$

**139. (c)** The balanced equation for reaction of ammonia with Chlorine.

$$NH_3 + 3Cl_2 \longrightarrow NCl_3 + 3HCl$$

Thus, the ratio of moles of $NH_3$ to $3Cl_2$ is 1 : 3.

**140. (a)** Generally, paramagnetism is shown by lanthanoid ions. The paramagnetism arises due to presence of unpaired electrons in *f*-orbital.

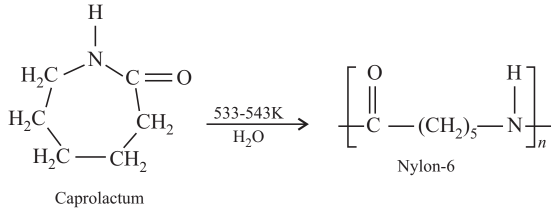
| | |
|---|---|
| $Lu^{3+}: [Xe]4f^{14}$ | (Diamagnetic) |
| $Ce^{3+}: [Xe]4f^{1}$ | (Paramagnetic) |
| $Eu^{3+}: [Xe]4f^{6}$ | (Paramagnetic) |
| $Yb^{3+}: [Xe]4f^{13}$ | (Paramagnetic) |

$Lu^{3+}$ ($4f^{14}$) does not have unpaired electrons. Hence, it does not exhibit paramagnetism.

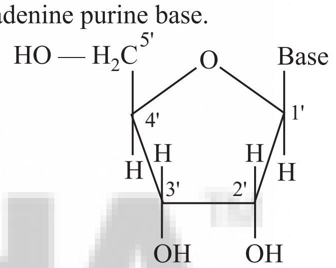
**141. (b)** Ligands have been arranged in an experimentally determined series based on the absorption of light by their complexes in increasing order of filled strength. The series is known as spectro chemical series.

$$I^- < Br^- < SCN^- < Cl^- < F^- < OH^- < C_2O_4^{2-}$$
$$< H_2O < NCS^- < EDTA^{4-} < NH_3 < en < CN^- < CO$$

**142. (c)** Nylon-6 is a condensation homopolymer. Obtained by heating caprolactum with $H_2O$ at high temperature.



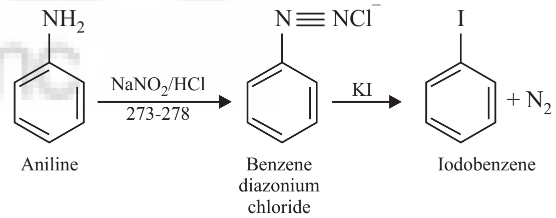

**143. (a)** Nucleoside consists of a pentose sugar and nitrogenous base. In this, base unit is attached at 1' position of sugar whereas in nucleotide phosphate group, is attached to s carbon of ribose sugar (option b). Option c is adenine purine base.



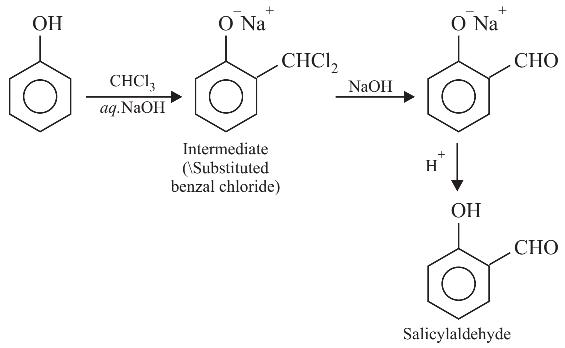
Structure of nucleoside

**144. (b)** Sulphapyridine is a sulphanilamide antibacterial substance used to treat skin infections.

**145. (c)**

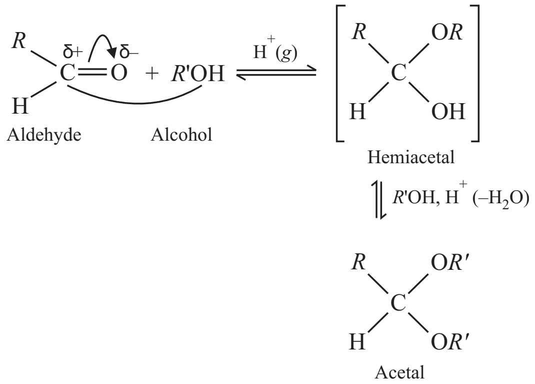



**146. (d) Reimer-Tiemann reaction :**




Hence, substituted benzal chloride is intermediate in Reimer-Tiemann reaction.

**147. (a)** Acetal are organic compounds formed by addition of alcohol molecules to aldehyde molecules.




**148. (b)** $CH_3CH_2COOH \xrightarrow[\Delta]{P_2O_5\cdot H^+} \underset{(X)}{(CH_3CH_2CO)_2O}$

$$\xrightarrow{H_2O} \underset{(Y)}{CH_3CH_2COOH} \xrightarrow{SOCl_2}$$

$$\underset{(Z)}{CH_3CH_2COCl} + SO_2 + HCl$$

**149. (b)** $\underset{1°\text{ amide}}{H_3CCONH_2} + Br_2 + 4NaOH \xrightarrow[\text{degradation}]{\text{Hoffmann}}$

$$\underset{(Y)}{CH_3NH_2} + Na_2CO_3 + 2NaBr + 2H_2O$$

Hofmann degradation of primary amides yields primary amine with one carbon atom less than that in amide on reactants side.

**150. (a)** Number of radial nodes = $n - l - 1$

For $3p$-orbital, $n = 3$, $l = 1$

$\therefore$ Number of radial nodes = 3 – 2 – 1 = 0.

**151. (a)**

| | Name | Frequency (in Hz) |
|---|---|---|
| (i) | X-rays | $10^{17}$ to $10^{20}$ |
| (ii) | UV rays | $10^{15}$ to $10^{16}$ |
| (iii) | IR rays | $10^{12}$ to $10^{14}$ |
| (iv) | Radiowaves | $10^5$ to $10^8$ |

**152. (a)** Enthalpy change accompanying when an electron is added to a gaseous atom in its ground state to convert it into a negative ion is called electron gain enthalpy.

It is a direct measure of the ease with which an atom attracts an electron to form anion.

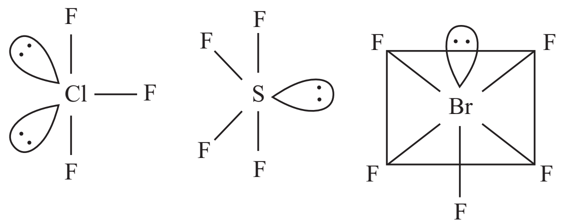
$$X(g) + e^- \longrightarrow X^-(g);$$

**153. (b)** If an element has 24 electrons in +2 oxidation state, then it should have 26 electrons in its normal state. Hence, its atomic number will be 26.

Atomic number of iron (Fe) is 26 and $Fe^{2+}$ have 4 unpaired electrons in $3d$-orbital.

$${}_{26}Fe^{2+} = 1s^2 \cdot 2s^2 2p^6 \cdot 3s^2 3p^6 \cdot 3d^6 4s^0$$

**154. (d)**




| $ClF_3$ | $SF_4$ | $BrF_5$ |
|---|---|---|
| ($sp^3d$ hyb.) | ($sp^3d$ hyb.) | ($sp^3d^2$ hyb.) |
| 3 - bond pairs | 4 - bond pairs | 5 - bond pairs |
| 2 - lone pairs | 1 - lone pair | 1 - lone pair |

**155. (a)** $N_2(14) = 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2$

$$\approx \pi 2p_y^2, \sigma 2p_z^2$$

Bond order $= \dfrac{N_b - N_a}{2} = \dfrac{10-4}{2} = 3.$

**156. (b)** A-(V), B-(III), C-(I), D-(II)

| List I | | List II | |
|---|---|---|---|
| (A) | Viscosity | (V) | $kgm^{-1}s^{-1}$ |
| (B) | Ideal gas behaviour | (III) | Compressibility factor |
| (C) | Liquefaction of gases | (I) | Critical temperature |
| (D) | Charles' law | (II) | Isobars |

**157. (c)** Most probable speed $\mu_{mp} = \sqrt{\dfrac{2RT}{M}}$

$= \sqrt{\dfrac{2RT}{32}}$ (M. Mass of $O_2$ = 32)

$= \sqrt{\dfrac{RT}{16}}$

**158. (b)** As per convention of significant figures, during addition, the resulting answer should not have more digits to the right of the decimal point than either of the original number.

12.11 + 18.0 + 11.012 = 31.122.

18.0 has only one digit after the decimal point, thus the result should be reported only upto one digit after the decimal point which is 31.1.

**159. (c)**

| Element | % amount | At. wt. | No. of moles | Simple molar ratio |
|---|---|---|---|---|
| C | 52.14 | 12 | $\dfrac{52.14}{12} = 4.34$ | $\dfrac{4.34}{2.17} = 2$ |
| H | 13.13 | 1 | $\dfrac{13.13}{\ }$ 13.13 | $\dfrac{13.13}{2.17} = 6$ |

| | | | | |
|---|---|---|---|---|
| O | 34.73 | 16 | $\frac{34.73}{16} = 2.17$ | $\frac{2.17}{2.17} = 1$ |

$\therefore$ Empirical formula = $C_2H_6O$

Empirical formula mass = 2(12) + 6(1) + 1(16) = 46

Molecular mass = 46

$$\because \quad n = \frac{\text{Molecular mass}}{\text{Empirical formula mass}} = \frac{46}{46} = 1$$

$\therefore$ Molecular formula = (Empirical formula)$_n$

= $(C_2H_6O)_1$ or $C_2H_6O$.

**160.** **(b)** The variable that depends only upon the initial and final states of a system, but not on the path are known as state functions, e.g. internal energy, entropy and free energy. Work depends on path of the system hence it is a path function.