TS/EAMCET Solved Paper 2020

Held on September 10

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 marks.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. Let $f : [0, 10] \to [1, 20]$ be a function defined

as $f(x) = \begin{cases} \frac{60-5x}{3}, & 0 \le x \le 6 \\ 10, & 6 \le x \le 7 \\ 31-3x, & 7 \le x \le 10 \end{cases}$, then f is

(a) bijective function
(b) one-one but not onto function
(c) onto but not one-one function
(d) neither one-one nor onto function

2. The domain of the function,

$f(x) = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$ is

(a) [0, 1] (b) [1, 4] (c) [4, 5] (d) $(-\infty, \infty)$

3. Let the greatest common divisor of m, n be 1. If

$\frac{1}{1.7} + \frac{1}{7.13} + \frac{1}{13.19} + ...$ upto 20 terms $= \frac{m}{n}$, then $5m + 2n =$

(a) 325 (b) 330 (c) 342 (d) 337

4. If A, B are two non singular matrices of order 3, $|B| = k$, a positive integer, then match the items of list-I with the items of list-II.

List-I	List-II
(A) $\|k^{-1}A^{-1}\|$	I. $BA^k + A^kB$
(B) $\|\text{Adj}\,(A^{-1})\|$	II. $\frac{B\text{Adj}(B)}{\|B\|}$
(C) $BAB^{-1} = I$, $\Rightarrow BA^kB^{-1} =$	III. $\frac{1}{\|B\|^3\|A\|}$
(D) Adj (Adj (A^{-1}))=	IV. $\frac{1}{\|A\|}(A^{-1})$
	V. $\frac{1}{\|A\|^2}$

The correct answer is

	A	B	C	D
(a)	III	V	II	IV
(b)	III	IV	I	II
(c)	I	V	II	IV
(d)	III	IV	II	I

5. All the real values of p, q so that the system of equations:

$2x + py + 6z = 8$
$x + 2y + qz = 5$
$x + y + 3z = 4$

may have no solution are

(a) $p = 2, q \ne 3$ (b) $p = 2, q = \frac{15}{2}$
(c) $p \ne 2, q = 3$ (d) $p = 3, q = \frac{15}{4}$

6. If p and q are two distinct real values of λ for which the system of equations

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$
$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$
$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$

has non-zero solution, then $p^2 + q^2 - pq =$

(a) 15 (b) 9 (c) 3 (d) 6

7. Let $z = x + iy$ be a complex number,

$A = \{z/|z| \le 2\}$ and $B = \{z / (1 - i)z + (1 + i)\bar{z} \ge 4\}$

Then which one of the following options belongs to $A \cap B$?

(a) $\sqrt{3} + \frac{1}{2}i$ (b) $\frac{1}{2} + \frac{i}{2}$
(c) $\sqrt{2} + \frac{i}{2}$ (d) $2 + 2i$

8. The solutions of the equation $z^2(1 - Z^2) = 16$, $z \in \boldsymbol{C}$, lie on the curve

(a) $|z| = 1$ (b) $|z| = \dfrac{2}{|z|}$

(c) $|z|^2 = 3|z| + 2$ (d) $|z| = 2$

9. If $z, \bar{z}, -z, -\bar{z}$ forms a rectangle of area $2\sqrt{3}$ square units, then one such z is

(a) $-\sqrt{\ }$ (b) $\dfrac{\sqrt{5}+\sqrt{3i}}{4}$

(c) $\dfrac{3}{2}+\dfrac{\sqrt{3i}}{2}$ (d) $\dfrac{\sqrt{3}+\sqrt{11i}}{2}$

10. $\left(\dfrac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^8 + \left(\dfrac{1+\cos\theta - i\sin\theta}{1+\cos\theta + i\sin\theta}\right)^{16} =$

(a) $2\cos 8\theta$ (b) $2\cos 16\theta$

(c) $2\sin 8\theta$ (d) $2\sin 16\theta$

11. Let S be the set of all possible integral values of λ in the interval $(-3,7)$ for which the roots of the quadratic equation $\lambda x^2 + 13 + 7 = 0$ are all rational numbers. Then the sum of the elements in S is

(a) 4 (b) 2 (c) 3 (d) 1

12. α is the maximum value of $1 - 2x - 5x^2$ and β is the minimum value of $x^2 - 2x + r$. If $5\alpha x^2 + \beta x + 6 > 0$ for all real values x, then the interval in which r lies is

(a) $(0, 5)$ (b) $(-5, \infty)$ (c) $(-\infty, 7)$ (d) $(-11, 13)$

13. For the equation $x^4 + x^3 - 4x^2 + x - 1 = 0$ the ratio of the sum of the squares of all the roots to the product of the distinct roots is

(a) 1 : 4 (b) 3 : 5 (c) 9 : 1 (d) 4 : 3

14. If $\alpha_1, \beta_1, \gamma_1, \delta_1$ are the roots of the equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ and $\alpha_2, \beta_2, \gamma_2, \delta_2$ are the roots of the equation $ex^4 + dx^3 + cx^2 + bx + a = 0$ such that $0 < \alpha_1 < \beta_1 < \gamma_1 < \delta_1$, $0 < \alpha_2 < \beta_2 < \gamma_2 < \delta_2$, $\alpha_1 - \delta_2 = 2 = \beta_1 - \gamma_2$; $\gamma_1 - \beta_2 = \delta_1 - \alpha_2 = 4$, then $a + b + c + d + e =$

(a) 10 (b) 12 (c) 6 (d) 8

15. The total number of three digit and five digit integers which can be formed by using the digits 0, 1, 2, 3, 4, 5 but using each digit not more than once in each number is

(a) 100 (b) 600 (c) 700 (d) 800

16. At an election a voter may vote for any number of candidates not exceeding the number to be elected. If 4 candidates are to be elected out of the 12 contested in the election and voter votes for at least one candidate, then the number of ways in which a voter can vote is

(a) 793 (b) 298 (c) 781 (d) 1585

17. Let $x \in \mathbf{R}$ be so small that the powers of x beyond two are insignificant and negligibly small. For such x, if $(1 - x)^3 (2 + x)^6$ is approximated by $a + bx + cx^2$, then $a + b + c =$

(a) -80 (b) 144 (c) 80 (d) 127

18. For $0 < x < 1$, the expansion of $\left(1+\dfrac{1}{x}\right)^{\frac{1}{2}}$ is

(a) $1+\dfrac{1}{2x}-\dfrac{1}{2!}\left(\dfrac{1}{2x}\right)^2+\dfrac{1.3}{3!}\left(\dfrac{1}{2x}\right)^3-\dfrac{1.3.5}{4!}\left(\dfrac{1}{2x}\right)^4+\ldots\infty$

(b) $\dfrac{1}{\sqrt{x}}+\dfrac{1}{2}\sqrt{x}-\dfrac{1}{2!}\dfrac{x\sqrt{x}}{2^2}+\dfrac{1.3}{3!}\dfrac{x^2\sqrt{x}}{2^3}-\ldots\infty$

(c) $1+\dfrac{1}{\sqrt{x}}+\dfrac{1}{2}x\sqrt{x}+\dfrac{1}{2!}\dfrac{x^2\sqrt{x}}{2^3}+\dfrac{1.3}{3!}\dfrac{x^3\sqrt{x}}{2^4}+\ldots\infty$

(d) $\dfrac{1}{\sqrt{x}}+\dfrac{1}{2x\sqrt{x}}-\dfrac{1}{2!}\left(\dfrac{1}{2x}\right)^2\dfrac{1}{\sqrt{x}}+\dfrac{1.3}{3!}\left(\dfrac{1}{2x}\right)^3\dfrac{1}{\sqrt{x}}-\ldots\infty$

19. If $\dfrac{4x^2+5x^4+7}{(x^2+1)(x^4+x^2+1)} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx^3+Dx^2+Ex+F}{x^4+x^2+1}$, then

$B + 2(D + F + E) - C \cdot A =$

(a) 0 (b) 3 (c) 1 (d) -3

20. $\sin^4\dfrac{\pi}{8}+\cos^4\dfrac{3\pi}{8}-\sin^4\dfrac{3\pi}{8}+\sin^4\dfrac{5\pi}{8}+\cos^4\dfrac{7\pi}{8}-\sin^4\dfrac{7\pi}{8}=$

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{2}$ (c) 0 (d) $\dfrac{3}{4}$

21. $\operatorname{cosec}^{-1}\left[\left(\dfrac{\tan^2\left(\dfrac{\alpha-\pi}{4}\right)-1}{\tan^2\left(\dfrac{\alpha-\pi}{4}\right)+1}+\cos\dfrac{\alpha}{2}.\cot 5\alpha\right)\sec\dfrac{11\alpha}{2}\right]$

(a) 2α (b) 5α (c) $\dfrac{\pi}{2}-4\alpha$ (d) $\dfrac{5}{2}\alpha$

22. **Assertion :** If A = 15°, B = 17° and C = 13°, then cot 2A + cot 2B + cot 2C = cot 2A cot 2B cot 2C

Reason : In a ΔPQR,

$\tan\dfrac{P}{2}\tan\dfrac{Q}{2}+\tan\dfrac{Q}{2}\tan\dfrac{R}{2}+\tan\dfrac{P}{2}\tan\dfrac{R}{2}=1$

The correct option among the following is

(a) (A) is true, (R) is true and (R) is the correct explanation for (A)

(b) (A) is true, (R) is true but (R) is not the correct explanation for (A)

(c) (A) is true but (R) is false

(d) (A) is false but (R) is true

23. The solution set of the trigonometric equation $\tan\theta + 5\cot\theta = \sec\theta$ is

(a) $\left\{\dfrac{\theta}{\theta} = 2n\pi \pm \dfrac{\pi}{3}, n \in \boldsymbol{Z}\right\}$

(b) $\left\{\frac{\theta}{\theta} = n\pi + (-1)^n \frac{\pi}{2}, n \in \boldsymbol{Z}\right\}$

(c) $\left\{\frac{\theta}{\theta} = n\pi + \frac{\pi}{6}, n \in \boldsymbol{Z}\right\}$

(d) ϕ

24. If $\tan^{-1}\frac{1}{5} + \frac{1}{2}\sec^{-1}x + \tan^{-1}\frac{1}{8} = \frac{\pi}{8}$, then $x^2 =$

(a) $\frac{12}{7}$ (b) $\frac{50}{49}$ (c) $\frac{13}{12}$ (d) –

25. **Assertion :** $\text{cosec h}^{-1}(3) = \log\left(\frac{1+\sqrt{10}}{3}\right)$

Reason : $e^{\text{cosec h}^{-1}x}$ is a root of the quadratic equation $xp^2 - 2p - x = 0$

The correct option among the following is

(a) (A) is true, (R) is true and (R) is the correct explanation for (A)
(b) (A) is true, (R) is true but (R) is not the correct explanation for (A)
(c) (A) is true but (R) is false
(d) (A) is false but (R) is true

26. In a ΔABC if $\angle$A = 3 $\angle$B, CA = 9 and BC = 16, then the length of AB is

(a) $\frac{5}{3}$ (b) $\frac{7}{3}$ (c) 2 (d) $\frac{35}{3}$

27. In $\Delta ABC, \frac{1+\cos C}{r_1+r_2} + \frac{1+\cos A}{r_2+r_3} + \frac{1+\cos B}{r_1+r_3} =$

(a) $\frac{2}{3R}$ (b) $\frac{R}{2}$ (c) $\frac{3}{2R}$ (d) $\frac{6R}{5}$

28. In a triangle ABC, if
cos A cos B + sin A sin B sin C = 1, then $a : b : c =$

(a) $1:1:\sqrt{2}$ (b) 1 : 1 : 1
(c) $\sqrt{2}:1:1$ (d) $1:\sqrt{2}:1$

29. Let $\boldsymbol{OA} = \boldsymbol{a}$, $\boldsymbol{OB} = \boldsymbol{b}$ be two non collinear vectors, $\boldsymbol{OP} = x_1\boldsymbol{a} + y_1\boldsymbol{b}$, $\boldsymbol{OQ} = x_2\boldsymbol{a} + y_2\boldsymbol{b}$ and $\boldsymbol{A'O} = \boldsymbol{OA}$, $\boldsymbol{B'O} = \boldsymbol{OB}$. If $x_1 = \frac{-3}{4}, x_2 = \frac{1}{3}, y_1 = \frac{7}{4}, y_2 = \frac{5}{3}$, then

(a) P lies inside the ΔA'OB and Q lies outside the ΔAOB
(b) P lies outside the ΔAOB' and Q lies on the ΔA'OB'
(c) P lies inside the ΔAOB and Q lies outside the ΔAOB'
(d) P lies on the ΔA'OB and Q lies outside the ΔAOB

30. The position vector of a point P is $2\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}}$ and $\boldsymbol{a} = -\hat{\boldsymbol{i}} - 2\hat{\boldsymbol{k}}, \boldsymbol{b} = \hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + 2\hat{\boldsymbol{k}}$ are two vectors which determine a plane π. The equation of a line through P normal to b and lying on the plane π is

(a) $\boldsymbol{r} = 2\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}} + \lambda(-\hat{\boldsymbol{i}} + 5\hat{\boldsymbol{j}} - 2\hat{\boldsymbol{k}})$
(b) $\boldsymbol{r} = 2\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}} + \lambda(\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}})$
(c) $\boldsymbol{r} = 2\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}} + \lambda(-2\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}})$
(d) $\boldsymbol{r} = 2\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}} + \lambda(-3\hat{\boldsymbol{i}} + 4\hat{\boldsymbol{j}} - 5\hat{\boldsymbol{k}})$

31. In a quadrilateral $ABCD$, the point P divides DC in the ratio 1 : 3 internally and Q is the mid-point of AC.
If $\boldsymbol{AB} + \boldsymbol{AD} + \boldsymbol{BC} - 2\boldsymbol{DC} = \lambda\boldsymbol{PQ}$, then the value of λ is

(a) –2 (b) 2 (c) 4 (d) –4

32. $\boldsymbol{p} = 2\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}, \boldsymbol{q} = \hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} - \hat{\boldsymbol{k}}$. If the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are the orthogonal projections of $\boldsymbol{p}$ on $\boldsymbol{q}$ and $\boldsymbol{q}$ on $\boldsymbol{p}$ respectively, then $\frac{\boldsymbol{a}\times\boldsymbol{b}}{\boldsymbol{a}.\boldsymbol{b}} =$

(a) $\frac{2\hat{\boldsymbol{i}} + 3\hat{\boldsymbol{j}} + 5\hat{\boldsymbol{k}}}{19\sqrt{2}}$ (b) $\frac{2\hat{\boldsymbol{i}} + 3\hat{\boldsymbol{j}} + 5\hat{\boldsymbol{k}}}{\sqrt{38}}$

(c) $\frac{2\hat{\boldsymbol{i}} + 3\hat{\boldsymbol{j}} + 5\hat{\boldsymbol{k}}}{2}$ (d) $\frac{3\hat{\boldsymbol{i}} - 2\hat{\boldsymbol{j}}}{13}$

33. Let $\boldsymbol{a} = 2\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}} + 4\hat{\boldsymbol{k}}, \boldsymbol{b} = 7\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} - 3\hat{\boldsymbol{k}}, \boldsymbol{c} = \hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}$. The vector $\boldsymbol{x}$ such that $\boldsymbol{x}.\boldsymbol{c}$ = 60 and perpendicular to both $\boldsymbol{a},\boldsymbol{b}$ is

(a) $14\hat{\boldsymbol{i}} - 6\hat{\boldsymbol{j}} - 12\hat{\boldsymbol{k}}$ (b) $\hat{\boldsymbol{i}} + 34\hat{\boldsymbol{j}} + 25\hat{\boldsymbol{k}}$
(c) $4\hat{\boldsymbol{i}} - 21\hat{\boldsymbol{j}} - 12\hat{\boldsymbol{k}}$ (d) $6\hat{\boldsymbol{i}} - 6\hat{\boldsymbol{j}} + 28\hat{\boldsymbol{k}}$

34. The shortest distance between the line
$\boldsymbol{r} = 2\hat{\boldsymbol{i}} - 2\hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}} + \lambda(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} + 4\hat{\boldsymbol{k}})$ and the plane
$\boldsymbol{r}.(\hat{\boldsymbol{i}} + 5\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}) = 5$ is

(a) $\frac{1}{3\sqrt{3}}$ (b) $\frac{5}{3\sqrt{3}}$ (c) $\frac{10}{3\sqrt{3}}$ (d) $\frac{11}{3\sqrt{3}}$

35. For the following frequency distribution, the variance is approximately equal to

Class Interval	0-5	5-10	10-15	15-20	20-25
Frequency	4	1	10	3	2

(a) 33.1 (b) 30.55 (c) 34.75 (d) 37.50

36. If the mean of the discrete distribution 8, 9, 6, 5, x, 4, 6, 5 is 6, then its standard deviation (nearest to two decimal places) is

(a) 2.50 (b) 1.58 (c) 0.51 (d) 1.41

37. If A and B are events of a sample space such that
$P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{2}{3}$, then $P(\overline{A} \cap B)$ is

(a) $\frac{5}{12}$ (b) $\frac{3}{8}$ (c) $\frac{4}{5}$ (d) $\frac{5}{4}$

38. Let X and Y be two events of a sample space such that $P(X) = \frac{1}{3}, P\left(\frac{X}{Y}\right) = \frac{1}{2}$ and $P\left(\frac{Y}{X}\right) = \frac{2}{5}$ then

(a) $P(X \cap Y) = \frac{1}{5}$ (b) $P(X \cup Y) = \frac{2}{5}$

(c) $P(Y) = \frac{1}{6}$ (d) $P\left(\frac{\overline{X}}{Y}\right) = \frac{1}{2}$

39. Let A and B be not mutually exclusive events. If $P(A)=\frac{4}{9}, P(A\cap\bar{B})=\frac{3}{7}$ then $P\left(\frac{B}{A}\right)=$

(a) 0 (b) $\frac{1}{28}$ (c) $\frac{3}{13}$ (d) $\frac{4}{7}$

40. If 20% of the bolts produced by a machine are defective then the probability that out of 4 bolts chosen at random, less than 2 bolts will be defective, is

(a) 0.2048 (b) 0.4096 (c) 0.8192 (d) 0.1024

41. In a book consisting of 600 pages, there are 60 typographical errors. The probability that a randomly chosen page will contain at most two errors, is

(a) $\frac{1}{5}\sqrt{e}$ (b) $\frac{1}{e^{0.1}}\left(\frac{221}{200}\right)$

(c) $\frac{1}{e^{0.1}}\left(\frac{111}{200}\right)$ (d) $\frac{1}{5}e^{0.1}$

42. If M is the foot of the perpendicular drawn from the origin O on to the variable line L, passing through a fixed point (a, b), then the locus of the mid-point of OM is

(a) $x^2+y^2=a^2+b^2$ (b) $2x^2+2y^2-ax-by=0$

(c) $ax+by=0$ (d) $2x^2+2y^2-ay-bx=0$

43. When the origin is shifted to the point $\left(\frac{3}{2},\frac{3}{2}\right)$ by the translation of coordinate axes, then the transformed equation of $32x^2+8xy+32y^2-108x-108y+99=0$ is

(a) $72X^2+56Y^2-63=0$

(b) $X^2-14XY-7Y^2-2=0$

(c) $32X^2-16XY+32Y^2-225=0$

(d) $32X^2+8XY+32Y^2-63=0$

44. A line L_1 passing through A(3, 4) and having slope 1 cuts another line L_2 passing through C at B, such that AB = AC. If the equation of line BC is $2x-y+4=0$, then the equation of AC is

(a) $7x-y-17=0$ (b) $x-y+1=0$

(c) $x-7y+25=0$ (d) $2x+3y-18=0$

45. Angles made with the X -axis by the two lines passing through the point P(1, 2) and cutting the line $x+y=4$ at a distance $\frac{\sqrt{6}}{3}$ units from the point P are

(a) $\frac{\pi}{5}$ and $\frac{3\pi}{10}$ (b) $\frac{\pi}{6}$ and $\frac{\pi}{3}$

(c) $\frac{\pi}{12}$ and $\frac{5\pi}{12}$ (d) $\frac{\pi}{8}$ and $\frac{3\pi}{8}$

46. The straight lines $x+3y-9=0$, $4x+5y-1=0$, $px+qy+10=0$ are concurrent, if the line $5x+6y+10=0$ passes through the point

(a) $(a,-p)$ (b) (q,p) (c) $(p,-q)$ (d) (p,q)

47. For $l\in\mathbf{R}$, the equation $(2l-3)x^2+2lxy-y^2=0$ represents a pair of lines

(a) only when $l=0$

(b) for all values of $l\in\mathbf{R}-(-3,1)$

(c) for all values of $l\in(-3,1)$

(d) for all values of $l\in\mathbf{R}$

48. The centroid of the triangle formed by the lines $x+y=1$ and $2y^2-xy-6x^2=0$ is

(a) (0, 0) (b) $\left(\frac{5}{9},\frac{11}{9}\right)$

(c) $\left(\frac{-5}{9},\frac{11}{9}\right)$ (d) $\left(\frac{5}{9},\frac{-11}{9}\right)$

49. Two points from the set of concyclic points of the circle passing through (I, 1), (2, – 1), (3,2) is

(a) $\left(\frac{5}{2}+\sqrt{\frac{5}{2}},\frac{1}{2}+\sqrt{\frac{5}{2}}\right),\left(\frac{5}{2},\frac{1}{2}+\sqrt{\frac{5}{2}}\right)$

(b) $\left(\frac{5}{2}+\sqrt{\frac{5}{2}},\frac{1}{2}\right),\left(\frac{5+\sqrt{5}}{2},\frac{1+\sqrt{5}}{2}\right)$

(c) $\left(\frac{5+\sqrt{5}}{2},\frac{1+\sqrt{5}}{\sqrt{2}}\right)\left(\frac{5}{2}+\sqrt{\frac{5}{2}}+\frac{1+\sqrt{5}}{4}\right)$

(d) $\left(\frac{5}{2}-\frac{\sqrt{5}}{2},\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\left(\frac{5}{2}-\frac{\sqrt{5}}{2},\frac{1}{2}+\frac{\sqrt{5}}{2}\right)$

50. If the polar of a point P with respect to a circle of radius r which touches the coordinate axes and lies in the first quadrant is $x+2y=4r$, then the point P is

(a) $(r, 2r)$ (b) $(2r, r)$

(c) $(2r, 3r)$ (d) $(-r, 4r)$

51. If the circles $x^2+y^2-2x-2(3+\sqrt{7})y+8+6\sqrt{7}=0$ and $x^2+y^2-8x-6y+k^2=0, k\in\boldsymbol{Z}$, have exactly two common tangents, then the number of possible values of k is

(a) 8 (b) 5

(c) 9 (d) 11

52. The circle S = 0 cuts the circles $C_1=x^2+y^2-8x-2y+16=0$ and $C_2=x^2+y^2-4x-4y-1=0$ orthogonally. If the common chord of S = 0 and $C_1=0$ is $2x+13y-15=0$, then the centre of S = 0 is

(a) $\left(\frac{-11}{3},\frac{7}{6}\right)$ (b) $\left(\frac{11}{3},\frac{-7}{6}\right)$

(c) $\left(\frac{2}{13},\frac{11}{15}\right)$ (d) $\left(\frac{11}{15},\frac{-2}{13}\right)$

53. The equation of the circle passing through the points of intersection of the two orthogonal circles

$S_1=x^2+y^2+kx-4y-1=0$,

$S_2=3x^2+3y^2-14x+23y-15=0$ and passing through the point (–1, –1) is

(a) $x^2+y^2-8x-2y-12=0$

(b) $3x^2+3y^2+18x-12y=0$

(c) $5x^2+5y^2-22x+15y-17=0$

(d) $x^2+y^2-5x+14y+7=0$

54. Consider the parabola $y^2 + 2x + 2y - 3 = 0$ and match the items of List-I with those of the List-II.

List-I	List-II
A. $2x - 5 = 0$	I. Vertex
B. $\left(-,\ 1\right)$	II. Focus
C. $y + 1 = 0$	III. Equation of directrix
D. $(2, -1)$	IV. Equation of the axis
	V. Equation of the Latus rectum

The correct answer is

	A	B	C	D
(a)	III	II	IV	I
(b)	V	I	IV	II
(c)	III	II	V	I
(d)	IV	I	III	II

55. The normal at a point on the parabola $y^2 = 4x$ passes through (5, 0). If there are two more normals to this parabola which pass through (5, 0), the centroid of the triangle formed by the feet of these three normals is

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) (4, 0) (c) (0, 2) (d) (2, 0)

56. The eccentricity of an ellipse passing through $(3\sqrt{2}, \sqrt{10})$ with foci at (–4, 0) and (4, 0) is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{\sqrt{2}}{3}$ (d) $\frac{1}{\sqrt{3}}$

57. If the product of the lengths of the perpendiculars drawn from the foci to the tangent $y = \frac{-3}{4}x + 3\sqrt{2}$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is 9, then the eccentricity of that ellipse

(a) $\frac{\sqrt{2}}{3}$ (b) $\frac{\sqrt{5}}{6}$ (c) $\frac{1}{9}$ (d) $\frac{\sqrt{7}}{4}$

58. The equation of the hyperbola, whose eccentricity is $\sqrt{2}$ and whose foci are 16 units apart, is

(a) $9x^2 - 4y^2 = 36$ (b) $2x^2 - 3y^2 = 7$
(c) $x^2 - y^2 = 16$ (d) $x^2 - y^2 = 32$

59. If the points A(–1,0, 7), B(3, 2, t), C(5, k, –2) are collinear, then the ratio in which the point P(t, $k - 2t$, $t + k$) divides the line segment BC is

(a) –2 : 3 (b) –1 : 2 (c) 4 : 3 (d) 1 : 1

60. The direction cosines l, m, n of two lines are satisfying $3l + m + 5n = 0$ and $6mn - 2nl + 5\,lm = 0$. If θ is the angle between those lines then |cosθ| =

(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{6}$ (d) $\frac{1}{\sqrt{3}}$

61. A tetrahedron has vertices O(0,0,0), A(l, 2, 1), B(2, 1, 3), C(–1, 1, 2). If θ is the angle between the faces OAB and ABC. then cosθ =

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{19}{35}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{17}{31}$

62. If $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...\infty$ and $\lim_{x\to\infty} \frac{\log(1+x)^{1+x}}{x^2} - \frac{1}{x} = k$, then $12k =$

(a) 1 (b) 3 (c) 6 (d) 9

63. If $f(x) = \begin{cases} k, & \text{for } x = 1 \\ \frac{(9x-1)(\sqrt{x}-1)}{3x^2+2x-5}, & \text{for } x \neq 1 \end{cases}$ is continuous on $[0, \infty)$, then $k =$

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

64. Let $g(x) \neq 0$, $g'(x) \neq 0$, $f(x) \neq 0$, $f'(x) \neq 0$. If $F(x) = f(x)\,g(x)$, $G(x) = f'(x)\,g'(x)$ and $F'(x) = G(x)\,H(x) = F(x)\,K(x)$, then $H(x) + K(x) =$

(a) $\frac{f'}{f} + \frac{f}{f'} + \frac{g}{g'}$ (b) $\frac{f'}{f} + \frac{g}{g'} + \frac{g'}{g}$

(c) $\frac{f'g' + fg}{ff'gg'}$ (d) $\frac{f'}{f} + \frac{g}{g'} + \frac{f}{f'} + \frac{g'}{g}$

65. If $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \log\sqrt{1-x^2}$, then $\frac{dy}{dx} =$

(a) $\frac{\sin^{-1}x}{1-x^2}$ (b) $\frac{\sin^{-1}x}{(1-x^2)^{3/2}}$

(c) $\frac{x}{1-x^2}$ (d) $\frac{x\sin^{-1}x}{\sqrt{1-x^2}} - \frac{2x}{\sqrt{1-x^2}}$

66. Let $f(x)$ and $g(x)$ be twice differentiable functions such that $f(x) = x^2 + g'(1)\,x + g''(2)$ and $g(x) = f(1)\,x^2 + xf'(x) + f''(x)$. Then $f(x) - g(x) =$

(a) $2x + 5$ (b) $3x^2 + 6x + 1$
(c) $x^2 - 6x + 2$ (d) $x^2 - 2$

67. If the area of a circle increases at the rate of $\frac{1}{\sqrt{\pi}}$ sq. unit /sec, then the rate (in units/sec) at which the perimeter of the circle changes. when perimeter is $\sqrt{\pi}$ units, is

(a) 2 (b) 4 (c) $\frac{1}{\sqrt{\pi}}$ (d) $\sqrt{\pi}$

68. Let a be a fixed positive real number and n be an arbitrary constant. For the curve $y = \frac{x^n}{a^{n-1}}$, if the length of the subnormal at any point (α,β) is proportional to a^2, then $n =$

(a) 2 (b) 1 (c) 0 (d) $\frac{3}{2}$

69. In each of the choices given below a function and an interval are given, The correct choice having a function and the associated interval for which the Lagrange's mean value theorem is not valid is

(a) $|x| : [1, 5]$ (b) $\log x : [1, e]$

(c) $\dfrac{2x-1}{3x-4} : [1, 2]$ (d) $(x-2)^2 (x-4)^2 : [2, 4]$

70. Let $P(x)$ be a polynomial of degree 3 having extreme value at $x = 1$. If $\lim\limits_{x\to 0}\left(\dfrac{P(x)+4}{x^2}+2\right) = 6$, then $\left(\dfrac{dP}{dx}\right)_{x=\frac{1}{2}} =$

(a) 2 (b) 0 (c) –2 (d) 4

71. $\displaystyle\int \frac{y^2+\sqrt[3]{y^4}+\sqrt[6]{y^2}}{y\left(1+\sqrt[3]{y^2}\right)}\,dy =$

(a) $\dfrac{3}{4}\sqrt[3]{y^4}+3\tan^{-1}(\sqrt[3]{y})+C$

(b) $\dfrac{3}{2}y^{2/3}+6\tan^{-1}(\sqrt[6]{y^2})+C$

(c) $\dfrac{2}{3\sqrt[3]{y^2}}+6\log(1+y^2)+C$

(d) $\dfrac{3}{1+y}+\tan^{-1}(\sqrt[3]{y^2})+C$

72. For $k \in (1,\infty)$, $\displaystyle\int \frac{1}{1+k\cos x}\,dx =$

(a) $\dfrac{2}{\sqrt{1+k^2}}\tan^{-1}\left(\sqrt{\dfrac{1-k}{1+k}}\tan\dfrac{x}{2}\right)+C$

(b) $\dfrac{1}{\sqrt{k^2-1}}\log\left(\dfrac{\sqrt{k+1}+\sqrt{k-1}\tan\dfrac{x}{2}}{\sqrt{k+1}-\sqrt{k-1}}\right)+C$

(c) $\dfrac{1}{\sqrt{k^2+1}}\log^{-1}\left(\dfrac{\sqrt{k+1}+\sqrt{k-1}\tan\dfrac{x}{2}}{\sqrt{k+1}-\sqrt{k-1}\tan\dfrac{x}{2}}\right)+C$

(d) $\dfrac{1}{\sqrt{k^2-1}}\tan^{-1}\left(\dfrac{\sqrt{k-1}\cos\dfrac{x}{2}+\sqrt{k-1}\sin\dfrac{x}{2}}{\sqrt{k+1}\cos\dfrac{x}{2}-\sqrt{k-1}\sin\dfrac{x}{2}}\right)+C$

73. $\displaystyle\int e^{-3x}(x^2+\sin 4x)\,dx =$

(a) $-e^{-3x}\left(\dfrac{x^2}{3}+\dfrac{2x}{9}+\dfrac{2}{27}+\dfrac{3}{25}\sin 4x+\dfrac{4}{25}\cos 4x\right)+C$

(b) $-e^{-3x}\left(\dfrac{x^2}{3}-\dfrac{2x}{9}+\dfrac{2}{27}+\dfrac{3}{25}\sin 4x+\dfrac{4}{25}\cos 4x\right)+C$

(c) $-e^{-3x}\left(\dfrac{x^2}{3}+\dfrac{2x}{9}+\dfrac{2}{27}+\dfrac{3}{25}\sin 4x-\dfrac{4}{25}\cos 4x\right)+C$

(d) $-e^{-3x}\left(\dfrac{x^2}{3}-\dfrac{2x}{9}+\dfrac{2}{27}+\dfrac{3}{25}\sin 4x-\dfrac{4}{25}\cos 4x\right)+C$

74. If $\displaystyle\int \frac{2x^{12}+5x^9}{(1+x^3+x^5)^3}\,dx = \frac{x^m}{l(1+x^3+x^5)^r}+C$ then $\dfrac{m-l}{r} =$

(a) 3 (b) 4 (c) 5 (d) 6

75. $\lim\limits_{n\to\infty}\left[\left(1+\dfrac{1}{n^2}\right)\left(1+\dfrac{2^2}{n^2}\right)\ldots\left(1+\dfrac{n^2}{n^2}\right)\right]^{1/n} =$

(a) e (b) $2e$ (c) $2e^{\frac{\pi-2}{2}}$ (d) $2e^{\frac{\pi-4}{2}}$

76. $\displaystyle\int_{\pi/4}^{\pi/2} \frac{3dx}{1+e^{\sqrt{8}\sin\left(x-\frac{3\pi}{8}\right)}} =$

(a) $\dfrac{3\sqrt{2}}{4}\pi$ (b) $\dfrac{3}{4}\pi$ (c) $\dfrac{\pi}{8}$ (d) $\dfrac{3}{8}\pi$

77. If the area of the region bounded by $y = \cos x$, $y = \sin x$, $x = \dfrac{\pi}{4}$ and $x = \pi$ is bisected by the line $x = a$, then $\sin\left(a+\dfrac{\pi}{4}\right) =$

(a) $\dfrac{\sqrt{2}}{2+\sqrt{2}}$ (b) $\dfrac{\sqrt{3}+1}{2}$ (c) $\dfrac{\sqrt{2}-1}{2\sqrt{2}}$ (d) $\dfrac{\sqrt{3}+1}{2\sqrt{2}}$

78. If the family of curves $y = ae^{4x} + be^{-x}$, where a, b are arbitrary constants represents the general solution of the differential equation $f\left(x, y\dfrac{dy}{dx}, \dfrac{d^2y}{dx^2}\right) = 0$, then $\dfrac{df}{dx} =$

(a) $\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}-4y$ (b) $\dfrac{d^3y}{dx^3}-3\dfrac{d^2y}{dx^2}-4\dfrac{dy}{dx}$

(c) $\dfrac{d^3y}{dx^3}-\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}+2$ (d) $\dfrac{d^3y}{dx^3}-\dfrac{d^2y}{dx^2}+3$

79. If the length of the sub tangent at any point $p(x, y)$ on a curve $f(x, y) = 0$ is $x + 7y^2$, then $f(x, y) =$

(a) $xy + cy - 7x$ (b) $\dfrac{x}{y}+7x-c$

(c) $7y^2 + cy - x$ (d) $7xy + cy - x$

80. If the general solution of the differential equation $(y - x + 1)\,dy - (y + x + 2)dx = 0$ is $f(x, y, c) = 0$, then the value of c such that $f(1, 1\ c) = 0$ is

(a) 4 (b) –4 (c) 2 (d) 1

PHYSICS

81. The nuclear forces are

(a) long range repulsive forces
(b) long range attractive forces
(c) short range attractive forces
(d) short range repulsive forces

82. Due to an explosion underneath water, a bubble started oscillating. If this oscillation has time period T, which is proportional to $p^{\alpha}S^{\beta}E^{\gamma}$, where p is static pressure, S is density of water and E is total energy of explosion. Determine α, β and γ.

(a) $\alpha=-\frac{3}{2},\beta=\frac{1}{3},\gamma=-\frac{5}{6}$

(b) $\alpha=-\frac{5}{6},\beta=\frac{1}{2},\gamma=\frac{1}{3}$

(c) $\alpha=\frac{1}{2},\beta=-\frac{5}{6},\gamma=\frac{7}{4}$

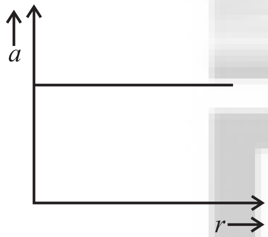
(d) $\alpha=\frac{1}{3},\beta=\frac{3}{2},\gamma=\frac{4}{3}$

83. A car travelling at 15 m/s overtake another car travelling at 10 m/s. Assuming, each car is 4 m long. What is the time taken during the overtake?

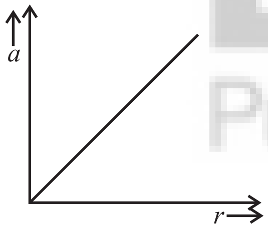
(a) 1.6 s (b) 0.8 s (c) 0.6 s (d) 0.4 s

84. If a body moving in a circular path maintains constant speed of 10 ms^{-1}, then which of the following correctly describes the relation between acceleration (a) and radius (r)?

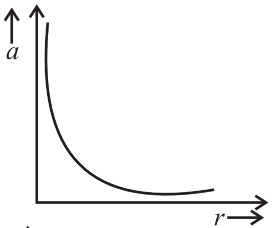
(a)



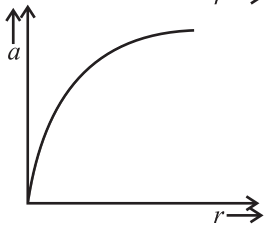
(b)



(c)



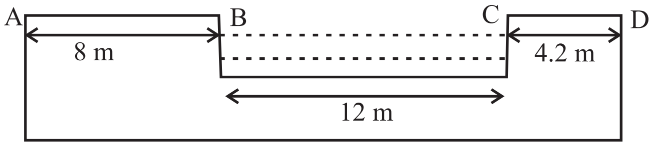
(d)



85. If $0.5\hat{i}+0.8\hat{j}+c\hat{k}$ is a unit vector, then c is

(a) $\sqrt{0.89}$ (b) 0.2 (c) 0.3 (d) $\sqrt{0.11}$

86. A projectile is launched from point A of the given landscape with a water body as shown in the diagram. The launching angle is 15°. From the following, identify the right initial velocity of the projectile with which it will fall somewhere in between the points C and D. [Assume, g =10 m/s^2]



(a) 21.5 m/s (b) 22.5 m/s

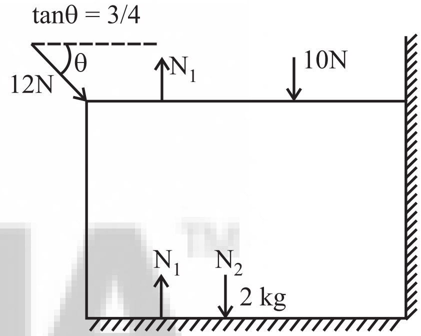
(c) 19.5 m/s (d) 24.0 m/s

87. When a bullet is fired from a rifle its momentum becomes 20 kg ms^{-1}. If the velocity of the bullet is 1000 ms^{-1}, then what is its mass?

(a) 30 g (b) 5 kg (c) 20 g (d) 500 g

88. A block is between two surfaces as shown in the figure. Find the normal reaction at both surfaces.

[Assume, g = 10 m/s^2]



(a) N_1 = 37.2 N and N_2 = 9.6 N

(b) N_1 = 382 N and N_2 = 8.6 N

(c) N_1 = 40 N and N_2 = 4N

(d) N_1 = 37.5 N and N_2 = 9.9 N

89. When a body is acted upon by a resultant force, then the work done by the resultant force is equal to

(a) its initial kinetic energy

(b) its initial potential energy

(c) change in the kinetic energy

(d) change in the potential energy

90. A force acts on a body of mass 10 kg, resulting in its displacement given as $x=\left(\frac{t^3}{25}\right)$m, where t is the time in seconds.

The work done by the force in 5 s is

(a) 620 J (b) 333 J (c) 524 J (d) 60 J

91. A bullet of mass 25 g moves horizontally at a speed of 250 m/s is fired into a wooden block of mass 1 kg suspended by a long string. The bullet crosses the block and emerges on the other side. If the centre of the mass of the block rises through a height of 20 cm. The speed of the bullet as it emerges from the block is (take, g = 10 m/s^2)

(a) 300 m/s (b) 220 m/s (c) 150 m/s (d) 170 m/s

92. A circular hole of radius 3 cm is cut out from a uniform circular disc of radius 6 cm. The centre of the hole is at 3 cm, from the centre of the original disc. The distance of centre of gravity of the resulting flat body from the centre of the original disc is

(a) 0.5 cm (b) 1 cm (c) 1.5 cm (d) 0.75 cm

93. For a particle executing SHM, determine the ratio of average acceleration of the particle between extreme position and equilibrium position w.r.t. the maximum acceleration.

(a) $\frac{4}{\pi}$ (b) $\frac{2}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{1}{2\pi}$

94. Choose the correct statement.

(a) Acceleration due to gravity increases with increasing altitude.

(b) Acceleration due to gravity is independent of mass of earth.

(c) A geostationary satellite can have a time period less than 24 h.

(d) Acceleration due to gravity decreases with increasing depth assuming earth to be a sphere of uniform density.

95. A slab of side 50 cm and thickness 10 cm is subjected to a shearing force of 10^5 N on its narrow edge. If the lower edge is riveted to the floor and upper edge is displaced by 0.2 mm, then shear modulus of the material of the slab is

(a) 6 GPa (b) 5 GPa (c) 4 GPa (d) 4.5 GPa

96. A meniscus drop of radius 1 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expended if surface tension of mercury is 435×10^{-3} N/m.

(a) 54.1×10^{-3} J (b) 64.1×10^{-3} J
(c) 74.1×10^{-3} J (d) 84.1×10^{-3} J

97. The specific heat of helium at constant volume is 12.6 J mol^{-1} K^{-1}. The specific heat of helium at constant pressure in J mol^{-1} K^{-1} is approximately (assume, the universal gas constant, R = 8.314 J mol^{-1} K^{-1})

(a) 12.6 (b) 16.8 (c) 18.9 (d) 20.9

98. A composite slab is prepared with two different materials A and B. The relation between their coefficient of thermal conductivity and thickness is given as $K_A = \frac{K_B}{2}$ and $X_A = 2X_B$, respectively. If the temperature of faces of A and B are 75°C and 50°C respectively, what will be the temperature of common surface?

(a) 75°C (b) 50°C (c) 55°C (d) 125°C

99. Work done on heating one mole of monoatomic gas adiabatically through 20°C is W. Then, the work done on heating 6 moles of rigid diatomic gas through the same change in temperature

(a) 9W (b) 10W (c) 12W (d) 8W

100. If a gas has n degrees of freedom, then the ratio of $\frac{C_P}{C_V}$ is

(a) $\frac{n+2}{n}$ (b) $\frac{2n+1}{n}$ (c) $\frac{n+2}{2n}$ (d) $\frac{n+4}{2n}$

101. A bus moving with an uniform speed of 72 km/h towards a building blows a horn of frequency 1.7 kHz. If speed of sound in air is 340 m/s, what will be the frequency of echo heard by bus driver?

(a) 1.8 kHz (b) 2.0 kHz (c) 1.6 kHz (d) 1.4 kHz

102. If the image of an object is at the focal point f to the right side of a convex lens, the position of the object on the left of the lens is at

(a) f (b) $2f$ (c) $< f$ (d) ∞

103. On using red light (λ = 6600 Å) in Young's double slit experiment, 60 fringes are observed in the field of view. If violet light (λ = 4400 Å) is used, the number of fringes observed will be

(a) 30 (b) 120 (c) 60 (d) 90

104. Young's double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe width recorded are β_G, β_R, β_B respectively, then

(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$
(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$

105. If a proton is moved against the coulomb force of an electric field, then

(a) work is done by the electric field

(b) energy is used from some outside source

(c) the strength of the field is decreased

(d) the strength of the field is increased

106. Assume each oil drop consists of a capacitance of C. If combine n drops to form a bigger drop, then the capacitance of bigger drop C' would be

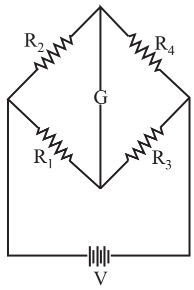
(a) $C' = \frac{2n^{1/3}}{3}C$ (b) $C' = \frac{5n^{1/3}}{4}C$

(c) $C' = \frac{n^{1/3}}{5}C$ (d) $C' = C.n^{1/3}$

107. A conductor of length 100 cm and area of cross-section 1 mm^2 carries a current of 5A. If the resistivity of the material of the conductor is 3.0×10^{-8} Ω-m, then the electric field across the conductor is

(a) 0.15 V/m (b) 0.015 V/m
(c) 1.5 V/m (d) 0.0015 V/m

108. If the Wheatstone's bridge with four resistors R_1, R_2 and R_3, R_4 is balanced, then the correct expression is



(a) $\frac{R_2}{R_1} = \frac{R_4}{R_3}$ (b) $\frac{R_2}{R_3} = \frac{R_1}{R_4}$

(c) $R_1R_2 = R_3R_4$ (d) $R_1 + R_2 = R_3 + R_4$

109. A circular coil of 10 turns and radius 10 cm is placed in a uniform magnetic field of 0.1 T normal to the plane of the coil. If the current in the coil is 5A, then the magnitude of the torque on the coil is

(a) 500 πN-m (b) 0.05 πN-m
(c) 0.005 πN-m (d) zero

110. A 50 cm long solenoid has winding of 400 turns. What current must pass through it to produce a magnetic field of induction $4\pi \times 10^{-3}$ T at the centre ?

(a) 10.5 A (b) 12.5 A (c) 25.0 A (d) 20.0 A

111. If relative permeability of iron is 5500, then its susceptibility is

(a) 5500×10^{7} (b) 5500×10^{-7}
(c) 5501 (d) 5499

112. A moving coil galvanometer of resistance 100Ω is used as an ammeter using a resistance 0.1Ω. The maximum deflection current in the galvanometer is 100 μA. Find the minimum current in the circuit, so that ammeter shows maximum deflection?

(a) 100.1 mA (b) 1000.1 mA
(c) 10.01 mA (d) 1.01 mA

113. In CR-circuit the growth of charge on the capacitor is

(a) more rapid if the CR is smaller
(b) more rapid if the CR is larger
(c) independent of CR
(d) independent of time

114. What is the amplitude of the electric field in a parallel beam of light intensity $\left(\frac{15}{\pi}\right)\frac{W}{m^2}$?

$$\left[\text{Assume, } \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \frac{\text{Nm}^2}{\text{C}^2}\right]$$

(a) 60 N/C (b) 50 N/C (c) 40 N/C (d) 30 N/C

115. Photons of energy 2.4 eV and wavelength λ fall on a metal plate and release photoelectrons with a maximum velocity *v*. By decreasing λ by 50%, the maximum velocity of photoelectrons becomes 3*v*. The work function of the material of the metal plate is

(a) 2.1 eV (b) 1.7 eV (c) 2.8 eV (d) 2.0 eV

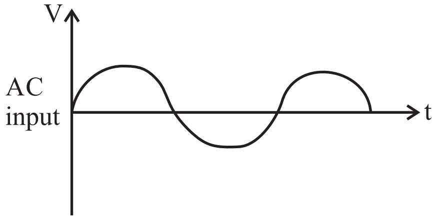
116. The ratio of maximum to minimum wavelength in Balmer series of an hydrogenic atom is

(a) $\frac{9}{5}$ (b) $\frac{12}{7}$ (c) $\frac{9}{7}$ (d) $\frac{14}{9}$

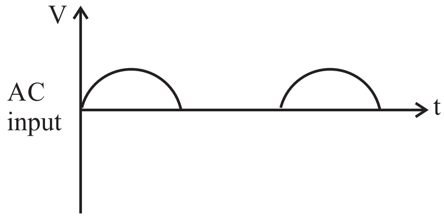
117. Alpha rays emitted from a radioactive substance are

(a) negatively charged particles
(b) doubly ionised helium atoms
(c) ionised hydrogen nuclei
(d) uncharged particles

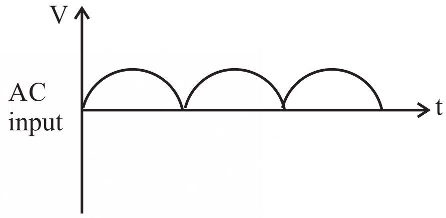
118. Which of the following depicts the output of the full wave rectifier with capacitor filter for the following AC input?



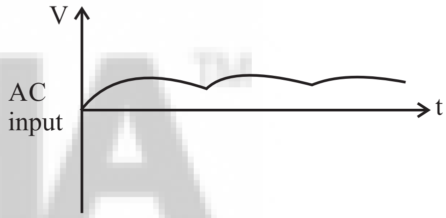
(a)



(b)

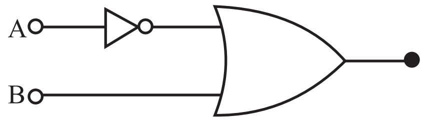


(c)



(d)

119. The Boolean expression of the circuit given in figure is



(a) $Y = A + \overline{B}$ (b) $Y = \overline{A + B}$
(c) $Y = \overline{A} + B$ (d) $Y = A + B$

120. A message signal of frequency 10 kHz and peak voltage of 15 V is used to modulate a carrier frequency of 1 MHz and peak voltage of 30V. Determine the modulation index.

(a) 0.5 (b) 0.6 (c) 0.7 (d) 0.8

CHEMISTRY

121. The degeneracy of the level of hydrogen atoms that contain the energy of $\left(\frac{-R_H}{16}\right)$ is

(a) 4 (b) 16 (c) 9 (d) 12

122. Wavelength of H^+ ion with kinetic energy 1.65 eV is (mass of proton $=1.6726 \times 10^{-27}$ kg)
(a) 1.22 nm (b) 0.22 nm
(c) 0.022 nm (d) 0.122 nm

123. **Assertion :** Mg^{2+} and Al^{3+} are isoelectronic but the magnitude of ionic radius of Al^{3+} is less than that in Mg^{2+}.
Reason : The effective nuclear charge on the outermost electrons in $A1^{3+}$ is greater than that in Mg^{2+}.
The correct option among the following is
(a) A is true, R is true and R is the correct explanation for A.
(b) A is true, R is true but R is not the correct explanation for A.
(c) A is true but R is false.
(d) A is false but R is true.

124. The successive ionisation energy values for an element 'X' are given below:
(i) 1st ionisation energy = 410 kJ mol^{-1}
(ii) 2nd ionisation energy = 820 kJ mol^{-1}
(iii) 3rd ionisation energy = 1100 kJ mol^{-1}
(iv) 4th ionisation energy = 1500 kJ mol^{-1}
(v) 5th ionisation energy = 3200 kJ mol^{-1}
The number of valence electron in 'X' will be
(a) 5 (b) 4 (c) 2 (d) 3

125. Match the following :

	List-I		List-II
(a)	BF_3	I.	Tetrahedral
(b)	CIF_3	II.	Trigonal planar
(c)	NH_3	III.	T-shape
(d)	NH_4^+	IV.	Trigonal pyramidal

The correct answer is

	A	B	C	D
(a)	III	II	IV	I
(b)	III	II	I	IV
(c)	II	III	IV	I
(d)	II	III	I	IV

126. Which of the following molecules does not exist according to molecular orbital theory?
(a) Li_2 (b) Be_2 (c) B_2 (d) C_2

127. Root mean square (rms) speed of O_2 is 500 m/s at a constant temperature. Calculate the rms speed and the average kinetic energy of H_2 at the same temperature. (Consider, R = 833 $JK^{-1}mol^{-1}$)
(a) 500 m/s and 4.0 kJ/mol
(b) 2000 m/s and 4.0 kJ/mol
(c) 500 m/s and 4.7 kJ/mol
(d) 2000 m/s and 4.7 kJ/mol

128. Which of the following describes an ideal gas?
(i) The volume occupied by a gas molecule is negligible.
(ii) The collision between ideal gases are elastic.
(iii) Particles are very small compared to the distance between each other.
(a) (i) and (ii) only (b) (i) and (iii) only
(c) (ii) and (iii) only (d) (i), (ii) and (iii) only

129. What is the % strength of 22.4 volume of H_2O_2 solution?
(a) 3.4% (b) 2.5% (c) 5% (d) 6.8%

130. $KMnO_4$ oxidises $C_2H_2O_4$ to form CO_2. In which of the following, the reaction will be faster?
(a) Aq.HCl solution (b) Aq. NaOH solution
(c) Aq. NaCl solution (d) Aq. $NaHCO_3$ solution

131. ΔH and ΔS for a reaction are +30.0 kJ mol^{-1} and 0.06 kJK^{-1} mol^{-1} at 1 atm pressure. The temperature at which free energy change is equal to zero and nature of the reaction below this temperature are
(a) 500°C and non-spontaneous
(b) 227°C and non-spontaneous
(c) 400°C and spontaneous
(d) 127°C and spontaneous

132. The vapour density of N_2O_4 in $N_2O_4 \rightleftharpoons 2NO_2$ is 40. The degree of dissociation is
(a) 1.25 (b) 2.50 (c) 1.50 (d) 0.15

133. What is the equilibrium constant (K_c) for the given reaction?
$$N_2 + O_2 \rightleftharpoons 2NO$$
Where the equilibrium concentration of N_2, O_2 and NO are found to be 4×10^{-3}, 3×10^{-3} and 3×10^{-3} M respectively.
(a) 0.750 (b) 0.622 (c) 9×10^{-3} (d) 12.8×10^{-6}

134. Hard water contains ion of
(a) zinc (b) magnesium and calcium
(c) iron (d) iron and manganese

135. Predict the feasibility of the given reactions in aqueous solution
(i) $Be(OH)_2 + 2OH^- \longrightarrow [Be(OH)_4]^{2-}$
(ii) $Be(OH)_2 + 2H^+ \longrightarrow [Be(OH)_4]^{2+}$
(a) Only (i) is feasible (b) Only (ii) is feasible
(c) (i) and (ii) are feasible (d) (i) and (ii) are not feasible

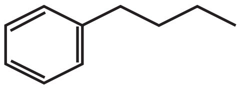
136. What is the nature of the bonding in anhydrous $AlCl_3$ and hydrated $AlCl_3$ respectively?
(a) Ionic and ionic (b) Ionic and covalent
(c) Covalent and ionic (d) Covalent and covalent

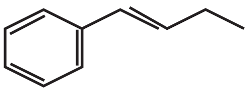
137. Which of the following elements reacts with water?
(a) C (b) Ge (c) Sn (d) Pb

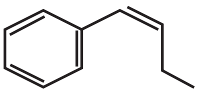
138. Biochemical oxygen demand (BOD) is a measure of organic materials present in water, BOD value less than 5 ppm indicates a water sample to be
(a) rich in dissolved oxygen
(b) poor in dissolved oxygen
(c) highly polluted
(d) not suitable for aquatic life

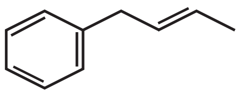
139. Sodium fusion extract of aniline when heated with ferrous sulphate solution and then acidified with concentrated H_2SO_4 from which of the following complexes?
(a) $[Fe(CN)_6]^{4-}$ (b) $Fe_4[Fe(CN)_6]_3 \cdot xH_2O$
(c) $[Fe(SCN)]^{2+}$ (d) $[Fe(CN)_5NOS]^{4-}$

140. Ethyl phenyl acetylene (l-phenyl-l-butyne) on reduction with partially deactiveated palladised charcoal (Lindlar's catalyst) gives

(a) 

(b) 

(c) 

(d) 

141. Which one of the following methods is suitable to generate aromatic compound(s) from linear aliphatic saturated hydrocarbons with at least six carbon atoms?

(a) Heating at 773 K
(b) Mo_2O_3, 773 K, 10-20 atm
(c) Anhyd. $AlCl_3$, conc. HCl, Δ
(d) Cu, 523 K, 100 atm

142. Copper crystallises in ccp arrangement and accepted value of metal ion radius was found to be 1.14 Å. Calculate the density of copper in grams per cubic centimetre. (Atomic weight of copper is 64, $N_A = 6 \times 10^{23}$)

(a) 6.67 (b) 7.80 (c) 8.90 (d) 10.00

143. A^{2+}, B^{2+} and C^- form an ionic complex like $A_{x-2}[B(C)_x]_2$. If the complex is 75% dissociated in a solvent with i = 4, the coordination number of B is

(a) 3 (b) 4 (c) 5 (d) 6

144. The freezing point of equimolal aqueous solution will be highest for

(a) $BaCl_2$ (b) $Ca(NO_3)_2$ (c) urea (d) Na_2SO_4

145. The standard electrode potentials of Ag^+/Ag is + 0.80 V and Cu^+/Cu is +0.34 V. If these electrodes are connected through a salt-bridge, which of the following statements is correct?

(a) Silver electrode acts as anode and E°_{cell} is –0.34 V.
(b) Copper electrode acts as anode and E°_{cell} is + 0.46 V.
(c) Silver electrode acts as a cathode and E°_{cell} is –0.34 V.
(d) Copper electrode acts as cathode and E°_{cell} is + 0.46 V.

146. For a zero order reaction, the plot of concentration of reactant vs time is (Hint: Consider the intercept on the concentration axis)

(a) linear with +ve slope and non zero +ve intercept
(b) linear with –ve slope and non zero +ve intercept
(c) linear with –ve slope and zero intercept
(d) linear with +ve slope and zero intercept

147. The gold numbers of gelatin, haemoglobin and sodium acetate are 5×10^{-3}, 5×10^{-2} and 7×10^{-1}, respectively. The protective actions will be in the order

(a) Gelatin < haemoglobin < sodium acetate
(b) Gelatin > haemoglobin > sodium acetate
(c) Haemoglobin > gelatin > sodium acetate
(d) Sodium acetate > gelatin > haemoglobin

148. Which one of the following ores does not contain iron"

(a) Hematite (b) Magnetite
(c) Calamine (d) Siderite

149. When copper metal is treated with cold and dilute nitric acid, it forms

(a) NO (b) N_2O (c) N_2O_5 (d) NO_3

150. Which one of the following is not a colourless compound?

(a) NO (b) N_2O_4 (c) N_2O (d) NO_2

151. Which of the following statement is not true about interstitial complexes?

(a) Small atom like C, H or N are trapped inside crystal lattice
(b) They are usually non-stoichiometric
(c) They generally· retain metallic conductivity
(d) They are chemically very active

152. Which of the following molecules is colourless?

(a) $CuSO_4 . 5H_2O$ (crystal)
(b) $CuSO_4$ (anhydrous)
(c) $[Cu(NH_3)_4]^{2+}$ (aq)
(d) $[CuCl_4]^{2-}$(aq)

153. Which of the following vinyl derivatives is the most reactive towards anionic polymerisation?

(a) $CH_2 = CH - CH_2 - CH_3$
(b) $CH_2 = CH - CH_2 - OE$
(c) $CH_2 = CH - Cl$
(d) $CH_2 = CH - C \equiv N$

154. Amino acids containing heterocyclic ring are

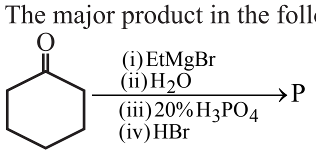
(i) Histidine (ii) Valine
(iii) Arginine (iv) Proline

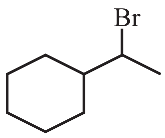
(a) (i), (iv) (b) (ii), (iii)
(c) (i), (iii) (d) (ii), (iv)

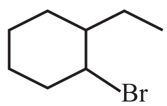
155. Which among the following is an arsenic based antibiotic drug, for which Paul Ehrlich was awarded Noble prize in 1908.

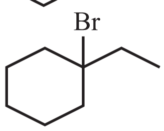
(a) Salvarsan (b) Penicillin
(c) Prontosil (d) Sulphapyridine

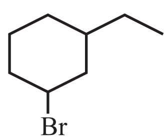
156. The major product in the following reaction sequence is



(a) 

(b) 

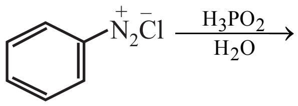
(c) 

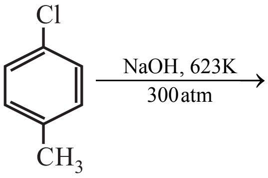
(d) 

157. Which of the following gives alcohol/phenol products?

(i) $C_2H_5CO_2H \xrightarrow{LiAlH_4}$

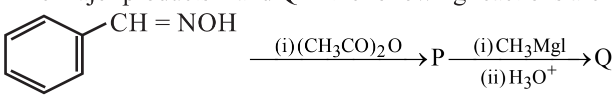
(ii) $C_2H_5Br \xrightarrow[\text{then } H_2O]{\text{Mg, dry ether}}$

(iii) 

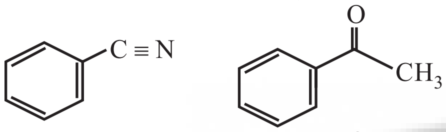
(iv) 

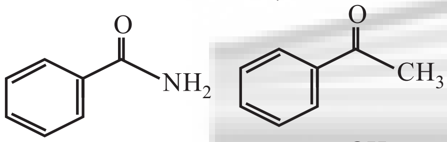
(a) (i) and (ii) (b) (ii) and (iii)
(c) (i) and (iv) (d) (i) and (iii)

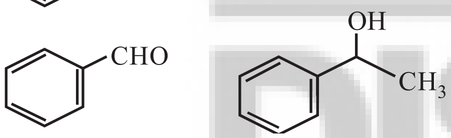
158. The major products P and Q in the following reactions are

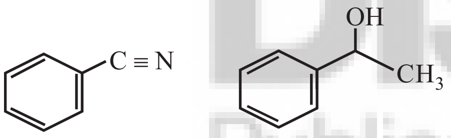


P **Q**

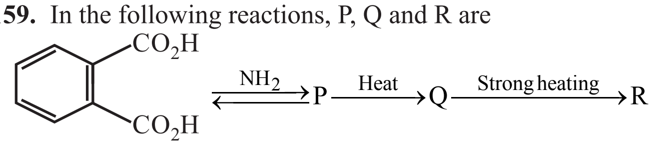
(a) 

(b) 

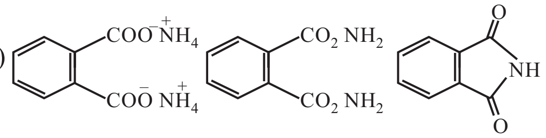
(c) 

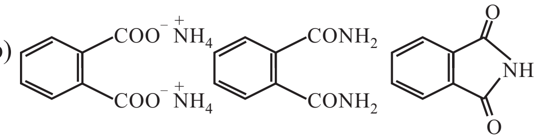
(d) 

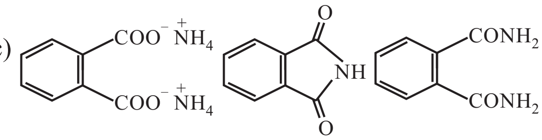
159. In the following reactions, P, Q and R are

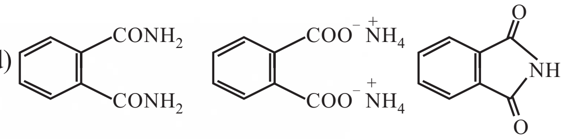


P **Q** **R**

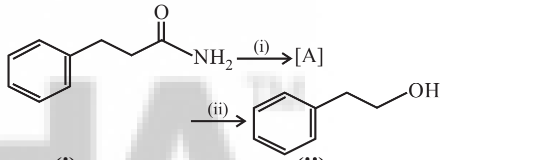
(a) 

(b) 

(c) 

(d) 

160. Following transformation can be accomplished by



	(i)	**(ii)**
(a)	$LiAlH_4$	Pyridinium dichromate
(b)	$Br_2/4KOH$	$NaNO_2.HCl$
(c)	$Br_2/3KOH$	Alc. $KMnO_4$
(d)	$NaNO_2$, HCl	$LiAlH_4$

ANSWER KEY

1	(c)	**2**	(b)	**3**	(c)	**4**	(a)	**5**	(c)	**6**	(b)	**7**	(a)	**8**	(d)	**9**	(a)	**10**	(b)
11	(a)	**12**	(d)	**13**	(c)	**14**	(d)	**15**	(c)	**16**	(a)	**17**	(a)	**18**	(a)	**19**	(a)	**20**	(d)
21	(b)	**22**	(a)	**23**	(d)	**24**	(b)	**25**	(a)	**26**	(d)	**27**	(c)	**28**	(a)	**29**	(a)	**30**	(a)
31	(d)	**32**	(c)	**33**	(b)	**34**	(c)	**35**	(c)	**36**	(b)	**37**	(a)	**38**	(d)	**39**	(b)	**40**	(c)
41	(b)	**42**	(b)	**43**	(d)	**44**	(a)	**45**	(c)	**46**	(a)	**47**	(b)	**48**	(c)	**49**	(b)	**50**	(c)
51	(c)	**52**	(a)	**53**	(c)	**54**	(a)	**55**	(d)	**56**	(b)	**57**	(d)	**58**	(d)	**59**	(b)	**60**	(c)
61	(b)	**62**	(c)	**63**	(d)	**64**	(d)	**65**	(b)	**66**	(d)	**67**	(a)	**68**	(c)	**69**	(c)	**70**	(a)
71	(a)	**72**	(b)	**73**	(a)	**74**	(b)	**75**	(d)	**76**	(d)	**77**	(a)	**78**	(b)	**79**	(c)	**80**	(c)
81	(c)	**82**	(b)	**83**	(a)	**84**	(c)	**85**	(d)	**86**	(a)	**87**	(c)	**88**	(a)	**89**	(c)	**90**	(*)
91	(d)	**92**	(b)	**93**	(b)	**94**	(d)	**95**	(b)	**96**	(a)	**97**	(d)	**98**	(c)	**99**	(b)	**100**	(a)
101	(a)	**102**	(d)	**103**	(d)	**104**	(d)	**105**	(b)	**106**	(d)	**107**	(a)	**108**	(a)	**109**	(d)	**110**	(b)
111	(d)	**112**	(a)	**113**	(a)	**114**	(a)	**115**	(a)	**116**	(a)	**117**	(b)	**118**	(c)	**119**	(c)	**120**	(a)
121	(b)	**122**	(c)	**123**	(a)	**124**	(b)	**125**	(c)	**126**	(b)	**127**	(b)	**128**	(d)	**129**	(d)	**130**	(a)
131	(a)	**132**	(d)	**133**	(a)	**134**	(b)	**135**	(c)	**136**	(c)	**137**	(c)	**138**	(a)	**139**	(b)	**140**	(c)
141	(b)	**142**	(a)	**143**	(c)	**144**	(c)	**145**	(b)	**146**	(b)	**147**	(b)	**148**	(c)	**149**	(a)	**150**	(d)
151	(d)	**152**	(b)	**153**	(d)	**154**	(a)	**155**	(a)	**156**	(c)	**157**	(c)	**158**	(a)	**159**	(b)	**160**	(b)

Hints & Solutions

MATHEMATICS

1. **(c)** We have, $f(x)=\begin{cases}\dfrac{60-5x}{3}, & 0\le x\le 6\\ 10, & 6\le x\le 7\\ 31-3x, & 7\le x\le 10\end{cases}$

Now, $f(x)=\dfrac{60-5x}{3}\Rightarrow f(x_1)=f(x_2)\Rightarrow x_1=x_2$

$\Rightarrow$ One-one

$f(x) = 31 - 3x \Rightarrow f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \Rightarrow$ One-one

But $f(x) = 10 \Rightarrow$ not one-one

So, $f(x)$ is not one-one.

Again,

$\Rightarrow\ 0 \le x \le 6$

$-30 \le -5x \le 0$

$60 - 30 \le 60 - 5x \le 60$

$\dfrac{30}{3}\le\dfrac{60-5x}{3}\le\dfrac{60}{3}$

$\Rightarrow\ 10\le\dfrac{60-5x}{3}\le 20$ ...(i)

and $7 \le x \le 10$

$-30 \le -3x \le -21 \Rightarrow 1 \le 31 - 3x \le 10$ ...(ii)

from eq. (i) & (ii)

$\therefore$ Range of $f(x) = [1, 20]$

and co-domain of $f(x) = [1, 20]$

$\therefore$ Range of $f(x)$ = co-domain of $f(x)$

So, $f(x)$ is onto but not one-one

2. **(b)** We have given that,

$$f(x)=\sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$$

For $f(x)$ to be defined,

$\log_{10}\left(\dfrac{5x-x^2}{4}\right)\ge 0$ and $\dfrac{5x-x^2}{4}>0\Rightarrow\dfrac{5x-x^2}{4}\ge 1$

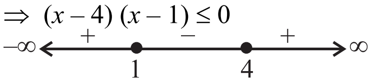
$\Rightarrow\ 5x - x^2 \ge 4 \Rightarrow x^2 - 5x + 4 \le 0$

$x^2 - 4x - x + 4 \le 0$

$x(x - 4) - 1(x - 4) \le 0$

$(x - 4)(x - 1) \le 0$

$\Rightarrow\ (x - 4)(x - 1) \le 0$



$\therefore\ x \in [1, 4]$

So, domain of $f(x)$ is $[1, 4]$.

3. **(c)** Given,

$$\frac{1}{1.7}+\frac{1}{7.13}+\frac{1}{13.19}+...+\frac{1}{[1+(20-1)6][7+(20-1)6]}=\frac{m}{n}$$

$$\frac{1}{1.7}+\frac{1}{7.13}+\frac{1}{13.19}+...+\frac{1}{115\times121}=\frac{m}{n}$$

$$\frac{1}{6}\left[\frac{6}{1.7}+\frac{6}{7.13}+\frac{6}{13.19}+...+\frac{6}{115\times121}\right]=\frac{m}{n}$$

$$\frac{1}{6}\left[\frac{7-1}{1\times7}+\frac{13-7}{7\times13}+\frac{19-13}{13\times19}+...+\frac{121-115}{115\times121}\right]=\frac{m}{n}$$

$$\frac{1}{6}\left[\left(\frac{1}{1}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{13}\right)+\left(\frac{1}{13}-\frac{1}{19}\right)+...+\left(\frac{1}{115}-\frac{1}{121}\right)\right]=\frac{m}{n}$$

$$\frac{1}{6}\left[1-\frac{1}{121}\right]=\frac{20}{121}=\frac{m}{n}\qquad\therefore m=20,\ n=121$$

Thus, $5m + 2n = 5 \times 20 + 2 \times 121 = 100 + 242 = 342$

4. **(a)** We have, the matrices A and B are non-singular of order 3 and |B| = k, a positive integer, so

$|k^{-1}A^{-1}|=(k^{-1})^3|A^{-1}|=\dfrac{1}{k^3|A|}$ $\left[\because |A^{-1}|=\dfrac{1}{|A|}\right]$

$=\dfrac{1}{|B|^3|A|}$ $(\because |B| = K)$

$\because |\text{adj}(A^{-1})| = |A^{-1}|^{3-1} = |A^{-1}|^2=\dfrac{1}{|A|^2}$

[$\because$ |adj**(A)**| = $|A|^{n-1}$, where n is the order of matrix A.]

Now, since it is given that $BAB^{-1} = I$

Multiplying B^{-1} both the sides,

$AB^{-1} = B^{-1}$ $(\because B^{-1}B = I\ \&\ AI = A)$

$\therefore\ BA^kB^{-1} = BA^{k-1}(AB^{-1}) = BA^{k-1}B^{-1}$

$= BA^{k-2}(AB^{-1}) = BA^{k-2}B^{-1}$

$\therefore\ BA^kB^{-1} = BAB^{-1} = I$

and, $B\dfrac{\text{adj }(B)}{|B|}=BB^{-1}=I$

Therefore, $BA^kB^{-1}=B\dfrac{\text{adj }(B)}{|B|}$, if $BAB^{-1}=I$

Now, the adj (adj (A^{-1})) = $|A^{-1}|^{3-2}(A^{-1})$

[$\because$ adj (adj A) = $|A|^{n-2}$ A where n is the order of matrix A]

$\therefore$ adj (adj A) $=\dfrac{1}{|A|}(A^{-1})$

5. **(c)** Given that,

$2x + py + 6z = 8$

$x + 2y + qz = 5$

$x + y + 3z = 4$

Expressing above equations in matrix form, we get,

AX = B

Where, $A=\begin{bmatrix}2 & p & 6\\ 1 & 2 & q\\ 1 & 1 & 3\end{bmatrix}, B=\begin{bmatrix}8\\5\\4\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$

For no solution, det **(A)** = 0

$\Rightarrow \begin{vmatrix} 2 & p & 6 \\ 1 & 2 & q \\ 1 & 1 & 3 \end{vmatrix} = 0$

$\Rightarrow 2(6-q) - p(3-q) + 6(1-2) = 0$

$\Rightarrow 12 - 2q - 3p + pq - 6 = 0$

$\Rightarrow 3p - 6 + 2q - pq = 0$

$\Rightarrow (p-2)(3-q) = 0 \Rightarrow p = 2, 3$

But for p = 2, both equations $2x + py + 6z = 8$ and $x + y + 3z = 4$ become same.

So, $p \neq 2, q = 3$

6. **(b)** We have,

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$

Expressing above equations in matrix form, we get $AX = 0$

Where, $A = \begin{bmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

For non-zero solution, det **(A)** = 0

$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0$

On applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ 0 & \lambda-3 & -\lambda+3 \\ -\lambda+3 & 0 & \lambda-3 \end{vmatrix} = 0$

$\Rightarrow (\lambda - 1)[(\lambda - 3)^2] + (-\lambda + 3)((3\lambda + 1)(-\lambda + 3) - 2\lambda(\lambda - 3)) = 0$

$\Rightarrow (\lambda - 1)(\lambda - 3)^2 - (\lambda - 3)[(\lambda - 3)(-3\lambda - 1 - 2\lambda)) = 0$

$\Rightarrow (\lambda - 3)^2[\lambda - 1 + 5\lambda + 1] = 0$

$\Rightarrow (\lambda - 3)^2(6\lambda) = 0 \Rightarrow \lambda = 3, 0$

As p & q are two real distinct values of λ so,

$\therefore\ p = 3, q = 0$

$\therefore\ p^2 + q^2 - pq = 9 + 0 - 0 = 9$

7. **(a)** A complex number, $z = z + iy$

Given, $|z| \leq 2$

$\Rightarrow \sqrt{x^2 + y^2} \leq 2 \Rightarrow x^2 + y^2 \leq 4$ ...(i)

Also given, $(1-i)z + (1+i)\bar{z} \geq 4$

$\Rightarrow (1 - i)(x + iy) + (1 + i)(x - iy) \geq 4$

$\Rightarrow x + iy - ix + y + x - iy + ix + y \geq 4$

$\Rightarrow 2x + 2y \geq 4 \Rightarrow x + y \geq 2$ ...(ii)

So, $A \cap B = \{|z| \leq 2\} \cap \{(1 - i)z + (1 + i)\bar{z} \geq 4\}$

$A \cap B = \{x^2 + y^2 \leq 4\} \cap \{x + y \geq 2\}$

Now, check all the options satisfying the condition $A \cap B$,

$\therefore\ z = \sqrt{3} + \frac{1}{2}i$

8. **(a)** Given equation, $z^2(1 - z^2) = 16, z \in \mathbf{C}$

Now, let $z^2 = w = r(\cos\theta + i\sin\theta\}$ where $r > 0$.

$\therefore\ 1 - z^2 = \frac{16}{z^2} \Rightarrow z^2 + \frac{16}{z^2} = 1$

Modulas of $z^2 \Rightarrow r$

$\therefore$ Modulus of $\frac{16}{z^2} \Rightarrow \frac{16}{r}$

$\Rightarrow \left(r + \frac{16}{r}\right)\cos\theta + i\left(r - \frac{16}{r}\right)\sin\theta = 1$

On comparing real and imaginary parts, we get

$\left(r + \frac{16}{r}\right)\cos\theta = 1$ and $\left(r - \frac{16}{r}\right)\sin\theta = 0$

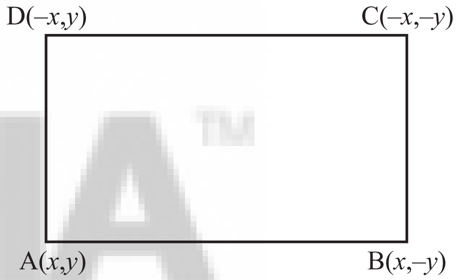
$\cos\theta = 1 \Rightarrow r + \frac{16}{r} = 1$ (not possible)

or $r - \frac{16}{r} = 0 \Rightarrow r^2 = 16 \Rightarrow r = 4$

$\Rightarrow$ Modulus of $z^2 = |z|^2 = r = 4 \Rightarrow |z| = 2$

9. **(a)** Let a complex number, $z = x + iy \Rightarrow \bar{z} = \bar{x} - iy$

Then, vertices of rectangle for $z, \bar{z}, -z - \bar{z}$ are (x, y), $(x, -y)$ $(x, -y)$, $(-x, y)$.



Now, Area of rectangle = $(2x)(2y) = 4xy$

It is given that,

Area $= 2\sqrt{3} = 4xy \Rightarrow 2xy = \sqrt{3}$

$\therefore\ x = \frac{1}{2}, y = \sqrt{3}\ \ \therefore z = \frac{1}{2} + \sqrt{3}i$

10. **(b)** Given that,

$\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^8 + \left(\frac{1 + \cos\theta - i\sin\theta}{1 + \cos\theta + i\sin\theta}\right)^{16}$

$= \left(\frac{\cos\theta + i\sin\theta}{i(\cos\theta - i\sin\theta)}\right)^8 + \left(\frac{2\cos^2\frac{\theta}{2} - i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} + i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)^{16}$

$(\because 2\cos^2\theta = 1 + \cos\theta$ & $\sin^2\theta = 2\sin\theta\cos\theta)$

$= \frac{1}{i^8}\left(\frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}\right)^8 + \left(\frac{\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}\right)^{16}$

By using De Moivre's theorem, we get

$= \frac{\cos 8\theta + i\sin 8\theta}{\cos 8\theta - i\sin 8\theta} + \frac{\cos 8\theta - i\sin 8\theta}{\cos 8\theta + i\sin 8\theta}$

$= (\cos 8\theta + i \sin 8\theta)(\cos 8\theta - i\sin 8\theta)^{-1}$
$+ (\cos 8\theta - i \sin 8\theta (\cos 8\theta + i\sin 8\theta)^{-1}$
$= (\cos 8\theta + i \sin 8\theta)(\cos 8\theta + i (\sin 8\theta) +$
$(\cos 8\theta - i \sin 8\theta)(\cos 8\theta - i \sin 8\theta)$
$= (\cos 8\theta + i \sin 8\theta)^2 + (\cos 8\theta - i\sin 8\theta)^2$
$= \cos^2 8\theta + i^2\sin^2 8\theta + 2i \cos 8\theta \sin 8\theta + \cos^2 8\theta$
$+ i^2 \sin^2 8\theta - 2i \cos 8\theta \sin 8\theta$
$= 2(\cos^2 8\theta - \sin^2 8\theta) = 2\cos 16\theta$
$(\because \cos^2\theta - \sin^2\theta = \cos^2\theta)$

11. **(a)** Given that,
$\lambda x^2 + 13x + 7 = 0$
$\therefore$ $D = (13)^2 - 4(\lambda)(7) = 169 - 28\lambda$
For rational roots, D should be a perfect square.
So, $\lambda \in (-3, 7)$ has values $-2, 0, 6$ so that D become perfect square.
$\therefore$ Required sum of elements in S = $-2 + 0 + 6 = 4$

12. **(d)** Given, $f(x) = 1 - 2x - 5x^2$

$$= -5\left[x^2 + \frac{2}{5}x - \frac{1}{5}\right] = -5\left[\left(x + \frac{1}{5}\right)^2 - \frac{1}{5} - \frac{1}{25}\right]$$

$$= -5\left(x + \frac{1}{5}\right)^2 + \frac{6}{5}$$

$\therefore$ $f(x) \le \frac{6}{5}$

$\therefore$ Maximum value of $f(x) = \alpha = \frac{6}{5}$

Also given, $g(x) = x^2 - 2x + r = (x-1)^2 + r - 1$
$\therefore$ $g(x) \ge (r - 1)$
$\therefore$ Minimum value of given $= \beta = r - 1$
So, $5ax^2 + \beta x + 6 = 6x^2 + (r-1)x + 6$
Since, $5\alpha x^2 + \beta x + 6 > 0$ and coefficient of $x^2 > 0$
$D < 0 \Rightarrow (r-1)^2 - 4 \times 6 \times 6 < 0$
$(r + 11)(r - 13) < 0$
$\therefore$ $r \in (-11, 13)$

13. **(c)** $x^4 + x^3 - 4x^2 + x - 1 = 0$
$(x-1)(x^3 + 2x^2 - 2x - 1) = 0$
$(x-1)(x-1)(x^2 + 3x + 1) = 0$

$$(x-1)(x-1)\left[x - \left(\frac{-3+\sqrt{5}}{2}\right)\right]\left[x - \left(\frac{-3-\sqrt{5}}{2}\right)\right] = 0$$

$\therefore$ Roots of the equation are $x = 1, 1$

$$\frac{-3+\sqrt{5}}{2}, \frac{-3-\sqrt{5}}{2}$$

$\therefore$ Required ratio of roots

$$= \frac{(1)^2 + (1)^2 + \left(\frac{-3+\sqrt{5}}{2}\right)^2 + \left(\frac{-3-\sqrt{5}}{2}\right)^2}{1 \times \left(\frac{-3+\sqrt{5}}{2}\right)\left(\frac{-3-\sqrt{5}}{2}\right)}$$

$$= \frac{1 + 1 + \frac{5+9-6\sqrt{5}}{4} + \frac{5+9+6\sqrt{5}}{4}}{\frac{9-5}{4}}$$

$$= \frac{8+5+9-6\sqrt{5}+5+9+6\sqrt{5}}{4} = 9$$

$\therefore$ Ratio = 9 : 1

14. **(d)** We have,
$\alpha_1, \beta_1, \gamma_1, \delta_1$ are the roots of equation
$ax^4 + bx^3 + cx^2 + dx + e = 0$ ...(i)
$\alpha_2, \beta_2, \gamma_2, \delta_2$ are the roots of equation
$ex^4 + dx^3 + cx^2 + bx + a = 0$ ...(ii)
Also, $\delta_1 > \gamma_1 > \beta_1 > \alpha_1 > 0$
and $\delta_2 > \gamma_2 > \beta_2 > \alpha_2 > 0$
$\therefore$ Clearly, from the above equations (i) & (ii)
$\therefore$ δ_1 and α_2 are reciprocal
Similarly γ_1 and β_2,
γ_2 and β_1 and α_1 and δ_2 are reciprocal
$\alpha_1\delta_2 = 1, \beta_1\gamma_2 = 1, \beta_2\gamma_1 = 1, \alpha_2\delta_1 = 1$
$\alpha_1 - \delta_2 = 2$

$$\therefore \quad \alpha_1 - \frac{1}{\alpha_1} = 2 \Rightarrow \alpha_1^2 - 2\alpha_1 - 1 = 0$$

and $\delta_1 - \alpha_2 = 4$

$$\delta_1 - \frac{1}{\delta_1} = 4 \Rightarrow \delta_1^2 - 4\delta_1 - 1 = 0$$

As it is given that α_1 and δ_1 are roots of the equation
$ax^4 + bx^3 + cx^2 + dx + e = 0$
$\therefore$ $(x^2 - 2x - 1)(x^2 - 4x - 1) = ax^4 + bx^3 + cx^2 + dx + e$
Put $x = 1$,
$a + b + c + d + c = (1 - 2 - 1)(1 - 4 - 1)$
$\therefore$ $a + b + c + d + e = 8$

15. **(c)** Given numbers are 0, 1, 2, 3, 4, 5
Total number of three digit numbers
$= 5 \times 5 \times 4 = 100$
Total number of five digit numbers
$= 5 \times 5 \times 4 \times 3 \times 2 = 600$
$\therefore$ Total numbers = 100 + 600 = 700

16. **(a)** Total contested candidates = 12
Number of ways of selections
4 candidates are to be elected and voter votes for at least one candidate then total
$= {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + {}^{12}C_4$

$$= 12 + \frac{12 \times 11}{2 \times 1} + \frac{12 \times 11 \times 10}{3 \times 2 \times 1} + \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}$$

$= 12 + 66 + 220 = 495 = 793$

17. **(a)** We have, $(1 - x)^3 = 1 - 3x + 3x^2$
$(2 + x)^6 = {}^6C_0 2^6 x^0 + {}^6C_1 2^5 x^1 + {}^6C_2 2^4 x^2$
$= 64 + 192x + 240x^2$
neglecting higher terms in both the expansions
Now,
$(1 - x)^3 (2 + x)^6 = (1 - 3x + 3x^2)(64 + 192x + 240x^2)$
$= 64 + 192x + 240x^2 - 192x - 576x^2 + 192x^2$
(neglecting higher term)
$= -144x^2 + 64$
Comparing with the given equation, we get
$\therefore$ $a = 64, b = 0, c = -144$
$\therefore$ $a + b + c = 64 + 0 - 144 = -80$

18. (a) We have

$$\left(1+\frac{1}{x}\right)^{1/2}=1+\frac{1}{2}\left(\frac{1}{x}\right)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}\left(\frac{1}{x}\right)^2+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{1}{x}\right)^3$$

$$+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)}{4!}\left(\frac{1}{x}\right)^4 x\ldots$$

$$=1+\frac{1}{2x}-\frac{1}{2!}\left(\frac{1}{2x}\right)^2+\frac{1.3}{3!}\left(\frac{1}{2x}\right)^3-\frac{1.3.5}{4!}\left(\frac{1}{2x}\right)^4+\ldots.\infty$$

19. (a) It is given that,

$$\frac{4x^2+5x^4+7}{(x^2+1)(x^4+x^2+1)}=\frac{Ax+B}{x^2+1}+\frac{Cx^3+Dx^2+Ex+F}{x^4+x^2+1}$$

$\Rightarrow\ 4x^2+5x^4+7=(Ax+B)\,(x^4+x^2+1)+(Cx^3+Dx^2+Ex+F)\,(x^2+1)$

$\Rightarrow\ 5x^4+4x^2+7=Ax^5+Ax^3+Ax+Bx^4+Bx^2+B+Cx^5+Cx^3+Dx^4+Dx^2+Ex^3+Ex+Fx^2+F$

$\Rightarrow\ 5x^4+4x^2+7=(A+C)\,x^5+(B+D)\,x^4+(A+C+E)\,x^3+(B+D+F)\,x^2+(A+E)\,x+(B+F)$

On comparing both the sides, we get

$A+C=0$

$B+D=5$

$A+C+E=0$

$B+D+F=4$

$A+E=0$

$B+F=7$

On solving the equations given above, we get

$A=C=E=0,\ B=8,\ D=-3,\ F=-1$

$\therefore\ B+2\,(D+F+E)-C\,.\,A$

$=8+2\,(-3-1+0)-0\times 0=0$

20. (d) Given that,

$$\sin^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}-\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}-\sin^4\frac{7\pi}{8}$$

$$=\sin^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}-\sin^4\frac{3\pi}{8}+\sin^4\left(\pi-\frac{3\pi}{8}\right)+\cos^4\frac{7\pi}{8}-\sin^4\left(\pi-\frac{\pi}{8}\right)$$

$$=\sin^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}-\sin^4\frac{3\pi}{8}+\sin^4\frac{3\pi}{8}+\cos^4\frac{7\pi}{8}-\sin^4\frac{\pi}{8}$$

$$=\cos^4\frac{3\pi}{8}+\cos^4\frac{7\pi}{8}=\cos^4\frac{3\pi}{8}+\cos^4\left(\pi-\frac{\pi}{8}\right)$$

$$=\cos^4\frac{3\pi}{8}+\left(-\cos\frac{\pi}{8}\right)^4=\cos^4\frac{3\pi}{8}+\cos^4\frac{\pi}{8}$$

$$=\left(\cos^2\frac{3\pi}{8}\right)^2+\left(\cos^2\frac{\pi}{8}\right)^2=\left(\frac{1+\cos\frac{3\pi}{4}}{2}\right)^2+\left(\frac{1+\cos\frac{\pi}{4}}{2}\right)^2$$

$$=\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]$$

$$=\frac{1}{4}\left[1+\frac{1}{2}-\frac{2}{\sqrt{2}}+1+\frac{1}{2}+\frac{2}{\sqrt{2}}\right]=\frac{3}{4}$$

21. (b) It is given that,

$$\operatorname{cosec}^{-1}\left[\left(\frac{\tan^2\left(\frac{\alpha-\pi}{4}\right)-1}{\tan^2\left(\frac{\alpha-\pi}{4}\right)+1}+\cos\frac{\alpha}{2}\cos 5\alpha\right)\sec\frac{11\alpha}{2}\right]$$

$$=\operatorname{cosec}^{-1}\left[\left(-\cos 2\left(\frac{\alpha-\pi}{4}\right)+\cos\frac{\alpha}{2}\cot 5\alpha\right)\sec\frac{11\alpha}{2}\right]$$

$$=\operatorname{cosec}^{-1}\left[\left(-\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\frac{\cos 5\alpha}{\sin 5\alpha}\right)\sec\frac{11\alpha}{2}\right]$$

$$=\operatorname{cosec}^{-1}\left[\left(\frac{-\sin 5\alpha\sin\frac{\alpha}{2}+\cos 5\alpha\cos\frac{\alpha}{2}}{\sin 5\alpha}\right)\sec\frac{11\alpha}{2}\right]$$

$$=\operatorname{cosec}^{-1}\left[\frac{\cos\left(5\alpha+\frac{\alpha}{2}\right)}{\sin 5\alpha}.\sec\frac{11\alpha}{2}\right]$$

$$=\operatorname{cosec}^{-1}\left[\operatorname{cosec}\ 5\alpha.\cos\frac{11\alpha}{2}\sec\frac{11\alpha}{2}\right]$$

$$=\operatorname{cosec}^{-1}\left[\operatorname{cosec}\ 5\alpha\cos\frac{11\alpha}{2}\times\frac{1}{\cos\frac{11\alpha}{2}}\right]$$

$=\operatorname{cosec}^{-1}(\operatorname{cosec}\ 5\alpha)=5\alpha$

22. (a) Reason : In ΔPQR,

$P+Q+R=180°$

$$\Rightarrow\ \frac{P}{2}+\frac{Q}{2}+\frac{R}{2}=\frac{180}{2}\ \Rightarrow\ \frac{P}{2}+\frac{Q}{2}=90°-\frac{R}{2}$$

$$\Rightarrow\ \tan\left(\frac{P}{2}+\frac{Q}{2}\right)=\tan\left(90-\frac{R}{2}\right)$$

$$\Rightarrow\ \frac{\tan\frac{P}{2}+\tan\frac{Q}{2}}{1-\tan\frac{P}{2}\tan\frac{Q}{2}}=\cot\frac{R}{2}=\frac{1}{\tan\frac{R}{2}}$$

$$\Rightarrow\ \left(\tan\frac{P}{2}+\tan\frac{Q}{2}\right)\tan\frac{R}{2}=1-\tan\frac{P}{2}\tan\frac{Q}{2}$$

$\Rightarrow \tan\frac{P}{2}\tan\frac{Q}{2}+\tan\frac{Q}{2}\tan\frac{R}{2}+\tan\frac{R}{2}\tan\frac{P}{2}=1$

So, reason is true

Assertion : Given that

$A = 15°, B = 17°, C = 13°$

$\Rightarrow A + B + C = 45°$

$\Rightarrow 2A + 2B + 2C = 90°$

$\therefore$ Let, $\frac{P}{2}=2A, \frac{Q}{2}=2B, \frac{R}{2}=2C$

By using the expression derived in reason

$\therefore$ tan 2 A tan 2B + tan 2B tan 2C + tan 2C tan 2A = 1

$$\Rightarrow \frac{1}{\cot 2A\cot 2B}+\frac{1}{\cot 2B\cot 2C}+\frac{1}{\cot 2C\cot 2A}=1$$

$$\Rightarrow \frac{\cot 2C+\cot 2A+\cot 2B}{\cot 2A\cot 2B\cot 2C}=1$$

$\Rightarrow$ cot 2A + cot 2B + cot 2C = cot 2 A cot 2 B cot 2C

$\therefore$ Assertion is true.

23. **(d)** Given, tan θ + 5 cot θ = sec θ

$$\Rightarrow \frac{\sin\theta}{\cos\theta}+\frac{5\cos\theta}{\sin\theta}=\frac{1}{\cos\theta} \Rightarrow \frac{\sin^2\theta+5\cos^2\theta}{\cos\theta\sin\theta}=\frac{1}{\cos\theta}$$

$\Rightarrow \sin^2\theta + 5(1-\sin^2\theta) = \sin\theta$

$\Rightarrow 4\sin^2\theta + \sin\theta - 5 = 0$

$\Rightarrow (4\sin\theta + 5)(\sin\theta - 1) = 0$

$\sin\theta = \frac{-5}{4}, 1$

$\Rightarrow \sin\theta = 1$ and $\sin\theta \neq -\frac{5}{4}$ $\quad [\because -1 \le \sin\theta \le 1]$

Hence, solution set of trigonometric equation.

$\theta \in \phi$

24. **(b)** It is given that,

$$\tan^{-1}\frac{1}{5}+\frac{1}{2}\sec^{-1}x+\tan^{-1}\frac{1}{8}=\frac{\pi}{8}$$

$$\Rightarrow \left(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}\right)+\frac{1}{2}\sec^{-1}x=\frac{\pi}{8}$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\times\frac{1}{8}}\right]+\frac{1}{2}\sec^{-1}x=\frac{\pi}{8}$$

$$\Rightarrow \tan^{-1}\frac{13}{39}+\frac{1}{2}\sec^{-1}x=\frac{\pi}{8}$$

$$\Rightarrow 2\tan^{-1}\frac{13}{39}+\sec^{-1}x=\frac{\pi}{4}$$

$$\Rightarrow 2\tan^{-1}\frac{1}{3}+\sec^{-1}x=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}}+\sec^{-1}x=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{3}{4}+\sec^{-1}x=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{3}{4}+\tan^{-1}\sqrt{x^2-1}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{3}{4}+\sqrt{x^2-1}}{1-\frac{3}{4}\sqrt{x^2-1}}\right]=\frac{\pi}{4}$$

$$\Rightarrow \frac{3+4\sqrt{x^2-1}}{4-3\sqrt{x^2-1}}=\tan\left(\frac{\pi}{4}\right)=1$$

$$\Rightarrow 3+4\sqrt{x^2-1}=4-3\sqrt{x^2-1} \Rightarrow 7\sqrt{x^2-1}=1$$

$$\Rightarrow \sqrt{x^2-1}=\frac{1}{7} \Rightarrow x^2-1=\frac{1}{49} \Rightarrow x^2=\frac{50}{49}$$

25. **(a)** **Reason :** The equation is, $xp^2 - 2p - x = 0$

$$\Rightarrow p=\frac{2\pm\sqrt{4+4x^2}}{2x}=\frac{1\pm\sqrt{1+x^2}}{x}$$

$$=\frac{1+\sqrt{1+x^2}}{x}, \frac{1-\sqrt{1+x^2}}{x}$$

$$=e^{\ln\left(\frac{1+\sqrt{1+x^2}}{x}\right)}, e^{\ln\left(\frac{1-\sqrt{1+x^2}}{x}\right)}$$

We know that cosec $h^{-1}x = \ln\left(\frac{1+\sqrt{1+x^2}}{x}\right)$

$\therefore e^{\text{cosec}\,h^{-1}x}$ is a solution of given equation.

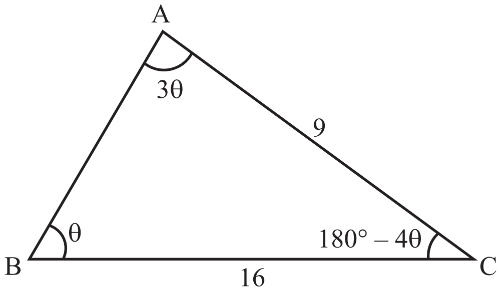
Assertion :

$$\text{cosech}^{-1}(3)=\ln\left(\frac{1+\sqrt{1+3^2}}{3}\right)=\ln\left(\frac{1+\sqrt{10}}{3}\right)$$

26. **(d)** In ΔABC, let $\angle B = \theta \Rightarrow \angle A = 3\theta$

$\angle A + \angle B + \angle C = 180°$

$\therefore \angle C = 180° - 4\theta$



In ΔABC, we know

$$\frac{AB}{\sin C}=\frac{BC}{\sin A}=\frac{AC}{\sin B}$$

$$\Rightarrow \frac{AB}{\sin(180°-4\theta)}=\frac{16}{\sin 3\theta}=\frac{9}{\sin\theta}$$

$$\Rightarrow \frac{AB}{\sin 4\theta}=\frac{16}{\sin 3\theta}=\frac{9}{\sin\theta}$$

Now, $\dfrac{16}{\sin 3\theta} = \dfrac{9}{\sin\theta} \Rightarrow 16\sin\theta = 9\sin 3\theta$

$\Rightarrow$ $16\sin\theta = 9(3\sin\theta - 4\sin^3\theta)$

$\Rightarrow$ $16 = 27 - 36\sin^2\theta \Rightarrow 36\sin^2\theta = 11$

$\Rightarrow$ $\sin\theta = \dfrac{\sqrt{11}}{6} \Rightarrow \cos\theta = \dfrac{5}{6}$

Now, $\sin 4\theta = 2\sin 2\theta\ \cos 2\theta$

$= 2 \times 2\sin\theta\cos\theta(1 - 2\sin^2\theta)$

$= 4 \times \dfrac{\sqrt{11}}{6} \times \dfrac{5}{6}\left(1 - 2 \times \dfrac{11}{36}\right) = \dfrac{35\sqrt{11}}{162}$

Again, we have

$\dfrac{AB}{\sin 4\theta} = \dfrac{9}{\sin\theta}$

$\Rightarrow$ $AB = \dfrac{9\sin 4\theta}{\sin\theta} = 9 \times \dfrac{35\sqrt{11}}{162} \times \dfrac{6}{\sqrt{11}} = \dfrac{35}{3}$

27. **(c)** In ΔABC, we know that

$r_1 + r_2 = 4R\left(\sin\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2} + \sin\dfrac{B}{2}\cos\dfrac{A}{2}\cos\dfrac{C}{2}\right)$

$= 4R\cos\dfrac{C}{2}\left[\sin\dfrac{A}{2}\cos\dfrac{B}{2} + \sin\dfrac{B}{2}\cos\dfrac{A}{2}\right]$

$= 4R\cos\dfrac{C}{2}\sin\left(\dfrac{A+B}{2}\right) = 4R\cos\dfrac{C}{2}\sin\left(\dfrac{\pi}{2} - \dfrac{C}{2}\right)$

$\left(\because A + B + C = \pi \Rightarrow \dfrac{A+B}{2} = \dfrac{\pi}{2} - \dfrac{C}{2}\right)$

$= 4R\cos\dfrac{C}{2}.\cos\dfrac{C}{2} = 2R\left(2\cos^2\dfrac{C}{2}\right) = 2R(1 + \cos C)$

$\therefore$ $\dfrac{1 + \cos C}{r_1 + r_2} = \dfrac{1}{2R}$

Similarly,

$\dfrac{1 + \cos A}{r_2 + r_3} = \dfrac{1}{2R'}\dfrac{1 + \cos B}{r_1 + r_3} = \dfrac{1}{2R}$

Hence, $\dfrac{1 + \cos C}{r_1 + r_2} + \dfrac{1 + \cos A}{r_2 + r_3} + \dfrac{1 + \cos B}{r_1 + r_3} = \dfrac{3}{2R}$

28. **(a)** In ΔABC we have

$\cos A\cos B + \sin A\sin B\sin C = 1$

$\cos A\cos B + \sin A\sin B - \sin A\sin B + \sin A\sin B\sin C = 1$

$\cos(A - B) - \sin A\sin B(1 - \sin C) = 1$

$1 - \cos(A - B) + \sin A\sin B(1 - \sin C) = 0$

$\left(\because 1 - \cos\theta = 2\sin\dfrac{2\theta}{2}\right)$

$2\sin^2\left(\dfrac{A-B}{2}\right) + \sin A\sin B(1 - \sin C) = 0$

As we know that,

$0 \le \sin A, \sin B, \sin C \le 1$

$\therefore$ $\sin\left(\dfrac{A-B}{2}\right) = 0$ and $\sin A\sin B(1 - \sin C) = 0$

$A - B = 0$ and $\sin C = 1$ $\quad [\because \sin A, \sin B \neq 0]$

$\Rightarrow$ $A = B$ and $C = 90°$

$\therefore$ $A = B = 45°, C = 90°$

Now, according to sine formulae.

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\dfrac{a}{\sin 45°} = \dfrac{b}{\sin 45°} = \dfrac{c}{\sin 90°}$

$a : b : c = \dfrac{1}{\sqrt{2}} : \dfrac{1}{\sqrt{2}} : 1 = 1 : 1 : \sqrt{2}$

29. **(a)** We have, $OA = a$, $OB = b$ be two non-collinear vectors, $OP = x_1a + y_1b$, $OQ = x_2a + y_2b$

where, $x_1 = \dfrac{-3}{4}, x_2 = \dfrac{1}{3}, y_1 = \dfrac{7}{4}, y_2 = \dfrac{5}{3}$

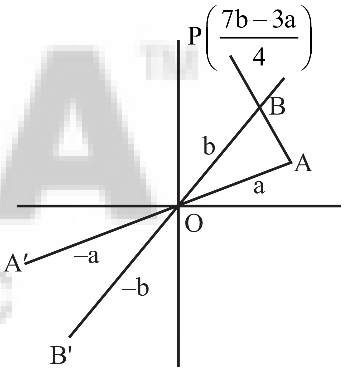
$OP = \dfrac{-3}{4}a + \dfrac{7}{4}b = \dfrac{-3a + 7b}{4} = \dfrac{7b - 3a}{7 - 3}$

$\therefore$ OP divides AB externally in ratio $m : n = 7 : 3$.

$OQ = \dfrac{1}{3}a + \dfrac{5}{3}b = \dfrac{a + 6b}{3}$

Also, A'O = OA = a, OA' = $-a$

B'O = OB = b, OB' = $-b$



Clearly we can observe from the figure that, p lies inside the ΔA'OB and Q lies outside the ΔAOB.

30. **(a)** Given,

Position vector of a point P is $(2\hat{i} + \hat{j} + 3\hat{k})$

$a = -\hat{i} - 2\hat{k}.b = \hat{i} + \hat{j} + 2\hat{k}$

Equation of plane, $(r - p).(a \times b) = 0$

$(r - 2\hat{i} + \hat{j} + 3\hat{k}).(a \times b) = 0$

$\therefore$ Normal of plane $= a \times b$

Equation of line through $P(2\hat{i} + \hat{j} + 3\hat{k})$ and normal to b and lying on the plane is (i.e. Direction of line $= ((a \times b) \times b)$

$r = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda((a \times b) \times b)$

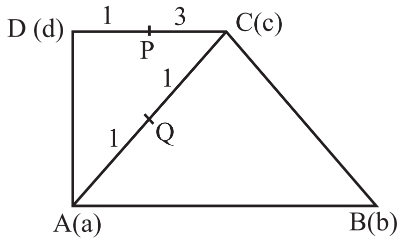
$r = 2\hat{i} + \hat{j} + 3\hat{k} + \lambda((b.a)b - (b.b)a)$

$\because (a \times b) \times b = (b.a)\,b - (b.b)\,a$

$r = 2\hat{i} + \hat{j} + 3\hat{k} + \lambda[(-5)(\hat{i} + \hat{j} + 2\hat{k}) - 6(-\hat{i} - 2\hat{k})]$

$r = 2\hat{i} + \hat{j} + 3\hat{k} + \lambda(-\hat{i} + 5\hat{j} - 2\hat{k})$

31. **(d)** In quadrilateral $ABCD$, the point P divides DC in the ratio 1 : 3 internally & Q is the mid point of AC.



We have, $OP = \dfrac{c+3d}{4}$ and $OQ = \dfrac{a+c}{2}$

It is given that

$AB + AD + BC - 2DC = \lambda PQ$

$(b - a) + (d - a) + (c - b) - 2\,(c - d)$

$= \lambda\left[\dfrac{a+c}{2} - \dfrac{c-3d}{4}\right]$

$-2a - c + 3d = \dfrac{\lambda}{4}[2a + c - 3d]$

$\therefore \quad \dfrac{2\lambda}{4} = -2 \Rightarrow \lambda = -4$

32. **(c)** It is given that, $p = 2\hat{i} - 3\hat{j} + \hat{k}, q = \hat{i} + \hat{j} - \hat{k}$

$\therefore \quad p.q = (2\hat{i} - 3\hat{j} + \hat{k}).(\hat{i} + \hat{j} - \hat{k})$

$= 2 - 3 - 1 = -2$

$|p| = \sqrt{(2)^2 + (-3)^2 + (1)^2} = \sqrt{4+9+1} = \sqrt{14}$

$|q| = \sqrt{(1)^2 + (1)^2 + (-1)^2} = \sqrt{1+1+1} = \sqrt{3}$

Now, a = orthogonal projections of p on q

$= \dfrac{p.q}{|q|^2} q = \dfrac{-2}{3}(\hat{i} + \hat{j} - \hat{k})$

b = orthogonal projections of q on p

$= \dfrac{q.p}{|p|^2} p = \dfrac{-2}{14}(2\hat{i} - 3\hat{j} + \hat{k})$

Now, $a \times b = \dfrac{4}{42}(\hat{i} + \hat{j} - \hat{k}) \times (2\hat{i} - 3\hat{j} + \hat{k})$

$= \dfrac{4}{42}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -3 & 1 \end{vmatrix} = \dfrac{4}{42}[\hat{i}(1-3) - \hat{j}(1+\hat{i}) + \hat{k}(-3-2)]$

$= \dfrac{4}{42}(-2\hat{i} - 3\hat{j} - 5\hat{k})$

and $a.b = \dfrac{4}{42}(\hat{i} + \hat{i} - \hat{k})(2\hat{i} - 3\hat{j} + \hat{k})$

$= \dfrac{4}{42}[(1,1,-1).(2,-3,k)] \;= \dfrac{4}{42}(2-3-1) = \dfrac{-8}{42}$

$\therefore \quad \dfrac{a \times b}{a.b} = \dfrac{\dfrac{4(-2\hat{i} - 3\hat{j} - 5\hat{k})}{42}}{\dfrac{-8}{42}} = \dfrac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2}$

33. **(a)** Given,

$a = 2\hat{i} - 3\hat{j} + 4\hat{k}, b = 7\hat{i} + 2\hat{j} - 3\hat{k}, c = \hat{i} + \hat{j} + \hat{k}$

Since, vector x is perpendicular to both vectors a and b.

$\therefore \quad x = \lambda(a \times b) = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 7 & 2 & -3 \end{vmatrix} = \lambda(\hat{i} + 34\hat{j} + 25\hat{k})$

Now, we have $x.c = 60$

$\Rightarrow \quad (\lambda\hat{i} + 34\lambda\hat{j} + 25\lambda\hat{k}).(\hat{i} + \hat{j} + \hat{k}) = 60$

$(\lambda, 34\lambda, 25\lambda).(1,1,1) = 60$

$\Rightarrow \quad \lambda + 34\lambda + 25\lambda = 60 \Rightarrow 60\lambda = 60 \Rightarrow \lambda = 1$

$\therefore \quad x = \hat{i} + 34\hat{j} + 25\hat{k}$

34. **(c)** We have,

$L : r = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$

$\pi : r.(\hat{i} + 5\hat{j} + \hat{k}) = 5$

Now, $(\hat{i} - \hat{j} + 4\hat{k}).(\hat{i} + 5\hat{j} + \hat{k}) = 1 - 5 + 4 = 0$

This means line is parallel to plane.

$\therefore$ Shortest distance between line and plane = Distance of point $(2\hat{i} - 2\hat{j} + 3\hat{k})$ from plane

$D = \left|\dfrac{(2\hat{i} - 2\hat{j} + 3\hat{k}).(\hat{i} + 5\hat{j} + \hat{k}) - 5}{|\hat{i} + 5\hat{j} + \hat{k}|}\right|$

$D = \left|\dfrac{2 - 10 + 3 - 5}{\sqrt{1+25+1}}\right| = \dfrac{10}{\sqrt{27}} = \dfrac{10}{3\sqrt{3}}$

35. **(c)** For the following frequency distribution,

CI	f_i	x_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-5	4	2.5	10	6.25	25
5-10	1	7.5	7.5	56.25	56.25
1015	10	12.5	125	156.25	1562.5
15-20	3	17.5	52.5	306.25	918.75
20-25	2	22.5	45	506.25	1012.50
Total	$\Sigma f_i = 20$		240		3575

Now, variance, $V(x) = \dfrac{1}{N}\Sigma f_i x_i^2 - \left(\dfrac{1}{N}\Sigma f_i x_i\right)^2$

$= \dfrac{1}{20} \times 3575 - \left(\dfrac{1}{20} \times 240\right)^2 = 178.75 - 144 = 34.75$

36. **(b)** For the given distribution : 8, 9, 6, 5, x, 4, 6, 5

$\therefore \quad$ Mean $= \dfrac{8+9+6+5+x+4+6+5}{8} = 6$

$\Rightarrow$ 43 + x = 48 $\Rightarrow x = 5$

$\therefore$ So, x_i = 8, 9, 6, 5, 5, 4, 6, 5

$\Rightarrow \quad x_i^2 = 64, 81, 36, 25, 25, 16, 36, 25$

$\therefore \quad \Sigma x_i^2 = 64+81+36+25+25+16+36+25$

$= 308$

Standard deviation,

$\therefore \quad SD(x) = \sqrt{\frac{1}{N}\Sigma x_i^2 - (\bar{x})^2}$

$= \sqrt{\frac{1}{8}\times 308 - (6)^2} = \sqrt{38.5-36} = \sqrt{2.5} = 1.58$

37. **(a)** It is given that, $P(A\cup B) = \frac{3}{4}, P(A\cap B) = \frac{1}{4}$ and

$P(\bar{A}) = \frac{2}{3}$

We know that, $P(\bar{A}) = 1 - P(A)$

$\therefore$ P(**A**) = 1/3

$\Rightarrow P(\bar{A}\cap B) = P(A\cup B) - P(A)$

$= \frac{3}{4} - \frac{1}{3} = \frac{5}{12}$

38. **(d)** For two events x and y, given that

$P(X) = \frac{1}{3}, P\left(\frac{X}{Y}\right) = \frac{1}{2}$ and $P\left(\frac{Y}{X}\right) = \frac{2}{5}$

Now, $P\left(\frac{Y}{X}\right) = \frac{P(Y\cap X)}{P(X)} = \frac{2}{5}$

$\Rightarrow P(Y\cap X) = \frac{2}{5}\times\frac{1}{3} = \frac{2}{15}$

Again, $P\left(\frac{X}{Y}\right) = \frac{P(X\cap Y)}{P(Y)} = \frac{1}{2}$

$\frac{1}{2} = \frac{\frac{2}{15}}{P(Y)} \Rightarrow P(Y) = \frac{4}{15}$

Also, $P(X\cup Y) = P(X) + P(Y) - P(X\cap Y)$

$= \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{5}{15} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$

$\therefore \quad P\left(\frac{\bar{X}}{Y}\right) = \frac{P(\bar{X}\cap Y)}{P(Y)} = \frac{P(Y) - P(X\cap Y)}{P(Y)}$

$= 1 - \frac{P(X\cap Y)}{P(Y)} = 1 - \frac{\frac{2}{15}}{\frac{4}{15}} = 1 - \frac{2}{15}\times\frac{15}{4} = 1 - \frac{1}{2} = \frac{1}{2}$

39. **(b)** Given that, A & B are not mutually exclusive events.

$P(A) = \frac{4}{9}, P(A\cap\bar{B}) = \frac{3}{7}$

Now, $P(A\cap\bar{B}) = P(A) - P(A\cap B)$

$\Rightarrow P(A\cap B) = P(A) - P(A\cap\bar{B})$

$= \frac{4}{9} - \frac{3}{7} = \frac{28-27}{63} = \frac{1}{63}$

$\therefore \quad P\left(\frac{B}{A}\right) = \frac{P(B\cap A)}{P(A)} = \frac{\left(\frac{1}{63}\right)}{\left(\frac{4}{9}\right)} = \frac{1}{28}$

40. **(c)** We have, probability of defective bolt,

$P = \frac{20}{100} = \frac{1}{5}$

Probability of non defective bolt,

$q = 1 - p = 1 - \frac{1}{5}$

$q = \frac{4}{5}$

Total chosen bolts $n = 4$

$\therefore$ Required probability $= P(x<2)$

$= P(x=0) + P(x=1)$

$= {}^4C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^4 + {}^4C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^3$

$= \left(\frac{4}{5}\right)^4 + \left(\frac{4}{5}\right)^4 = 2\left(\frac{4}{5}\right)^4 = 0.8192$

41. **(b)** We have, $\lambda = \frac{60}{600} = 0.1$

It follows poisson's distribution.

$\therefore \quad P(X=x) = \frac{e^{-\lambda}(\lambda)^x}{x!} = \frac{e^{-0.1}(0.1)^x}{x!}$

Required probability

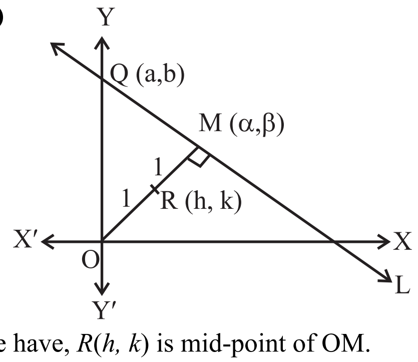
$= P(X=0) + P(X=1) + P(X=2)$

$= e^{-0.1}\left[\frac{(0.1)^0}{0!} + \frac{(0.1)^1}{1!} + \frac{(0.1)^2}{2!}\right]$

$= e^{-0.1}\left[1 + \frac{1}{10} + \frac{1}{200}\right] = e^{-0.1}\left[\frac{200+20+1}{200}\right]$

$= \frac{1}{e^{0.1}}\left(\frac{221}{200}\right)$

42. **(b)**



We have, $R(h, k)$ is mid-point of OM.

$\therefore$ Coordinates of $R = (h, k)$

$= \left(\frac{0+\alpha}{2}, \frac{0+\beta}{2}\right) \Rightarrow \alpha = 2h, \beta = 2k$

$\Rightarrow$ Coordinates of M are $(2h, 2k)$.

Now, slope of OM $= \frac{2k-0}{2h-0} = \frac{k}{h}$

and slope of MQ $= \frac{2k-b}{2h-a}$

Since, OM and MQ are perpendicular to each other

$\therefore \quad \frac{k}{h} \times \frac{2k-b}{2h-a} = -1 \Rightarrow 2k^2 - bk = -2h^2 + ah$

$\Rightarrow 2h^2 + 2k^2 - ah - bk = 0$

$\therefore$ Locus of $R(h, k)$ is $2x^2 + 2y^2 - ax - by = 0$

43. **(d)** When the origin is shifted to the point $\left(\frac{3}{2}, \frac{3}{2}\right)$. Then substituting $x = X + \frac{3}{2}, y = Y + \frac{3}{2}$ in the equation

$32x^2 + 8xy + 32y^2 - 108x - 108y + 99 = 0$

We get

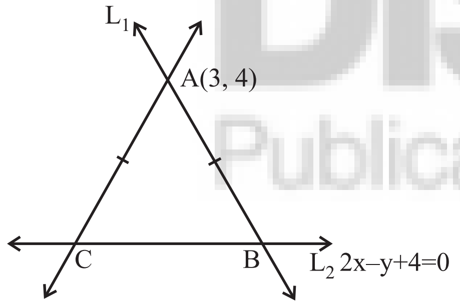
$$32\left(X + \frac{3}{2}\right)^2 + 8\left(X + \frac{3}{2}\right)\left(Y + \frac{3}{2}\right) + 32\left(Y + \frac{3}{2}\right)^2 - 108\left(X + \frac{3}{2}\right) - 108\left(Y + \frac{3}{2}\right) + 99 = 0$$

$$\Rightarrow 32\left[X^2 + 3X + \frac{9}{4}\right] + 8\left(\frac{2X+3}{2}\right)\left(\frac{2Y+3}{2}\right) + 32\left[Y^2 + 3Y + \frac{9}{4}\right] - 108X - 162 - 108Y - 162 + 99 = 0$$

$\Rightarrow 32X^2 + 96X + 72 + 2(4XY + 6X + 6Y + 9) + 32Y^2 + 96Y + 72 - 108X - 162 - 108Y - 162 + 99 = 0$

$\Rightarrow 32X^2 + 32Y^2 + 8XY - 63 = 0$

44. **(a)** Given, slope of $L_1 = 1$



Equation of line L_1:

$y - 4 = 1(x - 3)$

$y - 4 = x - 3$

$x - y + 1 = 0$

Let slope of AC is m

Since $AC = BC \Rightarrow \Delta ABC$ is isosceles

$\therefore \quad \angle C = \angle B$

$\tan C = \tan B$

$\left|\frac{m-2}{1+2m}\right| = \left|\frac{2-1}{1+2\times 1}\right| \quad \left[\because \text{slope of } BC = \frac{-2}{-1} = 2\right]$

$\Rightarrow \frac{m-2}{1+2m} = \pm\frac{1}{3}$

$\Rightarrow \frac{m-2}{1+2m} = \frac{1}{3}$ or $\frac{m-2}{1+2m} = \frac{-1}{3} \Rightarrow m = 7$ or $m = 1$

But for $m = 1$, AC become L_1

$\therefore m = 7$

Equation of line AC is

$y - 4 = 7(x - 3)$

$y - 4 = 7x - 21$

$7x - y - 17 = 0$

45. **(c)** Let slope of line $= m$

$\therefore$ Equation of line is

$y - 2 = m(x - 1)$

$y - 2 = mx - m$

$mx - y + (2 - m) = 0$

On solving $mx - y + (2 - m)$ and $x + y = 4$, we get the coordinates $\left(\frac{m+2}{m+1}, \frac{3m+2}{m+1}\right)$.

Now, it is given that

$$\text{Distance} = \sqrt{\left(\frac{m+2}{m+1} - 1\right)^2 + \left(\frac{3m+2}{m+1} - 2\right)^2} = \frac{\sqrt{6}}{3}$$

$\Rightarrow \frac{1}{(m+1)^2} + \frac{m^2}{(m+1)^2} = \frac{6}{9} \Rightarrow \frac{1+m^2}{(m+1)^2} = \frac{2}{3}$

$\Rightarrow 3 + 3m^2 = 2m^2 + 4m + 2$

$\Rightarrow m^2 - 4m + 1 = 0$

$\Rightarrow m = 2 + \sqrt{3}, 2 - \sqrt{3}$

$\tan\alpha = 2 - \sqrt{3} \Rightarrow \alpha = \frac{\pi}{12}$

$\tan\beta = 2 + \sqrt{3} \Rightarrow \beta = \frac{5\pi}{12}$

46. **(a)** It is given that lines $x + 3y - 9 = 0$, $4x + 5y - 1 = 0$, $px + qy + 10 = 0$ are concurrent if

$$\begin{vmatrix} 1 & 3 & -9 \\ 4 & 5 & -1 \\ p & q & 10 \end{vmatrix} = 0$$

$\Rightarrow 50 + q - 3(40 + p) - 9(4q - 5p) = 0$

$\Rightarrow 50 + q - 120 - 3p - 36q + 45p = 0$

$\Rightarrow -6p + 5q + 10 = 0$

So, $5x + 6y + 10 = 0$ passes through $(q, -p)$.

47. **(b)** The equation $(2l - 3)x^2 + 2lxy - y^2 = 0$ will represent a pair of real and distinct lines, if

$h^2 > ab$

Here, $a = 2l - 3$

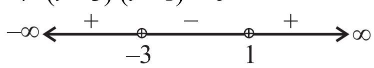
$h = 2l$,

$b = -1$

$l^2 - (2l - 3)(-1) > 0$

$\Rightarrow l^2 + 2l - 3 > 0$

$\Rightarrow (l + 3)(l - 1) > 0$



$\therefore \quad l \in R - (-3, 1)$

48. **(c)** Given that, the pair of lines

$2y^2 - xy - 6x^2 = 0$

$\Rightarrow 2y^2 - 4xy + 3xy - 6x^2 = 0$

$\Rightarrow 2y(y - 2x) + 3x(y - 2x) = 0$

$\Rightarrow (y - 2x)(2y + 3x) = 0$

$\therefore$ Equations of sides of triangle are $x + y = 1$, $y - 2x = 0$, $2y + 3x = 0$

On solving these equations, we get the vertices of triangle as $\left(\frac{1}{3}, \frac{2}{3}\right)$, (0, 0) and (–2, 3).

$$\therefore \text{ Centroid } = \left(\frac{\frac{1}{3}+0-2}{3}, \frac{\frac{2}{3}+0+3}{3}\right) = \left(\frac{-5}{9}, \frac{11}{9}\right)$$

49. **(b)** Equation of circle passing through the points (1, 1) (2, –1) and (3, 2) is

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 1+1 & 1 & 1 & 1 \\ 4+1 & 2 & -1 & 1 \\ 9+4 & 3 & 2 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 2 & 1 & 1 & 1 \\ 5 & 2 & -1 & 1 \\ 13 & 3 & 2 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$R_3 \to R_3 - R_4$

$$\begin{vmatrix} x^2+y^2-2 & x-1 & y-1 & 0 \\ -3 & -1 & 2 & 0 \\ -8 & -1 & -3 & 0 \\ 13 & 3 & 2 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x^2+y^2-2 & x-1 & y-1 \\ -3 & -1 & 2 \\ -8 & -1 & -3 \end{vmatrix} = 0$$

$(x^2 + y^2 - 2)(3 + 2) - (x - 1)(9 + 16) + (y - 1)(3 - 8) = 0$

$5(x^2 + y^2 - 2) - 25(x - 1) - 5(y - 1) = 0$

$x^2 + y^2 - 2 - 5(x - 1) - (y - 1) = 0$

$x^2 + y^2 - 5x - y + 4 = 0$

Now, by checking all the options we see

$\left(\frac{5}{2}+\sqrt{\frac{5}{2}}, \frac{1}{2}\right), \left(\frac{5+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$ passes through above derived circle.

50. **(c)** Equation of circle of radius r which touches the coordinate axes and lies in the first quadrant is

$(x - r)^2 + (y - r)^2 = r^2$

$x^2 + y^2 - 2xr - 2yr - r^2 = 0$

Now, polar of P (x_1, y_1) with respect to the above circle,

$xx_1 + yy_1 - (x + x_1) r - (y + y_1) r - r^2 = 0$

$(x_1 - r) x + (y_1 - r) y = (x_1 + y_1 - r)r$

But it is given that polar of P with respect to above circle is $x + 2y = 4r$

$$\therefore \frac{x_1 - r}{1} = \frac{y_1 - r}{2} = \frac{x_1 + y_1 - r}{4}$$

$\Rightarrow x_1 - r = \frac{x_1 + y_1 - r}{4}$ and $x_1 - r = \frac{y_1 - r}{2}$

$\Rightarrow 4x_1 - 4r = x_1 + y_1 - r$ and $2x_1 - 2r = y_1 - r$

$\Rightarrow 3x_1 - 3r = y_1$ ... (i)

and $2x_1 - r = y_1$ ...(ii)

From equation (i) & (ii) we get

$\therefore 3x_1 - 3r = 2x_1 - r$

$x_1 = 2r$ and $y_1 = 2x_1 - r = 4r - r = 3r$

Thus, P is $(2r, 3r)$

51. **(c)** The centres of the circles and

$x^2 + y^2 - 2x - 2(3+\sqrt{3})y + 8 + 6\sqrt{7} = 0$

$x^2 + y^2 - 8x - 6y + k^2 = 0$ are $C_1(1, 3+\sqrt{7})$

and $C_2(4, 3)$ and corresponding radii are

$r_1 = \sqrt{1^2 + (3+\sqrt{7})^2 - (8+6\sqrt{7})} = 3$

and $r_2 = \sqrt{4^2 + 3^2 - k^2} = \sqrt{25 - k^2}$

Now, $C_1C_2 = \sqrt{(4-1)^2 + (3-3-\sqrt{7})^2} = 4$

Clearly, $C_1C_2 < r_1 + r_2$

$\Rightarrow 4 < 3 + \sqrt{25 - k^2} \Rightarrow 1 < \sqrt{25 - k^2}$

$\Rightarrow 1 < 25 - k^2 \Rightarrow k^2 < 24$

$\therefore k = 0, \pm 1, \pm 2, \pm 3, \pm 4$ $[\because k \in Z]$

Total number of possible values of k are **9.**

52. **(a)** Let the equation of circle S is

$S = x^2 + y^2 + 2gx + 2fy + c = 0$

Given,

$C_1 = x^2 + y^2 - 8x - 2y + 16 = 0$

$C_2 = x^2 + y^2 - 4x - 4y - 1 = 0$

S cuts orthogonally to C_1 and C_2, we get

$\therefore -8g - 2f = C + 16$ ...(i)

$-4g - 4f = C - 1$ ...(ii)

From eqs. (i) and (ii),

$-4g + 2f = 17$ ...(iii)

Common chord of S and C_1 is

$(2g + 8) x + (2f + 2) y + c - 16 = 0$ ...(iv)

Given common chord is

$2x + 13y - 15 = 0$ ...(v)

Eqs. (iv) and (v) are coincide

$$\therefore \frac{2g+8}{2} = \frac{2f+2}{13} = \frac{c-16}{-15}$$

$\Rightarrow 13g - 2f = -50$ ...(vi)

Solving eqs. (iii) and (vi), we get

$-4g + 2f = 17$

$13g - 2f = -50$

$g = \frac{11}{3}\, f = \frac{-7}{6}$

$\therefore$ Centre of circle, $S\left(\frac{-11}{3}, \frac{7}{6}\right)$

53. **(c)** Given that,

$S_1 : x^2 + y^2 + kx - 4y - 1 = 0$

$S_2 : 3x^2 + 3y^2 - 14x + 23y - 15 = 0$

$= x^2 + y^2 - \frac{14}{3}x + \frac{23}{3}y - 5 = 0$

Since, S_1 and S_2 circles are orthogonal to each other,

$\therefore\ 2\left[\left(\frac{-k}{2}\right)\left(\frac{7}{3}\right) + (2)\left(\frac{-23}{6}\right)\right] = -1 - 5$

$\frac{7k}{6} = 3 - \frac{46}{6} \Rightarrow \frac{7k}{6} = \frac{18-46}{6}$

$7k = -28 \Rightarrow k = -4$

$\therefore\ S_1 : x^2 + y^2 - 4x - 4y - 1 = 0$

$S_2 : x^2 + y^2 - \frac{14}{3}x + \frac{23}{3}y - 5 = 0$

Thus, equation of circle passing through intersection of S_1 and S_2 is

$x^2 + y^2 - 4x - 4y - 1 + \lambda\left(x^2 + y^2 - \frac{14x}{3} + \frac{23}{3}y - 5\right) = 0$

Since above circle passes through (–1, –1), so

$1 + 1 + 4 + 4 - 1 + \lambda\left(1 + 1 + \frac{14}{3} - \frac{23}{3} - 5\right) = 0$

$9 - 6\lambda = 0 \Rightarrow \lambda = \frac{3}{2}$

$\therefore$ Equation of required circle is

$x^2 + y^2 - 4x - 4y - 1 + \frac{3}{2}\left(x^2 + y^2 - \frac{14x}{3} + \frac{23y}{3} - 5\right) = 0$

$\Rightarrow 5x^2 + 5y^2 - 22x + 15y - 17 = 0$

54. **(a)** It is the given that the equation of parabola is

$y^2 + 2x + 2y - 3 = 0$

$\Rightarrow (y + 1)^2 - 1 + 2x - 3 = 0$

$\Rightarrow (y + 1)^2 = -2(x - 2)$

$\therefore$ Vertex = (2, –1)

Equation of the Axis : $y + 1 = 0$

Equation of the Focus :

$(-a + 2, -1) = \left(-\frac{1}{2} + 2, -1\right) = \left(\frac{3}{2}, -1\right)$

Education of the directrix : $x - 2 = \frac{1}{2} \Rightarrow x = \frac{5}{2} \Rightarrow 2x - 5 = 0$

55. **(d)** Given that, $y^2 = 4x \therefore a = 1$

We know that, the centroid of the triangle formed by the conormal points on a parabola lies on its axis and its coordinates are $\left(\frac{2}{3}(h - 2a), 0\right)$, where (h, k) is the point through which all normals passes.

$\therefore$ Centroid of triangle $= \left(\frac{2}{3}(5 - 2 \times 1), 0\right) = (2, 0)$

56. **(b)** Let the equation of ellipse,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It passes through the point $(3\sqrt{2}, \sqrt{10})$

$\therefore\ \frac{18}{a^2} + \frac{10}{b^2} = 1$ ...(i)

Also, foci are (4, 0) and (–4, 0)

$\therefore\ ae = 4 \Rightarrow a^2e^2 = 16$

$\Rightarrow a^2\left(1 - \frac{b^2}{a^2}\right) = 16 \Rightarrow a^2 - b^2 = 16 \Rightarrow b^2 = a^2 - 16$

From solving eqs. (i) and (ii), we get

$\frac{8}{a^2} + \frac{10}{a^2 - 16} = 1$

$\frac{18(a^2 - 16) + 10a^2}{a^2(a^2 - 16)} = 1$

$\Rightarrow 18a^2 - 288 + 10a^2 = a^4 - 16a^2$

$\Rightarrow a^4 - 44a^2 + 288 = 0$

$\Rightarrow (a^2 - 36)(a^2 - 8) = 0 \Rightarrow a^2 = 36, 8$

$\Rightarrow a^2 = 36$ $[\because a^2 \neq 8 \text{ as } b^2 = a^2 - 16]$

We know that, $a^2e^2 = 16$

$\therefore$ Eccentricity, $e^2 = \frac{16}{a^2} = \frac{16}{36} \Rightarrow e = \frac{4}{6} = \frac{2}{3}$

57. **(d)** We know that, the product of the length of the perpendicular drawn from the foci to the tangent of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $b^2 \Rightarrow b^2 = 9$

Given, equation of tangent

$y = \frac{-3}{4}x + 3\sqrt{2}$

$\therefore\ 3\sqrt{2} = \pm\sqrt{a^2\left(\frac{-3}{4}\right)^2 + b^2}$

$18 = \frac{9}{16}a^2 + 9 \Rightarrow a^2 = 16$

Eccentricity,

$\therefore\ e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

58. **(d)** Let the equation of hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

We have, eccentricity, $e = \sqrt{2}$ and distance between two foci.

$2ae = 16 \Rightarrow 2a(\sqrt{2}) = 16 \Rightarrow a = \frac{16}{2\sqrt{2}} = 4\sqrt{2}$

Now,

$b^2 = a^2(e^2 - 1)$

$b^2 = a^2(2-1)$

$b = a = 4\sqrt{2}$

$\therefore$ Equation of hyperbola is

$$\frac{x^2}{32} - \frac{y^2}{32} = 1 \Rightarrow x^2 - y^2 = 32$$

59. **(b)** Given that,

$A(-1, 0, 7)$, $B(3, 2, t)$ and $C(5, k, -2)$ are collinear.

$$\therefore \quad \frac{3+1}{5-3} = \frac{2-0}{k-2} = \frac{t-7}{-2-t}$$

$$\Rightarrow 2 = \frac{2}{k-2} = \frac{t-7}{-2-t} \Rightarrow k = 3, t = 1$$

$\therefore B = (3, 2, 1)$, $C = (5, 3, -2)$ and $P(1, 1, 4)$

Let P divides BC in the ratio $\lambda : 1$

$$\underset{B(3,2,1)}{\bullet} \overset{\lambda}{\text{———}} \underset{P(1,1,4)}{|} \overset{1}{\text{———}} \underset{C(5,1,-2)}{\bullet}$$

$$\text{Coordinates of } P = \frac{\lambda x_3 + x_2}{\lambda+1}, \frac{\lambda y_3 + y_2}{\lambda+1}, \frac{\lambda z_3 + z_2}{\lambda+1}$$

$$(1, 1, 4) = \frac{5\lambda+3}{\lambda+1}, \frac{\lambda+2}{\lambda+1} = \frac{-2\lambda+1}{\lambda+1}$$

$$\therefore \quad \frac{5\lambda+3}{\lambda+1} = 1$$

$$\Rightarrow \quad 5\lambda + 3 = \lambda + 1 \Rightarrow 4\lambda = -2 \Rightarrow \lambda = -\frac{1}{2}$$

$\therefore$ Required ratio $= -1 : 2$

60. **(c)** $3l + m + 5n = 0$

$\Rightarrow m = -3l - 5n$ ...(i)

and $6mn - 2nl + 5lm = 0$

$\Rightarrow 6n(-3l - 5n) - 2nl + 5l(-3l - 5n) = 0$ [From eq.(i)]

$\Rightarrow -18nl - 30n^2 - 2nl - 15l^2 - 25nl = 0$

$\Rightarrow -15l^2 - 30n^2 - 45nl = 0$

$\Rightarrow l^2 + 3nl + 2n^2 = 0$

$\Rightarrow (l + n)(l + 2n) = 0 \Rightarrow l = -n, -2n$

when $l = -n$

$-3n + m + 5n = 0 \Rightarrow m = -2n$

So, direction ratio's are

$-n, -2n, n$ or $1, 2, -1$

When $l = -2n$

$-6n + m + 5n = 0 \Rightarrow m = n$

So, direction ratio's are

$-2n, n, n$ or $-2, 1, 1$

$$\therefore \cos\theta = \frac{1\times(-2) + 2\times1 + (-1)\times1}{\sqrt{1+4+1}\sqrt{4+1+1}} = \frac{-1}{6}$$

$$\therefore |\cos\theta| = \frac{1}{6}$$

61. **(b)** Equation of plane OAB through the vertices O (0, 0, 0) A (1, 2, 1) and B (2, 1, 3) is given by,

$$\begin{vmatrix} x-0 & y-0 & z-0 \\ 1-0 & 2-0 & 1-0 \\ 2-0 & 1-0 & 3-0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x & y & z \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 0$$

$x(6-1) - y(3-2) + z(1-4) = 0$

$5x - y - 3z = 0$ ...(i)

Equation of plane ABC through the vertices A(1, 2, 1), B(2, 1, 3), C(–1, 1, 2) is given by,

$$\begin{vmatrix} x-1 & y-2 & z-1 \\ 2-1 & 1-2 & 3-1 \\ -1-1 & 1-2 & 2-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-2 & z-1 \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = 0$$

$(x-1)(-1+2) - (y-2)(1+4) + (z-1)(-1-2) = 0$

$x - 5y - 3z + 12 = 0$ ...(ii)

Then angle between planes represented by eqs. (i) and (ii) is given by.

$$\cos\theta = \frac{(5)(1) + (-1)(-5) + (-3)(-3)}{\sqrt{25+1+9}\sqrt{1+25+9}} = \frac{19}{35}$$

62. **(c)** Given, $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...\infty$

and $\lim\limits_{x\to0} \frac{\log(1+x)^{1+x}}{x^2} - \frac{1}{x} = k$

$$\lim_{x\to0} \frac{(1+x)\log(1+x) - x}{x^2} = k$$

$$\lim_{x\to0} \frac{\log(1+x) + 1 - 1}{2x} = k \quad \text{[Using L' Hospital Rule]}$$

$$\frac{1}{2}\lim_{x\to0} \frac{\log(1+x)}{x} = k \quad \left[\because \lim_{x\to0}\frac{\log(1+x)}{x} = 1\right]$$

$$k = \frac{1}{2} \Rightarrow 12k = 12\times\frac{1}{2} = 6$$

63. **(d)** We have, $f(x) = \begin{cases} k, & \text{For } x = 1 \\ \frac{(9x-1)(\sqrt{x}-1)}{3x^2+2x-5}, & \text{For } x \neq 1 \end{cases}$

$f(x)$ is continuous at $x = 1$

$$\therefore \quad k = \lim_{x\to1} \frac{(9x-1)(\sqrt{x}-1)}{3x^2+2x-5}$$

$$\Rightarrow \quad k = \lim_{x\to1} \frac{9x^{3/2} - 9x - \sqrt{x} + 1}{3x^2+2x-5}$$

$$\Rightarrow \quad k = \lim_{x\to1} \frac{9\left(\frac{3}{2}x^{1/2}\right) - 9 - \frac{1}{2\sqrt{x}}}{6x+2}$$

$$\Rightarrow \quad k = \frac{\frac{27}{2} - 9 - \frac{1}{2}}{6+2} = \frac{13-9}{8} = \frac{1}{2}$$

64. **(d)** Given that, $F(x) = f(x)\,g(x)$

$\Rightarrow F'(x) = f'(x)\,g(x) + f'(x)\,g'(x)$ and

Also given, $G(x) = f'(x)\,g'(x)$,

Now, $f'(x) = G(x)\, H(x)$

$$H(x) = \frac{F'(x)}{G(x)} = \frac{f'(x)g(x) + f(x)g'(x)}{f'(x)g'(x)} = \frac{g}{g'} + \frac{f}{f'} \quad ...(i)$$

Again, $F'(x) = F(x)\, K(x)$

$$\Rightarrow K(x) = \frac{F'(x)}{F(x)} = \frac{f'(x)g(x) + f(x)g'(x)}{f(x)g(x)} = \frac{f'}{f} + \frac{g'}{g} \quad ...(ii)$$

On adding eq. (i) and eq. (ii), we get

$$\therefore \quad H(x) + K(x) = \frac{f'}{f} + \frac{g}{g'} + \frac{f}{f'} + \frac{g'}{g}$$

65. **(b)** Given, $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \log\sqrt{1-x^2}$

Differentiating w.r.t. x, we get

$$\therefore \quad \frac{dy}{dx} = \frac{\left[1.\sin^{-1}x + \frac{x}{\sqrt{1-x^2}}\right]\sqrt{1-x^2} - x\sin^{-1}x\left(\frac{1}{2\sqrt{1-x^2}}(-2x)\right)}{(1-x^2)} + \frac{1}{2}\frac{1}{(1-x^2)}(-2x)$$

$$= \frac{1}{1-x^2}\left[\sqrt{1-x^2}\sin^{-1}x + x + \frac{x^2\sin^{-1}x}{\sqrt{1-x^2}} - x\right]$$

$$= \frac{1}{1-x^2}\left[\frac{(1-x^2)\sin^{-1}x + x^2\sin^{-1}x}{\sqrt{1-x^2}}\right] = \frac{\sin^{-1}x}{(1-x^2)^{3/2}}$$

66. **(d)** Given that,

$f(x) = x^2 + xg'(1) + g''(2)$

$\Rightarrow f'(x) = 2x + g'(1)$ ...(i)

Put $x = 1$

$f'(1) = 2 + g'(1)$ ...(ii)

$\Rightarrow f''(x) = 2 \Rightarrow f'''(x) = 0$

$g(x) = f(1)\, x^2 + xf'(x) + f''(x)$

$\Rightarrow g'(x) = 2xf(1) + f'(x) + xf''(x) + f'''(x)$

Put $x = 1$

$\Rightarrow g'(1) = 2f(1) + f'(1) + 0$ ...(iii)

From Eq. (ii) and Eq. (iii), we get

$g'(1) = 2f(1) + 2 + g'(1) + 2$

$2f(1) + 4 = 0 \Rightarrow f(1) = -2$ ...(iv)

Now differentiating $g'(x)$ again,

$g''(x) = 2f(1) + f''(x) + f''(x) + xf'''(x)$

$g''(x) = 2f(1) + 4 = -4 + 4 = 0$

We know that, $f(x) = x^2 + xg'(1) + g'(2) = x^2 + xg'(1)$

Put $x = 1$

$f(1) = 1 + g'(1) \Rightarrow -2 = 1 + g'(1)$

$\Rightarrow g'(1) = -3$

$\therefore \quad f(x) = x^2 + x(-3) = x^2 - 3x$

$\therefore \quad g'(x) = f(1)\, x^2 + xf'(x) + f'(x)$

$= -2x^2 + x(2x - 3) + 2$

$= -2x^2 + 2x^2 - 3x + 2 = 2 - 3x$

Now,

$\therefore \quad f(x) - g(x) = x^2 - 3x - 2 + 3x = x^2 - 2$

67. **(a)** Let A and P are area and perimeter of circle.

We know, $A = \pi r^2$

$$\frac{dA}{dt} = \frac{1}{\sqrt{\pi}} \Rightarrow \frac{d}{dt}(\pi r^2) = \frac{1}{\sqrt{\pi}}$$

$$\Rightarrow 2\pi r\frac{dr}{dt} = \frac{1}{\sqrt{\pi}} \Rightarrow \frac{dr}{dt} = \frac{1}{2\pi r\sqrt{\pi}}$$

Now, $P = 2\pi r$

$$\Rightarrow \frac{dP}{dr} = 2\pi \times \frac{dr}{dt} = 2\pi \times \frac{1}{2\pi r\sqrt{\pi}} = \frac{1}{r\sqrt{\pi}} \quad \left[\because \frac{dr}{dt} = \frac{1}{2\pi r\sqrt{\pi}}\right]$$

$$= \frac{1}{\frac{1}{2\sqrt{\pi}} \times \sqrt{\pi}} \quad \left[\because P = \sqrt{\pi} \Rightarrow 2\pi r = \sqrt{\pi} \Rightarrow r = \frac{1}{2\sqrt{\pi}}\right]$$

$= 2$ unit/sec.

68. **(c)** Given, eqn. of curve

$$y = \frac{x^n}{a^{n-1}} \quad \therefore \frac{dy}{dx} = \frac{nx^{n-1}}{a^{n-1}}$$

$$\therefore \quad \text{Length of subnormal } = y\frac{dy}{dx} = \frac{x^n}{a^{n-1}} \times n\frac{x^{n-1}}{a^{n-1}}$$

$$= n\frac{x^{2n-1}}{a^{2n-2}}$$

$$\therefore \quad \text{Length of subnormal at } (\alpha, \beta) = \frac{n\alpha^{2n-1}}{a^{2n-2}}$$

Now, we have given that

Length of subnormal is proportional to a^2

$$\frac{n\alpha^{2n-1}}{a^{2n-2}} \alpha a^2$$

$$\therefore \quad \frac{n\alpha^{2n-1}}{a^{2n-2}} \alpha \frac{1}{a^{-2}}$$

$\therefore \quad 2n - 2 = -2 \Rightarrow n = 0$

69. **(c)** Let $f(x) = \frac{2x-1}{3x-4}, [1, 2]$

$$f(x) = \frac{2x-1}{3\left(x - \frac{4}{3}\right)}$$

Since, $f(x)$ is not defined at $x = \frac{4}{3} \in [1, 2]$

So, Lagrange's mean value theorem is not applicable on $f(x)$ on $[1, 2]$

70. **(a)** Let polynomial of degree 3, $P(x) = ax^3 + bx^2 + cx + d$.

Differentiating w.r.t. x, we get

$\therefore \quad P'(x) = 3ax^2 + 2bx + c$

Since, $P(x)$ has extreme value at $x = 1$

$\therefore \quad P'(1) = 0 \Rightarrow 3a + 2b + c = 0$ ...(i)

Now, it is given that

$$\lim_{x\to0}\left(\frac{P(x)+4}{x^2}+2\right)=6$$

$$\Rightarrow \lim_{x\to0}\frac{ax^3+bx^2+cx+d+4}{x^2}=4$$

$$\Rightarrow \lim_{x\to0} ax+b+\frac{c}{x}+\frac{d+4}{x^2}=4$$

Since, value of limit is finite

$\therefore$ $c=0$ and $d+4=0 \Rightarrow c=0, d=-4$

$\therefore$ $\lim_{x\to0} ax+b=4 \Rightarrow b=4$

Putting $b=4$, $c=0$ in eq. (i), we get

$3a+8=0$

$$\Rightarrow a=\frac{-8}{3} \quad \therefore P(x)=\frac{-8}{3}x^3+4x^2-4$$

$$\Rightarrow \frac{dP(x)}{dx}=-8x^2+8x$$

$$\therefore \left.\frac{dP(x)}{dx}\right|_{x=1/2}=\frac{-8}{4}+\frac{8}{2}=2$$

71. **(a)** Let $I=\int\frac{y^2+\sqrt[3]{y^4}+\sqrt[6]{y^2}}{y(1+\sqrt[3]{y^2})}dy$

$$=\int\frac{y^2+y^{4/3}+y^{1/3}}{y(1+y^{2/3})}dy$$

$$=\int\frac{y^{4/3}(y^{2/3}+1)+y^{1/3}}{y(1+y^{2/3})}dy$$

$$=\int\left(y^{1/3}+\frac{y^{-2/3}}{1+y^{2/3}}\right)dy$$

$$=\int y^{1/3}\,dy+\int\frac{y^{-2/3}}{1+(y^{1/3})^2}dy$$

$$=\frac{y^{4/3}}{4/3}+3\int\frac{d(y^{1/3})}{1+(y^{1/3})^2}$$

$$=\frac{3}{4}\sqrt[3]{y^4}+3\tan^{-1}y^{1/3}+C$$

$$=\frac{3}{4}\sqrt[3]{y^4}+3\tan^{-1}\sqrt[3]{y}+C$$

72. **(b)** For $k\in(\mathrm{I},\infty)$

$$I=\int\frac{1}{1+k\cos x}dx=\int\frac{1}{1+k\left(\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$=\int\frac{\left(1+\tan^2\frac{x}{2}\right)}{1+\tan^2\frac{x}{2}+k-k\tan^2\frac{x}{2}}dx$$

$$=\int\frac{\sec^2\frac{x}{2}}{(k+1)-(k-1)\tan^2\frac{x}{2}}dx$$

Let $\tan\frac{x}{2}=t\Rightarrow\sec^2\frac{x}{2}.\frac{1}{2}dx=dt\Rightarrow\sec^2\frac{x}{2}dx=2dt$

$$\therefore\ I=2\int\frac{dt}{(k+1)-(k-1)t^2}=\frac{2}{k-1}\int\frac{dt}{\left(\sqrt{\frac{k+1}{k-1}}\right)^2-t^2}$$

$$=\frac{2}{k-1}.\frac{1}{2\sqrt{\frac{k+1}{k-1}}}\log\left[\frac{\sqrt{\frac{k+1}{k-1}}+t}{\sqrt{\frac{k+1}{k-1}}-t}\right]+C$$

$$I=\frac{1}{\sqrt{k^2-1}}\log\left[\frac{\sqrt{k+1}+\sqrt{k-1}\tan\frac{x}{2}}{\sqrt{k+1}-\sqrt{k-1}\tan\frac{x}{2}}\right]+C$$

73. **(a)** Let $I=\int e^{-3x}(x^2+\sin4x)dx$

$$=\int e^{-3x}x^2dx+\int e^{-3x}\sin4x\,dx$$

Using integration by part,

$$\int uvdx=u\int vdx-\int\left(\frac{d}{du}u\int vdx\right)dx$$

$$=x^2.\frac{e^{-3x}}{-3}-\int-\frac{1}{3}e^{-3x}.2xdx+\frac{e^{-3x}}{9+16}[-3\sin4x-4\cos4x]$$

$$=-\frac{1}{3}x^2e^{-3x}+\frac{2}{3}\left[x\frac{e^{-3x}}{-3}-\int-\frac{1}{3}e^{-3x}.1dx\right]+\frac{e^{-3x}}{25}[-3\sin4x-4\cos4x]+C$$

$$=-\frac{1}{3}x^2e^{-3x}-\frac{2}{9}xe^{-3x}-\frac{2}{27}e^{-3x}-\frac{3}{25}e^{-3x}\sin4x-\frac{4}{25}e^{-3x}\cos4x+C$$

$$I=-e^{-3x}\left[\frac{x^2}{3}+\frac{2x}{9}+\frac{2}{27}+\frac{3}{25}\sin4x+\frac{4}{25}\cos4x\right]+C$$

74. **(b)** Let $I=\int\frac{2x^{12}+5x^9}{(1+x^3+x^5)^3}dx$

$$=\int\frac{2x^{12}+5x^9}{\left[x^5\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3\right]}dx$$

$$= \int \frac{2x^{12}+5x^9}{x^{15}\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx$$

$$= \int \frac{(2x^{12}+5x^9)x^{-15}}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx$$

$$= \int \frac{2x^{-3}+5x^{-6}}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx$$

Let $1+\frac{1}{x^2}+\frac{1}{x^5}=t \Rightarrow \left(-\frac{2}{x^3}-\frac{5}{x^6}\right)dx = dt$

$\therefore \quad I = -\int \frac{dt}{t^3} = \frac{1}{2t^2}+C = \frac{1}{2\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2}+C$

$$I = \int \frac{x^{10}}{2(x^5+x^3+1)^2}+C$$

Comparing with $\frac{x^m}{l(1+x^3+x^5)^r}+C$

$\therefore \quad m = 10, l = 2, r = 2$

$\therefore \quad \frac{m-l}{r} = \frac{10-2}{2} = 4$

75. **(d)** Let $L = \lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)...\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}$

Taking log both the sides,

$\therefore \quad \log L = \lim_{n\to\infty}\frac{1}{n}$

$$\left[\log\left(1+\frac{1}{n^2}\right)+\log\left(1+\frac{2^2}{n^2}\right)+...\log\left(1+\frac{n^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\left(\frac{r}{n}\right)^2\right) = \int_0^1 \log(1+x^2)dx$$

$$= \left[x\log(1+x^2)\right]_0^1 - \int_0^1 x.\frac{1}{1+x^2}.2x\,dx$$

$$= \log 2 - 2\int_0^1 \frac{x^2}{1+x^2}dx = \log 2 - 2\int_0^1\left(1-\frac{1}{1+x^2}\right)dx$$

$$= \log 2 - 2[x-\tan^{-1}x]_0^1 = \log 2 - 2\left[1-\frac{\pi}{4}\right]$$

$$= \log 2 - 2 + \frac{\pi}{2} = \log 2 - 2 + \frac{\pi}{2}$$

$$L = e^{\log 2 - 2 + \frac{\pi}{2}} = e^{\log 2}e^{\frac{\pi}{2}-2} = 2e^{\frac{\pi-4}{2}}$$

76. **(d)** Let $I = \int_{\pi/4}^{\pi/2}\frac{3dx}{1+e^{\sqrt{8}\sin\left(x-\frac{3\pi}{8}\right)}}$ ...(i)

$$= \int_{\pi/4}^{\pi/2}\frac{3dx}{1+e^{\sqrt{8}\sin\left(\frac{\pi}{2}+\frac{\pi}{4}-x-\frac{3\pi}{8}\right)}}$$

$$\left[\text{using } \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

$$= \int_{\pi/4}^{\pi/2}\frac{3dx}{1+e^{\sqrt{8}\sin\left(\frac{3\pi}{8}-x\right)}} = \int_{\pi/4}^{\pi/2}\frac{3dx}{1+e^{-\sqrt{8}\sin\left(\pi-\frac{3\pi}{8}\right)}}$$

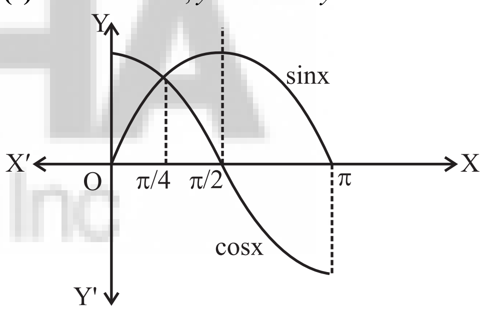
$$= \int_{\pi/4}^{\pi/2}\frac{3e^{\sqrt{8}\sin\left(x-\frac{3\pi}{8}\right)}dx}{e^{\sqrt{8}\sin\left(x-\frac{3\pi}{8}\right)}+1} \quad ...(ii)$$

On adding Eq. (i) and Eq. (ii), we get

$$2I = 3\int_{\pi/4}^{\pi/2}1dx = 3[x]_{\pi/4}^{\pi/2} = 3\left[\frac{\pi}{2}-\frac{\pi}{4}\right] = \frac{3\pi}{4}$$

$\therefore \quad I = \frac{3\pi}{8}$

77. **(c)** Given curves, $y = \sin x$ & $y = \cos x$



According to the given data

$$\int_{\frac{\pi}{4}}^{a}(\sin x-\cos x)dx = \int_a^{\pi}(\sin x - \cos x)dx$$

$$[-\cos x - \sin x]_{\frac{\pi}{4}}^{a} = [-\cos x - \sin x]_a^{\pi}$$

$$-\cos a - \sin a + \cos\frac{\pi}{4}+\sin\frac{\pi}{4}$$
$$= -\cos\pi - \sin\pi + \cos a + \sin a$$

$$-\cos a - \sin a + \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}} = 1 + \cos a + \sin a$$

$$\sqrt{2}-1 = 2(\cos a + \sin a)$$

$$\frac{\sqrt{2}-1}{2} = \sin a + \cos a$$

$$\frac{\sqrt{2}-1}{2\sqrt{2}} = \frac{1}{\sqrt{2}}\sin a + \frac{1}{\sqrt{2}}\cos a$$

$$\frac{\sqrt{2}-1}{2\sqrt{2}} = \sin a.\cos\frac{\pi}{4} + \cos a \sin\frac{\pi}{4}$$

$$\frac{\sqrt{2}-1}{2\sqrt{2}} = \sin\left(a + \frac{\pi}{4}\right)$$

78. **(b)** Given, family of curves

$$y = ae^{4x} + be^{-x} \quad \text{...(i)}$$

where a and b are arbitary constants

$$\frac{dy}{dx} = 4ae^{4x} - be^{-x} \quad \text{...(ii)}$$

and $\frac{d^2y}{dx^2} = 16ae^{4x} + be^{-x}$ ...(iii)

By adding eqs. (i) and (ii), we get

$$y + \frac{dy}{dx} = 5ae^{4x}$$

$$\Rightarrow ae^{4x} = \frac{1}{5}\left(y + \frac{dy}{dx}\right)$$

From eqs. (i) and (iv), we get

$$y = \frac{1}{5}y + \frac{1}{5}\frac{dy}{dx} + be^{-x} \Rightarrow be^{-x} = \frac{4}{5}y - \frac{1}{5}\frac{dy}{dx} \quad \text{...(v)}$$

From eqs. (iii), (iv) and (v), we get

$$\frac{d^2y}{dx^2} = \frac{16}{5}y + \frac{16}{5}\frac{dy}{dx} + \frac{4}{5}y - \frac{1}{5}\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = 4y + 3\frac{dy}{dx} \Rightarrow \frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$$

$$\therefore\ f = \frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y \quad \therefore\ \frac{df}{dx} = \frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} - 4\frac{dy}{dx}$$

79. **(c)** Given, $f(x, y) = 0$

We know that,

Length of subtangent $= \dfrac{y}{\frac{dy}{dx}} \Rightarrow x + 7y^2 = \dfrac{y}{\frac{dy}{dx}}$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x+7y^2} \Rightarrow \frac{dx}{dy} = \frac{x+7y^2}{y} \quad \Rightarrow \frac{dx}{dy} - \frac{1}{y}x = 7y$$

$$\therefore\ \text{IF} = e^{\int P(y)dy} = e^{\int -\frac{1}{y}dy} = e^{-\log y} = \frac{1}{y}$$

Here, $P(y) = \dfrac{-1}{y}$

$Q(y) = 7y$

So, solution is given by

$$x.\frac{1}{y} = \int 7y.\frac{1}{y}dy + C \Rightarrow \frac{x}{y} = 7y + C$$

$x = 7y^2 + cy \Rightarrow 7y^2 + cy - x = 0$

$\therefore\ f(x, y) : 7y^2 + cy - x = 0$

80. **(c)** Given differential equation,

$(y - x + 1)\, dy - (y + x + 2)\, dx = 0$

$ydy - xdy + dy - ydx - xdx + 2dx = 0$

$ydy + dy - (xdy + ydx) - xdx + 2dx = 0$

On integrating both the sides, we get

$$\frac{y^2}{2} + y - xy - \frac{x^2}{2} + 2x = C$$

$$f(x, y, c) = \frac{y^2}{2} - \frac{x^2}{2} - xy + 2x + y - C = 0$$

At $x = 1$,

$y = 1 \Rightarrow f(1, 1, C) = 0$

$$\therefore\ \frac{1}{2} - \frac{1}{2} - 1 + 2 + 1 - C = 0 \Rightarrow C = 2$$

PHYSICS

81. **(c)** Nuclear forces are responsible for holding protons inside the nucleus inspite of repulsion between them.Thus, it is attractive and short ranged in nature.

82. **(b)** Let time period

$T \propto P^{\alpha} S^{\beta} E^{\gamma}$ or $T = kP^{\alpha} S^{\beta} E^{\gamma}$

By substituting the following dimensions

$[P] = [ML^{-1}T^{-2}]$,

$[S] = [ML^{-3}]$,

$[E] = [ML^2T^{-2}]$

We get $[M^0L^0T^1] = k\, [ML^{-1}T^{-2}]^{\alpha}\, [ML^{-3}]^{\beta}\, [ML^2T^{-2}]^{\gamma}$

Comparing powers of similar terms of both sides we have

$\alpha + \beta + \gamma = 0$ ...(i)

$-\alpha - 3\beta + 2\gamma = 0$ ...(ii)

$-2\alpha - 2\gamma = 1$ ...(iii)

Solving, we get $\alpha = \dfrac{-5}{6}, \beta = \dfrac{1}{2}, \gamma = \dfrac{1}{3}$

83. **(a)** Relative velocity of car A w.r.t. car B is

$V_{AB} = V_A - V_B = 15 - 10 = 5$ m/s

Distance travelled by car A in overtaking car B

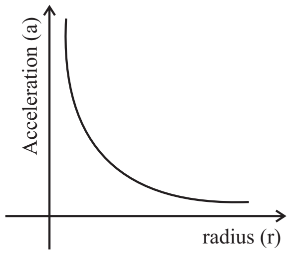
$s = 4m + 4m = 8m$

Time taken in overtaking this distance

$$t = \frac{s}{V_{AB}} = \frac{8}{5} = 1.6\,\text{s}$$

84. **(c)** The magnitude of acceleration of the body moving over a circular path is

$$a = \frac{v^2}{r} \Rightarrow a \propto \frac{1}{r}$$



85. **(d)** Given,
unit vector is $0.5\hat{i}+0.8\hat{j}+c\hat{k}$

$\Rightarrow \sqrt{(0.5)^2+(0.8)^2+c^2}=1$

$\Rightarrow 0.89+c^2=1$

$\Rightarrow c^2=0.11 \Rightarrow c=\sqrt{0.11}$

86. **(a)** For the projectile to fall between C and D range must be greater than 20 m and less than 24.2 m.

Range, $R=\dfrac{u^2\sin 2\theta}{g}$

$\Rightarrow R=\dfrac{u^2\times\sin(2\times 15°)}{10}=\dfrac{u^2}{2}$

$\Rightarrow 20\le R\le 24.2$

$\Rightarrow 200\le u^2\times\dfrac{1}{2}\le 242$

$\Rightarrow 400\le u^2\le 484$

$\Rightarrow$ 20 m/s $\le u\le$ 22 m/s

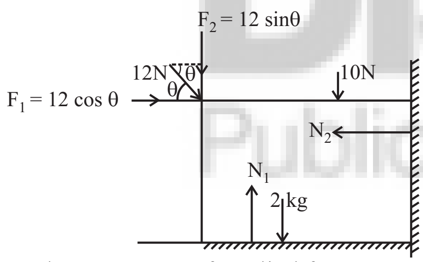
So, option (a) is best choice.

87. **(c)** Given,
Velocity of the bullet, v = 1000 ms^{-1}
Momentum of bullet after firing, p = 20 kg ms^{-1}
Linear momentum, $p = mv$

$\Rightarrow m=\dfrac{p}{v}=\dfrac{20}{1000}\text{kg}$

$\Rightarrow m=\dfrac{20}{1000}\times 1000\text{ g} \Rightarrow m=20\text{g}$

88. **(a)**



Horizontal component, of applied force

$F_1=12\times\cos\theta=12\times\dfrac{4}{5}=\dfrac{48}{5}\text{N}$

Vertical component, of applied force

$F_2=12\times\sin\theta=12\times\dfrac{3}{5}=\dfrac{36}{5}\text{N}$

So, total applied force on ground is

$10+\dfrac{36}{5}+2\times 10=37.2\text{N}$

So, reaction N_1 of ground = 37.2 N upwards
Also, total force horizontally on wall is

$F_1=\dfrac{48}{5}=9.6\text{N}$

So, reaction N_2 of wall = 9.6 N towards left

89. **(c)** From the work energy theorem,
Work done by net
force on the body = change in kinetic energy

90. (*) Given
Mass of the body, m = 10 kg

Displacement, $x=\dfrac{t^3}{25}$

So, velocity, $v=\dfrac{dx}{dt}=\dfrac{3t^2}{25}\text{ms}^{-1}$

Acceleration, $a=\dfrac{dv}{dt}=\dfrac{6t}{25}\text{ms}^{-2}$

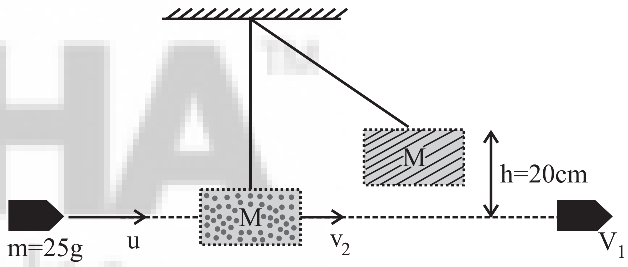
Force = $ma=10\left(\dfrac{6t}{25}\right)=\dfrac{12t}{5}\text{N}$

Work done in displacing object by displacement dx is
$dW = F.\,dx = ma\,vdt$ $\quad(\because dx = vdt)$

$W=\int_0^5 dW=\int_0^5 \dfrac{12t}{5}\times\dfrac{3t^2}{25}dt$

$=\dfrac{36}{5\times 25}\times\left[\dfrac{t^4}{4}\right]_0^5=\dfrac{36\times(5)^4}{5\times 25\times 4}=9\times 5=45\text{J}$

91. **(d)** From the given situation,
Initial momentum of the bullet = mu
Let final velocities of bullet and block are v_1 and v_2 respectively.



If the system rises up to height h then by conservation of energy

$\dfrac{1}{2}Mv_2^2=Mgh \Rightarrow v_2=\sqrt{2gh}$

$\Rightarrow v_2=\sqrt{2\times 10\times 20\times 10^{-2}}$ = 2m/s

By the conservation of linear momentum
$mu = mv_1 + Mv_2$

$\Rightarrow \dfrac{25}{1000}\times 250=\dfrac{25}{1000}\times v_1+1\times 2$

$\Rightarrow \dfrac{25}{4}=\dfrac{25}{1000}\times v_1+2 \Rightarrow v_1=170\,\text{ms}^{-1}$

92. **(b)** Given, radius of circular hole, r = 3 cm
Radius of circular disc, R = 6 cm
Mass of disc, $M = \pi R^2 m$
Mass of hole, $M' = \pi r^2 m$
If we assume the centre of mass of hole and disc to be at centre, then
The x-coordinate of the centre of mass of the remaining portion of the disc will be

$x_{CM}=\dfrac{Mx_1-M'x_2}{M-M'}=\dfrac{M\times 0-M'\times 3}{\pi R^2 m-\pi r^2 m}$

$$= \frac{-3M'}{\pi m(R^2 - r^2)} = \frac{-3\pi r^2 m}{\pi m(R^2 - r^2)}$$

$$= -\frac{-3r^2}{R^2 - r^2} = \frac{-3\times 3^2}{6^2 - 3^2} = -1 \text{ cm}$$

$\Rightarrow x_{CM} = -1$ cm

Hence, distance of centre of gravity of the resulting flat body from the centre of the original disc will be 1 cm left side.

93. **(b)** Let displacement, $x = a \sin\omega t$

Velocity, $v = \frac{dx}{dt} = a\omega \cos\omega t$

Acceleration, $a = \frac{d^2x}{dt^2} = -a\omega^2 \sin\omega$

Magnitude of acceleration, $|a| = A\omega^2 \sin\omega t$

Average acceleration of particle between extreme and equilibrium position

$$A_{avg} = \frac{\int_0^{\pi/4} a\omega^2 \sin\omega t\, dt}{\int_0^{\pi/4} dt} = \frac{\frac{a\omega^2}{\omega}\left[-\frac{\cos 2\pi t}{T}\right]_{}^{\frac{\pi}{4}}}{[t]^{\pi/4}}$$

$$= \omega^2 a \times \frac{2}{\pi}$$

Maximum acceleration, $A_{max} = \omega^2 a$

$$\therefore \frac{A_{avg}}{A_{max}} = \frac{\omega^2 a \times 2}{\omega^2 a \times \pi} = \frac{2}{\pi}$$

94. **(d)** Acceleration due to gravity at height h from the surface of the earth,

$$g_h = \frac{GM}{(R+h)^2}$$

$\therefore$ Acceleration due to gravity decreases with altitude.

Acceleration due to gravity on the surface of earth.

$$g = \frac{GM}{R^2}$$

Clearly, g depends on M.

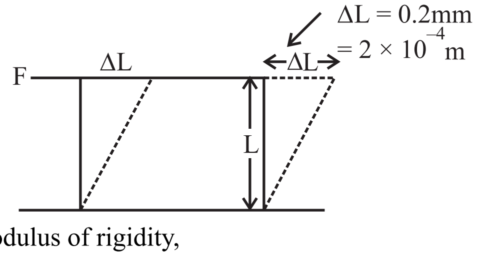
Geostationary satellite has a time period of 24 h.

g at depth d is

$$g_d = g\left(1 - \frac{d}{R}\right)$$

g decreases with increasing depth.

95. **(b)** Let ΔL is the displacement of the upper edge of the slab.



Modulus of rigidity,

$$\eta = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} = \frac{10^5}{\frac{50}{100}\times\frac{10}{100}} \times \frac{50\times 10^{-2}}{0.2\times 10^{-3}}$$

$$= \frac{2\times 10^6 \times 50 \times 10^{-2}}{0.2\times 10^{-3}} = 5 \times 10^9 \text{ N/m}^2 = 5 \text{ GPa}$$

96. **(a)** Given, Number of smaller droplets, $n = 10^6$

Surface tension, $T = 435 \times 10^{-3}$ Nm^{-1} Radius of meniscus, $R = 1$ cm $= 10^{-2}$ m

Work done in splitting a large drop of radius R in n smaller drops of equal size is

W = T × Increment in surface area

Here, T = surface tension.

$\therefore$ $W = (4\pi R^2)T.\ (n^{1/3} - 1)$

$= 4\pi \times (10^{-2})^2 \times 435 \times 10^{-3} \times ((10^6)^{1/3} - 1) = 54.1 \times 10^{-3}$ J

97. **(d)** Given,

Specific heat at constant volume of helium,

$C_v = 12.6$ J mol^{-1} K^{-1}

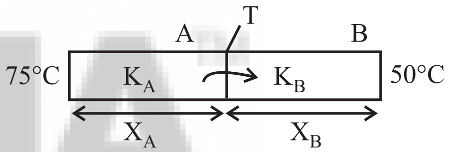
From R = 8.314 J mol^{-1} K^{-1}

From Mayer's relation, $C_p - C_v = R$

$\Rightarrow$ $C_p = C_v + R = 12.6 + 8.314$

$= 20.914 \approx 20.9$ J mol^{-1} K^{-1}

98. **(c)** The given situation is shown below



In steady state, both slab have same rate of heat flow

$$\therefore \frac{dQ}{dt} = \frac{K_A(75°C - T)}{X_A} = \frac{K_B(T - 50°C)}{X_B}$$

$\because$ $K_B = 2K_A$ and $X_A = 2X_B$

$$\therefore \frac{K_B(75°C - T)}{2\times 2X_B} = \frac{K_B(T - 50°C)}{X_B}$$

$\Rightarrow$ 75°C – T = 4T – 200°C

$\Rightarrow$ 5T = 275°C $\Rightarrow$ T = 55°C

99. **(b)** Change in temperature $\Delta T = 20°C$

Work done in an adiabatic change is

$$\Delta W = \frac{nR\Delta T}{\gamma - 1}$$

For monoatomic gas, $\gamma = \frac{5}{3}$

$n = 1$

$$\therefore W = \frac{1\times R\times 20}{\left(\frac{5}{3} - 1\right)} = 30R \quad ...(i)$$

For diatomic gas, $\gamma = \frac{7}{5}$ $n = 6$

$$\therefore \Delta W = \frac{6\times R\times 20}{\left(\frac{7}{5} - 1\right)} = \frac{5\times 6\times R\times 20}{2}$$

$= 10 \times 30R = 10W$ [From Eq. (i)]

100. (a) For a gas with n degree of freedom.

Specific heat at constant volume, $C_v = \frac{n}{2}R$

Specific heat at constant pressure,

$$C_p = \left(\frac{n}{2}+1\right).R$$

$$\therefore \quad \frac{C_p}{C_v} = \frac{\left(\frac{n}{2}+1\right)R}{\frac{n}{2}.R} = \left(\frac{n+2}{n}\right)$$

101. (a) From the Doppler's effect, apparent frequency of reflected sound from building is given by

$$n' = n\left(\frac{v+v}{v-v_b}\right) \quad \text{... (i)}$$

Here, speed of sound, $v = 340$ m/s,

v_b = speed of bus = **7**.2 km/h = 20 ms^{-1}

and n = original frequency produced by horn = 1.7 kHz

Substituting the respective values we get

$$n' = 1.7\left(\frac{340+20}{340-20}\right) = 1.9125 \text{ kHz} \approx 1.8 \text{ kHz}$$

102. (d) Here,

Image distance, $v = +f$

From the lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{f} - \frac{1}{f} = \frac{1}{u}$$

$$\Rightarrow \frac{1}{u} = 0 \Rightarrow u = \infty$$

So, object is at ∞ distance on right side of lens.

103. (d) If n fringes are visible in a field of view with light of wavelength λ_1 while n_2 with light of wavelength λ_2 in the same field, then

$n_1\lambda_1 = n_2\lambda_2$

Here, $\lambda_1 = 6600$Å,

$\lambda_2 = 4400$Å

$n_1 = 60$

$$\therefore \quad n_2 = \frac{n_1\lambda_1}{\lambda_2} = \frac{60 \times 6600}{4400} = \frac{60 \times 6}{4} = 15 \times 6 = 90 \text{ fringes}$$

104. (d) Fringe width, $\beta = \frac{\lambda D}{d}$

$\Rightarrow \beta \propto \lambda$

As $\lambda_R > \lambda_G > \lambda_B \Rightarrow \beta_R > \beta_G > \beta_B$

105. (b) Due to field, a proton experiences a force in the direction of field.

To move proton against the field an external agency is required to overcome this attractive force on the proton.

106. (d) Let r and R be the radius of small drop and big drop respectively.

Capacitance of each oil drop is

C = $4\pi\varepsilon r$

As volume remains same

$\therefore$ n × volume of 1 small drop = volume of 1 large drop

$$\Rightarrow n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow R = n^{1/3}.r$$

So, capacitance of large drop is

$$C' = 4\pi\varepsilon R = 4\pi\varepsilon.n^{\frac{1}{3}}.r = n^{\frac{1}{3}}C$$

107. (a) Given, current carried by conductor, I = 5A

Area of cross-section of conductor, A = 1 mm^2

= 1 × 10^{-6} m relation between current density (J) and electric field (E) is given by

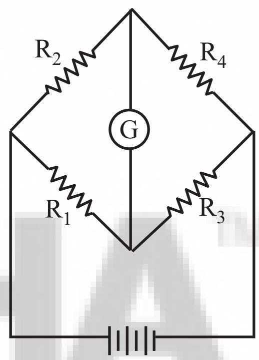
$$\text{J} = \frac{\text{E}}{\rho}$$

where, ρ = resistivity

$$\Rightarrow \text{E} = \text{J}.\rho = \frac{\text{I}}{\text{A}}.\rho \qquad \left(\because \text{J} = \frac{I}{A}\right)$$

$$= \frac{5 \times 3 \times 10^{-8}}{1 \times 10^{-6}} = 0.15 \text{ V/m}$$

108. (a)



From the balancing condition of Wheatstone bridge,

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

109. (d) Magnitude of torque on the coil is given as

$\tau = NIAB\sin\theta$

where, θ is the angle between normal to the loop and direction of magnetic field.

Here, $\theta = 0$ So, $\tau = 0$

110. (b) Given,

Length of solenoid, L = 50 cm

No. of turns, N = 400

Magnetic field at the centre of solenoid, B = $4\pi \times 10^{-3}$ T

Number of turns per unit length,

$$n = \frac{\text{N}}{\text{L}}$$

$$= \frac{400 \times 100}{50} = 800 \text{ turns/m}$$

Magnetic field at the centre of solenoid,

B = $\mu_0 n$I

$\Rightarrow 4\pi \times 10^{-3} = 4\pi \times 10^{-7} \times 800 \times \text{I}$

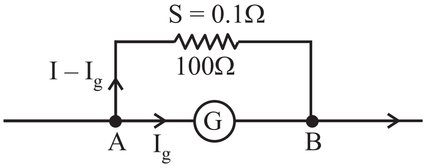
$$\Rightarrow I = \frac{4\pi \times 10^{-3}}{4\pi \times 10^{-7} \times 800} \Rightarrow I = \frac{10000}{800} = 12.5\text{A}$$

111. **(d)** Relation between relative permeability (μ_r) and susceptibility (χ_B) is

$\mu_r = 1 + \chi_B \Rightarrow \chi_B = \mu_r - 1$

$= 5500 - 1 = 5499$

112. **(a)** Here G and S are parallel to each other hence both will have equal potential difference.



$I_gG = (I - I_g)S \Rightarrow S = \dfrac{I_gG}{(I - I_g)}$

$$S = \frac{(100\times10^{-6})100}{(I - 100\times10^{-6})}$$

$\Rightarrow I - 100\times10^{-6} = 10^{-1} \Rightarrow I = 0.1 + 100\times10^{-6}$

$= 0.1 + 0.0001 = 0.1001$ A $= 100.1$ mA

So, circuit current is 100.1 mA.

113. **(a)** In state of charging, Charge on capacitor at any instant

$Q = Q_0(1 - e^{-t/RC})$

If CR is smaller then time constant for CR circuit is small and capacitor will charge rapidly.

114. **(a)** Given,

Intensity of parallel beam of light, $I = \dfrac{15}{\pi}\,\text{W/m}^2$

Intensity of parallel beam of light (I) is given by

$I = \dfrac{1}{2}\varepsilon_0 E_0^2 c$ ...(i)

where, E_0 = amplitude of electric field.

$$\Rightarrow E_0^2 = \frac{2\times I}{\varepsilon_0\times c} = \frac{2\times4\pi\times I}{4\pi\varepsilon_0\times c}$$

$$= \frac{2\times4\pi\times\frac{15}{\pi}\times9\times10^9}{3\times10^8} \Rightarrow E_0^2 = 3600 \Rightarrow E_0 = 60 \text{ N/C}$$

115. **(a)** From the Einstein's photoelectric equation,

$K_{max} = \dfrac{1}{2}mv^2 = \dfrac{hc}{\lambda} - \phi_0$ ...(i)

Where v is the velocity of emitted electrons and ϕ_0 is the work function of metal.

When wavelength is reduced by 50%

New wavelength $\lambda' = \dfrac{\lambda}{2}$

Maximum velocity $v' = 3v$

$$\Rightarrow \frac{1}{2}m(3v)^2 = \frac{hc}{\frac{\lambda}{2}} - \phi_0$$

$$\Rightarrow 9\left(\frac{1}{2}mv^2\right) = \frac{2hc}{\lambda} - \phi_0 \quad \text{...(ii)}$$

From eqs. (i) and (ii), we have

$$9\left(\frac{hc}{\lambda} - \phi_0\right) = \frac{2hc}{\lambda} - \phi_0$$

$$7\frac{hc}{\lambda} = 8\phi_0 \Rightarrow \phi_0 = \frac{7}{8}\frac{hc}{\lambda}$$

$\therefore$ work function $\phi_0 = \dfrac{7}{8}\times2.4 = 2.1\text{eV}$

116. **(a)** According to Bohr's theory the wavelength of radiation emitted from hydrogen atom is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For minimum wavelength of Balmer series, $n_2 = \infty$ and $n_1 = 2$

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$

$$\Rightarrow \frac{1}{\lambda_{min}} = \frac{R}{4} \Rightarrow \lambda_{min} = \frac{4}{R} \quad \text{...(i)}$$

For maximum wavelength of Balmer series $n_2 = 3$, $n_1 = 2$

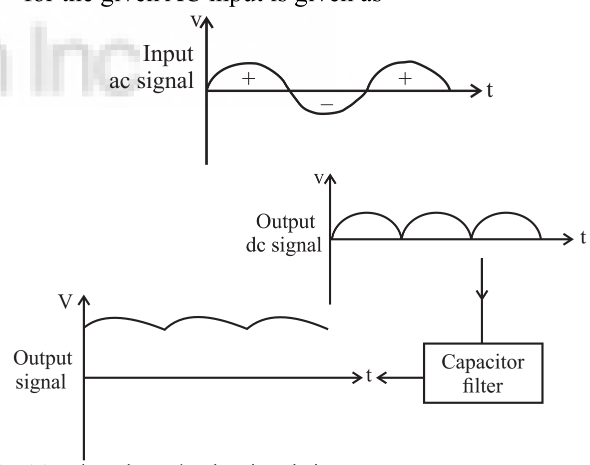
$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{5}{36}\right) \Rightarrow \lambda_{max} = \frac{36}{5R} \quad \text{...(ii)}$$

$\therefore$ From eq. (i) and (ii), we have

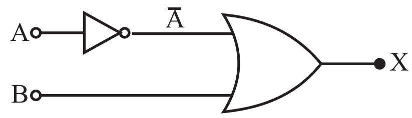
$$\Rightarrow \frac{\lambda_{max}}{\lambda_{min}} = \frac{36/5R}{4/R} = \frac{36}{20} = \frac{9}{5}$$

117. **(b)** α-rays are nucleus of ionised helium atom consisting of 2 neutrons and 2 protons. Hence a α-particle is a doubly ionised helium-atom.

118. **(c)** The output of full wave rectifier with capacitor filter for the given AC input is given as



119. **(c)** The given logic circuit is



$\therefore$ Boolean expression of the given logic circuit is given as $X = \overline{A} + B$

120. **(a)** Modulation index

$$\mu = \frac{\text{Amplitude of message signal}}{\text{Amplitude of carrier wave}} = \frac{A_m}{A_c} = \frac{15\text{V}}{30\text{V}} = 0.5$$

CHEMISTRY

121. **(b)** Given, $E = \frac{-R_H}{16}$

We know that, $E = \frac{-R_H}{n^2} = \frac{-R_H}{16}$, $\therefore n = 4$

Thus, degeneracy for $n = 4$ will be

$$\text{For } n = 4;\ l = \begin{matrix} 0, & 1, & 2, & 3 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 4s & 4p & 4d & 4f \end{matrix}$$

(i) For $4s$; $l = 0$, $m = 0$ (degeneracy = 1)

(ii) For $4p$; $l = 1$ $m = -1, 0 + 1$ (degeneracy = 3)

(iii) For $4d$; $l = 2$, $m = -2, -1, 0, +1, +2$ (degeneracy = 5)

(iv) For $4f$; $l = 3$, $m = -3 - 2, -1, 0, +1, +2, +3$ (degeneracy = 7)

Hence, total degeneracy of orbital for $n = 4$ will be 16.

122. **(c)** Given, kinetic energy of H^+ ion (K.E.) = 1.65 eV

Mass of proton (m) = 1.6726×10^{-27} kg

We calculate, the wavelength of H^+ ion as follows:

$$\lambda = \frac{h}{\sqrt{2mK.E}}$$

$$= \frac{6.62 \times 10^{-34}\,\text{kg m}^2\text{s}^{-1}}{\sqrt{2 \times 1.67 \times 10^{-27}\,\text{kg} \times 1.65 \times 1.6 \times 10^{-19}\,\text{kg m}^2 s^{-2}}}$$

= 0.220 Å = 0.022 nm

123. **(a)** Greater the electrostatic attraction between nucleus and valence electron, smaller will be the size of the atom/ion. Thus, Al^{3+} has lower ionic radii than Mg^{2+} due to higher nuclear charge.

124. **(b)** Removal of the 5th electron requires almost more than double the energy required to remove the 4th electron. This implies that the first four electrons are removed from the valence shell after that element acquires stable E.C. Hence, number of valence electrons in the atom 'X' will be four.

125. **(c)** A-(II) B - (III) C - (IV) D - (I)

BF_3	sp^2	Trigonal planar
ClF_3	sp^3d	T-shap
NH_3	sp^3	Trigonal pyramidal
NH_4^+	sp^3	Tetrahedral

126. **(b)** Bond order = $\frac{1}{2}[N_b - N_a] = \frac{1}{2}(4-4) = 0$

Be_2 molecules does not exist according to molecular orbital theory. Because bond order of Be_2 is zero. Hence, it does not exist.

127. **(b)** For H_2 gas, $u_{\text{rms}}(H_2) = \sqrt{\frac{3RT}{M_{H_2}}}$

For O_2 gas, $u_{\text{rms}} = \sqrt{\frac{3RT}{M_{O_2}}}$

$$\frac{u_{\text{rms}}(\text{H}_2)}{u_{\text{rms}}(\text{O}_2)} = \sqrt{\frac{3RT}{M_{H_2}}} \times \sqrt{\frac{M_{O_2}}{3RT}}$$

(Temperature constant, R = **8.**33 JK^{-1} mol^{-1})

$$u_{\text{rms}(H_2)} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} \times u_{\text{rms}(\text{O}_2)} = \sqrt{\frac{32}{2}} \times 500 = 2000 \text{ m/s}$$

Average kinetic energy of H_2 can be calculate as

$$= \frac{1}{2}mu^2 = \frac{1}{2} \times 2 \times (2000)^2$$

= 4000 J mol^{-1} or 4 kJ mol^{-1}

128. **(d)** All gases are made up of a very large number of extremely small identical particles called molecules. The molecules are separated from one another by large space. Hence, the actual volume occupied by the molecules is negligible as compared to the total volume of the gas. The collisions of gas particles with one another and with the walls of the container are perfectly elastic.

129. **(d)** $\underset{\substack{2\,\text{mol} \\ 68\text{g}}}{2H_2O_2} \longrightarrow 2H_2O + \underset{\substack{1\,\text{mol} \\ 22400\,\text{mL at STP}}}{O_2}$

Since 22400 mL O_2 is obtained by 68 g H_2O_2

$\therefore$ 22.4 mL O_2 is obtained by

$$= \frac{68 \times 22.4}{22400} = 0.068\,\text{g}\,H_2O_2$$

or 1 mL H_2O_2 solution = 0.068 g

$\therefore$ 100 mL H_2O_2 solution (% strength)

= 0.0680 × 100 = 6.8%

130. **(a)** The reaction will faster in aqueous HCl because chlorine ion present in HCl get oxidised into chlorine gas by $KMnO_4$, so the amount of $KMnO_4$ will be more as it oxidises both oxalic acid and chlorine ion when HCl is present.

131. **(a)** Given,

$\Delta H = +30.0$ kJ mol^{-1}; $\Delta S = 0.06$ kJ K^{-1} mol^{-1}

We know that, $\Delta G = \Delta H - T\Delta S$

If, $\Delta G = 0 \Rightarrow 0 = \Delta H - T\Delta S$

Hence, $-\Delta H = -T\Delta S$

$$T = \frac{-\Delta H}{-\Delta S} = \frac{30\,\text{kJ mol}^{-1}}{0.06\,\text{Jk}^{-1}\,\text{mol}^{-1}} = 500\text{K}$$

and nature of reaction will be non-spontaneous.

below 500K (500 – 273 = 227°C)

132. **(d)** Given, vapour density (d) = 40

$$N_2O_4 \rightleftharpoons 2NO_2$$

Molar mass 92

$$D = \frac{\text{M.M.}(N_2O_4)}{2} = \frac{92}{2} = 46$$

$\therefore$ α (Degree of dissociation) $= \frac{D-d}{(n-1)d}$

$$\alpha = \frac{46-40}{(2-1)40} \quad (n = \text{number of } NO_2 \text{ at equilibrium})$$

$$\alpha = \frac{6}{40} = 0.15$$

133. (a) $N_2 + O_2 \rightleftharpoons 2NO \; K_C = ?$

K_C for the above reaction can be given as,

$$K_C = \frac{[NO]^2}{[N_2][O_2]}$$

$$K_C = \frac{[3\times10^{-3}]^2}{[4\times10^{-3}][3\times10^{-3}]} \Rightarrow K_C = \frac{3\times10^{-3}}{4\times10^{-3}} = 0.75$$

134. (b) Hardness of water is due to the presence of bicarbonate, chlorides and sulphates of Ca and Mg.
Ca^{2+} + and Mg^{2+} ion present in hard water react with soap to form precipitate of Ca and Mg salt of fatty acids and hence, no lather is produced.

135. (c) Due to small size and high ionisation enthalpy of Be, Be $(OH)_2$ is amphoteric.
Therefore, it dissolves both in acid and bases.
$Be(OH)_2 + 2HCl + 2H_2O \rightarrow [Be(H_2O)_4]Cl_2$
$Be(OH)_2 + NaOH \rightarrow Na_2BeO_2 + 2H_2O$
or $Be(OH)_2 + 2\overset{\ominus}{O}H \rightarrow [Be(OH)4]^{2-}$
Hence, both reaction are feasible.

136. (c) Many simple compounds of elements such as $AlCl_3$, $GaCl_3$ and $InCl_3$ are covalent in anhydrous form, but in aqueous solution, these are ionic in nature. In anhydrous condition, the (charge/radius) ratio i.e. polarisability of Al^{3+} is high and hence, according to Fajans' rule, Al^{3+} polarises Cl^- ions to large extent, thus exhibits covalent character in the compound, i.e. $AlCl_3$ behaves as a covalent compound in anhydrous conditions.
In aqueous medium the ions get hydrated, because the amount of hydration enthalpy released exceeds, the sum total of ionisation enthalpy required.
Since, the (charge/radius ratio of hydrated aluminium ion is much smaller as compared to that of Al^{3+}, the tendency of $[Al(H_2O)_6]^{3+}$ to polarise hydrated $Cl^{\ominus}$ ion decreases and the resulting hydrated compound is ionic in nature.

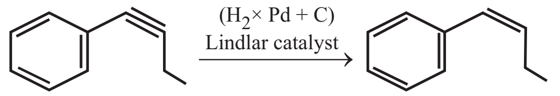
137. (c) For the most, group 14 elements do not react with water. But tin on heating with steam, reacts with water to form tin dioxide SnO_2 and hydrogen.

$$Sn + 2H_2O \xrightarrow{\Delta} SnO_2 + 2H_2$$

138. (a) The amount of oxygen required by bacteria to decompose the organic matter present in a certain volume of a sample of water, is called biochemical oxygen demand (BOD).
Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more. It means BOD value less than 5 ppm indicates a water sample to be rich in dissolved oxygen.

139. (b) The carbon and nitrogen present in the organic compound on fusion with sodium metal gives sodium cyanide (NaCN). This is converted into sodium ferrocyanide by the addition of sufficient quantity of ferrous sulphate. Ferric ions produced during the process react with ferrocyanide to form the prussian blue precipitate of ferric ferrocyanide. The prussian blue colour is due to formation of ferric ferrocyanide, $Fe_4[Fe(CN)_6]_3 \cdot xH_2O$

140. (c) Lindlar catalyst is partially deactivated catalyst composed of $BaSO_4$ coated with Pd poisoned with quinoline. It reduces $(C \equiv C)$ to $(C = C)$ bond, occurs *via* *syn*-addition and gives *cis*-alkene.



1-phenyl-1-butyne 1-phenyl-1-butene (cis)

141. (b) Alkane with six or more C-atoms, when heated under pressure in the presence of a catalyst Mo_2O_3 at 773 K at 10-20 atm. First cyclise and then aromatise to give benzene.

142. (a) Given,
Radius = 1.41 Å = 141 pm
Atomic weight of copper = 64
Avogadro's number = 6×10^{23}, Density = ?
Here, edge length of unit cell in ccp can be calculate as

$$r = \frac{a}{2\sqrt{2}} \Rightarrow a = r \times 2\sqrt{2}$$

a = 141 × 2 × 1.414
a = 398.74 pm or 398.74×10^{-10} cm
Volume of unit cell $(a^3) = (398.74 \times 10^{-10}\text{ cm})^3$
For ccp, Z = 4,

$$\rho = \frac{Z \times M}{a^3 \times N_a} \Rightarrow \rho = \frac{4\times64}{(398.74\times10^{-10}\text{cm})^3 \times 6.0\times10^{23}}$$

$\rho = 6.73 \approx 6.67$
Hence, option (a) is correct

143. (c) $A_{x-2}[B(C)_x]_2 \longrightarrow (x-2)A^{2+} + 2[B(C)_x]^{-(x-2)}$
$(1-\alpha)\; 2\alpha \;\; (x-2)\alpha$
$i = 1 - \alpha + (x-2)\alpha + 2\alpha$
$4 = 1 - 0.75 + (x-2)\, 0.75 + 2\,(0.75)$
$x = 5$
$\alpha = 0.75,\; i = 4$

144. (c) The freezing point among the following equimolal aqueous solutions will be highest for urea because urea is not an electrolyte does not dissociate. So, it has the minimum number of particles and therefore, it shows minimum depression in freezing point. So, it has the maximum freezing point.

145. (b) Electrode with higher reduction potential will act as cathode.
Hence, silver electrode act as a cathode. Because reduction potential of silver electrode (E°) is +80 V. Which is more than given reduction potential of Cu (+34 V).

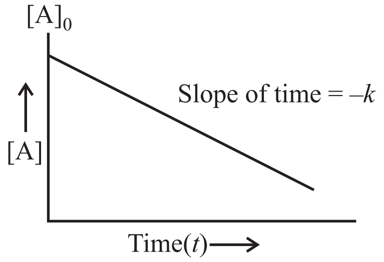
$$E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode}$$

$= +0.80 - (+0.34) = +0.46$ V

146. (b) Any reaction of zero order must obey the following equation:

$[A] = -kt + [A]_0$ or $[A] = [A]_0 + kt$

Compare this equation with straight line ($y = mx + c$) equation, the plot of $[A]$ *versus* t will be straight line with slope $= -k$ and intercept on the concentration axis $= [A]_0$ as shown in figure.



Plot of [A] *versus* t for a reaction of zero order.

This shows that, the concentration of reactant decreases linearly with time.

147. **(b)** The lyophilic colloids differ in their protective power. The protective power is measured in terms of gold number. It may be noted that smaller the value of the gold number, greater will be the protecting power of protective colloids. Gold number of gelatin, haemoglobin and sodium acetate are 5×10^{-3}, 5×10^{-2} and 7×10^{-1}, respectively, hence, the protective actions will be in the order :

gelatin > Haemoglobin > sodium acetate

148. **(c)** Calamine ($ZnCO_3$) in the ore of zinc (Zn) and rest of all are ores of iron.

Hematite (Fe_2O_3), Magnetite (Fe_3O_4)

Siderite ($FeCO_3$)

149. **(a)** When copper metal is treated with cold and dilute nitric acid. It yields copper nitrate, water and nitric oxide.

$$3Cu + 8HNO_3\underset{(cold)}{(dil.)} \longrightarrow 3Cu(NO_3)_2 + 4H_2O + 2NO$$

150. **(d)** NO_2 (Nitrogen dioxide) is a brown gas, highly reactive and paramagnetic. Oxidation state of N in NO_2 is + 4.

151. **(d)** Interstitial compounds or non stoichiometric compounds are formed, when small atoms like C, H, N are trapped inside the crystal lattice of metals.

They are very hard and rigid and have high melting point, which are higher than those of the pure metals.

They exhibit metallic conductivity like that of the pure metals. They aquire chemical inertness.

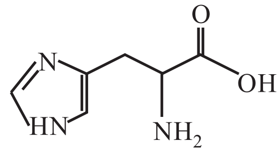
152. **(b)** Hydrated copper sulphate $CuSO_4 . 5H_2O$ is blue in colour, because the ligand (water) molecules facilitates *d-d-* transition.

Anhydrous copper sulphate $CuSO_4$ is colourless. In the absence of ligand (water) molecules, splitting of *d*-orbitals is not possible. Hence, *d-d* transition is not possible. Hence, option **(b)** is correct.

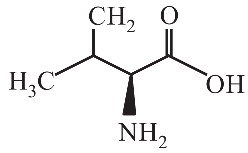
153. **(d)** Anionic polymerisation are anionic initiator generates a carbanionic intermediate and such polymerisation is termed anionic addition polymerisation. It takes place easily with the monomer having electron withdrawing groups such as phenyl, nitrile etc, that are capable of stabilising the propagating species.

Hence, $CH_2 = CH — CN$ is the most reactive towards anionic polymerisation.

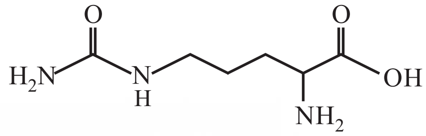
154. **(a)** (i) Histidine



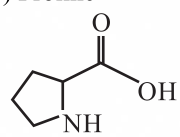
(ii) Valine



(iii) Arginine



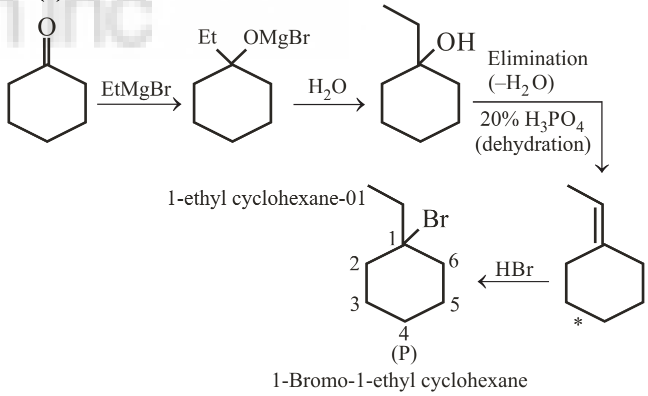
(iv) Proline



Histidine and proline contain heterocyclic ring.

155. **(a)** Paul Ehrlich a German bacteriologist got Nobel prize in 1908 for discovery of arsenic based medicine, Salvarsan (arsphenamine) which is used for the treatment of syphilis. Although Salvarsan is toxic to human being, but its effect on the bacteria, spirochete (Causing syphilis) is much greater than on human beings.

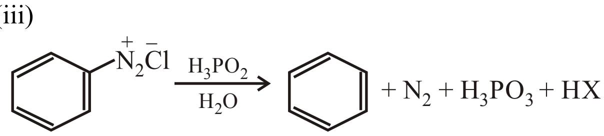
156. **(c)**

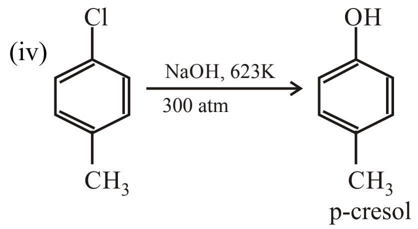


157. **(c)** (i) $C_2H_5COOH \xrightarrow{LiAlH_4} \underset{Ethanol}{C_2H_5CH_2OH}$

(ii) $C_2H_5Br \xrightarrow[\text{then } H_2O]{\text{Mg. dry ether}} \underset{Ethane}{C_2H_6} + Mg(Br)(OH)$

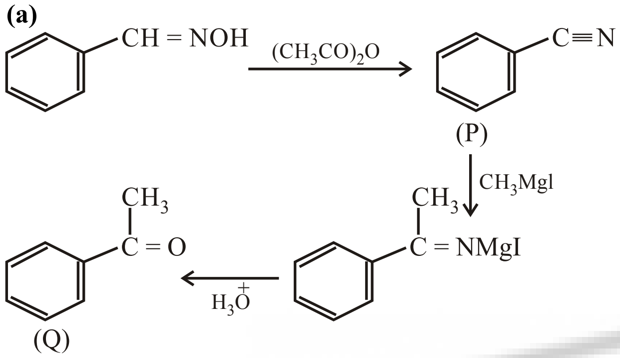
(iii)



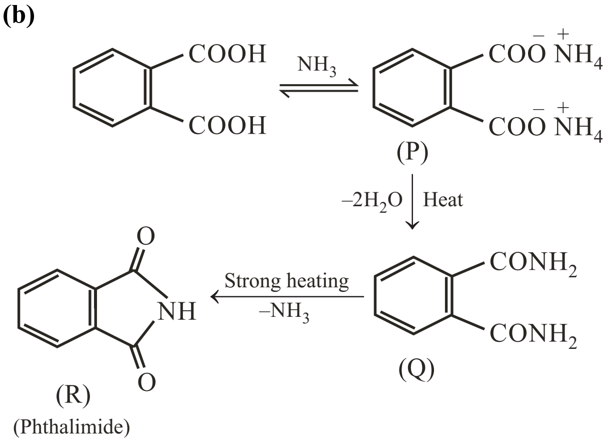


Hence, only reaction (i) and (iv) gives alcohol and phenol respectively.

158. (a)



159. (b)



160. (b)