TS/EAMCET Solved Paper 2018

Held on May 5

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. Let $X = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in R \right\}$, Define

$f : X \to R$ by $f(A) = \text{dct}(A)$, $\forall A \in X$. Then f is

(a) one one but not onto (b) onto but not one-one

(c) one-one and onto (d) neither one-one nor onto

2. Let $x \neq 0, |x| < \frac{1}{2}$ and

$f(x) = 1 + 2x + 4x^2 + 8x^3 + \ldots$. Then, $f^{-1}(x) =$

(a) $\frac{x-1}{2x}$ (b) $\frac{x-1}{2}$ (c) $\frac{x-1}{x}$ (d) $1 - 2x$

3. For all positive integers k, if the greatest divisor of $25^k + 12k - 1$ is d, then $4\sqrt{d}$ =

(a) 36 (b) 8 (c) 20 (d) 24

4. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, then $(AA')' =$

(a) $\begin{bmatrix} 14 & 32 & 50 \\ 32 & 122 & 194 \\ 14 & 32 & 50 \end{bmatrix}$ (b) $\begin{bmatrix} 14 & 50 & 32 \\ 32 & 122 & 194 \\ 50 & 194 & 122 \end{bmatrix}$

(c) $\begin{bmatrix} 14 & 32 & 50 \\ 32 & 194 & 122 \\ 50 & 149 & 256 \end{bmatrix}$ (d) $\begin{bmatrix} 14 & 32 & 50 \\ 32 & 77 & 122 \\ 50 & 122 & 194 \end{bmatrix}$

5. If $\Delta_1 = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+c & 1 \end{vmatrix}$ then

$\frac{\Delta_1}{\Delta_2} =$

(a) $ab + bc + ca$ (b) abc

(c) $2(ab + bc + ca)$ (d) $(a + b + c)^2$

6. If $x = a, y = b, z = c$ is the solution of the system of simultaneous linear equations $x + y + z = 4$, $x - y + z = 2$, $x + 2y + 2z = 1$, then $ab + bc + ca =$

(a) 0 (b) – 25

(c) 1 (d) – 4

7. The common roots of the equations

$z^3 + 2z^2 + 2z + 1 = 0$ and $z^{2018} + z^{2017} + 1 = 0$

satisfy the equation

(a) $z^2 - z + 1 = 0$ (b) $z^4 + z^2 + 1 = 0$

(c) $z^8 + z^3 + 1 = 0$ (d) $z^{12} + z^6 - 1 = 0$

8. The area (in sq. units) of the triangle whose vertices are the points represented by the complex numbers 0, z, ze^{i} $(0 < \alpha < \pi)$ is

(a) $\frac{1}{2}|z|^2$ (b) $\frac{1}{2}|z|^2 \sin\alpha$

(c) $\frac{1}{2}|z|^2 \sin\alpha\cos\alpha$ (d) $\frac{1}{2}|z|^2 \cos\alpha$

9. If $z+\frac{1}{z}=1$, then $\frac{(z^{2\theta}+1)(z^{4\theta}+1)(z^{6\theta}+1)}{z^{6\theta}}=$

(a) -2 (b) 2 (c) 1 (d) -1

10. If $\omega_0, \omega_1, ..., \omega_{n-1}$ are the nth roots of unity, then $(1+2\omega_0)(1+2\omega_1)(1+2\omega_2)...(1+2\omega_{n-1})=$

(a) $1+(-1)^n 2^n$ (b) $1+2^n$
(c) $(-1)^n+2^n$ (d) $1+(-1)^{n-1}2^n$

11. If $k \in R$, then roots of $(x-2)(x-3)=k^2$ are always

(a) real and distinct (b) real and equal
(c) complex number (d) rational numbers

12. If $x^2-3ax+14=0$ and $x^2+2ax-16=0$ have a common root, then a^4+a^2.

(a) 2 (b) 90 (c) 6 (d) 20

13. If $\alpha_1, \alpha_2, ..., \alpha_n$ are roots of $x^n+px+q=0$, then $(\alpha_n-\alpha_1)(\alpha_n-\alpha_2).....(\alpha_n-\alpha_{n-1})=$

(a) $n\alpha_a^{n-1}+q$ (b) $\alpha_1^2+\alpha_2^2+...+\alpha_{n-1}^2$
(c) $\alpha_n^{n-1}+p$ (d) $n\alpha_{n-}^{n-1}+p$

14. All the roots of the equation $x^5+15x^4+94x^3+305x^2+507x+353=0$ are increased by some real number k in order to eliminate the 4th degree term from the equation. Now, the coefficient of x in the transformed equation is

(a) 2 (b) 1 (c) 6 (d) 0

15. The number of ways in which four letters can be put in four addressed envelopes so that no letter goes into envelope meant for it is

(a) 8 (b) 12 (c) 16 (d) 9

16. If the integer represented by 100! has $K\pi$ consecutive zeroes at the end, then $K=$

(a) 24 (b) 36 (c) 64 (d) 128

17. If n is a positive integer and the coefficient of x^{10} in the expansion of $(1+x)^{15}$ is equal to the coefficient of x^5 in the expansion of $(1-x)^{-n}$, then $n=$

(a) 15 (b) 12 (c) 11 (d) 10

18. If $x=\frac{2.5}{3.6}-\frac{2.5.8}{3.6.9}\left(\frac{2}{5}\right)+\frac{2.5.8.11}{3.6.9.12}\left(\frac{2}{5}\right)^2-...\infty$, then

$7^2(12x+55)^3=$

(a) $3^8 5^3$ (b) $3^8 5^5$ (c) $3^3 5^5$ (d) $3^3 5^8$

19. If F_1 and F_2 are irreducible factors of x^4+x^2+1 with real coefficients and $\frac{x^4-2x^2+3x-4}{x^4+x^2+1}=\frac{Ax+B}{F_1}+\frac{Cx+D}{F_2}$,

then $A+B+C+D=$

(a) -2 (b) 1 (c) -3 (d) -4

20. The number of all the possible integral values of $n>2$ such that $\sin\frac{\pi}{2n}+\cos\frac{\pi}{2n}=\frac{\sqrt{n}}{2}$ is

(a) 5 (b) 4 (c) 3 (d) infinity

21. If α and β are angles in the first quadrant such that $\tan\alpha=\frac{1}{7}$ and $\sin\beta=\frac{1}{\sqrt{10}}$, then $\alpha+2\beta=$

(a) $30°$ (b) $45°$ (c) $75°$ (d) $90°$

22. $\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)=$

(a) $\frac{-1}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3\sqrt{3}}{8}$ (d) 1

23. If $0<\theta<\frac{\pi}{2}$, then solution of the equation $\sin\theta-3\sin 2\theta+\sin 3\theta=\cos\theta-3\cos 2\theta+\cos 3\theta$ is

(a) $\frac{\pi}{16}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{6}$

24. $\sin^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{63}{16}\right)=$

(a) 2π (b) π (c) 0 (d) $-\pi$

25. If $\sin h\, x=\frac{3}{4}$ and $\cos h\, y=\frac{5}{3}$, then $x+y=$

(a) log 2 (b) log 6 (c) log 3 (d) log 5

26. In a Δ ABC, if $a=5$, $b=6$, $c=7$, then the length of the median drawn from B is

(a) $2\sqrt{7}$ (b) $2\sqrt{6}$ (c) $\sqrt{7}$ (d) $\sqrt{6}$

27. If Δ ABC, if $\cot\frac{A}{2}:\cot\frac{B}{2}:\cot\frac{C}{2}=4:3:2$, then $a:b:c=$

(a) 2 : 3 : 4 (b) 6 : 5 : 7 (c) 4 : 5 : 6 (d) 5 : 6 : 7

28. If $\Delta\, ABC$, if $\cos A\cos B+\sin A\sin B\sin C=1$ and $C=\frac{\pi}{2}$, then $A:B=$

(a) 1 : 4 (b) 1 : 3 (c) 1 : 2 (d) 1 : 1

29. If a, b, c are distinct real numbers and P, Q, R are three points whose position vectors are respectively $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$, $b\hat{\mathbf{i}}+c\hat{\mathbf{j}}+a\hat{\mathbf{k}}$ and $c\hat{\mathbf{i}}+a\hat{\mathbf{j}}+b\hat{\mathbf{k}}$, then $\angle QRP=$

(a) $\cos^{-1}(a+b+c)$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\cos^{-1}\left(\frac{a^2+b^2+c^2}{abc}\right)$

30. Let $\mathbf{a} = \sin^2 x\hat{\mathbf{i}} + \cos^2 x\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $(x \in R)$. If the pairs of vectors $\mathbf{a}\,.\,\hat{\mathbf{i}}; \mathbf{a}\,.\,\hat{\mathbf{j}}$ and $\mathbf{a}\,.\,\hat{\mathbf{k}}$ are adjacent sides of 3 distinct parallelograms and A is the sum of the squares of areas of these parallelograms, then A lies in the interval

(a) (0, 1) (b) [3, 4] (c) [0, 2] (d) [1, 2]

31. **Assertion : a, b, c, d** are position vectors of 4 points such that $2\mathbf{a} - 3\mathbf{b} + 7\mathbf{c} - 6\mathbf{d} = 0 \Rightarrow$ **a, b, c, d** are coplanar.

Reason : Vector equation of the plane passing through three points whose position vectors are **a, b, c** is $\mathbf{r} = (1 - x - y)\,\mathbf{a} + x\mathbf{b} + y\mathbf{c}$. Which of the following is true?

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)

(b) Both (A) and (R) are true and (R) is not the correct explanation of (A)

(c) (A) is true but (R) is false

(d) (A) is false, but (R) is true

32. If $|\,\mathbf{a}\,| = 4$, $|\,\mathbf{b}\,| = 5$, $|\,\mathbf{a} - \mathbf{b}\,| = 3$ and θ is the angle between the vectors **a** and **b**, then $\tan^2\theta =$

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$

(c) $\frac{16}{9}$ (d) $\frac{9}{16}$

33. If **e** is the unit vector perpendicular to the plane determined by the points $2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $-\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$. If $\mathbf{a} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$, then the projection vector of **a** on **e** is

(a) $\frac{11}{14}(-2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ (b) $\frac{1}{3}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

(c) $\frac{1}{7}(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$ (d) $\frac{1}{\sqrt{14}}(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

34. If $\mathbf{a} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$, $\mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $\mathbf{c} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and $\mathbf{d} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, then $(a \times b) \times (c \times d) =$

(a) $-7\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ (b) $8\hat{\mathbf{i}} - 36\hat{\mathbf{j}} + 60\hat{\mathbf{k}}$

(c) $5\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (d) $-8\hat{\mathbf{i}} - 36\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$

35. The mean and standard deviation of a distribution of weights of a group of 20 boys are 40 kg and 5 kg respectively.

If two boys of weights 43 kg and 37 kg are exclude from this group, then the variance of the distribution of weights of the remaining group of boys is

(a) 26.18 (b) 5.27

(c) 26.78 (d) 5.17

36. Consider the following data:

	Group I	Group II	Group III
Number of Observation	50	60	90
Mean	113	120	115
Standard deviation	6	8	7

With respect to the consistencies of the above groups, the increasing order of them is

(a) I, III, II (b) II, I, III (c) III, II, I (d) I, II, III

37. In a battery manufacturing factory, machines P, Q and R manufacture 20%, 30% and 50% respectively of the total output. The chances that a defective battery is produced by these machines are 1%, 1.5% and 2% respectively. If a battery is selected as random from production, then the probability that it is defective is

(a) $\frac{69}{2000}$ (b) $\frac{33}{2000}$ (c) $\frac{1}{40}$ (d) $\frac{29}{2000}$

38. Suppose A and B are events of a random experiment such that $P(A) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{5}$ and $P(A \cup B) = \frac{3}{5}$.

Then, match the items of List-I with the items of List-II

	List-I		List-II
A.	$P\left(\frac{A}{B}\right)$	(i)	$\frac{2}{15}$
B.	$P(\overline{B})$	(ii)	$\frac{4}{15}$
C.	$P(A \cap \overline{B})$	(iii)	$\frac{8}{15}$
D.	$P(B \cap \overline{A})$	(iv)	$\frac{2}{3}$
		(v)	$\frac{3}{7}$

	A	B	C	D		A	B	C	D
(a)	(iv)	(i)	(ii)	(iii)	(b)	(v)	(i)	(ii)	(iii)
(c)	(iv)	(ii)	(i)	(v)	(d)	(v)	(iii)	(i)	(ii)

39. In a test, a student either guesses or copies or knows the answer to a multiple choice question with four choices having one correct answer. The probability that he guesses the answer is $\frac{1}{3}$ and the probability that he copies it is $\frac{1}{12}$. The probability that his answer is correct given that he copied it is $\frac{1}{6}$. The probability that he knew the answer, given that he has correctly answered it, is

(a) $\frac{6}{7}$ (b) $\frac{15}{49}$ (c) $\frac{7}{12}$ (d) $\frac{10}{13}$

40. The probability that a mechanic making an error while using a machine on the nth day is given by $P(E_n) = \frac{1}{2^n}$.

If he has operated the machine for 4 days, the probability that he had not made a mistake on 3 of 4 days is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{243}{512}$ (d) $\frac{343}{1024}$

41. If the probability of a bad reaction from a vaccination is 0. 01, then the probability that exactly two out of 300 people will get bad reaction is

(a) $\frac{7}{2e^3}$ (b) $\frac{9}{2e^3}$ (c) $\frac{7}{e^3}$ (d) $\frac{9}{e^3}$

42. If $A = (1, 2)$, $B = (2, 1)$ and P is a variable point satisfying the condition $|PA - PB| = 3$, then the locus of P is

(a) $8x^2 + 2xy + 8y^2 + 27x + 27y + 45 = 0$
(b) $4x^2 + xy + 4y^2 - 27x - 27y + 90 = 0$
(c) $32x^2 + 8xy + 32y^2 - 108x - 108y + 99 = 0$
(d) $8x^2 - 2xy + 8y^2 - 27x - 27y + 45 = 0$

43. For $a \neq b \neq c$, if the lines $x + 2ay + a = 0$
$x + 3by + b = 0$ and $x + 4cy + c = 0$ are concurrent, then a, b, c are in

(a) Arithmetic progression
(b) Geometric progression
(c) Harmonic progression
(d) Arithmetico geometric progression

44. A point moves in the XY-plane such that the sum of its distance from two mutually perpendicular lines is always equal to 3. The area enclosed by the locus of that point is (in sq. units)

(a) 27 (b) 18 (c) 9 (d) $\frac{9}{2}$

45. The equation of two altitudes of an equilateral triangle are $\sqrt{3}x - y + 8 - 4\sqrt{3} = 0$ and $\sqrt{3}x + y - 12 - 4\sqrt{3} = 0$. The equation of the third altitude is

(a) $\sqrt{3}x + y = 4$ (b) $y = 10$
(c) $x = 10$ (d) $x - \sqrt{3}y = 4$

46. If $P_1, P_2, P_3, ..., P_n$ are n points on the line $y = x$ all lying in the first quadrant, such that $(OP_n) = n(OP_{n-1})$ (O is origin), $OP_1 = 1$ and $P_n = (2520\sqrt{2}, 2520\sqrt{2})$, then $n =$

(a) 5 (b) 6 (c) 7 (d) 8

47. The straight line $x + y + 1 = 0$ bisects an angle between the pair of lines of which one is $2x + 3y - 4 = 0$. Then, the equation of the other line is

(a) $3x - 2y + 9 = 0$ (b) $3x - 2y - 9 = 0$
(c) $3x + 2y + 9 = 0$ (d) $x - y - 1 = 0$

48. The combined equation of the pair of straight lines passing through the point of intersection of the pair of lines $x^2 + 4xy - 3y^2 - 4x - 10y + 3 = 0$ and having slopes $\frac{1}{2}$ and $-\frac{1}{3}$ is

(a) $x^2 - y^2 - 8x - 2y + 15 = 0$
(b) $x^2 + 7xy + 12y^2 - x - 4y = 0$
(c) $x^2 + 7xy + 10y^2 - x - 8y - 2 = 0$
(d) $x^2 + xy - 6y^2 - 7x - 16y + 6 = 0$

49. If a circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 with radius 5 such that the common chord is of maximum length and has a slope equal to $\frac{3}{4}$ then the centre of the circle C_2 is

(a) $\left(-\frac{9}{5}, \frac{12}{5}\right)$ (b) $\left(\frac{9}{5}, \frac{12}{5}\right)$
(c) $\left(-\frac{5}{9}, \frac{6}{5}\right)$ (d) $\left(\frac{7}{5}, -\frac{12}{5}\right)$

50. The equation of the circle which touches the circle $x^2 + y^2 - 6x + 6y + 17 = 0$ externally and having the lines $x^2 - 3xy - 3x + 9y = 0$ as two normals, is

(a) $x^2 + y^2 - 2x + 5y - 1 = 0$
(b) $x^2 + y^2 + 2x + 3y + 1 = 0$
(c) $x^2 + y^2 - 6x - 2y + 1 = 0$
(d) $x^2 + y^2 + 4x - 3y + 3 = 0$

51. Let A be the centre of the circle
$x^2 + y^2 - 2x - 4y - 20 = 0$. If the tangents drawn at the points $B(1, 7)$ and $D(4, -2)$ on the given circle meet at the point C, then the area of the quadrilateral $ABCD$ is

(a) 60 (b) 65
(c) 70 (d) 75

52. Let $x - 4 = 0$ be the radical axis of two circles which are intersecting orthogonally. If $x^2 + y^2 = 36$ is one of those circles, then the other circle is

(a) $x^2 + y^2 - 16x + 36 = 0$
(b) $x^2 + y^2 - 18x + 36 = 0$
(c) $x^2 + y^2 - 18x + 24 = 0$
(d) $x^2 + y^2 - 6x + 8y + 36 = 0$

53. The length of common chord of the circles
$x^2 + y^2 - 6x - 4y + 13 - c^2 = 0$ and
$x^2 + y^2 - 4x - 6y + 13 - c^2 = 0$ is

(a) $\sqrt{4c^2 - 2}$ (b) $\frac{1}{2}\sqrt{4c^2 - 2}$
(c) $\sqrt{c^2 - 2}$ (d) $\sqrt{4c^2 - 1}$

54. If P is (3, 1) and Q is a point on the curve $y^2 = 8x$, then the locus of the mid-point of the line segment PQ is
(a) $4y^2 - 12x - 6y + 21 = 0$
(b) $4y^2 - 16x - 4y + 25 = 0$
(c) $4y^2 + 8x - 3y - 18 = 0$
(d) $4y^2 - 12x + 8y - 15 = 0$

55. Let $P(2, 4)$, $Q(18, -12)$ be the points on the parabola $y^2 = 8x$. The equation of straight line having slope $\frac{1}{2}$ and passing through the point of intersection of the tangents to the parabola drawn at the points P and Q is
(a) $2x - y = 1$ (b) $2x - y = 2$
(c) $x - 2y = 1$ (d) $x - 2y = 2$

56. Let A be a vertex of the ellipse
$S = \frac{x^2}{4} + \frac{y^2}{9} - 1 = 0$ and F be a focus of the ellipse
$S' = \frac{x^2}{9} + \frac{y^2}{4} - 1 = 0$. Let P be a point on the major axis of the ellipse $S' = 0$, which divides $\overline{OF}$ in the ratio 2 : 1 (O is the origin). If the length of the chord of the ellipse $S = 0$ through A and P is $\frac{3\sqrt{101}}{k}$, then $k =$

(a) 5 (b) 4 (c) 7 (d) 8

57. Tangents are drawn to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ at all the four ends of its latusrectum. Then the area (in sq. units) of the quadrilateral formed by these tangents is

(a) $\frac{125}{6}$ (b) $\frac{250}{3}$ (c) $\frac{80}{3}$ (d) $\frac{260}{3}$

58. The lines of the form $x \cos \phi + y \sin \phi = P$ are chords of the hyperbola $4x^2 - y^2 = 4a^2$ which subtend a right angle at the centre of the hyperbola. If these chords touch a circle with centre at (0, 0), then the radius of that circle is

(a) $\frac{2a}{\sqrt{3}}$ (b) $\frac{a}{\sqrt{3}}$ (c) $\sqrt{2}a$ (d) $\frac{a}{\sqrt{2}}$

59. Let $A(3, 2, -4)$ and $B(9, 8, -10)$ be two points. Let P_1 divides AB in the ratio 1 : 2 and P_2 divide AB in the ratio 2 : 1. If the point $P(\alpha, \beta, \gamma)$ divides $P_1 P_2$ in the ratio 1 : 1, then $\alpha + 2\beta + 2\gamma =$
(a) 1 (b) 2 (c) 3 (d) 4

60. If the direction cosines of the two lines satisfy the equation $l + m + n = 0$, $2lm + 2ln - mn = 0$, then the acute angle between these lines is

(a) $\cos^{-1}\left(\frac{1}{3}\right)$ (b) 30°
(c) $\cos^{-1}\left(\frac{2}{3}\right)$ (d) 60°

61. If the equation of the plane passing through the point (2, – 1, 3) and perpendicular to the planes $3x - 2y + z = 9$ and $x + y + z = 9$ is $x + by + cz + d = 0$, then $d =$

(a) $\frac{11}{3}$ (b) 0 (c) 3 (d) $\frac{1}{3}$

62. $\lim_{x \to a} \frac{\sqrt{a + 2x} - \sqrt{3a}}{\sqrt{x} - \sqrt{a}} =$

(a) $-\frac{5}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

63. If a function $f(x)$ defined by

$$f(x) = \begin{cases} ax + b, & x \le -1 \\ 2x^2 + 2bx - \frac{a}{2}, & -1 < x < 1 \\ 7, & x \ge 1 \end{cases}$$ is continuous

on R, then $(a, b) =$
(a) (– 22, – 3) (b) (22, – 3)
(c) (11, – 6) (d) (– 22, – 6)

64. The derivative of $y = (\sin x)^{x^2}$ with respect to x is

(a) $(\sin x)^{x^2} \log(\sin x)$
(b) $x^2 (\sin x)^{x^2 - 1}$
(c) $2x(\sin x)^{x^2} \cos x + 2x(\sin x)^{x^2} \log(\sin x)$
(d) $x^2 (\sin x)^{x^2 - 1} \cos x + 2x(\sin x)^{x^2} \log(\sin x)$

65. If $y = \dfrac{(x+1)^2\sqrt{x-1}}{(x+4)^3 e^x}$ then $\dfrac{dy}{dx} =$

(a) $\dfrac{(x+1)^3\sqrt{x-1}}{(x+4)^3 e^x}\left[\dfrac{2}{x+1}+\dfrac{1}{2(x-1)}-\dfrac{3}{x+4}-1\right]$

(b) $\dfrac{(x+1)\ \sqrt{x-1}}{(x\quad 4)\ e}\left[\dfrac{2}{x+1}+\dfrac{1}{2(x-1)}+\dfrac{3}{x+4}-\quad\right]$

(c) $\dfrac{(x+1)^2\sqrt{x-1}}{(x+4)^3 e^x}\left[\dfrac{2}{x+1}+\dfrac{1}{2(x-1)}-\dfrac{3}{x+4}-1\right]$

(d) $\dfrac{(x+1)\sqrt{x-1}}{(x+4)^2 e^x}\left[\dfrac{2}{x+1}+\dfrac{1}{x-1}-\dfrac{3}{4+x}-1\right]$

66. The slope of the tangent to the curve $f(x) = \tan h^{-1}(\sin x)$ at $x = \pi$ is

(a) 1 (b) 0 (c) – 1 (d) – 2

67. The approximation value of

$y = (1.01)^3 + 2(1.01)^{\frac{3}{2}} + 5$ is

(a) 8.06 (b) 8.04 (c) 8.02 (d) 8.16

68. If $y = 2x$ is a tangent to the curve $y^2 = ax^3 + b$ at (1, 2), then $(a, b) =$

(a) (8, 4) (b) $\left(\dfrac{2}{3}, 1\right)$ (c) $\left(\dfrac{8}{3}, \dfrac{4}{3}\right)$ (d) $\left(\dfrac{8}{3}, \dfrac{2}{3}\right)$

69. An angle between the curves $x^2 - y^2 = 4$ and $x^2 + y^2 = 4\sqrt{2}$ is

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{6}$

70. Let $f(x)$ be continuous on [0, 4], differentiable on (0, 4), $f(0) = 4$ and $f(4) = -2$. If $g(x) = \dfrac{f(x)}{x+2}$ then the value of $g'(c)$ for some Lagrange's constant $c \in (0, 4)$ is

(a) $\dfrac{1}{2}$ (b) $\dfrac{5}{12}$ (c) $-\dfrac{5}{12}$ (d) $-\dfrac{7}{12}$

71. $\int (\sqrt{\tan x} + \sqrt{\cot x})\, dx =$

(a) $\sqrt{2}\sin^{-1}(\sin x + \cos x) + c$

(b) $\sqrt{2}\cos^{-1}(\sin x + \cos x) + c$

(c) $\sqrt{2}\cos^{-1}(\sin x - \cos x) + c$

(d) $\sqrt{2}\sin^{-1}(\sin x - \cos x) + c$

72. $\int \dfrac{dx}{(2ax + x)^{\frac{3}{2}}} =$

(a) $\dfrac{-1}{a^2}\dfrac{(x+a)}{\sqrt{2ax+x^2}} + c$ (b) $\dfrac{-(x+a)}{\sqrt{2ax+x^2}} + c$

(c) $\dfrac{1}{2a^2}\dfrac{(x+a)}{\sqrt{2ax+x^2}} + c$ (d) $\dfrac{-1}{a}\dfrac{(x+a)}{\sqrt{2ax+x^2}} + c$

73. If $\int \dfrac{2\,dx}{\sqrt{\cot^2 x - \tan^2 x}} = -\sqrt{f(x)} + c$, then $f(x)$

(a) $\cot x$ (b) $\sin 2x$ (c) $\cos 2x$ (d) $\tan x$

74. $\int \dfrac{3^x}{\sqrt{9^x - 1}}\, dx =$

(a) $\dfrac{1}{\log 3}\log|3^x + \sqrt{9^x - 1}| + c$

(b) $\dfrac{1}{\log 3}\log|3^x - \sqrt{9^x - 1}| + c$

(c) $\dfrac{1}{\log 9}\log|3^x - \sqrt{9^x - 1}| + c$

(d) $\dfrac{1}{\log 9}\log|9^x - \sqrt{9^x - 1}| + c$

75. If $f(x) = \int_1^x \dfrac{1}{2 + t^x}\, dt$, then

(a) $\dfrac{1}{18} < \quad (2) < \dfrac{1}{3}$ (b) $f(2) < \dfrac{1}{2}$ (or) $f(2) > 2$

(c) $f(2) < \dfrac{1}{3}$ (d) $f(2) > \dfrac{1}{3}$

76. If $I_w = \int_{\pi/2}^{\infty} e^{-x}\cos^x x\, dx$, then $\dfrac{I_{2018}}{I_{2016}} =$

(a) $\dfrac{2018 \times 2019}{(2017)^2 + 1}$ (b) $\dfrac{2018 \times 2017}{(2018)^2 + 1}$

(c) $\dfrac{(2018)(2016)}{(2017)^2 + 1}$ (d) $\dfrac{(2018)(2017)}{(2019)^2 + 1}$

77. The area bounded by the curves $y = 2x^2$, $y = \max.\{x - [x] + |x|\}$ and the lines $x = 0$, $x = 2$ (in sq. units), is

(a) 2 (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{4}{3}$

78. The differential equation corresponding to the family of curves given by $y = a + be^{2x} + ce^{-3x}$ is

(a) $y_3 - y_2 + 6y_1 = 0$ (b) $y_3 + y_2 - 6y_1 = 0$

(c) $y_3 - 6y_2 - y_1 = 0$ (d) $y_3 + 6y_2 - y_1 = 0$

79. The general solution of the differential equation $x^2y\,dx - (x^3 + y^3)dy = 0$ is

(a) $y^3 = 3x^3 \log(cx)$ (b) $c(x^3 - y^3) = x^2$

(c) $\log|y| - \frac{x^3}{3y^3} = 0$ (d) $y^2 - x^2 = c^2(y^2 - x^2)$

80. The general solution of the differential equation $(1 + y^2)\,dx = (\tan^{-1} y - x)dy$ is

(a) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + c$

(b) $xy + \tan^{-1} y = c$

(c) $2\tan^{-1} y = (y^2 - 1)x + c$

(d) $xe^{\tan^{-1} y} = e^{\tan^{-1} y}(\tan^{-1} y - 1) + c$

PHYSICS

81. Choose the incorrect statement from the following:

(a) Strong nuclear force is a short range force.

(b) Weak nuclear force is weakest among gravitational. electromagnetic, weak and strong nuclear forces.

(c) Electromagnetic force is a long range force.

(d) Gravitational force acts on all objects.

82. If V_0 is the volume of a standard unit cell of germanium crystal containing N_0 atoms, then the expression for the mass m of a volume V in terms of V_0, N_0, M_{mol} and N_A is, [here, M is the molar mass of germanium and N_A is the Avogadro's constant]

(a) $M\frac{V}{V_0}\frac{N_0}{N_A}$ (b) $\frac{N_A}{N_0}\frac{V_0}{V}M$

(c) $M\frac{V}{V_0}\frac{N_0}{N_A}$ (d) $M\frac{V_0}{V}\frac{N_0}{N_A}$

83. A stone is dropped from a height of 100 m, while another one is projected vertically upwards from the ground with a velocity of 25 m/s at the same time.

The time in seconds after which they will have the same height is (acceleration due to gravity, $g = 10\ \text{ms}^{-1}$)

(a) 4 (b) 5 (c) 6 (d) 7

84. A car starts from rest and moves with a constant acceleration of 5 m/s^2 for 10 s before the driver applies the brake. It then decelerates for 5 s before coming to rest, then the average speed of the car over the entire journey of the car is

(a) 23 m/s (b) 30 m/s (c) 33 m/s (d) 25 m/s

85. A particle moves in a circle with speed v varying with time as $v(t) = 2t$. The total acceleration of the particles after it completes 2 rounds of cycle is

(a) 16π (b) $2\sqrt{1 + 64\pi^2}$

(c) $2\sqrt{1 + 49\pi}$ (d) 14π

86. A small object is thrown at an angle 45° to the horizontal with an initial velocity v_0. The velocity is averaged for first $\sqrt{2}$ s and the magnitude of average velocity comes out to be same as that of initial velocity, *i.e.* $|v_0|$. The magnitude $|v_0|$ will be (take, $g = 10$ m/s^2)

(a) 3 m/s (b) $3\sqrt{2}$ m/s (c) 4 m/s (d) 5 m/s

87. Consider a wheel rotating around a fixed axis. If the rotation angle θ varies with time as $\theta = at^2$, then the total acceleration of a point A on the rim of the wheel is (v being the tangential velocity)

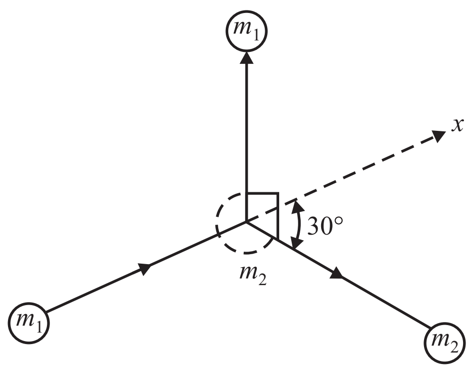
(a) $\frac{v}{t}\sqrt{1 + 4a^2t^4}$ (b) $\frac{v}{t}$

(c) $\frac{v}{t}(1 + 4a^2t^4)$ (d) $\sqrt{(1 + 4a^2t^4)}$

88. A block of mass 4 m traveling at a velocity v_1 in x-direction on a frictionless horizontal plane make a head-on collision with another block of mass 2 m travelling in opposite direction with a velocity v_2. After collision, both the blocks travel as a single block along x-direction with a final velocity $5v_2$. The ratio of velocity $\frac{v_1}{v_2}$ is

(a) 2 (b) 3 (c) 5 (d) 8

89. A particles of mass m_1 moving along the X-axis collides with a stationary particle of mass m_2 and deviates by an angle 30° to the X-axis as shown in the figure. If the percentage change in kinetic energy of the combined system of there two particles reduces by 50%, then the ratio of the masses $\frac{m_2}{m_1}$ is



(a) 8 (b) 6 (c) $\frac{8}{7}$ (d) $\frac{1}{6}$

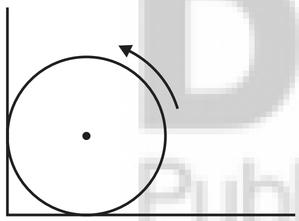
90. Collision takes place between two solid spheres denoted as 1 and 2. The initial velocities of the sphere are u_1 = 3 m/s and u_2 = 1.5 m/s and the final velocities are v_1 = 2.5 m/s and v_2 = 3.5 m/s. The coefficient of restitution between the materials of the sphere is nearly

(a) 0.67 (b) 0.78 (c) 0.83 (d) 0.96

91. A 30 kg boy stands at the far edge of a floating plank, whose near edge is against the shore of a river. The plank is 10 m long and weighs 10 kg. If the boy walks to the near edge of the plank, how far from the shore does the plank move

(a) 7 m (b) 8 m (c) 7.5 m (d) 15 m

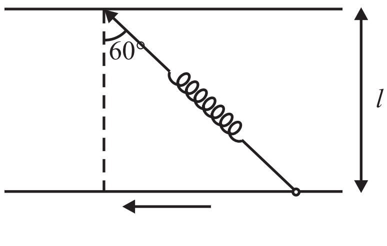
92. A uniform cylinder of radius 1 m, mass 1 kg spins about its axis with an angular velocity 20 rad/s. At certain moment, the cylinder is placed into a corner as shown in the figure. The coefficient of friction between the horizontal wall and the cylinder is μ, whereas the vertical wall is frictionless. If the number of rounds made by the cylinder is 5 before it stops, then the value of μ is (acceleration due to gravity, $g = 10\ \text{m/s}^2$)



(a) $\frac{3}{\pi}$ (b) $\frac{2}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{0.4}{\pi}$

93. A spring has a natural length l with one end fixed to the ceiling. The other end is fitted with a smooth ring which can slide on a horizontal rod fixed at distance l below the ceiling.

Initially, the spring makes an angle of 60° with the vertical, when system is released from test. Find the angle of the spring with the vertical, when, the velocity of the ring reaches half of the maximum velocity, which the ring can attain during the motion.



(a) 30° (b) $\cos^{-1}\left(\frac{2}{2+\sqrt{3}}\right)$

(c) $\cos^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$ (d) 45°

94. From the pole of the earth, a body of mass m is imparted a velocity v_0 directed vertically up. If M is the mass of the earth, R its radius and g is the free-fall acceleration on its surface, then the height h to which the body will ascent is (neglect air resistance)

(a) $\frac{Rv_0^2}{(2gR - v_0^2)}$ (b) $\frac{Rv_0^2}{2gR}$

(c) R (d) $\frac{Rv_0^2}{(2gR + v_0^2)}$

95. Young's modulus experiment is performed on a steel wire of 1 m length and 8 min diameter. The mass required to be added in the experiment to produce 5 min elongation of the wire is ($Y_{\text{steel}} = 2 \times 10^9\ \text{Nm}^{-2}$, $g = 10\ \text{m/s}^2$)

(a) 25 kg (b) 50 kg (c) 250 kg (d) 500 kg

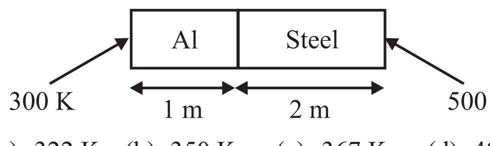
96. What is the rate at which a trapped bubble of 2 mm diameter rises slightly through a solution of density 13.6 × 10^3 kg/m^3 and coefficient of viscosity 1.5 centipoise? Assume, the density of air is negligibility and $g = 10\ \text{m/s}^2$.

(a) 20 m/s (b) 2 m/s (c) 0.2 m/s (d) 0.02 m/s

97. An electric heater with constant heat supply rate is used to convert a certain amount of liquid ammonia to saturated vapour at high pressure. The heater takes 14 minutes to bring the liquid at 15°C to the boiling point of 50 °C and 92 minutes to convert the liquid at the boiling point wholly to vapour. If the specific heat capacity of liquid ammonia is 4.9 kJ/kg K, the latent heat of vaporisation of ammonia in kJ/kg is

(a) 557 (b) 981 (c) 1127 (d) 2250

98. An aluminum rod of length 1 m and a steel rod of length 2 m both having same cross-sectional area, are soldered together end-to-end. The thermal conductivity of aluminum rod and steel rod is 200 $\text{Js}^{-1}\ \text{m}^{-1}\ \text{K}^{-1}$ and 50 $\text{Js}^{-1}\ \text{m}^{-1}\ \text{K}^{-1}$ respectively. The temperatures of the free ends are maintained at 300 K and 500 K. What is the temperature of the junction?



(a) 322 K (b) 350 K (c) 367 K (d) 400 K

99. One mole of the ideal gas goes through the process $p = p_0\left[1-\alpha\left(\frac{V}{V_0}\right)^3\right]$, where p and V are pressure and volume, p_0, V_0 and α are constants. If the maximum attainable temperature of the gas is $\left(\frac{3}{4}\right)\frac{p_0V_0}{R}$, then the value of α is

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 4

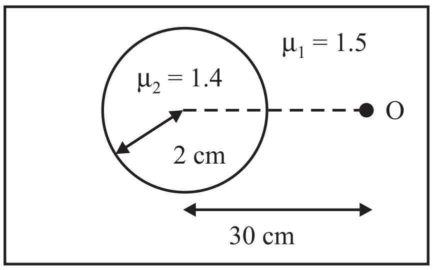
100. A gas mixture contains n_1 moles of a monoatomic gas and n_2 moles of gas of rigid diatomic molecules. Each molecule in monoatomic and diatomic gas has 3 and 5 degree of freedom respectively. If the adiabatic exponent $\left(\frac{C_p}{C_v}\right)$ for this gas mixture is 1.5, then the ratio $\frac{n_1}{n_2}$ will be

(a) 1 (b) 1.5 (c) 2 (d) 2.5

101. A wire of length 50 cm and weighing 10 gm is attached to a spring at one end and to a fixed wall at the other end. The spring has a spring constant of 50 N/m and is stretched by 1 cm. If a wave pulse is produced on the string near the wall, then how much time will it take to reach the spring?

(a) 0.1 s (b) 0.2 s (c) 0.3 s (d) 0.4 s

102. Consider a point object situated at a distance of 30 cm from the centre of sphere of radius 2 cm and refractive index 1.5 as shown in the figure. If the refractive index of the region surrounding this sphere is 1.4, then the position of the image due to refraction by sphere with respect to the centre is



(a) 30 cm (b) 45 cm (c) ∞ (d) 28 cm

103. At what distance from a biconvex lens of the focal length F, must be placed an object for the distance between the object and its real image to be minimal?

(a) $2F$ (b) F (c) $\frac{F}{2}$ (d) $4F$

104. In an experiment, light passing through two slits separated by a distance of 0.3 mm is projected on to a screen placed at 1 m from the plane of slits. It is observed that the distance between the central fringe and the adjacent bright fringe is 1.9 mm. The wavelength of light in mm is

(a) 450 (b) 495 (c) 530 (d) 570

105. A solid sphere of radius r_1 = 1 cm carries charge distributed uniformly over it with density $\rho_1 = -3$ C/cm^3. It is surrounded by a concentric spherical shell of radius r_2 = 2 cm carrying uniform charge density $\rho_2 = \frac{1}{2}$ C/cm^2.

If E_d denotes the magnitude of the electric field at distance d from the common centre of the sphere, then

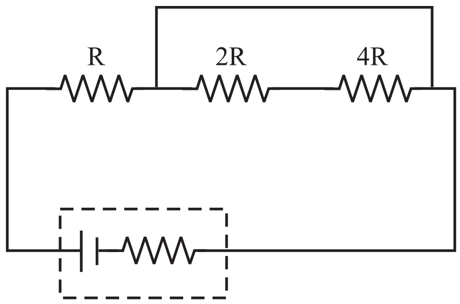
(a) $E_d = \frac{1}{3\varepsilon_0 d^2}$, $d \le 1$ cm (b) $E_d = \frac{1}{\varepsilon_0 d^2}$, $d \le 1$ cm

(c) $E_d = \frac{d}{3\varepsilon_0}$ $d \le 1$ cm (d) $E_d = \frac{d}{\varepsilon_0}$, $d \le 1$ cm

106. Two isolated concentric, conducting spherical shells have radii R and $2R$ and uniform charges q and $2q$ respectively. If V_1 and V_2 are potentials at points located at distance $3R$ and $\frac{R}{2}$, respectively, from the centre of shells. Then the ratio of $\left(\frac{V_2}{V_1}\right)$ will be

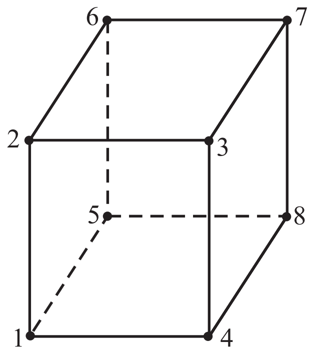
(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) 0

107. A battery with internal resistance of 4 Ω is connected to a circuit consisting three resistances, R, $2R$ and $4R$ (see following figure). If the power generated in the circuit is highest, then the magnitude of R must be



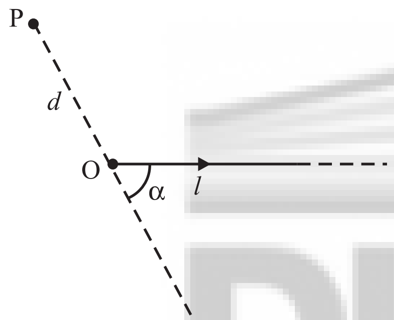
(a) 4 Ω (b) 7 Ω (c) 10 Ω (d) 14 Ω

108. If the resistance of each edge of a cube shaped wire frame as shown in figure below is R, then the resistance between points 1 and 7 is



(a) $\frac{5R}{6}$ (b) $\frac{R}{6}$ (c) $5R$ (d) $\frac{6}{5}R$

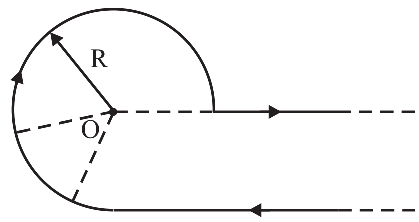
109. A steady current l flows through a wire with one end at O and the other and extending upto infinity as shwon in the figure. The magnetic field at a point P, located at a distance d from O is



(a) $\frac{\mu_0 l}{4\pi d \cos\alpha}(1-\sin\alpha)$ (b) $\frac{\mu_0 l}{2\pi d \cos\alpha}(1-\sin\alpha)$

(c) $\frac{\mu_0 l}{4\pi d}$ (d) $\frac{\mu_0 l}{4\pi d \sin\alpha}(1-\cos\alpha)$

110. The magnetic induction at point O of the given infinitely long current carrying wire shown in the figure below is



(a) $\frac{\mu_0 l}{4\pi R}\left(1-\frac{3\pi}{2}\right)$ (b) $\frac{\mu_0 l}{2R(1+\pi)}$

(c) $\frac{\mu_0 l}{4\pi R}\left[1+\frac{3\pi}{2}\right]$ (d) $\frac{\mu_0 l}{4\pi R}$

111. At a location, the horizontal components of the earth's magnetic field is 0.3 G in the magnetic meridian and the dip angle is 60°. The earth's magnetic field at this location in G is

(a) 0.3 (b) 0.6 (c) 0.9 (d) 1.2

112. A rectangular loop of wire is placed in the XY-plane with its side of length 3 cm parallel to the X-axis and the side of length 4 cm parallel to the Y-axis. It is moving in the positive X-direction with the speed 10 cm/s. A magnetic field exists in the space with its direction parallel to the Z-axis. The field decreases by 2×10^{-3} T/cm along the positive X-axis and increases in time by 2×10^{-2} T/s. The induced emf in the wire is

(a) -4.8×10^{-5} V (b) 4.8×10^{-5} V

(c) 0 (d) 3.6×10^{-5} V

113. A coil has inductance of 0.4 H and resistance of 8 Ω. It is connected to an AC source with peak emf 4 V and frequency $\frac{30}{\pi}$ Hz. The average power dissipated in the circuit is

(a) 1 W (b) 0.5 W (c) 0.3 W (d) 0.1 W

114. A laser beam is operating at 100 mW. The amount of energy stored by 90 cm length of this laser beam will be

(a) 2×10^{-10} J (b) 3×10^{-10} J

(c) 8×10^{-11} J (d) 6×10^{-11} J

115. A photon of energy 4 eV imparts all its energy to an electron that leaves a metal surface with 1.1 eV of kinetic energy. The work function of the metal is

(a) 2.9 eV (b) 5.1 eV (c) 3.64 eV (d) 4.4 eV

116. Consider an electron revolving in a circular orbit of hydrogen atom, whose quantum number is n = 2. The velocity of the electron in that orbit is

(a) 1.1×10^6 m/s (b) 2.2×10^7 m/s

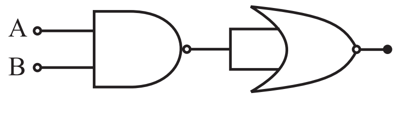
(c) 4.4×10^6 m/s (d) 2.2×10^5 m/s

117. The half-life of ${}^{209}_{84}P_0$ is 103 years. The time it takes for 100 g sample of ${}^{209}_{84}P_0$ to decay to 3.125 g is

(a) 3296 years (b) $103\sqrt{2}$ years

(c) 1648 years (d) 515 years

118. The logic operation performed by the following circuit is



(a) NOR (b) AND (c) NAND (d) OR

119. Which of the following statements is true?

(a) A solid is insulator or samiconductor, it its conduction band is partially filled.

(b) A solid is necessarily an insulator, if its conduction band is empty.

(c) A solid is necessarily a semiconductor, if its conduction band is empty.

(d) A solid is a conductor, it its conduction band is partially filled.

120. A transmitting and receiving antenna have heigh of d metres each. The maximum distance between them for satisfactory communication in Line-of-Sight mode (LOS) is $2d$ kilometers. If the radius of earth is 6400 km, then the value of d is

(a) 3.2 m (b) 6.4 m
(c) 12.8 m (d) 16.0 m

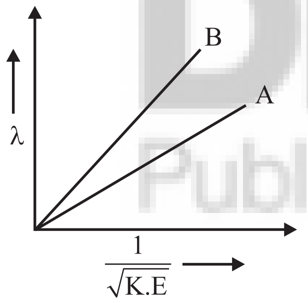
CHEMISTRY

121. In a photoelectric effect experiment the kinetic energy of an emitted electron is 1.986×10^{-19} J, when a radiation of frequency 1.0×10^{15} s^{-1} hits the metal. What is the threshold frequency of the metal (in s^{-1})?

(Plank's constant = 6.62×10^{-34} J s)

(a) 7.0×10^{14} (b) 5.8886×10^{14}
(c) 7.0×10^{-15} (d) 7.0×10^{15}

122. The following plot represent the de-Broglie wavelength as a function of the kinetic energy (K.E.) of two particles A and B. Identify the correct relation.



(a) $m_A = m_B$ (b) $m_A < m_B$
(c) $m_A > m_B$ (d) $m_A = m_B = 0$

123. The correct option for the first ionisation enthalpy (in kJ mol^{-1}) of Li, Na, K and Cs respectively is

(a) 496, 520, 419, 374 (b) 374, 419, 496, 520
(c) 520, 496, 419, 374 (d) 374, 419, 520, 496

124. Which of the following statements about BF_4^- and AlF_6^{3-} are correct?

(i) B and Al differ in their oxidation states
(ii) B and Al differ in their covalency
(iii) B obeys the octet rule.
(iv) B and Al are in diagonal relationship.

(a) (i), (ii) (b) (ii), (iii), (iv)
(c) (i), (ii), (iii) (d) (ii), (iii)

125. **Statement (I) :** CO_2 has no dipole moment, whereas SO_2 and H_2O have dipole moment,

Statement (II) : Which of the following is correct ? $SnCl_2$ is ionic, whereas $SnCl_4$ is covalent.

(a) Both (I) and (II) are not correct
(b) (I) is correct but (II) is not correct
(c) Both (I) and (II) are correct
(d) (I) is correct but (II) is correct

126. **Assertion :** Xe atoms in XeF_2 are d^2sp^3 hybridised.

Reason : XeF_2 molecule does not follow octet rule.

Which of the following is correct?

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A)
(c) (A) is true, but (R) is false
(d) (A) is false but (R) is true

127. 12 cm^3 of $SO_2(g)$ diffused through a porous membrane in 1 minute. Under similar conditions 120 cm^3 of another gas diffused in 5 minutes. The molar mass of the gas in g mol^{-1} is

(a) 32 (b) 18
(c) 44 (d) 16

128. 1 mole of gas A and 1 mole of gas B at 27°C were pumped into a 24.6 L. Volume pre-evacuated isolated flask. The catalyst coated inside the flask catalyses the following reaction $A(g) + B(g) \longrightarrow 2D(g)$. The kinetic energy of D is 98.03 L atm. Calculate the pressure realised at the end of the reaction.

(a) 1.66 atm (b) 2.66 atm
(c) 5.33 atm (d) 4.33 atm

129. 28 g KOH is required to completely neutralise CO_2 produced on heating 60 g of impure $CaCO_3$. The percentage purity of $CaCO_3$ is approximately (molar masses of KOH and $CaCO_3$ are 56 and 100 g mol^{-1}, respectively)

(a) 41.6 (b) 40
(c) 20.8 (d) 83.3

130. Which one of following is a disproportionation reaction?

(a) $2AgNO_3(aq) + Cu(s) \longrightarrow Cu(NO_3)_2(aq) + 2Ag(s)$
(b) $3AgNO_3(aq) + K_3PO_4(aq) \longrightarrow Ag_3PO_4(s) + 3KNO_3(aq)$
(c) $4KClO_3(s) \xrightarrow{\Delta} KCl(s) + 3KClO_4(s)$
(d) $4Fe(s) + 3O_2(g) \longrightarrow 2Fe_2O_3$

131. The standard enthalpy of formation of $CO(g)$, $CO_2(g)$, $N_2O(g)$ and $N_2O_4(g)$ are respectively – 10, – 393, 81 and – 10 kJ mol^{-1}. Enthalpy change (in kJ) of the following reaction is

$N_2O_4(g) + 3CO\ (g) \longrightarrow N_2O(g) + 3CO_2(g)$

(a) – 1058 (b) + 1058 (c) – 957 (d) + 957

132. Consider the following reaction in a 1 L closed vessel.

$N_2 + 3H_2 \rightleftharpoons 2NH_3$

If all the species; N_2, H_2 and NH_3 are in 1 mol in the beginning of the reaction and equilibrium is attained after unreacted N_2 is 0.7 mol. What is the value of equilibrium constant?

(a) 3600.00 (b) 3657.14

(c) 2657.14 (d) 1828.57

133. If the solubility product of $Ni(OH)_2$ is 4.0×10^{-15} the solubility (in mol L^{-1}) is

(a) 5.0×10^{-5} (b) 4.0×10^{-5}

(c) 2.0×10^{-5} (d) 1.0×10^{-5}

134. Identify the reactions in which H_2 is liberated?

(i) $Zn + NaOH(aq) \longrightarrow$

(ii) $HCOOH \xrightarrow[\text{Conc. } H_2SO_4]{373\text{ K}}$

(iii) $CH_4(g) + H_2O(g) \xrightarrow[\text{Ni}]{127_0\text{ K}}$

(iv) $Zn + H^+ (aq) \longrightarrow$

(v) $C(s) + H_2O\ (g) \xrightarrow{1270\text{ K}}$

(a) (i), (iii), (iv), (v) (b) (i), (ii), (iii), (iv)

(c) (ii), (iii), (iv), (v) (d) (i), (ii), (iii), (v)

135. BeH_2 can be prepared by the reaction of

(a) $BeCl_2$ with $LiAlH_4$ (b) Be with H_2

(c) Be with water (d) Be with liquid ammonia

136. $AlCl_3$ in water at pH < 7 forms

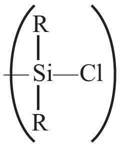
(a) tetrahedral $Al(OH)_4^-$ ions

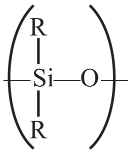
(b) octahedral $Al(OH)_6^{3-}$ ions

(c) square planar $Al(OH)_4^-$ ions

(d) octahedral $Al(OH_2)_4^{3+}$ ions

137. Which of the following is known as silicone?

(a) Polymer of 

(b) Polymer of 

(c) Polymer of SiO_2

(d) Polymer of $[SiO_4]^{4-}$

138. Identify the reactions that occur in photochemical smog.

(i) $CH_2 = O + H_2 \longrightarrow CH_3OH$

(ii) $NO_2(g) \xrightarrow{h\nu} NO(g) + O(g)$;

$O(g) + O_2(g) \longrightarrow O_3(g)$

(iii) $3CH_4 + 2O_3 \longrightarrow 3CH_2 = O + 3H_2O$

(iv) $NO(g) + O_3(g) \longrightarrow NO_2(g) + O_2(g)$

(a) (ii), (iii) (iv) (b) (i), (ii), (iii)

(c) (i), (ii), (iv) (d) (i), (iii), (iv)

139. IUPAC name of isoprene is

(a) 1, 3-butadiene

(b) 2, 3-dimethylbutadiene

(c) 2-methyl-1, 3-butadiene

(d) 1, 3-dimethybutadiene

140. Identify the correct catalyst and reaction conditions for the controlled oxidation of methane to (i) methanol (X), (ii) methanal (Y) and ethane to (iii) ethanoic acid (Z)

	(X)	(Y)	(Z)
(a)	Mo_2O_3/Δ	$(CH_3COO)_2$ Mn/Δ	Cu/523K/100 atm
	Cu/523K/100 atm		
(b)	Cu/523K/100 atm	Mo_2O_3/Δ	$(CH_3COO)_2$ Mn/Δ
(c)	$(CH_3COO)_2Mn/\Delta$		Mo_2O_3/Δ
(d)	Mo_2O_3/Δ	Cu/523K/ 100 atm	$(CH_3COO)_2Mn/\Delta$

141. The major product (P) formed in the below reaction is

$H — C{\equiv}C—CH_2 — CH{=}CH_2 \xrightarrow[CCl_4,\ 253K]{Br_2\ (1\text{ mol})} P$

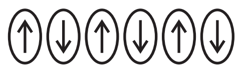
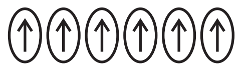
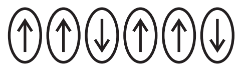
(a) $H — C{\equiv}C—CH_2— CH(Br) — CH_2(Br)$

(b) $CH(Br)_2 — C(Br)_2 — CH_2 — CH{=}CH_2$

(c) $CH_2(Br) — CH(Br) — CH_2 — CH{=}CH_2$

(d) $CH_2{=}CH(Br) — CH(Br) — CH{=}CH$

142. Match the following:

	List-I		List-II
A		(i)	Ferromagnetism
B		(ii)	Antiferromagnetism
C		(iii)	Ferrimagnetism

The correct answer is:

	A	B	C		A	B	C
(a)	(i)	(iii)	(ii)	(b)	(ii)	(i)	(ii)
(c)	(ii)	(i)	(iii)	(d)	(i)	(ii)	(iii)

143. How many grams of glucose must be added to 0.5 litre of a solution so that its osmotic pressure is same as that of a solution of 9.2 g of glucose dissolved in a litre?

(a) 1.15 (b) 9.22 (c) 2.31 (d) 4.60

144. Molarity of a 50 mL H_2SO_4 solution is 10.0 M. If the density of the solution is 1.4 g/cc, calculate its molarity

(a) 7.14 (b) 8.00 (c) 10.0 (d) 0.500

145. Limiting molar conductivity of Mg^{2+} and Cl^- ions in water is 106.0 and 76.3 S cm^2 mol^{-1}. The limiting molar conductivity of magnesium chloride (in S cm^2 mol^{-1}) in water is

(a) 182.3 (b) 258.6 (c) 288.3 (d) 364.6

146. A particular reaction has a rate constant 1.15×10^{-3} s^{-1}. How long does it take for 6 g of the reactant of reduce to 3 g? (log 2 = 0.301)

(a) 301 s (b) 603 s (c) 840 s (d) 15 s

147. If the value of $\frac{1}{n}$ is equal to 1 in Freundlich adsorption isotherm, then $\frac{x}{m}$ = (x = mass of absorbate, m = mass of the absorbent, p = pressure of the gas)

(a) $\frac{K}{p}$ (b) Kp (c) K (d) 0

148. What is the slag formed during the extraction of iron?

(a) MgO (b) $FeSiO_3$ (c) $CaSiO_3$ (d) $MgSiO_3$

149. What is the chemical formula of hypophosphorus acid?

(a) H_3PO_3 (b) H_3PO_2 (c) H_3PO_4 (d) $H_4P_2O_6$

150. Which of the following ions possesses S—O—S bond?

(a) $S_2O_3^{2-}$ (b) SO_4^{2-} (c) $S_2O_8^{2-}$ (d) $S_2O_7^{2-}$

151. Which of the elements posses only one electron in 5d-orbital?

(a) ^{69}Tm, ^{61}Pm (b) ^{59}Pr, ^{71}Lu

(c) ^{57}La, ^{61}Pm (d) ^{57}La, ^{71}Lu

152. The electronic configuration of Cr in $Cr(CO)_6$ as calculated using crystal field theory is

(a) $t_{2g}^4 e_g^0$ (b) $t_{2g}^3 e_g^1$ (c) $t_{2g}^6 e_g^0$ (d) $t_{2g}^4 e_g^2$

153. Match the following:

	List-I		List-II
A.	Natural rubber	(i)	β-glucose
B.	Cellulose	(ii)	Isoprene
C.	Nylon-6	(iii)	Tetrafluoroethylene
D.	Teflon	(iv)	Caprolactam
		(v)	Hexarnethylenediamine, adipic acid

The correct answer is

	A	B	C	D		A	B	C	D
(a)	(ii)	(i)	(iv)	(iii)	(b)	(ii)	(iv)	(i)	(iii)
(c)	(iv)	(i)	(ii)	(iii)	(d)	(ii)	(iii)	(iv)	(v)

154. Identify the non-reducing sugar from the following:

(a) Maltose (b) Sucrose (c) Lactose (d) Glucose

155. Identify the correct set of functional groups present in aspartame, an artificial sweetener.

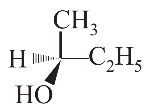
(a) $-COOCH_3$ $-NH_2$ $-\overset{O}{\overset{\|}{C}}-NH-$ $-\overset{O}{\overset{\|}{C}}-OC_2H_5$

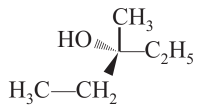
(b) $-COOH$ $-NH_2$ $-\overset{O}{\overset{\|}{C}}-NH-$ $-\overset{O}{\overset{\|}{C}}-OCH_3$

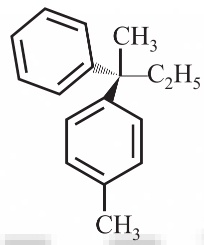
(c) $-CONH_2$ $-NH-$ $-CO-$ $-COOH$

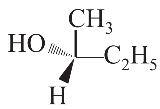
(d) $-CHO$ $-CN$ $-CH$ $-COOCH_3$

156. Which of the following molecules is not chiral?

(a) 

(b) 

(c) 

(d) 

157. The major product obtained in the reaction of bromobenzene with Mg in dry ether followed by the reaction with benzonitrile and hydrolysis is

(a) acetophenone (b) benzophenone

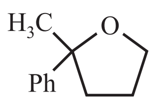
(c) phenyl benzoate (d) benzoic acid

158. The major is

$$CH_3-\overset{O}{\overset{\|}{C}}-CH_2-CH_2-CH_2-(Br) \xrightarrow[(C_2H_5)_2O]{PhMgBr} [P] \xrightarrow{-MgBr_2} Z$$

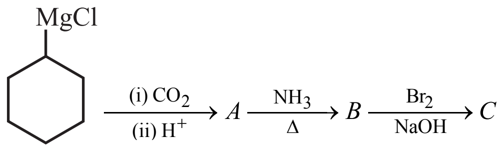
(a) $CH_3-\underset{Ph}{\underset{|}{\overset{OH}{\overset{|}{C}}}}-CH_2-CH_2-CH_3$

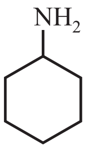
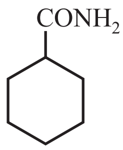
(b) $CH_3-\underset{Ph}{\underset{|}{\overset{OH}{\overset{|}{C}}}}-CH_2-CH_2-CH_2Br$

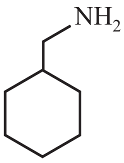
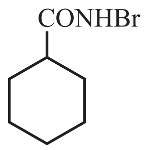
(c) 

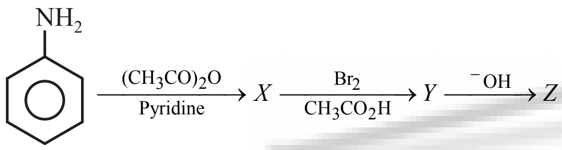
(d) $CH_3-\overset{O}{\overset{\|}{C}}-CH_2-CH_2-CH_2-Ph$

159. In the below given synthetic sequence, the product "*C*" is

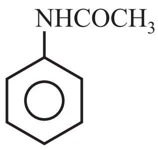
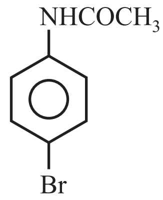
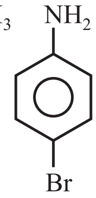
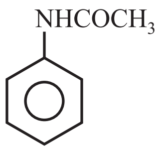
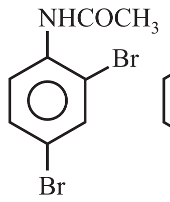
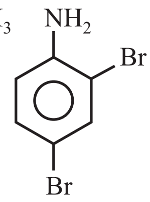
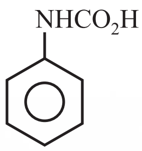
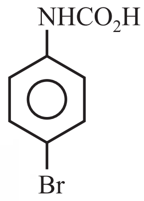
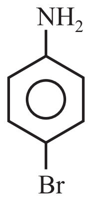
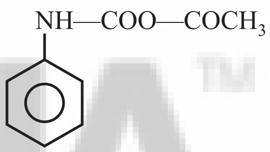
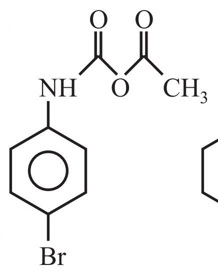
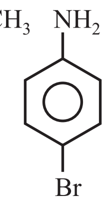


(a)  (b) 

(c)  (d) 

160. 

What are structures of *X*, *Y* and *Z* in the above given reaction sequence?

	X	Y	Z
(a)			
(b)			
(c)			
(d)			

ANSWER KEY

1	(b)	**2**	(a)	**3**	(d)	**4**	(d)	**5**	(a)	**6**	(b)	**7**	(b)	**8**	(b)	**9**	(b)	**10**	(d)
11	(a)	**12**	(b)	**13**	(d)	**14**	(d)	**15**	(d)	**16**	(a)	**17**	(c)	**18**	(d)	**19**	(c)	**20**	(c)
21	(b)	**22**	(a)	**23**	(c)	**24**	(b)	**25**	(b)	**26**	(a)	**27**	(d)	**28**	(d)	**29**	(c)	**30**	(b)
31	(a)	**32**	(d)	**33**	(a)	**34**	(b)	**35**	(c)	**36**	(Bonus)	**37**	(b)	**38**	(d)	**39**	(a)	**40**	(c)
41	(b)	**42**	(c)	**43**	(c)	**44**	(b)	**45**	(b)	**46**	(c)	**47**	(c)	**48**	(d)	**49**	(a)	**50**	(c)
51	(d)	**52**	(b)	**53**	(a)	**54**	(b)	**55**	(d)	**56**	(c)	**57**	(b)	**58**	(a)	**59**	(b)	**60**	(d)
61	(a)	**62**	(d)	**63**	(a)	**64**	(d)	**65**	(c)	**66**	(c)	**67**	(a)	**68**	(c)	**69**	(b)	**70**	(d)
71	(d)	**72**	(a)	**73**	(c)	**74**	(a)	**75**	(a)	**76**	(b)	**77**	(a)	**78**	(b)	**79**	(c)	**80**	(d)
81	(b)	**82**	(c)	**83**	(a)	**84**	(d)	**85**	(b)	**86**	(d)	**87**	(a)	**88**	(d)	**89**	(a)	**90**	(a)
91	(c)	**92**	(c)	**93**	(*)	**94**	(a)	**95**	(b)	**96**	(a)	**97**	(c)	**98**	(a)	**99**	(c)	**100**	(a)
101	(a)	**102**	(a)	**103**	(a)	**104**	(d)	**105**	(d)	**106**	(a)	**107**	(a)	**108**	(a)	**109**	(d)	**110**	(c)
111	(b)	**112**	(c)	**113**	(d)	**114**	(b)	**115**	(a)	**116**	(a)	**117**	(d)	**118**	(b)	**119**	(b)	**120**	(c)
121	(a)	**122**	(c)	**123**	(c)	**124**	(d)	**125**	(c)	**126**	(d)	**127**	(d)	**128**	(b)	**129**	(a)	**130**	(c)
131	(a)	**132**	(b)	**133**	(d)	**134**	(a)	**135**	(a)	**136**	(d)	**137**	(b)	**138**	(a)	**139**	(c)	**140**	(b)
141	(a)	**142**	(c)	**143**	(d)	**144**	(N)	**145**	(b)	**146**	(b)	**147**	(b)	**148**	(c)	**149**	(b)	**150**	(d)
151	(d)	**152**	(c)	**153**	(a)	**154**	(b)	**155**	(b)	**156**	(b)	**157**	(b)	**158**	(c)	**159**	(a)	**160**	(a)

Hints & Solutions

MATHEMATICS

1. **(b)** Given,

$$X=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix}: a, b, c, d \in R\right\}$$

$f(A) = \det(A) \Rightarrow \quad f(A) = \det(X)$

$$= \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

For any real values of a, b, c and d. Matrix formation is possible, therefore we get the pre-image of every real number. So this function is onto.
But for the different values of a, b, c and d we get same results so function is not one-one.
Hence, f is onto but not one-one.

2. **(a)**

3. **(d)** Given expression is
$25^k + 12k - 1$, for all positive integer k.
Put $k = 1$, $25^1 + 12(1) - 1 = 36$
Put $k = 2$, $25^2 + 12(2) - 1 = 624 + 24 - 1 = 648$
Since, HCF of 36 and 648 is 36.
So, greatest divisor is 36.
$\therefore \quad d = 36$
Now, $4\sqrt{d} = 4\sqrt{36} = 4 \times 6 = 24$

4. **(d)** Given that,

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow \quad A'=\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$\text{Now, } A \,.\, A' = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 4+10+18 & 7+16+27 \\ 4+10+18 & 16+25+36 & 28+40+54 \\ 7+16+27 & 28+40+54 & 49+64+81 \end{bmatrix}$$

$$\text{So, } AA' = \begin{bmatrix} 14 & 32 & 50 \\ 32 & 77 & 122 \\ 50 & 122 & 194 \end{bmatrix}$$

$$\Rightarrow \quad (AA)' = \begin{bmatrix} 14 & 32 & 50 \\ 32 & 77 & 122 \\ 50 & 122 & 194 \end{bmatrix}$$

5. **(a)**

6. **(b)** Given system of equations are:
$x + y + z = 4$, $x - y + z = 2$ and $x + 2y + 2z = 1$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = 1(-2-2) - 1(2-1) + (2+1)$$

$$= -4 - 1 + 3 = -2$$

$$\Delta_1 = \begin{vmatrix} 4 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = 4(-2-2) - 1(4-1) + 1(4+1)$$

$$= 4(-4) - 3 + 5 = -16 - 3 + 5 = -14$$

$$\Delta_2 = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 1(4-1) - 4(2-1) + 1(1-2)$$

$$= 3 - 4 - 1 = -2$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 4 \\ 1 & -1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 1(-1-4) - 1(1-2) + 4(2+1)$$

$$= (-5) + 1 + 12 = 8$$

$$\therefore x = \frac{\Delta_1}{\Delta} = \frac{-14}{-2} = 7,$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-2}{-2} = 1 \text{ and } z = \frac{\Delta_3}{\Delta} = \frac{8}{-2} = -4$$

So, $a = 7$, $b = 1$, $c = -4$
Now, $ab + bc + ca$
$= 7(1) + 1(-4) + (-4)(7) = 7 - 4 - 28 = -25$

7. **(b)**

8. **(b)** Let $z = x + iy$
Given that vertices of triangle are 0, $z = (x + iy)$ and
$ze^{i\alpha} = (x + iy)(\cos\alpha + i\sin\alpha)$
$= (x\cos\alpha - y\sin\alpha) + i(y\cos\alpha + x\sin\alpha)$
$\therefore$ Area of triangle

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ x\cos\alpha - y\sin\alpha & y\cos\alpha + x\sin\alpha & 1 \end{vmatrix}$$

$$= \frac{1}{2}[(xy\cos\alpha + x^2\sin\alpha) - (xy\cos\alpha - y^2\sin\alpha)]$$

$$= \frac{1}{2}[xy\cos\alpha + x^2\sin\alpha - xy\cos\alpha + y^2\sin\alpha]$$

$$= \frac{1}{2}(x^2 + y^2)\sin\alpha$$

$$= \frac{1}{2}(z)^2\sin\alpha \qquad [\because \ |z| = \sqrt{x^2 + y^2}]$$

9. **(b)**

10. **(d)** Given that, $\omega_0, \omega_1, \omega_2, \ldots, \omega_{n-1}$ are nth root of unity, so
$x^n - 1 = (x - \omega_0)(x - \omega_1)(x - \omega_2) \ldots (x - \omega_{n-1}) \ldots$(i)
put $x = \dfrac{-1}{2}$ in eqn. (i), we get

$$\left(\frac{-1}{2}\right)^n - 1 = \left(\frac{-1}{2} - \omega_0\right)\left(\frac{-1}{2} - \omega_1\right)\cdots\left(\frac{-1}{2} - \omega_{n-1}\right)$$

$\Rightarrow \left(\frac{-1}{2}\right)^n - 1$

$= \left(\frac{-1}{2}\right)^n (1+2\omega_0)(1+2\omega_1)\ldots(1+2\omega_{n-1})$

$\Rightarrow 1 - (-1)^n (2)^n = (1 + 2\omega_0)(1 + 2\omega_1) \ldots (1 + 2\omega_{n-1})$
$\Rightarrow 1 + (-1)^{n-1} 2^n = (1 + 2\omega_0)(1 + 2\omega_1) \ldots (1 + 2\omega_{n-1})$

11. **(a)** Given quadratic equation is,
$(x - 2)(x - 3) = k^2, k \in R$
$\Rightarrow x^2 - 5x + 6 - k^2 = 0$
Now, discriminant
$D = (-5)^2 - 4(1)(6 - k^2)$ $[\because D = b^2 - 4ac]$
$= 25 - 24 + 4k^2 = 1 + 4k^2 > 0$
Hence, the roots are real and distinct for $k \in R$.

12. **(b)** Given that,
$x^2 - 3ax + 14 = 0$
and $x^2 + 2ax - 16 = 0$ have a common root.
Let the common roots is α.
Then, $\alpha^2 - 3a\alpha + 14 = 0$
and $\alpha^2 + 2a\alpha - 16 = 0$
by cross multiplication method

$$\frac{\alpha^2}{48a - 28a} = \frac{\alpha}{14+16} = \frac{1}{2a+3a}$$

$$\Rightarrow \frac{\alpha^2}{20a} = \frac{\alpha}{30} = \frac{1}{5a}$$

So, $\frac{\alpha^2}{20a} = \frac{\alpha}{30}$ and $\frac{\alpha}{30} = \frac{1}{5a}$

$\Rightarrow \alpha = \frac{20a}{30}$

and $\alpha = \frac{30}{5a}$

$\Rightarrow \alpha = \frac{2}{3}a$ and $\alpha = \frac{6}{a}$

So, $\frac{2a}{3} = \frac{6}{a}$

$\Rightarrow 2a^2 = 18 \Rightarrow a^2 = 9 \quad \Rightarrow \quad a = \pm 3$
Now, $a^4 + a^2 = (\pm 3)^4 + (\pm 3)^2 = 81 + 9 = 90$

13. **(d)** Given that $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ are roots of given equation $x^n + px + q = 0$
So, $x^n + px + q = (x - \alpha_1)(x - \alpha_2)(x - \alpha_3) \ldots (x - \alpha_n)$

$$\Rightarrow \frac{x^n + px + q}{x - \alpha_n} = (x - \alpha_1)(x - \alpha_2)(x - \alpha_3)$$

Taking $\lim\limits_{x \to a_n}$ both side

$$\therefore \lim_{x \to a_n} \frac{x^n + px + q}{x - \alpha_n}$$

$$= \lim_{x \to a_n} (x - \alpha_1)(x - \alpha_2)(x - \alpha_3)$$

[By L'Hopital Rule]

$$\lim_{x \to a_n} \frac{nx^{n-1} + P}{1} = (\alpha_n - \alpha_1)(\alpha_n - \alpha_2) \ldots (\alpha_n - \alpha_{n-1})$$

$\Rightarrow na_n^{\,n-1} + p = (\alpha_n - \alpha_1)(\alpha_n - \alpha_2) \ldots (\alpha_n - \alpha_{n-1})$

14. **(d)**

15. **(d)** 4 letters in 4 addressed envelopes = 4! ways
Then, the number of ways in which no letters goes into envelope meant for it

$$= 4! - \sum_{k=1}^{4} {}^4C_k$$

$$\left[\because \text{ required number of ways} = n! - \sum_{k=1}^{n} {}^nC_k\right]$$

$= 24 - [{}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4]$
$= 24 - [4 + 6 + 4 + 1]$
$= 24 - 15 = 9$
Hence, required number of ways = 9.

16. **(a)** The number of zeroes in 10! depends on the number of pairs of 5 and 2.
So, the numbers containing factors 5 is
5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95 and 100.
Number of factors containing factor 5 = 20
But in 25, 50, 75, 100 contain 5 twice.
So, total number of factors = 20 + 4 = 24
Hence, the entire product contain 10 is 24 times.

17. **(c)**

18. **(d)** Given that

$$x = \frac{2 \cdot 5}{3 \cdot 6} - \frac{2 \cdot 5 \cdot 8}{3 \cdot 6 \cdot 9}\left(\frac{2}{5}\right) + \frac{2 \cdot 5 \cdot 8 \cdot 11}{3 \cdot 6 \cdot 9 \cdot 12}\left(\frac{2}{5}\right)^2 - \ldots \infty$$

$$= \frac{2 \cdot 5}{3^2 \cdot 2!} - \frac{2 \cdot 5 \cdot 8}{3^3 \cdot 3!}\left(\frac{2}{5}\right) + \frac{2 \cdot 5 \cdot 8 \cdot 11}{3^4 \cdot 4!}\left(\frac{2}{5}\right)^2 - \ldots \infty$$

multiply by $\left(\frac{2}{5}\right)^2$ on both sides,

$$\frac{4}{25}x = \frac{2 \cdot 5}{3^2 \cdot 2!}\left(\frac{2}{5}\right)^2 - \frac{2 \cdot 5 \cdot 8}{3^2 \cdot 2!}\left(\frac{2}{5}\right)^3 + \ldots \infty$$

$$\frac{4}{25}x - \frac{2}{3 \cdot 1!}\left(\frac{2}{5}\right) + 1$$

$$= 1 - \frac{2}{3 \cdot 1!}\left(\frac{2}{5}\right) + \frac{2 \cdot 5}{3^2 \cdot 2!}\left(\frac{2}{5}\right)^2 - \frac{2 \cdot 5 \cdot 8}{3^3 \cdot 3!}\left(\frac{2}{5}\right)^3 + \ldots \infty$$

$$\frac{4x}{25} - \frac{4}{15} + 1 = \left(1 + \frac{2}{5}\right)^{-\frac{2}{3}}$$

$$\Rightarrow \frac{4x}{25} + \frac{11}{15} = \left(\frac{7}{5}\right)^{-\frac{2}{3}} \Rightarrow \frac{12x + 55}{75} = \left(\frac{5}{7}\right)^{\frac{2}{3}}$$

cube both sides,

$$\Rightarrow \frac{(12x + 55)^3}{(75)^3} = \frac{5^2}{7^2} \qquad \Rightarrow 7^2(12x + 55)^3 = 75^3 \cdot 5^2$$

$= 3^3 \cdot 5^6 \cdot 5^2 = 3^3 \cdot 5^8$

19. **(c)** Since,
$x^4 + x^2 + 1 = x^4 + x^2 + x^2 - x^2 + 1$
$= x^4 + 2x^2 + 1 - x^2$
$= (x^2 + 1)^2 - (x)^2 = (x^2 + x + 1)(x^2 - x + 1)$
Now, by partial fraction,

$$\frac{x^3 - 2x^2 + 3x - 4}{x^4 + x^2 + 1} = \frac{Ax + B}{x^2 + x + 1} + \frac{Cx + D}{x^2 - x + 1}$$

$\Rightarrow\ x^3 - 2x^2 + 3x - 4 = (Ax + B)$
$(x^2 - x + 1) + (Cx + D)(x^2 + x + 1)$

$\Rightarrow\ x^3 - 2x^2 - 3x - 4 = (A + C)x^3 + x^2(B - A + C + D)$
$+ x(A - B + C + D) + (B + D)$

Comparing coefficient of all like terms, we get

$A + C = 1$ …(i)
$B - A + C + D = -2$
$A - B + C + D = 3$
$B + D = -4$ …(ii)

Adding Eqs. (i) and (ii), we get

$A + B + C + D = 1 + (-4) = -3$

20. (c)

21. (b) Given that, $\tan\alpha = \frac{1}{7}$

$\Rightarrow \alpha = \tan^{-1}\frac{1}{7}$ and $\sin\beta = \frac{1}{\sqrt{10}}$

$\Rightarrow \tan\beta = \frac{1}{3}$ $\left[\because\ \tan\beta = \frac{\sin\beta}{\sqrt{1-\sin^2\beta}}\right]$

$\Rightarrow \beta = \tan^{-1}\frac{1}{3}$

Now, $\alpha + 2\beta = \tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{1}{3}$

$= \tan^{-1}\frac{1}{7} + \tan^{-1}\left(\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right)$

$\left[\because\ 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$= \tan^{-1}\frac{1}{7} + \tan^{-1}\left(\frac{6}{8}\right) = \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{3}{4}$

$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}}\right) = \tan^{-1}\left(\frac{\frac{25}{28}}{\frac{25}{28}}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

$= 45°$ $[\because\ \tan^{-1}(1) = 45°]$

22. (a) Let $A = \frac{\pi}{7} \Rightarrow \pi = 7A$

$\cos\frac{\pi}{7} - \cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7} = \cos A\cos 2A\cos 4A$

$= \cos A\cos 2A\cos 2^2A$

Now, we know that

$\cos A\cos 2A\cos 2^2A\cos 2^3A \ldots \cos^{n-1}A$

$= \frac{\sin 2^n A}{2^n\sin A}$

$\therefore\ \cos A\cos 2A\cos 2^2A = \frac{\sin 2^3 A}{2^3\sin A}$

$= \frac{\sin 8A}{8\sin A} = \frac{\sin(7A + A)}{8\sin A} = \frac{\sin(\pi + A)}{8\sin A}$ $[\because\ 7A = \pi]$

$= \frac{-\sin A}{8\sin A}$ $[\because\ \sin(\pi + \theta) = -\sin\theta]$

$= -\frac{1}{8}$

23. (c) We have

$\sin\theta - 3\sin 2\theta + \sin 3\theta = \cos\theta - 3\cos 2\theta + \cos 3\theta$

$\Rightarrow (\sin\theta + \sin 3\theta) - 3\sin 2\theta - (\cos\theta + \cos 3\theta) + 3\cos 2\theta = 0$

$\Rightarrow 2\sin\left(\frac{\theta+3\theta}{2}\right)\cos\left(\frac{\theta-3\theta}{2}\right) - 3\sin 2\theta$
$- 2\cos\left(\frac{\theta+3\theta}{2}\right)\cos\left(\frac{\theta-3\theta}{2}\right) + 3\cos 2\theta = 0$

$\Rightarrow 2\sin 2\theta\cos\theta - 3\sin 2\theta - 2\cos 2\theta\cos\theta + 3\cos 2\theta = 0$

$\Rightarrow (2\cos\theta - 3)(\sin 2\theta - \cos 2\theta) = 0$

$\Rightarrow 2\cos\theta - 3 = 0 \quad \Rightarrow \cos\theta = \frac{3}{2}$

which is not possible.

$\therefore\ \sin 2\theta - \cos 2\theta = 0$

$\Rightarrow \sin 2\theta = \cos 2\theta \Rightarrow \tan 2\theta = 1$

$\Rightarrow \tan 2\theta = \tan\frac{\pi}{4}$ $\left[\because\ 0 < \theta < \frac{\pi}{4}\right]$

$\Rightarrow 2\theta = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{8}$

24. (b)

25. (b) Given that

$\sin h\,x = \frac{3}{4} \Rightarrow x = \sin h^{-1}\left(\frac{3}{4}\right)$

$\Rightarrow x = \log\left(\frac{3}{4} + \sqrt{\left(\frac{3}{4}\right)^2 + 1}\right)$

$[\because\ \sin h^{-1} x = \log(x + \sqrt{x^2+1})]$

$= \log\left(\frac{3}{4} + \sqrt{\frac{9}{16}+1}\right) = \log\left(\frac{3}{4} + \sqrt{\frac{25}{16}}\right)$

$= \log\left(\frac{3}{4} + \frac{5}{4}\right) \Rightarrow x = \log 2$

and $\cos h\,y = \frac{5}{3} \Rightarrow y = \cos h^{-1}\left(\frac{5}{3}\right)$

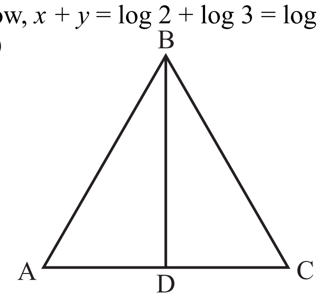
$[\because\ \cos h^{-1} x = \log(x + \sqrt{x^2-1})]$

$= \log\left(\frac{5}{3} + \sqrt{\left(\frac{5}{3}\right)^2 - 1}\right) = \log\left(\frac{5}{3} + \sqrt{\frac{25}{9} - 1}\right)$

$= \log\left(\frac{5}{3} + \frac{4}{3}\right) = \log\left(\frac{9}{3}\right) \Rightarrow y = \log 3$

Now, $x + y = \log 2 + \log 3 = \log 6$

26. (a)



In ΔABC, BD is the median

Given that, $a = 5$, $b = 6$ and $c = 7$

We know that, length of median

$BD = \frac{1}{2}\sqrt{2c^2+2a^2-b^2} = \frac{1}{2}\sqrt{[2(7)^2+2(5)^2-6^2]}$

$= \frac{1}{2}\sqrt{98+50-36} = \frac{1}{2}\sqrt{98+14} \quad \Rightarrow BD = 2\sqrt{7}$

27. **(d)**

28. **(d)** Given that

$\cos A \cos B + \sin A \sin B \sin C = 1$ and $C = \frac{\pi}{2}$

$\Rightarrow \sin C = \sin\frac{\pi}{2} = 1$

$\therefore \cos A \cos B + \sin A \sin B(1) = 1$

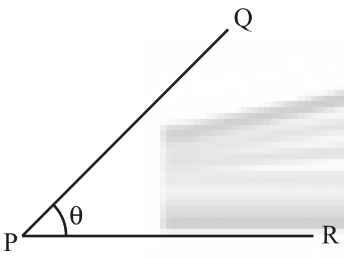
$\Rightarrow \cos(A-B) = 1$

$[\because \cos(A+B) = \cos A \cos B - \sin A \sin B]$

$\Rightarrow \cos(A-B) = \cos 0° \Rightarrow A - B = 0 \Rightarrow A = B \Rightarrow \frac{A}{B} = 1$

Hence, $A : B = 1 : 1$

29. **(c)** $\overrightarrow{PQ} = (b\hat{i}+c\hat{j}+a\hat{k}) - (a\hat{i}+b\hat{j}+c\hat{k})$



$= (b-a)\hat{i}+(c-b)\hat{j}+(a-c)\hat{k}$

Similarly,

$\overrightarrow{PR} = (c-a)\hat{i}+(a-b)\hat{j}+(b-c)\hat{k}$

Now,

$\overrightarrow{PQ}\cdot\overrightarrow{PR} = [(b-a)\hat{i}+(c-b)\hat{j}+(a-c)\hat{k}]$

$\cdot[(c-a)\hat{i}+(a-b)\hat{j}+(b-c)\hat{k}]$

$= (b-a)(c-a)+(c-b)(a-b)+(a-c)(b-c)$

$= a^2+b^2+c^2-ab-bc-ca$

$|\overrightarrow{PQ}| = \sqrt{(b-a)^2+(c-b)^2+(a-c)^2}$

$|\overrightarrow{PR}| = \sqrt{(c-a)^2+(a-b)^2+(b-c)^2}$

Let $\angle QPR = \theta$

$\therefore \cos\theta = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|} = \frac{a^2+b^2+c^2-ab-bc-ca}{(a-b)^2+(b-c)^2+(c-a)^2}$

$= \frac{a^2+b^2+c^2-ab-bc-ca}{2a^2+2b^2+2c^2-2ab-2bc-2ca}$

$= \frac{a^2+b^2+c^2-ab-bc-ca}{2(a^2+b^2+c^2-ab-bc-ca)}$

$\cos\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{3}$

Hence, $\angle QPR = \frac{\pi}{3}$

30. **(b)**

31. **(a)** Given that equation of plane passing through three non-collinear points ***a, b, c*** is

$r = (1-x-y)\boldsymbol{a} + x\boldsymbol{b} + y\boldsymbol{c}$ …(i)

If ***a, b, c, d*** are coplanar, then d should satisfy equation (i) for some x, y

$\boldsymbol{d} = (1-x-y)\boldsymbol{a} + x(\boldsymbol{b}) + y\boldsymbol{c}$ …(ii)

$2\boldsymbol{a} - 3\boldsymbol{b} + 7\boldsymbol{c} = 6\boldsymbol{d}$ (Given)

$\Rightarrow \frac{2\boldsymbol{a}-3\boldsymbol{b}+7\boldsymbol{c}}{6} = \boldsymbol{d}$

By substituting the value of ***d*** in equation (ii), we get

$\frac{2\boldsymbol{a}-3\boldsymbol{b}+7\boldsymbol{c}}{6} = (1-x-y)\boldsymbol{a} + x(\boldsymbol{b}) + y\boldsymbol{c}$

On comparing both sides, we get

$\Rightarrow 1-x-y = \frac{1}{3}$ ...(iii)

$x = -\frac{1}{2}, y = \frac{7}{6}$

Substituting value of x and y in equation (iii)

$1-\left(-\frac{1}{2}\right)-\frac{7}{6} = \frac{1}{3}$ satisfy the Eq. (iii)

Hence, ***d*** lie on plane $\Rightarrow$ ***a, b, c, d*** are coplanar.

Hence, A is correct, R is correct and R is correct explanation for A.

32. **(d)** We have

$|\boldsymbol{a}| = 4, |\boldsymbol{b}| = 5, |\boldsymbol{a}-\boldsymbol{b}| = 3$

Now, $|\boldsymbol{a}-\boldsymbol{b}|^2 = |\boldsymbol{a}|^2 + |\boldsymbol{b}|^2 - 2\boldsymbol{a}\cdot\boldsymbol{b}$

$\Rightarrow (3)^2 = 4^2 + 5^2 - 2a\cdot b$

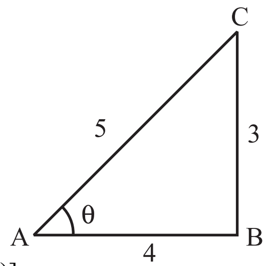
$\Rightarrow 9 = 16 + 25 - 2|\boldsymbol{a}||\boldsymbol{b}|\cos\theta$

$\Rightarrow 2\cdot 4\cdot 5\cos\theta = 32$

$\Rightarrow \cos\theta = \frac{4}{5}$

So, $\tan\theta = \frac{3}{4}$

$\Rightarrow \tan^2\theta = \frac{9}{16}$



33. **(a)** $e = [(2\hat{i}\times\hat{j}\times\hat{k}) - (\hat{i}-\hat{j}+\hat{k})]$

$\times[(2\hat{i}\times\hat{j}\times\hat{k}) - (-\hat{i}+\hat{j}-\hat{k})]$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 3 & 0 & 2 \end{vmatrix} = 4\hat{i}-2\hat{j}-6\hat{k}$

$\hat{e} = \frac{4\hat{i}-2\hat{j}-6\hat{k}}{2\sqrt{14}} = \frac{1}{\sqrt{14}}(2\hat{i}-\hat{j}-3\hat{k})$

Now, $a = 2\hat{i}-3\hat{j}+6\hat{k}$

Projection of a on $e = a\cdot e = \frac{4}{\sqrt{14}} + \frac{3}{\sqrt{14}} - \frac{18}{\sqrt{14}}$

$[\because a\cdot b = a_1a_2 + b_1b_2 + c_1c_2]$

$= \frac{-11}{\sqrt{14}}$

Then, the projection vector of a on e is

$= \frac{-11}{\sqrt{14}}\left(\frac{1}{\sqrt{14}}(2\hat{i}-\hat{j}-3\hat{k})\right) = \frac{11}{14}(-2\hat{i}+\hat{j}+3\hat{k})$

34. **(b)** Given that

$a = 2\hat{i} + \hat{j} - 3\hat{k}$, $b = \hat{i} - 2\hat{j} + 3\hat{k}$, $c = -\hat{i} + \hat{j} - 4\hat{k}$

$d = \hat{i} + \hat{j} + 2\hat{k}$

$$\therefore\ a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= \hat{i}(3-6) - \hat{j}(6+3) + \hat{k}(4-1) = -3\hat{i} - 9\hat{j} - 5\hat{k}$$

$$\text{and } c \times d = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -4 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(2+4) - \hat{j}(-2+4) + \hat{k}(-1-1) = 6\hat{i} - 2\hat{j} - 2\hat{k}$$

Now, $(a \times b) \times (c \times d)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -9 & -3 \\ 6 & -2 & -2 \end{vmatrix}$$

$$= \hat{i}(18-10) - \hat{j}(6+30) + \hat{k}(6+54)$$

$$= 8\hat{i} + 36\hat{j} + 60\hat{k}$$

35. **(c)** Given that, $n = 20$

Mean = 40

$$\text{Mean} = \frac{\text{Sum of weights}}{n} \Rightarrow 40 = \frac{\text{Sum of weights}}{20}$$

Sum of weights = 20 × 40 = 800

Now, two boys, are excluded of weight 43 kg and 73 kg.

$$\text{New mean} = \frac{\text{Remaining weight}}{20-2} = \frac{800-43-73}{18}$$

$\overline{X}_{\text{new}}$ = new mean = 38

Now, standard deviation, $\sigma = 5$

$\therefore$ Variance $= \sigma^2 = 5^2 = 25$

$$\frac{\Sigma x^2}{n} - (\overline{X})^2 = 25$$

$$\Rightarrow \frac{\Sigma x^2}{20} - (40)^2 = 25 \Rightarrow \frac{\Sigma x^2}{20} = 1600 + 25$$

$\Rightarrow \Sigma x^2 = 1625 \times 20 \Rightarrow \Sigma x^2 = 32500$

When two boys are removed

$\Sigma x^2_{\text{new}} = 32500 - (43)^2 - (73)^2 = 25322$

So, new variance

$$\sigma^2_{\text{new}} = \frac{\Sigma x^2_{\text{new}}}{18} - (\overline{X}_{\text{new}})^2 = \frac{25322}{18} - (38)^2$$

$\sigma^2_{\text{new}} = 26.78$

36. **(Bonus)** Given that

	Group I	Group II	Group III
n	50	60	90
Mean	113	120	115
SD	6	8	7

$$\text{Coefficient of variation (CV)} = \frac{\sigma}{x} \times 100\%$$

For group-I

$$CV_1 = \frac{\sigma_1}{\overline{x}_1} \times 100\% = \frac{6}{113} \times 100\%$$

$CV_1 = 5309\%$

For group-II

$$CV_2 = \frac{8}{120} \times 100\% = 6.66\%$$

For group III

$$CV_3 = \frac{7}{115} \times 100 = 6.086\%$$

We know that, the smaller the CV, the higher is the consistency

Since, $CV_1 < CV_3 < CV_2$.

$\therefore$ Increasing order of consistencies II < III < I.

37. **(b)**

38. **(d)** We have

$$P(A) = \frac{1}{3},\ P(A \cap B) = \frac{1}{3},\ P(A \cup B) = \frac{1}{3}$$

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{3}{5} = \frac{1}{3} + P(B) - \frac{1}{5}$$

$$P(B) = \frac{3}{5} + \frac{1}{5} - \frac{1}{3} = \frac{9+3-5}{15} \Rightarrow P(B) = \frac{7}{15}$$

$$A:\ P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{5}}{\frac{7}{15}} = \frac{3}{7}$$

$$B:\ P(\overline{B}) = 1 - P(B) = 1 - \frac{7}{15} = \frac{8}{15}$$

$$C:\ P(A \cap \overline{B}) = P(A) - P(A \cap B) = \frac{1}{3} - \frac{1}{5}$$

$$= \frac{5-3}{15} = \frac{2}{15}$$

$$D:\ P(B \cap \overline{A}) = P(B) - P(A \cap B) = \frac{7}{15} - \frac{1}{5}$$

$$= \frac{7-3}{15} = \frac{4}{15}$$

39. **(a)**

40. **(c)** The probability of a mechanic making an error while using a machine on nth day is $P(E_n) = \frac{1}{2^n}$.

Since, machine operated for 4 days, so probability of making an error for 1st day, 2nd, 3rd and 4th day is

$$P(E_1) = \frac{1}{2},\ P(E_2) = \frac{1}{4},\ P(E_3) = \frac{1}{8},\ P(E_4) = \frac{1}{16}$$

respectively.

Now, the probability that it had not make a mistake on 3 out of 4 is

$$= P(E_1 \cap \overline{E}_2 \cap \overline{E}_3 \cap \overline{E}_4) + P(\overline{E}_1 \cap E_2 \cap \overline{E}_3 \cap \overline{E}_4)$$
$$+ P(\overline{E}_1 \cap \overline{E}_2 \cap E_3 \cap \overline{E}_4) + P(\overline{E}_1 \cap \overline{E}_2 \cap \overline{E}_3 \cap E_4)$$

$$= \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{7}{8}\cdot\frac{5}{16}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{7}{8}\cdot\frac{15}{16}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{8}\cdot\frac{15}{16}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{7}{8}\cdot\frac{1}{16}$$

$$= \frac{315}{1024}+\frac{105}{1024}+\frac{45}{1024}+\frac{21}{1024} = \frac{486}{1024}=\frac{243}{512}$$

41. **(b)** Given that probability of a bad reaction from a vaccination is 0.01.

$\therefore$ $P = 0.01$ and $n = 300$

Mean $(\mu) = np = 0.01 \times 300 = 3$

Apply poission distribution.

$P(X=r) = \frac{e^{-\mu}\cdot\mu^r}{r!}$ $P(X=2) = \frac{e^{-3}\cdot 3^2}{2!} = \frac{9}{2e^3}$

42. **(c)** Let $P(h, k)$ be the variable point.

Given that, $A(1, 2)$, $B(2, 1)$ and $|PA - PB| = 3$

$\Rightarrow \sqrt{(h-1)^2+(k-2)^2} - \sqrt{(h-2)^2+(k-1)^2} = 3$

$\Rightarrow \sqrt{(h-1)^2+(k-2)^2} = 3+\sqrt{(h-2)^2+(k-1)^2}$

Squaring both side, we get

$\Rightarrow (h-1)^2+(k-2)^2$

$= 9 + (h-2)^2 + (k-1)^2 + 6\sqrt{(h-2)^2+(k-1)^2}$

$\Rightarrow h^2 + 1 - 2h + k^2 + 4 - 4k$

$= 9 + h^2 + 4 - 4h + k^2 + 1 - 2k + 6\sqrt{(h-2)^2+(k-1)^2}$

$\Rightarrow 2h - 2k - 9 = 6\sqrt{(h-2)^2+(k-1)^2}$

Again, squaring both sides, we get

$\Rightarrow (2h - 2k - 9)^2 = 36[(h-2)^2 + (k-1)^2]$

$\Rightarrow 4h^2 + 4k^2 + 81 - 8hk + 36k - 36h$

$= 36[h^2 + 4 - 4h + k^2 + 1 - 2k]$

$\Rightarrow 32h^2 + 32k^2 + 8hk - 108h - 108k + 99 = 0$

Hence, locus of $P(h, k)$ is

$32x^2 + 32y^2 + 8xy - 108x - 108y + 99 = 0$.

43. **(c)** Lines

$x + 2ay + a = 0$

$x + 3by + b = 0$

and $x + 4cy + c = 0$ are concurrent

If $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

$\Rightarrow (3b - 2a)(c - a) - (b - a)(4c - 2a) = 0$

$3bc - 3ab - 2ac + 2a^2 - 4bc + 2ab + 4ac - 2a^2 = 0$

$\Rightarrow ab + bc = 2ca$

$\Rightarrow \frac{1}{a}+\frac{1}{c} = \frac{2}{b}$

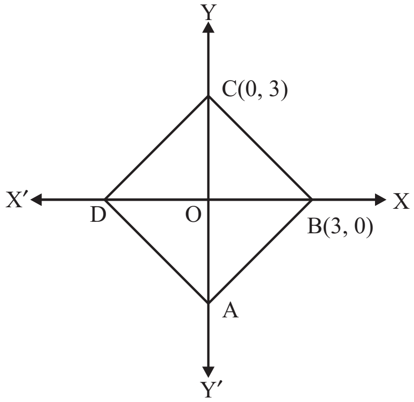
Hence, a, b and c are in H.P.

44. **(b)** Let, $P(x, y)$ be variable point.

Given that, sum of its distance from XX' and YY' in equal to 3.

$\therefore |x| + |y| = 3$

This equation form square $ABCD$, coordinate of B and C are (3, 0) and (0, 3).



Area of square $ABCD$

$= 4\times\frac{1}{2}\times OB\times OC$

$= 2 \times 3 \times 3 = 18$ units

45. **(b)** Equation of altitudes are

$\sqrt{3}x - y + 8 - 4\sqrt{3} = 0$ ...(i)

and $\sqrt{3}x + y - 12 - 4\sqrt{3} = 0$ ...(ii)

Adding Eqs. (i) and (ii),

$2\sqrt{3}x - 4 - 8\sqrt{3} = 0$

$\Rightarrow 2\sqrt{3}x = 4 + 8\sqrt{3}$

$\Rightarrow x = \frac{4(1+2\sqrt{3})}{2\sqrt{3}} \Rightarrow x = \frac{2(1+2\sqrt{3})}{\sqrt{3}}$

Subtracting Eqs. (i) and (ii)

$2y - 20 = 0$

$\Rightarrow 2y = 20 \Rightarrow y = 10$

Point of intersection of altitudes is $\left(\frac{2(1+2\sqrt{3})}{\sqrt{3}}, 10\right)$ the third altitude also passes through this point.

So, by checking options we get, equation of third altitude is $y = 10$.

46. **(c)**

47. **(c)** Let slope of required line = m

$\therefore$ Angle between $x + y + 1 = 0$ and new line is

$\tan\theta = \left|\frac{m+1}{1-m}\right|$ ...(i)

$\left[\because \text{ Angle between two lines, } \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|\right]$

And angle between $x + y + 1$ and $2x + 3y - 4 = 0$

$$\tan\theta = \left|\frac{\frac{-2}{3}+\frac{1}{3}}{1+\frac{2}{3}\times 1}\right| = \left|\frac{\frac{1}{3}}{\frac{3+2}{3}}\right|$$

$\Rightarrow \tan\theta = \left|\frac{1}{5}\right|$ ...(ii)

Since $x + y + 1 = 0$ bisects an angle between $2x + 3y - 4 = 0$ and new line.

$\therefore$ Equate equations (i) and (ii)

$\frac{m+1}{1-m} = \left(\frac{1}{5}\right)$ and $\frac{m+1}{1-m} = \frac{-1}{5}$

$\Rightarrow 5m + 5 = 1 - m$ or $5m + 5 = -1 + m$

$\Rightarrow 6m = -4$ or $4m = -6$

$\Rightarrow m = \frac{-2}{3}$ or $m = \frac{-3}{2}$

$\therefore$ Slope of new line is $\frac{-3}{2}$.

On solving $x + y + 1 = 0$ and $2x + 3y - 4 = 0$ we get point of intersection.

So, coordinate of point of intersection is $(-7, 6)$.

Equation of new line

$y - y_1 = m(x - x_1)$

$\Rightarrow (y - 6) = \frac{-3}{2}(x + 7)$

$\Rightarrow 2y - 12 = -3x - 21$

$\Rightarrow 3x + 2y + 9 = 0$

48. **(d)**

49. **(a)** We have

$C_1 : x^2 + y^2 = 16$

radius = 4 units, centre (0, 0)

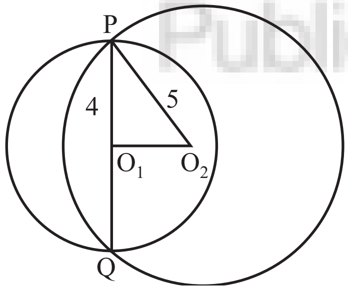
The maximum length of chord is equal to the diameter of circle c_1 = 8 units.

The equation of chord passing through (0, 0) and slope $\frac{3}{4}$ is $y = \frac{3}{4}x$

$\Rightarrow 3x - 4y = 0$

Since, the centre of circle C_2 must be lie on the line perpendicular to the chord.

Therefore, the coordinate of the centre of circle can be written as $(3a, -4a)$.



In ΔPO_1O_2

$(O_1O_2)^2 = 5^2 - 4^2$

$\Rightarrow O_1O_2 = 3$

$O_1O_2 \perp PQ$, so distance of $(3a, -4a)$ from $3x - 4y$ is 3 units

$$\left|\frac{+3(3a) - 4(-4a)}{\sqrt{3^2 + (-4)^2}}\right| = 3$$

$\Rightarrow \left|\frac{-25a}{5}\right| = 3 \Rightarrow a = \pm\frac{3}{5}$

If $a = \frac{3}{5}$

Then, $O_2\left(\frac{9}{5}, -\frac{12}{5}\right)$ when $a = -\frac{3}{5}$

Then, coordinate of $O_2\left(\frac{-9}{5}, \frac{12}{5}\right)$.

50. **(c)** Given that equation of normals

$x^2 - 3xy - 3x + 9y = 0$

$\Rightarrow x(x - 3y) - 3(x - 3y) = 0$

$\Rightarrow (x - 3y)(x - 3) = 0$

So, equation of normals is $x - 3y = 0$ and $x - 3 = 0$.

Since, two normal of circle intersect at centre.

On solving $x - 3y = 0$ and $x - 3 = 0$, we get

Centre = (3, 1)

Now, equation of given circle

$x^2 + y^2 - 6x + 6y + 17 = 0$

Centre (3, – 3) and radius

$= \sqrt{3^2 + (-3)^2 - 17}$

$r_1 = \sqrt{1} = 1$

Given that, circles touches externally, so sum of radius = distance between centres (3, 1) and (3, – 3).

$r_1 + r_2 = \sqrt{(3-3)^2 + (1-(-3))^2}$

$1 + r_2 = 4$

$r_2 = 3$

Equation of circle with centre (3, 1) and radius 3 units is

$(x - 3)^2 + (y - 1)^2 = 3^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 2y + 1 = 9$

$\Rightarrow x^2 + y^2 - 6x - 2y + 1 = 0$

51. **(d)**

52. **(b)** Given that equation of one circle

$x^2 + y^2 = 36$

$\Rightarrow x^2 + y^2 - 36 = 0$ …(i)

and radical axis of two circle is $x - 4 = 0$

So, equation of other circle is

$x^2 + y^2 - 36 + k(x - 4) = 0$

$x^2 + y^2 + kx - 4k - 36 = 0$ …(ii)

We know that two circles

$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and

$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

intersect orthogonally if $2(g_1g_2 + f_1f_2) = c_1 + c_2$

Since, both circles are intersecting orthogonally,

$\therefore -4k - 36 - 36 = 0$

$-4k = 72$

$k = -18$

So, equation of required circle

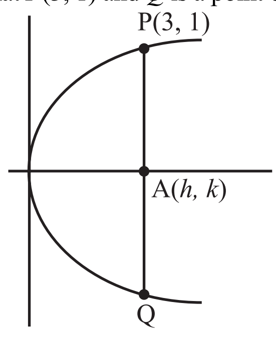
$x^2 + y^2 - 18x - 4(-18) - 36 = 0$

$\Rightarrow x^2 + y^2 - 18x + 72 - 36 = 0$

$\Rightarrow x^2 + y^2 - 18x + 36 = 0$

53. **(a)**

54. **(b)** Given that $P(3, 1)$ and Q is a point of parabola $y^2 = 8x$



Let $A(h, k)$ is the mid-point of PQ.
$\therefore$ $Q(2h-3, 2k-1)$ since point Q lie on parabola, so $(2k-1)^2 = 8(2h-3)$
$\Rightarrow 4k^2 - 4k + 1 = 16h - 24$
$\Rightarrow 4k^2 - 4k - 16h + 25 = 0$
Hence, locus $R(h, k)$ is $4y^2 - 4y - 16x + 25 = 0$.

55. **(d)** Given that, equation of parabola
$y^2 = 8x$
Equation of tangent at (2, 4)
$yy' = 4(x + x_1)$
$\Rightarrow 4y = 4(x + 2)$
$\Rightarrow y = x + 2$ …(i)
Equation of tangent at (18, –12)
$-12y = 4(x + 18)$
$\Rightarrow x + 3y + 18 = 0$ …(ii)
On solving equations (i) and (ii), we get
Tangents intersects at (– 6, – 4)
Required equation of line passes through (–6, –4) and slope $\frac{1}{2}$

$$y - (-4) = \frac{1}{2}[x - (-6)] \Rightarrow 2y + 8 = x + 6 \Rightarrow x - 2y = 2$$

56. **(c)**

57. **(b)** We have equation of ellipse

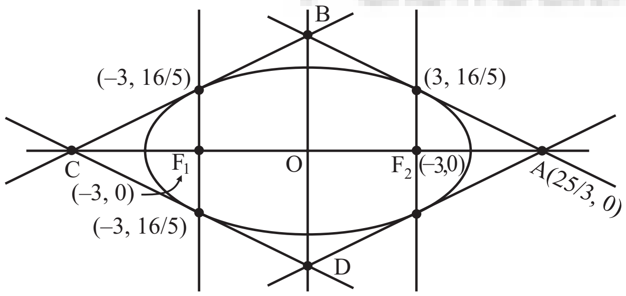
$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \Rightarrow \frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$$

So, $a = 5$, $b = 4$

Eccentricity, $c = \sqrt{1 - \left(\frac{16}{25}\right)}$ $\left[\because\ c = \sqrt{1 - \frac{b^2}{a^2}}\right]$

$= \sqrt{\frac{25-16}{25}} \Rightarrow e = \frac{3}{5}$ $\therefore$ Foci $= (\pm ae, 0) = (\pm 3, 0)$

Ends of latusrectum $= \left(\pm ae, \pm\frac{b^2}{a}\right) = \left(\pm 3, \pm\frac{16}{5}\right)$



Equation of tangent at $\left(3, \frac{16}{5}\right)$

$$\frac{3x}{25} + \frac{16}{5}\frac{y}{16} = 0 \Rightarrow \frac{3x}{25} + \frac{y}{5} = 0$$

It cuts the coordinate axis is $A\left(\frac{25}{3}, 0\right)$ and $B(0, 5)$.

Now, area of $\Delta OAB = \frac{1}{2} \times \frac{25}{3} \times 5 = \frac{125}{6}$ unit^2

$\therefore$ Area of quadrilateral $ABCD$

$$= 4 \times \frac{125}{6} = \frac{250}{3} \text{ unit}^2$$

58. **(a)** Given that
$x \cos\phi + y \sin\phi = P$ subtends a right angle at the centre (0, 0) of hyperbola $4x^2 - y^2 = 4a^2$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{4a^2} = 1$$

Now, with help of $x \cos\phi + y \sin\phi = P$
We make the equation of hyperbola in homogeneous form.

$$\frac{x^2}{a^2} - \frac{y^2}{4a^2} = \left(\frac{x\cos\phi + y\sin\phi}{P}\right)^2$$

$$\Rightarrow x^2\left(\frac{1}{a^2} - \frac{\cos^2\phi}{P^2}\right) + y^2\left(\frac{-1}{4a^2} - \frac{\sin^2\phi}{P^2}\right) - \frac{2xy\cos\phi\sin\phi}{p^2} = 0$$

$\therefore$ Coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow \frac{1}{a^2} - \frac{\cos^2\phi}{p^2} - \frac{1}{4a^2} - \frac{\sin^2\phi}{p^2} = 0$$

$$\Rightarrow \frac{1}{a^2} - \frac{1}{4a^2} - \left(\frac{\cos^2\phi + \sin^2\phi}{p^2}\right) = 0$$

$$\Rightarrow \left(\frac{4-1}{4a^2}\right) - \frac{1}{p^2} = 0 \Rightarrow \frac{3}{4a^2} = \frac{1}{p^2}$$

$$\Rightarrow p^2 = \frac{4a^2}{3} \Rightarrow p = \frac{2}{\sqrt{3}}a$$

Since, P is also length of perpendicular from (0, 0) to line.
$\therefore$ radius of circle $= P = \frac{2a}{\sqrt{3}}$.

59. **(b)** Given that P_1 divide AB in 1 : 2.
So, coordinate of

$$P_1\left(\frac{2\times3+9}{3}, \frac{2\times2+1\times8}{3}, \frac{2\times(-4)+1(-10)}{3}\right)$$

$\Rightarrow P_1(5, 4, -6)$
Since, P_2 divide AB in 2 : 1.
So, coordinate of $P_2\left(\frac{3+18}{3}, \frac{2+16}{3}, \frac{-4-20}{3}\right)$

$= P_2(7, 6, -8)$
Since, P divides P_1P_2 in 1 : 1.
So, coordinate of $P\left(\frac{5+7}{2}, \frac{4+6}{2}, \frac{-6-8}{2}\right)$

$P(6, 5, -7)$
But given that coordinate of $P(\alpha, \beta, \gamma)$
So, $\alpha = 6$, $\beta = 5$ and $\gamma = -7$
Hence, $\alpha + 2\beta + 2\gamma = 6 + 2(5) + 2(-7)$
$= 6 + 10 - 14 = 2$

60. **(d)**

61. **(a)** Required plane passes through (2, – 1, 3).
Let a, b and c are Dr's of normal to required plane.
Since required plane perpendicular to both given planes

$$\therefore \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -3\hat{i} - 2\hat{j} + 5\hat{k}$$

$\therefore$ Equation of required plane

$-3(x-2)-2(y+1)+5(z-3)=0$

$\Rightarrow x+\frac{2}{3}y-\frac{5}{3}z+\frac{11}{3}=0$

By comparing $ax+by+cz+d=0$, we get $d=\frac{11}{3}$

62. **(d)** $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3a}}{\sqrt{x}-\sqrt{a}}$

$$=\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3a}}{\sqrt{x}-\sqrt{a}}\times\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}\times\frac{\sqrt{a+2x}+\sqrt{3a}}{\sqrt{a+2x}+\sqrt{3a}}$$

$$=\lim_{x\to a}\frac{(a+2x)-3a}{x-a}\times\frac{\sqrt{x}+\sqrt{a}}{\sqrt{a+2x}+\sqrt{3a}}$$

$$=\lim_{x\to a}\frac{2x-2a}{x-a}\times\frac{\sqrt{x}+\sqrt{a}}{\sqrt{a+2x}+\sqrt{3a}}$$

$$=2\lim_{x\to a}\frac{(x-a)}{x-a}\times\frac{\sqrt{x}+\sqrt{a}}{\sqrt{a+2x}+\sqrt{3a}}$$

$$=2\left(\frac{\sqrt{a}+\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}\right)=2\frac{2\sqrt{a}}{2\sqrt{3a}}=\frac{2}{\sqrt{3}}$$

63. **(a)**

64. **(d)** Given that,

$y=(\sin x)^{x^2}$

Taking log both sides, we get

$\log y=x^2\log\sin x$

Differentiate w.r.t. 'x' on both sides

$$\frac{1}{y}\frac{dy}{dx}=x^2\left[\frac{1}{\sin x}\cos x\right]+2x\log\sin x$$

$$\Rightarrow\frac{dy}{dx}=y\left(\frac{x^2\cos x}{\sin x}+2x\log\sin x\right)$$

$$=(\sin x)^{x^2}\left(\frac{x^2\cos x}{\sin x}+2x\log\sin x\right)$$

$$\therefore\quad\frac{dy}{dx}=x^2\cos x\,(\sin x)^{x^2-1}+2x\log\sin x\,(\sin x)^{x^2}$$

65. **(c)** Given that, $y=\frac{(x+1)^2\sqrt{x-1}}{(x+4)^3e^x}$

Taking log both sides, we get

$$\log y=\log\left(\frac{(x+1)^2\sqrt{x-1}}{(x+4)^3e^x}\right)$$

$$\Rightarrow\log y=2\log(x+1)+\frac{1}{2}\log(x-1)-3\log(x+4)-x\log x$$

$$\Rightarrow\log y=2\log(x+1)+\frac{1}{2}\log(x-1)-3\log(x+4)-x$$

Differentiate w.r.t. 'x' on both sides

$$\Rightarrow\frac{1}{y}\frac{dy}{dx}=\frac{2}{x+1}+\frac{1}{2(x-1)}-\frac{3}{x+4}-1$$

$$\Rightarrow\frac{dy}{dx}=y\left(\frac{2}{x+1}+\frac{1}{2(x-1)}-\frac{3}{x+4}-1\right)$$

$$=\frac{(x+1)^2\sqrt{x-1}}{(x+4)^5e^4}\left(\frac{2}{x+1}+\frac{1}{2(x-1)}-\frac{3}{x+4}-1\right)$$

66. **(c)** Given that, $f(x)=\tan h^{-1}(\sin x)$

Let $f(x)=y$

$y=\tan h^{-1}(\sin x)$

$\tan h\,y=\sin x$ ...(i)

Differentiate w.r.t. 'x' on both sides

$$\sec h^2\,y\frac{dy}{dx}=\cos x\Rightarrow\frac{dy}{dx}=-\frac{\cos x}{\sec h^2\,y}$$

$$\frac{dy}{dx}=\frac{\cos x}{1-\tan h^2\,y}\qquad[\because\sec h^2 y=1-\sin h^2 y]$$

Put $x=\pi$

$$\left.\frac{dy}{dx}\right|_{x=\pi}=\frac{(-1)}{1-(0)}$$

$$\left.\frac{dy}{dx}\right|_{x=\pi}=-1$$

Hence, slope of tangent to the curve is -1.

67. **(a)** Let $y=f(x)=x^3+2x^{3/2}+5$

Differentiate w.r.t. 'x' on both sides,

$$f'(x)=3x^2+2\cdot\frac{3}{2}x^{1/2}=3x^2+3x^{1/2}$$

Let $x=1$ and $\Delta x=0.01$

So, $f(1)=1^3+2(1)^{3/2}+5=1+2+5=8$

Since, $\Delta y=f'(x)\cdot\Delta x=(3(1)^2+3(1)^{1/2})\cdot 0.01$

$=6\times 0.01=0.06$

So, $f(x+\Delta x)=f(x)+\Delta y$

$=f(1)+\Delta y=8+0.06=8.06$

68. **(c)** We have,

$y^2=ax^3+b$ …(i)

Differentiate w.r.t. 'x' on both sides

$$2y\frac{dy}{dx}=3ax^2$$

$$\frac{dy}{dx}=\frac{3ax^2}{2y}$$

Slope at (1, 2)

$$\left.\frac{dy}{dx}\right|_{(1,2)}=\frac{3a(1)^2}{2(2)}$$

$$\left.\frac{dy}{dx}\right|_{(1,2)}=\frac{3a}{4}\qquad\text{…(ii)}$$

Given that equation of tangent $y=2x$

$\therefore$ Slope of tangent $=2$ …(iii)

From equations (ii) and (iii),

$$\frac{3a}{4}=2,\ a=\frac{8}{3}$$

Put in equation (i),

$$y^2=\frac{8}{3}x^3+b$$

Since, curve passes through (2, 2)

$$2^2 = \frac{8}{3}(1)^3 + b \Rightarrow b = 4 - \frac{8}{3} = \frac{12-8}{3} \Rightarrow b = \frac{4}{3}$$

So, $(a, b) = \left(\frac{8}{3}, \frac{4}{3}\right)$

69. **(b)**

70. **(d)** Given that, $f(x)$ be continuous on [0, 4], differentiable on (0, 4), $F(0) = 4$ and $F(4) = -2$

Since, $g(x) = \frac{f(x)}{x+2}$

At $x = 0$, $g(0) = \frac{f(0)}{0+2} = \frac{4}{2} = 2$

At $x = 4$, $g(4) = \frac{f(4)}{4+2} = \frac{-2}{6} = \frac{-1}{3}$

Now, by Lagrange's theorem

$$g'(c) = \frac{g(4)-g(0)}{4-0} = \frac{\frac{-1}{3}-2}{4} = \frac{-7}{3\times 4} \Rightarrow g'(c) = \frac{-7}{12}$$

71. **(d)** $I = \int \sqrt{\tan x} + \sqrt{\cot x}\, dx$

$$= \int \left(\sqrt{\frac{\sin x}{\cos x}} + \sqrt{\frac{\cos x}{\sin x}}\right) dx = \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}$$

multiply and divide by $\sqrt{2}$

$$\sqrt{2}\int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}\, dx$$

$$= \sqrt{2}\int \frac{\sin x + \cos x}{\sqrt{1-1+2\sin x\cos x}}\, dx$$

$$= \sqrt{2}\int \frac{\sin x + \cos x}{\sqrt{1-(1-2\sin x\cos x)}}\, dx$$

$$= \sqrt{2}\int \frac{\sin x + \cos x}{\sqrt{1-(\sin^2 x+\cos^2 x-2\sin x\cos x)}}\, dx$$

$$= \sqrt{2}\int \frac{\sin x + \cos x}{1-(\sin x-\cos x)^2}\, dx$$

Let $(\sin x - \cos x) = t$

$\Rightarrow (\cos x + \sin x)\, dx = dt$

$$= \sqrt{2}\int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2}\sin^{-1} t$$

$$= \sqrt{2}\sin^{-1}(\sin x - \cos x) + c$$

72. **(a)**

73. **(c)** $I = \int \frac{2\,dx}{\sqrt{\cot^2 x - \tan^2 x}}\, dx = \int \frac{2\,dx}{\sqrt{\frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x}}}$

$$= \int -\frac{2\sin x\cos x}{\sqrt{\cos^4 x - \sin^4 x}}\, dx \quad [\because \sin 2x = 2\sin x\cos x]$$

$$= \int \frac{\sin 2x\, dx}{\sqrt{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}}$$

$[\because \sin^2 x + \cos^2 x = 1$ and $\cos^2 x - \sin^2 x = \cos 2x]$

$$= \int \frac{\sin 2x\, dx}{\sqrt{\cos 2x}}$$

Let $\cos 2x = 1$

$\Rightarrow -2\sin 2x\, dx = dt$

$$\Rightarrow -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2}\frac{t^{-1/2-1}}{-\frac{1}{2}+1} + c$$

$$\Rightarrow -\frac{1}{2}\frac{t^{1/2}}{\frac{1}{2}} + c = -\sqrt{t} + c$$

Put $t = \cos 2x = -\sqrt{\cos 2x} + c$

Compare with $-\sqrt{f(x)} + c$

So, $f(x) = \cos 2x$

74. **(a)** $I = \int \frac{3^x}{\sqrt{9^x - 1}}\, dx = \int \frac{3^x}{(3^x)^2 - 1}$

Let $3^x = t \Rightarrow 3^x \log 3\, dx = dt$

$$\therefore \frac{1}{\log 3}\int \frac{dt}{\sqrt{t^2-1}} = \frac{1}{\log 3}\log|t + \sqrt{t^2-1}| + c$$

Put $t = 3^x = \frac{1}{\log 3}\log|3^x + \sqrt{9^x - 1}| + c$

75. **(a)** Given that,

$$f(x) = \int_1^2 \frac{1}{2+t^4}\, dt$$

So, $f(2) = \int_1^2 \frac{1}{2+t^4}\, dt$

We have, $f(2) > \int_1^2 \min.\left(\frac{1}{2+t^4}\right) dt$

$$> \int_1^2 \frac{1}{2+2^4}\, dt > \int_1^2 \frac{1}{18}\, dt > \frac{1}{18}[t]_1^2 \Rightarrow f(2) > \frac{1}{18}$$

and $f(2) < \int_1^2 \max\left(\frac{1}{2+t^4}\right) dt < \int_1^2 \frac{1}{2+(1)^4}\, dt$

$$< \int_1^2 \frac{1}{3}\, dt < \frac{1}{3}[t]_1^2 \Rightarrow f(2) < \frac{1}{3}$$

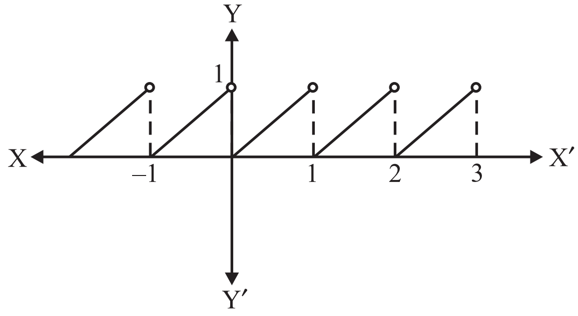
$$\frac{1}{18} < f(2) < \frac{1}{3}$$

76. **(b)**

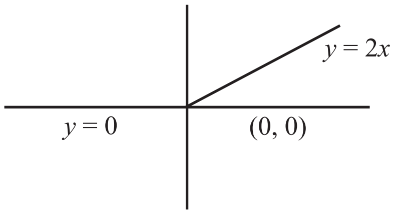
77. **(a)** Given that curves $y = 2x^2$

$y = \max\{x - [x], x + |x|\} = \max\{\{x\}, x + |x|\}$

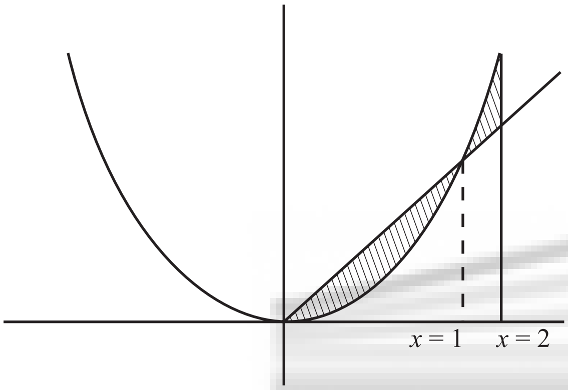
Since, graph of $\{x\}$



and graph of $x + |x| = \begin{cases} 0, & x < 0 \\ 2x, & x \geq 0 \end{cases}$



Now, draw the combine graph $y = 2x^2$ and $y = \max\{\{x\}, x + |x|\} = |x|$ and lines $x = 0$, $x = 2$ is



$\therefore$ Required area

$$= \int_0^1 (2x - 2x^2)\,dx + \int_1^2 (2x^2 - 2x)\,dx$$

$$= \left[\frac{2x^2}{2} - \frac{2x^3}{3}\right]_0^1 + \left[\frac{2x^3}{3} - \frac{2x^2}{3}\right]_1^2$$

$$= \left(1 - \frac{2}{3}\right) + \left(\frac{16}{3} - 4 - \frac{2}{3} + 1\right)$$

$$= \frac{1}{3} + \frac{14}{3} - 3 = 5 - 2 = 2 \text{ units}$$

78. **(b)** Given that

$y = a + be^{2x} + ce^{-3x}$ ...(i)

Differentiate w.r.t. 'x' on both sides

$y_1 = 2be^{2x} - 3ce^{-3x}$ ...(ii)

Again, differentiate w.r.t. 'x' on both sides

$y_2 = 4be^{2x} + 9ce^{-3x}$ ...(iii)

Again, differentiate w.r.t. 'x' on both sides

$y_3 = 8be^{2x} - 27ce^{-3x}$ ...(iv)

Now adding equations (iii) and (iv) and subtracting 6 × equation (ii),

$y_2 + y_3 - 6y_1 = (4be^{2x} + 9ce^{-3x}) + (8be^{2x} - 27ce^{-3x}) - 6(2be^{2x} - 3ce^{-3x})$

$= (12be^{2x} - 18ce^{-3x}) - (12be^{2x} - 18ce^{-3x})$

$= 0$

79. **(c)**

80. **(d)** Given that,

$(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$

$$\Rightarrow \frac{dx}{dy} = \frac{\tan^{-1} y - x}{1 + y^2}$$

$$\Rightarrow \frac{dy}{dx} + \frac{x}{1 + y^2} = \frac{\tan^{-1} y}{1 + y^2}$$

This is in the form linear differential equation of

$$\frac{dx}{dy} + Px = Q$$

$$\therefore \quad P = \frac{1}{1 + y^2} \text{ and } Q = \frac{\tan^{-1} y}{1 + y^2}$$

$$IF = e^{\int P\,dy} = e^{\int \frac{1}{1+y^2}\,dy}$$

$$IF = e^{\tan^{-1} x}$$

Now, solution of linear differentiate equation is

$x \cdot IF = \int Q \times IF + C$

$$x \cdot e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1 + y^2} \cdot e^{\tan^{-1} y}\,dy$$

Let $\tan^{-1} y = t$

$$\frac{1}{1 + y^2}\,dy = dt$$

$$= \int t \cdot e^t\,dt = t\int e^t - \int (e^t - 1)\,dt$$

$= te^t - e^t + c = e^t(t - 1) + c$

Putting $t = \tan^{-1} y$

$$x \cdot e^{\tan^{-1} y} = e^{\tan^{-1} y}(\tan^{-1} y - 1) + c$$

PHYSICS

81. **(b)** Order of magnitude of given forces is,
Strong nuclear force > Electromagnetic force > Weak nuclear force > Gravitational force

82. **(c)** Number of atoms in volume V

= Number of unit cells × Number of atoms in 1 unit cell

$$= \frac{V}{V_0} \times N_0$$

Number of moles in volume V

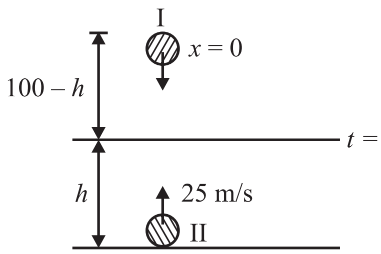
$$= \frac{\text{Number of atoms}}{\text{Avagadro number}} = \frac{V}{V_0} \times N_0 \times \frac{1}{N_A}$$

Mass n of given sample volume

= Number of moles × Molar mass

$$\Rightarrow m = \frac{V}{V_0} \times \frac{N_0}{N_A} \times M$$

83. **(a)**



Let at $t = t$ both stone have same height.

Using, $h = ut + \frac{1}{2}gt^2 \quad \Rightarrow h = 25t - \frac{1}{2}gt^2$ ...(i)

$100 - h = \frac{1}{2}gt^2$ ...(ii)

Adding (i) and (ii), we get

$100 = 25t \Rightarrow t = 4$ sec.

84. **(d)** At $t = 0$

Initial speed, $u = 0$, using $S = ut + \frac{1}{2}at^2$

Displacement, $s_1 = \frac{1}{2}a_1t^2 = \frac{1}{2} \times 5 \times (10)^2 = 250$ m

Final velocity, $v_1 = u + a_1 t$

$\Rightarrow v_1 = 0 + 5 \times 10 = 50\ \text{ms}^{-1}$

For $t = 10$ s to $t = 15$ s

$v_1 = 50\ \text{ms}^{-1}$, $v_2 = 0\ \text{ms}^{-1}$, $t = 5$ s

Using $v = u + at$

$\Rightarrow \quad a_2 = \frac{0-50}{5} = -10\ \text{ms}^{-2}$

And by $v^2 - u^2 = 2as$, we have

$\Rightarrow \quad v_2^2 - v_1^2 = 2(a_2)s_2$

$\Rightarrow \quad s_2 = \frac{50 \times 50}{2 \times 10} = 125$ m

So, average speed is given as

$$v_{avg} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{250+125}{10+5} = \frac{375}{15} = 25\ \text{ms}^{-1}$$

85. **(b)** Velocity = acceleration × time

$v = at$

$\Rightarrow \omega r = at \qquad (\because v = \omega r)$

$\Rightarrow \omega = \frac{at}{r}$

$\Rightarrow d\theta = \frac{a}{r} t\, dt \qquad \left(\because \omega = \frac{d\theta}{dt}\right)$

$\Rightarrow \int_0^{4\pi} d\theta = \frac{a}{r}\int_0^t t\, dt$

$\Rightarrow 4\pi = \frac{at^2}{2r} \Rightarrow t^2 = \frac{8\pi r}{a}$

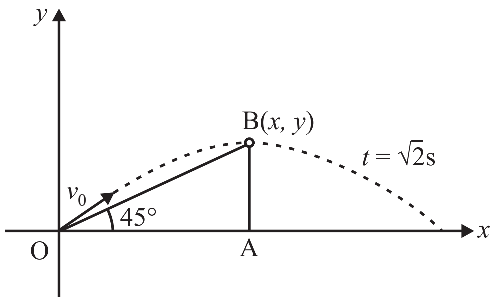
Radial acceleration, $a_r = \frac{v^2}{r} = \frac{a^2t^2}{r} = \frac{a^2}{r} \times \frac{8\pi r}{a} = 8\pi a$

$a_t = \frac{dv}{dt} = \frac{d}{dt}2t = 2$

So $a_{net} = \sqrt{a_t^2 + a_r^2} = \sqrt{a^2 + (8\pi a)^2}$

$= a\sqrt{1+64\pi^2} = 2\sqrt{1+64\pi^2}$

86. **(d)**



Let object is at $B(x, y)$ after $t = \sqrt{2}$ s

Then, $x = u_x \times t = v_0 \cos 45° \times \sqrt{2} = \frac{v_0}{\sqrt{2}} \times \sqrt{2} = v_0$

and $y = u_y t - \frac{1}{2}a_y t^2$

$= v_0 (\sin 45°)\sqrt{2} - \frac{1}{2}(10) \times (\sqrt{2})^2$

$= (v_0 - 10)$ m

Displacement OB of particle is

$OB = \sqrt{OA^2 + AB^2} = \sqrt{v_0^2 + (v_0 - 10)^2}$

So, $v_{avg} = \frac{OB}{t} = v_0$ or $OB = v_0 t$

$\Rightarrow \sqrt{v_0^2 + (v_0-10)^2} = v_0\sqrt{2}$

$\Rightarrow v_0^2 + (v_0 - 10)^2 = 2v_0^2 \Rightarrow v_0 = 5\ \text{ms}^{-1}$

87. **(a)** So, tangential linear acceleration of particle is

$a_t = \alpha r = \frac{d^2\theta}{dt^2} r = 2ar \qquad \left(\because \alpha = \frac{d^2\theta}{dt^2}\right)$

$v = \omega r = \frac{d\theta}{dt} r = 2rat \qquad \left(\because \omega = \frac{d\theta}{dt}\right)$

Also, normal or radial acceleration of particle is

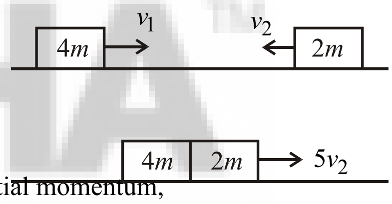
$a_n = \frac{v^2}{r} = \frac{4a^2t^2r^2}{r} = 4a^2t^2r$

Total acceleration of particle is

$a_{total} = \sqrt{a_t^2 + a_n^2} = \sqrt{4a^2r^2 + 16a^4t^4r^2}$

$= 2ar\sqrt{1+4a^2t^4} = \frac{v}{t}\sqrt{1+4a^2t^4}$

88. **(d)**



Initial momentum,

$P_i = 4mv_1 - 2mv_2$

Final momentum,

$P_f = 6m\,(5v_2) = 30mv_2$

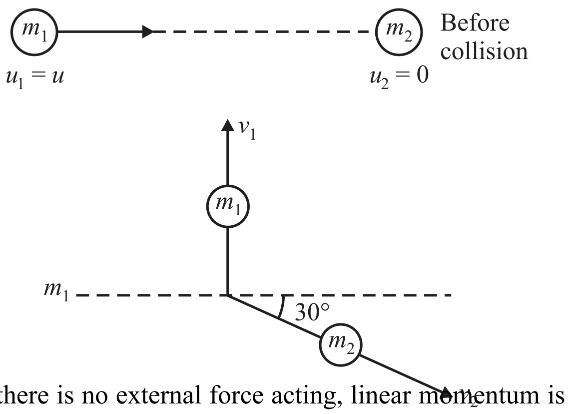
By law of conservation of momentum

$P_i = P_f$

$\Rightarrow 4mv_1 - 2mv_2 = 30mv_2 \Rightarrow 4v_1 = 32v_2$

$\Rightarrow \frac{v_1}{v_2} = \frac{32}{4} = 8$ or $\frac{v_1}{v_2} = 8$

89. **(a)**



As there is no external force acting, linear momentum is conserved in both x and y-directions

$m_1 u = m_2 v_2 \cos 30°$ (Along x) …(i)

$$m_1 u = \frac{m_2 v_2 \sqrt{3}}{2}$$

and $m_1 v_1 = m_2 v_2 \sin 30°$ (Along y) …(ii)

$$m_1 v_1 = \frac{m_2 v_2}{2}$$

Also, loss of K.E. = 50%

$$\frac{50}{100} = \frac{KE_i - KE_f}{KE_i}$$

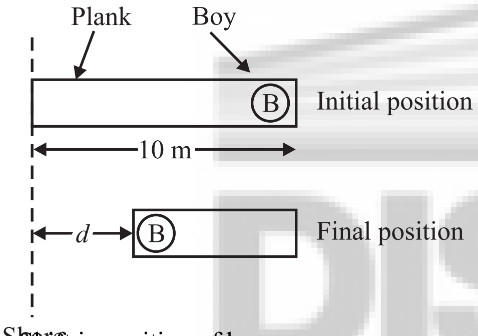
$$\Rightarrow 0.50 = \frac{\frac{1}{2}m_1u^2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2}{\frac{1}{2}m_1u^2} \quad \ldots\text{(iii)}$$

Solving Eqs. (i), (ii) and (iii), we get $\frac{m_2}{m_1} = 8$

90. **(a)** Coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{3.5 - 2.5}{3 - 1.5} = \frac{1}{1.5} \quad \frac{2}{3} = 0.67$$

91. **(c)**



dx_1 = Shift in position of boy

dx_2 = Shift in position of plank

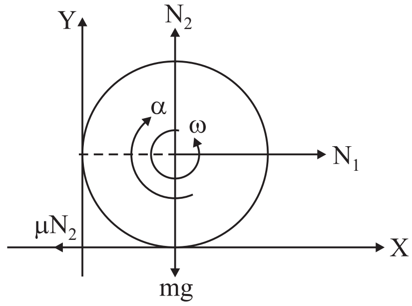
$$dx_{COM} = \frac{m_1 dx_1 + m_2 dx_2}{m_1 + m_2}$$

$|m_1 dx_1| = |m_2 dx_2|$ $[\because dx_{COM} = 0]$

$30(10 - d) = 10d$ $[\because dx_1 = 10 - d$ and $dx_2 = d]$

$d = 7.5$ m

92. **(c)**



Torque about centre of cylinder is

$$\tau = \mu N_2 R = \frac{mR^2}{2}\alpha$$

$$\Rightarrow \mu\, mg\, R = \frac{mR^2}{2}\alpha \qquad \Rightarrow \alpha = \frac{2\mu g}{R}$$

Now using,

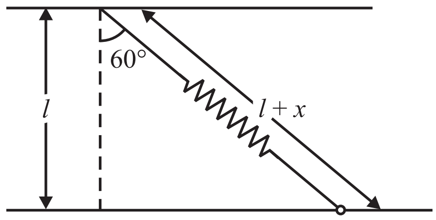
$$\omega^2 = \omega_0^2 - 2\alpha\Delta\theta$$

$$\Rightarrow 0 = (20)^2 - 2\left(\frac{2 \times \mu \times 10}{1}\right)(10\pi) \qquad [\because \Delta\theta = 5 \times 2\pi]$$

So, $(20)^2 = 40 \times 10\pi(\mu)$

$$\Rightarrow \mu = \frac{1}{\pi}$$

93. **(*)** From diagram,



$$\cos 60° = \frac{l}{l + x} \Rightarrow \frac{1}{2} = \frac{l}{l + x} \Rightarrow x = l$$

Extension in spring = l

At mean position, if velocity is v and there is no friction, then

$$\frac{1}{2}mv_m^2 = \frac{1}{2}kx^2 \Rightarrow v_m^2 = \frac{k}{m}l^2$$

If angle with vertical is θ when velocity is $v' = \frac{v}{2}$

$$\cos\theta = \frac{l}{l + x'} \Rightarrow x' = \left[\frac{l}{\cos\theta'} - l\right]$$

Then, $\frac{1}{2}mv'^2 = \frac{1}{2}kx'^2$

$$\frac{1}{2}m\left(\frac{v_m}{2}\right)^2 = \frac{1}{2}k\left(\frac{l}{\cos\theta'} - l\right)^2$$

Substituting value of v^2, we have

$$\frac{1}{2}m\left(\frac{hl^2}{4m}\right) = \frac{1}{2}k\left(\frac{l}{\cos\theta'} - l\right)^2$$

$$\Rightarrow \frac{l}{2} = \frac{l}{\cos\theta'} - l \Rightarrow \frac{3l}{2} = \frac{l}{\cos\theta'}$$

$$\cos\theta' = \frac{2}{3} \Rightarrow \theta' = \cos^{-1}\frac{2}{3}$$

(No option is matching)

94. **(a)** Let m be the mass of the body and h be the height it reaches.

Applying law of conservation of M.E.

$K_i + P_i = K_f + P_f$

$$\Rightarrow -\frac{GMm}{R} + \frac{1}{2}mV_0^2 = -\frac{GMm}{R + h}$$

$$\Rightarrow \frac{v_0^2}{2Gm} = \frac{1}{R} - \frac{1}{R + h}$$

$$\Rightarrow \frac{R(R + h)}{h} = \frac{2gR^2}{v_0^2} \qquad (\because Gm = gR^2)$$

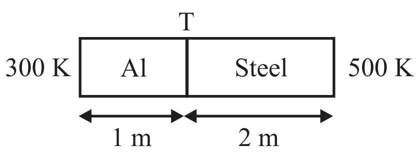
$$\Rightarrow h = \frac{R}{\left(\frac{2gR}{v_0^2} - 1\right)} \text{ or } h = \frac{Rv_0^2}{2gR - v_0^2}$$

95. **(b)** Given,
Length of wire, $l = 1m$
Radius of wire, $r = \frac{d}{2} = \frac{8}{2}$ mm = 4 mm
Elongation of the wire, $l = ?$
Pressure, $\Delta P = \frac{mg}{\pi r^2}$
Young's modulus, $Y = \frac{mgL}{\pi r^2 \Delta l} \Rightarrow m = \frac{Y\pi r^2 \Delta l}{gL}$
$$= \frac{2\times10^9 \times 314 \times (4\times10^{-3})^2 \times (5\times10^{-3})}{10\times1}$$
$$= \frac{2\times314\times16\times5}{10} \simeq 50 \text{ kg}$$

96. **(a)** Given,
Diameter of bubble, d = 2mm
Radius of bubble, $r = \frac{d}{2} = \frac{2}{2}\text{mm} = 1\text{mm}$
Velocity of small bubbles rising through a fluid is given by
$$v_{\text{terminal}} = \frac{2r^2(\rho_L - \rho_a)g}{9\eta}$$
$$\Rightarrow v = \frac{2\times(1\times10^{-3})^2 \times 136\times10^4 \times 10}{9\times1.5\times10^{-3}} = 20 \text{ ms}^{-1}$$

97. **(c)** Given, let Q be the rate of heat supplied and M be the mass of liquid ammonia.
$Q \times 14 = M \times 4.9 \times (50 - 15)$
$Q \times 92 = ML$
Dividing these, we get
$$\frac{92}{14} = \frac{L}{4.9\times(50-15)}$$
or $L = \frac{92\times4.9\times35}{14} = 1127$ kJ/kg

98. **(a)** In series
$\frac{Q}{t}$ = cons.



Let T be the temperature of the junction.
So, using formula for heat exchange rate
$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{l}$, we have,
$$\frac{Q}{t} = \frac{k_s A_s (500 - T)}{2} = \frac{k_{Al}A_{Al}(T - 300)}{1}$$
As area of both rods is same, so
$$\frac{k_s(500-T)}{2} = \frac{k_{Al}(T-300)}{1}$$
$$\Rightarrow \frac{50(500-T)}{2} = \frac{200(T-300)}{1}$$
$\Rightarrow T = 322.2 \simeq 322\ K$

99. **(c)** For one mole of ideal gas, $n = 1$
Using ideal gas equation $PV = RT$
$$T = \frac{PV}{R}$$
$$\Rightarrow T = \frac{p_0 V}{R} - \frac{\alpha p_0 V^4}{V_0^3 R} \qquad \ldots\text{(i)}$$
For maximum T, $= \frac{dT}{dV} = 0$
$$\Rightarrow \frac{dT}{dV} = \frac{p_0}{R} - \frac{4\alpha p_0}{RV_0^3}V^3 \Rightarrow \frac{p_0}{R} = \frac{4\alpha p_0}{RV_0^3}V^3$$
$$\Rightarrow V^3 = \frac{V_0^3}{4\alpha} \Rightarrow V = \frac{V_0}{(4\alpha)^{1/3}}$$
Substituting in Eq. (i), we get
$$T_{\max} = \frac{p_0V_0}{R(4\alpha)^{1/3}} - \frac{\alpha p_0 V_0^4}{V_0^3 R(4\alpha)^{4/3}}$$
$$= \frac{p_0V_0}{R(4\alpha)^{1/3}} - \frac{p_0V_0}{R4^{4/3}\alpha^{1/3}}$$
As we know, $T_{\max} = \frac{3p_0V_0}{4R}$, we get
$$\frac{3}{4} = \frac{1}{4^{1/3}\alpha^{1/3}} - \frac{1}{4^{4/3}\alpha^{1/3}}$$
Now from options, we can check that only $\alpha = \frac{1}{4}$, balances the equation. So, $\alpha = \frac{1}{4}$.

100. **(a)** Given,
For monoatomic, $C_{p_1} = \frac{5}{2}R$ and $C_{v_1} = \frac{3}{2}R$
For diatomic, $C_{p_2} = \frac{7}{2}R, C_{v_2} = \frac{5}{2}R$
Using
$$\gamma_{\text{mix}} = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1C_{V_1} + n_2C_{V_2}}$$
$$\gamma_{\text{mix}} = \frac{\frac{n_1}{n_2}C_{p_1} + C_{p_2}}{\frac{n_1}{n_2}C_{V_1} + C_{V_2}}$$
Now given for mixture,
$\gamma_{\text{mix}} = 1.5$
$$\text{So, } 1.5 = \frac{\frac{n_1}{n_2}\left(\frac{5}{2}\right) + \frac{7}{2}}{\frac{n_1}{n_2}\left(\frac{3}{2}\right) + \frac{5}{2}} \Rightarrow \frac{n_1}{n_2} = 1$$

101. **(a)** Given,
Length of wire, $l = 50$ cm
Spring constant, $k = 50$ N/m
Extension in the wire, $x = 1$ cm
Tension in wire, $T = kx$

$\Rightarrow T = 50 \times \frac{1}{100} = \frac{1}{2}$ N

Mass per unit length of string $\mu = \frac{m}{l}$

$= 10 \times 10^{-3} = 50 \times 10^{-2}$

Speed of wave is given as

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{0.5}{\left(\frac{1}{50}\right)}} = \sqrt{25}\ \text{ms}^{-1} = 5\ \text{ms}^{-1}$$

Time required by pulse to reach other end is

$$t = \frac{l}{v} = \frac{50 \times 10^{-2}\ \text{m}}{5\ \text{ms}^{-1}} = \frac{1}{10}\ \text{s} = 0.1\ \text{s}$$

102. (a) For 1st surface, $\mu_1 = 1.5$, $\mu_2 = 1.4$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Here, $R = +2$ cm

$u = -30$ cm

$$\Rightarrow \frac{1.4}{v} - \frac{15}{-30} = \frac{1.5 - 1.4}{2} \Rightarrow \frac{1.4}{v} = \frac{0.1}{2} - \frac{1.5}{30}$$

$$\Rightarrow \frac{1.4}{v} = \frac{3-3}{30} \Rightarrow v = \infty$$

For 2nd surface

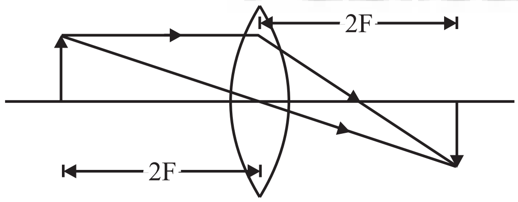
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$u = \infty \Rightarrow R = -2$ cm

$$\frac{1.5}{v} - \frac{1.4}{\infty} = \frac{-0.1}{-2} \Rightarrow \frac{1.5}{v} = \frac{1}{20}$$

$\Rightarrow v = 20 \times 1.5 = 30$ cm

103. (a) Minimum distance of an object and its real image is $4F$. For this, object must be placed at distance $2F$ from lens i.e., at the centre of curvature of lens.



104. (d) Given,

Distance between slits, $d = 0.3$ mm

Distance between slits and screen, $D = 1$m

Fringe width, $\beta = 1.9$ mm

Fringe width, $\beta = \frac{\lambda D}{d}$

$$\Rightarrow \lambda = \frac{\beta d}{D} \Rightarrow \lambda = \frac{1.9 \times 10^{-3} \times 0.3 \times 10^{-3}}{1}$$

$\Rightarrow \lambda = 57 \times 10^{-8}$ m $= 570 \times 10^{-9}$ m $= 570$ nm

105. (d) Using Gauss's law

$$\oint E.ds = \frac{q_{\text{enclosed}}}{\epsilon_0} \Rightarrow E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\text{But}\rho = \frac{q}{\frac{4}{3}\pi r^3} = \frac{3q}{4\pi r^3}$$

$$\Rightarrow E_d \times 4\pi r^2 = \frac{\rho 4\pi r^3}{3\epsilon_0} \Rightarrow E_d = \frac{\rho r}{3\epsilon_0} = \frac{(-3)(1)}{3\epsilon_0} = \frac{-1}{\epsilon_0}$$

$$E_d = \frac{(-3)(d)}{3\epsilon_0} = -\frac{d}{\epsilon_0}$$

106. (a) For point, $r = \frac{R}{2}$

Potential, $V_1 = \frac{kq}{R} + \frac{k2q}{2R} = \frac{2kq}{R}$

For point $r = 3R$

Potential, $V_2 = \frac{k(3q)}{3R} = \frac{kq}{R}$

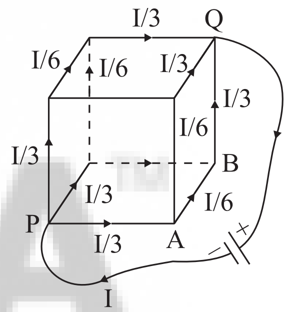
So, $\frac{V_1}{V_2} = 2$

107. (a) For maximum power

Net external resistance = Total internal resistance

Due to short circuiting $2R$ and $4R$ are useless. Hence net external resistance is 4Ω. Hence, $R = 4\Omega$.

108. (a) Now, we apply Kirchhoff's loop rule in the *PABQ*

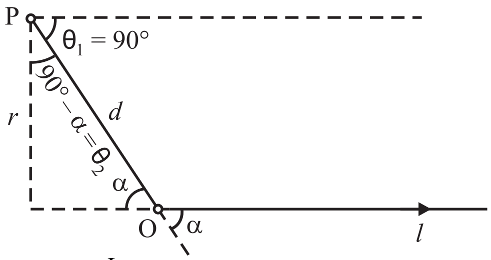


$$\Sigma V = 0 = \frac{-I}{3}R - \frac{I}{6}R - \frac{I}{3}R + V$$

$$V = IR\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{3}\right) \Rightarrow V = IR\left(\frac{2}{3} + \frac{1}{6}\right)$$

$$\Rightarrow V = IR\left(\frac{5}{6}\right) = IR_{eq} \Rightarrow R_{eq} = \frac{5}{6}R$$

109. (d)



As $B = \frac{\mu_0}{4\pi} \times \frac{I}{r} \times (\sin\theta_1 - \sin\theta_2)$

Here, $r = d \sin\alpha$

$\sin\theta_1 = \sin 90° = 1$ [$\because$ Wire is infinite towards right]

$\sin\theta_2 = \sin(90 - \alpha) = \cos\alpha$

So, $B = \frac{\mu_0 I}{4\pi d \sin\alpha}(1 - \cos\alpha)$

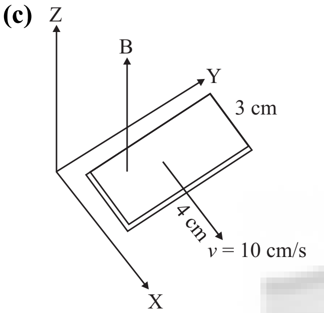
110. (c) Magnetic field at point O = Magnetic field due to wire + Magnetic field due to loop

$$= \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{2R}\left(\frac{3}{4}\right) = \frac{\mu_0 I}{4\pi R} + \frac{3}{8}\frac{\mu_0 I}{R} = \frac{\mu_0 I}{4\pi R}\left(1 + \frac{3\pi}{2}\right)$$

111. (b) Given,
Horizontal component of earth's magnetic field, $B_H = 0.3G$
Angle of dip, $\delta = 60°$
We know that, $\beta_H = B \cos \delta$
where B = Net magnetic field
δ = Angle of dip
$\therefore \; 0.3 = B \cos 60°$

$$\Rightarrow 0.3 = B \times \frac{1}{2} \text{ or } B = 0.6\ G$$

112. (c)



Magnetic field is a function of x and t,

$$\text{then, } dB = \frac{\partial B}{\partial x} dx + \frac{\partial B}{\partial x} dt$$

$$\text{Then, } \frac{dB}{dt} = \frac{\partial B}{\partial x} V + \frac{\partial B}{\partial t} \quad \ldots(i)$$

Induced emf in wire = Rate of change of flux

$$= \frac{d}{dt}(BA) = A\left\{\left(\frac{d}{dt}B\right) + v\frac{d}{dx}B\right\} \quad \text{[From (i)]}$$

$= 12 \times 10^{-4} (2 \times 10^{-2} + 10 \times 10^{-2} \times 2 \times 10^{-3})$
$= 24 \times 10^{-5}$ V
$= 0.000024 \text{ V} \simeq 0$ V

113. (d) Given,
Inductance, L = 0.4H
Resistance, R = 8Ω
Peak emf voltage, $V_{max} = 4V$
Power dissipated, $P_{avg} = V_{rms} I_{rms} \cos \phi$

$$= \frac{V_{max}}{\sqrt{2}} \times \left(\frac{V_{max}}{\sqrt{2}}\right) \times \frac{1}{Z} \times \frac{R}{Z} \quad \left[\because \cos \phi = \frac{R}{Z}\right]$$

$$= \frac{V_{max}^2}{2} \times \frac{R}{Z^2} = \frac{V_{max}^2}{2} \times \frac{R}{(\sqrt{X_L^2 + R^2})} \quad \left[\because Z = \sqrt{X_L^2 + R^2}\right]$$

$$= \frac{(4)^2}{2} \times \frac{8}{((0.4 \times 60)^2 + 8^2)} \quad [\because X_L = W_L]$$

$$= \frac{16 \times 8}{2 \times (24^2 + 8^2)} = \frac{64}{640} = \frac{1}{10} = 0.1 \text{ W}$$

114. (b) Energy of LASER beam is given

$$E = \frac{Il}{C} = \frac{100 \times 10^{-3} \times 90 \times 10^{-2}}{3 \times 10^8}$$

$= 3 \times 10^{-10}$ J

115. (a) Given,
Energy of photon, E = 4eV
Kinetic energy, KE = 1.1 eV
Work function, ϕ = ?
Using Einstein's photoelectric equation
$E = \phi + K.E \Rightarrow \phi = E - K.E$
$= 4 \text{ eV} - 1.1 \text{ eV} = 2.9$ eV

116. (a) $v_2 = \frac{v_1}{n} = \frac{2.2 \times 10^6}{2} = 1.1 \times 10^6 \text{ ms}^{-1}$

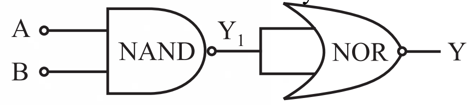
117. (d) Given,
Half life of sample = 103 years
Initial amount, N_0 = 100g
Final amount, N = 3.125g
Using

$$N = \frac{N_0}{2^n}, \; n = \text{no. of half life}$$

$$\Rightarrow 2^n = \frac{N_0}{N} = \frac{100}{3.125} \Rightarrow 2^n = 32 \Rightarrow n = 5$$

So, time taken = 103 × 5 = 515 years.

118. (b)



The gate at right is basically NOT gate.
So, NAND + NOT → AND
So, net output resembles an AND gate.

119. (b) For an insulator, conduction band must be vacant.
For a semiconductor, conduction band is partially vacant.
For a conductor, conduction band is partially filled.
(Note: option (b), is also correct).

120. (c) Given,
Radius of earth, R = 6400 km
For satisfactory communication, maximum distance

$$d_{max} = \sqrt{2Rh_T} + \sqrt{2Rh_R} \quad \ldots(i)$$

$$2d = \sqrt{2Rh_T} + \sqrt{2Rh_R} \quad \ldots(ii)$$

Substituting values in Eq. (iii), we get

$$2d \times 1000 = 2\sqrt{2 \times 6400 \times 1000 \times d}$$

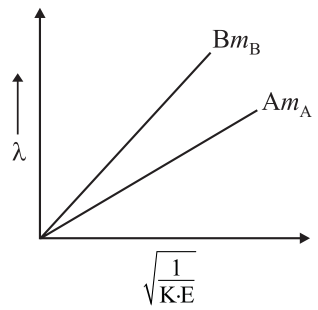
$$\Rightarrow (1000)^2 d^2 = 2 \times 6400 \times 1000 \times d$$

$$\Rightarrow d = \frac{2 \times 6400 \times 1000}{1000 \times 1000} \text{ m} = 12.8 \text{ m}$$

CHEMISTRY

121. (a) $h\nu - h\nu_0 = KE \Rightarrow h(\nu - \nu_0) = KE$
6.62×10^{-34} J-s $(1 \times 10^{18} \text{ s}^{-1} - \nu_0) = 1.986 \times 10^{-19}$ J
$\nu_0 = 7.0 \times 10^{14} \text{ s}^{-1}$

122. (c)



$h\nu = - \quad \Rightarrow h\dfrac{c}{\lambda} = \dfrac{1}{2}mv^2$

$\Rightarrow \lambda = \dfrac{hc}{\frac{1}{2}mv^2} \Rightarrow \lambda \; \alpha \; \dfrac{1}{m}$ as $\lambda_A < \lambda_B \Rightarrow m_A > m_B$

123. (c) First ionisation enthalpy decreases with increase in size. Hence, Li > Na > K > Cs.

124. (d) Covalency of $B = 4$, Covalency of Al = 6
In BF_4^-, B contains 8 electron in their octet.
Therefore, statement (ii) and (iii) are correct.

125. (c) $\overset{\mu_2}{\longleftarrow}\!\!+ \quad +\!\!\overset{\mu_1}{\longrightarrow}$ μ net = 0
O = C = O (Non-polar)

$\propto$ net $\neq 0$ polar $\qquad \propto$ net $\neq 0$ polar

$SnCl_2$ is ionic whereas $SnCl_4$ is covalent due to smaller size of Sn^{4+}.

126. (d) XeF_2 is sp^3d hybridised. In XeF_2 molecule, Xe has 10 electrons.

127. (d) Rate $(r) \; \alpha \; \dfrac{1}{\sqrt{M}}$ (where M is the molar mass)

$$\Rightarrow \frac{r_{(SO_2)}}{r_g} = \frac{(V/t)_{(SO_2)}}{(V/t)_g} = \frac{\left(\frac{1}{\sqrt{M}}\right)_{(SO_2)}}{\left(\frac{1}{\sqrt{M}}\right)_g}$$

$$\text{or, } \frac{V_{(SO_2)} \times t_g}{V_g \times t_{(SO_2)}} = \sqrt{\frac{M_g}{M_{(SO_2)}}}$$

$$\text{Thus, } \frac{12 \times 300}{120 \times 60} = \sqrt{\frac{M_g}{64}} \text{ or } \frac{1}{4} = \text{——}$$

$$\therefore \quad M_g = \frac{64}{4} = 16$$

128. (b) $\therefore \quad K.E. = \dfrac{3}{2}PV$

$98.03 \text{ L atm} = \dfrac{3}{2} \times P \times 24.6 \text{ L} \Rightarrow P = 2.66 \text{ atm}$

129. (a) $CaCO_3 \xrightarrow{\Delta} CO_2$
$2KOH + CO_2 \longrightarrow K_2CO_3 + H_2O$
$\because$ 112 g (56 × 2) of KOH will neutralise
= 100 g of $CaCO_3$

$\therefore$ 28 g KOH will neutralise $= \dfrac{100 \times 28}{112} = 25$ g of $CaCO_3$

$\because$ 60 g (impure) of $CaCO_3$ has = 25 g pure $CaCO_3$
$\therefore$ 100 g (impure) $CaCO_3$ has pure $CaCO_3$
$= \dfrac{25 \times 100}{60} = 41.6$ g

130. (c) $4K\overset{+5}{Cl}O_3 \xrightarrow{\Delta} 4K\overset{-1}{Cl} + 3K\overset{+7}{Cl}O_4$

Oxidation

Reduction

131. (a) $N_2O_4 + 3CO \longrightarrow N_2O + 3CO_2$
ΔH (enthalpy change)
$= \Sigma\Delta H_{N_2O} + 3\Delta H_{CO_2} - (\Sigma\Delta H_{N_2O_4} + 3\Delta H_{CO})$
$= [81 + (3 \times -393) - (-10 + (3 \times -10)]$
$\Delta H = -1058$ kJ/mol

132. (b)

	N_2	+ $3H_2$	$\rightleftharpoons$ $2NH_3$
Initial moles	1	1	1
At equilibrium	$1-x$	$1-3x$	$1+2x$

Given, $1 - x = 0.7$ mol $\Rightarrow x = 0.3$ mol
Therefore, concentration of N_2, H_2 and NH_3 at equilibrium will be
$[N_2] = [0.7]$
$[H_2] = 1 - (3 \times 0.3) = [0.1]$
$[NH_3] = 1 + 2x = 1 + (2 \times 0.3) = [1.6]$
According to law of equilibrium constant (K_c)

$$K_c = \frac{[NH_3]^2}{[N_2],[H_2]^3} = \frac{[1.6]^2}{[0.7],[0.1]^3}$$

$$K_c = \frac{2.56}{0.0007} = 3657.14$$

133. (d) $Ni(OH)_2 \rightleftharpoons \underset{s \text{ mol/L}}{Ni^{2+}} + \underset{2s \text{ mol/L}}{2OH^-}$

$K_{sp} = (s)(2s)^2$
$K_{sp} = 4s^3$

$$s = 3\sqrt{\frac{K_{sp}}{4}} = 3\sqrt{\frac{4 \times 10^{-15}}{4}} = 1 \times 10^{-5} \text{ mol/L}$$

134. (a) (i) $Zn + 2NaOH(aq) \longrightarrow Na_2ZnO_2 + H_2$
(iii) $CH_4(g) + H_2O(g) \xrightarrow[Ni]{1270\text{ K}} CH_3OH + H_2$
(iv) $Zn + 2H^+(aq) \longrightarrow Zn^{2+} + H_2$
(v) $C + H_2O \xrightarrow{1270\text{ K}} CO + H_2$

135. (a) $BeCl_2 \xrightarrow{LiAlH_4} BeH_2 + 2HCl$

136. (d) $AlCl_3$ from $[Al(H_2O)_6]^{3+}$ in acidic water.
$AlCl_3 + 6H_2O \longrightarrow [Al(H_2O)_6]^{3+} + 3Cl^-$

137. (b) Silicones have $\text{-(}R_2SiO\text{)-}$ repeating unit.

138. (a)

139. (c) $\overset{1}{C}H_2{=}\overset{2}{C}(CH_3){-}\overset{3}{C}H{=}\overset{4}{C}H_2$
(Isoprene) (2-methyl-1, 3-butadiene)

140. (b) $CH_3{-}H \xrightarrow[100\text{ atm}]{Cu/523\text{ K}} CH_3{-}OH$

$CH_3{-}H \xrightarrow{MO_2O_3/\Delta} CH_2{=}O + H_2O$

$CH_3{-}CH\langle^{H}_{H} \xrightarrow{Mn(CH_3COO)_2} CH_3{-}COOH + H_2O$

141. (a) Bromine added to opposite faces of the double bond (*anti* addition).

$CH{\equiv}C{-}CH_2{-}CH{=}CH_2 \xrightarrow[CCl_4\ (253\text{ K})]{Br_2\ (1\text{ mol})} CH{\equiv}C{-}CH_2{-}CH(Br){-}CH_2(Br)$

142. (c)

143. (d) For $\pi_1 = \pi_2$

$$\Rightarrow (CRT)_1 = (CRT)_2 \Rightarrow \left[\frac{w}{M \times V}\right]_1 = \left[\frac{w}{M \times V}\right]_2$$

$\therefore \left[\frac{w}{V}\right]_1 = \left[\frac{w}{V}\right]_2 \Rightarrow \frac{w_1}{0.5} = \frac{9.2}{1}$

$\therefore w_1 = 9.2 \times 0.5 \Rightarrow w_1 = 4.6$ g

144. (N) Relation between molarity, molality and density can be given as

$$m = \frac{1000 \times M}{M \times M_w - 1000 \times d} = \frac{1000 \times 10}{10 \times 98 - 1000 \times 1.4}$$

= 23.8 m

145. (b) Molar conductivity of $MgCl_2$

$= \Lambda^0_m$ of $Mg^{2+} + 2\Lambda^0_m$ of Cl^-

$= (106 + 2 \times 76.3)$ S cm^2/mol = 258.6 S cm^2/mol

146. (b) Initial amount of reactant, $(a) = 6$ g

Final amount of reactant, $(x) = 3$ g

From rate constant equation,

$$k = \frac{2.303}{t}\log\frac{a}{a-x} \Rightarrow 1.15 \times 10^{-3} = \frac{2.303}{t}\log\frac{6}{3}$$

$$\Rightarrow t = \frac{2.303}{1.15 \times 10^{-3}}\log\frac{6}{3} \Rightarrow t = 603 \text{ s}$$

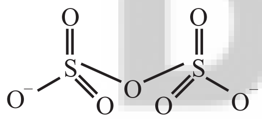
147. (b) $\frac{x}{m} = K \cdot p^{1/n}$

When, $1/n = 1$, Then, $\frac{x}{m} = K \cdot p$

148. (c) $\underset{\text{Flux}}{CaO} + \underset{\text{Gangue}}{SiO_2} \longrightarrow \underset{\text{Slag}}{CaSiO_3}$

149. (b)

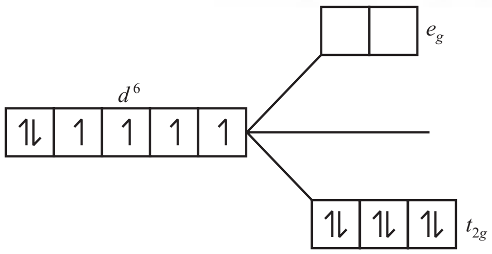
150. (d) $S_2O_7^{2-}$ (Pyrosulphate ion) possesses S—O—S bond.



151. (d) Electronic configuration of $_{57}$La = [Xe]$5d^4 6s^2$

Electronic configuration of $_{31}$Lu = [Xe]$4f^{14}5d^1$

152. (c) Cr(z = 24) [Ar]$3d^5\ 4s^1$ [$\because$ CO is strong field ligand]



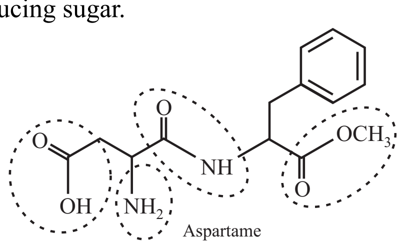
Electronic configuration of Cr in $[Cr(CO)_6]$ is $t^6_{2g}e^0_g$.

153. (a)

154. (b) Sucrose is made up of two monosaccharide glucose and fructose.

Carbonyl groups of glucose and fructose are involved in glycosidic bond formation. Hence, sucrose is a non-reducing sugar.

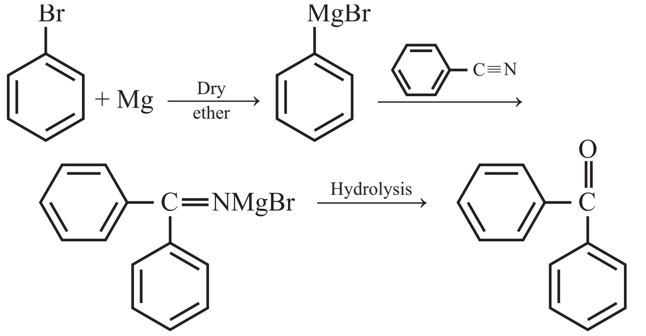
155. (b)



Aspartame

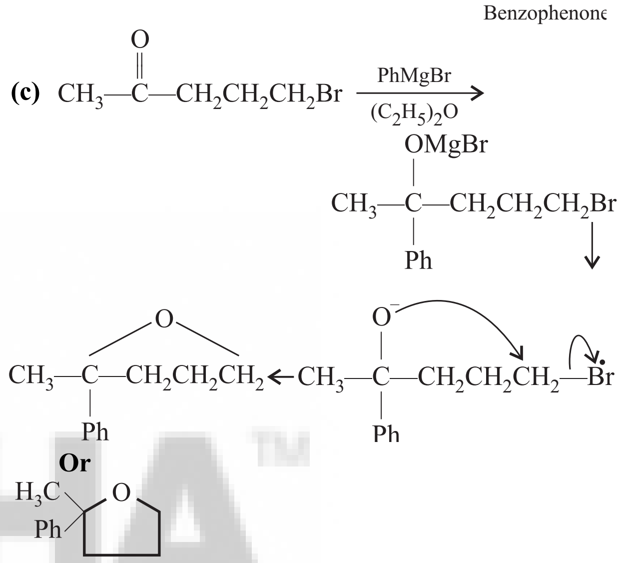
156. (b) It is not chiral due to presence of two $-C_2H_5$ groups.

157. (b)

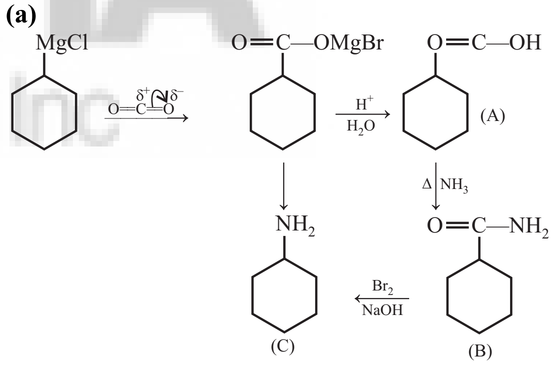


Benzophenone

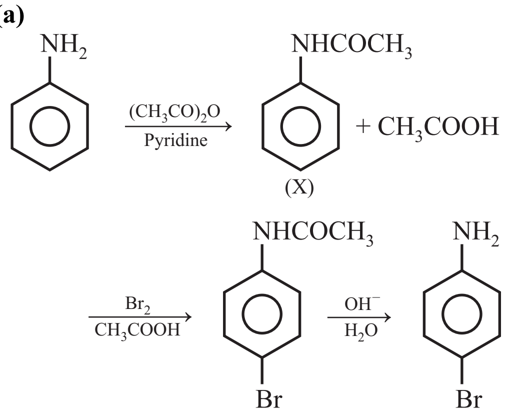
158. (c)



159. (a)



160. (a)



p-bormoaniline