TS/EAMCET Solved Paper 2017

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 marks.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. If tan20° = , then $\dfrac{\tan 160° - \tan 110°}{1 + (\tan 160°)(\tan 110°)} =$

(a) $\dfrac{1+\lambda^2}{2\lambda}$ (b) $\dfrac{1+\lambda^2}{\lambda}$ (c) $\dfrac{1-\lambda^2}{\lambda}$ (d) $\dfrac{1-\lambda^2}{2\lambda}$

2. Consider the circle $x^2 + y^2 - 6x + 4y = 12$. The equations of a tangent of this circle that is parallel to the line $4x + 3y + 5 = 0$ is

(a) $4x + 3y + 10 = 0$ (b) $4x + 3y - 9 = 0$

(c) $4x + 3y + 9 = 0$ (d) $4x + 3y - 31 = 0$

3. The mean deviation from the mean 10 of the data 6, 7, 11, 12, 13, α, 12, 16 is

(a) 3.5 (b) 3.25

(c) 3 (d) 3.75

4. Match the following

List I	**List II**
I. $\int_{-1}^{1} x\,\lvert x\rvert\,dx$	(A) $\dfrac{\pi}{2}$
II. $\int_0^{\frac{\pi}{2}} \left(1 + \log\left(\dfrac{4+3\sin x}{4+3\cos x}\right)\right) dx$	
	(B) $\int_0^{\frac{a}{2}} f(x)\,dx$
III. $\int_0^a f(x)\,dx$	(C) $\int_0^a [f(x) + f(-x)]\,dx$
IV. $\int_{-a}^{a} f(x)\,dx$	(D) 0
	(E) $\int_0^a f(a-x)\,dx$

(a) I-(D), II-(A), III-(E), IV-(C)

(b) I-(D), II-(A), III-(C), IV-(B)

(c) I-(D), II-(C), III-(A), IV-(E)

(d) I-(A), II-(D), III-(B), IV-(C)

5. If f is differentiable, $f(x+y) = f(x)\,f(y)$ for all $x, y \in IR$, $f(3) = 3$, $f'(0) = 11$, then $f'(3) =$

(a) $\dfrac{3}{11}$ (b) $\dfrac{11}{3}$ (c) 8 (d) 33

6. $\int_0^{\pi} \dfrac{x\,dx}{4\cos^2 x + 9\sin^2 x} =$

(a) $\dfrac{\pi^2}{12}$ (b) $\dfrac{\pi^2}{4}$ (c) $\dfrac{\pi^2}{6}$ (d) $\dfrac{\pi^2}{3}$

7. The probability distribution of a random variable X is given below

$X = k$	0	1	2	3	4
$P(X = k)$	0.1	0.4	0.3	0.2	0

The variance of X is

(a) 1.6 (b) 0.24 (c) 0.84 (d) 0.75

8. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & -1 & 4 \end{bmatrix}$, $A = B + C$, $B = B^T$ and $C = -C^T$, then $C =$

(a) $\begin{bmatrix} 0 & 0.5 & 0 \\ -0.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & -0.5 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -0.5 & 0.5 \\ 0.5 & 0 & 0 \\ -0.5 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0.5 & 0 \\ -0.5 & 0 & 0.5 \\ 0 & -0.5 & 0 \end{bmatrix}$

9. If **a** is a unit vector, then $|\mathbf{a}\times\hat{i}|^2+|\mathbf{a}\times\hat{j}|^2+|\mathbf{a}\times\hat{k}|^2=$

(a) 2 (b) 4 (c) 1 (d) 0

10. A bag contains 5 red balls, 3 black balls and 4 white balls. Three balls are drawn at random. The probability that they are not of same colour is

(a) $\frac{37}{44}$ (b) $\frac{31}{44}$ (c) $\frac{21}{44}$ (d) $\frac{41}{44}$

11. The radical centre of the circles $x^2+y^2-4x-6y+5=0$, $x^2+y^2-2x-4y-1=0$ and $x^2+y^2-6x-2y=0=0$ lies on the line

(a) $x+y-5=0$ (b) $2x-4y+7=0$
(c) $4x-6y+5=0$ (d) $18x-12y+1=0$

12. If $\operatorname{cosec}\theta-\cot\theta=2017$, then quadrant in which θ lies is

(a) I (b) IV (c) III (d) II

13. If $\int e^{2x} f'(x)\,dx = g(x)$, then

$\int (e^{2x} f(x)+e^{2x} f'(x))\,dx =$

(a) $\frac{1}{2}[e^{2x} f(x)-g(x)]+C$

(b) $\frac{1}{2}[e^{2x} f(x)+g(x)]+C$

(c) $\frac{1}{2}[e^{2x} f(2x)+g(x)]+C$

(d) $\frac{1}{2}[e^{2x} f'(x)+g(x)]+C$

14. If $A=(5, 3)$, $B=(3, -2)$ and a point P is such that the area of the traingle PAB is 9, then the locus of P represents

(a) a circle
(b) a pair of coincident lines
(c) a pair of parallel lines
(d) a pair of perpendicular lines

15. A straight line makes an intercept on the Y-axis twice as long as that on X-axis and is at unit distance from the origin. Then the line is represented by the equations

(a) $2x+3y=\pm\sqrt{5}$ (b) $x+y=\pm 2$
(c) $x+y=\pm 2$ (d) $2x+y=\pm\sqrt{5}$

16. Let S and S' be the foci of an ellipse and B be one end of its minor axis. If SBS' is a isosceles right angled triangle then the eccentricity of the ellipse is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$

17. For the parabola $y^2+6y-2x=-5$
I. the vertex is $(-2, -3)$
II. the directrix is $y+3=0$
Which of the following is correct?

(a) Both I and II are correct
(b) I is true, II is false
(c) Both I and II are false
(d) I is false, II is true

18. If $\frac{x^2+5}{(x^2+1)(x-2)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+1}$, then $A+B+C=$

(a) –1 (b) $\frac{2}{5}$ (c) $\frac{-3}{5}$ (d) 0

19. If the conjugate of $(x+iy)(1-2i)$ is $(1+i)$, then

(a) $x+iy=1-i$ (b) $x+iy=\frac{1-i}{1-2i}$
(c) $x-iy=\frac{1-i}{1+2i}$ (d) $x-iy=\frac{1-i}{1+i}$

20. $\int x^4 e^{2x}\,dx =$

(a) $\frac{e^{2x}}{4}(2x^4-4x^3+6x^2-6x+3)+C$

(b) $\frac{e^{2x}}{2}(2x^4-4x^3+6x^2-6x+3)+C$

(c) $\frac{e^{2x}}{8}(2x^4+4x^3+6x^2+6x+3)+C$

(d) $-\frac{e^{2x}}{4}(2x^4+4x^3+6x^2+6x+3)+C$

21. The sides of a triangle are in the ratio $1:\sqrt{3}:2$. Then the angles are in the ratio

(a) 1 : 2 : 3 (b) 1 : 2 : 4 (c) 1 : 4 : 5 (d) 1 : 3 : 5

22. The sum of the complex roots of the equations $(x-1)^3+64=0$ is

(a) 6 (b) 3 (c) $6i$ (d) $3i$

23. The area of the region bounded by the curves $x=y^2-2$ and $x=y$ is

(a) $\frac{9}{4}$ (b) 9
(c) $\frac{9}{2}$ (d) $\frac{9}{7}$

24. If $\mathbf{a}=\mathbf{x}\hat{i}+\mathbf{y}\hat{j}+\mathbf{z}\hat{k}$, then

$(\mathbf{a}\times\hat{i})\cdot(\hat{i}+\hat{j})+(\mathbf{a}\times\hat{j})\cdot(\hat{j}+\hat{k})+(\mathbf{a}\times\hat{k})\cdot(\hat{k}+\hat{i})=$

(a) $x-y+z$ (b) $x+y+z$
(c) $x+y-z$ (d) $-x+y+z$

25. If the imaginary part of $\frac{2z+1}{iz+1}$ is –2, then the locus of the point representing z in the complex plane is

(a) a circle (b) a parabola
(c) a straight line (d) an ellipse

26. Let $f:(-1,1)\to IR$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. If $g(x)=(f(2f(x)+2))^2$, then $g'(0) =$

(a) 0 (b) –2 (c) 4 (d) –4

27. If the perpendicular distance between the point (1, 1) to the line $3x + 4y + c = 0$ is 7, then the possible values of c are

(a) –35, 42 (b) 35, 28
(c) 42, –28 (d) 28, –42

28. The solution of $\frac{dy}{dx}=\frac{x+y}{x-y}$ is

(a) $\tan^{-1}\left(\frac{y}{x}\right)=\log\sqrt{x^2+y^2}+C$

(b) $\tan^{-1}\left(\frac{y}{x}\right)=\log\sqrt{x^2-y^2}+C$

(c) $\sin^{-1}\left(\frac{y}{x}\right)=\log\sqrt{x^2+y^2}+C$

(d) $\cos^{-1}\left(\frac{y}{x}\right)=\log\sqrt{x^2-y^2}+C$

29. If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\frac{d^2y}{dx^2}=$

(a) $-\frac{b^4}{a^2y^3}$ (b) $\frac{b^2}{ay^2}$ (c) $\frac{-b^3}{a^2y^3}$ (d) $\frac{b^3}{a^2y^2}$

30. $\lim_{y\to1}\left(\frac{1}{y^2-1}-\frac{2}{y^4-1}\right)=$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) 0

31. The solution of $(y-3x^2)dx+x\,dy=0$ is

(a) $y(x)=\sin x+\frac{1}{x^2}+C$ (b) $y(x)=\cos x-\frac{1}{x^2}+C$
(c) $y(x)=x^2+\frac{C}{x}$ (d) $y(x)=\sqrt{x}+\frac{C}{x}$

32. If the coefficients of $(2r + 1)^{th}$ term and $(r + 1)^{th}$ term in the expansion of $(1 + x)^{42}$ are equal then r can be

(a) 12 (b) 14 (c) 16 (d) 20

33. A point on the plane that passes through the points (1, –1, 6), (0, 0, 7) and perpendicular to the plane $x - 2y + z = 6$ is

(a) (1, –1, 2) (b) (1, 1, 2)
(c) (–1, 1, 2) (d) (1, 1, –2)

34. If the slope of the tangent of the curve $y=ax^3+bx+4$ at (2, 14) is 21, then the values of a and b respectively

(a) 2, –3 (b) 3, –2 (c) –3, –2 (d) 2, 3

35. The probability distribution of a random variable X is given below

x	1	2	3	4	5	6
$P(X = x)$	a	ea	a	b	b	0.3

If mean of X is 4.2, then a and b are respectively equal to

(a) 0.3, 0.2 (b) 0.1, 0.4 (c) 0.1, 0.2 (d) 0.2, 0.1

36. Let $f(x)$ be a quadratic expression such that $f(0)+f(1)=0$. If $f(-2) = 0$, then

(a) $f\left(\frac{-2}{5}\right)=0$ (b) $f\left(\frac{2}{5}\right)=0$
(c) $f\left(\frac{-3}{5}\right)=0$ (d) $f\left(\frac{3}{5}\right)=0$

37. The equation of tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a, b) is

(a) $\frac{x}{a}=-\frac{y}{b}$ (b) $\frac{x}{a}+\frac{y}{b}=2$
(c) $\frac{x}{a}=\frac{y}{b}$ (d) $\frac{x}{a}+\frac{y}{b}=n$

38. If the line $x + y + k = 0$ is a normal to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ then $k =$

(a) $\pm\frac{\sqrt{5}}{13}$ (b) $\pm\frac{13}{\sqrt{5}}$
(c) $\pm\frac{13}{5}$ (d) $\pm\frac{5}{13}$

39. The product of all the real roots of $x^2-8x+9-\frac{8}{x}+\frac{1}{x^2}=0$ is

(a) 2 (b) 1 (c) 3 (d) 7

40. If $\Delta=\begin{vmatrix}1&5&6\\0&1&7\\0&0&1\end{vmatrix}$ and $\Delta'=\begin{vmatrix}1&0&1\\3&0&3\\4&6&100\end{vmatrix}$, then

(a) $\Delta^2-3\Delta'=0$

(b) $(\Delta+\Delta')^2-3(\Delta+\Delta')+2=0$

(c) $(\Delta+\Delta')^2+3(\Delta+\Delta')+5=0$

(d) $\Delta+3\Delta'+1=0$

41. A village has 10 players. A team of 6 players is to be formed. 5 members are chosen out of these 10 players and then the captain is chosen from the remaining players. Then total number of ways of choosing such teams is

(a) 1260 (b) 210 (c) $({}^{10}C_6)\ 5!$ (d) $({}^{10}C_5)\ 6$

42. The equation of the straight line passing through the point of intersection of $5x - 6y - 1$, $3x + 2y + 5 = 0$ and perpendicular to the line $3x - 5y + 11 = 0$ is

(a) $5x + 3y + 18 = 0$ (b) $-5x - 3y + 18 = 0$

(c) $5x + 3y + 8 = 0$ (d) $5x + 3y - 8 = 0$

43. An integer is chosen from $\{2k / -9 \le k \le 10\}$. The probability that it is divisible by both 4 and 6 is

(a) $\frac{1}{10}$ (b) $\frac{1}{20}$

(c) $\frac{1}{4}$ (d) $\frac{3}{20}$

44. $\int \frac{dx}{x(x^4+1)} =$

(a) $\frac{1}{4}\log\left(\frac{x^4+1}{x^4}\right)+C$ (b) $\frac{1}{4}\log\left(\frac{x^4}{x^4+1}\right)+C$

(c) $\frac{1}{4}\log(x^4+1)+C$ (d) $\frac{1}{4}\log\left(\frac{x^4}{x^4+2}\right)+C$

45. $\sin^{-1}\frac{\sqrt{3}}{2}+\sin^{-1}\sqrt{\frac{2}{3}} =$

(a) $\sin^{-1}\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}$

(b) $\pi - \sin^{-1}\left(\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\right)$

(c) $-\pi - \sin^{-1}\left(\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\right)$

(d) $\pi + \sin^{-1}\left(\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\right)$

46. α and β are the roots of $x^2 + 2x + c = 0$. If $\alpha^3 + \beta^3 = 4$, then the value of c is

(a) –2 (b) 3 (c) 2 (d) 4

47. If the slope of the tangent of the circle $S \equiv x^2 + y^2 - 13 = 0$ at (2, 3) is m, then the point $\left(m, \frac{-1}{m}\right)$ is

(a) an external point with respect to the circle $S = 0$

(b) an internal point with respect to the circle $S = 0$

(c) the centre of the circle $S = 0$

(d) a point on the circle $S = 0$

48. Using the letters of the word TRICK, a five letter word with distinct letters is formed such that C is in the middle. In how many ways this is possible?

(a) 6 (b) 120 (c) 24 (d) 72

49. The angle between the curves $x^2 = 8y$ and $xy = 8$ is

(a) $\tan^{-1}\left(\frac{-1}{3}\right)$ (b) $\tan^{-1}(-3)$

(c) $\tan^{-1}(-\sqrt{3})$ (d) $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

50. $f:(-\infty,0] \to [0,\infty)$ is defined as $f(x) = x^2$. The domain and range of its inverse is

(a) Domain $(f^{-1}) = [0, \infty)$. Range of $(f^{-1}) = (-\infty, 0]$

(b) Domain of $(f^{-1}) = [0, \infty)$. Range of $(f^{-1}) = (-\infty, \infty]$

(c) Domain of $(f^{-1}) = [0, \infty)$. Range of $(f^{-1}) = (0, \infty)$

(d) f^{-1} does not exist

51. If **a**, **b** and **c** are unit vectors such that **a** + **b** + **c** = 0 and $(\mathbf{a}, \mathbf{b}) = \frac{\pi}{3}$, then $|\mathbf{a}\times\mathbf{b}| + |\mathbf{b}\times\mathbf{c}| + |\mathbf{c}\times\mathbf{a}| =$

(a) $\frac{3}{2}$ (b) 0

(c) $\frac{3\sqrt{3}}{2}$ (d) 3

52. The differential equaiton of the simple harmonic motion given by $x = A\cos(nt+\alpha)$ is

(a) $\frac{d^2x}{dt^2} - n^2x = 0$ (b) $\frac{d^2x}{dt^2} + n^2x = 0$

(c) $\frac{dx}{dt} - \frac{d^2x}{dt^2} = 0$ (d) $\frac{d^2x}{dt^2} - \frac{dx}{dt} + nx = 0$

53. If **a** and **b** are unit vectors and α is the angle between them, then **a** + **b** is a unit vector when $\cos\alpha =$

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) $-\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{2}$

54. A parallelogram has vertices A (4, 4, –1), B(5, 6, –1), C(6, 5, 1) and $D(x, y, z)$. Then the vertex D is

(a) (5, 1, 0) (b) (–5, 0, 1) (c) (5, 3, 1) (d) (5, 1, 3)

55. If $2x^2 - 10xy + 2\lambda y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, then point of intersection of those lines is

(a) (2, –3) (b) (5, –16)

(c) $\left(-10, \frac{-7}{2}\right)$ (d) $\left(-10, \frac{-3}{2}\right)$

56. If rank of $\begin{pmatrix} x & x & x \\ x & x^2 & x \\ x & x & x+1 \end{pmatrix}$ is 1, then

(a) $x = 0$ or $x = 1$ (b) $x = 1$
(c) $x = 0$ (d) $x \neq 0$

57. If the vectors $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$, $\mathbf{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\mathbf{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$ are coplanar, then $x =$

(a) 1 (b) 2 (c) 0 (d) –2

58. In order to eliminate the first degree terms from the equation
$4x^2 + 8xy + 10y^2 - 8x - 44y + 14 = 0$ the point to which the origin has to be shifted is

(a) (–2, 3) (b) (2, –3) (c) (1, –3) (d) (–1, 3)

59. Two circles of equal radius a cut orthogonally. If their centres are (2, 3) and (5, 6) then radical axis of these circles passes through the point

(a) $(3a, 5a)$ (b) $(2a, a)$ (c) $\left(a, \frac{5a}{3}\right)$ (d) (a, a)

60. If $\tan\theta_1 = k\cot\theta_2$, then $\frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)} =$

(a) $\frac{1+k}{1-k}$ (b) $\frac{1-k}{1+k}$
(c) $\frac{k+1}{k-1}$ (d) $\frac{k-1}{k+1}$

61. Let $\mathbf{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\mathbf{b} = \hat{i} + 3\hat{j} + 2\hat{k}$. Then the volume of the parallelopiped having coterminous edges as **a, b** and **c**, where **c** is the vector perpendicular to the plane of **a, b** and | **c** | = 2 is

(a) $2\sqrt{195}$ (b) 24
(c) $\sqrt{200}$ (d) $\sqrt{195}$

62. The local maximum of $y = x^3 - 3x^2 + 5$ is attained at

(a) $x = 0$ (b) $x = 2$ (c) $x = 1$ (d) $x = -1$

63. In the expansion of $(1 + x)^n$, the coefficients of pth and $(p + 1)$th terms are respectively p and q then $p + q =$

(a) $n + 3$ (b) $n + 2$ (c) n (d) $n + 1$

64. If $f(x) = \begin{cases} \sin x & \text{if } x \leq 0 \\ x^2 + a^2 & \text{if } 0 < x < 1 \\ bx + 2 & \text{if } 1 \leq x \leq 2 \\ 0 & \text{if } x > 2 \end{cases}$ is continuous on IR, then $a + b + ab =$

(a) –2 (b) 0 (c) 2 (d) –1

65. If $\cosh^{-1} x = 2\log_c(\sqrt{2}+1)$, then $x =$

(a) 1 (b) 2 (c) 4 (d) 3

66. For any integer $n \geq 1$, $\sum_{K=1}^{n} K(K+2) =$

(a) $\frac{n(n+1)(n+2)}{6}$ (b) $\frac{n(n+1)(2n+7)}{6}$
(c) $\frac{n(n+1)(2n+1)}{6}$ (d) $\frac{n(n-1)(2n+8)}{6}$

67. The foci of the ellipse
$25x^2 + 4y^2 + 100x - 4y + 100 = 0$ are

(a) $\left(\frac{5 \pm \sqrt{21}}{10}, -2\right)$ (b) $\left(-2, \frac{5 \pm \sqrt{21}}{10}\right)$
(c) $\left(\frac{2 \pm \sqrt{21}}{10}, -2\right)$ (d) $\left(-2, \frac{2 \pm \sqrt{21}}{10}\right)$

68. $\left[\frac{1+\cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)}{1+\cos\left(\frac{\pi}{12}\right) - i\sin\left(\frac{\pi}{12}\right)}\right]^{72} =$

(a) 0 (b) –1 (c) 1 (d) $\frac{1}{2}$

69. If the range of the function $f(x) = -3x - 3$ is {3, –6, –9, –18}, then which of the following elements is not in the domain of f?

(a) –1 (b) –2 (c) 1 (d) 2

70. In ΔABC, if $a = 1, b = 2, \angle C = 60°$ then $4\Delta^2 + c^2 =$

(a) 6 (b) 3 (c) $\frac{\sqrt{3}}{2}$ (d) 9

71. If the magnitudes of **a**, **b** and **a + b** are respectively 3, 4 and 5, then the magnitude of (**a – b**) is

(a) 3 (b) 4 (c) 6 (d) 5

72. If $\int f(x)\cos x\,dx = \frac{1}{2}(f(x))^2 + C$ and $f(0) = 0$, then $f'(0) = ?$

(a) 1 (b) –1 (c) 0 (d) 2

73. If α and β are the roots of the equation $ax^2 + bx + c = 0$ and the equation having roots $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$ is $px^2 + qx + r = 0$, then $r =$

(a) $a + 2b$ (b) $ab + bc + ca$
(c) $a + b + c$ (d) abc

74. If $A\left(\frac{\pi}{3}\right)$, $B\left(\frac{\pi}{6}\right)$ are the points on the circle represented in parametric from with centre (0, 0) and radius 12 then the length of the chord AB is

(a) $6(\sqrt{6}-\sqrt{2})$ (b) $6(\sqrt{6}-\sqrt{3})$
(c) $\sqrt{2}(\sqrt{3}-1)$ (d) $6(\sqrt{3}-1)$

75. If the pair of straight lines $xy - x - y + 1 = 0$ and the line $x + ay - 3 = 0$ are concurrent, then the acute angle between the pair of lines $ax^2 - 13xy - 7y^2 + x + 23y - 6 = 0$ is

(a) $\cos^{-1}\left(\frac{5}{\sqrt{218}}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$

(c) $\cos^{-1}\left(\frac{5}{\sqrt{173}}\right)$ (d) $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

76. The number of solutions of $\cos 2\theta = \sin\theta$ in $(0, 2\pi)$ is

(a) 4 (b) 3 (c) 2 (d) 5

77. The lengths of the sides of a triangle are 13, 14 and 15. If R and r respectively denote circumradius and inradius of that triangle, then $8R + r =$

(a) 84 (b) $\frac{65}{8}$ (c) 4 (d) 69

78. If A and B are variances of the 1^{st} 'n' even numbers and 1^{st} 'n' odd numbers respectively then

(a) $A = B$ (b) $A > B$ (c) $A < B$ (d) $A = B - 1$

79. If the line $x - y = -4k$ is a tangent to the parabola $y^2 = 8x$ at P, then the perpendicular distance of normal at P from $(k, 2k)$ is

(a) $\frac{5}{2\sqrt{2}}$ (b) $\frac{7}{2\sqrt{2}}$

(c) $\frac{9}{2\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

80. If A and B are events having probabilities, $P(A) = 0.6$, $P(B) = 0.4$ and $P(A \cap B) = 0$, then probability that neither A nor B occurs is

(a) $\frac{1}{4}$ (b) 1

(c) $\frac{1}{2}$ (d) 0

PHYSICS

81. A force **F** is applied on a square plate of length L. If the percentage error in the determination of L is 3% and in F is 4%, then permissible error in the calculation of pressure is

(a) 13% (b) 10% (c) 7% (d) 12%

82. A positive charge Q is placed on a conducting spherical shell with inner radius R_1 and outer radius R_2. A particle with charge q is placed at the center of the spberical cavity. The magnitude of the electric field at a point in the cavity, a distance r from center is

(a) zero (b) $\frac{Q}{4\pi\varepsilon r^2}$ (c) $\frac{Q}{4\pi\varepsilon_0 r^2}$ (d) $\frac{(q+Q)}{4\pi\varepsilon_0 r^2}$

83. A swimmer wants to cross a 200 m wide river which is flowing at a speed of 2 m/s. The velocity of the swimmer with respect to the river is 1 m/s. How far from the point directly opposite to the starting point does the swimmer reach the opposite bank?

(a) 200 m (b) 400 m (c) 600 m (d) 800 m

84. A coil having n turns and resistance $R\Omega$ is connected with a galvanometer of resistance $4R\Omega$. This combination is moved in time t seconds from a magnetic flux ϕ_1 Weber to ϕ_2 Weber. The induced current in the circuit is

(a) $\frac{\phi_2 - \phi_1}{5Rnt}$ (b) $-\frac{n(\phi_2 - \phi_1)}{5Rt}$

(c) $-\frac{(\phi_2 - \phi_1)}{Rnt}$ (d) $-\frac{n(\phi_2 - \phi_1)}{Rt}$

85. A simple pendulum of length 1 m is freely suspended from the ceiling of an elevalor. The time period of small oscillations as the elevator moves up with an acceleration of 2 m/s^2 is (use g = 10 m/s^2)

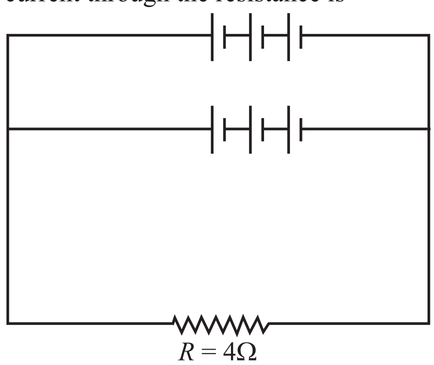
(a) $\frac{\pi}{\sqrt{5}}$ s (b) $\sqrt{\frac{2}{5}}\pi$ s (c) $\frac{\pi}{\sqrt{2}}$ s (d) $\frac{\pi}{\sqrt{3}}$ s

86. Consider a metal ball of radius r moving at a constant velocity v in a uniform magnetic field of induction $\overline{B}$. Assuming that the direction of velocity forms an angle α with the direction of $\overline{B}$, the maximum potential difference between points on the ball is

(a) $r|\overline{B}||\overline{v}|\sin\alpha$ (b) $|\overline{B}||\overline{v}|\sin\alpha$

(c) $2r|\overline{B}||\overline{v}|\sin\alpha$ (d) $2r|\overline{B}||\overline{v}|\cos\alpha$

87. Each of the six ideal batteries of emf 20 V is connected to an external resistance of 4Ω as shown in the figure. The current through the resistance is



(a) 6 A (b) 3 A (c) 4 A (d) 5 A

88. The energy that should be added to an electron to reduce ils de-Broglie wavelength from 1 nm to 0.5 nm is

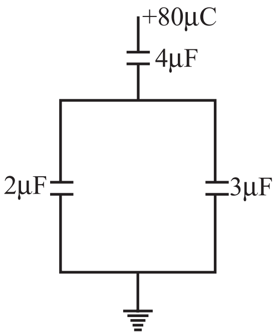
(a) four-limes the Initial energy

(b) equal to the initial energy

(c) two-times the initial energy

(d) three-times the initial energy

89. In the given circuit, a charge of +80 μC is given to upper plate of a 4μF capacitor. At steady state, the charge on the upper plate of the 3μF capacitor is



(a) 60 μC (b) 48 μC (c) 80 μC (d) 0 μC

90. The Young's modulus of a material is 2×10^{11} N/m^2 and its elastic limit is 1×10^8 N/m^2. For a wire of 1 m length of this material, the maximum elongation achievable is

(a) 0.2 mm (b) 0.3 mm (c) 0.4 mm (d) 0.5 mm

91. A wooden box lying at rest on an inclined surface of a wet wood is held at static equilibrium by a constant force **F** applied perpendicular to the incline. If the mass of the box is 1 kg, the angle of inclination is 30° and the coefficient of static friction between the box and the inclined plane is 0.2, the minimum magnitude of **F** is (Use g =10 m /s^2)

(a) 0 N, as 30° is less than angle of repose

(b) ≥ 1 N

(c) ≥ 3.3 N

(d) ≥ 16.3 N

92. A meter scale made of steel, reads accurately at 25°C. Suppose in an experiment an accuracy of 0.06 mm in 1 m is reqnired, the range of temperature in which the experiment can be performed with this meter scale is (Coefficient of linear expansion of steel is 11×10^{-6}/°C)

(a) 19°C to 31°C (b) 25°C to 32°C

(c) 18°C to 25°C (d) 18°C to 32°C

93. Consider a solenoid carrying current supplied by a DC source with a constant emf containing iron core inside it. When the core is pulled out of the solenoid, the change in current will

(a) remain same (b) decrease

(c) increase (d) modulate

94. A parallel beam of light of intensity I_0 is incident on a coated glass plate. If 25% of the incident light is reflected from the upper surface and 50% of light is reflected from the lower surface of the glass plate, the ratio of maximum to minimum intensity in the interference region of the reflected light is

(a) $\left(\dfrac{\frac{1}{2}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ (b) $\left(\dfrac{\frac{1}{4}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$

(c) $\dfrac{5}{8}$ (d) $\dfrac{8}{5}$

95. A thermocol box has a total wall area (including the lid) of 1.0 m^2 and wall thickness of 3 cm. It is filled with ice at 0°C. If the average temperature outside the box is 30°C throughout the day, the amount of ice that melts in one day is
[Use $K_{\text{thermocol}}$ = 0.03 W/mK, $L_{\text{fusion (ice)}}$ = 3.00×10^5 J/kg]

(a) 1 kg (b) 2.88 kg

(c) 25.92 kg (d) 8.64 kg

96. Which of the following is emitted, when $^{239}_{94}\text{Pu}$ decays into $^{235}_{92}\text{U}$?

(a) Gamma ray (b) Neutron

(c) Electron (d) Alpha particle

97. An AC generator 10 V (rms) at 300 rad/s is connected in series with a 50 Ω resistor, a 400mH inductor and a 200 μF capacitor. The rms voltage across the inductor is

(a) 2.5 V (b) 3.4 V (c) 6.7 V (d) 10.8 V

98. A wire has resistance of 3.1 Ω at 30°C and 4.5Ω at 100°C. The temperature coefficient of resistance of the wire is

(a) 0.0012°C^{-1} (b) 0.0024°C^{-1}

(c) 0.0032°C^{-1} (d) 0.0064°C^{-1}

99. An object is thrown vertically upward with a speed of 30 m/s. The velocity of the object half-a-second before it reaches the maximum height is

(a) 4.9 m/s (b) 9.8 m/s (c) 19.6 m/s (d) 25.1 m/s

100. An electron collides wilh a hydrogen atom in its ground state and excites it to n = 3 state. The energy given to the hydrogen atom in this inelastic collision (neglecting the recoil of hydrogen atom) is

(a) 10.2 eV (b) 12.1 eV

(c) 12.5 eV (d) 13.6 eV

101. Consider the motion of a particle described by $x = a \cos t$, $y = a \sin t$ and $z = t$. The trajectory traced by the particle as a function of time is

(a) helix (b) circular

(c) elliptical (d) straight line

102. Consider a reversible engine of coefficiency $\frac{1}{6}$. When the temperature of the sink is reduced by 62°C, its efficiency gets doubled, The temperature of the source and sink respectively are

(a) 372 K and 310 K (b) 273 K and 300 K

(c) 99°C and 10°C (d) 200°C and 37°C

103. Consider a light source placed at a distance of 1.5 m along the axis facing the convex side of a spherical mirror of radius of curvature 1 m. The position (s'), nature and magnification (m) of the image are

(a) $s' = 0.375$ m, virtual, upright, $m = 0.25$
(b) $s' = 0.375$ m, real, inverted, $m = 0.25$
(c) $s' = 3.75$ m, virtual, inverted, $m = 2.5$
(d) $s' = 3.75$ m, real, upright, $m = 2.5$

104. An office room contains about 2000 moles of air, The change in the internal energy of this much air when it is cooled from 34°C to 24°C at a constant pressure of 1.0 atm is
[Use $\gamma_{air} = 1.4$ and universal gas constant = 8.314 J/mol-K]

(a) -1.9×10^5 J (b) $+1.9 \times 10^5$ J
(c) -42×10^5 J (d) $+0.7 \times 10^5$ J

105. A ball is throw at a speed of 20 m/s at an angle of 30° with the horizontal. The maximum height reached by the ball is
(Use $g = 10$ m/s^2)

(a) 2 m (b) 3 m (c) 4 m (d) 5 m

106. A horizontal pipeline carrying gasoline has a cross-sectional diameter of 5 mm. 1f the viscosity and density of the gasoline are 6×10^{-3} Poise and 720 kg/m^3 respectively, the velocity after which the flow becomes turbulent is

(a) > 1.66 m/s (b) > 3.33 m/s
(c) > 1.6×10^{-3} m/s (d) > 0.33 m/s

107. A piece of copper and a piece of germanium are cooled from room temperature to 80K, Then, which one of the following is correct?

(a) Resistance of each will increase
(b) Resistance of each will decrease
(c) Resistance of copper will decrease while that of germanium will increase
(d) Resistance of copper will increase while that of germanium will decrease

108. A beam of light propagating at an angle α_1 from a medium 1 through to another medium 2 at an angle α_2. If the wavelength of light in medium 1 is λ_1, then the wavelength of light in menium 2, (λ_2), is

(a) $\frac{\sin\alpha_2}{\sin\alpha_1}\lambda_1$ (b) $\frac{\sin\alpha_1}{\sin\alpha_2}\lambda_2$

(c) $\left(\frac{\alpha_1}{\alpha_2}\right)\lambda_1$ (d) λ_1

109. An amplitude modulated signal consists of a message signal of frequency 1 KHz and peak voltage of 5V, modulating a carrier frequency of 1 MHz and peak voltage of 15 V. The cor eet description of this signal is

(a) $5[1+3\sin(2\pi 10^6 t)]\sin(2\pi 10^3 t)$

(b) $15\left[1+\frac{1}{3}\sin(2\pi 10^3 t)\right]\sin(2\pi 10^6 t)$

(c) $[5+15\sin(2\pi 10^3 t)]\sin(2\pi 10^6 t)$

(d) $[15+5\sin(2\pi 10^6 t)]\sin(2\pi 10^3 t)$

110. Which of the following principles is being used in Sonar Technology?

(a) Newton's laws of motion
(b) Reflection of electromagnetic waves
(c) Law's of thermodynamics
(d) Reflection of ultrasonic waves

111. A particle of mass M is moving in a horizontal circle of radius R with uniform speed v. When the particle moves from one point to a diametrically opposite point, its

(a) momentum does not change
(b) momentum changes by $2Mv$
(c) kinetic energy changes by $\frac{Mv^2}{4}$
(d) kinetic energy changes by Mv^2

112. A billiard ball of mass M, moving with velocity v_1 collides with another ball of the same mass but at rest. If the collision is elastic, the angle of divergence after the collision is

(a) 0° (b) 30° (c) 90° (d) 45°

113. A planet of mass m moves in a elliptical orbit around an unknown star of mass M such that its maximum and minimum distances from the star are equal to r_1 and r_2 respectively. The angular momentum of the planet relative to the centre of the star is

(a) $m\sqrt{\frac{2GMr_1r_2}{r_1+r_2}}$ (b) 0

(c) $m\sqrt{\frac{2GM(r_1+r_2)}{r_1r_2}}$ (d) $\sqrt{\frac{2GMmr_1}{(r_1+r_2)r_2}}$

114. Consider a frictionless ramp on which a smooth object is made to slide down from an initial beight h. The distance d necessary to stop the object on a flat track (of coefficient of friction μ), kept at the ramp end is

(a) h / μ (b) μh
(c) $\mu^2 h$ (d) $h^2 \mu$

115. A generator with a circular coil of 100 turns of area 2×10^{-2} m^2 is immersed in a 0.01 T magnetic field and rotated at a frequency of 50 Hz, The maximum emf which is produced during a cycle is

(a) 6.28 V (b) 3.44 V (c) 10 V (d) 1.32 V

116. A sound wave of frequency v Hz initially travels a distance of 1 km in air. Then, it gets reflected into a water reservoir of depth 600 m. The frequency of the wave at the bottom of the reservoir is

(V_{air} = 340 m/s V_{water} = 1484 m /s)

(a) $> v$ Hz

(b) $< v$ Hz

(c) v Hz

(d) 0 (the sound wave gets attenuated by water completely)

117. Which of the following statement is not true?

(a) the resistance of an intrinsic semiconductor decreases with increase in temperature

(b) doping pure Si with trivalent impurities gives p-type semiconductor

(c) the majority carriers in n-type semiconductors are holes

(d) a $p-n$ junction can act as a semiconductor diode

118. The deceleration of a car traveling on a straight highway is a function of its instantaneous velocity v given by $\omega = a\sqrt{v}$, , where a is a constant. If the initial velocity of the car is 60 km/h, the distance of the car will travel and the time it takes before it stops are

(a) $\frac{2}{3}$ m, $\frac{1}{2}$ s (b) $\frac{3}{2a}$ m, $\frac{1}{2a}$ s

(c) $\frac{3a}{2}$ m, $\frac{a}{2}$ s (d) $\frac{2}{3a}$ m, $\frac{2}{a}$ s

119. A current carrying wire in its neighbourhood produces

(a) electric field

(b) electric and magnetic fields

(c) magnetic field

(d) no field

120. Consider a particle on which constant forces $\mathbf{F}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$ N and $\mathbf{F}_2 = 4\hat{i} - 5\hat{j} - 2\hat{k}$ N act together resulting in a displacement from position $\mathbf{r}_1 = 20\hat{i} + 15\hat{j}$ cm to $\mathbf{r}_2 = 7\hat{k}$ cm. The total work done on the particle is

(a) – 0.48 J (b) + 0.48 J (c) – 4.8 J (d) + 4.8 J

CHEMISTRY

121. Which of the following conditions are correct for real solutions showing negative deviation from Raoult's law?

(a) $\Delta H_{Mix} < 0; \Delta V_{Mix} > 0$

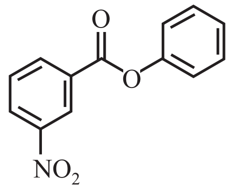
(b) $\Delta H_{Mix} > 0; \Delta V_{Mix} > 0$

(c) $\Delta H_{Mix} > 0; \Delta V_{Mix} < 0$

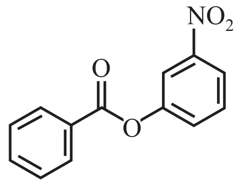
(d) $\Delta H_{Mix} < 0; \Delta V_{Mix} < 0$

122. Nitration of phenyl benzoate yields the product

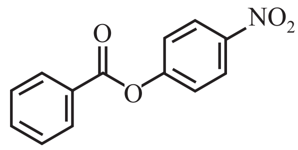
(a)



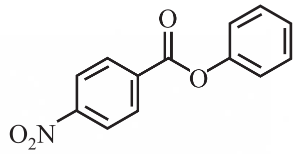
(b)



(c)



(d)



123. The electronic configuration of $_{59}$Pr (praseodimium) is

(a) $[_{54}Xe]4f^2 5d^1 6s^2$ (b) $[_{54}Xe]4f^1 5d^1 6s^2$

(c) $[_{54}Xe]4f^3 6s^2$ (d) $[_{54}Xe]4f^3 5d^2$

124. Which of the following is the most basic oxide?

(a) SO_3 (b) SeO_3

(c) PoO (d) TeO

125. The element that forms stable compounds in low oxidation state is

(a) Mg (b) Al

(c) Ga (d) Tl

126. Atomic radius (pm) of Al, Si, N and F respectively is

(a) 117, 143, 64, 74 (b) 143, 117, 74, 64

(c) 143, 47, 64, 74 (d) 64, 74, 117, 143

127. Reaction of calgon with hard water containing Ca^{2+} ions produce

(a) $[Na_2CaP_6O_{18}]^{2-}$ (b) $Ca_2(PO_4)_3$

(c) $CaCO_3$ (d) $CaSO_4$

128. Which of the following statement(s) is/are true

(a) The pressure of a fixed amount of an ideal gas is proportional to its temperature only

(b) Frequency of collisions increases in proportion to the square root of temperature

(c) The value of van der Waal's constant 'a' is smaller for ammonia than for nitrogen

(d) If a gas is expanded at constant temperature, the kinetic energy of the molecules decrease

129. Conversion of esters to aldehydes can be accomplished by

(a) Stephen reduction
(b) Rosenmund reduction
(c) Reduction with lithium aluminium hydride
(d) Reduction with disobutyl aluminium hydride

130. Consider the following electrode processes of a cell,

$$Cl^- \rightarrow \frac{1}{2}Cl_2 + e^-,\ MCl + e^- \rightarrow M + Cl^-$$

If EMF of this cell is –1140V and $E°$ value of the cell is –0.55 V at 298 K, the value of the equilibrium constant of the sparingly soluble salt MCl is in the order of

(a) 10^{-10} (b) 10^{-8} (c) 10^{-7} (d) 10^{-11}

131. Which of the following is true for spontaneous adsorption of H_2 gas without dissociation on solid surface

(a) Process is exothermic and $\Delta S < 0$
(b) Process is endothermic and $\Delta S > 0$
(c) Process is exothermic and $\Delta S > 0$
(d) Process is endothermic and $\Delta S < 0$

132. Consider the single electrode process $4H^+ + 4e^- \rightleftharpoons 2H_2$ catalysed the platinum black electrode in HCl electrolyte. The potential of the electrode is –0.059 V. SHE. What is the concentration of the acid in the hydrogen half cell if the H_2 pressure is 1 bar?

$$4H^+ + 4e^- \rightleftharpoons 2H_2$$

(a) 1 M (b) 10 M (c) 0.1 M (d) 0.01 M

133. Which of the following elements has the lowest melting point?

(a) Sn (b) Pb (c) Si (d) Ge

134. The number of complementary hydrogen bond(s) between a guanine and cytosine pair is

(a) 2 (b) 1 (c) 4 (d) 3

135. Given ΔH_r° for CO_2 (g), CO (*g*) and H_2O (*g*) are –393.5, –110.5 and –241.8 kJ mol^{-1}, respectively. The ΔH_r° (in kJ mol^{-1}) for the reaction

$$CO_2(g) + H_2(g) \longrightarrow CO(g) + H_2O(g) \text{ is}$$

(a) 524.1 (b) –262.5
(c) –41.7 (d) 41.2

136. Which among the following is the strongest acid?

(a) HF (b) HCl
(c) HBr (d) HI

137. The species having pyramidal shape according to VSEPR theory is

(a) SO_3 (b) BrF_3
(c) SiO_3^{2-} (d) OSF_2

138. The bonding in diborane (B_2H_6) can be described by

(a) 4 two centre - two electron bonds and 2 three - centre - two electron bonds
(b) 3 two centre - two electron bonds and 3 three - centre - two electron bonds
(c) 2 two centre - two electron bonds and 4 three - centre - two electron bonds
(d) 4 two centre - two electron bonds and 4 three - centre - two electron bonds

139. The monomers of buna-S rubber are

(a) Isoprene and butadiene
(b) Butadiene and phenol
(c) Styrene and butadiene
(d) Vinyl chloride and sulphur

140. Heating a mixture of Cu_2O and Cu_2S will give

(a) CuO + CuS (b) $Cu + SO_3$
(c) $Cu + SO_2$ (d) $Cu(OH)_2 + CuSO_4$

141. Which of the following corresponds to the energy of the possible excited state of hydrogen?

(a) –13.6 eV (b) 13.6 eV
(c) –3.4 eV (d) 3.4 eV

142. Which of the following are the correct representations of a zero order reaction, where *A* represents the reactant?

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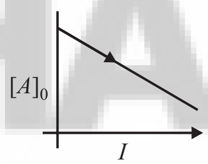
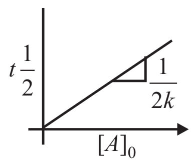
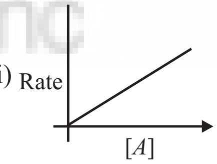
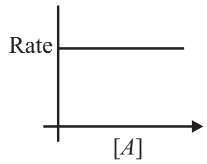


(ii)


(iii)


(iv)


(a) i, ii, iii (b) i, ii, iv (c) ii, iii, iv (d) i, iii, ii

143. The set representing the right order of ionic radius is

(a) $Li^+ > Na^+ + > Mg^{2+} > Be^{2-}$
(b) $Mg^{2-} > Be^{2+} > Li^- > Na^+$
(c) $Na^+ > Mg^{2+} > Li^+ > Be^{2+}$
(d) $Na^+ > Li^+ > Mg^{2+} > Be^{2+}$

144. Which one of the following statement is correct for d^4 ions [P = pairing energy]

(a) When $\Delta_0 > P$, low-spin complex form
(b) When $\Delta_0 < P$, low-spin complex form
(c) When $\Delta_0 > P$, high-spin complex form
(d) When $\Delta_0 < P$, both high-spin and low-spin complexes form

145. The reactivity of alkyl bromides.

I. CH_3CH_2Br II. $CH_3-CH(CH_3)-Br$

III. $CH_3-C(CH_3)_2-Br$ IV. CH_3Br

Towards iodide ion in dry acetone decrease in the order

(a) IV > I > II > III (b) I > IV > II > III

(c) III > II > I > IV (d) III > II > IV > I

146. Optically active $CH_3-CH_2-CH(OH)-CH_3$ was found to have lost its optically activity after standing in water containing a few drops of acids, mainly due to the formation of

(a) $CH_3-CH_2-CH=CH_2$

(b) $CH_3-CH=CH-CH_3$

(c) $CH_3-CH(CH_3)-CH_2-OH$

(d) $CH_3-CH_2-CH_2-CH_2-OH$

147. Commercially available H_2SO_4 is 98 g by weight of H_2SO_4 and 2 g by weight of water. It's density is 1.38 g cm^{-3}. Calculate the molality (m) of H_2SO_4 (molar mass of H_2SO_4 is 98 g mol^{-1})

(a) 500 m (b) 20 molal (c) 50 m (d) 200 m

148. Cyclohexylamine and aniline can be distinguished by

(a) Hinsberg test (b) Carbylamine test

(c) Lassaigne test (d) Azo dye test

149. .... is a potent vasodilator.

(a) Histamine (b) Serotonin

(c) Codeine (d) Cimetidine

150. Standard enthalpy (heat) of formation of liquid water at 25°C is around

$$H_2(g)+\frac{1}{2}O_2(g)\longrightarrow H_2O(l)$$

(a) –237 kJ/mol (b) 237 kJ/mol

(c) –286 kJ/mol (d) 286 kJ/mol

151. The alcohol that reacts faster with Lucas reagent is

(a) $CH_3-CH_2-CH_2-CH_2-OH$

(b) $CH_3-CH_2-CH(OH)-CH_3$

(c) $CH_3-CH(CH_3)-CH_2-OH$

(d) $CH_3-C(CH_3)_2-OH$

152. Balance the following equation by choosing the correct option.

$$x KNO_3 + y C_{12}H_{22}O_{11} \longrightarrow p N_2 + q CO_2 + r H_2O + s K_2CO_3$$

	x	y	p	q	r	s
(a)	36	55	24	24	5	48
(b)	48	5	24	36	55	24
(c)	24	24	36	55	48	5
(d)	24	48	36	24	5	55

153. Which of the following element is purified by vapour phase refining?

(a) Fe (b) Zr

(c) Cu (d) Au

154. When helium gas is allowed to expand into vacuum, heating effect is observed. The reason for this is (assume He as a non ideal gas)

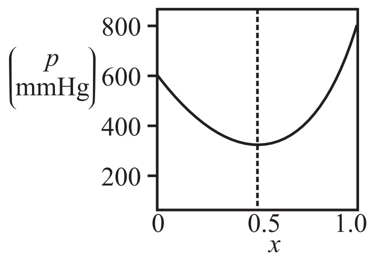
(a) He is an inert gas

(b) The inversion temperature of helium is very high

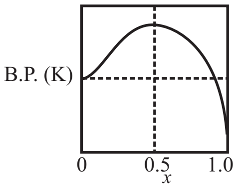
(c) The inversion temperature of helium is very low

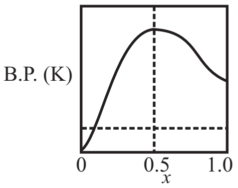
(d) He has the lowest boiling point.

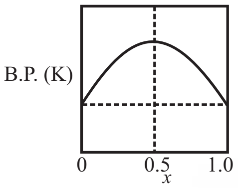
155. The vapour pressure of a non-ideal two component solution is given below

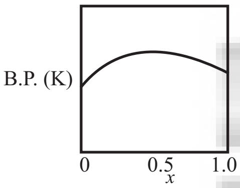


Identify the correct $T–X$ curve for the same mixture,

(a) 

(b) 

(c) 

(d) 

156. Cyclopentadienyl anion is
(a) benzenoid and aromatic
(b) non-benzenoid and aromatic
(c) non-benzenoid and non-aromatic
(d) non-benzenoid and anti-aromatic

157. Oxidation of cyclohexene in presence of acidic potassium permaganate leads to
(a) glutaric acid (b) adipic acid
(c) pimelic acid (d) succinic acid

158. How many emission spectral lines are possible when hydrogen atom is excited to n^{th} energy level?
(a) $\frac{n(n+1)}{2}$ (b) $\frac{(n+1)}{2}$
(c) $\frac{(n-1)n}{2}$ (d) $\frac{n^2}{2}$

159. The bond length (pm) of F_2, H_2, Cl_2 and I_2, respectively is
(a) 144, 74, 199, 267 (b) 74, 144, 199, 267
(c) 74, 267, 199, 144 (d) 144, 74, 267, 199

160. The number of tetrahedral and octahedral voids in CCP unit cell are respectively
(a) 4, 8 (b) 8, 4 (c) 12, 6 (d) 6, 12

ANSWER KEY

1	(d)	**2**	(d)	**3**	(b)	**4**	(a)	**5**	(d)	**6**	(a)	**7**	(c)	**8**	(b)	**9**	(a)	**10**	(d)
11	(d)	**12**	(d)	**13**	(a)	**14**	(c)	**15**	(d)	**16**	(a)	**17**	(b)	**18**	(c)	**19**	(b)	**20**	(a)
21	(a)	**22**	(a)	**23**	(c)	**24**	(b)	**25**	(b)	**26**	(d)	**27**	(d)	**28**	(a)	**29**	(a)	**30**	(a)
31	(c)	**32**	(b)	**33**	(b)	**34**	(a)	**35**	(c)	**36**	(d)	**37**	(b)	**38**	(b)	**39**	(b)	**40**	(b)
41	(a)	**42**	(c)	**43**	(d)	**44**	(b)	**45**	(b)	**46**	(c)	**47**	(b)	**48**	(c)	**49**	(b)	**50**	(a)
51	(c)	**52**	(b)	**53**	(a)	**54**	(c)	**55**	(c)	**56**	(c)	**57**	(d)	**58**	(a)	**59**	(c)	**60**	(b)
61	(a)	**62**	(a)	**63**	(d)	**64**	(d)	**65**	(d)	**66**	(b)	**67**	(b)	**68**	(c)	**69**	(a)	**70**	(a)
71	(d)	**72**	(a)	**73**	(c)	**74**	(a)	**75**	(b)	**76**	(b)	**77**	(d)	**78**	(a)	**79**	(c)	**80**	(d)
81	(b)	**82**	(c)	**83**	(b)	**84**	(b)	**85**	(d)	**86**	(c)	**87**	(*)	**88**	(d)	**89**	(b)	**90**	(d)
91	(d)	**92**	(a)	**93**	(c)	**94**	(a)	**95**	(d)	**96**	(d)	**97**	(d)	**98**	(d)	**99**	(a)	**100**	(b)
101	(a)	**102**	(a)	**103**	(a)	**104**	(c)	**105**	(d)	**106**	(c)	**107**	(c)	**108**	(a)	**109**	(b)	**110**	(d)
111	(b)	**112**	(a)	**113**	(a)	**114**	(a)	**115**	(a)	**116**	(c)	**117**	(c)	**118**	(d)	**119**	(c)	**120**	(a)
121	(d)	**122**	(c)	**123**	(c)	**124**	(c)	**125**	(d)	**126**	(b)	**127**	(a)	**128**	(b)	**129**	(d)	**130**	(a)
131	(a)	**132**	(c)	**133**	(a)	**134**	(d)	**135**	(d)	**136**	(d)	**137**	(d)	**138**	(a)	**139**	(c)	**140**	(c)
141	(c)	**142**	(b)	**143**	(d)	**144**	(a)	**145**	(a)	**146**	(b)	**147**	(a)	**148**	(d)	**149**	(a)	**150**	(c)
151	(d)	**152**	(b)	**153**	(b)	**154**	(c)	**155**	(a)	**156**	(b)	**157**	(b)	**158**	(c)	**159**	(b)	**160**	(b)

Hints & Solutions

MATHEMATICS

1. **(d)** We have, $\tan 20° = \lambda$

$$\therefore \frac{\tan 160° - \tan 110°}{1 + (\tan 160°)(\tan 110°)}$$

$$= \frac{\tan(180° - 20°) - \tan(90° + 20°)}{1 + (\tan(180° - 20°))(\tan(90° + 20°))}$$

$$= \frac{-\tan 20° + \cot 20°}{1 + \tan 20° \cot 20°}$$

$[\because \tan(180° - \theta) = -\tan\theta; \tan(90° + \theta) = -\cot\theta]$

$$= \frac{-\tan 20° + \dfrac{1}{\tan 20°}}{1 + \tan 20° \dfrac{1}{\tan 20°}} = \frac{-\lambda + 1/\lambda}{1+1} = \frac{-\lambda^2 + 1}{2\lambda} = \frac{1 - \lambda^2}{2\lambda}.$$

2. **(d)** Equation of circle,

$$x^2 + y^2 - 6x + 4y = 12$$

$$x^2 + y^2 - 6x + 4y + 13 = 12 + 13$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 = 25$$

$$(x-3)^2 + (y+2)^2 = (5)^2$$

Equation of tangent whose slope is m,

$$y + 2 = m(x-3) \pm 5\sqrt{m^2 + 1} \quad \text{...(i)}$$

Now, this tangent is parallel to line $4x + 3y + 5 = 0$

$\therefore$ Slope of line is $-\frac{4}{3}$

Put the value of $m = -\frac{4}{3}$ in Eq. (i), we get

$$y + 2 = -\frac{4}{3}(x-3) \pm 5\sqrt{\left(\frac{-4}{3}\right)^2 + 1}$$

$$y + 2 = \frac{-4}{3}(x-3) \pm 5\left(\frac{5}{3}\right)$$

$$3y + 6 = -4x + 12 \pm 25$$

$$4x + 3y = 6 \pm 25$$

Hence, equation of tangent is

$4x + 3y - 31 = 0$ or $4x + 3y + 19 = 0$.

3. **(b)** We have mean $(\bar{x}) = 10$

$$\therefore \bar{x} = \frac{6 + 7 + 10 + 12 + 13 + \alpha + 12 + 16}{8}$$

$$10 = \frac{76 + \alpha}{8} \Rightarrow \alpha = 4$$

Mean Deviation

$$MD(\bar{x}) = \frac{[|6-10| + |7-10| + |10-10| + |12-10| + |13-10| + |4-10| + |12-10| + |16-10|]}{8}$$

$$= \frac{4 + 3 + 0 + 2 + 3 + 6 + 2 + 6}{8}$$

$$MD(\bar{x}) = \frac{26}{8} = 3.25.$$

4. **(a)** I. $\int_{-1}^{1} x|x|\,dx = 0$ $\quad [\because x|x|$ is an odd function$]$

II. Let

$$I = \int_0^{\pi/2}\left[1 + \left(\log\frac{4 + 3\sin x}{4 + 3\cos x}\right)\right]dx \quad \text{...(i)}$$

$$I = \int_0^{\pi/2}\left[1 + \left[\log\frac{4 + 3\sin\left(\frac{\pi}{2} - x\right)}{4 + 3\cos\left(\frac{\pi}{2} - x\right)}\right]\right]dx$$

$$I = \int_0^{\pi/2}\left[1 + \left(\log\frac{4 + 3\cos x}{4 + 3\sin x}\right)\right]dx \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2}\left(2 + \log\frac{4 + 3\sin x}{4 + 3\cos x} + \log\frac{4 + 3\cos x}{4 + 3\sin x}\right)dx$$

$$2I = \int_0^{\pi/2}\left(2 + \log\frac{(4 + 3\sin x)(4 + 3\cos x)}{(4 + 3\cos x)(4 + 3\sin x)}\right)dx$$

$$2I = \int_0^{\pi/2}(2 + \log 1)\,dx = \int_0^{\pi/2} 2\,dx = \pi$$

$$I = \frac{\pi}{2}$$

III. Property of Integral,

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

IV. Property of Integral,

$$\int_{-a}^{a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(-x)\,dx$$

$$= \int_0^a [f(x) + f(-x)]\,dx$$

5. **(d)** It is given that, $f(x + y) = f(x)f(y)$

Differentiating w.r.t. x, we get

$$f'(x + y) = f'(x)f(y)$$

Put $x = 0$, $y = 3$, we get

$$f'(0 + 3) = f'(0)f(3)$$

$$f'(3) = f'(0) \cdot f(3)$$

$$f'(3) = 11 \times 3 = 33 \quad [\because f'(0) = 11,\ f(3) = 3]$$

6. **(a)** Let, $I = \int_0^{\pi} \frac{x\,dx}{4\cos^2 x + 9\sin^2 x}$ ...(i)

$$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{4\cos^2(\pi - x) + 9\sin^2(\pi - x)}$$

$$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{4\cos^2 x + 9\sin^2 x} \quad ...(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{\pi\,dx}{4\cos^2 x + 9\sin^2 x}$$

$$2I = \int_0^{\pi} \frac{\pi \sec^2 x\,dx}{4 + 9\tan^2 x}$$

$$2I = \int_0^{\pi/2} \frac{2\pi \sec^2 x\,dx}{4 + 9\tan^2 x}$$

$$\left[\because \int_0^{2a} f(x)dx - 2\int_0^{a} f(x)\,dx \Rightarrow f(2a - x) = f(x)\right]$$

$$I = \frac{\pi}{9}\int_0^{\pi/2} \frac{\sec^2 x\,dx}{\frac{4}{9} + \tan^2 x}$$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$x = 0 \Rightarrow t = 0,\ x = \frac{\pi}{2} \Rightarrow t = \infty$$

$$I = \frac{\pi}{9}\int_0^{\infty} \frac{dt}{\left(\frac{2}{3}\right)^2 + t^2}$$

$$I = \frac{\pi}{9} \times \frac{3}{2}\left[\tan^{-1}\frac{3t}{2}\right]_0^{\infty}$$

$$I = \frac{\pi}{9} \times \frac{3}{2} \times \frac{\pi}{2} = \frac{\pi^2}{12}$$

7. **(c)** Given that,

X	$P(X)$	P_iX_i	$P_iX_i^2$
0	0.1	0	0
1	0.4	0.4	0.4
2	0.3	0.6	1.2
3	0.2	0.6	1.8
4	0	0	0
		$\Sigma P_iX_i = 1.6$	$\Sigma P_iX_i^2 = 3.4$

Variance $(\sigma^2) = \Sigma P_iX_i^2 - (\Sigma P_iX_i)^2$

$= 3.4 - (1.6)^2 = 3.4 - 2.56 = 0.84.$

8. **(b)** Given,

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & -1 & 4 \end{bmatrix},\ A^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & 0 & 4 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 4 & -1 \\ 2 & -1 & 8 \end{bmatrix},\ A - A^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -0.5 \\ 1 & -0.5 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & -0.5 & 0 \end{bmatrix}$$

$A = B + C$

$$\therefore C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & -0.5 & 0 \end{bmatrix} = -C^T.$$

9. **(a)** Given, **a** is a unit vector.

$$[\mathbf{a} \times \hat{i}]^2 + [\mathbf{a} \times \hat{j}]^2 + [\mathbf{a} \times \hat{k}]^2$$

$$(|\mathbf{a}||\hat{i}|\sin\alpha)^2 + (|\mathbf{a}||\hat{j}|\sin\beta)^2 + (|\mathbf{a}||\hat{k}|\sin\gamma)^2$$

$$= \sin^2\alpha + \sin^2\beta + \sin^2\gamma \quad [\because |\mathbf{a}| = |\hat{i}| = |\hat{j}| = |\hat{k}| = 1]$$

$$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$$

$$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$

$$= 3 - 1 = 2. \quad [\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1]$$

10. **(d)** Red balls = 5, Black balls = 3, White balls = 4

Total numbers of balls = (5 + 3 + 4) = 12

Three balls are drawn at random = ${}^{12}C_3 = 220$

Three balls are drawn at random of same colour

$= {}^5C_3 + {}^3C_3 + {}^4C_3 = 10 + 1 + 4 = 15$

Probability of are not same colour

= 1 – Probability of same colour

$$= 1 - \frac{15}{220} = 1 - \frac{3}{44} = \frac{41}{44}$$

11. **(d)** We have,

$$S_1 = x^2 + y^2 - 4x - 6y + 5 = 0$$

$$S_2 = x^2 + y^2 - 2x - 4y - 1 = 0$$

$$S_3 = x^2 + y^2 - 6x - 2y = 0$$

$$S_1 - S_2 = 0$$

$$\Rightarrow x^2 + y^2 - 4x - 6y + 5 - x^2 - y^2 + 2x + 4y + 1 = 0$$

$$\Rightarrow 2x + 2y - 6 = 0 \Rightarrow x + y - 3 = 0 \quad ...(i)$$

$S_2 - S_3 =$

$$\Rightarrow x^2 + y^2 - 2x - 4y - 1 - x^2 - y^2 + 6x + 2y = 0$$

$$\Rightarrow 4x - 2y - 1 = 0 \quad ...(ii)$$

On solving Eqs. (i) and (ii), we get

$$x = \frac{7}{6},\ y = \frac{11}{6}$$

$\left(\frac{7}{6}, \frac{11}{6}\right)$ satisfies the equations $18x - 12y + 1 = 0$.

12. **(d)** Given that,

$\operatorname{cosec}\theta - \cot\theta = 2017$ ...(i)

As we know,

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1$

$\therefore \quad \operatorname{cosec}\theta + \cot\theta = \dfrac{1}{2017}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2\operatorname{cosec}\theta = 2017 + \dfrac{1}{2017}$

$\Rightarrow \quad \operatorname{cosec}\theta = \dfrac{1}{2}\left[2017 + \dfrac{1}{2017}\right] > 0$

θ lie in Ist or IInd quadrant.

On subtracting Eq. (i) from Eq. (ii), we get

$2\cot\theta = \dfrac{1}{2017} - 2017 \;\Rightarrow\; \cot\theta = \dfrac{1}{2}\left(\dfrac{1}{2017} - 2017\right) < 0$

$\therefore$ θ lie in IInd and IIIrd quadrant.

Hence, θ lies in IInd quadrant.

13. **(a)** Given that,

$\int e^{2x} f'(x)\,dx = g(x)$

Let $I = \int (e^{2x} f(x) + e^{2x} f'(x))\,dx$

$= f(x)\int e^{2x}dx - \int f'(x)\int e^{2x}(dx)\,dx + \int e^{2x} f'(x)\,dx$

[Using Integration by parts]

$= \dfrac{f(x)e^{2x}}{2} - \dfrac{1}{2}\int e^{2x} f'(x)\,dx + \int e^{2x} f'(x)\,dx$

$= \dfrac{e^{2x}}{2} f(x) - \dfrac{1}{2}\int e^{2x} f'(x)\,dx$

$= \dfrac{1}{2}[e^{2x} f(x) - \int e^{2x} f'(x)\,dx]$

$= \dfrac{1}{2}[e^{2x} f(x) - g(x)] + C.$

14. **(c)** Let $P(x, y)$, $A(5, 3)$ and $B(3, -2)$.

Area of $\Delta PAB = \dfrac{1}{2}\left\|\begin{matrix} x & y & 1 \\ 5 & 3 & 1 \\ 3 & -2 & 1 \end{matrix}\right\|$

$9 = \dfrac{1}{2}|x(3+2) - y(5-3) + (-10-9)|$

$18 = |5x - 2y - 19|$

$5x - 2y - 19 = |18|$

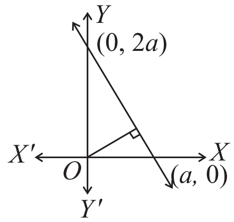
$5x - 2y - 19 = \pm 18 \Rightarrow 5x - 2y = 19 \pm 18$

$5x - 2y = 19 + 18$ or $5x - 2y = 19 - 18$

$5x - 2y = 1$ or $5x - 2y = 37$

Hence, it represents a pair of parallel lines.

15. **(d)** Let equation of line in the intercept form.



$\dfrac{x}{a} + \dfrac{y}{2a} = 1$

$\Rightarrow \quad 2x + y = 2a$

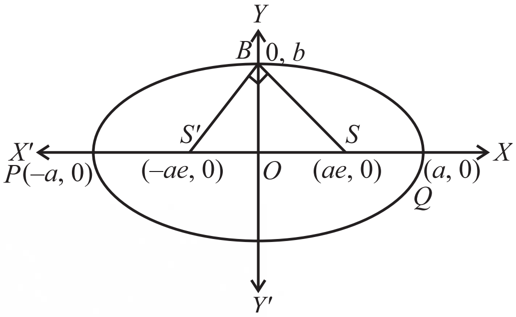
Distance of line from origin,

$D = \left|\dfrac{2a}{\sqrt{(2)^2 + (1)^2}}\right| = \left|\dfrac{2a}{\sqrt{5}}\right| = 1.$

$\therefore \quad 2a = \pm\sqrt{5}.$

Hence, equation of line is $2x + y = \pm\sqrt{5}$

16. **(a)**



SBS' is an isosceles right angle triangle.

$\therefore \quad SS'^2 = SB^2 + S'B^2$

$(2ae)^2 = b^2 + (ae)^2 + b^2 + (ae)^2$

$4(ae)^2 = 2(b^2 + (ae)^2)$

$a^2e^2 = b^2 \Rightarrow e^2 = \dfrac{b^2}{a^2}$ ...(i)

We know that, $e^2 = 1 - \dfrac{b^2}{a^2}$

From Eq. (ii), $e^2 = 1 - e^2 \Rightarrow 2e^2 = 1 \Rightarrow e = \dfrac{1}{\sqrt{2}}$

17. **(b)** Equation of parabola,

$y^2 + 6y - 2x = -5$

$y^2 + 6y + 9 = 2x - 5 + 9$

$(y+3)^2 = 2x + 4 = 2(x+2) \;\therefore\; 4a = 2 \Rightarrow a = \dfrac{1}{2}$

Vertex = (–2, –3)

Equations of directrix

$x + 2 = -\dfrac{1}{2} \Rightarrow x + 2 + \dfrac{1}{2} = 0$

$x + \dfrac{5}{2} = 0 \Rightarrow 2x + 5 = 0.$

18. **(c)** Given that, $\dfrac{x^2+5}{(x^2+1)(x-2)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+1}$

$x^2 + 5 = A(x^2+1) + (Bx+C)(x-2)$

$x^2 + 5 = Ax^2 + A + Bx^2 - 2Bx + Cx - 2C$

$x^2 + 5 = (A+B)x^2 + x(-2B+C) + A - 2C$

Equating the coefficient of x^2, x and constant terms, we get
$1 = A + B, 0 = -2B + C, 5 = A - 2C$
Solving these equations, we get

$$A = \frac{9}{5}, B = -\frac{4}{5}, C = -\frac{8}{5}$$

$$\therefore \quad A + B + C = \frac{9}{5} - \frac{4}{5} - \frac{8}{5} = \frac{9-4-8}{5} = \frac{-3}{5}$$

19. **(b)** It is given that,
Conjugate of $(x+iy)(1-2i)$ is $1 + i$.

i.e. $(x-iy)(1+2i) = 1+i \quad \Rightarrow \quad x - iy = \frac{1+i}{1+2i}$

Taking conjugate on both the sides, we get

$$x + iy = \frac{1-i}{1-2i}$$

20. **(a)** Let $I = \int x^4 e^{2x}\, dx$

$$I = x^4 \int e^{2x}\, dx - \int \frac{dx^4}{dx} \cdot \int e^{2x} \cdot dx \cdot dx + C$$

$$= \frac{x^4 \cdot e^{2x}}{2} - \frac{1}{2} \int 4x^3 e^{2x} dx + C$$

$$= \frac{x^4 e^{2x}}{2} - 2\left[\frac{x^3 e^{2x}}{2} - \int \frac{3x^2 e^{2x}}{2} dx\right] + C$$

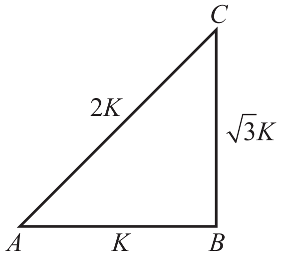
$$= \frac{x^4 e^{2x}}{2} - x^3 e^{2x} + 3\left[\frac{x^2 e^{2x}}{2} - \frac{1}{2}\int 2x e^{2x}\, dx\right] + C$$

$$= \frac{x^4 e^{2x}}{2} - x^3 e^{2x} + \frac{3}{2} x^2 e^{2x} - 3\left[\frac{x e^{2x}}{2} - \int \frac{e^{2x}}{2} dx\right] + C$$

$$= \frac{x^4 e^{2x}}{2} - x^3 e^{2x} + \frac{3}{2} x^2 e^{2x} - \frac{3}{2} x e^{2x} + \frac{3e^{2x}}{4} + C$$

$$= \frac{e^{2x}}{4}[2x^4 - 4x^3 + 6x^2 - 6x + 3] + C.$$

21. **(a)** We have,
Ratio of the sides of a triangle are $1 : \sqrt{3} : 2$
Let the sides are $k, \sqrt{3}k, 2k$.



Since, this triangle is a right angled triangle.

$$\therefore \quad \sin A = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2} \Rightarrow A = 60°$$

$$\sin C = \frac{k}{2k} = \frac{1}{2} \Rightarrow C = 30°$$

and $B = 90°$
$\therefore$ Ratio of angles are $30° : 60° : 90° = 1 : 2 : 3$.

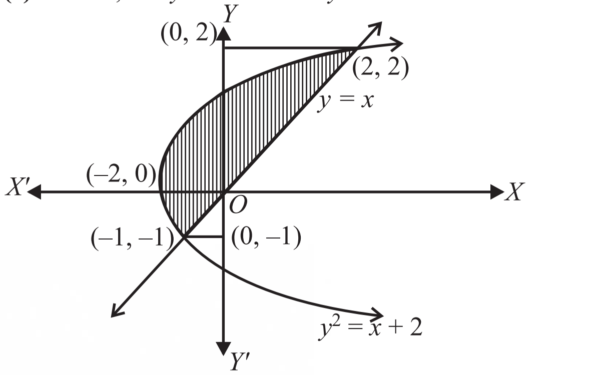
22. **(a)** Given equation is,
$(x-1)^3 + 64 = 0$; $(x-1)^3 = -64$

$(x-1)^3 = (-4)^3$; $x - 1 = -4, -4w, -4w^2$
$x = -3, -4w+1, -4w^2+1$

Sum of complex roots are
$-4w + 1 - 4w^2 + 1 = -4(w + w^2) + 2$

$= -4(-1) + 2 = 4 + 2 = 6 \quad [\because 1 + w + w^2 = 0]$

23. **(c)** Given, $x = y^2 - 2$ and $x = y$.



On solving, $x = y^2 - 2$ and $x = y$, we get $(-1, -1)$ and $(2, 2)$.
Area of the shaded region,

$$A = \int_{-1}^{2} y\, dy - \int_{-1}^{2} (y^2 - 2)\, dy$$

$$= \left[\frac{y^2}{2} - \frac{y^3}{3} + 2y\right]_{-1}^{2} = \left(\frac{4}{2} - \frac{8}{3} + 4\right) - \left(\frac{1}{2} + \frac{1}{3} - 2\right)$$

$$= \frac{10}{3} + \frac{7}{6} = \frac{27}{6} = \frac{9}{2}.$$

24. **(b)** Given that, $\mathbf{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$(\mathbf{a} \times \hat{i})(\hat{i} + \hat{j}) + (\mathbf{a} \times \hat{j})(\hat{j} + \hat{k}) + (\mathbf{a} \times \hat{k})(\hat{k} + \hat{i})$

$= (\mathbf{a} \times \hat{i})\hat{i} + (\mathbf{a} \times \hat{i})\hat{j} + (\mathbf{a} \times \hat{j})\hat{j} + (\mathbf{a} \times \hat{j})\hat{k}$
$+ (\mathbf{a} \times \hat{k})\hat{k} + (\mathbf{a} \times \hat{k})\hat{i}$

$= [\mathbf{a}\, \hat{i}\, \hat{i}] + [\mathbf{a}\, \hat{i}\, \hat{j}] + [\mathbf{a}\, \hat{j}\, \hat{j}] + [\mathbf{a}\, \hat{j}\, \hat{k}] + [\mathbf{a}\, \hat{k}\, \hat{k}] + [\mathbf{a}\, \hat{k}\, \hat{i}]$
$(\because (a \times b)c = [a\ b\ c])$

$= [\mathbf{a}\, \hat{i}\, \hat{j}] + [\mathbf{a}\, \hat{j}\, \hat{k}] + [\mathbf{a}\, \hat{k}\, \hat{i}]$
$[\because [\mathbf{a}\, \hat{i}\, \hat{i}] = [\mathbf{a}\, \hat{j}\, \hat{j}] = [\mathbf{a}\, \hat{k}\, \hat{k}] = 0]$

$= \mathbf{a} \cdot (\hat{i} \times \hat{j}) + \mathbf{a} \cdot (\hat{j} \times \hat{k}) + \mathbf{a} \cdot (\hat{k} \times \hat{i})$

$= \mathbf{a} \cdot \hat{k} + \mathbf{a} \cdot \hat{i} + \mathrm{a} \cdot \hat{j} = \mathbf{a} \cdot (\hat{i} + \hat{j} + \hat{k})$

$= (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = x + y + z.$

25. **(b)** Let a complex number, $z = x + iy$

$$\therefore \quad \frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{2x+1+iy}{(-y+1)+ix}$$

$$= \frac{[(2x+1)+iy][(-y+1)-ix]}{[(-y+1)+ix][(-y+1)-ix]}$$

$$= \frac{(2x+1)(-y+1)-(2x+1)xi+y(-y+1)j+xy}{(-y+1)^2+x^2}$$

$$= \frac{(2x+1)(-y+1)+xy+(-y^2+y-2x^2-x)i}{(-y+1)^2+x^2}$$

According to the data,

Imaginary part of $\frac{2z+1}{iz+1} = -2$

$\therefore \quad \frac{-y^2+y-2x^2-x}{(-y+1)^2+x^2} = -2$

$\Rightarrow \ -y^2+y-2x^2-x = -2(-y+1)^2-2x^2$

$\Rightarrow \ -y^2+y-x = -2y^2-2+4y$

$\Rightarrow \ y^2-3y-x+2=0$

which represent the equations of parabola.

26. **(d)** Given that, $g(x) = (f(2f(x)+2))^2$

Differentiating w.r.t. x,

$g'(x) = 2(f(2f(x)+2)\cdot f'(2f(x)+2)f'(x)\cdot 2$

Put $x = 0$,

$g'(0) = 2(f(2f(0)+2)\cdot f'(2f(0)+2)\cdot 2f'(0)$

$g'(0) = 2(f(2(-1)+2)\cdot f'(2(-1)+2\cdot 2(1)$

$[\because f(0) = -1,\ f'(0) = 1]$

$g'(0) = 2[f(-2+2)\cdot f'(-2+2)]\cdot 2$

$g'(0) = 4f(0)\cdot f'(0)$

$g'(0) = 4\times(-1)(1) = -4.$

27. **(d)** Given that,

Distance from (1, 1) to $3x + 4y + c = 0$ is 7

$\therefore \quad 7 = \left|\frac{3(1)+4(1)+c}{\sqrt{3^2+4^2}}\right|$

$7 = \left|\frac{7+c}{5}\right|$

$35 = |7 + c|$

$7 + c = \pm 35$

$c = -7 \pm 35$

$c = -7 + 35, -7 - 35$

$\therefore \quad c = 28, -42$

28. **(a)** It is given that,

$$\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$$

Put $v = \frac{y}{x} \Rightarrow y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \therefore \ v + x\frac{dv}{dx} = \frac{1+v}{1-v}$

$x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v} = \frac{1+v^2}{1-v}$

$\frac{1-v}{1+v^2}dv = \frac{dx}{x}$

$\int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$

$\tan^{-1} v - \frac{1}{2}\log|1+v^2| = \log x + C$

$\tan^{-1}\frac{y}{x} = \log x + \frac{1}{2}\log\left(1+\frac{y^2}{x^2}\right) + C$

$\tan^{-1}\frac{y}{x} = \log\frac{x(\sqrt{x^2+y^2})}{x} + C$

$\therefore \quad \tan^{-1}\frac{y}{x} = \log\sqrt{x^2+y^2} + C$

This is the required solution.

29. **(a)** Given that, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Put $x = a\cos\theta,\ y = b\sin\theta$

On differentiating w.r.t. θ, we get

$\frac{dx}{d\theta} = -a\sin\theta,\ \frac{dy}{d\theta} = b\cos\theta$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{b}{a}\cot\theta$

On differentiating w.r.t. x we get

$\frac{d^2y}{dx^2} = -\frac{b}{a}(-\text{cosec}^2\theta)\frac{d\theta}{dx}$

$\frac{d^2y}{dx^2} = \frac{b\,\text{cosec}^2\theta}{-a^2\sin\theta} \qquad \left(\because \frac{dx}{d\theta} = -a\sin\theta\right)$

$\frac{d^2y}{dx^2} = -\frac{b}{a^2\sin^3\theta}$

$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3} \qquad \left[\because \sin\theta = \frac{y}{b}\right]$

30. **(a)** Given, $\lim_{y\to 1}\left(\frac{1}{y^2-1} - \frac{2}{y^4-1}\right)$

$= \lim_{y\to 1}\left(\frac{y^2+1-2}{y^4-1}\right)$

$= \lim_{y\to 1}\left(\frac{y^2-1}{(y^2-1)(y^2+1)}\right)$

$= \lim_{y\to 1}\frac{1}{y^2+1} = \frac{1}{1+1} = \frac{1}{2}.$

31. **(c)** We have, $(y-3x^2)dx + x\,dy = 0$

$\Rightarrow\ y\,dx - 3x^2\,dx + x\,dy = 0\ \Rightarrow\ y\,dx + x\,dy = 3x^2\,dx$

$\Rightarrow\ dxy = 3x^2\,dx$

On integrating both sides, we get

$xy = x^3 + C \Rightarrow y = x^2 + \frac{C}{x}$

This is the required solution.

32. **(b)** In the expansion of $(1+x)^{42}$

$T_{2r\ 1}\quad {}^{42}C_{2t}\,x^2$

$T_{r+1} = {}^{42}C_t\,x^t$

Coefficient of $(2r+1)^{th}$ = Coefficient of $(r+1)^{th}$ term

$\therefore\ {}^{42}C_{2r} = {}^{42}C_r$

$2r + r = 42 \qquad [\because {}^nC_x = {}^nC_y \Rightarrow x + y = n]$

$\therefore\ r = 14.$

33. **(b)** Equation of plane passes through the points (1, – 1, 6), (0, 0, 7) and perpendicular to the plane $x - 2y + z = 6$ is

$$\begin{vmatrix} x-1 & y+1 & z-6 \\ 0-1 & 0+1 & 7-6 \\ 1 & -2 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y+1 & z-6 \\ -1 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 0$$

$(x-1)(1+2) - (y+1)(-1-1) + (z-6)(2-1) = 0$

$3(x-1) - (-2)(y+1) + 1(z-6) = 0$

$3x - 3 + 2y + 2 + z - 6 = 0\ ;\ 3x + 2y + z - 7 = 0$

Now, put all the points given in the options and verify it. The plane passes through (1, 1, 2).

34. **(a)** Given curve $y = ax^3 + bx + 4$ is passes through (2, 14).

$\therefore\ 14 = a(2)^3 + b(2) + 4\ ;\ 14 = 8a + 2b + 4$

$5 = 4a + b$ ...(i)

Slope of tangent to the curve, $y = ax^3 + bx + 4$

i.e. $\frac{dy}{dx} = 3ax^2 + b$

At point (2, 14),

$\left(\frac{dy}{dx}\right)_{(2,\,14)} = 3a(2)^2 + b$

$21 = 12a + b$ ...(ii)

$\left[\because \left(\frac{dy}{dx}\right)_{(2,\,14)} = 21\right]$

On solving Eqs. (i) and (ii), we get

$a = 2, b = -3.$

35. **(c)** We have

x	$P(x)$	P_ix_i
1	a	a
2	a	$2a$
3	a	$3a$
4	b	$4b$
5	b	$5b$
6	0.3	1.8

$\Sigma P_i = a + a + a + b + b + 0.3$

$1 = 3a + 2b + 0.3$

$3a + 2b = 0.7$ ...(i)

$\Sigma P_ix_i = a + 2a + 3a + 4b + 5b + 1.8$

$42 = 6a + 9b + 1.8$

$2a + 3b = 0.8$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = 0.1, b = 0.2.$

36. **(d)** Let quadratic eqatuion $f(x)$ as,

$f(x) = ax^2 + bx + c$

At $x = 0 \Rightarrow f(0) = c$, at $x = 1 \Rightarrow f(1) = a + b + c$

$\therefore\ f(0) + f(1) = 0$

$c + a + b + c = 0$

$a + b + 2c = 0$ ...(i)

Put $x = -2$,

$f(-2) = a(-2)^2 + b(-2) + c$

$0 = 4a - 2b + c$

$\Rightarrow\ 4a - 2b + c = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$\frac{a}{5} = \frac{b}{7} = \frac{c}{-6}$

Let $a = 5k,\ b = 7k,\ c = -6k$

$\therefore\ f(x) = k(5x^2 + 7x - 6) \Rightarrow 5x^2 + 7x - 6 = 0$

$(x + 2)(5x - 3) = 0$

$x = -2,\ x = \frac{3}{5}$

Hence, according to the given options $f\left(\frac{3}{5}\right) = 0.$

37. **(b)** Given equation of curve,

$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$

On differentiating w.r.t. x, we get

$\frac{nx^{n-1}}{a^n} + \frac{ny^{n-1}}{b^n}\frac{dy}{dx} = 0\ ;\ \frac{dy}{dx} = \frac{-b^n x^{n-1}}{a^n y^{n-1}}$

At point (a, b), $\left(\frac{dy}{dx}\right)_{(a,\,b)} = \frac{-b^n a^{n-1}}{a^n b^{n-1}} = \frac{-b}{a}$

Now, equation of tangent,

$$y-b=\frac{-b}{a}(x-a)$$

$$ay-ab=-bx+ab$$

$$bx+ay=2ab$$

$$\frac{bx}{ab}+\frac{ay}{ab}=\frac{2ab}{ab}$$

$$\frac{x}{a}+\frac{y}{b}=2.$$

38. **(b)** As we know that the equation of normal of hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,$$

$$\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2-b^2$$

So, equation of normal is

$$\frac{9x}{x_1}+\frac{4y}{y_1}=9+4$$

$$\frac{9x}{x_1}+\frac{4y}{y_1}=13$$

Since, line $x + y = k$ is normal to the given hyperbola

$$\frac{\frac{9}{x_1}}{1}=\frac{\frac{4}{y_1}}{1}=\frac{13}{k}$$

$$\frac{9}{x_1}=\frac{4}{y_1}=\frac{13}{k}$$

$\therefore\quad x_1=\frac{9k}{13}$ and $y_1=\frac{4k}{13}$

As (x_1, y_1) lie on the hyperbola,

$$\therefore\quad \frac{\left(\frac{9k}{13}\right)^2}{9}-\frac{\left(\frac{4k}{13}\right)^2}{4}=1$$

$$\frac{9k^2}{169}-\frac{4k^2}{169}=1$$

$$5k^2 = 169 \Rightarrow k=\pm\frac{13}{\sqrt{5}}$$

39. **(b)** We have,

$$x^2-8x+9-\frac{8}{x}+\frac{1}{x^2}=0$$

$$\Rightarrow\quad x^4-8x^3+9x^2-8x+1=0$$

Comparing the above equation with $ax^4+bx^3+cx^2+dx+e=0$

Now, products of all roots $=\frac{e}{a}=1.$

40. **(b)** Given, $\Delta=\begin{vmatrix}1 & 5 & 6\\ 0 & 1 & 7\\ 0 & 0 & 1\end{vmatrix}$

= 1 $(1 - 0) - 5\,(0 - 0) + 6\,(0 - 0) = 1\,(1) - 5(0) + 6(0)$

$\therefore\quad \Delta = 1.$

Now, $\Delta'=\begin{vmatrix}1 & 0 & 1\\ 3 & 0 & 3\\ 4 & 6 & 100\end{vmatrix}$

$= 1(0 - 18) - 0(300 - 12) + 1(18 - 0)$

$= 1\,(-18) - 0(288) + 1(18) = -18 - 0 + 18$

$\therefore\quad \Delta' = 0$

So, $(\Delta+\Delta')^2-3(\Delta+\Delta')+2$

$= (1 + 0)^2 - 3(1 + 0) + 2 = 1 - 3 + 2 = -2 + 2 = 0.$

41. **(a)** Total players in village = 10,

Number of players in a team = 6

Total number of ways of choosing such terms

$$={}^{10}C_5\cdot{}^{5}C_1=\frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1}\times5$$

$= 10 \times 3 \times 7 \times 6 = 1260.$

42. **(c)** Point of intersection of two lines i.e. $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ is $(-1, -1)$.

And slope of line $3x - 5y + 11 = 0$ is $\frac{3}{5}$

Equation of the line passing through $(-1, -1)$ and perpendicular to $3x - 5y + 11 = 0$ is

$$(y+1)=-\frac{5}{3}(x+1)$$

$$3(y+1)=-5x-5$$

$$3y+3=-5x-5$$

$$5x + 3y + 8 = 0.$$

43. **(d)** Let $S=\{2k\,|-9\le k\le10\}$ = {–18, –16, –14, ..., 0, 2, 4, 6, ... 20}

Total number of possible outcomes, $n = 20$

Favourable outcomes are –12, 0 and 12.

So, number of favourable outcomes, $r = 3$

$\therefore$ Required probability, $P=\frac{r}{n}=\frac{3}{20}.$

44. **(b)** Let, $I=\int\frac{dx}{x(x^4+1)}=\int\frac{x^4+1-x^4}{x(x^4+1)}dx$

$$=\int\frac{x^4+1}{x(x^4+1)}dx-\int\frac{x^4}{x(x^4+1)}dx$$

$$=\int\frac{1}{x}dx-\int\frac{x^3}{x^4+1}dx$$

$$=\log|x|-\int\frac{x^3}{x^4+1}dx+C$$

$$= \log|x| - \frac{1}{4}\int \frac{4x^3}{x^4+1}dx + C$$

$$= \log|x| - \frac{1}{4}\log|x^4+1| + C$$

$$= \frac{1}{4}\log|x^4| - \frac{1}{4}\log|x^4+1| + C$$

$$= \frac{1}{4}[\log|x^4| - \log|x^4+1|] + C \quad = \frac{1}{4}\log\left|\frac{x^4}{x^4+1}\right| + C$$

45. **(b)** Given that, $\sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\sqrt{\frac{2}{3}}$

$$= \pi - \sin^{-1}\left[\frac{\sqrt{3}}{2}\sqrt{1-\left(\sqrt{\frac{2}{3}}\right)^2} + \sqrt{\frac{2}{3}}\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}\right]$$

As $0 < \frac{\sqrt{3}}{2} = 0.866, \sqrt{\frac{2}{3}} = 0.816 \le 1$ and

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\sqrt{\frac{2}{3}}\right)^2 = 1.4167 > 1$$

$$[\because \sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}x\sqrt{1-y^2} + y\sqrt{1-x^2}, \text{ if } 0 < x, y \le 1 \text{ and } x^2+y^2 > 1]$$

$$= \pi - \sin^{-1}\left[\frac{\sqrt{3}}{2}\sqrt{1-\frac{2}{3}} + \sqrt{\frac{2}{3}}\sqrt{1-\frac{3}{4}}\right]$$

$$= \pi - \sin^{-1}\left[\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{3}} + \frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{1}{2}\right]$$

$$= \pi - \sin^{-1}\left[\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\right]$$

46. **(c)** Given, α and β are the roots of $x^2 + 2x + c = 0$

$\therefore$ Sum of roots, $\alpha + \beta = -\frac{b}{a}$

$$\alpha + \beta = \frac{-2}{1} = -2 \quad \text{...(i)}$$

and product of roots, $\alpha\beta = \frac{c}{a}$

$$\alpha\beta = \frac{c}{1} = c \quad \text{...(ii)}$$

It is given that, $\alpha^3 + \beta^3 = 4$

$$(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta) = 4$$

$$(\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta] = 4$$

$(-2)[(-2)^2 - 3\times c] = 4$ [From Eqs. (i) and (ii)]

$(-2)[4 - 3c] = 4$

$4 - 3c = -2$

$-3c = -6 \Rightarrow c = 2.$

47. **(b)** Equations of circle is $S \equiv x^2 + y^2 - 13 = 0$

On differentiating it w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$$

Now, slope of tangent, $m = \left.\frac{dy}{dx}\right|_{\text{at }(2,3)} = \frac{-2}{3}$

$$\therefore \left(m, -\frac{1}{m}\right) = \left(-\frac{2}{3}, \frac{3}{2}\right)$$

On substituting this point in LHS of Eq. (i),

$$\text{LHS} = \frac{4}{9} + \frac{9}{4} - 13 = \frac{16+81-468}{36} = \frac{-371}{36} < 0$$

$\therefore \left(m, -\frac{1}{m}\right)$ is an internal point with respect to the circle $S = 0$.

48. **(c)** According to the given data,

		C		

If we fix C in the middle, then the rest 4 letters can be arranged in ${}^4P_4$ ways.

$\therefore$ ${}^4P_4 = 4! = 24$ ways.

49. **(b)** Given equation of the curves are

$x^2 = 8y$ ...(i)

and $xy = 8$ ...(ii)

On solving both these curves, $x = 4$

On substitution $x = 4$ in Eq. (ii), we get $y = 2$

Thus, the point of intersection of given curves is (4, 2).

Now, let m_1 and m_2 be the slope of tangent to the curve $x^2 = 8y$ and $xy = 8$ at point (4, 2) respectively.

Then,

$$x^2 = 8y \Rightarrow 2x = 8\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x}{4}$$

$$\therefore m_1 = \frac{4}{4} = 1$$

and $xy = 8 \Rightarrow x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

$$\therefore m_2 = \frac{-2}{4} = -\frac{1}{2}$$

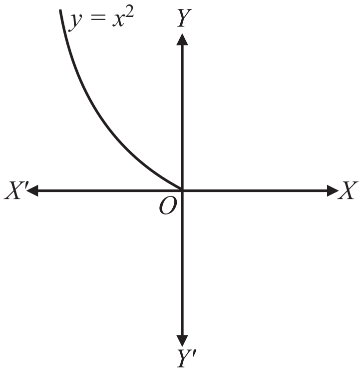
Now, the angle between the curves,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{-1}{2} - 1}{1 - \frac{1}{2}}\right| = \left|\frac{\frac{-3}{2}}{\frac{1}{2}}\right|$$

$$\tan\theta = -3$$

$$\therefore \theta = \tan^{-1}(-3).$$

50. **(a)** $f:(-\infty, 0] \to [0, \infty)$ and $f(x) = x^2$.



Since, each line parallel to x-axis cuts the above curve at maximum one point, therefore f is one-one. Also from the graph it is clear that range $f = [0, \infty)$

Therefore f is onto also.

Thus, f is invertible function.

Hence, $f^{-1}:[0, \infty) \to (-\infty, 0]$

$\Rightarrow$ Domain $(f^{-1}) = [0, \infty)$ and Range of $(f^{-1}) = (-\infty, 0]$.

51. **(c)** Given that, **a, b, c** are unit vector

$\Rightarrow |\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$ and $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0,\ \theta = \frac{\pi}{3}$

Now, $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$

$\mathbf{a} \times (\mathbf{a} + \mathbf{b} + \mathbf{c}) = 0$

$\mathbf{a} \times \mathbf{a} + \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c} = 0$

$\mathbf{a} \times \mathbf{b} = \mathbf{c} \times \mathbf{a}$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{c} \times \mathbf{a}|$

Similarly, $|\mathbf{a} \times \mathbf{b}| = |\mathbf{b} \times \mathbf{c}|$

$\therefore\ |\mathbf{a} \times \mathbf{b}| + |\mathbf{b} \times \mathbf{c}| + |\mathbf{c} \times \mathbf{a}| = 3|\mathbf{a} \times \mathbf{b}|$

$= 3|\mathbf{a}||\mathbf{b}|\sin\theta$

$= 3 \times 1 \times 1 \times \sin\frac{\pi}{3} = \frac{3\sqrt{3}}{2}$

52. **(b)** Given, $x = A\cos(nt + \alpha)$

$$A = \frac{x}{\cos(nt + \alpha)} \quad \text{...(i)}$$

On differentiating x w.r.t. 't', we get

$$\frac{dx}{dt} = -A\sin(nt + \alpha)\frac{d}{dt}(nt + \alpha)$$

$$= -A\sin(nt + \alpha)(n)$$

$$\frac{dx}{dt} = -\frac{xn\sin(nt + \alpha)}{\cos(nt + \alpha)} \quad \text{[from Eq. (i)]}$$

$$\frac{dx}{dt} = -nx\tan(nt + \alpha)$$

Again, differentiating w.r.t. 't', we get

$$\frac{d^2x}{dt^2} = -\left[nx\frac{d}{dt}\tan(nt + \alpha) + \tan(nt + \alpha)\frac{d}{dt}nx\right]$$

$$= -\left[nx\sec^2(nt + \alpha)\frac{d}{dt}(nt + \alpha) + \tan(nt + \alpha)\frac{dx}{dt} \times n\right]$$

$$= -[nx\sec^2(nt + \alpha)n + n\tan(nt + \alpha) \times -nx\tan(nt + \alpha)]$$

[from Eq. (ii)]

$$= -[n^2x\sec^2(nt + \alpha) - n^2x\tan^2(nt + \alpha)]$$

$$= -n^2x[\sec^2(nt + \alpha) - \tan^2(nt + \alpha)]$$

$$= -n^2x \times 1 \qquad [\because \sec^2\theta - \tan^2\theta = 1]$$

$$\Rightarrow \frac{d^2x}{dt^2} = -n^2x$$

$$\therefore \frac{d^2x}{dt^2} + n^2x = 0$$

This is the required differential equation.

53. **(a)** Given that,

$|\mathbf{a}| = |\mathbf{b}| = 1$ and α is the angle between **a** and **b**.

$$\cos\alpha = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

$$\cos\alpha = \mathbf{a} \cdot \mathbf{b} \quad \text{...(i)}$$

Now, $\mathbf{a} + \mathbf{b}$ is also unit vector, then

$|\mathbf{a} + \mathbf{b}| = 1$

$|\mathbf{a} + \mathbf{b}|^2 = 1$

$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = 1$

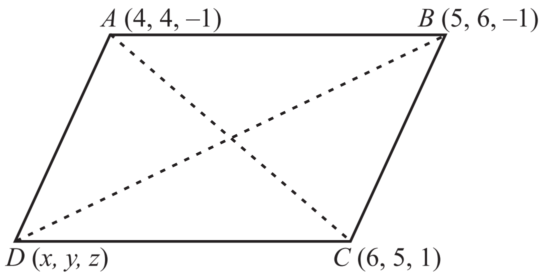
$|\mathbf{a}|^2 + 2\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2 = 1$

$1 + 2\cos\alpha + 1 = 1$ [using Eq. (i)]

$1 + 2\cos\alpha = 0$

$\cos\alpha = -\frac{1}{2}$

54. **(c)** Given, a parallelogram has vertices A (4, 4, –1), B (5, 6, – 1), C (6, 5, 1) and D (x, y, z)



We know that diagonals of parallelogram $ABCD$ bisects each other.

$\therefore$ Mid point of AC = Mid-Point of BD

$$\left(\frac{4+6}{2}, \frac{4+5}{2}, \frac{-1+1}{2}\right)=\left(\frac{x+5}{2}, \frac{y+6}{2}, \frac{z-1}{2}\right)$$

$$\left(\frac{10}{2}, \frac{9}{2}, 0\right)=\left(\frac{x+5}{2}, \frac{y+6}{2}, \frac{z-1}{2}\right)$$

On comparing both sides, we get

$$\frac{x+5}{2}=\frac{10}{2}, \frac{y+6}{2}=\frac{9}{2} \text{ and } \frac{z-1}{2}=0$$

$x+5=10,\ y+6=9$ and $z-1=0$

$\therefore x = 5, y = 3$ and $z = 1$

Thus, $D(x, y, z) = (5, 3, 1)$.

55. **(c)** Given, $2x^2-10xy+2\lambda y^2+5x-16y-3=0$

On comparing the given equation with

$ax^2+2hxy+by^2+2gx+2fy+c=0$, we get

$a=2, b=2\lambda, h=-5, g=\frac{5}{2}, f=-8, c=-3$

Since, the given equation represents a pair of straight lines, therefore

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0;\ \begin{vmatrix} 2 & -5 & 5/2 \\ -5 & 2\lambda & -8 \\ 5/2 & -8 & -3 \end{vmatrix}=0$$

$$2(-6\lambda-64)+5(15+20)+\frac{5}{2}(40-5\lambda)=0$$

$$-12\lambda-128+175+100-\frac{25}{2}\lambda=0$$

$$-\frac{49}{2}\lambda=-147 \Rightarrow \lambda=6.$$

Now, the point of intersection of given lines is given by

$$\left(\frac{bg-fh}{h^2-ab}, \frac{af-gh}{h^2-ab}\right)=\left(\frac{30-40}{25-24}, \frac{-16\times\frac{25}{4}}{25-24}\right)$$

$$=\left(-10, \frac{-7}{2}\right)$$

56. **(c)** Given Matrix, $A=\begin{pmatrix} x & x & x \\ x & x^2 & x \\ x & x & x+1 \end{pmatrix}$

Since, rank of A = 1, therefore atleast one determinant of order 1 should be non-zero and all the determinants of order 2 and 3 should be zero.

If $x = 0$, then $A=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, which have non-zero determinant of order 1 only.

$\therefore x$ can take value 0 only.

57. **(d)** We have, $\mathbf{a}=\hat{i}+\hat{j}+\hat{k},\ \mathbf{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\mathbf{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$

Since, the given vectors are coplanar, therefore

$[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix}=0$$

$$1(1-2x+4)-1(-1-2x)+1(x-2+x)=0$$

$$5-2x+1+2x+2x-2=0$$

$2x + 4 = 0$

$\therefore\ x = -2.$

58. **(a)** Given, $4x^2+8xy+10y^2-8x-44y+14=0$

On comparing the given equation with

$ax^2+2hxy+by^2+2gx+2fy+c=0$, we get

$a = 4, h = 4, b = 10, g = -4, f = -22$ and $c = 14$

For removal of first degree terms, shift the origin to the point,

$$\left(\frac{bg-fh}{h^2-ab}, \frac{af-gh}{h^2-ab}\right)=\left(\frac{-40+88}{16-40}, \frac{-88+16}{16-40}\right)$$

$$=\left(\frac{48}{-24}, \frac{-72}{-24}\right)=(-2, 3).$$

59. **(c)** Let S_1 and S_2 be the circle with centre (2, 3) and (5, 6) with radius a respectively.

Then $S_1 \equiv (x-2)^2+(y-3)^2-a^2=0$

and $S_2 \equiv (x-5)^2+(y-6)^2-a^2=0$

Now, radical axis of these circle is given by $S_1 - S_2 = 0$.

$$(x-2)^2+(y-3)^2-a^2-(x-5)^2-(y-6)^2+a^2=0$$

$$x^2-4x+4+y^2-6y+9-x^2+10x-25-y^2+12y-36=0$$

$$4-4x+9-6y-25+10x-36+12y=0$$

$$13-4x-6y-25+10x-36+12y=0$$

$$6x+6y-48=0$$

$x + y = 8$ ...(i)

Since, the given circle cut orthogonally, therefore

$$2g_1g_2+2f_1f_2=c_1+c_2$$

$2(-2)(-5) + 2(-3)(-6) = ((-2)^2 + (-3)^2 - a^2) + ((-5)^2 + (-6)^2 - a^2)$

$[\because$ radius $=\sqrt{g^2+f^2-c}$ $\therefore a^2=g^2+f^2-c$ $\Rightarrow c=g^2+f^2-a^2]$

$20 + 36 = (13 - a^2) + (61 - a^2)$

$56 = 74 - 2a^2$

$2a^2 = 18 \Rightarrow a^2 = 9 \Rightarrow a = 3$

Hence, option (c) satisfy the Eq. (i)

60. **(b)** Given, $\tan\theta = \cot\theta$

$\Rightarrow \tan\theta_1 \tan\theta_2 = k$

Now, $\dfrac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)} = \dfrac{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2}{\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2}$

$= \dfrac{\cos\theta_1\cos\theta_2[1-\tan\theta_1\tan\theta_2]}{\cos\theta_1\cos\theta_2[1+\tan\theta_1\tan\theta_2]}$

$= \dfrac{1-\tan\theta_1\tan\theta_2}{1+\tan\theta_1\tan\theta_2} = \dfrac{1-k}{1+k}$

61. **(a)** Given, $\mathbf{a} = 2\hat{i} + \hat{j} - 3\hat{k}$

$\mathbf{b} = \hat{i} + 3\hat{j} + 2\hat{k}$

According to the data, **c** is parallel to **a** × **b**

Here, $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & 3 & 2 \end{vmatrix}$

$= \hat{i}(2+9) - \hat{j}(4+3) + \hat{k}(6-1)$

$= 11\hat{i} - 7\hat{j} + 5\hat{k} = \mathbf{d}$ (say)

$|\mathbf{a}\times\mathbf{b}| = \sqrt{(11)^2 + (-7)^2 + (5)^2} = \sqrt{195}$

Now, $\hat{\mathbf{d}} = \dfrac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}\times\mathbf{b}|} = \dfrac{11\hat{i} - 7\hat{j} + 5\hat{k}}{\sqrt{195}}$

Thus, $\mathbf{c} = |\mathbf{c}|\hat{\mathbf{d}} = \dfrac{2}{\sqrt{195}}(11\hat{i} - 7\hat{j} + 5\hat{k})$

$= \dfrac{22}{\sqrt{195}}\hat{i} - \dfrac{14}{\sqrt{195}}\hat{j} + \dfrac{10}{\sqrt{195}}\hat{k}$

Hence, volume of the parallelopiped = [**a** **b** **c**]

$= \begin{vmatrix} 2 & 1 & -3 \\ 1 & 3 & 2 \\ \frac{22}{\sqrt{195}} & \frac{-14}{\sqrt{195}} & \frac{10}{\sqrt{195}} \end{vmatrix}$

$= \dfrac{2}{\sqrt{195}}\begin{vmatrix} 2 & 1 & -3 \\ 1 & 3 & 2 \\ 11 & -7 & 5 \end{vmatrix}$

$= \dfrac{2}{\sqrt{195}}[2(29) - 1(-17) - 3(-40)]$

$= \dfrac{2}{\sqrt{195}}[58 + 17 + 120] = \dfrac{2}{\sqrt{195}}\times 195 = 2\sqrt{195}$

62. **(a)** Given, curve $y = x^3 - 3x^2 + 5$ ...(i)

On differentiating w.r.t. 'x', we get

$\dfrac{dy}{dx} = 3x^2 - 6x$ ...(ii)

For local maxima or local minima, $\dfrac{dy}{dx} = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0$ or $x = 2$

Now, differentiating Eq. (ii) w.r.t. 'x' we get

$\dfrac{d^2y}{dx^2} = 6x - 6 \Rightarrow \left(\dfrac{d^2y}{dx^2}\right)_{x=0} = -6 < 0$

$\therefore$ $x = 0$ is a point of local maxima.

and $\left(\dfrac{d^2y}{dx^2}\right)_{x=2} = 6\times 2 - 6 = 12 - 6 = 6 > 0$

$\therefore$ $x = 2$ is a point of local minima.

63. **(d)** In the expansion of $(1+x)^n$.

General term,

$T_{r+1} = {}^nC_r(1)^{n-r}x^r = {}^nC_r\,x^r$

$\therefore$ Coefficient of $(r+1)$th term is ${}^nC_r$

Similarly, coefficient of pth term

$\therefore\quad p = {}^nC_{p-1}$ ...(i)

and coefficient of $(p+1)$th term

$\therefore\quad q = {}^nC_p$ ...(ii)

On dividing Eq. (i) by Eq. (ii), we get

$\dfrac{p}{q} = \dfrac{{}^nC_{p-1}}{{}^nC_p} = \dfrac{\dfrac{n!}{(p-1)!(n-p+1)!}}{\dfrac{n!}{(p)!\,(n-p)!}}$

$= \dfrac{(p)!\,(n-p)!}{(n-p+1)!\,(p-1)!} = \dfrac{p(p-1)!\,(n-p)!}{(n-p+1)(n-p)!(p-1)!}$

$\dfrac{p}{q} = \dfrac{p}{n-p+1}$

$\dfrac{1}{q} = \dfrac{1}{n-p+1}$

$n - p + 1 = q$

$p + q = n + 1$

64. **(d)** Given,

$f(x) = \begin{cases} \sin x, & \text{if } x \le 0 \\ x^2 + a^2, & \text{if } 0 < x < 1 \\ bx + 2, & \text{if } 1 \le x \le 2 \\ 0, & \text{if } x > 2 \end{cases}$ is continuous on IR

$\therefore\quad \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

and $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$\Rightarrow \lim_{x\to 0^-} \sin x = \lim_{x\to 0^+} x^2 + a^2$

$\Rightarrow 0 = 0 + a^2 \Rightarrow a = 0$

and $\lim_{x\to 2^-} bx + 2 = \lim_{x\to 2^+} 0$

$\Rightarrow 2b + 2 = 0 \Rightarrow \quad b = -1$

Now, $a + b + ab = 0 + (-1) + 0 = -1$.

65. **(d)** Given, $\cosh^{-1} x = 2\log_e(\sqrt{2}+1)$

$\log_e(x+\sqrt{x^2-1}) = \log_e(\sqrt{2}+1)^2$

$x+\sqrt{x^2-1} = (\sqrt{2}+1)^2 = 2+1+2\sqrt{2}$

$x+\sqrt{x^2-1} = 3+2\sqrt{2} = 3+\sqrt{8}$

On comparing rational and irrational part, we get $x = 3$.

66. **(b)** $\sum_{k=1}^{n} k(k+2) = \sum_{k=1}^{n} (k^2+2k) = \sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k$

$= \frac{n(n+1)(2n+1)}{6} + 2\frac{n(n+1)}{2}$

$= n(n+1)\left[\frac{(2n+1)}{6}+1\right] = n(n+1)\left[\frac{2n+7}{6}\right]$

$\therefore \sum_{k=1}^{n} k(k+2) = \frac{n(n+1)(2n+7)}{6}$

67. **(b)** Given equation of ellipse

$25x^2 + 4y^2 + 100x - 4y + 100 = 0$

It can be rewritten as,

$((5x)^2 + 2(5)(10)x + 10^2) + ((2y)^2 - 2(2)(1)y + 1^2) - 10^2 - 1^2 + 100 = 0$

$(5x+10)^2 + (2y-1)^2 = 1$

$25(x+2)^2 + 4\left(y-\frac{1}{2}\right)^2 = 1$

$\frac{(x+2)^2}{(1/5)^2} + \frac{(y-1/2)^2}{(1/2)^2} = 1$, which is of the form

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, where $a < b$

Here, $a = \frac{1}{5}, b = \frac{1}{2}$

Now, $e = \sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{4}{25}} = \sqrt{\frac{21}{25}} = \frac{\sqrt{21}}{5}$

$\therefore$ foci are

$\left(-2, \frac{1}{2} \pm be\right) = \left(-2, \frac{1}{2} \pm \frac{\sqrt{21}}{10}\right) = \left(-2, \frac{5\pm\sqrt{21}}{10}\right)$

68. **(c)** Given, $\left[\frac{\left(1+\cos\frac{\pi}{12}\right)+i\sin\frac{\pi}{12}}{\left(1+\cos\frac{\pi}{12}\right)-i\sin\frac{\pi}{12}}\right]^{72}$

$= \left[\frac{2\cos^2\frac{\pi}{24} + i2\sin\frac{\pi}{24}\cos\frac{\pi}{24}}{2\cos^2\frac{\pi}{24} - i2\sin\frac{\pi}{24}\cos\frac{\pi}{24}}\right]^{72}$

$= \left[\frac{2\cos\frac{\pi}{24}\left(\cos\frac{\pi}{24}+i\sin\frac{\pi}{24}\right)}{2\cos\frac{\pi}{24}\left(\cos\frac{\pi}{24}-i\sin\frac{\pi}{24}\right)}\right]^{72}$

$= \left(\frac{\left(\cos\frac{\pi}{24}+i\sin\frac{\pi}{24}\right)}{\left(\cos\frac{\pi}{24}-i\sin\frac{\pi}{24}\right)}\right)^{72}$

$= \left(\frac{\left(\cos\frac{\pi}{24}+i\sin\frac{\pi}{24}\right)}{\left(\cos\frac{\pi}{24}-i\sin\frac{\pi}{24}\right)} \times \frac{\left(\cos\frac{\pi}{24}+i\sin\frac{\pi}{24}\right)}{\left(\cos\frac{\pi}{24}+i\sin\frac{\pi}{24}\right)}\right)^{72}$

$= \left(\frac{\left(\cos\frac{\pi}{24}+i\sin\frac{\pi}{24}\right)^2}{\cos^2\frac{\pi}{24} - i^2\sin^2\frac{\pi}{24}}\right)^{72}$

$= \left(\frac{\left(\cos\frac{\pi}{24}+i\sin\frac{\pi}{24}\right)^2}{\cos^2\frac{\pi}{24} + \sin^2\frac{\pi}{24}}\right)^{72}$

$= \left(\cos\frac{\pi}{24}+i\sin\frac{\pi}{24}\right)^{144} \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= \cos\left(\frac{\pi}{24}\times 144\right) + i\sin\left(\frac{\pi}{24}\times 144\right)$

[By Demoivre's theorem]

$= \cos 6\pi + i\sin 6\pi = 1$

$\left[\begin{array}{l}\because \sin n\pi = 0 \ \forall\ n \in Z \\ \text{and } \cos n\pi = \begin{cases} 1, & \text{if } n \text{ is even} \\ -1, & \text{if } n \text{ is odd}\end{cases}\end{array}\right]$

69. **(a)** Given function, $f(x) = -3x - 3$

Range of $f = \{3, -6, -9, -18\}$

When $f(x) = 3$,

$$3 = -3x - 3 \Rightarrow 6 = -3x \Rightarrow x = -2$$

When $f(x) = -6$,

$$-6 = -3x - 3 \Rightarrow -3 = -3x \Rightarrow x = 1$$

When $f(x) = -9$,

$$-9 = -3x - 3 \Rightarrow -6 = -3x \Rightarrow x = 2$$

When $f(x) = -18$,

$$-18 = -3x - 3 \Rightarrow -15 = -3x \Rightarrow x = 5$$

Thus, domain of $f = \{-2, 1, 2, 5\}$

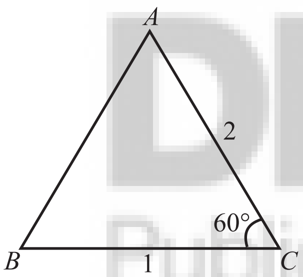
Hence, -1 cannot be in the domain of f.

70. **(a)** Area of triangle,

$$\Delta = \frac{1}{2} \cdot a \cdot b \sin c$$

$$\Delta = \frac{1}{2} \cdot 2 \cdot 1 \cdot \sin 60°$$

$$\Delta = \frac{\sqrt{3}}{2}$$



According to the sine rule,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{1} = \frac{\sin B}{2} = \frac{\sin 60°}{c}$$

$$\frac{\sin A}{\sin B} = \frac{1}{2} = \frac{\sin 60°}{c}$$

$$\therefore \quad \frac{1}{2} = \frac{\frac{\sqrt{3}}{2}}{c} \Rightarrow c = \sqrt{3}$$

Thus, $4\Delta^2 + c^2 = 4 \times \left(\frac{\sqrt{3}}{2}\right)^2 + (\sqrt{3})^2$

$$= 4 \times \frac{3}{4} + 3 = 6.$$

71. **(d)** Given that, $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and $|\mathbf{a} + \mathbf{b}| = 5$

Since, $|\mathbf{a} + \mathbf{b}| = 5$

$\therefore \quad |\mathbf{a} + \mathbf{b}|^2 = 25$

$|\mathbf{a}|^2 + 2\,\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2 = 25$

$9 + 2\,\mathbf{a} \cdot \mathbf{b} + 16 = 25$

$\mathbf{a} \cdot \mathbf{b} = 0$

Now, consider $|\mathbf{a} - \mathbf{b}|^2 = |\mathbf{a}|^2 - 2\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2$

$= 9 - 0 + 16 = 25$

$\therefore \quad |\mathbf{a} - \mathbf{b}| = 5.$

72. **(a)** Given that, $\int f(x) \cos x \, dx = \frac{1}{2}(f(x))^2 + C$

On differentiating w.r.t. 'x', we get

$$f(x) \cos x = \frac{1}{2} \times 2 f(x) \cdot f'(x)$$

$$f(x) \cos x = f(x) \cdot f'(x)$$

$f'(x) = \cos x$

Put $x = 0$, $f'(0) = \cos 0 = 1$

73. **(c)** It is given that,

α, β are the roots of the equation $ax^2 + bx + c = 0$

$$\therefore \quad \alpha + \beta = -\frac{b}{a}, \ \alpha\beta = \frac{c}{a}$$

Similarly, whose roots are $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$, equation will be

$$x^2 - \left(\frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta}\right)x + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1-\beta}{\beta}\right) = 0$$

$$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta} - 2\right)x + \left(\frac{1-(\alpha+\beta)+\alpha\beta}{\alpha\beta}\right) = 0$$

$$x^2 - \left(\frac{\alpha+\beta-2\alpha\beta}{\alpha\beta}\right)x + \left(\frac{1-(\alpha+\beta)+\alpha\beta}{\alpha\beta}\right) = 0$$

$$\alpha\beta x^2 - (\alpha+\beta-2\alpha\beta)x + (1-(\alpha+\beta)+\alpha\beta) = 0$$

$$\frac{c}{a}x^2 - \left(-\frac{b}{a} - \frac{2c}{a}\right)x + \left(1 + \frac{b}{a} + \frac{c}{a}\right) = 0$$

$$cx^2 + (b+2c)x + (a+b+c) = 0$$

Comparing the above equation with $px^2 + qx + r = 0$, we get $r = a + b + c$.

74. **(a)** Centre and radius of the circle are (0, 0) and 12.

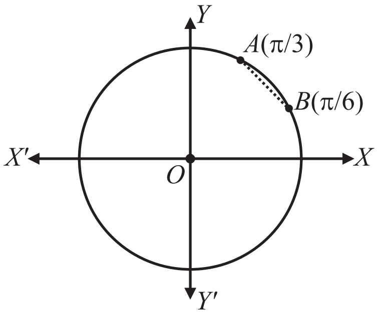
Parametric equations.

$$x = 12\cos\theta, \ y = 12\sin\theta$$

Now, coordinates of point A,

$$x = 12\cos\frac{\pi}{3}, \ y = 12\sin\frac{\pi}{3}; \quad x = 12 \cdot \frac{1}{2} \cdot y = 12 \cdot \frac{\sqrt{3}}{2}$$

$$x = 6, \ y = 6\sqrt{3} \ \text{ i.e. } \ A \equiv (6, 6\sqrt{3})$$



and coordinates of point B,

$$x = 12\cos\frac{\pi}{6},\ y = 12\sin\frac{\pi}{6}$$

$$\Rightarrow\ x = 12\cdot\frac{\sqrt{3}}{2},\ y = 12\cdot\frac{1}{2}\ \Rightarrow\ x = 6\sqrt{3},\ y = 6$$

i.e. $B \equiv (6\sqrt{3}, 6)$

Now, length of chord

$$AB = \sqrt{(6\sqrt{3}-6)^2 + (6-6\sqrt{3})^2}$$

$$= \sqrt{2\times 6^2(\sqrt{3}-1)^2} = 6\sqrt{2}(\sqrt{3}-1) = 6(\sqrt{6}-\sqrt{2})$$

75. **(b)** Given equations of pair of straight lines,

$xy - x - y + 1 = 0$

$x(y-1) - 1(y-1) = 0$

$(x-1)(y-1) = 0$

$x = 1$ and $y = 1$.

Thus, three concurrent lines are can be given as,

$x = 1$ ...(i)

$y = 1$ ...(ii)

and $x + ay - 3 = 0$ ...(iii)

Since, Eqs. (i) and (ii), intersect at only point, namely (1, 1), therefore this point also satisfy the Eq. (iii) we get

$1 + a - 3 = 0$

$\Rightarrow\ a = 2$

Now, the pair of lines

$ax^2 - 13xy - 7y^2 + x + 23y - 6 = 0$ becomes

$2x^2 - 13xy - 7y^2 + x + 23y - 6 = 0$

The acute angle between these lines,

$$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{2\sqrt{\left(-\frac{13}{2}\right)^2+14}}{-5}\right|$$

$$= \left|\frac{2\sqrt{\frac{169+56}{4}}}{-5}\right| = \left|\frac{2\sqrt{\frac{225}{4}}}{-5}\right| = \left|\frac{2\times\frac{15}{2}}{-5}\right| = |-3| = 3.$$

$$\theta = \tan^{-1}(3) = \cos^{-1}\left(\frac{1}{\sqrt{1+3^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$$

76. **(b)** Given, $\cos 2\theta = \sin\theta$

$1 - 2\sin^2\theta = \sin\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(\sin\theta + 1)(2\sin\theta - 1) = 0$

$\sin\theta = -1$ or $\sin\theta = \frac{1}{2}$

$\theta = \frac{3\pi}{2}$ or $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad [\because \theta \in (0, 2\pi)]$

Thus, number of solutions of the given equation is 3.

77. **(d)** Let $a = 13$, $b = 14$ and $c = 15$. Then

$$S = \frac{a+b+c}{2} = \frac{13+14+15}{2} = 21$$

Area of triangle,

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21\times 8\times 7\times 6}$$

$$= \sqrt{7\times 3\times 4\times 2\times 7\times 3\times 2} = 7\times 3\times 4 = 84.$$

Now, as we know $R = \frac{abc}{4\Delta}$ and $r = \frac{\Delta}{s}$

$\therefore\ R = \frac{13\times 14\times 15}{4\times 84}$ and $r = \frac{84}{21}$

$R = \frac{65}{8}$ and $r = 4$

So, $8R + r = 8\left(\frac{65}{8}\right) + 4 = 65 + 4 = 69.$

78. **(a)** As we know that the variance of n numbers which are in A.P. whose first terms is 'a' and common difference is d.

$$\text{Variance} = \sigma^2 = \frac{d^2(n^2-1)}{12}$$

$$\therefore\ \text{Var}(\sigma_1) = \frac{2^2(n^2-1)}{12} = \frac{(n^2-1)}{3} = A$$

$$\text{Var}(\sigma_2) = \frac{2^2(n^2-1)}{12} = \frac{(n^2-1)}{3} = B$$

$\therefore\ A = B.$

79. **(c)** It is given that $x - y = -4k$ or $y = x + 4k$ is a tangent to the parabola $y^2 = 8x$, therefore

$4k = \frac{2}{1}$ $\quad [\because \text{Here } 4a = 8 \Rightarrow a = 2]$

$\Rightarrow\ k = \frac{1}{2}$

Also point of contact p is $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = (2, 4)$

Now, equation of normal

$$(y-4)=\frac{-4}{2(2)}(x-2)$$

$y-4=-1\,(x-2)$

$y-4=-x+2$

$x+y=6$

$x+y-6=0.$

The perpendicular distance of normal from $(k, 2k)$ i.e. $\left(\frac{1}{2},1\right)$,

$$D=\frac{\left|\frac{1}{2}+1-6\right|}{\sqrt{1^2+1^2}}=\frac{\left|\frac{3}{2}-6\right|}{\sqrt{2}}=\frac{9}{2\sqrt{2}}$$

80. **(d)** Given that $P(A) = 0.6$, $P(B) = 0.4$ and $P(A\cap B)=0$,

then probability that neither A nor B occurs $=P(\bar{A}\cap\bar{B})$

$=P(\overline{A\cup B})=1-P(A\cup B)$

$=1-P(A)-P(B)+P(A\cap B)$

$[\because P(A\cup B)=P(A)+P(B)-P(A\cap B)]$

$= 1 - 0.6 - 0.4 + 0 = 1 - 1 = 0.$

PHYSICS

81. **(b)** Pressure, $P=\frac{F}{L^2}$ $[\because$ Area of square $= L^2]$

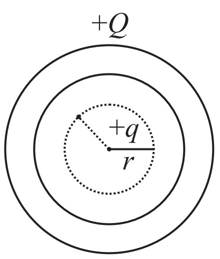
$$\frac{\Delta P}{P}=\frac{\Delta F}{F}+\frac{2\Delta L}{L}$$

$$\left(\frac{\Delta P}{P}\times 100\right)=\left(\frac{\Delta F}{F}\times 100\right)+2\left(\frac{\Delta L}{L}\times 100\right)$$

$= 4\% + 2(3\%) = 10\%.$

82. **(c)** According the Gauss's law, $\oint_S E ds=\frac{Q_{inside}}{\varepsilon_0}$

Here, Gaussion surface is a sphere of radius r as shown in figure.

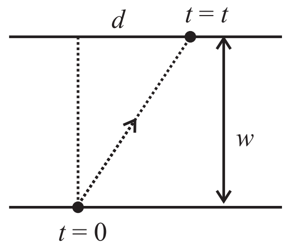


$$E\oint_S ds=\frac{q}{\varepsilon_0}$$

$$E(4\pi r^2)=\frac{q}{\varepsilon_0}$$

Electric field, $E=\frac{q}{4\pi r^2\varepsilon_0}$

83. **(b)** Given,
Width of river $(w) = 200$ m
Velocity of river, $V_{stream} = 2$ m/s
Velocity of man w.r.t. river, $V_{man} = 1$ m/s
We know that,



$$\frac{w}{v_{man}}=\frac{d}{v_{stream}}=t$$

$$d=\frac{w\times v_{stream}}{v}$$

$$d=\frac{200\times 2}{1}=400\text{ m.}$$

84. **(b)** Current $(i)=\frac{e}{R_{eq}}=\frac{-\Delta\phi}{R_{eq}\Delta t}$ $\left[\because e=\frac{-\Delta\phi}{\Delta t}\right]$

Total resistance of the combination,

$R_{eq}=R+4R=5R$

Given that ϕ_1 and ϕ_2 are the values of magnetic flux at $t = 0$ and $t = t$ respectively.

$\therefore\quad i=-\frac{n(\phi_2-\phi_1)}{5R(\Delta t)}$

Induced current, $i=-\frac{n(\phi_2-\phi_1)}{5Rt}$

85. **(d)** Given,
Length of simple pendulum, $l = 1$m.
Acceleration of an elevator, $a = 2\text{m/s}^2$.
Acceleration due to gravity, $g = 10\text{m/s}^2$
We know that

$$T=2\pi\sqrt{\frac{l}{g+a}}=2\pi\sqrt{\frac{1}{10+2}}$$

$$=2\pi\sqrt{\frac{1}{12}}=\pi\sqrt{\frac{4}{12}}\quad T=\frac{\pi}{\sqrt{3}}\text{ s}$$

86. **(c)** Given,
Radius of metal ball $= r$, Velocity of ball $= v$,
Angle between velocity (v) and magnetic field $(B) = \alpha$
We know that,

$e=Bl\,v\sin\alpha,$

Here, $l = 2r$

$\therefore\quad e=2r\,|B|\,|v|\sin\alpha$

87. **(*)** Equivalent emf of the circuit

$E_{eq} = 3E \,||\, 3E = 3E$

$\Rightarrow\ E_{eq} = 3E = 3 \times 20 = 60$

External resistance, $R = 4\Omega$

$\therefore$ Main current flowing through the load

$$i=\frac{3E}{R}=\frac{60}{4}=15\text{ A}$$

88. **(d)** de-Broglie wavelength, $\lambda=\frac{h}{\sqrt{2mk}}\Rightarrow\lambda\propto\frac{1}{\sqrt{\text{K.E.}}}$

Given, Initial de-Broglie wavelength, $\lambda_1 = 1$nm

Final de-Broglie wavelength, $\lambda_2 = 0.5$ nm

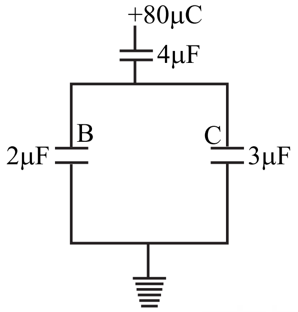
$\therefore$ Kinetic energy (KE) $\propto \frac{1}{\lambda^2}$ $\quad \therefore \frac{KE_2}{KE_1} = \left(\frac{\lambda_1}{\lambda_2}\right)^2$

$\Rightarrow \frac{KE_2}{KE_1} = \left(\frac{1}{0.5}\right)^2 \Rightarrow KE_2 = 4KE_1$

Hence, the kinetic energy is increases three times.

$\therefore \Delta KE = 3KE_1$

89. **(b)**



Let Q_B and Q_C be the charges on plates B and C respectively.

$Q_B + Q_C = 80$ ...(i)

The potential across plates B and C will be equal

$\therefore \frac{Q_B}{C_B} = \frac{Q_C}{C_C}$

$\Rightarrow \frac{Q_B}{2} = \frac{Q_C}{3} \Rightarrow \frac{80 - Q_C}{2} = \frac{Q_C}{3}$

$\Rightarrow 240 - 3Q_C = 2Q_C \Rightarrow Q_C = 48\mu C$

Since the lower plate of capacitor C is connected to ground. So, upper plate of capacitor C will be positive.

$\therefore$ Charge on upper plate of $3\mu F = +48\mu C$

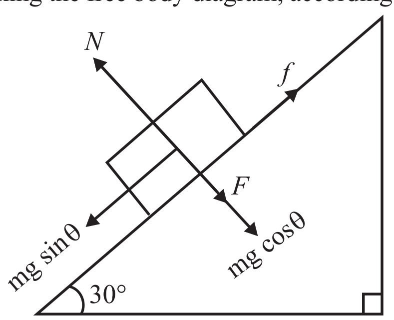
90. **(d)** Given,

Young's modulus of material, $Y = 2 \times 10^{11}$ N/m^2

$$Y = \frac{\text{Stress}}{\text{Strain}} \Rightarrow Y = \frac{\text{Stress}}{\frac{\Delta L}{l}} = \frac{\Delta P}{\frac{\Delta L}{l}}$$

$$\Delta L = \frac{\Delta P \times l}{Y} = \frac{1 \times 10^8 \times 1}{2 \times 10^{11}} = 0.5 \times 10^{-3} = 0.5 \text{ mm}$$

91. **(d)** Making the free body diagram, according to question,



$mg\sin\theta = \mu(F + mg\cos\theta)$

$\Rightarrow g\sin\theta = \mu F_{min} + \mu g\cos\theta$

Here, mass, m = 1kg

Accelerating due to gravity, g = 10m/s^2

Coefficient of static friction, $\mu = 0.2$

$$\Rightarrow 10 \times \frac{1}{2} = 0.2\ F_{min} + 0.2 \times 10 \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow 5 = 0.2F_{min} + 0.2 \times 5\sqrt{3} \Rightarrow F_{min} = 16.33 \text{ N.}$$

92. **(a)** Given,

Coefficient of linear expansion, $\alpha = 11 \times 10^{-6}$/°C.

Length of scale, l = 1m

Change in length,

$$\Delta l = l\alpha\Delta T \Rightarrow \Delta T = \frac{\Delta l}{l\alpha}$$

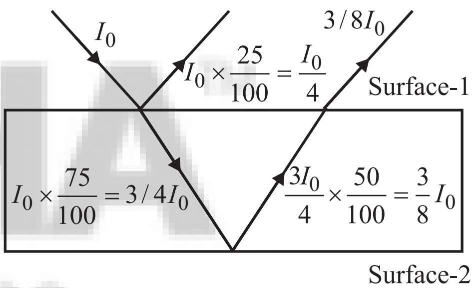
$$\Rightarrow \Delta T = \frac{6 \times 10^{-5}}{1 \times 11 \times 10^{-6}} = 5.45°C.$$

So, the range of temperature in which the experiment can be performed will be 19°C to 31°C.

93. **(c)** As core is pulled out $\phi \downarrow\downarrow$ i.e. $\frac{\Delta\phi}{\Delta t}$ is –ve.

So, $e = -\frac{\Delta\phi}{\Delta t}$ is +ve i.e. induced emf will support current and hence overall current increases.

94. **(a)**



Intensity of reflected light from upper surface

$I_1 = \frac{I_0}{4}$

Intensity of reflected light from lower surface

$I_2 = \frac{3}{8}I_0$

We know that,

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \left(\frac{\frac{1}{2} + \sqrt{\frac{3}{8}}}{\frac{1}{2} - \sqrt{\frac{3}{8}}}\right)^2$$

95. **(d)** Given, Area of the thermocal box, A = 1cm^2.

Thermal conductivity of thermocal box, K = 0.03W/mk

Thickness of wall, $l = 3 \times 10^{-2}$ m

$\Delta T = 30 - 0 = 30°C$

L_{Fusion} (ice) $= 3 \times 10^5$ J/kg

In one day t = 24 × 60 × 60 s using

$$\frac{Q}{t} = \frac{KA\Delta T}{l} \Rightarrow \frac{m \times L_{fusion\ (ice)}}{t} = \frac{KA}{l}\Delta T$$

$$\Rightarrow \frac{m\times 3\times 10^5}{24\times 60\times 60}=\frac{0.03\times 1}{3\times 10^{-2}}\times 30$$

$$\Rightarrow m=\frac{0.03\times 1\times 30\times 24\times 60\times 60}{3\times 10^{-2}\times 3\times 10^5}$$

$$\Rightarrow m=\frac{77760}{9000}=8.64 \text{ kg.}$$

96. **(d)** $^{239}_{94}\text{Pu}\xrightarrow{\alpha}\,^{235}_{92}\text{U}$

When α-particle is emitted that atomic mass is decreases by four and atomic number is decreases by two. So option (d) is correct.

97. **(d)** Given,
Inductance, L = 400 mH
Capacitance, C = 200 μF
Resistance, R = 50 Ω
Inductive reactance,

$$X_L=\omega L=200\times 400\times 10^{-3}=80\ \Omega$$

Capacitive reactance,

$$X_C=\frac{1}{\omega C}=\frac{1}{200\times 200\times 10^{-6}}=25\ \Omega$$

We know that, Impedance,

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{(50)^2+(80-25)^2}$$

$$=\sqrt{(50)^2-(55)^2}=\sqrt{2500+3025}$$

$$Z=\sqrt{5525}=74.3\Omega$$

Current, $I=\frac{E}{Z}=\frac{10}{74.3}=0.13549$ A

RMS voltage across Inductor,

$$E_L=IX_L=0.13459\times 80=10.76 \text{ V or } 10.8 \text{ V.}$$

98. **(d)** Given,
Resistance, R_2 = 4.5Ω at T = 100°C
Resistance, R_1 = 3.1Ω at T = 30°C

$$R_2=R_1[1+\alpha(T_2-T_1)]$$

$$\Rightarrow \frac{R_2-R_1}{R_1(T_2-T_1)}=\alpha \Rightarrow \alpha=\frac{4.5-3.1}{3.1\times 70}=0.0064°\text{C}^{-1}$$

99. **(a)** Velocity of an object half a second before maximum height = Velocity of an object half a second after maximum height (return journey)

V = 0 + gt (∵ u = 0)

$$=0+9.8\times\frac{1}{2}=4.9 \text{ m/s.}$$

100. **(b)** Energy given to hydrogen atom
= Difference in energy level from n_1 = 1 to n_2 = 3 states

$$=13.6\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$=13.6\left(\frac{1}{1^2}-\frac{1}{3^2}\right)=13.6\times\frac{8}{9}=12.08=12.1 \text{ eV.}$$

101. **(a)** Given,
We have $x^2+y^2=a^2\cos^2 t+a^2\sin^2 t$

$\Rightarrow x^2+y^2=a^2$ which is the equation of circle,

and $\frac{dz}{dt}=1=$ Const.

It means particle moves along z direction with constant speed.
Thus, the particle will move along z direction with constant speed tracing a circular path in xyz plane.
Hence, path is helix.

102. **(a)** In first case, efficiency of heat engine is given by

$$1-\frac{T_2}{T_1}=\frac{1}{6} \qquad ...(i)$$

Here, T_1 = temperature of source
T_2 = temperature of sink
In second case

$$1-\frac{T_2-62}{T_1}=\frac{2}{6} \Rightarrow 1-\frac{T_2}{T_1}-\frac{62}{T_1}=\frac{2}{6}$$

$$\Rightarrow \frac{1}{6}-\frac{62}{T_1}=\frac{2}{6} \Rightarrow T_1=372 \text{ K.}$$

From (i), T_2 = 310 K.

103. **(a)** Given, Distance of light source from mirror, u = 1.5 m
Radius of curvature, R = 1m

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u} \qquad \left[\because f=\frac{R}{2}\right]$$

$$\therefore \frac{2}{R}=\frac{1}{v}+\frac{1}{u} \Rightarrow \frac{2}{1}=\frac{1}{v}+\frac{1}{\frac{-3}{2}}$$

$$\Rightarrow \frac{2}{1}=\frac{1}{v}-\frac{2}{3}\Rightarrow\frac{1}{v}=2+\frac{2}{3}$$

$$\Rightarrow \frac{1}{v}=\frac{8}{3}\Rightarrow v=\frac{3}{8}=0.375 \text{ m.}$$

The image formed at right side of mirror, so the image is virtual and upright.

$$\text{Magnification } (m)=\frac{-v}{u}=\frac{\frac{-3}{8}}{\frac{-3}{2}}=\frac{1}{4}=0.25.$$

104. **(c)** Given, Number of moles of air in room, n = 2000
Difference of temperature, ΔT = 24°C – 34°C = –10°C
Change in internal energy,

$$\Delta U=nC_v\Delta T$$

$$C_v=\frac{R}{Y-1}=\frac{8314}{1.4-1}=\frac{8314}{0.4}$$

$$dU=2\times 10^3\times\frac{8314}{0.4}\times(-10)=-4.2\times 10^5 \text{ J}$$

105. **(d)** Given,
Initial speed of projectile u = 20m/s
Angle of projectile, θ = 30°
Maximum height,

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{(20)^2(\sin 30)^2}{2\times 10}$$

$$\Rightarrow H = \frac{20\times 20\times\left(\frac{1}{2}\right)^2}{20} = 5 \text{ m.}$$

106. **(c)** As Diameter of pipe (d) = 5 mm = 5×10^{-3} m
Density of gasoline (ρ) = 720 kg/m^3
Viscocity of gas0oline $(\eta) = 6\times 10^{-3}$ Poise
The flow of water becomes turbulent after critical velocity.
Critical velocity,

$$v_c = \frac{\eta}{\rho d} = \frac{6\times 10^{-3}}{720\times 5\times 10^{-3}}$$

$$= \frac{6\times 10^{-3}}{3600\times 10^{-3}} = 1.66\times 10^{-3} \text{ m/s.}$$

107. **(c)** As $R_{\text{Conductor}} \propto T$ and $R_{\text{Semi conductor}} \propto \frac{1}{T}$.
So, resistance of Cu will decrease and Ge will increase.

108. **(a)** From Snell's law, $\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{\lambda_1}{\lambda_2}$

$$\Rightarrow \lambda_2 = \lambda_1 \frac{\sin\alpha_2}{\sin\alpha_1}$$

109. **(b)** Given,
Peak voltage of message signal, $A_m = 5$
Peak voltage of carrier signal, $A_C = 15$
Modulated wave signal = $[A_C + A_m \sin(2\pi f_m t)] \sin(2\pi f_C t)$

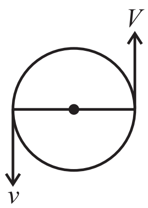
$$C_m(t) = A_C\left[1 + \frac{A_m}{A_C}\sin(2\pi f_m)t\right]\sin 2\pi f_C t$$

$$= 15\left[1 + \frac{5}{15}\sin(2\pi\times 10^3)t\right]\sin 2\pi\times 10^6 t$$

$$= 15\left[1 + \frac{1}{3}\sin(2\pi 10^3 t)\right]\sin(2\pi 10^6 t)$$

110. **(d)** SONAR technology uses the reflection of ultrasonic waves.

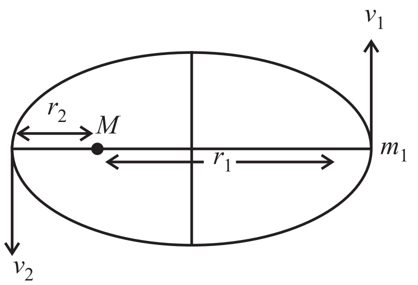
111. **(b)**



Change in momentum, $Mv - (-Mv) = 2Mv$.

112. **(a)** In an elastic collision, when a body collide with another identical body at rest, then they merely exchange their velocities and kinetic energies. Line of motion is same.
So, after collision, $\theta_{\text{diversion}} = 0°$.

113. **(a)**



As angular momentum of planet is conserved.

So, $\vec{L_1} = \vec{L_2} \Rightarrow mv_1r_1 = mv_2r_2$

$$\Rightarrow v_2 = \frac{v_1r_1}{r_2} \quad \text{...(i)}$$

Using the law of conservation of total mechanical energy.

$$-\frac{-GMm}{r_1} + \frac{1}{2}mv_1^2 = -\frac{GMm}{r_2} + \frac{1}{2}mv_2^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii) we get

$$v_1 = \sqrt{\frac{2GMr_2}{(r_1+r_2)r_1}}$$

Angular momentum, $L = mv_1r_1$

$$= m\left(\sqrt{\frac{2GMr_2}{(r_1+r_2)r_1}}\right)\times r_1 \Rightarrow L = m\sqrt{\frac{2GMr_1r_2}{r_1+r_2}}$$

114. **(a)** Let the distance object slides on rough surface bed.
$W_{\text{all forces}} = \Delta K$
Work done by friction, $W_f = \mu mgd$
Work done by gravitational force, $W_g = mgh$

$$\Rightarrow W_f + W_g = \Delta K \Rightarrow -\mu mgd + mgh = 0$$

$$\Rightarrow \mu mgd = mgh \Rightarrow d = \frac{h}{\mu}$$

115. **(a)** Given, magnetic field, B = 0.01T
Area, A = 2×10^{-2}m^2

$$Emf,\ e = -N\frac{d\phi}{dt} = -N\frac{d}{dt}(BA\sin\omega t) = -NBA\omega\cos\omega t$$

So, $e_{\max} = NBA\omega = NBA(2\pi f)$

$$= 100\times 0.01\times 2\times 10^{-2}\times 2\times\frac{22}{7}\times 50 = 6.28 \text{ V.}$$

116. **(c)** The frequency of sound wave does not change with medium.

117. **(c)** The majority charge carriers in n-type semiconductor are electrons and that in p-type are holes.

118. **(d)** Deceleration $\omega = -a\sqrt{v}$

But, $\omega = \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = -a\sqrt{v}$

$$\Rightarrow \frac{-dv}{\sqrt{v}} = a\cdot dt \Rightarrow \int_{v_1}^{0}\frac{dv}{\sqrt{v}} = \int a\ dt$$

$$\Rightarrow \left|2\sqrt{v}\right|_{v_1}^{0} = at \Rightarrow t = \frac{2}{a}\sqrt{v_1}$$

Again, $\frac{-dv}{dt} = a\sqrt{v} \Rightarrow \frac{dv}{dx}\cdot\frac{dx}{dt} = -a\sqrt{v}$

$\Rightarrow \frac{dv}{dx} \cdot v = -a\sqrt{v} \Rightarrow dv\sqrt{v} = -a \cdot dx$

$\Rightarrow \int_{v_0}^{0} \sqrt{v}\, dv = -a\int_{0}^{s} ds$

After solving, we get

$s = \frac{2}{3a} \cdot v_0^{\frac{3}{2}}.$

119. **(c)** A current carrying conductor produces magnetic field in its neighbourhood. It does not produce electric field as current carrying wire is overall neutral.

120. **(a)** Given,

Total force on particle,

$F = F_1 + F_2$

$= \hat{i} + 2\hat{j} + 3\hat{k} + 4\hat{i} - 5\hat{j} - 2\hat{k} = (5\hat{i} - 3\hat{j} + \hat{k})\,N$

Displacement of particle,

$s = r_2 - r_1 = 7\hat{k} - 20\hat{i} - 15\hat{j}$

$= (-20\hat{i} - 15\hat{j} + 7\hat{k})$ cm

We know that

Work $(w) = F \cdot s$

$= (5\hat{i} - 3\hat{j} + \hat{k})(-20\hat{i} - 15\hat{j} + 7\hat{k}) \times 10^{-2}$

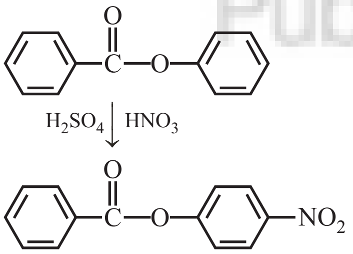
$= (-100 + 45 + 7) \times 10^{-2} = -0.48$ J.

CHEMISTRY

121. **(d)** Conditions for real solutions showing negative deviation from Raoult's law are as follows :

$\Delta H_{mix} < 0$ and $\Delta V_{mix} < 0$

122. **(c)**



123. **(c)**

124. **(c)** As we go down the group, basic nature of oxide increases. Hence, PoO is the most basic oxide.

125. **(d)** Stability of compound is low oxi dation state increases down the group.

126. **(b)** The size of element decreases from left to right in a period while increases down the group.

127. **(a)** $Na_2(Na_4P_6O_{18}) \longrightarrow 2Na^+ + Na_4P_6O_{18}^{2-}$

Calgon (Complex anion)

$Ca^{2+} + Na_4P_5O_{18}^{2-} \longrightarrow 2Na^+ + CaNa_2P_6O_{18}^{2-}$

128. **(b)**

129. **(d)** DIBAL–H is a strong, bulky reducing agent.

130. **(a)** $MCl + e^- \longrightarrow M + Cl^-$ cathode (reduction)

$Cl^- \longrightarrow \frac{1}{2}Cl_2 + e^-$ anode (oxidation)

$MCl \longrightarrow M + \frac{1}{2}Cl_2$ (Cell reaction)

$E_{cell} = E^0_{cell} - \frac{0.059}{n}\log K_c$

$\Rightarrow -1.140 = -0.55 - \frac{0.059}{1}\log K_c$

$\Rightarrow -0.59 = -0.059\log K_c \Rightarrow \log K_c = 10$

$\therefore K_c = 10^{10}$

K_{sp} is for $M + \frac{1}{2}Cl_2 \longrightarrow MCl$

$\therefore K_{sp} = \frac{1}{K_c} = \frac{1}{10^{10}} = 10^{-10}.$

131. **(a)**

132. **(c)** $4H^+ + 4e^- \rightleftharpoons 2H_2$

$E = E^\circ - \frac{0.059}{n}\log\frac{(pH_2)^2}{[H^+]^4}$

$\Rightarrow -0.059 = 0 - \frac{0.059}{4}\log\frac{1}{[H^+]^4}$

$\Rightarrow [H^+] = 10^{-1} = 0.1$ M.

133. **(a)** The melting point decreases down the group but Pb > Sn.

134. **(d)**

135. **(d)** $CO_2(g) + H_2(g) \longrightarrow CO(g) + H_2O(g)$

$\Delta H_r^\circ = (\Delta H)_{products} - (\Delta H)_{reactants}$

Hence,

$\Delta H_r^\circ = [\Delta H_f(CO(g)) + \Delta H_f H_2O(g)] - [\Delta H_f CO_2(g) + \Delta H_f H_2(g)]$

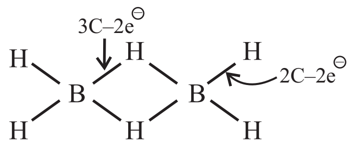
$\therefore \Delta H_r^\circ = -110.5 - 241.8 + 393.5 = 41.2.$

136. **(d)** As the size of halogen increase, bond length increases and thus, bond energy decreases.

Hence, HI releases $H^\oplus$ ions easily therefore it is the strongest acid.

137. **(d)** OSF_2, central atom sulphur contains 3 *b.p.* and 1 *l.p.* of e^-, hence OSF_2 is pyramidal.

138. **(a)** Diborane molecular formula B_2H_6.



139. **(c)**

140. **(c)** $2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2$

This reaction is self reduction.

141. (c) $E_n = -13.6\left(\frac{Z^2}{n^2}\right)$ eV

For first excited state, $n = 2$

$E_2 = \frac{-13.6}{2^2} = \frac{-13.6}{4} = -3.4$ eV.

142. (b) In zero order reaction, $t_{1/2} \propto \frac{[A]_0}{2k}$

143. (d) Down the group size increases and as charge on cation increases, size decrease.

144. (a) When value of Δ_0 is greater than P, pairing of electrons take place.

145. (a) In dry acetone, reaction follows S_N2 mechanism. Hence,

$CH_3Br > CH_3CH_2Br > CH_3—CH(CH_3)—Br > CH_3—C(CH_3)_2—Br$

146. (b) $CH_3—CH_2—CH(OH)—CH_3 \xrightarrow[-H_2O]{\text{Acid}} CH_3—CH=CH—CH_3$

147. (a) Given weight of solute 98 g. molar mass of solute 98 g mol^{-1} weight of solvent = 2 g

$$\text{Molality} = \frac{\text{Weight of } H_2SO_4}{\text{Molecular weight of } H_2SO_4} \times \frac{100}{\text{Weight of water}}$$

$$= \frac{98}{98} \times \frac{1000}{2} = 500 \text{ m.}$$

148. (d) Cyclohexylamine is not a aromatic amine and azo dye test is given by aromatic amine only.

149. (a) **150. (c)**

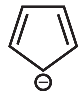
151. (d) Tertiary alcohol reacts fastest with Lucas reagent.

152. (b) $48\,KNO_3 + 5\,C_{12}H_{22}O_{11} \longrightarrow 24\,N_2 + 36\,CO_2 + 55\,H_2O + 24\,K_2CO_3$

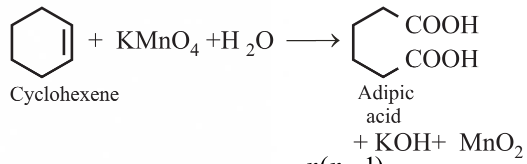
153. (b)

154. (c) Helium gas exhibit heating effect during Joule Thomson expansion due to low inversion temperature.

155. (a) As vapour pressure increases, boiling point will decrease.

156. (b) 

Cyclopentadienyl anion. It has $(4n + 2)\pi e^-$. Therefore, aromatic but structure is non benzenoid.

157. (b) 

158. (c) Number of spectral lines $= \frac{n(n-1)}{2}$

159. (b) As the size of halogen atom increase, the bond length increase from F_2 to I_2.

160. (b) If there are n atoms in packing then,

Number of octahedral voids = n

Number of tetrahedral voids = $2n$

In CCP, $N_{eff} = 4$.

$\therefore$ no. of octahedral voids = 4

no. of tetrahedral voids = 2 × 4 = 8.