TS/EAMCET Solved Paper 2021
Held on August 4

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. Each question is of 1 mark.
3. There are three parts in the question paper consisting of Mathematics (80 Questions), Physics (40 Questions) and Chemistry (40 Questions).
4. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
5. All calculations / written work should be done in the rough sheet provided .

MATHEMATICS

1. If $f: R \to R$ is defined as $f(x + y) = f(x) + f(y)$, $\forall\ x, y \in R$ and $f(1) = 10$, then $\sum_{r=1}^{n} (f(r))^2$ is equal to

(a) $\frac{7}{2}n(n+1)$ (b) $5n(n+1)$

(c) $\frac{50}{3}n(n+1)(2n+1)$ (d) $\frac{100}{4}n^2(n+1)^2$

2. If $f: R \to R$ is defined as $f(x) = \frac{3^x + 3^{-x}}{2}$, $\forall\ x \in R$ and it satisfies $f(x + y) + f(x - y) = af(x)f(y)$, then a is equal to

(a) 2 (b) 1 (c) 4 (d) 8

3. For all $n \in N$, $\frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \ldots + \frac{1}{(3n-1)(3n+2)}$ is equal to

(a) $\frac{n}{6n+4}$ (b) $\frac{n^2}{6n+4}$

(c) $\frac{1}{2} \cdot \frac{n^2}{6n+4}$ (d) $\frac{n}{6n^2+4}$

4. If the system of equations $3x - 2y + z = 0$, $\lambda x - 14y + 15z = 0$ and $x + 2y - 3z = 0$ has a solution other than $x = y = z = 0$, then λ is equal to

(a) 1 (b) 2 (c) 3 (d) 5

5. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = K(a - b)(b - c)(c - a)$, then K is equal to

(a) – 1 (b) 1 (c) 2 (d) 3

6. If $x = \alpha$, $y = \beta$ and $z = \gamma$ is the unique solution of the system of equations $5x - 7y + 3z = 0$, $7x + 10y - 8z = 3$ and $2x + 3y - 4z + 4 = 0$, then β is equal to

(a) $\frac{1}{2}$ (b) 2 (c) – 2 (d) $\frac{-1}{2}$

7. If $(a + ib)^{\frac{1}{4}} = 2 + 3i$, then $3b - 2a$ is equal to

(a) – 22 (b) – 122 (c) – 598 (d) – 698

8. ω is a complex cube root of unity. Match the items of List-I to the items of List-II.

List-I		List-II	
(A)	$\omega^{1010} + \omega^{2020}$	(i)	0
(B)	$(1 + \omega - \omega^2)(1 - \omega + \omega^2)$	(ii)	1
(C)	$(2 + \omega^2 + \omega^4)^5$	(iii)	– 1
(D)	$(3 + 5\omega + 3\omega^2)^3$	(iv)	4
		(v)	8

The correct match is

	A	B	C	D
(a)	(iii)	(iv)	(i)	(v)
(b)	(i)	(iv)	(ii)	(v)
(c)	(iii)	(iv)	(ii)	(v)
(d)	(iii)	(i)	(ii)	(iv)

9. If the roots of the equation $(z - 4)^3 = 8i$ are $a - 2i$, $b + i$ and $c + i$, then $\sqrt{abc}$ is equal to

(a) $13\sqrt{3}$ (b) $4\sqrt{13}$ (c) $2\sqrt{13}$ (d) $5\sqrt{3}$

10. If $\frac{\alpha}{\alpha+1}$ and $\frac{\beta}{\beta+1}$ are the roots of the quadratic equation $x^2 + 7x + 3 = 0$, then the equation having roots α and β is

(a) $3x^2 - x - 3 = 0$ (b) $11x^2 + 13x + 3 = 0$

(c) $13x^2 + 11x + 13 = 0$ (d) $11x^2 + 3x + 13 = 0$

11. If $y = \frac{x^2+14x+9}{x^2+2x+3}$, $\forall\ x \in R$, then the interval of maximum length in which y lies is

(a) $[-5, 4]$ (b) $[-4, 5]$

(c) $\left[\frac{1}{3}, 3\right]$ (d) $\left[\frac{-1}{3}, 3\right]$

12. The equation whose roots are squares of the roots of $x^4 - 2x^3 + 6x - 21 = 0$ is

(a) $x^4 - 4x^3 - 18x^2 - 36x + 441 = 0$

(b) $x^4 + 18x^3 - 4x^2 + 36x + 441 = 0$

(c) $x^4 - 2x^3 + 4x^2 + 6x + 441 = 0$

(d) $x^4 + 3x^3 - 5x^2 + 6x + 441 = 0$

13. If α, β and γ are the roots of $x^3 - x + 1 = 0$, then $\frac{1+\alpha}{1-\alpha}+\frac{1+\beta}{1-\beta}+\frac{1+\gamma}{1-\gamma}$ is equal to

(a) 1 (b) 0 (c) 2 (d) – 2

14. ${}^{34}C_5 + \sum_{r=0}^{4} {}^{(38-r)}C_4$ is equal to

(a) $22 \times {}^{39}C_4$ (b) ${}^{39}C_4$

(c) ${}^{3\times 39}C_5$ (d) ${}^{39}C_5$

15. If ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11 : 52$, then r is equal to

(a) 3 (b) 5 (c) 7 (d) 9

16. The value of the numerically greatest term in the expansion of $(2x + 3y)^{11}$, when $x = \frac{1}{2}$ and $y = \frac{1}{3}$ is

(a) 462 (b) ${}^{11}C_5\left(\frac{2}{3}\right)^6$

(c) ${}^{11}C_6\left(\frac{3}{2}\right)^5$ (d) 576

17. The coefficient of x^4 in the expansion of $(1 - x - x^2 + x^3)^6$ is

(a) 120 (b) 15 (c) – 75 (d) – 60

18. The sum of the coefficients in the expansion of $\left(1+\frac{x}{2}\right)^{12}$ is

(a) 0 (b) 2^{11} (c) $\left(\frac{3}{2}\right)^{12}$ (d) 2^{12}

19. For any quadratic polynomial $f(x)$, it is true that

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2,$$

where a is any real number.

If $\frac{3x^2+4x+7}{(x-2)^3} = \frac{A}{(x-2)^3}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)}$ and $g(x) = 3x^2 + 4x + 7$, then $A + B + C$ is equal to

(a) $g(2)+g'(2)+g''(2)$ (b) $g''(2)+2g(2)+\frac{g'(1)}{2!}$

(c) $g(2)+g'(2)+\frac{g''(2)}{2!}$ (d) $2g(2)+2g'(2)+\frac{g''(2)}{2!}$

20. $\cot\frac{\pi}{16}\cdot\cot\frac{2\pi}{16}\cdot\cot\frac{3\pi}{16}\cdot\cot\frac{4\pi}{16}\cdot\cot\frac{5\pi}{16}\cdot\cot\frac{6\pi}{16}\cdot\cot\frac{7\pi}{16}$ is equal to

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

21. Let ACB be a triangle with right-angle at C. Let $AB = 29$ units, $BC = 21$ units and $\angle ABC = \theta$. Then, $\cos^2\theta - \sin^2\theta$ is equal to

(a) 1 (b) $\frac{41}{841}$ (c) $\frac{40}{441}$ (d) $\frac{41}{800}$

22. $\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}$ is equal to

(a) cot θ (b) cos 2θ (c) tan θ (d) tan 2θ

23. Let $y = 4\sin^2\theta - \cos 2\theta$. If l and m are the minimum and maximum values of y respectively, then

(a) $lm = \frac{m}{l}$ (b) $lm = \frac{l}{m}$

(c) $l + m = \frac{l}{m}$ (d) $\frac{lm}{l-m} = 1 + m$

24. If $\cos\theta = -\frac{1}{\sqrt{2}}$ and $\tan\theta = 1$, then the general value of θ is

(a) $2n\pi + \frac{\pi}{4}$, $n = 0, 1, 2, 3, \ldots$

(b) $(2n+1)\pi + \frac{\pi}{4}$, $n = 0, 1, 2, 3, \ldots$

(c) $n\pi + \frac{\pi}{4}$, $n = 0, 1, 2, 3, \ldots$

(d) $n\pi \pm \frac{\pi}{4}$, $n = 0, 1, 2, 3, \ldots$

25. If $\sin^{-1} x < \cos^{-1} x$, then

(a) $-1 \le x < \frac{1}{\sqrt{2}}$ (b) $-\sqrt{3} \le x < -1$

(c) $\frac{1}{\sqrt{2}} < x \le 1$ (d) $1 < x < \sqrt{3}$

26. $\coth^{-1}(2) + \operatorname{cosech}^{-1}(-2\sqrt{2})$ is equal to

(a) $\log\sqrt{\frac{3}{2}}$ (b) $\log\sqrt{6}$ (c) $\log\frac{3}{\sqrt{2}}$ (d) $\log\frac{3}{2}$

27. In ΔABC, $\angle B = \frac{\pi}{4}$ and $\angle C = \frac{\pi}{3}$. If the area of the triangle is $54 + 18\sqrt{3}$ sq units, then a is equal to

(a) $\sqrt{3}+1$ (b) $2(\sqrt{3}+1)$

(c) $4(\sqrt{3}+1)$ (d) $6(\sqrt{3}+1)$

28. If a, b and c are the sides of a ΔABC and exradii r_1, r_2 and r_3 are 12, 6 and 4 respectively. Then, $a + 2b + 3c$ is equal to
(a) 24 (b) 44 (c) 30 (d) 54

29. In a ΔABC, if $a : b : c = 4 : 5 : 6$, then $\frac{1}{4R}[r_1 + r_2 + r_3]$ is equal to
(a) $\frac{71}{64}$ (b) $\frac{4}{5}$ (c) $\frac{81}{84}$ (d) $\frac{7}{9}$

30. In a ΔABC, with usual notation, if $a = 12$, $b = 16$ and $c = 20$, then the ratio of the exradii of the triangle opposite to the angles in the order $\angle C$, $\angle B$ and $\angle A$ is
(a) 3 : 4 : 5 (b) 6 : 3 : 2
(c) 12 : 7 : 5 (d) 2 : 3 : 5

31. If the vectors $-3\hat{i} + 4\hat{j} + \lambda\hat{k}$ and $\mu\hat{i} + 8\hat{j} + 6\hat{k}$ are collinear, then $\lambda - \mu$ is equal to
(a) 0 (b) – 3 (c) 6 (d) 9

32. The angle between the vectors $2\hat{k} - 3\hat{j}$ and $\hat{i} - 2\hat{k}$ is
(a) $\cos^{-1}\left(\frac{8}{\sqrt{65}}\right)$ (b) $\cos^{-1}\left(\frac{-4}{\sqrt{65}}\right)$
(c) $\cos^{-1}\left(\frac{2}{\sqrt{65}}\right)$ (d) $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$

33. Three vectors **a**, **b** and **c** satisfy **a** + **b** + **c** = 0. If | **a** | = 3, | **b** | = 4, | **c** | = 2, then
a · b + b · c + c · a + 2(| **a** | + | **b** | + | **c** |) is equal to
(a) $\frac{-7}{2}$ (b) $\frac{7}{2}$ (c) $\frac{-11}{2}$ (d) $\frac{11}{2}$

34. (**a** + 2**b** – **c**) · {(**a – b**) × (**a – b – c**)} is equal to
(a) $2[a\ bc]$ (b) $[a\ bc]$ (c) $3[a\ b\ c]$ (d) $[a\ b\ c]^2$

35. If **a** = $\hat{i} + \hat{j} + \hat{k}$, **c** = $\hat{j} - \hat{k}$ are given vectors, then a vector **b** satisfying the equations **a** × **b** = **c** and **a** · **b** = 3 is
(a) $5\hat{i} + 2\hat{j} + 2\hat{k}$ (b) $\frac{5}{2}\hat{i} + \hat{j} + \hat{k}$
(c) $\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$ (d) $\hat{i} + \frac{2}{5}\hat{j} + \frac{2}{5}\hat{k}$

36. Let a = $2\hat{i} - 3\hat{j} + \hat{k}$, b = $\hat{i} + 2\hat{j} - 3\hat{k}$, c = $\hat{i} - \hat{j}$ and **d** = $\hat{i} + \hat{j} + x\hat{k}$. If (**a** × **b**) × **c** is perpendicular to **d**, then x is equal to
(a) $\frac{3}{2}$ (b) 2 (c) $\frac{2}{3}$ (d) 1

37. If 10 is the mean of the data 2, 3, 5, 18, 17, 15, 13, x, 9 and 7, then the mean deviation of this data about its mean is
(a) 4.7 (b) 4.8 (c) 4.9 (d) 5.0

38. *A, B* and *C* are aiming to shoot a balloon. *A* will succeed 4 times out of 6 attempts. The chance of *B* to shoot the balloon is 3 out of 5 and that of *C* is 2 out of 3. If the three aim to shoot the balloon simultaneously. Then, the probability that at least two of them hit the balloon is
(a) $\frac{5}{9}$ (b) $\frac{9}{25}$ (c) $\frac{4}{9}$ (d) $\frac{32}{45}$

39. ω is a complex cube root of unity. When an unbiased die is thrown 3 times, if β_1, β_2 and β_3 are the numbers appeared on the die, then the probability that β_1, β_2 and β_3 satisfy $\omega^{\beta_1} + \omega^{\beta_2} = -\omega^{\beta_3}$ is
(a) $\frac{212}{513}$ (b) $\frac{1}{3}$ (c) $\frac{3}{5}$ (d) $\frac{2}{9}$

40. If $P\left(\frac{A}{B}\right) = \frac{3}{10}, P\left(\frac{B}{A}\right) = \frac{4}{5}$ and $P(A \cup B) = KP(B)$, then $\frac{1}{K}$ is equal to
(a) $\frac{40}{49}$ (b) $\frac{40}{43}$ (c) $\frac{100}{101}$ (d) 1

41. If the probability that an individual will suffer a bad reaction from an injection is 0.001, then the probability that out of 2000 individuals, exactly 3 individuals suffer a bad reaction is
(a) $\frac{4}{3e^2}$ (b) $\frac{2}{e^2}$ (c) $\frac{2}{3e^2}$ (d) $\frac{4}{5e^2}$

42. A fair coin is tossed 15 times. The probability that the tail will appear atleast thrice is
(a) $1 - \frac{10^5}{2^{15}}$ (b) $1 - \frac{121}{2^{15}}$ (c) $1 - \frac{1}{2^{15}}$ (d) $1 - \frac{16}{2^{15}}$

43. A rod of length 6 units slides with its ends on the coordinate axes. The locus of the mid-point of the rod is
(a) $x^2 + y^2 = 9$ (b) $x + y = 3$
(c) $x^2 + y^2 = 36$ (d) $x + y = 6$

44. The transformed equation of the curve $2x^2 + y^2 - 3x + 5y - 8 = 0$, when the origin is translated to the point (– 1, 2) is
(a) $2x^2 + y^2 - 7x + 9y + 11 = 0$
(b) $2x^2 + y^2 + 7x + 9y + 11 = 0$
(c) $2x^2 + y^2 - x + y + 11 = 0$
(d) $2x^2 + y^2 + 7x - 9y + 11 = 0$

45. The point on the line $4x - y - 2 = 0$ which is equidistant from the point (– 5, 6) and (3, 2) is
(a) (2, 6) (b) (4, 14) (c) (1, 2) (d) (3, 10)

46. The incentre of the triangle having the vertices $(1, \sqrt{3})$, (0, 0) and (2, 0) is
(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$
(c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

47. If $P'(a, b)$ is the image of the point $P(-1, 2)$ with respect to the line $x - 2y + 3 = 0$, then the length of the perpendicular from P' on to the line $2x + y - 7 = 0$ is
(a) $\frac{3}{\sqrt{5}}$ units (b) 5 units
(c) $\frac{7}{\sqrt{5}}$ units (d) 7 units

48. If the vertices of a ΔABC are $A(1, 7)$, $B(-5, -1)$ and $C(7, 4)$, then the equation of a bisector of $\angle ABC$ is

(a) $7x - 9y + 26 = 0$ (b) $9x - 7y + 38 = 0$

(c) $7x + 9y + 44 = 0$ (d) $9x + 7y + 52 = 0$

49. The combined equation of the two diameters of a circle which divide the circle into four sectors is $ax^2 + 2hxy + by^2 = 0$. If the area of the bigger sector is 5 times the area of the smaller sector, then $\frac{|a+b|}{\sqrt{(a-b)^2 + 4h^2}}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

50. If $9x^2 - 24xy + 16y^2 + \alpha x + \beta y + 6$ represents a pair of parallel lines of 1 unit apart and one of those lines passes through (1, 1), then $\frac{\alpha}{\beta}$ is equal to

(a) $\frac{2}{3}$ (b) 1 (c) $\frac{-3}{2}$ (d) $\frac{-3}{4}$

51. If (a, b) is the centre of the circle passing through the vertices of the triangle formed by $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$, then (a, b) is

(a) $(-17, -16)$ (b) $\left(\frac{17}{2}, \frac{19}{2}\right)$

(c) (17, 18) (d) $\left(\frac{-17}{2}, \frac{-19}{2}\right)$

52. The locus of the mid-points of the chords of the circle $x^2 - 2x + y^2 = 0$ drawn from a point (0, 0) on it is

(a) $x^2 + y^2 - x = 0$ (b) $2x^2 + y - 2 = 0$

(c) $y^2 + x - 1 = 0$ (d) $x^2 + 2x + y - 3 = 0$

53. The number of possible common tangents that can be drawn to the circles

$x^2 + y^2 + 4x - 6y - 3 = 0$

and $x^2 + y^2 + 4x - 2y + 1 = 0$ is

(a) 4 (b) 3 (c) 1 (d) 0

54. The equation of the circle passing through (1, 1) and through the points of intersection of the circles

$x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is

(a) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

(b) $2x^2 + 2y^2 + 15x - 19y = 0$

(c) $4x^2 + 4y^2 + 25x + 12y - 45 = 0$

(d) $4x^2 + 4y^2 + 13x - 30y + 9 = 0$

55. If the circles $(x - 2)^2 + (y - 3)^2 = 25$ and $25x^2 + 25y^2 - 40x - 70y - 160 = 0$ touch internally at (α, β), then $\alpha + \beta$ is equal to

(a) 0 (b) -2 (c) -1 (d) 1

56. A point on the parabola whose focus and vertex are respectively at $\left(\frac{5}{4}, -2\right)$ and $(1, -2)$ is

(a) (4, 0) (b) (15, 2) (c) $(3, -1)$ (d) (10, 1)

57. The parametric equations of the parabola $y^2 - 4x - 8y - 12 = 0$ are

(a) $x = 7 + 2t$ and $y = -4 + t^2$

(b) $x = -7 + 2t$ and $y = 4 + 2t$

(c) $x = -7 + t^2$ and $y = -4 + 2t$

(d) $x = -7 + t^2$ and $y = 4 + 2t$

58. A focus of an ellipse having eccentricity $\frac{1}{2}$ is at (0, 0) and a directrix is the line $x = 4$. Then, the equation of one such ellipse is

(a) $\frac{9x^2}{64} + \frac{3y^2}{16} = 1$ (b) $\frac{(2x+1)^2}{32} + \frac{y^2}{16} = 1$

(c) $\frac{(3x+4)^2}{64} + \frac{y^2}{32} = 1$ (d) $(3x + 4)^2 + 12y^2 = 64$

59. The area (in sq. units) of the quadrilateral formed by joining the focii of the two ellipses $\frac{x^2}{9} + \frac{y^2}{5} = 1$ and $\frac{x^2}{5} + \frac{y^2}{9} = 1$ is

(a) 4 (b) 2 (c) 6 (d) 8

60. The lines $x \cos \alpha + y \sin \alpha = P$, $\alpha \in R$ are chords of the hyperbola $\frac{x^2}{9} - \frac{y^2}{36} = 1$ and they subtend a right angle at the centre of the hyperbola. The locus of the poles of these lines with respect to the given hyperbola is

(a) $x^2 - 16y^2 = 108$ (b) $16x^2 - y^2 = 108$

(c) $16x^2 + y^2 = 108$ (d) $x^2 + 16y^2 = 108$

61. If $\left(\frac{9}{4}, \frac{5}{4}, \frac{15}{4}\right)$ is the centroid of a tetrahedron whose vertices are $(a, 2, 1)$, $(1, b, 4)$, $(4, 0, c)$ and $(1, 1, 7)$, then

(a) $a = b = c$ (b) $a = b = c + 1$

(c) $b = c = a + 1$ (d) $a = c = b + 1$

62. The direction cosines of the line making angles $\frac{\pi}{4}, \frac{\pi}{3}$ and $\theta\left(0 < \theta < \frac{\pi}{2}\right)$ respectively with X, Y and Z-axes, are

(a) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{\sqrt{3}}{2}$

(c) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}$

63. The point on the plane $2x - 2y + 4z + 5 = 0$ that is nearer to $\left(1, \frac{3}{2}, 2\right)$ is

(a) $\left(0, \frac{5}{2}, 0\right)$ (b) $\left(-5, \frac{-5}{2}, 0\right)$

(c) $\left(0, 0, \frac{-5}{4}\right)$ (d) $\left(-\frac{1}{2}, 0, -1\right)$

64. $\lim_{x \to 1} \frac{\log x}{(1-x)}$ is equal to

(a) 1 (b) -1 (c) 0 (d) $\frac{-1}{2}$

65. Let $f(x) = \frac{2-\sqrt{x+4}}{\sin 2x}$, $x \neq 0$. In order that $f(x)$ is continuous at $x = 0, f(0)$ is to be defined as

(a) $\frac{-1}{8}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{8}$

66. If $f(x) = \begin{cases} 1+\cos x, & x \le 0 \\ a - x, & 0 < x \le 2 \\ x^2 - b^2, & x > 2 \end{cases}$ is continuous everywhere, then $a^2 + b^2$ is equal to

(a) 4 (b) 8 (c) 6 (d) 12

67. If $x = 5(1 - \sin t)$, $y = 5(t + \cos t)$, then $\frac{dx}{dy}$ is equal to

(a) $\frac{\sin t - 1}{\cos t}$ (b) $\frac{\cos t}{\sin t - 1}$

(c) $\tan \frac{t}{2}$ (d) $\frac{\cos \frac{t}{2} - \sin \frac{t}{2}}{\cos \frac{t}{2} + \sin \frac{t}{2}}$

68. Let $g : [-2, 2] \to R$ and $f : [-2, 2] \to R$ are two functions defined as

$g(x) = \begin{cases} -1, & \text{if } -2 \le x < 0 \\ x^2 - 1, & \text{if } 0 \le x \le 2 \end{cases}$ and

$f(x) = |g(x)| + g(|x|) + 2$. In the interval $(-2, 2)$, f is not differentiable at x is equal to

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) -1

69. Consider the following statements:

I. If a function is differentiable at a point 'p', then it is not continuous at 'p'.

II. If a function is not continuous at $x = a$, then it is not differentiable at $x = a$.

III. If $f(x) = |x|$, then $f(x)$ is not differentiable but continuous on R.

IV. If $f(x) = x - [x]$, then $f'(1) = 1$.

Which of the above statements are (is) correct?

(a) Only II (b) II and III (c) Only III (d) III and IV

70. If $x^2 + xy + y^2 = k$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{-6k}{(x+2y)^3}$ (b) $\frac{-6k}{(x+2y)^2}$

(c) $\frac{x^2+xy+y^2}{(2x+y)^2}$ (d) 0

71. The approximate value of $(8.01)^{4/3} + (8.01)^2$ upto 3 decimal places is

(a) 80.116 (b) 80.216 (c) 80 (d) 80.186

72. If the tangent at a point P on the curve $y = 4x^4 + x$ is perpendicular to the tangent to the same curve at (0, 0), then the point P is

(a) $\left(\frac{-1}{2}, \frac{-1}{4}\right)$ (b) $\left(\frac{1}{2}, \frac{3}{4}\right)$

(c) (1, 5) (d) $(-1, 3)$

73. The angle between the curves $2x^2 + y^2 = 20$ and $4y^2 - x^2 = 8$ at a point where they intersect in the IVth quadrant is

(a) 0 (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

74. If the function $f(x) = x(x+3)e^{\frac{-x}{2}}$ satisfies all the conditions of Rolle's theorem in $[-3, 0]$, then a root of $f'(x) = 0$ is

(a) 3 (b) -1 (c) -2 (d) -3

75. $\int \frac{dx}{\sin x + \cos x}$ is equal to

(a) $\frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) \right| + C$

(b) $\frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) \right| + C$

(c) $\frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{4} + \frac{\pi}{2}\right) \right| + C$

(d) $\frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{8} + \frac{\pi}{2}\right) \right| + C$

76. $\int (x+2)\sqrt{x+3}\, dx$ is equal to

(a) $\frac{2}{15}\sqrt{x+3}\,(3x^2 - 13x + 12) + C$

(b) $\frac{2}{15}\sqrt{x+3}\,(3x^2 + 13x + 12) + C$

(c) $\frac{2}{5}\sqrt{x+3}\,(3x^2 - 12x + 13) + C$

(d) $\frac{2}{5}\sqrt{x+3}\,(3x^2 + 12x + 13) + C$

77. $\int \frac{(1-\cos x)^{2/7}}{(1+\cos x)^{9/7}} dx$ is equal to

(a) $\frac{7}{11}\left(\tan \frac{x}{2}\right)^{\frac{11}{7}} + C$ (b) $\frac{7}{11}\left(\tan \frac{x}{2}\right)^{\frac{7}{11}} + C$

(c) $\frac{7}{11}\left(\cot \frac{x}{2}\right)^{\frac{11}{7}} + C$ (d) $\frac{11}{7}\left(\cot \frac{x}{2}\right)^{\frac{7}{11}} + C$

78. $\int_5^9 \frac{\log 3x^2}{\log 3x^2 + \log(588 - 84x + 3x^2)} dx$ is equal to

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) 4

79. $\int_{-1}^{1} \frac{\log(1+x)}{1+x^2}dx = \int_0^1 \frac{\log(1+x)}{1+x^2}dx + \int_0^1 f(x)\,dx$, then $f(x)$ is equal to

(a) $\frac{\log(1+x)}{1+x^2}$ (b) $-\frac{\log(1+x)}{1+x^2}$

(c) $\frac{\log(1-x)}{1+x^2}$ (d) 0

80. The general solution of

$\frac{dy}{dx} = x + \sin x \cos y + x\cos y + \sin x$ is

(a) $\tan\frac{x}{2} = \frac{y^2}{2} - \cos y + C$

(b) $\tan\frac{y}{2} = \frac{x^2}{2} - \cos x + C$

(c) $\sec^2\frac{y}{2} = \frac{x^2}{2} - \cos x + C$

(d) $\tan\frac{y}{2} = \frac{x^2}{2} + \cos x + C$

PHYSICS

81. Which of the following statement is incorrect?

(a) Conservation laws have deep connection with symmetrics of nature.

(b) Weak nuclear force is weakest among all fundamental forces of nature.

(c) A conservation law is hypothesis based on observations and experiments.

(d) In a nuclear process mass gets converted to energy of *vice-versa*.

82. The dimensions of σb^4, where σ is Stefan's constant and b is Wien's constant are

(a) $[M^0L^0T^0]$ (b) $[ML^4T^{-3}]$

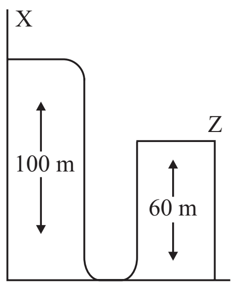
(c) $[ML^{-2}T]$ (d) $[ML^6T^{-3}]$

83. A car travels in a straight line along a road. Its distance x from a stop sign is given as a function of t by the equation $x(t) = \alpha t + \beta t^3$, where $\alpha = 2.0$ m/s, $\beta = 0.01$ m/s^3. Calculate the average velocity of the car in the time interval $t = 2.00$ s to 4.00 s.

(a) 2.28 m/s (b) 4.94 m/s

(c) 3.34 m/s (d) 4.12 m/s

84. A car driver is trying to jump across a path as shown in figure by driving horizontally off a cliff X at the speed 10 m/s. When he touches peak Z (ignore air resistance). What would be speed? (Use, $g = 10$ m/s^2)



(a) 30 m/s

(b) 40 m/s

(c) 15 m/s

(d) 50 m/s

85. Find the angle between the two vectors $\mathbf{a} = 3\hat{i} + 2\hat{j} + 5\hat{k}$, $\mathbf{b} = 5\hat{i} + 3\hat{j} + \hat{k}$.

(a) $\cos^{-1}\left(\frac{26}{\sqrt{1330}}\right)$ (b) $\sin^{-1}\left(\frac{26}{\sqrt{1330}}\right)$

(c) $\cos^{-1}\left(\frac{26}{\sqrt{1335}}\right)$ (d) $\tan^{-1}\left(\frac{26}{\sqrt{1330}}\right)$

86. A particle moves over an xy-plane with a constant acceleration $\mathbf{a} = (4\hat{i} + 4\hat{j})$ m/s^2. At time $t = 0$, the velocity is $(4\hat{i})$ m/s^2. The speed of the particle when it is displaced by 6.0 m parallel to the X-axis is

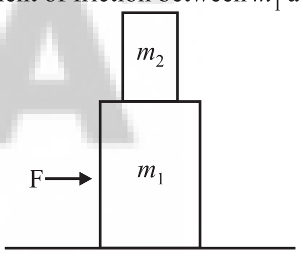
(a) $4\sqrt{5}$ m/s (b) $\sqrt{60}$ m/s

(c) $3\sqrt{10}$ m/s (d) $\sqrt{20}$ m/s

87. Block A of mass 3 kg rests on another block B of mass 7 kg. The coefficient of friction between A and B is 0.4, while the coefficient of friction between B and the horizontal floor on which B rests is 0.55. Find the force of friction between A and B, when a horizontal force of 50 N is applied on the block B. (Use $g = 10$ m/s^2)

(a) 0 (b) 5 N (c) 4 N (d) 12 N

88. What is the maximum force F that can be applied on block m_1, so that both m_1 and m_2 will move together? There is no friction between m_1 and the horizontal table. The coefficient of friction between m_1 and m_2 is μ



(a) $\mu m_2 g$ (b) $\mu(m_1 + m_2)g$

(c) $\mu\frac{m_1m_2}{(m_1+m_2)}g$ (d) $\mu m_1 g$

89. The potential energy of a particle in a central field has the form $U(r) = \frac{1}{r^2} - \frac{1}{r}$, where r is the distance from the centre of the field. The magnitude of the maximum attractive force (in N) is

(a) $\frac{1}{27}$ (b) $\frac{1}{9}$ (c) $\frac{1}{3}$ (d) 1

90. Consider a rocket is being fired. The kinetic energy of the rocket is increased by 16 times whereas its total mass is reduced by half through the burning of fuel. The factor by which its momentum increases is

(a) 8 (b) $2\sqrt{2}$ (c) 4 (d) $4\sqrt{2}$

91. A rigid body of mass m and radius R rolls without slipping on an inclined plane of inclination θ, under gravity. Match the type of body in Column I with magnitude of the force of friction Column II.

	Column I		Column II
(A)	For ring	(i)	$\frac{mg\sin\theta}{2.5}$
(B)	For solid sphere	(ii)	$\frac{mg\sin\theta}{3}$
(C)	For solid cylinder	(iii)	$\frac{mg\sin\theta}{3.5}$
(D)	For hollow cylinder	(iv)	$\frac{mg\sin\theta}{2}$

Codes:

	A	B	C	D
(a)	(iv)	(iii)	(ii)	(iv)
(b)	(i)	(ii)	(iv)	(iv)
(c)	(ii)	(i)	(iv)	(iii)
(d)	(ii)	(iv)	(i)	(iii)

92. A small disc of mass 500 g and radius 5 cm rolls down an inclined plane without slipping. Speed of its centre of mass when it reaches the bottom of the inclined plane depends on

(a) mass and radius
(b) mass and height of the incline
(c) height of the incline
(d) height of the incline and acceleration due to gravity

93. A body of mass 1 kg is executing simple harmonic motion (SHM). Its displacement y (in cm) at time t given by $y = \left[6\sin\left(100t + \frac{\pi}{4}\right)\right]$ cm. Its maximum kinetic energy is

(a) 1.8 J (b) 18 J (c) 180 J (d) 0.18 J

94. What is the change in mass of a body, when taken 64 km below the surface of the earth? [take, radius of the earth as 6400 km]

(a) Increases by 2% (b) Remains constant
(c) Increases by 1% (d) Decreases by 1%

95. Consider a rod of length 1.0 m with a cross-sectional area of 0.50 cm^2. The rod supports a 500 kg platform that hangs attached to the rod's lower end. What is elongation of the rod under the stress ignoring the weight of the rod? Consider the Young's modulus to be 10^{11} Pa and $g = 10$ m/s^2.

(a) 2 mm (b) 0.5 mm (c) 1.5 mm (d) 1 mm

96. Water is pumped steadily out of a flooded basement, at the speed of 10 m/s through a hose (tube) of radius 1 cm, passing through a window 3 m above the water level. The power of the pump is

(Assume $g = 10$ m/s^2, density of water = 1000 kg/m^3)

(a) 80π W (b) 30π W (c) 50π W (d) 90π W

97. A U-shaped tube is partially filled with an incompressible liquid of density 1.2 g/cm^3.

Oil which does not mix with the liquid is next poured into left side of the U-tube until the liquid rises by 15 cm on the right side of U-tube. If the density of the oil is 0.9 g/cm^3, the oil level will stand higher than the liquid level of right side of U-tube by

(a) 15 cm (b) 10 cm (c) 12 cm (d) 9 cm

98. The ratio of linear expansivity to the co-efficient of a real expansion of a rectangular sheet of a solid is

(a) 2 (b) 0.5 (c) 1 (d) 1.5

99. Two rods whose lengths are l_1 and l_2 with heat conductivity coefficients K_1 and K_2 are placed end-to-end. The heat conductivity coefficients of a uniform rod of length $l_1 + l_2$ whose conductivity is same as that of the system of these two rods is

(a) $\frac{(l_1 + l_2)K_1K_2}{K_2l_1 + K_1l_2}$ (b) $\frac{(l_1 + l_2)K_1K_2}{K_1l_1 + K_2l_2}$

(c) $\frac{K_1l_1 + K_2l_2}{(l_1 + l_2)K_1K_2}$ (d) $\frac{K_1l_2 + K_2l_1}{(l_1 + l_2)K_1K_2}$

100. Consider a two stage Carnot engine. In the first stage heat Q_1 is absorbed at temperature T and heat Q_2 is expelled at temperature αT (where $\alpha < 1$). In the second stage heat Q_2 is absorbed at temperature αT and heat Q_3 is expelled at temperature $\beta T(\beta < \alpha)$. The efficiency of the Carnot engine will be

(a) $1 - \alpha - \beta$ (b) $1 - \alpha$
(c) $1 - \beta$ (d) $1 - \alpha\beta$

101. A diatomic gas of volume 2 m^3 at a pressure 2×10^5 N/m^2, is compressed adiabatically to a volume 0.5 m^3. The work done in this process is [Use $4^{1.4} = 6.96$]

(a) 2.96×10^5 J (b) -2.96×10^5 J
(c) -7.4×10^5 J (d) 7.4×10^5 J

102. As per kinetic theory of gases which of the following statement(s) is/are true?

I. Temperature of a gas is a measure of average kinetic energy of a molecule.
II. Temperature of a gas depends on the nature of the gas.
III. Heavier molecule has lower average speed.
IV. Lighter molecule has lower average speed.

(a) I and II (b) II and III
(c) I and III (d) II and IV

103. A wave is represented by the equation $y = (0.02)\sin(5\pi x - 20t)$ m. The minimum distance between the two particles always having the same speed is

(Assume x and t are in SI units)

(a) 0.02 m (b) 0.4 m
(c) 0.8 m (d) 0.2 m

104. A screen is placed 90 cm from an object. The image is formed by using a convex lens twice on the screen by putting the lens at two different locations separated by 20 cm. The focal length of the lens is approximately equal to

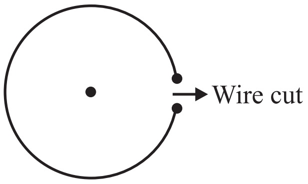
(a) 21.38 cm (b) 30.0 cm
(c) 35.0 cm (d) 24 cm

105. What would be the angular separation between the consecutive bright fringes in Young's double slit experiment with blue-green light of wavelength 400 nm? The separation between the slits is 0.001 m.

(a) 4×10^{-4} rad (b) 3×10^{-4} rad
(c) 2×10^{-4} rad (d) 1×10^{-4} rad

106. A circular wire loop of radius 10 cm carries a total charge of 10^{-5} C distributed uniformly over its length. A small length of 3.14×10^{-6} m of wire is cut off. The magnitude to electric field at the centre due to the remaining wire is

$$\left(\text{Assume, } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ SI units}\right)$$



(a) 30 N/C (b) 40 N/C (c) 35 N/C (d) 45 N/C

107. Consider the charged cylindrical capacitor.
The magnitude of electric field **E** in its annual region

(a) varies as $\frac{1}{r}$, where r is the distance from its axis.
(b) is zero.
(c) is same throughout and $|\mathbf{E}| > 0$.
(d) varies as $\frac{1}{r^2}$, where r is the distance from its axis.

108. A spherical capacitor consists of two concentric spherical conductors. Find the capacitance of the spherical capacitor if the outer radius is 2R and the inner radius is R.

(a) $4\pi\varepsilon_0 R$ (b) $8\pi\varepsilon_0 R$ (c) $\frac{8\pi\varepsilon_0}{R}$ (d) $\frac{4\pi\varepsilon_0}{R}$

109. A wire of length l carries a current I along the X-axis. The magnetic force acting on the wire is given by
$\mathbf{F} = IB_0 l(\hat{k} - \hat{j})T$, where B_0 is a constant. The existing magnetic field **B** is

(a) $B_0\hat{i}$ (b) $B_0(\hat{i} + \hat{j} - \hat{k})$
(c) $B_0(\hat{i} + \hat{j} + \hat{k})$ (d) $B_0(\hat{i} - \hat{j} - \hat{k})$

110. Two long wires with no contact are placed perpendicular to each other, i_1 and i_2 are currents flowing through these wires, respectively. The magnetic force on a small length dl of the second wire situated at a distance l from first wire is proportional to

(a) $i_1 i_2$ (b) i (c) $\frac{1}{i_1 i_2}$ (d) i^2

111. A solenoid has a core of a material with relative permeability 501. The windings of the solenoid are insulated from the core and carry a current of 2.5 A. If the number of turns are 900 per metre. The magnetisation (in A/m) is

(a) 1.12×10^6 (b) 2.8×10^6
(c) 2.25×10^6 (d) 1.69×10^6

112. A long straight solenoid with cross-sectional radius a and number of turns per unit length n has a current varying with time as $I(As^{-1})$. The magnitude of the electric field as a function of distance r from the solenoid axis is

(a) $\frac{n\mu_0 a^2 l}{2r}$ (b) $\frac{\mu_0 ln}{2a}$ (c) $\frac{na^2 l}{2\mu_0 r}$ (d) $\frac{\mu_0 la}{2n}$

113. An alternating current is given by $i = (2 \sin \omega t + 6 \cos \omega t)$ A. The rms current (in A) is

(a) $2\sqrt{5}$ (b) $2\sqrt{10}$ (c) $\sqrt{5}$ (d) $10\sqrt{2}$

114. For an EM wave, the electric and magnetic fields are 300 V/m and 7.9 A/m, respectively. The maximum rate of energy flow is

(a) $2730 \frac{W}{m^2}$ (b) $2790 \frac{W}{m^2}$
(c) $2370 \frac{W}{m^2}$ (d) $2390 \frac{W}{m^2}$

115. Which of the following particle has the shortest de-Broglie wavelength?

(a) Proton (b) Electron
(c) α-particle (d) X-rays

116. Hydrogen atom in the ground state absorbs ΔE amount of energy. If the orbital angular momentum of the electron is increased by $\frac{h}{2\neq}$ (h = Planck's constant), then the magnitude of ΔE is

(a) 12.09 eV (b) 12.75 eV
(c) 10.2 eV (d) 13.6 eV

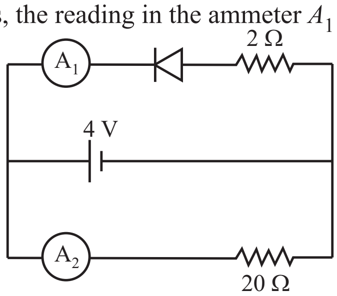
117. In a nuclear reactor, the main purpose of the moderator is to

(a) initiate that fission process by giving away its neutron
(b) slow down the fast neutrons
(c) cool down the excess of heat generated in the reactor
(d) absorb excess of neutrons and control the reaction rate

118. If the temperature of the semi-conductor is increased, which of the following is correct statement?

(a) It's resistance increases.
(b) The number of electrons in valence band increases.
(c) The number of electrons in conduction band increases.
(d) The number of holes in valence band decreases.

119. Two ammeters A_1 and A_2 are connected as shown in the given figure. By neglecting the internal resistance of the ammeters, the reading in the ammeter A_1 is



(a) $2A$ (b) 0 (c) $1A$ (d) $4A$

120. A message signal of frequency f_m is used to modulate a carrier of frequency f_c. If the side bands are f_1 and f_2, then the ratio $\frac{f_c}{f_m}$ is

(a) $\frac{f_1+f_2}{f_2-f_1}$ (b) $\frac{(f_1+f_2)^2}{f_1 f_2}$

(c) $\frac{f_1-f_2}{f_2+f_1}$ (d) $\frac{f_1 f_2}{(f_1+f_2)^2}$

CHEMISTRY

121. Heisenberg's uncertainty principle is in general significant to
(a) planets
(b) cricket ball of 500 g
(c) cars
(d) micro particles having a very high speed

122. Which of the following relations is correct, if the wavelength (λ) is equal to the distance travelled by the electron in one second?
h is the Planck's constant and *m* is the mass of electron

(a) $\lambda = \frac{h}{p}$ (b) $\lambda = \frac{h}{m}$ (c) $\lambda = \sqrt{\frac{h}{p}}$ (d) $\lambda = \sqrt{\frac{h}{m}}$

123. The set of amphoteric oxides among the given oxides are Ga_2O_3, As_4O_{10}, Sb_4O_{10}, B_2O_3, Tl_2O
(a) Tl_2O, B_2O_3 (b) Sb_4O_{10}, B_2O_3, Ga_2O_3
(c) Ga_2O_3, Tl_2O, As_4O_{10} (d) Ga_2O_3, AS_4O_{10}, Sb_4O_{10}

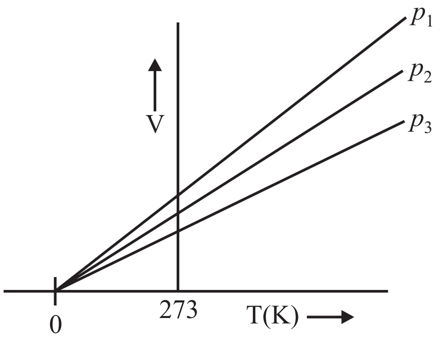
124. The first ionisation energies (in kJ mol^{-1}) of four consecutive elements of the second period are given in the options. The first ionisation energy of nitrogen is
(a) 1086 (b) 1402 (c) 1681 (d) 1314

125. Highest covalent character is found in which of the following?
(a) CaF_2 (b) $CaCl_2$
(c) $CaBr_2$ (d) CaI_2

126. The correct sequence of bond order is
(a) $O_2^+ > O_2^- > O_2 > O_2^{2-}$ (b) $O_2^+ > O_2 > O_2^- > O_2^{2-}$
(c) $O_2^+ > O_2 > O_2^{2-} > O_2^-$ (d) $O_2^+ > O_2^{2-} > O_2^- > O_2$

127. The most probable velocity of a gas at 7200 K is equal to the RMS velocity of He gas at 27°C. The gas is
(a) O_2 (b) CO (c) N_2 (d) SO_2

128. A plot of volume of the gas *versus* *T* (K) is shown below. Which of the options is correct for the plot?



(a) $p_1 < p_2 < p_3$ (b) $p_3 < p_2 < p_1$
(c) $p_1 = p_2 \neq p_3$ (d) $p_1 = p_2 = p_3 = 0$ at 273 K

129. In the balanced equation of the following reaction, the ratio of $\frac{a}{b}$ is a

$$a\text{CaCO}_3 + b\text{H}_3\text{PO}_4 \longrightarrow p\text{Ca}_3(\text{PO}_4)_2 + q\text{CO}_2 + r\text{H}_2\text{O}$$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{7}{5}$

130. The number of grams of oxygen in 32.2 g of Na_2SO_4. 10 H_2O is approximately
(a) 32.2 g (b) 22.4 g (c) 11.2 g (d) 64.4 g

131. Enthalpy of hydrogenation of one mole of benzene to cyclohexane is
[Resonance energy of benzene = – 150.4 kJ/mol, enthalpy of hydrogenation of cyclohexene = – 119.5 kJ/mol]
(a) – 208.1 kJ/mol (b) – 358.1 kJ/mol
(c) + 150.4 kJ/mol (d) – 269.9 kJ/mol

132. Match the following columns:

	Column-I (Acid)		Column-II [K_a (ionisation constant)]
A.	HCN	1.	6.8×10^{-4}
B.	$H_2C_2O_4$	2.	8.9×10^{-8}
C.	H_2S	3.	4.9×10^{-10}
D.	Niacin	4.	5.6×10^{-2}
		5.	1.5×10^{-5}

The correct match is

Codes:

	A	B	C	D
(a)	1	3	4	5
(b)	5	2	3	4
(c)	2	3	4	5
(d)	3	4	2	5

133. If the molar concentrations of base and its conjugate acid are same, then pOH of the buffer solution is
(a) same as pK_b of base (b) same as pK_a of base
(c) same as pK_a of acid (d) same as pK_b of acid

134. H_2 + CO + alkene $\xrightarrow{\text{Catalyst}}$ 1° alcohol. What is the stable intermediate and the nature of the reaction?
(a) Acid, reduction (b) Aldehyde, oxidation
(c) Aldehyde, reduction (d) Alcohol, oxidation

135. The correct order of electrical conductivity of alkali metals ions in their aqueous solution for Cs^+, K^+, Na^+ and Li^+ is
(a) $Cs^+ > K^+ > Na^+ > Li^+$ (b) $K^+ > Cs^+ > Li^+ > Na^+$
(c) $Cs^+ > K^+ > Li^+ > Na^+$ (d) $Li^+ > Na^+ > K^+ > Cs^+$

136. Which of the following complex ions does not exist?
(a) $[B(H_2O)_6]^{3+}$ (b) $[Al(H_2O)_6]^{3+}$
(c) $[Ga(H_2O)_6]^{3+}$ (d) $[In(H_2O)_6]^{3+}$

137. Which of the following statements are correct?

A. The isotope of carbon containing 7 neutrons has natural abundance of 1.1%

B. Among the IVA group elements, Sn has the lowest melting point

C. Silicon is the 2nd (by mass) most abundant element in the Earth's crust

D. Element carbon shows the highest electrical resistivity among the 14 group elements.

(a) A, C and D (b) A, B and C
(c) B, C and D (d) A, B, C and D

138. Which of the following chemicals can be used as dry cleaning agents?

A. $Cl_2C=CCl_2$ B. CO_2(liq.)
C. H_2O_2 D. CH_3CHO

(a) A, B, C and D (b) A, B and C
(c) B, C and D (d) A and B

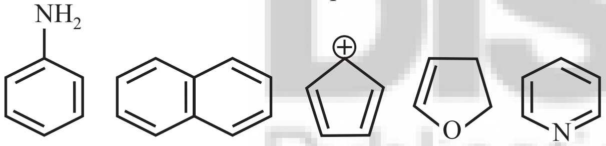
139. The number of *sp* and sp^2 carbon in hepta-1, 3-dien-5-yne, respectively, are

(a) 2, 4 (b) 4, 3
(c) 2, 2 (d) 2, 5

140. When propyne is passed through a red hot iron tube at 873 K, the reaction gives product having molecular formula of

(a) C_7H_8 (b) C_9H_2
(c) C_8H_{10} (d) C_6H_6

141. Total number of aromatic compounds from below is



(a) 2 (b) 3 (c) 4 (d) 5

142. A compound made up of elements *A* and *B* (with a general formula A_xB_y), where *B* form a hcp lattice and *A* occupy 2/3rd of the tetrahedral voids. The formula of the compound is

(a) A_2B_3 (b) A_3B_4
(c) A_4B_3 (d) A_3B_2

143. If the density of a 2M solution of ethylene glycol in water is 1.11 g/mL, the molality (in 'm') of the solution is approximately

(a) 1.92 (b) 1.57
(c) 2.05 (d) 2.15

144. Given,

Sol *A* Phenol and aniline

Sol *B* Chloroform and acetone

Which of the following is correct as per Raoult's law?

(a) Sol *A* shows – ve and B shows + ve deviation
(b) Both solutions *A* and B show – ve deviation
(c) Sol *A* shows + ve and B shows – ve deviation
(d) Both solutions *A* and B show + ve deviation

145. Salts of *A* (atomic weight 8), *B* (atomic weight 18) and *C* (atomic weight 50) were electrolysed under identical conditions using the same quantity of electricity. It was found that 2.4 g of *A* was deposited, the weight of *B* and *C* deposited are 1.8 g and 7.5 g, respectively. The valences of *A, B* and C are, respectively,

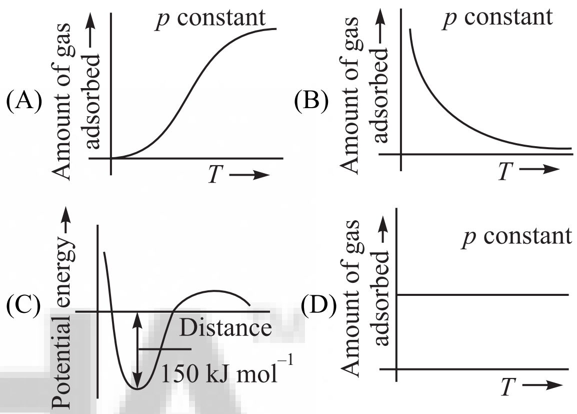
(a) 3, 1 and 2 (b) 1, 2 and 3
(c) 1, 3 and 2 (d) 3, 2 and 1

146. What will be the overall order of a reaction for which the rate expression is given as

$$\text{Rate} = k[A]^{\frac{1}{2}}[B]^{\frac{3}{2}}$$

(a) second order (b) first order
(c) zero order (d) third order

147. Which of the following options are correct?



(a) A and C represent physisorption
(b) A and D represent physisorption
(c) A and C represent chemisorption
(d) B and C represent chemisorption

148. The main products formed when copper metal is reacted with concentrated HNO_3 are

(a) $Cu(NO_3)_2 : NO$ (b) $Cu(NO_3)_2 : H_2$
(c) $Cu(NO_3)_2 : NO_2$ (d) $Cu(NO_3) : NO$

149. **Assertion (A):** SF_6 is highly stable.

Reason (R): SF_6 is a gas.

The correct option among the following is

(a) (A) is true, (R) is true and (R) is the correct explanation for (A)
(b) (A) is true, (R) is true but (R) is not the correct explanation for (A)
(c) (A) is true but (R) is false
(d) (A) is false but (R) is true

150. Complete hydrolysis of XeF_4 and XeF_6 gives its oxides P and Q, respectively. Identify P and Q.

	P	Q
(a)	XeO_2	XeO_3
(b)	XeO	XeO_2
(c)	XeO_3	XeO_3
(d)	XeO_2	XeO_2

151. Match the following:

	Column-I (Reaction)		Column-II (Main product)
A.	$4FeCl_3 + 3K_4[Fe(CN)_6] \rightarrow$	1.	$Fe_4[Fe(CN)_6]_3$
B.	$ZnCl_2 + 4NaOH \rightarrow$	2.	$FeCl_2$
C.	$2FeCl_3 + H_2S \rightarrow$	3.	$Zn(OH)Cl$
		4.	$Fe_3[Fe(CN)_6]_3$
		5.	Na_2ZnO_2
		6.	FeS

The correct match is

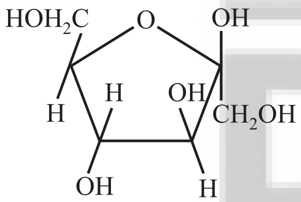
Codes:

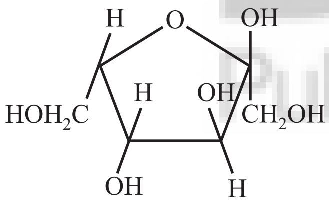
	A	B	C
(a)	1	3	6
(b)	1	5	2
(c)	4	5	2
(d)	4	3	6

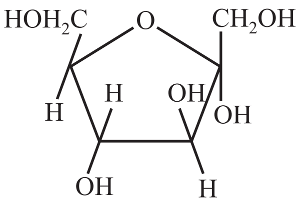
152. The oxidation number of central metal in $[Pt(NH_3)_2Cl(NO_2)]$ and $[CoCl_2(en)_2]^{\oplus}$ respectively, are

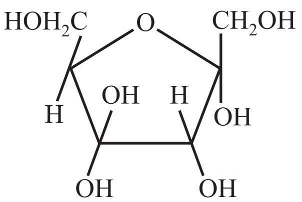
(a) + 2 : + 1 (b) + 2 : + 2
(c) + 2 : + 3 (d) + 3 : + 2

153. The structure of α-D-fructofuranose is

(a) 

(b) 

(c) 

(d) 

154. $CH_3—CH_2—Br + Nu^{\ominus} \rightarrow CH_3—CH_2—Nu + Br^{\ominus}$

The decreasing order of the reaction rate with nucleophile ($Nu^{\ominus}$) is

$Nu^{\ominus}$ = (I) $PhO^{\ominus}$: (II) $CH_3COO^{\ominus}$: (III) $OH^{\ominus}$: (IV) $CH_3O^{\ominus}$

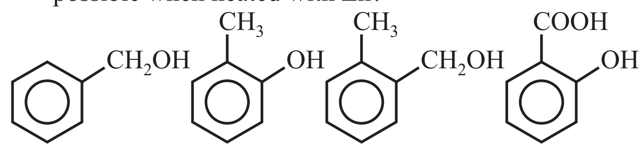
(a) IV > III > I > II (b) IV > III > II > I
(c) I > II > III > IV (d) III > IV > II > I

155. Identify Z in the following reaction

$$CH_3CH_2OH \xrightarrow{PBr_3} X \xrightarrow{Alc. KOH.} Y \xrightarrow[(ii)\ H_2O,\ heat]{(i)\ H_2SO_4,\ RT} Z$$

(a) $CH_2{=}CH_2$
(b) CH_3CH_2CH
(c) $CH_3CH_2—O—CH_2CH_3$
(d) $CH_3CH_2—SO_3H$

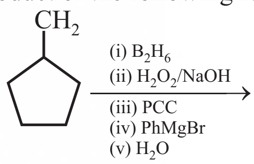
156. In which of the following compounds deoxygenation is possible when heated with Zn?

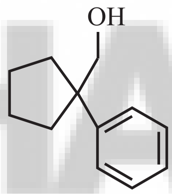
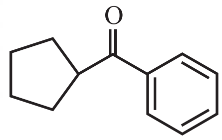
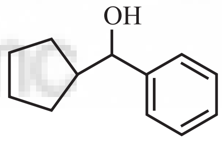
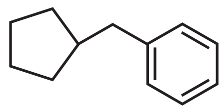


(A) (B) (C) (D)

(a) A, B and C (b) A, C and D
(c) B and D (d) B and C

157. The major product of the following reactions is



(a) 
(b) 
(c) 
(d) 

158. Match the following:

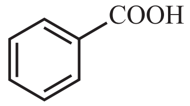
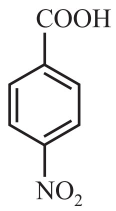
	Column-I (Reaction of carbonyl compound with)		Column-II (Product)
A.	Hydroxylamine	1.	Hydrazone
B.	Alcohol	2.	Schiff's base (Substituted imine)
C.	Hydrazine	3.	Oxime
D.	Amine	4.	Ketal

The correct match is

Codes:

	A	B	C	D
(a)	3	4	1	2
(b)	3	2	1	4
(c)	1	4	3	2
(d)	1	2	3	4

159. Match the following:

Column-I (Acid)		Column-II (pK_a Value)	
A.	CH_3COOH	1.	0.23
B.	F_3CCOOH	2.	3.41
C.		3.	4.19
D.		4.	4.76

The correct match is

Codes:

	A	B	C	D
(a)	4	1	3	2
(b)	1	4	2	3
(c)	4	1	2	3
(d)	4	3	2	1

160. Products that are formed in the given reaction including by products are

$CH_3CH_2CH_2CH_2CONH_2 + Br_2 + 4NaOH \rightarrow$

(a) $CH_3CH_2CH_2CH_2CH_2NH_2 + Na_2CO_3 + 2NaBr + 2H_2O$

(b) $CH_3CH_2CH_2CH_2NH_2 + Na_2CO_3 + 2NaBr + 2H_2O$

(c) $CH_3CH_2CH_2CH_2CH_2NH_2 + 2NaHCO_3 + Br_2 + 2H_2O$

(d) $CH_3CH_2CH_2CH_2NH_2 + 2Na_2CO_3 + Br_2 + 2H_2O$

ANSWER KEY

1.	(c)	**2.**	(a)	**3.**	(a)	**4.**	(d)	**5.**	(b)	**6.**	(b)	**7.**	(b)	**8.**	(c)	**9.**	(c)	**10.**	(b)
11.	(a)	**12.**	(a)	**13.**	(a)	**14.**	(d)	**15.**	(c)	**16.**	(a)	**17.**	(d)	**18.**	(c)	**19.**	(c)	**20.**	(b)
21.	(b)	**22.**	(c)	**23.**	(a)	**24.**	(b)	**25.**	(a)	**26.**	(a)	**27.**	(d)	**28.**	(b)	**29.**	(a)	**30.**	(b)
31.	(d)	**32.**	(b)	**33.**	(b)	**34.**	(c)	**35.**	(c)	**36.**	(d)	**37.**	(b)	**38.**	(d)	**39.**	(d)	**40.**	(b)
41.	(a)	**42.**	(b)	**43.**	(a)	**44.**	(a)	**45.**	(b)	**46.**	(d)	**47.**	(bonus)	**48.**	(a)	**49.**	(b)	**50.**	(d)
51.	(b)	**52.**	(a)	**53.**	(c)	**54.**	(a)	**55.**	(b)	**56.**	(d)	**57.**	(d)	**58.**	(d)	**59.**	(d)	**60.**	(c)
61.	(d)	**62.**	(a)	**63.**	(a)	**64.**	(b)	**65.**	(a)	**66.**	(b)	**67.**	(b)	**68.**	(b)	**69.**	(b)	**70.**	(a)
71.	(d)	**72.**	(a)	**73.**	(b)	**74.**	(c)	**75.**	(a)	**76.**	(b)	**77.**	(a)	**78.**	(a)	**79.**	(c)	**80.**	(b)
81.	(b)	**82.**	(b)	**83.**	(a)	**84.**	(a)	**85.**	(a)	**86.**	(a)	**87.**	(a)	**88.**	(b)	**89.**	(a)	**90.**	(b)
91.	(a)	**92.**	(c)	**93.**	(b)	**94.**	(d)	**95.**	(d)	**96.**	(a)	**97.**	(b)	**98.**	(b)	**99.**	(a)	**100.**	(c)
101.	(c)	**102.**	(c)	**103.**	(d)	**104.**	(a)	**105.**	(a)	**106.**	(d)	**107.**	(a)	**108.**	(b)	**109.**	(c)	**110.**	(a)
111.	(a)	**112.**	(a)	**113.**	(a)	**114.**	(c)	**115.**	(c)	**116.**	(c)	**117.**	(b)	**118.**	(c)	**119.**	(b)	**120.**	(a)
121.	(d)	**122.**	(d)	**123.**	(d)	**124.**	(b)	**125.**	(d)	**126.**	(b)	**127.**	(d)	**128.**	(a)	**129.**	(b)	**130.**	(b)
131.	(a)	**132.**	(d)	**133.**	(a)	**134.**	(c)	**135.**	(a)	**136.**	(a)	**137.**	(d)	**138.**	(b)	**139.**	(a)	**140.**	(b)
141.	(b)	**142.**	(c)	**143.**	(c)	**144.**	(b)	**145.**	(c)	**146.**	(a)	**147.**	(c)	**148.**	(c)	**149.**	(b)	**150.**	(c)
151.	(b)	**152.**	(c)	**153.**	(c)	**154.**	(a)	**155.**	(b)	**156.**	(c)	**157.**	(c)	**158.**	(a)	**159.**	(a)	**160.**	(d)

Hints & Solutions

MATHEMATICS

1. **(c)** Since $\sum_{r=1}^{n}[f(r)]^2 = [f(1)]^2 + [f(2)]^2 + [f(3)]^2 + \ldots + [f(n)]^2$

Given, $f(x+y) = f(x) + f(y)$ and $f(1) = 10 = 1 \times 10$

Let $f(2) = f(1+1) = f(1) + f(1) = 10 + 10 = 2 \times 10$

$f(3) = f(2+1) = f(2) + f(1) = 20 + 10 = 3 \times 10$

$f(4) = f(3+1) = f(3) + f(1) = 30 + 10 = 4 \times 10$

$\ldots f(n) = n \cdot 10$

Now,

$= (1 \cdot 10)^2 + (2 \cdot 10)^2 + (3 \cdot 10)^2 + \ldots + (n \cdot 10)^2$

$= 10^2(1^2 + 2^3 + 3^2 + \ldots + n^2)$

$= 100 \times \frac{n(n+1)(2n+1)}{6} \quad \left[\because \; \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}\right]$

$= \frac{50}{3} n(n+1)(2n+1)$

2. **(a)** Given $f(x) = \frac{3^x + 3^{-x}}{2}, x \in R$

$f(x+y) = \frac{3^{x+y} + 3^{-(x+y)}}{2}$

$f(x-y) = \frac{3^{x-y} + 3^{-(x-y)}}{2}$

Hence, $f(x+y) + f(x-y)$

$= \frac{1}{2}[3^{x+y} + 3^{-(x+y)} + 3^{x-y} + 3^{-(x-y)}]$

Given $f(x+y) + f(x-y) = Qf(x) \cdot f(y)$

$\Rightarrow \; a = 2$

3. **(a)** Here difference between 2.5 is 3.

Use trick, $\frac{1}{D}\left[\frac{1}{F} - \frac{1}{L}\right]$ [F = First, L = Last]

$\frac{1}{2 \cdot 5} = \frac{1}{3}\left(\frac{1}{2} - \frac{1}{5}\right), \frac{1}{5 \cdot 8} = \frac{1}{3}\left(\frac{1}{5} - \frac{1}{8}\right), \ldots$

$= \frac{1}{3}\left(\frac{1}{2} - \frac{1}{3n+2}\right) = \frac{1}{3} \cdot \frac{3n+2-2}{2(3n+2)} = \frac{n}{6n+4}$

4. **(d)** Given, system of equations

$3x - 2y + z = 0$

$\lambda x - 14y + 15z = 0$

$x + 2y - 3z = 0$

has a solution other than $x = y = z = 0$, then this system of equation will have non-trivial solutions as $|A| = 0$.

i.e., $AX = 0$

can be written as $\begin{bmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{vmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & -3 \end{vmatrix} = 0$

$3(42 - 30) + 2(-3\lambda - 15) + 1(2\lambda + 14) = 0$

$\Rightarrow \; 36 - 6\lambda - 30 + 2\lambda + 14 = 0$

$\Rightarrow \; -4\lambda + 20 = 0 \Rightarrow \; \lambda = 5$

5. **(b)** $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

It has a standard result as $(a-b)(b-c)(c-a)$

$\therefore$ so $K = 1$.

6. **(b)** Matrix representation of given system of equations,

$\begin{bmatrix} 5 & -7 & 3 \\ 7 & 10 & -8 \\ 2 & 3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -4 \end{bmatrix}$

By Cramer's rule

$x = \alpha = \frac{\Delta_1}{\Delta}; y = \beta = \frac{\Delta_2}{\Delta}$ and $z = \gamma = \frac{\Delta_3}{\Delta}$

Hence, $\Delta = \begin{vmatrix} 5 & -7 & 3 \\ 7 & 10 & -8 \\ 2 & 3 & -4 \end{vmatrix}$

$= 5(-40 + 24) + 7(-28 + 16) + 3(21 - 20) = -161$

and $\Delta_2 = \begin{vmatrix} 5 & 0 & 3 \\ 7 & 3 & -8 \\ 2 & -4 & -4 \end{vmatrix}$

$= 5(-12 - 32) - 0 + 3(-28 - 6)$

$= 5 \times (-44) + 3 \times (-34) = -322 \; \therefore \; \beta = \frac{-322}{-161} = 2$

7. **(b)** Given $(a + ib)^{1/4} = 2 + 3i$

or $a + ib = (2 + 3i)^4$

$= 2^4 + {}^4C_1 2^3(3i) + {}^4C_2 2^2(3i)^2 + {}^4C_3 2(3i)^3 + (3i)^4$

$= 16 + 4 \cdot 8 \cdot 3i + 6 \cdot 4 \cdot 9 \cdot (-1) + 4 \cdot 2 \cdot 27(-i) + 81$

$= 16 + 96i - 216 - 216i + 81 = -119 - 120i$

$\therefore \quad a = -119, b = -120$

Now, $3b - 2a = 3(-120) - 2(-119)$

$= -360 + 238 = -122$

8. **(c)** (A) $\omega^{1010} + \omega^{2020} = \omega^{336 \times 3 + 2} + \omega^{673 \times 3 + 1}$

$= (\omega^3)^{336} \cdot \omega^2 + (\omega^3)^{673} \cdot \omega$

$= \omega^2 + \omega = -1$ $[\because \omega^3 = 1]$

(B) $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$ $\because (1 + \omega + \omega^2 = 0)$

$= (-\omega^2 - \omega^2)(-\omega - \omega)$

$= (-2\omega^2)(-2\omega) = 4\omega^3 = 4$

(C) $(2 + \omega^2 + \omega^4)^5 = (2 + \omega^2 + \omega)^5$

$[\because 1 + \omega + \omega^2 = 0]$

$= (1 + 1 + \omega + \omega^2)^5 = (1 + 0)^5 = 1$

(D) $(3 + 5\omega + 3\omega^2)^3 = (3 + 2\omega + 3(\omega + \omega^2))^5$

$[\because \omega + \omega^2 = 1]$

$= (3 + 2\omega - 3)^3 = (2\omega)^3 = 8$

9. **(c)** Given $(z-4)^3 = 8i \Rightarrow z - 4 = (8i)^{1/3}$

$z = 4 + 2(i)^{1/3} \Rightarrow i = e^{i\pi/2} = e^{i\left(\frac{\pi}{2} + 2n\pi\right)}$

$[\because e^{i\theta} = \cos\theta + i\sin\theta]$

$i^{1/3} = e^{i\left(\frac{\pi}{6} + \frac{2n\pi}{3}\right)}$, $n = 0, 1, 2$

$$\begin{cases} e^{i\frac{\pi}{6}} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + i\frac{1}{2} \\ e^{i\left(\frac{\pi}{6} + \frac{2\pi}{3}\right)} = e^{i\frac{5\pi}{6}} = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = \frac{-\sqrt{3}}{2} + i\frac{1}{2} \\ e^{i\left(\frac{\pi}{6} + \frac{4\pi}{3}\right)} = e^{i\frac{3\pi}{2}} = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = 0 - i \end{cases}$$

For $n = 0$, $e^{i\frac{\pi}{6}} = \frac{\sqrt{3}}{2} + i\frac{1}{2} \Rightarrow n = 1$, $e^{i\frac{5\pi}{6}} = -\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}$

$n = 2$, $e^{i\frac{3\pi}{2}} = 0 - i$

$\therefore z = 4 + 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = (4 + \sqrt{3}) + i = b + i$

$z = 4 + 2\left(\frac{-\sqrt{3}}{2} + \frac{i}{2}\right) = (4 - \sqrt{3}) + i = c + i$

$z = 4 + 2(0 - i) = 4 - 2i = a - 2i$

$\Rightarrow a = 4$, $b = 4 + \sqrt{3}$ and $c = 4 - \sqrt{3}$

$\therefore \sqrt{abc} = \sqrt{4(4+\sqrt{3})(4-\sqrt{3})} = \sqrt{4(16-3)} = 2\sqrt{13}$

10. **(b)** Let $f(x) = x^2 + 7x + 3 = 0$

Roots are $\frac{\alpha}{\alpha+1}$ and $\frac{\beta}{\beta+1}$.

Equation having roots α and β is $f\left(\frac{x}{x+1}\right) = 0$

$\left(\frac{x}{x+1}\right)^2 + 7\left(\frac{x}{x+1}\right) + 3 = 0$

$\Rightarrow x^2 + 7x(x+1) + 3(x+1)^2 = 0$

$\Rightarrow x^2 + 7x^2 + 7x + 3(x^2 + 2x + 1) = 0$

$\Rightarrow 11x^2 + 13x + 3 = 0$

11. **(a)** Let $\frac{x^2 + 14x + 9}{x^2 + 2x + 3} = y$

$\Rightarrow x^2 + 14x + 9 = yx^2 + 2yx + 3y$

$\Rightarrow (y-1)x^2 + (2y - 14)x + (3y - 9) = 0$

$\because x \in R \Rightarrow$ Discriminant ≥ 0

$\Rightarrow (2y - 14)^2 - 4(y-1)(3y-9) \geq 0$

$\Rightarrow 4[(y-7)^2 - 3(y-1)(y-3)] \geq 0$

$\Rightarrow [y^2 - 14y + 49 - 3(y^2 - 14y + 3)] \geq 0$

$\Rightarrow -2y^2 - 2y + 40 \geq 0$ or $y^2 + y - 20 \leq 0$

or $(y-4)(y+5) \leq 0 \Rightarrow y \in [-5, 4]$

12. **(a)** Let $f(x) = x^4 - 2x^3 + 6x - 21 = 0$ ...(i)

and their roots are α, β, γ and δ.

Let $\alpha^2 = x$ or $\alpha = \sqrt{x}$ and Put $\alpha = \sqrt{x}$ in eqn. (i)

$(\sqrt{x})^4 - 2(\sqrt{x})^3 + 6\sqrt{x} - 21 = 0$

$\Rightarrow x^2 - 2x\sqrt{x} + 6\sqrt{x} - 21 = 0$

$\Rightarrow x^2 - 21 = 2x\sqrt{x} - 6\sqrt{x}$

$\Rightarrow x^2 - 21 = 2\sqrt{x}(x-3)$

On squaring both side,

$\Rightarrow (x^2 - 21)^2 = 4x(x-3)^2$

$\Rightarrow x^4 - 42x^2 + 441 = 4x(x^2 - 6x + 9)$

$\Rightarrow x^4 - 42x^2 + 441 = 4x^3 - 24x^2 + 36x$

$\Rightarrow x^4 - 4x^3 - 18x^2 - 36x + 441 = 0$

13. **(a)** α, β and γ are the roots of $x^3 - x + 1 = 0$ ...(i)

We have to find a polynomial whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$

Let $x = \frac{1+\alpha}{1-\alpha}$ or $\alpha = \frac{x-1}{x+1}$

Substituting $\alpha = \frac{x-1}{x+1}$ in Eq. (i) we have

$\left(\frac{x-1}{x+1}\right)^3 - \left(\frac{x-1}{x+1}\right) + 1 = 0$

$\Rightarrow (x-1)^3 - (x-1)(x+1)^2 + (x+1)^3 = 0$

$\Rightarrow x^3 - 1 + 3x - 3x^2 - (x^2 - 1)(x + 1) + (x^3 + 1 + 3x + 3x^2) = 0$

$\Rightarrow x^3 - 1 + 3x - 3x^2 - x^3 - x^2 + x + 1 + x^3 + 1 + 3x + 3x^2) = 0$

$\Rightarrow x^3 - x^2 + 7x + 1 = 0$

$\therefore$ Sum of roots $= \frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma} = \frac{-(-1)}{1} = 1$

14. **(d)** ${}^{34}C_5 + \sum_{r=0}^{4} {}^{(38-r)}C_4$

$= {}^{34}C_5 + {}^{38}C_4 + {}^{37}C_4 + {}^{36}C_4 + {}^{35}C_4 + {}^{34}C_4$

$= ({}^{34}C_5 + {}^{34}C_4) + {}^{35}C_4 + {}^{36}C_4 + {}^{37}C_4 + {}^{38}C_4$

$[\because \quad {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}]$

$= ({}^{35}C_5 + {}^{35}C_4) + {}^{36}C_4 + {}^{37}C_4 + {}^{38}C_4$

$= ({}^{36}C_5 + {}^{36}C_4) + {}^{37}C_4 + {}^{38}C_4 = ({}^{37}C_5 + {}^{37}C_4) + {}^{38}C_4$

$= {}^{38}C_5 + {}^{38}C_4 = {}^{39}C_5$

15. **(c)** ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11 : 52$

or $(21 - r)(20 - r)(19 - r) = 52 \times 2 \times 21$

$\Rightarrow (21 - r)(20 - r)(19 - r) = 14 \times 13 \times 12$

$\Rightarrow (21 - r)(20 - r)(19 - r) = (21 - 7)(20 - 7)(19 - 7)$

$\Rightarrow r = 7$

16. **(a)** Greatest term of $(1 + x)^n = {}^nC_{\frac{n+1}{2}}$ (When n is odd)

Greatest term $= {}^{11}C_{\frac{11+1}{2}} = T_6$

$\therefore \ T_6 = T_{5+1} = {}^{11}C_5 = \dfrac{11 \times 10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4 \times 5} = 462$

17. **(d)** Given $(1 - x - x^2 + x^3)^6$ it can be written as $[(1 - x)(1 - x^2)]^6 = (1 - x)^{12}(1 + x)^6$

In the expansion of $(1 - x)^{12}$, coefficient are of the form $(-1)^r\, {}^{12}C_r$ and $(1 + x)^{12}$, coefficient are of the form ${}^6C_r$.

Coefficient of x^4 in expansion of $(1 - x - x^2 + x^3)^6$

$= {}^{12}C_0 \times {}^6C_4 - {}^{12}C_1 \times {}^6C_3 + {}^{12}C_2 \times {}^6C_2 - {}^{12}C_3 \times {}^6C_1 + {}^{12}C_4 \times {}^6C_0$

$= 15 - 240 + 990 - 1320 + 495 = -60$

18. **(c)** Sum of coefficient in the expansion of

$\left(1 + \dfrac{x}{2}\right)^{12} = \left(1 + \dfrac{1}{2}\right)^{12} = \left(\dfrac{3}{2}\right)^{12}$ [Put variable = 1]

[$\because$ sum of coefficients in the expansion of $(1 + x)^n = 2^n$ which can be obtained by putting $x = 1$]

19. **(c)** $\dfrac{3x^2 + 4x + 7}{(x-2)^3} = \dfrac{A}{(x-2)^3} + \dfrac{B}{(x-2)^2} + \dfrac{C}{(x-2)}$

$f(x) = f(a) + f'(a)(x - a) + \dfrac{f''(a)}{2!}(x - a)^2$

On solving $a = 2$ and $g(x) = 3x^2 + 4x + 7$

$\therefore \ A = g(2), B = g'(2), C = \dfrac{g''(2)}{2!}$

$\Rightarrow A + B + C = g(2) + g'(2) + \dfrac{g''(2)}{2!}$

20. **(b)** Given $\cot\dfrac{\pi}{16} \cdot \cot\dfrac{2\pi}{16} \cdot \cot\dfrac{3\pi}{16} \cdot \cot\dfrac{4\pi}{16} \cdot \cot\dfrac{5\pi}{16} \cdot \cot\dfrac{6\pi}{16} \cdot \cot\dfrac{7\pi}{16}$

$= \left(\cot\dfrac{\pi}{16} \cdot \cot\dfrac{7\pi}{16}\right) \cdot \left(\cot\dfrac{2\pi}{16} \cdot \cot\dfrac{6\pi}{16}\right) \cdot \left(\cot\dfrac{3\pi}{16} \cdot \cot\dfrac{5\pi}{16}\right) \cdot \cot\dfrac{4\pi}{16}$

$= \left[\cot\dfrac{\pi}{16}\cot\left(\dfrac{\pi}{2} - \dfrac{\pi}{16}\right)\right] \cdot \left[\cot\dfrac{2\pi}{16} \cdot \cot\left(\dfrac{\pi}{2} - \dfrac{2\pi}{16}\right)\right] \cdot \left[\cot\dfrac{3\pi}{16} \cdot \cot\left(\dfrac{\pi}{2} - \dfrac{3\pi}{16}\right)\right] \cdot \cot\dfrac{\pi}{4}$

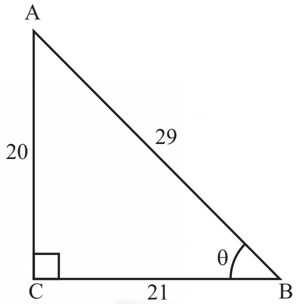
$= \left(\cot\dfrac{\pi}{16} \cdot \tan\dfrac{\pi}{16}\right) \cdot \left(\cot\dfrac{2\pi}{16} \cdot \tan\dfrac{2\pi}{16}\right) \left(\because \omega + \left(\dfrac{\pi}{2} - \theta\right) = \tan\theta\right) \cdot \left(\cot\dfrac{3\pi}{16} \cdot \cot\dfrac{3\pi}{16}\right) \cdot 1$

$= 1 \cdot 1 \cdot 1 \cdot 1 = 1$

21. **(b)** $AB = 29$ units, $BC = 21$ units, then

$AC = \sqrt{29^2 - 21^2} = 20$ units

By Pythagoras theorem



$\cos\theta = \dfrac{BC}{AB} = \dfrac{21}{29}$ and $\sin\theta = \dfrac{AC}{AB} = \dfrac{20}{29}$

$\therefore \ \cos^2\theta - \sin^2\theta = \left(\dfrac{21}{29}\right)^2 - \left(\dfrac{20}{29}\right)^2 = \dfrac{41}{841}$

22. **(c)** $\dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$

$= \dfrac{1 - (1 - 2\sin^2\theta) + \sin 2\theta}{1 + (2\cos^2\theta - 1) + \sin 2\theta}$

$= \dfrac{2\sin^2\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + 2\sin\theta\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta}\left(\dfrac{\sin\theta + \cos\theta}{\sin\theta + \cos\theta}\right) = \tan\theta$

23. **(a)** Given $y = 4\sin^2\theta - \cos 2\theta$

or $= 4\sin^2\theta - (1 - 2\sin^2\theta)$

$= 4\sin^2\theta - 1 + 2\sin^2\theta$

or $= 6\sin^2\theta - 1$ $[\because \quad 0 \le \sin^2\theta \le 1]$

Then, $0 \le 6\sin^2\theta \le 6$

or $\Rightarrow -1 \le 6\sin^2\theta - 1 \le 5$ or $-1 \le y \le 5$

Hence, minimum value of $y = -1$

$\Rightarrow l = -1$

Maximum value of $y = 5 \quad \Rightarrow m = 5$

$lm = -5, \ \dfrac{m}{l} = -5 \ \therefore \ lm = \dfrac{m}{l}$

24. **(b)** $\cos\theta = \frac{-1}{\sqrt{2}}$ and $\tan\theta = 1$

[$\because$ In IIIrd quadrant, cosine is negative and tangent is positive]

$\Rightarrow \cos\theta = \cos\left(\pi + \frac{\pi}{4}\right)$ and $\tan\theta = \tan\left(\pi + \frac{\pi}{4}\right)$

$\Rightarrow \cos\theta = -\cos\frac{\pi}{4}$ and $\tan\theta = \tan\frac{\pi}{4}$

$\Rightarrow \theta = (2n+1)\pi + \frac{\pi}{4}, n = 0, 1, 2, 3, \ldots$

25. **(a)** $\sin^{-1}x < \cos^{-1}x$

$\Rightarrow \sin^{-1}x < \frac{\pi}{2} - \sin^{-1}x \quad \left[\because \sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}\right]$

$\Rightarrow 2\sin^{-1}x < - \Rightarrow \sin^{-1}x < \frac{\pi}{4}$

$\Rightarrow x < \sin\frac{\pi}{4} \Rightarrow x < \frac{1}{\sqrt{2}}$

For $\sin^{-1}(x)$; $1 \le x \le 1$

Then $-1 \le x < \frac{1}{\sqrt{2}}$

26. **(a)** $\coth^{-1}(x) = \frac{1}{2}\log\left(\frac{x+1}{x-1}\right), x < -1$ or $x > 1$

and $\text{cosech}^{-1}(x) = \log\left(\frac{1}{x} + \sqrt{\frac{1}{x^2}+1}\right), x \ne 0$

$\therefore \coth^{-1}(2) + \text{cosech}^{-1}(-2\sqrt{2})$

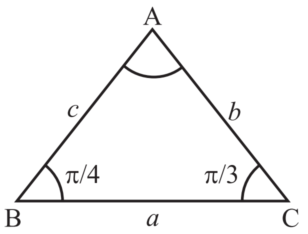
$= \frac{1}{2}\log\left(\frac{2+1}{2-1}\right) + \log\left(\frac{-1}{2\sqrt{2}} + \sqrt{\frac{1}{(-2\sqrt{2})^2}+1}\right)$

$= \frac{1}{2}\log 3 + \log\left(\frac{-1}{2\sqrt{2}} + \frac{3}{2\sqrt{2}}\right)$

$= \log\sqrt{3} + \log\frac{1}{\sqrt{2}} = \log\sqrt{\frac{3}{2}}$

27. **(d)** Given $\angle B = \frac{\pi}{4}$, $\angle C = \frac{\pi}{3}$

Let ΔABC



$\angle A = \pi - \frac{\pi}{4} - \frac{\pi}{3} = \frac{5\pi}{12} \quad (\because \angle A + \angle B + \angle C = \pi)$

By sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\Rightarrow b = a\frac{\sin B}{\sin A} = \frac{a\sin\frac{\pi}{4}}{\sin\frac{5\pi}{12}}$

and $\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}a^2\frac{\sin B}{\sin A}\sin C$

$\therefore \frac{1}{2}a^2\frac{\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = 54 + 18\sqrt{3}$

$\Rightarrow$ or $a^2 = 36(\sqrt{3} \quad 1) \quad \therefore$ or $a = 6(\sqrt{3}+1)$

28. **(b)** Given $r_1 = 12, r_2 = 6, r_3 = 4$

$\frac{1}{r} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{1}{2} \quad \left[\because \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right]$

$\therefore r = 2 \quad \left[\because a = \sqrt{(r_1 - r)(r_2 + r_3)}\right]$

$a^2 = (r_1 - r)(r_2 + r_3) = (12 - 2)(6 + 4) = 100 \Rightarrow a = 10$

Similarly,

$b^2 = (r_2 - r)(r_1 + r_3) = (6 - 2)(12 + 4) = 64 \Rightarrow b = 8$

and, $c^2 = (r_3 - r)(r_1 + r_2) = (4 - 2)(12 + 6) = 36 \Rightarrow c = 6$

$\therefore a + 2b + 3c = 10 + 2 \times 8 + 3 \times 6 = 44$

29. **(a)** We know that,

$r_1 + r_2 + r_3 - r = 4R \therefore$ or $r_1 + r_2 + r_3 = 4R + r$ …(i)

Let $\frac{a}{4} = \frac{b}{5} = \frac{c}{6} = \lambda$

$\Rightarrow a = 4\lambda, b = 5\lambda, c = 6\lambda$

$s = \frac{4\lambda + 5\lambda + 6\lambda}{2} = \frac{15\lambda}{2} \quad \left(\because s = \frac{a+b+c}{2}\right)$

Area $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{15\lambda}{2}\left(\frac{15\lambda}{2} - 4\lambda\right)\left(\frac{15\lambda}{2} - 5\lambda\right)\left(\frac{15\lambda}{2} - 6\lambda\right)}$

$= 15\sqrt{7}\frac{\lambda^2}{4}$

$\therefore r = \frac{\Delta}{s} = \frac{15\sqrt{7}\frac{\lambda^2}{4}}{\frac{15\lambda}{2}} = \sqrt{7}\frac{\lambda}{2}$ …(ii)

and $R = \frac{abc}{4\Delta} = \frac{(4\lambda)\cdot(5\lambda)(6\lambda)}{4 \times 15\sqrt{7}\frac{\lambda^2}{4}} = \frac{8\lambda}{\sqrt{7}}$ …(iii)

Now, $\frac{1}{4R}[r_1 + r_2 + r_3] = \frac{4R + r}{4R}$ [From Eq. (i)]

$= 1 + \frac{r}{4R} = 1 + \frac{\sqrt{7}\cdot\frac{\lambda}{2}}{4 \times \frac{8\lambda}{\sqrt{7}}} = 1 + \frac{7}{64} = \frac{71}{64}$

30. **(b)** Given $a = 12, b = 16, c = 20$

$$s = \frac{a+b+c}{2} = \frac{12+16+20}{2} = 24$$

$$\Delta = \sqrt{24(24-12)(24-16)(24-20)}$$

$$= \sqrt{24\times 12\times 8\times 4} = 96$$

$$r_1 = \frac{\Delta}{s-a} = \frac{96}{24-12} = \frac{96}{12} = 8$$

$$r_2 = \frac{\Delta}{s-b} = \frac{96}{24-16} = \frac{96}{8} = 12$$

$$r_3 = \frac{96}{24-20} = \frac{96}{4} = 24$$

Let $r_1 : r_2 : r_3 = 8 : 12 : 24$

But ratio of external radii of the triangle opposite to the angles in the order $\angle C, \angle B, \angle A$ is $= 24 : 12 : 8$.

$= 6 : 3 : 2$

31. **(d)** Given vectors $(-3i + 4\hat{j} + \lambda\hat{k})$ and $(\mu\hat{i} + 8\hat{j} + 6\hat{k})$ are collinear, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then $\frac{-3}{\mu} = \frac{4}{8} = \frac{\lambda}{6}$

$$= \frac{-3}{\mu} = \frac{1}{2} \text{ and } \frac{\lambda}{6} = \frac{1}{2} \Rightarrow \mu = -6, \lambda = 3 \therefore \lambda - \mu = 9$$

32. **(b)** $\cos\theta = \frac{0.1 + (-3)(0) + 2(-2)}{\sqrt{0^2 + (-3)^2 + 2^2}\sqrt{1^2 + 0^2 + (-2)^2}} = \frac{-4}{\sqrt{65}}$

$$\theta = \cos^{-1}\left(\frac{-4}{\sqrt{65}}\right)$$

33. **(b)** Given $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ $\qquad \{(a)^2 = \bar{a}\cdot\bar{a}\}$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = (\vec{a} + \vec{b} + \vec{c})(\vec{a} + \vec{b} + \vec{c})$

$\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow 3^2 + 4^2 + 2^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow 29 + 2\,(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = \frac{-29}{2}$

$\therefore\ \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} + 2|\vec{a}| + |\vec{b}| + |\vec{c}|$

$= \frac{-29}{2} + 2\,.(3+4+2) = \frac{-29+36}{2} = \frac{7}{2}$

34. **(c)** $(\vec{a} + 2\vec{b} - \vec{c})\cdot[(\vec{a} - \vec{b})\times(\vec{a} - \vec{b} - \vec{c})]$

$= (\vec{a} + 2\vec{b} - \vec{c})\cdot(\vec{a}\times\vec{a} - \vec{a}\times\vec{b} - \vec{a}\times\vec{c}$
$\qquad - \vec{b}\times\vec{a} + \vec{b}\times\vec{b} + \vec{b}\times\vec{c})$

$= (\vec{a} + 2\vec{b} - \vec{c})\cdot(0 - \vec{a}\times\vec{b} - \vec{a}\times\vec{c} + \vec{a}\times\vec{b} + 0 + \vec{b}\times\vec{c})$

$= (\vec{a} + 2\vec{b} - \vec{c})\cdot(\vec{b}\times\vec{c} - \vec{a}\times\vec{c})$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) - \vec{a}\cdot(\vec{a}\times\vec{c}) + 2\vec{b}\cdot(\vec{b}\times\vec{c}) - 2\vec{b}\cdot(\vec{a}\times\vec{c})$
$\qquad - \vec{c}\cdot(\vec{b}\times\vec{c}) + \vec{c}\cdot(\vec{a}\times\vec{c})$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) + 2\vec{b}\cdot(\vec{c}\times\vec{a})$

$= [\vec{a}\,\vec{b}\,\vec{c}] + 2[\vec{b}\,\vec{c}\,\vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}] + 2[\vec{a}\,\vec{b}\,\vec{c}] = 3[\vec{a}\,\vec{b}\,\vec{c}]$

35. **(c)** $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = \hat{j} - \hat{k}$

Given, $\vec{a}\times\vec{b} = \vec{c}$ and $\vec{a}\cdot\vec{b} = 3$ $\because\ \vec{a}\times\vec{b} = \vec{c}$

$\Rightarrow \vec{a}\times(\vec{a}\times\vec{b}) = \vec{a}\times\vec{c} \Rightarrow (\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b} = \vec{a}\times\vec{c}$

$\Rightarrow 3\vec{a} - |\vec{a}|^2\,\vec{b} = \vec{a}\times\vec{c}$ $\qquad$ ...(i)

$$\vec{a}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = \hat{i}(-1-1) - \hat{j}(-1-0) + \hat{k}(1-0)$$

$= -2\hat{i} + \hat{j} + \hat{k}\,;\ |\vec{a}|^2 = 1^2 + 1^2 + 1^2 = 3$

From eq. (i), $3(\hat{i} + \hat{j} + \hat{k}) - 3\vec{b} = -2\hat{i} + \hat{j} + \hat{k}$

$3\hat{i} + 3\hat{j} + 3\hat{k} + 2\hat{i} - \hat{j} - \hat{k} = 3\vec{b}$

or $\vec{b} = \frac{5\hat{i}}{3} + \frac{2\hat{j}}{3} + \frac{2\hat{k}}{3}$

36. **(d)** $(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$

$= [(2\hat{i} - 3\hat{j} + \hat{k})\cdot(\hat{i} - \hat{j})(\hat{i} + 2\hat{j} - 3\hat{k})]$
$\qquad - [(\hat{i} + 2\hat{j} - 3\hat{k})\cdot(\hat{i} - \hat{j})(2\hat{i} - 3\hat{j} + \hat{k})]$

$= 5(\hat{i} + 2\hat{j} - 3\hat{k}) + (2\hat{i} - 3\hat{j} + \hat{k}) = 7\hat{i} + 7\hat{j} - 14\hat{k}$

$(\vec{a}\times\vec{b})\times\vec{c} \perp \vec{d}$

$\Rightarrow [(\vec{a}\times\vec{b})\times\vec{c}]\cdot\vec{d} = 0 \Rightarrow (7\hat{i} + 7\hat{j} - 14\hat{k})\cdot(\hat{i} + \hat{j} + x\hat{k}) = 0$

$\Rightarrow 7 + 7 - 14x = 0$ or $x = 1$

37. **(b)** $\bar{x} = 10$

$$\frac{2+3+5+18+17+15+13+x+9+7}{10} = 10$$

or $x + 89 = 100$ or $x = 11$

Deviation from mean $|x_i - \bar{x}|$, we get

8, 7, 5, 8, 7, 5, 3, 1, 1, 3

Hence, mean deviation $= \frac{\Sigma|x_i - \bar{x}|}{n} = \frac{48}{10} = 4.8$

38. **(d)** Probability of hitting the balloon by A i.e.,

$P(A) = \frac{4}{6} = \frac{2}{3}$, while $P(\bar{A}) = \frac{1}{3}$

Probability of hitting the balloon by B i.e., $P(B) = \frac{3}{5}$

While $P(\bar{B}) = \frac{2}{5}$

Probability of hitting the balloon by C i.e., $P(C) = \frac{2}{3}$

While $P(\bar{C}) = \frac{1}{3}$

Probability of hitting the balloon by at least two of A, B and C

$= P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C) + P(A \cap B \cap C)$

$= P(A) \times P(B) \times P(C') + P(A) \times P(B') \times P(C) + P(A') \times P(B) \times P(C) + P(A) \times P(B) \times P(C)$

$$= \frac{2}{3} \times \frac{3}{5} \times \left(1 - \frac{2}{3}\right) + \frac{2}{3} \times \left(1 - \frac{3}{5}\right) \times \frac{2}{3} + \left(1 - \frac{2}{3}\right) \times \frac{3}{5} \times \frac{2}{3} + \frac{2}{3} \times \frac{3}{5} \times \frac{2}{3}$$

$$= \frac{6}{45} + \frac{8}{45} + \frac{6}{45} + \frac{12}{45} = \frac{32}{45}$$

39. **(d)** Total number of ways of throwing an unbiased die three times $= 6^3 = 216$

$\because$ $\beta_1, \beta_2, \beta_3$ satisfies

$\omega^{\beta_1} + \omega^{\beta_2} = -\omega^{\beta_3}$

or $\omega^{\beta_1} + \omega^{\beta_2} + \omega^{\beta_3} = 0$

Here order matters

Hence $\beta_1, \beta_2, \beta_3$ is a permutation of the numbers of the form $3k$, $3n + 1$, $3m + 2$ from the set (1, 2, 3, 4, 5, 6).

Thus, $\beta_1, \beta_2, \beta_3$ can be chosen in 2^3 ways.

Hence, required probability $= \frac{2^3}{6^3} 3! = \frac{2}{9}$

40. **(b)** Given $P(A/B) = \frac{3}{10}$ and $P(B/A) = \frac{4}{5}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Hence, $P(A \cap B) = \frac{3}{10} P(B)$ and $P(A \cap B) = \frac{4}{5} P(A)$

$$\Rightarrow \frac{3}{10} P(B) = \frac{4}{5} P(A) \text{ or } \frac{P(A)}{P(B)} = \frac{3}{8}$$

Now, $P(A \cup B) = kP(B)$ or $k = \frac{P(A \cup B)}{P(B)}$

$$\text{Hence, } = \frac{P(A) + P(B) - P(A \cap B)}{P(B)}$$

$$= \frac{P(A)}{P(B)} + 1 - P(A/B) = \frac{3}{8} + 1 - \frac{3}{10}$$

$$k = \frac{43}{40} \quad \text{or} \quad \frac{1}{k} = \frac{40}{43}$$

41. **(a)** $p = 0.001$, $n = 2000$

$\because$ $\lambda = np \Rightarrow \lambda = 2$

$$\because \quad p(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$p(x = 3) = \frac{e^{-2} 2^3}{3!} = \frac{8}{6e^2} = \frac{4}{3e^2}$$

42. **(b)** Probability of getting tail $P = \frac{1}{2}$

Then, probability of getting head $q = \frac{1}{2}$, $n = 15$

Here probability of occuring tail only thrice

So, $P(x \geq 3) = 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$

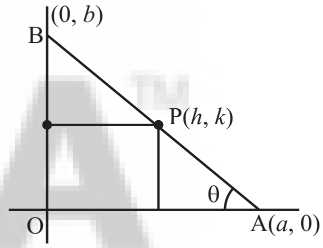
$$= 1 - ({}^{15}C_0 + {}^{15}C_1 + {}^{15}C_2) \frac{1}{2^{15}}$$

[by binomial distribution]

$$= 1 - (1 + 15 + 105) \frac{1}{2^{15}} = 1 - \frac{121}{2^{15}}$$

43. **(a)** Given $AB = 6$

Let $A(a, 0)$, $B(0, b)$ and $P(h, k)$ be the mid-point of AB. and $\angle OAB = \theta$, $\theta \in \left(0, \frac{\pi}{2}\right)$



In ΔOAB

$\sin\theta = \frac{k}{3}$ and $\cos\theta = \frac{h}{3}$

Hence, $\cos^2\theta + \sin^2\theta = 1 \Rightarrow k^2 + h^2 = 9$

$\Rightarrow x^2 + y^2 = 9$ is the locus of the midpoint of rod.

44. **(a)** Let $x = X - 1$

$y = Y + 2$

According to question

$2(X - 1)^2 + (Y + 2)^2 - 3(X - 1) + 5(Y + 2) - 8 = 0$

or $2(X^2 - 2X + 1) + (Y^2 + 4Y + 4) - 3X + 3 + 5Y + 10 - 8 = 0$

or $2X^2 + Y^2 - 7X + 9Y + 11 = 0$

45. **(b)** Let the point be $P(a, b)$ which lies on line

$4x - y - 2 = 0$ …(i)

and point P is equidistant from $A(-5, 6)$ and $B(3, 2)$.

Then, $PA = PB$ or $PA^2 = PB^2$

$\Rightarrow (a + 5)^2 + (b - 6)^2 = (a - 3)^2 + (b - 2)^2$

$\Rightarrow a^2 + 10a + 25 + b^2 - 12b + 36$

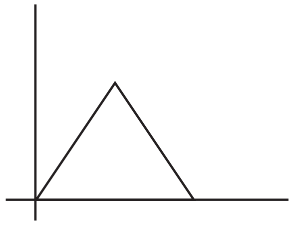
$= a^2 - 6a + 9 + b^2 - 4b + 4 \Rightarrow 16a - 8b + 48 = 0$

$\Rightarrow 2a - b + 6 = 0$ …(ii)

On solving equations (i) and (ii), we get $a = 4$, $b = 14$

46. **(d)** On solving

$OA = 2$, $OB = 2$ and $AB = 2$



Let vertices of the given triangle are (0, 0), $(1, \sqrt{3})$ and (2, 0); $OA = OB = AB$

$\therefore$ Triangle is equilateral.

In centre $\left(\frac{0+2+1}{3}, \frac{0+0+\sqrt{3}}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$

47. **(Bonus)**

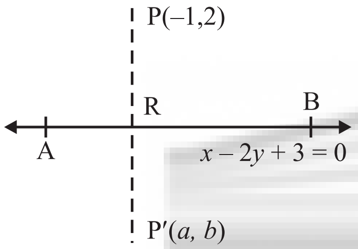


Image of (–1, 2) w.r.t. $x - 2y + 3 = 0$

$$\frac{a+1}{1} = \frac{b-2}{-2} = \frac{-2(-1-4+3)}{5} \Rightarrow a = \frac{-1}{5}, b = \frac{12}{5}$$

Length of perpendicular drawn from P' to $2x + y - 7 = 0$

$$= \left|\frac{2\left(-\frac{1}{5}\right) + \frac{12}{5} - 7}{\sqrt{5}}\right| = \sqrt{5}$$

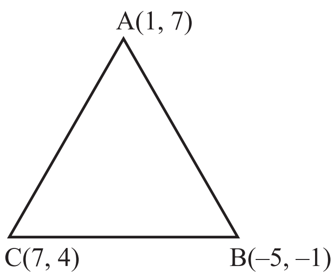
48. **(a)** Let a triangle ABC.

Angle bisector of $\angle ABC$ is given as

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}} \quad \because \angle B < \pi$$

$\Rightarrow \frac{\angle B}{2}$ will be an acute angle and here $a_1a_2 + b_1b_2 > 0$

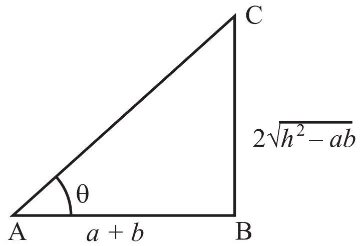
So, we use – ve sign.



$$\Rightarrow \frac{4x - 3y + 17}{5} = -\frac{5x - 12y + 13}{13} \Rightarrow 7x - 9y + 26 = 0$$

49. **(b)** We have,

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$



Then by Pythagoras theorem

$$\cos\theta = \frac{|a+b|}{\sqrt{(a-b)^2 + 4h^2}} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

50. **(d)** Given $9x^2 - 24xy + 16y^2 + \alpha x + \beta y + 6 = 0$

On comparing with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$a = 9, b = 16, c = 6$

$h = -12, g = \frac{\alpha}{2}, f = \frac{\beta}{2}$

Hence, given equation will represent pair of parallel lines, if $h^2 = ab$ and $af^2 = bg^2$ Now, we have $\frac{\alpha}{\beta} = \pm\frac{3}{4}$

51. **(b)** Given, sides of triangles

$x + y = 6$ …(i)

$2x + y = 4$ …(ii)

$x + 2y = 5$ …(iii)

on solving equations find vertices, $A(-2, 8)$, $B(7, -1)$ and $C(1, 2)$

Let coordinate of circumcentre be $O(h, k)$

Then $OA = OB = OC$

$OA = OB \Rightarrow OA^2 = OB^2$

$\Rightarrow (h+2)^2 + (k-8)^2 = (h-7)^2 + (k+1)^2$

$\Rightarrow h^2 + 4 + 4h + k^2 + 64 - 16k = h^2 + 49 - 14h + k^2 + 1 + 2k$ ...(iv)

or $h - k + 1 = 0 \Rightarrow OB = OC \Rightarrow OB^2 = OC^2$

$\Rightarrow (h-7)^2 + (k+1)^2 = (h-1)^2 + (k-2)^2$

$\Rightarrow -12h + 6k + 45 = 0$ ...(v)

From eqs. (iv) and (v), we get

$h = \frac{17}{2}, k = \frac{19}{2}$

$\therefore$ Circumcentre $\left(\frac{17}{2}, \frac{19}{2}\right)$

52. **(a)** Given $x^2 - 2x + y^2 = 0$ is a circle

or $(x-1)^2 + y^2 = 1$

The chord with midpoint $P(x_1, y_1)$ is $S_1 = S_2$

or $x_1x + y_1y - (x + x_1) = x_1^2 + y_1^2 - 2x_1$

it passes through (0, 0) $\Rightarrow x^2 + y^2 - x = 0$

53. **(c)** Number of possible tangents are depend on conditions

(i) $|C_1 C_2| > r_1 + r_2$ (ii) $|C_1 C_2| = r_1 - r_2$

Centre, $C_2(-2, 1)$

Radius, $r_2 = \sqrt{(-2)^2 + 1^2 - 1} = 2$

Distance between centres $C_1 C_2 = \sqrt{0^2 + 2^2} = 2$

Difference in radius $|r - r_2| = 2$

$\because C_1 C_2 = |r_1 - r_2|$

Figure

$\therefore$ Circle touches each other internally at one point

$\Rightarrow$ Number of tangents = 1

54. **(a)** Let $S_1 \equiv x^2 + y^2 + 13x - 3y = 0$ and

$S_2 \equiv 2x^2 + 2y^2 + 4x - 7y - 25 = 0$

Equation of circle passing through point of intersection of S_1 and S_2 is given by $S_1 + \lambda S_2 = 0$

$$(x^2 + y^2 + 13x - 3y) + \lambda(2x^2 + 2y^2 + 4x - 7y - 25) = 0 \quad \text{...(i)}$$

This circle is passing through (1, 1)

$$\therefore [1^2 + 1^2 + 13(1) - 3(1)] + \lambda[2(1)^2 + 2(1)^2 + 4(1) - 7(1) - 25] = 0$$

$\Rightarrow \lambda = 1/2$ From eq. (i), we get

$4x^2 + 4y^2 + 30x - 13y - 25 = 0$

55. **(b)** Let $S_1 \equiv (x - 2)^2 + (y - 3)^2 = 25$

Centre $C_1(2, 3)$ and radius $r_1 = 5$

and $S_2 \equiv 25x^2 + 25y^2 - 40x - 70y - 160 = 0$

Centre $C_2\left(\frac{4}{5}, \frac{7}{5}\right)$ and radius $r_2 = 3$

When two circles touch internally then coordinate of point of contact is $(x, y) \equiv \left(\frac{r_1x_2 - r_2x_1}{r_1 - r_2}, \frac{r_1y_2 - r_2y_1}{r_1 - r_2}\right)$

$$(\alpha, \beta) = \left(\frac{5 \times \frac{4}{5} - 3 \times 2}{5 - 3}, \frac{5 \times \frac{7}{5} - 3 \times 3}{5 - 3}\right)$$

$= (-1, -1)$

$\alpha + \beta = (-1) + (-1) = -2$

56. **(d)** Let $U : (h, k)$ and $S : ((h + a), k)$

Then, equation of parabola

$(y - k)^2 = 4a(x - h)$ ...(i)

Given $h = 1, k = -2$ and $S, \left(\frac{5}{4}, -2\right) = (h + a, k)$

$\Rightarrow h + a = \frac{5}{4} \Rightarrow a = \frac{5}{4} - 1 = \frac{1}{4}$

Putting values in Eq. (i), we get

$(y - (-2))^2 = 4 \cdot \frac{1}{4}(x - 1)$ or $(y + 2)^2 = (x - 1)$

From options

It is clear point (10, 1) satisfies this equation.

$\therefore$ Options (d) is correct.

57. **(d)** Given equation of parabola,

$y^2 - 4x - 8y - 12 = 0$ or $y^2 - 8y + 16 = 4x + 28$

$(y - 4)^2 = 4(x + 7)$

Comparing with $Y^2 = 4aX$ We know,

$Y = y - 4, a = 1, X = x + 7$

parametric form of $Y^2 = 4aX$

$X = at^2, Y = 2at$ or, $x + 7 = t^2, y - 4 = 2t$

or, $x = -7 + t^2, y = 4 + 2t$

58. **(d)** According to the definition of an ellipse.

$$\sqrt{x^2 + y^2} = \frac{1}{2}\left|\frac{x - 4}{\sqrt{1}}\right|$$

On squaring both side

$$\left(x + \frac{4}{3}\right)^2 + \frac{4}{3}y^2 = \frac{64}{9} \Rightarrow (3x + 4)^2 + 12y^2 = 64$$

59. **(d)** For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Foci $(\pm ae, 0)$, when $a > b$ and $b^2 = a^2(1 - e^2)$ and $(0, \pm be)$, when $b > a$ and $a^2 = b^2(1 - e^2)$

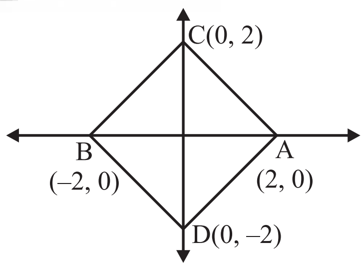
Let $S \equiv \frac{x^2}{9} + \frac{y^2}{5} = 1$

Foci $(\pm ae, 0) \Rightarrow \pm 3 \times \frac{2}{3}, 0 \Rightarrow (\pm 2, 0)$

Hence, Foci (2, 0), (– 2, 0) For $\frac{x^2}{5} + \frac{y^2}{9} = 1$

Foci $(0, \pm be) \Rightarrow (0, \pm 2)$ or (0, 2) and (0, – 2)

Let $A(2, 0), B(-2, 0), C(0, 2), D(0, -2)$



Area of quadrilateral $ADBC = 4 \times$ area of ΔOAC

$= 4 \times \frac{1}{2} \times 2 \times 2 = 8$ sq. units

60. **(c)** Let $P(x_1, y_1)$ be the pole of chord of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{36} = 1 \quad \text{...(i)}$$

Then, equation of chord

$$\frac{xx_1}{9} - \frac{yy_1}{36} = 1 \quad \text{...(ii)}$$

Homogenization of eq. (i) and (ii)

$$\frac{x^2}{9}-\frac{y^2}{36}=\left(\frac{xx_1}{9}-\frac{yy_1}{36}\right)^2$$

Here, chord substends right angle at the centre.

$\Rightarrow$ Coefficient of x^2 + Coefficient of $y^2 = 0$

$$\therefore\ \frac{1}{9}-\frac{x_1^2}{9^2}-\frac{1}{36}-\frac{y_1^2}{36^2}=0$$

$$\Rightarrow\ \frac{x_1^2}{9^2}+\frac{y_1^2}{36^2}=\frac{1}{9}-\frac{1}{36}\ \Rightarrow\ 16{x_1}^2+{y_1}^2=108$$

Hence, locus is, $16x^2 + y^2 = 108$.

61. **(d)** Vertices of tetrahedron $(a, 2, 1)$, $(1, b, 4)$, $(4, 0, c)$, $(1, 1, 7)$

Centroid

$$\left(\frac{a+1+4+1}{4},\frac{2+b+0+1}{4},\frac{1+4+c+7}{4}\right)=\left(\frac{9}{4},\frac{5}{4},\frac{15}{4}\right)$$

On comparing,

$$\Rightarrow\ \frac{a+6}{4}=\frac{9}{4},\frac{b+3}{4}=\frac{5}{4},\frac{c+12}{4}=\frac{15}{4}$$

$\Rightarrow a = 3, b = 2, c = 3$ or $a = c = b + 1$

62. **(a)** Let $\alpha=\frac{\pi}{4},\beta=\frac{\pi}{3}$ and $\gamma=\theta$ Then direction cosines are

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1\Rightarrow\cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{3}+\cos^2\theta=1$$

$$\frac{1}{2}+\frac{1}{4}+\cos^2\theta=1\Rightarrow\Rightarrow\ \cos^2\theta=\frac{1}{4}\quad\Rightarrow\ \cos\theta=\pm\frac{1}{2}$$

Hence direction cosines are $\cos\frac{\pi}{4},\cos\frac{\pi}{3},\cos\frac{\pi}{3}$

$$\Rightarrow\ \frac{1}{\sqrt{2}},\frac{1}{2},\frac{1}{2}$$

63. **(a)** Given, plane $2x - 2y + 4z + 5 = 0$

Direction ratios of the normal = 2, – 2, 4

$\because$ Equation of line passing through $\left(1,\frac{3}{2},2\right)$

and direction ratios are 2, – 2, 4 is given by

$$\frac{x-1}{2}=\frac{y-\frac{3}{2}}{-2}=\frac{z-2}{4}=\lambda\ \text{(say)}$$

$$x=2\lambda+1,\ y=-2\lambda+\frac{3}{2},\ z=4\lambda+2$$

$\because$ This point lies on plane

$\therefore$ Satisfy the eqn. of plane

$$\Rightarrow\ 2(2\lambda+1)-2\left(-2\lambda+\frac{3}{2}\right)+4(4\lambda+2)+5=0$$

$$24\lambda+12=0\quad\text{or}\quad\lambda=\frac{-1}{2}$$

Hence, required point

$$\left(2\left(\frac{-1}{2}\right)+1,-2\left(\frac{-1}{2}\right)+\frac{3}{2},4\left(\frac{-1}{2}\right)+2\right)=\left(0,\frac{5}{2},0\right)$$

64. **(b)** $\lim\limits_{x\to1}\frac{\log x}{1-x}$, when $x\to1$ [0/0, form]

$$=\lim_{x\to1}\frac{1/x}{-1}=\lim_{x\to1}\left(\frac{-1}{x}\right)=-1\ \text{[Using L' Hospital rule]}$$

65. **(a)** $f(0)=\lim\limits_{x\to0}f(x)$

$$=\lim_{x\to0}\frac{2-\sqrt{x+4}}{\sin 2x}\qquad\text{[0/0 form]}$$

$$=\lim_{x\to0}\frac{-\frac{1}{2\sqrt{x+4}}}{2\cos 2x}\qquad\text{[Using L' Hospital rule]}$$

$$=\lim_{x\to0}\frac{-1}{4\sqrt{x+4}\,\cos 2x}=\frac{-1}{4\cdot\sqrt{4}\cdot1}=\frac{-1}{8}$$

66. **(b)** Since, given function $f(x)$ is continuous everywhere

Hence, LHL at $(x = 0)$ = RHL at $(x = 0) = f(0)$

and LHL at $(x = 2)$ = RHL at $(x = 2) = f(2)$

$$\lim_{x\to0^-}f(x)=\lim_{x\to0^+}f(x)=2$$

$$\lim_{x\to0^-}(1+\cos x)=\lim_{x\to0^+}(a-x)=2$$

$\Rightarrow 1 + 1 = a - 0 = 2 \Rightarrow a = 2$

$$\lim_{x\to2^-}f(x)=\lim_{x\to2^+}f(x)=a-2$$

$$\Rightarrow\ \lim_{x\to2^-}(a-x)=\lim_{x\to2^+}(x^2-b^2)=2-2$$

or $2 - 2 = 2^2 - b^2 = 0 \Rightarrow b^2 = 4\ \therefore\ a^2 + b^2 = 4 + 4 = 8$

67. **(b)** Do it yourself.

68. **(b)** Given $g(x)=\begin{cases}-1, & -2\le x<0\\ x^2-1, & 0\le x\le 2\end{cases}$

$$|g(x)|=\begin{cases}1, & -2\le x<0\\ 1-x^2, & 0\le x<1\\ x^2-1, & 1\le x\le 2\end{cases}$$

$|g(x)| = \{x^2 - 1, 0 \le x \le 2$

Let's check differentiability at $x = 1$

LHD

$$f'(1)=\lim_{h\to0}\frac{f(1-h)-f(1)}{h}$$

$$=\lim_{h\to0}\frac{(1-(1-h)^2)-(1-1)}{h}=-2$$

Again at $x = 1$ RHD

$$f'(1)=\lim_{h\to0}\frac{f(1+h)-f(1)}{h}=\frac{(1+h)^2-1-(1-1)}{h}=2$$

Hence, $f(x)$ is not differentiable at $x = 1$.

69. **(b)** It is obvious.

1. If $f(x)$ is differentiable at $x = a$, then it will be continuous at $x = a$.

2. If $f(x)$ is not continuous at $x = a$, then it is not differentiable at $x = a$.

$|x|$ is continuous on R but it is not differentiable at $x = 0$

(d) $f(x) = x - [x] = \{x\}$ [fractional part function]

Then, $f(x) = \{x\}, x \varepsilon [0, 1)$

Hence, $f(1) \neq 1$

70. **(a)** Given $x^2 + xy + y^2 = k$

Differentiating with respect to x

$$\frac{dy}{dx} = \frac{-2x - y}{x + y} \Rightarrow \frac{d^2y}{dx^2} = \frac{3x\frac{dy}{dx} - 3y}{(x+2y)^2}$$

$$= \frac{-3x(2x+y) - 3y(x+2y)}{(x+2y)^3} \Rightarrow \frac{d^2y}{dx} = \frac{-6k}{(x+2y)^3}$$

71. **(d)** Given $(8.01)^{4/3} + (8.01)^2$

$= (8 + 0.01)^{4/3} + (8 + 0.01)^2$

$$= 8^{4/3}\left(1 + \frac{0.01}{8}\right)^{4/3} + 8^2\left(1 + \frac{0.01}{8}\right)^2$$

Using binomial expansion of any index.

$= 16(1 + 0.00125)^{4/3} + 64(1 + 0.00125)^2$

$$= 16\left(1 + \frac{4}{3} \times 0.00125\right) + 64(1 + 2 \times 0.00125)$$

Neglecting higher terms

$$= 16 \times \frac{3.005}{3} + 64 \times 1.0025 = 80.186$$

72. **(a)** Given equation of curve,

$y = 4x^4 + x$ …(i)

Tangent at P is $\frac{dy}{dx} = 16x^3 + 1$

According to question. It is perpendicular to tangent at (0, 0).

Slope of tangent at (0, 0) $m_2 = \left(\frac{dy}{dx}\right)_{(0,0)} = 1$

According to question, Hence $16x^3 + 1 = -1$

$x^3 = \frac{-1}{\theta}$ or $x = \frac{-1}{2}$ Putting it in equation of curve

$$\therefore\ y = 4\left(\frac{-1}{2}\right)^4 + \left(\frac{-1}{2}\right) = \frac{-1}{4}$$

$$y = \left(-\frac{1}{2}\right)^4 + \left(-\frac{1}{2}\right) = \frac{-1}{4} \qquad \therefore\ P\left(\frac{-1}{2}, \frac{-1}{4}\right)$$

73. **(b)** Given curves are

$2x^2 + y^2 = 20$ …(i)

and $4y^2 - x^2 = 8$ …(ii)

Hence point of intersection is $(\pm 2\sqrt{2}, \pm 2)$

Since, coordinates are in IV quadrants hence point $(2\sqrt{2}, -2)$ (say)

Now, slopes of curve (i) and (ii) $m_1 = \frac{-2x}{y}$ and $m_2 = \frac{x}{4y}$

Putting values of x and y from the point $P(2\sqrt{2}, -2)$

$$m_1 = \frac{-2 \cdot 2\sqrt{2}}{-2} = 2\sqrt{2} \text{ and } m_2 = \frac{2\sqrt{2}}{4(-2)} = -\frac{\sqrt{2}}{4}$$

Now, $m_1 m_2 = 2\sqrt{2} \times \frac{-\sqrt{2}}{4} = -1$

Hence, angle between the curve is $\frac{\pi}{2}$.

74. **(c)** According to R.T theorem function should be continuous and differentiable in (– 3, 0)

$f(x) = x(x+3)e^{-x/2} = (x^2 + 3x)e^{-x/2}$

$$f'(x) = (2x+3)e^{-x/2} + (x^2+3x)e^{-x/2} \cdot \left(\frac{-1}{2}\right) = 0$$

$$e^{-x/2}\left\{(2x+3) - \frac{(x^2+3x)}{2}\right\} = 0$$

Here $e^{-x/2} \neq 0$ Hence, $(x^2 - x - 6) = 0$

$\Rightarrow x^2 - x - 6 = 0 \Rightarrow (x+2)(x-3) = 0$

$\Rightarrow x = -2 \in [-3, 0]$

75. **(a)** $$\int \frac{dx}{\sin x + \cos x} = \int \frac{dx}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sin\left(x + \frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}}\int \operatorname{cosec}\left(x + \frac{\pi}{4}\right)dx$$

$$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} + \frac{\pi}{8}\right)\right| + C$$

76. **(b)** $I = \int (x+2)\sqrt{x+3}\,dx = \int \{(x+3) - 1\}(x+3)^{1/2}\,dx$

$= \int \{(x+3)^{3/2} - (x+3)^{1/2}\}\,dx$

$\Rightarrow \int (x+3)^{3/2}\,dx - \int (x+3)^{1/2}\,dx$

On solving, $= \frac{2}{15}\sqrt{x+3}\,(3x^2 + 13x + 12) + C$

77. **(a)** $$\int \frac{(1-\cos x)^{2/7}}{(1+\cos x)^{9/7}}dx = \int \frac{2^{2/7}\sin^{4/7}(x)}{2^{9/7}\cos^{18/7}(x)}dx$$

$$\Rightarrow \frac{1}{2}\int \tan^{4/7}\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)dx$$

Let $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2}\,dx = 2\,dt$

$$= \int t^{4/7}\,dt = \frac{7}{11}t^{11/7} + C = \frac{7}{11}\tan^{11/7}\frac{x}{2} + C$$

78. **(a)** Let $I = \int_5^9 \frac{\log 3x^2\, dx}{\log 3x^2 + \log(588 - 84x + 3x^2)}$ ...(i)

$= \int_5^9 \frac{\log 3x^2\, dx}{\log 3x^2 + \log 3(14-x)^2}$

$= \int_5^9 \frac{\log 3(14-x)^2\, dx}{\log 3(14-x)^2 + \log 3(14-(14-x))^2}$

$$\left[\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$I = \int_5^9 \frac{\log 3(14-x)^2\, dx}{\log 3(14-x)^2 + \log 3x^2}$ ...(ii)

Adding Eqs. (i) and (ii), we get

$2I = \int_5^9 \frac{\log 3x^2 + \log 3(14-x)^2}{\log 3(14-x)^2 + \log 3x^2}\,dx$

$= \int_5^9 dx = 9 - 5 = 4 \qquad I = 2$

79. **(c)** $\int_{-1}^1 \frac{\log(1+x)}{1+x^2}\,dx$

$= \int_0^1 \left[\frac{\log(1+x)}{1+x^2}\right] dx + \int_0^1 [f(x)]\,dx$

According to property

$\int_{-a}^a f(x)\,dx = \int_0^a (f(x) + f(-x))\,dx$

$= \int_0^a \left(\frac{\log(1+x)}{1+x^2} + \frac{\log(1-x)}{1+x^2}\right) dx$

Hence, $f(x) = \frac{\log(1-x)}{1+x^2}$

80. **(b)** Given, $\frac{dy}{dx} = x + \sin x \cdot \cos y + x \cos y + \sin x$

Hence $\frac{dy}{dx} = x(1 + \cos y) + \sin x(1 + \cos y)$

or $\frac{dy}{dx} = (x + \sin x)(1 + \cos y)$ or $\frac{dy}{1+\cos y} = (x + \sin x)\,dx$

$\frac{dy}{2\cos^2 \frac{y}{2}} = (x + \sin x)\,dx \Rightarrow \frac{1}{2}\sec^2 \frac{y}{2}\,dy = (x + \sin x)\,dx$

Integrating both sides, we get

$\tan \frac{y}{2} = \frac{x^2}{2} - \cos x + C$

PHYSICS

81. **(b)** Gravitational force is the weakest among all the fundamental forces of nature. As we know that, the strength of fundamental force i.e., in order of

(i) Strong nuclear force = 1

(ii) Electromagnetic force = $\frac{1}{137}$

(iii) Weak nuclear force = 10^{-6}

(iv) Gravitational forces = 6×10^{-39}

82. **(b)** As $\sigma = \frac{\rho}{AeT^4}$

and $b^4 = \lambda^4 T^4$ $[\because \quad b = \lambda T]$

So, $\sigma b^4 = \frac{\rho}{Ae\cancel{T^4}} \times \lambda^4 \cancel{T^4}$

$[\sigma b^4] = \left[\frac{\rho\lambda^4}{Ae}\right] = \frac{ML^2T^{-3}L^4}{L^2} = MT^{-3}L^4$

83. **(a)** $x = x(t_2) - x(t_1)$

$x = \alpha(t_2 - t_1) + \beta(t_2^3 - t_1^3)$, where $t_2 = 4$ sec, $t_1 = 2$ sec

$= 2(4-2) + \frac{1}{100}(4^3 - 2^3) = 4 + \frac{1}{100}(56) = 4.56$ m

$\therefore \quad v_{av} = \frac{x}{t_2 - t_1} = \frac{4.56}{4-2} = \frac{4.56}{2} = 2.28$ m/s

84. **(a)** $V_y = \sqrt{2g(h_1 - h_2)}$ $[\because \quad u_y = 0]$

$= \sqrt{2 \times 10(100-60)} = \sqrt{800} = 20\sqrt{2}\ \text{ms}^{-1}$

$\therefore \quad v = 10\hat{i} + 20\sqrt{2}\hat{j}\ |v| = \sqrt{10^2 + (20\sqrt{2})^2} = \sqrt{900}$

$= 30\ \text{ms}^{-1}$

85. **(a)** Angle between two vector $\vec{a}$ and $\vec{b}$ is given by

$\cos\theta = \frac{a \cdot b}{|a||b|}$

$\Rightarrow \cos\theta = \frac{(3\hat{i} + 2\hat{j} + 5\hat{k}) \cdot (5\hat{i} + 3\hat{j} + \hat{k})}{\sqrt{3^2 + 2^2 + 5^2}\sqrt{5^2 + 3^2 + 1^2}}$

$\Rightarrow \cos\theta = \frac{15 + 6 + 5}{\sqrt{9 + 4 + 25}\sqrt{25 + 9 + 1}} = \frac{26}{\sqrt{1330}}$

$\therefore \theta = \cos^{-1}\left(\frac{26}{\sqrt{1330}}\right)$

86. **(a)** Acceleration of particle,

$a = (4\hat{i} + 4\hat{j})\ \text{ms}^{-2}$ given

For displacement along X-axis

$v_x^2 - u_x^2 = 2a_x s_x$

$\Rightarrow v_x^2 - 16 = 2 \times 4 \times 6 \Rightarrow v_x = \sqrt{64} = 8\ \text{ms}^{-1}$

Now, $t = \frac{v_x - u_x}{a_x}$ where, t is time taken

$\therefore \ t = \frac{8-4}{4} = 2$ s

Again $v_y = u_y + a_y t$

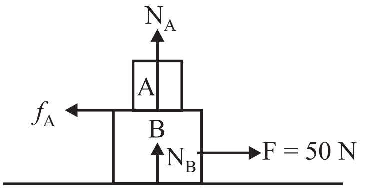
$\therefore\ v_y = 0 + 4 \times 1 = 4\ \text{ms}^{-1}$

$\therefore\ v = v_x\hat{i} + v_y\hat{j} = 8\hat{i} + 4\hat{j}$

Now, resultant velocity vector

$|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{8^2 + 4^2} = \sqrt{80} = 4\sqrt{5}\ \text{ms}^{-1}$

87. **(a)** Let f_A, f_B be the friction forces between blocks A and B and B and floor respectively.



From block B diagram

$f_{\lim} = \mu_B N_B$ …(i)

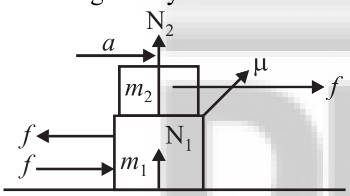
where, N_B be the normal reaction on block B

$= \mu_B(m_A + m_B)g$

$= 0.55(3 + 7)\ 10 = 55$ N

as $f_{\lim} > F_{app}$ (50 N) so block B will not move w.r.t. A so $f_A = 0$

88. **(b)** The FBD of given system is



Both block will move together when they have same acceleration (a) as

$f = m_2 a$ [For m_2]

$\mu m_2 g = m_2 a \Rightarrow a = \mu g$ For m_1

$F - f = m_1 a$

$F = f + m_1 a = m_2\mu g + m_1\mu g = \mu g(m_1 + m_2)$

89. **(a)** Given, $U(r) = \dfrac{1}{r^2} - \dfrac{1}{r}$

$$F = -\frac{\partial U}{\partial r}$$

$$\therefore\ F = -\frac{\partial}{\partial r}\left(\frac{1}{r^2} - \frac{1}{r}\right) = (2r^{-3} - r^{-2}) \quad \text{…(i)}$$

Now, for F_{max} i.e., maximum value of force

$$\frac{dF}{dr} = 0 \ \therefore\ \frac{\partial}{\partial r}(2r^{-3} - r^{-2}) = 0$$

$\Rightarrow -6r^{-4} + 2r^{-3} = 0 \qquad \Rightarrow 1 = 3r^{-1}$

$\Rightarrow r = 3$

Putting the value of r in equation (i), we get

$$F = \frac{2}{r^3} - \frac{1}{r^2}\bigg|_{r=3} = \frac{2}{3^2} - \frac{1}{3^2} = -\frac{1}{27}$$

90. **(b)** As, $\text{KE} = \dfrac{p^2}{2m}$

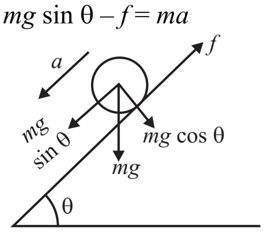
$$\frac{\text{KE}_1}{\text{KE}_2} = \left(\frac{p_1}{p_2}\right)^2 \times \left(\frac{m_2}{m_1}\right) \Rightarrow \frac{1}{16} = \left(\frac{p_1}{p_2}\right)^2 \times \left(\frac{1}{2}\right)$$

$$\Rightarrow \frac{p_1}{p_2} = \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}} \Rightarrow p_2 = 2\sqrt{2}p_1$$

91. **(a)** Moment of inertia,

$I = nmR^2$

From figure, $mg \sin\theta - f = ma$ …(i)



where, f is friction force and a is acceleration

$\tau = I\alpha = fR$ …(ii)

$$I\frac{a}{R} = fR \quad nmR^2 \cdot \frac{a}{R} = fR \quad \Rightarrow nma = f$$

$mg \sin\theta - nma = ma$ [From eq. (ii)]

$$\Rightarrow a = \frac{g \sin\theta}{n+1}$$

$$\therefore f = nma = nm\frac{g\sin\theta}{n+1}$$

(A) For ring, $I = mR^2$

Here, $n = 1$

and $f_{\text{ring}} = \dfrac{mg \sin\theta}{1+1} = \dfrac{mg\sin\theta}{2}$

(B) For solid sphere, $I = \dfrac{2}{5}mR^2$ Here, $n = \dfrac{2}{5}$

$$f_{\text{solid sphere}} = \frac{\frac{2}{5}mg\sin\theta}{\frac{2}{5}+1} = \frac{mg\sin\theta}{7/2} = \frac{mg\sin\theta}{3.5}$$

(C) For solid cylinder, $I = \dfrac{mR^2}{2}$ Here, $n = \dfrac{1}{2}$

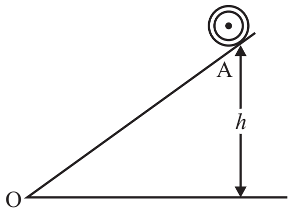
$$f_{\text{solid cylinder}} = \frac{\frac{1}{2}mg\sin\theta}{\frac{1}{2}+1} = \frac{mg\sin\theta}{3}$$

(D) For hollow cylinder $I = MR^2$

Here, $n = 1$

$$f_{\text{hollow cylinder}} = \frac{mg\sin\theta}{1+1} = \frac{mg\sin\theta}{2}$$

92. **(c)** The given situation is shown in figure.



According to conservation of mechanical energy.

$M{\cdot}E_i = M{\cdot}E_f$

$\Rightarrow U_i + (K_r)_{initial} + K_i = U_f + (K_r)_f + K_f$

$\Rightarrow mgh + 0 + 0 = 0 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$\Rightarrow mgh + \frac{1}{2}\pm\omega^2 + \frac{1}{2}mv^2$

$\therefore mgh = \frac{1}{2}\times\frac{1}{2}mR^2\times\frac{v^2}{R^2} + \frac{1}{2}mv^2$

$\left[\text{For disc } I = \frac{1}{2}mR^2\right]$

$\Rightarrow v = \sqrt{\frac{4}{3}gh} \quad \therefore v \propto \sqrt{h}$

$\therefore$ So speed depends on height of incline plane.

93. **(b)** Maximum velocity,

$V_{max} = \omega A = 100 \times 6 \times 10^{-2} = 6$ m/s

$\therefore$ Maximum kinetic energy,

$K_{max} = \frac{1}{2}m\,V_{max}^2 = \frac{1}{2}\times 1\times 6^2 = 18$ J

94. **(b)** Mass is invariant quantity. So mass remains constant when taken 64 km below the surface of the earth.

95. **(d)** Let Δl be the elongation in rod.

As we know yong's modulus

$Y = \frac{\text{Stress}}{\text{Strain}} \Rightarrow Y = \frac{F\cdot l}{A\Delta l}$

$\therefore \Delta l = \frac{Fl}{AY} = \frac{mgl}{AY}$

$\therefore \Delta l = \frac{500\times 10\times 1}{0.5\times 10^{-4}\times 10^{11}} = 1\times 10^{-3}$ m = 1 mm

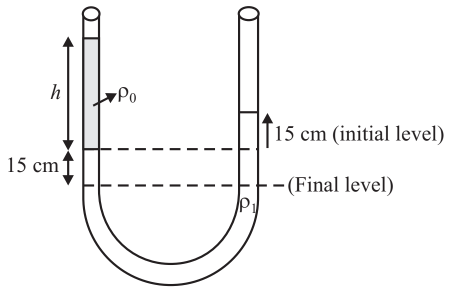
96. **(a)** Work done by pump, = W = total energy = $mgh + \frac{1}{2}mv^2$

$\therefore$ Power, $P = \frac{W}{t} = \frac{mgh + \frac{1}{2}mv^2}{t} = \frac{m}{t}\left(gh + \frac{v^2}{2}\right)$

$= \frac{\rho A\cdot l}{t}\left(gh + \frac{v^2}{2}\right) = \rho A v\left(gh + \frac{v^2}{2}\right) \qquad \left[\because \frac{l}{t} = v\right]$

$= 1000\times\pi\left(\frac{1}{100}\right)^2\times 10\left(10\times 3 + \frac{100}{2}\right) = 80\pi$ W

97. **(b)**



At same level, pressure in same fluid do not change.

$\therefore \rho_0 + \rho_0 g(h + 15) = \rho_0 + \rho_l g(15 + 15)$

where, ρ_0 is atmospheric pressure

$\therefore 0.9(h + 15) = \rho_l 30$

$\Rightarrow 0.9(h + 15) = 1.2 \times 30$

$\Rightarrow 3h = 75 \Rightarrow h = 25$ cm

$\therefore$ Difference in level of oil and liquid

= 25 – 15 = 10 cm

98. **(b)** Linear and areal expansion

$l_f = l_0(1 + \alpha\Delta T)$

and, $A_f = A_0(1 + \beta\Delta T)$ …(i)

Let l_0 and b_0 be length and width of rectangular sheet.

So, $A_f = l_0(1 + \alpha\Delta T)\, b_0(1 + \alpha\Delta T)$

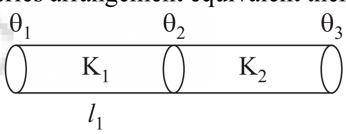
$= l_0 b_0 (1 + \alpha\Delta T)^2 = l_0 b_0 (1 + 2\alpha\Delta T)$

$A_f = A_0(1 + 2\alpha\Delta T)$ …(ii)

Comparing (i) and (ii), we get

$\beta = 2\alpha \Rightarrow \frac{\alpha}{\beta} = 0.5$

99. **(a)** In series arrangement equivalent thermal resistance,



$R_{eq} = R_1 + R_2$

or, $\frac{l_1 + l_2}{KA} = \frac{l_1}{K_1 A} + \frac{l_2}{K_2 A}$

or, $\frac{l_1 + l_2}{K} = \frac{l_1}{K_1} + \frac{l_2}{K_2}$

or, $K = \frac{(l_1 + l_2)K_1K_2}{K_2 l_1 + K_1 l_2}$

100. **(c)** The total work done by double stage carnot engine

$= W_1 + W_2 = (Q_1 - Q_2) + (Q_2 - Q_3)$

So, efficiency $\eta = \frac{W_1 + W_2}{Q_1}$

$\Rightarrow \eta = \frac{(Q_1 - Q_2) + (Q_2 - Q_3)}{Q_1} = \frac{Q_1 - Q_3}{Q_1}$

$\Rightarrow \eta = 1 - \frac{Q_3}{Q_1}$ …(i)

Now, $\frac{Q_1}{T_1}=\frac{Q_2}{T_2}=\frac{Q_3}{T_3}$ or, $\frac{Q_3}{Q_1}=\frac{T_3}{T_1}$ ...(ii)

From Eqs. (i) and (ii),

$\eta = 1-\frac{T_3}{T_1}$

As $T_1 = T$ and $T_3 = \beta T$

So, $\eta = 1-\frac{\beta T}{T}=1-\beta$

101. (c) In adiabatic process, pV^γ = constant

$\therefore p_2 = p_1\left(\frac{V_1}{V_2}\right)^\gamma = 2\times10^5\left(\frac{2}{0.5}\right)^{1.4}$

$= 2 \times 10^5 \times 4^{1.4} = 2 \times 10^5 \times 6.96 = 13.92 \times 10^5$ N/m^2

Therefore, work done in adiabatic condition,

$W = \frac{p_1V_1 - p_2V_2}{\gamma-1}$

$= \frac{2\times10^5\times2-13.92\times10^5\times0.5}{1.4-1} = -7.4 \times 10^5$ J

102. (c) Since, average kinetic energy of gases,

$KE_{av} = \frac{3}{2}K_BT \quad \therefore KE_{av} \propto T$

RMS speed of gas $v_{rms} = \sqrt{\frac{3RT}{M_O}}$,

$\because \quad v_{rms} \propto \frac{1}{\sqrt{M_O}}$ M_O = Molar mass

As M_O increases v_{rms} decreases.

103. (d) Given, wave equation, $y = 0.02 \sin(5\pi x - 20t)$ m

Since particle in same or opposite phase have same speed

And minimum distance

$= \frac{\text{wavelength }(\lambda)}{2}$

$\therefore \lambda = \frac{2\pi}{k}=\frac{2\pi}{5\pi}=0.4$ m $\quad (\because k = \frac{2\pi}{\lambda})$

$\therefore$ Minimum separation $= \frac{0.4}{2}=0.2$ m

104. (a) Separation between object and screen D = 90 cm and lens displacement, x = 20 cm

Since, $f = \frac{D^2-x^2}{4D}$

where, f is focal length

$\therefore f = \frac{90^2-20^2}{4\times90}=\frac{770}{36}=21.38$ cm

105. (a) For maxima, $d \sin\theta_n = n\lambda$

$\Rightarrow d\,\theta_n = n\lambda \Rightarrow \theta_n = \frac{n\lambda}{d}$

Then $\theta_{n+1}-\theta_n = \frac{\lambda}{d}$

So, $\Delta\theta = \frac{\lambda}{d}=\frac{400\times10^{-9}}{10^{-3}}=4\times10^{-4}$ rad

106. (d) Electric field due to dq = charge on arc length x

$E = \frac{kdq}{r^2}$ ...(i)

$\therefore dq = \frac{q}{2\pi r}x = \frac{10^{-5}}{2\pi\times10\times10^{-2}}\times314\times10^{-6} = \frac{1}{2}\times10^{-10}$ C

Putting this value of dq in eq. (i) we get

$E = \frac{9\times10^9\times\frac{1}{2}\times10^{-10}}{(10\times10^{-2})^2} = 4.5 \times 10^{9-10+2} = 45$ N/C

107 (a) From Gauss's law $\int E\cdot dS = \frac{q_{\text{enclosed}}}{\varepsilon_0}$

where, E = electric field,

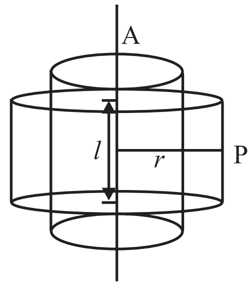
dS = surface area

ε_0 = free space permittivity

$\Rightarrow E\cdot 2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{1}{2\pi\varepsilon_0 r}$

where, λ = charge per unit length,

and r = distance from axis of cylinder.



$\therefore E \propto \frac{1}{r}$

108. (b) Using capacitance of spherical capacitor

$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

$\therefore C = 4\pi\varepsilon_0\left(\frac{R\cdot2R}{2R-R}\right)=4\pi\varepsilon_0\frac{2R^2}{R}$

or, $C = 8\pi\varepsilon_0 R$

109. (c) Force, $\vec{F} = I(\vec{l}\times\vec{B})$

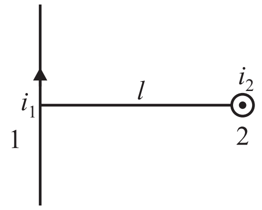
$\Rightarrow I_0Bl(\hat{k}-\hat{j}) = I(l\hat{i}\times\vec{B})$

where $\vec{B} = B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$

Solving, we get

$B_x = B_y = B_z = B_0 \Rightarrow \vec{B} = B_0(\hat{i}+\hat{j}+\hat{k})$

110. (a)



Magnetic field produced by first wire 1 on second wire 2.

$$B_1 = \frac{\mu_0}{2\pi} \cdot \frac{i_1}{l}$$

$\therefore$ Magnetic force on small length dl of the second wire 2,

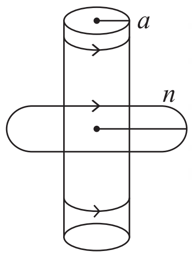
$$F = i_2 B_1 dl = i_2 \cdot \frac{\mu_0}{2\pi} \frac{i_1}{l} \cdot dl$$

$$\Rightarrow F = \frac{\mu_0 dl}{2\pi l} \cdot i_1 i_2 \quad \therefore F \propto i_1 i_2$$

111. (a) Magnetisation, $H = \frac{B}{\mu_0} = \frac{n\mu I}{\mu_0} = \frac{n\mu_r \mu_0 I}{\mu_0}$

$\Rightarrow H = n\mu_r I = 900 \times 501 \times 2.5 = 1.12 \times 10^6$ A/m

112. (a)



Since, $\oint E \cdot dl = +\frac{d\phi}{dt}$ $\left[\because |l| = \frac{d\phi}{dt}\right]$

$$E \cdot 2\pi r = \frac{B \cdot A}{t} = \frac{n\mu_0 I \cdot t \cdot \pi a^2}{t}$$

$$\therefore E = \frac{n\mu_0 I a^2}{2r}$$

113. (a) As $i = (2 \sin \omega t + 6 \cos \omega t)$A

$$i = \sqrt{2^2 + 6^2}\left[\frac{2}{\sqrt{2^2+6^2}} \sin \omega t + \frac{2}{\sqrt{2^2+6^2}} \cos \omega t\right]$$

$= \sqrt{40}\,[\cos\theta \sin \omega t + \sin\theta \cos \omega t]$,

where $\cos\theta = \frac{2}{\sqrt{40}}$ and $\sin\theta = \frac{6}{\sqrt{40}}$

$i = \sqrt{40} \sin(\omega t + \theta)$

$$\therefore i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{\sqrt{40}}{\sqrt{2}} = 2\sqrt{5}\text{A}$$

114. (c) Maximum rate of energy
$S = BE = 7.9 \times 300 = 2370$ W/m^2

115. (c) Using de-Broglie hypothesis

$$\lambda = \frac{h}{mv} \quad \therefore \lambda \propto \frac{1}{m}$$

Since, α-particle is the heavier most.
$\therefore$ λ of α-particle will be least.

116. (c) Angular momentum $L = \frac{nh}{2\pi}$

$$L_g = 1 \cdot \frac{h}{2\pi}$$

$$L_f = \frac{h}{2\pi} + \frac{h}{2\pi} = 2\frac{h}{2\pi}$$

So, final orbit level, $n_j = 2$

$$\Delta E = 13.6 \times 1^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 13.6 \times \frac{3}{4}$$

$= 3.4 \times 3 = 10.2$ eV

117. (b) Moderators are used to slow down the neutrons produced by fission reaction, to sustain control chain reaction.

118. (c) With increase of temperature, its (semiconductors) covalent bonds start breaking and its resistivity starts decreasing. So, conductivity increases and hence, number of electrons in conduction band increases.

119. (b) According to given circuit diagram, diode is in reverse bias, hence no current will flow through upper part of circuit.
So, A_1 will read zero current.

120. (a) Lower side band frequency, $f_1 = f_c - f_m$ ...(i)
and upper side band frequency, $f_2 = f_c + f_m$ ...(ii)
Adding eqs. (i) and (ii), we get

$$f_c = \frac{f_1 + f_2}{2}$$

And subtracting Eq. (ii) from (i), we get

$$f_1 - f_2 = -2f_m \Rightarrow f_m = \frac{f_1 - f_2}{-2}$$

$$\Rightarrow f_m = \frac{f_2 - f_1}{2} \quad \therefore \frac{f_c}{f_m} = \frac{f_1 + f_2}{f_2 - f_1}$$

CHEMISTRY

121. (d)

122. (d) According to de-Broglie,

$$\lambda = \frac{h}{mv}$$

If wavelength (λ) is equal to distance travelled by the electron in one second. i.e., $\lambda = v$
Put in Eq. (i), we get

$$\lambda = \frac{h}{m\lambda} \Rightarrow \lambda^2 = \frac{h}{m} \Rightarrow \lambda = \sqrt{\frac{h}{m}}$$

123. (d) Ga_2O_3, As_4O_{10}, Sb_4O_{10} and Al_2O_3 are amphoteric oxides because they react with acid as well as base.

124. (b) Option (c) has the highest ionisation energy $I{\cdot}E_1$ 1681 kJ mol^{-1} among the given value hence it cannot be the $I{\cdot}E_1$ of N. It is the $I{\cdot}E_1$ of 'F'.

Option (a) has the lowest $I \cdot E_1$ (1086 kJ mol^{-1}). Hence, it cannot be $I \cdot E_1$ of N.

If we compare (b) and (d) then (b) has higher $I \cdot E_1$ then (d) which can be compared with I·E of N and O. Since nitrogen has half filled stable.

Electronic configuration so, it must have higher $I \cdot E_1$ than that of oxygen. Hence, (b) is correct answer i.e., 1402 kJ mol^{-1} is $I \cdot E_1$ of nitrogen.

125. (d) If size of cation is same then size of anion will affect the polarisation.

Bigger the size of the anion, greater the polarisability, greater the covalent character.

Among the given ionic compound, the iodide anion has the largest size, it undergoes polarisation to larger extent. Hence, CaI_2 has the largest covalent character.

126. (b) Bond order (BO)

$$= \frac{\text{(Number of electrons in antibonding molecular orbital)} - \text{(Number of electrons in bonding molecular orbital)}}{2}$$

Here, antibonding molecular orbital electrons denoted by (*)

$O^+_2 = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2\ \sigma^*\ 2s^2 \sigma\ 2p_x^2\ (\pi 2p_y^2 = \pi 2p_z^2)\ (\pi^* 2p_y^1 = \pi^* 2p_z^0)$

$BO = \frac{1}{2}(10-5) = 2.5$

$O_2 = \sigma 1s^2 \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_x^2\ (\pi 2p_x^2 = \pi 2p_z^2)\ (\pi^* 2p_z^2 = \pi^* 2p_z^1)$

$BO = \frac{1}{2}(10-6) = 2$

$O^-_2 = \sigma 1s^2 \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_x^2\ (\pi 2p_x^2 = \pi 2p_y^2)\ (\pi^* 2p_y^2 = \pi^* 2p_z^1)$

$BO = \frac{1}{2}(10-7) = 1.5$

$O^{2-}_2 = \sigma 1s^2 \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_x^2\ (\pi 2p_x^2 = \pi 2p_y^2)\ (\pi^* 2p_y^2 = \pi^* 2p_z^2)$

$BO = \frac{1}{2}(10-8) = 1$ $\therefore$ Correct order of bond order is

$O^+_2 > O_2 > O^-_2 > O_2^{2-}$.

127. (d) Given, most probable velocity at 7200 K = RMS velocity of He gas at 300 K

$$\sqrt{\frac{2RT_1}{M_1}} = \sqrt{\frac{3RT_2}{M_2}} \Rightarrow \sqrt{\frac{2RT_1}{M_1}} = \sqrt{\frac{3RT_2}{4}}$$

($\therefore$ Atomic mass of He = 4 amu)

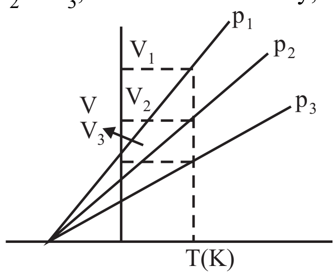
$$\sqrt{\frac{2 \times R \times 7200}{M_1}} = \sqrt{\frac{3 \times R \times 300}{4}} \Rightarrow \frac{2 \times 7200}{M_1} = \frac{900}{4}$$

$$M_1 = \frac{2 \times 7200 \times 4}{900} = 64 \text{ (Mass of } SO_2)$$

128. (a) If we keep the temperature constant

At a particular temperature, pV = constant

As $V_1 > V_2 > V_3$, therefore we can say,



Since, for ideal gas p is inversely proportional to volume V. Thus, p_1 will be less than p_2 is lesser than p_3.

So, $p_1 > p_2 > p_3$

129. (b) The given reaction is an acid-base reaction (neutralisation reaction)

$$\underset{\text{Calcium carbonate base}}{3CaCO_3} + \underset{\text{Phosphoric acid}}{2H_3PO_4} \longrightarrow \underset{\text{Calcium phosphate}}{Ca_3(PO_4)_2} + 3CO_2 + 3H_2O$$

Here, $a = 3$, $b = 2$, $p = 1$, $q = 3$, $r = 3$

Now ratio of $\frac{a}{b}$ is $\frac{3}{2}$.

130. (b) Molar mass of $Na_2SO_4 \cdot 10H_2O$

= 46 + 32 + 64 + 180 = 322 g/mol

$$\text{Moles} = \frac{\text{Mass of compound}}{\text{Molar mass of compound}} = \frac{32.2}{322} = 0.1 \text{ mol}$$

Number of oxygen in one mole of $Na_2SO_4 \cdot 10H_2O$ = 4 + 10 = 14

Now, moles of oxygen = 0.1 × 14 = 1.4

So, mass of oxygen = moles × molar mass

= 16 × 1.4 = 22.4 g

131. (a) The hydrogenation of benzene forms cyclohexane.

$$\underset{\text{Benzene}}{C_6H_6} + 3H_2 \longrightarrow \underset{\text{Cyclohexane}}{C_6H_{12}}$$

Enthalpy of hydrogenation of cyclohexane = – 119.5 kJ/mol

$\therefore$ Calculated enthalpy of benzene

= 3 × (–119.5) = – 358.5 kJ/mol

Resonance energy of benzene = – 150.4 kJ/mol

As we know,

Actual value of enthalpy

= Calculated enthalpy – Resonance energy

$\therefore$ Actual value of enthalpy = (–358.5) – (– 150.4)

= – 208.1 kJ/mol

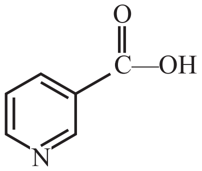
Hence, the enthalpy of hydrogenation of benzene is – 208.1 kJ/mol.

132. (d) A – (3), B – (4), C – (2) and D – (5).

Greater the value of $[H^+]$ ion, greater the value of ionisation constant, K_a and *vice-versa*.

Reaction, $HA \rightarrow H^+ + A^-$

$$K_a = \frac{[H^+][A^-]}{[HA]} \Rightarrow K_a \propto [H^+]$$

$H_2C_2O_4$ > Niacine  > H_2S > HCN

Value of K_a 5.6×10^{-2} 1.5×10^{-5} 8.9×10^{-8} 4.9×10^{-10}

133. **(a)** If [Conjugate acid] = [Base]

According to Handerson Hassel balch equation for buffer solution.

$$pOH = pK_b + \log \frac{[\text{Conjugate acid}]}{[\text{Base}]}$$

Then, [$pOH = pK_b$]

So, pOH of the buffer solution is same as pK_b of acid.

134. **(c)** Alkenes combine with CO and H_2 in presence of catalyst to form aldehyde which can be further reduced to 1° alcohols by hydrogen.

$$R-CH=CH_2 + H_2 + CO \xrightarrow[{[Co_2(CO)_8]}]{\text{Catalyst}} \underset{\substack{\text{Butanal} \\ \text{(intermediate)}}}{RCH_2CH_2CHO}$$

$$\xrightarrow{H_2} \underset{\substack{\text{(Butanol)} \\ 1° \text{ alcohol}}}{RCH_2CH_2CH_2OH}$$

135. **(a)** As we move down the group, the atomic and ionic size of alkali metals increases. As the size increases, the extent of hydration decreases in aqueous solution.

The electrical conductivity or ionic mobility is inversely proportional to extent of hydration.

So, extent of hydration $Li^+ > Na^+ > K^+ > Cs^+$.

Hence, the increasing order of electrical conductivity of alkali metals ions in their aqueous solution is

$Cs^+ > K^+ > Na^+ > Li^+$

136. **(a)** Among B, Al, Ga and In, Boron atom (B) is smallest in size. It can not expand its valency or coordination more than 4 due to absence of *d*-orbital.

137. **(d)** (a) Isotope of carbon, i.e. ^{13}C is a natural, stable isotope of carbon with a nucleus containing six protons and seven neutrons. It make up about 1.1% of all natural carbon on Earth.

(b) On moving from top to bottom in group IV A, the melting point decreases. The decreasing order of melting point

$$\underset{4373\,K}{C} > \underset{1693\,K}{Si} > \underset{1218\,K}{Ge} > \underset{505\,K}{Pb} > \underset{600\,K}{Sn}$$

(c) On Earth, oxygen is most common element making up about 47% of Earth's mass. Silicon is second making up 28% followed by aluminium (8%), iron (5%) etc.

(d) The resistivity of semiconductors depends strongly on the presence of impurities in the material.

Carbon resistivity (Ωm) = $3 \times 10^5 - 60 \times 10^{-5}$

Hence, all the statements are correct.

138. **(b)** $Cl_2C{=}CCl_2$, CO_2(liquid), H_2O_2

But acetaldehyde is not used for dry purpose due to its decomposition property.

139. **(a)** Hepta-1, 3-dien-5-yne (IUPAC name)

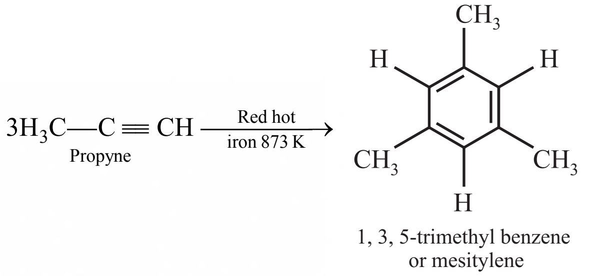
Structure:

$$\underset{sp^3}{H_3C}-\underset{sp}{C}\equiv\underset{sp}{C}-\overset{sp^2}{\underset{H}{C}}=\overset{sp^2}{\underset{H}{C}}-\underset{sp^2}{C}=\underset{sp^2}{CH_2}$$

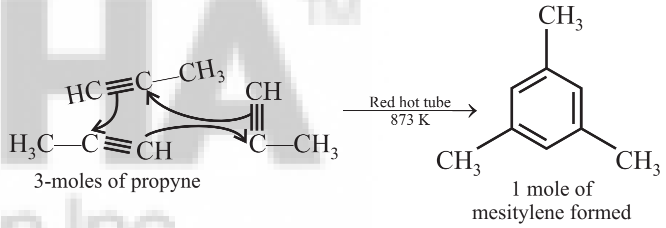
There are 2*sp* carbons and 4sp^2 carbons present in hepta-1, 3-dien-5-yne structure.

140. **(b)** Propyne on the reaction in red hot iron tube at 873 K, gives 1, 3, 5 trimethyl benzene or mesitylene.

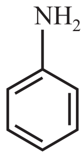
Reaction is as follows:



Mechanism:

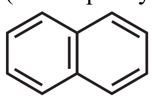


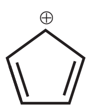
141. **(b)** Aromatic compounds are compounds that consist of conjugated planar ring system, delocalised π-electron cloud in place of individual alternating double or single bond compound which follow Huckel's rule or $(4n + 2)\pi$ rule are called aromatic compound.



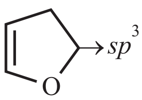
→ $(4n + 2)\pi$ rule follow → Aromatic

(due to phenyl group attached to an amino group)

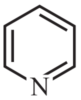
 → $(4n+2)\pi$ rule follow → $10e^-$ (Aromatic)



→ $4n\pi$ rule → $4e^-$ (Non-aromatic or anti aromatic)



→ Non-aromatic (due to sp^3 carbon).



$\rightarrow (4n + 2)\pi$ rule follow (Aromatic) due to $6e^-$

Hence, total number of aromatic compound is 3.

142. (c) Effective number of atoms in HCP (n) unit cell

$= \frac{1}{6} \times 12 + \frac{1}{2} \times 2 + 1 \times 3 = 2 + 1 + 3 = 6$

Number of tetrahedral voids = $2n = 2 \times 6 = 12$

Number of atoms of A = $\frac{2}{3} \times 12 = 8.$

So, formula of the compound is A_8B_6 or A_4B_3.

143. (c) As we know, the relation between molarity (M) and molality (m).

here, m = molality

molarity = 2 (given)

density of solution in g/mL (d) = 1.11 g/mL

Molar mass of ethylene glycol $\Rightarrow$ 62 g/mol

$\because \quad m = \frac{1000 \times M}{(1000 \times d) - (M \times \text{molar mass})}$

$m = \frac{1000 \times 2}{(1000 \times 1.11) - (2 \times 62)}$ g/mol

$= \frac{2000}{1110 - 124} = \frac{2000}{976} = 2.05$

Hence, molality of the solution is 2.05m.

144. (b) Both the solutions i.e. A and B, shows negative deviation.

In case of negative deviations from Raoult's law, A-B forces are stronger than A-A and B-B forces.

In phenol and aniline amount of H-bonding increase due to increase in number of hydrogen atoms of aniline.

Similarly solution of chloroform and acetone have an attractive interaction between them which results in the formation of hydrogen bonding. That's why they show negative deviation from Raoult's law.

145. (c) According to Faraday' s law,

$W = \frac{E \times Q}{96500} \qquad \left(E = \frac{\text{Molarmass}}{\text{Valence}}\right)$

Here, W = weight of substance discharged at an electrode.

Q = Electricity Charge (Coulomb)

Let the valence of A, B and C are a, b and c respectively.

For A, $2.4 = \frac{8 \times Q}{a \times 96500}$

For B, $1.8 = \frac{18 \times Q}{b \times 96500}$

For C, $7.5 = \frac{50 \times Q}{c \times 96500}$

For ratio of A, B and C, we get

$a : b : c = \left(\frac{8}{2.4}\right) : \left(\frac{18}{1.8}\right) : \left(\frac{50}{7.5}\right)$

$= 3.33 : 10 : 6.66 = 1 : 3 : 2$

By solving equation

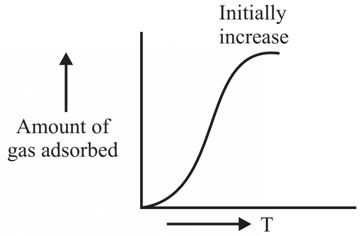
$a = 1, b = 3, c = 2$

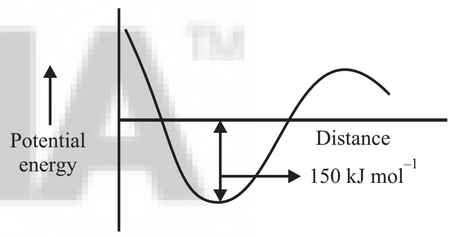
valence of A, B and C are 1, 3 and 2.

146. (a) The overall order of the reaction is = $\frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$

Given rate expression is for second order reaction.

147. (c) Chemisorption requires high activation energy. So, it is referred to as activated adsorption. In chemisorption, adsorption first increases and then decreases with an increase in temperature. The initial increase is due to heat supplied, which act as activation energy required in chemisorption.



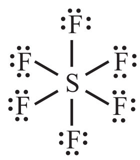


148. (c) Copper is oxidised by concentrated nitric acid to produce $Cu(NO_3)_2$ ions. The nitric acid is reduced to form nitrogen dioxide (NO_2) which is a poisonous brown gas with an irritating odour.

$$\underset{\text{Metal}}{Cu(s)} + \underset{\text{Conc. nitric acid}}{4HNO_3(aq)} \longrightarrow Cu(NO_3)_2(aq) + \underset{\text{Nitrogen dioxide}}{2NO_2(g)} + \underset{\text{Water}}{2H_2O(l)}$$

149. (b) SF_6 is extremely stale and chemically inert gas for stearic reasons. In SF_6, the six F atoms protect the sulphur atoms from attack by reagent to such an extent that even thermodynamically most favourable reactions like hydrolysis do not occur. S is completely blocked by fluorine atoms from all directions.

SF_6 is a gas comprising of one sulphur and 6 fluoride atoms.



150. (c) The complete hydrolysis of XeF_4 and XeF_6 produces XeO_3 which is xenon trioxide. This xenon trioxide is highly explosive and acts as a powerful oxidising agent in solution.

Complete hydrolysis of XeF_4

$$\underset{\text{Xenon tetrafluoride}}{6XeF_4} + 12H_2O \longrightarrow \underset{\text{Xenon}}{4Xe} + \underset{\text{Xenon trioxide (P)}}{2XeO_3} + \underset{\text{Hydrogen fluoride}}{24HF} + 3O_2$$

XeF_4 on partial hydrolysis produces $XeOF_2$ (xenon oxyfluoride).

Complete hydrolysis of XeF_6

$$\underset{\text{Xenon hexafluoride}}{XeF_6} + 3H_2O \longrightarrow \underset{\text{Xenon trioxide (Q)}}{XeO_3} + \underset{\text{Hydrogen fluoride}}{6HF}$$

151. (b) (A) If we add ferric chloride over potassium ferrocyanide, it forms a white precipitate that will become blue.

The reaction is:

$$\underset{\text{Ferric chloride}}{4FeCl_3} + \underset{\text{Potassium ferrocyanide}}{3K_4[Fe(CN)_6]} \longrightarrow \underset{\text{White precipitate}}{Fe_4[Fe(CN)_6]_3} + \underset{\text{Potassium chloride}}{3KCl}$$

$$\underset{\text{Zinc chloride}}{ZnCl_2} \xrightarrow{NaOH} \underset{\text{White ppt.}}{Zn(OH)_2} \xrightarrow[NaOH]{Excess} \underset{\text{ppt. dissolved (V)}}{Na_2ZnO_2}$$

(C) Yellow coloured ferric chloride reacts with hydrogen sulphide to form green coloured ferrous chloride.

In this reaction, ferric chloride is reduced and H_2S is oxidised.

$$\underset{\text{Ferric chloride}}{2FeCl_3} + \underset{\text{Hydrogen sulphide}}{H_2S} \longrightarrow \underset{\text{Ferrous chloride (green) (II)}}{2FeCl_2} + \underset{\text{Hydrochloric acid}}{2HCl} + S$$

152. (c) $[Pt(NH_3)_2Cl(NO_2)]$

Let x be the oxidation state of platinum.

$x + 2(0) + (-1) + (-1) = 0$

$x - 2 = 0 \Rightarrow x = +2$

$[CoCl_2(en)_2]^+$

Let y be the oxidation state of cobalt.

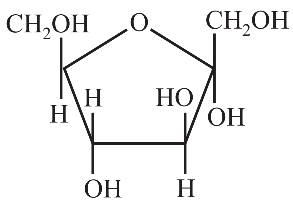
$y + 2(-1) + 0 = +1$

$y - 2 = +1 \Rightarrow y = +1 + 2 \Rightarrow y = +3$

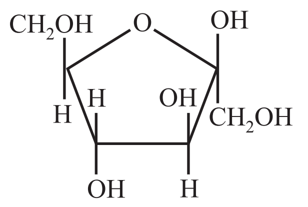
So, oxidation number of central metal in $[Pt(NH_3)_2Cl(NO_2)]$ is +2 and in $[CoCl_2(en)_2]^+$ is +3.

153. (c) α-D-fructofuranose: It's a structure is analogous to the cyclic structure called as furan, which is a 5 membered ring. It is an enantiomer of an α-L-fructofuranose.

It is non-reducing sugar.



α-D-fructofuranose



β-D-fructofuranose

154. (a) $CH_3—CH_2—Br + Nu^{\ominus} \longrightarrow CH_3—CH_2—Nu + Br^{\ominus}$

Above reaction is known as nucleophilic substitution reaction. Bimolecular nucleophilic substitution reaction follows second-order kinetics. The rate of reaction depends on the concentration of two first-order reactants.

Hydroxide ion (OH^-) is stronger nucleophilie than acetate ion as in acetate ion, the negative charge is involved in resonance with carboxylic group.

Methoxide ion (CH_3O^-) is more nucleophilic than hydroxide ion (OH^-) due to +I effect of methyl group.

$PHO^{\ominus}$ is very weak nucleophile because the negative charge is in resonance with ring and charge further decreases the order is

$$\underset{(iv)}{CH_3O^{\ominus}} > \underset{(iii)}{OH^{\ominus}} > \underset{(i)}{PhO^{\ominus}} > \underset{(ii)}{CH_3COO^{\ominus}}$$

155. (b)

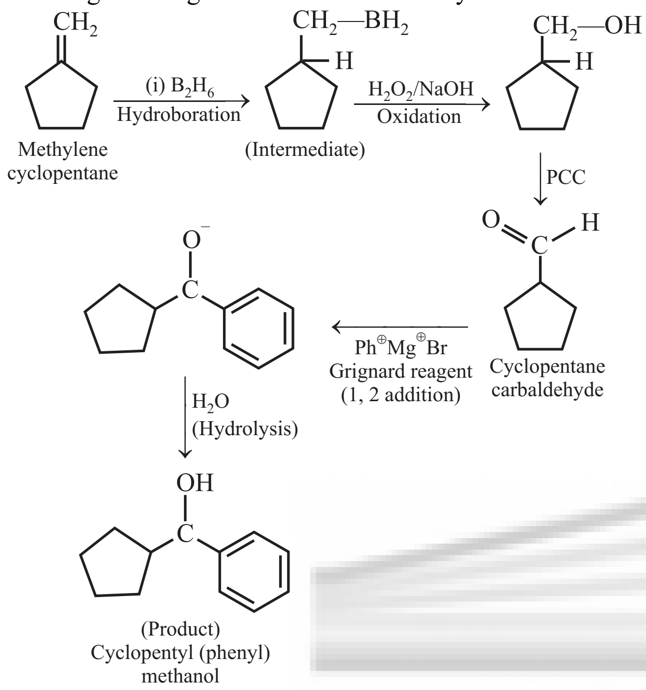
$$\underset{\text{Ethanol}}{CH_3CH_2OH} \xrightarrow{PBr_3} \underset{\text{Bromoethane (X)}}{CH_3CH_2\text{–}Br} \xrightarrow[\text{Dehydro halogenation}]{\text{Alc. KOH}} \underset{\text{Ethene (Y)}}{CH_2{=}CH_2} \xrightarrow{(i)\ H_2SO_4} CH_3CH_2—HSO_4 \xrightarrow{(ii)\ H_2O} \underset{\text{Ethanol (Z)}}{CH_3CH_2OH}$$

In first step, nucleophilic substitution reaction occur to form bromoethane (X). In next step, dehydrohalogenation reaction gives alkene i.e. ethene (Y). In final step, hydrolysis will be occur to form ethanol (Z).

156. (c) Phenol when treated with zinc dust, undergo reduction to form benzene. Only phenol gives deoxygenation reaction by using zinc with supply of heat.

∴ (B) o-cresol and (D) 2-hydroxy benzoic acid give deoxygenaration reaction as they contain aromatic alcohol group (—OH).

157. (c) In given reaction, first two step is hydroboration-oxidation reaction which form alcohol. PCC is used for oxidation of alcohol into aldehyde. PhMgBr is used as a Grignard reagent to attack in a carbonyl carbon.



158. (a) A – (3), B – (4), C – (1), D – (2)

159. (a) A – (4), B – (1), C – (3), D – (2)

Electron donating group attached to the carboxylic group increasing the value of pK_a. More electron negative atoms having least pK_a value. Smaller the pK_a value, stronger the acid.

Acidic order

F_3C—COOH > $NO_2C_6H_5COOH$ > C_6H_5COOH > CH_3COOH

(B) (D) (C) (A)

Order of pK_a value

(A) > (C) > (D) > (B)

160. (d) Given reaction is Hofmann bromide degradation reaction.

An amide reacts with bromine and aqueous solution of sodium hydroxide which produces primary amine. This is a degradation reaction as primary amine in the product has one carbon lesser than primary amide (in reactant).

$$\underset{\text{Pentanamide}}{CH_3CH_2CH_2CH_2CONH_2} + Br_2 + \underset{\text{Sodium hydroxide}}{4NaOH}$$

$$\rightarrow \underset{\text{Butan-1-amine}}{CH_3CH_2CH_2CH_2NH_2} + \underset{\text{Sodium carbonate}}{Na_2CO_3} + \underset{\text{Sodium bromide}}{2NaBr} + 2H_2O$$