All India 2020

CBSE Board Solved Paper

Time Allowed : 3 Hours *Maximum Marks : 80*

General Instructions:

(i) This question paper comprises **four** sections – **A, B, C** and **D.**
This question paper carries **40** questions. **All** questions are compulsory.

(ii) **Section A** – Question no. **1** to **20** comprises of **20** questions of **one** mark each.

(iii) **Section B** – Question no. 2**1** to **26** comprises of **6** questions of **two** marks each.

(iv) **Section C** – Question no. 2**7** to **34** comprises of **8** questions of **three** marks each.

(v) **Section D** – Question no. **35** to **40** comprises of **6** questions of **four** marks each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions of **one** mark, **2** questions of **two** marks, **3** questions of **three** marks and **3** questions of **four** marks. You have to attempt only **one** of the choices in such questions.

(vii) In addition to this separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is not permitted.

SECTION - A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option.

1. The sum of exponents of prime factors in the prime-factorisation of 196 is

(a) 3 (b) 4

(c) 5 (d) 2

2. Euclid's division Lemma states that for two positive integers a and b, there exists unique integer q and r satisfying $a = bq + r$, and

(a) $0 < r < b$ (b) $0 < r \leq b$

(c) $0 \leq r < b$ (d) $0 \leq r \leq b$

3. The zeroes of the polynomial $x^2 - 3x - m(m + 3)$ are

(a) $m, m + 3$ (b) $-m, m + 3$

(c) $m, -(m + 3)$ (d) $-m, -(m + 3)$

4. The value of k for which the system of linear equations $x + 2y = 3$, $5x + ky + 7 = 0$ is inconsistent is

(a) $-\frac{14}{3}$ (b) $\frac{2}{5}$

(c) 5 (d) 10

5. The roots of the quadratic equation $x^2 - 0.04 = 0$ are

(a) ± 0.2 (b) ± 0.02

(c) 0.4 (d) 2

6. The common difference of the A.P. $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p} A$, ... is

(a) 1 (b) $\frac{1}{p}$

(c) –1 (d) $-\frac{1}{p}$

7. The n^{th} term of the A.P. a, $3a$, $5a$, ..., is

(a) na (b) $(2n - 1)a$

(c) $(2n + 1)a$ (d) $2na$

8. The point P on x-axis equidistant from the points A(–1, 0) and B(5, 0) is

(a) (2, 0) (b) (0, 2)

(c) (3, 0) (d) (2, 2)

9. The co-ordinates of the point which is reflection of point (–3, 5) in x-axis are

(a) (3, 5) (b) (3, –5)

(c) (–3, –5) (d) (–3, 5)

10. If the point P(6, 2) divides the line segment joining A(6, 5) and B(4, y) in the ratio 3 : 1, then the value of y is

(a) 4 (b) 3

(c) 2 (d) 1

In Question numbers 11 to 15, fill in the blanks. Each question is of 1 mark.

11. In fig. 1, MN || BC and AM : MB = 1 : 2, then

$\frac{ar(\Delta AMN)}{ar(\Delta ABC)} =$ ____________

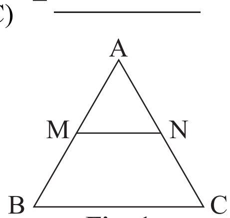


Fig.-1

12. In given Fig. 2, the length PB = __________ cm.

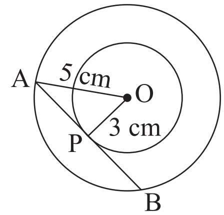


Fig.-2

13. In ΔABC, AB = $6\sqrt{3}$ m, AC = 12 cm and BC = 6 cm, then ∠B = __________.

OR

Two triangles are similar if their corresponding sides are __________.

14. The value of (tan 1° tan 2° .... tan 89°) is equal to _____________.

15. In fig. 3, the angles of depressions from the observing positions O_1 and O_2 respectively of the object A are __________, __________.

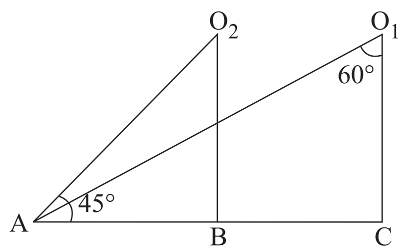


Fig.-3

Question numbers 16 to 20 are short answer type questions of 1 mark each.

16. If sin A + $\sin^2$A = 1, then find the value of the expression ($\cos^2$A + $\cos^4$A).

17. In fig.4 is a sector of circle of radius 10.5 cm. Find the perimeter of the sector. $\left(\text{Take } \pi = \frac{22}{7}\right)$

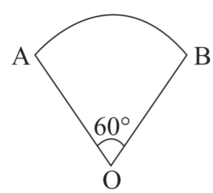


Fig.-4

18. If a number x is chosen at random from the numbers –3, –2, –1, 0, 1, 2, 3, then find the probability of $x^2 < 4$.

OR

What is the probability that a randomly taken leap year has 52 Sundays?

19. Find the class-marks of the classes 10-25 and 35-55.

20. A die is thrown once. What is the probability of getting a prime number.

SECTION - B

Question numbers 21 to 26 carry 2 marks each.

21. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students :

$2x + 3$, $3x^2 + 7x + 2$, $4x^3 + 3x^2 + 2$, $x^3 + \sqrt{3x} + 7$, $7x + \sqrt{7}$, $5x^3 - 7x + 2$, $2x^2 + 3 - \frac{5}{x}$, $5x - \frac{1}{2}$, $ax^3 + bx^2 + cx + d$, $x + \frac{1}{x}$

Answer the following questions:

(i) How many of the above ten, are not polynomials ?

(ii) How many of the above ten, are quadratic polynomials ?

22. In fig. 5, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

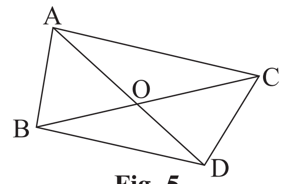


Fig.-5

OR

In fig. 6, if AD ⊥ BC, then prove that $AB^2 + CD^2 = BD^2 + AC^2$

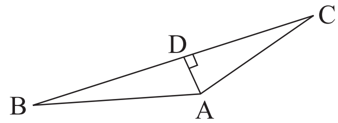


Fig.-6

23. Prove that $1 + \frac{\cot^2 \alpha}{1 + \text{cosec}\, \alpha} = \text{cosec}\, \alpha$

OR

Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

24. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. (Use $\pi = \frac{22}{7}$)

25. A child has a die whose six faces show the letters as shown below :

A	B	C	D	E	A

The die is thrown once. What is the probability of getting (i) A, (ii) D ?

26. Compute the mode for the following frequency distribution :

Size of items (in cm)	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24	24 – 28
Frequency	5	7	9	17	12	10	6

SECTION - C

Question numbers 27 to 34 carry 3 marks each.

27. If $2x + y = 23$ and $4x - y = 19$, find the value of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$.

OR

Solve for x : $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, x # –4, 7.

28. Show that the sum of all terms of an A.P. whose first term is a, the second term is b and the last term is c is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$.

OR

Solve the equation :

$1 + 4 + 7 + 10 + ... + x = 287$.

29. In a flight of 600 km, an aircraf was slowed down due to bad weather. The average speed of the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight.

30. If the mid-point of the line segment joining the points A(3, 4) and B(k, 6) is P(x, y) and $x + y - 10 = 0$, find the value of k.

OR

Find the area of triangle ABC with A(1, –4) and the mid-points of sides through A being (2, –1) and (0, –1).

31. In Fig. 7, if ΔABC ~ ΔDEF and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle.

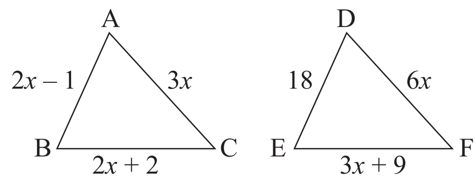


Fig.-7

32. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that

$$AQ = \frac{1}{2}(BC + CA + AB)$$

33. If $\sin\theta + \cos\theta = \sqrt{2}$, prove that $\tan\theta + \cot\theta = 2$.

34. The area of a circular play ground is 22176 cm^2. Find the cost of fencing this ground at the rate of ₹50 per metre.

SECTION - D

Question numbers 35 to 40 carry 4 marks each.

35. Prove that $\sqrt{5}$ is an irrational number.

36. It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter sused for four hours and the pipe of smaller diameter for 9 hous, only half of the pool can be filled. How long would it take for each pire to fill the pool separately ?

37. Draw a circle of radius 2 cm with centre O and take a point P outside the circle such that OP = 6.5 cm. From P, draw two tangents to the circle.

OR

Construct a triangle with sides 5 cm, 6 cm and 7 cm and then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the first triangle.

38. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

39. Find the area of the shaded region in fig. 8, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

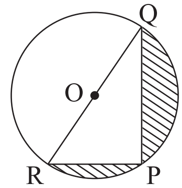


Fig.-8

OR

Find the curved surface area of the frustum of a cone, the diameters of whose circular ends are 20 m and 6 m and its height is 24 m.

40. The mean of the following frequency distribution is 18. The frequency *f* in the class interval 19 – 21 is missing. Determine *f*.

Class interval	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Frequency	3	6	9	13	f	5	4

OR

The following table gives production yield per hectare of wheat of 100 farms of a village :

Production yield	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70
No. of farms	4	6	16	20	30	24

Change the distribution to a 'more than' type distribution and draw its ogive.

Solutions

SECTION - A

1. **(b)** $196 = 2^2 \cdot 7^2$, sum of exponents = 2 + 2 = 4 **(1 Mark)**

2. **(c)** $0 \le r < b$ **(1 Mark)**

3. **(b)** $x^2 - (m+3)x + mx - m(m+3) = 0$

$\Rightarrow x[x - (m+3)] + m[x - (m+3)] = 0$

$\Rightarrow (x+m)[x - (m+3)] = 0$

$\therefore x + m = 0$ | $x - (m+3) = 0$

$x = -m$ | $x = m + 3$

(1 Mark)

4. **(d)** $\frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7}$ $\left[\because \text{For inconsistent } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\right]$

$\Rightarrow k = 10$ **(1 Mark)**

5. **(a)** $x^2 - 0.04 = 0$

$\Rightarrow x^2 = 0.04$

$\Rightarrow x = \pm 0.2$ **(1 Mark)**

6. **(c)** $d = \frac{1-p}{p} - \frac{1}{p} = \frac{1-p-1}{p} = \frac{-p}{p} = -1$

(1 Mark)

7. **(b)** $a_n = a + (n-1)d = a + (n-1)2a$

$[\because d = 3a - a = 2a]$

$= a + 2an - 2a = 2an - a = (2n-1)a$

(1 Mark)

8. **(a)** $P(x, 0) = \left(\frac{5-1}{2}, 0\right) = (2, 0)$

[$\because$ A and B both lies on x-axis] **(1 Mark)**

> ***Note***
>
> *Three or more points lies in same line are called collinear.*

9. **(c)**

> ***Note***
>
> *For reflection of a point with respect to x-axis change sign of y-coordinate and with respect to y-axis change sign of x-coordinate.*

(1 Mark)

10. **(d)** $P(6, 2) = \left(\frac{4\times 3 + 1\times 6}{3+1}, \frac{3\times y + 1\times 5}{3+1}\right)$

$\because 6 \neq \frac{18}{4}$ (Question is wrong)

$2 = \frac{3y+5}{4} \Rightarrow 3y + 5 = 8$

$3y = 3 \Rightarrow y = 1$ **(1 Mark)**

11. $\left[\frac{\mathbf{1}}{\mathbf{9}}\right]$ $\frac{AM}{AB} = \frac{AM}{AM+BM} = \frac{1}{1+2} = \frac{1}{3}$

$\therefore \frac{ar(\Delta AMN)}{ar(\Delta ABC)} = \left(\frac{AM}{AB}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

(1 Mark)

12. **[4 cm]**

$PB = AP = \sqrt{5^2 - 3^2}$ $(\because OP \perp AB)$

$= \sqrt{25-9} = 4$ cm **(1 Mark)**

13. **[90°]**

$\because AB^2 + BC^2 = 108 + 36 = 144 = AC^2$

So, AC is hypotenuse and $\angle B = 90°$.

(1 Mark)

OR

[Proportional] **(1 Mark)**

14. **[1]** $\tan 1° \cdot \tan 2° \ldots \tan 45° \ldots \tan 89°$

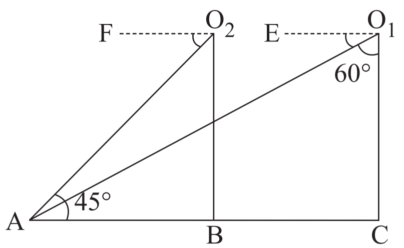
$= \cot(90° - 1°) \cdot \cot(90 - 2°) \ldots \tan 45° \ldots \tan 88° \cdot \tan 89°$

$= \cot 89° \cdot \cot 88° .... \tan 45° .... \tan 88° \cdot \tan 89°$

$= 1 \times 1 \times 1 ..... \times 1$ $[\because \tan\theta \cdot \cot\theta = 1]$

$= 1$ **(1 Mark)**

15. [30°, 45°]



(1 Mark)

Depression angle at $O_1 = 90° - 60° = 30°$

Depression angle at $O_2 = 45° = \angle BAO_2$ **(1 Mark)**

16. $\sin A + \sin^2 A = 1$ ...(i)

$\sin A = 1 - \sin^2 A = \cos^2 A$

$\therefore$ $\cos^2 A + \cos^4 A = \cos^2 A + (\cos^2 A)^2$

$= \sin A + \sin^2 A = 1$ [From (i)]

(1 Mark)

17. Length of arc AB $= \frac{60°}{360°} \times 2\pi \times 10.5$

$= \frac{1}{6} \times 2 \times \frac{22}{7} \times \frac{105}{10} = 11$ cm

Perimeter of the sector = Length of arc AB + $2r$

$= 11 + 2 \times 10.5 = 11 + 21 = 32$ cm **(1 Mark)**

18. Total numbers = 7

$\because$ $x^2 < 4 \Rightarrow -2 < x < 2$

i.e. $x = -1, 0, 1$

$\therefore$ Number of favourable outcomes = 3

$P(x^2 < 4) = \frac{3}{7}$ **(1 Mark)**

OR

Total number of days in leap year = 366

So, a leap year with have 52 weeks and other two days can be, (Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun).

Total outcomes = 7

Number of out comes (Not Sunday) = 5

P(Exactly 52 Sundays) $= \frac{5}{7}$ **(1 Mark)**

19. Class Marks $= \frac{U + L}{2}$

Class Marks of (10 – 25) $= \frac{10 + 25}{2} = 17.5$

Class Marks of (35 – 55) $= \frac{35 + 55}{2} = 45$

(1 Mark)

20. A die is thrown once

$\therefore$ Number of total outcomes = 6

Prime number = 2, 3, 5

$\therefore$ P(Prime number) $= \frac{3}{6} = \frac{1}{2}$ **(1 Mark)**

SECTION - B

21.

Polynomials	**Not polynomials**	**Quadratic polynomials**
$2x + 3$ $3x^2 + 7x + 2$ $4x^3 + 3x^2 + 2$ $7x + \sqrt{7}$ $5x^3 - 7x + 2$ $5x - \frac{1}{2}$ $ax^3 + bx^2 + cx + d$	$x^3 + \sqrt{3x} + 7$ $2x^2 + 3 - \frac{5}{x}$ $x + \frac{1}{x}$	$3x^2 + 7x + 2$

(1 Mark)

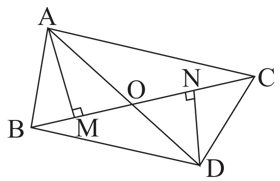
(i) Number of not polynomials = 3

(ii) Number of quadratic polynomials = 1

(1 Mark)

22. Construction: Draw AM ⊥ BC and DN ⊥ BC.

Proof: In ΔAMO and ΔDNO



$\angle AMO = \angle DNO = 90°$ (By construction)

$\angle AOM = \angle DON$ (V.O.A)

$\therefore$ By AA-similarity

$\Delta AMO \sim \Delta DNO$

$$\therefore \quad \frac{OA}{OD} = \frac{AM}{DN} \quad ...(i) \quad \textbf{(1 Mark)}$$

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN} = \frac{AM}{DN}$$

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{OA}{OD} \quad \text{[From (i)]} \quad \textbf{(1 Mark)}$$

Hence, proved.

OR

Given: In ΔABC, $AD \perp BC$

To prove: $AB^2 + CD^2 = BD^2 + AC^2$

Proof: In right ΔABD

$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = AB^2 - BD^2$ ...(i) **(½ Mark)**

In right ΔADC

$AC^2 = AD^2 + CD^2$

$\Rightarrow AD^2 = AC^2 - CD^2$ ...(ii) **(½ Mark)**

From (i) and (ii)

$AB^2 - BD^2 = AC^2 - CD^2$

$AB^2 + CD^2 = AC^2 + BD^2$ **(1 Mark)**

Hence, proved.

23. $$\text{L.H.S.} = 1 + \frac{\cot^2 \alpha}{1 + \text{cosec } \alpha} = 1 + \frac{\text{cosec}^2 \alpha - 1}{\text{cosec } \alpha + 1}$$

(1 Mark)

$$= 1 + \frac{(\text{cosec } \alpha + 1)(\text{cosec } \alpha - 1)}{\text{cosec } \alpha + 1}$$

$= 1 + \text{cosec } \alpha - 1 = \text{cosec } \alpha$ = R.H.S. **(1 Mark)**

Hence, proved.

OR

L.H.S.$= \tan^4\theta + \tan^2\theta = (\tan^2\theta)^2 + \tan^2\theta$

$= (\sec^2\theta - 1)^2 + \sec^2\theta - 1$ **(1 Mark)**

$= \sec^4\theta - 2\sec^2\theta + 1 + \sec^2\theta - 1$

$= \sec^4\theta - \sec^2\theta$ = R.H.S. **(1 Mark)**

Hence, proved.

24. Since height of cylinder = radius (i.e., $h = r$)

Volume $= \pi r^2 h$ **(1 Mark)**

$$25\frac{1}{7} = \frac{22}{7} \times h^2 \times h$$

$$\frac{176}{7} = \frac{22}{7} \times h^3$$

$$h^3 = \frac{176}{7} \times \frac{7}{22} = 8$$

$h = 2$ cm **(1 Mark)**

25. Total outcomes (No. of faces) = 6

(i) Number of A = 2

$$\therefore \quad P(A) = \frac{\text{No. of favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{2}{6} = \frac{1}{3} \quad \textbf{(1 Mark)}$$

(ii) Number of D = 1

$$\therefore \quad P(D) = \frac{\text{No. of favourable outcomes}}{\text{Total outcomes}} = \frac{1}{6}$$

(1 Mark)

Note

The outcome that we are looking in an experiment is a favourable outcome.

26. Maximum frequency = 17

$\therefore$ Model class = 12 – 16

$\therefore$ $L = 12$, $F_0 = 9$, $F_1 = 17$, $F_2 = 12$ and $h = 4$

$$\text{Mode} = L + \frac{F_1 - F_0}{2F_1 - F_0 - F_2} \times h \quad \textbf{(1 Mark)}$$

$$= 12 + \frac{17 - 9}{2 \times 17 - 12 - 9} \times 4$$

$= 12 + \dfrac{32}{34-21} = 12 + \dfrac{32}{13}$

$= 12 + 2.46 = 14.46$ **(1 Mark)**

SECTION - C

27. $2x + y = 23$ ...(i)

$4x - y = 19$ ...(ii)

Adding (i) and (ii)

$$\begin{array}{l} 2x + y = 23 \\ 4x - y = 19 \\ \hline 6x \quad = 42 \end{array}$$

$x = \dfrac{42}{6} = 7$ **(1 Mark)**

putting $x = 7$ in (i) we get

$2(7) + y = 23$

$y = 23 - 14 = 9$ **(1 Mark)**

$\therefore \quad 5y - 2x = 5(9) - 2(7) = 45 - 14 = 31$

and $\dfrac{y}{x} - 2 = \dfrac{9}{7} - 2 = \dfrac{9-14}{7} = \dfrac{-5}{7}$ **(1 Mark)**

Note

Mostly use ellimination method to solve two linear equations of two variables.

OR

$\dfrac{1}{x+4} - \dfrac{1}{x-7} = \dfrac{11}{30}$

$\Rightarrow \quad \dfrac{(x-7)-(x+4)}{(x+4)(x-7)} = \dfrac{11}{30}$

$\Rightarrow \quad \dfrac{x-7-x-4}{x^2-3x-28} = \dfrac{11}{30}$ **(1 Mark)**

$\Rightarrow \quad \dfrac{-11}{x^2-3x-28} = \dfrac{11}{30}$

$\Rightarrow \quad x^2 - 3x - 28 = -30$

$\Rightarrow \quad x^2 - 3x + 2 = 0$

$\Rightarrow \quad x^2 - 2x - x + 2 = 0$ **(1 Mark)**

$\Rightarrow \quad x(x-2) - 1(x-2) = 0$

$\Rightarrow \quad (x-1)(x-2) = 0$

$\Rightarrow \quad x - 1 = 0 \quad | \quad x - 2 = 0$

$x = 1 \quad | \quad x = 2$ **(1 Mark)**

28. First term $= a$

Second term $= b$

Last term $(a_n) = c$

$d = b - a$

$\because \quad a_n = a + (n-1)d$ **(1 Mark)**

$c = a + (n-1)(b-a)$

$\dfrac{c-a}{b-a} = n - 1$ **(1 Mark)**

$n = \dfrac{c-a}{b-a} + 1 = \dfrac{c-a+b-a}{b-a}$

$\Rightarrow \quad n = \dfrac{c+b-2a}{b-a}$

$S_n = \dfrac{n}{2}(a+l) = \dfrac{(b+c-2a)}{2(b-a)}(a+c)$

$[\because l = \text{last term}]$ **(1 Mark)**

Hence, proved.

OR

$\because \quad a_2 - a_1 = 4 - 1 = 3$

$a_3 - a_2 = 7 - 4 = 3$

Since, $a_2 - a_1 = a_3 - a_2$

$\therefore$ Given sequence are in A.P. whose $a = 1$, $d = 3$, $l = x$ and $S_n = 287$

$a_n = a + (n-1)d$

$x = 1 + (n-1)3$

$n - 1 = \dfrac{x-1}{3}$

$n = \dfrac{x-1}{3} + 1 = \dfrac{x-1+3}{3} = \dfrac{x+2}{3}$ **(1 Mark)**

$S_n = \dfrac{n}{2}(a+l)$ **(1 Mark)**

$287 = \dfrac{(x+2)}{2 \times 3}(1+x)$

$287 \times 6 = (x+2)(1+x)$

$1722 = x^2 + 3x + 2$

$x^2 + 3x - 1720 = 0$

$x^2 + 43x - 40x - 1720 = 0$

$x(x + 43) - 40(x + 43) = 0$

$(x - 40)(x + 43) = 0$

$x - 40 = 0 \quad | \quad x + 43 = 0$

$x = 40 \quad | \quad x = -43$

(Negative value is not possible because each terms is positive)

Hence, $x = 40$ **(1 Mark)**

Note

When each terms of A.P multiply by 2 then it is also are in A.P.

29. Let duration of flight = t hr.

Distance = 600 km

$\therefore$ Average speed = $\frac{600}{t}$

New duration of flight = $t + \frac{30}{60} = \left(t + \frac{1}{2}\right)$ hr.

New speed = $\frac{600}{t + \frac{1}{2}} = \frac{1200}{2t + 1}$ **(1 Mark)**

According to question,

$$\frac{600}{t} - \frac{1200}{2t+1} = 200$$ **(1 Mark)**

$\Rightarrow \quad \frac{-1200t + 600(2t+1)}{t(2t+1)} = 200$

$\Rightarrow \quad \frac{1200t - 1200t + 600}{2t^2 + t} = 200$

$\Rightarrow \quad 600 = 200(2t^2 + t)$

$\Rightarrow \quad 2t^2 + t = \frac{600}{200} = 3$

$\Rightarrow \quad 2t^2 + t - 3 = 0$

$\Rightarrow \quad 2t^2 + 3t - 2t - 3 = 0$

$\Rightarrow \quad t(2t + 3) - 1(2t + 3) = 0$

$\Rightarrow \quad (2t + 3)(t - 1) = 0$

$\therefore \quad t - 1 = 0 \quad | \quad 2t + 3 = 0$

$t = 1 \quad | \quad t = \frac{-3}{2} = -1\frac{1}{2}$

$\because$ Time is not negative

$\therefore$ Duration of flight = 1 h **(1 Mark)**

30. Since, mid-point of A(3, 4) and B(k, 6) is P(x, y).

$\therefore \quad P(x, y) = \left(\frac{3+k}{2}, \frac{4+6}{2}\right) = \left(\frac{3+k}{2}, 5\right)$

$\Rightarrow \quad x = \frac{3+k}{2}, y = 5$ **(1 Mark)**

Putting values of x and y in

$x + y - 10 = 0$

$\frac{3+k}{2} + 5 - 10 = 0$

$\Rightarrow \quad \frac{3+k}{2} = 5 \quad \Rightarrow \quad 3 + k = 10$

$\Rightarrow \quad k = 10 - 3 \quad \Rightarrow \quad k = 7$ **(2 Marks)**

OR

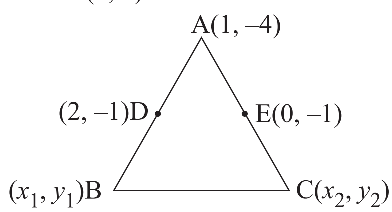
$D(2, -1) = \left(\frac{x_1 + 1}{2}, \frac{y_1 - 4}{2}\right)$

$\Rightarrow \quad 2 = \frac{x_1 + 1}{2} \quad | \quad -1 = \frac{y_1 - 4}{2}$

$4 = x_1 + 1 \quad | \quad -2 = y_1 - 4$

$x_1 = 3 \quad | \quad y_1 = 2$

$\therefore \quad$ B(3, 2) **(½ Mark)**



(½ Mark)

$E(0, -1) = \left(\frac{x_2 + 1}{2}, \frac{y_2 - 4}{2}\right)$

$0 = \frac{x_2 + 1}{2} \quad | \quad -1 = \frac{y_2 - 4}{2}$

$x_2 + 1 = 0 \quad | \quad -2 = y_2 - 4$

$x_2 = -1 \quad | \quad y_2 = 2$

$\therefore \quad$ C(–1, 2) **(1 Mark)**

Area of ΔABC [where A(1, –4), B(3, 2), C(–1, 2)]

$= \frac{1}{2} |[1(2 - 2) + 3(2 + 4) - 1(-4 - 2)]|$

$= \frac{1}{2} |0 + 18 + 6| = \frac{1}{2} \times 24 = 12$

= 12 sq unit. **(1 Mark)**

31. $\because \quad \Delta ABC \sim \Delta DEF$

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ **(1 Mark)**

$\Rightarrow \quad \frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$

$\Rightarrow \quad \frac{2x-1}{18} = \frac{3x}{6x}$

$\Rightarrow \quad \frac{2x-1}{18} = \frac{1}{2}$

$\Rightarrow \quad 2x - 1 = \frac{1}{2} \times 18$

$\Rightarrow \quad 2x = 9 + 1$

$\Rightarrow \quad 2x = 10$

$\Rightarrow \quad x = 5$

$\frac{2x+2}{3x+9} = \frac{3x}{6x}$

$\frac{2x+2}{3x+9} = \frac{1}{2}$

$4x + 4 = 3x + 9$

$4x - 3x = 9 - 4$

$x = 5$

In ΔABC

$AB = 2x - 1 = 10 - 1 = 9$ cm

$BC = 2x + 2 = 10 + 2 = 12$ cm

$AC = 3x = 15$ cm **(1 Mark)**

In ΔDEF

$DE = 18$ cm, $EF = 3x + 9 = 15 + 9 = 24$ cm

$DF = 6x = 30$ cm **(1 Mark)**

Note

When two triangle are similar then their corresponding angles are equal and corresponding sides are proportional.

32. Given: A circle touches the side BC of a ΔABC at P and extended sides AB and AC at Q and R.

To prove: $AQ = \frac{1}{2}(BC + CA + AB)$

Proof: Lengths of tangents drawn to a circle from an external point are equal.

$\therefore \quad AQ = AR$ ...(i) [Tangents from A]

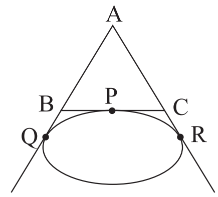
$BP = BQ$ ...(ii) [Tangents from B]

$CP = CR$ ...(iii) [Tangents from C]

(1 Mark)

$R.H.S. = \frac{1}{2}(BC + CA + AB)$

$= \frac{1}{2}(BP + PC + CA + AB)$



(1 Mark)

$= \frac{1}{2}(AB + BQ + AC + CR)$ [from (ii) & (iii)]

$= \frac{1}{2}[AQ + AR] = \frac{1}{2}[AQ + AQ]$ [from (i)]

$= \frac{1}{2}[2AQ] = AQ = L.H.S.$ **(1 Mark)**

Hence, proved.

Note

An excircle of the triangle is a circle lying outside the triangle, tangent to one of its sides and tangent to the extensions of the other two. The length of tangents to the extensions side is equal to the semi perimeter of triangle.

33. $\sin\theta + \cos\theta = \sqrt{2}$

Squaring both sides

$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$

$\sin^2\theta + \cos^2\theta + 2\sin\theta \times \cos\theta = 2$

$1 + 2\sin\theta \cdot \cos\theta = 2 \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$2\sin\theta \cdot \cos\theta = 1$

$\sin\theta \cdot \cos\theta = \frac{1}{2}$ ...(i) **(1 Mark)**

$L.H.S. = \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

(1 Mark)

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta} = \frac{1}{\frac{1}{2}}$ [from (i)]

$= 2 = R.H.S.$ **(1 Mark)**

Hence, proved.

34. Area of circular play ground = πr^2 **(1 Mark)**

$$22176 = \frac{22}{7} \times r^2$$

$$r^2 = \frac{22176 \times 7}{22} = 7056$$

$r = \sqrt{7056} = 84$ cm **(1 Mark)**

Length of fencing (circumference of circle)

$$= 2\pi r = 2 \times \frac{22}{7} \times 84 = 528 \text{ cm}$$

Cost of fencing = 528 × 0.5 = ₹ 264 **(1 Mark)**

SECTION - D

35. Let $\sqrt{5}$ is rational number

$\therefore \quad \sqrt{5} = \frac{p}{q}$,

where p and q are coprime integers and $q \neq 0$. **(1 Mark)**

$$\sqrt{5} = \frac{p}{q}$$

$\Rightarrow \quad p = \sqrt{5}q$ **(1 Mark)**

Squaring both sides

$p^2 = 5q^2$ ...(i)

So, p^2 is divisible by 5

Then p is also divisible by 5

Let $p = 5$ m

Putting in (i)

$(5m)^2 = 5q^2$

$25m^2 = 5q^2$

$q^2 = 5m^2$

So, q^2 is divisible by 5 **(1 Mark)**

Then q is also divisible by 5

Thus, p and q both divisible by 5 but p and q are coprime integers.

By contradiction.

$\sqrt{5}$ is irrational number. **(1 Mark)**

Hence, proved.

36. Let pipe of larger diameter (p_1) fill the pool in x hours.

$\therefore p_1$ fill the pool in 1 hour = $\frac{1}{x}$ part **(1 Mark)**

Both pipe p_1 and p_2 fill the pool in 12 hours.

$\therefore$ Both pipe p_1 and p_2 fill the pool in 1 hour $= \frac{1}{12}$ part.

$\therefore$ p_2 fill the pool in 1 hour = $\frac{1}{12} - \frac{1}{x}$.

(1 Mark)

According to question,

$$4\left(\frac{1}{x}\right) + 9\left(\frac{1}{12} - \frac{1}{x}\right) = \frac{1}{2}$$

$$\frac{4}{x} + \frac{9}{12} - \frac{9}{x} = \frac{1}{2} \Rightarrow \frac{4}{x} - \frac{9}{x} = \frac{1}{2} - \frac{3}{4}$$

$$\Rightarrow \frac{4-9}{x} = \frac{2-3}{4} \Rightarrow \frac{-5}{x} = \frac{-1}{4}$$

$x = 20$

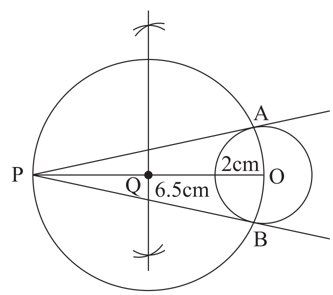
$\therefore$ Time taken by p_1 = 20 hours **(1 Mark)**

Time taken by

$$p_2 = \frac{1}{\frac{1}{12} - \frac{1}{x}} = \frac{1}{\frac{1}{12} - \frac{1}{20}} = \frac{60}{5-3}$$

$$= \frac{60}{2} = 30 \text{ hours}$$ **(1 Mark)**

37.



(2 Marks)

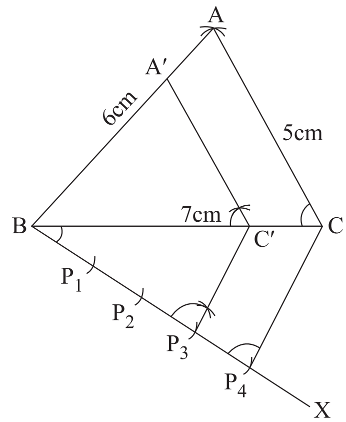
Step of construction:

1. Draw a circle with radius 2 cm. Mark centre O.
2. Draw line segment OP = 6.5 cm.
3. Draw perpendicular bisector of OP.
4. Put the compasses at midpoint Q and draw a circle with radius equal to OQ, which intersect circle at A and B.

5. Join AP and PB, which is required tangents.

(2 Marks)

OR



(2 Marks)

Steps of construction:

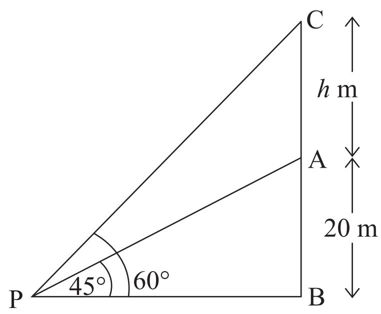
1. Draw BC = 7 cm.
2. From B mark at arc 6 cm and from C mark at arc 5 cm which intersect at A.
3. Join AB and AC, triangle ABC in formed.
4. Draw acute ∠CBX and mark four equal arc at arm BX name it P_1, P_2, P_3 and P_4.
5. Join P_4C, at P_3 draw $P_3C' \parallel P_4C$ which intersect BC at C′.
6. At C′, draw C′A′ ∥ CA.

Then required triangle A′BC′ is constructed.

(2 Marks)

38. AB be the building of height 20 m

Let height of tower AC is h m.



(1 Mark)

In right ΔAPB.

$$\tan 45° = \frac{AB}{PB} \qquad \left[\tan\theta = \frac{P}{B}\right]$$

$$1 = \frac{20}{PB}$$

PB = 20 m **(1 Mark)**

In right ΔPCB

$$\tan 60° = \frac{BC}{PB}, \qquad \sqrt{3} = \frac{20+h}{20}$$

$$20\sqrt{3} = 20 + h$$

$$h = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1) \text{ m}$$

(2 Marks)

Note

The angle of elevation of a point viewed, is the angle formed by the line of sight with horizontal when it is above horizontal level.

39. In ΔRPQ, ∠P = 90°

$\therefore\ RQ^2 = PR^2 + PQ^2 = (7)^2 + (24)^2 = 49 + 576$

$RQ^2 = 625$

$RQ = \sqrt{625} = 25$

(Negative values is not possible)

(1 Mark)

Radius of circle $(r) = \frac{RQ}{2} = \frac{25}{2}$

∴ Area of the shaded region = Area of semicircle – Area ΔRPQ

$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times \text{base} \times \text{height}$ **(1 Mark)**

$= \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} - \frac{1}{2} \times 24 \times 7$

$= 245.54 - 84 = 161.54 \text{ cm}^2$ **(2 Marks)**

Note

Angle in semicircle is right angle.

OR

$r_1 = \frac{20}{2} = 10$ m

$r_2 = \frac{6}{2} = 3\text{m}$

$h = 24$ m

$l = \sqrt{(r_1 - r_2)^2 + h^2}$ **(1 Mark)**

$= \sqrt{(10-3)^2 + (24)^2}$

$= \sqrt{49+576} = \sqrt{625} = 25$ **(1 Mark)**

Curved surface area of frustum $= \pi(r_1 + r_2)l$

(1 Mark)

$= \frac{22}{7} \times (10 + 3) \times 25$

$= \frac{22}{7} \times 13 \times 25 = 1021.43 \text{ cm}^2$ **(1 Mark)**

40.

Class-interval	11–13	13–15	15–17	17–19	19–21	21–23	23–25	Total
Class-Mark (x_i)	12	14	16	18	20	22	24	
Frequency (f_i)	3	6	9	13	f	5	4	$40 + f$
$x_i f_i$	36	84	144	234	$20f$	110	96	$704 + 20f$

(1 Mark)

Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i}$ **(1 Mark)**

$18 = \frac{704 + 20f}{40 + f}$

$18(40 + f) = 704 + 20f$

$720 + 18f = 704 + 20f$

$720 - 704 = 20f - 18f$

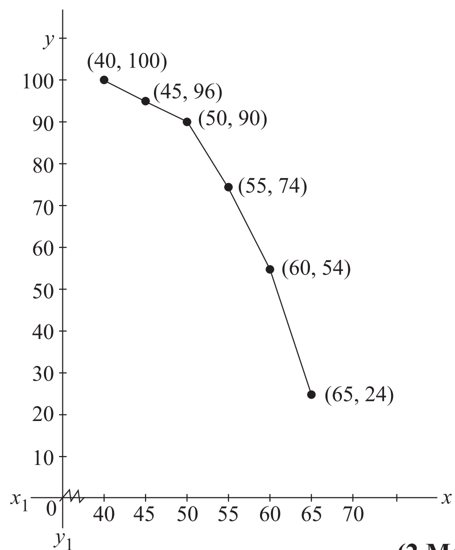
$16 = 2f$

$f = \frac{16}{2} = 8$ **(2 Marks)**

OR

Production Yield	No. of farms
More than 40	100
More than 45	96
More than 50	90
More than 55	74
More than 60	54
More than 65	24

(2 Marks)



(2 Marks)

Note

We can find the median from ogive. x-coordinate of intersection of more than and less than ogive is called median.