All India CBSE Board 2020 Solved Paper

GENERAL INSTRUCTIONS

Read the following instructions very carefully and strictly follow them:

(i) This question paper comprises **four** Sections A, B, C and D. This question paper carries **36** questions. **All** questions are compulsory.

(ii) **Section A:** Questions no. **1** to **20** comprises of **20** questions of **1** mark each.

(iii) **Section B:** Questions no. **21** to **26** comprises of **6** questions of **2** marks each.

(iv) **Section C:** Questions no. **27** to **32** comprises of **6** questions of **4** marks each.

(v) **Section D:** Questions no. **33** to **36** comprises of **4** questions of **6** marks each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is **not** permitted.

SECTION A

Question numbers 1 to 20 carry 1 mark each.

Question numbers 1 to 10 are multiple choice type questions. Select the correct option.

1. If A is a square matrix of order 3 and |A| = 5, then the value of |2A′| is

(a) –10 (b) 10

(c) –40 (d) 40

2. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to

(a) I (b) 0

(c) I – A (d) I + A

3. The principal value of $\tan^{-1}\left(\tan \frac{3\pi}{5}\right)$ is

(a) $\frac{2\pi}{5}$ (b) $\frac{-2\pi}{5}$

(c) $\frac{3\pi}{5}$ (d) $\frac{-3\pi}{5}$

4. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

(a) 0 (b) 1

(c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$

5. The vector equation of the line passing through the point (–1, 5, 4) and perpendicular to the plane $z = 0$ is

(a) $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$

(b) $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$

(c) $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$

(d) $\vec{r} = \lambda\hat{k}$

6. The number of arbitrary constants in the particular solution of a differential equation of second order is (are)

(a) 0 (b) 1

(c) 2 (d) 3

7. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\, dx$ is equal to

(a) –1 (b) 0

(c) 1 (d) 2

8. The length of the perpendicular drawn from the point (4, –7, 3) on the y-axis is

(a) 3 units (b) 4 units

(c) 5 units (d) 7 units

9. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'|A)$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{3}{4}$ (d) 1

10. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 5). If the maximum value of $z = ax + by$, where $a, b > 0$ occurs at both (2, 4) and (4, 0), then

(a) $a = 2b$ (b) $2a = b$

(c) $a = b$ (d) $3a = b$

Fill in the blanks in question numbers 11 to 15.

11. A relation R in a set A is called ________, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

12. The greatest integer function defined by

$f(x) = [x]$, $0 < x < 2$ is not differentiable at $x =$ ________.

13. If A is a matrix of order 3×2, then the order of the matrix A′ is _______.

OR

A square matrix A is said to be skew-symmetric, if ________.

14. The equation of the normal to the curve $y^2 = 8x$ at the origin is ________.

OR

The radius of a circle is increasing at the uniform rate of 3 cm/sec. At the instant when the radius of the circle is 2 cm, its area increases at the rate of _________ cm^2/s.

15. The position vectors of two points A and B are $\overrightarrow{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\overrightarrow{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is _________.

Question numbers 16 to 20 are very short answer type questions.

16. If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then find A (adj A).

17. Find : $\int x^4 \log x\, dx$

OR

Find: $\int \frac{2x}{\sqrt[3]{x^2+1}} dx$

18. Evaluate: $\int_1^3 |2x-1|\, dx$

19. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

20. Find: $\int \frac{dx}{\sqrt{9-4x^2}}$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. Prove that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x$, $\frac{1}{\sqrt{2}} \le x \le 1$.

OR

Consider a bijective function $f = R_+ \to (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$, where R_+ is the set of all positive real numbers. Find the inverse function of f.

22. If $x = at^2$, $y = 2at$, then find $\frac{d^2y}{dx^2}$.

23. Find the points on the curve $y = x^3 - 3x^2 - 4x$ at which the tangent lines are parallel to the line $4x + y - 3 = 0$.

24. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

OR

Find the volume of the parallelopiped whose adjacent edges are represented by $2\vec{a}$, $-\vec{b}$ and $3\vec{c}$, where

$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$, and $\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$.

25. Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

26. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?

SECTION C

Question number 27 to 32 carry 4 marks each.

27. Let N be the set of natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad = bc for all $a, b, c, d \in$ N. Show that R is an equivalence relation.

28. If $y = e^{x^2 \cos x} + (\cos x)^x$, then find $\frac{dy}{dx}$.

29. Find: $\int \sec^3 x\, dx$

30. Find the general solution of the differential equation

$y\, e^y\, dx = (y^3 + 2x\, e^y)dy$.

OR

Find the particular solution of the differential equation

$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$, given that $y = \frac{\pi}{4}$ at $x = 1$.

31. A furniture trader deals in only two items – chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1000 and a table costs him ₹ 2000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

32. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

OR

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

SECTION D

Question numbers 33 to 36 carry 6 marks each.

33. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations:

$x + 2y - 3z = 6$

$3x + 2y - 2z = 3$

$2x - y + z = 2$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$= (a - b)(b - c)(c - a)(a + b + c)(a^2 + b^2 + c^2)$.

34. Using integration, find the area of the region bounded by the triangle whose vertices are (2, –2), (4, 5) and (6, 2).

35. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone, and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

36. Find the equation of the plane that contains the point A(2, 1, –1) and is perpendicular to the line of intersection of the planes $2x + y - z = 3$ and $x + 2y + z = 2$. Also find the angle between the plane thus obtained and the y-axis.

OR

Find the distance of the point P(–2, –4, 7) from the point of intersection Q of the line $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 6$. Also write the vector equation of the line PQ.

Solutions

SECTION A

1. **(d)** $|A| = 5$

We know that $|A| = |A|'$

$\therefore \quad |A|' = 5$

$|2A'| = 2^n |A|'$

$= 2^3 (5)$ $\quad (n = 3)$

$= 8 \times 5 = 40$

2. **(a)** $A^2 = A$ $\quad$ (given)

$(I - A)^3 + A = I^3 - A^3 - 3IA(I - A) + A$

$[\because (a - b)^3 = a^3 - b^3 - 3ab(a - b)]$

$= I - A^2.A - 3I^2A + 3IA^2 + A$ $\quad (\because I^3 = I)$

$= I - A.A - 3IA + 3A^2 + A$

$(\because I^2 = I \text{ and } IA = A)$

$= I - A^2 - 3A + 3A + A$ $\quad (\because IA = A)$

$= I - A + A = I$

3. **(b)** Let $y = \tan^{-1}\left(\tan\frac{3\pi}{5}\right)$

$\tan y = \tan\frac{3\pi}{5}$

$\tan y = \tan\left(\pi - \frac{2\pi}{5}\right)$

$\tan y = -\tan\left(\frac{2\pi}{5}\right)$ $\quad [\because \tan(\pi - \theta) = -\tan\theta]$

$\tan y = \tan\left(-\frac{2\pi}{5}\right)$ $\quad [\because \tan(-\theta) = -\tan\theta]$

$y = -\frac{2\pi}{5}$

4. **(c)** Projection of $\vec{a}$ on $\vec{b} = 0$ $\quad$ (given)

Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$\therefore \quad \frac{\vec{a}.\vec{b}}{|\vec{b}|} = 0$

$\Rightarrow \quad \vec{a}.\vec{b} = 0$

$\Rightarrow \quad (\hat{i} - 2\hat{j} + 3\hat{k}).(2\hat{i} + \lambda\hat{k}) = 0$

$\Rightarrow \quad 2 - 2(0) + 3(\lambda) = 0$

$\Rightarrow \quad 2 + 3\lambda = 0$

$\Rightarrow \quad 3\lambda = -2$

$\Rightarrow \quad \lambda = \frac{-2}{3}$

5. **(b)** Line is perpendicular to $z = 0$ plane.

So it will be along $\hat{k}$ vector and also it passes through $(-1, 5, 4)$

$\therefore \quad \vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda\hat{k}$

$\vec{r} = -\hat{i} + 5\hat{j} + \hat{k}(4 + \lambda)$

6. **(a)** In particular solution of any differential equation, arbitrary constants have to be removed by substituting some particular values. So 0 is answer.

7. **(d)** $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\,dx = [\tan x]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$ $\quad \left[\because \int \sec^2\theta d\theta = \tan\theta\right]$

$= \tan\frac{\pi}{4} - \tan\left(-\frac{\pi}{4}\right)$

$= 1 - \left(-\tan\frac{\pi}{4}\right)$ $\quad [\therefore \tan(-\theta) = -\tan\theta]$

$= 1 + 1 = 2$

8. **(c)** Let point on y-axis be $A(0, y, 0)$ and point $B(4, -7, 3)$

As the line is perpendicular from B to A.

So coordinate of y will be same $y = -7$

$\therefore A(0, -7, 0)$

Distance = $\sqrt{(x_1-x_1)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

AB = $\sqrt{(0-4)^2+(-7+7)^2+(0-3)^2}$

AB = $\sqrt{16+0+9} = \sqrt{25} = 5$

AB = 5 units

9. **(c)** $P\left(\frac{B'}{A}\right) = \frac{P(B'\cap A)}{P(A)} = \frac{P(B')P(A)}{P(A)}$

[$\therefore$ A and B are independent elements]

= P(B′) = 1 – P(B)

$= 1 - \frac{1}{4}$ $\left[\therefore\ P(B) = \frac{1}{4} \text{ (given)}\right]$

$= \frac{3}{4}$

10. **(a)** Z = $ax + by$

Maximum value at (2, 4) is

Z = $a(2) + b(4) = 2a + 4b$ ...(1)

Maximum value at (4, 0)

Z = $a(4) + b(0)$

Z = $4a$ ...(2)

Maximum value occur at both points.

So it should have equal value

$2a + 4b = 4a$ [from (1) and (2)]

$4b = 4a - 2a$

$4b = 2a$

$2b = a$

$\Rightarrow\ a = 2b$

11. **[Symmetric]**

12. **[1]** At 1 L.H.D. = $\lim_{h\to 0}\frac{f(x)-f(x-h)}{h}$

$= \lim_{h\to 0}\frac{f(1)-f(1-h)}{h} = \lim_{h\to 0}\frac{[1]-[(1-h)]}{h}$

$= \lim_{h\to 0}\frac{1-0}{h} = \lim_{h\to 0}\frac{1}{h} = \frac{1}{0}$, Not defined.

R.H.D. = $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{[(1+h)]-[1]}{h}$

$= \lim_{h\to 0}\frac{1-1}{h} = \lim_{h\to 0}\frac{0}{h} = \lim_{h\to 0} 0 = 0$

L.H.D. ≠ R.H.D

$\therefore f(x)$ is not differentiable at $x = 1$.

13. **[2 × 3]**

OR

[A^T = –A]

14. **[y = 0]** $y^2 = 8x$

Differentiating on both sides

$\frac{d}{dx}(y^2) = \frac{d}{dx}(8x)$

$2y\frac{dy}{dx} = 8(1)$

$\frac{dy}{dx} = \frac{8}{2y} = \frac{4}{y}$

Slope of normal = $\frac{-1}{\frac{dy}{dx}} = \frac{-1}{\frac{4}{y}} = \frac{-y}{4}$

at origin (0, 0), slope of normal = 0

$\therefore$ Equation of normal

$\frac{y-0}{x-0} = 0$

$y = 0$

OR

[12π]

Area of circle = πr^2

A = πr^2

$\frac{dA}{dt} = \pi(2r)\frac{dr}{dt}$ $\left[\because \frac{dx^n}{dx} = nx^{n-1}\right]$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$ ...(1)

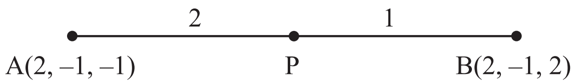
As radius is increasing at uniform rate of 3 cm/sec.

$\frac{dr}{dt} = 3$ cm/sec [From (1)]

$\frac{dA}{dt} = 2\pi r(3) = 6\pi r$

At $r = 2$, $\frac{dA}{dt} = 6\pi(2) = 12\pi$ cm^2/sec. (given)

15. $[2\hat{i} - \hat{j} + \hat{k}]$ Point A(2, –1, –1), Point B(2, –1, 2)



By section formula

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

$$= \left(\frac{4+2}{2+1}, \frac{-2-1}{2+1}, \frac{4-1}{2+1}\right) = (2, -1, 1)$$

Position vector, $\overrightarrow{OP} = 2\hat{i} - \hat{j} + \hat{k}$

16. $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$

$|A| = 2(10 - 9) = 2$

$A^{-1} = \frac{1}{|A|}\text{adj}(A)$

$\text{adj}(A) = |A|A^{-1}$

Multiplying both side by A

$A(\text{adj } A) = |A|AA^{-1}$

$A(\text{adj } A) = |A|I$ $\qquad [\because AA^{-1} = I]$

$$= 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

17. $\int x^4 \log x \, dx$

$$\because \int u\,v\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$$

$$\int \log x \cdot x^4\, dx = \log x\int x^4\,dx - \int\left[\frac{d}{dx}\log x\int x^4 dx\right]dx$$

$$= \log x\left(\frac{x^5}{5}\right) - \int \frac{1}{x}\frac{x^5}{5}dx \qquad \left[\because \frac{d}{dx}\log x = \frac{1}{x}\right]$$

$$\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + C\right]$$

$$= \frac{x^5}{5}\log x - \frac{1}{5}\int x^4 dx + C = \frac{x^5}{5}\log x - \frac{1}{5}\times\frac{x^5}{5} + C$$

$$= \frac{x^5}{5}\log x - \frac{x^5}{25} + C = \frac{x^5}{5}\left[\log x - \frac{1}{5}\right] + C$$

OR

$$\int \frac{2x}{(x^2+1)^{3/2}}dx$$

Let $x^2 + 1 = t$

$2x\,dx = dt$

$$\int \frac{dt}{t^{3/2}} = \frac{t^{-3/2+1}}{-\frac{3}{2}+1} + C \qquad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + C\right]$$

$$= \frac{t^{-1/2}}{-\frac{1}{2}} + C = \frac{-2}{t^{1/2}} + C = \frac{-2}{\sqrt{x^2+1}} + C$$

18. $\int_1^3 |2x - 1|\,dx$

$2x - 1 > 0$ for [1, 3]

$$\therefore \int_1^3 |2x-1|\,dx = \int_1^3 (2x-1)dx$$

$$= \left[\frac{2x^2}{2} - x\right]_1^3 \qquad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1}\right]$$

$$= [x^2 - x]_1^3 = (9 - 3) - (1 - 1) = 6$$

19. 2 cards drawn one by one without replacement

Number of red cards = 26

Number of black cards = 26

Total number of cards = 52

P(RB) = Probability of drawn card is first red and then black

$$P(RB) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(RB) = \frac{26}{52}\times\frac{26}{51}$$

P(BR) = Probability when first card is black and second is red

$$P(BR) = \frac{26}{52}\times\frac{26}{51}$$

$$\text{Hence } P(B \text{ or } R) = \frac{26\times 26}{52\times 51} + \frac{26\times 26}{52\times 51}$$

$$= \frac{2\times 26\times 26}{52\times 51} = \frac{26}{51}$$

20. Let $I = \int \frac{dx}{\sqrt{9-4x^2}}$

$$I = \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{3}{2}\right)^2 - x^2}}$$

$$\because \quad \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\therefore \quad I = \frac{1}{2}\sin^{-1}\left(\frac{x}{3/2}\right) + C = \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + C$$

SECTION B

21. L.H.S. = $\sin^{-1}(2x\sqrt{1-x^2})$, $\frac{1}{\sqrt{2}} \le x \le 1$

Let $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$, $0 \le \theta \le \frac{\pi}{4}$

$\therefore$ L.H.S. = $\sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}(2\sin\theta\cos\theta)$ $\quad [\because \sin^2\theta + \cos^2\theta = 1]$

$= \sin^{-1}(\sin 2\theta)$ $\quad [\because \sin 2\theta = 2\sin\theta.\cos\theta]$

$= 2\theta$

$= 2\cos^{-1}x$ = R.H.S.

OR

$f : R_+ \to (7, \infty)$

$f(x) = 16x^2 + 24x + 7$

$= 16x^2 + 24x + 7 + 9 - 9$

$= (4x)^2 + 2 \times 3 \times 4x + (3)^2 - 2$

$= (4x + 3)^2 - 2 > 0, \forall x \in R_+$

Then, function is strictly increasing in R_+

$\therefore$ $f(x)$ is injective

Now, $f(x) = (4x + 3)^2 - 2$

$\therefore$ $(4x + 3)^2 = f(x) + 2$ $\quad [\therefore f(x) + 2 \in (9, \infty)]$

$4x + 3 = \sqrt{f(x)+2}$

$$x = \frac{\sqrt{f(x)+2}-3}{4}$$

For every $f(x) \in (7, \infty)$, there exist one $x \in R_+$.

$\therefore$ $f(x)$ is surjective

$\therefore$ Function is bijective, so inverse of the function is

$$f^{-1}(x) = \frac{\sqrt{x+2}-3}{4}$$

22. $x = at^2$

$$\frac{dx}{dt} = a(2t) = 2at \qquad \left[\because \frac{dx^n}{dx} = nx^{n-1}\right]$$

$y = 2at$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow \frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$$

$$\frac{dy}{dx} = \frac{1}{t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx}$$

$$= \frac{-\frac{1}{t^2}}{2at} = -\frac{1}{2at^3} \qquad \left[\because \frac{dx}{dt} = 2at\right]$$

$$= \left[\frac{-1}{2a\left(\frac{x}{a}\right)^{3/2}}\right] = \frac{-a^{1/2}}{2x^{3/2}} \qquad \left[\because t = \left(\frac{x}{a}\right)^{1/2}\right]$$

23. $y = x^3 - 3x^2 - 4x$

$$\frac{dy}{dx} = 3x^2 - 6x - 4$$

Slope of tangent = $\frac{dy}{dx} = 3x^2 - 6x - 4$ ...(1)

Given that tangent is parallel to $4x + y - 3 = 0$

$4x + y - 3 = 0$

$y = -4x + 3$

Slope, $m = -4$ ....(2)

$[\therefore y = mx + C]$

From (1) and (2),

$3x^2 - 6x - 4 = -4$

(Slope of parallel lines are equal)

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0, 2$

$y = x^3 - 3x^2 - 4x$

when $x = 0, y = 0$ (0, 0)

when $x = 2, y = 2^3 - 3(2)^2 - 4(2) = -12$ (2, –12)

$\therefore$ Required points are (0, 0) and (2, –12)

24. Let $\hat{C}$ be a vector perpendicular to $\vec{a}$ and $\vec{b}$

$$\hat{C} = \vec{a} \times \vec{b}$$

$$\hat{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix}$$

$$= \hat{i}(12+12) - \hat{j}(10+14) + \hat{k}(30-42)$$

$$= 24\hat{i} - 24\hat{j} - 12\hat{k} = 12(2\hat{i} - 2\hat{j} - \hat{k})$$

Unit vector $\hat{C} = \dfrac{\vec{C}}{|\vec{C}|}$

$$|\vec{C}| = \sqrt{(12)^2[2^2 + (-2)^2 + (-1)^2]}$$

$$= 12\sqrt{4+4+1} = 12\sqrt{9} = 36$$

$$\hat{C} = \frac{12}{36}(2\hat{i} - 2\hat{j} - \hat{k})$$

$$\hat{C} = \frac{1}{3}(2\hat{i} - 2\hat{j} - \hat{k})$$

$$\hat{C} = \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$$

OR

Vector length of sides of parallelopiped are

$$\vec{l_1} = 2\vec{a} = 2(\hat{i} - \hat{j} + 2\hat{k}) = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\vec{l_2} = -\vec{b} = -(3\hat{i} + 4\hat{j} - 5\hat{k}) = -3\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\vec{l_3} = 3\vec{c} = 3(2\hat{i} - \hat{j} + 3\hat{k}) = 6\hat{i} - 3\hat{j} + 9\hat{k}$$

Volume = $|(\vec{l_1} \times \vec{l_2}) \cdot \vec{l_3}|$

$$\vec{l_1} \times \vec{l_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 4 \\ -3 & -4 & 5 \end{vmatrix}$$

$$= \hat{i}(-10+16) - \hat{j}(10+12) + \hat{k}(-8-6)$$

$$= 6\hat{i} - 22\hat{j} - 14\hat{k}$$

$$|(\vec{l_1} \times \vec{l_2}) \cdot \vec{l_3}| = |(6\hat{i} - 22\hat{j} - 14\hat{k}) \cdot (6\hat{i} - 3\hat{j} + 9\hat{k})|$$

$$|(\vec{l_1} \times \vec{l_2}) \cdot \vec{l_3}| = |36 + 66 - 126| = 24$$

Volume is 24 cubic units.

25. $x = -y = kz$

$$\frac{x}{1} = \frac{y}{-1} = \frac{z}{\frac{1}{k}}$$

Vector, $\vec{a} = \hat{i} - \hat{j} + \frac{1}{k}\hat{k}$ ...(1)

$x - 2 = 2y + 1 = -z + 1$

$$\frac{x-2}{1} = \frac{y+\frac{1}{2}}{\frac{1}{2}} = \frac{z-1}{-1}$$

Vector $\vec{b} = \hat{i} + \frac{1}{2}\hat{j} - \hat{k}$ ...(2)

For two lines to be perpendicular

$$\vec{a} . \vec{b} = 0$$

$$\left(\hat{i} - \hat{j} + \frac{1}{k}\hat{k}\right) . \left(\hat{i} + \frac{1}{2}\hat{j} - \hat{k}\right) = 0$$

$$1 - \frac{1}{2} - \frac{1}{k} = 0$$

$$\frac{1}{2} = \frac{1}{k}$$

$$k = 2$$

26. Probability of green signal = 30% = $\frac{30}{100}$

P(green signal) = 0.3

$\because$ P(G) = 0.3

Probability of other signals, P(other) = 1 – 0.3 = 0.7

P(X) = Probability of getting green signal on two consecutive days out of 3.

= P(G) P(G) P(Other) + P(Other) P(G) P(G)

= (0.3)(0.3)(0.7) + (0.7)(0.3)(0.3)

= 2(0.3)(0.3)(0.7)

= 2 × 0.09 × 0.7

= 0.126

SECTION C

27. $(a, b)\ R(c, d) \Leftrightarrow ad = bc$ (given relation)

(i) Reflexivity

Since $ab = ba$ $\because ab \in N$

$(a, b)\ R\ (a, b)$ is true

Relation R is reflexive.

(ii) Symmetry

Let $(a, b)\ R\ (c, d)$

$ad = bc$

But $bc = cb$ and $ad = da$

$\because$ $cb = da \Rightarrow cb = ad$

$(c, d)\ R\ (a, b)$

$\therefore (a, b)\ R\ (c, d) \Leftrightarrow (c, d)\ R\ (a, b)$

Relation is symmetric.

(iii) Transitivity

Let $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

$ad = bc$ and $cf = de$

$(ad)\ (cf) = (bc)\ (de)$

$(af)\ (dc) = (be)\ (dc)$

$af = be$

$\Rightarrow (a, b)\ R\ (e, f)$

$\therefore (a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

$\Rightarrow (a, b)\ R\ (e, f)$

$\Rightarrow$ Relation is transitive

Relation is symmetric, equivalence and transitive.

$\therefore$ Relation is equivalence relation.

28. $y = e^{x^2 \cos x} + (\cos x)^x$

$$y = e^{x^2 \cos x} + e^{\log(\cos x)^x}$$

$$y = e^{x^2 \cos x} + e^{x \log(\cos x)}$$

By chain rule

$$\frac{d}{dx} f(g(x)) = \frac{df}{dg}\frac{dg}{dx}$$

$$\frac{dy}{dx} = \frac{d}{dx}(e^{x^2 \cos x}) + \frac{d}{dx}(e^{x \log \cos x})$$

$$= e^{x^2 \cos x}(2x\cos x - x^2 \sin x)$$

$$+ e^{x\log(\cos x)}\left[\log(\cos x) - \frac{x}{\cos x}\sin x\right]$$

$$\left[\because \frac{d}{dx}e^x = e^x, \frac{d}{dx}\log x = \frac{1}{x}, \frac{d}{dx}\cos x = -\sin x\right]$$

$$= e^{x^2 \cos x}(2x\cos x - x^2 \sin x)$$

$$+ (\cos x)^x[\log\cos x - x\tan x]$$

29. Let $\quad I = \int \sec^3 x\, dx$

$$I = \int \sec x \sec^2 x\, dx$$

$$\int u\,v\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$$

$$I = \sec x\int \sec^2 x\,dx - \int\left(\frac{d}{dx}\sec x\int \sec^2 x\right)dx$$

$$= \sec x \tan x - \int \sec x \tan x \tan x dx$$

$$\left[\because \int \sec^2 x\,dx = \tan x + C, \frac{d}{dx}\sec x = \sec x \tan x\right]$$

$$= \sec x \tan x - \int \sec x \tan^2 x\,dx$$

$$= \sec x \tan x - \int \sec x(\sec^2 x - 1)dx$$

$$[\because \sec^2\theta = 1 + \tan^2\theta]$$

$$I = \sec x \tan x - \int \sec^3 dx + \int \sec x\,dx$$

$$= \sec x \tan x - I + \log|\sec x + \tan x| + C$$

$$\left[\because \int \sec x = \log|\sec x + \tan x| + C\right]$$

$$2I = \sec x \tan x + \log|\sec x + \tan x| + C$$

$$I = \frac{1}{2}\sec x.\tan x + \frac{1}{2}\log|\sec x + \tan x| + C$$

30. $ye^y\,dx = (y^3 + 2xe^y)dy$

$$\frac{dx}{dy} = \frac{y^3 + 2xe^y}{ye^y}$$

$$\frac{dx}{dy} = \frac{y^2}{e^y} + \frac{2x}{y}$$

$$\frac{dx}{dy} - \frac{2}{y}x = y^2 e^{-y}$$

This is a linear differential equation of the form

$$\frac{dx}{dy} + Px = Q$$

where $P = \frac{-2}{y}$, $Q = y^2e^{-y}$

$\therefore$ $$\text{IF} = e^{\int P\,dy} = e^{\int \frac{-2}{y}dy} = e^{-2\log y}$$

$$\left[\because \int \frac{1}{y}dy = \log y\right]$$

$$\text{IF} = e^{\log y^{-2}} = y^{-2} = \frac{1}{y^2} \qquad \left[\because e^{\log m^n} = m^n\right]$$

$$x.\,(\text{I.F.}) = \int \text{I.F.}\times Q\; dy + C$$

$$x \times \frac{1}{y^2} = \int \frac{1}{y^2}(y^2e^{-y})dy + C$$

$$\frac{x}{y^2} = \int e^{-y}dy + C$$

$$\frac{x}{y^2} = -e^{-y} + C \qquad \left[\because \int e^{-y}dx = -e^{-y}\right]$$

$$\frac{x}{y^2} + e^{-y} = C$$

OR

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

$$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right) \qquad ...(1)$$

Let $z = \frac{y}{x}$

Given $y = \frac{\pi}{4}$ at $x = 1$

$\therefore$ $$z = \frac{\pi}{4}$$

$z = \frac{y}{x}$, Differentiating w.r.t. x

$$\frac{dz}{dx} = \frac{1}{x}\frac{dy}{dx} + y\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$\frac{dz}{dx} = \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2}$$

$$\frac{dz}{dx} = \frac{1}{x}\left(\frac{dy}{dx} - \frac{y}{x}\right)$$

$$\frac{dz}{dx} = \frac{1}{x}\left(-\tan\left(\frac{y}{x}\right)\right) \qquad \text{[from (1)]}$$

$$\frac{dz}{dx} = \frac{-\tan z}{x}$$

$$\frac{-dz}{\tan z} = \frac{dx}{x} \Rightarrow -\cot z\,dz = \frac{dx}{x}$$

Integrating on both sides

$$\int \cot z\,dz = \int \frac{1}{x}dx$$

$$\log|\sin z| = -\log|x| + \log|c| \qquad \left[\because \int \cot x\,dx = \log|\sin x|\right]$$

$$\log|\sin z| = \log\left|\frac{c}{x}\right|$$

$$\sin z = \frac{c}{x}$$

Putting $x = 1$, $y = \frac{\pi}{4}$

$$\sin\frac{\pi}{4} = c$$

$$c = \frac{1}{\sqrt{2}}$$

$$\sin z = \frac{1}{\sqrt{2}x}$$

$$z = \sin^{-1}\left(\frac{1}{\sqrt{2}x}\right)$$

$$\frac{y}{x} = \sin^{-1}\left(\frac{1}{\sqrt{2}x}\right)$$

31. Let the number of chairs be x and the number of tables be y

Maximize profit Z = $150x + 250y$

Subjected to: $1000x + 2000y \le 50000$

$x + 2y \le 50$

$x + y \le 35, x \ge 0, y \ge 0$

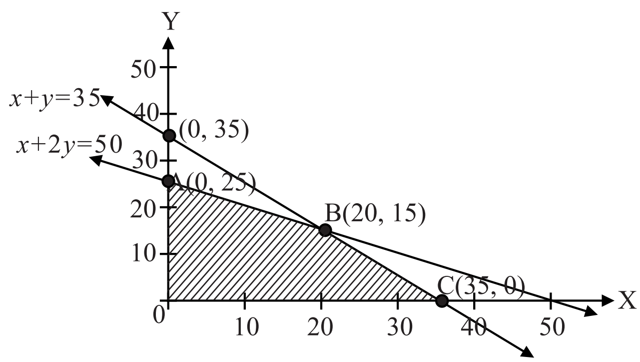
$x + y = 35$		
x	0	35
y	35	0

$x + 2y = 50$		
x	0	50
y	25	0

Intersecting point

$$x + y = 35$$
$$x + 2y = 50$$
$$- \quad - \quad -$$
$$-y = -15$$
$$y = 15$$

$$x + 15 = 35$$
$$x = 20, B(20, 15)$$



OABCO is required region

	x	y	z
O(0, 0)	0	0	0
A(0, 25)	0	25	6250
C(35, 0)	35	0	5250
B(20, 15)	20	15	6750 (Max.)

Maximum profit is ₹ 6750 when number of chairs are 20 and tables are 15.

32.

Bag I
Red = 3 Black = 5

Bag II
Red = 4 Black = 3

1 ball is transferred randomly from bag I to bag II.

Let E_1: transferred ball is black

E_2: transferred ball in red

F: Ball drawn from bag II is black

$$P(E_1) = \frac{5}{8} \qquad P(E_2) = \frac{3}{8}$$

$$P(F/E_1) = \frac{4}{8} \qquad P(F/E_2) = \frac{3}{8}$$

By Baye's theorem:

$$P(E_1/F) = \frac{P(F/E_1)\cdot P(E_1)}{P((F/E_1)\cdot P(E_1) + P(F/E_2)\cdot P(E_2)}$$

$$= \frac{\frac{5}{8}\times\frac{4}{8}}{\frac{5}{8}\times\frac{4}{8}+\frac{3}{8}\times\frac{3}{8}} = \frac{20}{29}$$

OR

Red = 5 White = 2 Black = 3

Let X denote the number of white balls when 3 balls are drawn one by one without replacement.

So, X can be 0, 1, 2

$P(x = 0)$ = Probability when none of the balls are white

$$P(x = 0) = \frac{{}^8C_3}{{}^{10}C_3}$$

$$\left[\because \text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}\right]$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

$$P(x = 0) = \frac{\frac{8\times7\times6\times5!}{5!\;3!}}{\frac{10\times9\times8\times7!}{3!\;7!}}$$

$$= \frac{8\times7\times6}{10\times9\times8} = \frac{7}{15}$$

$P(x = 1)$ = Probability when 1 of 3 balls in white

$$P(x = 1) = \frac{{}^8C_2 \times {}^2C_1}{{}^{10}C_3}$$

$$= \frac{\frac{8\times7\times6!}{6!\times2!}\times\frac{2!}{1!\,1!}}{\frac{10\times9\times8\times7!}{7!\times3!}} = \frac{7}{15}$$

P(x = 2) = Probability when two balls out of 3 are white

$$P(x=2) = \frac{{}^{8}C_1 \times {}^{2}C_2}{{}^{10}C_3}$$

$$= \frac{\frac{8\times7!}{1!\times7!}\times\frac{2!}{0!\,2!}}{\frac{10\times9\times8\times7!}{7!\,3!}} = \frac{1}{15}$$

∴ Probability distribution of X is

X	0	1	2	Total
P(X = x)	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$	1
XP(X)	0	$\frac{7}{15}$	$\frac{2}{15}$	$\frac{9}{15}$
X^2P(X)	0	$\frac{7}{15}$	$\frac{4}{15}$	$\frac{11}{15}$

$$\text{Mean} = \Sigma X\,P(X) = \frac{9}{15} = \frac{3}{5} = 0.6$$

$$\text{Variance} = \Sigma X^2\,P(X) - [\Sigma X\,P(X)]^2$$

$$= \frac{11}{15} - \frac{9}{25} = \frac{55-27}{75}$$

$$= \frac{28}{75} = 0.37.$$

SECTION D

33. $$A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$$

$$|A| = 1(2-2) - 2(3+4) - 3(-3-4)$$

$$= -14 + 21 = 7$$

∵ $|A| \neq 0$, A^{-1} exist.

$$A_{11} = (-1)^2\begin{vmatrix} 2 & -2 \\ -1 & 1 \end{vmatrix} = 2-2 = 0$$

$$A_{12} = (-1)^3\begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = (-1)[3+4] = -7$$

$$A_{13} = (-1)^4\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = [-3-4] = -7$$

$$A_{21} = (-1)^3\begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix} = (-1)[2-3] = 1$$

$$A_{22} = (-1)^4\begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} = 1+6 = 7$$

$$A_{23} = (-1)^5\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = (-1)[-1-4] = 5$$

$$A_{31} = (-1)^4\begin{vmatrix} 2 & -3 \\ 2 & -2 \end{vmatrix} = -4+6 = 2$$

$$A_{32} = (-1)^5\begin{vmatrix} 1 & -3 \\ 3 & -2 \end{vmatrix} = -1(-2+9) = -7$$

$$A_{33} = (-1)^6\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = 2-6 = -4$$

$$\text{adj } A = \begin{bmatrix} 0 & -7 & -7 \\ 1 & 7 & 5 \\ 2 & -7 & -4 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}A$$

$$= \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ 7 & 5 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{7} & \frac{2}{7} \\ -1 & 1 & -1 \\ 1 & \frac{5}{7} & -\frac{4}{7} \end{bmatrix}$$

Given equation:

$x + 2y - 3z = 6$

$3x + 2y - 2z = 3$

$2x - y + z = 2$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

AX = B

Multiply A^{-1} on both sides

$$(A^{-1}A)X = A^{-1}B$$

$$IX = A^{-1}B \qquad [\because A^{-1}A = I]$$

$$X = A^{-1}B$$

$$X = \begin{bmatrix} 0 & \frac{1}{7} & \frac{2}{7} \\ -1 & 1 & -1 \\ 1 & \frac{5}{7} & -\frac{4}{7} \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{3}{7}+\frac{4}{7} \\ -6+3-2 \\ 6+\frac{15}{7}-\frac{8}{7} \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}$$

$$\therefore \quad x = 1, y = -5, z = 7$$

OR

L.H.S.

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$R_1 \to R_1 - R_3, R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} (b+c+a+b)(b+c-a-b) & (a+c)(a-c) & b(c-a) \\ (c+a+a+b)(c+a-a-b) & (b+c)(b-c) & a(c-b) \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$\Delta = \begin{vmatrix} (c-a)(2b+a+c) & (a+c)(a-c) & b(c-a) \\ (c-b)(2a+c+b) & (b+c)(b-c) & a(c-b) \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$\Delta = (c-a)(b-c)\begin{vmatrix} a+c+2b & -a-c & b \\ -2a-c-b & b+c & -a \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$R_1 \to R_1 + R_2$

$$\Delta = (c-a)(b-c)\begin{vmatrix} -a+b & b-a & b-a \\ -2a-b-c & b+c & -a \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$\Delta = (a-b)(c-a)(b-c)\begin{vmatrix} -1 & -1 & -1 \\ -2a-b-c & b+c & -a \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$C_1 \to C_1 - C_3, C_2 \to C_2 - C_3$

$$\Delta = (a-b)(c-a)(b-c)\begin{vmatrix} 0 & 0 & -1 \\ -a-b-c & a+b+c & -a \\ (a+b)^2-ab & c^2-ab & ab \end{vmatrix}$$

$$\Delta = (a-b)(b-c)(c-a)\{[(a+b+c)(c^2-ab)] + (a+b+c)[(a+b)^2-ab]\}$$

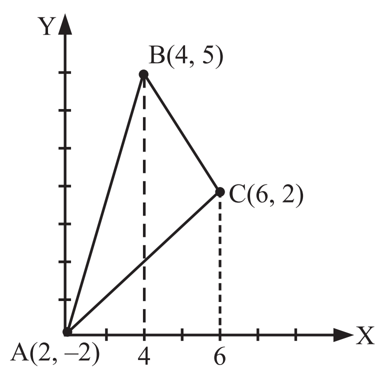
$$= (a-b)(b-c)(c-a)(a+b+c)[c^2-ab+a^2+2ab+b^2-ab]$$

$$\Delta = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

= R.H.S.

Hence Proved.

34. A(2, –2) B(4, 5) C(6, 2)



Equation of line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Equation of line AB:

$$y - 5 = \frac{-2-5}{2-4}(x-4)$$

$$y - 5 = \frac{-7}{-2}(x-4)$$

$$y = \frac{7}{2}x - \frac{28}{2} + 5$$

$$y = \frac{7}{2}x - 9$$

Equation of line BC:

$$y - 5 = \frac{2-5}{6-4}(x-4)$$

$$y = \frac{-3}{2}x + \frac{12}{2} + 5$$

$$y = \frac{-3}{2}x + 6 + 5$$

$$y = \frac{-3}{2}x + 11$$

Equation of line AC:

$$y - 2 = \frac{-2-2}{2-6}(x-6)$$

$$y - 2 = \frac{4}{4}(x-6)$$

$$y - 2 = x - 6$$

$$y = x - 4$$

Area of ΔABC

$$= \int_2^4 \text{AB}\,dx + \int_4^6 (\text{BC})dx - \int_2^6 (\text{CA})dx$$

$$= \int_2^4 \left(\frac{7}{2}x - 9\right)dx + \int_4^6 \left(\frac{-3}{2}x + 11\right)dx - \int_2^6 (x-4)dx$$

$$= \left[\frac{7x^2}{4} - 9x\right]_2^4 + \left[\frac{-3x^2}{4} + 11x\right]_4^6 - \left[\frac{x^2}{2} - 4x\right]_2^6$$

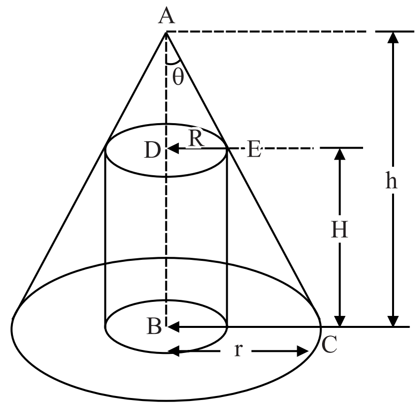
= [(28 – 36) – (7 – 18)] + [(–27 + 66) – (–12 + 44)] – [(18 – 24) – (2 – 8)]

= [(–8) – (–11)] + [39 – (32)] – [(–6) – (–6)]

= 3 + 7 – 0 = 10 sq. units

Area of ΔABC = 10 sq. units

35.



Given height of cone be h and radius of cone be r

Let height of cylinder be H and radius of cylinder be R

Volume of cylinder $V = \pi R^2 H$ ...(1)

In right ΔADE

$$\tan\theta = \frac{R}{h-H}$$

In right ΔABC

$$\tan\theta = \frac{r}{h}$$

$$\frac{r}{h} = \frac{R}{h-H}$$

$$rh - rH = hR$$

$$R = r - \frac{rH}{h}$$

$$R = r\left(1 - \frac{H}{h}\right) \quad ...(2)$$

From (1)

$$V = \pi r^2 \left(1 - \frac{H}{h}\right)^2 H$$

Differentiate both sides w.r.t. H

$$\frac{dV}{dH} = \pi r^2 \left[\left(1 - \frac{H}{h}\right)^2 + H2\left(1 - \frac{H}{h}\right)\left(\frac{-1}{h}\right)\right]$$

$$= \pi r^2 \left(1 - \frac{H}{h}\right)\left[1 - \frac{H}{h} - \frac{2H}{h}\right]$$

$$= \pi r^2\left(1-\frac{H}{h}\right)\left[1-\frac{3H}{h}\right]=0$$

$$\because \quad 1-\frac{H}{h}\neq 0$$

$$\therefore \quad 1-\frac{3H}{h}=0 \Rightarrow H=\frac{h}{3} \quad ...(3)$$

$$\therefore \quad \frac{d^2V}{dH^2} = \pi r^2\left[\frac{-1}{h}\left(1-\frac{3H}{h}\right)+\left(1-\frac{H}{h}\right)\left(\frac{-3}{h}\right)\right]$$

Put $H=\frac{h}{3}$

$$= \pi r^2\left[0+\left(1-\frac{1}{3}\right)\left(\frac{-3}{h}\right)\right]$$

$$= \pi r^2\left(\frac{-2}{h}\right)<0$$

$\therefore$ Volume is maximum when $H=\frac{h}{3}$.

Volume of cylinder = $\pi r^2\left(1-\frac{H}{h}\right)^2 H$

Putting $H=\frac{h}{3}$

$$= \pi r^2\left(1-\frac{1}{3}\right)^2\cdot\frac{h}{3}$$

$$= \pi r^2\frac{4}{9}h$$

$$= \frac{4}{9}\pi r^2 h$$

$$= \frac{4}{9}\times \text{volume of cone.}$$

Hence proved.

36. Plane passing through A(2, 1, –1) and perpendicular to line of intersection of 2 planes

$2x + y - z = 3$ and $x + 2y + z = 2$

Since plane is perpendicular to the line of intersection. Therefore, normal of the required plane is parallel to the line of intersection and perpendicular to a normal of both given plane.

$\vec{n}_1 \to$ Normal vector of first given plane.

$\vec{n}_2 \to$ Normal vector of second given plane.

$\vec{N} \to$ Normal vector of required plane.

$\vec{N} \parallel \vec{n}_1\times\vec{n}_2 \quad (\because \vec{N}\perp n_1, \vec{N}\perp n_2)$

Normal vector, $\vec{N} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ 1 & 2 & 1\end{vmatrix} = 3\hat{i}-3\hat{j}+3\hat{k}$

$\therefore$ Direction ratios of normal are 3, –3, 3

Equation of plane passing through (x_1, y_1, z_1) is given by

$A(x-x_1)+B(y-y_1)+c(z-z_1)=0$

Where A, B, C are the direction ratios of normal to the plane.

$A(x-2)+B(y-1)+C(z+1)=0$

$3(x-2)-3(y-1)+3(z+1)=0$

$(x-2)-(y-1)+(z+1)=0$

$x-2-y+1+z+1=0$

$x-y+z=0$

$$\cos\theta = \frac{|\vec{N}\cdot\vec{b}|}{|\vec{N}||\vec{b}|}$$

Direction vector of y axis is $b = 0\hat{i}+\hat{j}+0\hat{k}$

$$\cos\theta = \frac{|(3\hat{i}-3\hat{j}+3\hat{k})(0\hat{i}+\hat{j}+0\hat{k})|}{\sqrt{3^2+3^2+3^2}\sqrt{1^2}}$$

$$\cos\theta = \frac{|-1|}{\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

OR

$\vec{r} = (3\hat{i}-2\hat{j}+6\hat{k})+\lambda(2\hat{i}-\hat{j}+2\hat{k})$ line equation

$\vec{r}.(\hat{i}-\hat{j}+\hat{k})=6$ plane equation

From line equation

$x = 3 + 2\lambda$

$y = -2 - \lambda$

$z = 6 + 2\lambda$

Equation of plane can be written as

$$x - y + z = 6$$

$(3 + 2\lambda) - (-2 - \lambda) + (6 + 2\lambda) = 6$

$3 + 2\lambda + 2 + \lambda + 6 + 2\lambda = 6$

$$5\lambda + 11 = 6$$

$$\lambda = \frac{-5}{5} = -1$$

$$\lambda = -1$$

Point of intersection

$$x = 3 + 2(-1) = 1$$

$$y = -2 + 1 = -1$$

$$z = 6 - 2 = 4$$

Q(1, –1, 4)

P(–2, –4, 7)

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(-2-1)^2 + (-4+1)^2 + (7-4)^2}$$

$$= \sqrt{3^2 + 3^2 + 3^2} = 3\sqrt{3}$$

Vector equation of PQ

$$\vec{r} = (\hat{i} - \hat{j} + 4\hat{k}) + \lambda\left[(1+2)\hat{i} + (-1+4)\hat{j} + (4-7)\hat{k}\right]$$

$$\vec{r} = (\hat{i} - \hat{j} + 4\hat{k}) + \lambda(3i + 3\hat{j} - 3\hat{k})$$

$$= (\hat{i} - \hat{j} + 4\hat{k}) + \lambda(i + \hat{j} - \hat{k})$$