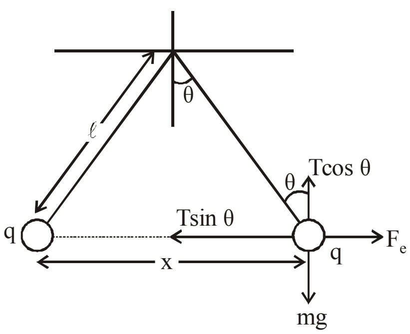
Mock Test-1

ANSWER KEY																			
1	(4)	**16**	(1)	**31**	(1)	**46**	(1)	**61**	(3)	**76**	(3)
2	(4)	**17**	(3)	**32**	(1)	**47**	(1)	**62**	(2)	**77**	(1)
3	(3)	**18**	(1)	**33**	(2)	**48**	(1)	**63**	(2)	**78**	(4)
4	(2)	**19**	(1)	**34**	(2)	**49**	(2)	**64**	(2)	**79**	(4)
5	(4)	**20**	(2)	**35**	(4)	**50**	(3)	**65**	(3)	**80**	(1)
6	(1)	**21**	(170)	**36**	(3)	**51**	(1.24)	**66**	(1)	**81**	(0)
7	(3)	**22**	(6.67)	**37**	(4)	**52**	(2.25)	**67**	(3)	**82**	(0.80)
8	(4)	**23**	(6.275)	**38**	(3)	**53**	(0.5)	**68**	(1)	**83**	(0.25)
9	(1)	**24**	(20)	**39**	(3)	**54**	(1.75)	**69**	(1)	**84**	(0.33)
10	(1)	**25**	(0.25)	**40**	(3)	**55**	(5.97)	**70**	(4)	**85**	(7.00)
11	(1)	**26**	(4)	**41**	(4)	**56**	(400)	**71**	(1)	**86**	(1.00)
12	(2)	**27**	(40.05)	**42**	(4)	**57**	(210)	**72**	(2)	**87**	(3.00)
13	(2)	**28**	(5.3)	**43**	(4)	**58**	(0.04)	**73**	(1)	**88**	(1.50)
14	(2)	**29**	(23)	**44**	(2)	**59**	(6)	**74**	(4)	**89**	(14.00)
15	(2)	**30**	(2)	**45**	(3)	**60**	(0.241)	**75**	(1)	**90**	(3.20)

Solutions

PHYSICS

1. (4)



In equilibrium, $F_e = T\sin\theta$

$mg = T\cos\theta$

$$\tan\theta = \frac{F_e}{mg} = \frac{q^2}{4\pi\epsilon_0 x^2 \times mg}$$

also $\tan\theta \approx \sin = \frac{x/2}{\ell}$

Hence, $\frac{x}{2\ell} = \frac{q^2}{4\pi\epsilon_0 x^2 \times mg}$

$$\Rightarrow x^3 = \frac{2q^2\ell}{4\pi\epsilon_0 mg}$$

$$\therefore x = \left(\frac{q^2\ell}{2\pi\epsilon_0 mg}\right)^{1/3}$$

Therefore $x \propto \ell^{1/3}$

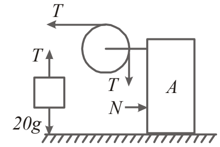
2. (4) **Case I :**

$T - N = 40a$

and $20g - T = 20a$

Also $N = 20a$

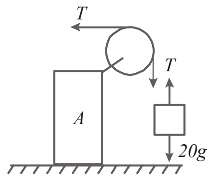
After simplifying, we get



$$a = \frac{g}{4}$$

Acceleration of block B, $= \sqrt{2}a = \frac{g}{2\sqrt{2}}$.

Case II :



$T = 40\,a$

and $20\,g - T = 20\,a$

After simplifying above equation, we get

$a = g/3$

$$\text{Ratio} = \frac{g/2\sqrt{2}}{g/3} = \frac{3}{2\sqrt{2}}.$$

3. (3) $V_{in} = \frac{-GM}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right]$,

$$V_{surface} = \frac{-GM}{R}, V_{out} = \frac{-GM}{r}$$

4. (2) $T\downarrow$ (300K to 70K)

$T\downarrow$ $R_{metal}\downarrow$ $R\uparrow$ semi-conductor
(Al) (Si)

5. (4) When two rods are connected in series

$$Q = \frac{A(T_1 - T_2)t}{\frac{d_1}{K_1} + \frac{d_2}{K_1}} = \frac{A(T_1 - T_2)t}{(d_1 + d_2)/K}$$

$$\therefore \frac{d_1 + d_2}{K} = \frac{d_1}{K_1} + \frac{d_2}{K_2};$$

$$\therefore K = \frac{(d_1 + d_2)}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

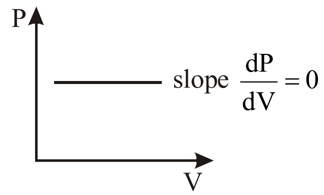
6. (1) When the ring rotates about its axis with a uniform frequency f Hz, the current flowing in the ring is

$$I = \frac{q}{T} = qf$$

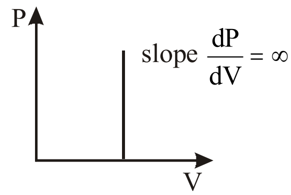
Magnetic field at the centre of the ring is

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 qf}{2R}$$

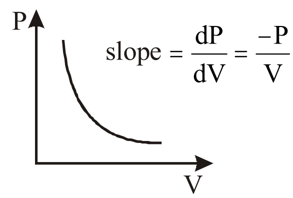
7. (3) P-V indicator diagram for isobaric



P-V indicator diagram for isochoric process



P-V indicator diagram for isothermal process



8. (4) Let 'S' be the distance between two ends 'a' be the constant acceleration

As we know $v^2 - u^2 = 2aS$

or, $aS = \frac{v^2 - u^2}{2}$

Let v_c be velocity at mid point.

Therefore, $v_c^2 - u^2 = 2a\frac{S}{2}$

$$v_c^2 = u^2 + aS$$

$$v_c^2 = u^2 + \frac{v^2 - u^2}{2}$$

$$v_c = \sqrt{\frac{u^2 + v^2}{2}}$$

9. (1) Angular retardation,

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi(0 - 900/60)}{60} = -\frac{\pi}{2}\text{rad/s}^2.$$

10. (1) Given : $k_A = 300\,\text{N/m}$, $k_B = 400\,\text{N/m}$

Let when the combination of springs is compressed by force F. Spring A is compressed by x_A. Therefore compression in spring B

$x_B = (8.75 - x_A)$ cm

$F = 300 \times x_A = 400(8.75 - x_A)$

Solving we get, $x_A = 5$ cm

$x_B = 8.75 - 5 = 3.75$ cm

$$\frac{E_A}{E_B} = \frac{\frac{1}{2}k_A(x_A)^2}{\frac{1}{2}k_B(x_B)^2} = \frac{300 \times (5)^2}{400 \times (3.75)^2} = \frac{4}{3}$$

11. (1) From $F = \frac{R}{t^2}v(t) \Rightarrow m\frac{dv}{dt} = \frac{R}{t^2}v(t)$

Integrating both sides $\int \frac{dv}{v(t)} = \int \frac{Rdt}{mt^2}$

$\text{In } v = -\frac{R}{mt}$

$\therefore \ln v \propto \frac{1}{t}$

12. (2) In reverse bias on p-n junction when high voltage is applied, electric break down of junction takes place, resulting large increase in reverse current. This high voltage applied is called zener voltage.

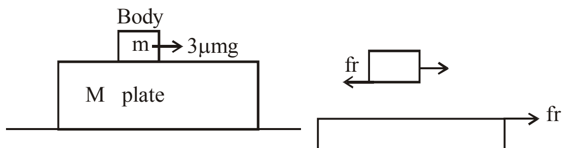
13. (2) If in nuclear reaction binding energy per nucleon increases, energy is released.

14. (2) Let initial amount be 100 gm.

			disintegrated	Left
100 gm	$\xrightarrow{\text{5 days}}$	$\frac{100 \times 10}{100}$	10	90
90	$\xrightarrow{\text{Next 5 days}}$	$\frac{90 \times 10}{100}$	9	81
81	$\xrightarrow{\text{Next 5 days}}$	$\frac{81 \times 10}{100}$	8.1	≈ 73
73	$\xrightarrow{\text{Next 5 days}}$	$\frac{73 \times 10}{100}$	7.3	≈ 66

15. (2) As $\mu_v > \mu_r$ therefore, $v_v < v_r$.

16. (1) $a = \frac{\mu mg}{M}$



fr = μmg = Ma

17. (3) Einstein equation KE_{max} = E – Work function;

$\frac{1}{2}mv^2 = E - W$

Using this concept,

$$\frac{\frac{1}{2}mV_1^2\,\text{max}}{\frac{1}{2}mV_2^2\,\text{max}} = \frac{1 - .5}{2.5 - .5} = \frac{1}{4} \text{ or } \frac{V_1\,\text{max}}{V_2\,\text{max}} = \frac{1}{2}$$

18. (1) Velocity of electron in n^{th} orbit of hydrogen atom is given by :

$$V_n = \frac{2\pi KZe^2}{nh}$$

Substituting the values we get,

$$V_n = \frac{2.2 \times 10^6}{n}\text{m/s} \quad \text{or} \quad V_n \propto \frac{1}{n}$$

As principal quantum number increases, velocity decreases.

19. (1) From $\eta = 1 - \frac{T_2}{T_1}$, $\frac{T_2}{T_1} = 1 - \eta = 1 - \frac{1}{6} = \frac{5}{6}$ ...(i)

In 2nd case :

$$\frac{T_2 - 62}{T_1} = 1 - \eta' = 1 - \frac{2}{6} = \frac{2}{3} \quad .....(ii)$$

Using (i), $T_2 - 62 = \frac{2}{3}T_1 = \frac{2}{3} \times \frac{6}{5}T_2 = \frac{4}{5}T_2$

or $\frac{1}{5}T_2 = 62,\ T_2 = 310\,K = 310 - 273 = 37°C$

$$T_1 = \frac{6}{5}T_2 = \frac{6}{5}\times 310 = 372K$$

$$= 372 - 273 = 99°C$$

20. (2) Given,
$V_0 = 283$ volt, $\omega = 320$, $R = 5\,\Omega$, $L = 25$ mH, C = 1000 μF
$X_L = \omega L = 320 \times 25 \times 10^{-3} = 8\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{320\times 1000\times 10^{-6}} = 3.1\,\Omega$$

Total impedance of the circuit :

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{25 + (4.9)^2} = 7\,\Omega$$

Phase difference between the voltage and current

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{4.9}{5} \approx 1 \Rightarrow \phi = 45°$$

21. (170) From Doppler's effect

$$f(\text{direct}) = f\left(\frac{340}{340-5}\right) = f_1$$

$$f(\text{by wall}) = f\left(\frac{340}{340+5}\right) = f_2$$

Beats = $(f_1 - f_2)$

$$5 = f\left(\frac{340}{340-5} - \frac{340}{340+5}\right)$$

$$\Rightarrow f = 170 \text{ Hz.}$$

22. (6.67) In balance position of bridge,

$$\frac{P}{Q} = \frac{l}{(100-l)}$$

Initially neutral position is 60 cm from A, so

$$\frac{4}{60} = \frac{Q}{40} \Rightarrow Q = \frac{16}{6} = \frac{8}{3}\,\Omega$$

Now, when unknown resistance R is connected in series to P, neutral point is 80 cm from A then,

$$\frac{4+R}{80} = \frac{Q}{20}$$

$$\frac{4+R}{80} = \frac{8}{60}$$

$$R = \frac{64}{6} - 4 = \frac{64-24}{6} = \frac{40}{6}\,\Omega$$

Hence, the value of unknown resistance R is $\frac{20}{3}\,\Omega$

23. (6.275) Pitch = 1 mm
Number of divisions on circular scale = 200

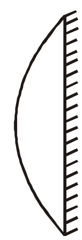
$$L.C = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

$$= \frac{1\text{ mm}}{200} = 0.005 \text{ mm} = 0.0005 \text{ cm}$$

Diameter of the wire = (Main scale reading + Circular scale reading × L.C.) – zero error
= 6 mm + 45 × 0.005 – (– 0.05)
= 6 mm + 0.225 mm + 0.05 mm = 6.275 mm

24. (20) The silvered plano convex lens behaves as a concave mirror; whose focal length is given by

$$\frac{1}{F} = \frac{2}{f_1} + \frac{1}{f_m}$$



If plane surface is silvered

$$f_m = \frac{R_2}{2} = \frac{\infty}{2} = \infty$$

$$\therefore \frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{\mu - 1}{R}$$

$$\therefore \frac{1}{F} = \frac{2(\mu-1)}{R} + \frac{1}{\infty} = \frac{2(\mu-1)}{R}$$

$$\Rightarrow F = \frac{R}{2(\mu-1)}$$

Here R = 20 cm, μ = 1.5

$$\therefore F = \frac{20}{2(1.5-1)} = 20\text{cm}$$

25. (0.25) Resistors 4 Ω, 6 Ω and 12 Ω are connected in parallel, its equivalent resistance (*R*) is given by

$$\frac{1}{R}=\frac{1}{4}+\frac{1}{6}+\frac{1}{12} \Rightarrow R=\frac{12}{6}=2\Omega$$

Again R is connected to 1.5 V battery whose internal resistance $r = 1\ \Omega$.

Equivalent resistance now,

$R' = 2\Omega + 1\Omega = 3\Omega$

Current, $I_{total} = \frac{V}{R'} = \frac{1.5}{3} = \frac{1}{2}$ A

$I_{total} = \frac{1}{2} = 3x + 2x + x = 6x \Rightarrow x = \frac{1}{12}$

$\therefore$ Current through 4Ω resistor $= 3x$

$= 3 \times \frac{1}{12} = \frac{1}{4}$ A

Therefore, rate of Joule heating in the 4Ω resistor

$= I^2R = \left(\frac{1}{4}\right)^2 \times 4 = \frac{1}{4} = 0.25$W

26. (4) Stress $= \frac{\text{Normal force}}{\text{Area}} = \frac{N}{A} = \frac{N}{(2\pi a)b}$

Stress = B×strain

$$\frac{N}{(2\pi a)b} = B\frac{2\pi a\Delta a \times b}{\pi a^2 b}$$

$$\Rightarrow N = B\frac{(2\pi a)^2 \Delta ab^2}{\pi a^2 b}$$

Force needed to push the cork.

$f = \mu N = \mu 4\pi b \Delta a B = (4\pi\mu Bb)\Delta a$

27. (40.05) $I_m = \frac{V_m}{R_f + R_L} = \frac{25}{(10+1000)} = 24.75$ mA

$I_{dc} = \frac{I_m}{\pi} = \frac{24.75}{3.14} = 7.87$ mA

$I_{rms} = \frac{I_m}{2} = \frac{24.75}{2} = 12.37$ mA

$P_{dc} = I_{dc}^2 \times R_L = (7.87 \times 10^{-3})^2 \times 10^3 = 61.9$ mW

$P_{ac} = I_{rms}{}^2(R_f + R_L) = (12.37 \times 10^{-3})^2 \times (10 + 1000)$

$= 154.54$ mW

Rectifier efficiency

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100 = \frac{61.9}{154.54} \times 100 = 40.05\%$$

28. (5.3) This is a example of uniform circular motion.

$\omega = \frac{2\pi}{T} = 2\pi n = 2\pi \times \frac{7}{100} = 0.44$ rad / sec;

$V = R\omega = 0.44 \times 12 = 5.3$ cm/sec.

29. (23) Given, $B = 4 \times 10^{-5}$ T

$R_E = 6.4 \times 10^6$ m

Dipole moment of the earth M = ?

$B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$

$$4\times10^{-5} = \frac{4\pi\times10^{-7}\times M}{4\pi\times\left(6.4\times10^6\right)^3}$$

$\therefore$ $M \cong 10^{23}$ Am^2

30. (2) Total energy, $E = \frac{1}{2}m\omega^2 a^2$;

K.E. $= \frac{3E}{4} = \frac{1}{2}m\omega^2(a^2 - y^2)$.

So, $\frac{3}{4} = \frac{a^2 - y^2}{a^2}$ or $y^2 = \frac{a^2}{4}$ or $y = \frac{a}{2}$.

CHEMISTRY

31. (1) The order of stability of resonating structures: carrying no charge > carrying minimum charge and each atom having octet complete.

32. (1) Bond order in N_2 and O_2^{2+} is 3 (calculated by energy level diagram)

33. (2) In A—O—H, if EN of 'A' is 2.1 then it will be neutral, as $X_A - X_0 = X_0 - X_H$. (where X is EN)

34. (2) Correct order is B < Be < O < N.

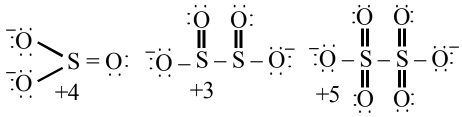
35. (4) In disproportionation reaction, one element of a compound will simultaneously get reduced and oxidised. In ClO_4^-, oxidation number of Cl is + 7 and it can not increase it further. So, ClO_4^- will not get oxidised and so will not undergo disporportionation reaction.

36. (3) $K_a = \frac{[H_3O^+][F^-]}{[HF][H_2O]}$ and

$K_b = \frac{[HF][OH^-]}{[F^-][H_2O]}$.

Therefore, $K_a \times K_b = [H_3O^+][OH^-] = K_w$.

37. (4) The chemical bond method gives the O.N.



38. (3) As Sb_2S_3 is a negative sol, so $Al_2(SO_4)_3$ will be the most effective coagulant due to higher positive charge on Al (Al^{3+}) – Hardy-Schulze rule.

39. (3) *o*-Nitrophenol is not sufficiently strong acid so as to react with $NaHCO_3$.

40. (3) Because the layer of Al_2O_3 (oxide) is inert, insoluble and impervious.

41. (4) Urotropine is used as antibiotic for urinary tract infection.

42. (4) To convert covalent compounds into ionic compounds such as NaCN, Na_2S, NaX, etc.

43. (4)

$$Zn^{2+} + 2NH_4OH \longrightarrow \underset{\text{White ppt.}}{Zn(OH)_2} + 2NH_4^+$$

$$Zn(OH)_2 + 2NH_4OH \longrightarrow \underset{\text{Soluble}}{(NH_4)_2ZnO_2} + 2H_2O$$

$$(NH_4)_2ZnO_2 + H_2S \longrightarrow \underset{\text{White ppt.}}{ZnS} + 2NH_4OH$$

44. (2) $5Fe^{2+} + MnO_4^- + 8H^+ \longrightarrow Mn^{2+} + 4H_2O + 5Fe^{3+}$

$5NO_2^- + 2MnO_4^- + 6H^+ \longrightarrow 2Mn^{2+} + 5NO_3^- + 3H_2O$

45. (3) Formaldehyde can not produce iodoform, as only those compound which contains either $CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-$ group or $CH_3-\underset{\displaystyle O}{\underset{\|}{C}}-$ group on reaction with potassium iodide and sod. hypochlorite yield iodoform.

46. (1) $2Al + 6HCl \xrightarrow{\Delta,\ air} 2AlCl_3 + 3H_2$

$2Al + 3Cl_2 \longrightarrow 2AlCl_3$

$Al_2O_3 + 3C + 3Cl_2 \xrightarrow{1000°C} \underset{\text{vapours}}{2AlCl_3} + 3CO$

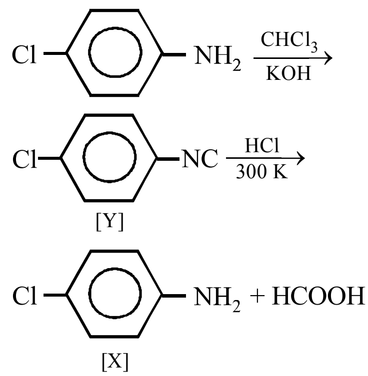
↓ Cooling

Anhydrous $AlCl_3$

$AlCl_3.6H_2O \xrightarrow{\Delta} Al(OH)_3 + 3HCl + 3H_2O$

Thus $AlCl_3$ cannot be obtained by this method.

47. (1)



48. (1) Let the rate law be $r = [A]^x[B]^y$

Divide (3) by (1) $\frac{0.10}{0.10} = \frac{[0.024]^x[0.035]^y}{[0.012]^x[0.035]^y}$

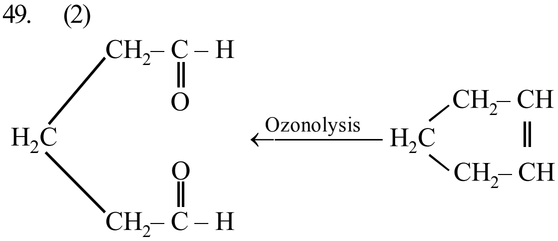
$\therefore 1 = [2]^x, x = 0$

Divide (2) by (3) $\frac{0.80}{0.10} = \frac{[0.024]^x[0.070]^y}{[0.024]^x[0.035]^y}$

$\therefore 8 = (2)^y, y = 3$

Hence rate equation, $R = K[A]^0[B]^3 = K[B]^3$

49. (2)



50. (3) $2H_2O_2 \xrightarrow{\Delta} 2H_2O + O_2$

$2KI + H_2O_2 \longrightarrow 2KOH + I_2$

$$2KMnO_4 + 3H_2SO_4 + 5H_2O_2 \longrightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 5O_2$$

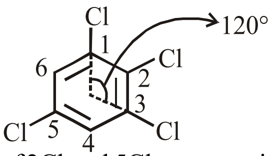
51. (1.24) Energy $= N_A h\nu$

$495.5 = 6.023 \times 10^{23} \times 6.6 \times 10^{-34} \times \nu$

$$\nu = \frac{495.5 \times 10^3}{6.023 \times 10^{23} \times 6.6 \times 10^{-34}} = 12.4 \times 10^{14}$$

$= 1.24 \times 10^{15}\ s^{-1}$.

52. (2.25)



Dipole moments of 2Cl and 5Cl are vectorically cancelled.
It is due 1 Cl and 3 Cl

$\mu^2 = \mu_1^2 + \mu_2^2 + 2\mu_1\mu_2 \cos\theta$

$= (1.5)^2 + (1.5)^2 + 2 \times 1.5 \times 1.5 \cos 120°$

$= 2.25 + 2.25 + 4.5 \times -\frac{1}{2}$

$= 2.25 + 2.25 - 2.25$

$= 2.25$ D

$\therefore \mu = 2.25$ D

53. (0.5) Moles of H_2SO_4 in 98 mg of H_2SO_4

$= \frac{1}{98} \times 0.098 = 0.001$

Moles of H_2SO_4 remove

$= \frac{3.01 \times 10^{20}}{6.02 \times 10^{23}} = 0.5 \times 10^{-3} = 0.0005$

Moles of H_2SO_4 left $= 0.001 - 0.0005$
$= 0.5 \times 10^{-3}$

54. (1.75) According to Boyle's law

$$\frac{V_1}{V_2} = \frac{P_2}{P_1}; \ \frac{750}{V_2} = \frac{360}{840}$$

$V_2 = 1750$ mL $= 1.750$ L

55. (5.97) Isoelectric point (pI)

$$= \frac{pK_{a_1} + pK_{a_2}}{2} = \frac{2.34 + 9.60}{2} = 5.97$$

56. (400) Let bond energy of A_2 be x then bond energy of AB is also x and bond energy of B_2 is $x/2$.

Enthalpy of formation of AB is – 100 kJ/mol

$A_2 + B_2 \rightarrow 2AB$;

$\frac{1}{2}A_2 + \frac{1}{2}B_2 \rightarrow AB$; $\Delta_f H = -100$ kJ mol^{-1}

or $-100 = \left(\frac{x}{2} + \frac{x}{4}\right) - x$

$\therefore -100 = \frac{2x + x - 4x}{4}$

$x = 400$ kJ mol^{-1}

57. (210) Osmotic pressure (π) of isotonic solutions are equal. For solution of unknown substance C_1(concentration).

$C_1 = \frac{5.25 / M}{V}$

Where M represents molar mass.
For solution of urea, C_2 (concentration)

$= \frac{1.5 / 60}{V}$

Given, $\pi_1 = \pi_2$

$\because \pi = CRT$

$\therefore C_1RT = C_2RT$ or $C_1 = C_2$ or

$\frac{5.25 / M}{V} = \frac{1.5 / 60}{V}$

$\therefore M = 210$ g/mol

58. (0.04) $\Delta G_3^\circ = \Delta G_1^\circ + \Delta G_2^\circ$

$-n_3FE_3^\circ = -n_1FE_1^\circ - n_2FE_2^\circ$

$E_3^\circ = \frac{n_1E_1^\circ + n_2E_2^\circ}{n_3}$

$E_3^\circ = \frac{1 \times 0.77 + 2(-0.44)}{3}$

$= \frac{0.77 - 0.88}{3} = -\frac{0.11}{3} \approx -0.04$ V

59. (6) $2MnO_2 + 4KOH + O_2 \longrightarrow 2K_2MnO_4 + 2H_2O$

Oxidation number of Mn in K_2MnO_4 is 6

K_2MnO_4; $2 + x - 8 = 0$

$x = +6$

60. (0.241) Solid AB crystallizes as NaCl structure, so it has coordination number 6 and its r^+/r^- ranges from 0.414 – 0.732.
For maximum radius of anion, we have to take the lower limit of the range 0.414 – 0.732. So, $\frac{r^+}{r^-} = 0.414$

$$\Rightarrow r^- = \frac{0.100}{0.414} \text{nm} = 0.241 \text{ nm}$$

MATHEMATICS

61. (3) $\frac{2}{\cos 2\alpha} = \frac{\tan^2\beta + 1}{\tan\beta} = \frac{1}{\sin\beta \cos\beta}$

$$\Rightarrow \sin 2\beta = \cos 2\alpha = \sin(90 - 2\alpha)$$

$$\Rightarrow \alpha + \beta = \frac{\pi}{4}$$

62. (2) $\frac{^nC_r}{^nC_{r-1}} = \frac{\lfloor n}{\lfloor r \lfloor n-r} \cdot \frac{\lfloor r-1 \lfloor n-r+1}{\lfloor n} = \frac{\lfloor n-r+1}{\lfloor n-r}$

$$= \frac{\lfloor n-r+1}{\lfloor n-r} = n - r + 1$$

$$\therefore \sum_{r=1}^{5} = n + (n-1) + (n-2) + (n-3) + (n-4)$$
$$= 5n - 10 = 5(n-2)$$

63. (2) $^{14}C_7 + \sum_{i=1}^{3} {}^{17-i}C_6$

$$= {}^{14}C_7 + {}^{14}C_6 + {}^{15}C_6 + {}^{16}C_6$$
$$= {}^{15}C_7 + {}^{15}C_6 + {}^{16}C_6$$
$$= {}^{16}C_7 + {}^{16}C_6 = {}^{17}C_7$$

64. (2) Given relation is $e^y(x+1) = 1$.
Differentiating both sides w.r.t. x, we get

$$(x+1)e^y \frac{dy}{dx} + e^y \cdot 1 = 0$$

$$\therefore \frac{dy}{dx} = -\frac{1}{x+1} \quad \text{...(i)}$$

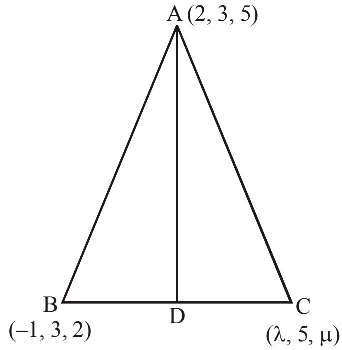
Differentiating again w.r.t. x both sides, we get

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\frac{1}{x+1}\right)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} \quad \text{...(ii)}$$

From (i) and (ii), we get $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

65. (3) Since AD is the median



$$\therefore D = \left(\frac{\lambda - 1}{2}, 4, \frac{\mu + 2}{2}\right)$$

Now, dr's of AD is

$$a = \left(\frac{\lambda - 1}{2} - 2\right) = \frac{\lambda - 5}{2}$$

$$b = 4 - 3 = 1, \quad c = \frac{\mu + 2}{2} - 5 = \frac{\mu - 8}{2}$$

Also, a, b, c are dr's
$\therefore$ $a = kl, b = km, c = kn$ where $l = m = n$ and $l^2 + m^2 + n^2 = 1$

$$\Rightarrow l = m = n = \frac{1}{\sqrt{3}}$$

Now, $a = 1, b = 1$ and $c = 1$
$\Rightarrow \lambda = 7$ and $\mu = 10$

66. (1) $a_1 = 2, b_1 = 2, c_1 = -1$ and $a_2 = 1, b_2 = 2, c_2 = 2$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{2 + 4 - 2}{\sqrt{4+4+1}\sqrt{1+4+4}} = \pm\frac{4}{9}.$$

67. (3) Contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$
$\therefore$ contrapositive of $(p \vee q) \Rightarrow r$ is
$\sim r \Rightarrow \sim (p \vee q)$ i.e. $\sim r \Rightarrow (\sim p \wedge \sim q)$

68. (1) Let the co-ordiante of other ends are (x, y, z).
The centre of sphere is C(3, 6, 1)

Therefore, $\frac{x+2}{2}=3 \Rightarrow x=4$

$\frac{y+3}{2}=6 \Rightarrow y=9$ and $\frac{z+5}{2}=1 \Rightarrow z=-3$

69. (1) $I=\int \frac{dx}{(x-\beta)\sqrt{(x-\alpha)(\beta-x)}}$

Put $x=\alpha \sin^2\theta+\beta\cos^2\theta$

$dx=2(\alpha-\beta)\sin\theta\cos\theta\, d\theta$

Also, $(x-\alpha)=(\beta-\alpha)\cos^2\theta$

$(x-\beta)=(\alpha-\beta)\sin^2\theta$

$\therefore I=\int \frac{2(\alpha-\beta)\sin\theta\cos\theta\, d\theta}{(\alpha-\beta)\sin^2\theta(\beta-\alpha)\sin\theta\cos\theta}$

$=\frac{2}{\beta-\alpha}\int\frac{d\theta}{\sin^2\theta}=\frac{2}{\beta-\alpha}\int \text{cosec}^2\theta\, d\theta$

$=\frac{2}{\beta-\alpha}(-\cot\theta)+C=\frac{2}{\alpha-\beta}\cot\theta+C$

Now, $x=\alpha\sin^2\theta+\beta\cos^2\theta$

$\Rightarrow x\,\text{cosec}^2\theta=\alpha+\beta\cot^2\theta$

$\Rightarrow x(1+\cot^2\theta)=\alpha+\beta\cot^2\theta$

$\therefore \cot\theta=\sqrt{\frac{x-\alpha}{\beta-x}}$;

$\therefore I=\frac{2}{\alpha-\beta}\sqrt{\frac{x-\alpha}{\beta-x}}+C$

70. (4) Given planes are
$P: x+y-2z+7=0$
$Q: x+y+2z+2=0$
and $R: 3x+3y-6z-11=0$
Consider Plane P and R.
Here $a_1=1, b_1=1, c_1=-2$
and $a_2=3, b_2=3, c_2=-6$
Since, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{1}{3}$
therefore P and R are parallel.

71. (1) $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+....\infty=\frac{\pi^4}{90}$

$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+.....+\infty+\frac{1}{2^4}\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+....\infty\right)$

$=\frac{\pi^4}{90}$

$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\frac{1}{7^4}+....+\infty+\frac{1}{16}\times\frac{\pi^4}{90}=\frac{\pi^4}{90}$

$\therefore \frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\frac{1}{7^4}+....+\infty=\frac{\pi^4}{90}-\frac{1}{16}\left(\frac{\pi^4}{90}\right)$

$=\frac{15}{16}\left(\frac{\pi^4}{90}\right)=\frac{\pi^4}{96}.$

72. (2) We have, $f(x)=\exp\left(\sqrt{5x-3-2x^2}\right)$

i.e., $f(x)=e^{\sqrt{5x-3-2x^2}}$

For domain of $f(x)$, $5x-3-2x^2$ should be +ve.

i.e., $5x-3-2x^2\ge 0$

$\Rightarrow 2x^2-5x+3\le 0$

(By taking –ve sign common)

$\Rightarrow 2x(x-1)-3(x-1)\le 0$

$\Rightarrow (2x-3)(x-1)\le 0$

$\Rightarrow 2x-3\le 0$ or $x-1\ge 0$

$\Rightarrow x\le\frac{3}{2}$ or $x\ge 1$

$\therefore 1\le x\le\frac{3}{2}$ i.e., $x\in\left[1,\frac{3}{2}\right]$

Hence, domain of the given function is $\left[1,\frac{3}{2}\right]$.

73. (1) Given determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(n-1)x & \cos nx & \cos(n+1)x \\ \sin(n-1)x & \sin nx & \sin(n+1)x \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 1+a^2-2a\cos x & a & a^2 \\ 0 & \cos nx & \cos(n+1)x \\ 0 & \sin nx & \sin(n+1)x \end{vmatrix}=0$$

By applying $C_1\to C_1+C_3-2\cos x\, C_2$

By expanding

$(1 + a^2 - 2a\cos x)\,[\cos nx \sin(n+1)x - \sin nx \cos(n+1)x] = 0$

Now, $(1 + a^2 - 2a\cos x)\sin(n+1-n)x = 0$

$\Rightarrow (1+a^2-2a\cos x)\sin x = 0$

$\sin x = 0$ or $\cos x = \dfrac{1+a^2}{2a}$

As $a \neq 1$ $\therefore \left(\dfrac{1+a^2}{2a}\right) > 1$

$\Rightarrow \cos x > 1$ It is not possible.

$\therefore \sin x = 0$

74. (4) We have, $a = \cos\left(\dfrac{2\pi}{7}\right) + i\sin\left(\dfrac{2\pi}{7}\right)$

$\Rightarrow a^7 = \left[\cos\left(\dfrac{2\pi}{7}\right) + i\sin\left(\dfrac{2\pi}{7}\right)\right]^7$

$= \cos 2\pi + i\sin 2\pi = 1$ ...(i)

Let $S = \alpha + \beta = (a + a^2 + a^4) + (a^3 + a^5 + a^6)$

$[\because \alpha = a + a^2 + a^4, \beta = a^3 + a^5 + a^6]$

$\Rightarrow S = a + a^2 + a^3 + a^4 + a^5 + a^6$

$= \dfrac{a(1-a^6)}{1-a}$

$\Rightarrow S = \dfrac{a-a^7}{1-a} = \dfrac{a-1}{1-a} = -1$ ...(ii)

Let $P = \alpha\beta = (a + a^2 + a^4)(a^3 + a^5 + a^6)$

$= a^4 + a^6 + a^7 + a^5 + a^7 + a^8 + a^7 + a^9 + a^{10}$

$= a^4 + a^6 + 1 + a^5 + 1 + a + 1 + a^2 + a^3$ [from Eq. (i)]

$= 3 + (a + a^2 + a^3 + a^4 + a^5 + a^6) = 3 + S$

$= 3 - 1 = 2$ [from Eqn. (ii)]

Required equation is, $x^2 - Sx + P = 0$

$\Rightarrow x^2 + x + 2 = 0$

75. (1) We have,

$$AB = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta\cos^2\phi + \cos\theta\cos\phi\sin\theta\sin\phi & \cos^2\theta\cos\phi\sin\phi + \cos\theta\sin\theta\sin^2\phi \\ \cos\theta\sin\theta\cos^2\phi + \sin^2\theta\cos\phi\sin\phi & \cos\theta\cos\phi\sin\theta\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$$

$$= \cos(\theta-\phi)\begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi \\ \sin\theta\cos\phi & \sin\theta\sin\phi \end{bmatrix}$$

Since, $AB = 0$, $\therefore \cos(\theta-\phi) = 0$

$\therefore \theta-\phi$ is an odd multiple of $\dfrac{\pi}{2}$

76. (3) $f(x) = xe^{x(1-x)}, x \in R$

$f'(x) = e^{x(1-x)}.\left[1 + x - 2x^2\right]$

$= -e^{x(1-x)}.\left[2x^2 - x - 1\right]$

$= -2e^{x(1-x)}.\left[\left(x+\dfrac{1}{2}\right)(x-1)\right]$

$f'(x) = -2e^{x(1-x)}.A$

where $A = \left(x+\dfrac{1}{2}\right)(x-1)$

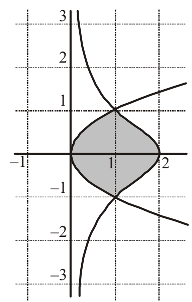
Now, exponential function is always +ve and $f'(x)$ will be opposite to the sign of A

which is –ve in $\left[-\dfrac{1}{2}, 1\right]$

Hence, $f'(x)$ is +ve in $\left[-\dfrac{1}{2}, 1\right]$

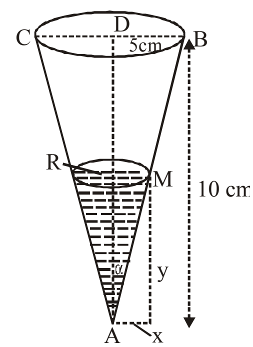
$\therefore$ $f(x)$ is increasing on $\left[-\dfrac{1}{2}, 1\right]$

77. (1) Solving curves, we get point of intersections $(1, \pm 1)$. Required area



$$= 2\int_0^1 \left(\frac{2}{1+y^2} - y^2\right)dy = \pi - \frac{2}{3}.$$

78. (4) Let y be the level of water at time t and x the radius of the surface and V, the volume of water.



We know that the volume of cone

$= \frac{1}{3}\pi$ (radius)$^2 \times$ height

$\therefore\ V = \frac{1}{3}\pi x^2 y$. Let $\angle BAD = \alpha$

$\Rightarrow \tan\alpha = \frac{BD}{AD} = \frac{5}{10} = \frac{1}{2}$.

Again, from right angled ΔAMR, we have

$\tan\alpha = \frac{MR}{AR} = \frac{x}{y}; \Rightarrow \frac{1}{2} = \frac{x}{y};\ \therefore x = \frac{y}{2}$.

$\therefore V = \frac{1}{3}\pi x^2 y = \frac{1}{3}\pi\left(\frac{y}{2}\right)^2 . y = \frac{\pi}{12}y^3$ ......(1).

By question, the rate of change of volume

$= \frac{dV}{dt} = 4$ cub.cm./min.

We have to find out the rate of increase of water-level i.e. $\frac{dy}{dt}$.

Differentiating (1) with respect to t, we get

$\frac{dV}{dt} = \frac{\pi}{12}.3y^2.\frac{dy}{dt};\ \therefore 4 = \frac{\pi}{4}y^2.\frac{dy}{dt};\ \therefore\ \frac{dy}{dt} = \frac{16}{\pi y^2}$.

When y = 6 cm,

$\frac{dy}{dt} = \frac{16}{\pi 6^2} = \frac{4}{9\pi}$ cub.cm./min.

79. (4) Slope of the tangent to $4x^2 - 9y^2 = 36$ is given by

$8x - 18y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{4x}{9y}$ or $m_1 = \frac{4x}{9y}$

Slope of the straight line, $5x + 2y - 10 = 0$ is

$m_2 = -\frac{5}{2}$

Therefore, for the perpendicularity, $m_1 m_2 = -1$

Now, $\frac{4x}{9y} \times \frac{-5}{2} = -1 \Rightarrow y = \frac{10x}{9}$.

Putting $y = \frac{10x}{9}$ in $4x^2 - 9y^2 = 36$

gives imaginary roots resulting in no tangents.

80. (1) $I = \int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx$

Put $e^{2x} = t \Rightarrow 2e^{2x}dx = dt$

When $x = \log\sqrt{\pi/2}$, $t = e^{2\log\sqrt{\pi/2}}$

$= e^{\log\pi/2} = \frac{\pi}{2}$

When $x = \log\sqrt{\pi}$, $t = e^{2\log\sqrt{\pi}} = \pi$

$\therefore\ I = \int_{\frac{\pi}{2}}^{\pi}\frac{1}{2}\sec^2\left(\frac{1}{3}t\right)dt$

$= \frac{1}{2}.\frac{1}{\frac{1}{3}}\left[\tan\frac{t}{3}.\right]_{\pi/2}^{\pi}$

$= \frac{3}{2}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{6}\right]$

$= \frac{3}{2}\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = \sqrt{3}$

81. (0) If the G.P be $a, ar, ar^2, ...$ then $a_n = ar^{n-1}$

$$D = \begin{vmatrix} \log a + (n-1)\log r & \log a + n\log r & \log a + (n+1)\log r \\ \log a + n\log r & \log a + (n+1)\log r & \log a + (n+2)\log r \\ \log a + (n+1)\log r & \log a + (n+2)\log r & \log a + (n+3)\log r \end{vmatrix}$$

$R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$ gives,

$$= \begin{vmatrix} \log a + (n-1)\log r & \log a + n\log r & \log a + (n+1)\log r \\ \log r & \log r & \log r \\ \log r & \log r & \log r \end{vmatrix}$$

= 0, since R_2 and R_3 are identical.

82. (0.80) Total no. of arrangements of the letters of the word UNIVERSITY is $\frac{10!}{2!}$.

No. of arrangements when both I's are together = 9!

So. the no. of ways in which 2 I's do not together $= \frac{10!}{2!} - 9!$

$\therefore$ Required probability

$= \frac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}} = \frac{10! - 9!\,2!}{10!}$

$= \frac{10 \times 9! - 9!.2!}{10!} = \frac{9![10-2]}{10 \times 9!}$

$= \frac{8}{10} = \frac{4}{5} = 0.80$

83. (0.25) $n(S)$ = the area of the circle of radius r

$n(E)$ = the area of the circle of radius $\frac{r}{2}$

$\therefore$ The probability $= \frac{n(E)}{n(S)} = \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}.$

84. (0.33) $P(\bar{E}E) + P(\bar{E}\bar{E}\bar{E}\bar{E}E) + \ldots\infty$

$$= \frac{5}{6}\times\frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \left(\frac{5}{6}\right)^8 \frac{1}{6} + \ldots\infty$$

$$= \frac{5}{36}\left[1 + \left(\frac{5}{6}\right)^3 + \ldots\ldots\infty\right] = \frac{30}{91}$$

85. (7.00) Given equation of ellipse is

$$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$$

eccentricity $= e = \sqrt{1 - \frac{b^2}{16}}$

foci: $\pm ae = \pm 4\sqrt{1 - \frac{b^2}{16}}$

Equation of hyperbola is $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$$\Rightarrow \frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1$$

Eccentricity $= e = \sqrt{1 + \frac{81}{25} \times \frac{25}{144}} = \sqrt{1 + \frac{81}{144}}$

$$= \sqrt{\frac{225}{144}} = \frac{15}{12}$$

foci: $\pm ae = \pm\frac{12}{5} \times \frac{15}{12} = \pm 3$

Since, foci of ellipse and hyperbola coincide

$$\therefore \pm 4\sqrt{1 - \frac{b^2}{16}} = \pm 3 \Rightarrow b^2 = 7$$

86. (1.00) For $x > 10$, $f(x) = x - 2$.

Therefore, $g(x) = x - 2 - 2 = x - 4$

$\therefore g'(x) = 1$.

87. (3.00) $\because$ A.M. $\geq$ G.M. $\Rightarrow \frac{a+b+c}{3} \geq (abc)^{1/3}$

$\Rightarrow \quad a + b + c \geq 3(abc)^{1/3}$ (1)

But given : ab^2c^3, $a^2b^3c^4$, $a^3b^4c^5$ are also in AP.

($\because abc \neq 0$)

$\Rightarrow \quad 2abc = 1 + a^2b^2c^2 \quad \Rightarrow \quad (abc - 1)^2 = 0$

$\therefore \quad abc = 1$

Now from equation (1), $a + b + c \geq 3\,(1)^{1/3}$

$\Rightarrow (a + b + c) \geq 3$

Hence, minimum value of is $a + b + c$ is 3.

88. (1.50) Sum of coefficients is obtained by simply putting x = 1 in the expression

So, sum of coefficients $= \left(\frac{2}{3}\right)^{199} \times \left(\frac{3}{2}\right)^{200}$

$$= \left(\frac{3}{2}\right)^{-199} \times \left(\frac{3}{2}\right)^{200} = \frac{3^{200-199}}{2^{200-199}} = \frac{3}{2} = 1.50$$

89. (14.00) Since $0 < y < x < 2y$

$\therefore \quad y > \frac{x}{2} \quad \Rightarrow \quad x - y < \frac{x}{2}$

$\therefore \quad x - y < y < x < 2x + y$

Hence median $= \frac{y + x}{2} = 10$

$\Rightarrow x + y = 20$ ...(i)

And range $= (2x + y) - (x - y) = x + 2y$

But range = 28

$\therefore x + 2y = 28$ ...(ii)

From equations (i) and (ii),

$x = 12, y = 8$

$\therefore$ Mean

$$= \frac{(x - y) + y + x + (2x + y)}{4} = \frac{4x + y}{4}$$

$$= x + \frac{y}{4} = 12 + \frac{8}{4} = 14$$

90. (3.20) $\int \frac{dx}{\cos^3 x\sqrt{4\sin x \cos x}}$

$$= \int \frac{dx}{2\cos^4 x\sqrt{\tan x}}$$

Let $\tan x = t^2 \Rightarrow \sec^2 x = 1 + t^4$

$\sec^2 x\, dx = 2t\, dt$

$$= \int \frac{\sec^4 x\, dx}{2\sqrt{\tan x}} = \int \frac{\sec^2 x(\sec^2 x\, dx)}{2\sqrt{\tan x}}$$

$$= \int \frac{(1 + t^4)2t\; dt}{2t} = \int (1 + t^4)\; dt$$

$$= t + \frac{t^5}{5} + k$$

$$= \sqrt{\tan x} + \frac{1}{5}\tan^{5/2} x + k\left[t = \sqrt{\tan x}\right]$$

$$A = \frac{1}{2},\; B = \frac{5}{2},\; C = \frac{1}{5}$$

$$A + B + C = \frac{16}{5} = 3.20$$