3 Chapter

Trigonometric Functions

Topic-1: Trigonometric Ratios, Domain and Range of Trigonometric Functions, Trigonometric Ratios of Allied Angles

1 MCQs with One Correct Answer

1. The value of $36\,(4\cos^2 9° - 1)(4\cos^2 27° - 1)$ $(4\cos^2 81° - 1)(4\cos^2 243° - 1)$ is **[Main April 8, 2023 (II)]**

(a) 54 (b) 18 (c) 27 (d) 36

2. $\alpha = \sin 36°$ is a root of which of the following equation **[Main June 27, 2022 (II)]**

(a) $10x^4 - 10x^2 - 5 = 0$ (b) $16x^4 + 20x^2 - 5 = 0$

(c) $16x^4 - 20x^2 + 5 = 0$ (d) $16x^4 - 10x^2 + 5 = 0$

3. The value of

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

[Main Aug. 26, 2021 (II)]

(a) $\frac{1}{4\sqrt{2}}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{8\sqrt{2}}$

4. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals: **[Main Jan. 9, 2019 (I)]**

(a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

(b) $13 - 4\cos^6\theta$

(c) $13 - 4\cos^2\theta + 6\cos^4\theta$

(d) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

5. If $2\cos\theta + \sin\theta = 1$ $\left(\theta \neq \frac{\pi}{2}\right)$, then $7\cos\theta + 6\sin\theta$ is equal to: **[Main Online April 11, 2014]**

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{11}{2}$ (d) $\frac{46}{5}$

6. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as : **[2013]**

(a) $\sin A \cos A + 1$ (b) $\sec A \operatorname{cosec} A + 1$

(c) $\tan A + \cot A$ (d) $\sec A + \operatorname{cosec} A$

7. Given both θ and ϕ are acute angles and $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to **[2004S]**

(a) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ (c) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (d) $\left(\frac{5\pi}{6}, \pi\right]$

8. If $\tan\theta = -\frac{4}{3}$, then $\sin\theta$ is **[1979]**

(a) $-\frac{4}{5}$ but not $\frac{4}{5}$ (b) $-\frac{4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) None of these

3 Numeric/ New Stem Based Questions

9. The value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$ is _____. **[Main April 6, 2023 (II)]**

6 MCQs with One or More than One Correct Answer

10. Which of the following number(s) is/are rational? **[1998 - 2 Marks]**

(a) $\sin 15°$ (b) $\cos 15°$

(c) $\sin 15° \cos 15°$ (d) $\sin 15° \cos 75°$

7 Match the Following

11. In this questions there are entries in columns 1 and 2. Each entry in column I is related to exactly one entry in column II. Write the correct letter from column 2 against the entry number in column 1 in your answer book. $\frac{\sin 3\alpha}{\cos 2\alpha}$ is

[1992 - 2 Marks]

Column I	Column II
(A) Positive	(p) $\left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$
(B) Negative	(q) $\left(\frac{14\pi}{48}, \frac{18\pi}{48}\right)$
	(r) $\left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$
	(s) $\left(0, \frac{\pi}{2}\right)$

10 Subjective Problems

12. Find the range of values of t for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. **[2005 - 2 Marks]**

Topic-2: Trigonometric Identities, Greatest and Latest Value of Trignometric Expressions

1 MCQs with One Correct Answer

1. The number of elements in the set $S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^2\theta + 2 = 0\}$ is **[Main April 11, 2023 (I)]**

(a) 10 (b) 8 (c) 9 (d) 12

2. $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ is equal to

[Main April 10, 2023 (I)]

(a) 3 (b) 2 (c) 4 (d) 1

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$, for some m, such that the range of f is [0, 2]. Then the value of m is _____

(a) 5 (b) 3 (c) 2 (d) 4

[Main Jan. 25, 2023 (II)]

4. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to **[Main July 25, 2022 (II)]**

(a) $\frac{3}{16}$ (b) $\frac{1}{16}$ (c) $\frac{1}{32}$ (d) $\frac{9}{32}$

5. If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha+\beta)$ and the quadrant in which $\alpha+\beta$ lies, respectively are

[Main June 28, 2022 (II)]

(a) $-\frac{1}{7}$ and IVth quadrant (b) 7 and Ist quadrant

(c) –7 and IVth quadrant (d) $\frac{1}{7}$ and Ist quadrant

6. The value of $\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$ is equal to:

[Main June 27, 2022 (I)]

(a) -1 (b) $-\frac{1}{2}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{4}$

7. $16\sin(20°)\sin(40°)\sin(80°)$ is equal to :

[Main June 26, 2022 (II)]

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) 3 (d) $4\sqrt{3}$

8. The value of $2\sin(12°) - \sin(72°)$ is : **[Main June 25, 2022 (II)]**

(a) $\frac{\sqrt{5}(1-\sqrt{3})}{4}$ (b) $\frac{1-\sqrt{5}}{8}$

(c) $\frac{\sqrt{3}(1-\sqrt{5})}{2}$ (d) $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

9. If $0 < \theta, \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty} \cos^{2n}\theta, y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi$ then: **[Main Feb. 25, 2021 (I)]**

(a) $xyz = 4$ (b) $xy - z = (x+y)z$

(c) $xy + yz + zx = z$ (d) $xy + z = (x+y)z$

10. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then : **[Main Sep. 05, 2020 (II)]**

(a) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$ (b) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

(c) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$ (d) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

11. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is : **[Main April 9, 2019 (II)]**

(a) $\frac{3}{4} + \cos 20°$ (b) 3/4

(c) $\frac{3}{2}(1 + \cos 20°)$ (d) 3/2

12. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to : **[Main 2014(S), Jan. 11, 2019 (I)]**

(a) $\frac{1}{12}$ (b) $\frac{1}{4}$ (c) $\frac{-1}{12}$ (d) $\frac{5}{12}$

13. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is : **[Main 2017]**

(a) $-\frac{7}{9}$ (b) $-\frac{3}{5}$ (c) $\frac{1}{3}$ (d) $\frac{2}{9}$

14. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to **[Adv. 2016]**

(a) $3 - \sqrt{3}$ (b) $2\left(3 - \sqrt{3}\right)$

(c) $2\left(\sqrt{3} - 1\right)$ (d) $2\left(2 - \sqrt{3}\right)$

15. If m and M are the minimum and the maximum values of $4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x, x \in R$, then $M - m$ is equal to : **[Main Online April 9, 2016]**

(a) $\frac{9}{4}$ (b) $\frac{15}{4}$ (c) $\frac{7}{4}$ (d) $\frac{1}{4}$

16. If $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta = \frac{1}{2}$ and θ is the the arithmetic mean of α and β, then $\sin 2\theta + \cos 2\theta$ is equal to : **[Main Online April 11, 2015]**

(a) $\frac{3}{5}$ (b) $\frac{7}{5}$ (c) $\frac{4}{5}$ (d) $\frac{8}{5}$

17. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then **[2006 - 3M, –1]**

(a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$

(c) $t_3 > t_1 > t_2 > t_4$ (d) $t_2 > t_3 > t_1 > t_4$

18. The values of $\theta \in (0, 2\pi)$ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, are **[2006 - 3M, –1]**

(a) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (b) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$

(c) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (d) $\left(\frac{41\pi}{48}, \pi\right)$

19. If $\alpha + \beta = \pi/2$ and $\beta + \gamma = \alpha$, then $\tan\alpha$ equals **[2001S]**

(a) $2(\tan\beta + \tan\gamma)$ (b) $\tan\beta + \tan\gamma$

(c) $\tan\beta + 2\tan\gamma$ (d) $2\tan\beta + \tan\gamma$

20. The maximum value of $(\cos\alpha_1).(\cos\alpha_2)\ldots(\cos\alpha_n)$, under the restrictions $0 \le \alpha_1, \alpha_2, \ldots, \alpha_n \le \frac{\pi}{2}$ and $(\cot\alpha_1).(\cot\alpha_2) \ldots (\cot\alpha_n) = 1$ is **[2001S]**

(a) $1/2^{n/2}$ (b) $1/2^n$

(c) $1/2n$ (d) 1

21. Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ is **[2000S]**

(a) ≥ 0 only when $\theta \ge 0$ (b) ≤ 0 for all real θ

(c) ≥ 0 for all real θ (d) ≤ 0 only when $\theta \le 0$

22. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$ **[1995S]**

(a) 11 (b) 12

(c) 13 (d) 14

23. Let $0 < x < \frac{\pi}{4}$ then $(\sec 2x - \tan 2x)$ equals **[1994]**

(a) $\tan\left(x - \frac{\pi}{4}\right)$ (b) $\tan\left(\frac{\pi}{4} - x\right)$

(c) $\tan\left(x + \frac{\pi}{4}\right)$ (d) $\tan^2\left(x + \frac{\pi}{4}\right)$

24. Given $A = \sin^2\theta + \cos^4\theta$ then for all real values of θ

(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 1$ **[1980]**

(c) $\frac{13}{16} \le A \le 1$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

25. If $\alpha + \beta + \gamma = 2\pi$, then **[1979]**

(a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$

(c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(d) None of these

2 Integer Value Answer/ Non-Negative Integer

26. Let α and β be real numbers such that $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$. If $\sin(\alpha + \beta) = \frac{1}{3}$ and $\cos(\alpha - \beta) = \frac{2}{3}$, then the greatest integer less than or equal to

$$\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}\right)^2$$

is _____. **[Adv. 2022]**

27. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is _____. **[Main Feb. 24, 2021 (I)]**

28. The maximum value of the expression

$$\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$$ is _____. **[2010]**

3 Numeric/ New Stem Based Questions

29. If m and n respectively are the numbers of positive and negative value of θ in the interval $[-\pi, \pi]$ that satisfy the equation $\cos 2\theta \cos\frac{\theta}{2} = \cos 3\theta \cos\frac{9\theta}{2}$, then mn is equal to _____. **[Main Jan. 25, 2023 (II)]**

30. If $\sin^2(10°)\sin(20°)\sin(40°)\sin(50°)\sin(70°) = \alpha - \frac{1}{16}\sin(10°)$, then $16 + \alpha^{-1}$ is equal to _____. **[Main June 26, 2022 (I)]**

31. Let $f : [0, 2] \to \mathbf{R}$ be the function defined by

$$f(x) = (3 - \sin(2\pi x))\sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right).$$

If $\alpha, \beta \in [0, 2]$ are such that $\{x \in [0, 2] : f(x) \ge 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is _____ **[Adv. 2020]**

32. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1 + \cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1 - \cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$, $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to _____. **[Main Jan. 8, 2020 (II)]**

4 Fill in the Blanks

33. If $A > 0, B > 0$ and $A + B = \pi/3$, then the maximum value of $\tan A \tan B$ is _____. **[1993 - 2 Marks]**

34. If $K = \sin(\pi/18)\sin(5\pi/18)\sin(7\pi/18)$, then the numerical value of K is _____. **[1993 - 2 Marks]**

35. The value of

$$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$$ is equal to _____ **[1991 - 2 Marks]**

36. Suppose $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx$ is an identity in x, where $C_0, C_1, \ldots C_n$ are constants, and $C_n \ne 0$. then the value of n is _____ **[1981 - 2 Marks]**

5 True / False

37. If $\tan A = (1 - \cos B)/\sin B$, then $\tan 2A = \tan B$. **[1983 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

38. Let $f(x) = x \sin \pi x$, $x > 0$. Then for all natural numbers n, $f'(x)$ vanishes at [Adv. 2013]

(a) A unique point in the interval $\left(n, n+\frac{1}{2}\right)$

(b) A unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

(c) A unique point in the interval (n, n + 1)

(d) Two points in the interval (n, n + 1)

39. Let $\theta, \varphi \in [0, 2\pi]$ be such that $2\cos\theta(1-\sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi - 1, \tan(2\pi-\theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$, then φ cannot satisfy [2012]

(a) $0 < \varphi < \frac{\pi}{2}$ (b) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$

(c) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ (d) $\frac{3\pi}{2} < \varphi < 2\pi$

40. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then [2009]

(a) $\tan^2 x = \frac{2}{3}$ (b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

(c) $\tan^2 x = \frac{1}{3}$ (d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

41. For a positive integer n, let $f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\ldots(1+\sec 2^n\theta)$. [1999 - 3 Marks]

Then

(a) $f_2\left(\frac{\pi}{16}\right) = 1$ (b) $f_3\left(\frac{\pi}{32}\right) = 1$

(c) $f_4\left(\frac{\pi}{64}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$

42. The minimum value of the expression $\sin\alpha + \sin\beta + \sin\gamma$, where α, β, γ are real numbers satisfying $\alpha + \beta + \gamma = \pi$ is [1995]

(a) positive (b) zero

(c) negative (d) –3

43. Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval [1994]

(a) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (b) $\left(-1, \frac{5\pi}{6}\right)$

(c) (–1, 2) (d) $\left(\frac{\pi}{6}, 2\right)$

44. The expression $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$ is equal to [1986 - 2 Marks]

(a) 0 (b) 1

(c) 3 (d) $\sin 4\alpha + \cos 6\alpha$

(e) none of these

45. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is equal to [1984 - 3 Marks]

(a) $\frac{1}{2}$ (b) $\cos\frac{\pi}{8}$ (c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

10 Subjective Problems

46. In any triangle ABC, prove that [2000 - 3 Marks]

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}.$$

47. Prove that $\sum_{k=1}^{n-1}(n-k)\cos\frac{2k\pi}{n} = -\frac{n}{2}$, where $n \geq 3$ is an integer. [1997 - 5 Marks]

48. Prove that the values of the function $\frac{\sin x \cos 3x}{\sin 3x \cos x}$ do not lie between $\frac{1}{3}$ and 3 for any real x. [1997 - 5 Marks]

49. Find the smallest positive number p for which the equation $\cos(p \sin x) = \sin(p\cos x)$ has a solution $x \in [0, 2\pi]$. [1995 - 5 Marks]

50. Determine the smallest positive value of x (in degrees) for which $\tan(x + 100°) = \tan(x + 50°)\tan(x)\tan(x - 50°)$. [1993 - 5 Marks]

51. If $\exp\{(\sin^2 x + \sin^4 x + \sin^6 x + \ldots\ldots\ldots \infty)\ln 2\}$ satisfies the equation $x^2 - 9x + 8 = 0$, find the value of $\frac{\cos x}{\cos x + \sin x}$, $0 < x < \frac{\pi}{2}$. [1991 - 4 Marks]

52. ABC is a triangle such that

$$\sin(2A + B) = \sin(C - A) = -\sin(B + 2C) = \frac{1}{2}.$$

If A, B and C are in arithmetic progression, determine the values of A, B and C. [1990 - 5 Marks]

53. Prove that $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$ [1988 - 2 Marks]

54. Show that $16\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)\cos\left(\frac{16\pi}{15}\right) = 1$ [1983 - 2 Marks]

55. Without using tables, prove that

$$(\sin 12°)(\sin 48°)(\sin 54°) = \frac{1}{8}.$$ [1982 - 2 Marks]

Topic-3: Solutions of Trignometric Equations

1 MCQs with One Correct Answer

1. If $\tan 15° + \frac{1}{\tan 75°} + \frac{1}{\tan 105°} + \tan 195° = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is: **[Main Jan. 30, 2023 (I)]**

(a) 4 (b) $4 - 2\sqrt{3}$ (c) 2 (d) $5 - \frac{3}{2}\sqrt{3}$

2. If the solution of the equation $\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1$, $x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right)$, where α β are integers, then $\alpha + \beta$ is equal to: **[Main Jan. 30, 2023 (I)]**

(a) 3 (b) 5 (c) 6 (d) 4

3. The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = 1$ **[Main Jan. 29, 2023 (II)]**

(a) $[-2, -1]$ (b) $\left[-2, -\frac{3}{2}\right]$

(c) $\left[-1, -\frac{1}{2}\right]$ (d) $\left[-\frac{3}{2}, -1\right]$

4. The number of elements in the set

$S = \left\{x \in \mathbb{R} : 2\cos\left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x}\right\}$ is :

[Main July 29, 2022 (II)]

(a) 1 (b) 3 (c) 0 (d) infinite

5. Let $S = \left\{\theta \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right)\sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}$

Then **[Main July 27, 2022 (II)]**

(a) $S = \left\{\frac{\pi}{12}\right\}$ (b) $S = \left\{\frac{2\pi}{3}\right\}$

(c) $\sum_{\theta \in S} \theta = \frac{\pi}{2}$ (d) $\sum_{\theta \in S} \theta = \frac{3\pi}{4}$

6. Let $S = \left\{\theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\right\}$. Then

$n(S) + \sum_{\theta \in S}\left(\sec\left(\frac{\pi}{4} + 2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right)\right)$ is equal to:

[Main July 26, 2022 (I)]

(a) 0 (b) -2 (c) -4 (d) 12

7. The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is: **[Main July 25, 2022 (I)]**

(a) 4 (b) 6 (c) 8 (d) 12

8. Let $S = \left\{\theta \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}$.

If $T = \sum_{\theta \in S} \cos 2\theta$, then $T + n(S)$ is equal

[Main June 24, 2022 (I)]

(a) $7 + \sqrt{3}$ (b) 9 (c) $8 + \sqrt{3}$ (d) 10

9. The number of solutions of the equation $\cos\left(x + \frac{\pi}{3}\right)\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4}\cos^2 2x$, $x \in [-3\pi, 3\pi]$ is :

[Main June 24, 2022 (II)]

(a) 8 (b) 5 (c) 6 (d) 7

10. If n is the number of solutions of the equation $2\cos x\left(4\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) - 1\right) = 1$, $x \in [0, \pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is : **[Main Sep. 1, 2021 (II)]**

(a) $(3, 13\pi/9)$ (b) $(2, 2\pi/3)$

(c) $(2, 8\pi/9)$ (d) $(3, 5\pi/3)$

11. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval: **[Main Sep. 02, 2020 (II)]**

(a) $\left(-\frac{5}{4}, -1\right)$ (b) $\left[-1, -\frac{1}{2}\right]$

(c) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$ (d) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$

12. If $[x]$ denotes the greatest integer $\le x$, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$

$[\cot\theta]x + y = 0$ **[Main April 12, 2019 (II)]**

(a) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(b) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(c) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(d) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

13. Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is: **[Main April 9, 2019 (I)]**

(a) $\frac{13\pi}{6}$ (b) $\frac{5\pi}{3}$ (c) 2π (d) π

14. If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0, \pi]$ is $k\pi$, then k is equal to : **[Main 2018]**

(a) $\frac{13}{9}$ (b) $\frac{8}{9}$ (c) $\frac{20}{9}$ (d) $\frac{2}{3}$

15. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \text{cosec}\, x + 2(\tan x - \cot x) = 0$ in the set S is equal to **[Adv. 2016]**

(a) $-\frac{7\pi}{9}$ (b) $-\frac{2\pi}{9}$ (c) 0 (d) $\frac{5\pi}{9}$

16. The number of $x \in [0, 2\pi]$ for which

$$\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1 \text{ is}$$

[Main Online April 9, 2016]

(a) 2 (b) 6 (c) 4 (d) 8

17. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has **[Adv. 2014]**

(a) infinitely many solutions
(b) three solutions
(c) one solution
(d) no solution

18. The number of values of α in $[0, 2\pi]$ for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$, is: **[Main Online April 9, 2014]**

(a) 6 (b) 4 (c) 3 (d) 1

19. The number of solutions of the pair of equations

$2\sin^2\theta - \cos 2\theta = 0$

$2\cos^2\theta - 3\sin\theta = 0$

in the interval $[0, 2\pi]$ is **[2007 - 3 Marks]**

(a) zero (b) one (c) two (d) four

20. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$ where $\alpha, \beta \in [-\pi, \pi]$. Pairs of α, β which satisfy both the equations is/are **[2005S]**

(a) 0 (b) 1 (c) 2 (d) 4

21. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is **[2002S]**

(a) 4 (b) 8 (c) 10 (d) 12

22. In a triangle PQR, $\angle R = \pi/2$. If $\tan(P/2)$ and $\tan(Q/2)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$ then. **[1999 - 2 Marks]**

(a) $a + b = c$ (b) $b + c = a$
(c) $a + c = b$ (d) $b = c$

23. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if **[1996 - 1 Mark]**

(a) $x + y \neq 0$ (b) $x = y, x \neq 0$
(c) $x = y$ (d) $x \neq 0, y \neq 0$

24. The general values of θ satisfying the equation $2\sin^2\theta - 3\sin\theta - 2 = 0$ is **[1995S]**

(a) $n\pi + (-1)^n \pi/6$ (b) $n\pi + (-1)^n \pi/2$

(c) $n\pi + (-1)^n 5\pi/6$ (d) $n\pi + (-1)^n 7\pi/6$

25. Let n be a positive integer such that

$\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$. Then **[1994]**

(a) $6 \leq n \leq 8$ (b) $4 < n \leq 8$
(c) $4 \leq n \leq 8$ (d) $4 < n < 8$

26. Number of solutions of the equation **[1993 - 1 Mark]**

$\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is :

(a) 0 (b) 1 (c) 2 (d) 3

27. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ In the variable x, has real roots. Then p can take any value in the interval **[1990 - 2 Marks]**

(a) $(0, 2\pi)$ (b) $(-\pi, 0)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

28. The general solution of

$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is **[1989 - 2 Marks]**

(a) $n\pi + \frac{\pi}{8}$ (b) $\frac{n\pi}{2} + \frac{\pi}{8}$

(c) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (d) $2n\pi + \cos^{-1}\frac{3}{2}$

29. The value of the expression $\sqrt{3}\,\text{cosec}\, 20° - \sec 20°$ is equal to **[1988 - 2 Marks]**

(a) 2 (b) $2\sin 20°/\sin 40°$
(c) 4 (d) $4\sin 20°/\sin 40°$

30. The general solution of the trigonometric equation $\sin x + \cos x = 1$ is given by : **[1981 - 2 Marks]**

(a) $x = 2n\pi$; $n = 0, \pm 1, \pm 2 \ldots$

(b) $x = 2n\pi + \pi/2$; $n = 0, \pm 1, \pm 2 \ldots$

(c) $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$, where $n = 0, \pm 1, \pm 2 \ldots$

(d) none of these

31. The equation $2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2}$; $0 < x \leq \frac{\pi}{2}$ has **[1980]**

(a) no real solution (b) one real solution
(c) more than one solution (d) none of these

2 Integer Value Answer/ Non-Negative Integer

32. The number of distinct solutions of the equation

$$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

in the interval $[0, 2\pi]$ is **[Adv. 2015]**

33. The positive integer value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$$ is **[2011]**

34. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart . If the chords subtend at the center , angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of $[k]$ is **[2010]**

[**Note :** $[k]$ denotes the largest integer less than or equal to k]

35. The number of values of θ in the interval, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is **[2010]**

36. The number of all possible values of θ where $0 < \theta < \pi$, for which the system of equations

$$(y+z)\cos 3\theta = (xyz)\sin 3\theta$$

$$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is **[2010]**

3 Numeric/ New Stem Based Questions

37. Let $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$.

Then $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to

[Main Jan. 24, 2023 (II)]

38. The number of elements in the set

$$S = \{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\}$$

is _____. **[Main June 29, 2022 (I)]**

39. The number of integral values of 'k' for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is _____.

[Main Feb. 26, 2021 (I)]

40. The number of distinct solutions of the equation,

$\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is _____. **[Main Jan. 9, 2020 (I)]**

41. Let a, b, c be three non-zero real numbers such that the equation : $\sqrt{3}a\cos x + 2b\sin x = c, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is _____. **[Adv. 2018]**

4 Fill in the Blanks

42. The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are ..., ..., and _____ .

[1997 - 2 Marks]

43. General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is_____ . **[1996 - 1 Mark]**

44. The sides of a triangle inscribed in a given circle subtend angles α, β and γ at the centre. The minimum value of the arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right)$, $\cos\left(\beta + \frac{\pi}{2}\right)$ and $\cos\left(\gamma + \frac{\pi}{2}\right)$ is equal to _____ **[1987 - 2 Marks]**

45. The set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \geq 0$, is _____ . **[1987 - 2 Marks]**

46. The solution set of the system of equations $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x and y are real, is _____ .

[1987 - 2 Marks]

5 True / False

47. There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$. **[1984 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

48. Let α and β be non–zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true? **[Adv. 2017]**

(a) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

(b) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$

(c) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

(d) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

49. The number of points in $(-\infty\ \infty)$, for which $x^2 - x\sin x - \cos x = 0$, is **[Adv. 2013]**

(a) 6 (b) 4 (c) 2 (d) 0

50. For $0 < \theta < \frac{\pi}{2}$, the solution (s) of

$$\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$$

is (are) **[2009]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{12}$ (d) $\frac{5\pi}{12}$

7 Match the Following

53. Consider the following lists: **[Adv. 2022]**

List-I		List-II	
(I)	$\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P)	has two elements
(II)	$\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q)	has three elements
(III)	$\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R)	has four elements
(IV)	$\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S)	has five elements
		(T)	has six elements

The correct option is:

(a) (I) → (P); (II) → (S); (III) → (P); (IV) → (S) (b) (I) → (P); (II) →(P); (III) → (T); (IV) → (R)

(c) (I) → (Q); (II) → (P); (III) → (T); (IV) → (S) (d) (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)

54. Let $f(x) = \sin(\pi\cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order.

$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$

$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$

Column - I contains the sets X, Y, Z and W. Column - II contains some information regarding these sets. **[Adv. 2019]**

Column I		Column II	
(I)	X	(p)	$\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(q)	an arithmetic progression
(III)	Z	(r)	NOT an arithmetic progression
(IV)	W	(s)	$\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(t)	$\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(u)	$\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination?

(a) (IV), (p), (r), (s) (b) (III), (p), (q), (u) (c) (III), (r), (u) (d) (IV), (q), (t)

51. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is **[1998 - 2 Marks]**

(a) 0 (b) 5 (c) 6 (d) 10

52. The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2\cos(2x) + a_3\sin^2(x) = 0$ for all x is **[1987 - 2 Marks]**

(a) zero (b) one

(c) three (d) infinite

55. Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order.

$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$

$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$

Column - I contains the sets X, Y, Z and W. Column - II contains some information regarding these sets.

[Adv. 2019]

Column I	Column II
(I) X	(p) $\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II) Y	(q) an arithmetic progression
(III)Z	(r) NOT an arithmetic progression
(IV) W	(s) $\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
	(t) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(u) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination?

(a) (I), (q), (u) (b) (I), (p), (r) (c) (II), (r), (s) (d) (II), (q), (t)

10 Subjective Problems

56. Find all values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$.

[1996 - 2 Marks]

57. Find the values of $x \in (-\pi, +\pi)$ which satisfy the equation $8^{(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+.....)} = 4^3$ **[1984 - 2 Marks]**

Answer Key

Topic-1 : Trigonometric Ratios, Domain and Range of Trigonometric Functions, Trigonometric Ratios of Allied Angles

1.	(d)	**2.**	(c)	**3.**	(c)	**4.**	(b)	**5.**	(d)	**6.**	(b)	**7.**	(b)	**8.**	(b)				
9.	(4)	**10.**	(c)	**11.**	(A → r; B → p)														

Topic-2 : Trigonometric Identities, Greatest and Least Value of Trigonometric Expressions

1.	(c)	**2.**	(a)	**3.**	(a)	**4.**	(b)	**5.**	(a)	**6.**	(b)	**7.**	(b)	**8.**	(d)	**9.**	(d)	**10.**	(d)
11.	(b)	**12.**	(a)	**13.**	(a)	**14.**	(c)	**15.**	(b)	**16.**	(b)	**17.**	(b)	**18.**	(a)	**19.**	(c)	**20.**	(a)
21.	(c)	**22.**	(c)	**23.**	(b)	**24.**	(b)	**25.**	(a)	**26.**	(1)	**27.**	(9)	**28.**	(2)	**29.**	(25)	**30.**	(80)
31.	(1)	**32.**	(1)	**33.**	(1/3)	**34.**	(1/8)	**35.**	(1/64)	**36.**	(6)	**37.**	(True)	**38.**	(b, c)	**39.**	(a, c, d)		
40.	(a, b)	**41.**	(a, b, c, d)			**42.**	(c)	**43.**	(d)	**44.**	(b)	**45.**	(c)						

Topic-3 : Solutions of Trigonometric Equations

1.	(a)	**2.**	(d)	**3.**	(d)	**4.**	(a)	**5.**	(c)	**6.**	(c)	**7.**	(c)	**8.**	(b)	**9.**	(d)	**10.**	(a)
11.	(b)	**12.**	(a)	**13.**	(c)	**14.**	(a)	**15.**	(c)	**16.**	(d)	**17.**	(d)	**18.**	(c)	**19.**	(c)	**20.**	(d)
21.	(b)	**22.**	(a)	**23.**	(b)	**24.**	(d)	**25.**	(d)	**26.**	(c)	**27.**	(d)	**28.**	(b)	**29.**	(c)	**30.**	(c)
31.	(a)	**32.**	(8)	**33.**	(7)	**34.**	(3)	**35.**	(3)	**36.**	(3)	**37.**	(2)	**38.**	(32)	**39.**	(11)	**40.**	(8)
41.	(0.5)	**42.**	$-\frac{\pi}{2}, \frac{\pi}{2}, 0$			**43.**	$n\pi, n\pi \pm \frac{\pi}{3}$			**44.**	$-\frac{\sqrt{3}}{2}$	**45.**	$\left[0, \frac{\pi}{6}\right] \cup \left\{\frac{\pi}{2}\right\} \cup \left[\frac{5\pi}{6}, \pi\right]$					**46.**	φ
47.	(False)	**48.**	(a, c)	**49.**	(c)	**50.**	(c, d)	**51.**	(c)	**52.**	(d)	**53.**	(b)	**54.**	(a)	**55.**	(d)		