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Trigonometric Functions



Topic-1: Trigonometric Ratios, Domain and Range of Trigonometric Functions, Trigonometric Ratios of Allied Angles



1 MCQs with One Correct Answer

1. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as : [2013]
- (a) $\sin A \cos A + 1$ (b) $\sec A \cosec A + 1$
(c) $\tan A + \cot A$ (d) $\sec A + \cosec A$
2. Given both θ and ϕ are acute angles and $\sin \theta = \frac{1}{2}$,
 $\cos \phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to [2004S]
- (a) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ (c) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right)$ (d) $\left(\frac{5\pi}{6}, \pi\right)$
3. If $\tan \theta = -\frac{4}{3}$, then $\sin \theta$ is [1979]
- (a) $-\frac{4}{5}$ but not $\frac{4}{5}$ (b) $-\frac{4}{5}$ or $\frac{4}{5}$
(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) None of these



6 MCQs with One or More than One Correct Answer

4. Which of the following number(s) is/are rational? [1998 - 2 Marks]
- (a) $\sin 15^\circ$ (b) $\cos 15^\circ$
(c) $\sin 15^\circ \cos 15^\circ$ (d) $\sin 15^\circ \cos 75^\circ$



7 Match the Following

5. In this questions there are entries in columns 1 and 2. Each entry in column I is related to exactly one entry in column II. Write the correct letter from column 2 against the entry number in column 1 in your answer book.

$\frac{\sin 3\alpha}{\cos 2\alpha}$ is

[1992 - 2 Marks]

Column I

(A) Positive

(B) Negative

Column II

(p) $\left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$

(q) $\left(\frac{14\pi}{48}, \frac{18\pi}{48}\right)$

(r) $\left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$

(s) $\left(0, \frac{\pi}{2}\right)$



10 Subjective Problems

6. Find the range of values of t for which $2 \sin t$
 $= \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. [2005 - 2 Marks]



Topic-2: Trigonometric Identities, Greatest and Latest Value of Trigonometric Expressions



1 MCQs with One Correct Answer

1. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to [Adv. 2016]

- (a) $3 - \sqrt{3}$ (b) $2(3 - \sqrt{3})$
 (c) $2(\sqrt{3} - 1)$ (d) $2(2 - \sqrt{3})$

2. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then [2006-3M,-1]
 (a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$
 (c) $t_3 > t_1 > t_2 > t_4$ (d) $t_2 > t_3 > t_1 > t_4$

3. The values of $\theta \in (0, 2\pi)$ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, are [2006-3M,-1]

- (a) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (b) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
 (c) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (d) $\left(\frac{41\pi}{48}, \pi\right)$

4. If $\alpha + \beta = \pi/2$ and $\beta + \gamma = \alpha$, then $\tan \alpha$ equals [2001S]
 (a) $2(\tan\beta + \tan\gamma)$ (b) $\tan\beta + \tan\gamma$
 (c) $\tan\beta + 2\tan\gamma$ (d) $2\tan\beta + \tan\gamma$
5. The maximum value of $(\cos \alpha_1).(\cos \alpha_2) \dots (\cos \alpha_n)$, under the restrictions

- $0 \leq \alpha_1, \alpha_2, \dots, \alpha_n \leq \frac{\pi}{2}$ and $(\cot \alpha_1).(\cot \alpha_2) \dots (\cot \alpha_n) = 1$ is [2001S]

- (a) $1/2^{n/2}$ (b) $1/2^n$
 (c) $1/2n$ (d) 1

6. Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ is [2000S]
 (a) ≥ 0 only when $\theta \geq 0$ (b) ≤ 0 for all real θ
 (c) ≥ 0 for all real θ (d) ≤ 0 only when $\theta \leq 0$

7. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$ [1995S]

- (a) 11 (b) 12
 (c) 13 (d) 14

8. Let $0 < x < \frac{\pi}{4}$ then $(\sec 2x - \tan 2x)$ equals [1994]

- (a) $\tan\left(x - \frac{\pi}{4}\right)$ (b) $\tan\left(\frac{\pi}{4} - x\right)$
 (c) $\tan\left(x + \frac{\pi}{4}\right)$ (d) $\tan^2\left(x + \frac{\pi}{4}\right)$

9. Given $A = \sin^2 \theta + \cos^4 \theta$ then for all real values of θ

- (a) $1 \leq A \leq 2$ (b) $\frac{3}{4} \leq A \leq 1$ [1980]
 (c) $\frac{13}{16} \leq A \leq 1$ (d) $\frac{3}{4} \leq A \leq \frac{13}{16}$

10. If $\alpha + \beta + \gamma = 2\pi$, then [1979]

- (a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2} \tan\frac{\beta}{2} \tan\frac{\gamma}{2}$
 (b) $\tan\frac{\alpha}{2} \tan\frac{\beta}{2} + \tan\frac{\beta}{2} \tan\frac{\gamma}{2} + \tan\frac{\gamma}{2} \tan\frac{\alpha}{2} = 1$
 (c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2} \tan\frac{\beta}{2} \tan\frac{\gamma}{2}$
 (d) None of these

2 Integer Value Answer/ Non-Negative Integer

11. Let α and β be real numbers such that

$-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$. If $\sin(\alpha + \beta) = \frac{1}{3}$ and $\cos(\alpha - \beta)$

$= \frac{2}{3}$, then the greatest integer less than or equal to

$$\left(\frac{\sin \alpha}{\cos \beta} + \frac{\cos \beta}{\sin \alpha} + \frac{\cos \alpha}{\sin \beta} + \frac{\sin \beta}{\cos \alpha} \right)^2$$

is _____. [Adv. 2022]

12. The maximum value of the expression

$$\frac{1}{\sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta} \text{ is } _____.$$
 [2010]

3 Numeric/ New Stem Based Questions

13. Let $f : [0, 2] \rightarrow \mathbf{R}$ be the function defined by

$$f(x) = (3 - \sin(2\pi x)) \sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right).$$

If $\alpha, \beta \in [0, 2]$ are such that

$\{x \in [0, 2] : f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is _____ [Adv. 2020]



4 Fill in the Blanks

14. If $A > 0, B > 0$ and $A + B = \pi/3$, then the maximum value of $\tan A \tan B$ is _____. [1993 - 2 Marks]

15. If $K = \sin(\pi/18)\sin(5\pi/18)\sin(7\pi/18)$, then the numerical value of K is _____. [1993 - 2 Marks]

16. The value of

$\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \sin \frac{7\pi}{14} \sin \frac{9\pi}{14} \sin \frac{11\pi}{14} \sin \frac{13\pi}{14}$ is equal to _____ [1991 - 2 Marks]

17. Suppose $\sin^3 x \sin 3x = \sum_{m=0}^n C_m \cos mx$ is an identity in x,

where C_0, C_1, \dots, C_n are constants, and $C_n \neq 0$. then the value of n is _____ [1981 - 2 Marks]



5 True / False

18. If $\tan A = (1 - \cos B) / \sin B$, then $\tan 2A = \tan B$. [1983 - 1 Mark]



6 MCQs with One or More than One Correct Answer

19. Let $f(x) = x \sin \pi x$, $x > 0$. Then for all natural numbers n, $f'(x)$ vanishes at [Adv. 2013]

- (a) A unique point in the interval $\left(n, n + \frac{1}{2}\right)$
 (b) A unique point in the interval $\left(n + \frac{1}{2}, n + 1\right)$
 (c) A unique point in the interval $(n, n + 1)$
 (d) Two points in the interval $(n, n + 1)$

20. Let $\theta, \phi \in [0, 2\pi]$ be such that $2 \cos \theta (1 - \sin \phi)$

$$= \sin^2 \theta \left(\tan \frac{\theta}{2} + \cot \frac{\theta}{2} \right) \cos \phi - 1, \tan(2\pi - \theta) > 0 \text{ and}$$

$$-1 < \sin \theta < -\frac{\sqrt{3}}{2}, \text{ then } \phi \text{ cannot satisfy} \quad [2012]$$

- (a) $0 < \phi < \frac{\pi}{2}$ (b) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$
 (c) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ (d) $\frac{3\pi}{2} < \phi < 2\pi$

21. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then [2009]

(a) $\tan^2 x = \frac{2}{3}$ (b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

(c) $\tan^2 x = \frac{1}{3}$ (d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

22. For a positive integer n, let $f_n(\theta) = \left(\tan \frac{\theta}{2} \right) (1 + \sec \theta) (1 + \sec 2\theta) (1 + \sec 4\theta) \dots (1 + \sec 2^n \theta)$. Then [1999 - 3 Marks]

(a) $f_2 \left(\frac{\pi}{16} \right) = 1$ (b) $f_3 \left(\frac{\pi}{32} \right) = 1$

(c) $f_4 \left(\frac{\pi}{64} \right) = 1$ (d) $f_5 \left(\frac{\pi}{128} \right) = 1$

23. The minimum value of the expression $\sin \alpha + \sin \beta + \sin \gamma$, where α, β, γ are real numbers satisfying $\alpha + \beta + \gamma = \pi$ is [1995]

- (a) positive (b) zero
 (c) negative (d) -3

24. Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval [1994]

(a) $\left(\frac{\pi}{6}, \frac{5\pi}{6} \right)$ (b) $\left(-1, \frac{5\pi}{6} \right)$

(c) $(-1, 2)$ (d) $\left(\frac{\pi}{6}, 2 \right)$

25. The expression $3 \left[\sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4 (3\pi + \alpha) \right] - 2 \left[\sin^6 \left(\frac{\pi}{2} + \alpha \right) + \sin^6 (5\pi - \alpha) \right]$ is equal to [1986 - 2 Marks]

- (a) 0 (b) 1
 (c) 3 (d) $\sin 4\alpha + \cos 6\alpha$
 (e) none of these

26. $\left(1 + \cos \frac{\pi}{8} \right) \left(1 + \cos \frac{3\pi}{8} \right) \left(1 + \cos \frac{5\pi}{8} \right) \left(1 + \cos \frac{7\pi}{8} \right)$ is equal to [1984 - 3 Marks]

- (a) $\frac{1}{2}$ (b) $\cos \frac{\pi}{8}$ (c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

10 Subjective Problems

27. In any triangle ABC, prove that [2000 - 3 Marks]

$$\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}.$$

28. Prove that $\sum_{k=1}^{n-1} (n-k) \cos \frac{2k\pi}{n} = -\frac{n}{2}$, where $n \geq 3$ is an integer. [1997 - 5 Marks]

29. Prove that the values of the function $\frac{\sin x \cos 3x}{\sin 3x \cos x}$ do not lie between $\frac{1}{3}$ and 3 for any real x. [1997 - 5 Marks]

30. Find the smallest positive number p for which the equation $\cos(p \sin x) = \sin(p \cos x)$ has a solution $x \in [0, 2\pi]$.

[1995 - 5 Marks]

31. Determine the smallest positive value of x (in degrees) for which $\tan(x + 100^\circ) = \tan(x + 50^\circ) \tan(x) \tan(x - 50^\circ)$.

[1993 - 5 Marks]

32. If $\exp\{(\sin^2 x + \sin^4 x + \sin^6 x + \dots \infty) \ln 2\}$ satisfies the equation $x^2 - 9x + 8 = 0$, find the value of $\frac{\cos x}{\cos x + \sin x}$, $0 < x < \frac{\pi}{2}$.

[1991 - 4 Marks]

33. ABC is a triangle such that

$$\sin(2A + B) = \sin(C - A) = -\sin(B + 2C) = \frac{1}{2}.$$

If A , B and C are in arithmetic progression, determine the values of A , B and C .

[1990 - 5 Marks]

34. Prove that $\tan \alpha + 2 \tan 2\alpha + 4 \tan 4\alpha + 8 \cot 8\alpha = \cot \alpha$

[1988 - 2 Marks]

35. Show that $16 \cos\left(\frac{2\pi}{15}\right) \cos\left(\frac{4\pi}{15}\right) \cos\left(\frac{8\pi}{15}\right) \cos\left(\frac{16\pi}{15}\right) = 1$

[1983 - 2 Marks]

36. Without using tables, prove that

$$(\sin 12^\circ)(\sin 48^\circ)(\sin 54^\circ) = \frac{1}{8}.$$

[1982 - 2 Marks]



Topic-3: Solutions of Trigonometric Equations



1 MCQs with One Correct Answer

1. Let $S = \left\{ x \in (-\pi, \pi) : x \neq 0, \pm \frac{\pi}{2} \right\}$. The sum of all distinct

solutions of the equation $\sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

[Adv. 2016] (a) $-\frac{7\pi}{9}$ (b) $-\frac{2\pi}{9}$ (c) 0 (d) $\frac{5\pi}{9}$

2. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

[Adv. 2014] (a) infinitely many solutions (b) three solutions (c) one solution (d) no solution

3. The number of solutions of the pair of equations

$$\begin{aligned} 2\sin^2 \theta - \cos 2\theta &= 0 \\ 2\cos^2 \theta - 3\sin \theta &= 0 \end{aligned}$$

in the interval $[0, 2\pi]$ is

[2007 - 3 Marks] (a) zero (b) one (c) two (d) four

4. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$ where $\alpha, \beta \in [-\pi, \pi]$. Pairs of α, β which satisfy both the equations is/ are

M[2005S] (a) 0 (b) 1 (c) 2 (d) 4

5. The number of integral values of k for which the equation $7 \cos x + 5 \sin x = 2k + 1$ has a solution is

[2002S] (a) 4 (b) 8 (c) 10 (d) 12

6. In a triangle PQR , $\angle R = \pi/2$. If $\tan(P/2)$ and $\tan(Q/2)$

are the roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$) then.

[1999 - 2 Marks]

(a) $a + b = c$ (b) $b + c = a$
(c) $a + c = b$ (d) $b = c$

7. $\sec^2 \theta = \frac{4xy}{(x+y)^2}$ is true if and only if

[1996 - 1 Mark]

(a) $x + y \neq 0$ (b) $x = y, x \neq 0$
(c) $x = y$ (d) $x \neq 0, y \neq 0$

8. The general values of θ satisfying the equation $2\sin^2 \theta - 3\sin \theta - 2 = 0$ is

[1995S]

(a) $n\pi + (-1)^n \pi/6$ (b) $n\pi + (-1)^n \pi/2$
(c) $n\pi + (-1)^n 5\pi/6$ (d) $n\pi + (-1)^n 7\pi/6$

9. Let n be a positive integer such that

$$\sin \frac{\pi}{2n} + \cos \frac{\pi}{2n} = \frac{\sqrt{n}}{2}. \text{ Then}$$

[1994]

(a) $6 \leq n \leq 8$ (b) $4 < n \leq 8$
(c) $4 \leq n \leq 8$ (d) $4 < n < 8$

10. Number of solutions of the equation

$\tan x + \sec x = 2 \cos x$ lying in the interval $[0, 2\pi]$ is :

[1993 - 1 Mark]

(a) 0 (b) 1 (c) 2 (d) 3

11. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$

In the variable x , has real roots. Then p can take any value in the interval

[1990 - 2 Marks]

(a) $(0, 2\pi)$ (b) $(-\pi, 0)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

12. The general solution of $\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$ is

[1989 - 2 Marks]

(a) $n\pi + \frac{\pi}{8}$ (b) $\frac{n\pi}{2} + \frac{\pi}{8}$
(c) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (d) $2n\pi + \cos^{-1} \frac{3}{2}$

13. The value of the expression $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$ is equal to [1988 - 2 Marks]
 (a) 2 (b) $2 \sin 20^\circ / \sin 40^\circ$
 (c) 4 (d) $4 \sin 20^\circ / \sin 40^\circ$
14. The general solution of the trigonometric equation $\sin x + \cos x = 1$ is given by : [1981 - 2 Marks]
 (a) $x = 2n\pi ; n=0, \pm 1, \pm 2 \dots$
 (b) $x = 2n\pi + \pi/2 ; n = 0, \pm 1, \pm 2 \dots$
 (c) $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$, where $n = 0, \pm 1, \pm 2 \dots$
 (d) none of these
15. The equation $2\cos^2 \frac{x}{2} \sin^2 x = x^2 + x^{-2} ; 0 < x \leq \frac{\pi}{2}$ has [1980]
 (a) no real solution
 (b) one real solution
 (c) more than one solution
 (d) none of these



2 Integer Value Answer/Non-Negative Integer

16. The number of distinct solutions of the equation $\frac{5}{4} \cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is [Adv. 2015]
17. The positive integer value of $n > 3$ satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is [2011]
18. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of $[k]$ is [2010]
 [Note : $[k]$ denotes the largest integer less than or equal to k]

19. The number of values of θ in the interval, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan \theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is [2010]
20. The number of all possible values of θ where $0 < \theta < \pi$, for which the system of equations $(y+z)\cos 3\theta = (xyz)\sin 3\theta$
 $x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$
 $(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$ have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is [2010]



3 Numeric/New Stem Based Questions

21. Let a, b, c be three non-zero real numbers such that the equation : $\sqrt{3}a \cos x + 2b \sin x = c, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is _____. [Adv. 2018]



4 Fill in the Blanks

22. The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are ..., ..., and _____. [1997 - 2 Marks]
23. General value of θ satisfying the equation $\tan^2 \theta + \sec 2\theta = 1$ is _____. [1996 - 1 Mark]
24. The sides of a triangle inscribed in a given circle subtend angles α, β and γ at the centre. The minimum value of the arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right), \cos\left(\beta + \frac{\pi}{2}\right)$ and $\cos\left(\gamma + \frac{\pi}{2}\right)$ is equal to _____. [1987 - 2 Marks]
25. The set of all x in the interval $[0, \pi]$ for which $2 \sin^2 x - 3 \sin x + 1 \geq 0$, is _____. [1987 - 2 Marks]
26. The solution set of the system of equations $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x and y are real, is _____. [1987 - 2 Marks]



5 True / False

27. There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4 \theta - 2 \sin^2 \theta - 1 = 0$. [1984 - 1 Mark]



6 MCQs with One or More than One Correct Answer

28. Let α and β be non-zero real numbers such that $2(\cos \beta - \cos \alpha) + \cos \alpha \cos \beta = 1$. Then which of the following is/are true? [Adv. 2017]
- (a) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3} \tan\left(\frac{\beta}{2}\right) = 0$
 (b) $\sqrt{3} \tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$
 (c) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3} \tan\left(\frac{\beta}{2}\right) = 0$
 (d) $\sqrt{3} \tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

29. The number of points in $(-\infty, \infty)$, for which $x^2 - x \sin x - \cos x = 0$, is [Adv. 2013]
 (a) 6 (b) 4 (c) 2 (d) 0

30. For $0 < \theta < \frac{\pi}{2}$, the solution (s) of

$$\sum_{m=1}^6 \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$$

is (are) [2009]

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{12}$ (d) $\frac{5\pi}{12}$
31. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3 \sin^2 x - 7 \sin x + 2 = 0$ is [1998 - 2 Marks]
 (a) 0 (b) 5 (c) 6 (d) 10
32. The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2 \cos(2x) + a_3 \sin^2(x) = 0$ for all x is [1987 - 2 Marks]
 (a) zero (b) one (c) three (d) infinite



7 Match the Following

33. Consider the following lists:

List-I

- (I) $\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$
 (II) $\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3} \tan 3x = 1\right\}$
 (III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2 \cos(2x) = \sqrt{3}\right\}$
 (IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$

List-II

- (P) has two elements
 (Q) has three elements
 (R) has four elements
 (S) has five elements
 (T) has six elements

The correct option is:

- (a) (I) \rightarrow (P); (II) \rightarrow (S); (III) \rightarrow (P); (IV) \rightarrow (S)
 (b) (I) \rightarrow (P); (II) \rightarrow (P); (III) \rightarrow (T); (IV) \rightarrow (R)
 (c) (I) \rightarrow (Q); (II) \rightarrow (P); (III) \rightarrow (T); (IV) \rightarrow (S)
 (d) (I) \rightarrow (Q); (II) \rightarrow (S); (III) \rightarrow (P); (IV) \rightarrow (R)

34. Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order.

$$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$$

Column - I contains the sets X, Y, Z and W. Column - II contains some information regarding these sets.

[Adv. 2019]

Column I

- (I) X
 (II) Y
 (III) Z
 (IV) W

Column II

- (p) $\supseteq \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi \right\}$
 (q) an arithmetic progression
 (r) NOT an arithmetic progression
 (s) $\supseteq \left\{ \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \right\}$
 (t) $\supseteq \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \pi \right\}$
 (u) $\supseteq \left\{ \frac{\pi}{6}, \frac{3\pi}{4} \right\}$

Which of the following is the only CORRECT combination?

- (a) (IV), (p), (r), (s) (b) (III), (p), (q), (u)
 (c) (III), (r), (u) (d) (IV), (q), (t)

35. Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order.

$$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$$

Column - I contains the sets X, Y, Z and W. Column - II contains some information regarding these sets.

[Adv. 2019]

Column I	Column II
(I) X	(p) $\supseteq \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi \right\}$
(II) Y	(q) an arithmetic progression
(III) Z	(r) NOT an arithmetic progression
(IV) W	(s) $\supseteq \left\{ \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \right\}$

$$(t) \supseteq \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \pi \right\}$$

$$(u) \supseteq \left\{ \frac{\pi}{6}, \frac{3\pi}{4} \right\}$$

Which of the following is the only CORRECT combination?

- (a) (I), (q), (u) (b) (I), (p), (r)
 (c) (II), (r), (s) (d) (II), (q), (t)



10 Subjective Problems

36. Find all values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation $(1 - \tan \theta)(1 + \tan \theta) \sec^2 \theta + 2 \tan^2 \theta = 0$. [1996 - 2 Marks]
37. Find the values of $x \in (-\pi, +\pi)$ which satisfy the equation $8^{(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+\dots)} = 4^3$ [1984 - 2 Marks]



Answer Key

Topic-1 : Trigonometric Ratios, Domain and Range of Trigonometric Functions,

Trigonometric Ratios of Allied Angles

1. (b) 2. (b) 3. (b) 4. (c)
 5. (A \rightarrow r; B \rightarrow p)

Topic-2 : Trigonometric Identities, Greatest and Least Value of Trigonometric Expressions

1. (c) 2. (b) 3. (a) 4. (c) 5. (a) 6. (c) 7. (c) 8. (b) 9. (b) 10. (a)
 11. (1) 12. (2) 13. (1) 14. $\frac{1}{3}$ 15. $\frac{1}{8}$ 16. $\frac{1}{64}$ 17. (6) 18. (True) 19. (b, c) 20. (a, c, d)
 21. (a, b) 22. (a, b, c, d) 23. (c) 24. (d) 25. (b) 26. (c)

Topic-3 : Solutions of Trigonometric Equations

1. (c) 2. (d) 3. (c) 4. (d) 5. (b) 6. (a) 7. (b) 8. (d) 9. (d) 10. (c)
 11. (d) 12. (b) 13. (c) 14. (c) 15. (a) 16. (8) 17. (7) 18. (3) 19. (3) 20. (3)
 21. (0.5) 22. $-\frac{\pi}{2}, \frac{\pi}{2}, 0$ 23. $n\pi, n\pi \pm \frac{\pi}{3}$ 24. $-\frac{\sqrt{3}}{2}$ 25. $\left[0, \frac{\pi}{6}\right] \cup \left\{\frac{\pi}{2}\right\} \cup \left[\frac{5\pi}{6}, \pi\right]$ 26. φ
 27. (False) 28. (a, c) 29. (c) 30. (c, d) 31. (c) 32. (d) 33. (b) 34. (a) 35. (d)