$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp (\Delta y/y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\Delta x/x \ll 1$, $\Delta y/y \ll 1$. Therefore, the higher powers of these quantities are neglected. **[Adv. 2018]**

25. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa ($\Delta a/a \ll 1$, then what is the error Δr in determining r?

(a) $\frac{\Delta a}{(1+a)^2}$ (b) $\frac{2\Delta a}{(1+a)^2}$ (c) $\frac{2\Delta a}{(1-a^2)}$ (d) $\frac{2a\Delta a}{(1-a^2)}$

26. In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For $|x| \ll 1$, $\ln(1+x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ, in s^{-1}, is

(a) 0.04 (b) 0.03 (c) 0.02 (d) 0.01

10 Subjective Problems

27. If n^{th} division of main scale coincides with $(n+1)^{th}$ divisions of vernier scale. Given one main scale division is equal to 'a' units. Find the least count of the vernier.

[2003 - 2 Marks]

Topic-4: Miscellaneous (Mixed Concepts) Problems

8 Comprehension/Passage Based Questions

Passage

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency 'ω_p' which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals. **[2011]**

1. Taking the electronic charge as 'e' and the permittivity as 'ε_0'. Use dimensional analysis to determine the correct expression for ω_p.

(a) $\sqrt{\frac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\frac{m\varepsilon_0}{Ne}}$ (c) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$

2. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}\ m^{-3}$. Taking $\varepsilon_0 = 10^{-11}$ and mass $m \approx 10^{-30}$, where these quantities are in proper SI units.

(a) 800 nm (b) 600 nm (c) 300 nm (d) 200 nm

Answer Key

Topic-1 : Unit of Physical Quantities

1. (a, b, c, d) **2.** A → p, q; B → r, s; C → r, s; D → r, s

Topic-2 : Dimensions of Physical Quantities

1. (a) **2.** (c) **3.** (b) **4.** (d) **5.** (c) **6.** (d) **7.** (c) **8.** (4) **9.** (3) **14.** (b, d)
15. (a, b) **16.** (a, b, c) **17.** (b, d) **18.** (a, c) **19.** (a, c, d) **20.** (b, c) **21.** (a, b, c) **22.** (a, d) **23.** (c) **25.** (c)
26. (d)

Topic-3 : Errors in Measurements & Experimental Physics

1. (c) **2.** (b) **3.** (b) **4.** (b) **5.** (a) **6.** (c) **7.** (d) **8.** (b) **9.** (d) **10.** (c)
11. (a) **12.** (a) **13.** (1) **14.** (4) **15.** (4) **16.** (1.39) **17.** (3.00) **18.** ($2.66\ g\ cm^{-3}$)
19. ($1.09 \times 10^{10}\ N/m^2$) **20.** ($2.6\ cm^2$) **21.** (a, b, d) **22.** (b, c) **23.** (d) **24.** (a, c) **25.** (b) **26.** (c)

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. (c) **2.** (b)