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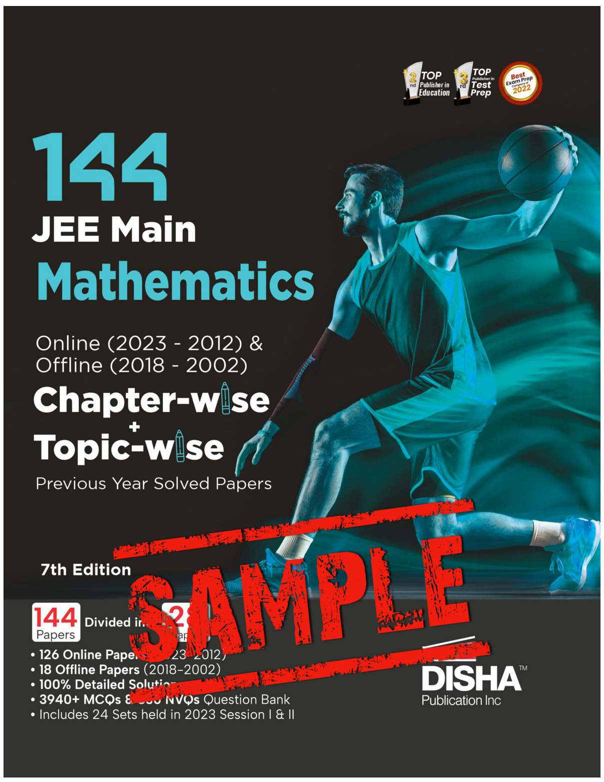

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5 Complex Numbers and Quadratic Equations

Topic 1 Integral Powers of Iota, Algebraic Operations of Complex Numbers, Conjugate, Modulus and Argument or Amplitude of a Complex Number

1. If the set $\left\{\mathrm{Re}\left(\dfrac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right): z \in \mathbb{C}, \mathrm{Re}(z)=3\right\}$ is equal to the interval $(\alpha, \beta]$, then $24(\beta-\alpha)$ is equal to **[April 15, 2023 (I)]**

(a) 36 (b) 42 (c) 27 (d) 30

2. Let $S=\left\{z \in C: \bar{z}=i\left(z^2+\mathrm{Re}(\bar{z})\right)\right\}$. Then $\sum_{z \in S}|z|^2$ is equal to **[April 13, 2023 (II)]**

(a) $\dfrac{7}{2}$ (b) 4 (c) $\dfrac{5}{2}$ (d) 3

3. For $a \in C$, let $A=\{z \in C: \mathrm{Re}(a+\bar{z})>\mathrm{Im}(\bar{a}+z)\}$ and $B=\{z \in C: \mathrm{Re}(a+\bar{z})<\mathrm{Im}(\bar{a}+z)\}$. Then among the two statements :

(S1) : If Re (A), Im (A) > 0, then the set A contains all the real numbers.

(S2) : If Re (A), Im (A) < 0, then the set B contains all the real numbers. **[April 11, 2023 (II)]**

(a) Only (S1) is true (b) both are false

(c) Only (S2) is true (d) Both are true

4. Let $S=\left\{z \in C-\{i, 2i\}: \dfrac{z^2+8iz-15}{z^2-3iz-2} \in R\right\}$. If $\alpha-\dfrac{13}{11}i \in S, \alpha \in \mathbb{R}-\{0\}$, then $242\alpha^2$ is equal to **[NA, April 11, 2023 (II)]**

5. Let the complex number $z = x + iy$ be such that $\dfrac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to: **[April 10, 2023 (I)]**

(a) $\dfrac{3}{2}$ (b) $\dfrac{4}{3}$

(c) $\dfrac{2}{3}$ (d) $\dfrac{3}{4}$

6. Let $S=\left\{z=x+iy: \dfrac{2z-3i}{4z+2i} \text{ is a real number}\right\}$. Then which of the following is NOT correct? **[April 10, 2023 (II)]**

(a) $y+x^2+y^2 \neq -\dfrac{1}{4}$

(b) $x = 0$

(c) $(x, y)=\left(0,-\dfrac{1}{2}\right)$

(d) $y \in\left(-\infty,-\dfrac{1}{2}\right) \cup\left(-\dfrac{1}{2}, \infty\right)$

7. Let $A=\left\{\theta \in(0,2\pi): \dfrac{1+2i\sin\theta}{1-i\sin\theta} \text{ is purely imaginary}\right\}$. Then the sum of the elements in A is **[April 8, 2023 (II)]**

(a) π (b) 2π

(c) 4π (d) 3π

8. Let $a \neq b$ be two non-zero real numbers. Then the number of elements in the set $X=\{z \in C: \mathrm{Re}(az^2+bz)=a \text{ and } \mathrm{Re}(bz^2+az)=b\}$ is equal to **[April 6, 2023 (II)]**

(a) 1 (b) 3

(c) 0 (d) 2

9. Let $z = 1 + i$ and $z_1=\dfrac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$, then $\dfrac{12}{\pi}\arg(z_1)$ is equal to _______. **[NA, Jan. 30, 2023 (I)]**

10. For two non-zero complex number z_1 and z_2, if $\mathrm{Re}(z_1 z_2) = 0$ and $\mathrm{Re}(z_1 + z_2) = 0$, then which of the following are possible? **[Jan. 29, 2023 (I)]**

(A) $\mathrm{Im}(z_1) > 0$ and $\mathrm{Im}(z_2) > 0$

(B) $\mathrm{Im}(z_1) < 0$ and $\mathrm{Im}(z_2) > 0$

(C) $\mathrm{Im}(z_1) > 0$ and $\mathrm{Im}(z_2) < 0$

(D) $\mathrm{Im}(z_1) < 0$ and $\mathrm{Im}(z_2) < 0$

Choose the correct answer from the options given below:

(a) B and D (b) B and C

(c) A and B (d) A and C

11. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set $S = \left\{ z \in C : |z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2 \right\}$ represents a **[Jan. 25, 2023 (I)]**
(a) straight line with sum of its intercepts on the coordinate axes equals 14
(b) hyperbola with the length of the transverse axis 7
(c) straight line with the sum of its intercepts on the coordinate axes equals –18
(d) hyperbola with eccentricity 2

12. Let S be the set of all (α, β), $\pi < \alpha, \beta < 2\pi$, for which the complex number $\dfrac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely imaginary and $\dfrac{1 - i\cos\beta}{1 + 2i\sin\beta}$ is purely real. Let $Z\alpha\beta = \sin 2\alpha + i\cos 2\beta$, $(\alpha, \beta) \in S$. Then $\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta} + \dfrac{1}{i\overline{Z}\alpha\beta}\right)$ is equal to **[July 27, 2022 (II)]**
(a) 3 (b) $3i$ (c) 1 (d) 2–1

13. If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z-i|-|z+5i|=0$, then **[July 26, 2022 (II)]**
(a) $x + 2y - 4 = 0$ (b) $x^2 + y - 4 = 0$
(c) $x + 2y + 4 = 0$ (d) $x^2 - y + 3 = 0$

14. For $z \in C$ if the minimum value of $\left(|z - 3\sqrt{2}| + |z - p\sqrt{2}i|\right)$ is $5\sqrt{2}$, then a value of p is **[July 25, 2022 (II)]**
(a) 3 (b) $\dfrac{7}{2}$ (c) 4 (d) $\dfrac{9}{2}$

15. The real part of the complex number $\dfrac{(1+2i)^8 \cdot (1-2i)^2}{(3+2i)\cdot\overline{(4-6i)}}$ is equal to: **[June 30, 2022 (I)]**
(a) $\dfrac{500}{13}$ (b) $\dfrac{110}{13}$ (c) $\dfrac{55}{6}$ (d) $\dfrac{550}{13}$

16. Let z_1 and z_2 be two complex numbers such that $\overline{z}_1 = i\overline{z}_2$ and $\arg\left(\dfrac{z_1}{\overline{z}_2}\right) = \pi$. Then **[June 25, 2022 (II)]**
(a) $\arg z_2 = \dfrac{\pi}{4}$ (b) $\arg z_2 = -\dfrac{3\pi}{4}$
(c) $\arg z_1 = \dfrac{\pi}{4}$ (d) $\arg z_1 = -\dfrac{3\pi}{4}$

17. The least positive integer n such that $\dfrac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$ is a positive integer, is ______. **[NA, Aug. 26, 2021 (II)]**

18. If the real part of the complex number $z = \dfrac{3 + 2i\cos\theta}{1 - 3i\cos\theta}, \theta \in \left(0, \dfrac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to______. **[NA, July 27, 2021 (II)]**

19. Let a complex number z, $|z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\dfrac{|z| + 11}{(|z| - 1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to. **[2019(S), March 16, 2021(I)]**
(a) 8 (b) 7 (c) 6 (d) 5

20. Let z and w be two complex numbers such that $w = z\overline{z} - 2z + 2, \left|\dfrac{z+i}{z-3i}\right| = 1$ and Re(w) has minimum value. Then, the minimum value of $n \in N$ for which w^n is real, is equal to **[NA, March 16, 2021 (I)]**

21. The least value of $|z|$ where z is complex number which satisfies the inequality $\exp\left(\dfrac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7} + 9i\right|, i = \sqrt{-1}$, is equal to: **[March 16, 2021 (II)]**
(a) 2 (b) 8 (c) 3 (d) $\sqrt{5}$

22. If the least and the largest real values of α, for which the equation $z + \alpha|z - 1| + 2i = 0$ ($z \in C$ and $i = \sqrt{-1}$) has a solution, are p and q respectively, then $4(p^2 + q^2)$ is equal to ________. **[2018(s), NA, Feb. 24, 2021(I)]**

23. If z_1, z_2 are complex numbers such that $\text{Re}(z_1) = |z_1 - 1|$, $\text{Re}(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \dfrac{\pi}{6}$, then $\text{Im}(z_1 + z_2)$ is equal to : **[Sep. 03, 2020 (II)]**
(a) $\dfrac{2}{\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\dfrac{\sqrt{3}}{2}$ (d) $\dfrac{1}{\sqrt{3}}$

24. Let z be a complex number such that $\left|\dfrac{z-i}{z+2i}\right| = 1$ and $|z| = \dfrac{5}{2}$. Then the value of $|z + 3i|$ is : **[Jan. 9, 2020 (I)]**
(a) $\sqrt{10}$ (b) $\dfrac{7}{2}$ (c) $\dfrac{15}{4}$ (d) $2\sqrt{3}$

25. If $\dfrac{3 + i\sin\theta}{4 - i\cos\theta}, \theta \in [0, 2\pi]$, is *a* real number, then an argument of $\sin\theta + i\cos\theta$ is: **[2019(s), Jan. 7, 2020 (II)]**
(a) $\pi - \tan^{-1}\left(\dfrac{4}{3}\right)$ (b) $\pi - \tan^{-1}\left(\dfrac{3}{4}\right)$
(c) $-\tan^{-1}\left(\dfrac{3}{4}\right)$ (d) $\tan^{-1}\left(\dfrac{4}{3}\right)$

26. The equation $|z - i| = |z - 1|, i = \sqrt{-1}$, represents: **[April 12, 2019 (I)]**
(a) a circle of radius $\dfrac{1}{2}$.
(b) the line through the origin with slope 1.
(c) a circle of radius 1.
(d) the line through the origin with slope – 1.

27. Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number n. Then : **[April 12, 2019 (II)]**
(a) $n = 20$ and $\text{Re}(z) = -10$
(b) $n = 40$ and $\text{Re}(z) = 10$
(c) $n = 40$ and $\text{Re}(z) = -10$
(d) $n = 20$ and $\text{Re}(z) = 10$

28. If $a > 0$ and $z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to : **[April 10, 2019 (I)]**
(a) $-\frac{1}{5}-\frac{3}{5}i$ (b) $-\frac{3}{5}-\frac{1}{5}i$
(c) $\frac{1}{5}-\frac{3}{5}i$ (d) $-\frac{1}{5}+\frac{3}{5}i$

29. If $\frac{z-\alpha}{z+\alpha} (\alpha \in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is : **[Jan. 12, 2019 (I)]**
(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\sqrt{2}$

30. For all complex numbers z of the form $1 + i\alpha$, $\alpha \in R$, if $z^2 = x + iy$, then **[Online April 19, 2014]**
(a) $y^2 - 4x + 2 = 0$ (b) $y^2 + 4x - 4 = 0$
(c) $y^2 - 4x - 4 = 0$ (d) $y^2 + 4x + 2 = 0$

31. Let $z \neq -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number. Then $z+\frac{1}{z}$ is: **[Online April 9, 2013(S) & 12, 2014]**
(a) zero
(b) any non-zero real number other than 1.
(c) any non-zero real number.
(d) a purely imaginary number.

32. If z_1, z_2 and z_3, z_4 are 2 pairs of complex conjugate numbers, then $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$ equals: **[Online April 11, 2014]**
(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

33. Let w (Im w ≠ 0) be a complex number. Then the set of all complex number z satisfying the equation $w - \overline{w}z = k(1-z)$, for some real number k, is **[Online April 9, 2014]**
(a) $\{z : |z| = 1\}$ (b) $\{z : z = \bar{z}\}$
(c) $\{z : z \neq 1\}$ (d) $\{z : |z| = 1, z \neq 1\}$

34. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals: **[2013]**
(a) $-\theta$ (b) $\frac{\pi}{2}-\theta$ (c) θ (d) $\pi - \theta$

35. Let $a = \text{Im}\left(\frac{1+z^2}{2iz}\right)$, where z is any non-zero complex number. The set $A = \{a : |z| = 1 \text{ and } z \neq \pm 1\}$ is equal to: **[Online April 23, 2013]**
(a) $(-1, 1)$ (b) $[-1, 1]$ (c) $[0, 1)$ (d) $(-1, 0]$

36. $|z_1 + z_2|^2 + |z_1 - z_2|^2$ is equal to **[Online May 26, 2012]**
(a) $2(|z_1| + |z_2|)$ (b) $2(|z_1|^2 + |z_2|^2)$
(c) $|z_1||z_2|$ (d) $|z_1|^2 + |z_2|^2$

37. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals **[2010]**
(a) 1 (b) 2 (c) ∞ (d) 0

38. The conjugate of a complex number is $\frac{1}{i-1}$ then that complex number is **[2008]**
(a) $\frac{-1}{i-1}$ (b) $\frac{1}{i+1}$ (c) $\frac{-1}{i+1}$ (d) $\frac{1}{i-1}$

39. If $z = x - i\,y$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p}+\frac{y}{q}\right)\Big/(p^2+q^2)$ is equal to **[2004]**
(a) -2 (b) -1 (c) 2 (d) 1

40. Let z and w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $\arg zw = \pi$. Then arg z equals **[2002(S), 2004]**
(a) $\frac{5\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

41. If $\left(\frac{1+i}{1-i}\right)^x = 1$ then **[2003]**
(a) $x = 2n+1$, where n is any positive integer
(b) $x = 4n$, where n is any positive integer
(c) $x = 2n$, where n is any positive integer
(d) $x = 4n+1$, where n is any positive integer.

42. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$ and $Arg(z) - Arg(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to **[2003]**
(a) -1 (b) 1 (c) $-i$ (d) i

43. If $|z - 4| < |z - 2|$, its solution is given by **[2002]**
(a) $\text{Re}(z) > 0$ (b) $\text{Re}(z) < 0$
(c) $\text{Re}(z) > 3$ (d) $\text{Re}(z) > 2$

Topic 2 Rotational Theorem, Square Root of a Complex Number, Cube Roots of Unity, Geometry of Complex Numbers, De-moiver's Theorem, Powers of Complex Numbers

44. Let $\omega = z\bar{z} + k_1 z + k_2 iz + \lambda(1+i)$, $k_1, k_2 \in \mathbb{R}$. Let $Re(\omega) = 0$ be the circle C of radius 1 in the first quadrant touching the line y = 1 and the y-axis. If the curve Im (ω) = 0 intersects C at A and B, then $30(AB)^2$ is equal to ______. **[NA, April 13, 2023 (I)]**

45. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to **[April 11, 2023 (I)]**

(a) $-\pi + \tan^{-1}\frac{33}{5}$ (b) $-\pi - \tan^{-1}\frac{33}{5}$

(c) $-\pi + \tan^{-1}\frac{8}{9}$ (d) $\pi - \tan^{-1}\frac{8}{9}$

46. If for $z = \alpha + i\beta, |z+2| = z + 4(1+i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation **[April 8, 2023 (I)]**

(a) $x^2 + 7x + 12 = 0$ (b) $x^2 + 3x - 4 = 0$

(c) $x^2 + 2x - 3 = 0$ (d) $x^2 + x - 12 = 0$

47. If the center and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α, β) and γ, then $3(\alpha + \beta + \gamma)$ is equal to **[Feb. 1, 2023 (I)]**

(a) 12 (b) 11 (c) 9 (d) 10

48. For all $z \in C$ on the curve $C_1 : |z| = 4$, let the locus of the point $z + \frac{1}{z}$ be the curve C_2. Then **[Jan. 31, 2023 (I)]**

(a) the curves C_1 and C_2 intersect at 4 points
(b) the curves C_1 lies inside C_2
(c) the curves C_1 and C_2 intersect at 2 points
(d) the curves C_2 lies inside C_1

49. The complex number $z = \dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ is equal to: **[Jan. 31, 2023 (II)]**

(a) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$ (b) $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$

(c) $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ (d) $\sqrt{2}\, i\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$

50. Let $\alpha = 8 - 14i$, $A = \left\{z \in \mathbb{C} : \dfrac{\alpha z - \bar{\alpha}\bar{z}}{z^2 - (\bar{z})^2 - 112i} = i\right\}$ and $B = \{z \in \mathbb{C} : |z + 3i| = 4\}$. Then $\sum_{z \in A \cap B} (\text{Re } z - \text{Im } z)$ is equal to ______. **[NA, Jan. 29, 2023 (II)]**

51. Let $\alpha_1, \alpha_2, ., \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \ge |\alpha_2| \ge .. \ge |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to______. **[NA, Jan. 29, 2023 (II)]**

52. Let $S = \left\{\alpha : \log_2\left(9^{2\alpha-4} + 13\right) - \log_2\left(\frac{5}{3}.3^{2\alpha-4} + 1\right) = 2\right\}$.

Then the maximum value of β for which the equation $x^2 - 2\left(\sum_{\alpha \in s}\alpha\right)^2 x + \sum_{\alpha \in s}(\alpha+1)^2\beta = 0$ has real roots is ________. **[NA, Jan. 25, 2023 (I)]**

53. Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right| = 2, z \ne -i$. Then z lies on the circle of radius 2 and centre **[Jan. 25, 2023 (II)]**

(a) (2, 0) (b) (0, 0) (c) (0, 2) (d) (0, .2)

54. Let $p, q \in R$ and $(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq), i = \sqrt{-1}$. Then $p + q + q^2$ and $p - q + q^2$ are roots of the equation. **[Jan. 24, 2023 (I)]**

(a) $x^2 + 4x - 1 = 0$ (b) $x^2 - 4x + 1 = 0$

(c) $x^2 + 4x + 1 = 0$ (d) $x^2 - 4x - 1 = 0$

55. The value of $\left(\dfrac{1 + \sin\frac{2\pi}{9} + i\cos\frac{2\pi}{9}}{1 + \sin\frac{2\pi}{9} - i\cos\frac{2\pi}{9}}\right)^3$ is **[Sep. 02, 2020 (I), Jan. 24, 2023 (II)]**

(a) $\frac{-1}{2}(1 - i\sqrt{3})$ (b) $\frac{1}{2}(1 - i\sqrt{3})$

(c) $\frac{-1}{2}(\sqrt{3} - i)$ (d) $\frac{1}{2}(\sqrt{3} + i)$

56. If $z = 2 + 3i$, then $z^5 + (\bar{z})^5$ is equal to : **[July 29, 2022 (I)]**

(a) 244 (b) 224 (c) 245 (d) 265

57. Let $S = \{z = x + iy : |z - 1 + i| \ge |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of x, for which $w = 2x + iy \in S$ for some $y \in \mathbb{R}$, is **[July 29, 2022 (II)]**

(a) $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$ (b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

(c) $\left(-\sqrt{2}, \frac{1}{2}\right]$ (d) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

58. If $z \ne 0$ be a complex number such that $\left|z - \frac{1}{z}\right| = 2$, then the maximum value of $|z|$ is: **[July 29, 2022 (II)]**

(a) $\sqrt{2}$ (b) 1 (c) $\sqrt{2} - 1$ (d) $\sqrt{2} + 1$

59. Let $S_1=\left\{z_1\in C:|z_1-3|=\frac{1}{2}\right\}$ and $S_2=\{z_2\in C:|z_2-|z_2+1||=|z_2+|z_2-1||\}$. Then, for $z_1\in S_1$ and $z_2\in S_2$, the least value of $|z_2-z_1|$ is : **[July 28, 2022(I)]**

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

60. Let $z = a + ib, b\neq 0$ be complex numbers satisfying $z^2=\bar{z}.2^{1-|z|}$. Then the least value of $n\in N$, such that $z^n=(z+1)^n$, is equal to ____. **[NA, July 28, 2022(II)]**

61. Let the minimum value v_0 of $v=|z|^2+|z-3|^2+|z-6i|^2$, $z\in\mathbb{C}$ is attained at $z=z_0$. Then $|2z_0^2-\bar{z}_0^3+3|^2+v_0^2$ is equal to **[July 27, 2022 (I)]**

(a) 1000 (b) 1024 (c) 1105 (d) 1196

62. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, Re $(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true? **[July 26, 2022 (I)]**

(a) $\arg z_2=\pi-\tan^{-1}3$ (b) $\arg(z_1-2z_2)=-\tan^{-1}\frac{4}{3}$

(c) $|z_2|=\sqrt{10}$ (d) $|2z_1-z_2|=5$

63. For $n\in\mathbf{N}$, let $S_n=\left\{z\in \boldsymbol{C}:|z-3+2i|=\frac{n}{4}\right\}$ and $T_n=\left\{z\in\boldsymbol{C}:|z-2+3i|=\frac{1}{n}\right\}$. Then the number of elements in the set $\{n\in\mathbf{N}:S_n\cap T_n=\phi\}$ is: **[July 25, 2022 (I)]**

(a) 0 (b) 2 (c) 3 (d) 4

64. Let $S=\{z\in C:|z-2|\le 1,\ z(1+i)+\bar{z}(1-i)\le 2\}$. Let $|z-4i|$ attains minimum and maximum values, respectively, at $z_1\in S$ and $z_2\in S$. If $5(|z_1|^2+|z_2|^2)=\alpha+\beta\sqrt{5}$, where α and β are integers, then the value of $\alpha+\beta$ is equal to _____. **[NA, June 29, 2022 (I)]**

65. Let arg (z) represent the principal argument of the complex number z. The $|z|=3$ and $\arg(z-1)-\arg(z+1)=\frac{\pi}{4}$ intersect: **[June 29, 2022 (II)]**

(a) Exactly at one point (b) Exactly at two points

(c) Nowhere (d) At infinitely many points.

66. The number of elements in the set $\{z=a+ib\in\mathbb{C}: a, b\in\mathbb{Z}$ and $1<|z-3+2i|<4\}$ is _______ **[NA, June 28, 2022 (I)]**

67. Sum of squares of modulus of all the complex numbers z satisfying $\bar{z}=iz^2+z^2-z$ is equal to ________. **[NA, June 28, 2022 (II)]**

68. The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z}=iz^2$ is : **[June 27, 2022 (I)]**

(a) $\frac{3\sqrt{3}}{4}$ (b) $\frac{3\sqrt{3}}{2}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

69. Let $A=\left\{z\in C:\left|\frac{z+1}{z-1}<1\right|\right\}$ and $B=\left\{z\in C:\arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}\right\}$

Then $A\cap B$ is : **[2021(S), June 26, 2022 (I)]**

(a) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

(b) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

(c) an empty set

(d) a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

70. If $z^2+z+1=0$, $z\in C$, then $\left|\sum_{n=1}^{15}\left(z^n+(-1)^n\frac{1}{z^n}\right)^2\right|$ is equal to ______ . **[NA, June 26, 2022 (II)]**

71. Let $A=\{z\in C:1\le|z-(1+i)|\le 2\}$ and $B=\{z\in A:|z-(1-i)|=1\}$. Then, B : **[June 24, 2022 (I)]**

(a) is an empty set
(b) contains exactly two elements
(c) contains exactly three elements
(d) is an infinite set

72. Let $S=\{z\in\mathbb{C}:|z-3|\le 1$ and $z(4+3i)+\bar{z}(4-3i)\le 24\}$. If $\alpha+i\beta$ is the point in S which is closest to $4i$, then $25(\alpha+\beta)$ is equal to _________. **[NA, June 24, 2022 (II)]**

73. If for the complex numbers z satisfying $|z-2-2i|\le 1$, the maximum value of $|3iz+6|$ is attained at $a+ib$, then $a+b$ is equal to ________. **[2014(S), NA, Sep. 1, 2021 (II)]**

74. If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z-(3+3i)|$ is : **[2019(S), Aug. 27, 31, 2021 (II)]**

(a) $2\sqrt{2}-1$ (b) $3\sqrt{2}$ (c) $6\sqrt{2}$ (d) $2\sqrt{2}$

75. Let z_1 and z_2 be two complex numbers such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z - 3| = \mathrm{Re}(z)$. Then the imaginary part of $z_1 + z_2$ is equal to ________. **[NA, Aug. 27, 2021 (II)]**

76. Let $z = \frac{1 - i\sqrt{3}}{2}, i = \sqrt{-1}$. Then the value of

$$21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \ldots + \left(z^{21} + \frac{1}{z^{21}}\right)^3$$

is __________. **[NA, Aug. 26, 2021 (I)]**

77. Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is ________. **[NA, March 18, 2021 (I)]**

78. Let a complex number be $w = 1 - \sqrt{3}\, i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to : **[March 18, 2021 (II)]**

(a) $\frac{1}{2}$ (b) 2 (c) 4 (d) $\frac{1}{4}$

79. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to _______. **[NA, March 18, 2021 (II)]**

80. Let S_1, S_2 and S_3 be three sets defined as

$S_1 = \{z \in \mathbb{C} : |z - 1| \le \sqrt{2}\}$
$S_2 = \{z \in \mathbb{C} : \mathrm{Re}((1 - i)z\} \ge 1\}$
$S_3 = \{z \in \mathbb{C} : \mathrm{Im}(z) \le 1\}$

Then the set $S_1 \cap S_2 \cap S_3$ **[March 17, 2021 (II)]**

(a) Has infinitely many elements
(b) Is a singleton
(c) Has exactly three elements
(d) Has exactly two elements

81. Let z be those complex numbers which satisfy $|z + 5| \le 4$ and $z(1+i) + \bar{z}(1-i) \ge -10$, $i = \sqrt{-1}$. If the maximum value of $|z + 1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is _____ . **[NA, Feb. 26, 2021 (II)]**

82. The sum of 162[th] power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is _________. **[NA, Feb. 26, 2021 (I)]**

83. Let $i = \sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$, and $n = [|k|]$ be the greatest integral part of $|k|$.

Then $\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to ________. **[NA, Feb.24, 2021(II)]**

84. Let $z = x + iy$ be a non-zero complex number such that $z^2 = i\,|z|^2$, where $i = \sqrt{-1}$, then z lies on the: **[Sep. 06, 2020 (II)]**

(a) line, $y = -x$ (b) imaginary axis
(c) line, $y = x$ (d) real axis

85. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is : **[Sep. 05, 2020 (II)]**

(a) -2^{15} (b) $2^{15}\, i$ (c) $-2^{15}\, i$ (d) 6^5

86. If the four complex numbers $z, \bar{z}, \bar{z} - 2\mathrm{Re}(\bar{z})$ and $z - 2\mathrm{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to : **[Sep. 05, 2020 (I)]**

(a) $4\sqrt{2}$ (b) 4 (c) $2\sqrt{2}$ (d) 2

87. If a and b are real numbers such that $(2+\alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$, then $a + b$ is equal to : **[Sep. 04, 2020 (II)]**

(a) 9 (b) 24 (c) 33 (d) 57

88. If $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1, (m, n \in \mathbf{N})$, then the greatest common divisor of the least values of m and n is _________. **[NA, Sep. 03, 2020 (I)]**

89. The imaginary part of $(3+2\sqrt{-54})^{1/2} - (3-2\sqrt{-54})^{1/2}$ can be : **[Sep. 02, 2020 (II)]**

(a) $-\sqrt{6}$ (b) $-2\sqrt{6}$ (c) 6 (d) $\sqrt{6}$

90. If z be a complex number satisfying $|\mathrm{Re}(z)| + |\mathrm{Im}(z)| = 4$, then $|z|$ cannot be: **[Jan. 9, 2020 (II)]**

(a) $\sqrt{\frac{17}{2}}$ (b) $\sqrt{10}$ (c) $\sqrt{7}$ (d) $\sqrt{8}$

91. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$ and $b = \sum_{k=0}^{100}\alpha^{3k}$, then a and b are the roots of the quadratic equation: **[Jan. 8, 2020 (II)]**

(a) $x^2 + 101x + 100 = 0$ (b) $x^2 - 102x + 101 = 0$
(c) $x^2 - 101x + 100 = 0$ (d) $x^2 + 102x + 101 = 0$

92. If $\mathrm{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x, y) lies on a: **[Jan. 7, 2020 (I)]**

(a) circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$.
(b) straight line whose slope is $-\frac{2}{3}$.
(c) straight line whose slope is $\frac{3}{2}$.
(d) circle whose diameter is $\frac{\sqrt{5}}{2}$.

93. Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5+3z}{5(1-z)}$, then : **[April 09, 2019 (II)]**
(a) $5\ \text{Re}(\omega) > 4$ (b) $4\ \text{Im}(\omega) > 5$
(c) $5\ \text{Re}(\omega) > 1$ (d) $5\ \text{Im}(\omega) < 1$

94. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}, (i = \sqrt{-1})$, then $(1 + iz + z^5 + iz^8)^9$ is equal to: **[April 08, 2019 (II)]**
(a) 0 (b) 1
(c) $(-1 + 2i)^9$ (d) -1

95. Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x+iy}{27}\left(i = \sqrt{-1}\right)$, where x and y are real numbers then $y - x$ equals : **[Jan. 11, 2019 (I)]**
(a) 91 (b) -85 (c) 85 (d) -91

96. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of z, then: **[Jan. 10, 2019 (II)]**
(a) $I(z) = 0$ (b) $R(z) > 0$ and $I(z) > 0$
(c) $R(z) < 0$ and $I(z) > 0$ (d) $R(z) = -$ (c)

97. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then: **[Jan. 10 2019 (II)]**
(a) $\text{Re}(z) = 0$ (b) $|z| = \sqrt{\frac{5}{2}}$
(c) $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$ (d) $\text{Im}(z) = 0$

98. The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$, is **[Online April 16, 2018]**
(a) 2 (b) 6 (c) 5 (d) 3

99. The point represented by $2 + i$ in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south–westwards direction. Then its new position in the Argand plane is at the point represented by : **[Online April 9, 2016]**
(a) $1 + i$ (b) $2 + 2i$ (c) $-2 - 2i$ (d) $-1 - i$

100. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a: **[2015]**
(a) circle of radius 2.
(b) circle of radius $\sqrt{2}$.
(c) straight line parallel to x-axis
(d) straight line parallel to y-axis.

101. If z is a non-real complex number, then the minimum value of $\frac{Im\, z^5}{(Im\, z)^5}$ is : **[Online April 11, 2015]**
(a) -1 (b) -4 (c) -2 (d) -5

102. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies : **[2012]**
(a) either on the real axis or on a circle passing through the origin.
(b) on a circle with centre at the origin
(c) either on the real axis or on a circle not passing through the origin.
(d) on the imaginary axis.

103. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals **[2011]**
(a) (1, 1) (b) (1, 0) (c) (–1, 1) (d) (0, 1)

104. If $|z + 4| \leq 3$, then the maximum value of $|z + 1|$ is **[2007]**
(a) 6 (b) 0 (c) 4 (d) 10

105. If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on **[2005]**
(a) an ellipse (b) a circle
(c) a straight line (d) a parabola

106. If z_1 and z_2 are two non- zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to **[2005]**
(a) $\frac{\pi}{2}$ (b) $-\pi$ (c) 0 (d) $\frac{-\pi}{2}$

107. If the cube roots of unity are $1, \omega, \omega^2$ then the roots of the equation $(x-1)^3 + 8 = 0$, are **[2005]**
(a) $-1, -1 + 2\omega, -1 - 2\omega^2$
(b) $-1, -1, -1$
(c) $-1, 1 - 2\omega, 1 - 2\omega^2$
(d) $-1, 1 + 2\omega, 1 + 2\omega^2$

108. If $|z^2 - 1| = |z|^2 + 1$, then z lies on **[2004]**
(a) an ellipse (b) the imaginary axis
(c) a circle (d) the real axis

109. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be **[2002]**
(a) an ellipse (b) a hyperbola
(c) a circle (d) none of these

Topic 3 Solutions of Quadratic Equations, Sum and Product of Roots, Nature of Roots, Relation Between Roots and Co-efficients, Formation of an Equation with Given Roots

110. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then $\alpha^{14} + \beta^{14}$ is equal to **[April 13, 2023 (II)]**

(a) $-64\sqrt{2}$ (b) $-128\sqrt{2}$
(c) –64 (d) –128

111. Let α, β be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\dfrac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$ is equal to **[April 12, 2023 (I)]**

(a) 729 (b) 72 (c) 81 (d) 9

112. If a and b are the roots of equation $x^2 - 7x - 1 = 0$, then the value of $\dfrac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is equal to ______. **[NA, April 11, 2023 (I)]**

113. The number of points, where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1, x \in \mathbb{R}$ cuts x-axis, is equal to **[NA, April 11, 2023 (II)]**

114. Let $S = \left\{x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10\right\}$ and $\beta = \sum_{x \in S} \tan^2\left(\frac{x}{3}\right)$, then $\frac{1}{6}(\beta - 14)^2$ is equal to **[April 10, 2023 (II)]**

(a) 32 (b) 8 (c) 64 (d) 16

115. Let α, β, γ be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to **[April 8, 2023 (I)]**

(a) 21 (b) $\frac{169}{8}$ (c) 19 (d) $\frac{155}{8}$

116. The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is: **[April 6, 2023 (I)]**

(a) $9 + \sqrt{3}$ (b) $11 + \sqrt{3}$
(c) $9 - \sqrt{3}$ (d) $11 - \sqrt{3}$

117. The number of intergral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval (1, 2) and its other root lies in the interval (2, 3), is : **[Feb. 1, 2023 (II)]**

(a) 1 (b) 3 (c) 0 (d) 2

118. Let a, b be two real numbers such that ab < 0. If the complex number $\frac{1 + ai}{b + i}$ is of unit modulus and a + ib lies on the circle $|z - 1| = |2z|$, then a possible value of $\frac{1 + (a)}{4b}$, where [t] is greatest integer function, is: **[Feb. 1, 2023 (II)]**

(a) 1 (b) $-\frac{1}{2}$ (c) – 1 (d) $\frac{1}{2}$

119. The number of real roots of the equation $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$, is: **[Jan. 31, 2023 (I)]**

(a) 0 (b) 1 (c) 3 (d) 2

120. Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation : **[Jan. 29, 2023 (I)]**

(a) $7x^2 + 245x - 250 = 0$ (b) $7x^2 - 245x + 250 = 0$
(c) $49x^2 - 245x + 250 = 0$ (d) $49x^2 + 245x + 250 = 0$

121. Let $a \in R$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}} x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is _____. **[NA, Jan. 25, 2023 (II)]**

122. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has: **[Jan. 24, 2023 (I)]**

(a) exactly two solutions in (–∞,∞)
(b) no solution
(c) a unique solution in (–∞,1)
(d) a unique solution in (–∞,∞)

123. The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is **[NA, Jan. 24, 2023 (II)]**

124. Let $S = \{\theta \in (0, 2\pi) : 7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2\}$. Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0, \theta \in S$, is________. **[NA, July 29, 2022 (I)]**

125. Let α, β (α > β) be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n$, $n \in \mathbb{N}$, then $\dfrac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to _________. **[2003(S), 2015(S), 2018(S), 2019(S), 2020(S), 2021(S), NA, July 29, 2022 (II)]**

126. The sum of all real values of x for which $\dfrac{3x^2 - 9x + 17}{x^2 + 3x + 10} = \dfrac{5x^2 - 7x + 19}{3x^2 + 5x + 12}$ is equal to _______. **[NA, July 28, 2022(I)]**

127. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + \sqrt{6} = 0$ and $\frac{1}{\alpha^2}+1, \frac{1}{\beta^2}+1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a+b-2)x + (a+b+2) = 0$ are: **[28 July, 2022(II)]**

(a) non-real complex numbers
(b) real and both negative
(c) real and both positive
(d) real and exactly one of them is positive

128. Let $S = \{z \in \mathbb{C} : z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} (\text{Re}(z) + \text{Im}(z))$ is equal to __________. **[NA, 27 July, 2022 (I)]**

129. If α, β are the roots of the equation

$$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right)x + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log 3)^{\frac{2}{3}}} - 1\right) = 0$$

then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, **[July 27, 2022 (II)]**

(a) $3x^2 - 20x - 12 = 0$ (b) $3x^2 - 10x - 4 = 0$
(c) $3x^2 - 10x + 2 = 0$ (d) $3x^2 - 20x + 16 = 0$

130. The minimum value of the sum of the squares of the roots of $x^2 + (3 - a)x + 1 = 2a$ is: **[July 26, 2022 (II)]**

(a) 4 (b) 5 (c) 6 (d) 8

131. If α, β, γ, δ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to: **[July 25, 2022 (I)]**

(a) –4 (b) –1 (c) 1 (d) 4

132. Let α and β be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to: **[June 29, 2022 (I)]**

(a) 50 (b) 250 (c) 1250 (d) 1500

133. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to: **[June 29, 2022 (II)]**

(a) 1 (b) α (c) $1+\alpha$ (d) $1+2\alpha$

134. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is ________. **[2021(S), NA, June 28, 2022 (I)]**

135. Let $f(x)$ be a quadratic polynomial such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is –1, then the sum of the roots of $f(x) = 0$ is equal to **[June 28, 2022 (II)]**

(a) $\frac{11}{3}$ (b) $\frac{7}{3}$ (c) $\frac{13}{3}$ (d) $\frac{14}{3}$

136. Let α, β be the roots of the equation $x^2 - 4\lambda x + 5 = 0$ and α, γ be the roots of the equation $x^2 - (3\sqrt{2} + 2\sqrt{3})x + 7 + 3\lambda\sqrt{3} = 0$. If $\beta + \gamma = 3\sqrt{2}$, then $(\alpha + 2\beta + \gamma)^2$ is equal to _______. **[NA, June 27, 2022 (II)]**

137. The sum of the cubes of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ is ______. **[NA, June 26, 2022 (I)]**

138. Let p and q be two real numbers such that $p + q = 3$ and $p^4 + q^4 = 369$. Then $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2}$ is equal to _______. **[NA, June 26, 2022 (II)]**

139. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to : **[June 24, 2022 (I)]**

(a) 18 (b) 24 (c) 36 (d) 96

140. The sum of all the real roots of the equation $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$ is **[June 24, 2022 (II)]**

(a) $\log_e 3$ (b) $-\log_e 3$ (c) $\log_e 6$ (d) $-\log_e 6$

141. The sum of all integral values of k (k ≠ 0) for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is _____. **[NA, Aug. 26, 2021 (I)]**

142. Let α, β be two roots of the equation $x^2 + (20)^{1/4} x + (5)^{1/2} = 0$. Then $\alpha^8 + \beta^8$ is equal to **[July 20(S), July 27, 2021 (I)]**

(a) 10 (b) 100 (c) 50 (d) 160

143. The number of real solutions of the equation, $x^2 - |x| - 12 = 0$ is: **[July 25, 2021 (II)]**

(a) 2 (b) 3 (c) 1 (d) 4

144. The number of solutions of the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x + 1)^2 - 4 = 0$, $x > 0$, is **[NA, July 20, 2021 (II)]**

145. The value of $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots\infty}}}}$ is equal to **[March 17(s), March 18, 2021 (I)]**

(a) $2+\sqrt{3}$ (b) $3+2\sqrt{3}$
(c) $4+\sqrt{3}$ (d) $1.5+\sqrt{3}$

146. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is ________. **[NA, Feb. 26, 2021 (I)]**

147. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of p_n^2 is ________. **[NA, Feb. 26, 2021 (II)]**

148. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to: **[Feb. 25, 2021 (II)]**

(a) -3 (b) -7 (c) 7 (d) 3

149. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \ldots\infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of

$$\frac{2\sin x}{\sin x + \sqrt{3}\cos x}\left(0 < x < \frac{\pi}{2}\right) \text{ is}$$ **[Feb. 24, 2021(I)]**

(a) $2\sqrt{3}$ (b) $\frac{3}{2}$ (c) $\sqrt{3}$ (d) $\frac{1}{2}$

150. The number of the real roots of the equation

$(x+1)^2 + |x-5| = \frac{27}{4}$ is ________.

[2014(S), NA, Feb. 24, 2021 (II)]

151. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is:

[Sep. 06, 2020 (I)]

(a) 2 (b) 3 (c) 1 (d) 4

152. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to: **[Sep. 06, 2020 (II)]**

(a) $2\alpha(\alpha + 1)$ (b) $-2\alpha(\alpha + 1)$
(c) $2\alpha(\alpha - 1)$ (d) $2\alpha^2$

153. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is : **[Sep. 05, 2020 (I)]**

(a) $\frac{5}{9}$ (b) $\frac{25}{81}$ (c) $\frac{5}{27}$ (d) $\frac{25}{9}$

154. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to :

[Sep. 05, 2020 (II)]

(a) $\frac{27}{32}$ (b) $\frac{1}{24}$ (c) $\frac{3}{8}$ (d) $\frac{27}{16}$

155. Let $u = \frac{2z+i}{z-ki}$, $z = x + iy$ and $k > 0$. If the curve represented by Re(u) + Im(u) = 1 intersects the y-axis at the points P and Q where $PQ = 5$, then the value of k is : **[Sep. 04, 2020 (I)]**

(a) 3/2 (b) 1/2 (c) 4 (d) 2

156. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to :

[Sep. 03, 2020 (I)]

(a) $\frac{9}{4}(9+q^2)$ (b) $\frac{9}{4}(9-q^2)$
(c) $\frac{9}{4}(9+p^2)$ (d) $\frac{9}{4}(9-p^2)$

157. The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is :

[Sep. 03, 2020 (II)]

(a) (0, 2) (b) (2, 4] (c) (1, 3] (d) (–3, –1)

158. The least positive value of 'a' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is ________.

[NA, Jan. 8, 2020 (I)]

159. If the equation, $x^2 + bx + 45 = 0$ ($b \in R$) has conjugate complex roots and they satisfy $|z + 1| = 2\sqrt{10}$, then:

[Jan. 8, 2020 (I)]

(a) $b^2 - b = 30$ (b) $b^2 + b = 72$
(c) $b^2 - b = 42$ (d) $b^2 + b = 12$

160. Let α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\,.\,\lambda\tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is:

[Jan. 7, 2020 (I)]

(a) $10\sqrt{2}$ (b) 10 (c) 5 (d) $5\sqrt{2}$

161. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0, \theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12})(\alpha - \beta)^{24}}$ is equal to : **[April 10, 2019 (I)]**

(a) $\frac{2^{12}}{(\sin\theta - 4)^{12}}$ (b) $\frac{2^{12}}{(\sin\theta + 8)^{12}}$
(c) $\frac{2^{12}}{(\sin\theta - 8)^{6}}$ (d) $\frac{2^{6}}{(\sin\theta + 8)^{12}}$

162. The number of real roots of the equation

$5 + |2^x - 1| = 2^x(2^x - 2)$ is: **[April 10, 2019 (II)]**

(a) 3 (b) 2 (c) 4 (d) 1

163. Let $p, q \in R$. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then: **[April 9, 2019 (I)]**

(a) $p^2 - 4q + 12 = 0$ (b) $q^2 - 4p - 16 = 0$
(c) $q^2 + 4p + 14 = 0$ (d) $p^2 - 4q - 12 = 0$

164. If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is:

[2003(S), Jan 10, 2019 II(S), April 09, 2019 (II)]

(a) $10\sqrt{5}$ (b) $8\sqrt{3}$ (c) $8\sqrt{5}$ (d) $4\sqrt{3}$

165. The sum of the solutions of the equation $\left|\sqrt{x} - 2\right| + \sqrt{x}\left(\sqrt{x} - 4\right) + 2 = 0$, $(x > 0)$ is equal to:

[April 8, 2019 (I)]

(a) 9 (b) 12 (c) 4 (d) 10

166. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is : **[April 8, 2019 (I)]**

(a) 2 (b) 5 (c) 4 (d) 3

167. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m-4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is : **[Jan. 12, 2019 (I)]**

(a) $2-\sqrt{3}$ (b) $4-3\sqrt{2}$
(c) $-2+\sqrt{2}$ (d) $4-2\sqrt{3}$

168. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is : **[Jan. 11, 2019 (I)]**

(a) –81 (b) 100 (c) 144 (d) – 300

169. Consider the quadratic equation $(c-5)x^2 - 2cx + (c-4) = 0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is: **[Jan. 10, 2019 (I)]**

(a) 18 (b) 12 (c) 10 (d) 11

170. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is: **[Jan. 09, 2019 (II)]**

(a) 3 (b) 2 (c) 4 (d) 5

171. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval: **[Jan. 09, 2019 (II)]**

(a) (– 5, – 4) (b) (4, 5)
(c) (5, 6) (d) (3, 4)

172. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6i\, z_0^{81} - 3i\, z_0^{93}$, then arg z is equal to: **[Jan. 09, 2019 (II)]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) 0

173. Let p, q and r be real numbers ($p \neq q$, $r \neq 0$), such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to. **[Online April 16, 2018]**

(a) $p^2 + q^2 + r^2$ (b) $p^2 + q^2$
(c) $2(p^2 + q^2)$ (d) $\frac{p^2+q^2}{2}$

174. If an angle A of a Δ ABC satisfies 5 cos A + 3 = 0, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are. **[Online April 16, 2018]**

(a) sin A, sec A (b) sec A, tan A
(c) tan A, cos A (d) sec A, cot A

175. If tan A and tan B are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$ then the value of $3\sin^2(A+B) - 10\sin(A+B).\cos(A+B) - 25\cos^2(A+B)$ is **[Online April 15, 2018]**

(a) 25 (b) – 25 (c) – 10 (d) 10

176. If $f(x)$ is a quadratic expression such that $f(1) + f(2) = 0$, and – 1 is a root of $f(x) = 0$, then the other root of $f(x) = 0$ is **[Online April 15, 2018]**

(a) $-\frac{5}{8}$ (b) $-\frac{8}{5}$ (c) $\frac{5}{8}$ (d) $\frac{8}{5}$

177. If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + ... + (x+\overline{n-1})(x+n) = 10n$ has two consecutive integral solutions, then n is equal to: **[2017]**

(a) 11 (b) 12 (c) 9 (d) 10

178. The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is : **[2016(S), Online April 9, 2017]**

(a) 16 (b) 14 (c) – 4 (d) – 5

179. Let p(x) be a quadratic polynomial such that p(0)=1. If p(x) leaves remainder 4 when divided by x–1 and it leaves remainder 6 when divided by x + 1; then : **[Online April 8, 2017]**

(a) p(2) = 11 (b) p(2) = 19
(c) p(–2) = 19 (d) p(–2) = 11

180. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1$, $\left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to : **[Online April 10, 2016]**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $2\sqrt{2}$ (d) 2

181. If the two roots of the equation, $(a-1)(x^4 + x^2 + 1) + (a+1)(x^2 + x + 1)^2 = 0$ are real and distinct, then the set of all values of 'a' is : **[Online April 11, 2015]**

(a) $\left(0, \frac{1}{2}\right)$ (b) $\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$
(c) $\left(-\frac{1}{2}, 0\right)$ (d) $(-\infty, -2) \cup (2, \infty)$

182. If $2 + 3i$ is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0$, $k \in R$, then the real root of this equation : **[Online April 10, 2015]**

(a) exists and is equal to $-\frac{1}{2}$.
(b) exists and is equal to $\frac{1}{2}$.
(c) exists and is equal to 1.
(d) does not exist.

183. If $a \in R$ and the equation

$$-3(x-[x])^2+2(x-[x])+a^2=0$$

(where $[x]$ denotes the greatest integer $\le x$) has no integral solution, then all possible values of a lie in the interval: **[2014]**

(a) $(-2,-1)$ (b) $(-\infty,-2)\cup(2,\infty)$

(c) $(-1,0)\cup(0,1)$ (d) $(1,2)$

184. The equation $\sqrt{3x^2+x+5}=x-3$, where x is real, has; **[Online April 19, 2014]**

(a) no solution (b) exactly one solution

(c) exactly two solution (d) exactly four solution

185. If α and β are roots of the equation,

$x^2-4\sqrt{2}kx+2e^{4\ln k}-1=0$ for some k, and $\alpha^2+\beta^2=66$, then $\alpha^3+\beta^3$ is equal to: **[Online April 11, 2014]**

(a) $248\sqrt{2}$ (b) $280\sqrt{2}$ (c) $-32\sqrt{2}$ (d) $-280\sqrt{2}$

186. If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation,

$ax^2+bx+1=0$ ($a \ne 0$, a, b $\in$ R), then the equation,

$x(x+b^3)+(a^3-3abx)=0$ as roots : **[Online April 9, 2014]**

(a) $\alpha^{3/2}$ and $\beta^{3/2}$ (b) $\alpha\beta^{1/2}$ and $\alpha^{1/2}\beta$

(c) $\sqrt{\alpha\beta}$ and $\alpha\beta$ (d) $\alpha^{-\frac{3}{2}}$ and $\beta^{-\frac{3}{2}}$

187. If p and q are non-zero real numbers and $\alpha^3+\beta^3=-p, \alpha\beta=q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta},\frac{\beta^2}{\alpha}$ is : **[Online April 25, 2013]**

(a) $px^2-qx+p^2=0$ (b) $qx^2+px+q^2=0$

(c) $px^2+qx+p^2=0$ (d) $qx^2-px+q^2=0$

188. If α and β are roots of the equation $x^2+px+\frac{3p}{4}=0$, such that $|\alpha-\beta|=\sqrt{10}$, then p belongs to the set : **[Online April 22, 2013]**

(a) $\{2,-5\}$ (b) $\{-3,2\}$ (c) $\{-2,5\}$ (d) $\{3,-5\}$

189 If a complex number z statisfies the equation

$z+\sqrt{2}|z+1|+i=0$, then $|z|$ is equal to : **[Online April 22, 2013]**

(a) 2 (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) 1

190. Let $p, q, r \in R$ and $r > p > 0$. If the quadratic equation $px^2+qx+r=0$ has two complex roots α and β, then $|\alpha|+|\beta|$ is **[Online May 19, 2012]**

(a) equal to1

(b) less than 2 but not equal to 1

(c) greater than 2

(d) equal to 2

191. If the sum of the square of the roots of the equation $x^2-(\sin\alpha-2)x-(1+\sin\alpha)=0$ is least, then α is equal to **[Online May 12, 2012]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

192. The value of k for which the equation $(k-2)x^2+8x+k+4=0$ has both roots real, distinct and negative is **[Online May 7, 2012]**

(a) 6 (b) 3 (c) 4 (d) 1

193. Let for a $\ne a_1 \ne 0$, $f(x)=ax^2+bx+c$,

$g(x)=a_1x^2+b_1x+c_1$ and $p(x)=f(x)-g(x)$.

If $p(x)=0$ only for $x=-1$ and $p(-2)=2$, then the value of $p(2)$ is : **[2011 RS]**

(a) 3 (b) 9 (c) 6 (d) 18

194. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4,3). Rahul made a mistake in writing down coefficient of x to get roots (3,2). The correct roots of equation are : **[2011 RS]**

(a) 6, 1 (b) 4, 3 (c) – 6, – 1 (d) – 4, – 3

195. Let α, β be real and z be a complex number. If $z^2+\alpha z+\beta=0$ has two distinct roots on the line Re $z=1$, then it is necessary that : **[2011]**

(a) $\beta\in(-1,0)$ (b) $|\beta|=1$

(c) $\beta\in(1,\infty)$ (d) $\beta\in(0,1)$

196. If the roots of the equation $bx^2+cx+a=0$ be imaginary, then for all real values of x, the expression $3b^2x^2+6bcx+2c^2$ is : **[2009]**

(a) less than $4ab$ (b) greater than $-4ab$

(c) less than $-4ab$ (d) greater than $4ab$

197. If the difference between the roots of the equation $x^2+ax+1=0$ is less than $\sqrt{5}$, then the set of possible values of a is **[2007]**

(a) $(3,\infty)$ (b) $(-\infty,-3)$ (c) $(-3,3)$ (d) $(-3,\infty)$

198. If the roots of the quadratic equation $x^2+px+q=0$ are $\tan 30°$ and $\tan 15°$, respectively, then the value of $2+q-p$ is **[2005(S), 2006]**

(a) 2 (b) 3 (c) 0 (d) 1

199. If $z^2+z+1=0$, where z is complex number, then the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\ldots\ldots+\left(z^6+\frac{1}{z^6}\right)^2$ is **[2006]**

(a) 18 (b) 54 (c) 6 (d) 12

200. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals **[2005]**

(a) -2 (b) 3 (c) 2 (d) 1

201. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is **[2004]**

(a) 4 (b) 12 (c) 3 (d) $\frac{49}{4}$

202. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its root are **[2004]**

(a) $-1, 2$ (b) $-1, 1$ (c) $0, -1$ (d) $0, 1$

203. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$ **[2002]**

(a) is always positive (b) is always negative

(c) does not exist (d) none of these

204. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then **[2002]**

(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$

(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

205. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is **[2002]**

(a) $3x^2 - 19x + 3 = 0$ (b) $3x^2 + 19x - 3 = 0$

(c) $3x^2 - 19x - 3 = 0$ (d) $x^2 - 5x + 3 = 0.$

Topic 4 Condition for Common Roots, Maximum and Minimum value of Quadratic Expression, Quadratic Expression in two Variables, Solution of Quadratic Inequalities

206. The number of real roots of the equation $x|x| - 5|x+2| + 6 = 0$, is **[NA, April 15, 2023 (I)]**

(a) 5 (b) 3 (c) 6 (d) 4

207. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x+2| - 4 = 0$ respectively, where $[x]$ denotes the greatest integer $\leq x$. Then $m^2 + mn + n^2$ is equal to ____. **[NA, April 8, 2023 (II)]**

208. Let $S = \left\{x : x \in \mathbb{R}\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10\right\}$.

Then $n(S)$ is equal to **[Feb. 1, 2023 (I)]**

(a) 4 (b) 0 (c) 6 (d) 2

209. The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0$, $x \in \mathbb{R}$ has: **[Jan. 31, 2023 (II)]**

(a) Two solutions and both are negative

(b) No solution

(c) Four solutions two of which are negative

(d) Two solutions and only one of them is negative

210. If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real roots is $\frac{3}{\sqrt{2\beta}}$ then β is equal to ______. **[NA, Jan. 30, 2023 (II)]**

211. Let $\lambda \in R$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = \{x + 1 : x \text{ is an integer solution of } E\}$ is_______. **[NA, Jan. 24, 2023 (I)]**

212. If for some p, q, r $\in \mathbf{R}$, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2 + r^2}{p^2}$ is equal to ______ . **[NA July 26, 2022 (I)]**

213. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to _______. **[2020, NA, Aug. 26, 2021 (II)]**

214. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is: **[Feb. 25, 2021 (I)]**

(a) 2 (b) 3 (c) 4 (d) 0

215. Let $a, b \in$ R, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to : **[2009(S) Jan. 9, 2020 (II)]**

(a) 25 (b) 26 (c) 28 (d) 24

216. If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to: **[Jan. 10, 2019 (I)]**

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $\frac{7}{4}$ (d) $\frac{3}{2}$

217. If $\lambda \in$ R is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is **[Online April 15, 2018]**

(a) 20 (b) $2\sqrt{5}$ (c) $2\sqrt{7}$ (d) $4\sqrt{2}$

218. If $|z - 3 + 2i| \leq 4$ then the difference between the greatest value and the least value of $|z|$ is **[Online April 15, 2018]**

(a) $\sqrt{13}$ (b) $2\sqrt{13}$ (c) 8 (d) $4 + \sqrt{13}$

219. If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from -1, then $|b|$ is equal to : **[Online April 9, 2016]**

(a) 2 (b) 3 (c) $\sqrt{3}$ (d) $\sqrt{2}$

220. If non-zero real numbers b and c are such that min f(x) > max g(x), where $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ $(x \in R)$; then $\left|\frac{c}{b}\right|$ lies in the interval: **[Online April 19, 2014]**

(a) $\left(0, \frac{1}{2}\right)$ (b) $\left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ (c) $\left[\frac{1}{\sqrt{2}}, \sqrt{2}\right]$ (d) $\left(\sqrt{2}, \infty\right)$

221. If equations $ax^2 + bx + c = 0$ $(a, b, c \in R, a \neq 0)$ and $2x^2 + 3x + 4 = 0$ have a common root, then a : b : c equals: **[2013(s), Online April 9, 2014]**

(a) 1 : 2 : 3 (b) 2 : 3 : 4 (c) 4 : 3 : 2 (d) 3 : 2 : 1

222. The least integral value α of x such that $\frac{x-5}{x^2+5x-14} > 0$, satisfies : **[Online April 23, 2013]**

(a) $\alpha^2 + 3\alpha - 4 = 0$ (b) $\alpha^2 - 5\alpha + 4 = 0$

(c) $\alpha^2 - 7\alpha + 6 = 0$ (d) $\alpha^2 + 5\alpha - 6 = 0$

223. The values of 'a' for which one root of the equation $x^2 - (a+1)x + a^2 + a - 8 = 0$ exceeds 2 and the other is lesser than 2, are given by : **[2006(S), Online April 9, 2013]**

(a) $3 < a < 10$ (b) $a \geq 10$

(c) $-2 < a < 3$ (d) $a \leq -2$

224. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|z|$ is equal to: **[2009]**

(a) $\sqrt{5}+1$ (b) 2 (c) $2+\sqrt{2}$ (d) $\sqrt{3}+1$

225. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is **[2006]**

(a) $\frac{1}{4}$ (b) 41 (c) 1 (d) $\frac{17}{7}$

226. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval **[2005]**

(a) (5, 6] (b) (6, ∞) (c) (–∞, 4) (d) [4, 5]

ANSWER KEYS

1	(d)	24	(b)	47	(a)	70	(2)	93	(c)	116	(a)	139	(b)	162	(d)	185	(d)	208	(a)
2	(b)	25	(b)	48	(a)	71	(d)	94	(d)	117	(a)	140	(b)	163	(d)	186	(a)	209	(a)
3	(b)	26	(b)	49	(a)	72	(80)	95	(a)	118	(Bonus)	141	(66)	164	(c)	187	(b)	210	(13)
4	(1680)	27	(c)	50	(14)	73	(5)	96	(a)	119	(b)	142	(c)	165	(d)	188	(c)	211	(5)
5	(d)	28	(a)	51	(9)	74	(d)	97	(Bonus)	120	(c)	143	(a)	166	(c)	189	(c)	212	(272)
6	(c)	29	(a)	52	(25)	75	(6)	98	(d)	121	(45)	144	(1)	167	(b)	190	(c)	213	(18)
7	(c)	30	(b)	53	(d)	76	(13)	99	(a)	122	(d)	145	(d)	168	(d)	191	(d)	214	(b)
8	(Bonus)	31	(c)	54	(b)	77	(6)	100	(a)	123	(b)	146	(1)	169	(d)	192	(b)	215	(a)
9	(9)	32	(a)	55	(c)	78	(a)	101	(b)	124	(16)	147	(324)	170	(a)	193	(d)	216	(c)
10	(b)	33	(d)	56	(a)	79	(0)	102	(a)	125	(16)	148	(b)	171	(b)	194	(a)	217	(b)
11	(a)	34	(c)	57	(b)	80	(a)	103	(a)	126	(6)	149	(d)	172	(a)	195	(c)	218	(b)
12	(c)	35	(a)	58	(d)	81	(48)	104	(a)	127	(b)	150	(2)	173	(b)	196	(b)	219	(c)
13	(c)	36	(b)	59	(c)	82	(3)	105	(c)	128	(0)	151	(a)	174	(b)	197	(c)	220	(d)
14	(c)	37	(a)	60	(6)	83	(310)	106	(c)	129	(b)	152	(b)	175	(b)	198	(b)	221	(b)
15	(d)	38	(c)	61	(a)	84	(c)	107	(c)	130	(c)	153	(b)	176	(d)	199	(d)	222	(d)
16	(c)	39	(a)	62	(d)	85	(c)	108	(b)	131	(b)	154	(d)	177	(a)	200	(d)	223	(c)
17	(6)	40	(c)	63	(d)	86	(c)	109	(b)	132	(a)	155	(d)	178	(c)	201	(d)	224	(a)
18	(1)	41	(b)	64	(26)	87	(a)	110	(d)	133	(a)	156	(d)	179	(c)	202	(c)	225	(b)
19	(b)	42	(a)	65	(c)	88	(4)	111	(c)	134	(2)	157	(c)	180	(a)	203	(a)	226	(c)
20	(4)	43	(c)	66	(40)	89	(b)	112	(51)	135	(a)	158	(8)	181	(b)	204	(a)		
21	(c)	44	(24)	67	(2)	90	(c)	113	(2)	136	(98)	159	(a)	182	(b)	205	(a)		
22	(10)	45	(d)	68	(a)	91	(b)	114	(a)	137	(36)	160	(b)	183	(c)	206	(b)		
23	(b)	46	(a)	69	(b)	92	(d)	115	(c)	138	(4)	161	(b)	184	(a)	207	(9)		