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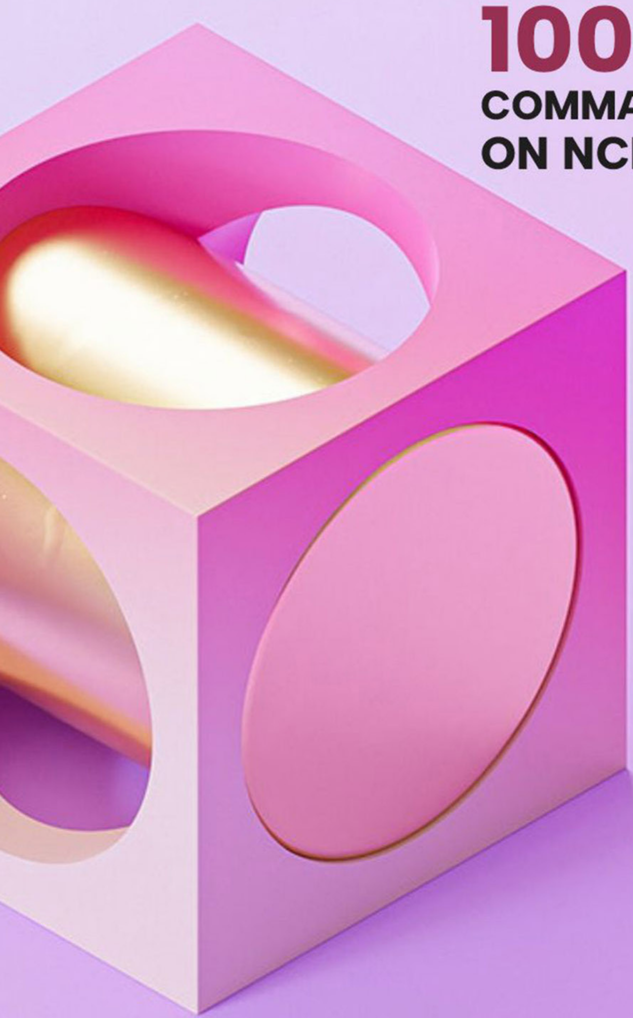

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Contents of Free Sample Book

Class XII

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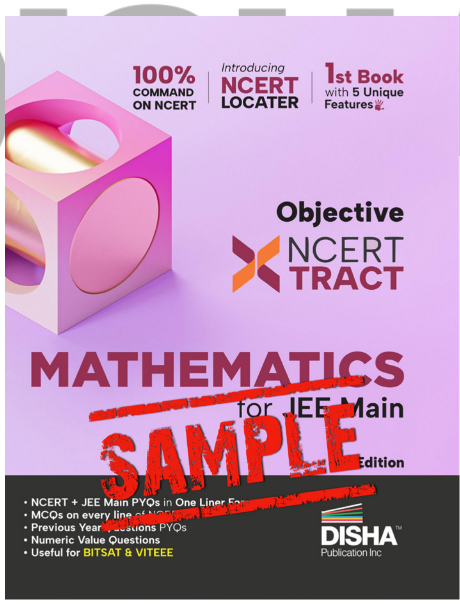

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Class XIth

A1–A198

Note : * The four Exercises in each of the chapters are :

- Exercise 1 : NCERT Based Topic-wise MCQs
- Exercise 2 : NCERT Exemplar & Past Years JEE Main
- Exercise 3 : Skill Enhancer MCQs
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Class XIIth B1–B198

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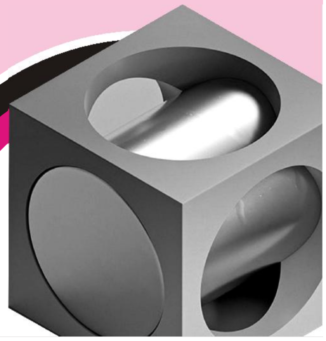
Mock Tests

CHAPTER 7

Integrals



Trend Analysis JEE MAIN

	JEE	Remarks
Number of Questions from 2022-16	14	More important for JEE Main.
Weightage	6.83%	

			JEE	
Year	Topic Name	Concept Used	No. of Questions	Difficulty Level
2022	Evaluation of definite integral/ limit of the sum	Properties of definite integral / limit of sum	2	Average
2021	Partial Fraction/ Evaluation of definite integral	Partial fraction/ properties of definite integral	2	Average/ Difficult
2020	Evaluation of definite integral	Integration by substitution	1	Average
2019	Evaluation of definite integral / indefinite integral	Properties of definite integral / integration by substitution	4	Easy/ Average
2018	Evaluation of definite integral / indefinite integral	Properties of definite integral / integration by substitution	2	Easy/ Average
2017	Evaluation of definite integral/ definite integral	Properties of definite integral / integration by substitution	2	Easy
2016	Indefinite integral	Integration by substitution	1	Difficult

NCERT ONE-LINERS
(Important Points to Remember)

7.1 Introduction

- Integral Calculus is motivated by the problem of defining and calculating the area of the region bounded by the graph of the functions.
- The functions that could possibly have given function as a derivative are called anti derivatives (or primitive) of the function. Further, the formula that gives all these anti derivatives is called the ***indefinite integral*** of the function and such process of finding anti derivatives is called integration. Such type of problems arise in many practical situations. e.g., if we know the instantaneous velocity of an object at any instant, then there arises a natural question, i.e., can we determine the position of the object at any instant?
- Integral calculus arises out of the efforts of solving the problems of the following types:
  - the problem of finding a function whenever its derivative is given,
  - the problem of finding the area bounded by the graph of a function under certain conditions.
- Indefinite and definite integrals, which together constitute the ***Integral Calculus***.
- There is a connection, known as the ***Fundamental Theorem of Calculus***, between indefinite integral and definite integral which makes the definite integral as a practical tool for science and engineering.

7.2 Integration as an Inverse Process of Differentiation

- Integration is the inverse process of differentiation. We are given the derivative of a function and asked to find its primitive, i.e., the original function. Such a process is called ***integration or anti differentiation.***
- Anti derivatives (or integrals) of the above cited functions are not unique. Actually, there exist infinitely many anti derivatives of each of these functions which can be obtained by choosing C arbitrarily from the set of real numbers. C is the ***parameter*** by varying which one gets different anti derivatives (or integrals) of the given function.
- Any arbitrary real number C, (also called ***constant of integration***)
- $\int f(x)dx$ which will represent the entire class of anti derivatives read as the indefinite integral of **f** with respect to **x**. Symbolically, we write $\int f(x)dx = F(x) + C$. JEE M 2020

- Some symbols/terms/phrases are given below.

Symbols/Terms/ Phrases	Meaning
$\int f(x)dx$	Integral of f with respect to x
$f(x)$ in $\int f(x)\,dx$	Integrand
x in $\int f(x)\,dx$	Variable of integration
Integrate	Find the integral
An integral of f	A function F such that $F'(x) = f(x)$
Integration	The process of finding the integral
Constant of Integration	Any real number C, considered as constant function

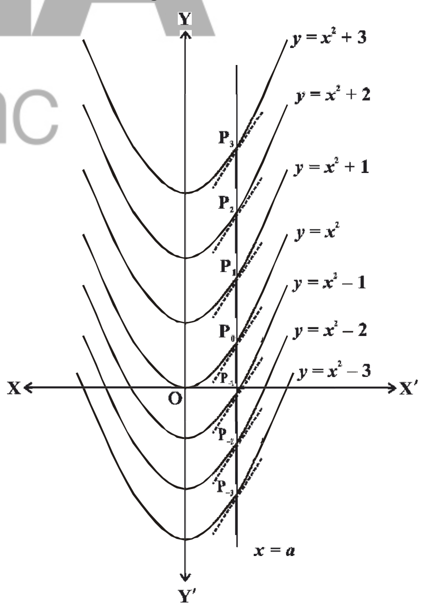
- The formulae for the derivatives of many important functions are:

	Derivatives	**Integrals (Anti derivatives)**
(i)	$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$;	$\int x^n dx = \frac{x^{n-1}}{n+1} + C, n \neq -1$
	Particularly, we note that	
	$\frac{d}{dx}(x) = 1$;	$\int dx = x + C$
(ii)	$\frac{d}{dx}(\sin x) = \cos x$;	$\int \cos x\, dx = \sin x + C$
(iii)	$\frac{d}{dx}(-\cos x) = \sin x$;	$\int \sin x\, dx = -\cos x + C$
(iv)	$\frac{d}{dx}(\tan x) = \sec^2 x$;	$\int \sec^2 x\, dx = \tan x + C$
(v)	$\frac{d}{dx}(-\cot x) = \text{cosec}^2 x$;	$\int \text{cosec}^2 x\, dx = -\cot x + C$
(vi)	$\frac{d}{dx}(\sec x) = \sec x \tan x$;	$\int \sec x \tan x\, dx = \sec x + C$
(vii)	$\frac{d}{dx}(-\text{cosec}\, x) = \text{cosec}\, x \cot x$;	$\int \text{cosec}\, x \cot x\, dx = -\text{cosec}\, x + C$
(viii)	$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$;	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
(ix)	$\frac{d}{dx}(-\cos^{-1} x) = \frac{1}{\sqrt{1-x^2}}$;	$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$
(x)	$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$;	$\int \frac{dx}{1-x^2} = \tan^{-1} x + C$
(xi)	$\frac{d}{dx}(-\cot^{-1} x) = \frac{1}{1+x^2}$;	$\int \frac{dx}{1+x^2} = -\cot^{-1} x + C$
(xii)	$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$;	$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$
(xiii)	$\frac{d}{dx}(-\text{cosec}^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$;	$\int \frac{dx}{x\sqrt{x^2-1}} = -\text{cosec}^{-1} x + C$
(xiv)	$\frac{d}{dx}(e^x) = e^x$;	$\int e^x dx = e^x + C$
(xv)	$\frac{d}{dx}(\log\|x\|) = \frac{1}{x}$;	$\int \frac{1}{x} dx = \log\|x\| + C$
(xvi)	$\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x$;	$\int a^x dx = \frac{a^x}{\log a} + C$ **JEE M 2019**

- In practice, we normally do not mention the interval over which the various functions are defined. However, in any specific problem one has to keep it in mind.

Geometrical interpretation of indefinite integral:

- $\int 2x\, dx = x^2 + C = F_C(x)$ (say), implies that



- The tangents to all the curves $y = F_C(x)$, $C \in \mathbf{R}$, at the points of intersection of the curves by the line $x = a$, $(a \in \mathbf{R})$, are parallel.

- The different values of C will correspond to different members of this family and these members can be obtained by shifting any one of the curves parallel to itself.

Some properties of indefinite integral:

- The process of differentiation and integration are inverses of each other in the sense of the following results:

$$\frac{d}{dx}\int f(x)\,dx = f(x)$$

and $\int f'(x)\,dx = f(x)+C$, where C is any arbitrary constant.

- Two indefinite integrals with the same derivative lead to the same family of curves and so they are equivalent.

$\int f(x)dx$ and $\int g(x)dx$ are equivalent.

$\int f(x)dx = \int g(x)dx$

- $\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$
- For any real number k, $\int k\, f(x)\,dx = k\int f(x)\,dx$
- A finite number of functions $f_1, f_2, ..., f_n$ and the real numbers, $k_1, k_2, ..., k_n$ giving

$$\int [k_1 f_1(x) + k_2 f_2(x) + ... + k_n f_n(x)]\,dx$$

$$= k_1\int f_1(x)\,dx + k_2\int f_2(x)\,dx + ... + k_n\int f_n(x)\,dx.$$

- To find an anti derivative of a given function, we search intuitively for a function whose derivative is the given function. The search for the requisite function for finding an anti derivative is known as integration by the method of inspection.
- We shall write only one constant of integration in the final answer.
- Determines a specific value of C giving unique anti derivative of the given function.
- Sometimes, F is not expressible in terms of elementary functions viz., polynomial, logarithmic, exponential, trigonometric functions and their inverses etc. We are therefore blocked for finding $\int f(x)dx$. For example, it is not possible to find $\int e^{-x^2}$ by inspection since we can not find a function whose derivative is e^{-x^2}.
- When the variable of integration is denoted by a variable other than x, the integral formulae are modified accordingly.

Comparison between differentiation and integration:

- Both are operations on functions.
- Both satisfy the property of linearity.
- We have already seen that all functions are not differentiable. Similarly, all functions are not integrable.
- The derivative of a function, when it exists, is a unique function. The integral of a function is not so. However, they are unique upto an additive constant.
- When a polynomial function P is differentiated, the result is a polynomial whose degree is 1 less than the degree of P. When a polynomial function P is integrated, the result is a polynomial whose degree is 1 more than that of P.
- We can speak of the derivative at a point. We never speak of the integral at a point, we speak of the integral of a function over an interval on which the integral is defined.
- The derivative of a function has a geometrical meaning, namely, the slope of the tangent to the corresponding curve at a point. Similarly, the indefinite integral of a function represents geometrically, a family of curves placed parallel to each other having parallel tangents at the points of intersection of the curves of the family with the lines orthogonal (perpendicular) to the axis representing the variable of integration.
- The derivative is used for finding some physical quantities like the velocity of a moving particle, when the distance traversed at any time t is known. Similarly, the integral is used in calculating the distance traversed when the velocity at time t is known.

7.3 Methods of Integration

Integration by substitution:

- Integral $\int f(x)dx$ can be transformed into another form by changing the independent variable x to t by substituting $x = g(t)$ then $dx \equiv g'(t)dt$

$$I = \int f(x)dx = \int f(g(t))g'(t)dt$$

- We make a substitution for a function whose derivative also occurs in the integrand as illustrated in the following examples.
- Some important integrals involving trigonometric functions.
  - (i) $\int \tan x\,dx = \log|\sec x| + C$
  - (ii) $\int \cot x\,dx = \log|\sin x| + C$
  - (iii) $\int \sec x\,dx = \log|\sec x + \tan x| + C$
  - (iv) $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + C$ **JEE M 2021**

- For integral of $I = \int \frac{P(x)}{[f(x)]^n}dx$, we convert it in the form

$$I = \int \frac{P\left(\frac{1}{x}\right)}{f\left(\frac{1}{x}\right)^n}dx \text{ and } P\left(\frac{1}{x}\right) = A\int\left(\frac{1}{x}\right) + B$$ **JEE M 2016**

then integrate by Substitution Method.

Integration using trigonometric identities:

- When the integrand involves some trigonometric functions, we use some known identities to find the integral.

(i) $\sin^2 x = \frac{1-\cos 2x}{2}$

(ii) $\cos^2 x = \frac{1+\cos 2x}{2}$

(iii) $\sin^3 x = \frac{3\sin x + \sin 3x}{4}$

(iv) $\cos^3 x = \frac{3\cos x + \cos 3x}{4}$

(v) $\sin x \cdot \sin y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$

(vi) $\cos x \cdot \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$

(vii) $\sin x \cdot \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$

7.4 Integrals of Some Particular Functions

(i) $\displaystyle\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

(ii) $\displaystyle\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

(iii) $\displaystyle\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

(iv) $\displaystyle\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + C$

(v) $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

(vi) $\displaystyle\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + C$

(vii) To find the integral $\displaystyle\int \frac{dx}{ax^2+bx+c}$, we write

$$ax^2+bx+c = a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right] =$$

$$a\left[\left(x+\frac{b}{2a}\right)^2+\left(\frac{c}{a}-\frac{b^2}{4a^2}\right)\right]$$

Put $x+\dfrac{b}{2a} = t$ so that $dx = dt$ and writing

$\dfrac{c}{a}-\dfrac{b^2}{4a^2} = \pm k^2$. The integral reduced to the form

$\displaystyle\frac{1}{a}\int \frac{dt}{t^2 \pm k^2}$.

(viii) To find the integral of the type $\displaystyle\int \frac{dx}{\sqrt{ax^2+bx+c}}$, proceeding as in (7).

(ix) To find the integral of the type $\displaystyle\int \frac{px+q}{ax^2+bx+c}dx$, where p, q, a, b, c are constants, we are to find real numbers A, B such that

$$px+q = A\frac{d}{dx}(ax^2+bx+c)+B = A\,(2ax+b)+B$$

To determine A and B, we equate from both sides the coefficients of x and the constant terms. A and B are thus obtained and hence the integral is reduced to one of the known forms.

JEE M 2019

(x) For the evaluation of the integral of the type

$\displaystyle\int \frac{(px+q)\,dx}{\sqrt{ax^2+bx+c}}$, we proceed as in (9).

- $\displaystyle I_n = \int \tan^n x\,dx = \frac{\tan^{n-2}x}{n-1} - I_{n-2}$ where $n \in N$.

JEE M 2017

7.5 Integration by Partial Fractions

- A rational function is defined as the ratio of two polynomials in the form $\dfrac{P(x)}{Q(x)}$, where P (x) and Q(x) are polynomials in x and Q$(x) \neq 0$.
- If the degree of P(x) is less than the degree of Q(x), then the rational function is called proper, otherwise, it is called improper.
- The improper rational functions can be reduced to the proper rational functions by long division process. Thus, if $\dfrac{P(x)}{Q(x)}$ is improper, then $\dfrac{P(x)}{Q(x)} = T(x)+\dfrac{P_1(x)}{Q(x)}$, where T$(x)$ is a polynomial in x and $\dfrac{P_1(x)}{Q(x)}$ is a proper rational function.
- A proper rational function which we shall consider here for integration purposes will be those whose denominators can be factorised into linear and quadratic factors.
- It is always possible to write the integrand as a sum of simpler rational functions by a method called partial fraction decomposition.

S. No.	Form of the rational function	Form of the partial fraction
1.	$\dfrac{px+q}{(x-a)(x=b)}$, $a \neq b$	$\dfrac{A}{x-a}+\dfrac{B}{x-b}$
2.	$\dfrac{px+q}{(x-a)^2}$	$\dfrac{A}{x-a}+\dfrac{B}{(x-a)^2}$
3.	$\dfrac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\dfrac{A}{x-a}+\dfrac{B}{x-b}+\dfrac{C}{x-c}$
4.	$\dfrac{px^2+qx+r}{(x-a)^2(x-b)}$	$\dfrac{A}{x-a}+\dfrac{B}{(x-a)^2}+\dfrac{C}{x-b}$
5.	$\dfrac{px^2+qx+r}{(x-a)(x^2+bx+c)}$ where x^2+bx+c cannot be factorised further.	$\dfrac{A}{x-a}+\dfrac{Bx+C}{x^2+bx+c}$

- Write the integrand as a sum of simpler rational functions by a method partical fraction decomposition.

$\dfrac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \dfrac{A}{x-a}+\dfrac{Bx+c}{x^2+bx+c}$ where x^2+bx+c cannot be factorised further JEE M 2021

7.6 Integration by Parts

- If u and v are any two differentiable functions of a single variable x (say). Then, by the product rule of differentiation, we have

$$\frac{d}{dx}(uv) = u\frac{dv}{dx}+v\frac{du}{dx}$$

If we take f as the first function and g as the second function.

- Then this preference in this order can be decided by the word ILATE, where
  I → stands for Inverse function.
  L → stands for Logarithmic function.
  A → stands for Algebraic function.
  T → stands for Trigonometric function.
  E → stands for Exponential function.
  The formula may be stated as follows:
- "The integral of the product of two functions = (first function) × (integral of the second function) – Integral of [(differential coefficient of the first function) × (integral of the second function)]"
- It is worth mentioning that integration by parts is not applicable to product of functions in all cases.
- Observe that while finding the integral of the second function, we did not add any constant of integration.
- Usually, if any function is a power of x or a polynomial in x, then we take it as the first function. However, in cases where other function is inverse trigonometric function or logarithmic function, then we take them as first function.

Integral of the type $\int e^x[f(x)+f'(x)]dx$:

- $\int e^x[f(x)+f'(x)]\,dx = e^x f(x) + C$ JEE M 2021

Integrals of some more types:

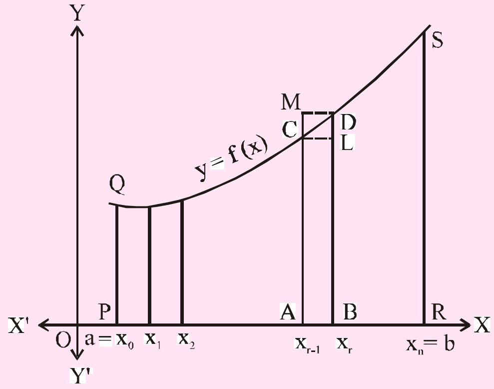
- $I = \int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$
- $\int\sqrt{x^2+a^2}\,dx = \frac{1}{2}x\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$
- $\int\sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

7.7 Definite Integral as the Limit of Sum

- The definite integral has a unique value.
- A definite integral is denoted by $\int_a^b f(x)\,dx$, where a is called the lower limit of the integral and b is called the upper limit of the integral.
- The definite integral is introduced either as the limit of a sum or if it has an anti derivative F in the interval $[a, b]$, then its value is the difference between the values of F at the end points, i.e., $F(b) - F(a)$.

Definite integral as the limit of a sum:

- The definite integral $\int_a^b f(x)\,dx$ is the area bounded by the curve $y = f(x)$, the ordinates $x = a$, $x = b$ and the x-axis.



$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\,[f(a)+f(a+h)+\ldots+f(a+(n-1)h]$$

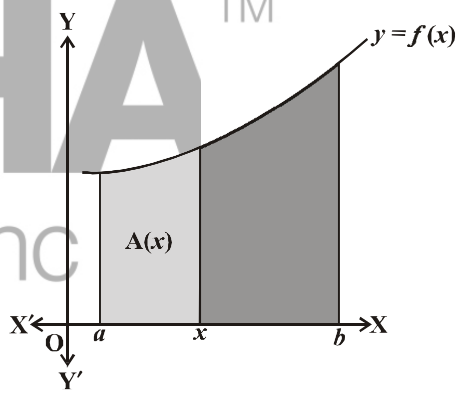
or $$\int_a^b f(x)\,dx = (b-a)\lim_{x\to\infty}\frac{1}{n}[f(a)+f(a+h)+\ldots+f(a+(n-1)h]$$

Where $h = \frac{b-a}{n} \to 0$ as $n\to\infty$ JEE M 2019

- The value of the definite integral of a function over any particular interval depends on the function and the interval, but not on the variable of integration that we choose to represent the independent variable.
- The variable of integration is called a *dummy variable.*

7.8 Fundamental Theorem of Calculus

Area function:



- The area of this shaded region is a function of x. We denote this function of x by $A(x)$. We call the function $A(x)$ as Area function and is given by

$$A(x) = \int_a^x f(x)\,dx$$

First fundamental theorem of integral calculus:

- **Theorem 1** Let f be a continuous function on the closed interval $[a, b]$ and let A (x) be the area function. Then $A'(x) = f(x)$, for all $x \in [a, b]$.

Second fundamental theorem of integral calculus:

- **Theorem 2** Let f be continuous function defined on the closed interval $[a, b]$ and F be an anti derivative of f. Then

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a).$$ JEE M 2020

- This theorem is very useful, because it gives us a method of calculating the definite integral more easily.
- The crucial operation in evaluating a definite integral is that of finding a function whose derivative is equal to the integrand. This strengthens the relationship between differentiation and integration.
- In $\int_a^b f(x)\,dx$, the function f needs to be well defined and continuous in $[a, b]$. Steps for calculating $\int_a^b f(x)\,dx$.
- Find the indefinite integral $\int f(x)dx$. Let this be F(x). There is no need to keep integration constant C
- Evaluate $F(b) - F(a) = [F(x)]_a^b$, which is the value of $\int_a^b f(x)\,dx$.

7.9 Evaluation of Definite Integrals by Substitution

- To evaluate $\int_a^b f(x)\,dx$, by substitution, the steps could be as follows:
- Consider the integral without limits and substitute, $y = f(x)$ or $x = g(y)$ to reduce the given integral to a known form.
- Integrate the new integrand with respect to the new variable without mentioning the constant of integration.
- Resubstitute for the new variable and write the answer in terms of the original variable.
- Find the values of answers obtained in (3) at the given limits of integral and find the difference of the values at the upper and lower limits.
- For integral of function $\dfrac{1}{x^p\left(1+x^2\right)^{\frac{m}{n}}}$, where q is multiple of n. We convert it in the form of $\dfrac{1}{x^p + \dfrac{2m}{n}\left(\dfrac{1}{x^2}+1\right)^{\frac{m}{n}}}$ and substitute $1+\dfrac{1}{x^q} = u$. **JEE M 2020**
- In order to quicken this method, we can proceed as follows: After performing steps 1, and 2, there is no need of step 3. Here, the integral will be kept in the new variable itself, and the limits of the integral will accordingly be changed, so that we can perform the last step.

7.10 Some Properties of Definite Integrals

- Some important properties of definite integrals. These will be useful in evaluating the definite integrals more easily.

P_0 : $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

P_1 : $\int_a^b f(x)\,dx - \int_b^a f(x)\,dx$. In particular, $\int_a^a f(x)\,dx = 0$

P_2 : $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

P_3 : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

P_4 : $\int_0^a f(x)\,dx = \int_0^a f(a-x)dx$

(Note that P_4 is a particular case of P_3)

P_5 : $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

P_6 : $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if $(2a-x) = f(x)$ and 0 if $f(2a-x) = -f(x)$

P_7 : (i) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)dx$, if f is an even function, i.e., if $f(-x) = f(x)$.

(ii) $\int_{-a}^a f(x)\,dx = 0$, if f is an odd function, i.e., if $f(-x) = -f(x)$.

- If $f(x) = [x]$ = greatest integer fuction such that $f(x) \le x$ then

$$I = \int_0^n [x]dx = \int_0^1 0dx + \int_1^2 1dx + \int_2^3 2dx + \ldots\ldots + \int_{[n]}^n [n]dx.$$

JEE M 2021

- Use the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and after simplification adding to previous integral. **JEE M 2019**
- Use the property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ and after simplification adding to previous integral **JEE M 2018**

Tips/Tricks/Techniques ONE-Liners
(Exam Special)

- **Some Important Substitutions are:**

Function	Substitutions
$\sqrt{a^2-x^2}$	$x = a\sin\theta$ or $x = a\cos\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$

- $\int e^{ax}\sin bx\,dx = \dfrac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + c$

$$= \frac{e^{ax}}{\sqrt{a^2+b^2}}\sin\left(bx - \tan^{-1} b/a\right) + c$$

- $\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$

  $= \frac{e^{ax}}{\sqrt{a^2+b^2}} \cos(bx - \tan^{-1} b/a) + c$

- **Differentiation of integration (leibnithz's rule) :**

  $$\frac{d}{dt}\left[\int_{\phi(t)}^{\Psi(t)} f(x)dx\right] = f\{\psi(t)\}\,\psi'(t) - f\{\phi(t)\}\,\phi'(t)$$

- $\int_a^b f(x)dx = (b-a)\int_0^1 f((b-a)x+a)dx.$
- If f (x) is defined on [a, b], then

  $$\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|\,dx$$

- **Reduction formulae for definite integration:**

  (i) $\int_0^\infty e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2}$

  (ii) $\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}$

  (iii) $\int_0^\infty e^{-ax}x^n\,dx = \frac{n!}{a^n+1}$

- If $\int f(x)dx = \phi(x)$, then $\int f(ax+b)dx = \frac{1}{a}\phi(ax+b)$
- $\int (ax+b)^n dx = \frac{1}{a}\cdot\frac{(ax+b)^{n+1}}{n+1} + c,\quad n \ne -1$
- $\int \frac{1}{ax+b}dx = \frac{1}{a}\log|ax+b| + c$
- $\int e^{ax+b}dx = \frac{1}{a}e^{ax+b} + c$
- $\int a^{bx+c}dx = \frac{1}{b}\cdot\frac{a^{bx+c}}{\log a} + c,\ a > 0$ and $a \ne 1$
- $\int \frac{f'(x)}{f(x)}dx = \log\{f(x)\} + c$
- $\int \{f(x)\}^n f'(x)\,dx = \frac{\{f(x)\}^{n+1}}{n+1},\quad n \ne -1$
- $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$
- $\int [xf'(x) + f(x)]dx = x f(x) + c$

Exercise 1 : NCERT Based Topic-wise MCQs

7.2 Integration as an Inverse Process of Differentiation

1. $\int x^{51}(\tan^{-1}x + \cot^{-1}x)\,dx$ **NCERT** Page-296

   (a) $\frac{x^{52}}{52}(\tan^{-1}x + \cot^{-1}x) + c$

   (b) $\frac{x^{52}}{52}(\tan^{-1}x - \cot^{-1}x) + c$

   (c) $\frac{\pi x^{52}}{104} + \frac{\pi}{2} + c$

   (d) $\frac{x^{52}}{52} + \frac{\pi}{2} + c$

2. $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}dx =$ **NCERT** Page-295

   (a) $e.3^{-3x} + c$ (b) $e^3 \log x + c$

   (c) $\frac{x^3}{3} + c$ (d) None of these

3. Evaluate : $\int (x^2+x)^2 dx$ **NCERT** Page-296

   (a) $\frac{x^5}{5} + \frac{x^3}{3} + \frac{x^4}{2}$ (b) $\frac{x^5}{5} + \frac{x^3}{3} + \frac{x^4}{2} + c$

   (c) $5x^5 + 3x^5 + 8x^4$ (d) $5x^5 + 3x^3 + 8x^4 + c$

4. Evaluate: $\int \frac{x^2 - 5x + 6}{x-2}dx$ **NCERT** Page-296

   (a) $\frac{x^2}{2} - 3x$ (b) $x - 3$

   (c) $\frac{x^2}{2} - 3x + c$ (d) $\frac{x^2}{2} + 3x + c$

5. Evaluate: $\int \frac{x^2 + x + 1}{\sqrt{x}}dx$ **NCERT** Page-296

   (a) $\frac{5}{2}x^{5/2} + \frac{3}{2}x^{3/2} + \frac{1}{2}x^{1/2} + c$

   (b) $\frac{2}{5}x^{5/2} + \frac{3}{2}x^{3/2} + 2x^{1/2} + c$

   (c) $x^{3/2} + x^{1/2} + x^{-1/2}$

   (d) $\frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} + c$

6. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x\cos^2 x}dx =$ **NCERT** Page-295

   (a) tan x + cot x + c (b) cosec x + sce x + c

   (c) tan x + sec x + c (d) tan x + cosec x + c

7. For $I(x)=\int \frac{\sec^2 x - 2022}{\sin^{2022} x}dx$, if $I\left(\frac{\pi}{4}\right)$, then NCERT Page-350

(a) $3^{1010} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

(b) $3^{1010} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

(c) $3^{1011} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

(d) $3^{1011} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

8. $\int x^x (1 + \log x) dx$ is equal to NCERT Page-300

(a) $x^x + C$ (b) $x^{2x} + C$

(c) $x^x \log x + C$ (d) $1/2\,(1 + \log x)^2 + C$

9. If the integral $\int \frac{\cos 8x + 1}{\cot 2x - \tan 2x} dx = A \cos 8x + k$, where k is an arbitrary constant, then A is equal to : NCERT Page-301

(a) $-\frac{1}{16}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

10. Let $\int \frac{x^{1/2}}{\sqrt{1-x^3}} dx = \frac{2}{3} gof(x) + C$, then NCERT Page-304

(a) $f(x) = \sqrt{x}$ (b) $f(x) = x^{3/2}$ and $g(x) = \sin^{-1} x$

(c) $f(x) = x^{2/3}$ (d) None of these

11. $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx =$ NCERT Page-303

(a) $-3(\tan x)^{1/3} + c$ (b) $-3(\tan x)^{-1/3} + c$

(c) $3(\tan x)^{-1/3} + c$ (d) $(\tan x)^{-1/3} + c$

12. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals NCERT Page-304

(a) $\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$ (b) $\log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

(c) $\frac{1}{2}\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$ (d) $\frac{1}{2}\log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

13. Let $g : (0, \infty) \to R$ be a differentiable function such that

$$\int \left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2} \right) dx = \frac{xg(x)}{e^x + 1} + c,$$

for all $x > 0$, where c is an arbitrary constant. Then

(a) g is decreasing in $\left(0, \frac{\pi}{4}\right)$ NCERT Page-325

(b) g' is increasing in $\left(0, \frac{\pi}{4}\right)$

(c) $g + g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$

(d) $g - g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$

14. $\int \frac{(x^2+1)e^x}{(x+1)^2} dx = f(x)e^x + C$, where C is a constant, then $\frac{d^3 f}{dx^3}$ at x = 1 is equal to: NCERT Page-326

(a) $-\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $-\frac{3}{2}$ (d) $\frac{3}{2}$

15. The integral $\int \frac{\left(1 - \frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1 + \frac{2}{\sqrt{3}}\sin 2x\right)} dx$ is equal to NCERT Page-??

(a) $\frac{1}{2}\log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)}{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)} \right| + C$

(b) $\frac{1}{2}\log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)}{\tan\left(\frac{x}{2} + \frac{\pi}{3}\right)} \right| + C$

(c) $\log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)}{\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)} \right| + C$

(d) $\frac{1}{2}\log_e \left| \frac{\tan\left(\frac{x}{2} - \frac{\pi}{12}\right)}{\tan\left(\frac{x}{2} - \frac{\pi}{6}\right)} \right| + C$

16. $\int \frac{1}{[(x-1)^3 (x+2)^5]^{1/4}} dx$ is equal to NCERT Page-304

(a) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$ (b) $\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{1/4} + C$

(c) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$ (d) $\frac{1}{3}\left(\frac{x+2}{x-1}\right)^{1/4} + C$

17. $\int \frac{2dx}{(e^x + e^{-x})^2} =$ NCERT Page-304

(a) $\frac{-e^{-x}}{(e^x + e^{-x})} + C$ (b) $\frac{-1}{(e^x + e^{-x})} + C$

(c) $\frac{1}{(e^x + 1)^2} + C$ (d) $\frac{1}{(e^x + e^{-x})} + C$

18. If $\int x^{13/2}.(1+x^{5/2})^{½}\,dx$ NCERT Page-304
$= A(1+x^{5/2})^{7/2} + B(1+x^{5/2})^{5/2} + C(1+x^{5/2})^{3/2} + D$, then

(a) $A=-\frac{4}{35}, B=-\frac{8}{25}, C=\frac{4}{15}$
(b) $A=\frac{4}{35}, B=-\frac{8}{25}, C=-\frac{4}{15}$
(c) $A=\frac{4}{35}, B=-\frac{8}{25}, C=\frac{4}{15}$
(d) None of these

19. $\int (27e^{9x}+e^{12x})^{1/3}dx$ is equal to NCERT Page-304

(a) $(1/4)(27+e^{3x})^{1/3}+C$ (b) $(1/4)(27+e^{3x})^{2/3}+C$
(c) $(1/3)(27+e^{3x})^{4/3}+C$ (d) $(1/4)(27+e^{3x})^{4/3}+C$

20. $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$ equals NCERT Page-304

(a) $-\cot(ex^x)+C$ (b) $\tan(xe^x)+C$
(c) $\tan(e^x)+C$ (d) $\cot(e^x)+C$

21. Evaluate: $\int 2^{2^{2^x}} 2^{2^x} 2^x\,dx$ NCERT Page-304

(a) $\frac{1}{(\log 2)^3}2^{2^{2^x}}+C$ (b) $\frac{1}{(\log 2)^3}2^{2^x}+C$
(c) $\frac{1}{(\log 2)^2}2^{2^x}+C$ (d) $\frac{1}{(\log 2)^4}2^{2^{2x}}+C$

22. $\int \frac{10x^9+10^x\log_e 10}{10^x+x^{10}}dx$ is equal to NCERT Page-304

(a) $10^x - x^{10}+C$ (b) $10^x + x^{10}+C$
(c) $(10^x - x^{10})^{-1}+C$ (d) $\log_e(\log^x + x^{10})+C$

23. $\int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$ is equal to NCERT Page-304

(a) $\frac{1}{8}(x^2-1)+k$ (b) $\frac{1}{2}x^2+k$
(c) $\frac{1}{2}x+k$ (d) None of these

24. $\int e^{3\log x}(x^4+1)^{-1}dx$ is equal to NCERT Page-305

(a) $\log(x^4+1)+C$ (b) $\frac{1}{4}\log(x^4+1)+C$
(c) $-\log(x^4+1)+C$ (d) None of these

25. If $\int \sin^3 x\cos^5 x\,dx = A\sin^4 x + B\sin^6 x + C\sin^8 x + D$.
Then NCERT Page-303

(a) $A=\frac{1}{4},\ B=-\frac{1}{3},\ C=\frac{1}{8},\ D\in R$
(b) $A=\frac{1}{8},\ B=\frac{1}{4},\ C=\frac{1}{3},\ D\in R$
(c) $A=0,\ B=-\frac{1}{6},\ C=\frac{1}{8},\ D\in R$
(d) None of these

26. $\int\left(x+\frac{1}{x}\right)^{n+5}\left(\frac{x^2-1}{x^2}\right)dx$ is equal to : NCERT Page-303

(a) $\frac{\left(x+\frac{1}{x}\right)^{n+6}}{n+6}+c$ (b) $\left[\frac{x^2+1}{x^2}\right]^{n+6}(n+6)+c$
(c) $\left[\frac{x}{x^2+1}\right]^{n+6}(n+6)+c$ (d) None of these

27. $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$ is equal to NCERT Page-304

(a) $\frac{1}{2}\sin 2x+C$ (b) $-\frac{1}{2}\sin 2x+C$
(c) $-\frac{1}{2}\sin x+C$ (d) $-\sin^2 x+C$

28. If $\int \cos^n x\sin x\,dx = -\frac{\cos^6 x}{6}+C$, then n =
NCERT Page-303

(a) 0 (b) 1 (c) 2 (d) 5

29. Evaluate: $\int \sin^3 x\cos^3 x\,dx$ NCERT Page-304

(a) $\frac{1}{32}\left\{\frac{3}{2}\cos 2x-\frac{1}{6}\cos 6x\right\}+C$
(b) $\frac{1}{32}\left\{-\frac{3}{2}\cos 2x+\frac{1}{6}\cos 6x\right\}+C$
(c) $\frac{1}{32}\left\{-\frac{3}{2}\cos 2x-\frac{1}{6}\cos 6x\right\}+C$
(d) None of these

30. Evaluate: $\int \frac{1}{\sqrt{\sin^3 x\cos^5 x}}dx$ NCERT Page-303

(a) $\frac{2}{\sqrt{\tan x}}-\frac{2}{3}(\tan x)^{3/2}+C$
(b) $-\frac{2}{\sqrt{\tan x}}+\frac{2}{3}(\tan x)^{3/2}+C$
(c) $-\frac{2}{\sqrt{\tan x}}-\frac{2}{3}(\tan x)^{3/2}+C$
(d) None of these

31. $\int \frac{1}{x\log x\log(\log x)}dx =$ NCERT Page-303

(a) $\log\log(x)+c$ (b) $\log[\log(\log x)]+c$
(c) $-\log\log(\log x)+c$ (d) none of these

32. If $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log\sin(x-\alpha)+C$, then value of (A, B) is NCERT Page-303

(a) $(-\cos\alpha, \sin\alpha)$ (b) $(\cos\alpha, \sin\alpha)$
(c) $(-\sin\alpha, \cos\alpha)$ (d) $(\sin\alpha, \cos\alpha)$

33. $\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx$ is equal to NCERT Page-303

(a) $\tan^{-1}x^3 + C$ (b) $\frac{1}{6}(\tan^{-1}x^3)^2 + C$

(c) $-\frac{1}{2}(\tan^{-1}x^3)^2 + C$ (d) $\frac{1}{2}(\tan^{-1}x^2)^3 + C$

34. Value of $\int \frac{(x-x^3)^{1/3}}{x^4} dx$ is NCERT Page-304

(a) $\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}} + C$ (b) $\frac{-3}{8}\left(\frac{1}{x^2}-1\right)^{\frac{4}{3}} + C$

(c) $\frac{-3}{8}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}} + C$ (d) None of these

35. The value of $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} dx$ is NCERT Page-304

(a) $\frac{1}{2}\log\left|e^x + e^{-x}\right| + C$ (b) $2\log\left|e^{2x}+e^{-2x}\right| + C$

(c) $\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right| + C$ (d) None of these

7.4 Integrals of Some Particular Functions

36. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\, dx = g(x) + c$, $g(1) = 0$, then $g\left(\frac{1}{2}\right)$ is equal to : NCERT Page-314

(a) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) + \frac{\pi}{3}$ (b) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{\pi}{3}$

(c) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) - \frac{\pi}{3}$ (d) $\frac{1}{3}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) - \frac{\pi}{6}$

37. Evaluate: $= \int \frac{\tan x \cdot \sec x}{\sqrt{\sec^2 x + 4}} dx$ NCERT Page-314

(a) $\log\left|\sec x - \sqrt{\sec^2 x + 4}\right| + c$

(b) $\log\left|\sec x + \sqrt{\sec^2 x + 4}\right| + c$

(c) $\log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + c$

(d) $\frac{1}{2}\tan^{-1}\left(\frac{\sec x}{2}\right) + c$

38. Evaluate: $\int \frac{x}{1-x^4} dx$ NCERT Page-315

(a) $\frac{1}{4}\log\left|\frac{1+x^2}{1-x^2}\right| + c$ (b) $\frac{1}{4}\log\left|\frac{1-x^2}{1+x^2}\right| + c$

(c) $\frac{1}{2}\log\left|\frac{1+x^2}{1-x^2}\right| + c$ (d) $\frac{1}{2}\log\left|\frac{1-x^2}{1+x^2}\right| + c$

39. Value of $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$ if $(\beta > \alpha)$ is NCERT Page-314

(a) $2\sin^{-1}\sqrt{\frac{\beta-\alpha}{x-\alpha}} + C$ (b) $2\sin^{-1}\sqrt{\frac{x-\alpha}{\beta-\alpha}} + C$

(c) $2\sin^{-1}\sqrt{\frac{x+\alpha}{\beta-\alpha}} + C$ (d) None of these

40. Evaluate: $\int \frac{x^3+x}{x^4-9} dx$ NCERT Page-314

(a) $\frac{1}{4}\log\left|x^4-9\right| + \frac{1}{12}\log\left|\frac{x^2+3}{x^2-3}\right| + C$

(b) $\frac{1}{4}\log\left|x^4-9\right| - \frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right| + C$

(c) $\frac{1}{4}\log\left|x^4-9\right| + \frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right| + C$

(d) None of these

41. Evaluate: $\int \frac{1}{\sqrt{9+8x-x^2}} dx$ NCERT Page-314

(a) $-\sin^{-1}\left(\frac{x-4}{5}\right) + C$ (b) $-\sin^{-1}\left(\frac{x+4}{5}\right) + C$

(c) $\sin^{-1}\left(\frac{x-4}{5}\right) + C$ (d) None of these

42. The value of $\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})} dx$ is equal to NCERT Page-336

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

43. $\int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1+\sqrt[3]{x})} dx$ is equal to NCERT Page-312

(a) $\frac{3}{2}x^{2/3} + 6\tan^{-1}x^{1/6} + C$

(b) $\frac{3}{2}x^{2/3} - 6\tan^{-1}x^{1/6} + C$

(c) $-\frac{3}{2}x^{2/3} + 6\tan^{-1}x^{1/6} + C$

(d) None of these

44. Evaluate: $\int \frac{1}{1+3\sin^2 x+8\cos^2 x}dx$ NCERT Page-313

(a) $\frac{1}{6}\tan^{-1}(2\tan x)+C$ (b) $\tan^{-1}(2\tan x)+C$

(c) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$ (d) None of these

45. Value of $\int \frac{x^2+1}{x^4+x^2+1}dx$ is NCERT Page-313

(a) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{x^2-1}\right)+C$ (b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+C$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2+1}{\sqrt{3}x}\right)+C$ (d) None of these

46. Value of $\int \frac{dx}{\sqrt{x(a-x)}}$ is NCERT Page-314

(a) $2\sin^{-1}\sqrt{\frac{x}{a}}+c$ (b) $-2\sin^{-1}\sqrt{\frac{x}{a}}+c$

(c) $2\sin^{-1}\frac{\sqrt{x}}{a}+c$ (d) None of these

47. Value of $\int \frac{dx}{4\sin^2 x+4\sin x\cos x+5\cos^2 x}$ is NCERT Page-314

(a) $\frac{-1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right)+C$

(b) $\frac{1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right)+C$

(c) $\frac{1}{22}\tan^{-1}\left(\frac{\tan x+2}{2}\right)+C$

(d) None of these

48. If $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$ NCERT Page-313

$= A\sqrt{x^2-9x+20}+B\log\left|x+\sqrt{x^2-9x+20}-\frac{9}{2}\right|+C$

Then the values of A and B are

(a) 6, 34 (b) 3, 9

(c) 12, 17 (d) None of these

49. $\int \frac{dx}{x\sqrt{1-x^3}} = a\ln\left|\frac{\sqrt{1-x^3}+b}{\sqrt{1-x^3}+1}\right|+k$, then: NCERT Page-313

(a) $b=1, a=1$ (b) $b=-1$, and $a=\frac{1}{3}$

(c) $b=1$, $a=-\frac{2}{3}$ (d) $b=1$ and $a=-\frac{1}{3}$

50. $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$ equal to NCERT Page-314

(a) $\frac{1}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$ (b) $\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$

(c) $\frac{2}{3}\sin^{-1}\sqrt{\frac{x}{a}}+C$ (d) None of these

51. The value of $\int\sqrt{\frac{a-x}{a+x}}dx$ is NCERT Page-315

(a) $a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{x^2-a^2}+C$

(b) $a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}+C$

(c) $a\sin^{-1}\left(\frac{a}{x}\right)+\sqrt{x^2-a^2}+C$

(d) None of these

52. If $\int \frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{3}+c$, then a = NCERT Page-315

(a) 3 (b) 4 (c) 6 (d) 8

7.5 Integration by Partial Fractions

53. $\int\left(\frac{ax+b}{cx+d}\right)dx=\frac{3x}{2}-\log\sqrt{x+2}+k$, NCERT Page-315

Then $\frac{ab}{cd}=$

(a) $\frac{2}{7}$ (b) $\frac{15}{8}$ (c) $\frac{3}{8}$ (d) $\frac{8}{15}$

54. $\int \frac{x}{(x-1)(x^2+4)}dx.$ is equal to NCERT Page-313

(a) $\frac{1}{5}\log|x-1|-\frac{1}{10}\log(x^2+4)+\frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right)+C$

(b) $\frac{2}{5}\log|x-1|+\log(x^2+4)-\tan^{-1}\left(\frac{x}{2}\right)+C$

(c) $\frac{4}{5}(\log|x-1)+\log(x^2+4)-2\tan^{-1}\left(\frac{x}{2}\right)+C$

(d) None of these

55. Integration of $\int \frac{dx}{(x-1)(x+2)(2x+3)}$ is NCERT Page-314

(a) $\log|x+2|-\frac{1}{3}\log|x-1|+2\log|2x+3|+C$

(b) $\frac{1}{15}\log|x-1|+\frac{1}{3}\log|x+2|-\frac{2}{5}\log|2x+3|+C$

(c) $\frac{4}{5}\log|x-1|-\frac{1}{15}\log(x-1)-\frac{1}{3}\log|x+2|+C$

(d) None of these

56. If $\int \frac{3x+1}{(x-3)(x-5)}dx = \int \frac{-5}{(x-3)}dx + \int \frac{B}{(x-5)}dx$, then the value of B is NCERT Page-314

(a) 3 (b) 4 (c) 6 (d) 8

57. Evaluate: $\int \frac{1}{(x-1)(x^2+2x+1)}dx$ NCERT Page-314

(a) $\frac{1}{4}\log\left|\frac{x+1}{x-1}\right| + \frac{1}{2(x+1)} + c$

(b) $\frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + \frac{1}{4(x+1)} + c$

(c) $\frac{1}{4}\log\left|\frac{x-1}{x+1}\right| + \frac{1}{2(x+1)} + c$

(d) $\frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2(x+1)} + c$

58. Value of $\int \frac{x^2+1}{(x-1)(x-2)}dx$ is NCERT Page-313

(a) $x + \log\left[\frac{(x-2)^5}{(x-1)^2}\right] + C$ (b) $x + \log\left[\frac{(x-1)^2}{(x-2)^5}\right] + C$

(c) $x - \log\left[\frac{(x-2)^5}{(x-1)^2}\right] + C$ (d) None of these

59. If $\int \frac{1}{(\sin x+4)(\sin x-1)}dx$

$= A\frac{1}{\left(\tan\frac{x}{2}-1\right)} + B\tan^{-1}\left[f(x)\right] + C_1$, then NCERT Page-314

(a) $A = -\frac{1}{5}$, $B = \frac{-2}{5\sqrt{15}}$, $f(x) = \frac{4\tan x+3}{\sqrt{15}}$

(b) $A = -\frac{1}{5}$, $B = \frac{1}{\sqrt{15}}$, $f(x) = \frac{4\tan\left(\frac{x}{2}\right)+1}{\sqrt{15}}$

(c) $A = \frac{2}{5}$, $B = -\frac{2}{5}$, $f(x) = \frac{4\tan x+1}{5}$

(d) $A = \frac{2}{5}$, $B = \frac{-2}{5\sqrt{15}}$, $f(x) = \frac{4\tan\left(\frac{x}{2}\right)+f}{\sqrt{15}}$

60. Evaluate: $\int \frac{1-\cos x}{\cos x(1+\cos x)}dx$ NCERT Page-314

(a) $\log|\sec x+\tan x| - 2\tan(x/2) + C$

(b) $\log|\sec x-\tan x| - 2\tan(x/2) + C$

(c) $\log|\sec x+\tan x| + 2\tan(x/2) + C$

(d) None of these

61. If $\int \frac{3x+4}{x^3-2x-4}dx = \log|x-2| + k\log f(x) + c$, then NCERT Page-315

(a) $f(x) = |x^2+2x+2|$ (b) $f(x) = x^2+2x+2$

(c) $k = -\frac{1}{2}$ (d) All of these

62. $\int \frac{x^3-1}{x^3+x}dx$ is equal to NCERT Page-315

(a) $x - \log x + \log(x^2+1) - \tan^{-1}x + C$

(b) $x - \log x + \frac{1}{2}\log(x^2+1) - \tan^{-1}x + C$

(c) $x + \log x + \frac{1}{2}\log(x^2+1) + \tan^{-1}x + C$

(d) None of these

7.6 Integration by Parts

63. If $\int x\log\left(1+\frac{1}{x}\right)dx = f(x)\log(x+1) + g(x)x^2 + Lx + C$, then NCERT Page-324

(a) $f(x) = \frac{1}{2}x^2$ (b) $g(x) = \log x$

(c) $L = 1$ (d) None of these

64. $\int \tan^{-1}\sqrt{x}\,dx$ is equal to NCERT Page-325

(a) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(b) $x\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(c) $\sqrt{x} - x\tan^{-1}\sqrt{x} + C$

(d) $\sqrt{x} - (x+1)\tan^{-1}\sqrt{x} + C$

65. The value of $\int \cos(\log x)\,dx$ is : NCERT Page-325

(a) $\frac{1}{2}[\sin(\log x) + \cos(\log x)] + C$

(b) $\frac{x}{2}[\sin(\log x) + \cos(\log x)] + C$

(c) $\frac{x}{2}[\sin(\log x) - \cos(\log x)] + C$

(d) $\frac{1}{2}[\sin(\log x) - \cos(\log x)] + C$

66. If $\int \frac{e^x(1+\sin x)dx}{1+\cos x} = e^x f(x) + C$, then f(x) is equal to NCERT Page-324

(a) $\sin\frac{x}{2}$ (b) $\cos\frac{x}{2}$ (c) $\tan\frac{x}{2}$ (d) $\log\frac{x}{2}$

67. $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$ is equal to **NCERT Page-324**

(a) $-e^x \tan\left(\frac{x}{2}\right)+C$ (b) $-e^x \cot\left(\frac{x}{2}\right)+C$

(c) $-\frac{1}{2}e^x \tan\left(\frac{x}{2}\right)+C$ (d) $\frac{1}{2}e^x \cot\left(\frac{x}{2}\right)+C$

68. $\int\sqrt{4x^2+9}\ dx$ is equal to **NCERT Page-325**

(a) $\frac{x}{2}\sqrt{4x^2+9}-\frac{9}{4}\log|2x+\sqrt{4x^2+9}|+c$

(b) $\frac{x}{2}\sqrt{4x^2+9}+\frac{9}{4}\log|2x+\sqrt{4x^2+9}|+c$

(c) $-\frac{x}{2}\sqrt{4x^2+9}-\frac{9}{4}\log|2x+\sqrt{4x^2+9}|+c$

(d) $\frac{x}{2}\sqrt{4x^2+9}+\frac{9}{4}\sin^{-1}(4x^2+9)|+c$

69. Evaluate: $\int\sqrt{7x-10-x^2}dx$ **NCERT Page-324**

(a) $\frac{1}{4}(2x-7)\sqrt{7x-10-x^2}+\frac{9}{8}\sin^{-1}\left(\frac{2x-7}{3}\right)+c$

(b) $-\frac{1}{4}(2x-7)\sqrt{7x-10-x^2}-\frac{9}{8}\sin^{-1}\left(\frac{2x-7}{3}\right)+c$

(c) $-\frac{1}{4}(2x-7)\sqrt{7x-10-x^2}+\frac{9}{8}\sin^{-1}\left(\frac{2x-3}{7}\right)+c$

(d) None of these.

70. $\int\sin^{-1}x dx$ is equal to **NCERT Page-325**

(a) $x\sin^{-1}x+\sqrt{1-x^2}+c$ (b) $x\sin^{-1}x-\sqrt{1-x^2}+c$

(c) $\cos^{-1}x+c$ (d) $\frac{1}{\sqrt{1-x^2}}+c$

71. Find $\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$. **NCERT Page-327**

(a) $x+\sqrt{1-x^2}\sin^{-1}x+c$ (b) $x-\sqrt{1-x^2}\sin^{-1}x+c$

(c) $\sqrt{1-x^2}+\sin^{-1}x+c$ (d) $\sqrt{1-x^2}-\sin^{-1}x+c$

72. Find $\int xe^x dx$ **NCERT Page-325**

(a) $e^x(x-1)+c$ (b) $e^x(x+1)+c$

(c) $x(e^x-1)+c$ (d) $x(e^x+1)+c$

73. Find $\int \log x\, dx$ **NCERT Page-324**

(a) $x(\log x+1)+c$ (b) $x(\log x-1)+c$

(c) $\log x+x+c$ (d) $\frac{1}{x}+c$

74. $\int\sqrt{x^2-8x+7}\ dx=$ **NCERT Page-326**

(a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log x-4+\sqrt{x^2-8x+7}+c$

(b) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log[x-4+\sqrt{x^2-8x+7}]+c$

(c) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log[x-4+\sqrt{x^2-8x+7}]+c$

(d) None of these

75. $\int x\sin x\sec^3 x\ dx=$ **NCERT Page-326**

(a) $\frac{1}{2}[\sec^2 x-\tan x]+c$ (b) $\frac{1}{2}[x\sec^2 x-\tan x]+c$

(c) $\frac{1}{2}[x\sec^2 x+\tan x]+c$ (d) $\frac{1}{2}[\sec^2 x+\tan x]+c$

76. $\int x\sec^2 x\, dx=$ **NCERT Page-326**

(a) $\tan x+\log\cos x+c$ (b) $x\tan x+\log\sec x+c$

(c) $x\tan x+\log\cos x+c$ (d) $\frac{x^2}{2}\sec^2 x+\log\cos x+c$

77. $\int e^x(1+\tan x+\tan^2 x)dx=$ **NCERT Page-325**

(a) $e^x\sin x+c$ (b) $e^x\cos x+c$

(c) $e^x\tan x+c$ (d) $e^x\sec x+c$

78. $\int x^3e^{ax}dx$ is **NCERT Page-324**

(a) $\frac{1}{a^3}e^{ax}\left(3a^2x^2-6ax+6\right)+c$

(b) $\frac{1}{a^4}e^{ax}\left(a^3x^3-3a^2x^2+6ax-6\right)+c$

(c) $\frac{1}{a^5}e^{ax}\left(a^4x^4-5a^3x^3+3a^2x^2-6ax+6\right)+c$

(d) None of these

79. $\int\sin 2x.\log\cos x\, dx$ is equal to **NCERT Page-324**

(a) $\cos^2 x\left(\frac{1}{2}+\log\cos x\right)+C$

(b) $\cos^2 x.\log\cos x+C$

(c) $\cos^2 x\left(\frac{1}{2}-\log\cos x\right)+C$

(d) None of these

80. $\int\sin 2x.\log\cos x\, dx$ is equal to **NCERT Page-325**

(a) $\cos^2 x\left(\frac{1}{2}+\log\cos x\right)+k$

(b) $\cos^2 x.\log\cos x+k$

(c) $\cos^2 x\left(\frac{1}{2}-\log\cos x\right)+k$

(d) None of these

7.7 Definite Integral as the Limit of Sum

81. Evaluate $\int_1^2 x^2 dx$ as limit of sums. NCERT Page-332

(a) 1 (b) $\frac{7}{3}$ (c) $\frac{1}{3}$ (d) 0

7.8 Fundamental Theorem of Calculus

82. Value of $\int_0^4 \frac{1}{\sqrt{x^2+2x+3}}dx$ is NCERT Page-334

(a) $\log\left(\frac{1+\sqrt{3}}{5+3\sqrt{3}}\right)$ (b) $\log\left(\frac{5-3\sqrt{3}}{1-\sqrt{3}}\right)$

(c) $\log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$ (d) None of these

83. $A_n = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x}dx;\ B_n = \int_0^{\pi/2}\left(\frac{\sin nx}{\sin x}\right)^2 dx;$

For $n \in \mathbf{N}$, then NCERT Page-335

(a) $A_{n+1} = A_n$, $B_{n+1} - B_n = A_{n+1}$

(b) $B_{n+1} = B_n$

(c) $A_{n+1} - A_n = B_{n+1}$

(d) None of these

84. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0)=1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$, is NCERT Page-334

(a) $e+\frac{e^2}{2}+\frac{5}{2}$ (b) $e-\frac{e^2}{2}-\frac{5}{2}$

(c) $e+\frac{e^2}{2}-\frac{3}{2}$ (d) $e-\frac{e^2}{2}-\frac{3}{2}$

85. Let f(x) be defined by $f(x) = \int_1^x t(t^2-3t+2)dt,\ 1 \le x \le 3,$ then range of f (x) is NCERT Page-335

(a) [0, 4] (b) $\left[-\frac{1}{4}, 4\right]$

(c) $\left[-\frac{1}{4}, 2\right]$ (d) None of these

86. If $\int_0^{\pi/4} \sec^2\theta \sin\theta d\theta = \int_0^k \frac{dx}{(\sqrt{x}+\sqrt{x+k})}$, then k = NCERT Page-334

(a) $\frac{9}{16}$ (b) $\frac{9}{25}$ (c) $\frac{16}{25}$ (d) $\frac{25}{16}$

87. $\int_{1/4}^{1/2} \frac{dx}{\sqrt{x-x^2}}$ is equal to : NCERT Page-335

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) 0

88. $\int_1^2 e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx =$ NCERT Page-336

(a) $e\left(\frac{e}{2}-1\right)$ (b) $e(e-1)$

(c) 0 (d) None of these

89. $\int_0^1 x^2 e^x dx =$ NCERT Page-336

(a) $e-2$ (b) $e+2$

(c) e^2-2 (d) e^2

90. The value of the integral NCERT Page-335

$\int_1^3\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$ is equal to

(a) π (b) 2π (c) 4π (d) None

91. If $a_n = \int_0^{\frac{\pi}{2}} \frac{\sin^2 nx}{\sin x}dx$ then NCERT Page-335

$a_2-a_1,\ a_3-a_2,\ a_4-a_3$,............ are in

(a) A.P. (b) GP.

(c) H.P. (d) None of these

92. $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ is NCERT Page-334

(a) $\frac{\pi ab}{a+b}$ (b) $\frac{\pi}{2(a+b)}$

(c) $\frac{\pi}{2ab(a+b)}$ (d) $\frac{\pi(a+b)}{2ab}$

7.9 Evaluation of Definite Integrals by Substitution

93. $\int_0^{\pi/2}(\sqrt{\tan x}+\sqrt{\cot x})\,dx =$ NCERT Page-338

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\pi\sqrt{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{\sqrt{2}}{\pi}$

94. If $\int_{\log 2}^x \frac{du}{(e^u-1)^{1/2}} = \frac{\pi}{6}$, then e^x = NCERT Page-337

(a) 1 (b) 2 (c) 4 (d) –1

95. The correct value of $\int_0^{\pi/2}\sin x \sin 2x$ is NCERT Page-338

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

96. $\int_0^{\pi/2}\sqrt{\cos\theta}\,\sin^3\theta\,d\theta =$ **NCERT** Page-337

(a) $\frac{20}{21}$ (b) $\frac{8}{21}$ (c) $\frac{-20}{21}$ (d) $\frac{-8}{21}$

97. If $f(\alpha)=\int_1^{\alpha}\frac{\log_{10}t}{1+t}dt, \alpha>0,$ then $f(e^3)+f(e^{-3})$ is equal to:

(a) 9 (b) $\frac{9}{2}$ **NCERT** Page-336

(c) $\frac{9}{\log_e(10)}$ (d) $\frac{9}{2\log_e(10)}$

98. The integral $\int_0^{\frac{\pi}{2}}\frac{1}{3+2\sin x+\cos x}dx$ is equal to:

NCERT Page-345

(a) $\tan^{-1}(2)$ (b) $\tan^{-1}(2)-\frac{\pi}{4}$

(c) $\frac{1}{2}\tan^{-1}(2)-\frac{\pi}{8}$ (d) $\frac{1}{2}$

99. If $[t]$ denotes the greatest integer $\le t$, then the value of $\int_0^1[2x-|3x^2-5x+2|+1]\,dx$ is: **NCERT** Page-351

(a) $\frac{\sqrt{37}+\sqrt{13}-4}{6}$ (b) $\frac{\sqrt{37}-\sqrt{13}-4}{6}$

(c) $\frac{-\sqrt{37}-\sqrt{13}+4}{6}$ (d) $\frac{-\sqrt{37}+\sqrt{13}+4}{6}$

100. The minimum value of the twice differentiable function $f(x)=\int_0^x e^{x-t}f'(t)dt-\left(x^2-x+1\right)e^x, x\in R,$ is :

NCERT Page-340

(a) $-\frac{2}{\sqrt{e}}$ (b) $-2\sqrt{e}$ (c) $-\sqrt{e}$ (d) $\frac{2}{\sqrt{e}}$

101. Let $I_n(x)=\int_0^x\frac{1}{\left(t^2+5\right)^n}dt, n=1,2,3,\ldots.$ Then:

NCERT Page-340

(a) $50I_6-9I_5=xI'_5$ (b) $50I_6-11I_5=xI'_5$
(c) $50I_6-9I_5=I'_5$ (d) $50I_6-11I_5=I'_5$

102. $I=\int_{\pi/4}^{\pi/3}\left(\frac{8\sin x-\sin 2x}{x}\right)dx.$ Then **NCERT** Page-340

(a) $\frac{\pi}{2}<I<\frac{3\pi}{4}$ (b) $\frac{\pi}{5}<I<\frac{5\pi}{12}$

(c) $\frac{5\pi}{12}<I<\frac{\sqrt{2}}{3}\pi$ (d) $\frac{3\pi}{4}<I<\pi$

103. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined as $f(x)=a\sin\left(\frac{\pi[x]}{2}\right)+[2-x], a\in\mathbb{R},$ where [t] is the greatest integer less than or equal to t. If $\lim_{x\to-1}f(x)$ exists, then the value of $\int_0^4 f(x)dx$ is equal to: **NCERT** Page-351

(a) -1 (b) -2 (c) 1 (d) 2

104. Evaluate: $\int_0^{\pi/2}\frac{\cos x}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx$ **NCERT** Page-339

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$ (c) $3+\sqrt{3}$ (d) $3-\sqrt{3}$

105. $\int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx$ is equal to: **NCERT** Page-340

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{3\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{3}}$

106. Evaluate: $\int_0^{\pi}\frac{1}{5+4\cos x}dx$ **NCERT** Page-338

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

107. Let $F(x)=f(x)+f\left(\frac{1}{x}\right)$, where $f(x)=\int_l^x\frac{\log t}{1+t}dt,$ Then $F(e)$ equals **NCERT** Page-338

(a) 1 (b) 2 (c) 1/2 (d) 0

108. $\int_1^{e^{37}}\frac{\pi\sin(\pi\log_e x)}{x}dx$ is equal to **NCERT** Page-337

(a) 2 (b) 20 (c) $\frac{2}{\pi}$ (c) 2π

109. $\int_0^1\tan^{-1}x\,dx=$ **NCERT** Page-338

(a) $\frac{\pi}{4}-\frac{1}{2}\log 2$ (b) $\pi-\frac{1}{2}\log 2$

(c) $\frac{\pi}{4}-\log 2$ (d) $\pi-\log 2$

7.10 Some Properties of Definite Integrals

110. The value of the integral $\int_{-2}^{2}\frac{\left|x^3+x\right|}{(e^{x|x|}+1)}dx$ is equal to:

NCERT Page-346

(a) $5e^2$ (b) $3e^{-2}$ (c) 4 (d) 6

111. Let the slope of the tangent to a curve $y=f(x)$ at (x, y) be given by 2 tan x (cos x – y). If the curve passes through the point, $\left(\frac{\pi}{4},0\right)^{9.5}$, then the value of $\int_0^{\pi/2}y\,dx$ is equal to

(a) $(2-\sqrt{2})+\frac{\pi}{\sqrt{2}}$ (b) $2-\frac{\pi}{\sqrt{2}}$ **NCERT** Page-344

(c) $(2+\sqrt{2})+\frac{\pi}{\sqrt{2}}$ (d) $2+\frac{\pi}{\sqrt{2}}$

112. Let $f:\mathbf{R}\to\mathbf{R}$ be continuous function satisfying $f(x)+f(x+k)=n$, for all $x\in\mathbf{R}$ where $k>0$ and n is a positive integer. If $I_1=\int_0^{4nk}f(x)dx$ and $I_2=\int_{-k}^{3k}f(x)dx$, then **NCERT** Page-351

(a) $I_1 + 2I_2 = 4nk$ (b) $I_1 + 2I_2 = 2nk$
(c) $I_1 + nI_2 = 4n^2k$ (d) $I_1 + nI_2 = 6n^2k$

113. Let $f : \mathbf{R} \to \mathbf{R}$ be a differentiable function such that

$$f\left(\frac{\pi}{4}\right) = \sqrt{2},\ f\left(\frac{\pi}{2}\right) = 0 \text{ and } f'\left(\frac{\pi}{2}\right) = 1 \text{ and let}$$

$$g(x) = \int_x^{\pi/4} (f'(t)\sec t + \tan t \sec t\, f(t))\, dt \text{ for}$$

$x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then $\lim\limits_{x \to \left(\frac{\pi}{2}\right)^{-1}} g(x)$ is equal to

NCERT Page-351

(a) 2 (b) 3 (c) 4 (d) – 3

114. Let $[t]$ denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 [-8x^2 + 6x - 1]dx$ is equal to

(a) –1 (b) $-\frac{5}{4}$ **NCERT** Page-345

(c) $\frac{\sqrt{17}-13}{8}$ (d) $\frac{\sqrt{17}-16}{8}$

115. If $\int_0^2 \left(\sqrt{2x} - \sqrt{2x - x^2}\right)dx =$ **NCERT** Page-346

$$\int_0^1 \left(1 - \sqrt{1-y^2} - \frac{y^2}{2}\right)dy + \int_1^2 \left(2 - \frac{y^2}{2}\right)dy + I$$

(a) $\int_0^1 (1 + \sqrt{1-y^2})dy$

(b) $\int_0^1 \left(\frac{y^2}{2} - \sqrt{1-y^2} + 1\right)dy$

(c) $\int_0^1 (1 - \sqrt{1-y^2})dy$

(d) $\int_0^1 \left(\frac{y^2}{2} + \sqrt{1-y^2} + 1\right)dy$

Exercise 2 : NCERT Exemplar & Past Years JEE Main

NCERT Exemplar Questions

1. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta} dx$ is equal to **NCERT** Page-296

(a) $2(\sin x + x\cos\theta) + C$ (b) $2(\sin x - x\cos\theta) + C$
(c) $2(\sin x + 2x\cos\theta) + C$ (d) $2(\sin x - 2x\cos\theta) + C$

2. $\frac{dx}{\sin(x-a)\sin(x-b)}$ is equal to **NCERT** Page-295

(a) $\sin(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right| + C$

(b) $\operatorname{cosec}(b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + C$

(c) $\operatorname{cosec}(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right| + C$

(d) $\sin(b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + C$

3. $\int \tan^{-1}\sqrt{x}\, dx$ is equal to **NCERT** Page-296

(a) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$
(b) $x\tan^{-1}\sqrt{x} - \sqrt{x} + C$
(c) $\sqrt{x} - x\tan^{-1}\sqrt{x} + C$
(d) $\sqrt{x} - (x+1)\tan^{-1}\sqrt{x} + C$

4. $\int \frac{x^9}{(4x^2+1)^6} dx$ is equal to **NCERT** Page-295

(a) $\frac{1}{5x}\left(4 + \frac{1}{x^2}\right)^{-5} + C$ (b) $\frac{1}{5}\left(4 + \frac{1}{x^2}\right)^{-5} + C$

(c) $\frac{1}{10x}\left(\frac{1}{x} + 4\right)^{-5} + C$ (d) $\frac{1}{10}\left(\frac{1}{x^2} + 4\right)^{-5} + C$

5. If $\int \frac{dx}{(x+2)(x^2+1)} = a\log|1+x^2| + b\tan^{-1}x + \frac{1}{5}\log|x+2| + C$, then **NCERT** Page-297

(a) $a = \frac{-1}{10}, b = \frac{-2}{5}$ (b) $a = \frac{1}{10}, b = -\frac{2}{5}$

(c) $a = \frac{-1}{10}, b = \frac{2}{5}$ (d) $a = \frac{1}{10}, b = \frac{2}{5}$

6. $\int \frac{x^3}{x+1}$ is equal to **NCERT** Page-298

(a) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1-x| + C$

(b) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1-x| + C$

(c) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1+x| + C$

(d) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1+x| + C$

7. $\int \frac{x + \sin x}{1 + \cos x}$ is equal to **NCERT** Page-300

(a) $\log|1 + \cos x| + C$ (b) $\log|x + \sin x| + C$

(c) $x - \tan\frac{x}{2} + C$ (d) $x \cdot \tan\frac{x}{2} + C$

8. If $\int \frac{x^3dx}{\sqrt{1+x^2}} = a\left(1+x^2\right)^{3/2} + b\sqrt{1+x^2} + C$, then

NCERT Page-314

(a) $a=\frac{1}{3}, b=1$ (b) $a=\frac{-1}{3}, b=1$

(c) $a=\frac{-1}{3}, b=-1$ (d) $a=\frac{1}{3}, b=-1$

9. $\int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ is equal to NCERT Page-325

(a) 1 (b) 2

(c) 3 (d) 4

10. $\int_0^{\pi/2} \sqrt{1-\sin 2x}$ is equal to NCERT Page-326

(a) $2\sqrt{2}$ (b) $2\left(\sqrt{2}+1\right)$

(c) 2 (d) $2\left(\sqrt{2}-1\right)$

11. $\int_0^{\pi/4} \cos x e^{\sin x} dx$ is equal to NCERT Page-327

(a) $e+1$ (b) $e-1$

(c) e (d) $-e$

12. $\int \frac{x+3}{(x+4)^2} e^x\, dx$ is equal to NCERT Page-326

(a) $e^x\left(\frac{1}{x+4}\right)+C$ (b) $e^{-x}\left(\frac{1}{x+4}\right)+C$

(c) $e^{-x}\left(\frac{1}{x-4}\right)+C$ (d) $e^{2x}\left(\frac{1}{x-4}\right)+C$

Past Years JEE Main

13. The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to :

NCERT Page-303 | 2016, A

(a) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$ (b) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

(c) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$ (d) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

where C is an arbitrary constant.

14. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to :

NCERT Page-345 | 2017, A

(a) –1 (b) –2 (c) 2 (d) 4

15. Let $I_n = \int \tan^n x\, dx, (n > 1)$. $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is constant of integration, then the ordered pair (a, b) is equal to : NCERT Page-303 | 2017, A

(a) $\left(-\frac{1}{5}, 0\right)$ (b) $\left(-\frac{1}{5}, 1\right)$

(c) $\left(\frac{1}{5}, 0\right)$ (d) $\left(\frac{1}{5}, -1\right)$

16. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2}dx$$ is equal to:

NCERT Page-304 | 2018, S

(a) $\frac{-1}{3(1+\tan^3 x)}+C$ (b) $\frac{1}{1+\cot^3 x}+C$

(c) $\frac{-1}{1+\cot^3 x}+C$ (c) $\frac{1}{3(1+\tan^3 x)}+C$

(where C is a constant of integration)

17. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}dx$ is : NCERT Page-346 | 2018, A

(a) $\frac{\pi}{2}$ (b) 4π (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

18. The value of $\int_0^{\pi} |\cos x|^3\, dx$ is: NCERT Page-341 | 2019, C

(a) 0 (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-4}{3}$

19. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral

$$\int x\sqrt{\frac{2\sin(x^2-1) - \sin 2\left(x^2-1\right)}{2\sin(x^2-1) + \sin 2\left(x^2-1\right)}}dx$$ is equal to:

NCERT Page-305 | 2019, C

(a) $\log_e\left|\frac{1}{2}\sec^2\left(x^2-1\right)\right|+c$

(b) $\frac{1}{2}\log_e|\sec(x^2-1)|+c$

(c) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(d) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$

(where c is a constant of integration)

20. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx$ is equal to:

NCERT Page-304 | 2019, C

(a) $-3\tan^{-1/3} x + C$ (c) $-\frac{3}{4}\tan^{-4/3} x + C$

(c) $-3\cot^{-1/3} x + C$ (d) $3\tan^{-1/3} x + C$

(Here C is a constant of integration)

21. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x}dx$ is: NCERT Page-342 | 2019, C

(a) $\frac{\pi-2}{8}$ (b) $\frac{\pi-1}{4}$ (c) $\frac{\pi-2}{4}$ (d) $\frac{\pi-1}{2}$

22. If $\int \frac{\cos x\, dx}{\sin^3 x(1+\sin^6 x)^{2/3}} = f(x)(1+\sin^6 x)^{1/\lambda} + c$ where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to:

NCERT Page-305 | 2020, C

(a) $-\frac{9}{8}$ (b) 2 (c) $\frac{9}{8}$ (d) –2

23. If $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx$ $= \alpha \log_e |1 + \tan x| + \beta \log_e |1 - \tan x + \tan^2 x| + \gamma \tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$, when C is constant of integration, then the value of $18(\alpha + \beta + \gamma^2)$ is : NCERT Page-325 | 2021, C

24. If [x] is the greatest integer ≤ x, then $\pi^2 \int_0^2 \left(\sin \frac{\pi x}{2}\right)(x-[x])^{[x]} dx$ is equal to :

NCERT Page-333 | 2021, S

(a) $2(\pi-1)$ (b) $4(\pi-1)$ (c) $4(\pi+1)$ (d) $2(\pi+1)$

25. If $a = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2n}{n^2+k^2}$ and $f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}}$, $x \in (0,1)$, then: NCERT Page-333 | 2022, C

(a) $2\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$ (b) $f\left(\frac{a}{2}\right) f'\left(\frac{a}{2}\right) = \sqrt{2}$

(c) $\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$ (d) $f\left(\frac{a}{2}\right) = \sqrt{2} f'\left(\frac{a}{2}\right)$

26. If $n(2n+1)\int_0^1 (1-x^n)^{2n} dx = 1177\int_0^1 (1-x^n)^{2n+1} dx$, then $n \in N$ is equal to ________. NCERT Page-336 | 2022, S

Exercise 3 : Skill Enhancer MCQs

1. $\int (x+3)\sqrt{3-4x-x^3}\, dx$ is equal to

(a) $-\frac{1}{3}(3-4x-x^3)^{3/2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + \frac{(x+2)\sqrt{3-4x-x^2}}{2} + C$

(b) $\frac{1}{3}(3-4x-x^3)^{3/2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{7}\right) + \frac{(x+2)\frac{2}{\sqrt{3-4x-x^2}}}{2} + C$

(c) $\frac{1}{3}(3-4x-x^3)^{3/2} + \frac{7}{2}\sin^{-1}\left(\frac{x+7}{2}\right) + \frac{(x+7)\sqrt{3-4x-x^2}}{2} + C$

(d) $\frac{1}{3}(3-4x-x^3)^{3/2} + \frac{7}{2}\log\left[x+7+\frac{2}{\sqrt{3-4x-x^2}}\right] + C$

2. For $0 < x < 1$, let

$f(x) = \lim_{n\to\infty}(1+x)(1+x^2)(1+x^4).....(1+x^{2^n})$

then $\int \frac{f(x)}{1-x}\log_e x\, dx$ equals

(a) $\log_e\left(\frac{x}{1-x}\right) + c$

(b) $-\log_e\left(\frac{x}{1-x}\right) + \frac{\log_e x}{1-x} + c$

(c) $\frac{\log_e x}{1-x} + \log_e(1-x) + c$

(d) $x\log_e x + \log_e(1-x) + c$

3. $\int \frac{x^2-1}{x\sqrt{(x^2+\alpha x+1)(x^2+\beta x+1)}} dx =$

(a) $2\log\left\{\frac{\sqrt{x^2+\alpha x+1}+\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right\} + c$

(b) $2\log\left\{\frac{\sqrt{x^2+\alpha x+1}-\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right\} + c$

(c) $\log\{\sqrt{x^2+\alpha x+1}-\sqrt{x^2+\beta x-1}\} + c$

(d) None of these

4. If $\int \frac{x}{x^8+1} dx = \frac{1}{k_1}\tan^{-1}\left(\frac{x^4-1}{k_2 x^2}\right) - \frac{1}{2}\int \frac{x^5-x}{x^8+1} + C$, then value of $k_1 k_2$ is

(where C is an integration constant)

(a) 8 (b) 2 (c) $4\sqrt{2}$ (d) 4

5. If $\int \frac{dx}{x^2(x^n+1)^{(n-1)/n}} = -[f(x)]^{1/n} + C$, then f(x) is

(a) $(1+x^n)$ (b) $1+x^{-n}$

(c) $x^n + x^{-n}$ (d) None of these

6. The value of $\int_a^b \frac{dx}{1+x^{2n+1}+\sqrt{1+x^{4n+2}}}$ where $n \in N$ and a and b are local maximum and minimum point of $g(x) = \int_0^x (t+2016)^7(t+2014)^{110}(t-2016)^5 dt$ respectively, is

(a) 0 (b) 2016 (c) 2014 (d) $2n+1$

7. If $\alpha, \beta(\beta > \alpha)$ are the roots of $f(x) \equiv ax^2 + bx + c = 0,\ a \neq 0$ and $f(x)$ is an even function and $I = \int_{\alpha}^{\beta} \frac{e^{f\left(\frac{f(x)}{x-\alpha}\right)}}{e^{f\left(\frac{f(x)}{x-\alpha}\right)} + e^{f\left(\frac{f(x)}{x-\beta}\right)}}$, then $|I|$ is equal to

(a) $\left|\frac{b}{a}\right|$ (b) $\left|\frac{b}{2a}\right|$

(c) $\frac{\sqrt{b^2-4ac}}{|2a|}$ (d) None of these

8. $f : [0, 1] \to R$ is a function with continuous second derivative and $f'(x) \in (0,1]\ \forall x \in [0,1]$.

Let $\int_0^x f(t)\,dt = g(x), f(0) = g(0)$ and

$\int_0^x (f(t))^3\,dt = h(x)$, then

(a) $h(x) \leq g(x)$ (b) $(g(x))^2 \geq h(x)$

(c) $2g(x) \geq (f(x))^2$ (d) $f(1) = 1$

9. If p, q, r, s are in arithmetic progression and $f(x) = \begin{vmatrix} p+\sin x & q+\sin x & p-r+\sin x \\ q+\sin x & r+\sin x & -1+\sin x \\ r+\sin x & s+\sin x & s-q+\sin x \end{vmatrix}$ such that

$\int_0^2 f(x)dx = -4$, then the common difference of the progression is

(a) ± 1 (b) $\frac{1}{2}$

(c) ± 2 (d) None of these

10. Let $g(x) = \int_0^x f(t)dt$, where f is such that

$\frac{1}{2} \leq f(t) \leq 1$, for $t \in [0,1]$ and $0 \leq f(t) \leq \frac{1}{2}$, for $t \in [1,2]$.

Then $g(2)$ satisfies the inequality

(a) $-\frac{3}{2} \leq g(2) < \frac{1}{2}$ (b) $0 \leq g(2) < 2$

(c) $\frac{3}{2} < g(2) \leq \frac{5}{2}$ (d) $2 < g(2) < 4$

11. Let $f : R \to R$ be a differentiable function and $f(1) = 4$. Then the value of $\lim_{x \to 1} \frac{\int_4^{f(x)} 2t\,dt}{x-1}$, if $f'(1) = 2$ is

(a) 16 (b) 8 (c) 4 (d) 2

12. If $\lim_{x \to 0} \frac{f(x)}{x^2}$ exists finitely and $\lim_{x \to 0}\left(1 + x + \frac{f(x)}{x}\right)^{1/x} = e^3$, then $\int f(x)\log_e x\,dx$ is equal to

(a) $\frac{2}{3}x^3\left(\log_e x - \frac{1}{3}\right) + c$ (b) $\frac{x^3}{3}\left(\log_e x - \frac{1}{3}\right) + c$

(c) $\frac{2}{3}x^3(\log_e x + 1) + c$ (d) $\frac{2}{3}x^3(\log_e x - 1) + c$

13. If $f : \mathbf{R} \to \mathbf{R}$ is a function satisfying the following:

(i) $f(-x) = -f(x)$ (ii) $f(x+1) = f(x) + 1$

(iii) $f\left(\frac{1}{x}\right) = \frac{f(x)}{x^2}\ \forall\ x \neq 0$ then $\int e^x f(x)dx$ is equal to

(a) $e^x(x-1) + c$ (b) $e^x \log x + c$

(c) $\frac{e^x}{x} + c$ (d) $\frac{e^x}{x+1} + c$

14. If $\int \frac{\operatorname{cosec}^2 x - 2010}{\cos^{2010} x}dx = -\frac{f(x)}{(g(x))^{2010}} + C$; where $f\left(\frac{\pi}{4}\right) = 1$; then the number of solutions of the equation $\frac{f(x)}{g(x)} = \{x\}$ in $[0, 2\pi]$ is/are : (where $\{.\}$ represents fractional part function)

(a) 0 (b) 1 (c) 2 (d) 3

15. $\int \frac{1}{x}\{\log_{ex} e \cdot \log_{e^2 x} e \cdot \log_{e^3 x} e\}\,dx$ is equal to

(a) $\frac{1}{2}\log\{\log_e e^2 x\} - \log\{\log_e e^3 x\} + \log\{\log_e e^4 x\} + C$

(b) $\frac{1}{2}\log\{\log_e x\} - \log\{\log_e x\} + \log\{\log_e x\} + C$

(c) $\frac{1}{2}\log\{\log_e ex\} - \log\{\log_e e^2 x^2\} + \log\{\log_e e^3 x^3\} + C$

(d) $\frac{1}{2}\log\{\log_e ex\} - \log\{\log_e e^2 x\} + \frac{1}{2}\log\{\log_e e^3 x\} + C$

Exercise 4 : Numeric Value Answer Questions

1. If $\int_0^{\sqrt{3}} \frac{15x^3}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}dx = \alpha\sqrt{2} + \beta\sqrt{3}$, where α, β are integers, then $\alpha + \beta$ is equal to

2. The value of the integral $\int_0^{\frac{\pi}{2}} 60\frac{\sin(6x)}{\sin x}dx$ ______.

3. Let $f(x) = \min\{[x-1], [x-2], \ldots, [x-10]\}$ where $[t]$ denotes the greatest integer $\leq$ t. Then is equal to

$$\int_0^{10} f(x)dx + \int_0^{10}(f(x))^2 dx + \int_0^{10}|f(x)|dx \text{ \_\_\_\_\_\_.}$$

4. Let f be a differentiable function satisfying $f(x) = \frac{2}{\sqrt{3}}\int_0^{\sqrt{3}} f\left(\frac{\lambda^2 x}{3}\right)d\lambda, x > 0$ and $f(1) = \sqrt{3}$ If $y = f(x)$ passes through the point $(\alpha, 6)$, then α is equal to ______.

5. Let f be a twice differentiable function on R. If $f'(0) = 4$ and $f(x) + \int_0^x (x-t)f'(t)dt = (e^{2x} + e^{-2x})\cos 2x + \frac{2}{a}x$, then $(2a+1)^5 a^2$ is equal to _____

6. Let $a_n = \int_{-1}^{n}\left(1 + \frac{x}{2} + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^{n-1}}{n}\right)dx$ for $n \in N$. Then the sum of all the elements of the set $\{n \in N : a_n \in (2, 30)\}$ is _______

7. If $\int_0^x f(t)\,dt = x^2 + \int_x^1 t^2 f(t)\,dt$, if $f'(1/2)$ is $\frac{24}{5^k}$, then the value of k is

8. $I_n = \int_0^{\pi/4} \tan^n x\,dx$ then $\lim_{n\to\infty} n[I_n + I_{n+2}]$ is equal to

9. Let f be a positive function.

Let $I_1 = \int_{1-k}^{k} x f\left[x(1-x)\right]dx$,

$I_2 = \int_{1-k}^{k} f\left[x(1-x)\right]dx$, where $2k - 1 > 0$. Then $\frac{I_1}{I_2}$ is

10. Let $f : R \to R$ and $g : R \to R$ be continuous functions. Then the value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[f(x) + f(-x)\right]\left[g(x) - g(-x)\right]dx \text{ is}$$

11. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$, where C is a constant of integration. If the function $f(x)$ is equal to $-\frac{C}{2x^3}$. Then, the value of C is

12. The value of the integral $\int_0^1 x\cot^{-1}(1 - x^2 + x^4)dx$ is equal to $\frac{\pi}{4} - \frac{k}{2}$ ln2, (natural log ln) then k is

Answer Keys

Exercise - 1 : (NCERT Based Topic-wise MCQs)															
1	(a)	13	(d)	25	(a)	37	(b)	49	(b)	61	(d)	73	(b)	85	(c)
2	(c)	14	(b)	26	(a)	38	(a)	50	(b)	62	(b)	74	(c)	86	(a)
3	(a)	15	(a)	27	(b)	39	(b)	51	(b)	63	(d)	75	(b)	87	(a)
4	(c)	16	(a)	28	(d)	40	(c)	52	(a)	64	(a)	76	(c)	88	(a)
5	(b)	17	(a)	29	(b)	41	(c)	53	(b)	65	(b)	77	(c)	89	(a)
6	(a)	18	(c)	30	(b)	42	(c)	54	(a)	66	(c)	78	(b)	90	(a)
7	(a)	19	(d)	31	(b)	43	(a)	55	(b)	67	(b)	79	(c)	91	(c)
8	(a)	20	(b)	32	(b)	44	(c)	56	(d)	68	(b)	80	(c)	92	(c)
9	(a)	21	(a)	33	(b)	45	(b)	57	(c)	69	(a)	81	(b)	93	(b)
10	(b)	22	(d)	34	(b)	46	(a)	58	(a)	70	(b)	82	(c)	94	(c)
11	(b)	23	(b)	35	(c)	47	(b)	59	(d)	71	(b)	83	(a)	95	(d)
12	(c)	24	(b)	36	(a)	48	(a)	60	(a)	72	(a)	84	(d)	96	(b)

Exercise - 1 (contd.)			
97	(d)	109	(a)
98	(b)	110	(d)
99	(a)	111	(b)
100	(a)	112	(c)
101	(a)	113	(b)
102	(c)	114	(c)
103	(b)	115	(c)
104	(a)		
105	(a)		
106	(a)		
107	(c)		
108	(a)		

Exercise - 2 : (NCERT Exemplar & Past Years JEE Main)																	
1	(a)	4	(d)	7	(d)	10	(d)	13	(d)	16	(a)	19	(c,d)	22	(d)	25	(c)
2	(c)	5	(c)	8	(d)	11	(b)	14	(c)	17	(c)	20	(a)	23	(3)	26	(24)
3	(a)	6	(d)	9	(a)	12	(a)	15	(c)	18	(b)	21	(b)	24	(b)		

Exercise - 3 : (Skill Enhancer MCQs)															
1	(a)	3	(a)	5	(b)	7	(c)	9	(a)	11	(a)	13	(a)	15	(d)
2	(b)	4	(a)	6	(b)	8	(b)	10	(b)	12	(a)	14	(a)		

Exercise - 4 : (Numeric Value Answer Questions)											
1	(10)	3	(385)	5	(8)	7	(2)	9	(0.5)	11	(1)
2	(104)	4	(12)	6	(5)	8	(1)	10	(0)	12	(1)

Chapter

7 Integrals

EXERCISE - 1

1. **(a)** $\int x^{51}(\tan^{-1}x+\cot^{-1}x)\,dx$

$= \int x^{51}.\frac{\pi}{2}dx \quad \left\{\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right\}$

$= \frac{\pi x^{52}}{104}+c=\frac{x^{52}}{52}(\tan^{-1}x+\cot^{-1}x)+c\,.$

2. **(c)** $\int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx=\int \frac{x^5-x^4}{x^3-x^2}dx$

$= \int \frac{x^4(x-1)}{x^2(x-1)}dx=\int x^2 dx=\frac{x^3}{3}+c.$

3. **(a)** $\int (x^4+x^2+2x^3)\,dn=\frac{x^5}{5}+\frac{x^3}{3}+\frac{x^4}{2}+c.$

4. **(c)** $\int \frac{(x-3)(x-2)}{x-2}dx=\int (x-3)\,dx=\frac{x^2}{2}-3x+c$

5. **(b)** $\int \left(x^{2-\frac{1}{2}}+x^{1-\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx=\int \left(x^{\frac{3}{2}}+x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx$

$= \frac{2}{5}x^{5/2}+\frac{2}{3}x^{3/2}+2x^{1/2}+c$

6. **(a)** Consider $\int \frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx$

$= \int \frac{\sin^2 x\ dx}{\sin^2 x\ \cos^2 x}-\int \frac{\cos^2 x\ dx}{\sin^2 x\cos^2 x}$

$= \int \sec^2 x\,dx-\int \text{cosec}^2 x\,dx$

Let $I=\int (\sec^2 x-\text{cosec}^2 x)dx$

$= \tan x-(-\cot x)+c=\tan x+\cot x+c.$

7. **(a)** Given integral is

$I(x)=\int \underset{\text{II}}{\sec^2 x}.\overset{\text{I}}{\sin^{-2022}x}dx-2022\int \sin^{-2022}x dx$

$= \tan x.(\sin x)^{-2022}+\int (2022)\tan x.(\sin x)^{-2023}x dx$

$-2022\int (\sin x)^{-2022}dx$

$I(x)=(\tan x)(\sin x)^{-2022}+C \quad ....(i)$

At $x=\pi/4$, $2^{1011}=\left(\frac{1}{\sqrt{2}}\right)^{-2022}+C$

so, C = 0

Hence $I(x)=\frac{\tan x}{(\sin x)^{2022}}$

Put $x=\frac{\pi}{6}$ in the above expression,

$I\left(\frac{\pi}{6}\right)=\frac{1}{\sqrt{3}\left(\frac{1}{2}\right)^{2022}}=\frac{2^{2022}}{\sqrt{3}}$

$I\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^{2022}}=\frac{2^{2022}}{(\sqrt{3})^{2021}}=\frac{1}{3^{1010}}I\left(\frac{\pi}{6}\right)$

$3^{1010}I\left(\frac{\pi}{3}\right)=I\left(\frac{\pi}{6}\right)$

8. **(a)** $I=\int x^x(1+\log x)dx$

Put $x^x=t$, then $x^x(1+\log x)\,dx=dt$

$\therefore \quad I=\int dt \Rightarrow I=t+C \Rightarrow I=x^x+C.$

9. **(a)** Let $I=\int \frac{\cos 8x+1}{\cot 2x-\tan 2x}dx$

Now, $D^r=\cot 2x-\tan 2x=\frac{\cos 2x}{\sin 2x}-\frac{\sin 2x}{\cos 2x}$

$= \frac{\cos^2 2x-\sin^2 2x}{\sin 2x\cos 2x}=\frac{2\cos 4x}{\sin 4x}$

$\therefore \quad I=\int \frac{2\cos^2 4x}{\frac{2\cos 4x}{\sin 4x}}dx=\int \frac{2\cos^2 4x\,.\sin 4x}{2\cos 4x}dx$

$= \frac{1}{2}\int \sin 8x\,dx=-\frac{1}{2}\frac{\cos 8x}{8}+k=-\frac{1}{16}.\cos 8x+k$

Now, $-\frac{1}{16}.\cos 8x+k=A\cos 8x+k \Rightarrow A=-\frac{1}{16}$

10. **(b)** Put $x^{3/2}=t \Rightarrow \frac{3}{2}x^{1/2}dx=dt$

$\therefore$ integral is

$\int \frac{\frac{2}{3}dt}{\sqrt{1-t^2}}=\frac{2}{3}\sin^{-1}t+C=\frac{2}{3}\sin^{-1}(x^{3/2})+C$

11. **(b)** $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx = \int \frac{dx}{\sin^{4/3} x \cos^{2/3} x}$

Multiplying N^r and D^r by $\cos^2 x$, we get

{ Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$}

$$= \int \frac{\sec^2 x\, dx}{\tan^{4/3} x} = \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{(-1/3)} + c = -3(\tan x)^{-1/3} + c.$$

12. **(c)** $I = \int \frac{dx}{\cos x + \sqrt{3} \sin x} \Rightarrow I = \int \frac{dx}{2\left[\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right]}$

$$= \frac{1}{2}\int \frac{dx}{\left[\sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x\right]} = \frac{1}{2}.\int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)}$$

$$\Rightarrow \quad I = \frac{1}{2}.\int \operatorname{cosec}\left(x + \frac{\pi}{6}\right) dx$$

$$\because \int \operatorname{cosec} x\, dx = \log|(\tan x/2)| + C$$

$$\therefore \; I = \frac{1}{2}.\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$$

13. **(d)** Given integral is

$$\int\left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2}\right)dx = \frac{xg(x)}{e^x + 1} + c$$

On differentiating both sides w.r.t. x, we get

$$\left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2}\right)$$

$$= \frac{(e^x + 1)\,(g(x) + xg'(x)) - e^x.x.g(x)}{(e^x + 1)^2}$$

$(e^x + 1)\, x\, (\cos x - \sin x) + g(x)\,(e^x + 1 - xe^x)$

$= (e^x + 1)\,(g(x) + xg'(x)) - e^x.\, x\, g(x)$

$\Rightarrow g'(x) = \cos x - \sin x$ ...(i)

Take integral both sides.

$\Rightarrow g(x) = \sin x + \cos x + C$

Take $g(x) = 0$; then $x = \frac{\overline{x}}{4}$

So, $g(x)$ is increasing in $(0, \pi/4)$

Again, differentiate w.r.t. x in equation (i),

$g''(x) = -\sin x - \cos x < 0$

$\Rightarrow g'(x)$ is decreasing function

Let $r(x) = g(x) + g'(x) = 2\cos x + C$

$\Rightarrow r'(x) = g'(x) + g''(x) = -2\sin x < 0$

$\Rightarrow r$ is decreasing

Let $l(x) = g(x) - g'(x) = 2\sin x + C$

Differentiate w.r.t. x

$\Rightarrow l'(x) = g'(x) - g''(x) = 2\cos x > 0$

Take $l'(x) = 0$; $\cos x = 0$; $x = \frac{\pi}{2}$

$\Rightarrow l$ is increasing

Therefore, $l(x)$ is increasing at $\left(0, \frac{\pi}{2}\right)$

14. **(b)** Given integral is

$$\int\left(\frac{x^2+1}{(x+1)^2}\right)e^x.dx. = \int\left(\frac{x^2-1+2}{(x+1)^2}\right)e^x dx$$

$$= \int\left(\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right)e^x dx = \int (f(x) + f'(x))e^x dx$$

$= f(x)\, e^x + c$

After comparing the integral the function is

$$f(x) = \frac{x-1}{x+1}$$

Then, $f'(x) = \frac{2}{(x+1)^2}$

Differentiate again two times w.r.t. x.

$$f''(x) = \frac{-4}{(x+1)^3} \Rightarrow f'''(x) = \frac{12}{(x+1)^4}$$

Put $x = 1$ in above equation.

$$f'''(1) = \frac{12}{16} = \frac{3}{4}$$

15. **(a)** Given integral is $I = \int \frac{\left(1 - \frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1 + \frac{2}{\sqrt{3}}\sin 2x\right)} dx$

$$I = \frac{\sqrt{3}}{2}\int \frac{\left(1 - \frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(\frac{\sqrt{3}}{2} + \sin 2x\right)} dx$$

$$I = \int \frac{\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)(\cos x - \sin x)}{\sin\left(\frac{\pi}{3}\right) + \sin 2x} dx$$

$$I = \int \frac{\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\sin x\right)}{2\sin\left(x + \frac{\pi}{6}\right)\cos\left(x - \frac{\pi}{6}\right)} dx$$

$$I = \int \frac{\left(\cos\left(x - \frac{\pi}{6}\right) - \sin\left(x + \frac{\pi}{6}\right)\right)}{2\sin\left(x + \frac{\pi}{6}\right)\cos\left(x - \frac{\pi}{6}\right)} dx$$

$$\frac{1}{2}\left(\int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)} - \int \frac{dx}{\cos\left(x - \frac{\pi}{6}\right)}\right)$$

$$\frac{1}{2}\left(\int \operatorname{cosec}\left(x + \frac{\pi}{6}\right)dx - \int \sec\left(x - \frac{\pi}{6}\right)dx\right)$$

$$\frac{1}{2}\ln\left|\frac{\tan\left(\frac{x}{2} + \frac{x}{12}\right)}{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)}\right|$$

16. (a) $\int \frac{dx}{\{(x-1)^3(x+2)^5\}^{1/4}} = \int \frac{dx}{\left(\frac{x-1}{x+2}\right)^{3/4} \cdot (x+2)^2}$

Put $\frac{x-1}{x+2} = t \Rightarrow \frac{3}{(x+2)^2}dx = dt$

$\therefore$ Integral $= \int \frac{dt}{3t^{3/4}} = \frac{4}{3}t^{\frac{1}{4}} + c = \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$

17. (a) $\int \frac{2dx}{(e^x + e^{-x})^2} = \int \frac{2e^{2x}}{(e^{2x}+1)^2}dx$

$= -\frac{1}{(e^{2x}+1)} + c = -\frac{e^{-x}}{e^x + e^{-x}} + c$

18. (c)

19. (d) Let $I = \int (27e^{9x} + e^{12x})^{1/3}dx$

$= \int e^{3x}(27 + e^{3x})^{1/3}dx$

Put $27 + e^{3x} = t \Rightarrow 3e^{3x}dx = dt$

$\therefore \quad I = \frac{1}{3}\int t^{1/3}dt = \frac{1}{3} \times \frac{t^{4/3}}{4/3} + C \quad = \frac{1}{4}(27 + e^{3x})^{4/3} + C$

20. (b) $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$

Let $xe^x = t$

$\Rightarrow \quad (xe^x + e^x) = \frac{dt}{dx} \quad \Rightarrow \quad dx = \frac{dt}{e^x(x+1)}$

$\therefore \quad \int \frac{e^x(1+x)}{\cos^2(e^x x)}dx = \int \frac{e^x(1+x)}{\cos^2 t} \times \frac{dt}{e^x(1+x)}$

$= \int \frac{1}{\cos^2 t}dt = \int \sec^2 t\, dt = \tan t + C = \tan(xe^x) + C$

21. (a) Let $I = \int 2^{2^{2^x}} 2^{2^x} 2^x dx$

Let $2^{2^{2^x}} = t \Rightarrow 2^{2^{2^x}} 2^{2^x} 2^x (\log 2)^3 dx = dt$

$\Rightarrow I = \int \frac{1}{(\log 2)^3}dt = \frac{1}{(\log 2)^3}t + C = \frac{1}{(\log 2)^3}2^{2^{2^x}} + C$

22. (d) Put $10^x + x^{10} = t$

$\therefore \quad (10^x \log_e 10 + 10x^9)dx = dt$

$\therefore \quad \int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}}dx = \int \frac{dt}{t}$

$= \log_e t + c = \log_e(10^x + x^{10}) + C$

23. (b) Put $x = \cos 2\theta$

$\therefore \quad I = \int \cos\{2\tan^{-1}\tan\theta\}(-2\sin 2\theta)d\theta$

$= -\int \sin 4\theta\, d\theta = \frac{1}{4}\cos 4\theta + c$

$= \frac{1}{4}(2x^2 - 1) + c = \frac{1}{2}x^2 + k$

24. (b) $\int e^{3\log x}(x^4+1)^{-1}dx = \int e^{\log x^3}\frac{1}{x^4+1}dx$

$= \int \frac{x^3}{x^4+1}dx = \frac{1}{4}\log(x^4+1) + C$

[since $e^{\log_e x^3} = x^3$]

25. (a) $I = \int \sin^3 x.\cos^5 x\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

$I = \int \sin^3 x\,.\cos^4 x\,.\cos x\; dx = \int t^3(1-t^2)^2 dt$

$= \int (t^3 - 2t^5 + t^7)\, dt = \frac{1}{4}t^4 - \frac{2}{6}t^6 + \frac{1}{8}t^8 + C$

$= \frac{1}{4}\sin^4 x - \frac{1}{3}\sin^6 x + \frac{1}{8}\sin^8 x + C$

26. (a) $I = \int \left(x + \frac{1}{x}\right)^{n+5}\left(\frac{x^2-1}{x^2}\right)dx$

Put $x + \frac{1}{x} = t \Rightarrow \left(1 - \frac{1}{x^2}\right)dx = dt$

$\Rightarrow \left(\frac{x^2-1}{x^2}\right)dx = dt$

$\therefore \quad I = \int t^{n+5}\, dt = \frac{t^{n+6}}{n+6} + c = \frac{\left(x+\frac{1}{x}\right)^{n+6}}{n+6} + c$

27. (b) $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x\cos^2 x}dx$

$= \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{1 - 2\sin^2 x\cos^2 x}dx$

$= \int \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x)}{1 - 2\sin^2 x\cos^2 x}dx$

$= \int \frac{1.(\sin^2 x - \cos^2 x)[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x]}{1 - 2\sin^2 x\cos^2 x}$

$= \int \frac{(\sin^2 x - \cos^2 x)(1 - 2\sin^2 x\cos^2 x)}{1 - 2\sin^2 x\cos^2 x}dx$

$= -\int \cos 2x\, dx = -\frac{1}{2}\sin 2x + C$

28. (d) Let $I = \int \cos^n x \sin x\, dx$

Put $\cos x = t$

$-\sin x\, dx = dt$

$\therefore \quad I = -\int t^n dt = -\frac{t^{n+1}}{n+1} + C$

$= -\frac{\cos^{n+1} x}{n+1} + C = -\frac{\cos^6 x}{6} + C$

$\therefore \quad n + 1 = 6 \quad$ or $\quad n = 5$

29. **(b)** Let $I=\int \sin^3 x\cos^3 x\,dx$. Then,

$$I=\frac{1}{8}\int(2\sin x\cos x)^3\,dx$$

$$\Rightarrow\quad I=\frac{1}{8}\int\sin^3 2x\,dx \Rightarrow I=\frac{1}{8}\int\frac{3\sin 2x-\sin 6x}{4}dx$$

$$\Rightarrow\quad I=\frac{1}{32}\int(3\sin 2x-\sin 6x)\,dx$$

$$=\frac{1}{32}\left\{-\frac{3}{2}\cos 2x+\frac{1}{6}\cos 6x\right\}+C$$

30. **(b)** Let $I=\int\frac{1}{\sqrt{\sin^3 x\cos^5 x}}dx$

$$\Rightarrow\quad I=\int\frac{1}{\sin^{3/2}x\cos^{5/2}x}dx \Rightarrow I=\int\frac{\sec^4 x}{\tan^{3/2}x}dx$$

[Dividing numerator and denominator by $\cos^4 x$]

$$\Rightarrow\quad I=\int\frac{(1+\tan^2 x)}{\tan^{3/2}x}\sec^2 x\,dx$$

Putting $\tan x=t\Rightarrow \sec^2 x\,dx=dt$

$$I=\int\frac{1+t^2}{t^{3/2}}dt \Rightarrow I=\int\left(t^{-3/2}+t^{1/2}\right)dt=\frac{-2}{\sqrt{t}}+\frac{t^{3/2}}{3/2}+C$$

$$=-\frac{2}{\sqrt{\tan x}}+\frac{2}{3}(\tan x)^{3/2}+C$$

31. **(b)**

32. **(b)** $\int\frac{\sin x}{\sin(x-\alpha)}dx=\int\frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)}dx$

$$=\int\frac{\sin(x-\alpha)\cos\alpha+\cos(x-\alpha)\sin\alpha}{\sin(x-\alpha)}dx$$

$$=\int\{\cos\alpha+\sin\alpha\cot(x-\alpha)\}dx$$

$$=(\cos\alpha)x+(\sin\alpha)\log\sin(x-\alpha)+C$$

$\therefore$ $A=\cos\alpha$, $B=\sin\alpha$

33. **(b)**

34. **(b)** $\int\frac{(x-x^3)^{1/3}}{x^4}\,dx=\int\frac{1}{x^3}\left(\frac{1}{x^2}-1\right)^{1/3}dx$

$$=\frac{-1}{2}\int t^{1/3}\,dt\qquad\left[\text{Putting }\frac{1}{x^2}-1=t\Rightarrow\frac{-2}{x^3}dx=dt\right]$$

$$=\frac{-1}{2}\cdot\frac{t^{4/3}}{4/3}+C=\frac{-3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+C$$

35. **(c)** $\int\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$

Let $e^{2x}+e^{-2x}=t$

$$\Rightarrow\quad 2e^{2x}-2e^{-2x}=\frac{dt}{dx}\Rightarrow dx=\frac{dt}{2\left(e^{2x}-e^{-2x}\right)}$$

$$\therefore\int\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx=\int\frac{e^{2x}-e^{-2x}}{t}=\frac{dt}{2\left[e^{2x}-e^{-2x}\right]}$$

$$=\frac{1}{2}\int\frac{1}{t}dt=\frac{1}{2}\log|t|+C=\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$$

36. **(a)** $\int\frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx=g(x)+c$

Put $x=\cos 2t$

$dx=-2\sin 2t\,.\,dt$

$$=\int\frac{1}{\cos 2t}\tan t(-4\sin t\cos t)\,dt$$

$$=\int\frac{1}{\cos 2t}(-4\sin^2 t)\,dt\quad=-2\int\frac{1-\cos 2t}{\cos 2t}dt$$

$$=-\frac{2}{2}\ln|\sec 2t+\tan 2t|+2t+c$$

$$=\ln|\sec 2t-\tan 2t|+2t+c$$

$$=\ln\left|\frac{1-\sin 2t}{\cos 2t}\right|+\cos^{-1}x+c$$

Let $g(x)=\ln\left|\frac{1-\sqrt{1-x^2}}{x}\right|+\cos^{-1}x+c$ (say)

$\therefore\ g(1)=0\Rightarrow c=0$

Now $g(x)=\ln\left|\frac{1-\sqrt{1-x^2}}{x}\right|+\cos^{-1}x$

$$g\left(\frac{1}{2}\right)=\ln|2-\sqrt{3}|+\frac{\pi}{3}$$

After checking options we get

$$g\left(\frac{1}{2}\right)=\ln\left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right|+\frac{\pi}{3}$$

37. **(b)** Let $\sec x=t\Rightarrow\tan n.\sec x\,dx=dt$

$$\therefore I=\int\frac{1}{t^2+(2)^2}dt=\log\left|t+\sqrt{t^2+(2)^2}\right|+c$$

$$=\log\left|\sec x+\sqrt{\sec^2 x+4}\right|+c$$

38. **(a)** $I=\int\frac{x}{1-(x^2)^2}dx\cdot$ Let $x^2=t\Rightarrow x\,dx=\frac{1}{2}dt$

$$I=\frac{1}{2}\int\frac{1}{1-t^2}dt=\frac{1}{2}\times\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|+c$$

$$=\frac{1}{4}\log\left|\frac{1+x^2}{1-x^2}\right|+c$$

39. **(b)** Put $x-\alpha=t^2\Rightarrow dx=2t\,dt$

$$\therefore I=2\int\frac{t\,dt}{\sqrt{t^2(\beta-\alpha-t^2)}}$$

$$=2\int\frac{dt}{\sqrt{(\beta-\alpha)-t^2}}=2\sin^{-1}\frac{t}{\sqrt{\beta-\alpha}}+C$$

$$=2\sin^{-1}\sqrt{\frac{x-\alpha}{\beta-\alpha}}+C$$

40. **(c)** Let $I = \int \frac{x^3 + x}{x^4 - 9} dx$. Then,

$I = \int \frac{x^3}{x^4 - 9} dx + \int \frac{x}{x^4 - 9} dx = I_1 + I_2 + C$(say), where

$I_1 = \int \frac{x^3}{x^4 - 9} dx$ and $I_2 = \int \frac{x}{x^4 - 9} dx$

Putting $x^4 - 9 = t$ in $I_1 \Rightarrow 4x^3 dx = dt$, we get

$$I_1 = \frac{1}{4}\int \frac{1}{t} dt = \frac{1}{4}\log|t| = \frac{1}{4}\log\left|x^4 - 9\right|$$

$$I_2 = \int \frac{x}{x^4 - 9} dx = \int \frac{x}{\left(x^2\right)^2 - 3^2} dx$$

Putting $x^2 = t \Rightarrow 2x\, dx = dt$, we get

$$I_2 = \frac{1}{2}\int \frac{dt}{t^2 - 3^2} = \frac{1}{2}\cdot\frac{1}{2\times 3}\log\left|\frac{t-3}{t+3}\right| = \frac{1}{12}\log\left|\frac{x^2 - 3}{x^2 + 3}\right|$$

Hence, $I = \frac{1}{4}\log\left|x^4 - 9\right| + \frac{1}{12}\log\left|\frac{x^2 - 3}{x^2 + 3}\right| + C$

41. **(c)** Let $I = \int \frac{1}{\sqrt{9 + 8x - x^2}} dx$. Then,

$$I = \int \frac{1}{\sqrt{-\left\{x^2 - 8x - 9\right\}}} dx$$

$$I = \int \frac{1}{\sqrt{-\left\{x^2 - 8x + 16 - 25\right\}}} dx$$

$$\Rightarrow I = \int \frac{1}{\sqrt{-\left\{(x-4)^2 - 5^2\right\}}} dx = \int \frac{1}{\sqrt{5^2 - (x-4)^2}} dx$$

$$= \sin^{-1}\left(\frac{x-4}{5}\right) + C$$

42. **(c)** Let $I = \int_0^\pi \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x} + e^{-\cos x})} dx$ ...(i)

Applying identity $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$

$I = \int_0^\pi \frac{e^{-\cos x}\sin x}{(1+\cos^2 x)(e^{-\cos x} + e^{\cos x})} dx$ ...(ii)

Add equations (i) and (ii), we get

$$2I = \int_0^\pi \frac{\sin x}{1+\cos^2 x} dx$$

On putting $\cos x = t$, we get

$$2I = 2\int_0^1 \frac{dt}{1+t^2} = (\tan^{-1} t)_0^1 = \frac{\pi}{4}$$

43. **(a)** Put $x = z^6 \Rightarrow dx = 6z^5\, dz$

$$\therefore \int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1+\sqrt[3]{x})} dx = \int \frac{(z^6 + z^4 + z)6z^5 dz}{z^6(1+z^2)}$$

$$= 6\int \frac{z^5 + z^3 + 1}{z^2 + 1} dz = 6\int \left(z^3 + \frac{1}{z^2+1}\right) dz$$

$$= \frac{3}{2}z^4 + 6\tan^{-1} z + C = \frac{3}{2}x^{2/3} + 6\tan^{-1} x^{1/6} + C$$

44. **(c)** $I = \int \frac{1}{1 + 3\sin^2 x + 8\cos^2 x} dx$

Dividing the numerator and denominator by $\cos^2 x$, we get

$$I = \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x + 8} dx$$

$$\Rightarrow I = \int \frac{\sec^2 x}{1 + \tan^2 x + 3\tan^2 x + 8} dx = \int \frac{\sec^2 x}{4\tan^2 x + 9} dx$$

Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$, we get

$$I = \int \frac{dt}{4t^2 + 9} = \frac{1}{4}\int \frac{dt}{t^2 + (3/2)^2} = \frac{1}{4}\times\frac{1}{3/2}\tan^{-1}\left(\frac{t}{3/2}\right) + C$$

$$\Rightarrow I = \frac{1}{6}\tan^{-1}\left(\frac{2t}{3}\right) + C = \frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$$

45. **(b)** $I = \int \frac{1 + 1/x^2}{x^2 + 1 + 1/x^2} dx = \int \frac{d\,(x - 1/x)}{(x - 1/x)^2 + 3}$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{x - 1/x}{\sqrt{3}} + C$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{(x^2 - 1)}{\sqrt{3}x}\right) + C$$

46. **(a)** Let $x = a\sin^2\theta$

then $dx = 2a\sin\theta\,\cos\theta\, d\theta$

$$\therefore I = \int \frac{2a\sin\theta.\cos\theta}{\sqrt{a\sin^2\theta.a\cos^2\theta}} d\theta$$

$$= 2\int d\theta = 2\theta + c$$

$$= 2\sin^{-1}\left(\sqrt{x/a}\right) + c$$

47. **(b)** After dividing by $\cos^2 x$ to numerator and denominator of integration

$$I = \int \frac{\sec^2 x\, dx}{4\tan^2 x + 4\tan x + 5}$$

$$= \int \frac{\sec^2 x\, dx}{(2\tan x + 1)^2 + 4} = \frac{1}{22}\tan^{-1}\left(\frac{2\tan x + 1}{2}\right) + C$$

48. **(a)** Let $6x+7=\lambda\frac{d}{dx}(x-5)(x-4)+\mu$

i.e. $6x+7=\lambda\,(2x-9)+\mu$ which

gives $\lambda=3$ and $\mu=34$

$$\therefore \int\frac{6x+7}{\sqrt{(x-5)(x-4)}}dx=\int\frac{3(2x-9)+34}{x^2-9x+20}dx$$

$$=3\int(2x-9)(x^2-9x+20)^{-\frac{1}{2}}dx+34\int\frac{dx}{\sqrt{x^2-9x+20}}$$

$$=3\int\frac{(x^2-9x+20)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+34\int\frac{dx}{\sqrt{x^2-9x+\frac{81}{4}-\frac{1}{4}}}$$

$$=6\sqrt{x^2-9x+20}+34\int\frac{dx}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$

$$=6\sqrt{x^2-9x+20}$$

$$+34\log\left\{\left|x-\frac{9}{2}+\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right|\right\}+C$$

$$=6\sqrt{x^2-9x+20}+34\log\left|x+\sqrt{x^2-9x+20}-\frac{9}{2}\right|+C$$

$\therefore$ A = 6, B = 34.

49. **(b)** $I=\int\frac{dx}{x\sqrt{1-x^3}}=\int\frac{x^2dx}{x^3\sqrt{1-x^3}}$

Put $1-x^3=t^2\Rightarrow -3x^2dx=2t\,dt$

$$I=-\frac{2}{3}\int\frac{t\,dt}{\left(1-t^2\right).t}=\frac{2}{3}\int\frac{dt}{t^2-1}=\frac{2}{3}.\frac{1}{2}\,ln\left|\frac{t-1}{t+1}\right|+C$$

$$=\frac{1}{3}ln\left[\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right]+C\ \therefore a=\frac{1}{3},\ b=-1$$

50. **(b)** We have, $I=\int\frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$

$$I=\int\frac{\sqrt{x}}{\sqrt{\left(a^{3/2}\right)^2-\left(x^{\frac{3}{2}}\right)^2}}dx$$

Put, $x^{\frac{3}{2}}=t$

$\Rightarrow \frac{3}{2}x^{\frac{1}{2}}dx=dt\quad\Rightarrow\sqrt{x}\,dx=\frac{2}{3}dt$

$$\therefore I=\frac{2}{3}\int\frac{dt}{\sqrt{\left(a^{\frac{3}{2}}\right)^2-t^2}}=\frac{2}{3}\sin^{-1}\left(\frac{t}{a^{\frac{3}{2}}}\right)+C$$

$$=\frac{2}{3}\sin^{-1}\left(\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}}\right)+C=\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$$

51. **(b)** $I=\int\sqrt{\frac{a-x}{a+x}}dx=\int\sqrt{\frac{a-x}{a+x}\times\frac{a-x}{a-x}}dx=\int\frac{a-x}{\sqrt{a^2-x^2}}dx$

$$\Rightarrow I=\int\frac{a}{\sqrt{a^2-x^2}}dx-\int\frac{x}{\sqrt{a^2-x^2}}dx$$

$$\Rightarrow I=a\int\frac{1}{\sqrt{a^2-x^2}}dx+\frac{1}{2}\int\frac{-2x}{\sqrt{a^2-x^2}}dx$$

Putting $a^2-x^2=t$, and $-2x\,dx=dt$, we get

$$I=a\sin^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}\int\frac{dt}{\sqrt{t}}=a\sin^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}\left(\frac{t^{1/2}}{1/2}\right)+C$$

$$\Rightarrow I=a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{t}+C=a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}+C$$

52. **(a)** $\because\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}+c$

But it is given that

$$\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{3}+c$$

$\therefore$ a = 3

53. **(b)**

54. **(a)** $\int\frac{x}{(x-1)(x^2+4)}dx$

Let $\frac{x}{(x-1)(x^2+4)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+4}$ ...(i)

or $x=A(x^2+4)+(Bx+C)(x-1)$ ...(ii)

Solving, we obtain

$$A=\frac{1}{5},\ B=-\frac{1}{5}\text{ and }C=\frac{4}{5}.$$

Substituting the values of A, B, and C in equation (i), we obtain

$$\frac{x}{(x-1)(x^2+4)}=\frac{1}{5(x-1)}+\frac{-\frac{1}{5}x+\frac{4}{5}}{x^2+4}$$

$$=\frac{1}{5(x-1)}-\frac{1}{5}\frac{(x-4)}{(x^2+4)}$$

or $I=\frac{1}{5}\int\frac{1}{x-1}dx-\frac{1}{5}\int\frac{x-4}{x^2+4}dx$

$$= \frac{1}{5}\int \frac{1}{x-1}dx - \frac{1}{10}\int \frac{2x}{x^2+4}dx + \frac{4}{5}\int \frac{1}{x^2+4}dx$$

$$= \frac{1}{5}\log|x-1| - \frac{1}{10}\log(x^2+4) + \frac{4}{5}\times\frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

$$= \frac{1}{5}\log|x-1| - \frac{1}{10}\log(x^2+4) + \frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right) + C$$

55. **(b)** Let $\frac{1}{(x-1)(x+2)(2x+3)} = \frac{A_1}{x-1} + \frac{A_2}{x+2} + \frac{A_3}{2x+3}$

$$A_1 = \frac{1}{(1+2)(2.1+3)} = \frac{1}{15};\ A_2 = \frac{1}{(-2-1)(-2.2+3)} = \frac{1}{3}$$

$$A_3 = \frac{1}{\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}+2\right)} = \frac{1}{-\frac{5}{4}} = -\frac{4}{5}$$

Now $I = \int \frac{dx}{(x-1)(x+2)(2x+3)}$

$$= \int\left[\frac{1}{15(x-1)} + \frac{1}{3(x+2)} - \frac{4}{5(2x+3)}\right]dx$$

$$= \frac{1}{15}\log|x-1| + \frac{1}{3}\log|x+2| - \frac{2}{5}\log|2x+3| + C$$

56. **(d)** We have,

$$\frac{3x+1}{(x-3)(x-5)} = \frac{-5}{x-3} + \frac{B}{x-5}$$

$3x + 1 = -5(x - 5) + B(x - 3)$

Put $x = 5$

$3(5) + 1 = B(5 - 3)$

$16 = 2B$ or $B = 8$

57. **(c)** Let $\frac{1}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$

Comparing both sides we get $A = \frac{1}{4}, B = -\frac{1}{4}$ and $C = -\frac{1}{2}$

$$\therefore I = \frac{1}{4}\int\frac{1}{x-1}dx \frac{1}{4}\int\frac{1}{x+1}dx - \frac{1}{2}\int\frac{1}{(x+1)^2}dx$$

58. **(a)** Here since the highest powers of x in numerator and denominator are equal and coefficients of x^2 are also equal, therefore

$$\int\frac{x^2+1}{(x-1)(x-2)} \equiv 1 + \frac{A}{x-1} + \frac{B}{x-2}$$

On solving we get $A = -2, B = 5$

Thus $\int\frac{x^2+1}{(x-1)(x-2)} \equiv 1 - \frac{2}{x-1} + \frac{5}{x-2}$

The above method is used to obtain the value of constant corresponding to non-repeated linear factor in the denominator.

Now, $I = \int\left(1 - \frac{2}{x-1} + \frac{5}{x-2}\right)dx$

$= x - 2\log(x-1) + 5\log(x-2) + C$

$$= x + \log\left[\frac{(x-2)^5}{(x-1)^2}\right] + C$$

59. **(d)** We have, $I = \int\frac{1}{(\sin x+4)(\sin x-1)}dx$

$$= \frac{1}{5}\int\frac{(\sin x+4)-(\sin x-1)}{(\sin x+4)(\sin x-1)}dx$$

$$= \frac{1}{5}\int\frac{1}{\sin x-1}dx - \frac{1}{5}\int\frac{1}{\sin x+4}dx$$

$$= \frac{1}{5}\int\frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2}-1-\tan^2\frac{x}{2}}dx$$

$$-\frac{1}{5}\int\frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2}+4+4\tan^2\frac{x}{2}}dx$$

Put, $\tan\frac{x}{2} = t$

$\Rightarrow \sec^2\frac{x}{2}dx = 2\,dt$

$$\therefore I = \frac{1}{5}\int\frac{2dt}{2t-1-t^2} - \frac{1}{5}\int\frac{2dt}{\left[2t+4\left(1+t^2\right)\right]}$$

$$\therefore I = -\frac{2}{5}\int\frac{dt}{t^2-2t+1} - \frac{1}{10}\int\frac{dt}{t^2+\frac{1}{2}t+1}$$

$$= -\frac{2}{5}\int\frac{1}{(t-1)^2}dt - \frac{1}{10}\int\frac{dt}{\left(t+\frac{1}{4}\right)^2+\left(\frac{\sqrt{15}}{4}\right)^2}$$

$$= \frac{2}{5}.\frac{1}{t-1} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4t+1}{\sqrt{15}}\right) + C$$

$$= \frac{2}{5}\frac{1}{\tan\frac{x}{2}-1} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}\right) + C \quad \text{...(i)}$$

But, given that

$$I = A\frac{1}{\left(\tan\frac{x}{2}-1\right)} + B\tan^{-1}\left[f(x)\right] + C \quad \text{...(ii)}$$

From eqs. (i) and (ii), we get

$$A = \frac{2}{5},\ B = \frac{-2}{5\sqrt{15}},\ f(x) = \frac{4\tan\frac{x}{2}+1}{\sqrt{15}}$$

60. **(a)** Let $I = \int \frac{1-\cos x}{\cos x(1+\cos x)} dx$

Let $\cos x = y \Rightarrow \frac{1-\cos x}{\cos x(1+\cos x)} = \frac{1-y}{y(1+y)}$

Now $\frac{1-y}{y(1+y)} = \frac{A}{y} + \frac{B}{1+y}$ ...(i)

$\Rightarrow 1 - y = A(1 + y) + By$

Put $y = 0$ in (i), we get $A = 1$.

Put $y = -1$ in (i), we get $B = -2$ ...(ii)

Substituting the values of A and B in (i), we obtain

$$\frac{1-y}{y(1+y)} = \frac{1}{y} - \frac{2}{1+y}$$

$\Rightarrow \frac{1-\cos x}{\cos x(1+\cos x)} = \frac{1}{\cos x} - \frac{2}{1+\cos x}$ $[\because y = \cos x]$

$\therefore I = \int \frac{1-\cos x}{\cos x(1+\cos x)} dx = \int \frac{1}{\cos x} dx - \int \frac{2}{1+\cos x} dx$

$\Rightarrow I = \int \sec x \, dx - \int \frac{1}{\cos^2(x/2)} dx$

$= \int \sec x \, dx - \int \sec^2(x/2) dx$

$\Rightarrow I = \log|\sec x + \tan x| - 2\tan(x/2) + C$

61. **(d)** $\frac{3x+4}{x^3-2x-4} = \frac{3x+4}{(x-2)(x^2+2x+2)}$

$= \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+2}$

$\Rightarrow 3x + 4 = A(x^2 + 2x + 2) + (Bx + C)(x - 2)$

$\therefore A + B = 0$

$2A - 2B + C = 3$

$2A - 2C = 4$

$\Rightarrow A = 1, B = C = -1$

$\therefore \int \frac{3x+4}{x^3-2x-4} dx = \int \frac{dx}{x-2} - \frac{1}{2}\int \frac{2x+2}{x^2+2x+2} dx$

$= \log_e|x-2| - \frac{1}{2}\log|x^2+2x+2| + C$

$\Rightarrow k = -\frac{1}{2}$ and $f(x) = |x^2 + 2x + 2|$

62. **(b)** We have, $\frac{x^3-1}{x^3+x} = 1 - \frac{1}{x} + \frac{x}{x^2+1} - \frac{1}{x^2+1}$

[Partial fraction]

$\therefore \int \frac{x^3-1}{x^2+x} dx = x - \log x + \frac{1}{2}\log(1+x^2) - \tan^{-1} x + C$

63. **(d)**

64. **(a)** We have, $I = \int 1.\tan^{-1}\sqrt{x} \, dx$

Using by parts,

$I = \tan^{-1}\sqrt{x}.(x) - \int \frac{1}{1+x} \times \frac{1}{2\sqrt{x}} \times x \, dx$

$= x\tan^{-1}\sqrt{x}. - \int \frac{x}{(1+x)2\sqrt{x}} dx$

$= x\tan^{-1}\sqrt{x} - \int\left(\frac{1+x}{(1+x)2\sqrt{x}} - \frac{1}{(1+x)2\sqrt{x}}\right)dx$

$= x\tan^{-1}\sqrt{x} - \int \frac{dx}{2\sqrt{x}} + \int \frac{dx}{2\sqrt{x}(1+x)}$

$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$

$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

65. **(b)**

66. **(c)** $\int e^x \frac{(1+\sin x)}{(1+\cos x)} dx = \int e^x\left[\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right]dx$

$= \frac{1}{2}\int e^x \sec^2\frac{x}{2} dx + \int e^x \tan\frac{x}{2} dx$

$= e^x \tan\frac{x}{2} + C$

But $I = e^x f(x) + C$ (given)

$\therefore f(x) = \tan\frac{x}{2}$

67. **(b)** $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx = \int e^x\left(\frac{1-\sin x}{2\sin^2\frac{x}{2}}\right)dx$

$= \int e^x\left(\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right)dx$

$= \frac{1}{2}\int e^x \operatorname{cosec}^2\frac{x}{2} dx - e^x\cot\frac{x}{2} - \frac{1}{2}\int e^x \operatorname{cosec}^2\frac{x}{2} dx + C$

$= -e^x\cot\frac{x}{2} + C$

68. **(b)** $I = 2\int\sqrt{x^2+\frac{9}{4}}dx = 2\int\sqrt{x^2+\left(\frac{3}{2}\right)^2}dx$

$= \frac{x}{2}\sqrt{4x^2+9} + \frac{9}{4}\log|2x+\sqrt{4x^2+9}| + C$

69. **(a)** $I = \int\sqrt{-(x^2-7x+10)dx} = \int\sqrt{\left(\frac{3}{2}\right)^2 - \left(x-\frac{7}{2}\right)^2} dx$

$= \frac{1}{4}(2x-7)\sqrt{7x-10-x^2} + \frac{9}{8}\sin^{-1}\left(\frac{2x-7}{3}\right) + c$

70. **(b)** Let $\sin^{-1}x = \theta \Rightarrow x = \sin\theta$

$\Rightarrow dx = \cos\theta \, d\theta$

$\therefore I = \int \theta\cos\theta d\theta = \theta\int\cos\theta d\theta + \int\left(\frac{d\theta}{d\theta}\int\cos\theta d\theta\right)d\theta$

$= \theta\sin\theta + \int\sin\theta d\theta = \theta\sin\theta - \cos\theta + c$

$= x\sin^{-1}x - \sqrt{1-x^2} + c.$

71. **(b)** Let $\sin^{-1}x = t \Rightarrow \frac{1}{\sqrt{1-x^2}}dx = dt$

and x = sint

$\therefore I = \int + \sin t dt = t\int \sin t dt - \int\left(\frac{dt}{dt}\int \sin t dt\right)dt$

$= -t\cos t + \int \cos t dt = \theta - t\cos t + \sin t + c$

$= x - \sqrt{1-x^2}\sin^{-1}x + c$

72. **(a)** $I = \int xe^x dx = x\int e^x dx - \int\left(\frac{dx}{dx}\int e^x dx\right)dx$

$= xe^x - \int e^x dx = xe^x - e^x + c$

73. **(b)** $I = \int 1\cdot\log x\,dx = \log x\int 1dx - \int\left(\frac{d\log x}{dx}\int 1dx\right)dx$

$= x\log x - \int 1dx = x\log x - x + c$

74. **(c)** $\int\sqrt{x^2-8x+7}\,dx = \int\sqrt{(x-4)^2-(3)^2}\,dx$

Now apply formula of $\int\sqrt{x^2-a^2}\,dx.$

75. **(b)** $\int x\sin x\sec^3 x\,dx = \int x\sin x\frac{1}{\cos^3 x}dx$

$= \int x\tan x.\sec^2 x\,dx$

Now put $\tan x = t \Rightarrow \sec^2 x dx = dt$ and $x = \tan^{-1}t,$ then it reduces to

$\int \tan^{-1}t.tdt = \frac{t^2}{2}\tan^{-1}t - \frac{1}{2}t + \frac{1}{2}\tan^{-1}t$

$= \frac{x(\sec^2 x-1)}{2} - \frac{1}{2}\tan x + \frac{1}{2}x = \frac{1}{2}[x\sec^2 x - \tan x] + c.$

76. **(c)** Let $I = \int x\sec^2 x\,dx$

$\Rightarrow\quad I = x\int\sec^2 x\,dx - \int\left[\frac{d}{dx}(x)\int\sec^2 x\right]dx$

$\Rightarrow\quad I = x\tan x - \int\tan x\,dx$

$\Rightarrow\quad I = x\tan x - (-\log\cos x) + c$

where 'c' is a constant of integration.

$\Rightarrow\quad I = x\tan x + \log\cos x + c$

77. **(c)** Let the given integration be

$I = \int e^x(1+\tan^2 x + \tan x)\,dx$

$\Rightarrow\quad I = \int e^x(\sec^2 x + \tan x)\,dx$

$= \int e^x\tan x\,dx + \int e^x\sec^2 x\,dx$

$= \tan x.\int e^x dx - \int\left(\frac{d}{dx}\tan x\int e^x\right)dx + \int e^x\sec^2 x\,dx$

$= \tan x.e^x - \int\sec^2 x\,e^x dx + \int e^x\sec^2 x\,dx$

$= \tan x\,e^x + c$

78. **(b)** Let $I_3 = \int x^3 e^{ax}dx = \frac{1}{a}x^3e^{ax} - \frac{3}{a}I_2$

$= \frac{1}{a}\left(x^3e^{ax}\right) - \frac{3}{a}\left[\frac{1}{a}x^2e^{ax} - \frac{2}{a}I_1\right]$

$= \frac{1}{a}x^3e^{ax} - \frac{3}{a^2}x^2e^{ax} + \frac{6}{a^2}\left[\frac{1}{a}xe^{ax} - \frac{1}{a}I_0\right]$

$= \frac{1}{a}x^3e^{ax} - \frac{3}{a^2}x^2e^{ax} + \frac{6}{a^3}xe^{ax} - \frac{6}{a^3}\int e^{ax}dx$

$\left[\because I_0 = \int x^0e^{ax}dx = \int e^{ax}dx\right]$

$= \frac{1}{a}x^3e^{ax} - \frac{3}{a^2}x^2e^{ax} + \frac{6}{a^3}xe^{ax} - \frac{6}{a^4}e^{ax} + c$

$= \frac{1}{a^4}e^{ax}\left[a^3x^3 - 3a^2x^2 + 6ax - 6\right] + c$

79. **(c)** $I = \int 2\sin x.\cos x.\log\cos x\,dx$

put log cos x = t $\quad\therefore -\frac{\sin x}{\cos x}dx = dt$

$I = \int 2\sin x.\cos x.t\frac{\cos x}{-\sin x}dt$

$= -2\int\cos^2 x.t\,dt = -2\int te^{2t}dt$

$= -te^{2t} + \frac{1}{2}e^{2t} + C$

$e^{2t}\left(\frac{1}{2}-t\right) + k = \cos^2 x\left\{\frac{1}{2} - \log\cos x\right\} + C$

80. **(c)** $I = \int 2\sin x.\cos x.\log\cos x\,dx$

put log cos x = t

$\therefore -\frac{\sin x}{\cos x}dx = dt$

$I = \int 2\sin x.\cos x.t\frac{\cos x}{-\sin x}dt$

$= -2\int\cos^2 x.t\,dt = -2\int te^{2t}dt$

$= -2\left[t.\frac{e^{2t}}{2} - \int\frac{e^{2t}}{2}.dt\right] = -te^{2t} + \frac{1}{2}e^{2t} + k$

$= e^{2t}\left(\frac{1}{2}-t\right) + k = \cos^2 x\left\{\frac{1}{2} - \log\cos x\right\} + k$

81. **(b)** Here, a = 1, b = 2, $f(x) = x^2$, $b - a = 1 = nh$

$\therefore\quad \int_1^2 x^2dx = \lim_{h\to\infty}\sum_{r=0}^{n-1}f(a+rh)$

$= \lim_{h\to 0}\left[h\left\{1^2 + (1+h)^2 + (1+2h)^2 + ... + (1+(n-1)h)^2\right\}\right]$

$$= \lim_{h\to 0}\left[h\left\{\left(1^2+1^2+n\text{ times}\right)+h^2(1^2+2^2+\ldots\ldots +(n-1)^2+2h\left(1+2+\ldots+(n-1)\right)\right\}\right]$$

$$= \lim_{h\to 0} h\left\{n+h^2\frac{(n-1)n(2n-1)}{6}+2h\frac{(n-1)n}{2}\right\}$$

$$= \lim_{h\to 0}\left\{nh+\frac{(nh-h)nh(2nh-h)}{6}+\frac{2(hn)(nh-h)}{2}\right\}$$

$$= 1+\frac{1}{3}+1=\frac{7}{3} \qquad (\text{as } n\to\infty,\ h\to 0)$$

82. (c) We know that,

$$\int_0^4 \frac{1}{\sqrt{x^2+2x+3}}dx = \int_0^4 \frac{1}{\sqrt{(x+1)^2+(\sqrt{2})^2}}dx$$

$$= \left[\log\left|x+1+\sqrt{(x+1)^2+(\sqrt{2})^2}\right|\right]_0^4$$

$$= \left[\log\left|x+1+\sqrt{x^2+2x+3}\right|\right]_0^4$$

$$= \log(5+\sqrt{16+8+3}) - \log(1+\sqrt{3})$$

$$= \log(5+3\sqrt{3}) - \log(1+\sqrt{3})$$

$$= \log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$$

83. (a) We have $A_{n+1}-A_n = \int_0^{\frac{\pi}{2}} \frac{\sin(2n+1)x-\sin(2n-1)x}{\sin x}dx$

$$= \int_0^{\frac{\pi}{2}} \frac{2\cos 2nx \sin x}{\sin x}dx = 2\int_0^{\frac{\pi}{2}}\cos 2n\pi\, dx = 0$$

Again $B_{n+1}-B_n = \int_0^{\frac{\pi}{2}} \frac{\sin^2(n+1)x-\sin^2 nx}{\sin^2 x}dx$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin(2n+1)x\sin x}{\sin^2 x}dx = A_{n+1}$$

84. (d) Given $f'(x)=f(x) \Rightarrow \frac{f'(x)}{f(x)}=1$

Integrating; $\log f(x)=x+c \Rightarrow f(x)=e^{x+c}$

$f(0)=1 \Rightarrow f(x)=e^x$

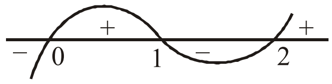
$$\therefore \int_0^1 f(x)g(x)dx = \int_0^1 e^x(x^2-e^x)dx$$

$$= \int_0^1 x^2e^x dx - \int_0^1 e^{2x}dx$$

$$= \left[x^2e^x\right]_0^1 - 2\left[xe^x-e^x\right]_0^1 - \frac{1}{2}\left[e^{2x}\right]_0^1$$

$$= e-\left[\frac{e^2}{2}-\frac{1}{2}\right]-2\left[e-e+1\right] = e-\frac{e^2}{2}-\frac{3}{2}$$

85. (c) $f'(x)=x(x^2-3x+2)=x(x-1)(x-2)$



F(x) decrease in [1, 2] and increases in [2, 3]

$\therefore$ Minimum of $f(x)=f(2)$ and Maximum of $f(x)=\text{Max}\{f(1), f(3)\}$

Now $f(x)=\left[\frac{t^4}{4}-t^3+t^2\right]_1^x = \frac{x^4}{4}-x^3+x^2-\frac{1}{4}$

$\therefore$ $f(1)=0,\ f(2)=-\frac{1}{4}$ and $f(3)=2$

$\therefore$ Range of $f(x)=\left[-\frac{1}{4}, 2\right]$

86. (a) $\int_0^{\pi/4}\sec^2\theta\sin\theta d\theta = \int_0^{\pi/4}\sec\theta\tan\theta d\theta$

$$= [\sec\theta]_0^{\pi/4} = \sqrt{2}-1$$

Now $\int_0^k \frac{dx}{(\sqrt{x}+\sqrt{x+k})} = \int_0^k \frac{\sqrt{x}-\sqrt{x+k}}{-k}dx$

$$= -\frac{1}{k}\cdot\frac{2}{3}\left[x^{3/2}-(x+k)^{3/2}\right]_0^k$$

$$= -\frac{2}{3k}\left[k^{3/2}-(2k)^{3/2}+k^{3/2}\right]$$

$$= -\frac{2}{3}k^{1/2}\left[2-2\sqrt{2}\right] = \frac{4}{3}k^{1/2}(\sqrt{2}-1)$$

So, $\sqrt{2}-1=\frac{4}{3}k^{1/2}(\sqrt{2}-1) \Rightarrow k=\frac{9}{16}$

87. (a) Let $I=\int_{1/4}^{1/2}\frac{dx}{\sqrt{x-x^2}}$

$$\therefore I=\int_{1/4}^{1/2}\frac{dx}{\sqrt{\frac{1}{4}-\frac{1}{4}+x-x^2}}$$

$$\Rightarrow I=\int_{1/4}^{1/2}\frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}}$$

$$\Rightarrow \; I=\left[\sin^{-1}\left(\frac{x-1/2}{1/2}\right)\right]_{1/4}^{1/2}=[\sin^{-1}(2x-1)]\Big|_{1/4}^{1/2}$$

$$\Rightarrow \; I=\sin^{-1}(1-1)-\sin^{-1}\left(2.\frac{1}{4}-1\right)$$

$$\Rightarrow \; I=0-\sin^{-1}\left(-\frac{1}{2}\right)=\sin^{-1}\left(\frac{1}{2}\right)=\sin^{-1}\left(\sin\frac{\pi}{6}\right)$$

$$\Rightarrow \; I=\frac{\pi}{6}$$

88. **(a)** Let $I=\int_1^2 e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx$

Consider $\int e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx=\int e^x.\frac{1}{x}dx-\int\frac{e^x}{x^2}dx$

We know $\int e^x(f(x)+f'(x))dx=e^x f(x)$

$$\therefore \; I=\int_1^2 e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx=\left[\frac{1}{x}e^x\right]_1^2$$

$$=\frac{1}{2}e^2-\frac{1}{1}e^1=e\left[\frac{e}{2}-1\right]$$

89. **(a)** Let $I=\int_0^1 x^2 e^x\,dx$

$$=x^2\int_0^1 e^x dx-\int_0^1\left[\frac{d}{dx}(x^2)\int e^x\right]dx$$

$$\Rightarrow \; I=\left[x^2.e^x\right]_0^1-\int_0^1 2x.e^x\,dx$$

$$=1.e^1-2\int_0^1 x e^x\,dx=e-2\left[x\int_0^1 e^x dx-\int_0^1 e^x dx\right]$$

$$=e-2\left[xe^x-e^x\right]_0^1=e-2[e-e-(0-1)]=e-2$$

90. **(a)** $\int_1^3\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$

$$=\int_1^3\left(\tan^{-1}\frac{x}{x^2+1}+\cot^{-1}\frac{x}{x^2+1}\right)dx=\int_1^3\frac{\pi}{2}dx=\pi$$

[As $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$]

91. **(c)** $a_n-a_{n-1}=\int_0^{\frac{\pi}{2}}\frac{\sin^2 nx-\sin^2(n-1)x}{\sin x}\,dx$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin\{nx+(n-1)x\}\sin\{nx-(n-1)x\}}{\sin x}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin(2nx-x)\sin x}{\sin x}dx=\int_0^{\frac{\pi}{2}}\sin(2n-1)x\,dx$$

$$=-\left[\frac{\cos(2n-1)x}{2n-1}\right]_0^{\frac{\pi}{2}}=-\left[\frac{\cos(2n-1)\frac{\pi}{2}}{2n-1}-\frac{\cos 0}{2n-1}\right]=\frac{1}{2n-1}$$

$\therefore a_2-a_1=\frac{1}{3},\; a_3-a_2=\frac{1}{5},\; a_4-a_3=\frac{1}{7}$ ....etc.

$\therefore a_2-a_1,\; a_3-a_2,\; a_4-a_3$ .... form an H.P.

92. **(c)** $\int_0^{\infty}\frac{dx}{(x^2+a^2)(x^2+b^2)}=\frac{1}{b^2-a^2}\int_0^{\infty}\frac{(x^2+b^2)-(x^2+a^2)}{(x^2+a^2)(x^2+b^2)}$

$$=\frac{1}{b^2-a^2}\int_0^{\infty}\left[\frac{1}{x^2+a^2}-\frac{1}{x^2+b^2}\right]dx$$

$$=\frac{1}{b^2-a^2}\left[\frac{1}{a}\tan^{-1}\frac{x}{a}-\frac{1}{b}\tan^{-1}\frac{x}{b}\right]_0^{\infty}$$

$$=\frac{1}{b^2-a^2}\left[\frac{\pi}{2a}-\frac{\pi}{2b}\right]=\frac{\pi}{2ab(a+b)}$$

93. **(b)** Let $I=\int_0^{\pi/2}\sqrt{\tan x}+\sqrt{\cot x}\,dx$

$$=\int_0^{\pi/2}\frac{\sin x+\cos x}{\sqrt{\sin x\cos x}}dx$$

$$=\int_0^{\pi/2}\frac{\sqrt{2}(\sin x+\cos x)}{\sqrt{\sin 2x}}dx$$

$$=\int_0^{\pi/2}\frac{\sqrt{2}(\sin x+\cos x)}{\sqrt{1-(\sin x-\cos x)^2}}dx$$

Put, $\sin x-\cos x=t$

$\Rightarrow (\cos x+\sin x)\,dx=dt$

$$=\int_{-1}^{1}\frac{\sqrt{2}}{\sqrt{1-t^2}}dt=\sqrt{2}\sin^{-1}t\Big|_{-1}^{1}=\sqrt{2}\left[\frac{\pi}{2}+\frac{\pi}{2}\right]=\pi\sqrt{2}.$$

94. **(c)** $\int_{\log 2}^{x}\frac{du}{(e^u-1)^{1/2}}=\frac{\pi}{6}$

Put $e^u-1=t^2 \Rightarrow e^u du=2t\,dt$

$$\Rightarrow \int_1^{\sqrt{e^x-1}}\frac{2t}{1+t^2}dt=\frac{\pi}{6}$$

$$\Rightarrow 2(\tan^{-1}t)_1^{\sqrt{e^x-1}}=\frac{\pi}{6}\Rightarrow \tan^{-1}\sqrt{e^x-1}-\frac{\pi}{4}=\frac{\pi}{12}$$

$$\Rightarrow \sqrt{e^x-1}=\tan\frac{\pi}{3}\Rightarrow\sqrt{e^x-1}=\sqrt{3}\Rightarrow e^x=4.$$

95. **(d)** Let $I=\int_0^{\pi/2}\sin x\sin 2x\,dx = 2\int_0^{\pi/2}\sin^2 x\cos x dx$

Put $t=\sin x \Rightarrow dt=\cos x\,dx$

Now, $I=2\int_0^1 t^2 dt = \frac{2}{3}[t^3]^1_0 = \frac{2}{3}$

96. **(b)** Let $I=\int_0^{\pi/2}\sqrt{\cos\theta}\sin^3\theta\,d\theta$

Put $t=\cos\theta \Rightarrow dt=-\sin\theta\,d\theta$, then

$$I=-\int_1^0 t^{1/2}(1-t^2)dt=\int_0^1(t^{1/2}-t^{5/2})dt$$

$$I=\left[\frac{2}{3}t^{3/2}-\frac{2}{7}t^{7/2}\right]_0^1=\frac{8}{21}.$$

97. **(d)** $f(e^3)=\int_1^{e^3}\frac{\ln t}{\ln 10(1+t)}dt$ .....(i)

$$f(\alpha)=\int_1^{\alpha}\frac{\ln t}{(\ln 10)(1+t)}dt$$

Let $t=\frac{1}{x}\Rightarrow x=\frac{1}{t}$

$$dt=\frac{-1}{x^2}dx$$

$$=\int_1^{\frac{1}{\alpha}}\frac{-\ln x}{(\ln 10)\left(1+\frac{1}{x}\right)}\left(-\frac{1}{x^2}\right)dx$$

$$\Rightarrow f(\alpha)=\frac{1}{\ln 10}\int_1^{\frac{1}{\alpha}}\frac{\ln x}{x(x+1)}dx$$

$$\Rightarrow f(e^{-3})=\frac{1}{\ln 10}\int_1^{e^3}\frac{\ln t}{t(t+1)}dt \quad ....(ii)$$

Add equation (i) and (ii)
$f(e^3)+f(e^{-3})$

$$=\left(\frac{1}{\ln 10}\right)\int_1^{e^3}\frac{\ln t}{(1+t)}\left[1+\frac{1}{t}\right]dt$$

$$=\left(\frac{1}{\ln 10}\right)\int_1^3\frac{\ln t}{t}dt$$

Let $\ln t = r$; $\frac{dt}{t}=dr$

$$=\frac{1}{\ln 10}\int_0^3 r\,dr=\left(\frac{1}{\ln 10}\right)\left(\frac{r^2}{2}\right)\Bigg|_0^3=\left(\frac{1}{\log 10}\right)\frac{9}{2}=\frac{9}{2\log_e 10}$$

98. **(b)** Now we are given that the integral

$$I=\int_0^{\frac{\pi}{2}}\frac{dx}{3+2\sin x+\cos x}=\int_0^{\frac{\pi}{2}}\frac{\sec^2\frac{x}{2}.dx}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}$$

Put $\tan\frac{x}{2}=t\Rightarrow\frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx=dt$

$$\text{Now } I=\int_0^1\frac{dt}{(t+1)^2+1}=\tan^{-1}(x+1)\Big|_0^1=\tan^{-1}2-\frac{\pi}{4}$$

99. **(a)** Given integral is

$$I=\int_0^1[2x-|3x^2-3x-2x+2|+1]dx$$

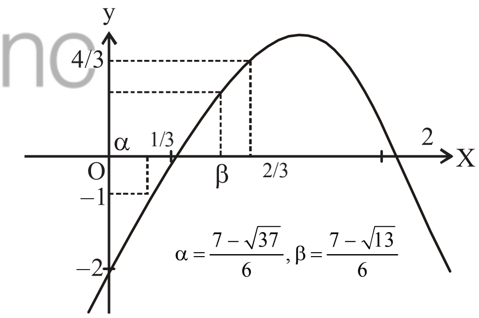
$$I=\int_0^1[2x-|(3x-2)(x-1)|]dx+\int_0^1 1dx$$

when $x<\frac{2}{3}$, then $(3x^2-5x-12)<0$ in second integral.

$$I=\int_0^{\frac{2}{3}}\left[\left(2x-\left(3x^2-5x+2\right)\right)\right]dx+\int_{\frac{2}{3}}^1\left(2x+\left(3x^2-5x+2\right)\right)dx+1$$

$$I=\int_0^{\frac{2}{3}}\left[-3x^2+7x-2\right]dx+\int_{\frac{2}{3}}^1\left(3x^2-3x+2\right)dx+1$$

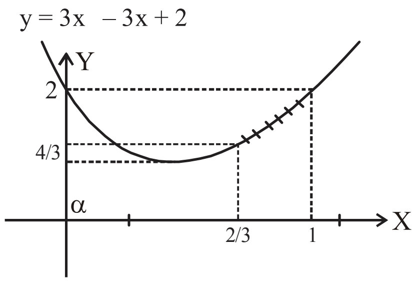
$y=-3x^2+7x-2$



$$\int_0^{\alpha}(-2)dx+\int_{\alpha}^{\frac{1}{3}}(-1)dx+\int_{\frac{1}{3}}^{\beta}0\,dx+\int_{\beta}^{\frac{2}{3}}1.dx$$

$$=-2\alpha-\left(\frac{1}{3}-\alpha\right)+\frac{2}{3}-\beta=-\alpha-\beta+\frac{1}{3}$$

$y=3x^2-3x+2$



When $x \in \left(\frac{2}{3}, 1\right)$ $3x^2 - 3x + 2 \in \left(\frac{4}{3}, 2\right)$ $[3x^2 - 3x + 2] = 1$

Therefore, $\int_{\frac{2}{3}}^{1} \left[3x^2 - 3x + 2\right] dx = 1\left(1 - \frac{2}{3}\right) = \frac{1}{3}$

Hence $I = \left(\frac{1}{3} - (\alpha + \beta)\right) + \left(\frac{1}{3}\right) + 1$

$$= \frac{5}{3} - \left(\frac{7 - \sqrt{37}}{6} + \frac{7 - \sqrt{13}}{6}\right)$$

$$= \frac{-2}{3} + \frac{\sqrt{37} + \sqrt{13}}{6} = \frac{\sqrt{37} + \sqrt{13} - 4}{6}$$

100. (a) $f(x) = e^x \cdot \int_0^x \frac{f'(t)}{e^t} dt$

$$f'(x) = e^x \cdot \int_0^x \frac{f'(t)}{e^t} dt + e^x \cdot \frac{f'(x)}{e^x}$$

$$-\left[(2x - 1) \cdot e^x + \left(x^2 - x + 1\right) \cdot e^x\right]$$

Now $\int_0^x \frac{f'(t)}{e^t} dt = x^2 + x$

Differentiate on both sides w.r.t. 'x'.

$\Rightarrow \frac{f'(x)}{e^x} = 2x + 1$

$\Rightarrow f'(x) = (2x + 1) \cdot e^x$

Now $f'(x) = 0 \Rightarrow x = -\frac{1}{2}$

$f(x) = (2x + 1) \cdot e^x - 2e^x + C$

$\therefore f(0) = -1$

$-1 = 1 - 2 + C$

$\Rightarrow C = 0$

Now $f(x) = e^x (2x - 1)$

$\Rightarrow f\left(-\frac{1}{2}\right) = \frac{-2}{\sqrt{e}}$

101. (a) Given that $I_n(x) = \int_0^x \frac{dt}{(t^2 + 5)^n}$

Applying integral by parts

$$I_n(x) = \left[\frac{t}{(t^2 + 5)^n}\right]_0^x - \int_0^x -n(t^2 + 5)^{-n-1} \cdot 2t^2$$

$$I_n(x) = \frac{x}{(x^2 + 5)^n} + 2n \int_0^x \frac{t^2}{(t^2 + 5)^{n+1}} dt$$

$$I_n(x) = \frac{x}{(x^2 + 5)^n} + 2n \int_0^x \frac{(t^2 + 5) - 5}{(t^2 + 5)^{n+1}} dt$$

$$I_n(x) = \frac{x}{(x^2 + 5)^n} + 2n\, I_n(x) - 10n\, I_{n+1}(x)$$

$$10n\, I_{n+1}(x) + (1 - 2n) I_n(x) = \frac{x}{(x^2 + 5)^n} \quad \text{...(i)}$$

$$I_5 \frac{d(I_5)}{dx} = \frac{1}{(x^2 + 5)^n}$$

Put n = 5 in (i) we get

$50\, I_6 - 9\, I_5 = x\, I'_5$

102. (c) Consider

Let $f(x) = 8\sin x - \sin 2x$

$\Rightarrow \quad f'(x) = 8\cos x - 2\cos 2x$

$\Rightarrow \quad f''(x) = -8\sin x + 4\sin 2x$

$= -8\sin x\,(1 - \cos x)$

$\therefore \quad f''(x) < 0 \; x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$\therefore \quad f'(x)$ is $\downarrow$ function

$$f'\left(\frac{\pi}{3}\right) < f'(x) < d'\left(\frac{\pi}{4}\right)$$

$$5 < f'(x) < \frac{8}{\sqrt{2}}$$

$$5 < f'(x) < 4\sqrt{2}$$

$$5x < f(x) < 4\sqrt{2}x$$

or $\quad 5 < \frac{f(x)}{x} < 4\sqrt{2}$

$$\int_{\pi/4}^{\pi/3} 5 < \int \frac{f(x)}{x} < \int_{\pi/4}^{\pi/3} 4\sqrt{2}$$

$$\int_{\pi/4}^{\pi/3} 5 < \int \frac{8\sin x - \sin 2x}{x} < \int_{\pi/4}^{\pi/3} 4\sqrt{2}$$

$$5[x]_{\pi/4}^{\pi/3} < \int \frac{(x)}{x} < \left[4\sqrt{2}\right]_{\pi/4}^{\pi/3}$$

$$\frac{5\pi}{12} < I < \frac{\sqrt{2}\pi}{3}$$

103. (b) $\lim_{x \to -1^+} a\sin\left(\pi\frac{[x]}{2}\right)+[2-x]=-a+2$

$\lim_{x \to -1^-} a\sin\left(\pi\frac{[x]}{2}\right)+[2-x]=0+3=0$

$\lim_{x \to -1} g(x)$ exist when $a=-1$ Now,

$$\int_0^4 g(x)dx=\int_0^1 g(x)dx+\int_1^2 g(x)dx+\int_2^3 g(x)dx+\int_3^4 g(x)dx$$

$$=\int_0^1(0+1)dx+\int_1^2(-1+0)dx+\int_2^3(0-1)dx+\int_3^4(1-2)dx$$

$= 1 - 1 - 1 - 1 = -2$

104. (a) We have,

$$I=\int_0^{\pi/2}\frac{\cos x}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx$$

$$=\int_0^{\pi/2}\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx=\int_0^{\pi/2}\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}dx$$

Let $\cos\frac{x}{2}+\sin\frac{x}{2}=t$. Then,

$$\frac{1}{2}\left(-\sin\frac{x}{2}+\cos\frac{x}{2}\right)dx=dt$$

$$\Rightarrow \left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)dx=2dt$$

Also, $x=0 \Rightarrow t=1$ and $x=\frac{\pi}{2} \Rightarrow t=\sqrt{2}$

$$\therefore\ I=\int_1^{\sqrt{2}}\frac{2dt}{t^2}=2\int_1^{\sqrt{2}}\frac{1}{t^2}dt$$

$$=2\left[-\frac{1}{t}\right]_1^{\sqrt{2}}=2\left[-\frac{1}{\sqrt{2}}+1\right]=\left(2-\sqrt{2}\right)$$

105. (a) $I=\int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx$

Put $e^{2x}=t \Rightarrow 2e^{2x}dx=dt$

When $x=\log\sqrt{\pi/2}$, $t=e^{2\log\sqrt{\pi/2}}=e^{\log\pi/2}=\frac{\pi}{2}$

When $x=\log\sqrt{\pi}$, $t=e^{2\log\sqrt{\pi}}=\pi$

$$\therefore I=\int_{\frac{\pi}{2}}^{\pi}\frac{1}{2}\sec^2\left(\frac{1}{3}t\right)dt=\frac{1}{2}\cdot\frac{1}{\frac{1}{3}}\left[\tan\frac{t}{3}.\right]_{\pi/2}^{\pi}$$

$$=\frac{3}{2}\left[\tan\frac{\pi}{3}-\tan\frac{\pi}{6}\right]=\frac{3}{2}\left[\sqrt{3}-\frac{1}{\sqrt{3}}\right]=\sqrt{3}$$

106. (a) We have, $I=\int_0^{\pi}\frac{1}{5+4\cos x}dx$

$$=\int_0^{\pi}\frac{1}{5+4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$=\int_0^{\pi}\frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)+4\left(1-\tan^2\frac{x}{2}\right)}dx$$

$$=\int_0^{\pi}\frac{1+\tan^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx=\int_0^{\pi}\frac{\sec^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx$$

Let $\tan\frac{x}{2}=t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx=dt$

Also, $x=0 \Rightarrow t=0$ and $x=\pi \Rightarrow t=\infty$

$$\therefore\quad I=\int_0^{\infty}\frac{dt}{9+t^2}$$

$$\therefore\quad I=2\int_0^{\infty}\frac{dt}{3^2+t^2}$$

$$\therefore\quad I=\frac{2}{3}\left[\tan^{-1}\frac{t}{3}\right]_0^{\infty}=\frac{2}{3}\left[\tan^{-1}\infty-\tan^{-1}0\right]$$

$$=\frac{2}{3}\left(\frac{\pi}{2}-0\right)=\frac{\pi}{3}$$

107. (c) Given $F(x)=f(x)+f\left(\frac{1}{x}\right)$, where

$$f(x)=\int_1^x\frac{\log t}{1+t}dt$$

$$\therefore F(e)=f(e)+f\left(\frac{1}{e}\right)$$

$$\Rightarrow F(e)=\int_1^e\frac{\log t}{1+t}dt+\int_1^{1/e}\frac{\log t}{1+t}dt \qquad \text{...(i)}$$

Now for solving, $I=\int_1^{1/e}\frac{\log t}{1+t}dt$

$\therefore$ Put $\frac{1}{t}=z \Rightarrow -\frac{1}{t^2}dt=dz \Rightarrow dt=-\frac{dz}{z^2}$

and limit for $t=1 \Rightarrow z=1$ and for $t=1/e \Rightarrow z=e$

$$\therefore\quad I=\int_1^e\frac{\log\left(\frac{1}{z}\right)}{1+\frac{1}{z}}\left(-\frac{dz}{z^2}\right)=\int_1^e\frac{(\log 1-\log z).z}{z+1}\left(-\frac{dz}{z^2}\right)$$

$$= \int_1^e -\frac{\log z}{(z+1)}\left(-\frac{dz}{z}\right) = \int_1^e \frac{\log z}{z(z+1)}dz$$

$$\therefore\ I = \int_1^e \frac{\log t}{t(t+1)}dt$$

Equation (i) becomes: $F(e) = \int_1^e \frac{\log t}{1+t}dt + \int_1^e \frac{\log t}{t(1+t)}dt$

$$= \int_1^e \frac{t.\log t + \log t}{t(1+t)}dt = \int_1^e \frac{(\log t)(t+1)}{t(1+t)}dt$$

$$\Rightarrow\ F(e) = \int_1^e \frac{\log t}{t}dt$$

Let $\log t = x$ $\quad\therefore\ \frac{1}{t}dt = dx$

[for limit $t = 1, x = 0$ and $t = e, x = \log e = 1$]

$$\therefore\ F(e) = \int_0^1 x\,dx \quad F(e) = \left[\frac{x^2}{2}\right]_0^1 \Rightarrow F(e) = \frac{1}{2}$$

108. **(a)**

109. **(a)** Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

Also as $x = 0, \theta = 0$ and $x = 1, \theta = \frac{\pi}{4}$

Therefore, $\int_0^1 \tan^{-1} x dx = \int_0^{\pi/4} \theta \sec^2\theta d\theta$

$$= \frac{\pi}{4} - \log\sqrt{2} = \frac{\pi}{4} - \frac{1}{2}\log 2.$$

110. **(d)** Given function is

$$f(x) = \frac{|x^3+x|}{\left(e^{x|x|}+1\right)}dx$$

Apply property

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx. \& \int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$$

$$I = \int_{-2}^{2} \frac{|x^3+x|}{\left(e^{x|x|}+1\right)}dx \quad ...(i)$$

$$I = \int_{-2}^{2} \frac{|x^3+x|}{\left(e^{-x|x|}+1\right)}dx \quad ...(ii)$$

Add (i) & (ii)

$$2I = 2\int_0^2 \left(\frac{|x^3+x|}{\left(e^{x|x|}+1\right)} + \frac{|-x^3-x|}{(e^{-x|-x|}+1)}\right)dx.$$

$$I = \int_0^2 \left(\frac{|x^3+x|}{(e^{x|x|}+1)} + \frac{|-x^3-x|}{(e^{-x|-x|}+1)}\right)dx$$

$$I = \int_0^2 \left(\frac{|x^3+x|}{(e^{x|x|}+1)} + \frac{|x^3+x|}{(e^{-x|x|}+1)}\right)dx$$

$$I = \int_0^2 \left(\frac{x^3+x}{(e^{x^2}+1)} + \frac{x^3+x}{(e^{-x^2}+1)}\right)dx$$

$$I = \int_0^2 \left(\frac{x^3+x}{1+e^{x^2}} + \frac{e^{x^2}(x^3+x)}{1+e^{x^2}}\right)dx = \int_0^2 (x^3+x)dx$$

$$= \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^2 = 4 + 2 = 6.$$

111. **(b)** Given slope of the tangent to a curve $y = f(x)$ at (x, y) be given by

$$\frac{dy}{dx} = 2\tan x\cos x - 2\tan x \cdot y$$

$$\Rightarrow \frac{dy}{dx} + (2\tan x)\, y = 2\sin x$$

Now integrating factor $= e^{\int 2\tan x dx} = \frac{1}{\cos^2 x}$

$\therefore$ solution is given by

$$y\left(\frac{1}{\cos^2 x}\right) = \int \frac{2\sin x}{\cos^2 x}dx$$

$$\Rightarrow\ y\sec^2 x = \frac{2}{\cos x} + C$$

$$\Rightarrow\ y = 2\cos x + C\cos^2 x \quad ...(i)$$

$\because$ curve passes through $\left(\frac{\pi}{4}, 0\right)$

$\therefore$ from eqn. (i)

$$0 = \sqrt{2} + \frac{C}{2} \Rightarrow C = -2\sqrt{2}$$

Now $f(x) = 2\cos x - 2\sqrt{2}\cos^2 x$ is required curve

$$\therefore \int_0^{\pi/2} ydx = 2\int_0^{\pi/2} \cos x dx - 2\sqrt{2}\int_0^{\pi/2} \cos^2 x dx$$

$$= \left[2\sin x\right]_0^{\pi/2} - 2\sqrt{2}\left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_0^{\pi/2}$$

$= 2 - \frac{\pi}{\sqrt{2}}$ is required answer

112. **(c)** We are given that

$f(x) + f(x+k) = n\ \forall x \in R$ ...(i)

or $\Rightarrow f(x+k) + f(x+2k) = n$ ...(ii)

$\Rightarrow f(x+2k) + f(x+3k) = n$

Now from (i)

$f(x+k) = n - f(x)$

Now from eq (ii)

$n - f(x) + f(x+2k) = n$

$\Rightarrow\ f(x) = f(x+2k)\ \forall x \in R$

$\Rightarrow\ f(x)$ is periodic with period $2k$

$$I_1 = \int_0^{4nk} f(x)dx = 2n\int_0^{2k} f(x)dx$$

$$I_2 = \int_{-k}^{3k} f(x)dx = 2\int_0^{2k} f(x)dx$$

$\because$ we are given that

$f(x) + f(x + k) = n$

$$\Rightarrow \int_0^k f(x)dx + \int_0^k f(x+k)dx = nk$$

$$\Rightarrow \int_0^k f(x)dx + \int_k^{2k} f(x)dx = nk$$

$$\Rightarrow \int_0^{2k} f(x)dx = nk$$

$\Rightarrow$ $I_1 = 2n^2k$, $I_2 = 2nk$

$\Rightarrow$ $I_1 + nI_2 = 4n^2k$

113. (b) We are given that

$$g(x) = \int_x^{\pi/4} (f'(t)\sec t + \tan t \sec t f(t))\, dt$$

$$\Rightarrow g(x) = \int_x^{\pi/4} d(f(t)\cdot \sec t) = f(t)\sec t\Big|_x^{\pi/4}$$

$$\Rightarrow g(x) = f\left(\frac{\pi}{4}\right)\sec\frac{\pi}{4} - f(x)\cdot \sec x$$

$$\Rightarrow g(x) = 2 - f(x)\sec x = 2 - \left(\frac{f(x)}{\cos x}\right)$$

Now $\lim\limits_{x\to\left(\frac{\pi}{2}\right)^-} g(x) = 2 - \lim\limits_{x\to\left(\frac{\pi}{2}\right)^-}\left(\frac{f(x)}{\cos x}\right)\left[\frac{0}{0}\text{form}\right]$

By L'Hopital Rule

$$= 2 - \lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{f'(x)}{(-\sin x)}$$

$$= 2 + \frac{f'\left(\frac{\pi}{2}\right)}{\sin\frac{\pi}{2}} = 2 + \frac{1}{1} = 3$$

114. (c) Given integral is $\int_0^1 [-8x^2 + 6x - 1]dx$, Take $f(x) = 0$,

Then, $x = \frac{1}{4}, \frac{1}{2}$

Put $x = 1$ in $f(x)$ then $y = -3$.

Now, find the values of x for the values of $y = -1$ & -2

$$= \int_0^{1/4} -1dx + \int_{1/4}^{1/2} 0dx + \int_{1/2}^{3/4} -1dx + \int_{3/4}^{\frac{3+\sqrt{17}}{8}} -2dx + \int_{\frac{3+\sqrt{17}}{8}}^{1} -3dx$$

$$= -[x]_0^{1/4} + 0 - [x]_{1/2}^{3/4} + -2[x]_{3/4}^{\frac{3+\sqrt{17}}{8}} - 3[x]_{\frac{3+\sqrt{17}}{8}}^{1}$$

Apply the limit,

$$= -\left(\frac{1}{4} - 0\right) - \left(\frac{3}{4} - \frac{1}{2}\right) - 2\left(\frac{3+\sqrt{17}}{8} - \frac{3}{4}\right) - 3\left(1 - \frac{3+\sqrt{17}}{8}\right)$$

$$= -\frac{1}{4} - \frac{1}{4} + \frac{-6-2\sqrt{17}}{8} + \frac{3}{2} - 3 + \frac{9+3\sqrt{17}}{8}$$

$$= \frac{\sqrt{17}-13}{8}$$

115. (c) $\int_0^2 \sqrt{2x}\, dx - \int_0^2 \sqrt{1-(x-1)^2}\, dx$

$$= \int_0^2 \left(1 - \frac{y^2}{2}\right)dy - \int_0^1 \sqrt{1-y^2}\, dy + 1 + \mathrm{I}$$

$$= \frac{8}{3} - 2\int_0^1 \sqrt{1-y^2}\,dy = \frac{2}{3} + 1 - \int_0^1 \sqrt{1-y^2}\,dy + I$$

$$\Rightarrow I = 1 - \int_0^1 \sqrt{1-y^2}\, dy$$

$$\Rightarrow I = \int_0^1 \left(1 - \sqrt{1-y^2}\right)dy$$

EXERCISE - 2

1. (a) Suppose

$$I = \int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta}\, dx$$

$$= \int \frac{\left(2\cos^2 x - 1\right) - \left(2\cos^2\theta - 1\right)}{\cos x - \cos\theta}\, dx$$

$$= 2\int \frac{(\cos x + \cos\theta)(\cos x - \cos\theta)}{(\cos x - \cos\theta)}\, dx$$

$$= 2\int (\cos x + \cos\theta)\, dx$$

$$= 2(\sin x + x\cos\theta) + C$$

2. (c) Suppose,

$$I = \int \frac{dx}{\sin(x-a)\sin(x-b)}$$

$$= \frac{1}{\sin(b-a)}\int \frac{\sin(b-a)}{\sin(x-a)\sin(x-b)}\, dx$$

$$= \frac{1}{\sin(b-a)}\int \frac{\sin[(x-a)-(x-b)]}{\sin(x-a)\sin(x-b)}\, dx$$

$$= \frac{1}{\sin(b-a)}\int \frac{\sin(x-a)\cos(x-b) - \cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)}\, dx$$

$$= \frac{1}{\sin(b-a)}\int\left(\cot(x-b)-\cot(x-a)\right)dx$$

$$= \frac{1}{\sin(b-a)}\left(\log|\sin(x-b)|-\log|\sin n(x-a)|\right)+C$$

$$= \operatorname{cosec}(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+C$$

3. **(a)** Suppose, $I = \int 1 . \tan^{-1}\sqrt{x}\,dx$

$$= x\tan^{-1}\sqrt{x} - \frac{1}{2}\int\frac{2\sqrt{x}}{(1+x)}dx$$

$$= x\tan^{-1}\sqrt{x} - \int\frac{t^2}{1+t^2}dt \quad \left[\text{Let } x = t^2 \Rightarrow dx = 2t\,dt\right]$$

$$= x\tan^{-1}\sqrt{x} - \int\left(1-\frac{1}{1+t^2}\right)dt$$

$$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}t + C$$

$$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$$

4. **(d)** We have, $\mathrm{I} = \int\frac{\mathrm{x}^9}{\left(4\mathrm{x}^2+1\right)^6}\mathrm{dx}$

$$= \int\frac{\mathrm{x}^9\,\mathrm{dx}}{\mathrm{x}^{12}\left(4+\frac{1}{\mathrm{x}^2}\right)^6} = \int\frac{\mathrm{dx}}{\mathrm{x}^3\left(4+\frac{1}{\mathrm{x}^2}\right)^6}$$

Put $4+\frac{1}{\mathrm{x}^2} = \mathrm{t} \Rightarrow \frac{-2}{\mathrm{x}^3}\mathrm{dx} = \mathrm{dt}$

$$\mathrm{I} = -\frac{1}{2}\int\frac{\mathrm{dt}}{\mathrm{t}^6} = \frac{-1}{2}\int\mathrm{t}^{-6}\mathrm{dt}$$

$$= -\frac{1}{2}\frac{\mathrm{t}^{-5}}{-5}+\mathrm{C} = \frac{1}{10}\left(4+\frac{1}{\mathrm{x}^2}\right)^{-5}+\mathrm{C}$$

5. **(c)** Since, $\int\frac{dx}{(x+2)\left(x^2+1\right)}$

$$= a\log|1+x^2| + b\tan^{-1}x + \frac{1}{5}\log\ |x+2| + C$$

Suppose

$$I = \int\frac{dx}{(x+2)\left(x^2+1\right)}$$

Let, $\frac{1}{(x+2)\left(x^2+1\right)} = \frac{A}{x+2}+\frac{Bx+C}{x^2+1}$

$\therefore$ $1 = \mathrm{A}(\mathrm{x}^2+1) + (\mathrm{Bx}+\mathrm{C})(\mathrm{x}+2)$

$\Rightarrow$ $1 = (\mathrm{A}+\mathrm{B})\mathrm{x}^2 + (2\mathrm{B}+\mathrm{C})\mathrm{x} + \mathrm{A} + 2\mathrm{C}$

So, $\mathrm{A}+\mathrm{B} = 0,\ \mathrm{A}+2\mathrm{C} = 1,\ 2\mathrm{B}+\mathrm{C} = 0$

$\Rightarrow$ $A = \frac{1}{5}, B = -\frac{1}{5}$ and $C = \frac{2}{5}$

Therefore, $\int\frac{dx}{(x+2)\left(x^2+1\right)} = \frac{1}{5}\int\frac{1}{x+2}dx + \int\frac{-\frac{1}{5}x+\frac{2}{5}}{x^2+1}dx$

$$= \frac{1}{5}\int\frac{1}{x+2}dx - \frac{1}{5}\int\frac{x}{1+x^2}dx + \frac{1}{5}\int\frac{2}{1+x^2}dx$$

$$= \frac{1}{5}\log|x+2| - \frac{1}{10}\log|1+x^2| + \frac{2}{5}\tan^{-1}x + C$$

Hence, $a = \frac{-1}{10}, b = \frac{2}{5}$

6. **(d)** Suppose, $I = \int\frac{x^3}{x+1}dx$

$$= \int\left[\left(x^2-x+1\right)-\frac{1}{(x+1)}\right]dx$$

$$= \frac{x^3}{3}-\frac{x^2}{2}+x-\log|x+1|+C$$

7. **(d)** Suppose, $I = \int\frac{x+\sin x}{1+\cos x}dx$

$$I = \int\frac{x}{1+\cos x}dx + \int\frac{\sin nx}{1+\cos x}dx$$

$$= \int\frac{x}{2\cos^2(x/2)}dx + \int\frac{2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)}dx$$

$$= \frac{1}{2}\int x\sec^2(x/2)\,dx + \int\tan(x/2)\,dx$$

$$= \frac{1}{2}\left[2x\tan(x/2) - \int 2\tan\frac{x}{2}dx\right] + \int\tan\frac{x}{2}dx$$

$$= x.\tan\frac{x}{2}+C$$

8. **(d)** Suppose, $I = \int\frac{x^3}{\sqrt{1+x^2}}dx$

$$= a\left(1+x^2\right)^{3/2} + b\sqrt{1+x^2} + C$$

Now $I = \int\frac{x^3}{\sqrt{1+x^2}}dx = \int\frac{x^2 . x}{\sqrt{1+x^2}}dx$

Let $1+x^2 = t^2$

So $2x\,dx = 2t\,dt$

Therefore,

$$I = \int\frac{t\left(t^2-1\right)}{t}dt = \frac{t^3}{3}-t+C$$

$$= \frac{1}{3}\left(1+x^2\right)^{3/2} - \sqrt{1+x^2} + C$$

Hence, $a = \frac{1}{3}$ and $b = -1$

9. **(a)** Suppose,

$$I=\int_{-\pi/4}^{\pi/4}\frac{dx}{1+\cos 2x}=\int_{-\pi/4}^{\pi/4}\frac{dx}{2\cos^2 x}$$

$$=\frac{1}{2}\int_{-\pi/4}^{\pi/4}\sec^2 x\,dx=\int_0^{\pi/4}\sec^2 x\,dx$$

$$=[\tan x]_0^{\pi/4}=1$$

$$\left[\int_{-a}^{a}f(x)dx=2\int_0^a f(x)dx \text{ if } f(-x)=f(x)\right]$$

10. **(d)** Suppose, $I=\int_0^{\pi/2}\sqrt{1-\sin 2x}\,dx$

$$=\int_0^{\pi/4}\sqrt{(\cos x-\sin x)^2}\,dx+\int_{\pi/4}^{\pi/2}\sqrt{(\sin x-\cos x)^2}\,dx$$

$$\left[\begin{array}{l}\because \cos x\ge\sin x \text{ for } x\in\left[0.\frac{\pi}{4}\right]\\ \text{and } \sin x\ge\cos x \text{ for } x\in\left[\frac{\pi}{4}.\frac{\pi}{2}\right]\end{array}\right]$$

$$=[\sin x+\cos x]_0^{\pi/4}+[-\cos x-\sin x]_{\pi/4}^{\pi/2}$$

$$=\left(\frac{1}{\sqrt2}+\frac{1}{\sqrt2}\right)-0+1+\left(-0-1+\frac{1}{\sqrt2}+\frac{1}{\sqrt2}\right)$$

$$=2\left(\sqrt2-1\right)$$

11. **(b)** Suppose, $I=\int_0^{\pi/2}\cos x\, e^{\sin x}dx$

Let $\sin x=t\Rightarrow\cos x\,dx=dt$

$x\to0\Rightarrow t\to0$

and $x\to\frac{\pi}{2}\Rightarrow t\to1$

So $I=\int_0^1 e^t dt=\left[e^t\right]_0^1=e-1$

12. **(a)** Suppose,

$$I=\int\frac{x+3}{(x+4)^2}e^x dx=\int\frac{(x+4)1}{(x+4)^2}e^x dx$$

$$=\int\frac{e^x}{(x+4)}-\int\frac{e^x}{(x+4)^2}dx$$

$$=\int e^x\left(\frac{1}{(x+4)}-\frac{1}{(x+4)^2}\right)dx$$

$$=e^x\frac{1}{(x+4)}+C\quad\left[\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C\right]$$

13. **(d)** $\int\frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx=\int\frac{\frac{2}{x^3}+\frac{5}{x^6}\,dx}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}$

Dividing by x^{15} in numerator and denominator

Substitute $1+\frac{1}{x^2}+\frac{1}{x^5}=t$

$$\Rightarrow\left(\frac{-2}{x^3}-\frac{5}{x^6}\right)dx=dt\quad\Rightarrow\left(\frac{2}{x^3}+\frac{5}{x^6}\right)dx=-dt$$

This gives,

$$\int\frac{\frac{2}{x^3}+\frac{5}{x^6}dx}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}=\int\frac{-dt}{t^3}=\frac{1}{2t^2}+C$$

$$=\frac{1}{2\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2}+C=\frac{x^{10}}{2(x^5+x^3+1)^2}+C$$

14. **(c)** $I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{dx}{1+\cos x}$ ...(i)

$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{dx}{1-\cos x}$ ...(ii)

Adding (i) and (ii)

$$2I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{2}{\sin^2 x}dx\Rightarrow I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\operatorname{cosec}^2 x\,dx$$

$$I=-(\cot x)_{\pi/4}^{3\pi/4}=-\left[\cot\frac{3\pi}{4}-\cot\frac{\pi}{4}\right]=2$$

15. **(c)** $I_n=\int\tan^n x\,dx, n>1$ Let $I=I_4+I_6$

$$=\int(\tan^4 x+\tan^6 x)dx=\int\tan^4 x\sec^2 x\,dx$$

Let $\tan x=t\Rightarrow\sec^2 x\,dx=dt$

$$\therefore I=\int t^4 dt=\frac{t^5}{5}+C=\frac{1}{5}\tan^5 x+C$$

$\Rightarrow$ On comparing, we have $a=\frac{1}{5}$, $b=0$

16. **(a)** Let $I=\int\frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$

$$=\int\frac{\sin^2 x\cos^2 x}{[(\sin^2 x+\cos^2 x)(\sin^3 x+\cos^3 x)]^2}dx$$

$$=\int\frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx=\int\frac{\tan^2 x\cdot\sec^2 x}{(1+\tan^3 x)^2}dx$$

Now, put $(1 + \tan^3 x) = t$

$\Rightarrow \quad 3\tan^2 x \sec^2 x \, dx = dt$

$$\therefore \quad I = \frac{1}{3}\int \frac{dt}{t^2} = -\frac{1}{3t} + C = \frac{-1}{3(1+\tan^3 x)} + C$$

17. **(c)** Let, $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx$ ...(i)

Using, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get :

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{-x}} dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get;

$$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x \, dx \Rightarrow 2I = 2 \cdot \int_0^{\pi/2} \sin^2 x \, dx$$

$$\Rightarrow \quad 2I = 2 \times \frac{\pi}{4} \Rightarrow I = \frac{\pi}{4}$$

18. **(b)** $I = \int_0^{\pi} |\cos x|^3 \, dx$

$$= 2\int_0^{\pi/2} \cos^3 x \, dx$$

$$= \frac{2}{4}\int_0^{\pi/2} (3\cos x + \cos 3x) dx$$

$[\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta]$

$$= \frac{1}{2}\left[3\sin x + \frac{\sin 3x}{3}\right]_0^{\pi/2} = \frac{1}{2}\left(3 - \frac{1}{3}\right) = \frac{4}{3}$$

19. **(c, d)** Consider the given integral

$$I = \int x\sqrt{\frac{2\sin(x^2-1) - 2\sin(x^2-1)\cos(x^2-1)}{2\sin(x^2-1) + 2\sin(x^2-1)\cos(x^2-1)}} dx$$

$(\therefore \sin 2\theta = 2\sin\theta\cos\theta)$

$$\Rightarrow \quad I = \int x\sqrt{\frac{1-\cos(x^2-1)}{1+\cos(x^2-1)}} dx$$

$$\Rightarrow \quad I = \int x\left|\tan\left(\frac{x^2-1}{2}\right)\right| dx,$$

Now let $\frac{x^2-1}{2} = t \Rightarrow \frac{2x}{2}dx = dt$

$$\therefore \quad I = \int |\tan(t)| \, dt = \ln|\sec t| + C$$

or $I = \ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c = \frac{1}{2}\ln\left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$

20. **(a)** $I = \int \sec^{\frac{2}{3}} x . \operatorname{cosec}^{\frac{4}{3}} dx$

$$I = \int \frac{\sec^2 x dx}{\tan^{\frac{4}{3}} x}$$

Put $\tan x = z$

$\Rightarrow \sec^2 x \, dx = dz$

$$\Rightarrow I = \int z^{-\frac{4}{3}} \cdot dz = \frac{z^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)} + C \Rightarrow I = -3(\tan x)^{\frac{-1}{3}} + C$$

21. **(b)** Let $I = \int_0^{\pi/2} \frac{\sin^3 x dx}{\sin x + \cos x}$ ...(i)

Use the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\cos^3 x \, dx}{\sin x + \cos x} \quad \text{...(ii)}$$

Adding equations (i) and (ii), we get

$$2I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sin(2x)\right) dx$$

$$\Rightarrow I = \frac{1}{2}\left[x + \frac{1}{4}\cos 2x\right]_0^{\pi/2} \Rightarrow I = \frac{\pi - 1}{4}$$

22. **(d)** Let $I = \int \frac{\cos x \, dx}{\sin^3 x(1+\sin^6 x)^{2/3}}$

$= f(x)\,(1 + \sin^6 x)^{1/\lambda} + c$ ...(i)

If $\sin x = t$

then, $\cos x \, dx = dt$

$$I = \int \frac{dt}{t^3(1+t^6)^{\frac{2}{3}}} = \int \frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$$

Put $1 + \frac{1}{t^6} = r^3 \Rightarrow \frac{dt}{t^7} = \frac{-1}{2}r^2 dr$

$$-\frac{1}{2}\int \frac{r^2 dr}{r^2} = -\frac{1}{2}r + c$$

$$= -\frac{1}{2}\left(\frac{\sin^6 x + 1}{\sin^6 x}\right)^{\frac{1}{3}} + c$$

$$= -\frac{1}{2\sin^2 x}(1+\sin^6 x)^{\frac{1}{3}} + c$$

$f(x) = -\frac{1}{2}\operatorname{cosec}^2 x$ and $\lambda = 3$ [from eqn. (i)]

$$\therefore \quad \lambda f\left(\frac{\pi}{3}\right) = -2$$

23. **(3)** $I=\int \frac{\frac{\sin x}{\cos^3 x}}{1+\tan^3 x}dx$

$=\int \frac{\tan x.\sec^2 x}{(\tan x+1)(1+\tan^2 x-\tan x)}dx$

Let tan x = $t \Rightarrow \sec^3x.dx = dt$

$=\int \frac{t}{(t+1)(t^2-t+1)}dt$

$\text{Let} \frac{t}{(t+1)(t^2-t+1)}=\frac{A}{t+1}+\frac{B(2t-1)}{t^2-t+1}+\frac{C}{t^2-t+1}$

$\Rightarrow A(t^2-t+1)+B(2t-1)(t+1)+C(t+1)=t$

$\Rightarrow t^2(A+2B)+t(-A+B+C)+A-B+C=t$

$\therefore A + 2B = 0$ ... (i)
$-A + B + C = 1$ ... (ii)
$A - B + C = 0$ ... (iii)

$\Rightarrow C=\frac{1}{2} \Rightarrow A-B=-\frac{1}{2}$ ... (iv)

Solving equations (i) and (iv), we get

$B=\frac{1}{6}, A=-\frac{1}{3}$

$\therefore I=-\frac{1}{3}\int\frac{dt}{1+t}+\frac{1}{6}\int\frac{2t-1}{t^2-t+1}dt+\frac{1}{2}\int\frac{dt}{t^2-t+1}$

$=-\frac{1}{3}\log_e|(1+\tan x)|+\frac{1}{6}\log_e|\tan^2 x-\tan x+1|$

$+\frac{1}{2}.\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\left(\tan x-\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right)$

$=-\frac{1}{3}\log_e|(1+\tan x)|+\frac{1}{6}\log_e|\tan^2 x-\tan x+1|$

$+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$

$\alpha=-\frac{1}{3},\beta=\frac{1}{6},\gamma=\frac{1}{\sqrt{3}}$

$\text{So}, 18(\alpha+\beta+\gamma^2)=18\left(-\frac{1}{3}+\frac{1}{6}+\frac{1}{3}\right)=3$

24. **(b)** $\pi^2\left[\int_0^1 \sin\frac{\pi x}{2}dx+\int_1^2 \sin\frac{\pi x}{2}(x-1)dx\right]$

$=\pi^2\left[\left(-\frac{2}{\pi}\left(\cos\frac{\pi x}{2}\right)\right)_0^1+\left((x-1)\left(-\frac{2}{\pi}\cos\frac{\pi x}{2}\right)\right)_1^2-\int_1^2-\frac{2}{\pi}\cos\frac{\pi x}{2}dx\right]$

$=\pi^2\left[0+\frac{2}{\pi}+\frac{2}{\pi}+\frac{2}{\pi}.\frac{2}{\pi}\left(\sin\frac{\pi x}{2}\right)_1^2\right]$

$= 4\pi - 4 = 4(\pi - 1)$

25. **(c)** Given $a=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2n}{k^2+n^2}$

$a=\int_0^1\frac{2}{x^2+1}dx$

$a=2\tan^{-1}\Big|_0^1=2(\tan^{-1}(1)-\tan^{-1}(0))$

$a=2\times\frac{\pi}{4}=\frac{\pi}{2}$

Take $f(x)=\sqrt{\frac{1-\cos x}{1+\cos x}}=\left|\tan\frac{x}{2}\right|$

Diff. w.r.t. x

$f'(x)=\frac{1}{2}\sec^2\frac{x}{2}\Big|_{x=\frac{\pi}{4}}=\frac{1}{2}\sec^2\left(\frac{\pi}{8}\right)=\left|\frac{\sqrt{2}}{\sqrt{2}+1}\right.$

$f'\left(\frac{\pi}{4}\right)=\sqrt{2}f\left(\frac{\pi}{4}\right).$

26. **(24)** Given expression is

$n(2n+1)\int_0^1(1-x^n)^{2n}dx=1177\int_0^1(1-x^n)^{2n+1}dx$

$\frac{\int_0^1(1-x^n)^{2n}dx}{\int_0^1(1-x^n)^{2n+1}dx}=\frac{1177}{n(2n+1)}$ ...(i)

Let $I_1=\int_0^1(1-x^n)^{2n}dx,\ I_2=\int_0^1(1-x^n)^{2n+1}dx.$

$\Rightarrow\ I_2=(1-x^n)^{2n+1}.$

$x\Big|_0^1-\int_0^1(2x+1)(1-x^n)^{2n}(-nx^{n-1})x\,dx$

$I_2 = -n(2n+1)\{I_2 - I_1\}$
$(2n^2 + n + 1)I_2 = n(2n + 1)I_1$

$\frac{2n^2+n+1}{n(2n+1)}=\frac{1177}{n(2n+1)}$ {from (i)}

$\Rightarrow\ 2n^2 + n - 1176 = 0$
$2n^2 + 49n - 48n - 1176 = 0$
$(n - 24)(2n + 49) = 0 = 7\ \Rightarrow\ n = 24.$

EXERCISE - 3

1. **(a)** Let $x + 3 = A\frac{d(3-4x-x^2)}{dx}+B$

$= A(-4-2x) + B$

On comparing we get $A=-\frac{1}{2}$ and B = 1

$$\therefore\ \text{I} = \int -\frac{1}{2}(-4-2x)\sqrt{3-4x-x^2}\,dx + \int\sqrt{3-4x-x^2}\,dx$$

$$= -\frac{1}{2}\times\frac{2}{3}(3-4x-x^2)^{3/2} + \int\sqrt{\left(\sqrt{7}\right)^2-(x+2)^2}\ dx$$

$$= -\frac{1}{3}(3-4x-x^2)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + \text{C}$$

2. **(b)** We have

$$f(x) = \lim_{n\to\infty}\frac{1}{1-x}[(1-x)(1+x)(1+x^2)....(1+x^{2^n})]$$

$$= \lim_{n\to\infty}\frac{1}{1-x}[(1-x^2)(1+x^2)(1+x^4)....(1+x^{2^n})]$$

$$= \lim_{n\to\infty}\frac{1}{1-x}[(1-x^4)(1+x^4)....(1+x^{2^n})]$$

$$= \lim_{n\to\infty}\left(\frac{1-x^{2^{n+1}}}{1-x}\right) = \frac{1}{1-x}$$

Therefore $f(x) = \dfrac{1}{1-x}$

Let $\quad I = \displaystyle\int\frac{f(x)}{1-x}\log_e x\,dx = \int\frac{\log_e x}{(1-x)^2}dx$

$$= \left(\frac{1}{1-x}\right)\log_e x - \int\frac{1}{1-x}\cdot\frac{1}{x}dx$$

$$= \frac{\log_e x}{1-x} - \int\left(\frac{1}{1-x}+\frac{1}{x}\right)dx$$

$$= \frac{\log_e x}{1-x} - \log_e\left(\frac{x}{1-x}\right) + c$$

3. **(a)** We have,

$$\int\frac{x^2-1}{x\sqrt{(x^2+\alpha x+1)(x^2+\beta x+1)}}dx$$

$$= \int\frac{1-\frac{1}{x^2}}{\sqrt{\left(x+\frac{1}{x}+\alpha\right)\left(x+\frac{1}{x}+\beta\right)}}dx$$

$$= \int\frac{d\left(x+\frac{1}{x}\right)}{\sqrt{\left(x+\frac{1}{x}\right)^2+(\alpha+\beta)\left(x+\frac{1}{x}\right)+\alpha\beta}}$$

$= \displaystyle\int\frac{1}{\sqrt{t^2+(\alpha+\beta)t+\alpha\beta}}dt$, where $t = x+\dfrac{1}{x}$

$$= \int\frac{1}{\sqrt{\left(t+\frac{\alpha+\beta}{2}\right)^2-\left(\frac{\alpha-\beta}{2}\right)^2}}dt$$

$$= \log\left|\left(t+\frac{\alpha+\beta}{2}\right)+\sqrt{\left(t+\frac{\alpha+\beta}{2}\right)^2-\left(\frac{\alpha-\beta}{2}\right)^2}\right| + C_1$$

$$= \log\left|t+\frac{\alpha+\beta}{2}+\sqrt{t^2+(\alpha+\beta)t+\alpha\beta}\right| + C_1$$

$$= \log\{\sqrt{t+\alpha}+\sqrt{t+\beta}\}^2 + C_1 - \log 2$$

$$= 2\log\{\sqrt{t+\alpha}+\sqrt{t+\beta}\} + C_1 - \log 2$$

$$= 2\log\left\{\frac{\sqrt{x^2+\alpha x+1}+\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right\} + C$$

4. **(a)** Let $x^2 = t \Rightarrow 2xdx = \text{dt}$

$$\frac{1}{2}\int\frac{dt}{t^4+1} = \frac{1}{4}\int\frac{(t^2+1)-(t^2-1)}{t^4+1}dt$$

$$= \frac{1}{4}\int\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt - \frac{1}{2}\int\frac{x^5-x}{x^8+1}dx$$

$$= \frac{1}{4}\int\frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+2} - \frac{1}{2}\int\frac{x^5-x}{x^8+1}dx$$

$$= \frac{1}{4\sqrt{2}}\tan^{-1}\left(\frac{x^4-1}{\sqrt{2}x^2}\right) - \frac{1}{2}\int\left(\frac{x^5-x}{x^8+1}\right)dx + C$$

5. **(b)** We have $\displaystyle\int\frac{dx}{x^2(x^n+1)^{(n-1)/n}}$

$$= \int\frac{dx}{x^2.x^{n-1}\left(1+\frac{1}{x^n}\right)^{(n-1)/n}} = \int\frac{dx}{x^{n+1}(1+x^{-n})^{(n-1)/n}}$$

Put $1 + x^{-n} = t$

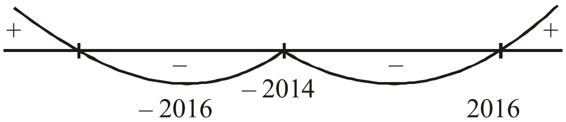
$\therefore\ -nx^{-n-1}dx = dt$ or $\dfrac{dx}{x^{n+1}} = -\dfrac{dt}{n}$ then

$$\int\frac{dx}{x^2(x^n+1)^{(n-1)/n}} = -\frac{1}{n}\int\frac{dt}{t^{(n-1)/n}} = -\frac{1}{n}\int t^{\frac{1}{\text{n}}-1}dt$$

$$= -\frac{1}{n}\frac{t^{1/n-1+1}}{1/n-1+1} + c = -t^{1/.n} + c = -(1+x^{-n})^{1/n} + c$$

6. **(b)** $g(x) = \displaystyle\int_0^x (t+2016)^7\,(t+2014)^{110}\,(t-2016)^5\ \text{dt}$

$g'(x) = (x + 2016)^7\,(x + 2014)^{110}\,(x - 2016)^5$



at $x = -2016$ is local max.
$x = 2016$ is local min.

$$I = \int_{-2016}^{2016} \frac{dx}{1+x^{2n+1}+\sqrt{1+x^{4n+2}}}$$

$$I = \int_{-2016}^{2016} \frac{1+x^{2n+1}\sqrt{1+x^{4n+2}}}{2x^{2n+1}}\,\mathrm{dx}$$

$$I = \int_{-2016}^{2016} \frac{1-\sqrt{1+x^{4n+2}}}{2x^{2n+1}}\,\mathrm{dx} + \frac{1}{2}\int_{-2016}^{2016} 1\,dx$$

$$= 0 + \frac{1}{2}[2016 + 2016] = 2016$$

7. **(c)** $$I = \int_{\alpha}^{\beta} \frac{e^{f\left(\frac{a(x-\alpha)(x-\beta)}{x-\alpha}\right)}}{e^{f\left(\frac{a(x-\alpha)(x-\beta)}{x-\alpha}\right)} e^{f\left(\frac{a(x-\alpha)(x-\beta)}{x-\beta}\right)}}dx$$

$$= \int_{\alpha}^{\beta} \frac{e^{f(a(x-\beta))}}{e^{f(a(x-\beta))+e^{f(a(x-\alpha))}}}dx \quad \text{... (i)}$$

$$= \int_{\alpha}^{\beta} \frac{e^{f(a(\alpha+\beta-x-\beta))}}{e^{f(a(\alpha+\beta-x-\beta))}+e^{f(a(\alpha+\beta-x-\alpha))}}dx$$

$$I = \int_{\alpha}^{\beta} \frac{e^{f(a(x-\alpha))}}{e^{f(a(x-\alpha)}+e^{f(a(x-\beta))}}dx \quad \text{... (ii)}$$

$$2I = \int_{\alpha}^{\beta} dx \Rightarrow I = \frac{|\alpha-\beta|}{2} = \frac{\sqrt{b^2-4ac}}{2|a|}$$

8. **(b)** Let $p(x) = (g(x))^2 - h(x)$
$p'(x) = f(x)(2g(x) - (f(x))^2)$
Let $q(x) = 2g(x) - (f(x))^2$
$q'(x) = 2f(x)(1 - f(x)) \geq 0 \ \forall x$

$\Rightarrow \quad q(x) \geq 0 \forall x$

$\Rightarrow \quad p'(x) \geq 0 \forall x$

$\Rightarrow \quad p(x) \geq 0 \forall x \in [0,1]$

9. **(a)** Let $q = p + d, r = p + 2d, s = p + 3d$

$$\therefore\ f(x) = \begin{vmatrix} p+\sin x & p+d+\sin x & -2d+\sin x \\ p+d+\sin x & p+2d+\sin x & -1+\sin x \\ p+2d+\sin x & p+3d+\sin x & 2d+\sin x \end{vmatrix}$$

Applying $R \to R_1 + R_3 - 2R_2$, we get

$$f(x) = \begin{vmatrix} 0 & 0 & 2 \\ p+d+\sin x & p+2d+\sin x & -1+\sin x \\ p+2d+\sin x & p+3d+\sin x & 2d+\sin x \end{vmatrix}$$

$$= 2[(p+d+\sin x)(p+3d+\sin x)$$
$$-(p+2d+\sin x)^2 = -2d^2$$

Given $\int_0^2 f(x)dx = -4$

$\Rightarrow \quad \int_0^2 (-2d^2)dx = -4 \Rightarrow d^2 = 1 \Rightarrow d = \pm 1$

10. **(b)** $g(x) = \int_0^x f(t)dt$

$\Rightarrow \quad g(2) = \int_0^2 f(t)dt = \int_0^1 f(t)dt + \int_1^2 f(t)dt$

Now, $\frac{1}{2} \leq f(t) \leq 1$ for $t \in [0,1]$

We get $\int_0^1 \frac{1}{2}dt \leq \int_0^1 f(t)dt \leq \int_0^1 1dt$
(applying line integral on inequality)

$\Rightarrow \quad \frac{1}{2} \leq \int_0^1 f(t)dt \leq 1$ ... (i)

Again, $0 \leq f(t) \leq \frac{1}{2}$ for $t \in [1,2]$

We get $\int_1^2 0dt \leq \int_1^2 f(t)dt \leq \int_1^2 \frac{1}{2}dt$
(applying line integral on inequality)

$\Rightarrow \quad 0 \leq \int_1^2 f(t)dt \leq \frac{1}{2}$ … (ii)

From (i) and (ii), we get $\frac{1}{2} \leq \int_0^1 f(t)dt + \int_1^2 f(t)dt \leq \frac{3}{2}$

or $\quad \frac{1}{2} \leq g(2) \leq \frac{3}{2}$

$\Rightarrow \quad 0 \leq g(2) \leq 2$ is the most appropriate solution.

11. **(a)** $$\lim_{x\to1} \frac{\int_4^{f(x)} 2t\,dt}{x-1} = \lim_{x\to1} \frac{(t^2)|_4^{f(x)}}{x-1}$$

$$= \lim_{x\to1} \frac{(f(x))^2-4^2}{x-1}$$

$$= \lim_{x\to1} \frac{2f(x).f'(x)}{1} \quad \text{(L-Hospital rule)}$$
$= 2f(1) . f'(1) = 2 \times 4 \times 2 = 16$

12. **(a)** We have $\lim_{x\to 0}\left(1+x+\frac{f(x)}{x}\right)^{1/x}=e^3$

$$\Rightarrow \lim_{x\to 0}\left(1+x\left(1+\frac{f(x)}{x^2}\right)\right)^{1/x}=e^3$$

$$\Rightarrow \lim_{x\to 0}\left[\left(1+\frac{x^2+f(x)}{x}\right)^{\frac{x}{x^2+f(x)}}\right]^{\frac{x^2+f(x)}{x}\frac{1}{x}}=e^3$$

$$\Rightarrow \frac{x^2+f(x)}{x^2}=3 \Rightarrow f(x)=2x^2$$

Therefore,

$$\int f(x)\log_e x\,dx=2\int x^2\log_e x\,dx$$

$$=2\left[\frac{x^3}{3}\log_e x-\int\frac{x^3}{3}\cdot\frac{1}{x}dx\right] \text{(By Parts)}$$

$$=\frac{2}{3}x^3\log_e x-\frac{2}{9}x^3+c=\frac{2}{3}x^3\left(\log_e x-\frac{1}{3}\right)+c$$

13. **(a)** First, we determine $f(x)$.

$f(0)=f(-0)=-f(0)\Rightarrow f(0)=0$

and $0=f(0)=f(1+(-1))=f(-1)+1\Rightarrow f(-1)=-1$

Therefore $f(0)=0$ and $f(-1)=-1$. Let $x\neq 0$ and -1. From (ii), we have

$$f\left(\frac{1}{x}+1\right)=f\left(\frac{1}{x}\right)+1=\frac{f(x)}{x^2}+1 \quad \text{[By (iii)]}$$

Therefore $f\left(\frac{1+x}{x}\right)=\frac{f(x)}{x^2}+1$ ...(i)

Again

$$f\left(\frac{x}{1+x}\right)=f\left(\frac{1}{\left(\frac{1+x}{x}\right)}\right)=\frac{f\left(\frac{1+x}{x}\right)}{\left(\frac{1+x}{x}\right)^2}$$

$$=\left(\frac{x}{1+x}\right)^2\left(\frac{f(x)}{x^2}+1\right) \quad \text{[By Eq. (i)]}$$

Therefore

$$(1+x)^2 f\left(\frac{x}{1+x}\right)=f(x)+x^2 \quad \text{...(ii)}$$

Also

$$(1+x)^2 f\left(\frac{x}{1+x}\right)=(1+x)^2 f\left(1-\frac{1}{x+1}\right)$$

$$=(1+x)^2 f\left(\frac{-1}{x+1}+1\right)=(1+x)^2\left[-f\left(\frac{1}{x+1}\right)+1\right]$$

$[\because f(-x)=-f(x)]$

$$=(1+x)^2\left[\frac{-f(x+1)}{(x+1)^2}+1\right] \quad \left(\because f\left(\frac{1}{x}\right)=\frac{f(x)}{x^2}\right)$$

$$=-f(x+1)+(x+1)^2=-(f(x)+1)+(x+1)^2$$

$$=-f(x)+x^2+2x \quad \text{...(iii)}$$

Therefore from eqs. (ii) and (iii), we have

$$f(x)+x^2=-f(x)+x^2+2x$$

$$\Rightarrow 2f(x)=2x\Rightarrow f(x)=x$$

Thus $f(0)=0, f(-1)=-1$ and $f(x)=x\ \forall\ x\neq 0,-1$.

So $f(x)=x\ \forall\ x\in\mathbf{R}$.

Hence $\int e^x f(x)\,dx=\int e^x x\,dx$

$$=xe^x-\int e^x\,dx$$

$$=xe^x-e^x+c=e^x(x-1)+c$$

14. **(a)** $\int \sec^{2010}x\,\text{cosec}^2 x\,dx-\int 2010\sec^{2010}x\,dx$

$$=\int \sec^{2010}x(-\cot x)-\int 2010\sec^{2010}x\cdot\tan x\cdot(-\cot x)-\int 2010\sec^{2010}x\,dx$$

$$=-\frac{\cot x}{(\cos x)^{2010}}+2010\int\sec^{2010}x\,dx-2010\int\sec^{2010}x\,dx+C$$

$$=\frac{-\cot x}{(\cos x)^{2010}}+C \quad \therefore \frac{f(x)}{g(x)}=\frac{1}{\sin x}=\{x\}$$

No solution.

15. **(d)** We have,

$$\int\frac{1}{x}[\log_{ex}e\cdot\log_{e^2x}\cdot\log_{e^3x}e]\,dx$$

$$=\int\frac{1}{x\,log_e ex\,log_e e^2x.log_e e^3x}dx$$

$$=\int\frac{1}{x(\log_e e+\log_e x)(\log_e e^2+\log_e x)(\log_e e^3+\log_e x)}dx$$

$$=\int\frac{d(\log_e x)}{(1+\log_e x)(2+\log_e x)(3+\log_e x)}$$

$$=\int\frac{1}{(1+t)(2+t)(3+t)}dt, \text{ where } t=\log_e x$$

$$=\int\left(\frac{1}{2}\cdot\frac{1}{1+t}-\frac{1}{2+t}+\frac{1}{2}\cdot\frac{1}{3+t}\right)dt$$

$$=\frac{1}{2}\log|1+\log_e x|-\log|2+\log_e x|+\frac{1}{2}\log|3+\log_e x|+C$$

$$=\frac{1}{2}\log\{\log_e ex\}-\log\{\log_e e^2x\}+\frac{1}{2}\log\{\log_e e^3x\}+C$$

EXERCISE - 4

1. **(10)** Put $1 + x^2 = t^2$

$\Rightarrow$ $2x\,dx = 2t\,dt$

$x\,dx = t\,dt$

Now $\displaystyle\int_1^2 \frac{15(t^2-1)\,t\,dt}{\sqrt{t^2+t^3}} = 15\int_1^2 \frac{t(t^2-1)}{t\sqrt{1+t}}dt$

Put $1 + t = u^2$

$\Rightarrow$ $dt = 2u\,du$

$$15\int_{\sqrt{2}}^{\sqrt{3}} \frac{(u^2-1)^2-1}{u}\times 2u\,du$$

$$30\int_{\sqrt{2}}^{\sqrt{3}} (u^4-2u^2)\,du$$

$$30\left(\frac{u^5}{5}-\frac{2u^3}{3}\right)_{\sqrt{2}}^{\sqrt{3}}$$

$$30\left[\frac{1}{5}\left(\sqrt{3}^5-\sqrt{2}^5\right)-\frac{2}{3}\left(\sqrt{3}^3-\sqrt{2}^3\right)\right]$$

$$30\left[\frac{1}{5}\left(9\sqrt{3}-4\sqrt{2}\right)-\frac{2}{3}\left(3\sqrt{3}-2\sqrt{2}\right)\right]$$

$$30\left[-\frac{1}{5}\times\sqrt{3}+\frac{8}{15}\sqrt{2}\right]$$

$$-6\sqrt{3}+16\sqrt{2}=\alpha\sqrt{2}+\beta\sqrt{3}$$

$\Rightarrow$ $\alpha = 16, \beta = -6$

$\therefore$ $\alpha + \beta = 10$

2. **(104)**

$$I = 60\int_0^{\pi/2}\left(\frac{\sin 6x-\sin 4x}{\sin x}+\frac{\sin 4x-\sin 2x}{\sin x}+\frac{\sin 2x}{\sin x}\right)dx$$

$$\left(\because \sin x-\sin y = 2\sin\frac{(x+y)}{2}\cdot\cos\frac{(x-y)}{2}\right)$$

$$I = 60\int_0^{\pi/2}(2\cos 5x+2\cos 3x+2\cos x)\,dx$$

$$I = 60\left(\frac{2}{5}\sin 5x+\frac{2}{3}\sin 3x+2\sin x\right)\Bigg|_0^{\pi/2} = 104$$

3. **(385)** Given function is $f(x) = [x] - 10$

Take, $\displaystyle\int_0^{10} f(x)\cdot dx = -10-9-8-.....-1$

$= -\dfrac{10\cdot 11}{2} = -55$

Take,

$$\int_0^{10}\left(f(x)\right)^2 dx = 10^2+9^2+8^2+...+1^2 = \frac{10\cdot 11\cdot 21}{6} = 385$$

Take, $\displaystyle\int_0^{10}\left|f(x)\right| = 10+9+8+...+1 = \frac{10\cdot 11}{2} = 55$

$= -55 + 385 + 55 = 385$

4. **(12)** Let $\dfrac{\lambda^2 x}{3} = \ell$

$$\Rightarrow \frac{2\lambda x}{3}d\lambda = d\ell \Rightarrow d\lambda = \frac{3}{2}\cdot\frac{1\sqrt{x}}{x\cdot\sqrt{3}\sqrt{\ell}}d\ell$$

$$\Rightarrow d\lambda = \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{x}}\cdot\frac{dt}{\ell}$$

So, $f(x) = \dfrac{1}{\sqrt{x}}\displaystyle\int_0^x \frac{f(t)}{\sqrt{t}}dt$

$$\Rightarrow \sqrt{x}\cdot f'(x)+\frac{f(x)}{2\sqrt{x}} = \frac{f(x)}{\sqrt{x}}$$

$$\Rightarrow \sqrt{x}\cdot f'(x) = \frac{f(x)}{2\sqrt{x}} \Rightarrow \frac{dy}{y} = \frac{dx}{2x}$$

Take integral both sides,

$$\Rightarrow \ln y = \frac{1}{2}\ln x + c \Rightarrow f(x) = \sqrt{x}$$

Here, $f(1) = \sqrt{3} \Rightarrow y = \sqrt{3x}$

So, $f(x) = \sqrt{3x}$

Now, $f(\alpha) = 6 \Rightarrow 36 = 3\alpha \Rightarrow \alpha = 12$

5. **(8)** Given integral is

$$f(x)+\int_0^x (x-t)f'(t)\,dt = \left(e^{2x}+e^{-2x}\right)$$

$\cos 2x + \dfrac{2x}{a}$ ...(i)

Here $f(0) = 2$ ...(ii)

Differentiate equation (i) w.r.t. x,

$$f'(x)+\int_0^x f'(t)\,dt + xf'(x) - xf'(x) = 2\left(e^{2x}-e^{-2x}\right)$$

$$\cos 2x - 2\left(e^{2x}+e^{2x}\right)\sin 2x + \frac{2}{a}$$

$$\Rightarrow f(x)+f(x)-f(0) = 2\left(e^{2x}-e^{2x}\right)\cos 2x - 2\left(e^{2x}+e^{-2x}\right)$$

$$\sin 2x + \frac{2}{a}$$

Replace x by 0 we get :

$$\Rightarrow 4 = \frac{2}{a} \Rightarrow a = \frac{1}{2}.$$

Therefore, $(2a+1)^5.a^2 = 2^5.\dfrac{1}{2^2} = 2^3 = 8$

6. **(5)** Given integral is

$$a_n = \int_{-1}^{n}\left(1+\frac{x}{2}+\frac{x^2}{3}+....+\frac{x^{n-1}}{n}\right)dx$$

$$=\left[x+\frac{x^2}{2^2}+\frac{x^3}{3^2}+....+\frac{x^n}{n^2}\right]_{-1}^{n}$$

Then, $a_n = \frac{n+1}{1^2}+\frac{n^2-1}{2^2}+\frac{n^3+1}{3^2}+\frac{n^4-1}{4^2}+....+\frac{n^n+(-1)^{n+1}}{n^2}$

Here $a_1 = 2, a_2 = \frac{2+1}{1}+\frac{2^2-1}{2} = 3+\frac{3}{2} = \frac{9}{2}$

$a_3 = 4+2+\frac{28}{9} = \frac{100}{9}$

$a_4 = 5+\frac{15}{4}+\frac{65}{9}+\frac{255}{16} > 31.$

$\therefore$ The required set is {2, 3}. $\because$ $a_n = \in (2,30)$

Hence, sum of elements = 5.

7. **(2)** $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$

$\Rightarrow f(x) = 2x - x^2 f(x)$

$\Rightarrow f(x) = \frac{2x}{1+x^2}$

$\Rightarrow f'(x) = \frac{2(1-x^2)}{(1+x^2)^2}$

Then,

$$f'(1/2) = \frac{2\left(1-\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2} = \frac{3}{2}\times\frac{16}{25} = \frac{24}{25}$$

$\Rightarrow \frac{24}{5^k} = \frac{24}{25} \quad \Rightarrow \frac{24}{5^k} = \frac{24}{5^2} \quad \Rightarrow k = 2$

8. **(1)** $I_n + I_{n+2} = \int_0^{\pi/4} \tan^n x(1+\tan^2 x)dx$

$$= \int_0^{\pi/4} \tan^n x \sec^2 x\, dx = \left[\frac{\tan^{n+1} x}{n+1}\right]_0^{\pi/4}$$

$$= \frac{1-0}{n+1} = \frac{1}{n+1}$$

$\therefore I_n + I_{n+2} = \frac{1}{n+1} \Rightarrow \lim_{n\to\infty} n\,[I_n + I_{n+2}]$

$$= \lim_{n\to\infty} n.\frac{1}{n+1} = \lim_{n\to\infty}\frac{n}{n+1} = \lim_{n\to\infty}\frac{n}{n\left(1+\frac{1}{n}\right)} = 1$$

9. **(0.5)** Given f is a function and

$$I_1 = \int_{1-k}^{k} x\, f\left[x(1-x)\right]dx$$

$$I_2 = \int_{1-k}^{k} f\left[x(1-x)\right]dx$$

Now, $I_1 = \int_{1-k}^{k} x\, f\left[x(1-x)\right]dx$ ...(i)

$\int_{1-k}^{k}(1-x)\, f\left[(1-x)x\right]dx$ ...(ii)

$\left[\text{Using Property} \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$

Adding (i) and (ii)

$$2I_1 = \int_{1-k}^{k} f\left[x(1-x)\right]dx = I_2$$

$\therefore \frac{I_1}{I_2} = \frac{1}{2} = 0.5$

10. **(0)** We have, $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[f(x)+f(-x)\right]\left[g(x)-g(-x)\right]dx$

Let $F(x) = \left(f(x)+f(-x)\right)\left(g(x)-g(-x)\right)$, then

$F(-x) = \left(f(-x)+f(x)\right)\left(g(-x)-g(x)\right)$

$= -\left[f(x)+f(-x)\right]\left[g(x)-g(-x)\right]$

$= -F(x)$

$\therefore$ $F(x)$ is an odd function.

Using the property $\int_{-a}^{a} f(x)\,dx = 0$,

if $f(-x) = f(x)$, we get $I = 0$

11. **(1)** Let, $\int\frac{dx}{x^3(1+x^6)^{\frac{2}{3}}} = \int\frac{dx}{x^7(1+x^{-6})^{\frac{2}{3}}}$

Put $1 + x^{-6} = t^3 \Rightarrow -6^{-7}\,dx = 3t^2\,dt$

$\Rightarrow \frac{dx}{x^7} = \left(-\frac{1}{2}\right)t^2 dt$

Now, $I = \int\left(-\frac{1}{2}\right)\frac{t^2 dt}{t^2} = -\frac{1}{2}t + C$

$$= -\frac{1}{2}(1+x^{-6})^{\frac{1}{3}} + C = -\frac{1}{2}\frac{(1+x^6)^{\frac{1}{3}}}{x^2} + C$$

$$= -\frac{1}{2x^3}x(1+x^6)^{\frac{1}{3}} + C$$

Hence, $f(x) = -\frac{1}{2x^3} \Rightarrow \frac{-C}{2x^3} = \frac{-1}{2x^3} \Rightarrow C = 1$

12. **(1)** $\int_0^1 x\cot^{-1}\left(1-x^2+x^4\right)dx=\int_0^1 x\tan^{-1}\left(\frac{1}{1+x^4-x^2}\right)$

$=\int_0^1 x\tan^{-1}\left(\frac{x^2-\left(x^2-1\right)}{1+x^2\left(x^2-1\right)}\right)dx$

$=\frac{1}{2}\int_0^1 1\tan^{-1}t^2dt-\frac{1}{2}\int_{-1}^0 1\tan^{-1}k\ dk$

Put $x^2=t\Rightarrow 2xdx=dt$ in the first integral
and $x^2-1=k\Rightarrow 2xdx=dk$ in the second integral.

$=\frac{1}{2}\int_0^1 1\tan^{-1}tdt-\frac{1}{2}\int_0^1 1\tan^{-1}kdk$

$=\frac{1}{2}\left(t\tan^{-1}t\Big|_0^1-\int_0^1\frac{t}{1+t^2}dt\right)$

$-\frac{1}{2}\left(k\tan^{-1}k\Big|_0^1-\int_{-1}^0\frac{k}{1+k^2}dk\right)$

$=\frac{1}{2}\left(\frac{\pi}{4}-\left(\frac{1}{2}\ln\left(1+t^2\right)\Big|_0^1\right)\right)$

$-\frac{1}{2}\left(-\frac{\pi}{4}-\left(\frac{1}{2}\ln\left(1+k^2\right)\Big|_{-1}^0\right)\right)$

$=\left(\frac{\pi}{8}-\frac{1}{4}\ln 2\right)-\left(\frac{-\pi}{8}-\frac{1}{4}10-\ln 2\right)=\frac{\pi}{4}-\frac{1}{2}\ln 2$

$\Rightarrow\frac{\pi}{4}-\frac{1}{2}\text{In}2=\frac{\pi}{4}-\frac{k}{2}\text{In}2$

$\Rightarrow k=1$