Electric field on axial line of an electric dipole

Resultant electric field intensity at the point P is

$E_p = E_A + E_B$

The vectors E_A and E_B are collinear and opposite.

$\therefore \quad E_p = E_B - E_A$

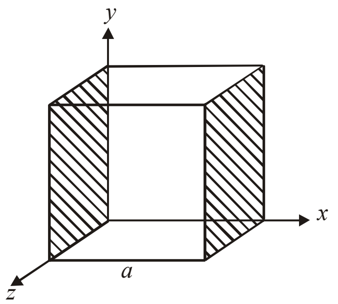
Here, $E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(x+l)^2} \Rightarrow E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(x-l)^2}$

$\therefore E_p = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(x-l)^2} - \frac{q}{(x+l)^2}\right] = \frac{1}{4\pi\varepsilon_0}\frac{4qlx}{(x^2-l^2)^2}$

Hence, $E_p = \frac{1}{4\pi\varepsilon_0}\frac{4px}{(x^2-l^2)^2}$ (2½ marks)

If dipole is short, $2l << x$, then $E_p = \frac{2px}{4\pi\varepsilon_0 x^3}$

(b) The electric field has only x component, for faces normal to x direction, the angle between E and Δs is $\pm\frac{\pi}{2}$. Therefore, the flux is separately zero for each face of the cube except the two shaded ones.



The magnitude of the electric field at the left face is $E_L = 0$ (As x = 0 at the left face)

The magnitude of the electric field at the right face is $E_R = 2a$ (As $x = a$ at the right face)

Their corresponding fluxes are

$\phi_L = \vec{E}_L \cdot \Delta\vec{S} = 0$

$\phi_R = \vec{E}_L \cdot \Delta\vec{S} = E_R \Delta S \cos\theta = E_R \Delta S \ (\because \theta = 0°)$

$\Rightarrow \phi_R = E_R a^2$

Net flux (ϕ) through the cube $= \phi_L + \phi_R = 0 + E_R a^2 = E_R a^2$

$\phi = 2a(a^2) = 2a^3$

From, Gauss's law

$\phi = \frac{q}{\varepsilon_\circ} \Rightarrow q = \phi\varepsilon_0$ (2½ marks)

$\therefore q = 2a^3\varepsilon_0$

32. Phase difference between voltage and current,

$\tan\phi = \frac{X_L - X_C}{R}$ ..... (i)

and, $I_0 = \frac{V_0}{2} = \frac{V_0}{\sqrt{(X_L - X_C)^2 + R^2}}$

$\therefore$ Expression of AC, $I = I_0 \sin(\omega t = \phi)$ (2 marks)

Condition for resonance

Inductive reactance must be equal to capacitive reactance

i.e., $X_L = X_C$

As, $X_L = X_C$ (1 mark)

$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

where, ω_0 = resonant angular frequency.

Impedance becomes minimum and equal to ohmic resistance

i.e., $Z = Z_{\text{minimum}} = R$

AC becomes maximum,

$\therefore I_{\max} = \frac{V_{\max}}{Z_{\min}} = \frac{V_{\max}}{R}$ (2 marks)

Voltage and current arrives in same phase.

OR

(a) The power is defined as the rate of at which work is being done in the circuit.

When $V = V_0 \sin\omega t$ is applied to a series LCR circuit.

Current is $I = I_0 \sin(\omega t + \phi)$

$$I_0 = \frac{V_0}{Z} \text{ and } \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

Instantaneous power supplied by the source is

$P = VI = (V_0 \sin\omega t) \times (I_0 \sin(\omega t + \phi))$

$P = V_0 I_0 \sin\omega t \sin(\omega t + \phi)$

$P = \frac{V_0 I_0}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$

$P_{av} = \frac{V_0 I_0}{2}[\cos\phi - 0]$ $\{\because < \cos(2\omega t) = 0\}$

The average power $P_{av} = V_{\text{rms}} I_{\text{rms}} \cos\phi$

$= \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cdot \cos\phi$ (1 mark)

In this expression $\cos\phi$ is known as the power factor.

Case I : For pure inductive circuit or pure capacitive circuit, the phase difference between current and voltage is $\frac{\pi}{2}$.

$\therefore \quad \phi = \frac{\pi}{2}, \cos\phi = 0$

Therefore, $P_{av} = 0$. Thus no power is dissipated in the circuit. This current is sometimes referred to as wattless current and such a circuit is called wattless circuit. (1 mark)

Case II : For power dissipated at resonance in an LCR circuit,

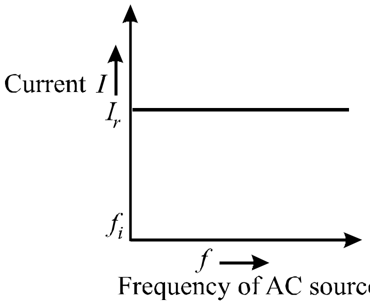
$X_C - X_L = 0, \phi = 0$

$\therefore \quad \cos\phi = 1$

So, maximum power is dissipated in the circuit. (1 mark)

(b) Let initially I_r current is flowing in all the three circuits. If frequency of applied *AC* source is increased then, the change in current will occur in following manner:

Circuit containing resistance R only There will not be any effect in the current, on changing the frequency of *AC* source.



(1 mark)

where, f_i = initial frequency of *AC* source.

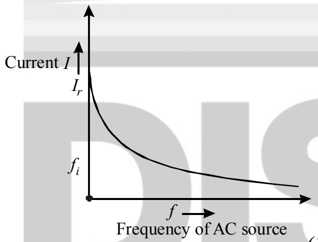
There is no effect on current with the increase in frequency.

AC circuit containing inductance only With the increase of frequency of AC source, inductive reactance increase as

$$I = \frac{V_{\text{rms}}}{X_L} = \frac{V_{\text{rms}}}{2\pi f L}$$

For given circuit,

$$I \propto \frac{1}{f}$$



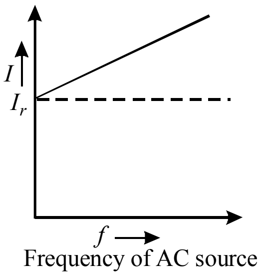
(1 mark)

Current decreases with the increase of frequency.

AC circuits containing capacitor only

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$\text{Current, } I = \frac{V_{\text{rms}}}{X_C} = \frac{V_{\text{rms}}}{\left(\frac{1}{2\pi f C}\right)}$$



(1 mark)

$$I = 2\pi f C V_{\text{rms}}$$

For given circuit, $I \propto f$

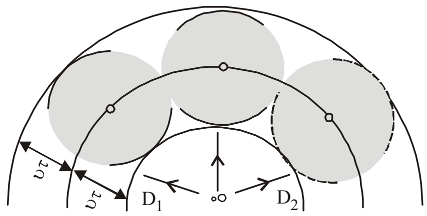
Current increases with the increase of frequency.

33. (a) A **wavefront** is defined as the continuous locus of all the particles of a medium, which are vibrating in the same phase or it is a surface of constant phase. (½ mark)

Huygens' principle:

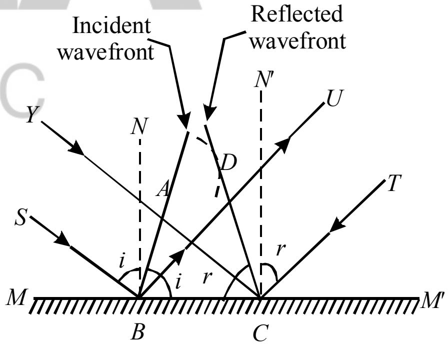
(1) Every points on the given wavefront (called primary wavefront) acts as a fresh source of new disturbance (secondary wavelets), which travel in all directions with the velocity of light in the medium.

(2) A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant. This is called secondary wavefront. (1 mark)



The above model has one shortcoming: we also have a backwave which is shown as D_1D_2 in figure. Huygens argued that the amplitude of the secondary wavelets is maximum in the forward direction and zero in the backward direction, by making this assumption, Huygens could explain the absence of the backwave.

Let a plane wavefront *AB* is incident on the plane mirror *MM*′. As per Huygen's wave theory, every point on wavefront again behaves like a light source and emits secondary wavelets. In the time taken by the wave to reach from *A* to *C*, the secondary wavelets from *B* gets spread over a hemisphere of radius.



where, c is velocity of light and *t* is the time taken by wave in going from *A* to *C*. The tangent plane *CD* drawn from the point *C* over this hemisphere of radius ct gives new reflected wavefront *CD* corresponding to incident wavefront A*B*.

Let *i* and *r* be angles of incidence and reflection respectively.

Now, in ΔABC and ΔDCB

$\angle BAC = \angle CDB$	[each 90°, ray ⊥ wavefront]
$BC = BC$	(common)
$AC = DB$	[From Eq. (i)]
$\Rightarrow \quad \Delta ABC \cong \Delta DCB$	(RHS congruence)
$\Rightarrow \quad \angle ABC = \angle DCB$	

or $i = r$

$$\left[\because SB \perp AB \Rightarrow \angle NBA = 90° - i \text{ and } BN \perp BC \Rightarrow \angle ABC = i\right]$$

Similarly, $\angle N'CT = \angle DCB = r$

$\Rightarrow$ Angle of incidence = Angle of reflection

Also, incident ray, reflected ray and normal meet at one point on a plane.

Thus, laws of reflection are verified using Huygen's principle. (2 marks)

(b) As the number of point sources increases, their contribution towards intensity also increases. Intensity varies as square of the slit width. Thus, when the width of the slit is made double the original width, intensity will get four times of its original value.

Width of central maximum is given by, $\beta = \frac{2D\lambda}{b}$

So, with the increase in size of slit, the width of central maxima decreases. Hence, double the size of the slit would results as half the width of the central maxima. (1 mark)

(c) The waves diffracted from the edge of the circular obstacle interfere constructively at the centre of the shadow producing a bright spot. (½ mark)

OR

(i) (a) From the fringe width expression,

$$\beta = \frac{\lambda D}{d}$$

With the decrease in separation between two slits, the fringe-width d increases. (1 mark)

(b) For interference fringes to be seen,

$$\frac{s}{S} < \frac{\lambda}{d}$$

Condition should be satisfied

where, s = size of the source,

S = distance of the source from the plane of two slits.

As the source-slit-width increase, fringe pattern gets less and less sharp.

When the source-slit is so wide, the above condition does not satisfied and the interference pattern disappears.

(1 mark)

(ii) Intensity at a point is given by,

$I = 4I' \cos^2 \phi/2$ (1 mark)

where, ϕ = phase difference,

I' = intensity produced by each one of the individual sources.

At central maxima, $\phi = 0$, the intensity at the central maxima, $I = I_0 = 4I'$

or $I' = \frac{I_0}{4}$ .... (i)

As, path difference $= \frac{\lambda}{3'}$

Phase difference,

$$\phi' = \frac{2\pi}{\lambda} \times \text{ path difference } = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$ (1 mark)

Now, intensity at the point,

$$I'' = 4I'\cos^2 \frac{1}{2}\left(\frac{2\pi}{3}\right) = 4I'\cos^2 \frac{\pi}{3} = 4I' \times \frac{1}{4} = I'$$ (1 mark)

or $I'' = \frac{I_0}{4}$ [From eq. (i)]