





























$$\text{Average atomic wt.} = \frac{m_1 x_1}{x_1} + \frac{m_2 x_2}{x_2}$$

$$\text{or Average atomic wt.} = \frac{x \times 10.01 + (100 - x) \times 11.01}{100}$$

$$10.81 = \frac{x \times 10.01 + (100 - x) \times 11.01}{100}$$

$$x = 20$$

$$\therefore \% \text{ of isotope with atomic wt. } 10.01 = 20$$

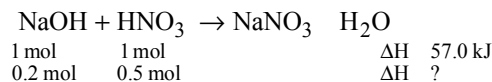
$$\% \text{ of isotope with atomic wt. } 11.01 = 100 - x = 80$$

$$57. \quad (80) \quad \frac{P^\circ - P}{P} = \frac{n_2}{n_1} \cdot \frac{n_2}{n_2}$$

$$\frac{640 - 600}{640} = \frac{2.5/x}{39/78}$$

$$x = \frac{640 \times 78 \times 2.5}{39 \times 40} = 80$$

58. (11.4) Given



Given heat of neutralisation of strong acid by strong base = 57.0 kJ

$\therefore$  0.2 mole NaOH is limiting reagent.

$\therefore$  Heat of neutralization =  $0.2 \times 57 = 11.4$  kJ

59. (3) Calculating the oxidation state of nitrogen in given molecules;

Oxidation state of N in  $\text{NH}_3$  is  $x + 3 \times (+1) = 0$  or  $x = -3$

Oxidation state of N in  $\text{NaNO}_3$  is  $1 + x + 3 \times (-2) = 0$  or  $x = +5$

Oxidation state of N in  $\text{NaN}_3$  is  $+1 + 3x = 0$  or  $x = -\frac{1}{3}$

Oxidation state of N in  $\text{Mg}_3\text{N}_2$  is  $3 \times 2 + 2x = 0$  or  $x = -3$

Thus 3 molecules (i.e.  $\text{NH}_3$ ,  $\text{NaN}_3$  and  $\text{Mg}_3\text{N}_2$ ) have nitrogen in negative oxidation state.

60. (18) Reaction Rate  $r_1 = k[A]^2[B]$

Now increase conc. of A by three times and conc. of B by two times. Then new rate

$$r_2 = k[3A]^2[2B]$$

$$\frac{r_1}{r_2} = \frac{k[A]^2[B]}{k[3A]^2[2B]} = \frac{1}{3^2} \times \frac{1}{2} = \frac{1}{18}$$

$$r_2 = 18 \times r_1$$

Hence rate increases by 18 times.

### MATHEMATICS

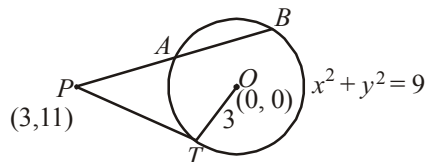
$$61. \quad (3) \quad \lim_{x \rightarrow 0} (\sin x)^{1/x} = \lim_{x \rightarrow 0} \left( \frac{1}{x} \right)^{\sin x}$$

$$0 \lim_{x \rightarrow 0} e^{\log \left( \frac{1}{x} \right)^{\sin x}} \left\{ \begin{array}{l} \text{as, (decimal)}^\infty \rightarrow 0 \\ \lim_{x \rightarrow 0} (\sin x)^{1/x} \rightarrow 0 \end{array} \right\}$$

$\lim_{x \rightarrow 0} \frac{\log \left( \frac{1}{x} \right)}{\csc x}$ , applying L-Hospital's rule, we get

$$\lim_{x \rightarrow 0} \frac{x \left( -\frac{1}{x^2} \right)}{-\csc x \cot x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \tan x = e^0 = 1$$

62. (2) Given circle,  $x^2 + y^2 = 9$



$$PA \times PB = PT^2$$

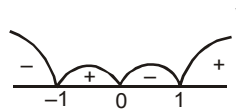
[By Geometry]

$$PA \times PB = (3)^2 + (11)^2 - 9 = 121$$

63. (1)  $f(x) = \frac{3}{4-x^2} \log_{10}(x^3 - x)$

$$4 - x^2 \neq 0; \quad x^3 - x > 0;$$

$$x \neq \pm\sqrt{4} \text{ and } -1 < x < 0 \text{ or } 1 < x < \infty$$



$$\therefore D = (-1, 0) \cup (1, \infty) - \sqrt{4}$$

$$D = (-1, 0) \cup (1, 2) \cup (2, \infty).$$

64. (1) We have

$$\frac{1}{2} \begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = 28$$

$$\Rightarrow 5k^2 + 13k - 46 = 0 \text{ or } 5k^2 + 13k + 66 = 0$$

$$\text{Now, } 5k^2 + 13k - 46 = 0$$

$$\Rightarrow k = \frac{-13 \pm \sqrt{1089}}{10} \therefore k = \frac{-23}{5}; k = 2$$

since k is an integer,  $\therefore k = 2$

$$\text{Also } 5k^2 + 13k + 66 = 0$$

$$\Rightarrow k = \frac{-13 \pm \sqrt{-1151}}{10}. \text{ So no real solution exist}$$

For orthocentre  $BH \perp AC$

$$\therefore \left( \frac{\beta - 2}{\alpha - 5} \right) \left( \frac{8}{-4} \right) = -1 \Rightarrow \alpha - 2\beta = 1 \quad \dots(i)$$

Also  $CH \perp AB$

$$\therefore \left( \frac{\beta - 2}{\alpha - 2} \right) \left( \frac{8}{3} \right) = -1$$

$$\Rightarrow 3\alpha + 8\beta = 10 \quad \dots(ii)$$

Solving (i) and (ii), we get  $\alpha = 2, \beta = \frac{1}{2}$  orthocentre is  $\left( 2, \frac{1}{2} \right)$ .

$$65. (1) \vec{a} - \vec{b} \cdot \left( \vec{c} - \frac{\vec{a} \cdot \vec{b}}{2} \right) = \vec{b} \cdot \vec{c} - \vec{b} \cdot \left( \frac{\vec{a} \cdot \vec{b}}{2} \right) - \vec{a} \cdot \vec{c} \quad \frac{\vec{a}}{2} \cdot \vec{a} \cdot \vec{b}$$

$$\text{and } |\vec{a} - \vec{c}| = |\vec{b} - \vec{c}| \Rightarrow |\vec{a} - \vec{c}|^2 = |\vec{b} - \vec{c}|^2$$

$$\therefore \vec{a} \cdot \vec{b} = 2\vec{c}$$

$$\text{Therefore, } (\vec{b} - \vec{a}) \cdot \left( \vec{c} - \frac{\vec{a} \cdot \vec{b}}{2} \right) = 0.$$

66. (2) Let  $\alpha, \beta$  be the roots of the equation

$$x^2 - a - 2x - a - 1 = 0,$$

$$\text{then } \alpha + \beta = a - 2, \alpha\beta = -a - 1$$

$$\therefore z = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (a - 2)^2 + 2(a - 1) = a^2 - 2a + 2$$

$$\frac{dz}{da} = 2a - 2 = 0 \Rightarrow a = 1$$

$$\frac{d^2z}{da^2} = 2 > 0, \text{ so } z \text{ has minima at } a = 1.$$

So  $\alpha^2 + \beta^2$  has least value for  $a = 1$ . This is because we have only one stationary value at which we have minima. Hence  $a = 1$ .

67. (2) We have

$$\sin(300^\circ) = \sin(270^\circ + 30^\circ) = \sin(3 \times 90^\circ + 30^\circ)$$

$$= -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\tan(330^\circ) = \tan(270^\circ + 60^\circ) = \tan(3 \times 90^\circ + 60^\circ)$$

$$= -\cot 60^\circ = -\frac{1}{\sqrt{3}}$$

$$\sec(420^\circ) = \sec(4 \times 90^\circ + 60^\circ) = \sec 60^\circ = 2$$

$$\tan(135^\circ) = \tan(90^\circ + 45^\circ) = -\cot 45^\circ = -1$$

$$\sin(210^\circ) = \sin(2 \times 90^\circ + 30^\circ)$$

$$= -\sin 30^\circ = -\frac{1}{2}$$

$$\sec(315^\circ) = \sec(270^\circ + 45^\circ)$$

$$= \operatorname{cosec} 45^\circ = \sqrt{2}$$

$$\text{Now, } \frac{\sin 300 \cdot \tan 330 \cdot \sec 420}{\tan 135 \cdot \sin 210 \cdot \sec 315}$$

$$\frac{-\frac{\sqrt{3}}{2} \times \left(-\frac{1}{\sqrt{3}}\right) \times 2}{-1 \times \left(-\frac{1}{2}\right) \times \sqrt{2}} = \sqrt{2}$$

68. (4) Given differential equation is

$$\log\left(\frac{dy}{dx}\right) = 3x + 4y, y(0) = 0$$

$$\Rightarrow \frac{dy}{dx} = e^{3x + 4y} = e^{3x} \cdot e^{4y}$$

$$\Rightarrow \int e^{-4y} dy = \int e^{3x} dx$$

$$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$$

By using  $y = 0$  when  $x = 0$ , we get  $c = -\frac{7}{12}$

$$\therefore \text{Particular solution is } 4e^{3x} - 3e^{-4y} = 7$$

69. (2) We have,  $f(x) = \frac{1}{2}x - 1$  for  $0 \leq x \leq \pi$

$$\therefore [f(x)] = \begin{cases} -1, & 0 \leq x < 2 \\ 0, & 2 \leq x \leq \pi \end{cases}$$

$$\Rightarrow \tan [f(x)] = \begin{cases} \tan(-1), & 0 \leq x < 2 \\ \tan 0, & 2 \leq x \leq \pi \end{cases}$$

$$\therefore \lim_{x \rightarrow 2^-} \tan [f(x)] = -\tan 1 \text{ and } \lim_{x \rightarrow 2^+} \tan [f(x)] = 0$$

So,  $\tan f(x)$  is not continuous at  $x = 2$

$$\text{Now } f(x) = \frac{1}{2}x - 1 \Rightarrow f(x) = \frac{x-2}{2}$$

$$\Rightarrow \frac{1}{f(x)} = \frac{2}{x-2}$$

Clearly,  $\frac{1}{f(x)}$  is not continuous at  $x = 2$ .

So,  $\tan [f(x)]$  and  $\left[\frac{1}{f(x)}\right]$  are both discontinuous at  $x = 2$ .

$$70. (2) \frac{a - bx}{a - bx} \cdot \frac{b - cx}{b - cx} \cdot \frac{c - dx}{c - dx}$$

$$\Rightarrow \frac{a}{bx} \cdot \frac{b}{cx} \cdot \frac{c}{dx}$$

(using componendo and dividendo)

$$\Rightarrow \frac{a}{b} \cdot \frac{b}{c} \cdot \frac{c}{d}$$

$$71. (3) \int_2^4 \{|x-2| + |x-3|\} dx$$

$$= \int_2^3 \{(x-2) - (x-3)\} dx + \int_3^4 \{(x-2) + (x-3)\} dx$$

$$= \int_2^3 dx + \int_3^4 (2x-5) dx = x \Big|_2^3 + \left[ \frac{2x^2}{2} - 5x \right]_3^4$$

$$= (3-2) + [(16-20) - (9-15)] = 1 - 4 + 6 = 3$$

72. (4) Given  $x = \frac{1}{x} + 2 \cos \theta$  .....(i)  
Cubing on both side of equation (i), we get

$$\left(x - \frac{1}{x}\right)^3 = 8 \cos^3 \theta$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 3x \frac{1}{x^2} - 3x^2 \frac{1}{x} = 8 \cos^3 \theta$$

[Using  $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$ ]

$$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 8 \cos^3 \theta$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 6 \cos \theta = 8 \cos^3 \theta$$

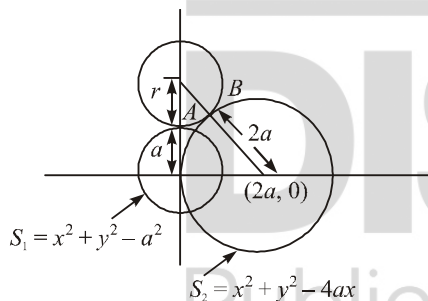
[From (i)]

$$\therefore x^3 - \frac{1}{x^3} = 8 \cos^3 \theta + 6 \cos \theta$$

73. (1) Let centre of the circle is,  $C = (h, k)$  and radius =  $r$

Co-ordinates of  $A \equiv \left[\frac{ah}{a-r}, \frac{ak}{a-r}\right]$

Co-ordinates of  $B \equiv \left[\frac{2ar - 2ah}{2a-r}, \frac{2ak}{2a-r}\right]$



Putting co-ordinates of  $A$  and  $B$  in  $S_1, S_2$  respectively and eliminating  $r$ ,  
 $12x^2 - 4y^2 - 24ax + 9a^2 = 0$ .

74. (1) Given equation of line is  
 $x - 5 = \frac{1}{4}(y - 3) - \frac{1}{9}(z - 6)$   
or  $\frac{x - 5}{1} = \frac{y - 3}{4} = \frac{z - 6}{-9} = \lambda$  (say)  
 $x = \lambda + 5, y = 4\lambda + 3, z = -9\lambda + 6$   
 $(x, y, z) \equiv (\lambda + 5, 4\lambda + 3, -9\lambda + 6)$  .....(i)  
Let it is foot of perpendicular  
So, d.r.'s of  $\perp$  line is  
 $(\lambda + 5 - 2, 4\lambda + 3 - 4, -9\lambda + 6 + 1)$   
 $\equiv (\lambda - 7, 4\lambda - 7, -9\lambda + 7)$   
D.r.'s of given line is  $(1, 4, -9)$  and both lines are  $\perp$   
 $\therefore (\lambda - 7) \cdot 1 + (4\lambda - 7) \cdot 4 + (-9\lambda + 7) \cdot (-9) = 0$   
 $\Rightarrow 98\lambda = 98 \Rightarrow \lambda = 1$   
 $\therefore$  Point is  $(-4, 1, -3)$ . [Substituting  $\lambda = 1$  in (i)]

75. (1)  $\because r = \max_{i \neq j} |x_i - x_j|$  and  $S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

Now, consider  $(x_i - \bar{x})^2 = \left(x_i - \frac{x_1 + x_2 + \dots + x_n}{n}\right)^2$

$$\frac{1}{n^2} [(x_i - x_1)(x_i - x_2) \dots (x_i - x_{i-1}) \dots (x_i - x_n)]^2$$

$$\leq \frac{1}{n^2} [(n-1)r]^2$$

[ $\because |x_i - x_j| \leq r$ ]

$$\Rightarrow (x_i - \bar{x})^2 \leq r^2 \Rightarrow \sum_{i=1}^n (x_i - \bar{x})^2 \leq nr^2$$

$$\Rightarrow \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \leq \frac{nr^2}{(n-1)} \Rightarrow S^2 \leq \frac{nr^2}{(n-1)}$$

$$\Rightarrow S \leq r \sqrt{\frac{n}{n-1}}$$

76. (2) If  $\vec{a} \cdot \vec{b} \cdot \vec{c} = 0, |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$

$$\Rightarrow \vec{a} \cdot \vec{b} \cdot \vec{c} = 0 \Rightarrow \vec{a} \cdot \vec{b} = -\vec{c} \cdot \vec{c}$$

$$\Rightarrow a^2 \cdot b^2 - 2\vec{a} \cdot \vec{b} \cos \theta = c^2$$

$$\Rightarrow 3^2 + 5^2 + 2 \times 3 \times 5 \cos \theta = 7^2$$

$$\Rightarrow 9 + 25 + 30 \cos \theta = 49$$

$$\Rightarrow 30 \cos \theta = 49 - 34$$

$$\Rightarrow 30 \cos \theta = 15 \Rightarrow \cos \theta = \frac{15}{30}$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \cos \theta = \cos \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

77. (1) If  $z = x + iy$  is a complex number satisfying the given conditions, then

$$a^2 - 3a + 2 = |z + \sqrt{2}| = |z + i\sqrt{2} + \sqrt{2} - i\sqrt{2}|$$

$$\leq |z + i\sqrt{2}| + \sqrt{2} |1 - i| < a^2 + 2$$

$$\Rightarrow -3a < 0 \Rightarrow a > 0 \quad \dots(i)$$

Since  $|z + \sqrt{2}| = a^2 - 3a + 2$  represents a circle with centre at  $A(-\sqrt{2}, 0)$  and radius  $\sqrt{a^2 - 3a - 2}$ , and  $|z + i\sqrt{2}| < a^2$  represents the interior of the circle with centre at  $B(0, -\sqrt{2})$  and radius  $a$ .

Therefore there will be a complex number satisfying the given condition and the given inequality if the distance  $AB$  is less than the sum or difference of the radii of the two circles, i.e., if



$$\sqrt{(-\sqrt{2}-0)^2 + (0-\sqrt{2})^2} = \sqrt{a^2-3a+2} = a$$

$$\Rightarrow 2 - a = \sqrt{a^2-3a+2} \Rightarrow 4 - a^2 = 4a - a^2 - 3a + 2$$

$$\Rightarrow -a < -2 \text{ or } 7a < -2 \Rightarrow a > 2 \text{ or } a < -\frac{2}{7}$$

But  $a > 0$  from (i), therefore  $a > 2$ .

78. (2) We define the following events:  
 $A_1$  : Selecting a pair of consecutive letter from the word LONDON.  
 $A_2$  : Selecting a pair of consecutive letters from the word CLIFTON  
 $E$  : Selecting a pair of letters 'ON'.

Then  $P(A_1 \cap E) = \frac{2}{5}$ ; as there are 5 pairs of consecutive letters out of which 2 are ON.

$P(A_2 \cap E) = \frac{1}{6}$ ; as there are 6 pairs of consecutive letters of which one is ON.  
 $\therefore$  the required probability is

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1 \cap E)}{P(A_1 \cap E) + P(A_2 \cap E)} = \frac{\frac{2}{5}}{\frac{2}{5} + \frac{1}{6}} = \frac{12}{17}$$

79. (1)  $\int f(x) \sin x \cos x \, dx = \frac{1}{2(b^2 - a^2)} \log f(x) + C$  therefore

$$f(x) \sin x \cos x = \frac{1}{2(b^2 - a^2)} \cdot \frac{1}{f(x)} f'(x)$$

[by differentiating both the sides]

$$\Rightarrow 2(b^2 - a^2) \sin x \cos x = \frac{f'(x)}{f(x)^2}$$

$$\int (2b^2 \sin x \cos x - 2a^2 \sin x \cos x) \, dx = \int \frac{f'(x)}{f(x)^2} \, dx$$

[by integrating both the sides]

$$\Rightarrow -b^2 \cos^2 x - a^2 \sin^2 x - C = -\frac{1}{f(x)}$$

80. (3) Let  $L = \lim_{x \rightarrow 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$   
 Using L.H. Rule, we get

$$L = \lim_{x \rightarrow 0} \frac{f'(x^2) \cdot 2x - f'(x)}{f'(x)}$$

$$= \lim_{x \rightarrow 0} \frac{f'(x^2) \cdot 2x}{f'(x)} - 1 \quad \left[ \because f'(a) = 0, f \text{ being strictly increasing} \right]$$

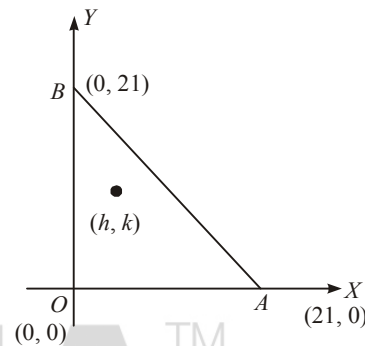
$$= 0 - 1 = -1$$

81. (879) Number of white balls = 10  
 Number of green balls = 9  
 and number of black balls = 7  
 $\therefore$  Required ways  
 $= (10+1)(9+1)(7+1) - 1$   
 $= 11 \cdot 10 \cdot 8 - 1 = 879$

[ $\because$  The total number of ways of selecting one or more items from  $p$  identical items of one kind,  $q$  identical items of second kind;  $r$  identical items of third kind is  $(p+1)(q+1)(r+1) - 1$ ]

82. (190) Equation of  $AB$  is,

$$\frac{x}{21} + \frac{y}{21} = 1 \Rightarrow x + y = 21$$



- Let  $(h, k)$  be any point inside the  $\Delta OAB$  then  
 $h < 21, h > 0$  ... (i)  
 $k < 21, k > 0$  ... (ii)  
 and  $h + k < 21$  ... (iii)

For integral values of  $(h, k)$  satisfying (i), (ii) and (iii) simultaneously, let  
 Total number of points  
 $h = 1$  then  $k = 1, 2, 3 \dots 19$  19  
 $h = 2$  then  $k = 1, 2, 3 \dots 18$  18  
 $h = 3$  then  $k = 1, 2, 3 \dots 17$  17  
 $\vdots$   
 $h = 19$  then  $k = 1$  1  
 $\therefore$  Total number of integral points  
 $= 19 + 18 + 17 + \dots + 1$   
 $= \frac{19 \times 20}{2} = 190$

83. (60) Let  $z = \frac{1-i\sqrt{3}}{1+i\sqrt{3}} \times \frac{(1-i\sqrt{3})}{(1-i\sqrt{3})}$

$$\frac{1+3i^2-2\sqrt{3}i}{1-3i^2} = \frac{1-3-2\sqrt{3}i}{1+3} \quad [\because i^2 = -1]$$

$$= \frac{-1-2\sqrt{3}i}{4} = -\frac{1}{4} - \frac{\sqrt{3}}{2}i$$

$$\therefore \tan \theta = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} \quad \left[ \begin{array}{l} \therefore \tan \theta = \frac{y}{x} \\ \text{where } z = x + iy \end{array} \right]$$

$$\therefore \text{Argument of } z = \theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3} = 60^\circ = k^\circ$$

$$\Rightarrow k = 60$$

84. (97)  $t_{r+1} = {}^{100}C_r (\sqrt[8]{5})^{100-r} \cdot (\sqrt[6]{2})^r$

As 2 and 5 are coprime,  $t_{r+1}$  will be rational if  $100 - r$  is a multiple of 8 and  $r$  is a multiple of 6.

Also  $0 \leq r \leq 100$

$$\therefore r = 0, 6, 12, \dots, 96$$

$$\therefore 100 - r = 4, 10, 16, \dots, 100 \quad \dots\dots (i)$$

But  $100 - r$  is to be a multiple of 8.

$$\text{So, } 100 - r = 0, 8, 16, 24, \dots, 96 \quad \dots\dots (ii)$$

The common terms in (i) and (ii) are 16, 40, 64, and 88

$$\therefore r = 84, 60, 36, 12 \text{ give rational terms.}$$

$$\therefore \text{the number of irrational terms} = 101 - 4 = 97$$

85. (12)  $2|x| + 3|y| \leq 6$  can be written as

$$l_1 : 2x - 3y = 6, \text{ points are } (3, 0), (0, -2)$$

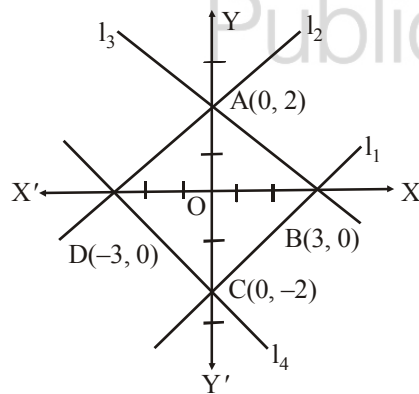
$$l_2 : -2x - 3y = 6, \text{ points are } (-3, 0), (0, 2)$$

$$l_3 : 2x + 3y = 6, \text{ points are } (3, 0), (0, 2)$$

$$l_4 : -2x + 3y = 6, \text{ points are } (-3, 0), (0, -2)$$

Required area = area of quadrilateral ABCD

$$= \text{area}(\triangle ADC) + \text{area}(\triangle ABC)$$



$$= \frac{1}{2} AC \times DO + \frac{1}{2} \times BO \times AC$$

$$\frac{1}{2} AC (BO + OD) = \frac{1}{2} \times 4(3 + 3)$$

$$= 2 \times 6 = 12 \text{ square units.}$$

86. (0.5)  $\cos^{-1} \sqrt{p} = \frac{\pi}{4}, \cos^{-1} \sqrt{1-p}$  and  $\cos^{-1} \sqrt{1-q} = \frac{\pi}{4}$

$$\Rightarrow \sqrt{1-q} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \Rightarrow q = 1 - \frac{1}{2} = 0.5$$

87. (1.25)  $1 - x + 1 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$

$$\Rightarrow 1 - 2 + 5x - 3 + 3 - x - 2 = 0 \Rightarrow x = \frac{5}{4}$$

$$\Rightarrow x + (2 + 5x + 3) + (-2)(3 + x + 2) = 0 \Rightarrow x = \frac{5}{4}$$

88. (2)  $f(x) = \begin{cases} 2 + x^3, & -3 < x \leq -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$

$$\Rightarrow f'(x) = \begin{cases} 3 + 2x^2, & -3 < x \leq -1 \\ \frac{2}{3} x^{-1/3}, & -1 < x < 2 \end{cases}$$

For  $-3 < x \leq -1, f'(x) > 0$ .

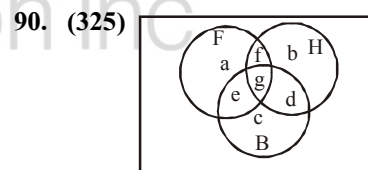
But  $-1 < x < 2, f'(x) < 0$

So, we have one local max. at  $x = -1$

Further for  $0 < x < 2, f'(x) < 0$

$\therefore$  also, we have one local minima.

89. (0.25)  $P(\text{tail in 3rd}) \cdot P(\text{tail in 4th}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$



$$a + e + f + g = 285, b + d + f + g = 195$$

$$c + d + e + g = 115, e + g = 45, f + g = 70, d + g = 50$$

$$a + b + c + d + e + f + g = 500 - 50 = 450$$

$$\text{We obtain } a + f = 240, b + d = 125, c + e = 65$$

$$a + e = 215, b + f = 145; b + c + d = 165$$

$$a + c + e = 255; a + b + f = 335$$

Solving we get

$$b = 95, c = 40, a = 190, d = 30, e = 25, f = 50 \text{ and } g = 20$$

$$\text{Desired quantity} = a + b + c = 325$$