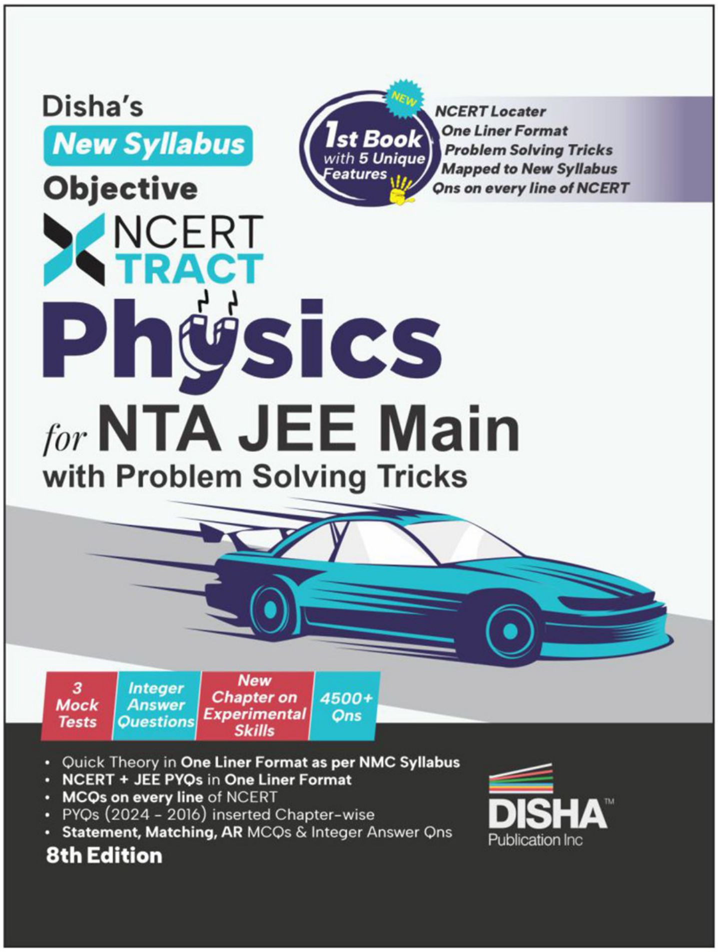
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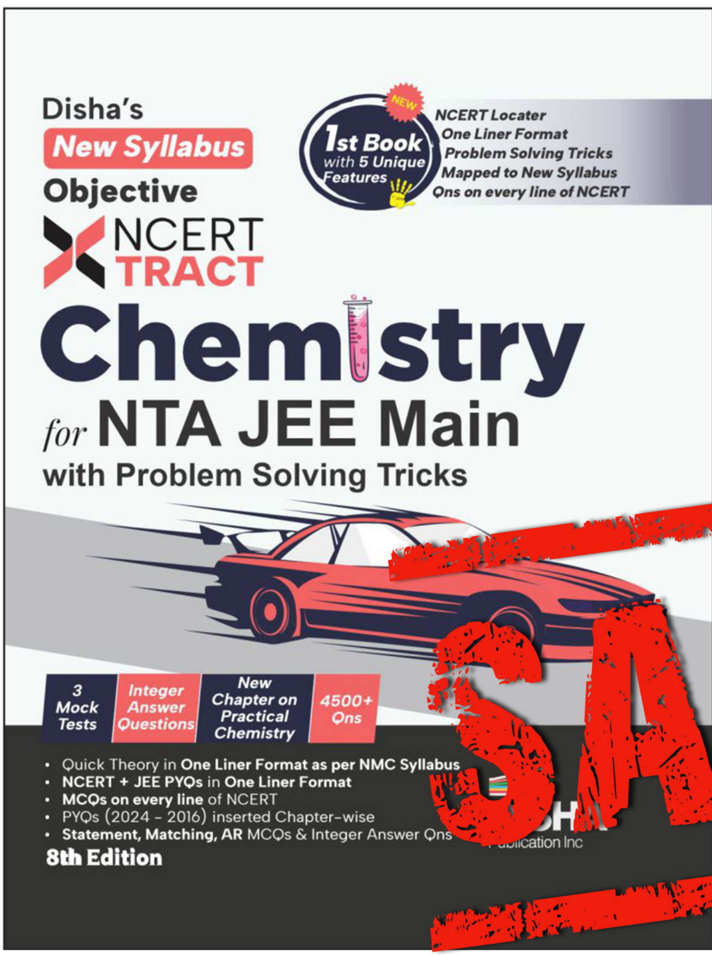
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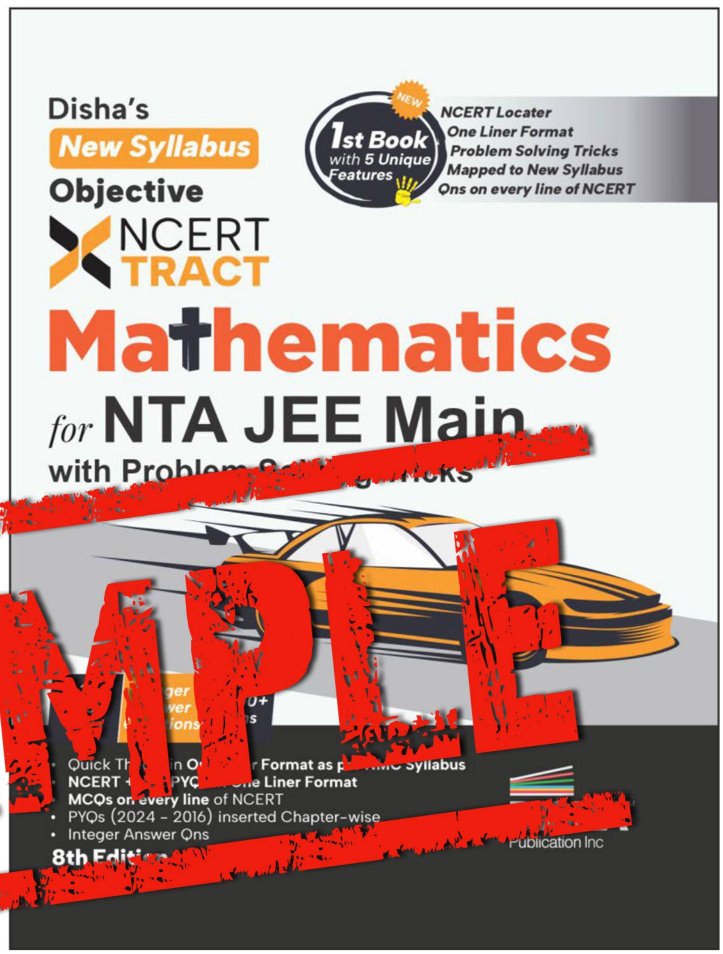
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Free Sample Contents

Class XI

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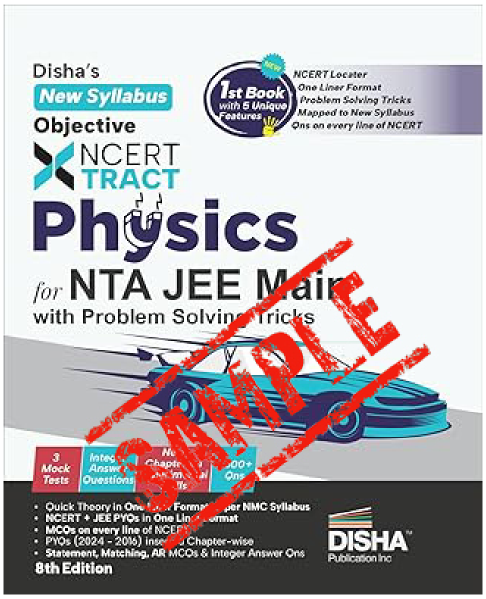

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Class XII

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NOTE* These Topics are in new NCERT, but not in the new JEE (M) 2024 Syllabus. These Topics have been retained in the book so as to match NCERT and any future amendments in JEE (M). Questions on these Topics have also been marked with a * in the respective Exercises of the Chapters.

5 Work, Energy and Power

Trend Analysis JEE Main

	JEE	Remarks
Number of Questions from 2024-17	10	Higher weightage chapter for JEE(M)
Weightage	4.9%	

		JEE		
Year	Topic Name	Concept Used	No. of Ques.	Difficulty Level
2024	Kinetic Energy	$E = \frac{p^2}{2m}$	1	Easy
2023	Power, Kinetic energy	$P = \frac{W}{t} = \frac{mgh}{t}$ $K.E = \frac{p^2}{2m}$	2	Easy (1) Average(1)
2022	Work energy theorem	$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	1	Average
2021	Collision	Collision in two dimension	1	Average
2020	Power	Power, $P = F \times V$	1	Average
2019	Work done by force	Work done in lifting hanging part	1	Difficult
2018	Collisions	Conservation of momentum & elastic collision	2	Average
2017	Work energy theorem	Work energy theorem/ Conservation of energy	2	Average

NCERT ONE-LINERS
(Important Points to Remember)

5.1 Introduction

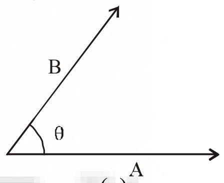
- Energy is our capacity to do work. In Physics too, the term 'energy' is related to work in this sense, but the term 'work' is defined much more precisely.
- Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

The Scalar Product

- The scalar product or dot product of any two vectors **A** and **B**, denoted as **A·B** is defined as

  $$\mathbf{A}\cdot\mathbf{B} = A\,B\cos\theta$$

  where θ is the angle between the two vectors.

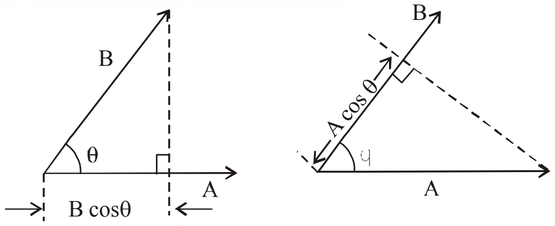


(a)

Fig.: (a) The scalar product of two vectors A and B is a scalar: A.B = AB cosθ.

- The dot product of **A** and **B** is a scalar quantity. Each vector, **A** and **B**, has a direction but their scalar product does not have a direction.

  Two vectors $\vec{A}$ and $\vec{B}$ are said to be orthogonal if $\vec{A}\cdot\vec{B} = 0$
- **B** cos θ is the projection of **B** onto **A** and **A** cos θ is the projection of **A** onto **B.**



(b) (c)

Fig.: (b) B cosθ is the projection of B onto A. (c) A cosθ is the projection of A onto B.

- The scalar product follows the **commutative law**:

  $$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$$
- Scalar product obeys the **distributive law**:

  $$\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$$
- For unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ we have

  $$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$

  $$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0$$

5.2 Notions of Work and Kinetic Energy: The Work-energy Theorem

- The change in kinetic energy of a particle is equal to the work done on it by the net force.

$$K_f - K_i = w$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \mathbf{ma.d} = \mathbf{F \cdot d}$$

This is known as **work energy theorem**.

5.3 Work

- The **work** done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement. Thus $W = (F\cos\theta)d = \mathbf{F \cdot d}$

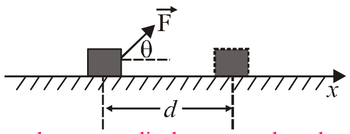


Fig.: An object undergoes a displacement d under the influence of the force F.

No work is done if :

- the displacement is zero. A weightlifter holding a 150kg mass steadily on his shoulder for 30 s does no work on the load during this time.
- the force is zero. A block moving on a smooth horizontal table is not acted upon by a horizontal force (since there is no friction), but may undergo a large displacement.
- the force and displacement are mutually perpendicular. This is so since, for $\theta = \pi/2$ rad (= 90°), $\cos(\pi/2) = 0$.

- In many examples the frictional force opposes displacement and $\theta = 180°$. Then the work done by friction is negative (cos 180° = –1).

5.4 Kinetic Energy

- If an object of mass m has velocity **v**, its kinetic energy K is

$$K = \frac{1}{2}m\,\mathbf{v}\cdot\mathbf{v} = \frac{1}{2}mv^2$$

Relation between kinetic energy (K) and momentum (p) is given by

$$K = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{p^2}{m}$$ JEE M 2021

- Kinetic energy is a scalar quantity. The kinetic energy of an object is a measure of the work an object can do by the virtue of its motion.
- The kinetic energy of a fast flowing stream has been used to grind corn. Sailing ships employ the kinetic energy of the wind.

5.5 Work Done by a Variable Force

- For a varying force the work done can be expressed as a definite integral of force over displacement.
- The work done by a variable force is

$$W = \int_{x_i}^{x_f} F(x)\,dx$$ JEE M 2022

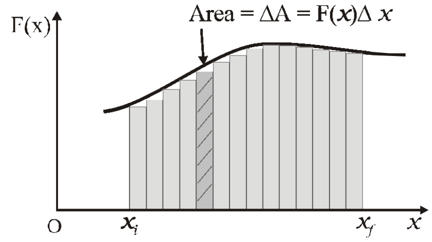


Fig.: The shaded rectangle represented the work done by the varying force F(x), over the small displacement Δx, ΔW = F(x) Δx.

5.6 The Work-Energy Theorem for a Variable Force

- The time rate of change of kinetic energy is

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = m\frac{dv}{dt}v$$

$$= F\,v \text{ (from Newton's Second Law)} = F\frac{dx}{dt}$$

Thus $dK = Fdx$

Integrating from the initial position (x_i) to final position (x_f), we have

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} Fdx$$

where, K_i and K_f are the initial and final kinetic energies corresponding to x_i and x_f.

or $K_f - K_i = \int_{x_i}^{x_f} Fdx = W$

5.7 The Concept of Potential Energy

- Potential energy is the 'stored energy' by virtue of the position or configuration of a body.
- Gravitational potential energy of an object, as a function of the height h, is denoted by V(h) and it is the negative of work done by the gravitational force in raising the object to that height.

$V(h) = mgh$ JEE M 2023

- If h is taken as a variable, the gravitational force F equals the negative of the derivative of $V(h)$ with respect to h.

$$F = -\frac{d}{dh}V(h) = mg$$

The negative sign indicates that the gravitational force is downward.

- Change in potential energy $(V_f - V_i) = \Delta V = V_f - V_i$

$$= \int_{x_i}^{x_f} F(x)dx = -W_{\text{conservative}}$$

- The work done by a **conservative force** such as gravity depends on the initial and final positions only.
- If the work done or the kinetic energy did depend on other factors such as the velocity or the particular path taken by the object, the force would be called **non-conservative**.
- When work is done upon a system by a conservative force then its potential energy increases.

5.8 The Conservation of Mechanical Energy

- Suppose that a body undergoes displacement Δx under the action of a conservative force F. Then from the *WE* theorem we have,

$$\Delta K = F(x)\,\Delta x$$

- If the force is conservative, the potential energy function $V(x)$ can be defined such that

$$\Delta V = -F(x)\,\Delta x$$

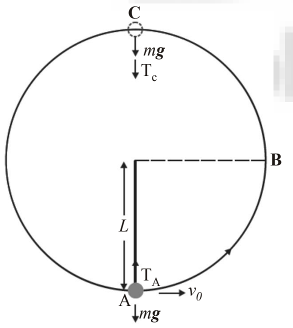
The above equations imply that

$$\Delta K + \Delta V = 0$$
$$\Delta(K + V) = 0$$

- Individually the kinetic energy (K) and potential energy V(x) may vary from point to point but the sum is a constant.
- The work done by the **conservative force** depends only on the end points. This can be seen from the relation,
Work done $W = K_f - K_i = V(x_i) - V(x_f)$
which depends on the end points.
- Work done by this force in a closed path is zero.
- The total mechanical energy of a system is conserved if the forces, doing work on it, are conservative.

Vertical Circular Motion

- At the maximum height from the ground. The energy is purely potential (E = mgH). It is partially converted to kinetic at height h (E = mgh + ½ mv^2) and is fully kinetic at ground level (k = ½ mv^2).



- The potential energy of the system to be zero at the lowest point A. Thus, at A :

$$E = \frac{1}{2}mv_0^2;\ T_A - mg = \frac{mv_0^2}{L}\ \text{[Newton's Second Law]}$$

where T_A is the tension in the string at A.
- At the highest point C, the string slackens, as the tension in the string (T_C) becomes zero.
Thus, at C

$$E = \frac{1}{2}mv_c^2 + 2mgL;\ mg = \frac{mv_c^2}{L}\ \text{[Newton's Second Law]}$$

where v_C is the speed at C. $E = \frac{5}{2}mgL$

Equating this to the energy at A

$$\frac{5}{2}mgL = \frac{m}{2}v_0^2$$

or, $v_0 = \sqrt{5gL}$

It is clear; $v_c = \sqrt{gL}$
At B, the energy is

$$E = \frac{1}{2}mv_B^2 + mgL$$

Equating this to the energy at A and employing the result namely $v_0^2 = 5gL$

$$\frac{1}{2}mc_B^2 + mgL = \frac{1}{2}mv_0^2 = \frac{5}{2}mgL \quad \therefore\ v_B = \sqrt{3gL}$$

The ratio of the kinetic energies at B and C is :

$$\frac{K_B}{K_C} = \frac{\frac{1}{2}mv_B^2}{\frac{1}{2}mv_C^2} = \frac{3}{1}$$

At point C, the string becomes slack and the velocity of the bob is horizontal and to the left.
- When one end of a string of length l is connected to a particle of mass m and the other end to a small peg on a smooth horizontal surface. Centripetal force for circular motion will be provided by the tension in string.

5.9 The Potential Energy of a Spring

- Force law for the spring is called Hooke's law and is mathematically stated as

$$F_s = -kx$$

The constant k is called the spring constant.
- The spring is said to be stiff if k is large and soft if k is small.
- If the extension is x_m, then work done by the spring force is

$$W_s = \int_0^{x_m} F_s\,dx = -\int_0^{x_m} kx\,dx = -\frac{k x_m^2}{2}$$ **JEE M 2021**

$$\Rightarrow V_f - V_i = -W = \frac{kx_m^2}{2}$$

$$\Rightarrow V = \frac{kx_m^2}{2} \qquad \text{[putting } V_i = 0\ \&\ V_f = V\text{]}$$

$V = \frac{1}{2}kx_m^2$ is the potential energy stored in spring in extension or compression x_m.

- If the block is moved from an initial displacement x_i to a final displacement x_f, the work done by the spring force W_s is

$$W_s = -\int_{x_i}^{x_f} k\,x\,dx = \frac{k\,x_i^2}{2} - \frac{k\,x_f^2}{2}$$

- If the block is pulled from x_i and allowed to return to x_i;

$$W_s = -\int_{x_i}^{x_f} k\,x\,dx = \frac{k\,x_i^2}{2} - \frac{k\,x_i^2}{2} = 0$$

The **work done by the spring force in a cyclic process is zero**.

- Spring force (i) is position dependent only as first stated by Hooke ($F_s = -kx$); (ii) does work which only depends on the initial and final positions.
- The spring force is a **conservative force**.
- The potential energy $V(x)$ of the spring is zero when block and spring system is in the equilibrium position.

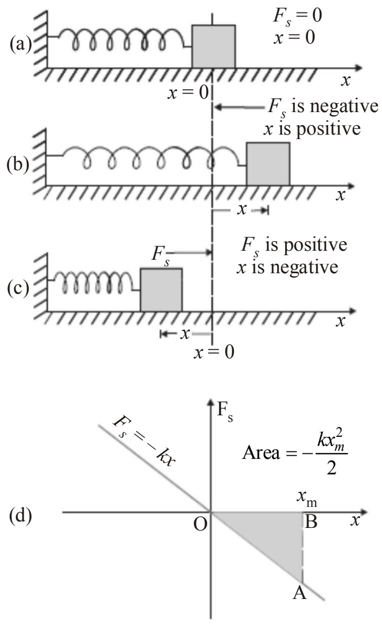
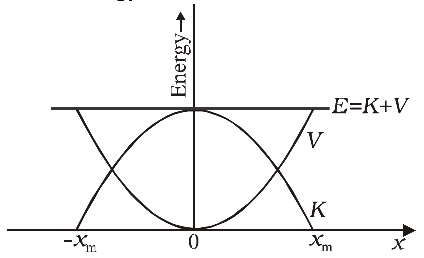


Fig.: Illustration of the spring force with a block attached to the free end of the spring. (a) The spring force F_s is zero when the displacement x from the equilibrium position is zero. (b) For the stretched spring $x > 0$ and $F_s < 0$ (c) For the compressed spring $x < 0$ and $F_s > 0$.(d) The plot of F_s versus x. The area of the shaded triangle represents the work done by the spring force. Due to the opposing signs of F_s and x, this work done is negative, $W_s = -kx_m^2 / 2$.

- The kinetic energy (K) of spring gets converted to potential energy (V) and vice versa, however, the total mechanical energy remains constant.



5.10 Power

- **Power** is defined as the time rate at which work is done or energy is transferred.
- The **average power** of a force is defined as the ratio of the work, W, to the total time t taken

$$P_{av} = \frac{W}{t}$$

JEE M 2023

- The **instantaneous power** is defined as the limiting value of the average power as time interval approaches zero,

$$P = \frac{dW}{dt}$$

- The work dW done by a force F for a displacement **dr** is $dW = \mathbf{F}\cdot d\mathbf{r}$.
- The instantaneous power can also be expressed as

$$P = F \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \mathbf{v}$$

where **v** is the instantaneous velocity when the force is **F**.

- There is another unit of power, namely the **horse-power** (hp)

1 hp = 746 W

This unit is still used to describe the output of automobiles, motorbikes, etc.

- 1 kilowatt hour (kWh) of energy. = 3.6×10^6 J

Our electricity bills carry the energy consumption in units of kWh.

Power on turbine, $P = \frac{d(mgh)}{dt}$

The energy radiated by 100 kW transmitter in

1 hour = 100 kW × 1hr = 100 kWh

$= 100 \times 3.6 \times 10^6 J = 36 \times 10^7 J$

5.11 Collisions

- In **collision** a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particles change.

Elastic and Inelastic Collisions

- In all collisions the total linear momentum is conserved; the initial momentum of the system is equal to the final momentum of the system.
- The total kinetic energy of the system is not necessarily conserved. The impact and deformation during collision may generate heat and sound. Part of the initial kinetic energy is transformed into other forms of energy.
- A useful way to visualise the deformation during collision is in terms of a 'compressed spring'. If the 'spring' connecting the two masses regains its original shape without loss in energy, then the initial kinetic energy is equal to the final kinetic energy but the kinetic energy during the collision time Δt is not constant. Such a collision is called an **elastic collision**.
- A collision in which the two particles move together after the collision is called a **completely inelastic collision.**
- The intermediate case where the deformation is partly relieved and some of the initial kinetic energy is lost is more common and is appropriately called an **inelastic collision**.

Collisions in One Dimension

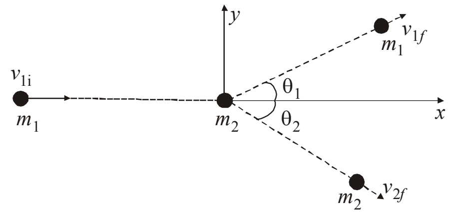


Fig.: Collision of mass m_1 with a stationary mass m_2.

- A **completely inelastic collision** in one dimension.

  $\theta_1 = \theta_2 = 0$

  $m_1 v_{1i} = (m_1 + m_2) v_f$ (momentum conservation)

  $$v_f = \frac{m_1}{m_1 + m_2} v_{1i}$$

  The loss in kinetic energy on collision is

  $$\Delta K = \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2) v_f^2$$

  $$= \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} \frac{m_1^2}{m_1 + m_2} v_{1i}^2$$

  $$= \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} v_{1i}^2$$

  the momentum and kinetic energy conservation equations are

  $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$ ...(i)

  $m_1 v_{1i}^2 = m_1 v_{1f}^2 + m_1 v_{2f}^2 \therefore v_{2f} = v_{1i} + v_{1f}$

  Substituting this in Eq. (i)

  $$v_{1f} = \frac{(m_1 - m_2)}{m_1 + m_2} v_{1i} \text{ and } v_{2f} = \frac{2 m_1 v_{1i}}{m_1 + m_2}$$

- Coefficient of restitution, $e = \dfrac{\text{velocity of seperation}}{\text{velacity of approach}}$

  JEE M 2019

Case I: If the two masses are equal

$v_{1f} = 0$

$v_{2f} = v_{1i}$

Case II: If one mass dominates, e.g. $m_2 >> m_1$

$v_{1f} \simeq -v_{1i} \; v_{2f} \simeq 0$

The heavier mass is undisturbed while the lighter mass reverses its velocity.

- If the initial velocities and final velocities of both the bodies are along the same straight line, then it is called a **one-dimensional collision**, or **head-on collision**.

  When a body of mass m, collides with a body of mass m_2 in elastic and head on collision, energy lost by body of mass m $= \dfrac{4 m_1 m_2}{(m_1 + m_2)^2}$

Collisions in Two Dimensions

- Linear momentum is conserved. The *x*- and *y*-component equations are

  $m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$

  $0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2$

  If, the collision is elastic,

  $$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

- In collision particles may or may not come in real touch.

  In scattering, the velocities and directions in which the two particles go away depend on their initial velocities as well as the type of interactions between them, their masses shapes and sizes.

Tips/Tricks/Techniques ONE-LINERS
(Exam Special)

- If work is done on a body, its kinetic or potential energy increases. If work is done by the body, its potential or kinetic energy decreases.
- In the elastic collisions, the kinetic or mechanical energy is not converted into any other form of energy.
- The force involved in an inelastic collision is non-conservative in nature.
- If collision is head on the colliding bodies move along the same straight line before and after collision.
- If collision is oblique, the colliding bodies move at certain angles before and/or after the collisions.
- In static and dynamic equilibrium, work done is zero.
- Work done by a man holding the weight at fixed position is zero.
- Work done by a force depends on the frame of reference.
- Only tangential component of force is responsible for power dissipation. Power dissipated by radial component is zero. For example power dissipated by centripetal force is zero.
- In a perfectly elastic collision, in one dimension, when masses of the colliding particles are equal, their velocities get exchanged after collision.

- If a body starts rotating after collision, then both linear momentum and angular momentum are conserved.
- In the elastic collisions the forces involved are conservative.
- If the speed of water flowing through a pipe is v, then power is proportional to v^3.
- If n bullets are fired from a machine gun, each having kinetic energy k, then power of the machine gun will be nk.
- Work done by a body of mass m against friction on a rough horizontal surface of coefficient of friction μ is μmgx. Here, x is the distance moved by the body.
- Let h_1, h_2 .... h_n be the heights of the body to which the body rebounds again and again, then

$$e = \sqrt{\frac{h_1}{h}} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{h_3}{h_2}}$$

Clearly $h_1 = e^2h$; $h_2 = e^4h$ and similarly, after nth rebound $h_n = (e^{2n})h$

- The velocity of an object depends on the choice of reference frame. Thus, kinetic energy also depends on the choice of the reference frame.
- Potential energy depends on the reference level. In case of spring, it is advised to assume zero potential energy at the natural length of the spring. In case of gravity any convenient level can be chosen as reference frame.
- If a ball is dropped from a height h_0 on a horizontal floor, then time taken by the ball to stop bouncing is given by

$$T = \left(\frac{1+e}{1-e}\right)\sqrt{\frac{2h_0}{g}}$$ (Here, e = coefficient of restitution)

- An engine pulls a train of mass m with constant velocity. If the rails are on a plane surface and there is no friction, the power dissipated by the engine is zero.
- When the momentum of a body is made n times then its kinetic energy is increased by factor n^2.
- Slope of work time graph = power.
- Area under power time curve = work done by/on the body.
- If a chain of length L and mass M is held on a frictionless table with $(1/n)^{th}$ of its length is hanging over the edge, then work done in pulling the hanging portion on the table is given by

$$W = \frac{MgL}{2n^2}$$

- The phrase 'calculate the work done' is incomplete. We should refer to the work done by a specific force or a group of forces on a given body over a certain displacement.
- Work done is a scalar quantity. It can be positive or negative unlike mass and kinetic energy which are positive scalar quantities. The work done by the friction or viscous force on a moving body is negative.
- For two bodies, the sum of the mutual forces exerted between them is zero from Newton's Third Law,

$$\mathbf{F}_{12} + \mathbf{F}_{21} = 0$$

But the sum of the work done by the two forces need not *always cancel, i.e.*

$$W_{12} + W_{21} \neq 0$$

However, it may sometimes be true.

- The work done by a force can be calculated sometimes even if the exact nature of the force is not known.
- The WE theorem is not independent of Newton's Second Law. The WE theorem may be viewed as a scalar form of the Second Law. The principle of conservation of mechanical energy may be viewed as a consequence of the WE theorem for conservative forces.
- The WE theorem holds in all inertial frames. It can also be extended to noninertial frames provided we include the pseudoforces in the calculation of the net force acting on the body under consideration.
- The potential energy of a body subjected to a conservative force is always undetermined upto a constant. For example, the point where the potential energy is zero is a matter of choice. For the gravitational potential energy mgh, the zero of the potential energy is chosen to be the ground. For the spring potential energy $kx^2/2$, the zero of the potential energy is the equilibrium position of the oscillating mass.
- Every force encountered in mechanics does not have an associated potential energy. For example, work done by friction over a closed path is not zero and no potential energy can be associated with friction.
- During a collision : (a) the total linear momentum is conserved at each instant of the collision ; (b) the kinetic energy conservation (even if the collision is elastic) applies after the collision is over and does not hold at every instant of the collision. In fact the two colliding objects are deformed and may be momentarily at rest with respect to each other.

Exercise 1 : NCERT Based Topic-wise MCQs

5.1 Introduction

1. Let $A = \hat{i}A\cos\theta + \hat{j}A\sin\theta$ be any vector. Another vector B, which is normal to A can be expressed as
NCERT Page-115 / N-72
(a) $\hat{i}\,B\cos\theta - \hat{j}\,B\sin\theta$ (b) $\hat{i}\,B\cos\theta + \hat{j}B\sin\theta$
(c) $\hat{i}\,B\sin\theta - \hat{j}\,B\cos\theta$ (d) $\hat{i}\,B\sin\theta + \hat{j}\,B\cos\theta$

2. If $\vec{A} = 4\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$ then cosine of angle between $\vec{A}$ and $\vec{A} + \vec{B}$ is NCERT Page-115 / N-72
(a) $\frac{9\sqrt{2}}{5}$ (b) $\frac{7}{5\sqrt{2}}$ (c) $\frac{5\sqrt{2}}{49}$ (d) $\frac{5\sqrt{2}}{28}$

3. A particle moves from position $3\hat{i} + 2\hat{j} - 6\hat{k}$ to $14\hat{i} + 13\hat{j} + 9\hat{k}$ due to a uniform force of $(4\hat{i} + \hat{j} + 3\hat{k})\,N$. If the displacement is in metre then work done will be
NCERT Page-115 / N-72
(a) 300 J (b) 250 J (c) 100 J (d) 0

4. If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$, then the value of α is NCERT Page-115 / N-72
(a) 1/2 (b) –1/2 (c) 1 (d) –1

5.2 Notions of Work and Kinetic Energy: The Work-energy Theorem

5. According to work-energy theorem, the work done by the net force on a particle is equal to the change in its
NCERT Page-116 / N-73
(a) kinetic energy (b) potential energy
(c) linear momentum (d) angular momentum

6. A body starts from rest and acquires a velocity V in time T. The work done on the body in time t will be proportional to
NCERT Page-116 / N-73
(a) $\frac{V}{T}t$ (b) $\frac{V^2}{T}t^2$ (c) $\frac{V^2}{T^2}t$ (d) $\frac{V^2}{T^2}t^2$

5.3 Work

7. When the force retards the motion of body, the work done is
NCERT Page-117 / N-74
(a) zero
(b) negative
(c) positive
(d) Positive or negative depending upon the magnitude of force and displacement

8. A man pushes a wall and fails to displace it, he does
NCERT Page-117 / N-74
(a) negative work
(b) positive but not maximum work
(c) no work at all
(d) maximum positive work

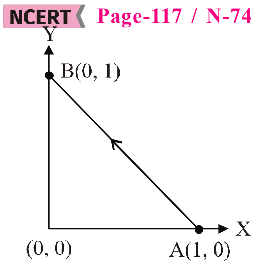
9. A boy carrying a box on his head is walking on a level road from one place to another is doing no work. This statement is
NCERT Page-117 / N-74
(a) correct (b) incorrect
(c) partly correct (d) cannot say

10. No work is done if NCERT Page-117 / N-74
(a) displacement is zero
(b) force is zero
(c) force and displacement are mutually perpendicular
(d) All of these

11. A particle is taken round a circle by application of force. The work done by the force is NCERT Page-117 / N-74
(a) positive non–zero (b) negative non–zero
(c) Zero (d) None of these

12. A porter lifts a heavy suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the work done by the porter in lowering the suitcase. (take g = 9.8 ms^{-2})
NCERT Page-117 / N-74
(a) –62720.0 J (b) –627.2 J
(c) +627.2 J (d) 784.0 J

13. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is: (all quantities are in SI units)
NCERT Page-117 / N-74
(a) 2J
(b) $\frac{1}{2}$J
(c) 1J
(d) $\frac{3}{2}$J



14. If W represents the work done, then match the two columns:
NCERT Page-117 / N-74

	Column I		Column II
(A)	Force is always along the velocity	(1)	W = 0
(B)	Force is always perpendicular to velocity	(2)	W < 0
(C)	Force is always perpendicular to acceleration	(3)	W > 0
(D)	The object is stationary but the point of application of the force moves on the object		

(a) (A)→(1); (B)→(2); C→(3); (D)→(2)
(b) (A)→(3); (B)→(1); C→(2,3); (D)→(1)
(c) (A)→(2); (B)→(3); C→(1); (D)→(2)
(d) (A)→(1); (B)→(2); C→(3); (D)→(1)

15. A particle moving in the xy plane undergoes a displacement of $\vec{s} = (2\hat{i} + 3\hat{j})$ while a constant force $\vec{F} = (5\hat{i} + 2\hat{j})$ N acts on the particle. The work done by the force F is
NCERT Page-117 / N-74
(a) 17 joule (b) 18 joule (c) 16 joule (d) 15 joule

16. A particle describe a horizontal circle of radius 0.5 m with uniform speed. The centripetal force acting is 10 N. The work done in describing a semicircle is NCERT Page-117 / N-74
(a) zero (b) 5 J (c) 5π J (d) 10π J

17. A force acts on a 30 g particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. The work done during the first 4 seconds is NCERT Page-118 / N-74
(a) 576mJ (b) 450mJ (c) 490mJ (d) 530mJ

18. A cord is used to lower vertically a block of mass M, a distance d at a constant downward acceleration of g/4. The work done by the cord on the block is NCERT Page-117 / N-74
(a) $Mg\frac{d}{4}$ (b) $3Mg\frac{d}{4}$ (c) $-3Mg\frac{d}{4}$ (d) Mg d

19. If a motorcyclist skids and stops after covering a distance of 15 m. The stopping force acting on the motorcycle by the road is 100 N, then the work done by the motorcycle on the road is NCERT Page-117 / N-74
(a) 1500 J (b) –1500 J (c) 750J (d) Zero

20. A ball moves in a frictionless inclined table without slipping. The work done by the table surface on the ball is NCERT Page-117 / N-74
(a) positive (b) negative
(c) zero (d) None of these

21. A uniform force of $(3\hat{i} + \hat{j})$ newton acts on a particle of mass 2 kg. The particle is displaced from position $(2\hat{i} + \hat{k})$ meter to position $(4\hat{i} + 3\hat{j} - \hat{k})$ meter. The work done by the force on the particle is NCERT Page-117 / N-74
(a) 6 J (b) 13 J (c) 15 J (d) 9 J

22. A boy pushes a toy box 2.0 m along the floor by means of a force of 10 N directed downward at an angle of 60º to the horizontal. The work done by the boy is NCERT Page-117 / N-74
(a) 6 J (b) 8 J (c) 12 J (d) 12 J

23. A body moves a distance of 10 m along a straight line under the action of a force of 5 newtons. If the work done is 25 joules, the angle which the force makes with the direction of motion of body is NCERT Page-117 / N-74
(a) 0º (b) 30º (c) 60º (d) 90º

5.4 Kinetic Energy

24. A light and a heavy body have equal momentum. Which one has greater K.E.? NCERT Page-117 / N-74
(a) The lighter body (b) The heavier body
(c) Both have equal K.E. (d) Data given is incomplete

25. A particle of mass *m* has momentum *p*. Its kinetic energy will b NCERT Page-117 / N-74
(a) mp (b) p^2m (c) $\frac{p^2}{m}$ (d) $\frac{p^2}{2m}$

26. Kinetic energy, with any reference, must be NCERT Page-117 / N-74
(a) zero (b) positive
(c) negative (d) both (b) and (c)

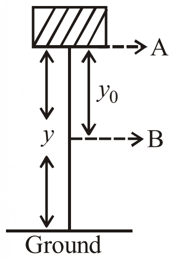
27. If the momentum of a body is increased by 50%, then the percentage increase in its kinetic energy is NCERT Page-117 / N-74
(a) 50% (b) 100% (c) 125% (d) 200%

28. Four particles given, have same momentum. Which has maximum kinetic energy NCERT Page-117 / N-74
(a) Proton (b) Electron (c) Deutron (d) α-particles

29. The K.E. acquired by a mass m in travelling a certain distance d, starting form rest, under the action of a constant force is directly proportional to NCERT Page-117 / N-74
(a) m (b) $\sqrt{m}$
(c) $\frac{1}{\sqrt{m}}$ (d) independent of m

30. A running man has half the kinetic energy of that of a boy of half of his mass. The man speeds up by 1m/s so as to have same K.E. as that of the boy. The original speed of the man will be NCERT Page-117 / N-74
(a) $\sqrt{2}$ m/s (b) $(\sqrt{2}-1)$m/s
(c) $\frac{1}{(\sqrt{2}-1)}$m/s (d) $\frac{1}{\sqrt{2}}$m/s

31. In the given figure, the block of mass m is dropped from the point 'A'. The expression for kinetic energy of block when it reaches point 'B' is : NCERT Page-117 / N-74



(a) $\frac{1}{2}mgy_0^2$
(b) $\frac{1}{2}mgy^2$
(c) $mg(y - y_0)$
(d) mgy_0

32. Two bodies A and B having masses in the ratio of 3 : 1 possess the same kinetic energy. The ratio of linear momentum of B to A is NCERT Page-117 / N-74
(a) 1 : 3 (b) 3 : 1 (c) $1:\sqrt{3}$ (d) $\sqrt{3}:1$

33. A small body is projected in a direction inclined at 45° to the horizontal with kinetic energy *K*. At the top of its flight, its kinetic energy will be NCERT Page-117 / N-74
(a) Zero (b) *K*/2 (c) *K*/4 (d) $K/\sqrt{2}$

34. A body accelerates uniformly from rest to a velocity of 1 ms^{-1} in 15 seconds. The kinetic energy of the body will be $\frac{2}{9}$ J when 't' is equal to [Take mass of body as 1 kg] NCERT Page-117 / N-74
(a) 4s (b) 8s (c) 10s (d) 12s

35. A bomb of mass 9 kg explodes into the pieces of masses 3 kg and 6 kg. The velocity of mass 3 kg is 16 m/s. The kinetic energy of mass 6 kg in joule is NCERT Page-117 / N-74
(a) 96 (b) 384
(c) 192 (d) 768

36. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range NCERT Page-117 / N-74
(a) 200 J - 500 J (b) 2×10^5 J - 3×10^5 J
(c) 20, 000 J - 50,000 J (d) 2,000 J - 5, 000 J

5.5 Work Done by a Variable Force

37. A particle moves under the effect of a force F = cx from x = 0 to x = x_1, the work done in the process is

NCERT Page-118 / N-75

(a) cx_1^2 (b) $\frac{1}{2}cx_1^2$ (c) $2\,cx_1^2$ (d) zero

38. Calculate the work done on the tool by $\vec{F}$ if this displacement is along the straight line y = x that connects these two points. NCERT Page-118 / N-75

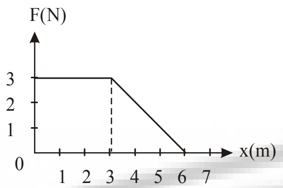
(a) 2.50 J (b) 500 J (c) 50.6 J (d) 2 J

39. Calculate the work done on the tool by $\vec{F}$ if the tool is first moved out along the x-axis to the point x = 3.00m, y = 0 and then moved parallel to the y-axis to x = 3.00m, y = 3.00 m.

NCERT Page-118 / N-75

(a) 67.5 J (b) 85 J (c) 102 J (d) 7.5 J

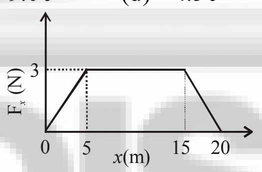
40. A force F acting on an object varies with distance x as shown here. The force is in N and x in m. The work done by the force in moving the object from x = 0 to x = 6 m is



NCERT Page-119 / N-76

(a) 18.0 J (b) 13.5 J (c) 9.0 J (d) 4.5 J

41. A force F_x acts on a particle such that its position x changes as shown in the figure. The work done by the particle as it moves from x = 0 to 20 m is

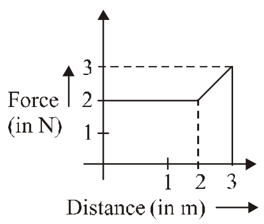


NCERT Page-119 / N-76

(a) 37.5 J (b) 10 J (c) 45 J (d) 22.5 J

42. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is :

NCERT Page-118 / N-75



(a) 4 J (b) 2.5 J (c) 6.5 J (d) 5 J

5.6 The Work-energy Theorem for a Variable Force

43. A body of mass 0.5 kg travels on straight line path with velocity $v = (3x^2 + 4)$ m/s. The net workdone by the force during its displacement from $x = 0$ to $x = 2$ m is :

NCERT Page-119 / N-76

(a) 64 J (b) 60 J (c) 120 J (d) 128 J

44. A body of mass 20kg, moving in x-direction with a constant speed of 20 m/s, is subjected to a retarding force $F = 0.2x$ J/m during its travel from x = 20 m to 30 m. Its final kinetic energy will be NCERT Page-119 / N-76

(a) 50 J (b) 475 J

(c) 4000 J (d) 3250 J

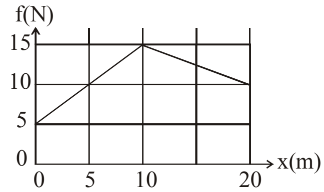
45. Figure here shows the frictional force versus displacement for a particle in motion. The loss of kinetic energy in travelling over s = 0 to 20 m will be NCERT Page-119 / N-76

(a) 225 J

(b) 200 J

(c) 250 J

(d) 750 J



5.7 The Concept of Potential Energy

46. Work done by a conservative force is positive if

(a) P.E. of the body increases NCERT Page-121 / N-78

(b) P.E. of the body decreases

(c) K.E. of the body increases

(d) K.E. of the body decreases

47. For a conservative force in one dimension, potential energy function V(x) is related to the force F(x) as

NCERT Page-121 / N-78

(a) $F(x) = \frac{-dV(x)}{dx}$ (b) $F(x) = \frac{dV(x)}{dx}$

(c) $F(x) = V(x)\,dx$ (d) $F(x) = \int \frac{-dV(x)}{dx}$

48. One man takes 1 min. to raise a box to a height of 1 metre and another man takes 1/2 min. to do so. The energy of the

NCERT Page-120 / N-77

(a) two is different (b) two is same

(c) first is more (d) second is more

49. A ball dropped from a height of 2m reaches to a height of 1.5m before hitting the ground. Then the percentage of potential energy lost is NCERT Page-120 / N-77

(a) 25 (b) 30 (c) 50 (d) 100

50. The potential energy of a system increases if work is done

NCERT Page-121 / N-78

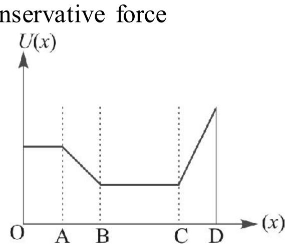
(a) upon the system by a non conservative force

(b) by the system against a conservative force

(c) by the system against a non conservative force

(d) upon the system by a conservative force

51. The figure gives the potential energy function $U(x)$ for a system in which a particle is in one-dimensional motion. In which region the magnitude of the force on the particle is greatest :



NCERT Page-121 / N-78

(a) *OA* (b) *AB* (c) *BC* (d) *CD*

52. A rod of mass m and length ℓ is made to stand at an angle of 60° with the vertical. Potential energy of the rod in this position is NCERT Page-120 / N-77

(a) $mg\ell$ (b) $\frac{mg\ell}{2}$ (c) $\frac{mg\ell}{3}$ (d) $\frac{mg\ell}{4}$

53. The potential energy of a conservative system is given by $U = ay^2 - by$, where y represents the position of the particle and a as well as b are constants. What is the force acting on the system? NCERT Page-121 / N-78

(a) $-ay$ (b) $-by$
(c) $2ay - b$ (d) $b - 2ay$

54. A body falls freely under gravity. Its velocity is v when it has lost potential energy equal to U. What is the mass of the body? NCERT Page-120, 121 / N-77, 78

(a) U^2/v^2 (b) $2U^2/v^2$ (c) $2U/v^2$ (d) U/v^2

5.8 The Conservation of Mechanical Energy

55. Total ....*X*.... energy of a system is conserved, if the forces, doing work on it, are .....*Y*..... . NCERT Page-121 / N-79

Here, *X* and *Y* refer to

(a) conservative, mechanical
(b) mechanical, conservative
(c) mechanical, non-conservative
(d) kinetic, conservative

56. A metallic wire of length L metre extends by ℓ metre when stretched by suspending a weight Mg from it. The mechanical energy stored in the wire is NCERT Page-121 / N-79

(a) $2 Mg\ell$ (b) $Mg\ell$ (c) $\frac{Mg\ell}{2}$ (b) $\frac{Mg\ell}{4}$

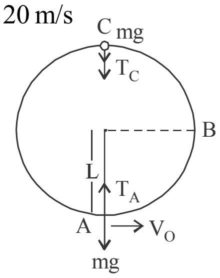
57. A steel ball of mass 5g is thrown downward with velocity 10 m/s from height 19.5 m. It penetrates sand by 50 cm. The change in mechanical energy will be ($g = 10\ m/s^2$) NCERT Page-121 / N-79

(a) 1J (b) 1.25 J (c) 1.5 J (d) 1.75 J

58. A ball is allowed to fall from a height of 10 m. If there is 40% loss of energy due to air friction, then velocity of the ball when it hit the ground is NCERT Page-121 / N-79

(a) $\sqrt{190}$ m/s (b) $\sqrt{180}$ m/s
(c) $\sqrt{150}$ m/s (d) $\sqrt{120}$ m/s

59. Figure shows a bob of mass *m* suspended from a string of length L. The velocity is V_0 at A, then the potential energy of the system is ________ at the lowest point A.



NCERT Page-122 / N-79

(a) $\frac{1}{2}mv_0^2$ (b) mgh (c) $\frac{-1}{2}mv_0^2$ (d) zero

60. Calculate the K.E and P.E. of the ball half way up, when a ball of mass 0.1 kg is thrown vertically upwards with an initial speed of 20 ms^{-1}. NCERT Page-121, 122 / N-79

(a) 10 J, 20 J (b) 10 J, 10 J
(c) 15 J, 8 J (d) 8 J, 16 J

61. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is NCERT Page-121 / N-79

(a) 20 m/s (b) 40 m/s (c) $10\sqrt{30}$ m/s (d) 10 m/s

62. A particle is moving in a circle of radius r under the action of a force $F = \alpha r^2$ which is directed towards centre of the circle. Total mechanical energy (kinetic energy + potential energy) of the particle is (take potential energy = 0 for r = 0) NCERT Page-121 / N-79

(a) $\frac{1}{2}\alpha r^3$ (b) $\frac{5}{6}\alpha r^3$ (c) $\frac{4}{3}\alpha r^3$ (d) αr^3

63. When a body is projected vertically up from the ground with certain velocity, its potential energy and kinetic energy at a point A are in the ratio 2 : 3. If the same body is projected with double the previous velocity, then at the same point A the ratio of its potential energy to kinetic energy is NCERT Page-121 / N-79

(a) 9 : 1 (b) 2 : 9 (c) 1 : 9 (d) 9 : 2

64. A mass m is revolving in a vertical circle at the end of a string of length 20 cm. By how much does the tension of the string at the lowest point exceed the tension at the topmost point? NCERT Page-122 / N-79

(a) 2 m g (b) 4 m g (c) 6 m g (d) 8 m g

65. A particle tied to a string describes a vertical circular motion of radius r continually. If it has a velocity $\sqrt{3\ gr}$ at the highest point, then the ratio of the respective tensions in the string holding it at the highest and lowest points is NCERT Page-122 / N-79

(a) 4 : 3 (b) 5 : 4 (c) 1 : 4 (d) 3 : 2

66. A particle of mass '*m*' tied to a string of length 'ℓ' and whirled in a horizontal circle. If the particles moves in circle with speed '*v*' then the net force on the particle will be (*T* = tensaion in the string) NCERT Page-122 / N-79

(a) $T+\frac{mv^2}{l}$ (b) $T-\frac{mv^2}{r}$ (c) T (d) zero

67. A body of mass 0.4 kg is whirled in a vertical circle making 2 rev/sec. If the radius of the circle is 1.2 m, then tension in the string when the body is at the top of the circle, is NCERT Page-122 / N-79

(a) 41.56 N (b) 89.86 N (c) 109.86 N (d) 115.86 N

68. A bucket tied at the end of a 1.6 m long string is whirled in a vertical circle with constant speed. What should be the minimum speed so that the water from the bucket does not spill when the bucket is at the highest position? NCERT Page-122 / N-79

(a) 4 m/sec (b) 6.25 m/sec
(c) 16 m/sec (d) None of these

5.9 The Potential Energy of a Spring

69. The work done in stretching a spring of force constant k from length ℓ_1 and ℓ_2 is **NCERT** Page-124 / N-81

(a) $k(\ell_2^2 - \ell_1^2)$ (b) $\frac{1}{2}k(\ell_2^2 - \ell_1^2)$

(c) $k(\ell_2 - \ell_1)$ (d) $\frac{k}{2}(\ell_2 + \ell_1)$

70. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is **NCERT** Page-124 / N-81

(a) 18.75 J (b) 25.00 J (c) 6.25 J (d) 12.50 J

71. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be: **NCERT** Page-123 / N-80

(a) n (b) $\frac{1}{n^2}$ (c) $\frac{1}{n}$ (d) n^2

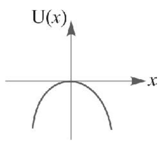
72. The ...X... energy $V(x)$ of the spring is said to be zero when block and spring system is in the ...Y... position.
Here, X and Y refer to **NCERT** Page-124 / N-81

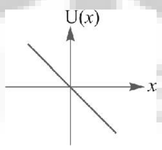
(a) potential, equilibrium (b) kinetic, equilibrium
(c) mechanical, equilibrium (d) vibrational, left

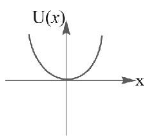
73. If stretch in a spring of force constant k is tripled then the ratio of elastic potential energy in the two cases will be **NCERT** Page-124 / N-81

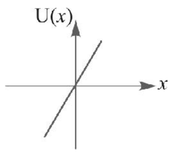
(a) 9 : 1 (b) 1 : 6 (c) 3 : 1 (d) 1 : 3

74. A particle is placed at the origin and a force $F = kx$ is acting on it (where k is positive constant). If $U(0) = 0$, the graph of $U(x)$ versus x will be (where U is the potential energy function) : **NCERT** Page-124 / N-81

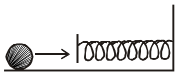
(a) 

(b) 

(c) 

(d) 

75. A mass of m kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant k = 50 N/m. If the maximum compression of the spring is 0.15 m, the value of mass m is **NCERT** Page-124 / N-81



(a) 0.5 kg (b) 0.15 kg (c) 0.12 kg (d) 1.5 kg

76. Two spring P and Q of force constant k_p and $k_q \left(k_q = \frac{k_p}{2}\right)$ are stretched by applying forces of equal magnitude. If the energy stored in Q is E, then the energy stored in P is **NCERT** Page-123 / N-80

(a) E (b) 2 E (c) E / 8 (d) E / 2

77. Two springs have their force constant as k_1 and k_2 ($k_1 > k_2$). When they are stretched by the same force **NCERT** Page-123 / N-80

(a) no work is done in case of both the springs.
(b) equal work is done in case of both the springs
(c) more work is done in case of second spring
(d) more work is done in case of first spring.

78. One end of a light spring of spring constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. In a displacement, the work done by the spring is 1/2 k x^2. The possible cases are **NCERT** Page-123 / N-80

(a) the spring was initially compressed by a distance x, was finally in its natural length
(b) it was initially stretched by a distance x and was finally in its natural length
(c) it was initially in its natural length and finally in a compressed position
(d) it was initially in its natural length and finally in the stretched position

79. If the extension in a spring is increased to 4 times then the potential energy **NCERT** Page-124 / N-81

(a) remains the same (b) becomes 4 times
(c) becomes one fourth (d) becomes 16 times

80. A long string is stretched by 2 cm and the potential energy is V. If the spring is stretched by 10 cm, its potential energy will be **NCERT** Page-124 / N-81

(a) V / 25 (b) V/5 (c) 5 V (d) 25 V

5.10 Power

81. At time t = 0s particle starts moving along the x-axis. If its kinetic energy increases uniformly with time t, the net force acting on it must be proportional to **NCERT** Page-128 / N-83

(a) $\sqrt{t}$ (b) constant (c) t (d) $\frac{1}{\sqrt{t}}$

82. If a force F is applied on a body and it moves with a velocity V, the power will be **NCERT** Page-128 / N-83

(a) F × v (b) F/v
(c) F/v^2 (d) F × v^2

83. Sand is being dropped from a stationary dropper at a rate of 0.5 kgs^{-1} on a conveyor belt moving with a velocity of 5 ms^{-1}. The power needed to keep belt moving with the same velocity will be : **NCERT** Page-128 / N-83

(a) 1.25 W (b) 2.5 W (c) 6.25 W (d) 12.5 W

84. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is given by **NCERT** Page-128 / N-83

(a) $\frac{mv^2}{T^2} \cdot t^2$ (b) $\frac{mv^2}{T^2} \cdot t$ (c) $\frac{1}{2}\frac{mv^2}{T^2} \cdot t^2$ (d) $\frac{1}{2}\frac{mv^2}{T^2} \cdot t$

85. A car of mass m starts from rest and accelerates so that the instantaneous power delivered to the car has a constant magnitude p_o. The instantaneous velocity of this car is proportiourl to: NCERT Page-128 / N-83

(a) $t^2 p_o$ (b) $t^{1/2}$ (c) $t^{-1/2}$ (d) $\frac{t}{\sqrt{m}}$

86. A vehicle is moving with a uniform velocity on a smooth horizontal road, then power delivered by its engine must be NCERT Page-128 / N-83

(a) uniform (b) increasing
(c) decreasing (d) zero

87. How much water, a pump of 2 kW can raise in one minute to a height of 10 m, take g = 10 m/s^2? NCERT Page-128 / N-83

(a) 1000 (b) 1200 (c) 100 (d) 2000

88. The engine of a vehicle delivers constant power. If the vehicle is moving up the inclined plane then, its velocity, NCERT Page-128 / N-83

(a) must remain constant
(b) must increase
(c) must decrease
(d) may increase, decrease or remain same.

89. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to NCERT Page-128 / N-83

(a) $t^{3/4}$ (b) $t^{3/2}$ (c) $t^{1/4}$ (d) $t^{1/2}$

90. A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest NCERT Page-128 / N-83

(a) at the highest position of the body
(b) at the instant just before the body hits the earth
(c) it remains constant all through
(d) at the instant just after the body is projected

91. A body of mass 10 kg moves with a velocity v of 2 m/s along a circular path of radius 8 m. The power produced by the body will be NCERT Page-128 / N-83

(a) 10 J/s (b) 98 J/s (c) 49 J/s (d) zero

92. Johnny and his sister Jane race up a hill. Johnny weighs twice as much as jane and takes twice as long as jane to reach the top . Compared to Jane NCERT Page-128 / N-83

(a) Johnny did more work and delivered more power.
(b) Johnny did more work and delivered the same amount of power.
(c) Johnny did more work and delivered less power
(d) Johnny did less work and johnny delivered less power.

93. A constant power P is applied to a car starting from rest. If v is the velocity of the car at time t, then NCERT Page-128 / N-83

(a) $v \propto t$ (b) $v \propto \frac{1}{t}$
(c) $v \propto \sqrt{t}$ (d) $v \propto \frac{1}{\sqrt{t}}$

94. If two persons A and B take 2 seconds and 4 seconds respectively to lift an object to the same height h, then the ratio of their powers is NCERT Page-128 / N-83

(a) 1 : 2 (b) 1 : 1
(c) 2 : 1 (d) 1: 3

95. A 10 H.P. motor pumps out water from a well of depth 20 m and fills a water tank of volume 22380 litres at a height of 10 m from the ground. The running time of the motor to fill the empty water tank is (g = 10ms^{-2}) NCERT Page-128 / N-83

(a) 5 minutes (b) 10 minutes
(c) 15 minutes (d) 20 minutes

96. If a machine gun fires n bullets per second each with kinetic energy K, then the power of the machine gun is NCERT Page-128 / N-83

(a) nK^2 (b) $\frac{K}{n}$ (c) n^2K (d) nK

97. A particle of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = \left(4 + \hat{i} + 6\hat{j}\right)N$ where $\hat{i}$ and $\hat{j}$ are unit vectors along x and y axis. What power will be developed by the force at the time t? NCERT Page-128 / N-83

(a) $(4t^2 + 6 + 4)W$ (b) $(4t^3 + 6 + 5)W$
(c) $(4t + 6t^2)W$ (d) $(4t^3 + 6t^4)W$

98. A force applied by an engine of a train of mass 2.05 × 10^6 kg changes its velocity from 5m/s to 25 m/s in 5 minutes. The power of the engine is NCERT Page-128 / N-83

(a) 1.025 MW (b) 2.05 MW (c) 5 MW (d) 6 MW

99. A 10 m long iron chain of linear mass density 0.8 kg m^{-1} is hanging freely from a rigid support. If g = 10 ms^{-2}, then the power required to left the chain upto the point of support in 10 second NCERT Page-128 / N-83

(a) 10 W (b) 20W (c) 30 W (d) 40 W

5.11 Collisions

100. Which one of the following statements is true? NCERT Page-129 / N-84

(a) Momentum is conserved in elastic collisions but not in inelastic collisions
(b) Total kinetic energy is conserved in elastic collisions but momentum is not conserved in elastic collisions
(c) Total kinetic energy is not conserved but momentum is conserved in inelastic collisions
(d) Kinetic energy and momentum both are conserved in all types of collisions

101. When after collision the deformation is not relived and the two bodies move together after the collision, it is called NCERT Page-129 / N-84

(a) elastic collision
(b) inelastic collision
(c) perfectly inelastic collision
(d) perfectly elastic collision

102. In an inelastic collision, which of the following does not remain conserved? NCERT Page-129 / N-84

(a) Momentum
(b) kinetic energy
(c) Total energy
(d) Neither momentum nor kinetic energy

103. In case of elastic collision, at the time of impact. NCERT Page-129 / N-84

(a) total K.E. of colliding bodies is conserved.
(b) total K.E. of colliding bodies increases
(c) total K.E. of colliding bodies decreases
(d) total momentum of colliding bodies decreases.

104. In elastic collision, 100% energy transfer takes place when **NCERT** Page-130 / N-84

(a) $m_1 = m_2$ (b) $m_1 > m_2$ (c) $m_1 < m_2$ (d) $m_1 = 2m_2$

105. If two equal masses ($m_1 = m_2$) collide elastically in one dimension, where m_2 is at rest and m_1 moves with a velocity u_1, then the final velocities of two masses are **NCERT** Page-130 / N-84

(a) $V_1 = 0; V_2 = u_1$ (b) $V_1 = V_2 = 0$
(c) $V_1 = 0$ and $V_2 = -u_1$ (d) $V_1 = -u_1; V_2 = 0$

106. A particle A suffers an oblique elastic collision with a particle B that is at rest initially. If their masses are the same, then after collision **NCERT** Page-130, 131 / N-84, 85

(a) they will move in opposite directions
(b) A continues to move in the original direction while B remains at rest
(c) they will move in mutually perpendicular directions
(d) A comes to rest and B starts moving in the direction of the original motion of A

107. A metal ball of mass 2 kg moving with a velocity of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after the collision, the two balls move together, the loss in kinetic energy due to collision is **NCERT** Page-129, 130 / N-84, 85

(a) 140 J (b) 100 J (c) 60 J (d) 40 J

108. Two particles having the position $\vec{r}_1 = (3\hat{i} + 5\hat{j})$ m and $\vec{r}_2 = (-5\hat{i} - 3\hat{j})$ m move with velocities $\vec{V}_1 = (4\hat{i} + 3\hat{j})$ m/s and $\vec{V}_2 = (a\hat{i} + 7\hat{j})$ m/s. If the particles collide, then value of a must be

(a) 8 (b) 6
(c) 4 (d) 2

109. A mass of 20 kg moving with a speed of 10m/s collides with another stationary mass of 5 kg. As a result of the collision, the two masses stick together. The kinetic energy of the composite mass will be **NCERT** Page-129 / N-84

(a) 600 (b) 800
(c) 1000 (d) 1200

110. A mass m moving horizontally (along the x-axis) with velocity v collides and sticks to mass of 3m moving vertically upward (along the y-axis) with velocity 2v. The final velocity of the combination is **NCERT** Page-131 / N-85

(a) $\frac{1}{4}v\hat{i} + \frac{3}{2}v\hat{j}$ (b) $\frac{1}{3}v\hat{i} + \frac{2}{3}v\hat{j}$

(c) $\frac{2}{3}v\hat{i} + \frac{1}{3}v\hat{j}$ (d) $\frac{3}{2}v\hat{i} + \frac{1}{4}v\hat{j}$

111. A body of mass m moving with velocity v collides head on with another body of mass 2m which is initially at rest. The ratio of K. E. of colliding body before and after collision will be **NCERT** Page-130 / N-85

(a) 1 : 1 (b) 2 : 1
(c) 4 : 1 (d) 9 : 1

112. An object of mass 2.0 kg makes an elastic collision with another object of mass M at rest and continues to move in the original direction but with one-fourth of its original speed. What is the value of M? **NCERT** Page-131 / N-86

(a) 0.75 kg (b) 1.0 kg
(c) 1.2 kg (d) None of these

113. A bullet of mass 20g and moving with 600 m/s collides with a block of mass 4 kg hanging with the string. What is velocity of bullet when it comes out of block, if block rises to height 0.2 m after collision? **NCERT** Page-129, 130 / N-84, 85

(a) 200 m/s (b) 150 m/s
(c) 400 m/s (d) 300 m/s

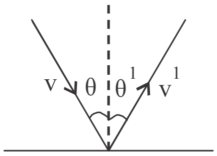
114. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is **NCERT** Page-129 / N-84

(a) 0.16 J (b) 1.00 J (c) 0.67 J (d) 0.34 J

115. If a shell fired from a cannon, explodes in mid air, then **NCERT** Page-129 / N-84

(a) its total kinetic energy increases
(b) its total momentum increases
(c) its total momentum decreases
(d) None of these

116. A ball of mass *m* hits the floor making an angle θ as shown in the figure. If e is the coefficient of restitution, then which relation is true, for the velocity component before and after collision?



NCERT Page-129, 130 / N-84, 85

(a) $V^1 \sin\theta = V \sin\theta$ (b) $V^1 \sin\theta' = -\sin\theta$
(c) $V^1 \cos\theta' = V \cos\theta$ (d) $V^1 \cos\theta' = -V \cos\theta$

117. Before a rubber ball bounces off from the floor, the ball is in contact with the floor for a fraction of second. Which of the following statements is correct? **NCERT** Page-129, 131 / N-84, 85

(a) Conservation of energy is not valid during this period
(b) Conservation of energy is valid during this period
(c) As ball is compressed, kinetic energy is converted to compressed potential energy
(d) None of these

118. A bag of sand of mass 9.8 kg is suspended by a rope. A bullet of 200 g travelling with speed 10 ms^{-1} gets embedded in it, then loss of kinetic energy will be **NCERT** Page-129, 130 / N-84, 85

(a) 4.9 J (b) 9.8 J (c) 14.7 (d) 19.6 J

119. A object of mass m_1 collides with another object of mass m_2, which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is: **NCERT** Page-129, 130 / N-84, 85

(a) 2 : 1 (b) 3 : 1
(c) 1 : 2 (d) 1 : 1

120. Two billiard balls of mass 0.05 kg each moving in opposite directions with 10 ms^{-1} collide and rebound with the same speed. If the time duration of contact is t = 0.005 s, then what is the force exerted on the ball due to each other? **NCERT** Page-129, 130 / N-84, 85

(a) 100 N (b) 200 N (c) 300 N (d) 4000 N

Exercise 2 : NCERT Exemplar & Past Years JEE Main

NCERT Exemplar Questions

1. An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is, because

NCERT Page-117 / N-74

(a) the two magnetic forces are equal and opposite, so they produce no net effect
(b) the magnetic forces do not work on each particle
(c) the magnetic forces do equal and opposite (but non-zero) work on each particle
(d) the magnetic forces are necessarily negligible

2. A proton is kept at rest. A positively charged particle is released from rest at a distance d in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a positron. In the same time t, the work done on the two moving charged particles is

NCERT Page-117 / N-74

(a) same as the same force law is involved in the two experiments
(b) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens
(c) more for the case of a positron, as the positron moves away a larger distance
(d) same as the work done by charged particle on the stationary proton

3. A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is

NCERT Page-117 / N-74

(a) constant and equal to mg in magnitude
(b) constant and greater than mg in magnitude
(c) variable but always greater than mg
(d) at first greater than mg and later becomes equal to mg

4. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is

NCERT Page-117 / N-74

(a) + 2000 J (b) – 200 J (c) zero (d) – 20,000 J

5. A body is falling freely under the action of gravity alone in vaccum. Which of the following quantities remain constant during the fall?

NCERT Page-121 / N-78

(a) Kinetic energy
(b) Potential energy
(c) Total mechanical energy
(d) Total linear momentum

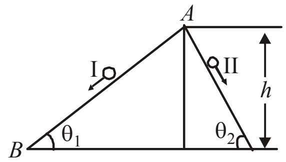
6. During inelastic collision between two bodies, which of the following quantities always remain conserved?

NCERT Page-129 / N-84

(a) Total kinetic energy (b) Total mechanical energy
(c) Total linear momentum (d) Speed of each body

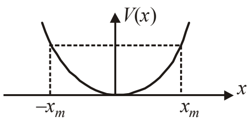
7. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure. Which of the following statement is correct?

NCERT Page-121 / N-78



(a) Both the stones reach the bottom at the same time but not with the same speed
(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II
(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I
(d) Both the stones reach the bottom at different times and with different speeds

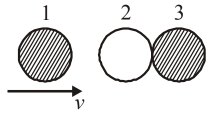
8. The potential energy function for a particle executing linear SHM is given by $V(x) = \frac{1}{2}kx^2$ where k is the force constant of the oscillator (Fig.). For $k = 0.5$ N/m, the graph of $V(x)$ versus x is shown in the figure. A particle of total energy E turns back when it reaches $x = \pm x_m$. If V and K indicate the PE and KE, respectively of the particle at $x = +x_m$, then which of the following is correct?



NCERT Page-124 / N-81

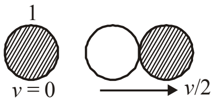
(a) $V = O,\ K = E$ (b) $V = E,\ K = O$
(c) $V < E,\ K = O$ (d) $V = O,\ K < E$

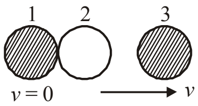
9. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v as shown in figure.

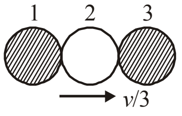


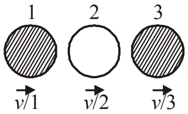
If the collision is elastic, which of the following (figure) is a possible result after collision?

NCERT Page-129, 130 / N-84, 85

(a)


(b)


(c)


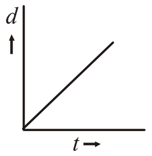
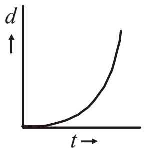
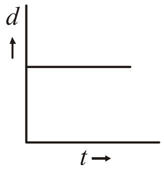
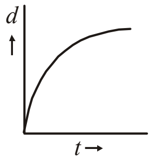
(d)


10. A body of mass 0.5 kg travels in a straight line with velocity $v = a\,x^{3/2}$ where $a = 5\ m^{-1/2}s^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is

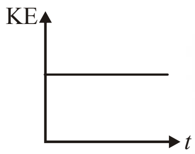
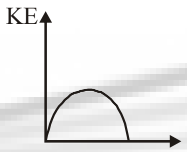
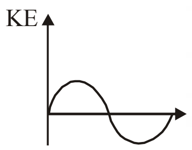
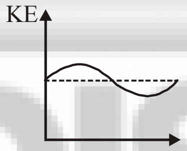
NCERT Page-118 / N-75

(a) 15 J (b) 50 J (c) 10 J (d) 100 J

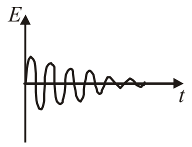
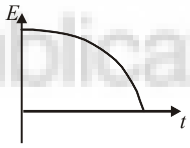
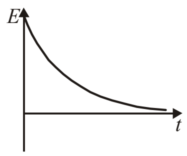
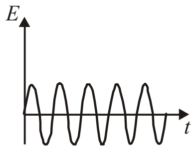
11. A body is moving unidirectionally under the influence of a source of constant power supplying energy. Which of the diagrams shown in figure correctly shown the displacement-time curve for its motion? NCERT Page-128 / N-83

(a) 
(b) 
(c) 
(d) 

12. Which of the diagrams shown in figure most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit? NCERT Page-117 / N-74

(a) 
(b) 
(c) 
(d) 

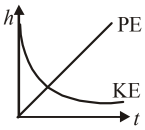
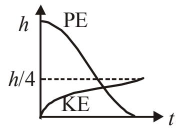
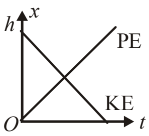
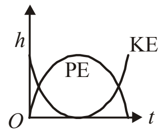
13. Which of the diagrams shown in figure represents variation of total mechanical energy of a pendulum oscillating in air as function of time? NCERT Page-121 / N-78

(a) 
(b) 
(c) 
(d) 

14. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 rev/min, its kinetic energy would be NCERT Page-117 / N-74

(a) $250\,\pi^2$ (b) $100\,\pi^2$
(c) $5\,\pi^2$ (d) 0

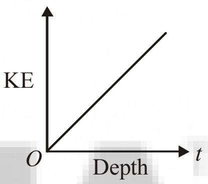
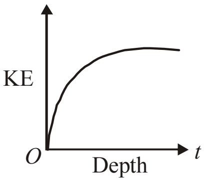
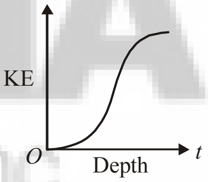
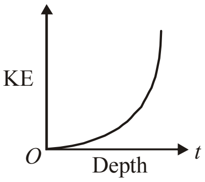
15. A raindrop falling from a height h above ground, attains a near terminal velocity when it has fallen through a height $(3/4)h$. Which of the diagrams shown in figure correctly shows the change in kinetic and potential energy of the drop during its fall up to the ground? NCERT Page-121 / N-78

(a) 
(b) 
(c) 
(d) 

16. In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1 m s^{-1} at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m s^{-2}, the kinetic energy of the shotput when it just reaches the ground will be NCERT Page-121 / N-78

(a) 2.5 J (b) 5.0 J
(c) 52.5 J (d) 155.0 J

17. Which of the diagrams in figure correctly shows the change in kinetic energy of an iron sphere falling freely in a lake having sufficient depth to impart it a terminal velocity? NCERT Page-117 / N-74

(a) 
(b) 
(c) 
(d) 

18. A cricket ball of mass 150 g moving with a speed of 126 km/h hits at the middle of the bat, held firmly at its position by the batsman. The ball moves straight back to the bowler after hitting the bat. Assuming that collision between ball and bat is completely elastic and the two remain in contact for 0.001s, the force that the batsman had to apply to hold the bat firmly at its place would be NCERT Page-129 / N-84

(a) 10.5 N (b) 21 N
(c) 1.05×10^4 N (d) 2.1×10^4 N

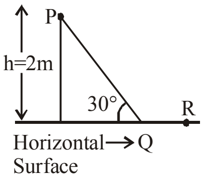
Past Years JEE Main

19. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed **v**. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to : NCERT Page-129 / N-84 | JEE M 2015, C

(a) 56% (b) 62% (c) 44% (d) 50%

20. A point particle of mass m, moves long the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The value of the coefficient of friction μ and the distance x (= QR), are, respectively close to :

NCERT Page-116, 117 / N-74 | JEE M 2016, A, BN



(a) 0.29 and 3.5 m
(b) 0.29 and 6.5 m
(c) 0.2 and 6.5 m
(d) 0.2 and 3.5 m

21. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take g = 9.8 ms^{-2} :

NCERT Page-120, 128 / N-83 | JEE M 2016, S

(a) 9.89×10^{-3} kg
(b) 12.89×10^{-3} kg
(c) 2.45×10^{-3} kg
(d) 6.45×10^{-3} kg

22. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 second will be

NCERT Page-119 / N-76 | JEE M 2017, C

(a) 9 J (b) 18 J (c) 4.5 J (d) 22 J

23. A body of mass m = 10^{-2} kg is moving in a medium and experiences a frictional force F = $-kv^2$. Its intial speed is v_0 = 10 ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be:

NCERT Page-119 / N-76 | JEE M 2017, C

(a) 10^{-4} kg m^{-1}
(b) 10^{-1} kg m^{-1} s^{-1}
(c) 10^{-3} kg m^{-1}
(d) 10^{-3} kg s^{-1}

24. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is:

NCERT Page-129, 130 / N-84 | JEE M 2018, A

(a) $\frac{v_0}{4}$ (b) $\sqrt{2}v_0$ (c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

25. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively:

NCERT Page-129, 130 / N-84 | JEE M 2018, S

(a) (·89, ·28) (b) (·28, ·89) (c) (0, 0) (d) (0, 1)

26. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be:

NCERT Page-116 / N-73 | JEE M 2019, S

(a) $\left(\frac{1}{n}\right)^{th}$ (b) $\frac{MgL}{n^2}$ (c) $\frac{2MgL}{n^2}$ (d) nMgL

27. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, g = 10 ms^{-2})

NCERT Page-128 / N-83 | JEE M 2020, A

(a) 1.7 ms^{-1} (b) 1.9 mjs^{-1} (c) 1.5 ms^{-1} (d) 2.0 ms^{-1}

28. A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is $x : y$, where x is __________.

NCERT Page-131 / N-85 | JEE M 2021, A

29. A particle of mass 500 gm is moving in a straight line with velocity $\upsilon = b\,x^{5/2}$. The work done by the net force during its displacement from $x = 0$ to $x = 4$ m is : (Take $b = 0.25\ m^{-3/2}\ s^{-1}$).

NCERT Page-118 / N-75 | JEE M 2022, A

(a) 2 J (b) 4 J (c) 8 J (d) 16 J

30. The ratio of powers of two motors is $\frac{3\sqrt{x}}{\sqrt{x}+1}$, that are capable of raising 300 kg water in 5 minutes and 50 kg water in 2 minutes respectively from a well of 100 m deep. The value of x will be

NCERT Page-128 / N-83 | JEE M 2023, S

(a) 2 (b) 4 (c) 2.4 (d) 16

31. Two bodies are having kinetic energies in the ratio 16 : 9. If they have same linear momentum, the ratio of their masses respectively is:

NCERT Page-117 / N-74 | JEE M 2023, A

(a) 4 : 3 (b) 3 : 4 (c) 16 : 9 (d) 9 : 16

32. Two bodies of mass 4 g and 25 g are moving with equal kinetic energies. The ratio of magnitude of their linear momentum is :

JEE M 2024

(a) 3 : 5
(b) 5 : 4
(c) 2 : 5
(d) 4 : 5

Exercise 3 : Matching, Statement & Assertion-Reason Type

Match the Following

1. A small block of mass 200g is kept at the top of a an incline which is 10 m long and 3.2 m high. Match the columns

Column I	Column II
(A) Work done, to lift the block from the ground and put it at the top	(1) 6.4 J
(B) Work done to slide the block up the incline	(2) 7.2 J
(C) the speed of the block at the ground when left from the top of the incline to fall vertically	(3) 4 m/s
(D) The speed of the block at the ground when side along the incline	(4) 8 m/s

(a) (A)→(2); (B)→(3); C→(1); (D)→(4)
(b) (A)→(1); (B)→(1); C→(3); (D)→(3)
(c) (A)→(4); (B)→(3); C→(2); (D)→(2)
(d) (A)→(1); (B)→(3); C→(1); (D)→(2)

2. If W represents the work done, then match the two columns:

Column I	Column II
(A) Force is always along the velocity	(1) W = 0
(B) Force is always perpendicular to velocity	(2) W < 0
(C) Force is always opposite to velocity	(3) W > 0
(D) The object is stationary but the point of application of the force moves on the object	

(a) (A)→(1); (B)→(2); C→(3); (D)→(2)
(b) (A)→(3); (B)→(1); C→(2); (D)→(1)
(c) (A)→(2); (B)→(3); C→(1); (D)→(2)
(d) (A)→(1); (B)→(2); C→(3); (D)→(1)

3.

Column I	Column II
(A) Kinetic Energy	(1) Stretched spring
(B) Potential Energy	(2) Watt
(C) Collision	(3) Elastic or inelastic
(D) Power	(4) A boy running on the roof

(a) (A)→(2); (B)→(3); C→(1); (D)→(4)
(b) (A)→(1); (B)→(1); C→(3); (D)→(3)
(c) (A)→(4); (B)→(3); C→(2); (D)→(2)
(d) (A)→(4); (B)→(1); C→(3); (D)→(2)

Two-Statement Type Questions

Directions: *Read the statements carefully and answer the question on the basis of following options.*

(a) Both statement I and II are correct.
(b) Both statement I and II are incorrect.
(c) Statement I is correct but statement II is incorrect.
(d) Statement II is correct but statement I is incorrect.

4. **Statement I:** The work done in moving a body over a closed loop is zero for every force in nature.
Statement II : Work done depends on nature of force.

5. **Statement I :** If collision occurs between two elastic bodies their kinetic energy decreases during the time of collision.
Statement II : During collision intermolecular space decreases and hence elastic potential energy increases.

6. **Statement I :** In an elastic collision of two billiard balls, the total kinetic energy is conserved during the short time of collision of the balls (*i.e.*, when they are in contact).
Statement II : Energy spent against friction follow the law of conservation of energy.

Four/Five Statement Type Questions

7. Which of the following statements are incorrect?
I. If there were no friction, work need to be done to move a body up an inclined plane is zero.
II. Kinetic energy. $Ek = \frac{1}{2} mv^2$
III. As the angle of inclination is increased, the normal reaction on the body placed on it increases.
IV. A duster weighing 0.5 kg is pressed against a vertical board with a force of 11 N. If the coefficient of friction is 0.5, the work done in rubbing it upward through a distance of 10 cm is 0.55J.
(a) I and II (b) I, II and IV
(c) I, III and IV (d) I, II, III and IV

8. Which of the following statements are incorrect ?
I. If there were no friction, work need to be done to move a body up an inclined plane is zero.
II. If there were no friction, moving vehicles could not be stopped even by locking the brakes.
III. As the angle of inclination is increased, the normal reaction on the body placed on it increases.
IV. A duster weighing 0.5 kg is pressed against a vertical board with a force of 11 N. If the coefficient of friction is 0.5, the work done in rubbing it upward through a distance of 10 cm is 0.55J.
(a) I and II (b) I, II and IV
(c) I, III and IV (d) I, II, III and IV

9. A force $F(x)$ is conservative, if
I. there is change in kinetic energy over a round trip.
II. it depends only on the end points.
III. work done by $F(x)$ in a closed path is zero.
IV. it depends on the path taken.
Which of the following option is correct ?
(a) Only I (b) I and III
(c) II and IV (d) II and III

10. Consider the following statements and select the correct statements.
I. Area under force- displacement curve with proper algebraic sign represents work done by the force.

II. Conservation of mechanical energy is a consequence of work energy theorem for conservative forces
III. Work energy theorem holds in all inertial frames
IV. Area under force-displacement gives kinetic energy.

(a) I and II (b) II and III
(c) I and IV (d) I, II and III

11. In elastic collision,
I. initial kinetic energy is equal to the final kinetic energy.
II. kinetic energy during the collision time Δt is constant.
III. total momentum is conserved.
IV. kinetic energy during the collision time Δt is not constant.

Which of the above statements is/are correct ?

(a) Only I (b) I, III and IV
(c) Only III (d) Only II

Assertion & Reason Questions

Directions : *These questions consist of two statements, each printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.*

(a) If both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) If both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
(c) If the Assertion is correct but Reason is incorrect.
(d) If the Assertion is incorrect and Reason is correct.

12. Assertion : A force applied on the body always does work on the body.

Reason : If a force applied on a body displaces the body along the direction of force work done will be minimum.

13. Assertion : The change in kinetic energy of a particle is equal to the work done on it by the net force.

Reason : Change in kinetic energy of particle is equal to the work done only in case of a system of one particle.

14. Assertion: A light body and heavy body have same momentum. Then they have same kinetic energy.

Reason: Kinetic energy does not depend on mass of the body.

15. Assertion : When a machine gun fires n bullets per second each with kinetic energy K, the power of a gun is $P = nK$

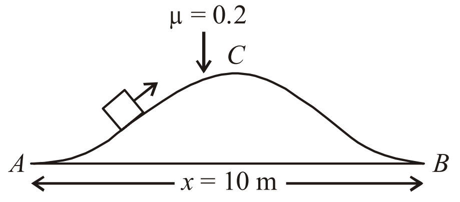
Reason : Power P = work done / time also work done = change in kinetic energy.

16. Assertion : A point particle of mass m moving with speed V collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{M}{M+m}\right)$.

Reason : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

Exercise 4 : Skill Enhancer MCQs

1. A block of mass 1 kg is pulled along the curve path ACB by a tangential force as shown in figure. The work done by the frictional force when the block moves from A to B is



(a) 5 J (b) 10 J
(c) 20 J (d) None of these

2. A bullet looses $\left(\frac{1}{n}\right)^{th}$ of its velocity passing through one plank. The number of such planks that are required to stop the bullet can be:

(a) $\frac{n^2}{2n-1}$ (b) $\frac{2n^2}{n-1}$ (c) infinite (d) n

3. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it untill the string makes an angle of 45° with the initial vertical direction is :

(a) $Mg(\sqrt{2}-1)$ (b) $Mg(\sqrt{2}+1)$
(c) $Mg\sqrt{2}$ (d) $\frac{Mg}{\sqrt{2}}$

4. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$ where k is a constant. The power delivered to the particles by the force acting on it is

(a) $2\pi mk^2r^2t$ (b) mk^2r^2t
(c) $\frac{(mk^4r^2t^5)}{3}$ (d) zero

5. A man of mass m on an initially stationary boat gets off the boat by jumping to the left in an exactly horizontal direction. Immediately after the jump, the boat of mass M, is observed to be moving to the right at speed v. How much work did the man do during the jump (both on his own body and on the boat)

(a) $\frac{1}{2}(M+m)v^2$ (b) $\frac{1}{2}\left(M+\frac{M^2}{m}\right)v^2$
(c) $\frac{1}{2}\left(\frac{Mm}{M+m}\right)v^2$ (d) None of these

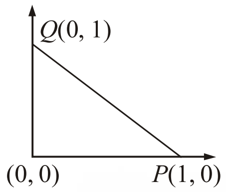
6. The potential energy of particle in a force field is $U = \frac{A}{r^2} - \frac{B}{r}$, where A and B are positive constants and r is the distance of particle from the centre of the field. For stable equilibrium, the distance of the particle is

(a) $B/2A$ (b) $2A/B$ (c) A/B (d) B/A

7. An electric pump is used to fill an overhead tank of capacity $9 m^3$ kept at a height of 10m above the ground. If the pump takes 5 minutes to fill the tank by consuming 10 kW power the efficiency of the pump should be ($g = 10\ ms^{-2}$)

(a) 60% (b) 40% (c) 20% (d) 30%

8. Consider a force $F = -x\hat{i} + y\hat{j}$. The workdone by this force in moving a particle from point $P(1, 0)$ to $Q(0, 1)$ along the line segment is (all quantities are in SI units).



(a) $\frac{1}{2}$J (b) 1J (c) $\frac{3}{2}$J (d) zero

9. A car of mass m starts from rest and accelerates so that the instantaneous power delivered to the car has a constant magnitude P_0. The instantaneous velocity of this car is proportional to :

(a) $t^2 P_0$ (b) $t^{1/2}$ (c) $t^{-1/2}$ (d) $\frac{t}{\sqrt{m}}$

10. A body of mass 5 kg explodes at rest into three fragments with masses in the ratio 1 : 1 : 3. The fragments with equal masses fly in mutually perpendicular directions with speeds of 21 m/s. The velocity of heaviest fragment in m/s will be

(a) $7\sqrt{2}$ (b) $5\sqrt{2}$ (c) $3\sqrt{2}$ (d) $\sqrt{2}$

11. A stationary particle explodes into two particles of masses m_1 and m_2 which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies E_1/E_2 is

(a) $m_1 v_2 / m_2 v_1$ (b) m_2/m_1
(c) m_1/m_2 (d) 1

12. A bullet is fired and gets embedded in block kept on table. If table is frictionless, then

(a) kinetic energy gets conserved
(b) potential energy gets conserved
(c) momentum gets conserved
(d) both (a) and (c)

13. The speed of an object of mass m dropped from an inclined plane (frictionless), at the bottom of the plane, depends on:

(a) height of the plane above the ground
(b) angle of inclination of the plane
(c) mass of the object
(d) All of these

14. If two particles are brought near one another, the potential energy of the system will

(a) increase (b) decrease
(c) remains the same (d) equal to the K.E

15. A crate is pushed horizontally with 100 N across a 5 m floor. If the frictional force between the crate and the floor is 40 N, then the kinetic energy gained by the crate is

(a) 200 J (b) 240 J (c) 250 J (d) 300 J

16. A 2 kg block slides on a horizontal floor with a speed of 4m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15N and spring constant is 10,000 N/m. The spring compresses by

(a) 8.5 cm (b) 5.5 cm (c) 2.5 cm (d) 11.0 cm

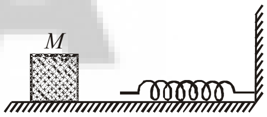
17. The kinetic energy of particle moving along a circle of radius R depends upon the distance covered S and is given by K = aS where a is a constant. Then the force acting on the particle is

(a) $\frac{aS}{R}$ (b) $\frac{2(aS)^2}{R}$ (c) $\frac{aS^2}{R^2}$ (d) $\frac{2aS}{R}$

18. A uniform chain of length 2 m and mass 0.1 kg overhangs a smooth table with its two third part lying on the table. Find the kinetic energy of the chain as it completely slips-off the table.

(a) $\frac{8}{9}$J (b) $\frac{12}{5}$J (c) $\frac{3}{7}$J (d) $\frac{11}{3}$J

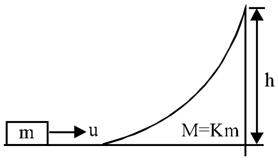
19. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is



(a) $\frac{kL^2}{2M}$ (b) $\sqrt{Mk}\ L$

(c) $\frac{ML^2}{k}$ (d) zero

20. A block of mass m is moved towards a movable wedge of mass M = km and height h with velocity u (All the surface are smooth).



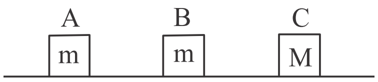
If the block just reaches the top of the wedge, the value of u is

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{2ghK}{1+K}}$

(c) $\sqrt{\frac{2gh(1+K)}{K}}$ (d) $\sqrt{2gh\left(1-\frac{1}{K}\right)}$

Exercise 5 : Numeric Value Answer Questions

1. There block A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an inital speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically and $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. If the value of M/m is $\frac{4}{x}$. Find the value of x.



2. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done (in joule) by this force in first 5 seconds?

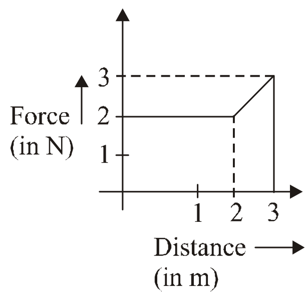
3. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height (in metre) to which the combined system reaches above the top of the building before falling below is: (g = 10 ms^{-2})

4. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy (in joule) at the end of the displacement?

5. A body of mass 1 kg falls freely from a height of 100m, on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x cm. Given that $g = 10\ ms^{-2}$, the value of x will be close to :

6. An alpha-particle of mass m suffers one-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is x m. Find the value of x.

7. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy (in joule) of the particle after it has travelled 3 m is :



8. A ball is dropped from a height of 100 m. After striking the earth, 20% of its energy is lost. To what height (in metre) the ball will rise?

9. A force of 5 N making an angle θ with the horizontal, acting on an object displaces it by 0.4m along the horizontal direction. If the object gains kinetic energy of 1J, the horizontal component of the force (in newton) is

10. A motor of 100 H.P. moves a load with a uniform speed of 72 km/h. The forward thrust (in newton) applied by the engine of the car is

11. Four smooth steel balls of equal mass at rest are free to move along a straight line without friction. The first ball is given a velocity of 0.4 m/s. It collides head on with the second elastically, the second one similarly with the third and so on. The velocity (in m/s) of the last ball is

12. A shell of mass 20 kg at rest explodes into two fragments whose masses are in the ratio 2 : 3. The smaller fragment moves with a velocity of 6 m/s. The kinetic energy (in joule) of the larger fragment is

13. A ball drops from a celling of a room and after rebounding twice from the floor reaches a height equal to half that of the celling. If the coefficient of restitution is $\left(\frac{1}{2}\right)^{1/x}$. Find the value of x.

14. A block of mass 10 kg is moving along x-axis under the action of force F = 5x N. The work done by the force in moving the block from x = 2 m to 4 m will be ________ J.

15. A particle of mass 10 g moves in a straight line with retardation 2x, where x is the displacement in SI unts. Its loss of kinetic energy for above displacement is $\left(\frac{10}{x}\right)^{-n}$ J. The value of n will be ______.

Answer Keys

Exercise 1 : (NCERT Based Topic-wise MCQs)																			
1	(c)	**13**	(c)	**25**	(d)	**37**	(b)	**49**	(a)	**61**	(b)	**73**	(a)	**85**	(b)	**97**	(b)	**109**	(b)
2	(b)	**14**	(b)	**26**	(b)	**38**	(c)	**50**	(d)	**62**	(b)	**74**	(a)	**86**	(d)	**98**	(b)	**110**	(a)
3	(c)	**15**	(c)	**27**	(c)	**39**	(a)	**51**	(d)	**63**	(c)	**75**	(a)	**87**	(b)	**99**	(d)	**111**	(d)
4	(b)	**16**	(a)	**28**	(b)	**40**	(b)	**52**	(d)	**64**	(c)	**76**	(d)	**88**	(a)	**100**	(c)	**112**	(d)
5	(a)	**17**	(a)	**29**	(d)	**41**	(c)	**53**	(d)	**65**	(c)	**77**	(c)	**89**	(b)	**101**	(c)	**113**	(a)
6	(d)	**18**	(c)	**30**	(c)	**42**	(c)	**54**	(c)	**66**	(c)	**78**	(a)	**90**	(b)	**102**	(b)	**114**	(c)
7	(b)	**19**	(d)	**31**	(d)	**43**	(b)	**55**	(b)	**67**	(a)	**79**	(d)	**91**	(d)	**103**	(c)	**115**	(a)
8	(c)	**20**	(c)	**32**	(c)	**44**	(d)	**56**	(c)	**68**	(a)	**80**	(d)	**92**	(b)	**104**	(a)	**116**	(a)
9	(a)	**21**	(d)	**33**	(b)	**45**	(a)	**57**	(b)	**69**	(b)	**81**	(d)	**93**	(a)	**105**	(a)	**117**	(b)
10	(d)	**22**	(c)	**34**	(c)	**46**	(b)	**58**	(d)	**70**	(a)	**82**	(a)	**94**	(c)	**106**	(c)	**118**	(b)
11	(c)	**23**	(c)	**35**	(c)	**47**	(a)	**59**	(d)	**71**	(c)	**83**	(d)	**95**	(c)	**107**	(c)	**119**	(b)
12	(b)	**24**	(a)	**36**	(d)	**48**	(b)	**60**	(b)	**72**	(a)	**84**	(b)	**96**	(d)	**108**	(a)	**120**	(b)
Exercise 2 : (NCERT Exemplar & Past Years JEE Main)																			
1	(b)	**5**	(c)	**9**	(b)	**13**	(c)	**17**	(b)	**21**	(b)	**25**	(a)	**29**	(d)				
2	(c)	**6**	(c)	**10**	(b)	**14**	(a)	**18**	(c)	**22**	(c)	**26**	(b)	**30**	(d)				
3	(d)	**7**	(c)	**11**	(b)	**15**	(b)	**19**	(a)	**23**	(a)	**27**	(b)	**31**	(d)				
4	(c)	**8**	(b)	**12**	(d)	**16**	(d)	**20**	(a)	**24**	(b)	**28**	(1)	**32**	(c)				
Exercise 3 : (Matching, Statement & Assertion-Reason Type)																			
1	(b)	**3**	(d)	**5**	(b)	**7**	(c)	**9**	(d)	**11**	(b)	**13**	(c)	**15**	(a)				
2	(b)	**4**	(b)	**6**	(d)	**8**	(c)	**10**	(d)	**12**	(d)	**14**	(d)	**16**	(a)				
Exercise 4 : (Skill Enhancer MCQs)																			
1	(c)	**3**	(a)	**5**	(b)	**7**	(d)	**9**	(b)	**11**	(b)	**13**	(a)	**15**	(d)	**17**	(d)	**19**	(b)
2	(a)	**4**	(b)	**6**	(b)	**8**	(b)	**10**	(a)	**12**	(c)	**14**	(a)	**16**	(b)	**18**	(a)	**20**	(c)
Exercise 5 : (Numeric Value Answer Questions)																			
1	(1)	**3**	(40)	**5**	(4)	**7**	(6.5)	**9**	(2.5)	**11**	(0.4)	**13**	(4)	**15**	(2)				
2	(900)	**4**	(15)	**6**	(4)	**8**	(80)	**10**	(3730)	**12**	(96)	**14**	(30)						

5 Work, Energy and Power

EXERCISE-1

1. **(c)** The dot product should be zero.
2. **(b)**
3. **(c)** $S = \vec{r}_2 - \vec{r}_1 = (14\hat{i}+13\hat{j}+9\hat{k})-(3\hat{i}+2\hat{j}-6\hat{k})$

   $W = \vec{F}\cdot\vec{S} = (4\hat{i}+\hat{j}+3\hat{k})\cdot(11\hat{i}+11\hat{j}+15\hat{k})$

   $= 4\times 11 + 1\times 11 + 3\times 15 = 100\text{J}$
4. **(b)** For two vectors to be perpendicular to each other

   $\vec{A}\cdot\vec{B} = 0$

   $(2\hat{i}+3\hat{j}+8\hat{k})\cdot(4\hat{j}+4\hat{i}+\alpha\hat{k}) = 0$

   $-8 + 12 + 8\alpha = 0$ or $\alpha = -\frac{4}{8} = -\frac{1}{2}$
5. **(a)** Work done by the net force = change in kinetic energy of the particle.
   This is according to work energy theorem.
6. **(d)** Work done on the body is gain in the kinetic energy. Acceleration of the body is a = V/T.

   Velocity acquired in time t is $v = at = \frac{V}{T}t$

   K.E. acquired $\propto v^2$. That is work done $\propto \frac{V^2t^2}{T^2}$
7. **(b)** When force retards motion *i.e.*, F –(ve) so, work done –(ve)
8. **(c)** When a man pushes a wall and fails to displace it, then displacement of wall = 0
   $\therefore$ Work done by man = $F \times 0 = 0$
   Therefore, man does no work at all.
9. **(a)** When a person carrying load on his head moves over a horizontal road, work done against gravitational force is zero.
10. **(d)** $W = FS\cos\theta$
    $\therefore$ If $F = 0$; $W = 0$
    If $S = 0$; $W = 0$
    & if $\theta = 90°$; $\cos 90° = 0$ $\therefore$ $W = 0$.
11. **(c)** Displacement of the particle when it takes a complete round the circular path is zero.
    $\therefore$ Work done = force × displacement
    $W = F \times 0 = 0$
    Therefore, work done by the force is zero.
12. **(b)** From work-energy theorem,

    $W_{Porter} + W_{mg} = \Delta K.E. = 0$ ($\because$ velocity constant)

    or, $W_{Porter} = -W_{mg} = -mgh$

    $\therefore$ $W_{Porter} = -80\times 9.8\times\frac{80}{100} = -627.2\text{J}$
13. **(c)** Work done, $W = \int \vec{F}\cdot\vec{ds} = (-x\hat{i}+y\hat{j})\cdot(d\times\hat{i}+dy\hat{j})$

    $\Rightarrow W = -\int_1^0 x\,dx + \int_0^1 y\,dy = \left(0+\frac{1}{2}\right)+\frac{1}{2} = 1J$
14. **(b)** (A)→(3); (B)→(1); C→(2,3); (D)→(1)
15. **(c)** $W = \vec{F}.\vec{s} = (5\hat{i}+2\hat{j}).(2\hat{i}+3\hat{j}) = 10+6 = 16$ J.
16. **(a)** W = F s cos 90º = zero
17. **(a)** $x = 3t - 4t^2 + t^3$ $\frac{dx}{dt} = 3 - 8t + 3t^2$

    Acceleration $= \frac{d^2x}{dt^2} = -8 + 6t$

    Acceleration after 4 sec = $-8 + 6 \times 4 = 16$
    Displacement in 4 sec = $3\times 4 - 4\times 4^2 + 4^3 = 12$ m
    $\therefore$ Work = Force × displacement
    = Mass × acc. × disp. = $3\times 10^{-3}\times 16\times 12 = 576$ mJ
18. **(c)** As the cord is trying to hold the motion of the block, work done by the cord is negative.

    $W = -M(g-a)d = -M\left(g-\frac{g}{4}\right)d = \frac{-3Mgd}{4}$
19. **(d)** Though an equal and opposite force acts on the road but since road does not undergo any displacement, hence no work is done on the road.
20. **(c)** Motion without slipping implies pure rolling. During pure rolling work done by friction force is zero.
21. **(d)** Given : $\vec{F} = 3\hat{i}+\hat{j}$

    $\vec{r}_1 = (2\hat{i}+\hat{k})$, $\vec{r}_2 = (4\hat{i}+3\hat{j}-\hat{k})$

    $\vec{r} = \vec{r}_2 - \vec{r}_1 = (4\hat{i}+3\hat{j}-\hat{k}) - (2\hat{i}+\hat{k})$

    or $\vec{r} = 2\hat{i}+3\hat{j} - 2\hat{k}$

    So work done by the given force $w = \vec{f}.\vec{r}$

    $= (3\hat{i}+\hat{j}).(2\hat{i}+3\hat{j}-2\hat{k}) = 6+3 = 9\text{J}$
22. **(c)** $W = F s\cos\theta = 10\times 2\cos 60º = 10$ J.
23. **(c)** $W = F s\cos\theta$, $\cos\theta = \frac{W}{Fs} = \frac{25}{5\times 10} = \frac{1}{2}$, $\theta = 60º$.
24. **(a)** Since momentum of both bodies are equal

    So $p_1 = p_2 \Rightarrow \frac{M_1}{M_2} = \frac{u_2}{u_1} \Rightarrow u_2 > u_1$ (let $M_1 > M_2$)

    so $\frac{E_{k_1}}{E_{k_2}} = \frac{P_1^2/2M_1}{P_2^2/2M_2} = \frac{M_2}{M_1} \Rightarrow E_{k_1} < E_{k_2}$

    It means that light body has greater kinetic energy, if they have equal momentum.
25. **(d)** Let the velocity of the particle be *v m/s*.
    Momentum of the particle (p) = mv
    Kinetic energy of the particle

    $(E) = \frac{1}{2}mv^2 = \frac{1}{2}.\frac{(mv)^2}{m} \Rightarrow E = \frac{p^2}{2m}$

26. **(b)** K. E = $\frac{1}{2}mv^2$ It is always positive

27. **(c)** Initial momentum (p_1) = p; Final momentum (p_2) = 1.5 p and initial kinetic energy (K_1) = K.

Kinetic energy $(K) = \frac{p^2}{2m} \propto p^2$

or, $\frac{K_1}{K_2} = \left(\frac{p_1}{p_2}\right)^2 = \left(\frac{p}{1.5p}\right)^2 = \frac{1}{2.25}$ or, $K_2 = 2.25\,K$.

Therefore, increase in kinetic energy is 2.25 K – K = 1.25 K or 125%.

28. **(b)** $E = \frac{P^2}{2m} \therefore E \propto \frac{1}{m}$ [If P = constant]

i.e., the lightest particle will possess maximum kinetic energy and in the given option mass of electron is minimum.

29. **(d)** $\text{K.E.} = \frac{1}{2}\text{mv}^2$

Further, $v^2 = u^2 + 2as = 0 + 2ad = 2ad \; = 2(F/m)d$

Hence, $\text{K.E.} = \frac{1}{2}m \times 2(F/m)d = Fd$

or, K.E. acquired = Work done = F × d = constant.

i.e., it is independent of mass m.

30. **(c)** Let m = mass of boy, M = mass of man

v = velocity of boy, V = velocity of man

$\frac{1}{2}MV^2 = \frac{1}{2}\left[\frac{1}{2}mv^2\right]$ ...(i)

$\frac{1}{2}M(V+1)^2 = 1\left[\frac{1}{2}mv^2\right]$ ...(ii)

Putting $m = \frac{M}{2}$ and solving $V = \frac{1}{\sqrt{2}-1}$

31. **(d)** By law of conservation of mechanical energy

$\Delta k = -\Delta U$

$\Rightarrow k_f - k_i = U_i - U_f$

$\Rightarrow k_f = mgy - mg[y - y_0]$

$[\because k_i = 0,\ U_i = mgy \text{ and } U_f = mg(y - y_0)]$

$\Rightarrow k_f = mgy_0$

32. **(c)** As $\frac{1}{2}m_A v_A^2 = \frac{1}{2}m_B v_B^2$

$\frac{v_A}{v_B} = \sqrt{\frac{m_B}{m_A}}$;

$\frac{P_B}{P_A} = \frac{m_B v_B}{m_A v_A} = \frac{m_B}{m_A}\sqrt{\frac{m_A}{m_B}} = \sqrt{\frac{m_B}{m_A}} = \frac{1}{\sqrt{3}}$

33. **(b)** At the top of flight, horizontal component of velocity $= u \cos 45° = u/\sqrt{2}$

$\therefore \text{K.E.} = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}\left(\frac{mu^2}{2}\right) = \frac{1}{2}K.$

34. **(c)** The uniform acceleration is $a = \frac{1-0}{15} = \frac{1}{15}\ \text{ms}^{-2}$

Let v be the velocity at kinetic energy $\frac{2}{9}$ J

therefore $\frac{1}{2} \times 1 \times v^2 = \frac{2}{9}$ or $v = \frac{2}{3}\ \text{ms}^{-1}$

Using v = u + at

$\frac{2}{3} = 0 + \frac{1}{15} \times t \Rightarrow t = 10s$

35. **(c)** $v_2 = \frac{-m_1 v_1}{m_2} = \frac{-3}{6} \times 16 = -8\ \text{m/s}$

$E_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2} \times 6(-8)^2 = 192\ \text{J}$

36. **(d)** The average speed of the athelete

$v = \frac{100}{10} = 10\ \text{m/s}$

$\therefore \text{K.E.} = \frac{1}{2}mv^2$

If mass is 40 kg then, $\text{K.E.} = \frac{1}{2} \times 40 \times (10)^2 = 2000\ \text{J}$

If mass is 100 kg then,

$\text{K.E.} = \frac{1}{2} \times 100 \times (10)^2 = 5000\ \text{J}$

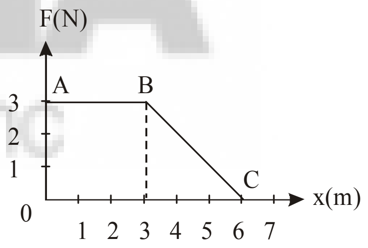
37. **(b)** $W = \int_0^{x_1} F\,dx = \int_0^{x_1} c\,x\,dx = \left[\frac{1}{2}cx^2\right]_0^{x_1}$

$= \frac{1}{2}c(x_1^2 - 0) = \frac{1}{2}cx_1^2$

38. **(c)**

39. **(a)**

40. **(b)**



Work done = area under F-x graph

= area of trapezium OABC = $\frac{1}{2}(3+6)(3) = 13.5$ J

41. **(c)** W = area of $F - x$ graph

= area of Δ + area of rectangle + area of Δ

$= \frac{5 \times 3}{2} + 10 \times 3 + \frac{5 \times 3}{2} = 45\ J$

42. **(c)** We know area under F-x graph gives the work done by the body

$\therefore W = \frac{1}{2} \times (3+2) \times (3-2) + 2 \times 2 = 2.5 + 4 = 6.5\ \text{J}$

Using work energy theorem,

Δ K.E = work done

$\therefore \Delta$K.E = 6.5 J

43. **(b)** By work-energy thorem

$W = \Delta K$

$\Rightarrow W = \frac{1}{2}m\,(v_f^2 - v_i^2) \Rightarrow W = \frac{1}{2}\times 0.5\times(16^2 - 4^2)$

$\Rightarrow W = \frac{1}{4}\times 240 \Rightarrow W = 60$ J

44. **(d)**

45. **(a)** Loss in K.E = Area under the curve $F - x$ = work done

46. **(b)**

47. **(a)** Conservative force is negative gradient of potential

$F(x) = \frac{-dV(x)}{dx}$

48. **(b)** Energy required = mgh

In both cases, h is the same. Hence energy both is the same.

49. **(a)** $U_1 = mgh_1$ and $U_2 = mgh_2$

% energy lost = $\frac{U_1 - U_2}{U_1}\times 100$

$= \frac{mgh_1 - mgh_2}{mgh_1}\times 100 = \left(\frac{h_1 - h_2}{h_1}\right)\times 100$

$= \frac{2 - 1.5}{2}\times 100 = 25\%$

50. **(d)** When work is done upon a system by a conservative force then its potential energy increases.

51. **(d)** $|F| = \frac{dU}{dx}$, which is greatest in the reagion *CD*.

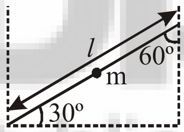
52. **(d)** For any uniform rod, the mass is supposed to be concentrated at its centre.

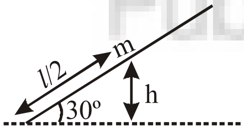
$\therefore$ height of the mass from ground is, h = (l/2) sin 30º

$\therefore$ Potential energy of the rod

$= m\times g\times\frac{\ell}{2}\sin 30^\circ$

$= m\times g\times\frac{\ell}{2}\times\frac{1}{2} = \frac{mg\ell}{4}$





53. **(d)** $F = -\frac{dU}{dy} = b - 2ay$

54. **(c)** $U = (1/2)Mv^2$

55. **(b)** The principle of conservation of total mechanical energy can be stated as, the total mechanical energy of a system is conserved if the forces, doing work on it, are conservative.

56. **(c)** Weight Mg moves the centre of gravity of the spring through a distance $\frac{(0+\ell)}{2} = \ell/2$

$\therefore$ Mechanical energy stored = Work done = Mg ℓ/2.

57. **(b)** $v^2 = u^2 + 2gh = (10)^2 + 2\times 10\times 19.5 = 490$

K.E. at the ground

$= \frac{1}{2}mv^2 = \frac{1}{2}\times\frac{5}{1000}\times 490 = \frac{49}{40}$ J

$P.E. = mgh = \frac{5}{1000}\times 10\times\left(\frac{-50}{100}\right) = -\frac{1}{40}$ J

$\therefore$ Change in energy $= \frac{49}{40} - \left(-\frac{1}{40}\right) = \frac{50}{40} = 1.25$J

58. **(d)**

59. **(d)** At the lowest point, h = 0 $\therefore$ P.E. = 0 (gravitational P.E.). There is no work done on the **bob** by the tension as it is perpendicular to the displacement.

$\therefore$ Potential energy is associated only to the gravitational force.

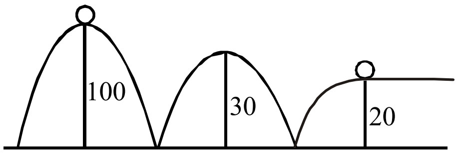
60. **(b)** Total energy at the time of projection

$= \frac{1}{2}m\,v^2 = \frac{1}{2}\times 0.1(20)^2 = 20$J

Half way up, P.E. becomes half the P.E. at the top i.e.

$P.E. = \frac{20}{2} = 10$J $\quad\therefore$ K.E. = 20 – 10 = 10J.

61. **(b)**



Using conservation of energy,

$m(10\times 100) = m\left(\frac{1}{2}v^2 + 10\times 20\right)$

or $\frac{1}{2}v^2 = 800$ or $v = \sqrt{1600} = 40$ m/s

62. **(b)** As we know, dU = F·dr

$U = \int_0^r \alpha r^2 dr = \frac{ar^3}{3}$ ...(i)

As, $\frac{mv^2}{r} = \alpha r^2 = m^2v^2 = m\alpha r^3$

or, $2m(KE) = \frac{1}{2}\alpha r^3$ ...(ii)

Total energy = Potential energy + kinetic energy

Now, from eqn (i) and (ii)

Total energy = K.E. + P.E. $= \frac{\alpha r^3}{3} + \frac{\alpha r^3}{2} = \frac{5}{6}\alpha r^3$

63. **(c)** Let *E* be the total energy then

$\frac{P.E}{K.E} = \frac{mgh}{E - mgh} = \frac{2}{3} \Rightarrow E = \frac{5}{2}$ mgh

When velocity is double then inital energy becomes 4*E*.

So, $\frac{mgh}{4E - mgh} = NL = \frac{mgh}{10mgh - mgh}$

On solving we get $\frac{P.E}{K.E} = \frac{1}{9}$.

64. **(c)** The tension T_1 at the topmost point is given by

$T_1 = \frac{m\,v_1^2}{20} - mg$

Centrifugal force acting outward while weight acting downward.

The tension T_2 at the lowest point $T_2 = \frac{mv_2^2}{20} + mg$

Centrifugal force and weight (both) acting downward

$T_2 - T_1 = \frac{mv_2^2 - mv_1^2}{20} + 2mg$

$v_1^2 = v_2^2 - 2gh$ or $v_2^2 - v_1^2 = 2g(40) = 80g$

$\therefore\ T_2 - T_1 = \frac{80mg}{20} + 2mg = 6mg$

65. **(c)** Tension at the highest point

$T_{top} = \frac{mv^2}{r} - mg = 2mg \quad (\because v_{top} = \sqrt{3gr})$

Tension at the lowest point

$T_{bottom} = 2mg + 6mg = 8mg$

$\therefore \frac{T_{top}}{T_{bottom}} = \frac{2mg}{8mg} = \frac{1}{4}.$

66. **(c)** Tension in the string, $T = \frac{mv^2}{\ell}$

67. **(a)** Given : Mass (m) = 0.4 kg

Its frequency (n) = 2 rev/sec

Radius (r) =1.2 m. We know that linear velocity of the body (v) = ωr = (2πn)r

$= 2 \times 3.14 \times 1.2 \times 2 = 15.08$ m/s.

Therefore, tension in the string when the body is at the top of the circle (T)

$= \frac{mv^2}{r} - mg = \frac{0.4 \times (15.08)^2}{2} - (0.4 \times 9.8)$

$= 45.78 - 3.92 = 41.56N$

68. **(a)** Since water does not fall down, therefore the velocity of revolution should be just sufficient to provide centripetal acceleration at the top of vertical circle. So,

$v = \sqrt{(gr)} = \sqrt{\{10 \times (1.6)\}} = \sqrt{(16)} = 4$ m/sec.

69. **(b)** $W = \frac{1}{2}k\ell_2^2 - \frac{1}{2}k\ell_1^2 = \frac{1}{2}k(\ell_2^2 - \ell_1^2)$

70. **(a)** $W_1 = \frac{1}{2} \times 5 \times 10^3 (0.05)^2 \Rightarrow W_2 = \frac{1}{2} \times 5 \times 10^3 (0.10)^2$

$\therefore\ \Delta W = \frac{1}{2} \times 5 \times 10^3 \times 0.15 \times 0.05 = 18.75J.$

71. **(c)** $l_1 + l_2 = l$ and $l_1 = nl_2$

$\therefore\ l_1 = \frac{nl}{n+1}$ and $l_2 = \frac{l}{n+1}$

As $k \propto \frac{1}{l}$, $\therefore\ \frac{k_1}{k_2} = \frac{l/(n+1)}{(nl)/(n+1)} = \frac{1}{n}$

72. **(a)** We define the potential energy V(x) of the spring to be zero when block and spring system is in the equilibrium position.

73. **(a)** For a given spring, $u = \frac{1}{2}kx^2$

$\therefore\ \frac{u_2}{u_1} = \frac{\frac{1}{2}Kx_2^2}{\frac{1}{2}Kx_1^2} = \frac{(3x)^2}{x^2} = 9:1$

74. **(a)** $U = -\int_0^x Fdx = -\int_0^x kxdx = -\frac{1}{2}kx^2.$

It is correctly drawn in (a)

75. **(a)** $\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow mv^2 = kx^2$ or $m \times (1.5)^2 = 50 \times (0.15)^2$

$\therefore\ m = 0.5$ kg

76. **(d)**

77. **(c)** From Hooke's law

$F \propto x \Rightarrow F = kx.$, where k is spring constant

Since force is same in stretching for both spring so $F = k_1x_1 = k_2x_2 \Rightarrow x_1 < x_2$ because $k_1 > k_2$

so work done in case of first spring is $W_1 = \frac{1}{2}k_1x_1^2$ and work done in case of second spring is

$W_2 = \frac{1}{2}k_2x_2^2$ so $\frac{W_1}{W_2} = \frac{x_1}{x_2} \Rightarrow W_1 < W_2$

It means that more work is done in case of second spring (work done on spring is equal to stored elastic potential energy of the spring)

78. **(a)** Stored elastic potential energy of spring =½kx^2 where x is compression or elongation of spring from its natural length. In this position the spring can do work on the block tied to it, which is equal to ½kx^2, so both option (a) & b are correct.

79. **(d)** P.E. = $\frac{1}{2}kx^2$

$\therefore$ If x = 4x, then P.E = $\frac{1}{2}k(16x^2) = 16\left(\frac{1}{2}kx^2\right)$

80. **(d)** $V = \frac{1}{2}k(x)^2 = \frac{1}{2}k(2)^2$ or $k = \frac{2V}{4} = \frac{V}{2}$

$V' = \frac{1}{2}k(10)^2 = \frac{1}{2} \times \left(\frac{V}{2}\right)(10)^2 = 25V$

81. **(d)** Given, $\frac{dk}{dt}$ = constant

$\Rightarrow\ k \propto t \ \Rightarrow\ v \propto \sqrt{t}$

Also, $P = Fv = \frac{dk}{dt}$ = constant

$\Rightarrow\ F \propto \frac{1}{v} \ \Rightarrow F \propto \frac{1}{\sqrt{t}}$

82. **(a)**

83. **(d)** $F_{thrust} = V_{rel}\frac{dm}{dt}$

$= 5 \times 0.5 = 2.5\,N$

So, Power = Force × Velocity

$= 2.5 \times 5 = 12.5$ watt.

84. **(b)** u = 0; v = u + aT; v = aT

Instantaneous power = F × v = m. a. at = m.a^2.t

$\therefore$ Instantaneous power = $m\frac{v^2}{T^2}t$

85. **(b)** Constant power of car $P_0 = F.V = ma.v$

$P_0 = m\frac{dv}{dt}.v$

$P_0 dt = mvdv$ Integrating

$P_0 .t = \frac{mv^2}{2}$ $v = \sqrt{\frac{2P_0 t}{m}}$

$\because$ P_0, m and 2 are constant $\therefore$ $v \propto \sqrt{t}$

86. **(d)**

87. **(b)** $P = \frac{W}{t}$. Here, P = 2kW = 2000 W.

W = Mgh = M × 10 × 10 = 100 M and t = 60 s.

This gives, M = 1200 kg

Its volume = 1200 litre as 1 litre of water contains 1 kg of its mass.

88. **(a)**

89. **(b)** We know that F × v = Power

$\therefore F \times v = c$ where c = constant

$\therefore m\frac{dv}{dt} \times v = c$ $\left(\therefore F = ma = \frac{mdv}{dt}\right)$

$\therefore m\int_0^v vdv = c\int_0^t dt$ $\therefore \frac{1}{2}mv^2 = ct$

$\therefore v = \sqrt{\frac{2c}{m}} \times t^{1/2}$

$\therefore \frac{dx}{dt} = \sqrt{\frac{2c}{m}} \times t^{1/2}$ where $v = \frac{dx}{dt}$

$\therefore \int_0^x dx = \sqrt{\frac{2c}{m}} \times \int_0^t t^{1/2} dt$

$x = \sqrt{\frac{2c}{m}} \times \frac{2t^{3/2}}{3}$ $\Rightarrow$ $x \propto t^{3/2}$

90. **(b)** Power exerted by a force is given by

P = F.v

When the body is just above the earth's surface, its velocity is greatest. At this instant, gravitational force is also maximum. Hence, the power exerted by the gravitational force is greatest at the instant just before the body hits the earth.

91. **(d)** The power of body is given by = $\vec{F} \cdot \vec{v}$ as the body is moving in circular path, centripetal force and velocity are at 90°, or power = 0.

92. **(b)** The work is done against gravity so it is equal to the change in potential energy. W = E_p = mgh

For a fixed height, work is proportional to weight lifted. Since Johnny weighs twice as much as Jane he works twice as hard to get up the hill.

Power is work done per unit time. For Johnny this is W/Δt. Jane did half the work in half the time, (1/2 W)/(1/2 Δt) = W/Δt which is the same power delivered by Johnny.

93. **(a)** Power, P = F.v = $m\frac{dv}{dt}.v$

As P is constant, $\frac{v}{dt}$ = constant

$\Rightarrow v \propto dt \Rightarrow v \propto t$

94. **(c)** Power = $\frac{\text{work done}}{\text{time}}$

Therefore power of A, $P_A = \frac{mgh}{t_A}$

and power of B, $P_B = \frac{mgh}{t_B}$

$\therefore$ $\frac{P_A}{P_B} = \frac{t_B}{t_A} = \frac{4}{2} = 2:1$

95. **(c)** Volume of water to raise = 22380 l = $22380 \times 10^{-3} m^3$

$P = \frac{mgh}{t} = \frac{V\rho gh}{t} \Rightarrow t = \frac{V\rho gh}{P}$

$t = \frac{22380 \times 10^{-3} \times 10^3 \times 10 \times 10}{10 \times 746} = 15$ min

96. **(d)** Power = $\frac{\text{total work done}}{\text{time}}$

$= \frac{\frac{1}{2}Mv^2}{t} = \frac{1}{2}(mv^2)n \left(\because \frac{M}{t} = mn\right)$

$= kn\left[\because K.E. K = \frac{1}{2}mv^2\right]$

97. **(b)** $\vec{F} = \frac{md\vec{v}}{dt} = 4t\hat{i} + 6t^2\hat{j}$ $(\because m = 1 \text{ kg})$

$\int_0^v dv = \int_0^t (4t\hat{i} + 6t^2\hat{j})dt \Rightarrow \vec{v} = t^2\hat{i} + t^3\hat{j}$

Power, $P = \vec{F} \cdot \vec{v}$

$= (4t\hat{i} + 6t^2 j) \cdot (t^2\hat{i} + t^3\hat{j})$

$= (4t^3 + 6t^5)$W

98. **(b)** Power = $\frac{\text{Work done}}{\text{Time}} = \frac{\frac{1}{2}m\left(v^2 - u^2\right)}{t}$

$P = \frac{1}{2} \times \frac{2.05 \times 10^6 \times \left[(25)^2 - \left(5^2\right)\right]}{5 \times 60}$

P = 2.05 × 10^6 W = 2.05 MW

99. **(d)** $m = 10 \times 0.8\text{kg} = 8\text{kg}$

height of iron chain = 5m

$P = \frac{mgh}{t} = \frac{8 \times 10 \times 5}{10} W = 40W$

100. **(c)** The law of conservation of momentum is true in all type of collisions, but kinetic energy is conserved only in elastic collision. The kinetic energy is not conserved in inelastic collision but the total energy is conserved in all type of collisions.

101. **(c)** In a perfectly inelastic collision, the two bodies move together as one body.

102. **(b)** In an inelastic collision, momentum remains conserved, but K.E is changed.

103. **(c)**

104. **(a)** During elastic collision between two equal masses, the velocities get exchanged. Hence energy transfer is maximum when $m_1 = m_2$.

105. (a) In an elastic collision

$$V_1 = \frac{(m_1 - m_2)}{m_1 + m_2}u_1 \,;\, V_2 = \frac{2m_1u_1}{m_1 + m_2}$$

$\therefore$ if $m_1 = m_2$, then $V_1 = 0$; and $V_2 = \frac{2\cancel{m_1}v_1}{2\cancel{m_1}} = u_1$

106. (c)

107. (c) Apply conservation of momentum,

$$m_1v_1 = (m_1 + m_2)v;\; v = \frac{m_1 v_1}{(m_1 + m_2)}$$

Here $v_1 = 36$ km/hr = 10 m/s, $m_1 = 2$ kg, $m_2 = 3$ kg

$$v = \frac{10 \times 2}{5} = 4 \text{ m/s}$$

$$\text{K.E. (initial)} = \frac{1}{2} \times 2 \times (10)^2 = 100\,\text{J}$$

$$\text{K.E. (Final)} = \frac{1}{2} \times (3+2) \times (4)^2 = 40\text{ J}$$

Loss in K.E. = 100 – 40 = 60 J

Alternatively use the formula

$$-\Delta E_k = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2$$

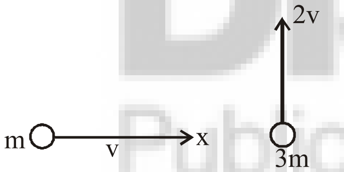
108. (a)

109. (b) $m_1v_1 + m_2v_2 = (m_1 + m_2)v_{sys.}$

$$20 \times 10 + 5 \times 0 = (20+5)v_{sys} \Rightarrow v_{sys} = 8m/s$$

$$\text{K. E. of composite mass } = \frac{1}{2}(20+5) \times (8)^2 = 800J$$

110. (a) As the two masses stick together after collision, hence it is inelastic collision. Therefore, only momentum is conserved.



$$\therefore mv\hat{i} + 3m(2v)\hat{j} = (4m)\vec{v}$$

$$\vec{v} = \frac{v}{4}\hat{i} + \frac{6}{4}v\hat{j} = \frac{v}{4}\hat{i} + \frac{3}{2}v\hat{j}$$

111. (d) K. E. of colliding body before collision $= \frac{1}{2}mv^2$

After collision its velocity becomes

$$v' = \frac{(m_1 - m_2)}{(m_1 + m_2)}v = \frac{m}{3m}v = \frac{v}{3}$$

$$\therefore \text{ K. E. after collision } = \frac{1}{2}\frac{mv^2}{9}$$

$$\text{Ratio of kinetic energy} = \frac{K.E._{before}}{K.E._{after}} = \frac{\frac{1}{2}mv^2}{\frac{1mv^2}{2 \times 9}} = 9:1$$

112. (d) For the object of mass 2.0 kg.

$$\frac{\Delta k}{k} = \frac{k - k/4}{k} = \frac{3}{4}$$

Kinetic energy transferred

$$\frac{\Delta k}{k} = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

Here, $m_1 = 2.0$ kg, $m_2 = M$

$$\therefore \frac{3}{4} = \frac{4 \times 2M}{(2+M)^2} \Rightarrow M = \frac{2}{3}\text{kg or 6kg}$$

113. (a) Initial, $\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times \frac{20}{1000} \times 600 \times 600 = 3600\,\text{J}$

Change in K.E. = P.E.

$$\frac{1}{2}m(v^2 - v'^2) = mgh$$

$$\Rightarrow 3600 - \frac{1}{2} \times \frac{20}{1000} \times v_1^2 = 4 \times 10 \times 80$$

$$\Rightarrow v_1 = 200\,\text{m/s}$$

114. (c) Initial kinetic energy of the system

$$\text{K.E}_i = \frac{1}{2}mu^2 + \frac{1}{2}M(0)^2 = \frac{1}{2} \times 0.5 \times 2 \times 2 + 0 = 1\text{J}$$

For collision, applying conservation of linear momentum

$m \times u = (m + M) \times v$

$$\therefore\; 0.5 \times 2 = (0.5 + 1) \times v$$

$$\Rightarrow v = \frac{2}{3}\text{m/s}$$

Final kinetic energy of the system is

$$\text{K.E}_f = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(0.5+1) \times \frac{2}{3} \times \frac{2}{3} = \frac{1}{3}\text{J}$$

$$\therefore \text{ Energy loss during collision } = \left(1 - \frac{1}{3}\right)\text{J} = 0.67\text{J}$$

115. (a)

116. (a) As the floor exerts a force on the ball along the normal, & no force parallel to the surface, therefore the velocity component along the parallel to the floor remains constant. Hence $V \sin\theta = V^1 \sin\theta^1$.

117. (b) The law of conservation of energy is valid at any instant & in all circumstances.

118. (b) As no external force is acting on system so, $P_i = P_f$

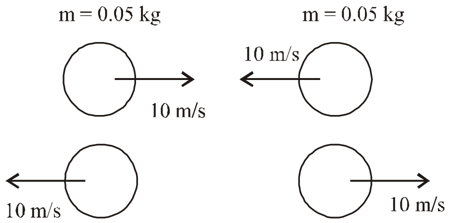
$0.2 \times 10 = 10 \times v \Rightarrow v = 0.2$ m/sec

$$\text{Loss in K.E.} = \frac{1}{2} \times (0.2) \times 10^2 - \frac{1}{2} \times 10(0.2)^2$$

$$= \frac{1}{2} \times 10 \times (0.2)\,[10 - 0.2] = 9.8\text{ J}$$

119. (b)

120. (b)



Chnage in momentum of any one ball

$$|\Delta\vec{P}| = 2 \times 0.05 \times 10 = 1$$

$$|\vec{F}_{av}| = \frac{|\Delta\vec{P}|}{\Delta t} = \frac{1}{0.005} = \frac{1000}{5} = 200\text{ N}$$

EXERCISE-2

1. **(b)** When electron and proton are moving under influence of their mutual forces, then according to the flemings left hand rule, the direction of force acting on a charge particle is perpendicular to the direction of motion.

In magnetic field, work-done = $F. s.\ \cos\theta$

$= F . s. \cos 90° = 0.$

So magnetic forces do not work on moving charge particle.

2. **(c)** Forces between two protons is same as that of between proton and a positron.

As positron is much lighter than proton, it moves away through much larger distance compared to proton.

Work done = Force × Distance

As forces are same in case of proton and positron but distance moved by positron is larger, hence, work done on positron will be more than proton.

3. **(d)** When the man squatting on the ground he is tilted somewhat, hence he also has to apply frictional force besides his weight.

R (reactional force) = friction force (f) + mg *i.e.*, $R > mg$

When the man does not squat and gets straight up in that case friction $(f) \approx 0$

R (Reactional force) $\approx mg$

Hence, the reaction force (R) is larger when squatting and become equal to mg when no squatting.

4. **(c)** According to the question, work done by the frictional force on the cycle is :

$= 200 \times 10 = -2000$ J

As the road is not moving, hence work done by the cycle on the road is zero.

5. **(c)** As the body is falling freely under gravity and no external force act on body in vaccum so law of conservation, the potential energy decreases and kinetic energy increases because total mechanical energy (PE + KE) of the body and earth system will be remain constant.

6. **(c)** According to the question, consider the two bodies as system, the total external force on the system will be zero.

Hence, in an inelastic collision KE does not conserved but total linear momentum of the system remain conserved.

7. **(c)** As the (inclined surface) are frictionless, hence, mechanical energy will be conserved. As both the tracks having common height, h (and no external force acts on system).

KE & PE of stone I at top = KE + PE at bottom of I.

From conservation of mechanical energy,

$$0 + \frac{1}{2}mv_1^2 = mgh + 0 \Rightarrow v_1 = \sqrt{2gh} \text{ similarly } v_2 = \sqrt{2gh}$$

Hence, speed is same for both stones.

For stone I, acceleration along inclined plane $a_1 = g \sin\theta_1$

Similarly, for stone II $a_2 = g \sin\theta_2$

$\sin\theta_1 < \sin\theta_2$ Thus, $\theta_2 > \theta_1$ hence $a_2 > a_1$.

a_2 is greater than a_1 and both length for track II is also less hence, stone II reaches earlier than stone I.

8. **(b)** Total Mechanical energy is E = PE + KE at any instant.

When particle is at $x = x_m$ i.e., at extreme position, partical returns back and its velocity become zero for an instant. Hence, at $x = x_m$; $x = 0$, K.E. = 0.

From Eq. (i), $E = \text{PE} + 0 = \text{PE} = V(x_m) = \frac{1}{2}kx_m^2$

but at mean position at origin $V(x_m) = 0$.

9. **(b)** If two bodies of equal masses collides elastically, their velocities are interchanged.

When ball 1 collides with ball-2, then velocity of ball-1, v_1 becomes zero and velocity of ball-2, v_2 becomes v, *i.e.*, similarly then its own all momentum is mV.

So, $v_1 = 0 \Rightarrow v_2 = v, P_1 = 0, P_2 = mV$

Now ball 2 collides to ball 3 and its transfer it's momentum is mV to ball 3 and itself comes in rest.

So, $v_2 = 0 \Rightarrow v_3 = v, P_2 = 0, P_3 = mV$

So, ball 1 and ball 2, become in rest and ball 3 move with velocity v in forward direction.

10. **(b)** As we know that,

$$\text{W.D.} = \int_{x_1}^{x_2} \vec{F} \cdot \overrightarrow{dx} = \int_{x_1}^{x_2} m\vec{a}_0 \cdot \overrightarrow{dx}$$

As given that, $m = 0.5$ kg, $a = 5\ \text{m}^{-1/2}\ \text{s}^{-1}$,

$v = ax^{3/2}$

We also know that Acceleration,

$$a_0 = \frac{dv}{dt} = v \cdot \frac{dv}{dx} = ax^{3/2}\frac{d}{dx}(ax^{3/2})$$

$$= ax^{3/2} \times a \times \frac{3}{2} \times x^{1/2} = \frac{3}{2}a^2x^2$$

Now, force $= ma_0 = m\frac{3}{2}a^2x^2$

From (i), work done $= \int_{x=0}^{x=2} F dx$

$$= \int_0^2 \left[\frac{3}{2}ma^2x^2\right] dx = \frac{3}{2}ma^2 \times \left(\frac{x^3}{3}\right)_0^2 = \frac{1}{2}ma^2 \times 8 = 50 \text{ J}$$

11. **(b)** As given that power = constant

As we know that power (P)

$$P = \frac{dW}{dt} = \frac{\vec{F} \cdot \overrightarrow{dx}}{dt} = \frac{F\ dx}{dt}$$

As the body is moving unidirectionally.

Hence, $F \cdot dx = Fdx \cos 0° = Fdx$

$$P = \frac{Fdx}{dt} = \text{constant} \quad (\because P = \text{constant by question})$$

$L^2 \propto T^3 \Rightarrow L \propto T^{3/2} \Rightarrow$ Displacement (d) $\propto t^{3/2}$

Verifies the graph (b).

12. **(d)** 13. **(c)**

14. **(a)** As given that, mass (m) = 5 kg, n = 300 revolution

Radius (R) = 1 m; t = 60 sec

$\omega = \left(\frac{2\pi n}{t}\right) = (300 \times 2 \times \pi)$ rad / 60s = 10 π rad/s

linear speed (v) = ωR = (10π × 1) ⇒ v = 10π m/s

$$KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times (10\pi)^2 = 250\pi^2 J$$

So, verifies the option (a).

15. **(b)** P.E. is maximum when drop start falling at $t = 0$ as it fall is P.E. decrease gradually to zero. So, it rejects the graph (a), (c) and (d).

K.E. at t = 0 is zero as drop falls with zero velocity, its velocity increases (gradually), hence, first KE also increases. After sometime speed (velocity) is constant this is called terminal velocity, so, KE also become constant. It happens when it falls $\left(\frac{3}{4}\right)$ height or remains at $\left(\frac{4}{4}\right)$ from ground, then PE decreases continuously as the drop is falling continuously.

The variation in PE and KE is best represented by (b).

16. **(d)** As given that, h = 1.5m, v = 1m/s, m = 10kg, g = 10 ms^{-2}

By the law of conservation of mechanical energy as no force acts on shotput after thrown.

$$(PE)_i + (KE)_i = (PE)_f + (KE)_f$$

$$mgh_i + \frac{1}{2}mv_i^2 = 0 + (KE)_f$$

$$(KE)_f = mgh_i + \frac{1}{2}mv_i^2$$

Total energy when it reaches ground, so

$$(KE)_f = 10 \times 10 \times 1.5 + \frac{1}{2} \times 10 \times (1)^2$$

E = 150 + 5 = 155 J.

17. **(b)** First velocity of the iron sphere

$V = \sqrt{2gh}$ after sometime its velocity becomes constant, called terminal velocity. Hence, according first KE increases and then becomes constant due to resistance of sphere and water which is represented by (b).

18. **(c)** As given that,

$$m = 150 \text{ g} = \frac{150}{1000} \text{ kg} = 0.15 \text{ kg}$$

Δt = time of contact = 0.001 s

$$u = 126 \text{ km/h} = \frac{126 \times 1000}{60 \times 60} \text{ m/s} = 35 \text{ m/s}$$

$$v = -126 \text{km/h} = -126 - \times \frac{5}{8} = -35 \text{ m/s}$$

So, final velocity is acc. to initial force applied by batsman.

So, change in momentum of the ball

$$\Delta p = m(v - u) = \frac{3}{20}(-35 - 35) = -\frac{21}{2} \text{ kg-m/s}$$

As we know that, force

$$F = \frac{\Delta p}{\Delta t} = \frac{-21/2}{0.001} \text{ N} = -1.05 \times 10^4 \text{ N}$$

Hence negative sign shown that direction of force will be opposite to initial velocity which taken positive direction. Hence verify the option (c).

19. **(a)**

20. **(a)** Loss in P.E. = Work done against friction from P → Q + work done against friction from Q → R

mgh = μ(mg cos θ) PQ + μmg (QR)

h = μ cos θ × PQ + μ(QR)

$2 = 2\sqrt{3}\,\mu + \mu x$ --- (i) $\left[\sin 30° = \frac{2}{PQ}\right]$

Also work done P → Q = work done Q → R

$\therefore\ 2\sqrt{3}\,\mu = \mu x$ $\therefore\ x \approx 3.5$m

From (i) $2 = 2\sqrt{3}\,\mu + 2\sqrt{3}\,\mu = 4\sqrt{3}\,\mu$

$$\mu = \frac{2}{4\sqrt{3}} = \frac{1}{2 \times 1.732} = 0.29$$

21. **(b)** $n = \frac{W}{\text{input}} = \frac{mgh \times 1000}{\text{input}} = \frac{10 \times 9.8 \times 1 \times 1000}{\text{input}}$

$$\text{Input} = \frac{98000}{0.2} = 49 \times 10^4 J$$

$$\text{Fat used} = \frac{49 \times 10^4}{3.8 \times 10^7} = 12.89 \times 10^{-3} \text{kg.}$$

22. **(c)** Using, $F = ma = m\frac{dV}{dt}$

$6t = 1.\frac{dV}{dt}$ [∵ m = 1 kg given]

$\int_0^v dV = \int 6t\,dt \Rightarrow V = 6\left[\frac{t^2}{2}\right]_0^1 = 3\,\text{ms}^{-1}$ [t = 1 sec]

From work-energy theorem,

$$W = \Delta KE = \frac{1}{2}m\left(V^2 - u^2\right) = \frac{1}{2} \times 1 \times 9 = 4.5 \text{ J}$$

23. **(a)** Let V_f is the final speed of the body.

From questions,

$$\frac{1}{2}mV_f^2 = \frac{1}{8}mV_0^2 \Rightarrow V_f = \frac{V_0}{2} = 5\,m/s$$

$$F = m\left(\frac{dV}{dt}\right) = -kV^2 \quad \therefore\ (10^{-2})\frac{dV}{dt} = -kV^2$$

$$\int_{10}^{5} \frac{dV}{V^2} = -100K\int_0^{10} dt \Rightarrow \frac{1}{5} - \frac{1}{10} = 100K(10)$$

or, $K = 10^{-4}\,kgm^{-1}$

24. **(b)**

25. **(a)** For collision of neutron with deuterium:

Applying conservation of momentum :

$mv + 0 = mv_1 + 2mv_2$ .....(i)

$v_2 - v_1 = v$ .....(ii)

$\because$ Collision is elastic, e = 1

From eqn (i) and eqn (ii) $v_1 = -\dfrac{v}{3}$

$$P_d = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{8}{9} = 0.89$$

Now, For collision of neutron with carbon nucleus

Applying Conservation of momentum

$mv + 0 = mv_1 + 12mv_2$ ....(iii)

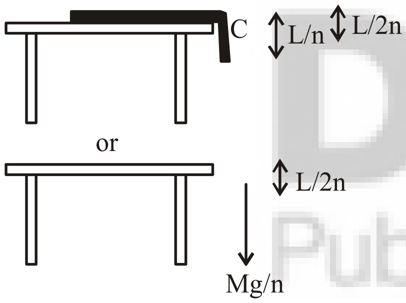
$v = v_2 - v_1$ ....(iv)

From eqn (iii) and eqn (iv)

$$v_1 = -\frac{11}{13}v$$

$$P_c = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{11}{13}v\right)^2}{\frac{1}{2}mv^2} = \frac{48}{169} \approx 0.28$$

26. **(b)** Length of hanging part = L/n
Mass of hanging part = M/n
Weight of hanging part = Mg/n
Let 'C' be the centre of mass of the hanging part.



$$\therefore W = \left(\frac{Mg}{n}\right)\left(\frac{L}{n}\right)$$

$$\therefore W = \frac{MgL}{n^2}$$

The hanging part can be assumed to be a particle of weight Mg/n at a distance L/n below the table top. The work done in lifting it to the table top is equal to increase in its potential energy.

27. **(b)** Total force required to lift maximum load capacity against frictional force = 400 N

$F_{total} = Mg$ + friction

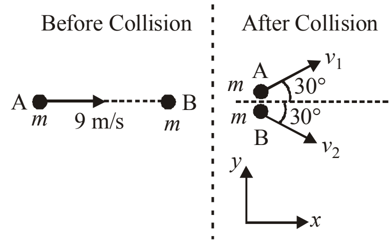
$= 2000 \times 10 + 4000 = 20,000 + 4000 = 24000\ N$

Using power, $P = F \times v$

$60 \times 746 = 24000 \times v \Rightarrow v = 1.86$ m/s $\approx 1.9\ m/s$

Hence speed of the elevator at full load is close to 1.9 ms^{-1}

28. **(1)**



From conservation of momentum along y-axis.

$\vec{P}_{iy} = \vec{P}_{fy}$

$0 + 0 = mv_1 \sin 30^\circ\ \hat{j} + mv_2 \sin 30^\circ(-\hat{j})$

$mv_2 \sin 30^\circ = mv_1 \sin 30^\circ$

$v_2 = v_1$ or $\dfrac{v_1}{v_2} = 1$

29. **(d)** By work – energy theorem

$$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2} \times 0.5 \times \left(b^2.4^5\right)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4^2} \times 4^5 = 16 \text{ J}$$

30. **(d)** Power, $P = \dfrac{W}{t} = \dfrac{mgh}{t}$ $\quad \therefore \dfrac{P_1}{P_2} = \dfrac{\frac{m_1gh}{t_1}}{\frac{m_2gh}{t_2}} = \dfrac{m_1}{t_1}\dfrac{t_2}{m_2}$

$$\therefore \frac{P_1}{P_2} = \frac{300 \times 2}{5 \times 50} = \frac{12}{5} = \frac{3\sqrt{x}}{\sqrt{x}+1}$$

So, $12\sqrt{x} + 12 = 15\sqrt{x} \Rightarrow 3\sqrt{x} = 12 \qquad \therefore x = 16$

31. **(d)** K.E. $= \dfrac{P^2}{2m}$ $\therefore \dfrac{K_1}{K_2} = \dfrac{p_1^2}{2m_1} \times \dfrac{2m_2}{p_2^2} = \dfrac{m_2}{m_1} = \dfrac{16}{9}$ $\therefore \dfrac{m_1}{m_2} = \dfrac{9}{16}$

32. **(c)** Using, $P = \sqrt{2mE}$

Here $E_1 = E_2 \Rightarrow \dfrac{P_1^2}{2m_1} = \dfrac{P_2^2}{2m_2} \Rightarrow \dfrac{P_1}{P_2} = \sqrt{\dfrac{m_1}{m_2}} = \sqrt{\dfrac{4}{25}} = \dfrac{2}{5}$

EXERCISE - 3

1. **(b)** (A) → (1); (B) → (1); (C) → (3); (D) → (3)
2. **(b)** (A)→(3); (B)→(1); C→(2); (D)→(1)
3. **(d)** (A)→(4); (B)→(1); C→(3); (D)→(2)
4. **(b)** In close loop, s = 0, and so $W = Fs$ = 0.
5. **(b)**
6. **(d)** The billiard balls in an elastic collision are in a deformed state. Their total energy is partly kinetic and partly potential. So K.E. is less than the total energy. The energy spent against friction is dissipated as heat which is not available for doing work.
7. **(c)** If there were no friction, moving vehicles could not be stopped by looking the brakes. Vehicles are stopped by air friction only. So, this statement is correct.
8. **(c)** If there were no friction, moving vehicles could not be stopped by looking the brakes. Vehicles are stopped by air friction only. So, this statement is correct.
9. **(d)** force depends only on the end points.
This can be seen from the relation,
$W = K_f - K_i = V(X_i) - V(X_f)$
which depends on the end points.

A third definition states that the work done by this force in a closed path is zero. This is once again apparent from Eq. $K_i + v(X_i) = K_f + v(X_f)$, since $X_i = X_f$

10. **(d)**

11. **(b)** In elastic collision, total momentum and kinetic energy will remain conserved.

12. **(d)** Work done may be zero, even F is not zero. also, $W = Fs\cos 0° = Fs$ (maximum). [when applied force and displacement is in the same direction]

13. **(c)** Change in kinetic energy = work done by net force. This relationship is valid for particle as well as system of particles.

14. **(d)** $\text{K.E.} = \dfrac{P^2}{2m} \Rightarrow E \propto \dfrac{1}{m}$ when P constant

15. **(a)** $\text{Power} = \dfrac{W}{t} = \dfrac{K}{1/n} = nK$

16. **(a)** Maximum energy loss $= \dfrac{P^2}{2m} - \dfrac{P^2}{2(m+M)}$

$$\left[\because \text{K.E.} = \frac{P^2}{2m} = \frac{1}{2}mv^2\right]$$

$$= \frac{P^2}{2m}\left[\frac{M}{(m+M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\}$$

Reason is a case of perfectly inelastic collision.

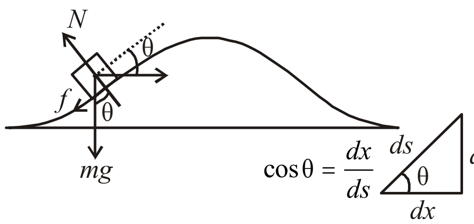
By comparing the equation given in Assertion with above equation, we get

$$f = \left(\frac{M}{m+M}\right)$$

EXERCISE-4

1. **(c)** Work done by friction

$$= \int \vec{F}\cdot\vec{ds} = \int_0^x \mu mg\cos\theta \frac{dx}{\cos\theta} = \mu mg\,x = 20\text{ J}$$



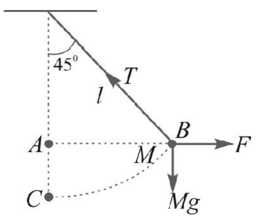
2. **(a)**

3. **(a)** By using work-energy theorem, $W_{all} = \Delta K$, we have Work done by F + work done by Mg = 0

$F(AB) - Mg(AC) = 0$

$$F = Mg\left(\frac{AC}{AB}\right) = \left[\frac{l - l/\sqrt{2}}{l/\sqrt{2}}\right]$$

$$= Mg(\sqrt{2} - 1).$$



4. **(b)** The centripetal acceleration

$$a_c = k^2 r t^2 \text{ or } \frac{v^2}{r} = k^2 r t^2 \quad \therefore \quad v = krt$$

So, tangential acceleration, $a_t = \dfrac{dv}{dt} = kr$

Work is done by tangential force.

Power $= F_t \cdot v \cdot \cos 0° = (ma_t)(krt) = (mkr)(krt) = mk^2r^2t$

5. **(b)** The required work done by man = kinetic energy of man + kinetic energy of boat

$$= \frac{1}{2}\frac{p^2}{M} + \frac{1}{2}\frac{p^2}{m} \quad (\text{where } p = Mv)$$

$$\therefore \; W = \frac{1}{2}\left(\frac{M^2}{M} + \frac{M^2}{m}\right)v^2 = \frac{1}{2}\left(M + \frac{M^2}{m}\right)v^2$$

6. **(b)**

7. **(d)** $P_{out} = \dfrac{mgh}{t} = \dfrac{9000\times10\times10}{5\times60} = 3000\ W$

$P_{in} = 10 \times 10^3$ W.

$$\therefore \; \eta = \frac{P_{out}}{P_{in}}\times100 = \frac{3000}{10\times10^3}\times100 = 30\%$$

8. **(b)** Work done, $W = \int \vec{F}\cdot\vec{ds} = \left(-x\hat{i} \times y\hat{j}\right)\cdot\left(x\hat{i} + y\hat{j}\right)$

$$\Rightarrow W = \int_1^0 x\,dx + \int_0^1 y\,dy = \left(0 + \frac{1}{2}\right) + \frac{1}{2} = 1\text{ J.}$$

9. **(b)** Constant power of car $P_0 = F \cdot V = ma \cdot v$

$$P_0 = m\frac{dv}{dt}\cdot v$$

$P_0 dt = mv dv$. Integrating $P_0 \cdot t = \dfrac{mv^2}{2}$

$$v = \sqrt{\frac{2P_0 t}{m}} \quad \because P_0, m \text{ and } 2 \text{ are constant} \quad \therefore \; v \propto \sqrt{t}$$

10. **(a)** Masses of the pieces are 1, 1, 3 kg. Hence $(1\times21)^2 + (1\times21)^2 = (3\times V)^2$

That is, $V = 7\sqrt{2}$ m/s

11. **(b)**

12. **(c)** Only momentum is conserved. Some kinetic energy is lost when bullet penetrates the block.

13. **(a)** If an object of mass m is released from rest from top of a smooth inclined plane, its speed at the bottom is $\sqrt{2gh}$, independent of angle θ and mass.

14. **(a)**

15. **(d)** Here, F = 100 N, d = 5 m,

frictional force $f_r = 40$ N

$\therefore F - f_r = ma$

$100 - 40 = ma$

Now kinetic energy gained is = ma × d

= 60 × 5 = 300 J

16. (b) Let the blow compress the spring by x before stopping. Kinetic energy of the block = (P.E of compressed spring) + work done against function.

$$\frac{1}{2}\times 2\times(4)^2=\frac{1}{2}\times 10,000\times x^2+(+15)\times x$$

$$10,000\,x^2+30x-32=0$$

$$\Rightarrow 5000x^2+15x-16=0$$

$$\therefore\ x=-\frac{15\pm\sqrt{(15)^2-4\times(5000)(-16)}}{2\times 5000}=0.055\text{m}=5.5\text{cm}.$$

17. (d) Centripetal force

$$=\frac{mv^2}{R}=\left(\frac{1}{2}mv^2\right)\frac{2}{R}=\frac{2K}{R}=\frac{2aS}{R}$$

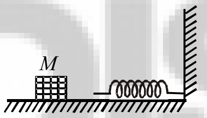
18. (a) $U_1=\int_0^{\ell/3}-\frac{m}{\ell}gxdx=-\frac{1}{18}mg\ell;$

$$U_2=\int_0^{\ell}-\frac{m}{\ell}gxdx=-\frac{1}{2}mg\ell$$

loss in P.E. $=U_1-U_2=\frac{4}{9}mgl$

$$=\frac{4}{9}\times 0.1\times 10\times 2=\frac{8}{9}\text{J}=\text{Final K.E.}$$

19. (b) $\frac{1}{2}Mv^2=\frac{1}{2}kL^2$



$$\Rightarrow v=\sqrt{\frac{k}{M}}.L$$

Momentum $=M\times v=M\times\sqrt{\frac{k}{M}}.L=\sqrt{kM}.L$

20. (c)

EXERCISE-5

1. (1) Kinetic energy of block A

$$k_1=\frac{1}{2}mv_0^2$$

$\therefore$ From principle of linear momentum conservation

$$mv_0=(2m+M)v_f\ \Rightarrow v_f=\frac{mv_0}{2m+M}$$

According to question, of $\frac{5}{6}$th the initial kinetic energy is lost in whole process.

$$\therefore\ \frac{k_i}{k_f}=6\ \Rightarrow\frac{\frac{1}{2}mv_0^2}{\frac{1}{2}(2m+M)\left(\frac{mv_0}{2m+M}\right)^2}=6$$

$$\Rightarrow\frac{2m+M}{m}=6\ \therefore\ \frac{M}{m}=4$$

2. (900) Position, $x=3t^2+5$

$\therefore$ Velocity, $v=\frac{dx}{dt}$

$$\Rightarrow v=\frac{d(3t^2+5)}{dt}\quad\Rightarrow v=6t+0$$

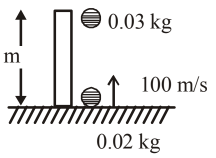
At t = 0 v = 0

And, at t = 5 sec v = 30 m/s

According to work-energy theorem, w = ΔKE

or, $W=\frac{1}{2}mv^2-0=\frac{1}{2}(2)(30)^2=900\text{J}$

3. (40)



Time taken for the particles to collide,

$$t=\frac{d}{v_{rel}}=\frac{100}{100}=1\text{s}$$

Speed of wood just before collision = gt = 10m/s and speed of bullet just before collision = v – gt = 100 – 10 = 90 m/s

$$S=100\times 1-\frac{1}{2}\times 10\times 1=95\text{ m}$$

Now, using conservation of linear momentum just before and after the collision

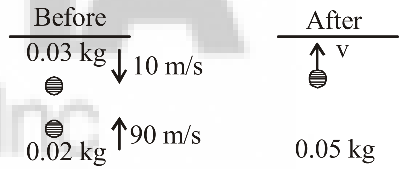
$-(0.03)(10)+(0.02)(90)=(0.05)v$

$\Rightarrow 150=5v$

$\therefore\ v=30$ m/s

Max. height reached by body

$$h=\frac{v^2}{2g}=\frac{30\times 30}{2\times 10}=45\text{m}$$



$\therefore$ Height above tower = 40 m

4. (15) Work done = $\vec{F}.\vec{d}=(3\hat{i}-12\hat{j}).(4\hat{i})=12\text{J}$

From work energy theorem,

$w_{net}=\Delta\text{K.E.}=k_f-k_i$

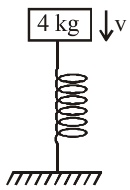
$\Rightarrow 12=k_f-3$

$\therefore K_f=15\text{J}$

5. (4) Velocity of 1 kg block just before it collides with 3 kg block $=\sqrt{2gh}=\sqrt{2000}$ m/s

Using principle of conservation of linear momentum just before and just after collision, we get

$$1\times\sqrt{2000}=4v\Rightarrow v=\frac{\sqrt{2000}}{4}\text{ m/s}$$



initial compression of spring

$1.25\times 10^6\,x_0=30\Rightarrow x_0\approx 0$

using work energy theorem,

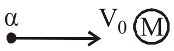
$W_g + W_{sp} = \Delta KE$

$\Rightarrow\ 40 \times x + \frac{1}{2} \times 1.25 \times 10^6 (0^2 - x^2)\ = 0 - \frac{1}{2} \times 4 \times v^2$

solving x ≈ 4 cm

6. **(4)** Using conservation of momentum,

$mv_0 = mv_2 - mv_1$



After collision

$\frac{1}{2} mv_1^2 = 0.36 \times \frac{1}{2} mv_0^2$

$\Rightarrow v_1 = 0.6 v_0$

The collision is elastic. So,

$\frac{1}{2} MV_2^2 = 0.64 \times \frac{1}{2} mv_0^2$ [∴ M = mass of nucleus]

$\Rightarrow V_2 = \sqrt{\frac{m}{M}} \times 0.8 V_0$

$mV_0 = \sqrt{mM} \times 0.8 V_0 - m \times 0.6 V_0$

$\Rightarrow 1.6 m = 0.8\sqrt{mM}$

$\Rightarrow 4m^2 = mM$

$\therefore M = 4m$

7. **(6.5)** We know area under F-x graph gives the work done by the body

$\therefore\ W = \frac{1}{2} \times (3+2) \times (3-2) + 2 \times 2$

$= 2.5 + 4$

$= 6.5$ J

Using work energy theorem,

Δ K.E = work done

∴ Δ K.E = 6.5 J

8. **(80)** $mgh^1 = \frac{80}{100} \times mg \times 100 \Rightarrow h^1 = 80m$

9. **(2.5)** Work done on the body = K. E. gained by the body

$Fs\cos\theta = 1 \Rightarrow F\cos\theta = \frac{1}{s} = \frac{1}{0.4} = 2.5N$

10. **(3730)** Forward thrust, $F = \frac{P}{v} = \frac{100 \times 746}{20} = 3730$ N.

11. **(0.4)** In an elastic head-on collision, of two equal masses their kinetic energies or velocities are exchanged. Hence when the first ball collides with the second ball at rest, the second ball attains the speed of 0.4 m/s and the first ball comes to rest. This process continues. Thus the velocity of the last ball is 0.4 ms^{-1}.

12. **(96)**

13. **(4)** $h_n = e^{2n} h_0\ \Rightarrow \frac{h}{2} = e^4 h\ \Rightarrow e = \left(\frac{1}{2}\right)^{1/4}$

14. **(30)** Work done $W = \int F dx$

$= \int_2^4 5x dx = 5\left[\frac{x^2}{2}\right]_2^4 = \frac{5}{2}[16 - 4] = 30J$

15. **(2)** Given,

Mass of particle, m = 10 g

Retardation, a = 2x

Loss of K.E = work done against retarding force.

$\therefore \Delta KE = \int_0^x madx = \int_0^x m2xdx = mx^2 = \frac{10}{1000} \times x^2 J$

$= \left(\frac{10}{x}\right)^{-2} J$

$\therefore n = 2$

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Class XI

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Note : The four Exercises in each of the chapters are :

- Exercise 1 : NCERT Based Topic-wise MCQs
- Exercise 2 : NCERT Exemplar Past Years NEET
- Exercise 3 : Matching Statements & Assertion Reason Type
- Exercise 4 : Skill Enhancer MCQs
- Exercise 5 : Numeric Value Answer Question

Mock Tests

NOTE* These Topics are in new NCERT, but not in the new JEE Main 2024 Syllabus. These Topics have been retained in the book so as to match NCERT and any future amendments in JEE Main. Questions on these Topics have also been marked with a * in the respective Exercises of the Chapters.

6 Equilibrium

Trend Analysis JEE Main

	JEE Main	Remarks
Number of Questions from 2024-17	10	Minimum three questions have been asked every year in JEE Main.
Weightage	4.1%	

		JEE Main		
Year	Topic Name	Concept Used	No. of Ques.	Difficulty Level
2024	Acid, base and salt	Acid / Basic nature of slat	1	Easy
2023	Relationship between k, Q and G / Acid, Bases and salt / Solubility Equilibria of sparingly soluble salt.	Gibbs energy / Lewis acid and lewis base / solubility product.	1	Average
2022	Law of mass action, equilibrium constant and its application / Common ion effect, salt hydrolysis, Buffer solution	Equilibrium constant / K_p & K_c / Buffer solution	2	Easy
2021	Ionisation of weak acids and bases and relation between K_a and K_b	Ionisation of weak acid and bases.	1	Average
2020	Acid, bases and salt/ solubility of sparingly soluble salt/ Relation between eqilibrium constant and Gibbs energy	pH of the solution /Solubility / Relation between free energy and equilibrium constant	1	Difficult
2019	Buffer solutions, solubility of sparingly solubles alt/acid, bases and salt	Buffer solutions/K_{sp}/ ionisation of acid and bases/acid and bases	2	Average
2018	Law of chemical equilibrium/acid, bases and salt/ application/factor affecting eqilibria/ buffer solution/solubility of sparingly soluble salt	Equilibrium constant/pH of the solution/Le-chatelier principle/K_{sp}	1	Average
2017	Buffer solutions, solubility of sparingly soluble salt/law of chemical equilibrium/ acid, bases and salt	buffer solution/solubility product/equilibrium constant/ ionisation of acid and bases/acid and bases	1	Average

NCERT ONE-LINERS
(Important Points to Remember)

6.0 Introduction

- The mixture of reactants and products in the equilibrium state is called an **equilibrium mixture**.
- Equilibrium can be established for both physical processes and chemical reactions.
- Dynamic equilibrium and the rates of the forward and reverse reactions become equal.
- The equilibrium involving ions in aqueous solutions which is called as **ionic equilibrium**.

6.1 Equilibrium in Physical Processes

- The most familiar examples are **phase transformation processes,** e.g., solid $\rightleftharpoons$ liquid; liquid $\rightleftharpoons$ gas; solid $\rightleftharpoons$ gas
- **Solid-Liquid Equilibrium :** Ice and water are in equilibrium only at particular temperature and pressure. For any pure substance at atmospheric pressure, the temperature at which the solid and liquid phases are at equilibrium is called the normal melting point or normal freezing point of the substance.
- **Liquid-Vapour Equilibrium:** $H_2O(l) \rightleftharpoons H_2O$ (vap)

  Water and water vapour are in equilibrium position at atmospheric pressure (1.013 bar) and at 100°C in a closed vessel. The boiling point of water is 100°C at 1.013 bar pressure. For any pure liquid at one atmospheric pressure (1.013 bar), the temperature at which the liquid and vapours are at equilibrium is called normal boiling point of the liquid. Boiling point of the liquid depends on the atmospheric pressure.

- **Solid – Vapour Equilibrium :** The systems where solids sublime to vapour phase. If we place solid iodine in a closed vessel, after sometime the vessel gets filled up with violet vapour and the intensity of colour increases with time.
- Solid iodine sublimes to give iodine vapour and the iodine vapour condenses to give solid iodine. The equilibrium can be represented as,

$$I_2(\text{solid}) \rightleftharpoons I_2(\text{vapour})$$

- **Equilibrium Involving Dissolution of Solid or Gases in Liquids Solids in liquids:** Saturated solution depends upon the temperature. In a saturated solution, a dynamic equilibrium exits between the solute molecules in the solid state and in the solution:

$$\text{Sugar (solution)} \rightleftharpoons \text{Sugar (solid)}$$

- **Gases in liquids:**
  - When a soda water bottle is opened, some of the carbon dioxide gas dissolved in it fizzes out rapidly. The phenomenon arises due to difference in solubility of carbon dioxide at different pressures. There is equilibrium between the molecules in the gaseous state and the molecules dissolved in the liquid under pressure i.e.,
  - This equilibrium is governed by Henry's law, **which states that the mass of a gas dissolved in a given mass of a solvent at any temperature is proportional to the pressure of the gas above the solvent**. This amount decreases with increase of temperature.
- **General Characteristics of Equilibria Involving Physical Processes:**
  - Equilibrium is possible only in a closed system at a given temperature.
  - Both the opposing processes occur at the same rate and there is a dynamic but stable condition.
  - All measurable properties of the system remain constant.
  - When equilibrium is attained for a physical process, it is characterised by constant value of one of its parameters at a given temperature. Following table lists such quantities.

Process	Conclusion
Liquid $\rightleftharpoons$ Vapour H_2O (l) $\rightleftharpoons$ H_2O (g)	p_{H_2O} constant at given temperature
Solid $\rightleftharpoons$ Liquid H_2O (s) $\rightleftharpoons$ H_2O (l)	Melting point is fixed at constant pressure
Solute(s) $\rightleftharpoons$ Solute (solution) Sugar(s) $\rightleftharpoons$ Sugar (solution)	Concentration of solute in solution is constant at a given temperature
Gas(g) $\rightleftharpoons$ Gas (aq) CO_2(g) $\rightleftharpoons$ CO_2(aq)	[gas(aq)]/[gas(g)] is constant at a given temperature [CO_2(aq)]/[CO_2(g)] is constant

- The magnitude of such quantities at any stage indicates the extent to which the physical process has proceeded before reaching equilibrium.

6.2 Equilibrium in Chemical Processes – Dynamic Equilibrium

- When the rates of the forward and reverse reactions become equal, the concentrations of the reactants and the products remain constant. This is the stage of chemical equilibrium. This equilibrium is dynamic in nature as it consists of a forward reaction in which the reactants give product(s) and reverse reaction in which product(s) gives the original reactants.

$$A + B \rightleftharpoons C + D$$

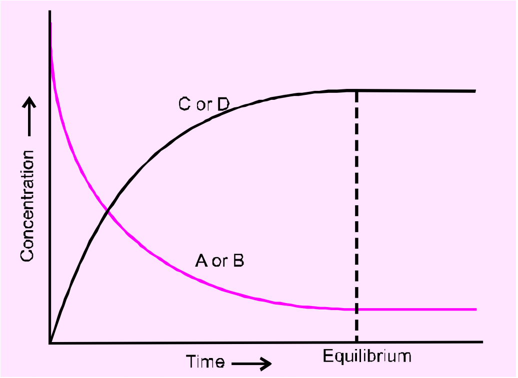


Fig.: *Attainment of chemical equilibrium.*

6.3 Law of Chemical Equilibrium and Equilibrium Constant

- At General reversible reaction:

$$A + B \rightleftharpoons C + D$$

Where A and B are the reactants, C and D are the products in the balanced chemical equation.

Equilibrium equation

$$K_c = \frac{[C][D]}{[A][B]}$$

JEE M 2016

- At a given temperature, the product of concentrations of the reaction products raised to the respective stoichiometric coefficient in the balanced chemical equation divided by the product of concentrations of the reactants raised to their individual stoichiometric coefficients has a constant value. This is known as the Equilibrium Law or Law of Chemical Equilibrium.
- The equilibrium constant for a general reaction,
  $aA + bB \rightleftharpoons cC + dD$
  is expressed as,
  $K_c = [C]^c[D]^d / [A]^a[B]^b$
  where [A], [B], [C] and [D] are the equilibrium concentrations of the reactants and products.

Table: Relations between Equilibrium Constants for a General Reaction and its Multiples.

Chemical equation	Equilibrium constant
$aA + bB \rightleftharpoons cC + dD$	K_c
$cC + dD \rightleftharpoons aA + bB$	$K'_c = (1/K_c)$
$naA + nbB \rightleftharpoons ncC + ndD$	$K''_c = (K_c^n)$

6.4 Homogeneous Equilibria

- In a homogeneous system, all the reactants and products are in the same phase. For example,

  $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$,

  $CH_3COOC_2H_5(aq) + H_2O(l) \rightleftharpoons CH_3COOH(aq) + C_2H_5OH(aq)$

  $Fe^{3+}(aq) + SCN^-(aq) \rightleftharpoons Fe(SCN)^{2+}(aq)$

  all the reactants and products are in homogeneous solution phase.

- **Equilibrium Constant in Gaseous Systems:** JEE M 2021

  For a general reaction

  $aA + bB \rightleftharpoons cC + dD$

  $$K_p = \frac{(P_C^c)(P_D^d)}{(P_A^a)(P_B^b)} = K_c(RT)^{\Delta n}$$

  where Δn = (number of moles of gaseous products) – (number of moles of gaseous reactants) in the balanced chemical equation.

6.5 Heterogeneous Equilibria

- Equilibrium in a system having more than one phase is called heterogeneous equilibrium.
- Heterogeneous equilibria often involve pure solids or liquids.
- For the existence of heterogeneous equilibrium pure solids or liquids must also be present (however small the amount may be) at equilibrium, but their concentrations or partial pressures do not appear in the expression of the equilibrium constant. In the reaction,

  $$CaCO_3(s) \overset{\Delta}{\rightleftharpoons} CaO(s) + CO_2(g)$$

  $$K_c = \frac{[CaO(s)][CO_2(g)]}{[CaCO_3(s)]}$$

  Since $[CaCO_3(s)]$ and $[CaO(s)]$ are both constant.

  $K'_c = [CO_2(g)]$ or $K_p = P_{CO_2}$

6.6 Applications of Equilibrium Constants

- **Important features of equilibrium constants as follows:**
  - Expression for equilibrium constant is applicable only when concentrations of the reactants and products have attained constant value at equilibrium state.
  - The value of equilibrium constant is independent of initial concentrations of the reactants and products.
  - Equilibrium constant is temperature dependent having one unique value for a particular reaction represented by a balanced equation at a given temperature.
  - The equilibrium constant for the reverse reaction is equal to the inverse of the equilibrium constant for the forward reaction.
  - The equilibrium constant *K* for a reaction is related to the equilibrium constant of the corresponding reaction, whose equation is obtained by multiplying or dividing the equation for the original reaction by a small integer.
- **Predicting the Extent of a Reaction:**
  - If $K_c > 10^3$, products predominate over reactants, i.e., if K_c is very large, the reaction proceeds nearly to completion.
  - If $K_c < 10^{-3}$, reactants predominate over products, i.e., if consider K_c is very small, the reaction proceeds rarely.
  - If K_c is in the range of 10^{-3} to 10^3, appreciable concentrations of both reactants and products are present.

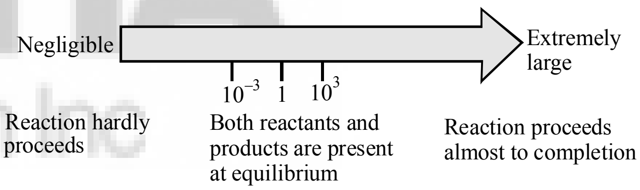


Fig.: *Dependence of extent of reaction on K_c*

- **Predicting the Direction of the Reaction:**

  For a general reaction:

  $aA + bB \rightleftharpoons cC + dD$

  $Q_c = [C]^c[D]^d / [A]^a[B]^b$

  Then,
  - If $Q_c > K_c$, the reaction will proceed in the direction of reactants (reverse reaction). JEE M 2015
  - If $Q_c < K_c$, the reaction will proceed in the direction of the products (forward reaction).
  - If $Q_c = K_c$, the reaction mixture is already at equilibrium.

6.7 Relationship between Equilibrium Constant K, Reaction Quotient Q and Gibbs Energy G

- The value of K_c for a reaction does not depend on the rate of the reaction. It is directly related to the thermodynamics of the reaction and in particular, to the change in Gibbs energy, ΔG. If,
  - ΔG is negative, then the reaction is spontaneous and proceeds in the forward direction.
  - ΔG is positive, then reaction is considered non-spontaneous.
  - ΔG is 0, reaction has achieved equilibrium.
- A mathematical expression of this thermodynamic view of equilibrium can be described by the following equation:

$$\Delta G = \Delta G^0 + RT \ln Q \quad \text{....(i)}$$

where, G° is standard Gibbs energy.

At equilibrium, when ΔG = 0 and Q = K_c, the equation (i) becomes,

$$\ln K = -\Delta G^\circ / RT \quad \text{....(ii)}$$

Taking antilog of both sides, we get,

$$K = e^{-\Delta G^\circ/RT} \quad \text{....(iii)}$$

Hence, using the equation (iii), the reaction spontaneity can be interpreted in terms of the value of ΔG°.

  - If ΔG° < 0, then –ΔG°/RT is positive, and $e^{-\Delta G^\circ/RT}$ >1, making K >1, which implies a spontaneous reaction or the reaction which proceeds in the forward direction to such an extent that the products are present predominantly.
  - If ΔG° > 0, then –ΔG°/RT is negative, and $e^{-\Delta G^\circ/RT}$ < 1, that is , K < 1, which implies a non-spontaneous reaction or a reaction which proceeds in the forward direction to such a small degree that only a very minute quantity of product is formed.

6.8 Factors Affecting Equilibria

- ***Le Chatelier's principle***. It states that a change in any of the factors that determine the equilibrium conditions of a system will cause the system to change in such a manner so as to reduce or to counteract the effect of the change. This is applicable to all physical and chemical equilibria.
- **Effect of Concentration Change:** The concentration stress of an added/removed reactant/ product is relieved by net reaction in the direction that consumes/replenishes the added/ removed substance.
- **Effect of Pressure Change:** A pressure change obtained by changing the volume can affect the yield of products in case of a gaseous reaction where the total number of moles of gaseous reactants and total number of moles of gaseous products are different.

Upon increasing the pressure, the equilibrium will shift in the direction in which the number of moles of gas is low and vice-versa.
- **Effect of Inert Gas Addition:** If the volume is kept constant and an inert gas such as argon is added which does not take part in the reaction, the equilibrium remains undisturbed. The reaction quotient changes only if the added gas is a reactant or product involved in the reaction.
- **Effect of Temperature Change:**
  - The equilibrium constant for an exothermic reaction (negative Δ*H*) decreases as the temperature increases.
  - The equilibrium constant for an endothermic reaction (positive Δ*H*) increases as the temperature increases. Temperature changes affect the equilibrium constant and rates of reactions.
- **Effect of a Catalyst:**
  - A catalyst increases the rate of the chemical reaction by making available a new low energy pathway for the conversion of reactants to products.
  - It increases the rate of forward and reverse reactions.
  - Catalyst lowers the activation energy for the forward and reverse reactions by exactly the same amount.
  - Catalyst does not affect the equilibrium composition of a reaction mixture.

6.9 Ionic Equilibrium in Solution

- The substances, which conduct electricity in their aqueous solutions are called *electrolytes*.
- Faraday classified electrolytes into strong and weak electrolytes. Strong electrolytes on dissolution in water are ionized almost completely, while the weak electrolytes are only partially dissociated.
- There is almost 100% ionization in case of sodium chloride as compared to less than 5% ionization of acetic acid which is a weak electrolyte. It should be noted that in weak electrolytes, equilibrium is established between ions and the unionized molecules. This type of equilibrium involving ions in aqueous solution is called **ionic equilibrium**.

6.10 Acids, Bases and Salts

- **Arrhenius Concept of Acids and Bases:**
According to Arrhenius theory, acids are substances that dissociates in water to give hydrogen ions $H^+(aq)$ and bases are substances that produce hydroxyl ions OH^- (aq).

$$HX(aq) \rightarrow H^+(aq) + X^-(aq)$$
$$MOH(aq) \rightarrow M^+(aq) + OH^-(aq)$$

- **The Brönsted-Lowry Acids and Bases:**

  According to Brönsted-Lowry theory, acid is a substance that is capable of donating a hydrogen ion H^+ and bases are substances capable of accepting a hydrogen ion, H^+.

  adds proton

  $$NH_3(aq) + H_2O(l) \rightleftharpoons NH_4^+(aq) + OH^-(aq)$$

  base   acid   conjugate acid   conjugate base

  loses proton

  - The acid-base pair that differs only by one proton is called a **conjugate acid-base pair**.
  - If Brönsted acid is a strong acid then **its conjugate base is a weak base and viceversa**.
- **Lewis Acids and Bases:**
  - G.N. Lewis in 1923 defined an acid as a species which accepts electron pair and base which donates an electron pair.
  - In Lewis concept many acids do not have proton.
  - BF_3 does not have a proton but still acts as an acid and reacts with NH_3 by accepting its lone pair of electrons. The reaction can be represented by,

    $$BF_3 + :NH_3 \rightarrow BF_3:NH_3$$
  - Electron deficient species like $AlCl_3$, Co^{3+}, Mg^{2+}, etc. can act as Lewis acids while species like H_2O, NH_3, OH^- etc. which can donate a pair of electrons, can act as Lewis bases.

6.11 Ionization of Acids and Bases

- Strong acids like perchloric acid ($HClO_4$), hydrochloric acid (HCl), hydrobromic acid (HBr), hyrdoiodic acid (HI), nitric acid (HNO_3) and sulphuric acid (H_2SO_4) are termed strong because they are almost completely dissociated into their constituent ions in an aqueous medium, thereby acting as proton (H^+) donors.
- Strong bases like lithium hydroxide (LiOH), sodium hydroxide (NaOH), potassium hydroxide (KOH), caesium hydroxide (CsOH) and barium hydroxide $Ba(OH)_2$ are almost completely dissociated into ions in an aqueous medium giving hydroxyl ions, OH^-.
- **The Ionization Constant of Water and its Ionic Product:**

  The value of K_w at 298K,

  $K_w = [H_3O^+][OH^-] = (1 \times 10^{-7})^2 = 1 \times 10^{-14}\ M^2$

  The value of K_w is temperature dependent as it is an equilibrium constant.
- **The pH Scale:** Hydronium ion concentration in molarity is more conveniently expressed on a logarithmic scale known as the **pH scale**.

  $$pH = -\log\{[H^+] / mol\ L^{-1}\}$$

  Thus, at 25°C,

  Acidic solution pH < 7; Basic solution pH > 7;
  Neutral solution pH = 7
  $pK_w = pH + pOH = 14$
- **Ionization Constants of Weak Acids:**
  - $HX(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + X^-(aq)$
    $K_a = c^2\alpha^2 / c(1-\alpha) = c\alpha^2/(1-\alpha)$
    K_a is called the **dissociation or ionization constant** of acid HX.
  - At a given temperature T, K_a is a measure of the strength of the acid HX i.e., larger the value of K_a, the stronger is the acid. K_a is a dimensionless quantity with the understanding that the standard state concentration of all species is 1M.
    $pK_a = -\log(K_a)$
- **Ionization of Weak Bases:**
  $MOH(aq) \rightleftharpoons M^+(aq) + OH^-(aq)$
  $K_b = (c\alpha)^2 / c(1-\alpha) = c\alpha^2/(1-\alpha)$
  $pK_b = -\log(K_b)$
- **Relation between K_a and K_b:**
  $K_a \times K_b = K_w$
  $pK_a + pK_b = pK_w = 14$ (at 298K)
- **Di- and Polybasic Acids and Di- and Polyacidic Bases:**
  - Some of the acids like oxalic acid, sulphuric acid and phosphoric acids have more than one ionizable proton per molecule of the acid. Such acids are known as polybasic or polyprotic acids.
  - Higher order ionization constants (K_{a2}, K_{a3}) are smaller than the lower order ionization constant (K_{a1}) of a polyprotic acid.
- **Factors Affecting Acid Strength:** The extent of dissociation of an acid depends on the **strength** and **polarity** of the H-A bond.

  Size increases →
  $HF << HCl << HBr << HI$
  Acid strength increases →

  Electronegativity of central atom increases →
  $CH_4 < NH_3 < H_2O < HF$
  Acid strength increases →
- **Common ion effect in the Ionisation of Acids and Bases:**
  Common ion effect. It can be defined as a shift in equilibrium on adding a substance that provides more of an ionic species already present in the dissociation equilibrium.
- **Hydrolysis of Salts and the pH of their Solutions:**
  - Salts formed by the reactions between acids and bases in definite proportions, undergo ionization in water.
  - The process of interaction between water and cations/anions or both of salts is called hydrolysis. The pH of the solution gets affected by this interaction.

- **Consider the types of hydrolysis of the salts :**
  - salts of weak acid and strong base e.g., CH_3COONa.
  - salts of strong acid and weak base e.g., NH_4Cl.
  - salts of weak acid and weak base, e.g., CH_3COONH_4.
- Degree of hydrolysis is independent of concentration of solution, and pH of such solutions is determined by their pK values:

$$pH = 7 + \frac{1}{2}(pK_a - pK_b)$$ JEE M 2017

- The pH of solution can be greater than 7, if the difference is positive and it will be less than 7, if the difference is negative.

6.12 Buffer Solutions

- The solutions which resist change in pH on dilution or with the addition of small amounts of acid or alkali are called Buffer Solutions.

Designing Buffer Solution:

$$pH = pK_a + \log\frac{[\text{Conjugate base, } A^-]}{[\text{Acid, HA}]}$$

- **Henderson–Hasselbalch equation:**

$$pH = pK_a + \log\frac{[\text{Salt}]}{[\text{Acid}]}$$

$$pOH = pK_b + \log\frac{[\text{Conjugate acid, } BH^+]}{[\text{Base, B}]}$$

JEE M 2019

6.13 Solubility Equilibria of Sparingly Soluble Salts

- A solid salt of the general formula $M_x^{p+} X_y^{q-}$ with molar solubility S in equilibrium with its saturated solution may be represented by the equation:

$$M_xX_y(s) \rightleftharpoons xM^{p+}(aq) + yX^{q-}(aq)$$

(where $x \times p^+ = y \times q^-$)

And its solubility product constant is given by:

$K_{sp} = [M^{p+}]^x[X^{q-}]^y = (xS)^x(yS)^y$

$= x^x \cdot y^y \cdot S^{(x+y)}$

$S^{(x+y)} = K_{sp} / x^x \cdot y^y$

$S = (K_{sp} / x^x \cdot y^y)^{1/x+y}$ JEE M 2023

- **Common Ion Effect on Solubility of Ionic Salts:**
  - If we increase the concentration of any one of the ions, it should combine with the ion of its opposite charge and some of the salt will be precipitated till $K_{sp} = Q_{sp}$. Similarly, if the concentration of one of the ions is decreased, more salt will dissolve to increase the concentration of both the ions till $K_{sp} = Q_{sp}$.
  - If we take a saturated solution of sodium chloride and pass HCl gas through it, then sodium chloride is precipitated due to increased concentration (activity) of chloride ion available from the dissociation of HCl. Sodium chloride thus obtained is of very high purity.

Tips/Tricks/Techniques ONE-Liners
(Exam Special)

Factors Affecting Equilibrium Concentrations:

Temperature

- An increase in temperature for exothermic reactions decreases the concentration of products at equilibrium and vice-versa.
- An increase in temperature for endothermic reactions increases the concentration of products at equilibrium and vice-versa.

Pressure

- An increase in pressure to an equilibrium favours the reaction where number of moles show a decrease and vice-versa.

Effect or pressure (or volume)

Effect of reaction	Effect of pressure (or volume)
I. $\Delta n = 0$	Equilibrium is not affected
II. $\Delta n = +ve$	Increase in P (or decrease in V) shifts the equilibrium to left
III. $\Delta n = -ve$	Increase in P (or decrease in V) shifts the equilibrium to right

- Variation of equilibrium constant with variation of the reaction equation (K = equilibrium constant for original reaction) is given as:

When the equation	the change in equilibrium constant
Reversed	1/K
Divided by 2	$\sqrt{K}$
Multiplied by 2	K^2
Divided into two steps	$K = K_1 \times K_2$

- Buffer capacity $= \dfrac{\text{Moles of acid or a base added to 1 litre of buffer}}{\text{Change in pH}}$

Exercise 1 : NCERT Based Topic-wise MCQs

6.0 Introduction

1. Which of the following is not a general characteristic of equilibria involving physical processes? NCERT Page-192 / N-168

(a) Equilibrium is possible only in a closed system at a given temperature.
(b) All measurable properties of the system remain constant.
(c) All the physical processes stop at equilibrium.
(d) The opposing processes occur at the same rate and there is dynamic but stable condition.

6.1 Equilibrium in Physical Processes

2. A small amount of acetone is taken in a watch glass and it is kept open in atmosphere. Which statement is correct for the given experiment? NCERT Page-193 / N-170

(a) The rate of condensation from vapour to liquid state is higher than the rate of evaporation.
(b) The rate of condensation from vapour to liquid state is equal to the rate of evaporation.
(c) The rate of condensation from vapour to liquid state is much less than the rate of evaporation.
(d) The rate of condensation from vapour to liquid state is equal or less than the rate of evaporation.

6.2 Equilibrium in Chemical Processes – Dynamics Equilibrium

3. A reaction is said to be in equilibrium when NCERT Page N-168

(a) the rate of transformation of reactant to products is equal to the rate of transformation of products to the reactants.
(b) 50% of the reactants are converted to products.
(c) the reaction is near completion and all the reactants are converted to products.
(d) the volume of reactants is just equal to the volume of the products.

4. Which of the following is not true about a reversible reaction?

(a) The reaction does not proceed to completion
(b) It cannot be influenced by a catalyst
(c) Number of moles of reactants and products is always equal
(d) It can be attained only in a closed container

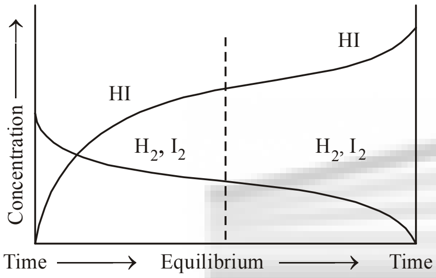
5. If a system is at equilibrium, the rate of forward to the reverse reaction is : NCERT Page-196 / N-172

(a) less (b) equal
(c) high (d) at equilibrium

6. If the synthesis of ammonia from Haber's process is carried out with exactly the same starting conditions (of partial pressure and temperature) but using D_2 (deuterium) in place of H_2. Then NCERT Page-196 / N-172

(a) the equilibrium will be disturbed
(b) the composition of reaction mixture will remain same at equilibrium.
(c) Use of isotope in reaction will not produce ammonia.
(d) At equilibrium rate of forward reaction will be greater than the rate of reverse reaction

7. Consider the following graph and mark the correct statement. NCERT Page-198 / N-174



(a) Chemical equilibrium in the reaction, $H_2 + I_2 \rightleftharpoons 2HI$ can be attained from either directions.
(b) Equilibrium can be obtained when H_2 and I_2 are mixed in an open vessel.
(c) The concentrations of H_2 and I_2 keep decreasing while concentration of HI keeps increasing with time.
(d) We can find out equilibrium concentration of H_2 and I_2 from the given graph.

6.3 Law of Chemical Equilibrium and Equilibrium Constant

8. The equilibrium constant for the reversible reaction

$N_2 + 3H_2 \rightleftharpoons 2NH_3$ is K and for reaction

$\frac{1}{2}N_2 + \frac{3}{2}H_2 \rightleftharpoons NH_3$, the equilibrium constant is K'

The K and K' will be related as: NCERT Page N-176

(a) $K \times K' = 1$ (b) $K = K'$
(c) $K' = \sqrt{K}$ (d) $K = \sqrt{K'}$

9. The rate constant for forward and backward reaction of hydrolysis of ester are 1.1×10^{-2} and 1.5×10^{-3} per minute respectively. Equilibrium constant for the reaction

$CH_3COOC_2H_5 + H^+ \rightleftharpoons CH_3COOH + C_2H_5OH$ is

NCERT Page N-175

(a) 4.33 (b) 5.33
(c) 6.33 (d) 7.33

10. If the equilibrium constant for $A \rightleftharpoons B + C$ is $K_{eq}^{(1)}$ and that of $B + C \rightleftharpoons P$ is $K_{eq}^{(2)}$, the equilibrium constant for $A \rightleftharpoons P$ is : NCERT Page N-176

(a) $K_{eq}^{(1)} / K_{eq}^{(2)}$ (b) $K_{eq}^{(2)} - K_{eq}^{(1)}$
(c) $K_{eq}^{(1)} + K_{eq}^{(2)}$ (d) $K_{eq}^{(1)} K_{eq}^{(2)}$

11. In the following equilibrium reaction

$2A \rightleftharpoons B + C,$

the equilibrium concentrations of A, B and C are 1×10^{-3} M, 2×10^{-3} M and 3×10^{-3} M respectively at 300 K. The value of K_c for this equilibrium at the same temperature is

NCERT Page-199 / N-175

(a) $\frac{1}{6}$ (b) 6
(c) $\frac{1}{36}$ (d) 36

12. Given the reaction between 2 gases represented by A_2 and B_2 to give the compound $AB(g)$. NCERT Page-202 / N-175

$A_2(g) + B_2(g) \rightleftharpoons 2\,AB(g).$

At equilibrium, the concentration

of $A_2 = 3.0 \times 10^{-3}$ M

of $B_2 = 4.2 \times 10^{-3}$ M

of $AB = 2.8 \times 10^{-3}$ M

If the reaction takes place in a sealed vessel at 527 °C, then the value of K_c will be :

(a) 2.0 (b) 1.9
(c) 0.62 (d) 4.5

13. In $A + B \rightleftharpoons C$. The unit of equilibrium constant is :

NCERT Page N-180

(a) Litre mol^{-1} (b) Mol litre
(c) Mol litre^{-1} (d) No unit

14. For the following three reactions (i), (ii) and (iii), equilibrium constants are given: NCERT Page-200 / N-176

(i) $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g); K_1$

(ii) $CH_4(g) + H_2O(g) \rightleftharpoons CO(g) + 3H_2(g); K_2$

(iii) $CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g); K_3$

(a) $K_1\sqrt{K_2} = K_3$ (b) $K_2K_3 = K_1$
(c) $K_3 = K_1 K_2$ (d) $K_3 \cdot K_2^3 = K_1^2$

15. K_1, K_2 and K_3 are the equilibrium constants of the following reactions (I), (II) and (III) respectively:

(I) $N_2 + 2O_2 \rightleftharpoons 2NO_2$ NCERT Page N-176 & 177

(II) $2NO_2 \rightleftharpoons N_2 + 2O_2$

(III) $NO_2 \rightleftharpoons \frac{1}{2}N_2 + O_2$

The correct relation from the following is

(a) $K_1 = \frac{1}{K_2} = \frac{1}{K_3}$ (b) $K_1 = \frac{1}{K_2} = \frac{1}{(K_3)^2}$

(c) $K_1 = \sqrt{K_2} = K_3$ (d) $K_1 = \frac{1}{K_2} = K_3$

6.4 Homogeneous Equilibria

16. In which of the following equilibrium K_c and K_p are not equal? NCERT Page N-178

(a) $2NO(g) \rightleftharpoons N_2(g) + O_2(g)$

(b) $SO_2(g) + NO_2(g) \rightleftharpoons SO_3(g) + NO(g)$

(c) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

(d) $2C(s) + O_2(g) \rightleftharpoons 2CO_2(g)$

17. For the reaction $C(s) + CO_2(g) \rightleftharpoons 2CO(g)$, the partial pressures of CO_2 and CO are 2.0 and 4.0 atm respectively at equilibrium. The K_p for the reaction is. NCERT Page-202 / N-178

(a) 0.5 (b) 4.0
(c) 8.0 (d) 32.0

18. The K_p/K_c ratio will be highest in case of NCERT Page N-178

(a) $CO(g) + \frac{1}{2}O_2(g) \rightleftharpoons CO_2(g)$

(b) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

(c) $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

(d) $7H_2(g) + 2NO_2(g) \rightleftharpoons 2NH_3(g) + 4H_2O(g)$

19. Steam reacts with iron at high temperature to give hydrogen gas and Fe_3O_4 (s). The correct expression for the equilibrium constant is NCERT Page-202 / N-178

(a) $\frac{(p_{H_2})^2}{(p_{H_2O})^2}$ (b) $\frac{(p_{H_2})^4}{(p_{H_2O})^4}$

(c) $\frac{(p_{H_2})^4[Fe_3O_4]}{(p_{H_2O})^4[Fe]}$ (d) $\frac{[Fe_3O_4]}{[Fe]}$

20. For the reversible reaction, NCERT Page-202 / N-178

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$ at 500°C, the value of K_p is 1.44×10^{-5} when partial pressure is measured in atmospheres. The corresponding value of K_c, with concentration in mol litre^{-1}, is

(a) $\frac{1.44 \times 10^{-5}}{(0.082 \times 500)^{-2}}$ (b) $\frac{1.44 \times 10^{-5}}{(8.314 \times 773)^{-2}}$

(c) $\frac{1.44 \times 10^{-5}}{(0.082 \times 773)^{2}}$ (d) $\frac{1.44 \times 10^{-5}}{(0.082 \times 773)^{-2}}$

21. Two moles of PCl_5 were heated in a closed vessel of 2L. At equilibrium 40% of PCl_5 is dissociated into PCl_3 and Cl_2. The value of equilibrium constant is NCERT Page N-175

(a) 0.53 (b) 0.267
(c) 2.63 (d) 5.3

22. PCl_5 is dissociating 50% at 250°C at a total pressure of P atm. If equilibrium constant is K_p, then which of the following relation is numerically correct ? NCERT Page N-178

(a) $K_p = 3P$ (b) $P = 3K_p$

(c) $P = \frac{2K_p}{3}$ (d) $K_p = \frac{2P}{3}$

6.5 Heterogeneous Equilibria

23. The thermal dissociation of calcium carbonate showing heterogeneous equilibrium is NCERT Page-204 / N-180

$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$

For this reactions which of the following is/are true

(i) $K_c = [CO_2(g)]$

(ii) $K_p = p_{CO2}$

(iii) $[CaCO_3(s)]$ and $[CaO(s)]$ are both constant

(iv) $[CO_2(g)]$ is constant

(a) (i), (ii) and (iv) (b) (i), (ii) and (iii)
(c) (ii) and (iv) (d) (i), (iii) and (iv)

24. Unit of equilibrium constant for the given reaction is

$Ni(s) + 4CO(g) \rightleftharpoons Ni(CO)_4(g)$ NCERT Page N-180

(a) $(mol/L)^{-3}$ (b) $(mol/L)^3$
(c) $(mol/L)^{-4}$ (d) $(mol/L)^4$

25. For a chemical reaction ; NCERT Page-202 / N-178

$A(g) + B(l) \rightleftharpoons D(g) + E(g)$

Hypothetically at what temperature, $K_p = K_c$

(when, R = 0.08 l-atm/mol-K)

(a) $T = 0$ K (b) $T = 1$K
(c) $T = 12.5$ K (d) $T = 273$ K

26. Equilibrium constant (K) for the reaction

$Ni(s) + 4CO(g) \rightleftharpoons Ni(CO)_4(g)$ can be written in terms of

(1) $Ni(s) + 2CO_2(g) + 2C(s) \rightleftharpoons Ni(CO)_4(g)$; equilibrium constant $= K_1$.

(2) $CO_2(g) + C(s) \rightleftharpoons 2CO(g)$; equilibrium constant $= K_2$.

What is the relation between K, K_1 and K_2?

NCERT Page-201 / N-176

(a) $K = (K_1)/(K_2)^2$ (b) $K = (K_1 . K_2)$

(c) $K = (K_1)(K_2)^2$ (d) $K = K_1/K_2$

27. For the decomposition of the compound, represented as

$NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$

the $K_p = 2.9 \times 10^{-5}$ atm^3. NCERT Page-202 / N-178

If the reaction is started with 1 mol of the compound, the total pressure at equilibrium would be :

(a) 1.94×10^{-2} atm (b) 5.82×10^{-2} atm

(c) 7.66×10^{-2} atm (d) 38.8×10^{-2} atm

6.6 Applications of Equilibrium Constants

28. In a reversible chemical reaction having two reactants, in equilibrium if the concentration of the reactants are doubled then the equilibrium constant will NCERT Page N-175

(a) Also be doubled (b) Be halved

(c) Become one-fourth (d) Remain the same

29. If K_c is in the range of .... appreciable concentrations of both reactants and products are present.

NCERT Page N-182

(a) 10^{-4} to 10^4 (b) 10^{-3} to 10^3

(c) 10^3 to 10^6 (d) 10^{-5} to 10^3

30. The reaction quotient (Q) for the reaction

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$ NCERT Page-206 / N-182

is given by $Q = \dfrac{[NH_3]^2}{[N_2][H_2]^3}$. The reaction will proceed from right to left if

(a) $Q = 0$ (b) $Q = K_c$ (c) $Q < K_c$ (d) $Q > K_c$

where K_c is the equilibrium constant

31. On doubling P and V with constant temperature the equilibrium constant will NCERT Page N-186

(a) remain constant (b) become double

(c) become one-fourth (d) None of these

32. The reaction quotient Q is used to NCERT Page-206 N-182

(a) predict the extent of a reaction on the basis of its magnitude

(b) predict the direction of the reaction

(c) calculate equilibrium concentrations

(d) calculate equilibrium constant

6.7 Relationship Between Equilibrium Constant K, Reaction Quotient Q and Gibbs Energy G

33. Using the equation ($K = e^{-\Delta G°/RT}$), the reaction spontaneity can be interpreted in terms of the value of $\Delta G°$ is/are NCERT Page-208 / N-184

(a) If $\Delta G° < 0$, then $-\Delta G°/RT$ is positive, and $e^{-\Delta G°/RT} >$ 1 making $K > 1$, which implies a spontaneous reaction or the reaction which proceeds in the forward direction to such an extent that the products are present predominantly.

(b) If $\Delta G° > 0$, then $-\Delta G°/RT$ is negative, and $e^{-\Delta G°/RT} <$ 1 making $K < 1$, which implies a non-spontaneous reaction or a reaction which proceeds in the forward direction to such a small degree that only a very minute quantity of product is formed.

(c) Both (a) and (b)

(d) None of the above

34. The correct relationship between free energy change in a reaction and the corresponding equilibrium constant, K_c is NCERT Page-208 / N-184

(a) $\Delta G = RT \ln K_c$ (b) $-\Delta G = RT \ln K_c$

(c) $\Delta G° = RT \ln K_c$ (d) $-\Delta G° = RT \ln K_c$

6.8 Factors Affecting Equilibria

35. If for the reaction NCERT Page-211 / N-187

$N_2 + 3H_2 \rightleftharpoons 2NH_3$, $\Delta H = -92.38$ kJ/mol, then what happens if the temperature is increased?

(a) Reaction proceed forward

(b) Reaction proceed backward

(c) No effect on the formation of product

(d) None of these

36. The equilibrium which remains unaffected by pressure change is NCERT Page-210 / N-186

(a) $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

(b) $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

(c) $2O_3(g) \rightleftharpoons 3O_2(g)$

(d) $2NO_2(g) \rightleftharpoons N_2O_4(g)$

37. Which one of the following information can be obtained on the basis of Le Chatelier principle? NCERT Page N-185

(a) Dissociation constant of a weak acid

(b) Entropy change in a reaction

(c) Equilibrium constant of a chemical reaction

(d) Shift in equilibrium position on changing value of a constraint

38. According to Le-chatelier's principle, adding heat to a solid $\rightleftharpoons$ liquid equilibrium will cause the

NCERT Page-211 / N-187

(a) temperature to increase
(b) temperature to decrease
(c) amount of liquid to decrease
(d) amount of solid to decrease

39. Which of the following reaction will be favoured at low pressure?

NCERT Page-210 / N-186

(a) $H_2 + I_2 \rightleftharpoons 2HI$

(b) $N_2 + 3H_2 \rightleftharpoons 2NH_3$

(c) $PCl_5 \rightleftharpoons PCl_3 + Cl_2$

(d) $N_2 + O_2 \rightleftharpoons 2NO$

40. For the manufacture of ammonia by the reaction

$$N_2 + 3H_2 \rightleftharpoons 2NH_3 + 2\,kcal$$

the favourable conditions are

NCERT Page N-186 & 187

(a) Low temperature, low pressure and catalyst
(b) Low temperature, high pressure and catalyst
(c) High temperature, low pressure and catalyst
(d) High temperature, high pressure and catalyst

41. In a vessel N_2, H_2 and NH_3 are at equilibrium. Some helium gas is introduced into the vessel so that total pressure increases while temperature and volume remain constant. According to Le Chatelier's principle, the dissociation of NH_3

NCERT Page N-187

(a) increases
(b) decreases
(c) remains unchanged
(d) equilibrium is disturbed

42. In a two-step exothermic reaction

$$A_2(g) + B_2(g) \underset{\text{Step 1}}{\rightleftharpoons} 3C(g) \underset{\text{Step 2}}{\rightleftharpoons} D(g)$$

Steps 1 and 2 are favoured respectively by

NCERT Page-210 & 211 / N-186 & 187

(a) high pressure, high temperature and low pressure, low temperature
(b) high pressure, low temperature and low pressure, high temperature
(c) low pressure, high temperature and high pressure, high temperature
(d) low pressure, low temperature and high pressure, low temperature

43. The exothermic formation of ClF_3 is represented by the equation :

NCERT Page-211 / N-187

$$Cl_2(g) + 3F_2(g) \rightleftharpoons 2ClF_3(g); \Delta H = -329\,kJ$$

Which of the following will increase the quantity of ClF_3 in an equilibrium mixture of Cl_2, F_2 and ClF_3?

(a) Adding F_2
(b) Increasing the volume of the container
(c) Removing Cl_2
(d) Increasing the temperature

44. Suitable conditions for melting of ice :

NCERT Page N-186 & 187

(a) high temperature and high pressure
(b) high temperature and low pressure
(c) low temperature and low pressure
(d) low temperature and high pressure

6.9 Ionic Equilibrium in Solution

45. In weak electrolytes, equilibrium is established between ions and the unionized molecules. This type of equilibrium

NCERT Page-212 / N-188

(a) is due to complete ionization of electrolyte.
(b) is called non ionic equilibrium.
(c) is called physical equilibrium.
(d) involves ions in aqueous solution.

6.10 Acids, Bases and Salts

46. A base, as defined by Bronsted theory, is a substance which can

NCERT Page-214 / N-190

(a) lose a pair of electrons
(b) donate protons
(c) gain a pair of electrons
(d) accept protons

47. Which of the following molecules acts as a Lewis acid ?

NCERT Page-216 / N-192

(a) $(CH_3)_2O$
(b) $(CH_3)_3P$
(c) $(CH_3)_3N$
(d) $(CH_3)_3B$

48. Which one of the following molecular hydrides acts as a Lewis acid?

NCERT Page N-192

(a) NH_3
(b) H_2O
(c) B_2H_6
(d) CH_4

49. Water is well known amphoprotic solvent. In which chemical reaction water is behaving as a base only?

NCERT Page-215 / N-191

(a) $H_2SO_4 + H_2O \longrightarrow H_3O^+ + HSO_4^-$

(b) $H_2O + H_2O \longrightarrow H_3O^+ + OH^-$

(c) $H_2O + NH_2^- \longrightarrow NH_3 + OH^-$

(d) $H_2O + NH_3 \longrightarrow NH_4^+ + OH^-$

50. Which one of the following is the correct statement ?

NCERT Page-216 / N-192

(a) HCO_3^- is the conjugate base of CO_3^{2-}.
(b) NH_2^- is the conjugate acid of NH_3.
(c) H_2SO_4 is the conjugate acid of HSO_4^-.
(d) NH_3 is the conjugate base of NH_2^-.

51. Three reactions involving $H_2PO_4^-$ are given below:

(i) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$

(ii) $H_2PO_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$

(iii) $H_2PO_4^- + OH^- \rightarrow H_3PO_4 + O^{2-}$

In which of the above does $H_2PO_4^-$ act as an acid ?

NCERT Page N-192

(a) (ii) only
(b) (i) and (ii)
(c) (iii) only
(d) (i) only

52. Which equilibrium can be described as an acid-base reaction using the Lewis acid-base definition but not using the Bronsted-Lowry definition? NCERT Page N-190, 191 & 192

(a) $2NH_3 + H_2SO_4 \rightleftharpoons 2NH_4^+ + SO_4^{2-}$

(b) $NH_3 + CH_3COOH \rightleftharpoons NH_4^+ + CH_3COO^-$

(c) $H_2O + CH_3COOH \rightleftharpoons H_3O^+ + CH_3COO^-$

(d) $[Cu(H_2O)_4]^{2+} + 4\,NH_3 \rightleftharpoons [Cu(NH_3)_4]^{2+} + 4H_2O$

53. Among boron trifluoride, stannic chloride and stannous chloride, Lewis acid is represented by

NCERT Page-216 / N-192

(a) only boron trifluoride
(b) boron trifluoride and stannic chloride
(c) boron trifluoride and stannous chloride
(d) all three

54. An acid/ base dissociation equilibrium is dynamic involving a transfer of proton in forward and reverse directions. Now, with passage of time in which direction equilibrium is favoured? NCERT Page N-193

(a) in the direction of stronger base and stronger acid
(b) in the direction of formation of stronger base and weaker acid
(c) in the direction of formation of weaker base and weaker acid
(d) in the direction of formation of weaker base and stronger acid

55. In HS^-, I^-, RNH_2 and NH_3, order of proton accepting tendency will be NCERT Page N-193

(a) $I^- > NH_3 > RNH_2 > HS^-$

(b) $HS^- > RNH_2 > NH_3 > I^-$

(c) $RNH_2 > NH_3 > HS^- > I^-$

(d) $NH_3 > RNH_2 > HS^- > I^-$

56. Conjugate acid of NH_2^- is : NCERT Page-215 / N-191

(a) NH_4^+ (b) NH_3

(c) NH_2 (d) NH

6.11 Ionization of Acids and Bases

57. The value of the ionic product of water

NCERT Page-217 / N-193

(a) depends on volume of water
(b) depends on temperature
(c) changes by adding acid or alkali
(d) always remains constant

58. What is the approximate pH of a 1×10^{-3} M NaOH solution?

NCERT Page N-194

(a) 3 (b) 11
(c) 7 (d) 1×10^{-11}

59. Calculate the pOH of a solution at 25°C that contains 1×10^{-10} M of hydronium ions, *i.e.*, H_3O^+.

NCERT Page N-194

(a) 4.000 (b) 9.0000
(c) 1.000 (d) 7.000

60. A weak acid, H*A*, has a K_a of 1.00×10^{-5}. If 0.100 mole of this acid dissolved in one litre of water, the percentage of acid dissociated at equilbrium is closest to

NCERT Page-219 / N-195

(a) 1.00% (b) 99.9%
(c) 0.100% (d) 99.0%

61. A monobasic weak acid solution has a molarity of 0.005 and pH of 5. What is the percentage ionization in this solution? NCERT Page N-195

(a) 2.0 (b) 0.2
(c) 0.5 (d) 0.25

62. Equimolar solutions of HF, HCOOH and HCN at 298 K have the values of K_a as 6.8×10^{-4}, 1.8×10^{-4} and 4.8×10^{-9} respectively. What is the observed trend of dissociation constants in successive stages ? NCERT Page N-195

(a) HF > HCN > HCOOH (b) HF > HCOOH > HCN
(c) HCN > HF > HCOOH (d) HCOOH > HCN > HF

63. Which of the following pK_a value represents the strongest acid ? NCERT Page-220 / N-196

(a) 10^{-4} (b) 10^{-8}
(c) 10^{-5} (d) 10^{-2}

64. The dissociation constant of two acids HA_1 and HA_2 are 3.14×10^{-4} and 1.96×10^{-5} respectively. The relative strength of the acids will be approximately

NCERT Page-223 / N-199

(a) 1 : 4 (b) 4 : 1
(c) 1 : 16 (d) 16 : 1

65. At 298K a 0.1 M CH_3COOH solution is 1.34% ionized. The ionization constant K_a for acetic acid will be

NCERT Page-219 / N-195

(a) 1.82×10^{-5} (b) 18.2×10^{-5}

(c) 0.182×10^{-5} (d) None of these

66. In qualitative analysis, in III group NH_4Cl is added before NH_4OH because NCERT Page N-200 & 201
(a) to increase the concentration of NH_4^+ ions
(b) to increase concentration of Cl^- ions
(c) to reduce the concentration of OH^- ions
(d) to increase concentration of OH^- ions

67. The solubility of AgI in NaI solution is less than that in pure water because : NCERT Page N-200 & 201
(a) the temperature of the solution decreases
(b) solubility product to AgI is less than that of NaI
(c) of common ion effect
(d) AgI forms complex with NaI

68. Which of the following statements about pH and H^+ ion concentration is incorrect? NCERT Page N-194
(a) Addition of one drop of concentrated HCl in NH_4OH solution decreases pH of the solution.
(b) A solution of the mixture of one equivalent of each of CH_3COOH and NaOH has a pH of 7
(c) pH of pure neutral water is not zero
(d) A cold and concentrated H_2SO_4 has lower H^+ ion concentration than a dilute solution of H_2SO_4

69. The pH of a 10^{-3} M HCl solution at 25°C if it is diluted 1000 times, will be – NCERT Page-218 / N-194
(a) 3 (b) zero
(c) 5.98 (d) 6.02

70. How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2 ? NCERT Page N-194
(a) 0.1 L (b) 0.9 L
(c) 2.0 L (d) 9.0 L

71. The pH value of a 10 M solution of HCl is
NCERT Page-218 / N-194 & 195
(a) less than 0 (b) equal to 0
(c) equal to 1 (d) equal to 2

72. What is the H^+ ion concentration of a solution prepared by dissolving 4 g of NaOH (Atomic weight of Na = 23 amu) in 1000 ml? NCERT Page N-194
(a) 10^{-10} M (b) 10^{-4} M
(c) 10^{-1} M (d) 10^{-13} M

73. Calculate the pH of a solution obtained by diluting 1 mL of 0.10 M weak monoacidic base to 100 mL at constant temperature if K_b of the base is 1×10^{-5} ?
NCERT Page N-197
(a) 8 (b) 9
(c) 10 (d) 11

74. At 25°C, the dissociation constant of a base, *B*OH, is 1.0×10^{-12}. The concentration of hydroxyl ions in 0.01 M aqueous solution of the base would be
NCERT Page-221 / N-197
(a) 1.0×10^{-5} mol L^{-1} (b) 1.0×10^{-6} mol L^{-1}
(c) 2.0×10^{-6} mol L^{-1} (d) 1.0×10^{-7} mol L^{-1}

75. Given NCERT Page-222 & 223 / N-198 & 199

$$HF + H_2O \xrightarrow{K_a} H_3O^+ + F^-$$

$$F^- + H_2O \xrightarrow{K_b} HF + OH^-$$

Which of the following reaction is correct
(a) $K_b = K_w$ (b) $K_b = \dfrac{1}{K_w}$
(c) $K_a \times K_b = K_w$ (d) $\dfrac{K_a}{K_b} = K_w$

76. Equal volumes of three acid solutions of pH 3, 4 and 5 are mixed in a vessel. What will be the H^+ ion concentration in the mixture ? NCERT Page N-194
(a) 1.11×10^{-4} M (b) 3.7×10^{-4} M
(c) 3.7×10^{-3} M (d) 1.11×10^{-3} M

77. Which of the following statements are correct ?
(i) Ionic product of water $(K_w) = [H^+][OH^-] = 10^{-14}\ M^2$
(ii) At 298K $[H^+] = [OH^-] = 10^{-7}$ M
(iii) K_w does not depends upon temperature
(iv) Molarity of pure water = 55.55 M NCERT Page N-193
(a) (i), (ii) and (iii) (b) (i), (ii) and (iv)
(c) (i) and (iv) (d) (ii) and (iii)

78. Arrange the following solutions in the decreasing order of pOH : NCERT Page N-194
(A) 0.01 M HCl (B) 0.01 M NaOH
(C) 0.01 M CH_3COONa (D) 0.01 M NaCl
(a) (A) > (C) > (D) > (B) (b) (A) > (D) > (C) > (B)
(c) (B) > (C) > (D) > (A) (d) (B) > (D) > (C) > (A)

79. 100 mL of 0.04 N HCl aqueous solution is mixed with 100 mL of 0.02 N NaOH solution. The pH of the resulting solution is: NCERT Page N-194
(a) 1.0 (b) 1.7
(c) 2.0 (d) 2.3

80. At 100°C the K_w of water is 55 times its value at 25°C. What will be the pH of neutral solution? (log 55 = 1.74)
NCERT Page-217 / N-194
(a) 6.13 (b) 7.00
(c) 7.87 (d) 5.13

81. Values of dissociation constant, K_a are given as follows :

Acid	Ka
HCN	6.2×10^{-10}
HF	7.2×10^{-4}
HNO_2	4.0×10^{-4}

Correct order of increasing base strength of the base CN^-, F^- and NO_2^- will be : NCERT Page N-195
(a) $F^- < CN^- < NO_2^-$ (b) $NO_2^- < CN^- < F^-$
(c) $F^- < NO_2^- < CN^-$ (d) $NO_2^- < F^- < CN^-$

82. If degree of dissociation of pure water at 100°C is 1.8×10^{-8}, then the dissociation constant of water will be (density of H_2O = 1 g/cc) NCERT Page-219 / N-195

(a) 1×10^{-12} (b) 1×10^{-14}

(c) 1.8×10^{-12} (d) 1.8×10^{-14}

83. Ionisation constant of CH_3COOH is 1.7×10^{-5} if concentration of H^+ ions is 3.4×10^{-4}M, then find out initial concentration of CH_3COOH molecules NCERT Page-219 / N-195

(a) 3.4×10^{-4} M (b) 3.4×10^{-3} M

(c) 6.8×10^{-3} M (d) 6.8×10^{-4} M

84. The dissociation constants for acetic acid and HCN at 25°C are 1.5×10^{-5} and 4.5×10^{-10} respectively. The equilibrium constant for the equilibrium

$CN^- + CH_3COOH \rightleftharpoons HCN + CH_3COO^-$ would be: NCERT Page-219 / N-195

(a) 3.3×10^{-5} (b) 3.3×10^{-4}

(c) 3.3×10^{4} (d) 3.3×10^{5}

85. K_{a_1}, K_{a_2} and K_{a_3} are the respective ionization constants for the following reactions (A), (B), and (C).

(A) $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$

(B) $HC_2O_4 \rightleftharpoons H^+ + HC_2O_4^{2-}$

(C) $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$

The relationship between K_{a_1}, K_{a_2} and K_{a_3} is given as NCERT Page N-199 & 200

(a) $K_{a_3} = K_{a_1} + K_{a_2}$ (b) $K_{a_3} = K_{a_1} + K_{a_2}$

(c) $K_{a_3} = K_{a_1} / K_{a_2}$ (d) $K_{a_3} = K_{a_1} \times K_{a_2}$

6.12 Buffer Solutions

86. The pK_a of a weak acid, HA, is 4.80. The pK_b of a weak base, BOH, is 4.78. The pH of an aqueous solution of the correspondng salt, BA, will be NCERT Page-227 / N-202

(a) 9.58 (b) 4.79 (c) 7.01 (d) 9.22

87. A buffer solution is prepared in which the concentration of NH_3 is 0.30M and the concentration of NH_4^+ is 0.20 M. If the equilibrium constant, K_b for NH_3 equals 1.8×10^{-5}, what is the pH of this solution ? (log 2.7 = 0.433). NCERT Page N-203

(a) 9.08 (b) 9.43 (c) 11.72 (d) 8.73

88. What is $[H^+]$ in mol/L of a solution that is 0.20 M in CH_3COONa and 0.10 M in CH_3COOH ? K_a for CH_3COOH = 1.8×10^{-5}. NCERT Page-227 / N-203

(a) 3.5×10^{-4} 10^{-4} (b) 1.1×10^{-5}

(c) 1.8×10^{-5} (d) 9.0×10^{-6}

89. Which of the following pairs constitutes a buffer? NCERT Page N-202 & 203

(a) NaOH and NaCl (b) HNO_3 and NH_4NO_3

(c) HCl and KCl (d) HNO_2 and $NaNO_2$

90. A buffer solution is prepared by mixing 0.1 M ammonia and 1.0 M ammonium chloride. At 298 K, the pK_b of NH_4OH is 5.0. The pH of the buffer is NCERT Page-227 N-203

(a) 10.0 (b) 9.0

(c) 6.0 (d) 8.0

91. The pK_a of a weak acid (HA) is 4.5. The pOH of an aqueous buffer solution of HA in which 50% of the acid is ionized is NCERT Page N-203

(a) 7.0 (b) 4.5 (c) 2.5 (d) 9.5

92. Buffer solutions have constant acidity and alkalinity because NCERT Page N-202 & 203

(a) these give unionised acid or base on reaction with added acid or alkali.

(b) acids and alkalies in these solutions are shielded from attack by other ions.

(c) they have large excess of H^+ or OH^- ions

(d) they have fixed value of pH

93. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions? NCERT Page N-204

(a) 9 (b) 10 (c) 11 (d) 8

94. When a buffer solution of sodium acetate and acetic acid, is diluted with water : NCERT Page N-203

(a) Acetate ion concentration increases

(b) H^+ ion concentration increases

(c) OH^- ion conc. increases

(d) H^+ ion concentration remains unaltered

95. A student needs to prepare a buffer solution of propanoic acid and its sodium salt with pH 4. The ratio of $\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$ required to make buffer is ....

Given : $K_a(CH_3CH_2COOH) = 1.3 \times 10^{-5}$ NCERT Page N-202 & 203

(a) 0.03 (b) 0.13 (c) 0.23 (d) 0.33

6.13 Solubility Equilibria of Sparingly Soluble Salts

96. The K_{sp} for $Cr(OH)_3$ is 1.6×10^{-30}. The solubility of this compound in water is : NCERT Page-228 / N-204

(a) $\sqrt[4]{1.6 \times 10^{-30}}$ (b) $\sqrt[4]{1.6 \times 10^{-30} / 27}$

(c) $1.6 \times 10^{-30/27}$ (d) $\sqrt{1.6 \times 10^{-30}}$

97. The product of ionic concentration in a saturated solution of an electrolyte at a given temperature is constant and is known as NCERT Page N-204
(a) Ionic product of the electrolyte
(b) Solubility product
(c) Ionization constant
(d) Dissociation constant

98. If s and S are respectively solubility and solubility product of a sparingly soluble binary electrolyte then : NCERT Page N-204
(a) $s = S$ (b) $s = S^2$
(c) $s = S^{1/2}$ (d) $s = \frac{1}{2}S$

99. pH of a saturated solution of $Ba(OH)_2$ is 12. The value of solubility product (K_{sp}) of $Ba(OH)_2$ is : NCERT Page-228 / N-204
(a) 3.3×10^{-7} (b) 5.0×10^{-7}
(c) 4.0×10^{-6} (d) 5.0×10^{-6}

100. What is the molar solubility of $Fe(OH)_3$ if $K_{sp} = 1.0 \times 10^{-38}$? NCERT Page N-204
(a) 3.16×10^{-10} (b) 1.386×10^{-10}
(c) 1.45×10^{-9} (d) 1.12×10^{-11}

101. The solubility of AgCl will be maximum in which of the following ? NCERT Page N-204
(a) 0.01 M KCl (b) 0.01 M HCl
(c) 0.01 M $AgNO_3$ (d) Deionised water

102. The K_{sp} for bismuth sulphide (Bi_2S_3) is 1.08×10^{-73}. The solubility of Bi_2S_3 in mol L^{-1} at 298 K is NCERT Page-228 / N-204
(a) 1.0×10^{-15} (b) 2.7×10^{-12}
(c) 3.2×10^{-10} (d) 4.2×10^{-8}

103. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is NCERT Page N-204
(a) 1.2×10^{-10} g (b) 1.2×10^{-9} g
(c) 6.2×10^{-5} g (d) 5.0×10^{-8} g

104. In qualitative analysis, the metals of Group I can be separated from other ions by precipitating them as chloride salts. A solution initially contains Ag^+ and Pb^{2+} at a concentration of 0.10 M. Aqueous HCl is added to this solution until the Cl^- concentration is 0.10 M. What will the concentrations of Ag^+ and Pb^{2+} be at equilibrium? (K_{sp} for AgCl = 1.8×10^{-10}, K_{sp} for $PbCl_2 = 1.7 \times 10^{-5}$) NCERT Page N-204
(a) $[Ag^+] = 1.8 \times 10^{-7}$ M ; $[Pb^{2+}] = 1.7 \times 10^{-6}$ M
(b) $[Ag^+] = 1.8 \times 10^{-11}$ M ; $[Pb^{2+}] = 8.5 \times 10^{-5}$ M
(c) $[Ag^+] = 1.8 \times 10^{-9}$ M ; $[Pb^{2+}] = 1.7 \times 10^{-3}$ M
(d) $[Ag^+] = 1.8 \times 10^{-11}$ M ; $[Pb^{2+}] = 8.5 \times 10^{-4}$ M

105. The solubility of $Ca(OH)_2$ in water is :
[Given : The solubility product of $Ca(OH)_2$ in water = 5.5×10^{-6}] NCERT Page-228 / N-204
(a) 1.77×10^{-2} (b) 1.11×10^{-2}
(c) 1.77×10^{-6} (d) 1.11×10^{-6}

Exercise 2 : NCERT Exemplar & Past Years JEE Main

NCERT Exemplar Questions

1. We know that the relationship between K_c and K_p is
$$K_p = K_c (RT)^{\Delta n}$$
What would be the value of Δn for the reaction?
$NH_4Cl(s) \rightleftharpoons NH_3(g) + HI(g)$ NCERT Page-202 / N-178
(a) 1 (b) 0.5 (c) 1.5 (d) 2

2. For the reaction, $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$, the standard free energy is $\Delta G° > 0$. The equilibrium constant (K) would be NCERT Page-208 / N-184
(a) $K = 0$ (b) $K > 1$ (c) $K = 1$ (d) $K < 1$

3. Which of the following is not a general characteristic of equilibria involving physical processes?
(a) Equilibrium is possible only in a closed system at a given temperature. NCERT Page-193 / N-169
(b) All measurable properties of the system remain constant.
(c) All the physical processes stop at equilibrium.
(d) The opposing processes occur at the same rate and there is dynamic but stable condition.

4. PCl_5, PCl_3, and Cl_2 are at equilibrium at 500 K in a closed container and their concentrations are 0.8×10^{-3} mol L^{-1}, 1.2×10^{-3} mol L^{-1} and 1.2×10^{-3} mol L^{-1} respectively. The value of K_c for the reaction NCERT Page-199 / N-175
$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$ will be
(a) 1.8×10^3 mol L^{-1} (b) 1.8×10^{-3}
(c) 1.8×10^{-3} mol^{-1} L (d) 0.55×10^4

5. The ionisation constant of an acid, K_a is the measure of strength of an acid. The K_a values of acetic acid, hypochlorous acid and formic acid are 1.74×10^{-5}, 3.0×10^{-8} and 1.8×10^{-4} respectively. Which of the following orders of pH of 0.1 mol dm^{-3} solutions of these acids is correct?
(a) Acetic acid > hypochlorous acid > formic acid
(b) Hypochlorous acid > acetic acid > formic acid
(c) Formic acid > hypochlorous acid > acetic acid
(d) Formic acid > acetic acid > hypochlorous acid

6. K_{a_1}, K_{a_2} and K_{a_3} are the respective ionisation constants for the following reactions. **NCERT** Page-223 / N-199

$$H_2S \rightleftharpoons H^+ + HS^-$$
$$HS^- \rightleftharpoons H^+ + S^{2-}$$
$$H_2S \rightleftharpoons 2H^+ + S^{2-}$$

The correct relationship between K_{a_1}, K_{a_2} and K_{a_3} is
(a) $K_{a_3} = K_{a_1} \times K_{a_2}$ (b) $K_{a_3} = K_{a_1} + K_{a_2}$
(c) $K_{a_3} = K_{a_1} - K_{a_2}$ (d) $K_{a_3} = K_{a_1} / K_{a_2}$

7. Acidity of BF_3 can be explained on the basis of which of the following concepts? **NCERT** Page-216 / N-192
(a) Arrhenius concept
(b) Bronsted Lowry concept
(c) Lewis concept
(d) Bronsted Lowry as well as Lewis concept

8. Which of the following options will be correct for the stage of half completion of the reaction $A \rightleftharpoons B$? **NCERT** Page-208 / N-184
(a) $\Delta G^\ominus = 0$ (b) $\Delta G^\ominus > 0$
(c) $\Delta G^\ominus < 0$ (d) $\Delta G^\ominus = -RT \ln K$

9. At 500 K, equilibrium constant, K_c, for the following reaction is 5. **NCERT** Page-200 / N-176

$$\frac{1}{2}H_2(g) + \frac{1}{2}I_2(g) \rightleftharpoons HI(g)$$

What would be the equilibrium constant K_c for the reaction ?

$$2HI(g) \rightleftharpoons H_2(g) + I_2(g)$$

(a) 0.04 (b) 0.4 (c) 25 (d) 2.5

10. In which of the following reactions, the equilibrium remains unaffected on addition of small amount of argon at constant volume? **NCERT** Page-211 / N-187
(a) $H_2(g) + I_2 \rightleftharpoons 2HI(g)$
(b) $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$
(c) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$
(d) The equilibrium will remain unaffected in all the three cases

11. When hydrochloric acid is added to cobalt solution at room temperature, the following reaction takes place and thus the reaction mixture becomes blue. On cooling the mixture it becomes pink. On the basis of this information mark the correct answer. **NCERT** Page-212 N-188

$$\underset{\text{Pink}}{[Co(H_2O)_6]^{3+}}(aq) + 4Cl^-(aq) \rightleftharpoons \underset{\text{Blue}}{[CoCl_4]^{2-}}(aq) + 6H_2O(l)$$

(a) $\Delta H > 0$ for the reaction
(b) $\Delta H < 0$ for the reaction
(c) $\Delta H = 0$ for the reaction
(d) The sign of ΔH cannot be predicted on the basis of this information

12. The pH of neutral water at 25°C is 7.0. As the temperature increases, ionisation of water increases, however, the concentration of H^+ ions and OH^- ions are equal. What will be the pH of pure water at 60°C? **NCERT** Page-217 / N-193
(a) Equal to 7.0 (b) Greater than 7.0
(c) Less than 7.0 (d) Equal to zero

13. Which of the following will produce a buffer solution when mixed in equal volumes? **NCERT** Page N-202
(a) 0.1 mol dm^{-3} NH_4OH and 0.1 mol dm^{-3} HCl
(b) 0.05 mol dm^{-3} NH_4OH and 0.1 mol dm^{-3} HCl
(c) 0.1 mol dm^{-3} NH_4OH and 0.05 mol dm^{-3} HCl
(d) 0.1 mol dm^{-3} CH_3COONa and 0.1 mol dm^{-3} NaOH

14. In which of the following solvent silver chloride is most soluble?
(a) 0.1 mol dm^{-3} $AgNO_3$ solution
(b) 0.1 mol dm^{-3} HCl solution
(c) H_2O
(d) Aqueous ammonia

15. What will be the value of pH of 0.01 mol dm^{-3} CH_3COOH ($K_a = 1.74 \times 10^{-5}$)? **NCERT** Page-219 / N-195
(a) 3.4 (b) 3.6 (c) 3.9 (d) 3.0

16. K_a for CH_3COOH is 1.8×10^{-5} and K_b for NH_4OH is 1.8×10^{-5}. The pH of ammonium acetate will be **NCERT** Page-226 / N-202
(a) 7.005 (b) 4.75
(c) 7.0 (d) Between 6 and 7

17. On increasing the pressure, in which direction will the gas phase reaction proceed to re-establish equilibrium, is predicted by applying the Le-Chatelier's principle. Consider the reaction, **NCERT** Page N-186

$$N_2(g) + 3H_2 \rightleftharpoons 2NH_3(g)$$

Which of the following is correct, if the total pressure at which the equilibrium is established, is increased without changing the temperature?
(a) K will remain same
(b) K will decrease
(c) K will increase
(d) K will increase initially and decrease when pressure is very high

18. Which of the following statements is incorrect?

NCERT Page-210 / N-186

(a) In equilibrium mixture of ice and water kept in perfectly insulated flask, mass of ice and water does not change with time
(b) The intensity of red colour increases when oxalic acid is added to a solution containing iron (III) nitrate and potassium thiocyanate
(c) On addition of catalyst the equilibrium constant value is not affected
(d) Equilibrium constant for a reaction with negative ΔH value decreases as the temperature increases.

Past Years JEE Main

19. Given below are two statements :

Statement (I) : Aqueous solution of ammonium carbonate is basic. NCERT Page-225 / N-201 | JEE M 2024, C

Statement (II) : Acidic/basic nature of salt solution of a salt of weak acid and weak base depends on K_a and K_b value of acid and the base forming it.

In the light of the above statements, choose the most appropriate answer from the options given below :
(a) Both Statement I and Statement II are correct
(b) Statement I is correct but Statement II is incorrect
(c) Both Statement I and Statement II are incorrect
(d) Statement I is incorrect but Statement II is correct

20. 25.0 mL of 0.050 M $(Ba(NO_3)_2$ is mixed with 25.0 mL of 0.020 M NaF. K_{sp} of BaF_2 is 0.5×10^{-6} at 298 K. The ratio of $[Ba^{2+}][F^-]^2$ and K_{sp} is ________ (Nearest integer)

NCERT Page-228 | JEE M NV, 2023, A

21. 20 mL of 0.1 M NH_4OH is mixed with 40 mL of 0.05 M HCl. The pH of the mixture is nearest to:
(Given : $K_b\,(NH_4OH) = 1 \times 10^{-5}$, $\log 2 = 0.30$, $\log 3 = 0.48$, $\log 5 = 0.69$, $\log 7 = 0.84$, $\log 11 = 1.04$)

NCERT Page N-202 | JEE M 2022, A

(a) 3.2 (b) 4.2
(c) 5.2 (d) 6.2

22. The equilibrium constant for the reversible reaction

$2A(g) \rightleftharpoons 2B(g) + C(g)$ is K_1 and for the reaction

$\frac{3}{2}A(g) \rightleftharpoons \frac{3}{2}B(g) + \frac{3}{4}C(g)$ is K_2

K_1 and K_2 are related as:

NCERT Page-201 / N-176 | JEE M 2022, A

(a) $K_1 = \sqrt{K_2}$ (b) $K_2 = \sqrt{K_1}$
(c) $K_2 = K_1^{3/4}$ (d) $K_1 = K_2^{3/4}$

23. 0.01 moles of a weak acid HA ($K_a = 2.0 \times 10^{-6}$) is dissolved in 1.0 L of 0.1 M HCl solution. The degree of dissociation of HA is ______ $\times 10^{-5}$ (Round off to the Nearest Integer).
[Neglect volume change on adding HA.
Assume degree of dissociation <<1]

NCERT Page N-194 | JEE M NV, 2021, S

24. Two solutions, A and B, each of 100 L was made by dissolving 4g of NaOH and 9.8 g of H_2SO_4 in water, respectively. The pH of the resultant solution obtained from mixing 40 L of solution A and 10 L of solution. B is _______. NCERT Page N-194 JEE M NV, 2020, S

25. 20 mL of 0.1 M H_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resultant mixture is:
[pK_b of $NH_4OH = 4.7$]. NCERT Page-227 / N-194 | JEE M 2019, A
(a) 5.2 (b) 9.0
(c) 5.0 (d) 9.4

26. Which amongst the following is the strongest acid?

NCERT Page N-194 JEE M 2019, C

(a) $CHBr_3$ (b) CHI_3
(c) $CH(CN)_3$ (d) $CHCl_3$

27. Which of the following salts is the most basic in aqueous solution? NCERT Page-226 / N-202 | JEE M 2018, S
(a) $Al(CN)_3$
(b) CH_3COOK
(c) $FeCl_3$
(d) $Pb(CH_3COO)_2$

28. pK_a of a weak acid (*HA*) and pK_b of a weak base (*BOH*) are 3.2 and 3.4, respectively. The pH of their salt (*AB*) solution is NCERT Page-226 / N-202 | JEE M 2017, A
(a) 7.2 (b) 6.9
(c) 7.0 (d) 1.0

29. The equilibrium constant at 298 K for a reaction $A + B \rightleftharpoons C + D$ is 100. If the initial concentration of all the four species were 1 M each, then equilibrium concentration of *D* (in mol L^{-1}) will be:

NCERT Page-199 / N-175 | JEE M 2016, S

(a) 1.818 (b) 1.182
(c) 0.182 (d) 0.818

Exercise 3 : Matching, Statement & Assertion Reason Type

Match the Followings

1. Match Column-I with Column-II.

Column-I	Column-II
(A) Liquid $\rightleftharpoons$ Vapour	(p) Saturated solution
(B) Solid $\rightleftharpoons$ Liquid	(q) Boiling point
(C) Solid $\rightleftharpoons$ Vapour	(r) Sublimation point
(D) Solute (s) $\rightleftharpoons$ Solute (solution)	(s) Melting point

(a) A – (p) ; B – (r) ; C – (q) ; D – (s)
(b) A – (q) ; B – (s) ; C – (r) ; D – (p)
(c) A – (s) ; B – (q) ; C – (p) ; D – (r)
(d) A – (r) ; B – (s) ; C – (q) ; D – (p)

2. Match Column-I with Column-II.

Column-I	Column-II
(A) Hydrochloric acid	(p) Lemon and orange
(B) Acetic acid	(q) Tamarind paste.
(C) Citric and ascorbic acids	(r) Digestive juice
(D) Tartaric acid	(s) Constituent of vinegar

(a) A – (q), B – (r), C – (p), D – (s)
(b) A – (r), B – (s), C – (p), D – (q)
(c) A – (s), B – (p), C – (q), D – (r)
(d) A – (r), B – (p), C – (s), D – (q)

3. Match Column-I with Column-II.

Column-I	Column-II
(A) $HClO_4$	(p) Strong base
(B) HNO_2	(q) Strong acid
(C) NH_2^-	(r) Weak base
(D) HSO_4^-	(s) Weak acid

(a) A – (s), B – (q), C – (p), D – (r)
(b) A – (q), B – (s), C – (p), D – (r)
(c) A – (r), B – (p), C – (q), D – (s)
(d) A – (s), B – (q), C – (p), D – (r)

Two-Statement Type Questions

DIRECTIONS : *Read the statements carefully and answer the question on the basis of following options.*

(a) Both statement I and II are correct.
(b) Both statement I and II are incorrect.
(c) Statement I is correct but statement II is incorrect.
(d) Statement II is correct but statement I is incorrect.

4. **Statement I:** The extent of dissociation of an acid depends on the strength and polarity of the H — *A* bond (where *A* is an electronegative element.)

Statement II: As the strength of H — *A* bond increases, the energy required to break the bond decreases.

5. **Statement I:** Water and water vapour remain in equilibrium position at atmospheric pressure (1.013 bar) and at 100°C in a closed vessel.

Statement II: The boiling point of water is 100°C at 1.013 bar pressure

6. **Statement I:** The value of equilibrium constant is independent of initial concentrations of the reactants and products.

Statement II: Equilibrium constant is temperature dependent. The equilibrium constant for the reverse reaction is equal to the inverse of the equilibrium constant for the forward reaction.

7. **Statement I:** The numerical value of the equilibrium constant for a reaction indicates the extent of the reaction.

Statement II: An equilibrium constant give information about the rate at which the equilibrium is reached.

Four / Five Statement Type Questions

8. Read the following statements and choose the correct option

(i) Most of the acids taste sour
(ii) Acids turns blue litmus paper into red
(iii) Bases turns red litmus paper blue
(iv) Bases taste bitter and feel soapy

(a) (i), (ii) and (iv) are correct
(b) (i), (iii) and (iv)
(c) (i), (ii) and (iii) are correct
(d) All statements are correct

9. You must have seen that when a soda water bottle is opened, some of the carbon dioxide gas dissolved in it fizzes out rapidly. There is equilibrium between the molecules in the gaseous state and the molecules dissolved in the liquid under pressure *i.e.*,

$$CO_2(gas) \rightleftharpoons CO_2(\text{in solution})$$

Which of the following statements is/are correct regarding this?

(i) The phenomenon arises due to difference in solubility of carbon dioxide at different pressures.
(ii) This equilibrium is governed by Henry's law.
(iii) The amount of CO_2 gas dissolved in liquid increases with decrease of temperature.
(iv) The amount of CO_2 gas dissolved in liquid decreases with increase of temperature.

(a) only (iii) is correct
(b) (i) , (iii) and (iv) are correct
(c) (i), (ii) and (iii) are correct
(d) All are correct

Assertion & Reason Questions

DIRECTIONS : *These questions consist of two statements, each printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.*

(a) If both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) If both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
(c) If the Assertion is correct but Reason is incorrect.
(d) If the Assertion is incorrect and Reason is correct.

10. **Assertion :** K_p can be less than, greater than or equal to K_c.
Reason : Relation between K_p and K_c depends on the change in number of moles of gaseous reactants and products (Δn).

11. **Assertion :** Equilibrium constant for the reverse reaction is the inverse of the equilibrium constant for the reaction in the forward direction.
Reason : Equilibrium constant depends upon the direction in which the reaction follows.

12. **Assertion :** On increasing pressure there occurs a decrease in melting point of ice.
Reason : On melting, ice contracts.

Exercise 4 : Skill Enhancer MCQs

1. Phosphorus pentachloride dissociates as follows, in a closed reaction vessel

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

If total pressure at equilibrium of the reaction mixture is P and degree of dissociation of PCl_5 is x, the partial pressure of PCl_3 will be

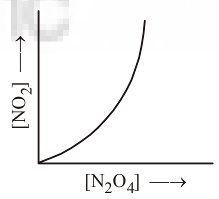
(a) $\left(\frac{x}{x-1}\right)P$ (b) $\left(\frac{x}{1-x}\right)P$

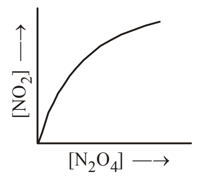
(c) $\left(\frac{x}{x+1}\right)P$ (d) $\left(\frac{2x}{1-x}\right)P$

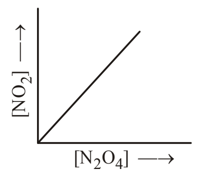
2. Consider the following equilibrium

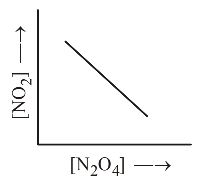
$$N_2O_4(g) \rightleftharpoons 2NO_2(g)$$

Then the select the correct graph, which shows the variation in concentrations of NO_2 against concentrations of N_2O_4:

(a) 

(b) 

(c) 

(d) 

3. Equilibrium constant for the reaction,
$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$, follows the equation

$\ln K_p = 7 - \frac{8400}{T}$, where T = absolute temperature. Find the equilibrium temperature if decomposition of $CaCO_3$

produces CO_2 gas having partial pressure equal to atmospheric pressure.

(a) 1200°C (b) 927 K
(c) 927°C (d) None

4. An amount of solid NH_4HS is placed in a flask already containing ammonia gas at a certain temperature and 0.50 atm pressure. Ammonium hydrogen sulphide decomposes to yield NH_3 and H_2S gases in the flask. When the decomposition reaction reaches equilibrium, the total pressure in the flask rises to 0.84 atm? The equilibrium constant for NH_4HS decomposition at this temperature is

(a) 0.11 (b) 0.17 (c) 0.18 (d) 0.30

5. For a reaction at equilibrium

$$A(g) \rightleftharpoons B(g) + \frac{1}{2}C(g)$$

the relation between dissociation constant (K), degree of dissociation (α) and equilibrium pressure (p) is given by:

(a) $K = \dfrac{\alpha^{\frac{1}{2}} p^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(b) $K = \dfrac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$

(c) $K = \dfrac{(\alpha p)^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(d) $K = \dfrac{(\alpha p)^{\frac{3}{2}}}{(1+\alpha)(1-\alpha)^{\frac{1}{2}}}$

6. 8 mole of a gas AB_3 are introduced into a 1.0 dm^3 vessel. It dissociates as $2AB_3(g) \rightleftharpoons A_2(g) + 3B_2(g)$

At equilibrium, 2 mole of A_2 is found to be present. The equilibrium constant for the reaction is

(a) 2 $mol^2 L^{-2}$ (b) 3 $mol^2 L^{-2}$
(c) 27 $mol^2 L^{-2}$ (d) 36 $mol^2 L^{-2}$

7. For the equilibrium reaction

$$2NOBr(g) \rightleftharpoons 2NO(g) + Br_2(g)$$

If $P_{Br_2} = \frac{P}{9}$, where P is total pressure at equilibrium. The ratio $\frac{K_p}{P}$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{1}{81}$

(c) $\frac{1}{9}$ (d) $\frac{1}{27}$

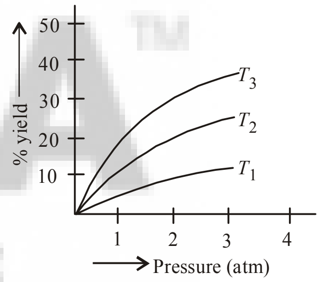
8. For a reversible gaseous reaction, $N_2 + 3H_2 \rightleftharpoons 2NH_3$ at equilibrium, if some moles of H_2 are replaced by same number of moles of T_2 (T is tritium, isotope of H and assume isotopes do not have different chemical properties) without affecting other parameter, then :

(a) the sample of ammonia obtained after sometime will be radioactive.
(b) moles of N_2 after the change will be different as compared to moles of N_2 present before the change
(c) the value of K_p or K_c will change
(d) the average molecular mass of new equilibrium will be same as that of old equilibrium

9. The preparation of $SO_3(g)$ by reaction

$$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$$

is an exothermic reaction. If the preparation follows the following temperature-pressure relationship for its % yield, then for temperatures T_1, T_2 and T_3. The correct option is:



(a) $T_3 > T_2 > T_1$

(b) $T_1 > T_2 > T_3$

(c) $T_1 = T_2 = T_3$

(d) Nothing could be predicted about temperature through given information.

10. Heat of neutralization of a strong acid HA and a weaker acid HB with KOH are – 13.7 and – 12.7 k cal mol^{-1}. When 1 mole of KOH was added to a mixture containing 1 mole each of HA and HB, the heat change was – 13.5 kcal. In what ratio is the base distributed between HA and HB ?

(a) 3 : 1
(b) 1 : 3
(c) 4 : 1
(d) 1 : 4

11. For the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$, the value of K_p is 1.7×10^3 at 500 K and 1.7×10^4 at 600 K. Which of the following is/are correct?
   (a) The proportions of NO_2 in the equilibrium mixture is increased by decrease in pressure.
   (b) The standard enthalpy change for the forward reaction in negative.
   (c) Units of K_p is atm^{-1}.
   (d) At 500 K the degree of dissociation of N_2O_4 decreases by 50% by increasing the pressure by 100%.
12. For the complexation reaction
   $$Ag^+(aq) + 2NH_3(aq) \rightleftharpoons [Ag(NH_3)_2^+](aq),$$
   the rates of forward and reverse reactions are given by :
   $(rate)_f = 1.0 \times 10^6\ L^2\ mol^{-2}\ s^{-1}[Ag^+][NH_3]^2$
   $(rate)_r = 2.0 \times 10^{-2}\ s^{-1}\ [Ag(NH_3)_2]^+$
   The instability constant of the complex is :
   (a) 5.0×10^9 (b) 2.0×10^{-4}
   (c) 2.0×10^{-4} (d) 2.0×10^{-8}
13. The 0.001M Solution of $Mg(NO_3)_2$ is adjusted to pH 9, K_{sp} of $Mg(OH)_2$ is 8.9×10^{-12}. At this pH
   (a) $Mg(OH)_2$ will be precipitated
   (b) $Mg(OH)_2$ is not precipitated
   (c) $Mg(OH)_3$ will be precipitated
   (d) $Mg(OH)_3$ is not precipitated
14. Calculate $\Delta_r G$ for the reaction at 27°C
   $$H_2(g) + 2Ag^+(aq) \rightleftharpoons 2A(s) + 2H^+(aq)$$
   Given : $P_{H_2} = 0.5$ bar; $[Ag^+] = 10^{-5}$ M;
   $[H^+] = 10^{-3}$ M; $\Delta_f G°\ [Ag^+(aq)] = 77.1$ kJ/mol
   (a) – 154.2 kJ/mol (b) – 178.9 kJ/mol
   (c) – 129.5 kJ/mol (d) None of these
15. A solution is saturated with respect to $SrCO_3$ and SrF_2. The $[CO_3^{2-}]$ was found to be 1.2×10^{-3} M. The concentration of F^- in the solution would be
   Given K_{sp} of $SrCO_3 = 7.0 \times 10^{-10}\ M^2$,
   K_{sp} of $SrF_2 = 7.9 \times 10^{-10}\ M^3$,
   (a) 1.3×10^{-3} M (b) 2.6×10^{-2} M
   (c) 3.7×10^{-2} M (d) 5.8×10^{-7} M

Exercise 5 : Numeric Value Answer Questions

1. Equilibrium constant K_p for the reaction $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$ is 0.82 atm at 727 °C. If 1 mole of $CaCO_3$ is placed in a closed container of 20L and heated to this temperature, what amount of $CaCO_3$ in grams would dissociate at equilibrium ?
2. An amount of solid NH_4HS is placed in a flask already containing ammonia gas at a certain temperature and 1.0 atm pressure. Ammonium hydrogen sulphide decomposes to yield NH_3 and H_2S gases in the flask. When the decomposition reaction reaches equilibrium, the total pressure in the flask rises to 2 atm. What will be the equilibrium constant for NH_4HS decomposition at this temperature?
3. The heat of neutralisation of strong base and strong acid is 57.0 kJ. Calculate the heat released when 0.5 mole of HNO_3 is added to 0.20 mole of NaOH solution.
4. Calculate the pH of a solution obtained by diluting 1 mL of 0.10 M weak monoacidic base to 100 mL at constant temperature if K_b of the base is 1×10^{-5}.
5. 28 g N_2 and 6.0 g of H_2 are heated over catalyst in a closed one litre flask of 450 °C. The entire equilibrium mixture required 500 mL of 1.0 M H_2SO_4 for neutralisation. Calculate the value of K_c in $L^2\ mol^2$ for the given reaction.
   $$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$
6. At 600K, 2 mol of NO are mixed with 1 mol of O_2.
   $$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$$
   The reaction occurring as above comes to equilibrium under a total pressure of 1 atm. Analysis of the system shows that 0.6 mol of oxygen are present at equilibrium. The equilibrium constant for the reaction is ________.
   (Nearest integer)
7. pH value of 0.001 M NaOH solution is______.
8. The pK_a of HCOOH is 3.8 and pK_b of NH_3 is 4.8, find the pH of aqueous solution of 1M $HCOONH_4$.
9. In the reaction $AB(g) \rightleftharpoons A(g) + B(g)$ at 30° C, K_p for the dissociation equilibrium is 1.6×10^{-3} atm. If the total pressure at equilibrium is 1 atm, then calculate the percentage dissociation of AB.
10. Calculate the pH at the equivalence point when a solution of 0.01 M CH_3COOH is titrated with a solution of 0.01 M NaOH. pK_a of CH_3COOH is 4.74.

11. The value of K_p for the equilibrium reaction

$$N_2O_4(g) \rightleftharpoons 2NO_2(g) \text{ is } 2.$$

Calculate the percentage dissociation of $N_2O_4(g)$ at a pressure of 0.5 atm.

12. A buffer solution is prepared by mixing 10 mL of 1.0 M CH_3COOH and 20 mL of 0.5 M CH_3COONa and then diluted to 100 mL with distilled water. If pK_a of CH_3COOH is 4.76, what is the pH of the buffer solution?

13. At a certain temperature and 2 atm pressure equilibrium constant (K_p) is 25 for the reaction

$$SO_2(g) + NO_2(g) \rightleftharpoons SO_3(g) + NO(g)$$

Initially if we take 2 moles of each of the four gases and 2 moles of inert gas, what would be the equilibrium partial pressure of NO_2 in atm?

14. Find the pH of a 2 litre solution which is 0.1 M each with respect to CH_3COOH and $(CH_3OO)_2Ba$. ($K_a = 1.8 \times 10^{-5}$)

15. If the solubility product of PbS is 8×10^{-28}, then the solubility of PbS in pure water at 298 K is $x \times 10^{-16}$ mol L^{-1}. The value of x is ________. (Nearest integer)

[Given $\sqrt{2} = 1.41$]

16. What can be the maximum possible molarity of Co^{2+} ions in 0.1 M HCl saturated with H_2S ($K_a = 4 \times 10^{-21}$)? Given that K_{sp} for CoS is 2×10^{-21} and concentration of saturated $H_2S = 0.1$ M.

Answer Keys

Exercise - 1 : (NCERT Based Topic-wise MCQs)																			
1	(c)	**12**	(c)	**23**	(b)	**34**	(d)	**45**	(d)	**56**	(b)	**67**	(c)	**78**	(b)	**89**	(d)	**100**	(b)
2	(c)	**13**	(a)	**24**	(a)	**35**	(b)	**46**	(d)	**57**	(b)	**68**	(b)	**79**	(c)	**90**	(a)	**101**	(d)
3	(a)	**14**	(c)	**25**	(c)	**36**	(a)	**47**	(d)	**58**	(b)	**69**	(c)	**80**	(a)	**91**	(d)	**102**	(a)
4	(c)	**15**	(b)	**26**	(a)	**37**	(d)	**48**	(c)	**59**	(a)	**70**	(d)	**81**	(c)	**92**	(a)	**103**	(b)
5	(b)	**16**	(d)	**27**	(b)	**38**	(d)	**49**	(a)	**60**	(a)	**71**	(a)	**82**	(d)	**93**	(b)	**104**	(c)
6	(b)	**17**	(c)	**28**	(d)	**39**	(c)	**50**	(c)	**61**	(b)	**72**	(d)	**83**	(c)	**94**	(d)	**105**	(b)
7	(a)	**18**	(c)	**29**	(b)	**40**	(b)	**51**	(a)	**62**	(b)	**73**	(c)	**84**	(c)	**95**	(b)		
8	(c)	**19**	(b)	**30**	(d)	**41**	(c)	**52**	(d)	**63**	(b)	**74**	(d)	**85**	(d)	**96**	(b)		
9	(d)	**20**	(d)	**31**	(a)	**42**	(d)	**53**	(d)	**64**	(b)	**75**	(c)	**86**	(c)	**97**	(b)		
10	(d)	**21**	(b)	**32**	(b)	**43**	(a)	**54**	(c)	**65**	(a)	**76**	(b)	**87**	(b)	**98**	(c)		
11	(b)	**22**	(b)	**33**	(c)	**44**	(a)	**55**	(c)	**66**	(c)	**77**	(b)	**88**	(d)	**99**	(b)		
Exercise - 2 : (NCERT Exemplar & Past Years JEE Main)																			
1	(d)	**5**	(b)	**9**	(a)	**13**	(c)	**17**	(a)	**21**	(c)	**25**	(b)	**29**	(a)				
2	(d)	**6**	(a)	**10**	(d)	**14**	(d)	**18**	(b)	**22**	(c)	**26**	(c)						
3	(c)	**7**	(c)	**11**	(a)	**15**	(a)	**19**	(a)	**23**	(2)	**27**	(b)						
4	(b)	**8**	(a)	**12**	(c)	**16**	(c)	**20**	(5)	**24**	(10.60)	**28**	(b)						
Exercise - 3 : (Matching, Statement & Assertion-Reason Type)																			
1	(b)	**3**	(b)	**5**	(a)	**7**	(c)	**9**	(d)	**11**	(a)								
2	(b)	**4**	(c)	**6**	(a)	**8**	(d)	**10**	(a)	**12**	(a)								
Exercise - 4 : (Skill Enhancer MCQs)																			
1	(c)	**3**	(c)	**5**	(b)	**7**	(b)	**9**	(b)	**11**	(a)	**13**	(b)	**15**	(c)				
2	(b)	**4**	(a)	**6**	(c)	**8**	(a)	**10**	(c)	**12**	(d)	**14**	(c)						
Exercise - 5 : (Numeric Value Answer Questions)																			
1	(20)	**3**	(11.4)	**5**	(0.592)	**7**	(11)	**9**	(4)	**11**	(71)	**13**	(0.134)	**15**	(282)				
2	(0.75)	**4**	(10)	**6**	(2)	**8**	(6.5)	**10**	(8.22)	**12**	(4.76)	**14**	(5.0)	**16**	(0.05)				

6 Equilibrium

EXERCISE - 1

1. **(c)** All the physical processes stops at equilibrium is not a general characteristic of equilibria.
2. **(c)** When the watch glass is open to the atmosphere, the rate of evaporation remains constant but the molecules are dispersed into large volume of the room. As a consequence the rate of condensation from vapour to liquid state is much less than the rate of evaporation.
3. **(a)** A reaction is said to be in equilibrium when rate of forward reaction is equal to the rate of backward reaction.
4. **(c)** In reversible reaction; number of moles of reactants and products are always unequal.
5. **(b)** At equilibrium, the rate of forward and backward reactions is equal.
6. **(b)** The reaction mixtures starting either with H_2 or D_2 reach equilibrium with the same composition, except that D_2 and ND_3 are present instead of H_2 and NH_3.
7. **(a)** Equilibrium can be attained by either side of the reactions of equilibrium.
8. **(c)** $N_2 + 3H_2 \rightleftharpoons 2NH_3$

   $\therefore \quad K = [NH_3]^2/[N_2]\,[H_2]^3$ .... (i)

   $\frac{1}{2}N_2 + \frac{3}{2}H_2 \rightleftharpoons NH_3$

   $\therefore \quad K' = \dfrac{[NH_3]}{[N_2]^{1/2}[H_2]^{3/2}}$ .... (ii)

   Dividing equation (i) by equation (ii), we get

   $K' = \sqrt{K}$
9. **(d)** Rate constant of forward reaction (K_f) = 1.1×10^{-2} and rate constant of backward reaction (K_b) = 1.5×10^{-3} per minute.

   Equilibrium constant (K_c) $= \dfrac{K_f}{K_b} = \dfrac{1.1\times10^{-2}}{1.5\times10^{-3}} = 7.33$
10. **(d)** $A \rightleftharpoons B + C\ ;\ K_{eq}^{(1)}$ ...(i)

    $B + C \rightleftharpoons P\ ;\ K_{eq}^{(2)}$ ...(ii)

    On adding equations (i) and (ii), we get

    $A \rightleftharpoons P$

    $K_{eq}\text{ (overall)} = K_{eq}^{(1)} \cdot K_{eq}^{(2)}$
11. **(b)** Given reaction, $2A \rightleftharpoons B + C$

    $K_c = \dfrac{[B][C]}{[A]^2}$

    $K_c = \dfrac{2\times10^{-3}\times3\times10^{-3}}{(10^{-3})^2} = 6$
12. **(c)** $A_2 + B_2 \rightleftharpoons 2AB \quad K_c = \dfrac{[AB]^2}{[A_2][B_2]}$

    $K_c = \dfrac{(2.8\times10^{-3})^2}{3\times10^{-3}\times4.2\times10^{-3}} = \dfrac{(2.8)^2}{3\times4.2} = 0.62$
13. **(a)** For $A + B \rightleftharpoons C$, $\Delta n = 1 - 2 = -1$ (assuming A, B and C are gases)

    Unit of $K_c = \left[\dfrac{mol}{litre}\right]^{\Delta n} = \left[\dfrac{mol}{litre}\right]^{-1}$ = Litre mol^{-1}
14. **(c)** Reaction (iii) can be obtained by adding reactions (i) and (ii) therefore, $K_3 = K_1 . K_2$

    Hence, (c) is the correct answer.
15. **(b)** (I) $N_2 + 2O_2 \xrightleftharpoons{K_1} 2NO_2$

    $K_1 = \dfrac{[NO_2]^2}{[N_2][O_2]^2}$ ...(i)

    (II) $2NO_2 \xrightleftharpoons{K_2} N_2 + 2O_2$

    $K_2 = \dfrac{[N_2][O_2]^2}{[NO_2]^2}$ ...(ii)

    (III) $NO_2 \xrightleftharpoons{K_3} \frac{1}{2}N_2 + O_2$

    $K_3 = \dfrac{[N_2]^{1/2}[O_2]}{[NO_2]}$

    $\therefore \quad (K_3)^2 = \dfrac{[N_2][O_2]^2}{[NO_2]^2}$ ...(iii)

    $\therefore$ from equations (i), (ii) and (iii)

    $K_1 = \dfrac{1}{K_2} = \dfrac{1}{(K_3)^2}$

16. **(d)** $2C(s) + O_2(g) \rightleftharpoons 2CO_2(g)$

$\Delta n = 2 - 1 = +1 \neq 0 \quad \therefore \; K_c$ and K_p are not equal.

17. **(c)** $K_p = \dfrac{(P_{CO})^2}{(P_{CO_2})}; K_p = \dfrac{4 \times 4}{2} = 8;\ C(s) = 1;$

The concentration of solids and liquids are taken as unity.

18. **(c)** Using the relation $K_p = K_c . (RT)^{\Delta n}$, we get

$$\frac{K_p}{K_c} = (RT)^{\Delta n}$$

Thus $\dfrac{K_p}{K_c}$ will be highest for the reaction having highest value of Δn.

The Δn values for various reactions are

(a) $\Delta n = 1 - \left(1 + \dfrac{1}{2}\right) = -\dfrac{1}{2}$

(b) $\Delta n = 2 - (1 + 1) = 0$

(c) $\Delta n = (1 + 1) - 1 = 1$

(d) $\Delta n = (2 + 4) - (7 + 2) = -3$

Thus maximum value of $\Delta n = 1$

19. **(b)** $3Fe(s) + 4H_2O\ (\text{steam}) \rightleftharpoons Fe_3O_4(s) + 4H_2(g)$

$K_p = \dfrac{(p_{H_2})^4}{(p_{H_2O})^4}$ only gaseous products and reactants.

20. **(d)** $K_c = \dfrac{K_p}{(RT)^{\Delta n}} = \dfrac{1.44 \times 10^{-5}}{(0.082 \times 773)^{-2}}$

(R in L. atm. K^{-1} $mole^{-1}$).

21. **(b)** $\underset{a(1-x)}{PCl_5} \rightleftharpoons \underset{ax}{PCl_3} + \underset{ax}{Cl_2}$

$a = 2, x = 0.4, V = 2\ L$

$\therefore\ [PCl_5] = \dfrac{2(1-0.4)}{2} = 0.6 \text{ mol } L^{-1}$

$[PCl_3] = [Cl_2] = \dfrac{2 \times 0.4}{2} = 0.4 \text{ mol } L^{-1}$

$\therefore\ K_c = \dfrac{0.4 \times 0.4}{0.6} = 0.267$

22. **(b)**

	PCl_5	$\rightleftharpoons$	PCl_3	+	Cl_2
Moles at equilibrium	$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$
Mole fraction at equilibrium	$\frac{1}{3}$		$\frac{1}{3}$		$\frac{1}{3}$
Partial pressure at equilibrium	$\frac{P}{3}$		$\frac{P}{3}$		$\frac{P}{3}$

$$K_p = \frac{\frac{P}{3} \times \frac{P}{3}}{P/3} = \frac{P}{3} \Rightarrow P = 3K_p$$

23. **(b)** For the reaction

$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$

On the basis of the stoichiometric equation, we can write,

$K_c = [CaO(s)][CO_2(g)]/[CaCO_3(s)]$

Since $[CaCO_3(s)]$ and $[CaO(s)]$ are both constant, therefore, modified equilibrium constant for the thermal decomposition of calcium carbonate will be

$K_c = [CO_2(g)]$

$K_p = [p_{CO_2}(g)]$

24. **(a)** $K = \dfrac{[Ni(CO)_4]}{[CO]^4} = \dfrac{\text{mol } L^{-1}}{(\text{mol } L^{-1})^4} = (\text{mol } l^{-1})^{-3}$

25. **(c)** As $K_p = K_c (RT)^{\Delta n_g}$

Here $\Delta n_g = 1$

So, $K_p = K_c$ when $RT = 1 \Rightarrow 0.08 \times T = 1$

Thus, $T = 12.5$ K

26. **(a)** $K_1 = \dfrac{[Ni(CO)_4]}{[CO_2]^2}\ ;\ K_2 = \dfrac{[CO]^2}{[CO_2]}$

$$K = \frac{[Ni(CO)_4]}{[CO]^4} \Rightarrow K = \frac{[Ni(CO)_4]}{[CO_2]^2} \times \left(\frac{[CO_2]}{[CO]^2}\right)^2$$

$$K = \frac{K_1}{K_2^2}$$

27. **(b)** $NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$

$$K_P = \frac{(P_{NH_3})^2 \times (P_{CO_2})}{P_{NH_2COONH_4}(s)} = (P_{NH_3})^2 \times (P_{CO_2})$$

As evident by the reaction, NH_3 and CO_2 are formed in molar ratio of 2 : 1. Thus if P is the total pressure of the system at equilibrium, then

$P_{NH_3} = \dfrac{2 \times P}{3};\ P_{CO_2} = \dfrac{1 \times P}{3}$

$$K_P = \left(\frac{2P}{3}\right)^2 \times \frac{P}{3} = \frac{4P^3}{27}$$

Given $K_P = 2.9 \times 10^{-5}$

$\therefore\quad 2.9 \times 10^{-5} = \dfrac{4P^3}{27}$

$P^3 = \dfrac{2.9 \times 10^{-5} \times 27}{4}$

$P = \left(\dfrac{2.9 \times 10^{-5} \times 27}{4}\right)^{\frac{1}{3}} = 5.82 \times 10^{-2}$ atm

28. **(d)** Equilibrium constant (K) is independent of concentrations of reactants and products.

29. **(b)** If K_c is in the range of 10^{-3} to 10^3 appreciable concentrations of both reactants and products are present.

30. **(d)** For reaction to proceed from right to left $Q > K_c$ *i.e.*, the reaction will be fast in backward direction.

31. **(a)** Equilibrium constant is not effected by change in conditions like *P* and *V*. These changes can change only the time required to attain equilibrium.

32. **(b)** The equilibrium constant helps in predicting the direction in which a given reaction will proceed at any stage. For this purpose, we calculate the reaction quotient *Q*. The reaction quotient Q(Q_c with molar concentration and Q_p with partial pressures) is defined in the same way as the equilibrium constant K_c except that the concentrations in Q_c are not necessary equilibrium values.

33. **(c)** Both (a) and (b) are correct for the equation,

$K = e^{-\Delta G°/RT}$

34. **(d)** $\Delta G = 0$ and $\Delta G° = -RT \ln K$

35. **(b)** Reaction proceed backward according to Le-chatelier's principle.

36. **(a)** For $\Delta n = 0$, no effect of pressure.

37. **(d)** According to Le-chatelier's principle, whenever a constraint is applied to a system in equilibrium, the system tends to readjust so as to nullify the effect of the constraint.

38. **(d)** Solid $\rightleftharpoons$ Liquid

It is an endothermic process. So when temperature is raised, more liquid is formed. Hence, adding heat will shift the equilbrium in the forward direction.

39. **(c)** As in this, no. of moles are increasing hence, low pressure will favour the forward direction.

$\Delta n = (1 + 1) - 1 = 1$

40. **(b)** The most favourable conditions are :

(i) High pressure ($\Delta n < 0$)

(ii) Low temperature (Exothermic reaction)

(iii) Catalyst Fe in presence of Mo.

41. **(c)** By increasing pressure at constant volume, molar concentration remains same. Thus, the equilibrium position remains same.

42. **(d)** $A_2(g) + B_2(g) \underset{\text{step-1}}{\rightleftharpoons} 3C(g) \underset{\text{step-2}}{\rightleftharpoons} D(g)$

since the steps 1 and 2 are exothermic hence low temprature will favour both the reactions. In step - 1 moles are increasing hence low pressure will favour it. In step 2, moles are decreasing, hence high pressure will favour it.

43. **(a)** The reaction given is an exothermic reaction thus according to Le-chatalier's principle, lowering of temperature, addition of F_2 and / or Cl_2 favour the forward direction and hence, the production of ClF_3.

44. **(a)** $\underset{(19.66\ cm^3/mol)}{\text{Ice}} \rightleftharpoons \underset{(18.02\ cm^3/mol)}{\text{Water}}$; $\Delta H = +ve$

(i) When temperature is increased, the ice-water system will try to decrease this effect by decreasing the temperature. This can be done by forming more water as this process absorbs energy.

(ii) When external pressure is increased, the ice-water system oppose the external pressure by increasing its own pressure. It is like pushing a piston in a gaseous system, that is in equilibrium, the system work to push back the piston.

Now to increase the pressure on the external agent, the ice-water system can decrease its volume by forming more water.

45. **(d)** The equilibrium is established between ions and unionised molecules involves ions in aqueous solution.

46. **(d)** Base accepts protons and acid donates protons.

47. **(d)** $(CH_3)_3$ B - is an electron deficient, thus behave as a lewis acid.

48. **(c)** Boron in B_2H_6 is electron deficient

49. **(a)** Bronsted base is a substance which accepts proton. In option (a), H_2O is accepting proton, *i.e.*, acting as a base.

50. **(c)** HSO_4^- accepts a proton to form H_2SO_4.

Thus, H_2SO_4 is the conjugate acid of HSO_4^-.

$$\underset{\text{base}}{HSO_4^-} \xrightarrow{+H^+} \underset{\text{conjugate acid of } HSO_4^-}{H_2SO_4}$$

51. **(a)** (i) $\underset{acid_1}{H_3PO_4} + \underset{base_2}{H_2O} \longrightarrow \underset{acid_2}{H_3O^+} + \underset{base_1}{H_2PO_4^-}$

(ii) $\underset{acid_1}{H_2PO_4^-} + \underset{base_2}{H_2O} \longrightarrow \underset{base_1}{HPO_4^{2-}} + \underset{acid_2}{H_3O^+}$

(iii) $\underset{base_1}{H_2PO_4^-} + \underset{acid_2}{OH^-} \longrightarrow \underset{acid_1}{H_3PO_4} + \underset{base_2}{O^{2-}}$

Hence, only in (ii) reaction $H_2PO_4^-$ is acting as an acid.

52. **(d)** $[Cu(H_2O)_4]^{2+} + 4NH_3 \rightleftharpoons [Cu(NH_3)_4]^{2+} + 4H_2O$ involves lose and gain of electrons. H_2O is coordinated to Cu by donating electrons (LHS). It is then removed by withdrawing electrons.

53. **(d)** BF_3 is an electron deficient molecule and thus, it can accept a pair of electrons.

In stannic chloride and stannous chloride, the Sn atom has vacant *d*-orbitals. Sn can increase its covalency to 6 by accepting pair of electrons. Thus $SnCl_2$ and $SnCl_4$ can also act as Lewis acids.

54. **(c)** Dissociation of a weak acid in water :

$$\underset{\text{acid}}{HA(aq)} + \underset{\text{base}}{H_2O(l)} \rightleftharpoons \underset{\text{conjugate acid}}{H_3O^+(aq)} + \underset{\text{conjugate base}}{A^-(aq)}$$

If HA is a stronger acid than H_3O^+, then HA will donate protons and not H_3O^+, and the solution will mainly contain A^- and H_3O^+ ions. Hence, the equilibrium moves in the direction of formation of weaker acid and weaker base. Similarly, this is true for dissociation of a weak base also.

55. **(c)** Strong base has higher tendency to accept the proton. Increasing order of base and hence the order of accepting tendency of proton is

$I^- < HS^- < NH_3 < RNH_2$

56. **(b)** Because NH_3 after losing a proton (H^+) gives NH_2^-

$$NH_3 + H_2O \rightleftharpoons NH_2^- + H_3O^+$$

(Conjugate acid-base pair differ only by a proton)

57. **(b)** The value of ionic product of water changes with the temperature.

58. **(b)** Given $[OH^-] = 10^{-3}$

$\therefore$ pOH = 3

$\because$ pH + pOH = 14

$\therefore$ pH = 14 – 3 = 11

59. **(a)** Given $[H_3O^+] = 1 \times 10^{-10}$ M

at 25° $[H_3O^+][OH^-] = 10^{-14}$

$$\therefore [OH^-] = \frac{10^{-14}}{10^{-10}} = 10^{-4}$$

Now, $[OH^-] = 10^{-pOH} = 10^{-4}$

$\therefore$ pOH = 4

60. **(a)** Given $K_a = 1.00 \times 10^{-5}$, $C = 0.100$ mol for a weak electrolyte, degree of dissociation

$$(\alpha) = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{1 \times 10^{-5}}{0.100}} = 10^{-2} = 1\%$$

61. **(b)** $HA \longrightarrow H^+ + A^-$

$$K_a = \frac{[H^+][A^-]}{[HA]}, \quad \because [H^+] = 10^{-pH}$$

$\therefore$ $[H^+] = 10^{-5}$; and at equilibrium $[H^+] = [A^-]$

$$\therefore K_a = \frac{10^{-5} \times 10^{-5}}{0.0015} = 2 \times 10^{-8}$$

$$\alpha = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{2 \times 10^{-8}}{.005}} = \sqrt{4 \times 10^{-6}} = 2 \times 10^{-3}$$

Percentage ionization = 0.2

62. **(b)** Acidic strength $\propto \sqrt{K_a}$

63. **(b)** $pK_a = -\log K_a$

Smaller the value of pK_a, stronger will be acid

$\therefore$ Acid having pK_a value of 10^{-8} is strongest acid.

64. **(b)** $$\frac{\alpha_1}{\alpha_2} = \sqrt{\frac{K_{a_1}}{K_{a_2}}} = \sqrt{\frac{3.14 \times 10^{-4}}{1.96 \times 10^{-5}}} = 4:1$$

65. **(a)** $$K = c\alpha^2 = 0.1 \times \left(\frac{1.34}{100}\right)^2 = 1.8 \times 10^{-5}$$

66. **(c)** Due to common ion effect addition of NH_4Cl in group (III) suppresses the ionisation of NH_4OH with the result, concentration of OH^- decreases.

67. **(c)** Solubility of weak electrolyte decreases in solvent having common ion. So solubility of AgI in NaI solution is less than in pure water because of common ion effect.

68. **(b)** CH_3COOH is weak acid while NaOH is strong base, so one equivalent of NaOH can not be neutralized with one equivalent of CH_3COOH. Hence the solution of one equivalent of each does not have pH value as 7. Its pH will be towards basic side as NaOH is a strong base. Hence, conc. of OH^- will be more than the conc. of H^+.

69. **(c)** On dilution $[H^+] = 10^{-6}$ M $= 10^{-6}$ mol

Now dissociation of water cannot be neglected,

Total $[H^+] = 10^{-6} + 10^{-7} = 11 \times 10^{-7}$

$pH = -\log [H^+] = -\log (11 \times 10^{-7}) = 5.98$

70. **(d)** $\because$ pH = 1 ; $H^+ = 10^{-1} = 0.1$ M

pH = 2 ; $H^+ = 10^{-2} = 0.01$ M

$\therefore M_1 = 0.1 \quad V_1 = 1$

$M_2 = 0.01 \quad V_2 = ?$

From $M_1V_1 = M_2V_2$

$0.1 \times 1 = 0.01 \times V_2$

$V_2 = 10$ litres

$\therefore$ Volume of water added = 10 – 1 = 9 litres

71. **(a)** Molarity (M) = 10M. HCl is a strong acid and it is completely dissociated in aqueous solutions as :

$$HCl\,(10) \rightleftharpoons H^+(10) + Cl^-.$$

So, for every moles of HCl, there is one H^+. Therefore, $[H^+] = [HCl]$ or $[H^+] = 10$.

$pH = -\log[H^+] = -\log[10] = -1$.

72. **(d)** No. of moles of NaOH $= \frac{4}{40} = 0.1$

[Molecular weight of NaOH = 40]

No. of moles of $OH^- = 0.1$

Concentration of $OH^- = \frac{0.1}{1 \text{ litre}} = 0.1 \text{mol / L}$

As we know that, $[H^+][OH^-] = 10^{-14}$

$\therefore$ $[H^+] = 10^{-13}$ $\quad (\because OH^- = 10^{-1})$

73. **(c)** $M_1V_1 = M_2V_2$

$1 \times 0.10 = M_2 \times 100 \Rightarrow M_2 = 0.001 = 10^{-3}$

$$\underset{\substack{C \\ C(1-\alpha)}}{BOH} \rightleftharpoons \underset{\substack{0 \\ C\alpha}}{B^+} + \underset{\substack{0 \\ C\alpha}}{OH^-}$$

$$K_b = \frac{C\alpha \times C\alpha}{C(1-\alpha)} \Rightarrow K_b = C\alpha^2 \Rightarrow \alpha = \sqrt{K_b / C}$$

$(\because 1 - \alpha \approx 1)$

$$[OH^-] = C\alpha = \sqrt{\frac{K_b}{C}} \times C = \sqrt{K_b C}$$

$$= \sqrt{10^{-5} \times 10^{-3}} = 10^{-4}$$

$\therefore$ pH + pOH = 14

$\Rightarrow \therefore$ pH = 14 – 4 = 10

74. **(d)** Given $K_b = 1.0 \times 10^{-12}$

[BOH] = 0.01 M ; $[OH^-]$ = ?

$$\begin{array}{lccc} & BOH & \rightleftharpoons B^+ & + \quad OH^- \\ t=0 & c & 0 & 0 \\ t_{eq} & C(1-x) & Cx & Cx \end{array}$$

$$K_b = \frac{C^2x^2}{C(1-x)} = \frac{Cx^2}{(1-x)} \Rightarrow 1.0 \times 10^{-12} = \frac{0.01x^2}{(1-x)}$$

On calculation, we get, $x = 1.0 \times 10^{-5}$

Now, $[OH^-] = Cx = 0.01 \times 10^{-5} = 1 \times 10^{-7}$ mol L^{-1}

75. **(c)** $K_a = \frac{[H_3O^+][F^-]}{[HF]}$ ...(i)

$K_b = \frac{[HF][OH^-]}{[F^-]}$ ...(ii)

From (i) and (ii), $K_aK_b = [H_3O^+][OH^-] = K_w$
(ionic product of water)

76. **(b)** $[H_3O]^+$ for a solution having pH = 3 is given by

$[H_3O]^+ = 1\times10^{-3}$ moles/litre [$\therefore$ $[H_3O]^+ = 10^{-pH}$]

Similarly for solution having pH = 4,

$[H_3O]^+ = 1 \times 10^{-4}$ moles/ litre and for pH = 5

$[H_3O^+] = 1\times10^{-5}$ moles/ litre

Let the volume of each solution in mixture be IL, then total volume of mixture solution L = (1 + 1 + 1) L = 3L

Total $[H_3O]^+$ ion present in mixture solution $= (10^{-3} + 10^{-4} + 10^{-5})$ moles

Then $[H_3O]^+$ ion concentration of mixture solution

$$= \frac{10^{-3} + 10^{-4} + 10^{-5}}{3} M = \frac{0.00111}{3} M$$

$= 0.00037\ M = 3.7 \times 10^{-4}$ M.

77. **(b)** K_w depends upon temperature as it is an equilibrium constant.

78. **(b)** The solution with lowest pH will have the highest value of pOH.

$[H^+]$ order	:	(A) > (D) > (C) > (B)
pH order	:	(B) > (C) > (D) > (A)
pOH order	:	(A) > (D) > (C) > (B)

79. **(c)** Number of meq. of the acid = 0.04 × 100 = 4

Number of meq. of the base = 0.02 × 100 = 2

$\therefore$ Number of meq. of the acid left on mixing = 4 – 2 = 2

Total volume of the solution = 200 mL

$\therefore$ No. of meq of the acid present in 1000 mL of the solution = 10

or No. of eq. of the acid in 1000 mL of the solution

$$= \frac{10}{1000} = 0.01$$

Since the acid is monobasic and completely ionises in solution

0.01 N HCl = 0.01 M HCl

Thus $[H^+] = 0.01$

$\therefore$ pH = – log (0.01) = – (– 2) = 2

80. **(a)** K_w at 25°C = 1×10^{-14}

At 25ºC

$K_w = [H^+][OH^-] = 10^{-14}$

At 100°C (given)

$K_w = [H^+][OH^-] = 55 \times 10^{-14}$

$\because$ for a neutral solution

$[H^+] = [OH^-]$

$\therefore$ $[H^+]^2 = 55 \times 10^{-14}$

or $[H^+] = (55 \times 10^{-14})^{1/2}$

$\because$ pH = – log $[H^+]$

On taking log on both side

$-\log[H^+] = -\log(55 \times 10^{-14})^{1/2}$

$$pH = -\frac{1}{2}\log 55 + 7\log 10$$

pH = – 0.87 + 7 = 6.13

81. **(c)** Higher the value of K_a lower will be the value of pK_a *i.e.*, higher will be the acidic nature. Further since, CN^-, F^- and NO_2^- are conjugate base of the acids HCN, HF and HNO_2 respectively hence, the correct order of base strength will be

$F^- < NO_2^- < CN^-$

($\because$ stronger the acid weaker will be its conjugate base)

82. **(d)** As, molarity, $= \frac{\text{wt. of solute per litre of solution}}{\text{Mol. wt. of solute}}$

Molarity of $H_2O = \frac{1000}{18}$ mole/litre

$$\begin{array}{ccc} H_2O & \rightleftharpoons H^+ & + \quad OH^- \\ c(1-\alpha) & c\alpha & c\alpha \end{array}$$

Thus, $K_a = \frac{c\alpha^2}{1-\alpha} = c\alpha^2 = 1.8 \times 10^{-14}$

83. **(c)** $CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

$$K_a = \frac{[CH_3COO^-][H^+]}{[CH_3COOH]}$$

Given that,

$[CH_3COO^-] = [H^+] = 3.4\times10^{-4}$ M

K_a for $CH_3COOH = 1.7 \times 10^{-5}$

CH_3COOH is weak acid, so in it $[CH_3COOH]$ is equal to initial concentration. Hence

$$1.7\times10^{-5} = \frac{(3.4\times10^{-4})(3.4\times10^{-4})}{[CH_3COOH]}$$

$$[CH_3COOH] = \frac{3.4\times10^{-4} \times 3.4\times10^{-4}}{1.7\times10^{-5}} = 6.8 \times 10^{-3} M$$

84. **(c)** Given, $CH_3COOH \rightleftharpoons CH_3COO^- + H^+$;

$K_{a_1}, = 1.5 \times 10^{-5}$ ....(i)

$HCN \rightleftharpoons H^+ + CN^-$; $K_{a_2} = 4.5 \times 10^{-10}$

or $H^+ + CN^- \rightleftharpoons HCN$;

$$K_{a_2}^{'} = \frac{1}{K_{a_2}} = \frac{1}{4.5 \times 10^{-10}} \quad ...(ii)$$

$\therefore$ From (i) and (ii), we find that the equilibrium constant (K_a) for the reaction,

$CN^- + CH_3COOH \rightleftharpoons CH_3COO^- + HCN$, is

$$K_a = K_{a_1} \times K_{a_2}^{'} = \frac{1.5 \times 10^{-5}}{4.5 \times 10^{-10}} = \frac{1}{3} \times 10^5 = 3.33 \times 10^4$$

85. **(d)** $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^- \quad K_{a_1}$ ...(i)

$H_2C_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2-} \quad K_{a_2}$ ...(ii)

$H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-} \quad K_{a_3}$ ...(iii)

addition of eq. (i) with eq. (ii) gives eq (iii),

So $K_{a_3} = K_{a_1} \times K_{a_2}$

86. **(c)** In aqueous solution BA(salt) hydrolyses to give

$$BA + H_2O \rightleftharpoons \underset{\text{Base}}{BOH} + \underset{\text{acid}}{HA}$$

Now pH is given by

$$pH = \frac{1}{2} pK_w + \frac{1}{2} pKa - \frac{1}{2} pK_b$$

substituting given values, we get

$$pH = \frac{1}{2}(14 + 4.80 - 4.78) = 7.01$$

87. **(b)** Given $[NH_3] = 0.3$ M, $[NH_4^+] = 0.2$ M,

$K_b = 1.8 \times 10^{-5}$.

$$pOH = pK_b + \log \frac{[salt]}{[base]}$$

$[pK_b = -\log K_b; pK_b = -\log 1.8 \times 10^{-5}]$

$\therefore \quad pK_b = 4.74$

$$pOH = 4.74 + \log \frac{0.2}{0.3} = 4.74 + 0.3010 - 0.4771 = 4.56$$

$pH = 14 - 4.56 = 9.43$

88. **(d)** $pH = pK_a + \log \left[\frac{Salt}{Acid}\right]$

$$\log [H^+] = \log K_a - \log \left[\frac{Salt}{Acid}\right]$$

$$\log [H^+] = \log K_a + \log \left[\frac{Acid}{Salt}\right]$$

$$[H^+] = K_a \left[\frac{Acid}{Salt}\right] = 1.8 \times 10^{-5} \times \frac{0.1}{0.2} = 9 \times 10^{-6}\ M$$

89. **(d)** HNO_2 is a weak acid and $NaNO_2$ is salt of that weak acid and strong base (NaOH).

90. **(a)** The pOH of a buffer consisting of NH_3 (*i.e.*, NH_4OH) and salt NH_4Cl (salt) is given by the equation

$$pOH = pK_b + \log \frac{[ammonia]}{[salt]} = 5.0 + \log \frac{[0.1]}{[1.0]}$$

$= 5.0 - \log 10 = 5 - 1 = 4.0$

$\therefore \quad pH = 14 - pOH = 14 - 4.0 = 10$

91. **(d)** For acidic buffer $pH = pK_a + \log \left[\frac{salt}{acid}\right]$

or $\quad pH = pK_a + \log \frac{[A^-]}{[HA]}$

Given $pK_a = 4.5$ and acid is 50% ionised.

$[HA] = [A^-]$ (when acid is 50% ionised)

$\therefore \quad pH = pK_a + \log 1$

$\therefore \quad pH = pK_a = 4.5$

$pOH = 14 - pH = 14 - 4.5 = 9.5$

92. **(a)** Lets take an example of an acidic buffer CH_3COOH and CH_3COONa.

$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$;

$CH_3COONa \rightleftharpoons CH_3COO^- + Na^+$

when few drops of HCl are added to this buffer, the H^+ of HCl immediatly combine with CH_3COO^- ions to form undissociated acetic acid molecules. Thus there will be no appreciable change in its pH value. Like wise if few drops of NaOH are added, the OH^- ions will combine with H^+ ions to form unionised water molecule. Thus pH of solution will remain constant.

93. **(b)** $Mg(OH)_2 \rightleftharpoons Mg^{2+} + 2OH^-$

$K_{sp} = [Mg^{++}][OH^-]^2$

$1.0 \times 10^{-11} = 10^{-3} \times [OH^-]^2$

$$[OH^-] = \sqrt{\frac{10^{-11}}{10^{-3}}} = 10^{-4}$$

$\therefore \quad pOH = 4$

$\therefore \quad pH + pOH = 14$

$\therefore \quad pH = 10$

94. **(d)** pH or $[H^+]$ of a buffer does not change with dilution.

95. **(b)** According to Henderson-Hasselbalch equation

$$pH = pK_a + \log \frac{[Salt]}{[Acid]}$$

$$4 = 5 - \log 1.3 + \log \frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$$

$$\log \frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]} = \log 1.3 - 1 = \log \frac{1.3}{10}$$

$$\therefore \ \log A - \log(B) = \log\left(\frac{A}{B}\right)$$

$$\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]} = 0.13$$

96. **(b)** $Cr(OH)_3(s) \rightleftharpoons \underset{s}{Cr^{3+}}(aq.) + \underset{3s}{3OH^-}(aq.)$

$(s)\ (3s)^3 = K_{sp}$

$27s^4 = K_{sp}$

$$s = \left(\frac{K_{sp}}{27}\right)^{1/4} = \left(\frac{1.6 \times 10^{-30}}{27}\right)^{1/4}$$

97. **(b)** Solubility product is the product of ionic concentration in a saturated solution of an electrolyte at a given temperature.

98. **(c)** Let binary electrolyte be AB

$\underset{s}{AB} \rightleftharpoons \underset{s}{A^+} + \underset{s}{B^-}$

Hence, solubility product of AB

$K_{sp} = [A^+]\,[B^-]$

$S = [s]\,[s] \Rightarrow s = S^{½}$

99. **(b)** Given pH = 12

or $[H^+] = 10^{-12}$

Since, $[H^+]\,[OH^-] = 10^{-14}$

$$\therefore \quad [OH^-] = \frac{10^{-14}}{10^{-12}} = 10^{-2}$$

$Ba(OH)_2 \rightleftharpoons \underset{s}{Ba^{2+}} + \underset{2s}{2OH^-}$

$[OH^-] = 10^{-2}$

$2s = 10^{-2}$

$$s = \frac{10^{-2}}{2}$$

$$K_{sp} = 4s^3 = 4 \times \left(\frac{10^{-2}}{2}\right)^3 = 5 \times 10^{-7}$$

100. **(b)** $K_{sp} = [Fe^{3+}].[3OH^-]$

So molar solubility of $Fe^{3+} = s$ and $[3OH^-] = 3s$

$Fe(OH)_3 \rightleftharpoons \underset{[s]}{Fe^{3+}} + \underset{[3s]}{3OH^-}$

$1.0 \times 10^{-38} = [s]\,[3s]^3$

$1.0 \times 10^{-38} = s^4 \times 27$

$$s^4 = \frac{1.0 \times 10^{-38}}{27}$$

$s^4 = 3.703 \times 10^{-40}$

$$s = \left(3.703 \times 10^{-40}\right)^{\frac{1}{4}} = 1.386 \times 10^{-10}$$

101. **(d)** In deionized water no common ion effect will take place so solubility of AgCl will be maximum in it.

$KCl \longrightarrow K^+ + Cl^-$

$HCl \longrightarrow H^+ + Cl^-$

$AgNO_3 \longrightarrow Ag^+ + NO_3^-$

102. **(a)** $Bi_2S_3 \rightleftharpoons \underset{2s}{2Bi^{3+}} + \underset{3s}{3S^{2-}}$

$K_{sp} = (2s)^2\,(3s)^3 = 108\,(s)^5$

$$(s)^5 = \frac{1.08 \times 10^{-73}}{108} \Rightarrow s = 10^{-15}$$

103. **(b)** $AgBr \rightleftharpoons Ag^+ + Br^-$

$K_{sp} = [Ag^+]\,[Br^-]$

For precipitation to occur

Ionic product > Solubility product

$$[Br^-] = \frac{K_{sp}}{[Ag^+]} = \frac{5 \times 10^{-13}}{0.05} = 10^{-11}$$

i.e., precipitation just starts when 10^{-11} moles of KBr is added to 1L $AgNO_3$ solution

$\therefore$ Number of moles of Br^- needed from KBr = 10^{-11}

$\therefore$ Mass of KBr = $10^{-11} \times 120 = 1.2 \times 10^{-9}$ g

104. **(c)** $K_{sp} = [Ag^+]\,[Cl^-]$ for $AgCl \rightleftharpoons Ag^+ + Cl^-$

$1.8 \times 10^{-10} = [Ag^+]\,[0.1]$

$[Ag^+] = 1.8 \times 10^{-9}$ M

$K_{sp} = [Pb^{2+}]\,[Cl^-]^2$ for $PbCl_2 \rightleftharpoons Pb^{2+} + 2Cl^-$

$1.7 \times 10^{-5} = [Pb^{+2}]\,[0.1]^2$

$[Pb^{2+}] = 1.7 \times 10^{-3}$ M

105. **(b)** Let s be the solubility of $Ca(OH)_2$ in water

$Ca(OH)_2 \rightleftharpoons \underset{s}{Ca^{2+}} + \underset{2s}{2OH^-}$

$K_{sp} = [Ca^{2+}][OH^-]^2 = s \times (2s)^2$

$\Rightarrow 5.5 \times 10^{-6} = 4s^3$

$$\Rightarrow s^3 = \frac{5.5}{4} \times 10^{-6}$$

$$s = \left[\frac{5.5}{4}\right]^{\frac{1}{3}} \times 10^{-2} = 1.11 \times 10^{-2}$$

EXERCISE - 2

1. **(d)** Relationship between K_c and K_p is given as :

$$K_p = K_c\,(RT)^{\Delta n}$$

where, Δn = (number of moles of gaseous products) – (number of moles of gaseous reactants)

For given reaction,

$NH_4Cl\,(s) \rightleftharpoons NH_3\,(g) + HCl\,(g)$

$\Delta n = 2 - 0 = 2$

2. **(d)** The relationship between $\Delta G°$ and K is :
$\Delta G° = -RT \ln K$
When $G° > 0$ it implies $\Delta G°$ must be positive, which will be possible when $\ln K$ is negative *i.e.*, $K < 1$.

3. **(c)** At the stage of equilibria physical processes does not stop but forward and reverse process occur at the same rate.

4. **(b)** For the given reaction,
$PCl_5 \rightleftharpoons PCl_3 + Cl_2$
At 500 K in a closed container,
$[PCl_5] = 0.8 \times 10^{-3}$ mol L^{-1}
$[PCl_3] = 1.2 \times 10^{-3}$ mol L^{-1}
$[Cl_2] = 1.2 \times 10^{-3}$ mol L^{-1}

$$K_c = \frac{[PCl_3][Cl_2]}{[PCl_5]} = \frac{(1.2\times10^{-3})\times(1.2\times10^{-3})}{(0.8\times10^{-3})}$$

$= 1.8 \times 10^{-3}$

5. **(b)** With the increase in acidity or K_a value of the given acids pH decreases, hence the order of pH value of the acids will be :

hypochlorous acid > acetic acid > formic acid
(3.8×10^{-8}) (1.74×10^{-5}) (1.8×10^{-4})

6. **(a)** For the reaction,
$H_2S \rightleftharpoons H^+ + HS^-$

$$K_{a_1} = \frac{[H^+][HS^-]}{[H_2S]} \quad \ldots(i)$$

For the reaction,
$HS^- \rightleftharpoons H^+ + S^{2-}$

$$K_{a_2} = \frac{[H^+][S^{2-}]}{[HS^-]} \quad \ldots(ii)$$

$$K_{a_3} = \frac{[H^+]^2[S^{2-}]}{[H_2S]} \quad \ldots(iii)$$

Hence, $K_{a_3} = K_{a_1} \times K_{a_2}$

7. **(c)** According to GN Lewis, an acid is a species which accepts an electron pair and base which donates an electron pair. Since BF_3 is an electron deficient species, hence, it is a Lewis acid.

8. **(a)** $\Delta G° = -RT \ln K$
At the stage of half completion of the reaction,
$[A] = [B]$ and $K = [B]/[A]$
Therefore, $K = 1$
As we know that
$\Delta G^\ominus = -RT \ln K$
$\therefore \quad \Delta G^\ominus = 0$

9. **(a)** For the reaction, $\frac{1}{2}H_2(g) + \frac{1}{2}I_2(g) \rightleftharpoons HI(g)$

$$K_c = \frac{[HI]}{[H_2]^{1/2}[I_2]^{1/2}} = 5$$

Thus, for the reaction,
$2HI(g) \rightleftharpoons H_2(g) + I_2(g)$

$$K'_c = \frac{[H_2][I_2]}{[HI]^2}$$

$$K'_c = \left(\frac{1}{K_c}\right)^2 = \left(\frac{1}{5}\right)^2 = \frac{1}{25} = 0.04$$

10. **(d)** Addition of inert gas at constant volume does not affect any equilibrium.

11. **(a)** For the given reaction,

$$\underset{\text{Pink}}{[Co(H_2O)_6]^{3+}}(aq) + 4Cl^-(aq) \rightleftharpoons \underset{\text{Blue}}{[CoCl_4]^{2-}}(aq) + 6H_2O(l)$$

Equilibrium shifts to backward direction on cooling where as on heating, it shifts to forward direction. Thus, reaction is endothermic. *i.e.*, $\Delta H > 0$.

12. **(c)** pH of neutral water at 25°C = 7.0
At 25°C, $[H^+] = [OH^-] = 10^{-7}$ and
$K_w = [H^+][OH^-] = 10^{-14}$
On heating, K_w increases, *i.e.*, $[H^+][OH^-] > 10^{-14}$
As $[H^+] = [OH^-]$
$\therefore \quad [H^+]^2 > 10^{-14}$.
or, $[H^+] > 10^{-7}$ M
$\therefore \quad pH < 7$
As temperature increases, pH of pure water decreases hence, it will become less than 7 at 60°C.

13. **(c)** When the concentration of weak base (NH_4OH) is higher than the strong acid (HCl) a mixture of weak base and its conjugate acid is obtained, which acts as basic buffer.

14. **(d)** AgCl, react with aqueous ammonia to form a complex, $[Ag(NH_3)_2]^+ Cl^-$.

15. **(a)** Given, $K_a = 1.74 \times 10^{-5}$
Concentration CH_3COOH $(C) = 0.01$ mol dm^{-3}
$[H^+] = \sqrt{K_a \cdot C} = \sqrt{1.74\times10^{-5}\times0.01} = 4.17\times10^{-4}$
$pH = -\log[H^+] = -\log(4.17 \times 10^{-4}) = 3.4$

16. **(c)** Given K_a for $CH_3COOH = 1.8 \times 10^{-5}$
K_b for $NH_4OH = 1.8 \times 10^{-5}$.

Ammonium acetate is a salt of weak acid and weak base. For such salts

$$pH = 7 + \frac{1}{2}(pK_a - pK_b)$$

$$= 7 + \frac{[-\log 1.8 \times 10^{-5}] - [-\log 1.8 \times 10^{-5}]}{2}$$

$$= 7 + \frac{4.74 - 4.74}{2} = 7.00$$

17. **(a)** In the reaction $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

If the total pressure at which the equilibrium is established, is increased without changing the temperature, K will remain same but the equilibrium concentration will change.

18. **(b)** $Fe^{3+} + SCN^- \rightleftharpoons \underset{\text{(Red)}}{[Fe(SCN)]^{2+}}$

When oxalic acid is added to a solution containing iron (III) nitrate and potassium thiocynate then, equilibrium shifts towards backward direction and intensity of red colour decreases.
Oxalic acid reacts with Fe^{3+} ions to form a stable complex ion $[Fe(C_2O_4)_3]^{3-}$. Thus, decreasing the concentration of Fe^{3+} ions.

19. **(a)** $(NH_4)_2CO_3$ is a salt of NH_4OH and H_2CO_3. Its aqueous solution is basic as K_b of NH_4OH is greater than K_a of H_2CO_3.
pH of salt of weak acid and weak base depends on K_a and K_b value of constituent acid and the base.

20. **(5)** $[Ba^{2+}] = \frac{25 \times 0.05}{50} = 0.025$ M

$$[F^-] = \frac{25 \times 0.02}{50} = 0.01\text{ M} \Rightarrow [Ba^{+2}][F^-]^2 = 25 \times 10^{-7}$$

$$\text{Ratio} = \frac{[Ba^{+2}][F^-]^2}{K_{sp}} = 5 \Rightarrow K_{sp} = 5 \times 10^{-7} \text{ (given)}$$

21. **(c)** C – conc. of salt, NH_4Cl, produced by neutralization of NH_4OH and HCl.
NH_4Cl, no. of moles = 2m mol = 2×10^{-3} mol

$$[NH_4Cl] = \frac{2\text{m mol}}{(20+40)\text{ml}} = \frac{1}{30}\text{M}$$

$$pH = 7 - \frac{1}{2}pK_b - \frac{1}{2}\log C$$

$$= 7 - \frac{1}{2} \times (+5) - \frac{1}{2}\log\left(\frac{1}{30}\right) = 7 - \frac{5}{2} + \frac{1.48}{2} = 5.24$$

22. **(c)** $2A(g) \rightleftharpoons 2B(g) + C(g) \quad K_1$ ...(i)

$\frac{3}{2}A(g) \rightleftharpoons \frac{3}{2}B(g) + \frac{3}{4}C(g) \quad K_2$ ...(ii)

eq. (ii) is $\frac{3}{4}$ times of eq. (i), hence, $K_2 = (K_1)^{\frac{3}{4}}$

23. **(2)** $\underset{}{HCl} \longrightarrow \underset{0.1\text{ M}}{H^+} + \underset{0.1\text{ M}}{Cl^-}$

$\underset{0.01(1-\alpha)}{HA} \rightleftharpoons \underset{0.1 + 0.01\alpha}{H^+} + \underset{0.01\alpha}{A^-} \quad K_a = 2.0 \times 10^{-6}$

$$K_a = \frac{[H^+][A^-]}{[HA]}$$

$$2 \times 10^{-6} = \frac{0.1 \times 0.01\alpha}{0.01(1-\alpha)} \simeq \frac{0.1 \times 0.01\alpha}{0.01}$$

$\alpha = 2.0 \times 10^{-5}$

24. **(10.60)**

$$M_{H_2SO_4} = \frac{9.8}{98 \times 100} = 10^{-3}\text{ M}$$

$$M_{NaOH} = \frac{4}{40 \times 100} = 10^{-3}\text{ M}$$

After neutralisation $[OH^-]$ can be calculated as

$$[OH^-] = \frac{(40 \times 10^{-3}) - (2 \times 10^{-3} \times 10)}{50} = \frac{20}{50} \times 10^{-3}$$

$$[OH^-] = \frac{2}{5} \times 10^{-3}$$

pOH = 3.397
pH = 14 – pOH = 14 – 3.397 = 10.603

25. **(b)** mmol of $H_2SO_4 = 20 \times 0.1 = 2$
mmol of $NH_4OH = 30 \times 0.2 = 6$

$$H_2SO_4 + 2NH_4OH \rightarrow (NH_4)_2SO_4 + 2H_2O$$

	H_2SO_4	$2NH_4OH$	$(NH_4)_2SO_4$
Initial	2 mmol	6 mmol	0
Final	(2–2) = 0 mmol	(6 – 2 × 2) = 2 mmol	2 mmol

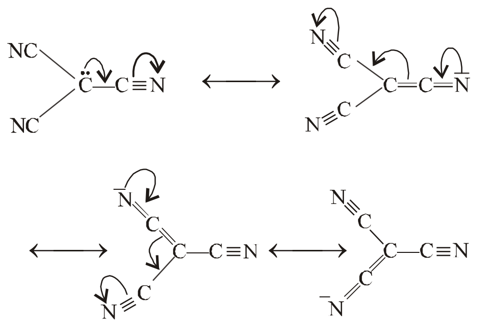
$[NH_4OH]$left = 2 m mol
$[(NH_4)_2SO_4]$ = 2 m mol
$[\overset{+}{N}H_4]$ = 2 × 2 = 4 m mol
Total Volume = 30 + 20 = 50 mL

$$pOH = pK_b + \log\left[\frac{\text{Salt}}{\text{Base}}\right] = 4.7 + \log\frac{4/50}{2/50}$$

$= 4.7 + \log 2 = 5$
pH = 14 – pOH $\Rightarrow$ pH = 14 – 5 = 9

26. **(c)** Due to the resonance stabilisation of the conjugate base, $CH(CN)_3$ is the strongest acid amongst the given compounds.

$$CH(CN)_3 \rightleftharpoons H^+ + \bar{C}(CN)_3$$



The conjugate bases of $CHBr_3$ and CHI_3 are stabilised by inductive effect of halogens. This is why, they are less stable. Also, the conjugate base of $CHCl_3$ involves back-bonding between 2*p* and 3*p* orbitals.

27. **(b)** CH_3COOK is a salt of weak acid (CH_3COOH) and strong base (KOH)

$$CH_3COOK + H_2O \longrightarrow \underset{\text{Weak acid}}{CH_3COOH} + \underset{\text{Strong base}}{KOH}$$

Hence, nature of solution is basic

28. **(b)** Given, $pK_a\,(HA) = 3.2$

$pK_b\,(BOH) = 3.4$

The salt (*AB*) given is a salt of weak acid and weak base. Hence the pH can be calculated by the formula

$$\therefore\ pH = 7 + \frac{1}{2}pK_a - \frac{1}{2}pK_b = 7 + \frac{1}{2}(3.2) - \frac{1}{2}(3.4) = 6.9$$

29. **(a)** Given,

	A	B	C	D
No. of moles initially	1	1	1	1
At equilibrium	$1-x$	$1-x$	$1+x$	$1+x$

($A + B \rightleftharpoons C + D$)

$$\therefore\ K_c = \left(\frac{1+x}{1-x}\right)^2 = 100 \quad \therefore\ \frac{1+x}{1-x} = 10$$

On solving, $x = 0.81$

$[D]_{\text{At eqm.}} = 1 + x = 1 + 0.81 = 1.81$

EXERCISE - 3

1. **(b)**

(A) Liquid $\rightleftharpoons$ Vapour, equilibrium exists at the boiling point.

(B) Solid $\rightleftharpoons$ Liquid, equilibrium exists at the melting point.

(C) Solid $\rightleftharpoons$ Vapour, equilibrium exists at the sublimation point.

(D) Solute $\rightleftharpoons$ Solute (solution), equilibrium exists in a saturated solution.

2. **(b)** A – (r), B – (s), C – (p), D – (q)

3. **(b)** $HClO_4$ is a strong acid

HNO_2 is a weak acid.

NH_2^- is a very good proton acceptor and thus, it is a strong base.

H_2SO_4 is a strong acid hence its conjugate base (HSO_4^-) will be a weak base.

4. **(c)** Bond energy being directly related to bond strength; increases with increase in bond strength.

(ii) It is not always ture.

5. **(a)** Boiling point depends on the altitude of the place; at high altitude the boiling point decreases.

6. **(a)** Equilibrium constant is temperature dependent having one unique value for a particular reaction represented by a balanced equation at a given temperature.

7. **(c)** An equilibrium constant does not give any information about the rate at which the equilibrium is reached.

8. **(d)** All statements are correct.

9. **(d)** (i) Solubility is high at high pressure. As the water bottle is opened, solubility decreases.

(ii) The given equilibrium is governed by Henry's law.

(iii), (iv) The amount of CO_2 gas dissolved in liquid decreases with increase in temperature or it increases with decrease in temperature.

10. **(a)** $K_p = K_c(RT)^{\Delta n}$

11. **(a)** Let us consider,

$$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)\ ;\ K_C = \frac{[HI]^2}{[H_2][I_2]}$$

For reverse reaction $2HI(g) \rightleftharpoons H_2(g) + I_2(g)$;

$$\text{Then, } K'_C = \frac{[H_2][I_2]}{[HI]^2} = \frac{1}{K_C}$$

12. **(a)** On increasing pressure, water-ice equilibrium shifts towards water side since the volume of the system decreases on melting of ice. For ice-water equilibrium again, temperature will have to be lowered. In other words melting point of ice decreases.

EXERCISE - 4

1. **(c)** $\underset{1-x}{PCl_5(g)} \rightleftharpoons \underset{x}{PCl_3(g)} + \underset{x}{Cl_2(g)}$

Total moles after dissociation

$1 - x + x + x = 1 + x$

$$p_{PCl_3} = \text{mole fraction of } PCl_3 \times \text{Total pressure} = \left(\frac{x}{1+x}\right)P$$

2. **(b)** $K_c = \dfrac{[NO_2]^2}{[N_2O_4]}$

If $[NO_2] \rightarrow$ at *y*-axis

$[N_2O_4] \rightarrow$ at *x*-axis

$\Rightarrow\ y = K_c x^2$

Parabolic curve.

3. **(c)** $CaCO_3(s) \rightleftharpoons CaO\,(s) + CO_2(g)$

$K_P = P_{CO_2} = 1$ atm (given)

$$\therefore \text{ By } \ln K_p = 7 - \frac{8400}{T} \Rightarrow 0 = 7 - \frac{8400}{T} \qquad [\ln 1 = 0]$$

$$\Rightarrow T = \frac{8400}{7} = 1200\text{ K} = (1200 - 273) = 927\,°C$$

4. **(a)** $NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$

	$NH_3(g)$	$H_2S(g)$
Start	0.5 atm	0 atm
At equib.	$0.5 + x$ atm	x atm

Then $0.5 + x + x = 2x + 0.5 = 0.84$ (given)

$\Rightarrow x = 0.17$ atm.

$p_{NH_3} = 0.5 + 0.17 = 0.67$ atm ; $p_{H_2S} = 0.17$ atm

$K = p_{NH_3} \times p_{H_2S} = 0.67 \times 0.17 = 0.1139 = 0.11\ atm^2$

5. **(b)** $A(g) \rightleftharpoons B(g) + \frac{1}{2} C(g)$

	A(g)	B(g)	C(g)
Initial :	P_i	0	0
At eq. :	$P_i(1-\alpha)$	$P_i \cdot \alpha$	$P_i \frac{\alpha}{2}$

Equilibrium pressure $(p) = P_i\left(1-\alpha+\alpha+\frac{\alpha}{2}\right)$

$$p = P_i \times \left(1+\frac{\alpha}{2}\right)$$

$$\Rightarrow \therefore P_A = \left(\frac{1-\alpha}{1+\frac{\alpha}{2}}\right)p$$

$$P_B = \left(\frac{\alpha}{1+\frac{\alpha}{2}}\right)p;\ P_C = \left(\frac{\frac{\alpha}{2}}{1+\frac{\alpha}{2}}\right)p$$

$$\therefore K = \frac{P_C^{\frac{1}{2}} \times P_B}{P_A}$$

$$\Rightarrow K = \frac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$$

6. **(c)** $2AB_3(g) \rightleftharpoons A_2(g) + 3B_2(g)$

	$2AB_3(g)$	$A_2(g)$	$3B_2(g)$
Initial	8	0	0
At eqm.	$\frac{8-2\alpha}{1}$	$\frac{\alpha}{1}$	$\frac{3\alpha}{1}$

Since volume = 1 dm^3 = 1 L

At eq. $[A_2] = 2$ mol $= \alpha$

$\therefore$ $[AB_3] = 8 - 2 \times 2 = 4$ M

$[A_2] = 2$ M

$[B_2] = 3 \times 2 = 6$ M

$$K = \frac{[B_2]^3[A_2]}{[AB_3]^2} = \frac{6\times6\times6\times2}{4\times4} = 27\ \text{mol}^2\text{L}^{-2}$$

7. **(b)** $2NOBr(g) \rightleftharpoons 2NO(g) + Br_2(g)$

$2NOBr(g)$	$2NO(g)$	$Br_2(g)$
x	$\frac{2P}{9}$	$\frac{P}{9}$

Since total pressure is P.

So, $x + \frac{2P}{9} + \frac{P}{9} = P$

$$\Rightarrow x = \frac{6P}{9}$$

$$\therefore K_p = \frac{p_{NO}^2 \times p_{Br_2}}{p_{NOBr}^2} = \frac{\left(\frac{2P}{9}\right)^2 \times \frac{P}{9}}{\left(\frac{6P}{9}\right)^2}$$

$$\therefore \frac{K_p}{P} = \frac{1}{81}$$

8. **(a)**

(a) The sample of ammonia obtained will be a mixture of NH_3, NT_3, NH_2T and NHT_2. Thus, this mixture will be radioactive.

(b) The position of equilibrium will not change by using isotopes.

(c) The value of equilibrium constant also remains same.

(d) Now the sample will contain tritium containing forms of ammonia and hydrogen (H_2, HT and T_2). Thus, the average molecular mass will increase.

9. **(b)** For exothermic reaction, yield $\uparrow$ as temperature $\downarrow$ *i.e.*, $T_1 > T_2 > T_3$

10. **(c)** Let x mole of KOH be neutralized by the strong acid HA. Then, moles neutralized by H$B = 1 - x$

Hence, $-13.7 \times x + (-12.7) \times (1-x) = -13.5$

$$\Rightarrow x = 0.8;\quad \frac{x}{1-x} = \frac{0.8}{0.2} = 4$$

11. **(a)** $\Delta n = 2 - 1 = 1$

That is, with the decrease of pressure, reaction shifts towards right, *i.e.*, proportions of NO_2 increases. Statement (a) is correct.

(b) Value of K_p increases by increasing the temperature. Thus, forward reaction increases. So, the reaction is endothermic or ΔH is positive.

(c) Unit of $K_p = (\text{atm})^{\Delta n}$ = atm.

(d) By increasing the pressure, $[N_2O_4]$ will increase.

12. **(d)** $Ag^+_{(aq)} + 2NH_{3(aq)} \rightleftharpoons [Ag(NH_3)_2]^+_{(aq)}$

$$\text{Instability constant} = \frac{1}{\text{formation constant}}$$

$$= \frac{K_r}{K_f} = \frac{2.0\times10^{-2}}{1.0\times10^{6}} = 2.0\times10^{-8}$$

13. **(b)** $pH = 9; [H^+] = 10^{-9}; [OH^-] = 10^{-5};$

$[Mg^{2+}] = 1 \times 10^{-3}; [Mg^{++}][OH^-]^2 = 1 \times 10^{-13}$

given K_{sp} of $Mg(OH)_2 = 8.9 \times 10^{-12}$ which is more than 1×10^{-13}. Hence, $Mg(OH)_2$ will not precipitate.

14. **(c)** $\Delta_r G° = \Delta G_f(\text{Products}) - \Delta G_f(\text{Reactants})$

$= 0 - 77.1 \times 2 = -154.2$ kJ/mol

$$Q = \frac{[H^+]^2}{P_{H_2} \cdot [Ag^+]^2} = \frac{10^{-6}}{0.5 \times (10^{-10})} = 2 \times 10^4$$

$$\Delta_n G = \Delta_r G° + RT \ln Q$$

$$\Delta_r G = -154.2 + \frac{8.314 \times 300 \ln(2 \times 10^4)}{1000} = -129.5 \text{ kJ/mol}$$

15. **(c)** The two K_{sp} values do not differ very much. So it is a case of simultaneous equilibria, where the concentration of any species can not be neglected.

$$\frac{[Sr^{2+}][F^-]^2}{[Sr^{2+}][CO_3^{2-}]} = \frac{K_{sp_{SrF_2}}}{K_{sp_{SrCO_2}}} = \frac{7.9 \times 10^{-10}}{7.0 \times 10^{-10}} = 1.128$$

$\therefore$ $[F^-]^2 = 1.128 \times 1.2 \times 10^{-3} = 13.5 \times 10^{-4}$

$\therefore$ $[F^-] = (13.5 \times 10^{-4})^{1/2} = 3.674 \times 10^{-2} \approx 3.7 \times 10^{-2}$ M.

EXERCISE - 5

1. **(20)** $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$

$K_p = P_{CO_2} = 0.82$ atm ;

$$n_{CO_2} = \frac{PV}{RT} = \frac{0.82 \times 20}{0.082 \times 1000} = 0.2 \text{ mole}$$

Mole of $CaCO_3$ dissociated = $n_{CO_2} = 0.2$

Amount dissociated $= 0.2 \times 100 = 20$ g

2. **(0.75)** $NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$

	$NH_3(g)$	$H_2S(g)$
Initial	1.0 atm	0 atm
At eqm.	1.0 + x atm	x atm

Then $1.0 + x + x = 2x + 1.0 = 2.0$ (given)

$\Rightarrow x = 0.5$ atm

$p_{NH_3} = 1.0 + 0.5 = 1.5$ atm ; $p_{H_2S} = 0.5$ atm^2

$K = p_{NH_3} \times p_{H_2S} = 1.5 \times 0.5$ atm^2 $= 0.75$ atm^2

3. **(11.4)** Given;

$NaOH + HNO_3 \rightarrow NaNO_3 + H_2O$

$NaOH$	HNO_3	
1 mole	1 mole	$\Delta H = 57.0$ kJ
0.2 mole	0.5 mole	$\Delta H = ?$

Given heat of neutralisation of strong acid by strong base = 57.0 kJ

$\because$ 0.2 mole NaOH is limiting reagent.

$\therefore$ Heat of neutralization $= 0.2 \times 57 = 11.4$ kJ

4. **(10)** $M_1V_1 = M_2V_2$

$1 \times 0.10 = M_2 \times 100$

$M_2 = 0.001 = 10^{-3}$

$BOH(g) \rightleftharpoons B^+ + OH^-$

	BOH	B^+	OH^-
	C	0	0
	C (1 – α)	Cα	Cα

$$K_b = \frac{C\alpha \times C\alpha}{C(1-\alpha)}$$

$K_b = C\alpha^2$ $\quad (\because 1 - \alpha \approx 1)$

$\alpha = \sqrt{K_b / C}$

$$[OH^-] = C\alpha = \sqrt{\frac{K_b}{C}} \times C = \sqrt{K_b C}$$

$= \sqrt{10^{-5} \times 10^{-3}} = 10^{-4}$

$\therefore$ pH + pOH = 14

$\therefore$ pH = 14 – 4 = 10

5. **(0.592)** Moles of $N_2 = \frac{28}{28} = 1$, Moles of $H_2 = \frac{6}{2} = 3$

Moles of H_2SO_4 required $= \frac{500 \times 1}{1000} = 0.5$

Moles of NH_3 neutralised by $H_2SO_4 = 1.0$

$\left(2NH_3 + H_2SO_4 \longrightarrow (NH_4)_2SO_4\right)$

Hence 1 mole of NH_3 by the reaction between N_2 and H_2.

	N_2	$3H_2$	$2NH_3$
initial conc.	1	3	0
at equilibrium conc.	1 – 0.5	3 – 0.5 × 3	1

$$K_c = \frac{1 \times 1}{0.5 \times (1.5)^3} = 0.592 \text{ mol}^{-2} L^2$$

6. **(2)** $2NO + O_2 \rightleftharpoons 2NO$

	$2NO$	O_2	$2NO$
Initial:	2	1	—
Change:	2–2x	1–x	2x
Equilibrium:	1.2	0.6	0.8

$$K_p = \frac{\left(\frac{0.8}{2.6}\right)^2}{\left(\frac{1.2}{2.6}\right)^2 \left(\frac{0.6}{2.6}\right)} = 1.925$$

7. **(11)** 0.001 M NaOH

$[OH^-] = 10^{-3}$

pOH = – log $[OH^-] \Rightarrow$ pOH = 3

pH + pOH = 14 $\Rightarrow$ pH = 11

8. **(6.5)** $HCOONH_4$ is a salt of weak acid and weak base;

$$pH = \frac{1}{2} pK_w + \frac{1}{2} pK_a - \frac{1}{2} pK_b$$

$$\therefore pH = \frac{1}{2} \times 14 + \frac{1}{2} \times 3.8 - \frac{1}{2} \times 4.8 \text{ ; } pH = 6.5$$

9. **(4)** $AB(g) \longrightarrow A(g) + B(g)$

Applying law of mass action

$$K_p = \frac{\alpha^2 p}{1-\alpha^2} \quad \text{(given p = 1 atm)}$$

$$\therefore \frac{\alpha^2}{1-\alpha} = 1.6 \times 10^{-3} \Rightarrow \alpha^2 = 1.6 \times 10^{-3} \quad (\because \alpha <<< 1)$$

$$\Rightarrow \alpha = \sqrt{1.6 \times 10^{-3}} \Rightarrow \alpha = 0.04$$

% age dissociation = 4%.

10. **(8.22)** $CH_3COOH + NaOH \rightleftharpoons CH_3COONa + H_2O$

Let acid be = V mL

V mL of 0.01 M CH_3COOH will require V mL of 0.01 M NaOH. But CH_3COONa formed will make solution alkaline due to hydrolysis.

$$CH_3COONa + H_2O \rightleftharpoons CH_3COOH + NaOH$$

$$[CH_3COONa] = \frac{0.01}{2} = 0.005\,M$$

Using equation for pH of salt of weak acid and strong base.

$$pH = 7 + \frac{pK_a}{2} + \frac{\log C}{2} = 7 + \frac{4.74}{2} + \frac{\log 0.005}{2}$$

$= 8.22$

11. **(71)** $N_2O_4(g) \rightleftharpoons 2NO_2(g)$

	N_2O_4 (g)	$2NO_2$ (g)
Initial moles	1	0
Moles at eqm.	(1 – α)	2α

(α = degree of dissociation)

Total number of moles at equilibrium

= (1 – α) + 2α

= (1 + α)

$$p_{N_2O_4} = \frac{(1-\alpha)}{(1+\alpha)} \times P$$

$$p_{NO_2} = \frac{2\alpha}{(1+\alpha)} \times P$$

$$K_P = \frac{(p_{NO_2})^2}{P_{N_2O_4}} = \frac{\left(\frac{2\alpha}{(1+\alpha)} \times P\right)^2}{\left(\frac{1-\alpha}{1+\alpha}\right) \times P} = \frac{4\alpha^2 P}{1-\alpha^2}$$

Given, $K_p = 2$, P = 0.5 atm

$$\therefore K_P = \frac{4\alpha^2 P}{1-\alpha^2} = \frac{4\alpha^2 \times 0.5}{1-\alpha^2}$$

$\alpha = 0.707 \approx 0.71$

$\therefore$ Percentage dissociation = 0.71 × 100 = 71

12. **(4.76)** Total volume = 100 mL

$$[acid] = 10\,mL \times \frac{1.0}{100} = 0.1$$

$$[salt] = 20\,mL \times \frac{0.5}{100} = 0.1$$

$$\text{pH of acidic buffer} = pK_a + \log\frac{[salt]}{[acid]}$$

$$= 4.76 + \log\frac{0.1}{0.1} = 4.76$$

13. **(0.134)** $SO_2(g) + NO_2(g) \rightleftharpoons SO_3(g) + NO(g)$

	SO_2 (g)	NO_2 (g)	SO_3 (g)	NO (g)
Initial moles	2	2	2	2
at eqm.	2 – x	2 – x	2 + x	2 + x

$(\because Q_p < K_p)$

Total no. of moles of gases at equilibrium

= 8 + 2 = 10

$$K_p = \frac{P_{SO_3} \cdot P_{NO}}{P_{SO_2} \cdot P_{NO_2}}$$

$$\Rightarrow 25 = \frac{\left(\frac{2+x}{10} \times P\right)^2}{\left(\frac{2-x}{10} \times P\right)^2}$$

$$\Rightarrow 5 = \frac{2+x}{2-x}; \quad x = 1.33$$

$$\text{Partial Pressure of } NO_2 = \frac{2-x}{10} \times P_{total}$$

$$= \frac{2-1.33}{10} \times 2$$

$$= \frac{0.67}{10} \times 2 = 0.134\text{ atm}$$

14. **(5.0)** $pH = pK_a + \log\frac{[CH_3COO^-]}{[CH_3COOH]}$

$pK_a = -\log(1.8 \times 10^{-5}) = 4.7447$

$[CH_3COO^-] = 2 \times [(CH_3COO)_2Ba] = 0.2\,M$

$[CH_3COOH] = 0.1\,M$

$$pH = 4.7447 + \log\frac{0.2}{0.1} = 5.046 \approx 5.0$$

15. **(282)** $K_{sp} = S^2$

$$S = \sqrt{K_{sp}} = \sqrt{8 \times 10^{-28}} = 2\sqrt{2} \times 10^{-14} = 2.82 \times 10^{-14}$$

$= 282 \times 10^{-16}$

16. **(0.05)** $K_a(H_2S) = 4 \times 10^{-21}$

$$= \frac{[H^+]^2[S^{2-}]}{[H_2S]} = \frac{0.1^2 \times [S^{2-}]}{0.1}$$

$$\Rightarrow [S^{2-}] = 4 \times 10^{-20}\,M$$

$K_{sp}(CoS) = 2 \times 10^{-21} = [Co^{2+}][S^{2-}]$

$= [Co^{2+}] \times 4 \times 10^{-20} \Rightarrow [Co^{2+}] = 0.05\,M$

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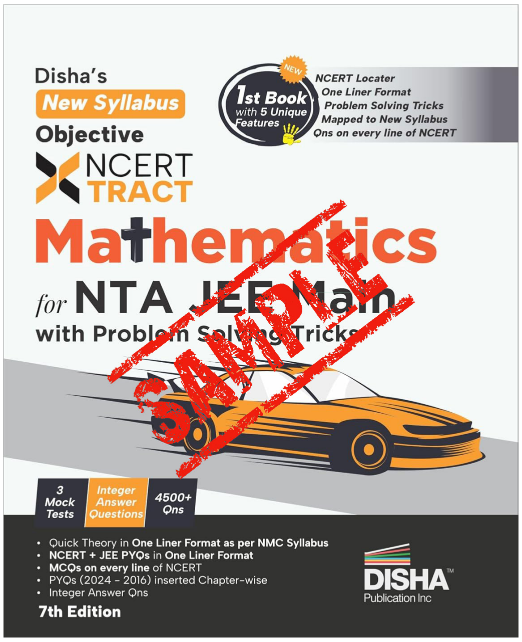
Class XII

This sample book is prepared from the book **"Disha's New Syllabus Objective NCERT Xtract Mathematics for NTA JEE Main 7th Edition | Useful for BITSAT, VITEEE & Advanced |MCQs/ NVQs of NCERT, Tips on your Fingertips, Previous Year Questions PYQs".**



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Class XIth

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Note : * The four Exercises in each of the chapters are :
- Exercise 1 : NCERT Based Topic-wise MCQs
- Exercise 2 : NCERT Exemplar & Past Years JEE Main
- Exercise 3 : Skill Enhancer MCQs
- Exercise 4 : Numeric Value Answer Questions

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Class XIIth

Note : * The four Exercises in each of the chapters are :
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Mock Tests

4 Complex Numbers and Quadratic Equations

Trend Analysis JEE MAIN

	JEE	Remarks
Number of Questions from 2024-17	14	Important chapter, average two questions have been asked in each year.
Weightage	5.95%	

			JEE	
Year	Topic Name	Concept Used	No. of Ques.	Difficulty Level
2024	Geometry of complex number/ Quadratic equation	Distance from point/ Cube roots of unity	2	Average
2023	Algebraic operations and modulus of complex numbers	Rationalisation/ Modulus of complex numbers	2	Easy/ Average
2022	Complex number / Quadratic equation	Multiplication of complex numbers/ Roots of quadratic equation	2	Easy/ Average
2021	Argument of complex Numbers / Solution of quadratic equations	Properties of argument / Graph and wavy curve	2	Easy / Average
2020	Quadratic equations / Equation of circle in complex form	Relation between roots and coefficient / Properties of complex number	2	Average
2019	Quadratic equation / Equation of circle in complex form	Relation between roots and coefficient / Properties of complex number	2	Easy / Average
2018	Algebraic operations of complex numbers	Rationalisations of complex numbers	1	Easy
2017	Quadratic Equation	Relation between roots and coefficient and sum of special series	1	Difficult

NCERT ONE-LINERS
(Important Points to Remember)

4.1 Introduction

- In earlier classes, we have studied linear equation in one and two variables and quadratic equations in one variable. We have seen that the equation $x^2 + 1 = 0$ has no real solution as $x^2 + 1 = 0$ gives $x^2 = -1$ and square of every real number is non-negative. So, we need to extend the real number system to a larger system so that we can find the solution of the equation $x^2 = -1$.
- In fact, the main objective is to solve the equation $ax^2 + bx + c = 0$, where $D = b^2 - 4ac < 0$, which is not possible in the system of real numbers.

4.2 Complex Numbers

- The main objective of complex numbers is to solve the equation $ax^2 + bx + c = 0$, where $D = b^2 - 4ac < 0$, which is not possible in the system of real numbers. **JEE M 2022**
- Let us denote $\sqrt{-1}$ by the symbol i. Then, we have $i^2 = -1$. This means that i is a solution of the equation $x^2 + 1 = 0$.
- A number of the form a + ib, where a and b are real numbers, is defined to be a **complex number**. For example, $2 + i3, (-1) + i\sqrt{3}, 4 + i\left(\frac{-1}{11}\right)$ are complex numbers.
- For the complex number $z = a + ib$, a is called **the real part**, denoted by $Re\, z$ and b is called **the imaginary part** denoted by $Im\, z$ of the complex number z. For example, if $z = 2 + i5$, then $Re\, z = 2$ and $Im\, z = 5$. **JEE M 2019**

Equality of Complex Numbers

- Two complex numbers $z_1 = a + ib$ and $z_2 = c + id$ are equal if $a = c$ and $b = d$.

4.3 Algebra of Complex Numbers

Addition of complex numbers: Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the sum $z_1 + z_2$ is defined as follows: $z_1 + z_2 = (a + c) + i(b + d)$, which is again a complex number.

The addition of complex numbers satisfy the following properties:

- **The closure law:** The sum of two complex numbers is a complex number, *i.e.*, $z_1 + z_2$ is a complex number for all complex numbers z_1 and z_2.
- **The commutative law:** For any two complex numbers z_1 and z_2, $z_1 + z_2 = z_2 + z_1$
- **The associative law:** For any three complex numbers z_1, z_2, z_3, $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$
- **The existence of additive identity:** There exists the complex number $0 + i0$ (denoted as 0), called the **additive identity** or the **zero complex number**, such that, for every complex number z, $z + 0 = z$.
- **The existence of additive inverse:** To every complex number $z = a + ib$, we have the complex number $-a + i(-b)$ (denoted as $-z$), called **the additive inverse or negative of z**. We observe that $z + (-z) = 0$ (the additive identity).
- **Difference of two complex numbers:** Given any two complex numbers z_1 and z_2, the difference $z_1 - z_2$ is defined as follows: $z_1 - z_2 = z_1 + (-z_2)$.

Multiplication of two complex numbers : Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the product $z_1 z_2$ is defined as follows: $z_1 z_2 = (ac - bd) + i(ad + bc)$.

The multiplication of complex numbers possesses the following properties, which we state without proofs:

- Complex number, $z = x + iy$

  Then, rationalize the given complex number,

  $$z = x + iy = \frac{a+ib}{c+id} \times \frac{c-id}{c-id}$$

  Compare both the sides & determine the values of x and y to identify the given curve. **JEE M** 2021, 2023

- **The closure law:** The product of two complex numbers is a complex number, the product $z_1 z_2$ is a complex number for all complex numbers z_1 and z_2.
- **The commutative law:** For any two complex numbers z_1 and z_2, $z_1 z_2 = z_2 z_1$
- **The associative law:** For any three complex numbers z_1, z_2, z_3, $(z_1 z_2) z_3 = z_1 (z_2 z_3)$.
- **The existence of multiplicative identity:** There exists the complex number $1 + i0$ (denoted as 1), called **the multiplicative identity** such that $z_1 = z$, for every complex number z.
- **The existence of multiplicative inverse:** For every non-zero complex number $z = a + ib$ or $a + bi (a \neq 0, b \neq 0)$, we have the complex number

  $\frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}$ (denoted by $\frac{1}{z}$ or z^{-1}), called the *multiplicative inverse* of z such that $z\frac{1}{z} = 1$ (the multiplicative identity).
- **The distributive law:** For any three complex numbers z_1, z_2, z_3,

  (a) $z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$

  (b) $z_3 (z_1 + z_2) = z_1 z_3 + z_2 z_3$
- **Division of two complex numbers:** Given any two complex numbers z_1 and z_2, where $z_2 \neq 0$, the quotient $\frac{z_1}{z_2}$ is defined by $\frac{z_1}{z_2} = z_1 \frac{1}{z_2}$

Power of i: We know that,

- $i^3 = i^2 i = (-1)\, i = -i$, $\quad i^4 = (i^2)^2 = (-1)^2 = 1$
- $i^5 = (i^2)^2 i = (-1)^2 i = i$, $\quad i^6 = (i^2)^3 = (-1)^3 = -1$
- $i^{-1} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{-1} = -i$, $\quad i^{-2} = \frac{1}{i^2} = \frac{1}{-1} = -1$,
- $i^{-3} = \frac{1}{i^3} = \frac{1}{-i} \times \frac{i}{i} = \frac{i}{1} = i$, $\quad i^{-4} = \frac{1}{i^4} = \frac{1}{1} = 1$
- In general, for any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$

The square roots of a negative real number:

- Note that $i^2 = -1$ and $(-i)^2 = i^2 = -1$

  Therefore, the square roots of -1 are i, $-i$. However, by the symbol $\sqrt{-1}$, we would mean i only.
- Now, we can see that i and $-i$, both are the solutions of the equation $x^2 + 1 = 0$.
- Generally, if a is a positive real number,

  $\sqrt{-a} = \sqrt{a}\sqrt{-1} = \sqrt{a}\, i.$

Identities

- $(z_1 + z_2)^2 = z_1^2 + z_2^2 + 2z_1 z_2$,
- $(z_1 - z_2)^2 = z_1^2 - 2z_1 z_2 + z_2^2$
- $(z_1 + z_2)^3 = z_1^3 + 3z_1^2 z_2 + 3z_1 z_2^2 + z_2^3$
- $(z_1 - z_2)^3 = z_1^3 - 3z_1^2 z_2 + 3z_1 z_2^2 - z_2^3$
- $z_1^2 - z_2^2 = (z_1 + z_2)(z_1 - z_2)$

4.4 The Modulus and the Conjugate of a Complex Number

- **Modulus of complex number:** Let $z = a + ib$ be a complex number. Then, the modulus of z, denoted by $|z|$, is defined to be the non-negative real number $\sqrt{a^2 + b^2}$, *i.e.*, $|z| = \sqrt{a^2 + b^2}$

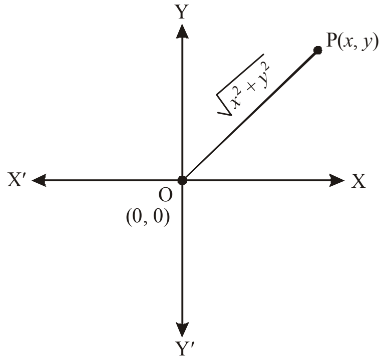
- When $K = \frac{a+bz}{c+dz} \Rightarrow$ calculate z,
  Use the value of z in |z| > t (OR) |Z| < t
  $z\bar{z} = |z|^2$ JEE M 2019
- **Conjugate of complex number:** Let $z = a + ib$ be a complex number. Then, the conjugate of z denoted as $\bar{z}$, is the complex number $a - ib$, i.e., $\bar{z} = a - ib$.
- For a given complex number z,
  Use the property of conjugate complex number,
  $z + \bar{z} = 0$ JEE M 2019

Properties:

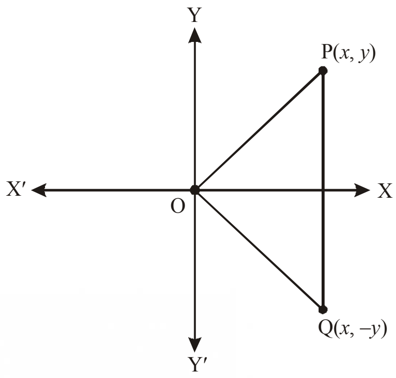
- $z^{-1} = \frac{\bar{z}}{|z|^2}$
- $z\,\bar{z} = |z|^2$
- $|z_1 z_2| = |z_1||z_2|$
- $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ provided $|z_2| \neq 0$
- $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$
- $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$ provided $z_2 \neq 0$.
- For an equation,
  y (y – a) = |y – b| + |y – c| c > b
  Case :I When y > c , if D < 0, No real roots will be obtained.
  Case :II When b ≤ y ≤ c
  Case :III When y ≤ b JEE M 2020

4.5 Argand Plane and Polar Representation

- **Argand plane:** The plane having a complex number assigned to each of its point is called **the complex plane** or **the Argand plane**.
- Let $z = x + iy$ be the complex number. Then, the modulus of complex number $z = x + iy = \sqrt{x^2 + y^2}$ is the distance between the point P(x, y) and the origin O (0, 0) (Fig.).

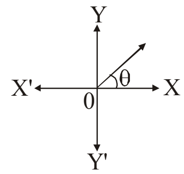
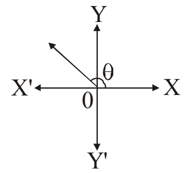
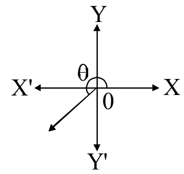
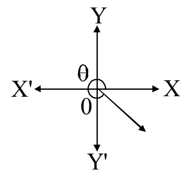


- The points on the x-axis corresponds to the complex numbers of the form $a + i0$ and the points on the y-axis corresponds to the complex numbers of the form $0 + ib$.
- The x-axis and y-axis in the Argand plane are called, respectively, **the real** axis and the **imaginary axis**.
- The representation of a complex number $z = x + iy$ and its conjugate $z = x - iy$ in the Argand plane are, respectively, the points P(x, y) and Q(x, –y).
- Geometrically, the point (x, –y) is the mirror image of the point (x, y) on the real axis (Fig.)

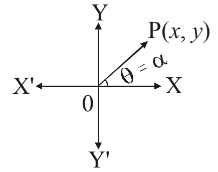
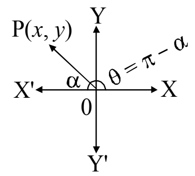
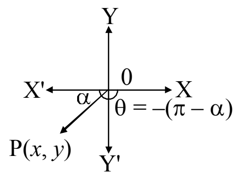
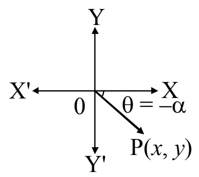


Polar Representation of a Complex Number:

- Let the point P represent the non-zero complex number $z = x + iy$. The point P is uniquely determined by the ordered pair of real numbers (r, θ), called **the polar coordinates of the point P**.
- We consider the origin as the pole and the positive direction of the x-axis as the initial line.
- We have, $x = r\cos\theta$, $y = r\sin\theta$ and therefore, $z = r(\cos\theta + i\sin\theta)$. The latter is said to be the polar form of the complex number. JEE M 2021
- Here, $r = \sqrt{x^2 + y^2} = |z|$ is the modulus of z and θ is called **the argument (or amplitude)** of z which is denoted by arg z.
- **General argument:** When $0 \leq \theta < 2\pi$.

Quadrant	Signs of x and y	Argument	Graph
I.	$x > 0, y > 0$	$0 < \theta < \frac{\pi}{2}$	
II.	$x < 0, y > 0$	$\frac{\pi}{2} < \theta < \pi$	
III.	$x < 0, y < 0$	$0 < \theta < \frac{3\pi}{2}$	
IV.	$x > 0, y < 0$	$\frac{3\pi}{2} < \theta < 2\pi$	

- **Principal argument:** The value of argument *i.e.*, θ, such that $-\pi < \theta \le \pi$ is called the principal argument.

Quadrant	Signs of x and y	Argument	Graph
I.	$x > 0, y > 0$	$\theta = \alpha$ and $0 < \theta < \frac{\pi}{2}$	
II.	$x < 0, y > 0$	$\theta = \pi - \alpha$ and $\frac{\pi}{2} < \theta < \pi$	
III.	$x < 0, y < 0$	$\theta = -(\pi - \alpha) = \alpha - \pi$ and $-\pi < \theta < -\frac{\pi}{2}$	
IV.	$x > 0, y < 0$	$\theta - \alpha$ and $-\frac{\pi}{2} < \theta < 0$	

4.6 Quadratic Equation

- To find the number of real roots of an equation, $f(x) = ax^n + bx^{n-1} + ... + cx + d = 0$
Find $f'(x) = 0$, with the help of $f'(x)$ check where the slope is changing and with the help of this, draw the graph where the curve intersects x-axis at various points.
JEE M 2021
- Let us consider the quadratic equation $ax^2 + bx + c = 0$ with real coefficients a, b, c and $a \neq 0$. Also, let us assume that $b^2 - 4ac < 0$.
- Therefore, the solutions to the above equation are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{4ac - b^2}i}{2a}$$

- For a quadratic equation, $ax^2 + bx + c = 0$
Roots = α, β; $\alpha + \beta = -b/a$; $\alpha\beta = c/a$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$ JEE M 2020

- **Fundamental Theorem of Algebra:**
"A polynomial equation has at least one root."
- As a consequence of this theorem, the following result, which is of immense importance, is arrived at:
"A polynomial equation of degree n has n roots."

- For a quadratic equation,
$ax^2 + bx + c = 0$; Roots = α, β
Roots of quadratic equation always satisfies the quadratic equation.
$\therefore a\alpha^2 + b\alpha + c = 0$ and
$a\beta^2 + b\beta + c = 0$ JEE M 2019
- For a quadratic equation, $ax^2 + bx + c = 0$
Roots = α, β; $\alpha + \beta = -b/a$, $\alpha\beta = c/a$
For function to be greatest, $f'(x) = 0$, $f''(x) < 0$
$|\alpha^3 - \beta^3| = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)$ JEE M 2019, 2023

- Solution of quadratic equation,

$ax^2 + bx + c = 0$; solutions $\alpha, \beta = \frac{-b \pm \sqrt{D}}{2a}$

$D = b^2 - 4ac$

For $(ax^2 + bx + c)^{(dx^2 + ex + f)} = K$
Case :I $ax^2 + bx + c = K$ and $dx^2 + ex + f$ can be any real number $\Rightarrow$ find value of x
Case :II $ax^2 + bx + c = -K$ and $dx^2 + ex + f$ has to be an even number $\Rightarrow$ find value of x
Case :III $ax^2 + bx + c$ can be any real number and $dx^2 + ex + f = 0$
Rationalize the given Complex Number

$$Z = \frac{a + ib}{c + id} \times \frac{c - id}{c - id}$$

Complex number to be purely imaginary, it's real part should be zero, $\therefore$ Re $(z) = 0$ JEE M 2016

Tips/Tricks/Techniques ONE-Liners
(Exam Special)

- **Properties of conjugate complex number**
Let $z = a + ib \Rightarrow \bar{z} = a - ib$, then
(a) $\overline{(\bar{z})} = z$ (b) $z + \bar{z} = 2a = 2$ Re (z) = purely real
(c) $z - \bar{z} = 2ib = 2i$ Im(z) = purely imaginary
(d) $z\bar{z} = a^2 + b^2 = |z|^2$ (e) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
(f) $\overline{z^n} = (\bar{z})^n$
(g) $|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)} = (z_1 + z_2)(\bar{z}_1 + \bar{z}_2)$
$= |z_1|^2 + |z_2|^2 + z_1\bar{z}_2 + \bar{z}_1 z_2$
(h) $z + \bar{z} = 0$ or $z = -\bar{z} \Rightarrow z = 0$ or z is purely imaginary
(i) $z = \bar{z} \Rightarrow z$ is purely real
(j) $z_1\bar{z}_2 + z_2\bar{z}_1 = 2\,\text{Re}(z_1 \cdot z_2)$
- **Properties of modulus of a complex number**
(a) $|z| \geq 0$ and $|z| = 0$ if and only if $z = 0$, i.e., $x = 0, y = 0$
(b) $-|z| \leq$ Re$(z) \leq |z|$ (c) $-|z| \leq$ Im$(z) \leq |z|$
(d) $|z| = |\bar{z}| = |-z| = |-\bar{z}|$ (e) $z\bar{z} = |z|^2$
(f) $|z^2| = |z|^2$ or $|z^n| = |z|^n, n \in N$
also $|z_1 z_2 ...... z_n| = |z_1|\,|z_2| ....|z_n|$
(g) $|z| = 1 \Leftrightarrow \bar{z} = \frac{1}{z}$ (h) $z^{-1} = \frac{\bar{z}}{|z|^2}$
(i) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$
(j) $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\text{Re}(z_1\bar{z}_2)$
- **Properties of argument of a complex number**
(a) arg (any real positive number) = 0
(b) arg (any real negative number) = π
(c) arg $(z - \bar{z}) = \pm\pi/2$
(d) arg $(z_1 . z_2)$ = arg (z_1) + arg (z_2)
(e) arg $\left(\frac{z_1}{z_2}\right)$ = arg (z_1) – arg (z_2)
(f) arg $(\bar{z})$ = – arg (z) = arg $(1/z)$
(g) arg $(-z)$ = arg $(z) \pm \pi$
(h) arg $(z^n) = n$ arg (z)

(i) $\arg(iy) = \begin{cases} \frac{\pi}{2}, & \text{if } y > 0 \\ -\frac{\pi}{2} & \text{if } y < 0 \end{cases}$

(j) $\arg(z) + \arg(\bar{z}) = 0$

- **Distance formula :** Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ be two complex numbers represented by points P and Q respectively in Argand Plane, then

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = |(x_2 - x_1) + i(y_2 - y_1)| = |z_2 - z_1|$$ JEE M 2024

- **Section formula :** If the line segment joining A (z_1) and B (z_2) is divided by the point P (z) internally in the ratio $m_1 : m_2$ then

$$z = \frac{m_1 z_2 + m_2 z_1}{m_1 + m_2}$$

But if P divides AB externally in the ratio $m_1 : m_2$, then

$$z = \frac{m_1 z_2 - m_2 z_1}{m_1 - m_2}$$

If P is mid point of AB, then $z = \frac{z_1 + z_2}{2}$

- **Area of a triangle :** Area of triangle ABC with vertices A (z_1), B (z_2) and C (z_3) is given by

$$\Delta = \frac{1}{4}\left\| \begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix} \right\|$$

or $\Delta = \frac{1}{4}\left|z_1(\bar{z}_2 - \bar{z}_3) - z_2(\bar{z}_1 - \bar{z}_3) + z_3(\bar{z}_1 - \bar{z}_2)\right|$

- **Equation of a circle :** The equation of a circle with centre z_0 and radius r is $|z - z_0| = r$

The general equation of a circle is $z\bar{z} + a\bar{z} + \bar{a}z + b = 0$, where a is a complex number and b is real number.

- **Condition for common roots**

Only one root common : For the two given equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ then

The condition for only one Root common is $(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$

Both roots are common : Required condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- **Nature of the factors of the quadratic expression**

For $ax^2 + bx + c = 0$ the factors are

(a) Real and different, if $b^2 - 4ac > 0$

(b) Rational and different, if $b^2 - 4ac$ is a prefect square.

(c) Real and equal, if $b^2 - 4ac = 0$

(d) Imaginary, if $b^2 - 4ac < 0$

- The square root of i is $\pm\left(\frac{1+i}{\sqrt{2}}\right)$ (Here $b = 1$)
- The square root of $-i$ is $\pm\left(\frac{1-i}{\sqrt{2}}\right)$ (Here $b = -1$)
- $\arg z_1 - \arg z_2 = \arg\left(\frac{z_1}{z_2}\right)$
- If z_1, z_2, z_3 are collinear, which implies $\theta = 0$, therefore, $\frac{z_3 - z_1}{z_2 - z_1}$ is purely real.
- If z_1, z_2, z_3 are such that PR ⊥ PQ, which implies $\theta = \pi/2$, so $\frac{z_3 - z_1}{z_2 - z_1}$ is purely imaginary.
- **Maximum and minimum value of quadratic expression**

In a quadratic expression $ax^2 + bx + c$

(a) If $a > 0$, quadratic expression has least value at $x = -\frac{b}{2a}$.

This least value is given by $\frac{4ac - b^2}{4a} = -\frac{D}{4a}$

(b) If $a < 0$, quadratic expression has greatest value at $x = -\frac{b}{2a}$. This greatest value is given by $\frac{4ac - b^2}{4a} = -\frac{D}{4a}$

Exercise 1 : NCERT Based Topic-wise MCQs

4.1 Introduction

1. The main objective of complex number is to solve the equation $ax^2 + bx + c = 0$, where $D = b^2 - 4ac$ satisfy which of the following option? NCERT Page-98/N-76

(a) $D > 0$ (b) $D < 0$ (c) $D = 0$ (d) $D \neq 0$

4.2 Complex Numbers

2. For a complex number z, its Re(z) = 0 and Im(z) = 2. Then what will be the complex number z ? NCERT Page-98/N-76

(a) 0 (b) i (c) 2i (d) –2i

3. If $4x + i(3x - y) = 3 + i(-6)$, where x and y are real numbers, then the values of x and y are NCERT Page-99/N-77

(a) $x = \frac{3}{5}$ and $y = \frac{33}{4}$ (b) $x = \frac{3}{4}$ and $y = \frac{22}{3}$

(c) $x = \frac{3}{4}$ and $y = \frac{33}{4}$ (d) $x = \frac{3}{4}$ and $y = \frac{33}{5}$

4. For a complex number z = – 4i + 7, what is the Re (z) and Im (z) respectively? NCERT Page-98/N-76

(a) –4, 7 (b) 7, –4 (c) 4, –7 (d) –7, 4

4.3 Algebra of Complex Numbers

5. If $z = 5i\left(\frac{-3}{5}i\right)$, then z is equal to 3 + bi. The value of 'b' is **NCERT Page-101/N-77**

(a) 1 (b) 2 (c) 0 (d) 3

6. The real part of the complex number $\frac{(1+2i)^8 \cdot (1-2i)^2}{(3+2i)\cdot\overline{(4-6i)}}$ is equal to: **NCERT Page-110/N-81**

(a) $\frac{500}{13}$ (b) $\frac{110}{13}$ (c) $\frac{55}{6}$ (d) $\frac{550}{13}$

7. Let S be the set of all (α, β), $\pi < \alpha, \beta < 2\pi$, for which the complex number $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely imaginary and $\frac{1-i\cos\beta}{1+2i\sin\beta}$ is purely real. Let $Z\alpha\beta = \sin 2\alpha + i\cos 2\beta$, $(\alpha, \beta) \in S$. Then $\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta} + \frac{1}{i\bar{Z}\alpha\beta}\right)$ is equal to **NCERT Page-111/N-76**

(a) 3 (b) $3i$ (c) 1 (d) 2–1

8. If $z = i^9 + i^{19}$, then z is equal to a + ai. The value of 'a' is **NCERT Page-103/N-79**

(a) 0 (b) 1 (c) 2 (d) 3

9. Value of $\left(\frac{2i}{1+i}\right)^2$ is **NCERT Page-103/N-81**

(a) i (b) $2i$ (c) $1-i$ (d) $1-2i$

10. If $\left(\frac{1-i}{1+i}\right)^{100} = a+ib$ then **NCERT Page-103/N-81**

(a) $a = 2, b = -1$ (b) $a = 1, b = 0$
(c) $a = 0, b = 1$ (d) $a = -1, b = 2$

11. $1 + i^2 + i^4 + i^6 + ... + i^{2n}$ is **NCERT Page-104/N-79**

(a) positive (b) negative
(c) 0 (d) cannot be determined

12. Value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}} - 1$ is **NCERT Page-100/N-79**

(a) –2 (b) 0 (c) –1 (d) 1

13. If $z = x - i\,y$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p}+\frac{y}{q}\right)\Big/(p^2+q^2)$ is equal to **NCERT Page-102/N-79**

(a) –2 (b) –1 (c) 2 (d) 1

14. If z = 1 + i, then the multiplicative inverse of z^2 is (where, $i = \sqrt{-1}$) **NCERT Page-102/N-80**

(a) 2i (b) 1–i (c) $-\frac{i}{2}$ (d) $\frac{i}{2}$

15. The multiplicative inverse of $\frac{3+4i}{4-5i}$ is **NCERT Page-102/N-82**

(a) $\frac{8}{25} - \frac{31}{25}i$ (b) $-\frac{8}{25} - \frac{31}{25}i$
(c) $-\frac{8}{25} + \frac{31}{25}i$ (d) None of these

16. If z(2 – i) = (3 + i), then z^{20} is equal to **NCERT Page-105/N-81**

(a) 2^{10} (b) -2^{10} (c) 2^{20} (d) -2^{20}

17. If $(x + iy)^{\frac{1}{3}} = a + ib$, where x, y, a, b ∈ R, then $\frac{x}{a} - \frac{y}{b} =$ **NCERT Page-102/N-81**

(a) $a^2 - b^2$ (b) $-2(a^2 + b^2)$
(c) $2(a^2 - b^2)$ (d) $a^2 + b^2$

18. If z = 2 –3i, then value of $z^2 - 4z + 13$ is **NCERT Page-100/N-80**

(a) 0 (b) 1 (c) 2 (d) 3

19. If $z = i^{-39}$, then simplest form of z is equal to $a + i$. The value of 'a' is **NCERT Page-103/N-79**

(a) 0 (b) 1 (c) 2 (d) 3

20. If $z_1 = 2 + 3i$ and $z_2 = 3 + 2i$, then $z_1 + z_2$ equals to a + ai. Value of 'a' is equal to **NCERT Page-102/N-77**

(a) 3 (b) 4 (c) 5 (d) 2

21. If $z_1 = 2 + 3i$ and $z_2 = 3 - 2i$, then $z_1 - z_2$ equals to –1 + bi. The value of 'b' is **NCERT Page-102/N-79**

(a) 1 (b) 2 (c) 3 (d) 5

22. The value of $(1 + i)^4\left(1+\frac{1}{i}\right)^4$ is **NCERT Page-103/N-81**

(a) 12 (b) 2 (c) 8 (d) 16

23. Evaluate: $(1 + i)^6 + (1 - i)^3$. **NCERT Page-102/N-81**

(a) –2 – 10i (b) 2 – 10i
(c) –2 + 10i (d) 2 + 10i

24. The value of $\frac{i^{4n+1} - i^{4n-1}}{2}$. is **NCERT Page-100/N-79**

(a) i (b) 2 i (c) –i (d) –2i

25. Value of $i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3}$ is **NCERT Page-100/N-79**

(a) 0 (b) 1 (c) 2 (d) 3

26. The value of $(1 + i)^5 \times (1 - i)^5$ is **NCERT Page-98/N-81**

(a) –8 (b) 8i (c) 8 (d) 32

4.4 The Modulus and the Conjugate of a Complex Number

27. Let z_1 and z_2 be two complex numbers such that $\bar{z}_1 = i\bar{z}_2$ and $\arg\left(\frac{z_1}{\bar{z}_2}\right) = \pi$. Then **NCERT Page-107/N-81**

(a) $\arg z_2 = \frac{\pi}{4}$ (b) $\arg z_2 = -\frac{3\pi}{4}$
(c) $\arg z_1 = \frac{\pi}{4}$ (d) $\arg z_1 = -\frac{3\pi}{4}$

28. If $z = 2 + 3i$, then $z^5 + (\bar{z})^5$ is equal to :

NCERT Page-103/N-81

(a) 244 (b) 224 (c) 245 (d) 265

29. Let $S = \{z = x + iy : |z-1+i| \geq |z|, |z| < 2, |z+i| = |z-1|\}$. Then the set of all values of x, for which $w = 2x + iy \in S$ for some $y \in \mathbb{R}$, is

NCERT Page-102/N-82

(a) $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$ (b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

(c) $\left(-\sqrt{2}, \frac{1}{2}\right]$ (d) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

30. The value of $(z+3)(\bar{z}+3)$ is equivalent to

NCERT Page-102/N-82

(a) $|z + 3|^2$ (b) $|z - 3|$

(c) $z^2 + 3$ (d) None of these

31. What is the conjugate of $\dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$?

NCERT Page-103/N-81

(a) –3i (b) 3i (c) $\frac{3}{2}i$ (d) $-\frac{3}{2}i$

32. If $(x + iy)(2 - 3i) = 4 + i$, then

NCERT Page-102/N-82

(a) $x = -14/13, y = 5/13$ (b) $x = 5/13, y = 14/13$

(c) $x = 14/13, y = 5/13$ (d) $x = 5/13, y = -14/13$

33. $i^{57} + \dfrac{1}{i^{25}}$, when simplified has the value

NCERT Page-103/N-82

(a) 0 (b) $2i$ (c) $-2i$ (d) 2

34. The modulus of $\dfrac{(1+i\sqrt{3})(2+2i)}{(\sqrt{3}-i)}$ is

NCERT Page-104/N-81

(a) 2 (b) 4 (c) $3\sqrt{2}$ (d) $2\sqrt{2}$

35. $\left(\dfrac{1}{1-2i}+\dfrac{3}{1+i}\right)\left(\dfrac{3+4i}{2-4i}\right)$ is equal to :

NCERT Page-104/N-81

(a) $\frac{1}{2}+\frac{9}{2}i$ (b) $\frac{1}{2}-\frac{9}{2}i$ (c) $\frac{1}{4}-\frac{9}{4}i$ (d) $\frac{1}{4}+\frac{9}{4}i$

36. The real part of $\dfrac{(1+i)^2}{(3-i)}$ is

NCERT Page-103/N-76

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$

(c) $-\frac{1}{3}$ (d) None of these

37. If $z_1 = 6 + 3i$ and $z_2 = 2 - i$, then $\dfrac{z_1}{z_2}$ is equal to $\dfrac{1}{a}(9 + 12i)$.

The value of 'a' is

NCERT Page-104/N-82

(a) 1 (b) 2 (c) 4 (d) 5

38. If $z = \dfrac{7-i}{3-4i}$, then $|z|^{14} =$

NCERT Page-105/N-81

(a) 2^7 (b) $2^7 i$ (c) -2^7 (d) $-2^7 i$

39. If $z_1 = 6 + 3i$ and $z_2 = 2 - i$, then $\dfrac{z_1}{z_2}$ is equal to

NCERT Page-99/N-81

(a) $\frac{1}{5}(9+12i)$ (b) $9 + 12i$

(c) $3 + 2i$ (d) $\frac{1}{5}(12+9i)$

40. If $(1 - i)^n = 2^n$, then the value of n is

NCERT Page-103/N-81

(a) 1 (b) 2

(c) 0 (d) None of these

41. If $\dfrac{c+i}{c-i} = a + ib$, where a, b, c are real, then $a^2 + b^2$ is equal to:

NCERT Page-102/N-81

(a) 7 (b) 1

(c) c^2 (d) $-c^2$

42. If $|z - 4| < |z - 2|$, its solution is given by

NCERT Page-102

(a) $Re(z) > 0$ (b) $Re(z) < 0$

(c) $Re(z) > 3$ (d) $Re(z) > 2$

43. Modulus of $z = \dfrac{(1+i\sqrt{3})(\cos\theta + i\sin\theta)}{2(1-i)(\cos\theta - i\sin\theta)}$ is

NCERT Page-102/N-81

(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

44. If $z_1 = 2 - i$ and $z_2 = 1 + i$, then value of $\left|\dfrac{z_1+z_2+1}{z_1-z_2+1}\right|$ is

NCERT Page-104/N-81

(a) 2 (b) 2i (c) $\sqrt{2}$ (d) $\sqrt{2}i$

45. If $\dfrac{(1+i)^3}{(1-i)^3} - \dfrac{(1-i)^3}{(1+i)^3} = x + iy$

NCERT Page-102/N-81

(a) $x = 0, y = -2$ (b) $x = -2, y = 0$

(c) $x = 1, y = 1$ (d) $x = -1, y = 1$

46. Additive inverse of $1 - i$ is

NCERT Page-99/N-77

(a) $0 + 0i$ (b) $-1 - i$

(c) $-1 + i$ (d) None of these

47. If z is a complex number such that $z^2 = (\bar{z})^2$, then NCERT Page-102/N-81
(a) z is purely real
(b) z is purely imaginary
(c) either z is purely real or purely imaginary
(d) None of these

48. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, then $(x^2 + y^2)^2 =$ NCERT Page-105/N-82
(a) $\frac{a^2+b^2}{c^2+d^2}$ (b) $\frac{a+b}{c+d}$
(c) $\frac{c^2+d^2}{a^2+b^2}$ (d) $\left(\frac{a^2+b^2}{c^2+d^2}\right)^2$

49. The conjugate of the complex number $\frac{2+5i}{4-3i}$ is equal to : NCERT Page-103/N-81
(a) $\frac{7-26i}{25}$ (b) $\frac{-7-26i}{25}$
(c) $\frac{-7+26i}{25}$ (d) $\frac{7+26i}{25}$

50. If z = 2 + i, then $(z-1)(\bar{z}-5)+(\bar{z}-1)(z-5)$ is equal to NCERT Page-104/N-81
(a) 2 (b) 7 (c) –1 (d) –4

51. If |z| = 1, (z ≠ –1) and z = x + iy, then $\left(\frac{z-1}{z+1}\right)$ is NCERT Page-103/N-81
(a) purely real (b) purely imaginary
(c) zero (d) undefined

52. If $\bar{z}$ be the conjugate of the complex number z, then which of the following relations is false? NCERT Page-103/N-81
(a) $|z| = |\bar{z}|$ (b) $z \cdot \bar{z} = |\bar{z}|^2$
(c) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ (d) arg z = arg $\bar{z}$

53. (x – iy) (3 + 5i) is the conjugate of (–6 – 24i), then x and y are NCERT Page-103/N-82
(a) x = 3, y = –3 (b) x = –3, y = 3
(c) x = –3, y = –3 (d) x = 3, y = 3

54. If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then NCERT Page-104/N-81
(a) |z| = 0 (b) |z| = 1 (c) |z| > 1 (d) |z| < 1

55. If $x + iy = \frac{a+ib}{a-ib}$, then $x^2 + y^2 =$ NCERT Page-104/N-82
(a) 1 (b) 2 (c) 0 (d) 4

4.5 Argand Plane and Polar Representation

56. The amplitude of $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right)$ is NCERT Page-106/N-84
(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{\pi}{15}$

57. If $Z = \frac{1-i}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$, then polar form of Z is NCERT Page-106/N-83
(a) $\sqrt{2}\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$ (b) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$
(c) $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ (d) $\sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$

58. The modulus of the complex number z such that |z + 3 – i| = 1 and arg(z) = π is equal to NCERT Page-106/N-83
(a) 3 (b) 2 (c) 9 (d) 4

59. arg $\bar{z}$ + arg z; z ≠ 0 is equal to : NCERT Page-106/N-83
(a) $\frac{\pi}{4}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

60. If $z_1 = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$ and $z_2 = \sqrt{3}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$, then $|z_1 z_2|$ is equal to $\sqrt{m}$. Value of m is NCERT Page-108/N-84
(a) 6 (b) 3 (c) 2 (d) 5

61. The plane having a complex number assigned to each of its point is called .... NCERT Page-104/N-84
(a) Complex plane (b) Argand plane
(c) Cartesian plane (d) Both (a) & (b)

62. The y-axis in Argand plane is known as ... NCERT Page-104/N-84
(a) Argand axis (b) Imaginary axis
(c) Real axis (d) Cartesian axis

63. The polar form of the complex number $(i^{25})^3$ is NCERT Page-105/N-83
(a) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (b) $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$
(c) $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ (d) $\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$

64. If $z_1 = \sqrt{3} + i\sqrt{3}$ and $z_2 = \sqrt{3} + i$, then in which quadrant $\left(\frac{z_1}{z_2}\right)$ lies? NCERT Page-105/N-83
(a) I (b) II (c) III (d) IV

65. The modulus and amplitude of $\dfrac{1+2i}{1-(1-i)^2}$ are

NCERT Page-106/N-83

(a) $\sqrt{2}$ and $\dfrac{\pi}{6}$ (b) 1 and 0

(c) 1 and $\dfrac{\pi}{3}$ (d) 1 and $\dfrac{\pi}{4}$

66. Let z be any complex number such that | z | = 4 and arg (z) = $\dfrac{5\pi}{6}$, then value of z is NCERT Page-106/N-84

(a) $-2\sqrt{3}-2i$ (b) $2\sqrt{3}-i$

(c) $\sqrt{2}+3i$ (d) $-2\sqrt{3}+2i$

67. If $z=\dfrac{3-i}{2+i}+\dfrac{3+i}{2-i}$, then value of arg (zi) is

NCERT Page-107/N-84

(a) 0 (b) $\dfrac{\pi}{6}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

68. The complex number $\left(\dfrac{1+2i}{1-i}\right)$ lies in:

NCERT Page-107/N-83

(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

69. Amplitude of $\dfrac{1+\sqrt{3}i}{\sqrt{3}+1}$ is : NCERT Page-108/N-83

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

70. Represent z = $1+i\sqrt{3}$ in the polar form.

NCERT Page-107/N-84

(a) $\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}$ (b) $\cos\dfrac{\pi}{3}-i\sin\dfrac{\pi}{3}$

(c) $2\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)$ (d) $4\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)$

71. The argument of the complex number $\left(\dfrac{i}{2}-\dfrac{2}{i}\right)$ is equal to NCERT Page-106/N-83

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{3\pi}{4}$ (c) $\dfrac{\pi}{12}$ (d) $\dfrac{\pi}{2}$

4.6 Quadratic Equation

72. Let α, β be the roots of the equation $x^2-\sqrt{2}x+\sqrt{6}=0$ and $\dfrac{1}{\alpha^2}+1, \dfrac{1}{\beta^2}+1$ be the roots of the equation $x^2+ax+b=0$. Then the roots of the equation $x^2-(a+b-2)x+(a+b+2)=0$ are: NCERT Page-109

(a) non-real complex numbers
(b) real and both negative
(c) real and both positive
(d) real and exactly one of them is positive

73. If α, β are the roots of the equation

$$x^2-\left(5+3^{\sqrt{\log_3 5}}-5^{\sqrt{\log_5 3}}\right)+3\left(3^{(\log_3 5)^{\frac{1}{3}}}-5^{(\log 3)^{\frac{2}{3}}}-1\right)=0$$

then the equation, whose roots are $\alpha+\dfrac{1}{\beta}$ and $\beta+\dfrac{1}{\alpha}$,

NCERT Page-109

(a) $3x^2-20x-12=0$ (b) $3x^2-10x-4=0$
(c) $3x^2-10x+2=0$ (d) $3x^2-20x+16=0$

74. Let α and β be the roots of the equation $x^2+(2i-1)=0$. Then, the value of $|\alpha^8+\beta^8|$ is equal to:

NCERT Page-108

(a) 50 (b) 250 (c) 1250 (d) 1500

75. A polynomial equation of degree n has ——— roots.

NCERT Page-108

(a) n! (b) n + 1 (c) n (d) n – 1

76. For getting complex roots of quadratic equation, D = b^2 – 4ac should be ——— NCERT Page-108

(a) D > 0 (b) D < 0 (c) D = 0 (d) D ≠ 0

77. The solutions of the quadratic equation $ax^2+bx+c=0$, where $a, b, c \in R$, $a \neq 0$, $b^2-4ac<0$, are given by x = ?

NCERT Page-108

(a) $\dfrac{b\pm\sqrt{4ac-b^2}i}{2a}$ (b) $\dfrac{-b\pm\sqrt{4ac+b^2}i}{2a}$

(c) $\dfrac{-b\pm\sqrt{4ac-b^2}i}{2a}$ (d) $\dfrac{-b\pm\sqrt{4ab-c^2}i}{2a}$

78. If the ratio of the roots of $x^2+bx+c=0$ and $x^2+qx+r=0$ be the same, then NCERT Page-109

(a) $r^2c=b^2q$ (b) $r^2b=c^2q$
(c) $rb^2=cq^2$ (d) $rc^2=bq^2$

79. Sum of all real roots of the equation $|x-2|^2+|x-2|-2=0$ is NCERT Page-110

(a) 2 (b) 4 (c) 5 (d) 6

80. If the roots of $4x^2+5k=(5k+1)x$ differ by unity, then the negative value of k is NCERT Page-109

(a) –3 (b) –5 (c) $-\dfrac{1}{5}$ (d) $-\dfrac{3}{5}$

81. If the roots of the equation $x^2-5x+16=0$ are α, β and the roots of equation $x^2+px+q=0$ are $\alpha^2+\beta^2$, $\dfrac{\alpha\beta}{2}$, then NCERT Page-110

(a) p = 1, q = –56 (b) p = –1, q = –56
(c) p = 1, q = 56 (d) p = –1, q = 56

82. If the sum of the roots of the equation $x^2+px+q=0$ is three times their difference, then which one of the following is true? NCERT Page-110

(a) $9p^2=2q$ (b) $2q^2=9p$
(c) $2p^2=9q$ (d) $9q^2=2p$

83. If α, β be the roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha - 35)^3 \cdot (2\beta - 35)^3$ is equal to **NCERT Page-110**

(a) 1 (b) 64
(c) 8 (d) None of these

84. If the roots of the equation $ax^2 + bx + c = 0$ are α, β, then the value of $\alpha\beta^2 + \alpha^2\beta + \alpha\beta$ will be **NCERT Page-110**

(a) $\frac{c(a-b)}{a^2}$ (b) 0
(c) $-\frac{bc}{a^2}$ (d) None of these

85. If α, β are the roots of $(x - a)(x - b) = c$, $c \neq 0$, then the roots of $(x - \alpha)(x - \beta) + c = 0$ shall be **NCERT Page-109**

(a) a, c (b) b, c
(c) a, b (d) a + c, b + c

86. If one root of $ax^2 + bx + c = 0$ be square of the other, then the value of $b^3 + ac^2 + a^2c$ is **NCERT Page-108**

(a) 3abc (b) –3abc
(c) 0 (d) None of these

87. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} =$ **NCERT Page-109**

(a) $\frac{2}{a}$ (b) $\frac{2}{b}$ (c) $\frac{2}{c}$ (d) $-\frac{2}{a}$

88. If $x^2 + x + 1 = 0$, then what is the value of x ? **NCERT Page-109**

(a) $\frac{1+\sqrt{3}i}{2}$ (b) $\frac{-1\pm\sqrt{3}i}{2}$
(c) $\frac{-1\pm\sqrt{3}i}{3}$ (d) $\frac{-1\pm\sqrt{2}i}{2}$

89. The solution of $\sqrt{3x^2 - 2} = 2x - 1$ are : **NCERT Page-109**

(a) (2, 4) (b) (1, 4) (c) (3, 4) (d) (1, 3)

90. If α, β are roots of the equation $x^2 - 5x + 6 = 0$, then the equation whose roots are $\alpha + 3$ and $\beta + 3$ is **NCERT Page-109**

(a) $2x^2 - 11x + 30 = 0$ (b) $-x^2 + 11x = 0$
(c) $x^2 - 11x + 30 = 0$ (d) $2x^2 - 5x + 30 = 0$

91. For the equation $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$, if the product of roots is zero, then sum of roots is **NCERT Page-109**

(a) $-\frac{2bc}{b+c}$ (b) $\frac{2ca}{c+a}$ (c) $\frac{bc}{c+a}$ (d) $\frac{-bc}{b+c}$

92. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$ **NCERT Page-109**

(a) is always positive (b) is always negative
(c) does not exist (d) None of these

93. $2x^2 - (p + 1)x + (p - 1) = 0$. If $\alpha - \beta = \alpha\beta$, then what is the value of p? **NCERT Page-109**

(a) 1 (b) 2 (c) 3 (d) –2

94. If p and q are the roots of the equation $x^2 + px + q = 0$, then **NCERT Page-108**

(a) $p = 1, q = -2$ (b) $p = 0, q = 1$
(c) $p = -2, q = 0$ (d) $p = -2, q = 1$

95. The roots of the given equation $(p - q)x^2 + (q - r)x + (r - p) = 0$ are : **NCERT Page-108**

(a) $\frac{p-q}{r-p}, 1$ (b) $\frac{q-r}{p-q}, 1$
(c) $\frac{r-p}{p-q}, 1$ (d) None of these

96. If α, β are the roots of $ax^2 + bx + c = 0$, then $\alpha\beta^2 + \alpha^2\beta + \alpha\beta$ equals **NCERT Page-108**

(a) $\frac{c(a-b)}{a^2}$ (b) 0
(c) $\frac{-bc}{a^2}$ (d) abc

97. The roots of equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ is **NCERT Page-108**

(a) one (b) two
(c) infinite (d) None of these

98. Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$. **NCERT Page-108**

(a) $\pm\frac{\sqrt{19}}{5}i$ (b) $\pm\frac{\sqrt{19}i}{2}$
(c) $\frac{-1\pm\sqrt{19}i}{2\sqrt{5}}$ (d) $\frac{-1\pm\sqrt{19}i}{\sqrt{5}}$

99. If α and β are the roots of the equation $x^2 + 2x + 4 = 0$, then $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ is equal to **NCERT Page-108**

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 32 (d) $\frac{1}{4}$

100. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$ equals **NCERT Page-108**

(a) $\frac{ac}{b}$ (b) 1 (c) $\frac{ab}{c}$ (d) $\frac{b}{ac}$

101. Roots of $x^2 + 2 = 0$ are $\pm\sqrt{n}\,i$. The value of n is **NCERT Page-108**

(a) 1 (b) 2 (c) 3 (d) 4

102. A value of k for which the quadratic equation $x^2 - 2x(1 + 3k) + 7(2k + 3) = 0$ has equal roots is NCERT Page-108

(a) 1 (b) 2 (c) 3 (d) 4

103. The roots of the equation $3^{2x} - 10.3^x + 9 = 0$ are NCERT Page-108

(a) 1, 2 (b) 0, 2 (c) 0, 1 (d) 1, 3

104. The equation whose roots are twice the roots of the equation, $x^2 - 3x + 3 = 0$ is: NCERT Page-108

(a) $4x^2 + 6x + 3 = 0$ (b) $2x^2 - 3x + 3 = 0$
(c) $x^2 - 3x + 6 = 0$ (d) $x^2 - 6x + 12 = 0$

105. The roots of the equation $4^x - 3 . 2^{x+3} + 128 = 0$ are NCERT Page-108

(a) 4 and 5 (b) 3 and 4 (c) 2 and 3 (d) 1 and 2

106. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots , then the value of 'q' is NCERT Page-108

(a) 4 (b) 12 (c) 3 (d) $\frac{49}{4}$

107. For the equation $3x^2 + px + 3 = 0$, $p > 0$, if one of the root is square of the other, then p is equal to NCERT Page-108

(a) 1/3 (b) 1 (c) 3 (d) 2/3

108. The number of real roots of $\left(x+\frac{1}{x}\right)^3 + \left(x+\frac{1}{x}\right) = 0$ is NCERT Page-108

(a) 0 (b) 2 (c) 4 (d) 6

109. If the roots of the equation $\frac{a}{x-a} + \frac{b}{x-b} = 1$ are equal in magnitude and opposite in sign, then NCERT Page-109

(a) a = b (b) a + b = 1
(c) a – b = 1 (d) a + b = 0

110. Find the value of a such that the sum of the squares of the roots of the equation $x^2 - (a - 2)x - (a + 1) = 0$ is least. NCERT Page-109

(a) 4 (b) 2 (c) 1 (d) 3

111. If α, β are the roots of the equation (x – a) (x – b) = 5, then the roots of the equation (x – α) (x – β) + 5 = 0 are NCERT Page-108

(a) a, 5 (b) b, 5 (c) a, α (d) a, b

112. If the roots of the equations $px^2 + 2qx + r = 0$ and $qx^2 - 2\left(\sqrt{pr}\right)x + q = 0$ be real, then NCERT Page-109

(a) p = q (b) $q^2 = pr$
(c) $p^2 = qr$ (d) $r^2 = pq$

113. If a > 0, b > 0, c > 0, then both the roots of the equation $ax^2 + bx + c = 0$. NCERT Page-108

(a) Are real and negative (b) Have negative real parts
(c) Are rational numbers (d) None of these

114. If a and b are the odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0$, $a \neq 0$, will be NCERT Page-108

(a) rational (b) irrational
(c) non-real (d) equal

115. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then (p, q) = NCERT Page-109

(a) (–4, 7) (b) (4, –7) (c) (4, 7) (d) (–4, –7)

116. If the sum of the roots of the equation $x^2 + px + q = 0$ is equal to the sum of their squares, then NCERT Page-108

(a) $p^2 - q^2 = 0$ (b) $p^2 + q^2 = 2q$
(c) $p^2 + p = 2q$ (d) None of these

117. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then NCERT Page-108

(a) a < 2 (b) 2 ≤ a ≤ 3
(c) 3 < a ≤ 4 (d) a > 4

118. If the equation $(m - n)x^2 + (n - l)x + l - m = 0$ has equal roots, then l, m and n satisfy NCERT Page-109

(a) $2l = m + n$ (b) $2m = n + l$
(c) $m = n + l$ (d) $l = m + n$

119. If the product of the roots of the equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ be 2, then the sum of roots is NCERT Page-109

(a) 1 (b) –1 (c) 2 (d) –2

Exercise 2 : NCERT Exemplar & Past Years JEE Main

NCERT Exemplar Questions

1. sin x + i cos 2x and cos x – i sin 2x are conjugate to each other for NCERT Page-107/N-81

(a) x = nπ (b) $x = \left(n+\frac{1}{2}\right)\frac{\pi}{2}$
(c) x = 0 (d) No value of x

2. The real value of α for which the expression $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely real is NCERT Page-107/N-76

(a) $(n+1)\frac{\pi}{2}$ (b) $(2n+1)\frac{\pi}{2}$
(c) $n\pi$ (d) None of these

3. If $z = x + iy$ lies in the third quadrant, then $\frac{\bar{z}}{z}$ also lies in the third quadrant, if **NCERT Page-98/N-81**
(a) $x > y > 0$ (b) $x < y < 0$
(c) $y < x < 0$ (d) $y > x > 0$

4. The value of $(z + 3)(\bar{z} + 3)$ is equivalent to **NCERT Page-99/N-82**
(a) $|z+3|^2$ (b) $|z-3|$
(c) $z^2 + 3$ (d) None of these

5. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then **NCERT Page-111/N-81**
(a) $x = 2n + 1$ (b) $x = 4n$
(c) $x = 2n$ (d) $x = 4n + 1$
where, $n \in N$

6. A real value of x satisfies the equation $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta$ $(\alpha, \beta \in R)$, if $\alpha^2 + \beta^2$ is equal to **NCERT Page-112**
(a) 1 (b) -1 (c) 2 (d) -2

7. Which of the following is correct for any two complex numbers z_1 and z_2? **NCERT Page-103/N-81**
(a) $|z_1 z_2| = |z_1||z_2|$
(b) $\arg(z_1 z_2) = \arg(z_1) \cdot \arg(z_2)$
(c) $|z_1 + z_2| = |z_1| + |z_2|$
(d) None

8. If $x, y \in R$, then $x + iy$ is a non-real complex number, if **NCERT Page-99**
(a) $x = 0$ (b) $y = 0$ (c) $x \neq 0$ (d) $y \neq 0$

9. If $a + ib = c + id$, then **NCERT Page-99/N-81**
(a) $a^2 + c^2 = 0$ (b) $b^2 + c^2 = 0$
(c) $b^2 + d^2 = 0$ (d) $a^2 + b^2 = c^2 + d^2$

10. The complex number z which satisfies the condition $\left|\frac{i+z}{i-z}\right| = 1$ lies on **NCERT Page-111**
(a) circle $x^2 + y^2 = 1$ (b) the x-axis
(c) the y-axis (d) the line $x + y = 1$

11. If z is a complex number, then **NCERT Page-112**
(a) $|z^2| > |z|$ (b) $|z^2| = |z|^2$
(c) $|z^2| < |z|^2$ (d) $|z^2| \geq |z|^2$

12. $|z_1 + z_2| = |z_1| + |z_2|$ is possible, if **NCERT Page-111**
(a) $z_2 = \bar{z}_1$ (b) $z_2 = \frac{1}{z_1}$
(c) $\arg(z_1) = \arg(z_2)$ (d) $|z_1| = |z_2|$

13. The real value of θ for which the expression $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number is **NCERT Page-111/N-76**
(a) $n\pi + \frac{\pi}{4}$ (b) $n\pi + (-1)^n \frac{\pi}{4}$
(c) $2n\pi \pm \frac{\pi}{2}$ (d) None of these

14. The value of arg (x), when $x < 0$ is **NCERT Page-105/N-81**
(a) 0 (b) $\frac{\pi}{2}$
(c) π (d) None of these

15. If $f(z) = \frac{7-z}{1-z^2}$, where $z = 1 + 2i$, then $|f(z)|$ is equal to **NCERT Page-102/N-82**
(a) $\frac{|z|}{2}$ (b) $|z|$
(c) $2|z|$ (d) None of these

Past Years JEE Main

16. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is : **NCERT Page-107 | 2016, C**
(a) 6 (b) 5 (c) 3 (d) -4

17. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is: **NCERT Page-108/N-76 | 2016, C**
(a) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

18. If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + ... + (x+\overline{n-1})(x+n) = 10n$ has two consecutive integral solutions, then n is equal to : **NCERT Page-111 | 2017, S**
(a) 11 (b) 12 (c) 9 (d) 107

19. The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$, is **NCERT Page-111/N-81, 2018, A**
(a) 2 (b) 6 (c) 5 (d) 3

20. All the points in the set $S = \left\{\frac{\alpha+i}{\alpha-i} : \alpha \in R\right\} (i = \sqrt{-1})$ lie on a: **NCERT Page-111 | 2019, A**
(a) straight line whose slope is 1.
(b) circle whose radius is 1.
(c) circle whose radius is $\sqrt{2}$.
(d) straight line whose slope is –1.

21. Let $p, q \in R$. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then: **NCERT Page-110 | 2019, A**
(a) $p^2 - 4q + 12 = 0$ (b) $q^2 - 4p - 16 = 0$
(c) $q^2 + 4p + 14 = 0$ (d) $p^2 - 4q - 12 = 0$

22. Let α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\,.\,\lambda\tan x = (1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha+\beta) = 50$, then a value of λ is:
NCERT Page-112 | 2020, S

(a) $10\sqrt{2}$ (b) 10 (c) 5 (d) $5\sqrt{2}$

23. If $\text{Re}\left(\dfrac{z-1}{2z+i}\right) = 1$, where $z = x+iy$, then the point (x, y) lies on *a*:
NCERT Page-112 | 2020, A

(a) circle whose centre is at $\left(-\dfrac{1}{2}, -\dfrac{3}{2}\right)$.

(b) straight line whose slope is $-\dfrac{2}{3}$.

(c) straight line whose slope is $\dfrac{3}{2}$.

(d) circle whose diameter is $\dfrac{\sqrt{5}}{2}$.

24. If $S = \left\{z \in \mathbb{C} : \dfrac{z-i}{z+2i} \in \mathbb{R}\right\}$, then :
NCERT Page-112 | 2021, C

(a) S contains exactly two elements
(b) S contains only one element
(c) S is a circle in the complex plane
(d) S is a straight line in the complex plane

25. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is ________.
NCERT Page-111 | 2021, C

(a) 1 (b) 2 (c) 3 (d) 4

26. The real part of the complex number $\dfrac{(1+2i)^8 \cdot (1-2i)^2}{(3+2i)\cdot(4-6i)}$ is equal to:
NCERT Page-103/N-76| 2022, C

(a) $\dfrac{500}{13}$ (b) $\dfrac{110}{13}$ (c) $\dfrac{55}{6}$ (d) $\dfrac{550}{13}$

27. If for some p, q, r $\in$ **R**, not all have same sign, one of the roots of the equation $(p^2+q^2)x^2 - 2q(p+r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\dfrac{q^2+r^2}{p^2}$ is equal to ______ .
NCERT Page-111 | 2022, C

28. The complex number $z = \dfrac{i-1}{\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}}$ is equal to:
NCERT Page-110/N-81 | 2023, A

(a) $\sqrt{2}\left(\cos\dfrac{5\pi}{12} + i\sin\dfrac{5\pi}{12}\right)$ (b) $\cos\dfrac{\pi}{12} - i\sin\dfrac{\pi}{12}$

(c) $\sqrt{2}\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12}\right)$ (d) $\sqrt{2}\,i\left(\cos\dfrac{5\pi}{12} - i\sin\dfrac{5\pi}{12}\right)$

29. Let $S = \left\{z \in C : \bar{z} = i\left(z^2 + \text{Re}(\bar{z})\right)\right\}$. Then $\sum_{z\in S} |z|^2$ is equal to
NCERT Page-112 | 2023, A

(a) $\dfrac{7}{2}$ (b) 4 (c) $\dfrac{5}{2}$ (d) 3

30. If $S = \{z \in C : |z-i| = |z+i| = |z-1|\}$, then, n(S) is:
NCERT Page-112 | 2024, S

(a) 1 (b) 0 (c) 3 (d) 2

31. If α satisfies the equation $x^2 + x + 1 = 0$ and $(1+\alpha)^7 = A + B\alpha + C\alpha^2$, A, B, C ≥ 0, then $5(3A - 2B - C)$ is equal to _____.
NCERT Page-112 | 2024, C

Exercise 3 : Skill Enhancer MCQs

1. Number of positive integral values of 'x', for which $||x^2 - 2x| - |3x - 20|| = |x^2 + x - 20|$, is
(a) 5 (b) 6 (c) 7 (d) infinite

2. If the complex number is $(1 + ri)^3 = \lambda\,(1 + i)$, when $i = \sqrt{-1}$, for some real λ, the value of r can be

(a) $\cos\dfrac{\pi}{5}$ (b) $\text{cosec}\,\dfrac{3\pi}{2}$

(c) $\cos\dfrac{\pi}{12}$ (d) $\text{cosec}\,\dfrac{\pi}{12}$

3. Which of the following represent the subset of set of complex number z satisfying $\log_{1/3}(\log_{1/2}(|z|^2 + 4|z| + 3)) > 0$,
(a) $[-1,3]$ (b) $\{z : \text{Re}(z) \geq 1\}$
(c) $\{z : i\,(z) \leq 2\}$ (d) None of these

4. The equation $2\cos^2\dfrac{x}{2}\sin^2 x = x^2 + \dfrac{1}{x^2}$, $0 \leq x \leq \dfrac{\pi}{2}$ has
(a) one real solution
(b) no real solution
(c) more than one real solution
(d) None of these

5. If *a*, *b*, *c* are positive rational numbers such that $a > b > c$ and the quadratic equation $(a+b-2c)x^2 + (b+c-2a)x + (c+a-2b) = 0$ has a root in the interval $(-1, 0)$, then which of the following is not correct?
(a) $c + a < 2b$
(b) Both roots of the given equation are rational
(c) The equation $ax^2 + 2bx + c = 0$ has both negative real roots
(d) None of these

6. Which of the following is/are value of

$\sin \ln (i^i)^{i^2} + \cos \ln (i^i)^{i^2}$?

(a) -1 (b) 1
(c) 0 (d) None of these

7. Let z and w be two complex numbers such that

$|z| \le 1, |w| \le 1, |z - iw| = |z - i\bar{w}| = 2$, then how many complex numbers z can satisfy

(a) 2 (b) 3
(c) 4 (d) None of these

8. If $|z| = \max \{|z-1|, |z+1|\}$ then

(a) $|z+\bar{z}| = \frac{1}{2}$ (b) $z + \bar{z} = 1$

(c) $|z+\bar{z}| = 1$ (d) None of these

9. Given that the two curves $\arg(z) = \frac{\pi}{6}$ and $|z - 2\sqrt{3}i| = r$ intersect in two distinct points then ([r] represents integral part of r) which of the following is not correct?

(a) $r > 3$ (b) $r = 6$ (c) $0 < r < 3$ (d) $[r] \ne 2$

10. Consider the quadratic equation $(a + c - b)x^2 + 2cx + (b + c - a) = 0$, where a, b, c are distinct real numbers and $a + c - b \ne 0$. Suppose that both the roots of the equation are rational, then

(a) a, b and c are rational (b) $\frac{c}{(a-b)}$ is rational

(c) $\frac{b}{(c-a)}$ is rational (d) None of these

11. z_1, z_2, z_3, z_4 correspond to the points A, B, C, D lie on a circle. If $z_1 + z_2 + z_3 + z_4 = 0$, then how many of the following statements are correct ?

(i) ABCD is necessarily rectangle
(ii) ABCD is necessarily a rhombus
(iii) Angle ABC must be right angle.

(a) 0 (b) 1
(c) 2 (d) None of these

12. If $z = \frac{\pi}{4}(1+i)^4\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right)$, where $i = \sqrt{-1}$, then

$\left(\frac{|z|}{\text{amp}(z)}\right)$ equals to

(a) 1 (b) 2 (c) 3 (d) 4

13. Let $r_k > 0$ and $z_k = r_k(\cos \alpha_k + i \sin \alpha_k)$ for $k = 1, 2, 3$ be such that $\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$. Let A_k be the point in the complex plane given by $w_k = \frac{\cos 2\alpha_k + i \sin 2\alpha_k}{z_k}$ for k = 1, 2, 3. The origin, O is the

(a) orthocentre of $\Delta A_1A_2A_3$
(b) circumcentre of $\Delta A_1A_2A_3$
(c) centroid of $\Delta A_1A_2A_3$
(d) None

Exercise 4 : Numeric Value Answer Questions

1. Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$). Then | z | is equal to ________.

2. If $\frac{z-\alpha}{z+\alpha}(\alpha \in R)$ is a purely imaginary number and |z| = 2, then a value of α is ________.

3. Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then the minimum value of $|z_1 - z_2|$ is ________.

4. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is ________.

5. All the points in the set $S = \left\{\frac{\alpha+i}{\alpha-i} : \alpha \in R\right\} (i = \sqrt{-1})$ lie on a circle, then it's radius is equal to ________.

6. If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest and the absolute difference of the cubes of its roots is equal to $a\sqrt{5}$, then a is ________.

7. If $|z_1| = 2, |z_2| = 3, |z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$, then $|8z_2z_3 + 27z_3z_1 + 64z_1z_2|$ is equal to ________.

8. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$ and $Arg(z) - Arg(\omega) = \frac{\pi}{2}$, and $\bar{z}\omega$ is equal to i^k, where k is smallest natural number then k is ________.

9. If α, β are the roots of the equations $x^2 - 2x - 1 = 0$, then what is the value of $\alpha^2 \beta^{-2} + \alpha^{-2} \beta^2$ ________.

10. If $z = x + iy$, $z^{1/3} = a - ib$, then $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$ where k is equal to ________.

11. If one root of the equations $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ ($a, b, c \in R$) is common, then the value of $\left(\frac{a^3 + b^3 + c^3}{abc}\right)^3$ is ________.

12. If $x^2 - hx - 21 = 0, x^2 - 3hx + 35 = 0 (h > 0)$ has a common root, then the value of h is equal to ________.

Answer Keys

Exercise - 1 : (NCERT Based Topic-wise MCQs)																			
1	(b)	**13**	(a)	**25**	(a)	**37**	(d)	**49**	(c)	**61**	(d)	**73**	(b)	**85**	(c)	**97**	(b)	**109**	(d)
2	(c)	**14**	(c)	**26**	(d)	**38**	(a)	**50**	(d)	**62**	(b)	**74**	(a)	**86**	(a)	**98**	(c)	**110**	(c)
3	(c)	**15**	(b)	**27**	(c)	**39**	(a)	**51**	(b)	**63**	(b)	**75**	(c)	**87**	(d)	**99**	(d)	**111**	(d)
4	(b)	**16**	(b)	**28**	(a)	**40**	(c)	**52**	(d)	**64**	(a)	**76**	(b)	**88**	(b)	**100**	(d)	**112**	(b)
5	(c)	**17**	(b)	**29**	(b)	**41**	(b)	**53**	(a)	**65**	(b)	**77**	(c)	**89**	(d)	**101**	(b)	**113**	(b)
6	(d)	**18**	(a)	**30**	(a)	**42**	(c)	**54**	(b)	**66**	(d)	**78**	(c)	**90**	(c)	**102**	(b)	**114**	(a)
7	(c)	**19**	(a)	**31**	(c)	**43**	(c)	**55**	(a)	**67**	(d)	**79**	(b)	**91**	(a)	**103**	(b)	**115**	(a)
8	(a)	**20**	(c)	**32**	(b)	**44**	(c)	**56**	(c)	**68**	(b)	**80**	(c)	**92**	(a)	**104**	(d)	**116**	(c)
9	(b)	**21**	(d)	**33**	(a)	**45**	(a)	**57**	(b)	**69**	(c)	**81**	(b)	**93**	(b)	**105**	(b)	**117**	(a)
10	(b)	**22**	(d)	**34**	(d)	**46**	(c)	**58**	(a)	**70**	(c)	**82**	(c)	**94**	(a)	**106**	(d)	**118**	(b)
11	(d)	**23**	(a)	**35**	(d)	**47**	(c)	**59**	(c)	**71**	(d)	**83**	(b)	**95**	(c)	**107**	(c)	**119**	(b)
12	(a)	**24**	(a)	**36**	(d)	**48**	(a)	**60**	(a)	**72**	(b)	**84**	(a)	**96**	(a)	**108**	(a)		
Exercise - 2 : (NCERT Exemplar & Past Years JEE Main)																			
1	(d)	**5**	(b)	**9**	(d)	**13**	(c)	**17**	(b)	**21**	(d)	**25**	(d)	**29**	(b)				
2	(c)	**6**	(a)	**10**	(b)	**14**	(c)	**18**	(a)	**22**	(b)	**26**	(d)	**30**	(a)				
3	(b)	**7**	(a)	**11**	(b)	**15**	(a)	**19**	(d)	**23**	(d)	**27**	(272)	**31**	(5)				
4	(a)	**8**	(d)	**12**	(c)	**16**	(c)	**20**	(b)	**24**	(d)	**28**	(a)						
Exercise - 3 : (Skill Enhancer MCQs)																			
1	(a)	**3**	(d)	**5**	(d)	**7**	(a)	**9**	(c)	**11**	(c)	**13**	(c)						
2	(b)	**4**	(b)	**6**	(b)	**8**	(d)	**10**	(b)	**12**	(d)								
Exercise - 4 : (Numeric Value Answer Questions)																			
1	(1.67)	**3**	(0)	**5**	(1)	**7**	(96)	**9**	(34)	**11**	(27)								
2	(2)	**4**	(4)	**6**	(8)	**8**	(2)	**10**	(4)	**12**	(4)								

4 Complex Numbers and Quadratic Equations

EXERCISE - 1

1. **(b)**
2. **(c)**
3. **(c)** We have, $4x + i(3x - y) = 3 + i(-6)$
   Now, equating the real and the imaginary parts of above equation, we get :
   $4x = 3$ and $3x - y = -6$
   $$\Rightarrow x = \frac{3}{4} \text{ and } 3 \times \frac{3}{4} - y = -6$$
   $$\text{or } \frac{9}{4} + 6 = y \Rightarrow \frac{9+24}{4} = y \therefore y = \frac{33}{4}$$
   hence, $x = \frac{3}{4}$ and $y = \frac{33}{4}$
4. **(b)**
5. **(c)** $z = 5i\left(\frac{-3}{5}i\right) = -3i^2 = -3(-1) = 3 \equiv 3 + 0i$
   Hence, b = 0
6. **(d)** We have $\dfrac{(1+2i)^8 \cdot (1-2i)^2}{(3+2i)\cdot(4-6i)}$
   $$\Rightarrow \frac{(1+2i)^2 (1-2i)^2 (1+2i)^6}{26i}$$
   $$\Rightarrow \frac{25(1+2i)^6}{26i} \Rightarrow (-i)\frac{25}{26}(7+24i)(3-4i)$$
   $$\Rightarrow \frac{(72-28)25}{26} = \frac{44 \times 25}{26} = \frac{550}{13}$$
7. **(c)** Given interval is $\pi < \alpha, \beta < 2\pi$ and expression
   $$\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$$
   Now, $\dfrac{(1 - i\sin\alpha)(1 - 2i\sin\alpha)}{1 + 4\sin^2\alpha}$ = Purely imaginary
   $$\Rightarrow \frac{1 - 2\sin^2\alpha}{1 + 4\sin^2\alpha} = 0 \Rightarrow \sin^2\alpha = \frac{1}{2} \Rightarrow \alpha = \left[\frac{5\pi}{4}, \frac{7\pi}{4}\right]$$
   Take, $\dfrac{1 - i\cos\beta}{1 + 2i\sin\beta)}$ = Purely real
   $$\frac{(1 - i\cos\beta)(1 - 2i\cos\beta)}{1 + 4\cos^2\beta} = \text{Purely real}$$
   $$\Rightarrow 3\cos\beta = 0 \text{ so, } \beta = \frac{2\pi}{2}$$
   Take, $Z_{\alpha\beta} = \sin\frac{5\pi}{2} + i\cos 5\pi = 1 - i$
   or $Z_{\alpha\beta} = \sin\frac{7\pi}{2} + i\cos 3\pi = -1 - i$
   Required value
   $$= \left[i(1-i) + \frac{1}{i(1+i)}\right] + \left[i(-1-i) + \frac{1}{i(-1+i)}\right]$$
   $$= i(-2i) + \frac{1}{1}\frac{2i}{(-2)} \Rightarrow 2 - 1 = 1$$
8. **(a)** $z = i^9 + i^{19} = (i^4)^2 . i + (i^4)^4 . i^3$
   $= i + i^3 = i - i = 0 \equiv 0 + 0\,i$
   Hence, a = 0
9. **(b)** $\left(\dfrac{2i}{1+i}\right)^2 = \dfrac{4i}{1+i^2+2i} = \dfrac{-4}{1-1+2i} = \dfrac{-4}{2i} = \dfrac{-2}{i} = 2i$
10. **(b)** $\dfrac{1-i}{1+i} = \dfrac{(1-i)(1-i)}{(1+i)(1-i)} = \dfrac{(1-i)^2}{1-i^2} = \dfrac{1+i^2-2i}{2} = -i$
    $$\therefore (-i)^{100} = (i)^{100} = (i^4)^{25} = 1$$
    $$\Rightarrow 1 = a + ib \Rightarrow a = 1, b = 0$$
11. **(d)** Given expression $= 1 + i^2 + i^4 + ... + i^{2n}$
    $= 1 - 1 + 1 - 1 + ... + (-1)^n$, which cannot be determined unless n is known.
12. **(a)** Given expression
    $$= \frac{i^{10}\left(i^{582} + i^{580} + i^{578} + i^{576} + i^{574}\right)}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1$$
    $= i^{10} - 1 = (i^2)^5 - 1 = (-1)^5 - 1 = -1 - 1 = -2$
13. **(a)** $z^{\frac{1}{3}} = p + iq \Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p + iq)$
    $$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$$
    $$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2$$
    $$y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2$$
    $$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2 \quad \therefore \left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2) = -2$$

14. **(c)** Let $z = 1 + i$ then $z^2 = (1 + i)^2$
$= 1^2 + i^2 + 2.1.i = 1 + i^2 + 2i$
$= 1 - 1 + 2i = 2i$

Now, $2i \times -\dfrac{i}{2} \Rightarrow -i^2 = 1$

Hence, $-\dfrac{i}{2}$ is multiplicative inverse of z^2.

15. **(b)** Let $z = \dfrac{3+4i}{4-5i} \times \dfrac{4+5i}{4+5i} = -\dfrac{8}{41} + \dfrac{31}{41}i$

Then, $\bar{z} = -\dfrac{8}{41} - \dfrac{31}{41}i$

and $|z| = \sqrt{\left(-\dfrac{8}{41}\right)^2 + \left(\dfrac{31}{41}\right)^2} = \dfrac{5}{\sqrt{41}}$

$\therefore$ Multiplicative inverse of z

$= \dfrac{\bar{z}}{|z|^2} = \dfrac{-\frac{8}{41} - \frac{31}{41}i}{\frac{25}{41}} = -\dfrac{8}{25} - \dfrac{31}{25}i$

16. **(b)** We have, $z(2 - i) = (3 + i)$
$\Rightarrow z = \left(\dfrac{3+i}{2-i}\right) \times \left(\dfrac{2+i}{2+i}\right) = \dfrac{5+5i}{5}$
$\Rightarrow z = 1 + i$
$\Rightarrow z^2 = 2i \Rightarrow z^{20} = -2^{10}$

17. **(b)** $(x + iy)^{\frac{1}{3}} = a + ib \Rightarrow x + iy = (a + ib)^3$
$\Rightarrow x + iy = a^3 - ib^3 + i3a^2 b - 3ab^2$
$= a^3 - 3ab^2 + i(3a^2 b - b^3)$
$\Rightarrow x = a^3 - 3ab^2$ and $y = 3a^2 b - b^3$

So, $\dfrac{x}{a} - \dfrac{y}{b} = a^2 - 3b^2 - 3a^2 + b^2$
$= -2a^2 - 2b^2 = -2(a^2 + b^2)$

18. **(a)** We have, $z = 2 - 3i$
$\Rightarrow z - 2 = -3i \Rightarrow (z - 2)^2 = (-3i)^2$
$\Rightarrow z^2 - 4z + 4 = 9i^2 \Rightarrow z^2 - 4z + 13 = 0$

19. **(a)** $z = i^{-39} = \dfrac{1}{i^{39}} = \dfrac{1}{\left(i^4\right)^9} \cdot \dfrac{1}{i^3} = 1 \cdot \dfrac{1}{i^3}$

$= \dfrac{1}{i^2 \cdot i} = \dfrac{-1}{i} = i \equiv 0 + i$
Hence, value of a = 0.

20. **(c)** $z_1 = 2 + 3i, z_2 = 3 + 2i$
$z_1 + z_2 = (2 + 3i) + (3 + 2i) = 5 + 5i$
Hence, a = 5

21. **(d)** $z_1 = 2 + 3i$ and $z_2 = 3 - 2i$
$z_1 - z_2 = (2 + 3i) - (3 - 2i) = -1 + 5i \equiv -1 + bi$
Hence, b = 5.

22. **(d)** $(1 + i)^4 \times \left(1 + \dfrac{1}{i}\right)^4 = (1 + i)^4 \times (1 - i)^4$

$= (1 - i^2)^4 = (1 + 1)^4 = 2^4 = 16.$

23. **(a)** $(1 + i)^6 = \{(1 + i)^2\}^3 = (1 + i^2 + 2i)^3$
$= (1 - 1 + 2i)^3 = 8i^3 = -8i$ and $(1 - i)^3$
$= 1 - i^3 - 3i + 3i^2$
$= 1 + i - 3i - 3 = -2 - 2i$
$\therefore$ $(1 + i)^6 + (1 - i)^3 = -8i - 2 - 2i = -2 - 10i$

24. **(a)** $\dfrac{i^{4n+1} - i^{4n-1}}{2} = \dfrac{i^{4n}\, i - i^{4n}\, i^{-1}}{2}$

$= \dfrac{i - \frac{1}{i}}{2} = \dfrac{i^2 - 1}{2i} = \dfrac{-2}{2i} = i$

25. **(a)** Consider $i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3}$
$= (i^4)^k + (i^4)^k . i + (i^4)^k . i^2 + (i^4)^k . i^3$
$= 1 + i + i^2 + i^3 = 1 + i - 1 - i = 0$

26. **(d)** $(1 + i)^5 (1 - i)^5 = (1 - i^2)^5 = 2^5 = 32$

27. **(c)** $\overline{z}_1 = i\overline{z}_2 \Rightarrow z_1 = -iz_2$ ...(i) $\{z\bar{z} = |z|^2\}$

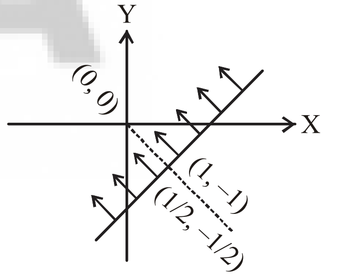
Take, $\arg\left(\dfrac{z_1}{\overline{z}_2}\right) = \pi \Rightarrow \arg\left(-i\dfrac{z_2}{\overline{z}_2}\right) = \pi$ $[\arg(z_2) = \theta]$

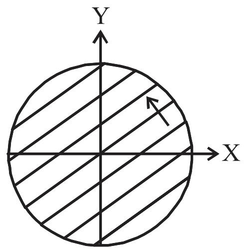
$-\dfrac{\pi}{2} + \theta + \theta = \pi \Rightarrow 2\theta = \dfrac{3\pi}{2}$

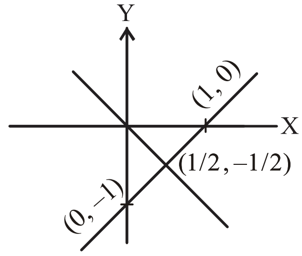
$\arg(z_2) = \theta = \dfrac{3\pi}{4}, \arg z_1 = \dfrac{\pi}{4}$

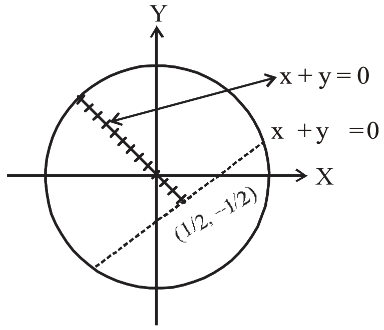
28. **(a)** Now $z^5 + \left(\bar{z}\right)^5 = (2 + 3i)^5 + (2 - 3i)^5$
$= 2({}^5C_0 2^5 + {}^5C_2 2^3 (3i)^2 + {}^5C_4 2^1 (3i)^4)$
$= 2(32 + 10 \times 8(-9) + 5 \times 2 \times 81) = 244$

29. **(b)** Given expression is
$|z - 1 + i| \geq |z|$; $|z| < 2$; $|z + i| = |z - 1|$









Hence, $w = 2x + iy \in S$

$2x \le \frac{1}{2} \quad \therefore x \le \frac{1}{4}$

Now, $(2x)^2 + (2x)^2 < 4$

$x^2 < \frac{1}{2} \Rightarrow x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Therefore, $x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{4}\right)$

30. (a) $(z+3)(\bar{z}+3) = z\bar{z} + 3z + 3\bar{z} + (3)^2$

$= |z|^2 + 3\left(\frac{z+\bar{z}}{2}\right) \times 2 + 3^2 \quad \left[\because |z|^2 = z\bar{z}\right]$

$= |z|^2 + 2 \times 3 \times (\text{Re}(z)) + 3^2$

$= |z|^2 + 2\text{Re}(3z) + (3)^2 = |z+3|^2$

31. (c) Let $z = \frac{\sqrt{5+12i} + \sqrt{5-12i}}{\sqrt{5+12i} - \sqrt{5-12i}} \times \frac{\sqrt{5+12i} + \sqrt{5-12i}}{\sqrt{5+12i} + \sqrt{5-12i}}$

$= \frac{5+12i+5-12i+2\sqrt{25+144}}{5+12i-5+12i}$

$= \frac{3}{2i} = \frac{3i}{-2} = 0 - \frac{3}{2}i$

Therefore, the conjugate of $z = 0 + \frac{3}{2}i$

32. (b) $x + iy = \frac{4+i}{2-3i} = \frac{(4+i)(2+3i)}{13} = \frac{5+14i}{13}$

$\therefore x = 5/13,\ y = 14/13$

33. (a) $i^{57} + \frac{1}{i^{25}} = (i^4)^{14}.i + \frac{1}{(i^4)^6.i} = i + \frac{1}{i} = i - i = 0$

34. (d) $\frac{(1+i\sqrt{3})(2+2i)}{\sqrt{3}-i} = \frac{2+2\sqrt{3}i+2i-2\sqrt{3}}{\sqrt{3}-i}$

$= \frac{(2-2\sqrt{3})+(2\sqrt{3}+2)i}{\sqrt{3}-i} \times \frac{\sqrt{3}+i}{\sqrt{3}+i}$

$= \frac{2\sqrt{3}-6+2i-2\sqrt{3}i+6i+2\sqrt{3}i-2\sqrt{3}-2}{3+1}$

$= \frac{8i-8}{4} = -2+2i$

$\therefore$ Modulus $= \sqrt{(-2)^2+(2)^2} = 2\sqrt{2}.$

35. (d) Let $z = \left(\frac{1}{1-2i} + \frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$

$= \left[\frac{1+i+3-6i}{(1-2i)(1+i)}\right]\left[\frac{3+4i}{2-4i}\right] = \left[\frac{4-5i}{3-i}\right]\left[\frac{3+4i}{2-4i}\right]$

$= \left[\frac{32+i}{2-14i}\right] = \frac{32+i}{2-14i} \times \frac{2+14i}{2+14i}$

$= \frac{64+448i+2i-14}{4+196} = \frac{50+450i}{200} = \frac{1}{4} + \frac{9}{4}i$

36. (d) $(1+i)^2 = 1 + i^2 + 2i = 2i$

$\therefore \frac{(1+i)^2}{3-i} = \frac{2i(3+i)}{3^2-i^2} = \frac{6i-2}{10} = \frac{-1+3i}{5}$

$\therefore$ Real part $= \frac{-1}{5}.$

37. (d) $\frac{z_1}{z_2} = \frac{6+3i}{2-i} = \frac{6+3i}{2-i} \times \frac{2+i}{2+i} = \frac{12+6i+6i-3}{4-i^2}$

$= \frac{9+12i}{5} = \frac{1}{5}(9+12i) \equiv \frac{1}{a}(9+12i)$

Hence, $a = 5$.

38. (a) $z = \frac{7-i}{3-4i} = \frac{7-i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{21+4+i(28-3)}{25}$

$= 1 + i$

$\therefore |z| = |1+i| = \sqrt{2}$

$\therefore |z|^{14} = (\sqrt{2})^{14} = 2^7$

39. (a) Let $z_1 = 6 + 3i$ and $z_2 = 2 - i$

Then, $\frac{z_1}{z_2} = (6+3i)\frac{1}{2-i} = \frac{(6+3i)(2+i)}{(2-i)(2+i)}$

$= (6+3i)\left(\frac{2}{2^2+(-1)^2} + i\frac{1}{2^2+(-1)^2}\right)$

$= (6+3i)\left(\frac{2}{5} + i\frac{1}{5}\right) = (6+3i)\frac{(2+i)}{5}$

$= \frac{1}{5}[12 - 3 + i(6+6)] = \frac{1}{5}(9+12i)$

40. (c) $(1-i)^n = 2^n$

Take modulus, both the side $|(1-i)^n| = |2^n|$

$|1-i|^n = |2|^n$

$\Rightarrow \left[\sqrt{1^2+(-1)^2}\right]^n = 2^n \Rightarrow (\sqrt{2})^n = 2^n \Rightarrow 2^{\frac{n}{2}} = 2^n$

$\Rightarrow \frac{n}{2} = n \Rightarrow n = 0$

41. **(b)** Given: $\frac{c+i}{c-i}=a+ib$

Then, $a+ib=\frac{c+i}{c-i}\times\frac{c+i}{c+i}=\frac{c^2-1+2ic}{c^2+1}$

$\Rightarrow\ a=\frac{c^2-1}{c^2+1}$ and $b=\frac{2c}{c^2+1}$

$\Rightarrow\ (a^2+b^2)=\frac{(c^2-1)^2+4c^2}{(c^2+1)^2}$

$=\frac{c^4+1-2c^2+4c^2}{(c^2+1)^2}=\frac{(c^2+1)^2}{(c^2+1)^2}=1.$

42. **(c)** Given $|z-4|<|z-2|$ Let $z=x+iy$
$\Rightarrow |(x-4)+iy)|<|(x-2)+iy|$
$\Rightarrow (x-4)^2+y^2<(x-2)^2+y^2$
$\Rightarrow x^2-8x+16<x^2-4x+4 \Rightarrow 12<4x$
$\Rightarrow\ x>3 \Rightarrow\ \text{Re}(z)>3$

43. **(c)** $|z|=\frac{|1+i\sqrt{3}|\,|\cos\theta+i\sin\theta|}{2|1-i|\,|\cos\theta-i\sin\theta|}=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}$

44. **(c)** $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|=\left|\frac{2-i+1+i+1}{2-i-(1+i)+1}\right|$

$[\because z_1=2-i \text{ and } z_2=1+i]$

$=\left|\frac{4}{2-i-1-i+1}\right|=\left|\frac{4}{2-2i}\right|=\left|\frac{2}{1-i}\right|=\frac{2}{|1-i|}$

$\left[\because \left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}\right]$

$=\frac{2}{\sqrt{(1)^2+(-1)^2}}$ $\quad\left[\because |z|=\sqrt{a^2+b^2}\right]$

$=\frac{2}{\sqrt{2}}=\sqrt{2}.$

45. **(a)** $\frac{(1+i)^3}{(1-i)^3}-\frac{(1-i)^3}{(1+i)^3}=x+iy$

$\Rightarrow\ \frac{\left(1+i^2+2i\right)^3-\left(1+i^2-2i\right)^3}{\left(1-i^2\right)^3}=x+iy$

$\Rightarrow\ \frac{8i^3+8i^3}{2^3}=x+iy \Rightarrow 2i^3=x+iy \Rightarrow -2i=x+iy$

$\Rightarrow\ x=0,\ y=-2$

46. **(c)** If $z=x+iy$ is the additive inverse of $1-i$, then $(x+iy)+(1-i)=0$
$\Rightarrow x+1=0,\ y-1=0 \Rightarrow x=-1,\ y=1$
$\therefore$ The additive inverse of $1-i$ is $z=-1+i$
Trick: Since $(1-i)+(-1+i)=0$.

47. **(c)** Let $z=x+iy$, then its conjugate $\bar{z}=x-iy$
Given that $z^2=(\bar{z})^2$
$\Rightarrow\ x^2-y^2+2ixy=x^2-y^2-2ixy \Rightarrow 4ixy=0$
If $x\neq 0$, then $y=0$ and if $y\neq 0$, then $x=0$.

48. **(a)** $x+iy=\sqrt{\frac{a+ib}{c+id}} \Rightarrow x-iy=\sqrt{\frac{a-ib}{c-id}}$

Also, $x^2+y^2=(x+iy)(x-iy)=\sqrt{\frac{a^2+b^2}{c^2+d^2}}$

$\Rightarrow\ (x^2+y^2)^2=\frac{a^2+b^2}{c^2+d^2}.$

49. **(c)** Let $z=\frac{2+5i}{4-3i}$ Rationalize $=\frac{2+5i}{4-3i}\times\frac{4+3i}{4+3i}$

$=\frac{8+26i-15}{(4)^2-(3i)^2}=\frac{8+26i-15}{16+(9)}=\frac{-7+26i}{25}$

50. **(d)** $(z-1)(\bar{z}-5)+(\bar{z}-1)(z-5)=2\,\text{Re}\left[(z-1)(\bar{z}-5)\right]$

$\left[\because z_1\bar{z}_2+z_2\bar{z}_1=2\,\text{Re}(z_1 z_2)\right]$

$=2\,\text{Re}[(1+i)(-3-i)]=2(-2)=-4$ [Given $z=2+i$]

51. **(b)** $z=x+iy$
$\Rightarrow |z|^2=x^2+y^2=1$ ... (i)

Now, $\left(\frac{z-1}{z+1}\right)=\frac{(x-1)+iy}{(x+1)+iy}\times\frac{(x+1)-iy}{(x+1)-iy}$

$=\frac{\left(x^2+y^2-1\right)+2iy}{(x+1)^2+y^2}=\frac{2iy}{(x+1)^2+y^2}$ [By equation (i)]

Hence, $\left(\frac{z-1}{z+1}\right)$ is purely imaginary.

52. **(d)** Let $z=x+iy$, $\bar{z}=x-iy$

Since $\arg(z)=\theta=\tan^{-1}\frac{y}{x}$

$\arg(\bar{z})=\theta=\tan^{-1}\left(\frac{-y}{x}\right)$

Thus, $\arg(z)\neq\arg(\bar{z})$.

53. **(a)** $(x-iy)(3+5i)=3x+5xi-3yi-5yi^2$
$=3x+(5x-3y)i+5y$ $[\because i^2=-1]$
$=(3x+5y)+(5x-3y)i$ ...(i)
Given, $(x-iy)(3+5i)=-6+24i$
[using equation (i), and $z=(a+ib)$
$\Rightarrow \bar{z}=(a-ib)$]
On comparing the real and imaginary parts of both sides, we get
$3x+5y=-6$ and $5x-3y=24$
Solving the above equations by substitution or elimination method, we get
$x=3,\ y=-3$

54. **(b)** Let $z=x+iy$, then $\frac{z-1}{z+1}=\frac{x-1+iy}{x+1+iy}$

$= \frac{(x-1+iy)}{(x+1+iy)} \times \frac{(x+1-iy)}{(x+1-iy)}$

$= \frac{\left[(x^2-1) - i(x-1)y + i(x+1)y + y^2\right]}{\left[(x+1)^2 + y^2\right]}$

For purely imaginary, real (z) = 0

$\Rightarrow \quad x^2 + y^2 = 1, |z| = 1.$

55. **(a)** We have, $x + iy = \frac{a+ib}{a-ib}$ ... (i)

Its conjugate, $x - iy = \frac{a-ib}{a+ib}$ ... (ii)

Multiply (i) and (ii),

$(x + iy)(x - iy) = \frac{a+ib}{a-ib} \times \frac{a-ib}{a+ib}$

$x^2 + y^2 = 1.$

56. **(c)** $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right) = 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} + i\,2\sin^2\frac{\pi}{10}$

$= 2\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$

For amplitude, $\tan\theta = \frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}} = \tan\frac{\pi}{10} \Rightarrow \theta = \frac{\pi}{10}.$

57. **(b)** Given that :

$Z = \frac{1-i}{\frac{1}{2} + \frac{\sqrt{3}}{2}i} = \frac{2(i-1)}{1+i\sqrt{(3)}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$= \frac{2(i + \sqrt{3} - 1 + i\sqrt{3})}{1+3} = \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}i$

Now, put $\frac{\sqrt{3}-1}{2} = r\cos\theta,\ \frac{\sqrt{3}+1}{2} = r\sin\theta$

Squaring and adding, we obtain

$r^2 = \left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}+1}{2}\right)^2 = \frac{2\times 4}{4} = 2$

Hence, $r = \sqrt{2}$ which gives :

$\cos\theta = \frac{\sqrt{3}-1}{2\sqrt{2}},\ \sin\theta = \frac{\sqrt{3}+1}{2\sqrt{2}}$

Therefore, $\theta = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$

Hence, the polar form is $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right).$

58. **(a)** Let z = x + iy

$\therefore \quad |z + 3 - i| = |(x+3) + i(y-1)| = 1$

$\Rightarrow \quad \sqrt{(x+3)^2 + (y-1)^2} = 1$ ... (i)

$\because \quad \arg z = \pi \Rightarrow \tan^{-1}\frac{y}{x} = \pi$

$\Rightarrow \quad \frac{y}{x} = \tan\pi = 0 \Rightarrow \quad y = 0$ ... (ii)

From equations (i) and (ii), we get

$x = -3, y = 0 \therefore z = -3$

$\Rightarrow \quad |z| = |-3| = 3$

59. **(c)** Let $z = r(\cos\theta + i\sin\theta)$

Then $r = |z|$ and $\theta = \arg(z)$

Now $z = r(\cos\theta + i\sin\theta)$

$\Rightarrow \bar{z} = r(\cos\theta - i\sin\theta)$

$= r[\cos(-\theta) + i\sin(-\theta)]$

$\therefore \arg(\bar{z}) = -\theta \Rightarrow \arg(\bar{z}) = -\arg(z)$

$\Rightarrow \arg(\bar{z}) + \arg(z) = 0.$

60. **(a)** $z_1 = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right] = \sqrt{2}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right] = 1 + i$

$|z_1| = \sqrt{2}$

and $z_2 = \sqrt{3}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right] = \sqrt{3}\left[\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$

$|z_2| = \sqrt{\frac{3}{4} + \frac{9}{4}} = \sqrt{3}$

$|z_1 z_2| = |z_1||z_2| = \sqrt{2}\cdot\sqrt{3} = \sqrt{6}$

61. **(d)**

62. **(b)**

63. **(b)** $z = (i^{25})^3 = (i)^{75} = i^{4\times 18+3} = (i^4)^{18}(i)^3$

$= i^3 = -i = 0 - i$

Polar form of $z = r(\cos\theta + i\sin\theta)$

$= 1\left\{\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right\} = \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

64. **(a)** $\frac{z_1}{z_2} = \frac{\sqrt{3} + i\sqrt{3}}{\sqrt{3} + i} = \left(\frac{3+\sqrt{3}}{4}\right) + \left(\frac{3-\sqrt{3}}{4}\right)i$

which is represented by a point in first quadrant.

65. **(b)** Suppose, $z = \frac{1+2i}{1-(1-i)^2}$

$= \frac{1+2i}{1-(1^2 + i^2 - 2i)} = \frac{1+2i}{1+2i}$

$= 1 = 1 + 0\,.\,i$

$|z| = \sqrt{(\text{Real part})^2 + (\text{Img. Part})^2}$

and $\text{amp}(z) = \tan^{-1}\left[\frac{\text{Img. part}}{\text{Real part}}\right]$

$\therefore \quad |z| = 1$ and $\text{amp}(z) = \tan^{-1}\left(\frac{0}{1}\right) = 0$

66. **(d)** Let $z = r(\cos\theta + i\sin\theta)$. Then $r = 4,\ \theta = \frac{5\pi}{6}$

$\therefore z = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 4\left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = -2\sqrt{3} + 2i$

67. **(d)** $z = \frac{3-i}{2+i} + \frac{3+i}{2-i} = \frac{(3-i)(2-i)+(3+i)(2+i)}{(2+i)(2-i)}$

$\Rightarrow z = 2 \Rightarrow (iz) = 2i$, which is the positive imaginary quantity

$\therefore\ \arg(iz) = \frac{\pi}{2}$

68. **(b)** Let $z = \frac{1+2i}{1-i}$ be the given complex number.

$\Rightarrow\ z = \frac{1+2i}{1-i} \times \frac{1+i}{1+i} = \frac{1+i+2i+2i^2}{1-i^2} = \frac{-1}{2} + \frac{3}{2}i$

$\Rightarrow\ (x, y) = \left(\frac{-1}{2}, \frac{3}{2}\right)$ which lies in IInd quadrant.

69. **(c)** Let $r(\cos\theta + i\sin\theta) = \frac{1+i\sqrt{3}}{\sqrt{3}+1} = \frac{1}{\sqrt{3}+1} + i\frac{\sqrt{3}}{\sqrt{3}+1}$

$\Rightarrow\ r\cos\theta = \frac{1}{\sqrt{3}+1};\ r\sin\theta = \frac{\sqrt{3}}{\sqrt{3}+1}$

$\Rightarrow\ \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$.

70. **(c)** Let $1 = r\cos\theta,\ \sqrt{3} = r\sin\theta$

By squaring and adding, we get

$r^2(\cos^2\theta + \sin^2\theta) = 4$

i.e, $r = \sqrt{4} = 2$

Therefore, $\cos\theta = \frac{1}{2},\ \sin\theta = \frac{\sqrt{3}}{2}$,

which gives $\theta = \frac{\pi}{3}$

Therefore, required polar form is :

$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$.

71. **(d)** Since $\left(\frac{i}{2} - \frac{2}{i}\right) = \frac{i}{2} - \frac{2i}{i^2} = \frac{i}{2} + 2i = \frac{5}{2}i$

So, argument is $\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{\frac{5}{2}}{0}\right) = \frac{\pi}{2}$.

72. **(b)** For $x^2 + ax + b = 0$

Sum of roots $= a = \frac{-1}{\alpha^2} - \frac{1}{\beta^2} - 2$

Product of roots $= b = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + 1 + \frac{1}{\alpha^2\beta^2}$

$a + b = \frac{1}{(\alpha\beta)^2} - 1 = \frac{1}{6} - 1 = -\frac{5}{6}$ $\qquad [\because \alpha\beta = \sqrt{6}]$

$x^2 - \left(-\frac{5}{6} - 2\right)x + \left(2 - \frac{5}{6}\right) = 0$

$\Rightarrow 6x^2 + 17x + 7 = 0$

$x = -\frac{7}{3},\ x = -\frac{1}{2}$ are the roots

Both roots are real and negative.

73. **(b)** Given expression is $x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_3 5}}\right)$

$+3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$

Take, $3^{\sqrt{\log_3 5}} = 3^{\sqrt{\log_3 5}\cdot\sqrt{\log_3 5}\sqrt{\log_3 5}} = 3^{\sqrt{\log_3 5}\cdot\sqrt{\log_3 5}}$

$= 3^{(\log_3 5)^{\sqrt{\log_5 3}}} = 5^{\sqrt{\log_5 3}}$

$3^{\sqrt[3]{\log_3 5}} = 3^{\log_3 5\cdot\sqrt[3]{(\log_5 3)^2}} = \left(3^{\log_3 5}\right)^{(\log_5 3)^{2/3}}$

$= 5^{(\log_5 3)^{2/3}}$

After putting the value the equation is $x^2 - 5x - 3 - 0$ and roots are α & β.

Then, $\alpha + \beta = 5;\ \alpha\beta = -3$.

New roots are $\alpha + \frac{1}{\beta}$ & $\beta + \frac{1}{\alpha}$

$\Rightarrow \frac{\alpha\beta+1}{\beta}$ & $\frac{\alpha\beta+1}{\alpha} \Rightarrow \frac{-2}{\beta}$ & $\frac{-2}{\alpha}$

Let $\frac{-2}{\alpha} = p \Rightarrow \alpha = \frac{-2}{p}$

As $\alpha^2 - 5\alpha - 3 = 0$

$\Rightarrow \left(\frac{-2}{p}\right)^2 - 5\left(\frac{-2}{p}\right) - 3 = 0$

$\Rightarrow \frac{4}{p^2} + \frac{10}{p} - 3 = 0$

$4 + 10p - 3p^2 = 0$

$\Rightarrow 3p^2 - 10p - \alpha = 0$.

Now replace 'p' by 'x' to get required equation.

$3x^2 - 10x - 4 = 0$.

74. **(a)** $\because x^2 + (2i - 1) = 0$

$\Rightarrow X^2 = 1 - 2i$

Since, α, β are the roots of $x^2 + (2i - 1) = 0$

$\Rightarrow \alpha^2 = 1-2i, \beta^2 = 1-2i$

$|\alpha^8+\beta^8| = |2\alpha^8| = 2|\alpha^2|^4 = 2\sqrt{5}^4 = 50$

75. **(c)**

76. **(b)**

77. **(c)** Quadratic equation

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \text{ or } x = \frac{-b \pm \sqrt{4ac-b^2 i}}{2a}$$

78. **(c)** Let α, β be the roots of $x^2 + bx + c = 0$ and α', β' be the roots of $x^2 + qx + r = 0$.

Then, $\alpha + \beta = -b$, $\alpha\beta = c$, $\alpha' + \beta' = -q$, $\alpha'\beta' = r$

It is given that $\frac{\alpha}{\beta} = \frac{\alpha'}{\beta'} \Rightarrow \frac{\alpha+\beta}{\alpha-\beta} = \frac{\alpha'+\beta'}{\alpha'-\beta'}$

$$\Rightarrow \frac{(\alpha+\beta)^2}{(\alpha-\beta)^2} = \frac{(\alpha'+\beta')^2}{(\alpha'-\beta')^2} \Rightarrow \frac{b^2}{b^2-4c} = \frac{q^2}{q^2-4r}$$

$\Rightarrow b^2 r = q^2 c$.

79. **(b)** **Case I :** $x-2>0$, Putting $x-2=y$, $y>0$ $x>2$

$\therefore y^2+y-2=0 \Rightarrow y=-2, 1 \Rightarrow x=0, 3$

But $0<2$, Hence $x=3$ is the real root.

Case II : $x-2<0 \Rightarrow x<2, y<0$

$y^2-y-2=0 \Rightarrow y=2, -1 \Rightarrow x=4, x=1$

Since $4 \not< 2$, only $x=1$ is the real root.

Hence the sum of the real roots = 3 + 1 = 4

80. **(c)** $4x^2 + 5k = (5k+1)x$

$\Rightarrow 4x^2 - (5k+1)x + 5k = 0; (\alpha-\beta) = 1$

$\therefore \alpha+\beta = \frac{(5k+1)}{4}$ and $\alpha\beta = \frac{5k}{4}$

Now, $\alpha-\beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$$\Rightarrow \alpha-\beta = \sqrt{\frac{(5k+1)^2}{16} - \frac{4\cdot 5k}{4}} = 1$$

$\therefore 25k^2 - 70k - 15 = 0$

$\Rightarrow (5k+1)(k-3) = 0 \Rightarrow k = -\frac{1}{5}, 3$.

81. **(b)** Since roots of the equation $x^2 - 5x + 16 = 0$ are α, β.

$\Rightarrow \alpha+\beta = 5$ and $\alpha\beta = 16$ and $\alpha^2+\beta^2+\frac{\alpha\beta}{2} = -p$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta + \frac{\alpha\beta}{2} = -p \Rightarrow 25 - 32 + 8 = -p$

$\Rightarrow p = -1$ and $(\alpha^2+\beta^2)\left(\frac{\alpha\beta}{2}\right) = q$

$\Rightarrow [(\alpha+\beta)^2 - 2\alpha\beta]\left[\frac{\alpha\beta}{2}\right] = q \Rightarrow q = [25-32]\frac{16}{2} = -56$

So, $p = -1$, $q = -56$.

82. **(c)** Let α, β are roots of $x^2 + px + q = 0$

So, $\alpha+\beta = -p$ and $\alpha\beta = q$

Given that $(\alpha+\beta) = 3(\alpha-\beta) = -p$

$\Rightarrow \alpha-\beta = \frac{-p}{3}$

Now, $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$\Rightarrow \frac{p^2}{9} = p^2 - 4q$ or $2p^2 = 9q$.

83. **(b)** Since α, β are the roots of the equation $2x^2 - 35x + 2 = 0$

Also, $\alpha\beta = 1$

$\therefore 2\alpha^2 - 35\alpha = -2$ or $2\alpha - 35 = \frac{-2}{\alpha}$

$2\beta^2 - 35\beta = -2$ or $2\beta - 35 = \frac{-2}{\beta}$

Now, $(2\alpha-35)^3(2\beta-35)^3 = \left(\frac{-2}{\alpha}\right)^3\left(\frac{-2}{\beta}\right)^3$

$= \frac{8\cdot 8}{\alpha^3\beta^3} = \frac{64}{1} = 64$.

84. **(a)** $\alpha+\beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, $\alpha\beta^2 + \alpha^2\beta + \alpha\beta = \alpha\beta(\beta+\alpha) + \alpha\beta$

$= \alpha\beta(1+\alpha+\beta) = \frac{c}{a}\left\{1+\left(-\frac{b}{a}\right)\right\} = \frac{c(a-b)}{a^2}$.

85. **(c)** Given $(x-a)(x-b) = c$

$\therefore \alpha+\beta = a+b$ and $\alpha\beta = ab-c$

Now, given equation $(x-\alpha)(x-\beta) + c = 0$

$\Rightarrow x^2 - (\alpha+\beta)x + \alpha\beta + c = 0$

If its roots be p and q, then

$p + q = (\alpha+\beta) = a + b$

$pq = \alpha\beta + c = ab - c + c = ab$

So, it can be given by $x^2 - (a+b)x + ab = 0$

So, its roots will be a and b.

86. **(a)** Let α, α^2 be the two roots. Then,

$\alpha + \alpha^2 = -\frac{b}{a}$ ... (i)

and $\alpha\cdot\alpha^2 = \frac{c}{a}$ ... (ii)

On cubing both sides of (i),

$\alpha^3 + \alpha^6 + 3\alpha\alpha^2(\alpha+\alpha^2) = -\frac{b^3}{a^3}$

$\Rightarrow \frac{c}{a} + \frac{c^2}{a^2} + 3\frac{c}{a}\left(-\frac{b}{a}\right) = -\frac{b^3}{a^3}$ [by (i) and (ii)]

$\Rightarrow b^3 + ac^2 + a^2c = 3abc$.

87. **(d)** $\alpha+\beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$ and $\alpha^2+\beta^2 = \frac{(b^2-2ac)}{a^2}$

Now, $\frac{\alpha}{a\beta+b} + \frac{\beta}{a\alpha+b} = \frac{\alpha(a\alpha+b)+\beta(a\beta+b)}{(a\beta+b)(a\alpha+b)}$

$$= \frac{a(\alpha^2+\beta^2)+b(\alpha+\beta)}{\alpha\beta a^2+ab(\alpha+\beta)+b^2} = \frac{a\frac{(b^2-2ac)}{a^2}+b\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)a^2+ab\left(-\frac{b}{a}\right)+b^2}$$

$$= \frac{b^2-2ac-b^2}{a^2c-ab^2+ab^2} = \frac{-2ac}{a^2c} = -\frac{2}{a}.$$

88. **(b)** $b^2-4ac = 1^2-4\times1\times1$ $[\because a=1, b=1, c=1]$

$b^2-4ac = 1-4 = -3$

$\therefore$ the solutions are given by

$$x = \frac{-1\pm\sqrt{-3}}{2\times1} = \frac{-1\pm\sqrt{3}i}{2}$$

89. **(d)** Given $\sqrt{3x^2-2} = 2x-1$

squaring both the sides

$\Rightarrow 3x^2-2 = 4x^2+1-4x \Rightarrow x^2-4x+3=0$

$\Rightarrow (x-3)(x-1)=0 \Rightarrow x=1, 3.$

90. **(c)** Let $\alpha+3 = x \therefore \alpha = x-3$ (replace x by $x-3$)

So the required equation

$(x-3)^2-5(x-3)+6=0$

$\Rightarrow x^2-6x+9-5x+15+6=0 \Rightarrow x^2-11x+30=0$

91. **(a)** $\frac{1}{x+a}-\frac{1}{x+b}=\frac{1}{x+c}$

$$\frac{b-a}{x^2+(b+a)x+ab}=\frac{1}{x+c}$$

or $x^2+(a+b)x+ab=(b-a)x+(b-a)c$

or $x^2+2ax+ab+ca-bc=0$

Since product of the roots = 0

$ab+ca-bc=0 \Rightarrow a=\frac{bc}{b+c}.$

Thus, sum of roots $= -2a = \frac{-2bc}{b+c}$

92. **(a)** Product of real roots $= \frac{9}{t^2} > 0, \ \forall\ t \in R$

$\therefore$ Product of real roots is always positive.

93. **(b)** $2x^2-(p+1)x+(p-1)=0$

Given $\alpha-\beta=\alpha\beta \Rightarrow (\alpha+\beta)^2-4\alpha\beta=\alpha^2\beta^2$

$$\Rightarrow \frac{(p-1)^2}{4}=\frac{(p+1)^2}{4}-\frac{4(p-1)}{2}$$

$\Rightarrow\ 2(p-1)=p \Rightarrow p=2.$

94. **(a)** $p+q=-p$ and $pq=q \Rightarrow q(p-1)=0$

$\Rightarrow q=0$ or $p=1$.

If $q=0$, then $p=0$. i.e. $p=q$

$\therefore p=1$ and $q=-2$.

95. **(c)** Given equation is

$(p-q)x^2+(q-r)x+(r-p)=0$

By using formula for finding the roots

viz: $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, we get

$$x=\frac{(r-q)\pm\sqrt{(q-r)^2-4(r-p)(p-q)}}{2(p-q)}$$

$$\Rightarrow\ x=\frac{(r-q)\pm(q+r-2p)}{2(p-q)}=\frac{r-p}{p-q},1$$

96. **(a)** Given. $ax^2+bx+c=0$ and α, β are roots of given equation

$\therefore\ \alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$ ...... (i)

Now, $\alpha\beta^2+\alpha^2\beta+\alpha\beta=\alpha\beta(\beta+\alpha)+\alpha\beta$

$=\frac{c}{a}.\left(-\frac{b}{a}\right)+\frac{c}{a}$ [Using equation (i)]

$$=-\frac{cb}{a^2}+\frac{c}{a}=\frac{-cb+ac}{a^2}=\frac{c(a-b)}{a^2}$$

97. **(b)** Consider the given equation

$$x-\frac{2}{x-1}=1-\frac{2}{x-1}$$

By taking L.C.M, we get

$\frac{x(x-1)-2}{x-1}=\frac{x-1-2}{x-1} \Rightarrow x(x-1)-2=x-3$

$\Rightarrow\ x^2-x-2=x-3 \Rightarrow\ x^2-2x+1=0$

$\Rightarrow\ (x-1)^2=0 \Rightarrow x=1, 1$

Thus, the given equation has two roots.

98. **(c)** Here, $b^2-4ac=1^2-4\times\sqrt{5}\times\sqrt{5}=1-20=-19$

Therefore, the solutions are

$$\frac{-1\pm\sqrt{-19}}{2\sqrt{5}}=\frac{-1\pm\sqrt{19}i}{2\sqrt{5}}$$

99. **(d)** Given equation is $x^2+2x+4=0$

Since α, β are roots of this equation

$\therefore\ \ \alpha+\beta=-2$ and $\alpha\beta=4$

Now, $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}=\frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{(\alpha\beta)^3}$

$$=\frac{(-2)((\alpha+\beta)^2-3\alpha\beta)}{4\times4\times4}=\frac{-2(4-12)}{4\times4\times4}=\frac{1}{4}$$

100. **(d)** Since α, β are roots of the equation $ax^2+bx+c=0$

$\therefore\ \ \alpha+\beta=\frac{-b}{a},\ \alpha\beta=\frac{c}{a}$ ...(i)

Now, $\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}=\frac{a(\alpha+\beta)+2b}{(a\alpha+b)(a\beta+b)}$

$=\frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$

$=\frac{a\left(-\frac{b}{a}\right)+2b}{a^2\cdot\frac{c}{a}+ab\left(-\frac{b}{a}\right)+b^2}=\frac{b}{ac}.$ [using (i)]

101. (b) $x^2+2=0 \Rightarrow x^2=-2 \Rightarrow x=\pm\sqrt{-2}=\pm\sqrt{2}\,i$

102. (b) Given equation is $x^2-2x(1+3k)+7(2k+3)=0$
Since, it has equal roots.
$\therefore$ Discriminant D = 0
$\Rightarrow b^2-4ac=0 \Rightarrow 4(1+3k)^2-4\times 7(2k+3)=0$
$\Rightarrow 1+9k^2+6k-14k-21=0$
$\Rightarrow 9k^2-8k-20=0 \Rightarrow 9k^2-18k+10k-20=0$
$\Rightarrow 9k(k-2)+10(k-2)=0 \Rightarrow k=\frac{-10}{9},\ 2$
Only k = 2 satisfy given equation.

103. (b) Given equation is $3^{2x}-10.3^x+9=0$ can be written as $(3^x)^2-10(3^x)+9=0$
Let $a=3^x$, then it reduces to the equation
$a^2-10a+9=0 \Rightarrow (a-9)(a-1)=0$
$\Rightarrow a=9, 1$
Now, $a=3^x \Rightarrow 9=3^x \Rightarrow 3^2=3^x \Rightarrow x=2$
and $1=3^x \Rightarrow 3^0=3^x \Rightarrow x=0$
Hence, roots are 0, 2.

104. (d) The given equation is $x^2-3x+3=0$
Let a, b be the roots of the given equation then,
$a+b=3,\ ab=3$
We know, $(a-b)^2=(a+b)^2-4ab=9-12$
$\Rightarrow a-b=\sqrt{3}i$ So, $a=\frac{3+\sqrt{3}i}{2}$
and $b=\frac{3-\sqrt{3}i}{2}$
If A and B are the roots of the new equation which are double of the founded roots then
$A=3+\sqrt{3}i$ and $B=3-\sqrt{3}i$
So, $A+B=6$ and $AB=9+3=12$
Thus the new equation is : $x^2-6x+12=0$

105. (b) We have, $4^x-3.2^{x+3}+128=0$
$\Rightarrow 2^{2x}-3.2^x.2^3+128=0$
$\Rightarrow 2^{2x}-24.2^x+128=0$
$\Rightarrow y^2-24y+128=0$ where $2^x=y$
$\Rightarrow (y-16)(y-8)=0 \Rightarrow y=16, 8$
$\Rightarrow 2^x=16$ or $2^x=8 \Rightarrow x=4$ or 3

106. (d) 4 is a root of $x^2+px+12=0$
$\Rightarrow 16+4p+12=0 \Rightarrow p=-7$
Now, the equation $x^2+px+q=0$ has equal roots.
$\therefore p^2-4q=0 \Rightarrow q=\frac{p^2}{4}=\frac{49}{4}$

107. (c) Let α, α^2 be the roots of $3x^2+px+3$.
$\therefore \alpha+\alpha^2=-p/3$ and $\alpha^3=1$
$\Rightarrow (\alpha-1)(\alpha^2+\alpha+1)=0$
$\Rightarrow \alpha=1$ or $\alpha^2+\alpha=-1$
If $\alpha=1$, $p=-6$ which is not possible as $p>0$
If $\alpha^2+\alpha=-1 \Rightarrow -p/3=-1 \Rightarrow p=3$.

108. (a) We have, $\left(x+\frac{1}{x}\right)^3+\left(x+\frac{1}{x}\right)=0$
$\Rightarrow \left(x+\frac{1}{x}\right)\left[\left(x+\frac{1}{x}\right)^2+1\right]=0$
$\Rightarrow$ either $x+\frac{1}{x}=0$
$\Rightarrow x^2=-1 \Rightarrow x=\pm i$
or $\left(x+\frac{1}{x}\right)^2+1=0 \Rightarrow x^2+\frac{1}{x^2}+3=0$
$\Rightarrow x^4+3x^2+1=0$
$\Rightarrow x^2=\frac{-3\pm\sqrt{5}}{2}<0$
$\therefore$ There is no real root.

109. (d) Given equation is $\frac{a}{x-a}+\frac{b}{x-b}=1$
$\Rightarrow a(x-b)+b(x-a)=(x-a)(x-b)$
$\Rightarrow x^2-x(a+b)+ab=ax-ab+bx-ab$
$\Rightarrow x^2-2x(a+b)+3ab=0$
So, sum of roots $=\alpha+(-\alpha)=2(a+b)$
or $a+b=0$.

110. (c) Let α, β be the roots of the equation.
$\therefore \alpha+\beta=a-2$ and $\alpha\beta=-(a+1)$
Now, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(a-2)^2+2(a+1)$
$=(a-1)^2+5$
$\therefore \alpha^2+\beta^2$ will be minimum if $(a-1)^2=0$, i.e. a = 1.

111. (d) Since α, β are roots of the equation
$(x-a)(x-b)=5$ or $x^2-(a+b)x+(ab-5)=0$
$\therefore \alpha+\beta=a+b$ or $a+b=\alpha+\beta$
and $\alpha\beta=ab-5$ or $ab=\alpha\beta+5$ ... (i)
Taking another equation
$(x-\alpha)(x-\beta)+5=0$
or $x^2-(\alpha+\beta)x+(\alpha\beta+5)=0$
or $x^2-(a+b)x+ab=0$ [using (i)]
$\therefore$ Its roots are a, b.

112. (b) Equations $px^2+2qx+r=0$ and
$qx^2-2(\sqrt{pr})x+q=0$ have real roots, then from first
$4q^2-4pr\ge 0 \Rightarrow q^2-pr\ge 0$
$\Rightarrow q^2\ge pr$ ... (i)
and from second
$4(pr)-4q^2\ge 0$ (for real root)
$\Rightarrow pr\ge q^2$ ... (ii)
From (i) and (ii), we get result $q^2=pr$.

113. (b) The roots of the equations are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(i) Let $b^2 - 4ac > 0$, $b > 0$
Now, if $a > 0$, $c > 0$, $b^2 - 4ac < b^2$
$\Rightarrow$ the roots are negative.
(ii) Let $b^2 - 4ac < 0$, then the roots are given by

$$x = \frac{-b \pm i\sqrt{(4ac - b^2)}}{2a}, \quad (i = \sqrt{-1})$$

which are imaginary and have negative real part. $[\because b > 0]$

$\therefore$ In each case, the roots have negative real part.

114. (a) Given equation $2ax^2 + (2a + b)x + b = 0$, $(a \neq 0)$
Now, its discriminant $D = B^2 - 4AC$
$= (2a + b)^2 - 4.2ab = (2a - b)^2$
Hence, D is a perfect square. So, given equation has rational roots.

115. (a) Since $2 + i\sqrt{3}$ is a root, therefore, $2 - i\sqrt{3}$ will be other root. Now sum of the roots = 4 = –p and product of roots = 7 = q. Hence (p, q) = (–4, 7).

116. (c) Let the roots be α and $\beta \Rightarrow \alpha + \beta = -p$, $\alpha\beta = q$
Given, $\alpha + \beta = \alpha^2 + \beta^2$
But $\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta \Rightarrow -p = (-p)^2 - 2q$
$\Rightarrow p^2 - 2q = -p \Rightarrow p^2 + p = 2q$

117. (a) Given equation is $x^2 - 2ax + a^2 + a - 3 = 0$
If roots are real, then $D \geq 0$
$\Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0$
$\Rightarrow -a + 3 \geq 0$
$\Rightarrow a - 3 \leq 0 \Rightarrow a \leq 3$
As roots are less than 3, hence $f(3) > 0$.
$9 - 6a + a^2 + a - 3 > 0 \Rightarrow a^2 - 5a + 6 > 0$
$\Rightarrow (a - 2)(a - 3) > 0 \Rightarrow$ either $a < 2$ or $a > 3$
Hence, $a < 2$ satisfy all.

118. (b) As sum of coefficients is zero, hence one root is 1 and other root is $\frac{l - m}{m - n}$.

Since roots are equal, $\therefore \frac{l - m}{m - n} = l \Rightarrow 2m = n + l$.

119. (b) It is given that: $\alpha\beta = 2 \Rightarrow \frac{3a + 4}{a + 1} = 2$

$\Rightarrow 3a + 4 = 2a + 2 \Rightarrow a = -2$

Also, $\alpha + \beta = -\frac{2a + 3}{a + 1}$

Putting this value of a, we get sum of roots

$$= -\frac{2a + 3}{a + 1} = -\frac{-4 + 3}{-2 + 1} = -1.$$

EXERCISE - 2

1. (d) Let $z = \sin x + i \cos 2x$
According to the given condition,
$\bar{z} = \cos x - i \sin 2x$
$\therefore$ $\sin x - i \cos 2x = \cos x - i \sin 2x$
$\Rightarrow$ $(\sin x - \cos x) + i(\sin 2x - \cos 2x) = 0$
On equating real and imaginary parts, we get
$\sin x - \cos x = 0$, $\sin 2x - \cos 2x = 0$
$\Rightarrow$ $\tan x = 1$ and $\tan 2x = 1$

$\Rightarrow$ $x = \frac{\pi}{4}$ and $2x = \frac{\pi}{4} \Rightarrow x = \frac{\pi}{4}$ and $x = \frac{\pi}{8}$

which is not possible.

2. (c) Let, $z = \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha} = \frac{(1 - i\sin\alpha)(1 - 2i\sin\alpha)}{(1 + 2i\sin\alpha)(1 - 2i\sin\alpha)}$

$$= \frac{1 - i\sin\alpha - 2i\sin\alpha + 2i^2\sin^2\alpha}{1 - 4i^2\sin^2\alpha} = \frac{1 - 3i\sin\alpha - 2\sin^2\alpha}{1 + 4\sin^2\alpha}$$

$$= \frac{1 - 2\sin^2\alpha}{1 + 4\sin^2\alpha} - \frac{3i\sin\alpha}{1 + 4\sin^2\alpha}$$

As, z is a purely real.

So, $\frac{-3\sin\alpha}{1 + 4\sin^2\alpha} = 0 \Rightarrow \sin\alpha = 0 \Rightarrow \alpha = n\pi$

3. (b) As, $z = x + iy$ lies in the third quadrant.
So, $x < 0$ and $y < 0$.

$$\because \frac{\bar{z}}{z} = \frac{x - iy}{x + iy} = \frac{(x - iy)(x - iy)}{(x + iy)(x - iy)} = \frac{x^2 - y^2 - 2ixy}{x^2 + y^2}$$

$$= \frac{\bar{z}}{z} = \frac{x^2 - y^2}{x^2 + y^2} - \frac{2ixy}{x^2 + y^2}$$

As, $\frac{\bar{z}}{z}$ also lies in the third quadrant.

Therefore, $\frac{x^2 - y^2}{x^2 + y^2} < 0$ and $\frac{-2xy}{x^2 + y^2} < 0$

$\Rightarrow$ $x^2 - y^2 < 0$ and $-2xy < 0$ $[\because x^2 + y^2 > 0]$
$\Rightarrow$ $x^2 < y^2$ and $xy > 0$. Hence, $x < y < 0$

4. (a) Suppose, $z = x + iy$
Now, $(z + 3)(\bar{z} + 3) = (x + iy + 3)(x + 3 - iy)$
$= (x + 3)^2 - (iy)^2 = (x + 3)^2 + y^2$ $[i^2 = -1]$
$= |x + 3 + iy|^2 = |z + 3|^2$

5. (b) Since, $\left(\frac{1 + i}{1 - i}\right)^x = 1 \Rightarrow \left[\frac{(1 + i)(1 + i)}{(1 - i)(1 + i)}\right]^x = 1$

$\Rightarrow \left[\frac{1 + 2i + i^2}{1 - i^2}\right]^x = 1 \Rightarrow \left[\frac{2i}{1 + 1}\right]^x = 1 \Rightarrow \left[\frac{2i}{2}\right]^x = 1$

$\Rightarrow$ $i^x = 1$
$\Rightarrow i^x = \{i^{4n}\}$ $[\because i^2 = -1\ i^{4n} = 1$ where $n \in N]$
Hence, $x = 4n$

6. (a) Since, $\left(\frac{3 - 4ix}{3 + 4ix}\right) = \alpha - i\beta\ (\alpha, \beta \in R)$

$$\Rightarrow (\alpha - i\beta) = \frac{(3 - 4ix)(3 - 4ix)}{(3 + 4ix)(3 - 4ix)} = \frac{9 + 16i^2x^2 - 24ix}{9 - 16i^2x^2}$$

$$\Rightarrow \alpha - i\beta = \frac{9 - 16x^2 - 24ix}{9 + 16x^2} \quad [\because i^2 = -1]$$

$\Rightarrow \quad \alpha - i\beta = \dfrac{9-16x^2}{9+16x^2} - \dfrac{i(24x)}{9+16x^2}$

$\alpha^2+\beta^2 = \left(\dfrac{9-16x^2}{9+16x^2}\right)^2 + \left(\dfrac{24x}{9+16x^2}\right)^2$

$= \dfrac{81+256x^4-288x^2+576x^2}{(9+16x^2)^2}$

$= \dfrac{81+256x^4+288x^2}{(9+16x^2)^2} = \dfrac{(9+16x^2)^2}{(9+16x^2)^2} = 1$

7. **(a)** (a) $|z_1 z_2| = |z_1||z_2|$
(b) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
(c) $|z_1 + z_2| \neq |z_1| + |z_2|$

8. **(d)** Since, $x, y \in R$
So, $x + iy$ is non-real complex number if $y \neq 0$.

9. **(d)** Since, $a + ib = c + id$ So, $a = c$ and $b = d$
Therefore, $a^2 + b^2 = c^2 + d^2$

10. **(b)** Given, $\left|\dfrac{i+z}{i-z}\right| = 1$

Let z = x + iy

$\therefore \quad \left|\dfrac{i+x+iy}{i-(x+iy)}\right| = 1 \Rightarrow \left|\dfrac{x+i(1+y)}{-x+i(1-y)}\right| = 1$

$\Rightarrow \quad \sqrt{x^2+(1+y)^2} = \sqrt{(-x)^2+(1-y)^2}$
$\Rightarrow \quad x^2 + 1 + y^2 + 2y = x^2 + 1 + y^2 - 2y$
$\Rightarrow \quad 4y = 0 \Rightarrow y = 0$
Hence, z lies on x-axis.

11. **(b)** Here, z is a complex number. Let $z = x + iy$
$|z| = |x + iy|$
$\Rightarrow \quad |z|^2 = x^2 + y^2$ ... (i)
Now, $z^2 = (x+iy)^2 = x^2 + i^2y^2 + i(2xy)$
$\Rightarrow \quad z^2 = x^2 - y^2 + i2xy$
$\Rightarrow \quad |z^2| = \sqrt{(x^2-y^2)^2+(2xy)^2}$
$\Rightarrow \quad |z^2| = \sqrt{x^4+y^4+2x^2y^2} = \sqrt{(x^2+y^2)^2}$
$\Rightarrow \quad |z^2| = x^2 + y^2$ ... (ii)
From Eqs. (i) and (ii) we get: $|z|^2 = |z^2|$

12. **(c)** Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$,
$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$
$\because \quad |z_1 + z_2| = |z_1| + |z_2|$ [given]
$\Rightarrow \quad |(r_1\cos\theta_1 + r_2\cos\theta_2) + i(r_1\sin\theta_1 + r_2\sin\theta_2)| = r_1 + r_2$
$\Rightarrow \quad \sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1-\theta_2)} = r_1 + r_2$
$\Rightarrow \quad r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1-\theta_2) = r_1^2 + r_2^2 + 2r_1r_2$
$\Rightarrow \quad \cos(\theta_1-\theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0 \Rightarrow \theta_1 = \theta_2$
$\Rightarrow \quad \arg(z_1) = \arg(z_2)$.

13. **(c)** $\dfrac{1+i\cos\theta}{1-2i\cos\theta} = \dfrac{(1+i\cos\theta)(1+2i\cos\theta)}{(1-2i\cos\theta)(1+2i\cos\theta)}$

$= \dfrac{1+i\cos\theta+2i\cos\theta+2i^2\cos^2\theta}{1-4i^2\cos^2\theta}$

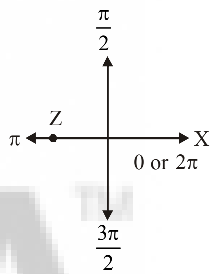
$= \dfrac{1+3i\cos\theta-2\cos^2\theta}{1+4\cos^2\theta}$ $[\because i^2 = -1]$

For real values of the given expression,

$\dfrac{3\cos\theta}{1+4\cos^2\theta} = 0 \Rightarrow \cos\theta = 0 \Rightarrow \cos\theta = \cos\dfrac{\pi}{2}$

$\Rightarrow \quad \theta = 2n\pi \pm \dfrac{\pi}{2}$

14. **(c)** Suppose, $z = x + 0i$ where $x < 0$
As, $z = x + 0i$ lies on the negative side of x-axis.
So, $\arg(z) = \pi$



15. **(a)** Since, $z = 1 + 2i \Rightarrow |z| = \sqrt{1^2+2^2} = \sqrt{5}$

Here, $f(z) = \dfrac{7-z}{1-z^2} = \dfrac{7-1-2i}{1-(1+2i)^2} = \dfrac{6-2i}{1-1-4i^2-4i}$

$= \dfrac{(3-i)(2+2i)}{(2-2i)(2+2i)} = \dfrac{6-2i+6i-2i^2}{4-4i^2} = \dfrac{8+4i}{8}$

So, $f(z) = 1 + \dfrac{1}{2}i$

$\therefore \quad |f(z)| = \sqrt{1+\dfrac{1}{4}} = \dfrac{\sqrt{5}}{2} = \dfrac{|z|}{2}$

16. **(c)** $(x^2-5x+5)^{x^2+4x-60} = 1$
Case I
$x^2 - 5x + 5 = 1$ and $x^2 + 4x - 60$ can be any real number
$\Rightarrow x = 1, 4$
Case II
$x^2 - 5x + 5 = -1$ and $x^2 + 4x - 60$ has to be an even number
$\Rightarrow x = 2, 3$
where 3 is rejected because for x = 3, $x^2 + 4x - 60$ is odd.

Case III

x^2-5x+5 can be any real number and

$x^2+4x-60=0 \Rightarrow x=-10, 6$

$\Rightarrow$ Sum of all values of $x=-10+6+2+1+4=3$

17. (b) Rationalizing the given expression

$$\frac{(2+3i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$$

For the given expression to be purely imaginary, real part of the above expression should be equal to zero.

$$\Rightarrow \frac{2-6\sin^2\theta}{1+4\sin^2\theta}=0 \quad \Rightarrow \sin^2\theta=\frac{1}{3} \quad \Rightarrow \sin\theta=\pm\frac{1}{\sqrt{3}}$$

18. (a) We have $\sum_{r=1}^{n}(x+r-1)(x+r)=10n$

$$\sum_{r=1}^{n}(x^2+xr+(r-1)x+r^2-r=10n$$

$$\Rightarrow \sum_{r=1}^{n}(x^2+(2r-1)x+r(r-1)=10n$$

$$\Rightarrow nx^2+\{1+3+5+....+(2n-1)\}x+\{1.2+2.3+....+(n-1)n\}=10n$$

$$\Rightarrow nx^2+n^2x+\frac{(n-1)n(n+1)}{3}=10n$$

$$\Rightarrow x^2+nx+\frac{n^2-31}{3}=0$$

Let α and $\alpha+1$ be its two solutions

($\because$ it has two consequtive integral solutions)

$\Rightarrow \alpha+(\alpha+1)=-n$

$$\Rightarrow \alpha=\frac{-n-1}{2} \quad ...(i)$$

$$\text{Also } \alpha(\alpha+1)=\frac{n^2-31}{3} \quad ...(ii)$$

Putting value of (i) in (ii), we get

$$-\left(\frac{n+1}{2}\right)\left(\frac{1-n}{2}\right)=\frac{n^2-31}{3} \Rightarrow n^2=121 \Rightarrow n=11$$

19. (d) Let $l=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$.

$$\therefore l=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1+i\sqrt{3}}\right)=\left(\frac{-2+i2\sqrt{3}}{4}\right)=\left(\frac{1-i\sqrt{3}}{-2}\right)$$

$$\text{Also, } l=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1-i\sqrt{3}}{1-i\sqrt{3}}\right)$$

$$=\left(\frac{4}{-2-i2\sqrt{3}}\right)=\left(\frac{-2}{1+i\sqrt{3}}\right)$$

$$\text{Now, } \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^3=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$$

$$=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{-2}{1+i\sqrt{3}}\right)\times\left(\frac{1-i\sqrt{3}}{-2}\right)=1$$

$\therefore$ least positive integer n is 3.

20. (b) Let $z\in S$, then $z=\frac{\alpha+i}{\alpha-i}$

Since, z is a complex number and let $z=x+iy$

$$\text{Then, } x+iy=\frac{(\alpha+i)^2}{\alpha^2+1} \text{ (by rationalisation)}$$

$$\Rightarrow x+iy=\frac{(\alpha^2-1)}{\alpha^2+1}+\frac{i(2\alpha)}{\alpha^2+1}$$

Then compare both sides

$$x=\frac{\alpha^2-1}{\alpha^2+1} \quad ...(i)$$

$$y=\frac{2\alpha}{\alpha^2+1} \quad ...(ii)$$

Now squaring and adding equations (i) and (ii)

$$\Rightarrow x^2+y^2=\frac{(\alpha^2-1)^2}{(\alpha^2+1)^2}+\frac{4\alpha^2}{(\alpha^2+1)^2}=1$$

21. (d) Since, $2-\sqrt{3}$ is a root of the quadratic equation $x^2+px+q=0$

$\therefore 2+\sqrt{3}$ is the other root

$$\Rightarrow x^2+px+q=[x-(2-\sqrt{3})][x-(2+\sqrt{3})]$$

$$=x^2-(2+\sqrt{3})x-(2-\sqrt{3})x+(2^2-(\sqrt{3})^2)$$

$$=x^2-4x+1$$

Now, by comparing $p=-4, q=1$

$\Rightarrow$ $p^2-4q-12=16-4-12=0$

22. (b) $(k+1)\tan^2 x-\sqrt{2}\lambda\tan x+(k-1)=0$

$$\tan\alpha+\tan\beta=\frac{\sqrt{2}\lambda}{k+1} \quad \text{[Sum of roots]}$$

$$\tan\alpha\cdot\tan\beta=\frac{k-1}{k+1} \quad \text{[Product of roots]}$$

$$\therefore \quad \tan(\alpha+\beta)=\frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}}=\frac{\sqrt{2}\lambda}{2}=\frac{\lambda}{\sqrt{2}}$$

$$\tan^2(\alpha+\beta)=\frac{\lambda^2}{2}=50 \quad \Rightarrow \lambda=10.$$

23. (d) $\because$ $z=x+iy$

$$\left(\frac{z-1}{2z+i}\right)=\frac{(x-1)+iy}{2(x+iy)+i}$$

$$=\frac{(x-1)+iy}{2x+(2y+1)i}\times\frac{2x-(2y+1)i}{2x-(2y+1)i}$$

$$\text{Re}\left(\frac{z+1}{2z+i}\right)=\frac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2}=1$$

$$\Rightarrow \left(x+\frac{1}{2}\right)^2+\left(y+\frac{3}{4}\right)^2=\left(\frac{\sqrt{5}}{4}\right)^2.$$

24. **(d)** Given $\frac{z-i}{z+2i} \in R$

Then, $\arg\left(\frac{z-i}{z+2i}\right)$ is 0 or π

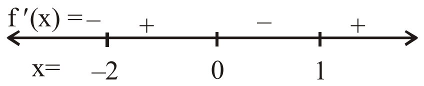
$\Rightarrow$ S is straight line in complex.

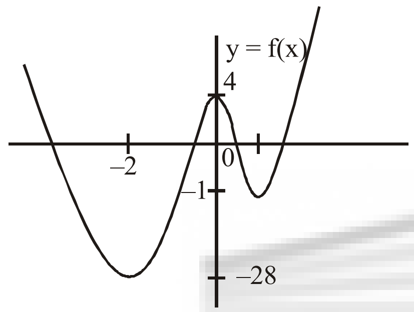
25. **(d)** Consider the given equation

$3x^4 + 4x^3 - 12x^2 + 4 = 0$

Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 4$

$\Rightarrow f'(x) = 12x(x^2 + x - 2) = 12x(x+2)(x-1)$





So, graph of equation intersects the x-axis at 4 distinct points $\Rightarrow$ 4 distinct real roots

26. **(d)** We have $\frac{(1+2i)^8 \cdot (1-2i)^2}{(3+2i)\cdot\overline{(4-6i)}}$

$\Rightarrow \frac{(1+2i)^2(1-2i)^2(1+2i)^6}{26i}$

$\Rightarrow \frac{25(1+2i)^6}{26i} \Rightarrow (-i)\frac{25}{26}(7+24i)(3-4i)$

$\Rightarrow$ Real part $= \frac{(72-28)25}{26} = \frac{44\times 25}{26} = \frac{550}{13}$

27. **(272)** Given equation $(p^2+q^2)x^2 - 2q(p+r)x + q^2 + r^2 = 0$

$(px-q)^2 + (qx-r)^2$ ...(i)

Here, $x^2 + 2x - 8 = 0$

$x^2 + 4x - 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, 2.$

From (i),

$x = \frac{q}{p}, \frac{r}{q}$

Put $x = -4$

$x = \frac{q}{p} = \frac{r}{q} = -4$

$q = -4p, r = -4q$

$r = 16p$

take square both sides,

$q^2 = 16p^2, r^2 = 256p^2$

$\Rightarrow \frac{q^2 + r^2}{p^2} = \frac{16p^2 + 256p^2}{p^2} = 272.$

28. **(a)** Let $z_1 = i - 1 = -1 + i$

$r = \sqrt{1+1} = \sqrt{2}\ ;\ \theta = \frac{3\pi}{4}$

$= \pi - \tan^{-1}(1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\therefore z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}} = \frac{\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$

$= \sqrt{2}\left[\cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)\right]$

$= \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

29. **(b)** Let $Z = x + iy, x, y \in R$

$x - iy = i(x^2 - y^2 + (2xy)i + x)$

$x = -2xy$ ......(i)

$-y = -y^2 + x^2 + x$ ......(ii)

$\Rightarrow x = 0,\ y = -\frac{1}{2}$ (from (i))

If $x \neq 0$, then $y = 0, 1$

If $y = -\frac{1}{2}$, then $x = \frac{1}{2}, -\frac{3}{2}$

then $Z = 0 + i0, 0 + i, \frac{1}{2} - \frac{i}{2}, -\frac{3}{2} - \frac{i}{2}$

So, $\sum_{z\in S} |z|^2 = 0 + 1 + \frac{1}{2} + \frac{10}{4} = 4$

30. **(a)**

31. **(5)**

EXERCISE - 3

1. **(a)** $(x^2 - 2x)(3x - 20) \le 0 \quad \Rightarrow x \in (-\infty, 0) \cup \left[2, \frac{20}{3}\right]$

2. **(b)** $\therefore (1 + ri)^3 = \lambda(1 + i)$

$\Rightarrow \quad 1 - r^3 i + 3ri - 3r^2 = \lambda + i\lambda$

On comparing real and imaginary parts. we get

$1 - 3r^2 = \lambda$ and $-r^3 + 3r = \lambda$

Then $-r^3 + 3r = 1 - 3r^2 \Rightarrow r^3 - 3r^2 - 3r + 1 = 0$

$\Rightarrow \quad (r^3 + 1) - 3r(r + 1) = 0 \Rightarrow (r + 1)(r^2 - r + 1 - 3r) = 0$

$\Rightarrow \quad (r + 1)(r^2 - 4r + 1) = 0 \therefore r = -1, 2 \pm \sqrt{3}$

$\Rightarrow \quad r = \text{cosec}\ \frac{3\pi}{2}$

3. **(d)** $\log_{1/3}\{\log_{1/2}(|z|^2 + 4|z| + 3)\} > 0$

$\Rightarrow \quad 0 < \log, (|z|^2 + 4|z| + 3) < 1 \Rightarrow |z|^2 + 4|z| + 3 > 1/2$

$\Rightarrow \quad 2|z|^2 + 8|z| + 5 > 0$, which is always true

$\therefore$ All (a), (b), (c) represent subsets of set of complex numbers set.

4. **(b)** We have for any real x, $\cos^2\frac{x}{2} \le 1$ and $\sin^2 x \le 1$

$\therefore\ 2\cos^2\frac{x}{2}\sin^2 x \le 2 \Rightarrow \text{LHS} \le 2$

Again for any real x,

$$x^2+\frac{1}{x^2}=x^2+\frac{1}{x^2}-2+2=\left(x-\frac{1}{x}\right)^2+2\ge 2$$

$\therefore$ RHS ≥ 2 Thus, the given equation can have solution only if LHS = RHS = 2

$$\Rightarrow 2\cos^2\frac{x}{2}\sin^2 x = x^2+\frac{1}{x^2}=2$$

Now, $x^2+\frac{1}{x^2}=2 \Rightarrow x=\pm 1$

but for $x=\pm 1$, $\cos^2\frac{x}{2}\sin^2 x \ne 1$.

That is, LHS and RHS cannot be simultaneously equal to 2 for any value of x.

$\therefore$ The equation has no real solution.

5. **(d)** $a > b > c$ ...(i)

and given equation is

$(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)=0$ ...(ii)

$\because$ Equation (ii) has a root in the interval (–1, 0)

$\therefore\ f(-1)f(0)<0 \Rightarrow (2a-b-c)(c+a-2b)<0$ ...(iii)

From (i), $a>b \Rightarrow a-b>0$ and

$a>c \Rightarrow a-c>0$

$\therefore\ 2a-b-c>0$ ...(iv)

From (iii) and (iv), $c+a-2b<0$ or $c+a<2b$. Option (a) is correct. Again, the sum of coefficients of the equation = 0, that is one root is 1 and the other root is $\frac{c+a-2b}{a+b-2c}$, which is a rational number as *a*, *b*, *c* are rational. Hence, both the roots of the equation are rational .

$\Rightarrow$ (b) is correct. Further, the discriminant of equation $ax^2+2bx+c=0$ is $D=4b^2-4ac$.

As deduced earlier, $c+a<2b$

$\Rightarrow 4b^2>(c+a)^2 \Rightarrow 4b^2>c^2+a^2+2ac$

$\Rightarrow 4b^2-4ac>c^2+a^2-2ac=(c-a)^2$

$\Rightarrow 4b^2-4ac>0 \Rightarrow D>0$. Also, each of *a*,*b*,*c* are positive.

$\therefore$ The equation $ax^2+2bx+c=0$ has real and negative roots. So (c) is also correct.

6. **(b)** Let $z=\sin\ln\,(i^i)^{i^2}+\cos\ln\,(i^i)^{i^2}$

$$=\sin\left\{\ln\left(e^{\frac{\pi}{2}}\right)^{-1}\right\}+\cos\left\{\ln\left(e^{-\frac{\pi}{2}}\right)^{-1}\right\}=\sin\frac{\pi}{2}+\cos\frac{\pi}{2}=1$$

7. **(a)** From the given condition we can infer that *z*, *w*, $-iw$ and $i\bar{w}$ lie inside or on the boundary of the circle $|z|\le 1$. $|z+iw|=|z-i\bar{w}|=2$ Since we get $-iw, i\bar{w}$ are the other end points of the diameter having one end point as *z*. So, $-iw = i\bar{w} = -z$.

Therefore, $|z+z|=2$ or $|z|=1$

but, $-z=-iw=i\bar{w} \Rightarrow w=-iz$ and $\bar{w}=iz$

$\therefore\ \bar{w}=i\bar{z}$ and $\bar{w}=iz\ \therefore\ i\bar{z}=iz \Rightarrow z=\bar{z}$

z is real and $|z|=1$, we get $z=1,-1$.

8. **(d)** If $|z-1|>|z+1|$, then max. $\{|z-1|, |z+1|\}=|z-1|$

$\Rightarrow$ If $|z|^2+1-z-\bar{z}>|z|^2+1+z+\bar{z}$ then $|z|=|z-1|$

$\Rightarrow$ If $z+\bar{z}<0$ then $|z|^2=|z|^2+1-z-\bar{z}$

$\Rightarrow$ If $z+\bar{z}<0$ then $z+\bar{z}=1$,

which is not possible.

Again if $|z+1|>|z-1|$ then max. $\{|z-1|, |z+1|\}=|z+1|$

$\Rightarrow$ If $|z|^2+1+z+\bar{z}>|z|^2+1-z-\bar{z}$ then $|z|=|z+1|$

$\Rightarrow$ If $z+\bar{z}>0$ then $|z|^2=|z|^2+1+z+\bar{z}$

$\Rightarrow$ If $z+\bar{z}>0$ then $z+\bar{z}=-1$ Not possible again.

Therefore the given result cannot hold.

9. **(c)** Let $z=x+iy$; $\arg(z)=\frac{\pi}{6} \Rightarrow \frac{y}{x}=\tan\frac{\pi}{6}$ and $x>0, y>0 \Rightarrow y=\frac{1}{\sqrt{3}}x$ and $x>0, y>0$ which is a portion of a straight line.

Also, $|z-2\sqrt{3}\,i|=r$, represents a circle with centre at $(0, 2\sqrt{3})$ and radius *r*. The straight line will intersect the circle if the perpendicular distance from the centre on the line < *r*

$$\Rightarrow \left|\frac{0-2\sqrt{3}.\sqrt{3}}{2}\right|<r \Rightarrow r>3. \text{ Therefore } [r]\ge 3.$$

10. **(b)** $(a+c-b)x^2+2cx+(b+c-a)=0$; $a, b, c \in \mathbb{R}$ and distinct and $(a+c-b)\ne 0$, (given both roots α and β rational).

$$\Rightarrow \alpha+\beta=\frac{2c}{b-c-a}\in\mathbb{Q} \text{ and } \alpha\beta=\frac{b+c-a}{a+c-b}\in\mathbb{Q}$$

$$\Rightarrow \frac{2}{\frac{b-a}{c}-1}\in\mathbb{Q} \Rightarrow \frac{b-a}{c}\in\mathbb{Q} \Rightarrow \frac{c}{a-b}\in\mathbb{Q}$$

Now, consider the following counter example

Let $c=3, a=\sqrt{3}, b=2+\sqrt{3}$

Clearly, $D=4(2)^2=16$ i.e., perfect square of rational

and $\frac{c}{a-b}=\frac{3}{2}\in\mathbb{Q}, \frac{b}{c-a}=\frac{2+\sqrt{3}}{3-\sqrt{3}}\notin\mathbb{Q}$

Now, given equation becomes $x^2 + 6x + (5) = 0$

$\Rightarrow \quad (x+1)(x+5) = 0$

Clearly, both roots are rational.

Thus if both roots are rational, then it is not necessary that a, b, c are rational and $\frac{b}{c-a}$ is rational.

11. **(c)** Given that $z_1 + z_2 + z_3 + z_4 = 0$

$\Rightarrow \quad \frac{z_1 + z_2}{2} = -\frac{z_3 + z_4}{2}$

$\frac{z_1 + z_2}{2}$ represents mid point E of AB

And $\frac{z_3 + z_4}{2}$ represents mid point F of DC

ABCD is a parallelogram inscribed in a circle and hence a rectangle

Then ABCD is necessarily rectangle, so statement (i) and (iii) are correct.

12. **(d)** $\because (1+i)^4 = [(1+i)^2]^2 = (1 + i^2 + 2i)^2$

$= (1 - 1 + 2i)^2 = 4i^2 = -4$ ...(i)

$\& \quad \frac{1-\sqrt{\pi} i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi} i}$

$= \frac{(1-\sqrt{\pi} i)(\sqrt{\pi}-i)}{\pi+1} + \frac{(\sqrt{\pi}-i)(1-\sqrt{\pi} i)}{1+\pi}$

$= \frac{\sqrt{\pi} - i - \pi i - \sqrt{\pi} + \sqrt{\pi} - \pi i - i - \sqrt{\pi}}{\pi+1}$

$= \frac{-2\pi i - 2i}{\pi+1} = -2i$ ...(ii)

Given,

$z = \frac{\pi}{4}(1+i)^4 \left(\frac{1-\sqrt{\pi} i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi} i} \right) = \frac{\pi}{4}(-4)(-2i) = 2\pi i$

[From Eqs. (i) and (ii)]: Now, $\left(\frac{|z|}{\text{amp}(z)} \right) = \frac{2\pi}{\pi/2} = 4.$

13. **(c)** $w_k = \frac{(\cos\alpha_k + i\sin\alpha_k)^2}{z_k} = \frac{z_k^2}{r_k^2 z_k} = \frac{z_k^2}{z_k \bar{z}_k z_k} = \frac{1}{\bar{z}_k}$

We have

$w_1 + w_2 + w_3 = \frac{1}{\bar{z}_1} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3} = \overline{\left(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right)} = 0$

$\therefore$ The origin O is the centroid of $\Delta A_1A_2A_3$.

EXERCISE - 4

1. **(1.67)** Since, $|z| + z = 3 + i$

Let $z = a + ib$, then

$|z| + z = 3 + i \Rightarrow \sqrt{a^2 + b^2} + a + ib = 3 + i$

Compare real and imaginary coefficients on both sides

$b = 1, \ \sqrt{a^2 + b^2} + a = 3$

$\sqrt{a^2 + 1} = 3 - a$

$a^2 + 1 = a^2 + 9 - 6a$

$6a = 8 \Rightarrow a = \frac{4}{3}$

Then, $|z| = \sqrt{\left(\frac{4}{3}\right)^2 + 1} = \sqrt{\frac{16}{9} + 1} = \frac{5}{3}$

2. **(2)** Let $t = \frac{z - \alpha}{z + \alpha}$

$\because$ t is purely imaginary number.

$\therefore \quad t + \bar{t} = 0 \quad \Rightarrow \quad \frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\alpha}{\bar{z}+\alpha} = 0$

$\Rightarrow \quad (z-\alpha)(\bar{z}+\alpha) + (\bar{z}-\alpha)(z+\alpha) = 0$

$\Rightarrow \quad \bar{z} z - \alpha^2 + z\bar{z} - \alpha^2 = 0$

$\Rightarrow \quad z\bar{z} - \alpha^2 = 0 \Rightarrow |z|^2 - \alpha^2 = 0$

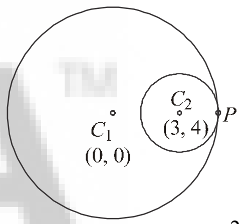
$\Rightarrow \quad \alpha^2 = 4 \Rightarrow \alpha = \pm 2$

3. **(0)** $|z_1| = 9, |z_2 - 3 - 4i| = 4$

z_1 lies on a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$

z_2 lies on a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$

So, minimum value of $|z_1 - z_2|$ is zero at point of contact (i.e. A)



4. **(4)** The given quadratic equation is $x^2 - 2x + 2 = 0$

Then, the roots of the this equation are $\frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$

Now, $\frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = i$

or $\frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = i$

So, $\frac{\alpha}{\beta} = \pm i$

Now, $\left(\frac{\alpha}{\beta}\right)^n = 1 \ (\pm i)^n = 1$

n must be a multiple of 4.

Hence, the required least value of $n = 4$.

5. **(1)** Let $z \in S$ then $z = \frac{\alpha + i}{\alpha - i}$

Since, z is a complex number and let $z = x + iy$

Then, $x + iy = \frac{(\alpha + i)^2}{\alpha^2 + 1}$ (by rationalisation)

$\Rightarrow x+iy=\dfrac{(\alpha^2-1)}{\alpha^2+1}+\dfrac{i(2\alpha)}{\alpha^2+1}$

Then compare both sides

$x=\dfrac{\alpha^2-1}{\alpha^2+1}$ ...(i)

$y=\dfrac{2\alpha}{\alpha^2+1}$ ...(ii)

Now squaring and adding equations (i) and (ii)

$\Rightarrow x^2+y^2=\dfrac{(\alpha^2-1)^2}{(\alpha^2+1)^2}+\dfrac{4\alpha^2}{(\alpha^2+1)^2}=1$

6. **(8)** Sum of roots $=\dfrac{3}{m^2+1}$

$\because$ sum of roots is greatest. $\therefore m=0$

Hence equation becomes $x^2-3x+1=0$

Now, $\alpha+\beta=3,\ \alpha\beta=1\Rightarrow|-\alpha-\beta|=\sqrt{5}$

$\left|\alpha^3-\beta^3\right|=\left|(\alpha-\beta)\left(\alpha^2+\beta^2+\alpha\beta\right)\right|=\sqrt{5}(9-1)=8\sqrt{5}$

7. **(96)** We have,

$|8z_2z_3+27z_3z_1+64z_1z_2|$

$=\left|z_1z_2z_3\left(\dfrac{8}{z_1}+\dfrac{27}{z_2}+\dfrac{64}{z_3}\right)\right|$

$=|z_1z_2z_3|\left|\dfrac{8}{z_1}+\dfrac{27}{z_2}+\dfrac{64}{z_3}\right|$

$=|z_1||z_2||z_3|\left|\dfrac{8\bar{z}_1}{|z_1|^2}+\dfrac{27\bar{z}_2}{|z_2|^2}+\dfrac{64\bar{z}_3}{|z_3|^2}\right|$

$=24|2\bar{z}_1+3\bar{z}_2+4\bar{z}_3|$

$=24|\overline{2z_1+3z_2+4z_3}|$

$=24\times4=96$

8. **(2)** $|\bar{z}\omega|=|\bar{z}||\omega|=|z||\omega|=|z\omega|=1$

$\text{Arg}(\bar{z}\omega)=\arg(\bar{z})+\arg(\omega)$

$=-\arg(z)+\arg\omega=-\dfrac{\pi}{2}\quad\therefore\bar{z}\omega=-1$

9. **(34)** Since, α and β are the roots of the equation $x^2-2x-1=0$, then

Sum of roots, $\alpha+\beta=2$ and

product of the roots $\alpha\beta=-1$

Since, $(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$

$\Rightarrow 4=\alpha^2+\beta^2-2\ \Rightarrow\ \alpha^2+\beta^2=6$

Now, $\alpha^2\beta^{-2}+\alpha^{-2}\beta^2=\dfrac{\alpha^2}{\beta^2}+\dfrac{\beta^2}{\alpha^2}=\dfrac{\alpha^4+\beta^4}{(\alpha\beta)^2}$

$\Rightarrow(\alpha^2+\beta^2)^2=6^2\ \Rightarrow\ \alpha^4+\beta^4+2\alpha^2\beta^2=36$

$\Rightarrow\alpha^4+\beta^4+2=36\ \Rightarrow\ \alpha^4+\beta^4=34$ ...(i)

$\Rightarrow\dfrac{\alpha^4+\beta^4}{(\alpha\beta)^2}=\dfrac{34}{(-1)^2}=34$

10. **(4)** $z^{1/3}=a-ib\Rightarrow z=(a-ib)^3$

$\therefore\quad x+iy=a^3+ib^3-3ia^2b-3ab^2.$

$\Rightarrow\quad x=a^3-3ab^2\Rightarrow\dfrac{x}{a}=a^2-3b^2$

and $y=b^3-3a^2b\Rightarrow\dfrac{y}{b}=b^2-3a^2$

So, $\dfrac{x}{a}-\dfrac{y}{b}=4(a^2-b^2)$

11. **(27)** Applying condition for common root, we get

$a^3+b^3+c^3=3abc\Rightarrow\left(\dfrac{a^3+b^3+c^3}{abc}\right)^3=27$

12. **(4)** Subtracting, we get $2hx=56$ or $hx=28$

Putting in any, $x^2=49$

$\therefore\left[\dfrac{28}{h}\right]^2=7^2\Rightarrow h=4(h>0)$