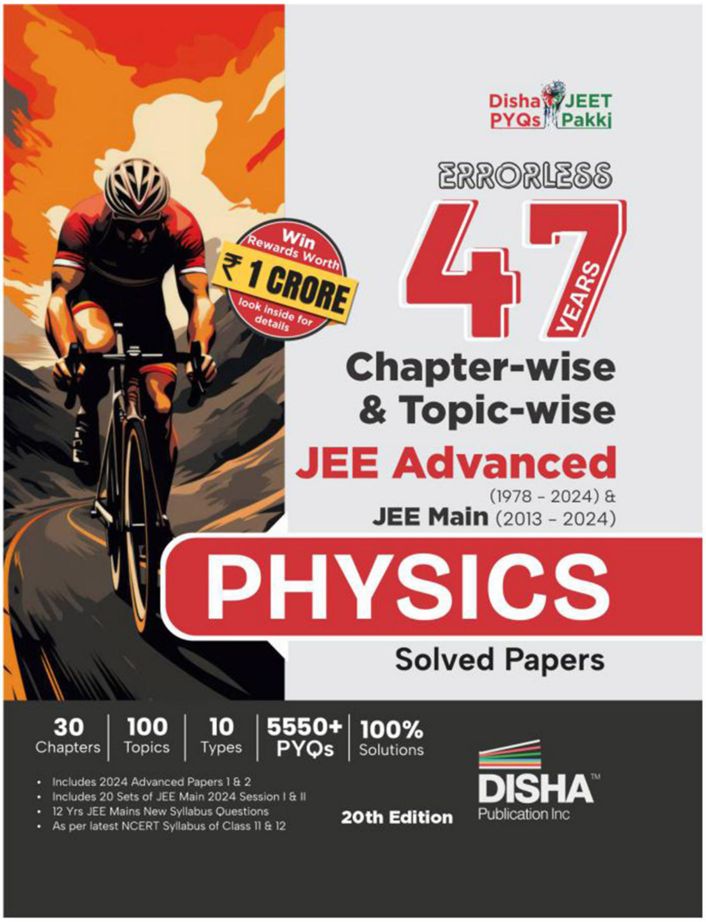
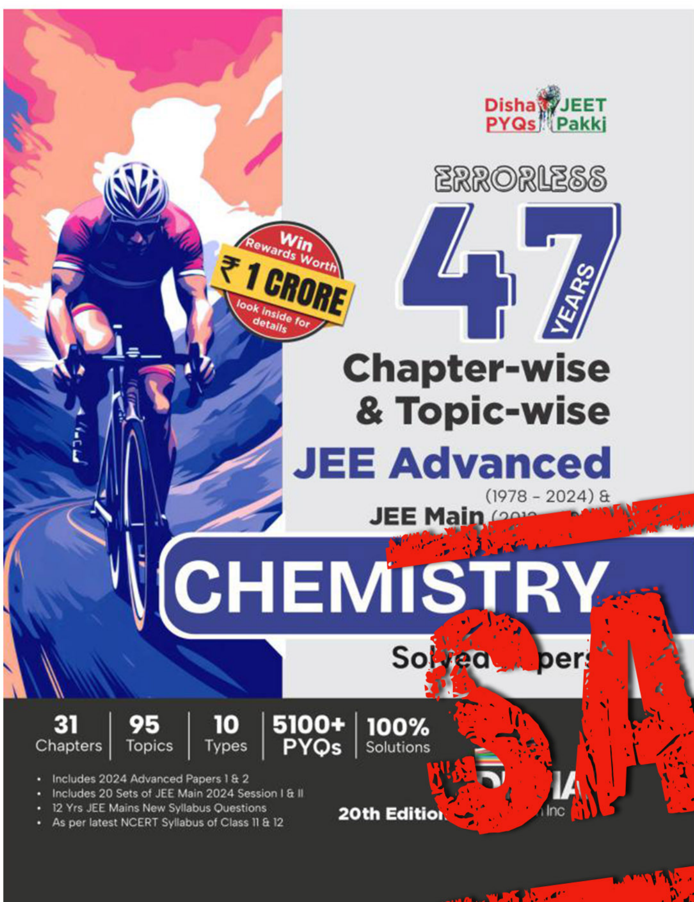

Disha PYQs JEET Pakki
ERRORLESS
47 YEARS
Chapter-wise
& Topic-wise
JEE Advanced
(1978 - 2024) &
JEE Main (2013 - 2024)
PHYSICS
Solved Papers
Win Rewards Worth ₹ 1 CRORE look inside for details
30 Chapters
100 Topics
10 Types
5550+ PYQs
100% Solutions
Includes 2024 Advanced Papers 1 & 2
Includes 20 Sets of JEE Main 2024 Session I & II
12 Yrs JEE Mains New Syllabus Questions
As per latest NCERT Syllabus of Class 11 & 12
20th Edition
DISHA Publication Inc


Disha PYQs JEET Pakki
ERRORLESS
47 YEARS
Chapter-wise
& Topic-wise
JEE Advanced
(1978 - 2024) &
JEE Main
CHEMISTRY
Win Rewards Worth ₹ 1 CRORE look inside for details
31 Chapters
95 Topics
10 Types
5100+ PYQs
100% Solutions
Includes 2024 Advanced Papers 1 & 2
Includes 20 Sets of JEE Main 2024 Session I & II
12 Yrs JEE Mains New Syllabus Questions
As per latest NCERT Syllabus of Class 11 & 12
20th Edition
SAMPLE


Disha PYQs JEET Pakki
ERRORLESS
47 YEARS
Chapter-wise
& Topic-wise
(1978 - 2024) &
Papers
Win Rewards Worth ₹ 1 CRORE look inside for details
Chapters
Includes 20 Sets of JEE Main 2024 Session I & II
12 Yrs JEE Mains New Syllabus Questions

Win Rewards Worth ₹ 1 CRORE look inside for details

Disha PYQs JEET Pakki

ERRORLESS

47 YEARS

Chapter-wise & Topic-wise JEE Advanced (1978 – 2024) & JEE Main (2013 – 2024)

PHYSICS

Solved Papers

SAMPLE

30 Chapters	100 Topics	10 Types	5550+ PYQs	100% Solutions

- Includes 2024 Advanced Papers I & 2
- Includes 20 Sets of JEE Main 2024 Session I & II
- 12 Yrs JEE Mains New Syllabus Questions
- As per latest NCERT Syllabus of Class 11 & 12

20th Edition

DISHA™ Publication Inc

DISHA Publication Inc.

A-23 FIEE Complex, Okhla Phase II

New Delhi-110020

Tel: 49842349/ 49842350

Edited By
Sanjeev Kumar Jha
Govind Thakur

Typeset By
DISHA DTP Team

Write To Us At

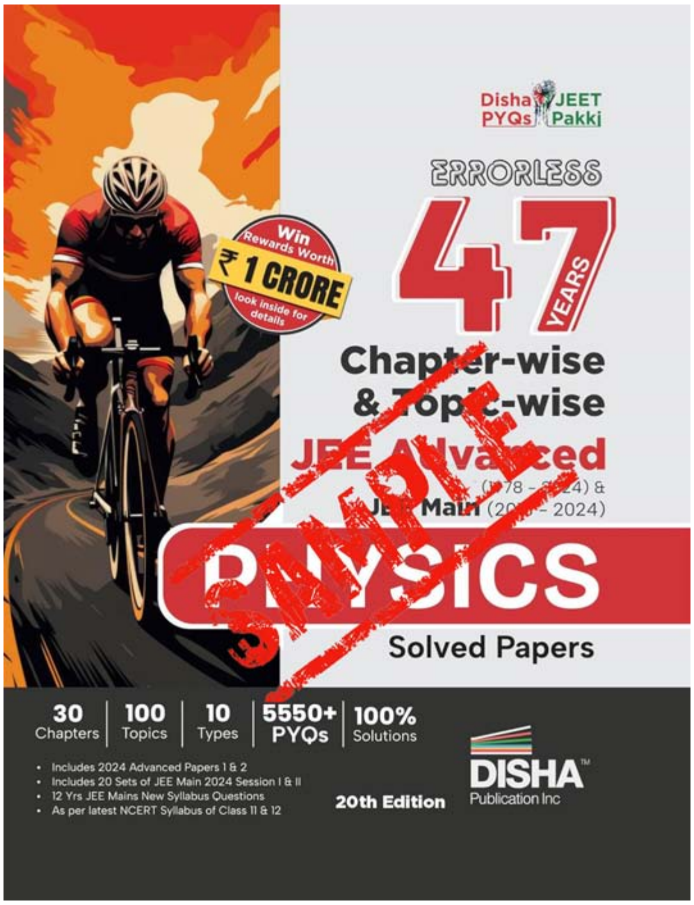
feedback_disha@aiets.co.in

www.dishapublication.com

DISHA™ Publication Inc

Free Sample Contents

This sample book is prepared from the book **"Errorless 47 Years Chapter-wise & Topic-wise JEE Advanced (1978 - 2024) & JEE Main (2013 - 2024) PHYSICS Solved Papers 20th Edition | PYQ Question Bank in NCERT Flow for JEE 2025".**



ISBN - 978-9362251558

MRP- 950/-

In case you like this content, you can buy the **Physical Book** or **E-book** using the ISBN provided above.

The book & e-book are available on all leading online stores.

Contents

A1 – A424

B1 – B408

Class XIIth B1–B188

Hints & Solutions (Class XIIth) B189–B408

5 Work, Energy and Power

Topic-1: Work

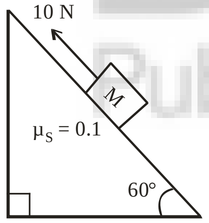
1 MCQs with One Correct Answer

1. A particle of mass m moves on a straight line with its velocity increasing with distance according to the equation $v = \alpha\sqrt{x}$, where α is a constant. The total work done by all the forces applied on the particle during its displacement from x = 0 to x = d, will be: **[April 9, 2024 (I)]**

(a) $\frac{m}{2\alpha^2 d}$ (b) $\frac{md}{2\alpha^2}$ (c) $\frac{m\alpha^2 d}{2}$ (d) $2m\alpha^2 d$

2. A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 60° by a force of 10 *N* parallel to the inclined surface as shown in figure. When the block is pushed up by 10 m along inclined surface, the work done against frictional force is : **[Jan. 30, 2024 (II)]**

$[g = 10\ m/s^2]$



(a) $5\sqrt{3}$ J (b) 5 J (c) 5×10^3 J (d) 10 J

3. A block of mass 100 kg slides over a distance of 10 m on a horizontal surface. If the co-efficient of friction between the surfaces is 0.4, then the work done against friction (in J) is: **[Jan. 29, 2024 (I)]**

(a) 4000 (b) 3900 (c) 4200 (d) 4500

4. A body of mass 0.5 kg travels on straight line path with velocity $v = (3x^2 + 4)$ m/s. The net workdone by the force during its displacement from $x = 0$ to $x = 2$ m is : **[Jan. 29, 2023 (II)]**

(a) 64 J (b) 60 J (c) 120 J (d) 128 J

5. A particle of mass 500 gm is moving in a straight line with velocity $\upsilon = b\,x^{5/2}$. The work done by the net force during its displacement from $x = 0$ to $x = 4$ m is :
(Take $b = 0.25\ m^{-3/2}\ s^{-1}$). **[June 29, 2022 (I), Similar July 25, 2022 (I)]**

(a) 2 J (b) 4 J (c) 8 J (d) 16 J

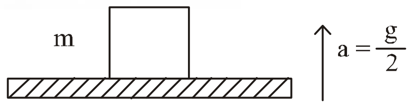
6. A porter lifts a heavy suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the work done by the porter in lowering the suitcase. (take $g = 9.8\ ms^{-2}$) **[July 22, 2021 (II)]**

(a) –62720.0 J (b) –627.2 J
(c) +627.2 J (d) 784.0 J

7. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box ? **[4 Sep. 2020 (II)]**

(a) 3280 J (b) 2780 J (c) 5690 J (d) 5250 J

8. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in fig. work done by normal reaction on block in time t is: **[10 Jan. 2019 I]**



(a) $-\frac{mg^2t^2}{8}$ (b) $\frac{mg^2t^2}{8}$
(c) 0 (d) $\frac{3mg^2t^2}{8}$

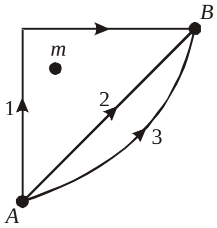
9. The work done on a particle of mass *m* by a force,

$$K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i} + \frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$$

(*K* being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius *a* about the origin in the *x* – *y* plane is **[Adv. 2013]**

(a) $\frac{2K\pi}{a}$ (b) $\frac{K\pi}{a}$ (c) $\frac{K\pi}{2a}$ (d) 0

10. If W_1, W_2 and W_3 represent the work done in moving a particle from *A* to *B* along three different paths 1,2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3 **[2003S]**



(a) $W_1 > W_2 > W_3$

(b) $W_1 = W_2 = W_3$

(c) $W_1 < W_2 < W_3$

(d) $W_2 > W_1 > W_3$

11. A force $F = -K(y\hat{i} + x\hat{j})$ (where K is a positive constant) acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive x axis to the point $(a, 0)$, and then parallel to the y axis to the point (a, a), The total work done by the force F on the particle is **[1998S - 2 Marks]**

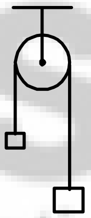
(a) $-2Ka^2$ (b) $2Ka^2$ (c) $-Ka^2$ (d) Ka^2

12. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is **[1985 - 2 Marks]**

(a) MgL (b) $MgL/3$ (c) $MgL/9$ (d) $MgL/18$

2 Integer Value Answer

13. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking g = 10 m/s^2, find the work done **(in joules)** by the string on the block of mass 0.36 kg during the first second after the system is released from rest. **[2009]**



3 Numeric Answer/ New Stem Based Questions

14. To maintain a speed of 80 km/h by a bus of mass 500 kg on a plane rough road for 4 km distance, the work done by the engine of the bus will be ________ KJ. [The coefficient of friction between tyre of bus and road is 0.04]. **[Main April 12, 2023 (I)]**

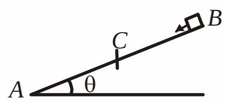
15. A small particle moves to position $5\hat{i} - 2\hat{j} + \hat{k}$ from its initial position $2\hat{i} + 3\hat{j} - 4\hat{k}$ under the action of force $5\hat{i} + 2\hat{j} + 7\hat{k}$ N. The value of work done will be ______ J. **[Main Feb. 1, 2023 (I)]**

16. A force F = (5 + 3y^2) acts on a particle in the y-direction, where F is Newton and y is in meter. The work done by the force during a displacement from y = 2m to y = 5m is _______ J. **[Main Feb. 1, 2023 (II)]**

17. Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angle 45° and 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is ________ . **[Main Aug. 27, 2021 (I), Similar April 9, 2024 (II)]**

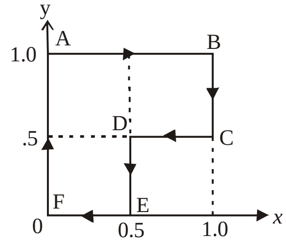
18. A force of F = (5y + 20) $\hat{j}$ N acts on a particle. The work done by this force when the particle is moved from y = 0 m to y = 10 m is _______ J. **[Main 2014 (S) July 25, 2021 (II), Similar April 15, 2023 (I)]**

19.



A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If $BC = 2AC$, the coefficient of friction is given by $\mu = k \tan\theta$. The value of k is _________. **[Main 2 Sep. 2020 (I)]**

20. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a forced $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})N$, where x and y are in meter and $\alpha = -1Nm^{-1}$. The work done on the particle by this force $\vec{F}$ will be ___ Joule. **[Adv. 2019]**



Topic-2: Energy

1 MCQs with One Correct Answer

1. Three bodies A, B and C have equal kinetic energies and their masses are 400 g, 1.2 kg and 1.6 kg respectively. The ratio of their linear momenta is : **[April 8, 2024 (I)]**

(a) $1:\sqrt{3}:2$ (b) $1:\sqrt{3}:\sqrt{2}$

(c) $\sqrt{2}:\sqrt{3}:1$ (d) $\sqrt{3}:\sqrt{2}:1$

2. A stationary particle breaks into two parts of masses m_A and m_B which move with velocities v_A and v_B respectively. The ratio of their kinetic energies $(K_B : K_A)$ is : **[April 8, 2024 (I)]**

(a) $v_B : v_A$ (b) $m_B : m_A$

(c) $m_B v_B : m_A v_A$ (d) 1 : 1

3. A bullet of mass 50 g is fired with a speed 100 m/s on a plywood and emerges with 40 m/s. The percentage loss of kinetic energy is : **[April 6, 2024 (I)]**

(a) 32% (b) 44% (c) 16% (d) 84%

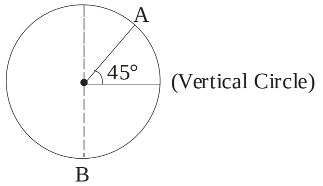
4. Four particles A, B, C, D of mass $\frac{m}{2}$, m, 2m, 4m, have same momentum, respectively. The particle with maximum kinetic energy is : **[April 6, 2024 (I)]**

(a) D (b) C (c) A (d) B

5. When kinetic energy of a body becomes 36 times of its original value, the percentage increase in the momentum of the body will be : **[April 6, 2024 (II)]**

(a) 500% (b) 600% (c) 6% (d) 60%

6. A body of m kg slides from rest along the curve of vertical circle from point A to B in friction less path. The velocity of the body at B is : **[April 4, 2024 (II)]**



(given, R = 14 m, g = 10 m/s^2 and $\sqrt{2} = 1.4$)

(a) 19.8 m/s (b) 21.9 m/s (c) 16.7 m/s (d) 10.6m/s

7. A simple pendulum of length 1 m has a wooden bob of mass 1kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of 2×10^2 ms^{-1}. The bullet gets embedded into the bob. The height to which the bob rises before swinging back is. (use g = 10m/s^2) **[Feb. 1, 2024 (I)]**

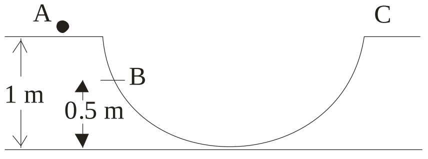
(a) 0.30 m (b) 0.20 m (c) 0.35 m (d) 0.40m

8. An artillery piece of mass M_1 fires a shell of mass M_2 horizontally. Instantaneously after the firing, the ratio of kinetic energy of the artillery and that of the shell is : **[Jan. 31, 2024 (I)]**

(a) $\frac{M_1}{(M_1+M_2)}$ (b) $\frac{M_2}{(M_1+M_2)}$

(c) $\frac{M_1}{M_2}$ (d) $\frac{M_2}{M_1}$

9. A particle is placed at the point A of a frictionless track ABC as shown in figure. It is gently pushed towards right. The speed of the particle when it reaches the point B is: (Take g = 10 m/s^2). **[Jan. 30, 2024 (I)]**



(a) 20 m/s (b) $\sqrt{10}$m/s (c) $2\sqrt{10}$m/s (d) 10 m/s

10. The potential energy function (in J) of a particle in a region of space is given as U = (2x^2 + 3y^3 + 2z). Here x, y and z are in meter. The magnitude of x - component of force (in N) acting on the particle at point P (1, 2, 3) m is : **[Jan. 29, 2024 (I)]**

(a) 6 (b) 2 (c) 8 (d) 4

11. Two bodies of mass 4 g and 25 g are moving with equal kinetic energies. The ratio of magnitude of their linear momentum is : **[Jan. 27, 2024 (I)]**

(a) 3 : 5 (b) 5 : 4 (c) 2 : 5 (d) 4 : 5

12. A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle (θ) of thread deflection in the extreme position will be : **[Jan. 27, 2024 (II)]**

(a) $\tan^{-1}(\sqrt{2})$ (b) $2\tan^{-1}\left(\frac{1}{2}\right)$

(c) $\tan^{-1}\left(\frac{1}{2}\right)$ (d) $2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

13. Two bodies are having kinetic energies in the ratio 16 : 9. If they have same linear momentum, the ratio of their masses respectively is: **[April 13, 2023 (I)]**

(a) 4 : 3 (b) 3 : 4 (c) 16 : 9 (d) 9 : 16

14. A small block of mass 100 g is tied to a spring of spring constant 7.5 N/m and length 20 cm. The other end of spring is fixed at a particular point A. If the block moves in a circular path on a smooth horizontal surface with constant angular velocity 5 rad/s about point A, then tension in the spring is **[April 6, 2023 (I)]**

(a) 1.5 N (b) 0.75 N (c) 0.25 N (d) 0.50 N

15. A body of mass 200 g is tied to a spring of spring constant 12.5 N/m, while the other end of spring is fixed at point *O*. If the body moves about *O* in a circular path on a smooth horizontal surface with constant angular speed 5 rad/s, then the ratio of extension in the spring to its natural length will be : **[Jan. 24, 2023 (II)]**

(a) 1 : 2 (b) 1 : 1 (c) 2 : 3 (d) 2 : 5

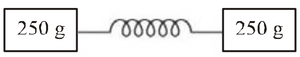
16. A ball is projected with kinetic energy E, at an angle of 60° to the horizontal. The kinetic energy of this ball at the highest point of its flight will become : **[July 29, 2022 (I)]**

(a) Zero (b) $\frac{E}{2}$ (c) $\frac{E}{4}$ (d) E

17. If momentum of a body is increased by 20%, then its kinetic energy increases by : **[July 29, 2022 (II), Similar Main April 8, 2023 (I)]**

(a) 36% (b) 40% (c) 44% (d) 48%

18. As per the given figure, two blocks each of mass 250 g are connected to a spring of spring constant 2 Nm^{-1}. If both are given velocity *v* in opposite directions, then maximum elongation of the spring is : **[July 26, 2022 (I)]**



(a) $\frac{v}{2\sqrt{2}}$ (b) $\frac{v}{2}$ (c) $\frac{v}{4}$ (d) $\frac{v}{\sqrt{2}}$

19. A body of mass 8 kg and another of mass 2 kg are moving with equal kinetic energy. The ratio of their respective momenta will be : **[July 26, 2022 (II)]**

(a) 1 : 1 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

20. Two bodies A and B of masses 5 kg and 8 kg are moving such that the momentum of body B is twice that of the body A. The ratio of their kinetic energies will be: **[June 30, 2022 (I)]**

(a) 4 : 5 (b) 2 : 5 (c) 5 : 4 (d) 5 : 2

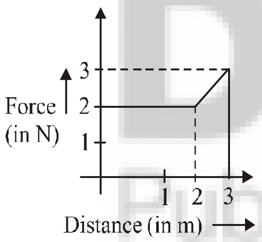
21. An object is thrown vertically upwards. At its maximum height, which of the following quantity becomes zero ? **[June 26, 2022 (I)]**
(a) Momentum (b) Potential energy
(c) Acceleration (d) Force

22. A particle experiences a variable force $\vec{F} = (4x\hat{i} + 3y^2\hat{j})$ in a horizontal x-y plane. Assume distance in meters and force is newton. If the particle moves from point (1, 2) to point (2, 3) in the x–y plane, the kinetic energy changes by **[June 24, 2022 (I)]**
(a) 50.0 J (b) 12.5 J (c) 25.0 J (d) 0 J

23. A body of mass 'm' dropped from a height 'h' reaches the ground with a speed of $0.8\sqrt{gh}$. The value of workdone by the air-friction is : **[Sep. 1, 2021 (II)]**
(a) –0.68 mgh (b) mgh
(c) 1.64 mgh (d) 0.64 mgh

24. If the Kinetic energy of a moving body becomes four times its initial Kinetic energy, then the percentage change in its momentum will be : **[July 20, 2021 (II)]**
(a) 100% (b) 200% (c) 300% (d) 400%

25. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is : **[8 April 2019 I, 7 Jan. 2020 II]**



(a) 4 J (b) 2.5 J (c) 6.5 J (d) 5 J

26. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be: **[12 April 2019 II]**
(a) n (b) $\frac{1}{n^2}$ (c) $\frac{1}{n}$ (d) n^2

27. A body of mass 1 kg falls freely from a height of 100m, on a platform of mass 3 kg which is mounted on a spring having spring constant k = 1.25×10^6 N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that g = 10 ms^{-2}, the value of x will be close to : **[11 April 2019 I]**
(a) 40 cm (b) 4 cm (c) 80 cm (d) 8 cm

28. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{\text{th}}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be: **[9 April 2019 I]**
(a) $\frac{MgL}{2n^2}$ (b) $\frac{MgL}{n^2}$ (c) $\frac{2MgL}{n^2}$ (d) $nMgL$

29. A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by: **[9 April 2019 II]**
(a) $\frac{v^2}{g}$ (b) $\frac{2v^2}{7g}$ (c) $\frac{2v^2}{5g}$ (d) $\frac{v^2}{2g}$

30. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement? **[10 Jan. 2019 II]**
(a) 9 J (b) 12 J (c) 10 J (d) 15 J

31. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds? **[9 Jan. 2019 II]**
(a) 850 J (b) 950 J (c) 875 J (d) 900 J

32. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is: **[2018]**
(a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$ (c) zero (d) $-\frac{3}{2}\frac{k}{a^2}$

33. Two particles of the same mass m are moving in circular orbits because of force, given by $F(r) = \frac{-16}{r} - r^3$. The first particle is at a distance r = 1, and the second, at r = 4. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to **[Online April 16, 2018]**
(a) 10^{-1} (b) 6×10^{-2} (c) 6×10^2 (d) 3×10^{-3}

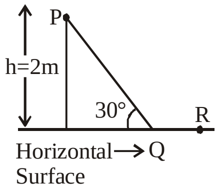
34. A body of mass m = 10^{-2} kg is moving in a medium and experiences a frictional force F = –kv^2. Its intial speed is $v_0 = 10$ ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be: **[2017]**
(a) 10^{-4} kg m^{-1} (b) 10^{-1} kg m^{-1} s^{-1}
(c) 10^{-3} kg m^{-1} (d) 10^{-3} kg s^{-1}

35. An object is dropped from a height h from the ground. Every time it hits the ground it looses 50% of its kinetic energy. The total distance covered as t → ∞ is **[Online April 8, 2017]**
(a) 3h (b) ∞ (c) $\frac{5}{3}$h (d) $\frac{8}{3}$h

36. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 second will be **[2017]**
(a) 9 J (b) 18 J (c) 4.5 J (d) 22 J

37. A point particle of mass m, moves long the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest from the point P and it comes to rest at a

point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.
The value of the coefficient of friction μ and the distance x (= QR), are, respectively close to : **[2016]**



(a) 0.29 and 3.5 m
(b) 0.29 and 6.5 m
(c) 0.2 and 6.5 m
(d) 0.2 and 3.5 m

38. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m, 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take g = 9.8 ms^{-2} : **[2016]**
(a) 9.89×10^{-3} kg (b) 12.89×10^{-3} kg
(c) 2.45×10^{-3} kg (d) 6.45×10^{-3} kg

39. A particle is moving in a circle of radius r under the action of a force F = αr^2 which is directed towards centre of the circle. Total mechanical energy (kinetic energy + potential energy) of the particle is (take potential energy = 0 for r = 0) : **[Online April 11, 2015]**

(a) $\frac{1}{2}\alpha r^3$ (b) $\frac{5}{6}\alpha r^3$ (c) $\frac{4}{3}\alpha r^3$ (d) αr^3

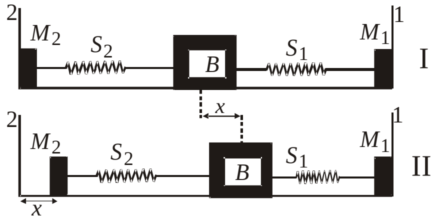
40. A bullet looses $\left(\frac{1}{n}\right)^{th}$ of its velocity passing through one plank. The number of such planks that are required to stop the bullet can be: **[Online April 19, 2014]**

(a) $\frac{n^2}{2n-1}$ (b) $\frac{2n^2}{n-1}$ (c) infinite (d) n

41. Two springs of force constants 300 N/m (Spring A) and 400 N/m (Spring B) are joined together in series. The combination is compressed by 8.75 cm. The ratio of energy stored in A and B is $\frac{E_A}{E_B}$. Then $\frac{E_A}{E_B}$ is equal to : **[Online April 9, 2013]**

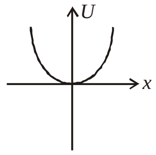
(a) $\frac{4}{3}$ (b) $\frac{16}{9}$ (c) $\frac{3}{4}$ (d) $\frac{9}{16}$

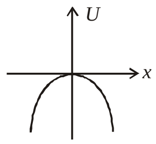
42. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$, respectively (see fig. I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio y/x is – **[2008]**

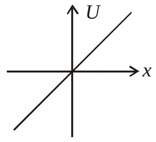


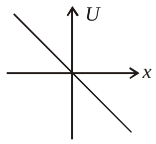
(a) 4
(b) 2
(c) 1/2
(d) 1/4

43. A particle is acted by a force $F = kx$, where k is a +ve constant. Its potential energy at $x = 0$ is zero. Which curve correctly represents the variation of potential energy of the block with respect to x **[2004S]**

(a) 

(b) 

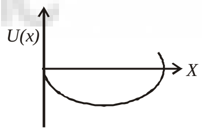
(c) 

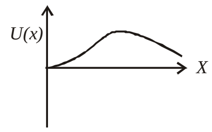
(d) 

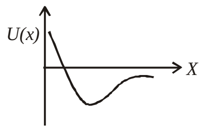
44. An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is **[2002S]**

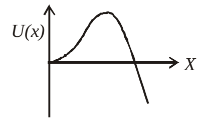
(a) $\frac{4Mg}{k}$ (b) $\frac{2Mg}{k}$ (c) $\frac{Mg}{k}$ (d) $\frac{Mg}{2k}$

45. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is **[2002S]**

(a) 

(b) 

(c) 

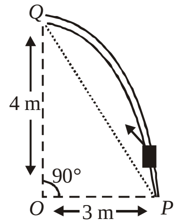
(d) 

46. A spring of force-constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force-constant of **[1999S - 2 Marks]**
(a) $(2/3)k$ (b) $(3/2)k$ (c) $3k$ (d) $6k$

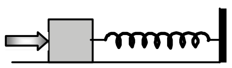
47. Two masses of 1 gm and 4 gm are moving with equal kinetic energies. The ratio of the magnitudes of their linear momenta is **[1980]**
(a) 4 : 1 (b) $\sqrt{2}$: 1 (c) 1 : 2 (d) 1 : 16

2 Integer Value Answer

48. Consider an elliptical shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictionless losses, the kinetic energy of the block when it reaches Q is $(n \times 10)$ joules. The value of n is (take acceleration due to gravity = 10 ms^{-2}) **[Adv. 2014]**



49. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is **[2011]**



3 Numeric Answer

50. A body of mass 5 kg is moving with a momentum of 10 kg ms^{-1}. Now a force of 2 N acts on the body in the direction of its motion for 5 s. The increase in the kinetic energy of the body is ______ J. **[Main April 8, 2023 (II)]**

51. A particle of mass 10 g moves in a straight line with retardation 2x, where x is the displacement in SI unts. Its loss of kinetic energy for above displacement is $\left(\frac{10}{x}\right)^{-n}$ J. The value of n will be _____. **[Main April 6, 2023 (I)]**

52. A body is dropped on ground from a height 'h_1' and after hitting the ground, it rebounds to a height 'h_2'. If the ratio of velocities of the body just before and after hitting ground is 4, then percentage loss in kinetic energy of the body is $\frac{x}{4}$. The value of x is______. **[Main April 6, 2023 (II)]**

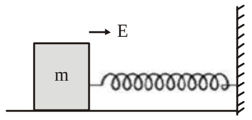
53. A block is fastened to a horizontal spring. The block is pulled to a distance x = 10 cm from its equilibrium position (at x = 0) on a frictionless surface from rest. The energy of the block at x = 5 cm is 0.25 J. The spring constant of the spring is ______ Nm^{-1}. **[Main Feb. 1, 2023 (II)]**

54. A lift of mass M = 500 kg is descending with speed of 2 ms^{-1}. Its supporting cable begins to slip thus allowing it to fall with a constant acceleration of 2 ms^{-2}. The kinetic energy of the lift at the end of fall through to a distance of 6 m will be ____kJ. **[Main Jan. 31, 2023 (I)]**

55. A 0.4 kg mass takes 8s to reach ground when dropped from a certain height 'P' above surface of earth. The loss of potential energy in the last second of fall is ____ J. [Take g = 10 m/s^2] **[Main Jan. 29, 2023 (I)]**

56. A spherical body of mass 2 kg starting from rest acquires a kinetic energy of 10000 J at the end of 5th second. The force acted on the body is ____ N. **[Main Jan. 24, 2023 (I)]**

57. A block of mass 'm' (as shown in figure) moving with kinetic energy E compresses a spring through a distance 25 cm when, its speed is halved. The value of spring constant of used spring will be nE Nm^{-1} for n = ___________. **[Main July 28, 2022 (I)]**



Smooth surface

58. An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 40,000 kg is moving with a speed of 72 kmh^{-1} when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, the spring constant is _______ $\times 10^5$ N/m. **[Main Sep. 1, 2021 (II)]**

59. A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, then the fractional change in the kinetic energy will be x : 4 where x =__________. **[NA, Aug. 31, 2021 (I)]**

60. A uniform chain of length 3 meter and mass 3 kg overhangs a smooth table with 2 meter laying on the table. If k is the kinetic energy of the chain in joule as it completely slips off the table, then the value of k is _________ . (Take g = 10 m/s^2) **[Main Aug. 26, 2021 (I)]**

61. A ball of mass 4 kg, moving with a velocity of 10ms^{-1}, collides with a spring of length 8 m and force constant 100 Nm^{-1}. The length of the compressed spring is x m. The value of x, to the nearest integer, is ________. **[Main March 18, 2021 (II)]**

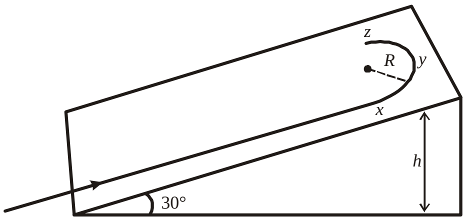
62. Two particles having masses 4 g and 16 g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their momentum is n : 2. The value of n will **[Feb. 25, 2021 (II)]**

63. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies $(K.E.)_A : (K.E.)_B$ will be $\frac{A}{1}$, so the value of A will be ________. **[Feb. 24, 2021 (II)]**

64. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F (in N) is (g = 10 ms^{-2}) __________. **[Main 3 Sep. 2020 (I)]**

6 MCQs with One or More than One Correct Answer

65. A student skates up a ramp that makes an angle 30° with the horizontal. He/she starts (as shown in the figure) at the bottom of the ramp with speed v_0 and wants to turn around over a semicircular path xyz of radius R during which he/she reaches a maximum height h (at point y) from the ground as shown in the figure. Assume that the energy loss is negligible and the force required for this turn at the highest point is provided by his/her weight only. Then (g is the acceleration due to gravity) **[Adv. 2020]**



(a) $v_0^2 - 2gh = \frac{1}{2}gR$

(b) $v_0^2 - 2gh = \frac{\sqrt{3}}{2}gR$

(c) the centripetal force required at points x and z is zero

(d) the centripetal force required is maximum at points x and z

66. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true? **[Adv. 2018]**

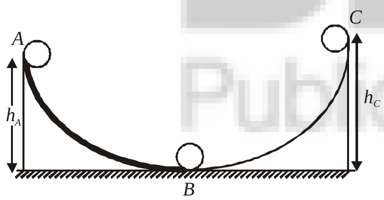
(a) The force applied on the particle is constant

(b) The speed of the particle is proportional to time

(c) The distance of the particle from the origin increases linearly with time

(d) The force is conservative

67. A small ball starts moving from A over a fixed track as shown in the figure. Surface AB has friction. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C, respectively. Then **[2006 - 5M, –1]**



(a) $h_A > h_C;\ K_B > K_C$ (b) $h_A > h_C;\ K_C > K_A$

(c) $h_A = h_C;\ K_B = K_C$ (d) $h_A < h_C;\ K_B > K_C$

68. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is **[1998S - 2 Marks]**

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$

(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

69. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that : **[1987 - 2 Marks]**

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant.

(d) it moves in a circular path.

7 Match the Following

70. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 constants). Match the potential energies in column I to the corresponding statement(s) in column II.

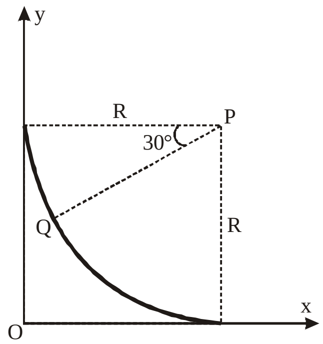
Column I	Column II
(A) $U_1(x) = \frac{U_0}{2}\left[1 - \left(\frac{x}{a}\right)^2\right]^2$	(p) The force acting on the particle is zero at $x = a$
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(q) The force acting on the particle is zero at $x = 0$
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(r) The force acting on the particle is zero at $x = -a$
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(s) The particle experiences an attractive force towards $x = 0$ in the region $\|x\| < a$
	(t) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$

8 Comprehension/Passage Based Questions

Passage

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J.

(Take the acceleration due to gravity, $g = 10\ ms^{-2}$) **[Adv. 2013]**



71. The magnitude of the normal reaction that acts on the block at the point Q is

(a) 7.5 N (b) 8.6 N (c) 11.5 N (d) 22.5 N

72. The speed of the block when it reaches the point Q is

(a) $5\ ms^{-1}$ (b) $10\ ms^{-1}$ (c) $10\sqrt{3}\ ms^{-1}$ (d) $20\ ms^{-1}$

9 Statement/Assertion and Reason Type Questions

73. **STATEMENT–1 :** A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

STATEMENT–2 : The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination. **[2007]**

(a) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1

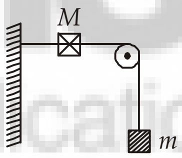
(b) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation for Statement–1

(c) Statement–1 is True, Statement–2 is False

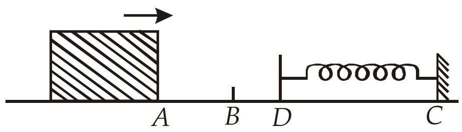
(d) Statement–1 is False, Statement–2 is True

10 Subjective Problems

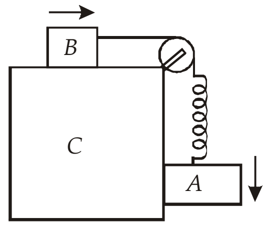
74. A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2m from the wall, has a point mass M = 2kg attached to it at a distance of 1m from the wall. A mass m = 0.5 kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released? **[1985 - 6 Marks]**



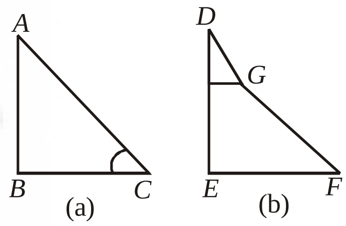
75. A 0.5 kg block slides from the point A (see Fig) on a horizontal track with an initial speed of 3 m/s towards a weightless horizontal spring of length 1 m and force constant 2 Newton/m. The part AB of the track is frictionless and the part BC has the coefficients of static and kinetic friction as 0.22 and 0 2 respectively. If the distances AB and BD are 2 m and 2.14 m respectively, find the total distance through which the block moves before it comes to rest completely.
(Take $g = 10 m/s^2$) **[1983 - 7 Marks]**



76. Two blocks A and B are connected to each other by a string and a spring ; the string passes over a frictionless pulley as shown in the figure. Block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C, both with the same uniform speed. The coefficient of friction between the surfaces of blocks is 0.2. Force constant of the spring is 1960 newtons/m. If mass of block A is 2 Kg., calculate the mass of block B and the energy stored in the spring. **[1982 - 5 Marks]**



77. In the figures (a) and (b) AC, DG and GF are fixed inclined planes, $BC = EF = x$ and $AB = DE = y$. A small block of mass M is released from the point A. It slides down AC and reaches C with a speed V_c. The same block is released from rest from the point D. It slides down DGF and reaches the point F with speed V_F. The coefficients of kinetic frictions between the block and both the surface AC and DGF are μ. **[1980]**



Calculate V_C and V_F.

78. When a ball is thrown up, the magnitude of its momentum decreases and then increases. Does this violate the conservation of momentum principle? **[1979]**

79. A spring of force constant k is cut into three equal parts. What is force constant of each part? **[1978]**

80. A bullet is fired from a rifle. If the rifle recoils freely, determine whether the kinetic energy of the rifle is greater than, equal or less than that of the bullet. **[1978]**

Topic-3: Power

1 MCQs with One Correct Answer

1. A body is moving unidirectionally under the influence of a constant power source. Its displacement in time t is proportional to : **[April 5, 2024 (II)]**

(a) t^2 (b) $t^{2/3}$ (c) $t^{3/2}$ (d) t

2. A body of mass 2 kg begins to move under the action of a time dependent force given by $\vec{F} = \left(6t\hat{i} + 6t^2\hat{j}\right)$ N . The power developed by the force at the time t is given by: **[Jan. 31, 2024 (II)]**

(a) $(6t^4 + 9t^5)$ W (b) $(3t^3 + 6t^5)$ W
(c) $(9t^5 + 6t^3)$ W (d) $(9t^3 + 6t^5)$ W

3. The ratio of powers of two motors is $\frac{3\sqrt{x}}{\sqrt{x}+1}$, that are capable of raising 300 kg water in 5 minutes and 50 kg water in 2 minutes respectively from a well of 100 m deep. The value of x will be **[April 13, 2023 (I)]**

(a) 2 (b) 4
(c) 2.4 (d) 16

4. Sand is being dropped from a stationary dropper at a rate of 0.5 kgs^{-1} on a conveyor belt moving with a velocity of 5 ms^{-1}. The power needed to keep belt moving with the same velocity will be : **[July 27, 2022 (I)]**
(a) 1.25 W (b) 2.5 W (c) 6.25 W (d) 12.5 W

5. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration (a) is varying with time t as a = k^2rt^2 where k is a constant. The power delivered to the particle by the force acting on it is given as **[June 28, 2022 (I)]**
(a) zero (b) $mk^2r^2t^2$ (c) mk^2r^2t (d) mk^2rt

6. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, g = 10 ms^{-2})
[7 Jan. 2020 (I), Similar Main April 10, 2023 (II)]
(a) 1.7 ms^{-1} (b) 1.9 ms^{-1} (c) 1.5 ms^{-1} (d) 2.0 ms^{-1}

7. A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by n^2Rt^2 where n is a constant. The power delivered to the particle by the force acting on it, is :
[Online April 10, 2016]
(a) $\frac{1}{2}Mn^2R^2t^2$ (b) Mn^2R^2t
(c) MnR^2t^2 (d) MnR^2t

8. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be most likely proportional to
[Online April 25, 2013]
(a) v^4 (b) v^2 (c) v^3 (d) v

9. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$ where k is a constant. The power delivered to the particles by the force acting on it is:
[1994 - 1 Mark]
(a) $2\pi\, mk^2r^2t$ (b) mk^2r^2t
(c) $\frac{(mk^4r^2t^5)}{3}$ (d) zero

10. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to
[Main 1984- 2 Marks; June 28, 2022 (I)]
(a) $t^{1/2}$ (b) $t^{3/4}$ (c) $t^{3/2}$ (d) t^2

11. If a machine is lubricated with oil **[1980]**
(a) the mechanical advantage of the machine increases.
(b) the mechanical efficiency of the machine increases.
(c) both its mechanical advantage and efficiency increase.
(d) its efficiency increases, but its mechanical advantage decreases.

2 Integer Value Answer

12. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is **[Adv. 2013]**

3 Numeric Answer

13. A block of mass 5 kg starting from rest pulled up on a smooth incline plane making an angle of 30° with horizontal with an affective acceleration of 1 ms^{-2}. The power delivered by the pulling force at t = 10 s from the start is _____ W. [Use g = 10 ms^{-2}] **[Main April 10, 2023 (I)]**
(calculate the nearest integer value)

14. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = (t\hat{i} + 3t^2\hat{j})N$ where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The power developed by above force, at the time t = 2s. will be ________ W.
[Main Jan. 24, 2023 (II)]

15. A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) ________.
[Main 5 Sep. 2020 (II)]

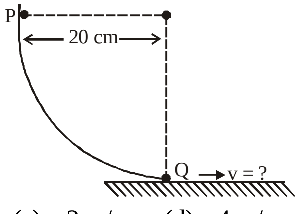
Topic-4: Collisions

1 MCQs with One Correct Answer

1. If a rubber ball falls from a height h and rebounds upto the height of h/2. The percentage loss of total energy of the initial system as well as velocity ball before it strikes the ground, respectively, are : **[April 4, 2024 (I)]**
(a) 50%, $\sqrt{\frac{gh}{2}}$ (b) 50%, $\sqrt{gh}$
(c) 40%, $\sqrt{2gh}$ (d) 50%, $\sqrt{2gh}$

2. A particle of mass m moving with velocity v collides with a stationary particle of mass 2m. After collision, they stick together and continue to move together with velocity
[April 10, 2023 (I)]
(a) v (b) $\frac{v}{2}$ (c) $\frac{v}{3}$ (d) $\frac{v}{4}$

3. As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest.
Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be : (g = 10 m/s^2)
[Jan. 30, 2023 (I)]



(a) 0 (b) 0.25 m/s (c) 2 m/s (d) 4 m/s

4. A ball of mass 200 g rests on a vertical post of height 20 m. A bullet of mass 10 g, travelling in horizontal direction, hits the centre of the ball. After collision both travels independently. The ball hits the ground at a distance 30 m and the bullet at a distance of 120 m from the foot of the

post. The value of initial velocity of the bullet will be (if g = 10 m/s^2) : **[Jan. 30, 2023 (I)]**
(a) 120 m/s (b) 60 m/s (c) 400 m/s (d) 360 m/s

5. A bag of sand of mass 9.8 kg is suspended by a rope. A bullet of 200 g travelling with speed 10 ms^{-1} gets embedded in it, then loss of kinetic energy will be **[July 25, 2022 (II)]**
(a) 4.9 J (b) 9.8 J (c) 14.7 (d) 19.6 J

6. Two billiard balls of mass 0.05 kg each moving in opposite directions with 10 ms^{-1} collide and rebound with the same speed. If the time duration of contact is t = 0.005 s, then what is the force exerted on the ball due to each other? **[July 25, 2022 (II)]**
(a) 100 N (b) 200 N (c) 300 N (d) 4000 N

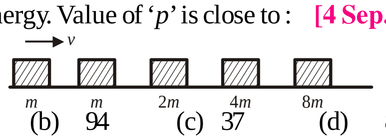
7. A body of mass M at rest explodes into three pieces, in the ratio of masses 1 : 1 : 2. Two smaller pieces fly off perpendicular to each other with velocities of 30 ms^{-1} and 40 ms^{-1} respectively. The velocity of the third piece will be : **[June 29, 2022 (I)]**
(a) 15 ms^{-1} (b) 25 ms^{-1} (c) 35 ms^{-1} (d) 50 ms^{-1}

8. What percentage of kinetic energy of a moving particle is transferred to a stationary particle when it strikes the stationary particle of 5 times its mass? (Assume the collision to be head-on elastic collision) **[June 27, 2022 (I)]**
(a) 50.0% (b) 66.6% (c) 55.5% (d) 33.3%

9. A block moving horizontally on a smooth surface with a speed of 40 m/s splits into two parts with masses in the ratio of 1:2. If the smaller part moves at 60 m/s in the same direction, then the fractional change in kinetic energy is : **[Aug. 31, 2021 (II)]**
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

10. A object of mass m_1 collides with another object of mass m_2, which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is: **[March 18, 2021 (II)]**
(a) 2 : 1 (b) 3 : 1 (c) 1 : 2 (d) 1 : 1

11. Blocks of masses *m*, 2*m*, 4*m* and 8*m* are arranged in a line on a frictionless floor. Another block of mass *m*, moving with speed *v* along the same line (see figure) collides with mass *m* in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass 8*m* starts moving the total energy loss is *p*% of the original energy. Value of '*p*' is close to : **[4 Sep. 2020 (I)]**



(a) 77 (b) 94 (c) 37 (d) 87

12. A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take $g = 10$ m/s^2. Assume there is no rotational motion and losss of energy after the collision is negligiable.] **[3 Sep. 2020 (II)]**
(a) 20 J (b) 21 J (c) 19 J (d) 23 J

13. A particle of mass *m* with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3 m at rest. It moves with a velocity $v\hat{j}$ after collision, then, *v* is given by : **[2 Sep. 2020 (I)]**
(a) $v = \sqrt{\frac{2}{3}}u$ (b) $v = \frac{u}{\sqrt{3}}$ (c) $v = \frac{u}{\sqrt{2}}$ (d) $v = \frac{1}{\sqrt{6}}u$

14. Two particles of equal mass *m* have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is: **[9 Jan. 2020 I, Similar Sep. 12, 2020 (I)]**
(a) $\frac{1}{3}mu^2$ (b) $\frac{1}{8}mu^2$ (c) $\frac{3}{4}mu^2$ (d) $\sqrt{\frac{2}{3}}mu^2$

15. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is : **[12 April 2019 I]**
(a) 0.28 ms^{-1} (b) 0.20 ms^{-1}
(c) 0.47 ms^{-1} (d) 0.14 ms^{-1}

16. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body? **[9 April 2019 I]**
(a) 1.0 kg (b) 1.5 kg
(c) 1.8 kg (d) 1.2 kg

17. A particle of mass '*m*' is moving with speed '2*v*' and collides with a mass '2*m*' moving with speed '*v*' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass '*m*', which move at angle 45° with respect to the original direction. **[9 April 2019 II]**
The speed of each of the moving particle will be:
(a) $\sqrt{2}\,v$ (b) $2\sqrt{2}\,v$ (c) $v/(2\sqrt{2})$ (d) $v/\sqrt{2}$

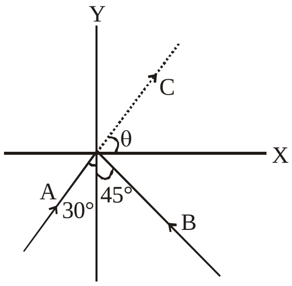
18. An alpha-particle of mass m suffers 1-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is **[12 Jan. 2019 II]**
(a) 2m (b) 3.5m (c) 1.5m (d) 4m

19. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is: **[2018]**
(a) $\frac{v_0}{4}$ (b) $\sqrt{2}v_0$ (c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

20. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly: **[2018]**

(a) 2.35×10^3 N/m^2 (b) 4.70×10^3 N/m^2
(c) 2.35×10^2 N/m^2 (d) 4.70×10^2 N/m^2

21. Two particles A and B of equal mass M are moving with the same speed v as shown in the figure. They collide completely inelastically and move as a single particle C. The angle θ that the path of C makes with the X-axis is given by: **[Online April 9, 2017]**



(a) $\tan\theta = \dfrac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$

(b) $\tan\theta = \dfrac{\sqrt{3}-\sqrt{2}}{1-\sqrt{2}}$

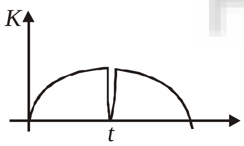
(c) $\tan\theta = \dfrac{1-\sqrt{2}}{\sqrt{2}(1+\sqrt{3})}$

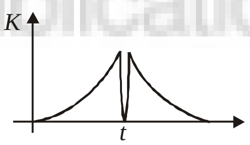
(d) $\tan\theta = \dfrac{1-\sqrt{3}}{1+\sqrt{2}}$

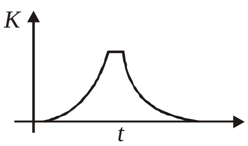
22. A bullet of mass 4 g is fired horizontally with a speed of 300 m/s into 0.8 kg block of wood at rest on a table. If the coefficient of friction between the block and the table is 0.3, how far will the block slide approximately? **[Online April 12, 2014]**

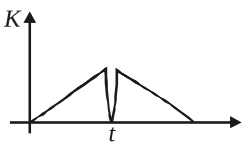
(a) 0.19 m (b) 0.379 m (c) 0.569 m (d) 0.758 m

23. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figure are only illustrative and not to the scale. **[Adv. 2014]**

(a)


(b)


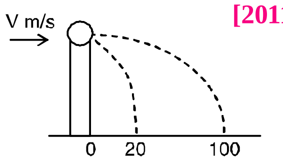
(c)


(d)


24. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is **[Adv. 2013]**

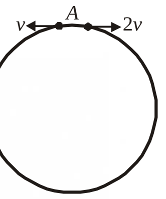
(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{4}+\alpha$ (c) $\dfrac{\pi}{2}-\alpha$ (d) $\dfrac{\pi}{2}$

25. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The velocity V of the bullet is **[2011]**



(a) 250 m/s
(b) $250\sqrt{2}$ m/s
(c) 400 m/s
(d) 500 m/s

26. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A? **[2009]**



(a) 4 (b) 3 (c) 2 (d) 1

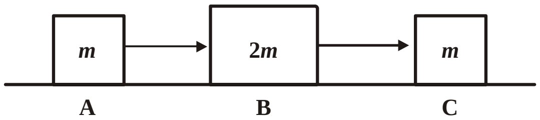
27. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time $t = 0$. They collide at time t_o. Their velocities become $\vec{v}_1{}'$ and $\vec{v}_2{}'$ at time $2t_o$ while still moving in air. The value of $\left|(m_1\vec{v}_1{}' + m_2\vec{v}_2{}') - (m_1\vec{v}_1 + m_2\vec{v}_2)\right|$ is **[2001S]**

(a) zero (b) $(m_1 + m_2)gt_o$

(c) $\dfrac{1}{2}(m_1 + m_2)gt_o$ (d) $2(m_1 + m_2)gt_o$

2 Integer Value Answer

28. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. There after, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed **(in m/s)** of the object C. **[2009]**



29. A bob of mass m, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\dfrac{l_1}{l_2}$ is **[Adv. 2013]**

3 Numeric Answer

30. A body starts falling freely from height H hits an inclined plane in its path at height h. As a result of this perfectly elastic impact, the direction of the velocity of the body becomes horizontal. The value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground is _____. **[Jan. 31, 2024 (I)]**

31. A ball is dropped from a height of 20 m. If the coefficient of restitution for the collision between ball and floor is 0.5, after hitting the floor, the ball rebounds to a height of_________m. **[Main Jan. 31, 2023 (II)]**

32. A body of mass 1 kg collides head on elastically with a stationary body of mass 3 kg. After collision, the smaller body reverses its direction of motion and moves with a speed of 2m/s. The initial speed of the smaller body before collision is _______ ms^{-1}. **[Main Jan. 25, 2023 (II)]**

33. A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is $x : y$, where x is __________. **[Feb. 24, 2021 (I)]**

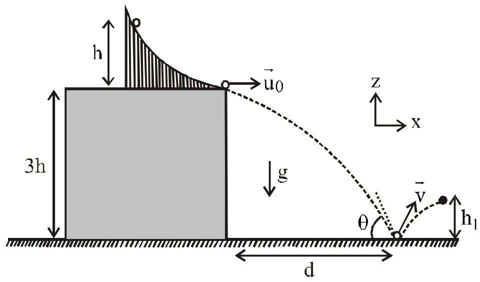
34. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is ______. **[Main 6 Sep. 2020 (I)]**

35. A body A, of mass $m = 0.1$ kg has an initial velocity of $3\hat{i}$ ms^{-1}. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}$ ms^{-1}. After collision, A moves with a velocity $\vec{v} = 4(\hat{i} + \hat{j})$. The energy of B after collision is written as $\frac{x}{10}$ J. The value of x is _______. **[Main 8 Jan. 2020 I]**

36. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in *metres*, is ___________. **[Adv. 2018]**

6 MCQs with One or More than One Correct Answer

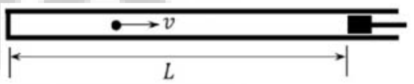
37. A slide with a frictionless curved surface, which becomes horizontal at its lower end, is fixed on the terrace of a building of height 3h from the ground, as shown in the figure. A spherical ball of mass m is released on the slide from rest at a height h from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_0 = u_0\hat{x}$ and falls on the ground at a distance d from the building making an angle θ with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height h_1. The acceleration due to gravity is g and the coefficient of restitution of the ground is $1/\sqrt{3}$. Which of the following statement(s) is(are) correct? **[Adv. 2023]**



(a) $\vec{u}_0 = \sqrt{2gh}\,\hat{x}$ (b) $\vec{v} = \sqrt{2gh}(\hat{x} - \hat{z})$

(c) $\theta = 60°$ (d) $d/h_1 = 2\sqrt{3}$

38. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L}v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct ? **[Adv. 2019]**



(a) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

(b) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

(c) The rate at which the particle strikes the piston is v/L

(d) After each collision with the piston, the particle speed increases by 2 V.

39. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules.
Which of the following options is/are true? **[Adv. 2017]**

(a) The pressure difference between the leading and trailing faces of the plate is proportional to uv

(b) The resistive force experienced by the plate is proportional to v

(c) The plate will continue to move with constant non-zero acceleration, at all times

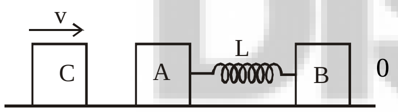
(d) At a later time the external force F balances the resistive force

40. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses? **[2010]**
(a) Total momentum of the system is 3 kg ms^{-1}
(b) Momentum of 5 kg mass after collision is 4 kg ms^{-1}
(c) Kinetic energy of the centre of mass is 0.75 J
(d) Total kinetic energy of the system is 4J

41. The balls, having linear momenta $\vec{p}_1 = \vec{p}i$ and $\vec{p}_2 = -\vec{p}i$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}_1'$ and $\vec{p}_2'$ be their final momenta. The following option (s) is (are) NOT ALLOWED for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2. **[2008]**

(a) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j}$
(b) $\vec{p}_1' = c_1\hat{k}$, $\vec{p}_2' = c_2\hat{k}$
(c) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$
(d) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j}$, $\vec{p}_2' = a_2\hat{i} + b_1\hat{j}$

42. Two blocks *A* and *B*, each of mass *m*, are connected by a massless spring of natural length *L* and spring constant *K*. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in fig.. A third identical block *C*, also of mass *m*, moves on the floor with a speed *v* along the line joining *A* and *B*, and collides elastically with *A*. Then **[1993-2 Marks]**



(a) the kinetic energy of the *A*-*B* system, at maximum compression of the spring, is zero.
(b) the kinetic energy of the *A*-*B* system, at maximum compression of the spring, is $mv^2/4$.
(c) the maximum compression of the spring is $v\sqrt{(m/K)}$
(d) the maximum compression of the spring is $v\sqrt{(m/2K)}$

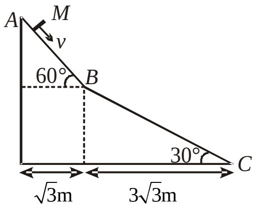
43. A shell is fired from a cannon with a velocity *v* (m/sec.) at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (in m/sec.) of the other piece immediately after the explosion is **[1986 - 2 Marks]**
(a) $3v\cos\theta$
(b) $2v\cos\theta$
(c) $\frac{3}{2}v\cos\theta$
(d) $\sqrt{\frac{3}{2}}v\cos\theta$

44. A ball hits the floor and rebounds after an inelastic collision. In this case **[1986 - 2 Marks]**
(a) the momentum of the ball just after the collision is the same as that just before the collision.
(b) the mechanical energy of the ball remains the same in the collision
(c) the total momentum of the ball and the earth is conserved
(d) the total energy of the ball and the earth is conserved

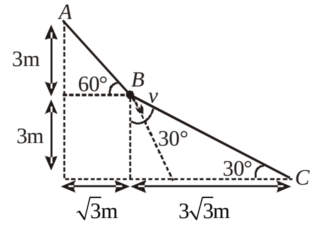
8 Comprehension/Passage Based Questions

Passage

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point *B*. The block is initially at rest at *A*. Assume that collisions between the block and the incline are totally inelastic ($g = 10$ m/s^2). **[2008]**



45. The speed of the block at point *B* immediately after it strikes the second incline is –



(a) $\sqrt{60}$ m/s
(b) $\sqrt{45}$ m/s
(c) $\sqrt{30}$ m/s
(d) $\sqrt{15}$ m/s

46. The speed of the block at point *C*, immediately before it leaves the second incline is
(a) $\sqrt{120}$ m/s (b) $\sqrt{105}$ m/s (c) $\sqrt{90}$ m/s (d) $\sqrt{75}$ m/s

47. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point *B*, immediately after it strikes the second incline is –
(a) $\sqrt{30}$ m/s (b) $\sqrt{15}$ m/s (c) 0 (d) $-\sqrt{15}$ m/s

9 Statement/Assertion and Reason Type Questions

48. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A : Body 'P' having mass M moving with speed 'u' has head-on collision elastically with another body 'Q' having mass 'm' initially at rest. If m << M, body 'Q' will have a maximum speed equal to '2u' after collision.
Reason R : During elastic collision, the momentum and kinetic energy are both conserved. **[Feb. 24, 2021 (II)]**
In the light of the above statements, choose the most appropriate answer from the options given below :
(a) Both A and R are correct and R is the correct explanation of A
(b) A is not correct but R is correct
(c) A is correct but R is not correct
(d) Both A and R are correct but R is NOT the correct explanation of A

49. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[2013]**
Statement - I: A point particle of mass m moving with speed υ collides with stationary point particle of mass M. If the

maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$

then $f = \left(\frac{m}{M+m}\right)$.

Statement - II: Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(a) Statement - I is true, Statment - II is true, Statement - II is the correct explanation of Statement - I.

(b) Statement-I is true, Statment - II is true, Statement - II is **not** the correct explanation of Statement - II.

(c) Statement - I is true, Statment - II is false.

(d) Statement - I is false, Statment - II is true.

50. **STATEMENT–1 :** In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision. **[2007]**

STATEMENT–2 : In an elastic collision, the linear momentum of the system is conserved.

(a) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1.

(b) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation for Statement–1

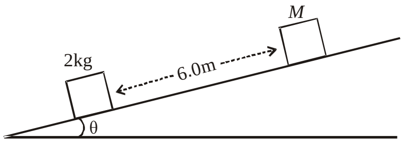
(c) Statement–1 is True, Statement–2 is False.

(d) Statement–1 is False, Statement–2 is True.

10 Subjective Problems

51. Two blocks of mass 2 kg and M are at rest on an inclined plane and are separated by a distance of 6.0 m as shown in Figure. The coefficient of friction between each of the blocks and the inclined plane is 0.25. The 2 kg block is given a velocity of 10.0 m/s up the inclined plane. It collides with M, comes back and has a velocity of 1.0 m/s when it reaches its initial position. The other block M after the collision moves 0.5 m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M. **[1999 - 10 Marks]**

[Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10\,m/s^2$.]

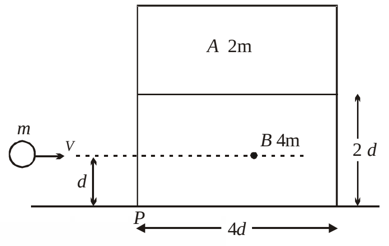


52. A cart is moving along $+x$ direction with a velocity of 4 m/s. A person on the cart throws a stone with a velocity of 6 m/s relative to himself. In the frame of reference of the cart the stone is thrown in y-z plane making an angle of 30° with vertical z-axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from the branch of a tree by means of a string of length L. A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine : **[1997 - 5 Marks]**

(i) The speed of the combined mass immediately after the collision with respect to an observer on the ground,

(ii) The length L of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass.

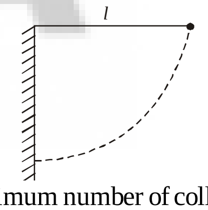
53. A block 'A' of mass 2m is placed on another block 'B' of mass 4m which in turn is placed on a fixed table. The two blocks have a same length 4d and they are placed as shown in fig. The coefficient of friction (both static and kinetic) between the block 'B' and table is μ. There is no friction between the two blocks. A small object of mass m moving horizontally along a line passing through the centre of mass (cm.) of the block B and perpendicular to its face with a speed v collides elastically with the block B at a height d above the table. **[1991 - 4 + 4 Marks]**



(a) What is the minimum value of v (call it v_0) required to make the block A topple ?

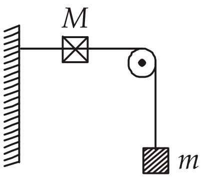
(b) If $v = 2v_0$, find the distance (from the point P in the figure) at which the mass m falls on the table after collision. (Ignore the role of friction during the collision).

54. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position (see fig.) and released. The ball hits the wall, the coefficient of restitution being $\frac{2}{\sqrt{5}}$. **[1987 - 7 Marks]**



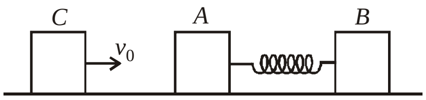
What is the minimum number of collisions after which the amplitude of oscillations becomes less than 60 degrees ?

55. A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2m from the wall, has a point mass $M = 2$kg attached to it at a distance of 1m from the wall. A mass $m = 0.5$ kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released ? **[1985 - 6 Marks]**



56. A ball of mass 100 gm is projected vertically upwards from the ground with a velocity of 49 m/sec. At the same time another identical ball is dropped from a height of 98 m to fall freely along the same path as that followed by the first ball. After some time the two balls collide and stick together and finally fall to the ground. Find the time of flight of the masses. **[1985 - 8 Marks]**

57. Two bodies A and B of masses m and 2 m respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with velocity v_0 along the line joining A and B and collides elastically with A as shown in Fig.



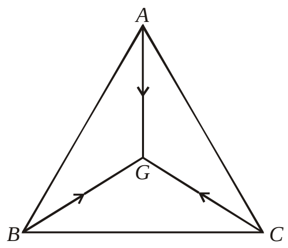
At a certain instant of time t_0 after collision, it is found that the instantaneous velocities of A and B are the same. Further at this instant the compression of the spring is found to be x_0. Determine (i) the common velocity of A and B at time t_0; and (ii) the spring constant. **[1984- 6 Marks]**

58. A bullet of mass M is fired with a velocity 50 m/s at an angle with the horizontal. At the highest point of its trajectory, it collides head-on with a bob of mass 3M suspended by a massless string of length 10/3 metres and gets embedded in the bob. After the collision, the string moves through an angle of 120°. Find

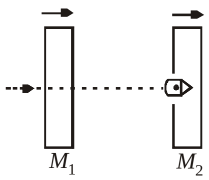
(i) the angle θ ;

(ii) the vertical and horizontal coordinates of the initial position of the bob with respect to the point of firing of the bullet. Take g =10 m /s^2

59. Three particles A, B and C of equal mass move with equal speed V along the medians of an equilateral triangle as shown in figure. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed V. What is the velocity of C ? **[1982 - 2 Marks]**



60. A 20 gm bullet pierces through a plate of mass M_1 = 1 kg and then comes to rest inside a second plate of mass M_2 = 2.98 kg. as shown. It is found that the two plates initially at rest, now move with equal velocities. Find the percentage loss in the initial velocity of the bullet when it is between M_1 and M_2. Neglect any loss of material of the plates due to the action of the bullet. **[1979]**



61. A body of mass m moving with velocity V in the X-direction collides with another body of mass M moving in Y-direction with velocity v. They coalesce into one body during collision. Calculate : **[1978]**

(i) the direction and magnitude of the momentum of the final body.

(ii) the fraction of initial kinetic energy transformed into heat during the collision in terms of the two masses.

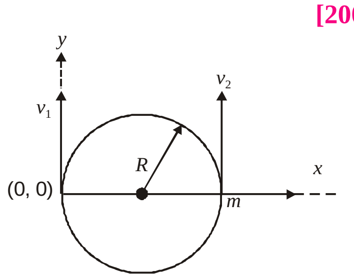
Topic-5: Miscellaneous (Mixed Concepts) Problems

3 Numeric Answer

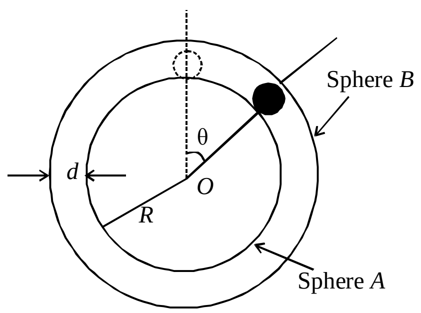
1. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4$ kg is at rest on this surface. An impulse of 1.0 Ns is applied to the block at time $t = 0$ so that it starts moving along the x -axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4s$. The displacement of the block, in *metres*, at t = τ is __________. Take $e^{-1} = 0.37$. **[Adv. 2017]**

10 Subjective Problems

2. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point (2R, 0) at time t = 0 and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time t = 0. Calculate the linear momentum of the particle w.r.t. the man as a function of time. **[2003 - 2 Marks]**



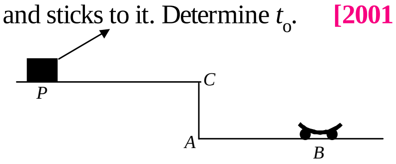
3. A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ (shown in the figure). **[2002 - 5 Marks]**



(a) Express the total normal reaction force exerted by the sphere on the ball as a function of angle θ.

(b) Let N_A and N_B denote the magnitudes of the normal reaction forces on the ball exerted by the sphere A and B, respectively. **Sketch** the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two **separate** graphs in your answer book, taking $\cos\theta$ on the horizontal axes.

4. A car P is moving with a uniform speed of $5\sqrt{3}$ m/s towards a carriage of mass 9 kg at rest kept on the rails at a point B as shown in figure. The height AC is 120 m. Cannon balls of 1 kg are fired from the car with an initial velocity 100 m/s at an angle 30^o with the horizontal. The first cannon ball hits the stationary carriage after a time t_o and sticks to it. Determine t_o. **[2001 - 10 Marks]**



At t_o, the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second ball also hits and sticks to the carriage, what will be the horizontal velocity of the carriage just after the second impact?

Answer Key

Topic-1 : Work

1. (c)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (d)	**8.** (d)	**9.** (d)	**10.** (b)
11. (c)	**12.** (d)	**13.** (8)	**14.** (784)	**15.** (40)	**16.** (132)	**17.** (2)	**18.** (450)	**19.** (3)	**20.** (0.75)

Topic-2 : Energy

1. (a)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (d)	**9.** (b)	**10.** (d)
11. (c)	**12.** (b)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (b)
21. (a)	**22.** (c)	**23.** (a)	**24.** (a)	**25.** (c)	**26.** (c)	**27.** (None)	**28.** (a)	**29.** (c)	**30.** (d)
31. (d)	**32.** (c)	**33.** (b)	**34.** (a)	**35.** (a)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (b)	**40.** (a)
41. (a)	**42.** (c)	**43.** (b)	**44.** (b)	**45.** (d)	**46.** (b)	**47.** (c)	**48.** (5)	**49.** (4)	**50.** (30)
51. (2)	**52.** (375)	**53.** (50)	**54.** (7)	**55.** (300)	**56.** (40)	**57.** (24)	**58.** (16)	**59.** (1)	**60.** (40)
61. (6)	**62.** (1)	**63.** (2)	**64.** (150)	**65.** (a, d)	**66.** (a, b, d)		**67.** (a, b, d)	**68.** (d)	**69.** (c, d)
70. $A \to p, q, r, t$; $B \to q, s$; $C \to p, q, r, s$; $D \to p, r, t$					**71.** (b)	**72.** (b)	**73.** (c)		

Topic-3 : Power

1. (c)	**2.** (d)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (c)
11. (b)	**12.** (5)	**13.** (300)	**14.** (100)	**15.** (18)					

Topic-4 : Collisions

1. (d)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (c)	**10.** (b)
11. (b)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (d)	**17.** (b)	**18.** (d)	**19.** (b)	**20.** (a)
21. (a)	**22.** (b)	**23.** (b)	**24.** (a)	**25.** (d)	**26.** (c)	**27.** (d)	**28.** (4)	**29.** (5)	**30.** (2)
31. (5)	**32.** (4)	**33.** (1)	**34.** (120)	**35.** (1)	**36.** (30.00)	**37.** (a, c, d)		**38.** (a, d)	
39. (a, b, d)		**40.** (a, c)	**41.** (a, d)	**42.** (b, d)	**43.** (a)	**44.** (c, d)	**45.** (b)	**46.** (b)	**47.** (c)
48. (a)	**49.** (d)	**50.** (d)							

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1. (6.30)

5 Work, Energy and Power

Topic-1: Work

1. **(c)** Given, $v = \alpha\sqrt{x}$
at $x = 0$; $v = 0$
at $x = d$; $v = \alpha\sqrt{d}$
Using work energy theorem
Work done = Change in kinetic energy
$= \frac{1}{2}m\left(\alpha\sqrt{d}\right)^2 - \frac{1}{2}m(0)^2$
$= \frac{m\alpha^2 d}{2}$

2. **(b)** Mass of black, m = 1 kg
Force of parallel inclined surface, F = 10 N
Work done against frictional force
$= \mu_s N \times 10 = \mu_s Mg \times 10 = 0.1 \times 5 \times 10 = 5$ J

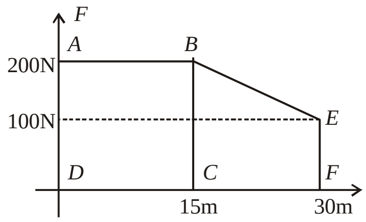
3. **(a)** Using $v^2 - u^2 = 2as \Rightarrow \frac{u^2}{2a} = s$
$u^2 = 2 \times \mu gs$ [$\because$ $v = 0$ & $a = \mu g$]
$u^2 = 2 \times (.4) \times 10 \times 10 \Rightarrow u^2 = 80$
$W_f = \Delta k$
$= 0 - \frac{1}{2} \times 100 \times 80$
$\therefore W_f = 4000$ J

4. **(b)** By work-energy thorem
$W = \Delta K$
$\Rightarrow W = \frac{1}{2}m\,(v_f^2 - v_i^2) \Rightarrow W = \frac{1}{2} \times 0.5 \times (16^2 - 4^2)$
$\Rightarrow W = \frac{1}{4} \times 240 \Rightarrow W = 60$ J

5. **(d)** By work – energy theorem
$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2} \times 0.5 \times \left(b^2.4^5\right)$
$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4^2} \times 4^5 = 16$ J

6. **(b)** From work-energy theorem,
$W_{Porter} + W_{mg} = \Delta K.E. = 0$ ($\because$ velocity constant)
or, $W_{Porter} = -W_{mg} = -mgh$
$\therefore$ $W_{Porter} = -80 \times 9.8 \times \frac{80}{100} = -627.2$J

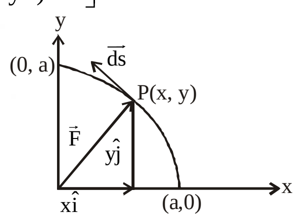
7. **(d)** The given situation can be drawn graphically as shown in figure.
Work done = Area under *F-x* graph
= Area of rectangle *ABCD* + Area of trapezium *BCFE*



$W = (200 \times 15) + \frac{1}{2}(100 + 200) \times 15 = 3000 + 2250$
$\Rightarrow W = 5250$ J

8. **(d)** Here, $N - mg = ma = \frac{mg}{2} \Rightarrow N = \frac{3mg}{2}$
N = normal reaction
Now, work done by normal reaction 'N' on block in time t, $W = \vec{N}\vec{S} = \left(\frac{3mg}{2}\right)\left(\frac{1}{2}g/_2 t^2\right)$
or, $W = \frac{3mg^2t^2}{8}$

9. **(d)** Radius of circular path = a
The equation of circle is $x^2 + y^2 = a^2$
Given : force
$\vec{F} = K\left[\frac{x\hat{i}}{(x^2+y^2)^{3/2}} + \frac{y\hat{j}}{(x^2+y^2)^{3/2}}\right]$
$\vec{F} = K\left[\frac{x\hat{i}}{(a^2)^{3/2}} + \frac{y\hat{j}}{(a^2)^{3/2}}\right]$
$\vec{F} = \frac{K}{a^3}\left[x\hat{i} + y\hat{j}\right]$



The force acts radially outwards as shown in the figure and the displacement is tangential to the circular path. Here the angle between the force which acts radially outwards and displacement which is tangential to the circular path is 90°
$\therefore$ Work done, $W = FS\cos\theta = 0$

10. **(b)** In a conservative field work done does not depend on the path *i.e.*, path independent. The gravitational field is a conservative field.
$\therefore$ $W_1 = W_2 = W_3$

11. **(c)** $dW = \vec{F}\,.\,d\vec{S}$ and $\vec{F} = -K(y\hat{i} + x\hat{j})$ given
$d\vec{S} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$
$dW = -K(y\,\hat{i} + x\,\hat{j})\,.\,(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$
$= -K(ydx + xdy)$
for (0, 0) to (a, 0)
$x \Rightarrow 0 \to a$
y = 0 and dy = 0

So, dW = 0 ⇒ W = 0

for (a, 0) to (a, a)

y ⇒ 0 to a

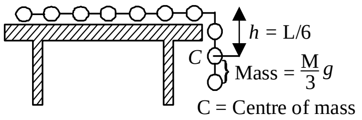
x = a, dx = 0

So, dW = – Ka dy

$$W = \int_0^a -Ka\,dy = -Ka^2$$

Hence total work done by the force on the particle, $W = -Ka^2$

12. **(d)** The work done in bringing the mass up will be equal to the change in potential energy of the mass.



i.e., W = Change in potential energy

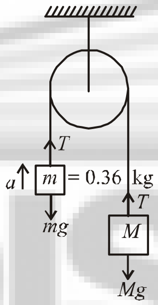
$$= mgh = \frac{M}{3} \times g \times \frac{L}{6} = \frac{MgL}{18}$$

13. **(8)** When the system is released,

$T - mg = ma$ ...(i)

$Mg - T = Ma$ ...(ii)

From eq. (i) & (ii)



$$a = \frac{(M-m)g}{M+m} = g/3$$

and $T = 4\,mg/3$

For block m = 0.36 kg

$u = 0$, $a = g/3$, $t = 1$, $s = ?$

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times \frac{g}{3} \times 1^2 = g/6 \quad (m = 0.72\text{ kg})$$

∴ Work done by the string on m

$$T\,s \cos 0° = 4\frac{mg}{3} \times \frac{g}{6} \times 1 = \frac{4 \times 0.36 \times 10 \times 10}{3 \times 6} = 8\text{J}$$

14. **(784)** Given,

Mass of bus, m = 500 kg.

Distance Travelled, x = 4 km

Coefficient of friction $\mu = 0.04$

Work done by engine = –Work done by friction = – [–μmgx]

$= 0.04 \times 500 \times 9.8 \times 4 \times 10^3 = 784$ kJ

15. **(40)** $W = \vec{F}.\left(\vec{r}_f - \vec{r}_1\right)$

$$= \left(5\hat{i} + 2\hat{j} + 7\hat{k}\right).\left(\left(5\hat{i} - 2\hat{j} + \hat{k}\right) - \left(2\hat{i} + 3\hat{j} - 4\hat{k}\right)\right)$$

W = 40 J

16. **(132)** We know that

$$W = \int F dy = \int_2^5 (5 + 3y^2)dy$$

$$= [5y + y^3]_2^5 = [(25 + 125) - (10 + 8)] = 150 - 18 = 132\text{ J}$$

17. **(2)** Work done by A = Work done by B

F_A dcos 45° = F_B dcos 60°

$$\Rightarrow F_A \times \frac{1}{\sqrt{2}} = F_B \times \frac{1}{2} \Rightarrow \frac{F_A}{F_B} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \Rightarrow x = 2$$

18. **(450)** Given,

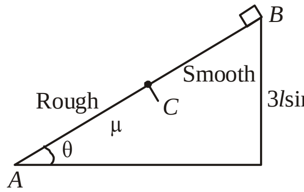
Force, $F = (5y + 20)\hat{j}$

Work done, $W = \int F.dy$

$$\Rightarrow \quad W = \int_0^{10} (5y + 20)dy = \left[\frac{5y^2}{2} + 20y\right]_0^{10}$$

$$= \frac{5}{2} \times 100 + 20 \times 10 = 450\text{J}$$

19. **(3)** If $AC = l$ then according to question, $BC = 2l$ and $AB = 3l$.



Here, work done by all the forces is zero.

$$W_{\text{friction}} + W_{mg} = 0$$

$$mg(3l)\sin\theta - \mu mg\cos\theta(l) = 0$$

$$\Rightarrow \mu mg\cos\theta l = 3mgl\sin\theta \Rightarrow \mu = 3\tan\theta = k\tan\theta$$

$\therefore k = 3$

20. **(0.75)** Given : Force, $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$

and $\alpha = -1\text{ Nm}^{-1}$

We know that $dW = \vec{F}\cdot d\vec{r} = \left(\alpha y\,\hat{i} + 2\alpha x\,\hat{j}\right)\cdot\left(dx\,\hat{i} + dy\,\hat{j}\right)$

∴ dW = αydx + 2αxdy

Work done

From A → B dy = 0, as y = 1

$$\therefore W_1 = \int_0^1 \alpha y\,dx = \alpha\int_0^1 dx = \alpha$$

From B → C dx = 0, as x = 1

$$\therefore W_2 = \int_1^{0.5} 2\alpha n\,dy = \int_1^{0.5} 2\alpha dy = 2\alpha(-0.5) = -\alpha$$

From C → D dy = 0, as y = 0.5

$$\therefore W_3 = \int_1^{0.5} \alpha \times 0.5\,dx = -\frac{\alpha}{4}$$

From D → E dx = 0, as x = 0.5

$$\therefore W_4 = \int_{0.5}^{0} 2\alpha \times 0.5\,dy = -\frac{\alpha}{2}$$

From E → F dy = 0, as y = 0

$$\therefore W_5 = \int \alpha \times 0 \times dx = 0$$

From F → A dx = 0 as x = 0

$$\therefore W_6 = \int 2\alpha \times 0 dx = 0$$

∴ Total work done W = $W_1 + W_2 + W_3 + W_4 + W_5 + W_6$

$$= \alpha - \alpha - \frac{\alpha}{4} - \frac{\alpha}{2} = -\frac{3\alpha}{4} = \frac{-3(-1)}{4} = 0.75\text{J}$$

Topic-2: Energy

1. **(a)** Relation between kinetic energy and linear momentum is

$P = \sqrt{2m\,KE}$

$\therefore\ P \propto \sqrt{m}$ ($\because$ KE is same)

$\therefore P_A : P_B : P_C = \sqrt{400} : \sqrt{1200} : \sqrt{1600} = 1 : \sqrt{3} : 2$

2. **(a)** From the conservation of momentum

$|P_A| = |P_B|$

$\therefore\quad m_A v_A = m_B v_B$

$\therefore\quad \dfrac{m_B}{m_A} = \dfrac{v_A}{v_B}$

$$\frac{K_A}{K_B} = \frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \frac{P_A^2 m_B}{P_B^2 m_A} = \frac{v_A}{v_B}\ (\because P_A = P_B)$$

$$\Rightarrow \frac{K_B}{K_A} = \frac{v_B}{v_A}$$

3. **(d)** Kinetic energy is $KE = \dfrac{1}{2}mv^2$

Initial kinetic energy $KE_i = \dfrac{1}{2}(0.05)(100)^2$

$K_f = \dfrac{1}{2}(0.05)(40)^2$

Percentage loss of kinetic energy is

$$S = \frac{KE_f - KE_i}{KE_i} \times 100$$

$$= \frac{\frac{1}{2}0.05\,(40)^2 - \frac{1}{2}0.05\,(100)^2}{\frac{1}{2}0.05\,(100)^2} \times 100$$

$$= \frac{|1600 - 100 \times 100|}{100} = 84\%$$

4. **(c)** Kinetic energy is

$$K = \frac{p^2}{2m} \Rightarrow k \propto \frac{1}{\sqrt{m}}$$

Particle of mass $\dfrac{m}{2}$ will have maximum kinetic energy.

5. **(a)** Kinetic energy is given as, $KE = \dfrac{P^2}{2m}$

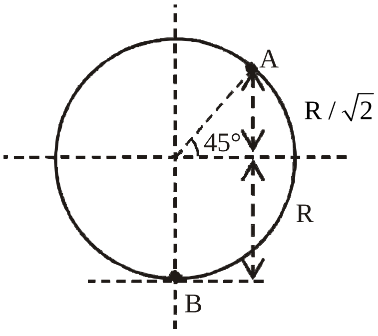
Given $\therefore KE_f = 36\,KE_i$

$$\frac{P_f^2}{2m} = \frac{P_i^2}{2m}(36) \Rightarrow P_f = 6P_i$$

Percentage increase in momentum $= \dfrac{P_f - P_i}{P_i} \times 100\%$

$$= \frac{6P_i - P_i}{P_i} \times 100\% = 500\%$$

6. **(b)** From work energy theorem



$W_{mg} = K_B - K_A$

$$\Rightarrow mg \times \left(\frac{R}{\sqrt{2}} + R\right) = \frac{1}{2}mv_B^2 - 0 \qquad \{v_A = 0 \text{ rest}\}$$

$$\Rightarrow mgR\frac{\left(\sqrt{2}+1\right)}{\sqrt{2}} = \frac{1}{2}mv_B^2 \Rightarrow \sqrt{gR\frac{2\left(\sqrt{2}+1\right)}{\sqrt{2}}} = v_B$$

$$\Rightarrow \sqrt{\frac{10 \times 14 \times 2(2.4)}{1.4}} = v_B$$

$\therefore v_B = 21.9$ m/s

7. **(b)** Using conservation of linear momentum:

$mu = (M + m)v$

$$v = \frac{mu}{M+m} = \frac{0.01 \times 200}{1.01} = 1.98\,m/s.$$

Using law of conservation of energy

$$h = \frac{v^2}{2g} = \frac{(1.98)^2}{2 \times 9.8} = 0.20m$$

8. **(d)** By conservation of linear momentum

$|\vec{P}_1| = |\vec{P}_2|$

Kinetic energy, $KE = \dfrac{P^2}{2M}$ ($\because$ P is same)

$$\therefore \frac{KE_1}{KE_2} = \frac{P^2/2M_1}{P^2/2M_1} = \frac{M_2}{M_1}$$

9. **(b)** From the conservation of energy

$KE_A + U_A = KE_B + U_B$

$0 + mg(1) = \dfrac{1}{2}mv^2 + mg \times 0.5$

$v = \sqrt{g} = \sqrt{10}$ m/s

10. **(d)** Potential Energy, $U = 2x^2 + 3y^3 + 2z$

The x-component of force is

$$F_x = \frac{\partial U}{\partial x} = -4x$$

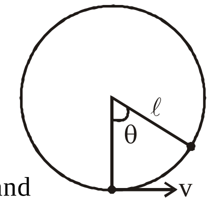
At x = 1, $|F_x| = |-4 \times 1| = 4N$

11. **(c)** Using, $P = \sqrt{2mE}$

Here $E_1 = E_2 \Rightarrow \dfrac{P_1^2}{2m_1} = \dfrac{P_2^2}{2m_2} \Rightarrow \dfrac{P_1}{P_2} = \sqrt{\dfrac{m_1}{m_2}} = \sqrt{\dfrac{4}{25}} = \dfrac{2}{5}$

12. (b) Here,
loss in kinetic energy = gain in potential energy

$\Rightarrow \frac{1}{2}mv^2 = mg\ell(1-\cos\theta)$



$\Rightarrow \frac{v^2}{\ell} = 2g(1-\cos\theta)$

Acceleration at lowest point = $\frac{v^2}{\ell}$ and at extreme point = g sin θ

Hence, $\frac{v^2}{\ell}$ = g sin θ ∴ sin θ = 2(1 – cos θ)

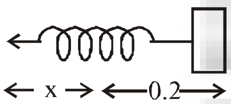
$\Rightarrow \tan\frac{\theta}{2} = \frac{1}{2} \Rightarrow \theta = 2\tan^{-1}\left(\frac{1}{2}\right)$

13. (d) K.E. $= \frac{P^2}{2m}$

$\therefore \frac{K_1}{K_2} = \frac{p_1^2}{2m_1} \times \frac{2m_2}{p_2^2} = \frac{m_2}{m_1} = \frac{16}{9}$

$\therefore \frac{m_1}{m_2} = \frac{9}{16}$

14. (b)



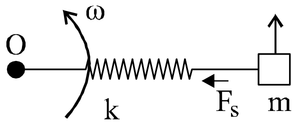
Given,
Mass of block, m = 100 g = 0.1 kg
Spring constant, k = 7.5 N/m
Let x be the extension in length of spring.
Radius of circle r = 0.2 + x
Tension, $T = kx = m\omega^2 r$
$\Rightarrow 7.5x = 0.1\,(25)\,(0.2 + x)$

$\Rightarrow \frac{15}{2}x = \frac{5}{2}\left(x + \frac{1}{5}\right) \Rightarrow 3x = \frac{5x+1}{5}$

$\Rightarrow x = \frac{1}{10} = 0.1$

∴ Tension in spring = kx = 7.5 × 0.1 = 0.75N

15. (c)

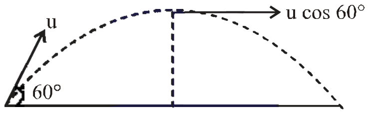


Natural length = L_0
Extension = x
$F_{spring} = F_{centripetal}$

$\Rightarrow kx = m\,(L_0 + x)\omega^2 \Rightarrow 12.5x = \frac{1}{5}(L_0 + x)25$

$\Rightarrow 1.5x = L_0 \Rightarrow \frac{x}{L_0} = \frac{2}{3}$

16. (c)



At maximum height, we only have horizontal component of velocity. So, Velocity $v = u\cos 60° = \frac{u}{2}$

∴ K.E at top most point $= \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{E}{4}$

17. (c) Momentum of a body is increased by

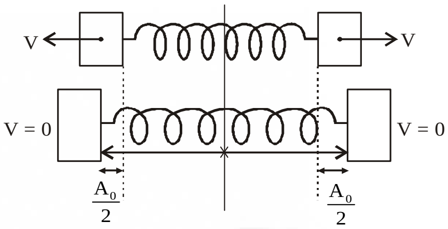
$P' = P + \frac{20}{100}P = 1.2\,P$

Percentage change in KE $= \frac{K' - K}{K} \times 100$

$= \left(\frac{\frac{P'^2}{2m} - \frac{P^2}{2m}}{\frac{P^2}{2m}}\right) \times 100 = [(1.2)^2 - 1] \times 100 = 44\%$

18. (b) Given, spring constant of spring, $K = 2Nm^{-1}$

Mass of block, m = 250 g $= \frac{250}{1000}g = \frac{1}{4}kg$



Using energy conservation

$\frac{1}{2}mv^2 \times 2 = \frac{1}{2}kx^2 \Rightarrow \frac{1}{4}v^2 = \frac{1}{2} \times 2 \times x^2$

$\therefore \quad x = \frac{v}{2}$

19. (b) K.E. and momentum P relation, K.E $= \frac{P^2}{2m}$

$K_1 = \frac{P_1^2}{2(8)}; K_2 = \frac{P_2^2}{2(2)}$

Given, $K_1 = K_2$

So, $\frac{P_1^2}{16} = \frac{P_2^2}{4} \Rightarrow P_1^2 = 4P_2^2 \Rightarrow \frac{P_1}{P_2} = 2$

20. (b) Kinetic energy, $K.E = \frac{p^2}{2m}$

$\frac{K.E_1}{K.E_2} = \left(\frac{P_1}{P_2}\right)^2 \times \left(\frac{m_2}{m_1}\right) = \left(\frac{1}{2}\right)^2 \times \frac{8}{5} = \frac{2}{5}$

21 (a) At maximum height, $v = 0$
$\Rightarrow mv = 0 \Rightarrow P = 0$

22. (c) By work-energy theorem,

$\Delta k = W_{\text{all forces}} = \int \vec{F}.\,d\vec{r}$

$= \int (4x\hat{i} + 3y^2\hat{j}).(dx\hat{i} + dy\,\hat{j}) = \int_1^2 4x\,dx + \int_2^3 3y^2 dy$

$= 4\left[\frac{x^2}{2}\right]_1^2 + 3\left[\frac{y^3}{3}\right]_2^3 = 2[2^2 - 1^2] + [3^3 - 2^3]$

$= 6 + 19 = 25$ J.

23. **(a)** Work done by air friction = Final kinetic energy - Initial potential energy $W_{\text{air-friction}} = \frac{1}{2}mv^2 - mgh$

$= \frac{1}{2}m\left(8\sqrt{gh}\right)^2 - mgh$

$W_{\text{air-friction}} = \frac{64}{2}mgh - mgh = -0.68\ mgh$

24. **(a)** Relation between kinetic energy and linear momentum is given as

$K.E. = K = \frac{P^2}{2m} \Rightarrow P \propto \sqrt{K}$

$\frac{P_2}{P_1} = \sqrt{\frac{K_2}{K_1}} \Rightarrow \frac{P_2}{P_1} = \sqrt{\frac{4K}{K}} \Rightarrow \frac{P_2}{P_1} = 2$

$\Rightarrow \frac{P_2 - P_1}{P_1}\% = \left(\frac{P_2}{P_1} - 1\right) \times 100 = (2-1) \times 100 = 100$

$\Rightarrow \frac{\Delta P}{P_1}\% = 100\%$

25. **(c)** We know area under F-x graph gives the work done by the body

$\therefore\ W = \frac{1}{2} \times (3+2) \times (3-2) + 2 \times 2 = 2.5 + 4 = 6.5\ J$

Using work energy theorem,
Δ K.E = work done $\quad\therefore \Delta K.E = 6.5\ J$

26. **(c)** $l_1 + l_2 = l$ and $l_1 = nl_2$

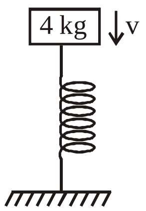
$\therefore\ l_1 = \frac{nl}{n+1}$ and $l_2 = \frac{l}{n+1}$

As $k \propto \frac{1}{l}$, $\therefore\ \frac{k_1}{k_2} = \frac{l/(n+1)}{(nl)/(n+1)} = \frac{1}{n}$

27. **(None)** Velocity of 1 kg block just before it collides with 3 kg block $= \sqrt{2gh} = \sqrt{2000}$ m/s

Using principle of conservation of linear momentum just before and just after collision, we get

$1 \times \sqrt{2000} = 4v \Rightarrow v = \frac{\sqrt{2000}}{4}$ m/s



Initial compression of spring
$1.25 \times 10^6\ x_0 = 30 \Rightarrow x_0 \approx 0$
using work energy theorem,
$W_g + W_{sp} = \Delta KE$

$\Rightarrow\ 40 \times x + \frac{1}{2} \times 1.25 \times 10^6 (0^2 - x^2)$

$= 0 - \frac{1}{2} \times 4 \times v^2$

solving $x \approx 2$ cm

28. **(a)** $W = u_f - u_i = 0 - \left(-\frac{mg}{n} \times \frac{L}{2n}\right) = \frac{MgL}{2n^2}$.

29. **(c)** $mv = (m+M)V'$

or $v = \frac{mv}{m+M} = \frac{mv}{m+4m} = \frac{v}{5}$

Using conservation of ME, we have

$\frac{1}{2}mv^2 = \frac{1}{2}(m+4m)\left(\frac{v}{5}\right)^2 + mgh$ or $h = \frac{2}{5}\frac{v^2}{g}$

30. **(d)** Work done = $\vec{F}.\vec{d} = \left(3\vec{i} - 12\vec{j}\right).\left(4\vec{i}\right) = 12J$

From work energy theorem,
$W_{net} = \Delta K.E. = k_f - k_i \Rightarrow 12 = k_f - 3$
$\therefore K_f = 15J$

31. **(d)** Position, $x = 3t^2 + 5$

$\therefore$ Velocity, $v = \frac{dx}{dt} \Rightarrow v = \frac{d\left(3t^2 + 5\right)}{dt}$

$\Rightarrow v = 6t + 0$

At t = 0 $\quad v = 0$
And, at t = 5 sec $\quad v = 30$ m/s
According to work-energy theorem, w = ΔKE

or, $W = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(2)(30)^2 = 900J$

32. **(c)** $F = -\frac{\partial u}{\partial r}\hat{r} = \frac{K}{r^3}\hat{r}$

Since particle is moving in circular path

$F = \frac{mv^2}{r} = \frac{K}{r^3} \Rightarrow mv^2 = \frac{K}{r^2} \quad \therefore\ K.E. = \frac{1}{2}mv^2 = \frac{K}{2r^2}$

Total energy = P.E. + K.E.

$= -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{Zero} \qquad (\because P.E. = -\frac{K}{2r^2}$ given)

33. **(b)** As the particles moving in circular orbits, So

$\frac{mv^2}{r} = \frac{16}{r} + r^2$

Kinetic energy, $KE_0 = \frac{1}{2}mv^2 = \frac{1}{2}[16 + r^4]$

For first particle, r = 1, $K_1 = \frac{1}{2}m(16+1)$

Similarly, for second particle, r = 4, $K_2 = \frac{1}{2}m(16+256)$

$\therefore\ \frac{K_1}{K_2} = \frac{\frac{16+1}{2}}{\frac{16+256}{2}} = \frac{17}{272} \simeq 6 \times 10^{-2}$

34. **(a)** Let V_f is the final speed of the body.
From questions,

$\frac{1}{2}mV_f^2 = \frac{1}{8}mV_0^2 \quad \Rightarrow \quad V_f = \frac{V_0}{2} = 5\,m/s$

$F = m\left(\frac{dV}{dt}\right) = -kV^2 \quad \therefore \quad (10^{-2})\frac{dV}{dt} = -kV^2$

$\int_{10}^{5}\frac{dV}{V^2} = -100K\int_{0}^{10}dt$

$\frac{1}{5} - \frac{1}{10} = 100K(10) \qquad$ or, $\quad K = 10^{-4}\ kgm^{-1}$

35. **(a)** $h \propto \frac{V^2}{2g} \qquad h \propto K.E.$

As K.E. becomes half after every collision. So height will also become half.

So, total distance = h + 2 $\left(\frac{h}{2}+\frac{h}{4}+....\right)$

$$= h + 2h\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right) = 3h$$

36. **(c)** Using, F = ma = $m\frac{dV}{dt}$

$6t = 1.\frac{dV}{dt}$ [$\because$ $m = 1$ kg given]

$\int_0^v dV = \int 6t\,dt$ $V = 6\left[\frac{t^2}{2}\right]_0^1 = 3\,\text{ms}^{-1}$ [$\because$ $t = 1$ sec given]

From work-energy theorem,

$W = \Delta KE = \frac{1}{2}m\left(V^2 - u^2\right) = \frac{1}{2}\times 1\times 9 = 4.5$ J

37. **(a)** Work done by friction at QR = μmg$x$

In triangle, sin 30° = $\frac{1}{2} = \frac{2}{PQ} \Rightarrow PQ = 4m$

Work done by friction at PQ = μmg × Cos 30° × 4

= μmg × $\frac{\sqrt{3}}{2}$ × 4 = $2\sqrt{3}$ μmg

Since work done by friction on parts PQ and QR are equal,

μmg x = $2\sqrt{3}$ umg $\Rightarrow x = 2\sqrt{3} \cong 3.5m$

Using work energy theorem mg sin 30° × 4 = $2\sqrt{3}$ μmg + μmg$x$

$\Rightarrow 2 = 4\sqrt{3}\,\mu \Rightarrow \mu = 0.29$

38. **(b)** $n = \frac{W}{\text{input}} = \frac{mgh\times 1000}{\text{input}} = \frac{10\times 9.8\times 1\times 1000}{\text{input}}$

Input = $\frac{98000}{0.2}$ = 49 × 10^4J

Fat used = $\frac{49\times 10^4}{3.8\times 10^7}$ = 12.89 × 10^{-3}kg.

39. **(b)** As we know, dU = F.dr

$U = \int_0^r \alpha r^2 dr = \frac{ar^3}{3}$ ...(i)

As, $\frac{mv^2}{r} = \alpha r^2 \Rightarrow m^2v^2 = m\alpha r^3$

or, 2m(KE) = $\frac{1}{2}\alpha r^3$ ...(ii)

Total energy = Potential energy + kinetic energy

Now, from eqn (i) and (ii)

Total energy = K.E. + P.E.

$= \frac{\alpha r^3}{3} + \frac{\alpha r^3}{2} = \frac{5}{6}\alpha r^3$

40. **(a)** Let u be the initial velocity of the bullet of mass m. After passing through a plank of width x, its velocity decreases to v.

$\therefore$ $u - v = \frac{u}{n}$ or, $v = u - \frac{u}{n} = \frac{u(n-1)}{n}$

If F be the retarding force applied by each plank, then using work – energy theorem,

$Fx = \frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}mu^2 - \frac{1}{2}mu^2\frac{(n-1)^2}{n^2}$

$= \frac{1}{2}mu^2\left[\frac{1-(n-1)^2}{n^2}\right]$ $Fx = \frac{1}{2}mu^2\left(\frac{2n-1}{n^2}\right)$

Let P be the number of planks required to stop the bullet.

Total distance travelled by the bullet before coming to rest = Px

Using work-energy theorem again,

$F(Px) = \frac{1}{2}mu^2 - 0$

or, $P(Fx) = P\left[\frac{1}{2}mu^2\frac{(2n-1)}{n^2}\right] = \frac{1}{2}mu^2$ $\therefore$ $P = \frac{n^2}{2n-1}$

41. **(a)** Given : $k_A = 300\,N/m$, $k_B = 400\,N/m$

Let when the combination of springs is compressed by force F. Spring A is compressed by x. Therefore compression in spring B

$x_B = (8.75 - x)$ cm. In series force is same across both spring

So, F = 300 × x = 400(8.75 – x)

Solving we get, x = 5 cm

$x_B = 8.75 - 5 = 3.75$cm

$$\frac{E_A}{E_B} = \frac{\frac{1}{2}k_A(x_A)^2}{\frac{1}{2}k_B(x_B)^2} = \frac{300\times(5)^2}{400\times(3.75)^2} = \frac{4}{3}$$

42. **(c)** Heres when the block B is displaced towards wall 1, only spring S_1 is compressed and S_2 is in its natural state as the other end of S_2 is free.

Therefore the energy stored in the system = $\frac{1}{2}k_1x^2$.

When the block is released, it will come back to the equilibrium position, gain momentum, overshoot to equilibrium position and move towards wall 2. As this happens, the spring S_1 comes to its natural length and S_2 gets compressed. The P.E. stored in the spring S_1 gets stored as the P.E. of spring S_2 when the block B reaches its extreme position after compressing S_2 by y.

It is because no friction anywhere.

So, energy is conserved

$\therefore$ $\frac{1}{2}k_1\,x^2 = \frac{1}{2}k_2\,y^2 \Rightarrow \frac{1}{2}\times\,kx^2 = \frac{1}{2}\times 4\,ky^2$

$\Rightarrow$ $x^2 = 4y^2$ $\therefore \frac{y}{x} = \frac{1}{2}$

43. **(b)** For conservative forces $\Delta U = -W$

$\Delta U = -\int_0^x F dx$ or $\Delta U = -\int_0^x k\,xdx$

$\Rightarrow$ $U_{(x)} - U_{(0)} = -\frac{kx^2}{2}$ ($\because$ $U_{(0)} = 0$)

$\therefore$ $U_{(x)} = -\frac{kx^2}{2}$ $\Rightarrow$ $x^2 = \frac{-2U_x}{k}$

It represents a parabola below x – axis symmetrical

44. **(b)** Let x be the maximum extension of the string. Here mechanical energy is conserved, so decrease in the gravitational potential energy of spring mass system (Mgx)

= gain in spring elastic potential energy $\left(\frac{1}{2}kx^2\right)$

$Mgx = \frac{1}{2}kx^2$ $\Rightarrow$ $x = \frac{2Mg}{k}$

45. **(d)**

46. **(b)** When spring is cut into pieces then the length of longer piece of spring = $\frac{2L}{3}$

Here the original length of spring be L and spring constant is K (given)

For a spring, $K \times L$ = constant $[\because K = YA/l]$

Let K' be the spring cosntant of longer piece of spring

$$\therefore \quad K \times L = \frac{2L}{3} \times K' \Rightarrow K' = \frac{3}{2}K$$

47. **(c)** K.E. = $\frac{p^2}{2m}$; [K.E. = Kinetic energy; P = momentum]

$$KE_1 = KE_2 \quad \therefore \frac{p_1^2}{m_1} = \frac{p_2^2}{m_2}$$

$$\therefore \quad \frac{p_1^2}{p_2^2} = \frac{m_l}{m_2} \Rightarrow \frac{p_l}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

48. **(5)** Work done = Increase in P.E. + gain in K.E.

$F \times d = mgh$ + gain in K.E.

$18 \times 5 = 1 \times 10 \times 4$ + gain in K.E.

$\therefore$ Gain in K.E. = 50 J = 10n $\therefore n = 5$

49. **(4)** Here, loss in K.E. of the block = gain in P.E. of the spring + work done against friction

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu mg.x \Rightarrow v = \frac{2\,\mu mgx + kx^2}{m}.$$

$$v = \sqrt{\frac{2 \times 0.1 \times 0.18 \times 10 \times 0.6 + 2 \times 0.6 \times 0.6}{0.18}}$$

$$\therefore v = \frac{4}{10} = \frac{N}{10} \qquad \therefore N = 4$$

50. **(30)** Given,

Mass of body M = 5 kg

initial momentum P_i = 10 kg m/s

Impulse = $F\Delta t = \Delta P = P_f - P_i$

$2 \times 5 = P_f - 10$

final momentum P_f = 20 kg m/s

Increase in KE = $KE_f - KE_i$ $\left(\because KE = \frac{P^2}{2m}\right)$

$$= \frac{P_f^2}{2m} - \frac{P_i^2}{2m} = \frac{400}{2\times 5} - \frac{100}{2 \times 5} = 40 - 10 = 30J$$

51. **(2)** Given,

Mass of particle, m = 10 g

Retardation, a = 2x

Loss of K.E = work done against retarding force.

$$\therefore \Delta KE = \int_0^x madx = \int_0^x m2xdx = mx^2 = \frac{10}{1000} \times x^2 J = \left(\frac{10}{x}\right)^{-2} J$$

$\therefore$ n = 2

52. **(375)** Let V_1 be the velocity before hitting the floor and V_2 be the velocity just after hitting the floor.

$$\frac{V_1}{V_2} = 4 \Rightarrow V_1 = 4V_2 \qquad KE_{before} = \frac{1}{2}mV_1^2$$

$$KE_{after} = \frac{1}{2}mV_2^2 = \frac{1}{2}\frac{m.V_1^2}{16}$$

$$\Delta KE = KE_{after} - KE_{before} = \frac{1}{2}\frac{mV_1^2}{16} - \frac{1}{2}mV_1^2$$

$$\Delta KE = \frac{1}{2}mV_1^2\left(\frac{1}{16} - 1\right) = \frac{-15}{32}mV_1^2$$

The percentage loss in kinetic energy is given by

$$\frac{\Delta KE}{KE_{before}} \times 100\% = \frac{-15}{16} \times 100 = \frac{-375}{4}\%$$

The value of x is 375.

53. **(50)** We have $M.E_i = M.E_f$

$$\Rightarrow K_i + P_i = M.E_f \Rightarrow 0 + \frac{1}{2}kA^2 = 0.25$$

$$\Rightarrow \frac{1}{2} \times k \times 0.1^2 = 0.25 \Rightarrow k \times 0.01 = 0.5$$

$$\Rightarrow k = \frac{0.50}{0.01} = 50 \text{ N/m}$$

54. **(7)** $V^2 = u^2 + 2as = 2^2 + 2 \times 2 \times 6 = 28$

So, K.E = $\frac{1}{2}mV^2 = \frac{1}{2} \times 500 \times 28 = 7000J = 7$ kJ

55. **(300)** Given, mass of the body, m = 0.4 kg

Displacement in n^{th} sec.

$$D_n = u + \frac{1}{2}g(2n - 1)$$

$$\Rightarrow \quad D_8 = 0 + \frac{1}{2} \times 10 \times (2 \times 8 - 1) \Rightarrow D_8 = 5 \times 15$$

$\Delta U = mg\,D_8 = 0.4 \times 10 \times 5 \times 15 \Rightarrow \Delta U = 20 \times 15 = 300$

56. **(40)** We have, $\frac{1}{2}mv_f^2 = 10000$ J

$$\Rightarrow \frac{1}{2} \times 2 \times v_f^2 = 10000 \text{ J} \Rightarrow v_f^2 = 10000 \text{ J} \Rightarrow v_f = 100 \text{ m/s}$$

So, $a = \frac{v_f - v_i}{t} = \frac{100 - 0}{5} = 20 \text{ m/s}^2$

$F = ma = 20 \times 2 = 40$ N

57. **(24)** Using work-energy theorem, $W_{net} = (K_f - K_i)$

$$\Rightarrow \quad \frac{1}{2}Kx^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 - \frac{1}{2}mv^2 = \frac{E}{4} - E$$

$$\Rightarrow \quad \frac{1}{2}Kx^2 = \frac{3E}{4} \Rightarrow K = \frac{3E}{2x^2} \Rightarrow K = \frac{3E}{2 \times \left(\frac{1}{4}\right)} = 24E$$

So, value of spring constant of used spring is 24 times of kinetic energy

$\therefore$ n = 24

58. **(16)** Given,

Mass of engine - wagon system, m = 40,000 kg

Velocity, v = 72 × 5/18 = 20m/s

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2} \times (40{,}000) \times (20)^2 = 8000000 \text{ J}$$

As 90% of K.E of system lost in friction, only 10% is transfered to spring.

$$\therefore \frac{1}{2}Kx^2 = \frac{10}{100} \times 8000000 \Rightarrow \frac{1}{2} \times K \times 1 \times 1 = 800000$$

$$\Rightarrow K = 16 \times 10^5 \text{ N/m}$$

59. **(1)** From conservation of linear momentum

$P_{initial} = P_{final}$ (and $P = mv$)

$$m \times 40 = \frac{m}{2} \times v + \frac{m}{2} \times 60 \Rightarrow 40 = \frac{v}{2} + 30$$

$\therefore v = 20$

Initial kinetic energy, $E_1 = \frac{1}{2} m \times (40)^2 = 800m$

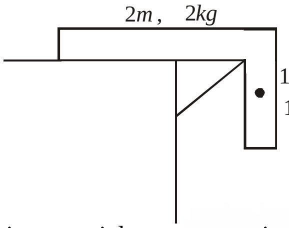
Final kinetic energy, E_2

$$= \frac{1}{2}\frac{m}{2}.(20)^2 + \frac{1}{2}.\frac{m}{2}(60)^2 = 1000m$$

$$\Delta E = E_2 - E_1 = 1000m - 800m = 200m$$

$$\frac{\Delta E}{E_1} = \frac{200m}{800m} = \frac{1}{4} = \frac{x}{4} \Rightarrow x = 1$$

60. **(40)**



Loss in potential energy = gain in kinetic energy

Take zero potential energy at table, initial potential energy

$$= -1 \times 10 \times \frac{1}{2} = -5J$$

Final potential energy $= -3 \times 10 \times \frac{3}{2} = -45J$

Change in potential energy = –5–(–45)J = 40J

$\therefore \quad k = 40$

61. **(6)** Here kinetic energy of ball is equal to P.E. stored in spring i.e., $\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta x)^2$

$$\Rightarrow \frac{1}{2} \times 4 \times (10)^2 = \frac{1}{2} \times 100 \times (\Delta x)^2 \Rightarrow \Delta x = 2 \text{ m}$$

Therefore length of the compressed spring

$x = 8 - 2 = 6$ m

62. **(1)** From $P = \sqrt{2mk} \Rightarrow P \propto \sqrt{m}$ as k is equal for two particles.

$$\therefore \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{4}{16}} = \frac{1}{2} \qquad \therefore n = 1.$$

63. **(2)** Kinetic energy of a body, $K = \frac{P^2}{2m}$

$$\therefore K \propto \frac{1}{m} \quad (P_A = P_B) \qquad \text{Now, } \frac{K_A}{K_B} = \frac{m_B}{m_A} = \frac{2}{1}.$$

64. **(150)** From work energy theorem,

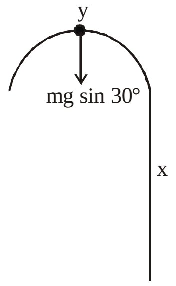
$$W = F \cdot s = \Delta KE = \frac{1}{2}mv^2 \qquad \text{Here } V^2 = 2gh$$

$$\therefore F \cdot s = F \times \frac{2}{10} = \frac{1}{2} \times \frac{15}{100} \times 2 \times 10 \times 20 \therefore F = 150 \text{ N.}$$

65. **(a, d)** At point Y the centripetal force provided by the component of weight mg

$$\therefore mg \sin 30° = \frac{mv^2}{R}$$

$$\therefore v^2 = \frac{gR}{2} \qquad ..(ii)$$



Now by the energy conservation between bottom point and point Y

$$\frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv^2$$

$$\therefore v^2 = v_0^2 - 2gh \qquad ...(i)$$

$\therefore$ From eq. (i)

$$\frac{gR}{2} = v_0^2 - 2gh$$

Hence option (a) is correct.

At point x and z of circula path, the points are at same height but less then h. So the velocity more than at point y.

So required centripetal force $= \frac{mv^2}{r}$ is maximum at points x and y.

66. **(a, b, d)** $G \frac{dk}{dt} = \gamma t$ and $k = \frac{1}{2}mV^2 \quad \therefore \frac{d}{dt}\left(\frac{1}{2}mV^2\right) = \gamma t$

$$\Rightarrow \frac{m}{2} \times 2V\frac{dV}{dt} = \gamma t \qquad \therefore mV\frac{dV}{dt} = \gamma t$$

$$\therefore m\int_0^V VdV = \gamma\int_0^t tdt \qquad \Rightarrow \frac{mV^2}{2} = \frac{\gamma t^2}{2}$$

$$\therefore V = \sqrt{\frac{\gamma}{m}} \times t. \text{ i.e., } V \propto t;\ V = \frac{ds}{dt} = \sqrt{\frac{\gamma}{m}}t \Rightarrow s = \sqrt{\frac{\gamma}{m}}\frac{t^2}{2}$$

So V is proportional to 't' and distance cannot be proportional to 't'.

$$\text{Now } F = ma = m\frac{dV}{dt} = m\frac{d}{dt}\left[\sqrt{\frac{\gamma}{m}} \times t\right] = m\sqrt{\frac{\gamma}{m}} = \sqrt{\gamma m} = \text{constant}$$

Since force applied is constant and displacement between any two points on x-axis will also be constant, thus work done will be independent of path. Hence force is conservative in nature.

67. **(a, b, d)** From figure given in question,

Potential energy of the ball at point A = mgh_A

Potential energy of the ball at point B = 0

Potential energy of the ball at point C = mgh_C

Total energy at point A, $E_A = K_A + mgh_A$

Total energy at point B, $E_B = K_B$

Total energy at point C, $E_C = K_C + mgh_C$

As body rolls between A and B and between B and C there is no friction. So energy should be conserved here

By law of conservation of energy.

$E_A = E_B = E_C$ $\qquad$ As $E_A = E_C$

$K_A + mgh_A = K_C + mgh_C$

So, If $h_A > h_C \Rightarrow K_A < K_C$. So option (b) is correct

If $h_A < h_C \Rightarrow K_A > K_C$

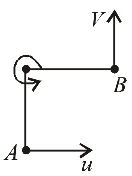
Doesn't matter if $h_A > h_C$ or $h_A < h_C$, we will always have $K_B > K_C$ because $E_A = E_B = E_C$. So option (a) and (d) is also correct.

68. **(d)** From principle of conservation of energy

$$(K.E.)_B + (P.E.)_B = (K.E.)_A + (P.E.)_A,$$

$$\text{or, } \frac{1}{2}mv^2 + mgL = \frac{1}{2}mu^2 + 0$$

$$\Rightarrow v = \sqrt{u^2 - 2gL} \quad ... (i)$$



Change in velocity $|\Delta \vec{v}|$

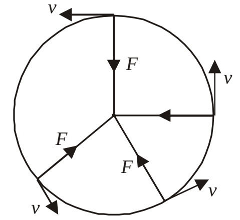
$$= |\vec{v} - \vec{u}| = \sqrt{v^2 + u^2}$$

Putting the value of v from eq. (i)

$$|\Delta \vec{v}| = \sqrt{2(u^2 - g\ell)}$$

69. **(c, d)** As particle is acted upon by a force of constant magnitude and is perpendicular to the velocity of the particle so it is a case of uniform circulat motion.

The force is constant in magnitude, this show the speed is not changing and hence kinetic energy will remain constant.
The velocity and acceleration changes continuously due to change in the direction.

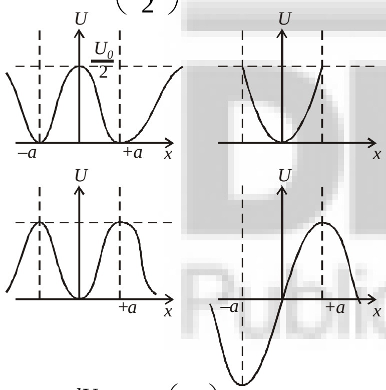


70. **A → p, q, r, t; B → q, s; C → p, q, r, s; D → p, r, t**
For A

$$Fx = -\frac{dU}{dx} = \frac{-d}{dx}\frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2 = \frac{-2U_0}{a^4}(x-a)x(x+a)$$

F = 0 at $x = 0, x = a, x = -a$
and $U = 0$ at $x = -a, x = a$
For attractive force, Fα – x. So (s) does not apply.

At x = – a, $\frac{d^2U}{dx^2}$ is +ve, which implies curve is having minima at this point and will oscillate about this point as it is minimum P.E. and total energy $\left(\frac{U_0}{4}\right)$ of particle is less than maximum P.E. $\left(\frac{U_0}{2}\right)$



For B $F_x = \frac{-dU}{dx} = -U_0\left(\frac{x}{a^2}\right)$

For C $F_x = \frac{-dU}{dx} = U_0\frac{e^{-x^2/a^2}}{a^3}x(x-a)(x+a)$

For D $F_x = -\frac{dU}{dx} = \frac{U_0}{2a^3}[(x-a)(x+a)]$

71. **(b)** As $W_{all\ forces} = \Delta K \Rightarrow W_{mg} + W_{fr} = \frac{1}{2}mv^2 - 0$

$\Rightarrow mgh - 150 = \frac{1}{2}mv^2 \Rightarrow mgR\sin 30° - 150 = \frac{1}{2}mv^2$

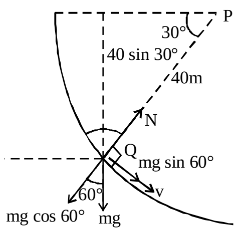
$\Rightarrow 1 \times 10 \times 40 \times \frac{1}{2} - 150 = \frac{v^2}{2} \Rightarrow v = 10$ m/s

$N - mg\cos\theta$ will provide required centripetal force

$N - mg\cos\theta = \frac{mv^2}{R}$

$N = mg\cos\theta + \frac{mv^2}{R}$

$= 1 \times 10 \times \frac{1}{2} + \frac{1\times(10)^2}{40} = 7.5$ N



72. **(b)** As discussed earlier, we get v = 10 m/s.

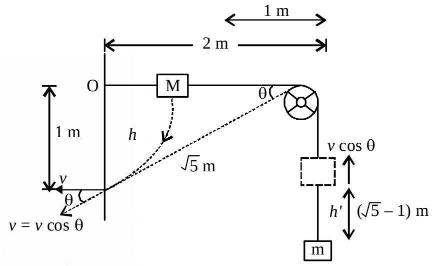
73. **(c)** In the first case the mechanical energy is completely converted into heat becuase of friction. *i.e.*, Decrease in mechanical energy = $\frac{1}{2}mv^2$.

While is second case, a part of mechanical energy is converted into heat due to fiction but another part of mechanical energy is retained in the form of potential energy of the block. *i.e.*,

Decrease in mechanical energy = $\frac{1}{2}mv^2 - mgh$

Therefore statement 1 is correct.
Statement 2 is wrong. The coefficient of friction between the block and the surface does not depend on the angle of inclination.

74. Let point mass hit the wall with the speed v.
Then, velocity of mass m at this instant = $v\cos\theta = \frac{2}{\sqrt{5}}v$.
Further M will fall a distance of 1 m while m will rise up by $(\sqrt{5}-1)$ m.
From energy conservation, Decrease in P.E. of M = increase in P.E. of m + increase in K.E. of both the blocks.



$$Mgh = mgh' + \frac{1}{2}mv^2 + \frac{1}{2}m(v\cos\theta)^2$$

or, $(2)(9.8)(1) = (0.5)(9.8)(\sqrt{5}-1) + \frac{1}{2}\times 2\times v^2 + \frac{1}{2}\times 0.5\times\left(\frac{2v}{\sqrt{5}}\right)^2$

Solving we get, v = 3.29 m/s.

75. K.E. of block = work against friction + P.E. of spring

$\frac{1}{2}mv^2 = \mu_k mg(BD + x) + \frac{1}{2}kx^2$

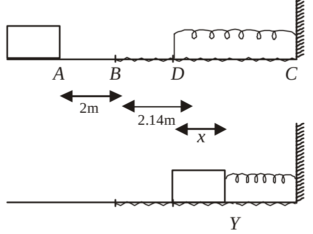
$\frac{1}{2}mv^2 = \mu_k mg(2.14 + x) + \frac{1}{2}kx^2$

$\frac{1}{2}\times 0.5\times 3^2 = 0.2\times 0.5\times 9.8(2.14 + x) + \frac{1}{2}2\times x^2$

$2.14 + x + x^2 = 2.25 \therefore x^2 + x - 0.11 = 0$

On solving, we get $x = \frac{1}{10} = 0.1$

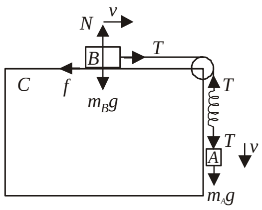
The spring gets compressed by 0.1 m
Restoring force at $Y = kx = 2 \times 0.1 = 0.2\ N$
Frictional force at $Y = \mu_s mg = 0.22 \times 0.5 \times 9.8 = 1.078\ N$
Since frictional force > restoring force, the body will stop here *i.e.*, after compressing the spring by x
∴ The total distance travelled
$= AB + BD + x = 2 + 2.14 + 0.1 = 4.24$ m.



76. Here the net force acting on A and B is zero. Since the blocks A and B are moving with constant velocity.

Let the extension of the spring be x.

There will be no friction force between block A and C



$\because$ $f = \mu N$. Here there is no normal reaction on A because A is not pushing C so frictional force between block A and C,

$f = \mu N = 0$

Applying $F_{net} = ma$ on A, $m_A g - T = m_A \times 0$

$\therefore \quad T = m_A g$ ... (i)

Applying $F_{net} = ma$ on B,

$T - f = m_B \times 0$

$\therefore \quad T = f = \mu N$

$= \mu m_B g$ ... (ii)

From (i) and (ii)

$$\mu m_B g = m_A g \Rightarrow m_B = \frac{m_A}{\mu} = \frac{2}{0.2} = 10\,\text{kg}$$

Here the force acting on the spring is the tension equal to restoring force

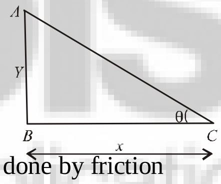
$\therefore \quad T = kx \Rightarrow x = \frac{T}{k} \qquad \therefore \; x = \frac{m_a g}{k} = \frac{19.6}{k}$

Energy stored in spring

$$U = \frac{1}{2}kx^2 = \frac{1}{2} \times k \times \frac{19.6}{k} \times \frac{19.6}{k} = \frac{19.6 \times 19.6}{2 \times 1960} = 0.098\,\text{J}$$

77. Here, K.E. at C = Loss in P.E. – work done by friction

In both cases work done by friction = μmgx



or, $\frac{1}{2}mv_c^2 = mg\,y - \mu mgx$

$= gy - \mu gx$

$\therefore \quad v_c = \sqrt{2g(y - \mu x)}$

Now, K.E. at F = loss in P.E. – work done by friction

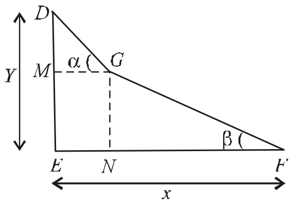
$$\frac{1}{2}mv_f^2 = mgy - (\mu\, mg \cos\alpha)DG - (\mu\, mg \cos\beta)GF$$

$$\Rightarrow \frac{1}{2}mv_f^2 = mgy - \mu\, mg\,(DG\cos\alpha + GF\cos\beta)$$

$$\Rightarrow \frac{1}{2}mv_F^2 = mgy - \mu mgx$$

$$\Rightarrow \quad \frac{1}{2}v_F^2 = gy - \mu gx$$

$\therefore \quad v_F = \sqrt{2g(y - \mu x)}$



i.e., $v_C = v_F = \sqrt{2gy - 2\mu gx}$

78. No. when a ball is thrown up the gravitational pull of earth, is acting on the ball which is responsible for the change in momentum. As $F_{net} \neq 0$, so momentum will not be conserved.

79. For a spring, (spring constant) k × (length) l = Constant

If length is made one third, *i.e.*, $\frac{l}{3}$ the spring constant becomes three times *i.e.*, 3K.

80. Using Kinetic energy (K.E.) and momentum relation,

K.E. = $\frac{p^2}{2m}$ For equal value of p, K.E. $\propto \frac{1}{\text{mass}}$

Topic-3: Power

1. **(c)** Given, P = costant $\Rightarrow$ FV = constant = K

$$\Rightarrow m\frac{dV}{dt}V = \text{constant} = K$$

$$\Rightarrow \int_0^V V\,dV = (K)\int_0^t dt \Rightarrow \left(\frac{V^2}{2}\right) = kt \Rightarrow V \propto t^{1/2}$$

$$\Rightarrow \frac{ds}{dt} \propto t^{1/2} \Rightarrow \int_0^S ds = C\int_0^t t^{1/2}dt \Rightarrow S = C \times \frac{2}{3}t^{3/2}$$

$\therefore \; s \propto t^{3/2}$

2. **(d)** Given,

Mass of the body, m = 2 kg

Force, $\vec{F} = ma = (6t\hat{i} + 6t^2\hat{j})\,N$

$$\vec{a} = \frac{\vec{F}}{m} = 3t\hat{i} + 3t^2\hat{j} \Rightarrow \frac{d\vec{v}}{dt} = 3t\hat{i} + 3t^2\hat{j}$$

$$\Rightarrow v = \frac{3t^2}{2}\hat{i} + t^3\hat{j}$$

$$P = \vec{F}.\vec{v} = (6t\hat{i} + 6t^2\hat{j}).(\frac{3t^2}{2}\hat{i} + t^3\hat{j}) = (9t^3 + 6t^5)W$$

3. **(d)** Power, $P = \frac{W}{t} = \frac{mgh}{t}$

$$\therefore \frac{P_1}{P_2} = \frac{\frac{m_1 gh}{t_1}}{\frac{m_2 gh}{t_2}} = \frac{m_1}{t_1}\frac{t_2}{m_2}$$

$$\therefore \frac{P_1}{P_2} = \frac{300 \times 2}{5 \times 50} = \frac{12}{5} = \frac{3\sqrt{x}}{\sqrt{x}+1}$$

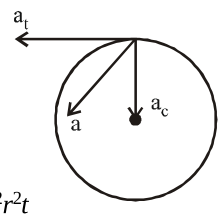
So, $12\sqrt{x} + 12 = 15\sqrt{x} \Rightarrow 3\sqrt{x} = 12$

$\therefore x = 16$

4. **(d)** $F_{thrust} = V_{rel}\frac{dm}{dt} = 5 \times 0.5 = 2.5\,N$

So, Power = Force × Velocity = 2.5 × 5 = 12.5 watt.

5. **(c)** $a_c = k^2rt^2 = \frac{v^2}{r} \Rightarrow v = krt$



$a_t = \frac{dv}{dt} = kr$

Now, $P = (\vec{F_r} + \vec{F_t}).\vec{v}$

$= 0 + F_t v = ma_t v = mkr\,(krt) = mk^2r^2t$

6. **(b)** Total force required to lift maximum load capacity against frictional force = 400 N

$F_{total} = Mg$ + friction

$= 2000 \times 10 + 4000 = 20{,}000 + 4000 = 24000\,N$

Using power, $P = F \times v$

$60 \times 746 = 24000 \times v \Rightarrow v = 1.86\text{ m/s} \approx 1.9\,m/s$

Hence speed of the elevator at full load is close to 1.9 ms^{-1}

7. **(b)** Centripetel acceleration $a_c = n^2Rt^2$

$$a_c = \frac{v^2}{R} = n^2Rt^2$$

$v^2 = n^2R^2t^2 \qquad v = nRt$

Here power is delivered by tangential force only because power by centripetal force is zero. $\left[\text{Since } \vec{F}_c \perp \vec{V}\right]$

$$a_t = \frac{dv}{dt} = nR$$

Power = $ma_tv = m\,nR\,nRt = Mn^2R^2t$.

8. **(c)** Force exerted by wind, $F = m\frac{dv}{dt} + v\frac{dm}{dt}$

as, v = cons.

$$F = v\frac{dm}{dt} = v\frac{d}{dt}(v_0 \times \rho),\ v_0 = \text{volume} = v\rho\frac{d}{dt}(Al)$$

$= v\rho A\frac{dl}{dt}$ So, power = Fv

$= v\rho Av$ $= \rho Av^3$

$= \rho Av^2$ i.e. power $\propto v^3$

9. **(b)** The centripetal acceleration $a_c = k^2\ r\ t^2$

or, $\frac{v^2}{r} = k^2rt^2 \qquad \therefore \quad V = krt$

Now, $a_t = \frac{dv}{dt} = kr$

Power, $P = F_tv = ma_tv$

$\therefore$ Power = mk^2r^2t

10. **(c)** Power, $P = \frac{w}{t} = \frac{E}{t} = \text{constt} \quad \therefore \quad \frac{\frac{1}{2}mv^2}{t} = \text{constt}$

From work-energy theorem, not work done = change in kinetic energy.

$\Rightarrow \frac{v^2}{t} = \text{constt}(k) \therefore \quad v = kt^{1/2}$ and $\frac{ds}{dt} = kt^{1/2}$

or, $ds = kt^{1/2}\ dt$

By integrating, we get $\Rightarrow\ s = \frac{2kt^{3/2}}{3} + C \quad \Rightarrow\ s \propto t^{3/2}$

i.e., Distance moved $S \propto t^{3/2}$

11. **(b)** If machine is lubricated with oil friction is reduced.

Mechanical efficiency = $\frac{\text{Output work}}{\text{Input energy}}$

Due to less friction output work will increase. Thus the mechanical efficiency increases.

Mechanical advantage, M.A. = $\frac{\text{load}}{\text{effort}}$

12. **(5)** Using, work – energy theorem, Δ K.E. = W = P × t

$$\frac{1}{2}mv^2 = P \times t \therefore v = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2 \times 0.5 \times 5}{0.2}} = 5ms^{-1}$$

13. **(300)** Given, acceleration of block, $a = 1ms^{-2}$

Mass of block, m = 5 kg

$\therefore\ v = u + at = 0 + 1(10) = 10$ m/s

$F - mg\sin\theta = ma \Rightarrow F - 5g\sin 30° = 5a$

$\Rightarrow F = 5 + 25 = 30N$

Power P = Fv = 300 W

14. **(100)** We have $\vec{F} = t\hat{i} + 3t^2\hat{j}$

$$\Rightarrow \frac{md\vec{v}}{dt} = t\hat{i} + 3t^2\hat{j} \Rightarrow 1.d\vec{v} = \left(t\hat{i} + 3t^2\hat{j}\right)dt$$

$$\Rightarrow \int_0^{\vec{v}} d\vec{v} = \int_0^t tdt\hat{i} + \int_0^t 3t^2dt\hat{j} \Rightarrow \vec{v} = \frac{t^2}{2}\hat{i} + t^3\hat{j}$$

And, Power = $\vec{F}.\vec{V} = \frac{t^3}{2} + 3t^5$

At t = 2, power = $\frac{8}{2} + 3 \times 32 = 100$ w

15. **(18)** Given, Mass of the body, $m = 2$ kg

Power delivered by engine, $P = 1$ J/s

Time, $t = 9$ seconds

Power, $P = Fv$

$\Rightarrow P = mav \qquad [\because F = ma]$

$\Rightarrow m\frac{dv}{dt}v = P \qquad \left(\because a = \frac{dv}{dt}\right)$

$\Rightarrow v\ dv = \frac{P}{m}dt$

Integrating both sides we get

$$\Rightarrow \int_0^v v\,dv = \frac{P}{m}\int_0^t dt \Rightarrow \frac{v^2}{2} = \frac{Pt}{m} \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$$

$\Rightarrow \frac{dx}{dt} = \sqrt{\frac{2P}{m}}t^{1/2} \qquad \left(\because v = \frac{dx}{dt}\right)$

$$\Rightarrow \int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\ dt$$

$$\therefore \text{ Distance, } x = \sqrt{\frac{2P}{m}}\frac{t^{3/2}}{3/2} = \sqrt{\frac{2P}{m}} \times \frac{2}{3}t^{3/2}$$

$$\Rightarrow x = \sqrt{\frac{2 \times 1}{2}} \times \frac{2}{3} \times 9^{3/2} = \frac{2}{3} \times 27 = 18m$$

Topic-4: Collisions

1. **(d)** Using $v^2 - u^2 = 2gh$

Velocity just before collision $= \sqrt{2gh}$

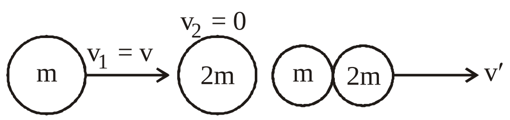
Velocity just after collision $= \sqrt{2g\left(\frac{h}{2}\right)}$

$$\therefore \Delta KE = \frac{1}{2}m(2gh) - \frac{1}{2}mgh = \frac{1}{2}mgh$$

$\therefore$ % Loss in energy

$$= \frac{\Delta KE}{KE_i} \times 100 = \frac{\frac{1}{2}mgh}{\frac{1}{2}mg2h} \times 100 = 50\%$$

2. **(c)**



Applying conservation of linear momentum

$\Rightarrow \vec{P}_i = \vec{P}_f \quad (\because P = mv)$

$$mv_1 + 2mv_2 = (m+2m)v'$$

$$mv + 2m \times 0 = (3m)v' \Rightarrow mv = 3mv' \Rightarrow v' = \frac{v}{3}$$

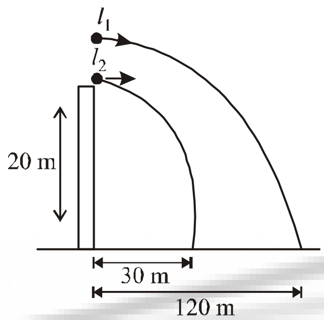
3. **(c)** The velocities will be interchanged after collision.
$O = v^2 - 2gh$

$v = \sqrt{2gh}$

Speed of P just before collision = $\sqrt{2gh}$

$= \sqrt{2 \times 10 \times 0.2} = 2\text{m/s}$ $\quad (\because h = 20\text{ cm} = 0.2\text{ m})$

4. **(d)** From equation of motion $h = ut + \frac{1}{2}gt^2$



$h = \frac{1}{2}gt^2$

$t = \sqrt{\frac{2h}{g}}$; $v_1 = \frac{30}{\sqrt{\frac{2h}{g}}}$,

$v_2 = \frac{120}{\sqrt{\frac{2h}{g}}}$

apply the conservation of momentum $Mu = m_1v_1 + m_2v_2$

$$(0.01)u = (0.2)\frac{30\sqrt{g}}{\sqrt{2h}} + (0.01)\frac{120\sqrt{g}}{\sqrt{2h}}$$

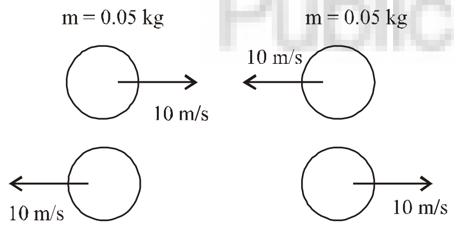
$u = 300 + 60 = 360\text{ ms}^{-1}$

5. **(b)** As no external force is acting on system so, $P_i = P_f$
$0.2 \times 10 = 10 \times v \Rightarrow v = 0.2$ m/sec

Loss in K.E. $= \frac{1}{2} \times (0.2) \times 10^2 - \frac{1}{2} \times 10(0.2)^2$

$= \frac{1}{2} \times 10 \times (0.2)[10 - 0.2] = 9.8$ J

6. **(b)**



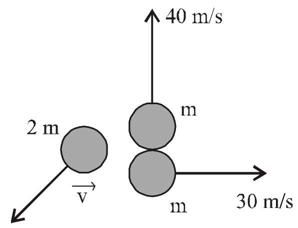
Chnage in momentum of any one ball

$|\Delta \vec{P}| = 2 \times 0.05 \times 10 = 1$

$$|\vec{F}_{av}| = \frac{|\Delta \vec{P}|}{\Delta t} = \frac{1}{0.005} = \frac{1000}{5} = 200\text{ N}$$

7. **(b)** By law of conservation of momentum

$|\vec{P}_i| = |\vec{P}_f|$



$0 = m\left(30\hat{i} + 40\hat{j}\right) + 2m\vec{v}$

$\vec{v} = -15\hat{i} - 20\hat{j}$

So, $|\vec{v}| = \sqrt{-15^2 + (-20)^2}$

$= \sqrt{625} = 25$ m/s.

8. **(c)** Let the velocity of striking particle be u_0
Then, $mu_0 = mv_1 + 5mv_2$
$u_0 = v_1 + 5v_2$ ...(i)
as, collision is elastic
So, $e = 1$

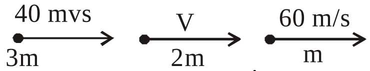
$\Rightarrow \frac{v_2 - v_1}{u_0} = 1$

$\Rightarrow v_2 - v_1 = u_0$ ...(ii)
Adding (i) and (ii), we get

$2u_0 = 6v_2 \Rightarrow v_2 = u_0/3$

$$\text{So, } \%\ \Delta K.E_2 = \frac{\frac{1}{2}(5m)\left(\frac{u_0}{3}\right)^2 - 0}{\frac{1}{2}mu_0^2} \times 100 = \frac{500}{9} \simeq 55.6\%$$

9. **(c)**

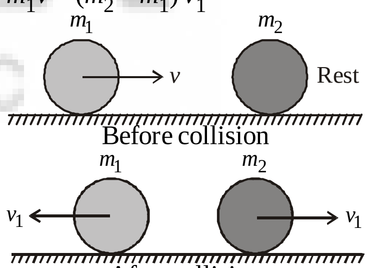


Using momentum conservation
$3m \times 40 = 2mv + m60 \Rightarrow v = 30$ m/s
Fractional charge in K.E is given by

$$\frac{\Delta KE}{K.E} = \frac{\text{Final K.E} - \text{Initial K.E}}{\text{Initial K.E}} = \frac{\frac{1}{2}m60^2 + \frac{1}{2}2m \times 30^2 - \frac{1}{2}3m\,40^2}{\frac{1}{2}3m\,40^2}$$

$$= \frac{60^2 + 2 \times 30^2 - 3 \times 40^2}{3 \times 40^2} = \frac{1}{8}$$

10. **(b)** After the collision the objects move in opposite direction let with velocity v_1 then from law of conservation of momentum $P_i = P_f$
$m_1v = (m_2 - m_1)v_1$



After collision

$\Rightarrow \quad v_1 = \frac{m_1 v}{(m_2 - m_1)}$ . . . (i)

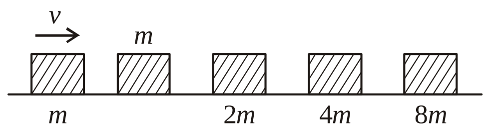
If collision is elastic then, $e = 1 = \frac{v_1 - (-v_1)}{v - 0}$

also, $2v_1 = v \Rightarrow v_1 = \frac{v}{2}$ . . . (ii)

From equation (i) & (ii)

$\frac{1}{2} = \frac{m_1}{m_2 - m_1}$ $\quad \therefore \quad \frac{m_2}{m_1} = \frac{3}{1}$

11. **(b)** According to the question, all collisions are perfectly inelastic, so after the final collision, all blocks are moving together.



Let the final velocity be v', using momentum conservation

$$mv = 16mv' \Rightarrow v' = \frac{v}{16}$$

Now initial energy $E_i = \frac{1}{2}mv^2$

Final energy : $E_f = \frac{1}{2} \times 16m \times \left(\frac{v}{16}\right)^2 = \frac{1}{2}\frac{mv^2}{16}$

Energy loss : $E_i - E_f = \frac{1}{2}mv^2 - \frac{1}{2}m\frac{v^2}{16}$

$$\Rightarrow \frac{1}{2}mv^2\left[1 - \frac{1}{16}\right] \Rightarrow \frac{1}{2}mv^2\left[\frac{15}{16}\right]$$

The total energy loss is P% of the original energy.

$$\therefore \%P = \frac{\text{Energy loss}}{\text{Original energy}} \times 100$$

$$= \frac{\frac{1}{2}mv^2\left[\frac{15}{16}\right]}{\frac{1}{2}mv^2} \times 100 = 93.75\%$$

Hence, value of P is close to 94.

12. **(b)** Given,
Mass of block, $m_1 = 1.9$ kg
Mass of bullet, $m_2 = 0.1$ kg
Velocity of bullet, $v_2 = 20$ m/s
Let v be the velocity of the combined system. It is an inelastic collision.
Using conservation of linear momentum

$$m_1 \times 0 + m_2 \times v_2 = (m_1 + m_2)v$$

$$\Rightarrow 0.1 \times 20 = (0.1 + 1.9) \times v \Rightarrow v = 1 \text{ m/s}$$

Using work energy theorem
Work done = Change in Kinetic energy
Let K be the Kinetic energy of combined system.

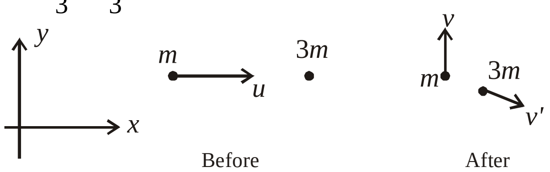
$$(m_1 + m_2)gh = K - \frac{1}{2}(m_1 + m_2)v^2$$

$$\Rightarrow 2 \times g \times 1 = K - \frac{1}{2} \times 2 \times 1^2 \Rightarrow K = 21 \text{ J}$$

13. **(c)** From conservation of linear momentum

$$mu\hat{i} + 0 = mv\hat{j} + 3m\vec{v}'$$

$$\vec{v}' = \frac{u}{3}\hat{i} - \frac{v}{3}\hat{j}$$



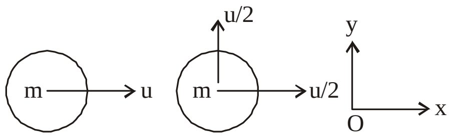
Before collision After collision

From kinetic energy conservation,

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)\left(\left(\frac{u}{3}\right)^2 + \left(\frac{v}{3}\right)^2\right)$$

or, $mu^2 = mv^2 + \frac{mu^2}{3} + \frac{mv^2}{3}$ $\therefore v = \frac{u}{\sqrt{2}}$

14. **(b)**



x-direction

$$mu + \frac{mu}{2} = 2mv_x' \Rightarrow V_x' = \frac{3u}{4}$$

y-direction $0 + \frac{mu}{2} = 2mv_y' \Rightarrow v_y' = \frac{u}{4}$

$$K.E._i = \frac{1}{2}m\,u^2 + \frac{1}{2}m\left[\left(\frac{u}{2}\right)^2 + \left(\frac{u}{2}\right)^2\right]$$

$$= \frac{1}{2}mu^2 + \frac{mu^2}{4} = \frac{3mu^2}{4}$$

$$K.E._f = \frac{1}{2}(2m)\left(v_x'\right)^2 + \frac{1}{2}(2m)\left(v_y'\right)^2$$

$$= \frac{1}{2}2m\left[\left(\frac{3u}{4}\right)^2 + \left(\frac{u}{4}\right)^2\right] = \frac{5}{8}mu^2$$

$$\therefore \text{Loss in } KE = KE_f - KE_i = mu^2\left(\frac{6}{8} - \frac{5}{8}\right) = \frac{mu^2}{8}$$

15. **(b)** $v_{s,\,m} = v_s - v_m \Rightarrow 0.7 = v_s - v_m$
$P_i = P_f$
or $0 = 20(0.7 - v) = 50v$
or $v = 0.2$ m/s

16. **(d)** By law of conservation of momentum

$$2u = 2\frac{u}{4} + mv \Rightarrow \frac{3u}{2} = mv$$

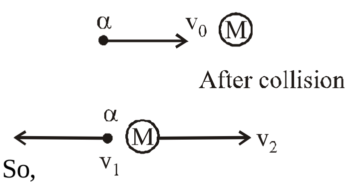
Now, $e = \frac{v - \frac{u}{4}}{u} \Rightarrow u = v - \frac{u}{4}$ $[\because e = 1] \Rightarrow \frac{5u}{4} = v$

$$\Rightarrow \frac{5mu}{4} = \frac{3u}{2} \Rightarrow m = \frac{6}{5} = 1.2 \text{ kg}$$

17. **(b)** $m(2v) + 2mv = 0 + 2mv' \cos 45°$ or $v' = 2\sqrt{2}v$

18. **(d)** Using conservation of momentum,

$$mv_0 = mv_2 - mv_1$$



$$\frac{1}{2}mv_1^2 = 0.36 \times \frac{1}{2}mv_0^2$$

$$\Rightarrow v_1 = 0.6v_0$$

The collision is elastic. So,

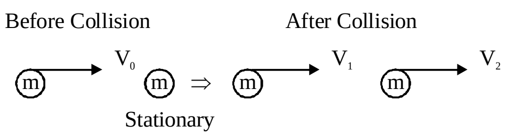
$$\frac{1}{2}Mv_2^2 = 0.64 \times \frac{1}{2}mv_0^2 \; [\therefore \text{M = mass of nucleus}]$$

$$\Rightarrow v_2 = \sqrt{\frac{m}{M}} \times 0.8v_0$$

$$mv_0 = \sqrt{mM} \times 0.8v_0 - m \times 0.6v_0$$

$$\Rightarrow 1.6m = 0.8\sqrt{mM} \Rightarrow 4m^2 = mM \qquad \therefore M = 4m$$

19. **(b)**



$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow \quad v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \qquad \text{....(i)}$$

From momentum conservation

$$mv_0 = m(v_1 + v_2) \qquad \text{....(ii)}$$

Squarring both sides,

$$(v_1 + v_2)^2 = v_0{}^2 \Rightarrow v_1{}^2 + v_2{}^2 + 2v_1v_2 = v_0{}^2$$

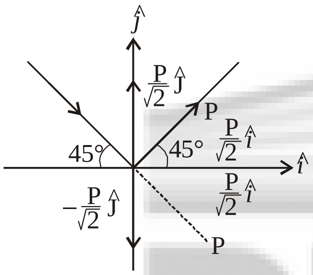
$$2v_1v_2 = -\frac{v_0^2}{2}$$

$$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 = \frac{3}{2}v_0^2 + \frac{v_0^2}{2}$$

Solving we get relative velocity between the two particles

$$v_1 - v_2 = \sqrt{2}v_0$$

20. **(a)** Change in momentum



$$\Delta P = \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{i} - \frac{P}{\sqrt{2}}\hat{i}$$

$$\Delta P = \frac{2P}{\sqrt{2}}\hat{j} = I_H \text{ molecule} \quad \Rightarrow \quad I_{wall} = -\frac{2P}{\sqrt{2}}\hat{j}$$

Pressure, P

$$= \frac{F}{A} = \frac{\sqrt{2}P}{A}n \qquad (\because n = \text{no. of particles})$$

$$= \frac{\sqrt{2} \times 3.32 \times 10^{-27} \times 10^3 \times 10^{23}}{2 \times 10^{-4}} = 2.35 \times 10^3 \text{N/m}^2$$

21. **(a)** For particle C,

According to law of conservation of linear momentum, verticle component,

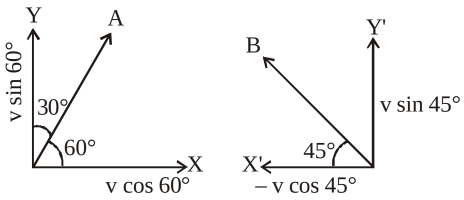
$$2\,mv' \sin\theta = mv \sin 60° + mv \sin 45°$$

$$2mv'\sin\theta = \frac{mv}{\sqrt{2}} + \frac{mv\sqrt{3}}{2} \qquad \text{...... (i)}$$

Horizontal component,

$$2\,mv' \cos\theta = mv \sin 60° - mv \cos 45°$$

$$2mv'\cos\theta = \frac{mv}{2} + \frac{mv}{\sqrt{2}} \qquad \text{...... (ii)}$$



For particle A **For particle B**

Dividing eq^n (i) by eq^n (ii),

$$\tan\theta = \frac{\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}}{\frac{1}{2} - \frac{1}{\sqrt{2}}} = \frac{\sqrt{2} + \sqrt{3}}{1 - \sqrt{2}}$$

22. **(b)** Given, $m_1 = 4g$, $u_1 = 300$ m/s

$m_2 = 0.8$ kg $= 800$ g, $u_2 = 0$ m/s

From law of conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Let the velocity of combined system = v m/s

then,

$$4 \times 300 + 800 \times 0 = (800 + 4) \times v \Rightarrow v = \frac{1200}{804} = 1.49 \text{ m/s}$$

Now, $\mu = 0.3$ (given)

$a = \mu g \;\Rightarrow a = 0.3 \times 10$ (take $g = 10$ m/s^2)

$= 3$ m/s^2

then, from $v^2 = u^2 + 2as$

$$(1.49)^2 = 0 + 2 \times 3 \times s$$

$$s = \frac{(1.49)^2}{6} \qquad s = \frac{2.22}{6} = 0.379 \text{ m}$$

23. **(b)** As tennis ball is dropped, so initial velocity $u = 0$

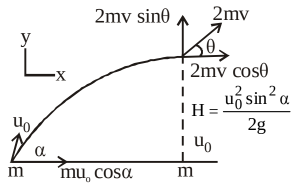
$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m[u + at]^2 = \frac{1}{2}m[0 + gt]^2$$

$$\therefore \; K.E = \frac{1}{2}mg^2t^2 \qquad \therefore \; K.E \propto t^2 \qquad \text{...(i)}$$

i.e., The relation between k and t is parabolic.

First the kinetic energy will increase as per eq (i). As the balls touches the ground it starts deforming and loses its K.E., when the deformation is maximum, K.E. = 0. As the ball moves up it loses K.E. and gain gravitational potential energy in the same time interval. These characteristics are best illustrated by kt graph shown in (b).

24. **(a)** Height, $h = \dfrac{u_0^2 \sin^2 \alpha}{2g}$



using $v^2 - u^2 = 2gh$

$$v_1^2 - u_0^2 = 2(-g)\left[\frac{u_0^2 \sin^2 \alpha}{2g}\right]$$

$$\Rightarrow \quad v_1^2 = u_0^2(1 - \sin^2\alpha) = u_0^2 \cos^2\alpha$$

$\Rightarrow \quad v_1 = u_0 \cos\alpha$

Applying conservation of linear momentum in Y-direction

$2mv \sin\theta = mv_1 = mu_0\cos\alpha$ ...(i)

Applying conservation of linear momentum in X-direction

$2mv \cos\theta = mu_0 \cos\alpha$ ...(ii)

Dividing (i) and (ii) we get

$$\tan\theta = 1 \qquad \therefore \; \theta = 45° = \frac{\pi}{4}$$

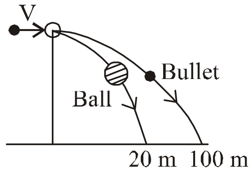
25. **(d)** Let after 't' time both ball and bullet hit the ground.

Then, $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 5}{10}} = 1$ sec.

After collision let V_{ball} be velocity of ball and V_{bullet} be velocity of bullet.

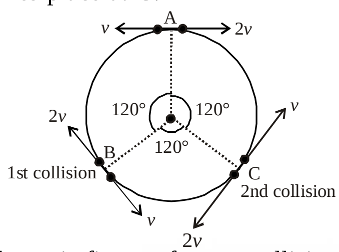
So, $20 = V_{ball} \times 1 \Rightarrow V_{ball} = 20$ m/s

$100 = V_{bullet} \times 1 \Rightarrow V_{bullet} = 100$ m/s



By law of conservation of momentum
$0.01\,V = 0.01 \times 100 + 0.2 \times 20$
$\Rightarrow 0.01\,V = 1 + 4$
$\Rightarrow V = \dfrac{5}{0.01} = 500$ m/s.

26. **(c)** According to question, between collision, the particles move with constant speed.
At first collision one particle having speed $2v$ will rotate $2 \times 120° = 240°$ while other particle having speed v will rotate 120°. Hence, first collision takes place at B. At first collision, they will exchange their velocities as the collision is elastic and the particles have equal masses. Again second collision takes place at C.



Now, as shown in figure, after two collisions they will again reach at point A.

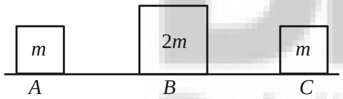
27. **(d)** $F_{ext} = (m_1 + m_2)g$ and $\Delta t = 2t_0$

$F_{ext} = \dfrac{\Delta p}{\Delta t}$

$\therefore\ \Delta p = F_{ext}\ \Delta t = \left(m_1\vec{v}_1{}' + m_2\vec{v}_2{}'\right) - \left(m_1\vec{v}_1 + m_2\vec{v}_2\right)$
$= (m_1 + m_2)\,g \times 2t_0$

28. **(4)** Hence, Just after collision with A velocity of B

$$v_B = \frac{(m_B - m_A)u_B}{m_B + m_A} + \frac{2m_A u_A}{m_A + m_B} = \frac{0 + 2m \times 9}{m + 2m} = 6\text{m/s}$$

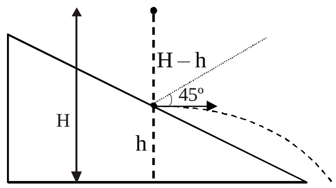


The collision between B and C is completely inelastic
$\therefore\ m_B v_B = (m_B + m_c)v$
$\Rightarrow\ v = \dfrac{6 \times 2m}{2m + m} = 4$m/s.

29. **(5)** Due to elastic head on collision of equal mass m of bob, velocity at the highest point of bob tied to string ℓ_1 is acquired by the bob tied to string ℓ_2.

$\therefore\ \sqrt{g\ell_1} = \sqrt{5g\ell_2} \qquad \Rightarrow\ \dfrac{\ell_1}{\ell_2} = 5$

30. **(2)**



T = Total time of flight

$$T = \sqrt{\frac{2h}{g}} + \sqrt{\frac{2(H-h)}{g}}$$

For maximum time $= \dfrac{dT}{dh} = 0$

$$\therefore \sqrt{\frac{2}{g}}\left(\frac{-1}{2\sqrt{H-h}} + \frac{1}{2\sqrt{h}}\right) = 0$$

$\Rightarrow 2\sqrt{h} = 2\sqrt{H-h} \ \Rightarrow \sqrt{H-h} = \sqrt{h} \ \Rightarrow H - h = h$

$\Rightarrow h = \dfrac{H}{2} \Rightarrow \dfrac{H}{h} = 2$

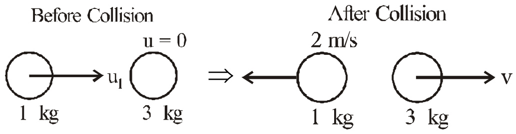
31. **(5)** $e = \dfrac{V_{separation}}{V_{approach}} = \dfrac{V_{sep}}{\sqrt{2 \times 10 \times 20}} = \dfrac{V_{sep}}{20} \Rightarrow 0.5 = \dfrac{V_{sep}}{20}$

$\Rightarrow\ V_{sep} = 10$ m/s

Let 'h' be the maximum height reached

Then, $\quad h = \dfrac{V_{sep}^2}{2g} = \dfrac{10^2}{2 \times 10} = 5$m

32. **(4)**

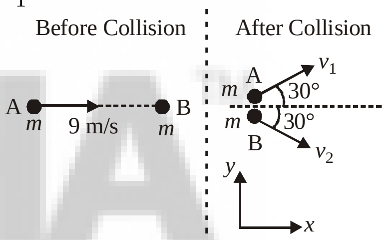


$1 \times u_1 + 3 \times 0 = -2 + 3v \qquad \Rightarrow\ u_1 = -2 + 3v \quad ...(1)$

$1 = \dfrac{v + 2}{u_1} \qquad \Rightarrow\ v + 2 = u_1$

Using equation (1)
$v + 2 = -2 + 3v$
$2v = 4$
$v = 2$
Put this in equation (1)
$\Rightarrow\ u_1 = -2 + 3v = -2 + 3 \times 2 = 4$ m/s

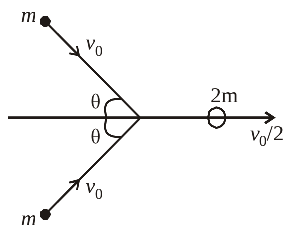
33. **(1)**



From conservation of momentum along y-axis.
$\vec{P}_{iy} = \vec{P}_{fy}$
$0 + 0 = mv_1 \sin 30°\ \hat{j} + mv_2 \sin 30°(-\hat{j})$

$mv_2 \sin 30° = mv_1 \sin 30° \quad v_2 = v_1$ or $\dfrac{v_1}{v_2} = 1$

34. **(120)**



Momentum conservation along x direction,

$2mv_0 \cos\theta = 2m\dfrac{v_0}{2} \Rightarrow \cos\theta = \dfrac{1}{2}$ or $\theta = 60°$

Hence angle between the initial velocities of the two bodies $= \theta + \theta = 60° + 60° = 120°$.

35. **(1)** For elastic collision $KE_i = KE_f$

$\dfrac{1}{2}m \times 25 + \dfrac{1}{2} \times m \times 9 = \dfrac{1}{2}m \times 32 + \dfrac{1}{2}mv_B^2$

$34 = 32 + V_B^2 \Rightarrow V_B = \sqrt{2}$

$KE_B = \dfrac{1}{2}mv_B^2 = \dfrac{1}{2} \times 0.1 \times 2 = 0.1J = \dfrac{1}{10}J \qquad \therefore\ x = 1$

36. **(30.00)** Maximum height,

$$H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow 120 = \frac{u^2\left(\frac{1}{2}\right)}{2g} \quad \therefore \quad u^2 = 480\text{ g}$$

Upon hitting the ground, it loses half of its kinetic energy

$$\therefore \quad K.E_{initial} = \frac{1}{2}mu^2 = 240\text{ mg}$$

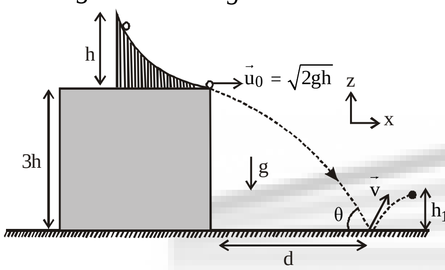
$$K.E_{final} = \frac{1}{2}(240\text{ mg}) = 120\text{ mg}$$

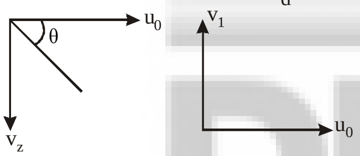
$$\therefore \quad \frac{1}{2}mv^2 = 120\text{ mg} \qquad \therefore v^2 = 240\text{ g}$$

After the bounce, the maximum height the ball reaches

$$\therefore \quad H' = \frac{v^2 \sin^2\theta}{2g} = \frac{240g \times \left(\frac{1}{4}\right)}{2g} = 30\text{ m}$$

37. **(a, c, d)**





Before collision After collision

From $v^2 - u^2 = 2gh$

$$0^2 - u_0^2 = 2gh \quad \therefore u_0 = \sqrt{2gh} \quad \therefore \vec{u}_0 = \sqrt{2gh}\,\hat{x}$$

and $v_z = \sqrt{2g(3h)}$

$$\therefore \tan\theta = \frac{v_z}{u_0} = \frac{\sqrt{2g(3h)}}{\sqrt{2gh}} = \sqrt{3} \quad \therefore \theta = 60°$$

$$\text{Distance } d = u_0 t = \sqrt{2gh} \times \sqrt{\frac{2\times 3h}{g}} = 2\sqrt{3}h$$

After collision only velocity along z–direction will change

$$v_1 = ev_z = \frac{1}{\sqrt{3}} \times \sqrt{2g(3h)} = \sqrt{2gh}$$

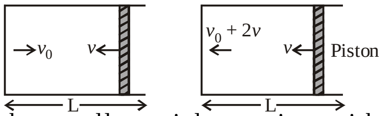
$$\therefore \vec{v} = v_1\hat{k} + u_0\hat{i} = \sqrt{2gh}\,\hat{k} + \sqrt{2gh}\,\hat{i} = \sqrt{2gh}\left[\hat{i} + \hat{k}\right]$$

$$\text{Height } h_1 = \frac{{v_1}^2}{2g} = \frac{\left(\sqrt{2gh}\right)^2}{2g} = h \quad \therefore d/h_1 = \frac{2\sqrt{3}h}{h} = 2\sqrt{3}$$

Therefore options (a, c, d) are correct.

38. **(a, d)**

Before collision After collision



When the small particle moving with velocity v_0 undergoes an elastic collision with the heavy movable piston moving with velocity v, it acquires a new velocity $v_0 + 2v$. So, the increase in velocity after every collision is $2v$.

Time period of collision when the piston is at a distance 'L' from the closed end

$$T = \frac{\text{distance}}{\text{speed}} = \frac{2L}{v'}$$

Where v' is the speed of the particle at that time.

$\therefore$ Frequency or rate at which the particle strikes the piston

$$= \frac{v'}{2L}$$

The rate of change of speed of the particle

$$= \frac{dv'}{dt} = (\text{frequency}) \times 2v \quad \therefore \quad dv' = \frac{v'}{2L}2v\,dt$$

$$\therefore \quad \frac{dv'}{v'} = \frac{v\,dt}{L} = \frac{-dL}{L}$$

Where dL is the distance travelled by the piston in time dt. The minus sign indicates decrease in 'L' with time.

$$\therefore \quad \int_{V_0}^{V} \frac{dv'}{v'} = -\int_{L_0}^{x} \frac{dL}{L}$$

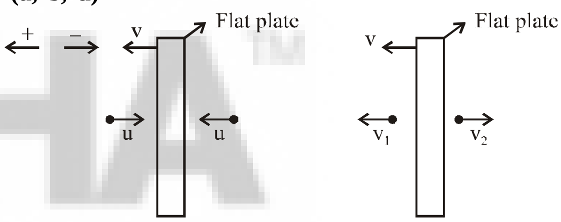
$$\therefore \quad \ell n\frac{v'}{v_0} = -\ell n\frac{L}{L_0} \qquad \text{or} \qquad |v'| = \frac{v_0 L_0}{L}$$

$$\text{when } L = \frac{L_0}{2} \text{ we have } |V'| = \frac{v_0 L_0}{L_0/2} = 2V_0$$

$$\therefore \quad K.E_{L_{0/2}} = \frac{1}{2}m(2v_0)^2$$

$$\therefore \quad K.E_{L_0} = \frac{1}{2}mv_0^2 \qquad \therefore \quad \frac{K.E_{L_{0/2}}}{K.E_{L_0}} = 4$$

39. **(a, b, d)**



Before collision Just after collision

For left gaseous particle

$$1 = \frac{v_1 - v}{v + u}$$

$\therefore v_1 = u + 2v$

$\therefore \Delta v_1 = 2u + 2v$

For right gaseous particle

$$1 = \frac{v + v_2}{u - v}$$

$\therefore v_2 = u - 2v$

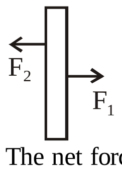
and $\Delta v_2 = 2u - 2v$

$$\text{Now } F_1 = \frac{dp_1}{dt} = \rho A(u + v)(2u + 2v)$$

$$\text{and } F_2 = \frac{dp_2}{dt} = \rho A(u - v)(2u - 2v)$$

$\therefore F_1 = 2\rho A(u + v)^2$ and $F_2 = 2\rho A(u - v)^2$

ΔF is the net force due to the air molecules on the plate.



$$\Delta F = F_1 - F_2 = 8\rho Auv \quad \therefore \quad P = \frac{\Delta F}{A} = 8\rho uv$$

The net force $F_{net} = F - \Delta F = ma$

$\therefore F - 8\rho Auv = ma$

Due to viscosity, plate will eventually reach terminal velocity. So now plate will move with constant velocity.

40. **(a, c)** According to law of conservation of linear momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$1 \times u_1 + 5 \times 0 = 1(-2) + 5(v_2)$

$\Rightarrow u_1 = -2 + 5v_2$ ...(i)

The coefficient of restituition

$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 1 = \frac{v_2 - (-2)}{u_1 - 0} \Rightarrow u_1 = v_2 + 2 \qquad ...(ii)$$

From eq (i) & (ii) $u_1 = 3$ m/s and $v_2 = 1$ m/s

Hence total momentum of the system = 3 kg m/s and K.E.cm = 0.75 J

41. **(a, d)** From law of conservation of linear momentum

The initial linear momentum of the system, $p\hat{i} - p\hat{i} = 0$

$\therefore$ Final linear momentum should also be zero *i.e.*, $p_1' + p_2' = 0$

Option a :

$p_1' + p_2' = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + c_1\hat{k}$ = Final momentum.

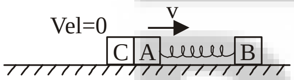
It is given that a_1, b_1, c_1, a_2, b_2 and c_2 have non-zero values. If $a_1 = x$ and $a_2 = -x$. Also if $b_1 = y$ and $b_2 = -y$ then the $\hat{i}$ and $\hat{j}$ components become zero. But the third term having $\hat{k}$ component is non-zero. This gives a definite final momentum to the system which violates conservation of linear momentum, so this is a wrong option.

Option d:

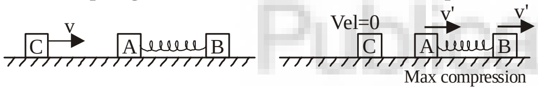
$p_1' + p_2' = (a_1 + a_2)\hat{i} + 2b_1\hat{j} \neq 0$ because $b_1 \neq 0$

Following the same reasoning as above the option d is also wrong.

42. **(b, d)** Just after the collision of *C* with *A*, *C* stops and *A* acquires a velocity *v* because of head-on elastic collision between identical masses.



When *A* starts moving towards right, the spring suffer a compression due to which *B* also starts moving towards right. The compression of the spring continues till there is relative velocity between *A* and *B*. When this relative velocity becomes zero, both *A* and *B* move with the same velocity *v*' and the spring is in a state of maximum compression say *x*.



From principle of conservation of linear momentum

$mv = mv' + mv' \Rightarrow v' = \frac{v}{2}$

$\therefore$ K.E. of *A* – *B* system at maximum compression,

$$\frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 = mv'^2 = \frac{mv^2}{4} \quad \left(\because v' = \frac{v}{2}\right)$$

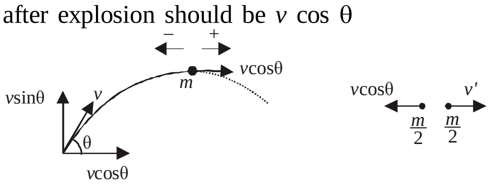
Applying energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 + \frac{1}{2}Kx^2$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{4}mv^2 + \frac{1}{2}Kx^2$$

$$\Rightarrow \frac{1}{2}Kx^2 = \frac{1}{4}mv^2 \quad \therefore \quad x = v\sqrt{\frac{m}{2K}}$$

43. **(a)** Let *v*' be the speed of other piece of shell after the collision. As one piece retraces its path, the speed of this piece just after explosion should be *v* cos θ



Applying conservation of linear momentum at the highest point

$$m\,(v\cos\theta) = \frac{m}{2} \times v' - \frac{m}{2} \times v\cos\theta$$

$\therefore \quad v' = 3v\cos\theta$

44. **(c, d)** In inelastic collision only momentum of the system may remain conserved.

(a) is incorrect because the momentum of ball changes in magnitude as well as direction.

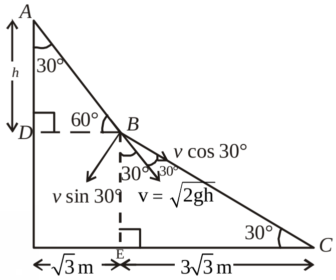
(b) is incorrect because on collision, some mechanical energy is converted into heat, sound energy.

(c) is correct because for earth + ball system the impact force is an internal force.

(d) is correct. Total energy of the ball and the earth is conserved.

45. **(b)** As the inclined plane is frictionless,

The K. E . at B = P.E. at *A*



$$\frac{1}{2}mv^2 = mgh$$

$$\Rightarrow v = \sqrt{2gh}$$

In Δ *ADB*, tan 60° = $\frac{h}{\sqrt{3}}$

$\therefore h = 3$ m

$\therefore v = \sqrt{6g} = \sqrt{60}$ m/s

This is the velocity of the block just before collision. This velocity makes an angle of 30° with the vertical. Also in right angled triangle *BEC*, ∠*EBC*= 60°. Therefore v makes an angle of 30° with the second inclined plane *BC*. The component of v along *BC* is *v* cos 30°.

It is given that the collision at *B* is perfectly inelastic therefore the impact forces act normal to the plane such that the vertical component of velocity becomes zero. The component of velocity along the incline *BC* remains unchanged and is equal to *v* cos 30°

$$= \sqrt{60} \times \frac{\sqrt{3}}{2} = \sqrt{\frac{180}{4}} = \sqrt{45}\text{ m/s}$$

46. **(b)** In Δ *BCE*, tan30° $= \frac{BE}{CE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BE}{3\sqrt{3}} \Rightarrow BE = 3\text{m}$

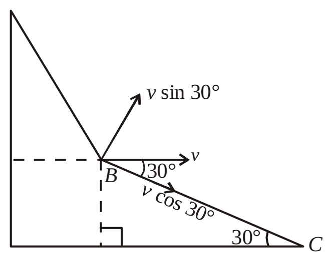
From mechanical energy conservation principle,

Mechanical energy at *B* = mechanical energy at *C*

$$\frac{1}{2}M\left(\sqrt{45}\right)^2 + M \times 10 \times 3 = \frac{1}{2}Mv_c^2$$

$$45 + 60 = v_c^2 \qquad \therefore v_c = \sqrt{105}\text{ m/s}$$

47. **(c)** The velocity of the block along *BC* just before collision is v cos 30°. The impact forces act perpendicular to the surface so the component of velocity along the incline remains unchanged.

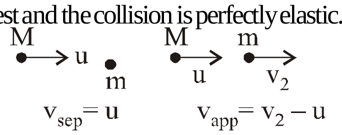


Just after collision

Also since the collision is elastic, the vertical component of velocity (*v* sin 30°) before collision changes in direction, the magnitude remaining the same as shown in the figure. So the rectangular components of velocity after collision are as shown in the figure. This means that the final velocity of the block should be horizontal making an angle 30º with *BC*. Therefore the vertical component of the final velocity of the block is zero.

48. **(a)** As the body Q was at rest and the collision is perfectly elastic.

$$e = \frac{v_{sep}}{v_{app}} = 1$$



So, $\frac{u}{v_2 - u} = 1 \Rightarrow v_2 = 2u$

During elastic collision, the momentum and kinetic energy both are conserved.

49. **(d)** Maximum energy loss $= \frac{P^2}{2m} - \frac{P^2}{2(m+M)}$

$$\left[\because \text{K.E.} = \frac{P^2}{2m} = \frac{1}{2}mv^2\right]$$

$$= \frac{P^2}{2m}\left[\frac{M}{(m+M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\}$$

Statement II is a case of perfectly inelastic collision.

By comparing the equation given in statement I with above equation, we get

$$f = \left(\frac{M}{m+M}\right) \text{ instead of } \left(\frac{m}{M+m}\right)$$

Hence statement I is wrong and statement II is correct.

50. **(d)** **Statement 1 :** For an elastic collision, the coefficient of restitution = 1

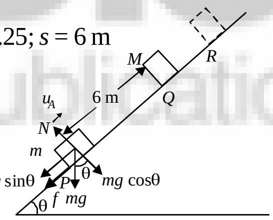
$$e = \frac{|v_2 - v_1|}{|u_1 - u_2|} \Rightarrow |v_2 - v_1| = |u_1 - u_2|$$

$\Rightarrow$ Relative velocity after collision is equal to relative velocity before collision. But in the statement relative speed is given.

Statement 2 : Linear momentum remains conserved in an elastic collision. This statement is true.

51. **From *P* to *Q*.**

Given : $u = 10$ m/s and $\mu = 0.25$; $s = 6$ m



$$a = -\left[\frac{mg\sin\theta + f}{m}\right]$$

$$= -\left[\frac{mg\sin\theta + \mu mg\cos\theta}{m}\right]$$

$= -[g\sin\theta + \mu g\cos\theta] = -g[\sin\theta + \mu\cos\theta]$

$= -10[0.05 + 0.25 \times 0.99] = -2.99$ m/s^2

Using, $v^2 - u^2 = 2as \Rightarrow v^2 = 100 + 2(-2.99) \times 6 \Rightarrow v = 8$ m/s

Hence velocity of mass m just before collision = 8 m/s.

The velocity of mass M just before collision = 0 m/s (given).

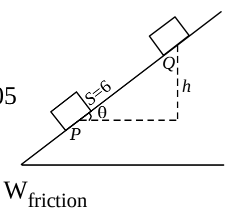
Let v_1 be the velocity of mass m after collision and v_2 be the velocity of mass M after collision. **Body of mass *M* moving from *Q* to *R* and coming to rest.** After collision,

$u = v_2 \Rightarrow v = 0$

$a = -2.99$ m/s^2 $\Rightarrow s = 0.5$

$v^2 - u^2 = 2as \Rightarrow (0)^2 - v_2^2 = 2(-2.99) \times 0.5 \Rightarrow v_2 = 1.73$ m/s

Body of mass *m* moving from *Q* to *P* after collision



$$\sin\theta = \frac{h}{6}$$

$h = 6\sin\theta = 6\sin 60° = 6 \times 0.05$

$u = v_1$

$v = +1$ m/s

(K.E. + P.E.)$_{\text{initial}}$ = (K.E. + P.E.)$_{\text{final}}$ + W$_{\text{friction}}$

$$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv^2 + 0 + \mu mgs$$

$$\frac{1}{2}v_1^2 + 10 \times (6 \times 0.05) = \frac{1}{2}(1)^2 + 0.25 \times 10 \times 6$$

$v_1 = -5$ m/s

$\therefore$ **Coefficient of restitution**

$$e = \left|\frac{\text{relative velocity of separation}}{\text{relative velocity of approach}}\right| = \left|\frac{-5-1.73}{8-0}\right| = 0.84$$

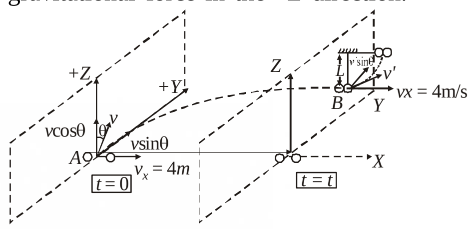
Applying conservation of linear momentum before and after collision,

$mv + M \times 0 = m \times v_1 + m \times v_2$

$2 \times 8 + M \times 0 = 2 \times (-5) + M(1.73)$

$\therefore$ $M = \frac{26}{1.73} = 15.02$ kg

52. When the stone reaches the point B, the component of velocity in the $+Z$ direction ($v\cos\theta$) becomes zero due to the gravitational force in the $-Z$ direction.



The stone has two velocities at B

v_x in the $+X$ direction (4 m/s)

$v\sin\theta$ in the $+Y$ direction (6 sin 30° = 3 m/s)

Resultant velocity of the stone

$$v' = \sqrt{(v_x)^2 + (v\sin\theta)^2} = \sqrt{4^2 + 3^2} = 5 \text{ m/s}$$

(i) Applying conservation of linear momentum at B for collision with a mass of equal magnitude.

Since, the collision is completely inelastic the two masses will stick together. v is the velocity of the two masses just after collision.]

$m \times 5 = 2m \times v$

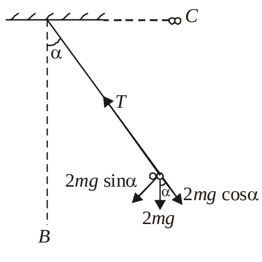
$\therefore$ $v = 2.5$ m/s

(ii) When the string is undergoing circular motion, at any arbitrary position, $T - 2mg\cos\alpha = \frac{2mv^2}{\ell}$

According to question, $T = 0$ when $\alpha = 90°$

$\therefore$ $0 - 0 = \frac{2mv^2}{\ell} \Rightarrow v = 0$

i.e., in the horizontal position, $v = 0$



Applying energy conservation from B to C,

$$\frac{1}{2}2mv^2 = 2mg\ell$$

$$\Rightarrow \ell = \frac{v^2}{2g} = \frac{(2.5)^2}{2 \times 9.8} = 0.318 \text{ m}$$

53. Object of mass m collides with block B of mass $4m$.

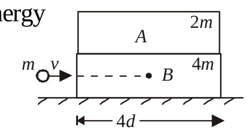
Since the collision is elastic in nature applying conservation of linear momentum

$mv = (4m)u + mv'$

where u is the velocity of mass 4m after collision and v' is the velocity of mass m

$\Rightarrow$ $v' = v - 4u$ ... (i)

Applying conservation of kinetic energy



Also, $\frac{1}{2}mv^2 = \frac{1}{2}(4m)u^2 + \frac{1}{2}mv'^2$

$\Rightarrow$ $v^2 = 4u^2 + v'^2$ ... (ii)

From eq. (i) & (ii)

$$v^2 = 4u^2 + (v-4u)^2 \Rightarrow u = \frac{2v}{5}$$

block B starts moving but the block A remains at rest. As there is no friction between A and B

For block A to topple, block B should move a distance $2d$. Let the retardation produced in B due to friction force between B and the table be a

$$F = \mu N \Rightarrow (4\,m)a = \mu\,(6\,mg) \Rightarrow a = 1.5\,\mu g$$

For the motion of B,

$$u = \frac{2v}{5},\ v = 0,\ s = 2\text{d},\ a = -1.5\,\mu g$$

Now, $v^2 - u^2 = 2as \Rightarrow (0)^2 - \left(\frac{2v}{5}\right)^2 = 2(-1.5\mu g)\,2d$

$$\Rightarrow \quad v = \frac{5}{2}\sqrt{6\mu \text{gd}}$$

For elastic collision between two bodies

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2}$$

Here $m_1 = m$, $m_2 = 4m$, $u_1 = 5\sqrt{6\mu gd}$, $u_2 = 0$

$$\Rightarrow v_1 = \frac{(m-4m)5\sqrt{6\mu gd}}{m+4m} + 0 = -3\times 5\frac{\sqrt{6\mu gd}}{5} = -3\sqrt{6\mu gd}$$

The negative sign shows that the mass m rebounds. It then follows a projectile motion and its path's parabolic.

$u_y = 0$, $s_y = d$, $a_y = g$, $t_y = ?$

For vertical motion,

$$S = ut + \frac{1}{2}at^2 \Rightarrow d = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2d}{g}}$$

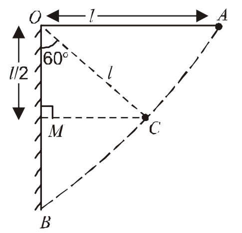
The horizontal distance travelled by mass m during this time t from P leftwards

$$x = 3\sqrt{6\mu gd} \times \sqrt{\frac{2d}{g}} = 6\sqrt{3\mu d^2} = 6d\sqrt{3\mu}$$

54. Initially when the bob of pendulum is at A, its P.E. = mgl

When the bob released from A and strikes to the wall at B, P.E. changes to K.E. and when it is at position 'C' the angular amplitude is 60°.

In ΔOCM



$$\cos 60° = \frac{OM}{\ell} \Rightarrow OM = \frac{\ell}{2}$$

The velocity of bob at B,

$$mg\ell = \frac{1}{2}mv_B^2 \quad \Rightarrow \quad v_B = \sqrt{2g\ell}$$

Let after n collisions, the angular amplitude is 60° when the bob again moves towards the wall from C, the velocity v'_B before collision

$$mg\frac{\ell}{2} = \frac{1}{2}mv'^2_B \quad \Rightarrow \quad v'_B = \sqrt{g\ell}$$

This means that the velocity of the bob should reduce from $\sqrt{2g\ell}$ to $\sqrt{g\ell}$ due to collisions with walls.

The final velocity after n collisions is $\sqrt{g\ell}$

$$\therefore \quad e^n(\sqrt{2g\ell}) = \sqrt{g\ell}$$

where e is coefficient of restitution.

$$\left(\frac{2}{\sqrt{5}}\right)^n \times \sqrt{2g\ell} = \sqrt{g\ell} \Rightarrow \left(\frac{2}{\sqrt{5}}\right)^n = \frac{1}{\sqrt{2}}$$

Taking log on both sides we get

$$n \log\left(\frac{2}{\sqrt{5}}\right) = \log\frac{1}{\sqrt{2}} \quad \Rightarrow \quad n = 3.1$$

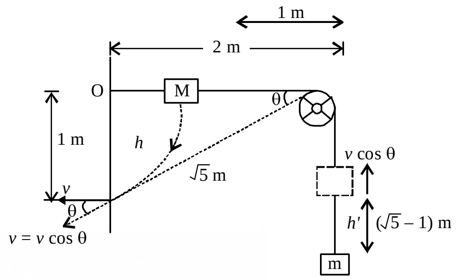
Hence, number of collisions = 4.

55. Let point mass hit the wall with the speed v.

Then, velocity of mass m at this instant = $v\cos\theta = \frac{2}{\sqrt{5}}v$.

Further M will fall a distance of 1 m while m will rise up by $(\sqrt{5} - 1)$ m.

From energy conservation, Decrease in P.E. of M = increase in P.E. of m + increase in K.E. of both the blocks.



$$Mgh = mgh' + \frac{1}{2}mv^2 + \frac{1}{2}m(\text{v}\cos\theta)^2$$

or, $(2)(9.8)(1) = (0.5)(9.8)\,(\sqrt{5}-1) + \frac{1}{2}\times 2\times v^2$

$$+\frac{1}{2}\times 0.5\times\left(\frac{2v}{\sqrt{5}}\right)^2$$

Solving we get, $v = 3.29$ m/s.

56. Initial position of C.M.

$$x_1 = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{100\times 0 + 100\times 98}{200} = 49\text{ m}$$

100 g vel = 0 (dropped)

Initial velocity of C.M

$$u_c = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{100\times 49 + 100\times 0}{200} = 24.5\text{ ms}^{-1}$$

Acceleration of C.M

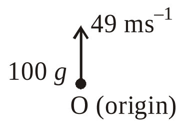
$a_c = -9.8\text{ ms}^{-2}$

Displacement of C.M is

$s_c = -49$ m

Applying

$$S = ut + \frac{1}{2}at^2$$



$-49 = 24.5\,t - 4.9\,t^2$

$t^2 - 5t - 10 = 0$

$$t = \frac{5\pm\sqrt{25+40}}{2} = \frac{5\pm\sqrt{65}}{2} = 6.53\text{ s}$$

57. The collision between C and A is elastic and their masses are equal so they will exchange their velocities A was initially at rest, therefore the result of collision will be that C will come to rest and A will initially start moving with a velocity v_0. But since A is connected to B with a spring, the spring will get compressed.

Let v' be the common velocity of A and B.

At $t = t_0$, the velocities of A and B become same.
Applying energy conservation;

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv'^2 + \frac{1}{2}2mv'^2 + \frac{1}{2}kx_0^2$$

where x_0 is the compression in the spring at $t = t_0$

$$\therefore \quad v_0^2 = 3v'^2 + \frac{k}{m}x_0^2 \qquad \text{... (i)}$$

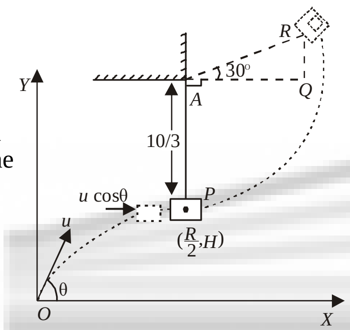
Applying momentum conservation,

$$mv_0 = mv' + 2mv' \qquad \therefore \; v' = \frac{v_0}{3} \qquad \text{... (ii)}$$

From eq. (i) and (ii)

58. At the highest point P, the velocity of the bullet = $u \cos \theta$
In $\Delta\, AQR$
(i) From figure,

$$\sin 30° = \frac{QR}{10/3}, \; QR = \frac{5}{3}$$



Now from, conservation of linear momentum at the highest point - P
$M (u \cos \theta) + 3M \times 0$
$= (M + 3M)\, v$

$$v = \frac{Mu\cos\theta}{4M} = \frac{u\cos\theta}{4}$$

From energy conservation principle $K.E.$ of the bullet-mass system at P = $P.E.$ of the bullet-mass system at R

$$\frac{1}{2}(4M)v^2 = (4M)\,gh$$

$$\frac{1}{2}(4M)\frac{u^2\cos^2\theta}{16} = 4Mg \times \left(\frac{10}{3} + \frac{5}{3}\right)$$

$$\cos^2\theta = \frac{9.8 \times 5 \times 2 \times 16}{50 \times 50} \;\Rightarrow\; \theta = 37°$$

($\because\; u = 50$ m/s given)

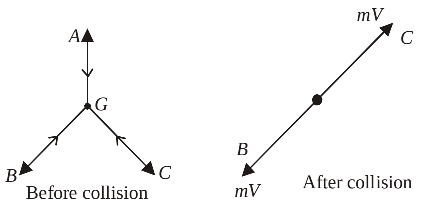
(ii) Vertical component,

$$\frac{R}{2} = \frac{u^2 \sin 2\theta}{2g} = \frac{50 \times 50 \sin 2 \times 37°}{2 \times 9.8} = 122.6\,\text{m}$$

Horizontal component,

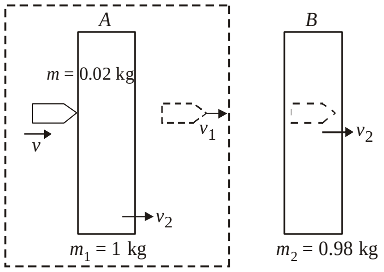
$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{50 \times 50 \times (\sin 37°)^2}{2 \times 9.8} = 46\,\text{m}$$

59.



Before collision by symmetry, the net momentum of the system is zero.
After collision, momentum is also zero as no esternal force is acting on the system. A comes to rest so B and C should have equal and opposite momenta, so velocity of C should be same in magnitude *i.e.*, V but in opposite direction of velocity of B.

60. Applying conservation of linear momentum for the system of bullet and plate A, $mv = mv_1 + m_1v_2$
or, $\quad 0.02v = 0.02\, v_1 + 1 \times v_2 \qquad$ ...(i)



Again applying conservation of linear momentum for collision at B, $mv_1 = (m + m_2)v_2$
or, $\quad 0.02\, v_1 = (2.98 + 0.02)\, v_2 = 3v_2$

$$\Rightarrow \quad v_2 = \frac{0.02 v_1}{3} \qquad \text{... (ii)}$$

From eq. (i) and (ii)

$$0.02\, v = 0.02\, v_1 + \frac{0.02 v_1}{3} \quad\Rightarrow\quad v = \frac{4}{3}v_1 \Rightarrow \frac{v}{v_1} = \frac{4}{3}$$

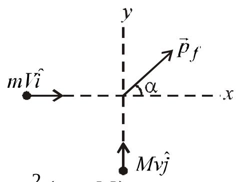
$$\frac{v_1}{v} = \frac{3}{4} \Rightarrow 1 - \frac{v_1}{v} = 1 - \frac{3}{4} = \frac{1}{4} = 0.25 \;\Rightarrow\; \frac{v - v_1}{v} = 0.25$$

$$\therefore \quad \%\text{ loss in velocity, } \frac{v - v_1}{v} \times 100 = \frac{1}{4} \times 100 = 25\%$$

61. (i) According to consevation of linear momentum
$\vec{p}_f = \vec{p}_i = mV\hat{i} + Mv\hat{j}$ (Magnitude of the momentum of final body)

$$\therefore \quad p_f = \sqrt{m^2V^2 + M^2v^2}$$

$$\Rightarrow \quad \tan\alpha = \frac{Mv}{mV} \Rightarrow \alpha = \tan^{-1}\frac{Mv}{mV}$$



which gives the direction of the momentum.

$$\text{(ii)} \quad \frac{\text{K.E}_i - \text{K.E}_f}{\text{K.E}_i} = 1 - \frac{\text{K.E}_f}{\text{K.E}_i} = 1 - \frac{\frac{p_f^2}{(m+M)}}{\frac{1}{2}mV^2 + \frac{1}{2}Mv^2}$$

$$= 1 - \frac{m^2V^2 + M^2v^2}{(m+M)(mV^2 + Mv^2)}$$

$$= \frac{m^2V^2 + mMv^2 + mMV^2 + M^2v^2 - m^2V^2 - M^2v^2}{(m+M)(mV^2 + Mv^2)}$$

$$\frac{\Delta K.E.}{K.E_i} = \frac{mM(v^2 + V^2)}{(m+M)(mV^2 + Mv^2)}$$

Topic-5: Miscellaneous (Mixed Concepts) Problems

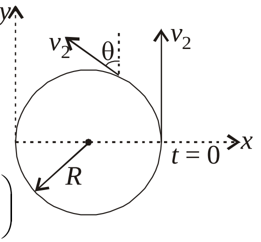
1. **(6.30)** Given: mass $m = 0.4$ kg; impulse J = 1.0 Ns $V_{(t)} = v_0e^{-t/\tau}$; $\tau = 4$s and $e^{-1} = 0.37$.
Impulse = Change in linear momentum

$$\text{or,} \quad \text{J} = \text{mV}_0 \Rightarrow \quad \text{V}_0 = \frac{J}{m} = \frac{1}{0.4} = 2.5\,\text{ms}^{-1}$$

$$\text{Also } (\text{V}_t) = v_0e^{-t/\tau} \quad \therefore \; \frac{ds}{dt} = v_0e^{-t/\tau} \quad \Rightarrow ds = v_0e^{-t/\tau}\,dt$$

$$\therefore\; s = v_0\int_0^{\tau} e^{-t/\tau}dt = v_0\tau(1 - e^{-1}) = 2.5 \times 4 \times 0.63 = 6.30\,\text{m}$$

2. Angular speed of particle about centre of the circular path



$\omega = \frac{v_2}{R}, \theta = \omega t = \frac{v_2}{R}t$

$v_p = \left(-v_2 \sin\theta \hat{i} + v_2 \cos\theta \hat{j}\right)$

or

$v_p = \left(-v_2 \sin\frac{v_2}{R}t\hat{i} + v_2 \cos\frac{v_2}{R}t\hat{j}\right)$

and $v_m = v_1\hat{j}$

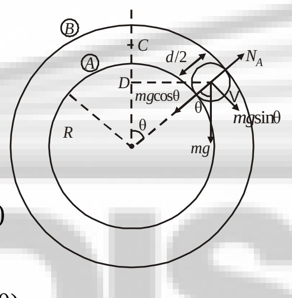
$\therefore$ Linear momentum of particle w.r.t. man as a function of time

$L_{pm} = m(v_p - v_m)$

$= m\left[\left(-v_2 \sin\frac{v_2}{R}t\right)\hat{i} + \left(v_2 \cos\frac{v_2}{R}t - v_1\right)\hat{j}\right]$

3. (a) For circular motion of the ball, the necessary centripetal force is provided by ($mg \cos\theta - N$).

$mg\cos\theta - N = \frac{mv^2}{\left(R + \frac{d}{2}\right)}$ ... (i)



N = normal reaction at angle θ

According to energy conservation

$\frac{1}{2}mv^2 = mg\left(R + \frac{d}{2}\right)(1 - \cos\theta)$

$\Rightarrow \quad v^2 = 2g\left(R + \frac{d}{2}\right)(1 - \cos\theta)$

Putting this value of v^2 in eq. (i)

$N = mg(3\cos\theta - 2)$

(b) The ball will lose contact with the inner sphere when $N = 0$

or $3\cos\theta - 2 = 0$ or $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

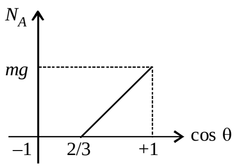
After this it makes contact with outer sphere and normal reaction starts acting towards the centre.

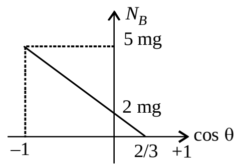
So for $\theta \le \cos^{-1}\left(\frac{2}{3}\right)$ $N_B = 0$ and $N_A = mg(3\cos\theta - 2)$

and for $\theta \ge \cos^{-1}\left(\frac{2}{3}\right)$

$N_A = 0$ and $N_B = mg(2 - 3\cos\theta)$

The $N_A - \cos\theta$ and $N_B - \cos\theta$ graphs are as follows.



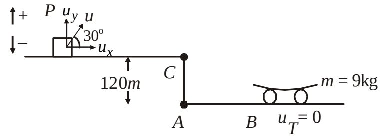


4. Consider the vertical motion of the cannon ball

$u_y = +100 \sin 30°$ (vertical component of velocity)

$s_y = -120$ m (vertical displacement)

$a_y = -10$ m/s^2



$t = t_0$ (Time of first impact on carriage)

$s_y = u_y t + \frac{1}{2}a_y t_0^2 \quad \therefore -120 = 50\,t_0 - 5\,t_0^2$

$\Rightarrow \quad t_0^2 - 10\,t_0 - 24 = 0$

$\therefore \quad t_0 = -\frac{(-10) \pm \sqrt{100 - 4(1)(-24)}}{2} = 12$ or -2 [Not valid]

$\therefore \quad t_0 = 12$ sec.

Horizontal component of velocity of the cannon ball remains the same

$\therefore \quad u_x = 100\cos 30° + 5\sqrt{3} = 55\sqrt{3}$

$\therefore$ Applying conservation of linear momentum to the cannon ball-trolley system in horizontal direction.

$mu_x + M \times 0 = (m + M)\,v_x$

$\therefore \quad v_x = \frac{mu_x}{m + M}$ (where m = mass of cannon ball, M = mass of trolley, v_x = velocity of the cannon ball-trolley system)

$\therefore \quad v_x = \frac{1 \times 55\sqrt{3}}{1 + 9} = 5.5\sqrt{3}\,\text{ms}^{-1}$

Horizontal distance covered by the car

$P = 12 \times 5\sqrt{3} = 60\sqrt{3}$ m ($\because$ Second ball was projected after 12 second.)

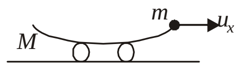
Since the second ball also struck the trolley,

$\therefore$ in time 12 seconds, the trolley covers a distance of $60\sqrt{3}$ m.

For trolley after 12 seconds;

$u = 5\sqrt{3}$ m/s, $v = ?$, $t = 12$ s

$s = 60\sqrt{3}$ m,



$s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad 60\sqrt{3} = 5.5\sqrt{3} \times 12 + \frac{1}{2} \times a \times 144$

$\therefore \quad a = -\sqrt{3}/12\,\text{ms}^{-2} \quad \therefore v = u + at = 5\sqrt{3}$ m/s.

$v = u + at_0 = 5.5\sqrt{3} - \left(\frac{\sqrt{3}}{12}\right) \times 12$

$v = 4.5\sqrt{3}\,\text{ms}^{-1}$

To find the final velocity v_f of the carriage after the second impact.

Applying conservation of linear momentum in the horizontal direction

$mu_x + (M + m)\,v_x = (M + 2m)\,v_f$

$1 \times 55\sqrt{3} + (9 + 1)\,4.5\sqrt{3} = (9 + 2)\,v_f \therefore v_f = 15.75$ m/s

Disha PYQs JEET Pakki

ERRORLESS 47 YEARS Chapter-wise & Topic-wise JEE Advanced (1978 – 2024) & JEE Main (2013 – 2024)

CHEMISTRY

Solved Papers

31 Chapters	95 Topics	10 Types	5100+ PYQs	100% Solutions

- Includes 2024 Advanced Papers 1 & 2
- Includes 20 Sets of JEE Main 2024 Session I & II
- 12 Yrs JEE Mains New Syllabus Questions
- As per latest NCERT Syllabus of Class 11 & 12

20th Edition

DISHA™
Publication Inc

DISHA Publication Inc.

A-23 FIEE Complex, Okhla Phase II

New Delhi-110020

Tel: 49842349/ 49842350

Edited By

Kalpana Bhargav
Mohammad Salman
Gaurav Kumar

Typeset By

DISHA DTP Team

Write To Us At

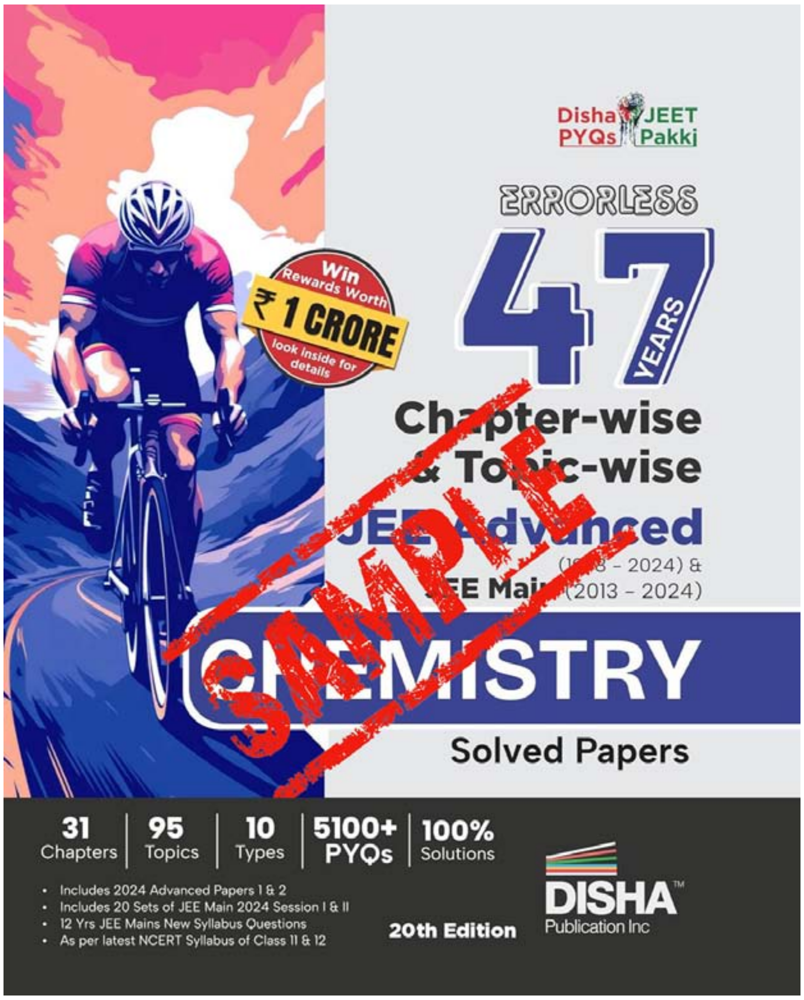
feedback_disha@aiets.co.in

www.dishapublication.com

DISHA™ Publication Inc

Free Sample Contents

This sample book is prepared from the book **"Errorless 47 Years Chapter-wise & Topic-wise JEE Advanced (1978 - 2024) & JEE Main (2013 - 2024) CHEMISTRY Solved Papers 20th Edition | PYQ Question Bank in NCERT Flow for JEE 2025".**



ISBN - 978-9362259547

MRP- 900/-

In case you like this content, you can buy the **Physical Book** or **E-book** using the ISBN provided above.

The book & e-book are available on all leading online stores.

Contents

A1 – A268

Class XIth — A1 – A120

Hints & Solutions (Class XIth) — A121–A268

B1 – B400

Class XII[th] B1 – B188

Hints & Solutions (Class XII[th]) B189–B400

5 Thermodynamics

Topic-1: Thermodynamics

1 MCQs with One Correct Answer

1. Choose the correct option for free expansion of an ideal gas under adiabatic condition from the following: **[Main Feb. 1, 2024 (I)]**

(a) $q = 0, \Delta T \neq 0, w = 0$ (b) $q = 0, \Delta T < 0, w \neq 0$
(c) $q = \neq 0, \Delta T = 0, w = 0$ (d) $q = 0, \Delta T = 0, w = 0$

2. Which of the following is **not** correct? **[Main Jan. 29, 2024 (I)]**

(a) ΔG is zero for a reversible reaction
(b) ΔG is negative for a spontaneous reaction
(c) ΔG is positive for a spontaneous reaction
(d) ΔG is positive for a non-spontaneous reaction

3. What happens when methane undergoes combustion in systems A and B respectively? **[Main April 13, 2023 (II)]**

Adiabatic system — System A
Diathermic container — System B

(a)

System A	System B
Temperature rises	Temperature remains same

(b)

System A	System B
Temperature falls	Temperature rises

(c)

System A	System B
Temperature falls	Temperature remains same

(d)

System A	System B
Temperature remains same	Temperature rises

4. Which of the following relations are correct?

(A) $\Delta U = q + p\Delta V$ (B) $\Delta G = \Delta H - T\Delta S$

(C) $\Delta S = \frac{q_{rev}}{T}$ (D) $\Delta H = \Delta U - \Delta nRT$

Choose the most appropriate answer from the options given below : **[Main Jan. 29, 2023 (II)]**

(a) C and D only (b) B and C only
(c) A and B only (d) B and D only

5. Which of the following relation is not correct? **[Main July 28, 2022 (I)]**

(a) $\Delta H = \Delta U - P\Delta V$ (b) $\Delta U = q + W$
(c) $\Delta S_{sys} = \Delta S_{surr} \geq 0$ (d) $\Delta G = \Delta H - T\Delta S$

6. Five moles of an ideal gas at 1 bar and 298 K is expanded into vacuum to double the volume. The work done is : **[Main Sep. 04, 2020 (II)]**

(a) $C_V(T_2 - T_1)$ (b) $-RT(V_2 - V_1)$
(c) $-RT \ln V_2/V_1$ (d) zero

7. The true statement amongst the following is : **[Main Jan. 09, 2020 (II)]**

(a) Both ΔS and S are functions of temperature.
(b) Both S and ΔS are not functions of temperature.
(c) S is not a function of temperature but ΔS is a function of temperature.
(d) S is a function of temperature but ΔS is not a function of temperature.

8. Among the following, the set of parameters that represents path functions, is: **[Main April 9, 2019 (I)]**

(A) $q + w$ (B) q (C) w (D) $H - TS$
(a) (B) and (C) (b) (B), (C) and (D)
(c) (A) and (D) (d) (A), (B) and (C)

9. 5 moles of an ideal gas at 100 K are allowed to undergo reversible compression till its temperature becomes 200 K. If $C_V = 28\ J\ K^{-1}\ mol^{-1}$, calculate ΔU and ΔPV for this process. ($R = 8.0\ J\ K^{-1}\ mol^{-1}$) **[Main April 8, 2019 (II)]**

(a) $\Delta U = 14$ kJ; $\Delta(PV) = 18$ kJ
(b) $\Delta U = 14$ kJ; $\Delta(PV) = 0.8$ kJ
(c) $\Delta U = 14$ kJ; $\Delta(PV) = 4$ kJ
(d) $\Delta U = 14$ kJ; $\Delta(PV) = 8.0$ kJ

10. At 320 K, a gas A_2 is 20% dissociated to A(g). The standard free energy change at 320 K and 1 atm in J mol^{-1} is approximately: **[Main Online April 16, 2018]**

($R = 8.314 JK^{-1}\ mol^{-1}$; ln 2 = 0.693; ln 3 = 1.098)

(a) 1844 (b) 2068 (c) 4281 (d) 4763

11. A gas undergoes change from state A to state B. In this process, the heat absorbed and work done by the gas is 5 J and 8 J, respectively. Now gas is brought back to A by another process during which 3 J of heat is evolved. In this reverse process of B to A : **[Main Online April 9, 2017]**
(a) 10 J of the work will be done by the gas.
(b) 6 J of the work will be done by the gas.
(c) 10 J of the work will be done by the surrounding on gas.
(d) 6 J of the work will be done by the surrounding on gas.

12. The standard state Gibbs free energies of formation of C(graphite) and C(diamond) at $T = 298$ K are
$\Delta_f G^0$ [C(graphite)] = 0 kJ mol^{-1}
$\Delta_f G^0$ [C(diamond)] = 2.9 kJ mol^{-1}
The standard state means that the pressure should be 1 bar, and substance should be pure at a given temperature. The conversion of graphite [C(graphite)] to diamond [C(diamond)] reduces its volume by 2×10^{-6} m^3 mol^{-1}. If C(graphite) is converted to C(diamond) isothermally at T = 298 K, the pressure at which C(graphite) is in equilibrium with C(diamond), is
[Useful information : 1 J = 1 kg m^{2}s^{-2}; 1 Pa = 1 kg m^{-1} s^{-2}; 1 bar = 10^5 Pa] **[Adv. 2017]**
(a) 14501 bar (b) 58001 bar
(c) 1450 bar (d) 29001 bar

13. If 100 mole of H_2O_2 decomposes at 1 bar and 300 K, the work done (kJ) by one mole of O_2(g) as it expands against 1 bar pressure is : **[Main Online April 10, 2016]**
$2H_2O_2(l) \rightleftharpoons 2H_2O(l) + O_2(g)$ $(R = 83\ JK^{-1}\ mol^{-1})$
(a) 124.50 (b) 249.00 (c) 498.00 (d) 62.25

14. One mole of an ideal gas at 300 K in thermal contact with surroundings expands isothermally from 1.0 L to 2.0 L against a constant pressure of 3.0 atm. In this process, the change in entropy of surroundings (ΔS_{surr}) in JK^{-1} is
(1 L atm = 101.3 J) **[Adv. 2016]**
(a) 5.763 (b) 1.013
(c) –1.013 (d) –5.763

15. The ($S°$) of the following substances are:
CH_4 (g) 186.2 JK^{-1} mol^{-1}; O_2 (g) 205.2 JK^{-1} mol^{-1}
CO_2 (g) 213.6 JK^{-1} mol^{-1}; H_2O (l) 69.9 JK^{-1} mol^{-1}
The entropy change ($\Delta S°$) for the reaction
$CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O$ (l) is:
[Main Online April 12, 2014]
(a) – 312.5 J K^{-1} mol^{-1} (b) – 242.8 J K^{-1} mol^{-1}
(c) – 108.1 J K^{-1} mol^{-1} (d) – 37.6 J K^{-1} mol^{-1}

16. For the process **[Adv. 2014]**
$H_2O(l) \rightarrow H_2O(g)$
at T = 100°C and 1 atmosphere pressure, the correct choice is
(a) $\Delta S_{system} > 0$ and $\Delta S_{surroundings} > 0$
(b) $\Delta S_{system} > 0$ and $\Delta S_{surroundings} < 0$
(c) $\Delta S_{system} < 0$ and $\Delta S_{surroundings} > 0$
(d) $\Delta S_{system} < 0$ and $\Delta S_{surroundings} < 0$

17. Which of the following statements/relationships is **not** correct in thermodynamic changes?
[Main Online April 23, 2013]
(a) $\Delta U = 0$ (isothermal reversible expansion of a gas)
(b) $w = -nRT \ln \frac{V_2}{V_1}$ (isothermal reversible expansion of an ideal gas)
(c) $w = nRT \ln \frac{V_2}{V_1}$ (isothermal reversible expansion of an ideal gas)
(d) For a system of constant volume, heat involved directly changes to internal energy.

18. For the process $H_2O(l)$ (1 bar, 373 K) $\rightarrow H_2O(g)$ (1 bar, 373 K), the correct set of thermodynamic parameters is **[2007]**
(a) $\Delta G = 0, \Delta S = +ve$ (b) $\Delta G = 0, \Delta S = -ve$
(c) $\Delta G = +ve, \Delta S = 0$ (d) $\Delta G = -ve, \Delta S = +ve$

19. The value of $\log_{10} K$ for a reaction $A \rightleftharpoons B$ is
(Given : $\Delta_r H°_{298K} = -54.07$ kJ mol^{-1}, $\Delta_r S°_{298K}$ = 10 JK^{-1} mol^{-1} and R = 8.314 JK^{-1} mol^{-1};
$2.303 \times 8.314 \times 298 = 5705$) **[2007]**
(a) 5 (b) 10 (c) 95 (d) 100

20. A mono-atomic ideal gas undergoes a process in which the ratio of P to V at any instant is constant and equals to 1. What is the molar heat capacity of the gas
[2006 - 3M; –1]
(a) $\frac{3R}{2}$ (b) $2R$ (c) 0 (d) $\frac{5R}{2}$

21. When one mole of monoatomic ideal gas at T K undergoes adiabatic change under a constant external pressure of 1 atm, volume changes from 1 litre to 2 litre. The final temperature in Kelvin would be **[2005S]**
(a) $\frac{T}{2^{(2/3)}}$ (b) $T + \frac{2}{3} \times 0.0821$
(c) T (d) $T - \frac{2}{3} \times 0.0821$

22. The enthalpy of vapourization of liquid is 30 kJ mol^{-1} and entropy of vapourization is 75 J mol^{-1} K. The boiling point of the liquid at 1 atm is **[2004S]**
(a) 250 K (b) 400 K
(c) 450 K (d) 600 K

23. Two moles of an ideal gas is expanded isothermally and reversibly from 1 litre to 10 litres at 300 K. The enthalpy change (in kJ) for the process is **[2004S]**
(a) 11.4 kJ (b) – 11.4 kJ
(c) 0 kJ (d) 4.8 kJ

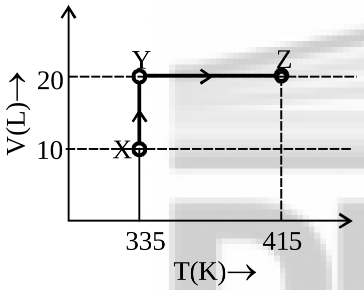
24. One mole of a non-ideal gas undergoes a change of state (2.0 atm, 3.0L, 95 K) $\rightarrow$ (4.0 atm, 5.0 L, 245 K) with a change in internal energy, ΔU = 30.0 L atm. The change in enthalpy (ΔH) of the process in L atm is **[2002S]**
(a) 40.0
(b) 42.3
(c) 44.0
(d) not defined, because pressure is not constant

25. Which one of the following statements is false? **[2001S]**
(a) Work is a state function.
(b) Temperature is a state function.
(c) Change in the state is completely defined when the initial and final states are specified.
(d) Work appears at the boundary of the system.

26. In thermodynamics, a process is called reversible when **[2001S]**
(a) surroundings and system change into each other.
(b) there is no boundary between system and surroundings.
(c) the surroundings are always in equilibrium with the system.
(d) the system changes into the surroundings spontaneously.

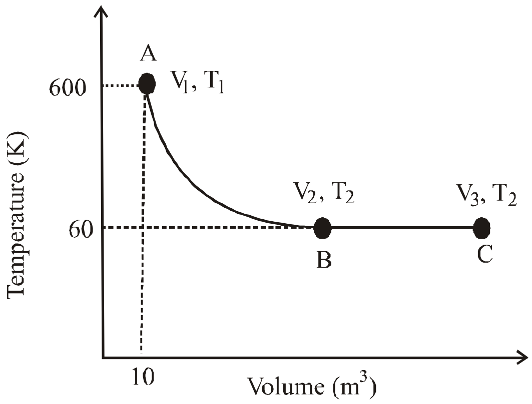
2 Integer Value Answer

27. Consider the following volume–temperature (V–T) diagram for the expansion of 5 moles of an ideal monoatomic gas.



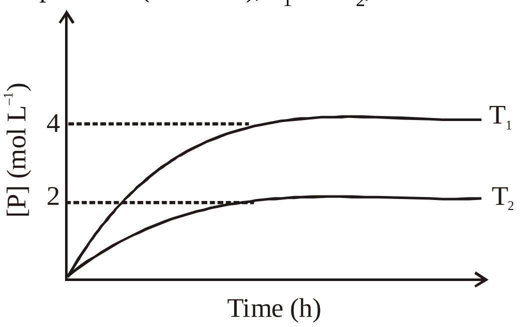
Considering only P-V work is involved, the total change in enthalpy (in Joule) for the transformation of state in the sequence **X→Y→Z** is ______. **[Adv. 2024]**
[Use the given data: Molar heat capacity of the gas for the given temperature range, $C_{V,m}$ = 12 J K^{-1} mol^{-1} and gas constant, R = 8.3 J K^{-1} mol^{-1}].

28. One mole of an ideal monoatomic gas undergoes two reversible processes ($A \rightarrow B$ and $B \rightarrow C$) as shown in the given figure: **[Adv. 2023]**



$A \rightarrow B$ is an adiabatic process. If the total heat absorbed in the entire process ($A \rightarrow B$ and $B \rightarrow C$) is $RT_2 \ln 10$, the value of $2 \log V_3$ is ______.
[Use, molar heat capacity of the gas at constant pressure, $C_{p,m} = \frac{5}{2}R$]

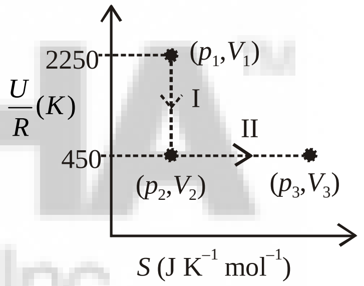
29. In a one–litre flask, 6 moles of A undergoes the reaction $A(g) \rightleftharpoons P(g)$. The progress of product formation at two temperatures (in Kelvin), T_1 and T_2, is shown in the figure:



If $T_1 = 2T_2$ and $\left(\Delta G_2^\circ - \Delta G_1^\circ\right) = RT_2 \ln x$, then the value of x is ______. **[Adv. 2023]**

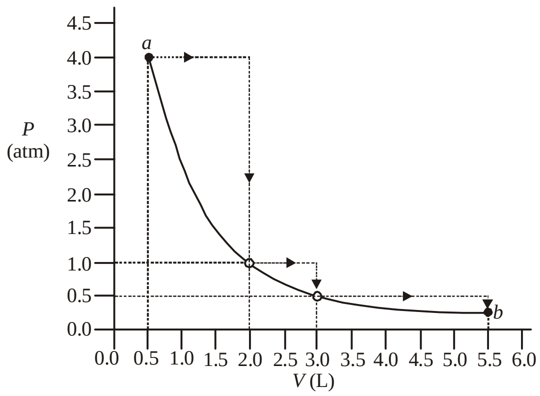
[ΔG_1° and ΔG_2° are standard Gibb's free energy change for the reaction at temperatures T_1 and T_2, respectively.]

30. One mole of an ideal gas at 900 K, undergoes two reversible processes, I followed by II, as shown below. If the work done by the gas in the two processes are same, the value of $\ln \frac{V_3}{V_2}$ is ______. **[Adv. 2021]**



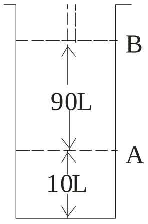
(U : internal energy, S : entropy, p: pressure, V : volume, R: gas constant)
(Given: molar heat capacity at constant volume, $C_{V,m}$ of the gas is $\frac{5}{2}R$)

31. One mole of an ideal gas is taken from a to b along two paths denoted by the solid and the dashed lines as shown in the graphs below. If the work done along the solid line path is w_s and that along the dotted line path is w_d, then the integer closest to the ratio w_d / w_s is : **[2010]**



3 Numeric/New Stem Based Questions

32. When $\Delta H_{vap} = 30$ kJ/mol and $\Delta S_{vap} = 75$ J mol^{-1} K^{-1}, then the temperature of vapour, at one atmosphere is ______K. **[Main April 9, 2024 (II)]**

33. 

Consider the figure provided. **[Main April 8, 2024 (I)]**

1 mol of an ideal gas is kept in a cylinder, fitted with a piston, at the position A, at 18°C. If the piston is moved to position B, keeping the temperature unchanged, then 'x' L atm work is done in this reversible process.

x = ______ L atm. (nearest integer)

[Given : Absolute temperature = °C + 273.15, R = 0.08206 L atm mol^{-1} K^{-1}]

34. $\Delta_{vap} H^{\ominus}$ for water is +40.49 kJ mol^{-1} at 1 bar and 100°C. Change in internal energy for this vapourisation under same condition is ______ kJ mol^{-1}. (Integer answer)

(Given R = 8.3 JK^{-1} mol^{-1}) **[Main April 8, 2024 (II)]**

35. An ideal gas, $\bar{C}v = \frac{5}{2}R$, is expanded adiabatically against a constant pressure of 1 atm untill it doubles in volume. If the initial temperature and pressure is 298 K and 5 atm, respectively then the final temperature is ________ K (nearest integer). **[Main April 6, 2024 (I)]**

[$\bar{C}v$ is the molar heat capacity at constant volume]

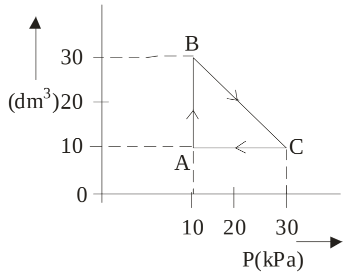
36. For the reaction at 298 K, $2A + B \rightarrow C$. $\Delta H = 400$ kJ mol^{-1} and $\Delta S = 0.2$ kJ mol^{-1} K^{-1}. The reaction will become spontaneous above______ K. **[Main April 6, 2024 (II)]**

37. Three moles of an ideal gas are compressed isothermally from 60 L to 20 L using constant pressure of 5 atm. Heat exchange Q for the compression is _____ Lit. atm. **[Main April 4, 2024 (II)]**

38. If 5 moles of an ideal gas expands from 10 L to a volume of 100 L at 300 K under isothermal and reversible condition then work, w, is –x J. The value of x is ______.

(Given R = 8.314 J K^{-1}1mol^{-1}) **[Main Jan. 31, 2024 (II)]**

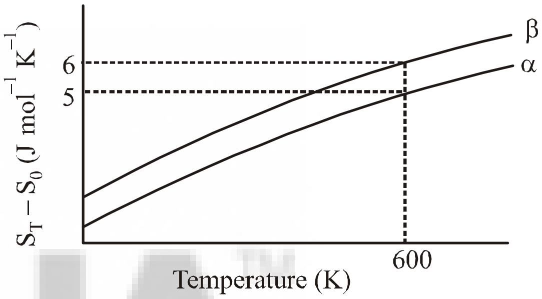
39. An ideal gas undergoes a cyclic transformation starting from the point A and coming back to the same point by tracing the path $A \rightarrow B \rightarrow C \rightarrow A$ as shown in the diagram above. The total work done in the process is ________ J. **[Main Jan. 30, 2024 (I)]**



40. If three moles of an ideal gas at 300 K expand isothermally from 30 dm^3 to 45 dm^3 against a constant opposing pressure of 80 kPa, then the amount of heat transferred is_______ J. **[Main Jan. 27, 2024 (I)]**

Qusetion Stem for Question no. 41 & 42

The entropy versus temperature plot for phases α and β at 1 bar pressure is given. S_T and S_0 are entropies of the phases at temperatures T and 0 K, respectively.



The transition temperature for α to β phase change is 600 K and $C_{p,\beta} - C_{p,\alpha} = 1$ J mol^{-1} K^{-1}. Assume $(C_{p,\beta} - C_{p,\alpha})$ is independent of temperature in the range of 200 to 700 K. $C_{p,\alpha}$ and $C_{p,\beta}$ are heat capacities of α and β phases, respectively.

41. The value of entropy change, $S_\beta - S_\alpha$ (in J mol^{-1} K^{-1}), at 300 K is _________. **[Adv. 2023]**

[Use: ln 2 = 0.69

Given: $S_\beta - S_\alpha = 0$ at 0 K]

42. The value of enthalpy change, $H_\beta - H_\alpha$ (in J mol^{-1}), at 300 K is ________. **[Adv. 2023]**

43. The total number of intensive properties from the following is _____

Volume, Molar heat capacity, Molarity, E^θ cell, Gibbs free energy change, Molar mass, Mole. **[Main April 11, 2023 (II)]**

44. The number of endothermic process/es from the following is ______ **[Main April 10, 2023 (II)]**

A. $I_2(g) \rightarrow 2I(g)$

B. $HCl(g) \rightarrow H(g) + Cl(g)$

C. $H_2O(l) \rightarrow H_2O(g)$

D. $C(s) + O_2(g) \rightarrow CO_2(g)$

E. Dissolution of ammonium chloride in water

45. When a 60 W electric heater is immersed in a gas for 100s in a constant volume container with adiabatic walls, the temperature of the gas rises by 5°C. The heat capacity of the given gas is_____J K^{-1} (Nearest integer) **[Main April 08, 2023 (I)]**

46. For complete combustion of ethene. **[Main April 08, 2023 (II)]**

$C_2H_4(g) + 3O_2(g) \rightarrow 2CO_2(g) + 2H_2O(l)$ the amount of heat produced as measured in bomb calorimeter is 1406 kJ mol^{-1} at 300K. The minimum value of $T\Delta S$ needed to reach equilibrium is (–) _________kJ. (Nearest integer)

Given : $R = 8.3\ JK^{-1}\ mol^{-1}$

47. When 2 litre of ideal gas expands isothermally into vacuum to a total volume of 6 litre, the change in internal energy is ______ J. (Nearest integer) **[Main Jan. 30, 2023 (I)]**

48. 1 mole of ideal gas is allowed to expand reversibly and adiabatically from a temperature of 27°C. The work done is 3 kJ mol^{-1}. The final temperature of the gas is _______K (Nearest integer). Given $C_v = 20\ J\ mol^{-1}K^{-1}$. **[Main Jan. 30, 2023 (II)]**

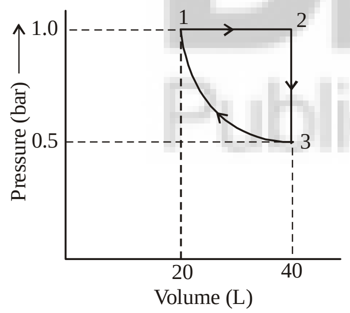
49. An athlete is given 100 g of glucose ($C_6H_{12}O_6$) for energy. This is equivalent to 1800 kJ of energy. The 50% of this energy gained is utilized by the athlete for sports activities at the event. In order to avoid storage of energy, the weight of extra water he would need to perspire is_______g (Nearest integer) **[Main Jan. 25, 2023 (I)]**

Assume that there is no other way of consuming stored energy.

Given : The enthalpy of evaporation of water is 45 kJ mol^{-1}

Molar mass of C, H & O are 12. 1 and 16 g mol^{-1}.

50. One mole of an ideal monoatomic gas is subjected to changes as shown in the graph. The magnitude of the work done (by the system or on the system) is _______ J (nearest integer). **[Main Jan. 24, 2023 (II)]**



Given : log 2 = 0.3, ln 10 = 2.3

51. 30.4 kJ of heat is required to melt one mole of sodium chloride and the entropy change at the melting point is 28.4 J K^{-1} mol^{-1} at 1 atm. The melting point of sodium chloride is _______ K (Nearest Integer) **[Main April 15, 2023 (I)]**

52. One mole of an ideal gas at 350K is in a 2.0 L vessel of thermally conducting walls, which are in contact with the surroundings. It undergoes isothermal reversible expansion from 2.0 L to 3.0 L against a constant pressure of 4 atm. The change in entropy of the surroundings (ΔS) is _________ J K^{-1} (Nearest integer)

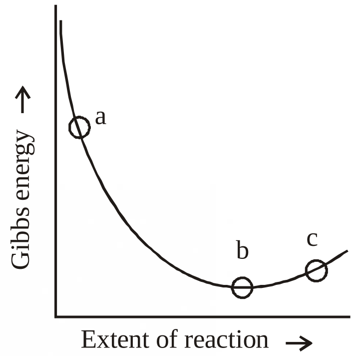
Given : $R = 8.314\ J\ K^{-1}\ Mol^{-1}$. **[Main April 12, 2023 (I)]**

53. For independent process at 300 K. **[Main Jan. 24, 2023 (I)]**

Process	ΔH/kJ mol^{-1}	ΔS/J K^{-1}
A	–25	–80
B	–22	40
C	25	–50
D	22	20

The number of non-spontaneous process from the following is _______.

54. Consider the graph of Gibbs free energy G vs Extent of reaction. The number of statement/s from the following which are true with respect to points (a), (b) and (c) is............. **[Main April 06, 2023 (I)]**



A. Reaction is spontaneous at (a) and (b)

B. Reaction is at equilibrium at point (b) and non–spontaneous at point (c)

C. Reaction is spontaneous at (a) and non–spontaneous at (c)

D. Reaction is non–spontaneous at (a) and (b)

55. Among the following the number of state variables is _________. **[Main July 28, 2022 (II)]**

Internal energy (U), Volume (V), Heat (q), Enthalpy (H)

56. The molar heat capacity for an ideal gas at constant pressure is 20.785 J K^{-1} mol^{-1}. The change in internal energy is 5000 J upon heating it from 300K to 500K. The number of moles of the gas at constant volume is ___ [Nearest integer]

(Given: $R = 8.314\ J\ K^{-1}\ mol^{-1}$) **[Main July 27, 2022 (I)]**

57. 17.0 g of NH_3 completely vapourises at – 33.42 °C and 1 bar pressure and the enthalpy change in the process is 23.4 kJ mol^{-1}. The enthalpy change for the vapourisation of 85 g of NH_3 under the same conditions is ________ kJ. **[Main June 29, 2022 (I)]**

58. When 5 moles of He gas expand isothermally and reversibly at 300 K from 10 litre to 20 litre, the magnitude of the maximum work obtained is ____ J. [nearest integer]

(Given: $R = 8.3\ J\ K^{-1}mol^{-1}$ and log 2 = 0.3010) **[Main June 27, 2022 (II)]**

59. A fish swimming in water body when taken out from the water body is covered with a film of water of weight 36 g. When it is subjected to cooking at 100 °C, then the internal energy for vaporization in kJ mol^{-1} is ________. [nearest integer]

[Assume steam to be an ideal gas. Given $\Delta_{vap}H^{\ominus}$ for water at 373 K and 1 bar is 41.1 kJ mol^{-1} ; $R = 8.31\ JK^{-1}mol^{-1}$] **[Main June 26, 2022 (II)]**

60. For the reaction $2NO_2(g) \rightleftharpoons N_2O_4$ (g), when $\Delta S = -176.0$ JK^{-1} and $\Delta H = -57.8$ kJ mol^{-1}, the magnitude of ΔG at 298 K for the reaction is_____ kJ mol^{-1}. (Nearest integer) **[Main Sep. 1, 2021 (II)]**

61. Consider the following cell reaction :

$$Cd(s) + Hg_2SO_4(s) + \frac{9}{5}H_2O(l) \rightleftharpoons CdSO_4.\frac{9}{5}H_2O(s) + 2Hg(l)$$

The value of E^0_{cell} is 4.315 V at 25°C. If $\Delta H° = -825.2$ kJ mol^{-1}, the standard entropy change $\Delta S°$ in J K^{-1} is ______. (Nearest integer)
[Given : Faraday constant = 96487 C mol^{-1}]
[Main Aug. 31, 2021 (I)]

62. At 25 °C, 50 g of iron reacts with HCl to form $FeCl_2$. The evolved hydrogen gas expands against a constant pressure of 1 bar. The work done by the gas during this expansion is ________ J. (Round off to the Nearest Integer).
[Given : R = 8.314 J mol^{-1} K^{-1}. Assume, hydrogen is an ideal gas]
[Atomic mass of Fe is 55.85 u] **[Main March 16, 2021 (II)]**

63. For the reaction $A(g) \rightarrow B(g)$, the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of Δ_rG for the reaction at 300 K and 1 atm in J mol^{-1} is $-xR$, where x is ________ (Rounded off to the nearest integer) (R = 8.31 J mol^{-1} K^{-1} and ln 10 = 2.3)
[Main February 24, 2021 (I)]

64. Tin is obtained from cassiterite by reduction with coke. Use the data given below to determine the minimum temperature (in K) at which the reduction of cassiterite by coke would take place. **[Adv. 2020]**

At 298 K: $\Delta_f H^0(SnO_2(s)) = -581.0$ kJ mol^{-1},

$\Delta_f H^0(CO_2(g)) = -394.0$ kJ mol^{-1},

$S^0(SnO_2(s)) = 56.0$ J K^{-1} mol^{-1},

$S^0(Sn(s)) = 52.0$ J K^{-1} mol^{-1},

$S^0(C(s)) = 6.0$ J K^{-1} mol^{-1},

$S^0(CO_2(g)) = 210.0$ J K^{-1} mol^{-1}.

Assume that the enthalpies and the entropies are temperature independent.

65. For a dimerization reaction, $2A(g) \rightarrow A_2(g)$,

at 298 K, $\Delta U^o = -20$ kJ mol^{-1}, $\Delta S^o = -30$ J K^{-1} mol^{-1},

then the ΔG^o will be ___________ J.

[Main Sep. 05, 2020 (II)]

66. The surface of copper gets tarnished by the formation of copper oxide. N_2 gas was passed to prevent the oxide formation during heating of copper at 1250 K. However, the N_2 gas contains 1 mole % of water vapour as impurity. The water vapour oxidises copper as per the reaction given below: $2Cu(s) + H_2O(g) \rightarrow Cu_2O(s) + H_2(g)$
p_{H2} is the minimum partial pressure of H_2 (in bar) needed to prevent the oxidation at 1250 K. The value of ln(p_{H2}) is_____.
(Given: total pressure = 1 bar, R (universal gas constant) = 8 J K^{-1} mol^{-1}, ln (10) = 2.3. Cu (s) and Cu_2O (s) are mutually immiscible.
At 1250 K : $2Cu(s) + ½O_2(g) \rightarrow Cu_2O(s)$; $\Delta G° = -78{,}000$ J mol^{-1}
$H_2(g) + ½O_2(g) \rightarrow H_2O(g)$; $\Delta G° = -1{,}78{,}000$J mol^{-1};
(G is the Gibbs energy)
[Adv. 2018]

67. For the reaction, $2CO + O_2 \longrightarrow 2CO_2$; $\Delta H = -560$kJ. Two moles of CO and one mole of O_2 are taken in a container of volume 1 L. They completely form two moles of CO_2, the gases deviate appreciably from ideal behaviour. If the pressure in the vessel changes from 70 to 40 atm, find the magnitude (absolute value) of ΔU at 500 K. (1 L atm = 0.1 kJ) **[2006 - 6M]**

68. A sample of argon gas at 1 atm pressure and 27 °C expands reversibly and adiabatically from 1.25 dm^3 to 2.50 dm^3. Calculate the enthalpy change in this process. $C_{V,m}$ for argon is 12.48 JK^{-1} mol^{-1}. **[2000 - 4 Marks]**

69. An athlete is given 100 g of glucose ($C_6H_{12}O_6$) of energy equivalent to 1560 kJ. He utilizes 50 percent of this gained energy in the event. In order to avoids storage of energy in the body, calculate the weight of water he would need to perspire. The enthalpy of evaporation of water is 44 kJ/mole. **[1989 - 2 Marks]**

4 Fill in the Blanks

70. When Fe(s) is dissolved in aqueous hydrochloric acid in a closed vessel, the work done is .... **[1997 - 1 Mark]**

71. Enthalpy is an .... property. **[1997 - 1 Mark]**

72. A system is said to be .... if it can neither exchange matter nor energy with the surroundings. **[1993 - 1 Mark]**

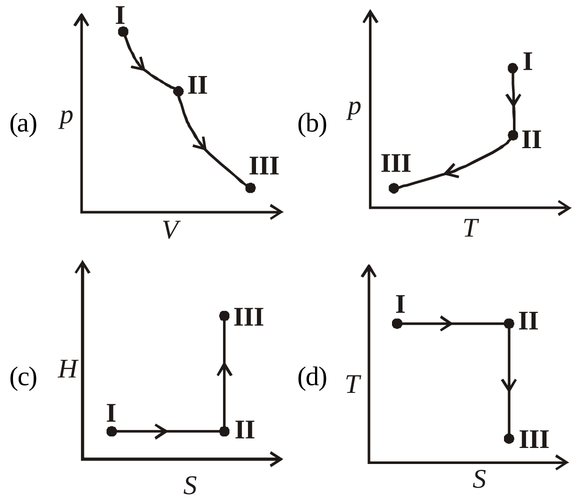
5 True / False

73. Heat capacity of a diatomic gas is higher than that of a monoatomic gas. **[1985 - ½ Mark]**

74. First law of thermodynamics is not adequate in predicting the direction of a process. **[1982 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

75. An ideal gas undergoes a reversible isothermal expansion from state I to state II followed by a reversible adiabatic expansion from state II to state III. The correct plot(s) representing the changes from state I to state III is (are) (p : pressure, V : volume, T : temperature, H : enthalpy, S : entropy) **[Adv. 2021]**



76. In thermodynamics, the $P - V$ work done is given by

$$w = -\int dV\ P_{\text{ext}}.$$

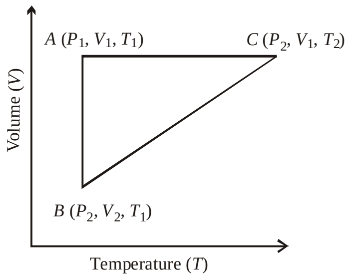
For a system undergoing a particular process, the work done is,

$$w = -\int dV\left(\frac{RT}{V-b} - \frac{a}{V^2}\right).$$

This equation is applicable to a **[Adv. 2020]**

(a) system that satisfies the van der Waals equation of state.
(b) process that is reversible and isothermal.
(c) process that is reversible and adiabatic.
(d) process that is irreversible and at constant pressure.

77. A reversible cyclic process for an ideal gas is shown below. Here, P, V, and T are pressure, volume and temperature, respectively. The thermodynamic parameters q, w, H and U are heat, work, enthalpy and internal energy, respectively. **[Adv. 2018]**



The correct option(s) is (are)

(a) $q_{AC} = \Delta U_{BC}$ and $w_{AB} = P_2(V_2 - V_1)$
(b) $w_{BC} = P_2(V_2 - V_1)$ and $q_{BC} = \Delta H_{AC}$
(c) $\Delta H_{CA} < \Delta U_{CA}$ and $q_{AC} = \Delta U_{BC}$
(d) $q_{BC} = \Delta H_{AC}$ and $\Delta H_{CA} > \Delta U_{CA}$

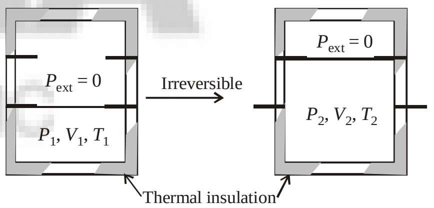
78. For a reaction taking place in a container in equilibrium with its surroundings, the effect of temperature on its equilibrium constant K in terms of change in entropy is described by **[Adv. 2017]**

(a) With increase in temperature, the value of K for exothermic reaction decreases because the entropy change of the system is positive
(b) With increase in temperature, the value of K for endothermic reaction increases because unfavourable change in entropy of the surroundings decreases
(c) With increase in temperature, the value of K for endothermic reaction increases because the entropy change of the system is negative
(d) With increase in temperature, the value of K for exothermic reaction decreases because favourable change in entropy of the surroundings decreases

79. An ideal gas is expanded from (P_1, V_1, T_1) to (P_2, V_2, T_2) under different conditions. The correct statement(s) among the following is (are) **[Adv. 2017]**

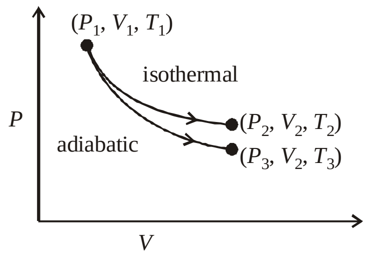
(a) The work done on the gas is maximum when it is compressed irreversibly from (P_2, V_2) to (P_1, V_1) against constant pressure P_1
(b) If the expansion is carried out freely, it is simultaneously both isothermal as well as adiabatic
(c) The work done by the gas is less when it is expanded reversibly from V_1 to V_2 under adiabatic conditions as compared to that when expanded reversibly from V_1 to V_2 under isothermal conditions
(d) The change in internal energy of the gas is (i) zero, if it is expanded reversibly with $T_1 = T_2$, and (ii) positive, if it is expanded reversibly under adiabatic conditions with $T_1 \neq T_2$

80. An ideal gas in a thermally insulated vessel at internal pressure = P_1, volume = V_1 and absolute temperature = T_1 expands irreversibly against zero external pressure, as shown in the diagram. The final internal pressure, volume and absolute temperature of the gas are P_2, V_2 and T_2, respectively. For this expansion, **[Adv. 2014]**



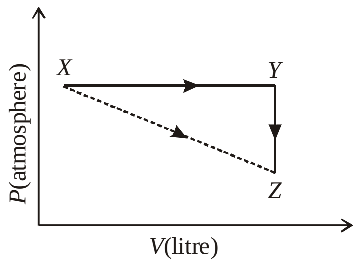
(a) $q = 0$
(b) $T_2 = T_1$
(c) $P_2V_2 = P_1V_1$
(d) $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

81. The reversible expansion of an ideal gas under adiabatic and isothermal conditions is shown in the figure. Which of the following statement(s) is (are) correct ? **[2012 - II]**



(a) $T_1 = T_2$
(b) $T_3 > T_1$
(c) $w_{\text{isothermal}} > w_{\text{adiabatic}}$
(d) $\Delta U_{\text{isothermal}} > \Delta U_{\text{adiabatic}}$

82. For an ideal gas, consider only P–V work in going from an initial state X to the final state Z. The final state Z can be reached by either of the two paths shown in the figure. Which of the following choice(s) is (are) correct?
[Take ΔS as change in entropy and w as work done]. **[2012]**



(a) $\Delta S_{X\to Z} = \Delta S_{X\to Y} + \Delta S_{Y\to Z}$
(b) $w_{X\to Z} = w_{X\to Y} + w_{Y\to Z}$
(c) $w_{X\to Y\to Z} = w_{X\to Y}$
(d) $\Delta S_{X\to Y\to Z} = \Delta S_{X\to Y}$

83. Among the following, the intensive property is (properties are) **[2010]**
(a) molar conductivity (b) electromotive force
(c) resistance (d) heat capacity

84. Among the following the state function(s) is (are) **[2009]**
(a) Internal energy
(b) Irreversible expansion work
(c) Reversible expansion work
(d) Molar enthalpy

85. The following is (are) endothermic reaction(s):
(a) Combustion of methane **[1999 - 3 Marks]**
(b) Decomposition of water
(c) Dehydrogenation of ethane to ethylene
(d) Conversion of graphite to diamond

86. Identify the intensive quantities from the following: **[1993 - 1 Mark]**
(a) Enthalpy (b) Temperature
(c) Volume (d) Refractive Index

7 Match the Following

87. Match List-I with List-II **[Main June 27, 2022 (I)]**

List-I	List-II
(A) Spontaneous process	(I) $\Delta H < 0$
(B) Process with $\Delta P = 0$, $\Delta T = 0$	(II) $\Delta G_{T,P} < 0$
(C) $\Delta H_{reaction}$	(III) Isothermal and isobaric process
(D) Exothermic process	(IV) [Bond energies of molecules in reactants] - [Bond energies of product molecules

Choose the correct answer from the options given below:
(a) (A) – (III), (B) – (II), (C) – (IV), (D) – (I)
(b) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)
(c) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)
(d) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

88. Match the thermodynamic processes given under Column-I with the expressions given under Column-II. **[Adv. 2015]**

Column-I	Column-II
(A) Freezing of water at 273 K and 1 atm	(p) $q = 0$
(B) Expansion of 1 mol of an ideal gas into a vacuum under isolated conditions	(q) $w = 0$
(C) Mixing of equal volumes of two ideal gases at constant temperature and pressure in an isolated container	(r) $\Delta S_{sys} < 0$
(D) Reversible heating of $H_2(g)$ at 1 atm from 300 K to 600 K, followed by reversible cooling to 300 K at 1 atm	(s) $\Delta U = 0$
	(t) $\Delta G = 0$

89. Match the transformations in **column I** with appropriate options in **column II** **[2011]**

Column-I	Column-II
(A) $CO_2(s) \to CO_2(g)$	(p) phase transition
(B) $CaCO_3(s) \to CaO(s) + CO_2(g)$	(q) allotropic change
(C) $2H^{\bullet} \to H_2(g)$	(r) ΔH is positive
(D) $P_{(white,\ solid)} \to P_{(red,\ solid)}$	(s) ΔS is positive
	(t) ΔS is negative

8 Comprehension/Passage Based Questions

Passage-I

The amount of energy required to break a bond is same as the amount of energy released when the same bond is formed. In gaseous state, the energy required for *homolytic cleavage* of a bond is called Bond Dissociation Energy (BDE) or Bond Strength. BDE is affected by s-character of the bond and the stability of the radicals formed. Shorter bonds are typically stronger bonds. BDEs for some bonds are given below :

$$H_3C-H(g) \longrightarrow H_3C^{\bullet}(g) + H^{\bullet}(g) \qquad \Delta H° = 105 \text{ kcal mol}^{-1}$$

$$Cl-Cl(g) \longrightarrow Cl^{\bullet}(g) + Cl^{\bullet}(g) \qquad \Delta H° = 58 \text{ kcal mol}^{-1}$$

$$H_3C-Cl(g) \longrightarrow H_3C^{\bullet}(g) + Cl^{\bullet}(g) \qquad \Delta H° = 85 \text{ kcal mol}^{-1}$$

$$H-Cl(g) \longrightarrow H^{\bullet}(g) + Cl^{\bullet}(g) \qquad \Delta H° = 103 \text{ kcal mol}^{-1}$$

90. Correct match of the C–H bonds (shown in bold) in Column J with their BDE in Column K is

Column J (Molecule)	Column K (BDE (kcal mol^{-1}))
(P) $H-CH(CH_3)_2$	(i) 132
(Q) $H-CH_2Ph$	(ii) 110
(R) $H-CH=CH_2$	(iii) 95
(S) $H-C\equiv CH$	(iv) 88

[Adv. 2021]
(a) P – iii, Q – iv, R – ii, S – i
(b) P – i, Q – ii, R – iii, S – iv
(c) P – iii, Q – ii, R – i, S – iv
(d) P – ii, Q – i, R – iv, S – iii

91. For the following reaction

$CH_4(g) + Cl_2(g) \xrightarrow{\text{light}} CH_3Cl(g) + HCl(g)$

the correct statement is **[Adv. 2021]**

(a) Initiation step is exothermic with $\Delta H^o = -58$ kcal mol^{-1}

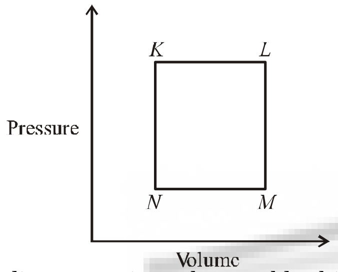
(b) Propagation step involving $^{\bullet}CH_3$ formation is exothermic with $\Delta H^o = -2$ kcal mol^{-1}.

(c) Propagation step involving CH_3Cl formation is endothermic with $\Delta H^o = +27$ kcal mol^{-1}.

(d) The reaction is exothermic with $\Delta H^o = -25$ kcal mol^{-1}.

Passage-II

A fixed mass '*m*' of a gas is subjected to transformation of states from *K* to *L* to *M* to *N* and back to *K* as shown in the figure **[Adv. 2013]**



92. The succeeding operations that enable this transformation of states are

(a) Heating, cooling, heating, cooling

(b) Cooling, heating, cooling, heating

(c) Heating, cooling, cooling, heating

(d) Cooling, heating, heating, cooling

93. The pair of isochoric processes among the transformation of states is

(a) *K* to *L* and *L* to *M* (b) *L* to *M* and *N* to *K*

(c) *L* to *M* and *M* to *N* (d) *M* to *N* and *N* to *K*

9 Statement / Assertion and Reason Type Questions

Each question contains **STATEMENT-I** and **STATEMENT-II**. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct. Mark your answer as

(a) If both Statement -I and Statement -II are correct.

(b) If both Statement -I and Statement -II are incorrect.

(c) If Statement -I is correct but Statement -II is incorrect.

(d) If Statement -I is incorrect but Statement -II is correct.

94. **Statement - I :** There is a natural asymmetry between converting work to heat and converting heat to work.

Statement - II : No process is possible in which the sole result is the absorption of heat form a reservoir and its complete conversion into work. **[2008S]**

Each Question contains Assertion and Reason statements. In the light of the given statements in the question choose the correct answer from the options given below.

(a) If both Assetion and Reason are correct and Reason is the correct explanation of the Assertion

(b) If both Assertion and Reason are correct but Reaon is not the correct explanation of the Assertion.

(c) If the Assertion is correct but the Reason is incorrect

(d) If the Assertion is incorrect but the Reason is correct

95. **Assertion :** The reduction of a metal oxide is easier if the metal formed is in liquid state than solid state.

Reason : The value of $\Delta G°$ becomes more on negative side as entropy is higher liquid state than solid state. **[Main July 28, 2022 (II)]**

96. **Assertion :** The heat absorbed during the isothermal expansion of an ideal gas against vacuum is zero.

Reason : The volume occupied by the molecules of an ideal gas is zero. **[2000S]**

10 Subjective Problems

97. An insulated container contains 1 mol of a liquid, molar volume 100 mL, at 1 bar. When liquid is steeply pressed to 100 bar, volume decreases to 99 mL. Find. ΔH and ΔU for the process. **[2004 - 2 Marks]**

98. In the following equilibrium $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ **[2004 - 2 Marks]**

When 5 moles of each is taken and the temperature is kept at 298 K, the total pressure was found to be 20 bar.

Given : $\Delta G_f^\circ(N_2O_4) = 100\text{kJ};\ \Delta G_f^\circ(NO_2) = 50$ kJ

(i) Find ΔG of the reaction at 298 K.

(ii) Find the direction of the reaction

99. C_v value of He is always $3R/2$ but C_v value of H_2 is $3R/2$ at low temperature and $5R/2$ at moderate temperature and more than $5R/2$ at higher temperature. Explain in two to three lines. **[2003 - 2 Marks]**

100. Two moles of a perfect gas undergo the following processes: **[2002 - 5 Marks]**

(a) a reversible isobaric expansion from (1.0 atm, 20.0 L) to (1.0 atm, 40.0 L);

(b) a reversible isochoric change of state from (1.0 atm, 40.0 L) to (0.5 atm, 40.0 L);

(c) a reversible isothermal compression from (0.5 atm, 40.0 L) to (1.0 atm, 20.0 L).

(i) Sketch with labels each of the processes on the same *P-V* diagram.

(ii) Calculate the total work (*W*) and the total heat change (*q*) involved in the above processes.

(iii) What will be the values of ΔU, ΔH and ΔS for the overall process?

101. When 1-pentyne (*A*) is treated with 4 N alcoholic KOH at 175 °C, it is converted slowly into an equilibrium mixture of 1.3% 1-pentyne (*A*), 95.2% 2-pentyne (*B*) and 3.5% of 1, 2-pentadiene (*C*). The equilibrium was maintained at 175 °C. Calculate $\Delta G°$ for the following equilibria :

$B \rightleftharpoons A \quad \Delta G_1^\circ = ? \qquad B \rightleftharpoons C \quad \Delta G_2^\circ = ?$

From the calculated value of ΔG_1° and ΔG_2° indicate the order of stability of (*A*), (*B*) and (*C*). Write a reasonable reaction mechanism showing all intermediates leading to (*A*), (*B*) and (*C*). **[2001 - 10 Marks]**

102. Show that the reaction $CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$ at 300 K, is spontaneous and exothermic, when the standard entropy change is -0.094 kJ mol^{-1} K^{-1}. The standard Gibbs free energies of formation for CO_2 and CO are -394.4 and -137.2 kJ mol^{-1}, respectively. **[2000 - 3 Marks]**

103. "The heat energy *q*, absorbed by a gas is ΔH", is true at what condition(s). **[1984 - 1 Mark]**

Topic-2: Thermochemistry

1 MCQs with One Correct Answer

1. Combustion of glucose ($C_6H_{12}O_6$) produces CO_2 and water. The amount of oxygen (in g) required for the complete combustion of 900 g of glucose is: [Molar mass of glucose in g mol^{-1} = 180] **[Main April 8, 2024 (I)]**
   (a) 480 (b) 960 (c) 800 (d) 32

2. Given **[Main April 10, 2023 (I)]**
   (A) $2CO(g) + O_2(g) \rightarrow 2CO_2(g)$ $\Delta H_1^\theta = -x$ kJ mol^{-1}
   (B) $C(graphite) + O_2(g) \rightarrow CO_2(g)$ $\Delta H_2^\theta = -y$ kJ mol^{-1}
   The ΔH^θ for the reaction
   $C(graphite) + \frac{1}{2} O_2(g) \rightarrow CO(g)$ is
   (a) $\frac{x-2y}{2}$ (b) $\frac{x+2y}{2}$ (c) $\frac{2x-y}{2}$ (d) $2y-x$

3. At 25°C and 1 atm pressure, the enthalpy of combustion of benzene (1) and acetylene (g) are –3268 kJ mol^{-1} and –1300 kJ mol^{-1}, respectively. The change in enthalpy for the reaction $3C_2H_2(g) \rightarrow C_6H_6(l)$, is **[Main June 25, 2022 (II)]**
   (a) +324 kJ mol^{-1} (b) +632 kJ mol^{-1}
   (c) –632 kJ mol^{-1} (d) –732 kJ mol^{-1}

4. At 25°C and 1 atm pressure, the enthalpies of combustion are as given below:

Substance	H_2	C (graphite)	$C_2H_6(g)$
$\frac{\Delta_c H°}{kjmo1^{-1}}$	–286.0	–394.0	–1560.0

   The enthalpy of formation of ethane is **[Main June 24, 2022 (II)]**
   (a) +54.0 kJ mol^{-1} (b) –68.0 kJ mol^{-1}
   (c) –86.0 kJ mol^{-1} (d) +97.0 kJ mol^{-1}

5. Lattice enthalpy and enthalpy of solution of NaCl are 788 kJ mol^{-1} and 4 kJ mol^{-1}, respectively. The hydration enthalpy of NaCl is : **[Main Sep. 05, 2020 (II)]**
   (a) –780 kJ mol^{-1} (b) 780 kJ mol^{-1}
   (c) –784 kJ mol^{-1} (d) 784 kJ mol^{-1}

6. If enthalpy of atomisation for $Br_2(l)$ is x kJ/mol and bond enthalpy for Br_2 is y kJ/mol, the relation between them: **[Main Jan. 09, 2020 (I)]**
   (a) is $x = y$ (b) does not exist
   (c) is $x > y$ (d) is $x < y$

7. The difference between ΔH and ΔU ($\Delta H - \Delta U$), when the combustion of one mole of heptane (I) is carried out at a temperature T, is equal to : **[Main April 10, 2019 (II)]**
   (a) $-4RT$ (b) $-3RT$ (c) $4RT$ (d) $3RT$

8. For silver, $C_p(JK^{-1} mol^{-1}) = 23 + 0.01T$. If the temperature ($T$) of 3 moles of silver is raised from 300 K to 1000 K at 1 atm pressure, the value of ΔH will be close to: **[Main April 8, 2019 (I)]**
   (a) 62 kJ (b) 16 kJ (c) 21 kJ (d) 13 kJ

9. The combustion of benzene (l) gives CO_2 (g) and H_2O (l). Given that heat of combustion of benzene at constant volume is –3263.9 kJ mol^{-1} at 25 °C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be : (R= 8.314 $JK^{-1} mol^{-1}$) **[Main 2018]**
   (a) 4152.6 (b) –452.46
   (c) 3260 (d) –3267.6

10. For which of the following reactions, ΔH is equal to ΔU? **[Main Online April 15, 2018 (I)]**
    (a) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$
    (b) $2HI(g) \rightarrow H_2(g) + I_2(g)$
    (c) $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$
    (d) $2NO_2(g) \rightarrow N_2O_4(g)$

11. Given
    $C(graphite) + O_2(g) \rightarrow CO_2(g)$; $\Delta_r H° = -393.5$ kJ mol^{-1}
    $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)$; $\Delta_r H° = -285.8$ kJ mol^{-1}
    $CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g)$; $\Delta_r H° = +890.3$ kJ mol^{-1}
    Based on the above thermochemical equations, the value of $\Delta_r H°$ at 298 K for the reaction,
    $C(graphite) + 2H_2(g) \rightarrow CH_4(g)$ will be : **[Main 2017]**
    (a) +74.8 kJ mol^{-1} (b) +144.0 kJ mol^{-1}
    (c) –74.8 kJ mol^{-1} (d) –144.0 kJ mol^{-1}

12. The heats of combustion of carbon and carbon monoxide are –393.5 and –283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is: **[Main 2016; Similar 2004]**
    (a) –676.5 (b) –110.5 (c) 110.5 (d) 676.5

13. The heat of atomization of methane and ethane are 360 kJ/mol and 620 kJ/mol, respectively. The longest wavelength of light capable of breaking the C – C bond is (Avogadro number $= 6.02 \times 10^{23}$, $h = 6.62 \times 10^{-34}$ J s) : **[Main Online April 10, 2015]**
    (a) 2.48×10^4 nm (b) 1.49×10^3 nm
    (c) 2.48×10^3 nm (d) 1.49×10^4 nm

14. For complete combustion of ethanol,
    $C_2H_5OH(l) + 3O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(l)$,
    the amount of heat produced as measured in bomb calorimeter, is 1364.47 kJ mol^{-1} at 25 °C. Assuming ideality, the enthalpy of combustion, $\Delta_c H$, for the reaction will be: (R = 8.314 kJ mol^{-1}) **[Main 2014]**
    (a) –1366.95 kJ mol^{-1} (b) –1361.95 kJ mol^{-1}
    (c) –1460.95 kJ mol^{-1} (d) –1350.50 kJ mol^{-1}

15. The standard enthalpy of formation of NH_3 is – 46.0 kJ/mol. If the enthalpy of formation of H_2 from its atoms is – 436 kJ/mol and that of N_2 is – 712 kJ/mol, the average bond enthalpy of N - H bond in NH_3 is: **[Main Online April 9, 2014]**
    (a) – 1102 kJ/mol (b) – 964 kJ/mol
    (c) + 352 kJ/mol (d) + 1056 kJ/mol

16. The standard enthalpies of formation of CO_2(g), H_2O(l) and glucose(s) at 25 °C are –400 kJ/mol, –300 kJ/mol and –1300 kJ/mol, respectively. The standard enthalpy of combustion per gram of glucose at 25 °C is **[Adv. 2013-I]**
    (a) +2900 kJ (b) –2900 kJ
    (c) –16.11 kJ (d) +16.11 kJ

17. Given that: **[Main Online April 25, 2013]**

(i) $\Delta_f H°$ of N_2O is 82 kJ mol^{-1}

(ii) Bond energies of $N \equiv N$, $N = N$, $O = O$ and $N = O$ are 946, 418, 498 and 607 kJ mol^{-1} respectively,

The resonance energy of N_2O is :

(a) –88kJ (b) –66kJ (c) –62kJ (d) –44kJ

18. The species which by definition has **ZERO** standard molar enthalpy of formation at 298 K is **[2010]**

(a) $Br_2(g)$ (b) $Cl_2(g)$ (c) $H_2O(g)$ (d) $CH_4(g)$

19. In a constant volume calorimeter, 3.5 g of a gas with molecular weight 28 was burnt in excess oxygen at 298.0 K. The temperature of the calorimeter was found to increase from 298.0 K to 298.45 K due to the combustion process. Given that the heat capacity of the calorimeter is 2.5 kJ K^{-1}, the numerical value for the enthalpy of combustion of the gas in kJ mol^{-1} is **[2009 - 6M]**

20. Which of the reaction defines ΔH_f°? **[2003S]**

(a) $C(\text{diamond}) + O_2(g) \longrightarrow CO_2(g)$

(b) $\frac{1}{2}H_2(g) + \frac{1}{2}F_2(g) \longrightarrow HF(g)$

(c) $N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$

(d) $CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$

21. The ΔH_f^0 for $CO_2(g)$, $CO(g)$ and $H_2O(g)$ are –393.5, –110.5 and –241.8 kJ mol^{-1} respectively. The standard enthalpy change (in kJ) for the reaction

$CO_2(g) + H_2(g) \rightarrow CO(g) + H_2O(g)$ is **[2000S]**

(a) 524.1 (b) 41.2 (c) –262.5 (d) –41.2

22. For which change $\Delta H \neq \Delta E$: **[1995S]**

(a) $H_2(g) + I_2(g) \rightarrow 2HI(g)$

(b) $HCl + NaOH \rightarrow NaCl$

(c) $C(s) + O_2(g) \rightarrow CO_2(g)$

(d) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$

23. The difference between heats of reaction at constant pressure and constant volume for the reaction :

$2C_6H_6(l) + 15O_2(g) \rightarrow 12CO_2(g) + 6H_2O(l)$ at 25 °C in kJ is **[1991 - 1 Mark]**

(a) –7.43 (b) +3.72 (c) –3.72 (d) +7.43

3 Numeric/New Stem Based Questions

24. The heat of combustion of solid benzoic acid at constant volume is –321.30 kJ at 27°C. The heat of combustion at constant pressure is (–321.30 – xR) kJ, the value of x is …………… **[Main April 5, 2024 (I)]**

25. Combustion of 1 mole of benzene is expressed at

$C_6H_6(1) + \frac{15}{2}O_2(g) \rightarrow CO_2(g) + 3H_2O(1)$.

The standard enthalpy of combustion of 2 mol of benzene is – 'x' kJ. x = _____. **[Main April 5, 2024 (II)]**

(1) standard Enthalpy of formation of 1 mol of $C_6H_6(1)$, for the reaction $6C(\text{graphite}) + 3H_2(g) \rightarrow C_6H_6(1)$ is 48.5 kJ mol^{-1}.

(2) Standard Enthalpy of formation of 1 mol of $CO_2(g)$, for the reaction $C(\text{graphite}) + O_2(g) \rightarrow CO_2(g)$ is –393.5 kJ mol^{-1}.

(3) Standard and Enthalpy of formation of 1 mol of $H_2O(1)$, for the reaction $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(1)$ is –286 kJ mol^{-1}.

26. The enthalpy of formation of ethane (C_2H_6) from ethylene by addition of hydrogen where the bond-energies of C – H, C – C, C = C, H – H are 414 kJ, 347 kJ, 615 kJ and 435 kJ respectively is – _____ kJ. **[Main April 4, 2024 (I)]**

27. Two reactions are given below: **[Main Jan. 30, 2024 (II)]**

$$2Fe(s) + \frac{3}{2}O_2(g) \rightarrow Fe_2O_3(s), DH^o = -822 kJ/mol$$

$$C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g), DH^o = -110 kJ/mol$$

Then enthalpy change for following reaction $3C(s) + Fe_2O_3(s) \rightarrow 2Fe(s) + 3CO(g)$ is __________ kJ/mol.

28. Standard enthalpy of vapourisation for CCl_4 is 30.5 kJ mol^{-1}. Heat required for vapourisation of 284g of CCl_4 at constant temperature is _____ kJ.

(Given molar mass in g mol^{-1} ; C = 12, Cl = 35.5)

[Main Jan. 29, 2024 (II)]

29. $A_2 + B_2 \rightarrow 2AB. \Delta H_f^0 = -200$ kJmol^{-1}

AB, A_2 and B_2 are diatomic molecule. If the bond enthalpies of A_2, B_2 and AB are in the ratio 1 : 0.5 : 1, then the bond enthalpy of A_2 is ______ kJmol^{-1} (Nearest integer)

[Main April 13, 2023 (I)]

30. Solid fuel used in rocket is a mixture of Fe_2O_3 and Al (in ratio 1 : 2). The heat evolved (kJ) per gram of the mixture is ________ (Nearest integer)

Given : $\Delta H_f^\theta(Al_2O_3) = -1700$ kJ mol^{-1}

$\Delta H_f^\theta(Fe_2O_3) = -840$ kJ mol^{-1}

Molar mass of Fe, Al and O are 56, 27 and 16 g mol^{-1} respectively. **[Main April 11, 2023 (I)]**

31. Consider the following date **[Main April 06, 2023 (II)]**

Heat of combustion of $H_2(g) = -241.8$ kJ mol^{-1}

Heat of combustion of $C(s) = -393.5$ kJ mol^{-1}

Heat of combustion of $C_2H_5OH(l) = -1234.7$ kJ mol^{-1}

________ kJ mol^{-1} (Nearest integer)

32. At 25ºC, the enthalpy of the following processes are given:

$H_2(g) + O_2(g)$	$\rightarrow$	$2OH(g)$	$\Delta H^o = 78$ kJ mol^{-1}
$H_2(g) + 1/2\, O_2(g)$	$\rightarrow$	$H_2O(g)$	$\Delta H^o = -242$ kJ mol^{-1}
$H_2(g)$	$\rightarrow$	$2H(g)$	$\Delta H^o = 436$ kJ mol^{-1}
$1/2\, O_2(g)$	$\rightarrow$	$O(g)$	$\Delta H^o = 249$ kJ mol^{-1}

What would be the value of X for the following reaction? _______ (Nearest integer)

$H_2O(g) \rightarrow H(g) + OH(g)\ \Delta H^o = X$ kJ mol^{-1}

[Main Feb. 01, 2023 (I)]

33. The enthalpy change for the conversion of $\frac{1}{2}Cl_2(g)$ to $Cl^-(aq)$ is (-) ___________ kJ mol^{-1} (Nearest integer)

Given : $\Delta_{dis}H°_{Cl_{2(g)}} = 240$ kJmol^{-1}. **[Main Jan. 31, 2023 (I)]**

$\Delta_{eg}H^\circ_{Cl_{(g)}} = -350$ kJmol^{-1}, $\Delta_{hyd}H^\circ_{Cl^-_{(g)}} = -380$ kJmol^{-1}

34. Enthalpies of formation of $CCl_4(g)$, $H_2O(g)$, $CO_2(g)$ and $HCl(g)$ are –105, –242, –394 and –92 KJ mol^{-1} respectively. The magnitude of enthalpy of the reaction given below is ______ kJ mol^{-1} (nearest integer) $CCl_4(g) + 2H_2O(g) \rightarrow CO_2(g) + 4HCl(g)$ **[Main Jan. 31, 2023 (II)]**

35. 28.0 L of CO_2 is produced on complete combustion of 16.8 L gaseous mixture of ethene and methane at 25°C and 1 atm. Heat evolved during the combustion process is ________ kJ. Given : $\Delta H_C(CH_4) = -900$ kJ mol^{-1} $\Delta H_C(C_2H_4) = -1400$ kJ mol^{-1} **[Main Jan. 25, 2023 (II)]**

36. 2 mol of Hg(g) is combusted in a fixed volume bomb calorimeter with excess of O_2 at 298 K and 1 atm into HgO(s). During the reaction, temperature increases from 298.0 K to 312.8 K. If heat capacity of the bomb calorimeter and enthalpy of formation of Hg(g) are 20.00 kJ K^{-1} and 61.32 kJ mol^{-1} at 298 K, respectively, the calculated standard molar enthalpy of formation of HgO(s) at 298 K is X kJ mol^{-1}. The value of |X| is _______.
[Given: Gas constant R = 8.3 J K^{-1} mol^{-1}] **[Adv. 2022]**

37. When 600 mL of 0.2 M HNO_3 is mixed with 400 mL of 0.1M NaOH solution in a flask, the rise in temperature of the flask is _______ × 10^{-2} °C.
(Enthalpy of neutralisation = 57 kJ mo1^{-1} and Specific heat of water = 4.2 JK^{-1} g^{-1})
(Neglect heat capacity of flask) **[Main July 29, 2022 (I)]**

38. While performing a thermodynamics experiment, a student made the following observations,
$HCl + NaOH \rightarrow NaCl + H_2O$, $\Delta H = -57.3$ kJ mol^{-1}
$CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$
$\Delta H = -55.3$ kJ mol^{-1} **[Main July 25, 2022 (II)]**
The enthalpy of ionization of CH_3COOH as calculated by the student is _____ kJ mol^{-1}, (nearest integer)

39. The Born-Haber cycle for KCl is evaluated with the following data :
$\Delta_f H°$ for KCl = –436.7 kJ mol^{-1}; $\Delta_{sub}H°$ for K = 89.2 kJ mol^{-1};
$\Delta_{ionization}$ H° for K = 419.0 kJ mol^{-1}; $\Delta_{electron\ gain}$ H° for $Cl_{(g)}$ = –348.6 kJ mol^{-1}; $\Delta_{bond}H°$ for Cl_2 = 243.0 kJ mol^{-1}
The magnitude of lattice enthalpy of KCl in kJ mol^{-1} is _______ (Nearest integer) **[Main Aug. 26, 2021 (I)]**

40. At 298 K, the enthalpy of fusion of a solid (X) is 2.8 kJ mol^{-1} and the enthalpy of vaporisation of the liquid (X) is 98.2 kJ mol^{-1}. The enthalpy of sublimation of the substance (X) in kJ mol^{-1} is ________ . (in nearest integer)
[Main July 25, 2021 (I)]

41. The average S-F bond energy in kJ mol^{-1} of SF_6 is ____________. (Rounded off to the nearest integer)
[Given : The values of standard enthalpy of formation of SF_6(g), S(g) and F(g) are – 1100, 275 and 80 kJ mol^{-1} respectively.] **[Main Feb. 26, 2021 (II)]**

42. The internal energy change (in J) when 90 g of water undergoes complete evaporation at 100 °C is ________ .
(Given : ΔH_{vap} for water at 373 K = 41 kJ/ mol, R = 8.314 JK^{-1} mol^{-1}) **[Main Sep. 02, 2020 (I)]**

43. Diborane is a potential rocket fuel which undergoes combustion according to the reaction. **[2000 - 2 Marks]**
$B_2H_6(g) + 3O_2(g) \rightarrow B_2O_3(s) + 3H_2O(g)$
From the following data, calculate the enthalpy change for the combustion of diborane.

$$2B(s) + \frac{3}{2}O_2(g) \longrightarrow B_2O_3(s) \qquad \Delta H = -1273 \text{ kJ mol}^{-1}$$
$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(\ell) \qquad \Delta H = -286 \text{ kJ mol}^{-1}$$
$$H_2O(l) \longrightarrow H_2O(g) \qquad \Delta H = 44 \text{ kJ mol}^{-1}$$
$$2B(s) + 3H_2(g) \longrightarrow B_2H_6(g) \qquad \Delta H = 36 \text{ kJ mol}^{-1}$$

44. Estimate the average S–F bond energy in SF_6. The values of standard enthalpy of formation of SF_6(g), S(g) and F(g) are : – 1100, 275 and 80 kJ mol^{-1} respectively.
[1999 - 3 Marks]

45. From the following data, calculate the enthalpy change for the combustion of cyclopropane at 298 K. The enthalpy of formation of CO_2(g), H_2O(l) and propene(g) are – 393.5, – 285.8 and 20.42 kJ mol^{-1} respectively. The enthalpy of isomerisation of cyclopropane to propene is – 33.0 kJ mol^{-1}. **[1998 - 5 Marks]**

46. Compute the heat of formation of liquid methyl alcohol in kilojoules per mole, using the following data. Heat of vaporization of liquid methyl alcohol = 38 kJ/mol. Heat of formation of gaseous atoms from the elements in their standard states; H, 218 kJ/mol; C, 715 kJ/mol; O, 249kJ / mol. Average bond energies :
C – H = 415kJ/mol, C – O = 365 kJ/mol, O – H = 463 kJ/mol
[1997 - 5 Marks]

47. The standard molar enthalpies of formation of cyclohexane (l) and benzene (l) at 25° C are – 156 and + 49 kJ mol^{-1} respectively. The standard enthalpy of hydrogenation of cyclohexene (l) at 25° C is –119 kJ mol^{-1}. Use these data to estimate the magnitude of the resonance energy of benzene. **[1996 - 2 Marks]**

48. The polymerisation of ethylene to linear polyethylene is represented by the reaction **[1994 - 2 Marks]**
$$nCH_2 = CH_2 \longrightarrow \left[CH_2 - CH_2 \right]_n$$
where n has a large integral value. Given that the average enthalpies of bond dissociation for C = C and C– C at 298 K are + 590 and + 331 kJ mol^{-1}, respectively, calculate the enthalpy of polymerisation per mole of ethylene at 298 K.

49. Determine the enthalpy change of the reaction.
$C_3H_8(g) + H_2(g) \rightarrow C_2H_6(g) + CH_4(g)$, at 25 ºC, using the given heat of combustion values under standard conditions:

Compound	H_2(g)	CH_4(g)	C_2H_6(g)	C(graphite)
$\Delta H°$ (kJ/mol)	– 285.8	–890.0	–1560.0	–393.5

The standard heat of formation of C_3H_8(g) is –103.8 kJ/ mol. **[1992 - 3 Marks]**

50. A gas mixture of 3.67 litres of ethylene and methane on complete combustion at 25 ºC produces 6.11 litres of CO_2. Find out the amount of heat evolved on burning one litre of the gas mixture. The heats of combustion of ethylene and methane are –1423 and –891 kJ mol^{-1} at 25 ºC.
[1991 - 5 Marks]

51. The standard enthalpy of combustion at 25 ºC of hydrogen, cyclohexene (C_6H_{10}) and cyclohexane (C_6H_{12}) are –241, –3800 and –3920 kJ/mole respectively. Calculate the heat of hydrogenation of cyclohexene. **[1989 - 2 Marks]**

52. An intimate mixture of ferric oxide, Fe_2O_3, and aluminium, Al, is used in solid fuel rockets. Calculate the fuel value per gram and fuel value per cc of the mixture. Heats of formation and densities are as follows : **[1988 - 2 Marks]**
$H_f(Al_2O_3)$ = 399 kcal/mole; $H_f(Fe_2O_3)$ = 199 kcal/mole;
Density of Fe_2O_3 = 5.2 g/cc; Density of Al = 2.7 g/cc.

53. The standard molar heats of formation of ethane, carbon dioxide and liquid water are –21.1, –94.1 and –68.3 kcal respectively. Calculate the standard molar heat of combustion of ethane. **[1986 - 2 Marks]**

54. The bond dissociation energies of gaseous H_2, Cl_2 and HCl are 104, 58 and 103 kcal/mole respectively. Calculate the enthalpy of formation of HCl gas. **[1985 - 2 Marks]**

55. Given the following standard heats of reactions :
(i) heat of formation of water = –68.3 kcal;
(ii) heat of combustion of acetylene = –310.6 kcal;
(iii) heat of combustion of ethylene = –337.2 kcal;
Calculate the heat of reaction for the hydrogenation of acetylene at constant volume (25 ºC). **[1984 - 4 Marks]**

56. The molar heats of combustion of $C_2H_2(g)$, C(graphite) and $H_2(g)$ are 310.62 kcal, 94.05 kcal and 68.32 kcal, respectively. Calculate the standard heat of formation of $C_2H_2(g)$. **[1983 - 2 Marks]**

57. The enthalpy for the following reaction (ΔH^o) at 25 ºC are given below : **[1981 - 2 Marks]**

(i) $\frac{1}{2}H_2(g) + \frac{1}{2}O_2(g) \rightarrow OH(g)$ 10.06 kcal
(ii) $H_2(g) \rightarrow 2H(g)$ 104.18 kcal
(iii) $O_2(g) \rightarrow 2O(g)$ 118.32 kcal

Calculate the O–H bond energy in the hydroxyl radical.

4 Fill in the Blanks

58. The heat content of the products is more than that of the reactants in an .... reaction. **[1993 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

59. Choose the reaction(s) from the following options, for which the standard enthalpy of reaction is equal to the standard enthalpy of formation **[Adv. 2019]**

(a) $\frac{1}{8}S_8(s)+O_2(g)\rightarrow SO_2(g)$

(b) $2H_2(g)+O_2(g)\rightarrow 2H_2O(l)$

(c) $\frac{3}{2}O_2(g)\rightarrow O_3(g)$

(d) $2C(g)+3H_2(g)\rightarrow C_2H_6(g)$

10 Subjective Problems

60. In order to get maximum calorific output, a burner should have an optimum fuel to oxygen ratio which corresponds to 3 times as much oxygen as is required theoretically for complete combustion of the fuel. A burner which has been adjusted for methane as fuel (with *x* litre/hour of CH_4 and $6x$ litre/hour of O_2) is to be readjusted for butane, C_4H_{10}. In order to get the same calorific output, what should be the rate of supply of butane and oxygen ? Assume that losses due to incomplete combustion, *etc*, are the same for both the fuels and the gases behave ideally.
Heats of combustion :

$CH_4 = 809$ kJ / mol; $C_4H_{10} = 2878$ kJ / mol

[1993 - 3 Marks]

61. Using the data (all values are in kcal mol^{-1} at 25 ºC) given below, calculate the bond energy of C–C and C–H bonds. **[1990 - 5 Marks]**

$\Delta H^o_{combustion}$(ethane) = –372.0
$\Delta H^o_{combustion}$(propane) = –530.0
$\Delta H^o_{C(s) \rightarrow C(g)}$ = 172.0
Bond energy of H–H = 104.0
ΔH^o_f of $H_2O(l)$ = –68.0
ΔH^o_f of $CO_2(g)$ = –94.0

Answer Key

Topic-1 : Thermodynamics

1.	(d)	**2.**	(c)	**3.**	(a)	**4.**	(b)	**5.**	(a)	**6.**	(d)	**7.**	(a)	**8.**	(a)	**9.**	(c)	**10.**	(c)
11.	(d)	**12.**	(a)	**13.**	(a)	**14.**	(c)	**15.**	(b)	**16.**	(b)	**17.**	(c)	**18.**	(a)	**19.**	(b)	**20.**	(b)
21.	(a)	**22.**	(b)	**23.**	(c)	**24.**	(c)	**25.**	(a)	**26.**	(c)	**27.**	(8120)	**28.**	(7)	**29.**	(8)		
30.	(10)	**31.**	(2)	**32.**	(400)	**33.**	(55)	**34.**	(38)	**35.**	(274)	**36.**	(2000)	**37.**	(200)	**38.**	(28721)	**39.**	(200)
40.	(1200)	**41.**	(0.31)	**42.**	(300)	**43.**	(4)	**44.**	(4)	**45.**	(1200)	**46.**	(1411)	**47.**	(0)	**48.**	(150)	**49.**	(360)
50.	(620)	**51.**	(1070)	**52.**	(3)	**53.**	(2)	**54.**	(2)	**55.**	(3)	**56.**	(2)	**57.**	(117)	**58.**	(8630)		
59.	(38)	**60.**	(5)	**61.**	(25)	**62.**	(2218)	**63.**	(1380)	**64.**	(935.00)			**65.**	(–13538)				
66.	(–14.6)	**67.**	(557)	**68.**	(115.87)	**69.**	(319.1)	**70.**	(zero)	**71.**	(extensive)			**72.**	(isolated)				
73.	(True)	**74.**	(True)	**75.**	(a,b,d)	**76.**	(a,b,c)	**77.**	(b,c)	**78.**	(b,d)	**79.**	(a,b,c)	**80.**	(a,b,c)	**81.**	(a,c,d)	**82.**	(a,c)
83.	(a,b)	**84.**	(a,d)	**85.**	(b,c,d)	**86.**	(b,d)	**87.**	(b)	**88.**	A- (r, t); B -(p, q, s); C -(p, q, s); D- (p, q, s, t)								
89.	A – p, r, s ; B – r, s ; C – t ; D – p, q, t							**90.**	(a)	**91.**	(d)	**92.**	(c)	**93.**	(b)	**94.**	(a)	**95.**	(a)
96.	(b)																		

Topic-2 : Thermochemistry

1.	(b)	**2.**	(a)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(c)	**7.**	(a)	**8.**	(a)	**9.**	(d)
10.	(b)	**11.**	(c)	**12.**	(b)	**13.**	(b)	**14.**	(a)	**15.**	(c)	**16.**	(c)	**17.**	(a)	**18.**	(b)
19.	(9)	**20.**	(b)	**21.**	(b)	**22.**	(d)	**23.**	(a)	**24.**	(150)	**25.**	(6535)	**26.**	(125)	**27.**	(492)
28.	(56)	**29.**	(800)	**30.**	(4)	**31.**	(278)	**32.**	(499)	**33.**	(610)	**34.**	(173)	**35.**	(925)	**36.**	(90.39)
37.	(54)	**38.**	(2)	**39.**	(718)	**40.**	(101)	**41.**	(309)	**42.**	(189494)			**43.**	(–2035)		
44.	(309.16)			**45.**	(–2091)			**46.**	(–266)	**47.**	(–152)	**48.**	(–72)	**49.**	(–55.7)		
50.	(50.90)	**51.**	(–121)	**52.**	(3.94)	**53.**	(–372.0)			**54.**	(–22)	**55.**	(41.104)			**56.**	(54.20)
57.	(101.19)			**58.**	(endothermic)			**59.**	(a, c)								

5 Thermodynamics

Topic-1: Thermodynamics

1. **(d)** $\Delta U = q + w$

For a adiabatic process, $\boxed{q = 0}$

$\Delta U = w = -p_{ex}\,\Delta v$

For free expansion of ideal gas $p_{ex} = 0$

$\boxed{\Delta U = w = 0}$

As $\Delta U = C\,\Delta T = 0$, $\boxed{\Delta T = 0}$

2. **(c)** $\Delta G = 0$ for reversible reaction

ΔG = –ve for spontaneous reaction

ΔG = +ve for non-spontaneous reaction

Therefore option (c) is not correct.

3. **(a)** Adiabatic boundary does not allow heat exchange thus heat generated in container can't escape out thereby increasing the temperature.

In case of Diathermic container, heat flow can occur to maintain the constant temperature.

4. **(b)** (A) First law is given by

$\Delta U = Q + W$

If we apply reversible work at constant P $\Rightarrow \Delta U = Q - P\Delta V$

(D) $\Delta H = \Delta U + \Delta nRT$

5. **(a)** $H = U + PV$ (By definition)

$\Delta H = \Delta U + \Delta(PV)$ at constant pressure

$\Delta H = \Delta U + P\Delta V$

6. **(d)** In expansion against vacuum,

$P_{ext} = 0 \Rightarrow w = -P_{ext}\,\Delta V = 0$

7. **(a)** A system at higher temperature has greater entropy (randomness). S and ΔS are related with T as:

$$S_T = \int_0^T \frac{nC \cdot dT}{T} \text{ and } \Delta S = \int \frac{dq}{T}$$

Thus, both S and ΔS are function of temperature.

8. **(a)** We know that heat and work are not state functions but $q + w = \Delta U$ is a state function. $H - TS = G$ is also a state function.

9. **(c)** $\Delta U = n\,C_v\,\Delta T = 5 \times 28 \times 100 = 14$ kJ

$\Delta(PV) = nR\,(T_2 - T_1) = 5 \times 8 \times 100 = 4$ kJ

10. **(c)**

	A_2	$\rightleftharpoons$	$2A$
initial	1		0
final	$(1-0.2)$		(2×0.2)

The equilibrium constant,

$$K = \frac{[A]^2}{[A_2]}; \quad K = \frac{[0.4]^2}{[0.8]} = 0.2$$

$$\Delta G^\circ = -RT\ln K = -8.314\ \text{JK}^{-1}\ \text{mol}^{-1} \times 320\ \text{K} \times \ln 0.2$$

$$= 4281\ \text{J/mol}$$

11. **(d)** $A \xrightarrow{q=+5,\ w=8\ \text{J}} B$, $B \longrightarrow A$

$\Delta U_{AB} = q + w = +5 + (-8) = -3$ J

$q = -3,\ \Delta U_{BA} = +3$

$\Delta U_{BA} = q + w$

$\Rightarrow 3 = -3 + w \Rightarrow w = +6$ (work done on the system).

12. **(a)** C(graphite) $\rightarrow$ C(diamond) (Isothermally)

$\Delta_r G^\circ = \Delta G^\circ(\text{diamond}) - \Delta G^\circ(\text{graphite})$

$= 2.9 - 0 = 2.9$ kJ mol^{-1}

Gibbs free energy is the maximum useful work, then

$-\Delta G^\circ = w_{max} = \Delta PV$

$-2.9 \times 10^3 = -\Delta P \times 2 \times 10^{-6}$

$$\Delta P = \frac{2.9\times10^3}{2\times10^{-6}} = 1.45\times10^9\ \text{Pa} = 1.45\times10^9\times10^{-5}\ \text{bar}$$

$= 1.45\times 10^4$ bar = 14500 bar

$P = \Delta P + P_0 = 14500 + 1 = 14501$ bar

13. **(a)** $2H_2O_2(l) \rightleftharpoons 2H_2O(l) + O_2(g)$

$w = -P_{ext}(\Delta V) = -n_{O_2}RT$

$\because$ 100 mole H_2O_2 on decomposition give 50 mole O_2.

$\therefore w = -(50)(8.3)(300) = -124500$ J $= -124.5$ kJ.

14. **(c)** From 1st law of thermodynamics

$q_{sys} = \Delta U - w = 0 - [-P_{ext}.\Delta V]$

$= 3.0$ atm $\times$ (2.0 L – 1.0 L) = 3.0 L-atm

$$\therefore \Delta S_{surr} = \frac{(q_{rev})_{surr}}{T} = -\frac{q_{sys}}{T}$$

$$= -\frac{3.0\times101.3\ \text{J}}{300\ \text{K}} = -1.013\ \text{J/K}$$

15. **(b)** $\Delta S^\circ = S^\circ_{CO_2} + 2\times S^\circ_{H_2O} - \left(S^\circ_{CH_4} + 2\times S^\circ_{O_2}\right)$

$= (213.6 + 2 \times 69.9) - (186.2 + 2 \times 205.2)$

$= -242.8$ J K^{-1} mol^{-1}.

16. **(b)** Given conditions are boiling conditions for water due to which system is in equilibrium.

$H_2O(l) \rightleftharpoons H_2O(g)$

$\Delta S_{total} = 0$

$\Delta S_{system} + \Delta S_{surroundings} = 0$

$\Delta S_{system} = -\Delta S_{surroundings}$

For process, $\Delta S_{system} > 0$

$\Delta S_{surroundings} < 0$

17. **(c)** For isothermal reversible expansion.

$$w = -nRT\ln\frac{V_2}{V_1}$$

Trick: Either of option (b) or (c) must be not correct.

18. **(a)** Since, liquid is passing into gaseous phase so entropy will increase and at 373 K during the phase transformation, it remains at equilibrium. So, $\Delta G = 0$.

19. **(b)** $A \rightleftharpoons B$

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$, $\Delta G^\circ = -2.303\, RT\log_{10} K$

$-2.303\, RT \log_{10} K = \Delta H^\circ - T\Delta S^\circ$

$\Rightarrow \quad 2.303\, RT \log_{10} K = T\Delta S^\circ - \Delta H^\circ$

$$\log_{10} K = \frac{T\Delta S^\circ - \Delta H^\circ}{2.303RT} = \frac{298\times10+54.07\times1000}{2.303\times8.314\times298} = 10$$

20. **(b)** In general, the molar heat capacity for any process is given by

$C = C_v + \frac{R}{1-\gamma}$, when PV^γ = constant

Here $\frac{P}{V} = 1$, *i.e.* PV^{-1} = constant $\Rightarrow \gamma = -1$

For monoatomic gas, $C_v = \frac{3}{2}R$

$\therefore C = \frac{3}{2}R + \frac{R}{1-(-1)} = \frac{3}{2}R + \frac{R}{2} = \frac{4R}{2} = 2R$.

21. **(a)** $TV^{\gamma-1}$ = Constant ($\because$ change is adiabatic)

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

For monoatomic gas $\gamma = \frac{5}{3}$

$\therefore T_1V_1^{2/3} = T_2V_2^{2/3} \Rightarrow T(1)^{2/3} = T_2(2)^{2/3}$

$T_2 = \frac{T}{2^{(2/3)}}$

22. **(b)** $\Delta S = \frac{\Delta Q_{rev.}}{T}$; $75 = \frac{30\times10^3}{T}$ $\therefore T = 400$ K

23. **(c)** $\Delta H = nC_p\,\Delta T$ solution; since $\Delta T = 0$ so, $\Delta H = 0$

24. **(c)**

$\Delta H = \Delta U + P_2V_2 - P_1V_1$ Given, $\Delta U = 30.0$ L atm

$P_1 = 2.0$ atm, $V_1 = 3.0$ L, $T_1 = 95$ K

$P_2 = 4.0$ atm, $V_2 = 5.0$ L, $T_2 = 245$ K

$\Delta H = \Delta U + P_2V_2 - P_1V_1$

$= 30 + (4\times5) - (2\times3) = 30 + 20 - 6 = 44$ L atm.

25. **(a)** Work is not a state function because it depends upon the path followed.

26. **(c)** In a reversible process, the driving and the opposite forces are nearly equal, hence the system and the surroundings always remain in equilibrium with each other.

27. **(8120)** $X \to Y$ is an isothermal process an ideal gas:

$\Delta H = 0, \Delta U = 0$

$Y \to Z$ is an isochoric process $\Rightarrow \Delta V = 0$

$\therefore \quad W = 0$

$\Delta U = nC_{v\cdot m}(T_2 - T_1) = 5\times12\,(415-335) = 4800$ J

$\Delta H = \Delta U + \Delta(PV) = \Delta U + nR\Delta T$

$= 4800 + 5\times8.3\times(415-335) = 8120$ J

28. **(7)** For $A \to B$ (Reversible adiabatic)

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\Rightarrow 600(V_1)^{2/3} = 60(V_2)^{2/3} \quad \left(\because \gamma = \frac{5}{3}\right)$

$\Rightarrow 10 = \left(\frac{V_2}{10}\right)^{2/3}$

$V_{total} = E_{AB} + q_{BC} = 0 + q_{BC} = q_{BC}$

$q_{BC} = RT_2 \ln 10 \Rightarrow q_{BC} = 60\,R \ln 10 = 60\,R \ln \frac{V_3}{V_2}$

[$\because$ B $\to$ C is reversible isothermal]

$\Rightarrow 60\,R \ln 10 = 60\,R \ln\left(\frac{V_3}{10^{5/2}}\right)$

$\Rightarrow \log 10 = \log V_3 - \frac{5}{2} \Rightarrow \log V_3 = \frac{7}{2} \Rightarrow 2\log V_3 = 7$

29. **(8)**

At T_1 K:	$A(g) \rightleftharpoons P(g)$
t = 0	6
At eq, T_1K	$6 - x$ $x = 4$ (from plot)
At eq, T_2K	$6 - y$ $y = 2$ (from plot)

$\Rightarrow$ At T_1 K: $K_{P_1} = \frac{4}{2} = 2 \Rightarrow$ At T_2 K: $K_{P_2} = \frac{2}{4} = \frac{1}{2}$

Now, $\Delta G_2^\circ = -RT_2 \ln K_{P_2} = -RT_2 \ln\frac{1}{2} = RT_2 \ln 2$

$\Delta G_1^\circ = -RT_1 \ln K_{P_1} = RT_1 \ln 2$

Given : $\Delta G_2^\circ - \Delta G_1^\circ = RT_2 \ln 2 + RT_1 \ln 2$

$= RT_2 \ln 2 + 2\,RT_2 \ln 2$

$= 3RT_2 \ln 2 = RT_2 \ln x\ (T_1 = 2\,T_2) \Rightarrow x = 2^3 = 8$

30. **(10)**

Process (I) $\Rightarrow$ (Adiabatic reversible)

$\frac{\Delta U}{R} = 450 - 2250$

$\Delta U = -1800R$

$W_I = \Delta U = -1800\,R$

Process (II) $\Rightarrow$ (Reversible isothermal process)

$T_1 = 900$ K

Calculation of T_2 after reversible adiabatic process

$\Delta U = nC_v dT$

$\Rightarrow \quad -1800\,R = 1\times\frac{5}{2}R(T_2 - 900)$

$T_2 = 180$ K

$W_{II} = -nRT_2 \ln\frac{V_3}{V_2} = W_I$

$\Rightarrow \quad -1\times R\times180 \ln\frac{V_3}{V_2} = -1800R \Rightarrow \ln\frac{V_3}{V_2} = 10$

31. **(2)** $w_d = \left(-4\times\frac{3}{2}\right) + (-1\times1) + \left(-\frac{1}{2}\times\frac{5}{2}\right) = -\left(6+1+\frac{5}{4}\right)$

$w_d = -\frac{33}{4}$ L atm

$w_s = -2.303\,RT \log\frac{5.5}{1/2} = -2.303\,PV \log 11$

$w_s = -4.606\times1.04 = -4.8$ L atm

$$\frac{w_d}{w_s} = \frac{-\frac{33}{4}}{-4.8} = 1.72 \simeq 2.0$$

32. **(400)** At equilibrium $\Delta G_{PT} = 0$ and $\Delta G = \Delta H - T\Delta S$

$\Rightarrow \quad \Delta H_{vap} = T\Delta S_{vap} \Rightarrow 30\times1000 = T\times75 \Rightarrow T = 400$K

33. **(55)** Work done for reversible isothermal expansion is given as:

$$w = -nRT\ln\left(\frac{V_2}{V_1}\right) = -1 \times 0.8206 \times 291.15 \ln\left(\frac{100}{10}\right)$$

$= -55.0128$ L atm
Work done by system $\approx$ 55 L atm

34. **(38)** $H_2O(\ell) \rightleftharpoons H_2O(g)$ $\quad \Delta H^o_{vap} = 40.79$ kJ/mole

$$\Delta H^o_{vap} = \Delta U^o_{vap} + \Delta n_g RT$$

$$40.79 = \Delta U^o_{vap} + \frac{1 \times 8.3 \times 373.15}{1000}$$

$$\Delta U^o_{vap} = 40.70 - 3.0971 = 37.6929$$

$$\Delta U^o_{vap} \simeq 38$$

35. **(274)** $\Delta U = q + w$ where $(q = 0)$
$nC_v\Delta T = -P_{ext}(V_2 - V_1)$
$V_2 = 2V_1$ (given)

$$\frac{nRT_2}{P_2} = \frac{2nRT_1}{P_1}$$

$$P_1 = 5, T_1 = 298, \; P_2 = \frac{5T_2}{2 \times 298}$$

$$n\frac{5}{2}R(T_2 - T_1) = -1\left(\frac{nRT_2}{P_1} - \frac{nRT_1}{P_1}\right)$$

Put $T_1 = 298$ and $P_2 = \dfrac{5T_2}{2 \times 298}$

We get $T_2 = 274.16$ K
$T_2 \approx 274$ K

36. **(2000)** Applying Gibbs Helmhaltz equation $\Delta G = \Delta H - T\Delta S$
For reaction to be spontaneous $\Delta G = -$ ve
For limiting case $\Delta G = 0$
$\Delta H = T\Delta S$

$$T = \frac{\Delta H}{\Delta S} = \frac{400 \text{kJ/mol}}{0.2 \text{kJ/mol} - \text{K}}$$

$= 2000$ K
above 2000 K reaction become spontaneous.

37. **(200)**
For isothermal irreversible process, $\Delta U = 0 \Rightarrow q = -w$
$-w = -[-P_{ext}(V_2 - V_1)]$
$q = 5(20 - 60) = -200$ atm-L.

38. **(28721)**
In Isothermal and reversible expansion the work done is negative as the work is done by the system.

$$\therefore \quad W = -2.303 \; nRT \; \log\left(\frac{V_2}{V_1}\right)$$

$$= -2.303 \times 5 \times 8.314 \times 300 \log\left(\frac{100}{10}\right) = -28720.7 \approx -28721 \text{ J.}$$

39. **(200)** Work done is given by area enclosed by triangle ABC.

$$\text{Area of } \Delta ABC = \frac{1}{2} \times AC \times AB$$

$$= \frac{1}{2}(30 \text{ kPa} - 10 \text{kPa}) \times (30\text{dm}^3 - 10\text{dm}^3)$$

$$= \frac{1}{2}(20 \text{ kPa})(20\text{dm}^3) = 200 \text{ kPa dm}^3$$

$$= 200\left(\frac{1}{100}\text{bar}\right)(L) = 2 \text{ L bar} \qquad \left\{\begin{aligned} &1 \text{ kPa} = \frac{1}{100}\text{bar} \\ &1 \text{ dm}^3 = 1 \text{ L} \end{aligned}\right\}$$

$= 200$ J $\qquad$ {1 L bar = 100 J}

40. **(1200)**
Using, first law of thermodynamics,
$\Delta U = D + W$, $\Delta U = 0$: Process is isothermal
$Q = -W$, $W = -P_{ext}\Delta V$ (Irreversible)
$= -80 \times 10^3 (45 - 30) \times 10^{-3} = -1200$ J

41. **(0.31)**

$$\alpha \to \beta \text{ and } S^0_{\beta(600)} - S^0_{\alpha(600)} = 6 - 5 = 1 \text{ (from graph)}$$

$$S^0_{\alpha(600)} - S^0_{\alpha(300)} + C_{P(\alpha)} \ln\frac{600}{300}$$

$$S^0_{\beta(600)} = S^0_{\beta(300)} + C_{P(\beta)} \ln\frac{600}{300}$$

$$S^0_{\beta(600)} - S^0_{\alpha(600)} = S^0_{\beta(300)} - S^0_{\alpha(300)} + C_{P(\beta)} - C_{P(\alpha)} \ln 2$$

$$6 - 5 = S^0_{\beta(300)} - S^0_{\alpha(300)} + [1 \times \ln 2]$$

$$1 = S^0_{\beta(300)} - S^0_{\alpha(300)} + 0.69$$

$$\text{So } S^0_{\beta(300)} - S^0_{\alpha(300)} = 0.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

42. **(300)** As the phase transition temperature is 600 K
So at 600 K ; $\Delta G°_{rxn} = 0$
So $\Delta H°_{\text{reaction (600)}} = T\,\Delta S°_{\text{reaction (600)}}$
$\Delta H°_{(600)} = 600 \times 1 = 600$ Joule/mole
So, $\Delta H_{600} - \Delta H_{300} = \Delta C_P (T_2 - T_1)$
$= 1 \times 300 = 300$ J mol^{-1}
$\Rightarrow \Delta H_{300} = \Delta H_{600} - 300 = 600 - 300 = 300$ J mol^{-1}

43. **(4)** Extensive $\Rightarrow$ Mole, Volume, Gibbs free energy.
Intensive $\Rightarrow$ Molar mass, Molar heat capacity, Molarity, $E^{\ominus}$ cell.

44. **(4)** A $\to$ Endothermic (Atomisation)
B $\to$ Endothermic (Atomisation)
C $\to$ Endothermic (Vapourisation)
D $\to$ Exothermic (Combustion)
E $\to$ Endothermic (Dissolution)

45. **(1200)** Power of heater = 60 W
Total energy emitted = 60 × 100 = 6000 J
Heat capacity × temp difference = 6000

$$\text{Heat capacity} = \frac{6000}{5} = 1200 \text{JK}^{-1}$$

46. **(1411)** $C_2H_4(g) + 3O_2(g) \to 2CO_2(g) + 2H_2O(\ell)$
$\Delta U = -1406$ kJ mol^{-1}, T = 300 K
$\Delta H = \Delta U + \Delta n_g RT \Rightarrow \Delta n_g = \Delta n_p - \Delta n_R = 2 - 4 = -2$
$\Delta H = -1406 + (-2) \times 8.3 \times 300 = -1406 - 4.98$
$= -1410.98$ kJ mol^{-1} ≈ -1411 kJ mol^{-1}
$\Delta G = \Delta H - T\Delta S$, at equilibrium $\Delta G = 0$
$\therefore \Delta H = T\Delta S = -1411$ kJ mol^{-1}

47. **(0)** For an, ideal gas internal energy is a function of temperature and for isothermal process temperature is constant $\therefore$ The change in internal energy is zero.

48. **(150)** $q = 0, \Delta U = w$
$1 \times 20 \times [T_2 - 300] = -3000$
$T_2 - 300 = -150 \Rightarrow T_2 = 150$ K.

49. **(360)**
$C_6H_{12}O_6(s) + 6O_2 \rightarrow 6CO_2(g) + 6H_2O(l)$
50% energy used to convert $H_2O(l)$ into $H_2O(g)$

$$= \frac{1800}{2} = 900 \text{kJ}$$

The energy required to evaporate 1 mole of water = 45 kJ

$$\Rightarrow \quad 900 = n_{H_2O} \times 45 \Rightarrow n_{H_2O} = \frac{900}{45} = 20 \text{ mole}$$

$W_{H_2O} = 20 \times 18 = 360$ g

50. **(620)** $1 \rightarrow 2 \Rightarrow$ Isobaric process
$2 \rightarrow 3 \Rightarrow$ Isochoric process
$3 \rightarrow 1 \Rightarrow$ Isothermal process

$$W = W_{1\rightarrow2} + W_{2\rightarrow3} + W_{3\rightarrow1}$$

$$= \left(-P(V_2 - V_1) + 0 + \left[-P_1V_1 \ln\left(\frac{V_2}{V_1}\right)\right]\right)$$

$$= \left[-1 \times (40 - 20) + 0 + \left[-1 \times 20 \ln\left(\frac{20}{40}\right)\right]\right]$$

$= -20 + 20 \ln 2 = -20 + 20 \times 2.3 \times 0.3$
$= -6.2$ Lbar (1 bar = 100 J) $\Rightarrow |W| = 6.2$ L bar = 620 J

51 **(1070)** $\Delta S = \frac{\Delta H}{T_{mp}}$

$$28.4 = \frac{30.4 \times 1000}{T_{mp}}$$

$T_{mp} = 1070.422$ K.

52. **(3)** $\Delta S_{System} = nR\ln\left(\frac{V_2}{V_1}\right) = 1 \times 8.314 \ln\left(\frac{3}{2}\right)$

$\Delta S_{System} = 3.37$
$\Delta S_{Surr.} = 3.37$

53. **(2)** $\Delta G = \Delta H - T\Delta S$
A : ΔG (J mol^{-1}) $= -25 \times 10^3 + 80 \times 300 \Rightarrow \Delta G = -ve$
B : ΔG (J mol^{-1}) $= -22 \times 10^3 - 40 \times 300 \Rightarrow \Delta G = -ve$
C : ΔG (J mol^{-1}) $= 25 \times 10^3 + 300 \times 50 \Rightarrow \Delta G = +ve$
D : ΔG (J mol^{-1}) $= 22 \times 10^3 - 20 \times 300 \Rightarrow \Delta G = +ve$
$\Rightarrow$ Processes C and D are non-spontaneous.

54. **(2)** For, Spontaneous process $\Delta G < 0$
For, Equilibrium $\Delta G = 0$
For, Non-spontaneous process $\Delta G > 0$
Statement B and C are correct.

55. **(3)** State Variable is an independent variable of a state function.
Internal energy, volume and enthalpy are state variable.

56. **(2)** $C_{m,p} - C_{m,v} = R \Rightarrow 20.785 - C_{m,v} = 8.314$
$\Rightarrow C_{m,v} = 12.471$ J K^{-1} mol^{-1}
$\Delta U = n\, C_{m,v}\, \Delta T$

$$\Rightarrow n = \frac{5000}{12.471 \times (500 - 300)} \Rightarrow n = 2$$

57. **(117)** Molar mass of NH_3 = 17 g/mol

moles of NH_3 given $= \frac{17}{17} = 1$ mole.

For 1 mole enthalpy change = 23.4 kJ
5 moles enthalpy change = 23.4 × 5 = 117 kJ

58. **(8630)** $n = 5$ mol; $T = 300$ K; $V_1 = 10$ L; $V_2 = 20$ L
Work done in isothermal condition.

$$w_{rev} = -nRT \ln \frac{V_2}{V_1}$$

$$= -5 \times 8.3 \times 300 \ln \frac{20}{10} = -8630.38 \text{ J}$$

59. **(38)** $H_2O\,(l) \rightarrow H_2O(g)$

$$\Delta n_g = \sum np - \sum nR = 1 - 0 = 1$$

$\Delta H = \Delta U + \Delta n_g RT$
$\Delta U = \Delta H - \Delta n_g RT$

$$= 41.1 - \frac{1 \times 8.31 \times 373}{1000} \text{kJ/mol} = 38 \text{ kJ/mol}$$

60. **(5)** $\Delta G = \Delta H - T\Delta S$
$\Delta G = -57.8 - 298 \times (-176 \times 10^{-3}) = -5$ kJ mol^{-1}

61. **(25)** $\Delta G^\circ = -nFE^\circ = \Delta H^\circ - T\Delta S^\circ$

$$\Delta S^\circ = \frac{\Delta H^\circ + nFE^\circ}{T}$$

$$= \frac{(-825.2 \times 10^3) + (2 \times 96487 \times 4.315)}{298}$$

$= 25.11$ JK^{-1}mol^{-1} ≈ 25 JK^{-1}mol^{-1}

62. **(2218)** $Fe + 2HCl \longrightarrow FeCl_2 + H_2$

No. of moles of Fe $= \frac{50}{55.85}$ moles

No. of moles of H_2 produced $= \frac{50}{55.85}$ moles

Word done $= -P_{ext} \cdot \Delta V = -\Delta n_g RT$

$$= \frac{50}{55.85} \times 8.314 \times 298 \approx 2218 \text{ J}$$

63. **(1380)** $\Delta G^\circ = RT \ln K_p$

$$= -R(300)\ln(10)^2 = -R(300 \times 2 \times 2.3)$$

$\Delta G^\circ = -1380R.$

64. **(935.00)** $SnO_2(s) + C(s) \longrightarrow Sn(s) + CO_2(g)$
$\Delta_r H^o = [-394] - [-581] = 187$ kJ/mole $= 187 \times 10^3$ J/mol
$\Delta_r S^o = [52 + 210] - [56 + 6] = 200$ JK^{-1} mol^{-1}

$$T = \frac{\Delta_r H^o}{\Delta_r S^o} = \frac{187 \times 10^3}{200} = 935 \text{ K}$$

65. **(– 13538)**
From $\Delta H^\circ = \Delta U^\circ + \Delta n_g RT$

$\Delta H^\circ = -20 \times 1000 - 1 \times 8.314$ J / mol.K × 298 K
$= -22477.57$ J
$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = -22477.57 - (298 \times (-30))$
$= -13538$ J

66. **(– 14.6)**

(i) $2Cu(s)+\frac{1}{2}O_2(g)\longrightarrow Cu_2O(s): \Delta G^\circ = -78 \text{ kJ/mol}$

(ii) $H_2(g)+\frac{1}{2}O_2(g)\longrightarrow H_2O(g),\ \Delta G^\circ = -178 \text{ kJ/mol}$

(i) – (ii) then

$2Cu(s)+H_2O(g)\longrightarrow Cu_2O(s)+H_2(g)$

$\Delta G^\circ = -78 + 178 = 100$ kJ/mol $= 10^5$ J/mol

Now for the above reaction

$$\Delta G = \Delta G^\circ + RT\ln\left(\frac{P_{H_2}}{P_{H_2O}}\right)$$

To prevent the above reaction: $\Delta G \geq 0$

$$\Delta G^\circ + RT\ln\left(\frac{P_{H_2}}{P_{H_2O}}\right) \geq 0$$

$$\Rightarrow \quad 10^5 + 8\times 1250\ln\left(\frac{P_{H_2}}{P_{H_2O}}\right) \geq 0$$

$$\ln P_{H_2} \geq -10 + \ln P_{H_2O}$$

Now, $P_{H_2O} = X_{H_2O}\times P_{total} = 0.01\times 1 = 10^{-2}$

$\Rightarrow \quad \ln P_{H_2} \geq -10 - 2\ln 10$

$\Rightarrow \quad \ln P_{H_2} \geq -14.6$ (Given $\ln 10 = 2.3$)

$\therefore \quad$ Minimum $\ln P_{H_2} = -14.6$

67. **(557)**

$\Delta H = \Delta U + \Delta(PV) = \Delta U + V\Delta P \qquad (\because \Delta V = 0)$

or $\Delta U = \Delta H - V\Delta P = -560 - [1(40-70)\times 0.1]$

$= -560 + 3 = -557$ kJ mol^{-1}

So, the magnitude is **557** kJ mol^{-1}.

68. **(115.87)** $\quad C_p - C_v = R$

$\Rightarrow \quad C_p = 12.48 + 8.31 = 20.794$ J mol^{-1} K^{-1}

$$\gamma = \frac{C_p}{C_v} = \frac{20.794}{12.48} = 1.67$$

For a reversible adiabatic process,

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\Rightarrow T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = 300\left(\frac{1.25}{2.50}\right)^{1.67-1}$$

$\Rightarrow T_2 = 188.55$ K

Number of moles of gas,

$$n = \frac{PV_1}{RT_1} \Rightarrow n = \frac{1\times 1.25}{0.0821\times 300} = 0.05$$

Enthalpy change at constant pressure,

$\Delta H = n\cdot C_p\cdot \Delta T$

$= 0.05 \times 20.794 \times (300 - 188.55)$

$= 115.87$ Joule.

69. **(319.1)**

100 g of glucose = 1560 kJ

Energy utilised in body $= \frac{50}{100}\times 1560 = 780$ kJ

Energy to be given out = 1560 – 780 = 780 kJ

Enthalpy of evaporation of water = 44 kJ/mole = 44 kJ/18 g of water [1 mole H_2O = 18 g water]

Hence, amount water to be perspired to avoid storage of energy $= \frac{18}{44}\times 780 =$ **319.1 g**

70. **zero;** In a closed vessel, $\Delta V = 0$

71. **extensive** (because its value depends on quantity of substance)

72. **isolated**

73. **True;**

Heat capacity	**Monoatomic**	**Diatomic**
C_v	$3R/2$	$5R/2$
C_p	$5R/2$	$7R/2$

Thus, the heat capacity of diatomic gas is higher than that of a monoatomic gas.

74. **True;** It only tells that if the heat gained by one end would be exactly equal to heat lost by the other. It does not predict the direction.

75. **(a, b, d)** From state I to II (Reversible isothermal expansion):

$T \to$ constant, $\Delta V \to$ +ve, $\Delta S \to$ +ve, $\Delta H \to 0$

$\Delta P \to$ –ve

From state II to III (Reversible adiabatic expansion) :

$q \to 0$, $\Delta V \to$ +ve, $\Delta S \to$ constant

$\Delta H \to$ –ve, $\Delta P \to$ –ve, $\Delta T \to$ –ve

$\therefore$ Plots (a), (b), (d) are correct while (c) is wrong as from state II to III, H is decreasing.

76. **(a, b, c)**

P-V work done is applicable for reversible isobaric as well as isothermal and adiabatic process.

$w = -\int P_{ext}\cdot dV$

For van der Waals equation,

$$P_{ext} = P = \left(\frac{RT}{v-b} - \frac{a}{v^2}\right)$$

$$w = -\int dv\left(\frac{RT}{v-b} - \frac{a}{v^2}\right) \qquad \text{..... (i)}$$

Equation (i) is not applicable to irreversible process. Therefore work done is calculated assuming pressure is constant throughout the process.

77. **(b, c)**

A– C $\Rightarrow$ isochoric process

A– B $\Rightarrow$ isothermal process

B– C $\Rightarrow$ isobaric process

(a) $q_{AC} = \Delta U_{AC} = nC_{V,m}(T_2 - T_1) = \Delta U_{BC}$

$W_{AB} = -nRT_1\ln\left(\frac{V_2}{V_1}\right)$ (pressure is not constant)

(b) $W_{BC} = -P_2(V_1 - V_2) = P_2(V_2 - V_1)$

$q_{BC} = \Delta H_{BC} = nC_{P,m}(T_2 - T_1) = \Delta H_{AC}$

(c) $\Delta H_{CA} = nC_{P,m}(T_1 - T_2)$

(d) $\Delta U_{CA} = nC_{V,m}(T_1 - T_2)$

$\Delta H_{CA} < \Delta U_{CA}$ since both are negative ($T_1 < T_2$) and $C_{p,m} > C_{V,m}$

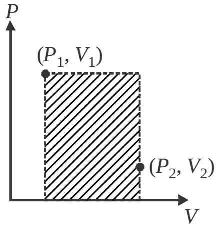
78. **(b, d)** $\Delta S_{surr} = \dfrac{\Delta H}{T_{surr}}$

For endothermic reaction, if T_{surr} increases, unfavourable change in entropy of the surroundings decreases.

For exothermic reaction, if T_{surr} increases, favourable change in entropy of the surroundings decreases.

79. **(a, b, c)**

(a)



During irreversible compression, maximum work is done on the gas (corresponding to shaded area) when $P_1 = P_2$

(d) When $T_1 = T_2 \Rightarrow \Delta U = nC_V\Delta T = 0$

In reversible adiabatic expansion, $T_2 < T_1$.

$\therefore$ ΔT = –ve and also ΔU = –ve

(b) In free expansion, $P_{ext} = 0$, $\therefore W = 0$

From I[st] law of thermodynamics, $\Delta U = q + W$ $\therefore \Delta U = q$

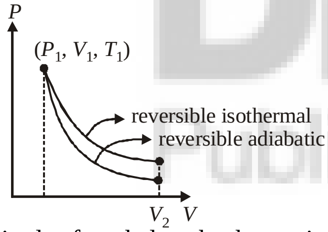
If expansion is carried out isothermally, $\Delta U = 0$

Hence $q = 0$, $\therefore$ It is adiabatic process.

If carried out adiabatically ($q = 0$), $\therefore \Delta U = 0$

$\therefore$ It is an isothermal process.

(c) During adiabatic expansion, the final temperature is less than the initial temperature. Therefore, the final volume in adiabatic expansion will also be less than the final volume in isothermal expansion. This can be graphically shown as:



The magnitude of work done by the gas is equal to the area under the curve. As seen from the figure, the area under curve in reversible isothermal is more. Hence, the magnitude of work done is lesser in adiabatic reversible expansion as compared to the corresponding work in isothermal expansion.

80. **(a, b, c)** Since the vessel is thermally insulated, $q = 0$

Further since, $P_{ext} = 0$, so $w = 0$, hence $\Delta U = 0$

Therefore, $\Delta T = 0$, $T_2 = T_1$, and $P_2V_2 = P_1V_1$

However, the process is adiabatic irreversible, so we can't apply $P_2V_2^{\gamma} = P_1V_1^{\gamma}$.

81. **(a, c, d)** $T_1 = T_2$ because process is isothermal.

Work done in adiabatic process is less than in isothermal process because area covered by isothermal curve is more than the area covered by the adiabatic curve.

In adiabatic process expansion occurs by using internal energy, hence, it decreases while in isothermal process temperature remains constant, that's why no change in internal energy.

82. **(a, c)** $\Delta S_{X\to Z} = \Delta S_{X\to Y} + \Delta S_{Y\to Z}$ [Entropy is a state function, hence additive]

$w_{X\to Y\to Z} = w_{X\to Y}$ [Work done in $Y \to Z$ is zero because it is an isochoric process].

83. **(a, b)** Mass independent properties (molar conductivity and electromotive force) are intensive properties. Resistance and heat capacity are mass dependent, hence extensive properties.

84. **(a, d)** Internal energy and molar enthalpy are state functions. Work (reversible or irreversible) is a path function.

85. **(b, c, d)** All combustion reactions are exothermic in nature.

(b) Decomposition reactions are endothermic in nature.

(c) $C_2H_6(g) \longrightarrow C_2H_4(g) + H_2(g)$

More stable compound is converting into less stable compound. Thus, reaction is endothermic.

(d) Graphite $\longrightarrow$ Diamond

More stable allotrope is converting into less stable allotrope. Thus, reaction is endothermic.

86. **(b, d)** Properties independent of mass are intensive properties. Hence, (b) and (d) which are independent of mass are the obvious choices.

87. **(b)** (A) For a spontaneous process Gibb's free energy value is negative at constant temperature and pressure i.e. $\Delta G_{T,P} < 0$.

(B) In isobaric pressure, pressure remains constant i.e., $\Delta P = 0$ while in isothermal process, temperature remains constant i.e., $\Delta T = 0$.

(C) $\Delta H_{reaction}$ = (Σ bond energies of reactants) – (Σ bond energies of products)

(D) In exothermic process, energy is released.

So, the value of enthalpy is negative i.e., $\Delta H < 0$.

88. **A- (r, t); B -(p, q, s); C -(p, q, s); D- (p, q, s, t)**

(A) $\to$ r, t

$H_2O\,(l) \rightleftharpoons H_2O\,(s)$

It is at equilibrium at 273 K and 1 atm.

So, ΔS_{sys} is negative.

As it is equilibrium process, so $\Delta G = 0$.

(B) $\to$ p, q, s

Expansion of 1 mole of an ideal gas in vacuum under isolated condition

Hence, $w = 0$

and $q_p = C_p dT$ ($\because dT = 0$) $\Rightarrow q = 0$

$\Delta U = C_v dT$ ($\because dT = 0$) $\Delta U = 0$

(C) $\to$ p, q, s

Mixing of two ideal gases at constant temperature

Hence, $\Delta T = 0$

$\therefore$ $q = 0$; $\Delta U = 0$ also $w = 0$ ($\Delta U = q + w$)

(D) $\to$ p, q, s, t

Reversible heating and cooling of gas follows same path; also initial and final position is same.

Hence, $\left.\begin{matrix} q = 0 \\ w = 0 \end{matrix}\right\}$ Path same

$\left.\begin{matrix} \Delta U = 0 \\ \Delta G = 0 \end{matrix}\right\}$ State function

89. **A – p, r, s ; B – r, s ; C – t ; D – p, q, t**

(A) $CO_2(s) \to CO_2(g)$

It is phase transition. The process is endothermic (sublimation). Gas is produced, so entropy increases.

(B) On heating $CaCO_3$ decomposes. So, process is endothermic.

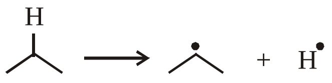
The entropy increases as gaseous product is formed.

(C) $2H\bullet \to H_2(g)$

Entropy decreases as number of gaseous particles decreases.

(D) The transition between different allotropes is considered as phase transtion. White and red P are allotropes.
Due to polymeric nature of red P, its entropy is less than that of white P. Red P is more stable than white. So ΔH is –ve.

90. **(a)** Stability of free radical $\propto \dfrac{1}{\text{Bond energy}}$

(P) 

2° free radical

(Q) $Ph - CH_2 - H \longrightarrow Ph - \dot{C}H_2 + H^{\bullet}$

Most stable due to resonance

(R) $CH_2 = CH - H \longrightarrow CH_2 = \dot{C}H + H^{\bullet}$

Less stable

(S) $CH \equiv C - H \longrightarrow CH \equiv \dot{C} + H^{\bullet}$

More % s-Character decreases stability of free radical

Q require least BDE and S required maximum BDE.
So, order of BDE is Q < P < R < S.

91. **(d)**

$$CH_4 \longrightarrow {}^{\bullet}CH_3 + H^{\bullet} \qquad \Delta H = 105 \text{ kcal/mol}$$
$$Cl_2 \longrightarrow Cl^{\bullet} + Cl^{\bullet} \qquad \Delta H = 58 \text{ kcal/mol}$$
$$Cl^{\bullet} + {}^{\bullet}CH_3 \longrightarrow CH_3 - Cl \qquad \Delta H = -85 \text{ kcal/mol}$$
$$Cl^{\bullet} + H^{\bullet} \longrightarrow HCl \qquad \Delta H = -103 \text{ kcal/mol}$$
$$CH_4 + Cl_2 \longrightarrow CH_3 - Cl + HCl \qquad \Delta H = -25 \text{ kcal/mol}$$

Initiation step is endothermic, hence option (a) is wrong.
Propagation step involving ${}^{\bullet}CH_3$ formation is endothermic, hence option (b) is wrong.
Propagation step involving CH_3Cl formation is exothermic, hence option (c) is wrong. So,overall raction is exothermic with $\Delta H^o = -25$kcal/mol, hence option (d) is correct.

92. **(c)** $K \to L \Rightarrow V$ increasing at constant P
Hence, T increases (Heating).
$L \to M \Rightarrow P$ decreasing at constant V
Hence, T decreases (Cooling),
$M \to N \Rightarrow V$ decreasing at constant P
Hence, T decreases (Cooling),
$N \to K \Rightarrow P$ increasing at constant V
Hence, T increases (Heating).

93. **(b)** L to M and N to K, both are having constant volume, therefore, these processes are isochoric.

94. **(a)** Statement I is true because it is not possible to convert whole of heat to work. For such a conversion, we need an efficiency of 100% but so far, we have not been able to get such a machine (carnot engine).
Statement II is true because it is not possible to convert the whole of heat absorbed from a reservoir into work. Some of the heat is always given to the sink.

95. **(a)** $\Delta G = \Delta H - T\Delta S \quad \because \quad \Delta S_g > \Delta S_l > \Delta S_s$
$\therefore$ on melting the entropy increases and ΔG becomes more negative and hence it becomes easier to reduce metal

96. **(b)** **Assertion :** For isothermal expansion,
$\Delta T = 0 \Rightarrow \Delta U = 0$
For an ideal gas, work done against vacuum is zero, *i.e.* $W = 0$
Now, $\Delta Q = \Delta U + W \Rightarrow \Delta Q = 0$. Thus, assertion is correct.
Reason : By kinetic theory of ideal gases, the volume occupied by the molecules of an ideal gas is zero.
Thus, reason is correct, but it is not the correct explanation of the assertion.

97. For adiabatic process, $W = P(V_2 - V_1)$
Here $P_1 = 1$ bar, $P_2 = 100$ bar, $V_1 = 100$ mL, $V_2 = 99$ mL; For adiabatic process, $q = 0 \ \therefore \ \Delta U = W$
$\Delta U = q + W = q - P(V_2 - V_1)$
$= 0 - 100\,(99 - 100) = \mathbf{100\ bar\ mL}$
$\Delta H = \Delta U + \Delta(PV) = \Delta U + (P_2V_2 - P_1V_1)$
$= 100 + [(100 \times 99) - (1 \times 100)]$
$= 100 + (9900 - 100) = \mathbf{9900\ bar\ mL}$

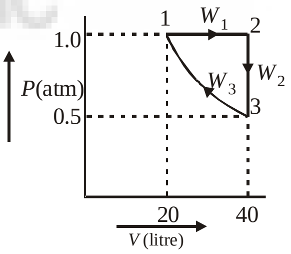
98. (i) $N_2O_4(g) \rightleftharpoons 2NO_2(g)$
Initially $p_{N_2O_4} = p_{NO_2} = 10$

Reaction quotient $= \dfrac{(p_{NO_2})^2}{p_{N_2O_4}} = \dfrac{100}{10} = 10$

$\Delta G^\circ = 2\Delta G^\circ_{f(NO_2)} - \Delta G^\circ_{f(N_2O_4)} = 100 - 100 = 0$
$\Delta G = \Delta G^o - 2.303\,RT \log K_p$
$= 0 - 2.303 \times 298 \ \log 10 = \mathbf{-56.0304\ L\ atm.}$
(ii) The negative value of ΔG indicates that the reaction is spontaneous and will lie in the right direction, (forward).

99. Helium molecule is monoatomic so it has just three degrees of freedom corresponding to the three translational motion at all temperature and hence C_v value is always 3/2 R. Hydrogen molecule is diatomic which are not rigidly held, so they vibrate about a well defined average separation. For hydrogen molecule, we have rotational and vibrational motion both besides translational motion. Contribution from vibrational motion is not appreciable at low temperature but increases from 0 to R on raising temperature.

100. (i)



(ii) $PV = nRT$

$T = \dfrac{PV}{nR} = \dfrac{0.5 \times 40}{2 \times 0.082} = 121.95\ \text{K}$

Total work $(W) = W_1 + W_2 + W_3$

$= -P\Delta V + 0 + 2.303nRT \log \dfrac{V_2}{V_1}$

$= -1 \times 20 + 2.303 \times 2 \times 0.082 \times 121.95 \log 2$
$= -20 + 13.87 = \mathbf{-6.13\ L\ atm}$
Since the system has returned to its initial state i.e. the process is cyclic, so $\Delta U = 0$
$\Delta U = q + W = 0$, so $q = -W = -(-6.13)$ L.atm $= \mathbf{620.7\ J}$
In a cyclic proces, sheat absorbed is completely converted into work.

(iii) Entropy is a state function and since the system has returned to its initial state, so $\boldsymbol{\Delta S = 0}$. Similarly $\boldsymbol{\Delta H = 0}$ and $\boldsymbol{\Delta U = 0}$ for the same reason.

101. $\Delta G^\circ = -2.303RT\log\frac{[\text{Product}]}{[\text{Reactant}]}$

Calculation of ΔG values :

Thus for the equilibrium $B \rightleftharpoons A$

$$\Delta G_1^\circ = (-2.303\times 8.314\times 448)\log\frac{1.3}{95.2}$$

or $\Delta G_1^\circ = \mathbf{15.992\ kJ\ mol^{-1}}$

Similarly for the equilibrium $B \rightleftharpoons C$

$$\Delta G_2^\circ = (-2.303\times 8.314\times 448)\log\frac{3.5}{95.2}$$

or $\Delta G_2^\circ = \mathbf{12.312\ kJ\ mol^{-1}}$

Similarly for equilibrium, $A \rightleftharpoons C$

$$\Delta G_3^\circ = -8.314\times 448\times 2.303\times\log_{10}\frac{3.5}{1.3} = -3.688\text{ kJ mole}^{-1}$$

Hence, we have that

$B \rightleftharpoons A,\ \Delta G_1^\circ = +15.992\text{ kJ mole}^{-1}$

$B \rightleftharpoons C,\ \Delta G_2^\circ = +12.312\text{ kJ mole}^{-1}$

$A \rightleftharpoons C,\ \Delta G_3^\circ = -3.688\text{ kJ mole}^{-1}$

Thus, the correct order of stability, $\boldsymbol{B > C > A}$

102. For following reaction

$$CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$$

$\Delta G°$ can be calculated as follows :

$$\Delta G° = \Delta G_p° - \Delta G°_R = \left[\Delta G°CO_2 - \left(\Delta G°CO + \frac{1}{2}\Delta G°O_2\right)\right]$$

$$= -394.4 - (-137.2 + \frac{1}{2}\times 0) = -257.2\text{ kJ mol}^{-1}$$

Since,

$\Delta G° = \Delta H° - T\Delta S°$ or $-257.2 = \Delta H° - 300(0.094)$

$\therefore \Delta H° = -285.4$ kJ/mol

$\Delta H°$ is negative, so the reaction is exothermic and since $\Delta G°$ is negative so the reaction is spontaneous.

103. If heat is absorbed at constant pressure, then

$q_p = \Delta E - (-P\Delta V)$ or $q_p = E_2 - E_1 - [-P(V_2 - V_1)]$

or $q_p = (E_2 + PV_2) - (E_1 + PV_1) = H_2 - H_1 = \Delta H$

Topic-2: Thermochemistry

1. (b) Combustion of glucose takes place as :

$C_6H_{12}O_6(s) + 6O_2(g) \to 6CO_2(g) + 6H_2O(\ell)$

Moles of glucose in 900g of glucose $= \frac{900}{180} = 5$ mol

1 mole glucose requires 6 moles of O_2

5 moles of glucose require 30 moles of O_2

Mass of O_2 required = 30 × 32 = 960 g.

2. (a) $C(\text{graphite}) + \frac{1}{2}O_2(g) \to CO(g)$ .....(i) ΔH

$C(\text{graphite}) + O_2(g) \to CO_2(g)$; (ii) $\Delta H_1 = -y$ kJ / mole

$CO_2(g) \to CO(g) + \frac{1}{2}O_2(g)$; (iii) $\Delta H_2 = \frac{x}{2}$ kJ / mole

eq. (i) = eq.(ii) + eq (iii) $\therefore \Delta H = \frac{x}{2} - y = \frac{x-2y}{2}$

3. (c) $\Delta_r H = \Sigma\Delta_c H(\text{Reactant}) - \Sigma\Delta_c H(\text{Product})$

$= 3\times(-1300) - (-3268) = -632\text{ kJ mol}^{-1}$

4. (c) $C_2H_6(g) + \frac{7}{2}O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(l)$

Heat of combustion

$= \Sigma\Delta_f H_{(\text{products})} - \Sigma\Delta_f H_{(\text{reactants})}$

$\Delta_c H(C_2H_6, g) = 2\Delta c H(C, \text{graphite}) + 3\Delta_c H(H_2, g) - \Delta_f H(O_2, g) - \Delta_f H(C_2H_6, g)$

$\Rightarrow\ -1560 = 2(-394) + 3(-286) - 0 - \Delta_f H(C_2H_6, g)$

$\Rightarrow\ \Delta_f H(C_2H_6, g) = -86\text{ KJ mol}^{-1}$

5. (c) $\Delta_{sol.}H° = \Delta_{lattice}H° + \Delta_{Hyd.}H°$

$4 = 788 + \Delta_{Hyd.}H°$; $\Delta_{Hyd.}H° = -784\text{ kJ mol}^{-1}$

6. (c) $\Delta H_{atomisation} = \Delta H_{vap}$ + Bond energy

Hence, $x > y$.

7. (a) $C_7H_{16}(l) + 11O_2(g) \xrightarrow{\Delta} 7CO_2(g) + 8H_2O(l)$

$\Delta H - \Delta U = \Delta n_g RT$

$\because \Delta n_g = 7 - 11 = -4 \quad \therefore \Delta H - \Delta U = -4RT$

8. (a) **Given:** $n = 3$

$T_1 = 300$; $T_2 = 1000$

$C_p = 23 + 0.01T$

The relation between ΔH and C_p is

$$\Delta H = \int_{T_1}^{T_2} nC_p dT \quad \text{...(i)}$$

After putting all variable values in eq. (i), we get

$$\Delta H = n\int_{300}^{1000}(23 + 0.01T)dT = 3\left[23T + \frac{0.01T^2}{2}\right]_{300}^{1000}$$

$= 3[23(1000 - 300)]\,[\frac{0.01}{2}(1000^2 - 300^2)]$

$= 61950\text{ J} = 61.95\text{ kJ} \approx 62\text{ kJ}$

9. (d) $C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$

$$\Delta n_g = 6 - \frac{15}{2} = -\frac{3}{2} \Rightarrow \Delta H = \Delta U + \Delta n_g RT$$

$$= -3263.9 + \left(-\frac{3}{2}\right)\times 8.314\times 10^{-3}\times 298$$

$= -3263.9 + (-3.71) = -3267.6\text{ kJ mol}^{-1}$

10. (b) $\Delta H = \Delta U + \Delta n_g RT$

$2HI(g) \to H_2(g) + I_2(g)$; $\Delta n_g = (1+1) - 2 = 0$

$\therefore\ \Delta H = \Delta U$

11. (c) $C(\text{graphite}) + O_2(g) \longrightarrow CO_2(g)$;

$\Delta_r H°_1 = -393.5\text{ kJ/mol}^{-1}$ ...(i)

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l);$

$\Delta_r H°_2 = -285.8$ kJ/mol^{-1} ...(ii)

$CO_2(g) + 2H_2O(l) \longrightarrow CH_4(g) + 2O_2(g);$

$\Delta_r H°_3 = +890.3$ kJ/mol^{-1} ...(iii)

C(graphite) + $2H_2(g) \longrightarrow CH_4(g); \Delta_r H = ?$ ...(iv)

[Eq. (i) + Eq. (iii)] + [2 × Eq. (ii)] = Eq (iv)

$\therefore [\Delta H_1 + \Delta H_3] + [2 \times \Delta H_2] = \Delta H$

$[(-393.5) + (890.3)] + [2(-285.8)] = -74.8$ kJ/mol

12. **(b)** Given

$C(s) + O_2(g) \to CO_2(g);\ \Delta H_1 = -393.5$ kJ mol^{-1} ...(i)

$CO(g) + \frac{1}{2}O_2(g) \to CO_2(g);\ \Delta H_2 = -283.5$ kJ mol^{-1} ...(ii)

$\therefore$ Heat of formation of CO = $\Delta H_1 - \Delta H_2$

$= -393.5 - (-283.5) = -110$ kJ

13. **(b)** In CH_4, $4 \times BE_{(C-H)} = 360$ kJ/mol

$\therefore\ BE_{(C-H)} = 90$ kJ/mol

In C_2H_6, $BE_{(C-C)} + 6 \times BE_{(C-H)} = 620$ kJ/mol

$\therefore\ BE_{(C-C)} = 620 - (6 \times 90) = 80$ kJ/mol

$\therefore\ BE_{(C-C)} = \frac{80 \times 10^3}{6.023 \times 10^{23}}$ J/mol

Now,

$E = \frac{hc}{\lambda}\ \therefore \lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^8 \times 6.023 \times 10^{23}}{80 \times 10^3}$

$\lambda = 1.49 \times 10^{-6}$ m $\quad (\because 1\text{ nm} = 10^{-9}\text{ m})$

$\therefore\ \lambda = 1.49 \times 10^3$ nm

14. **(a)** $C_2H_5OH(l) + 3O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(l)$

Bomb calorimeter gives ΔU of the reaction

Given, $\Delta U = -1364.47$ kJ mol^{-1}

$\Delta n_g = -1$

$\Delta H = \Delta U + \Delta n_g RT$

$= -1364.47 - \frac{1 \times 8.314 \times 298}{1000} = -1366.95$ kJ mol^{-1}

15. **(c)** Given $\frac{1}{2}N_2 + \frac{3}{2}H_2 \rightleftharpoons NH_3;$

$\Delta_f H = -46.0$ kJ / mol

$H + H \rightleftharpoons H_2;\ \Delta_f H = -436$ kJ / mol

$N + N \rightleftharpoons N_2;\ \Delta_f H = -712$ kJ / mol

$\Delta_f H(NH_3) = \frac{1}{2}\Delta H_{N-N} + \frac{3}{2}\Delta H_{H-H} - 3\Delta H_{N-H}$

$-46 = \frac{1}{2}(712) + \frac{3}{2}(436) - 3\Delta H_{N-H}$

$\Delta H_{N-H} = 352$ kJ/mol

16. **(c)** The standard enthalpy of the combustion of glucose can be calculated by the eqn.

$C_6H_{12}O_6(s) + 6O_2(g) \to 6CO_2(g) + 6H_2O(l)$

$\Delta H_C = 6 \times \Delta H_f(CO_2) + 6 \times \Delta H_f(H_2O) - \Delta H_f(C_6H_{12}O_6)$

$\Delta H° = 6(-400) + 6(-300) - (-1300) = -2900$ kJ/mol

For one gram of glucose, enthalpy of combustion

$\Delta H° = -\frac{2900}{180} = -16.11$ kJ/g

17. **(a)** $N_2(g) + \frac{1}{2}O_2 \to N_2O(g)$

$N \equiv N(g) + \frac{1}{2}(O = O) \to \overset{-}{N} = \overset{+}{N} = O(g)$

$\Delta_f H^0$ = [Energy required for breaking of bonds]

−[Energy released for forming of bonds]

$= (\Delta H_{N\equiv N} + \frac{1}{2}\Delta H_{O=O}) - (\Delta H_{N=N} + \Delta H_{N=O})$

$= (946 + \frac{1}{2} \times 498) - (418 + 607) = 170$ kJ mol^{-1}

Resonance energy = observed $\Delta_f H^o$ – calculated $\Delta_f H^o$

$82 - 170 = -88$ kJ mol^{-1}

18. **(b)** The species in its elemental form has zero standard molar enthalpy of formation at 298 K. At 298K, Cl_2 is gas while Br_2 is liquid.

19. **(9)** Energy released by combustion of 3.5 g gas

$= 2.5 \times (298.45 - 298)$ kJ

Energy released by 1 mole of gas

$= \frac{2.5 \times 0.45}{3.5/28} = 9$ kJmol^{-1}

20. **(b)** ΔH_f° is the enthalpy change when 1 mole of the substance is formed from its elements in their standard states. In (a) carbon is present in diamond however, standard state of carbon is graphite. Again, in (d) CO (g) is involved so it can't be the right option. Further in (c) 2 moles of NH_3 are generated. Hence, the correct option is (b).

21. **(b)** $CO_2(g) + H_2(g) \longrightarrow CO(g) + H_2O(g)$, $\Delta H = ?$

$\Delta H = \sum \Delta H_f$ (Product) $- \sum \Delta H_f$ (reactant)

Given, $\Delta H_f CO_2(g) = -393.5$ kJ/mol

$\Delta H_f CO(g) = -110.5$ kJ/mol

$\Delta H_f H_2O(g) = -241.8$ kJ/mol

$\therefore\ \Delta H = [\Delta H_f CO(g) + \Delta H_f H_2O(g)]$

$- [\Delta H_f CO_2(g) + \Delta H_f H_2(g)]$

$= [-110.5 + (-241.8)] - [-393.5 + 0]$

$[\because \Delta H_f(H_2)(g) = 0]$

$= 41.2$ kJ mol^{-1}

22. **(d)** $\Delta H = \Delta E + \Delta nRT$ For $\Delta H \neq \Delta E, \Delta n \neq 0$

Where Δn = no. of moles of gaseous products – no. of moles of gaseous reactants

(a) $\Delta n = 2 - 2 = 0$

(b) $\Delta n = 0$ ($\because$ they are either in solid or liquid state)

(c) $\Delta n = 1 - 1 = 0$ ($\because$ C is in solid state)

(d) $\Delta n = 2 - 4 = -2$

$\therefore$ (d) is correct answer

23. **(a)** Heat capacity at constant volume (q_v) = ΔE

Heat capacity of constant pressure (q_p) = ΔH

$\Delta H = \Delta E + \Delta nRT$ or $\Delta H - \Delta E = \Delta nRT$

Δn = no. of moles of gaseous products – no. of moles of gaseous reactants

$= 12 - 15 = -3$

$\Delta H - \Delta E = -3 \times 8.314 \times 298$ J $= -7.43$ kJ.

24. **(150)** Combustion taking place as :

$$C_6H_5COOH(s)+\frac{15}{2}O_2(g)\rightarrow 7CO_2(g)+3H_2O(\ell)$$

$$\Delta n_g=7-\frac{15}{2}=-\frac{1}{2}$$

$$\Delta H=\Delta U+\Delta n_gRT=-321.30-\frac{1}{2}R\times 300$$

$=(-321.30-150\ R)$ kJ

On comparing with $(-321.30-xR)=x=150$.

25. **(6535)** $C_6H_6(l)+\frac{15}{2}O_2(g)\rightarrow 6CO_2(g)+3H_2O(l)$

$\Delta_f H=H_P-H_R$

$\Delta H_c=6\Delta H_f(CO_2)+3\Delta H_f(H_2O)-\Delta H_f(C_6H_6)$

$=6\times(-393.5)+3(-286)-48.5=-3267.5$ kJ/mol

For 2 mol of benzene $=-6535$ kJ/mol; $x=6535$

26. **(125)** $C_2H_4(g)+H_2(g)\rightarrow C_2H_6(g)$

$\Delta_f H=[BE(C=C)+4BE(C-H)+BE(H-H)]$

$-[BE(C-C)+6BE(C-H)]$

$\Delta_f H=BE(C=C)+BE(H-H)-BE(C-C)$

$-2BE(C-H)=615+435-347-2\times 414=-125$ kJ

27. **(492)** Let's take:-

$2Fe(s)+\frac{3}{2}O_2(g)\longrightarrow Fe_2O_3(s)$ ... (1)

$C(s)+\frac{1}{2}O_2(g)\longrightarrow CO(g)$ ... (2)

$3C(s)+Fe_2O_3(s)\longrightarrow 2Fe(s)+3CO(g)$ ... (3)

Equation (3) can be obtained by subtracting (1) from (3) × (2).

Thus, $\Delta H_3^o=(3\times(-110))-(-822)=+492$ kJ mol^{-1}

28. **(56)** $\Delta H^0_{vap.}=30.5$ kJ mol^{-1}, $M_{CCl_4}=153.82$ g mol^{-1}

$\Rightarrow$ Heat required for 153.82 g CCl_4 = 30.5 kJ

$\Rightarrow$ Heat required for 284 g $CCl_4=\frac{30.5\times 284}{153.82}=56.32$ kJ.

29. **(800)** $A_2+B_2\rightarrow 2AB;\ \Delta H_f^o=-200$ kJ mol^{-1}

$\Rightarrow \Delta H_f^o(AB)=-200$ kJ mol^{-1}

$\therefore \Delta H_R^o$ for reaction $A_2+B_2\rightarrow 2AB$ is -400 kJ mol^{-1}

Given: Bond Enthalpy of A_2, B_2 and AB is 1 : 0.5 : 1

Assuming bond enthalpy of A_2 be x kJ mol^{-1}

$\therefore$ Bond enthalpy $B_2=0.5$ x kJ mol^{-1}

$\therefore$ Bond enthalpy AB = (x) kJ mol^{-1}

$$\begin{array}{ccccc} A_2 & + & B_2 & \longrightarrow & 2AB;\ \Delta H_R^o=-400\text{ kJ/mol} \\ \swarrow x & & \downarrow 0.5x & & \uparrow -2x \\ 2A & + & 2B & \longrightarrow & \end{array}$$

$-400=x+0.5x-2x$

$-400=-0.5x\quad \therefore x=800$ kJ/mol

30. **(4)**

$$\begin{array}{lccc} & Fe_2O_3 & +\ 2Al\longrightarrow & Al_2O_3+2Fe \\ \text{Molar mass:} & 160g & 27g & \end{array}$$

$$\left(\Delta H_f^o\right)_{reaction}=\left[\left(\Delta H_f^o\right)_{Al_2O_3}+2(\Delta H_f^o)_{Fe}\right]-\left[\left(\Delta H_f^o\right)_{Fe_2O_3}+2\left(\Delta H_f^o\right)_{Al}\right]$$

$=[-1700+0]-[-840+0]=-860$ kJ/mol

Total mass of mixture = Fe_2O_3 + Al (1 : 2 molar ratio)

$=(160)+(2\times 27)=214$ g/mol

Heat evolved per gram $=\frac{860}{214}=4$ kJ / g

31. **(278)** $2C(S)+3H_2(g)+\frac{1}{2}O_2(g)\rightarrow C_2H_5OH(l)$

$$\left(\Delta H_f\right)_{C_2H_5OH_{(l)}}=\sum\left(\Delta H_{comb}\right)_{reactant}-\sum\left(\Delta H_{comb}\right)_{product}$$

$=2\times(-393.5)+3(-241.8)-(-1234.7)$

$=-277.7$ kJ/mol ≈ 278 kJ/mol

32. **(499)** $H_2O\rightarrow H_2+\frac{1}{2}O_2;\ \Delta H_2^\circ=242$ kJ / mol ...(i)

$H_2\rightarrow 2H;\ \Delta H_3^\circ=436$ kJ / mol ...(ii)

$H_2+O_2\rightarrow 2OH;\ \Delta H_1^\circ=48$ kJ / mol ...(iii)

Eq. (ii) + Eq. (iii),

$2H_2+O_2\rightarrow 2OH+2H;\ \Delta H_5^\circ=\Delta H_3^\circ+\Delta H_1^\circ$

$\therefore H_2+\frac{1}{2}O_2\rightarrow OH+H;\ \frac{\Delta H_5^\circ}{2}=\frac{\Delta H_3^\circ+\Delta H_1^\circ}{2}=\Delta H_6^\circ$ ...(iv)

Eq. (i) + Eq.

(iv), $H_2O\rightarrow H_2+\frac{1}{2}O_2;\ \Delta H_2^\circ=242$ kJ / mol

$H_2+\frac{1}{2}O_2\rightarrow OH+H\ ;\ \Delta H_6^\circ=\frac{436+78}{2}=257$ kJ / mol

$H_2O\rightarrow H+OH;$

$X=242+257=499$ kJ / mol

33. **(610)** $\frac{1}{2}Cl_2(g)\rightarrow Cl(g)\rightarrow Cl^-(g)\rightarrow Cl^-(aq)$

$\Delta H_{reaction}=\frac{1}{2}\times 240+(-350)+(-380)=-610$

34. **(173)** $\Delta_r H=\Sigma H_p-\Sigma H_R$

$=[(-394+4\times(-92)]-[(-105)+(2\times -242)]=-173$ kJ/mol

35. **(925)** Let, Volume of C_2H_4 is x litre

$$\begin{array}{lcccc} & C_2H_4 & +\ 3O_2 & \rightarrow 2CO_2 & +\ 2H_2O \\ \text{Initial} & x & - & & \\ \text{Final} & - & & 2x & \end{array}$$

$$\begin{array}{lcccc} & CH_4 & +\ 2O_2 & \rightarrow CO_2 & +\ 2H_2O \\ \text{Initial} & (16.8-x) & & - & \\ \text{Final} & - & & (16.8-x) & \end{array}$$

Total volume of $CO_2=2x+16.8-x$

$\Rightarrow\quad 28=16.8+x$

$x=11.2$ L

Volume of $C_2H_4=4.2$ L, Volume of CH_4 $=(16.8-11.2)$L $=5.6$ L

$^nCH_4=\frac{5.6}{22.4}=0.25$ mol; $^nC_2H_4=\frac{11.2}{22.4}=0.5$ mol

$\therefore$ Heat evolved $=0.25\times(-900)+0.5(-1400)=-925$ kJ

36. **(90.39)** $2Hg(g)+O_2(g)\rightarrow 2HgO(s)$

$\Delta_r H^\circ=2\Delta_f H^\circ(HgO,s)-2\Delta_f H^\circ(Hg,g)-\Delta_f H^\circ(O_2,g)$

$= 2\Delta_f H°(HgO, s) - 2\Delta_f H°(Hg, g)$ ... (i)

$[\because \Delta_f H°(O_2, g) = 0]$

Now, $\Delta_r H°$ is the heat evolved by bomb calorimeter due to the occurrence of the reaction at constant volume.

$\therefore -(Q_v)_r = \Delta_r U$

$\therefore \Delta_r H = \Delta_r U + \Delta n_g RT = -C\Delta T + \Delta n_g RT$

[where C – the heat capacity of calorimeter = 20 kJ/K at 298 K]

$= -[20(312.8 - 298)] - 3\,RT$

$= -296 - 3 \times 8.3 \times 10^{-3} \times 298\ kJ = -303.42\ kJ$

Hence, from eq. (i)

$-303.42 = 2\Delta_f H°(HgO, s) - 2 \times 61.32$

or, $2\Delta_f H°(HgO, s) = -180.78\ kJ$

$\therefore$ Standard Molar Enthalpy of formation of HgO

$= \dfrac{-180.78}{2} = -90.39\ kJ \Rightarrow |X| = 90.39$

37. **(54)**

	HNO_3	NaOH	
	600 mL × 0.2 M	400 mL × 0.1 M	
	= 120 mmol	= 40 mmol	

$HNO_3 + NaOH \rightarrow NaNO_3 + H_2O$

Start	120	40	
End	80	0	40 mmol

$\Delta_r H = 40\ mmol \times (57 \times 10^3)\ J\ mol^{-1}$

$= 40 \times 10^{-3}\ mol \times 57 \times 10^3\ J\ mol^{-1} = 2280\ J$

$m\,S\Delta T = 2280 \Rightarrow 1000\ g \times 4.2 \times \Delta T = 2280$

$\Delta T = \dfrac{2280}{4.2} \times 10^{-3} = \dfrac{22800}{42} \times 10^{-3} = 542.86 \times 10^{-3}$

$= 54.286 \times 10^{-2}\ K = 54.286 \times 10^{-2}\ °C$

38. **(2)** $\Delta H_{neutaralization} = -57.3\ kJ/mol$

In case of acetic acid

$\Delta H = \Delta H_{ioni} + \Delta H_{neutralization}$ $-55.3 = \Delta H_{ioni} - 57.3$

$\Delta H_{ion_i} = 2\ kJ/mol$

39. **(718)**

$$\Delta_f H°_{KCl} = \Delta_{sub} H°_{(K)} + \Delta_{ionization} H°_{(K)} + \frac{1}{2}\Delta_{bond} H°_{(Cl_2)} + \Delta_{electron\ gain} H°_{(Cl)} + \Delta_{lattice} H°_{(KCl)}$$

$$\Rightarrow -436.7 = 89.2 + 419.1 + \frac{1}{2}(243.0) + \{-348.6\} + \Delta_{lattice} H°_{(KCl)}$$

$$\Rightarrow \Delta_{lattice} H°_{(KCl)} = -717.8\ kJ\ mol^{-1} \simeq 718\ kg\ mol^{-1}$$

40. **(101)** $\Delta H_{sub} = \Delta H_{fus.} + \Delta H_{vap.} = 2.8 + 98.2 = 101\ kJ/mol$

41. **(309)** $SF_6(g) \rightarrow S(g) + 6F(g)$

$\Delta H° = \Delta H_f°(S) + 6\Delta H_f°(F) - \Delta H_f°(SF_6)$

$= 275 + 6 \times 80 - (-1100) = 1855\ kJ\ mol^{-1}$

Also, $\Delta H° = 6\Delta H_{S-F}$

$\therefore \quad \Delta H_{S-F} = \dfrac{1855}{6} = 309.17 \simeq 309\ kJ\ mol^{-1}$

42. **(189494)**

$\Delta H = \Delta U + \Delta n_g RT$; $n = \dfrac{90}{18} = 5$ mol

$H_2O(l) \rightleftharpoons H_2O(g)$

$\Delta n = 1$

$41000 = \Delta U + 1 \times 8.314 \times 373 \Rightarrow \Delta U = 37898.875\ J$

For 5 moles, $\Delta U = 37898.87 \times 5 = 189494\ J$

43. **(–2035)**

The chemical reaction for combustion of diborane is

$B_2H_6(g) + 3O_2(g) \longrightarrow B_2O_3(s) + 3H_2O(g),\ \Delta H = ?$

For this the enthalpy change can be calculated in the following way.

$\Delta_c H = [\Delta_f H_{B_2O_3(s)} + 3\Delta_f H_{H_2O(g)}] - \Delta_f H_{B_2H_6(g)}$;

$(\because \Delta_f H_{O_2} = 2)$

$\Delta_f H_{H_2O(g)}$ can be obtained by adding $\Delta_r H_{H_2O(l)}$ and $\Delta_r H_{H_2O(g)}$, *i.e.* $-286 + 44 = -242\ kJ\ mol^{-1}$

$\Delta H = [-1273 + 3 \times (-242)] - 36\ kJ\ mol^{-1} = -1273 - 726 - 36$

$= \mathbf{-2035\ kJ\ mol^{-1}}$

44. **(309.16)**

Given $S(s) + 3F_2(g) \rightarrow SF_6(g)$; $\Delta H = -1100\ kJ$ .....(i)

$S(s) \rightarrow S(g)$; $\Delta H = 275\ kJ$ .....(ii)

$1/2\ F_2(g) \rightarrow F(g)$; $\Delta H = 80\ kJ$ .....(iii)

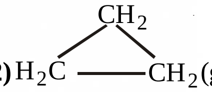
To get $SF_6(g) \rightarrow S(g) + 6F(g)$ we can proceed as

(ii) + 6 × (iii) - (i)

$\therefore SF_6(g) \rightarrow S(g) + 6F(g)$; $\Delta H = 1855\ kJ$

Thus, average bond energy for S-F bond

$= \dfrac{1855}{6} = \mathbf{309.16kJ}$

45. **(–2091.32)**  $(g) \rightarrow CH_3CH = CH_2(g)$;

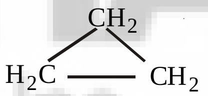
$\Delta H = -33.0\ kJ$ ...(i)

$C(s) + O_2(g) \rightarrow CO_2(g)$; $\Delta H = -393.5kJ$ ...(ii)

$H_2(g) + 1/2O_2(g) \rightarrow H_2O(l)$; $\Delta H = -285.8\ kJ$ ...(iii)

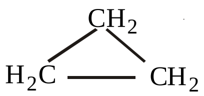
$3C(s) + 3H_2(g) \rightarrow CH_3–CH = CH_2(g)$; $\Delta H = 20.42\ kJ$ ...(iv)

The required reaction is

 $+ \dfrac{9}{2}O_2 \longrightarrow 3CO_2 + 3H_2O$; $\Delta H = ?$

To calculate the value of ΔH follow the following steps.

(i) + 3 × (ii) + 3 × (iii)] – (iv) :

 $+ (9/2)O_2 \rightarrow 3CO_2 + 3H_2O$;

$\Delta H = \mathbf{-2091.32\ kJ}$

46. **(–266)** The required thermochemical equation is

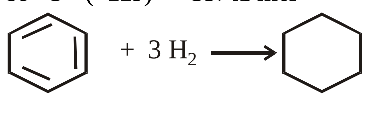
$C(g) + 4H(g) + O(g) \longrightarrow CH_3OH(l)$; $\Delta H_f = ?$

$$\Delta H_f = \left[\Delta H_{C(s)\rightarrow C(g)} + 2\Delta H_{H-H} + \frac{1}{2}\Delta H_{O=O}\right] - \left[3\Delta H_{C-H} + \Delta H_{C-O} + \Delta H_{O-H} + \Delta H_{vap.CH_3OH}\right]$$

$= [715 + 2 \times 436 + 249] - [3 \times 415 + 356 + 463 + 38]$

$= \mathbf{-266\ kJ\ mol^{-1}}$

47. **(–152)** Standard enthalpy of hydrogenation of cyclohexene ($-119 kJ\ mol^{-1}$) means the enthalpy of hydrogenation of one C = C double bond. Now benzene has three C = C double bonds, the enthalpy of the reaction would be $= 3 \times (-119) = -357\ kJ\ mol^{-1}$



Actual enthalpy of the reaction can be evaluated as follows.

$$\Delta H_{(\text{Reaction})} = \Delta H_f^\circ(\text{Product}) - \Delta H_f^\circ(\text{Reactants})$$

$$= -156 - (49 + 0) = -205 \text{ kJ mol}^{-1}$$

$\therefore$ Resonance energy = $\Delta H_{\text{Exp}} - \Delta H_{\text{cal}}$

$= -357 - (-205) =$ **-152 kJ mol^{-1}**

48. **(–72)** $nCH_2 = CH_2 \rightarrow (CH_2 - CH_2)_n$

During the polymerisation of ethylene, one mole of ethylene breaks *i.e.* one C = C double bond breaks and the two CH_2 – groups are linked with C – C single bonds thus, forming three single bonds (two single bonds are formed when each CH_2 – group of ethylene links with one CH_2 – group of another ethylene molecule). But in the whole unit of polymer, number of single C–Cbonds formed/mole of ethylene is 2.

$$-(CH_2 - CH_2)-(CH_2 - CH_2)-(CH_2 - CH_2)-(CH_2 - CH_2)-$$

e.g. Number of single bonds formed by 4 moles of ethylene = 8

Energy released = Energy due to formation of 2 C–C single bonds

= 2 × 331 = 662 kJ/mol of ethylene

Energy absorbed = Energy due to dissociation of 1 C = C double bond = 590 kJ/mol of ethylene

$\therefore$ Enthalpy of polymerisation/mol of ethylene or $\Delta H_{\text{polymerisation}}$ = 590 – 662 kJ/mol = **–72 kJ/mole**

49. **(–55.7)** From the given data, we can write :

(*i*) $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$; $\Delta H_1 = -285.8$ kJ/mol

(*ii*) $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$; $\Delta H_2 = -890$ kJ mol

(*iii*) $C_2H_6 + \frac{7}{2}O_2 \rightarrow 2CO_2 + 3H_2O$; $\Delta H_3 = -1560$ kJ/mol

(*iv*) $C(s) + O_2 \rightarrow CO_2$; $\Delta H_4 = -393.5$ kJ/mol

(*v*) $3C(s) + 4H_2 \rightarrow C_3H_8(g)$; $\Delta H_5 = -103.8$ kJ/mol

The required reaction is $C_3H_8(g) + H_2(g) \rightarrow C_2H_6(g) + CH_4(g)$, $\Delta H = ?$

It can be obtained by the following calculations.

3 × (*iv*)–(*v*) + 5(*i*) – (*iii*) – (*ii*)

In other words, $\Delta H = 3\Delta H_4 - \Delta H_5 + 5\Delta H_1 - \Delta H_2 - \Delta H_3$

$\therefore \Delta H = 3(-393.5) - (-103.8) + 5(-285.8) + 890 + 1560$

$= -2609.5 + 2553.8 =$ **–55.7 kJ/mol**

50. **(50.90)** Combustion of C_2H_4 and CH_4 takes place as follows:

$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O$
1 vol. 2 vol.

$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$
1 vol. 1 vol.

Let the vol. of CH_4 in mixture = x L

$\therefore$ Vol. of C_2H_4 in the mixture = $(3.67 - x)$ L

Vol. of CO_2 produced by x L of CH_4 = x L and

Vol. of CO_2 produced by $(3.67 - x)$ L of C_2H_4 = $2(3.67 - x)$ L

$\therefore$ Total vol. of CO_2 produced = $x + 2(3.67 - x)$

or $6.11 = x + 2(3.67 - x)$ or $x = 1.23$ L

$\therefore$ Vol. of CH_4 in the mixture = 1.23 L

and Vol. of C_2H_4 in the mixture = 3.67 – 1.23 = 2.44 L

Vol. of CH_4 per litre of the mixture $= \frac{1.23}{3.67} = 0.335$ L

Vol. of C_2H_4 per litre of the mixture $= \frac{2.44}{3.67} = 0.665$ L

Now, we know that volume of 1 mol. of any gas at

25 ºC (298 K) $= \frac{22.4 \times 298}{273} = 24.45$ L

[$\because$ Volume at NTP = 22.4 L]

Heat evolved due to combustion of 0.335 L of CH_4

$= -\frac{0.335 \times 891}{24.45} = -12.20$ kJ

[given, heat evolved by combustion of 1 L = 891 kJ]

Similarly, heat evolved due to combustion of 0.665 L of

$C_2H_4 = -\frac{0.665 \times 1423}{24.45} = -38.70$ kJ

$\therefore$ Total heat evolved = 12.20 + 38.70 = **50.90 kJ**

51. **(–121)** The required reaction is

$C_6H_{10}(g) + H_2(g) \rightarrow C_6H_{12}(g)$, $\Delta H_1 = ?$ ...(1)
Cyclohexene Cyclohexane

The given facts can be written as :

$H_2(g) + ½O_2(g) \rightarrow H_2O(l)$, $\Delta H_2 = -241$ kJ/mol ...(2)

$C_6H_{10}(g) + \frac{17}{2}O_2(g) \rightarrow 6CO_2(g) + 5H_2O$; $\Delta H_3 = -3800$ kJ/mol ...(3)

$C_6H_{12}(g) + 9O_2(g) \rightarrow 6CO_2(g) + 6H_2O$, $\Delta H_4 = -3920$ kJ/mol ...(4)

The required reaction (1) can be obtained by adding equations (2) and (3) and subtracing (4) from the sum of (2) and (3).

$C_6H_{10}(g) + H_2(g) \rightarrow C_6H_{12}(g)$

$\Delta H_1 = (\Delta H_2 + \Delta H_3) - \Delta H_4$

$= [-241 + (-3800)] - (-3920) = (-241 - 3800) - (-3920)$

$= -4041 + 3920 = -$ **121 kJ/mole**

52. **(3.94)**

$Fe_2O_3 + 2Al \rightarrow 2Fe + Al_2O_3$
2 × 56 + 48 = 160 2 × 27 = 54

Heat of reaction = 399 – 199 = 200 kcal [Al and Fe are in their standard states]

Total weight of reactants = 160 + 54 = 214 g

$\therefore$ Fuel value/gram $= \frac{200}{214} = 0.9346$ kcal/g

Volume of Al $= \frac{54}{2.7} = 20$ cc

Volume of $Fe_2O_3 = \frac{160}{5.2} = 30.77$ cc

Total volume = 20 + 30.77 = 50.77 cc

$\therefore$ Fuel value per cc $= \frac{200}{50.77} =$ **3.94 kcal/cc**

53. **(–372.0)** The required chemical reaction.

$2C_2H_6 + 7O_2 \longrightarrow 4CO_2 + 6H_2O$; $\Delta H = x$

Note that since 2 moles of ethane are reacting, the ΔH of the reaction will be ½ x.

The thermochemical equations for the given data are written as below.

(*i*) $C(s) + O_2(g) \longrightarrow CO_2(g)$; $\Delta H = -94.1$ kcal

(*ii*) $H_2(g) + ½O_2(g) \longrightarrow H_2O(g)$; $\Delta H = -68.3$ kcal

(*iii*) $2C(s) + 3H_2(g) \longrightarrow C_2H_6(g)$; $\Delta H = -21.1$ kcal

We know that $\Delta H = H_{\text{Products}} - H_{\text{Reactants}}$

$\Delta H = 4\Delta H_{CO_2} + 6\Delta H_{H_2O} - (2\Delta H_{C_2H_6} + 7\Delta H_{O_2})$

$\Delta H = 4 \times (-94.1) + 6 \times (-68.3) - (2 \times (-21.1) + 0)$

$= -376.4 - 409.8 + 42.2 = -744.0$ kcal/2 mole of ethane

= **–372.0 kcal/mole of ethane**

54. **(–22)**

Bond	H – H	Cl – Cl	H – Cl
ΔH disso.	104 kcal	58 kcal	103 kcal

Formation of hydrogen chloride can be represented as
$H-H+Cl-Cl \rightarrow 2H-Cl$
Thus, the reaction involves
Cleavage of one H – H bond, $\Delta H = 104$ kcal
Cleavage of one Cl – Cl bond, $\Delta H = 58$ kcal
Formation of two H – Cl bonds, $\Delta H = 2 \times (-103)$ kcal
$\therefore \Delta H$ of the reaction $= (104 + 58) - 2(103)$
$= 162 - 206 = -44$ kcal
Now, since the enthalpy of formation of a compound is the change in heat content accompanied in the formation of one mole of the compound, the enthalpy of formation of HCl gas $= -\dfrac{44}{2} =$ **– 22 kcal**

55. **(41.104)** The given data can be written as follows

(i) $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l); \quad \Delta H = -68.3$ kcal

(ii) $C_2H_2(g) + \frac{5}{2}O_2(g) \rightarrow H_2O(l) + 2CO_2(g);$
$\Delta H = -310.6$ kcal

(iii) $C_2H_4(g) + 3O_2(g) \rightarrow 2H_2O(l) + 2CO_2(g);$
$\Delta H = -337.2$ kcal

The required thermochemical equation is
$C_2H_2(g) + H_2(g) \rightarrow C_2H_4(g)$
The required equation can be obtained by subtracting equation (*iii*) from the sum of equations (*i*) and (*ii*), thus ΔH of the required equation can be calculated as below.
$\Delta H = [-68.3 + (-310.6)] - (-337.2)$
$= [-68.3 - 310.6] + 337.2 = -378.9 + 337.2 = -41.7$ kcal
ΔE, the heat of reaction for the hydrogenation of acetylene at constant volume is given by :
$\Delta E = \Delta H - \Delta nRT$
Here Δn = Moles of the gaseous products – Moles of the gaseous reactants
$= 1 - (1 + 1) = -1$
Substituting the values of ΔH, Δn, R and T in
$\Delta E = \Delta H - \Delta nRT = -41.7 - (-1 \times 2 \times 10^{-3} \times 298)$

$$\left[\because R = 2\text{cal / degree / mole } = 2\times 10^{-3}\text{ kcal / deg/ mole}\right]$$

$= -41.7 + 0.596 =$ **41.104 kcal**

56. **(54.20)** The required equation is :
$2C(s) + H_2(g) \rightarrow C_2H_2; \quad \Delta H = ?$
Write the thermochemical equations for the given data

(i) $C_2H_2(g) + \frac{5}{2}O_2(g) \rightarrow 2CO_2(g) + H_2O(l);$
$\Delta H = -310.62$ kcal

(ii) $C(s) + O_2(g) \rightarrow CO_2(g); \ \Delta H = -94.05$ kcal

(iii) $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l); \ \Delta H = -68.32$ kcal

For getting the above required reaction, we will have to
(a) Bring C_2H_2 in the product that can be done by reversing the equation (i) to give equation (iv).
(b) Multiply equation (ii) by 2 to get 2C atoms in the reactants and thus, equation (v) is obtained.
(c) Keep equation (iii) as such.
(d) Add equations (iv), (v) and (iii).

(iv) $2CO_2 + H_2O \rightarrow C_2H_2 + \frac{5}{2}O_2; \ \Delta H = 310.62$ kcal
(v) $2C + 2O_2 \rightarrow 2CO_2; \quad \Delta H = -188.10$ kcal
(iii) $H_2 + \frac{1}{2}O_2 \rightarrow H_2O; \quad \Delta H = -68.32$ kcal
On adding, $2C + H_2 \rightarrow C_2H_2; \ \Delta H = 54.20$ kcal
Hence, the standard heat of formation of $C_2H_2(g)$ = **54.20 kcal**

57. **(101.19)**
The required reaction in terms of dissociation energy is
$OH(g) \rightarrow O(g) + H(g); \quad \Delta H = ?$
This equation can be achieved by (a) reversing the equation (*i*), (b) dividing equation (*ii*) and (*iii*) each by 2, and (c) adding the three resulting equations.

$OH(g) \rightarrow \frac{1}{2}H_2(g) + \frac{1}{2}O_2(g); \ \Delta H = +10.06$ kcal
[Reversing eq (i)]

$\frac{1}{2}H_2(g) \rightarrow H(g) \quad \Delta H = -52.09$ kcal $\left[\frac{1}{2}\text{Eq (ii)}\right]$

$\frac{1}{2}O_2(g) \rightarrow O(g); \quad \Delta H = -59.16$ kcal $\left[\frac{1}{2}\text{Eq (iii)}\right]$

$OH(g) \rightarrow O(g) + H(g); \quad \Delta H = -101.19$ kcal (adding)
Thus, one mole of OH(g) needs 101.19 kcal of energy to break into oxygen and hydrogen gaseous atoms. Hence, the bond energy of O–H bond is **101.19 kcal**.

58. **endothermic**

59. **(a, c)** Enthalpy of formation is the enthalpy change for formation of 1 mole of substance from its elements present in the most stable natural form.

60. Combustion of CH_4 and C_4H_{10} takes place as follows
$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O, \Delta H = -809$ kJ mol^{-1}
$C_4H_{10} + 13/2O_2 \rightarrow 4CO_2 + 5H_2O, \Delta H = -2878$ kJ mol^{-1}
In order to get the same calorific output due to C_4H_{10}, the rate of supply of butane $= \dfrac{x \times 809}{2878} =$ **0.281 *x* L/hr**

Rate of supply of oxygen $= 0.28\,x \times \dfrac{13}{2} \times 3 =$ **5.481 *x* L/hr**

61. For C_3H_8: $3C + 4H_2 \rightarrow C_3H_8; \Delta H_1 = ?$
For C_2H_6: $2C + 3H_2 \rightarrow C_2H_6; \Delta H_2 = ?$
$\therefore \Delta H_1 = -[2(C–C) + 8(C–H)] + [3C_{s\rightarrow g} + 4(H–H)]$ ...(1)
$\therefore \Delta H_2 = -[1(C–C) + 6(C–H)] + [2C_{s\rightarrow g} + 3(H–H)]$ ...(2)
Let bond energy of C–C be x kcal and bond energy of C–H be y kcal
$\therefore$ By eq. (1) $\Delta H_1 = -(2x + 8y) + [3 \times 172 + 4 \times 104]$ ...(3)
$\Delta H_2 = -(x + 6y) + [2 \times 172 + 3 \times 104]$ ...(4)
Also given $C + O_2 \rightarrow CO_2; \Delta H = -94.0$ k cal ...(5)
$H_2 + ½O_2 \rightarrow H_2O; \Delta H = -68.0$ kcal ...(6)
$C_2H_6 + (7/2)O_2 \rightarrow 2CO_2 + 3H_2O; \Delta H = -372$ k cal ...(7)
$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O; \Delta H = -530$ k cal ...(8)
By inspection method : 2 × (5) + 3 × (6) – (7) gives
$2C + 3H_2 \rightarrow C_2H_6; \Delta H_2 = -20$ k cal ...(9)
and 3 × (5) + 4 × (6) – (8) gives
$3C + 4H_2 \rightarrow C_3H_8; \Delta H_1 = -20$ k cal ...(10)
$\therefore$ By eq. (3), (4), (9) and (10), $x + 6y = 676; 2x + 8y = 956$
$\therefore x = 82$ k cal and $y = 99$ k cal
Bond energy of C–C bond = **82 k cal**
and Bond energy of C–H bond = **99 k cal**

Disha PYQs JEET Pakki

ERRORLESS

47 YEARS

Chapter-wise & Topic-wise JEE Advanced (1978 – 2024) & JEE Main (2013 – 2024)

MATHEMATICS

Solved Papers

27 Chapters	72 Topics	10 Types	5200+ PYQs	100% Solutions

- Includes 2024 Advanced Papers 1 & 2
- Includes 20 Sets of JEE Main 2024 Session I & II
- 12 Yrs JEE Mains New Syllabus Questions
- As per latest NCERT Syllabus of Class 11 & 12

20th Edition

DISHA™ Publication Inc

DISHA Publication Inc.
A-23 FIEE Complex, Okhla Phase II
New Delhi-110020
Tel: 49842349/ 49842350

Edited By
Raghvendra Kumar Sinha

Typeset By
DISHA DTP Team

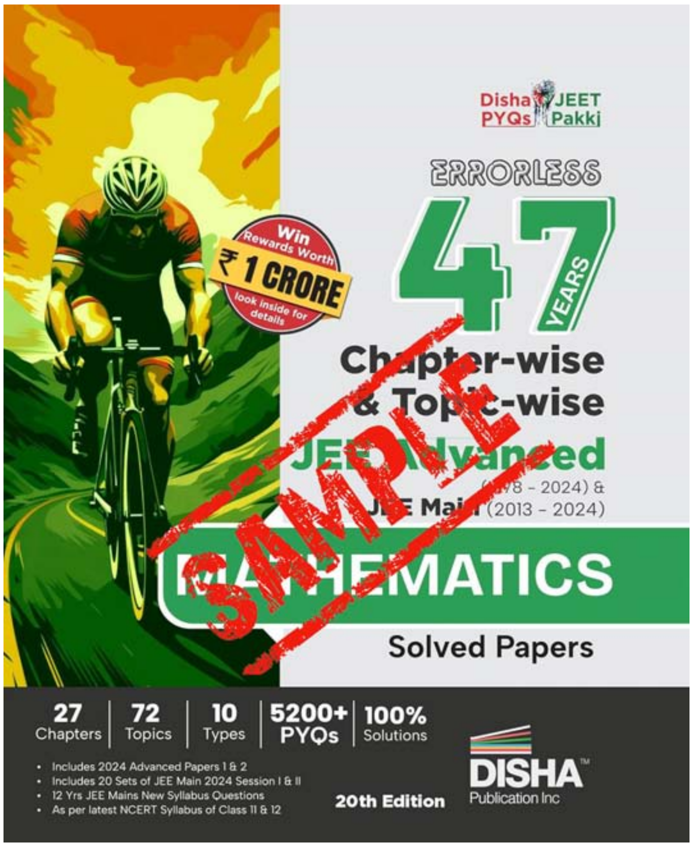
Write To Us At
feedback_disha@aiets.co.in

www.dishapublication.com

DISHA™ Publication Inc

Free Sample Contents

This sample book is prepared from the book **"Errorless 47 Years Chapter-wise & Topic-wise JEE Advanced (1978 - 2024) & JEE Main (2013 - 2024) MATHEMATICS Solved Papers 20th Edition | PYQ Question Bank in NCERT Flow for JEE 2025 ".**



ISBN - 978-9362257192

MRP- 900/-

In case you like this content, you can buy the **Physical Book** or **E-book** using the ISBN provided above.

The book & e-book are available on all leading online stores.

Contents

A1 – A348

Class XIth A1 – A116

Hints & Solutions (Class XIth) A117–A348

B1 – B448

Class XIIth B1 – B152

Hints & Solutions (Class XIIth) B153–B448

11 Three Dimensional Geometry

Topic-1: Direction Ratios & Direction cosines of a Line, Angle between two lines in terms of dc's and dr's, Projection of a Point on a Line

1 MCQs with One Correct Answer

1. Let P(x, y, z) be a point in the first octant, whose projection in the xy-plane is the point Q. Let OP = γ; the angle between OQ and the positive x-axis be θ; and the angle between OP and the positive z-axis be ϕ, where O is the origin. Then the distance of P from the x-axis is : **[April 8, 2024 (I)]**

(a) $\gamma\sqrt{1-\sin^2\phi\cos^2\theta}$ (b) $\gamma\sqrt{1+\cos^2\theta\sin^2\phi}$

(c) $\gamma\sqrt{1-\sin^2\theta\cos^2\phi}$ (d) $\gamma\sqrt{1+\cos^2\phi\sin^2\theta}$

2. Let P(3, 2, 3), Q(4, 6, 2) and R(7, 3, 2) be the vertices of ΔPQR. Then, the angle $\angle QPR$ is **[Jan. 29, 2024(II)]**

(a) $\cos^{-1}\left(\frac{7}{18}\right)$ (b) $\frac{\pi}{3}$

(c) $\cos^{-1}\left(\frac{1}{18}\right)$ (d) $\frac{\pi}{6}$

3. If two straight lines whose direction cosines are given by the relations $l + m - n = 0$, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is: **[June 27, 2022 (I)]**

(a) 6 (b) 4 (c) 3 (d) 2

4. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is: **[2013 (S), 2014 (S), 2018 (S) Feb. 25, 2021 (I)]**

(a) $\frac{3}{4}$ (b) $\frac{5}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{8}$

5. If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q(8, 0, 10), then distance of R from the origin is : **[April 08, 2019 (II)]**

(a) $2\sqrt{14}$ (b) $2\sqrt{21}$ (c) 6 (d) $\sqrt{53}$

6. ABC is triangle in a plane with vertices A (2, 3, 5), B (–1, 3, 2) and C (λ, 5, μ). If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3 + \mu^3 + 5)$ is: **[2013 (S), 2014 (S), Online April 10, 2016]**

(a) 1130 (b) 1348 (c) 1077 (d) 676

7. A line in the 3-dimensional space makes an angle θ $\left(0 < \theta \le \frac{\pi}{2}\right)$ with both the x and y axes. Then the set of all values of θ is the interval: **[Online April 9, 2014]**

(a) $\left(0, \frac{\pi}{4}\right]$ (b) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

(c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$

8. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are : **[2009, 2013 (S)]**

(a) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (b) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

(c) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (d) $6, -3, 2$

9. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals **[2007 (S), 2010]**

(a) 45° (b) 60° (c) 75° (d) 30°

10. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals **[2004]**

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

3 Numeric/ New Stem Based Questions

11. Consider a line L passing through the points P(1,2,1) and Q(2,1, –1). If the mirror image of the point A(2,2,2) in the line L is (α, β, γ), then $\alpha + \beta + 6\gamma$ is equal to ...... **[April. 4, 2024(II)]**

12. Let $P(-2, -1, 1)$ and $Q\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ be the vertices of the rhombus *PRQS*. If the direction ratios of the diagonal *RS* are $\alpha, -1, \beta$, where both α and β are integers of minimum absolute values, then $\alpha^2 + \beta^2$ is equal to ________. **[July 28, 2022(I)]**

Topic-2: Equation of a Straight Line in Cartesian and Vector Form, Angle Between two Lines, Distance Between two Parallel Lines

1 MCQs with One Correct Answer

1. Let the line L intersect the lines $x - 2 = -y = z - 1$, $2(x + 1) = 2(y - 1) = z + 1$ and be parallel to the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{2}$. Then which of the following points lies on L ? **[April 9, 2024 (I)]**

(a) $\left(-\frac{1}{3}, 1, 1\right)$ (b) $\left(-\frac{1}{3}, 1, -1\right)$

(c) $\left(-\frac{1}{3}, -1, -1\right)$ (d) $\left(-\frac{1}{3}, -1, 1\right)$

2. The shortest distance between the line

$$\frac{x-3}{4} = \frac{y+7}{-11} = \frac{z-1}{5} \text{ and } \frac{x-5}{3} = \frac{y-9}{-6} = \frac{z+2}{1} \text{ is :}$$

[April 9, 2024 (I)]

(a) $\frac{187}{\sqrt{563}}$ (b) $\frac{178}{\sqrt{563}}$

(c) $\frac{185}{\sqrt{563}}$ (d) $\frac{179}{\sqrt{563}}$

3. Consider the line L passing through the points (1, 2, 3) and (2, 3, 5). The distance of the point $\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$ is equal to: **[April 9, 2024 (II)]**

(a) 3 (b) 5

(c) 4 (d) 6

4. If the shortest distance between the lines

$$L_1 : \vec{r} = (2+\lambda)\hat{i} + (1-3\lambda)\hat{j} + (3+4\lambda)\hat{k}, \lambda \in \mathbb{R}$$

$$L_2 : \vec{r} = 2(1+\mu)\hat{i} + 3(1+\mu)\hat{j} + (5+\mu)\hat{k}, \mu \in \mathbb{R}$$

is $\frac{m}{\sqrt{n}}$, where gcd (m, n) = 1, then the value of m + n equals. **[April 8, 2024 (I)]**

(a) 384 (b) 387

(c) 377 (d) 390

5. If the shortest distance between the lines $\frac{x-\lambda}{2} = \frac{y-4}{3} = \frac{z-3}{4}$ and $\frac{x-2}{4} = \frac{y-4}{6} = \frac{z-7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of λ is: **[April 8, 2024 (II)]**

(a) $-\frac{13}{25}$ (b) $\frac{13}{25}$

(c) 1 (d) –1

6. The shortest distance between the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ is **[April. 6, 2024(I)]**

(a) $6\sqrt{3}$ (b) $4\sqrt{3}$

(c) $5\sqrt{3}$ (d) $8\sqrt{3}$

7. Let P (α, β, γ) be the image of the point Q(3, –3, 1) in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and R be the point (2, 5, –1). If the area of the triangle PQR is λ and $\lambda^2 = 14K$, then K is equal to: **[April 6, 2024(II)]**

(a) 36 (b) 72

(c) 18 (d) 81

8. Let d be the distance of the point of intersection of the lines $\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}$ and $\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}$ from the point (7, 8, 9). Then d^2+6 is equal to : **[April. 5, 2024(I)]**

(a) 72 (b) 69
(c) 75 (d) 78

9. If the line $\frac{2-x}{3}=\frac{3y-2}{4\lambda+1}=4-z$ makes a right angle with the line $\frac{x+3}{3\mu}=\frac{1-2y}{6}=\frac{5-z}{7}$, then $4\lambda+9\mu$ is equal to: **[April. 5, 2024(I)]**

(a) 13 (b) 4 (c) 5 (d) 6

10. Let (α, β, γ) be the point (8, 5, 7) in the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{5}$. Then $\alpha+\beta+\gamma$ is equal to **[April 5, 2024 (II)]**

(a) 16 (b) 18
(c) 14 (d) 20

11. Let the point, on the line passing through the points P(1, –2, 3) and Q(5, –4, 7), farther from the origin and at a distance of 9 units from the point P, be (α, β, γ). Then $\alpha^2+\beta^2+\gamma^2$ is equal to : **[April. 4, 2024(I)]**

(a) 155 (b) 150 (c) 160 (d) 165

12. Let P be the point of intersection of the lines $\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}$ and $\frac{x-3}{2}=\frac{y-3}{3}=\frac{z-3}{2}$. Then, the shortest distance of P from the line $4x=2y=z$ is **[April. 4, 2024(II)]**

(a) $\frac{5\sqrt{14}}{7}$ (b) $\frac{\sqrt{14}}{7}$
(c) $\frac{3\sqrt{14}}{7}$ (d) $\frac{6\sqrt{14}}{7}$

13. If the shortest distance between the lines $\frac{x-\lambda}{-2}=\frac{y-2}{1}=\frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1}=\frac{y-1}{-2}=\frac{z-2}{1}$ is 1, then the sum of all possible values of λ is:

(a) 0 (b) $2\sqrt{3}$ (c) $3\sqrt{3}$ (d) $-2\sqrt{3}$

[Feb. 1, 2024(I)]

14. Let P and Q be the points on the line $\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$ which are at a distance of 6 units from the point R (1, 2, 3). If the centroid of the triangle PQR is (α, β, γ) then $\alpha^2+\beta^2+\gamma^2$ is: **[Feb. 1, 2024(II)]**

(a) 18 (b) 24 (c) 26 (d) 36

15. If the mirror image of the point P(3, 4, 9) in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α, β, γ), then $14(\alpha+\beta+\gamma)$ is: **[Feb. 1, 2024(II)]**

(a) 102 (b) 138 (c) 132 (d) 108

16. The distance of the point Q(0, 2, – 2) form the line passing through the point P(5, – 4, 3) and perpendicular to the lines $\vec{r}=\left(-3\hat{i}+2\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+5\hat{k}\right)$, $\lambda \in \mathbb{R}$ and $\vec{r}=\left(\hat{i}-2\hat{j}+\hat{k}\right)+\mu\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$, $\mu \in \mathbb{R}$ is : **[Jan. 31, 2024(I)]**

(a) $\sqrt{54}$ (b) $\sqrt{86}$
(c) $\sqrt{74}$ (d) $\sqrt{20}$

17. Let (α, β, γ) be mirror image of the point (2, 3, 5) in the line $\frac{x-1}{2}-\frac{y-2}{3}-\frac{z-3}{4}$. Then $2\alpha+3\beta+4\gamma$ is equal to **[Jan. 31, 2024(II)]**

(a) 32 (b) 33 (c) 31 (d) 34

18. The shortest distance between lines L_1 and L_2, where $L_1:\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+4}{2}$ and L_2 is the line passing through the points A (–4, 4, 3). B (–1, 6, 3) and perpendicular to the line $\frac{x-3}{-2}=\frac{y}{3}=\frac{z-1}{1}$, is **[Jan. 31, 2024(II)]**

(a) $\frac{121}{\sqrt{221}}$ (b) $\frac{24}{\sqrt{117}}$
(c) $\frac{141}{\sqrt{221}}$ (d) $\frac{42}{\sqrt{117}}$

19. Let (α, β, γ) be the foot of perpendicular from the point (1, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. Then $19(\alpha+\beta+\gamma)$ is equal to: **[Jan. 30, 2024(I)]**

(a) 102 (b) 101 (c) 99 (d) 100

20. Let $L_1: \vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+2\hat{k}\right), \lambda \in R$

$L_2: \vec{r}=\left(\hat{j}-\hat{k}\right)+\mu\left(3\hat{i}+\hat{j}+p\hat{k}\right), \mu \in IR$, and

$L_3: \vec{r}=\delta\left(l\hat{i}+m\hat{j}+n\hat{k}\right), \delta \in IR$

be three lines such that L_1 is perpendicular to L_2 and L_3 is perpendicular to both L_1 and L_2. Then, the point which lies on L_3 is **[Jan. 30, 2024(II)]**

(a) (–1, 7, 4) (b) (–, 1–7, 4)
(c) (1, 7, –4) (d) (1, – 7, 4)

21. Let PQR be a triangle with R(–1, 4, 2). Suppose M (2, 1, 2) is the mid point of PQ. The distance of the centroid of ΔPQR from the point of intersection of the lines $\frac{x-2}{0}=\frac{y}{2}=\frac{z+3}{-1}$ and $\frac{x-1}{1}=\frac{y+3}{-3}=\frac{z+1}{1}$ is **[Jan. 29, 2024(I)]**

(a) $\sqrt{99}$ (b) 9 (c) $\sqrt{69}$ (d) 69

22. The distance, of the point (7, –2, 11) from the line $\frac{x-6}{1}=\frac{y-4}{0}=\frac{z-8}{3}$ along the line $\frac{x-5}{2}=\frac{y-1}{-3}=\frac{z-5}{6}$, is : **[Jan. 27, 2024(I)]**

(a) 12 (b) 14 (c) 18 (d) 21

23. If the shortest distance between the lines $\frac{x-4}{1}=\frac{y+1}{2}=\frac{z}{-3}$ and $\frac{x-\lambda}{2}=\frac{y+1}{4}=\frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is : **[Jan. 27, 2024(I)]**

(a) 5 (b) 8
(c) 7 (d) 10

24. Let the image of the point (1, 0, 7) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ be the point (α, β, γ). Then which one of the following point lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y - axis and z - axis respectively and an acute angle with x - axis? **[Jan. 27, 2024(II)]**

(a) $(1,-2,1+\sqrt{2})$ (b) $(1,2,1-\sqrt{2})$
(c) $(3,4,3-2\sqrt{2})$ (d) $(3,-4,3+2\sqrt{2})$

25. Let Q be the cube with the set of vertices $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1, x_2, x_3 \in \{0,1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0, 0, 0) and (1, 1, 1) is in S. For lines ℓ_1 and ℓ_2, let d(ℓ_1, ℓ_2) denote the shortest distance between them. Then the maximum value of d(ℓ_1, ℓ_2), as ℓ_1 varies over F and ℓ_2 varies over S, is **[Adv. 2023]**

(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{\sqrt{8}}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{12}}$

26. Let S be the set of all values of λ, for which the shortest distance between the lines $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ and $\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ is 13. Then $8\left|\sum_{\lambda \in S}\lambda\right|$ is equal to **[April 15, 2023 (I)]**

(a) 304 (b) 308 (c) 306 (d) 302

27. The shortest distance between the lines $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ is **[April 10, 2023 (I)]**

(a) 6 (b) 9 (c) 7 (d) 8

28. The shortest distance between the lines $\frac{x-4}{4}=\frac{y+2}{5}=\frac{z+3}{3}$ and $\frac{x-1}{3}=\frac{y-3}{4}=\frac{z-4}{2}$ is **[April 8, 2023 (I)]**

(a) $3\sqrt{6}$ (b) $6\sqrt{3}$
(c) $6\sqrt{2}$ (d) $2\sqrt{6}$

29. One vertex of a rectangular parallelopiped is at the origin O and the lengths of its edges along x, y and z axes are 3, 4 and 5 units respectively. Let P be the vertex (3, 4, 5). Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is: **[April 6, 2023 (I)]**

(a) $\frac{12}{\sqrt{5}}$ (b) $\frac{12}{5\sqrt{5}}$ (c) $12\sqrt{5}$ (d) $\frac{12}{5}$

30. The shortest distance between the lines $\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$ and $\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$ is **[Feb. 1, 2023 (I)]**

(a) $4\sqrt{3}$ (b) $7\sqrt{3}$ (c) $5\sqrt{3}$ (d) $6\sqrt{3}$

31. The line l_1 passes through the point (2, 6, 2) and is perpendicular to the plane 2x + y – 2z = 10. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2}=\frac{y+4}{-3}=\frac{z}{2}$ is: **[Jan. 30, 2023 (I)]**

(a) 7 (b) $\frac{19}{3}$ (c) $\frac{19}{3}$ (d) 9

32. Shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ is **[Jan. 29, 2023 (II)]**

(a) $2\sqrt{3}$ (b) $4\sqrt{3}$ (c) $3\sqrt{3}$ (d) $5\sqrt{3}$

33. The distance of the point P(4, 6, –2) from the line passing through the point (–3, 2, 3) and parallel to a line with direction ratios 3, 3, –1 is equal to : **[Jan. 25, 2023 (I)]**

(a) 3 (b) $\sqrt{6}$ (c) $2\sqrt{3}$ (d) $\sqrt{14}$

34. Consider the lines L_1 and L_2 given by

$$L_1 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2} \qquad L_2 : \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

A line L_3 having direction ratios 1, –1, –2, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is **[Jan. 25, 2023 (I)]**

(a) $2\sqrt{6}$ (b) $3\sqrt{2}$

(c) $4\sqrt{3}$ (d) 4

35. The shortest distance between the lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ is **[Jan. 25, 2023 (II)]**

(a) 2 (b) 3 (c) $\frac{5}{2}$ (d) $\frac{3}{2}$

36. The foot of perpendicular of the point (2, 0, 5) on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ). Then. Which of the following is NOT correct? **[Jan. 25, 2023 (II)]**

(a) $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$ (b) $\frac{\alpha}{\beta} = -8$

(c) $\frac{\beta}{\gamma} = -5$ (d) $\frac{\gamma}{\alpha} = \frac{5}{8}$

37. If the length of the perpendicular drawn from the point $P(a, 4, 2), a > 0$ on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ is $2\sqrt{6}$ units and $Q(\alpha_1, \alpha_2, \alpha_3)$ is the image of the point P in this line, then $a + \sum_{i=1}^{3} \alpha_i$ is equal to: **[July 27, 2022 (II)]**

(a) 7 (b) 8 (c) 12 (d) 14

38. The lines $\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$ **[Sep. 03, 2020 (I)]**

(a) do not intersect for any values of l and m

(b) intersect for all values of l and m

(c) intersect when $l = 2$ and $m = \frac{1}{2}$

(d) intersect when $l = 1$ and $m = 2$

39. If the length of the perpendicular from the point (β, 0, β) (β ≠ 0) to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to: **[April 10, 2019 (I)]**

(a) 1 (b) 2 (c) –1 (d) –2

40. If the angle between the lines, $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{5-x}{-2} = \frac{7y-14}{P} = \frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then P is equal to **[2005 (S), Online April 16, 2018]**

(a) $-\frac{7}{4}$ (b) $\frac{2}{7}$ (c) $-\frac{4}{7}$ (d) $\frac{7}{2}$

41. Equation of the line of the shortest distance between the lines $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ and $\frac{x-1}{0} = \frac{y+1}{-2} = \frac{z}{1}$ is: **[Online April 19, 2014]**

(a) $\frac{x}{1} = \frac{y}{-1} = \frac{z}{-2}$ (b) $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{-2}$

(c) $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{1}$ (d) $\frac{x}{-2} = \frac{y}{1} = \frac{z}{2}$

42. If two lines L_1 and L_2 in space, are defined by

$L_1 = \left\{x = \sqrt{\lambda}y + \left(\sqrt{\lambda} - 1\right),\ z = \left(\sqrt{\lambda} - 1\right)y + \sqrt{\lambda}\right\}$ and

$L_2 = \left\{x = \sqrt{\mu}y + \left(1 - \sqrt{\mu}\right),\ z = \left(1 - \sqrt{\mu}\right)y + \sqrt{\mu}\right\}$

then L_1 is perpendicular to L_2, for all non-negative reals λ and μ, such that : **[Online April 23, 2013]**

(a) $\sqrt{\lambda} + \sqrt{\mu} = 1$ (b) $\lambda \neq \mu$

(c) $\lambda + \mu = 0$ (d) $\lambda = \mu$

43. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is **[2004S]**

(a) 3/2 (b) 9/2 (c) – 2/9 (d) – 3/2

3 Numeric/ New Stem Based Questions

44. The square of the distance of the image of the point (6, 1, 5) in the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$, from the origin is ______. **[April 9, 2024 (II)]**

45. Let P(α, β, γ) be the image of the point Q(l, 6, 4) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Then $2\alpha+\beta+\gamma$ is equal to _______.

[April 8, 2024 (II)]

46. Let P be the point (10, –2, –1) and Q be the foot of the perpendicular drawn from the point R(1, 7, 6) on the line passing through the points (2, –5, 11) and (–6, 7, –5). Then the length of the line segment PQ is equal to ________.

[April. 6, 2024(I)]

47. If the shortest distance between the lines

$$\frac{x-\lambda}{3}=\frac{y-2}{-1}=\frac{z-1}{1} \text{ and } \frac{x+2}{-3}=\frac{y+5}{2}=\frac{z-4}{4} \text{ is } \frac{44}{\sqrt{30}},$$

then the largest possible value of $|\lambda|$ is equal to ________.

[April 6, 2024(II)]

48. Let the point (–1, α, β) lie on the line of the shortest distance between the lines $\frac{x+2}{-3}=\frac{y-2}{4}=\frac{z-5}{2}$ and $\frac{x+2}{-1}=\frac{y+6}{2}=\frac{z-1}{0}$. Then $(\alpha-\beta)^2$ is equal to _______.

[April 5, 2024 (II)]

49. If the shortest distance between the lines

$\frac{x+2}{2}=\frac{y+3}{3}=\frac{z-5}{4}$ and $\frac{x-3}{1}=\frac{y+2}{-3}=\frac{z+4}{2}$ is $\frac{38}{3\sqrt{5}}K$ and $\int_0^k\left[x^2\right]dx=\alpha-\sqrt{\alpha}$, where [x] denotes the greatest integer function, then $6\alpha^3$ is equal to _____

[April. 4, 2024(I)]

50. Let the line of the shortest distance between the lines

$L_1:\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ and

$L_2:\vec{r}=\left(4\hat{i}+5\hat{j}+6\hat{k}\right)+\mu\left(\hat{i}+\hat{j}+\hat{k}\right)$

intersects L_1 and L_2 at P and Q respectively. If (α, β, γ) is the mid point of line segment PQ, then $2(\alpha+\beta+\gamma)$ is equal to _______ .

[Feb. 1, 2024(I)]

51. Let Q and R be the feet of perpendiculars from the point P(a, a, a) on the lines x = y, z = 1 and x = – y, z = – 1 respectively. If ∠QPR is a right angle, then $12a^2$ is equal to ______

[Jan. 31, 2024(I)]

52. A line passes through A(4, –6, –2) and B(16, –2, 4). The point P(a, b, c) where a, b, c are non-negative integers, on the line AB lies at a distance of 21 units, from the point A. The distance between the points P(a, b, c) and Q(4, –12, 3) is equal to __________.

[Jan. 31, 2024(II)]

53. If d_1 is the shortest distance between the lines $x+1=2y=-12z$, $x=y+2=6z-6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$, $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}d_1}{d_2}$ is:

[Jan. 30, 2024(I)]

54. Let a line passing through the point (–1, 2, 3) intersect the lines $L_1:\frac{x-1}{3}=\frac{y-2}{2}=\frac{z+1}{-2}$ at $M(\alpha,\beta,\gamma)$ and $L_2:\frac{x+2}{-3}=\frac{y-2}{-2}=\frac{z-1}{4}$ at $N(a, b, c)$. Then, the value of $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}$ equals ________ .

[Jan. 30, 2024(II)]

55. A line with direction ratios 2, 1, 2 meets the lines x = y + 2 = z and x + 2 = 2y = 2z respectively at the points P and Q. If the length of the perpendicular from the point (1, 2, 12) to the line PQ is l, then l^2 is ______.

[Jan. 29, 2024(I)]

56. Let O be the origin, and M and N be the point on the lines $\frac{x-5}{4}=\frac{y-4}{1}=\frac{z-5}{3}$ and $\frac{x+8}{12}=\frac{y+2}{5}=\frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overrightarrow{OM}.\overrightarrow{ON}$ is equal to ______.

[Jan. 29, 2024(II)]

57. The lines $\frac{x-2}{2}=\frac{y}{-2}=\frac{z-7}{16}$ and $\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2}=\frac{y-1}{3}=\frac{z-1}{1}$ is l, then $14l^2$ is equal to_____.

[Jan. 27, 2024(II)]

58. Let a line ℓ pass through the origin and be perpendicular to the lines

$\ell_1:\vec{r}=\left(\hat{i}-11\hat{j}-7\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right),\ \lambda\in\mathbb{R}$

and $\ell_2:\vec{r}=\left(-\hat{i}+\hat{k}\right)+\mu\left(2\hat{i}+2\hat{j}+\hat{k}\right),\ \mu\in\mathbb{R}$.

If P is the point of intersection of ℓ and ℓ_1, and Q(α, β, λ) is the foot of perpendicular from P on ℓ_2, then $9(\alpha+\beta+\gamma)$ is equal to ______.

[April 11, 2023 (I)]

59. If the lines $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}$ and $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}$ intersect, then the magnitude of the minimum value of $8\alpha\beta$ is ________.

[April 6, 2023 (II)]

60. Let a line L pass through the point P(2, 3, 1) and be parallel to the line $x + 3y - 2z - 2 = 0 = x - y + 2z$.
If the distance of L from the point (5, 3, 8) is α, then $3\alpha^2$ is equal to _____.

[Jan. 30, 2023 (II)]

61. Let the co-ordinates of one vertex of ΔABC be A(0, 2, α) and the other two vertices lie on the line $\frac{x+\alpha}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. For $\alpha \in \mathbb{Z}$ if the area of ΔABC is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to _____.

[Jan. 29, 2023 (I)]

62. If the shortest distance between the line joining the points(1, 2, 3) and (2, 3, 4), and the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to _____.

[Jan. 25, 2023 (II)]

63. The shortest distance between the lines $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}$ and $\frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{0}$ is equal to ________.

[Jan. 24, 2023 (I)]

64. If the shortest between the lines

$$\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4} \text{ and}$$

$$\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$$

is 6, then the square of sum of all possible values of λ is

[Jan. 24, 2023 (II)]

65. Let Q and R be two points on the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point P(4, 2, 7). Then the square of the area of the triangle PQR is _______.

[July 26, 2022 (I)]

66. If the shortest distance between the lines
$\vec{r}_1=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right)$, $\lambda \in R$, $\alpha > 0$ and $\vec{r}_2=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$, $\mu \in R$ is 9, then α is equal to ______ .

[Jul, 20, 2021 (I)]

67. If the foot of the perpendicular drawn from the point (1, 0, 3) on *a* line passing through (α, 7, 1) is (5/3, 7/3, 17/3), then α is equal to _________.

[Jan. 07, 2020 (II)]

6 MCQs with One or More than One Correct Answer

68. Three lines $L_1 : \vec{r}=\lambda\hat{i}, \lambda \in R$

$L_2 : \vec{r}=\hat{k}+\mu\hat{j}, \mu \in R$ and

$L_3 : \vec{r}=\hat{i}+\hat{j}+v\hat{k}, v \in R$

are given. For which point(s) *Q* on L_2 can we find a point *P* on L_1 and a point *R* on L_3 so that *P*, *Q* and *R* are collinear?

[Adv. 2019]

(a) $\hat{k}-\frac{1}{2}\hat{j}$ (b) $\hat{k}$ (c) $\hat{k}+\hat{j}$ (d) $\hat{k}+\frac{1}{2}\hat{j}$

69. Let L_1 and L_2 denote the lines

$\vec{r}=\hat{i}+\lambda(-\hat{i}+2\hat{j}+2\hat{k}), \lambda \in R$

and $\vec{r}=\mu(2\hat{i}-\hat{j}+2\hat{k}), \mu \in R$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3 ? **[Adv. 2019]**

(a) $\vec{r}=\frac{2}{9}(4\hat{i}+\hat{j}+\hat{k})+t(2\hat{i}+2\hat{j}-\hat{k}), t \in R$

(b) $\vec{r}=\frac{2}{9}(2\hat{i}-\hat{j}+2\hat{k})+t(2\hat{i}+2\hat{j}-\hat{k}), t \in R$

(c) $\vec{r}=t(2\hat{i}+2\hat{j}-\hat{k}), t \in R$

(d) $\vec{r}=\frac{1}{3}(2\hat{i}+\hat{k})+t(2\hat{i}+2\hat{j}-\hat{k}), t \in R$

70. From a point $P(\lambda, \lambda, \lambda)$, perpendicular *PQ* and *PR* are drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1$. If *P* is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is/(are) **[Adv. 2014]**

(a) $\sqrt{2}$ (b) 1 (c) -1 (d) $-\sqrt{2}$

71. A line *l* passing through the origin is perpendicular to the lines **[Adv. 2013]**

$$l_1 : (3+t)\hat{i}+(-1+2t)\hat{j}+(4+2t)\hat{k}, \quad -\infty < t < \infty$$

$$l_2 : (3+2s)\hat{i}+(3+2s)\hat{j}+(2+s)\hat{k}, \quad -\infty < s < \infty$$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of *l* and l_1 is (are)

(a) $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$ (b) $(-1,-1,0)$

(c) $(1, 1, 1)$ (d) $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

7 Match the Following

72. Let $\gamma \in \mathbb{R}$ be such that the lines

$$L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3} \text{ and } L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma}$$

intersect. Let R_1 be the point of intersection of L_1 and L_2. Let O = (0, 0, 0), and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2. Match each entry in **List-I** to the correct entry in **List-II**.

List-I	List-II
(P) γ equals	(1) $-\hat{i}-\hat{j}+\hat{k}$
(Q) A possible choice for $\hat{n}$ is	(2) $\sqrt{\frac{3}{2}}$
(R) $\overrightarrow{OR_1}$ equals	(3) 1
(S) A possible value of $\overrightarrow{OR_1}.\hat{n}$ is	(4) $\frac{1}{\sqrt{6}}\hat{i}-\frac{2}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}$
	(5) $\sqrt{\frac{2}{3}}$

The correct option is **[Adv. 2024]**

(a) (P)→(3) (Q)→(4) (R)→(1) (S)→(2)
(b) (P)→(5) (Q)→(4) (R)→(1) (S)→(2)
(c) (P)→(3) (Q)→(4) (R)→(1) (S)→(5)
(d) (P)→(3) (Q)→(1) (R)→(4) (S)→(5)

73. Match the statement in **Column-I** with the values in **Column -II** **[2010]**

Column – I	Column – II
(A) A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length $PQ = d$, then d^2 is	(p) -4
(B) The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0
(C) Non-zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b}=0$. $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$ and $2\lvert\vec{b}+\vec{c}\rvert=\lvert\vec{b}-\vec{a}\rvert$. If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	(r) 4
(D) Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x)=\sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for $x \neq 0$	(s) 5
The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ is	(t) 6

Topic-3: Equation of a Plane in Different Forms, Equation of a Plane Passing Through the Intersection of two Given Planes, Projection of a Line on a Plane

1 MCQs with One Correct Answer

1. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1 ,–1) is **[2012]**

(a) $5x - 11y + z = 17$ (b) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(c) $x + y + z = \sqrt{3}$ (d) $x - \sqrt{2}y = 1 - \sqrt{2}$

2. The point P is the intersection of the straight line joining the points $Q(2, 3, 5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to QR, then the length of the line segment PS is **[2012]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

3. If the distance of the point $P(1, -2, 1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is **[2010]**

(a) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (b) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$

(c) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (d) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

4. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is **[2010]**

(a) $x + 2y - 2z = 0$ (b) $3x + 2y - 2z = 0$

(c) $x - 2y + z = 0$ (d) $5x + 2y - 4z = 0$

5. A line with positive direction cosines passes through the point $P(2, -1, 2)$ and makes equal angles with the coordinate axes. The line meets the plane

$$2x + y + z = 9$$

at point Q. The length of the line segment PQ equals **[2009]**

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

6. Let P (3, 2, 6) be a point in space and Q be a point on the line

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$$

Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is **[2009]**

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

7. A plane which is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, passes through (1, –2, 1). The distance of the plane from the point (1, 2, 2) is **[2006 - 3M, –1]**

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

8. A variable plane at a distance of the one unit from the origin cuts the coordinates axes at A, B and C. If the centroid $D(x, y, z)$ of triangle ABC satisfies the relation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$, then the value k is **[2005S]**

(a) 3 (b) 1 (c) $\frac{1}{3}$ (d) 9

9. The value of k such that $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, is **[2003S]**

(a) 7 (b) –7

(c) no real value (d) 4

2 Integer Value Answer/ Non-Negative Integer

10. Let P be the plane $\sqrt{3}x + 2y + 3z = 16$ and let $S = \left\{\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1 \text{ and the distance of } (\alpha, \beta, \gamma) \text{ from the plane P is } \frac{7}{2}\right\}$. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u} - \vec{v}| = |\vec{v} - \vec{w}| = |\vec{w} - \vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u}, \vec{v}$ and $\vec{w}$. Then the value of $\frac{80}{\sqrt{3}}V$ is **[Adv. 2023]**

11. Let P be a point in the first octant, whose image Q in the plane $x + y = 3$ (that is, the line segment PQ is perpendicular to the plane $x + y = 3$ and the mid-point of PQ lies in the plane $x + y = 3$) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is _____ . **[Adv. 2018]**

12. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then find $|d|$. **[2010]**

3 Numeric/ New Stem Based Questions

13. Three lines are given by $\vec{r} = \lambda\hat{i}, \lambda \in R$;

$\vec{r} = \mu(\hat{i} + \hat{j}), \mu \in R$ and $\vec{r} = v(\hat{i} + \hat{j} + \hat{k}), v \in R$.

Let the lines cut the plane $x + y + z = 1$ at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals ____. **[Adv. 2019]**

4 Fill in the Blanks

14. A nonzero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j},\ \hat{i}+\hat{k}$. The angle between $\vec{a}$ and the vector $\hat{i}-2\hat{j}+2\hat{k}$ is .... **[1996 - 2 Marks]**

15. The unit vector perpendicular to the plane determined by $P(1, -1, 2)$, $Q(2, 0, -1)$ and $R(0, 2, 1)$ is .... **[1983 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

16. Let $\mathbb{R}$ denote the three-dimensional space. Take two points $P = (1, 2, 3)$ and $Q = (4, 2, 7)$. Let dist (X, Y) denote the distance between two points X and Y in $\mathbb{R}^3$. Let

$S = \{X \in \mathbb{R}^3 : (\text{dist}(X, P))^2 - (\text{dist}(X, Q))^2 = 50\}$ and

$T = \{Y \in \mathbb{R}^3 : (\text{dist}(Y, Q))^2 - (\text{dist}(Y, P))^2 = 50\}$.

Then which of the following statements is (are) TRUE? **[Adv. 2024]**

(a) There is a triangle whose area is 1 and all of whose vertices are from S.

(b) There are two distinct points L and M in T such that each point on the line segment LM is also in T.

(c) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

(d) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

17. A straight line drawn from the point P (1, 3, 2), parallel to the line $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z-6}{1}$, intersects the plane $L_1: x-y+3z=6$ at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane $L_2: 2x-y+z=-4$ at the point R. Then which of the following statements is (are) TRUE? **[Adv. 2024]**

(a) The length of the line segment PQ is $\sqrt{6}$

(b) The coordinates of R are (1, 6, 3)

(c) The centroid of the triangle PQR is $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$

(d) The perimeter of the triangle PQR is $\sqrt{2}+\sqrt{6}+\sqrt{11}$

18. Let P_1 and P_2 be two planes given by **[Adv. 2022]**

P_1: $10x + 15y + 12z - 60 = 0$,

P_2: $-2x + 5y + 4z - 20 = 0$.

Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2 ?

(a) $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{5}$ (b) $\frac{x-6}{-5}=\frac{y}{2}=\frac{z}{3}$

(c) $\frac{x}{-2}=\frac{y-4}{5}=\frac{z}{4}$ (d) $\frac{x}{1}=\frac{y-4}{-2}=\frac{z}{3}$

19. Let S be the reflection of a point Q with respect to the plane given by

$\vec{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$

where t, p are real parameters and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i}+15\hat{j}+20\hat{k}$ and $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ respectively, then which of the following is/are TRUE ? **[Adv. 2022]**

(a) $3(\alpha+\beta) = -101$ (b) $3(\beta+\gamma) = -71$

(c) $3(\gamma+\alpha) = -86$ (d) $3(\alpha+\beta+\gamma) = -121$

20. Let L_1 and L_2 be the following straight line.

$$L_1 : \frac{x-1}{1}=\frac{y}{-1}=\frac{z-1}{3} \text{ and } L_2 : \frac{x-1}{-3}=\frac{y}{-1}=\frac{z-1}{1}.$$

Suppose the straight line

$$L : \frac{x-\alpha}{l}=\frac{y-1}{m}=\frac{z-\gamma}{-2}$$

lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE? **[Adv. 2020]**

(a) $\alpha - \gamma = 3$ (b) $l + m = 2$

(c) $\alpha - \gamma = 1$ (d) $l + m = 0$

21. Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2+\beta^2+\gamma^2 \neq 0$ and $\alpha+\gamma = 1$. Suppose the point (3, 2, –1) is the mirror image of the point (1, 0, –1) with respect to the plane $\alpha x + \beta y + \gamma z = \delta$.

Then which of the following statements is/are TRUE? **[Adv. 2020]**

(a) $\alpha + \beta = 2$ (b) $\delta - \gamma = 3$

(c) $\delta + \beta = 4$ (d) $\delta + \beta + \gamma = \delta$

22. Let $P_1: 2x + y - z = 3$ and $P_2: x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE? **[Adv. 2018]**

(a) The line of intersection of P_1 and P_2 has direction ratios 1, 2, – 1

(b) The line $\frac{3x-4}{9}=\frac{1-3y}{9}=\frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2

(c) The acute angle between P_1 and P_2 is 60°.

(d) If P_3 is the plane passing through the point (4, 2, – 2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$

23. Consider a pyramid OPQRS located in the first octant ($x \geq 0, y \geq 0, z \geq 0$) with O as origin, and OP and OR along the x–axis and the y–axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point, T of diagonal OQ such that TS = 3. Then **[Adv. 2016]**

(a) the acute angle between OQ and OS is $\frac{\pi}{3}$

(b) the equation of the plane containing the triangle OQS is $x - y = 0$

(c) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(d) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

24. In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie (s) on M? **[Adv. 2015]**

(a) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$ (b) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

(c) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$ (d) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

25. In R^3, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be the plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true? **[Adv. 2015]**

(a) $2\alpha + \beta + 2\gamma + 2 = 0$ (b) $2\alpha - \beta + 2\gamma + 4 = 0$

(c) $2\alpha + \beta - 2\gamma - 10 = 0$ (d) $2\alpha - \beta + 2\gamma - 8 = 0$

26. Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then α can take value(s) **[Adv. 2013]**

(a) 1 (b) 2 (c) 3 (d) 4

27. If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane (s) containing these two lines is (are) **[2012]**

(a) $y + 2z = -1$ (b) $y + z = -1$

(c) $y - z = -1$ (d) $y - 2z = -1$

28. Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2. Plane P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and that P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector $\vec{A}$ and a given vector $2\hat{i} + \hat{j} - 2\hat{k}$ is **[2006 - 5M, –1]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{4}$

7 Match the Following

29. Let ℓ_1 and ℓ_2 be the lines $\vec{r}_1 = \lambda(\hat{i} + \hat{j} + \hat{k})$ and $\vec{r}_2 = (\hat{j} - \hat{k}) + \mu(\hat{i} + \hat{k})$, respectively. Let X be the set of all the planes H that contain the line ℓ_1. For a plane H, let d(H) denote the smallest possible distance between the points of ℓ_2 and H. Let H_0 be a plane in X for which $d(H_0)$ is the maximum value of d(H) as H varies over all planes in X.

Match each entry in **List-I** to the correct entries in **List-II.**

List-I		List-II	
(P)	The value of $d(H_0)$ is	(1)	$\sqrt{3}$
(Q)	The distance of the point (0, 1, 2) from H_0 is	(2)	$\frac{1}{\sqrt{3}}$
(R)	The distance of origin from H_0 is	(3)	0
(S)	The distance of origin from the point of intersection of planes y = z, x = 1 and H_0 is	(4)	$\sqrt{2}$
		(5)	$\frac{1}{\sqrt{2}}$

The correct option is: **[Adv. 2023]**

(a) (P) → (2), (Q) → (4), (R) → (5), (S) → (1)

(b) (P) → (5), (Q) → (4), (R) → (3), (S) → (1)

(c) (P) → (2), (Q) → (1), (R) → (3), (S) → (2)

(d) (P) → (5), (Q) → (1), (R) → (4), (S) → (2)

30. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}, L_2 \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 = 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2.

Match List I with List II and select the correct answer using the code given below the lists : **[Adv. 2013]**

	List I		**List II**
P.	$a =$	1.	13
Q.	$b =$	2.	-3
R.	$c =$	3.	1
S.	$d =$	4.	-2

Codes:

	P	Q	R	S
(a)	3	2	4	1
(b)	1	3	4	2
(c)	3	2	1	4
(d)	2	4	1	3

31. Match the statements/expressions given in **Column-I** with the values given in **Column-II**. **[2009]**

	Column-I		**Column-II**
(A)	The number of solutions of the equation $x\, e^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p)	1
(B)	Value(s) of k for which the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	(q)	2
(C)	Value(s) of k for which $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x+1\rvert + \lvert x+2\rvert = 4k$ has integer solution(s)	(r)	3
(D)	If $y' = y + 1$ and $y(0) = 1$, then value(s) of $y(\ln 2)$	(s)	4
		(t)	5

32. Consider the following linear equations

$ax + by + cz = 0; \quad bx + cy + az = 0; \quad cx + ay + bz = 0$

Match the conditions/expressions in **Column I** with statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the *ORS*. **[2007]**

	Column I		**Column II**
(A)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p)	the equations represent planes meeting only at asingle point
(B)	$a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q)	the equations represent the line $x = y = z$.
(C)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r)	the equations represent identical planes.
(D)	$a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s)	the equations represent the whole of the three dimensional space.

33. Match the following : **[2006 - 6M]**

	Column I		**Column II**
(A)	Two rays $x + y = \lvert a\rvert$ and $ax - y = 1$ intersects each other in the first quadrant in the interval $a \in (a_0, \infty)$, the value of a_0 is	(p)	2
(B)	Point (α, β, γ) lies on the plane $x + y + z = 2$. Let $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\hat{k} \times (\hat{k} \times \vec{a}) = 0$, then $\gamma =$	(q)	$\frac{4}{3}$
(C)	$\left\lvert \int_0^1 (1 - y^2)\, dy \right\rvert + \left\lvert \int_1^0 (y^2 - 1)\, dy \right\rvert$	(r)	$\left\lvert \int_0^1 \sqrt{1 - x}\, dx \right\rvert + \left\lvert \int_{-1}^0 \sqrt{1 + x}\, dx \right\rvert$
(D)	If $\sin A \sin B \sin C + \cos A \cos B = 1$, then the value of $\sin C =$	(s)	1

8 Comprehension/Passage Based Questions

Passage

Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2} \quad L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

34. The unit vector perpendicular to both L_1 and L_2 is **[2008]**

(a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

35. The shortest distance between L_1 and L_2 is **[2008]**

(a) 0 (b) $\frac{17}{\sqrt{3}}$ (c) $\frac{41}{5\sqrt{3}}$ (d) $\frac{17}{5\sqrt{3}}$

36. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is **[2008]**

(a) $\frac{2}{\sqrt{75}}$ (b) $\frac{7}{\sqrt{75}}$ (c) $\pm\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$ (d) $\frac{23}{\sqrt{75}}$

9 Assertion and Reason/Statement Type Questions

37. Consider three planes

$P_1 : x - y + z = 1$ $\quad P_2 : x + y - z = -1$

$P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively.

statement - 1 : At least two of the lines L_1, L_2 and L_3 are non-parallel and

statement - 2 : The three planes doe not have a common point. **[2008]**

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is NOT** a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

38. Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

STATEMENT-1 : The parametric equations of the line of intersection of the given planes are $x = 3 + 14t$, $y = 1 + 2t$, $z = 15t$. **because**

STATEMENT-2 : The vector $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of given planes. **[2007 -3 marks]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

10 Subjective Problems

39. Find the equation of the plane containing the line $2x - y + z - 3 = 0$, $3x + y + z = 5$ and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, –1). **[2005 - 2 Marks]**

40. P_1 and P_2 are planes passing through origin. L_1 and L_2 are two line on P_1 and P_2 respectively such that their intersection is origin. Show that there exists points A, B, C, whose permutation A', B', C' can be chosen such that (i) A is on L_1, B on P_1 but not on L_1 and C not on P_1 (ii) A' is on L_2, B' on P_2 but not on L_2 and C' not on P_2 **[2004 - 4 Marks]**

41. A parallelepiped 'S' has base points A, B, C and D and upper face points A', B', C' and D'. This parallelepiped is compressed by upper face $A'B'C'D'$ to form a new parallelepiped 'T' having upper face points A'', B'', C'' and D''. Volume of parallelepiped T is 90 percent of the volume of parallelepiped S. Prove that the locus of 'A''', is a plane. **[2004 - 2 Marks]**

42. Find the equation of plane passing through (1, 1, 1) & parallel to the lines L_1, L_2 having direction ratios (1,0,–1), (1,–1,0). Find the volume of tetrahedron formed by origin and the points where these planes intersect the coordinate axes. **[2004 - 2 Marks]**

43. (i) Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).

(ii) If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it. **[2003 - 4 Marks]**

Answer Key

Topic-1 : Direction Ratios & Direction cosines of a Line, Angle between two lines in terms of dc's and dr's, Projection of a Point on a Line

1. (a) **2.** (b) **3.** (a) **4.** (b) **5.** (a) **6.** (b) **7.** (c) **8.** (b) **9.** (b)
10. (c) **11.** (6) **12.** (450)

Topic-2 : Equation of a Straight Line in Cartesian and Vector Form, Angle Between two Lines, Distance Between two Parallel Lines

1. (b) **2.** (a) **3.** (a) **4.** (b) **5.** (c) **6.** (b) **7.** (d) **8.** (c) **9.** (d) **10.** (c)
11. (a) **12.** (c) **13.** (b) **14.** (a) **15.** (d) **16.** (c) **17.** (b) **18.** (c) **19.** (b) **20.** (a)
21. (c) **22.** (b) **23.** (b) **24.** (c) **25.** (a) **26.** (c) **27.** (b) **28.** (a) **29.** (d) **30.** (d)
31. (d) **32.** (b) **33.** (d) **34.** (a) **35.** (a) **36.** (c) **37.** (b) **38.** (a) **39.** (c) **40.** (d)
41. (b) **42.** (d) **43.** (b) **44.** (62) **45.** (11) **46.** (13) **47.** (43) **48.** (25) **49.** (48) **50.** (21)
51. (12) **52.** (22) **53.** (16) **54.** (196) **55.** (65) **56.** (9) **57.** (108) **58.** (5) **59.** (18) **60.** (158)
61. (9) **62.** (18) **63.** (14) **64.** (384) **65.** (153) **66.** (6) **67.** (4) **68.** (a, d) **69.** (a, b, d)
70. (c) **71.** (b, d) **72.** (c) **73.** (A)-t; (B)-p, r; (C)-q, s; (D)-r

Topic-3 : Equation of a Plane in Different Forms, Equation of a Plane Passing Through the Intersection of two Given Planes, Projection of a Line on a Plane

1. (a) **2.** (a) **3.** (a) **4.** (c) **5.** (c) **6.** (a) **7.** (d) **8.** (d) **9.** (a) **10.** (45)
11. (8) **12.** (6) **13.** (0.75) **14.** $\left(\frac{\pi}{4} \text{ or } \frac{3\pi}{4}\right)$ **15.** $\pm\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$ **16.** (a, b, c) **17.** (a, c) **18.** (a. b) **19.** (a, b, c)
20. (a, b) **21.** (a, b, c) **22.** (c, d) **23.** (b, c, d) **24.** (a, b) **25.** (b, d) **26.** (a, d) **27.** (b, c) **28.** (b, d)
29. (b) **30.** (a) **31.** (A) $\to$ p; (B) $\to$ q, s; (C) $\to$ q, r, s, t; (D) $\to$ r **32.** (A) $\to$ r ; (B) $\to$ q; (C) $\to$ p ; (D) $\to$ s
33. (A) $\to$ (s); (B) $\to$ (p); (C) $\to$ (q),(r); (D) $\to$ (s) **34.** (b) **35.** (d) **36.** (c) **37.** (b) **38.** (d)

11 Three Dimensional Geometry

Topic-1: Direction Ratios & Direction cosines of a Line, Angle between two lines in terms of dc's and dr's, Projection of a Point on a Line

1. **(a)** Projection of P(x, y, z) on xy-plane = Q(x,y,0)

$x^2 + y^2 + z^2 = \gamma^2$

Now, $\overrightarrow{OQ} = x\hat{i} + y\hat{j}$

So, $\cos\theta = \dfrac{x}{\sqrt{x^2+y^2}}$ and $\cos\phi = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$

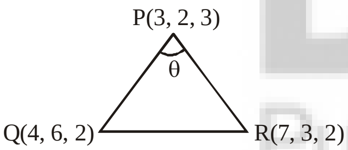
$\Rightarrow \sin^2\phi = \dfrac{x^2+y^2}{x^2+y^2+z^2}$

Distance of P from x-axis is $\sqrt{y^2+z^2}$

$\Rightarrow d = \sqrt{\gamma^2 - x^2} = \gamma\sqrt{1-\dfrac{x^2}{\gamma^2}}$ $\{\because x^2+y^2+z^2 = \gamma^2\}$

$\Rightarrow \gamma\sqrt{1-\cos^2\theta\sin^2\phi}$

2. **(b)** Given ΔPQR



Direction ratio of PR = (4, 1, –1)

Direction ratio of PQ = (1, 4, –1)

Now, $\cos\theta = \left|\dfrac{4+4+1}{\sqrt{18}.\sqrt{18}}\right| = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{3}$

3. **(a)** Given equations are

$l + m - n = 0 \Rightarrow n = l + m$

Now, put the value of n in another equation then,

$3l^2 + m^2 + cl\,(l + m) = 0$

$3l^2 + m^2 + cl^2 + clm = 0$

$(3 + c)l^2 + clm + m^2 = 0$

$(3+c)\left(\dfrac{l}{m}\right)^2 + c\left(\dfrac{l}{m}\right) + 1 = 0$ ...(i)

Given that lines are parallel.

Then, roots of (i) must be equal $\Rightarrow D = 0$

$c^2 - 4(3 + c) = 0 \Rightarrow c^2 - 4c - 12 = 0$

$(c - 6)\,(c + 2) = 0 \Rightarrow c = 6$ or $c = -2$.

Therefore, +ve value of $c = 6$.

4. **(b)** Given that,

$l + m - n = 0 \Rightarrow l = n - m$ ...(i)

and $l^2 + m^2 - n^2 = 0$ ...(ii)

Substitute l from (i) into (ii)

$\Rightarrow (n-m)^2 + m^2 - n^2 = 0$

$\Rightarrow 2m(m-n) = 0 \Rightarrow m = 0$ or $m = n$

Case-I : If $m = 0$,

$l = n$ [from (i)]

We know that,

$l^2 + m^2 + n^2 = 1 \Rightarrow l^2 = \dfrac{1}{2} \Rightarrow l_1, l_2 = \dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}}$

$\because l = n \Rightarrow n_1, n_2 = \dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}}$

$\therefore \left(\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right)$ or $\left(\dfrac{-1}{\sqrt{2}}, 0, \dfrac{-1}{\sqrt{2}}\right)$ are direction cosines of line L_1.

Case-II : If $m = n$

$\Rightarrow l = 0$ [from (i)]

$l^2 + m^2 + n^2 = 1 \Rightarrow m^2 = \dfrac{1}{2} \Rightarrow m_1, m_2 = \dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}}$

$\because m = n \Rightarrow n_1, n_2 = \dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}}$

$\therefore \left(0, \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ or $\left(0, \dfrac{-1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}}\right)$ are direction cosines of line L_2.

$\cos\alpha = l_1l_2 + m_1m_2 + n_1n_2 = 0 + 0 \pm \dfrac{1}{2} = \pm\dfrac{1}{2}$

$\cos^2\alpha = \dfrac{1}{4} \Rightarrow \sin^2\alpha = \dfrac{3}{4}$

So, $\sin^4\alpha + \cos^4\alpha = \dfrac{5}{8}$

5. **(a)** Here, P, Q, R are collinear

$\therefore \overrightarrow{PR} = \lambda\overrightarrow{PQ}$

$2\hat{i} + (y+3)\hat{j} + (z-4)\hat{k} = \lambda[6\hat{i} + 3\hat{j} + 6\hat{k}]$

$\Rightarrow 6\lambda = 2, y + 3 = 3\lambda, z - 4 = 6\lambda \Rightarrow \lambda = \dfrac{1}{3}, y = -2, z = 6$

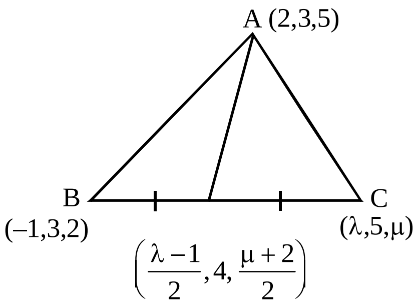
$\therefore$ Point $R\,(4, -2, 6)$

Now, OR = $\sqrt{(4)^2 + (-2)^2 + (6)^2} = \sqrt{56} = 2\sqrt{14}$

6. **(b)** DR's of AD are $\frac{\lambda-1}{2}-2, 4-3, \frac{\mu+2}{2}-5$

i.e. $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$

$\because$ This median is making equal angles with coordinate axes, therefore,



$\frac{\lambda-5}{2}=1=\frac{\mu-8}{2}$

$\Rightarrow \quad \lambda=7 \ \& \ \mu=10$

$\therefore \quad \lambda^3+\mu^3+5=1348$

7. **(c)** It makes θ with x and y-axis.

Let line makes α with z-axis. $\alpha \in \left[0.\frac{\pi}{2}\right]$

$l = \cos\theta$, $m = \cos\theta$, $n = \cos\alpha$

we have $l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\theta + \cos^2\theta + \cos^2\alpha = 1$

$\Rightarrow 2\cos^2\theta = 1 - \cos^2\alpha$

$\Rightarrow \cos^2\theta = \frac{\sin^2\alpha}{2} \Rightarrow \cos\theta = \frac{\sin\alpha}{\sqrt{2}}$

As $\alpha \in \left[0, \frac{\pi}{2}\right]$ then $\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

8. **(b)** Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be the initial and final points of the vector whose projections on the three coordinate axes are 6, – 3, 2 then

$x_2 - x_1, = 6$; $y_2 - y_1 = -3$; $z_2 - z_1 = 2$

So that direction ratios of $\overrightarrow{PQ}$ are 6, – 3, 2

$\therefore$ Direction cosines of $\overrightarrow{PQ}$ are

$$\Rightarrow \frac{6}{\sqrt{6^2+(-3)^2+2^2}}, \frac{-3}{\sqrt{6^2+(-3)^2+2^2}},$$

$$\frac{2}{\sqrt{6^2+(-3)^2+2^2}} = \frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$$

9. **(b)** As per question, direction cosines of the line :

$l=\cos 45°=\frac{1}{\sqrt{2}}$, $m=\cos 120°=\frac{-1}{2}$, $n=\cos\theta$

where θ is the angle, which line makes with positive z-axis.

We know that, $l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad \frac{1}{2}+\frac{1}{4}+\cos^2\theta=1$

$\Rightarrow \quad \cos^2\theta=\frac{1}{4}$

$\Rightarrow \quad \cos\theta=\frac{1}{2}=\cos\frac{\pi}{3}$ (θ being acute)

$\Rightarrow \quad \theta=\frac{\pi}{3}$

10. **(c)** As per question the direction cosines of the line are $\cos\theta, \cos\beta, \cos\theta$

$\therefore \quad \cos^2\theta+\cos^2\beta+\cos^2\theta=1$

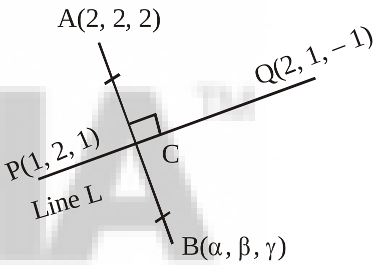
$\because \quad 2\cos^2\theta = 1 - \cos^2\theta$

$\Rightarrow \quad 2\cos^2\theta=\sin^2\beta=3\sin^2\theta$ (given)

$\Rightarrow \quad 2\cos^2\theta=3-3\cos^2\theta$

$\therefore \quad \cos^2\theta=\frac{3}{5}$

11. **(6)**



DR's of Line L = – 1 : 1 : 2

DR's of AB = $\alpha-2 : \beta-2 : \gamma-2$

$AB \perp L \Rightarrow 2-\alpha+\beta-2+2\gamma-4=0$

$2\gamma+\beta-\alpha=4$ ...(i)

Let C is mid-point of AB

$C\left(\frac{\alpha+2}{2}+\frac{\beta+2}{2}, \frac{\gamma+2}{2}\right)$

DR's of PC = $\frac{\alpha}{2} : \frac{\beta-2}{2} : \frac{\gamma}{2}$

Line PC & PQ are same.

$\Rightarrow \frac{-\alpha}{2}=\frac{\beta-2}{2}=\frac{\gamma}{2}=K$ (let)

$\alpha = -2K$

$\beta = 2K + 2$

$\gamma = 4K$

Use in (i)$\Rightarrow K=\frac{1}{6}$

$\therefore$ Value of $\alpha+\beta+6\gamma=24K+2=6$

12. **(450)** $RS \equiv (\alpha, -1, \beta)$

Now direction ratio's of $PQ \equiv \left(\frac{56}{17}+2, \frac{43}{17}+1, \frac{111}{17}-1\right)$

$\equiv \left(\frac{90}{17}, \frac{60}{17}, \frac{94}{17}\right)$

$\Rightarrow \frac{90}{17}\alpha + \frac{60}{17}(-1) + \frac{94}{17}\beta = 0 \Rightarrow 90\alpha + 94\beta = 60$

$\Rightarrow \beta = \frac{60-90\alpha}{94} \Rightarrow \beta = \frac{30(2-3\alpha)}{94}$

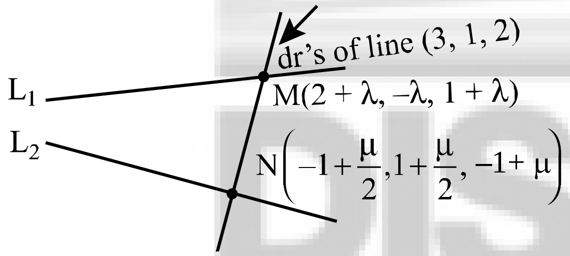
$\Rightarrow \beta = -30\frac{(3\alpha-2)}{94} \Rightarrow \beta = \frac{-15}{47}(3\alpha-2)$

$\Rightarrow \frac{\beta}{-15} = \frac{3\alpha-2}{47} \Rightarrow \beta = -15, \alpha = -15$

Now $\alpha^2 + \beta^2 = 225 + 225 = 450$

Topic-2: Equation of a Straight Line in Cartesian and Vector Form, Angle Between two Lines, Distance Between two Parallel Lines

1. **(b)**



$L_1 : \frac{x-2}{1} = \frac{y}{-1} = \frac{z-1}{1} = \lambda$

$L_2 : \frac{x+1}{\frac{1}{2}} = \frac{y-1}{\frac{1}{2}} = \frac{z+1}{1} = \mu$

dr's of line MN will be

$<3+\lambda-\frac{\mu}{2}, -1-\lambda-\frac{\mu}{2}, 2+\lambda-\mu>$ & it will be proportional to <3, 1, 2>

$\therefore \quad \frac{3+\lambda-\frac{\mu}{2}}{3} = \frac{-1-\lambda-\frac{\mu}{2}}{1} = \frac{2+\lambda-\mu}{2}$

On solving we get

$4\lambda + \mu = -6$ ...(i)

$4 + 3\lambda = 0$ ...(ii)

On solving (i) and (ii)

$\Rightarrow \quad \lambda = -\frac{4}{3} \,\&\, \mu = -\frac{2}{3}$

$\therefore$ Coordinate of M will be $\left(\frac{2}{3}, \frac{4}{3}, -\frac{1}{3}\right)$

and equation of required line will be.

$$\frac{x-\frac{2}{3}}{3} = \frac{y-\frac{4}{3}}{1} = \frac{z+\frac{1}{3}}{2} = k$$

So any point on this line will be

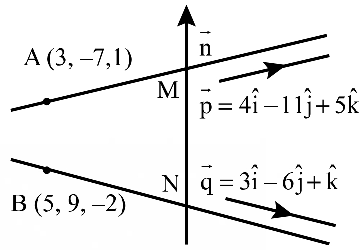
$\left(\frac{2}{3}+3k, \frac{4}{3}+k, -\frac{1}{3}+2k\right)$

$\because \frac{2}{3}+3k = -\frac{1}{3} \Rightarrow k = -\frac{1}{3}$

$\therefore$ Point lie on the line for

$k = -\frac{1}{3}$ is $\left(-\frac{1}{3}, 1, -1\right)$

2. **(a)**



$\vec{n} = \vec{p} \times \vec{q}$

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -11 & 5 \\ 3 & -6 & 1 \end{vmatrix} = 19\hat{i} + 11\hat{j} + 9\hat{k}$

S.d. = projection of $\overrightarrow{AB}$ on $\vec{n}$

$= \left|\frac{\overrightarrow{AB}\cdot\vec{n}}{|\vec{n}|}\right| = \left|\frac{(2\hat{i}+16\hat{j}-3k)\cdot(19\hat{i}+11\hat{j}+9\hat{k})}{\sqrt{361+121+81}}\right|$

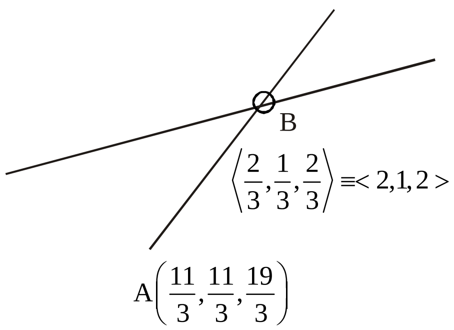
$= \frac{38+176-27}{\sqrt{563}}$

S.d. $= \frac{187}{\sqrt{563}}$

3. **(a)** $\frac{x-1}{2-1} = \frac{y-2}{3-2} = \frac{z-3}{5-3}$

$\Rightarrow \quad \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2} = \lambda$

$A \equiv \left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$



$B(1+\lambda, 2+\lambda, 3+2\lambda)$

D.R. of AB $= <\dfrac{3\lambda-8}{3}, \dfrac{3\lambda-5}{3}, \dfrac{6\lambda-10}{3}>$

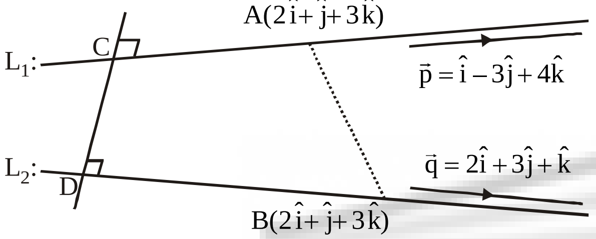
$\dfrac{3\lambda-8}{3\lambda-5}=\dfrac{2}{1}\Rightarrow 3\lambda-8=6\lambda-10$

$3\lambda=2$

$\lambda=\dfrac{2}{3}\Rightarrow B\left(\dfrac{5}{3},\dfrac{8}{3},\dfrac{13}{3}\right)$

$AB=\dfrac{\sqrt{36+9+36}}{3}=\dfrac{9}{3}=3$

4. **(b)** Let d be the shortest distance between L_1 and L_2.



$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 4\\ 2 & 3 & 1\end{vmatrix}=-15\hat{i}+7\hat{k}$

$d=\left|\dfrac{\overrightarrow{AB}.\vec{p}\times\vec{q}}{|\vec{p}\times\vec{q}|}\right|=\left|\dfrac{(0\hat{i}+2\hat{j}+2\hat{k}).(-15\hat{i}+7\hat{j}+9\hat{k})}{\sqrt{355}}\right|=\dfrac{32}{\sqrt{355}}$

$\therefore m+n=32+355=387$

5. **(c)** From the given information

$$\left.\begin{array}{l}\vec{r}_1=(\lambda\hat{i}+4\hat{j}+3\hat{k})+\alpha(2\hat{i}+3\hat{j}+4\hat{k})\\ \vec{r}_2=(2\hat{i}+4\hat{j}+7\hat{k})+\beta(2\hat{i}+3\hat{j}+4\hat{k})\end{array}\right\}\begin{array}{l}\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}\\ \vec{a}_1=\lambda\hat{i}+4\hat{j}+3\hat{k}\\ \vec{a}_2=2\hat{i}+4\hat{j}+7\hat{k}\end{array}$$

Shortest dist. $=\dfrac{|\vec{b}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{b}|}=\dfrac{13}{\sqrt{29}}$

$\dfrac{\left|(2\hat{i}+3\hat{j}+4\hat{k})\times((2-\lambda)\hat{i}+4\hat{k})\right|}{\sqrt{29}}=\dfrac{13}{\sqrt{29}}$

$\left|-8\hat{j}-3(2-\lambda)\hat{k}+12\hat{i}+4(2-\lambda)\hat{j}\right|=13$

$\left|12\hat{i}-4\lambda\hat{j}+(3\lambda-6)\hat{k}\right|=13$

$144+16\lambda^2+(3\lambda-6)^2=169$

$16\lambda^2+(3\lambda-6)^2=25$

$25\lambda^2-36\lambda+11=0\Rightarrow\lambda=1,\dfrac{11}{25}$

6. **(b)** Given lines are

$\dfrac{x-3}{2}=\dfrac{y+15}{-7}=\dfrac{z-9}{5}$ & $\dfrac{x+1}{2}=\dfrac{y-1}{1}=\dfrac{z-9}{-3}$

Shortest distance $=\dfrac{|(\bar{a}_2-\bar{a}_1).(\bar{b}_1\times\bar{b}_2)|}{|\bar{b}_1\times\bar{b}_2|}$

$a_1=3,-15,9$ $\quad b_1=2,-7,5$

$a_2=-1,1,9$ $\quad b_2=2,1,-3$

$a_2-a_1=-4,16,0$

$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}=\hat{i}(16)-\hat{j}(-16)+\hat{k}(16)$

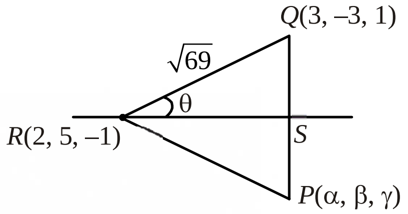
$=16(\hat{i}+\hat{j}+\hat{k})$

$|\vec{b}_1\times\vec{b}_2|=16\sqrt{3}$

$\therefore\ (\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)=16[-4+16]=(16)(12)$

$S.D.=\dfrac{(16)(12)}{16\sqrt{3}}=4\sqrt{3}$

7. **(d)**



$RQ=\sqrt{1+64+4}=\sqrt{69}$

$\overrightarrow{RQ}=\hat{i}-8\hat{j}+2\hat{k}$

Vector along $\overrightarrow{RS}=\hat{i}+\hat{j}-\hat{k}$

$\cos\theta=\dfrac{\overrightarrow{RQ}\cdot\overrightarrow{RS}}{|\overrightarrow{RQ}||\overrightarrow{RS}|}=\left|\dfrac{1-8-2}{\sqrt{69}\sqrt{3}}\right|=\dfrac{9}{3\sqrt{23}}$

$\cos\theta=\dfrac{3}{\sqrt{23}}$

$RS=\sqrt{69}\times\dfrac{3}{23}=3\sqrt{3}$

$\sin\theta=\dfrac{\sqrt{14}}{\sqrt{23}}=\dfrac{QS}{\sqrt{69}}$

$QS=\sqrt{42}$

area $=\dfrac{1}{2}.2QS.RS=\sqrt{42}.3\sqrt{3}$

$\lambda=9\sqrt{14}$

$\lambda^2=81.14=14k$

$k=81$

8. **(c)** Let $\dfrac{x+6}{3}=\dfrac{y}{2}=\dfrac{z+1}{1}=\lambda$ ... (i)

$x=3\lambda-6, y=2\lambda, z=\lambda-1$

and $\dfrac{x-7}{4}=\dfrac{y-9}{3}=\dfrac{z-4}{2}=\mu$ ... (ii)

$x=4\mu+7, y=3\mu+9, z=2\mu+4$

$3\lambda-6=4\mu+7\Rightarrow 3\lambda-4\mu=13$ ... (iii) $\times 2$

$2\lambda = 3\mu + 9 \Rightarrow 2\lambda - 3\mu = 9$ ... (iv) × 3

$6\lambda - 8\mu = 26$

$6\lambda - 9\mu = 27$

$-\quad+\quad-$

$\mu = -1$

$\Rightarrow 3\lambda - 4(-1) = 13 \Rightarrow 3\lambda = 9 \Rightarrow \lambda = 3$

$\because \lambda - 1 = 2\mu + 4$

$\Rightarrow 3 - 1 = 2(-1) + 4$

$\Rightarrow 2 = 2$ satisty

$\therefore$ point of intersection is (3, 6, 2)

$d^2 = 16 + 4 + 49 = 69$

Now, $d^2 + 6 = 69 + 6 = 75$

9. **(d)** $\dfrac{2-x}{3} = \dfrac{3y-2}{4\lambda+1} = 4 - z$

$$\Rightarrow \frac{x-2}{(-3)} = \frac{y-\frac{2}{3}}{\left(\frac{4\lambda+1}{3}\right)} = \frac{z-4}{(-1)}$$

$$\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7} \quad \text{... (i)}$$

$$\frac{x+3}{3\mu} = \frac{y-\frac{1}{2}}{(-3)} = \frac{z-5}{(-7)} \quad \text{... (ii)}$$

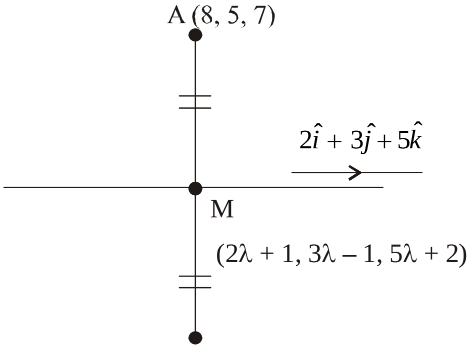
For right angle $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$$\Rightarrow (-3)(3\mu) + \left(\frac{4\lambda+1}{3}\right)(-3) + (-1)(-7) = 0$$

$-9\mu - 4\lambda - 1 + 7 = 0$

$4\lambda + 9\mu = 6$

10. **(c)** Let M be arbitrary point on the given line.



$\overrightarrow{AM}.(2\hat{i} + 3\hat{j} + 5\hat{k}) = 0$

$(2\lambda - 7)(2) + (3\lambda - 6)(3) + (5\lambda - 5)(5) = 0$

$38\lambda = 57$

$\lambda = \dfrac{3}{2}$

$M\left(4, \dfrac{7}{2}, \dfrac{19}{2}\right)$

$A'(0, 2, 12)$

$\therefore \quad \alpha + \beta + \gamma = 14$

11. **(a)** Line PQ, $\dfrac{x-1}{4} = \dfrac{y+2}{-2} = \dfrac{z-3}{4}$

Any point on PQ be $R(4\lambda+1, -2\lambda-2, 4\lambda+3)$

$PR = 9$ unit

$(PR)^2 = 81$

$(4\lambda + 1 - 1)^2 + (-2\lambda - 2 + 2)^2 + (4\lambda + 3 - 3)^2 = 81$

$\Rightarrow 16\lambda^2 + 4\lambda^2 + 16\lambda^2 = 81 \Rightarrow 36\lambda^2 = 81 \Rightarrow \lambda = \pm\dfrac{3}{2}$

$\therefore R$ can be (7, –5, 9) or (–5, 1, –3).

Distance from origin for both points be

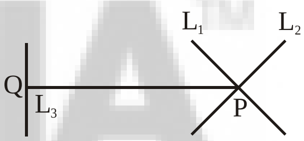
$\sqrt{49+25+81} = \sqrt{155}$ and $\sqrt{25+1+9} = \sqrt{35}$

$\therefore$ Distance of (7, –5, 9) is farthest from origin.

$\therefore (\alpha, \beta, \gamma) = (7, -5, 9)$

Now $7^2 + (-5)^2 + 9^2 = 155$.

12. **(c)**



$$L_1 = \frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1} = \lambda$$

$P(\lambda+2, 5\lambda+4, \lambda+2)$

$$L_2 = \frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2} = \mu$$

$P(2\mu+3, 3\mu+2, 2\mu+3)$

$\lambda + 2 = 2\mu + 3 \qquad 3\mu + 2 = 5\lambda + 4$

$\lambda = 2\mu + 1 \qquad 3\mu = 5\lambda + 2$

$3\mu = 5(2\mu + 1) + 2$

$3\mu = 10\mu + 7$

$\mu = -1 \quad \lambda = -1$

$P(1, -1, 1)$

$$L_3 = \frac{x}{\frac{1}{4}} = \frac{y}{\frac{1}{2}} = \frac{z}{1}$$

$$L_3 = \frac{x}{1} = \frac{y}{2} = \frac{z}{4} = k$$

Coordinates of $Q(k, 2k, 4k)$

DR's of PQ = <$k - 1, 2k + 1, 4k - 1$>

$PQ \perp$ to L_3

$(k-1)+2(2k+1)+4(4k-1)=0$

$k-1+4k+2+16k-4=0$

$k=\frac{1}{7}$, then the coordinate $Q\left(\frac{1}{7},\frac{2}{7},\frac{4}{7}\right)$

$$PQ=\sqrt{\left(1-\frac{1}{7}\right)^2+\left(-1-\frac{2}{7}\right)^2+\left(1-\frac{4}{7}\right)^2}$$

$$=\sqrt{\frac{36}{49}+\frac{81}{49}+\frac{9}{49}}=\frac{\sqrt{126}}{7}$$

$$PQ=\frac{3\sqrt{14}}{7}$$

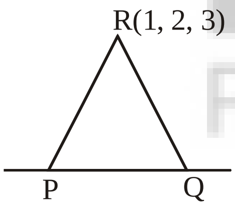
13. **(b)** Lines L_1 and L_2 are passing through $(\lambda, 2, 1)$ & $(\sqrt{3},1,2)$ respectively. Given, shortest distance = 1

$$\Rightarrow \frac{\begin{vmatrix}\sqrt{3}-\lambda & -1 & 1\\ -2 & 1 & 1\\ 1 & -2 & 1\end{vmatrix}}{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 1 & 1\\ 1 & -2 & 1\end{vmatrix}}=1$$

$$\Rightarrow \left|\frac{\sqrt{3}-\lambda}{\sqrt{3}}\right|=1\Rightarrow\lambda=0,\ 2\sqrt{3}$$

$\therefore$ Required sum = $2\sqrt{3}$

14. **(a)**



Let $A(8\lambda-3, 2\lambda+4, 2\lambda-1)$ be general point on given line

Let if AR = 6 then,

$\Rightarrow (8\lambda-4)^2+(2\lambda+2)^2+(2\lambda-4)^2=36$

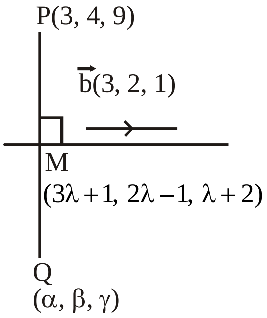
$\Rightarrow \lambda=0, 1$. Hence the co-ordinate of P.

and Q will be P(–3, 4, –1) & Q(5, 6, 1).

Centroid of $\Delta PQR=(1, 4, 1)\equiv(\alpha, \beta, \gamma)$

$\alpha^2+\beta^2+\gamma^2=18$

15. **(d)**



Clearly, $\overrightarrow{PM}.\vec{b}=0$

$\Rightarrow (3\lambda-2)+2(2\lambda-5)+(\lambda-7)=0$

$\Rightarrow 14\lambda=23\Rightarrow\lambda=\frac{23}{14}$

Hence co-ordinates of point M is $\left(\frac{83}{14},\frac{32}{14},\frac{51}{14}\right)$

$\therefore \frac{\alpha+3}{2}=\frac{83}{14}\Rightarrow\alpha=\frac{62}{7}$

$\frac{\beta+4}{2}=\frac{32}{14}\Rightarrow\beta=\frac{4}{7}\Rightarrow\frac{\gamma+9}{2}=\frac{51}{14}\Rightarrow\gamma=\frac{-12}{7}$

Hence, $14(\alpha+\beta+\gamma)=108$

16. **(c)** A vector in the direction of the required line can be obtained by cross product of

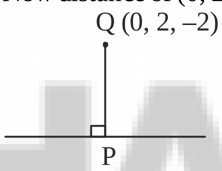
$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 5\\ -1 & 3 & 2\end{vmatrix}=-9\hat{i}-9\hat{j}+9\hat{k}$$

Required line

$\vec{r}=\left(5\hat{i}-4\hat{j}+3\hat{k}\right)+\lambda\left(-9\hat{i}-9\hat{j}+9\hat{k}\right)$

$\vec{r}=\left(5\hat{i}-4\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}+\hat{j}-\hat{k}\right)$

Now distance of (0, 2, –2)



P.V. of $P\equiv(5+\lambda)\hat{i}+(\lambda-4)\hat{j}+(3-\lambda)\hat{k}$

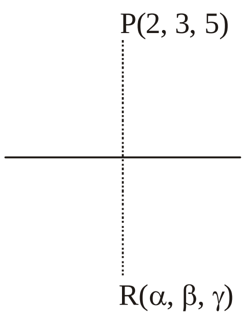
$\overrightarrow{QP}=(5+\lambda)\hat{i}+(\lambda-6)\hat{j}+(5-\lambda)\hat{k}$

$\overrightarrow{QP}.\left(\hat{i}+\hat{j}-\hat{k}\right)=0$

$5+\lambda+\lambda-6-5+\lambda=0 \Rightarrow \lambda=2$

$\left|\overrightarrow{QP}\right|=\sqrt{49+16+9}=\sqrt{74}$

17. **(b)** $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ ...(i)



$\because$ $\overrightarrow{PR}$ is perpendicular to Eq. (i)

$\Rightarrow \overrightarrow{PR}.(2,3,4)=0$

$(\alpha-2, \beta-3, \gamma-5).(2, 3, 4)=0$

$\Rightarrow 2\alpha+3\beta+4\gamma=4+9+20=33$

18. **(c)** $L_2 = \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$

$\vec{b}_1 = 2\hat{i} - 3\hat{j} + 2\hat{k}, \vec{b}_2 = 3\hat{i} + 2\hat{j}$

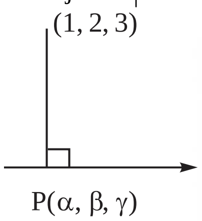
$\vec{b}_1 \times \vec{b}_2 = -4\hat{i} + 6\hat{j} + 13\hat{k}$

$$\therefore \quad S.D = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \frac{\begin{vmatrix} 5 & -5 & -7 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \frac{141}{|-4\hat{i} + 6\hat{j} + 13\hat{k}|} = \frac{141}{\sqrt{16+36+169}} = \frac{141}{\sqrt{221}}$$

19. **(b)**



Given: $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = k$ (say)

$\Rightarrow x = 5k - 3$

$y = 2k + 1,\ z = 3k - 4$

Let foot $P(5k-3,\ 2k+1,\ 3k-4)$

DR's of $AP: 5k-4,\ 2k-1,\ 3k-7$

DR's of Line : 5, 2, 3

Condition of perpendicular lines

$(25k - 20) + (4k - 2) + (9k - 21) = 0$

Then $k = \frac{43}{38}$

$\therefore (\alpha, \beta, \gamma) = \left(\frac{101}{38}, \frac{124}{38}, \frac{-23}{38}\right)$

Then $19(\alpha + \beta + \gamma) = 101$

20. **(a)** According to the question:

$L_1 \perp L_2$, $\quad L_3 \perp L_1$ and $L_3 \perp L_2$

$\therefore \quad 3 - 1 + 2p = 0$

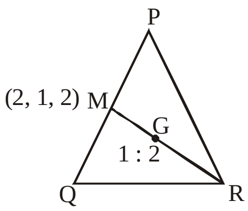
$p = -1$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 4\hat{k}$$

$\therefore \quad (-\delta, 7\delta, 4\delta)$ will lie on L_3

For $\delta = 1$ the point will be $(-1, 7, 4)$

21. **(c)**



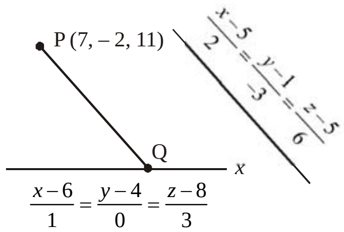
Centroid G divides MR in 1 : 2

G(1, 2, 2)

Point of intersection A of given lines is (2, –6, 0)

AG = $\sqrt{69}$

22. **(b)**



$\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3} = \lambda$

$Q(\lambda + 6, 4, 3\lambda + 8)$

D.r. of $PQ = \lambda - 1, 6, 3\lambda - 3$

D.r of line = 2, – 3, 6 $\therefore \frac{\lambda - 1}{2} = \frac{6}{-3} = \frac{3\lambda - 3}{6} \Rightarrow \lambda = -3$

$\therefore Q(3, 4, -1)$

PQ is = $\sqrt{(7-3)^2 + (4+2)^2 + (11+1)^2} = 14.$

23. **(b)** $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3};\ \frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$

the shortest distance between the lines

$$= \left|\frac{(\vec{a}_1 - \vec{a}_2)\cdot(\vec{b}_1 \times \vec{b}_2)}{(\vec{b}_1 \times \vec{b}_2)}\right| = \left|\frac{\begin{vmatrix} \lambda - 4 & 0 & 2 \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}{\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{matrix}}\right|$$

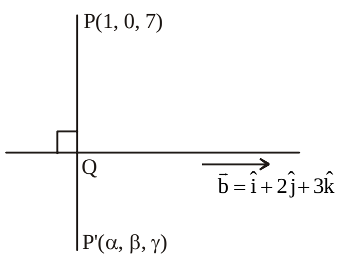
$$= \left|\frac{(\lambda - 4)(-10 + 12) - 0 + 2(4 - 4)}{|2\hat{i} - 1\hat{j} + 0\hat{k}|}\right|$$

$\frac{6}{\sqrt{5}} = \left|\frac{2(\lambda - 4)}{\sqrt{5}}\right| \quad \Rightarrow 3 = |\lambda - 4|$

$\lambda - 4 = \underline{+}3 \ \Rightarrow \lambda = 7, 1$

Sum of all possible values of λ is = 8

24. **(c)** $L_1 = \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$



$Q(\lambda, 16 + 2\lambda, 2 + 3\lambda)$

D. r of PQ = $(\lambda - 1), (1 + 2\lambda), (3\lambda - 5)$

D.r. of L_1 = 1, 2, 3

PQ is perpendicular to line L_1

$1(\lambda-1)+2(1+2\lambda)+3(3\lambda-5)=0$

$\Rightarrow \quad \lambda-1+4\lambda+2+9\lambda-15=0$

$14\lambda=14 \Rightarrow \lambda=1$. So, Q(1, 3, 5)

Q is midpoint of P & P′

$\frac{\alpha+1}{2}=1, \frac{\beta+0}{2}=3, \frac{\gamma+7}{2}=5$

$\therefore (\alpha, \beta, \gamma)=(1,6,3)$

Let required line having direction cosine (l, m, n)

$l^2+m^2+n^2=1 \Rightarrow l^2+\cos^2\frac{2\pi}{3}+\cos^2\frac{3\pi}{4}=1$

$\Rightarrow \quad l^2+\left(-\frac{1}{2}\right)^2+\left(-\frac{1}{\sqrt{2}}\right)^2=1 \Rightarrow l^2=\frac{1}{4}$

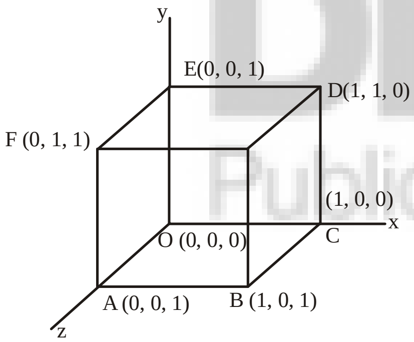
$\therefore \quad l=\frac{1}{2}$ [Line make acute angle with x-axis]

Equation of line passing through (1, 6, 3) will be

$\vec{r}=(\hat{i}+6\hat{j}+3\hat{k})+\mu\left(\frac{1}{2}\hat{i}-\frac{1}{2}\hat{j}-\frac{1}{\sqrt{2}}\hat{k}\right)$

Option (c) satisfying for $\mu=4$

25. **(a)**



Equation of face diagonal OD line is

$l_1: \vec{r}=\lambda\left(\hat{i}+\hat{j}\right)$

Equation of main diagonal BE is

$l_2: \vec{r}=\hat{j}+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$

Shortest distance = $\left|\frac{j.\left(\hat{i}+\hat{j}\right)\times\left(\hat{i}-\hat{j}+\hat{k}\right)}{\left(\hat{i}+\hat{j}\right)\times\left(\hat{i}-\hat{j}+\hat{k}\right)}\right|$

$=\left|\frac{\hat{j}.\left(\hat{i}-\hat{j}-2\hat{k}\right)}{\hat{i}-\hat{j}-2\hat{k}}\right|=\frac{1}{\sqrt{6}}$

In other case S.D is zero.

26. **(c)** Given the shortest distance = 13

$$\Rightarrow \frac{\left\|\begin{matrix}0 & 4 & 1\\ 3 & -4 & 0\\ 2\lambda & 3 & -12\end{matrix}\right\|}{\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 4 & 1\\ 3 & -4 & 0\end{matrix}\right\|}=13$$

$\Rightarrow \frac{|153+8\lambda|}{\left|4\hat{i}+3\hat{j}-12\hat{k}\right|}=13$

$\Rightarrow \frac{|153+8\lambda|}{13}=13 \Rightarrow |153+8\lambda|=169$

$\Rightarrow 153+8\lambda=169, -169 \Rightarrow \lambda=\frac{16}{8}, \frac{-322}{8}$

So, $8\left|\sum_{\lambda\in S}\lambda\right|=306$

27. **(b)** Given the lines

$\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ & $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$

$$\text{Shortest distance} = \frac{\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}}{\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{matrix}\right\|}$$

$$=\frac{\begin{vmatrix}6 & 1 & -8\\ 1 & -2 & 2\\ 1 & 2 & 0\end{vmatrix}}{\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 1 & 2 & 0\end{matrix}\right\|}=\frac{54}{6}=9$$

28. **(a)** Shortest distance

$=\left|\frac{[\vec{a}_2-\vec{a}_1\ \vec{b}_1\ \vec{b}_2]}{|\vec{b}_1\times\vec{b}_2|}\right|$

$\vec{a}_1=(4,-2,-3),\ \vec{b}_1=(4,5,3)$

$\vec{a}_2=(1,3,4),\ \vec{b}_2=(3,4,2)$

$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 5 & 3\\ 3 & 4 & 2\end{vmatrix}=\hat{i}(-2)-\hat{j}(-1)+\hat{k}(1)=(-2,1,1)$

$[\vec{a}_2-\vec{a}_1\ \vec{b}_1\ \vec{b}_2]=\begin{vmatrix}-3 & 5 & 7\\ 4 & 5 & 3\\ 3 & 4 & 2\end{vmatrix}=18$; $S_d=\frac{18}{\sqrt{6}}=3\sqrt{6}$

29. **(d)** Equation of line OP passes through (0, 0, 0) and (3, 4, 5) is $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

Equation of edge parallel to z-axis and passes through (3, 0, 5) & (3, 0, 0) is

$\frac{x-3}{0}=\frac{y-0}{0}=\frac{z-5}{1}$

$\text{S.D}=\frac{\left(\vec{a}_2-\vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}$

$$= \frac{\begin{vmatrix}3&0&5\\3&4&5\\0&0&1\end{vmatrix}}{\left\|\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&4&5\\0&0&1\end{vmatrix}\right\|} = \frac{3(4)}{|4\hat{i}-3\hat{j}|} = \frac{12}{5}$$

30. **(d)** We have shortest distance between two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{a_2} = \frac{z-z_1}{a_3} \text{ab} \;\&$$

$$\frac{x-x_2}{b_1} = \frac{y-y_2}{b_2} = \frac{z-z_2}{b_3}$$

$$\frac{\begin{vmatrix}x_1-x_2 & y_1-y_2 & z_1-z_2\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}}{\sqrt{(a_1b_3-a_3b_2)^2+(a_1b_3-a_3b_1)^2+(a_1b_2-a_2b_1)^2}}$$

$$= \frac{\begin{vmatrix}5-(3) & 2-(-5) & 4-1\\ 1 & 2 & -3\\ 1 & 4 & -5\end{vmatrix}}{\sqrt{(-10+12)^2+(-5+3)^2+(4-2)^2}}$$

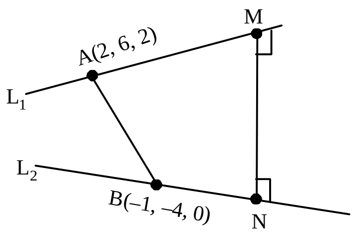
$$= \frac{\begin{vmatrix}8 & 7 & 3\\ 1 & 2 & -3\\ 1 & 4 & -5\end{vmatrix}}{\sqrt{(2)^2+(2)^2+(2)^2}} = \frac{16+14+6}{\sqrt{12}} = \frac{36}{\sqrt{12}} = \frac{36}{2\sqrt{3}}$$

$$= \frac{18}{\sqrt{3}} = 6\sqrt{3}$$

31. **(d)** Since the line ℓ, is given by

$$L_1 : \frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$$

and $L_2 : \dfrac{x+1}{2} = \dfrac{y+4}{-3} = \dfrac{z}{2}$



Now shortest distance $= \left|\dfrac{\overline{AB}\cdot\overline{MN}}{MN}\right|$

$$\overrightarrow{AB} = 3\hat{i}+10\hat{j}+2\hat{k}$$

$$\overrightarrow{MN} = \begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&1&-2\\2&-3&2\end{vmatrix} = -4\hat{i}-8\hat{j}-8\hat{k}$$

$$MN = \sqrt{16+64+64} = 12$$

So, required distance $= \left|\dfrac{-12-80-16}{12}\right| = 9$

32. **(b)** $\dfrac{x-1}{2} = \dfrac{y+8}{-7} = \dfrac{z-4}{5}$;

$\vec{a}_1 = \hat{i}-8\hat{j}+4\hat{k},\ \vec{b}_1 = 2\hat{i}-7\hat{i}+5\hat{k}$

$\dfrac{x-1}{2} = \dfrac{y-2}{1} = \dfrac{z-6}{-3}$;

$\vec{a}_2 = \hat{i}+2\hat{j}+6\hat{k},\ \vec{b}_2 = 2\hat{i}+\hat{j}-3\hat{k}$

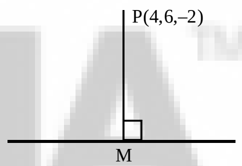
$$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-7&5\\2&1&-3\end{vmatrix}$$

$$= \hat{i}(16)-\hat{j}(-16)+\hat{k}(16) = 16\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$d = \frac{(\vec{a}_1-\vec{a}_2)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}$$

$$= \left|\frac{\left(-10\hat{j}-2\hat{k}\right)\cdot 16\left(\hat{i}+\hat{j}+\hat{k}\right)}{16\sqrt{3}}\right| = \left|\frac{-12}{\sqrt{3}}\right| = 4\sqrt{3}$$

33. **(d)**



Equation of line is $\dfrac{x+3}{3} = \dfrac{y-2}{3} = \dfrac{z-3}{-1} = \lambda$

$M(3\lambda-3, 3\lambda+2, 3-\lambda)$

D.R of PM $(3\lambda-7, 3\lambda-4, 5-\lambda)$

Since PM is perpendicular to line

$\Rightarrow 3(3\lambda-7)+3(3\lambda-4)-1(5-\lambda) = 0$

$\Rightarrow \lambda_2 = 2$

Put $\lambda = 2$ in cordinate M.

$\Rightarrow M(3, 8, 1) \Rightarrow PM = \sqrt{14}$

34. **(a)** Let $P(2\lambda+1, \lambda+3, 2\lambda+2)$ and $Q = (\mu+2, 2\mu+2, 3\mu+3)$

Take direction ratio's of PQ to get equation of line

$$\Rightarrow \frac{2\lambda-\mu-1}{1} = \frac{\lambda-2\mu+1}{-1} = \frac{2\lambda-3\mu-1}{-2}$$

By comparing the above equations we get

$\Rightarrow \lambda = \mu = 3 \Rightarrow P(7, 6, 8)$ and $Q(5, 8, 12)$

$PQ = 2\sqrt{6}$

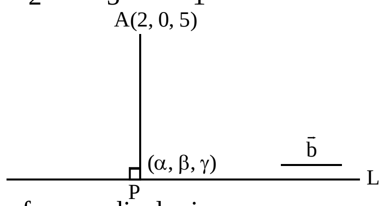
35. **(a)** $\dfrac{x+1}{1} = \dfrac{y}{\frac{1}{2}} = \dfrac{z}{\frac{-1}{12}}$ and $\dfrac{x}{1} = \dfrac{y+2}{1} = \dfrac{z-1}{\frac{1}{6}}$

$$\Rightarrow \text{shortest distance} = \frac{\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}}{|\vec{p}\times\vec{q}|}$$

$$\left\{\vec{p}\times\vec{q}\equiv\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \frac{1}{2} & \frac{-1}{12}\\ 1 & 1 & \frac{1}{6}\end{vmatrix}=\frac{1}{6}\hat{i}-\frac{1}{4}\hat{j}+\frac{1}{2}\hat{k}\text{ or }2\hat{i}-3\hat{j}+6\hat{k}\right\}$$

$$\text{S.D.}=\frac{\left(-\hat{i}+2\hat{j}-\hat{k}\right)\cdot\left(2\hat{i}-3\hat{j}+6\hat{k}\right)}{\sqrt{2^2+3^2+6^2}}=\left|\frac{-14}{7}\right|=2$$

36. **(c)** $L:\dfrac{x+1}{2}=\dfrac{y-1}{5}=\dfrac{z+1}{-1}=\lambda(\text{let})$



Let foot of perpendicular is

$P(2\lambda-1, 5\lambda+1, -\lambda-1)=(\alpha,\beta,\gamma)$

$\overrightarrow{PA}=\left(2-(2\lambda-1)\hat{i}+\left(0-(5\lambda+1)\right)\hat{j}+\left(5-(-\lambda-1)\right)\hat{k}\right)$

$\overrightarrow{PA}=(3-2\lambda)\hat{i}-(5\lambda+1)\hat{j}+(6+\lambda)\hat{k}$

Direction ratio of line $\Rightarrow\vec{b}=2\hat{i}+5\hat{j}-\hat{k}$

Now, $\overrightarrow{PA}.\vec{b}=\left|\overrightarrow{PA}\right|\left|\vec{b}\right|\cos 90^\circ=0$

$\Rightarrow 2(3-2\lambda)-5(5\lambda+1)-(6+\lambda)=0$

$\Rightarrow\lambda=\dfrac{-1}{6}$

$P(2\lambda-1,5\lambda+1,-\lambda-1)\equiv P(\alpha,\beta,\gamma)$

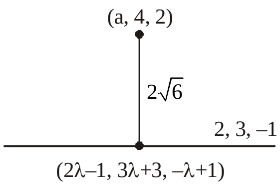
$\Rightarrow\alpha=2\left(-\dfrac{1}{6}\right)-1=-\dfrac{4}{3}\Rightarrow\alpha=-\dfrac{4}{3}$

$\Rightarrow\beta=5\left(-\dfrac{1}{6}\right)+1=\dfrac{1}{6}\Rightarrow\beta=\dfrac{1}{6}$

$\Rightarrow\gamma=-\lambda-1=\dfrac{1}{6}-1\Rightarrow\gamma=-\dfrac{5}{6}$

$\therefore$ Check options

37. **(b)**



Given line is $\dfrac{x+1}{2}=\dfrac{y-3}{3}=\dfrac{z-1}{-1}=\lambda$

$x=2\lambda-1, y=3\lambda+3, z=-\lambda+1.$

$(2\lambda-1-a)2+(3\lambda-1)3+(-\lambda-1)(-1)=0$

$\Rightarrow 4\lambda-2-2a+9\lambda-3+\lambda+1=0$

$\Rightarrow 14\lambda-4-2a=0\ \Rightarrow 7\lambda-2-a=0$ and,

$(2\lambda-1-a)^2+(3\lambda-1)^2+(\lambda+1)^2=24$

$\Rightarrow(5\lambda-1)^2+(3\lambda-1)^2+(\lambda+1)^2=24$

$\Rightarrow 35\lambda^2-14\lambda-21=0\Rightarrow(\lambda-1)(35\lambda+21)=0$

For, $\lambda=1\Rightarrow a=5$

Let $(\alpha_1,\alpha_2,\alpha_3)$ be reflection of point P

$\alpha_1+5=2$ $\quad\alpha_2+4=12$ $\quad\alpha_3+2=0$

$\alpha_1=-3$ $\quad\alpha_2=8$ $\quad\alpha_3=-2$

$a+\alpha_1+\alpha_2+\alpha_3=8$

$\therefore\ a+b+c=2$

38. **(a)** $L_1\equiv\vec{r}=(\hat{i}-\hat{j})+\ell(2\hat{i}+\hat{k})$

$L_2\equiv\vec{r}=(2\hat{i}-\hat{j})+m(\hat{i}+\hat{j}-\hat{k})$

Equating coeff. of $\hat{i}$, $\hat{j}$ and $\hat{k}$ of L_1 and L_2

$2l+1=m+2$ ...(i)

$-1=-1+m\Rightarrow m=0$ ...(ii)

$l=-m$ ...(iii)

$\Rightarrow m=l=0$, which is not satisfy eqn. (i) hence lines do not intersect for any value of l and m.

39. **(c)** Given, $\dfrac{x}{1}=\dfrac{y-1}{0}=\dfrac{z+1}{-1}=p$ (let) and point P $(\beta, 0, \beta)$

Any point on line A $=(p, 1, -p-1)$

Now, DR of AP are $<p-\beta, 1-0, -p-1-\beta>$

Which is perpendicular to line.

$\therefore (p-\beta)\,1+0.1-1\,(-p-1-\beta)=0$

$\Rightarrow p-\beta+p+1+\beta=0\Rightarrow p=\dfrac{-1}{2}$

$\therefore$ Point $A\left(\dfrac{-1}{2},1,-\dfrac{1}{2}\right)$

Given that distance AP $=\sqrt{\dfrac{3}{2}}\Rightarrow AP^2=\dfrac{3}{2}$

$\Rightarrow\left(\beta+\dfrac{1}{2}\right)^2+1+\left(\beta+\dfrac{1}{2}\right)^2=\dfrac{3}{2}$ or $2\left(\beta+\dfrac{1}{2}\right)^2=\dfrac{1}{2}$

$\Rightarrow\left(\beta+\dfrac{1}{2}\right)^2=\dfrac{1}{4}\Rightarrow\beta=0,-1,(\beta\neq 0)$

$\therefore\beta=-1$

40. **(d)** Let θ be the angle between the two lines

Here direction cosines of $\dfrac{x}{2}=\dfrac{y}{2}=\dfrac{z}{1}$ are 2, 2, 1

Also second line can be written as:

$$\frac{x-5}{2}=\frac{y-2}{\frac{P}{7}}=\frac{z-3}{4}$$

$\therefore$ its direction cosines are $2,\dfrac{P}{7},4$

$$\because\quad\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

$$\Rightarrow \frac{2}{3} = \left| \frac{(2\times 2)+\left(2\times\frac{P}{7}\right)+(1\times 4)}{\sqrt{2^2+2^2+1^2}\sqrt{2^2+\frac{P^2}{49}+4^2}} \right|$$

$$= \frac{4+\frac{2P}{7}+4}{3\times\sqrt{2^2+\frac{P^2}{49}+4^2}}$$

$$\Rightarrow \left(4+\frac{P}{7}\right)^2 = 20+\frac{P^2}{49} \Rightarrow 16+\frac{8P}{7}+\frac{P^2}{49} = 20+\frac{P^2}{49}$$

$$\Rightarrow \frac{8P}{7} = 4 \Rightarrow P = \frac{7}{2}$$

41. **(b)** Let equation of the required line be

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \quad \text{...(i)}$$

Given two lines

$$\frac{x}{1} = \frac{y}{-1} = \frac{z}{1} \quad \text{...(ii)}$$

and $\dfrac{x-1}{0} = \dfrac{y+1}{-2} = \dfrac{z}{1}$ ...(iii)

Since the line (i) is perpendicular to both the lines (ii) and (iii), therefore

$a - b + c = 0$ ...(iv)

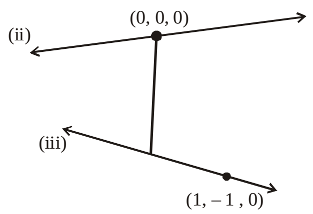
$-2b + c = 0$ ...(v)

From (iv) and (v) $c = 2b$ and $a + b = 0$, which are not satisfy by options (c) and (d). Hence options (c) and (d) are rejected.

Thus point (x_1, y_1, z_1) on the required line will be either (0, 0, 0) or (1, –1, 0).

Now foot of the perpendicular from point (0, 0, 0) to the line (iii)

$= (1, -2r - 1, r)$



The direction ratios of the line joining the points (0, 0, 0) and $(1, -2r - 1, r)$ are $1, -2r - 1, r$

Since, $a + b = 0$.

$\therefore 1 - 2r - 1 = 0 \Rightarrow r = 0$

Hence direction ratio are 1, – 1, 0

But $c = 2b$ i.e. $0 = 2(-1)$, which is not true.

Hence the shortest line does not pass through the point (0, 0, 0). Therefore option (a) is also rejected.

42. **(d)** For L_1,

$$x = \sqrt{\lambda}y + (\sqrt{\lambda}-1) \Rightarrow y = \frac{x-(\sqrt{\lambda}-1)}{\sqrt{\lambda}} \quad \text{...(i)}$$

$$z = (\sqrt{\lambda}-1)y + \sqrt{\lambda} \Rightarrow y = \frac{z-\sqrt{\lambda}}{\sqrt{\lambda}-1} \quad \text{...(ii)}$$

From (i) and (ii)

$$\frac{x-(\sqrt{\lambda}-1)}{\sqrt{\lambda}} = \frac{y-0}{1} = \frac{z-\sqrt{\lambda}}{\sqrt{\lambda}-1} \quad \text{...(iii)}$$

The equation (A) is the equation of line L_1.

Similarly equation of line L_2 is

$$\frac{x-(1-\sqrt{\mu})}{\sqrt{\mu}} = \frac{y-0}{1} = \frac{z-\sqrt{\mu}}{1-\sqrt{\mu}}$$

Since $L_1 \perp L_2$, therefore

$$\sqrt{\lambda}\sqrt{\mu} + 1\times 1 + (\sqrt{\lambda}-1)(1-\sqrt{\mu}) = 0$$

$$\Rightarrow \sqrt{\lambda} + \sqrt{\mu} = 0 \Rightarrow \sqrt{\lambda} = -\sqrt{\mu}$$

$$\Rightarrow \lambda = \mu$$

43. **(b)** Let $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4} = \lambda$

$\Rightarrow \quad x = 2\lambda+1, y = 3\lambda-1$ and $z = 4\lambda+1$

and $\dfrac{x-3}{1} = \dfrac{y-k}{2} = \dfrac{z}{1} = \mu$

$\Rightarrow \quad x = 3+\mu, y = k+2\mu$ and $z = \mu$

Since given lines intersect each other

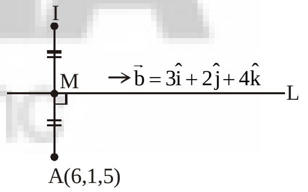
$\Rightarrow \quad 2\lambda+1 = 3+\mu$ ...(i)

$3\lambda-1 = 2\mu+k$ ...(ii)

$\mu = 4\lambda+1$ ...(iii)

Solving (i) and (iii) and putting the value of λ and μ in (ii) we get, $k = \dfrac{9}{2}$

44. **(62)**



Let $M(3\lambda+1, 2\lambda, 4\lambda+2)$ be the four of perpendicular from A on L.

$\overrightarrow{AM}\cdot\vec{b} = 0$

$\Rightarrow \quad 9\lambda-15+4\lambda-2+16\lambda-12 = 0$

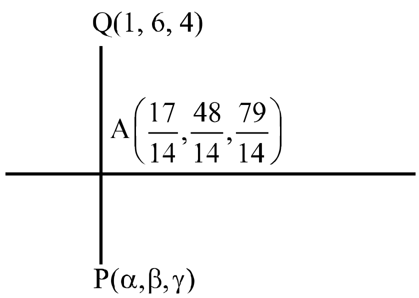
$\Rightarrow \quad 29\lambda = 29$

$\Rightarrow \quad \lambda = 1$

M (4, 2, 6), I = (2, 3, 7) I is the image of A in Line L.

Required Distance $= \sqrt{4+9+49} = \sqrt{62}$

45. **(11)**



Given, $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = t$

So, Direction ratio is $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

and A(t, 2t + 1, 3t + 2) will be a general point on given line.

$\therefore \quad \overrightarrow{QA} = (t-1)\hat{i} + (2t-5)\hat{j} + (3t-2)\hat{k}$

Now $\overrightarrow{QA}.\vec{b} = 0 \quad \{\because \overrightarrow{QA} \perp \vec{b}\}$

$\Rightarrow \quad (t-1) + 2(2t-5) + 3(3t-2) = 0$

$\Rightarrow \quad 14t = 17 \Rightarrow t = 17/14$

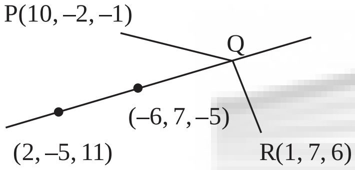
Hence point A is $A\left(\frac{17}{14}, \frac{48}{14}, \frac{79}{14}\right)$

Since A is mid point of P and Q

$\alpha = \frac{20}{14} \quad \beta = \frac{12}{14} \quad \gamma = \frac{102}{14}$

$2\alpha + \beta + \gamma = \frac{154}{14} = 11$

46. **(13)** P(10, –2, –1)



The given line : $\frac{x+6}{-8} = \frac{y-7}{12} = \frac{z+5}{-16} = \lambda$

Then co-ordinate of Q is,

$Q = (2\lambda - 6,\ 7 - 3\lambda,\ 4\lambda - 5)$

Now, $\overrightarrow{QR} = (2\lambda - 7,\ -3\lambda,\ 4\lambda - 11)$

Since, QR is perpendicular to the line

$\therefore \overrightarrow{QR} \cdot (dr's \text{ of line}) = 0$

$\Rightarrow 4\lambda - 14 + 9\lambda + 16\lambda - 44 = 0$

$\Rightarrow 29\lambda = 58 \Rightarrow \lambda = 2$

$\therefore \quad Q = (-2, 1, 3)$

$PQ = \sqrt{144 + 9 + 16} = \sqrt{169} = 13$

47. **(43)** $\bar{a}_1 = \lambda\hat{i} + 2\hat{j} + \hat{k}$

$\bar{a}_2 = -2\hat{} - 5\hat{} + 4\hat{k} \quad j \quad i$

$\bar{p} = 3\hat{i} - \hat{j} + \hat{k}$

$\vec{q} = -3\hat{i} + 2\hat{j} + 4\hat{k}$

$(\lambda + 2)\hat{i} + 7\hat{j} - 3\hat{k} = \bar{a}_1 - \bar{a}_2$

$\bar{p} \times \vec{q} = -6\hat{i} - 15\hat{j} + 3\hat{k}$

Shortest distance $= \left| \frac{(\bar{a}_2 - \bar{a}_1).(\bar{p} \times \vec{q})}{|\bar{p} \times \vec{q}|} \right|$

$\frac{44}{\sqrt{30}} = \frac{|-6\lambda - 12 - 105 - 9|}{\sqrt{(-6)^2 + (-15)^2 + 3^2}}$

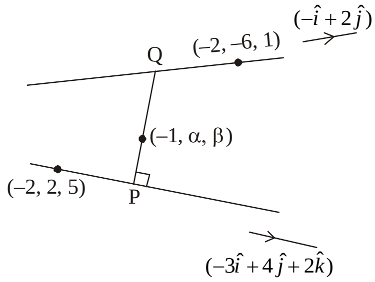
$\frac{44}{\sqrt{30}} = \frac{|6\lambda + 126|}{3\sqrt{30}}$

$132 = |6\lambda + 126|$

$\lambda = 1, \lambda = -43$

largest possible $|\lambda|$ is $|\lambda| = 43$

48. **(25)**



Coordinate of point P and Q, are

P (–3λ – 2, 4λ + 2, 2λ + 5)

Q (–μ – 2, 2μ – 6, 1)

dr′s of PQ = (3λ – μ, 2μ – 4λ – 8, –2λ – 4)

dr′s of PQ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -3 & 4 & 0 \end{vmatrix}$

$= (4\hat{i} + 2\hat{j} + 2\hat{k})$ OR (2, 1, 1)

$\frac{3\lambda - \mu}{2} = \frac{2\mu - 4\lambda - 8}{1} = \frac{-2\lambda - 4}{1}$

$\Rightarrow \mu = \lambda + 2 \;\&\; 7\lambda = \mu - 8$

$\lambda = -1 \qquad \mu = 1$

Q : (–3, –4, 1)

Equation of line PQ: $\frac{x+3}{2} = \frac{y+4}{1} = \frac{z-1}{1}$

$(-1, \alpha, \beta) \Rightarrow 1 = \frac{\alpha + 4}{1} = \frac{\beta - 1}{1}$

$\Rightarrow \quad \alpha = -3, \beta = 2$

$\therefore \quad (\alpha - \beta)^2 = 25$

49. **(48)** $\frac{38}{3\sqrt{5}}k = \frac{(5\hat{i} + 5\hat{j} - 9\hat{k})}{\sqrt{5}} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & -3 & 2 \end{vmatrix}$

$\Rightarrow \frac{38}{3\sqrt{5}}k = \frac{19}{\sqrt{5}}$

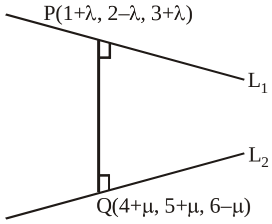
$\Rightarrow k = \frac{19}{\sqrt{5}} \times \frac{3\sqrt{5}}{38} \Rightarrow k = \frac{3}{2}$

$\int_0^{3/2} [x^2] dx = \int_0^1 0 dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{3/2} 2 dx$

$= 2 - \sqrt{2}$

$\alpha = 2 \Rightarrow 6\alpha^3 = 48.$

50. **(21)**



$\vec{b} = \hat{i} - \hat{j} + \hat{k}$ (Direction vector of L_1),

$\vec{d} = \hat{i} + \hat{j} - \hat{k}$ (Direction vector of L_2)

$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 0\hat{i} + 2\hat{j} + 2\hat{k}$ (Direction vector of Line perpendicular to L_1 and L_2)

DR's of PQ line

$= (0, 2, 2) = (3 + \mu - \lambda, 3 + \mu + \lambda, 3 - \mu - \lambda)$

$$\frac{3+\mu-\lambda}{0} = \frac{3+\mu+\lambda}{2} = \frac{3-\mu-\lambda}{2}$$

Solving above equations we get $\mu = -\frac{3}{2}$ and $\lambda = \frac{3}{2}$

point $P \equiv \left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right)$, $Q \equiv \left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$

Mid point of PQ $= \left(\frac{5}{2}, 2, 6\right) = (\alpha, \beta, \gamma)$

$2(\alpha + \beta + \gamma) = 5 + 4 + 12 = 21$

51. **(12)** $L_1 : \frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = r \;\to Q(r, r, 1)$

$L_2 : \frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0} = k \;\to R(k, -k, -1)$

Given P(a, a, a)

Dr's of $PQ = (a-r), (a-r), (a-1)$

Dr's of $L_1 = 1, 1, 0$

$\therefore \quad 1(a-r) + (a-r) = 0 \Rightarrow \quad a - r = 0$

$\Rightarrow \quad a = r$

Dr's of $PR = (a-k), (a+k), (a+1)$

Dr's of $L_2 = (1, -1, 0)$

$a - k - a - k = 0 \Rightarrow k = 0$

As $PQ \perp PR$

$(a-r)(a-k) + (a-r)(a+k) + (a-1)(a+1) = 0$

$a = 1$ or $-1 \Rightarrow 12a^2 = 12$

52. **(22)** Equation of line AB is

$$\frac{x-4}{12} = \frac{x+6}{4} = \frac{z+2}{6} = \lambda$$

$\therefore \quad P(12\lambda + 4, 4\lambda - 6, 6\lambda - 2)$

$PA = \sqrt{(12\lambda)^2 + (4\lambda)^2 + (6\lambda)^2} \Rightarrow 196\lambda^2 = 441$

$\Rightarrow \lambda = \pm\frac{3}{2}$

$$\frac{x-4}{\frac{6}{7}} = \frac{y+6}{\frac{2}{7}} = \frac{z+2}{\frac{3}{7}} = 21$$

$P\left(21 \times \frac{6}{7} + 4, \frac{2}{7} \times 21 - 6, \frac{3}{7} \times 21 - 2\right)$

$= P(22, 0, 7) = P(a, b, c)$

$\therefore \quad PQ = \sqrt{324 + 144 + 16} = 22$

53. **(16)** Let $L_1 = \frac{x+1}{1} - \frac{y}{1/2} = \frac{z}{-1/12}$,

$L_2 = \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$,

d_1 = shortest distance between lines L_1 & L_2

$$d_1 = \left|\frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|(\vec{b}_1 \times \vec{b}_2)|}\right|$$

$d_1 = 2$

$L_3 : \frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$, $L_4 : \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$

d_2= shortest distance between L_3 and L_4,

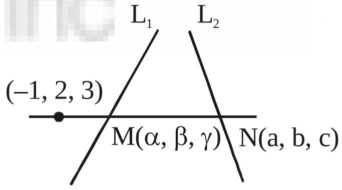
$d_2 = \frac{12}{\sqrt{3}}$

Now, $\frac{32\sqrt{3}d_1}{d_2} = \frac{32\sqrt{3} \times 2}{\frac{12}{\sqrt{3}}} = \frac{32 \times 3}{6} = 16$

54. **(196)**

$M(3\lambda + 1, 2\lambda + 2, -2\lambda - 1)$ Then, $\alpha + \beta + \gamma = 3\lambda + 2$

$N(-3\mu - 2, -2\mu + 2, 4\mu + 1)$. Then, $a + b + c = -\mu + 1$



Now, $\frac{3\lambda+2}{-3\mu-1} = \frac{2\lambda}{-2\mu} = \frac{-2\lambda-4}{4\mu-2}$

$\Rightarrow 3\lambda\mu + 2\mu = 3\lambda\mu + \lambda$

$\Rightarrow 2\mu = \lambda$ ...(i)

$2\lambda\mu - \lambda = \lambda\mu + 2\mu$

$\Rightarrow \lambda\mu = \lambda + 2\mu$ ...(ii)

$\Rightarrow \lambda\mu = 2\lambda$ ($\because$ from (i) and (ii))

$\Rightarrow \mu = 2 \quad (\lambda \neq 0)$

$\therefore \lambda = 4$

Now, $\alpha + \beta + \gamma = 14$

& $a + b + c = -1$

Then, $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2} = 196$

55. **(65)** Let P(t, t –2, t) and Q(2s – 2, s, s)
D.R's of PQ are 2, 1, 2

$$\frac{2s-2-t}{2} = \frac{s-t-2}{2} = \frac{s-t}{2}$$

$\Rightarrow$ t = 6 and s = 2
$\Rightarrow$ P(6, 4, 6) and Q (2, 2, 2)

PQ: $\frac{x-2}{2} = \frac{y-2}{1} = \frac{z-2}{2} = \lambda$

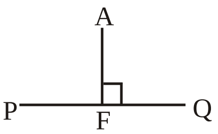
Let $F(2\lambda+2, \lambda+2, \lambda+2)$
A(1, 2, 12)

$\overrightarrow{AF}.\overrightarrow{PQ} = 0$ [AF $\perp$ PQ]

$\therefore \lambda = 2$

So, F(6, 4, 6) and AF = $\sqrt{65}$

$\Rightarrow l = \sqrt{65}$
$\Rightarrow l^2 = 65$



56. **(9)** $L_1: \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda$ (let)

dr's of $\vec{b}_1 = (4, 1, 3)$

$M(4\lambda+5, \lambda+4, 3\lambda+5)$

$L_2: \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \mu$ (let)

dr's of $\vec{b}_2 = (12, 5, 9)$

$N(12\mu-8, 5\mu-2, 9\mu-11)$

$MN = (4\lambda - 12\mu + 13, \lambda - 5\mu + 6, 3\lambda - 9\mu + 16)$ ...(i)

Now, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k}$ ...(ii)

From equation (i) and (ii)

$$\therefore \underset{\textbf{I}}{\frac{4\lambda-12\mu+13}{-6}} = \underset{\textbf{II}}{\frac{\lambda-5\mu+6}{0}} = \underset{\textbf{III}}{\frac{3\lambda-9\mu+16}{8}}$$

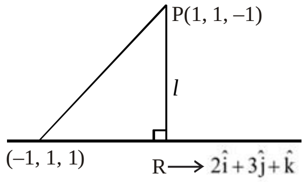
Solving I and II, we get
$\lambda - 5\mu + 6 = 0$ ...(iii)
Solving I and III, we get
$\lambda - 3\mu + 4 = 0$ ...(iv)
Solve (iii) and (iv), we get
$\lambda = -1, \mu = 1$
$\therefore$ M(1, 3, 2)
N(4, 3, –2)

$\therefore \overrightarrow{OM}.\overrightarrow{ON} = 4 + 9 - 4 = 9$

57. **(108)** $\frac{x-2}{1} = \frac{y}{-1} = \frac{z-7}{8} = \lambda$

$\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1} = k$

$\Rightarrow \lambda + 2 = 4k - 3$ and $-\lambda = 3k - 2$
$\Rightarrow k = 1, \lambda = -1$
Now, $8\lambda + 7 = k - 2 \Rightarrow -1 = -1$ satisfy
$\therefore$ P = (1, 1, –1)



Projection of $\overrightarrow{QP} = 2\hat{i} - 2\hat{k}$ on $2\hat{i} + 3\hat{j} + \hat{k}$ is

$|\overrightarrow{QR}| = \frac{4-2}{\sqrt{4+9+1}} = \frac{2}{\sqrt{14}}$ and $|\overrightarrow{QP}| = 2\sqrt{2}$

$\therefore l^2 = |\overrightarrow{PQ}|^2 - |\overrightarrow{QR}|^2 = 8 - \frac{4}{14} = \frac{108}{14}$

$\Rightarrow 14l^2 = 108$

58. **(5)** Let $\ell = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \gamma(a\hat{i} + b\hat{j} + c\hat{k})$

$= \gamma(a\hat{i} + b\hat{j} + c\hat{k})$

$= a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix} = -4\hat{i} - 5\hat{j} - 2\hat{k}$

$= \ell = y(-4\hat{i} + 5\hat{j} + 2\hat{k})$

and, $\ell_1 \ (1+\lambda)\hat{i} + (-11+2\lambda)\hat{j} + (-7+3\lambda)\hat{k}$
P is intersection of ℓ and ℓ_1
$-4\gamma = 1 + \lambda, 5\lambda = -1\lambda + 2\lambda, -2\lambda = -7 + 3\lambda$
By solving there equation $\gamma = -1$, P (4, –5, 2)
Let $Q(-1 + 2\mu, 2\mu, 1 + \mu)$
then, $\overrightarrow{PQ} = (5 - 2\mu, -5 - 2\mu, 1 - \mu)$

$\overrightarrow{PQ} \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 0 \Rightarrow -2 + 4\mu + 4\mu + 1 + \mu = 0$

$\mu = \frac{1}{9} \Rightarrow Q\left(\frac{-7}{9}, \frac{2}{9}, \frac{10}{9}\right)$

$9(\alpha + \beta + \gamma) = 9\left(\frac{-7}{9} + \frac{2}{9} + \frac{10}{9}\right) \Rightarrow \alpha + \beta + y = \frac{5}{9}$

59. **(18)** Given the lines $\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{\alpha}$

And $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$ intersect

So, point on first line (1,2,3) and point on second line (4,1,0).

Vector joining both points is $-3\hat{i}+\hat{j}+3\hat{k}$

Now vector along first line is $2\hat{i}+3\hat{j}+\alpha\hat{k}$

Also vector along second line is $5\hat{i}+2\hat{j}+\beta\hat{k}$

Now these three vectors must be coplanar

$$\Rightarrow \begin{vmatrix} 2 & 3 & \alpha \\ 5 & 2 & \beta \\ -3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2(6-\beta)-3(15+3\beta)+\alpha(11)=0$

$\Rightarrow \alpha-\beta=3 \Rightarrow \alpha=3+\beta$

Now, $8(3+\beta)\cdot\beta = 8\left(\beta^2+3\beta\right)$

$$=8\left(\beta^2+3\beta+\frac{9}{4}-\frac{9}{4}\right)=8\left(\beta+\frac{3}{2}\right)^2-18$$

So magnitude of minimum value = 18

60. **(158)** Since, $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = 4\hat{i}-4\hat{j}-4\hat{k}$

Now, equation of line is $\dfrac{x-2}{1}=\dfrac{y-3}{-1}=\dfrac{z-1}{-1}$

Let Q be (5, 3, 8) and foot of $\perp$ from Q on this line be R.

Now, R $\equiv$ (k + 2, –k + 3, –k + 1) and

DR of QR are (k – 3, –k, –k –7)

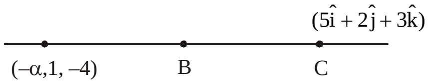
So, (1)(k – 3) + (–1)(–k) + (–1)(–k –7) = 0

$\Rightarrow k=-\dfrac{4}{3}$

$$\therefore \alpha^2=\left(\frac{13}{3}\right)^2+\left(\frac{4}{3}\right)^2+\left(\frac{17}{3}\right)^2=\frac{474}{9}$$

$\therefore 3\alpha^2=158$

61. **(9)** Since A $\cdot$ (0, 2, α)



$$\text{Now, } \left|\frac{1}{2}.2\sqrt{21}.\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & \alpha+4 \\ 5 & 2 & 3 \end{vmatrix}\frac{1}{\sqrt{25+4+9}}\right| = 21\sqrt{21}$$

$\Rightarrow \sqrt{(2\alpha+5)^2+(2\alpha+20)^2+(2\alpha-5)^2}=\sqrt{21}\sqrt{38}$

$\Rightarrow\quad 12\alpha^2+80\alpha+450=798$

$\Rightarrow\quad 12\alpha^2+80\alpha-348=0$

$\Rightarrow\quad \alpha=3 \Rightarrow \alpha^2=9$

62. **(18)** $\vec{P}=(2,3,4)-(1,2,3)=(1,1,1)$

$\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right) \qquad \vec{r}=\vec{a}+\lambda\vec{p}$

$\vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)+\mu\left(2\hat{i}-\hat{j}\right) \qquad \vec{r}=\vec{b}+\mu\vec{q}$

$$\vec{p}\times\vec{q}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix}=\hat{i}+2\hat{j}-3\hat{k}$$

$$d=\left|\frac{\left(\vec{b}-\vec{a}\right)\cdot\left(\vec{p}\times\vec{q}\right)}{|\vec{p}\times\vec{q}|}\right|$$

$$\Rightarrow d=\left|\frac{\left(-3\hat{j}-\hat{k}\right)\cdot\left(\hat{i}+2\hat{j}-3\hat{k}\right)}{\sqrt{14}}\right|$$

$$d=\left|\frac{0-6+3}{\sqrt{14}}\right|=\frac{3}{\sqrt{14}};\ \alpha=\frac{3}{\sqrt{14}}$$

Now, $28\alpha^2=28\times\dfrac{9}{14}=18$

63. **(14)** Shortest distance between the lines

Since $\vec{a}_1=(2,-1,6)$, $\vec{a}_2=(6,1,-8)$ and

$\vec{b}_1=(3,2,2)$, $\vec{b}_2=(3,-2,0)$

Now $(\vec{a}_2-\vec{a}_1)=(4,2,-14)$

$\vec{b}_1\times\vec{b}_2=(4,6,-12)$

So, shortest distance = $\left|\dfrac{(\vec{a}_2-\vec{a}_1)(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$$=\left|\frac{16+12+168}{\sqrt{16+36+144}}\right|=\left|\frac{196}{14}\right|=14$$

64. **(384)** Given the lines

$$\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$$

$$\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$$

Now, Vector along line of shortest distance

$$=\begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} \Rightarrow -\hat{i}+2\hat{j}-k(\text{its magnitude is}\sqrt{6})$$

$$\text{Now, } \frac{1}{\sqrt{6}}\begin{vmatrix} \sqrt{6}+\lambda & \sqrt{6} & -3\sqrt{6} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} \Rightarrow \pm 6$$

$\Rightarrow \lambda=-2\sqrt{6},10\sqrt{6}$

Then, square of sum of these values is 384

65. **(153)** Given line is $\dfrac{x+1}{2}=\dfrac{y+2}{3}=\dfrac{z-1}{2}$

let $\dfrac{x+1}{2}=\dfrac{y+2}{3}=\dfrac{z-1}{2}=\lambda$

$x=2\lambda-1,\ y=3\lambda-2,\ z=2\lambda+1$

let point Q(2λ – 1, 3λ – 2, 2λ + 1)

Then, distance of PQ = $\sqrt{26}$.

$(2\lambda-5)^2+(3\lambda-4)^2+(2\lambda-6)^2=26$

$4\lambda^2+25-20\lambda+9\lambda^2+16-24\lambda+4\lambda^2+36-24\lambda=26$

$17\lambda^2-68\lambda+51=0 \quad \Rightarrow 17(\lambda^2-4\lambda+3)=0$

$\Rightarrow \ \lambda^2-4\lambda+3=0 \Rightarrow (\lambda-1)(\lambda-3)=0$

$\Rightarrow \ \lambda=1,3$

Put λ in point Q.

$Q \to (2(1)-1, 3(1)-2, 2(1)+(1) \to (1, 1, 3)$

$R \to (2(3)-1, 3(3)-2, 2(3)+1) \to (5, 7, 7)$

Area of triangle PQR $= \frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right|$

$= \frac{1}{2}\left|(3\hat{i}+\hat{j}+4\hat{k})\times(\hat{i}\times 5\hat{j})\right|$

$= \sqrt{153}$

66. **(6)** If $\vec{r}=\vec{a}+\lambda\vec{b}$ and $\vec{r}=\vec{c}+\lambda\vec{d}$

$\therefore \ \vec{a}-\vec{c}=(\alpha+4)\hat{i}+2\hat{j}+3\hat{k}$

$$\frac{\vec{b}\times\vec{d}}{\left|\vec{b}\times\vec{d}\right|}=\frac{\left(2\hat{i}+2\hat{j}+\hat{k}\right)}{3}$$

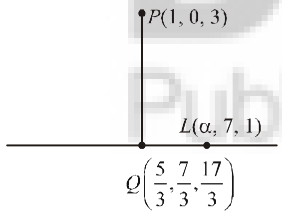
Then shortest distance between two lines is,

$$\frac{(\vec{a}-\vec{c}).\left(\vec{b}\times\vec{d}\right)}{\left|\vec{b}\times\vec{d}\right|}=9$$

$$\Rightarrow \quad \left((\alpha+4)\hat{i}+2\hat{j}+3\hat{k}\right).\frac{\left(2\hat{i}+2\hat{j}+\hat{k}\right)}{3}=9$$

$\Rightarrow \quad 2\alpha+15=27 \Rightarrow \alpha=6$

67. **(4)** Since, PQ is perpendicular to L



$$\therefore \left(1-\frac{5}{3}\right)\left(\alpha-\frac{5}{3}\right)+\left(\frac{-7}{3}\right)\left(7-\frac{7}{3}\right)+\left(3-\frac{17}{3}\right)\left(1-\frac{17}{3}\right)=0$$

$$\Rightarrow \quad \frac{-2\alpha}{3}+\frac{10}{9}-\frac{98}{9}+\frac{112}{9}=0$$

$$\Rightarrow \quad \frac{2\alpha}{3}=\frac{24}{9} \quad \Rightarrow \quad \alpha=4$$

68. **(a, d)** Let any point

$P(\lambda,0,0)$ on L_1, $Q(0,\mu,1)$ on L_2 and $R(1,1,v)$ on L_3

$\because$ P, Q, R are collinear, $\therefore \ \overrightarrow{PQ}\,||\,\overrightarrow{PR}$

$$\Rightarrow \frac{\lambda}{\lambda-1}=\frac{-\mu}{-1}=\frac{-1}{-v}$$

$$\Rightarrow \mu=\frac{\lambda}{\lambda-1}, v=\frac{\lambda-1}{\lambda}$$

Clearly from above that $\lambda\neq 0,1$

$$\therefore \ Q\left(0,\frac{\lambda}{\lambda-1},1\right)$$

(a) For $Q=\hat{k}-\frac{1}{2}\hat{j}$

$\frac{\lambda}{\lambda-1}=-\frac{1}{2}\Rightarrow 3\lambda=+1$, which is possible.

(b) For $Q=\hat{k}$

$\frac{\lambda}{\lambda-1}=0\Rightarrow\lambda=0$, not possible

(c) For $Q=\hat{k}+\hat{j}$

$\frac{\lambda}{\lambda-1}=1\Rightarrow\lambda=\lambda-1$, not possible

(d) For $Q=\hat{k}+\frac{1}{2}\hat{j}$

$\frac{\lambda}{\lambda-1}=\frac{1}{2}\Rightarrow 2\lambda=\lambda-1\Rightarrow\lambda=-1$, which is possible

Hence options (a) and (d) are correct and options (b) and (c) are incorrect.

69. **(a, b, d)**

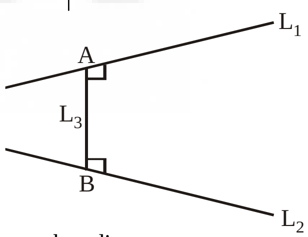
$L_1: \vec{r} = \hat{i}+\lambda(-i+2j+2\hat{k})$

$L_2: \vec{r} = \mu(2i-j+2\hat{k})$

Since L_3 being perpendicular to both L_1 and L_2, is the shortest distance line between L_1 & L_2.

$\therefore$ Direction vector of line L_3: $(-\hat{i}+2\hat{j}+2\hat{k}) \times (2\hat{i}-\hat{j}+2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{vmatrix} = 6\hat{i}+6\hat{j}-3\hat{k}$$



$\because$ L_1 and L_2 are skew lines

Let any point on L_1 and L_2 be

$A(1-\lambda, 2\lambda, 2\lambda)$ and $B(2\mu, -\mu, 2\mu)$.

$\therefore$ dr's of AB $= 2\mu+\lambda-1, -\mu-2\lambda, 2\mu-2\lambda$

$\because$ AB and L_3 are representing the same line

$$\therefore \quad \frac{2\mu+\lambda-1}{6}=\frac{-\mu-2\lambda}{6}=\frac{2\mu-2\lambda}{-3}$$

$\Rightarrow \quad 3\lambda+3\mu=1 \quad$ ...(i)

$6\lambda-3\mu=0 \quad$ ...(ii)

Solving (i) and (ii) we get : $\lambda=\frac{1}{9}$, $\mu=\frac{2}{9}$

$$\therefore \quad A\left(\frac{8}{9},\frac{2}{9},\frac{2}{9}\right) \text{ and } B\left(\frac{4}{9},\frac{-2}{9},\frac{4}{9}\right)$$

$\therefore$ Equation of L_3 is given by

$$\vec{r} = \frac{2}{9}(4\hat{i}+\hat{j}+\hat{k})+t(2i+2j-\hat{k})$$

$\therefore$ (a) is correct.

or $\quad \vec{r} = \frac{2}{9}(2\hat{i}-\hat{j}+2\hat{k})+t(2i+2j-\hat{k})$

$\therefore$ (b) is correct

Also mid-point of AB is $\left(\frac{2}{3}, 0, \frac{1}{3}\right)$

$\therefore$ L_3 can also be written as

$\vec{r} = \frac{1}{3}(2\hat{i}+\hat{k})+t(2\hat{i}+2\hat{j}-\hat{k})$, where $t \in R$

$\therefore$ (d) is correct.

Clearly (0, 0, 0) does not lie on

$$\vec{r} = \frac{2}{9}(4\hat{i}+\hat{j}+\hat{k})+t(2\hat{i}+2\hat{j}-\hat{k})$$

$\therefore$ $\vec{r} = t(2\hat{i}+2\hat{j}-\hat{k})$ can not describe the line L_3.

$\therefore$ (c) is incorrect.

70. **(c)** Given that lines are $x = y$, $z = 1$

$\Rightarrow L_1 = \frac{x-0}{1} = \frac{y-0}{1} = \frac{z-1}{0} = \alpha$ ...(i)

$\therefore Q(\alpha, \alpha, 1)$

and $y = -x$, $z = -1$

$\Rightarrow L_2 = \frac{x-0}{-1} = \frac{y-0}{1} = \frac{z+1}{0} = \beta$ ...(ii)

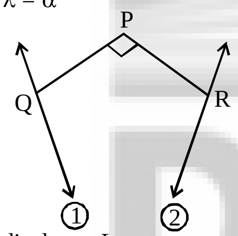
$\therefore R(-\beta, \beta, -1)$ (say)

Direction ratios of PQ are $\lambda - \alpha, \lambda - \alpha, \lambda - 1$

and direction ratios of PR are $\lambda + \beta, \lambda - \beta, \lambda + 1$

$\because PQ$ is perpendicular to L_1

$\therefore \lambda - \alpha = 0 \Rightarrow \lambda = \alpha$ ...(iii)



$\because$ PR is perpendicular to L_2

$\therefore -(\lambda + \beta) + \lambda - \beta = 0 \Rightarrow \beta = 0$

$\therefore$ dr's of PQ are $0, 0, \lambda - 1$

and dr's of PR are $\lambda, \lambda, \lambda + 1$

$\because \angle QPR = 90° \Rightarrow (\lambda - 1)(\lambda + 1) = 0 \Rightarrow \lambda = 1$ or -1

But for $\lambda = 1$, we get point Q itself

$\therefore$ we take $\lambda = -1$

71. **(b, d)** The given lines are

$\ell_1 : \left(3\hat{i}-\hat{j}+4\hat{k}\right)+t\left(\hat{i}+2\hat{j}+2\hat{k}\right)$

$\ell_2 : \left(3\hat{i}+3\hat{j}+2\hat{k}\right)+s\left(2\hat{i}+2\hat{j}+\hat{k}\right)$

Direction vector perpendicular to bo ℓ_1 and ℓ_2

$$\vec{b} = \ell_1 \times \ell_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i}+3\hat{j}-2\hat{k}$$

$\therefore$ $\ell : \frac{x}{2} = \frac{y}{-3} = \frac{z}{2} = \lambda$

Any point on ℓ_1 is $(t + 3, 2t - 1, 2t + 4)$ and any point on ℓ is $(2\lambda, -3\lambda, 2\lambda)$

$\therefore$ Let intersection point of ℓ and ℓ_1 be P.

$t + 3 = 2\lambda,\ 2t - 1 = -3\lambda,\ 2t + 4 = 2\lambda$

$\Rightarrow$ $t = -1,\ \lambda = 1$ $\therefore P(2, -3, 2)$

Any point Q on ℓ_2 is $(2s + 3, 2s + 3, s + 2)$

According to question $PQ = \sqrt{17}$

$\Rightarrow$ $(2s + 1)^2 + (2s + 6)^2 + s^2 = 17$

$\Rightarrow$ $9s^2 + 28s + 20 = 0 \Rightarrow s = -2, \frac{-10}{9}$

$\therefore$ Point Q can be $(-1, -1, 0)$ and $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

72. **(c)** Let $L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3} = \lambda$

and $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma} = \mu$

$x = \lambda - 11 = 3\mu - 16 \Rightarrow \lambda - 3\mu = -5$ ...(i)

$y = 2\lambda - 21 = 2\mu - 11 \Rightarrow 2\lambda - 2\mu = 10$ ...(ii)

$z = 3\lambda - 29 = \mu\gamma - 4 \Rightarrow 3\lambda - \mu\gamma = 25$ ...(iii)

from (i) & (ii)

$\lambda = 10, \mu = 5$

Now from (iii)

$3(10) - 5\gamma = 25$ $\therefore \gamma = 1$

So, $R_1 \equiv (-1, -1, 1)$

Now, $OR_1 = -\hat{i}-\hat{j}+\hat{k}$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = -4\hat{i}-(-8)\hat{j}-4\hat{k}$$

$$\vec{n} = -4\hat{i}+8\hat{j}+4\hat{k} = -4(\hat{i}-2\hat{j}+\hat{k})$$

$$\hat{n} = \pm\frac{4(\hat{i}-2\hat{j}+\hat{k})}{4\sqrt{6}} = \pm\frac{(\hat{i}-2\hat{j}+\hat{k})}{\sqrt{6}}$$

$$\overline{OR}.\hat{n} = \pm(-\hat{i}-\hat{j}+\hat{k})\left(\frac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{6}}\right)$$

$$= \pm\frac{2}{\sqrt{6}} = \pm\sqrt{\frac{4}{6}} = \pm\sqrt{\frac{2}{3}}$$

73. **(A) $\rightarrow$ t ; (B) $\rightarrow$ p,r ; (C) $\rightarrow$ q,s ; (D) $\rightarrow$ r**

Let the line through origin be L : $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ .... (i)

since line L intersects

$L_1 : \frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ .....(ii)

and $L_2 : \frac{x-8/3}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ ....(iii)

at P and Q,

$\therefore$ line L and L_1 coplaner.

$$\therefore \text{Using} \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

we get $\begin{vmatrix} 2 & 1 & -1 \\ a & b & c \\ 1 & -2 & 1 \end{vmatrix} = 0 \Rightarrow a + 3b + 5c = 0$ ...(iv)

Also L and L_2 coplaner

and $\begin{vmatrix} 8/3 & -3 & 1 \\ a & b & c \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow 3a + b - 5c = 0$ ...(v)

On solving (iv) and (v), we get

$\frac{a}{-15-5} = \frac{b}{15+5} = \frac{c}{1-9}$ or $\frac{a}{5} = \frac{b}{-5} = \frac{c}{2}$

Hence equation (i) becomes $\frac{x}{5} = \frac{y}{-5} = \frac{z}{2} = \lambda$

Any point on L, $P(5\lambda, -5\lambda, 2\lambda)$ which lies on (ii) also

$\therefore \frac{5\lambda - 2}{1} = \frac{-5\lambda - 1}{-2} = \frac{2\lambda + 1}{1} \Rightarrow \lambda = 1$

$\therefore$ P $(5, -5, 2)$

Also Any point on L, $Q(5\lambda, -5\lambda, 2\lambda)$ which lies on (iii) also

$\therefore \frac{5\lambda - 8/3}{3} = \frac{-5\lambda + 3}{-1} = \frac{2\lambda - 1}{1} \Rightarrow \lambda = 2/3$

$\therefore Q\left(\frac{10}{3}, \frac{-10}{3}, \frac{4}{3}\right)$

Hence $d^2 = PQ^2 = \left(\frac{25}{9} + \frac{25}{9} + \frac{4}{9}\right) = 6$

(B) $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \tan^{-1}\frac{3}{4}$

$\left[\because \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}\right]$

$\Rightarrow \tan^{-1}\left(\frac{x+3-x+3}{1+x^2-9}\right) = \tan^{-1}\left(\frac{3}{4}\right), x^2 - 9 > -1$

$\Rightarrow \frac{6}{x^2-8} = \frac{3}{4} \Rightarrow x^2 = 16$ or $x = 4, -4$

(C) $\because \vec{a} = \mu\vec{b} + 4\vec{c} \Rightarrow \vec{c} = \frac{\vec{a} - \mu\vec{b}}{4}$

Then $(\vec{b} - \vec{a}).(\vec{b} + \vec{c}) = 0$

$\Rightarrow (\vec{b} - \vec{a}).\left(\vec{b} + \frac{\vec{a} - \mu\vec{b}}{4}\right) = 0$

$\Rightarrow (\vec{b} - \vec{a}).\left(\frac{4-\mu}{4}\vec{b} + \frac{\vec{a}}{4}\right) = 0$

$\Rightarrow \frac{4-\mu}{4}|\vec{b}|^2 - \frac{|\vec{a}|^2}{4} = 0$

$\Rightarrow (4-\mu)|\vec{b}|^2 - |\vec{a}|^2 = 0$ ...(i)

Also,

$2|\vec{b} + \vec{c}| = |\vec{b} - \vec{a}| \Rightarrow 2^2\left|\frac{4-\mu}{4}\vec{b} + \frac{\vec{a}}{4}\right|^2 = |\vec{b} - \vec{a}|^2$

$\Rightarrow (4-\mu)^2|\vec{b}|^2 + |\vec{a}|^2 = 4|\vec{b}|^2 + 4|\vec{a}|^2 \quad [\because \vec{a}.\vec{b} = 0]$

$\Rightarrow \left[(4-\mu)^2 - 4\right]|\vec{b}|^2 = 3|\vec{a}|^2$ ...(ii)

From (i) and (ii), we get

$\frac{(4-\mu)^2 - 4}{4-\mu} = \frac{3}{1}$

$\Rightarrow \mu^2 - 8\mu + 12 = 12 - 3\mu \Rightarrow \mu^2 - 5\mu = 0$

$\Rightarrow \mu = 0$ or 5

(D) $I = \frac{2}{\pi}\int_{-\pi}^{\pi} \frac{\sin(9x/2)}{\sin(x/2)}dx = \frac{2}{\pi} \times 2\int_0^{\pi} \frac{\sin 9x/2}{\sin x/2}dx$

[$\because f(x)$ is even function]

Let $\frac{x}{2} = \theta \Rightarrow dx = 2d\theta$

Also at $x = 0, \theta = 0$ and at $x = \pi, \theta = \pi/2$

$\therefore I = \frac{8}{\pi}\int_0^{\pi/2} \frac{\sin 9\theta}{\sin\theta}d\theta$

$I = \frac{8}{\pi}\int_0^{\pi/2}\left[\frac{\sin 9\theta - \sin 7\theta}{\sin\theta} + \frac{(\sin 7\theta - \sin 5\theta)}{\sin\theta} + \frac{(\sin 5\theta - \sin 3\theta)}{\sin\theta} + \frac{(\sin 3\theta - \sin\theta)}{\sin\theta} + \frac{\sin\theta}{\sin\theta}\right]d\theta$

$= \frac{16}{\pi}\int_0^{\pi/2}(\cos 8\theta + \cos 6\theta + \cos 4\theta + \cos 2\theta)d\theta + \frac{8}{\pi}\int_0^{\pi/2} d\theta$

$= \frac{16}{\pi}\left[\frac{\sin 8\theta}{8} + \frac{\sin 6\theta}{6} + \frac{\sin 4\theta}{4} + \frac{\sin 2\theta}{2}\right]_0^{\pi/2} + \frac{8}{\pi}(\theta)_0^{\pi/2}$

$= 0 + \frac{8}{\pi}\left(\frac{\pi}{2} - 0\right) = 4$

Topic-3: Equation of a Plane in Different Forms, Equation of a Plane Passing Through the Intersection of two Given Planes, Projection of a Line on a Plane

1. **(a)** Equation of the plane passing through the intersection line of given planes is

$(x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$

or

$(1+\lambda)x + (2-\lambda)y + (3+\lambda)z + (-2-3\lambda) = 0$

$\because$ Its distance from the point (3, 1, –1) is $\frac{2}{\sqrt{3}}$

$\therefore \left|\frac{3(1+\lambda) + 1(2-\lambda) - 1(3+\lambda) + (-2-3\lambda)}{\sqrt{(1+\lambda)^2 + (2-\lambda)^2 + (3+\lambda)^2}}\right| = \frac{2}{\sqrt{3}}$

$\Rightarrow \left|\frac{-2\lambda}{\sqrt{3\lambda^2 + 4\lambda + 14}}\right| = \frac{2}{\sqrt{3}}$

$\Rightarrow 3\lambda^2 + 4\lambda + 14 = 3\lambda^2 \Rightarrow \lambda = -\frac{7}{2}$

$\therefore$ Required equation of plane is

$(x + 2y + 3z - 2) - \frac{7}{2}(x - y + z - 3) = 0$

or $5x - 11y + z = 17$

2. **(a)** Equation of st. line joining Q (2, 3, 5) and R (1, –1, 4) is

$\frac{x-2}{-1} = \frac{y-3}{-4} = \frac{z-5}{1} = \lambda$

Let $P(-\lambda + 2, -4\lambda + 3, -\lambda + 5)$

Since P also lies on $5x - 4y - z = 1$

$\therefore -5\lambda + 10 + 16\lambda - 12 + \lambda - 5 = 1$

$\Rightarrow 12\lambda = 8 \Rightarrow \lambda = \frac{2}{3}$ $\quad \therefore P = \left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$

Now let another point S on QR be

$(-\mu + 2, -4\mu + 3, -\mu + 5)$

Since S is the foot of perpendicular drawn from $T(2, 1, 4)$ to QR, where dr's of ST are μ, $4\mu - 2$, $\mu - 1$ and dr's of QR are $-1, -4, -1$

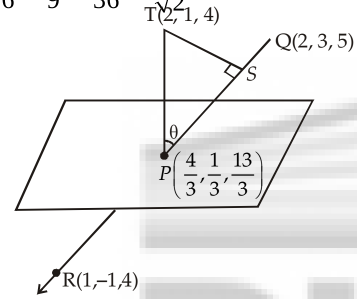
$\therefore \quad -\mu - 16\mu + 8 - \mu + 1 = 0 \Rightarrow 18\mu = 9 \Rightarrow \mu = \frac{1}{2}$

$\therefore \quad S = \left(\frac{3}{2}, 1, \frac{9}{2}\right)$

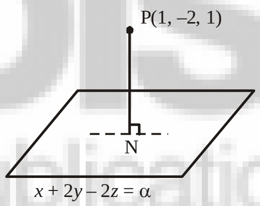
$\therefore$ Distance between P and S

$= \sqrt{\left(\frac{4}{3} - \frac{3}{2}\right)^2 + \left(\frac{1}{3} - 1\right)^2 + \left(\frac{13}{3} - \frac{9}{2}\right)^2}$

$= \sqrt{\frac{1}{36} + \frac{4}{9} + \frac{1}{36}} = \frac{1}{\sqrt{2}}$



3. **(a)** Since perpendicular distance of $x + 2y - 2z - \alpha = 0$ from the point (1, – 2, 1) is 5



$\therefore \left|\frac{1 - 4 - 2 - \alpha}{3}\right| = 5$

$\Rightarrow \frac{-5 - \alpha}{3} = 5 \text{ or } -5$

$\Rightarrow \alpha = -20 \text{ or } 10$

But $\alpha > 0 \Rightarrow \alpha = 10$

$\therefore$ Equation of plane : $x + 2y - 2z - 10 = 0$

We know that foot of perpendicular from point (x, y, z) to the plane $ax + by + cz + d = 0$ is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \frac{-(ax_1 + by_1 + cz_1 + d)}{(a^2 + b^2 + c^2)}$$

$$\therefore \quad \frac{x - 1}{1} = \frac{y + 2}{2} = \frac{z - 1}{-2} = \frac{-(1 - 4 - 2 - 10)}{9} = \frac{5}{3}$$

$\Rightarrow \quad x = \frac{8}{3}, y = \frac{4}{3}, z = -\frac{7}{3}$

$\therefore$ Foot of $\perp^r \equiv \left(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3}\right)$

4. **(c)** Equation of plane containing two lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is given by

$$\begin{vmatrix} x & y & z \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 8x - y - 10z = 0$$

Now equation of plane containing the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane $8x - y - 10z = 0$ is given by,

$$\begin{vmatrix} x & y & z \\ 2 & 3 & 4 \\ 8 & -1 & -10 \end{vmatrix} = 0$$

$\Rightarrow -26x + 52y - 26z = 0 \quad or \quad x - 2y + z = 0$

5. **(c)** Since line makes equal angle with coordinate axes and which has positive direction cosines

$\therefore$ D·c's = $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$\Rightarrow$ D·r's = 1, 1, 1

$\therefore$ Equation of line is

$\frac{x - 2}{1} = \frac{y + 1}{1} = \frac{z - 2}{1} = \lambda$

$\therefore Q(\lambda + 2, \lambda - 1, \lambda + 2)$ be any point on this line where it meets the plane $2x + y + z = 9$

$\Rightarrow \quad 2(\lambda + 2) + \lambda - 1 + \lambda + 2 = 9 \Rightarrow \lambda = 1$

$\therefore$ Q has coordintes (3, 0, 3)

$\therefore \quad PQ = \sqrt{(2 - 3)^2 + (-1 - 0)^2 + (2 - 3)^2} = \sqrt{3}$

6. **(a)** $\because \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$

$= (1 - 3\mu)\hat{i} + (-1 + \mu)\hat{j} + (2 + 5\mu)\hat{k}$

Let coordinates of Q be $(-3\mu + 1, \mu - 1, 5\mu + 2)$

$\therefore$ d.r's of $\overrightarrow{PQ} = -3\mu - 2, \mu - 3, 5\mu - 4$

Given that $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$

$\therefore \quad 1.(-3\mu - 2) - 4.(\mu - 3) + 3.(5\mu - 4) = 0$

$\Rightarrow \quad 8\mu = 2 \text{ or } \mu = \frac{1}{4}$

7. **(d)** We know that the equation of plane through the point (1, –2, 1) and perpendicular to the planes $2x - 2y + z = 0$ and $x - y + 2z = 4$ is

$$\begin{vmatrix} x - 1 & y + 2 & z - 1 \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 0 \Rightarrow x + y + 1 = 0$$

It's distance from the point (1, 2, 2) is

$\left|\frac{1 + 2 + 1}{\sqrt{2}}\right| = 2\sqrt{2}.$

8. **(d)** Let $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ be the eq^n of variable plane which meets the axes at A (a, 0, 0), B (0, b, 0) and C (0. 0, c).

$\therefore$ Centroid of ΔABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\Rightarrow x = \frac{a}{3}, y = \frac{b}{3}, z = \frac{c}{3}$

putting these values in

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k \Rightarrow \frac{9}{a^2}+\frac{9}{b^2}+\frac{9}{c^2}=k$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{z^2}=\frac{k}{9} \qquad ...(i)$$

Also given that the distance of plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ from (0, 0, 0) is 1 unit.

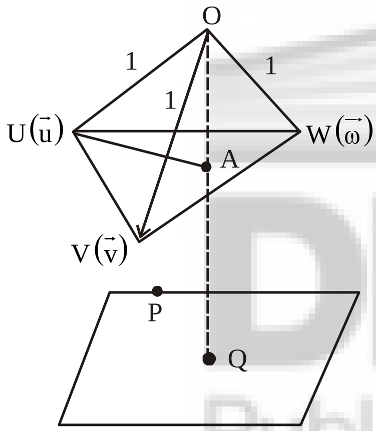
$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=1 \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=1$$

From (i) we get $\frac{k}{9}=1$ i.e. $k=9$

9. **(a)** Since the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x-4y+z=7,$ then the point (4, 2, k) on line also lie on the given plane and hence

$2\times4-4\times2+k=7 \Rightarrow k=7$

10. **(45)**



Given, $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}=|\vec{w}-\vec{u}|$

So, ΔUVW is one equilateral triangle

Given that distances of points U, V, W from plane $P=\frac{7}{2}\Rightarrow AQ=\frac{7}{2}$

Distance of plane P from origin

$$=\left|\frac{0+0+0-16}{\sqrt{3+4+9}}\right|=4=OQ$$

$\therefore$ OA = OQ – AQ = $4-\frac{7}{2}=\frac{1}{2}$

In ΔOAU, $UA=\sqrt{OV^2-OA^2}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}=R$

In ΔUVW, is circumcenter

US = R cos30° $\Rightarrow$ UV = 2 R cos 30° = $\frac{3}{2}$

$\therefore$ Ar $\Delta UVW=\frac{\sqrt{3}}{4}\left(\frac{3}{2}\right)^2=\frac{9\sqrt{3}}{16}$

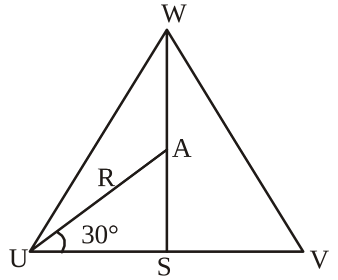
Volume of tetrahedron with coteminous edges

$\vec{u}, \vec{v}, \vec{w}=\frac{1}{3}$ (Ar ΔUVW) × OA

$$=\frac{1}{3}\times\frac{9\sqrt{3}}{16}\times\frac{1}{2}=\frac{3\sqrt{3}}{32}$$

$\therefore$ Volume of parallelopiped:

V = 6 × volume of tetrahedron = $\frac{6\times3\sqrt{3}}{32}=\frac{9\sqrt{3}}{16}$



Now, $\frac{80}{\sqrt{3}}\mathbf{V}=\frac{80}{\sqrt{3}}\times\frac{9\sqrt{3}}{16}=45$

11. **(8)** Let coordinates of P are (a, b, c).

So, coordinates of Q are (0, 0, c) and coordinates of R are (a, b, –c).

Given that, PQ is perpendicular to the plane x + y = 3.

So, PQ is parallel to the normal of given plane

i.e. $(a\hat{i}+b\hat{j})$ is parallel to $(\hat{i}+\hat{j})$ on comparing

$\Rightarrow$ a = b

As mid-point of PQ lies in the plane x + y = 3, so

$$\frac{a}{2}+\frac{b}{2}=3$$

$\Rightarrow$ a + b = 6 $\Rightarrow$ a = 3 = b

Therefore, distance of P from the x-axis

$=\sqrt{b^2+c^2}=5$ (given)

$\Rightarrow$ $b^2+c^2=25$

$\Rightarrow$ $c^2=25-9=16 \Rightarrow c=\pm4$

Hence, PR = |2c| = 8

12. **(6)** The equation of plane containing the given lines:

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}=0 \Rightarrow x-2y+z=0$$

$\therefore$ Distance between $x-2y+z=0$ and $Ax-2y+z=d$

= Perpendicular distance between parallel planes $(\because A=1)$

$$=\frac{|d|}{\sqrt{6}}=\sqrt{6} \Rightarrow |d|=6.$$

13. **(0.75)**

Given that lines are $\vec{r}=\lambda\hat{i}$ ......(i)

$\vec{r}=\mu(\hat{i}+\hat{j})$ ......(ii)

$\vec{r}=v(\hat{i}+\hat{j}+\hat{k})$ ....(iii)

These lines cut the plane $x+y+z=1$ at points A (λ, 0, 0), B $(\mu,\mu,0)$ and C (ν,ν,ν) respectively

Since, A lies on plane $\Rightarrow \lambda=1 \Rightarrow A$ (1, 0, 0)

Since, B lies on plane $\Rightarrow \mu+\mu=1 \Rightarrow \mu=\frac{1}{2}$

$$\Rightarrow B\left(\frac{1}{2},\frac{1}{2},0\right)$$

Since, C lies on plane $\Rightarrow \nu+\nu+\nu=1 \Rightarrow \nu=\frac{1}{3}$

$\Rightarrow C\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Area $(\Delta ABC) = \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = \frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{-1}{2} & \frac{1}{2} & 0\\ \frac{-2}{3} & \frac{1}{3} & \frac{1}{3}\end{vmatrix}$

$= \frac{1}{2}\left|\frac{1}{6}\hat{i}+\frac{1}{6}\hat{j}+\frac{1}{6}\hat{k}\right| = \frac{1}{2}\times\frac{1}{6}\sqrt{3} = \frac{\sqrt{3}}{12}$

$\therefore (6\Delta)^2 = 36\times\frac{3}{144} = \frac{3}{4} = 0.75$

14. Equation of plane containing vectors $\hat{i}$ and $\hat{i}+\hat{j}$ is

$\left[\vec{r}-\hat{i}\ \ \hat{i}\ \ \hat{i}+\hat{j}\right] = 0 \Rightarrow \begin{vmatrix}x-1 & y & z\\ 1 & 0 & 0\\ 1 & 1 & 0\end{vmatrix} = 0$

$\Rightarrow z = 0$ ....(i)

Similarly, equation of plane containing vectors $\hat{i}-\hat{j}$ and $\hat{i}+\hat{k}$ is

$[\hat{r}-(\hat{i}-\hat{j})\ \hat{i}-\hat{j}\ \hat{i}+\hat{k}] = 0 \Rightarrow \begin{vmatrix}x-1 & y+1 & z\\ 1 & -1 & 0\\ 1 & 0 & 1\end{vmatrix} = 0$

$\Rightarrow (x-1)(-1-0)-(y+1)(1-0)+z(0+1) = 0$
$\Rightarrow -x+1-y-1+z = 0$
$\Rightarrow x-y+z = 0$ ....(ii)

Let $\vec{a} = a\hat{i}+b\hat{j}+c\hat{k}$

Since $\vec{a}$ is parallel to (i) and (ii)
$\therefore c = 0$ and $a+b-c = 0 \Rightarrow a = -b$

$\therefore$ a vector in direction of $\vec{a}$ is $\hat{i}-\hat{j}$

Let θ is the angle between $\vec{a}$ and $\hat{i}-2\hat{j}+2\hat{k}$ then

$\cos\theta = \pm\frac{1.1+(-1)(-2)}{\sqrt{1+1}\sqrt{1+4+4}} = \pm\frac{3}{\sqrt{2}.3}$

$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \pi/4$ or $3\pi/4$

15. Unit vector perpendicular to plane, $\hat{n} = \pm\frac{\overrightarrow{PQ}\times\overrightarrow{PR}}{\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right|}$

$\overrightarrow{PQ} = \hat{i}+\hat{j}-3\hat{k}$; $\overrightarrow{PR} = -\hat{i}+3\hat{j}-\hat{k}$

$\therefore \overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -3\\ -1 & 3 & -1\end{vmatrix}$

$= (-1+9)\hat{i}-(-1-3)\hat{j}+(3+1)\hat{k} = 8\hat{i}+4\hat{j}+4\hat{k}$

$\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right| = \sqrt{64+16+16} = \sqrt{96} = 4\sqrt{6}$

$\hat{n} = \pm\left(\frac{8\hat{i}+4\hat{j}+4\hat{k}}{4\sqrt{6}}\right) = \pm\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$

16. **(a, b, c)** Let X = (x, y) S: $\{((x-1)^2+(y-2)^2+(z-3)^2) - ((x-4)^2+(y-2)^2+(z-7)^2) = 50\}$

$\Rightarrow$ S : $\{6x+8z-105 = 0\}$

Similarly T = $\{6x+8z-5 = 0\}$

Both S and T represents the equation of plane and parallel to each other.

Other Distance between plane $= \left|\frac{105-5}{\sqrt{36+64}}\right| = 10$ unit

So. S will contain a triangle of area 1. So (a) is correct.

Hence (b) and (c) are correct but (d) is incorrect.

17. **(a, c)** Equation of line parallel to $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1}$

through P(1, 3, 2) is $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-2}{1} = \lambda$ (let)

Now, putting any point $(\lambda+1, 2\lambda+3, \lambda+2)$ in plane L_1,
$\lambda+1-2\lambda-3+3(\lambda+2) = 6$
$\Rightarrow \lambda = 1$
So, point Q (2, 5, 3)
Equation of line through Q (2, 5, 3) perpendicular to L_1

is $\frac{x-2}{1} = \frac{y-5}{-1} = \frac{z-3}{3} = \mu$ (Let)

Putting any point $(\mu+2, -\mu+5, 3\mu+3)$ in plane L_2
$\Rightarrow \mu = -1$
So, point R (1, 6, 0)

(a) PQ $= \sqrt{1+4+1} = \sqrt{6}$
(b) R (1, 6, 0)

(c) Centroid $\left(\frac{4}{3},\frac{14}{3},\frac{5}{3}\right)$

(d) PQ + QR + PR $= \sqrt{6}+\sqrt{11}+\sqrt{13}$

18. **(a, b)** We have planes P_1 and P_2 given as
$P_1 : 10x+15y+12z-60 = 0$ and
$P_2 : -2x+5y+4z-20 = 0$
Thus, equation of pair of planes is
$S : (10x+15y+12z-60)(-2x+5y+4z-20) = 0$
Now we will obtain a general point of each line and we will solve it with S. If we get more than one value of variable λ, then the line can be the edge of given tetrahedron.

(a) From option we have $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5}$

Let $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5} = \lambda$
So, point is $(1, 1, 5\lambda+1)$
So, $(60\lambda-23)(20\lambda-17) = 0$

$\lambda = \frac{23}{60}$ and $\frac{17}{20}$
So, it can be the edge of tetrahedron.
(b) Similarly for option (b)
point is $(-5\lambda+6, 2\lambda, 3\lambda)$
So, $(16\lambda)(32\lambda-32) = 0$
$\Rightarrow \lambda = 0$ and 1
So, it can be the edge of tetrahedron.

(c) Similarly for option (c)
Point is $(-2\lambda, 5\lambda + 4, 4\lambda)$
So, $(103\lambda)(45\lambda) = 0$
$\lambda = 0$ only
So, it cannot be the edge of tetrahedron.
(d) Similarly for option (d)
Point is $(\lambda, -2\lambda + 4, 3\lambda)$
$\Rightarrow$ $(16\lambda)(-2\lambda) = 0 \Rightarrow \lambda = 0$ only
Hence, it cannot be the edge of tetrahedron.

19. **(a, b, c)** We are given that equation of plane is

$$\vec{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$$

This can be written as

$$\vec{r} = \hat{k} + t\left(-\hat{i} + \hat{j}\right) + p\left(-\hat{i} + \hat{k}\right)$$

Now, equation of plane in standard form is

$$\left[\vec{r} - \hat{k} - \hat{i} + \hat{j} - \hat{i} + \hat{k}\right] = 0$$

$\therefore$ $x + y + z = 1$ ...(i)
Coordinate of $Q = (10, 15, 20)$
Coordinate of $S = (\alpha, \beta, \gamma)$

$$\therefore \quad \frac{\alpha - 10}{1} = \frac{\beta - 15}{1} = \frac{\gamma - 20}{1} = \frac{-2(10 + 15 + 20 - 1)}{3}$$

$$\left[\begin{array}{l} \because \text{ point of reflection is given as } \dfrac{x - x_1}{a} \\ = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c} = \dfrac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2} \end{array}\right]$$

$$\therefore \quad \alpha - 10 = \beta - 15 = \gamma - 20 = -\frac{88}{3}$$

$$\therefore \quad \alpha = -\frac{58}{3}, \beta = -\frac{43}{3}, \gamma = -\frac{28}{3}$$

$\therefore$ $3(\alpha + \beta) = -101$, $3(\beta + \gamma) = -71$
$3(\gamma + \alpha) = -86$ and $3(\alpha + \beta + \gamma) = -129$

20. **(a, b)** The point of intersection of L_1 and L_2 is $(1, 0, 1)$
$\because$ Line L passes through the point of intersection $(1, 0, 1)$ of L_1 and L_2

$$\therefore \frac{1-\alpha}{\ell} = -\frac{1}{m} = \frac{1-\gamma}{-2} \quad ....(i)$$

$\because$ Line L_1 bisects the acute angle between the lines L_1 and L_2, then

$$\vec{r} = \hat{i} + \hat{k} + \lambda\left(\frac{\hat{i} - \hat{j} + 3\hat{k} - 3\hat{i} - \hat{j} + \hat{k}}{\sqrt{11}}\right)$$

$$\Rightarrow \vec{r} = \hat{i} + \hat{k} + t\left(\hat{i} + \hat{j} - 2\hat{k}\right)$$

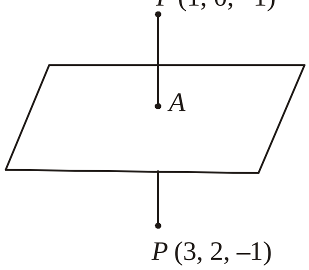
$$\Rightarrow \frac{\ell}{1} = \frac{m}{1} = \frac{-2}{-2} \Rightarrow \ell = m = 1$$

From (i), $\dfrac{1-\alpha}{1} = -1 \Rightarrow \alpha = 2$

and $\dfrac{1-\gamma}{-2} = -1 \Rightarrow \gamma = -1$

$\therefore$ $\alpha - \gamma = 2 - (-1) = 3$ and $l + m = 1 + 1 = 2$

21. **(a, b, c)**



Mid-point of PQ = A (2, 1, –1)
D.r's of PQ = 2, 2, 0
Since PQ perpendicular to plane and mid-point lies on plane
$\therefore$ Equation of plane :
$2(x - 2) + 2(y - 1) + 0(z + 1) = 0 \Rightarrow x - 2 + y - 1 = 0$
$\Rightarrow x + y = 3$ comparing with $\alpha x + \beta y + \gamma z = \delta$,
we get $\alpha = 1, \beta = 1, \gamma = 0$ and $\delta = 3$.
$\therefore$ option (a), (b), (c) are true.

22. **(c, d)**
(a) Direction vector of line of intersection of two planes will be given by $\vec{n}_1 \times \vec{n}_2$.

$$\therefore \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} + 3\hat{k}$$

$\therefore$ dr's of line of intersection of P_1 and P_2 are 1, – 1, 1
$\therefore$ (a) is not correct.
(b) The standard form of given line as

$$\frac{x - \frac{4}{3}}{3} = \frac{y - \frac{1}{3}}{-3} = \frac{z}{3}$$

$\because$ $1 \times 3 + (-1)(-3) + 1(3) = 9 \neq 0$
$\therefore$ This line is not perpendicular to line of intersection
$\therefore$ (b) is not correct.
(c) Let θ be the angle between P_1 and P_2 then

$$\cos\theta = \left|\frac{2 \times 1 + 1 \times 2 + (-1) \times 1}{\sqrt{6}\sqrt{6}}\right| = \frac{3}{6} = \frac{1}{2}$$

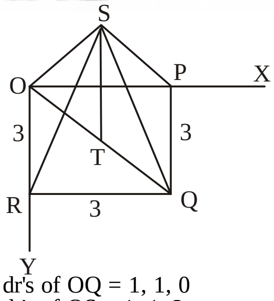
$\therefore$ $\theta = 60°$
Hence (c) is correct.
(d) Equation of plane P_3 :
$1(x - 4) - 1(y - 2) + 1(z + 2) = 0 \Rightarrow x - y + z = 0$

Distance of (2, 1, 1) from $P_3 = \dfrac{2 - 1 + 1}{\sqrt{1 + 1 + 1}} = \dfrac{2}{\sqrt{3}}$

$\therefore$ (d) is correct.

23. **(b, c, d)** According to question the coordinates of vertices of pyramid OPQRS will be

O(0, 0, 0), P (3, 0, 0), Q (3, 3, 0), R (0, 3, 0), $S\left(\dfrac{3}{2}, \dfrac{3}{2}, 3\right)$



dr's of OQ = 1, 1, 0
dr's of OS = 1, 1, 2
$\therefore$ acute angle between OQ and OS

$$= \cos^{-1}\left(\frac{2}{\sqrt{2} \times \sqrt{6}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \neq \frac{\pi}{3}$$

$\therefore$ (a) is not correct

$$\text{Eqn of plane OQS} = \begin{vmatrix} x & y & z \\ 1 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

$\Rightarrow 2x - 2y = 0$ or $x - y = 0$
$\therefore$ (b) is correct.

length of perpendicular from P (3, 0, 0) to plane x – y = 0 is = $\left|\frac{3-0}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$

$\therefore$ (c) is correct.

Eqn of RS : $\frac{x}{\frac{3}{2}} = \frac{y-3}{\frac{-3}{2}} = \frac{z}{3}$ or $\frac{x}{1} = \frac{y-3}{-1} = \frac{z}{2} = \lambda$

$\therefore$ Any point ON RS is N (λ, $-\lambda + 3$, 2λ)

Since ON is perpendicular to RS,

$\because$ ON $\perp$ RS $\Rightarrow 1\times\lambda - 1(-\lambda+3) + 2\times 2\lambda = 0$

$\Rightarrow \lambda = \frac{1}{2} \Rightarrow N\left(\frac{1}{2}, \frac{5}{2}, 1\right)$

$\therefore$ ON = $\sqrt{\frac{1}{4}+\frac{25}{4}+1} = \sqrt{\frac{15}{2}}$

$\therefore$ (d) is correct

24. **(a, b)** $\because$ All the points on L are at a constant distance from P_1 and P_2 that means L is parallel to both P_1 and P_2

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$\therefore \quad L : \frac{x}{1} = \frac{y}{-3} = \frac{z}{-5} = \lambda$ (say)

$\therefore$ Any point on line L is (λ, -3λ, -5λ)

Equation of line perpendicular to P_1 drawn from any point on L is

$$\frac{x-\lambda}{1} = \frac{y+3\lambda}{2} = \frac{z+5\lambda}{-1} = \mu$$

$\therefore \quad M(\mu+\lambda, 2\mu-3\lambda, -\mu-5\lambda)$

But M lies on P_1 so, it satisfy the eqn. of P_1.

$\therefore \quad \mu + \lambda + 4\mu - 6\lambda + \mu + 5\lambda + 1 = 0 \Rightarrow \mu = \frac{-1}{6}$

$\therefore \quad M\left(\lambda - \frac{1}{6}, -3\lambda \frac{-1}{3}, -5\lambda + \frac{1}{6}\right)$

For locus of M,

$x = \lambda - \frac{1}{6}$, $y = -3\lambda - \frac{1}{3}$, $z = 5\lambda + \frac{1}{6}$

$\Rightarrow \quad \frac{x+1/6}{1} = \frac{y+1/3}{-3} = \frac{z-1/6}{-5} = \lambda$

On checking the given point, we find $\left(0, \frac{-5}{6}, \frac{-2}{3}\right)$ and $\left(\frac{-1}{6}, \frac{-1}{3}, \frac{1}{6}\right)$ satisfy the above eqn.

25. **(b, d)** $P_3 : (x+z-1) + \lambda y = 0 \Rightarrow x + \lambda y + z - 1 = 0$

Distance of point (0, 1, 0) from P_3 :

$\left|\frac{\lambda-1}{\sqrt{2+\lambda^2}}\right| = 1 \Rightarrow \lambda^2 - 2\lambda + 1 = \lambda^2 + 2 \Rightarrow \lambda = \frac{-1}{2}$

Distance of point (α, β, γ) from P_3 :

$\left|\frac{\alpha+\lambda\beta+\gamma-1}{\sqrt{2+\lambda^2}}\right| = 2 \Rightarrow \frac{\alpha - \frac{1}{2}\beta + \gamma - 1}{\frac{3}{2}} = \pm 2$

$\Rightarrow \quad \alpha - \frac{1}{2}\beta + \gamma - 1 = \pm 3 \Rightarrow 2\alpha - \beta + 2\gamma - 2 = \pm 6$

$\Rightarrow \quad 2\alpha - \beta + 2\gamma - 8 = 0$ or $2\alpha - \beta + 2\gamma + 4 = 0$

26. **(a, d)** Given that L_1 and L_2 are coplanar, therefore

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$\Rightarrow (5-\alpha)[6 - 5\alpha + \alpha^2 - 2] = 0$

$\Rightarrow (5-\alpha)(\alpha-1)(\alpha-4) = 0 \Rightarrow \alpha = 1, 4, 5.$

27. **(b, c)** Given that lines are coplanar.

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0 \Rightarrow k = \pm 2$$

For $k = 2$, equation of the plane is given by

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & 2 & 2 \\ 5 & 2 & 2 \end{vmatrix} = 0 \Rightarrow y - z + 1 = 0$$

For $k = -2$, equation of the plane is given by

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & -2 & 2 \\ 5 & 2 & -2 \end{vmatrix} = 0 \Rightarrow y + z + 1 = 0$$

28. **(b, d)** Normal vector of plane P_1 is

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 3 \\ 0 & 4 & -3 \end{vmatrix} = -18\hat{i}$$

Normal vector of plane P_2 is

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 3 & 3 & 0 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 3\hat{k}$$

$\therefore \quad \vec{A}$ is parallel to $\pm(\hat{n}_1 \times \hat{n}_2) = \pm(-54\hat{j} + 54\hat{k})$

Now, angle between $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is given by

$$\cos\theta = \pm\frac{(-54\hat{j}+54\hat{k}).(2\hat{i}+\hat{j}-2\hat{k})}{54\sqrt{2}.3} = \pm\frac{1}{\sqrt{2}}$$

$\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

29. **(b)** For largest possible distance between plane H_o and l_2, the line l_2 must be parallel to plane H_o.

$\therefore$ H_o will be the plane containing the line l_1 and parallel to l_2

Normal vector $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{k}$

$\therefore$ H_o : x – z = c/(0, 0, 0) $\Rightarrow$ c = 0

$\therefore$ H_o : x – z = 0

(P) Distance of point (0, 1, –1) from H_o.

$$d(H_o) = \left|\frac{0-(1)}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$$

(Q) The distance of the point (0, 1, 2) from $H_o = \left|\frac{0-2}{\sqrt{2}}\right| = \sqrt{2}$

(R) The distance of origin from $H_o = \left|\frac{0}{\sqrt{2}}\right| = 0$

(S) Point of intersection of planes y = z, x = 1 and H_o is (1, 1, 1).

Distance $= \sqrt{1+1+1} = \sqrt{3}.$

30. **(a)** Let any point on L_1 is $(2\lambda + 1, -\lambda, \lambda - 3)$ and that on L_2 is $(\mu + 4, \mu - 3, 2\mu - 3)$

For point of intersection of L_1 and L_2

$2\lambda + 1 = \mu + 4, -\lambda = \mu - 3, \lambda - 3 = 2\mu - 3$

$\Rightarrow \lambda = 2, \mu = 1$

$\therefore$ Intersection point of L_1 and L_2 is $(5, -2, -1)$

Equation of plane passing through, $(5, -2, -1)$ and perpendicular to P_1 & P_2 is given by

$$\begin{vmatrix} x-5 & y+2 & z+1 \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = 0$$

$\Rightarrow x - 3y - 2z = 13$

$\therefore a = 1, b = -3, c = -2, d = 13$

or (P) $\to$ (3) (Q) $\to$ (2) (R) $\to$ (4) (S) $\to$ (1)

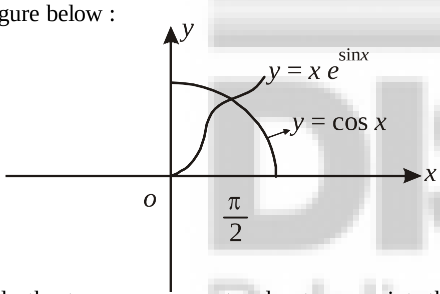
31. $\mathbf{A \to p; B \to q, s; C \to q, r, s, t; D \to r}$

(A) Let us consider two functions

$y = x e^{\sin x}$ and $y = \cos x$

The range of $y = xe^{\sin x}$ is $\left(0, \frac{\pi e}{2}\right)$ and

$\frac{dy}{dx} = e^{\sin x} + xe^{\sin x} \cos x \geq 0$, for $x \leftarrow \left(0, \frac{\pi}{2}\right)$, so, it is an increasing function on $\left(0, \frac{\pi}{2}\right)$. Their graph are as shown in the figure below :



Clearly the two curves meet only at one point, therefore the given equation has only one solution in $\left(0, \frac{\pi}{2}\right)$.

(B) Since given planes intersect in a straight-line

$$\therefore \begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow k(k-4) - 4(4-4) + 1(8-2k) = 0$

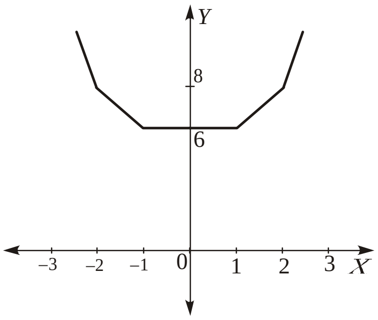
$\Rightarrow k^2 - 6k + 8 = 0 \Rightarrow (k-2)(k-4) = 0$

$\Rightarrow k = 2$ or 4

(C) We have $f(x) = |x-1| + |x-2| + |x+1| + |x+2|$

$$= \begin{cases} -4x, & x \leq -2 \\ -2x+4, & -2 < x \leq -1 \\ 6, & -1 < x \leq 1 \\ 2x+4, & 1 < x \leq 2 \\ 4x, & x \geq 2 \end{cases} \quad \left[\because |x-1| \begin{cases} x-1, \text{ is } x \geq 1 \\ -(x-1) \text{ is } x < 1 \end{cases}\right]$$

The graph of the above function is as given below



Clearly, from graph, $f(x) \geq 6$

$\Rightarrow 4k \geq 6 \Rightarrow k \geq \frac{3}{2}$

$\therefore k = 2, 3, 4, 5, 6, ...$

(D) $\int \frac{dy}{y+1} = \int dx \Rightarrow \ln|y+1| = x + c$

At $x = 0, y = 1 \Rightarrow c = \ln 2$

$\therefore \ln|y+1| = x + \ln 2 \Rightarrow y + 1 = 2e^x \Rightarrow y = 2e^x - 1$

$\therefore y(\ln 2) = 2e^{\ln 2} - 1 = 2 \times 2 - 1 = 3$

32. **(A) $\to$ r ; (B) $\to$ q; (C) $\to$ p ; (D) $\to$ s**

The determinant of the coefficient matrix of given equation, as

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= -\frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

(A) When $a + b + c \neq 0$ and

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

$\Rightarrow a = b = c$ (but $\neq 0$ as $a + b + c \neq 0$)

This equation represent identical planes.

(B) When $a + b + c = 0$ and

$a^2 + b^2 + c^2 - ab - bc - ca \neq 0$

$\Rightarrow \Delta = 0$ and a, b, c are not all equal.

$\therefore$ All equations are not identical but have infinite many solutions.

$\therefore ax + by = (a+b)z$ ... (i) (using $a+b+c=0$)

and $bx + cy = (b+c)z$ ... (ii)

On Solving eqn. (i) and (ii) we, get

$\Rightarrow (b^2 - ac)y = (b^2 - ac)z \Rightarrow y = z$

$\Rightarrow ax + by + cy = 0 \Rightarrow ax = ay \Rightarrow x = y \Rightarrow x = y = z$

$\therefore$ The equations represent the line $x = y = z$

(C) When $a + b + c \neq 0$ and

$a^2 + b^2 + c^2 - ab - bc - ca \neq 0$

$\Rightarrow \Delta \neq 0 \Rightarrow$ Equations have only trivial solution

i.e., $x = y = z = 0$

$\therefore$ the equations represents the three planes meeting at a single point namely origin.

(D) When $a + b + c = 0$ and

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow \quad a = b = c$ and $\Delta = 0 \Rightarrow \quad a = b = c = 0$

$\Rightarrow$ All equations are satisfied by all x, y, and z.

$\Rightarrow$ The equations represent the whole of the three dimensional space (all points in 3–D)

33. (A) → (s); (B) → (p); (C) → (q), (r); (D) → (s)

(A) $x + y = |a|$

$$\frac{ax - y = 1}{(1+a)x = 1 + |a|}$$

$$\Rightarrow x = \frac{1+|a|}{a+1} \Rightarrow y = \frac{a|a|-1}{a+1}$$

$\because$ Rays intersect each other in I quad i.e. $x > 0$. $y \ge 0$

$\Rightarrow a + 1 > 0$ and $a|a| - 1 > 0 \Rightarrow a > 1$

$\therefore a_0 = 1$ (A) → (s)

(B) Given that (α, β, γ) lies on the plane $x + y + z = 2$

$\Rightarrow \alpha + \beta + \gamma = 2$

Also $\hat{k} \times (\hat{k} \times \vec{a}) = (\hat{k}.\vec{a})\hat{k} - (\hat{k}.\hat{k})\vec{a}$

$$\Rightarrow \gamma\hat{k} - \alpha\hat{i} - \beta\hat{j} - \gamma\hat{k} = 0 \Rightarrow \alpha\hat{i} + \beta\hat{j} = 0$$

$\Rightarrow \alpha = 0 = \beta \Rightarrow \gamma = 2$ $\quad (\because \alpha + \beta + \gamma = 2)$

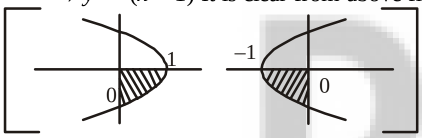
(B) → (p)

(C) $\left|\int_0^1 (1-y^2)dy\right| + \left|\int_0^1 (y^2-1)dy\right|$

$$= 2\left|\int_0^1 (1-y^2)dy\right| = \frac{4}{3}$$

$\because$ $y = \sqrt{1-x}$, $\Rightarrow y^2 = -(x-1)$ and $y = \sqrt{1+x}$

$\Rightarrow y^2 = (x+1)$ It is clear from above figure that



$$\left|\int_0^1 \sqrt{1-x}\,dx\right| + \left|\int_{-1}^0 \sqrt{1+x}\,dx\right| = 2\int_0^1 \sqrt{1-x}\,dx$$

$$= 2\int_0^1 \sqrt{x}\,dx \quad \left[\text{Using} \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \left[2.\frac{2}{3}x^{3/2}\right]_0^1 = \frac{4}{3}, \quad \text{(C) → (r) and (q)}$$

(D) Given that $\sin A \sin B \sin C + \cos A \cos B = 1$

We know that $\sin A \sin B \sin C + \cos A \cos B \le \sin A \sin B + \cos A \cos B = \cos(A - B)$

$\Rightarrow \cos(A - B) \ge 1 \Rightarrow \cos(A - B) = 1$

$\Rightarrow A - B = 0 \Rightarrow A = B$

$\therefore$ Given relation becomes $\sin^2 A \sin C + \cos^2 A = 1$

$\Rightarrow \sin C = 1$,

(D) → (s)

34. (b) Vector in the direction of $L_1 = \overrightarrow{b_1} = 3\hat{i} + \hat{j} + 2\hat{k}$

Vector in the direction of $L_2 = \overrightarrow{b_2} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ Vector perpendicular to both L_1 and L_2

$$= \overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}$$

$\therefore$ Required unit vector

$$= \hat{b} = \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{\sqrt{1+49+25}} = \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$$

35. (d) The shortest distance between L_1 and L_2 is

$$= \frac{(\vec{a}_2 - \vec{a}_1).\vec{b}_1 \times \vec{b}_2}{|\vec{b}_1 \times \vec{b}_2|} = (\vec{a}_2 - \vec{a}_1).\hat{b}$$

Since, $a_1 = -\hat{i} - 2\hat{j} - \hat{k}$ $\quad a_2 = 2\hat{i} - 2\hat{j} + 3\hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 4\hat{k}$ $\quad \therefore (\vec{a}_2 - \vec{a}_1).\hat{b}$

$$\therefore (3\hat{i} + 4\hat{k}) \cdot \left(\frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}\right) = \frac{-3+20}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}$$

36. (c) The plane passing through (–1, –2, –1) and having normal along $\vec{b}$ is

$-1(x + 1) - 7(y + 2) + 5(z + 1) = 0$

$\Rightarrow x + 7y - 5z + 10 = 0$

$\therefore$ Distance of point (1, 1, 1) from the above plane is

$$= \frac{1 + 7 \times 1 - 5 \times 1 + 10}{\sqrt{1+49+25}} = \frac{13}{\sqrt{75}}$$

37. (d) The given planes are

$P_1 : x - y + z = 1$ ...(1)

$P_2 : x + y - z = -1$ ...(2)

$P_3 : x - 3y + 3z = 2$ ...(3)

Since, line L_1 is intersection of planes P_2 and P_3.

$\therefore L_1$ is parallel to the vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -3 & 3 \end{vmatrix} = -4\hat{j} - 4\hat{k}$$

Line L_2 is intersection of P_3 and P_1

$\therefore L_2$ is parallel to the vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & -3 & 3 \end{vmatrix} = -2\hat{j} - 2\hat{k}$$

And line L_3 is intersection of P_1 and P_2

$\therefore L_3$ is parallel to the vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 2\hat{j} + 2\hat{k}$$

Clearly lines L_1, L_2 and L_3 are parallel to each other.

$\therefore$ Statement–1 is False

Also family of planes passing through the intersection of P_1 and P_2 is $P_1 + \lambda P_2 = 0$.

$$\Rightarrow x(1+\lambda) + y(\lambda - 1) + z(1-\lambda) + (\lambda - 1) = 0$$

The three planes have a common point

$$\frac{1+\lambda}{1} = \frac{\lambda - 1}{-3} = \frac{1-\lambda}{3} = \frac{1-\lambda}{2} \quad ...(1)$$

Taking $\frac{1+\lambda}{1} = \frac{1-\lambda}{2}$, we get $\lambda = -\frac{1}{3}$ and taking $\frac{1+\lambda}{1} = \frac{1-\lambda}{3}$, we get $\lambda = -\frac{2}{3}$.

$\therefore$ There is no value of λ which satisfies eq (1).

$\therefore$ The three planes do not have a common point.

$\Rightarrow$ Statement 2 is true.

$\therefore$ (d) is the correct option.

38. **(d)** The line of intersection of given plane is

$$3x - 6y - 2z - 15 = 0 = 2x + y - 2z - 5$$

For $z = 0$, we get $x = 3$ and $y = -1$

$\therefore$ Line passes through (3, –1, 0).

Direction vector of line is

$$\vec{b} = \vec{x}_1 \times \vec{x}_2$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= 14\hat{i} + 2\hat{j} + 15\hat{k}$$

$\therefore$ Eqn. of line is $\dfrac{x-3}{14} = \dfrac{y+1}{2} = \dfrac{z}{15} = t$

whose parametric form is

$$x = 3 + 14t, y = 2t - 1, z = 15t$$

$\therefore$ Statement-I is false

$\therefore$ Statement 2 is true.

39. Equation of plane containing line of intersection of two given planes is given by

$$(2x - y + z - 3) + \lambda(3x + y + z - 5) = 0$$

$$\Rightarrow (3\lambda + 2)x + (\lambda - 1)y + (\lambda + 1)z + (-5\lambda - 3) = 0$$

since distance of this plane from the pt. (2, 1, – 1) is $\dfrac{1}{\sqrt{6}}$

$$\therefore \left| \frac{(3\lambda+2)2 + (\lambda-1)1 + (\lambda+1)(-1) + (-5\lambda-3)}{\sqrt{(3\lambda+2)^2 + (\lambda-1)^2 + (\lambda+1)^2}} \right| = \frac{1}{\sqrt{6}}$$

$$\Rightarrow \left| \frac{\lambda - 1}{\sqrt{11\lambda^2 + 12\lambda + 6}} \right| = \frac{1}{\sqrt{6}}$$

Squaring both sides, we get

$$\frac{(\lambda-1)^2}{11\lambda^2 + 12\lambda + 6} = \frac{1}{6}$$

$$\Rightarrow 6\lambda^2 - 12\lambda + 6 - 11\lambda^2 - 12\lambda - 6 = 0$$

$$\Rightarrow 5\lambda^2 + 24\lambda = 0 \Rightarrow \lambda(5\lambda + 24) = 0$$

$\Rightarrow \lambda = 0$ or – 24/5

$\therefore$ The required equations of planes are

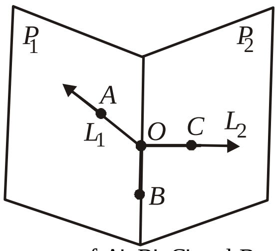
$$2x - y + z - 3 = 0$$

or $\left[3\left(\dfrac{-24}{5}\right) + 2\right]x + \left[-\dfrac{24}{5} - 1\right]y$

$$+\left[-\frac{24}{5} + 1\right]z - 5\left(\frac{-24}{5}\right) - 3 = 0$$

or $62x + 29y + 19z - 105 = 0$

40. Following fig. shows the possible situation for planes P_1 and P_2 and the lines L_1 and L_2



A corresponds to one of A', B', C' and B corresponds to one of the remaining of A', B', C' and C corresponds to third of A', B', C'. Hence six such permutations are possible

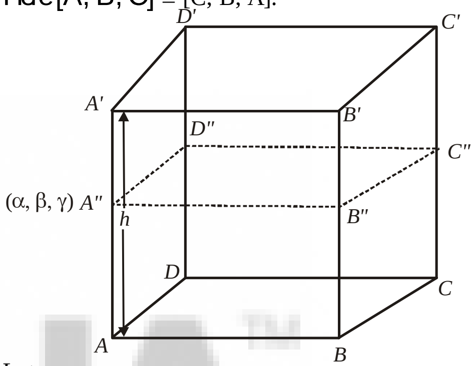
e.g., One of the permutations may A = A', B = B', C = C'

From the given conditions : A lies on L_1, B lies on the line of intersection of P_1 and P_2 and 'C' lies on the line L_2 on the plane P_2.

Now, A' lies on L_2 = C, B' lies on the line of intersection of P_1 and P_2 = B and C' lie on L_1 on plane P_1 = A.

Hence there exist a particular set [A', B', C'] which is the permutation of [A, B, C] such that both (i) and (ii) is satisfied. Here [A', B', C'] ≡ [C, B, A].

41.



Let

Let equation of plane $ABCD$ be

$ax + by + cz + d = 0$, h be the height of original parallelepiped S. and $A''(\alpha, \beta, \gamma)$

Then height of new parallelepipe dT is the length of perpendicular from A'' to $ABCD$

i.e. $\dfrac{a\alpha + b\beta + c\gamma + d}{\sqrt{a^2 + b^2 + c^2}}$

$$V_T = \frac{90}{100} V_s$$

$$\therefore (ar\ ABCD) \times \frac{a\alpha + b\beta + c\gamma + d}{\sqrt{a^2 + b^2 + c^2}}$$

= (ar $ABCD$) × h × 0.9

But given that,

$$\frac{a\alpha + b\beta + c\gamma + d}{\sqrt{a^2 + b^2 + c^2}} = 0.9h$$

$$\Rightarrow a\alpha + b\beta + c\gamma + (d - 0.9h\sqrt{a^2 + b^2 + c^2}) = 0$$

$\therefore$ Locus of $A''(\alpha, \beta, \gamma)$ is

$$ax + by + cz + (d - 0.9h\sqrt{a^2 + b^2 + c^2}) = 0$$

which is a plane parallel to $ABCD$. Hence proved.

42. Equation of plane through (1, 1, 1) is

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad (x-1)(0-1)-(y-1)(0+1)+(z-1)(-1-0)=0$

$\Rightarrow \quad -1(x-1)-1(y-1)-1(z-1)=0 \Rightarrow x+y+z=3$

$\Rightarrow \quad \dfrac{x}{3}+\dfrac{y}{3}+\dfrac{z}{3}=1$ ...(1)

$\therefore$ plane intersect the axes at

$A(3,0,0), B(0,3,0)$, and $C(0,0,3)$

$\therefore$ Vol.of tetrahedron $OABC$

$= \dfrac{1}{6} \times$ Area of base $\times$ altitude

$= \dfrac{1}{6} \times \text{Ar}(\Delta ABC) \times \text{length of } \perp^{\text{lar}} (0,0,0) \text{ to plane } (1)$

$= \dfrac{1}{6} \times \dfrac{1}{2}\left[\dfrac{\sqrt{3}}{4} \times |\overrightarrow{AB}|^2\right] \times \left[\left|\dfrac{-3}{\sqrt{1+1+1}}\right|\right]$

($\therefore \Delta ABC$ is an equilateral triangle)

$= \dfrac{1}{12} \times \dfrac{\sqrt{3}}{4} \times (3\sqrt{2})^2 \times \sqrt{3} = \dfrac{3 \times 18}{48} = \dfrac{9}{2}$ cubic units.

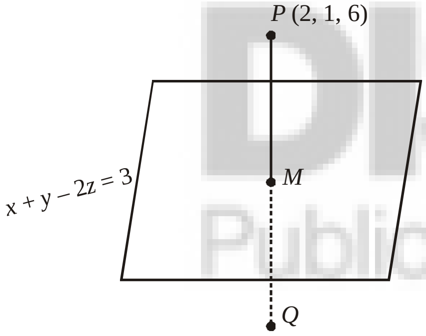
43. (i) Equation of plane passing through (2, 1, 0) , (5, 0, 1) and (4, 1, 1) is

$\begin{vmatrix} x-2 & y-1 & z-0 \\ 5-2 & 0-1 & 1-0 \\ 4-2 & 1-1 & 1-0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 3 & -1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad (x-2)(-1-0)-(y-1)(3-2)+z(0-(-2))=0$

$\Rightarrow \quad -x+2-y+1+2z=0 \Rightarrow x+y-2z=3$

(ii)



Eq^n of PQ passing through $P(2, 1, 6)$ and $\perp$ to plane $x+y-2z=3$, is given by

$\dfrac{x-2}{1} = \dfrac{y-1}{1} = \dfrac{z-6}{-2} = \lambda$

$\therefore \quad Q(\lambda+2, \lambda+1, -2\lambda+6)$

$\therefore \quad$ Mid. pt. of PQ

i.e. $\quad M\left(\dfrac{2+\lambda+2}{2}, \dfrac{1+\lambda+1}{2}, \dfrac{6-2\lambda+6}{2}\right)$

$= \left(\dfrac{\lambda+4}{2}, \dfrac{\lambda+2}{2}, \dfrac{12-2\lambda}{2}\right)$

But M lies on plane $x+y-2z=3$

$\therefore \quad \dfrac{\lambda+4}{2} + \dfrac{\lambda+2}{2} - (12-2\lambda) = 3$

$\Rightarrow \lambda+4+\lambda+2-24+4\lambda=6 \Rightarrow 6\lambda=24 \Rightarrow \lambda=4$

$\therefore \quad Q(4+2, 4+1, -8+6) = (6, 5, -2)$