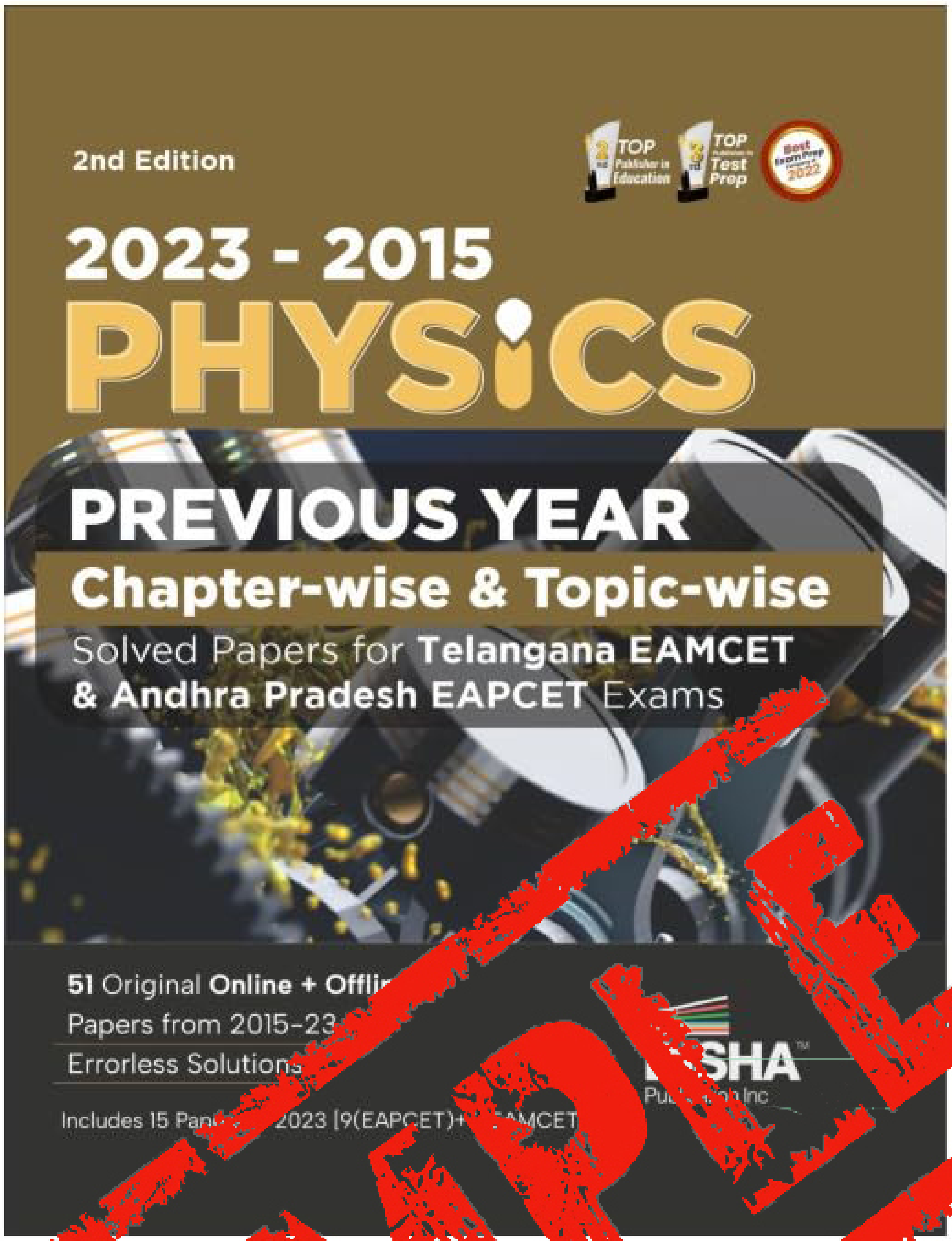

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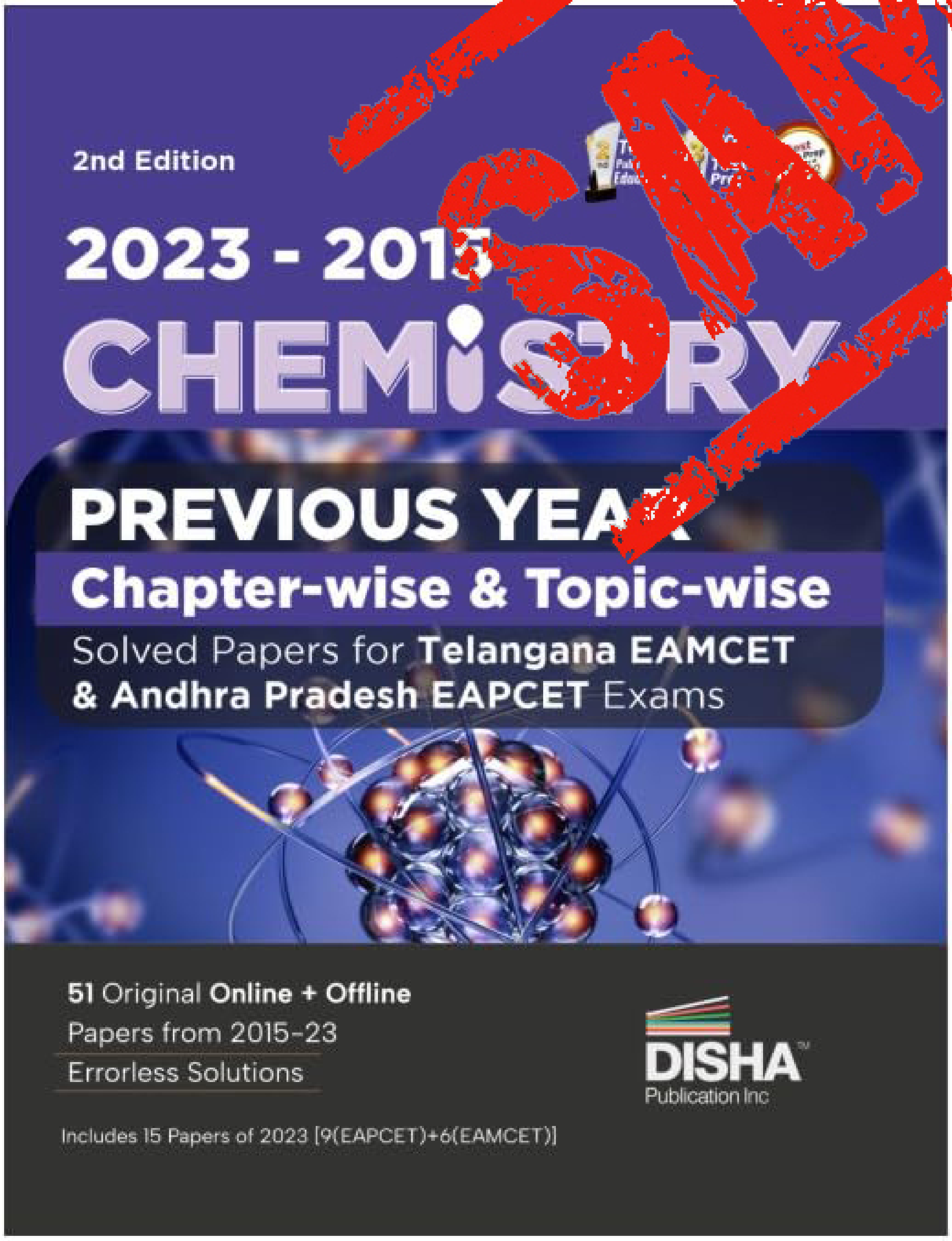

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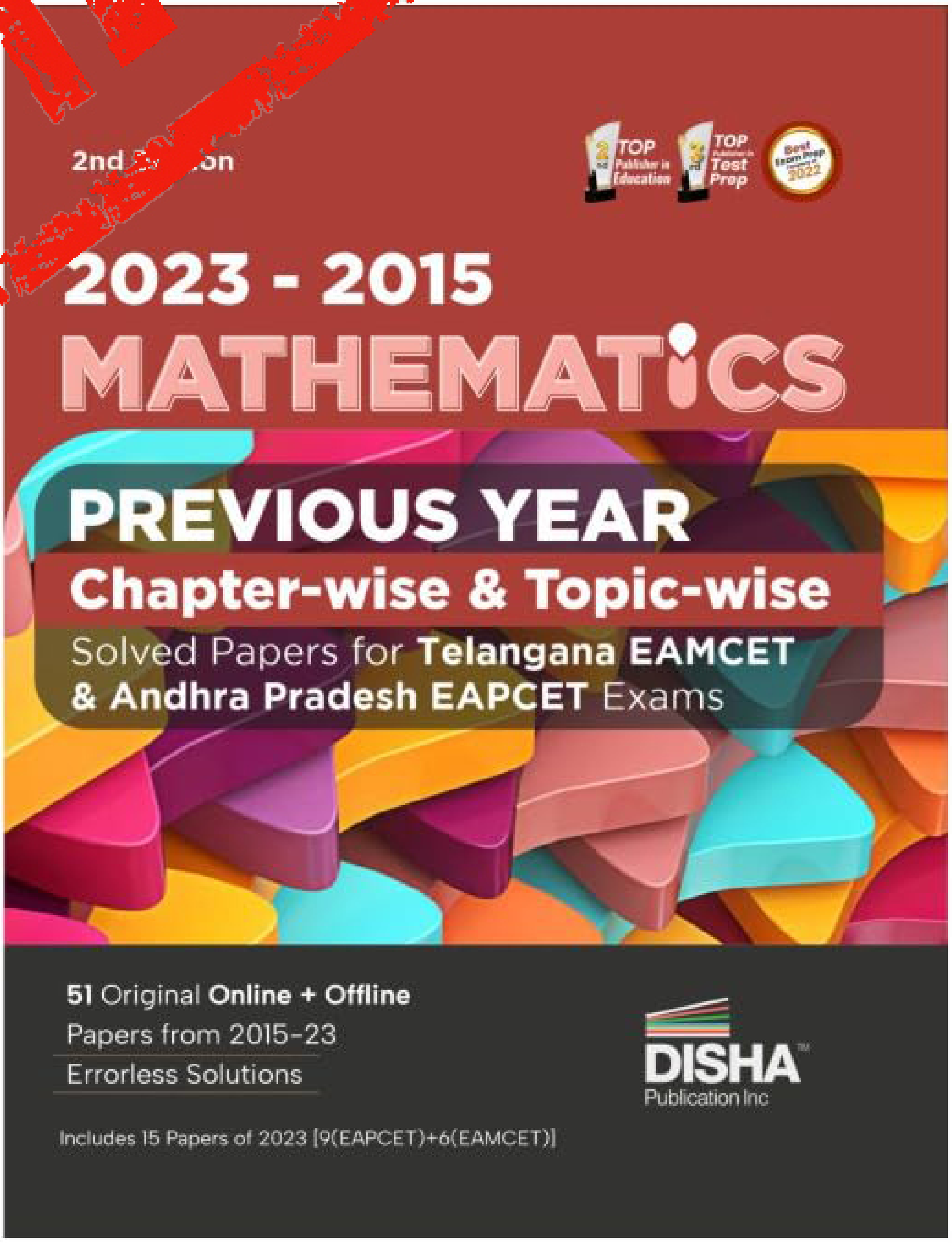

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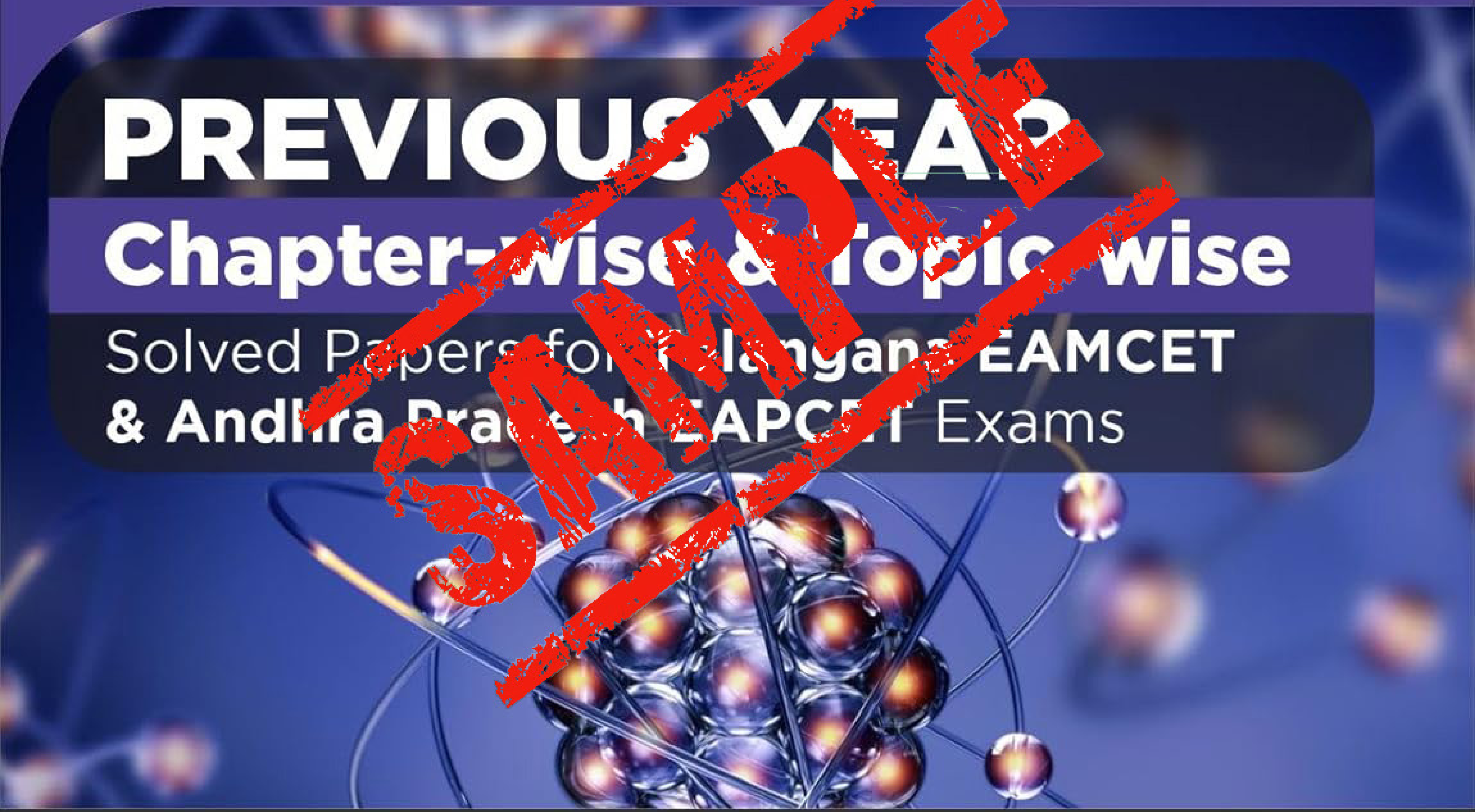

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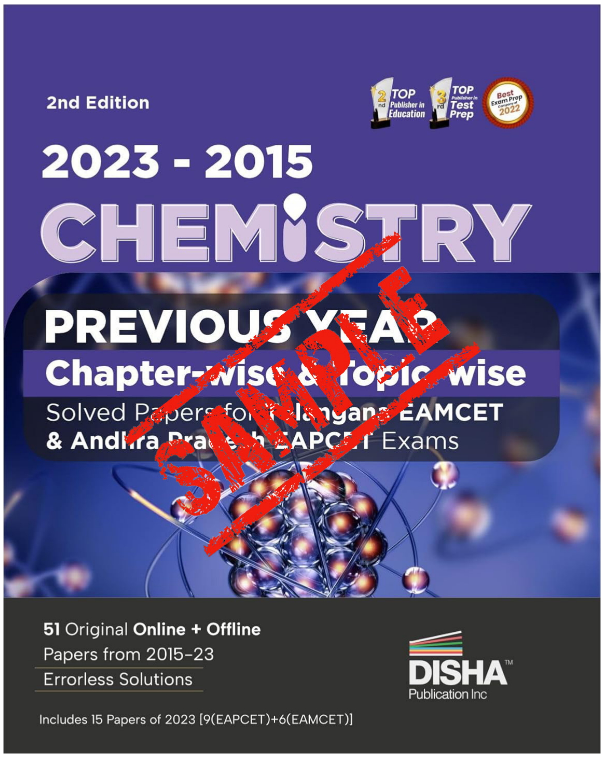

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7

Equilibrium

CHAPTER SYNOPSIS

- Equilibrium constant for gaseous homogenous equilibrium can be expressed in two ways K_P and K_C. This means value of equilibrium constant depends upon choice of standard state in which concentrations of reactants and products are expressed.
- The equilibrium constant for a reaction decreases with increase in temperature, if the reaction is exothermic and if the reaction is endothermic the equilibrium constant increases with increase in temperature
- Effect of pressure (or volume) at equilibrium

Reaction Type	**Effect of P or V**
n = 0	Equilibrium is not affected
n = +ve	Increase in P or decrease in V shift the equilibrium to left
n = -ve	Increase in P or decrease in volume shift the equilibrium to right

- Effect of addition of inert gas at equilibrium

Reaction Type	**Effect**
$\Delta n = 0$	Equilibrium is not affected at constant P or V
$\Delta n = +ve$	At constant V : Equilibrium is not affected
	At constant P : Equilibrium shifts to right
$\Delta n = -ve$	At constant V : Equilibrium is not affected
	At constant P : Equilibrium shifts to left

- Variation of equilibrium constant with variation of the reaction equation (K = equilibrium constant for original reaction) is given as below:

When the equation	**the change in equilibrium constant**
Reversed	1/K
Divided by 2	$\sqrt{K}$
Multiplied by 2	K^2
Divided into two steps	$K = K_1 \times K_2$

- The value of the equilibrium constant is not affected by the addition of a catalyst to the reaction. This is because the catalyst increases the speed of the forward reaction and the backward reaction to the same extent.
- K_P is related to K_C as $K_P = K_C (RT)^{\Delta n}$
- Relation between $\Delta G°$ and K

$\Delta G°$	lnK	R	Reaction
–ve	+ve	>1	Spontaneous
+ve	–ve	<1	Non-spontaneous
0	0	=1	Equilibrium

- Prediction of the direction of the reaction using following relation
  - If $Q_c < K_c$, net reaction goes from left to right
  - If $Q_c > K_c$, net reaction goes from right to left
  - If $Q_c = K_c$, no net reaction.
- $\Delta G°$ is related to K by the relation $\Delta G° = - RT \ln K$
- Ionic product of water is the product of concentration of H^+ ions and OH^- ions in pure water. It is constant at constant temperature.
- The ionic product of water increases with increase in temperature.
- Salt hydrolysis is reverse of neutralisation.
- The aqueous solution of salts of weak acid and strong base is alkaline in nature (pH > 7).
- The aqueous solution of strong acid and weak base is acidic in nature (pH < 7).
- The nature of salts of weak acid and weak base on hydrolysis depends upon the relative hydrolysis of the cation or anion of the salt.
- The aqueous solution of salts of strong acid and strong base is neutral.
- Solubility product is defined as the product of ionic concentration of sparingly soluble electrolyte in a saturated solution and is constant at constant temperature.

Topic 1 Law of Mass Action, Equilibrium Constant (K_c and K_p) and its Application

1. At 300 K, K_C for the reaction.
$A_2B_2(g) \rightleftharpoons A_2(g) + B_2(g)$ is 100 mol L^{-1}. What is its K_p (in atm) at the same temperature?
(R = 0,082 L atm mol^{-1} K^{-1}) **[AP/May 23, 2024 (I)]**
(a) 100 (b) 2460 (c) 4.06 (d) 246

2. At equilibrium for the reaction **[AP/May 22, 2024 (I)]**
$$A_2(g)+B_2(g) \rightleftharpoons 2AB(g)$$
the concentrations of A_2, B_2 and AB respectively are 1.5×10^{-3} M, 2.1×10^{-3} M and 1.4×10^{-3} M in a sealed vessel at 800 K. What will be K_p for the decomposition of AB at same temperature?
(a) 0.62 (b) 1.6 (c) 0.44 (d) 2.27

3. 15 moles of H_2 and 5.2 moles of I_2 are mixed and allowed to attain equilibrium at 773 K. At equilibrium, the number of moles of HI is found to be 10. The equilibrium constant for the dissociation of HI is **[AP/May 21, 2024 (II)]**
(a) 2×10^{-2} (b) 50
(c) 2×10^{-1} (d) 5.0

4. K_c for the reaction
$$A_2(g) \overset{T(K)}{\rightleftharpoons} B_2(g)$$
is 39.0. In a closed one litre flask, one mole of A_2(g) was heated to T(K). What are the concentrations of A_2(g) and B_2(g) (in mol L^{-1}) respectively at equilibrium?
[AP/May 21, 2024 (I)]
(a) 0.025, 0.975 (b) 0.975, 0.025
(c) 0.05, 0.95 (d) 0.02, 0.98

5. At T(K), the equilibrium constant for the reaction $H_2(g) + Br_2(g) \rightleftharpoons 2HBr(g)$ is 1.6×10^5. If 10 bar of HBr is introduced into a sealed vessel at T(K), the equilibrium pressure of HBr (in bar) is approximately
[AP/May 20, 2024 (II)]
(a) 10.20 (b) 10.95 (c) 9.95 (d) 11.95

6. K_c for the reaction, $A_2(g) \overset{T(K)}{\rightleftharpoons} B_2(g)$ is 99.0. In a 1 L closed flask two moles of B_2 (g) is heated to T (K). What is the concentration of B_2 (g) (in mol L^{-1}) at equilibrium?
[AP/May 20, 2024 (I)]
(a) 0.02 (b) 1.98 (c) 0.198 (d) 1.5

7. K_c for the following reaction is 99.0
$$A_2(g) \overset{T(K)}{\rightleftharpoons} B_2(g)$$
In a one litre flask, 2 moles of A_2 was heated to T(K) and the above equilibrium is reached. The concentrations at equilibrium of A_2 and B_2 are $C_1(A_2)$ and $C_2(B_2)$ respectively. Now, one mole of A_2 was added to flask and heated to T(K) to establish the equilibrium again. The concentrations of A_2 and B_2 are $C_3(A_2)$ and $C_4(B_2)$ respectively. What is the value of $C_3(A_2)$ in mol L^{-1}?
[AP/May 19, 2024 (II)]
(a) 1.98 (b) 0.01 (c) 0.03 (d) 2.97

8. At T (K), K_c for the reaction. $A_2(g) \rightleftharpoons B_2(g)$ is 99.0. Two moles of A_2 (g) was heated to T (K) in a 1L closed flask to reach the above equilibrium. What are the concentrations (in mol L^{-1}) of A_2 (g) and B_2(g) respectively at equilibrium? **[AP/May 18, 2024 (I)]**
(a) 1.86,0.0187 (b) 1.98,0.02
(c) 0.0187,1.86 (d) 0.02,1.98

9. At T(K) the equilibrium constants for the following two reactions are given below **[TS/May 11, 2024 (I)]**
$$2A(g) \rightleftharpoons B(g)+C(g);\ K_1 = 16$$
$$2B(g)+C(g) \rightleftharpoons 2D(g);\ K_2 = 25$$
What is the value of equilibrium constant (K) for the reaction given below at T(K)?
$$A(g)+\frac{1}{2}B(g) \rightleftharpoons D(g)$$
(a) 100 (b) 50 (c) 20 (d) 75

10. At T(K), K_c for the dissociation of PCl_5 is 2×10^{-2} mol L^{-1}. The number of moles of PCl_5 that must be taken in 1.0 L flask at the same temperature to get 0.2 mol of chlorine at equilibrium is **[TS/May 9, 2024 (II)]**
(a) 2.2 (b) 1.1 (c) 1.8 (d) 4.4

11. At T(K), K_c for the reaction
$AO_2(g)+BO_2(g) \rightleftharpoons AO_3(g)+BO(g)$ is 16. One mole each of reactants and products are taken in a 1 L flask and heated to T(K), and equilibrium is established. What is the equilibrium concentration of BO (in mol L^{-1})?
[TS/May 9, 2024 (I)]
(a) 1.6 (b) 0.4 (c) 1.2 (d) 0.8

12. At T(K), the K_p for the reaction $A_2B_6(g) \rightarrow A_2B_4(g) + B_2(g)$ is 0.04 atm. The equilibrium pressure (in atm) of A_2B_6)g) when it is placed in a flask at 4 atm pressure and allowed to come to above equilibrium is
[AP/May 19, 2023 (I)]
(a) 0.362 (b) 0.380 (c) 3.62 (d) 2.62

13. At 1000 K, the value of K_c for the below reaction is 10 mol L^{-1}. Value of K_p (in atm) is **[AP/May 18, 2023 (I)]**
$$A(g) \rightleftharpoons B(g)+C(g)$$
(given R = 0.082 atm L mol^{-1} K^{-1})
(a) 82 (b) 0.82 (c) 8.2 (d) 820

14. One mole of A(g) is heated to T(K) till the following equilibrium is obtained
$$A(g) \overset{T(K)}{\rightleftharpoons} B(g)$$
The equilibrium constant of this reaction is 10^{-1}. After reaching the equilibrium, 0.5 moles of A(g) is added and heated. The equilibrium is again established. The value of $\frac{[A]}{[B]}$ is **[AP/May 17, 2023 (II)]**
(a) 10^{-1} (b) 10 (c) 10^{-2} (d) 100

15. One mole $H_2O(g)$ and one mole CO(g) are taken in 1L flask and heated to 725K. At equilibrium, 40% (by mass) of water reacted with CO(g) as follows.

$H_2O(g)+CO(g) \rightleftharpoons H_2(g)+CO_2(g)$. Its K_p value is

[AP/May 17, 2023 (I); Similar to AP/May 15, 2023 (I)]

(a) 2.220 (b) 0.444 (c) 4.440 (d) 0.222

16. One mole of $PCl_5(g)$ was heated in a 1L closed flask at 500 K. At equilibrium, 0.1 mole of Cl_2 (g) was formed. What is its K_p (in atm)?

(Given R = 0.082 L atm mol^{-1} K^{-1}) **[AP/May 16, 2023 (I)]**

(a) 2.7×10^{-4} (b) 0.455

(c) 0.0111 (d) 90.0

17. At T(K), the equilibrium constant for the reaction a A(g) ż b B (g) is K_c. If the reaction takes place in the following form 2aA(g) → 2b B(g) its equilibrium constant is K'_c. The correct relationship between K_c and K_c is

[AP/May 15, 2023 (II)]

(a) $K'_c = (K_c)^2$ (b) $K'_c = (K_c)^{\frac{1}{2}}$

(c) $K'_c = (K_c)^{-1}$ (d) $K'_c = K_c$

18. At T (K), K_C value for the reaction

$\frac{1}{3}N_2(g)H_2(g) \rightleftharpoons \frac{2}{3}NH_3(g)$ is 50. The K_C value for the reaction $2NH_3(g) \rightleftharpoons N_2(g) + 3H_2(g)$ at the same temperature is **[TS/May 13, 2023 (II)]**

(a) 4×10^{-6} (b) 8×10^{-6}

(c) 6×10^{-6} (d) 8×10^{-3}

19. At T(K) when one mol of X and one mol of Y are heated in a 1 L flask, 0.5 moles of Z is formed at the equilibrium. The K_c value for the reaction is

$X(g) + Y(g) \rightleftharpoons Z(g) + A(g)$ **[TS/May 13, 2023 (I)]**

(a) 0.5 (b) 1.0 (c) 0.75 (d) 0.82

20. K_P/K_C for the reaction at T(K) is

[AP/July 8, 2022 (I); Similar to TS/July 18, 2022 (II)]

$$CO(g)+\frac{1}{2}O_2(g) \rightleftharpoons CO_2(g)$$

(a) $\sqrt{RT}$ (b) 2RT (c) RT (d) $1/\sqrt{RT}$

21. For ammonia formation from constituent elements, the expression for K_C is **[AP/July 6, 2022 (I)]**

(a) $K_C = \frac{[NH_3]^3}{[N_2]^3[H_2]^3}$ (b) $K_C = \frac{[NH_3]^2}{[N_2][H_2]^3}$

(c) $K_C = \frac{[NH_3]}{[N_2][H_2]}$ (d) $K_C = [NH_3]^2$

22. Calculate the value of the equilibrium constant (K_p) for the reaction of oxygen gas oxidising ammonia gas to nitric oxide and water vapour. The pressure of each gas at equilibrium is 0.5 atm.

[TS/July 20, 2022 (II); Similar to TS/July 19, 2022 (I)]

(a) 1.5 atm (b) 0.5 atm (c) 1 atm (d) 2.5 atm

23. For the formation of ammonia from its constituent elements (1 mole of N_2 and 3 moles of H_2) in a closed vessel of volume V(L), the value of K_C is [units of $K_C = mol^{-2}L^2$] **[TS/July 20, 2022 (I)]**

(a) $\frac{3x^2V^2}{9(1-x)^4}$ (b) $\frac{4xV^2}{9(1-x)^3}$

(c) $\frac{4x^2V^2}{27(1-x)^4}$ (d) $\frac{x^2V^2}{27(1-x)^3}$

24. For a reaction A(s) ~ B(s) + C(g) the set of all correct statements are **[TS/July 18, 2022 (I)]**

(A) K is independent of [A].

(B) K is dependent on partial pressure of C at a given temperature.

(C) ΔH will be independent of temperature.

(D) ΔH is independent of the catalyst addition.

(a) A, B, C, D (b) A, B only

(c) A, B, D only (d) A, B, C only

25. Identify the correct expression for the equilibrium constant of the following reaction. **[AP/Aug. 23, 2021 (I)]**

$$2X(g)+Y(g) \rightleftharpoons 3Z(g)$$

(a) $k = \frac{[X]^2[Y]}{[Z]^3}$ (b) $k = \frac{[Z]^3}{[X]^2[Y]}$

(c) $k = \frac{3[Z]}{2[X][Y]}$ (d) $k = [Z]^3 [X]^2[Y]$

26. Which among the following denotes the correct relationship between K_p and K_c for the reaction, $2A(g) \rightleftharpoons B(g)+C(g)$

[AP/Aug. 19, 2021 (II); Similar to TS/Aug. 6, 2021 (I)]

(a) $K_p > K_c$ (b) $K_c > K_p$

(c) $K_c = (K_p)^2$ (d) $K_p = K_c$

27. The equilibrium constant (K_p) for the formation of ammonia from its constituent elements at 27°C is 1.2×10^{-4} and at 127°C is 0.60×10^{-4}. Calculate the mean heat of formation of ammonia per mole in this temperature range. **[TS/Aug. 5, 2021 (I)]**

(a) – 82.64 cal (b) – 826.4 cal

(c) – 1652.8 cal (d) – 165.2 cal

28. For a reaction, 2A → B + C, K_c is 2×10^{-3}. At a given time, the reaction mixture has $[A] = [B] = [C] = 3 \times 10^{-4}$ M. Which of the following options is correct?

[TS/Aug. 5, 2021 (I)]

(a) The system is at equilibrium

(b) The reaction proceeds to the left

(c) The reaction proceeds to the right

(d) The reaction is complete

29. For a reversible reaction, if the concentration of the reactants is reduced to half, the equilibrium constant will be .... **[AP/Sept. 18, 2020 (I)]**

(a) doubled (b) halved
(c) reduced to one-fourth (d) remains same

30. What is the equilibrium constant (K_c) for the given reaction?

$$N_2 + O_2 \rightleftharpoons 2NO$$

Where the equilibrium concentration of N_2, O_2 and NO are found to be 4×10^{-3}, 3×10^{-3} and 3×10^{-3} M respectively.

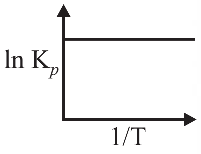
[TS/Sept. 10, 2020 (I); Similar to AP/Apr. 21, 2019 (II)]

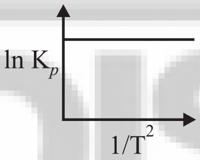
(a) 0.750 (b) 0.622
(c) 9×10^{-3} (d) 12.8×10^{-6}

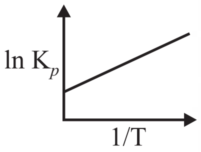
31. For the given equilibrium reaction, $2A(g) \rightleftharpoons 2B(g) + C(g)$ the equilibrium constant (K_C) at 1000 K is 4×10^{-4}. Calculate K_p for the reaction at 800 K temperature **[TS/Sept. 9, 2020 (I)]**

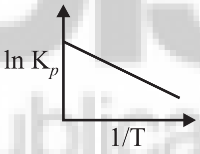
(a) 0.044 (b) 0.026 (c) 0.33 (d) 1

32. In which of the following plots, an endothermic reaction is correctly represented? **[TS/May 6, 2019 (I)]**

(a)


(b)


(c)


(d)


33. For the reaction, $0.5C(s) + 0.5CO_2(g) \rightleftharpoons CO(g)$ the equilibrium pressure is 12 atm. If CO_2 conversion is 50%, the value of K_p, in atm is **[TS/May 3, 2019 (I)]**

(a) 4 (b) 1 (c) 0.5 (d) 2

34. At 1000 K, the equilibrium constant. K_C for the reaction $2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g)$ is 4.0×10^{-6} mol L^{-1}. The K_P (in bar) at the same temperature is **[AP/2018]**

($R = 0.083$ L bar $K^{-1}mol^{-1}$)

(a) 3.32×10^{-6} (b) 3.32×10^{4}
(c) 3.32×10^{-4} (d) 3.32×10^{-3}

35. Consider the following reaction in a 1 L closed vessel.

$$N_2 + 3H_2 \rightleftharpoons 2NH_3$$

If all the species; N_2, H_2 and NH_3 are in 1 mol in the beginning of the reaction and equilibrium is attained after unreacted N_2 is 0.7 mol. What is the value of equilibrium constant? **[TS/May 5, 2018 (I)]**

(a) 3600.00 (b) 3657.14
(c) 2657.14 (d) 1828.57

36. The equilibrium constant (K_C) for the following equilibrium **[AP/2017]**

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$$

at 563 K is 100. At equilibrium, the number of moles of SO_3 in the 10 litre flask is twice the number of moles of SO_2, then the number of moles of oxygen is

(a) 0.4 (b) 0.3 (c) 0.2 (d) 0.1

37. (i) $H_3PO_4(aq) \rightleftharpoons H^+(aq) + H_2PO_4^-(aq)$

(ii) $H_2PO_4^-(aq) \rightleftharpoons H^+(aq) + HPO_4^{2-}(aq)$

(iii) $HPO_4^{2-}(aq) \rightleftharpoons H^+(aq) + PO_4^{3-}(aq)$

The equilibrium constants for the above reactions at a certain temperature are K_1, K_2 and K_3 respectively. The equilibrium constant for the reaction $H_3PO_4(aq) \rightleftharpoons 3H^+(aq) + PO_4^{3-}(aq)$ in terms of K_1, K_2 and K_3 is **[AP/2017]**

(a) $K_1 + K_2 + K_3$ (b) $\dfrac{K_1}{K_2 + K_3}$
(c) $\dfrac{K_3}{K_1 K_2}$ (d) $K_1 K_2 K_3$

38. At 400 K, in a 1.0 L vessel, N_2O_4 is allowed to attain equilibrium, $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ At equilibrium, the total pressure is 600 mm Hg, when 20% of N_2O_4 is dissociated. The value of K_p for the reaction is **[AP/2016]**

(a) 50 (b) 100 (c) 150 (d) 200

39. Which one of the following is correct? **[TS/2016]**

(a) The equilibrium constant (K_C) is independent of temperature.
(b) The value of K_C is independent of initial concentrations of reactants and products.
(c) At equilibrium, the rate of the forward reaction is twice the rate of the backward reaction.
(d) The equilibrium constant (K_c) for the reaction.

$$Ni(s) + 4CO(g) \rightleftharpoons Ni(CO)_4(g) \text{ is, } \frac{[Ni(CO)_4]}{[CO]}.$$

40. When one mole of *A* and one mole of *B* were heated in one litre flask at *T*(K), 0.5 mole of *C* was formed in the equilibrium,

$$A + B \rightleftharpoons C + D$$

the equilibrium constant, K_C is **[TS/2015]**

(a) 0.25 (b) 0.5 (c) 1 (d) 2

Topic 2 Relation between K, Q and G and Factors Effecting Equilibrium

41. At 300 K, $\Delta_r G^\circ$ for the reaction $A_2(g) \rightleftharpoons B_2(g)$ is –11.5 kJ mol^{-1}. The equilibrium constant at 300 K is approximately ($R = 8.314$ J mol^{-1} K^{-1}) **[AP/May 22, 2024 (II)]**

(a) 10 (b) 100 (c) 1000 (d) 25

42. Observe the following equilibrium

Fe^{3+} (aq) + SCN^- (aq) $\rightleftharpoons$ $[Fe(SCN)]^{2+}$ (aq)

yellow colourless deep red

Addition of aqueous oxalic acid solution to the above equilibrium **[AP/May 18, 2023 (II)]**

(a) Shifts the equilibrium towards the formation of $[Fe(SCN)]^{2+}$
(b) Deep red color increases
(c) Intensity of deep red color decreases
(d) No change in equilibrium

43. Observe the following equilibrium at T (K)

$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

Which one of the following does not disturb the above equilibrium? **[AP/May 16, 2023 (II)]**

(a) Addition of H_2 (g) (b) Removal of HI(g)
(c) Addition of I_2(g) (d) Addition of He(g)

44. At 780 K and 10 atmosphere pressure the equilibrium constant for the reaction $2A(g) \rightleftharpoons B(g) + C(g)$ is 3.52. At the same temperature and 7.04 atmosphere pressure, the equilibrium constant for the same reaction is **[TS/May 12, 2023 (II)]**

(a) 7.04 (b) 3.52 (c) 10.56 (d) 5.23

45. For a reaction

$$A(g)+\frac{1}{2}B(g) \rightleftharpoons C(g)+\text{heat}$$

favorable conditions for the reaction to occur in the forward direction are **[AP/July 5, 2022 (II)]**

(a) Low T and Low P (b) Low T and high P
(c) High T and Low P (d) High T and high P

46. Which of the following expression is correct **[AP/July 4, 2022 (II)]**

(a) $\Delta G = -RT \ln K$ (b) $\Delta G = \dfrac{1}{RT^2 \ln K}$

(c) $\Delta G^0 = -RT \ln K$ (d) $\Delta G^0 = -\dfrac{1}{RT^2 \ln K}$

47. The formation of ammonia from its constituent elements is an exothermic reaction. The effect of increase temperature on the reaction equilibrium is **[AP/July 4, 2022 (I)]**

(a) The rate of the forward reaction becomes zero
(b) No effect of temperature.
(c) Forward reaction is favored
(d) Backward reaction is favored

48. In which of the following reactions at equilibria, the position of the equilibrium shifts towards the products, if the total pressure is increased? **[TS/July 19, 2022 (I)]**

(i) $X_2(g)+3Y_2(g) \rightleftharpoons 2XY_3(g)$

(ii) $X_2(g)+Y_2(g) \rightleftharpoons 2XY(g)$

(iii) $X_2(g)+Z_2(g) \rightleftharpoons 2XZ(g)$

(iv) $X_2(g)+Y_4(g) \rightleftharpoons 2XY_2(g)$

(a) (ii) (b) (iii) (c) (i) (d) (iv)

49. When reaction is carried out at standard states then, at the equilibrium **[AP/Aug. 23, 2021 (II)]**

(a) $\Delta H° = 0$
(b) $\Delta S° = 0$
(c) equilibrium constant $(K) = 0$
(d) equilibrium constant $(K) = 1$

50. Le-Chateliers' principle is not applicable to **[AP/Aug. 20, 2021 (I)]**

(a) $H_2(g)+I_2(g) \rightleftharpoons 2HI(g)$
(b) $Fe(s)+S(s) \rightleftharpoons FeS(s)$
(c) $N_2(g)+3H_2(g) \rightleftharpoons 2NH_3(g)$
(d) $N_2(g)+O_2(g) \rightleftharpoons 2NO(g)$

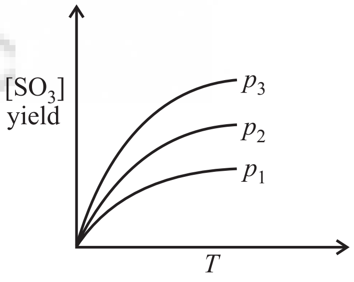
51. Standard entropies of X_2, Y_2 and XY_3 are 60, 40 and 50 JK^{-1} mol^{-1} respectively. At what temperature, the following reaction will be at equilibrium?

[given : $\Delta H° = -30$ kJ] **[AP/Aug. 19, 2021 (II)]**

$$\frac{1}{2}X_2+\frac{3}{2}Y_2 \rightleftharpoons XY_3$$

(a) 500 K (b) 750 K (c) 1000 K (d) 1250 K

52. For the reaction $SO_2(g)+\frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$, the percentage yield of product at different pressure is shown in the figure. Then, which among the following is true? **[AP/Aug. 19, 2021 (II)]**



(a) Pressure has no effect (b) $p_1 < p_2 < p_3$
(c) $p_1 > p_2 > p_3$ (d) $p_1 = p_2 = p_3 \neq 0$

53. For the formation of NH_3(g) from its constituent elements, the favourable conditions for its formation are **[TS/Aug. 6, 2021 (II)]**

(a) high pressure and low temperature
(b) high pressure and high temperature
(c) low pressure and high temperature
(d) low pressure and low temperature

54. Find the value of the equilibrium constant (K) of a reaction at 300 K, when standard Gibbs free energy change is –25 kJ mol^{-1}? (Consider R = 8.33 $mo1^{-1}K^{-1}$) **[TS/Sept. 11, 2020 (I)]**

(a) e^8 (b) e^9 (c) e^{10} (d) e^{11}

55. K_P for the conversion of oxygen to ozone at 400 K is 1.0×10^{-30}, its standard Gibbs energy change in kJ mol^{-1} is approximately **[AP/Apr. 22, 2019 (I)]**
(a) 229.8 (b) 114.9 (c) –229.8 (d) –114.9

56. On increasing temperature, the equilibrium constant of exothermic and endothermic reactions, respectively **[AP/2015]**
(a) increases and decreases
(b) decreases and increases
(c) increases and increases
(d) decreases and decreases

Topic 3 Theories of Acids and Bases, Ionic Product of Water and pH Scale

57. 100 mL of 0.1 M HA (weak acid) and 100 mL of 0.2 M NaA are mixed. What is the pH of resultant solution? (K_a of HA is 10^{-5}; log 2 = 0.3) **[AP/May 22, 2024 (II)]**
(a) 4.7 (b) 5.0 (c) 5.3 (d) 4.0

58. Which of the following when added to 20 mL of a 0.01 M solution of HCl would decrease its pH? **[AP/May 22, 2024 (I)]**
(a) 20 mL of 0.02 M HCl (b) 20 mL of 0.005 M HCl
(c) 20 mL of 0.01 M HCl (d) 40 mL of 0.005 M HCl

59. At 27°C. 100 mL of 0.4 M HCl is mixed with 100 mL of 0.5 M NaOH solution. To the resultant solution. 800 mL of distilled water is added. What is the pH of final solution? **[AP/May 20, 2024 (I)]**
(a) 12 (b) 2 (c) 1.3 (d) 1.0

60. Observe the following stoichiometric equation
$P_4 + 3\ OH^- + 3H_2O \rightarrow PH_3 + 3x^-$ **[AP/May 19, 2024 (II)]**
What is the conjugate acid of x^- ?
(a) Phosphorous acid
(b) Hypophosphorous acid
(c) Phosphoric acid
(d) Pyrophosphoric acid

61. What is the conjugate base of chloric acid? **[AP/May 19, 2024 (II)]**
(a) ClO_4^- (b) ClO^- (c) ClO_2^- (d) ClO_3^-

62. At 27°C, 100 mL of 0.5 M HCl is mixed with 100 mL of 0.4 M NaOH solution. To this resultant solution, 800 mL of distilled water is added. What is the pH of final solution? **[TS/May 10, 2024 (II)]**
(a) 12.0 (b) 2.0
(c) 1.3 (d) 1.0

63. Observe the following species **[TS/May 10, 2024 (I)]**
(i) NH_3 (ii) $AlCl_3$
(iii) $SnCl_4$ (iv) CO_2
(v) Ag^+ (vi) HSO_4^-
How many of the above species act as Lewis acids?
(a) 5 (b) 3 (c) 4 (d) 2

64. Observe the following species
$AlCl_3$, NH_3, H^+, Co^{3+}, ^-OH, Mg^{2+}, BF_3, Cl^-
How many Lewis acids are present in the above list?
[AP/May 18, 2023 (II); Similar to AP/May 17, 2023 (II)]
(a) 5 (b) 4 (c) 2 (d) 3

65. Given below are two statements
Statement I: The changes in pH with temperature are so small that we often ignore it
Statement II: When the hydrogen ion concentration changes by a factor of 100, the pH changes by one unit
In the light of above statements, identify the correct answer from the options given below **[AP/May 17, 2023 (I)]**
(a) Both statements I and II are correct.
(b) Both statements I and II are not correct.
(c) Statement I is correct but statement II is not correct.
(d) Statement I is not correct but statement II is correct.

66. Observe the following solutions
I. Black coffee II. 0.2M NaOH
III. Lemon juice IV. Lime water
V. Human Saliva VI. Tomato juice
The number of solutions with pH less than 7 is
[AP/May 16, 2023 (II); Similar to AP/May 15, 2023 (II)]
(a) 2 (b) 5 (c) 4 (d) 3

67. Conjugate acid and conjugate base of HCO_3^- are respectively **[AP/May 16, 2023 (I)]**
(a) H_2CO_3, $H_3CO_3^+$ (b) H_2CO_3, CO_3^{2-}
(c) CO_3^{2-}, H_2CO_3 (d) CO_3^{2-}, CO_2

68. A solution is prepared by mixing 10 mL of 1.0 M acetic acid and 20 mL of 0.5M sodium acetate and diluted to 100 mL. If the pK_a of acetic acid is 4.76, then the pH of the solution is **[TS/May 12, 2023 (I)]**
(a) 4.76 (b) 3.76 (c) 5.76 (d) 9.24

69. The conjugate base of H_3O^+ is **[AP/July 6, 2022 (I)]**
(a) H_2O (b) OH^- (c) H^+ (d) H^-

70. The pH of 0.01 N lime water is **[AP/July 4, 2022 (II)]**
(a) 13.09 (b) 10 (c) 12 (d) 9.8

71. Ammonia is a Lewis base because it is **[TS/July 20, 2022 (II)]**
(a) Electron pair donor
(b) Electron pair acceptor
(c) Proton donor
(d) Proton acceptor

72. How many of the following are diprotic acids?
Citric acid, Chromic acid, Oxalic acid, Pyrosulfuric acid, Sulfurous acid **[TS/July 20, 2022 (I)]**
(a) 2 (b) 5 (c) 4 (d) 3

73. Calculate the number of moles of NaOH required to completely neutralise 100 g of 118% oleum **[TS/July 18, 2022 (II)]**
(a) 2.4 (b) 1.2 (c) 4.8 (d) 8.4

74. The pH of pure water at 80 °C is **[TS/July 18, 2022 (I)]**
(a) 7.0 (b) ∞ (c) > 7.0 (d) < 7.0

75. Which of the following gas has the highest pH at 25°C? **[AP/Aug. 23, 2021 (I)]**
(a) Distilled H_2O (b) 1 M *aq* NH_3
(c) 1 M NaOH (d) 1 M HCl

76. The successive equilibrium constants for the stepwise dissociation of a tribasic acid are K_1, K_2 and K_3 respectively. The equilibrium constant for the overall dissociation is **[TS/Aug. 6, 2021 (I)]**

(a) $(K_1 + K_2 + K_3)$ (b) $\sqrt[3]{(K_1 + K_2 + K_3)}$

(c) $(K_1 \times K_2 \times K_3)^3$ (d) $K_1 \times K_2 \times K_3$

77. Strongest conjugate base among the following is **[AP/Sept. 18, 2020 (I)]**

(a) Cl^- (b) F^- (c) Br^- (d) I^-

78. At T (K), if the ionisation constant of ammonia in solution is 2.5×10^{-5}, the pH of 0.01 M ammonia solution and the ionisation constant of its conjugate acid respectively at that temperature are (log 2 = 0.30) **[AP/Apr. 22, 2019 (II)]**

(a) 10.7, 4.0×10^{-8} (b) 10.7, 4.0×10^{-10}

(c) 3.3, 4.0×10^{-8} (d) 3.3, 4.0×10^{-10}

79. The number of species of the following that can act both as Bronsted acids and bases is **[AP/Apr. 20, 2019 (I)]**

$HCl, ClO_4^-, {}^-OH, H^+, H_2O, HSO_4^-, SO_4^{2-}, H_2SO_4, Cl^-$

(a) 4 (b) 3 (c) 1 (d) 2

80. What is the order of relative basic strength of ClO_2^-, ClO_3^-, ClO_4^-? **[TS/May 6, 2019 (I)]**

(a) $ClO^-_2 > ClO^-_3 > ClO^-_4$

(b) $ClO^-_3 > ClO^-_2 > ClO^-_4$

(c) $ClO^-_4 > ClO^-_2 > ClO^-_3$

(d) $ClO^-_2 > ClO^-_4 > ClO^-_3$

81. pH of an aqueous solution of NH_4Cl is **[TS/2016]**

(a) 7 (b) > 7 (c) < 7 (d) 1

82. What is the pH of the NaOH solution when 0.04 g of it dissolved in water and made to 100 mL solution? **[AP/2015]**

(a) 2 (b) 1 (c) 13 (d) 12

Topic 4 Ionisation of Weak Acids and Bases and Relation between K_a and K_b

83. At 27°C. the degree of dissociation of HA (weak acid) in 0.5 M of its solution is 1%. The concentrations of H_3O^-, A^- and HA at equilibrium (in mol L^{-1}) are respectively **[AP/May 23, 2024 (I)]**

(a) 0.005, 0.005, 0.495 (b) 0.05, 0.05, 0.45

(c) 0.01, 0.01, 0.49 (d) 0.005, 0.495, 0.005

84. At 27°C, the degree of dissociation of weak acid (HA) in its 0.5M aqueous solution is 1%. Its K_a value is approximately **[AP/May 18, 2024 (I)]**

(a) 5×10^{-4} (b) 5×10^{-5}

(c) 5×10^{-6} (d) 5×10^{-8}

85. The pH of 0.01M BOH solution is 10. What is its degree of dissociation? **[AP/May 19, 2023 (I)]**
(Given K_b of BOH is 1×10^{-6})

(a) 10% (b) 5% (c) 2% (d) 1%

86. The K_a values of A, B and C are 1.8×10^{-4}, 5×10^{-10} and 3×10^{-8} respectively. The correct order of their acidic strength is **[AP/July 5, 2022 (II)]**

(a) B > A > C (b) B > C > A

(c) A > B > C (d) A > C > B

87. Equal volumes of 0.5 N acetic acid and 0.5 N sodium acetate are mixed. What is the pH of resultant solution? (pK_a of acetic acid = 4.75) **[AP/July 4, 2022 (I)]**

(a) 4.85 (b) 4.65 (c) 4.75 (d) 7.0

88. The pH of 10^{-8} M HCl solution is **[AP/Sept. 21, 2020 (II)]**

(a) 8 (b) – 8

(c) Between 7-8 (d) Between 6-7

89. Which acid among the following has the highest pK_a value? **[AP/Sept. 21, 2020 (I)]**

(a) HCl (b) HF (c) HI (d) HBr

90. The ionic product of water .... with increase in temperature. **[AP/Sept. 21, 2020 (I)]**

(a) remains constant (b) increases

(c) decreases (d) may increase or decrease

91. 30.0 mL of the given HCl solution requires 20.0 mL of 0.1 M sodium carbonate solution for complete neutralisation. What is the volume of this HCl solution required to neutralise 30.0 mL of 0.2 M NaOH solution? **[AP/Apr. 21, 2019 (II)]**

(a) 25 mL (b) 50 mL (c) 90 mL (d) 45 mL

92. If the pK_a of acetic acid and pK_b of dimethylamine are 4.76 and 3.26 respectively, the pH of dimethyl ammonium acetate solution is **[AP/2018]**

(a) 7.75 (b) 6.75 (c) 7.0 (d) 8.5

Topic 5 Common Ion Effect, Salt Hydrolysis, Buffer Solutions and Solubility Product

93. The solubility of barium phosphate of molar mass ‘M’ g mol^{-1} in water is x g per 100 mL at 298 K. Its solubility product is $1.08 \times \left(\frac{x}{M}\right)^a \times (10)^b$. The values of a and b respectively are **[AP/May 21, 2024 (II)]**

(a) 7, 5 (b) 5, 7 (c) 5, 5 (d) 7, 7

94. At T(K), the solubility product of AX is 10^{-10}. What is the molar solubility of AX in 0.1 M HX solution? **[AP/May 21, 2024 (I)]**

(a) 10^{-5} (b) 10^{-10} (c) 10^{-9} (d) 10^{-8}

95. Which of the following will make a basic buffer solution. **[AP/May 20, 2024 (II)]**

(a) 100 mL of 0.1 M CH_3COOH + 100 mL of 0.1 M NaOH

(b) l00 mL of 0.l M HCl + l00 mL of 0.1 M NaOH

(c) 50 mL of 0.1 M KOH + 25 mL of 0.1 M CH_3COOH

(d) 100 mL of 0.1 M HC1 + 200 mL of 0.l M NH_4OH

96. Observe the following solutions

(i) 1L of 10^{-6} M Ag NO_3

(ii) 1L of 10^{-7} M $AgNO_3$

(iii) 1L of 10^{-9} M Ag NO_3
(iv) 1L of 10^{-3} M $AgNO_3$
(v) 1L of 10^{-5} M NaCl

Which of the above two solutions when mixed will give a white precipitate, AgCl? **[AP/May 18, 2023 (I)]**
(Given K_{sp} of AgCl = 1×10^{-10})

(a) (i), (v) (b) (ii), (v)
(c) (iv), (v) (d) (iii), (v)

97. At 298 K the molar solubility of $Cd(OH)_2$ in 0.1 M KOH solution is $x \times 10^{-y}$. The values of x and y are respectively (at 298 K, K_{sp} of $Cd(OH)_2 = 2.5 \times 10^{-14}$) **[TS/May 14, 2023 (II)]**

(a) 2.5, 14 (b) 25, 13 (c) 25, 14 (d) 2.5, 16

98. Which of the following does not form a buffer solution? **[TS/May 14, 2023 (I)]**

(a) $NH_3 + HCl$ (2:1 mole ratio)
(b) $CH_3CO_2H + NaOH$ (2:1 mole ratio)
(c) $NaOH + CH_3COOH$ (1:1 mole ratio)
(d) $NH_4Cl + NH_3$ (1:1 mole ratio)

99. **Statement A :** pH of buffer increases with increasing temperature.
Statement B : The value of K_W of water decreases with decreasing temperature. **[AP/July 8, 2022 (I)]**

(a) A is correct, but B is wrong.
(b) Both A and B are correct
(c) Both A and B are wrong
(d) A is wrong but B is correct

100. At 25°C, the solubility product of MCl is 1×10^{-10}. What is its molar solubility in 0.1 M NaCl solution at same temperature? **[AP/July 7, 2022 (II)]**

(a) 0.1 M (b) 0.05 M (c) 10^{-9} M (d) 10^{-5} M

101. The solubility products of NiS, ZnS, CdS and HgS are 4.7×10^{-5}, 1.6×10^{-24}, 8×10^{-27} and 4×10^{-53} respectively. An aqueous solution contains Ni^{2+}, Zn^{2+}, Cd^{2+}, and Hg^{2+} of equal concentration. H_2S gas was passed into this solution very slowly. The first and the last ions that precipitate as sulphides are respectively **[AP/July 7, 2022 (II)]**

(a) Ni^{2+}, Hg^{2+} (b) Hg^{2+}, Cd^{2+}
(c) Zn^{2+}, Hg^{2+} (d) Hg^{2+}, Ni^{2+}

102. Match the following.

Metal sulfide	**Solubility product**
(A) PbS	(I) 4.0×10^{-53}
(B) HgS	(II) 8.0×10^{-28}
(C) MnS	(III) 1.6×10^{-24}
(D) ZnS	(IV) 2.5×10^{-13}

The correct answer is **[TS/July 19, 2022 (II)]**

	A	**B**	**C**	**D**
(a)	I	II	III	IV
(b)	II	I	IV	III
(c)	II	III	IV	I
(d)	III	IV	I	II

103. The solubility of AgBr(*s*), having solubility product 5×10^{-10} in 0.2 M NaBr solution equals **[AP/Aug. 20, 2021 (I); Similar to AP/Sep. 21, 2020 (I)]**

(a) 5×10^{-10} M (b) 25×10^{-10} M
(c) 0.5 M (d) 0.002 M

104. 100 g of a mixture of NaOH and Na_2SO_4 is neutralised by 100 mL of 0.5 M H_2SO_4.
What is the amount of Na_2SO_4 present in the mixture? **[TS/Aug. 6, 2021 (I)]**

(a) 82 g (b) 96 g (c) 88 g (d) 92 g

105. Match the following columns: **[TS/Aug. 4, 2021 (I)]**

Column-I (Acid)		**Column-II [K_a (ionisation constant)]**	
A.	HCN	1.	6.8×10^{-4}
B.	$H_2C_2O_4$	2.	8.9×10^{-8}
C.	H_2S	3.	4.9×10^{-10}
D.	Niacin	4.	5.6×10^{-2}
		5.	1.5×10^{-5}

The correct match is

Codes:

	A	**B**	**C**	**D**
(a)	1	3	4	5
(b)	5	2	3	4
(c)	2	3	4	5
(d)	3	4	2	5

106. If the molar concentrations of base and its conjugate acid are same, then pOH of the buffer solution is **[TS/Aug. 4, 2021 (I)]**

(a) same as pK_b of base (b) same as pK_a of base
(c) same as pK_a of acid (d) same as pK_b of acid

107. The solubility product of $Ni(OH)_2$ at 298 K is 2×10^{-15} $mol^3\ dm^{-9}$. The pH value if, its aqueous and saturated solution is **[AP/Sept. 17, 2020 (I)]**

(a) 5 (b) 7.5 (c) 9 (d) 13

108. What is the pH of 10 L of a buffer solution containing 0.01 M NH_4Cl and 0.1 M NH_4OH having pK_b of 5? **[TS/Sept. 11, 2020 (I)]**

(a) 8 (b) 7 (c) 10 (d) 5

109. At 25°C, the ionisation constant for anilinium hydroxide is 5.00×10^{-10} . The hydrolysis constant of anilinium chloride is **[TS/Sept. 9, 2020 (I)]**

(a) 2.00×10^{-5} (b) 4.00×10^{-3}
(c) 1.50×10^{-6} (d) 2.50×10^{-4}

110. The solubility product of a sparingly soluble salt A_2B is 3.2×10^{-11}. Its solubility in mol L^{-1} is **[AP/Apr. 21, 2019 (II); Similar to TS/May 3, 2019 (I)]**

(a) 4×10^{-4} (b) 2×10^{-4}
(c) 6×10^{-4} (d) 3×10^{-4}

111. In which of the following, the solubility of AgCl will be minimum? **[TS/May 4, 2019 (I)]**

(a) 0.1 M KNO_3 (b) 0.1 M KCl

(c) 0.2 M KNO_3 (d) Water

112. If the solubility product of $Ni(OH)_2$ is 4.0×10^{-15} the solubility (in mol L^{-1}) is **[TS/May 5, 2018 (I); Similar to TS/ 2015 (I)]**

(a) 5.0×10^{-5} (b) 4.0×10^{-5}

(c) 2.0×10^{-5} (d) 1.0×10^{-5}

113. In which of the following salts, only cationic hydrolysis is involved? **[AP/2016]**

(a) CH_3COONH_4 (b) CH_3COONa

(c) NH_4Cl (d) Na_2SO_4

114. The buffer system which helps to maintain the pH of blood between 7.26 to 7.42, is **[TS/2015]**

(a) H_2CO_3 / HCO_3^-

(b) NH_4OH / NH_4Cl

(c) CH_3COOH / CH_3COO^-

(d) CH_3COONH_4

7 Equilibrium

1. **(b)** $K_p = K_c \times (RT)^{\Delta n}$

Given,

$K_c = 100 \text{ mol L}^{-1}$

$R = 0.082 \text{ L atm mol}^{-1} \text{ K}^{-1}$

$T = 300 \text{ K}$

Δn = (No. of gaseous product) – (no. of gaseous reactant)

$= 2 - 1 = 1$

$\therefore K_p = 100 \text{ mol L}^{-1} \times (0.082 \text{ L atm mol}^{-1} \text{ k}^{-1} \times 300\text{K})^1$

$K_P = 2460 \text{ atm}$

2. **(b)** $A_2(g) + B_2(g) \rightleftharpoons 2\,AB(g)$

At equilibrium 1.5×10^{-3}M $\quad 2.1 \times 10^{-3}$M $\quad 1.4 \times 10^{-3}$M

$$K_c = \frac{[AB]^2}{[A_2][B_2]}$$

$$K_c = \frac{1.4\times10^{-3}\times1.4\times10^{-3}}{1.5\times10^{-3}\times2.1\times10^{-3}}$$

$K_c = 0.622$

$K_p = K_c X (RT)^{\Delta n}$

$\Delta n = 0$

$K_p = K_c = 0.622$

For the decomposition of AB$(K') = \dfrac{1}{K_p}$

$$= \frac{1}{0.622}$$

$K' = 1.60$

3. **(a)**

	H_2	+ I_2 $\rightleftharpoons$	$2HI$
Initial	15	5.2	O
At equilibrium	15 – x	5.2 – x	2x

Let 'V' is volume in liter

Given, 2x = 10

x = 5

Concentration of H_2 at equilibrium = 15 – 5

$[H_2] = 10/V$

$$[I_2] = \frac{0.2}{V}$$

$$[HI] = \left[\frac{10}{v}\right]$$

$$K_c = \frac{\left[\frac{10}{V}\right]^2}{\left[\frac{10}{V}\right]\times\left[\frac{0.2}{V}\right]}$$

$$K_c = \frac{10\times10}{10\times0.2} = 50$$

$\therefore \quad K_c^{'}$ for the dissociation of HI is $= \dfrac{I}{K_c}$

$$= \frac{1}{50} = 0.2\times10^{-1}$$

$= 2 \times 10^{-2}$

4. **(a)**

	$A_2(g)$	$\xrightleftharpoons{T(K)}$	$B_2(g)$
Initial	1		O
Equilibrium	1 – x		x

$$K_c = \frac{[B_2]}{[A_2]}$$

$$39 = \frac{\left[\frac{x}{1}\right]}{\left[\frac{1-x}{1}\right]}$$

or, 39 – 39x = x

$\Rightarrow \quad 40x = 39$

$\Rightarrow \quad x = \dfrac{39}{40}$

x = 0.975

Concentration of B_2 = 0.975

Concentration of A_2 = 1 – 0.975 = 0.025

5. **(c)** Given

$H_2(g) + Br_2(g) \rightleftharpoons 2HBr\,(g)$. $K_p = 1.6 \times 10^5$

Reverse reaction,

$$2HBr(g) \rightleftharpoons H_2(g) + Br_2(g),\ K_p^{'} = \frac{1}{K_p}$$

$$HBr(g) \rightleftharpoons \frac{1}{2}H_2(g) + \frac{1}{2}Br(g),\ K_p^{''} = \left(\frac{1}{K_p}\right)^{1/2}$$

At t = 0	10 bar	0	0
At eqm.	10 – x	$\frac{x}{2}$	$\frac{x}{2}$

$$\left(\frac{1}{K_p}\right)^{\frac{1}{2}} = \frac{p_{H_2}^{1/2} \times p_{Br}^{1/2}}{P_{HBr}}$$

$$\left(\frac{1}{1.6\times10^5}\right)^{1/2} = \frac{\left(\frac{x}{2}\right)^{1/2}\left(\frac{x}{2}\right)^{1/2}}{(10-x)}$$

$\therefore$ 10 > > x

$$2.5\times10^{-3} = \frac{x}{20}$$

$\Rightarrow x = 0.05 \Rightarrow p_{HBr} = 10 - 0.05 = 9.95$ bar.

6. **(b)** 1.98

$A_2(g) \overset{T(K)}{\rightleftharpoons} B_2(g)$, $K_c = 99.0$

moles of $B_2 = 2$

$$K_c = \frac{[B_2]^2}{[A_2]} \quad ...(1)$$

$$99 = \frac{[B_2]^2}{[A_2]}$$

Initial concentration of $A_2 = 0$

Initial concentration of $B_2 = 2$

Let x be the concentration of A_2 so the change in B_2 would be – 2x

Equilibrium concentration are

$[A_2] = x$, $[B_2] = 2 - 2x$

Putting the value in eq. (1)

$$99 = \frac{(2-2x)^2}{x}$$

$$99 = 4\frac{(1-x)^2}{x}$$

Let's simplify the equation – $[(a - b)^2 = a^2 - 2ab + b^2]$

$$99 = \frac{4\left(1-2x+x^2\right)}{x}$$

$99x = 4 - 8x + 4x^2$

$4x^2 - 107x + 4 = 0$

Solving the quadratic equation we use the Quadratic formula:-

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 4, b = -107, c = 4$$

$$x = \frac{-(-107) \pm \sqrt{(-107)^2 - 4(4)(4)}}{2 \times 4}$$

$$x = \frac{107 \pm \sqrt{11449 - 64}}{8}$$

$$x = \frac{107 \pm \sqrt{11\,385}}{8}$$

$$x = \frac{107 \pm 106.70}{8}$$

$$x = \frac{213.65}{8} \approx 26.71,$$

$$x = \frac{0.30}{x} \approx 0.030$$

Putting the value of x to finding the concentration of B_2

$B_2 = 2 - 2x$

$B_2 = 2 - 2 \times 0.03$

$B_2 = 2 = 0.06 = 1.9$

7. **(c)** Given, $K_c = 99.0$

$$A_2(g) \rightleftharpoons B_2(g)$$

t = 0	2 moles	0
t = Equilibrium,	0.02	1.98

On adding one mole of A_2

$$A_2(g) \rightleftharpoons B_2(g)$$

t = Equilibrium	0.02 +	0.01

Hence, $C_3(A_2) = 0.03$ moles.

8. **(d)** given $K_c = 99.0$

n = 2 moles of A_2

v = 1L

$$A_2 \rightleftharpoons B_2$$

Initial	2	0
At equilibrium,	2 – x	x

$$K_c = \frac{[\text{Product}]}{[\text{Reactant}]}$$

$$K_c = \frac{[x/1]}{\left[\frac{2-x}{1}\right]}$$

$$99 = \frac{x}{2-x}$$

$19 - 99x = x$

$$\therefore \ x = \frac{198}{100} = 1.98$$

concentration of B_2 at equilibrium is 1.98 mol L^{-1}

concentration of A_2 at equilibrium is (2 – 198)

= 0.02 mol L^{-1}

= 0.02 mo1 L^{-1}

9. **(c)** $2A(g) \rightleftharpoons B(g) + C(g)$; $K_1 = 16$

$$K_1 = \frac{[B][C]}{[A]^2} = 16 \qquad ...(i)$$

$2B(g) + C(g) \rightleftharpoons 2D(g)$; $K_2 = 25$

$$K_2 = \frac{[D]^2}{[C][B]^2} = 25 \qquad ...(ii)$$

$$\therefore \quad A + \frac{1}{2}B(g) \rightleftharpoons D \ ; K_3$$

$$K_3 = \frac{[D]}{[A][B]^{1/2}} \qquad ...(iii)$$

$$K_3 = \sqrt{K_2 \times K_1}$$

$$= \sqrt{\frac{[D]^2}{[C][B]^2} \times \frac{[B][C]}{[A]^2}} = \sqrt{25 \times 16}$$

$$\Rightarrow \quad \frac{[D]}{[B]^{1/2}[A]} = 4 \times 5 = 20$$

10. **(a)** 2.2

Given – $K_c = 2 \times 10^{-2}$ mol L^{-1}

$$PCl_5(g) \rightarrow PCl_3(g) + Cl_2(g)$$

moles at initial	(x)	0	0
moles at equilibrium	x – 0.2	0.2	0.2

$$\text{Equilibrium constant } K = \frac{[PCl_3][Cl_2]}{[PCl_5]}$$

$$2\times10^{-2}=\frac{[0.2]\times[0.2]}{[x-0.2]}$$

x = 2.2 moles

11. **(a)** $k_c=\frac{[AO_3][BO]}{[AO_2][BO_2]}=16$

Initial concentrations :-

$[AO_3] = [BO] = [AO_2] = [BO_2] = \frac{1.0\text{ mol}}{1.0\text{ L}} = 10$ M.

Conc.	$[AO_2]$	$[BO_2]$	$[AO_3]$	[BO]
Initial	1.0	1.0	1.0	1.0
At equilibrium	$1-\alpha$	$1-\alpha$	$1+\alpha$	$1+\alpha$

$$\frac{(1+\alpha)\times(1+\alpha)}{(1-\alpha)(1-\alpha)}=16$$

$$\frac{1+\alpha}{1-\alpha}=4$$

$\therefore\ 1+\alpha=4-4\alpha$

$5\alpha=3$

$\alpha=3/5=0.6$

$\therefore$ Equilibrium concentration of [BO] is = 1 + 0.6 = 1.6.

12. **(c)** $A_2B_6(g)\rightleftharpoons A_2B_4(g)+B_2(g)$

Initial	p = 4 atm	0.0	0.0
Equilibrium	$4-4\alpha$ $=4(1-\alpha)$	$p\alpha=4\alpha$	$p\alpha=4\alpha$

$$K_p=\frac{P(A_2B_4)\times P(B_2)}{P(A_2B_6)}=0.04\text{ atm}$$

$$=\frac{(4\alpha)(4\alpha)}{4(1-\alpha)}=\frac{4\alpha^2}{1-\alpha}\approx4\alpha^2$$

$\Rightarrow\ \alpha=0.1$

Therefore, $P(A_2B_6)$ at equilibrium = $4(1-\alpha)$

$=4(1-0.1)=3.62$

13. **(d)** $K_p=K_c(RT)^{\Delta n_g}$

$K_c=10\text{ mol L}^{-1}$, T = 1000 K,

$R=0.082\text{ L atm mol}^{-1}\text{ K}^{-1}$

$\Delta n_g=(1+1)-1=1$

$\Rightarrow K_p=(10)(0.082\times1000)^1=820$

14. **(b)** $K_C=\frac{[B]}{[A]}=10^{-1}$ or $\frac{1}{10}$

Since K_C remains constant at a given temperature, the value of $\frac{[B]}{[A]}$ will still be the same.

$$\Rightarrow\frac{[B]}{[A]}=\frac{1}{10}\Rightarrow\frac{[A]}{[B]}=10$$

15. **(b)** $K_p=K_c(RT)^{\Delta n_g}$

$H_2O(g)+CO(g)\rightleftharpoons H_2(g)+CO_2(g)$

$\Rightarrow\ \Delta n_g=(1+1)-(1+1)=0$

$\Rightarrow\ K_p=K_c$.

Now, 40% of mass of water reacted with CO.

$\Rightarrow\ \alpha=0.4$

	H_2O	CO	H_2	CO_2
Initial moles	1.0	1.0	0.0	0.0
At equilibrium	1 – 0.4 = 0.6	1 – 0.4 = 0.6	0.4	0.4

Thus, $[H_2O]=\frac{0.6}{1.0}=0.6$ M, [CO] = 0.6 M,

$[H_2]=0.4$ M, $[CO_2]=0.4$ M

$$\Rightarrow K_C=\frac{[H_2][CO_2]}{[H_2O][CO]}=\frac{(0.4)(0.4)}{(0.6)(0.6)}=0.444$$

Thus, $K_p=0.444$.

16. **(b)** $K_p=K_c(RT)^{\Delta n_g}$

$$PCl_5(g)\xrightarrow{\Delta}PCl_3(g)+Cl_2(g)$$

Initial	1	0	0
Equilibrium	1 – x	x	x

x = 0.1 (given) so 1 – x = 0.9

$$\Rightarrow K_c=\frac{x^2}{1-x}=\frac{(0.1)^2}{0.9}=0.011$$

Thus, $K_p=(0.011)(0.082\times500)^{(2-1)}=0.451$.

17. **(a)** $K_c=\frac{[B]^b}{[A]^a}$ and $K'_c=\frac{[B]^{2b}}{[A]^{2a}}\Rightarrow\ K'_c=(K_c)^2$

18. **(b)** $\frac{1}{3}N_2+H_2\rightleftharpoons\frac{2}{3}NH_3$ $K_C=50$ given. ...(1)

Multiply by 3 in eq^n (1)

$\therefore\ N_2+3H_2\rightleftharpoons2NH_3$. $\therefore K'_C=(50)^3$ ... (2)

Reverse eq. (2)

$\therefore\ 2NH_3\rightleftharpoons N_2+3H_2\ K_C'=\left(\frac{1}{50}\right)^3$

$\therefore\ K_C'=\frac{1}{50}\times\frac{1}{50}\times\frac{1}{50}=8\times10^{-6}$.

19. **(b)** $[X]=\frac{1\text{mol}}{1\text{L}}=1\text{M};[Y]=\frac{1\text{mol}}{1\text{L}}=1\text{M}$

$[Z]=\frac{0.5\text{ mol}}{1\text{L}}=0.5$ M

$X(g)+Y(g)\rightleftharpoons Z(g)+A(g)$

$$K_c=\frac{[Z][A]}{[X][Y]}$$

	X	Y	Z	A
Before	1	1	0	0
At equili.	$1-\alpha$	$1-\alpha$	0.5	α

$\Rightarrow\alpha=0.5$ and $[X]=[Y]=1-\alpha=1-0.5$

= 0.5 M. Also, [A] = 0.5 M.

$$\Rightarrow K_c = \frac{(0.5)(0.5)}{(0.5)(0.5)} = 1.0$$

20. **(d)** $K_p = K_c(RT)^{\Delta n}$

[Δn = Change in the no. gaseous molecules from reactant to product.]

In this equation,

$$\Delta n = \left(1 - 1 - \frac{1}{2}\right) = -\frac{1}{2} \;\therefore\; \frac{K_p}{K_c} = (RT)^{-1/2} = \frac{1}{\sqrt{RT}}$$

21. **(b)** $N_2 + 3H_2 \rightarrow 2NH_3; K_c = \dfrac{[NH_3]^2}{[N_2][H_2]^3}$

22. **(b)** $4NH_3(g) + 5O_2(g) \longrightarrow 4NO(g) + 6H_2O(g)$

$$K_p = \frac{(p_{NO})^4 (p_{H_2O})^6}{(P_{NH_3})^4 (p_{O_2})^5} = \frac{(0.5)^{10}}{(0.5)^9} = 0.5 \text{ atm}$$

23. **(c)** $N_2(g) \quad + \quad 3H_2(g) \rightleftharpoons 2NH_3(g)$

Initial : 1 mol 3 mol 0

At eqm. (1 – x) mol (3 – 3x) mol 2x mol

$$\therefore \quad K_C = \frac{[NH_3]^2}{[N_2][H_2]^3} = \frac{\left(\frac{2x^2}{v^2}\right)}{\left\{\frac{(1-x)}{v}\right\}\left\{\frac{(3-3x)^2}{v^3}\right\}}$$

$$= \frac{4x^2v^2}{(1-x)\{3(1-x)\}^3} = \frac{4x^2v^2}{27(1-x)^4}$$

24. **(c)** $K = \dfrac{[B][C]}{[A]}$; since A and B are solid, the modified

K = [C] partial pressure of C = p_C

25. **(b)** Given reaction is

$2X + Y \rightleftharpoons 3Z$

Equilibrium constant $k = \dfrac{[Z]^3}{[X]^2[Y]}$

26. **(d)** $2(A)g \rightleftharpoons B(g) + C(g)$

$\Delta n = 2 - 2 = 0$

$K_p = K_C (RT)^0 \Rightarrow K_P = K_C$

27. **(b)** $\log \dfrac{K_2}{K_1} = \dfrac{-\Delta H}{2.303R}\left[\dfrac{1}{T_1} - \dfrac{1}{T_2}\right]$

Substituting the values in above equation

$$\log \frac{0.6 \times 10^{-4}}{1.2 \times 10^{-4}} = \frac{-\Delta H}{2.303 \times 2}\left[\frac{1}{300} - \frac{1}{400}\right]$$

$$\Rightarrow \log 0.5 = -\frac{\Delta H}{4.606}\left[\frac{100}{120000}\right] \Rightarrow \Delta H = -1663.69$$

Mean heat of formation

$$= \frac{\Delta H}{2} = \frac{-1663.69}{2} \approx -826.43 \text{cal}$$

28. **(b)** $Q_c = \dfrac{[B][C]}{[A]^2}$

As, [A] = [B] = [C]

$$Q_c = \frac{[A][A]}{[A]^2} = 1 \Rightarrow Q_c > K_c$$

$\therefore$ The reaction will proceed in the reverse direction.

29. **(d)** The value of equilibrium constant does not depend on initial concentrations of reactants and products.

30. **(a)** $N_2 + O_2 \rightleftharpoons 2NO$

K_C for the above reaction can be given as,

$$K_C = \frac{[NO]^2}{[N_2][O_2]}$$

$$K_C = \frac{[3 \times 10^{-3}]^2}{[4 \times 10^{-3}][3 \times 10^{-3}]} \Rightarrow K_C = \frac{3 \times 10^{-3}}{4 \times 10^{-3}} = 0.75$$

31. **(b)** Given, $K_C = 4 \times 10^{-4}$

T = 1000 K

$K_p = K_C.(RT)^{\Delta n_g}$

Δn_g = moles of gaseous product (n_p) – moles of gaseous reactant (n_R)

= 2 – 1 = 1

$K_p = 4 \times 10^{-4} (0.0821 \times 800)^1$

$K_p = 0.026129.$

(b) For the reaction,

$CO_2(g) + C(s) \rightleftharpoons 2\,CO(g)$

$$k_p = \frac{(P_{CO})^2}{(P_{CO_2})}$$

$$k_p = \frac{(0.6)^2}{(0.15)} = 2.4$$

Also, $K_p = K_c.(RT)^{\Delta n}$

where, Δn

$$\Rightarrow \quad K_c = \frac{K_p}{RT} \Rightarrow K_c = \frac{2.4}{0.0821 \times 1000} = 0.0289$$

$\Rightarrow \quad K_c = 2.89 \times 10^{-2}$ (approx. closer to option b)

32. **(d)** $\ln K_p = -\dfrac{\Delta H}{RT} + \text{constant}$

$\Delta H > 0$ for endothermic reaction, thus, slope $\left(-\dfrac{\Delta H}{R}\right)$ is negative.

33. **(a)** For the reaction,

$0.5C(s) + 0.5CO_2(g) \rightleftharpoons CO(g)$

If 50% conversion of $CO_2(g)$ takes place.

We have concentration of $CO_2(g)$ = [0.25]

K_P is calculated only for gaseous species, thus

$$K_p = \frac{[\text{Product}]_{(g)}}{[\text{Reactant}]_{(g)}} = \frac{[1]}{[0.25]} = 4$$

Hence, option (a) is the correct answer.

34. **(c)** Given, $K_C = 4 \times 10^{-6}$ mol/L

($\Delta n = n_P - n_R \Rightarrow 3 - 2 = 1$)

$$K_p = K_C \times (RT)^{\Delta n_g}$$

$= 4 \times 10^{-6}$ mol L^{-1} × (0.083 L bar k^{-1} mol^{-1} × 1000 K)^1

$K_p = 3.32 \times 10^{-4}$ bar

35. **(b)**

$$\begin{array}{lccc} & N_2 & +\ 3H_2 \rightleftharpoons & 2NH_3 \\ \text{Initial moles} & 1 & 1 & 1 \\ \text{At equilibrium} & 1-x & 1-3x & 1+2x \end{array}$$

Given, $1 - x = 0.7$ mol $\Rightarrow x = 0.3$ mol

Therefore, concentration of N_2, H_2 and NH_3 at equilibrium will be

$[N_2] = [0.7]$

$[H_2] = 1 - (3 \times 0.3) = [0.1]$

$[NH_3] = 1 + 2x = 1 + (2 \times 0.3) = [1.6]$

According to law of equilibrium constant (K_c)

$$K_c = \frac{[NH_3]^2}{[N_2],[H_2]^3} = \frac{[1.6]^2}{[0.7],[0.1]^3}$$

$$K_c = \frac{2.56}{0.0007} = 3657.14$$

36. **(a)** Let, number of moles of $SO_2 = x$

moles of $SO_3 = 2x$

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$$

At equilibrium,

$$K_c = \frac{[SO_3]^2}{[SO_2]^2[O_2]}$$

$$100 = \frac{\left[\frac{2x}{10}\right]^2}{\left[\frac{x}{10}\right]^2\left[\frac{n_{O_2}}{10}\right]}$$

$$100 = \frac{40}{n_{O_2}}$$

$$n_{O_2} = \frac{40}{100} = 0.4$$

Hence, number of moles of oxygen = 0.4

37. **(d)** Required relation

$$H_3PO_4(aq.) \rightleftharpoons 3H^+(aq.) + PO_4^{3-}(aq.)$$

For reaction (i)

$$K_1 = \frac{\left[H^+\right]^-_{(aq)}\left[H_2PO_4^-\right]_{(aq)}}{H_3PO_{4(aq.)}} \quad \text{...(i)}$$

For reaction (*ii*)

$$K_2 = \frac{\left[H^+\right]_{(aq.)}\left[HPO_4^{2-}\right]_{(aq.)}}{\left[H_2PO_4^-\right]_{(aq.)}} \quad \text{...(ii)}$$

For reaction (iii)

$$K_3 = \frac{\left[H^+\right]_{(aq.)}\left[PO_4^{3-}\right]_{(aq.)}}{\left[HPO_4^{2-}\right]_{(aq.)}} \quad \text{...(iii)}$$

On multiplying (i), (ii), (iii)

$K = K_1 \times K_2 \times K_3$

38. **(b)**

$$\begin{array}{lcc} & N_2O_4 \rightleftharpoons & 2NO_2 \\ \text{Initial moles} & 1 & 0 \\ \text{At equilibrium} & (1-0.2) & 2 \times 0.2 = 0.4 \end{array}$$

Total moles = 0.4 + 0.8 = 1.2

$$K_P = \frac{\left(P_{NO_2}\right)^2}{\left(P_{N_2O_4}\right)} \Rightarrow K_P = \frac{\left(\frac{0.4}{1.2} \times 600\right)^2}{\left(\frac{0.8}{1.2} \times 600\right)} = 100$$

39. **(b)**

40. **(c)** Given,

$$\begin{array}{lcccc} & A & +\ B \rightleftharpoons & C\ + & D \\ \text{Initial moles} & 1 & 1 & 0 & 0 \\ \text{Moles at equili.} & (1-x) & (1-x) & x & x \\ & (1-0.5) & (1-0.5) & 0.5 & 0.5 \end{array}$$

$$\Rightarrow \quad K_C = \frac{[C][D]}{[A][B]} = \frac{0.5 \times 0.5}{0.5 \times 0.5} = 1$$

41. **(b)** We know that,

$\Delta_r G^\circ = -$ RTln K_{eq}

given,

$\Delta_r G^\circ = -$ 11.5 kJ mol^{-1}

T = 300 K

R = 8.314 J mol^{-1} K^{-1}

∴ -11.5×10^3 J mol^{-1}

$= -$ 8.314J mol^{-1} K^{-1} × 300 K × ln K_{eq}

or, $4.6106 = 2.303 \log_{10} K_{eq}$

∴ $\log_{10} K_{eq} = 2$

⇒ $K_{eq} = (10)^2$

$$\boxed{K_{eq} = 100}$$

42. **(c)** Addition of oxalic acid $H_2C_2O_4$ causes the oxalate ions $C_2O_4^{2-}$ to react with Fe^{3+} ions and form a complex that decreases the concentration of Fe^{3+} ions.

Thus, the equilibrium would shift towards the direction where the concentration of Fe^{3+} ions would increase which is towards left and this decreases the intensity of deep red colour.

43. **(d)** Addition of He gas (inert gas) at constant pressure or volume will not have any effect on the equilibrium of the given reaction as a change of pressure will not affect the number of gaseous moles that are equal on reactant and product side.

44. **(b)** The value of the equilibrium constant is dependent on temperature but is independent of the pressure. Thus, the value of the equilibrium constant will remain 3.52.

45. **(b)** $A(g) + \frac{1}{2}B(g) \rightleftharpoons C(g) + \text{heat};$

exothermic reaction

∴ Low temperature is favourable.

$$\Delta n = +1 - \left(1 + \frac{1}{2}\right) = 1 - \frac{3}{2} = -\frac{1}{2}$$

no. of gas molecules decreases in the forward reaction. So, high pressure is favourable for forward reaction.

46. **(c)** At equilibrium, the equilibrium constant = K and Gibb's Free Energy = $\Delta G = 0$

Non-equilibrium condition,

$\Delta G = \Delta G^\circ + RT \ln K$

or, $0 = \Delta G^\circ + RT \ln K$

or, $\Delta G^\circ = -RT \ln K$

47. **(d)** $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) + \Delta H$, Exothermic reaction.

According to Le Chatelier's principle, the reaction will proceed in backward reaction as the temperature increases.

48. **(c)** According to Le-chatelier's principle; on increasing the pressure, equilibrium shifts towards that direction where number of gaseous moles are minimum. In reaction (i) no. of moles in product side is less. Hence, on increasing pressure, equilibrium shift towards product side.

49. **(d)** As equilibrium $\Delta G = 0$

$\Delta G^\circ = -RT \ln K_c$

At equilibrium [Product] = [Reactant]

Hence, equilibrium constant

$K_c = 1$.

50. **(b)** Le-Chatelier principle is not applicable to pure solids and liquids because they experience negligible change in concentration during chemical equilibrium.

51. **(b)** $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \rightleftharpoons XY_3; \Delta H = -30kJ$

$X_2 + 3Y_2 \rightleftharpoons 2XY_3; \Delta H = -60kJ$

$\Delta S_r = 2 \times 50 - 3 \times 40 - 1 \times 60 = -80\ JK^{-1}\ mol^{-1}$

$\Delta G = \Delta H - T\Delta S$

When, $\Delta G = 0$

$0 = \Delta H - T\Delta S \Rightarrow \Delta H = T\Delta S$

$1000 \times (-60) = T \times (-80) \Rightarrow T = 750\ K$.

52. **(b)** $SO_2 + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

When pressure is increased forward reaction is favoured because product side has lower no. of moles.

From p_1 to p_3, yield of SO_3 is increasing because higher pressure favours forward reaction.

53. **(a)** Formation of NH_3 takes place according to Le-Chatelier's principle.

$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$

As number of moles of product (2) is less than number of moles of reactants (4).

So, increase in pressure will favour forward direction.

As the reaction is exothermic so exothermic decrease in temperature will favour forward direction.

Hence, high pressure and low temperature are favourable conditions for formation of NH_3.

54. **(c)** $\Delta G^\circ = -RT\ ln\ K$

$-25\ kJ\ mol^{-1} = (-8.314\ J\ mol^{-1}\ K^{-1} \times 300K)\ ln\ K$

$$\ln K = \frac{25 kJ\ mol^{-1}}{8.314 J\ mol^{-1}\ K^{-1} \times 300\ K}$$

$$= \frac{25 \times 10^3\ J\ mol^{-1}}{8.314\ J\ mol^{-1}\ K^{-1} \times 300\ K}$$

$\ln K = 10 \Rightarrow K = e^{10}$

Hence, option (c) is correct.

55. **(a)** $\Delta G^\circ = -RT\ lnK_p = -8.314 \times 400 \times 2.303 \times \log 10^{-30}$

$= 22976570\ Jmol^{-1} = 229.8\ Jmol^{-1}$

56. **(b)** For exothermic reaction, k_f decreases and k_b increases with increase of temperature so that K_{eq} decreases. For endothermic reaction, k_f increases and k_b decreases with increase of temperature so that K_{eq} increases.

57. **(c)** $pH = pK_a + \log\frac{[Salt]}{[Acid]}$

$pH = 5 + \log\frac{[0.2]}{[0.1]}$ $[pK_a = -\log K_a = -\log 10^{-5} = 5]$

$pH = 5 + \log 2$

$pH = 5 + 0.3$

$pH = 5.3$

58. **(a)** For an acid,

pH of an acid solution will be lower as the solution is more concentrated.

$pH = -\log[H^+]$

$$M_a = \frac{M_1V_1 + M_2V_2}{V_1 + V_2}$$

$$= \frac{2\times10^{-2}\times10^{-2} + 2\times10^{-2}\times2\times10^{-2}}{4\times10^{-2}} = 1.5\times10^{-2}$$

$$M_b = \frac{2\times10^{-2}\times10^{-2} + 2\times10^{-2}\times5\times10^{-3}}{4\times10^{-2}} = 0.75\times10^{-2}$$

$$M_c = \frac{2\times10^{-2}\times10^{-2} + 2\times10^{-2}\times1\times10^{-2}}{4\times10^{-2}}$$

$M_c = 1 \times 10^{-2}$

$$M_d = \frac{2\times10^{-2}\times10^{-2} + 4\times10^{-2}\times5\times10^{-3}}{6\times10^{-2}}$$

$$= \frac{4\times10^{-4}}{6\times10^{-2}}$$

$M_d = 0.66 \times 10^{-2}$

Order of concentration : $M_a > M_c > M_b > M_d$

Order of pH :- $M_d > M_b > M_c > M_a$

∴ Ma have the lowest pH.

59. **(a)** 12

Moles of HCl = 0.4m × 0.1L = 0.04 moles

For NaOH:-

Moles of NaOH = 0.5 M × 0.1 L = 0.05 moles

HCl and NaOH react in 1 : 1 Ratio

$HCl + NaOH \rightarrow NaCl + H_2O$

NaOH is in excess since 0.05 moles of NaOH react with 0.04 moles of HCl

0.05 moles – 0.04 moles = 0.01 moles of NaOH remaining

The total volume after mixing – 100ml of HCl + 100 ml
of NaOH + 800 ml of
distilled water
⇒ 1000 ml or 1 L

so the concentration of NaOH = $\frac{0.01\ moles}{1L}$ = 0.01 m

$pOH = -\log[OH^-]$

$pOH = -\log[0.01] = 2$

As we know pH + pOH = 14

pH = 14 – pOH = 14 – 2 = 12

60 **(b)** Balanced stoichiometric equation is:
$P_4 + 3OH^- + 3H_2O \rightarrow PH_3 + 3H_2PO_2^-$
conjugate of $H_2PO_2^-$ is H_3PO_2.
H_3PO_2 is hypophosphorous acid.

61. **(d)** $\underset{\text{chloric acid}}{HClO_3} \rightleftharpoons \underset{\text{Conjugate base}}{ClO_3^- + H^+}$

62. **(b)** Concentration $(C_1) = 0.5$ M, $V_1 = \frac{100}{1000}$
Concentration $(C_2) = 0.4$ M, $= 0.1$ L
$V_2 = 0.1$ L
$n_1(\text{mole}) = C_1 \times V_1$
$= 0.5 \times 0.1 = 0.05$
$n_2(\text{mole}) = C_2 \times V_2$
$= 0.4 \times 0.1 = 0.04$
$\therefore \quad n_3 = (n_2 - n_1)$ [for SA & SB]
$= 0.05 - 0.04$
$n_3 = 0.01$
$\therefore$ Final volume after adding water
$= 200 + 800 = 1000$ mL$= 1$ L
Final concentration $= \frac{0.01}{1} = 0.01$ mol / L
$\therefore \quad pH = -\log H^+$
$= -\log [10^{-2}]$
$pH = 2.$

63. **(c)** $O = C = O$ e^- deficiency due to attraction of more
$\leftarrow \quad \rightarrow$
electronegative atom oxygen.
$AlCl_3$ electron deficient Al
Ag^+ electron deficient
$SnCl_4$, Sn electron deficient
Thus 4 species act as Lewis acid.

64. **(a)** Lewis acids $= AlCl_3, H^+, Co^{3+}, Mg^{2+}, BF_3$.

65. **(c)** A small change in temperature has little impact on the pH of a solution as the concentration of the H^+ ions does not change much.
Thus, statement I is correct.
$pH_1 = -\log [H^+]_1$ so if $[H^+]_2 = 100\,[H^+]_1$
$\Rightarrow \quad pH_2 = -\log [H^+]_2 = -\log (100[H^+]_1)$
$= -\log (100) + [-\log [H^+]_1]$
$= -2 + pH_1 = pH_1 - 2$
Thus, pH changes by two units.
Therefore, statement II is incorrect.

66. **(c)** Acidic solutions (pH < 7) are Black coffee, Lemon juice, Human Saliva, Tomato juice.

67. **(b)** Conjugate acid of $HCO_3^- = H_2CO_3$ (has an extra H^+)
Conjugate base of $HCO_3^- = CO_3^{2-}$ (has an less H^+)

68. **(a)** $pH = pK_a + \log\frac{[\text{salt}]}{[\text{Acid}]}$

$[\text{Acid}] = [CH_3\,COOH] = \frac{10 \times 1}{100} = 0.1\,M$

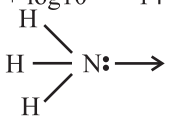
$[\text{Salt}] = [CH_3\,COO^-] = \frac{20 \times 0.5}{100} = 0.1\,M$

$\Rightarrow \quad pH = 4.76 + \log\frac{0.1}{0.1} = 4.76$

69. **(a)** $H_3O^+ \longrightarrow H^+ + \underset{\text{conjugate base}}{H_2O}$

70. **(c)** $\underset{0.01\,(N)}{Ca(OH)_2} \rightarrow Ca^{2+} + 2OH^-$
At 298K, $pH = 14 - pOH = 14 - [-\log (0.01)]$
$= 14 + \log 10^{-2} = 14 - 2 = 12$

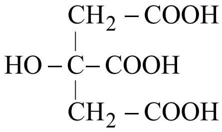
71. **(a)**



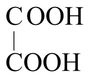
can donate this e^- pair.

72. **(c)**

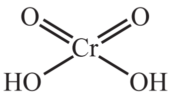
Citric acid:



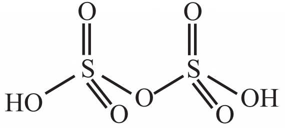
Oxalic acid



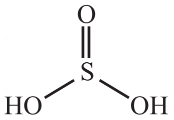
Chromic acid



Pyrosulfuric acid:



Sulfurous acid



73. **(a)** 118% oleum means 118 g f H_2SO_4 is obtained when we add 18 g of water in 100 g of oleum sample.
$2\,NaOH + H_2SO_4 \rightarrow Na_2\,SO_4 + 2H_2O$
2 moles of NaOH neutralise 1 mole of H_2SO_4
Moles of NaOH required to $= 2 \times \frac{118}{98}$
Complete neutralisation $= 2.4$ mol

74. **(d)** pH of water at 25°C is 7 as the dissociation of water generates 10^{-7}(M) of H^+ and OH^- each. As the temperature increases the dissociation of water increases. Therefore, the concentration of H^+ becomes more than 10^{-7} M. Hence, pH of the solution becomes < 7.0.

75. **(c)** Base have pH value more than seventh. Distilled H_2O is neutral, HCl is acidic, NaOH and NH_3 both are base but, NaOH is a strong base among the given options. Therefore, it has highest pH.

76. **(d)** The overall ionisation constants (K) will be product of ionisation constants of three steps, i.e.

$$K = K_1 \times K_2 \times K_3$$

$$H_3PO_4 \rightleftharpoons H^+ + H_2PO_4^-; K_1 = \frac{[H^+][H_2PO_4^-]}{[H_3PO_4]} \quad \text{...(i)}$$

$$H_2PO_4^- \rightleftharpoons H^+ + HPO_4^{2-}; K_2 = \frac{[H^+][HPO_4^-]}{[H_2PO_4^-]} \quad \text{...(ii)}$$

$$HPO_4^{2-} \rightleftharpoons H^+ + PO_4^{3-}; K_3 = \frac{[H^+][PO_4^{3-}]}{[HPO_4^{2-}]} \quad \text{...(iii)}$$

Adding Eqs. (i), (ii) and (iii), we get

$$H_3PO_4 \rightleftharpoons 3H^+ + PO_4^{3-} \quad \text{...(iv)}$$

$$K_1 \times K_2 \times K_3 = \frac{[H^+][H_2PO_4^-]}{[H_3PO_4]} \times \frac{[H^+][HPO_4^{2-}]}{[H_2PO_4^-]} \times \frac{[H^+][PO_4^{3-}]}{[HPO_4^{2-}]}$$

$$K_1 \times K_2 \times K_3 = \frac{[H^+]^3[PO_4^{3-}]}{[H_3PO_4]}$$

The above expression is the ionisation constant for overall reaction (iv).

77. **(b)** HF is weakest acid, thus, F^- is the strongest conjugate base.

78. **(b)** $K_a \times K_b = K_w = 10^{-14}$ (at 25°C)

$$\Rightarrow \quad K_a(NH_4^+).K_b(NH_3) = 10^{-14}$$

$$K_a(NH_4^+) \times 2.5 \times 10^{-5} = 10^{-14}$$

$$K_a(NH_4^+) = \frac{10^{-14}}{2.5 \times 10^{-5}} = 4 \times 10^{-10}$$

For pH of 0.01 M ammonia (NH_3)

$\therefore$ NH_4OH dissociate as :

At equilibrium $\underset{0.01(1-\alpha)}{NH_4OH} \rightleftharpoons \underset{0.01\alpha}{NH_4^+} + \underset{0.01\alpha}{OH^-}$

Also, $\alpha = \sqrt{\frac{K_b}{C}} = \sqrt{\frac{2.5 \times 10^{-5}}{0.01}} = 0.05$

and $[OH^-] = C\alpha = 0.01 \times 0.05 = 5.0 \times 10^{-4}$

$pOH = -\log[OH^-] = -\log(5.0 \times 10^{-4})$

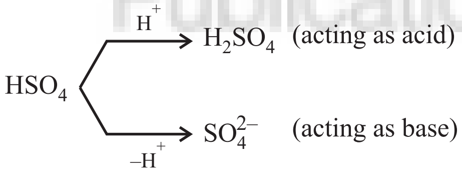
or, $pH = 14 - pOH = 14 - 3.3 \Rightarrow pH = 10.7$

79. **(d)**

Acid

$$H_2O + H_2O \longrightarrow H_3O^+ + OH^-$$ (Bronsted acid as well as base)

Base



80. **(a)** Order of acidic strength is: $HClO_4 > HClO_3 > HClO_2$

Thus, order of basic strength is: $ClO^-_2 > ClO^-_3 > ClO^-_4$

81. **(c)** Ammonium chloride NH_4Cl is a salt of weak base (NH_3) and strong acid (HCl). Hence, it is acidic in nature, i.e., pH < 7.

82. **(d)** Molecular mass of NaOH = 23 + 16 + 1 = 40

$$\text{Molarity} = \frac{\text{Mass of solute (in g)}}{\text{Molecular mass of the solute} \times \text{Volume of solution (in L)}} = \frac{0.04 \times 1000}{40 \times 100} = 10^{-2}$$

$OH^- = 10^{-2}$ mol/L

So, $pOH = -\log[OH^-]$

$pOH + pH = 14$ or $pH = 14 - 2 = 12$

83. **(a)** $HA \rightleftharpoons H_3O^+ + A^-$

0.5	0	0
0.5 – x	x	x

$[H^+] = c\alpha$

$= 0.5 \times (10^{-2})$

$= 0.005$

$\therefore$ $[H_3O]^+ = 0.005$

$[A^-] = 0.005$

$[HA] = 0.5 - x$

$= 0.5 - 0.005$

$= 0.495$

84. **(a)** degree of dissociation of weak acid (HA) = 1%

$\alpha = 1\%$

$\Rightarrow \frac{1}{100} = 0.01$

Dissociation constant (K_a) = $C\alpha^2$

where,

C(concentration) = 0.5 M

α (degree of dissociation) = 0.01

$\therefore K_a = 0.5 \times (0.01)^2$

$= 5 \times 10^{-1} \times (10^{-2})^2$

$= 5 \times 10^{-1} \times 10^{-4}$

$$\boxed{K_a = 5 \times 10^{-5}}$$

85. **(a)** $BOH \rightleftharpoons B^+ + OH^-$

$$K_b = \frac{[B^+][OH^-]}{[BOH]} = 1 \times 10^{-6}$$

$pH = 10 \Rightarrow pOH = 14 - 10 = 4 \Rightarrow [OH^-] = 10^{-4}$ M.

Now, $K_b = \frac{C\alpha^2}{1-\alpha} = \frac{[OH^-]\alpha^2}{1-\alpha}$

$$\Rightarrow \frac{K_b}{[OH^-]} = \frac{\alpha^2}{1-\alpha} \cong \alpha^2 = \frac{10^{-6}}{10^{-4}} = 10^{-2}$$

$\Rightarrow$ $\alpha = 10^{-1}$ or 0.1 or 10%

86. **(d)** Larger the K_a values, higher is the acidic strength.

87. **(c)** According to Henderson's equation-

$$pH = pK_a + \log\frac{[\text{salt}]}{[\text{acid}]} = 4.75 + \log\frac{0.5}{0.5} = 4.75$$

88. **(d)** Given, $[HCl] = 10^{-8}$ M

$[H^+] = 10^{-8} + 10^{-7} = 10^{-7}(0.1 + 1)$

10^{-8} M is a very low concentration and hence the protons from water are also considered.

89. **(b)** HF {pK_a = 3.1}

HCl {pK_a = – 6.0}

HBr {pK_a = – 9.0}

HI {pK_a = – 9.5}

Acidic nature of hydrogen halides incerease down the group –17.

Hence k_a = HF < HCl < HBr < HI

pK_a = HF > HCl > HBr > HI

90. **(b)** With an increase in temperature, there is an increase in dissociation of H_2O which gives rise to an increase ion the concentration of H^+ and OH^- ions. Hence, the ionic product of water increases with an increase in temperature.

91. **(d)** Applying, Volume of HCl solution (V_1) = 30 mL

Volume of sodium carbonate solution (Na_2CO_3)

(V_2) = 20 mL

Concentration of Na_2CO_3 solution (M_2) = 0.1 M

Volume of NaOH solution (V_3) = 30.0 mL
Concentration of NaOH solution (M_3) = 0.2 M
$N_1V_1\,(Na_2CO_3) \Rightarrow M_1 \times n_f \times v_1\,(HCl) = M_2 \times n_f \times v_2\,(Na_2CO_3)$
$N = M \times nf_i$ n_f of HCl = 1 n_f of Na_2CO_3 = 2
$M_1 \times 30 = 20 \times 2 \times 0.1 \quad \Rightarrow \quad M_1 = 0.133$
Now, volue of HCl solution required to neutralise 30 mL of 0.2 M NaOH can be calculated by
Applying $M_1V_1 = M_3V_3$
$0.133 \times V_1 = 0.2 \times 30 \quad \Rightarrow \quad V_1 = 45.1$ mL

92. **(a)** pH of the solution of salts of weak acid e.g. acetic acid and weak base e.g. dimethylamine can be calculated as

$$pH = 7 + \frac{1}{2}[pK_a - pK_b]$$

Here, $pK_a = 4.76$, $pK_b = 3.26$

$$= 7 + \frac{1}{2}[4.76 - 3.26] \Rightarrow 7 + \frac{1}{2}[1.50] \Rightarrow 7 + 0.75 = 7.75$$

93. **(b)** $Ba_3(PO_4)_2 \longrightarrow 3Ba^{2+} + 2PO_4^{3-}$

$$\text{Solubility of Barium} = \left(\frac{\frac{x}{M}}{10^{-1}}\right)$$

$$= \left(\frac{x}{M} \times 10\right) \text{mol / L}$$

$$Ba_3(PO_4)_2 \longrightarrow 3Ba^{2+} + 2PO_4^{3-}$$
$$\left(3\frac{x}{M} \times 10\right)\left(2\frac{x}{M} \times 10\right)$$

$$K_{sp} = \left(\frac{3x}{M} \times 10\right)^3 \times \left(\frac{2x}{M} \times 10\right)^2$$

$$= 27 \times \left(\frac{x}{M}\right)^3 \times 10^3 \times 4 \times \left(\frac{x}{M}\right)^2 \times 10^2$$

$$= 108 \times \left(\frac{x}{M}\right)^5 \times 10^5$$

$$K_{sp} = 1.08 \times \left(\frac{x}{M}\right)^5 \times 10^7$$

$\therefore$ a = 5
b = 7

94. **(c)** Solubility product (K_{sp}) of Ax = 10^{-10}
$[H^+] = [X^-] = 10^{-1}$ M
$[K_{sp}] = [H^+]\,[X^-]$
$10^{-10} = [10^{-1}] \times [S]$

$$\therefore \quad S = \frac{10^{-10}}{10^{-1}} = 10^{-9}$$

95. **(d)** Basic buffer solution is made by mixing a weak base with strong acid.

96. **(c)** A solution that has the concentration of Ag^+ or Cl^- greater than or equal to the solubility of Ag^+ or Cl^- from the value of K_{sp}, will precipitate out.
$K_{sp} = [Ag^+]\,[Cl^-] = (s)\,(s) = s^2 = 1 \times 10^{-10}\ M^2$
$\Rightarrow \quad s = 10^{-5}$ M
Thus, (iv) 1L of 10^{-3} M $AgNO_3$ and (v) 1L of 10^{-5} M NaCl will precipitate

97. **(b)** $KOH \rightarrow K^+ + OH^-$
$\qquad\qquad 0.1 \qquad 0.1$
$Cd(OH)_2 \rightleftharpoons Cd^{2+} + 2\,OH^-$
$\qquad\qquad S \qquad\quad 2S$
Solubility product $k_{sp} = [S]^1\,[2S + 0.1]^2$
$k_{sp} = [S]^1\,[0.1]^2 \;(\because 0.1 >>> 2S)$
$2.5 \times 10^{-14} = [S] \times 0.01$
$[S] = 2.5 \times 10^{-12}$
$[S] = 25 \times 10^{-13} = x \times 10^{-y}$
So, x = 25, y = 13.

98. **(c)** A buffer solution is formed when we have a mixture of a weak acid or base and its corresponding salt with a strong base or acid.
A 1 : 1 mixture of NaOH and CH_3 COOH will have unequal concentrations of CH_3COOH and CH_3COONa so it will not act as a buffer.

99. **(b)** Dissociation of electrolytes increases with increase in temperature: So, $K_w = [H^+][OH^-]$ also increases.
$\therefore$ pH of the buffer solution increases.

100. **(c)** $K_{sp} = 1 \times 10^{-10}$
$MCl \rightarrow M^+ + Cl^-$
In NaCl soln., total $[Cl^-] = (0.1 + s)M \;\; \therefore \;\; s(s + 0.1) = 10^{-10}$
or, $0.1s = 10^{-10}$ $\qquad [\because s + 0.1 \because 0.1 \text{ as } s <<< 0.1]$

$$\text{or,} \quad s = \frac{10^{-10}}{0.1} = 10^{-9}\,M$$

101. **(d)** Smaller the value of K_{sp} of the salt, quicker is the formation of percipitate.
Sequence of precipitation : HgS, CdS, ZnS, NiS.

102. **(b)** A - II, B - I, C - IV, D - III

103. **(b)** Given, $[K_{sp}]$ of AgBr = 5×10^{-10}
[NaBr] = 0.2 M; $[Na^+] = [Br^-] = 0.2$ M
Now, $[K_{sp}] = [Ag^+]\,[Br^-]$
$s(0.2) = 5^5 \times 10^{-10} \quad (s << 0.2)$

$$s = \frac{5 \times 10^{-10}}{0.2} = 25 \times 10^{-10}\ M$$

104. **(b)** For H_2SO_4
$100 \times 0.5 = 50$ m mol $= 50 \times 98 \times 10^{-3}$ g
$2NaOH + H_2SO_4 \longrightarrow Na_2SO_4 + 2H_2O$
$\;\; 2\times40g \qquad 98g$
$98g \equiv 2 \times 40g$

$$50 \times 98 \times 10^{-3} \equiv \frac{2 \times 40}{98} \times 50 \times 98 \times 10^{-3} = x = 4g \text{ NaOH}$$

Mass of Na_2SO_4 = 100 – 4g = 96g.

105. **(d)** A – (3), B – (4), C – (2) and D – (5).
Greater the value of $[H^+]$ ion, greater the value of ionisation constant, K_a and *vice-versa*.

106. **(a)** If [Conjugate acid] = [Base]
According to Handerson Hassel balch equation for buffer solution.

$$pOH = pK_b + \log\frac{[\text{Conjugate acid}]}{[\text{Base}]}$$

Then, [pOH = pK_b]

So, pOH of the buffer solution is same as pK_b of acid.

107. (c) $\underset{}{Ni(OH)_2} \rightleftharpoons \underset{s\ \text{mol/L}}{Ni^{2+}} + \underset{2s\ \text{mol/L}}{2OH^-}$

$\Rightarrow K_{sp} = [Ni^{2+}]\,[OH^-]^2 = s \times (2s)^2 = 4s^2$

$2 \times 10^{-15}\ (\text{mol/L})^3 = 4s^3$

$\Rightarrow s = \left(\dfrac{2}{4} \times 10^{-15}\right)^{1/3} = 7.9 \times 10^{-6} = 8 \times 10^{-6}$ mol/L

$[OH^-] = 2s = 2 \times 8 \times 10^{-6}$ mol/L

pOH = 6 – log 16 = 4.983 $\simeq$ 5

$\therefore$ pH = 14 – pOH = 14 – 5 = 9 at 25°C.

108. (c) Concentration of salt (NH_4Cl) = 0.01 M

Concentration of base (NH_4OH) = 0.1 M

$pK_b = 5$

$pOH = pK_b + \log \dfrac{[\text{Salt}]}{[\text{Base}]} \Rightarrow pOH = 5 + \log \dfrac{0.01}{0.1} = 4$

= pH = 14 – pOH = 14 – 4 = 10

109. (a) Given, K_b of $NH_4OH = 5.00 \times 10^{-10}$

$K_w = 10^{-14}$ at 298 K

$$K_h = \frac{K_w}{K_b} = \frac{10^{-14}}{5 \times 10^{-10}} = 2 \times 10^{-5}$$

110. (b) $A_2B \rightarrow \underset{2s}{2^+} + \underset{s}{B^{2-}}$

$sp = [A^+]\,[B^{2-}] = (2s)^2\,(s) = 4s^3$

Given, K_{sp} for $A_2B = 3.2 \times 10^{-11}$ $\therefore$ $3.2 \times 10^{-11} = 4s^3$

$s = \left(\dfrac{3.2}{4} \times 10^{-11}\right)^{1/3} = 8 \times 10^{-12}$

Thus $s = 2 \times 10^{-4}$

111. (b) Solubility of AgCl in 0.1 M KCl solution will be minimum because of common ion effect of Cl^-.

112. (d) $Ni(OH)_2 \rightleftharpoons \underset{s\ \text{mol/L}}{Ni^{2+}} + \underset{2s\ \text{mol/L}}{2OH^-}$

$K_{sp} = (s)(2s)^2$

$K_{sp} = 4s^3$

$s = \sqrt[3]{\dfrac{K_{sp}}{4}} = \sqrt[3]{\dfrac{4 \times 10^{-15}}{4}} = 1 \times 10^{-5}$ mol/L

113. (c) Dissolution $NH_4Cl \rightarrow NH_4^+ + Cl^-$

Cl^- will Cl^- (aq) and will not undergo hydrolysis.

Hydrolysis $NH_4^- + H_2O \rightleftharpoons NH_3 + H_3O^+$

114. (a) H_2CO_3 is a weak dibasic acid. It dissociates in following way

$$H_2CO_3\,(aq) + H_2O(l) \rightleftharpoons HCO_3^-\,(aq) + H_3O^+\,(aq.)$$

$$HCO_3^-\,(aq) + H_2O(l) \rightleftharpoons CO_3^{2-}\,(aq) + H_3O^+\,(aq.)$$

H_2CO_3 / HCO_3^- act as buffer system and helps to maintain pH of blood between 7.26 to 7.42.

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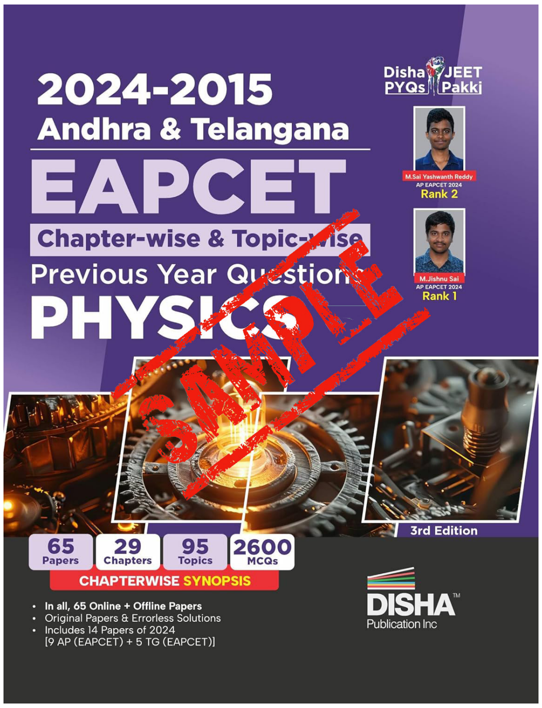

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CONTENTS

CLASS XII B1 - B224

Hints & Solutions (Class XII) B109 - B224

6

System of Particles and Rotational Motion

CHAPTER SYNOPSIS

- For a discrete system of N particles, the position vector of centre of mass, $\vec{R}_{cm} = \frac{\sum_{i=1}^{N} m_i r_i}{M}$
- For continuous mass distribution, $\vec{R}_{cm} = \frac{1}{M}\int \vec{r}\,dm$ where, dm = mass of small element located at position $\vec{r}$
- Velocity of CM of a two particles system, $\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$; $\vec{a}_{cm} = \frac{d\vec{V}_{cm}}{dt}$
- Net force is the force effective for C.M. but point of action of net force is not necessarily at the centre of mass.
- Point of action of resultant force is that point about which resultant torque due to individual forces becomes zero.
- If any physical quantity is the linear function of distance then only that physical quantity can be solved by considering total mass of system as point mass situated at the position of C.M.
- Freely placed system whenever rotates, it rotates, about its C.M.
- State of translatory motion of C.M. remains unaffected by internal forces.
- If C.M. of system is initially at rest and system is set free to move under the influence of resultant force of fixed line of action then system as a whole may move in translatory as well as in rotatory motion but C.M. of system will move in translatory motion only in the direction of resultant force.
- At the moment of minimum and maximum separation two bodies are at rest (along the line joining them) with respect to each other as well as with respect to C.M.
- The equations of motion for a body in rotational motion under constant acceleration.

  (i) $\omega_2 = \omega_1 + \alpha t$

  (ii) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

  (iii) $\omega_2^2 - \omega_1^2 = 2\alpha\ t$

  (iv) $\theta_{nth} = \omega_1 + \frac{1}{2}\alpha\ (2n-1)$
- The angular displacement of a body (which rotates with uniform angular acceleration) in the 1st, 2nd, 3rd second etc. will be in the ratio of 1 : 3 : 5 : 7........(2*n* – 1). i.e., $\theta_n \propto (2n-1)$.
- Moment of inertia of a body about the given axis of rotation,

  $I = m_1 r^2_1 + m_2 r_2^2 \ldots\ldots\ m_n r_n^2 = \sum_{i=1}^{n} m_i r_i^2$
- Perpendicular axes theorem for linear body. $I_z = I_x + I_y$

  Perpendicular axis theorem for three dimensional body $2I_0 = I_x + I_y + I_z$
- Theorem of parallel axes : $I = I_{cm} + Mr^2$
- Radius of gyration k is given by $I = Mk^2$or, $k = \sqrt{\frac{I}{M}}$

  When all the particles are of same mass $k = \sqrt{\frac{r_1^2 + r_2^2 + \ldots . r_n^2}{n}}$

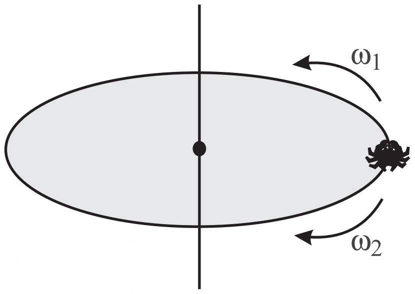
- Torque, $\vec{\tau} = (\vec{r} \times \vec{F}).\hat{n}$ where $\hat{n}$ is a unit vector, $|\vec{\tau}| = rF\sin\theta$
- Angular momentum $L = p\ (r\sin\theta) = \vec{r} \times \vec{p}$ (always applicable), $\vec{L} = I\vec{\omega}$ (only in rotatory motion)
- $\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$; $\vec{L} = I\vec{\alpha}$ (for rotatory motion only) ; $\frac{dL}{dt} = \frac{d}{dt} + \omega\frac{dI}{dt}$

  If I is constant $\frac{d\vec{L}}{dt} = I\vec{\alpha}$; torque, $\vec{\tau} = I\vec{\alpha}$ (applicable for rotatory motion with constant M.I.)

  If $\vec{\tau} = 0$; $\frac{d\vec{L}}{dt} = 0$; $d\vec{L} = 0$; $\vec{L}$ = constant (conservation of angular momentum)
- Work done by a torque w = $\tau\ \theta$
- Power of a torque p = $\tau\ \omega$
- Initially whole of the system is at rest and then start to rotate about common axis under the influence of torque due to internal forces only.

  $I_1\theta_1 = I_2\theta_2$; $I_1\omega_1 = I_2\omega_2$; $I_1\alpha_1 = I_2\alpha_2$; $\frac{\theta_1}{\theta_2} = \frac{\omega_1}{\omega_2} = \frac{\alpha_1}{\alpha_2} = \frac{I_2}{I_1}$

  Angular displacement (θ), angular velocity (ω) and angular acceleration (α) are real and have numerical values only. Direction of rotation must be opposite.



- Law of conservation of angular momentum; In the absence of any external torque L = Iω = constant or, $I_1\omega_1 = I_2\omega_2$
- Angular impulse (Axial vector) : A.I. = $\vec{\tau}\ (dt)$ (applicable only for constant torque)

  If torque is variable, A.I. = $\int_{t_1}^{t_2} \vec{\tau}\ (dt)$; $\frac{d\vec{L}}{dt} = \vec{\tau}$; $\int d\vec{L} = \int \vec{\tau}\ dt$

  Net change in angular momentum = net angular impulse
- Angular momentum of system having translatory as well as rotatory motion

  $\vec{L}_0 = \vec{L}_c + \vec{r}_{CM} \times \vec{p}_{CM}$; $L_0 = I_c\omega + (Mv_{CM})d$

  $(Mv_{CM})d \Rightarrow$ orbital angular momentum, $I_c\omega \Rightarrow$ spin angular momentum

  $I_c\omega$ and $(Mv_{CM}d)$ will be added or subtracted according to same or opposite sense of rotation.

  $L_x = L_e + (p_{CM})d$

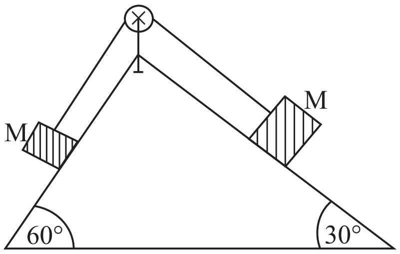
  Differentiating the equation with respect to time $\tau_x = \tau_c + (F_{C.M.})d$
- **26.** Rotational kinetic energy K. $E_{rot} = \frac{1}{2}I\omega^2$
- **27.** If a body is released from rest on rough inclined plane, then for pure rolling $\mu_r \geq \frac{n}{n+1}\tan\theta\left(I_c = nmr^2\right)$

  Rolling with sliding $0 < \mu_s < \left(\frac{n}{n+1}\right)\tan\theta$; $\frac{g\sin\theta}{n+1} < a < g\sin\theta$
- **28.** In case of pure rolling (without slipping) motion of a body, velocity $v = \left[\frac{2gh}{1+\frac{k^2}{R^2}}\right]^{½}$

  Acceleration $a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$; Time taken $t = \left[\frac{2s\left(1+\frac{k^2}{R^2}\right)}{g\sin\theta}\right]^{½}$

Topic 1 Centre of Mass, Centre of Gravity & Principle of Moments

1. Two blocks of equal masses are tied with a light string passing over a massless pulley (Assuming frictionless surfaces) acceleration of centre of mass of the two blocks is ($g = 10\ \text{ms}^{-2}$) **[AP/May 22, 2024 (II)]**



(a) $\frac{5(\sqrt{3}-1)}{2}$

(b) $\frac{5(\sqrt{3}-1)}{2\sqrt{2}}$

(c) $\frac{5(\sqrt{3}+1)}{2\sqrt{2}}$

(d) $\frac{5(\sqrt{3}-1)}{\sqrt{2}}$

2. Two blocks of masses m and 2m are connected by a massless string which passes over a fixed frictionless pulley. If the system of blocks is released from rest, the speed of the centre of mass of the system of two blocks after a time of 5.4 s is **[AP/May 19, 2024 (I)]**

(Acceleration due to gravity = $10\ \text{ms}^{-2}$)

(a) $6\ \text{ms}^{-1}$ (b) $8\ \text{ms}^{-1}$ (c) $4\ \text{ms}^{-1}$ (d) $12\ \text{ms}^{-1}$

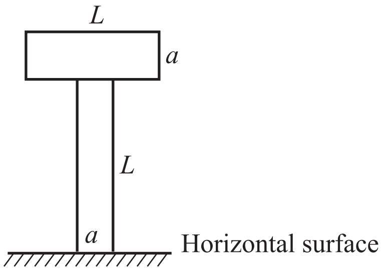
3. A metre scale is balanced on a knife edge as its centre. When two coins, each of mass 9 g are kept one above the other at the 10 cm mark, the scale is found to be balanced at 35 cm. The mass of the metre scale is **[TS/May 11, 2024 (I)]**

(a) 15 g (b) 30 g (c) 45 g (d) 60 g

4. A circular plate of radius r is removed from a uniform circular plate P of radius $4r$ to form a hole. If the distance between the centre of the hole formed and the centre of the plate P is $2r$, then the distance of the centre of mass of the remaining portion from the centre of the plate P is **[TS/May 10, 2024 (II)]**

(a) $\frac{r}{3}$ (b) $\frac{r}{15}$ (c) $\frac{2r}{15}$ (d) $2r$

5. An alphabet 'T' made of two similar thin uniform metal plates of each length 'L' and width 'a' is placed on a horizontal surface as shown in the figure. If the alphabet is vertically inverted, the shift in the position of its centre of mass from the horizontal surface is **[TS/May 10, 2024 (I)]**



(a) $\frac{L-a}{2}$

(b) $\frac{a-L}{2}$

(c) $L-\frac{a}{2}$

(d) $\frac{L}{2}-a$

6. Three particles A, B and C of masses m, $2m$ and $3m$ are moving towards north, south and east respectively. If the velocities of the particles A, B and C are $6\ \text{ms}^{-1}$, $12\ \text{ms}^{-1}$ and $8\ \text{ms}^{-1}$ respectively, then the velocity of the centre of mass of the system of particles is **[TS/May 9, 2024 (II)]**

(a) $7\ \text{ms}^{-1}$ (b) $5\ \text{ms}^{-1}$

(c) $26\ \text{ms}^{-1}$ (d) $8\ \text{ms}^{-1}$

7. Four identical particles each of mass 'm' are kept at the four corners of a square of side 'a'. If one of the particles is removed, the shift in the position of the centre of mass is **[TS/May 9, 2024 (I)]**

(a) $\sqrt{2}a$ (b) $\frac{3a}{\sqrt{2}}$ (c) $\frac{a}{\sqrt{2}}$ (d) $\frac{a}{3\sqrt{2}}$

8. Which of the following statements regarding centre of mass is NOT true? **[AP/May 18, 2023 (I)]**

(a) For two particles of equal mass, the centre of mass lies exactly midway between them

(b) For three non–linear particles of equal mass, the centre of mass coincides with the centroid of the triangle formed by the particles

(c) When the total external force on a system is zero, the velocity of the centre of mass of the system remains constant.

(d) For two particles of different masses, the centre of mass of the particles is nearer to the particle of lesser mass.

9. A particle of mass 10 g is moving towards east with a velocity of $10\ \text{ms}^{-1}$ and another particle of mass 15 g is moving towards north with a velocity of $5\ \text{ms}^{-1}$. The magnitude of the velocity of the centre of mass of the system of the two particles is **[AP/May 18, 2023 (I)]**

(a) $5\ \text{ms}^{-1}$ (b) $10\ \text{ms}^{-1}$

(c) $15\ \text{ms}^{-1}$ (d) $7.5\ \text{ms}^{-1}$

10. Two particles of masses 1 g and 2 g move towards each other with velocities $10\ \text{ms}^{-1}$ and $20\ \text{ms}^{-1}$ respectively. The velocity of the centre of mass of the system of the two particles is **[AP/May 16, 2023 (II)]**

(a) $5\ \text{ms}^{-1}$ (b) $10\ \text{ms}^{-1}$

(c) $15\ \text{ms}^{-1}$ (d) $20\ \text{ms}^{-1}$

11. A circular plate A of radius 1.5r is removed from one edge of a uniform circular plate B of radius 2r. The distance of centre of mass of the remaining portion from the centre of the plate B is **[AP/May 16, 2023 (I)]**

(a) $\frac{5r}{12}$ (b) $\frac{9r}{14}$ (c) $\frac{3r}{4}$ (d) $\frac{7r}{8}$

12. Two particles of masses 5 g and 3 g are separated by a distance of 40 cm. The centre of mass of the system of these two particles **[AP/May 15, 2023 (II)]**

(a) lies at a distance of 15 cm from 5 g particle

(b) lies at a distance of 25 cm from 5 g particle

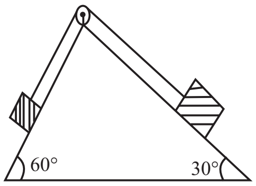
(c) lies at a distance of 10 cm from 3g particle

(d) lies at the mid point of the line joining the two particles

13. Two blocks of masses 2 kg and 1 kg are tied to the ends of a string which passes over a light frictionless pulley. The blocks are held at rest at the same horizontal level and then released suddenly. The distance traversed by their centre of mass in 2 seconds is **[TS/May 13, 2023 (II)]**
(Acceleration due to gravity = 10 ms^{-2})

(a) 1.42 m (b) 2.22 m (c) 3.12 m (d) 3.33 m

14. Two blocks of equal masses are tied to the ends of a light string. The string passes over a mass less pulley fixed on frictionless surface as shown in the figure. The acceleration of the centre of mass of the blocks is
(g – acceleration due to gravity) **[TS/May 13, 2023 (I)]**

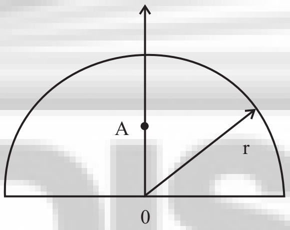


(a) $\left(\frac{\sqrt{3}-1}{4\sqrt{2}}\right)g$ (b) $\left(\frac{\sqrt{3}+1}{4\sqrt{2}}\right)g$

(c) $\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)g$ (d) $\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)g$

15. The centre of mass of a homogenous semi-circular plate of radius r is located at A as shown in the figure. The distance OA is

[AP/July 5, 2022 (II)]



(a) $\frac{r}{3}$ (b) $\frac{2r}{3}$ (c) $\frac{r}{2}$ (d) $\frac{4r}{5}$

16. Masses $m\left(\frac{1}{3}\right)^{N}\frac{1}{N}$ are placed at x = N, when N = 2, 3, 4, ... ∞. If the total mass of the system is M, then the centre of mass is **[AP/July 4, 2022 (I)]**

(a) $\frac{1}{6}\frac{m}{M}$ (b) $\frac{1}{5}\frac{m}{M}$ (c) $\frac{1}{3}\frac{m}{M}$ (d) $\frac{1}{2}\frac{m}{M}$

17. A metre stick is balanced on a knife edge at its centre. When four coins, each of mass 2g are put one on top of the other at 10.0 cm mark, the stick it found to be balanced at 46.0 cm mark. The mass of the metre stick is **[TS/July 19, 2022 (II)]**

(a) 66 g (b) 60 g (c) 72 g (d) 18 g

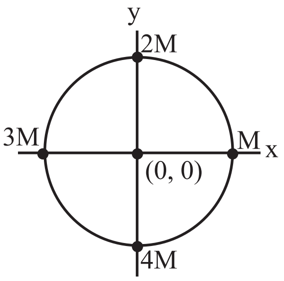
18. Four masses are arranged along a circle of radius 1 m as shown in the figure. The centre of mass of this system of masses is at **[TS/July 18, 2022 (I)]**

(a) $-\frac{1}{5}\hat{i}-\frac{1}{5}\hat{j}$

(b) $\frac{1}{5}\hat{i}+\hat{j}$

(c) $\hat{i}-\frac{1}{5}\hat{j}$

(d) $\frac{1}{5}\hat{i}+\frac{1}{5}\hat{j}$



19. The sum of moments of all the particles in a system about its centre of mass is always **[AP/Aug. 19, 2021 (I)]**

(a) minimum (b) zero

(c) maximum (d) infinite

20. A mass *m* is in rest on an inclined plane of mass *M* which is further resting on a smooth horizontal plane. Now, if the mass *m* starts moving under gravity, the position of centre of mass of system will **[AP/Sept. 18, 2020 (I)]**

(a) remain unchanged

(b) change along the horizontal direction

(c) move up in vertical direction

(d) move down in the vertical direction and changes along the horizontal

21. A bullet of mass 25 g moves horizontally at a speed of 250 m/s is fired into a wooden block of mass 1 kg suspended by a long string. The bullet crosses the block and emerges on the other side. If the centre of the mass of the block rises through a height of 20 cm. The speed of the bullet as it emerges from the block is (take, g = 10 m/s^2) **[TS/Sept. 10, 2020 (I)]**

(a) 300 m/s (b) 220 m/s

(c) 150 m/s (d) 170 m/s

22. A circular hole of radius 3 cm is cut out from a uniform circular disc of radius 6 cm. The centre of the hole is at 3 cm, from the centre of the original disc. The distance of centre of gravity of the resulting flat body from the centre of the original disc is **[TS/Sept. 10, 2020 (I)]**

(a) 0.5 cm (b) 1 cm

(c) 1.5 cm (d) 0.75 cm

23. A ball of mass 100 g is dropped at a time $t = 0$. A second ball of mass 200 g is dropped from the same point at $t = 0.2$ s. The distance between the centre of mass of two balls and the release point at $t = 0.4$ s is
(Assume, $g = 10$ m/s^2) **[TS/Sept. 9, 2020 (I)]**

(a) 0.4 m (b) 0.5 m (c) 0.6 m (d) 0.8 m

24. Three identical spheres each of diameter $2\sqrt{3}$ m are kept on a horizontal surface such that each sphere touches the other two spheres. If one of the sphere is removed, then the shift in the position of the centre of mass of the system is **[AP/Apr. 21, 2019 (I)]**

(a) 12 m (b) 1 m (c) 2 m (d) $\frac{3}{2}$m

25. A system of two particles is having masses m_1 and m_2. If the particle of mass m_1 is pushed towards the centre of mass of particles through a distance *d*, by what distance the particle of mass m_2 should be moved, so as to keep the centre of mass of particles at the original position?
[TS/May 12, 2023 (I) TS/2016]

(a) $\left(\frac{m_1}{m_1+m_2}d\right)$ (b) d

(c) $\left(\frac{m_1}{m_2}\right)d$ (d) $\left(\frac{m_2}{m_1}\right)d$

Topic 2 Angular Displacement, Velocity and Acceleration

26. A block (P) is rotating in contact with the vertical wall of a rotor as shown in figures A, B, C. The relation between angular velocities ω_A, ω_B and ω_C so that block does not slide down. **[AP/May 20, 2024 (II)]**

($R_A < R_B < R_C$ radii)

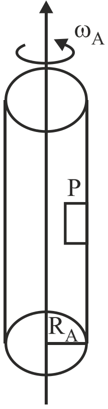


Fig A

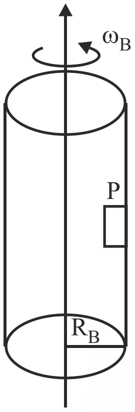


Fig B

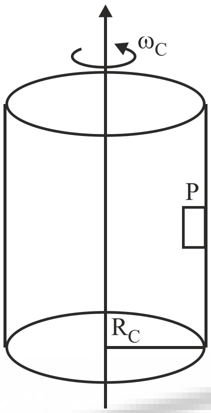


Fig C

(a) $\omega_A < \omega_B < \omega_C$
(b) $\omega_A = \omega_B = \omega_C$
(c) $\omega_C < \omega_B < \omega_A$
(d) $\omega_C = \omega_A + \omega_B$

27. A wheel of angular speed 600 rev/min is made to slow down at a rate of 2 rad s^{-2}. The number of revolutions made by the wheel before coming to rest is **[AP/May 18, 2024 (I)]**

(a) 157 (b) 314
(c) 177 (d) 117

28. A body rotating with uniform acceleration about its geometrical axis makes 8 rotations in the first 2 seconds. The number of rotations the body makes in the next 3 seconds is **[AP/May 17, 2023 (I)]**

(Initially the body is at rest)

(a) 50 (b) 25 (c) 42 (d) 21

29. A flywheel is rotating at a rate of 150 rev/minute. If it slows at constant retardation of π rads^{-2}, then the time required for the wheel to come to rest is **[AP/May 16, 2023 (I)]**

(a) 2.5 s (b) 5 s (c) 4 s (d) 6 s

30. The angular momentum of a solid cylinder rotating about its geometric axis with angular speed 40 rad s^{-1} is 2 kg m^2 s^{-1}. If the radius of the cylinder is 10 cm, the mass of the cylinder is **[AP/May 16, 2023 (I)]**

(a) 2 kg (b) 5 kg (c) 8 kg (d) 10 kg

31. The angular speed of a rigid body rotating about a fixed axis is (8 – 2t) rad s^{-1}. The angle through which the body rotates before it comes to rest is **[AP/May 15, 2023 (II)]**

(a) 8 rad (b) 12 rad
(c) 16 rad (d) 20 rad

32. A fan is rotating with an angular speed 300 rpm. The fan is switched off, and it takes 80 s to come to rest. Assuming constant angular deceleration, the number of revolutions made by the fan before it comes to rest is **[AP/July 6, 2022 (I)]**

(a) 400 (b) 200 (c) 300 (d) 314

33. An object undergoing simple harmonic motion takes 0.5 s to travel from one point of zero velocity to the next such point. The angular frequency of the motion is **[AP/July 4, 2022 (I)]**

(a) π rad s^{-1} (b) 2π rad s^{-1}
(c) 3π rad s^{-1} (d) $\frac{\pi}{2}$ rad s^{-1}

34. A spherical bob of mass 250g is attached to the end of a string having length 50 cm. The bob is rotated on a horizontal circular path about a vertical axis. The maximum tension that the string can bear is 72N. The maximum possible value of angular velocity of bob (in rad/s) is **[TS/July 20, 2022 (II)]**

(a) 18 (b) 24
(c) 28 (d) 32

35. A flywheel starts from rest and rotates at a constant acceleration of 2 rad s^{-2}. The number of revolutions that it makes in first 10 s is **[AP/Sept. 17, 2020 (I)]**

(a) 16 (b) 24
(c) 32 (d) 8

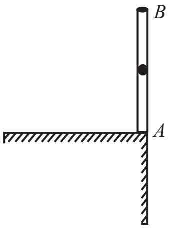
36. Consider a wheel rotating around a fixed axis. If the rotation angle θ varies with time as $\theta = at^2$, then the total acceleration of a point *A* on the rim of the wheel is (*v* being the tangential velocity) **[TS/May 5, 2018 (I)]**

(a) $\frac{v}{t}\sqrt{1+4a^2t^4}$ (b) $\frac{v}{t}$
(c) $\frac{v}{t}(1+4a^2t^4)$ (d) $\sqrt{(1+4a^2t^4)}$

37. The deceleration of a car traveling on a straight highway is a function of its instantaneous velocity *v* given by $\omega = a\sqrt{v}$, , where *a* is a constant. If the initial velocity of the car is 60 km/h, the distance of the car will travel and the time it takes before it stops are **[TS/2017]**

(a) $\frac{2}{3}$ m, $\frac{1}{2}$ s (b) $\frac{3}{2a}$ m, $\frac{1}{2a}$ s
(c) $\frac{3a}{2}$ m, $\frac{a}{2}$ s (d) $\frac{2}{3a}$ m, $\frac{2}{a}$ s

38. A rod *AB* of length 1 m is placed at the edge of a smooth table as shown. It is hit horizontally at point *B*. If the displacement of centre of mass in 1s is $5\sqrt{2}$ m, then the angular velocity of the rod is (Take, $g = 10$ ms^{-2})



[AP/Aug. 23, 2021 (I)]

(a) 30 rads^{-1} (b) 20 rads^{-1} (c) 10 rads^{-1} (d) 5 rads^{-1}

39. A ballet dancer suddenly folds her outstretched arms. Her angular velocity **[AP/Sept. 18, 2020 (I)]**

(a) increases (b) decreases

(c) remains the same (d) may increase or decrease

40. A flywheel of mass 1 kg and radius vector $\left(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}\right)$ m is at rest. When a force $\left(3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)$ N acts on it tangentially, it can rotate freely. Then, its angular velocity after 4.5 s is **[AP/Apr. 21, 2019 (I)]**

(a) $\frac{2}{9}\sqrt{261}$ rad s^{-1} (b) $\frac{3}{2}\sqrt{261}$ rad s^{-1}

(c) $\sqrt{261}$ rad s^{-1} (d) $\frac{5}{9}\sqrt{261}$ rad s^{-1}

41. A rigid metallic sphere is spinning around its own axis in the absence of external torque. If the temperature is raised, its volume increases by 9%. The change in its angular speed is

(a) increases by 9% (b) decreases by 9%

(c) increases by 6% (d) decreases by 6%

42. A uniform disc of mass 100 kg and radius 2 m is rotating at 1 rad/s about a perpendicular axis passing through its centre. A boy of mass 60 kg standing at the centre of the disk suddenly jumps to a point which is 1 m from the centre of the disc. The final angular velocity of the boy (in rad/s) is **[TS/May 4, 2018 (II)]**

(a) 0.77 (b) 0.5 (c) 41 (d) 2

Topic 3 Torque, Couple and Angular Momentum

43. A solid sphere of mass 50 kg and radius 20 cm is rotating about its diameter with an angular velocity of 420 rpm. The angular momentum of the sphere is **[AP/May 21, 2024 (I)]**

(a) 8.8 Js (b) 70.4 Js (c) 17.6 Js (d) 35.2 Js

44. A solid sphere of mass 2 kg is rolling without slipping on a horizontal surface with a velocity 5 ms^{-1}. The rotational kinetic energy of the sphere is **[AP/May 19, 2023 (I)]**

(a) 25 J (b) 12.5 J

(c) 10 J (d) 20 J

45. A constant torque acting on a uniform circular wheel changes its angular momentum from A_o to $4A_o$ in 4 seconds. The magnitude of the torque is **[TS/May 14, 2023 (II)]**

(a) $\frac{3A_o}{4}$ (b) A_o (c) $4A_o$ (d) $12A_o$

46. A particle performs uniform circular motion with an angular momentum L. If the frequency of the particle's motion is doubled and its kinetic energy is halved, then its angular momentum becomes **[TS/May 14, 2023 (II)]**

(a) 2L (b) 4L (c) $\frac{L}{2}$ (d) $\frac{L}{4}$

47. A particle of mass 'm' is moving along a line y = x + a with a constant velocity 'v'. The angular momentum of the particle about the origin is **[TS/May 13, 2023 (II)]**

(a) mva (b) $mva\sqrt{2}$ (c) $\frac{mva}{\sqrt{2}}$ (d) $\frac{mva}{x\sqrt{2}}$

48. A body is located at (1, 1, 1) m and experiences a force of 2 N in the direction $\hat{i}+\hat{j}$. The torque acting on the body in N-m is **[AP/July 8, 2022 (I)]**

(a) $\left(-\sqrt{2}\,\hat{i}+\sqrt{2}\,\hat{j}\right)$ (b) $\left(-\hat{i}+\hat{j}\right)$

(c) $\left(\hat{i}-\hat{j}\right)$ (d) $\left(\sqrt{2}\,\hat{i}+\sqrt{2}\,\hat{j}\right)$

49. The angular momentum of a wheel having a rotational inertia of 0.2 kg m^2 about its symmetric axis decreases from 4 to 2 kg m^2 s^{-1} in 4s. The average power of the wheel is **[AP/July 5, 2022 (II)]**

(a) 7.5 W (b) 15 W (c) 5 W (d) 12 W

50. **Assertion (A) :** Angular speed, linear speed and kinetic energy change with time but angular momentum remains constant for a planet orbiting the sun.

Reason (R) : Angular momentum is constant as no torque acts on the planet. **[AP/Aug. 23, 2021 (I)]**

(a) Both A and R are true and R is a correct explanation for A.

(b) Both A and R are true but R is not a correct explanation for A.

(c) A is true, R is false.

(d) A is false, R is true.

51. A wheel of mass 20 kg and radius 30 cm is rotating at an angular speed of 80 rev/min when the motor is turned off. Neglecting the friction at the axis, calculate the force that must be applied tangentially to the wheel to bring it to rest in 5 rev. **[TS/Aug. 6, 2021 (I)]**

(a) 1.06 πN (b) 2.06 πN

(c) 3.06 πN (d) 4.06 πN

52. A body of mass 5 kg acquires an acceleration of 10 rad s^{-2} due to an applied torque of 2 Nm. Its radius of gyrations is **[AP/Sept. 21, 2020 (I)]**

(a) 2.5 m (b) $\sqrt{2.5}$ m (c) $\sqrt{0.2}$ m (d) 0.2 m

53. A thin uniform circular disc of mass *M* and radius *R* is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of same thickness and radius but of mass $\frac{1}{8}M$ is placed gently on the first disc coaxially. The angular velocity of the system is now **[TS/2016]**

(a) $\frac{8}{9}\omega$ (b) $\frac{5}{9}\omega$

(c) $\frac{1}{3}\omega$ (d) $\frac{2}{9}\omega$

54. A particle of mass $m = 5$ units is moving with uniform speed $v = 3\sqrt{2}$ units in the XY-plane along the line $Y = X + 4$. The magnitude of the angular momentum about origin is **[AP/2015]**

(a) zero (b) 60 units
(c) 7.5 units (d) 40 units

Topic 4 Moment of Inertia, Rotational K.E.

55. The moment of inertia of a solid sphere about its diameter is 20 kg m^2 . The moment of inertia of a thin spherical shell having the same mass and radius about its diameter is **[AP/May 23, 2024 (I)]**

(a) 16.6 kg m^2 (b) 30.3 kg m^2
(c) 33.3 kg m^2 (d) 66.6 kg m^2

56. The moment of inertia of a rod about an axis passing through its centre and perpendicular to its length is $\frac{1}{12}ML^2$, where M is the mass and L is the length of the rod. The rod is bent in the middle so that the two halves make an angle of 60°. The moment of inertia of the bent rod about the same axis would be **[AP/May 22, 2024 (I)]**

(a) $\frac{1}{48}ML^2$ (b) $\frac{1}{12}ML^2$
(c) $\frac{1}{24}ML^2$ (d) $\frac{1}{8\sqrt{3}}ML^2$

57. Three particles of each mass 'm' are kept at the three vertices of an equilateral triangle of side '1'. The moment of inertia of system of the particles about any side of the triangle is **[AP/May 21, 2024 (II)]**

(a) $\frac{ml^2}{4}$ (b) ml^2 (c) $\frac{3}{4}ml^2$ (d) $\frac{2}{3}ml^2$

58. The moment of inertia of a solid sphere of mass 20 kg and diameter 20 cm about the tangent to the sphere is **[AP/May 20, 2024 (I)]**

(a) 0.24 kgm^2 (b) 0.14 kgm^2
(c) 0.28 kgm^2 (d) 0.08 kgm^2

59. The moments of inertia of a solid cylinder and a hollow cylinder of same mass and same radius about the axes of the cylinders are I_1 and I_2. The relation between I_1 and I_2 is **[AP/May 18, 2024 (I)]**

(a) $I_1 < I_2$ (b) $I_1 = I_2$
(c) $I_1 > I_2$ (d) $I_1 = I_2 = 0$

60. A solid sphere and a disc of same mass 'M' and radius 'R' are kept such that their curved surfaces are in contact and their centers lie along the same horizontal line. The moment of inertia of the two body system about an axis passing through their point of contact and perpendicular to the plane of the disc is **[TS/May 10, 2024 (I)]**

(a) $\frac{53MR^2}{20}$ (b) $\frac{39MR^2}{10}$ (c) $\frac{29MR^2}{10}$ (d) $\frac{9MR^2}{10}$

61. A thin uniform wire of mass 'm' and linear mass density 'ρ' is bent in the form of a circular loop. The moment of inertia of the loop about its diameter is **[TS/May 9, 2024 (II)]**

(a) $\frac{m^2}{4\pi^2\rho^2}$ (b) $\frac{m^3}{4\rho^2}$ (c) $\frac{m^3}{8\pi^2\rho^2}$ (d) $\frac{m^3}{8\rho^2}$

62. Moon revolves around the earth in an orbit of radius R with time period of revolution T. It also rotates about its own axis with a time period T. If mass of the moon is M and its radius is 'r', the total kinetic energy of the moon is **[TS/May 12, 2023 (II)]**

(a) $\frac{2M\pi^2R^2}{T^2}+\frac{4Mr^2\pi^2}{5T^2}$ (b) $\frac{M\pi^2R^2}{2T^2}$
(c) $\frac{4Mr^2\pi^2}{5T^2}$ (d) $\frac{M\pi^2R^2}{2T^2}+\frac{4Mr^2\pi^2}{5T^2}$

63. The ratio of the radii of two solid spheres of same mass is 2:3. The ratio of the moments of inertia of the spheres about their diameters is **[TS/May 14, 2023 (I)]**

(a) 4:9 (b) 2:3 (c) 8:27 (d) 16:81

64. A body is rolling without slipping on a horizontal plane. If the rotational kinetic energy of the body 50% of its total kinetic energy, then the body is **[TS/May 13, 2023 (I)]**

(a) Hollow sphere (b) Solid sphere
(c) Solid cylinder (d) Thin circular ring

65. The moment of inertia of a thin uniform rectangular plate of mass 'm', having length 'a' and width 'b' about an axis perpendicular to the plane of the plate and passing through one of its vertices is **[AP/July 8, 2022 (I)]**

(a) $\frac{2}{3}\text{mab}$ (b) $\frac{1}{3}\text{mab}$
(c) $\frac{2}{3}\text{m}\left(a^2+b^2\right)$ (d) $\frac{1}{3}\text{m}\left(a^2+b^2\right)$

66. A solid sphere of radius R has its outer half removed, so that its radius becomes (R/2). Then its moment of inertia about the diameter is **[AP/July 4, 2022 (II)]**

(a) becomes $\frac{1}{2}$ of its initial value.
(b) is unchanged.
(c) becomes $\frac{1}{16}$ of initial value.
(d) becomes $\frac{1}{32}$ of initial value.

67. A wheel having moment of inertia 40 kgm^2 about its axis, rotates at 50 rpm. The angular retardation required to stop this wheel in 90 s is .... $rads^{-2}$. **[AP/Aug. 23, 2021 (I)]**

(a) $\frac{\pi}{45}$ (b) $\frac{\pi}{30}$ (c) $\frac{\pi}{54}$ (d) $\frac{\pi}{24}$

68. Consider a disc of radius R and mass M. A hole of radius $\frac{R}{3}$ is created in the disk such that the center of the hole is $\frac{R}{3}$ away from centre of the disk. The moment of inertia of the system along the axis perpendicular to the disc passing through the centre of the disc is **[AP/July 4, 2022 (I)]**

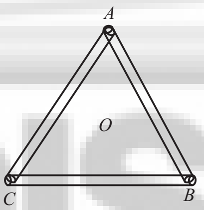
(a) $\frac{MR^2}{2}$ (b) $\frac{13}{27}MR^2$ (c) $\frac{1}{3}MR^2$ (d) $4\ MR^2$

69. Three point-masses m_1, m_2 and m_3 are located at the vertices of an equilateral triangle, having each side of length L. The moment of inertia of the system about an axis along an altitude of the triangle passing through m_1 is given by **[AP/Aug. 23, 2021 (I)]**

(a) $I = (m_1 + m_2 + m_3)L^2$ (b) $I = \left(m_1 + m_2\right)\frac{L^2}{2}$

(c) $I = (m_2 + m_3)L^2$ (d) $I = \left(m_2 + m_3\right)\frac{L^2}{4}$

70. Three rods each of mass 1 kg and length 2m are joined together end-to-end to form an equilateral triangle ABC. Find the moment of inertia of this system about an axis passing through its centre of mass and perpendicular to the plane of the triangle. **[AP/Aug. 23, 2021 (I)]**



(a) 4 kg-m^2 (b) 2 kg-m^2 (c) 3 kg-m^2 (d) 6 kg-m^2

71. Four spheres each of diameter $2a$ and mass m are placed in a way that their centres lie on the four corners of a square of side b. Moment of inertia of the system about an axis along one of the sides of the square is **[AP/Aug. 20, 2021 (I)]**

(a) $\frac{8}{5}ma^2$ (b) $\frac{4}{5}ma^2 + 5mb^2$

(c) $\frac{4}{5}ma^2 + 2mb^2$ (d) $\frac{8}{5}ma^2 + 2mb^2$

72. Which of the following type of wheels of same mass and radius will have largest moment of inertia? **[AP/Aug. 19, 2021 (I)]**

(a) Ring (b) Angular disc

(c) Solid disc (d) Cylindrical disc

73. The moment of inertia of a rectangular plate of mass M, length L and breadth B, about an axis passing through its centre and perpendicular to its plane is **[AP/Sept. 21, 2020 (I)]**

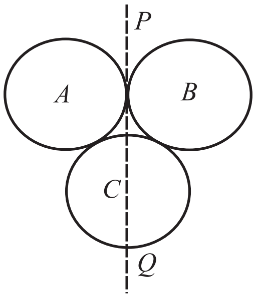
(a) $\frac{M\left(L+B\right)}{12}$ (b) $\frac{M\left(L^2\right)}{12}$

(c) $\frac{M\left(L^2+B^2\right)}{12}$ (d) $\frac{M\left(B^2\right)}{12}$

74. Four point masses, each of mass M are placed at the corners of a square of side L. The moment of inertia of the system about one of its diagonals is **[AP/Sept. 17, 2020 (I)]**

(a) $2ML^2$ (b) ML^2 (c) $4ML^2$ (d) $6ML^2$

75. Three identical uniform solid spheres each of mass m and radius r are joined as shown in the figure, with centres lying in the same plane. The moment of inertia of the system about an axis lying in that plane and passing through the centre of sphere C is **[AP/Sept. 17, 2020 (I)]**



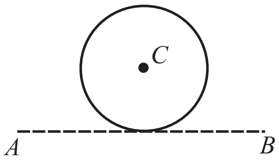
(a) $\frac{16}{5}mr^2$ (b) $\frac{12}{5}mr^2$ (c) $4\ mr^2$ (d) $\frac{3}{5}mr^2$

76. A rod of length L revolves in a horizontal plane about the axis passing through its centre and perpendicular to its length. The angular velocity of the rod is ω. If A is the area of cross-section of the rod and ρ is its density, then the rotational kinetic energy of the rod is **[TS/Sept. 11, 2020 (I)]**

(a) $\frac{1}{3}AL^3\rho\omega^2$ (b) $\frac{1}{2}AL^3\rho\omega^2$

(c) $\frac{1}{24}AL^3\rho\omega^2$ (d) $\frac{1}{18}AL^3\rho\omega^2$

77. A thin wire of length l having a linear density ρ is bent into a circular loop with C as its centre as shown in the figure. The moment of inertia of the loop about the line AB is **[AP/Apr. 22, 2019 (I)]**



(a) $\frac{5}{16}\frac{\rho l^3}{\pi^3}$ (b) $\frac{1}{16}\frac{\rho l^3}{\pi^3}$ (c) $\frac{1}{8}\frac{\rho l^3}{\pi^3}$ (d) $\frac{3}{8}\frac{\rho l^3}{\pi^2}$

78 A solid sphere of 100 kg and radius 10 m moving in a space becomes a circular disc of radius 20 m in one hour. Then the rate of change of moment of inertia in the process is **[AP/Apr. 20, 2019 (I)]**

(a) $\frac{40}{9}\text{kg m}^2\ \text{s}^{-1}$ (b) $\frac{10}{9}\text{kg m}^2\ \text{s}^{-1}$

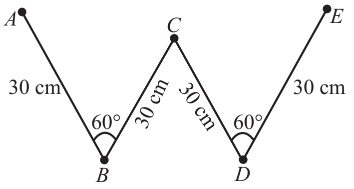
(c) $\frac{50}{9}\text{kg m}^2\ \text{s}^{-1}$ (d) $\frac{25}{9}\text{kg m}^2\ \text{s}^{-1}$

79. A semicircular plate of mass m has radius r and centre c. The centre of mass of the plate is at a distance x from its centre c. Its moment of inertia about an axis passing through its centre of mass and perpendicular to its plane is **[AP/Apr. 20, 2019 (I)]**

(a) $\frac{mr^2}{2}$ (b) $\frac{mr^2}{4}$

(c) $\frac{mr^2}{2} + mx^2$ (d) $\frac{mr^2}{2} - mx^2$

80. A uniform thin rod of 120 cm length and 1600 g mass is bent as shown in the figure. The moment of inertia of the bent rod about an axis passing through the point '*O*' and perpendicular to the plane of the paper is .... kg-m^2. **[AP/2017]**



(a) 0.084 (b) 0.360 (c) 0.018 (d) 0.120

81. Three identical uniform thin metal rods form the three sides of an equilateral triangle. If the moment of inertia of the system of these three rods about an axis passing through the centroid of the triangle and perpendicular to the plane of the triangle is *n* times. The moment of inertia of one rod separately about an axis passing through the centre of the rod and perpendicular to its length, the value of *n* is **[AP/2016]**

(a) 3 (b) 6 (c) 9 (d) 12

82. The moment of inertia of a solid cylinder of mass M, length $2R$ and radius R about an axis passing through the centre of mass and perpendicular to the axis of the cylinder is I_1 and about an axis passing through one end of the cylinder and perpendicular to the axis of the cylinder is I_2, then **[TS/2015]**

(a) $I_2 < I_1$ (b) $I_2 - I_1 = MR^2$

(c) $\frac{I_2}{I_1} = \frac{19}{12}$ (d) $\frac{I_2}{I_1} = \frac{7}{6}$

Topic 5 Rolling Motion

83. One ring, one solid sphere and one solid cylinder are rolling down on same inclined plane starting from rest. The radius of all the thee are equal, The object reaches down with maximum velocity is **[AP/May 23, 2024 (I)]**

(a) Solid cylinder (b) Solid sphere

(c) Ring (d) Solid sphere and Ring

84. A ring and a disc of same mass and same diameter are rolling without slipping. Their linear velocities are same, then the ratio of their kinetic energy is **[AP/May 22, 2024 (II)]**

(a) 0.75 (b) 1.33 (c) 0.5 (d) 2.66

85. A uniform rod of length '$2L$' is placed with one end in contact with the earth and is then inclined at an angle α to the horizontal and allowed to fall without slipping at contact point. When it becomes horizontal, its angular velocity will be **[AP/May 22, 2024 (I)]**

(a) $\sqrt{\frac{3g\sin\alpha}{2L}}$ (b) $\sqrt{\frac{2L}{3g\sin\alpha}}$

(c) $\sqrt{\frac{6g\sin\alpha}{L}}$ (d) $\sqrt{\frac{L}{g\sin\alpha}}$

86. A solid cylinder rolls down on an inclined plane of height 'h' and inclination 'θ'. The speed of the cylinder at the bottom is **[AP/May 21, 2024 (II)]**

(a) $\sqrt{\frac{gh}{2}}$ (b) $\sqrt{\frac{3gh}{2}}$ (c) $\sqrt{2gh}$ (d) $\sqrt{\frac{4gh}{3}}$

87. The masses of a solid cylinder and a hollow cylinder are 3.2 kg and 1.6 kg respectively. Both the solid cylinder and hollow cylinder start from rest from the top of an inclined plane and roll down without slipping. If both the cylinders have equal radius and the acceleration of the solid cylinder is 4 ms^{-2}, the acceleration of the hollow cylinder is **[AP/May 21, 2024 (I)]**

(a) 2 ms^{-2} (b) 9 ms^{-2} (c) 6 ms^{-2} (d) 3 ms^{-2}

88. A small disc is on the top of a smooth hemisphere of radius 'R'. The smallest horizontal velocity 'V' that should be imparted to the disc so that disc leaves the hemisphere surface without sliding down is (there is no friction) **[AP/May 20, 2024 (II)]**

(a) $V = \sqrt{g^2R}$ (b) $V = \sqrt{2gR}$

(c) $V = \sqrt{gR}$ (d) $V = \sqrt{g/R}$

89. A solid cylinder rolls down an inclined plane without slipping. If the translational kinetic energy of the cylinder is 140 J, the total kinetic energy of the cylinder is **[AP/May 19, 2024 (I)]**

(a) 105 J (b) 70 J (c) 210 J (d) 280 J

90. A body of mass '*m*' and radius '*r*' rolling horizontally with a velocity '*V*', rolls up an inclined plane to a vertical height $\frac{V^2}{g}$. The body is **[TS/May 11, 2024 (I)]**

(a) s sphere (b) a circular disc

(c) a circular ring (d) a solid cylinder

91. A hollow cylinder and a solid cylinder initially at rest at the top of an inclined plane are rolling down without slipping. If the time taken by the hollow cylinder to reach the bottom of the inclined plane is 2 s, the time taken by the solid cylinder to reach the bottom of the inclined plane is **[TS/May 10, 2024 (II)]**

(a) 2 s (b) 1.414 s (c) 1 s (d) 1.732 s

92. A solid sphere rolls down without slipping from the top of an inclined plane of height 28 m and angle of inclination 30°. The velocity of the sphere, when it reaches the bottom of the plane is (Acceleration due to gravity = 10 ms^{-2}) **[TS/May 9, 2024 (I)]**

(a) 20 ms^{-1} (b) 28 ms^{-1}

(c) 10 ms^{-1} (d) 14 ms^{-1}

93. A solid sphere is pushed on a horizontal surface such that it slides with a speed 3.5 ms^{-1} initially without rolling. The sphere will start rolling without slipping when its velocity becomes **[AP/May 17, 2023 (I)]**

(a) 2.5 ms^{-1} (b) 5 ms^{-1}

(c) 3.5 ms^{-1} (d) 7 ms^{-1}

94. A solid spherical ball is rolled up an inclined plane of angle of inclination 30° with an initial speed of 4 m/s at the bottom of the inclination. How far will the ball go up the plane.

(Use $g = 10\ m/s^2$) **[TS/July 20, 2022 (I)]**

(a) 56 cm (b) 112 cm
(c) 224 cm (d) 120 cm

95. A solid cylinder of mass m and radius R rolls down an inclined plane of height 30 m without slipping. The speed of its centre of mass when the cylinder reaches the bottom is [use $g = 10\ m/s^2$] **[TS/July 18, 2022 (II)]**

(a) 10 m/s (b) 20 m/s (c) 30 m/s (d) 40 m/s

96. A small disc of mass 500 g and radius 5 cm rolls down an inclined plane without slipping. Speed of its centre of mass when it reaches the bottom of the inclined plane depends on **[TS/Aug. 4, 2021 (I)]**

(a) mass and radius
(b) mass and height of the incline
(c) height of the incline
(d) height of the incline and acceleration due to gravity

97. If an energy of 684 J is needed to increase the speed of a flywheel from 180 rpm to 360 rpm, then find its moment of inertia. **[AP/Aug. 20, 2021 (I)]**

(a) $0.7\ kg/m^2$ (b) $1.28\ kg/m^2$
(c) $2.75\ kg/m^2$ (d) $7.28\ kg/m^2$

98. A sphere and a hollow cylinder without slipping, roll down two separate inclined planes *A* and *B*, respectively. They cover same distance in a given duration. If the angle of inclination of plane *A* is 30°, then the angle of inclination of plane *B* must be (approximately) **[AP/Aug. 20, 2021 (I)]**

(a) 60° (b) 53° (c) 45° (d) 37°

99. A solid cylinder of mass *M* and radius *R* rolls down an inclined plane of length *L* and height *h*, without slipping. Find the speed of its centre of mass when the cylinder reaches its bottom. **[AP/Sept. 21, 2020 (I)]**

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{3gh}{4}}$ (c) $\sqrt{\frac{4gh}{3}}$ (d) $\sqrt{4gh}$

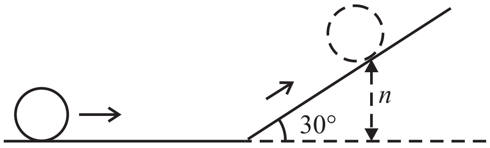
100. A circular ring of mass 10 kg rolls along a horizontal floor. The centre of mass of the ring has a speed 1.5 m/s. The work required to stop the ring is **[TS/May 4, 2019 (II)]**

(a) 10 J (b) – 6 J (c) 14.5 J (d) – 22.5 J

101. A thin circular disc of mass 12 kg and radius 0.5 m rotates with an angular velocity of 100 rad/s. The rotational kinetic energy of the disc is **[TS/May 6, 2019 (I)]**

(a) 12.2 kJ (b) 5.5 kJ
(c) 9.2 kJ (d) 7.5 kJ

102. A solid spherical ball rolls on a horizontal surface at 10 m/s and continues to roll up on an inclined surface as shown in the figure. If the mass of the ball is 11 kg and frictional losses are negligible, the value of *h* where the ball stop and starts rolling down the inclination is (Assume, $g = 10\ m/s^2$) **[TS/May 3, 2019 (II)]**



(a) 8 m (b) 6 m (c) 7 m (d) 10 m

103. Three bodies, a ring, a solid disc and a solid sphere roll down the same inclined plane without slipping. The radii of the bodies are identical and they start from rest. If V_S, V_R and V_D are the speeds of the sphere, ring and disc, respectively when they reach the bottom, then the correct option is **[TS/May 3, 2019 (I)]**

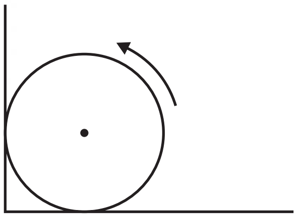
(a) $V_S > V_R > V_D$ (b) $V_D > V_S > V_R$
(c) $V_R > V_D > V_S$ (d) $V_S > V_D > V_R$

104. A solid sphere rolls down without slipping on a smooth inclined plane of inclination $\sin^{-1}(0.42)$. If the acceleration due to gravity is $10\ ms^{-2}$, the acceleration of the rolling sphere is **[AP/Apr. 22, 2019 (I)]**

(a) $1\ ms^{-2}$ (b) $2\ ms^{-2}$ (c) $3\ ms^{-2}$ (d) $4\ ms^{-2}$

105. A uniform cylinder of radius 1 m, mass 1 kg spins about its axis with an angular velocity 20 rad/s. At certain moment, the cylinder is placed into a corner as shown in the figure. The coefficient of friction between the horizontal wall and the cylinder is μ, whereas the vertical wall is frictionless. If the number of rounds made by the cylinder is 5 before it stops, then the value of μ is

(Acceleration due to gravity, $g = 10\ m/s^2$) **[TS/May 5, 2018 (I)]**



(a) $\frac{3}{\pi}$ (b) $\frac{2}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{0.4}{\pi}$

106. A solid sphere is projected up along an inclined plane of inclination 30° with the horizontal with a speed of $4\ ms^{-1}$. If it rolls without slipping, the maximum distance traversed by it is .... ($g = 10\ ms^{-2}$) **[AP/2017]**

(a) 2.24 m (b) 112 m (c) 1.12 m (d) 22.4 m

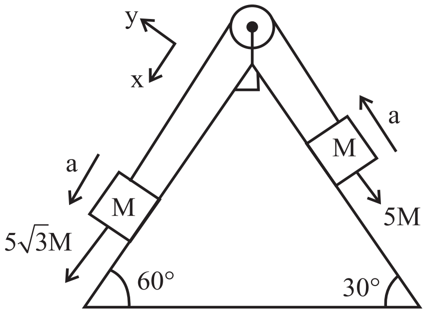
107. The kinetic energy of a circular disc rotating with a speed of 60 r.p.m. about an axis passing through a point on its circumference and perpendicular to its plane is (mass of circular disc = 5 kg, radius of disc = 1m) approximately. **[AP/2015]**

(a) 170 J (b) 160 J (c) 150 J (d) 140 J

6

System of Particles and Rotational Motion

1. **(b)**



The acceleration of the block is

$$a=\frac{\left(5\sqrt{3}-5\right)M}{(M+m)}=\frac{5}{2}\left(\sqrt{3}-1\right)\text{m/s}$$

∴ Acceleration of centre of mass is

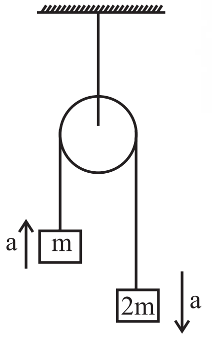
$$\vec{a}_{cm}=\frac{m_1\vec{a}_1+m_2\vec{a}_2}{m_1+m_2}=\frac{M\left(\vec{a}_1+\vec{a}_2\right)}{2M}=\frac{\vec{a}_1+\vec{a}_2}{2}$$

Also, $\vec{a}_1=a\hat{i},\vec{a}_2=a\hat{j}$

$$\therefore\ \vec{a}_{cm}=\frac{a}{2}\left(\hat{i}+\hat{j}\right)$$

$$\therefore\ a_{cm}=\frac{a\sqrt{2}}{2}=\frac{1}{\sqrt{2}}\cdot\frac{5}{2}\left(\sqrt{3}-1\right)=\frac{5\left(\sqrt{3}-1\right)}{2\sqrt{2}}$$

2. **(a)**



$$a=\left(\frac{m_2-m_1}{m_1+m_2}\right)g=\left(\frac{2m-m}{2m+m}\right)\times 10$$

$$=\frac{10}{3}\text{ms}^{-2}$$

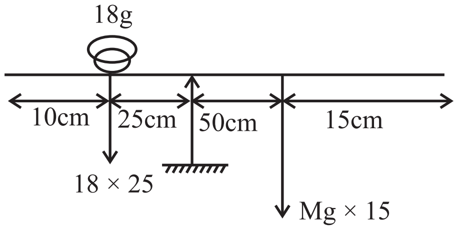
After time, t = 5.45

$$v_1=v_2=at=\frac{10}{3}\times 5.4=18\text{ ms}^{-1}$$

$$\therefore\ v_{cm}=\frac{m_2v_2-m_1v_1}{m_1+m_2}=\frac{18(2m-m)}{(m+2m)}$$

$$=6\text{ ms}^{-1}$$

3. **(b)** At equilibrium of meter scale,



$\Sigma\tau=0\ \Rightarrow\ 18g\times 25-Mg\times 15=0$

$$\therefore\ \text{Mass of scale, } M=\frac{18\times 25}{15}=30g$$

4. **(c)** Cut area,

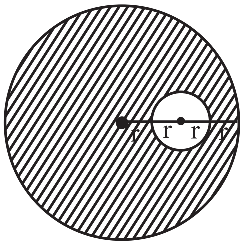
$A_C=\pi r^2\propto 1$

Remaining area,

$A_R=\pi\,(4r)^2-\pi r^2=15\,\pi r^2\propto 15$

Now, $A_C X_C=A_R X_R$

$$1\times 2r=15\,X_R\ \therefore\ X_R=\frac{2r}{15}$$

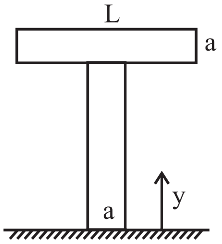


5. **(a)** $A_1=La, A_2=La$

$$y_1=\frac{L}{2},y_2=L+\frac{a}{2}$$

$$\therefore\ y_{cm}=\frac{A_1y_1+A_2A_2}{A_1+A_2}$$

$$=\frac{\frac{L}{2}+L+\frac{a}{2}}{2}=\left(\frac{3L}{4}+\frac{a}{4}\right)$$



Centre of mass from top,

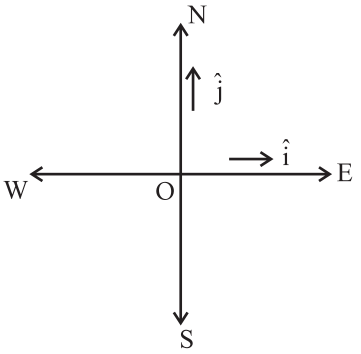
$$y'_{cm}=(L+a)-\left(\frac{3L}{4}+\frac{a}{4}\right)=\left(\frac{L}{4}+\frac{3a}{4}\right)$$

∴ Shift in centre of mass,

$$\Delta y_{cm}=y_{cm}-y'_{cm}=\left(\frac{3L}{4}+\frac{a}{4}\right)-\left(\frac{L}{4}+\frac{3a}{4}\right)$$

$$=\frac{L}{2}-\frac{a}{2}=\frac{L-a}{2}$$

6. **(b)**



For the particles A, B and C

$m_1 = m, m_2 = 2m, m_3 = 3m$

$\vec{v}_1 = 6\hat{j}\ ms^{-1}, \vec{v}_2 = -12\hat{j}\ ms^{-1}$

$\vec{v}_3 = 8\hat{i}\ ms^{-1}$

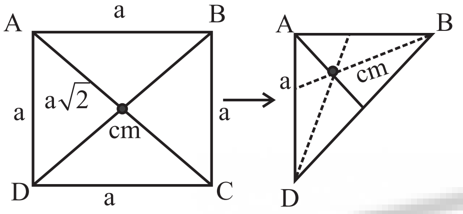
$\therefore$ Velocity of centre of mass is

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3}{m_1 + m_2 + m_3}$$

$$= \frac{m(6\hat{j}) + 2m(-12\hat{j}) + 3m(8\hat{i})}{m + 2m + 3m} = 4\hat{i} - 3\hat{j}$$

$$\therefore\ v_{cm} = \sqrt{(4)^2 + (-3)^2} = 5ms^{-1}$$

7. **(d)**



fig(i)
Square

fig(ii)
Isosceles
triangle

From figure (i) & (ii) of mass at point C is removed then the shift in the position of the centre of mass is $\frac{1}{3}\frac{a}{\sqrt{2}}$

8. **(d)** Option (d) is incorrect because the centre of mass of two particles is nearer to the particle of higher mass not lesser mass.

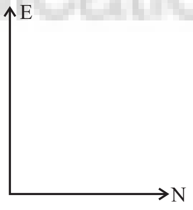
9. **(a)** Mass, $m_1 = 10\ g = 10 \times 10^{-3} = 10^{-2}$ Kg

velocity, $v_1 = 10$ m/s

Mass of 2nd particle, $M_2 = 15g = 15 \times 10^{-3}$ Kg

Velocity of IInd particle, $V_2 = 5$m/s

centre of mass makes 45° angle with north and east side.



Hence velocity of centre of mass is given as:

$$V_{1m} = \frac{m_1 v_1 \cos\theta_1 + m_2\theta, \sin\theta_1}{m_1 + m_2}$$

$$= \frac{10 \times 10 \times \cos 45 + 15 \times 5 \times \sin 45}{10 + 15}$$

$$= \frac{1}{\sqrt{2}}\left[\frac{175}{25}\right] = \frac{7}{2}(\sqrt{2})$$

$= 4.9$ m/s $\simeq 5$ m/s

10. **(b)** Masses, $m_1 = 1g$; $m_2 = 2g$

Velocity, $v_1 = 10$ m/s; v_2 20 m/s

The velocity of the centre of mass

$$V_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$= \frac{-1 \times 10 + 2 \times 20}{1 + 2} = \frac{30}{3} = 10 \text{ m/s}$$

11. **(b)** Radius of removed circular plate, A = 1.5 r

Radius of circular plate, B = 2r

Let mass of circular plate A, $M_1 = M$

So surface mass density, $\sigma = \frac{M}{\pi(2r)^2} = \frac{M}{4\pi r^2}$

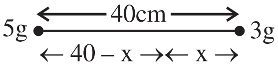
Then mass of removed circular plate, A

$$M_2 = \sigma A = \frac{M}{4\pi r^2} \times \pi(1.5r)^2 = \frac{M}{4} \times 1.5 \times 1.5 = 0.5625\ M$$

The distance of centre of mass of the remaining portion from the centre of the plate B is

$$X_{cm} = \frac{M_1x_1 - M_2x_2}{M_1 - M_2} = \frac{M \times 0 - 0.5625M \times 0.5g}{M - 0.5625M} = \frac{9r}{14}$$

12. **(b)** Suppose centre of mass is x cm away from 3g and assuming that 3g is at x = 0



$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{5 \times 40 + 3 \times 0}{5 + 3} = \frac{5 \times 40}{8} = 25 \text{ cm}$$

The centre of mass of the system of these two particles lies at a distance 25 cm from 5g particle.

13. **(b)** The mass of two blocks are $m_1 = 2$ kg and $m_2 = 1$ kg.

The acceleration of masses is given as

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)g = \left(\frac{2-1}{2+1}\right)g = \frac{g}{3}$$

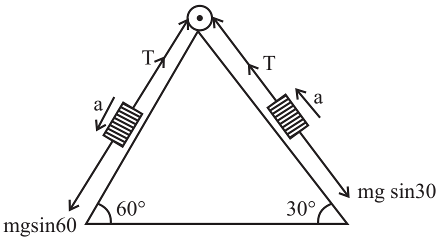
The acceleration of the centre of mass is given by

$$a_{cm} = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)a = \left(\frac{2-1}{2+1}\right) \times \frac{g}{3} = \frac{g}{9}$$

The distance travelled by their centre of mass is

$$S = \frac{1}{2}a_{cm}\ t^2 = \frac{1}{2} \times \frac{g}{9} \times 2^2 = \frac{2}{9}g = \frac{2}{9} \times 10 = 2.22 \text{ m.}$$

14. **(a)**



Since, mg sin60 > mg sin30. So, block will go left side.

T – mg sin30 = m a

$$T - \frac{mg}{2} = m\ a \qquad ...(i)$$

mg sin60 – T = m a

$$\frac{\sqrt{3}mg}{2} - T = m\ a \qquad ...(ii)$$

Adding equation (i) and (ii), we have

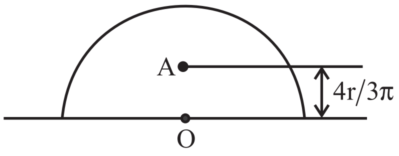
$$\frac{\sqrt{3}mg}{2} - \frac{mg}{2} = 2ma \Rightarrow a = \frac{(\sqrt{3}-1)g}{4}$$

The acceleration of the centre of mass of the block is

$$\vec{a}_{cm}=\frac{m_1\vec{a}_1+m_2\vec{a}_2}{m_1+m_2}=\frac{ma\hat{i}+ma\hat{j}}{2m}=\frac{a}{2}\left(\hat{i}+\hat{j}\right)$$

$$=\frac{\sqrt{2}}{2}\times\frac{\left(\sqrt{3}-1\right)}{4}g=\frac{\left(\sqrt{3}-1\right)g}{4\sqrt{2}}$$

15. **(None)** The COM of a homogeneous semi-circular plate is located at $\frac{4r}{3\pi}$ distance from 'O'.



So, $OA=\frac{4r}{3\pi}$

16. **(a)** $\overrightarrow{r_{cm}}=\frac{m_1\vec{r_1}+m_2\vec{r_2}+....}{m_1+m_2+.....}$

$$=\frac{\left[m\left(\frac{1}{3}\right)^2.\frac{1}{2}\right]\times2+\left[m\left(\frac{1}{3}\right)^3.\frac{1}{3}\right]\times3+\left[m\left(\frac{1}{3}\right)^4.\frac{1}{4}\right]\times4+...}{M}$$

$$=\frac{m\left(\frac{1}{3}\right)^2+m\left(\frac{1}{3}\right)^3+m\left(\frac{1}{3}\right)^4+...}{M}$$

$$=\frac{m}{M}.\frac{\left(\frac{1}{3}\right)^2}{1-\frac{1}{3}}=\frac{m}{M}\times\frac{\frac{1}{9}}{\frac{2}{3}}=\frac{m}{6M}$$

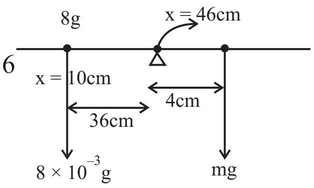
17. **(c)** As $J_{CW}=J_{ACW}$

$\Rightarrow$ mg × 4 = 8 × 10^{-3}g × 36

$\Rightarrow$ 4m = 8 × 10^{-3} × 36

$\Rightarrow$ m = 72 × 10^{-3} kg

= 72 gm



18. **(a)** We have

$$x_{cm}=\frac{\left(M\times1\right)+\left(2M\times0\right)+\left(4M\times0\right)+\left(3M\times-1\right)}{10M}$$

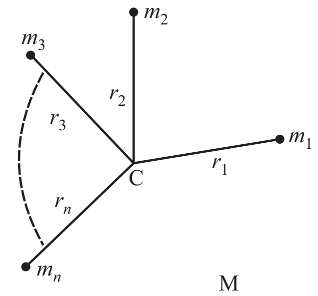
$$=\frac{-2M}{10M}=-\frac{1}{5}$$

and $y_{cm}=\frac{M\times0+\left(2M\times1\right)+\left(3M\times0\right)+\left(4M\times-1\right)}{10M}$

$$=-\frac{1}{5}$$

So, $\overrightarrow{x_{cm}}=-\frac{1}{5}\hat{i}-\frac{1}{5}\hat{j}$

19. **(b)** as $r=\frac{m_1r_1+m_2r_2+m_3r_3+....+m_nr_n}{M}$ ...(i)



If COM lies at origin then, $\vec{r}=0$

∴ From eq. (i), we get

$m_1r_1+m_2r_2+m_3r_3+...\ m_nr_n=0$ ...(ii)

Net moment of all particles in the system about centre of mass C.

$\tau_c=m_1gr_1+m_2gr_2+m_3gr_3+...\ mngrn$

$=g\ [m_1r_1+m_2r_2+m_3r_3+...\ mnrn]$

$=g\times0=0$ [from Eq. (ii)]

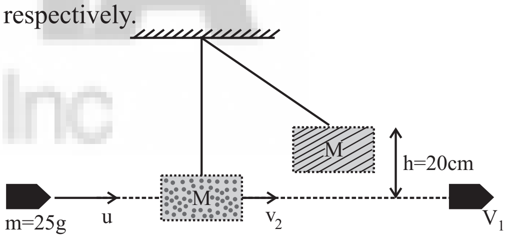
Hence, sum of moments of all the particles in a system about its COM is always zero.

20. **(c)** As there is zero net force on the system in horizontal direction because mass m starts moving under gravity but there is a net force in vertical direction so centre of mass of the system changes in vertical direction

21. **(d)** From the given situation,

Initial momentum of the bullet = mu

Let final velocities of bullet and block are v_1 and v_2 respectively.



If the system rises up to height h then by conservation of energy

$$\frac{1}{2}Mv_2^2=Mgh\Rightarrow v_2=\sqrt{2gh}$$

$$\Rightarrow\ v_2=\sqrt{2\times10\times20\times10^{-2}}=2\text{m/s}$$

By the conservation of linear momentum

$mu=mv_1+Mv_2$

$$\Rightarrow\ \frac{25}{1000}\times250=\frac{25}{1000}\times v_1+1\times2$$

$$\Rightarrow\ \frac{25}{4}=\frac{25}{1000}\times v_1+2\Rightarrow v_1=170\,\text{ms}^{-1}$$

22. **(b)** Given, radius of circular hole, r = 3 cm

Radius of circular disc, R = 6 cm

Mass of disc, $M=\pi R^2m$

Mass of hole, $M'=\pi r^2m$

If we assume the centre of mass of hole and disc to be at centre, then

The x-coordinate of the centre of mass of the remaining portion of the disc will be

$$x_{CM} = \frac{Mx_1 - M'x_2}{M - M'} = \frac{M \times 0 - M' \times 3}{\pi R^2 m - \pi r^2 m}$$

$$= \frac{-3M'}{\pi m(R^2 - r^2)} = \frac{-3\pi r^2 m}{\pi m(R^2 - r^2)}$$

$$= -\frac{-3r^2}{R^2 - r^2} = \frac{-3 \times 3^2}{6^2 - 3^2} = -1 \text{ cm}$$

$\Rightarrow$ $x_{CM} = -1$ cm

Hence, distance of centre of gravity of the resulting flat body from the centre of the original disc will be 1 cm left side.

23. **(a)** Height fallen by first ball in 0.4 s

$$x_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.4)^2 = 0.8\text{m}$$

Height fallen by second ball in 0.2 s

$$x_2 = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.2)^2 = 0.2\text{m}$$

Position of centre of mass is given by

$$CM \quad \frac{m_1x_1 \quad m_2x_2}{m_1 \quad m_2} = \frac{100 \times 0.8 + 200 \times 0.2}{100 + 200} = 0.4\,\text{m}$$

24. **(b)** The coordinates of centre of mass of the spheres is given by

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

As the spheres are identical,

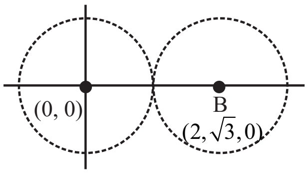
$\therefore$ $m_1 = m_1 = m_3 = m$ (say)

$$\Rightarrow x_{CM} = \frac{m}{3m}(0 + 2\sqrt{3} + \sqrt{3}) = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Similarly, $y_{CM} = \frac{m}{3m}(y_1 + y_2 + y_3) = \frac{(0+0+3)}{3} = i$

So, the centre of mass, $= (\sqrt{3}, 1)$

If one sphere is removed (say C), then



$$x'_{CM} = \frac{0 + 2\sqrt{2}}{2} = \sqrt{3}$$

$$y'_{CM} = 0$$

So, $C'_{CM} = (\sqrt{3}, 0)$

Hence, the centre of mass shifted by 1 m in $-y$ direction. The correct option is (b).

25. **(c)** Initial position of centre of mass,

$$r_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \qquad \text{...(i)}$$

Here, x_1 and x_2 are the distances of mass m_1 and m_2 from centre of mass respectively.

In second case,

$$r'_{cm} = \frac{m_1(x_1 + d) + m_2(x_2 + d')}{m_1 + m_1} \qquad \text{...(ii)}$$

Here, d and d' are the displacement of mass m_1 and m_2 respectively.

According to question, $r_{cm} = r'_{cm}$

$$\therefore \quad \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m_1(x_1 + d) + m_2(x_2 + d')}{m_1 + m_2}$$

$$\Rightarrow \quad d' = \frac{m_1}{m_2}d.$$

26. **(c)** For the block P,

$$\text{N} = \text{m}\,\omega_A^2\,\text{R}_A = \text{m}\omega_B^2\,\text{R}_B = \text{m}\omega_C^2\,\text{R}_C$$

$$\Rightarrow \text{R}_A : \text{R}_B : \text{R}_C = \frac{1}{\omega_A^2} : \frac{1}{\omega_B^2} : \frac{1}{\omega_C^2}$$

As, $\text{R}_A < \text{R}_B < \text{R}_C \Rightarrow \frac{1}{\omega_A^2} < \frac{1}{\omega_B^2} < \frac{1}{\omega_C^2}$

$\therefore$ $W_A > W_B > W_C$

27. **(a)** $w_0 = 600$ rev/min $= \frac{600 \times 2\pi}{60} = 20\pi$ rad/s

$\alpha = -2$ rad/s^2, $\omega = 0$

using, $\omega^2 = \omega_0{}^2 + 2\alpha\theta$

$\Rightarrow$ $0 = (20\pi)^2 - 2 \times 2 \times \theta$

$\therefore$ $\theta = 100\pi^2$

$\therefore$ Number of revolutions $= \frac{\theta}{2\pi} = \frac{100\pi^2}{2\pi} = 157$

28. **(a)** By equation of motion

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

Here, angular velocity, $\omega_0 = 0$

$$\theta = -\alpha$$

$$8 = \frac{1}{2}\alpha(2)^2 \qquad \text{... (1)}$$

$$\theta = \frac{1}{2}\alpha 5^2 \qquad \text{...(2)}$$

Divide equation (2) by (1)

$$\frac{\theta}{8} = \frac{\frac{1}{2}\alpha 5^2}{\frac{1}{2}\alpha 2^2} \Rightarrow \theta = \frac{25 \times 8}{4} = 50 \text{ rotations}$$

29. **(b)** Angular velocity of flywheel, $\omega = 150$ rev/minute $= \frac{150 \times 2\pi}{60} = 5\pi$ Rad/s

Angular acceleration, $\alpha = \pi$ rad/s^2

$\omega = \omega_0 + \alpha t$

The time required far the wheel to come to rest is

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{5\pi - 0}{\pi} = 5\,\text{s}$$

30. **(d)** Angular speed, $\omega = 40$ rad/s

Angular momentum, $L = 2$ kg $m^2 s^{-1}$

Radius of the cylinder, r = 10 cm = 0.10 m

Moment of inertia of solid cylinder, $I = \frac{mr^2}{2}$

Angular momentum, $L = I\,\omega$

$$2 = \frac{m\pi^2}{2} \times 40$$

$$2 = \frac{1}{2} \times m \times (0.10)^2 \times 40$$

$$m = \frac{2}{20 \times (0.1)^2} = 10 \text{ kg}$$

31. **(c)** Angular speed, $\omega = 8 - 2t$ rad/s

$$\frac{d\theta}{dt} = 8 - 2t \Rightarrow \int d\theta = \int (8 - 2t)\,dt$$

$$\theta = 8t - \frac{2t^2}{2} = 8t - t^2$$

When $\omega = 0$

$8 - 2t = 0 \Rightarrow t = 4$ s

$\theta = 8 \times 4 - 4^2 = 32 - 16 = 16$ rad

32. **(b)** As $\omega = \omega_0 + \alpha t$

$$\Rightarrow \quad f = f_0 + \frac{\alpha}{2\pi} t \;\Rightarrow\; 0 = 5 + \frac{\alpha}{2\pi} \times 80$$

$$\Rightarrow \quad \alpha = -\frac{10\pi}{80} = -\frac{\pi}{8} \text{rad/sec}^2$$

$$\text{So, } \theta = \frac{1}{2} \times \frac{\pi}{8} \times 80^2 = 400\pi$$

No. of revolution $= \frac{400\pi}{2\pi} = 200$

33. **(b)** To move from one point of zero velocity to next such point, time taken $= \frac{T}{2}$

So, $\frac{T}{2} = 0.5 \Rightarrow T = 1$ sec

$$\Rightarrow \frac{2\pi}{\omega} = 1 \Rightarrow \omega = 2\pi \text{ rad/s}$$

34. **(b)** We have

$$T_{\max} = m\omega_{\max}^2 R$$

$$\omega_{\max} = \sqrt{\frac{T_{\max}}{mR}} = \sqrt{\frac{72}{0.25 \times 0.5}} = 24 \text{ rad/s}$$

35. **(a)** Angular displacement in $t = 10$ s is given as

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad [\because \quad \omega_0 = 0 \text{ and } \alpha = 2 \text{ rad/s}^2]$$

$$= 0 \times 10 + \frac{1}{2} \times 2 \times 10^2 = 100 \text{ rad.}$$

Number of revolution $= \frac{\theta}{2\pi} = \frac{100}{2\pi}$

36. **(a)** So, tangential linear acceleration of particle is

$$at = \alpha r = \frac{d^2\theta}{dt^2} r = 2ar \qquad \left(\because \quad \alpha = \frac{d^2\theta}{dt^2}\right)$$

$$v = \omega r = \frac{d\theta}{dt} r = 2rat \qquad \left(\because \quad \omega = \frac{d\theta}{dt}\right)$$

Also, normal or radial acceleration of particle is

$$an = \frac{v^2}{r} = \frac{4a^2t^2r^2}{r} = 4a^2t^2r$$

Total acceleration of particle is

$$a_{\text{total}} = \sqrt{a_t^2 + a_n^2} = \sqrt{4a^2r^2 + 16a^4t^4r^2}$$

$$= 2ar\sqrt{1 + 4a^2t^4} = \frac{v}{t}\sqrt{1 + 4a^2t^4}$$

37. **(d)** Deceleration $\omega = -a\sqrt{v}$

$$\text{But, } \omega = \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = -a\sqrt{v}$$

$$\Rightarrow \quad \frac{-dv}{\sqrt{v}} = a \cdot dt \Rightarrow \int_{v_1}^{0} \frac{dv}{\sqrt{v}} = \int a\;dt$$

$$\Rightarrow \quad \left|2\sqrt{v}\right|_{v_1}^{0} = at \Rightarrow t = \frac{2}{a}\sqrt{v_1}$$

$$\text{Again, } \frac{-dv}{dt} = a\sqrt{v} \Rightarrow \frac{dv}{dx} \cdot \frac{dx}{dt} = -a\sqrt{v}$$

$$\Rightarrow \quad \frac{dv}{dx} \cdot v = -a\sqrt{v} \Rightarrow dv\sqrt{v} = -a \cdot dx$$

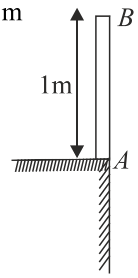
$$\Rightarrow \quad \int_{v_0}^{0} \sqrt{v}\,dv = -a\int_0^s ds$$

After solving, we get

$$s = \frac{2}{3a} \cdot v_0^{\frac{3}{2}}.$$

38. **(a)** Given, length of rod *AB*, *l* - 1 m

Net Displacemnt of centre of rod $\overline{\Delta \text{r}} = 5\sqrt{2}$m



$$\overline{\Delta r} = \Delta x\hat{\text{i}} + \Delta y\hat{\text{j}}$$

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\Delta y = ut + \frac{1}{2}gt^2$$

$$\Rightarrow \quad \Delta y = \frac{1}{2} \times 10 \times 1^2 = 5\text{m} \qquad [\text{As } u = 0, 2t = 1\text{s}]$$

$$\Delta r = \sqrt{4x^2 + \Delta y^2}$$

$$\Rightarrow \quad (5\sqrt{2})^2 = \Delta x^2 + \Delta y^2 \Rightarrow (5\sqrt{2})^2 = \Delta x^2 + 5^2$$

Displacement in *x* direction

$\Delta x^2 = 50 - 25$

$\Delta x = 5m$

$$v_x = \frac{x}{t} = 5\,m/s$$

Since, angular impulse = change in angular momentum

$J \times l/2 = I\omega$

$\Rightarrow \quad mv_x l / 2 = \dfrac{ml^2}{12}\omega$ [As horizental impulse]

$\omega = 6vx = 6 \times 5 = 30$ rads^{-1}

39. **(a)** From law of conservation of angular momentum, Iω = constant

When a ballet dancer suddenly folds her outstretched arms, then her moment of inertia decreases, hence her angular velocity will increases.

40. **(c)** Given, mass of flywheel, M = 1 kg,

Radius vectors $R = (2\hat{i} + \hat{j} + 2\hat{k})\,m$

Force applied, $F = (3\hat{i} + 2\hat{j} - 4\hat{k})$N

and time, $t = 4.5$ s

Magnitude of radius

(R) = $\sqrt{(2)^2 + (1)^2 + (2)^2} = \sqrt{9} = 3$ m

Similarly, F = $\sqrt{(3)^2 + (2)^2 + (-4)^2} = \sqrt{29}$N

Torque on the flywheel,

$\tau = I\alpha = F.R = \dfrac{MR^2}{2}\alpha$

$\Rightarrow \alpha = \dfrac{2F}{MR} = \dfrac{2\sqrt{29}}{1 \times 3} = \dfrac{2}{3}\sqrt{29}$

Now, using

$\omega = \omega_0 + \alpha t \;\Rightarrow \omega = 0 + \dfrac{2}{3}\sqrt{29} \times 4.5 \qquad (\because \omega_0 = 0)$

$\Rightarrow \omega = \sqrt{261}$ rad s^{-1}

Hence, the correct option is (c).

41. **(d)** Taken, $v = 1$

$v' = 1.09$ and $\Delta v = 0.09$

$\Rightarrow \quad \omega \propto \dfrac{1}{I} \qquad \left[\because \; J = I\omega \Rightarrow I \propto \dfrac{1}{\omega}\right]$

But $I \propto r^2$ and $\omega \propto v^{1/3}$

$\therefore \quad \omega \propto \dfrac{1}{v^{2/3}}$ (or $v^{-2/3}$)

$\Rightarrow \quad \omega = kv^{-2/3}$, k = cons

∴ Change in angular speed

$= \dfrac{\Delta\omega}{\omega} \times 100 = -\dfrac{2}{3}\dfrac{\Delta v}{v} \times 100$

$= -\dfrac{2}{3} \times 0.09 \times 100 = -6\%$

So decrease in angular speed is 6%

42. **(a)** Since no external torque, so angular momentum is conserved

$\therefore \quad I_1\omega_1 = I_2\omega_2$

$\Rightarrow \quad \left(\dfrac{1}{2}M_{\text{disc}} \times R_{\text{disc}}^2\right)\omega_1$

$= \left(\dfrac{1}{2}M_{\text{disc}}R_{\text{disc}}^2 + M_{\text{boy}}R_{\text{boy}}^2\right)\omega_2$

$\Rightarrow \quad \left(\dfrac{1}{2} \times 100 \times 2^2\right) \times 1 = \left(\dfrac{1}{2} \times 100 \times 2^2 + 60 \times 1^2\right) \times \omega_2$

Therefore final angular velocity,

$\omega_2 = \dfrac{200}{200 \quad 60} = 0.77$ rad-s^{-1}

43. **(d)** m = 50 kg, v = 20 cm = 0.2 m

$\omega = 420 \text{ rpm} = \dfrac{2\pi \times 420}{60}\text{rad/s} = 14\pi \text{ rad/s}$

The angular momentum of solid sphere,

$L = I.\omega = \dfrac{1}{5}mr^2\omega = \dfrac{2}{5} \times 50 \times 0.2 \times 0.2 \times 14\pi = 35.2$ Js.

44. **(c)** Mass of solid sphere, m = 2 kg

velocity, v = 5 m/s

Rotational kinetic energy is given by

K. E = $\dfrac{1}{2}$ I ω^2

(Here I = $\dfrac{2}{5}$ m r^2 ; v = rω)

$= \dfrac{1}{2} \times \dfrac{2}{5}mr^2 \times \dfrac{v^2}{r^2} = \dfrac{1}{5}mv^2 = \dfrac{1}{5} \times 2 \times 5^2 = 10$ J

45. **(a)** Torque $\tau = \dfrac{\Delta L}{\Delta t} = \dfrac{4A_0 - A_0}{4} = \dfrac{3}{4}A_0$

46. **(d)** Angular momentum, L = Iω and rotational kinetic energy $K = \dfrac{1}{2}I\omega^2 \;\therefore K = \dfrac{1}{2}L\omega$

$\therefore \; \dfrac{L_2}{L_1} = \dfrac{K_2}{K_1} \times \dfrac{\omega_1}{\omega_2} = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$

47. **(c)** Given, y = x + a

compairing with y = mx + c

m = 1, c = 2

tanθ = 1 = tan45°

θ = 45°

The velocity in vector form

$\vec{v} = v\cos\theta\hat{i} + v\sin\theta\hat{j} = v\cos 45\hat{i} + v\sin 45\hat{j}$

$= \dfrac{v}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$

Let us assume particle is at point (0, a)

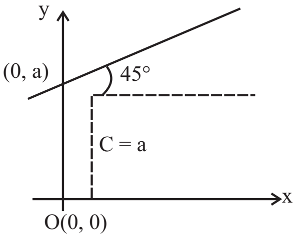
$\vec{r} = (0 - 0)\hat{i} + (a - 0)\hat{j} = a\hat{j}$

Angular momentum,

$\overline{L} = m\left(\vec{r} \times \vec{v}\right)$

$= m\left(a\hat{j} \times \dfrac{v}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)\right)$

$= \dfrac{mva}{\sqrt{2}}\left(-\hat{k}\right)$



48. **(a)** $\vec{F} = 2\cdot\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right) = \sqrt{2}\left(\hat{i}+\hat{j}\right)$

We have

$$\vec{j} = \vec{r}\times\vec{F} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ \sqrt{2} & \sqrt{2} & 0\end{vmatrix}$$

$$= \hat{i}\left(0-\sqrt{2}\right)+\hat{j}\left(\sqrt{2}-0\right)+\hat{k}\left(0\right) \quad = -\sqrt{2}\,\hat{i}+\sqrt{2}\,\hat{j}$$

49. **(a)** We have, $I = 0.2\ \text{kg/m}^2$

$L_f = 4\ \text{kg m}^2\text{s}^{-1}$

$L_i = 2\text{kg m}^2\text{s}^{-1}$

$$L_f = I\omega_f \Rightarrow \omega_f = \frac{4}{0.2} = 20\ \text{rad / s}$$

$$L_i = I\omega_i \Rightarrow \omega_i = \frac{2}{0.2} = 10\ \text{rad / s}$$

$$\text{So, power} = \frac{\text{work done}}{\text{time taken}} = \frac{\Delta K}{\Delta t}$$

$$= \frac{\frac{1}{2}\times 0.2\times 20^2 - \frac{1}{2}\times 0.2\times 10^2}{4} = 7.5\ \text{W}$$

50. **(a)** As we know that,

Gravitational torque on planet is zero due to sum.

$$\tau = \frac{dL}{dt} = 0$$

$\Rightarrow$ Angular momentum of planet remains constant, but linear speed keep chang as its distance term

51. **(a)** Initial angular speed,

$$\omega_0 = 80\text{rpm} = \frac{80\times 2\pi}{60} = \frac{8\pi}{3}\text{rad/s}$$

Angular displacement, $\theta = 2\pi n$ (number of n here revolution) $= 2\pi\times 5 = 10\pi$ rad

Final angular speed, $\omega = 0$

Using $\omega^2 = \omega_0^2 + 2\alpha\theta$

$$\Rightarrow \alpha = \frac{\omega^2-\omega_0^2}{2\theta}$$

$$\alpha = \frac{0-\left(\frac{8\pi}{3}\right)^2}{2\times 10\pi}\text{rad/s}^2 = -\frac{16\pi}{45}\text{rad/s}^2$$

Torque required to bring the wheel to rest.

$$\therefore \tau = I\alpha = FR$$

$$\Rightarrow F = \frac{I\alpha}{R} = \frac{\frac{1}{2}mR^2\alpha}{R} = \frac{1}{2}mR\alpha$$

$$\therefore \quad F = \frac{1}{2}\times 20\times 30\times 10^{-2}\times\frac{16\pi}{45}N = 1.06\pi N$$

52. **(d)** Given $m = 5$ kg

Angular acceleration, $\alpha = 10\ \text{rad s}^{-2}$

Torque, $\tau = 2$N–m

We know that,

From equations 2nd law of motion in retation

$\Rightarrow \quad \tau = I\alpha$ or $2 = I\times 10$

$\Rightarrow \quad I = 0.2\ \text{kg-m}^2$

Radius of gyration is related to moment of inertia as

$I = mk^2$

$0.2 = 5k^2$

$k = 0.2$ m

53. **(a)** Given, mass of first circular disc $(m_1) = M$

Radius of first circular disc $(r_1) = R$

Mass of second circular disc $(m_2) = \frac{1}{8}M$

Moment of inertia of circular disc,

$$I_1 = \frac{MR^2}{2} \qquad \text{...(i)}$$

Moment of inertia of combination of disc,

$$I_2 = \frac{MR^2}{2} + \frac{M}{8}\cdot\frac{R^2}{2} \Rightarrow I_2 = \frac{9MR^2}{8\times 2}$$

According to law of conservation of angular momentum, $I\omega$ = constant.

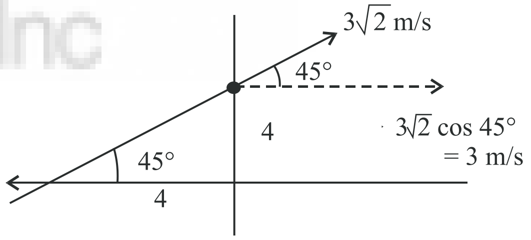
$$\therefore \quad I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \quad \frac{MR^2\omega_1}{2} = \frac{9MR^2}{8\times 2}\omega_2 \Rightarrow \omega_1 = \frac{9}{8}\omega_2$$

Given, $\omega_1 = \omega$

$$\therefore \quad \omega_2 = \frac{8}{9}\omega_1 = \frac{8}{9}\omega.$$

54. **(b)**



$L = m\,V\!\perp r$

$= 5\times 3\times 4 = 60$ units

55. **(c)** For solid sphere, $I = \frac{2}{5}mr^2 = 20$

$\therefore \quad mr^2 = 20\times\frac{5}{2} = 50$

For thin spherical shell,

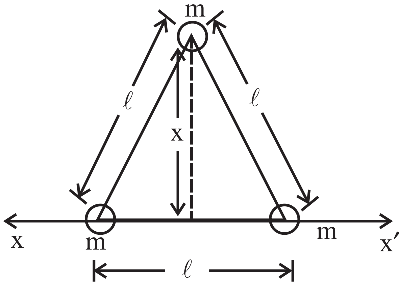
$$I = \frac{2}{3}mr^2 = \frac{2}{3}\times 50 = 33.3\ \text{kg m}^2$$

56. **(b)** The moment of inertia of bend rod is

$$I = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 + \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{24}ML^2 + \frac{1}{24}ML^2 = \frac{1}{12}ML^2$$

57. **(c)**



The moment of inertia of the system of particle about XX′ is

$I = mx^2 = m\,(l \sin 60°)^2 = \frac{3}{4} ml^2$

58. **(c)** m = 20 kg, d = 20 cm ∴ r = 10 cm

∴ The moment of inertia of a solid sphere about its tangent is

$$I = \frac{2}{5}mr^2 + mr^2 = \frac{7}{5}mr^2$$

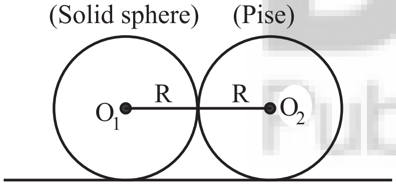
$$= \frac{7}{5} \times 20 \times 10 \times 10 \times 10^{-4} = 0.28 \text{ kg m}^2$$

59. **(a)** MOI of a solid cylinder, $I_1 = \frac{1}{2}mr^2$

MOI of a hollow cylinder, $I_2 = mr^2$

∴ $I_1 < I_2$

60. **(c)** $I_1 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$



$$I_2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

∴ Moment of inertia about point of contact,

$$I = I_1 + I_2 = \left(\frac{7}{5} + \frac{3}{2}\right)MR^2 = \frac{29MR^2}{10}$$

61. **(c)** About the diameter moment of inertia of the loop = moment of inertia of the ring $I = \frac{1}{2}mR^2$ where R = radius of the loop

Here, $\rho = \frac{m}{l}$ or, $\rho = \frac{m}{2\pi R} \Rightarrow R = \frac{m}{2\pi\rho}$

$$\therefore \quad I = \frac{1}{2}m\left(\frac{m}{2\pi\rho}\right)^2 = \frac{m^3}{8\pi^2\rho^2}$$

62. **(a)** Kinetic Energy of Moon = Transational Kinetic Energy + Rotational Kinetic Energy.

$$KE = KE_T + KE_R = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}M\left(R^2\omega^2\right) + \frac{1}{2}\left(\frac{2}{5}Mr^2\right)\left(\omega^2\right) \quad \left[\begin{array}{l} \omega = \frac{2\pi}{T} \\ I = \frac{2}{5}Mr^2 \end{array}\right]$$

$$= \frac{MR^2 \times 4\pi^2}{2 \times T^2} + \frac{Mr^2}{5} \times \frac{4\pi^2}{T^2}$$

$$\boxed{KE = \frac{2M\pi^2R^2}{T^2} + \frac{4Mr^2\pi^2}{5T^2}}$$

63. **(a)** $\frac{R_1}{R_2} = \frac{2}{3}$

Moment of inertia, $I = \frac{2}{5}MR^2$

$I \propto R^2$

$$\frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

64. **(d)** For without slipping

v = rω

Total energy,

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}m\,v^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}m\,r^2\omega^2$$

Rotational kinetic energy, $E_A = \frac{1}{2}I\omega^2$

Given, $E_R = \frac{50E}{100}$

$$\frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{1}{2}I\omega^2 + \frac{1}{2}m\omega^2r^2\right]$$

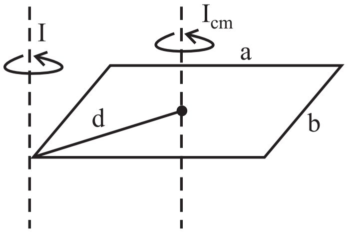
$$\frac{1}{2}I\omega^2 = \frac{1}{4}I\omega^2 + \frac{1}{4}m\omega^2r^2$$

$$I = \frac{1}{2}I + \frac{1}{2}m\,r^2$$

$$\frac{I}{2} = \frac{1}{2}m\,r^2 \Rightarrow I = m\,r^2$$

It's moment of inertia of thin circular ring.

65. **(d)**



By parallel axes theorem.

$I = I_{cm} + md^2$

$$= \frac{m}{12}\left(a^2 + b^2\right) + m\left(\frac{a^2 + b^2}{4}\right) = \frac{1}{3}m\left(a^2 + b^2\right)$$

66. **(d)** We have

$$I_{sphere} = \frac{2}{5}MR^2 = \frac{2}{5}\rho \times \frac{4}{3}\pi R^3 \times R^2$$

$I_{sphere} \propto R^5$.

$$\Rightarrow \frac{I_1}{I_2}=\left(\frac{R_1}{R_2}\right)^5=\left(\frac{R}{\frac{R}{2}}\right)^5=\frac{32}{1}$$

$$\Rightarrow I_2=\frac{I_1}{32}$$

67. **(c)** Given, moment of inertia, $I = 40$ kg-m^2

Initial angular frequency,

$\omega_0 = 50$ rpm

$$=\frac{50\times 2\pi}{60}=\frac{5\pi}{3}rads^{-1}$$

Final angular velocity, $\omega = 0$ rads^{-1}

From fitst eq^n of motion

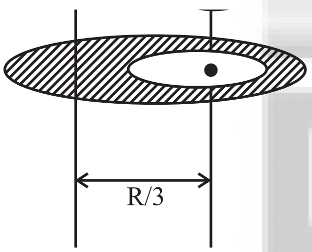
$\omega = \omega_0 + \alpha t$

Therefore,

$$\alpha=\frac{\omega-\omega_0}{t}=\frac{0-\frac{5\pi}{3}}{90}=\frac{-\pi}{3\times 18}=\frac{-\pi}{54}\text{rad } s^{-1}$$

Option (c) is correct

68. **(b)** We have, $\sigma=\frac{m}{4\pi R^2}$



Fill the hole with mass. Now consider the system as (complete disk + hole filled with negative mass)

About centre, $I_{net} = I_{disk} + I_{hole}$

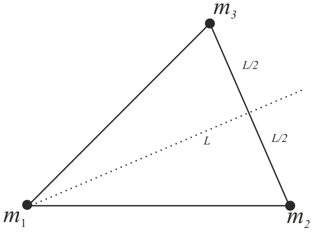
$$=\frac{1}{2}mR^2+\frac{1}{2}(-\sigma)\left[4\pi\left(\frac{R}{3}\right)^2\right]\left[\frac{R}{3}\right]^2-\sigma\left[4\pi\left(\frac{R}{3}\right)^2\right]\left[\frac{R}{3}\right]^2$$

$$=\frac{1}{2}mR^2-\frac{3}{2}\left(\frac{m}{4\pi R^2}\right)\left(4\pi\frac{R^2}{9}\right)\left(\frac{R^2}{9}\right)$$

$$=\frac{1}{2}mR^2-\frac{3}{2\times 81}mR^2=\frac{mR^2}{2}\left(1-\frac{1}{27}\right)$$

$$=\frac{mR^2}{2}\times\frac{26}{27}=\frac{13\ mR^2}{27}$$

69. **(d)** The given situation is shown below



Moment of inertia a particle $I = md^2$

where, d is the perpendicular distance of particle from axis choosen

$$I=m_2\left(\frac{L}{2}\right)^2+m_3\left(\frac{L}{2}\right)^2+m_1(0)^2$$

$$=\frac{m_2L^2}{4}+\frac{m_3L^2}{4}=(m_2+m_3)\frac{L^2}{4}$$

70. **(b)** Given, mass of rods, $m = 1$ kg

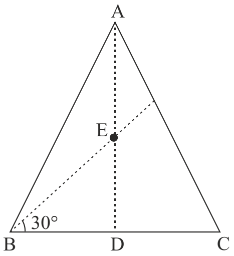
Length of rod, $l = 2$ m

Moment of inertia of rod about centre of rod,

$$I_{CM}=\frac{ml^2}{12}$$

D, E and F are mid points of side AC, AB and BC, O is centroid

$$\tan 30^\circ=\frac{ED}{BD}\Rightarrow\frac{1}{\sqrt{3}}=\frac{ED}{l/2}\quad\Rightarrow\quad ED=\frac{l}{2\sqrt{3}}=d$$



From parallel axis theorem,

$I = I_{cm} + md^2$

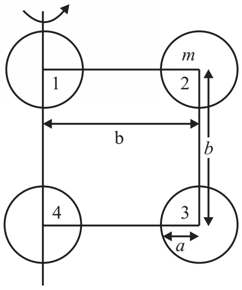
and for three rods,

$$I=3\left[\frac{ml^2}{12}+m\left(\frac{l}{2\sqrt{3}}\right)^2\right]$$

As $\left[d=\frac{l}{2\sqrt{3}}\right]$

$$=3\left[\frac{ml^2}{12}+\frac{ml^2}{12}\right]=\frac{ml^2}{2}=\frac{1\times 2^2}{2}=2\text{kg-m}^2$$

71. **(d)**



So, net moment of inertia on sphere

$I = (I_1 + I_4) + (I_2 + I_3)$

$$\Rightarrow I=2\times\left(\frac{2}{5}ma^2+mb^2\right)+2\times\left(\frac{2}{5}ma^2\right)$$

$$=\frac{8ma^2}{5}+2mb^2$$

72. **(a)** Moment of inertia (I) for following bodies are

(i) Ring = MR^2 (ii) Angular disc $=\frac{M}{2}(R^2-r^2)$

(iii) Solid disc = MR^2/2 (iv) Cylindrical disc = MR^2/2

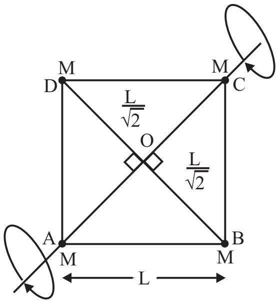
Clearly, MoI of ring is maximum

73. **(c)** Moment of inertia of a plate rectangular

$$I = \frac{M(L^2 + B^2)}{12}$$

This rexestt should be remenered by practice.

74. **(b)** The given masses are shown in figure.

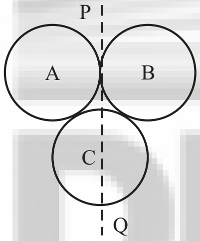


∴ Moment of inertia of given system about the diagonal *AC*

IAC = *IA* + *IB* + *IC* + *ID*

$$= 0 + M\left(\frac{L}{\sqrt{2}}\right)^2 + 0 + M\left(\frac{L}{\sqrt{2}}\right)^2 = \frac{ML^2}{2} + \frac{ML^2}{2} = ML^2$$

75. **(a)** Moment of inertia of each solid sphere,



$$I = \frac{2}{5}mr^2$$

where, *m* = mass and *r* = radius

The moment of inertia of the system about axis *PQ* as shown in the figure

IPQ = *IA* + *IB* + *IC*

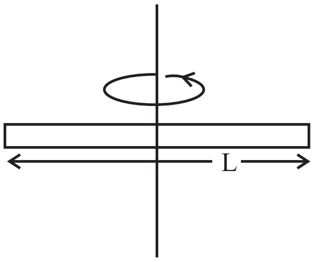
$$= (I_{cm} + mr^2) + (I_{cm} + mr^2) + \frac{2}{5}mr^2$$

$$= \left[\left(\frac{2}{5}mr^2\right) + mr^2\right] + \left[\left(\frac{2}{5}mr^2\right) + mr^2\right] + \left[\frac{2}{5}mr^2\right]$$

$$= \frac{16}{5}mr^2$$

76. **(c)** Rotational kinetic energy,

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{12}\right)\omega^2$$



$$K_{\text{rot}} = \frac{1}{24}ML^2\omega^2 \quad \text{...(i)}$$

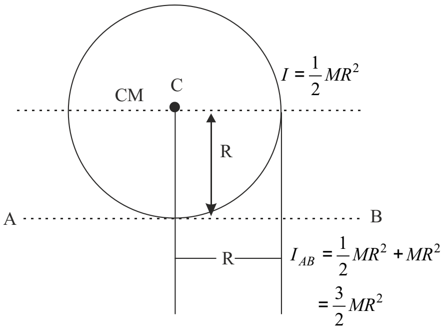
But M = volume × density = AL × ρ

M = ALρ ...(ii)

From Eqs. (i) and (ii), we have

$$K_{\text{rot}} = \frac{1}{24} \times AL\rho \times L^2\omega^2 = \frac{1}{24}AL^3\rho\omega^2$$

77. **(d)**



Mass of the wire, M = ρ*l* ...(i)

and l = 2πR or $r = \frac{1}{2\pi}$ ...(ii)

Now, from Eq. (i) and (ii) putting the value of *M* and *R* in Eq., we get

$$I = \frac{3}{2}(\rho l) \times \left(\frac{l}{2\pi}\right) \Rightarrow I = \frac{3\rho l^3}{8\pi^2}$$

78. **(a)** Given, mass of solid sphere or disc *M* = 100 kg

Radius of solid sphere, *R* = 10 m

Radius of circular disc, *r* = 20 m

and time = 1 hour = 60 minute = 60 × 60 sec

Moment of inertia of the solid sphere,

$$I_s = \frac{2}{5}MR^2 = \frac{2}{5} \times 100 \times (10)^2 = 4000 \text{ kg-m}^2$$

Similarly,

Moment of inertia of the disc,

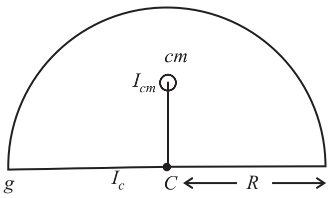
$$I_c = \frac{1}{2}Mr^2 = \frac{1}{2} \times 100 \times (20)^2 = 20{,}000 \text{ kg-m}^2$$

Rate of change of moment of inertia

$$= \frac{I_c - I_s}{t} = \frac{20000 - 4000}{60 \times 60} = \frac{16000}{60 \times 60} = \frac{160}{36}$$

$$= \frac{40}{9} \text{ kg-m}^2\text{s}^{-1}$$

79. **(d)** Using parallel axis theorem,



$$I_c = I_{cm} + mx^2$$

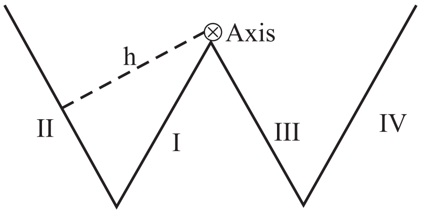
$$\frac{mR^2}{2} = I_{cm} + mx^2$$

Using $I_c = \frac{mR^2}{2}$

$$\Rightarrow I_{cm} = \frac{mR^2}{2} - mx^2$$

80. **(a)** There are four rods each of mass m = 400 g = 0.4 kg and each having a length of I = 30 cm = 0.3 m

So, we can calculate moment of inertia as follows



$$h^2 = l^2 - \left(\frac{l}{2}\right)^2 = \frac{3}{4}l^2$$

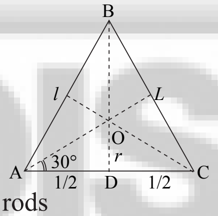
M.I. system = (MI of I) × 2 + (MI of II) × 2

$$= \frac{ml^2}{3} \times 2 + 2\left(\frac{ml^2}{12} + mh^2\right) = \frac{2}{3}ml^2 + 2\left(\frac{ml^2}{12} + \frac{3}{4}ml^2\right)$$

$$= \frac{7}{3}ml^2 = \frac{7}{3} \times 0.4 \times 0.3 \times 0.3 = 0.084 \text{ kg-m}^3$$

81. **(b)** Let the length of the rod be *l*.

The moment of inertia of one rod separately about an axis passing through the centre of the rod and perpendicular to its length, $I = I_{Ac} + mr^2$



$$\frac{1}{12}ml^2 + m\left(\frac{l}{2\sqrt{3}}\right)^2$$

$$= \frac{ml^2}{12} + \frac{ml^2}{12} = \frac{2ml^2}{12}$$

Moment of inertia of the system of 3rods

$$= 3 \times \frac{2ml^2}{12} = 6 \times \frac{ml^2}{12} = 6l$$

According to the question $\frac{6ml^2}{12} = l$

$$= n.\left(\frac{ml^2}{12}\right)$$

n = 6

82. **(b)** According to question,

By perpendicular axis theorem

$$I_2 = I_1 + MR^2$$

$$I_2 - I_1 = MR^2$$

83. **(b)** For rolling on inclined plane,

$$a = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}$$

$$\therefore \quad a_{\text{ring}} = \frac{g\sin\theta}{1+1} = \frac{1}{2}g\sin\theta = 0.5g\sin\theta$$

$$a_{\text{solid sphere}} = \frac{g\sin\theta}{1 + \frac{2}{5}} = \frac{5}{7}g\sin\theta = 0.71\,g\sin\theta$$

$$a_{\text{solid cylinder}} = \frac{g\sin\theta}{1 + \frac{1}{2}} = \frac{2}{3}g\sin\theta = 0.66g\sin\theta$$

Also, vμa

$\therefore \quad v_{\text{solid sphere}} > v_{\text{solid cylinder}} > v_{\text{ring}}$

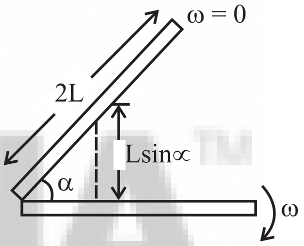
84. **(b)** For ring, $\frac{K^2}{R^2} = 1$

For disc, $\frac{K^2}{R^2} = \frac{1}{2}$

$$\frac{(\text{K.E})_{\text{ring}}}{(\text{K.E})_{\text{disc}}} = \frac{\left[\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)\right]_{\text{ring}}}{\left[\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)\right]_{\text{disc}}}$$

$$= \frac{(1+1)}{\left(1 + \frac{1}{2}\right)} = \frac{2 \times 2}{3} = 1.33$$

85. **(a)**



By conservation of energy,

Loss in PE = Gain in KE

$$\Rightarrow \quad mg(L\sin\mu) \ \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad mg\,L\sin\mu = \frac{1}{2}\left[\frac{1}{3}m(2L)^2\right]\omega^2$$

$$\therefore \quad \omega = \sqrt{\frac{3g\sin\alpha}{2L}}$$

86. **(d)** The speed of cylinder at the bottom of inclined plane is

$$v = \sqrt{\frac{2gh}{1 + \frac{K^2}{R^2}}} = \sqrt{\frac{2gh}{\left(1 + \frac{1}{2}\right)}} = \sqrt{\frac{4gh}{3}}$$

87. **(d)** For solid cylinder, $\frac{K^2}{R^2} = \frac{1}{2}$

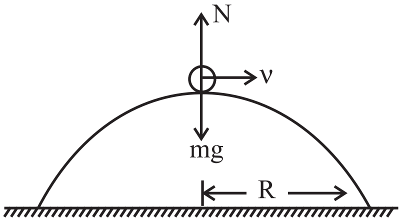
$$\therefore \quad a_1 = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}} = \frac{g\sin\theta}{1 + \frac{1}{2}} = \frac{2}{3}g\sin\theta = 4$$

$\therefore \quad g\sin\theta = 6$

For hollow cylinder, $\frac{K^2}{R^2} = 1$

$$\therefore \quad a_2 = \frac{g\sin\theta}{1+\frac{k^2}{R^2}} = \frac{1}{2}g\sin\theta = \frac{1}{2}\times 6 = 3\ \text{m/s}^2$$

88. **(c)**



At the top of hemisphere,

$$mg - N = \frac{mv^2}{R}$$

For the disc leaves the hemisphere,

N = 0

$$\therefore \ mg = \frac{mv^2}{R} \Rightarrow v = \sqrt{gR}$$

89. **(c)** For solid cylinder, $\frac{K^2}{R^2} = \frac{1}{2}$

$$(K.E)_{Total} = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$$

$$= (K.E)_T\left(1+\frac{K^2}{R^2}\right) = 140\left(1+\frac{1}{2}\right) = 210\ J$$

90. **(c)** For the rolling body,

Loss in KE = Gain in PE

$$\Rightarrow \ \frac{1}{2}mV^2\left(1+\frac{K^2}{R^2}\right) = mgh = mg\left(\frac{V^2}{g}\right) = mV^2$$

$$\therefore \ \frac{K^2}{R^2} = 1 \Rightarrow \text{The body is circular ring}$$

91. **(d)** For rolling on an inclined plane,

$$\text{Time, } t = \sqrt{\frac{2l}{a}} = \sqrt{\frac{2l}{g\sin\theta}\left(1+\frac{K^2}{R^2}\right)} \ \Rightarrow \ t \propto \sqrt{1+\frac{K^2}{R^2}}$$

$$\therefore \ \text{For hollow cylinder, } \frac{K^2}{R^2} = 1 \Rightarrow t_1 \propto \sqrt{2}$$

$$\text{For solid cylinder, } \frac{K^2}{R^2} = \frac{1}{2} \Rightarrow t_2 \propto \sqrt{\frac{3}{2}}$$

$$\therefore \ \frac{t_2}{t_1} = \frac{\sqrt{3}/2}{\sqrt{2}} \Rightarrow t_2 = \frac{\sqrt{3}}{2}t_1 = \frac{\sqrt{3}}{2}\times 2 = 1.732s$$

92. **(a)** Velocity of sphere rolls down without slipping

$$v = \sqrt{\frac{2gh}{1+\frac{k^2}{k^2}}} = \sqrt{\frac{2\times10\times28}{1+\frac{2}{5}}} = \sqrt{\frac{2\times10\times28\times5}{7}} = 20m/s$$

93. **(a)** Solid sphere with velocity, $v_1 = 3.5$ m/s

By conservation of angular momentum

$L_i = L_f$

$mv_1R = I\,\omega + mv_2R$

after some time sphere starts pure rolling

$v_2 = R\omega$

Moment of inertia of solid sphere, $I = \frac{2}{5}mR^2$

$$m\,v_1R = \frac{2}{5}mR^2 \times \frac{v_2}{R} + mv_2R$$

$$v_1 = \frac{2}{5}\ v_2 + v_2$$

$$\Rightarrow v_2 = \frac{5v_1}{7} = \frac{5}{7}\times 3.5 = 2.5\ \text{m/s}$$

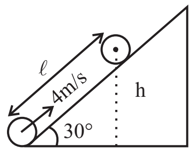
94. **(c)** As $P.E_i + (K.E)_i = (P.E)_f + (K.E)_f$

$$\Rightarrow \ 0 + \frac{1}{2}\times m\times 4^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\times\frac{4^2}{R^2} = mgh$$

$$\Rightarrow \ \frac{4^2}{2} + \frac{1}{5}\times 4^2 = 10h$$

$$\Rightarrow \ 8 + 3.2 = 10h$$

$$\Rightarrow \ 1.12 = h \Rightarrow h = 1.12m$$



$$\text{So, } \ell = \frac{h}{\sin 30°} = \frac{h}{1/2} = 2h = 2.24m = 224\ \text{cm}$$

95. **(b)** In rolling motion,

$$V = \sqrt{\frac{2gh}{1+\frac{I}{mr^2}}}$$

$$V = \sqrt{\frac{2\times10\times30}{1+\frac{1}{2}}} = \sqrt{\frac{600}{\frac{3}{2}}} = 20\ m/s$$

96. **(c)** The given situation is shown in figure.

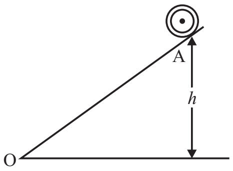
According to conservation of mechanical energy.

$M{\cdot}E_i = M{\cdot}E_f$

$$\Rightarrow \ U_i + (K_r)_{initial} + K_i = U_f + (K_r)_f + K_f$$

$$\Rightarrow mgh + 0 + 0$$

$$= \ 0 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$



$$\Rightarrow \ mgh + \frac{1}{2}\pm\omega^2 + \frac{1}{2}mv^2$$

$$\therefore \ mgh = \frac{1}{2}\times\frac{1}{2}mR^2\times\frac{v^2}{R^2} + \frac{1}{2}mv^2$$

$$\left[\text{For disc } I = \frac{1}{2}mR^2\right]$$

$$\Rightarrow \ v = \sqrt{\frac{4}{3}gh} \quad \therefore \ v \propto \sqrt{h}$$

∴ So speed depends on height of incline plane.

97. **(b)** Initial angular frequency, ωi = 180 rpm = 6π rad/sec
Final angular frequency, ωf = 360 rpm = 12π rad/sec
According to question,

$$\Delta E = \frac{1}{2}I(\omega_f^2 - \omega_i^2) \Rightarrow 684 = \frac{1}{2}I[(12\pi)^2 - (6\pi)^2]$$

$$\Rightarrow I = \frac{684 \times 2}{(144\pi^2 - 36\pi^2)} = 1.28 \text{ kg-m}^2$$

98. **(c)** For a rolling body on inclined plane,

$$a = \frac{mg\sin\theta}{m + \frac{I}{R^2}} = \frac{g\sin\theta}{1 + \frac{I}{mR}}$$

$\because$ Distance covered by body, $s = \frac{1}{2}at^2$

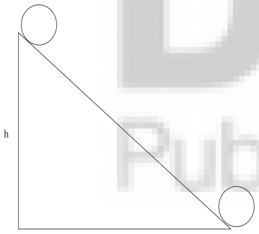
$\because$ $S_{\text{sphere}} = S_{\text{cylinder}}$

$$\frac{1}{2}a_{\text{sphere}} \times t^2 = \frac{1}{2} \times a_{\text{cylinder}} \times t^2 \Rightarrow a_{\text{sphere}} = a_{\text{cylinder}}$$

$$\Rightarrow \frac{g\sin 30^\circ}{1 + \frac{\frac{2}{5}mR^2}{mR^2}} = \frac{g\sin\theta}{1 + \frac{mR^2}{mR^2}} \Rightarrow \frac{\frac{1}{2}}{\frac{7}{5}} = \frac{\sin\theta}{2}$$

$$\Rightarrow \sin\theta = \frac{5}{7} \Rightarrow \theta = \sin^{-1}\left(\frac{5}{7}\right) = 45.6^\circ \approx 45^\circ$$

99. **(c)** Loss in granutational PE = increase in KE (Rotational + transtation)



Applying energy conservation between top and bottom

Hence, $Mgh = \frac{1}{2}\omega^2 + \frac{1}{2}Mv^2$ [v is velocity of centre]

$$= \frac{1}{2} \times \frac{MR^2}{2} \times \left(\frac{v}{R}\right)^2 + \frac{1}{2}Mv^2 \quad \left[\text{As } I = \frac{MR^2}{2} \text{ and } \omega = \frac{v}{R}\right]$$

$$= \frac{Mv^2}{4} + \frac{Mv^2}{2} \Rightarrow Mgh = \frac{3}{4}Mv^2 \Rightarrow v = \sqrt{\frac{4gh}{3}}$$

100. **(d)** According to work energy theorem, work required to stop the ring.

W = change in kinetic energy

$$= Kf - Ki = 0 - K_{\text{rotational}} + K_{\text{linear}} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$= \frac{1}{2}mR^2\left(\frac{v}{R}\right)^2 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$

$= 10 \times (1.5)^2 = 22.5$ J = – 22.5 J

101. **(d)** Mass of a circular disc, M = 12 kg
Radius (R) = 0.5 m
Angular velocity (ω) = 100 rad/s
$\therefore$ Rotational kinetic energy of the disc is equal to

$$\text{K.E.} = \frac{1}{2}I\omega^2$$

Using $I = \frac{1}{2}MR^2$, we get

$$\text{K.E.} = \frac{1}{2} \cdot \frac{1}{2}MR^2 \cdot \omega^2$$

Putting the values of M, ω 4R in the above relation,

$$\text{K.E.} = \frac{1}{4} \times 12 \times (0.5)^2 \times 100 \times 100$$

$= 7500$ J $= 7.5 \times 10^3$ 5 J = 7.5 kJ

102. **(c)** For rolling motion,

$$\text{KE}_{\text{rot}} = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)$$

Given, rolling velocity on horizontal surface,
v_H = 10 m/s, mass of ball, m = 11 kg and g = 10 m/s^2

$$\therefore \text{KE}_{\text{rot}} = \frac{1}{2} \times 11 \times (10)^2\left(1 + \frac{\frac{2}{5}R^2}{R^2}\right) = 11 \times 50 \times \frac{7}{5}\text{J}$$

$$\left(\because K_{\text{solid ball}} = \sqrt{\frac{2}{5}}R\right)$$

Now applying the law of energy conservation,
KEi + ui = KEf + uf

$$\Rightarrow 11 \times 50 \times \frac{7}{5} + 0 = 0 + mgh$$

$$\therefore h = \frac{50}{10} \times \frac{7}{5} = 7 \text{ m.}$$

103. **(d)** Velocity of body rolling down on inclined plane is given by

$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$, where v = velocity of body when it reaches the ground.

where, h = height of inclined plane, R = radius of body, and k = radius of gyration

Find a ring, $k^2 = R^2$

$$\therefore \text{Speed of the ring, } v_R = \sqrt{\frac{2gh}{1 + \frac{R^2}{R^2}}} = \sqrt{gh} \quad \ldots\text{(i)}$$

For a solid sphere, $k^2 = \frac{2R^2}{5}$

$$\therefore \text{Speed of the solid sphere, } v_s = \sqrt{\frac{2gh}{1 + \frac{2}{5}}}$$

$\Rightarrow \sqrt{\frac{10gh}{7}} = 1.19\sqrt{gh}$ …(ii)

For a solid disc, $k^2 = \frac{R^2}{2}$

∴ Speed of the solid disc, $v_D = \sqrt{\frac{2gh}{1+\frac{1}{2}}}$

$\Rightarrow \sqrt{\frac{4gh}{3}} = 1.15\sqrt{gh}$ …(iii)

From Eqs. (i), (ii) and (iii), it is clear that $V_S > V_D > V_R$.

104. **(c)** Here, $\theta = \sin^{-1}(0.42) \Rightarrow \sin\theta = 0.42$

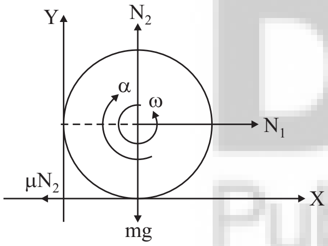
We have, $a = \frac{g\sin\theta}{1+\frac{1}{MR^2}}$

for a body rolling on inclined plane having inclinction 'θ'

But $I = \frac{2}{5}MR^2$

$$a = \frac{g\sin\theta}{1+\frac{\frac{2}{5}MR^2}{MR^2}} = \frac{5}{7}g\sin\theta = \frac{2}{5}\times10\times0.42 = 3m/s^2$$

105. **(c)**



Torque about centre of cylinder is

$$\tau = \mu N_2 R = \frac{mR^2}{2}\alpha$$

$\Rightarrow \mu\, mg\, R = \frac{mR^2}{2}\alpha \Rightarrow \alpha = \frac{2\mu g}{R}$

Now using,

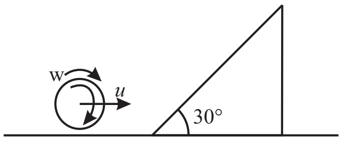
$\omega^2 = \omega_0^2 - 2\alpha\Delta\theta$

$\Rightarrow 0 = (20)^2 - 2\left(\frac{2\times\mu\times10}{1}\right)(10\pi)$ [∵ $\Delta\theta = 5\times2\pi$]

So, $(20)^2 = 40\times10\pi(\mu)$

$\Rightarrow \mu = \frac{1}{\pi}$

106. **(a)**



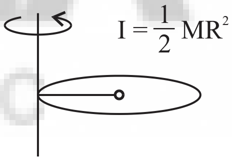
Given, $u = 4\ \text{ms}^{-1}$

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{10\times\sin30}{1+\frac{2}{5}} = \frac{5}{\frac{7}{5}} = \frac{25}{7}\ \text{m/s}^2$$

As $v^2 = u^2 + 2aS$

$$\Rightarrow |S| = \frac{u^2}{2a} = \frac{4^2}{2\times\frac{25}{7}} = \frac{16\times7}{50} = \frac{112}{50} = 2.24\ \text{m}$$

107. **(c)** $\text{KE} = \frac{1}{2}I\omega^2 = \frac{1}{2}\times\frac{3}{2}mr^2 4\pi^2 f^2$



$I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

$\Rightarrow$ KE = 3 $mr^2\times\pi^2 f^2$ = 3 × 5 × 1 × 10 × 1

KE = 150 J

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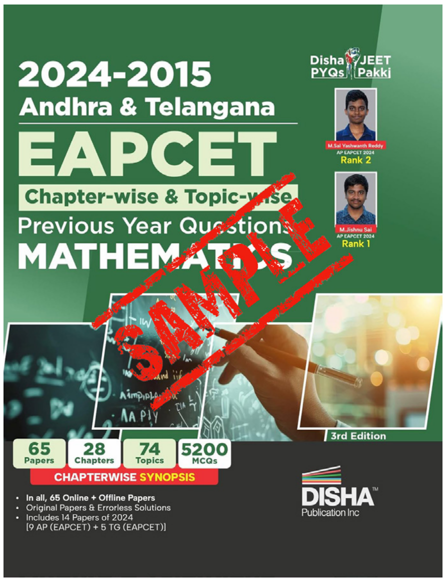

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5

Complex Numbers and Quadratic Equations

CHAPTER SYNOPSIS

- $\sqrt{-1} = i$
- For $z = x + iy, |z| = \sqrt{x^2 + y^2}; \bar{z} = x - iy$ & $\tan\theta = \frac{y}{x}$

  where q is the argument or amplitude of z.
- If z is the complex number, then $z^{-1} = \frac{\text{Re}(z)}{|z|^2} - i\frac{\text{Im}(z)}{|z|^2}$
- For z = x + iy,
  - its polar form is: $z = r(\cos\theta + i\sin\theta)$, where $r = |z|$.
  - its exponential form is $z = re^{i\theta}$.
- $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$. (Demoivre's Theorem)
- $\log_e(x + iy) = \log_e\sqrt{x^2 + y^2} + i \arg(z)$.
- $z_1 . z_2 = |z_1||z_2|(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$
- $\frac{z_1}{z_2} = \frac{|z_1|}{|z_1|}e^{i(\theta_1 - \theta_2)}$.
- $z^{1/n} = |z|^{1/n}$
- Cube roots of unity are 1, $\omega = \frac{-1 + i\sqrt{3}}{2}$ & $\omega^2 = \frac{-1 - i\sqrt{3}}{2}$.

  Also, (i) $1 + \omega + \omega^2 = 0$ (ii) $\omega^3 = 1$
- If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ be two complex numbers repersented by points P and Q respectively, then $PQ = |z_2 - z_1|$.
- Area of triangle ABC with vertices $A(z_1)$, $B(z_2)$ and $C(z_3)$ is given by $\Delta = \frac{1}{2}\begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix}$.
- Three points repersented by z_1, z_2 and z_3 are collinear, if $\begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix} = 0$
- Equation of a straight line passing through the point representing z_1 and z_2 is given by $\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$

Geometry of Complex Number

- $|z - z_1| + |z - z_2| = \lambda$, represents an ellipse if $|z_1 - z_2| < \lambda$, having the points z_1 and z_2 as its foci. And if $|z_1 - z_2| = \lambda$, then z lies on a line segment connecting z_1 and z_2.
- $|z - z_1| - |z - z_2| = \lambda$, represents a hyperbola if $|z_1 - z_2| > \lambda$, having the points z_1 and z_2 excluding the points between z_1 and z_2.
- The equation of a circle on a line segment joining points having affixes z_1 and z_2 as diameter is , $|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$
- If $z_1^2 + z_2^2 + 2z_1z_2\cos\theta = 0$, the points represented by z_1, z_2 and the origin form an isosceles triangle.
- Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively of an isosceles right angled triangle with right angle at C, then $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$.

- **Square Root of Complex Number**

  $$\sqrt{a + ib} = \pm\left(\sqrt{\frac{1}{2}\left(\sqrt{a^2 + b^2} + a\right)} + i\sqrt{\frac{1}{2}\left(\sqrt{a^2 + b^2} - a\right)}\right)$$
- For $ax^2 + bx + c = 0$; $D = b^2 - 4ac$
  - If D > 0; then its roots are real and different.
  - If D is a perfect square; then its roots are rational and different.
  - If D = 0; then its roots are real and equal.
  - If D < 0; then its roots are imaginary.
- For two quadratic equations; $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$.
  - If they have one root common, then $(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$.
  - If they have both roots common, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
- x^2 – (Sum of roots)x + (Product of roots) = 0
- If α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0$, Then, $\alpha+\beta+\gamma = \frac{-b}{a}$; $\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$ & $\alpha\beta\gamma = \frac{-d}{a}$.

Topic 1 Integral Powers of Iota, Algebraic Operations of Complex Numbers, Conjugate, Modulus and Argument or Amplitude of a Complex Number

1. $Arg\left[\frac{(1+i\sqrt{3})(-\sqrt{3}-i)}{(1-i)\,(-i)}\right]=$ **[AP/May 22, 2024 (II)]**

(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) $\frac{-\pi}{2}$

2. Imaginary part of $\frac{(1-i)^3}{(2-i)(3-2i)}$ is **[AP/May 21, 2024 (II)]**

(a) $\frac{22}{65}$ (b) $\frac{6}{65}$ (c) $-\frac{6}{65}$ (d) $-\frac{22}{65}$

3. The complex conjugate of (4 – 3i) (2 + 3i) (1 + 4i) is **[AP/May 21, 2024 (I)]**

(a) $7 + 74\,i$ (b) $-7 + 74\,i$

(c) $-7 - 74\,i$ (d) $7 - 74\,i$

4. If $z_1 = 10 + 6i$, $z_2 = 4 + 6i$ an z is any complex number such that the argument of $\frac{(z-z_1)}{(z-z_2)}$ is $\frac{\pi}{4}$,then **[AP/May 20, 2024 (II)]**

(a) $|z-7-9i| = 3\sqrt{2}$ (b) $|z-7-9i| = 2\sqrt{2}$

(c) $|z-3+9i| = 3\sqrt{2}$ (d) $|z+3-9i| = 2\sqrt{2}$

5. If the point p represents the complex number $z = x + iy$ in the argand plane and if $\frac{z+i}{z-1}$ is a purely imaginary number then the locus of p is **[AP/May 20, 2024 (I)]**

(a) $x^2 + y^2 + x - y = 0$ and $(x,y) \neq (1,0)$

(b) $x^2 + y^2 - x + y = 0$ and $(x,y) \neq (1,0)$

(c) $x^2 + y^2 - x + y = 0$ and $(x,y) = (1,0)$

(d) $x^2 + y^2 + x + y = 0$

6. If m,n are respectively the least positive and greatest negative integer values of k such that $\left(\frac{1-i}{1+i}\right)^k = -i$, then $m-n =$ **[AP/May 19, 2024 (II)]**

(a) 4 (b) 0 (c) 6 (d) 2

7. If real parts of $\sqrt{-5-12i}, \sqrt{5+12i}$ are positive values, the real part of $\sqrt{-8-6i}$ is a negative value and $a + ib = \frac{\sqrt{-5-12i}+\sqrt{5+12i}}{\sqrt{-8-6i}}$ then $2a + b =$ **[AP/May 18, 2024 (I)]**

(a) 3 (b) 2 (c) –3 (d) –2

8. If $z = \frac{(2-i)(1+i)^3}{(1-i)^2}$, then Arg ($z$) = **[TS/May 11, 2024 (I)]**

(a) $\tan^{-1}\left(\frac{1}{3}\right)-\pi$ (b) $\tan^{-1}\left(\frac{3}{4}\right)-\pi$

(c) $\pi-\tan^{-1}\left(\frac{3}{4}\right)$ (d) $\tan^{-1}\left(\frac{1}{3}\right)$

9. If $z = x + iy$ satisfies the equation $z^2 + az + a^2 = 0$, $a \in \mathbf{R}$, then **[TS/May 10, 2024 (II)]**

(a) $|z| = |a|$ (b) $|z-a| = |a|$

(c) $z = |a|$ (d) $z = a$

10. If Z_1, Z_2, Z_3 are three complex numbers with unit modulus such that $|Z_1-Z_2|^2+|Z_1-Z_3|^2=4$ then

$Z_1\overline{Z_2}+\overline{Z_1}Z_2+Z_1\overline{Z_3}+\overline{Z_1}Z_3=$ **[TS/May 10, 2024 (II)]**

(a) 0 (b) $|Z_2|^2+|Z_3|^2$

(c) $|Z_1|^2-|Z_2+Z_3|^2$ (d) 1

11. If $Z_1=\sqrt{3}+i\sqrt{3}$ and $Z_2=\sqrt{3}+i$, and $\left(\frac{Z_1}{Z_2}\right)^{50}=x+iy,$ then the point (x, y) lies in **[TS/May 10, 2024 (II)]**

(a) first quadrant (b) second quadrant

(c) third quadrant (d) fourth quadrant

12. If x and y are two positive real numbers such that $x+iy=\frac{13\sqrt{-5+12i}}{(2-3i)(3+2i)}$ then $13y - 26x =$ **[TS/May 10, 2024 (I)]**

(a) 28 (b) 39 (c) 42 (d) 54

13. If $\frac{(2-i)x+(1+i)}{2+i}+\frac{(1-2i)y+(1-i)}{1+2i}=1-2i,$ then $2x+4y=$ **[TS/May 9, 2024 (II)]**

(a) 5 (b) –2 (c) 1 (d) –1

14. If $z=1-\sqrt{3}i,$ then $z^3 - 3z^2 + 3z =$**[TS/May 9, 2024 (II)]**

(a) 0 (b) $1+3\sqrt{3}i$

(c) 1 (d) $2+3\sqrt{3}i$

15. In the Argand plane, the values of Z satisfying the equation |z – 1|=|i(z + 1)| lie on **[AP/May 18, 2023 (II)]**

(a) the Y - axis (b) a parabola

(c) a hyperbola (d) the X - axis

16. The modulus of the conjugate of $\mathrm{z}=\frac{-2+\mathrm{i}}{(1-2\mathrm{i})^2}$ is **[AP/May 17, 2023 (II)]**

(a) $\frac{1}{5}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{1}{25}$ (d) $\sqrt{5}$

17. If $z_1 = 2 + 5i$, $z_2 = -1 + 4i$ and $z_3 = 1$, then $\left|\frac{z_1-z_3}{z_3-z_2}\right|=$ **[AP/May 17, 2023 (II)]**

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $5\sqrt{2}$ (d) $4\sqrt{2}$

18. The locus of the variable point z = x + iy whose amplitude is always equal to θ, is **[AP/May 17, 2023 (II)]**

(a) $x^2 + y^2 = \tan^2\theta$ (b) $y = x\tan\theta$

(c) $\frac{x^2}{\sin^2\theta}+\frac{y^2}{\cos^2\theta}=1$ (d) $\frac{x^2}{\sin^2\theta}-\frac{y^2}{\cos^2\theta}=1$

19. For real numbers a and b, if 4a + i(3a – b) = b – 6i and $z = a + \frac{b}{4}i$, then $\frac{|z|}{a}=$ **[AP/May 17, 2023 (I)]**

(a) $2\sqrt{2}$ (b) $6\sqrt{2}$ (c) $\sqrt{2}$ (d) 2

20. If $z = (1 - i)^3 (x + i)$ is a purely imaginary number for $x = x_1$ and if z is a purely real number for $x = x_2$, then x_1x_2 = **[AP/May 17, 2023 (I)]**

(a) –1 (b) 0 (c) 1 (d) 2

21. If C is a point on the straight line joining the points A $(-2 + i)$ and B $(3 - 4i)$ in the Argand plane and $\frac{AC}{CB} = \frac{1}{2}$, then the argument of C is **[AP/May 16, 2023 (II)]**

(a) $\tan^{-1} 3$ (b) $\tan^{-1} 2 - \pi$
(c) $\tan^{-1} 2$ (d) $\pi - \tan^{-1} 3$

22. If $z_1 = 2 - 3i$ and the roots of the equation $z^3 + bz^2 + cz + d = 0$ are i, z_1 and $\bar{z}_1$, then b + c + d = **[AP/May 16, 2023 (I)]**

(a) 13 (b) 7 (c) 9 – 10i (d) 10 – 10i

23. If $z_1 = (2, -1)$ and $z_2 = (6, 3)$, then $\text{amp}\left(\frac{z_1 - z_2}{z_1 + z_2}\right) =$ **[AP/May 15, 2023 (I)]**

(a) $-\frac{3\pi}{4} - \tan^{-1}\left(\frac{1}{4}\right)$ (b) $\frac{\pi}{4}\tan^{-1}\left(\frac{1}{4}\right)$

(c) $\frac{3\pi}{4} + \tan^{-1}\left(\frac{1}{4}\right)$ (d) $\frac{\pi}{4} + \tan^{-1}\left(\frac{1}{4}\right)$

24. If $z = x + iy$ is a complex number such that $z\bar{z}^3 + \bar{z}z^3 = 350$ and x, y are integers, then |z| = **[TS/May 14, 2023 (II)]**

(a) $\sqrt{41}$ (b) 5 (c) 25 (d) $\sqrt{13}$

25. If $i = \sqrt{-1}$ then $\sum_{n=0}^{\infty}\left(\frac{i}{3}\right)^n =$ **[TS/May 14, 2023 (I)]**

(a) $\frac{9 - 3i}{10}$ (b) $9 - 3i$ (c) $9 + 3i$ (d) $\frac{9 + 3i}{10}$

26. If Z_1 and Z_2 are complex number such that $|Z_1 + Z_2| = |Z_1| + |Z_2|$ then the difference in the amplitudes of Z_1 and Z_2 is **[TS/May 12, 2023 (I)]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) 0

27. If $i = \sqrt{-1}$ then $1 + i^2 + i^4 + i^6 + ... + i^{2024} =$ **[TS/May 12, 2023 (I)]**

(a) i (b) –i (c) 1 (d) 1

28. Area of the triangle formed by the complex numbers z, iz and z + iz in the Argand diagram as vertices is **[AP/July 8, 2022 (I)]**

(a) $\frac{1}{2}.|z|^2$ (b) $\frac{1}{2}.z^2$ (c) z^2 (d) $|z|^2$

29. If the Arg z_1 and Arg z_2 are $\frac{\pi}{3}$ and $\frac{\pi}{5}$ respectively then the value of Arg z_1 + Argz_2 is **[AP/May 16, 2023 (I), (S), AP/July 8, 2022 (I)]**

(a) $\frac{11\pi}{15}$ (b) $\frac{6\pi}{15}$ (c) $\frac{2\pi}{15}$ (d) $\frac{8\pi}{15}$

30. $\left(\frac{1-i}{1+i}\right)^{2022} + \left(\frac{1+i}{1-i}\right)^{2021} =$ **[AP/May 15, 2023 (I), (S), AP/July 8, 2022 (I)]**

(a) –i (b) i (c) i + 1 (d) i – 1

31. $\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y}{3-i} = i \Rightarrow x + y =$ **[AP/July 7, 2022 (II)]**

(a) –1 (b) 1 (c) –2 (d) 2

32. If $(x + iy) = \left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$, then the true statement among the following is **[AP/July 6, 2022 (I)]**

(a) x < y (b) x > y (c) x ≠ 0 (d) x = y

33. The number of complex numbers z satisfying $\bar{z} = iz^2$ is **[AP/July 6, 2022 (I)]**

(a) 3 (b) 4 (c) 2 (d) 5

34. $\sum_{k=0}^{440} i^k = x + iy \Rightarrow x^{100} + x^{99}y + x^{242}y^2 + x^{97}y^3 =$ **[AP/July 4, 2022 (II)]**

(a) 0 (b) – 4 (c) 4 (d) 1

35. $\{x \in [0, 2\pi]/\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other$\}$ = **[TS/July 20, 2022 (I)]**

(a) $\left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi\right\}$

(b) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

(c) $\left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\right\}$

(d) ϕ

36. If $(2x - y + 1) + i(x - 2y - 1) = 2 - 3i$, then the multiplicative inverse of $(x - iy)$ is **[TS/July 19, 2022 (I)]**

(a) $\frac{15}{41} + \frac{12}{41}i$ (b) $\frac{6}{29} + \frac{15}{29}i$

(c) $\frac{15}{29} + \frac{6}{29}i$ (d) $\frac{12}{41} + \frac{15}{41}i$

37. If the point (x, y) satisfies the equation $\frac{x + i(x-2)}{3+i} - i = \frac{2y + i(1-3y)}{i-3}$, then x + y = **[TS/July 18, 2022 (II)]**

(a) 4 (b) 2 (c) 0 (d) –2

38. If $z \in C$, then the minimum value of $|z| + |2z - 3| + |z - 1|$ is **[AP/Aug. 23, 2021 (I)]**

(a) 2 (b) 1 (c) 3 (d) 0

39. Real part of $(\cos 4 + i\sin 4 + 1)^{2020}$ is

(a) $2^{2020}\cos^{2020}2\cos2020$ (b) $2^{2020}\cos^{2020}2\cos4040$
(c) $2^{1020}\cos^{2020}2\cos4040$ (d) $2^{2020}\cos^{2020}1\cos2020$

40. The locus of $z = x + iy$, such that $\text{Im}\left(\frac{z - 3i}{iz + 4}\right) = 0$ is

(a) $x^2 - y^2 + 7y - 12 = 0$ **[TS/Aug. 6, 2021 (I)]**
(b) $x^2 + y^2 - 7y + 12 = 0$
(c) $x^2 + y^2 - 7y + 12 = 0$ and $(x, y) \neq (0, 4)$
(d) $x^2 - y^2 + 7y - 12 = 0$ and $(x, y) \neq (0, 4)$

41. $\left(-i+\sqrt{3}\right)^{300}+\left(-i-\sqrt{3}\right)^{300}=$ **[AP/Sept. 21, 2020 (I)]**

(a) 2^{300} (b) 2^{301} (c) 2^{100} (d) -2^{300}

42. If $a+bi=\dfrac{i}{1-i}$, then $(a, b)=$ **[AP/Sept. 17, 2020 (I)]**

(a) $\left(\dfrac{-1}{2},\dfrac{-1}{2}\right)$ (b) $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$

(c) $\left(\dfrac{1}{2},\dfrac{-1}{2}\right)$ (d) $\left(\dfrac{-1}{2},\dfrac{1}{2}\right)$

43. The solutions of the equation $z^2(1-Z^2)=16$, $z \in \boldsymbol{C}$, lie on the curve **[TS/Sept. 10, 2020 (I)]**

(a) $|z|=1$ (b) $|z|=\dfrac{2}{|z|}$

(c) $|z|^2=3|z|+2$ (d) $|z|=2$

44. **Assertion :** If the arguments of $\bar{z}_1$ and z_2 are $\dfrac{\pi}{5}$ and $\dfrac{\pi}{3}$ respectively, then arg (z_1z_2) is $\dfrac{2\pi}{15}$.

Reason : For any complex number z, $\arg \bar{z}=\dfrac{\pi}{2}+\arg z$

The correct option among the following is **[TS/Sept. 9, 2020 (II)]**

(a) (A) is true, (R) is true and (R) is the correct explanation for (A)
(b) (A) is true (R) is true but (R) is not the correct explanation for (A)
(c) (A) is true but (R) is false
(d) (A) is false but (R) is true

45. If $A=\left\{z=x+iy\,/\text{ real part of }\dfrac{\bar{z}-1}{z-i}=2\right\}$, then the locus of the point $P(x, y)$ in the cartesian plane is **[TS/Sept. 9, 2020 (I)]**

(a) a pair of lines passing through (–1, –1)
(b) a circle of radius $\dfrac{1}{\sqrt{2}}$ and the centre $\left(\dfrac{-1}{2},\dfrac{3}{2}\right)$
(c) a pair of lines passing through (–1, –2)
(d) a circle of radius $\dfrac{1}{2}$

46. If $z=x+iy, x, y \in R, (x,y) \neq (0,-4)$ and $\operatorname{Arg}\left(\dfrac{2z-3}{z+4i}\right)=\dfrac{\pi}{4}$, then the locus of z is **[AP/Apr. 20, 2019 (I)]**

(a) $2x^2+2y^2+5x+5y-12=0$
(b) $2x^2-3xy+y^2+5x+y-12=0$
(c) $2x^2+3xy+y^2+5x+y+12=0$
(d) $2x^2+2y^2-11x+7y-12=0$

47. If $z=x+iy$, $x, y \in R$ and the imaginary part of $\dfrac{\bar{z}-1}{\bar{z}-i}$ is 1, then the locus of z is **[AP/Apr. 20, 2019 (I)]**

(a) $x+y+1=0$
(b) $x+y+1=0, (x, y) \neq (0,-1)$
(c) $x^2+y^2-x+3y+2=0$
(d) $x^2+y^2-x+3y+2=0, (x, y) \neq (0,-1)$

48. If z is a complex number such that $|z+4| \geq 3$, then the smallest value of $|z+3|$ is **[TS/May 4, 2019 (I)]**

(a) 3 (b) 1 (c) 2 (d) 0

49. The real part of z that satisfies $iz^4+1=0$ is **[TS/May 4, 2019 (I)]**

(a) $\sin\dfrac{\pi}{4}$ (b) $\cos\dfrac{\pi}{8}$ (c) 0 (d) -1

50. If x is real, then the interval in which no value of the expression $\dfrac{2(x^2+2x-11)}{2x-5}$ lies, is **[TS/May 4, 2019 (I)]**

(a) (2, 5) (b) (3, 6) (c) (3, 4) (d) (6, 8)

51. $i^2+i^3+\ldots+i^{4000}=$ **[TS/May 3, 2019 (I)]**

(a) 1 (b) 0 (c) i (d) $-i$

52. If the amplitude of $(z-1-2i)$ is $\dfrac{\pi}{3}$, then the locus of z is **[TS/May 3, 2019 (II)]**

(a) $y=\sqrt{3}x+(2-\sqrt{3})$ (b) $y=\sqrt{3}x-\sqrt{3}$

(c) $x=\sqrt{3}y+(2-\sqrt{3})$ (d) $y=\sqrt{3}x+2$

53. Imaginary part of $(\sqrt{3}-i)^{2016}+(-\sqrt{3}-i)^{2019}$ is **[TS/May 3, 2019 (II)]**

(a) 2^{2016} (b) -2^{2016} (c) -2^{2019} (d) 2^{2019}

54. Let $z=x+iy$ and a point P represent z in the Argand plane. If the real part of $\dfrac{\mathrm{z}-1}{\mathrm{z}+\mathrm{i}}$ is 1, then a point that lies on the locus of P is **[AP/2018]**

(a) (2016, 2017) (b) (–2016, 2017)
(c) (–2016, –2017) (d) (2016, –2017)

55. If $z+\dfrac{1}{z}=1$, then $\dfrac{(z^{2\theta}+1)\,(z^{4\theta}+1)\,(z^{6\theta}+1)}{z^{6\theta}}=$ **[TS/May 5, 2018 (I)]**

(a) -2 (b) 2 (c) 1 (d) -1

56. If $\dfrac{1-10i\cos\theta}{1-10\sqrt{3}\,i\sin\theta}$ is purely real then one of the values of θ is **[TS/May 4, 2018 (II)]**

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

57. If z and w are complex numbers such that $\bar{z}-i\bar{w}=0$ and $\arg(zw)=\dfrac{3\pi}{4}$, then $\arg z=$ **[TS/May 4, 2018 (II)]**

(a) $\dfrac{\pi}{16}$ (b) $\dfrac{\pi}{8}$ (c) $\dfrac{\pi}{4}$ (d) $\dfrac{3\pi}{4}$

58. The number of complex roots of the equation $x^{11}-x^7+x^4-1=0$ whose arguments lie in the first quadrant is **[TS/May 4, 2018 (II)]**

(a) 2 (b) 3 (c) 7 (d) 9

59. If z is a complex number with $|z| \geq 5$. Then the least value of $\left|z+\frac{2}{z}\right|$ is **[AP/2017]**

(a) $\frac{24}{5}$ (b) $\frac{26}{5}$ (c) $\frac{23}{5}$ (d) $\frac{29}{5}$

60. If $\log_{\frac{1}{\sqrt{3}}}\left\{\frac{|z|^2-|z|+1}{2+|z|}\right\} > -2$, then z lies inside **[AP/2017]**

(a) a triangle (b) an ellipse

(c) a circle (d) a square

61. If the conjugate of $(x+iy)(1-2i)$ is $(1+i)$, then **[TS/2017]**

(a) $x+iy=1-i$ (b) $x+iy=\frac{1-i}{1-2i}$

(c) $x-iy=\frac{1-i}{1+2i}$ (d) $x-iy=\frac{1-i}{1+i}$

62. $\frac{(1+i)^{2016}}{(1-i)^{2014}}$ is equal to **[AP/2016]**

(a) $-2i$ (b) $2i$ (c) 2 (d) -2

63. The minimum value of $|z-1|+|z-5|$ is **[TS/2016]**

(a) 5 (b) 4 (c) 3 (d) 2

Topic 2 Rotational Theorem, Square Root of a Complex Number, Cube Roots of Unity, Geometry of Complex Numbers, De-moivre's Theorem, Powers of Complex Numbers

64. ω is a complex cube root of unity and if Z is a complex number satisfying $|Z-1| \leq 2$ and $|\omega^2 Z - 1 - \omega| = a$, then the set of possible values of a is **[AP/May 23, 2024 (I)]**

(a) $0 \leq a \leq 2$ (b) $\frac{1}{2} \leq a \leq \frac{\sqrt{3}}{2}$

(c) $|\omega| \leq a \leq \frac{\sqrt{3}}{2}+2$ (d) $0 \leq a \leq 4$

65. If the roots of the equation $Z^3 + iZ^2 + 2i = 0$ are the vertices of a triangle ABC. then that triangle ABC is **[AP/May 23, 2024 (I)]**

(a) a right angled triangle

(b) an equilateral triangle

(c) an isosceles triangle

(d) a right angled isosceles triangle

66. (r, θ) denotes $r(\cos\theta + i\sin\theta)$. If $x = (l, \alpha)$,($y = (1, \beta)$, $z = (l, \gamma)$ and $x+y+z=0$ then $\sum \cos(2\alpha-\beta-\gamma) =$ **[AP/May 23, 2024 (I)]**

(a) 3 (b) 0 (c) 1 (d) -1

67. If P (x, y) represents the complex number $z = x+iy$ in the Argand plane and $Arg\left(\frac{z-3i}{z+4}\right)=\frac{\pi}{2}$, then the equation of the locus of P is **[AP/May 22, 2024 (II)]**

(a) $x^2+y^2+4x-3y=0$ and $3x-4y>0$

(b) $x^2+y^2+4x-3y+2=0$ and $3x-4y>0$

(c) $x^2+y^2+4x-3y=0$ and $3x-4y<0$

(d) $x^2+y^2+4x-3y+2=0$ and $3x-4y<0$

68. If Z is a complex number such that $|Z| \leq 3$ and $\frac{-\pi}{2} \leq amp\,Z \leq \frac{\pi}{2}$, then the area of the region formed by locus of Z is **[AP/May 22, 2024 (I)]**

(a) 9π (b) $\frac{9\pi}{2}$ (c) 3π (d) $\frac{9\pi}{4}$

69. The locus of the complex number Z such that $\arg\left(\frac{Z-1}{Z+1}\right)=\frac{\pi}{4}$ is **[AP/May 22, 2024 (I)]**

(a) a straight line (b) a circle

(c) a parabola (d) an ellipse

70. All the values of $(8i)^{\frac{1}{3}}$ are **[AP/May 22, 2024 (I)]**

(a) $\pm(\sqrt{3}+i), -2i$ (b) $\pm\sqrt{3}+i, -2i$

(c) $\pm\sqrt{3}-i, -2i$ (d) $\pm(2+i), i$

71. The square root of $7+24i$ **[AP/May 21, 2024 (II)]**

(a) $4-3i$ (b) $3+4i$ (c) $3-4i$ (d) $4+3i$

72. If n is an integer and $Z = \cos\theta + i\sin\theta$, $\theta \neq (2n+1)\frac{\pi}{2}$, then $\frac{1+Z^{2n}}{1-Z^{2n}} =$ **[AP/May 21, 2024 (II)]**

(a) $i\tan n\theta$ (b) $i\cot n\theta$

(c) $-i\tan n\theta$ (d) $-i\cot n\theta$

73. If the amplitude of $(Z-2)$ is $\frac{\pi}{2}$ then the locus of Z is **[AP/May 21, 2024 (I)]**

(a) $x=0, y>0$ (b) $x=2, y>0$

(c) $x>0, y=2$ (d) $x>0, y=0$

74. If ω is the cube root of unity $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}=$ **[AP/May 21, 2024 (I)]**

(a) 2 (b) -2 (c) 1 (d) -1

75. If $\frac{3-2i\sin\theta}{1+2i\sin\theta}$ is purely imaginary number, then $\theta =$ **[AP/May 20, 2024 (II)]**

(a) $2n\pi \pm \frac{\pi}{4}$ (b) $2n\pi \pm \frac{\pi}{2}$

(c) $n\pi \pm \frac{\pi}{3}$ (d) $n\pi \pm \frac{\pi}{6}$

76. If $\cos\alpha+\cos\beta+\cos\gamma = \sin\alpha+\sin\beta+\sin\gamma = 0$ then $(\cos^3\alpha+\cos^3\beta+\cos^3\gamma)^2+(\sin^3\alpha+\sin^3\beta+\sin^3\gamma)^2 =$ **[AP/May 20, 2024 (I)]**

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{9}{16}$ (d) $\frac{9}{8}$

77. If a complex number z is such that $\frac{z-2i}{z-2}$ purely imaginary number and the locus of z is a closed curve, then the area of the region bounded by that closed curve and lying in the first quadrant is **[AP/May 19, 2024 (II)]**

(a) 2π (b) $\frac{\pi}{2}$ (c) π (d) $\frac{\pi}{4}$

78. Real part of $\dfrac{(\cos a + i\sin a)^6}{(\sin b + i\cos b)^8}$ is **[AP/May 19, 2024 (II)]**

(a) $\sin(6a - 8b)$ (b) $\cos(6a - 8b)$

(c) $\sin(6a + 8b)$ (d) $\cos(6a + 8b)$

79. The set of all real values of c for which equation $z\overline{z} + (4-3i)\overline{z} + (4+3i)z + c = 0$ represents a circle is **[AP/May 18, 2024 (I)]**

(a) $[25, \infty)$ (b) $[-5, 5]$

(c) $(-\infty, -5] \cup [5, \infty)$ (d) $(-\infty, 25]$

80. If Z = $x + iy$ is a complex number, then the number of distinct solution of the equation $z^3 + \overline{z} = 0$ is **[AP/May 18, 2024 (I)]**

(a) 1 (b) 3 (c) Infinite (d) 5

81. $z = x + iy$ and the point P represents z in the Argand plane. If the amplitude of $\left(\dfrac{2z-i}{z+2i}\right)$ is $\dfrac{\pi}{4}$, then the equation of the locus of P is **[TS/May 11, 2024 (I)]**

(a) $2x^2 + 2y^2 - 3x + 3y - 2 = 0, (x, y) \neq (0, -2)$

(b) $2x^2 + 2y^2 + 5x + 3y - 2 = 0, (x, y) \neq (0, -2)$

(c) $2x^2 + 2y^2 + 3x + 3y - 2 = 0, (x, y) \neq (0, 2)$

(d) $2x^2 + 2y^2 - 5x + 3y - 2 = 0, (x, y) \neq (0, 2)$

82. α, β are the roots of the equation $x^2 + 2x + 4 = 0$. If the point representing α in the Argand diagram lies in the 2nd quadrant and $\alpha^{2024} - \beta^{2024} = ik, (i = \sqrt{-1})$, then $k =$ **[TS/May 11, 2024 (I)]**

(a) $-2^{2025}\sqrt{3}$ (b) $2^{2025}\sqrt{3}$

(c) $-2^{2024}\sqrt{3}$ (d) $2^{2024}\sqrt{3}$

83. If ω is the complex cube root of unity and

$$\left(\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}\right)^k + \left(\frac{a+b\omega+c\omega^2}{b+a\omega^2+c\omega}\right)^l = 2,$$

then $2k + l$ is always **[TS/May 10, 2024 (II)]**

(a) divisible by 2 (b) divisible by 6

(c) divisible by 3 (d) divisible by 5

84. With respect to the roots of the equation
$3x^3 + bx^2 + bx + 3 = 0$,
match the items of List-I with those of List-II.

	LIST-I		**LIST-II**
A.	All the roots are negative	I.	$(b-3)^2 = 36 + P^2$ for $P \in \mathbb{R}$
B.	Two roots are complex	II.	$-3 < b < 9$
C.	Two roots are positive	III.	$b \in (-\infty, -3) \cup (9, \infty)$
D.	All roots are real and distinct	IV.	$b = 9$
		V.	$b = -3$

[TS/May 10, 2024 (II)]

(a) A-V, B-III, C-I, D-II (b) A-IV, B-I, C-II, D-III

(c) A-V, B-II, C-III, D-I (d) A-IV, B-II, C-V, D-III

85. If $z = x + iy$ and if the point P represents z in the Argand plane, then the locus of z satisfying the equation $|z-1| + |z+i| = 2$ is **[TS/May 10, 2024 (I)]**

(a) $15x^2 - 2xy + 15y^2 - 16x + 16y - 48 = 0$

(b) $3x^2 + 2xy + 3y^2 - 4x - 4y = 0$

(c) $3x^2 - 2xy + 3y^2 - 4x + 4y = 0$

(d) $15x^2 + 2xy + 15y^2 + 16x - 16y - 48 = 0$

86. One of the values of $(-64i)^{5/6}$ is **[TS/May 10, 2024 (I)]**

(a) $32i$ (b) $16\sqrt{2}(1+i)$

(c) $32(1+i)$ (d) $16\sqrt{2}\,i$

87. α, β, γ are the roots of the equation $x^3 + 3x^2 - 10x - 24 = 0$. If $\alpha > \beta > \gamma$ and $\alpha^3 + 3\beta^2 - 10\gamma - 24 = 11k$, then $k =$ **[TS/May 10, 2024 (I)]**

(a) 1 (b) 11 (c) 5 (d) 55

88. The product of all the values of $(\sqrt{3} - i)^{2/5}$ is **[TS/May 9, 2024 (II)]**

(a) $2(\sqrt{3} - i)$ (b) $2(\sqrt{3} + i)$

(c) $2(1 - \sqrt{3}i)$ (d) $2(1 + \sqrt{3}i)$

89. The number of common roots among the 12th and 30th roots of unity is **[TS/May 9, 2024 (II)]**

(a) 12 (b) 9 (c) 8 (d) 6

90. If $\sqrt{5} - i\sqrt{15} = r(\cos\theta + i\sin\theta), -\pi < \theta < \pi$, then $r^2(\sec\theta + 3\operatorname{cosec}^2\theta) =$ **[TS/May 9, 2024 (I)]**

(a) 40 (b) 60 (c) 120 (d) 180

91. The point P denotes the complex number $z = x + iy$ in the Argand plane. If $\dfrac{2z-i}{z-2}$ is a purely real number, then the equation of the locus of P is **[TS/May 9, 2024 (I)]**

(a) $2x^2 + 2y^2 - 4x - y = 0$

(b) $x + 4y - 2 = 0$ and $(x, y) \neq (2, 0)$

(c) $x - 4y - 2 = 0$ and $(x, y) \neq (2, 0)$

(d) $x^2 + y^2 - 4x - 2y = 0$

92. x and y are two complex numbers such that $|x| = |y| = 1$. If $\operatorname{Arg}(x) = 2\alpha, \operatorname{Arg}(y) = 3\beta$ and $\alpha + \beta = \dfrac{\pi}{36}$, then $x^6y^4 + \dfrac{1}{x^6y^4} =$ **[TS/May 9, 2024 (I)]**

(a) 0 (b) –1 (c) 1 (d) $\dfrac{1}{2}$

93. The multiplicative inverse of z is **[AP/May 19, 2023 (I)]**

(a) $\dfrac{1}{z+\overline{z}}$ (b) $\dfrac{z}{|\overline{z}|}$ (c) $\dfrac{\overline{z}}{|z|^2}$ (d) $\dfrac{1}{\overline{z}}$

94. If $z_1 = 2 + 3i$, $z_2 = 4 - 5i$ and z_3 are are three points in the Argand plane such that $5z_1 + xz_2 + yz_3 = 0$ $(x, y \in \mathbb{R})$ and z_3 is the midpoint of the segment joining the points z_1 and z_2 then x + y = **[AP/May 19, 2023 (I)]**

(a) –5 (b) 0 (c) 4 (d) –1

95. If 1, ω, ω^2 are the cube roots of unity and $(x + y)(x\omega + y\omega^2)(x\omega^2 + y\omega) = f(x, y)$, then $f(2, 3) =$ **[AP/May 18, 2023 (II)]**

(a) 16 (b) 24 (c) 35 (d) 45

96. If ω is a complex cube root of unity, then
$\sin\left[\left(\omega^{10}+\omega^{23}\right)\pi-\frac{\pi}{4}\right]=$ **[AP/May 18, 2023 (I)]**
(a) $1/\sqrt{2}$ (b) 1/2 (c) 1 (d) $\sqrt{3}/2$

97. If 1, ω, ω^2 are the cube roots of unity, k is positive integer and $(1-\omega+\omega^2)^{3k}+(1-\omega^2+\omega)^{3k}=(1-\omega+\omega^2)^{3k+1}+(1+\omega-\omega^2)^{3k+1}$, then k = **[AP/May 17, 2023 (I)]**
(a) r, r, ∈ ℕ (b) 2r + 1, r ∈ ℕ
(c) 4r + 1, r ∈ ℕ (d) 3r, r ∈ ℕ

98. S = {z ∈ C/ |z – 1 + i| = 1} represents **[AP/May 15, 2023 (II)]**
(a) a circle with centre (–1, 1) and radius 1 unit
(b) a circle with centre (1, 2) and radius 5 units
(c) a circle with centre (1, –1) and radius 1 unit
(d) an ellipse with centre (1, –1)

99. If $\left|z-\frac{2}{z}\right|=2$, then the greatest value of |z| is **[AP/May 15, 2023 (II)]**
(a) $\sqrt{3}-1$ (b) $\sqrt{3}$ (c) $\sqrt{3}+1$ (d) $\sqrt{3}+2$

100. One of the 15th roots of –1 is **[AP/May 15, 2023 (II)]**
(a) cis 0 (b) $\text{cis}\frac{14\pi}{15}$ (c) $\text{cis}\frac{13\pi}{15}$ (d) $\text{cis}\frac{8\pi}{15}$

101. The number of all possible solutions of the equation $z^3+\bar{z}=0$ is **[AP/May 15, 2023 (I)]**
(a) 4 (b) 5 (c) 3 (d) 6

102. The locus of z such that $\left|\frac{z-i}{z+i}\right|=2$, where z = x + iy, is **[TS/May 14, 2023 (I)]**
(a) $3x^2+3y^2+10y+3=0$
(b) $3x^2-3y^2-10y-3=0$
(c) $3x^2+3y^2+10y-3=0$
(d) $x^2+y^2-5y+3=0$

103. One of the values of $\left(\sqrt{3}-i\right)^{\frac{2}{5}}$ is **[TS/May 13, 2023 (II)]**
(a) $2^{-}\left(1\quad\sqrt{3}i\right)$ (b) $2^{\frac{-3}{5}}\left(\sqrt{3}+i\right)$
(c) $2^{\frac{2}{5}}\left(\sqrt{3}-i\right)$ (d) $2^{\frac{-3}{5}}\left(1+\sqrt{3}\,i\right)$

104. If the value of $\sqrt{-5-12i}+\sqrt{7+24i}$ is a negative real number *k*, then *k* = **[TS/May 13, 2023 (I)]**
(a) – 5 (b) – 7 (c) – 6 (d) – 4

105. One of the values of $\left(\sqrt{3}-i\right)^{\frac{1}{6}}$ is **[TS/May 13, 2023 (I)]**
(a) $2^{\frac{1}{6}}\,cis\frac{61\pi}{36}$ (b) $2^{\frac{1}{6}}\,cis\frac{37\pi}{36}$
(c) $2^{\frac{1}{6}}\,cis\frac{59\pi}{36}$ (d) $2^{\frac{1}{6}}\,cis\frac{49\pi}{36}$

106. If x + iy = $\sqrt{\frac{3+i}{1+3i}}$, then $\left(x^2+y^2\right)^2=$ **[TS/May 12, 2023 (II)]**
(a) 0 (b) 1 (c) 2 (d) 3

107. If α, β are non-zero integers and z = (α + *i* β) (2 + 7*i*) is a purely imaginary number, then minimum value of $|z|^2$ is **[TS/May 12, 2023 (I)]**
(a) 0 (b) 2809 (c) 2808 (d) 1

108. If 1, ω, ω^2 are the cube roots of unity then the value of $(x+y)^2+(x\omega+y\omega^2)^2+(x\omega^2+y\omega)^2$ is **[AP/July 8, 2022 (I)]**
(a) $2x^2{\cdot}3y^2$ (b) 4xy
(c) 6xy (d) $2x^2{\cdot}2y^2$

109. Let z and w be two distinct non-zero complex numbers if $|z|^2w-|w|^2z=z-w$, then **[AP/July 7, 2022 (II)]**
(a) $w=\bar{z}^2$ (b) $z\bar{w}=2$ (c) $z\bar{w}=1$ (d) $w=\bar{z}$

110. If 1, ω, ω^2 denote the cube roots of unity, then, the value of $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$ is **[AP/July 7, 2022 (II)]**
(a) $32\,\omega^2$ (b) 32 ω (c) –32 (d) 32

111. The values of θ, for which $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is real are **[TS/May 14, 2023 (II), (S), AP/July 7, 2022 (II)]**
(a) $\theta=n\pi+\pi/3$ for $n\in\mathbb{Z}$
(b) $\theta=n\pi+\pi/4$ for $n\in\mathbb{Z}$
(c) $\theta=n\pi+\pi/2$ for $n\in\mathbb{Z}$
(d) θ = nπ for n ∈ ℤ

112. If $y^2+z^2=3yz$, $z^2+x^2=8zx$, $x^2+y^2=4xy$, then the value of $\frac{y^2}{xz}+\frac{xz}{y^2}$ is **[AP/July 7, 2022 (II)]**
(a) 2 (b) 3 (c) 4 (d) 5

113. z = cosθ + i sinθ ⇒ $z^r+\left(\bar{z}\right)^r=$ **[AP/July 6, 2022 (I)]**
(a) cos rθ (b) 2 cos rθ (c) sin rθ (d) 2 sin rθ

114. Multiplicative inverse of the complex number (sin θ, cosθ) is **[AP/July 4, 2022 (II)]**
(a) (sinθ, cosθ) (b) (sinθ, – cosθ)
(c) (cosθ, – sinθ) (d) (– cosθ, sinθ)

115. If $|x+iy|=\sqrt{x^2+y^2}$, then $\left|\left(1-\sqrt{3}i\right)^9+\left(\sqrt{3}+i\right)^9\right|=$ **[TS/July 20, 2022 (I)]**
(a) 2^9 (b) 2^{18} (c) 2^{10} (d) $2^{\frac{19}{2}}$

116. If 1, ω, ω^2 are the cube roots of unity and 1, α, α^2, α^3 are the fourth roots of unity in usual notation then $\alpha+\alpha\omega-\alpha^3\omega^2=$ **[TS/July 20, 2022 (I)]**
(a) 3 (b) 1 (c) 0 (d) –1

117. If *cis* α is the common value of $(-1)^{1/4}$ and $(-i)^{1/2}$ then tan α = **[TS/July 19, 2022 (I)]**
(a) –1 (b) 1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

118. One of the value of $\left(-32i\right)^{\frac{2}{5}}$ is **[TS/July 18, 2022 (II)]**

(a) $4\,cis\frac{2\pi}{5}$ (b) $4\,cis\frac{3\pi}{5}$

(c) $4\,cis\frac{4\pi}{5}$ (d) $4\,cis\frac{6\pi}{5}$

119. $\sqrt{(-3+4i)(8+6i)} =$ **[TS/July 18, 2022 (I)]**

(a) $\pm(1+2i)$ (b) $\pm(3+i)$

(c) $\pm(1+7i)$ (d) $\pm(7-i)$

120. If $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^m = 1, 2022 < m < 2029$, then $m =$ **[TS/July 18, 2022 (I)]**

(a) 2022 (b) 2024 (c) 2028 (d) 2026

121. If 1, ω, ω^2 are the cube roots of unity, $n \in \mathbb{N}$ and $n > 2$ then the least value of n such that $1 + \omega$ is a root of $x^n - x = 0$ is **[TS/July 18, 2022 (I)]**

(a) 3 (b) 5 (c) 7 (d) 4

122. Let $p(x)$ be a quadratic polynomial with real coefficients. If $p(x) = 0$ has only purely imaginary roots, then the zeroes of the polynomial $p(p(x))$ are

(a) only real numbers **[TS/July 18, 2022 (I)]**

(b) only purely imaginary numbers

(c) only rational numbers

(d) only complex numbers of the form $a + ib$ with $a \neq 0$ and $b \neq 0$

123. If ω is a root of the equation $x+\frac{1}{x}+1=0$, then

$$\begin{vmatrix} 1 & 1+\omega & 1+\omega+\omega^2 \\ 3 & 4+3\omega & 5+4\omega+3\omega^2 \\ 6 & 9+6\omega & 11+9\omega+6\omega^2 \end{vmatrix}$$ is equal to **[AP/Aug. 23, 2021 (I)]**

(a) 1 (b) – 1 (c) 0 (d) 1 + ω

124. If $a, b \in R$ and $i=\sqrt{-1}$, then the number of ordered pairs of real numbers (a, b) satisfying the condition $(a + bi)^3 = a - bi$ is **[AP/Aug. 23, 2021 (I)]**

(a) 3 (b) 2 (c) 4 (d) 5

125. $\dfrac{\left(\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}\right)^8}{\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)^8}$ is equal to **[AP/Aug. 23, 2021 (I)]**

(a) i (b) $-i$ (c) 1 (d) 2

126. If $z^2 + z + 1 = 0$, where z is a complex number, then $\left(z+\frac{1}{z}\right)^3+\left(z^4+\frac{1}{z^4}\right)^3$ is equal to **[AP/Aug. 23, 2021 (I)]**

(a) 1 (b) 0 (c) – 1 (d) – 2

127. The radius of the circle represented by $(1 + i)(1 + 3i)(1 + 7i) = x + iy$ is $\left(i=\sqrt{-1}\right)$.

[AP/Aug. 20, 2021 (I)]

(a) 1000 (b) $10\sqrt{10}$ (c) 10000 (d) 100

128. If 1, α_1, α_2, α_3 and α_4 are the roots of $z^5 - 1 = 0$ and ω is a cube roots of unity, then $(\omega - 1)(\omega - \alpha_1)(\omega - \alpha_2)(\omega - \alpha_3)(\omega - \alpha_4) + \omega$ is equal to **[AP/Aug. 20, 2021 (I)]**

(a) 0 (b) – 1 (c) – 2 (d) 1

129. If $a > 0$ and $z = x + iy$, then **[AP/Aug. 20, 2021 (I)]**

$\log_{\cos^2\theta}|z-a| > \log_{\cos^2\theta}|z-ai|, (\theta \in R)$ implies

(a) $x > y$ (b) $x < y$

(c) $x + y = \cos\theta$ (d) $x + y < 0$

130. $(\sin\theta - i\cos\theta)^3$ is equal to **[AP/Aug. 19, 2021 (I)]**

(a) $i^3(\cos 3\theta + i\sin 3\theta)$ (b) $\cos 3\theta + i\sin 3\theta$

(c) $\sin 3\theta - i\cos 3\theta$ (d) $(-i)^3(\cos 3\theta + i\sin 3\theta)$

131. If $(a+ib)^{\frac{1}{4}} = 2 + 3i$, then $3b - 2a$ is equal to **[TS/Aug. 4, 2021 (I)]**

(a) – 22 (b) – 122 (c) – 598 (d) – 698

132. ω is a complex cube root of unity. Match the items of List-I to the items of List-II. **[TS/Aug. 4, 2021 (I)]**

	List-I		**List-II**
(A)	$\omega^{1010} + \omega^{2020}$	(i)	0
(B)	$(1 + \omega - \omega^2)(1 - \omega + \omega^2)$	(ii)	1
(C)	$(2 + \omega^2 + \omega^4)^5$	(iii)	– 1
(D)	$(3 + 5\omega + 3\omega^2)^3$	(iv)	4
		(v)	8

The correct match is

	A	**B**	**C**	**D**
(a)	(iii)	(iv)	(i)	(v)
(b)	(i)	(iv)	(ii)	(v)
(c)	(iii)	(iv)	(ii)	(v)
(d)	(iii)	(i)	(ii)	(iv)

133. If the roots of the equation $(z - 4)^3 = 8i$ are $a - 2i$, $b + i$ and $c + i$, then $\sqrt{abc}$ is equal to **[TS/Aug. 4, 2021 (I)]**

(a) $13\sqrt{3}$ (b) $4\sqrt{13}$ (c) $2\sqrt{13}$ (d) $5\sqrt{3}$

134. Geometrically, the set $\{z \in \mathbf{C} : |z - 2 - 2i| \leq 1\}$ represents .... **[AP/Sept. 21, 2020 (I)]**

(a) a closed circular disc with centre at (–2, –2) and with radius 1

(b) a closed circular disc with centre at (2, 2) and with radius 1

(c) a closed circular disc with centre at (1, 1) and with radius 0.5

(d) a closed circular disc with centre at (–1, –1) and with radius 0.5

135. Let the complex numbers α and $\left(\frac{1}{\alpha}\right)$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha| =$ **[AP/Sept. 18, 2020 (I)]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{3}$

136. If $1, \alpha, \alpha^2, \ldots.., \alpha^{n-1}$ are the n^{th} roots of unity.

Then $\sum_{i=1}^{n-1} \frac{1}{2 - a^i}$ is equal to **[AP/Sept. 17, 2020 (I)]**

(a) $(n - 2)2^n$ (b) $\frac{(n-2)2^{n-1}+1}{2^n - 1}$

(c) $\frac{(n-2)2^{n-1}}{2^n - 1}$ (d) $\frac{1}{(n-2)2^n}$

137. For n ∈ **N**, If $A_n = \cos\left(\frac{\pi}{2^n}\right) + i\sin\left(\frac{\pi}{2^n}\right)$,

then $(A_1 A_2 A_3 A_4)^4 =$ **[TS/Sept. 11, 2020 (I)]**

(a) $\frac{-1-i}{\sqrt{2}}$ (b) 1 (c) 0 (d) $\frac{1-i}{\sqrt{2}}$

138. Let $A_r = \left(x + \frac{1}{x}\right)^3 . \left(x^2 + \frac{1}{x^2}\right)^3 . \left(x^3 + \frac{1}{x^3}\right)^3$ $\ldots..\left(x^r + \frac{1}{x^r}\right)^3$. If $x^2 + x + 1 = 0$, then

$\frac{1}{A_3} + \frac{1}{A_6} + \frac{1}{A_9} + \frac{1}{A_{12}} + \ldots.\infty =$ **[TS/Sept. 11, 2020 (I)]**

(a) $\frac{1}{6}$ (b) $\frac{2}{5}$ (c) 1 (d) $\frac{1}{7}$

139. Let $z = x + iy$ be a complex number,
$A = \{z/|z| \le 2\}$ and $B = \{z / (1 - i)\,z + (1 + i)\,\overline{z} \ge 4\}$
Then which one of the following options belongs to $A \cap B$? **[TS/Sept. 10, 2020 (I)]**

(a) $\sqrt{3} + \frac{1}{2}i$ (b) $\frac{1}{2} + \frac{i}{2}$ (c) $\sqrt{2} + \frac{i}{2}$ (d)$2 + 2i$

140. If $z, \overline{z}, -z, -\overline{z}$ forms a rectangle of area $2\sqrt{3}$ square units, then one such z is **[TS/Sept. 10, 2020 (I)]**

(a) $\frac{1}{2} + \sqrt{3}i$ (b) $\frac{\sqrt{5} + \sqrt{3}i}{4}$

(c) $\frac{3}{2} + \frac{\sqrt{3}i}{2}$ (d) $\frac{\sqrt{3} + \sqrt{11}i}{2}$

141. $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^8 + \left(\frac{1 + \cos\theta - i\sin\theta}{1 + \cos\theta + i\sin\theta}\right)^{16} =$

[TS/May 14, 2023 (II), (S), TS/Sept. 10, 2020 (I)]

(a) $2\cos 8\theta$ (b) $2\cos 16\theta$
(c) $2\sin 8\theta$ (d) $2\sin 16\theta$

142. If ω is a complex cube root of unity, then

$\left(\frac{1-\sqrt{3}i}{2}\right)^{2020} + \left(\frac{1+\sqrt{3}i}{2}\right)^{2026}$ **[TS/Sept. 9, 2020 (II)]**

$+ \sin\left(\sum_{j=1}^{6} (j + \omega)(j + \omega^2)\frac{3\pi}{152}\right) =$

(a) –2 (b) 2 (c) –1 (d) 0

143. If $z = e^{i\theta}$ and $\frac{3\cos 3\theta + 2\cos 2\theta + 5\cos 5\theta}{3\sin 3\theta + 2\sin 2\theta + 5\sin 5\theta}$

$= \frac{i\sum_{r=0}^{10} a_r z^r}{\sum_{r=0}^{10} b_r z^r}$ then $\frac{\left(\sum_{r=0}^{10} a_r + \sum_{r=0}^{10} b_r\right)}{10} =$

[TS/Sept. 9, 2020 (II)]

(a) 0 (b) 1 (c) 2 (d) 3

144. Let $z \in \boldsymbol{C}$ and $i = \sqrt{-1}$, if $a, b, c \in (0, 1)$ be such that $a^2 + b^2 + c^2 = 1$ and $b + ic = (1 + a)\,z$, then $\frac{iz}{iz}$

[TS/Sept. 9, 2020 (I)]

(a) $\frac{a+ib}{1+c}$ (b) $\frac{a-ib}{1+c}$ (c) $\frac{a-ib}{1-c}$ (d) $\frac{a+ib}{1-c}$

145. If ω is a complex cube root of unity, then $(1 - \omega + \omega^2)^6 + (1 - \omega^2 + \omega)^6 =$ **[TS/Sept. 9, 2020 (I)]**

(a) 0 (b) 6 (c) 64 (d) 128

146. If P, Q and R are points, respectively representing the complex numbers $z, ze^{\frac{i\pi}{3}}$ and $z\left(1 + e^{\frac{i\pi}{3}}\right)$ in argand plane, then the area of the triangle PQR, is

[AP/Apr. 22, 2019 (I)]

(a) $\sqrt{3}|z|^2$ (b) $\frac{\sqrt{3}}{2}|z|^2$ (c) $\frac{\sqrt{3}}{4}|z|^2$ (d) $2\sqrt{3}|z|^2$

147. $A(z_1)$ and $B(z_2)$ are two points in the argand plane. Then, the locus of the complex number z satisfying arg $\left(\frac{z - z_1}{z - z_2}\right) = 0$ or π, is **[AP/Apr. 22, 2019 (I)]**

(a) the circle with $\overline{AB}$ as a diameter
(b) the ellipse with A, B as extremities of the major axis
(c) the perpendicular bisector of $\overline{AB}$
(d) the straight line passing through the points A and B

148. If x is a cube root of unity other than 1, then

$\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \ldots + \left(x^{12} + \frac{1}{x^{12}}\right)^2 =$

[AP/Apr. 22, 2019 (I)]

(a) 12 (b) 64 (c) 24 (d) 0

149. If $z = x - iy$ and $z^{\frac{1}{3}} = a + ib$, then $\dfrac{\left(\dfrac{x}{a} + \dfrac{y}{b}\right)}{a^2 + b^2} =$ **[AP/Apr. 21, 2019 (I)]**

(a) –2 (b) –1 (c) 1 (d) 2

150. The equation whose solutions are the non-zero solutions of the equation $\overline{z} = iz^2$, is **[AP/Apr. 21, 2019 (I)]**

(a) $z^3 + i = 0$ (b) $z^3 + z + 1 = 0$

(c) $z^3 - i = 0$ (d) $z^3 + iz + 1 = 0$

151. $\sum_{k=1}^{6}\left(\sin\dfrac{2\pi k}{7} - i\cos\dfrac{2\pi k}{7}\right) =$ **[AP/Apr. 21, 2019 (I)]**

(a) –1 (b) 0 (c) i (d) $-i$

152. If ω represents a complex cube root of unity, then $\left(1+\dfrac{1}{\omega}\right)\left(1+\dfrac{1}{\omega^2}\right)+\left(2+\dfrac{1}{\omega}\right)\left(2+\dfrac{1}{\omega^2}\right)+\ldots+\left(n+\dfrac{1}{\omega}\right)\left(n+\dfrac{1}{\omega^2}\right) =$

[TS/May 14, 2023 (II), (S), AP/Apr. 20, 2019 (I)]

(a) $\dfrac{n(n^2+1)}{3}$ (b) $\dfrac{n(n^2+2)}{3}$

(c) $\dfrac{n(n^2-2)}{3}$ (d) $\dfrac{n^2(n-1)}{6}$

153. If ω is a complex cube root of unity, then $\sum_{r=1}^{9} r(r+1-\omega)(r+1-\omega^2) =$ **[AP/Apr. 20, 2019 (I)]**

(a) 5025 (b) 4020 (c) 2016 (d) 3015

154. The sum of the products of the non-conjugate roots of $i^{1/4}$ taken two at a time is **[TS/May 6, 2019 (I)]**

(a) 2 (b) 0 (c) – 1 (d) – 2

155. If $z = \cos\alpha + i\sin\alpha$; $0 < \alpha < \dfrac{\pi}{4}$, then $\left|\dfrac{1+z^4}{1-z^3}\right| =$

[TS/May 6, 2019 (I)]

(a) $\dfrac{\cos 2\alpha}{\sin\frac{3}{2}\alpha}$ (b) $\dfrac{\cos\alpha}{\sin\frac{3}{2}\alpha}$

(c) $\dfrac{\cos 2\alpha}{\sin\frac{\alpha}{2}}$ (d) $\dfrac{\cos\alpha}{\sin\frac{\alpha}{2}}$

156. If 1, ω, ω^2, …, ω^8 are the roots of the equation $x^9 - 1 = 0$, then $\sum_{r=1}^{8}(\omega^r)^{99} =$ **[TS/May 6, 2019 (I)]**

(a) 0 (b) 8 (c) 1 (d) ω

157. If $z = x + iy$ represents a point P in the argand plane, then the area of the region represented by the inequality $2 < |z - (1+i)| < 3$ is **[TS/May 4, 2019 (II)]**

(a) 49π (b) 36π (c) 25π (d) 5π

158. If P is a complex number whose modulus is one, then the equation $\left(\dfrac{1+iz}{1-iz}\right)^4 = P$ has **[TS/May 4, 2019 (II)]**

(a) real and equal roots

(b) real and distinct roots

(c) two real and two complex roots

(d) all complex roots

159. If the point $\left(\dfrac{k-1}{k}, \dfrac{k-2}{k}\right)$ lies on the locus of z satisfying the inequality $\left|\dfrac{z+3i}{3z+i}\right| < 1$, then the interval in which k lies is **[TS/May 3, 2019 (II)]**

(a) $(-\infty, 2) \cup (3, \infty)$ (b) $[2, 3]$

(c) $[1, 5]$ (d) $(-\infty, 1) \cup (5, \infty)$

160. If the complex cube roots of $(-i)$ are α, β, γ, then $\alpha^2 + \beta^2 + \gamma^2 =$ **[TS/May 3, 2019 (II)]**

(a) 1 (b) – 1 (c) $-i$ (d) 0

161. If $x + iy = (1+i)^6 - (1-i)^6$, then which one of the following is true? **[TS/May 3, 2019 (I)]**

(a) $x + y = 16$ (b) $x + y = -16$

(c) $x + y = -8$ (d) $x + y = 8$

162. If 1, ω and ω^2 are the cube roots of unity, then $(a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega) =$ **[TS/May 3, 2019 (I)]**

(a) $a^3 + b^3 + c^3$ (b) $a^3 + b^3 + c^3 - 3abc$

(c) $(a+b+c)^3 - 3abc$ (d) $a^3 + b^3 + c^3 + 3abc$

163. If $13e^{i\tan^{-1}\frac{5}{12}} = a + ib$, then the ordered pair $(a, b) =$ **[AP/2018]**

(a) (12, 5) (b) (5, 12) (c) (24, 10) (d) (10, 24)

164. If $z_1 = 1 - 2i$; $z_2 = 1 + i$ and $z_3 = 3 + 4i$, then $\left(\dfrac{1}{z_1} + \dfrac{3}{z_2}\right)\dfrac{z_3}{z_2} =$ **[AP/2018]**

(a) $13 - 6i$ (b) $13 - 3i$ (c) $6 - \dfrac{13}{2}i$ (d) $\dfrac{13}{2} - 3i$

165. The area (in sq. units) of the triangle whose vertices are the points represented by the complex numbers 0, z, ze^{i} $(0 < \alpha < \pi)$ is **[TS/May 5, 2018 (I)]**

(a) $\dfrac{1}{2}|z|^2$ (b) $\dfrac{1}{2}|z|^2 \sin\alpha$

(c) $\dfrac{1}{2}|z|^2 \sin\alpha\cos\alpha$ (d) $\dfrac{1}{2}|z|^2 \cos\alpha$

166. If $\omega_0, \omega_1, \ldots, \omega_{n-1}$ are the nth roots of unity, then $(1+2\omega_0)(1+2\omega_1)(1+2\omega_2)\ldots(1+2\omega_{n-1}) =$ **[TS/May 5, 2018 (I)]**

(a) $1 + (-1)^n 2^n$ (b) $1 + 2^n$

(c) $(-1)^n + 2^n$ (d) $1 + (-1)^{n-1} 2^n$

167. If α is a non-real root of $x^7 = 1$, then $\alpha(1 + \alpha)(1 + \alpha^2 + \alpha^4) =$ **[AP/2017]**

(a) 1 (b) 2 (c) – 1 (d) – 2

168. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is **[AP/2016]**

(a) 3 (b) 4 (c) 8 (d) 2

169. If 1, z_1, z_2, ..., z_{n-1} are the nth roots of unity, then $(1 - z_1)(1 - z_2) \ldots (1 - z_{n-1})$ is equal to **[AP/2016]**

(a) 0 (b) $n - 1$ (c) n (d) 1

170. The common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$, $z^{2014} + z^{2015} + 1 = 0$ are **[AP/2015]**

(a) ω, ω^2 (b) $1, \omega, \omega^2$

(c) $-1, \omega, \omega^2$ (d) $-\omega, -\omega^2$

171. If α, β are non-real cube roots of 2, then $\alpha^6 + \beta^6$ equals **[TS/2015]**

(a) 8 (b) 4 (c) 2 (d) 1

Topic 3 Solutions of Quadratic Equations, Sum and Product of Roots, Nature of Roots, Relation Between Roots and Co-efficients, Formation of an Equation with Given Roots

172. The cubic equation whose roots are the squares of the roots of the equation $12x^3 - 20x^2 + x + 3 = 0$ is

(a) $x^3 + 376x^2 - 121x - 9 = 0$ **[AP/May 23, 2024 (I)]**

(b) $144x^3 - 400x^2 + 12lx + 98 = 0$

(c) $144x^3 - 376x^2 + 12lx - 9 = 0$

(d) $x^3 + 400x^2 - 12lx - 98 = 0$

173. α, β, γ are the roots of the equation $x^3 + 3x^2 - 10x - 24 = 0$. If $\alpha(\beta + \gamma)$, $\beta(\gamma + \alpha)$ and $\gamma(\alpha + \beta)$ are the roots of the equation $x^3 + px^2 + qx + r = 0$. then $q =$ **[AP/May 23, 2024 (I)]**

(a) – 44 (b) –28 (c) 44 (d) 28

174. If 'a' is a rational number, then the roots of the equation $x^2 - 3ax + a^2 - 2a - 4 = 0$ are **[AP/May 22, 2024 (II)]**

(a) rational and equal numbers

(b) different real numbers

(c) different rational numbers only

(d) not real numbers

175. If α, β are the roots of the equation $x^2 - 6x - 2 = 0$, $\alpha > \beta$ and $a_n = \alpha^n - \beta^n$, $n > 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to **[AP/May 22, 2024 (I)]**

(a) 6 (b) 4 (c) 3 (d) 2

176. If both the roots of the equation $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ exceeds 3, then **[AP/May 22, 2024 (I)]**

(a) $a < \frac{3}{2}$ (b) $a > \frac{3}{2}$ (c) $a < \frac{5}{2}$ (d) $a > \frac{11}{9}$

177. If α and β are two distinct negative roots of $x^5 - 5x^3 + 5x^2 - 1 = 0$, then the equation of least degree with integer coefficients having $\sqrt{-\alpha}$ and $\sqrt{-\beta}$ as its roots is **[AP/May 22, 2024 (I)]**

(a) $x^2 - 3x + 1 = 0$ (b) $-x^4 + 5x^2 - 5x + 1 = 0$

(c) $-x^4 - 5x^2 + 5x + 1 = 0$ (d) $x^4 - 3x^2 + 1 = 0$

178. The equation $x^4 - x^3 - 6x^2 + 4x + 8 = 0$ has two equal roots. If α, β are the other two roots of this equation then $\alpha^2 + \beta^2 =$ **[AP/May 21, 2024 (II)]**

(a) 4 (b) 5 (c) 6 (d) 7

179. Roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are **[AP/May 21, 2024 (I)]**

(a) $\frac{a(b-c)}{c(a-b)}, 1$ (b) $\frac{b(c-a)}{c(a-b)}, 1$

(c) $\frac{c(a-b)}{a(b-c)}, 1$ (d) $\frac{c(a-b)}{b(c-a)}, 1$

180. If $(3 + i)$ is a root of $x^2 + ax + b = 0$ then $a =$ **[AP/May 21, 2024 (I)]**

(a) 3 (b) – 3 (c) 6 (d) – 6

181. If the roots of the equation $4x^3 - 12x^2 + 11x + m = 0$ are in arithmetic progression, then $m =$ **[AP/May 21, 2024 (I)]**

(a) – 3 (b) 1 (c) 2 (d) 3

182. If the sum of two roots of $x^3 + px^2 + qx - 5 = 0$ is equal to its third root, then $p(p^2 - 4q) =$ **[AP/May 20, 2024 (I)]**

(a) –20 (b) 20 (c) 40 (d) –40

183. $4 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + \ldots\infty}}} =$ **[AP/May 19, 2024 (II)]**

(a) $(2 + \sqrt{5}), (2 - \sqrt{5})$ (b) $2 + \sqrt{5}$

(c) $2 - \sqrt{5}$ (d) $2 + \sqrt{3}$

184. If α, β, γ are roots of the equation $x^3 + ax^2 + bx + c = 0$ then $\alpha^{-1} + \beta^{-1} + \gamma^{-1} =$ **[AP/May 19, 2024 (II)]**

(a) $\frac{a}{c}$ (b) $-\frac{b}{c}$ (c) $\frac{c}{a}$ (d) $\frac{b}{a}$

185. If the roots of the quadratic equation $x^2 - 35x + c = 0$ are in the ratio 2 : 3 and $c = 6$K, then K = **[AP/May 18, 2024 (I)]**

(a) 49 (b) 14 (c) 21 (d) 7

186. For real values of x and a, if the expression $\frac{x+a}{2x^2 - 3x + 1}$ assumes all real values, then **[AP/May 18, 2024 (I)]**

(a) $a < -1$ or $a > -\frac{1}{2}$ (b) $-1 < a < -\frac{1}{2}$

(c) $\frac{1}{2} < a < 1$ (d) $a < \frac{1}{2}$ or $a > 1$

187. If the sum of two roots α, β of the equation $x^4 - x^3 - 8x^2 + 2x + 12 = 0$ is zero and γ, δ $(\gamma > \delta)$ are its other roots, then $3\gamma + 2\delta =$ **[AP/May 18, 2024 (I)]**

(a) 0 (b) 1 (c) 3 (d) 5

188. If α is a root of the equation $x^2 - x + 1 = 0$, then

$$\left(\alpha + \frac{1}{\alpha}\right)^3 + \left(\alpha^2 + \frac{1}{\alpha^2}\right)^3 + \left(\alpha^3 + \frac{1}{\alpha^3}\right)^3 + \left(\alpha^4 + \frac{1}{\alpha^4}\right)^3 =$$

[TS/May 11, 2024 (I)]

(a) 0 (b) 1 (c) –3 (d) –9

189. α, β are the real roots of the equation $x^2 + ax + b = 0$. If $\alpha+\beta=\frac{1}{2}$ and $\alpha^3+\beta^3=\frac{37}{8}$, then $a-\frac{1}{b}=$ **[TS/May 11, 2024 (I)]**

(a) $\frac{-1}{6}$ (b) $\frac{3}{2}$ (c) $\frac{-3}{2}$ (d) $\frac{1}{6}$

190. If α, β, γ are the roots of the equation $4x^3 - 3x^2 + 2x - 1 = 0$, then $\alpha^3 + \beta^3 + \gamma^3 =$ **[TS/May 11, 2024 (I)]**

(a) $\frac{2}{27}$ (b) $\frac{1}{8}$ (c) $\frac{3}{64}$ (d) $\frac{27}{128}$

191. The roots of the equation $x^3 - 3x^2 + 3x + 7 = 0$ are α, β, γ and ω, ω^2 are complex cube roots of unity. If the terms containing x^2 and x are missing in the transformed equation when each one of these roots is decreased by h, then $\frac{\alpha-h}{\beta-h}+\frac{\beta-h}{\gamma-h}+\frac{\gamma-h}{\alpha-h}=$ **[TS/May 10, 2024 (II)]**

(a) $\frac{3}{\omega^2}$ (b) 3ω (c) 0 (d) $3\omega^2$

192. If α, β are the roots of the equation $x+\frac{4}{x}=2\sqrt{3}$, then $\frac{2}{\sqrt{3}}\left|\alpha^{2024}-\beta^{2024}\right|=$ **[TS/May 10, 2024 (I)]**

(a) 2^{2024} (b) 2^{2025} (c) 2^{2023} (d) 2^{1012}

193. α, β are the real roots of the equation $12x^{1/3}-25x^{1/6}+12=0$. If $\alpha > \beta$, then $\sqrt[6]{\frac{\alpha}{\beta}}=$ **[TS/May 10, 2024 (I)]**

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$ (c) $\frac{9}{8}$ (d) $\frac{16}{9}$

194. α is a root of the equation $\frac{x-1}{\sqrt{2x^2-5x+2}}=\frac{41}{60}$. If $-\frac{1}{2}<\alpha<0$, then $\alpha =$ **[TS/May 9, 2024 (II)]**

(a) $-\frac{5}{31}$ (b) $-\frac{7}{34}$ (c) $-\frac{9}{37}$ (d) $-\frac{11}{41}$

195. If α, β, γ are the roots of the equation $2x^3 - 5x^2 + 4x - 3 = 0$, then $\sum\alpha\beta(\alpha+\beta)=$ **[TS/May 9, 2024 (II)]**

(a) 8 (b) 4 (c) 2 (d) $\frac{1}{2}$

196. $\alpha, \beta, \gamma, 2, \varepsilon$ are the roots of the equation $x^5 + 4x^4 - 13x^3 - 52x^2 + 36x + 144 = 0$. If $\alpha < \beta < \gamma < 2 < \varepsilon$, then $\alpha + 2\beta + 3\gamma + 5\varepsilon =$ **[TS/May 9, 2024 (II)]**

(a) –1 (b) 25 (c) –36 (d) 48

197. If the quadratic equation $3x^2 + (2k + 1)x - 5k = 0$ has real and equal roots, then the value of k such that $-\frac{1}{2}<k<0$ is **[TS/May 9, 2024 (I)]**

(a) $\frac{-16+\sqrt{255}}{2}$ (b) $\frac{-16-\sqrt{255}}{2}$

(c) $-\frac{2}{3}$ (d) $-\frac{3}{5}$

198. If α, β, γ are the roots of the equation $2x^3 - 3x^2 + 5x - 7 = 0$, then $\sum\alpha^2\beta^2=$ **[TS/May 9, 2024 (I)]**

(a) $-\frac{17}{4}$ (b) $\frac{17}{4}$ (c) $-\frac{13}{4}$ (d) $\frac{13}{4}$

199. The sum of two roots of the equation $x^4 - x^3 - 16x^2 + 4x + 48 = 0$ is zero. If $\alpha, \beta, \gamma, \delta$ are the roots of this equation, then $\alpha^4+\beta^4+\gamma^4+\delta^4=$ **[TS/May 9, 2024 (I)]**

(a) 123 (b) 369 (c) 132 (d) 396

200. If the roots of the equation $6x^3 - 11x^2 + 6x - 1 = 0$ are in harmonic progression, then the roots of $x^3 - 6x^2 + 11x - 6 = 0$ will be in **[AP/May 19, 2023 (I)]**

(a) Geometric Progression
(b) Arithmetic Progression
(c) Harmonic Progression
(d) Arithmetico–Geometric Progression

201. If $\frac{k}{kx+3}+\frac{3}{3x-k}=\frac{12x+5}{(kx+3)(3x-k)}$ $\forall x\in R-\left\{\frac{\{3\}}{k},\frac{k}{3}\right\}$, then both the roots of the equation $kx^2 - 7x + 3 = 0$ are **[AP/May 19, 2023 (I)]**

(a) Rational numbers (b) Irrational numbers
(c) Complex numbers (d) Integers

202. The sum of the fourth powers of the roots of the equation $16x^2 - 10x + 1 = 0$ is **[AP/May 18, 2023 (II)]**

(a) $\frac{257}{4096}$ (b) $\frac{257}{2048}$ (c) $\frac{257}{1024}$ (d) $\frac{257}{512}$

203. If the equation having the roots as the values obtained by diminishing each root of the equation $x^3 - 3x^2 + 2x - 1 = 0$ by K is $x^3 - x - 1 = 0$, then K = **[AP/May 18, 2023 (I)]**

(a) 2 (b) –1 (c) 1 (d) –2

204. If α, β and γ are the roots of the equation $x^3 - ax^2 + bx - c = 0$ then, $\alpha^{-2} + \beta^{-2} + \gamma^{-2} =$ **[AP/May 17, 2023 (II)]**

(a) $\frac{b^2-3ac}{c^2}$ (b) $\frac{b^2-ac}{c^2}$

(c) $\frac{b^2-2ac}{c^2}$ (d) $\frac{b^2-4ac}{c^2}$

205. If a and b are the roots of the equation $y^2 + y + 1 = 0$, then the value of $a^4 + b^4 + a^{-1}b^{-1}$ is **[AP/May 16, 2023 (II)]**

(a) 1 (b) 0 (c) 5 (d) 2

206. If c and d are the roots of $x^2 + ax + b = 0$, then a root of $x^2 + (4c + a)x + (b + 2ac + 4c^2) = 0$ is **[AP/May 16, 2023 (II)]**

(a) d + 2c (b) d + c (c) d – c (d) d – 2c

207. If the roots of the equation $16x^3 - 44x^2 + 36x - 9 = 0$ are in harmonic progression, then its greatest root is **[AP/May 16, 2023 (I)]**

(a) $\frac{3}{4}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

208. If cot x cot y = a and x + y = $\frac{\pi}{6}$, then the quadratic equation satisfying cot x and cot y is **[AP/May 15, 2023 (II)]**

(a) $t^2 + (1 - a)\sqrt{3}t + a = 0$
(b) $\sqrt{3}t^2 + (1 - a)t + a\sqrt{3} = 0$
(c) $\sqrt{3}t^2 + (a - 1)t + a\sqrt{3} = 0$
(d) $t^2 + (a - 1)\sqrt{3}t + a = 0$

209. If the equation $x^4 + ax^3 + bx^2 + cx + d = 0$ has three equal roots, then that root is **[AP/May 15, 2023 (I)]**

(a) $\frac{6c - ab}{8b - 3a^2}$ (b) $\frac{ab - 6c}{8b + 3a^2}$
(c) $\frac{6c - ab}{3a^2 - 4b}$ (d) $\frac{6c - ab}{3a^2 - 8b}$

210. The sum of all the real values of x satisfying the equation $\left(x^2 - 7x + 11\right)^{x^2 - 6x - 7} = 1$ is **[TS/May 14, 2023 (II)]**

(a) 14 (b) 20 (c) 13 (d) 16

211. If $x^2 + 2px - 2p + 8 > 0$ for all real values of x, then the set of all possible values of p is **[TS/May 14, 2023 (I)]**

(a) (2, 4) (b) $(-\infty, -4)$ (c) $(2, \infty)$ (d) $(-4, 2)$

212. If α, β, γ, δ are the roots of the equation $x^4 + x^2 + 1 = 0$ such that $\alpha + \beta = -1$, $\gamma + \delta = 1$, $\alpha^2 = \beta$ and $\gamma^2 = -\delta$, then $\alpha^{2023} + \beta^{2023} + \gamma^{2022} + \delta^{2022} =$ **[TS/May 13, 2023 (II)]**

(a) 1 (b) 0 (c) $1+3\omega$ (d) $\omega - 2\omega^2$

213. If α is a multiple root of the equation $x^5 - 6x^4 + 11x^3 - 2x^2 - 12x + 8 = 0$ then $3\alpha^2 - 2\alpha + 1 =$ **[TS/May 13, 2023 (II)]**

(a) –2 (b) 1 (c) 0 (d) 9

214. If α, β, γ are the roots of the equation $x^3 - 3x^2 + 3x + 1 = 0$, then $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 =$ **[TS/May 13, 2023 (I)]**

(a) 9 (b) 15 (c) 8 (d) 20

215. If α and β are the roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15} =$ **[TS/May 12, 2023 (II)]**

(a) –512 (b) –256 (c) 256 (d) 512

216. If one root of the equation $4x^2 - 2x + k - 4 = 0$ is the reciprocal of the other, then the value of k is **[TS/May 12, 2023 (I)]**

(a) – 8 (b) 8 (c) – 4 (d) 4

217. If $x^2 + px + 1$ is a factor of $ax^3 + bx + c$, then **[AP/July 8, 2022 (I)]**

(a) $a^2 + c^2 = ab + 3$ (b) $a^2 - c^2 = ab$
(c) $a^2 - c^2 = -ab$ (d) $a^2 + c^2 = ab$

218. The quadratic equation whose sum of the roots is 11 and sum of squares of the roots is 61 is **[AP/July 8, 2022 (I)]**

(a) $x^2 + 11x - 30 = 0$ (b) $x^2 + 11x + 30 = 0$
(c) $x^2 - 11x - 30 = 0$ (d) $x^2 - 11x + 30 = 0$

219. The sum of the complex roots of the equation $x^4 - 2x^3 + x - 380 = 0$ is **[AP/July 8, 2022 (I)]**

(a) $-3i + 3$ (b) $3i - 3$ (c) –1 (d) 1

220. The product of real roots of the equation $4x^4 - 24x^3 + 57x^2 + 18x - 45 = 0$ if one of the root is $3 + i\sqrt{6}$ is **[AP/July 8, 2022 (I)]**

(a) –5/16 (b) 5/16 (c) 3/4 (d) –3/4

221. Let f : **R** → **R** be defined by f(x) = 2x + 3. If α, β are the roots of the equation $f\left(x^2\right) - 2f\left(\frac{x}{2}\right) - 1 = 0$ then $\alpha^2 + \beta^2 =$ **[AP/July 7, 2022 (II)]**

(a) 13 (b) 25 (c) 5 (d) 18

222. If α, β are the roots of $ax^2 + bx + c = 0$, then the quadratic equation whose roots are $\sqrt{5}\alpha, \sqrt{5}\beta$ is **[AP/July 7, 2022 (II)]**

(a) $ax^2 + \sqrt{5}bx + 5c = 0$ (b) $ax^2 + \sqrt{5}bx + \sqrt{5}c = 0$
(c) $ax^2 + 5bx + \sqrt{5}c = 0$ (d) $ax^2 + 5bx + 5c = 0$

223. If 2, 3, 6 are the roots of the polynomial $f(x) = x^3 + ax^2 + bx + c$, where a, b, c ∈ ℂ. Then the value of a – c is **[AP/July 7, 2022 (II)]**

(a) –11 (b) 36 (c) 25 (d) 11

224. $2\cot^2\theta - \cot\theta - 3 =$ **[AP/July 7, 2022 (II)]**

(a) $(2\cot\theta - 3)(\cot\theta + 1)$ (b) $(2\cot\theta - 1)(\cot\theta + 3)$
(c) $(2\cot\theta + 3)(\cot\theta - 1)$ (d) $(2\cot\theta + 1)(\cot\theta - 3)$

225. The number of distinct solutions of the equations $x^{11} - x^7 + x^4 - 1 = 0$ is **[AP/July 6, 2022 (I)]**

(a) 9 (b) 11 (c) 10 (d) 8

226. Let $a_n, a_{n-1}, ..., a_1, a_0 \in \mathbb{C}$ and $f(x) = a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$ is a polynomial. If the polynomial f(x) is monic then **[AP/July 6, 2022 (I)]**

(a) $a_n \neq 0$ (b) $a_n = 1$ (c) $a_n > 0$ (d) $a_n < 0$

227. The number of solutions of the equations $x + y + z = 12$; $x^2 + y^2 + z^2 = 50$; $x^3 + y^3 + z^3 = 216$ is **[AP/July 6, 2022 (I)]**

(a) 6 (b) 24 (c) 3 (d) 9

228. $\sum_{k=1}^{6} \sin\left(\frac{2\pi k}{7}\right) - i\cos\left(\frac{2\pi k}{7}\right) =$ **[AP/July 5, 2022 (II)]**

(a) 1 (b) $-i$ (c) i (d) –1

229. If $(x - iy)^{1/3} = a - ib$, then the value of $\frac{x}{2a} + \frac{y}{2b}$ is **[AP/July 5, 2022 (II)]**

(a) $2(a^2 - b^2)$ (b) $4(a^2 - b^2)$
(c) $a^2 - b^2$ (b) $\frac{1}{2}\left(a^2 - b^2\right)$

230. If $x = -5 + 2\sqrt{-4}$, then the value of $x^4 + 9x^3 + 35x^2 - x + 4$ is **[AP/July 5, 2022 (II)]**

(a) 80 (b) 160 (c) –160 (d) –80

231. α, β are the roots of $x^2 - 10x - 8 = 0$ with $\alpha > \beta$.
If $a_n = \alpha^n - \beta^n$ for $n \in \mathbb{N}$, then the value of $\dfrac{a_{10} - 8a_8}{5a_9}$ is **[AP/July 5, 2022 (II)]**
(a) –3 (b) 3 (c) –2 (d) 2

232. The number of real values of m so that the equation $x^2 + (2m+1)x + m = 0$ has equal roots is **[AP/July 5, 2022 (II)]**
(a) 1 (b) 0 (c) 2 (d) 3

233. The sum of the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is **[AP/July 4, 2022 (II)]**
(a) 4 (b) – 4 (c) 2 (d) – 2

234. If the difference between the roots of $x^2 + ax + b = 0$ and that of the roots of $x^2 + bx + a = 0$ is same and $a \neq b$, then **[AP/July 4, 2022 (II)]**
(a) $a - b - 4 = 0$ (b) $a - b + 4 = 0$
(c) $a + b + 4 = 0$ (d) $a + b - 4 = 0$

235. For what values of $a \in \mathbf{Z}$, the quadratic expression $(x + a)(x + 1991) + 1$ can be factorised as $(x + b)(x + c)$, where $b, c \in \mathbf{Z}$? **[AP/July 4, 2022 (II)]**
(a) 1990 (b) 1989 (c) 1991 (d) 1992

236. The number of integer solutions of the equation $|1 - i|^x = 2^x$ is **[AP/July 4, 2022 (I)]**
(a) 1 (b) 0 (c) 2 (d) 3

237. The number of positive real roots of the equation $3^{x+1} + 3^{-x+1} = 10$ is **[AP/July 4, 2022 (I)]**
(a) 3 (b) 2
(c) 1 (d) Infinitely many

238. The number of real roots of the equation $\sqrt{\dfrac{x}{1-x}} + \sqrt{\dfrac{1-x}{x}} = \dfrac{13}{6}$ is **[AP/July 4, 2022 (I)]**
(a) 1 (b) 2 (c) 3 (d) 4

239. If $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$ then the value of x is **[AP/July 4, 2022 (I)]**
(a) $\dfrac{7}{2}$ (b) $\dfrac{5}{2}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{3}{2}$

240. If the sum of two roots of the equation $x^3 - 7px^2 + 5qx - 6r = 0$ is zero, then **[TS/July 20, 2022 (II)]**
(a) $5p = \dfrac{6q}{7r}$ (b) $5q = \dfrac{6r}{7p}$
(c) $5r = \dfrac{6p}{7q}$ (d) $pqr = 35$

241. If α, β are the irrational roots of the equation $3p^2x^3 + px^2 + qx + 3 = 0$ when $p = 1$ and $q = -7$ then $|\alpha - \beta| =$ **[TS/July 20, 2022 (II)]**
(a) $\dfrac{3\sqrt{13}}{2}$ (b) $\dfrac{\sqrt{3}}{2}$ (c) $\dfrac{2\sqrt{13}}{3}$ (d) 4

242. If α, β are the roots of quadratic equation $x^2 + bx + c = 0$ such that $\alpha^2 + \beta^2 = 5$ and $\alpha^3 + \beta^3 = 9$, then $b + c =$ **[TS/July 20, 2022 (I)]**
(a) –5 (b) –1 (c) 1 (d) 5

243. If α, β, γ are the roots of the equation $x^3 - 9x^2 + 23x - 15 = 0$, then $\alpha^3 + \beta^3 + \gamma^3 =$ **[AP/May17, 2023, (I) (S); TS/July 20, 2022 (I)]**
(a) 36 (b) 92 (c) 153 (d) 244

244. If $\alpha, \beta, 2\beta$ are the real roots of the equation $x^3 - 9x^2 + k = 0$ and $k \in \mathbb{R} - \{0\}$, then $14\beta =$ **[TS/July 20, 2022 (I)]**
(a) 28 (b) 36 (c) 18 (d) 54

245. The sum of all distinct roots of the equation $x^5 - 3x^4 + 5x^3 - 5x^2 + 3x - 1 = 0$ is **[TS/July 20, 2022 (I)]**
(a) 1 (b) 2 (c) 3 (d) $2\sqrt{3}$

246. If α, β are the roots of the equation $x^2 - 2\sqrt{3}x + 4 = 0$ then $\alpha^6 + \beta^6 =$ **[TS/July 19, 2022 (I)]**
(a) 128 (b) –64 (c) 64 (d) –128

247. When $b = 17$, it is found that the roots of the equation $x^2 + bx + c = 0$ are –2 and –15. If α, β are the roots of the same equation when $b = 13$ then $|\alpha - \beta| =$ **[TS/July 19, 2022 (I)]**
(a) 7 (b) 13 (c) 17 (d) 30

248. If α, β, γ are the roots of the equation $5x^3 - 2x - 4 = 0$, then $\alpha^3 + \beta^3 + \gamma^3 =$ **[TS/July 19, 2022 (I)]**
(a) $\dfrac{12}{5}$ (b) $\dfrac{18}{29}$ (c) 4 (d) –4

249. If the roots of $x^5 - ax^4 + bx^3 - cx^2 + dx - 1 = 0$ are all positive such that their arithmetic mean and geometric mean are equal, then $a + b + c + d =$ **[TS/July 19, 2022 (I)]**
(a) 10 (b) 15 (c) 20 (d) 30

250. The equation of lowest degree with rational coefficients having roots $\sqrt{3} + \sqrt{2}i$ and $\sqrt{3} - \sqrt{2}$ is **[TS/May 19, 2023 (I) (S); TS/July 19, 2022 (I)]**
(a) $(x^4 - 2x^2 + 25)(x^4 - 10x^2 + 1) = 0$
(b) $\left(x^2 - 2\sqrt{3}x + 5\right)\left(x^2 - 2\sqrt{3}x + 1\right) = 0$
(c) $(x^4 - 2x^2 + 25)(x^4 + 10x^2 + 1) = 0$
(d) $(x^4 - 10x^2 + 1)(x^4 + 2x^2 + 25) = 0$

251. The number of non-real roots of the equation $x^{10} - 3x^8 + 5x^6 - 5x^4 + 3x^2 - 1 = 0$ is **[TS/July 19, 2022 (I)]**
(a) 8 (b) 6 (c) 4 (d) 2

252. If the quadratic equations $x^2 - 7x + 3c = 0$ and $x^2 + x - 5c = 0$ have a common root, then for non-zero real value of c the sign of the expression $x^2 - 3x + c$ is **[TS/July 18, 2022 (II)]**
(a) negative for all $x \in R$
(b) positive for all $x \in (1, 3)$
(c) negative for all $x \in (1, 3)$
(d) positive for all $x \in R$

253. If α, β, γ are the roots of the equation $x^3 + x^2 + x + r = 0$ and $\alpha^3 + \beta^3 + \gamma^3 = 5$, then $r =$ **[TS/July 18, 2022 (II)]**
(a) $\dfrac{-1}{2}$ (b) 1 (c) –1 (d) $\dfrac{1}{2}$

254. If $\frac{5}{2}$ is the sum of two roots of the equation $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$ then the sum of all non-real roots of the equation is **[TS/July 18, 2022 (II)]**

(a) does not exist (b) 0

(c) $\frac{5}{3}$ (d) $\frac{2}{5}$

255. If $1+\sqrt{2}$ and $2 - i$ are the roots of the equation $x^4 + bx^3 + cx^2 + dx + e = 0$ where b, c, d, e are rational numbers, then the roots of the equation $bx^2 + cx + d = 0$ are **[AP/May 18, 2023 (II), (S), TS/July 18, 2022 (II)]**

(a) real and different (b) real and equal

(c) purely imaginary (d) complex conjugate

256. Let the transformed equation of $2x^4 - 8x^3 + 3x^2 - 1 = 0$ so that the term containing the cubic power of x is absent be $2x^4 + bx^2 + cx + d = 0$. Then $b =$ **[TS/July 18, 2022 (II)]**

(a) -18 (b) -15 (c) -9 (d) -16

257. If tan 15° and tan 30° are the roots of the equation $x^2 + px + q = 0$, then $pq =$ **[TS/July 18, 2022 (II)]**

(a) $\frac{6\sqrt{3}+10}{\sqrt{3}}$ (b) $\frac{10-6\sqrt{3}}{3}$

(c) $\frac{10+6\sqrt{3}}{3}$ (d) $\frac{10-6\sqrt{3}}{\sqrt{3}}$

258. If α, β, γ are the roots of the equation $x^3 - 5x^2 - 2x + 24 = 0$ then $\frac{\beta\gamma}{\alpha}+\frac{\gamma\alpha}{\beta}+\frac{\alpha\beta}{\gamma}=$ **[TS/July 18, 2022 (I)]**

(a) 244 (b) $\frac{-1}{6}$ (c) 61 (d) $\frac{-61}{6}$

259. If α, β, γ are the roots of the equation $3x^3 - 26x^2 + 52x - 24 = 0$ such that α, β, γ are in geometric progression and $\alpha < \beta < \gamma$, then $3\alpha + 2\beta + \gamma =$ **[TS/July 18, 2022 (I)]**

(a) $\frac{68}{3}$ (b) $\frac{56}{3}$ (c) 12 (d) 24

260. If α, β, γ are the roots of the equation $4x^3 + 12x^2 - 7x + 165 = 0$ and $\alpha + 5, \beta + 5, \gamma + 5$ are the roots of the equation $ax^3 + bx^2 + cx + d = 0$ then the product of the roots of the second equation is **[TS/July 18, 2022 (I)]**

(a) 27 (b) 0 (c) -3 (d) $3\sqrt{5}+4$

261. If α, β and γ are the roots of the equation $x^3 - 6x^2 + 11x + 6 = 0$, then $\Sigma\alpha^2\beta + \Sigma\alpha\beta^2$ is equal to **[AP/Aug. 23, 2021 (I)]**

(a) 80 (b) 84 (c) 90 (d) -84

262. If α, β and γ are roots of the equation $x^3 + 4x - 19 = 0$. Then, the value of $\frac{\alpha^3}{19-4\alpha}+\frac{\beta^3}{19-4\beta}+\frac{\gamma^3}{19-4\gamma}$ is equal to **[AP/May 17, 2023 (I) (S); AP/Aug. 23, 2021 (I)]**

(a) 0 (b) 3 (c) -3 (d) 2

263. If one root of the equation $ix^2 - 2(i + 1)x + (2 - i) = 0$ is $(2 - i)$, then the other root is **[AP/Aug. 20, 2021 (I)]**

(a) $-i$ (b) $2 + i$ (c) i (d) $2 - i$

264. If α and β are the roots of the quadratic equation $x^2 + x + 1 = 0$, then the equation whose roots are α^{2021}, β^{2021} is given by **[AP/Aug. 20, 2021 (I)]**

(a) $x^2 - x + 1 = 0$ (b) $x^2 + x - 1 = 0$

(c) $x^2 - x - 1 = 0$ (d) $x^2 + x + 1 = 0$

265. If 2, 1 and 1 are roots of the equation $x^3 - 4x^2 + 5x - 2 = 0$, then the roots of

$$\left(x+\frac{1}{3}\right)^3 - 4\left(x+\frac{1}{3}\right)^2 + 5\left(x+\frac{1}{3}\right) - 2 = 0$$

[AP/Aug. 20, 2021 (I)]

(a) $\frac{7}{3},\frac{4}{3},\frac{4}{3}$ (b) $\frac{5}{3},\frac{2}{3},\frac{2}{3}$

(c) $\frac{-5}{3},\frac{-2}{3},\frac{-2}{3}$ (d) $\frac{-7}{3},\frac{-4}{3},\frac{-4}{3}$

266. If $(x^2 + 5x + 5)^{x + 5} = 1$, then the number of integers satisfying this equation is **[AP/Aug. 19, 2021 (I)]**

(a) 2 (b) 3 (c) 4 (d) 5

267. Let $f(x) = x^3 + ax^2 + bx + c$ be polynomial with integer coefficients. If the roots of $f(x)$ are integer and are in Arithmetic Progression, then a cannot take the value **[AP/Aug. 19, 2021 (I)]**

(a) -642 (b) 1214 (c) 1323 (d) 1626

268. If z_1 and z_2 are the roots of the equation $x^2 + 2x + 2 = 0$, then $\frac{-2^{11}\left(z_1+1+3i\right)^{11}}{2^5\left(z_2+1-3i\right)^{11}}$ is equal to **[TS/Aug. 6, 2021 (I)]**

(a) 64 (b) 32 (c) $16\sqrt{2}$ (d) $8\sqrt{2}$

269. If $\frac{\alpha}{\alpha+1}$ and $\frac{\beta}{\beta+1}$ are the roots of the quadratic equation $x^2 + 7x + 3 = 0$, then the equation having roots α and β is **[TS/Aug. 4, 2021 (I)]**

(a) $3x^2 - x - 3 = 0$ (b) $11x^2 + 13x + 3 = 0$

(c) $13x^2 + 11x + 13 = 0$ (d) $11x^2 + 3x + 13 = 0$

270. If α, β and γ are the roots of $x^3 - x + 1 = 0$, then $\frac{1+\alpha}{1-\alpha}+\frac{1+\beta}{1-\beta}+\frac{1+\gamma}{1-\gamma}$ is equal to **[TS/Aug. 4, 2021 (I)]**

(a) 1 (b) 0 (c) 2 (d) -2

271. If $2 + 4i$ is one of the roots of $x^2 + bx + c = 0$ with b, c $\in$ **R** then $(b, c) =$ **[AP/Sept. 21, 2020 (I)]**

(a) $(4, -20)$ (b) $(4, 20)$

(c) $(-4, -20)$ (d) $(-4, 20)$

272. If $(2 + i)$ is a root of the equation $x^3 - 5x^2 + 9x - 5 = 0$, then the other roots are **[AP/Sept. 21, 2020 (I)]**

(a) 1 and $(2 - i)$ (b) -1 and $(3 + i)$

(c) 0 and 1 (d) -1 and $(-2 + i)$

273. Solve the equation, $3^{x^2-x} = 25 - 4^{x^2-x}$ **[AP/Sept. 21, 2020 (I)]**

(a) -1 only (b) 2 only

(c) Both -1 and 2 (d) No solution

274. If α, β, γ are the roots of $f(x) = x^3 - 9x^2 + 26x - 24$, then $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ are the roots of **[AP/Sept. 21, 2020 (I)]**

(a) $24x^3 + 26x^2 + 9x - 1$ (b) $24x^3 - 26x^2 + 9x - 1$
(c) $24x^3 + 26x^2 - 9x - 1$ (d) $24x^3 - 26x^2 + 9x + 1$

275. What is the quotient of $x^3 - 5x^2 + 2x + 7$ when divided with $(x - 1)$? **[AP/Sept. 21, 2020 (I)]**

(a) $x^2 + 4x - 2$ (b) $x^2 - 4x + 2$
(c) $x^2 + 4x + 2$ (d) $x^2 - 4x - 2$

276. If α and β are non-real roots of $x^3 - x^2 - x - 2 = 0$, then $\alpha^{2020} + \beta^{2020} + \alpha^{2020} \cdot \beta^{2020 =}$ **[AP/Sept. 18, 2020 (I)]**

(a) 1 (b) 2020 (c) $1 + \alpha + \beta$ (d) -1

277. Find $\alpha^4 + \beta^4$ if α, β are the roots of equation $x^2 + x + 1 = 0$. **[AP/Sept. 18, 2020 (I)]**

(a) $\frac{1}{\alpha\beta}$ (b) $\frac{2}{\alpha\beta}$ (c) $\alpha\beta$ (d) $-\alpha\beta$

278. Which among the following equations has roots which are negative of the roots of the equation $x^3 - x^2 + x - 4 = 0$? **[AP/Sept. 18, 2020 (I)]**

(a) $x^3 - x^2 + x - 4 = 0$ (b) $x^3 + x^2 + x + 4 = 0$
(c) $x^3 - x^2 + x - 4 = 0$ (d) $x^3 - x^2 - x - 4 = 0$

279. If the sum of the roots of the quadratic equations is 1 and sum of the squares of the roots is 13, then find that equation. **[AP/Sept. 18, 2020 (I)]**

(a) $x^2 + x - 6 = 0$ (b) $x^2 - x + 6 = 0$
(c) $x^2 - x - 6 = 0$ (d) $x^2 + x + 6 = 0$

280. If $2\alpha = -1 - i\sqrt{3}$ and $2\beta = -1 + i\sqrt{3}$, then $5\alpha^4 + 5\beta^4 + 7\alpha^{-1}\beta^{-1}$ is equal to **[AP/Sept. 17, 2020 (I)]**

(a) -1 (b) -2 (c) 0 (d) 2

281. The roots of the equation $|x^2 - x - 6| = x + 2$ are **[AP/Sept. 17, 2020 (I)]**

(a) $-2, 1, 4$ (b) $0, 2, 4$
(c) $0, 1, 4$ (d) $-2, 2, 4$

282. If $x \in \mathbf{R}$, then one of the solutions of $\sqrt{x+1} - \left|\sqrt{x-1}\right| = \sqrt{4x-1}$ among the following is **[AP/Sept. 17, 2020 (I)]**

(a) $x = \frac{5}{4}$ (b) $x = \frac{-5}{4}$ (c) $x = 0$ (d) $x = 1$

283. If $f(x) \in \mathbf{Q}[x]$ be a non-zero polynomial such that all its roots are irrational, then the degree of $f(x)$ is **[AP/Sept. 17, 2020 (I)]**

(a) an even number (b) an odd number
(c) 0 (d) can't determine

284. Let S be the set of all possible integral values of λ in the interval $(-3,7)$ for which the roots of the quadratic equation $\lambda x^2 + 13x + 7 = 0$ are all rational numbers. Then the sum of the elements in S is **[TS/Sept. 10, 2020 (I)]**

(a) 4 (b) 2 (c) 3 (d) 1

285. For the equation $x^4 + x^3 - 4x^2 + x - 1 = 0$ the ratio of the sum of the squares of all the roots to the product of the distinct roots is **[TS/Sept. 10, 2020 (I)]**

(a) 1 : 4 (b) 3 : 5 (c) 9 : 1 (d) 4 : 3

286. If $\alpha_1, \beta_1, \gamma_1, \delta_1$ are the roots of the equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ and $\alpha_2, \beta_2, \gamma_2, \delta_2$ are the roots of the equation $ex^4 + dx^3 + cx^2 + bx + a = 0$ such that $0 < \alpha_1 < \beta_1 < \gamma_1 < \delta_1$, $0 < \alpha_2 < \beta_2 < \gamma_2 < \delta_2$, $\alpha_1 - \delta_2 = 2 = \beta_1 - \gamma_2; \gamma_1 - \beta_2 = \delta_1 - \alpha_2 = 4$, then $a + b + c + d + e =$ **[TS/Sept. 10, 2020 (I)]**

(a) 10 (b) 12 (c) 6 (d) 8

287. If $\frac{x^2 + ax + 3}{x^2 + x + 1}$ takes real values for all real values of x, then a lies in the interval **[TS/Sept. 9, 2020 (II)]**

(a) $(-2-\sqrt{11}, \sqrt{11}-2)$ (b) $(4, 3)$
(c) $(-2+\sqrt{2}, 2+\sqrt{2})$ (d) $(-1, 0)$

288. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 - 8x^3 + 11x^2 + 32x - 60 = 0$ and $\alpha < \beta < \gamma < \delta$, then $4\alpha + 3\beta + 2\gamma + \delta =$ **[TS/Sept. 9, 2020 (II)]**

(a) 0 (b) 1 (c) 9 (d) 10

289. Let $a, b \in \boldsymbol{R}$ and the roots α, β of the equation $z^2 + az + b = 0$ be complex. If the origin, α and β represent the vertices of an equilateral triangle on the Argand plane, then **[TS/Sept. 9, 2020 (II)]**

(a) $a = b$ (b) $a^2 = 3b$ (c) $a^2 = 4b$ (d) $a = 3b$

290. If e^{ix} is a solution of the equation $z^n + p_1 z^{n-1} + p_2 z^{n-2} + ... + p_n = 0$, where p_i are real $(i = 1, 2, 3, ..., n)$, then $p_n \sin nx + p_{n-1} \sin(n-1)x + ... + p_1 \sin x + 1 =$ **[TS/Sept. 9, 2020 (I)]**

(a) $\cos(n + 1)x$ (b) $\sin(n(n + 1))x$
(c) 1 (d) 0

291. Let α, β, γ be the roots of the equation $x^3 + px + q = 0$ and $f(x) = 3p^2x^2 + p^2x + 3q$. Then $\sum\alpha^2\beta + \sum\alpha^4 =$ **[TS/Sept. 9, 2020 (I)]**

(a) $f(1)$ (b) $f(-1)$ (c) $f(0)$ (d) $f(2)$

292. If α, β, γ are the roots of the equation $x^3 + ax^2 - bx + c = 0$, then $\sum\beta^2(\gamma + \alpha) =$ **[TS/Sept. 9, 2020 (I)]**

(a) $\frac{a^2 + b - c}{3ab}$ (b) $ac + b^3$

(c) $\frac{bc + a^2}{3ab}$ (d) $ab + 3c$

293. If $3x^2 - 7x + 2 = 0$ and $15x^2 - 11x + a = 0$ have a common root and a is a positive real number, then the sum of the roots of the equation $15x^2 - ax + 7 = 0$, is **[AP/Apr. 22, 2019 (I)]**

(a) $\frac{76}{15}$ (b) $\frac{38}{15}$ (c) $\frac{2}{15}$ (d) $\frac{36}{15}$

294. Let $\phi(x) = \frac{x}{(x^2 + 1)(x + 1)}$. If a, b and c are the roots of the equation $x^3 - 3x + \lambda = 0$, $(\lambda \neq 0)$. Then, $\phi(a)\,\phi(b)\,\phi(c) =$ **[AP/Apr. 22, 2019 (I)]**

(a) λ (b) $\frac{-\lambda}{(\lambda+2)(\lambda^2+16)}$

(c) $\frac{\lambda}{(\lambda+2)}$ (d) $\frac{\lambda}{(\lambda+2)(\lambda^2+16)}$

295. If $x, y \in R$ and $x^2 + y + 4i$ and $-3 + x^2 yi$ are conjugates to each other, then $(|x| + |y|)^2 =$ **[AP/Apr. 21, 2019 (I)]**

(a) 17 (b) 16 (c) 25 (d) 9

296. If α satisfies the equation $\sqrt{\frac{x}{2x+1}} + \sqrt{\frac{2x+1}{x}} = 2$, then the roots of the equation $\alpha^2x^2 + 4\alpha x + 3 = 0$ are **[AP/Apr. 21, 2019 (I)]**

(a) l, 3 (b) –1, 1 (c) 2, –3 (d) 3, 4

297. If α, β, γ are the roots of $x^3 - 6x^2 + 11x - 6 = 0$, then the equation having the roots $\alpha^2 + \beta^2$, $\beta^2 + \gamma^2$ and $\gamma^2 + \alpha^2$ is

(a) $x^3 - 28x^2 + 245x - 650 = 0$ **[AP/Apr. 21, 2019 (I)]**
(b) $x^3 - 28x^2 + 245x + 650 = 0$
(c) $x^3 + 28x^2 - 245x - 650 = 0$
(d) $x^3 + 28x^2 + 245x - 650 = 0$

298. If α and β are the roots of $x^2 + 7x + 3 = 0$ and $\frac{2\alpha}{3-4\alpha}, \frac{2\beta}{3-4\beta}$ are the roots of $ax^2 + bx + c = 0$ and GCD of a, b, c is 1, then $a + b + c =$ **[AP/Apr. 20, 2019 (I)]**

(a) 11 (b) 0 (c) 243 (d) 81

299. If α, β are the roots of $x^2 + bx + c = 0$, γ, δ are the roots of $x^2 + b_1x + c_1 = 0$ and $\gamma < \alpha < \delta < \beta$, then $(c - c_1)^2 <$ **[AP/Apr. 20, 2019 (I)]**

(a) $(b_1 - b)(bc_1 - b_1c)$ (b) 1
(c) $(b - b_1)^2$ (d) $(c - c_1)(b_1c - b_1c_1)$

300. The polynomial equation of degree 4 having real coefficients with three of its roots as $2 \pm \sqrt{3}$ and $1 + 2i$, is **[AP/Apr. 20, 2019 (I)]**

(a) $x^4 - 6x^3 - 14x^2 + 22x + 5 = 0$
(b) $x^4 - 6x^3 - 19x + 22x - 5 = 0$
(c) $x^4 - 6x^3 + 19x - 22x + 5 = 0$
(d) $x^4 - 6x^3 + 14x^2 - 22x + 5 = 0$

301. If $a_k = \cos\alpha_k + i\sin\alpha_k$, $k = 1, 2, 3$ and a_1, a_2, a_3 are the roots of the equation $x^3 + bx + c = 0$, then the real part of $b =$ **[TS/May 3, 2019 (I)]**

(a) – 1 (b) 1 (c) 0 (d) $\frac{2}{3}$

302. If $\sin 2\theta$ and $\cos 2\theta$ are solutions of $x^2 + bx - c = 0$, then **[TS/May 6, 2019 (I)]**

(a) $b^2 + 2c + 1 = 0$ (b) $b^2 + 2c - 1 = 0$
(c) $b^2 - 2c + 1 = 0$ (d) $b^2 - 2c - 1 = 0$

303. The sum of all the real numbers satisfying the equation $x^2 + |x - 3| = 4$ is **[TS/May 4, 2019 (II)]**

(a) 0 (b) 1 (c) 2 (d) –1

304. The solution set of the inequation $3^x + 3^{1-x} - 4 < 0$, is **[TS/May 3, 2019 (II)]**

(a) (0, 1) (b) (0, 2) (c) (1, 2) (d) (1, 3)

305. If $k \in R$, then roots of $(x - 2)(x - 3) = k^2$ are always **[TS/May 5, 2018 (I)]**

(a) real and distinct (b) real and equal
(c) complex number (d) rational numbers

306. If $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ are roots of $x^n + px + q = 0$, then $(\alpha_n - \alpha_1)(\alpha_n - \alpha_2) \ldots\ldots (\alpha_n - \alpha_{n-1}) =$ **[TS/May 5, 2018 (I)]**

(a) $n\alpha_a^{n-1} + q$ (b) $\alpha_1^2 + \alpha_2^2 + \ldots + \alpha_{n-1}^2$
(c) $\alpha_n^{n-1} + p$ (d) $n\alpha_{n-}^{n-1} + p$

307. All the roots of the equation $x^5 + 15x^4 + 94x^3 + 305x^2 + 507x + 353 = 0$ are increased by some real number k in order to eliminate the 4th degree term from the equation. Now, the coefficient of x in the transformed equation is **[TS/May 5, 2018 (I)]**

(a) 2 (b) 1 (c) 6 (d) 0

308. Given that the roots of $x^3 + 3px^2 + 3qx + r = 0$ are in harmonic progression. Then, **[TS/May 4, 2018 (II)]**

(a) $2q^3 = r(3pq - r)$ (b) $q^3 = r(3pq - r)$
(c) $q^3 = -r(3pq - r)$ (d) $q^3 = r(r + 3pq)$

309. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, then $\alpha^3 + \beta^3 + \gamma^3 =$ **[TS/May 4, 2018 (II)]**

(a) $p^3 - 3pq + r$ (b) $p^2 - 2pq + r$
(c) $3pq - 3r - p^3$ (d) $3pq + 3r + p^3$

310. If $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + px + q = 0$, then the value of $\sin^2(\alpha + \beta) + p\cos(\alpha + \beta)\sin(\alpha + \beta) + q\cos^2(\alpha + \beta)$ is **[AP/2017]**

(a) $p + q$ (b) p (c) q (d) $\frac{p}{p+q}$

311. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, then the value of $(1 + \alpha^2)(1 + \beta^2)(1 + \gamma^2)$ is **[TS/May 14, 2023 (II), (S), AP/2017]**

(a) $(r - p)^2 + (r - q)^2$ (b) $(1 + p)^2 + (1 + q)^2$
(c) $(r + p)^2 + (q + 1)^2$ (d) $(r - p)^2 + (q - 1)^2$

312. The sum of the complex roots of the equations $(x-1)^3 + 64 = 0$ is **[TS/2017]**

(a) 6 (b) 3 (c) $6i$ (d) $3i$

313. α and β are the roots of $x^2 + 2x + c = 0$. If $\alpha^3 + \beta^3 = 4$, then the value of c is **[TS/2017]**

(a) –2 (b) 3
(c) 2 (d) 4

314. If α and β are the roots of the equation $ax^2 + bx + c = 0$ and the equation having roots $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$ is $px^2 + qx + r = 0$, then $r =$ **[TS/2017]**

(a) $a + 2b$ (b) $ab + bc + ca$
(c) $a + b + c$ (d) abc

315. The product and sum of the roots of the equation $|x^2| - 5|x| - 24 = 0$ are respectively **[AP/2016]**

(a) –64, 0 (b) –24, 5 (c) 5, – 24 (d) 0, 72

316. If *a, b, c* are distinct and the roots of $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then *a, b* and *c* are in **[AP/2015]**

(a) arithmetic progression

(b) geometric progression

(c) harmonic progression

(d) arithmetico-geometric progression

317. If the roots of $x^3 - kx^2 + 14x - 8 = 0$ are in geometric progression, then *k* is equal to **[AP/2015]**

(a) –3 (b) 7 (c) 4 (d) 0

318. In ΔABC, the value of $\angle A$ is obtained from the equation $3\cos A + 2 = 0$. The quadratic equation, whose roots are $\sin A$ and $\tan A$, is **[TS/2015]**

(a) $3x^2 + \sqrt{5}x - 5 = 0$ (b) $6x^2 - \sqrt{5}x - 5 = 0$

(c) $6x^2 + \sqrt{5}x - 5 = 0$ (d) $6x^2 + \sqrt{5}x + 5 = 0$

Topic 4 Condition for Common Roots, Maximum and Minimum value of Quadratic Expression, Quadratic Expression in two Variables, Solution of Quadratic Inequalities

319. The set of all real values of *x* satisfying the inequality $\frac{7x^2 - 5x - 18}{2x^2 + x - 6} < 2$ is **[AP/May 23, 2024 (I)]**

(a) $\left(-\infty, -\frac{2}{3}\right] \cup [3, \infty)$ (b) $\left(-2, -\frac{2}{3}\right) \cup \left(\frac{3}{2}, 3\right)$

(c) $(-\infty, -2) \cup \left(\frac{3}{2}, \infty\right)$ (d) $\left[-\frac{2}{3}, \frac{3}{2}\right)$

320. The set of all values of *k* for which the inequality $x^2 - (3\text{k}+\text{l})x + 4\text{k}^2 + 3\text{k} - 3 > 0$ is true for all real values of *x* is **[AP/May 23, 2024 (I)]**

(a) $\left(-\frac{13}{7}, 1\right)$ (b) $\left(-1, \frac{13}{7}\right)$

(c) $\left(-\infty, -\frac{13}{7}\right) \cup (1, \infty)$ (d) $(-\infty, -1) \cup \left(\frac{13}{7}, \infty\right)$

321. The set of all real values '*a*' for which $-1 < \frac{2x^2 + ax + 2}{x^2 + x + 1} < 3$ holds for all real values of *x* is **[AP/May 22, 2024 (II)]**

(a) (–7, 5) (b) (5, ∞) (c) (1, 5) (d) (–∞, 1)

322. The quotient when $3x^5 - 4x^4 + 5x^3 - 3x^2 + 6x - 8$ is divided by $x^2 + x - 3$ is **[AP/May 22, 2024 (II)]**

(a) $3x^2 - 7x - 21$

(b) $3x^3 - 7x^2 + 21x - 45$

(c) $3x^4 - 7x^3 + 21x^2 - 45 + 114$

(d) $114x - 143$

323. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are the roots of $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$ then $\frac{1}{\alpha_1^2} + \frac{1}{\alpha_2^2} + \frac{1}{\alpha_3^2} + \frac{1}{\alpha_4^2} + \frac{1}{\alpha_5^2} =$ **[AP/May 22, 2024 (II)]**

(a) 15 (b) $\frac{1}{7}$ (c) 7 (d) 12

324. If the number of real roots of $x^9 - x^5 + x^4 - 1 = 0$ is *n*, the number of complex roots having argument on imaginary axis is *m* and the number of complex roots having argument in 2nd quadrant is k, then $m \cdot n \cdot k \cdot =$ **[AP/May 22, 2024 (I)]**

(a) 6 (b) 9 (c) 12 (d) 24

325. If *x* is real and α, β are maximum and minimum values of $\frac{x^2 - x + 1}{x^2 + x + 1}$ respectively then $\alpha + \beta =$ **[AP/May 21, 2024 (II)]**

(a) $\frac{10}{3}$ (b) $\frac{8}{3}$ (c) $\frac{4}{3}$ (d) $\frac{2}{3}$

326. If α is a common root of $x^2 - 5x + \lambda = 0$ and $x^2 - 8x - 2\lambda = 0$ $(\lambda \neq 0)$ and β, γ are the other roots of them, then $\alpha + \beta + \gamma + \lambda =$ **[AP/May 21, 2024 (II)]**

(a) 0 (b) – 1 (c) 1 (d) 2

327. The algebraic equation of degree 4 whose roots are the translates of the roots of the equation $x^4 + 5x^3 + 6x^2 + 7x + 9 = 0$ by – 1 is **[AP/May 21, 2024 (I)]**

(a) $x^4 + x^3 - 3x^2 + 6x + 4 = 0$

(b) $x^4 + 9x^3 + 27x^2 + 38x + 28 = 0$

(c) $x^4 + 5x^3 + 6x^2 + 7x + 9 = 0$

(d) $x^4 - 5x^3 + 6x^2 - 7x + 9 = 0$

328. If α and β are two double roots of $x^2 + 3(a + 3)x - 9a = 0$ for different values of α (α > β). then the minimum value of $x^2 + \alpha x - \beta = 0$ is **[AP/May 20, 2024 (I)]**

(a) $\frac{69}{4}$ (b) $-\frac{69}{4}$ (c) $-\frac{35}{4}$ (d) $\frac{35}{4}$

329. If $2x^2 + 3x - 2 = 0$ and $3x^2 + ax - 2 = 0$ have one common root, then the sum of all possible values of *a* is **[AP/May 20, 2024 (I)]**

(a) – 3.5 (b) 7.5 (c) –7.5 (d) –1.5

330. If $x^2 + 5ax + 6 = 0$ and $x^2 + 3ax + 2 = 0$ have a common root then that common root is **[AP/May 19, 2024 (II)]**

(a) 3 (or) –3 (b) 2 (or) –2

(c) 2 (or) –3 (d) –2 (or) 3

331. The equation $16x^4 + 16x^3 - 4x - 1 = 0$ has a multiple root. If α, β, γ, δ are the roots of this equation, then $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} + \frac{1}{\delta^4} =$ **[TS/May 11, 2024 (I)]**

(a) $\frac{1}{64}$ (b) $\frac{1}{32}$ (c) 32 (d) 64

332. The set of all real values of *x* for which the expansion of $\left(125x^2 - \frac{27}{x}\right)^{-2/3}$ is valid, is **[TS/May 11, 2024 (I)]**

(a) $\left(-\frac{3}{5}, \frac{3}{5}\right)$ (b) $\left(-\infty, -\frac{3}{5}\right) \cup \left(\frac{3}{5}, \infty\right)$

(c) $\left(-\frac{5}{3}, \frac{5}{3}\right)$ (d) $\left(-\infty, -\frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$

333. If the expression $7 + 6x - 3x^2$ attains its extreme value β at $x = \alpha$, then the sum of the squares of the roots of the equation $x^2 + \alpha x - \beta = 0$ is **[TS/May 10, 2024 (I)]**

(a) 21 (b) –19 (c) 19 (d) –21

334. If $4 + 3x - 7x^2$ attains its maximum value M at $x = \alpha$ and $5x^2 - 2x + 1$ attains its minimum value m at $x = \beta$, then $\frac{28(M-\alpha)}{5(m+\beta)} =$ **[TS/May 9, 2024 (II)]**

(a) 28 (b) 23 (c) 5 (d) 1

335. One of the roots of the equation $x^{14} + x^9 - x^5 - 1 = 0$ is **[TS/May 9, 2024 (I)]**

(a) $\frac{1+\sqrt{3}i}{2}$ (b) $\frac{\sqrt{5}-1}{4}+i\frac{\sqrt{10-2\sqrt{5}}}{4}$

(c) $\frac{1-\sqrt{3}i}{2}$ (d) $\frac{\sqrt{5}+1}{4}+i\frac{\sqrt{10-2\sqrt{5}}}{4}$

336. The equations $2x^2 + ax - 2 = 0$ and $x^2 + x + 2a = 0$ have exactly one common root. If $a \neq 0$, then one of the roots of the equation $ax^2 - 4x - 2a = 0$ is **[TS/May 9, 2024 (I)]**

(a) 2 (b) –2

(c) $\frac{-4+\sqrt{22}}{3}$ (d) $\frac{-2+\sqrt{22}}{3}$

337. The largest interval containing x for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is **[TS/May 13, 2023 (II), (S), AP/July 6, 2022 (I)]**

(a) $0 < x < 1$ (b) $-4 < x < 2$

(c) $-\infty < x < \infty$ (d) $-2^{10} < x < 2^{10}$

338. The number of solutions of the equations $x + y + z = 1; x^2 + y^2 + z^2 = 1; x^3 + y^3 + z^3 = 1$ is **[AP/July 5, 2022 (II)]**

(a) 6 (b) 3 (c) 9 (d) 12

339. **Statement (I) :** The set of solutions of $|x|^2 - 4|x| + 3 < 0$ is the interval (–3, 3).

Statement (II) : If $x < 3$ or $x > 5$ then $x^2 - 8x + 15 > 0$.

Which of the above statements is (are) true? **[TS/July 20, 2022 (II)]**

(a) Statement I is true, but Statement II is false
(b) Statement II is true, but Statement I is false
(c) Both Statement I and Statement II are true
(d) Both Statement I and Statement II are false

340. Let x be a real number. Match the following:

List-I	**List-II**
(A) The maximum value of $2x^2 + 4x + 5$	(I) –1
(B) The maximum value of $\frac{x^2+4x+1}{x^2+x+1}$	(II) 1
(C) If $1 \le \frac{3x^2-5x+6}{x^2+1} \le 2$, $\forall x \in [a, b]$ then $b =$	(III) 2
(D) If $1 \le \frac{3x^2-5x+6}{x^2+1} \le 2$, $\forall x \in [a, b]$ then $a =$	(IV) 3
	(V) 4

The correct answer is **[TS/July 19, 2022 (I)]**

	A	**B**	**C**	**D**
(a)	IV	III	II	V
(b)	IV	V	II	III
(c)	IV	III	V	II
(d)	III	V	IV	I

341. Let $f(x) = \frac{6x^2-18x+21}{6x^2-18x+17}$. If m is the maximum value of $f(x)$ and $f(x) > n\ \forall\ x \in \mathbb{R}$. Then $14m - 7n =$ **[TS/July 18, 2022 (II)]**

(a) –1 (b) 23 (c) 35 (d) 42

342. If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is divided by $x - 1$ and $x + 1$, the remainders are 5 and 19, respectively. If $f(x)$ is divided by $x - 2$. The remainder is **[AP/Aug. 23, 2021 (I)]**

(a) 8 (b) 5 (c) 10 (d) 12

343. Let a, b and c be positive real numbers.

If $\frac{x^2-bx}{ax-c} = \frac{m-1}{m+1}$ has two roots which are numerically equal but opposite in sign, then the value of m is **[AP/Aug. 23, 2021 (I)]**

(a) c (b) $\frac{1}{c}$ (c) $\frac{a+b}{a-b}$ (d) $\frac{a-b}{a+b}$

344. If $f(x) = 2x^3 + mx^2 - 13x + n$ and 2, 3 are the roots of the equation $f(x) = 0$, then the values of m and n are **[AP/Aug. 20, 2021 (I)]**

(a) – 5, – 30 (b) – 5, 30

(c) 5, 30 (d) 5, – 30

345. The sum of the roots of the equation **[AP/Aug. 19, 2021 (I)]**

$e^{4t} - 10e^{3t} + 29e^{2t} - 22e^t + 4 = 0$ is

(a) $\log_e 10$ (b) $2\log_e 2$ (c) $\log_2 29$ (d) $2\log_{10} 2$

346. Suppose, α is minimum value of $x^2 + bx + 5$ and β is maximum value of $-x^2 + ax + 5$. If $[\alpha, \beta]$ is the interval of maximum length for x in which $x^2 - 10x + 24 \le 0$, then a^2b^2 is equal to **[TS/Aug. 6, 2021 (II)]**

(a) 25 (b) 16 (c) 4 (d) 18

347. If the minimum value of the quadratic expression $x^2 + 5x - 2$ is M and it exists at a, then $\frac{M}{a}$ is equal to **[TS/Aug. 6, 2021 (II)]**

(a) 3.3 (b) $\frac{33}{5}$ (c) 2.5 (d) –0.25

348. Number of roots common to the equations $x^3 + x^2 - 2x - 2 = 0$ and $x^3 - x^2 - 2x + 2 = 0$ is **[AP/Sept. 18, 2020 (I)]**

(a) 1 (b) 2 (c) 3 (d) 0

349. If the quadratic equations $3x^2 - 7x + 2 = 0$ and $kx^2 + 7x - 3 = 0$ have a common root then the positive value of k is **[TS/Sept. 11, 2020 (I)]**

(a) 6 (b) 11/4 (c) 4 (d) 7/2

350. If the roots of the quadratic equation $ax^2 + bx + c = 0$ are imaginary, then for all real values of x, the minimum value of the expression $3a^2x^2 + 6abx + 2b^2$ is **[TS/Sept. 9, 2020 (I)]**

(a) $< 4ab$ (b) $> 4ac$ (c) $> -4ac$ (d) $< -4ab$

351. Let α, β be the roots of the equation $x^2 - |a|x - |b| = 0$ such that $|\alpha| < |\beta|$. If $|a| < \beta - 1$, then the positive root of $\log_{|\alpha|}\left(\frac{x^2}{\beta^2}\right) - 1 = 0$, is **[AP/Apr. 22, 2019 (I)]**

(a) $< |\alpha|$ (b) $< \alpha$ (c) $< \beta$ (d) $> \beta$

352. If $x \in \boldsymbol{R}$ and $1 \le \frac{3x^2 - 7x + 8}{x^2 + 1} \le 2$, then the minimum and maximum values of x are respectively. **[AP/Apr. 22, 2019 (I)]**

(a) 1, 2 (b) 5, 12 (c) 6, 10 (d) 1, 6

353. Let a, b and c be the sides of a scalane triangle. If λ is a real number such that the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then the interval in which λ lies is **[AP/Apr. 20, 2019 (I)]**

(a) $\left(-\infty, \frac{4}{3}\right)$ (b) $\left(\frac{5}{3}, \infty\right)$

(c) $\left(\frac{1}{3}, \frac{5}{3}\right)$ (d) $\left(\frac{4}{3}, \infty\right)$

354. The set of all values of 'a' for which the expression $\frac{ax^2 - 2x + 3}{2x - 3x^2 + a}$ assumes all real values for real values of x, is **[TS/May 3, 2019 (I)]**

(a) [2, 3] (b) $R - (2, 3)$ (c) ϕ (d) [1, 5]

355. If both the roots of the equation $x^2 - 4ax + 1 - 3a + 4a^2 = 0$ exceed 1, then a lies in the interval **[TS/May 3, 2019 (I)]**

(a) $\left(-\infty, \frac{7 - \sqrt{17}}{8}\right)$ (b) $\left(\frac{7 + \sqrt{17}}{8}, \infty\right)$

(c) $\left(\frac{7 - \sqrt{17}}{8}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{7 + \sqrt{17}}{8}\right)$

356. If the cubic equation $x^3 - ax^2 + ax - 1 = 0$ is identical with the cubic equation whose roots are the squares of the roots of the given cubic equation, then the non-zero real value of 'a' is **[TS/May 3, 2019 (I)]**

(a) $\frac{1}{2}$ (b) 2 (c) 3 (d) $\frac{7}{2}$

357. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha) =$ **[TS/May 3, 2019 (I)]**

(a) $p - qr$ (b) $r - pq$ (c) $q - rp$ (d) $r + pq$

358. The number of roots of the equation $\sqrt{2} + e^{\cos h^{-1} x} - e^{\sin h^{-1} x} = 0$, is **[TS/May 3, 2019 (I)]**

(a) 0 (b) 1 (c) 2 (d) 3

359. For real number x, if the minimum value of $f(x) = x^2 + 2bx + 2c^2$ is greater than the maximum value of $g(x) = -x^2 - 2cx + b^2$, then **[AP/2018]**

(a) $c^2 > 2b^2$ (b) $c^2 < 2b^2$

(c) $b^2 = 2c^2$ (d) $c^2 = 2b^2$

360. If a, b and c are the roots of $x^3 + qx + r = 0$, then $(a - b)^2 + (b - c)^2 + (c - a)^2 =$ **[AP/2018]**

(a) $-6q$ (b) $-4q$ (c) $6q$ (d) $4q$

361. If the sum of two roots of the equation $x^3 - 2px^2 + 3qx - 4r = 0$ is zero, then the value of r is **[AP/2018]**

(a) $\frac{3pq}{2}$ (d) $\frac{3pq}{4}$ (c) pq (d) $2pq$

362. The common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{2018} + z^{2017} + 1 = 0$ satisfy the equation **[TS/May 5, 2018 (I)]**

(a) $z^2 - z + 1 = 0$ (b) $z^4 + z^2 + 1 = 0$

(c) $z^8 + z^3 + 1 = 0$ (d) $z^{12} + z^6 - 1 = 0$

363. If $x^2 - 3ax + 14 = 0$ and $x^2 + 2ax - 16 = 0$ have a common root, then $a^4 + a^2$. **[TS/May 5, 2018 (I)]**

(a) 2 (b) 90 (c) 6 (d) 20

364. If α, β are the roots of $x^2 - x + 1 = 0$, then the quadratic equation whose roots are $\alpha^{2015}, \beta^{2015}$ is **[TS/2016]**

(a) $x^2 - x + 1 = 0$ (b) $x^2 + x + 1 = 0$

(c) $x^2 + x - 1 = 0$ (d) $x^2 - x - 1 = 0$

365. If α, β, γ are roots of $x^3 - 5x + 4 = 0$, then $(\alpha^3 + \beta^3 + \gamma^3)^2$ is equal to **[TS/2016]**

(a) 12 (b) 13 (c) 169 (d) 144

366. Suppose α, β, γ are roots of $x^3 + x^2 + 2x + 3 = 0$. If $f(x) = 0$ is a cubic polynomial equation whose roots are $\alpha + \beta, \beta + \gamma, \gamma + \alpha$, then $f(x)$ is equal to **[TS/2016]**

(a) $x^3 + 2x^2 - 3x - 1$ (b) $x^3 + 2x^2 - 3x + 1$

(c) $x^3 + 2x^2 + 3x - 1$ (d) $x^3 + 2x^2 + 3x + 1$

367. The smallest value of the constant $m > 0$ for which $f(x) = 9mx - 1 + \frac{1}{x} \ge 0$ for all $x > 0$, is **[TS/2015]**

(a) $\frac{1}{9}$ (b) $\frac{1}{16}$ (c) $\frac{1}{36}$ (d) $\frac{1}{81}$

5 Complex Numbers and Quadratic Equations

1. **(b)** Let $z = \left[\frac{(1+i\sqrt{3})(-\sqrt{3}-i)}{(1-i)(-i)}\right] = 2+2i$

Arg (z) lies in I$^{\text{st}}$ quadrant

$\tan\theta = \frac{2}{2} = 1 \Rightarrow \theta = n\pi + \frac{\pi}{4}$

For I$^{\text{st}}$ quadrant put $n = 0$, $\therefore$ arg $(z) = \frac{\pi}{4}$

2. **(d)** $Z = \frac{(1-i)^3}{(2-i)(3-2i)} = \frac{(1-i)^3}{4-7i} \times \frac{4+7i}{4+7i}$

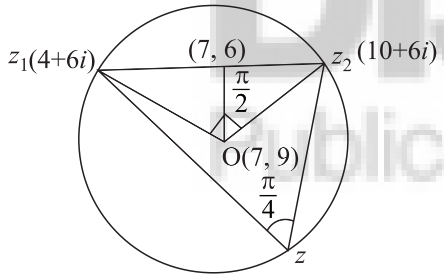
$\Rightarrow \; z = \frac{6-22i}{65} \;\Rightarrow\; \text{Img}(Z) = \frac{-22}{65}.$

3. **(c)** $(4 - 3i)\,(2 + 3i)\,(1 + 4i) = (4 - 3i)\,(-10 + 11i)$

$= -40 + 44i + 30i + 33 = -7 + 74i$

Complex conjugate $= -7 - 74i$

4. **(a)** $\arg\left(\frac{z-z_1}{z-z_2}\right) = \frac{\pi}{4}$



Locus of z lies on circle

Angle subtended at centre $= \frac{\pi}{2}$

Centre of circle is $(7 + 9i)$

Radius $= 3\sqrt{2}$

$|z - (7 + 9i)| = 3\sqrt{2}$

5. **(b)** $z = x + iy$, $p = (x, y)$

$$\frac{z+i}{z-1} = \frac{x+iy+i}{x+iy-1} = \frac{x+i(y+1)}{(x-1)+iy} \times \frac{(x-1)-iy}{(x-1)-iy}$$

$$= \frac{x^2+y^2-x+y}{(x-1)^2+y^2} + \frac{i(x-y+1)}{(x-1)^2+y^2}$$

Since $\frac{z+i}{z-1}$ is purely imaginary number

$\therefore \text{Re}\left(\frac{z+i}{z-1}\right) = 0 \Rightarrow \frac{x^2+y^2-x+y}{(x-1)^2+y^2} = 0$

$\Rightarrow x^2 + y^2 - x + y = 0$ and $(x, y) \neq (1, 0)$

6. **(a)** Given $\left(\frac{1-i}{1+i}\right)^k = -i \Rightarrow \left\{\frac{(1-i)^2}{2}\right\}^k = -i \Rightarrow (-i)^k = -i$

If $k = 1$ (least positive integer) $\Rightarrow (-i)^1 = -i$

If $k = -3$ (greatest negative integer) $\Rightarrow (-i)^{-3} = -i$

So, $m = 1$ and $n = -3 \Rightarrow m - n = 4.$

7. **(c)** Let $\sqrt{-5-12i} = a + bi, a > 0$

$\Rightarrow -5 - 12i = (a + bi)^2$

So $a^2 - b^2 = -5$ and $2ab = -12$

Now, $a^2 + b^2 = 13$

$\{a^2 - b^2 = -5,\ a^2 + b^2 = 13\} \Rightarrow 2a^2 = 8 \Rightarrow a = 2$

$(\because a > 0)$

and $b = \frac{-12}{4} = -3$. So, $\sqrt{-5-12i} = 2 - 3i$

Similarly, we get $\sqrt{5+12i} = 3 + 2i$ and $\sqrt{-8-6i} = -1 + 3i$

Now, $a + bi = \frac{\sqrt{-5-12i} + \sqrt{5+12i}}{\sqrt{-8-6i}}$

$= \frac{-8-14i}{10} = \frac{-4}{5} - \frac{7}{5}i \Rightarrow a = \frac{-4}{5}$ and $b = \frac{-7}{5}$

So, $2a + b = \frac{-8}{5} - \frac{7}{5} = \frac{-15}{5} = -3$

8. **(a)** $z = \frac{(2-i)(1+i)^3}{(1-i)^2} = \frac{(2-i)(1+i^3+3i(i+1))}{(1+i^2-2i)}$

$= \frac{(2-i)(-2+2i)}{-2i} = \frac{(1-i)(2-i)}{i}$

$= \frac{(1-i)(2i+1)}{-1} = (-3-i) = z$ ($\because$ lies in 3$^{\text{rd}}$ quadrant)

Arg (z) $= -\pi + \tan^{-1}\left(\frac{1}{3}\right).$

9. **(a)** $z = x + iy$

Put in $z^2 + az + a^2 = 0$

$\Rightarrow x^2 - y^2 + 2ixy + ax + iy + a^2 = 0$

$\Rightarrow x^2 - y^2 + ax + a^2 + i\,(2xy + ay) = 0$ ...(i)

Comparing imaginary part

$2xy + ay = 0 \Rightarrow x = \frac{-a}{2}$ $[\because\ y \neq 0]$

Comparing real part ; $x^2 - y^2 + ax + a^2 = 0$

$\Rightarrow \frac{a^2}{4} - y^2 - \frac{a^2}{2} + a^2 = 0 \Rightarrow y = \pm\frac{\sqrt{3}}{2}a$

$\therefore$ z = $\frac{-a}{2} \pm \frac{\sqrt{3}}{2}a\,i \Rightarrow |z| = \sqrt{\frac{a^2}{4} + \frac{3}{4}a^2}$

$\Rightarrow |z| = \sqrt{a^2} \Rightarrow |z| = |\mathrm{a}|$

10. **(a)** $|Z_1 - Z_2|^2 + |Z_1 - Z_3|^2 = 4$ $[\because Z\overline{Z} = |Z|^2]$

$\Rightarrow (Z_1 - Z_2)(\overline{Z_1 - Z_2}) + (Z_1 - Z_3)(\overline{Z_1 - Z_3}) = 4$

$\Rightarrow (Z_1 - Z_2)(\overline{Z}1 - \overline{Z}2) + (Z_1 - Z_3)(\overline{Z}1 - \overline{Z}3) = 4$

$\Rightarrow 2|Z_1|^2 + |Z_2|^2 + |Z_3|^2 - (Z_1\overline{Z}2 + \overline{Z}1Z_2 + Z_1\overline{Z}3$

$+ \overline{Z}1Z_3) = 4$

$\Rightarrow 4 - (Z_1\overline{Z}2 + \overline{Z}1Z_1 + Z_1\overline{Z}3 + \overline{Z}1Z_3) = 4$

$\Rightarrow Z_1\overline{Z}2 + \overline{Z}1Z_2 + Z_1\overline{Z}3 + \overline{Z}1Z_3) = 0$

11. **(a)** $Z_1 = \sqrt{3} + i\sqrt{3}$

$r_1 = \sqrt{6},\ \theta_1 = \tan^{-1}1 = \frac{\pi}{4} \Rightarrow Z_1 = \sqrt{6}e^{i\frac{\pi}{4}}$

$Z_2 = \sqrt{3} + i,\ r_2 = 2,\ \theta_2 = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$\Rightarrow Z_2 = 2e^{i\frac{\pi}{6}}$

Now, $\frac{Z_1}{Z_2} = \frac{\sqrt{6}e^{i\frac{\pi}{4}}}{2e^{i\frac{\pi}{4}}} = \frac{\sqrt{6}}{2}e^{i\frac{\pi}{12}}$

And $\left(\frac{Z_1}{Z_2}\right)^{50} = \left(\frac{\sqrt{6}}{2}\right)^{50} e^{i\frac{50\pi}{12}}$

$= \left(\frac{\sqrt{6}}{2}\right)^{50} e^{i\left(4\pi + \frac{2\pi}{12}\right)} = \left(\frac{\sqrt{6}}{2}\right)^{50} e^{i\frac{\pi}{6}}$

So, $\theta_{\left(\frac{Z_1}{Z_2}\right)^{50}} = \frac{\pi}{6}$

Hence, $\left(\frac{z_1}{z_2}\right)^{50}$ lies in Ist quadrant.

12. **(a)** $x + iy = \frac{13\sqrt{-5+12i}}{(2-3i)(3+2i)}$

$= \frac{13(2+3i)(3-2i)}{13 \times 13}\sqrt{-5+12i}$ ...(i)

$\Rightarrow \sqrt{-5+12i} = \pm(a+ib)$

(Using square root of complex number)

$a^2 = \frac{\sqrt{5^2+12^2} - 5}{2},\ b^2 = \frac{\sqrt{5^2+12^2} - (-5)}{2}$

$a = \pm 2,\ b = \pm 3$

$\therefore \sqrt{-5+12i} = \pm(2+3i)$ ...(ii)

= + (2 + 3i) (as x and y are positive)

From (i) and (ii),

$x + iy = \frac{(2+3i)(3-2i)(2+3i)}{13} = \frac{9+46i}{13}$

$\Rightarrow x = \frac{9}{13},\ y = \frac{46}{13} \Rightarrow 13y - 26x = 46 - 18 = 28.$

13. **(a)** $\frac{(2-i)x + (1+i)}{(2+i)} + \frac{(1-2i)y + (1-i)}{(1+2i)} = 1 - 2i$

$\Rightarrow \frac{(4 + i^2 - 4i)x + (2 - i + 2i - i^2)}{5}$

$+ \frac{(1 + 4i^2 - 4i)y + (1 - 2i - i + 2i^2)}{5} = 1 - 2i$

$\Rightarrow \frac{1}{5}(3x - 3y + 2) - \frac{2}{5}(2x + 2y + 1)i = 1 - 2i$

On compare we get

$3x - 3y + 2 = 5 \Rightarrow x - y = 1$ ...(i)

$2x + 2y + 1 = 5 \Rightarrow x + y = 2$ ...(ii)

On solving (i) and (ii), we get $x = \frac{3}{2}$ and $y = \frac{1}{2}$

Now, $2x + 4y = 2\left(\frac{3}{2}\right) + 4\left(\frac{1}{2}\right) = 5.$

14. **(b)** Given, $z = 1 - \sqrt{3}i$

$z^2 = 1 + 3i^2 - 2\sqrt{3}i = -2(1 + \sqrt{3}i)$

$z^3 = -2(1 + \sqrt{3}i)(1 - \sqrt{3}i) = -8$

Now, $z^3 - 3z^2 + 3z = -8 + 6 + 6\sqrt{3}i + 3 - 3\sqrt{3}i = 1 + 3\sqrt{3}i.$

15. **(a)** |z – 1| = |i (z + 1)| ⇒ |z – 1| = |i|.|z + 1|

$\Rightarrow |z - 1|^2 = |z + 1|^2 \Rightarrow |x + iy - 1|^2 = |x + iy + 1|^2$

$\Rightarrow x^2 + 1 - 2x = x^2 + 1 + 2x \Rightarrow 4x = 0 \Rightarrow x = 0$

which is equation of the y-axis.

16. **(b)** $\because z = \frac{-2+i}{(1-2i)^2}$

$z = \frac{(2-i)(3-4i)}{(3+4i)(3-4i)} \Rightarrow z = \frac{2-11i}{25}$ Then, $\overline{z} = \frac{2+11i}{25}$

$|\overline{z}| = \left|\frac{2}{25} + \frac{11}{25}i\right| = \sqrt{\frac{4}{625} + \frac{121}{625}} = \sqrt{\frac{125}{625}} = \sqrt{\frac{1}{5}}$

$\Rightarrow |\overline{z}| = \frac{1}{\sqrt{5}}$

17. **(a)** Given: $z_1 = 2 + 5i$, $z_2 = -1 + 4i$, $z_3 = i$

Now, $\dfrac{z_1 - z_3}{z_3 - z_2} = \dfrac{2+5i-i}{i-(-1+4i)} = \dfrac{2+4i}{1-3i}$

$\Rightarrow \dfrac{z_1 - z_3}{z_3 - z_2} = i - 1$

Then, $\left|\dfrac{z_1 - z_3}{z_3 - z_2}\right| = \sqrt{1+1} = \sqrt{2}$

18. **(b)** Given arg $(z) = \theta = \tan^{-1}\left(\dfrac{y}{x}\right) \Rightarrow \dfrac{y}{x} = \tan\theta$

$\Rightarrow y = x\tan\theta$

19. **(c)** $\because 4a + i(3a - b) = b - 6i$

Comparing both sides, we get :

$4a = b$

$3a - b = -6 \Rightarrow 3a - 4a = -6 \Rightarrow \text{a} = 6$

$\therefore \quad b = 24$

$z = 6 + \dfrac{24}{4}i = 6 + 6i$; $|z| = \sqrt{36+36} = 6\sqrt{2}$

$\dfrac{|z|}{a} = \dfrac{6\sqrt{2}}{6} = \sqrt{2}$

20. **(a)** $z = (1-i)^3(x+i) \Rightarrow z = (2-2x) - i(2+2x)$

If z is purely imaginary: $2 - 2x = 0$

$\Rightarrow x = 1 \ \therefore \ x_1 = 1$

If z is purely real:

$2 + 2x = 0 \Rightarrow x = -1; \therefore \quad x_2 = -1$

Now, $x_1x_2 = 1 \times (-1) = -1$

21. **(b)** Given the point A $(-2, 1)$, B $(3, -4)$

& $\dfrac{\text{AC}}{\text{BC}} = \dfrac{1}{2}$. So, $\text{C} \equiv \left(\dfrac{-1}{3}, \dfrac{-2}{3}\right)$

Now, argument of C = $\tan^{-1}\left(\dfrac{\frac{-2}{3}}{\frac{-1}{3}}\right) - \pi = \tan^{-1}(2) - \pi$

22. **(c)** Let $z_1 = 2 - 3i, z_2 = i$ and $z_3 = \overline{z}_1 = 2 + 3i$ are the root of the equation

$(z - z_1)(z - z_2)(z - z_3) = 0$

$\Rightarrow \quad (z - i)(z - (2-3i))(z - (2+3i)) = 0$

$\Rightarrow \quad z^3 + z^2(-4-i) + z(13+4i) - 13i = 0$ ...(i)

The given equation is

$z^3 + bz^2 + cz + d = 0$ ...(ii)

Comparing eqn. (i) with equation (ii), we get :-

$b = -4 - i$, $c = 13 + 4i$, $d = 0 - 13i$

Then, $b + c + d = 9 - 10i$.

23. **(a)** Given $z_1 = 2 - i, \quad z_2 = 6 + 3i$

$\text{amp}\left(\dfrac{z_1 - z_2}{z_1 + z_2}\right) = \text{amp}\left(\dfrac{-4-4i}{8+2i}\right)$

$= \left[\tan^{-1}(1) - \pi\right] - \left[\tan^{-1}\left(\dfrac{2}{8}\right)\right]$

$= \left(\dfrac{\pi}{4} - \pi\right) - \tan^{-1}\left(\dfrac{1}{4}\right) = -\dfrac{3\pi}{4} - \tan^{-1}\left(\dfrac{1}{4}\right)$

24. **(b)** $Z\overline{Z}^3 + \overline{Z}Z^3 = 350$; $Z \cdot \overline{Z}\,(\overline{Z}^2 + Z^2) = 350$

$|Z|^2\left((x - iy)^2 + (x + iy)^2\right) = 350$

$|Z|^2\,(x^2 - y^2 - 2xyi + x^2 - y^2 + 2xyi) = 350$

$2 \cdot |Z|^2\,(x^2 - y^2) = 350$

$|Z|^2\,(x^2 - y^2) = 175 = 25 \times 7$

$\therefore \ |Z|^2 = 25 \Rightarrow |Z| = 5$

25. **(d)** $\left(\dfrac{i}{3}\right)^0 + \left(\dfrac{i}{3}\right)^1 + \left(\dfrac{i}{3}\right)^2 + ...\infty$

$= 1 + \dfrac{i}{3} + \left(\dfrac{i}{3}\right)^2 + ...\infty = \dfrac{1}{1 - \frac{i}{3}} = \dfrac{3}{3-i}$

$= \dfrac{3}{3-i} \times \dfrac{3+i}{3+i} = \dfrac{3(3+i)}{10} = \dfrac{9+3i}{10}$

26. **(d)** $|Z_1 + Z_2| = |Z_1| + |Z_2|$ is only possible when Z_1 and Z_2 are collinear. Hence angle between them = 0.

27. **(c)** $1 + i^2 + i^4 + i^6 + ... + i^{2024}$; $i^{4n} = 1$; $i^{2n} = -1$

$(1 + i^2) + (i^4 + i^6) + ... + (i^{2020} + i^{2022}) + i^{2024}$

$= 0 + 0 + ... + 0 + 1 = 0 + 1 = 1.$

28. **(a)** Let $\text{z} = \text{x} + \text{iy}$; $\text{iz} = -\text{y} + \text{ix}$

and $\text{z} + \text{iz} = (\text{x} - \text{y}) + \text{i}(\text{x} + \text{y})$

$\therefore$ Area of triangle

$$\Delta = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ x-y & x+y & 1 \\ -y & x & 1 \end{vmatrix} = \frac{1}{2}(x^2 + y^2) = \frac{1}{2}|z|^2$$

29. **(c)** Given that Arg $z_1 = \left|\dfrac{\pi}{3}\right|$

Arg $z_2 = -$Arg $\overline{z}_2 = -\dfrac{\pi}{5}$

$\therefore$ Arg $z_1 +$ Arg $z_2 = \dfrac{\pi}{3} - \dfrac{\pi}{5} = \dfrac{2\pi}{15}$

30. **(d)** $\left(\dfrac{1-i}{1+i}\right)^{2022} + \left(\dfrac{1+i}{1-i}\right)^{2021}$

$= \left(\dfrac{-2i}{2}\right)^{2022} + \left(\dfrac{2i}{2}\right)^{2021} = (-i)^{2022} + (i)^{2021}$

$= (i)^{2022} + (i)^{2021} = (i)^{4\times505+2} + (i)^{4\times505+1}$

$= (i)^2 + (i)^1 = -1 + i = i - 1$

31. **(d)** $\frac{x(3-i+3i-i^2)-6i+2i^2+(3+i)[(2-3i)y+i]}{(3+i)(3-i)}=i$

$x(4 + 2i) – 6i \;– 2 + [16 – 9i + 2i – 3i^2]y + 3i + i^2]$
$= i(9 – i^2)$
$(4x + 9y) + i(2x – 7y) = 3i + 3$
Now, $4x + 9y = 3$ ...(i)
$2x – 7y = 13$ ...(ii)
Now, multiply equation (ii) by 2
$4x – 14y = 26$ ...(iii)
Subtract (i) from (iii),

$$\begin{array}{rcrcr} 4x & - & 14y & = & 26 \\ -4x & + & 9y & = & 3 \\ \hline & & -23y & = & 23 \\ & & y & = & -1 \end{array}$$

From (ii),
$2x – 7y = 13$; $2x – 7 \times (–1) = 13$
$2x + 7 = 13$; $2x = 6$; $x = 3$
Now, $x + y = 3 – 1 = 2$

32. **(b)** Given $(x + iy) = \left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$

$\Rightarrow\; x + iy = \left\{\left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)-\left(\frac{1-i}{1+i}\times\frac{1-i}{1+i}\right)\right\}$

$= \frac{1}{8}[–8i – 8i] = –2i$

$x + iy = 0 + (–2)i \Rightarrow\; x > y$

33. **(b)** Given equation is $\overline{z} = iz^2$ ...(i)

Let $z = x + iy \Rightarrow \overline{z} = x – iy$

Now, from equation (i)

$x – iy = (x + iy)^2$; $x – iy = i(x^2– y^2 + 2ixy)$

$x – iy = ix^2 – iy^2 – 2xy$

$\Rightarrow\; x + 2xy = 0$ and $y = y^2 – x^2$

On solving get (0, 0) (0, 1) $\left(\frac{-\sqrt{3}}{2},\frac{-1}{2}\right)\left(\frac{\sqrt{3}}{2},\frac{-1}{2}\right)$

are four solutions

34. **(d)** $\sum_{K=0}^{440} i^K = i^0+i^1+i^2+i^3+\ldots\ldots\ldots\ldots\ldots\ldots+i^{439}+i^{440}$

$\because\; i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0\ (n\in N)$
$\therefore\; i^1 + i^2 + i^3 + i^4 = i^5 + i^6 + i^7 + i^8 = \ldots\ldots\ldots\ldots\ldots = i^{437} + i^{438} + i^{439} + i^{440} = 0$

$\therefore \sum_{K\;0}^{440} i^K = i^0+0=1$

$\because\; x + iy = 1$
$\therefore\; x = 1, y = 0$
$x^{100} + x^{99}y + x^{242}y^2 + x^{97}\,y^3 = 1$

35. **(d)** Given, two complex numbers $z_1 = \sin x + i \cos 2x$ and $z_2 = \cos x – i \sin 2x$ with $x \in [0, 2\pi]$.

for $x = 0 \Rightarrow z_1 = i, z_2 = 1$

for $x=\frac{\pi}{6}\Rightarrow z_1=\frac{1}{2}+\frac{i}{2}, z_2=\frac{\sqrt{3}}{2}-\frac{\sqrt{3}i}{2}$

for $x=\frac{\pi}{4}\Rightarrow z_1=\frac{1}{\sqrt{2}}, z_2=\frac{1}{\sqrt{2}}-i$

for $x=\frac{\pi}{2}\;\Rightarrow z_1 = 1 – i, z_2 = 0$

for $x = \pi \Rightarrow z_1 = i, z_2 = –1$

for $x=\frac{3\pi}{2}\Rightarrow z_1=-1-i, z_2=-\frac{1}{z}$

for $z = 2\pi \Rightarrow z_1 = i, z_2 = 1 – 0 = 1$

So, there is not any value of x which gives two complex numbers.

36. **(d)** We have $(2x – y + 1) + i\,(x – 2y – 1) = 2 – 3i$.
Compare real and imaginary part,
$2x – y + 1 = 2$
$2x – y = –1$ ...(i)
$x – 2y – 1 = –3$
$x – 2y = –2$ ...(ii)
Multiply eq. (i) by 2.
$4x – 2y = 2$ ...(iii)
Subtract (ii) from (iii),

$4x – 2y = 2$, $3x = 4 \Rightarrow x = \frac{4}{3}$, From (i), $y = \frac{5}{3}$.

Now, $x – iy = \frac{4}{3}-\frac{5}{3}i$
Multiplicative inverse of $x – iy = \frac{12}{41}+\frac{15}{41}i$.

37. **(b)** $\left[\frac{x+i(x-2)}{3+i}\right]-i=\frac{2y+i(1-3y)}{i-3}$

$\Rightarrow\;\; (i – 3)\,[x + i\,(x – 2) – 3i + 1] = (3 + i)\,[2y + i\,(1 – 3y)]$
$\Rightarrow\;\; i\,(–2x + 16) + (–4x + 2) = (9y – 1) + i\,(3 – 7y)$
Now equating real and imaginary parts we have
$– 4x + 2 = 9y – 1$
$\Rightarrow\;\; 4x + 9y = 3$ ...(i)
$–2x + 16 = 3 – 7y$
$\Rightarrow\;\; 2x – 7y = 13$ ...(ii)
$\Rightarrow\;\; x = 3\ \&\ y = –1$
$\therefore\;\; x + y = 3 – 1 = 2$

38. **(a)** Given that, $\Rightarrow |z| + |2z – 3| + |z – 1|$
$|z| + |3 – 2z| + |z – 1|$
$\geq |z + z – 1 + 3 – 2z|$ [$\because\; |z_1 + z_2| \leq |z_1| + |z_2|$] ; $\geq |2|$
$\therefore$ Minimum value of $|z| + |2z – 3| + |z – 1|$ is 2.

39. **(b)** Let the real part of $(\cos 4 + i \sin 4 + 1)^{2020}$
$= (2 \cos^2 2 + i\, 2 \sin 2 \cos 2)^{2020}$
$= 2^{2020} \cos^{2020} (2) (\cos 4040 + i \sin 4040)$
$= 2^{2020} \cos^{2020} 2.\ \cos 4040$ $[z+\overline{z}=2\text{Re}(z)]$

40. **(c)** Given, $z = x + iy$ and $\text{Im}\left(\frac{z-3i}{iz+4}\right)=0$

$\Rightarrow \text{Im}\left[\frac{x+iy-3i}{i(x+iy)+4}\right]=0$

$$\Rightarrow \text{Im}\left[\frac{x-i(x^2+y^2-7y+12)}{(4-y)^2+x^2}\right]=0$$

$$\Rightarrow \frac{(x^2+y^2-7y+12)}{(4-y)^2+x^2}=0 \quad \left[\because (4-y)^2+x^2 \neq 0\right]$$

$\Rightarrow 4-y \neq 0$ and $x \neq 0$ $\Rightarrow y \neq 4$ and $x \neq 0$

$\Rightarrow$ $x_2 + y_2 - 7y + 12 = 0$ but $(x, y) \neq (0, 4)$.

41. **(b)** Given $(-i+\sqrt{3})^{300}+\left(-i-\sqrt{3}\right)^{300}$

$$(i-\sqrt{3})^{300}+\left(-i-\sqrt{3}\right)^{300}$$

$$=i^{300}(1+i\sqrt{3})^{300}+i^{300}(-1+i\sqrt{3})^{300}$$

$$=(-2w^2)^{300}+(2w)^{300} \quad \{\text{where } w \text{ is cube root of unity}\}$$

$$=2^{300}\left[\left(\omega^2\right)^{300}+\omega^{300}\right]=2^{300}\times 2=2^{301}$$

42. **(d)** Given $a+bi=\dfrac{i}{1-i}$

$$\Rightarrow \; a+bi=\frac{i(1+i)}{2}=-\frac{1}{2}+\frac{i}{2} \Rightarrow \; a=-\frac{1}{2}, b=\frac{1}{2}$$

43. **(a)** Given equation, $z^2(1 - z^2) = 16, z \in \mathbf{C}$

Now, let $z^2 = w = r\ (\cos\theta + \text{i} \sin\theta\}$ where $r > 0$.

$$\therefore \; 1-z^2=\frac{16}{z^2} \Rightarrow z^2+\frac{16}{z^2}=1$$

Modulas of $z^2 \Rightarrow r$

$$\therefore \; \text{Modulus of } \frac{16}{z^2} \Rightarrow \frac{16}{r}$$

$$\Rightarrow \left(r+\frac{16}{r}\right)\cos\theta+i\left(r-\frac{16}{r}\right)\sin\theta=1$$

On comparing real and imaginary parts, we get

$$\left(r+\frac{16}{r}\right)\cos\theta=1 \text{ and } \left(r-\frac{16}{r}\right)\sin\theta=0$$

$\cos\theta=1 \Rightarrow r+\dfrac{16}{r}=1$ (not possible)

$\Rightarrow r^2=16 \Rightarrow r=4$

$\Rightarrow$ Modulus of $z^2 = |z|^2 = r = 4 \; \Rightarrow |z| = 2$

44. **(c)** We have,

$$\arg\left(\overline{z}_1\right)=\frac{\pi}{5}, \arg\left(z_2\right)=\frac{\pi}{3}$$

$$\therefore \; \arg\left(z_1z_2\right)=\arg\left(z_2\right)-\arg(\overline{z}_1)=\frac{\pi}{3}-\frac{\pi}{5}=\frac{2\pi}{15}$$

$$\left[\because \arg\left(\overline{z}\right)=-\arg\left(z\right)\right]$$

(A) is true but (R) is false.

45. **(Bonus)** If $z = x + iy$, then $\overline{z}=x-iy$,

As $\dfrac{\overline{z}-1}{z-i}=2 \Rightarrow \left|\dfrac{\overline{z}-1}{z-i}\right|=2$ or, $\left|\dfrac{\overline{z}-1}{z-i}\right|=2$

$\Rightarrow$ $(x - 1)^2 + y^2 = 4\ [x^2 + (y - 1)^2]$

$$\Rightarrow \; x^2+y^2+\frac{2}{3}x-\frac{8}{3}y+1=0$$

So, the locus of point $P\ (x, y)$ in the cartesian plane is a circle of radius $\dfrac{\sqrt{10}}{3}i$ and the centre $\left(-\dfrac{1}{3},\dfrac{4}{3}\right)$

No option is correct.

46. **(a)** Let $z=x+iy,\ x, y \in R,\ (x,\ y) \neq (0,\ -4)$

$$\frac{2z-3}{z+4i}=\frac{(2x-3)+2iy}{x+i(y+4)}\times\frac{x-i(y+4)}{x-i(y+4)}$$

$$=\frac{(2x^2-3x+2y^2+8y)+i(12+3y-8x)}{x^2+(y+4)^2}$$

$$\text{So, } \arg\left(\frac{2z-3}{z+4i}\right)=\tan^{-1}\left(\frac{12+3y-8x}{2x^2-3x+2y^2+8y}\right)=\frac{\pi}{4}$$

(given)

$$\Rightarrow \; 2x^2+2y^2+5x+5y-12=0$$

47. **(d)** Let $z = x + iy$ then

$$\text{Now, } \frac{\overline{z}-1}{\overline{z}-i}=\frac{(x-1)-iy}{x-i(y+1)}\times\frac{x+i(y+1)}{x+i(y+1)}$$

$$=\frac{[x(x-1)+y(y+1)]+i[(y+1)(x-1)-xy]}{x^2+(y+1)^2}$$

$$\therefore \; \text{Im}\left(\frac{\overline{z}-1}{\overline{z}-i}\right)=\frac{xy-y+x-1-xy}{x^2+(y+1)^2}=1 \quad \text{(given)}$$

$$\Rightarrow \; x^2+y^2-x+3y+2=0, (x,\ y) \neq (0,\ -1)$$

48. **(c)** Given $|\,z + 4\,| \geq 3$

$|\,z + 3 + 1\,| \leq |\,z + 3\,| + 1$

$|\,z + 3\,| + 1 \geq 3 \quad \Rightarrow \quad |\,z + 3\,| \geq 2$

$\therefore$ Smallest value of $|\,z + 3\,| = 2$

49. **(b)** Given, $iz^4 + 1 = 0 \Rightarrow iz^4 = -1 = z^4 = \dfrac{-1}{i}$

$$\Rightarrow z^4=i \Rightarrow \; z=\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)^{1/4}$$

$$z=\cos\frac{\pi}{8}+i\sin\frac{\pi}{8} \quad \text{[By De-moiver's Theorem]}$$

$$\therefore \; \text{Re}\ (z)=\cos\frac{\pi}{8}$$

50. **(d)** Let to find range $y=\dfrac{2(x^2+2x-11)}{2x-5}$

$\Rightarrow$ $2x^2 + 2x(2 - y) - 22 + 5y = 0$

$\therefore$ $x \in R$, $\therefore$ $b^2 - 4ac \geq 0$

$\Rightarrow$ $4(2 - y)^2 + 4 \times 2(22 - 5y) \geq 0$

$4 + y^2 - 4y + 44 - 10y \geq 0$; $y^2 - 14y + 48 \geq 0$

$(y - 6)(y - 8) \geq 0$; $y \in (-\infty, 6] \cup [8, \infty)$

Hence, no value of y lie in (6, 8).

51. **(d)** $\because$ $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \forall\ n \in$ Integer

So, $i^2 + i^3 + i^4 + \ldots + i^{4000} = i - i + i^2 + i^3 + \ldots + i^{4000}$

$= -i + [i + i^2 + i^3 + \ldots + i^{4000}] = -i$

52. **(a)** Given $z - 1 - 2i$

Put $z = x + iy$

$\Rightarrow$ $x + iy - 1 - 2i = (x - 1) + (y - 2)i$

$$\arg(z-1-2i) = \tan^{-1}\left(\frac{y-2}{x-1}\right) = \frac{\pi}{3}$$

$$\left\{\because \tan\theta = \frac{y}{x}, \theta = \tan^{-1}\left(\frac{y}{x}\right)\right\}$$

$$\frac{y-2}{x-1} = \tan\frac{\pi}{3} = \sqrt{3} \Rightarrow \sqrt{3}(x-1) = y-2$$

$$\sqrt{3}x - y + 2 - \sqrt{3} = 0 \Rightarrow y = \sqrt{3}x + (2-\sqrt{3})$$

53. **(c)** We have,

$$(\sqrt{3}-i)^{2016} + (-\sqrt{3}-i)^{2019}$$

Convert the given expression into Euler form.

$$= 2^{2016}\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^{2016}$$

$$-2^{2019}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{2019}$$

Apply $(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$

$= 2^{2016}(\cos 336\pi - i\sin 336\pi)$

$$-2^{2019}\left(\cos 336\pi + \frac{\pi}{2}\right) + i\sin\left(336\pi + \frac{\pi}{2}\right)$$

$= 2^{2016}(1-0) - 2^{2019}(0+i) = 2^{2016} - 2^{2019}i$

$\therefore$ Imaginary part $= -2^{2019}$.

54. **(d)** Given $z = x + iy$ be a complex number,

Here $\dfrac{z-1}{z+i} = \dfrac{x+iy-1}{x+iy+i} = \dfrac{(x-1)+iy}{x+(y+1)i}$

$$= \frac{x(x-1)+y(y+1)}{x^2+(y+1)^2} + \frac{[xy-(x-1)(y+1)]i}{x^2+(y+1)^2}$$

Also given, $\text{Re}\left(\dfrac{z-1}{z+i}\right) = 1$; $\dfrac{x(x-1)+y(y+1)}{x^2+(y+1)^2} = 1$

$\therefore$ $x(x-1) + y(y+1) = x^2 + (y+1)^2$

$\Rightarrow$ $x^2 - x + y^2 + y = x^2 + y^2 + 2y + 1$

$\Rightarrow$ $-x + y = 2y + 1 \Rightarrow x + y + 1 = 0$

$\therefore$ $(2016, -2017)$ lies on $x + y + 1 = 0$

55. **(b)** $z + \dfrac{1}{z} = 1 \Rightarrow \dfrac{z^2+1}{z} = 1 \Rightarrow z^2 - z + 1 = 0$

$\therefore$ $z = \dfrac{+1 \pm \sqrt{1-4}}{2} \Rightarrow z = \dfrac{+1 \pm \sqrt{3}i}{2}$

$\Rightarrow z = -\omega$ and $-\omega^2$

Now, put $z = -\omega$

$$= \frac{(z^{20}+1)(z^{40}+1)(z^{60}+1)}{z^{60}}$$

$$= \frac{(\omega^2+1)(\omega+1)(1+1)}{1}$$

$= 2(1) = 2$ $\qquad [\because \omega^2 + \omega + 1 = 0]$

Put $z = -\omega^2$

$$\frac{(z^{20}+1)(z^{40}+1)(z^{60}+1)}{z^{60}}$$

$= 2(\omega)^3 + \omega + \omega^2 + 1$

$= 2(1) = 2$ $\qquad [\because \omega^2 + \omega + 1 = 0]$

56. **(a)** $\dfrac{1-10i\cos\theta}{1-10\sqrt{3}i\sin\theta} \times \dfrac{1+10\sqrt{3}i\sin\theta}{1+10\sqrt{3}i\sin\theta}$

(Rationalize numerator and denominator)

$$= \frac{1+100\sqrt{3}\sin\theta\cos\theta}{1+300\sin^2\theta} + i\frac{10\sqrt{3}\sin\theta - 10\cos\theta}{1+300\sin^2\theta}$$

$\therefore$ It is purely real imaginary part = 0

$\Rightarrow$ $1 + 300\sin^2\theta \neq 0$

$\therefore$ $10\sqrt{3}\sin\theta - 10\cos\theta = 0$

$\Rightarrow$ $\tan\theta = \dfrac{1}{\sqrt{3}} = \tan\left(\dfrac{\pi}{6}\right) \Rightarrow \theta = \dfrac{\pi}{6}$

57. **(b)** We given that, $\bar{z} - i\bar{\omega} = 0$

$\Rightarrow$ $i\bar{\omega} = \bar{z} \Rightarrow \bar{\omega} = \dfrac{1}{i}\bar{z}$

$\Rightarrow$ $\omega = -\dfrac{1}{i}z \Rightarrow \omega = iz$

$\because$ we given that, $\arg(z\omega) = \dfrac{3\pi}{4}$

$\therefore$ $\arg(z(iz)) = \dfrac{3\pi}{4} \Rightarrow \arg(iz^2) = \dfrac{3\pi}{4}$

$\Rightarrow$ $\arg(i) + \arg(z^2) = \dfrac{3\pi}{4}$

$[\because \arg(z_1 z_2) = \arg(z_1) + \arg(z_2)]$

$\Rightarrow$ $\dfrac{\pi}{2} + 2\arg(z) = \dfrac{3\pi}{4} \Rightarrow \arg(z) = \dfrac{\pi}{8}$.

58. **(a)** We given that

$x^{11} - x^7 + x^4 - 1 = 0$

$(x^7 + 1)(x^4 - 1) = 0$

$(x+1)(x^6 - x^5 + x^4 - x^3 + x^2 - x + 1)(x^2+1)(x-1) = 0$

Here $x = 1$ to $0 . i$

$\therefore$ $\arg(x) = \tan^{-1}\left|\dfrac{0}{1}\right|$ lie in Ist quadrant $x = 0 + 1.i$

$\therefore$ $\arg(x) = \tan^{-1}\left|\dfrac{1}{0}\right|$ lie in Ist quadrant

$\therefore$ Number of complex roots, whose argument lie on Ist quadrant = 2

59. **(c)** We have given, $|z| \geq 5$

Now, $\left|z + \dfrac{2}{z}\right| \geq |z| - \left|\dfrac{2}{z}\right| = \left||z| + \dfrac{2}{z}\right|$

$$= \left||z| + 2\left(\frac{-1}{|z|}\right)\right| \geq \left|5 - \frac{2}{5}\right|$$

$\therefore$ $\left|z + \dfrac{2}{z}\right| \geq \dfrac{23}{5}$

Thus, the least value of $\left|z + \dfrac{2}{z}\right|$ is $\dfrac{23}{5}$.

60. **(c)** Given that,

$$\log_{\frac{1}{\sqrt{3}}}\left\{\frac{|z|^2-|z|+1}{2+|z|}\right\}>-2$$

Since, $\log_a b > c \cdot b < a^c$ if $0<a<1$

$$\Rightarrow \frac{|z|^2-|z|+1}{2+|z|}<\left(-\frac{1}{\sqrt{3}}\right)^{-2}$$

$$\Rightarrow |z|^2-|z|+1<6+3|z| \Rightarrow |z|^2-4|z|+1<6$$

$$\Rightarrow (|z|-2)^2<9 \Rightarrow -3<|z|-2<3$$

$$\Rightarrow -1<|z|<6 \Rightarrow 0<|z|<5$$

61. **(b)** It is given that,

Conjugate of $(x+iy)(1-2i)$ is 1 + i.

i.e. $(x-iy)(1+2i)=1+i \Rightarrow x-iy=\dfrac{1+i}{1+2i}$

Taking conjugate on both the sides, we get

$$x+iy=\frac{1-i}{1-2i}$$

62. **(a)** *We have,*

$$\frac{(1+i)^{2016}}{(1-i)^{2014}}=\frac{\left[(1+i)^2\right]^{1008}}{\left[(1-i)^2\right]^{1007}}=\frac{(2i)^{1008}}{(-2i)^{1007}}$$

$$= -(2i)^{1008-1007}=-2i$$

63. **(b)** Given that,

$$|(z-1)-(z-5)|\le|(z-1)|+|(z-5)|$$

$$\therefore \quad |z-1-z+5|\le|z-1|+|z-5|$$

$$|4|\le|z-1|+|z-5|$$

Hence, $|z-1|+|z-5|\ge 4$

64. **(d)** Given, $|Z-1|\le 2$ and $|\omega^2 Z-1-\omega|=a$

$$\Rightarrow |\omega^2 Z+\omega^2|=a \qquad (\because 1+\omega+\omega^2=0)$$

$$\Rightarrow |\omega\ ||Z+1|=a \Rightarrow |Z-1+2|=a$$

$$\Rightarrow |Z-1|+2\ge a \Rightarrow 2+2\ge a$$

$$\Rightarrow 4\ge a \text{ and } a\ge 0$$

So, $0\le a\le 4$.

65. **(c)** Given, $Z^3+iZ^2+2i=0$

$$\Rightarrow (Z-i)(Z^2+2iZ-2)=0$$

$Z = i$ or $Z^2 + 2iZ - 2 = 0$

$$\Rightarrow Z=\frac{-2i\pm 2}{2} \Rightarrow z=-i+1, -i-1$$

Let $A(0, 1)$, $B(1, -1)$, $C(-1, -1)$

$AB=\sqrt{5}$; $BC=2$; $AC=\sqrt{5}$

Since, $AB^2+AC^2\ne BC^2$ and $AB=BC\ne BC$.

So, ΔABC is an isosceles triangle.

66. **(a)** Since (r, θ) denotes $r(\cos\theta + i\sin\theta)$

So, $x=(1,\alpha)=\cos\alpha+i\sin\alpha=e^{i\alpha}$

$y=(1,\beta)=\cos\beta+i\sin\beta=e^{i\beta}$

and $z=(1,\gamma)=\cos\gamma+i\sin\gamma=e^{i\gamma}$

Now, $x + y + z = 0$

$\Rightarrow x^3 + y^3 + z^3 = 3xyz$

$$\Rightarrow \frac{x^2}{yz}+\frac{y^2}{yz}+\frac{z^2}{xy}=3 \Rightarrow \frac{e^{2i\alpha}}{e^{i\beta}e^{i\gamma}}+\frac{e^{2i\beta}}{e^{i\gamma}e^{i\alpha}}+\frac{e^{2i\gamma}}{e^{i\alpha}e^{i\beta}}=3$$

$$\Rightarrow e^{i(2\alpha-\beta-\gamma)}+e^{i(2\beta-\alpha-\gamma)}+e^{i(2\gamma-\alpha-\beta)}=3$$

On comparing both sides

$$\sum\cos(2\alpha-\beta-\gamma)=3$$

67. **(c)** $\dfrac{z-3i}{z+4}=\dfrac{x+(y-3)i}{(x+4)+yi}\times\dfrac{(x+4)-yi}{(x+4)-yi}$

$$=\frac{x^2+y^2+4x-3y+(4x-3x)i}{(x+4)^2+y^2}$$

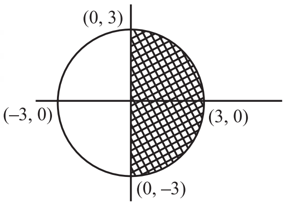
$$\text{Arg}\left(\frac{z-3i}{z+4}\right)=\frac{\pi}{2} \Rightarrow \tan\frac{\pi}{2}=\frac{4y-3x}{x^2+y^2+4x-3y}$$

$\Rightarrow x^2 + y^2 + 4x - 3y = 0$

Also, for I$^{\text{st}}$ quadrant, $4y - 3x > 0 \Rightarrow 3x - 4y < 0$

68. **(b)** $|Z|\le 3$,

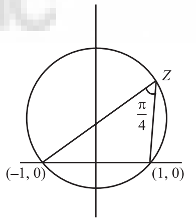
$$-\frac{\pi}{2}\le \text{amp}(Z)\le\frac{\pi}{2}$$



Required region is semicircle with radius 3.

Required area $=\dfrac{\pi\times 3^2}{2}=\dfrac{9\pi}{2}$.

69. **(b)**



Required locus is an arc of circle.

70. **(b)** $(8i)^{1/3}=(-8i^3)^{1/3}=-2i$

$$i=e^{i\frac{\pi}{2}}=e^{i\frac{5\pi}{2}}$$

$$(8i)^{1/3}=2e^{\frac{i\pi}{6}}=2\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)=\sqrt{3}+i$$

$$(8i)^{1/3}=2e^{i\frac{5\pi}{6}}=2\left(\frac{-\sqrt{3}}{2}+\frac{i}{2}\right)=-\sqrt{3}+i$$

$$(8i)^{1/3}=\pm\sqrt{3}+i,\ -2i.$$

71. **(d)** Let $\sqrt{7+24i} = x+iy \Rightarrow 7+24i = (x^2-y^2)+2xyi$

Equating real and imaginary parts

$x^2 - y^2 = 7$ and $2xy = 24 \Rightarrow y = \dfrac{12}{x}$

$\therefore\ x^2 - \left(\dfrac{12}{x}\right)^2 = 7 \Rightarrow x^4 - 7x^2 - 144 = 0$

$\Rightarrow x^2 = 16, -9 \Rightarrow x = \pm 4, y = \pm 3$

So, $(x + iy) = \pm (4 + 3i)$.

72. **(b)** $Z = \cos\theta + i\sin\theta$

$$\frac{1+Z^{2n}}{1-Z^{2n}} = \frac{1+(\cos\theta+i\sin\theta)^{2n}}{1-(\cos\theta+i\sin\theta)^{2n}} = \frac{1+\cos 2n\theta+i\sin 2n\theta}{1-\cos 2n\theta-i\sin 2n\theta}$$

$$= \frac{\cos n\theta}{i\sin n\theta}\left(\frac{\cos n\theta+i\sin n\theta}{-\cos n\theta-i\sin n\theta}\right) = i\cot n\theta.$$

73. **(b)** $z = x + \mathrm{i}y$

$z - 2 = x - 2 + iy$

$\arg(z-2) = \dfrac{y}{x-2} = \infty \Rightarrow x = 2; y > 0$

74. **(d)** $\dfrac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} + \dfrac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}$

$$= (1+\omega^2)\left(\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}\right) = -\frac{(a\omega+b\omega^2+c)}{(c+a\omega+b\omega^2)} = -1$$

75. **(c)** $\dfrac{3-2i\sin\theta}{1+2i\sin\theta} = \dfrac{(3-2i\sin\theta)(1-2i\sin\theta)}{1+4\sin^2\theta} = \dfrac{3-4\sin^2\theta}{1+4\sin^2\theta}$

From the given information real part = 0

$\Rightarrow \sin\theta = \dfrac{\sqrt{3}}{2}$

76. **(c)** Let $x = \cos\alpha + i\sin\alpha,\ y = \cos\beta + i\sin\beta$ and

$z = \cos\gamma + i\sin\gamma$

$\therefore\ x+y+z = 0 \quad \therefore x^3+y^3+z^3 = 3xyz$

$\Rightarrow (\cos\alpha+i\sin\alpha)^3 + (\cos\beta+i\sin\beta)^3 + (\cos\gamma+i\sin\gamma)^3$

$= 3(\cos\alpha+i\sin\alpha)(\cos\beta+i\sin\beta)(\cos\gamma+i\sin\gamma)$

$\Rightarrow (\cos 3\alpha+\cos 3\beta+\cos 3\gamma)+i(\sin 3\alpha+\sin 3\beta+\sin 3\gamma)$

$= 3\cos(\alpha+\beta+\gamma)+3i\sin(\alpha+\beta+\gamma)$

$\therefore\ \cos 3\alpha+\cos 3\beta+\cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$

$\Rightarrow 4(\cos^3\alpha+\cos^3\beta+\cos^3\gamma) = 3\cos(\alpha+\beta+\gamma)$

$\Rightarrow \cos^3\alpha+\cos^3\beta+\cos^3\gamma = \dfrac{3}{4}\cos(\alpha+\beta+\gamma)$

Similarly, $\sin^3\alpha+\sin^3\beta+\sin^3\gamma = \dfrac{-3}{4}\sin(\alpha+\beta+\gamma)$

Now, $(\cos^3\alpha+\cos^3\beta+\cos^3\gamma)^2+(\sin^3\alpha+\sin^3\beta+\sin^3\gamma)^2$

$= \dfrac{9}{16}\cos^2(\alpha+\beta+\gamma)+\dfrac{9}{16}\sin^2(\alpha+\beta+\gamma) = \dfrac{9}{16}$

77. **(c)** Given, $\dfrac{z-2i}{z-2}$ is purely imaginary.

$\Rightarrow \dfrac{z-2i}{z-2} + \left(\overline{\dfrac{z-2i}{z-2}}\right) = 2\,\mathrm{Re}\left(\dfrac{z-2i}{z-2}\right)$

$\Rightarrow \dfrac{z-2i}{z-2} + \dfrac{\overline{z}+2i}{\overline{z}-2} = 0$

$\Rightarrow 2|z|^2 - 2(z+\overline{z})+2i(z-\overline{z}) = 0$

$\Rightarrow x^2+y^2-2x-2y = 0 \quad (\because z = x+iy)$

$\Rightarrow (x-1)^2+(y-1)^2 = (\sqrt{2})^2.$

$\Rightarrow$ Required area $= \dfrac{\pi r^2}{2} = \dfrac{\pi\left(\sqrt{2}\right)^2}{2} = \pi$

78. **(d)** Since, $\dfrac{(\cos a+i\sin a)^6}{(\sin b+i\cos b)^8} = \dfrac{(e^{ia})^6}{(\cos b-i\sin b)^8}$

$$= \frac{e^{i6a}}{(e^{-ib})^8} = e^{i(6a+8b)} = \cos(6a+8b)+i\sin(6a+8b)$$

So, real part = $\cos(6a + 8b)$.

79. **(d)** Since, $z\overline{z}+(4-3i)\overline{z}+(4+3i)z+c = 0$ represents a circle. Now, general equation of circle is

$z\overline{z}+a\overline{z}+\overline{a}z+b = 0$

where, centre = $-a$ and radius = $\sqrt{|a|^2 - b}$

So, centre of given circle = $-4 + 3i$

and radius = $\sqrt{|-4+3i|^2 - \mathrm{C}} = \sqrt{5^2-\mathrm{C}} = \sqrt{25-\mathrm{C}}$

For existence of circle, radius $\geq 0 \Rightarrow \sqrt{25-\mathrm{C}} \geq 0$

$\Rightarrow 25 - \mathrm{C} \geq 0 \Rightarrow 25 \geq \mathrm{C} \Rightarrow \mathrm{C} = (-\infty, 25]$

80. **(d)** Given, $z^3 + \overline{z} = 0$, where $z = x + iy$ is a complex number

$\Rightarrow z^3 = -\overline{z} \ \Rightarrow |z^3| = |-\overline{z}| \Rightarrow |z|^3 = |z|$

$\Rightarrow |z|^3 - |z| = 0 \Rightarrow |z|(|z|^2 - 1) = 0$

$\Rightarrow |z| = 0$ or $|z|^2 = 1 \Rightarrow z\overline{z} = 1 \Rightarrow \overline{z} = \dfrac{1}{z}$

$\because z = 0$ or $z^3 + \dfrac{1}{z} = 0 \Rightarrow z = 0$ or $z^4 + 1 = 0$

So, total number of distinct solution of given equation is 5.

81. **(b)** $z = x + iy$

$\left(\dfrac{2z-i}{z+2i}\right) = \dfrac{2x+2iy-i}{x+iy+2i}$

$$= \frac{2x^2+(2y-1)(y+2)+i((2y-1)x-2x(y+2))}{x^2+(y+2)^2}$$

$$= \frac{(2x^2+2y^2+3y-2)-5ix}{x^2+(y+2)^2}$$

$\mathrm{Arg}\left(\dfrac{2z-i}{z+2i}\right) = 1 \Rightarrow \dfrac{-5x}{2x^2+2y^2+3y-2} = 1$

$\Rightarrow 2x^2 + 2y^2 + 3y + 5x - 2 = 0$

82. **(c)** $x^2 + 2x + 4 = 0$; $(x + 1)^2 + 3 = 0$

$x = -1 \pm \sqrt{3}i$

$\alpha = -1+\sqrt{3}i$ (α lies in 2nd quadrant)

$\alpha = 2\,\text{cis}\left(\frac{2\pi}{3}\right) \Rightarrow \beta = 2\ \text{cis}\left(\frac{-2\pi}{3}\right)$

$\alpha^{2024} = 2^{2024}\ \text{cis}\left(2024\times\frac{2\pi}{3}\right) = 2^{2024}\text{cis}\left(\frac{4\pi}{3}\right)$

$\beta^{2024} = 2^{2024}\text{cis}\left(\frac{-4\pi}{3}\right)$

$\alpha^{2024} - \beta^{2024} = 2^{2024}\left(\text{cis}\left(\frac{4\pi}{3}\right) - \text{cis}\left(\frac{-4\pi}{3}\right)\right)$

$= 2^{2024}\times -\left(\sqrt{3}i\right) = ik$

$k = -2^{2024}\sqrt{3}.$

83. **(c)** $\left(\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}\right)^k + \left(\frac{a+b\omega+c\omega^2}{b+a\omega^2+c\omega}\right)^l = 2$

$= (a+b\omega+c\omega^2)^k(\text{b}+a\omega^2+c\omega)^l + (a+b\omega+c\omega^2)^l$

$(b+a\omega^2+c\omega)^{\text{k}} = 2$

$\left(a+b\omega+c\omega^2\right)^{k+l}\frac{1}{\omega^{2k+l}}(b+a\omega^2+c\omega)^{k+l}$

$+\frac{1}{\omega^{2k+l}}(b+a\omega^2+c\omega)^{k+l} = 2(b+a\omega^2+c\omega)^{k+l}$

$\Rightarrow \frac{1}{\omega^{2k+l}} + \frac{1}{\omega^{2k+l}} = 2$

$\Rightarrow \omega^{2k+l} = 1 \Rightarrow 2k+l$ is divisible by 3

84. **(d)** If $b = 9$ then equation becomes
$3x^3 + 9x^2 + 9x + 3 = 0$ no change of sign
and all coefficient are +ve
So, all roots are negative $\therefore$ (A) → (iv)
If $b = -3$ then equation becomes
$\Rightarrow x^2(x-1) - 1(x-1) = 0 \Rightarrow (x^2-1)(x-1) = 0$
$\Rightarrow x = 1, 1, -1$
Two roots are positive $\therefore$ (C) → (v)
Hence option (d) is correct.

85. **(c)** $z = x + iy$ ...(i)
$|z-1| + |z+i| = 2$
$|(x-1)+iy| + |x+i(1+y)| = 2$ (from (i))

$\Rightarrow \sqrt{(x-1)^2+y^2} + \sqrt{x^2+(y+1)^2} = 2$

$\Rightarrow \sqrt{(x-1)^2+y^2} = 2 - \sqrt{x^2+(y+1)^2}$

Squaring both sides,

$\Rightarrow 2+x+y = 2\sqrt{x^2+(y+1)^2}$

Squaring both sides,

$4+x^2+y^2+4x+2xy+4y = 4x^2+4y^2+4+8y$

$\Rightarrow 3x^2+3y^2-2xy-4x+4y = 0.$

86. **(b)** $(-64i)^{5/6} = (64)^{5/6}\times(-i)^{5/6}$

$= 32\times\left(e^{\frac{3\pi}{2}+2k\pi}\right)^{5/6} = 32\times e^{\left(\frac{3\pi}{2}+2k\pi\right)\frac{5}{6}}$

Since it has 6 roots, $k = 0, 1, ...5$ (possible)
For $k = 3$, one of value of $(-64i)^{5/6}$ is

$(-64i)^{5/6} = 32e^{\left(\frac{25\pi}{4}\right)}$

$= 32(e^{(6\pi+\pi/4)})$ (1st quadrant $6\pi+\pi/4$)

$= 32\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 32\left(\frac{1}{\sqrt{2}} + i\times\frac{1}{\sqrt{2}}\right) = 16\sqrt{2}(1+i)$

87. **(c)** $x^3 + 3x^2 - 10x - 24 = 0$, α, β, γ are its roots
$x = 3$ is one of its root (by hit and trial)
On dividing by $(x-3)$, we get $(x-3)(x^2+6x+8) = 0$
Other roots are factors of $x^2 + 6x + 8 = (x+4)(x+2)$
i.e. $x = -4, x = -2 \Rightarrow \alpha = 3, \beta = -2, \gamma = -4$ $(\alpha > \beta > \gamma)$
$\alpha^3 + 3\beta^2 - 10\gamma - 24 = 55 \Rightarrow 55 = 11K \Rightarrow K = 5.$

88. **(c)** Let $z = (\sqrt{3}-i)^{2/5} = 2^{2/5}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

$= 2^{2/5}\cdot\left[e^{i\left(2n\pi-\frac{\pi}{6}\right)}\right]^{2/5} = 2^{2/5}e^{i\left(\frac{4}{5}n\pi-\frac{\pi}{15}\right)}$

Product of all values $= \left(2^{2/5}\right)^n e^{i\Sigma\left(\frac{4}{5}n\pi-\frac{\pi}{15}\right)}$

$= \left(2^{2/5}\right)^n e^{i\left(\frac{4}{5}\frac{n(n+1)}{2}\pi-\frac{n\pi}{15}\right)}$

Here power is $\frac{2}{5}$ *i.e.* 5 roots so total number of solutions is 5.

$\therefore$ Product of values $= (2^{2/5})^5\cdot e^{i\left(\frac{4}{5}\cdot\frac{5(5+1)}{2}\pi-\frac{5\pi}{15}\right)}$

$= 2^2\cdot e^{i\left(12\pi-\frac{\pi}{3}\right)}$

$= 4\left[\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right] = 4\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 2(1-\sqrt{3}i).$

89. **(d)** We know that n^{th} roots of unity $= e^{\frac{i2K\pi}{n}}$

$\therefore$ 12th roots of unity is $e^{i\frac{2K\pi}{12}} = e^{i\frac{K\pi}{6}}$

30th roots of unity is $e^{i\frac{2K\pi}{30}} = e^{i\frac{K\pi}{15}}$

Common values of $\frac{K\pi}{6}$ and $\frac{K\pi}{15}$ are

$0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\therefore$ Number of common roots = 6.

90. **(c)** $\sqrt{5}-i\sqrt{15}=2\sqrt{5}\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)$

$\cos\theta=\frac{1}{2}\Rightarrow\sec\theta=2\ \Rightarrow\operatorname{cosec}\theta=\frac{-2}{\sqrt{3}}$; $r=2\sqrt{5}$

$r^2(\sec\theta+3\operatorname{cosec}^2\theta)=20\times\left(2+3\times\frac{4}{3}\right)=120.$

91. **(b)** $\frac{2z-i}{z-2}=\frac{(2x+2iy-i)}{x+iy-2}=\frac{2x+i(2y-1)}{(x-2)+iy}$

For (*x, y*) = (*z,* 0). This is not defined.

$\frac{[2x+i(2y-1)][(x-2)-iy]}{(x-2)^2+y^2}$

$=\frac{2x(x-2)+y(2y-1)+i[(2y-1)(x-2)-2xy]}{(x-2)^2+y^2}$

Imaginary part = 0

$\Rightarrow\ \frac{2xy-4y-x+2-2xy}{(x-2)^2+y^2}=0\ \Rightarrow\ x+4y-2=0.$

92. **(c)** $x=e^{i2\alpha},\ y=e^{i3\beta}$

$x^6y^4=e^{12i(\alpha+\beta)}$; $\frac{1}{x^6y^4}=e^{-12i(\alpha+\beta)}$

$x^6y^4+\frac{1}{x^6y^4}=e^{12i(\alpha+\beta)}+e^{-12i(\alpha+\beta)}$

$=e^{i\frac{\pi}{3}}+e^{-i\frac{\pi}{3}}=2\cos\frac{\pi}{3}=1.$

93. **(c)** Let multiplicative inverse of z, be 'A'

hence z.A = 1

$\Rightarrow\quad A=\frac{1}{z}=\frac{1\cdot\bar{z}}{z\cdot\bar{z}}\Rightarrow A=\frac{\bar{z}}{z\,\bar{z}}=\frac{\bar{z}}{|z|^2}\qquad\{\because\ z\bar{z}=|z|^2\}$

94. **(a)** $z_3=\frac{z_1+z_2}{2}=3-i$; Now, $5z_1+xz_2+yz_3=0$

$\Rightarrow(10+4x+3y)+i\,(15-5x-y)=0$

$\Rightarrow 4x+3y=-10$ ... (i) and $15-5x-y=0$ ... (ii)

From equations (i) and (ii), we get $x=5,\ y=-10$

$\therefore\ x+y=5-10=-5$

95. **(c)** Since, $f(x, y)=(x+y)(x\omega+y\omega^2)(x\omega^2+y\omega)$

$\Rightarrow f(2,3)=(2+3)(2\omega+3\omega^2)(2\omega^2+3\omega)$

$=5(4\omega^3+6\omega^2+6\omega^4+9\omega^3)\qquad[\because\ \omega^3=1]$

$=5(13+6(\omega^2+\omega))=5(13+6\times(-1))=5\times7=35$

96. **(a)** $\sin\left[\left(\omega^{10}+\omega^{23}\right)\pi-\frac{\pi}{4}\right]=\sin\left[\left(\omega+\omega^2\right)\pi-\frac{\pi}{4}\right]$

$(\because\ \omega^3=1)$

$=\sin\left[-\pi-\frac{\pi}{4}\right]=-\sin\left[\frac{5\pi}{4}\right]\qquad(\because\ \omega^2+\omega+1=0)$

$=-\left(-\frac{1}{\sqrt{2}}\right)=\frac{1}{\sqrt{2}}$

97. **(a)** $\left(1-\omega+\omega^2\right)^{3k}+\left(1-\omega^2+\omega\right)^{3k}$

$=\left(1-\omega+\omega^2\right)^{3k+1}+\left(1+\omega-\omega^2\right)^{3k+1}$

$\Rightarrow\ (-2)^{3k}(\omega)^3+(-2)^{3k}(\omega^2)^3$

$=(-2\omega).(-2)^{3k}(\omega)^{3k}+(-2\omega^2).(-2)^{3k}\omega^{3k}$

$\Rightarrow\ 1+1=-2\omega-2\omega^2\ \Rightarrow\ 2=-2(\omega+\omega^2)$

$\Rightarrow\ 1=-1(-1)\ \Rightarrow\ 1=1$

$\therefore$ The given statement is true for all $k\in N$

$\therefore$ $k=r$, where $r\in N$

98. **(c)** Since $|z-1+i|=1\Rightarrow|z-(1-i)|=1$

So, S represent a circle with centre (1, –1) and radius 1

99. **(c)** Given $\left|z-\frac{2}{z}\right|=2\ \Rightarrow\left||z|-\frac{2}{|z|}\right|\le\left|z-\frac{2}{z}\right|=2$

$\Rightarrow|z|-\frac{2}{|z|}\le2\Rightarrow|z|^2-2|z|-2\le0$

$\Rightarrow|z|\le\frac{1\pm\sqrt{4+8}}{2}\le1+\sqrt{3}$

So max value of $|z|=\sqrt{3}+1$

100. **(c)** Since $(-1)^{1/15}=(-\cos2\pi+i\sin2\pi)^{1/15}$

$=\left(-\cos\frac{2\pi}{15}+i\sin\frac{2\pi}{15}\right)=\cos\left(\frac{13\pi}{15}\right)+i\sin\left(\frac{13\pi}{15}\right)$

$=\operatorname{cis}\left(\frac{13\pi}{15}\right)$

101. **(a)** Given that $z^3+\bar{z}=0\ \Rightarrow\ z^3=-\bar{z}$

$\Rightarrow|z|^3=|-\bar{z}|=|z|\ \Rightarrow\ |z|(|z|^2-1)=0$

$|z|=0$ or $|z|^2=1\Rightarrow\ z\bar{z}=1\ \Rightarrow\ \bar{z}=\frac{1}{z}$

$\therefore\ z^3+\bar{z}=0\ \Rightarrow\ z^3+\frac{1}{z}=0$

$\Rightarrow z^4+1=0\rightarrow$ 4 solution

$|z|=0\rightarrow$ one solution

Total 5 solution.

102. **(a)** $\left|\frac{z-i}{z+i}\right|=2\ \Rightarrow\ \left|\frac{z-i}{z+i}\right|^2=2^2$

$\Rightarrow\ \left(\frac{z-i}{z+i}\right)\overline{\left(\frac{z-i}{z+i}\right)}=2^2\ \Rightarrow\ \frac{z-\bar{z}+zi-\bar{z}i+1}{z\bar{z}-iz+i\bar{z}+1}=4$

$z=x+iy$

$\frac{x^2+y^2+1+i(2yi)}{x^2+y^2+1-i(2yi)}=4$

$3x^2+3y^2+10y+3=0$

103. **(d)** $Z=(\sqrt{3}-i)^{2/5} \Rightarrow Z= 2^{\frac{1}{5}}(1-\sqrt{3}i)^{\frac{1}{5}}$

$$= 2^{\frac{2}{5}}\left(\cos\left(\frac{3\pi}{2}+\frac{\pi}{6}\right)+i\sin\left(\frac{3\pi}{2}+\frac{\pi}{6}\right)\right)^{\frac{1}{5}}$$

$$= 2^{\frac{2}{5}}\left[\frac{1}{2}+i\frac{\sqrt{3}}{2}\right]=2^{-\frac{3}{5}}\left[1+i\sqrt{3}\right]$$

104. **(c)** $Z=\sqrt{-5-12i}+\sqrt{7+24i}$

$\because \quad \sqrt{a+ib}=\pm\left(\sqrt{\frac{|Z|+a}{2}}+i\frac{b}{|b|}\sqrt{\frac{|Z|-a}{2}}\right)$

$\therefore \quad \sqrt{-5-12i} = \pm(2-i\cdot 3)$

$\Rightarrow \quad \sqrt{7+24i} = \pm(4+3\cdot i)$

$\therefore \quad Z=\pm(2-3i)\pm(4+3i) = \pm 6$

$\because \quad Z<0 \Rightarrow Z=-6.$

105. **(c)** $Z=(\sqrt{3}-i)^{1/6} \Rightarrow Z^6=(\sqrt{3}-i)$

$\Rightarrow \; Z^6=2\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)$

$\Rightarrow \; Z^6=2\,cis\left(-\frac{\pi}{6}\right)$ or $2\,cis\left(2K\pi-\frac{\pi}{6}\right)$

$\Rightarrow \; Z=2^{1/6}cis\left(-\frac{\pi}{6}\right)^{1/6}$ or $2^{1/6}\,cis\left(2K\pi-\frac{\pi}{6}\right)^{1/6}$

$\Rightarrow \; Z=2^{1/6}\,cis\left(\frac{2K\pi}{6}-\frac{\pi}{36}\right)$

at $K=5 \Rightarrow Z=2^{1/6}\,cis\left(\frac{10\pi}{6}-\frac{\pi}{36}\right)$

$\Rightarrow \; Z=2^{1/6}\,cis\left(\frac{59\pi}{36}\right).$

106. **(b)** $x+iy=\sqrt{\frac{3+i}{1+3i}} = \sqrt{\frac{6-8i}{10}}=\sqrt{\frac{3}{5}-\frac{4}{5}i}$

$\therefore \quad (x+iy)^2=\frac{3}{5}-\frac{4}{5}i$

$\Rightarrow \; x^2-y^2+2xyi=\frac{3}{5}-\frac{4}{5}i$

$\Rightarrow \; x^2-y^2=\frac{3}{5},\ 2xy=\frac{-4}{5}$

$\Rightarrow \; (x^2+y^2)^2=(x^2-y^2)^2+4x^2y^2$

$$=\left(\frac{3}{5}\right)^2+\left(\frac{-4}{5}\right)^2=\frac{9}{25}+\frac{16}{25}=1.$$

107. **(b)** $Z=(\alpha+i\beta)(2+7i) = (2\alpha-7\beta)+(7\alpha+2\beta)i$

$\because$ Z is purely imaginary

$\Rightarrow$ Re $(Z)=0 \Rightarrow 2\alpha-7\beta=0 \Rightarrow 2\alpha=7\beta$

also $|Z|^2=(2\alpha-7\beta)^2+(7\alpha+2\beta)^2$

$$=\left(7\alpha+\frac{4\alpha}{7}\right)^2=\left(\frac{53\alpha}{7}\right)^2$$

For minimum value as integer, α must be integral multiple of 7.

$\therefore \quad \alpha=7$

$|Z|^2=(53)^2=2809.$

108. **(c)** Given that 1, ω, ω^2 are the cube roots of unity.

$\therefore \quad 1+\omega+\omega^2=0$ and $\omega^3=1.$

Now, $(x+y)^2+(x\omega+y\omega^2)^2+(x\omega^2+y\omega)^2$

$= x^2+y^2+2xy+x^2\omega^2+y^2\omega^4+2xy\omega^3 + x^2\omega^4+y^2\omega^2+2xy\omega^3$

$= x^2+y^2+6xy+x^2(\omega^2+\omega)+y^2(\omega+\omega^2)$

$= x^2+y^2+6xy-x^2-y^2=6xy$

109. **(c)** $|z|^2 w-|w|^2 z=z-w$

$\Rightarrow \quad zw\left(\bar{z}-\bar{w}\right)=z-w$ ...(i)

$\Rightarrow \quad \left[zw\left(\bar{z}-\bar{w}\right)\right]=\left(z-\bar{w}\right)$

$\Rightarrow \quad \bar{z}\,\bar{w}\left(z-w\right)=\bar{z}.\bar{w}$ ...(ii)

Multiply equations (i) and (ii)

$\Rightarrow \quad |z|^2\,|w|^2\,|z-w|^2=|z-w|^2$ $[\therefore\ |w|=|\bar{w}|]$

$\Rightarrow \quad |z\bar{w}|^2=1\Rightarrow|z\bar{w}|=1$

110. **(d)** Given 1.ω. ω^2 are cube roots of unity.

$(1-\omega+\omega^2)^5+(1+\omega.\ \omega^2)^5=(-\omega-\omega)^5+(-\omega^2-\omega^2)$

$=(-2\omega)^5+(-2\omega)^5=32\omega^6$

Here, $1+\omega+\omega^2=0$ and $\omega^3=1$

Then, $32\omega^6=32(\omega^3)^2=32(1)^2=32$

111. **(d)** Given expression is $\frac{3+2i\sin\theta}{1-2i\sin\theta}$

Here,

$$\frac{3+2i\sin\theta}{1-2i\sin\theta}\times\frac{1+2i\sin\theta}{1+2i\sin\theta}=\frac{3+6i\sin\theta+2i\sin\theta}{1-4i^2\sin^2\theta}$$

$$\Rightarrow \frac{3+8i\sin\theta+4\sin^2\theta}{1+4\sin^2\theta}$$

$$\Rightarrow \left(\frac{2}{1+4\sin^2\theta}+1\right)+i\left(\frac{8\sin\theta}{1+4\sin^2\theta}\right)$$

Here, given expression should be real for any value of θ. Then, imaginary part is 0.

$$\frac{8\sin\theta}{1+4\sin^2\theta}=0$$

Here, $(1+4\sin^2\theta)\neq 0$ then, $8\sin\theta=0$

$\sin\theta=0$

$\sin\theta=\sin(n\pi)$ for $n\in Z$

Then, $\theta=n\pi$ for $n\in Z$.

Here, given equation is $ax^2+bx+c=0$ with roots α & β.

112. **(c)** $y^2 + z^2 = 3yz \Rightarrow \frac{y}{z}+\frac{z}{y}=3$ ...(i)

$z^2 + x^2 = 8zx \Rightarrow \frac{z}{x}+\frac{x}{z}=8$ ...(ii)

$x^2 + y^2 = 4xy \Rightarrow \frac{x}{y}+\frac{y}{x}=4$ ...(iii)

Multiply equations (i) and (iii)

$$\left(\frac{y}{z}+\frac{z}{y}\right)\left(\frac{x}{y}+\frac{y}{x}\right)=12$$

$$\Rightarrow \frac{y^2}{xz}+\frac{zx}{yz}=12-8=4 \quad \text{[from (ii)]}$$

113. **(b)** We have $z = \cos\theta + i\sin\theta$

now $z^r = \cos(r\theta) + i\sin(r\theta)$

and $(\bar{z})^r = \cos(r\theta) - i\sin(r\theta)$

$\Rightarrow z^r + (\bar{z})^r = 2\cos(r\theta)$

114. **(b)** $Z = \sin\theta + i\cos\theta$

Multiplication inverse of Z is $\frac{1}{Z}$

$$\therefore \frac{1}{Z}=\frac{\sin\theta - i\cos\theta}{(\sin\theta + i\cos\theta)(\sin\theta - i\cos\theta)}$$

$= \sin\theta - i\cos\theta = \sin\theta + i(-\cos\theta)$

$\therefore$ Multiplicative inverse is $(\sin\theta, -\cos\theta)$

115. **(d)** Given $|x + iy| = \sqrt{x^2+y^2}$

Take $\left|\left(1-\sqrt{3}\,i\right)^9+\left(\sqrt{3}+i\right)^9\right|$

$$\left|\left(2\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)^9+\left(2\left(\cos\frac{\pi}{6}+i\sin\left(\frac{\pi}{6}\right)\right)\right)^9\right|$$

$$=(2)^9\,|-1-i|=(2)^9\times\sqrt{1+1}=2^{9+\frac{1}{2}}=2^{\frac{19}{2}}$$

116. **(c)** Given 1, ω, ω^2 are cube roots of unity and 1, α, α^2, α^3 are fourth roots of varity.

We know that four roots of unity are 1, i, –1, –i

Now, $\alpha + \alpha\omega - \alpha^3\omega^2 = i(1+\omega) + i\omega^2$

$= i(1 + \omega + \omega^2)$

Here, $1 + \omega + \omega^2 = 0$

$\alpha + \alpha\omega - \alpha^3\omega^2 = i.(0) = 0$

117. **(a)** We have $(-1)^{1/4}$ and $(-i)^{1/2}$.

$$Z_1=\left(\cos 4(-\pi)\pm i\sin 4(-\pi)\right)^{1/4}=\left(\cos\left(\frac{\pi}{4}\right)\pm\sin\left(\frac{\pi}{4}\right)i\right)$$

$$Z_2=\left(\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)i\right)^{1/2}=\left(\cos\frac{\pi}{4}-\sin\frac{\pi}{4}i\right)$$

Common value of Z_1 and Z_2 are $\cos\frac{\pi}{4}-\sin\frac{\pi}{4}i$.

$\cos\frac{\pi}{4}-\sin\frac{\pi}{4}i=\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i$. $\tan\alpha=\frac{y}{x}=-1.$

118. **(b)** $(-32i)^{2/5}=(32)^{2/5}(-i)^{2/5}$

$$=4\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)^{2/5}$$

$$=4\left(\cos\frac{3\pi}{5}+i\sin\frac{3\pi}{5}\right)=4cis\frac{3\pi}{5}$$

119. **(c)** We have $\sqrt{(-3+4i)(8+6i)}=\sqrt{-48+14i}$

$$\left\{\text{Using } \pm\left(\sqrt{\frac{1}{2}(\sqrt{a^2+b^2}+a)}+i\sqrt{\frac{1}{2}(\sqrt{a^2+b^2}-a)}\right)\right\}$$

$= \pm(1 + 7i)$

120. **(c)** We have $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^m=1$, where 2022 < m < 2029

$$\Rightarrow\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^m=1 \Rightarrow \cos\frac{m\pi}{3}+i\sin\frac{m\pi}{3}=1$$

as m = 2028,

LHS = RHS.

121. **(c)** We have $1 + \omega$ is a root of $x^n - x = 0$

$\because 1 + \omega + \omega^2 = 0 \Rightarrow 1 + \omega = -\omega^2$

$\Rightarrow (-\omega^2)^n - (-\omega^2) = 0$

$\omega^2((-1)^n\omega^{2n-2} + 1) = 0$

for n $\in$ even natural number, there is no solution

for n $\in$ odd natural number.

$-\omega^{2n-2} + 1 = 0 \Rightarrow \omega^{2n-2} = 1$

or $\omega^{2n-2} = \omega^{3k} \Rightarrow n=\frac{3k}{2}+1$

for n$\in$odd natural number. k$\in$even natural number

$\Rightarrow n = 7$ at $k = 4$

122. **(d)** Let $P(x) = ax^2 + bx + c$

for $P(x) = 0$

$ax^2 + bx + c = 0$ having roots $\alpha \pm i\beta$

Where $\alpha = 0 \Rightarrow 0 \pm i\beta$.

Now $P(P(x)) = a(P(x))^2 + b(P(x)) + c$

$= a(\pm i\beta)^2 + b(\pm i\beta) + c$

$\Rightarrow \alpha\beta \pm i(b\beta) + c \Rightarrow a + ib$

Hence, we get

Only complex numbers of form $a + ib$ with $a \neq 0, b \neq 0$

123. **(b)** ω is root of $x+\frac{1}{x}+1=0$, i.e. $x^2 + x + 1 = 0$

Roots are, $x=\frac{-1\pm i\sqrt{3}}{2}$

$\therefore \omega=\frac{-1+\sqrt{-3}}{2}$ and $\omega^2=\frac{-1-\sqrt{-3}}{2}$

Also, $\omega^3 = \omega.\omega^2 = 1$ and $1 + \omega + \omega^2 = 0$

Then, $\begin{vmatrix}1 & 1+\omega & 1+\omega+\omega^2\\ 3 & 4+3\omega & 5+4\omega+3\omega^2\\ 6 & 9+6\omega & 11+9\omega+6\omega^2\end{vmatrix}$ is written as,

$$= \begin{vmatrix} 1 & 1+\omega & 0 \\ 3 & 4+3\omega & 2+\omega \\ 6 & 9+6\omega & 5+3\omega \end{vmatrix}$$

Expand along row 1,

$(4 + 3\omega)(5 + 3\omega) – (2 + \omega)(9 + 6\omega)$

$– (1 + \omega)\{15 + 9\omega – 12 – 6\omega\}$

$= 20 + 27\omega + 9\omega^2 – 18 – 21\omega – 6\omega^2 – 3 – 6\omega – 3\omega^2 = – 1$

124. **(c)** We have, $(a + bi)^3 = a – bi$

$a^3 + i^3b^3 + 3a^2bi + 3ab^2i^2 = a – bi$

$(a^3 – 3ab^2) – i(b^3 – 3a^2b) = a – bi$

Equating the coefficient, we obtain

$a^3 – 3ab^2 = a$ and $b^3 – 3a^2b = b$

$a^2 – 3b^2 =1$ and $b^2 – 3a^2 = 1$

Using the above two equations, we get

$b^2 – 3 – 9b^2 = 1$ or $8b^2 + 4 = 0$

$\Rightarrow b = \pm\frac{i}{\sqrt{2}}$. Now, $a^2 = 1 + 3b^2 \Rightarrow a = \pm\frac{i}{\sqrt{2}}$

$\Rightarrow$ Possible ordered pairs are

$$\left(\frac{i}{\sqrt{2}},\frac{i}{\sqrt{2}}\right),\left(-\frac{i}{\sqrt{2}},-\frac{i}{\sqrt{2}}\right),\left(\frac{i}{\sqrt{2}},\frac{-i}{\sqrt{2}}\right),\left(-\frac{i}{\sqrt{2}},\frac{i}{\sqrt{2}}\right)$$

Hence, number of ordered pairs = 4

125. **(c)** Giventhat, $\frac{\left(\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}\right)^8}{\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)^8} = \frac{i^8\left(\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}\right)^8}{(-i)^8\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)^8}$

$$= \frac{\left(\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}\right)^8}{\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)^8} = \frac{\left(e^{-i\pi/8}\right)^8}{\left(e^{i\pi/8}\right)^8} = \frac{e^{-i\pi}}{e^{i\pi}}$$

$$= \frac{\{\cos\pi - i\sin\pi\}}{\{\cos\pi + i\sin\pi\}} = \frac{-1}{-1} = 1$$

126. **(d)** We have, $z^2 + z + 1 = 0 \Rightarrow z^2 + 1 = – z$

$$\left(z+\frac{1}{z}\right)^3+\left(z^4+\frac{1}{z^4}\right)^3=\left(\frac{z^2+1}{z}\right)^3+\left(\frac{z^8+1}{z^4}\right)^3$$

$$= -1+\left(\frac{z^8+1}{z^4}\right)^3 \quad \text{... (i)}$$

$\Rightarrow$ $z^2 + 1 = – z$, squaring on both the side,

$z^4 + 1 + 2z^2 = z^2 \Rightarrow z^4 + 1 = – z^2$

Again, squaring it $\Rightarrow z^8 + 1 + 2z^4 = z^4$ $\quad\because z^8 + 1 = – z^4$

Putting in Eq. (i), we get

$$\left(z+\frac{1}{z}\right)^3+\left(z^4+\frac{1}{z^4}\right)^3=-1+\left(-\frac{z^4}{z^4}\right)^3$$

$= – 1 + (–1) = – 2$

127. **(b)** We have,

$(1 + i) (1 + 3i) (1 + 7i) = x + iy$

$(–2 + 4i)(1 + 7i) = x + iy$

$–30 – 10i = x + iy$

$|x + iy| = \sqrt{30^2+10^2} = \sqrt{1000} = 10\sqrt{10}$

128. **(c)** We have given that

$1, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the roots of $z^5 – 1 = 0$

$\therefore$ We can write that

$(z – 1)(z – \alpha_1) (z – \alpha_2) (z – \alpha_3) (x – \alpha_4) = z^5 – 1$

Since, ω is the cube root of unity.

$(\omega – 1) (\omega – \alpha_1) (\omega – \alpha_2) (\omega – \alpha_3) (\omega – \alpha_4) = \omega^5 – 1$

$(\omega – 1) (\omega – \alpha_1) (\omega – \alpha_2) (\omega – \alpha_3) (\omega – \alpha_4) + \omega$

$= (\omega^5 – 1) + \omega = \omega^2 – 1 + \omega \qquad (\because \omega^2 + \omega + 1 = 0)$

$= – 1 – 1 = – 2$

129. **(a)** We have, $a > 0, z = x + iy$

$\log_{\cos^2\theta}|z-a| > \log_{\cos^2\theta}|z-ai|$

We know that, $0 < \cos^2\theta < 1$

So, $|z – a| < |z – ai|$

$(x – a)^2 + y^2 < x^2 + (y – a)^2$

$– 2ax < – 2ay \Rightarrow x > y$

130. **(d)** Given $(\sin\theta – i\cos\theta)^3$

$= (–i)^3 (\cos\theta + i\sin\theta)^3$

$= (–i)^3 (\cos 3\theta + i\sin 3\theta)$ [by De-Moivre theorem]

131. **(b)** Given $(a + ib)^{1/4} = 2 + 3i$

or $a + ib = (2 + 3i)^4$

$= 2^4 + {}^4C_1 2^3(3i) + {}^4C_2 2^2(3i)^2 + {}^4C_3 2(3i)^3 + (3i)^4$

$= 16 + 96i – 216 – 216i + 81 = – 119 – 120i$

$\therefore \quad a = – 119, b = – 120$

Now, $3b – 2a = 3(– 120) – 2(– 119) = – 122$

132. **(c)** (A) $\omega^{1010} + \omega^{2020} = \omega^{336\times3+2} + \omega^{673\times3+1}$

$= (\omega^3)^{336} \cdot \omega^2 + (\omega^3)^{673} \cdot \omega = \omega^2 + \omega = – 1 \quad [\because \omega^3 = 1]$

(B) $(1 – \omega + \omega^2) (1 + \omega – \omega^2) = 4\omega^3 = 4(\because 1 + \omega + \omega^2 = 0)$

(C) $(2 + \omega^2 + \omega^4)^5 = (1 + 1 + \omega + \omega^2)^5 = (1 + 0)^5 = 1$

(D) $(3 + 5\omega + 3\omega^2)^3 = (3 + 2\omega – 3)^3 = (2\omega)^3 = 8$

$[\because \omega + \omega^2 = 1]$

133. **(c)** $z = 4 + 2(i)^{1/3} \Rightarrow i = e^{i\pi/2} = e^{i\left(\frac{\pi}{2}+2n\pi\right)}$

$[\because \quad e^{i\theta} = \cos\theta + i\sin\theta]$

$i^{1/3} = e^{i\left(\frac{\pi}{6}+\frac{2n\pi}{3}\right)}$, $n = 0, 1, 2$

$$\left\{\begin{array}{l} e^{i\frac{\pi}{6}} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + i\frac{1}{2} \\ e^{i\left(\frac{\pi}{6}+\frac{2\pi}{3}\right)} = e^{i\frac{5\pi}{6}} = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = \frac{-\sqrt{3}}{2} + i\frac{1}{2} \\ e^{i\left(\frac{\pi}{6}+\frac{4\pi}{3}\right)} = e^{i\frac{3\pi}{2}} = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = 0 - i \end{array}\right\}$$

For $n = 0$, $e^{i\frac{\pi}{6}} = \frac{\sqrt{3}}{2} + i\frac{1}{2} \Rightarrow n = 1$, $e^{i\frac{5\pi}{6}} = -\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}$

$n = 2$, $e^{i\frac{3\pi}{2}} = 0 - i$

$\therefore \; z = 4 + 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = (4 + \sqrt{3}) + i = b + i$

$$z = 4 + 2\left(\frac{-\sqrt{3}}{2} + \frac{i}{2}\right) = (4 - \sqrt{3}) + i = c + i$$

$z = 4 + 2(0 - i) = 4 - 2i = a - 2i$

$\Rightarrow$ $a = 4$, $b = 4 + \sqrt{3}$ and $c = 4 - \sqrt{3}$

$$\therefore \quad \sqrt{abc} = \sqrt{4(4+\sqrt{3})\,(4-\sqrt{3})} = \sqrt{4(16-3)} = 2\sqrt{13}$$

134. **(b)** Given geometrical inequality is $|z - 2 - 2i| \le 1$

Let $z = x + iy$, then we get

$$\sqrt{(x-2)^2 + (y-2)^2} \le 1$$

$\Rightarrow$ $(x - 2)^2 + (y - 2)^2 \le 1$

The above inequality represents a closed circular disc with center at (2, 2) and with radius 1.

135. **(c)**

136. **(b)** Given $\sum_{i=1}^{n=1} \frac{1}{2 - a^i}$

$1, \alpha, \alpha^2, \ldots, \alpha^{n-1}$ are the nth root of unity.

$\because$ $x^n - 1 = 0$

$\Rightarrow$ $x^n - 1 = (x - 1)(x - \alpha)(x - \alpha^2) \ldots (x - \alpha^{n-1})$

$\Rightarrow$ $\log (x^n - 1) = \log (x - 1) + \log (x - \alpha)$
$+ \ldots \log (x - \alpha^{n-1})$

Differentiating w.r.t. 'x', we get

$$\Rightarrow \frac{nx^{n-1}}{x^n - 1} = \frac{1}{(x-1)} + \frac{1}{(x-\alpha)} + \ldots \frac{1}{(x-\alpha^{n-1})}$$

At ($x = 2$)

$$\Rightarrow \frac{n \cdot 2^{n-1}}{2^n - 1} = 1 + \frac{1}{2-\alpha} + \frac{1}{2-\alpha^2} + \ldots + \frac{1}{2-\alpha^{n-1}}$$

$$\Rightarrow \sum_{i=1}^{n=1} \frac{1}{2-a^i} = \left(\frac{n \cdot 2^{n-1}}{2^n - 1} - 1\right) = \frac{(n-2)2^{n-1} + 1}{2^n - 1}$$

137. **(d)** Given

$$A = \cos\left(\frac{\pi}{2^n}\right) + i\sin\left(\frac{\pi}{2^n}\right), n \in N \text{ or, } e^i\left(\frac{\pi}{2^n}\right)$$

Hence, $(A_1 A_2 A_3 A_4)^4 = e^{i\pi\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{g}+\frac{1}{16}\right)^4}$

$$= e^{i\frac{15\pi}{4}} = e^{i\left(4\pi - \frac{\pi}{4}\right)}$$

or, $\cos\left(4\pi - \frac{\pi}{4}\right) + i\sin\left(4\pi - \frac{\pi}{4}\right) = \cos\frac{\pi}{4} - i\sin\frac{\pi}{4} = \frac{1-i}{\sqrt{2}}$

138. **(d)** Given $x^2 + x + 1 = 0$, then

$$A_r = \left(x + \frac{1}{x}\right)^3 \left(x^2 + \frac{1}{x^2}\right)^3 \left(x^3 + \frac{1}{x^3}\right)^3 \ldots \left(x^r + \frac{1}{x^r}\right)^3$$

Here roots of $x^2 + x + 1 = 0$ are ω, ω^2

Hence $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$

$$\therefore \quad A_r = \left(\omega^1 + \frac{1}{\omega}\right)^3 \left(\omega^2 + \frac{1}{\omega^2}\right)^3 \left(\omega^3 + \frac{1}{\omega^3}\right)^3$$

$$A_3 = \left(\frac{1+\omega^2}{\omega}\right)^3 \left(\frac{1+\omega^4}{\omega^2}\right)^3 (1+1)^3$$

$\therefore$ $A_3 = (-1)\,(-1)\,(8) = 8$

Similarly, $A_6 = 8_2$, $A_9 = 8^3$ ... and so on.

$$\therefore \quad \frac{1}{A_3} + \frac{1}{A_6} + \frac{1}{A_9} + \frac{1}{A_{12}} + \ldots\infty$$

$$= \frac{1}{8} + \frac{1}{8^2} + \frac{1}{8^3} + \frac{1}{8^4} + \ldots\infty = \frac{\frac{1}{8}}{1 - \frac{1}{8}} = \frac{1}{7}$$

139. **(a)** A complex number, $z = x + iy$

Given, $|z| \le 2$

$\Rightarrow$ $\sqrt{x^2 + y^2} \le 2 \Rightarrow x^2 + y^2 \le 4$ ...(i)

Also given, $(1 - i)z + (1 + i)\overline{z} \ge 4$

$\Rightarrow$ $(1 - i)\,(x + iy) + (1 + i)\,(x - iy) \ge 4$

$\Rightarrow$ $2x + 2y \ge 4 \Rightarrow x + y \ge 2$ ...(ii)

So, $A \cap B = \{|z| \le 2\} \cap \{(1 - i)\,z + (1 + i)\,\overline{z} \ge 4\}$

$A \cap B = \{x^2 + y^2 \le 4\} \cap \{x + y \ge 2\}$

Now, check all the options satisfying the condition $A \cap B$,

$$\therefore \quad z = \sqrt{3} + \frac{1}{2}i$$

140. **(a)** Let a complex number, $z = x + iy \Rightarrow \overline{z} = \overline{x} - iy$

Then, vertices of rectangle for $z, \overline{z}, -z - \overline{z}$ are (x, y), $(x, -y)$ $(x, -y)$, $(-x, y)$.

Now, area of rectangle = $(2x)\,(2y) = 4xy$

It is given that,

Area $= 2\sqrt{3} = 4xy \Rightarrow 2xy = \sqrt{3}$

$$\therefore \; x = \frac{1}{2}, y = \sqrt{3} \quad \therefore z = \frac{1}{2} + \sqrt{3}i$$

141. **(b)** Given that,

$$\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^8 + \left(\frac{1+\cos\theta - i\sin\theta}{1+\cos\theta + i\sin\theta}\right)^{16}$$

$$= \frac{1}{i^8}\left(\frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}\right)^8 + \left(\frac{\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}\right)^{16}$$

By using De Moivre's theorem, we get

$$= \frac{\cos 8\theta + i\sin 8\theta}{\cos 8\theta - i\sin 8\theta} + \frac{\cos 8\theta - i\sin 8\theta}{\cos 8\theta + i\sin 8\theta}$$

$= (\cos 8\theta + i \sin 8\theta)^2 + (\cos 8\theta - i\sin 8\theta)^2$

$= \cos^2 8\theta + i^2\sin^2 8\theta + 2i \cos 8\theta \sin 8\theta + \cos^2 8\theta$
$+ i^2 \sin^2 8\theta - 2i \cos 8\theta \sin 8\theta$

$= 2\,(\cos^2 8\theta - \sin^2 8\theta) = 2 \cos 16\theta$

$(\because \cos^2\theta - \sin^2\theta = \cos^2\theta)$

142. (a) $$\because \quad \frac{1-\sqrt{3}i}{2} = -\left[\frac{-1+\sqrt{3}i}{2}\right] = -\omega$$

and $$\frac{1+\sqrt{3}i}{2} = -\left[\frac{-1-\sqrt{3}i}{2}\right] = -\omega^2$$

$$\therefore \quad \left(\frac{1-\sqrt{3}i}{2}\right)^{2020} + \left(\frac{1+\sqrt{3}i}{2}\right)^{2026}$$

$$= \omega^{2020} + \omega^{4052} = \omega + \omega^2 = -1 \qquad \ldots(i)$$

$$\because \quad \sum_{j=1}^{6} (j+\omega)(j+\omega^2)$$

$$= \sum_{j=1}^{6} (j^2 + j\omega^2 + j\omega + \omega^3)$$

$$= \sum_{j=1}^{6} (j^2 - j + 1) = \sum_{j=1}^{6} j^2 - \sum_{j=1}^{6} j + \sum_{j=1}^{6} 1$$

$$= 7 \times 13 - 3 \times 7 + 6 = 91 - 21 + 6 = 76$$

$$\therefore \quad \sin\left(\sum_{j=1}^{6} (j+\omega)(j+\omega^2)\frac{3\pi}{152}\right) = \sin\frac{3\pi}{2} = -1 \qquad \ldots(ii)$$

From (i) and (ii), we have

$$\left(\frac{1-\sqrt{3}i}{2}\right)^{2020} + \left(\frac{1+\sqrt{3}i}{2}\right)^{2026} + \sin\left(\sum_{j=1}^{6} (j+\omega)(j+\omega^2)\frac{3\pi}{152}\right)$$

$$= -1 - 1 = -2$$

143. (b) $\because$ For a complex number $z = e^{i\theta}$,

We have $$\frac{3\cos 3\theta + 2\cos 2\theta + 5\cos 5\theta}{3\sin 3\theta + 2\sin 2\theta + 5\sin 5\theta} = i\frac{\sum_{r=0}^{10} a_r z^r}{\sum_{r=0}^{10} b_r z^r}$$

$$\Rightarrow \frac{\sum_{r=0}^{10} z^r(a_r + b_r)}{\sum_{r=0}^{10} z^r(a_r - b_r)} = \frac{3e^{i3\theta} + 2e^{i2\theta} + 5e^{i5\theta}}{3e^{-i3\theta} + 2e^{-i2\theta} + 5e^{-i5\theta}}$$

$\Rightarrow a_0 + b_0 = a_1 + b_1 = a_4 + b_4 = a_6 + b_6$
$= a_7 + b_7 = a_8 + b_8$
$= a_9 + b_9 = a_{10} + b_{10} = 0$
and $a_2 + b_2 = 2$, $a_3 + b_3 = 3$, $a_5 + b_5 = 5$

$$\therefore \quad \frac{\sum_{r=0}^{10} (a_r + b_r)}{10} = \frac{2+3+5}{10} = 1$$

144. (a) Given C is a complex number

$$= \frac{1+iz}{1-iz} = \frac{(1+a-c)+ib}{(1+a+c)-ib}$$

$$= \frac{(1+a-c)+ib}{(1+a+c)-ib} \times \frac{(1+a+c)+ib}{(1+a+c)+ib} \qquad \left\{\text{Given, } z = \frac{b+ic}{1+a}\right\}$$

Here $iz = \dfrac{ib-c}{1+a} \Rightarrow -iz = \dfrac{-ib+c}{1+a}$

$$= \frac{(1+a+c+a+a^2+ac-c-ac-c^2-b^2) + ib(1+a-c+1+a+c)}{(1+a+c)^2+b^2}$$

$$= \frac{2(1+a)(a+ib)}{2(1+a)+2c(1+a)} = \frac{a+ib}{1+c}$$

145. (d) We have, $(1-\omega+\omega^2)^6 + (1-\omega^2+\omega)^6$
$= (-\omega-\omega)^6 + (-\omega^2-\omega^2)^6 \qquad [\because 1+\omega+\omega^2 = 0]$
$= (-2\omega)^6 + (-2\omega^2)^6$
$(-2)^6[\omega^6 + \omega^{12}]$
$= 2^6.2 = 2^7 = 128 \qquad [\because \omega^3 = 1]$

146. (c) Given points P, Q and R are z, $ze^{\frac{i\pi}{3}}$ and z $\left(1+e^{\frac{i\pi}{3}}\right)$

$$PQ = \left|ze^{i\pi/3} - z\right| = |z|\left|e^{i\pi/3} - 1\right|$$

$$= |z|\left|\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} - 1\right|$$

$$= |z|\left|2\sin\frac{\pi}{6}\right|\left|\sin\frac{\pi}{6} - i\cos\frac{\pi}{6}\right| = |z|.\left|2\times\frac{1}{2}\right||1|$$

$PQ = |z|$

Now, $QR = \left[z(1+e^{i\pi/3}) - 2e^{i\pi/3}\right]$

$QR = |z|$

Similarly, $PR = |z|$

So, $PQ = QR = PR$

$\Rightarrow \Delta PQR$ is equilateral triangle with side length ≥ 1.

Now, are a of $\Delta PQR = \dfrac{\sqrt{3}}{4}|z|^2$

147. (d) Let the point $z(x, y)$, $z_1(x_1, y_1)$ and $z_2(x_2, y_2)$
$\Rightarrow$ Here $z - z_1 = x - x_1 + i(y - y_1)$
and $\quad z - z_2 = x - x_2 + i(y - y_2)$

Given, $\dfrac{z-z_1}{z-z_2} = 0$ or π

$$= \frac{\left[(x-x_1)+i(y-y_1)\right]}{\left[(x-x_2)+i(y-y_2)\right]} \times \frac{\left[(x-x_2)-i(y-y_2)\right]}{\left[(x-x_2)-i(y-y_2)\right]} = 0$$

$$= \frac{(x-x_1)(x-x_2)+(y-y_1)(y-y_2) + i\left[(x-x_2)(y-y_1)-(x-x_1)(y-y_2)\right]}{(x-x_2)^2+(y-y_2)^2} = 0$$

$$= Arg\left(\frac{z-z_1}{z-z_2}\right) = 0 \text{ or } \pi$$

$$\Rightarrow \left[\frac{\left[(x-x_2)(y-y_1)\right]-\left[(x-x_1)(y-y_2)\right]}{\left[(x-x_1)(x-x_2)+(y-y_1)(y-y_2)\right]}\right] = 0$$

$\Rightarrow x(y_2 - y_1) + y(x_1 - x_2) + (x_2y_1 - x_1y_2) = 0$

It represents a straight line passing through the points A of B.

148. **(c)** Given, x is a cube root of unity other than 1 i.e. $x = \omega$ or ω^2

So, $\left(x+\frac{1}{x}\right)^2 + \left(x^2+\frac{1}{x^2}\right)^2 + \ldots + \left(x^{12}+\frac{1}{x^{12}}\right)^2$

$$= \left(\omega+\frac{1}{\omega}\right)^2 + \left(\omega^2+\frac{1}{\omega^2}\right)^2 + \ldots + \left(\omega^{12}+\frac{1}{\omega^{12}}\right)^2$$

$$= \left(\omega+\omega^2\right)^2 + \left(\omega^2+\omega\right)^2 + \left(1+1\right)^2 + \left(\omega+\omega^2\right)^2$$
$$+ \left(\omega^2+\omega\right)^2 + \left(1+1\right)^2 + \left(\omega+\omega^2\right)^2$$

$$= 8\left(\omega+\omega^2\right)^2 + 4\left(1+1\right)^2 \quad = 8\left(-1\right)^2 + 4\left(2\right)^2 = 24$$

149. **(a)** Given, $z = x - iy$ and $z^{\frac{1}{3}} = a + ib$

Cubing $(z^{1/3})^3 = (a+ib)^3$ $\quad [\because i^2 = -1]$

$\Rightarrow z = (a^3 - 3ab^2) - i\,(b^3 - 3a^2b)$

$\Rightarrow x - iy = (a^3 - 3ab^2) - i\,(b^3 - 3a^2b)$ $\quad [\because z = x - iy]$

Here, $x = a^3 - 3ab^2$ and $y = b^3 - 3a^2b$

Now, $\dfrac{\left(\dfrac{x}{a}+\dfrac{y}{b}\right)}{a^2+b^2} = \dfrac{a^2-3b^2+b^2-3a^2}{a^2+b^2}$

$$= \frac{-2a^2-2b^2}{a^2+b^2} = \frac{-2(a^2+b^2)}{a^2+b^2} = -2$$

150. **(a)** Given $\bar{z} = iz^2$

$\Rightarrow z = -i\bar{z}^2 \Rightarrow z = -i[i\,z^2]^2 \Rightarrow z = -i\,i^2z^4 \Rightarrow z = iz^4$

$\Rightarrow z^4 = \frac{1}{i}z \Rightarrow z\,(z^3 + i) = 0$

$\Rightarrow z^3 + i = 0$ $\quad (\because z \neq 0)$

151. **(c)** Given $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$

$$= -i\sum_{k=1}^{6}\left[\cos\left(\frac{2\pi k}{7}\right) + i\sin\left(\frac{2\pi k}{7}\right)\right]$$

$$= -i\left[\sum_{k=1}^{6} e^{\frac{i2\pi k}{7}}\right] \qquad \text{...(i)} \quad \{\because e^{i\theta} = \cos\theta + i\sin\theta\}$$

Now,

$$\left[1 + e^{\frac{i2\pi}{7}} + e^{\frac{i4\pi}{7}} + e^{\frac{i6\pi}{7}} + e^{\frac{i8\pi}{7}} + e^{\frac{i10\pi}{7}} + e^{\frac{i12\pi}{7}} = 0\right]$$

$$\left\{\because z^7 = 1 \text{ then, roots } 1, e^{\frac{i2\pi}{7}}, e^{\frac{i4\pi}{7}} \ldots, e^{\frac{i12\pi}{7}}\right\}$$

$$= 1 + \sum_{k=1}^{6} e^{i(2\pi k/7)} = 0 \sum_{k=1}^{6} e^{i(2\pi k/7)} = -1 \qquad \text{...(ii)}$$

From eqs. (i) and (ii), we get = (i)

152. **(b)** Given that ω is a complex cube root of unity

$\therefore \quad 1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$ $\quad$ ...(i)

$\because \quad \left(k+\frac{1}{\omega}\right)\left(k+\frac{1}{\omega^2}\right) = k^2 + k\left(\frac{1}{\omega}+\frac{1}{\omega^2}\right) + \frac{1}{\omega^3}$

$= k^2 - k + 1$ $\quad$ [From Eq. (i)]

Now, $\sum_{k=1}^{n}\left(k+\frac{1}{\omega}\right)\left(k+\frac{1}{\omega^2}\right)$

$$= \sum_{k=1}^{n}(k^2 - k + 1) = \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + n$$

$$= \frac{n(n+1)}{6}(2n-2) + n = \frac{n(n^2+2)}{3}$$

153. **(d)** Given that ω is a complex cube root of unity, then

$1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$ $\quad$ ...(i)

$\because \quad r(r+1-\omega)(r+1-\omega^2)$

$= r[(r+1)^2 - (\omega+\omega^2)(r+1) + \omega^3]$

$= r[(r+1)^2 + (r+1) + 1]$ $\quad$ [from Eq. (i)]

$= r^3 + 3r^2 + 3r$

Now,

$$\sum_{r=1}^{9} r(r+1-\omega)(r+1-\omega^2) = \sum_{r=1}^{9}(r^3 + 3r^2 + 3r)$$

$$= \left(\frac{9(10)}{2}\right)^2 + 3\frac{9(10)(19)}{6} + 3\frac{9\times 10}{2}$$

$= (45)^2 + (45\times 19) + (3\times 45) = 3015.$

154. **(b)** Given $z = i^{1/4}$. Hence, $z^4 = i = e^{i\pi/2}$

and its roots are $e^{i\left(\frac{\pi}{8}\right)}, e^{i\left(\frac{5\pi}{8}\right)}, e^{i\left(\frac{9\pi}{8}\right)}$ and $e^{i\left(\frac{13\pi}{8}\right)}$ and there is no pair of conjugate roots.

So, sum of products of the roots (which are non-conjugate) of equation $z^4 - i = 0$ is zero.

155. **(a)** Given $z = \cos\alpha + i\sin\alpha$, $0 < \alpha < \frac{\pi}{4}$

So, $\dfrac{1+z^4}{1-z^3} = \dfrac{1+(\cos\alpha + i\sin\alpha)^4}{1-(\cos\alpha + i\sin\alpha)^3}$

$$= \frac{1+\cos 4\alpha + i\sin 4\alpha}{1-\cos 3\alpha - i\sin 3\alpha} \qquad \text{(by De-Moivre's theorem)}$$

$$= \frac{2\cos^2 2\alpha + 2i\sin 2\alpha\cos 2\alpha}{2\sin^2\frac{3\alpha}{2} - 2i\sin\frac{3\alpha}{2}\cos\frac{3\alpha}{2}}$$

$$= \frac{2\cos 2\alpha}{2\sin\frac{3\alpha}{2}} \times \frac{(\cos 2\alpha + i\sin 2\alpha)}{\left(\sin\frac{3\alpha}{2} - i\cos\frac{3\alpha}{2}\right)}$$

$$\therefore \left|\frac{1+z^4}{1-z^3}\right| = \frac{\cos 2\alpha}{\sin\frac{3\alpha}{2}} \qquad \{\text{as } |\cos\theta + i\sin\theta| = 1\}$$

156. **(b)** Since 1, $\omega, \omega^2, \ldots, \omega^8$ are the roots of equation $x^9 - 1 = 0$.
Hence, $\omega^9 - 1 = 0$ and sum of roots = 0.
Hence $1 + \omega + \omega^2 + \ldots + \omega^8 = 0$ and $\omega^9 = 1$

Now, $\sum_{r=1}^{8} (\omega^r)^{99} = \sum_{r=1}^{8} (\omega^9)^{11r} = \sum_{r=1}^{8} (1)^{11r} = \sum_{r=1}^{8} 1 = 8$

157. **(d)** Given $2 < |z - (1 + i)| < 3$
Put $z = x + iy$
$2 < |(x - 1) - (y - 1)i| < 3$
$4 < (x - 1)^2 + (y - 1)^2 < 9$
Area of region = Area of largest circle – Area of smallest circle
$9\pi - 4\pi = 5\pi$

158. **(d)** We have, $\left(\frac{1+iz}{1-iz}\right)^4 = P \Rightarrow \left(\frac{z-i}{z+i}\right)^4 = P$

Take modulus both side

$$\Rightarrow \left|\frac{z-i}{z+i}\right|^4 = |P| \Rightarrow \left|\frac{z-i}{z+i}\right|^4 = 1$$

$\Rightarrow |z - i| = |z + i|$
$i\,z - i$ complex numbers lies on y-axis.
$\therefore$ z lies on perpendicular bisector of i and $-i$.
$\therefore$ z lies on y-axis.
$\therefore$ z has all complex roots.

159. **(d)** Given, $\left|\frac{z+3i}{3z+i}\right| < 1$

Put $z = x + iy$

$$\left|\frac{x + (y+3)i}{3(x+iy)+i}\right| < 1$$

$|x + (y + 3)i| < |3x + (3y + 1)i|$
$\Rightarrow$ $8x^2 + 8y^2 - 8 > 0$
$\Rightarrow$ $x^2 + y^2 - 1 > 0$ …(i)

Point $\left(\frac{k-1}{k}, \frac{k-2}{k}\right)$ lie on locus of z, then satisfy in equation (i)

$$\therefore \frac{(k-1)^2}{k^2} + \frac{(k-2)^2}{k^2} - 1 > 0$$

$\Rightarrow (k - 5)(k - 1) > 0 \Rightarrow$ $k \in (-\infty, 1) \cup (5, \infty)$

160. **(d)** Given, $z = (-i)^{1/3}$

$$z = \left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)^{1/3}$$

$$z = \cos\frac{2k\pi - \frac{\pi}{2}}{3} + i\sin\frac{2k\pi - \frac{\pi}{2}}{3} \quad \text{where, } k = 0, 1, 2$$

$e^{i\theta} = (\cos\theta + i\sin\theta)$; $\because z = e^{i}\left(\frac{4k-1}{6}\right)\pi$

Put $k = 0, 1, 2$
$z = e^{-i\pi/6}, e^{i\pi/2}, e^{i7\pi/6}$ $\therefore \alpha = e^{-i\pi/6}, \beta = e^{i\pi/2}, \gamma = e^{i7\pi/6}$

$$\alpha^2 = e^{\frac{-2i\pi}{6}}, \beta^2 = e^{-1}, \gamma^2 = e^{i14/6}$$

$$\alpha^2 + \beta^2 + \gamma^2 = e^{-\frac{i\pi}{3}} - 1 + e^{i}\frac{7\pi}{3}$$

Put $e^{i\theta} = \cos\theta + i\sin\theta$

$$= \cos\frac{\pi}{3} - i\sin\frac{\pi}{3} - 1 + \cos\frac{7\pi}{3} + i\sin\frac{7\pi}{3} = 0$$

161. **(b)** It is given that,
$x + iy = (1 + i)^6 - (1 - i)^6$

$$1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$(1 + i)^6 = 8\left(\cos\frac{6\pi}{4} \quad \sin\frac{6\pi}{4}\right)$$

Similarly,

$$(1 - i)^6 = 8\left(\cos\frac{6\pi}{4} - i\sin\frac{6\pi}{4}\right)$$

$$x + iy = 8\left[2i\sin\frac{3\pi}{2}\right] = -16i \qquad \left(\because \sin\frac{3\pi}{2} = -1\right)$$

$\therefore$ $x = 0, y = -16$
Hence, $x + y = 0 - 16 = -16$

162. **(b)** Here 1, ω and ω^2 are the cube roots of unity.
$(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega) = a^2 + b^2 + c^2 - ab - bc - ca$
and $a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$= (a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$
So, $(a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$
$= a^3 + b^3 + c^3 - 3abc$

163. **(a)** Given that,

$$13e^{i\tan^{-1}\frac{5}{12}} = a + ib$$

$$\Rightarrow 13\left[\cos\left(\tan^{-1}\frac{5}{12}\right) + i\sin\left(\tan^{-1}\frac{5}{12}\right)\right] = a + ib$$

$(\because e^{i\theta} = \cos\theta + i\sin\theta)$

$$\Rightarrow 13\left[\cos\left(\cos^{-1}\frac{12}{13}\right) + i\sin\left(\sin^{-1}\frac{5}{13}\right)\right] = a + ib$$

$$\Rightarrow 13\left[\frac{12}{13} + i\frac{5}{13}\right] = a + ib$$

$\Rightarrow$ $12 + 5i = a + ib$
On comparing both the sides, we get
$\therefore$ $a = 12, b = 5$
$\therefore$ $(a, b) = (12, 5)$

164. **(d)** Given that,
$z_1 = 1 - 2i, z_2 = 1 + i, z_3 = 3 + 4i$

Now, $\left(\frac{1}{z_1} + \frac{3}{z_2}\right)\frac{z_3}{z_2} = \left(\frac{1}{1-2i} + \frac{3}{1+i}\right)\left(\frac{3+4i}{1+i}\right)$

$$= \frac{32+i}{2(2+i)} = \frac{(32+i)}{2(2+i)} \times \frac{2-i}{2-i} = \frac{64+2i-32i+1}{2(4+1)}$$

$$= \frac{13}{2} - 3i$$

165. **(b)** Let $z = x + iy$

Given that vertices of triangle are 0, $z = (x + iy)$ and

$ze^{i\alpha} = (x + iy)(\cos\alpha + i\sin\alpha)$

$= (x\cos\alpha - y\sin\alpha) + i(y\cos\alpha + x\sin\alpha)$

$\therefore$ Area of triangle

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ x\cos\alpha - y\sin\alpha & y\cos\alpha + x\sin\alpha & 1 \end{vmatrix}$$

$$= \frac{1}{2}(x^2 + y^2)\sin\alpha$$

$$= \frac{1}{2}(z)^2 \sin\alpha \qquad [\because\ |z| = \sqrt{x^2 + y^2}]$$

166. **(d)** Given that, $\omega_0, \omega_1, \omega_2, \ldots, \omega_{n-1}$ are nth root of unity,

So, $x^n - 1 = (x - \omega_0)(x - \omega_1)(x - \omega_2) \ldots (x - \omega_{n-1}) \ldots$(i)

put $x = \frac{-1}{2}$ in eqn. (i), we get

$$\left(\frac{-1}{2}\right)^n - 1 = \left(\frac{-1}{2} - \omega_0\right)\left(\frac{-1}{2} - \omega_1\right) \cdots \left(\frac{-1}{2} - \omega_{n-1}\right)$$

$$\Rightarrow \left(\frac{-1}{2}\right)^n - 1$$

$\Rightarrow 1 - (-1)^n (2)^n = (1 + 2\omega_0)(1 + 2\omega_1) \ldots (1 + 2\omega_{n-1})$

$\Rightarrow 1 + (-1)^{n-1} 2^n = (1 + 2\omega_0)(1 + 2\omega_1) \ldots (1 + 2\omega_{n-1})$

167. **(c)** We have given that α is non-real root of $x^7 = 1$

$\therefore$ $\alpha^7 = 1$ and $\alpha \neq 1$

Now, $\alpha(1 + \alpha)(1 + \alpha^2 + \alpha^4)$

$= \alpha(1 + \alpha^2 + \alpha^4 + \alpha + \alpha^3 + \alpha^6)$

$= \alpha + \alpha^3 + \alpha^5 + \alpha^2 + \alpha^4 + \alpha^6$

$= \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6$

$$= \frac{\alpha\left(1-\alpha^6\right)}{1-\alpha} = \frac{\alpha - 1}{1-\alpha}\left[\because\ \alpha \neq 1 \text{ and } \alpha^7 = 1\right]$$

$$\therefore\ \alpha(1+\alpha)\left(1+\alpha^2+\alpha^4\right) = -1$$

168. **(d)** We have,

$|z_1| = 1, |z_2| = 2, |z_3| = 3$ and

$|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$

Since, we know $|z|^2 = z\,\bar{z}$

Now, $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$

$$\Rightarrow \left||z_3|^2 z_1z_2 + |z_2|^2 z_1z_3 + |z_1|^2 z_2z_3\right| = 12$$

$$\Rightarrow |z_1z_2z_3|\left|\bar{z}_1 + \bar{z}_2 + \bar{z}_3\right| = 12$$

$$\Rightarrow |z_1||z_2||z_3|\left|\bar{z}_1 + \bar{z}_2 + \bar{z}_3\right| = 12$$

$$\Rightarrow \left|\bar{z}_1 + \bar{z}_2 + \bar{z}_3\right| = 2$$

$$\Rightarrow |z_1 + z_2 + z_3| = 2$$

169. **(c)** We have given that, $z_1, z_2, z_3, \ldots\ldots z_{n-1}$ are the nth root of unity.

$\therefore$ $z^n - 1 = (z - 1)(z - z_1)(z - z_2) \ldots\ldots (z - z_{n-1})$

$z^n - 1 = (z - 1)(z - z_1)(z - z_2) \ldots\ldots (z - z_{n-1})$

$(z - 1)(z^{n-1} + z^{n-2} + \ldots\ldots (z - z_{n-1})$

$(z^{n-1} + {}^{n-2} + \ldots\ldots + z^2 + z + 1) = (z - z_1)$

$(z - z_1) = (z - z_{n-1})$

Put $z = 1$, we get

$(1 + 1 + \ldots. + n$ times)

$= (1 - z_1)(1 - z_2) + \ldots. (1 - z_{n-1})$

$n = (1 - z_1)(1 - z_2) \ldots (1 - z_{n-1})$

170. **(a)** Given equation, $(z + 1)(z^2 + z + 1) = 0$

Its roots are -1, ω and ω^2.

Let $f(z) = z^{2014} + z^{2015} + 1 = 0$

Put $z = -1$, ω and ω^2 respectively, we get

$f(-1) = (-1)^{2014} + (-1)^{2015} + 1 = 0 = 1 \neq 0$

Therefore, -1 is not a root of the equation $f(z) = 0$

Similarly,

Again, $f(\omega) = (\omega)^{2014} + (\omega)^{2015} + 1 = \omega^2 + \omega + 1 = 0$

Therefore, ω is a root of the equation $f(z) = 0$

$f(\omega^3) = (\omega^2)^{2014} + (\omega^3)^{2015} + 1 = 0$

Hence ω and ω^2 are the common roots of $z^{2014} + z^{2015} + 1 = 0$

171. **(a)** Given, α and β are non-real cube roots of 2.

$\therefore$ $\alpha = 2^{1/3}\omega$ and $\beta = 2^{1/3}\omega^2$

Now, $\alpha^6 + \beta^6 = (2^{1/3}\omega)^6 + (2^{1/3}\omega^2)^6$

$= 2^2\omega^6 + 2^2\omega^{12} = 4(\omega^3)^2 + 4(\omega^3)^4$

$= 4 + 4 = 8 \qquad [\because \omega^3 = 1]$

172. **(c)** Let α, β, γ be the roots of the equation

$12x^3 - 20x^2 + x + 3 = 0$

$$\because\ \alpha + \beta + \gamma = \frac{20}{12};\ \alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{12},\ \alpha\beta\gamma = \frac{-3}{12}$$

$$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = \frac{376}{144}$$

$$\text{and, } (\alpha\beta + \beta\gamma + \alpha\gamma)^2 = (\alpha\beta)^2 + (\beta\gamma)^2 + (\alpha\gamma)^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$$

$$\Rightarrow (\alpha\beta)^2 + (\beta\gamma)^2 + (\alpha\gamma)^2 = \frac{121}{144}$$

Now, required equation is

$$x^3 - (\alpha^2 + \beta^2 + \gamma^2)x^2 + (\alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2)x - \alpha^2\beta^2\gamma^2 = 0$$

$$\Rightarrow 144x^3 - 376x^2 + 121x - 9 = 0.$$

173. **(d)** Since, α, β, γ be the roots of the equation

$x^3 + 3x^2 - 10x - 24 = 0$

$\because\ \alpha\beta + \beta\gamma + \alpha\gamma = -10,\ \alpha\beta\gamma = 24,\ \alpha + \beta + \gamma = -3$

$$\text{Now, } (\alpha\beta + \beta\gamma + \alpha\gamma)^2 = \alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$$

$\Rightarrow\ \alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2=244$

Since, $q=\alpha\beta(\beta+\gamma)(\gamma+\alpha)+\beta\gamma(\gamma+\alpha)(\alpha+\beta)$
$+\alpha\gamma(\beta+\gamma)(\alpha+\beta)$

$=\alpha\beta\gamma(\alpha+\beta+\gamma)+\alpha^2\beta^2+\alpha\beta\gamma(\alpha+\beta+\gamma)$
$+\beta^2\gamma^2+\alpha\beta\gamma(\alpha+\beta+\gamma)+\alpha^2\gamma^2$

$=3\cdot24\cdot(-3)+244=-216+244=28.$

174. **(b)** $x^2-3ax+a^2-2a-4=0$
$D=9a^2-4a^2+8a+16=5a^2+8a+16>0$
So, roots are real and distinct.

175. **(c)** $\alpha^2-6\alpha-2=0\Rightarrow\alpha^{10}-2\alpha^8=6\alpha^9$ ...(i)
$\beta^2-6\beta-2=0\Rightarrow\beta^{10}-2\beta^8=6\beta^9$ ...(ii)
(i) – (ii)
$\alpha^{10}-\beta^{10}-2(\alpha^8-\beta^8)=6\cdot(\alpha^9-\beta^9)$
$\Rightarrow\ a_{10}-2a_8=6a_9\ \Rightarrow\ \frac{a_{10}-2a_8}{2a_9}=3.$

176. **(d)** $x^2-6ax+2-2a+9a^2=0$
$\because\ D>0\Rightarrow\ 8(a-1)>0\Rightarrow a>1$
$f(3)>0\Rightarrow\ 9a^2-20a+11>0$
$\Rightarrow\ (9a-11)(a-1)>0\Rightarrow\ a>\frac{11}{9}$
$f'(3)<0\Rightarrow 6-6a<0\Rightarrow a>1$
Condition for roots to be greater than 3 is $a>\frac{11}{9}$.

177. **(d)** $x^5-5x^3+5x^2-1=0$
By inspection $x=1$ is a root
$(x-1)(x^4+x^3-4x^2+x+1)=0$
$\Rightarrow\ (x-1)^3(x^2+3x+1)=0$
$\Rightarrow\ x=1,\ x=\frac{-3\pm\sqrt{5}}{2}\quad\therefore\ \alpha=\frac{-3+\sqrt{5}}{2}\quad\beta=\frac{-3-\sqrt{5}}{2}$
Since imaginary roots occurs in pair.
So, equation having roots $-i\sqrt{\alpha},\ i\sqrt{\alpha},\ i\sqrt{\beta},\ -i\sqrt{\beta}$
$(x-i\sqrt{\alpha})(x+i\sqrt{\alpha})(x-i\sqrt{\beta})(x+i\sqrt{\beta})=0$
$\Rightarrow\ x^4+(\alpha+\beta)x^2+\alpha\beta=0\ \Rightarrow\ x^4-3x^2+1=0.$

178. **(b)** $x^4-x^3-6x^2+4x+8=0$
By Hit and trial $x=2$ is a root of the equation
$\Rightarrow\ (x-2)(x^3+x^2-4x-4)=0$
Again $x=2$ is a root of the equation
$\Rightarrow\ (x-2)^2(x^2+3x+2)=0$
$\Rightarrow\ (x-2)^2(x-1)(x+2)=0\ \Rightarrow\ x=2,-2,1$
$\Rightarrow\ \alpha=-2,\beta=1\Rightarrow\ \alpha^2+\beta^2=5.$

179. **(c)** $a(b-c)x^2+b(c-a)x+c(a-b)=0$
$x=1$ satisfies his equation
Product of roots $=\frac{c(a-b)}{a(b-c)}=1\times\alpha$
$\alpha=\frac{c(a-b)}{a(b-c)}$ = other root
Roots are $\frac{c(a-b)}{a(b-c)}$ and 1

180. **(d)** Roots of equation are $(3+i)$ and $(3-i)$
Sum of roots $=-a=6\Rightarrow a=-6$

181. **(a)** $4x^3-12x^2+11x+m=0$
Let the roots be $A-d, A, A+d$
$3A=3\Rightarrow A=1$
and roots are $1-d, 1, 1+d$
$1-d+1+d+1-d^2=\frac{11}{4}\Rightarrow d=\pm\frac{1}{2}$
Product of roots $=\frac{1}{2}\times1\times\frac{3}{2}\Rightarrow\frac{3}{4}=\frac{-m}{4}\Rightarrow m=-3$

182. **(c)** $x^3+px^2+qx-5=0$
$\alpha+\beta+\gamma=-p$
$\Rightarrow 2\alpha=-p\ \Rightarrow\alpha=\frac{-p}{2}$ $[\because\beta+\gamma=\alpha]$
$\Rightarrow\frac{p^3}{8}+\frac{pq}{2}-5=0\Rightarrow p(p^2-4pq)=40$

183. **(b)** Let, $x=4+\frac{1}{x}\Rightarrow x^2-4x-1=0$
$\Rightarrow\ x=2\pm\sqrt{5}\ \Rightarrow\ x=2+\sqrt{5}$ and $x\neq2-\sqrt{5}$
(It is negative).

184. **(b)** Given α, β, γ are roots of $x^3+ax^2+bx+c=0$
So, $\alpha\cdot\beta\cdot\gamma=\frac{-c}{a}$ and $\alpha\beta+\beta\gamma+\alpha\gamma=\frac{b}{a}$
Now, $\alpha^{-1}+\beta^{-1}+\gamma^{-1}=\frac{\alpha\beta+\beta\gamma+\alpha\gamma}{\alpha\beta\gamma}=\frac{-b}{c}.$

185. **(a)** Let roots of $x^3-35x+c=0$ be $2t$ and $3t$
Now, $2t+3t=\frac{35}{1}\Rightarrow5t=35\Rightarrow t=7$
and $2t\times3t=c\Rightarrow c=6\times49$
$\Rightarrow6K=6\times49\Rightarrow K=49$

186. **(b)** Let $\frac{x+a}{2x^2-3x+1}=y$, where $y\in R$
$\Rightarrow x+a=2yx^2-3yx+y$
$\Rightarrow 2yx^2-(3y+1)x+y-a=0$
$(3y+1)^2-4.2y(y-a)\geq0$ $(\because\ x\in R)$
$\Rightarrow y^2+(8a+b)y+1\geq0$
$1>0$ and $(8a+b)^2-4.1.1<0\Rightarrow(2a+1)(a+1)<0$
$\Rightarrow -1<a<-\frac{1}{2}$

187. **(d)** Given α, β, γ and δ be the roots of

$x^4 - x^3 - 8x^2 + 2x + 12 = 0$

So, such that $\alpha + \beta = 0$

$x^4 - x^3 - 8x^2 + 2x + 12 = (x^2 + a)(x^2 - x + b)$

$= x^4 - x^3 + (a + b)x^2 - ax + ab$

After equating co-efficient of x and constant term, we get

$-a = 2 \Rightarrow a = -2 \Rightarrow b = -6$

So, $x^4 - x^3 - 8x^2 + 2x + 12 = (x^2 - 2)(x^2 - x - 6)$

$= (x^2 - 2)(x - 3)(x + 2)$

hence, $\gamma = 3, \delta = -2 \Rightarrow 3\gamma + 2\delta = 9 - 4 = 5 \quad (\because \gamma > \delta)$

188. **(d)** $x^2 - x + 1 = 0$

$$\alpha^2 - \alpha + 1 = 0 \Rightarrow \alpha + \frac{1}{\alpha} = 1$$

$$\Rightarrow \alpha^2 + \frac{1}{\alpha^2} + 2 = 1 \Rightarrow \alpha^2 + \frac{1}{\alpha^2} = -1$$

$$\Rightarrow \left(\alpha^3 + \frac{1}{\alpha^3}\right) = -2 \Rightarrow \left(\alpha^2 + \frac{1}{\alpha^2}\right) = -1 \Rightarrow \alpha^4 + \frac{1}{\alpha^4} = -1$$

$$\left(\alpha + \frac{1}{\alpha}\right)^3 + \left(\alpha^2 + \frac{1}{\alpha^2}\right)^3 + \left(\alpha^3 + \frac{1}{\alpha^3}\right)^3 + \left(\alpha^4 + \frac{1}{\alpha^4}\right)^3 = -9.$$

189. **(a)** $\alpha + \beta = -a = \frac{1}{2} \Rightarrow a = \frac{-1}{2}$

$$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = \frac{1}{2}\left((\alpha + \beta)^2 - 3\alpha\beta\right)$$

$$\frac{1}{2}\left(\frac{1}{4} - 3b\right) = \frac{37}{8} \Rightarrow b = -3 \therefore a - \frac{1}{b} = -\frac{1}{2} - \left(\frac{-1}{3}\right) = \frac{-1}{6}.$$

190. **(c)** α, β, γ are the roots of the equation

$4x^3 - 3x^2 + 2x - 1 = 0$

$$\Rightarrow 4\alpha^3 = 3\alpha^2 - 2\alpha + 1 \Rightarrow 4\beta^3 = 3\beta^2 - 2\beta + 1$$

$$\Rightarrow 4\gamma^3 = 3\gamma^2 - 2\gamma + 1$$

$$4(\alpha^3 + \beta^3 + \gamma^3) = 3(\alpha^2 + \beta^2 + \gamma^2) - 2(\alpha + \beta + \gamma) + 3$$

$$= 3\left[(\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)\right] - 2 \times \frac{3}{4} + 3$$

$$= 3\left(\frac{9}{16} - 2 \times \frac{1}{2}\right) - \frac{3}{2} + 3 = \frac{3}{2} - \frac{21}{16} = \frac{3}{16}$$

$$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = \frac{3}{64}.$$

191. **(d)** $x^3 - 3x^2 + 3x + 7 = 0 \Rightarrow (x + 1)(x^2 - 4x + 7) = 0$

$\Rightarrow x = -1, 1 - 2\omega, 1 - 2\omega^2$

Equation of roots $\alpha - h, \beta - h, \gamma - h$ is

$(x + 1 + h)(x - 1 + 2\omega + h)(x - 1 + 2\omega^2 + h) = 0$

$\Rightarrow x^3 + [3h - 1 + 2\omega + 2\omega^2]x^2 + [4 + 4\omega^2 h + 4\omega h - 2h + 3h^2 - 1]x + \text{constant term} = 0$

x^2 and x terms are missing

$\Rightarrow 3h - 1 + 2\omega + 2\omega^2 = 0 \Rightarrow h = 1$

Now, $\frac{\alpha - h}{\beta - h} + \frac{\beta - h}{\gamma - h} + \frac{\gamma - h}{\alpha - h} = \frac{-2}{-2\omega} + \frac{-2\omega}{-2\omega^2} + \frac{-2\omega^2}{-2}$

$$= \frac{1}{\omega} + \frac{1}{\omega} + \omega^2 = 3\omega^2$$

192. **(b)** $x^2 - 2\sqrt{3}x + 4 = 0$

Roots α, β are $\frac{2\sqrt{3} \pm \sqrt{-4}}{2} = \sqrt{3} \pm i$

$\alpha = \sqrt{3} + i, \beta = \sqrt{3} - i$

$$\frac{2}{\sqrt{3}}\left|(\sqrt{3} + i)^{2024} - (\sqrt{3} - i)^{2024}\right|$$

$$= \frac{(2)^{2025}}{\sqrt{3}}\left|\cos\frac{\pi}{6} \times 2024 + i\sin\frac{\pi}{6} \times 2024 - \cos\frac{\pi}{6} \times 2024 + i\sin\frac{\pi}{6} \times 2024\right|$$

$$= \frac{(2)^{2025}}{\sqrt{3}}\left|\frac{-1}{2} + i\left(\frac{-\sqrt{3}}{2}\right) + \frac{1}{2} + i\left(\frac{-\sqrt{3}}{2}\right)\right|$$

$$= \frac{2^{2025}}{\sqrt{3}} \times \sqrt{3} = 2^{2025}.$$

193. **(d)** $12x^{\frac{1}{3}} - 25x^{\frac{1}{6}} + 12 = 0$. Let $x^{1/6} = t$

$12t^2 - 25t + 12 = 0$, On solving, $t = \frac{4}{3}, \frac{3}{4}$

$$\alpha^{1/6} = \frac{4}{3}, \beta^{1/6} = \frac{3}{4} \Rightarrow \alpha = \left(\frac{4}{3}\right)^6, \beta = \left(\frac{3}{4}\right)^6$$

$$\therefore \left(\frac{\alpha}{\beta}\right)^{1/6} = \frac{16}{9}.$$

194. **(b)** Here, $\frac{x - 1}{\sqrt{2x^2 - 5x + 2}} = \frac{41}{60}$ (On squaring)

$$\Rightarrow \frac{x^2 - 2x + 1}{2x^2 - 5x + 2} = \frac{1681}{3600} \Rightarrow (34x + 7x)(7x + 34) = 0$$

$$\Rightarrow x = \frac{-7}{34} \text{ and } x = \frac{-34}{7}$$

Hence, $\alpha = \frac{-7}{34} \quad \left[\because \frac{-1}{2} < \alpha < 0\right]$

195. **(d)** Given that α, β, γ are roots of $2x^3 - 5x^2 + 4x - 3 = 0$

$$\therefore \alpha + \beta + \gamma = \frac{5}{2}; \alpha\beta + \beta\gamma + \gamma\alpha = 2; \alpha\beta\gamma = \frac{3}{2}$$

$$\Rightarrow \gamma = \frac{5}{2} - (\alpha + \beta) \Rightarrow \alpha\beta\left[\frac{5}{2} - (\alpha + \beta)\right] = \frac{3}{2}$$

$$\Rightarrow \frac{5}{2}\alpha\beta - \alpha\beta(\alpha + \beta) = \frac{3}{2} \qquad \text{...(i)}$$

Similarly, $\frac{5}{2}\beta\gamma - \beta\gamma(\beta + \gamma) = \frac{3}{2}$ ...(ii)

$$\frac{5}{2}\gamma\alpha-\gamma\alpha(\alpha+\gamma)=\frac{3}{2} \qquad ...(iii)$$

Adding equations (i), (ii) and (iii), we get

$$\frac{5}{2}[\alpha\beta+\beta\gamma+\gamma\alpha]-[\alpha\beta(\beta+\gamma)+\beta\gamma(\beta+\gamma)+\gamma\alpha(\alpha+\gamma)]=\frac{9}{2}$$

$$\Rightarrow \frac{5}{2}\times 2-\Sigma\alpha\beta(\alpha+\beta)=\frac{9}{2} \;\therefore\; \Sigma\alpha\beta(\alpha+\beta)=5-\frac{9}{2}=\frac{1}{2}.$$

196. **(a)** Given that α, β, γ, 2 and ε are roots of the equation $x^5+4x^4-13x^3-52x^2+36x+144=0$

By hit and trial $x=-2$, x = 3 and x = 2 are its roots.

∴ $(x-2)$, $(x+2)$, $(x-3)$ are factors

∴ $(x-2)(x+2)(x-3)=x^3-3x^2-4x+12$

By long division.

$$x^5+4x^4+(-13x^3)-52x^3-52x^2+36x+144$$

$$=(x^3-3x^2-4x+12)\times(x^2+7x+12)$$

Now, $x^2+7x+12=0$

$\Rightarrow (x+3)(x+4)=0$

$\Rightarrow x=-3,-4$

Hence roots are –2, 2, 3, –3, –4

Given that $\alpha<\beta<\gamma<2<\varepsilon$

∴ $\alpha=-4$, $\beta=-3$, $\gamma=-2$, $\varepsilon=3$

Now, $\alpha+2\beta+3\gamma+5\varepsilon=-4-6-6+15=-1$.

197. **(a)** $D=0$

$$(2k+1)^2+4\times5\times3k=0 \;\Rightarrow\; 4k^2+64k+1=0$$

$$\Rightarrow k=\frac{-64\pm\sqrt{64^2-16}}{8}=\frac{-64\pm4\sqrt{255}}{8}$$

$$\Rightarrow k=\frac{-16+\sqrt{255}}{2}\in\left(\frac{-1}{2},0\right).$$

198. **(a)** α, β, γ are the roots of the equation $2x^3-3x^2+5x-7=0$

$$\Rightarrow \alpha+\beta+\gamma=\frac{3}{2} \;\Rightarrow\; \alpha\beta+\beta\gamma+\gamma\alpha=\frac{5}{2} \;\Rightarrow\; \alpha\beta\gamma=\frac{7}{2}$$

By using identity

$$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$$

$$\therefore (\alpha\beta+\beta\gamma+\gamma\alpha)^2=(\alpha\beta)^2+(\beta\gamma)^2+(\gamma\alpha)^2 +2\alpha\beta^2\gamma+2\beta\gamma^2\alpha+2\gamma\alpha^2\beta$$

$$\Rightarrow (\alpha\beta)^2+(\beta\gamma)^2+(\gamma\alpha)^2=(\alpha\beta+\beta\gamma+\gamma\alpha)^2 -2\alpha\beta\gamma(\alpha+\beta+\gamma)$$

$$\Rightarrow \sum\alpha^2\beta^2=\left(\frac{5}{2}\right)^2-2\times\frac{7}{2}\times\frac{3}{2}=\frac{-17}{4}$$

199. **(b)** $\alpha+\beta=0 \Rightarrow \beta=-\alpha$

$$(x-\alpha)(x+\alpha)(x-\gamma)(x-\delta)=0$$

$$\Rightarrow x^4-(\delta+\gamma)x^3+(\gamma\delta-\alpha^2)x^2 +\alpha^2(\delta+\gamma)x-\alpha^2\gamma\delta=0$$

Comparing the coefficients, $\delta+\gamma=1$

$$\Rightarrow \alpha^2(\delta+\gamma)=4 \Rightarrow \alpha^2=4 \Rightarrow \alpha=2,\ \beta=-2$$

$$\Rightarrow \gamma\delta-\alpha=-16 \Rightarrow \gamma\delta=-12$$

$$\Rightarrow \gamma^2+\delta^2+2\gamma\delta=1$$

$$\Rightarrow \gamma^4+\delta^4+2\gamma^2\delta^2=625$$

$$\Rightarrow \delta^4+\gamma^4=625-288=337$$

$$\Rightarrow \alpha^4=16,\ \beta^4=16$$

$$\Rightarrow \alpha^4+\beta^4+\delta^4+\gamma^4=337+32=369.$$

200. **(b)** Consider $x^3-6x^2+11x-6=0$ ...(i)

Since x = 1, satisfies the above equation.

Hence (x – 1) will be a factor of equation (i)

$\Rightarrow (x^2-5x+6)(x-1)=0$

$\Rightarrow (x-3)(x-2)(x-1)=0 \Rightarrow x=1, 2, 3$

Since 2 – 1 = 1 = 3 – 2

Hence roots of equation is in Arithmetic progression.

201. **(a)** Given,

$$\frac{k}{(kx+3)}+\frac{3}{(3x-k)}=\frac{12x+5}{(kx+3)(3x-k)}$$

$\Rightarrow 6xk-k^2+9=12x+5$

Comparing both side, we get

$\Rightarrow 6k=12 \Rightarrow k=2$

Hence, $kx^2-7x+3=0$

$\Rightarrow 2x^2-7x+3=0$

$\Rightarrow (2x-1)(x-3)=0$

$\Rightarrow x=\frac{1}{2},3$

202. **(a)** $16x^2-10x+1=0$

$\Rightarrow 16x^2-8x-2x+1=0$

$\Rightarrow (2x-1)(8x-1)=0$

$\Rightarrow x=\frac{1}{2},\frac{1}{8}$

$$\left(\left(\frac{1}{2}\right)^4+\left(\frac{1}{8}\right)^4\right)=\frac{1}{16}+\frac{1}{4096}=\frac{257}{4096}$$

203. **(c)** Let α, β, γ be the root of $x^3-3x^2+2x-1=0$

Then, $\alpha+\beta+\gamma=3$

Roots of $x^3-x-1=0$ is : $\alpha-k$, $\beta-k$, $\gamma-k$

Then, $(\alpha-k)+(\beta-k)+(\gamma-k)=0$

$\Rightarrow \alpha+\beta+\gamma-3k=0$

$\Rightarrow 3-3k=0 \Rightarrow k=1$

204. **(c)** Since α, β, γ are the roots of the equation $x^3-ax^2+bx-c=0$

∴ $\alpha+\beta+\gamma=a$

$\alpha\beta+\beta\gamma+\gamma\alpha=b$

$\alpha\beta\gamma=c$

$$\text{Now, } \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{\beta^2\gamma^2+\alpha^2\gamma^2+\alpha^2\beta^2}{\alpha^2\beta^2\gamma^2}$$

$= \dfrac{(\alpha\beta+\beta\gamma+\gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha+\beta+\gamma)}{(\alpha\beta\gamma)^2}$

$= \dfrac{b^2 - 2\times c\times a}{c^2}$

205. (b) Given the equation $y^2 + y + 1 = 0$

Now, $a^4 + b^4 + a^{-1}b^{-1} = a^4 + b^4 + \dfrac{1}{ab}$

$= (a^2 + b^2)^2 - 2a^2b^2 + \dfrac{1}{1}$

$= ((a + b)^2 - 2ab)^2 - 2 \times 1 + 1$

$= ((-1)^2 - 2)^2 - 1 = 1 - 1 = 0$

206. (d) Given that c and d are the roots of $x^2 + ax + b = 0$

$\Rightarrow c + d = -a$ & $cd = b$ ... (i)

Now, $x^2 + (4c + a)x + (b + 2ac + 4c^2) = 0$

$\Rightarrow x^2 (4c - c - d)x + (b - 2c(c + d) + 4c^2) = 0$

$\Rightarrow (x + c)(x + 2c - d) = 0$

$\Rightarrow x = d - 2c.$

207. (b) The given equation is

$16x^3 - 44x^2 + 36x - 9 = 0$

Let α, β and γ are in H.P.

$\dfrac{2}{\beta} = \dfrac{1}{\alpha} + \dfrac{1}{\gamma}$

$\Rightarrow \dfrac{3}{\beta} = \dfrac{1}{\alpha} + \dfrac{1}{\gamma} + \dfrac{1}{\beta} \Rightarrow \dfrac{3}{\beta} = \dfrac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\gamma\beta}$

$\Rightarrow \beta = \dfrac{3}{4}$

Now, $\alpha+\beta+\gamma = \dfrac{11}{4} \Rightarrow \alpha+\gamma = \dfrac{11}{4} - \dfrac{3}{4} = 2$ ...(i)

And $\alpha\beta\gamma = \dfrac{9}{16} \Rightarrow \alpha\gamma = \dfrac{9}{16}\times\dfrac{4}{3} = \dfrac{3}{4}$ ...(ii)

Solving eqn. (i) and (ii) :

$\alpha = \dfrac{3}{2}, \gamma = \dfrac{1}{2}$

∴ Greatest root is $\alpha = \dfrac{3}{2}.$

208. (b) Given $x + y = \dfrac{\pi}{6} \Rightarrow \cot(x+y) = \cot\left(\dfrac{\pi}{6}\right)$

$\Rightarrow \dfrac{\cot x \cot y - 1}{\cot x + \cot y} = \sqrt{3} \Rightarrow \cot x + \cot y = \dfrac{1}{\sqrt{3}}(a-1)$

So, quadratic equation whose roots are $\cot x, \cot y$ is

$t^2 - (\cot x + \cot y)t + \cot x \cdot \cot y = 0$

$\Rightarrow t^2 - \dfrac{1}{\sqrt{3}}(a-1)t + a = 0 \Rightarrow \sqrt{3}t^2 + (1-a)t + \sqrt{3}a = 0$

209. (d) Let α, α, α, β be the roots of given equation

$x^4 + ax^3 + bx^2 + cx + d = 0$

Hence $3\alpha + \beta = -a$, $3\alpha(\alpha + \beta) = b$

$\alpha^2(\alpha + 3\beta) = -c$, $\alpha^3\beta = d$

Now consider $6c - ab = \alpha(3\alpha^2 - 6\alpha\beta + 3\beta^2)$ ...(i)

also consider $3a^2 - 8b = (3\alpha^2 - 6\alpha\beta + 3\beta^2)$ ...(ii)

dividing equation (i) by equation (ii),

$\Rightarrow \alpha = \dfrac{6c - ab}{3a^2 - 8b}$

210. (d) $f(x)^{g(x)} = 1$ is possible when

Case 1: $f(x) = 1$

$x^2 - 7x + 11 = 1$

$x = 2, 5$

Case 2: $g(x) = 0$

$x^2 - 6x - 7 = 0$

$x = -1, 7$

Case 3: $f(x) = -1$ and $g(x) =$ even

$x^2 - 7x + 11 = -1$

$x = 3, 4$

$g(3) = 3^2 - 6(3) - 7 = -16$ (even)

$g(4) = 4^2 - 6(4) - 7 = -15$ (odd)

∴ possible values of $x = -1, 2, 3, 5, 7$

sum = 16

211. (d) For any quadratic expression P(x)

$P(x) > 0 \ \forall x R \Rightarrow D < 0, a > 0$

$D = (2p)^2 - 4(8 - 2p) < 0$

$4p^2 - 32 + 8p < 0$; $(p + 4)(p - 2) < 0 \therefore p \in (-4, 2)$

212. (a) $x^4 + x^2 + 1 = 0$

$(x^2 + x + 1)(x^2 - x + 1) = 0$

$x^2 + x + 1 = 0$ have roots ω, ω^2

$x^2 - x + 1 = 0$ have roots $-\omega, -\omega^2$

$\omega + \omega^2 = -1 \Rightarrow \alpha = \omega, \beta = \omega^2$

$-\omega - \omega^2 = 1 \Rightarrow \gamma = -\omega, \delta = -\omega^2$

$\therefore \quad \alpha^{2023} + \beta^{2023} + \gamma^{2022} + \delta^{2022}$

$= (\omega)^{2023} + (\omega^2)^{2023} + (-\omega)^{2022} + (-\omega^2)^{2022}$

$= \omega + \omega^2 + 1 + 1 = 1.$

213. (d) $x^5 - 6x^4 + 11x^3 - 2x^2 - 12x + 8 = 0$

By observation, $x = \pm 1$ is a root

$\therefore \quad x^5 - 6x^4 + 11x^3 - 2x^2 - 12x + 8$

$= (x + 1)(x - 1)(x - 2)^3$

$\therefore \quad \alpha = 2$

$\Rightarrow 3\alpha^2 - 2\alpha + 1 = 3(2)^2 - 2(2) + 1 = 9.$

214. (b) $x^3 - 3x^2 + 3x + 1 = 0$

$\alpha+\beta+\gamma = 3$; $\alpha\beta+\beta\gamma+\gamma\alpha = 3$; $\alpha\beta\gamma = -1$

$\Rightarrow \quad \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$

$= (\alpha\beta+\beta\gamma+\gamma\alpha)^2 - 2(\alpha\gamma\beta^2 + \beta\alpha^2\gamma + \alpha\beta\gamma^2)$

$= (\alpha\beta+\beta\gamma+\gamma\alpha)^2 - 2(\alpha\beta\gamma)(\beta+\alpha+\gamma)$

$= (3)^2 - 2(-1)(3) = 15$

215. **(b)** $x^2 + 2x + 2 = 0$

$$x = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm 2i}{2}$$

$x = -1 \pm i,\ \alpha = -1 + i,\ \beta = -1 - i$

$$\alpha^{15} + \beta^{15} = (-1+i)^{15} + (-1-i)^{15}$$

$$= (\sqrt{2})^{15}\left[\left(\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{15} - \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{15}\right]$$

$$= (\sqrt{2})^{15}\left[\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)^{15} - \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{15}\right]$$

$$= (\sqrt{2})^{15}\left[e^{i\left(\frac{5\pi}{4}\right)} - e^{i\left(\frac{-\pi}{4}\right)}\right]$$

$$= (\sqrt{2})^{15}\left[\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} - \cos\left(\frac{-\pi}{4}\right) - i\sin\left(\frac{-\pi}{4}\right)\right]$$

$$= (\sqrt{2})^{15}\left[-\cos\frac{\pi}{4} - \cos\frac{\pi}{4}\right] = (\sqrt{2})^{15}(-\sqrt{2}) = -2^8.$$

216. **(b)** $4x^2 - 2x + K - 4 = 0$; Roots are α and $\frac{1}{\alpha}$

$$\therefore \quad \alpha \cdot \frac{1}{\alpha} = \frac{K-4}{4} \Rightarrow 1 = \frac{K-4}{4} \Rightarrow K = 8.$$

217. **(b)**

$$\begin{array}{r|l} & \qquad\qquad ax - ap \\ \hline x^2 + px + 1 & ax^3 + bx + c \\ & ax^3 + apx^2 + ax \\ & (-)\quad (-)\qquad (-) \\ \hline & -apx^2 + (b-a)x + c \\ & -apx^2 - ap^2x - ap \\ & (+)\qquad (+)\qquad (+) \\ \hline & (b - a + ap^2)\,x + ap + c \end{array}$$

Since $x^2 + px + 1$ is factor of $ax^3 + bx + c$

$\therefore ap + c = 0 \Rightarrow p = -\frac{c}{a}$ ....(i)

and $b - a + ap^2 = 0$

$\Rightarrow b - a + a\frac{c^2}{a^2} = 0$ [from (i)]

$\Rightarrow ab - a^2 + c^2 = 0 \Rightarrow ab = a^2 - c^2$

218. **(d)** Let α and β are roots of quadratic equation

$\therefore\ \alpha + \beta = 11$ and $\alpha^2 + \beta^2 = 61$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 61$

$\Rightarrow 121 - 2\alpha\beta = 61 \Rightarrow \alpha\beta = 30$

$\therefore$ Quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - 11x + 30 = 0$

219. **(d)** $x^4 - 2x^3 + x - 380 = 0$

Put x = 5

$5^4 - 2.(5)^3 + 5 - 380 \Rightarrow 0 = 0$

So, x = 5 one real root

Put n = –4

$(-4)^4 - 2(-4)^3 + (-4) - 380 = 0 \Rightarrow 0 = 0$

So, n = –4 is second real roots.

Let α and β are two other roots...

$$\therefore \alpha + \beta + 5 - 4 = -\frac{(-2)}{1} \Rightarrow \alpha + \beta = 1$$

220. **(d)** Given that $3 + i\sqrt{6}$ is one roots therefore $3 - i\sqrt{6}$ is also a root .Let α and β are other two real roots

$\therefore$ product of roots = $\frac{-45}{4}$

$$\Rightarrow \alpha.\beta(3 + i\sqrt{6})(3 - i\sqrt{6}) = \frac{-45}{4}$$

$$\Rightarrow \alpha.\beta(9 + 6) = \frac{-45}{4} \Rightarrow \alpha.\beta = \frac{-3}{4}$$

221. **(c)** Given function is $f(x) = 2x + 3$

then, $f(x^2) - 2f\left(\frac{x}{2}\right) - 1 = 0$

$2x^2 + 3 - 2\left(2 \times \frac{x}{2} + 3\right) - 1 = 0$

$\Rightarrow 2x^2 + 3 - 2x - 6 - 1 = 0 \Rightarrow 2x^2 - 2x - 4 = 0$

$\Rightarrow x + 1 = 0 \Rightarrow x = -1 \Rightarrow x - 2 = 0 \Rightarrow x = 2$

So, $\alpha^2 + \beta^2 = (-1)^2 + (2)^2 = 1 + 4 = 5$

222. **(a)** $\alpha + \beta = \frac{-b}{a}$; $\alpha\beta = \frac{c}{a}$

So, $\sqrt{5}(\alpha + \beta) = \frac{-\sqrt{5}b}{a}$; $5\alpha\beta = \frac{5c}{a}$

Required equation is $x^2 - 5x + P = 0$

$\Rightarrow ax^2 + \sqrt{5b} + 5c = 0$

223. **(c)** Given polynomial $f(x) = x^3 + ax^2 + bx + c$ with roots 2, 3, 6.

Let the roots of the cubic equation be α, β, y.

Then, $\alpha = 2$, $\beta = 3$, y = 6.

$\alpha + \beta + y = \frac{-b}{a} = -a \Rightarrow a = -11$

$\alpha\beta y = -c \Rightarrow 2 \times 3 \times 6 = -c \Rightarrow c = -36$

224. **(a)** Given $2\cot^2\theta - \cot\theta - 3$

$\Rightarrow 2\cot^2\theta - 3\cot\theta + 2\cot\theta - 3$

$\Rightarrow (2\cot\theta - 3)(\cot\theta + 1)$

225. **(c)** Given equation is $x^{11} - x^7 + x^4 - 1 = 0$

$= (x^4 - 1)(x^7 + 1) = 0$

Case (i): $x^4 - 1 = 0$ or $x^4 = 1$

$\Rightarrow \quad x^4 = (\cos 0^\circ + i\sin 0^\circ)$

$x = (\cos 2k\pi + i\sin 2k\pi)^{1/4}$

k = 0, 1, 2, 3

Case (ii): $x^7 + 1 = 0$

$$\Rightarrow x = \text{cis}\ \frac{(2k+1)\pi}{7}$$

Putting k = 0, 1, 2, 3, 4, 5, 6

Now values of x are

$$\text{cis}\,\frac{\pi}{7},\ \text{cis}\,\frac{3\pi}{7},\ \text{cis}\,\frac{5\pi}{7},\ \text{cis}\,\frac{9\pi}{7},\ \text{cis}\,\frac{11\pi}{7},\ \text{cis}\,\frac{13\pi}{7}$$

226. **(b)** We have $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots\ldots + a_1 x' + a_0$
Here polynomial f(x) is a monic polynomial so, leading coefficient should be 1. $\Rightarrow a_n = 1$

227. **(c)** We have $x + y + z = 12$ ...(i)
$x^2+y^2+z^2 = 50$ ....(ii)
and $x^3 + y^3 + z^3 = 216$ ....(iii)
$\because (x + y + z)^2 - (x^2 + y^2 + z^2) = 2(xy + yz + zx)$
$\Rightarrow xy + yz + zx = 47$
now $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
$\Rightarrow xyz = 60$
Now from equation (iii) $y^3 + z^3 = 216 - x^3$
$\Rightarrow (12 - x)\{(50 - x^2) - [47 - 12x + x^2]\} = 216 - x^3$
On solving, we get
$x^3 - 12x^2 + 47x - 60 = 0$ ...(iv)
by hit and trial
x = 3 is a root of equation (iv)
$\Rightarrow (x - 3)(x^2 - 9x + 20) = 0 \Rightarrow (x - 3)(x - 4)(x - 5) = 0$
$\Rightarrow x = 3, 4, 5$
Now no of solution are 3! =6

228. **(c)** $\displaystyle\sum_{k=1}^{6} -i\left[\cos\frac{2\pi k}{7} + i\sin = \frac{2\pi k}{7}\right]$

$\therefore \cos\theta + i\sin\theta = e^{i\theta}$

$$(-i)\sum_{k=1}^{6} e^{i\frac{2\pi k}{7}}$$

$$= -i\left[\frac{e^{i\frac{2\pi}{7}} - e^{i2\pi}}{1 - e^{i\frac{2\pi}{7}}}\right]\left\{\begin{aligned}&\because e^{i2\pi}\\ &= \cos 2\pi + i\sin 2\pi\\ &= 1\end{aligned}\right\}$$

$$= -i\left[\frac{e^{i\frac{2\pi}{7}} - 1}{1 - e^{i\frac{2\pi}{7}}}\right] = -i(-1) = i$$

229. **(a)** $(x - iy)^{1/3} = a - ib$
$(x - iy) = (a - ib)^3$
$x - iy = (a^3 - 3ab^2) + ib\,(b^2 - 3a^2)$
$x = a^3 - 3ab^2$

$\dfrac{x}{a} = a^2 - 3b^2$ ...(i)

$y = b\,(3a^2 - b^2)$

$\dfrac{y}{2b} = \dfrac{3a^2 - b^2}{2}$ ...(ii)

(i) + (ii)

$$\frac{x}{2a} + \frac{y}{2b} = \frac{a^2 - 3b^2 + 3a^2 - b^2}{2} = \frac{4a^2 - 4b^2}{2} = 2\left(a^2 - b^2\right)$$

230. **(c)** If $x = -5 + 2\sqrt{-4} \Rightarrow x + 5 = 4i$
$(x + 5)^2 = (4i)^2 \Rightarrow x^2 + 10x + 41 = 0$
Let $x^4 + 9x^3 + 35x^2 - x + 4 = (x^2 + 10x + 41)\,(p(x)) + R$
$\because x^2 + 10x + 41 = 0$
$\therefore x^2 + 9x^3 + 35x^2 - x + 4 = R$
i.e. Remainder when $x^4 + 9x^3 + 35x^2 - x + 4$ is divided by $x^2 + 10x + 41$
$\Rightarrow R = -160 \Rightarrow x^4 + 9x^3 + 35x^2 - x + 4 = -160$

231. **(d)** $x^2 - 10x - 8 = 0 \Rightarrow x^2 - 8 = 10x$
$\because \alpha, \beta$ are roots then
$\alpha^2 - 8 = 10\alpha$ ...(i)
$\beta^2 - 8 = 10\beta$ ...(ii)

$$\frac{a_{10} - 8a_8}{5a_9} = \frac{\alpha^{10} - \beta^{10} - 8\left(\alpha^8 - \beta^8\right)}{5\left(\alpha^9 - \beta^9\right)} \quad \text{[From (i) and (ii)]}$$

$$= \frac{\alpha^8 \cdot 10\alpha - \beta^8 \cdot 10\beta}{5\left(\alpha^9 - \beta^9\right)} = \frac{10\left(\alpha^9 - \beta^9\right)}{5\left(\alpha^9 - \beta^9\right)} = 2$$

232. **(b)** $x^2 + (2m + 1)x + m = 0$
For equal roots, D = 0
$(2m + 1)^2 - 4m = 0 \Rightarrow 4m^2 + 1 + 4m - 4m = 0$
$4m^2 + 1 = 0 \Rightarrow m^2 = -1/4 \ \therefore m^2 > 0$
$\therefore$ No real values of m exist.

233. **(a)** Let $|x - 2| = y \Rightarrow y^2 + y - 2 = 0$
$(y + 2)\,(y - 1) = 0 \Rightarrow y = -2, 1$
$\therefore |x - 2| = -2$ or $|x - 2| = 1$
not possible $\quad\therefore |x - 2| = 1$
$x - 2 = \pm 1 \Rightarrow x = 2 \pm 1$
$\therefore x = 1, 3 \ \therefore$ sum of real roots $= 1 + 3 = 4$

234. **(c)** Difference of roots
$= \sqrt{(\text{sum of roots}) - 4(\text{Products of roots})}$
$\therefore$ Equating for both equations,
$\sqrt{(-a)^2 - 4b} = \sqrt{(-b)^2 - 4a}$
Squaring both sides,
$(-a)^2 - 4b = (-b)^2 - 4a \Rightarrow a^2 - b^2 = 4b - 4a$
$(a + b)\,(a - b) = 4(b - a)\ (a \neq b)$
$\therefore a + b = -4 \Rightarrow a + b + 4 = 0$

235. **(b)** $(x + a)\,(x + 1991) + 1 = (x + b)\,(x + c)$
$x^2 + (1991 + a)\,x + (1991a + 1) = x^2 + (b + c)\,x + bx$
Consider quadratic equations
$(x + a)\,(x + 1991) = -1$
It is possible when
$(x + a = 1$ and $x + 1991 = -1)$ ...(i)
or $(x + a = -1)$ and $(x + 1991 = 1)$ ...(ii)
From (i)
$x + 1991 = -1 \Rightarrow x = -1992$
$x + a = +1 \Rightarrow a = +1 - x = +1 + 1992 = 1993$
From (ii)
$x + 1991 = 1 \Rightarrow x = -1990$
and $x + a = -1 \Rightarrow a = -1 - x = -1 + 1990$
$\therefore a = 1989$

236. **(a)** $|1 - i|\sqrt{(1)^2 + (-1)^2} = \sqrt{2}$
$\therefore |1 - i|^x = 2^x \Rightarrow \left(\sqrt{2}\right)^x = 2^x$

$\Rightarrow 2^{x/2} = 2^x \ \therefore \dfrac{x}{2} = x$
only possible if $x = 0$
$\therefore$ Number of integer solution = 1

237. **(c)** $3^{x+1} + 3^{-x+1} = 10 \Rightarrow 3^x \cdot 3 + 3^{-x} \cdot 3 = 10$

$$3^x \cdot 3 + \frac{3}{3^x} = 10$$

Let $3^x = y \Rightarrow 3y + \frac{3}{y} = 10$

$\Rightarrow (y-3)(3y-1) = 0 \Rightarrow y = \frac{1}{3}, 3$

$\therefore 3^x = \frac{1}{3}$ or $3^x = 3$, $3^x = 3^{-1}$ or $3^x = 3^1$

$\therefore x = -1$ or 1

$\therefore$ for positive values, there is only one solution $x = 1$

238. **(b)** Let $\sqrt{\frac{x}{1-x}} = y \Rightarrow y + \frac{1}{y} = \frac{13}{6}$

$\Rightarrow \frac{y^2+1}{y} = \frac{13}{6} \Rightarrow 6y^2 - 13y + 6 = 0$

$\Rightarrow 3y(2y-3) - 2(2y-3) = 0$

$\Rightarrow y = \frac{3}{2}, \frac{2}{3}; \sqrt{\frac{x}{1-x}} = \frac{3}{2}$ or $\sqrt{\frac{x}{1-x}} = \frac{2}{3}$

$x = \frac{9}{13}$ or $x = \frac{4}{13}$

$\therefore$ There are 2 real roots

239. **(d)** $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

$4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}} \Rightarrow 4^x + \frac{4^x}{2} = 3^x\left(\sqrt{3} + \frac{1}{3}\right)$

$\frac{3}{2} 4^x = 3^x \cdot \frac{4}{\sqrt{3}}$

$\left(\frac{3}{4}\right)^x = \frac{3\sqrt{3}}{8} = \frac{3^{3/2}}{4^{3/2}} \Rightarrow \left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^{3/2} \quad \therefore x = \frac{3}{2}$

240. **(b)** Here $\alpha + \beta + \gamma = 7p$. Now $\gamma = 7p$

$\alpha\beta + \beta\gamma + \gamma\alpha = 5q$

$\Rightarrow \alpha\beta + \gamma(\alpha + \beta) = 5q$

$\Rightarrow \alpha\beta = 5q$ & $\alpha\beta\gamma = 6r \Rightarrow 5q \times 7p = 6r$

$\Rightarrow 5q = \frac{6r}{7P}$

241. **(c)** Here we are given that $3p^3x^3 + px^2 + qx + 3 = 0$ when $p = 1$ and $q = -7$

Now $(x-1)(3x^2 + 4x - 3) = 0$

$\Rightarrow x = 1, x = \frac{2 \pm \sqrt{13}}{3}$

$\Rightarrow \alpha = \frac{2 \pm \sqrt{13}}{3}, \quad \beta = \frac{2 - \sqrt{13}}{3}$

$\Rightarrow |\alpha - \beta| = \left|\frac{2 + \sqrt{13} - 2 + \sqrt{13}}{3}\right| = \frac{2\sqrt{13}}{3}$

242. **(b)** Given quadratic equation is $x^2 + bx + c = 0$

Here, α, β are roots of equation.

$\alpha + \beta = -b$ ...(i)

Take square both sides, $\alpha^2 + \beta^2 + 2\alpha\beta = b^2$

$\alpha\beta = \frac{b^2 - 5}{2}$ ...(ii)

From (i) and take cube both sides,

$\alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta) = -b^3$

Here, $\alpha^3 + \beta^3 = 9$, $\alpha + \beta = -b$, $\alpha.\beta = \frac{b^2 - 5}{2}$

$9 + 3\left(\frac{b^2 - 5}{2}\right)(-b) = -b^3$

$\Rightarrow b^3 - 15b - 18 = 0$

Now, put $b = -3$ in above equation.

$\Rightarrow (-3)^3 + 45 - 18 = 0 \Rightarrow 45 - 45 = 0 \Rightarrow 0 = 0$

So, $(b + 3) = 0$ divides the above equation then, $b = -3$

From the given equation $x^2 + bx + c = 0$

$\Rightarrow \alpha\beta = c$

From (ii), $\alpha\beta = \frac{b^2 - 5}{2} = c$

Put $b = -3$ in above expression,

$\Rightarrow c = 2$

So, $b + c = -3 + 2 = -1$.

243. **(c)** Given equation is $x^3 - 9x^2 + 23x - 15 = 0$ with α, β and γ roots of equation.

$\alpha + \beta + \gamma = 9$ ...(i)

$\alpha\beta + \beta\gamma + \gamma\alpha = 23$ ...(ii)

$\alpha\beta\gamma = -(-15) = 15$ ...(iii)

Take square both sides to eq. (i),

$\alpha^2 + \beta^2 + \gamma^2 + 2(23) = 81$

$\alpha^2 + \beta^2 + \gamma^2 = 35$ ...(iv)

Now, $\alpha^3 + \beta^3 + \gamma^3 = (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha) + 3\alpha\beta\gamma$

$\alpha^3 + \beta^3 + \gamma^3 = (9)(35 - 23) + 3 \times 15$

$= 9 \times (12) + 45$

$= 108 + 45 = 153$

244. **(d)** Given equation $x^3 - 9x^2 + k = 0$ where $k \in R - \{0\}$.

$\alpha + \beta + 2\beta = -(-9) = 9$ ...(i)

$\alpha\beta + \beta(2\beta) + 2\beta(\alpha) = 0 \Rightarrow \alpha\beta + 2\beta^2 + 2\beta\alpha = 0$

$\Rightarrow 2\beta^2 + 3\beta\alpha = 0$ ...(ii)

$\Rightarrow 2\alpha\beta^2 = k$ ...(iii)

From (i), $\alpha + 3\beta = 9$

From (ii), $2\beta^2 + 3\alpha\beta = 0 \Rightarrow \beta(2\beta + 3\alpha) = 0$

$\beta = 0, 2\beta + 3\alpha = 0 \Rightarrow \alpha = \frac{-2\beta}{3}$

Put value of α in equation $\alpha + 3\beta = 9$.

$\Rightarrow \frac{-2\beta}{3} + 3\beta = 9 \Rightarrow \frac{-2\beta + 9\beta}{3} = 9$

$\Rightarrow \beta = \frac{27}{7}$ Now, $14\beta = 14 \times \frac{27}{7} = 54$

245. **(b)** Given $x^5 - 3x^4 + 5x^3 - 5x^2 + 3x - 1 = 0 = p(x)$ (say)

since $P(1) = 1 - 3 + 5 - 5 + 3 - 1 = 0$

so, $(x - 1)$ is the factor of given equation.

$\therefore \quad x^5 - 3x^4 + 5x^3 - 5x^2 + 3x - 1$

$= (x-1)(x^4 - 2x^3 + 3x^2 - 2x + 1)$

Now $x^4 - 2x^3 + 3x^2 - 2x + 1$

$= (x^4 + 1) - 2(x^3 + x) + 3x^2 = 0$

$(x^2 + x^{-2}) - 2(x + x^{-1}) + 3 = 0$

Let x + x–1 = t

take square both sides,

$$x^2 + \frac{1}{x^2} + 2 = t^2 \Rightarrow (x^2 + x^{-2}) = t^2 - 2.$$

Now, $(t^2 - 2) - 2t + 3 = 0 \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow (t - 1)^2 = 0$

$\Rightarrow \quad t = 1 \Rightarrow x^2 - x + 1 = 0$

so, $x = \frac{1+\sqrt{3}i}{2}, \frac{1-\sqrt{3}i}{2}$

Required sum of roots $= 1 + \frac{1+\sqrt{3}i}{2} + \frac{1-\sqrt{3}i}{2} = 2$

246. **(d)** Given equation $x^2 - 2\sqrt{3}x + 4 = 0$ with α & β roots of the equation.

$$\alpha + \beta = \frac{-b}{a} = 2\sqrt{3} \quad \text{...(i)}$$

$$\alpha\beta = \frac{c}{a} = 4 \quad \text{...(ii)}$$

Now, $(\alpha^6 + \beta^6) = (\alpha^2)^3 + (\beta^2)^3$

$= (\alpha^2 + \beta^2)\,[(\alpha^2 + \beta^2)^2 - 3\alpha^2\beta^2]$ ...(iii)

From (i), take square both sides,

$\alpha^2 + \beta^2 + 2\alpha\beta = 12$

$\alpha^2 + \beta^2 = 12 - 2 \times 4 = 12 - 8 = 4$

Put the values in eq. (iii),

$\alpha^6 + \beta^6 = (4)\,[(4)^2 - 3(4)^2] = 4[16 - 48]$

$= 4 \times (-32) = -128$

247. **(a)** Given quadratic equation $x^2 + bx + c = 0$ with $b = 17$

Then, $x^2 + 17x + c = 0$ and roots are –2 & –15.

$$\alpha + \beta = \frac{-b}{a} = -17\,;\ \alpha\beta = \frac{c}{a} \Rightarrow c = 30.$$

Quadratic equation is $x^2 + 17x + 30 = 0$.

Put $b = 13$ and keep fixed all other coefficients, then quadratic equation becomes $x^2 + 13x + 30 = 0$.

$\Rightarrow x = -10,\ -3$

So, $\alpha = -10, \beta = -3;\ |\alpha - \beta| = |-10 + 3| = |-7| = 7$

248. **(a)** Given quadratic equation $5x^3 - 2x - 4 = 0$.

Here, $a = 5$, $b = 0$, $c = -2$, $d = -4$ and α, β, γ are roots of equation.

$$\alpha + \beta + \gamma = \frac{-b}{a} = 0\,;\ \alpha\beta\gamma = \frac{-d}{a} = \frac{-(-4)}{5} = \frac{4}{5}$$

$$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = \frac{12}{5}$$

249. **(d)** Given equation is $x^5 - ax^4 + bx^3 - cx^2 + dx - 1 = 0$.

Here, geometrical mean and arithmetic mean are equal then all the roots are equal.

Let α is the root of the equation.

$$\frac{\alpha + \alpha + \alpha + \alpha + \alpha}{5} = \left(\alpha^5\right)^{1/5}$$

Here, sum of all roots is a then $\frac{a}{5} = \alpha \Rightarrow a = 5\alpha$ ...(i)

$$\left(\alpha\ \alpha\ \alpha\ \alpha\ \alpha\right)^{1/5} = \frac{-e}{a} = -(-1) = 1 \Rightarrow \alpha^{5 \times \frac{1}{5}} = 1 \Rightarrow \alpha = 1$$

From (i), a = 5.

Now, $\alpha\alpha + \alpha\alpha + \alpha\alpha + \ldots\ldots + \alpha\alpha$ (10 times) $= b$

$10\alpha^2 = b \Rightarrow b = 10$

$\alpha\alpha\alpha + \alpha\alpha\alpha + \ldots\ldots + 10$ times $= c$

$10\alpha^3 = c \Rightarrow c = 10$

$\alpha\alpha\alpha\alpha + \alpha\alpha\alpha\alpha + \ldots\ldots\ldots\ldots + 5$ times $= d$

$5\alpha^4 = d \Rightarrow d = 5$

Therefore, $a + d + c + b = 5 + 5 + 10 + 10 = 30$.

250. **(a)** Given roots are $\left(\sqrt{3} + \sqrt{2}i\right)$ and $\left(\sqrt{3} - \sqrt{2}\right)$.

The other roots are $\left(\sqrt{3} - \sqrt{2}i\right)$ and $\left(\sqrt{3} + \sqrt{2}\right)$.

$$\left(x - \left(\sqrt{3} + \sqrt{2}i\right)\right)\left(x - \left(\sqrt{3} - \sqrt{2}i\right)\right)\left(x - \left(\sqrt{3} - \sqrt{2}\right)\right)$$
$$\left(x - \left(\sqrt{3} + \sqrt{2}\right)\right) = 0$$

$$(x^2 - 2\sqrt{3}x + 5)\,(x^2 - 2\sqrt{3}x + 1) = 0$$

Put $x^2 = 2\sqrt{3}x + 5 = 0 \Rightarrow x^2 + 5 = 2\sqrt{3}x$

Take square both sides.

$x^4 + 25 + 10x^2 = 12x^2 \Rightarrow (x^4 - 2x^2 + 25) = 0$

$\Rightarrow (x^4 - 10x^2 + 1)\,(x^4 - 2x^2 + 25) = 0.$

251. **(a)** Given equation is $x^{10} - 3x^8 + 5x^6 - 5x^4 + 3x^2 - 1 = 0$

Let $f(x) = x^{10} - 3x^8 + 5x^6 - 5x^4 + 3x^2 - 1$.

It has a even degree 10 then the graph of the given function would be symmetrical about y-axis.

Put $x = 0, 1, -1$ in $f(x)$.

$f(0) = -1, f(1) = 0, f(-1) = 0$

So, there are only two real values which forms the symmetrical path of given function.

Non-real roots = Total roots – Real roots = 10 – 2 = 8.

Therefore, there are 8 non-real roots of the given equation.

252. **(d)** Now from given quadratic eqn. we have

$x^2 - 7x + 3c = 0$ ...(i)

$x^2 + x - 5c = 0$ ...(ii)

Now from eq (i), $c = \frac{x^2 - 7x}{-3}$ ...(iii)

and from eqn (ii), $c = \frac{x^2 + x}{5}$ ...(iv)

$\Rightarrow$ x = 0 or 4 (from eqn (iii) and (iv)

If $x = 0 \Rightarrow c = 0$ (not possible)

$\therefore$ we consider $x = 4 \Rightarrow c = +4$

$\therefore$ expression is: $x^2 - 3x + 4$

$$\Rightarrow \quad y = x^2 - 3x + 4 \Rightarrow y = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$$

$\Rightarrow$ Expression is +ve $\forall\ x \in |R$

253. **(c)** $\because\ (\alpha + \beta + \gamma)^3 = (\alpha^3 + \beta^3 + \gamma^3) + 3(\alpha + \beta)\,(\beta + \gamma)\,(\gamma + \alpha)$

$\because \quad \alpha + \beta + \gamma = -1;\ \alpha\beta + \beta\gamma + \gamma\alpha = 1;\ \alpha\beta\gamma = -r$

$\therefore \quad -1 = 5 + 3\,(-1 - \gamma)\,(\beta + \gamma)\,(\gamma + \alpha)$

$\Rightarrow \quad 2 = 1 + \gamma^2 + \gamma + \gamma^3 \Rightarrow \gamma^3 + \gamma^2 + \gamma - 1 = 0$

$\Rightarrow \quad r = -1$

254. **(c)** Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ are roots of the given equation

And $\alpha_1 + \alpha_2 = \frac{5}{2}$

and two roots of given equation are ±1.

Let $\alpha_3 = +1, \alpha_4 = -1$

∴ $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 = \frac{25}{6}$

$$\frac{5}{2} + \alpha_5 + \alpha_6 = \frac{25}{6} \Rightarrow \alpha_5 + \alpha_6 = \frac{25}{6} - \frac{5}{2} = \frac{25-15}{6}$$

$$\Rightarrow \alpha_5 + \alpha_6 = \frac{10}{6}$$

$$\Rightarrow \alpha_5 + \alpha_6 = \frac{5}{3}$$

255. **(b)** Given that $1+\sqrt{2}$ and $2 - i$ are the roots of given equation therefore $1-\sqrt{2}$ and $2 + i$ are also roots of equation.

∴ Equation is

$$\left[x^2 - \left(1+\sqrt{2}+1-\sqrt{2}\right)x + \left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)\right]$$

$$\left[x^2 - \left(2-i+2+i\right)x + \left(2-i\right)\left(2+i\right)\right] = 0$$

⇒ $(x^2 - 2x - 1)(x^2 - 4x + 5) = 0$

⇒ $x^4 - 6x^3 + 12x^2 - 6x - 5 = 0$

Compare with given equation, we get b = –6, c = 12, d = –6 and e = –5

∴ second equation $bx^2 + cx + d = 0$

Becomes $x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 = 0$

Hence, roots are real and equal.

256. **(c)** Given equation is $2x^4 - 8x^3 + 3x^2 - 1 = 0$

substitute x + h in place of x

∴ $2(x + h)^4 - 8(x + h)^3 + 3(x + h)^2 - 1 = 0$

$2(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4)$

$-8(x^3 + 3x^2h + 3xh^2 + h^3) + 3(x^2 + 2xh + h^2) - 1 = 0$

$2x^4 + (8h - 8)x^3 + (12h^2 - 24h + 3)x^2$

$+(8h^3 - 24h^2 + 6h)x + (2h^4 - 8h^3 + 3h^2 - 1) = 0$

Given that x^3 term is absent

$8h - 8 = 0 \Rightarrow h = 1$

Transformed equation will be

$2x^4 - 9x - 10x - 4 = 0 \Rightarrow b = -9$

257. **(b)** $\tan 15° = \tan(45-30)° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

∴ $\tan 15° + \tan 30° = -p$

$$\Rightarrow p = \frac{-4}{\sqrt{3}\left(\sqrt{3}+1\right)}$$

$q = \tan 15° \times \tan 30°$

$$= \frac{\sqrt{3}-1}{\sqrt{3}\left(\sqrt{3}+1\right)} \Rightarrow q = \frac{\sqrt{3}-1}{\sqrt{3}\left(\sqrt{3}+1\right)}$$

$$\Rightarrow p \cdot q = \frac{-4(\sqrt{3}-1)}{3(\sqrt{3}+1)^2} = \frac{10-6\sqrt{3}}{3} \Rightarrow p \cdot q = \frac{10-6\sqrt{3}}{3}$$

258. **(d)** We have equation

$x^3 - 5x^2 - 2x + 24 = 0$

Here, $\alpha + \beta + \gamma = 5$

$\alpha\beta + \beta\gamma + \gamma\alpha = -2, \alpha\beta\gamma = -24$

Here $\frac{\beta\gamma}{a} + \frac{\gamma\alpha}{b} + \frac{\alpha\beta}{g} = \frac{(\alpha\beta+\beta\gamma+\gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha+\beta+\gamma)}{\alpha\beta\gamma}$

$= \frac{-244}{24} = \frac{-61}{6}$

259. **(c)** We have equation $3x^3 - 26x^2 + 52x - 24 = 0$

having roots α, β and γ $\Rightarrow \alpha + \beta + \gamma = \frac{26}{3}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{52}{3}$; $\alpha\beta\gamma = 8$

α, β, γ are in G.P. and $\alpha < \beta < \gamma \Rightarrow \alpha.\gamma, = \beta^2$

On solving we get $\alpha = \frac{2}{3}$, $\beta = 2$ and $\gamma = 6$

$\Rightarrow 3\alpha + 2\beta + \gamma = 12$

260. **(b)** We have

$4x^3 + 12x^2 - 7x + 165 = 0$

having roots α, β and γ

now equation with roots α + 5, β + 5 and γ + 5 is,

$4x^3 - 48x^2 + 173x = 0$

Here product of roots will be = 0

261. **(b)** It is given that α, β and γ are the roots of

$x^3 - 6x^2 + 11x + 6 = 0$

Then we know that $\alpha + \beta + \gamma = 6$, $\alpha\beta + \beta\gamma + \gamma\alpha = 11$ and $\alpha\beta\gamma = -6$

$\Sigma\alpha^2\beta + \Sigma\alpha\beta^2$

$= \alpha^2\beta + \beta^2\gamma + \gamma^2\alpha + \alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2$

$= (\alpha + \beta + \gamma)(\alpha\beta + \beta\gamma + \gamma\alpha) - 3\alpha\beta\gamma$

$= (6)(11) - 3(-6) = 66 + 18 = 84$

262. **(b)** It is given that α, β and γ are the roots of

$x^3 + 4x - 19 = 0$

Then, $\alpha + \beta + \gamma = 0$ ...(i)

$\alpha\beta + \beta\gamma + \gamma\alpha = 4$ ...(ii)

$\alpha\beta\gamma = 19$ ...(iii)

On solving equations (i), (ii) and (iii), we get

$$\frac{\alpha^3}{19-4\alpha} = 1 \qquad ...(iv)$$

Similarly, from Eq. (ii) & (iii) we get

$$\frac{\beta^3}{19-4\beta} = 1 \qquad ...(v)$$

And now form Eq. (ii)

$$\gamma(\alpha+\beta) + \alpha\beta = 4 \Rightarrow \gamma(-\gamma) + \frac{19}{\gamma} = 4 \Rightarrow \frac{\gamma^3}{19-4\gamma} = 1$$

Adding Eqs. (iv), (v) and (vi), we get

Hence, $\frac{\alpha^3}{19-4\alpha} + \frac{\beta^3}{19-4\beta} + \frac{\gamma^3}{19-4\gamma} = 3$

263. **(a)** Given equation,

$ix^2 - 2(i + 1)x + (2 - i) = 0$

Let the other roots be k

$[x - (2 - i)][x - k] = 0$

$x^2 - (2 - i + k)x - (k(2 - i)) = 0$

{comparing with Eq. (i)}

$ix^2 - (2i + 1 + ki)x - [k(2i + 1)] = 0$

$2i + 1 + ki = 2i + 2$

$k = \frac{1}{i} = -i$

264. **(d)** Given quadratic equation, $x^2 + x + 1 = 0$

$\therefore$ Roots are $\alpha = \omega$ and $\beta = \omega^2$

where, $\omega = \frac{-1+\sqrt{3}i}{2}$ and $\omega^3 = 1$

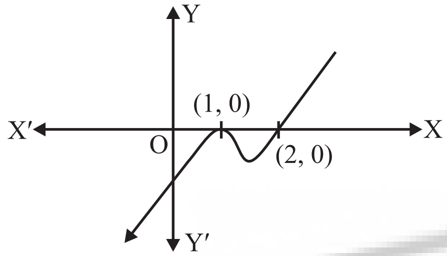
Now, $\alpha^{2021} = \omega^{2021} = \omega^{2019}\,\omega^2 = (\omega^3)^{673}\,\omega^2 = \omega^2$

and $\beta^{2021} = \omega^{4042} = \omega^{4041}\,\omega = (\omega^3)^{1347}\omega = \omega$

Hence, equation whose roots are α^{2021} and β^{2021} will be,

$(x - \omega)\,(x - \omega^2) = x^2 + x + 1$

265. **(b)**



This is the graph of $f(x) = x^3 - 4x^2 + 5x - 2$

Also given,

$$f\left(x+\frac{1}{3}\right) = \left(x+\frac{1}{3}\right)^3 - 4\left(x+\frac{1}{3}\right)^2 + 5\left(x+\frac{1}{3}\right) - 2 = 0$$

So, the graph of $f\left(x+\frac{1}{3}\right)$ will shift towards left by 1/3 units, so the new roots will also shift by 1/3 units towards left.

$\therefore$ New roots $= \left(1-\frac{1}{3}, 1-\frac{1}{3}, 2-\frac{1}{3}\right) = \left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}\right)$

266. **(b)** Given equation $(x^2 + 5x + 5)^{x+5} = 1$

If $x + 5 = 0 \Rightarrow x = -5$

and $x^2 + 5x + 5 = 1$

or $x^2 + 5x + 4 = 0$

$\Rightarrow x = -1, -4$

$x = -1, -4$ and -5 are the three integers satisfying given equation.

267. **(b)** Given $f(x) = x^3 + ax^2 + bx + c$

Let roots of $f(x)$ are α, β, γ

since roots are in AP. $\Rightarrow 2\beta = \alpha + \gamma$

Now, sum of roots $\alpha + \beta + \gamma = -$ a or $\beta = -\frac{a}{3}$

It is given that, roots are integers.

$\therefore$ β is an integer when a is multiple of 3.

Option (a), (c) and (d) are multiple of 3.

$\Rightarrow a \neq 1214$

268. **(a)** z_1 and z_2 are the roots of $x^2 + 2x + 2 = 0$

$\therefore z_1 + z_2 = -2 \Rightarrow z_2 = -2 - z_1$ ...(i)

Now, $2^6\left[\frac{-2^{11}(z_1+1+3i)^{11}}{2^{11}(z_2+1-3i)^{11}}\right]$ [From Eq. (i)]

$$2^6\left[\frac{-(z_1+1+3i)}{-2-z_1+1-3i}\right]^{11} = 2^6\left[\frac{-z_1-1-3i}{-z_1-1-3i}\right]^{11} = 2^6 = 64$$

269. **(b)** Let $f(x) = x^2 + 7x + 3 = 0$

Roots are $\frac{\alpha}{\alpha+1}$ and $\frac{\beta}{\beta+1}$.

Equation having roots α and β is $f\left(\frac{x}{x+1}\right) = 0$

$$\left(\frac{x}{x+1}\right)^2 + 7\left(\frac{x}{x+1}\right) + 3 = 0$$

$\Rightarrow 11x^2 + 13x + 3 = 0$

270. **(a)** α, β and γ are the roots of $x^3 - x + 1 = 0$ …(i)

We have to find a polynomial whose roots are

$\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$

Let $x = \frac{1+\alpha}{1-\alpha}$ or $\alpha = \frac{x-1}{x+1}$

Substituting $\alpha = \frac{x-1}{x+1}$ in Eq. (i) we have

$$\left(\frac{x-1}{x+1}\right)^3 - \left(\frac{x-1}{x+1}\right) + 1 = 0$$

$\Rightarrow x^3 - x^2 + 7x + 1 = 0$

$\therefore$ Sum of roots $= \frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma} = \frac{-(-1)}{1} = 1$

271. **(d)** Since $2 + 4i$ is one of the roots of $x^2 + bx + c = 0$ with b, c $\in$ R, so other root will be $2 - 4i$.

Now, the sum of roots $= -b$

$\Rightarrow$ $(2 + 4i) + (2 - 4i) = -b \Rightarrow b = -4$

and the product of roots = c

$\Rightarrow$ $(2 + 4i)\,(2 - 4i) = c \Rightarrow 4 + 16 = c \Rightarrow c = 20$

$\therefore$ $(b, c) = (-4, 20)$

272. **(a)** Since $2 + i$ is the root of the equation $x^3 - 5x^2 + 9x - 5 = 0$, so another non-real complex root will be $2 - i$.

Now, let the third root is α, so by product of roots, we have

$(2 + i)\,(2 - i)\,\alpha = 5 \Rightarrow \alpha = 1$

273. **(c)** Given equation $3^{x^2-x} = 25 - 4^{x^2-x}$

Can be written as

$$4^{x^2-x} + 3^{x^2-x} = 25 = 4^2 + 3^2$$

$\Rightarrow$ $x^2 - \mathrm{x} = 2 \Rightarrow x^2 - x - 2 = 0$

274. **(b)** Given that α, β, γ are roots of $f(x) = x^3 - 9x^2 + 26x - 24$ then the equation, where roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$. We can obtain $\alpha = \frac{1}{x}$

$\Rightarrow$ $24x^3 - 26x2 + 9x - 1$

275. **(d)** Hence, the given expression $x^3 - 5x^2 + 2x + 7$ on dividing by $x - 1$, We have

$$
\begin{array}{r l}
 & x^2-4x-2 \\
x-1\,\big) & x^3-5x^2+2x+7 \\
 & x^3-x^2 \\
 & -\quad + \\
\hline
 & \quad -4x^2+2x+7 \\
 & \quad -4x^2+4x \\
 & \quad +\quad - \\
\hline
 & \quad -2x+7 \\
 & \quad -2x+2 \\
 & \quad +\quad - \\
\hline
 & \quad 5
\end{array}
$$

$\therefore$ The required quotient is $(x^2 - 4x - 2)$

276. **(c)** Given equation, $x^3 - x^2 - x - 2 = 0$

$(x - 2)(x^2 + x + 1) = 0$

$a = 2,\ \dfrac{-1 \pm \sqrt{3}i}{2}$

$\therefore$ α and β are $\dfrac{-1 \pm \sqrt{3}i}{2}$

Non-real complex roots of this equation

So, let $\alpha = \omega$ and $\beta = \omega^2$, where $\omega^3 = 1$ and $\omega^2 + \omega + 1 = 0$.

$\therefore\ \alpha^{2020} + \beta^{2020} + \alpha^{2020}\ \beta^{2020}$

$= \omega^{2020} + \omega^{4040} + \omega^{2020}\,\omega^{4040}$

$= (\omega^3)^{673}\,\omega + (\omega^3)^{1346}\,\omega^2 + (\omega^3)^{673}\,\omega\,(\omega^3)^{1346}\,\omega^2$

$= \omega + \omega^2 + \omega^3 = 1 + \omega + \omega^2 = 1 + \alpha + \beta$

277. **(d)** The roots of the equation $x^2 + x + 1 = 0$ are α and β which are non-real roots of unity.

Thus, $\alpha^3 = \beta^3 = 1$ and $\alpha + \beta + 1 = 0$

$\therefore\ \alpha^4 + \beta^4 = \alpha^3 + \beta^3\beta = \alpha + \beta$

$\alpha^4 + \beta^4 = -1 = -\alpha\beta$

Since product of roots $\alpha\beta = 1$

278. **(b)** Let α is the root of the given equation,

$x^3 - x^2 + x - 4 = 0$

Now, we have given that root is the negative of the root of given equation so, put $(-\alpha) = x$, so we get the required equation,

$(-x)^3 - (-x)^2 + (-x) - 4 = 0$

$-x^3 - x^2 - x - 4 = 0 \Rightarrow x^3 + x^2 + x + 4 = 0$

279. **(c)** Let the roots of the quadratic equation are α and β then according to the given data,

$\alpha + \beta = 1$ and $\alpha^2 + \beta^2 = 13$

$$\alpha\beta = \frac{1}{2}[(\alpha+\beta)^2 - (\alpha^2+\beta^2)] = \frac{1}{2}[1-13] = -6$$

Hence, equation of required quadratic is

$x^2 - (\alpha + \beta)\,x + \alpha\beta = 0 \Rightarrow x^2 - x - 6 = 0$

280. **(d)** Given $2\alpha = -1 - i\sqrt{3},\ 2\beta = -1 + i\sqrt{3}$

Now, $5\alpha^4 + 5\beta^4 + 7\alpha^{-1}\beta^{-1} \Rightarrow 5(\alpha^4 + \beta^4) + \dfrac{7}{\alpha\beta}$

or $5[(\alpha^4 + \beta^4) - 2\alpha^2\beta^2] = +\dfrac{7}{\alpha\cdot\beta}$

$$= 5\left[\left\{\frac{1}{4}(2\alpha+2\beta)^2 - 2\alpha\beta\right\}^2 - 2\alpha^2\beta^2\right] + \frac{7}{\alpha\cdot\beta}$$

$$= 5\left[\left(\frac{1}{4}\times 4 - 2\right)^2 - 2\right] + 7 = 5[1-2] + 7 = 2$$

281. **(d)** **Case I:** When $x^2 - x - 6 \geq 0$ i.e., $x \in (-\infty, -2] \cup [3, \infty)$

$\therefore\ x^2 - x - 6 = x + 2$

$\Rightarrow x^2 - 2x - 8 = 0 \Rightarrow x = -2, 4$

Case II: When $x^2 - x - 6 < 0$ i.e., $x \in (-2, 3)$

$x^2 - x - 6 = -x - 2$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2 \Rightarrow x = 2$

Hence, $x = \{-2, 2, 4\}$.

282. **(a)** $\sqrt{x+1} - |\sqrt{x-1}| = \sqrt{4x-1}$

Here $x \geq 1$ to define each

Now, $\sqrt{x+1} - \sqrt{4x-1} = |\sqrt{x-1}|$

On squaring on both sides, we get

$\Rightarrow\ 4x + 1 = 2\sqrt{(x+1)(4x-1)}$

Again, squaring both the sides, we get

$\Rightarrow\ 16x^2 + 8x + 1 = 16x^2 + 12x - 4$

$\Rightarrow\ 4x = 5 \Rightarrow x = \dfrac{5}{4}$

283. **(a)** Since irrational roots exists in pair.

So, the degree of $f(x)$ will be an even number.

284. **(a)** Given that,

$\lambda x^2 + 13x + 7 = 0$

$\therefore$ D = $(13)^2 - 4(\lambda)(7) = 169 - 28\lambda$

For rational roots, D should be a perfect square.

So, $\lambda \in (-3, 7)$ has values $-2, 0, 6$ so that D become perfect square.

$\therefore$ Required sum of elements in S $= -2 + 0 + 6 = 4$

285. **(c)** $x^4 + x^3 - 4x^2 + x - 1 = 0$

$(x - 1)(x - 1)(x^2 + 3x + 1) = 0$

$$(x-1)(x-1)\left[x - \left(\frac{-3+\sqrt{5}}{2}\right)\right]\left[x - \left(\frac{-3-\sqrt{5}}{2}\right)\right] = 0$$

$\therefore$ Roots of the equation are $x = 1, 1$

$\dfrac{-3+\sqrt{5}}{2}, \dfrac{-3-\sqrt{5}}{2}$

$\therefore$ Required ratio of roots

$$= \frac{(1)^2 + (1)^2 + \left(\dfrac{-3+\sqrt{5}}{2}\right)^2 + \left(\dfrac{-3-\sqrt{5}}{2}\right)^2}{1 \times \left(\dfrac{-3+\sqrt{5}}{2}\right)\left(\dfrac{-3-\sqrt{5}}{2}\right)}$$

$$= \frac{8 + 5 + 9 - 6\sqrt{5} + 5 + 9 + 6\sqrt{5}}{4} = 9$$

$\therefore$ Ratio = 9 : 1

286. **(d)** We have,

$\alpha_1, \beta_1, \gamma_1, \delta_1$ are the roots of equation

$ax^4 + bx^3 + cx^2 + dx + e = 0$ ...(i)

$\alpha_2, \beta_2, \gamma_2, \delta_2$ are the roots of equation

$ex^4 + dx^3 + cx^2 + bx + a = 0$ ...(ii)

Also, $\delta_1 > \gamma_1 > \beta_1 > \alpha_1 > 0$

and $\delta_2 > \gamma_2 > \beta_2 > \alpha_2 > 0$

∴ Clearly, from the above equations (i) & (ii)

∴ δ_1 and α_2 are reciprocal

Similarly γ_1 and β_2,

γ_2 and β_1 and α_1 and δ_2 are reciprocal

$\alpha_1\delta_2 = 1, \beta_1\gamma_2 = 1, \beta_2\gamma_1 = 1, \alpha_2\delta_1 = 1$

$\alpha_1 - \delta_2 = 2$

$$\therefore \quad \alpha_1 - \frac{1}{\alpha_1} = 2 \Rightarrow \alpha_1^2 - 2\alpha_1 - 1 = 0$$

and $\delta_1 - \alpha_2 = 4$

$$\delta_1 - \frac{1}{\delta_1} = 4 \Rightarrow \delta_1^2 - 4\delta_1 - 1 = 0$$

As it is given that α_1 and δ_1 are roots of the equation

$ax^4 + bx^3 + cx^2 + dx + e = 0$

∴ $(x^2 - 2x - 1)(x^2 - 4x - 1) = ax^4 + bx^3 + cx^2 + dx + e$

Put $x = 1$,

$a + b + c + d + c = (1 - 2 - 1)(1 - 4 - 1)$

∴ $a + b + c + d + e = 8$

287. **(Bonus)** Let $\frac{x^2 + ax + 3}{x^2 + x + 1} = y$, where $y \in \boldsymbol{R}$

So, $(y - 1)x^2 + (y - a)x + (y - 3) = 0$

∵ $x \in \boldsymbol{R}$, so $D \ge 0$

∴ $(y - a)^2 - 4(y - 1)(y - 3) \ge 0$

⇒ $(y^2 - 2ay + a^2) - 4(y^2 - 4y + 3) \ge 0$

⇒ $-3y^2 + (16 - 2a)y + (a^2 - 12) \ge 0$

⇒ $3y^2 + (2a - 16)y + (12 - a^2) \le 0$

Which is not possible for every real value of y.

288. **(c)** Given that

$x^4 - 8x^3 + 11x^2 + 32x - 60 = 0$

∴ $(x + 2)(x - 2)(x - 3)(x - 5) = 0$

⇒ $\alpha = -2, \beta = 2, \gamma = 3, \delta = 5$

Therefore, $4\alpha + 3\beta + 2\gamma + \delta = 4(-2) + 3(2) + 2(3) + 5$

$= -8 + 6 + 6 + 5 = 9$

289. **(b)** Given that α and β are roots of equation

$z^2 + az + b = 0$

∴ $\alpha + \beta = -a$ and $\alpha\beta = b$

∵ origin α, β forms a equilateral triangle

$$\therefore \quad \beta = \alpha(\cos 60° + i \sin 60°) \Rightarrow \beta = \alpha\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$$

$$\Rightarrow 2\beta = \alpha + \sqrt{3}\alpha i \Rightarrow 2\beta - \alpha = \sqrt{3}\alpha i$$

⇒ $(2\beta - \alpha)^2 = 3\alpha^2 i^2$

⇒ $(\alpha + \beta)^2 - 2\alpha\beta = \alpha\beta \Rightarrow (-a)^2 = 3(b)$

⇒ $a^2 = 3b$

290. **(c)** We have

$z^n + p_1 z^{n-1} + p_2 z^{n-2} + ... + p_n = 0$ ...(i)

Given equation is polynomial in z

Hence dividing equation (i) by z^n, we get

⇒ $1 + p_1 z^{-1} + p_2 z^{-2} + ... + p_n z^{-n} = 0$

Now, $z = e^{ix}$ satisfied the above equation

∴ $1 + p_1 e^{-ix} + p_2 e^{-2ix} + ... + p_n e^{-inx} = 0$

Since, $e^{i\theta} = \cos 3\theta + i \sin\theta$

$$\Rightarrow (1 + p_1 \cos x + p_2 \cos x + ... + p_n \cos nx) - i(p_1 \sin x + p_2 \sin 2x + ... + p_n \sin nx) = 0$$

∴ $p_1 \sin x + p_2 \sin 2x + ... + p_n \sin nx = 0$

$$\Rightarrow p_n \sin nx + p_{n-1} \sin (n - 1)x + ... + p_2 \sin 2x + p_1 \sin x + 1 = 1$$

291. **(b)** We have, $x^3 + px + q = 0$

Since, α, β, γ are the roots of the given equation

$$\alpha + \beta + \gamma = -\frac{b}{a} = 0$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = p \; ; \; \alpha\beta\gamma = \frac{-d}{a} = -q$$

Let $S_1 = \alpha + \beta + \gamma = \Sigma\alpha$

$S_2 = \alpha\beta + \beta\gamma + \gamma\alpha = \Sigma\alpha\beta$ and $S_3 = \alpha\beta\gamma$

Now, $\Sigma\alpha^2 \beta = S_1S_2 - 3S_3 = (0)(p) - 3(-q) = 3q$

Again, $x^3 + px + q = 0$

⇒ $x^3 = -[px + q] \Rightarrow x^4 = -[px^2 + qx]$

∴ $\Sigma\alpha^4 = -[p\Sigma\alpha^2 + q\Sigma\alpha] = -[p((\Sigma\alpha)^2 - 2(\Sigma\alpha\beta)) + q\Sigma\alpha]$

$= -[p(S_1^2 - 2S_2) + qS_1]$

$= -[p(0 - 2p) + q \times 0] = 2p^2$

∴ $\Sigma\alpha^2\beta + \Sigma\alpha^4 = 3q + 2p^2$

Now, $f(x) = 3p^2x^2 + p^2x + 3q$

∴ $f(1) = 3p^2 + p^2 + 3q = 4p^2 + 3q$

$f(-1) = 3p^2 - p^2 + 3q = 2p^2 + 3q$

$f(0) = 3q \Rightarrow f(2) = 12p^2 + 2p^2 + 3q = 14p^2 + 3q$

∴ $\Sigma\alpha^2\beta + \Sigma\alpha^4 = f(-1)$

292. **(d)** We have, $x^3 + ax^2 - bx + c = 0$

$$\therefore \quad \text{Let } \Sigma\alpha = -\frac{(a)}{1} = -a$$

$$\Sigma\alpha\beta = \frac{(-b)}{1} = -b \qquad \left[\begin{array}{r} \text{Let } \Sigma\alpha = S_1 \text{(say)} \\ \Sigma\alpha\beta = S_2 \text{(say)} \\ \alpha\beta\gamma = S_3 \text{ (say)} \end{array}\right]$$

$$\alpha\beta\gamma = -\frac{(c)}{1} = -c$$

Now, $\Sigma\beta^2(\gamma + \alpha) = S_1S_2 - 3S_3$

$= (-a)(-b) - 3(-c) = ab + 3c$

293. **(c)** Given quadratic equations are

$3x^2 - 7x + 2 = 0$ ...(i)

and $15x^2 - 11x + a = 0$ ...(ii)

On comparing with $a_1x^2 + b_1x + c_1 = 0$ and

$a_2x^2 + b_2x + c_2 = 0$

Here, $a_1 = 3, b_1 = -7, c_1 = 2$

$a_2 = 15, b_2 = -11, c_2 = a$

Let α is a common root of the Eqs. (i) and (ii),

Then, α will satisfy both the equations.

∴ Common root is given by

$(2 \times 15 - a \times 3)^2 = (-7a + 22)(-33 + 105)$

⇒ $a^2 + 36a - 76 = 0 \Rightarrow (a + 38)(a - 2) = 0$

⇒ $a = 2$ [∵ $a > 0$]

Now, for equation $15x^2 - ax + 7 = 0$

$$\text{Sum of roots} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-(-a)}{15} = \frac{a}{15} = \frac{2}{15}$$

294. **(d)** Given, $\phi(x)=\dfrac{x}{(x^2+1)(x+1)}$

Now, $\phi(a)\ \phi(b)\ \phi(c)$

$$=\frac{abc}{(a^2+1)(a+1)(b^2+1)(b+1)(c^2+1)(c+1)}$$

or $\phi(a)\ \phi(b)\ \phi(c)$

$$=\frac{abc}{(1+a+b+c+ab+bc+ca+abc)(1+a^2+b^2+c^2+a^2b^2+b^2c^2+c^2a^2+(abc)^2)}$$

Also, given that a, b and c are roots of cubic equation $x^3-3x+\lambda=0$

$ab+bc+ca=-3$ ... (i)

$a+b+c=0$ ...(ii)

and $abc=-\lambda$ ...(iii)

Squaring Eq. (ii), we get, $(a+b+c)^2=0$

$a^2+b^2+c^2+2(ab+bc+ca)=0$

$a^2+b^2+c^2=6$

Similarly, $a^2b^2+b^2c^2+c^2a^2=9$ (by solving)

Put these values in $\phi(a)\ \phi(b)\ \phi(c)$, we get

$$\frac{-\lambda}{(1+0-3-\lambda)(1+6+9+\lambda^2)}$$

$$\frac{-\lambda}{(-2-\lambda)(\lambda^2+16)}=\frac{\lambda}{(\lambda+2)(\lambda^2+16)}$$

295. **(c)** Given x^2+y+4i and $-3+x^2yi$ are conjugate.

Therefore, $x^2+y+4i=-3-x^2yi$

$\Rightarrow (x^2+y)+4i=(-3)-x^2yi$

On comparing both sides, we get

$\Rightarrow x^2+y=-3$ ...(i)

and $4=-x^2y$

$\Rightarrow y=\dfrac{4}{-x^2}=-\dfrac{4}{x^2}$ ...(ii)

On putting the value of y in eq. (i), we get

$x^4+3x^2-4=0$

$x^2-1=0\Rightarrow x^2=1\Rightarrow x=\pm 1$

Put the value of $x=\pm 1$ in eq. (ii), we get

$y=\dfrac{-4}{(1)^2}=-4$

$\therefore\ x=\pm 1,\ y=-4$

Hence, $(|x|+|y|)^2=|x|^2+|y|^2+2\,|x|\,|y|$

296. **(a)** Given equation is $\sqrt{\dfrac{x}{2x+1}}+\sqrt{\dfrac{2x+1}{x}}=2$ ...(i)

Let $\sqrt{\dfrac{x}{2x+1}}=t$, Then, $t+\dfrac{1}{t}=2$

$\Rightarrow t^2-2t+1=0\Rightarrow t=1\Rightarrow \sqrt{\dfrac{x}{2x+1}}=1$

$\Rightarrow x=-1$

$\because$ α satisfies the eq. (i), therefore $\alpha=-1$

Now, $\alpha^2x^2+4ax+3=0$

Put the value of α

$(-1)^2x^2+4(-1)x+3=0\Rightarrow (x-1)(x-3)=0$

$x=1, 3$

Hence, the roots of the equation

$\alpha^2x^2+4ax+3=0$ are 1, 3.

297. **(a)** Given cubic equation is $x^3-6x^2+11x-6=0$ ...(i)

$\Rightarrow (x-1)(x-2)(x-3)=0$

$\Rightarrow x=1, 2, 3\equiv \alpha, \beta, \gamma$

Therefore, $\alpha^2+\beta^2=(1)^2+(2)^2=5=a$ (say)

$\beta^2+\gamma^2=(2)^2+(3)^2=13=b$ (say)

and $\gamma^2+\alpha^2=(3)^2+1=10=c$ (say)

Equation of the having the roots a, b, c

$x^3-(a+b+c)x^2+(ab+bc+ca)x-abc=0$

$\Rightarrow x^3-(5+13+10)x^2+(5\times 13+13\times 10$

$+10\times 5x-5\times 13\times 10=0$

$\Rightarrow x^3-28x^2+245x-650=0$

298. **(d)** Let $\dfrac{2\alpha}{3-4\alpha}=y\Rightarrow 2\alpha=3y-4\alpha y$

$\Rightarrow\ \alpha(2+4y)=3y\Rightarrow \alpha=\dfrac{3y}{2+4y}$

Given that α is root of quadratic equation $x^2+7x+3=0$,

$\therefore\ \left(\dfrac{3y}{2+4y}\right)^2+7\left(\dfrac{3y}{2+4y}\right)+3=0$

$\Rightarrow\ 47y^2+30y+4=0$ ...(i)

$\because\ y=\dfrac{2\alpha}{3-4\alpha}$ is root of quadratic equation

So, compare equation (i) with $ax^2+bx+c=0$.

$\therefore\ a=47$, $b=30$ and $c=4$ and GCD of 47, 30, 4 is 1.

$\therefore\ a+b+c=47+30+4=81$.

299. **(a)** Since it is given that roots are in order $\lambda<\alpha<\delta<\beta$

and $x^2+b_1x+c_1=0,\ x^2+bx+c=0$

For x-coordinate of point P, on subtracting given quadratic equations, we get

$x^2+b_1x+c_1=0\ \Rightarrow\ x^2+bx+c=0$

$(b_1-b)x+(c_1-c)=0\ \Rightarrow\ x=\left(\dfrac{c-c_1}{b_1-b}\right)$

Now, with respect to quadratic expression

$f(x)=x^2+bx+c$

$f\left(x=\dfrac{c-c_1}{b_1-b}\right)<0$

$\Rightarrow\ \left(\dfrac{c-c_1}{b_1-b}\right)^2+b\left(\dfrac{c-c_1}{b_1-b}\right)+c<0$

$\Rightarrow\ (c-c_1)^2<(b_1-b)(bc_1-cb_1)$

300. **(d)** Since it is given that, the polynomial equation of degree 4 having real three coefficients of its roots as $2\pm\sqrt{3}$ and $1+2i$, hence the remaining root is $1-2i$.

Now, the quadratic equation whose roots as $2\pm\sqrt{3}$ is $x^2-4x+1=0$, and the quadratic equation whose roots as $1\pm 2i$, is $x^2-2x+5=0$

So, the required polynomial equation is

$\Rightarrow\ x^4-6x^3+14x^2-22x+5=0$

301. **(c)** Given that $a_k = \cos\alpha_k + i \sin\alpha_k$

$\therefore a_1 = \cos\alpha_1 + i \sin\alpha_2$

$a_2 = \cos\alpha_2 + i \sin\alpha_2$

$a_3 = \cos\alpha_b + i \sin\alpha_3$

Also, given that a_1, a_2, a_3 are roots of $x^3 + bx + c = 0$

We have at least one root of cubic equation as real.

$\therefore \sin\alpha_1 = 0 \Rightarrow \cos\alpha_1 = 1$

$\therefore \alpha_1 = 0$

And other roots are congugate pair.

$\therefore a_3 = \cos\alpha_2 - i \sin\alpha_2$

Now, $a_1 + a_2 + a_3 = 0$

$1 + \cos\alpha_2 + i \sin\alpha_2 + \cos\alpha_2 - i \sin\alpha_2 = 0$

$$\Rightarrow \cos\alpha_2 = -\frac{1}{2} \Rightarrow \sin\alpha_2 = \frac{-\sqrt{3}}{2}$$

$$a_2 = \frac{-1}{2} - \frac{\sqrt{3}}{2} i \ ; \ a_3 = \frac{-1}{2} + \frac{\sqrt{3}}{2} i$$

Now, $a_1a_2 + a_2a_3 + a_3a_1 = b$

$$\Rightarrow \frac{-1}{2} - \frac{\sqrt{3}}{2} i + \frac{1}{4} + \frac{3}{4} - \frac{1}{2} + \frac{\sqrt{3}}{2} i = b$$

$\Rightarrow -1 + 1 = b \Rightarrow b = 0$

302. **(b)** Since $\sin 2\theta$, $\cos 2\theta$ are solutions of the equations. Hence roots of the equations are $\sin 2\theta$, $\cos 2\theta$.

$\sin 2\theta + \cos 2\theta = -b$ and $\sin 2\theta \cos 2\theta = -c$

$\because \quad (\sin 2\theta + \cos 2\theta)^2$

$= \sin^2 2\theta + \cos^2 2\theta + 2 \sin 2\theta \cos 2\theta$

$\Rightarrow (-b)^2 = 1 + 2(-c)$

$\Rightarrow b^2 + 2c - 1 = 0$

Hence, option (b) is correct.

303. **(b)** Given equation is $x^2 + |x - 3| = 4$

Case I: $x > 3$, then $(x - 3)$ is positive.

$x^2 + x - 7 = 0$

Apply quadratic formula,

$$x = \frac{-1 \pm \sqrt{29}}{2} \quad \therefore \text{ No solution}$$

Case II: $x < 3$, then $(x - 3)$ is negative

$x^2 - x - 1 = 0$

Apply quadratic formula $x = \dfrac{1 \pm \sqrt{5}}{2}$

$\therefore$ Sum of all real number satisfy ray the equal

$$\frac{1 + \sqrt{5}}{2} + \frac{1 - \sqrt{5}}{2} = 1$$

304. **(a)** Given inequality,

$$3^x + 3^{1-x} - 4 < 0 \Rightarrow 3^x + \frac{3}{3^x} - 4 < 0$$

$\Rightarrow 3^{2x} - 4.3^x + 3 < 0 \qquad [3^x > 0, \forall x \in R]$

Factorize the above equation,

$\Rightarrow (3^x - 1)(3^x - 3) < 0 \Rightarrow 3^x \in (1, 3)$

Compare the terms individually

$\Rightarrow x \in (0, 1)$.

305. **(a)** Given quadratic equation is,

$(x - 2)(x - 3) = k^2, k \in R \Rightarrow x^2 - 5x + 6 - k^2 = 0$

Now, discriminant

$D = (-5)^2 - 4(1)(6 - k^2) \qquad [\because D = b^2 - 4ac]$

$= 25 - 24 + 4k^2 = 1 + 4k^2 > 0$

Hence, the roots are real and distinct for $k \in R$.

306. **(d)** Given that $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ are roots of given equation $x^n + px + q = 0$

So, $x^n + px + q = (x - \alpha_1)(x - \alpha_2)(x - \alpha_3) \ldots (x - \alpha_n)$

$$\Rightarrow \frac{x^n + px + q}{x - \alpha_n} = (x - \alpha_1)(x - \alpha_2)(x - \alpha_3)$$

Taking $\lim\limits_{x \to a_n}$ both side

$$\therefore \lim_{x \to a_n} \frac{x^n + px + q}{x - \alpha_n}$$

$$= \lim_{x \to a_n} (x - \alpha_1)(x - \alpha_2)(x - \alpha_3) \quad \text{[By L'Hopital Rule]}$$

$$\lim_{x \to a_n} \frac{nx^{n-1} + P}{1} = (\alpha_n - \alpha_1)(\alpha_n - \alpha_2) \ldots (\alpha_n - \alpha_{n-1})$$

$\Rightarrow na_n^{n-1} + p = (\alpha_n - \alpha_1)(\alpha_n - \alpha_2) \ldots (\alpha_n - \alpha_{n-1})$

307. **(d)** Given that,

If all roots of the equation

$x^5 + 15x^4 + 94x^3 + 305x^2 + 507x + 353 = 0$

is increased by k, then transformed equation is

$(x - k)^5 + 15(x - k)^4 + 94(x - k)^3 + 305(x - k)^2$
$+ 507(x - k) + 353 = 0 \ \ldots\text{(i)}$

For eliminate 4th degree term, equate the coefficient of x^4 to zero.

Coefficient of $x^4 = -5k + 15 = 0 \Rightarrow k = 3$

Put value in Eq. (i), we get

$(x - 3)^5 + 15(x - 3)^4 + 94(x - 3)^3 + 305(x - 3)^2$
$+ 507(x - 3) + 353 = 0$

$\therefore$ Coefficient of x

$= {}^5C_4(-3)^4 + 15\,({}^4C_3(-3)^3) + 94\,({}^3C_2(-3)^2)$
$+ 305\,({}^2C_1(-3)) + 507$

$= 5(81) + 15[4(-27)] + 94[3 \times 9] + 305[2(-3)] + 507 = 0$

Hence, coefficient of $x = 0$.

308. **(a)** We given that, $x^3 + 3px^2 + 3qx + r = 0 \qquad \text{...(i)}$

And roots are in H.P.

So, let roots are [2, 3, 6] that is in HP.

So, x^3 – (sum of root)x^2
+ (sum of product of two-two roots)x
– (product of roots) = 0

$x^3 - (2 + 3 + 6)x^2 + [2 . 3 + 2.6 + 3.6]x - 2.3.6 = 0$

$\Rightarrow x^3 - 11x^2 + [36]x^2 - 36 = 0 \qquad \text{...(ii)}$

Comparing with Eq. (i), then

$$\Rightarrow p = -\frac{11}{3}, q = 12, r = -36$$

Now values of p, q, r is satisfy the option (a).

309. **(c)** Let α, β, γ are roots of $x^3 + px^2 + qx + r = 0$, we have $\alpha^3 + \beta^3 + \gamma^3 = ?$

Here $\alpha + \beta + \gamma = -p$

$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = q \quad \Rightarrow \alpha\beta\gamma = -r$

$\therefore [\alpha + \beta + \gamma]^3 = \alpha^3 + \beta^3 + \gamma^3 + 3(\alpha + \beta + \gamma)$
$(\alpha\beta + \beta\gamma + \gamma\alpha) + 3\alpha\beta\gamma$

$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = (\alpha + \beta + \gamma)^3 - 3(-p)(q) + 3(-r)$

$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = -p^3 + 3pq - 3r.$

310. **(c)** Given that, tan a and tan b are the roots of the equations $x^2+px+q=0$

$\therefore \quad \tan\alpha+\tan\beta=-p$ and $\tan\alpha\cdot\tan\beta=q$

Now, $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \;=\dfrac{p}{q-1}$

$$\sec(\alpha+\beta)=\sqrt{1+\tan^2(\alpha+\beta)}$$

$$\sec(\alpha+\beta)=\sqrt{1+\frac{p^2}{(q-1)^2}}$$

$$\therefore \quad \cos(\alpha+\beta)=\frac{1}{\sqrt{1+\dfrac{p^2}{(q-1)^2}}}$$

$$\sin^2(\alpha+\beta)+p\cos(\alpha+\beta)\sin(\alpha+\beta)$$
$$+q\cos^2(\alpha+\beta)$$

$$=\cos^2(\alpha+\beta)\left[\tan^2(\alpha+\beta)+p\tan(\alpha+\beta)+q\right]$$

$$=\frac{1}{1+\dfrac{p^2}{(q-1)^2}}\left[\frac{p^2}{(q-1)^2}+\frac{p^2}{q-1}+q\right]$$

$$=\frac{p^2+p^2q-p^2+q(q-1)^2}{p^2+(q-1)^2}=\frac{q\left\{p^2+(q-1)^2\right\}}{p^2+(q-1)^2}=q$$

311. **(d)** Given α, β and γ are the roots of the equations

$x^3+px^2+qx+r=0$

$\alpha+\beta+\gamma=-p$ ...(i)

$\alpha\beta+\beta\gamma+\gamma\alpha=q$ ...(ii)

and $\alpha\beta\gamma=-r$ ...(iii)

Now, $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)$

$$=1+(\alpha^2+\beta^2+\gamma^2)+\left((\alpha\beta)^2+(\beta\gamma)^2+(\gamma\alpha)^2\right.$$
$$\left.+(\alpha\beta\gamma)^2\right.$$

$$=1+\left[(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)\right]$$
$$+\left[(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)+(\alpha\beta\gamma)^2\right]$$

From Eq. (i), (ii) and (iii), we get

$$=1+\left[p^2-2q\right]+\left[q^2-2rp\right]+r^2 \;=(q-1)^2+(r-p)^2$$

312. **(a)** Given equation is,

$(x-1)^3+64=0$; $(x-1)^3=-64$

$x=-3,\,-4\omega+1,\,-4\omega^2+1$

Sum of complex roots are

$-4\omega+1-4\omega^2+1=-4(\omega+\omega^2)+2$

$=-4(-1)+2=4+2=6 \quad [\because 1+\omega+\omega^2=0]$

313. **(c)** Given, α and β are the roots of $x^2+2x+c=0$

$\alpha+\beta=\dfrac{-2}{1}=-2$ ...(i)

$\alpha\beta=\dfrac{c}{1}=c$ ...(ii)

It is given that, $\alpha^3+\beta^3=4$

$(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)=4$

$(-2)[(-2)^2-3\times c]=4$ [From Eqs. (i) and (ii)]

$\Rightarrow -3c=-6 \Rightarrow c=2.$

314. **(c)** Whose roots are $\dfrac{1-\alpha}{\alpha}$ and $\dfrac{1-\beta}{\beta}$, equation will be

$$x^2-\left(\frac{1-\alpha}{\alpha}+\frac{1-\beta}{\beta}\right)x+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1-\beta}{\beta}\right)=0$$

$$\alpha\beta x^2-(\alpha+\beta-2\alpha\beta)x+(1-(\alpha+\beta)+\alpha\beta)=0$$

$$\frac{c}{a}x^2-\left(-\frac{b}{a}-\frac{2c}{a}\right)x+\left(1+\frac{b}{a}+\frac{c}{a}\right)=0$$

$$cx^2+(b+2c)x+(a+b+c)=0$$

Comparing the above equation with $px^2+qx+r=0$, we get $r=a+b+c$.

315. **(a)** $(|x|-8)\,(|x|+3)=0$

$\Rightarrow$ $|x|-8$ or $|x|=-3$ is rejected. $\Rightarrow$ $|x|=8$ $\Rightarrow$ $x=\underline{+}8$

∴ The roots of this equation. $ax=8$ and -8.

Hence, product of roots $8\times(-8)=-64$

and sum of roots $=+8-8=0$

316. **(a)** Given that the roots of equation

$(b-c)x^2+(c-a)x+(a-b)=0$ are equal, so

$D=0$

$(c-a)^2-4\,(a-b)\,(b-c)=0 \Rightarrow [(c+a)-(2b)]^2=0$

$c+a-2b=0 \Rightarrow$ $2b=a+c$; a, b and c are in AP.

317. **(b)** Roots of the equation is in G.P. So let $\dfrac{a}{r}$, a and ar are the roots of equation.

Then, product of roots

$\dfrac{a}{r}\cdot a\cdot ar=\dfrac{D}{A} \Rightarrow \dfrac{a}{r}\cdot a\cdot ar=8 \Rightarrow a^3=8 \;\therefore\; a=2$

Therefore, at $a=2$ is the root of equation.

$\therefore \quad x^3-kx^2+14x-8=0$

$8-4k+28-8=0 \Rightarrow k=7$

318. **(c)** Given, $3\cos A+2=0$

$\Rightarrow \quad \cos A=\dfrac{-2}{3} \;\Rightarrow \sin A=\dfrac{\sqrt{5}}{3}$ and $\tan A=\dfrac{-\sqrt{5}}{2}$

Since, sin*A* and tan*A* are the roots of required equation.

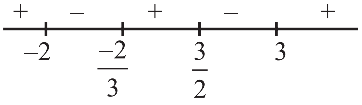
$x^2-(\sin A+\tan A)x+\sin A\tan A=0$

$$\Rightarrow \quad x^2+\frac{\sqrt{5}}{6}x-\frac{5}{6}=0$$

$$\Rightarrow \quad 6x^2+\sqrt{5}x-5=0$$

319. **(b)** Since, $\dfrac{7x^2-5x-18}{2x^2+x-6}<2$

$$\Rightarrow \frac{3x^2-7x-6}{(x+2)(2x-3)}<0 \Rightarrow \frac{(x-3)(3x+2)}{(x+2)(2x-3)}<0$$



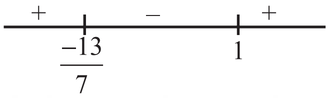
So, $x \in \left(-2, \dfrac{-2}{3}\right) \cup \left(\dfrac{3}{2}, 3\right)$.

320. **(c)** Since, $x^2-(3k+1)x+4k^2+3k-3>0\ \forall\ x \in \boldsymbol{R}$

So, $1 > 0$ and $b^2 - 4ac < 0$

$\Rightarrow\ \{-(3k+1)\}^2-4\times 1\times(4k^2+3k-3)<0$

$\Rightarrow\ (7k+13)(k-1)>0$



So, $k \in \left(-\infty, \dfrac{-13}{7}\right) \cup (1, \infty)$.

321. **(c)** $-1<\dfrac{2x^2+ax+2}{x^2+x+1}<3$

Consider $-1<\dfrac{2x^2+ax+2}{x^2+x+1} \Rightarrow 3x^2 + (a + 1)\,x + 3 > 0$

$\Rightarrow D < 0 \Rightarrow (a + 7)\,(a - 5) < 0 \Rightarrow a \in (-7, 5)$ ...(i)

Also, $\dfrac{2x^2+ax+2}{x^2+x+1}<3 \Rightarrow -x^2 + (a - 3)\,x - 1 < 0$

$\Rightarrow D < 0 \Rightarrow (a - 1)\,(a - 5) < 0 \Rightarrow a \in (1, 5)$ ...(ii)

So, from (i) and (ii), $a \in (1, 5)$

322. **(b)** $p(x) = 3x^5 - 4x^4 + 5x^3 - 3x^2 + 6x - 8$

$t(x) = x^2 + x - 3$

By dividing p(x) by t(x), the quotient is

$3x^3 - 7x^2 + 21x - 45$ and remainder is $114x - 143$.

So, $p(x) = t(x)\,(3x^3 - 7x^2 + 21x - 45) + 114x - 143$

Thus, quotient is $3x^3 - 7x^2 + 21x - 45$

323. **(c)** $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$ ...(i)

$\because\ a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$

Here $a_0 = -a_n$, $a_1 = -a_{n-1}$ is class II reciprocal equation.

So, eq (i) is an odd degree reciprocal equation

By inspection $x = 1$ is a root

Using synthetic division

1	1	–5	9	–9	5	–1
	0	1	–4	5	–4	1
	1	–4	5	–4	1	0

$x^4 - 4x^3 + 5x^2 - 4x + 1 = 0$ ...(ii)

Equation (ii) is an even degree reciprocal equation of class I.

$$\frac{x^4-4x^3+5x^2-4x+1}{x^2}=0 \Rightarrow \left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)+5=0 \quad ...(iii)$$

Let $x+\dfrac{1}{x}=a.$ So, Eq (iii) implies.

$(a^2 - 2) - 4a + 5 = 0 \Rightarrow (a - 3)\,(a - 1) = 0$

$\Rightarrow a = 1$ or $a = 3$

$\Rightarrow a = 1$ $\qquad a = 3$

$\Rightarrow x+\dfrac{1}{x}=1$ $\qquad x+\dfrac{1}{x}=3$

$\Rightarrow x^2 - x + 1 = 0$ $\qquad \Rightarrow x^2 - 3x + 1 = 0$

$x=\dfrac{1\pm i\sqrt{3}}{2}$ $\qquad x=\dfrac{3\pm\sqrt{5}}{2}$

So, $\alpha_1 = 1$, $\alpha_2=\dfrac{1+i\sqrt{3}}{2}, \alpha_3=\dfrac{1-i\sqrt{3}}{2}$

$\alpha_4=\dfrac{3+\sqrt{5}}{2}, \alpha_5=\dfrac{3-\sqrt{5}}{2}$

$$\frac{1}{\alpha_1^2}+\frac{1}{\alpha_2^2}+\frac{1}{\alpha_3^2}+\frac{1}{\alpha_4^2}+\frac{1}{\alpha_5^2}=1+\left(\frac{2}{1+i\sqrt{3}}\right)^2+$$

$$\left(\frac{2}{1-i\sqrt{3}}\right)^2+\left(\frac{2}{3+\sqrt{5}}\right)^2+\left(\frac{2}{3-\sqrt{5}}\right)^2$$

$$=1+\frac{4-12+4-12+36+20+36+20}{16} = 1 + 6 = 7$$

324. **(a)** $x^9 - x^5 + x^4 - 1 = 0 \Rightarrow (x^5 + 1)\,(x^4 - 1) = 0$

$\Rightarrow\ x^4 = 1\ \Rightarrow\ x = 1, -1, i, -i$

$\Rightarrow\ x^5 = -1\ \Rightarrow\ x = -1,\ e^{-\frac{i\pi}{5}}, e^{\frac{i\pi}{5}}, e^{-\frac{3\pi i}{5}}, e^{\frac{i3\pi}{5}}$

No. of real roots is $n = 3$, $m = 2$, $k = 1$

$\therefore\ m\cdot n\cdot k=3\times 2\times 1=6.$

325. **(a)** $\dfrac{x\quad - x +}{x\quad + x +}$

$$\Rightarrow\ \frac{dy}{dx}=\frac{(2x-1)(x^2+x+1)-(2x+1)(x^2-x+1)}{(x^2+x+1)^2}=0$$

$$\Rightarrow\ \frac{2x^2-2}{(x^2+x+1)^2}=0\ \Rightarrow\ x=\pm 1$$

Maximum value $(\alpha)=f(-1)=\dfrac{1+1+1}{1-1+1}=3$

Minimum value $(\beta)=f(1)=\dfrac{1}{3}\ \Rightarrow\ \alpha+\beta=\dfrac{10}{3}.$

326. **(c)** α is root of $x^2 - 5x + \lambda = 0$ and $x^2 - 8x - 2\lambda = 0$

$\Rightarrow\ \alpha^2-5\alpha+\lambda=0$ ...(i)

and $\alpha^2-8\alpha-2\lambda=0$ ...(ii)

Subtracting (ii) from (i), we get $3\alpha+3\lambda=0 \Rightarrow \alpha=-\lambda$

As α is root of $x^2 - 5x + \lambda = 0$

$\Rightarrow\ \lambda^2+5\lambda+\lambda=0 \Rightarrow \lambda^2+6\lambda=0 \Rightarrow \lambda=-6\ [\because \lambda \neq 0]$

$\Rightarrow\ \alpha = 6$

Now, α, β are roots of $x^2 - 5x + (-6) = 0$

$\Rightarrow\ \alpha + \beta = 5 \Rightarrow\ \beta = -1$

And, α, γ are roots of $x^2 - 8x + 12 = 0$

$\Rightarrow\ \alpha + \gamma = 8 \Rightarrow\ \gamma = 2$

So, $\alpha + \beta + \gamma + \lambda = 6 - 1 + 2 - 6 = 1$.

327. **(b)** Replacing 'x' by (x +1) we get,

$= (x + 1)^4 + 5\,(x + 1)^3 + 6\,(x + 1)^2 + 3\,(x + 1) + 9$

$\Rightarrow x^4 + 9x^3 + 27x^2 + 38x + 28 = 0$

328. **(b)** For double roots $D = b^2 - 4ac = 0$

$\Rightarrow 9(a+3)^2 + 36a = 0 \Rightarrow a^2 + 10a + 9 = 0$

$\Rightarrow (a+9)(a+1) = 0 \Rightarrow a = -1, -9$

For $a = -1$, equation becomes

$x^2 + 6x + 9 = 0 \Rightarrow x = -3$

For $a = -9$ equation becomes

$x^2 - 18x + 81 = 0 \Rightarrow x = 9\ \ \therefore \alpha = 9, \beta = -3\ (\because \alpha > \beta)$

Now, given equation becomes

$$x^2 + 9x + 3 = 0 \Rightarrow \left(x + \frac{9}{2}\right)^2 - \frac{69}{4} = 0$$

We know that $\left(x + \frac{9}{2}\right)^2 \geq 0\ \therefore\ \left(x + \frac{9}{2}\right)^2 - \frac{69}{4} \geq -\frac{69}{4}$

Hence, minimum value = $\frac{-69}{4}$

329. **(b)** $2x^2 + 3x - 2 = 0\quad \Rightarrow x = -2, \frac{1}{2}$

If $x = -2$ is the common root with $3x^2 + ax - 2 = 0$

then $3(-2)^2 + a(-2) - 2 = 0 \Rightarrow a = 5$

If $x = \frac{1}{2}$ is the common root with $3x^2 + ax - 2 = 0$

then $3\left(\frac{1}{2}\right)^2 + \frac{a}{2} - 2 = 0 \Rightarrow a = 2.5$

So, sum of all possible values of $a = 7.5$

330. **(b)** Let y be a common root of $x^2 + 5ax + 6 = 0$ and $x^2 + 3ax + 2 = 0$

So, $y^2 + 5ay + 6 = 0$ and $y^2 + 3ay + 2 = 0$

$\Rightarrow\ 5ay + 6 = 3ay + 2\ \Rightarrow\ 2ay = -4\ \Rightarrow\ ay = -2$

So, $y^2 + 5\cdot(-2) + 6 = 0 \Rightarrow y^2 - 4 = 0\ \Rightarrow\ y = \pm 2$.

331. **(d)** Given : $16x^4 + 16x^3 - 4x - 1 = 0 = f(x)$ has a multiple root.

Let $\alpha = \beta$ be the multiple root.

Since α is root of $f(x)\ \therefore f(\alpha) = 0$

$\Rightarrow\ 16\alpha^4 + 16\alpha^3 - 4\alpha - 1 = 0$ ...(i)

As α is multiple root

$\Rightarrow f'(\alpha) = 0$

Now, $f'(x) = 64x^3 + 48x^2 - 4$

$f'(\alpha) = 0$

$\Rightarrow\ 64\alpha^3 + 48\alpha^2 - 4 = 0\ \Rightarrow\ 16\alpha^3 + 12\alpha^2 - 1 = 0$ ...(ii)

Subtracting (ii) from (i),

$16\alpha^4 - 12\alpha^2 + 1 - 4\alpha - 1 = 0$

$\Rightarrow\ 16\alpha^4 - 12\alpha^2 - 4\alpha = 0\ \Rightarrow\ 4\alpha(4\alpha^3 - 3\alpha - 1) = 0$

$\alpha \neq 0$ as it doesn't satisfy given equation

$\Rightarrow\ 4\alpha^3 - 3\alpha - 1 = 0\ \Rightarrow\ (\alpha - 1)(2\alpha + 1)^2 = 0$

$\alpha \neq 1$ as it doesn't satisfy given equation

$\Rightarrow\ 2\alpha + 1 = 0 \Rightarrow \alpha = \frac{-1}{2}$

So, $\alpha = \frac{-1}{2}$ is a root of the given equation

By using synthetic division

$g(x) = 16x^3 + 8x^2 - 4x - 2$

Since, $\alpha = \frac{-1}{2}$ is multiple root

By using synthetic division again

$h(x) = 16x^2 - 4 = 0$

$\Rightarrow\ x^2 = \frac{4}{16} = \frac{1}{4}\ \Rightarrow\ x = \pm\frac{1}{2}$

Hence, $\alpha = \frac{-1}{2}, \beta = \frac{-1}{2}, \gamma = \frac{-1}{2}, \delta = \frac{1}{2}$

So, $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} + \frac{1}{\delta^4} = 2^4 + 2^4 + 2^4 + 2^4$

$= 16 + 16 + 16 + 16 = 64$.

332. **(b)** Given : $\left(125x^2 - \frac{27}{x}\right)^{\frac{-2}{3}} = \frac{x^{\frac{2}{3}}}{\sqrt[3]{(125x^3 - 27)^2}}$

For expansion to be valid, $x \neq 0$ and $125x^3 - 27 \neq 0$

$\Rightarrow\ x^3 \neq \frac{27}{125} \Rightarrow x \neq \frac{3}{5}$

$\therefore$ From given options domain will be $\left(-\infty, \frac{-3}{5}\right) \cup \left(\frac{3}{5}, \infty\right)$.

333. **(a)** Given expression is $7 + 6x - 3x^2 = f(x)$ ...(i)

Extreme value of $f(x)$ is $\beta = \frac{-D}{4a}$ which occurs at

$\alpha = \frac{-b}{2a}$

$a = -3, b = 6, c = 7$ (on comparing (i) with ax^2+bx+c)

$\beta = \frac{-(36+84)}{(4\times -3)} = 10\ ;\ \ \alpha = \frac{-6}{-6} = 1$

$x^2 + \alpha x - \beta = 0$ reduces to $x^2 + x - 10 = 0$ ...(ii)

Let α_1 and α_2 be roots of equation (ii)

$\alpha_1^2 + \alpha_2^2 = (\alpha_1 + \alpha_2)^2 - 2\alpha_1\alpha_2$

$\alpha_1^2 + \alpha_2^2 = (-1)^2 - 2\times(-10) = 1 + 20 = 21 \therefore \alpha_1^2 + \alpha_2^2 = 21$.

334. **(b)** Let $f(x) = 4 + 3x - 7x^2$

$$f'(x) = 3 - 14x = 0 \Rightarrow x = \frac{3}{14} = \alpha$$

$$f''(x) = -14 < 0$$

$\therefore f(x)$ is maximum at $x = \frac{3}{14}$ and maximum

Value is $f\left(\frac{3}{14}\right) = 4 + \frac{9}{14} - \frac{63}{196} = \frac{847}{196} = M$

And $g(x) = 5x^2 - 2x + 1$

$$g'(x) = 10x - 2 = 0 \Rightarrow x = \frac{1}{5} = \beta$$

$$g''(x) = 10 > 0$$

$\therefore g(x)$ is minimum at $x = \frac{1}{5}$ and minimum value is

$$g\left(\frac{1}{5}\right) = 5 \times \frac{1}{25} - \frac{2}{5} + 1 = \frac{4}{5} = m$$

Now, $\frac{28(M-\alpha)}{5(m+\beta)} = \frac{28\left(\frac{847}{196} - \frac{3}{14}\right)}{5\left(\frac{4}{5} + \frac{1}{5}\right)} = \frac{115}{5} = 23.$

335. **(d)** $x^{14} + x^9 - x^5 - 1 = 0$

$\Rightarrow (x^9 - 1)(x^5 + 1) = 0 \Rightarrow x^5 = -1$ or $x^9 = 1$

$\Rightarrow x^5 = \cos 180° + i \sin 180°$

$$\Rightarrow x = \cos\left(\frac{180}{5}\right) + i\sin\left(\frac{180}{5}\right) \Rightarrow x = \cos 36° + i \sin 36°$$

$$\Rightarrow x = \frac{\sqrt{5}+1}{14} + \frac{i\sqrt{10-2\sqrt{5}}}{4}.$$

336. **(d)** We know that condition of exactly one common root of quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ is

$$(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$$

Here, $a_1 = 2$, $b_1 = a$, $c_1 = -2$, $a_2 = 1$, $b_2 = 1$ and $c_2 = 2a$

$\therefore (-2-4a)^2 = (2a^2+2)(2-a)$

$\Rightarrow 2a(a^2 + 6a + 9) = 0 \Rightarrow 2a(a+3)^2 = 0$

$\therefore a = -3 \qquad (\because a \neq 0)$

$\therefore$ Given equation is

$-3x^2 - 4x + 6 = 0 \Rightarrow 3x^2 + 4x - 6 = 0$

$$\Rightarrow x = \frac{-4 \pm \sqrt{88}}{6} = \frac{-2 \pm \sqrt{22}}{3} \quad \therefore x = \frac{-2+\sqrt{22}}{3}.$$

337. **(c)** We have $f(x) = x^{12} - x^9 + x^4 - x + 1$

where, $f(x) > 0$

Case I: When $x \geq 1$

We get $f(x) \geq 1 \Rightarrow f(x) > 0\ \forall,\ x \geq 1$

Case II: When $x \leq 0$

$x^{12} + x^4 + 1 > 0$ and $-(x^9 + x)$ is also positive

$\Rightarrow f(x) > 0\ \ \forall\ x \leq 0$

Case III: When $0 < x < 1$

we have $f(x) = x^{12} - x^9 + x^4 - x + 1$

$\Rightarrow (x^8 + 1)(x^4 - x + 1)$

$\because \quad x^8 + 1 > 0$

now $x^4 < x \Rightarrow x^4 + 1 < x + 1$

$\Rightarrow x^4 + 1 - x < 1$ but $x^4 + 1 - x > 0$ as $x \in (0, 1)$

$\Rightarrow f(x) > 0$

from (I), (II), (III) cases

$x \in (-\infty, \infty)$

338. **(b)** $x + y + z = 1$; $x^2 + y^2 + z^2 = 1$; $x^3 + y^3 + z^3 = 1$

It is evident from all 3 that equation satisfies only if 2 of them are zero and third is 1.

There are 3 solution sets of (x, y, z)

(0, 0, 1), (0, 1, 0) and (1, 0, 0)

339. **(b)** I → $|x|^2 - 4|x| + 3 < 0$

$$\text{Now } |x|^2 - 4|x| + 3 = \begin{Bmatrix} x^2 - 4x + 3\ ; & x \geq 0 \\ x^2 + 4x + 3\ ; & x < 0 \end{Bmatrix}$$

When $x \geq 0$ then $x^2 - 4x + 3 < 0 \Rightarrow (x-1)(x-3) < 0$
$\Rightarrow x \in (1, 3)$
When $x < 0$ then $x^2 + 4x + 3 < 0$
$\Rightarrow (x+3)(x+1) < 0 \Rightarrow x \in (-3, -1)$
$\therefore$ Set of solutions $= (-3, -1) \cup (1, 3)$
⇒ Statement Ist is incorrect
II → Here if we take any value in $x < 3$ or $x > 5$ then it satisfy the given inequality $x^2 - 8x + 15 > 0$
⇒ Statement IInd is correct.

340. **(c)** From option (A) with equation $2x^2 + 4x + 5$.
$P(x) = 2x^2 + 4x + 5 = 2(x+1)^2 + 3$
Polynomial $P(x)$ has minimum value when term $(x+1)$ becomes 0.
Then the minimum will be 3 and maximum and define
So, (A) → option (IV)

From (C), $1 \leq \frac{3x^2 - 5x + 6}{x^2 + 1} \leq 2$

Multiply both sides by $(x^2 + 1)$
$x^2 + 1 \leq 3x^2 - 5x + 6 \leq 2x^2 + 2$

Take,

$$\begin{array}{l|l} 3x^2-5x+6\le 2x^2+2 & 3x^2-5x+6\ge x^2+1 \\ x^2-5x+4\le 0 & 2x^2-5x+5\ge 0 \\ (x-4)(x-1)\le 0 & x\in\left(-\infty,\dfrac{-5+\sqrt{15}}{2}\right] \\ x\le 4, x\ge 1 & \cup\left[\dfrac{5-\sqrt{15}}{2},\infty\right) \end{array}$$

So, $1 \le x \le 4$.

Therefore, option (C) → option (V)

341. **(b)** Let $y=\dfrac{6x^2-18x+21}{6x^2-18x+17}$

$\Rightarrow$ $x^2(6y-6)+x(18-18y)+17y-21=0$

$\because$ $x \in \mathbb{R}$

$\therefore$ $B^2-4AC\ge 0$

$\therefore$ $(18-18y)^2-4(6y-6)(17y-21)\ge 0$

$\Rightarrow$ $7y^2+22y+15\le 0 \Rightarrow (7y-15)(y-1)\le 0$

$\Rightarrow$ $y\in\left[1,\dfrac{15}{7}\right] \Rightarrow m=\dfrac{15}{7}, n=1$

$\therefore$ $14m-7n=14\times\dfrac{15}{7}-7=30-7=23$

342. **(c)** $f(x)=x^4-2x^3+3x^2-ax+b$

Given that, if $f(x)$ is divided by $x-1$ and $x+1$, Then reminder is 5 and 19. It means $f(1)=5$ and $f(-1)=19$, then

$f(1)=1-2+3-a+b=2-a+b=5$

$\Rightarrow$ $b-a=3$ ... (i)

and $f(-1)=19$

$\Rightarrow$ $b+a=13$ ... (ii)

Solving Eqs. (i) and (ii), we obtain $a=5$, $b=8$

$\Rightarrow f(x)=x^4-2x^3+3x^2-5x+8$

Put $x=2$, $f(2)=10$

$\therefore$ If $f(x)$ is divided by $x-2$, then the remainder is 10.

343. **(d)** We have, $\dfrac{x^2-bx}{ax-c}=\dfrac{m-1}{m+1}$ ...(i)

$x^2(m+1)-xb(m+1)+ax(m-1)+c(m-1)=0$

Let y and $-y$ are roots of Eq. (i), then

Sum of roots $\Rightarrow y+(-y)=\dfrac{b(m+1)+a(m-1)}{m+1}$

$\Rightarrow (a+b)m=-(b-a) \Rightarrow m=\dfrac{a-b}{a+b}$

344. **(b)** Given equation,

$f(x)=2x^3+mx^2-13x+n$

Let α, β, γ are the roots of $f(x)=0$

$\alpha+\beta+\gamma=-\dfrac{m}{2}=2+3+k$ ...(i)

$\alpha\beta+\beta\gamma+\gamma\alpha=-\dfrac{13}{2}=6+5k$ ...(ii)

$\alpha\beta\gamma=-\dfrac{n}{2}=6k$ ...(iii)

From Eq. (ii), $-13=12+10k$

$\Rightarrow k=\dfrac{-5}{2}$

$n=30$ and $m=-5$

345. **(b)** Given $e^{4t}-10e^{3t}+29e^{2t}-22e^{t}+4=0$ ...(i)

Let $e^t=x$

then, $x^4-10x^3+29x^2-22x+4=0$

Here, roots are x_1, x_2, x_3, x_4

Hence product of roots $x_1.x_2.x_3.x_4=4 \Rightarrow e^{t_1+t_2+t_3+t_4}=4$

$\Rightarrow t_1+t_2+t_3+t_4=\log_e 4=\log_e 2^2=2\log_e 2$

346. **(b)** For equation x^2+bx+5, the minimum value α occurs at $x=\dfrac{-b}{2a}$ *i.e.*, $x=\dfrac{-b}{2}$ and for maximum value β occurs at $x=\dfrac{-b}{2a}$, *i.e.*, $x=\dfrac{+a}{2}$

Put $x=\dfrac{-b}{2}$, then $\alpha=5-\dfrac{b^2}{4}$ ...(i)

Now, put $x=\dfrac{a}{2}$, then $\beta=5-\dfrac{a^2}{4}$ ...(ii)

Given, $x^2-10x+24\le 0$

$\Rightarrow (x-4)(x-6)\le 0 \Rightarrow 4\le x\le 6$

$\therefore$ $\alpha=4$ and $\beta=6$

From Eqs. (i) and (ii), we get

$b^2=4$ and $a^2=4$

Now, put the values of a^2 & b^2

$a^2b^2=16$

347. **(a)** For quadratic equation x^2+5x-2, the minimum value is M and its exists at a.

Here, $a>0$, then minimum value occurs at $x=\dfrac{-b}{2a}$

Then, $x=\dfrac{-5}{2}$

Now, put $x=\dfrac{-5}{2}$, then,

$M=\dfrac{-33}{4}$

Hence, $\dfrac{M}{a}=\dfrac{\frac{-33}{4}}{-5/2}=\dfrac{33}{4}\times\dfrac{2}{5}=3.3$

348. **(b)** Let α is the common root of the equation $x^3+x^2-2x-2=0$ and $x^3-x^2-2x+2=0$

So, α will satisfy both the equations

Now, $\alpha^3+\alpha^2-2\alpha-2=0$ ...(i)

and $\alpha^3-\alpha^2-2\alpha+2=0$ ...(ii)

On subtractiing Eq. (ii) from Eq. (i), we get

$2\alpha^2-2=0 \Rightarrow \alpha=\pm 1$.

349. **(a)** Given quadratic equations are

$3x^2-7x+2=0$ and $kx^2+7x-3=0$, so

Let α be the common root,

hence, $3\alpha^2-7\alpha+2=0$ ...(i)

and $k\alpha^2 + 7\alpha - 3 = 0$ ...(ii)

By cross multiplication, we get

$$\Rightarrow \quad \frac{\alpha^2}{7} = \frac{\alpha}{2k+9} = \frac{1}{7(k+3)}$$

k is positive, so

$49\ (k + 3) = 4k^2 + 36k + 81$

$\Rightarrow \quad 4k\ (k - 6) + 11\ (k - 6) = 0$

$\Rightarrow \quad k = 6$ [$\because$ k is positive]

350. **(c)** We have,

Equation $ax^2 + bx + c = 0$ has imaginary roots

$\therefore\ b^2 - 4ac < 0 \Rightarrow b^2 < 4ac$

Now, $f(x) = 3a^2x^2 + 6abx + 2b^2$

$$= 3a^2\left(x^2 + \frac{2b}{a}x\right) + 2b^2 \quad = 3a^2\left(x + \frac{b}{a}\right)^2 - b^2$$

$\therefore$ Minimum value of $f(x)$ is $-b^2 > -4ac$ [$\because\ b^2 < 4ac$]

351. **(d)** Given α, β are roots of equation $x^2 - |a|x - |b| = 0$

$$\text{and } \log_{|\alpha|}\left(\frac{x^2}{\beta^2}\right) = 1 \Rightarrow \frac{x^2}{\beta^2} = |\alpha| \Rightarrow x^2 = \beta^2|\alpha|$$

So, $\alpha + \beta = -|a| < 0$

So, $\beta < 0$ as $|\alpha| < |\beta|$

From Eq. (i), we get $x = \pm|\beta||\alpha|^{1/2}$

So, the positive root is $|\beta||\alpha|^{1/2} > \beta$ [as $\beta < 0$]

352. **(d)** Given, $1 \le \dfrac{3x^2 - 7x + 8}{x^2 + 1} \le 2 \quad \left[\because x^2 + 1 \ge 0\right]$

$\Rightarrow \quad x^2 + 1 \le 3x^2 - 7x + 8 \le 2(x^2 + 1)$

$\Rightarrow \quad 2x^2 - 7x + 7 \ge 0$ and $x^2 - 7x + 6 \le 0$

$\because \quad 2x^2 - 7x + 7 \ge 0;\ D = (7)^2 - 4.2.(7)$

$D < 0$ [$\because x \in R$]

$\Rightarrow$ Now $f(x) = 2x^2 - 7x + 7 > 0 \Rightarrow x^2 - 7x + 6 \le 0$

$\Rightarrow \quad (x - 1)(x - 6) \le 0 \Rightarrow x \in [1, 6]$

Hence, minimum and maximum values are 1 and 6.

353. **(a)** Given that roots of the quadratic equation are real, so $D \ge 0$

$\Rightarrow \quad 4(a+b+c)^2 - 4 \times 3\lambda(ab+bc+ca) \ge 0$

$\Rightarrow \quad \lambda \le \dfrac{(a+b+c)^2}{3(ab+bc+ca)}$. Now, for scalane triangle.

$\because$ For $\Delta ABC\ |b-c| < a, |c-a| < b$ and $|a-b| < c$

$\Rightarrow \quad (b-c)^2 + (c-a)^2 + (a-b)^2 < a^2 + b^2 + c^2$

$$\Rightarrow \quad \frac{(a+b+c)^2}{3(ab+bc+ca)} < \frac{4}{3} \qquad \therefore\ \lambda < \frac{4}{3}$$

354. **(c)** Let $y = \dfrac{ax^2 - 2x + 3}{2x - 3x^2 + a}$

$x^2(a + 3y) - 2x(y + 1) + 3 - ay = 0$

As $x \in R$, $D \ge 0$ for real values,

$\therefore\ 4(y + 1)^2 - 4(a + 3y)\ (3 - ay) \ge 0$

$y^2(3a + 1) + (a^2 - 7)y + 1 - 3a \ge 0$

As $y \in R$, $D \le 0$

$\therefore\ (a^2 - 7)^2 + 4(3a + 1)(3a - 1) \le 0$

$a^4 + 22a^2 + 45 \le 0 \Rightarrow [\because\ a^4 + 22a^2 + 45 > 0,\ \forall\ a \in R]$

$a \in \phi$

355. **(b)** We have $x^2 - 4ax + 1 - 3a + 4a^2 = 0$

When both roots greater than 1 then $D > 0$

$D = b^2 - 4ac$

$= 16a^2 - 4 \times 1 \times (4a^2 - 3a + 1) \Rightarrow\ a > \dfrac{1}{3}$.

So, $a \in \left(\dfrac{1}{3}, \infty\right)$

For $x = 1$, $f(1) = 1 - 4a \times 1 + 1 - 3a + 4a^2$

$4a^2 - 7a + 2 = 0$

$$a = \frac{7 \pm \sqrt{49 - 32}}{8} = \frac{7 \pm \sqrt{17}}{8}$$

Required interval $a \in \left(\dfrac{7 \pm \sqrt{17}}{8}, \infty\right)$

356. **(c)** Let α, β, γ are roots of equation

$x^3 - ax^2 + ax - 1 = 0$ …(i)

$\therefore\ \alpha + \beta + \gamma = a\ ;\ \alpha\beta + \beta\gamma + \gamma\alpha = a\ ;\ \alpha\beta\gamma = -1$

Cubic equation whose roots are α^2, β^2, γ^2 is

$x^2 - (\alpha^2 + \beta^2 + \gamma^2)x^2 + (\alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2)x$

$- \alpha^2\beta^2\gamma^2 = 0$ …(ii)

Eqs. (i) and (ii) are identical.

$$\therefore \quad \frac{a}{\alpha^2 + \beta^2 + \gamma^2} = \frac{a}{\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2} = \frac{1}{\alpha^2\beta^2\gamma^2}$$

$a = \alpha^2 + \beta^2 + \gamma^2$ [$\alpha\beta\gamma = -1$]

$a = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$a = a^2 - 2a \quad \Rightarrow\ a^2 = 3a$

$\Rightarrow\ a = 3$ [$\because$ a is non-zero real]

357. **(b)** We have $x^3 + px^2 + qx + r = 0$

$\alpha + \beta + \gamma = -p;\ \alpha\beta\gamma = -r\ ;\ \alpha\beta + \beta\gamma + \gamma\alpha = q$

So, $(\alpha + \beta)\ (\beta + \gamma)\ (\gamma + \alpha) = (-p - \gamma)\ (-p - \alpha)\ (-p - \beta)$

$= -p^3 - p^2(\alpha + \beta + \gamma) - p(\alpha\beta + \beta\gamma + \alpha\gamma) - \alpha\beta\gamma$

$= -p^3 + p^3 - p \times q + r\ = r - pq$

358. **(b)** We have $\sqrt{2} + e^{\cosh^{-1}x} - e^{\sinh^{-1}x} = 0$

$\cosh^{-1}x = \log\left(x + \sqrt{x^2 - 1}\right)$, $\sinh^{-1}x = \log\left(x + \sqrt{x^2 + 1}\right)$

$\Rightarrow \sqrt{2} + e^{\log(x + \sqrt{x^2 - 1})} - e^{\log(x + \sqrt{x^2 + 1})} = 0$

$\Rightarrow \sqrt{2} + x + \sqrt{x^2 - 1} - x - \sqrt{x^2 + 1} = 0 \Rightarrow 2\sqrt{2(x^2 - 1)} = 0$

$\Rightarrow x^2 - 1 = 0,\ \Rightarrow\ x = \pm 1 \Rightarrow x = 1.$

359. **(a)** We have, $f(x) = x^2 + 2bx + 2c^2$

$= x^2 + 2bx + b^2 + 2c^2 - b^2\ = (x + b)^2 + 2c^2 - b^2$

$\therefore$ Minimum value, $f(x) = 2c^2 - b^2$

Now, $g(x) = -x^2 - 2cx + b^2$

$= -[(x + c)^2 - b^2 - c^2] = -(x + c)^2 + b^2 + c^2$

$\therefore$ Maximum value, $g(x) = b^2 + c^2$

As given in the question,

Min $[f(x)] >$ Max $[g(x)]$

$2c^2 - b^2 > b^2 + c^2 \Rightarrow c^2 > 2b^2$

360. **(a)** Given Equation,

$x^3 + qx + r = 0$

Since, a, b and c are the roots of equation

$a + b + c = 0\ ;\ ab + bc + ca = q$ and $abc = -r$

As we have,

$a + b + c = 0$

$(a + b + c)^2 = 0$

$a^2 + b^2 + c^2 = -2q$

Now, $(a - b)^2 + (b - c)^2 + (c - a)^2$

$= 2(a^2 + b^2 + c^2) - 2(ab + bc + ca)$ [From Eq. (i)]

$= -4q - 2q = -6q$

361. (a) Let the two roots of equation is $a - a$

Now, sum of three roots = $2p$

So, third root will be $2p$,

Product of two consecutive roots = $3q$

$a \times (-a) + a \times (2p) + (-a) \times 2p = 3q$

$\Rightarrow -a^2 = 3q$

As we know, product of roots = $3q$

$a \times (-a) \times 2p = 4r$

$$\Rightarrow r = \frac{3pq}{2}$$

362. (b) Given that, $z^3 + 2z^2 + 2z + 1 = 0$

$\Rightarrow (z + 1)(z^2 + z + 1) = 0$

$\therefore z + 1 = 0$ or $z^2 + z + 1 = 0$

$z = -1$ or $z = \omega, \omega^2$

Hence, roots of $z^3 + 2z^2 + 2z + 1$ are $-1, \omega, \omega^2$

when $z = -1$

$z^{2018} + z^{2017} + 1 = (-1)^{2018} + (-1)^{2017} + 1$

$= +1 - 1 + 1 \neq 0$

when $z = w$,

$z^{2018} + z^{2017} + 1 = (\omega)^{2018} + (\omega)^{2017} + 1 = \omega^2 + \omega + 1 = 0$

$[\because \omega^2 + \omega + 1 = 0]$

when $z = \omega^2$; $z^{2018} + z^{2017} + 1 = (\omega^2)^{2018} + (\omega^2)^{2017} + 1$

$= \omega + \omega^2 + 1 = 0$

Thus, the common roots are ω and ω^2 by checking options $z^4 + z^2 + 1 = 0$

Put $z = \omega$; $\omega^4 + \omega^2 + 1 = \omega + \omega^2 + 1 = 0$ and put $z = \omega^2$

$(\omega^2)^4 + (\omega^2)^2 + 1 = \omega^8 + \omega^4 + 1 = \omega^2 + \omega + 1 = 0.$

Hence, $z^4 + z^2 + 1 = 0$ satisfy by the both common roots.

363. (b) Given that, $x^2 - 3ax + 14 = 0$

and $x^2 + 2ax - 16 = 0$ have a common root.

Let the common roots is α.

Then, $\alpha^2 - 3a\alpha + 14 = 0$

and $\alpha^2 + 2a\alpha - 16 = 0$

by cross multiplication method

$$\frac{\alpha^2}{48a - 28a} = \frac{\alpha}{14 + 16} = \frac{1}{2a + 3a}$$

$$\Rightarrow \alpha = \frac{2}{3}a \text{ and } \alpha = \frac{6}{a}. \text{ So, } \frac{2a}{3} = \frac{6}{a} \Rightarrow a = \pm 3$$

Now, $a^4 + a^2 = (\pm 3)^4 + (\pm 3)^2 = 81 + 9 = 90$

364. (b) We have, $x^2 - x + 1 = 0$

$x = \omega, \omega^2$

So, $\alpha = \omega$ and $\beta = \omega^2$

Now, $\alpha^{2015} = \omega^{2015} = \omega^{2013}\omega^2 = \omega^{3\times 671}\omega^2 = \omega^2$

$(\because \omega^3 = 1)$

$\beta^{2015} = (\omega^2)^{2015} = \omega^{4030} = \omega^{4029}\omega = \omega^{3\times 1343} = \omega$

$\therefore \alpha^{2015} + \beta^{2015} = \omega^2 + \omega = -1 \quad (\because \omega^2 + \omega + 1 = 0)$

and $\alpha^{2015} \cdot \beta^{2015} = \omega^2 \cdot \omega = \omega^3 = 1$

$\therefore$ Equation whose roots are α^{2015} and β^{2015} will be

$x^2 - (-1)x + 1 = 0$

$x^2 + x + 1 = 0$

365. (d) Given the roots of $x^3 - 5x + 4 = 0$ are α, β and γ.

$\therefore \alpha + \beta + \gamma = 0$; $\alpha\beta + \beta\gamma + \gamma\alpha = 5$ and $\alpha\beta\gamma = -4$

Since, $\alpha + \beta + \gamma = 0$

$\therefore \alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma = 3 \times (-4) = -12$

Therefore, $(\alpha^3 + \beta^3 + \gamma^3)^2 = (-12)^2 = 144$

366. (c) We have given the roots of $x^3 + x^2 + 2x + 3 = 0$ are α, β and γ.

$\therefore \alpha + \beta + \gamma = -1$ ...(i)

$\alpha\beta + \beta\gamma + \gamma\alpha = 2$ ...(ii)

and $\alpha\beta\gamma = -3$ ...(iii)

and it is also given that $(\alpha + \beta), (\beta + \gamma), (\gamma + \alpha)$ are roots of $f(x) = 0$, then the polynomial equation,

$$f(x) = x^3 - \{(\alpha+\beta) + (\beta+\gamma) + (\gamma+\alpha)\}x^2 + \{(\alpha+\beta)(\beta+\gamma)\} + \{(\beta+\gamma)(\gamma+\alpha) + (\alpha+\beta)(\gamma+\alpha)\}x + (\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$$

$\therefore (\alpha+\beta) + (\beta+\gamma) + (\gamma+\alpha) = 2(\alpha+\beta+\gamma) = -2$ ...(iv)

and $(\alpha+\beta)(\beta+\gamma) + (\beta+\gamma)(\gamma+\alpha) + (\alpha+\beta)(\gamma+\alpha)$

$= \alpha^2 + \beta^2 + \gamma^2 + 3(\alpha\beta + \beta\gamma + \gamma\alpha)$

By using Eq. (i), we get $\alpha + \beta + \gamma = -1$

On squaring both sides, we get $(\alpha + \beta + \gamma)^2 = 1$

$\alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) = 1 = \alpha^2 + \beta^2 + \gamma^2 = -3$

$\therefore (\alpha+\beta)(\beta+\gamma) + (\beta+\gamma)(\gamma+\alpha) + (\alpha+\beta)(\gamma+\alpha)$

$= -3 + 3(2) = -3 + 6 = 3.$ ...(v)

and $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha) = (-1-\gamma)(-1-\alpha)(-1-\beta)$

$= \{-1 - (-1) - 2 - (-3)\} = -1 + 1 - 2 + 3 = 1.$ ...(vi)

Hence, polynomial equation

$= x^3 - (-2)x^2 + 3x - 1$

$= x^3 + 2x^2 + 3x - 1$ [by Eqs. (iv), (v) and (vi)]

367. (c) We have, $f(x) = 9mx - 1 + \dfrac{1}{x} \geq 0$

For $f(x) \geq 0$, D should be less than or equal to 0.

$D = 1 - 36m \leq 0$

$$\therefore m \geq \frac{1}{36}$$