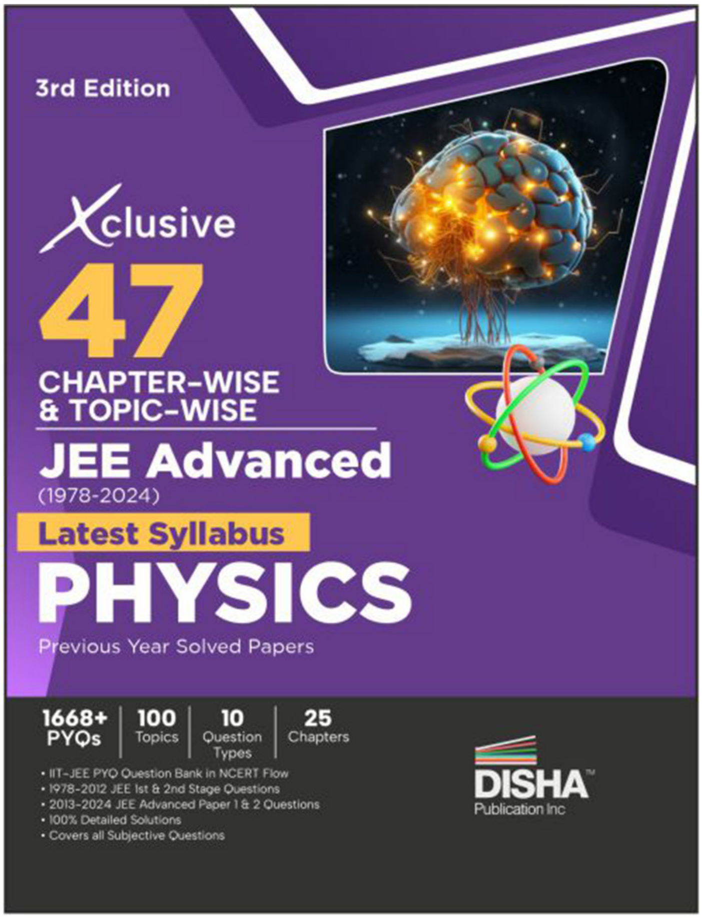
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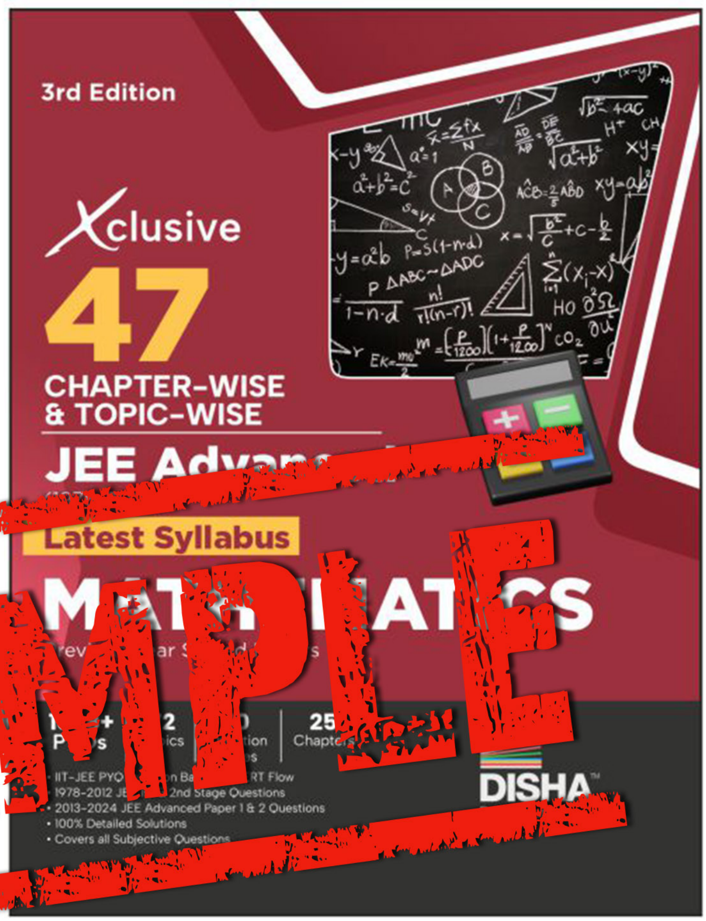
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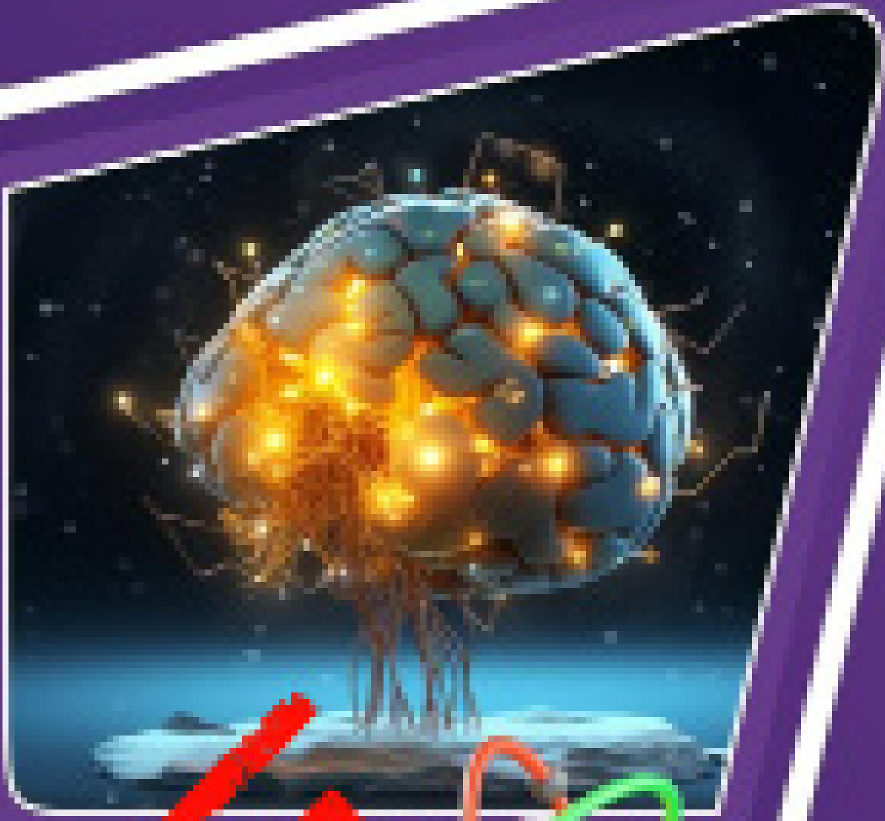

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Class XIIth B1 – B232

4 Laws of Motion

Topic-1: Ist, IInd & IIIrd Laws of Motion

1 MCQs with One Correct Answer

1. A block of mass 5 kg moves along the x-direction subject to the force $F = (-20x + 10)$ N, with the value of x in metre. At time $t = 0$ s, it is at rest at position $x = 1$ m. The position and momentum of the block at $t = \left(\frac{\pi}{4}\right)s$ are **[Adv. 2024]**

(a) –0.5 m, 5 kg m/s (b) 0.5 m, 0 kg m/s
(c) 0.5 m, –5 kg m/s (d) –1 m, 5 kg m/s

2. A particle of mass m is moving in the xy-plane such that its velocity at a point (x, y) is given as $\vec{v} = \alpha\left(y\hat{x} + 2x\hat{y}\right)$, where α is a non-zero constant. What is the force $\vec{F}$ acting on the particle? **[Adv. 2023]**

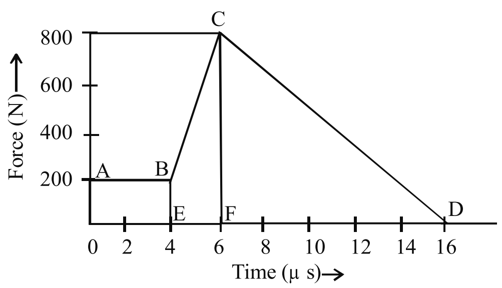
(a) $\vec{F} = 2m\alpha^2\left(x\hat{x} + y\hat{y}\right)$ (b) $\vec{F} = m\alpha^2\left(y\hat{x} + 2x\hat{y}\right)$
(c) $\vec{F} = 2m\alpha^2\left(y\hat{x} + x\hat{y}\right)$ (d) $\vec{F} = m\alpha^2\left(x\hat{x} + 2y\hat{y}\right)$

3. A particle moves in the X-Y plane under the influence of a force such that its linear momentum is $\vec{p}(t) = A\,[\hat{i}\cos(kt) - \hat{j}\sin(kt)]$, where A and k are constants. The angle between the force and the momentum is **[2007]**

(a) 0° (b) 30° (c) 45° (d) 90°

4 Fill in the Blanks

4. The magnitude of the force (in newtons) acting on a body varies with time t (in micro seconds) as shown in the fig AB, BC and CD are straight line segments. The magnitude of the total impulse of the force on the body from $t = 4$ μs to $t = 16$μs is ....Ns. **[1994 - 2 Marks]**



5 True / False

5. A rocket moves forward by pushing the surrounding air backwards. **[1980]**

6 MCQs with One or More than One Correct Answer

6. A reference frame attached to the earth **[1986 - 2 Marks]**

(a) is an inertial frame by definition.
(b) cannot be an inertial frame because the earth is revolving round the sun.
(c) is an inertial frame because Newton's laws are applicable in this frame.
(d) cannot be an inertial frame because the earth is rotating about its own axis.

9 Assertion and Reason Type Questions

7. **Statement-1 :** It is easier to pull a heavy object than to push it on a level ground and
Statement-2 : The magnitude of frictional force depends on the nature of the two surfaces in contact. **[2008]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement -1 is True, Statement-2 is False
(d) Statement -1 is False, Statement-2 is True

8. **Statement-1 :** A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.
Statement-2 : For every action there is an equal and opposite reaction. **[2007]**

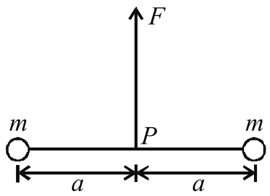
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

Topic-2: Motion of Connected Bodies, Pulley & Equilibrium of Forces

1 MCQs with One Correct Answer

1. Two particles of mass m each are tied at the ends of a light string of length $2a$. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance 'a' from the centre P (as shown in the figure).

Now, the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes $2x$, is **[2007]**



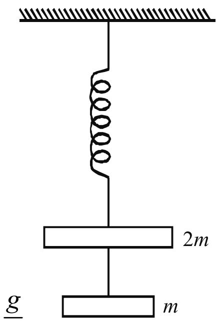
(a) $\frac{F}{2m}\frac{a}{\sqrt{a^2-x^2}}$

(b) $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$

(c) $\frac{F}{2m}\frac{x}{a}$

(d) $\frac{F}{2m}\frac{\sqrt{a^2-x^2}}{x}$

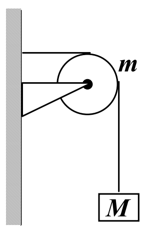
2. The string between blocks of mass m and 2m is massless and inextensible. The system is suspended by a massless spring as shown. If the string is cut find the magnitudes of accelerations of mass $2m$ and m (immediately after cutting) **[2006 - 3M, –1]**



(a) g, g (b) $g, \frac{g}{2}$

(c) $\frac{g}{2}, g$ (d) $\frac{g}{2}, \frac{g}{2}$

3. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by **[2001S]**



(a) $\sqrt{2}$ Mg

(b) $\sqrt{2}$ mg

(c) $\sqrt{(M+m)^2+m^2}\ g$

(d) $\sqrt{(M+m)^2+M^2}\ g$

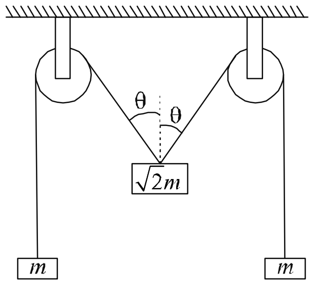
4. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be **[2001S]**

(a) 0^o

(b) 30^o

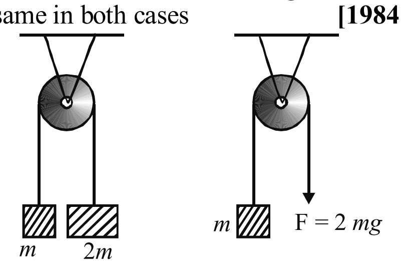
(c) 45^o

(d) 60^o



5 True / False

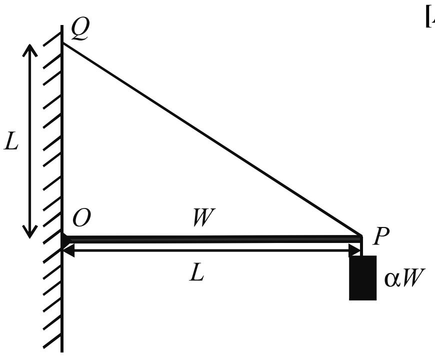
5. The pulley arrangements of Figs. (a) and (b) are identical. The mass of the rope is negligible. In (a) the mass m is lifted up by attaching a mass $2m$ to the other end of the rope. In (b), m is lifted up by pulling the other end of the rope with a constant downward force $F = 2\ mg$. The acceleration of m is the same in both cases **[1984 - 2 Marks]**



(a) (b)

6 MCQs with One or More than One Correct Answer

6. One end of a horizontal uniform beam of weight W and length L is hinged on a vertical wall at point O and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point Q, at a height L above the hinge at point O. A block of weight αW is attached at the point P of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of $(2\sqrt{2})$ W. Which of the following statement(s) is(are) correct ? **[Adv. 2021]**



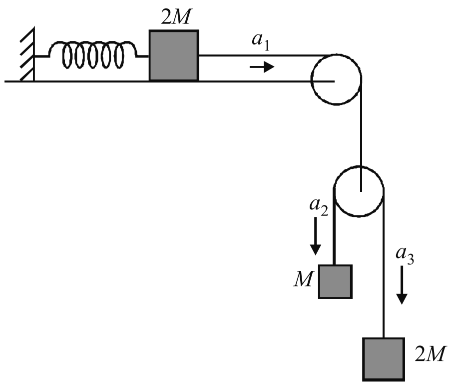
(a) The vertical component of reaction force at O does **not** depend on α

(b) The horizontal component of reaction force at O is equal to W for $\alpha = 0.5$

(c) The tension in the rope is $2W$ for $\alpha = 0.5$

(d) The rope breaks if $\alpha > 1.5$

7. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The acceleration of the blocks are a_1, a_2 and a_3 as shown in the figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct? [g is the acceleration due to gravity. Neglect friction] **[Adv. 2019]**



(a) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

(b) $x_0 = \frac{4Mg}{k}$

(c) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(d) $a_2 - a_1 = a_1 - a_3$

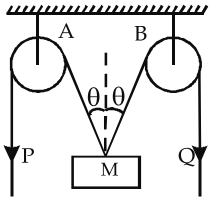
8. In the arrangement shown in the Fig, the ends P and Q of an unstretchable string move downwards with uniform speed U. Pulleys A and B are fixed. **[1982 -3 Marks]**
Mass M moves upwards with a speed

(a) 2Ucos θ

(b) U/ cos θ

(c) 2U / cos θ

(d) U cos θ

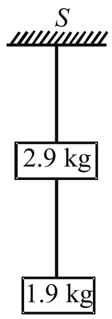


10 Subjective Problems

9. Two blocks of mass 2.9 kg and 1.9 kg are suspended from a rigid support S by two inextensible wires each of length 1 meter, see fig. The upper wire has negligible mass and the lower wire has a uniform mass of 0.2 kg/m. The whole system of blocks wires and support have an upward acceleration of 0.2 m/s^2. Acceleration due to gravity is 9.8 m/s^2. **[1989 - 6 Marks]**

(i) Find the tension at the mid-point of the lower wire.

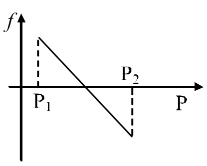
(ii) Find the tension at the mid-point of the upper wire.

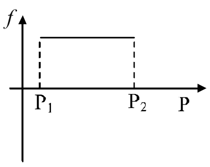


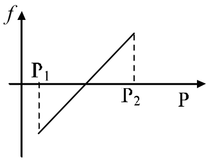
Topic-3: Friction

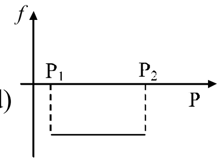
1 MCQs with One Correct Answer

1. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and tan θ > μ. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like **[2010]**

(a)


(b)


(c)


(d)


2. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then **[2009]**

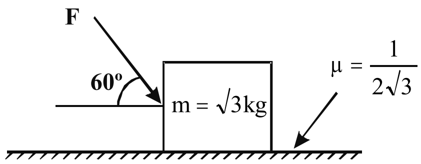
(a) at θ = 30°, the block will start sliding down the plane

(b) the block will remain at rest on the plane up to certain θ and then it will topple

(c) at θ = 60°, the block will start sliding down the plane and continue to do so at higher angles

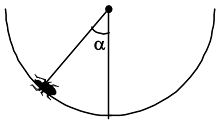
(d) at θ = 60°, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ.

3. What is the maximum value of the force F such that the block shown in the arrangement, does not move? **[2003S]**



(a) 20N (b) 10N (c) 12N (d) 15N

4. An insect crawls up a hemispherical surface very slowly (see fig.). The coefficient of friction between the insect and the surface is 1/3. If the line joining the center of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by **[2001S]**



(a) $\cot \alpha = 3$ (b) $\tan \alpha = 3$
(c) $\sec \alpha = 3$ (d) $\operatorname{cosec} \alpha = 3$

5. A block of mass 0.1 kg is held against a wall applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is : **[1994 - 1 Mark]**

(a) 2.5 N (b) 0.98 N (c) 4.9 N (d) 0.49 N

6. A block of mass 2 kg rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is **[1980]**

(a) 9.8 N (b) $0.7 \times 9.8 \times \sqrt{3}$ N
(c) $9.8 \times \sqrt{3}$ N (d) 0.7×9.8 N

2 Integer Value Answer

7. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N = 10 μ, then N is **[2011]**

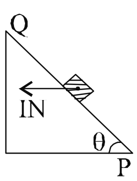
8. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6. If the acceleration of the truck is 5 m/s^2, the frictional force acting on the block is ..... newtons. **[1984 - 2 Marks]**

5 True / False

9. When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion. **[1981 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

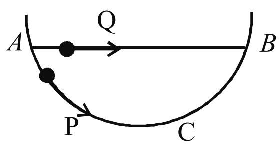
10. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. **[2012]**



The block remains stationary if (take $g = 10$ m/s^2)

(a) $\theta = 45°$
(b) $\theta > 45°$ and a frictional force acts on the block towards P.
(c) $\theta > 45°$ and a frictional force acts on the block towards Q.
(d) $\theta < 45°$ and a frictional force acts on the block towards Q.

11. A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at $t = 0$. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at $t = 0$ along the horizontal string AB, with the speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B. Then : **[1993-2 Marks]**

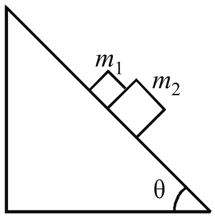


(a) $t_P < t_Q$
(b) $t_P = t_Q$
(c) $t_P > t_Q$
(d) $\dfrac{t_P}{t_Q} = \dfrac{\text{length of arc } ACB}{\text{length of arc } AB}$

7 Match the Following

12. A block of mass m_1 = 1 kg another mass m_2 = 2 kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List-I. The coefficient of friction between the block m_1 and plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In List-II expressions for the friction on block m_2 are given. Match the correct expression of the friction in List-II with the angles given in List-I, and choose the correct option. The acceleration due to gravity is denoted by g.
[**Useful information:** tan (5.5°) ≈ 0.1; tan (11.5°) ≈ 0.2; tan (16.5°) ≈ 0.3 **[Adv. 2014]**

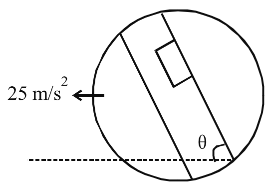
	List-I		**List-II**
P.	$\theta = 5°$	**1.**	$m_2 g \sin\theta$
Q.	$\theta = 10°$	**2.**	$(m_1 + m_2) g \sin\theta$
R.	$\theta = 15°$	**3.**	$\mu m_2 g \cos\theta$
S.	$\theta = 20°$	**4.**	$\mu(m_1 + m_2) g \cos\theta$



Code:
(a) P-1, Q-1, R-1, S-3 (b) P-2, Q-2, R-2, S-3
(c) P-2, Q-2, R-2, S-4 (d) P-2, Q-2, R-3, S-3

10 Subjective Problems

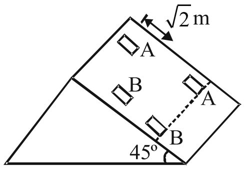
13. A circular disc with a groove along its diameter is placed horizontally on a rough surface.



A block of mass 1 kg is placed as shown. The co-efficient of friction between the block and all surfaces of groove and horizontal surface in contact is $\mu = \dfrac{2}{5}$.

The disc has an acceleration of 25 m/s^2 towards left. Find the acceleration of the block with respect to disc. Given $\cos\theta = \dfrac{4}{5}$, $\sin\theta = \dfrac{3}{5}$. **[2006 - 6M]**

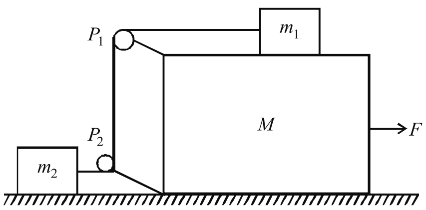
14. Two block A and B of equal masses are placed on rough inclined plane as shown in figure.



When and where will the two blocks come on the same line on the inclined plane if they are released simultaneously?

Initially the block A is $\sqrt{2}$ m behind the block B. Co-efficient of kinetic friction for the blocks A and B are 0.2 and 0.3 respectively (g =10 m/s^2). **[2004 - Marks]**

15. In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The coefficient of friction between M and ground is zero. The coefficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the strings are massless. The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2. The string is perfectly vertical between P_1 and P_2. An external horizontal force F is applied to the mass M. Take g = 10 m/s^2. **[2000 - 10 Marks]**

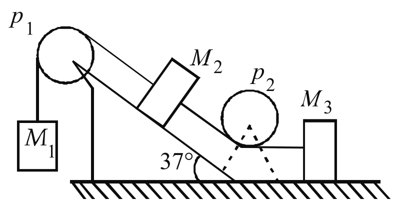


(a) Draw a free body diagram for mass M, clearly showing all the forces.

(b) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular F it is found that $f_1 = 2f_2$. Find f_1 and f_2. Write equations of motion of all the masses. Find F, tension in the string and acceleration of the masses.

16. A particle of mass m rests on a horizontal floor with which it has a coefficient of static friction μ . It is desired to make the body move by applying the minimum possible force F. Find the maguitude of F and the direction in which it has to be applied. **[1987 - 7 Marks]**

17. Masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and friction less pulleys P_1 and P_2 as shown in fig The masses move such that the portion of the string between P_1 and P_2 in parallel to the inclined plane and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surfaces is 0.25. The inclined plane makes an angle of 37° with the horizontal. **[1981- 6 Marks]**



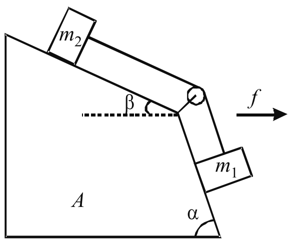
If the mass M_1 moves downwards with a uniform velocity, find

(i) the mass of M_1

(ii) The tension in the horizontal portion of the string (g = 9.8 m/sec^2, sin 37° $\simeq$ 3/5)

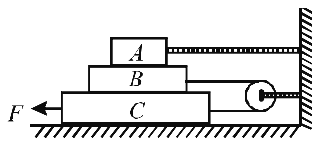
18. A horizontal uniform rope of length L, resting on a frictionless horizontal surface, is pulled at one end by force F. What is the tension in the rope at a distance l from the end where the force is applied? **[1978]**

19. Two cubes of masses m_1 and m_2 be on two frictionless slopes of block A which rests on a horizontal table.



The cubes are connected by a string which passes over a pulley as shown in the figure. To what horizontal acceleration f should the whole system (that is blocks and cubes) be subjected so that the cubes do not slide down the planes. What is the tension of the string in this situation? **[1978]**

20. In the diagram shown,

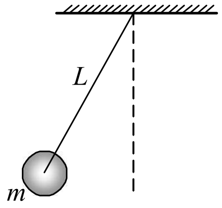


the blocks A, B and C weighs, 3 kg, 4 kg and 5 kg respectively. The coefficient of sliding friction between any two surface is 0.25. A is held at rest by a massless rigid rod fixed to the wall while B and C are connected by a light flexible cord passing around a frictionless pulley. Find the force F necessary to drag C along the horizontal surface to the left at constant speed. Assume that the arrangement shown in the diagram, B on C and A on B, is maintained all through. (g = 9.8 m/s^2) **[1978]**

Topic-4: Circular Motion & Banking of Road

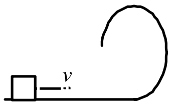
1 MCQs with One Correct Answer

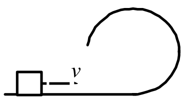
1. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is **[2011]**

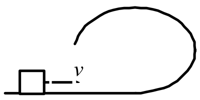


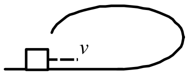
(a) 9 (b) 18 (c) 27 (d) 36

2. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in **[2001S]**

(a) 

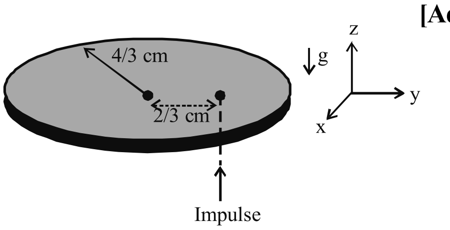
(b) 

(c) 

(d) 

2 Integer Value Answer

3. A thin circular coin of mass 5 gm and radius 4/3 cm is initially in a horizontal xy-plane. The coin is tossed vertically up (+z direction) by applying an impulse of $\sqrt{\frac{\pi}{2}} \times 10^{-2}$ N – s at a distance 2/3 cm from its center. The coin spins about its diameter and moves along the +z direction. By the time the coin reaches back to its initial position, it completes n rotations. The value of n is _____. [Given: The acceleration due to gravity g = 10 m s^{-2}] **[Adv. 2023]**



5 True / False

4. A simple pendulum with a bob of mass m swings with an angular amplitude of 40°. When its angular displacement is 20°, the tension in the string is greater than $mg \cos 20°$. **[1984 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

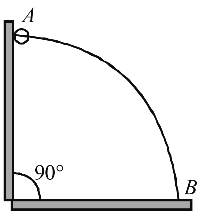
5. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is **[Adv. 2014]**

(a) always radially outwards

(b) always radially inwards

(c) radially outwards initially and radially inwards later

(d) radially inwards initially and radially outwards later



6. A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limit $-\phi$ and $+\phi$. For an angular displacement $\theta(|\theta| < \phi)$, the tension in the string and the velocity of the bob are T and V respectively. The following relations hold good under the above conditions : **[1986 - 2 Marks]**

(a) $T \cos \theta = Mg$.

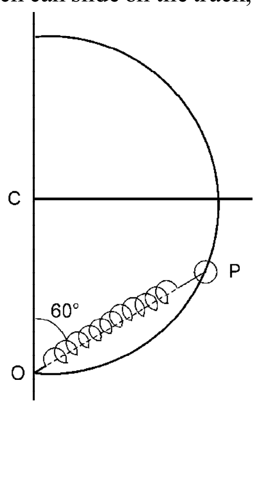
(b) $T - Mg \cos \theta = \frac{MV^2}{L}$

(c) The magnitude of the tangenial acceleration of the bob $|a_T| = g \sin \theta$

(d) $T = Mg \cos \theta$

10 Subject Problems

7. A smooth semicircular wire-track of radius R is fixed in a vertical plane. One end of a massless spring of natural length $3R/4$ is attached to the lowest point O of the wire-track. A small ring of mass m, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant $K = mg/R$. Consider the instant when the ring is released, and (i) draw the free body diagram of the ring, (ii) determine the tangential acceleration of the ring and the normal reaction. **[1996 - 5 Marks]**



Answer Key

Topic-1 : Ist, IInd & IIIrd Laws of Motion

1. (c) **2.** (a) **3.** (d) **4.** (0.005) **5.** False **6.** (b, d) **7.** (b) **8.** (b)

Topic-2 : Motion of Connected Bodies, Pulley & Equilibrium of Forces

1. (b) **2.** (c) **3.** (d) **4.** (c) **5.** False **6.** (a, b, d) **7.** (d) **8.** (b)

Topic-3 : Friction

1. (a) **2.** (b) **3.** (a) **4.** (a) **5.** (b) **6.** (a) **7.** (5) **8.** (5) **9.** False **10.** (a, c)
11. (a) **12.** (d)

Topic-4 : Circular Motion & Banking of Road

1. (d) **2.** (a) **3.** (30) **4.** False **5.** (d) **6.** (b, c)

4 Laws of Motion

Topic-1: Ist, IInd & IIIrd Laws of Motion

1. **(c)** Given mass of block = 5 kg moving along the x - direction subject to the force F = (–20x + 10)N with the value of x in metre.

Acceleration $a = \frac{F}{m}$ $\xleftarrow{F=(-20x+10)N}$ $\boxed{m = 5kg}$ t = 0; v = 0; x = 1m

$= \frac{-20x+10}{5} = -4x+2$

Also, $a = \frac{vdx}{dx} = -4x+2$

$\therefore \int_0^v vdv = \int_1^x (-4x+2)\,dx \rightarrow \frac{v^2}{2} = \left(-2x^2+2x\right)_1^x$

or, $v = -2\sqrt{x-x^2}$ [since particle starts moving in–ve x - direction]

$\therefore \frac{dx}{dt} = -2\sqrt{x-x^2} \Rightarrow \int_{x=1}^{x=x} \frac{dx}{\sqrt{x-x^2}} = -2\int_0^{\frac{\pi}{4}} dt$

or, $\sin^{-1}\,[2x-1]_1^x = -\frac{\pi}{2}$

$\therefore$ Position x = 0.5m

And since $v = -2\sqrt{x-x^2} = -2\sqrt{0.5-(0.5)^2} = -1m/s$

$\therefore$ Momentum P = mv = 5 (–1) = –5kg ms^{-1}

2. **(a)** $\because \vec{v} = \alpha\left(y\hat{x} + 2x\hat{y}\right)$

$\therefore a = \frac{d\vec{v}}{dt} = \alpha\left(\frac{dy}{dt}\hat{x} + 2\frac{dx}{dt}\hat{y}\right)$

$= \alpha\left(v_y\hat{x} + 2v_x\hat{y}\right) = \alpha\left(2x\alpha\hat{x} + 2\alpha y\hat{y}\right) = 2\alpha^2\left[x\hat{x} + y\hat{y}\right]$

3. **(d)** Given : momentum $\vec{p}(t) = A[\hat{i}\cos(kt) - \hat{j}\sin(kt)]$

And, force, $\vec{F} = \frac{d\vec{p}}{dt} = Ak\,[-\hat{i}\sin(kt) - \hat{j}\cos(kt)]$

Here, $\vec{F}.\vec{P} = 0$ But $\vec{F}.\vec{p} = Fp\cos\theta$

$\therefore \cos\theta = 0 \Rightarrow \theta = 90°$.

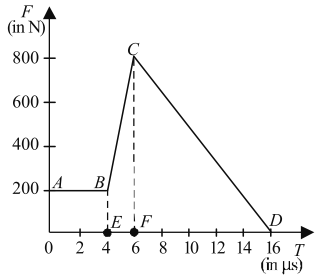
Hence, angle between the force momentum, θ = 90°

4. **(0.005)** Area under the $F - t$ graph gives the impulse imparted to the body.

The magnitude of total impulse of force on the body from $t = 4$ μs to $t = 16$ μs

= area ($BCDFEB$)

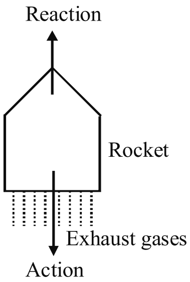
= area of $BCFEB$ + area $CDFC$



$= \frac{1}{2}(200+800)\times 2\times 10^{-6} + \frac{1}{2}\times 10\times 800\times 10^{-6}$

$= 0.001 + 0.004 = 0.005$ Ns

5. **False :** The forward motion of rocket is due to the exhaust gases, thrown backward not due to surrounding air pushing backwards.



Here exhaust gases thrown backwards is action and rocket moving forward is reaction.

This phenomenon takes place in the absence of air as well.

6. **(b, d)** Earth is an accelerated frame and hence, cannot be an inertial frame.

Earth is revolving round the sun and is rotating about its own axis.

7. **(b)** It is easier to pull a heavy object than to push it on a level ground. This is because the normal reaction in the case of pulling is less as compared by pushing. (f = μ N). Therefore the frictional force is small in case of pulling.

The magnitude of frictional force depends on the nature of the two surfaces in contact. But is not the currect explanation of statement-1.

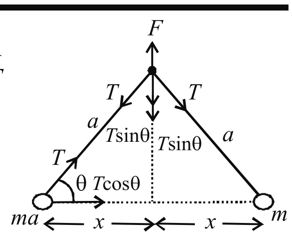
8. **(b)** Cloth can be pulled out without dislodging the dishes from the table because of inertia.

Law of inertia is the Newton's first law of motion.

For every action there is an equal and opposite reaction. This is Newton's third law of motion.

Topic-2: Motion of Connected Bodies, Pulley & Equilibrium of Forces

1. **(b)** From figure, acceleration of mass m is due to the force $T \cos\theta$



$\therefore \quad T\cos\theta = ma$

$\Rightarrow \quad a = \frac{T\cos\theta}{m}$ ... (i) also, $F = 2T\sin\theta \quad \Rightarrow \quad T = \dfrac{F}{2\sin\theta}$

Putting this value of T in eqn. (i)

$$a = \left(\frac{F}{2\sin\theta}\right)\frac{\cos\theta}{m}$$

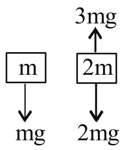
$$= \frac{F}{2m\tan\theta} = \frac{F}{2m}\frac{x}{\sqrt{a^2 - x^2}} \qquad \left[\because \tan\theta \frac{\sqrt{a^2 - x^2}}{x}\right]$$

2. **(c)** Before the string is cut the tension T has to hold both the masses $2m$ and m therefore,

$T = 3mg$

When the string is cut, the mass m is a freely falling body and its acceleration = acceleration due to gravity = g.

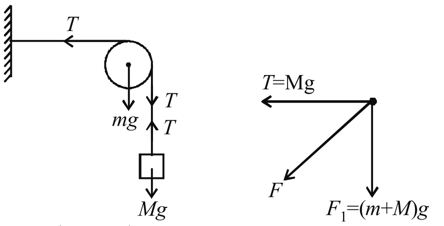
For mass $2m$, just after the string is cut, T remains $3mg$ because of the extension of string.



$\therefore \quad 3mg - 2mg = 2m \times a \quad \Rightarrow \quad \dfrac{g}{2} = a$

3. **(d)** At equilibrium $T = Mg$

F.B.D. of pulley



$F_1 = (m + M)\,g$

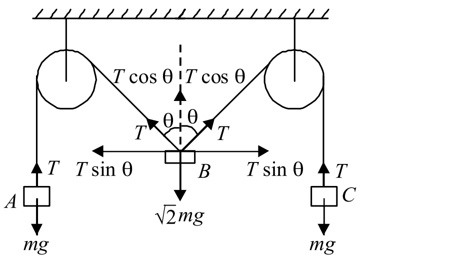
The resultant force on pulley

$F = \sqrt{F_1^2 + T^2} = [\sqrt{(m+M)^2 + M^2}\,]g$

As pulley is on rest. So force applied by clamp should be equal to 'F' and opposite to it.

4. **(c)** The tension in both strings will be same due to symmetry.

From figure,
T = mg



For equilibrium

$\sqrt{2}\,mg = T\cos\theta + T\cos\theta = 2T\cos\theta$

$\therefore \quad \sqrt{2}\,mg = 2(mg)\cos\theta$

$\therefore \quad \cos\theta = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

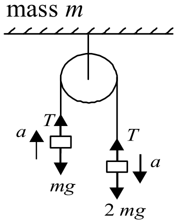
5. **False :** From FBD, shown in case (a) for mass m

$T - mg = ma$ ... (i)

For mass $2m$

Case (a)

$2mg - T = 2ma$ ... (ii)



From (i) and (ii)

$a = g/3$ ... (iii)

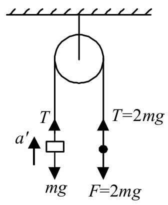
Case (b) $T - mg = ma'$

$\Rightarrow \quad 2mg - mg = ma'$

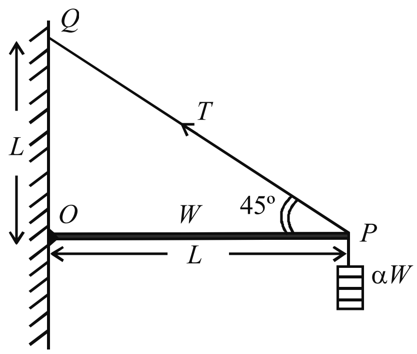
$[\because \; T = 2mg]$

$\therefore \quad a' = g$ ...(iv)

Hence, from eq (iii) & (iv) $a < a'$



6. **(a, b, d)**



Since $OQ = OP$

$\therefore \quad \angle P = \angle Q = 45°$

At equilibrium, about point 'O'

$$R_y + \frac{T}{\sqrt{2}} = W + \alpha W \qquad \text{. . . (i)}$$

and $R_x = \dfrac{T}{\sqrt{2}}$ . . . (ii)

Torque about point 'O' is zero

So, $W\dfrac{L}{2} + \alpha WL = \dfrac{T}{\sqrt{2}}L \;\therefore\; T = \sqrt{2}\left(\dfrac{W}{2} + \alpha W\right)$ . . . (iii)

$\therefore \quad R_x = \dfrac{T}{\sqrt{2}} = \left(\dfrac{W}{2} + \alpha W\right)$

Therefore for $\alpha = 0.5$

$R_x = \dfrac{W}{2} + \alpha W = \dfrac{W}{2} + 0.5W$

or $\quad R_x = W$

i.e., the horizontal component of reaction force at, 0, $R_x = W$ for $\alpha = 0.5$

Now torque about point P

$T_y L = W\dfrac{L}{2}$

$\Rightarrow \quad R_y = \dfrac{W}{2}$

The vertical component of reaction force at O does not depend on α

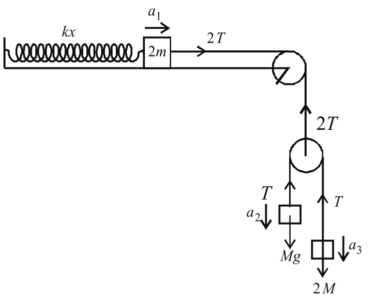
As per question, rope can sustain a maximum tension of $2\sqrt{2}\,W$

$\therefore \quad 2\sqrt{2}W = \sqrt{2}\left(\dfrac{W}{2} + \alpha W\right)$

$\Rightarrow \quad 2 = \frac{1}{2} + \alpha$

$\therefore \quad \alpha = \frac{3}{2}$

7. **(d)** According to constraint relation from figure,



$a_1 = \frac{a_2 + a_3}{2}$

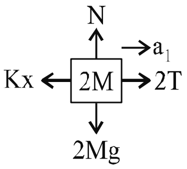
$\Rightarrow a_2 + a_3 = 2a_1$

$\Rightarrow \quad a_2 + a_3 = a_1 + a_1$

$\Rightarrow \quad a_1 - a_3 = a_2 - a_1$

$\Rightarrow$ Option (d) is correct

Let 'x' be the extension of the spring at a certain instant



$2T - Kx = 2Ma_1$

$2Mg - T = 2Ma_3$

$Mg - T = Ma_2$

On solving we get,

$$a_1 = \frac{4g}{7} - \frac{3kx}{14M} = \frac{-3K}{14M}\left(x - \frac{8mg}{3K}\right) \quad \text{...(i)}$$

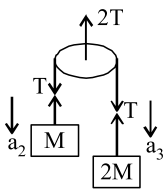
Comparing it with $a = -\omega^2(x - x_0)$

$\therefore \quad \omega^2 = \frac{3k}{14M} \quad \therefore \quad \omega = \sqrt{\frac{3k}{14M}}$

and $T = \frac{4Mg}{7} + \frac{2kx}{7}$ ...(ii)

For $a_1 = 0$ (Maximum extension of spring) we have from (i)

$$\frac{4g}{7} - \frac{3kx}{14M} = 0$$



$\therefore \quad 4g = \frac{3kx}{2M} \quad \therefore \quad x = \frac{8Mg}{3k}$

$\therefore \quad x_0 = 2x = \frac{16Mg}{3k}$

For $x = \frac{x_0}{4} = \frac{1}{4}\left(\frac{16Mg}{3k}\right) = \frac{4Mg}{3k}$

From eqn. (i) $a_1 = \frac{4g}{7} - \frac{3k}{14M} \times \frac{4Mg}{3x} = \frac{2g}{7}$

At $x = \frac{x_0}{2}$ particle is at mean position and its velocity $= A\omega$

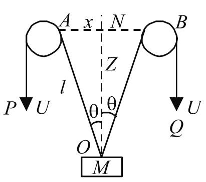
$= \frac{x_0}{2}\sqrt{\frac{3k}{14M}} = \frac{8Mg}{3k}\sqrt{\frac{3k}{14M}}$

8. **(b)** Here from figure, $AN = x$ (= constant as pulley A and B are fixed), $NO = z$. Then velocity of mass $m = \frac{dz}{dt}$. Also,

let $OA = \ell$ then $\frac{d\ell}{dt} = U$

From ΔANO

$x^2 + z^2 = \ell^2$ ...(i)

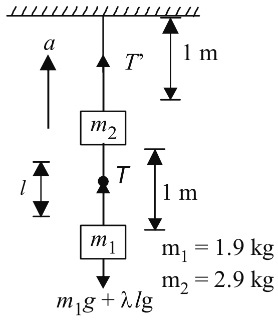


Differentiating equation (i) w.r.t to t

$0 + 2z\frac{dz}{dt} = 2\ell\frac{d\ell}{dt} \Rightarrow zv_M = \ell U$

$\Rightarrow \quad v_M = \frac{\ell}{z}U = \frac{U}{z/\ell} = \frac{U}{\cos\theta} \quad \left(\because \cos\theta = \frac{z}{\ell}\right)$

9. l = Mass of unit length of wire – 0.2 kg/m.



(i) Tension T at midpoint of lower wire :

l = Half-length = 0.5 m

$\therefore \quad T - (m_1 + \lambda l)g = (m_1 + \lambda l)a$

$T = (m_1 + \lambda l)(a + g)$

$= [1.9 + (0.2 \times 0.5)](0.2 + 9.8) = 2 \times 10 = 20$ N.

(ii) Tension T' at mid-point of upper wire :

$\therefore \; T' = [m_1 + (\lambda \times 2l) + m_2]a + [m_2g + \lambda \times 2lg + m_1g]$

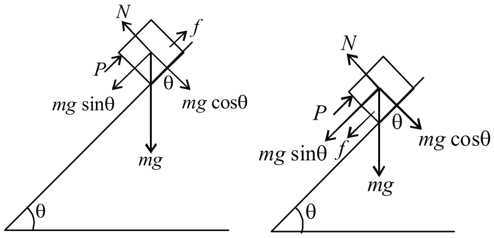
or $T' = [m_1 + (\lambda \times 2l) + m_2]\,(a + g)$

$= [1.9 + (0.2 \times 1) + 2.9][0.2 + 9.8] = 5 \times 10 = 50$N.

Topic-3: Friction

1. **(a)** According to question, $\tan\theta > \mu$, so block has a tendency to move down the incline. Force P is applied upwards along the incline to keep the block stationary. Here, at equilibrium $P + f = mg\sin\theta \Rightarrow f = mg\sin\theta - P$

Now as P increases, f decreases linearly with respect to P.



When $P = mg\sin\theta, f = 0$.

When force P is increased further, the block has a tendency to move upwards along the incline and hence frictional force acts downwards along the incline.

Here, at equilibrium $P = f + mg\sin\theta$

$\Rightarrow f = P - mg\sin\theta$

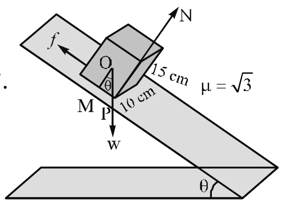
Now as P increases, f increases linearly w.r.t P.

Hence graph (a) correctly depicts the situation.

2. **(b)** Maximum angle not to slide the block, angle of inclination = angle of repose,

i.e., $\tan^{-1}\mu = \tan^{-1}\sqrt{3} = 60°$.

For the block to topple, the condition of the block has been shown in the figure.



In ΔPOM, $\tan\theta = \dfrac{PM}{OM} = \dfrac{10/2}{15/2} = \dfrac{5\,\text{cm}}{7.5\,\text{cm}} = \dfrac{2}{3}$

So, θ < 60°. From this we can conclude that the block will topple at lesser angle of inclination. Clearly the block will remain at rest on the plane up to a certain anlgle θ and then it will topple.

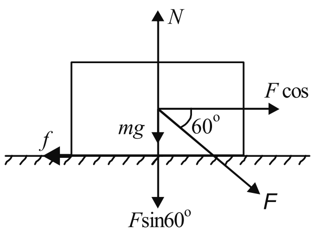
3. **(a)** Since the block is not moving forward for the maximum force F applied, therefore

$F\cos 60° = f = \mu N$

(Horizontal direction)

For vertical equilibrium of the block,

$N = mg + F\sin 60°$



$\therefore \quad F\cos 60° = \mu N = \mu\,[F\sin 60° + mg]$

$$\Rightarrow \quad F = \frac{\mu mg}{\cos 60° - \mu \sin 60°} = \frac{\frac{1}{2\sqrt{3}} \times \sqrt{3} \times 10}{\frac{1}{2} - \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{2}} = \frac{5}{\frac{1}{4}} = 20\,\text{N}$$

4. **(a)** The two forces acting on the insect are mg and N. Two components of mg are

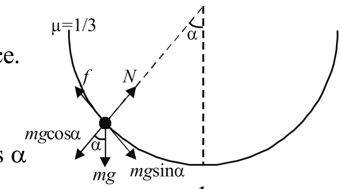
$mg\cos\alpha$ balances N.

$mg\sin\alpha$ is balanced by the frictional force.

$\therefore \quad N = mg\cos\alpha$

$\quad f = mg\sin\alpha$

But $f = \mu N = \mu\, mg\cos\alpha$



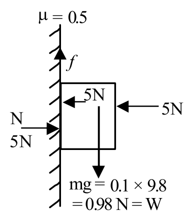
$\therefore \quad \mu\, mg\cos\alpha = mg\sin\alpha \Rightarrow \cot\alpha = \dfrac{1}{\mu} \Rightarrow \cot\alpha = 3$

5. **(b)** Limiting frictional force,

$f_l = \mu_s N = 0.5 \times 5 = 2.5\,N$.

For vertical equilibrium of the block,

Frictional force, $f = \text{mg} = 0.98\,N$.

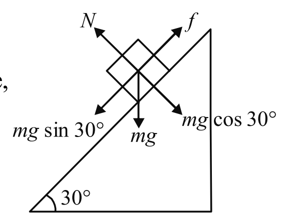


6. **(a)** The block is at rest.

For equilibrium, frictional force,

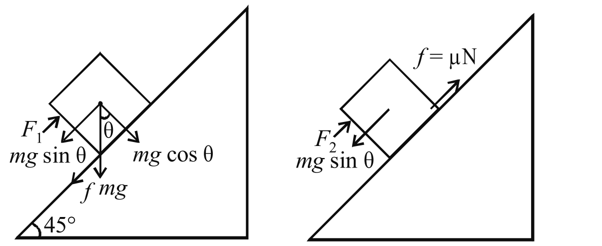
$f = \text{mg}\sin\theta = \text{mg}\sin 30°$

$= 2 \times 9.8 \times \dfrac{1}{2} = 9.8\text{N}$



7. **(5)**

Block moving upward | Block just remains stationary



For upward moving of block, pushing force $F_1 = mg\sin\theta + f$

$\therefore F_1 = mg\sin\theta + \mu\, mg\cos\theta = mg(\sin\theta + \mu\cos\theta)$

The force required to just prevent it from sliding down or block just remains stationary.

$F_2 = mg\sin\theta - \mu N = mg(\sin\theta - \mu\cos\theta)$

Given, $F_1 = 3F_2$

$\therefore \quad \sin\theta + \mu\cos\theta = 3(\sin\theta - \mu\cos\theta)$

$\therefore \quad 1 + \mu = 3(1 - \mu) \quad [\because \sin\theta = \cos\theta]$

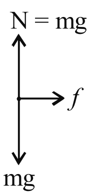
$\therefore \quad 4\mu = 2 \quad \Rightarrow \mu = 0.5$

$\therefore \quad N = 10\,\mu = 10 \times 0.5 = 5\,\text{N}$

8. **(5)** The frictional force is responsible to move the block of mass 1 kg with an acceleration of 5 m/s^2.

Therefore, frictional force,

$f = m \times a = 1 \times 5 = 5\,N$.



9. **False :** Friction force opposes the relative motion of the surface of contact.

As the feet pushes the surface in backward direction, so frictional force exerted by the surface on the person is in the direction of his motion.

10. **(a, c)** The various forces acting on the block are as shown in the figure.

When θ = 45º, sinθ = cosθ

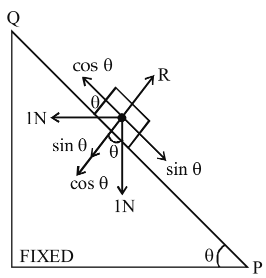
The block will remain stationary and the frictional force is zero.

When θ > 45º, sinθ > cosθ

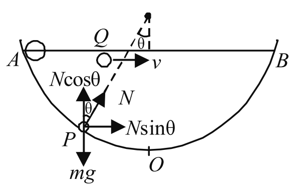
Therefore a frictional force acts towards Q.

When θ < 45º, cosθ > sinθ

Therefore a frictional force acts towards P.



11. **(a)** According to question, at A the horizontal speeds of both the masses is the same. As no force is acting in the horizontal direction the velocity of Q remains the same in horizontal.



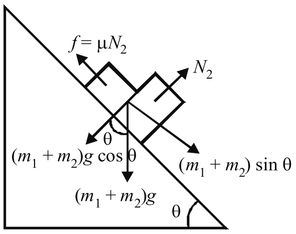
In case of P as shown in figure at any intermediate position, the horizontal velocity first increases due to $N\sin\theta$, reaches a max value at O and then decreases.

But, it always remains greater than v. So, $t_P < t_Q$.

12. (d) Block will not be slip or will be at rest if

$(m_1 + m_2)g \sin\theta \le \mu m_2 g \cos\theta$

$$\tan\theta \le \frac{\mu m_2 g}{(m_1+m_2)g}$$



$$\Rightarrow \tan\theta \le \frac{\mu m_2}{m_1+m_2}$$

$$\Rightarrow \tan\theta \le \frac{0.3\times 2}{1+2} \le \frac{1}{5}$$

$\Rightarrow$ $\tan\theta \le 0.2$ i.e., $\theta \le 11.5°$

i.e., If the angle $\theta < 11.5°$ the frictional force is less than

$\mu N_2 = \mu m_2 g = 0.3 \times 2 \times g = 0.6$ g

and is equal to $(m_1 + m_2)g \sin\theta$

Blocks will not slip on the inclined plane and friction is static.

At $\theta > 11.5°$ the bodies start moving on the inclined plane and friction is kinetic and equal to $\mu m_2 g \cos\theta$

13. Normal reaction, $N_1 = ma \sin\theta$ and $N_2 = mg$

Applying pseudo force ma and resolving it.

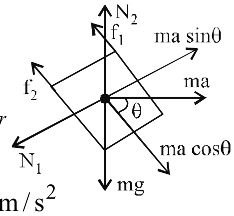
$F_{net} = ma_r$

$ma \cos\theta - (f_1 + f_2) = ma_r$

$ma \cos\theta - \mu N_1 - \mu N_2 = ma_r$

$ma \cos\theta - \mu ma \sin\theta - \mu\, mg = ma_r$

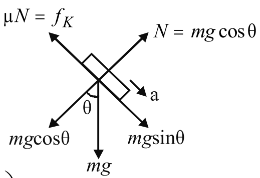
$\Rightarrow$ $a_r = a \cos\theta - \mu a \sin\theta - \mu g$



$$= 25\times\frac{4}{5} - \frac{2}{5}\times 25\times\frac{3}{5} - \frac{2}{5}\times 10 = 10\,\text{m/s}^2$$

14. Acceleration of block down the plane

$$a = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m}$$



$\therefore$ $a_A = g \sin\theta - \mu_{k,A}\, g \cos\theta$

$= g \sin 45° - \mu \cos 45°$

$$= 10\left(\frac{1}{\sqrt{2}}\right) - (0.2)(10)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}$$

And $a_B = g \sin\theta - \mu_{k,B}\, g \cos\theta$

$= g \sin 45°\ \mu_{kB}\ g \cos 45°$

$$= 10\left(\frac{1}{\sqrt{2}}\right) - (0.3)(10)\left(\frac{1}{\sqrt{2}}\right) = 3.5\sqrt{2}\ \text{m/s}^2$$

Let a_{AB} is relative acceleration of A w.r.t. B. Then

$a_{AB} = a_A - a_B$

The relative distance between A and B, L.

$$L = \frac{1}{2}a_{AB}t^2$$

or $$t^2 = \frac{2L}{a_{AB}} = \frac{2L}{a_A - a_B} = \frac{2(\sqrt{2})}{(4\sqrt{2}) - (3.5\sqrt{2})}$$

$\Rightarrow t^2 = 4$ or $t = 2s$.

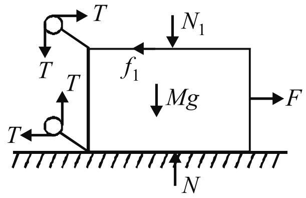
Distance moved by A during that time is given by

$$S_A = \frac{1}{2}a_4 t^2 = \frac{1}{2}\times 4.5\sqrt{2}\times 4 = 8\sqrt{2}\ \text{m}$$

Similarly for $B = 7\sqrt{2}$ m.

Hence both the blocks will come in line after A has travelled a distance $8\sqrt{2}$ m down the plane

15. (a) Free body diagram of mass M



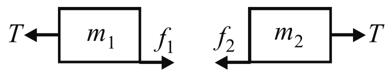
(b) The maximum value of force of friction between m_1 and M

$(f_1)_{max} = (0.3)(20)(10) = 60\,N$ ...(i)

The maximum value of force of friction between m_2 and M

$(f_2)_{max} = (0.3)(5)(10) = 15\,N$ ...(ii)

Forces on m_1 and m_2 in horizontal direction are as follows:



There are only two possibilities.

Case I Either both m_1 and m_2 will remain stationary (w.r.t. ground)

Case II both m_1 and m_2 will move (w.r.t. ground).

First case is possible when

or $T \le (f_1)_{max}$ or $T \le 60\,N$ and $T \le (f_2)_{max}$ or $T \le 15\,N$

These conditions will be satisfied when $T \le 15\,N$ say $T = 14$ then $f_1 = f_2 = 14\,N$.

Therefore the condition $f_1 = 2f_2$ will not be satisfied.

Thus m_1 and m_2 both can't remain stationary.

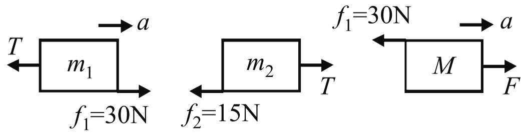
In the second case, when m_1 and m_2 both move

$f_2 = (f_2)\max = 15\,N$

$\therefore$ $f_1 = 2f_2 = 30\,N$

Since $f_1 < (f_1)_{max}$, there is no relative motion between m_1 and M, *i.e.*, all the masses move with same acceleration, say 'a'.

Free body diagrams and equations of motion are as follows:



For mass, m_1 : $30 - T = 20\,a$ ... (iii)

For mass, m_2 : $T - 15 = 5\,a$ ... (iv)

For mass, M : $F - 30 = 50\,a$ ... (v)

Adding eq. (iii) & (iv), we get.

acceleration $a = \frac{3}{5}\text{m/s}^2$.

From eq. (iv) $T - 15 = \frac{5\times 3}{5}$ $\Rightarrow$ T = 18 N

From eq. (v) $F - 30 = 50\times\frac{3}{5}$ $\Rightarrow$ F = 60 N

16. Let force F be applied to move the body at an angle θ to the horizontal.

The body will move when

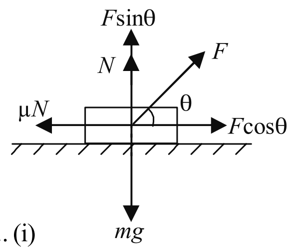
$F\cos\theta = \mu N$

from figure, normal reaction

$N = mg - F\sin\theta$

$F\cos\theta = \mu(mg - F\sin\theta)$

$$\Rightarrow F = \frac{\mu mg}{\cos\theta + \mu\sin\theta} \quad ...(i)$$



Differentiating the above equation w.r.t. θ, we get

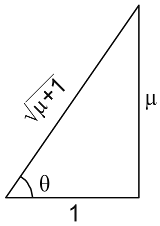
$$\frac{dF}{d\theta} = \frac{\mu mg}{(\cos\theta + \mu\sin\theta)^2}[-\sin\theta + \mu\cos\theta] = 0$$

$\therefore \quad \theta = \tan^{-1}\mu$

This is the angle for minimum force.

To find the minimum force substituting these values in equation (i)

$$\sin\theta = \frac{\mu}{\sqrt{\mu^2+1}}, \quad \cos\theta = \frac{1}{\sqrt{\mu^2+1}}$$



$$F = \frac{\mu mg}{\frac{1}{\sqrt{\mu^2+1}} + \frac{\mu}{\sqrt{\mu^2+1}} \times \mu}$$

$$\Rightarrow \quad F = \frac{\mu mg(\sqrt{\mu^2+1})}{\mu^2+1} = \frac{\mu mg}{\sqrt{\mu^2+1}}$$

$\Rightarrow \quad F = mg\sin\theta$

17. According to question, mass M_1 moves downwards with a uniform velocity i.e., net acceleration of the system is zero. Or net pulling force on the system is zero.

For equilibrium,

(a) $M_1 g = M_2 g \sin 37° + \mu M_2 g \cos 37° + \mu M_3 g$

or $M_1 = M_2 \sin 37° + \mu M_2 \cos 37° + \mu M_3$

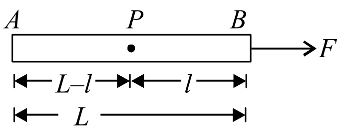
$$= (4)\left(\frac{3}{5}\right) + (0.25)(4)\left(\frac{4}{5}\right) + (0.25)(4) = 4.2\,\text{kg}$$

(b) Since, M_3 is moving with uniform velocity

$T = \mu_1 m_2 g = (0.25) \times 4 \times 9.8 = 9.8\text{ N}$

18. Let T be the tension in the rope at point P and a be the acceleration produced in the rope.

Mass per unit length of the rope is $\mu = \frac{F}{L}$



For the part AP;

$T = \mu(L-l)a$ ......(i)

For the part PB;

$F - T = \mu l a$ ......(ii)

$$F - T = \mu l\left[\frac{T}{\mu(L-l)}\right] \quad \text{[Using eq. (i)]}$$

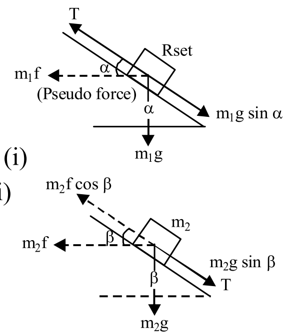
$$F - T = \frac{Tl}{L-l} \Rightarrow T\left[\frac{l}{L-l} + l\right] = F;$$

$$T\left[\frac{L}{L-l}\right] = F \Rightarrow T = F\left(\frac{L-l}{L}\right)$$

or, $T = F\left[1 - \frac{l}{L}\right]$

19. As cubes do not slide down the planes hence they have same acceleration.

Consider the FBD of the cubes along incline



$T + m_1 f\cos\alpha = m_1 g\sin\alpha \rightarrow$ (i)

$T + m_2 g\sin\beta = m_2 f\cos\beta \rightarrow$ (ii)

Eq (i) – Eq (ii)

$(m_1\cos\alpha + m_2\cos_\beta)f$

$= (m_1\sin\alpha + m_2\sin\beta)g$

$$\Rightarrow \quad f = \frac{(m_1\sin\alpha + m_2\sin\beta)}{(m_1\cos\alpha + m_2\cos\beta)}g$$

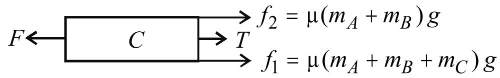
$$\text{from eq (i) } T = (m_1 g\sin\alpha) - (m_1\cos\alpha)\left[\frac{m_1\sin\alpha + m_2\sin\beta}{m_1\cos\alpha + m_2\cos\beta}\right]g$$

$$\text{or, } T = g\left[\frac{m_1^2\cos\alpha\sin\alpha + m_1 m_2\cos\beta\sin\alpha - m_1^2\cos\alpha\sin\alpha - m_1 m_2\cos\alpha\sin\beta}{(m_1\cos\alpha + m_2\cos\beta)}\right]$$

$$\text{or, } T = \frac{m_1 m_2[\cos\beta\sin\alpha - \sin\beta\cos\alpha]g}{(m_1\cos\alpha + m_2\cos\beta)}$$

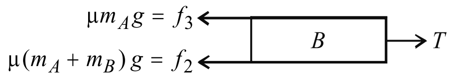
20. When force F is applied on block C will move towards left and the block 'B' will move towards right due to reaction of C on B, while block A always remains at rest.

The F.B.D. for mass C is



As C is moving with constant speed $F = f_1 + f_2 + T$ ... (i)

F.B.D. for mass B is



As B is moving with constant speed $f_2 + f_3 = T$ ... (ii)

Subtracting eq. (ii) from (i)

$F - (f_2 + f_3) = f_1 + f_2 + T - T = f_1 + f_2$

$\Rightarrow \quad F = f_1 + 2f_2 + f_3 = \mu(m_A + m_B + m_C)g + 2\mu(m_A + m_B)g + \mu m_A g$

$F = \mu(4m_A + 3m_B + m_C)g$

(Given: $m_A = 3$kg, $m_B = 4$kg, $M_e = 5$ kg and $\mu = 0.25$)

$= 0.25[4 \times 3 + 3 \times 4 + 5] \times 9.8 = 71.05\,N$

Hence, force necessary to drag, F = 71.05 N

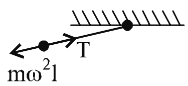
Topic-4: Circular Motion & Banking of Road

1. **(d)** $T\sin\theta = mR\omega^2$ ...(i)

$T\cos\theta = mg$ ...(ii)

Dividing (ii) by (i), we get

$$\tan\theta = \frac{\omega^2}{Rg} \Rightarrow \omega = \sqrt{Rg\tan\theta}$$



Clearly, ω is maximum, when $\tan\theta$ is maximum

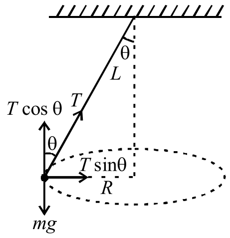
i.e. $\theta = 90°$

So, $T \sin 90° = mR\omega^2$

$T = mL\omega^2$ [Here, R = L]

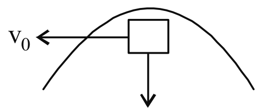
$$\Rightarrow \omega = \sqrt{\frac{T}{mL}} = \sqrt{\frac{324}{0.5 \times 0.5}}$$

$$= \frac{18}{0.5} = 36 \text{ rad/s}$$



2. **(a)** According to question, the speed with which the block enters the track is the same in all the tracks and the block rises to the same height so from law of conservation of energy, speed of the block at highest point will be same in all four cases.

Let the velocity at the highest point be v



N + mg

(N + mg) provides the centripetal force $\frac{mv^2}{R}$ to the body

$$N + mg = \frac{mv^2}{R}$$

or $$N = \frac{mv^2}{R} - mg$$

R (the radius of curvature) in first case is minimum.

Hence, normal reaction N will be maximum in first case.

3. **(30)** From impulse – momentum theorem,

$$J = MV_{CM} \Rightarrow V = \frac{J}{M} = \frac{\sqrt{\pi/2}}{100 \times \frac{5}{1000}} = \sqrt{2\pi}\ \text{m/s}$$

Total time taken, $t = \frac{2v}{g}$

$$= \frac{2 \times \sqrt{2\pi}}{g} = \frac{2 \times \sqrt{2\pi}}{10} = \frac{\sqrt{2\pi}}{5} \text{s}$$

By angular impulse – momentum theorem,

$$J \times \frac{R}{2} = I_c\omega = \left[\frac{1}{4}MR^2\right]\omega \qquad \therefore \omega = \frac{J \times \frac{R}{2}}{\frac{MR^2}{4}} = \frac{J \times 2}{MR}$$

$$= \frac{\frac{\sqrt{\pi/2}}{100} \times 2}{\frac{5}{1000} \times \frac{4}{3} \times \frac{1}{100}} = 2 \times 75\sqrt{2\pi} \text{ rad/s}$$

$$\because \theta = 2\pi n = \omega t \quad \therefore n = \frac{\omega t}{2\pi}$$

$$\therefore n = \frac{2 \times 75\sqrt{2\pi} \times \frac{\sqrt{2\pi}}{5}}{2\pi} = 30$$

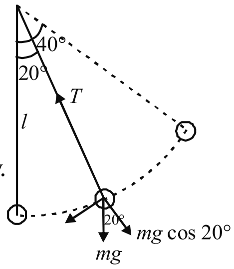
4. **False :** The angular amplitude of the pendulum is 40° given, when its angular displacement is 20° then

For equilibrium of the bob, $T - mg \cos 20° = \frac{mv^2}{l}$, where l is the length of the pendulum and v is the velocity of the bob.

$$\therefore \quad T = mg \cos 20° + \frac{mv^2}{l}$$

$\frac{mv^2}{l}$ is always a positive quantity.

Hence, clearly $T > mg \cos 20°$.



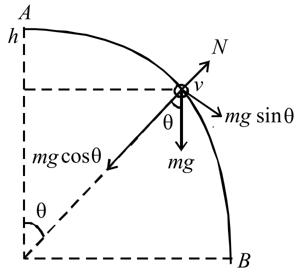
5. **(d)** Suppose 'N' is acting radially outward

Then, $mg\cos\theta - N = \frac{mv^2}{R}$

$$\Rightarrow N = mg\cos\theta - \frac{mv^2}{R} \quad \text{...(i)}$$

And by energy conservation,

$$\frac{1}{2}mv^2 = mg\left[R - R\cos\theta\right]$$



$$\therefore \frac{v^2}{R} = 2g\left(1 - \cos\theta\right)$$

Putting this value of $\frac{v^2}{R}$ in eqn. (i)

$$N = mg\cos\theta - m\left[2g - 2g\cos\theta\right]$$

$$\Rightarrow \quad N = mg\cos\theta - 2mg + 2mg\cos\theta$$

$$\Rightarrow \quad N = 3mg\cos\theta - 2mg \Rightarrow N = mg\left(3\cos\theta - 2\right)$$

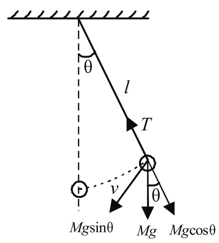
Clearly when $\cos\theta > \frac{2}{3}$, N is positive acts radially outwards

So, force on wire is inward and if $\cos\theta < \frac{2}{3}$ N acts radially inwards.

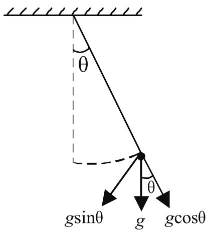
So, force on wire is outward.

6. **(b, c)** A long radius net force = centripetal force $\left(\frac{Mv^2}{\ell}\right)$.

And along tangent net force $= ma_t$ as the motion of a pendulum is the part of circular motion.



$$\therefore \quad T - Mg\cos\theta = \frac{Mv^2}{\ell}$$



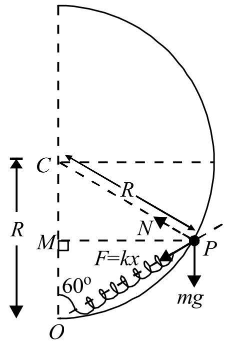
And, $ma_t = mg \sin\theta \Rightarrow a_t = g \sin\theta$

7. Radius of circle = R

In ΔOCP, $OC = CP = R$

$\therefore$ $\angle COP = \angle CPO = 60° \Rightarrow \angle OCP = 60°$

$\therefore$ ΔOCP is an equilateral triangle $\Rightarrow OP = R$



$\therefore$ Extension of string = $R - \dfrac{3R}{4} = \dfrac{R}{4} = x$

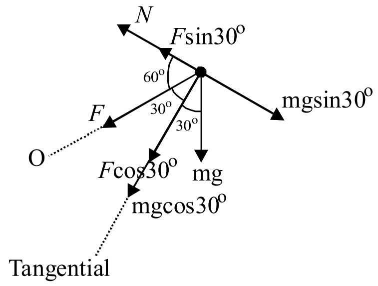
The forces acting are shown in the figure (i)

From FBD of the ring

Force in the tangential direction

$= F \cos 30° + mg \cos 30$

$= [kx + mg] \cos 30°$



$F_t = \dfrac{5mg}{8}\sqrt{3} \quad \therefore F_t = ma_t \quad \Rightarrow \quad a_t = \dfrac{5\sqrt{3}}{8} g$

Also, when the ring is just released

$N + F \sin 30° = mg \sin 30°$

$\Rightarrow \quad N = (mg - F) \sin 30° = \left(mg - \dfrac{mg}{4}\right) \times \dfrac{1}{2} = \dfrac{3mg}{8}$

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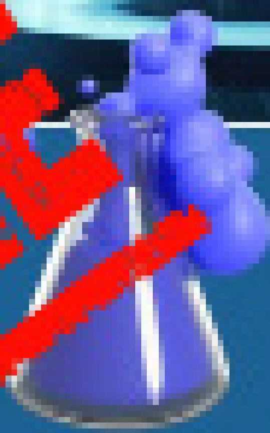

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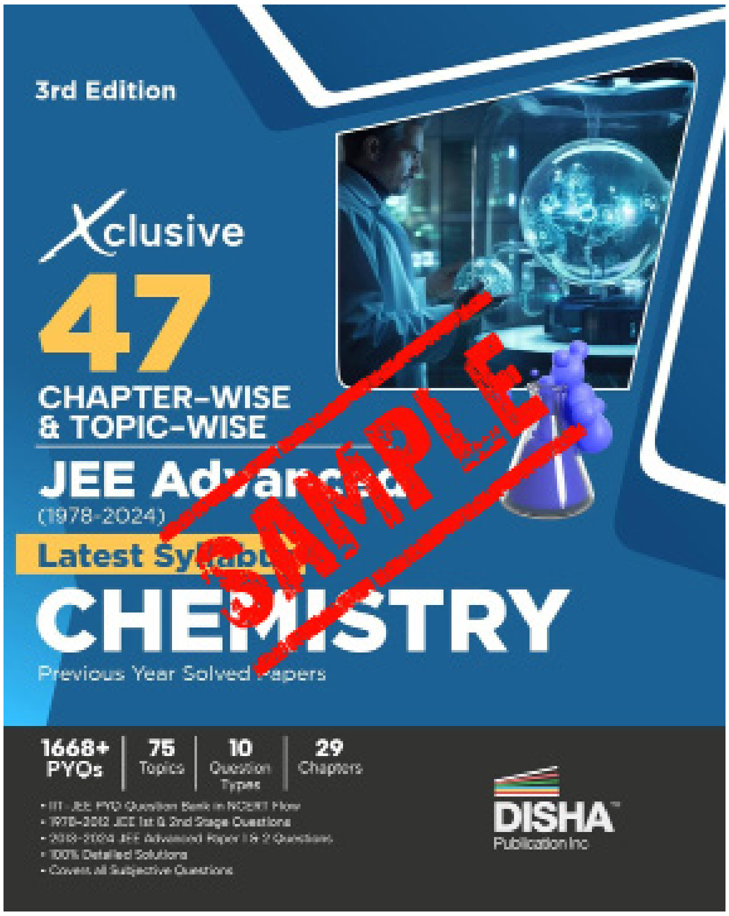

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Hints & Solutions (Class XII[th]) **B111–B262**

2 Structure of Atom

Topic-1: Different Atomic Models that Leads to Bohr Model

1 MCQs with One Correct Answer

1. According to Bohr's model, the highest kinetic energy is associated with the electron in the **[Adv. 2024]**
(a) First orbit of H atom (b) First orbit of He^+
(c) Second orbit of He^+ (d) Second orbit of Li^{2+}

2. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is Bohr radius] : **[2012]**
(a) $\frac{h^2}{4\pi^2 ma_0^2}$ (b) $\frac{h^2}{16\pi^2 ma_0^2}$
(c) $\frac{h^2}{32\pi^2 ma_0^2}$ (d) $\frac{h^2}{64\pi^2 ma_0^2}$

3. Given that the abundances of isotopes ^{54}Fe, ^{56}Fe and ^{57}Fe are 5%, 90% and 5%, respectively, the atomic mass of Fe is **[2009S]**
(a) 55.85 (b) 55.95 (c) 55.75 (d) 56.05

4. The radius of which of the following orbit is same as that of the first Bohr's orbit of hydrogen atom? **[2004S]**
(a) $He^+ (n = 2)$ (b) $Li^{2+} (n = 2)$
(c) $Li^{2+} (n = 3)$ (d) $Be^{3+} (n = 2)$

5. Rurtherford's experiment, which established the nuclear model of the atom, used a beam of **[2002S]**
(a) β -particles, which impinged on a metal foil and got absorbed
(b) γ -rays, which impinged on a metal foil and ejected electrons
(c) helium atoms, which impinged on a metal foil and got scattered
(d) helium nuclei, which impinged on a metal foil and got scattered

6. Which of the following does not characterise X-rays?
(a) The radiation can ionise gases **[1992 - 1 Mark]**
(b) It causes ZnS to fluorescence
(c) Deflected by electric and magnetic fields
(d) Have wavelengths shorter than ultraviolet rays

7. The wavelength of a spectral line for an electronic transition is inversely related to : **[1988 - 1 Mark]**
(a) the number of electrons undergoing the transition
(b) the nuclear charge of the atom
(c) the difference in the energy of the energy levels involved in the transition
(d) the velocity of the electron undergoing the transition.

8. The triad of nuclei that is isotonic is **[1988 - 1 Mark]**
(a) $^{14}_{6}C$, $^{15}_{7}N$, $^{17}_{9}F$ (b) $^{12}_{6}C$, $^{14}_{7}N$, $^{19}_{9}F$
(c) $^{14}_{6}C$, $^{14}_{7}N$, $^{17}_{9}F$ (d) $^{14}_{6}C$, $^{14}_{7}N$, $^{19}_{9}F$

9. The ratio of the energy of a photon of 2000 Å wavelength radiation to that of 4000 Å radiation is : **[1986 - 1 Mark]**
(a) 1/4 (b) 4 (c) 1/2 (d) 2

10. Rutherford's alpha particle scattering experiment eventually led to the conclusion that : **[1986 - 1 Mark]**
(a) mass and energy are related
(b) electrons occupy space around the nucleus
(c) neutrons are buried deep in the nucleus
(d) the point of impact with matter can be precisely determined.

11. Electromagnetic radiation with maximum wavelength is : **[1985 - 1 Mark]**
(a) ultraviolet (b) radiowave
(c) *X*-ray (d) infrared

12. The radius of an atomic nucleus is of the order of : **[1985 - 1 Mark]**
(a) 10^{-10} cm (b) 10^{-13} cm (c) 10^{-15} cm (d) 10^{-8} cm

13. Bohr model can explain : **[1985 - 1 Mark]**
(a) the spectrum of hydrogen atom only
(b) spectrum of an atom or ion containing one electron only
(c) the spectrum of hydrogen molecule
(d) the solar spectrum

14. Which electronic level would allow the hydrogen atom to absorb a photon but not to emit a photon? **[1984 - 1 Mark]**
(a) 3*s* (b) 2*p* (c) 2*s* (d) 1*s*

15. The increasing order (lowest first) for the values of e/m (charge/mass) for electron (e), proton (p), neutron (n) and alpha particle (α) is : **[1984 - 1 Mark]**
(a) e, p, n, α (b) n, p, e, α
(c) n, p, α, e (d) n, α, p, e

16. Rutherford's scattering experiment is related to the size of the **[1983 - 1 Mark]**
(a) nucleus (b) atom (c) electron (d) neutron

17. Rutherford's experiment on scattering of α-particles showed for the first time that the atom has **[1981 - 1 Mark]**
(a) electrons (b) protons
(c) nucleus (d) neutrons

18. The number of neutrons in dipositive zinc ion with mass number 70 is **[1979]**
(a) 34 (b) 36 (c) 38 (d) 40

2 Integer Value Answer

19. For He^+, a transition takes place from the orbit of radius 105.8 pm to the orbit of radius 26.45 pm.
The wavelength (in nm) of the emitted photon during the transition is ___. **[Adv. 2023]**
[**Use:** Bohr radius, a = 52.9 pm; Rydberg constant, $R_H = 2.2 \times 10^{-18}$ J; Planck's constant, $h = 6.6 \times 10^{-34}$ J s; Speed of light, $c = 3 \times 10^8$ m s^{-1}]

20. The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is **[2011]**

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

3 Numeric / New Stem Based Questions

21. Wavelength of high energy transition of H–atoms is 91.2nm. Calculate the corresponding wavelength of He atoms. **[2003 - 2 Marks]**

22. Calculate the wave number for the shortest wavelength transition in the Balmer series of atomic hydrogen. **[1996 - 1 Mark]**

23. According to Bohr's theory, the electronic energy of hydrogen atom in the n^{th} Bohr's orbit is given by $E_n = \dfrac{-21.76 \times 10^{-19}}{n^2}$ J. Calculate the longest wavelength of light that will be needed to remove an electron from the third Bohr orbit of the He^+ ion. **[1990 - 3 Marks]**

24. Calculate the wavelength in Angstrom of the photon that is emitted when an electron in the Bohr orbit, $n = 2$ returns to the orbit, $n = 1$ in the hydrogen atom. The ionization potential of the ground state hydrogen atom is 2.17×10^{-11} erg per atom. **[1982 - 4 Marks]**

25. The energy of the electron in the second and the third Bohr's orbits of the hydrogen atom is -5.42×10^{-12} erg and -2.41×10^{-12} erg respectively. Calculate the wavelength of the emitted radiation when the electron drops from the third to the second orbit. **[1981 - 3 Marks]**

4 Fill in the Blanks

26. The light radiations with discrete quantities of energy are called .... **[1993 - 1 Mark]**

27. Elements of the same mass number but of different atomic numbers are known as .... **[1983 - 1 Mark]**

28. Isotopes of an element differ in the number of .... in their nuclei. **[1982 - 1 Mark]**

29. The mass of a hydrogen atom is .... kg. **[1982 - 1 Mark]**

5 True / False

30. In a given electric field, β-particles are deflected more than α-particles in spite of α-particles having larger charge. **[1993 - 1 Mark]**

31. Gamma rays are electromagnetic radiations of wavelengths of 10^{-6} cm to 10^{-5} cm. **[1983 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

32. The energy of an electron in the first Bohr orbit of H atom is –13.6 eV. The possible energy value(s) of the excited state(s) for electrons in Bohr orbits of hydrogen is (are) **[1998 - 2 Marks]**
(a) – 3.4 eV (b) – 4.2 eV (c) – 6.8 eV (d) – 1.5 eV

33. The sum of the number of neutrons and proton in the isotope of hydrogen is : **[1986 - 1 Mark]**
(a) 6 (b) 2 (c) 4 (d) 3

34. When alpha particles are sent through a thin metal foil, most of them go straight through the foil because : **[1984 - 1 Mark]**
(a) alpha particles are much heavier than electrons
(b) alpha particles are positively charged
(c) most part of the atom is empty space
(d) alpha particle move with high velocity

35. Many elements have non-integral atomic masses because: **[1984 - 1 Mark]**
(a) they have isotopes
(b) their isotopes have non-integral masses
(c) their isotopes have different masses
(d) the constitutents, neutrons, protons and electrons, combine to give fractional masses

36. An isotope of ${}^{76}_{32}Ge$ is : **[1984 - 1 Mark]**
(a) ${}^{77}_{32}Ge$ (b) ${}^{77}_{33}As$ (c) ${}^{77}_{34}Se$ (d) ${}^{78}_{34}Se$

7 Match the Following

37. Consider the Bohr's model of a one – electron atom where the electron moves around the nucleus. In the following List–I contains some quantities for the n^{th} orbit of the atom and List–II contains options showing how they depend on n **[Adv. 2019]**

	List–I	List–II
(I)	Radius of the n^{th} orbit	(P) $\propto n^{-2}$
(II)	Angular momentum of the electron in the n^{th} orbit	(Q) $\propto n^{-1}$
(III)	Kinetic energy of the electron in the n^{th} orbit	(R) $\propto n^0$
(IV)	Potential energy of the electron in the n^{th} orbit	(S) $\propto n^1$
		(T) $\propto n^2$
		(U) $\propto n^{1/2}$

Which of the following options has the correct Combination considering List–I and List–II?
(a) (II), (R) (b) (II), (Q) (c) (I), (P) (d) (I), (T)

10 Subjective Problems

38. Calculate the energy required to excite one litre of hydrogen gas at 1 atm and 298 K to the first excited state of atomic hydrogen. The energy for the dissociation of H–H bond is 436 kJ mol^{-1}. **[2000 - 4 Marks]**

39. Consider the hydrogen atom to be a proton embedded in a cavity of radius a_0 (Bohr radius) whose charge is neutralised by the addition of an electron to the cavity in vacuum, infinitely slowly. Estimate the average total energy of an electron in its ground state in a hydrogen atom as the work done in the above neutralisation process. Also, if the magnitude of the average kinetic energy is half the magnitude of the average potential energy, find the average potential energy. **[1996 - 2 Marks]**

40. Iodine molecule dissociates into atoms after absorbing light of 4500Å. If one quantum of radiation is absorbed by each molecule, calculate the kinetic energy of iodine atoms. (Bond energy of I_2 = 240 kJ mol^{-1}) **[1995 - 2 Marks]**

41. Find out the number of waves made by a Bohr electron in one complete revolution in its 3rd orbit. **[1994 - 3 Marks]**

42. What transition in the hydrogen spectrum would have the same wavelength as the Balmer transition $n = 4$ to $n = 2$ of He^+ spectrum? **[1993 - 3 Marks]**

43. Estimate the difference in energy between 1st and 2nd Bohr orbit for a hydrogen atom. At what minimum atomic number, a transition from $n = 2$ to $n = 1$ energy level would result in the emission of X-rays with $\lambda = 3.0 \times 10^{-8}$m? Which hydrogen atom-like species does this atomic number correspond to ? **[1993 - 5 Marks]**

44. The electron energy in hydrogen atom is given by $E = (-21.7 \times 10^{-12})/n^2$ ergs. Calculate the energy required to remove an electron completely from the $n = 2$ orbit. What is the longest wavelength (in cm) of light that can be used to cause this transition? **[1984 - 3 Marks]**

45. Naturally occurring boron consists of two isotopes whose atomic weights are 10.01 and 11.01. The atomic weight of natural boron is 10.81. Calculate the percentage of each isotope in natural boron. **[1978]**

Topic-2: Advancement Towards Quantum Mechanical Model of Atom

1 MCQs with One Correct Answer

1. The wavelength associated with a golf ball weighing 200 g and moving at a speed of 5 m/h is of the order **[2001S]**

(a) 10^{-10} m (b) 10^{-20} m (c) 10^{-30} m (d) 10^{-40} m

2. Which of the following relates to photons both as wave motion and as a stream of particles? **[1992 - 1 Mark]**

(a) Inference (b) $E = mc^2$
(c) Diffraction (d) $E = hv$

2 Integer Value Answer

3. Consider a helium (He) atom that absorbs a photon of wavelength 330 nm. The change in the velocity (in cm s^{-1}) of He atom after the photon absorption is ___.
(Assume : Momentum is conserved when photon is absorbed.
Use : Planck constant = 6.6×10^{-34} J s, Avogadro number = 6×10^{23} mol^{-1}, Molar mass of He = 4 g mol^{-1}) **[Adv. 2021]**

4. The atomic masses of 'He' and 'Ne' are 4 and 20 a.m.u., respectively. The value of the de Broglie wavelength of 'He' gas at —73 °C is "M" times that of the de Broglie wavelength of 'Ne' at 727 °C. 'M' is **[Adv. 2013]**

4 Fill in the Blanks

5. Wave functions of electrons in atoms and molecules are called .... **[1993 - 1 Mark]**

6. The uncertainty principle and the concept of wave nature of matter were proposed by .... and .... respectively. (Heisenberg, Schrodinger, Maxwell, de Broglie) **[1988 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

7. Among the following, the correct statement(s) for electrons in an atom is(are) **[Adv. 2024]**

(a) Uncertainty principle rules out the existence of definite paths for electrons.
(b) The energy of an electron in 2s orbital of an atom is lower than the energy of an electron that is infinitely far away from the nucleus.
(c) According to Bohr's model, the most negative energy value for an electron is given by $n = 1$, which corresponds to the most stable orbit.
(d) According to Bohr's model, the magnitude of velocity of electrons increases with increase in values of n.

10 Subjective Problems

8. Find the velocity (ms^{-1}) of electron in first Bohr's orbit of radius a_0. Also find the de Broglie's wavelength (in m). Find the orbital angular momentum of 2p orbital of hydrogen atom in units of $h/2\pi$. **[2005 - 2 Marks]**

9. A ball of mass 100 g is moving with 100 ms^{-1}. Find its wavelength. **[2004 - 1 Mark]**

10. The Schrodinger wave equation for hydrogen atom is **[2004 - 2 Marks]**

$$\Psi_{2s} = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2 - \frac{r_0}{a_0}\right)e^{-r_0/a_0}$$

Where a_0 is Bohr's radius. If the radial node in 2s be at r_0, then find r_0 in terms of a_0.

Topic-3: Quantum Mechanical Model of Atom

1 MCQs with One Correct Answer

1. The number of radial nodes of 3*s* and 2*p* orbitals are respectively **[2005S]**
(a) 2, 0 (b) 0, 2 (c) 1, 2 (d) 2, 1

2. If the nitrogen atom has electronic configuration $1s^7$, it would have energy lower than that of the normal ground state configuration $1s^2 2s^2 2p^3$, because the electrons would be closer to the nucleus. Yet $1s^7$ is not observed because it violates. **[2002S]**
(a) Heisenberg uncertainty principle
(b) Hund's rule
(c) Pauli exclusion principle
(d) Bohr postulate of stationary orbits

3. The quantum numbers +1/2 and –1/2 for the electron spin represent **[2001S]**
(a) rotation of the electron in clockwise and anticlockwise direction respectively
(b) rotation of the electron in anticlockwise and clockwise direction respectively
(c) magnetic moment of the electron pointing up and down respectively
(d) two quantum mechanical spin states which have no classical analogue

4. The electronic configuration of an element is $1s^2$, $2s^2 2p^6$, $3s^2 3p^6 3d^5$, $4s^1$. This represents its **[2000S]**
(a) excited state (b) ground state
(c) cationic form (d) anionic form

5. The number of nodal planes in a p_x orbital is **[2000S]**
(a) one (b) two (c) three (d) zero

6. The electrons, identified by quantum numbers *n* and *l*, (i) $n = 4, l = 1$, (ii) $n = 4, l = 0$, (iii) $n = 3, l = 2$, and (iv) $n = 3, l = 1$ can be placed in order of increasing energy, from the lowest to highest, as **[1999 - 2 Marks]**
(a) (iv) < (ii) < (iii) < (i) (b) (ii) < (iv) < (i) < (iii)
(c) (i) < (iii) < (ii) < (iv) (d) (iii) < (i) < (iv) < (ii)

7. For a *d*-electron, the orbital angular momentum is **[1997 - 1 Mark]**
(a) $\sqrt{6}(h/2\pi)$ (b) $\sqrt{2}(h/2\pi)$
(c) $(h/2\pi)$ (d) $2(h/2\pi)$

8. The orbital angular momentum of an electron in 2*s* orbital is: **[1996 - 1 Mark]**
(a) $+\frac{1}{2}.\frac{h}{2\pi}$ (b) Zero (c) $\frac{h}{2\pi}$ (d) $\sqrt{2}.\frac{h}{2\pi}$

9. A 3*p* orbital has : **[1995S]**
(a) two non spherical nodes
(b) two spherical nodes
(c) one spherical and one non spherical node
(d) one spherical and two non spherical nodes

10. The correct set of quantum numbers for the unpaired electron of chlorine atom is : **[1989 - 1 Mark]**

	n	*l*	*m*
(a)	2	1	0
(b)	2	1	1
(c)	3	1	1
(d)	3	0	0

11. The correct ground state electronic configuration of chromium atom is : **[1989 - 1 Mark]**
(a) $[Ar]\,3d^5\,4s^1$ (b) $[Ar]\,3d^4\,4s^2$
(c) $[Ar]\,3d^6\,4s^0$ (d) $[Ar]\,4d^5\,4s^1$

12. The outermost electronic configuration of the most electronegative element is **[1988 - 1 Mark]**
(a) $ns^2\,np^3$ (b) $ns^2\,np^4$ (c) $ns^2\,np^5$ (d) $ns^2\,np^6$

13. The orbital diagram in which the Aufbau principle is violated is : **[1988 - 1 Mark]**

	2s	2p		
(a)	↑↓	↑↓	↑	
(b)	↑	↑↓	↑	↑
(c)	↑↓	↑	↑	↑
(d)	↑↓	↑↓	↑	↑

14. Which one of the following sets of quantum numbers represents an impossible arrangement? **[1986 - 1 Mark]**

	n	*l*	m_l	m_s
(a)	3	2	–2	½
(b)	4	0	0	½
(c)	3	2	–3	½
(d)	5	3	0	–½

15. Correct set of four quantum numbers for the valence (outermost) electron of rubidium ($Z = 37$) is : **[1984 - 1 Mark]**
(a) 5, 0, 0, +½ (b) 5, 1, 0, +½
(c) 5, 1, 1, +½ (d) 6, 0,0, +½

16. Any *p*-orbital can accommodate upto **[1983 - 1 Mark]**
(a) four electrons
(b) six electrons
(c) two electrons with parallel spins
(d) two electrons with opposite spins

17. The principal quantum number of an atom is related to the **[1983 - 1 Mark]**
(a) size of the orbital
(b) spin angular momentum
(c) orbital angular momentum
(d) orientation of the orbital in space

2 Integer Value Answer

18. Not considering the electronic spin, the degeneracy of the second excited state (n = 3) of H atom is 9, while the degeneracy of the second excited state of H^- is **[Adv. 2015]**

19. In an atom, the total number of electrons having quantum numbers $n = 4, |m_1| = 1$ and $m_s = -\frac{1}{2}$ is **[Adv. 2014]**

20. The maximum number of electrons that can have principal quantum number, $n = 3$, and spin quantum $m_s = -\frac{1}{2}$, is **[2011]**

3 Numeric / New Stem Based Questions

21. What is the maximum number of electrons that may be present in all the atomic orbitals with principal quantum number 3 and azimuthal quantum number 2? **[1985 - 2 Marks]**

4 Fill in the Blanks

22. The outermost electronic configuration of Cr is .... **[1994 - 1 Mark]**

23. The $2p_x$, $2p_y$ and $2p_z$ orbitals of atom have identical shapes but differ in their .... **[1993 - 1 Mark]**

24. When there are two electrons in the same orbital, they have .... spins. **[1982 - 1 Mark]**

5 True / False

25. The electron density in the XY plane in $3d_{x^2-y^2}$ orbital is zero. **[1986 - 1 Mark]**

26. The energy of the electron in the 3d-orbital is less than that in the 4s-orbital in the hydrogen atom. **[1983 - 1 Mark]**

27. The outer electronic configuration of the ground state chromium atom is $3d^4 4s^2$. **[1982 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

28. The ground state energy of hydrogen atom is –13.6 eV. Consider an electronic state ψ of He^+ whose energy, azimuthal quantum number and magnetic quantum number are –3.4 eV, 2 and 0, respectively. Which of the following statement(s) is (are) true from the state ψ? **[Adv. 2019]**
(a) It is a 4d state
(b) It has 3 radial nodes
(c) It has 2 angular nodes
(d) The nuclear charge experienced by the electron in this state is less than 2e, where e is the magnitude of the electronic charge

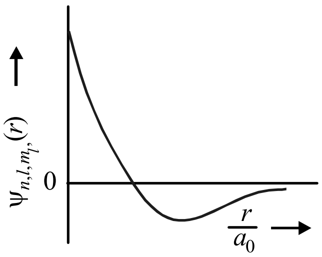
29. Ground state electronic configuration of nitrogen atom can be represented by **[1999 - 3 Marks]**
(a) ↑↓ ↑↓ ↑ ↑ ↑
(b) ↑↓ ↑↓ ↑ ↓ ↑
(c) ↑↓ ↑↓ ↑ ↓ ↓
(d) ↑↓ ↑↓ ↓ ↓ ↓

30. Which of the following satement(s) is (are) correct? **[1998 - 2 Marks]**
(a) The electronic configuration of Cr is [Ar] $3d^5 4s^1$. (Atomic Number of Cr = 24)
(b) The magnetic quantum number may have a negative value.
(c) In silver atom, 23 electrons have a spin of one type and 24 of the opposite type. (Atomic Number of Ag = 47)
(d) The oxidation state of nitrogen in HN_3 is – 3.

7 Match the Following

(Qs. 31-35) are based on the table, having 3 columns and 4 rows. Each question has four options (A), (B), (C) and (D). Only one of these four options is correct. By appropriately matching the information given in the three columns of the following table.

The wave function, ψ_{n,l,m_l} is a mathematical function whose value depends upon spherical polar coordinates (r, θ, ϕ) of the electron and characterized by the quantum numbers n, l and m_l. Here r is distance from nucleus, θ is colatitude and ϕ is azimuth. In the mathematical functions given in the table, Z is atomic number and a_0 is Bohr radius. **[Adv. 2017]**

Column-1	Column-2	Column-3
(I) 1s orbital	(i) $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\left(\frac{Zr}{a_0}\right)}$	(P) 
(II) 2s orbital	(ii) One radial node	(Q) Probability density at nucleus $\propto \frac{1}{a_0^3}$

(III)	$2p_z$ orbital	(iii)	$\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{5}{2}} re^{-\left(\frac{Zr}{2a_0}\right)} \cos\theta$	(R)	Probability density is maximum at nucleus
(IV)	$3d_{z^2}$ orbital	(iv)	xy-plane is a nodal plane	(S)	Energy needed to excite electron from $n = 2$ state to $n = 4$ state is $\frac{27}{32}$ times the energy needed to excite electron from $n = 2$ state to $n = 6$ state

31. For the given orbital in Column 1, the only CORRECT combination for any hydrogen-like species is
(a) (I) (ii) (S) (b) (IV) (iv) (R) (c) (II) (ii) (P) (d) (III) (iii) (P)

32. For hydrogen atom, the only CORRECT combination is
(a) (I) (i) (S) (b) (II) (i) (Q) (c) (I) (i) (P) (d) (I) (iv) (R)

33. For He^+ ion, the only INCORRECT combination is
(a) (I) (i) (R) (b) (II) (ii) (Q) (c) (I) (iii) (R) (d) (I) (i) (S)

34. Match the entries in Column I with the correctly related quantum number(s) in Column II. **[2008 - 6M]**

Column-I	**Column-II**
(A) Orbital angular momentum of the electron in a hydrogen-like atomic orbital	(p) Principal quantum number
(B) A hydrogen-like one-electron wave function obeying Pauli principle	(q) Azimuthal quantum number
(C) Shape, size and orientation of hydrogen- like atomic orbitals	(r) Magnetic quantum number
(D) Probability density of electron at the nucleus in hydrogen-like atom	(s) Electron spin quantum number

35. E_n = Total energy, K_n = Kinetic energy, V_n = Potential energy, r_n = Radius of n^{th} orbit Match the following :

Column-I	**Column-II**
(A) $V_n / K_n = ?$	(p) 0
(B) If radius of n^{th} orbit $\propto E_n^x$, $x = ?$	(q) –1
(C) Angular momentum in lowest orbital	(r) –2
(D) $\frac{1}{r_n} \propto Z^y$, $y = ?$	(s) 1

8 Comprehension/Passage Based Questions

The hydrogen-like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light the ion undergoes transition to a state S_2. The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom. **[2010]**

36. The state S_1 is :
(a) 1*s* (b) 2*s* (c) 2*p* (d) 3*s*

37. Energy of the state S_1 in units of the hydrogen atom ground state energy is :
(a) 0.75 (b) 1.50 (c) 2.25 (d) 4.50

38. The orbital angular momentum quantum number of the state S_2 is :
(a) 0 (b) 1 (c) 2 (d) 3

10 Subjective Problems

39. Give reasons why the ground state outermost electronic configuration of silicon is : **[1985 - 2 Marks]**

3*s* [↑↓] 3*p* [↑][↑][ ] and not 3*s* [↑↓] 3*p* [↑][↓][ ]

Answer Key

Topic-1 : Different Atomic Models that Leads to Bohr Model

1. (b) **2.** (c) **3.** (b) **4.** (d) **5.** (d) **6.** (c) **7.** (c) **8.** (a) **9.** (d) **10.** (b)
11. (b) **12.** (b) **13.** (b) **14.** (d) **15.** (d) **16.** (a) **17.** (c) **18.** (d) **19.** (30) **20.** (4)
21. (22.8) **22.** (27419) **23.** (2055) **24.** (1220) **25.** (660) **26.** (photons) **27.** (isobars)
28. (neutrons) **29.** (1.66×10^{-27} kg) **30.** True **31.** False **32.** (a, d) **33.** (b, d) **34.** (a, c) **35.** (a, c)
36. (b, d) **37.** (d)

Topic-2 : Advancement Towards Quantum Mechanical Model of Atom

1. (a) **2.** (d) **3.** (30) **4.** (5) **5.** (orbitals) **6.** (Heisenberg, de-Broglie) **7.** (a, b, c)

Topic-3 : Quantum Mechanical Model of Atom

1. (a) **2.** (c) **3.** (d) **4.** (b) **5.** (a) **6.** (a) **7.** (a) **8.** (b) **9.** (c) **10.** (c)
11. (a) **12.** (c) **13.** (b) **14.** (c) **15.** (a) **16.** (d) **17.** (a) **18.** (3) **19.** (6) **20.** (9)
21. (10) **22.** ($4s^1, 3d^5$) **23.** (orientation in space) **24.** (antiparallel; or opposite)
25. (False) **26.** (True) **27.** (False) **28.** (a, c) **29.** (a, d) **30.** (a, b, c) **31.** (c) **32.** (a) **33.** (c)
34. (A) - (q); (B) - (p, q, r, s); (C) - (p, q, r); (D) - (p, q, r) **35.** (A) - (r); (B) - (q); (C) - (p); (D) - (s) **36.** (b)
37. (c) **38.** (b)

2 Structure of Atom

Topic-1: Different Atomic Models that Leads to Bohr Model

1. **(b)** K.E. of electron in n^{th} Bohr's orbit is given by :

$$K.E. = 13.6\frac{Z^2}{n^2}\ \text{eV/atom}$$

$$n = 1\ (\text{H-atom}) \rightarrow K.E. \propto \frac{1^2}{1^2} = 1$$

$$n = 1\ (He^+\ \text{ion}) \rightarrow K.E. \propto \frac{2^2}{1^2} = 4$$

$$n = 2\ (He^+\ \text{ion}) \rightarrow K.E. \propto \frac{2^2}{2^2} = 1$$

$$n = 2\ (Li^{2+}\ \text{ion}) \rightarrow K.E. \propto \frac{3^2}{2^2} = \frac{9}{4}$$

Thus, K.E. is highest for first orbit of He^+.

2. **(c)** As per Bohr's postulate,

$$mvr = \frac{nh}{2\pi} \quad \text{So, } v = \frac{nh}{2\pi mr}$$

$$KE = \frac{1}{2}mv^2 \quad \text{So, } KE = \frac{1}{2}m\left(\frac{nh}{2\pi mr}\right)^2$$

Since, $r = \dfrac{a_o \times n^2}{z}$

So, for 2^{nd} Bohr orbit

$$r = \frac{a_o \times 2^2}{1} = 4a_o$$

$$KE = \frac{1}{2}m\left(\frac{2^2 h^2}{4\pi^2 m^2 \times (4a_o)^2}\right) = \frac{h^2}{32\pi^2 m a_o^2}$$

3. **(b)** Average atomic mass of Fe

$$= \frac{(54 \times 5) + (56 \times 90) + (57 \times 5)}{100} = 55.95$$

4. **(d)** $r_n = 0.529\dfrac{n^2}{Z}$ Å

For hydrogen, $n = 1$ and $Z = 1$; $\therefore\ r_H = 0.529$

For Be^{3+}, $n = 2$ and $Z = 4$;

$$\therefore\ r_{Be^{3+}} = \frac{0.529 \times 2^2}{4} = 0.529$$

5. **(d)** Rutherford's experiment was actually α-particle scattering experiment. α-Particle is doubly positively charged helium ion, i.e., He - nucleus.

6. **(c)** **X-rays** can ionise gases and cannot get deflected by electric and magnetic fields, wavelength of these rays is 150 to 0.1Å. Thus, the wavelength of X-rays is shorter than that of U.V. rays.

7. **(c)** Difference in the energy of the energy levels involved in the transition.

8. **(a)** Isotones have same number of neutrons. All atoms in triad (a) have same number of neutrons (i.e., $A - Z = 8$).

9. **(d)** $E = \dfrac{hc}{\lambda}$; $\lambda_1 = 2000$ Å; $\lambda_2 = 4000$ Å;

$$\text{so } \frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1} = \frac{4000}{2000} = 2$$

10. **(b)** Electrons in an atom occupy the extra nuclear region.

11. **(b)** The following is the increasing order of wavelength or decreasing order of energy of electromagnetic radiations :

- cosmic rays
- γ-rays
- X-rays
- UV-rays
- Visible
- Infra-red radiation
- Micro waves
- Radio waves

(↓ Increasing λ, Decreasing ν)

Among given choices, radiowaves have maximum wavelength.

12. **(b)** The radius of nucleus is of the order of 1.5×10^{-13} to 6.5×10^{-13} cm or 1.5 to 6.5 Fermi (1 Fermi = 10^{-13} cm)

13. **(b)** Bohr model can explain spectrum of atoms/ions containing one electron only.

14. **(d)** Energy is emitted when electron falls from higher energy level to lower energy level and energy is absorbed when electron moves from lower level to higher level. 1*s* is the lowest energy level of electron in an atom. $\therefore$ An electron in 1*s* level of hydrogen can absorb energy but cannot emit energy.

15. **(d)** $\frac{e}{m}$ for neutron $= \frac{0}{1} = 0$; α-particle $= \frac{2}{4} = 0.5$;

proton $= \frac{1}{1} = 1$; electron $= \frac{1}{1/1837} = 1837$

16. **(a)** According to Rutherford's experiment. "The central part consisting of whole of the positive charge and most of the mass, called nucleus, is extremely small in size compared to the size of the atom."

17. **(c)** Rutherford's scattering experiment led to the discovery of nucleus.

18. **(d)** No. of neutrons = Mass number – Atomic number $= 70 - 30 = 40$.

19. **(30)** For single electron system, $r_n = 52.9 \times \frac{n^2}{Z}$ pm

$105.8 = \frac{52.9 \times n_1^2}{2} \therefore n_1^2 = 4 \Rightarrow n_1 = 2$

$26.45 = \frac{52.9 \times n_2^2}{2} \therefore n_2 = 1$

So, transition is from 2 to 1.

Now $\frac{hc}{\lambda} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\lambda = 30 \times 10^{-9}$ m = 30 nm

20. **(4)** Energy associated with incident photon $= \frac{hc}{\lambda}$

$E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}}$ J

$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9} \times 1.6 \times 10^{-19}}$ eV = 4.16 eV

Photoelectric effect can take place only when $E_{photon} > \phi$

Thus, number of metals showing photoelectric effect will be 4 (*i.e.* Li, Na, K and Mg).

21. **(22.8)** For maximum energy, $n_1 = 1$ and $n_2 = \infty$

$\frac{1}{\lambda} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Since R_H is a constant and transition remains the same

$\frac{1}{\lambda} \propto Z^2$; $\frac{\lambda_{He}}{\lambda_H} = \frac{Z_H^2}{Z_{He}^2} = \frac{1}{4}$

Hence, $\lambda_{He} = \frac{1}{4} \times 91.2 =$ **22.8 nm**

22. **(27419)** The shortest wavelength transition in the Balmer series corresponds to the transition $n = 2 \rightarrow n = \infty$. Hence, $n_1 = 2, n_2 = \infty$

$\bar{\nu} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = (109677 \text{cm}^{-1}) \left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$

= **27419.25 cm⁻¹**

23. **(2055)** E_n of H $= \frac{-21.76 \times 10^{-19}}{n^2}$ J

$\therefore E_n$ of $He^+ = \frac{-21.76 \times 10^{-19}}{n^2} \times Z^2$ J

$\therefore E_3$ of $He^+ = \frac{-21.76 \times 10^{-19} \times 4}{9}$ J

Hence, energy equivalent to E_3 must be supplied to remove the electron from 3rd orbit of He^+. Wavelength corresponding to this energy can be determined by applying the relation.

$E = \frac{hc}{\lambda}$ or $\lambda = \frac{hc}{E} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8 \times 9}{21.76 \times 10^{-19} \times 4}$

$= 2055 \times 10^{-10}$ m = **2055 Å**

24. **(1220)** (i) Energy of n^{th} orbit $= E_n = \frac{E_1}{n^2}$

(ii) Difference in energy $= E_1 - E_2 = h\nu = \frac{hc}{\lambda}$

or $\lambda = \frac{hc}{E_1 - E_2}$

Given $E_1 = 2.17 \times 10^{-11}$

$\therefore$ Energy of second orbit $= E_2 = \frac{2.17 \times 10^{-11}}{2^2}$

$= 0.5425 \times 10^{-11}$ erg

$\Delta E = E_1 - E_2 = 2.17 \times 10^{-11} - 0.5425 \times 10^{-11}$

$= 1.6275 \times 10^{-11}$ erg

$\lambda = \frac{6.62 \times 10^{-27} \times 3 \times 10^{10}}{1.6275 \times 10^{-11}} = 12.20 \times 10^{-6}$ cm = **1220 Å**

25. **(660)** $\Delta E = E_3 - E_2 = h\nu = \frac{hc}{\lambda}$ or $\lambda = \frac{hc}{E_3 - E_2}$

Given $E_2 = -5.42 \times 10^{-12}$ erg, $E_3 = -2.41 \times 10^{-12}$ erg

$\therefore \quad \lambda = \frac{6.626 \times 10^{-27} \times 3 \times 10^{10}}{-2.41 \times 10^{-12} - (-5.42 \times 10^{-12})}$

$= \frac{19.878 \times 10^{-17}}{3.01 \times 10^{-12}} = 6.604 \times 10^{-5}$ cm = **660 nm**

26. **photons**

27. **isobars**

28. **neutrons**

29. **1.66×10^{-27} kg**

Mass of hydrogen atom

$$= \frac{\text{Atomic mass of hydrogen}}{\text{Avogadro number}} = \frac{1.008}{6.02 \times 10^{23}}$$

$= 0.166 \times 10^{-23}$ g $= 1.66 \times 10^{-27}$ kg

30. **True :** β-particles are deflected more than α-particles because they have very-very large *e/m* value as compared to α-particles due to the fact that electrons are much lighter than He^{2+} species.

31. **False :** Gamma rays are electromagnetic radiations of wavelengths 10^{-9} cm to 10^{-10} cm.

32. **(a, d)** The energy of an electron on Bohr orbits of hydrogen atoms is given by the expression

$$E_n = -\frac{\text{Constant}}{n^2}$$

Where n takes only integral values. For the first Bohr orbit, $n = 1$ and it is given that $E_1 = -13.6$ eV

Hence $E_n = -\frac{13.6\text{eV}}{n^2}$ of the given values of energy, only

– 3.4 eV and – 1.5 eV can be obtained by substituting $n = 2$ and 3 respectively in the above expression.

33. **(b, d)** In tritium (the isotope of hydrogen) nucleus there is one proton and 2 neutrons. $\therefore n + p = 3$. In deuterium nucleus there is one proton and one neutron $\therefore n + p = 2$.

34. **(a, c)** α-particles pass through because most part of the atom is empty.

35. **(a, c)** Because they have isotopes with different masses. The average atomic mass is the weighted mean of their presence in nature; e.g., Cl^{35} and Cl^{37} are present in ratio 3 : 1 in nature.

So $A = \frac{35 \times 3 + 37 \times 1}{4} = 35.5$

36. **(b, d)** ${}^{77}_{33}$As and ${}^{78}_{34}$Se have same number of neutrons ($= A - Z$) as ${}^{76}_{32}$Ge.

37. **(d)** $r \propto \frac{n^2}{Z}$ or $r = 0.529 \times \frac{n^2}{Z}$; (I), (T)

$|L| \propto n$ or $mvr = \frac{nh}{2\pi}$; (II), (S)

38. Determination of number of moles of hydrogen gas,

$$n = \frac{PV}{RT} = \frac{1 \times 1}{0.082 \times 298} = 0.0409$$

The concerned reaction is $H_2 \longrightarrow 2H$; $\Delta H = 436$ kJ mol^{-1}

Energy required to bring 0.0409 moles of hydrogen gas to atomic state $= 436 \times 0.0409 =$ **17.83 kJ**

Calculation of total number of hydrogen atoms in 0.0409 mole of H_2 gas.

1 mole of H_2 gas has 6.02×10^{23} molecules

0.0409 mole of H_2 gas $= \frac{6.02 \times 10^{23}}{1} \times 0.0409$ molecules

Since, 1 molecule of H_2 gas has 2 hydrogen atoms

$6.02 \times 10^{23} \times 0.0409$ molecules of H_2 gas

$= 2 \times 6.02 \times 10^{23} \times 0.0409 = 4.92 \times 10^{22}$ atoms of hydrogen

Since, energy required to excite an electron from the ground state to the next excited state is given by

$$E = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\text{eV}$$

$$= 13.6 \times \left(\frac{1}{1} - \frac{1}{4}\right) = 13.6 \times \frac{3}{4} = 10.2\text{eV} = 1.632 \times 10^{-21} \text{ kJ}$$

Therefore, energy required to excite 4.92×10^{22} electrons

$= 1.632 \times 10^{-21} \times 4.92 \times 10^{22}$ kJ $= 8.03 \times 10 = 80.3$ kJ

Therefore, total energy required $= 17.83 + 80.3 =$ **98.17 kJ**

39. Work done while bringing an electron infinitely slowly from infinity to proton of radius a_0 is given as follows

$$W = -\frac{e^2}{4\pi\varepsilon_0 . a_0}$$

This work done is equal to the total energy of an electron in its ground state in the hydrogen atom. At this stage, the electron is not moving and do not possess any K.E., so this total energy is equal to the potential energy.

$$\text{T.E.} = \text{P. E} + \text{K. E.} = \text{P. E.} = -\frac{\boldsymbol{e^2}}{\boldsymbol{4\pi\varepsilon_0 . a_0}} \quad ...(1)$$

In order the electron to be captured by proton to form a ground state hydrogen atom it should also attain

$$\text{K.E.} = \frac{e^2}{8\pi\varepsilon_0 a_0}$$

(It is given that magnitude of K.E. is half the magnitude of P.E. Note that P.E. is –ve and K.E is +ve)

$$\therefore \text{T.E} = \text{P. E.} + \text{K. E.} = -\frac{e^2}{4\pi\varepsilon_0 a_0} + \frac{e^2}{8\pi\varepsilon_0 a_0}$$

or $\quad \text{T.E.} = -\frac{e^2}{8\pi\varepsilon_0 a_0}$

$$\text{P.E.} = 2 \times \text{T.E.} = 2 \times \frac{-e^2}{8\pi \epsilon_0 a_0} \text{ or P.E.} = \frac{\boldsymbol{-e^2}}{\boldsymbol{4\pi \epsilon_0 a_0}}$$

40. Bond energy of $I_2 = 240$ kJ mol^{-1} $= 240 \times 10^3$ J mol^{-1}

$$= \frac{240 \times 10^3}{6.023 \times 10^{23}} \text{ J molecule}^{-1} = 3.984 \times 10^{-19} \text{ J molecule}^{-1}$$

Energy absorbed $= \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\,\text{Js}\times3\times10^{8}\,\text{ms}^{-1}}{4500\times10^{-10}\,\text{m}}$

$= 4.417\times10^{-19}\,\text{J}$

Kinetic energy = Absorbed energy – Bond energy

$\therefore$ Kinetic energy $= 4.417\times10^{-19} - 3.984\times10^{-19}\,\text{J}$

$= 4.33\times10^{-20}\,\text{J}$

$\therefore$ Kinetic energy of each atom of iodine

$= \frac{4.33\times10^{-20}}{2} = \mathbf{2.165\times10^{-20}\ J}$

41. Number of waves $= \frac{n(n-1)}{2}$

where n = Principal quantum number or number of orbit

Number of waves $= \frac{3(3-1)}{2} = \frac{3\times2}{2} = 3$

42. For He^+ ion, we have

$$\frac{1}{\lambda} = Z^2R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$= (2)^2R_H\left[\frac{1}{(2)^2} - \frac{1}{(4)^2}\right] = R_H\frac{3}{4} \quad ...(i)$$

Now for hydrogen atom $\frac{1}{\lambda} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ ...(ii)

Equating equations (i) and (ii), we get

$$\frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{3}{4}$$

Obviously, $n_1 = 1$ and $n_2 = 2$

Hence, the transition $\boldsymbol{n = 2}$ to $\boldsymbol{n = 1}$ in hydrogen atom will have the same wavelength as the transition, $n = 4$ to $n = 2$ in He^+ species.

43. $\Delta E = RhcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Here, $R = 1.0967\times10^7\,\text{m}^{-1}$

$h = 6.626\times10^{-34}$ J sec, $c = 3\times10^8$ m/sec

$n_1 = 1$, $n_2 = 2$ and for H-atom, $Z = 1$

$$E_2 - E_1 = 1.0967\times10^7\times6.626\times10^{-34}\times3\times10^8\left(\frac{1}{1} - \frac{1}{4}\right)$$

$$\Delta E = 1.0967\times6.626\times3\times\frac{3}{4}\times10^{-19}\,\text{J} = 16.3512\times10^{-19}\,\text{J}$$

$$= \frac{16.3512\times10^{-19}}{1.6\times10^{-19}}\,\text{eV} = \mathbf{10.22\ eV}$$

$$\Delta E = \frac{hc}{\lambda} = RhcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1} - \frac{1}{4}\right) = RZ^2\times\frac{3}{4}$$

Given, $\lambda = 3\times10^{-8}$ m

$$\therefore \frac{1}{3\times10^{-8}} = 1.0967\times Z^2\times\frac{3}{4}\times10^7$$

$$\therefore Z^2 = \frac{10^8\times4}{3\times3\times1.0967\times10^7} = \frac{40}{9\times1.0967} \approx 4 \quad \therefore Z = \mathbf{2}$$

So, it corresponds to He^+ which has 1 electron like hydrogen.

44. To calculate the energy required to remove electron from atom, $n = \infty$ is to be taken.

Energy of an electron in the n^{th} orbit of hydrogen is given by

$$E = -21.7\times10^{-12}\times\frac{1}{n^2}\ \text{ergs}$$

$$\therefore \Delta E = -21.7\times10^{-12}\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$

$$= -21.7\times10^{-12}\left(\frac{1}{4} - 0\right) = -21.7\times10^{-12}\times\frac{1}{4}$$

$= \mathbf{-5.42\times10^{-12}\ ergs}$

Now we know that $\Delta E = hv$

$$\therefore \quad \Delta E = \frac{hc}{\lambda} \quad \left(\because v = \frac{c}{\lambda}\right) \quad \text{or} \quad \lambda = \frac{hc}{\Delta E}$$

Substituting the values, $\lambda = \frac{6.627\times10^{-27}\times3\times10^{10}}{5.42\times10^{-12}}$

$= \mathbf{3.67\times10^{-5}\ cm}$

45. Let the % of isotope with At. wt. 10.01 = x

$\therefore$ % of isotope with At. wt. 11.01 = $(100 - x)$

At. wt. of boron $= \frac{x\times10.01 + (100-x)\times11.01}{100}$

$\Rightarrow 10.81 = \frac{x\times10.01 + (100-x)\times11.01}{100} \quad \therefore x = 20$

Hence, % of isotope with At. wt. 10.01 = 20%

$\therefore$ % of isotope with At. wt. 11.01 = 100 – 20 = **80%**.

Topic-2: Advancement Towards Quantum Mechanical Model of Atom

1. **(a)** According to de-Broglie's equation

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Given, $h = 6.6 \times 10^{-34}$ Js, $m = 200 \times 10^{-3}$ kg

$$v = \frac{5}{60 \times 60} \text{m/s} \quad \lambda = \frac{6.6 \times 10^{-34}}{200 \times 10^{-3} \times 5/(60 \times 60)}$$

$= 2.38 \times 10^{-10}$ m

2. **(d)** As packet of energy equal to $h\nu$; as wave having frequency ν.

3. **(30)** $p = \frac{h}{\lambda} \Rightarrow \frac{6.6 \times 10^{-34} \text{kgm}^2/\text{s}^2}{330 \times 10^{-9} \text{m}}$

$$= \frac{4 \times 10^{-3} \text{kg mol}^{-1}}{6 \times 10^{23} \text{mol}^{-1}} \times v \; (p = m \times v)$$

v = 0.3 m/s = 30 cm/s

4. **(5)** Since,

$$\lambda = \frac{h}{mV} = \frac{h}{\sqrt{2M\,K.E}} \;(\text{since K.E.} \propto T) \;\Rightarrow\; \lambda \propto \frac{1}{\sqrt{MT}}$$

For two gases,

$$\frac{\lambda_{He}}{\lambda_{Ne}} = \sqrt{\frac{M_{Ne}T_{Ne}}{M_{He}T_{He}}} = \sqrt{\frac{20}{4} \times \frac{1000}{200}} = 5$$

5. **orbitals**

6. **Heisenberg, de-Broglie**

7. **(a, b, c)**

(a) Uncertainty principle rules out existence of definite paths or trajectories of electron and other similar particles.

So, option (a) is correct.

(b) Shell or orbit more near to nucleus has less energy than far away.

So, option (b) is also correct.

(c) $E = -13.6 \frac{Z^2}{n^2}$ eV/atom

So, $n = 1$ has most negative energy.

So, option (c) is also correct.

(d) $V_e = V_0 \times \frac{Z}{n}$

When n increase velocity decreases.

So, option (d) is incorrect.

8. For hydrogen atom, $Z = 1, n = 1$

$$v = 2.18 \times 10^6 \times \frac{Z}{n} \text{ms}^{-1} = 2.18 \times 10^6 \text{ms}^{-1}$$

de Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.18 \times 10^6}$$

$= \mathbf{3.34 \times 10^{-10}}$ **m** = 3.3 Å

For $2p$, $l = 1$

$\therefore$ Orbital angular momentum $= \sqrt{l(l+1)}\frac{h}{2\pi} = \sqrt{2}\frac{h}{2\pi}$

9. $$\lambda = \frac{h}{mu} = \frac{6.627 \times 10^{-34}}{0.1 \times 100}$$

or $\lambda = \mathbf{6.627 \times 10^{-35}}$ **m =** $\mathbf{6.627 \times 10^{-25}}$ **Å**

10. ψ_{2s}^2 = probability of finding electron within $2s$ sphere

$\psi_{2s}^2 = 0$ (at node)

($\because$ probability of finding an electron is zero at node)

$$\therefore\; 0 = \frac{1}{32\pi}\left(\frac{1}{a_0}\right)^3 \left(2 - \frac{r_0}{a_0}\right)^2 \cdot e^{-\frac{2r_0}{a_0}}$$

(Squaring the given value of ψ_{2s})

or $\left[2 - \frac{r_0}{a_0}\right] = 0;\; \therefore\; 2 = \frac{r_0}{a_0};\; \mathbf{2a_0} = r_0$

Topic-3: Quantum Mechanical Model of Atom

1. **(a)** Number of radial nodes = $(n - l - 1)$

For $3s$: $n = 3, l = 0$ (Number of radial node = 2)

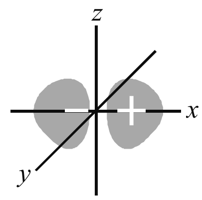
For $2p$: $n = 2, l = 1$ (Number of radial node = 0)

2. **(c)** Only two e^- can exist in the same orbital.

3. **(d)** The quantum numbers +1/2 and –1/2 for the electron spin can represent any one among clockwise or anticlockwise spin direction. But if one value represents clockwise spin then the other value will represent anticlockwise spin.

4. **(b)** $3d^5 4s^1$ system is more stable than $3d^4 4s^2$, hence former is the ground state configuration.

5. **(a)** p_x orbital being dumbell shaped, have number of nodal planes = 1, in yz plane. The electron density is only along x-axis (xy and zx planes), thus, in yz-plane, there will be zero electron density.



$2p_x$

6. **(a)** The two guiding rules to arrange the various orbitals in the increasing energy are:

(i) Energy of an orbital increases with increase in the value of $n + l$.

(ii) Of orbitals having the same value of $n + l$, the orbital with lower value of n has lower energy.

Thus, for the given orbitals, we have

(i) $n + l = 4 + 1 = 5$ (ii) $n + l = 4 + 0 = 4$
(iii) $n + l = 3 + 2 = 5$ (iv) $n + l = 3 + 1 = 4$
Hence, the order of increasing energy is
(iv) < (ii) < (iii) < (i)

7. (a) The expression for orbital angular momentum is

Angular momentum $= \sqrt{l(l+1)}\left(\frac{h}{2\pi}\right)$

For d orbital, $l = 2$.

Hence, $L = \sqrt{2(2+1)}\left(\frac{h}{2\pi}\right) = \sqrt{6}\left(\frac{h}{2\pi}\right)$

8. (b) Orbital angular momentum $(mvr) = \frac{h}{2\pi}\sqrt{l(l+1)}$

For 2s orbital, l (azimuthal quantum number) = 0
$\therefore$ Orbital angular momentum = 0.

9. (c) Total nodes $= n - l$
No. of radial nodes $= n - l - 1$
No. of angular nodes $= l$
For 3p sub-shell, $n = 3$, $l = 1$
$\therefore$ No. of radial nodes $= n - l - 1 = 3 - 1 - 1 = 1$
$\therefore$ No. of angular nodes $= l = 1$

10. (c) Electronic configuration of chlorine is [Ne] $3s^2, 3p^5$
$\therefore$ Unpaired electron is found in 3p sub-shell.
$\therefore n = 3, l = 1, m = 1$

11. (a) Exactly half filled orbitals are more stable than nearly half filled orbitals.
Cr (At. no. 24) has configuration [Ar] $3d^5, 4s^1$.

12. (c) Configuration ns^2, np^5 means it requires only one electron to attain nearest noble gas configuration. So, it will be most electronegative element among given choices.

13. (b) According to Aufbau principle, the orbital of lower energy (2s) should be fully filled before the filling of orbital of higher energy starts.

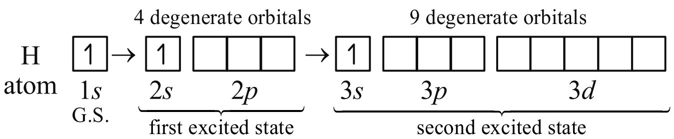
14. (c) If $l = 2, m \neq -3$, m will vary from –2 to +2. *i.e.* possible values of m are –2, –1, 0, + 1 and + 2.

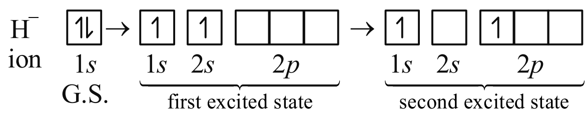
15. (a) Rb has the configuration : $1s^2\ 2s^2 2p^6\ 3s^2 3p^6 d^{10}\ 4s^2 p^6\ 5s^1$; so $n = 5$, $l = 0$, $m = 0$ and $s = +½$ is correct set of quantum numbers for valence shell electron of Rb.

16. (d) One p-orbital can accommodate up to two electrons with opposite spin while p-subshell can accommodate upto six electrons.

17. (a) The principal quantum number (n) is related to the size of the orbital (n = 1, 2, 3.....)

18. (3) In one electron system, all orbitals of a shell are degenerate.





In case of many electron system, different orbitals of a shell are non-degenerate. Hence, in the second excited state, only three p-orbitals (2p) are degenerate.

19. (6) $|m_l| = 1$ means m_l can be +1 and –1.

For n = 4, the total number of possible orbitals are :

4s	4p			4d					4f						
0	–1	0	+1	–2	–1	0	+1	+2	–3	–2	–1	0	+1	+2	+3

Thus, total number of orbitals having $|m_l| = 1$ is 6.
The number of electrons with $s = -1/2$ is 6.

20. (9)
Maximum number of orbitals when $n = 3$ is $n^2 = 3^2 = 9$

$\therefore$ Number of electrons with $m_s = -\frac{1}{2}$ will be 9.

21. (10) For n = 3 and l = 2 (*i.e.*, 3d orbital), the values of m varies from –2 to +2, *i.e.* –2, –1, 0, +1, +2 and for each 'm' there are 2 values of 's', *i.e.* +½ and –½.
$\therefore$ Maximum no. of electrons in all the five d-orbitals is **10**.

22. $4s^1$, $3d^5$;
The electronic configuration of Cr is : $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^1, 3d^5$.
$\therefore$ Outermost electronic configuration is $3d^5, 4s^1$.

23. orientation in space

24. antiparallel; or opposite

25. False : The orbital $3d_{x^2-y^2}$ lie along X and Y axis where electron density is maximum.

26. True : In case of hydrogen (single electron system), energy of electron depends only on the principal quantum number. Thus, 4s is in higher energy level than 3d.

27. False : The outer electronic configuration of the ground state chromium atom is $3d^5 4s^1$, as half filled orbitals are more stable than nearly half filled orbitals.

28. (a, c) Given, azimuthal quantum no. (l) = 2 (d-subshell)
Magnetic quantum no.(m) = 0(zero), which is for d_{z^2} orbital.

$$E = -13.6\frac{z^2}{n^2} = -13.6 \times \frac{2^2}{n^2} = -3.4$$

$$13.6 \times \frac{2^2}{n^2} = 3.4$$

$n^2 = 4^2 \Rightarrow n = 4$
Radial node $= n - l - 1 = 4 - 2 - 1 = 1$
Angular node $= l = 2$
Wave function corresponds to $\psi_{4,2,0}$. It represents $4d_{z^2}$-orbital which has only one radial node and two angular nodes. It experiences nuclear charge of 2e units.

29. **(a, d)** According to Hund's rule pairing of electrons starts only when each of the orbital in a sub shell has one electron each of parallel spin.

$\therefore$ (a) and (d) are correct ground state electronic configurations of nitrogen atom in ground state.

30. **(a, b, c)**

(a) $_{24}Cr = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^5, 4s^1 = [Ar]\,3d^5, 4s^1$

(b) For magnetic quantum number (m), negative values are possible.
For s - subshell, $l = 0$, hence $m = 0$
for p - subshell, $l = 1$, hence $m = -1, 0, +1$

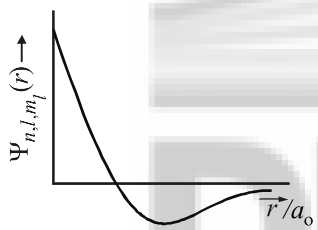
(c) $_{47}Ag = 1s^2, 2s^2\, 2p^6, 3s^2 3p^6 3d^{10}, 4s^2\, 4p^6\, 4d^{10}, 5s^1$
Hence, 23 electrons have a spin of one type and 24 of the opposite type.

(d) Oxidation state of N in HN_3 is $-1/3$.

31. **(c)** No. of radial nodes $= n - l - 1$, For $2s$ orbital, $n = 2$, $l = 0$

$\therefore$ No. of radial nodes $= 2 - 0 - 1 = 1$

The plotted graph is correct for $2s$-orbital, as wave function changes its sign at node.



32. **(a)** E.C. of H : $1s^1$; for $1s$ orbital

$$\psi_{n,l,m} \propto \left(\frac{Z}{a_o}\right)^{3/2} e^{-(Zr/a_o)}$$

For s-orbital, θ and ϕ cannot be a part of wave function expression. Hence, this is correct.

For $1s$ orbital of hydrogen like species: $E \propto -\frac{1}{n^2}$

Then, $E_4 - E_2 = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2 = \frac{3}{16}$

$E_6 - E_2 = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{6}\right)^2 = \frac{8}{36}$

$\therefore (E_4 - E_2) = \frac{27}{32} \times (E_6 - E_2)$

33. **(c)** In the wave function (ψ) expression for $1s$-orbital of He^+, there should be no angular part (θ).

34. **(A) - (q); (B) - (p, q, r, s); (C) - (p, q, r); (D) - (p, q, r)**

(A) Orbital angular momentum $L = \sqrt{l(l+1)}\frac{h}{2\pi}$, i.e., L depends on azimuthal quantum number only.

(B) To describe a hydrogen like one-electron wave function, three quantum numbers n, l and m are required. Further, to obey Pauli principle, fourth quantum number s is also required.

(C) To define size, shape and orientation of atomic orbitals, n, l and m are required respectively.

(D) Probability density (ψ^2) of an electron can be determined from the value of n, l and m.

35. **(A) - (r); (B) - (q); (C) - (p); (D) - (s)**

(A) $\frac{V_n}{K_n} = \frac{-Kze^2/r}{Kze^2/2r} = -2$; where $K = \frac{1}{4\pi\varepsilon_0}$

(B) $r_n = \frac{n^2 h^2 \varepsilon_0}{\pi m Z e^2}$, $E_n = \frac{me^4}{8\varepsilon_0^2 h^2 n^2}$

$\Rightarrow E_n \propto \frac{1}{n^2} \propto \frac{1}{r_n} \Rightarrow r_n \propto \frac{1}{E_n}$

$\Rightarrow r_n \propto (E_n)^{-1} \Rightarrow x = -1$

(C) Angular momentum of electron in lowest ($1s$) orbital

$= \sqrt{l(l+1)}\frac{h}{2\pi} = \sqrt{0(0+1)}\frac{h}{2\pi} = 0;$

(D) $r_n = \frac{a_0 n^2}{Z} \Rightarrow \frac{1}{r_n} \propto Z^1 \Rightarrow y = 1$

For 36-38 The spherically symmetric state S_1 of Li^{2+} with one radial node is $2s$. Upon absorbing light, the ion gets excited to state S_2, which also has one radial node. The energy of electron in S_2 is same as that of H-atom in its ground state.

$\therefore\ E_n = \frac{Z^2}{n^2}E_1$ where E_1 is the energy of H-atom in the ground state $= \frac{(3)^2 E_1}{n^2}$ for Li^{2+}

$E_n = E_1 \Rightarrow n = 3$

$\therefore$ State S_2 of Li^{2+} having one radial node is $3p$.

Orbital angular momentum quantum number of $3p$ is 1.

Energy of state $S_1 = \frac{(3)^2}{(2)^2}E_1 = 2.25\,E_1$

36. **(b)** **37.** **(c)** **38.** **(b)**

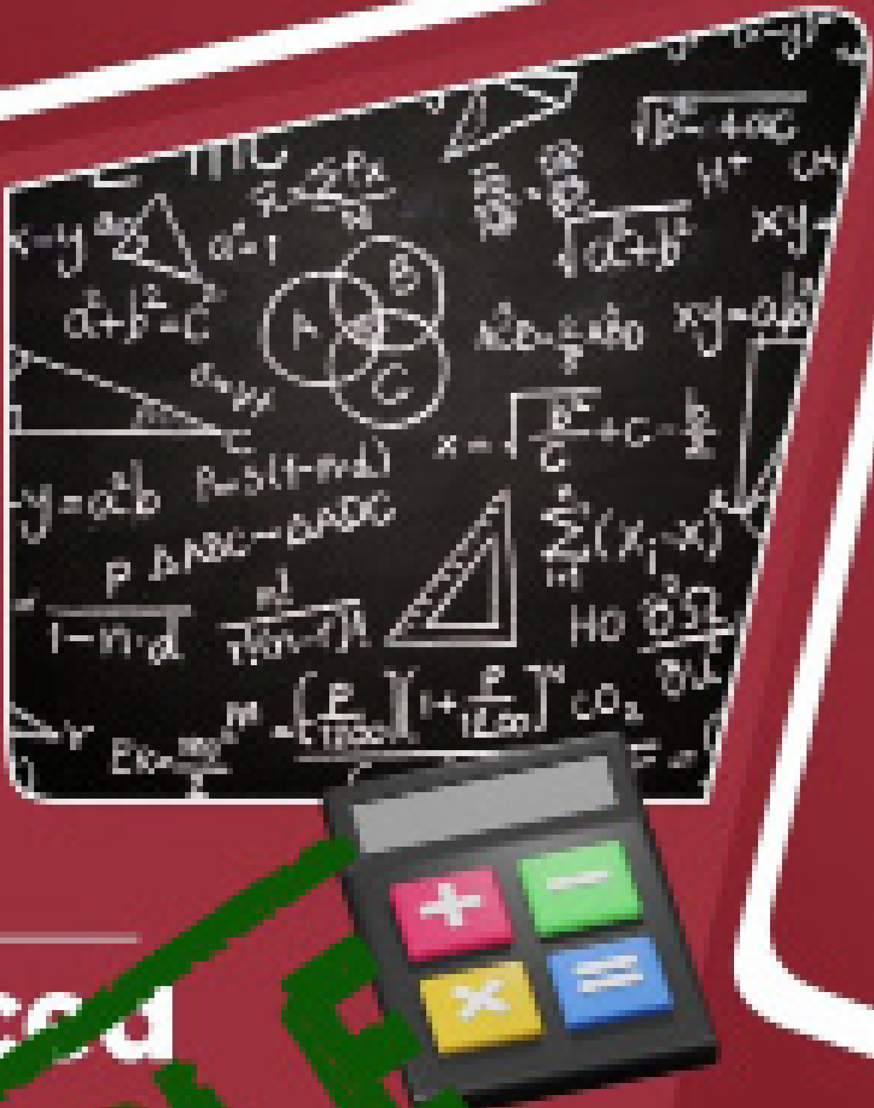
39. Ground state electronic configuration of Si

↑↓
$3s$

↑	↑	
$3p_x$	$3p_y$	$3p_z$

In a subshell, single e^- occupied orbitals must have parallel spins.

3rd Edition



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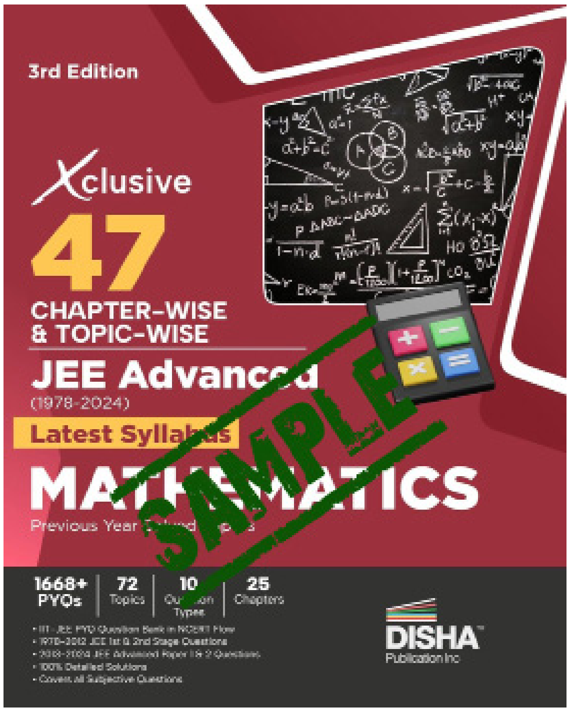

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Class XIth A1 – A180

Class XIIth B1 – B300

4 Complex Numbers and Quadratic Equations

Topic-1: Integral Powers of Iota, Algebraic Operations of Complex Numbers, Conjugate, Modulus and Argument or Amplitude of a Complex number

1 MCQs with One Correct Answer

1. If $\frac{w-\bar{w}z}{1-z}$ is purely real where $w=\alpha+i\beta, \beta\neq 0$ and $z\neq 1$, then the set of the values of z is **[2006 - 3M, –1]**

(a) $\{z : |z|=1\}$ (b) $\{z : z=\bar{z}\}$
(c) $\{z : z\neq 1\}$ (d) $\{z : |z|=1, z\neq 1\}$

2. For all complex numbers z_1, z_2 satisfying $|z_1|=12$ and $|z_2-3-4i|=5$, the minimum value of $|z_1-z_2|$ is **[2002S]**

(a) 0 (b) 2 (c) 7 (d) 17

3. If z_1, z_2 and z_3 are complex numbers such that **[2000S]**

$|z_1|=|z_2|=|z_3|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$, then $|z_1+z_2+z_3|$ is

(a) equal to 1 (b) less than 1
(c) greater than 3 (d) equal to 3

4. If $\arg(z)<0$, then $\arg(-z)-\arg(z)=$ **[2000S]**

(a) π (b) $-\pi$ (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

5. For positive integers n_1, n_2 the value of the expression $(1+i)^{n_1}+(1+i^3)^{n_1}+(1+i^5)^{n_2}+(1+i^7)^{n_2}$, where $i=\sqrt{-1}$ is a real number if and only if **[1996 - 1 Marks]**

(a) $n_1=n_2+1$ (b) $n_1=n_2-1$
(c) $n_1=n_2$ (d) $n_1>0, n_2>0$

6. Let z and ω be two complex numbers such that $|z|\leq 1$, $|\omega|\leq 1$ and $|z+i\omega|=|z-i\bar{\omega}|=2$ then z equals **[1995S]**

(a) 1 or i (b) i or $-i$
(c) 1 or -1 (d) i or -1

7. Let z and ω be two non zero complex numbers such that $|z|=|\omega|$ and $\text{Arg}\, z+\text{Arg}\,\omega=\pi$, then z equals **[1995S]**

(a) ω (b) $-\omega$ (c) $\bar{\omega}$ (d) $-\bar{\omega}$

8. The smallest positive integer n for which **[1980]**

$\left(\frac{1+i}{1-i}\right)^n=1$ is

(a) $n=8$ (b) $n=16$
(c) $n=12$ (d) none of these

2 Integer Value Answer/ Non-Negative Integer

9. Let $A=\left\{\frac{1967+1686\,i\sin\theta}{7-3\,i\cos\theta} : \theta\in\mathbb{R}\right\}$. If A contains exactly one positive integer n, then the value of n is **[Adv. 2023]**

10. For any integer k, let $\alpha_k=\cos\left(\frac{k\pi}{7}\right)+i\sin\left(\frac{k\pi}{7}\right)$, where $i=\sqrt{-1}$. The value of the expression $\frac{\sum_{k=1}^{12}|\alpha_{k+1}-\alpha_k|}{\sum_{k=1}^{3}|\alpha_{4k-1}-\alpha_{4k-2}|}$ is **[Adv. 2015]**

11. If z is any complex number satisfying $|z-3-2i|\leq 2$, then the minimum value of $|2z-6+5i|$ is **[2011]**

3 Numeric/ New Stem Based Questions

12. Let z be a complex number with non-zero imaginary part. If $\frac{2+3z+4z^2}{2-3z+4z^2}$ is a real number, then the value of $|z|^2$ is __________. **[Adv. 2022]**

13. Let $\bar{z}$ denote the complex conjugate of a complex number z and let $i=\sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation $\bar{z}-z^2=i\left(\bar{z}+z^2\right)$ is __________. **[Adv. 2022]**

4 Fill in the Blanks

14. If the expression **[1987 - 2 Marks]**

$$\frac{\left[\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)+i\tan(x)\right]}{\left[1+2\,i\,\sin\left(\frac{x}{2}\right)\right]}$$

is real, then the set of all possible values of x is ....

5 True / False

15. For complex number $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, we write $z_1 \cap z_2$, if $x_1 \le x_2$ and $y_1 \le y_2$. Then for all complex numbers z with $1 \cap z$, we have $\frac{1-z}{1+z} \cap 0$. **[1981 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

16. Let $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}, T_1 = \{(-1+\sqrt{2})^n : n \in \mathbb{N}\}$ and $T_2 = \{(1+\sqrt{2})^n : n \in \mathbb{N}\}$. Then which of the following statements is (are) TRUE? **[Adv. 2024]**

(a) $\mathbb{Z} \cup T_1 \cup T_2 \subset S$

(b) $T_1 \cap \left(0, \frac{1}{2024}\right) = \phi$, where ϕ denotes the empty set.

(c) $T_2 \cap (2024, \infty) \ne \phi$

(d) For any given $a, b \in \mathbb{Z}$, $\cos(\pi(a+b\sqrt{2})) + i\sin(\pi(a+b\sqrt{2})) \in \mathbb{Z}$ if and only if $b = 0$, where $i = \sqrt{-1}$.

17. Let $\bar{z}$ denote the complex conjugate of a complex number z. If z is a non-zero complex number for which both real and imaginary parts of $(\bar{z})^2 + \frac{1}{z^2}$ are integers, then which of the following is/are possible value(s) of $|z|$? **[Adv. 2022]**

(a) $\left(\frac{43+3\sqrt{205}}{2}\right)^{\frac{1}{4}}$ (b) $\left(\frac{7+\sqrt{33}}{4}\right)^{\frac{1}{4}}$

(c) $\left(\frac{9+\sqrt{65}}{4}\right)^{\frac{1}{4}}$ (d) $\left(\frac{7+\sqrt{13}}{6}\right)^{\frac{1}{4}}$

18. Let S be the set of all complex numbers z satisfying $|z^2 + z + 1| = 1$. Then which of the following statements is/are TRUE? **[Adv. 2020]**

(a) $\left|z + \frac{1}{2}\right| \le \frac{1}{2}$ for all $z \in S$

(b) $|z| \le 2$ for all $z \in S$

(c) $\left|z + \frac{1}{2}\right| \ge \frac{1}{2}$ for all $z \in S$

(d) The set S has exactly four elements

19. Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy$ $(x, y, \in \mathbb{R}, i = \sqrt{-1})$ of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE? **[Adv. 2018]**

(a) If L has exactly one element, then $|s| \ne |t|$

(b) If $|s| = |t|$, then L has infinitely many elements

(c) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(d) If L has more than one element, then L has infinitely many elements

20. For a non-zero complex number z, let arg(z) denote the principal argument with $-\pi < \arg(z) \le \pi$. Then, which of the following statement (s) is (are) **FALSE?** **[Adv. 2018]**

(a) $\arg(-1-i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(b) The function $f : \mathbb{R} \to (-\pi, \pi]$, defined by $f(t) = \arg(-1+it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$

(c) For any two non-zero complex numbers z_1 and z_2, $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π

(d) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition $\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$, lies on a straight line

21. Let a, b, x and y be real numbers such that a – b = 1 and y ≠ 0. If the complex number z = x + iy satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x ? **[Adv. 2017]**

(a) $-1+\sqrt{1-y^2}$ (b) $-1-\sqrt{1-y^2}$

(c) $1+\sqrt{1+y^2}$ (d) $1-\sqrt{1+y^2}$

22. Let z_1 and z_2 be two distinct complex numbers and let $z = (1-t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg (w) denotes the principal argument of a non-zero complex number w, then **[2010]**

(a) $|z - z_1| + |z - z_2| = |z_1 - z_2|$

(b) $\text{Arg}(z - z_1) = \text{Arg}(z - z_2)$

(c) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix}$

(d) $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

23. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x+iy$, then **[1998 - 2 Marks]**

(a) $x=3, y=2$ (b) $x=1, y=3$
(c) $x=0, y=3$ (d) $x=0, y=0$

24. The value of the sum $\sum_{n=1}^{13}(i^n+i^{n+1})$, where $i=\sqrt{-1}$, equals **[1998 - 2 Marks]**

(a) i (b) $i-1$ (c) $-i$ (d) 0

25. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$ is **[1987 - 2 Marks]**

(a) -1 (b) 0 (c) $-i$ (d) i
(e) None

26. If z_1 and z_2 are two nonzero complex numbers such that $|z_1+z_2|=|z_1|+|z_2|$, then Arg z_1 − Arg z_2 is equal to **[1987 - 2 Marks]**

(a) $-\pi$ (b) $-\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{2}$
(e) π

27. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1|=|z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1+z_2}{z_1-z_2}$ may be **[1986 - 2 Marks]**

(a) zero (b) real and positive
(c) real and negative (d) purely imaginary
(e) none of these.

28. If $z_1=a+ib$ and $z_2=c+id$ are complex numbers such that $|z_1|=|z_2|=1$ and $\text{Re}(z_1\bar{z}_2)=0$, then the pair of complex numbers $w_1=a+ic$ and $w_2=b+id$ satisfies – **[1985 - 2 Marks]**

(a) $|w_1|=1$ (b) $|w_2|=1$
(c) $\text{Re}(w_1\overline{w}_2)=0$ (d) none of these

7 Match the Following

29. Let z be a complex number satisfying $|z|^3+2z^2+4\bar{z}-8=0$, where $\bar{z}$ denotes the complex conjugate of z. Let the imaginary part of z be non-zero.
Match each entry in **List-I** to the correct entries in **List-II.**

List-I	List-II
(P) $|z|^2$ is equal to	(1) 12
(Q) $|z-\bar{z}|^2$ is equal to	(2) 4
(R) $|z|^2+|z+\bar{z}|^2$ is equal to	(3) 8
(S) $|z+1|^2$ is equal to	(4) 10
	(5) 7

The correct option is: **[Adv. 2023]**

(a) (P) → (1), (Q) → (3), (R) → (5), (S) → (4)
(b) (P) → (2), (Q) → (1), (R) → (3), (S) → (5)
(c) (P) → (2), (Q) → (4), (R) → (5), (S) → (1)
(d) (P) → (2), (Q) → (3), (R) → (5), (S) → (4)

30. Let $z_k=\cos\left(\frac{2k\pi}{10}\right)+i\sin\left(\frac{2k\pi}{10}\right)$; $k=1, 2, ..., 9.$ **[Adv. 2014]**

	List-I		List-II
P.	For each z_k there exists as z_j such that $z_k.\,z_j=1$	**1.**	True
Q.	There exists a $k\in\{1,2,...,9\}$ such that $z_1.z=z_k$ has no solution z in the set of complex numbers	**2.**	False
R.	$\frac{|1-z_1||1-z_2|....|1-z_9|}{10}$ equals	**3.**	1
S.	$1-\sum_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right)$ equals	**4.**	2

	P	Q	R	S
(a)	1	2	4	3
(b)	2	1	3	4
(c)	1	2	3	4
(d)	2	1	4	3

8 Comprehension/Passage Based Questions

PASSAGE-1

Let $S = S_1 \cap S_2 \cap S_3$, where

$S_1 = \{z \in \mathbb{C} : |z| < 4\}$, $S_2 = \left\{z \in \mathbb{C} : \text{Im}\left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right] > 0\right\}$

and $S_3 = \{z \in \mathbb{C} : \text{Re}\, z > 0\}$. **[Adv. 2013]**

31. Area of $S =$

(a) $\frac{10\pi}{3}$ (b) $\frac{20\pi}{3}$ (c) $\frac{16\pi}{3}$ (d) $\frac{32\pi}{3}$

32. $\min\limits_{z \in S} |1 - 3i - z| =$

(a) $\frac{2-\sqrt{3}}{2}$ (b) $\frac{2+\sqrt{3}}{2}$ (c) $\frac{3-\sqrt{3}}{2}$ (d) $\frac{3+\sqrt{3}}{2}$

PASSAGE-2

Let A, B, C be three sets of complex numbers as defined below

$A = \{z : \text{Im}\, z \geq 1\}$ **[2008]**

$B = \{z : |z - 2 - i| = 3\}$

$C = \{z : \text{Re}((1-i)z) = \sqrt{2}\}$

33. The number of elements in the set $A \cap B \cap C$ is

(a) 0 (b) 1 (c) 2 (d) ∞

34. Let z be any point in $A \cap B \cap C$.
Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

(a) 25 and 29 (b) 30 and 34
(c) 35 and 39 (d) 40 and 44

35. Let z be any point $A \cap B \cap C$ and let w be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between

(a) –6 and 3 (b) –3 and 6
(c) –6 and 6 (d) –3 and 9

10 Subjective Problems

36. If z_1 and z_2 are two complex numbers such taht $|z_1| < 1 < |z_2|$ then prove that $\left|\frac{1 - z_1\bar{z}_2}{z_1 - z_2}\right| < 1$. **[2003 - 2 Marks]**

37. Let $Z_1 = 10 + 6i$ and $Z_2 = 4 + 6i$. If Z is any complex number such that the argument of $\frac{(Z - Z_1)}{(Z - Z_2)}$ is $\frac{\pi}{4}$, then prove that $|Z - 7 - 9i| = 3\sqrt{2}$. **[1990 - 4 Marks]**

38. Show that the area of the triangle on the Argand diagram formed by the complex numbers z, iz and $z + iz$ is $\frac{1}{2}|z|^2$. **[1986 - 2½ Marks]**

39. Find the real values of x and y for which the following equation is satisfied $\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$ **[1980]**

40. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$. **[1979]**

41. Express $\frac{1}{1 - \cos\theta + 2i\sin\theta}$ in the form $x + iy$. **[1978]**

Topic-2: Rotational Theorem, Square Root of a Complex Number, Cube Roots of Unity, Geometry of Complex Numbers, De-moiver's Theorem, Powers of Complex Numbers

1 MCQs with One Correct Answer

1. Let $\theta_1, \theta_2, ..., \theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + ... + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}$, $z_k = z_{k-1}e^{i\theta_k}$ for $k = 2, 3, ..., 10$, where $i = \sqrt{-1}$. Consider the statements P and Q given below :

$P\; |z_2 - z_1| + |z_3 - z_2| + ... + |z_{10} - z_9| + |z_1 - z_{10}| \leq 2\pi$

$Q\; |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + ... + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \leq 4\pi$

[Adv. 2021]

(a) P is **TRUE** and Q is **FALSE**
(b) Q is **TRUE** and P is **FALSE**
(c) both P and Q are **TRUE**
(d) both P and Q are **FALSE**

2. Let S be the set of all complex numbers z satisfying $|z - 2 + i| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0 - 1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in S\right\}$, then the principal argument of $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$ is **[Adv. 2019]**

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

3. Let complex numbers α and $\frac{1}{\bar{\alpha}}$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha| =$ **[Adv. 2013]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{3}$

4. Let z be a complex number such that the imaginary part of z is non-zero and $a = z^2 + z + 1$ is real. Then a cannot take the value **[2012]**

(a) -1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

5. Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation : $z\bar{z}^3 + \bar{z}z^3 = 350$ is **[2009]**

(a) 48 (b) 32 (c) 40 (d) 80

6. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is **[2009]**

(a) $\frac{1}{\sin 2°}$ (b) $\frac{1}{3\sin 2°}$ (c) $\frac{1}{2\sin 2°}$ (d) $\frac{1}{4\sin 2°}$

7. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by **[2008]**

(a) $6 + 7i$ (b) $-7 + 6i$ (c) $7 + 6i$ (d) $-6 + 7i$

8. If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1 - z^2}$ lie on

(a) a line not passing through the origin **[2007 -3 marks]**
(b) $|z| = \sqrt{2}$
(c) the x-axis
(d) the y-axis

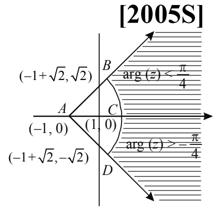
9. A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is **[2007 -3 marks]**

(a) $3e^{i\pi/4} + 4i$ (b) $(3 - 4i)e^{i\pi/4}$
(c) $(4 + 3i)e^{i\pi/4}$ (d) $(3 + 4i)e^{i\pi/4}$

10. a, b, c are integers, not all simultaneously equal and ω is cube root of unity ($\omega \neq 1$), then minimum value of $|a + b\omega + c\omega^2|$ is **[2005S]**

(a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

11. The locus of z which lies in shaded region (excluding the boundaries) is best represented by **[2005S]**



(a) $z : |z + 1| > 2$ and $|\arg(z+1)| < \pi/4$
(b) $z : |z - 1| > 2$ and $|\arg(z-1)| < \pi/4$
(c) $z : |z + 1| < 2$ and $|\arg(z+1)| < \pi/2$
(d) $z : |z - 1| < 2$ and $|\arg(z+1)| < \pi/2$

12. If $\omega (\neq 1)$ be a cube root of unity and $(1 + \omega^2)^n = (1 + \omega^4)^n$, then the least positive value of n is **[2004S]**

(a) 2 (b) 3 (c) 5 (d) 6

13. If $|z| = 1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), then Re(ω) is **[2003S]**

(a) 0 (b) $-\frac{1}{|z+1|^2}$
(c) $\left|\frac{z}{z+1}\right| \cdot \frac{1}{|z+1|^2}$ (d) $\frac{\sqrt{2}}{|z+1|^2}$

14. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, then the value of the det.

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is **[2002 - 2 Marks]**

(a) 3ω (b) $3\omega(\omega - 1)$
(c) $3\omega^2$ (d) $3\omega(1 - \omega)$

15. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is **[2001S]**

(a) of area zero (b) right-angled isosceles
(c) equilateral (d) obtuse-angled isosceles

16. Let z_1 and z_2 be n^{th} roots of unity which subtend a right angle at the origin. Then n must be of the form **[2001S]**

(a) $4k + 1$ (b) $4k + 2$
(c) $4k + 3$ (d) $4k$

17. If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to **[1999 - 2 Marks]**

(a) $1 - i\sqrt{3}$ (b) $-1 + i\sqrt{3}$
(c) $i\sqrt{3}$ (d) $-i\sqrt{3}$

18. If ω ($\neq 1$) is a cube root of unity and $(1 + \omega)^7 = A + B\omega$ then A and B are respectively **[1995S]**

(a) 0, 1 (b) 1, 1 (c) 1, 0 (d) – 1, 1

19. If a, b, c and u, v, w are complex numbers representing the vertices of two triangles such that $c = (1 - r)a + rb$ and $w = (1 - r)u + rv$, where r is a complex number, then the two triangles **[1985 - 2 Marks]**

(a) have the same area (b) are similar
(c) are congruent (d) none of these

20. The points $z_1, z_2, z_3 z_4$ in the complex plane are the vertices of a parallelogram taken in order if and only if **[1983 - 1 Mark]**

(a) $z_1 + z_4 = z_2 + z_3$ (b) $z_1 + z_3 = z_2 + z_4$
(c) $z_1 + z_2 = z_3 + z_4$ (d) None of these

21. If $z = x + iy$ and $\omega = (1 - iz)/(z - i)$, then $|\omega| = 1$ implies that, in the complex plane, **[1983 - 1 Mark]**
(a) z lies on the imaginary axis
(b) z lies on the real axis
(c) z lies on the unit circle
(d) None of these

22. The inequality $|z-4| < |z-2|$ represents the region given by **[1982 - 2 Marks]**
(a) $\text{Re}(z) \geq 0$ (b) $\text{Re}(z) < 0$
(c) $\text{Re}(z) > 0$ (d) none of these

23. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then **[1982 - 2 Marks]**
(a) $\text{Re}(z) = 0$ (b) $\text{Im}(z) = 0$
(c) $\text{Re}(z) > 0, \text{Im}(z) > 0$ (d) $\text{Re}(z) > 0, \text{Im}(z) < 0$

24. The complex numbers $z = x + iy$ which satisfy the equation $\left|\frac{z - 5i}{z + 5i}\right| = 1$ lie on **[1981 - 2 Marks]**
(a) the x-axis
(b) the straight line $y = 5$
(c) a circle passing through the origin
(d) none of these

25. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x-1)^3 + 8 = 0$ are **[1979]**
(a) $-1, 1 + 2\omega, 1 + 2\omega^2$ (b) $-1, 1 - 2\omega, 1 - 2\omega^2$
(c) $-1, -1, -1$ (d) None of these

2 Integer Value Answer/ Non-Negative Integer

26. For a complex number z, let $\text{Re}(z)$ denote the real part of z. Let S be the set of all complex numbers z satisfying $z^4 - |z|^4 = 4iz^2$, where $i = \sqrt{-1}$. Then the minimum possible value of $|z_1 - z_2|^2$, where $z_1, z_2 \in S$ with $\text{Re}(z_1) > 0$ and $\text{Re}(z_2) < 0$, is _____ **[Adv. 2020]**

27. Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\left\{\left|a + b\omega + c\omega^2\right|^2 : a, b, c \text{ distinct non-zero integers}\right\}$ equals _____. **[Adv. 2019]**

4 Fill in the Blanks

28. The value of the expression $1 \cdot (2-\omega)(2-\omega^2) + 2 \cdot (3-\omega)(3-\omega^2) + \ldots + (n-1).(n-\omega)(n-\omega^2)$, where ω is an imaginary cube root of unity, is..... **[1996 - 2 Marks]**

29. Suppose Z_1, Z_2, Z_3 are the vertices of an equilateral triangle inscribed in the circle $|Z| = 2$. If $Z_1 = 1 + i\sqrt{3}$ then $Z_2 =$ ...., $Z_3 =$ .... **[1994 - 2 Marks]**

30. $ABCD$ is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$. If the points D and M represent the complex numbers $1 + i$ and $2 - i$ respectively, then A represents the complex number ....or.......... **[1993 - 2 Marks]**

31. If a and b are the numbers between 0 and 1 such that the points $z_1 = a + i, z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle, then $a =$ ....and $b =$ .... **[1989 - 2 Marks]**

32. For any two complex numbers z_1, z_2 and any real number a and b. **[1988 - 2 Marks]**
$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 =$ ....

5 True / False

33. The cube roots of unity when represented on Argand diagram form the vertices of an equilateral triangle. **[1988 - 1 Mark]**

34. If three complex numbers are in A.P. then they lie on a circle in the complex plane. **[1985 - 1 Mark]**

35. If the complex numbers, Z_1, Z_2 and Z_3 represent the vertices of an equilateral triangle such that $|Z_1| = |Z_2| = |Z_3|$ then $Z_1 + Z_2 + Z_3 = 0$. **[1984 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

36. Let $a, b \in \mathbb{R}$ and $a^2 + b^2 \neq 0$.
Suppose $S = \left\{z \in C : Z = \frac{1}{a + ibt}, \in \mathbb{R}^+, t \neq 0\right\}$, where $i = \sqrt{-1}$. If $z = x + iy$ and $z \in S$, then (x, y) lies on **[JEE Adv. 2016]**
(a) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0, b \neq 0$
(b) the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for $a < 0, b \neq 0$
(c) the x-axis for $a \neq 0, b = 0$
(d) the y-axis for $a = 0, b \neq 0$

37. Let $w = \frac{\sqrt{3} + i}{2}$ and $P = \{w^n : n = 1, 2, 3, \ldots\}$. Further $H_1 = \left\{z \in \mathbb{C} : \text{Re}\, z > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in \mathbb{C} : \text{Re}\, z < \frac{-1}{2}\right\}$, where c is the set of all complex numbers. If $z_1 \in P \cap H_1, z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1 O z_2 =$ **[Adv. 2013]**
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

38. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals **[1998 - 2 Marks]**

(a) 128ω (b) -128ω (c) $128\omega^2$ (d) $-128\omega^2$

7 Match the Following

39. Match the statements in **Column I** with those in **Column II**. **[2010]**

[Note : Here z takes values in the complex plane and Im z and Re z denote , respectively, the imaginary part and the real part of z.]

Column I	Column II
(A) The set of points z satisfying $\lvert z - i\lvert z\rvert \rvert = \lvert z + i\lvert z\rvert\rvert$ is contained in or equal to	(p) an ellipse with eccentricity $\frac{4}{5}$
(B) The set of points z satisfying $\lvert z+4\rvert + \lvert z-4\rvert = 10$ is contained in or equal to	(q) the set of points z satisfying Im $z = 0$
(C) If $\lvert w\rvert = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to	(r) the set of points z satisfying $\lvert \text{Im } z\rvert \le 1$
(D) If $\lvert w\rvert = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to.	(s) the set of points z satisfying $\lvert \text{Re } z\rvert < 2$
	(t) the set of points z satisfying $\lvert z\rvert \le 3$

40. $z \neq 0$ is a complex number **[1992 - 2 Marks]**

Column I	Column II
(A) $\text{Re } z = 0$	(p) $\text{Re } z^2 = 0$
(B) $\text{Arg } z = \frac{\pi}{4}$	(q) $\text{Im } z^2 = 0$
	(r) $\text{Re } z^2 = \text{Im } z^2$

10 Subjective Problems

41. If one the vertices of the square circumscribing the circle $|z-1| = \sqrt{2}$ is $2 + \sqrt{3}\,i$. Find the other vertices of the square. **[2005 - 4 Marks]**

42. Find the centre and radius of circle given by

$$\left|\frac{z-\alpha}{z-\beta}\right| = k, k \neq 1$$

where, $z = x + iy$, $\alpha = \alpha_1 + i\alpha_2$, $\beta = \beta_1 + i\beta_2$ **[2004 - 2 Marks]**

43. Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$ where $|a_r| < 2$. **[2003 - 2 Marks]**

44. Let a complex number α, $\alpha \neq 1$, be a root of the equation $z^{p+q} - z^p - z^q + 1 = 0$, where p, q are distinct primes. Show that either $1 + \alpha + \alpha^2 + ... + \alpha^{p-1} = 0$ or $1 + \alpha + \alpha^2 + ... + \alpha^{q-1} = 0$, but not both together. **[2002 - 5 Marks]**

45. For complex numbers z and w, prove that $|z|^2\omega - |\omega|^2 z = z - \omega$ if and only if $z = \omega$ or $z\overline{\omega} = 1$. **[1999 - 10 Marks]**

46. Let z_1 and z_2 be roots of the equation $z^2 + pz + q = 0$, where the coefficients p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane. If $\angle AOB = \alpha \neq 0$ and $OA = OB$, where O is the origin, prove that $p^2 = 4q\cos^2\left(\frac{\alpha}{2}\right)$. **[1997 - 5 Marks]**

47. Find all non-zero complex numbers Z satisfying $\overline{Z} = iZ^2$. **[1996 - 2 Marks]**

48. If $|Z| \le 1, |W| \le 1,$ show that

$$|Z - W|^2 \le (|Z| - |W|)^2 + (Arg\,Z - Arg\,W)^2$$

[1995 - 5 Marks]

49. If $iz^3 + z^2 - z + i = 0,$ then show that $|z| = 1.$ **[1995 - 5 Marks]**

50. If $1, a_1, a_2 ......, a_{n-1}$ are the n roots of unity, then show that $(1 - a_1)(1 - a_2)(1 - a_3) ....(1 - a_{n-1}) = n$ **[1984 - 2 Marks]**

51. Prove that the complex numbers z_1, z_2 and the origin form an equilateral triangle only if $z_1^2 + z_2^2 - z_1 z_2 = 0.$ **[1983 - 3 Marks]**

52. Let the complex number z_1, z_2 and z_3 be the vertices of an equilateral triangle. Let z_0 be the circumcentre of the triangle. Then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$.
[1981 - 4 Marks]

53. If $x = a + b, y = a\gamma + b\beta$ and $z = a\beta + b\gamma$ where γ and β are the complex cube roots of unity, show that $xyz = a^3 + b^3$.
[1978]

Topic-3: Solutions of Quadratic Equations, Sum and Product of Roots, Nature of Roots, Relation Between Roots and Co-efficients, Formation of an Equation with Given Roots

1 MCQs with One Correct Answer

1. Suppose *a, b* denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose *c, d* denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of **[Adv. 2020]**
$ac(a-c)+ad(a-d)+bc(b-c)+bd(b-d)$ is
(a) 0 (b) 8000
(c) 8080 (d) 16000

2. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose θ_1 and B_1 are the roots of the equation $x^2 - 2x \sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals **[Adv. 2016]**
(a) $2(\sec\theta - \tan\theta)$ (b) $2\sec\theta$
(c) $-2\tan\theta$ (d) 0

3. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has **[Adv. 2014]**
(a) one purely imaginary root
(b) all real roots
(c) two real and two purely imaginary roots
(d) neither real nor purely imaginary roots

4. If $a \in \mathrm{R}$ and the equation
$$-3(x-[x])^2 + 2(x-[x]) + a^2 = 0$$
(where $[x]$ denotes the greatest integer $\le x$) has no integral solution, then all possible values of a lie in the interval: **[Main 2014]**
(a) $(-2,-1)$ (b) $(-\infty,-2)\cup(2,\infty)$
(c) $(-1,0)\cup(0,1)$ (d) $(1,2)$

5. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \ge 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is **[2011]**
(a) 1 (b) 2 (c) 3 (d) 4

6. Let (x_0, y_0) be the solution of the following equations
$(2x)^{\ell n 2} = (3y)^{\ell n 3}$; $3^{\ell n x} = 2^{\ell n y}$.
Then x_0 is **[2011]**
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 6

7. Let *p* and *q* be real numbers such that $p \ne 0, p^3 \ne q$ and $p^3 \ne -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is **[2010]**
(a) $(p^3+q)x^2 - (p^3+2q)x + (p^3+q) = 0$
(b) $(p^3+q)x^2 - (p^3-2q)x + (p^3+q) = 0$
(c) $(p^3-q)x^2 - (5p^3-2q)x + (p^3-q) = 0$
(d) $(p^3-q)x^2 - (5p^3+2q)x + (p^3-q) = 0$

8. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of *r* is **[2007 -3 marks]**
(a) $\frac{2}{9}(p-q)(2q-p)$ (b) $\frac{2}{9}(q-p)(2p-q)$
(c) $\frac{2}{9}(q-2p)(2q-p)$ (d) $\frac{2}{9}(2p-q)(2q-p)$

9. Let *a, b, c* be the sides of a triangle where $a \ne b \ne c$ and $\lambda \in R$. If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ are real, then **[2006 - 3M, –1]**
(a) $\lambda < \frac{4}{3}$ (b) $\lambda > \frac{5}{3}$
(c) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$ (d) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$

10. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between *p* and *q* is **[2004S]**
(a) $p^3 - q(3p-1) + q^2 = 0$
(b) $p^3 - q(3p+1) + q^2 = 0$
(c) $p^3 + q(3p-1) + q^2 = 0$
(d) $p^3 + q(3p+1) + q^2 = 0$

11. For the equation $3x^2+px+3=0, p>0$, if one of the root is square of the other, then p is equal to **[2000S]**

(a) 1/3 (b) 1 (c) 3 (d) 2/3

12. If $b>a$, then the equation $(x-a)(x-b)-1=0$ has **[2000S]**

(a) both roots in (a, b)
(b) both roots in $(-\infty, a)$
(c) both roots in $(b, +\infty)$
(d) one root in $(-\infty, a)$ and the other in $(b, +\infty)$

13. If α and β $(\alpha<\beta)$ are the roots of the equation $x^2+bx+c=0$, where $c<0<b$, then **[2000S]**

(a) $0<\alpha<\beta$ (b) $\alpha<0<\beta<|\alpha|$
(c) $\alpha<\beta<0$ (d) $\alpha<0<|\alpha|<\beta$

14. If the roots of the equation $x^2-2ax+a^2+a-3=0$ are real and less than 3, then **[1999 - 2 Marks]**

(a) $a<2$ (b) $2\le a\le 3$
(c) $3<a\le 4$ (d) $a>4$

15. Let $p, q\in\{1, 2, 3, 4\}$. The number of equations of the form $px^2+qx+1=0$ having real roots is **[1994]**

(a) 15 (b) 9
(c) 7 (d) 8

16. Let α, β be the roots of the equation $(x-a)(x-b)=c$, $c\ne 0$. Then the roots of the equation $(x-\alpha)(x-\beta)+c=0$ are **[1992 - 2 Marks]**

(a) a, c (b) b, c
(c) a, b (d) $a+c, b+c$

17. Let a, b, c be real numbers, $a\ne 0$. If α is a root of $a^2x^2+bx+c=0$. β is the root of $a^2x^2-bx-c=0$ and $0<\alpha<\beta$, then the equation $a^2x^2+2bx+2c=0$ has a root γ that always satisfies **[1989 - 2 Marks]**

(a) $\gamma=\frac{\alpha+\beta}{2}$ (b) $\gamma=\alpha+\frac{\beta}{2}$
(c) $\gamma=\alpha$ (d) $\alpha<\gamma<\beta$

18. The equation $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ has **[1984 - 2 Marks]**

(a) no root (b) one root
(c) two equal roots (d) infinitely many roots

19. If (x^2+px+1) is a factor of (ax^3+bx+c), then **[1980]**

(a) $a^2+c^2=-ab$ (b) $a^2-c^2=-ab$
(c) $a^2-c^2=ab$ (d) none of these

20. Both the roots of the equation $(x-b)(x-c)+(x-a)(x-c)+(x-a)(x-b)=0$ are always

(a) positive (b) real **[1980]**
(c) negative (d) none of these.

21. If ℓ, m, n are real, $\ell\ne m$, then the roots by the equation: $(\ell-m)x^2-5(\ell+m)x-2(\ell-m)=0$ are **[1979]**

(a) Real and equal (b) Complex
(c) Real and unequal (d) None of these

2 Integer Value Answer/ Non-Negative Integer

22. The product of all positive real values of x satisfying the equation $x^{\left(16(\log_5 x)^3-68\log_5 x\right)}=5^{-16}$ is ________ . **[Adv. 2022]**

23. For $x\in\mathbb{R}$, then number of real roots of the equation $3x^2-4|x^2-1|+x-1=0$ is _____. **[Adv. 2021]**

3 Numeric/ New Stem Based Questions

24. The smallest value of k, for which both the roots of the equation $x^2-8kx+16(k^2-k+1)=0$ are real, distinct and have values at least 4, is **[2009]**

4 Fill in the Blanks

25. If the product of the roots of the equation $x^2-3kx+2e^{2\ln k}-1=0$ is 7, then the roots are real for $k=$ .... **[1984 - 2 Marks]**

26. If $2+i\sqrt{3}$ is a root of the equation $x^2+px+q=0$, where p and q are real, then $(p, q)=($ .... , ... $)$. **[1982 - 2 Marks]**

5 True / False

27. If $a<b<c<d$, then the roots of the equation $(x-a)(x-c)+2(x-b)(x-d)=0$ are real and distinct. **[1984 - 1 Mark]**

28. The equation $2x^2+3x+1=0$ has an irrational root. **[1983 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

29. Let R^2 denote $R\times R$. Let $S=\{(a, b, c : a, b, c\in R$ and $ax^2+2bxy+cy^2>0$ for all $(x, y)\in R^2-\{(0,0)\}$.
Then which of the following statements is (are) TRUE? **[Adv. 2024]**

(a) $\left(2, \frac{7}{2}, 6\right)\in S$

(b) If $\left(3, b, \frac{1}{12}\right)\in S$, then $|2b|<1$.

(c) For any given $(a, b, c)\in S$, the system of linear equations $ax+by=1$ $by+cy=-1$ has a unique solution.

(d) For any given $(a, b, c)\in S$, the system of linear equations $(a+1)x+by=0$ $bx+(c+1)y=0$ has a unique solution.

30. If $3^x = 4^{x-1}$, then $x =$ **[Adv. 2013]**

(a) $\dfrac{2\log_3 2}{2\log_3 2 - 1}$ (b) $\dfrac{2}{2 - \log_2 3}$

(c) $\dfrac{1}{1 - \log_4 3}$ (d) $\dfrac{2\log_2 3}{2\log_2 3 - 1}$

8 Comprehension/Passage Based Questions

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For n = 0, 1, 2, ..., let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$ **[Adv. 2017]**

31. $a_{12} =$

(a) $a_{11} - a_{10}$ (b) $a_{11} + a_{10}$
(c) $2a_{11} + a_{10}$ (d) $a_{11} + 2a_{10}$

32. If $a_4 = 28$, then $p + 2q =$

(a) 21 (b) 14 (c) 7 (d) 12

9 Assertion and Reason/Statement Type Questions

33. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \dfrac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$

STATEMENT - 1 : $(p^2 - q)(b^2 - ac) \geq 0$

and

STATEMENT - 2 : $b \neq pa$ or $c \neq qa$ **[2008]**

(a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

(b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explanation for STATEMENT - 1

(c) STATEMENT - 1 is True, STATEMENT - 2 is False

(d) STATEMENT - 1 is False, STATEMENT - 2 is True

10 Subjective Problems

34. Let a and b be the roots of the equation $x^2 - 10cx - 11d = 0$ and those of $x^2 - 10ax - 11b = 0$ are c, d then the value of $a + b + c + d$, when $a \neq b \neq c \neq d$, is. **[2006 - 6M]**

35. If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in R$ then find the values of a for which equation has unequal real roots for all values of b. **[2003 - 4 Marks]**

36. If α, β are the roots of $ax^2 + bx + c = 0$, $(a \neq 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constant δ, then prove that $\dfrac{b^2 - 4ac}{a^2} = \dfrac{B^2 - 4AC}{A^2}$. **[2000 - 4 Marks]**

37. Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$, then show that $1 + \dfrac{c}{a} + \left|\dfrac{b}{a}\right| < 0$. **[1995 - 5 Marks]**

38. Solve $|x^2 + 4x + 3| + 2x + 5 = 0$ **[1988 - 5 Marks]**

39. For $a \leq 0$, determine all real roots of the equation $x^2 - 2a|x - a| - 3a^2 = 0$ **[1986 - 5 Marks]**

40. Solve for x; $(5 + 2\sqrt{6})^{x^2 - 3} + (5 - 2\sqrt{6})^{x^2 - 3} = 10$ **[1985 - 5 Marks]**

41. Solve the following equation for x : **[1978]**
$2\log_x a + \log_{ax} a + 3\log_{a^2x} a = 0$, $a > 0$

42. Solve for x : $\sqrt{x+1} - \sqrt{x-1} = 1$. **[1978]**

Topic-4: Condition for Common Roots, Maximum and Minimum value of Quadratic Equation, Quadratic Expression in two Variables, Solution of Quadratic Inequalities

1 MCQs with One Correct Answer

1. A value of b for which the equations
$x^2+bx-1=0$
$x^2+x+b=0$
have one root in common is **[2011]**
(a) $-\sqrt{2}$ (b) $-i\sqrt{3}$ (c) $i\sqrt{5}$ (d) $\sqrt{2}$

2. For all 'x', $x^2+2ax+10-3a>0$, then the interval in which 'a' lies is **[2004S]**
(a) $a<-5$ (b) $-5<a<2$
(c) $a>5$ (d) $2<a<5$

2 Integer Value Answer/ Non-Negative Integer

3. Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that $f(1) = -9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3+3ax^2+2bx=0$ where $i=\sqrt{-1}$. If $\alpha_1, \alpha_2, \alpha_3$ and α_4 are all the roots of the equation $f(x)=0$, then $|\alpha_1|^2+|\alpha_2|^2+|\alpha_3|^2+|\alpha_4|^2$ is equal to ______. **[Adv. 2024]**

4 Fill in the Blanks

4. If the quadratic equations $x^2+ax+b=0$ and $x^2+bx+a=0$ $(a \neq b)$ have a common root, then the numerical value of $a+b$ is .... **[1986 - 2 Marks]**

5 True / False

5. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then $P(x)Q(x)=0$ has at least two real roots. **[1985 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

6. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2-x+\alpha=0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1-x_2|<1$. Which of the following intervals is(are) a subset(s) of S? **[JEE Adv. 2015]**
(a) $\left(-\frac{1}{2},-\frac{1}{\sqrt{5}}\right)$ (b) $\left(-\frac{1}{\sqrt{5}},0\right)$
(c) $\left(0,\frac{1}{\sqrt{5}}\right)$ (d) $\left(\frac{1}{\sqrt{5}},\frac{1}{2}\right)$

7. If a, b, c, d and p are distinct real numbers such that $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)\le 0$ then a, b, c, d **[1987 - 2 Marks]**
(a) are in A. P. (b) are in G. P.
(c) are in H. P. (d) satisfy $ab = cd$
(e) satisfy none of these

8. For real x, the function $\frac{(x-a)(x-b)}{x-c}$ will assume all real values provided **[1984 - 3 Marks]**
(a) $a>b>c$ (b) $a<b<c$
(c) $a>c>b$ (d) $a<c<b$

10 Subjective Problems

9. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2+bx+c=0$. Express the roots of $a^3x^2+abcx+c^3=0$ in terms of α, β. **[2001 - 4 Marks]**

10. Find all real values of x which satisfy $x^2-3x+2>0$ and $x^2-3x-4\le 0$ **[1983 - 2 Marks]**

11. If α, β are the roots of $x^2+px+q=0$ and γ, δ are the roots of $x^2+rx+s=0$, then evaluate $(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)$ in terms of p, q, r and s.
Deduce the condition that the equations have a common root. **[1979]**

Answer Key

Topic-1 : Integral Powers of Iota, Algebraic Operations of Complex Numbers, Conjugate, Modulus and Argument or Amplitude of a Complex Number

1. (d) **2.** (b) **3.** (a) **4.** (a) **5.** (d) **6.** (c) **7.** (d) **8.** (d) **9.** (281) **10.** (4)

11. (5) **12.** (0.50) **13.** (4) **14.** $\left(2p\pi, n\pi+\frac{\pi}{4}\right)$ **15.** (True) **16.** (a, c, d) **17.** (a) **18.** (b, c) **19.** (a, c, d)

20. (a, b, d) **21.** (a, b) **22.** (a,c,d) **23.** (d) **24.** (b) **25.** (d) **26.** (c) **27.** (a, d) **28.** (a, b, c)

29. (b) **30.** (c) **31.** (b) **32.** (c) **33.** (b) **34.** (c) **35.** (d)

Topic-2 : Rotational Theorem, Square Root of a Complex Number, Cube Roots of Unity, Geometry of Complex Numbers, De-moiver's Theorem, Powers of Complex Numbers

1. (c) **2.** (d) **3.** (c) **4.** (d) **5.** (a) **6.** (d) **7.** (d) **8.** (d) **9.** (d) **10.** (b)

11. (a) **12.** (b) **13.** (a) **14.** (b) **15.** (c) **16.** (d) **17.** (c) **18.** (b) **19.** (b) **20.** (b)

21. (b) **22.** (d) **23.** (b) **24.** (a) **25.** (b) **26.** (8) **27.** (3) **28.** $\frac{1}{4}(n-1)n(n^2+3n+4)$

29. $-2, 1-i\sqrt{3}$ **30.** $3-i/2$ or $1-\frac{3}{2}i$ **31.** $2-\sqrt{3}, 2-\sqrt{3}$ **32.** $(a^2+b^2)(|z_1|^2+|z_2|^2)$ **33.** (True) **34.** (False)

35. (True) **36.** (a, c, d) **37.** (c, d) **38.** (d) **39.** $A\rightarrow$ (q, r); $B\rightarrow$ (p); $C\rightarrow$ (p, s, t); $D\rightarrow$ (q, r, s, t)

40. $A\rightarrow$ (q); $B\rightarrow$ (p)

Topic-3 : Solutions of Quadratic Equations, Sum and Product of Roots, Nature of Roots, Relation Between Roots and Co-efficients, Formation of an Equation with Given Roots

1. (d) **2.** (c) **3.** (d) **4.** (c) **5.** (c) **6.** (c) **7.** (b) **8.** (d) **9.** (a) **10.** (a)

11. (c) **12.** (d) **13.** (b) **14.** (a) **15.** (c) **16.** (c) **17.** (d) **18.** (a) **19.** (c) **20.** (b)

21. (c) **22.** (1) **23.** (4) **24.** (2) **25.** (2) **26.** (–4, 7) **27.** (True) **28.** (False) **29.** (a,b,c) **30.** (a,b,c)

31. (b) **32.** (d) **33.** (b)

Topic-4 : Condition for Common Roots, Maximum and Minimum value of Quadratic Equation, Quadratic Expression in two Variables, Solution of Quadratic Inequalities

1. (b) **2.** (b) **3.** (20) **4.** (1) **5.** (True) **6.** (a, d) **7.** (b) **8.** (c, d)

4 Complex Numbers and Quadratic Equations

Topic-1: Factorials and Permutations

1. **(d)** $\because \dfrac{w-\bar{w}z}{1-z}$ is purely real

$\therefore \overline{\left(\dfrac{w-\bar{w}z}{1-z}\right)}=\left(\dfrac{w-\bar{w}z}{1-z}\right) \Rightarrow \dfrac{\bar{w}-w\bar{z}}{1-\bar{z}}=\dfrac{w-\bar{w}z}{1-z}$

$\Rightarrow \bar{w}-\bar{w}z-w\bar{z}+wz\bar{z}=w-w\bar{z}-\bar{w}z+\bar{w}z\bar{z}$

$\Rightarrow w-\bar{w}=(w-\bar{w})|z|^2$

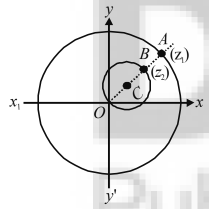
$\Rightarrow |z|^2=1$ ($\because w=\alpha+i\beta$ and $\beta\neq 0$)

$\Rightarrow |z|=1$ and also given that $z\neq 1$

$\therefore$ The required set is $\{z:|z|=1, z\neq 1\}=3\omega(\omega-1)$

2. **(b)** $|z_1|=12 \Rightarrow z_1$ lies on a circle with centre (0, 0) and radius 12 units.

And $|z_2-3-4i|=5 \Rightarrow z_2$ lies on a circle with centre (3, 4) and radius 5 units.



From figure, it is clear that $|z_1-z_2|$ i.e., distance between z_1 and z_2 will be min when they lie at A and B respectively i.e., O, C, B, A are collinear as shown.

Then $z_1-z_2=AB=OA-OB=12-2(5)=2$. As above is the minimum value, we must have $|z_1-z_2|\geq 2$.

3. **(a)** Given : $|z_1|=|z_2|=|z_3|=1$

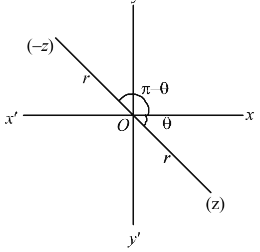
Now, $|z_1|=1 \Rightarrow |z_1|^2=1 \Rightarrow z_1\bar{z}_1=1$

Similarly $z_2\bar{z}_2=1,\ z_3\bar{z}_3=1$

Now, $\left|\dfrac{1}{z_1}+\dfrac{1}{z_2}+\dfrac{1}{z_3}\right|=1 \Rightarrow |\bar{z}_1+\bar{z}_2+\bar{z}_3|=1$

$\Rightarrow |\overline{z_1+z_2+z_3}|=1 \Rightarrow |z_1+z_2+z_3|=1$

4. **(a)** Given : arg $(z)<0$ (given) $\Rightarrow$ arg $(z)=-\theta$



Now, $z=r\cos(-\theta)+i\sin(-\theta)=r[\cos(\theta)-i\sin(\theta)]$

Again $-z=-r[\cos(\theta)-i\sin(\theta)]$

$=r[\cos(\pi-\theta)+i\sin(\pi-\theta)]$

$\therefore$ arg $(-z)=\pi-\theta$;

Thus arg $(-z)-\arg(z)=\pi-\theta-(-\theta)=\pi-\theta+\theta=\pi$

5. **(d)** $(1+i)^{n_1}+(1+i^3)^{n_1}+(1+i^5)^{n_2}+(1+i^7)^{n_2}$

$=(1+i)^{n_1}+(1-i)^{n_1}+(1+i)^{n_2}+(1-i)^{n_2}$

Using $1+i=\sqrt{2}(\cos\pi/4+i\sin\pi/4)$

and $1-i=\sqrt{2}(\cos\pi/4-i\sin\pi/4)$

We get the given expression as

$$=(\sqrt{2})^{n_1}\left[\cos\frac{n_1\pi}{4}+i\sin\frac{n_1\pi}{4}\right]+(\sqrt{2})^{n_2}\left[\cos\frac{n_2\pi}{4}+i\sin\frac{n_2\pi}{4}\right]+(\sqrt{2})^{n_2}\left[\cos\frac{n_2\pi}{4}-i\sin\frac{n_2\pi}{4}\right]$$

$$=(\sqrt{2})^{n_1}\left[2\cos\frac{n_1\pi}{4}\right]+(\sqrt{2})^{n_2}\left[2\cos\frac{n_2\pi}{4}\right]$$

= real number irrespective the values of n_1 and n_2

$\therefore$ (d) is the most appropriate answer.

6. **(c)** Given that $|z+i\omega|=|z-i\bar{\omega}|$

$\Rightarrow |z-(-i\omega)|=|z-(\overline{-i\omega})|$

$\Rightarrow$ z lies on perpendicular bisector of the line segment joining $(-i\omega)$ and $(-\overline{i\omega})$, which is real axis, $(-i\omega)$ and $(-\overline{i\omega})$ being mirror images of each other.

$\therefore$ Im$(z)=0$.

If $z=x$, then $|z|\leq 1 \Rightarrow x^2\leq 1 \Rightarrow -1\leq x\leq 1$

$\therefore$ (c) is the correct option.

7. **(d)** $\because |z|=|\omega|$ and $\arg z=\pi-\arg\omega$

Let $\omega=re^{i\theta}$ then $z=re^{i(\pi-\theta)}$

$\Rightarrow z=re^{i\pi}\cdot e^{-i\theta}$

$=(re^{-i\theta})(\cos\pi+i\sin\pi)=\bar{\omega}(-1)=-\bar{\omega}$

8. **(d)** $\dfrac{1+i}{1-i}=\dfrac{(1+i)^2}{(1-i)(1+i)}=\dfrac{1-1+2i}{2}=i$

Now $i^n=1 \Rightarrow$ the smallest positive integral value of n should be 4.

9. **(281)** is a positive integer

$$=\frac{281\,(49+18\sin\theta.\cos\theta+i\,(21\cos\theta+42\sin\theta))}{49+9\cos^2\theta}$$

for postive integer Im(z) = 0

$21\cos\theta+42\sin\theta=0$

$\Rightarrow \quad \tan\theta = -\frac{1}{2}, \sin 2\theta = \frac{-4}{5}, \cos^2\theta = \frac{4}{5}.$

Now Re (2) = $\frac{281\,(49 - 9\sin 2\theta)}{49 + 9\cos^2\theta}$

$= \frac{281\left(49 - 9 \times -\frac{4}{5}\right)}{49 + 9 \times \frac{4}{5}} = 281.$

10. **(4)** Given : $\alpha_k = \cos\frac{k\pi}{7} + i\sin\frac{k\pi}{7} = e^{\frac{i\pi k}{7}}$

$\alpha_{k+1} - \alpha_k = e^{\frac{i\pi(k+1)}{7}} - e^{\frac{i\pi k}{7}} = e^{\frac{i\pi k}{7}}(e^{i\pi/7} - 1)$

$|\alpha_{k+1} - \alpha_k| = |e^{i\pi/7} - 1|$

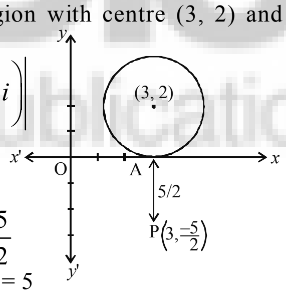
$\Rightarrow \quad \sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k| = 12|e^{i\pi/7} - 1|$

Similarly, $\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}| = 3|e^{i\pi/7} - 1|$

$\therefore \quad \frac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}|} = 4$

11. **(5)** Given : $|z - 3 - 2i| \le 2$,

which represents a circular region with centre (3, 2) and radius 2.



Now, $|2z - 6 + 5i| = 2\left|z - \left(3 - \frac{5}{2}i\right)\right|$

= 2 × distance of z from P

(where z lies in or on the circle)

Also min distance of z from $P = \frac{5}{2}$

$\therefore$ Minimum value of $|2z - 6 + 5i| = 5$

12. **(0.50)** Let $X = \frac{4z^2 + 3z + 2}{4z^2 - 3z + 2}$

It can be written as

$= 1 + \frac{6z}{4z^2 - 3z + 2}$

Now $X = 1 + \frac{6}{2\left(2z + \frac{1}{z}\right) - 3}$

$\because X \in R$, then $2z + \frac{1}{z} \in R$

$\Rightarrow \quad 2z + \frac{1}{z} = 2\bar{z} + \frac{1}{\bar{z}} \Rightarrow 2(z - \bar{z}) - \frac{z - \bar{z}}{|z|^2} = 0$

$\because \quad (z - \bar{z})\left(2 - \frac{1}{|z|^2}\right) = 0$

$\because \quad z \ne \bar{z}$ (given). So, $|z|^2 = \frac{1}{2}$

13. **(4)** Given, $\bar{z} - z^2 = i\left(\bar{z} + z^2\right)$

It can be written as $\bar{z}(1 - i) = z^2\,(1 + i)$

So $|\bar{z}||1 - i| = |z|^2|1 + i|$

$|z| = |z|^2 \Rightarrow |z| = 0$ or $|z| = 1$

Let arg $(z) = \alpha$. So from (i), we get

$2n\pi - \alpha - \frac{\pi}{4} = 2\alpha + \frac{\pi}{4}$

$\Rightarrow \quad \alpha = \frac{1}{3}\left(\frac{4n - 1}{2}\right)\pi = \frac{(4n - 1)\pi}{6}$

So we will get 3 distinct values of α. Hence there will be total 4 possible values of complex number z.

14. Let $z = \frac{\sin x/2 + \cos x/2 + i\tan x}{1 + 2i\sin x/2}$

$= \frac{(\sin x/2 + \cos x/2 + i\tan x)(1 - 2i\sin x/2)}{(1 + 2i\sin x/2)(1 - 2i\sin x/2)}$

$= \frac{[\sin x/2 + \cos x/2 + 2\sin x/2\tan x) + i(\tan x - 2\sin^2 x/2 - 2\sin x/2\cos x/2)]}{(1 + 4\sin^2 x/2)}$

But it is given that z is real.

$\therefore \; I_m(z) = 0$

$\Rightarrow \quad \tan x - 2\sin\frac{x}{2}\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) = 0$

$\Rightarrow \quad \frac{\sin x}{\cos x} - 2\sin^2 x/2 - 2\sin x/2\cos x/2 = 0$

$\Rightarrow \quad \frac{\sin x}{\cos x} - (1 - \cos x) - \sin x = 0$

$\Rightarrow \quad \sin x\left[\frac{1}{\cos x} - 1\right] - [1 - \cos x] = 0$

$\Rightarrow \quad \left(\frac{1 - \cos x}{\cos x}\right)\sin x - [1 - \cos x] = 0$

$\Rightarrow \quad (1 - \cos x)\left(\frac{\sin x}{\cos x} - 1\right) = 0$

$\Rightarrow \quad \cos x = 1 \quad \Rightarrow \quad x = 2n\pi$

and $\tan x = 1 \quad \Rightarrow \quad x = n\pi + \pi/4$

$\therefore \quad x = 2n\pi, n\pi + \pi/4$

15. (**True**) Let $z = x + iy$, then $1 \cap z \Rightarrow 1 \le x \,\&\, 0 \le y$ (by def .)

Consider,

$\frac{1 - z}{1 + z} = \frac{1 - (x + iy)}{1 + (x + iy)} = \frac{(1 - x) - iy}{(1 + x) + iy} \times \frac{(1 + x) - iy}{(1 + x) - iy}$

$= \frac{1 - x^2 - y^2}{(1 + x)^2 + y^2} - \frac{iy(1 - x + 1 + x)}{(1 + x)^2 + y^2}$

$= \frac{1 - x^2 - y^2}{(1 + x)^2 + y^2} - \frac{2iy}{(1 + x)^2 + y^2}$

$\frac{1-z}{1+z} \cap 0 \quad \Rightarrow \quad \frac{1-x^2-y^2}{(1+x)^2+y^2} \le 0$

and $\frac{-2y}{(1+x)^2+y^2} \le 0$

$\Rightarrow \quad 1-x^2-y^2 \le 0$ and $-2y \le 0$

$\Rightarrow \quad x^2+y^2 \ge 1$ and $y \ge 0$, which is true as

$x \ge 1$ and $y \ge 0$

Hence, the given statement is true $\forall\ z \in C$.

16. **(a, c, d)** (a) $S = \{a+b\sqrt{2} : a, b \in Z\}$

For $b = 0$; $Z \subset S$

$T_1 = (-1+\sqrt{2})^n = m+\sqrt{2}n, m, n \in Z$

$T_2 = (1+\sqrt{2})^n = m_1+\sqrt{2}\,n_1, m_1, n, \in Z$

For $n \in N$ elements of T_1 and T_2 are of the form $a+b\sqrt{2}$

Hence $Z \cup T_1 \cup T_2 \subset S$

(b) Now, $-1+\sqrt{2} < 1$ and its higher powers decreases

$\Rightarrow (-1+\sqrt{2})^n < 1$ and can be made in $\left(0, \frac{1}{2024}\right)$ for some higher *n*.

(c) $1+\sqrt{2} > 1$ and its higher power increases

$\Rightarrow \left(1+\sqrt{2}\right)^n$ can be made in $(2024, \infty)$ for some higher *n*.

(d) $\cos\pi(a+b\sqrt{2})+i\sin\pi(a+b\sqrt{2}) \in Z$ if

$a+b\sqrt{2}$ is an integer $\Rightarrow b = 0$

17. **(a)** Let $z = r.e^{i\theta} \Rightarrow \bar{z} = re^{-i\theta}$

$\therefore (\bar{z})^2+\frac{1}{z^2} = r^2e^{-2i\theta}+\frac{1}{r^2e^{2i\theta}} = \left(r^2+\frac{1}{r^2}\right)e^{-2/\theta} = a+ib$ (say),

where a, b ∈ Z

So, $\left(r^2+\frac{1}{r^2}\right)^2 = a^2+b^2 \Rightarrow r^8-(a^2+b^2-2)r^4+1 = 0$

$\Rightarrow r^4 = \frac{(a^2+b^2-2) \pm \sqrt{(a^2+b^2-2)^2-4}}{2}$

for option (a): $|z|^4 = \frac{43+3\sqrt{205}}{2}$

$\Rightarrow a^2+b^2 = 45$ i.e. $(a, b) = (\pm 6, \pm 3)$ or $(\pm 3, \pm 6)$

For option (b): $|z|^4 = \frac{7+\sqrt{33}}{4} \Rightarrow a^2+b^2 = \frac{11}{2}$

For option (c): $a^2+b^2 = \frac{13}{2}$

For option (d): $a^2+b^2 = \frac{13}{3}$

18. **(b, c)** $|z^2+z+1| = 1$

$\Rightarrow \left|\left(z+\frac{1}{2}\right)^2+\frac{3}{4}\right| \ge 1 \Rightarrow \left|\left(z+\frac{1}{2}\right)\right|^2 \ge \frac{1}{4} \Rightarrow \left|z+\frac{1}{2}\right| \ge \frac{1}{2}$

also $|(z^2+z)+1| = 1$

$\Rightarrow |z^2+z|-1 \le 1 \Rightarrow |z^2+z| \le 2$

$\Rightarrow ||z^2|-|z|| \le |z^2+z| \le 2 \Rightarrow |r^2-r| \le 2 \Rightarrow r = |z| \le 2;\ \forall\ z \in S$

Hence, set 'S' is infinite

19. **(a, c, d)**
We have,

$sz+t\bar{z}+r = 0$ ..... (i)

On taking conjugate

$\overline{sz}+\bar{t}z+\bar{r} = 0$ ..... (ii)

On solving Eqs. (i) and (ii), we get

$z = \frac{\bar{r}t-r\bar{s}}{|s|^2-|t|^2}$

(a) For unique solutions of *z*

$|s|^2-|t|^2 \ne 0 \Rightarrow |s| \ne |t|$

It is true.

(b) If $|s| = |t|$, then $\bar{r}t-r\bar{s}$ may or may not be zero.

So, *z* may have no solution.

$\therefore$ *L* may be an empty set.

It is false.

(c) If elements of set *L* represents line, then this line and given circle intersect at maximum two point.

Hence, it is true.

(d) In the case locus of *z* is a line, so *L* has infinite elements. Hence, it is true.

20. **(a, b, d)**

(a) $\arg(-1-i) = \frac{-3\pi}{4}$

$\therefore$ (a) is false

(b) $f(t) = \arg(-1+it) = \begin{cases} \pi-\tan^{-1}(t), t \ge 0 \\ -\pi+\tan^{-1}(t), t < 0 \end{cases}$

$\lim_{t\to 0^-} f(t) = -\pi$ and $\lim_{t \to 0^+} f(t) = \pi$

LHL ≠ RHL $\Rightarrow$ *f* is discontinuous at $t = 0$

$\therefore$ (b) is false.

(c) $\arg\left(\frac{z_1}{z_2}\right)-\arg z_1+\arg z_2$

$= 2n\pi+\arg z_1-\arg z_2-\arg z_1+\arg z_2$

$= 2n\pi$, multiple of 2π

$\therefore$ (c) is true.

(d) $\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$

$\Rightarrow \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)} = k, \quad k \in R$

$\Rightarrow \left(\dfrac{z-z_1}{z-z_3}\right) = k\left(\dfrac{z_2-z_1}{z_2-z_3}\right)$

$\Rightarrow$ z, z_1, z_2, z_3 are concyclic. i.e. z lies on a circle.

$\therefore$ (d) is false.

21. **(a, b)** $a - b = 1, y \neq 0$

$\text{Im}\left(\dfrac{az+b}{z+1}\right) = y$

$\Rightarrow \text{Im}\left[\dfrac{a(x+iy)+b}{(x+1)+iy} \times \dfrac{(x+1)-iy}{(x+1)-iy}\right] = y$

$\Rightarrow \dfrac{-(ax+b)y + ay(x+1)}{(x+1)^2+y^2} = y$

$\Rightarrow \dfrac{-axy - by + axy + ay}{(x+1)^2+y^2} = y$

$\Rightarrow a - b = (x+1)^2 + y^2$

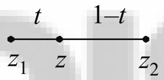
$\Rightarrow 1 = (x+1)^2 + y^2, \therefore x = -1 \pm \sqrt{1-y^2}$

22. **(a,c,d)** Given : $z = (1-t)\,z_1 + t\,z_2$, where $0 < t < 1$

$\Rightarrow z = \dfrac{(1-t)z_1 + tz_2}{(1-t)+t}$

$\Rightarrow$ z divides the join of z_1 and z_2 internally in the ratio $t : (1-t)$.

$\therefore$ z_1, z and z_2 are collinear



$\Rightarrow |z-z_1| + |z-z_2| = |z_1-z_2|$

Also $z = (1-t)z_1 + t\,z_2$

$\Rightarrow \dfrac{z-z_1}{z_2-z_1} = t$, which is purely real number

$\therefore \arg\left(\dfrac{z-z_1}{z_2-z_1}\right) = 0 \Rightarrow \arg(z-z_1) = \arg(z_2-z_1)$

Also $\dfrac{z-z_1}{z_2-z_1} = t \Rightarrow \dfrac{\bar{z}-\bar{z}_1}{\bar{z}_2-\bar{z}_1} = t$

$\Rightarrow \dfrac{z-z_1}{z_2-z_1} = \dfrac{\bar{z}-\bar{z}_1}{\bar{z}_2-\bar{z}_1}$

$\Rightarrow (z-z_1)(\bar{z}_2-\bar{z}_1) = (\bar{z}-\bar{z}_1)(z_2-z_1)$

$\Rightarrow \begin{vmatrix} z-z_1 & \bar{z}-\bar{z}_1 \\ z_2-z_1 & \bar{z}_2-\bar{z}_1 \end{vmatrix} = 0$

23. **(d)** Taking $-3i$ common from C_2, we get

$-3i\begin{vmatrix} 6i & 1 & 1 \\ 4 & -1 & -1 \\ 20 & i & i \end{vmatrix} = 0 \qquad (\because C_2 \equiv C_3)$

$\Rightarrow x = 0, \; y = 0$

24. **(b)** $\displaystyle\sum_{i=1}^{13}(i^n + i^{n+1}) = \sum_{i=1}^{13} i^n(1+i) = (1+i)\sum_{i=1}^{13} i^n$,

Which forms a G.P.

Sum of G.P. $= i\,(1+i)\,\dfrac{(1-i^{13})}{1-i} = i - 1$ as $i^{13} = i$

25. **(d)** Let $z = \cos\dfrac{2\pi}{7} + i\sin\dfrac{2\pi}{7}$

By DeMoivre's theorem,

$z^k = \cos\dfrac{2\pi k}{7} + i\sin\dfrac{2\pi k}{7}$

Now, $\displaystyle\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$

$\displaystyle = \sum_{k=1}^{6}(-i)\left(\cos\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}\right)$

$\displaystyle = (-i)\sum_{k=1}^{6} z^k = -i\,z\,\frac{(1-z^6)}{1-z} = -i\left(\frac{z-z^7}{1-z}\right)$

$= (-i)\left(\dfrac{z-1}{1-z}\right) = [\because z^7 = \cos 2\pi + i\sin 2\pi = 1]$

$= i\left(\dfrac{1-z}{1-z}\right) = i$

26. **(c)** Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

where $r_1 = |z_1|, r_2 = |z_2|, \theta_1 = \arg(z_1), \theta_2 = \arg(z_2)$

$\therefore z_1 + z_2 = r_1(\cos\theta_1 + i\sin\theta_1) + r_2(\cos\theta_2 + i\sin\theta_2)$

$= (r_1\cos\theta_1 + r_2\cos\theta_2) + i(r_1\sin\theta_1 + r_2\sin\theta_2)$

So, $|z_1 + z_2| = r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2 + 2r_1r_2\cos\theta_1\cos\theta_2$

$+ r_1^2\sin^2\theta_1 + r_2^2\sin^2\theta_2 + 2r_1r_2\sin\theta_1\sin\theta_2$

$= r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2)$

and $|z_1| + |z_2| = r_1 + r_2$

Given $|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow |z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$\Rightarrow r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1-\theta_2) = r_1^2 + r_2^2 + 2r_1r_2$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0$

$\therefore$ $\arg(z_1) = \arg(z_2)$

27. **(a, d)** Let $z_1 = a + ib$, $a > 0$ and $b \in R$; $z_2 = c + id$, $d < 0, c \in R$, then

$|z_1| = |z_2| \Rightarrow a^2 + b^2 = c^2 + d^2$

$\Rightarrow a^2 - c^2 = d^2 - b^2$ ....(i)

Now, $\dfrac{z_1+z_2}{z_1-z_2} = \dfrac{(a+c)+i(b+d)}{(a-c)+i(b-d)}$

$= \dfrac{[(a^2-c^2)+(b^2-d^2)] + i[(a-c)(b+d)-(a+c)(b-d)]}{(a-c)^2+(b-d)^2}$

$= \dfrac{i[(a-c)(b+d)-(a+c)(b-d)]}{(a-c)^2+(b-d)^2}$ [Using (i)]

Which is purely imaginary number or zero in case $a+c=b+d=0$.

28. **(a, b, c)** $z_1 = a + ib$ and $z_2 = c + id$.

Acc. to the ques, $|z_1|^2 = |z_2|^2 = 1$

$\Rightarrow a^2 + b^2 = 1$ and $c^2 + d^2 = 1$. ....(i)

Also Re $(z_1 \overline{z_2}) = 0 \Rightarrow ac + bd = 0$

$\Rightarrow \dfrac{a}{b} = \dfrac{-d}{c} = \alpha\ (say)$ ....(ii)

From (i) and (ii), we get

$b^2\alpha^2 + b^2 = c^2\alpha^2 + c^2 \Rightarrow b^2 = c^2$;

Similarly, $a^2 = d^2$

$\therefore\ |\omega_1| = \sqrt{a^2+c^2} = \sqrt{c^2+d^2} = 1$

and $|\omega_2| = \sqrt{b^2+d^2} = \sqrt{c^2+d^2} = 1$

Also, Re $(\omega_1\overline{\omega}_2) = ab + cd = (b\alpha)b + c(-c\alpha)$

$= \alpha(b^2 - c^2) = 0$

29. **(b)** Given, $|z|^3 + 2z^2 + 4\overline{z} - 8 = 0$ ...(i)

$|\overline{z}|^3 + 2\overline{z}^2 + 4z - 8 = 0$ [Conjugate both sides]

$2(z^2 - \overline{z}^2) + 4(\overline{z} - z) = 0$

$\Rightarrow 2(z-\overline{z})[z+\overline{z}-2] = 0$

$\because z = \overline{z}$ (Not possible) or $z + \overline{z} = 2$

$\therefore z = 1 + bi\ (b \neq 0) \Rightarrow \overline{z} = 1 - bi$

$(1+b^2)^{3/2} + 2(1-b^2+2bi) + 4(1-bi) - 8 = 0$ [from (i)]

$(1+b^2)^{3/2} - 2(1+b^2) = 0$

$\Rightarrow (1+b^2)\left(\sqrt{1+b^2} - 2\right) = 0$

$\because 1 + b^2 \neq 0 \Rightarrow \sqrt{1+b^2} - 2 = 0 \Rightarrow b^2 = 3$

(P) $|z|^2 = 1 + b^2 = 1 + 3 = 4$

(Q) $|z - \overline{z}|^2 = |1 + ib - 1 + ib|^2 = 4b^2 = 12$

(R) $|z|^2 + |z + \overline{z}|^2 = 4 + |1 + ib + 1 - ib|^2 = 4 + 4 = 8$

(S) $|z+1|^2 = |1+1+ib|^2 = 4 + b^2 = 4 + 3 = 7.$

30. **(c)** (P) $\to$ (1) : $z_k = \cos\dfrac{2k\pi}{10} + i\sin\dfrac{2k\pi}{10}$, $k = 1$ to 9

$\therefore\ z_k = e^{i\frac{2k\pi}{10}}$

Now $z_k.z_j = 1 \Rightarrow z_j = \dfrac{1}{z_k} = e^{-i\frac{2k\pi}{10}} = \overline{z_k}$

We know if z_k is 10th root of unity so will be $\overline{z}_k$.

$\therefore$ For every z_k, there exist $z_i = \overline{z}_k$

Such that $z_k.z_j = z_k.\overline{z}_k = 1$

Hence the statement is true.

(Q) $\to$ (2) $z_1 = z_k \Rightarrow z = \dfrac{z_k}{z_1}$ for $z_1 \neq 0$

$\therefore$ We can always find a solution of $z_1.z = z_k$

Hence the statement is false.

(R) $\to$ (3) : We know $z^{10} - 1 = (z-1)(z-z_1)....(z-z_9)$

$\Rightarrow (z-z_1)(z-z_2).....(z-z_9) = \dfrac{z^{10}-1}{z-1}$

$= 1 + z + z^2 + ...\ z^9$

For z = 1, we get $(1-z_1)(1-z_2).....(1-z_9) = 10$

$\therefore\ \dfrac{|1-z_1||1-z_2|.....|1-z_9|}{10} = 1$

(S) $\to$ (4) : 1, $Z_1, Z_2, ..., Z_9$ are 10th roots of unity.

$\therefore Z^{10} - 1 = 0$

From equation $1 + Z_1 + Z_2 + ... + Z_9 = 0$,

Re (1) + Re (Z_1) + Re (Z_2) + ... + Re(Z_9) = 0

$\Rightarrow$ Re (Z_1) + Re (Z_2) + ... Re(Z_9) = – 1

$\Rightarrow \displaystyle\sum_{K=1}^{9}\cos\frac{2k\pi}{10} = -1 \Rightarrow 1 - \sum_{K=1}^{9}\cos\frac{2k\pi}{10} = 2$

Hence (c) is the correct option.

For (Qs. 31-32)

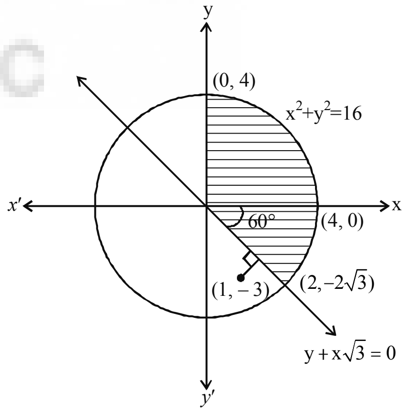
$S_1 : x^2 + y^2 < 16$

$S_2 :$ Im$\left[\dfrac{(x-1)+i(y+\sqrt{3})}{1-i\sqrt{3}}\right] > 0$

$\Rightarrow \sqrt{3}(x-1) + (y+\sqrt{3}) > 0 \Rightarrow y + \sqrt{3}x > 0$

$S_3 : x > 0$

Then S : $S_1 \cap S_2 \cap S_3$ is as shown in the figure given below.



31. **(b)** Area of shaded region

$= \dfrac{\pi}{4}\times 4^2 + \dfrac{\pi\times 4^2\times 60°}{360°} = 4\pi + \dfrac{8\pi}{3} = \dfrac{20\pi}{3}$

32. **(c)** $\min\limits_{z\in s}|1 - 3i - z|$ = min distance between z and (1, –3)

Clearly (from figure) minimum distance between z∈S and (1, –3)

from line $y + x\sqrt{3} = 0$ i.e. $\left|\dfrac{\sqrt{3}-3}{\sqrt{3+1}}\right| = \dfrac{3-\sqrt{3}}{2}$

For (Qs. 33 - 35)

Given : $A = \{z : \text{Im}(z) \geq 1\} = \{(x, y) : y \geq 1\}$

Clearly A is the set of all points lying on or above the line $y = 1$ in cartesian plane.

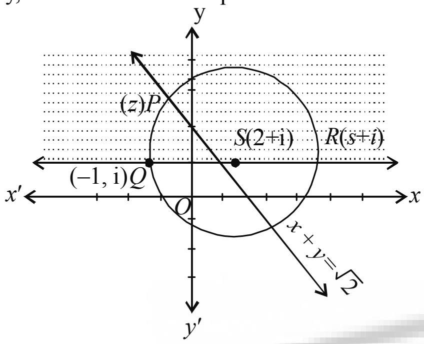
$B = \{z : |z - 2 - i| = 3\} = \{(x, y) : (x - 2)^2 + (y - 1)^2 = 9\}$

$\Rightarrow$ B is the set of all points lying on the boundary of the circle with centre (2, 1) and radius 3.

$C = \{z : \text{Re}[(1 - i)\, z] = \sqrt{2}\} = \{(x, y) : x + y = \sqrt{2}\}$

$\Rightarrow$ C is the set of all points lying on the straight line represented by $x + y = \sqrt{2}$.

Graphically, the three sets are represented as shown below :



33. **(b)** From graph $A \cap B \cap C$ consists of only one point P [the common point of the region $y \geq 1$, $(x - 2)^2 + (y - 1)^2 = 9$ and $x + y = \sqrt{2}$] $\therefore n(A \cap B \cap C) = 1$

34. **(c)** Since, z is a point of $A \cap B \cap C$ $\Rightarrow$ z represents the point P

$\therefore |z + 1 - i|^2 + |z - 5 - i|^2$

$\Rightarrow |z - (-1 + i)|^2 + |z - (5 + i)|^2$

$\Rightarrow PQ^2 + PR^2 = QR^2 = 6^2 = 36$, which lies between 35 and 39

$\therefore$ (c) is correct option.

35. **(d)** Given : $|w - 2 - i| < 3$

$\Rightarrow$ Distance between w and $2 + i$ i.e. S is smaller than 3.

$\Rightarrow$ w is a point lying inside the circle with centre S and radius 3.

$\Rightarrow$ Distance between z (i.e. the point P) and w should be smaller than 6 (the diameter of the circle)

i.e. $|z - w| < 6$

But we know that $||z| - |w|| < |z - w|$

$\Rightarrow ||z| - |w|| < 6 \Rightarrow -6 < |z| - |w| < 6$

$-3 < |z| - |w| + 3 < 9$

36. Given : $|z_1| < 1 < |z_2|$

Then $\left|\dfrac{1 - z_1\bar{z}_2}{z_1 - z_2}\right| < 1$ is true

if $|1 - z_1\bar{z}_2| < |z_1 - z_2|$ is true

or if $|1 - z_1\bar{z}_2|^2 < |z_1 - z_2|^2$ is true

or if $(1 - z_1\bar{z}_2)\overline{(1 - z_1\bar{z}_2)} < (z_1 - z_2)\,\overline{(z_1 - z_2)}$ is true

or if $(1 - z_1\bar{z}_2)\,(1 - \bar{z}_1 z_2) < (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$ is true

or if $1 - z_1\bar{z}_2 - \bar{z}_1 z_2 + z_1\bar{z}_1\, z_2\bar{z}_2 < z_1\bar{z}_1 - z_1\bar{z}_2 - \bar{z}_1 z_2 + z_2\bar{z}_2$ is true

or, if $1 + |z_1|^2 |z_2|^2 < |z_1|^2 + |z_2|^2$ is true

or, if $(1 - |z_1|^2)(1 - |z_2|^2) < 0$ is true.

which is obviously true

as $|z_1| < 1 < |z_2| \Rightarrow |z_1|^2 < 1 < |z_2|^2$

$\Rightarrow |1 - |z_1|^2 > 0$ and $(1 - |z_2|^2) < 0$

37. Given : $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$

Also $\arg\left(\dfrac{z - z_1}{z - z_2}\right) = \dfrac{\pi}{4}$

$\Rightarrow \arg(z - z_1) - \arg(z - z_2) = \dfrac{\pi}{4}$

$\Rightarrow \arg((x + iy) - (10 + 6i)) - \arg((x + iy) - (4 + 6i)) = \dfrac{\pi}{4}$

$\Rightarrow \arg[(x - 10) + i(y - 6)] - \arg[(x - 4) + i(y - 6)] = \dfrac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\dfrac{y - 6}{x - 10}\right) - \tan^{-1}\left(\dfrac{y - 6}{x - 4}\right) = \dfrac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\dfrac{\dfrac{y - 6}{x - 10} - \dfrac{y - 6}{x - 4}}{1 + \dfrac{(y - 6)^2}{(x - 4)(x - 10)}}\right) = \dfrac{\pi}{4}$

$\Rightarrow \dfrac{(x - 4)(y - 6) - (x - 10)\,(y - 6)}{(x - 4)\,(x - 10) + (y - 6)^2} = \tan\dfrac{\pi}{4}$

$\Rightarrow (x - 4 - x + 10)\,(y - 6) = (x - 4)\,(x - 10) + (y - 6)^2$

$\Rightarrow 6y - 36 = x^2 + y^2 - 14x - 12y + 40 + 36$

$\Rightarrow x^2 + y^2 - 14x - 18y + 112 = 0$

$\Rightarrow (x^2 - 14x + 49) + (y^2 - 18y + 81) = 18$

$\Rightarrow (x - 7)^2 + (y - 9)^2 = (3\sqrt{2})^2$

$\Rightarrow |(x + iy) - (7 + 9i)| = 3\sqrt{2}$

$\Rightarrow |z - (7 + 9i)| = 3\sqrt{2}$

38. Let $A = z = x + iy$, $B = iz = -y + ix$, $C = z + iz = (x - y) + i\,(x + y)$

Now, area of $\Delta ABC = \dfrac{1}{2}\begin{vmatrix} x & y & 1 \\ -y & x & 1 \\ x - y & x + y & 1 \end{vmatrix}$

On applying, $R_2 - R_1$, $R_3 - R_1$, we get

$\Delta = \dfrac{1}{2}\begin{vmatrix} x & y & 1 \\ -y - x & x - y & 0 \\ -y & x & 0 \end{vmatrix}$

$= \dfrac{1}{2}|-xy - x^2 + xy - y^2| = \dfrac{1}{2}|-x^2 - y^2|$

$= \dfrac{1}{2}|x^2 + y^2| = \dfrac{1}{2}|z|^2$

39. $\dfrac{(1 + i)x - 2i}{3 + i} + \dfrac{(2 - 3i)y + i}{3 - i} = i$

$\Rightarrow (4 + 2i)\,x - 6i - 2 + (9 - 7i)\,y + 3i - 1 = 10i$

$\Rightarrow$ $(4x + 9y - 3) + (2x - 7y - 3)\, i = 10i$

$\Rightarrow$ $4x + 9y - 3 = 0$ and $2x - 7y - 3 = 10$

On solving these two equations, we get $x = 3$, $y = -1$

40. Given : $x + iy = \sqrt{\dfrac{a+ib}{c+id}}$

$$\Rightarrow \quad (x+iy)^2 = \frac{a+ib}{c+id} \qquad ....(i)$$

Taking conjugate on both sides, we get

$$(x-iy)^2 = \frac{a-ib}{c-id} \qquad ....(ii)$$

On multiply (i) and (ii), we get

$$(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$$

41. $\dfrac{1}{1-\cos\theta + 2i\sin\theta}$

$$= \frac{1}{2\sin^2\theta/2 + 4i\sin\theta/2\cos\theta/2} = \frac{1}{2\sin\theta/2}\left[\frac{\sin\theta/2 - 2i\cos\theta/2}{(\sin\theta/2+2i\cos\theta/2)(\sin\theta/2-2i\cos\theta/2)}\right]$$

$$= \frac{1}{2\sin\theta/2}\left[\frac{\sin\theta/2 - 2i\cos\theta/2}{(\sin^2\theta/2 + 4\cos^2\theta/2}\right]$$

$$= \frac{1}{2\sin\theta/2}\left[\frac{2\sin\theta/2 - 4i\cos\theta/2}{1-\cos\theta+4+4\cos\theta}\right]$$

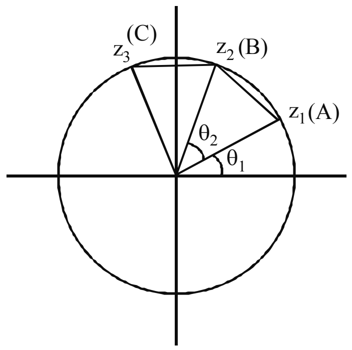
$$= \frac{2}{2\sin\theta/2}\left[\frac{2\sin\theta/2 - 2i\cos\theta/2}{5+3\cos\theta}\right]$$

$$= \left(\frac{1}{5+3\cos\theta}\right) + \left(\frac{-2\cot\theta/2}{5+3\cos\theta}\right)i$$

which is of the form $x + iy$.

Topic-2: Rotational Theorem, Square Root of a Complex Number, Cube Roots of Unity, Geometry of Complex Numbers, De-moiver's Theorem, Powers of Complex Numbers

1. **(c)**



Since, $|z_1| = |z_2| = ...|z_{10}| = 1$

$\theta_2 = \text{arc}(z_1 z_2)$

$|z_2 - z_1|$ = length of line AB $\leq$ length of arc AB

$|z_3 - z_2|$ = length of line BC $\leq$ length of arc BC

$\therefore$ Sum of length of these 10 lines $\leq$ Sum of length of arcs (*i.e.* 2π)

$$[\because \theta_1 + \theta_2 + \theta_3 + ... + \theta_{10} = 2\pi]$$

$\therefore$ $P : |z_2 - z_1| + |z_3 - z_2| + ... + |z_1 - z_{10}| \leq 2\pi$

P is true.

Now, $|z_2^2 - z_1^2| = |z_2 - z_1|\,|z_2 + z_1|$

We know that

$|z_2 + z_1| \leq |z_2| + |z_1| \leq 2$

$\therefore$ $|z_2^2 - z_1^2| + |z_3^2 - z_2^2| + ... + |z_1^2 - z_{10}^2| \leq$

$2\{|z_2 - z_1| + |z_3 - z_2| + ... + |z_1 - z_{10}|\} \leq 2(2\pi) \Rightarrow Q \leq 4\pi$

Q is also true.

2. **(d)** S : $|z - 2 + i| \geq \sqrt{5}$ represents boundary and outer region of circle with centre (2, –1) and radius $\sqrt{5}$ units.

$z_0 \in S$, such that $\dfrac{1}{|z_0 - 1|}$ is the maximum.

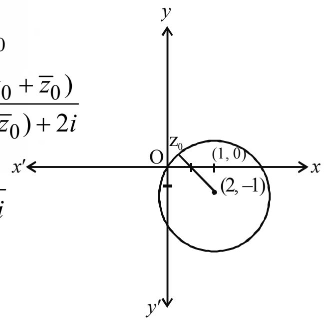
$\therefore$ $|z_0 - 1|$ is minimum

$z_0 \in S$ with $|z_0 - 1|$ as minimum will be a point on boundary of circle of region S which lies on radius of this circle, which passes through (1, 0).

$\therefore$ z_0, 1, $2 - i$ are collinear, or (x_0, y_0), (1, 0), (2, –1) are collinear.

$\therefore$ Using slopes of paralled lines,x'

$$\frac{y_0}{x_0 - 1} = \frac{-1}{2-1} \Rightarrow y_0 = 1 - x_0$$



Now, $\dfrac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i} = \dfrac{4 - (z_0 + \bar{z}_0)}{(z_0 - \bar{z}_0) + 2i}$

$$= \frac{4 - 2x_0}{2iy_0 + 2i} = \frac{4 - 2x_0}{2i - 2x_0 i + 2i}$$

$$= \frac{2(2 - x_0)}{2(2 - x_0)i} = \frac{1}{i} = -i$$

$$\therefore \quad Arg\left(\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 - 2i}\right) = Arg(-i) = \frac{-\pi}{2}$$

3. **(c)** Since, α lies on the circle $(x - x_0)^2 + (y - y_0)^2 = r^2$

$\therefore$ $|\alpha - z_0|^2 = r^2$

$\Rightarrow$ $(\alpha - z_0)(\bar{\alpha} - \bar{z}_0) = r^2$

$\Rightarrow$ $\alpha\bar{\alpha} - \alpha\bar{z}_0 - \bar{\alpha}z_0 + z_0\bar{z}_0 = r^2$

$$\Rightarrow |\alpha|^2 + |z_0|^2 - \alpha\bar{z}_0 - \bar{\alpha}z_0 = r^2 \qquad ...(i)$$

Also $\dfrac{1}{\alpha}$ lies on the circle $(x - x_0)^2 + (y - y_0)^2 = 4r^2$

$\therefore \quad \left|\frac{1}{\bar{\alpha}} - z_0\right|^2 = 4r^2 \Rightarrow \left(\frac{1}{\bar{\alpha}} - z_0\right)\left(\frac{1}{\alpha} - \bar{z}_0\right) = 4r^2$

$\Rightarrow \quad \frac{1}{\alpha\bar{\alpha}} - \frac{z_0}{\alpha} - \frac{\bar{z}_0}{\bar{\alpha}} + z_0\bar{z}_0 = 4r^2$

$\Rightarrow \quad \frac{1}{|\alpha|^2} - \frac{z_0\bar{\alpha}}{|\alpha|^2} - \frac{\bar{z}_0\alpha}{|\alpha|^2} + |z_0|^2 = 4r^2$

$\Rightarrow \quad 1 + |\alpha|^2 |z_0|^2 - z_0\bar{\alpha} - \bar{z}_0\alpha = 4r^2|\alpha|^2$ …(ii)

On subtracting equation (i) from (ii), we get

$1 - |\alpha|^2 + |z_0|^2 (|\alpha|^2 - 1) = r^2(4|\alpha|^2 - 1)$

or $\quad (|\alpha|^2 - 1)(|z_0|^2 - 1) = r^2(4|\alpha|^2 - 1)$

Using $|z_0|^2 = \frac{r^2+2}{2}$, we get

$(|\alpha|^2 - 1)\frac{r^2}{2} = r^2(4|\alpha|^2 - 1)$

$\Rightarrow \quad |\alpha|^2 - 1 = 8|\alpha|^2 - 2 \Rightarrow |\alpha| = \frac{1}{\sqrt{7}}$

4. **(d)** $\because \quad Im(z) \neq 0$

$\Rightarrow$ z is non real

and equation $z^2 + z + (1-a) = 0$

will have non real roots, if $D < 0$

$\Rightarrow \quad 1 - 4(1-a) < 0 \Rightarrow 4a < 3 \Rightarrow a < \frac{3}{4}$

$\therefore$ a can not take the value $\frac{3}{4}$.

5. **(a)** Given : $z = x + iy$, where x and y are integer

Also, $z\bar{z}^3 + \bar{z}z^3 = 350 \Rightarrow |z|^2(\bar{z}^2 + z^2) = 350$

$\Rightarrow \quad (x^2+y^2)(x^2-y^2) = 175$

$\Rightarrow \quad (x^2+y^2)(x^2-y^2) = 25 \times 7$ ...(i)

or $\quad (x^2+y^2)(x^2-y^2) = 35 \times 5$ ...(ii)

$\because$ x and y are integers,

$\therefore \; x^2 + y^2 = 25$ and $x^2 - y^2 = 7$ [From eq (i)]

$\Rightarrow \quad x^2 = 16$ and $y^2 = 9$

$\Rightarrow \quad x = \pm 4$ and $y = \pm 3$

$\therefore$ Vertices of rectangle are

(4, 3) , (4, – 3), (– 4, – 3) , (– 4, 3).

$\therefore$ Area of rectangle = 8 × 6 = 48 sq. units

Now from eq. (ii),

$x^2 + y^2 = 35$ and $x^2 - y^2 = 5$

$\Rightarrow x^2 = 20$, which is not possible for any integral value of x

6. **(d)** $z = \cos\theta + i\sin\theta$

$\Rightarrow \quad z^{2m-1} = (\cos\theta + i\sin\theta)^{2m-1}$

$= \cos(2m-1)\theta + i\sin(2m-1)\theta$

$\left[\begin{array}{l}\text{By De Moivre's theorem :}\\ (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta\end{array}\right]$

$\therefore \quad \text{Im}(z^{2m-1}) = \sin(2m-1)\theta$

$\therefore \quad \sum_{m=1}^{15} \text{Im}(z^{2m-1}) = \sum_{m=1}^{15} \sin(2m-1)\theta$

$= \sin\theta + \sin 3\theta + \sin 5\theta + \ldots\ldots + \text{upto 15 terms}$

$= \frac{\sin\left[15\left(\frac{2\theta}{2}\right)\right].\sin[\theta + 14\times\theta]}{\sin\theta}$

$\left[\begin{array}{l}\because \sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots. n \text{ terms}\\ = \frac{\sin(n\beta/2).\sin[\alpha+(n-1)\beta/2]}{\sin(\beta/2)}\end{array}\right]$

$= \frac{\sin 15\theta.\sin 15\theta}{\sin\theta} = \frac{\sin 30°.\sin 30°}{\sin 2°} = \frac{1}{4\sin 2°}$

7. **(d)** The initial position of point is $Z_0 = 1 + 2i$

$\therefore Z_1 = (1+5) + (2+3)i = 6 + 5i$

Now Z_1 is moved through a distance of $\sqrt{2}$ units in the direction $\hat{i} + \hat{j}$. (i.e. by $1 + i$)

$\therefore$ It becomes $Z_1' = Z_1 + (1+i) = 7 + 6i$

Now OZ_1' is rotated through an angle $\frac{\pi}{2}$ in anticlockwise direction, therefore $Z_2 = iZ_1' = -6 + 7i$

8. **(d)** Given : $|z| = 1$ and $z \neq \pm 1$

To find the locus of $\omega = \frac{z}{1-z^2}$

Now, $\omega = \frac{z}{1-z^2} = \frac{z}{z\bar{z} - z^2}$

$[\because |z| = 1 \Rightarrow |z|^2 = z\bar{z} = 1]$

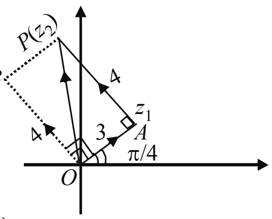
$= \frac{1}{\bar{z} - z}$ = purely imaginary number

$\therefore$ ω must lie on y-axis.

9. **(d)** $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$\Rightarrow \quad \overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$

$\Rightarrow \quad \overrightarrow{OP} = 3e^{i\pi/4} + 4e^{i(\pi/2+\pi/4)}$



$= 3e^{i\pi/4} + 4e^{i\pi/2}.e^{i\pi/4}$

$= 3e^{i\pi/4} + 4ie^{i\pi/4} = e^{i\pi/4}(3+4i)$.

10. **(b)** Given that a, b, c are integers not all equal and ω is cube root of unity $\neq 1$, then $|a + b\omega + c\omega^2|$

$= \left|a + b\left(\frac{-1+i\sqrt{3}}{2}\right) + c\left(\frac{-1-i\sqrt{3}}{2}\right)\right|$

$$= \left| \left(\frac{2a-b-c}{2} \right) + i \left(\frac{b\sqrt{3}-c\sqrt{3}}{2} \right) \right|$$

$$= \frac{1}{2}\sqrt{(2a-b-c)^2 + 3(b-c)^2}$$

$$= \sqrt{\frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]}$$

R.H.S. will be minimum when $a = b = c$, but according to the question, we cannot take $a = b = c$.

$\therefore$ The minimum value is obtained when any two are zero and third is a minimum magnitude integer i.e. 1.

$\therefore$ $b = c = 0$, $a = 1$; gives us the minimum value 1.

11. **(a)** In the figure, we see that.

$AB = AC = AD = 2$

$\therefore$ BCD is an arc of a circle with centre at A and radius 2. Shaded region is exterior part of this sector $ABCDA$.

$\therefore$ For any point represented by z on arc BCD we should have $|z - (-1)| = 2$

and for shaded region, $|z + 1| > 2$ ....(i)

For shaded region, we also have

$$-\pi/4 < \arg(z+1) < \pi/4$$

or $|\arg(z+1)| < \pi/4$ ...(ii)

From (i) and (ii), we get (a) is the correct option.

12. **(b)** $(1+\omega^2)^n = (1+\omega^4)^n$

$$\Rightarrow (-\omega)^n = (1+\omega)^n = (-\omega^2)^n \Rightarrow \omega^n = 1 \Rightarrow n = 3$$

13. **(a)** Given that $|z| = 1$ and $\omega = \frac{z-1}{z+1} (z \neq -1)$

Now we know that $z\bar{z} = |z|^2$

$\Rightarrow$ $z\bar{z} = 1$ (for $|z| = 1$)

$$\therefore \ \omega = \left(\frac{z-1}{z+1} \right) \times \frac{(\bar{z}+1)}{(\bar{z}+1)} = \frac{z\bar{z} + z - \bar{z} - 1}{z\bar{z} + z + \bar{z} + 1} = \frac{2iy}{2+2x}$$

[$\because$ $z\bar{z} = 1$ and taking $z = x + iy$ so that

$z + \bar{z} = 2x$ and $z - \bar{z} = 2iy$]

$\Rightarrow$ $\text{Re}(\omega) = 0$

14. **(b)** Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & -1-\omega & \omega^2 \\ 1 & \omega^2 & \omega 4 \end{vmatrix} = \begin{vmatrix} 3 & 0 & 0 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega 4 \end{vmatrix}$$

$$= 3[-\omega - 1 - \omega] = 3(\omega^2 - \omega)$$

15. **(c)** $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$

$$\Rightarrow \arg\left(\frac{z_1 - z_3}{z_2 - z_3} \right) = \arg\left(\frac{1 - i\sqrt{3}}{2} \right)$$

$\Rightarrow$ $\arg(\cos(-\pi/3) + i\sin(-\pi/3))$

$\Rightarrow$ angle between $(z_1 - z_3)$ and $(z_2 - z_3)$ is 60°.

and $\left| \frac{z_1 - z_3}{z_2 - z_3} \right| = \left| \frac{1 - i\sqrt{3}}{2} \right|$

$\Rightarrow$ $\left| \frac{z_1 - z_3}{z_2 - z_3} \right| = 1 \Rightarrow |z_1 - z_3| = |z_2 - z_3|$ (Imp Step)

$\Rightarrow$ The Δ with vertices z_1, z_2 and z_3 is isosceles with vertical angle 60°. Hence rest of the two angles should also be 60° each.

$\Rightarrow$ Required triangle is an equilateral triangle.

16. **(d)** Let $z = (1)^{1/n} = (\cos 2k\pi + i \sin 2k\pi)^{1/n}$

$$z = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}, k = 0, 1, 2, \ldots, n-1.$$

Let $z_1 = \cos\left(\frac{2k_1\pi}{n}\right) + i\sin\left(\frac{2k_1\pi}{n}\right)$

and $z_2 = \cos\left(\frac{2k_2\pi}{n}\right) + i \sin\frac{2k_2\pi}{n}$

be the two values of z. Such that they subtend right angle at origin.

$$\therefore \ \frac{2k_1\pi}{n} - \frac{2k_2\pi}{n} = \pm\frac{\pi}{2} \Rightarrow 4(k_1 - k_2) = \pm n$$

As k_1 and k_2 are integers and $k_1 \neq k_2$.

$\therefore$ $n = 4k, k \in I$

17. **(c)** $E = 4 + 5(\omega)^{334} + 3(\omega)^{365} = 4 + 5\omega + 3\omega^2$

$$= 1 + 2\omega + 3(1 + \omega + \omega^2) = 1 + (-1 + i\sqrt{3}) = i\sqrt{3}$$

18. **(b)** $(1+\omega)^7 = A + B\omega$

$\Rightarrow$ $(-\omega^2)^7 = A + B\omega$ ($\because$ $1 + \omega + \omega^2 = 0$)

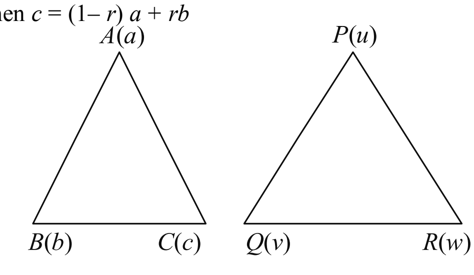
$\Rightarrow$ $-\omega^{14} = A + B\omega$

$\Rightarrow$ $-\omega^2 = A + B\omega$ ($\because$ $\omega^3 = 1$)

$\Rightarrow$ $1 + \omega = A + B\omega \Rightarrow A = 1, B = 1$

19. **(b)** Let ABC be the Δ whose vertices are represented by complex numbers a, b, c and PQR be the Δ with whose vertices are represented by complex numbers u, v, w.

Then $c = (1 - r) a + rb$



$\Rightarrow$ $c - a = r(b - a)$ $\Rightarrow$ $\frac{c-a}{b-a} = r$ ....(i)

$\Rightarrow$ $w = (1 - r) u + rv$ $\Rightarrow$ $\frac{w-u}{v-u} = r$ ....(ii)

From (i) and (ii), $\left|\frac{c-a}{b-a}\right| = \left|\frac{w-u}{v-u}\right|$

and $\arg\left(\frac{c-a}{b-a}\right) = \arg\left(\frac{w-u}{v-u}\right)$

$\Rightarrow \frac{AC}{AB} = \frac{PR}{PQ}$ and $\angle CAB = \angle RPQ$

$\Rightarrow \Delta ABC \sim \Delta PQR$

20. **(b)** If vertices of a parallelogram are z_1, z_2, z_3, z_4 then as diagonals bisect each other

$\therefore \frac{z_1+z_3}{2} = \frac{z_2+z_4}{2} \Rightarrow z_1 + z_3 = z_2 + z_4$

21. **(b)** $|\omega| = 1 \Rightarrow \left|\frac{1-iz}{z-i}\right| = 1$

$\Rightarrow |1-iz| = |z-i|$

$\Rightarrow |1-i(x+iy)| = |x+iy-i|$

$\Rightarrow |(y+1)-ix| = |x+i(y-1)|$

$\Rightarrow x^2+(y+1)^2 = x^2+(y-1)^2$

$\Rightarrow 4y = 0 \Rightarrow y = 0 \Rightarrow z$ lies on real axis

22. **(d)** $|z-4| < |z-2|$

$\Rightarrow |(x-4)+iy| < |(x-2)+iy|$

$\Rightarrow (x-4)^2+y^2 < (x-2)^2+y^2$

$\Rightarrow -8x+16 < -4x+4 \Rightarrow 4x-12 > 0$

$\Rightarrow x > 3 \Rightarrow \text{Re}(z) > 3$

23. **(b)** $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right) = -i\left(\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right) = i\omega$

$\frac{\sqrt{3}}{2}-\frac{i}{2} = i\left(\frac{-1}{2}-\frac{i\sqrt{3}}{2}\right) = i\omega^2$

$\therefore z = (-i\omega)^5+(i\omega^2)^5 = -i\omega^2+i\omega$

$= i(\omega-\omega^2) = i(i\sqrt{3}) = -\sqrt{3}$

$\Rightarrow \text{Re}(z) < 0$ and $\text{Im}(z) = 0$

24. **(a)** Since, $z = x+iy$ satisfies the equation $\left|\frac{z-5i}{z+5i}\right| = 1$

$\therefore |x+iy-5i| = |x+iy+5i|$

$\Rightarrow |x+(y-5)i| = |x+(y+5)i|$

$\Rightarrow x^2+(y-5)^2 = x^2+(y+5)^2$

$\Rightarrow x^2+y^2-10y+25 = x^2+y^2+10y+25$

$\Rightarrow 20y = 0 \Rightarrow y = 0$

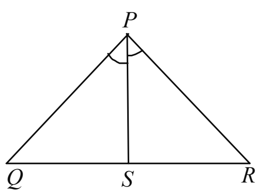
$\therefore$ 'a' is the correct alternative.

25. **(b)** $(x-1)^3 + 8 = 0$

$\Rightarrow (x-1)^3 = -8 = (-2)^3$

$\Rightarrow x-1 = -2$ or -2ω or $-2\omega^2$

$\Rightarrow x = -1, 1-2\omega, 1-2\omega^2$



26. **(8)** Let $z = x+iy$

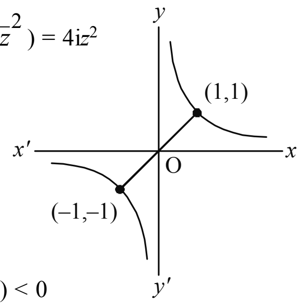
$z^4 - |z|^4 = 4iz^2$

$\Rightarrow z^4 - (z\bar{z})^2 = 4iz^2 \Rightarrow z^2(z^2-\bar{z}^2) = 4iz^2$

$\Rightarrow z = 0$ or $z^2 - (\bar{z})^2 = 4i$

$\Rightarrow 4ixy = 4i \Rightarrow xy = 1$

Locus of z is a rectangular hyperbola $xy = 1$



Given that $\text{Re}(z_1) > 0$ and $\text{Re}(z_2) < 0$

$\therefore |z_1-z_2|_{\min} = \sqrt{(1+1)^2+(1+1)^2} = \sqrt{8}$

$\Rightarrow |z_1-z_2|^2_{\min} = 8$

27. **(3)** a, b, c are distinct non-zero integers

Min. value of $|a+b\omega+c\omega^2|^2$ is to be found $|a+b\omega+c\omega^2|^2$

$= \left|a+b\left(\frac{-1+i\sqrt{3}}{2}\right)+c\left(\frac{-1-i\sqrt{3}}{2}\right)\right|^2$

$= \left|\frac{1}{2}(2a-b-c)+\frac{i\sqrt{3}}{2}(b-c)\right|^2$

$= \frac{1}{4}(2a-b-c)^2+\frac{3}{4}(b-c)^2$

$= \frac{1}{4}(4a^2+b^2+c^2-4ab+2bc-4ac+3b^2+3c^2-6bc)$

$= a^2+b^2+c^2-ab-bc-ca$

$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

For minimum value, let us consider $a = 3, \ b = 2, \ c = 1$

$\therefore$ minimum value $= \frac{1}{2}[1+1+4] = \frac{6}{2} = 3$

28. rth term of the given series

$= r[(r+1)-\omega][(r+1)-\omega^2]$

$= r[(r+1)^2-(\omega+\omega^2)(r+1)+\omega^3]$

$= r[(r+1)^2-(-1)(r+1)+1]$

$= r[(r^2+3r+3] = r^3+3r^2+3r$

$\therefore$ Sum of the given series $= \sum_{r=1}^{(n-1)}(r^3+3r^2+3r)$

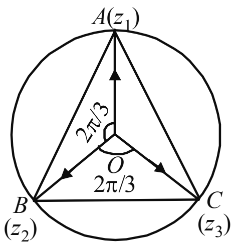
$= \frac{1}{4}(n-1)^2n^2+3.\frac{1}{6}(n-1)\ (n)\ (2n-1)\ +3.\frac{1}{2}(n-1)n$

$= (n-1)\ (n)\left[\frac{1}{4}(n-1)n+\frac{1}{2}\ (2n-1)+\frac{3}{2}\right]$

$= \frac{1}{4}(n-1)\ n[n^2-n+4n-2+6]$

$= \frac{1}{4}(n-1)n[n^2+3n+4]$

29. Let z_1, z_2, z_3 be the vertices A, B and C respectively of equilateral ΔABC, inscribed in a circle $|z| = 2$ with centre (0, 0) and rasius = 2



Given $z_1 = 1 + i\sqrt{3}$

$z_2 = e^{\frac{2\pi i}{3}} z_1$

$= \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)(1 + i\sqrt{3})$

$= \frac{-1-3}{2} = -2$ and $z_3 = e^{4(\pi/3)i} z_1$

$= \left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)(1 + i\sqrt{3})$

$= \left(\frac{-1-i\sqrt{3}}{2}\right)(1 + i\sqrt{3}) = \frac{-1-2i\sqrt{3}+3}{2} = 1 - i\sqrt{3}$

30. As D and m are represented by complex numbers $(1 + i)$ and $(2 - i)$ respectively

$\therefore$ $D \equiv (1,1)$ and $M \equiv (2, -1)$

We know that diagonals of rhombus bisect each other at right angles.

$\therefore$ AC passes through M and is $\perp$ to BD

$\therefore$ Eq. of AC in symmetric form can be written as

$$\frac{x-2}{2/\sqrt{5}} = \frac{y+1}{1/\sqrt{5}} = r$$

Now for pt. A, as

$$AM = \frac{1}{2}DM = \frac{1}{2}\sqrt{(2-1)^2 + (-1-1)^2} = \sqrt{5}/2$$

On putting $r = \pm\sqrt{5}/2$, we get

$$\frac{x-2}{2/\sqrt{5}} = \frac{y+1}{1/\sqrt{5}} = \pm\sqrt{5}/2 \Rightarrow x = \pm 1 + 2, y = \pm\frac{1}{2} - 1$$

$\Rightarrow$ $x = 3$ or $1, y = \frac{-1}{2}$ or $\frac{-3}{2}$

Therefore, point A is represented by $3 - i/2$ or $1 - (3/2)i$

31. Distance between two points represented by z_1 and z_2 $= |z_1 - z_2|$

Since $z_1 = a + i$, $z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle, therefore $|z_1 - z_3| = |z_2 - z_3| = |z_1 - z_2|$

$\Rightarrow$ $|a + i| = |1 + bi| = |(a - 1) + i(1 - b)|$

$\Rightarrow$ $a^2 + 1 = 1 + b^2 = (a - 1)^2 + (1 - b)^2$

$\Rightarrow$ $a^2 = b^2 = a^2 + b^2 - 2a - 2b + 1$

$\Rightarrow$ $a = b$ ....(i)

($\because$ $a \neq -b$ because $0 < a, b < 1$)

and $b^2 - 2a - 2b + 1 = 0$

$\Rightarrow$ $a^2 - 2a - 2b + 1 = 0$ ....(ii)

$\Rightarrow$ $a^2 - 2a - 2a + 1 = 0$ ($\because a = b$)

$\Rightarrow$ $a^2 - 4a + 1 = 0$

$\therefore$ $a = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$. But $0 < a, b < 1$

$\therefore$ $a = 2 - \sqrt{3}$ $\because b = a$ $\therefore$ $b = 2 - \sqrt{3}$

32. $|az_1 - bz_2|^2 + |bz_1 + az_2|^2$

$= a^2|z_1|^2 + b^2|z_2|^2 - 2ab\ \mathrm{Re}(z_1\bar{z}_2) + b^2|z_1|^2$

$+ a^2|z_2|^2 + 2ab\ \mathrm{Re}\ (z_1\bar{z}_2)$

$= (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

33. **(True)** $\because$ Cube roots of unity are 1, $\frac{-1+i\sqrt{3}}{2}, \frac{-1-\sqrt{3}}{2}$

$\therefore$ Vertices of triangle are

$$A(1,0), B\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right), C\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$$

$\Rightarrow$ $AB = BC = CA$, $\therefore$ Δ is equilateral.

34. **(False)** If z_1, z_2, z_3 are in A.P. then, $\frac{z_1 + z_3}{2} = z_2$

$\Rightarrow$ z_2 is mid pt. of line joining z_1 and z_3.

$\Rightarrow$ z_1, z_2, z_3 lie on a st. line

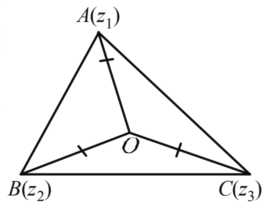
$\therefore$ Given statement is false

35. **(True)**

As $|z_1| = |z_2| = |z_3|$

$\therefore$ z_1, z_2, z_3 are equidistant from origin. Hence O is the circumcentre of ΔABC.

But ΔABC is equilateral and hence circumcentre and centriod of ΔABCcoincide.



$\therefore$ Centriod of $\Delta ABC = 0$

$\Rightarrow$ $\frac{z_1 + z_2 + z_3}{3} = 0$

$\Rightarrow$ $z_1 + z_2 + z_3 = 0$

$\therefore$ Statement is true.

36. **(a, c, d)** $z = \frac{1}{a + ibt} = x + iy$

$\Rightarrow x + iy = \frac{a - ibt}{a^2 + b^2t^2} \Rightarrow x = \frac{a}{a^2 + b^2t^2}$, $y = \frac{-bt}{a^2 + b^2t^2}$

$\Rightarrow x^2 + y^2 = \frac{1}{a^2 + b^2t^2} = \frac{x}{a} \Rightarrow x^2 + y^2 - \frac{x}{a} = 0$

$\therefore$ Locus of z is a circle with centre $\left(\frac{1}{2a}, 0\right)$ and radius $\frac{1}{2|a|}$

irrespective of 'a' +ve or –ve

Also for b = 0, a ≠ 0, we get, y = 0

$\therefore$ locus is x-axis

and for a = 0, b ≠ 0 we get x = 0

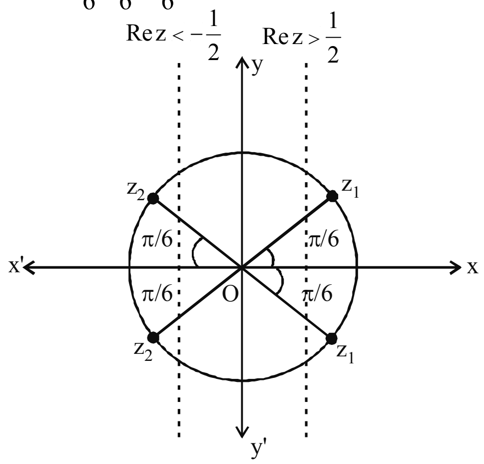
$\therefore$ locus is y-axis.

Hence, a, c, d are the correct options.

37. **(c, d)** We have $w = \frac{\sqrt{3} + i}{2} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$\Rightarrow w^n = \cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6}$

$\therefore$ P contains all those points which lie on unit circle and have arguments $\frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}$ and so on.



Since, $z_1 \in P \cap H_1$ and $z_2 \in P \cap H_2$, therefore z_1 and z_2 can have possible positions as shown in the figure.

$\therefore \angle z_1Oz_2$ can be $\frac{2\pi}{3}$ or $\frac{5\pi}{6}$.

38. **(d)** We have $(1+\omega-\omega^2)^7 = (-\omega^2-\omega^2)^7$

$= (-2)^7 (\omega^2)^7 = -128\omega^{14} = -128\omega^2$

39. **(A) → (q, r), B → (p), C → (p, s, t), D → (q, r, s, t)**

(A) → (q, r)

$\left|z - i|z|\right| = \left|z + i|z|\right|$

$\Rightarrow z$ is equidistant from two points $(0, |z|)$ and $(0, -|z|)$, which lie on imaginary axis.

$\therefore$ z must lie on real axis $\Rightarrow \text{Im}(z) = 0$. Also $|I_m(z)| \le 1$

(B) → p

Sum of distances of z from two points (–4, 0) and (4, 0) is 10 which is greater than 8.

$\therefore z$ traces an ellipse with $2a = 10$ and $2ae = 8$

$\Rightarrow e = \frac{4}{5}$

(C) → (p, s, t)

Let $\omega = 2(\cos\theta + i\sin\theta)$, then

$$z = \omega - \frac{1}{\omega} = 2(\cos\theta + i\sin\theta) - \frac{1}{2}(\cos\theta - i\sin\theta)$$

$\Rightarrow x + iy = \frac{3}{2}\cos\theta + i\frac{5}{2}\sin\theta$

Here, $|z| = \sqrt{\frac{9+25}{4}} = \sqrt{\frac{34}{4}} \le 3$ and $|R_e(z)| \le 2$

Also $x = \frac{3}{2}\cos\theta, y = \frac{5}{2}\sin\theta \Rightarrow \frac{4x^2}{9} + \frac{4y^2}{25} = 1$

Which is an ellipse with $e = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

(D) → (q, r, s, t)

Let $\omega = \cos\theta + i\sin\theta$ then $z = 2\cos\theta \Rightarrow \text{Im} z = 0$

Also $|z| \le 3$ and $|\text{Im}(z)| \le 1, |\text{R}_e(z)| \le 2$

40. **(A) → (q), (B) → (p)**

Given : $z \ne 0$ Let $z = a + ib$

Re (z) = 0 $\Rightarrow z = ib \Rightarrow z^2 = -b^2$

$\therefore Im\ (z)^2 = 0$

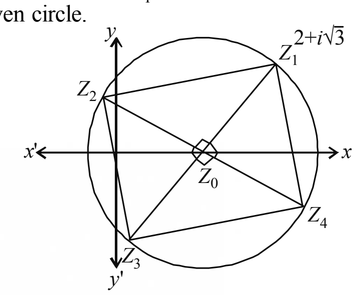
$\therefore$ (A) corresponds to (q)

Arg $z = \frac{\pi}{4} \Rightarrow a = b \Rightarrow z = a + ia$

$\Rightarrow z^2 = a^2 - a^2 + 2ia^2 \Rightarrow z^2 = 2ia^2 \Rightarrow \text{Re}\ (z)^2 = 0$

$\therefore$ (B) corresponds to (p).

41. The given circle is $|z-1| = \sqrt{2}$, where z_0=1 is the centre and $\sqrt{2}$ is radius of circle. z_1 is one of the vertex of square inscribed in the given circle.



Clearly z_2 can be obtained by rotating z_1 by an angle 90° in anticlockwise direction, about centre z_0

Thus, $z_2 - z_0 = (z_1 - z_0)\, e^{i\pi/2}$

or $z_2 - 1 = (2 + i\sqrt{3} - 1)i \Rightarrow z_2 = i - \sqrt{3} + 1$

$z_2 = (1 - \sqrt{3}) + i$

Again rotating z_2 by 90° about z_0, we get

$z_3 - z_0 = (z_2 - z_0)\ i$

$\Rightarrow z_3 - 1 = [(1 - \sqrt{3}) + i - 1]\ i = -\sqrt{3}i - 1$

$\Rightarrow z_3 = -i\sqrt{3}$

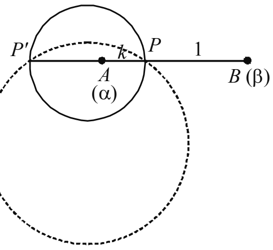
And similarly $1 = (-i\sqrt{3} - 1)\ i = \sqrt{3} - i$

$\Rightarrow z_4 = (\sqrt{3} + 1) - i$

Hence, remaining vertices are

$(1 - \sqrt{3}) + i, -i\sqrt{3}, (\sqrt{3} + 1) - i$

42. Given : $\left|\frac{z-\alpha}{z-\beta}\right| = k \Rightarrow |z - \alpha| = k\,|z - \beta|$



Let pt. A represents complex number α and B that of β, and P represents z. then $|z - \alpha| = k\,|z - \beta|$

$\Rightarrow$ z is the complex number whose distance from A is k times its distance from B.

i.e. $PA = k\ PB$

$\Rightarrow$ P divides AB in the ratio $k:1$ internally or externally (at P').

Then $P = \left(\dfrac{k\beta+\alpha}{k+1}\right)$ and $P' = \left(\dfrac{k\beta-\alpha}{k-1}\right)$

Now through PP' a number of circles can pass, but with given data we can find radius and centre of that circle for which PP' is diameter.

$\therefore$ Centre = mid. point of PP' $= \left(\dfrac{\dfrac{k\beta+\alpha}{k+1}+\dfrac{k\beta-\alpha}{k-1}}{2}\right)$

$= \dfrac{k^2\beta+k\alpha-k\beta-\alpha+k^2\beta-k\alpha+k\beta-\alpha}{2(k^2-1)}$

$= \dfrac{k^2\beta-\alpha}{k^2-1} = \dfrac{\alpha-k^2\beta}{1-k^2}$. Also radius $= \dfrac{1}{2}|PP'|$

$= \dfrac{1}{2}\left|\dfrac{k\beta+\alpha}{k+1}-\dfrac{k\beta-\alpha}{k-1}\right|$

$= \dfrac{1}{2}\left|\dfrac{k^2\beta+k\alpha-k\beta-\alpha-k^2\beta+k\alpha-k\beta+\alpha}{k^2-1}\right| = \dfrac{k|\alpha-\beta|}{|1-k^2|}$

43. Let us consider, $\sum_{r=1}^{n} a_r z^r = 1$ where $|a_r| < 2$

$\Rightarrow$ $a_1 z + a_2 z^2 + a_3 z^3 + ... + a_n z^n = 1$

$\Rightarrow$ $|a_1 z + a_2 z^2 + a_3 z^3 + ... + a_n z^n| = 1$ ....(i)

But we know that $|z_1+z_2| \le |z_1|+|z_2|$

$\therefore$ Using its generalised form, we get

$|a_1 z + a_2 z^2 + a_3 z^3 + ... + a_n z^n|$
$\le |a_1 z| + |a_2 z^2| + ... + |a_n z^n|$

$\Rightarrow$ $1 \le |a_1||z| + |a_2||z^2| + |a_3||z^3| + ... + |a_n||z^n|$ [using eq^n (i)]

But given that $|a_r| < 2 \;\forall\; r = 1,...,n$

$\therefore$ $1 < 2\,[\,|z| + |z|^2 + |z|^3 + ... + |z|^n\,]$ $[\because |z^n| = |z|^n]$

$\Rightarrow$ $1 < 2\left[\dfrac{|z|(1-|z|^n)}{1-|z|}\right] \Rightarrow 2\left[\dfrac{|z|-|z|^{n+1}}{1-|z|}\right] > 1$

$\Rightarrow$ $2\,[\,|z| - |z|^{n+1}] > 1-|z|$ $(\because 1-|z| > 0$ as $|z| < 1/3)$

$\Rightarrow$ $[|z|-|z|^{n+1}] > \dfrac{1}{2}-\dfrac{1}{2}|z|$

$\Rightarrow$ $\dfrac{3}{2}|z| > \dfrac{1}{2}+|z|^{n+1}$

$\Rightarrow$ $|z| > \dfrac{1}{3}+\dfrac{2}{3}|z|^{n+1} \Rightarrow |z| > \dfrac{1}{3}$

which is a contradiction as given that $|z| < \dfrac{1}{3}$

$\therefore$ There exist no such complex number.

44. The given equation can be written as

$(z^p-1)(z^q-1) = 0$

$\therefore$ $z = (1)^{1/p}$ or $(1)^{1/q}$ ....(i)

where p and q are distinct prime numbers.

Hence both the equations will have distinct roots and as $z \ne 1$, both will not be simultaneously zero for any value of z given by equations in (i)

Also $1+\alpha+\alpha^2+...+\alpha^{p-1} = \dfrac{1-\alpha^p}{1-\alpha} = 0 \;(\alpha \ne 1)$

or $1+\alpha+\alpha^2+...+\alpha^p = \dfrac{1-\alpha^q}{1-\alpha} = 0 \;(\alpha \ne 1)$

Because of (i) either $\alpha^p = 1$ and if $\alpha^q = 1$ but not both simultaneously as p and q are distinct primes.

45. $|z|^2\omega - |\omega|^2 z = z-\omega$ ..... (i)

$z\bar{z}\omega - \omega\bar{\omega}z = z-\omega$

$\Rightarrow$ $z\omega(\bar{z}-\bar{\omega}) = z-\omega.$

Taking modulus, $|z\cdot\omega||\bar{z}-\bar{\omega}| = z-\omega$

$|z\omega||z-\omega| = z-\omega.$

$\Rightarrow$ $|z-\omega|(|z\omega|-1) = 0$

If $|z-\omega| = 0$ then $z-\omega = 0$ $\therefore$ $z = \omega.$

If $|z\omega|-1 = 0$ then $z\omega = 1$ $\therefore$ $|z| = \dfrac{1}{|\omega|} = r$ (say)

Let $z = re^{i\theta}, \omega = \dfrac{1}{r}e^{i\phi}$

From (i) $r^2\left(\dfrac{1}{r}e^{i\phi}\right) - \dfrac{1}{r^2}(re^{i\theta}) = re^{i\theta} - \dfrac{1}{r}e^{i\phi}$

$\therefore$ $\left(r+\dfrac{1}{r}\right)e^{i\phi} = \left(r+\dfrac{1}{r}\right)e^{i\theta}$

$e^{i\phi} = e^{i\theta} \Rightarrow \theta = \phi$

$\therefore$ $z\bar{\omega} = (re^{i\theta})\left(\dfrac{1}{r}e^{-i\theta}\right) = 1$ $\therefore$ $z = \omega$ or $z\bar{\omega} = 1$

46. $z^2 + pz + q = 0$

$z_1 + z_2 = -p,\; z_1 z_2 = q$

By rotation through α in anticlockwise direction,

$z_2 = z_1 e^{i\alpha}$

$\dfrac{z_2}{z_1} = \dfrac{e^{i\alpha}}{1} = \dfrac{\cos\alpha + i\sin\alpha}{1}$

Add 1 in both sides to get $z_1 + z_2 = -p$

$\therefore$ $\dfrac{z_1+z_2}{z_1} = \dfrac{1+\cos\alpha+i\sin\alpha}{1} = 2\cos\dfrac{\alpha}{2}\left[\cos\dfrac{\alpha}{2}+i\sin\dfrac{\alpha}{2}\right]$

$\Rightarrow$ $\dfrac{(z_2+z_1)}{z_1} = 2\cos\dfrac{\alpha}{2}e^{i\alpha/2}$

On squaring, $(z_2+z_1)^2 = 4\cos^2(\alpha/2)z_1^2.e^{i\alpha}$

$= 4\cos^2\dfrac{\alpha}{2}z_1^2.\dfrac{z_2}{z_1} = 4\cos^2\dfrac{\alpha}{2}z_1 z_2$

$\Rightarrow$ $p^2 = 4q\cos^2\dfrac{\alpha}{2}$

47. Let $z = x+iy$ then $\bar{z} = iz^2$

$\Rightarrow$ $x - iy = i(x^2 - y^2 + 2ixy)$

$\Rightarrow$ $x - iy = i(x^2 - y^2) - 2xy$

$\Rightarrow$ $x(1+2y) = 0$; $x^2 - y^2 + y = 0$

$\Rightarrow$ $x = 0$ or $y = -\dfrac{1}{2}$ $\Rightarrow$ $x = 0,\; y = 0,\; 1$

or $y = -\dfrac{1}{2},\; x = \pm\dfrac{\sqrt{3}}{2}$

For non zero complex number z,

$$x=0, y=1;\ x=\frac{\sqrt{3}}{2},\ y=-\frac{1}{2};\ x=\frac{-\sqrt{3}}{2},\ y=-\frac{1}{2}$$

$$\therefore \quad z=i,\ \frac{\sqrt{3}}{2}-\frac{i}{2},\ -\frac{\sqrt{3}}{2}-\frac{i}{2}$$

48. Let $z = r_1(\cos\theta_1 + i\sin\theta_1)$ and $w = r_2(\cos\theta_2 + i\sin\theta_2)$
We have, $|z| = r_1$, $|w| = r_2$, $\arg(z) = \theta_1$ and $\arg(w) = \theta_2$
Given, $|z| \le 1$, $|w| < 1$

$\Rightarrow \quad r_1 \le 1$ and $r_2 \le 1$

Now, $z - w = (r_1\cos\theta_1 - r_2\cos\theta_2) + i(r_1\sin\theta_1 - r_2\sin\theta_2)$

$\Rightarrow \quad |z-w|^2 = (r_1\cos\theta_1 - r_2\cos\theta_2)^2 + (r_1\sin\theta_1 - r_2\sin\theta_2)^2$

$= r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2 - 2r_1r_2\cos\theta_1\cos\theta_2$

$\quad + r_1^2\sin^2\theta_1 + r_2^2\sin^2\theta_2 - 2r_1r_2\sin\theta_1\sin\theta_2$

$= r_1^2(\cos^2\theta_1 + \sin^2\theta_1) + r_2^2(\cos^2\theta_2 + \sin^2\theta_2)$

$\quad - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)$

$= r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)$

$= (r_1 - r_2)^2 + 2r_1r_2[1 - \cos(\theta_1 - \theta_2)]$

$= (r_1 - r_2)^2 + 4r_1r_2\sin^2\left(\frac{\theta_1 - \theta_2}{2}\right)$

$\le |r_1 - r_2|^2 + 4\left|\sin\left(\frac{\theta_1-\theta_2}{2}\right)\right|^2 \quad [\because r_1, r_2 \le 1]$

and $|\sin\theta| \le |\theta|,\ \forall\ \theta \in R$

Therefore, $|z-w|^2 \le |r_1 - r_2|^2 + 4\left|\frac{\theta_1-\theta_2}{2}\right|^2$

$\le |r_1 - r_2|^2 + |\theta_1 - \theta_2|^2$

$\Rightarrow \quad |z-w|^2 \le (|z| - |w|)^2 + (\arg z - \arg w)^2$

49. Dividing through out by i and knowing that $\frac{1}{i} = -i$, we get
$z^3 - iz^2 + iz + 1 = 0$
$\Rightarrow \quad z^2(z-i) + i(z-i) = 0 \quad$ as $\quad 1 = -i^2$
$\Rightarrow \quad (z-i)(z^2+i) = 0 \therefore z = i \quad$ or $\quad z^2 = -i$
$\therefore \quad |z| = |i| = 1$ or $|z^2| = |z|^2 = |-i| = 1 \Rightarrow |z| = 1$
Hence, in either case $|z| = 1$

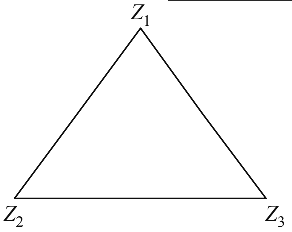
50. $1, a_1, a_2, ..., a_{n-1}$ are the n roots of unity. Therefore they are roots of eq. $x^n - 1 = 0$
Therefore by factor theorem,
$x^n - 1 = (x-1)(x-a_1)(x-a_2)....(x-a_{n-1}) \quad$ ....(i)

$$\Rightarrow \frac{x^n-1}{x-1} = (x-a_1)(x-a_2)....(x-a_{n-1}) \quad ....(ii)$$

On differentiating both sides of eq. (i), we get
$nx^{n-1} = (x-a_1)(x-a_2)...(x-a_{n-1}) + (x-1)(x-a_2)....(x-a_{n-1}) + ... + (x-1)(x-a_1)...(x-a_{n-2})$
For $x = 1$, we get $n = (1-a_1)(1-a_2)....(1-a_{n-1})$
[Since the terms except first, contain $(x-1)$ and hence become zero for $x = 1$]

51. We know that if z_1, z_2, z_3 are vertices of an equilateral triangle, then

$$\frac{z_1 - z_2}{z_3 - z_2} = \frac{z_3 - z_1}{z_2 - z_1}$$

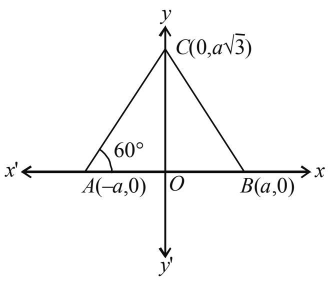


Here $z_3 = 0$,

$$\therefore \frac{z_1 - z_2}{-z_2} = \frac{-z_1}{z_2 - z_1}$$

$$\Rightarrow -z_1^2 - z_2^2 + 2z_1z_2 = z_1z_2,\ \therefore\ z_1^2 + z_2^2 - z_1z_2 = 0.$$

52.



Let us consider the equilateral triangle with each side of length $2a$ and having two of its vertices $A(-a,0)$ and $B(a, 0)$ on x-axis, then third vertex C will clearly lie on y-axis such that $OC = 2a\sin 60° = a\sqrt{3}$, $\therefore C = (0, a\sqrt{3})$.

Now if A, B and C are represented by complex number z_1, z_2, z_3 then $z_1 = -a$; $z_2 = a$; $z_3 = a\sqrt{3}\,i$

Since in an equilateral triangle, centriod and circumcentre coincide,

$$\therefore \text{ Circumcentre, } z_0 = \frac{z_1 + z_2 + z_3}{3}$$

$$\Rightarrow z_0 = \frac{-a + a + a\sqrt{3}\,i}{3} = \frac{ia}{\sqrt{3}}$$

Now, $z_1^2 + z_2^2 + z_3^2 = a^2 + a^2 - 3a^2 = -a^2$

and $3z_0^2 = (ia)^2 = -a^2$

$\therefore$ Clearly $3z_0^2 = z_1^2 + z_2^2 + z_3^2$

53. Since, β and γ are the complex cube roots of unity therefore, we can suppose $\beta = \omega$ and $\gamma = \omega^2$ so that $\omega + \omega^2 + 1 = 0$ and $\omega^3 = 1$.

Then $xyz = (a+b)(a\omega^2 + b\omega)(a\omega + b\omega^2)$

$= (a+b)(a^2\omega^3 + ab\omega^4 + ab\omega^2 + b^2\omega^3)$

$= (a+b)(a^2 + ab\omega + ab\omega^2 + b^2)$ (using $\omega^3 = 1$)

$= (a+b)(a^2 + ab(\omega + \omega^2) + b^2)$

$= (a+b)(a^2 - ab + b^2) = a^3 + b^3$

Topic-3: Solutions of Quadratic Equations, Sum and Product of Roots, Nature of Roots, Relation Between Roots and Co-efficients, Formation of an Equation with Given Roots

1. **(d)** Consider the quadratic polynomials in the form of equation
$x^2 + 20x - 2020 = 0 \quad$ ...(i)
$x^2 - 20x + 2020 = 0 \quad$ ...(ii)
Since, a and b are roots of the equation(i), then
$a + b = -20$, $ab = -2020$

$\because$ c and d are the roots of the equation (ii), then
$c + d = 20$, $cd = 2020$
Now,
$ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$
$= a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$
$= a^2(c+d) + b^2(c+d) - c^2(a+b) - d^2(a+b)$
$= (c+d)(a^2+b^2) - (a+b)(c^2+d^2)$
$= (c+d)((a+b)^2 - 2ab) - (a+b)((c+d)^2 - 2cd)$
$= 20[(20)^2 + 4040] + 20[(20)^2 - 4040]$
$= 20 \times 800 = 16000$

2. **(c)** $x^2 - 2x\sec\theta + 1 = 0 \Rightarrow x = \sec\theta \pm \tan\theta$
and $x^2 + 2x\tan\theta - 1 = 0 \Rightarrow x = -\tan\theta \pm \sec\theta$

$\because -\frac{\pi}{6} < \theta < -\frac{\pi}{12}$

$\Rightarrow \sec\frac{\pi}{6} > \sec\theta > \sec\frac{\pi}{12}$

and $-\tan\frac{\pi}{6} < \tan\theta < -\frac{\tan\pi}{12}$

Also $\tan\frac{\pi}{12} < -\tan\theta < \tan\frac{\pi}{6}$

Since, α_1, β_1 are roots of $x^2 - 2x\sec\theta + 1 = 0$
and $\alpha_1 > \beta_1$
$\therefore \alpha_1 = \sec\theta - \tan\theta$ and $\beta_1 = \sec\theta + \tan\theta$
Since, α_2, β_2 are roots of $x^2 + 2x\tan\theta - 1 = 0$
and $\alpha_2 > \beta_2$
$\therefore \alpha_2 = -\tan\theta + \sec\theta$, $\beta_2 = -\tan\theta - \sec\theta$
$\therefore \alpha_1 + \beta_2 = \sec\theta - \tan\theta - \tan\theta - \sec\theta = -2\tan\theta$

3. **(d)** Quadratic equation with real coefficients and purely imaginary roots can be considered as
$p(x) = x^2 + a = 0$ where $a > 0$ and $a \in R$
The $p[p(x)] = 0 \Rightarrow (x^2 + a)^2 + a = 0$

$\Rightarrow x^4 + 2ax^2 + (a^2 + a) = 0$

$$\Rightarrow x^2 = \frac{-2a \pm \sqrt{4a^2 - 4a^2 - 4a}}{2}$$

$\Rightarrow x^2 = -a \pm \sqrt{a}\, i$

$\Rightarrow x = \sqrt{-a \pm \sqrt{a}\, i} = \alpha \pm i\beta$, where $\alpha, \beta \neq 0$

$\therefore p[p(x)] = 0$, has complex roots which are neither purely real nor purely imaginary.

4. **(c)** Consider $-3(x - [x])^2 + 2[x - [x]] + a^2 = 0$
$\Rightarrow 3\{x\}^2 - 2\{x\} - a^2 = 0$ $\quad (\because x - [x] = \{x\})$

$\Rightarrow 3\left(\{x\}^2 - \frac{2}{3}\{x\}\right) = a^2, a \neq 0$

$\Rightarrow a^2 = 3\left(\{x\} - \frac{1}{3}\right)^2 - \frac{1}{3}$ $\quad (\because 0 \le \{x\} < 1)$

$\frac{-1}{3} \le \{x\} - \frac{1}{3} < \frac{2}{3}$; $0 \le 3\left(\{x\}\frac{-1}{3}\right) < \frac{4}{3}$

$-\frac{1}{3} \le 3\left(\{x\}\frac{-1}{3}\right) - \frac{1}{3} < 1$

For non-integral solution $0 < a^2 < 1$
$\Rightarrow a \in (-1, 0) \cup (0, 1)$

5. **(c)** $\because \alpha, \beta$ are the roots of $x^2 - 6x - 2 = 0$
$\therefore \alpha^2 - 6\alpha - 2 = 0$
$\Rightarrow \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$
$\Rightarrow \alpha^{10} - 2\alpha^8 = 6\alpha^9$ ...(i)
Similarly $\beta^{10} - 2\beta^8 = 6\beta^9$ ...(ii)
On subtracting (ii) from (i),
$\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8) = 6(\alpha^9 - \beta^9)$

$\Rightarrow a_{10} - 2a_8 = 6a_9 \Rightarrow \frac{a_{10} - 2a_8}{2a_9} = 3$

6. **(c)** Given : $(2x)^{\ell n2} = (3y)^{\ell n3}$
$\Rightarrow \ell n2.\ \ell n2x = \ell n3.\ \ell n3y$
$\Rightarrow \ell n2.\ \ell n2x = \ell n3.\ (\ell n3 + \ell ny)$ ...(i)
Also given : $3^{\ell nx} = 2^{\ell ny}$

$\Rightarrow \ell nx.\ \ell n3 = \ell ny.\ \ell n2 \Rightarrow \ell ny = \frac{\ell nx.\ell n3}{\ell n2}$ ...(ii)

From equation (i) and (ii), we get

$\ell n2.\ \ell n2x = \ell n3\left[\ell n3 + \frac{\ell nx.\ell n3}{\ell n2}\right]$

$\Rightarrow (\ell n2)^2\ \ell n2x = (\ell n3)^2\ \ell n2 + (\ell n3)^2\ \ell nx$
$\Rightarrow (\ell n2)^2\ \ell n2x = (\ell n3)^2(\ell n2 + \ell nx)$
$\Rightarrow (\ell n2)^2\ \ell n2x - (\ell n3)^2\ \ell n2x = 0$
$\Rightarrow [(\ell n2)^2 - (\ell n3)^2]\ \ell n2x = 0 \Rightarrow \ell n2x = 0$

$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

7. **(b)** Given : $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$

$\Rightarrow (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = q$

$\Rightarrow -p^3 - 3\alpha\beta(-p) = q \Rightarrow \alpha\beta = \frac{p^3 + q}{3p}$

Now for required quadratic equation,

Sum of roots $= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{p^2 - 2\left(\frac{p^3+q}{3p}\right)}{\frac{p^3+q}{3p}} = \frac{p^3 - 2q}{p^3 + q}$$

and Product of roots $= \frac{\alpha}{\beta}.\frac{\beta}{\alpha} = 1$

$\therefore$ Required equation is $x^2 - \left(\frac{p^3 - 2q}{p^3 + q}\right)x + 1 = 0$

$\Rightarrow (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

8. **(d)** Since α and β are the roots of $x^2 - px + r = 0$
$\therefore \alpha + \beta = p$ ....(i)
and $\alpha\beta = r$ ....(ii)

Also $\frac{\alpha}{2}$ and 2β are the roots of $x^2 - qx + r = 0$

$\therefore \quad \frac{\alpha}{2} + 2\beta = q \Rightarrow \alpha + 4\beta = 2q$ ....(iii)

Solving (i) and (iii) for α and β ,we get

$\beta = \frac{1}{3}(2q - p)$ and $\alpha = \frac{2}{3}(2q - q)$

On substituting the values of α and β, in equation (ii),

we get $\frac{2}{9}(2p-q)(2q-p) = r.$

9. **(a)** $\because$ a, b, c are sides of a triangle and $a \neq b \neq c$

$\therefore \quad |a-b| < |c| \Rightarrow a^2 + b^2 - 2ab < c^2$ ....(i)

Similarly,

$b^2 + c^2 - 2bc < a^2$ ....(ii) ; $c^2 + a^2 - 2ca < b^2$ ....(iii)

On adding, (i), (ii) and (iii) we get

$a^2 + b^2 + c^2 < 2(ab + bc + ca)$

$\Rightarrow \quad \frac{a^2 + b^2 + c^2}{ab + bc + ca} < 2$ ....(iv)

$\because$ Roots of the given equation are real

$\therefore \quad (a+b+c)^2 - 3\lambda(ab+bc+ca) \geq 0$

$\Rightarrow \quad \frac{a^2 + b^2 + c^2}{ab+bc+ca} \geq 3\lambda - 2$ ....(v)

From (iv) and (v), $3\lambda - 2 < 2 \Rightarrow \lambda < \frac{4}{3}.$

10. **(a)** $x^2 + px + q = 0$

Let roots be α and α^2, then

$\alpha + \alpha^2 = -p, \alpha\alpha^2 = q \Rightarrow \alpha = q^{1/3}$

$\therefore \quad (q)^{1/3} + (q^{1/3})^2 = -p$

On taking cube on both sides, we get

$q + q^2 + 3q(q^{1/3} + q^{2/3}) = -p^3$

$\Rightarrow \quad q + q^2 - 3pq = -p^3 \Rightarrow p^3 + q^2 - q(3p-1) = 0$

11. **(c)** Let α, α^2 be the roots of $3x^2 + px + 3 = 0$

$\therefore \quad \alpha + \alpha^2 = -p/3$ and $\alpha^3 = 1$

$\Rightarrow \quad (\alpha - 1)(\alpha^2 + \alpha + 1) = 0 \quad \Rightarrow \alpha = 1$ or $\alpha^2 + \alpha = -1$

If α = 1, then $p = -6$, which is not possible as $p > 0$

If $\alpha^2 + \alpha = -1 \Rightarrow -p/3 = -1 \Rightarrow p = 3.$

12. **(d)** Given : $(x - a)(x - b) - 1 = 0, b > a.$

or $\quad x^2 - (a + b)x + (ab - 1) = 0$

Let $f(x) = x^2 - (a + b)x + (ab - 1)$

$D = (a + b)^2 - 4(ab - 1)$

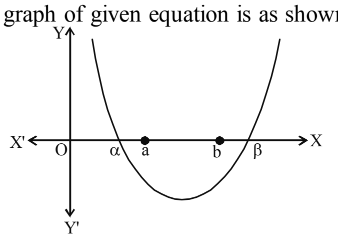
$= (a - b)^2 + 1 > 0$

Since coeff. of x^2 i.e. 1 > 0, $\therefore f(x)$ represents upward parabola, intersecting x-axis at two points corresponding to two real roots, D being +ve. Also $f(a) = f(b) = -1$

$\Rightarrow$ curve is below x-axis at a and b

$\therefore \quad a$ and b both lie between the roots.

Therefore, the graph of given equation is as shown.



It is clear from graph, that one root of the equation lies in $(-\infty, a)$ and other in (b, ∞).

13. **(b)** Given : $c < 0 < b$ and $\alpha + \beta = -b$ ....(i)

$\alpha\beta = c$ ....(ii)

From (ii), $c < 0 \Rightarrow \alpha\beta < 0 \Rightarrow$ Either α is -ve or β is - ve and second quantity is positive.

From (i), $b > 0 \Rightarrow -b < 0 \Rightarrow \alpha + \beta < 0$

$\Rightarrow$ the sum is negative

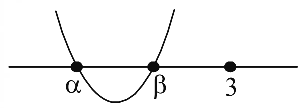
$\Rightarrow$ (Modules of nengative quantity) > (Modulus of positive quantity)

But given α < β. Therefore, it is clear that α is negative and β is positive and modulus of α is greater than modulus of β

$\Rightarrow \quad \alpha < 0 < \beta < |\alpha|$

14. **(a)** If both roots of a quadratic equation $ax^2 + bx + c = 0$ are less than k, then

$af(k) > 0, D \geq 0, \alpha + \beta < 2k.$



$f(x) = x^2 - 2ax + a^2 + a - 3 = 0,$

$f(3) > 0, \alpha + \beta < 6, D \geq 0.$

$\Rightarrow \quad a^2 - 5a + 6 > 0, a < 3, -4a + 12 \geq 0$

$\Rightarrow \quad a < 2$ or $a > 3, a < 3, a < 3 \Rightarrow a < 2.$

15. **(c)** For the equation $px^2 + qx + 1 = 0$ to have real roots

$D \geq 0 \Rightarrow q^2 \geq 4p$

If $p = 1$ then $q^2 \geq 4 \Rightarrow q = 2, 3, 4$

If $p = 2$ then $q^2 \geq 8 \Rightarrow q = 3, 4$

If $p = 3$ then $q^2 \geq 12 \Rightarrow q = 4$

If $p = 4$ then $q^2 \geq 16 \Rightarrow q = 4$

$\therefore$ Number of required equations = 7

16. **(c)** α, β are roots of the equation $(x - a)(x - b) = c, c \neq 0$

$\therefore \quad (x - a)(x - b) - c = (x - \alpha)(x - \beta)$

$\Rightarrow \quad (x - \alpha)(x - \beta) + c = (x - a)(x - b)$

$\Rightarrow$ Roots of $(x - \alpha)(x - \beta) + c = 0$ are a and b.

17. **(d)** If $f(\alpha)$ and $f(\beta)$ are of opposite signs then there must lie a value γ between α and β such that $f(\gamma) = 0$.

a, b, c are real numbers and $a \neq 0$.

Since α is a root of $a^2x^2 + bx + c = 0$

$\therefore \quad a^2\alpha^2 + b\alpha + c = 0$ ....(i)

Also β is a root of $a^2x^2 - bx - c = 0$

$\therefore \quad a^2\beta^2 - b\beta - c = 0$ .... (ii)

Now, let $f(x) = a^2x^2 + 2bx + 2c$

Then $f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c = a^2\alpha^2 + 2(b\alpha + c)$

$= a^2\alpha^2 + 2(-a^2\alpha^2)$ [using eq. (i)]

$= -a^2\alpha^2.$

and $f(\beta) = a^2\beta^2 + 2b\beta + 2c = a^2\beta^2 + 2(b\beta + c)$

$= a^2\beta^2 + 2(a^2\beta^2)$ [using eq. (ii)]

$= 3a^2\beta^2 > 0.$

Since $f(\alpha)$ and $f(\beta)$ are of opposite signs and γ is a root of equation $f(x) = 0$

$\therefore$ γ must lie between α and β

$\Rightarrow \alpha < \gamma < \beta$.

18. **(a)** Given : $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$

Clearly $x \neq 1$ for the given equation to be defined. If $x \neq 1$, we can cancel the common term $\frac{-2}{x-1}$ on both sides to get $x = 1$, but it is not possible. So given equation has no roots.

19. **(c)** Since, $(x^2 + px + 1)$ is a factor of $ax^3 + bx + c$, hence we can assume that zeros of $x^2 + px + 1$ are α, β and that of $ax^3 + bx + c$ be α, β, γ

$\therefore$ $\alpha + \beta = -p$ .... (i)

$\alpha\beta = 1$ .... (ii)

and $\alpha + \beta + \gamma = 0$ .... (iii)

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{b}{a}$ .... (iv)

$\alpha\beta\gamma = \frac{-c}{a}$ .... (v)

On solving (ii) and (v), we get $\gamma = -c/a$.

On solving (i) and (iii), we get $\gamma = p$

$\therefore$ $p = \gamma = -c/a$

Using equations (i) , (ii) and (iv), we get

$1 + \gamma(-p) = \frac{b}{a}$

$\Rightarrow 1 + \left(-\frac{c}{a}\right)\left(\frac{c}{a}\right) = \frac{b}{a}$ $(\because \gamma = p = -c/a)$

$\Rightarrow 1 - \frac{c^2}{a^2} = \frac{b}{a} \Rightarrow a^2 - c^2 = ab$

20. **(b)** Given :

$(x-b)(x-c) + (x-a)(x-c) + (x-a)(x-b) = 0$

$\Rightarrow 3x^2 - 2(a+b+c)x + (ab+bc+ca) = 0$

$D = 4(a+b+c)^2 - 12(ab+bc+ca)$

$= 4[a^2 + b^2 + c^2 - ab - bc - ca]$

$= 2[(a-b)^2 + (b-c)^2 + (c-a)^2] \geq 0 \quad \forall a, b, c$

$\therefore$ Roots of given equation are always real.

21. **(c)** ℓ, m, n are real, $\ell \neq m$

Given : $(\ell - m)x^2 - 5(\ell + m)x - 2(\ell - m) = 0$

$D = 25(\ell + m)^2 + 8(\ell - m)^2 > 0, \ell, m \in R$

$\therefore$ Roots are real and unequal.

22. **(1)** Taking log with base 5 on the both sides

$(16(\log_5 x)^3 - 68(\log_5 x))(\log_5 x) = -16$

Let $(\log_5 x) = t$

$16t^4 - 68t^2 + 16 = 0$

$\Rightarrow 4t^4 - 16t^2 - t^2 + 4 = 0$

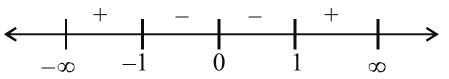
$\Rightarrow (4t^2 - 1)(t^2 - 4) = 0$

or $t = \pm\frac{1}{2}, \pm 2$

So $\log_5 x = \pm\frac{1}{2}$, or $\pm 2 \Rightarrow x = 5^{\frac{1}{2}}, 5^{\frac{-1}{2}}, 5^2, 5^{-2}$

Product $= 5^{\frac{1}{2}}.5^{\frac{-1}{2}}.5^2.5^{-2} = 1$

23. **(4)**



$3x^2 + x - 1 = 4|x^2 - 1|$

Case 1: If $x \in [-1, 1]$,

$3x^2 + x - 1 = -4x^2 + 4$

$\Rightarrow 7x^2 + x - 5 = 0 \because D = 141 > 0 \therefore$ Equation has two roots

Case 2: If $x \in (-\infty, -1] \cup [1, \infty)$

$3x^2 + x - 1 = 4x^2 - 4$

$\Rightarrow x^2 - x - 3 = 0 \because D = 13 > 0$

$\therefore$ Equation has two roots

So, total 4 roots.

24. **(2)** The given equation is

$$x^2 - 8kx + 16(k^2 - k + 1) = 0$$

$\because$ Both the roots are real and distinct.

$\therefore$ $D > 0 \Rightarrow (8k)^2 - 4 \times 16(k^2 - k + 1) > 0$

$\Rightarrow$ $k > 1$ ...(i)

$\because$ Both the roots are greater than or equal to 4

$\therefore$ $\alpha + \beta > 8$ and $f(4) \geq 0$

$\Rightarrow$ $k > 1$ ...(ii)

and $16 - 32k + 16(k^2 - k + 1) \geq 0$

$\Rightarrow k^2 - 3k + 2 \geq 0 \Rightarrow (k-1)(k-2) \geq 0$

$\Rightarrow k \in (-\infty, 1] \cup [2, \infty)$ ...(iii)

Combining (i), (ii) and (iii), we get $k \geq 2$

$\therefore$ Smallest value of $k = 2$.

25. The given equation : $x^2 - 3kx + 2e^{2\ln k} - 1 = 0$

$\Rightarrow x^2 - 3kx + (2k^2 - 1) = 0$

Now, product of roots $= 2k^2 - 1$

$\therefore 2k^2 - 1 = 7 \Rightarrow k^2 = 4 \Rightarrow k = 2, -2$

For real roots, $D \geq 0$

$\Rightarrow 9k^2 - 4(2k^2 - 1) \geq 0 \Rightarrow k^2 + 4 \geq 0$,

which is true for all k. Thus $k = 2, -2$

But for $k = -2$, $\ln k$ is not defined

We reject $k = -2$, we get $k = 2$.

26. Since, p and q are real and one root is $2 + i\sqrt{3}$, therefore other root should be $2 - i\sqrt{3}$

$\therefore$ $p = -$(sum of roots) $= -4$, $q =$ product of roots $= 4 + 3 = 7$

27. **(True)** $f(x) = (x-a)(x-c) + 2(x-b)(x-d)$.

$f(a) = +ve; f(b) = -ve; f(c) = -ve; f(d) = +ve$

$\therefore$ There exists two real and distinct roots one in the interval (a, b) and other in (c, d). True

28. **(False)** $2x^2 + 3x + 1 = 0 \Rightarrow x = -1, -1/2$; both are rationals

$\therefore$ Statement is false.

29. **(b,c,d)** Given that $ax^2 + 2bxy + cy^2 > 0$

and $y, x \in \mathbb{R} - \{0\}$

$\Rightarrow c\left(\frac{y}{x}\right)^2 + 2b\left(\frac{y}{x}\right) + a > 0 \Rightarrow c > 0, D < 0$

$4b^2 - 4ac < 0 \Rightarrow b^2 < ac$

(a) $\left(2, \frac{7}{2}, 6\right)$

$\left(\frac{7}{2}\right)^2 > 2 \times 6$

$\therefore$ Option (a) is incorrect

(b) If $\left(3, b, \frac{1}{12}\right) \in S$

$\Rightarrow b^2 < 3.\frac{1}{12} \Rightarrow b^2 < \frac{1}{4} \Rightarrow 4b^2 < 1$

$\Rightarrow |2b| < 1$ obtion (b) is correct

(c) $ax + by = 1$

$bx + cy = -1$

$D = \begin{vmatrix} a & b \\ b & c \end{vmatrix} = ac - b^2 \neq 0$

$\therefore$ unique solution option (c) is correct.

(d) $(a+1)x + by = 0$

$bx + (c+1)y = 0$

$D = \begin{vmatrix} (a+1) & b \\ b & (c+1) \end{vmatrix}$

$= (a+1)(c+1) - b^2 = ac - b^2 + a + c + 1$

Since $ac - b^2 > 0$

$\Rightarrow b^2 < ac \Rightarrow ac$ is + ve

$\Rightarrow a$ and c are positive then $(ac - b^2) + a + c + 1 > 0$

$\therefore$ unique solution

$\therefore$ ooption (d) is correct

30. **(a, b, c)**

$3^x = 4^{x-1} \Rightarrow x\log 3 = 2(x-1)\log 2$

$\Rightarrow x = \frac{2\log 2}{2\log 2 - \log 3}$

$\Rightarrow x = \frac{2\log_3 2}{2\log_3 2 - 1} = \frac{2}{2 - \log_2 3}$

Also $x = \frac{1}{1 - \frac{1}{2}\log_2 3} = \frac{1}{1 - \log_4 3}$

31. **(b)** $\alpha^2 = \alpha + 1$

$\beta^2 = \beta + 1$

$a_n = p\alpha^n + q\beta^n$

$= p(\alpha^{n-1} + \alpha^{n-2}) + q(\beta^{n-1} + \beta^{n-2})$

$= a_{n-1} + a_{n-2}$

$\therefore \quad a_{12} = a_{11} + a_{10}$

32. **(d)** $\alpha = \frac{1+\sqrt{5}}{2}, \beta = \frac{1-\sqrt{5}}{2}$

$a_4 = a_3 + a_2$

$= 2a_2 + a_1$

$= 3a_1 + 2a_0$

$28 = p(3\alpha + 2) + q(3\beta + 2)$

$28 = (p+q)\left(\frac{3}{2} + 2\right) + (p-q)\left(\frac{3\sqrt{5}}{2}\right)$

$\therefore \quad p - q = 0$ and $(p+q) \times \frac{7}{2} = 28$

$\Rightarrow \quad p + q = 8 \Rightarrow \quad p = q = 4$

$\therefore \quad p + 2q = 12$

33. **(b)** As $a, b, c, p, q, \in R$ and the two given equations have exactly one common root

$\Rightarrow$ Either both equations have real roots

or both eqations have imaginary roots

$\Rightarrow$ Either $D_1 \geq 0$ and $D_2 \geq 0$ or $D_1 \leq 0$ and $D_2 \leq 0$

$\Rightarrow p^2 - q \geq 0$ and $b^2 - ac \geq 0$

or $p^2 - q \leq 0$ and $b^2 - ac \leq 0$

$\Rightarrow (p^2 - q)(b^2 - ac) \geq 0$

$\therefore$ Statement 1 is true.

Also we have $\alpha\beta = q$ and $\frac{\alpha}{\beta} = \frac{c}{a}$

$\therefore \frac{\alpha\beta}{\alpha/\beta} = \frac{q}{c} \times a \Rightarrow \beta^2 = \frac{qa}{c}$

As $\beta \neq 1$ or $-1 \Rightarrow \beta^2 \neq 1 \Rightarrow \frac{qa}{c} \neq 1$ or $c \neq qa$

Again, as exactly one root α is common, and $\beta \neq 1$

$\therefore \alpha + \beta \neq \alpha + \frac{1}{\beta} \Rightarrow \frac{-2b}{a} \neq -2p \quad \Rightarrow b \neq ap$

$\therefore$ Statement 2 is correct.

But Statement 2 is not a correct explanation of Statement 1.

34. Roots of $x^2 - 10cx - 11d = 0$ are a and b

$\Rightarrow \quad a + b = 10c$ and $ab = -11d$

Similarly c and d are the roots of $x^2 - 10ax - 11b = 0$

$\Rightarrow c + d = 10a$ and $cd = -11b$

$\Rightarrow \quad a + b + c + d = 10(a + c)$ and $abcd = 121\,bd$

$\Rightarrow \quad b + d = 9(a + c)$ and $ac = 121$

Also we have $a^2 - 10ac - 11d = 0$ and $c^2 - 10ac - 11b = 0$

$\Rightarrow \quad a^2 + c^2 - 20ac - 11(b + d) = 0$

$\Rightarrow \quad (a + c)^2 - 22 \times 121 - 99(a + c) = 0$

$\Rightarrow \quad a + c = 121$ or -22

For $a + c = -22$, we get $a = c$

$\therefore$ Rejecting this value we have $a + c = 121$

$\therefore \quad a + b + c + d = 10(a + c) = 1210$

35. Given :

$x^2 + (a - b)x + (1 - a - b) = 0, a, b \in R$

For this equation to have unequal real roots for all value of b

if $D > 0$

$\Rightarrow \quad (a - b)^2 - 4(1 - a - b) > 0$

$\Rightarrow \quad a^2 + b^2 - 2ab - 4 + 4a + 4b > 0$

$\Rightarrow \quad b^2 + b(4 - 2a) + a^2 + 4a - 4 > 0$

Which is a quadratic expression in b, and it will be true for all $b \in R$, if discriminant of above equation is less than zero.

i.e., $(4 - 2a)^2 - 4(a^2 + 4a - 4) < 0$

$\Rightarrow \quad (2 - a)^2 - (a^2 + 4a - 4) < 0$

$\Rightarrow \quad 4 - 4a + a^2 - a^2 - 4a + a < 0$

$\Rightarrow \quad -8a + 8 < 0, \therefore a > 1$

36. We know $(\alpha-\beta)^2=[(\alpha+\delta)-(\beta+\delta)]^2$

$\Rightarrow \quad (\alpha+\beta)^2-4\alpha\beta=(\alpha+\delta+\beta+\delta)^2-4(\alpha+\delta)(\beta+\delta)$

$\Rightarrow \quad \frac{b^2}{a^2}-\frac{4c}{a}=\frac{B^2}{A^2}-\frac{4C}{A} \Rightarrow \frac{4ac-b^2}{a^2}=\frac{4AC-B^2}{A^2}$

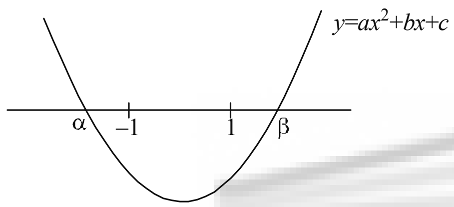
$\left[\text{Here } \alpha+\beta=-\frac{b}{a}, \alpha\beta=\frac{c}{a},\right.$

$\left.(\alpha+\delta)(\beta+\delta)=-\frac{B}{A} \text{ and } (\alpha+\delta)(\beta+\delta)=\frac{C}{A}\right]$

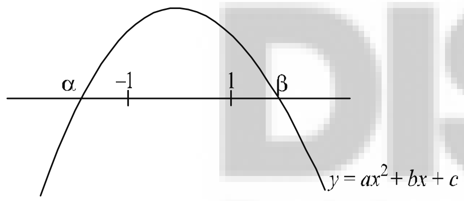
37. Given : For $a, b, c \in R$, $ax^2+bx+c=0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$. There may be two cases depending upon the value of a, as shown below.

In each of cases (i) and (ii) $af(-1) < 0$ and $af(1) < 0$

(i) If $a > 0$



(ii) If $a < 0$



$\Rightarrow \quad a(a-b+c) < 0$ and $a(a+b+c) < 0$

Dividing by a^2 (> 0), we get

$1-\frac{b}{a}+\frac{c}{a}<0$ ....(i)

and $1+\frac{b}{a}+\frac{c}{a}<0$ ....(ii)

On combining (i) and (ii) we get

$1+\left|\frac{b}{a}\right|+\frac{c}{a}<0$ or $1+\frac{c}{a}+\left|\frac{b}{a}\right|<0$

38. Given :

$|x^2+4x+3|+2x+5=0$

Here two cases are possible.

Case I : $x^2+4x+3\ge 0 \Rightarrow (x+1)(x+3)\ge 0$

$\Rightarrow \quad x\in(-\infty,-3]\cup[-1,\infty)$ ....(i)

Then the given equation becomes,

$\Rightarrow \quad x^2+6x+8=0$

$\Rightarrow \quad (x+4)(x+2)=0$, $\therefore \; x=-4,-2$

But $x = -2$ does not satisfy (i) and hence rejected.

$\therefore$ Solution is $x = -4$

Case II : $x^2 + 4x + 3 < 0$

$\Rightarrow \quad (x+1)(x+3) < 0$

$\Rightarrow \quad x \in (-3, -1)$ ....(ii)

Then the given equation becomes,

$-(x^2+4x+3)+2x+5=0$

$\Rightarrow -x^2-2x+2=0 \Rightarrow x^2+2x-2=0$

$\Rightarrow \quad x=\frac{-2\pm\sqrt{4+8}}{2} \quad \therefore \; x=-1+\sqrt{3}, -1-\sqrt{3}$

But $x=-1+\sqrt{3}$ does not satisfy (ii) and hence rejected.

$\therefore$ Solution is $x=-1-\sqrt{3}$

On combining solution in the two cases, we get the solutions: $x = -4, -1-\sqrt{3}$.

39. Given :

$x^2-2a|x-a|-3a^2=0$ .... (i)

Here two cases are possible.

Case I : $x-a>0$, then $|x-a|=x-a$

Hence, Eq. (i) becomes

$x^2-2a(x-a)-3a^2=0$

$\Rightarrow \quad x^2-2ax-a^2=0 \Rightarrow x=\frac{2a\pm\sqrt{4a^2+4a^2}}{2}$

$\therefore \quad x=a\pm a\sqrt{2}$

Case II : $x-a<0$, then $|x-a|=-(x-a)$

Hence, Eq. (i) becomes

$x^2+2a(x-a)-3a^2=0$

$\Rightarrow \quad x^2+2ax-5a^2=0 \Rightarrow \quad x=\frac{-2a\pm\sqrt{4a^2+20a^2}}{2}$

$\therefore \quad x=\frac{-2a\pm 2a\sqrt{6}}{2} \Rightarrow \quad x=-a\pm a\sqrt{6}$

Hence, the solution set is $\{a\pm a\sqrt{2}, -a\pm a\sqrt{6}\}$

40. Given, $(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10$ ....(i)

Put $y=(5+2\sqrt{6})^{x^2-3} \Rightarrow (5-2\sqrt{6})^{x^2-3}=\frac{1}{y}$

From Eq. (i), $y+\frac{1}{y}=10$

$\Rightarrow \quad y^2-10y+1=0 \Rightarrow \quad y=5\pm 2\sqrt{6}$

$\Rightarrow \quad (5+2\sqrt{6})^{x^2-3}=5+2\sqrt{6}$

or $\quad (5+2\sqrt{6})^{x^2-3}=5-2\sqrt{6}$

$\Rightarrow \quad x^2-3=1$ or $x^2-3=-1$

$\Rightarrow x=\pm 2$ or $x=\pm\sqrt{2} \Rightarrow x=\pm 2, \pm\sqrt{2}$

41. Given $a > 0$, so we have to consider two cases : $a \ne 1$ and $a = 1$.

Also it is clear that $x > 0$

and $x \ne 1$, $ax \ne 1$, $a^2x \ne 1$.

Case I : If $a > 0, \neq 1$
then given equation can be simplified as

$$\frac{2}{\log_a x}+\frac{1}{1+\log_a x}+\frac{3}{2+\log_a x}=0$$

Putting $\log_a x = y$, we get
$2(1+y)(2+y)+y(2+y)+3y(1+y)=0$

$\Rightarrow \quad 6y^2+11y+4=0 \Rightarrow y=-4/3$ and $-1/2$

$\Rightarrow \quad \log_a x=-4/3$ and $\log_a x=-1/2$

$\Rightarrow \quad x=a^{-4/3}$ and $x=a^{-1/2}$

Case II : If $a = 1$, then equation becomes
$2\log_x 1+\log_x 1+3\log_x 1=6\log_x 1=0$
which is true $\forall\, x>0, \neq 1$
Hence solution is $x>0, \neq 1$; if $a=1$,
and $x=a^{-1/2}, a^{-4/3}$, if $a>0, \neq 1$

42. $\sqrt{x+1}=1+\sqrt{x-1}$
Squaring both sides, we get
$x+1=1+x-1+2\sqrt{x-1} \Rightarrow 1=2\sqrt{x-1}$.
$\Rightarrow \quad 1=4(x-1) \Rightarrow \quad x=5/4$

Topic-4: Condition for Common Roots, Maximum and Minimum value of Quadratic Equation, Quadratic Expression in two Variables, Solution of Quadratic Inequalities

1. **(b)** Let α be the common root of given equations, then
$\alpha^2+b\alpha-1=0$ ...(i)
and $\alpha^2+\alpha+b=0$ ...(ii)
On subtracting (ii) from (i), we get
$(b-1)\alpha-(b+1)=0$

$\Rightarrow \quad \alpha=\dfrac{b+1}{b-1}$

Substituting this value of α in equation (i), we get

$$\left(\frac{b+1}{b-1}\right)^2+b\left(\frac{b+1}{b-1}\right)-1=0 \Rightarrow b^3+3b=0$$

$\Rightarrow \quad b=0, i\sqrt{3}, -i\sqrt{3}$

2. **(b)** $f(x)=ax^2+bx+c$ has same sign as that of a if $D<0$.
Since $x^2+2ax+10-3a>0\ \forall x$

$\therefore \quad D<0 \Rightarrow 4a^2-4(10-3a)<0 \Rightarrow a^2+3a-10<0$

$\Rightarrow \quad (a+5)(a-2)<0 \Rightarrow a\in(-5,2)$

3. **(20)** Given that $f(1)=-9 \Rightarrow 1+a+b+c=-9$ ...(i)
and $4x^3+3ax^2+2bx=0$
$\Rightarrow x=0$, or $4x^2+3ax+2b=0$ ...(ii)
$\Rightarrow \sqrt{3}i$ and $-\sqrt{3}i$ are roots of (ii)

$$\Rightarrow \sqrt{3}i-\sqrt{3}i=\frac{-3a}{4}, \sqrt{3}i(-\sqrt{3}i)=\frac{2b}{4}$$

$\Rightarrow a=0, b=6, c=-16$ from (i)
$\Rightarrow f(x)=0 \Rightarrow x^4+6x^2-16=0$

$$\Rightarrow x^2=\frac{-6\pm\sqrt{36+64}}{2}=-3\pm 5=2,-8$$

$x=-\sqrt{2},+\sqrt{2},-2\sqrt{2}i,2\sqrt{2}i$

$\Rightarrow |\alpha_1|^2+|\alpha_2|^2+|\alpha_3|^2+|\alpha_4|^2=20$

4. $\because$ x = 1, reduces both the equations to $1+a+b=0$
$\therefore$ 1 is the common root. for $a+b=-1$
$\therefore$ Numerical value of $a+b=1$

5. **(True)** $P(x).Q(x)=(ax^2+bx+c)(-ax^2+bx+c)$
$\Rightarrow \quad D_1=b^2-4ac$ and $D_2=b^2+4ac$
clearly, $D_1+D_2=2b^2\geq 0$
$\therefore$ Atleast one of D_1 and D_2 is positive. Hence, atleast two real roots. True

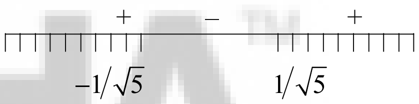
6. **(a, d)** Given, x_1 and x_2 are roots of $\alpha x^2-x+\alpha=0$.

$\therefore \quad x_1+x_2=\dfrac{1}{\alpha}$ and $x_1x_2=1$

Also, $|x_1-x_2|<1$
$\Rightarrow \quad |x_1-x_2|^2<1 \Rightarrow (x_1-x_2)^2<1$
or $\quad (x_1+x_2)^2-4x_1x_2<1$

$\Rightarrow \quad \dfrac{1}{\alpha^2}-4<1$ or $\dfrac{1}{\alpha^2}<5$

or $\quad 5\alpha^2-1>0$ or $(\sqrt{5}\alpha-1)(\sqrt{5}\alpha+1)>0$



$\therefore \quad \alpha\in\left(-\infty,-\dfrac{1}{\sqrt{5}}\right)\cup\left(\dfrac{1}{\sqrt{5}},\infty\right)$ ...(i)

Also, $D>0$

$\Rightarrow \quad 1-4\alpha^2>0$ or $\alpha\in\left(-\dfrac{1}{2},\dfrac{1}{2}\right)$ ...(ii)

From Eqs. (i) and (ii), we get

$$\alpha\in\left(-\frac{1}{2},\frac{-1}{\sqrt{5}}\right)\cup\left(\frac{1}{\sqrt{5}},\frac{1}{2}\right)$$

7. **(b)** Given : a, b, c, d, p are real and distinct numbers such that

$$(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)\leq 0$$

$$\Rightarrow (a^2p^2+b^2p^2+c^2p^2)-(2abp+2bcp+2cdp)+(b^2+c^2+d^2)\leq 0$$

$$\Rightarrow (a^2p^2-2abp+b^2)+(b^2p^2-2bcp+c^2)+(c^2p^2-2cdp+d^2)\leq 0$$

$\Rightarrow \quad (ap-b)^2+(bp-c)^2+(cp-d)^2\leq 0$
Since, LHS is the sum of perfect squares, therefore LHS can never be –ve.

$\therefore (ap-b)^2+(bp-c)^2+(cp-d)^2=0$
Which is possible only when each term is zero individually

i.e. $ap - b = 0;\ bp - c = 0;\ cp - d = 0$

$\Rightarrow \dfrac{b}{a} = p; \dfrac{c}{b} = p; \dfrac{d}{c} = p \Rightarrow \dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c} = p$

$\therefore$ a, b, c, d are in G.P.

8. **(c, d)** Let $y = \dfrac{(x-a)(x-b)}{(x-c)}$

$\Rightarrow (x-c)y = x^2 - (a+b)x + ab$

$\Rightarrow x^2 - (a+b+y)x + ab + cy = 0$

Here, $D = (a+b+y)^2 - 4(ab+cy)$

$= y^2 + 2y(a+b-2c) + (a-b)^2$

Since x is real and y assumes all real values.

$\therefore$ $D \geq 0$ for all real values of y

$\Rightarrow y^2 + 2y(a+b-2c) + (a-b)^2 \geq 0$

As we know that the sign of a quadratic polynomial is same as that of coefficient of y^2 if its descriminant < 0

$\therefore$ $4(a+b-2c)^2 - 4(a-b)^2 < 0$

$\Rightarrow 4(a+b-2c+a-b)(a+b-2c-a+b) < 0$

$\Rightarrow 16(a-c)(b-c) < 0$

$\Rightarrow 16(c-a)(c-b) < 0$ ....(i)

If $a < b$ then from inequation (i), we get $c \in (a, b)$

$\Rightarrow a < c < b$

If $a > b$ then from inequation (i), we get $c \in (b, a)$

$\Rightarrow a > c > b$

Thus, both (c) and (d) are the correct answer.

9. Given : $ax^2 + bx + c = 0$ .... (i)

and $a^3x^2 + abcx + c^2 = 0$ .... (ii)

$\therefore$ $\alpha + \beta = -\dfrac{b}{a},\ \alpha.\beta = \dfrac{c}{a}$

Divide the equation (ii) by a^3, we get

$x^2 + \dfrac{b}{a}.\dfrac{c}{a}.x + \left(\dfrac{c}{a}\right)^3 = 0$

$\Rightarrow x^2 - (\alpha + \beta).(\alpha\beta)x + (\alpha\beta)^3 = 0$

$\Rightarrow x^2 - \alpha^2\beta x - \alpha\beta^2 x + (\alpha\beta)^3 = 0$

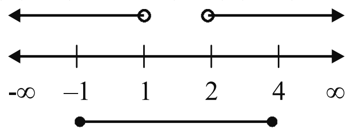
$\Rightarrow x(x - \alpha^2\beta) - \alpha\beta^2(x - \alpha^2\beta) = 0$

$\Rightarrow (x - \alpha^2\beta)(x - \alpha\beta^2) = 0$

$\Rightarrow x = \alpha^2\beta,\ \alpha\beta^2$

10. Given : $x^2 - 3x + 2 > 0,\quad x^2 - 3x - 4 \leq 0$

$\Rightarrow (x-1)(x-2) > 0$ and $(x-4)(x+1) \leq 0$



$\Rightarrow x \in (-\infty, 1) \cup (2, \infty)$ and $x \in [-1, 4]$

$\therefore$ Common solution = $[-1, 1) \cup (2, 4]$

11. $\because$ α, β are the roots of $x^2 + px + q = 0$

$\therefore$ $\alpha + \beta = -p,\ \alpha\beta = q$

$\because$ γ, δ are the roots of $x^2 + rx + s = 0$

$\therefore$ $\gamma + \delta = -r, \gamma\delta = s$

Now, $(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)$

$= [\alpha^2 - (\gamma + \delta)\alpha + \gamma\delta][\beta^2 - (\gamma + \delta)\beta + \gamma\delta]$

$= [\alpha^2 + r\alpha + s][\beta^2 + r\beta + s]$

$\because$ α, β are roots of $x^2 + px + q = 0$

$\therefore$ $\alpha^2 + p\alpha + q = 0$ and $\beta^2 + p\beta + q = 0$

$\Rightarrow \alpha^2 = -p\alpha - q$ and $\beta^2 = -p\beta - q$

$\therefore (\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta) = [\alpha^2 + r\alpha + s][\beta^2 + r\beta + s]$

$= [(r-p)\alpha + (s-q)][(r-p)\beta + (s-q)]$

$= (r-p)^2\alpha\beta + (r-p)(s-q)(\alpha + \beta) + (s-q)^2$

$= q(r-p)^2 - p(r-p)(s-q) + (s-q)^2$

Now if the equations $x^2 + px + q = 0$ and $x^2 + rx + s = 0$ have a common root say α, then $\alpha^2 + p\alpha + q = 0$ and $\alpha^2 + r\alpha + s = 0$

$\Rightarrow \dfrac{\alpha^2}{ps - qr} = \dfrac{\alpha}{q - s} = \dfrac{1}{r - p}$

$\Rightarrow \alpha^2 = \dfrac{ps - qr}{r - p}$ and $\alpha = \dfrac{q - s}{r - p}$

$\Rightarrow (q-s)^2 = (r-p)(ps - qr)$, which is the required condition.