HandBook Mathematics

with 4 Color FLASH CARDS for JEE, Class 11 & 12 & CUET

Rittik Baheti

Based on New Syllabus

Highly useful for Classes 11th & 12th

Concepts, Tips, Tricks for Faster Revision & Learning

2nd Edition

Features:-

- NCERT Simplified- All NCERT key Notes, Terms, Definitions and Formulae in a unique One Liner Format
- Short crisp notes with last minute tips and tricks to increase score
- **4 Color FLASH CARDS** for memorisation and simplifying complex concepts
- As per NCERT Topical flow

Also useful for State Boards

DISHA™
Publication Inc

DISHA Publication Inc.

A-23 FIEE Complex, Okhla Phase II
New Delhi-110020
Tel: 49842349/ 49842350

Author

Rittik Baheti

DISHA™ Publication Inc

Write To Us At

feedback_disha@aiets.co.in

www.dishapublication.com

Free Sample Contents

This sample book is prepared from the book **"HandBook of Mathematics with 4 Color Flash Cards for JEE, CBSE Class 11 & 12 & CUET 2nd Edition | Complete NCERT in One Liner Format | Engineering, CBSE XI & XII".**

ISBN - 978-9362255341

MRP- 325/-

In case you like this content, you can buy the **Physical Book** or **E-book** using the ISBN provided above.

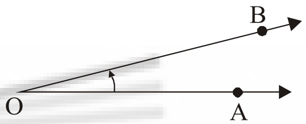
The book & e-book are available on all leading online stores.

Contents

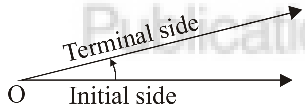
3 Trigonometric Functions

Angles

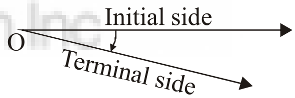
- An angle is a measure of rotation of a given ray about its initial point. Here OA is initial position and OB is the final position of the given ray.



- The original ray is called the initial side and the final position of the ray after rotation is called the terminal side of the angle.
- The point of rotation is called the vertex.
- If the direction of rotation is anticlockwise, the angle is said to be positive and if the direction of rotation is clockwise, then the angle is negative.



(i) Positive angle



(ii) Negative angle

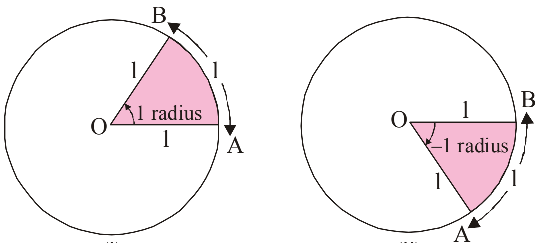
Angle Measurement

A. Degree measure

- If a rotation from the initial side to terminal side is $(1/360)^{th}$ of a revolution, the angle is said to have a measure of one degree, written as $1°$.
- A degree is divided into 60 minutes, written as $1'$, i.e. $1° = 60'$.
- A minute is divided into 60 seconds, written as $1'$, i.e. $1' = 60''$.

B. Radian measure

- Angle subtended at the centre by an arc of length 1 unit in a unit circle (circle of radius 1 unit) is said to have a measure of 1 radian.
- The figures show the angles whose measures are 1 radian, –1 radian



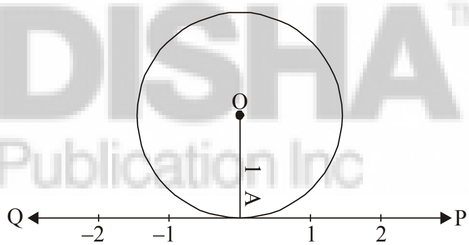
(i) (ii)

- One complete revolution of the initial side subtends an angle of 2π radian.
- A circle of radius r, an arc of length r subtends an angle whose measure is ℓ radian, an arc of length l will subtend an angle θ radian whose measure is

  $\theta = \ell / r$ radian, or $\boxed{\text{Angle} = \dfrac{\text{Arc}}{\text{Radius}}}$

C. Relation between radian and real numbers

- From the figure. If we rope the line AP along the circle in the anticlock wise direction, we find
- Every real number will correspond to a radian measure and conversely. Thus, radian measures and real numbers can be considered as one and the same.



D. Relation between degree and radian

- A circle subtends at the centre an angle whose radian measure is 2π and its degree measure is 360°.

  Hence, 2π radian = 360° or π radian = 180°
- Using approximate value of π as 22/7, we have

  1 radian = $180/\pi$ ° = 57° 16′ approximately. Also 1° = $\pi/180$ radian = 0.01746 radian approximately.

  The relation between degree measures and radian measure of some common angles are-

Degree	30°	45°	60°	90°	180°	270°	360°
Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{6}$	2π

E. Notational Convention

- In this convention, we generally omit the word ‘radian’, when we expressed an angle in radians.

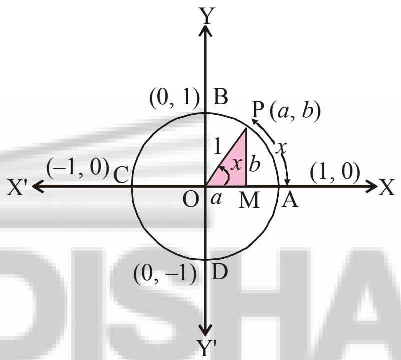
e.g. $45° = \frac{\pi}{4}$, $180° = \pi$

Note: Radian measure = $\frac{\pi}{180}$ × Degree measure

Degree measure = $\frac{180}{\pi}$ × Radian measure

Trigonometric Functions

- The extension of the definition of trigonometric ratios to any angle in terms of radian measure is studied as trigonometric functions.



- Let unit circle, Here we define cos x = a and sin x = b. Since ΔOMP is a right triangle, we have $OM^2 + MP^2 = OP^2$ or $a^2 + b^2 = 1$
- Thus, for every point on the unit circle, we have $a^2 + b^2 = 1$ or $\cos^2 x + \sin^2 x = 1$
  Since, one complet revolution subtends an angle of 2π radian at the centre of the circle, ΔAOB = π/2,
- All angles which are integral multiples of π/2 are called quadrantal angles.
- For quadrantal angles we have

$\cos 0° = 1$, $\sin 0° = 0$

$\cos \frac{\pi}{2} = 0$, $\sin \frac{\pi}{2} = 1$

$\cos \frac{3\pi}{2} = 0$ $\sin \frac{3\pi}{2} = -1$

- Thus,
  sin x = 0 implies x = nπ, where n is any integer
  cos x = 0 implies x = (2n + 1) π/2, where n is any integer.

Let us see the table:

	$0°$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	–1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined	0	not defined	0

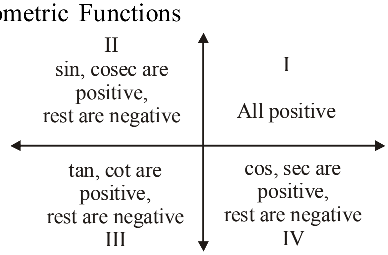
- **Basic Formulae**
  - $\sin\theta = \frac{1}{\operatorname{cosec}\theta},\ \theta \neq n\pi$
  - $\cos\theta = \frac{1}{\sec\theta},\ \theta \neq (2n+1)\frac{\pi}{2}.$
  - $\tan\theta = \frac{1}{\cot\theta},\ \theta \neq \frac{n\pi}{2}$
  - $\tan\theta = \frac{\sin\theta}{\cos\theta}, \theta \neq (2n+1)\frac{\pi}{2}.$
  - $\cot\theta = \frac{\cos\theta}{\sin\theta},\ \theta \neq n\pi.$

A. Sign of trigonometric functions

- In different quadrants from the values of sin x, cos x we can find the signs of other trigonometric functions as:

	I	*II*	*III*	*IV*
sin x	+	+	–	–
cos x	+	–	–	+
tan x	+	–	+	–
cosce x	+	+	–	–
sec x	+	–	–	+
cot x	+	–	+	–

- Sign of Trigonometric Functions



Note: Remember **ASTC** (After School to College)

B. Domain and range of trigonometric functions

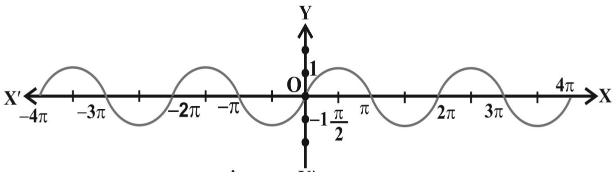
- By the definition of sine and cosine functions, we observe that they are defined for all real numbers.
- Thus, domain of $y = \sin x$ and $y = \cos x$ is the set of all real numbers and range is the interval $[-1, 1]$, i.e., $-1 \le y \le 1$.
- Since $\text{cosec } x = 1/\sin x$, the domain of $y = \text{cosec } x$ is the set $\{x : x \in R$ and $x \ne n\pi, n \in Z\}$ and range is the set $\{y : y \in R, y \ge 1$ or $y \le -1\}$.
- The domain of $y = \sec x$ is the set $\{x : x \in R$ and $x \ne (2n + 1)\pi/2, n \in Z\}$ and range is the set $\{y : y \in R, y \le -1$ or $y \ge 1\}$.
- The domain of $y = \tan x$ is the set $\{x : x \in R$ and $x \ne (2n + 1)\pi/2, n \in Z\}$ and range is the set of all real numbers.
- The domain of $y = \cot x$ is the set $\{x : x \in R$ and $x \ne n\pi, n \in Z\}$ and the range is the set of all real numbers.
- Let us discuss the behaviour of other trigonometric functions through the following table as below

Domain and Range of Trigonometric Functions

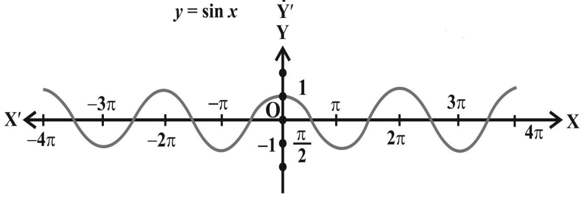
Function	Domain	Range
sin x	R	$[-1, 1]$
cos x	R	$[-1, 1]$
tan x	$R - \{(2n+1)\pi/2, n \in Z\}$	R
cosec x	$R - \{n\pi, n \in Z\}$	$(-\infty, -1] \cup [1, \infty)$
sec x	$R - \{(2n+1)\pi/2, n \in Z\}$	$(-\infty, -1] \cup [1, \infty)$
cot x	$R - \{n\pi, n \in Z\}$	R

	I quadrant	II quadrant	III quadrant	IV quadrant
sin	increases from 0 to 1	decreases from 1 to 0	decreases from 0 to –1	increases from –1 to 0
cos	decreases from 1 to 0	decreases from 0 to –1	increases from –1 to 0	increases from 0 to 1
tan	increases from 0 to ∞	increases from $-\infty$ to 0	increases from 0 to ∞	increases from $-\infty$ to 0
cot	decreases from ∞ to 0	decreases from 0 to $-\infty$	decreases from ∞ to 0	decreases from 0 to $-\infty$
sec	increases from 1 to ∞	increases from $-\infty$ to –1	decreases from –1 to $-\infty$	decreases from ∞ to 1
cosec	decreases from ∞ to 1	increases from 1 to ∞	increases from $-\infty$ to –1	decreases from –1 to $-\infty$

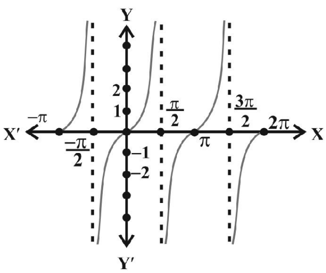
- Let us see graphs of the trigonometric functions-



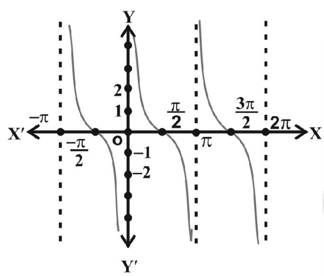
$y = \sin x$



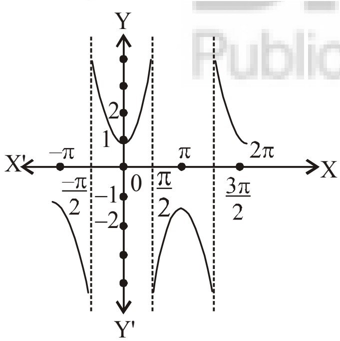
$y = \cos x$



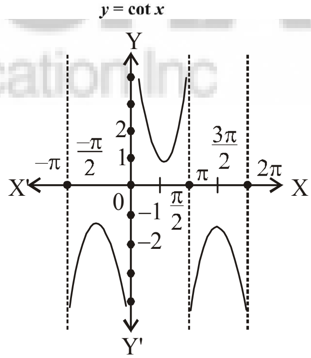
$y = \tan x$



$y = \cot x$



$y = \sec x$



$y = \operatorname{cosec} x$

Trigonometric Functions of Sum and Difference of Two Angles

- $\sin(-x) = -\sin x$
- $\cos(-x) = \cos x$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$

- $\cos(x - y) = \cos x \cos y + \sin x \sin y$
- $\cos(\pi/2 - x) = \sin x$
- $\sin(\pi/2 - x) = \cos x$
- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\sin(x - y) = \sin x \cos y - \cos x \sin y$
- $\cos(\pi/2 + x) = -\sin x$
- $\sin(\pi/2 + x) = \cos x$
- $\cos(\pi - x) = -\cos x$
- $\sin(\pi - x) = \sin x$
- $\cos(\pi + x) = -\cos x$
- $\sin(\pi + x) = -\sin x$
- $\cos(2\pi - x) = \cos x$
- $\sin(2\pi - x) = -\sin x$
- $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$
- $\cot(x + y) = \dfrac{\cot x \cot y - 1}{\cot y + \cot x}$
- $\cot(x - y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$
- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
  $= \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$
- $\sin 2x = 2 \sin x \cos x = \dfrac{2\tan x}{1 + \tan^2 x}$ $x \neq n\pi + \dfrac{\pi}{2}$, where n is an integer
- $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$ if $2x \neq n\pi + \dfrac{\pi}{2}$, where n is an integer
- $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$ if $3x \neq n\pi + \dfrac{\pi}{2}$, where n is an integer
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\cos 3x = 4\cos^3 x - 3\cos x$
- $\cos x + \cos y = 2\cos\dfrac{x + y}{2}\cos\dfrac{x - y}{2}$

- $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$
- $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$
- $2\cos x \cos y = \cos(x+y) + \cos(x-y)$
- $-2\sin x \sin y = \cos(x+y) - \cos(x-y)$
- $2\sin x \cos y = \sin(x+y) + \sin(x-y)$
- $2\cos x \sin y = \sin(x+y) - \sin(x-y)$.

Trigonometric Equations

- Equations involving trigonometric functions of a variable are called trigonometric equations.
- The solutions of a trigonometric equation for which $0 \le x < 2\pi$ are called principal solutions.
- The expression involving integer 'n' which gives all solutions of a trigonometric equation is called the general solution.

Principal Solutions

- The solutions of a trigonometric equation in the variable 'x' for which $0 \le x < 2\pi$ are called principal solutions. E.g., the principal solution of equation $\sin x = \frac{1}{2}$ are $x = \frac{\pi}{6}, \frac{5\pi}{6}$.

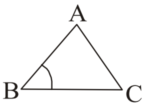
General Solutions

- $\sin x = 0 \Rightarrow x = n\pi, n \in Z$
- $\cos x = 0 \Rightarrow x = (2n+1)\pi/2, n \in Z$
- $\tan x = 0 \Rightarrow x = n\pi, n \in Z$
- $\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y, n \in Z$
- $\cos x = \cos y \Rightarrow x = 2n\pi \pm y, n \in Z$
- $\tan x = \tan y \Rightarrow x = n\pi + y, n \in Z$

Simple Applications of Sine and Cosine Formulae

- Sine Formulae: In any triangle, sides are proportional to the sines of the opposite angles, that is, in a triangle ABC.

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$



- Cosine Formulae: Let A, B and C be angles of a triangles and a, b and c be lengths of sides opposite to angles A, B and C respectively, then
  - $a^2 = b^2 + c^2 - 2bc \cos A$
  - $b^2 = c^2 + a^2 - 2ca \cos B$
  - $c^2 = a^2 + b^2 - 2ab \cos C$

Past Years ONE-LINERS JEE Main/Board

- $1 + \cos\theta = 2\cos^2\theta/2$
  $1 - \cos\theta = 2\sin^2\theta/2$
- $\sin 2\theta = 2\sin\theta.\cos\theta$
- $\tan(\alpha + 2\beta) = \dfrac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha \tan 2\beta}$
- Using A.M. and G. M. to find minimum or maximum value of Trigonomatric expressions where sin(x) > 0, cos (x) > 0. tan(x) > 0 etc. where A. M. ≥ G.M.
- To find number of solutions of trigonometric equations try to make dissimilar trigo function as similar, ie., Let
  $\Rightarrow \sin x = \cos 2x \Rightarrow \sin(x) - \cos(2x) = 0$
  $\Rightarrow \sin(x) - [1 - 2\sin^2(x)] = 0$
  $\Rightarrow 2\sin^2(x) + \sin(x) - 1 = 0$
- To find maximum value and minimum value of trigonometric expression (used concept)
  $-1 \le \sin x \le 1 \quad \Rightarrow \quad 0 \le \sin^2 x \le 1$
  $-1 \le \cos x \le 1 \quad \Rightarrow \quad 0 \le \cos^2 x \le 1$
- Used concept
  $$\cos(C) + \cos(D) = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

Tips/Tricks/Techniques ONE-LINERS
(Exam Special)

Periodic properties of trigonometric functions

(a) $\sin x$, $\cos x$, $\sec x$ and $\operatorname{cosec} x$ are periodic functions with fundamental period 2π.
(b) $\tan x$ and $\cot x$ are periodic functions with fundamental period π.
(c) $|\sin x|$, $|\cos x|$, $|\tan x|$, $|\cot x|$, $|\sec x|$, $|\operatorname{cosec} x|$ are periodic functions with fundamental period π.
(d) $\sin^n x$, $\cos^n x$, $\sec^n x$, $\operatorname{cosec}^n x$ are periodic functions with fundamental period 2π or π according as n is odd or even.
(e) $\tan^n x$ and $\cot^n x$ are periodic function with fundamental period π whether n is odd or even.

Conditional trigonometric identities

If A + B + C = 180° (or π), or A, B, C are angles of a triangle. Then,

(a) $\sin(A+B) = \sin(\pi - C) = \sin C$, etc .

(b) $\sin\left(\frac{A}{2}+\frac{B}{2}\right) = \sin\left(\frac{\pi}{2}-\frac{C}{2}\right) = \cos\frac{C}{2}$, etc

(c) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(d) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(e) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

(f) $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

(g) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

Properties of triangle

(a) Projection formula
  (i) $a = b\cos C + c\cos B$
  (ii) $b = c\cos A + a\cos C$
  (iii) $c = a\cos B + b\cos A$

(b) Tangent rule :

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\tan\left(\frac{B+C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$$

(c) Half angle formula :

  (i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

(ii) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$

(iii) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

(d) Area of a triangle :

$$\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \frac{1}{2}ab\sin C$$

Orthocentre of the triangle and pedal triangle

(a) The distances of the orthocentre of the triangle from the vertices are 2RcosA, 2RcosB, 2RcosC and its distances from the sides are 2RcosB cosC, 2RcosC cosA, 2RcosA cosB.

(b) Circumradius of the pedal triangle $= \frac{R}{2}$

(c) Area of the pedal triangle $= 2\Delta\cos A\cos B\cos C$.

(d) Circumcentre O, centroid G and orthocentre O' are collinear and G divides OO′ in the ratio 1 : 2.

(e) Distance between the circumcentre O and the incentre I is

$$OI = R\sqrt{1 - 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$$

Heights and distances

(a) The angle of elevation or depression is the angle between the line of observation and the horizontal line according as the object is at a higher or lower level than the observer.

(b) The angle of elevation or depression is always measured from horizontal line through the point of observation.