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Class XI

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4 Complex Numbers and Quadratic Equations

1. If α and β be the values of x in $m^2(x^2-x)+2mx+3=0$ and m_1 and m_2 be two values of m for which α and β are connected by the relation $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{4}{3}$. Then the value of $\frac{m_1^2}{m_2}+\frac{m_2^2}{m_1}$ is

(a) 6 (b) 68 (c) $\frac{3}{68}$ (d) $-\frac{68}{3}$

2. If a, b, c ∈ **R** and the equations $ax^2+bx+c=0$, $a\neq 0$, has real roots α and β satisfying $\alpha<-1$ and $\beta>1$, then $1+\frac{c}{a}+\left|\frac{b}{a}\right|$ is **Tricky**

(a) positive (b) negative
(c) zero (d) None

3. If the roots of $ax^2+bx+c=0$ are sin α and cos α for some α, then which one of the following is correct?

(a) $a^2+b^2=2ac$ (b) $b^2-c^2=2ab$
(c) $b^2-a^2=2ac$ (d) $b^2+c^2=2ab$

4. If $|z-2|=\min\{|z-1|,|z-5|\}$, where z is a complex number, then

(a) $\text{Re}(z)=\frac{3}{2}$ (b) $\text{Re}(z)=\frac{7}{2}$
(c) $\text{Re}(z)\in\left\{\frac{3}{2},\frac{7}{2}\right\}$ (d) None of these

5. Let x_1 and y_1 be real numbers. If z_1 and z_2 are complex numbers such that $|z_1|=|z_2|=4$, then $|x_1z_1-y_1z_2|^2+|y_1z_1+x_1z_2|^2=$

(a) $32(x_1^2+y_1^2)$
(b) $16(x_1^2+y_1^2)$
(c) $4(x_1^2+y_1^2)$
(d) $32(x_1^2+y_1^2)|z_1+z_2|^2$

6. If $z=\frac{\pi}{4}(1+i)^4\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i}+\frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right)$, then $\left(\frac{|z|}{\text{amp}(z)}\right)$ equals

(a) 1 (b) π (c) 3π (d) 4

7. If m_1, m_2, m_3 and m_4 respectively denote the moduli of the complex numbers $1+4i$, $3+i$, $1-i$ and $2-3i$, then the correct one, among the following is

(a) $m_1<m_2<m_3<m_4$
(b) $m_4<m_3<m_2<m_1$
(c) $m_3<m_2<m_4<m_1$
(d) $m_3<m_1<m_2<m_4$

8. If both the roots of $k(6x^2+3)+rx+2x^2-1=0$ and $6k(2x^2+1)+px+4x^2-2=0$ are common, then $2r-p$ is equal to

(a) –1 (b) 0 (c) 1 (d) 2

9. For the complex numbers z_1 and z_2 if $|1-\bar{z}_1z_2|^2-|z_1-z_2|^2=k(1-|z_1|^2)(1-|z_2|^2)$ then 'k' equals to

(a) 1 (b) –1 (c) 2 (d) –2

10. The principle value of the arg (z) and | z | of the complex number $z=1+\cos\left(\frac{11\pi}{9}\right)+i\sin\left(\frac{11\pi}{9}\right)$ are respectively.

(a) $\frac{11\pi}{8}, 2\cos\left(\frac{\pi}{18}\right)$ (b) $-\frac{7\pi}{18}, -2\cos\left(\frac{11\pi}{18}\right)$
(c) $\frac{2\pi}{9}, 2\cos\left(\frac{7\pi}{18}\right)$ (d) $-\frac{\pi}{9}, -2\cos\left(\frac{\pi}{18}\right)$

11. If $\lambda\neq\mu$ and $\lambda^2=5\lambda-3$, $\mu^2=5\mu-3$, then the equation whose roots are $\frac{\lambda}{\mu}$ and $\frac{\mu}{\lambda}$ is

(a) $x^2-5x+3=0$ (b) $3x^2+19x+3=0$
(c) $3x^2-19x+3=0$ (d) $x^2+5x-3=0$

12. If one root of the equation $(l-m)x^2+lx+1=0$ is double the other and l is real, then what is the greatest value of m?

(a) $-\frac{9}{8}$ (b) $\frac{9}{8}$ (c) $-\frac{8}{9}$ (d) $\frac{8}{9}$

13. What is $\frac{(1+i)^{4n+5}}{(1-i)^{4n+3}}$ equal to, where n is a natural number and $i=\sqrt{-1}$?
(a) 2 (b) $2i$ (c) -2 (d) i

14. If $(a+ib)(c+id)(e+if)(g+ih)=A+iB$, then $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2)=$
(a) A^2+B^2 (b) A^2-B^2
(c) A^2 (d) B^2

15. If centre of a regular hexagon is at origin and one of the vertex on argand diagram is $1+2i$, then its perimeter is **Critical Thinking**
(a) $2\sqrt{5}$ (b) $6\sqrt{2}$ (c) $4\sqrt{5}$ (d) $6\sqrt{5}$

16. If $|z_1|=|z_2|=..........|z_n|=1$, then the value of $|z_1+z_2+........z_n|-\left|\frac{1}{z_1}+\frac{1}{z_2}+........+\frac{1}{z_n}\right|$ is,
(a) 0 (b) 1 (c) -1 (d) None

17. The real roots of the equation $x^2+5|x|+4=0$ are
(a) $\{-1,-4\}$ (b) $\{1, 4\}$
(c) $\{-4, 4\}$ (d) None of these

18. The locus of a point in the Argand plane that moves satisfying the equation $|z-1+i|-|z-2-i|=3$:
(a) is a circle with radius 3 and centre at $z=\frac{3}{2}$
(b) is an ellipse with its foci at $1-i$ and $2+i$ and major axis = 3
(c) is a hyperbola with its foci at $1-i$ and $2+i$ and its transverse axis = 3
(d) None of the above

19. If both the roots of the equation $x^2-2kx+k^2-4=0$ lie between -3 and 5, then which one of the following is correct?
(a) $-2<k<2$ (b) $-5<k<3$
(c) $-3<k<5$ (d) $-1<k<3$

20. What is the value of $\left(-\sqrt{-1}\right)^{4n+3}+\left(i^{41}+i^{-257}\right)^9$, where $n\in N$?
(a) 0 (b) 1 (c) i (d) $-i$

21. The points z_1, z_2, z_3, z_4 in a complex plane are vertices of a parallelogram taken in order, then
(a) $z_1+z_4=z_2+z_3$ (b) $z_1+z_3=z_2+z_4$
(c) $z_1+z_2=z_3+z_4$ (d) None of these

22. Consider $f(x)=x^2-3x+a+\frac{1}{a}$, $a\in R-\{0\}$, such that $f(3)>0$ and $f(2)\le 0$. If α and β are the roots of equation $f(x)=0$ then the value of $\alpha^2+\beta^2$ is equal to
(a) greater than 11
(b) less than 5
(c) 5
(d) depends upon a and a cannot be determined.

23. If z in any complex number satisfying $|z-1|=1$, then which of the following is correct ?
(a) $\arg(z-1)=2\arg z$
(b) $2\arg(z)=\frac{2}{3}\arg(z^2-z)$
(c) $\arg(z-1)=\arg(z+1)$
(d) $\arg z=2\arg(z+1)$

24. If $y=2+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+.............\infty}}}$ then
(a) $y=6$ (b) $y=5$
(c) $y=\sqrt{6}$ (d) $y=\sqrt{5}$

25. If $0<a<b<c$ and the roots α, β of the equation $ax^2+bx+c=0$ are imaginary then incorrect statement is
(a) $|\alpha|=|\beta|$ (b) $|\alpha|>1$
(c) $|\beta|<1$ (d) None of these

26. What is $\left[\frac{\sin\frac{\pi}{6}+i\left(1-\cos\frac{\pi}{6}\right)}{\sin\frac{\pi}{6}-i\left(1-\cos\frac{\pi}{6}\right)}\right]^3$ **Tricky**
where $i=\sqrt{-1}$, equal to?
(a) 1 (b) -1 (c) i (d) $-i$

27. The value of the sum $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right)$; where $i=\sqrt{-1}$ is:

(a) i (b) –i (c) 0 (d) i – 1

28. If the roots of the equation $x^2 - ax + b = 0$ are real and differ by a quantity which is less than $c(c > 0)$, then b lies between

(a) $\frac{a^2-c^2}{4}$ and $\frac{a^2}{4}$ (b) $\frac{a^2+c^2}{4}$ and $\frac{a^2}{4}$

(c) $\frac{a^2-c^2}{2}$ and $\frac{a^2}{4}$ (d) None of these

29. For the equation $\left|x^2\right| + |x| - 6 = 0$, the roots are

(a) One and only one real number
(b) Real with sum one
(c) Real with sum zero
(d) Real with product zero

30. If z_1, z_2 and z_3 are complex numbers such that

$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1,$

then $|z_1 + z_2 + z_3|$ is

(a) equal to 1 (b) less than 1
(c) greater than 3 (d) equal to 3

31. If the roots of the equation $ax^2 - bx + c = 0$ are α, β then the roots of the equation $b^2cx^2 - ab^2x + a^3 = 0$ are **Tricky**

(a) $\frac{1}{\alpha^3+\alpha\beta}, \frac{1}{\beta^3+\alpha\beta}$

(b) $\frac{1}{\alpha^2+\alpha\beta}, \frac{1}{\beta^2+\alpha\beta}$

(c) $\frac{1}{\alpha^4+\alpha\beta}, \frac{1}{\beta^4+\alpha\beta}$

(d) None of these

32. What is the real part of $(\sin x + i \cos x)^3$ where $i=\sqrt{-1}$?

(a) $-\cos 3x$ (b) $-\sin 3x$
(c) $\sin 3x$ (d) $\cos 3x$

33. Suppose the quadratic equations $x^2 + px + q = 0$ and $x^2 + rx + s = 0$ are such that p, q, r, s are real and $pr = 2(q + s)$. Then **Critical Thinking**

(a) Both the equations always have real roots.
(b) At least one equation always has real roots
(c) Both the equation always have non real roots
(d) Atleast one equation always has real and equal roots.

34. If $z = 1 + i\tan\alpha\left(-\pi < \alpha < -\frac{\pi}{2}\right)$, then polar form of the complex number z is: **Toughnut**

(a) $\frac{1}{\cos\alpha}(\cos\alpha + i\sin\alpha)$

(b) $\frac{1}{-\cos\alpha}[\cos(\pi+\alpha) + i\sin(\pi+\alpha)$

(c) $\frac{1}{\cos\alpha}[\cos(2\pi+\alpha) + i\sin(2\pi+\alpha)]$

(d) None of these

35. If the roots of $ax^2 + bx + c = 0$ are the reciprocals of those of $\ell x^2 + mx + n = 0$ then $a : b : c =$

(a) $n : m : \ell$ (b) $\ell : m : n$
(c) $m : n : \ell$ (d) $n : \ell : m$

36. Let $\lambda \in \mathbf{R}$. If the origin and the non real roots of $2z^2 + 2z + \lambda = 0$ form the three vertices of an equilateral triangle in the argand plane. Then λ is

(a) 1 (b) $\frac{2}{3}$ (c) 2 (d) –1

37. If α, β be the roots of the equation $x^2 - px + q = 0$ and α_1, β_1 the roots of the equation $x^2 - qx + p = 0$, then the equation whose roots are

$\frac{1}{\alpha_1\beta}+\frac{1}{\alpha\beta_1}$ and $\frac{1}{\alpha\alpha_1}+\frac{1}{\beta\beta_1}$ is

(a) $pqx^2 - pqx + p^2 + q^2 + 4pq = 0$

(b) $p^2q^2x^2 - p^2q^2x + p^3 + q^3 - 4pq = 0$

(c) $p^3q^3x^2 - p^3q^3x + p^4 + q^4 - 4p^2q^2 = 0$

(d) $(p+q)x^2 - (p+q)x + p^2 + q^2 + pq = 0$

38. If m and n are the roots of the equation $(x + p)(x + q) - k = 0$, then the roots of the equation $(x - m)(x - n) + k = 0$ are

(a) P and q (b) $\frac{1}{p}$ and $\frac{1}{q}$

(c) $-p$ and $-q$ (d) $p + q$ and $p - q$

39. What is the argument of $(1 - \sin\theta) + i\cos\theta$?

Critical Thinking

(a) $\frac{\pi}{2} - \frac{\theta}{2}$ (b) $\frac{\pi}{2} + \frac{\theta}{2}$

(c) $\frac{\pi}{4} - \frac{\theta}{2}$ (d) $\frac{\pi}{4} + \frac{\theta}{2}$

40. If $\omega = \dfrac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on

(a) an ellipse (b) a circle

(c) a straight line (d) a parabola

41. z_1 and z_2 are the roots of $3z^2 + 3z + b = 0$. If O(0), $A(z_1)$, $B(z_2)$ form an equilateral triangle, then the value of b is

(a) -1 (b) 1

(c) 0 (d) does not exist

42. Let $A_0A_1A_2A_3A_4A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments A_0A_1, A_0A_2 and A_0A_4 is

(a) $\frac{3}{4}$ (b) $3\sqrt{3}$ (c) 3 (d) $\frac{3\sqrt{3}}{2}$

43. The values of k for which the equations $x^2 - kx - 21 = 0$ and $x^2 - 3kx + 35 = 0$ will have a common roots are:

(a) $k = \pm 4$ (b) $k = \pm 1$

(c) $k = \pm 3$ (d) $k = 0$

44. If α, β are real and α^2, β^2 are the roots of the equation $a^2x^2 - x + 1 - a^2 = 0 \left(\frac{1}{\sqrt{2}} < a < 1\right)$ and $\beta^2 \neq 1$, then $\beta^2 =$

(a) a^2 (b) $\frac{1-a^2}{a^2}$ (c) $1 - a^2$ (d) $1 + a^2$

45. If $|z| = \max\{|z-1|, |z+1|\}$ then

Critical Thinking

(a) $|z + \bar{z}| = \frac{1}{2}$ (b) $z + \bar{z} = 1$

(c) $|z + \bar{z}| = 1$ (d) None of these

46. The value of a for which the sum of the squares of the roots of the equation $2x^2 - 2(a-2)x - (a+1) = 0$ is least, is

(a) 1 (b) 3/2 (c) 2 (d) None

47. If $A = |x \in IR : x^2 + 6x - 7 < 0\}$ and $B = \{x \in IR : x^2 + 9x + 14 > 0\}$, then which of the following is/ are correct?

1. $(A \cap B) = (-2, 1)$
2. $(A \backslash B) = (-7, -2)$

Select the correct answer using the code given below:

(a) 1 only (b) 2 Only

(c) Both 1 and 2 (d) Neither 1 nor 2

48. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan 30° and tan 15°, respectively, then the value of $2 + q - p$ is

(a) 2 (b) 3 (c) 0 (d) 1

49. The greatest and the least absolute value of $z + 1$, where $|z + 4| \leq 3$ are respectively

(a) 6 and 0 (b) 10 and 6

(c) 4 and 3 (d) None of these

50. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$ and $Arg(z) - Arg(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to

(a) $-i$ (b) 1 (c) -1 (d) i.

51. If α, β, γ and a, b, c are complex numbers such that $\frac{\alpha}{a} + \frac{\beta}{b} + \frac{\gamma}{c} = 1 + i$ and $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} = 0$, then the value of $\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} + \frac{\gamma^2}{c^2}$ is equal to

(a) 0 (b) -1 (c) $2i$ (d) $-2i$

52. $\sum_{k=33}^{65} \left(\sin\frac{2k\pi}{8} - i\cos\frac{2k\pi}{8}\right)$ Toughnut

(a) $1 + i$ (b) $1 - i$ (c) $1 + \frac{i}{\sqrt{2}}$ (d) $\frac{1-i}{\sqrt{2}}$

53. If α, β are roots of $ax^2 + bx + b = 0$, then $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}}$ is ($b^2 \geq 4ab$, a and b are of same sign)

(a) 0 (b) 1 (c) 2 (d) $2\sqrt{\frac{b}{a}}$

54. Let $x + \frac{1}{x} = 1$ and a, b and c are distinct positive integers such that $\left(x^a + \frac{1}{x^a}\right) + \left(x^b + \frac{1}{x^b}\right) + \left(x^c + \frac{1}{x^c}\right) = 0$. Then the minimum value of $(a + b + c)$ is **Tricky**

(a) 7 (b) 8 (c) 9 (d) 10

55. If z is a complex number such that $z + |z| = 8 + 12i$, then the value of $|z^2|$ is equal to

(a) 228 (b) 144 (c) 121 (d) 169

56. A value of b for which the equations

$x^2 + bx - 1 = 0$

$x^2 + x + b = 0$

have one root in common is

(a) $-\sqrt{2}$ (b) $-i\sqrt{3}$ (c) $i\sqrt{5}$ (d) $\sqrt{2}$

57. The set of all real numbers x for which $x^2 - [x + 2] + x > 0$, is

(a) $(-\infty, -2) \cup (2, \infty)$

(b) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

(c) $(-\infty, -1) \cup (1, \infty)$

(d) $(\sqrt{2}, \infty)$

58. If n is a positive integer greater than unity and z is a complex satisfying the equation $z^n = (z+1)^n$, then

(a) $Re(z) < 2$ (b) $Re(z) > 0$

(c) $Re(z) = 0$ (d) z lies on $x = -\frac{1}{2}$

59. For what value of λ the sum of the squares of the roots of $x^2 + (2 + \lambda)x - \frac{1}{2}(1 + \lambda) = 0$ is minimum?

(a) 3/2 (b) 1 (c) –5/2 (d) 11/4

60. If $Re\left(\frac{z-1}{z+1}\right) = 0$, where $2 = x + iy$ is a complex number, then which one of the following is correct?

(a) $z = 1 + i$ (b) $|z| = 2$

(c) $z = 1 - i$ (d) $|z| = 1$

61. If $\frac{1}{2 - \sqrt{-2}}$ is one of the roots of $ax^2 + bx + c = 0$, where a, b, c are real, then what are the values of a, b, c respectively? **Critical Thinking**

(a) 6, –4, 1 (b) 4, 6, – 1

(c) 3, –2, 1 (d) 6, 4, 1

62. If $2x = 3 + 5i$, then what is the value of $2x^3 + 2x^2 - 7x + 72$?

(a) 4 (b) –4 (c) 8 (d) –8

63. If x be real and $b < c$, then $\frac{x^2 - bc}{2x - b - c}$ lies in

(a) (b, c)

(b) [b, c]

(c) $(-\infty, b] \cup [c, \infty)$

(d) $(-\infty, b) \cup (c, \infty)$

64. If the roots of the equations $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{pr}\,x + q = 0$ be real, then

(a) $p = q$ (b) $q^2 = pr$

(c) $p^2 = qr$ (d) $r^2 = pr$

65. The solution set of $\frac{x^2 - 3x + 4}{x + 1} > 1$, $x \in R$, is

(a) $(3, +\infty)$ (b) $(-1, 1) \cup (3, +\infty)$

(c) $[-1, 1] \cup [3, +\infty)$ (d) None of these

66. If z_1, z_2 are complex numbers such that $Re(z_1) = |z_1 - 2|$, $Re(z_2) = |z_2 - 2|$ and $\arg(z_1 - z_2) = \pi/3$, then $Im(z_1 + z_2) =$

(a) $2/\sqrt{3}$ (b) $4/\sqrt{3}$

(c) $2/\sqrt{3}$ (d) $\sqrt{3}$

67. Let z_1 and z_2 be complex numbers such that $z_1 + i(\overline{z}_2) = 0$ and $\arg(\overline{z}_1 z_2) = \frac{\pi}{3}$. Then $\arg(\overline{z}_1)$ is

(a) $\frac{\pi}{3}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{12}$

68. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is

Tricky

(a) 3 (b) 4 (c) 8 (d) 2

69. If $z = \sqrt{3} + i$, then the argument of z^2e^{z-i} is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $e^{\frac{\pi}{6}}$ (d) $\frac{\pi}{3}$

70. The solutions of the equation $z^2(1 - z^2) = 16$, $z \in \boldsymbol{C}$, lie on the curve

(a) $|z| = 1$ (b) $|z| = \frac{2}{|z|}$

(c) $|z|^2 = 3|z| + 2$ (d) $|z| = 2$

71. Let z and w be two distinct non-zero complex numbers if $|z|^2w - |w|^2z = z - w$, then

(a) $w = \bar{z}^2$ (b) $z\bar{w} = 2$

(c) $z\bar{w} = 1$ (d) $w = \bar{z}$

72. If $(x^2 + 5x + 5)^{x+5} = 1$, then the number of integers satisfying this equation is

(a) 2 (b) 3 (c) 4 (d) 5

73. Rational roots of the equation $2x^4 + x^3 - 11x^2 + x + 2 = 0$ are

(a) $\frac{1}{2}$ and 2 (b) $\frac{1}{2}, 2, \frac{1}{4}, -2$

(c) $\frac{1}{2}, 2, 3, 4$ (d) $\frac{1}{2}, 2, \frac{3}{4}, -2$

74. The solution set of the equation $pqx^2 - (p + q)^2 x + (p + q)^2 = 0$ is

(a) $\left\{\frac{p}{q}, \frac{q}{p}\right\}$ (b) $\left\{pq, \frac{p}{q}\right\}$

(c) $\left\{\frac{q}{p}, pq\right\}$ (d) $\left\{\frac{p+q}{p}, \frac{p+q}{q}\right\}$

75. The roots of the equation $|2x - 1|^2 - 3|2x - 1| + 2 = 0$ are

(a) $\left\{-\frac{1}{2}, 0, \frac{1}{2}\right\}$ (b) $\left\{-\frac{1}{2}, 0, \frac{3}{2}\right\}$

(c) $\left\{-\frac{3}{2}, \frac{1}{2}, 0, 1\right\}$ (d) $\left\{-\frac{1}{2}, 0, 1, \frac{3}{2}\right\}$

76. If one root of the equation $lx^2 + mx + n = 0$ is $\frac{9}{2}$ (l, m and n are positive integers) and $\frac{m}{4n} = \frac{l}{m}$, then $l + n$ is equal to

(a) 80 (b) 85 (c) 90 (d) 95

77. If the roots of $(a^2 + b^2)x^2 - 2(bc + ad)x + c^2 + d^2 = 0$ are equal, then

(a) $\frac{a}{b} = \frac{c}{d}$ (b) $\frac{a}{c} + \frac{b}{d} = 0$

(c) $\frac{a}{d} = \frac{b}{c}$ (d) $a + b = c + d$

78. If the point (x, y) satisfies the equation

$\frac{x + i(x-2)}{3+i} - i = \frac{2y + i(1-3y)}{i-3}$, then $x + y =$

(a) 4 (b) 2 (c) 0 (d) –2

79. $\left(\frac{1-i}{1+i}\right)^{2022} + \left(\frac{1+i}{1-i}\right)^{2021} =$

(a) –i (b) i (c) i + 1 (d) i – 1

80. If $\frac{1 - 10i\cos\theta}{1 - 10\sqrt{3}\, i \sin\theta}$ is purely real then one of the values of θ is

Critical Thinking

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

81. If $z, \bar{z}, -z, -\bar{z}$ forms a rectangle of area $2\sqrt{3}$ square units, then one such z is

(a) $\frac{1}{2} + \sqrt{3}i$ (b) $\frac{\sqrt{5} + \sqrt{3}i}{4}$

(c) $\frac{3}{2} + \frac{\sqrt{3}i}{2}$ (d) $\frac{\sqrt{3} + \sqrt{11}i}{2}$

82. Let $z \in \boldsymbol{C}$ and $i = \sqrt{-1}$, if $a, b, c \in (0, 1)$ be such that $a^2 + b^2 + c^2 = 1$ and $b + ic = (1 + a)z$, then $\frac{1+iz}{1-iz} =$

(a) $\frac{a+ib}{1+c}$ (b) $\frac{a-ib}{1+c}$

(c) $\frac{a-ib}{1-c}$ (d) $\frac{a+ib}{1-c}$

83. If the conjugate of $(x + iy)(1 - 2i)$ is $(1 + i)$, then

(a) $x + iy = 1 - i$ (b) $x + iy = \frac{1-i}{1-2i}$

(c) $x - iy = \frac{1-i}{1+2i}$ (d) $x - iy = \frac{1-i}{1+i}$

84. If $a+bi=\dfrac{i}{1-i}$, then $(a, b)=$

(a) $\left(\dfrac{-1}{2},\dfrac{-1}{2}\right)$ (b) $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$

(c) $\left(\dfrac{1}{2},\dfrac{-1}{2}\right)$ (d) $\left(\dfrac{-1}{2},\dfrac{1}{2}\right)$

85. The value of $\left(\sum_{m=1}^{2n+1} i^{2m}\right)^{\sum_{k=1}^{107} i^k}$ is

(a) i (b) $-i$ (c) -1 (d) 1

86. If $\dfrac{\omega-\bar{\omega}z}{1-z}$ is purely real where $\omega=\alpha+i\beta, \beta\neq 0$ and $z\neq 1$, then the set of the values of z is

(a) $\{z : |z| = 1\}$

(b) $\{z : z = \bar{z}\}$

(c) $\{z : z \neq 1\}$

(d) $\{z : |z| = 1, z \neq 1\}$

87. Let a complex function be defined as $g(z) = z - \bar{z}$. If $g(z\omega) = g(\bar{z}\,\omega)$, then

(a) z is purely real

(b) ω is purely real

(c) ω is purely imaginary

(d) atleast one of (a) or (c) is true

88. If $z = x + iy$, $x, y \in R$ and the imaginary part of $\dfrac{\bar{z}-1}{\bar{z}-i}$ is 1, then the locus of z is

(a) $x + y + 1 = 0$

(b) $x+y+1=0, (x, y)\neq(0,-1)$

(c) $x^2+y^2-x+3y+2=0$

(d) $x^2+y^2-x+3y+2=0, (x, y)\neq(0,-1)$

89. If C is a point on the straight line joining the points A (–2 + i) and B (3 – 4i) in the Argand plane and $\dfrac{AC}{CB}=\dfrac{1}{2}$ then the argument of C is

(a) $\tan^{-1} 3$ (b) $\tan^{-1} 2 - \pi$

(c) $\tan^{-1} 2$ (d) $\pi - \tan^{-1} 3$

90. Let z_1 and z_2 be complex numbers such that $z_1 + i(\bar{z}_2) = 0$ and $\arg(\bar{z}_1 z_2) = \dfrac{\pi}{3}$. Then $\arg(\bar{z}_1)$ is

(a) $\dfrac{\pi}{3}$ (b) π (c) $\dfrac{\pi}{2}$ (d) $\dfrac{5\pi}{12}$

91. If $a > 0$ and $z = x + iy$, then $\log_{\cos^2\theta}|z-a| > \log_{\cos^2\theta}|z-ai|, (\theta\in R)$ implies

Critical Thinking

(a) $x > y$ (b) $x < y$

(c) $x + y = \cos\theta$ (d) $x + y < 0$

92. If $z = \sqrt{3} + i$, then the argument of $z^2 e^{z-i}$ is equal to

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{6}$ (c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{3}$

93. The solutions of the equation $z^2(1 - z^2) = 16$, $z \in \boldsymbol{C}$, lie on the curve

Toughnut

(a) $|z| = 1$ (b) $|z| = \dfrac{2}{|z|}$

(c) $|z|^2 = 3|z| + 2$ (d) $|z| = 2$

94. If $z_1 = (2, -1)$ and $z_2 = (6, 3)$, then $\text{amp}\left(\dfrac{z_1 - z_2}{z_1 + z_2}\right) =$

(a) $-\dfrac{3\pi}{4} - \tan^{-1}\left(\dfrac{1}{4}\right)$ (b) $\dfrac{\pi}{4}\tan^{-1}\left(\dfrac{1}{4}\right)$

(c) $\dfrac{3\pi}{4} + \tan^{-1}\left(\dfrac{1}{4}\right)$ (d) $\dfrac{\pi}{4} + \tan^{-1}\left(\dfrac{1}{4}\right)$

95. If $(x^2 + 5x + 5)^{x+5} = 1$, then the number of integers satisfying this equation is

(a) 2 (b) 3 (c) 4 (d) 5

96. If the roots of $(a^2 + b^2)x^2 - 2(bc + ad)x + c^2 + d^2 = 0$ are equal, then

(a) $\dfrac{a}{b} = \dfrac{c}{d}$ (b) $\dfrac{a}{c} + \dfrac{b}{d} = 0$

(c) $\dfrac{a}{d} = \dfrac{b}{c}$ (d) $a + b = c + d$

97. The solution set of the equation $pqx^2 - (p + q)^2 x + (p + q)^2 = 0$ is

Tricky

(a) $\left\{\dfrac{p}{q},\dfrac{q}{p}\right\}$ (b) $\left\{pq,\dfrac{p}{q}\right\}$

(c) $\left\{\dfrac{q}{p}, pq\right\}$ (d) $\left\{\dfrac{p+q}{p},\dfrac{p+q}{q}\right\}$

98. If α, β are the irrational roots of the equation $3p^2x^3 + px^2 + qx + 3 = 0$ when $p = 1$ and $q = -7$ then $|\alpha - \beta| =$

(a) $\dfrac{3\sqrt{13}}{2}$ (b) $\dfrac{\sqrt{3}}{2}$ (c) $\dfrac{2\sqrt{13}}{3}$ (d) 4

99. The roots of the given equation $(p-q)x^2+(q-r)x+(r-p)=0$ are

(a) $\frac{p-q}{r-p},1$ (b) $\frac{q-r}{p-q},1$

(c) $\frac{r-p}{p-q},1$ (d) $1,\frac{q-r}{p-q}$

100. The product of real roots of the equation $4x^4-24x^3+57x^2+18x-45=0$ if one of the root is $3+i\sqrt{6}$ is

(a) –5/16 (b) 5/16 (c) 3/4 (d) –3/4

101. If α, β and γ are the roots of the equation $x^3-ax^2+bx-c=0$ then, $\alpha^{-2}+\beta^{-2}+\gamma^{-2}=$

Tricky

(a) $\frac{b^2-3ac}{c^2}$ (b) $\frac{b^2-ac}{c^2}$

(c) $\frac{b^2-2ac}{c^2}$ (d) $\frac{b^2-4ac}{c^2}$

102. If $\frac{\alpha}{\alpha+1}$ and $\frac{\beta}{\beta+1}$ are the roots of the quadratic equation $x^2+7x+3=0$, then the equation having roots α and β is

(a) $3x^2-x-3=0$

(b) $11x^2+13x+3=0$

(c) $13x^2+11x+13=0$

(d) $11x^2+3x+13=0$

103. If α, β are the roots of the equation $x^2-2\sqrt{3}x+4=0$ then $\alpha^6+\beta^6=$

Critical Thinking

(a) 128 (b) –64 (c) 64 (d) –128

104. If α, β, γ are the roots of the equation $x^3+x^2+x+r=0$ and $\alpha^3+\beta^3+\gamma^3=5$, then $r=$

(a) $\frac{-1}{2}$ (b) 1 (c) –1 (d) $\frac{1}{2}$

105. If α, β and γ are the roots of the equation $x^3-6x^2+11x+6=0$, then $\Sigma\alpha^2\beta+\Sigma\alpha\beta^2$ is equal to

(a) 80 (b) 84 (c) 90 (d) – 84

DIRECTIONS (Qs. 106-110): *Read the statements carefully and answer the question on the basis of following options.*

(a) Both statement I and II are correct.
(b) Both statement I and II are incorrect.
(c) Statement I is correct but Statement II is incorrect.
(d) Statement II is correct but Statement I is incorrect.

106. If α, β are the roots of the equation $6x^2+6px+p^2=0$, then

Statement I: The equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$ is $3x^2-4p^2x+p^4=0$.

Statement II: $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$.

107. **Statement I :** Equation $ix^2+(i-1)x-\left(\frac{1}{2}\right)-i=0$ has imaginary roots.

Statement II : If $a=i$, $b=i-1$ and $c=-\left(\frac{1}{2}\right)-i$, then $b^2-4ac<0$.

108. **Statement I :** Representation of $z=x+iy$ in terms of r and θ is called polar form of the complex number.

Statement II : $\arg\left(\frac{z_1}{z_2}\right)=\arg(z_1)-\arg(z_2)$.

109. **Statement I :** If the arguments of $\overline{z}_1$ and z_2 are $\frac{\pi}{5}$ and $\frac{\pi}{3}$ respectively, then $\arg(z_1z_2)$ is $\frac{2\pi}{15}$.

Statement II : For any complex number z, $\arg\overline{z}=\frac{\pi}{2}+\arg z$.

110. **Statement I:** $z_1(z_2\cdot z_3)=z_1\cdot z_2+z_1z_3$.

Statement II: $(z_2+z_3)z_1=z_1z_2+z_3z_1$.

Numeric Value Questions

111. Number of solutions of the equation, $z^3+\frac{3|z|^2}{z}=0$, where z is a complex number and $|z|=\sqrt{3}$ is ______.

112. Let z_0 be a root of the quadratic equation, $x^2+x+1=0$. If $z=3+6i\,z_0^{81}-3i\,z_0^{93}$, if arg z is equal to $\frac{\pi}{a}$, then a is ______.

113. If $f(z) = \dfrac{7-z}{1-z^2}$, where $z = 1 + 2i$, then $|f(z)|$ is equal to $\dfrac{|z|}{n}$. Find n ______.

114. Let z be a complex number such that $|z| + z = 3 + i$ $\left(\text{where } i = \sqrt{-1}\right)$. Then $|z|$ is equal to ______.

115. If $\dfrac{z-\alpha}{z+\alpha}(\alpha \in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is ______.

116. Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then the minimum value of $|z_1 - z_2|$ is ______.

117. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\dfrac{\alpha}{\beta}\right)^n = 1$ is ______.

118. All the points in the set $S = \left\{\dfrac{\alpha + i}{\alpha - i} : \alpha \in R\right\}(i = \sqrt{-1})$ lie on a circle, then it's radius is equal to ______.

Tricky

119. The number of total imaginary roots of the equation $x^2 - (5 + i)x + (18 - i) = 0$ is ________.

120. If $z_1, z_2 \in C$, $z_1^2 + z_2^2 \in R$, $z_1(z_1^2 - 3z_2^2) = 2$ and $z_2(3z_1^2 - z_2^2) = 11$, the value of $z_1^2 + z_2^2$ is _______.

Critical Thinking

ANSWER KEY																			
1	(d)	**14**	(a)	**27**	(d)	**40**	(c)	**53**	(d)	**66**	(b)	**79**	(d)	**92**	(d)	**105**	(b)	**118**	(1)
2	(b)	**15**	(d)	**28**	(a)	**41**	(b)	**54**	(c)	**67**	(d)	**80**	(a)	**93**	(d)	**106**	(a)	**119**	(2)
3	(c)	**16**	(a)	**29**	(c)	**42**	(c)	**55**	(d)	**68**	(d)	**81**	(a)	**94**	(a)	**107**	(a)	**120**	(5)
4	(c)	**17**	(d)	**30**	(a)	**43**	(a)	**56**	(b)	**69**	(d)	**82**	(a)	**95**	(b)	**108**	(a)		
5	(a)	**18**	(c)	**31**	(b)	**44**	(b)	**57**	(b)	**70**	(d)	**83**	(b)	**96**	(a)	**109**	(c)		
6	(d)	**19**	(d)	**32**	(b)	**45**	(d)	**58**	(d)	**71**	(c)	**84**	(d)	**97**	(d)	**110**	(d)		
7	(c)	**20**	(c)	**33**	(b)	**46**	(b)	**59**	(c)	**72**	(b)	**85**	(c)	**98**	(c)	**111**	(4)		
8	(b)	**21**	(b)	**34**	(b)	**47**	(c)	**60**	(d)	**73**	(a)	**86**	(d)	**99**	(c)	**112**	(4)		
9	(a)	**22**	(c)	**35**	(a)	**48**	(b)	**61**	(a)	**74**	(d)	**87**	(d)	**100**	(d)	**113**	(2)		
10	(b)	**23**	(a)	**36**	(b)	**49**	(a)	**62**	(a)	**75**	(d)	**88**	(d)	**101**	(c)	**114**	(3)		
11	(c)	**24**	(d)	**37**	(b)	**50**	(a)	**63**	(c)	**76**	(b)	**89**	(b)	**102**	(b)	**115**	(2)		
12	(b)	**25**	(c)	**38**	(c)	**51**	(c)	**64**	(b)	**77**	(a)	**90**	(d)	**103**	(d)	**116**	(0)		
13	(a)	**26**	(c)	**39**	(d)	**52**	(d)	**65**	(b)	**78**	(b)	**91**	(a)	**104**	(c)	**117**	(4)		

Hints & Solutions

1. **(d)** The given equation is

$m^2x^2 + (2m - m^2)x + 3 = 0$

$\therefore \alpha + \beta = -\dfrac{2m - m^2}{m^2} = \dfrac{m-2}{m}$ and $\alpha\beta = \dfrac{3}{m^2}$ Now

$\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{4}{3} \Rightarrow \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{4}{3} \Rightarrow \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \dfrac{4}{3}$

Substituting the values, we get

$$\frac{\left(\frac{m-2}{m}\right)^2 - 2.\frac{3}{m^2}}{\frac{3}{m^2}} = \frac{4}{3}$$

$\Rightarrow \dfrac{m^2 - 4m + 4 - 6}{3} = \dfrac{4}{3} \Rightarrow m^2 - 4m - 6 = 0$

m_1 and m_2 are roots of this equation, therefore $m_1 + m_2 = 4$ and $m_1m_2 = -6$

The given expression is, $\dfrac{m_1^2}{m_2} + \dfrac{m_2^2}{m_1} = \dfrac{m_1^3 + m_2^3}{m_1m_2}$

$= \dfrac{(m_1 + m_2)^3 - 3m_1m_2(m_1 + m_2)}{m_1m_2}$

$= \dfrac{(4)^3 - 3.(-6).(4)}{-6} = -\dfrac{68}{3}$

2. **(b)** $\alpha < -1$. Let $\alpha = -1 - p$

$\beta > 1$. Let $\beta = 1 + q$, $p > 0, q > 0$

Now $1 + \dfrac{c}{a} + \left|\dfrac{b}{a}\right| = 1 + \alpha\beta + |-\alpha - \beta|$

$= 1 + (1 + q)(-1 - p) + |-1 - p + 1 + q|$
$= 1 - (1 + p + q + pq) + |q - p|$

$$= \begin{cases} -p - q - pq + q - p = -2p - pq < 0 & \text{if } q > p \\ -p - q - pq + p - q = -2q - pq < 0 & \text{if } q < p \end{cases}$$

$\therefore \quad 1 + \dfrac{c}{a} + \left|\dfrac{b}{a}\right| < 0$

3. **(c)** Let $\sin\alpha$ and $\cos\alpha$ be the roots of $ax^2 + bx + c = 0$

Now, $\sin\alpha + \cos\alpha = \dfrac{-b}{a}$ and $\sin\alpha\cos\alpha = \dfrac{c}{a}$

Consider $\sin\alpha + \cos\alpha = \dfrac{-b}{a}$

Squaring both side, $(\sin\alpha + \cos\alpha)^2 = \dfrac{b^2}{a^2}$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = \dfrac{b^2}{a^2}$

$\Rightarrow 1 + \dfrac{2c}{a} = \dfrac{b^2}{a^2}$

$\Rightarrow \dfrac{a + 2c}{a} = \dfrac{b^2}{a^2} \Rightarrow a + 2c = \dfrac{b^2}{a}$

$\Rightarrow a^2 + 2ac = b^2 \Rightarrow b^2 - a^2 = 2ac$

4. **(c)** $|z - 2| = \min\{|z - 1|, |z - 5|\}$
i.e., $|z - 2| = |z - 1|$, where $|z - 1| < |z - 5|$

$\Rightarrow \quad \text{Re}(z) = \dfrac{3}{2}$ which satisfy $|z - 1| < |z - 5|$

Also, $|z - 2| = |z - 5|$, where $|z - 5| < |z - 1|$

$\Rightarrow \quad \text{Re}(z) = \dfrac{7}{2}$ which satisfy $|z - 5| < |z - 1|$

5. **(a)** $|x_1z_1 - y_1z_2|^2 + |y_1z_1 - x_1z_2|^2$
$= |x_1z_1|^2 + |y_1z_2|^2 - 2\text{Re}(x_1y_1z_1z_2)$
$\quad + |y_1z_1|^2 + |x_1z_2|^2 + 2\text{Re}(x_1y_1z_1z_2)$
$= x_1^2|z_1|^2 + y_1^2|z_2|^2 + y_1^2|z_1|^2 + x_1^2|z_2|^2$
$= x_1^2|z_1|^2 + y_1^2|z_2|^2 + y_1^2|z_1|^2 + x_1^2|z_2|^2$
$= 2(x_1^2 + y_1^2)(4^2) = 32(x_1^2 + y_1^2)$

6. **(d)** $z = \dfrac{\pi}{4}(1 + i)^4\left(\dfrac{1 - \sqrt{\pi}i}{\sqrt{\pi} + i} + \dfrac{\sqrt{\pi} - i}{1 + \sqrt{\pi}i}\right)$

$= \dfrac{\pi}{4}(1 + i)^4\left[\dfrac{1 + \pi + \pi + 1}{(\sqrt{\pi} + i)(1 + \sqrt{\pi}i)}\right] = \dfrac{\pi}{4}(1 + i)^4\dfrac{2}{i}$

$= \dfrac{\pi}{4}(2i)^2\dfrac{2}{i} = 2\pi i \;\therefore \left(\dfrac{|z|}{\text{amp}(z)}\right) = \dfrac{2\pi}{\frac{\pi}{2}} = 4$

7. **(c)** Let $z_1 = 1+4i, z_2 = 3+i, z_3 = 1-i$ and $z_4 = 2-3i$

$\therefore$ $m_1 = |z_1|, m_2 = |z_2|, m_3 = |z_3|$ and $m_4 = |z_4|$

$\Rightarrow$ $m_1 = \sqrt{17}$, $m_2 = \sqrt{10}$, $m_3 = \sqrt{2}$, and $m_4 = \sqrt{13}$,

$\Rightarrow$ $m_3 < m_2 < m_4 < m_1$.

8. **(b)** Given equation can be written as

$(6k+2)x^2 + rx + 3k - 1 = 0$ ...(i)

and $2(6k+2)x^2 + px + 2(3k-1) = 0$ ...(ii)

Condition for common root is

$$\frac{12k+4}{6k+2} = \frac{p}{r} = \frac{6k-2}{3k-1} = 2 \text{ or } 2r - p = 0$$

9. **(a)** $|1-\bar{z}_1 z_2|^2 - |z_1 - z_2|^2$

$= (1-\bar{z}_1 z_2)(1 - z_1\bar{z}_2) - (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$

$= 1 - z_1\bar{z}_2 - \bar{z}_1 z_2 + z_1\bar{z}_1 z_2\bar{z}_2 - (z_1\bar{z}_1 - z_1\bar{z}_2 - \bar{z}_1 z_2 + z_2\bar{z}_2)$

$= 1 + z_1\bar{z}_1 z_2\bar{z}_2 - z_1\bar{z}_1 - z_2\bar{z}_2$

$= 1 + |z_1|^2|z_2|^2 - |z_1|^2 - |z_2|^2$

$= (1-|z_1|^2)(1-|z_2|^2)$

$\Rightarrow$ $k = 1$

10. **(b)** $z = 1 + \cos\frac{11\pi}{9} + i\sin\frac{11\pi}{9}$

$\text{Re}(z) > 0$ and $\text{Im}(z) < 0$, so the number lies in the fourth quadrant. Also

$$z = 2\cos\frac{11\pi}{18}\left\{\cos\frac{11\pi}{18} + i\sin\frac{11\pi}{18}\right\}$$

$$= 2\cos\frac{11\pi}{18}\left\{\cos\left(-\frac{7\pi}{18}\right) + i\sin\left(-\frac{7\pi}{18}\right)\right\}$$

$$\therefore \arg(z) = -\frac{7\pi}{18}$$

$$|z| = \left|2\cos\frac{11\pi}{18}\right| = -2\cos\frac{11\pi}{18}$$

11. **(c)** λ and μ are the roots of $x^2 = 5x - 3$ or $x^2 - 5x + 3 = 0$

$\therefore$ $\lambda + \mu = 5$ and $\lambda\mu = 3$

$$\frac{\lambda}{\mu} + \frac{\mu}{\lambda} = \frac{(\lambda+\mu)^2 - 2\lambda\mu}{\lambda\mu} = \frac{19}{3} \quad = \frac{\lambda}{\mu}.\frac{\mu}{\lambda} = 1$$

$\therefore$ Desired equation is $x^2 - \frac{19}{3}x + 1 = 0$

or $3x^2 - 19x + 3 = 0$

12. **(b)** Given equation is

$(\ell - m)x^2 + \ell x + 1 = 0$

Roots are α, β.

$\because$ One root is double the other.

$\beta = 2\alpha$

Sum of roots $= \alpha + \beta$

$$3\alpha = \frac{-\ell}{\ell - m} \quad ; \quad \alpha(2\alpha) = \frac{1}{(\ell - m)}$$

$$\Rightarrow \alpha^2 = \frac{\ell^2}{9(\ell-m)^2} \quad ; \quad 2\alpha^2 = \frac{1}{\ell - m}$$

$$\Rightarrow 2\frac{\ell^2}{9(\ell-m)^2} = \frac{1}{(\ell-m)}$$

$$\Rightarrow \frac{2\ell^2}{9(\ell-m)} = 1$$

$\Rightarrow 2\ell^2 = 9(\ell - m) \Rightarrow 2\ell^2 - 9\ell + 9m = 0$

For ℓ to be real discriminant should be

$b^2 - 4ac \geq 0$

$81 - 4\times 2\times 9m \geq 0 \quad \therefore \quad m \leq \frac{9}{8}$.

13. **(a)** Given $\frac{(1+i)^{4n+5}}{(1-i)^{4n+3}}$

$$= \frac{(1+i)^{4n+3}.(1+i)^2}{(1-i)^{4n+3}} = \left(\frac{1+i}{1-i}\right)^{4n+3}.(1+i)^2$$

$$= \left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^{4n+3} \bullet (1+i^2+2i)$$

$$= \left[\frac{1+i^2+2i}{1+1}\right]^{4n+3}.2i = (i)^{4n+3}.2i = 2(i)^{4n+4}$$

$= 2.(i^{4(n+1)}) = 2$

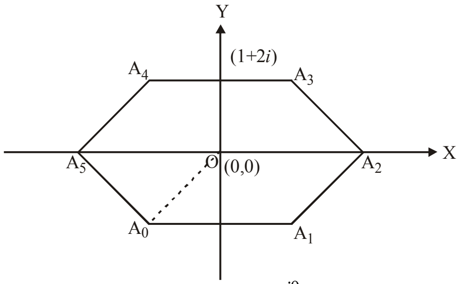
14. **(a)** $(a+ib)(c+id)(e+if)(g+ih) = A + iB$ ...(i)

$\Rightarrow$ $(a-ib)(c-id)(e-if)(g-ih) = A - iB$ ...(ii)

Multiplying (i) and (ii), we get

$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2 + B^2$

15. **(d)** Let the vertices be $z_0, z_1, \ldots, z_5$ w.r.t. centre O at origin and $|z_0| = \sqrt{5}$.



$\Rightarrow \; A_0A_1 = |z_1 - z_0| = |z_0 e^{i\theta} - z_0|$

$= |z_0|\,|\cos\theta + i\sin\theta - 1|$

$= \sqrt{5}\sqrt{(\cos\theta - 1)^2 + \sin^2\theta}$

$= \sqrt{5}\sqrt{2(1-\cos\theta)} = \sqrt{5}\;2\sin(\theta/2)$

$\Rightarrow A_0A_1 = \sqrt{5}.\,2\sin\left(\frac{\pi}{6}\right) = \sqrt{5}$

$\left(\because\;\theta = \frac{2\pi}{6} = \frac{\pi}{3}\right)$

Similarly, $A_1A_2 + A_2A_3 + A_3A_4 + A_4A_5 + A_5A_0 = 6\sqrt{5}$.
Hence the perimeter of, regular polygon is
$= A_0A_1 + A_1A_2 + A_2A_3 + A_3A_4 + A_4A_5 + A_5A_0 = 6\sqrt{5}$.

16. **(a)** $z_1\bar{z}_1 = z_2\bar{z}_2 = ... = z_n\bar{z}_n = 1$

$\Rightarrow \bar{z}_1 = \frac{1}{z_1},\; \bar{z}_2 = \frac{1}{z_2},\; \bar{z}_3 = \frac{1}{z_3}, ..., \bar{z}_n = \frac{1}{z_n}$

$\therefore \quad |z_1 + z_2 + ...+ z_n| - \left|\frac{1}{z_1} + \frac{1}{z_2} + ... + \frac{1}{z_n}\right|$

$= |z_1 + z_2 + ... + z_n| - |\bar{z}_1 + \bar{z}_2 + ... + \bar{z}_n| = 0$

17. **(d)** **Case 1 :** $x \geq 0$

$\therefore$ the equation becomes $x^2 + 5x + 4 = 0$

or $x = -1, -4$ but $x \geq 0$

$\therefore$ both values, non admissible :

Case 2 : $x \leq 0$

The eq^n becomes $x^2 - 5x + 4 = 0$ or $x = 1, 4$ both values are non admissible, $\therefore$ No real roots.

Alternatively, since $x^2 \geq 0$; $|x| \geq 0$

$\therefore\; x^2 + |x| + 4 > 0$ for all $x \in \mathbf{R}$

$\therefore\; x^2 + |x| + 4 \neq 0$ for any $x \in \mathbf{R}$

18. **(c)** The given eq. implies that the difference between the distances of the moving point from two fixed points (1 – i) and (2 + i) is constant using the property of the hyperbola that the difference between the focal distances of any point on the curve is constant, the locus in reference is therefore a hyperbola.

19. **(d)** $x^2 - 2kx + k^2 - 4 = 0$

$\Rightarrow \; (x-k)^2 - 2^2 = 0$

$\Rightarrow \; (x-k-2)(x-k+2) = 0$

$\Rightarrow \; x = k+2, k-2.$

$\Rightarrow \; k+2 < 5 \;\&\; k-2 > -3$

$\Rightarrow \; k < 3 \;\&\; k > -1$

$\Rightarrow \; -1 < k < 3$

20. **(c)** Consider

$(-\sqrt{-1})^{4n+3} + (i^{41} + i^{-257})^9$

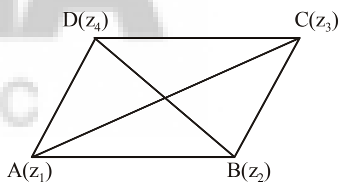
$= (-i)^{4n+3} + \left[(i^4)^{10}.i^1 + (i^3)^{-85}.i^{-2}\right]^9$

$= (-i)^{4n+3} + \left[i + \frac{1}{(i^3)^{85}}.\frac{1}{i^2}\right]^9$

$= (-i)^{4n+3} + \left(i + \frac{1}{i}\right)^9$

$= -(-1)^{4n+3}(i)^{4n}(i)^3 + (i-i)^9 = -(1)(-i) + 0 = i$

21. **(b)** Let z_1, z_2, z_3 and z_4 the points in complex plane be the vertices of a parallelogram taken in order.



Since the diagonals of a parallelogram bisect, hence the mid points of AC and BD must coincide i.e.,

$\frac{z_1 + z_3}{2} = \frac{z_2 + z_4}{2} \Rightarrow z_1 + z_3 = z_2 + z_4$

22. **(c)** $f(x) = x^2 - 3x + a + \frac{1}{a}$; $f(3) = 9 - 9 + a + \frac{1}{a} > 0$

$\Rightarrow \; a + \frac{1}{a} > 0 \;\Rightarrow a > 0$

$f(2) = 4 - 6 + a + \frac{1}{a} \leq 0 \Rightarrow \quad \frac{a^2 - 2a + 1}{a} \leq 0$

$\Rightarrow \; \frac{(a-1)^2}{a} \leq 0 \;\Rightarrow a = 1$

Therefore, $f(x) = x^2 - 3x + 2 = 0$ has roots 1 and 2.

$\therefore \quad \alpha^2 + \beta^2 = 5$

23. **(a)** Since $|z-1|=1$ $\therefore z-1=e^{i\theta}$, where $\arg|z-1|=\theta$

$\therefore z=1+\cos\theta+i\sin\theta$

$=2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right]$ $=2\cos\frac{\theta}{2}.e^{i\theta/2}$

Thus, $\arg(z-1)=2\arg z.$

24. **(d)** $y-2=\cfrac{1}{4+\left[\cfrac{1}{4+\cfrac{1}{4+.............\infty}}\right]}=\frac{1}{4+(y-2)}=\frac{1}{y+2}$

$\Rightarrow (y-2)(y+2)=1 \Rightarrow y^2-4=1 \Rightarrow y^2=5$

$\therefore y=\pm\sqrt{5}$ since $y>0$ $\therefore y=\sqrt{5}$

25. **(c)** Since the roots are imaginary $\therefore$ D < 0 and roots occur as conjugate pair, i.e. $\beta=\overline{\alpha}$

$\therefore |\beta|=|\overline{\alpha}|=|\alpha|$

Also, let $\alpha=\frac{-b+i\sqrt{4ac-b^2}}{2a}$

$\therefore |\alpha|=\sqrt{\frac{b^2}{4a^2}+\frac{4ac-b^2}{4a^2}}=\sqrt{\frac{c}{a}}$

$|\alpha|>1$ $(\because c>a)$

$\therefore |\alpha|=|\beta|>1$

26. **(c)** $\left[\frac{\sin\frac{\pi}{6}+i\left(1-\cos\frac{\pi}{6}\right)}{\sin\frac{\pi}{6}-i\left(1-\cos\frac{\pi}{6}\right)}\right]^3$

$=\left[\frac{2\sin\frac{\pi}{12}\cos\frac{\pi}{12}+i\left(2\sin^2\frac{\pi}{12}\right)}{2\sin\frac{\pi}{12}\cos\frac{\pi}{12}-i\left(2\sin^2\frac{\pi}{12}\right)}\right]^3$

$=\left[\frac{\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}}\right]^3=\left(\frac{e^{i\frac{\pi}{12}}}{e^{-i\frac{\pi}{12}}}\right)^3$

$=\left(e^{i\frac{\pi}{6}}\right)^3=e^{i\times 3\times\frac{\pi}{6}}=e^{i\frac{\pi}{2}}$

$=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=i$

27. **(d)** $\sum_{n=1}^{13}\left[i^n+i^{n+1}\right]=\sum_{n=1}^{13}i^n[1+i]$

$=(1+i)\left[i+i^2+i^3....i^{13}\right]=\frac{(1+i)}{(1-i)}i\left[1-i^{13}\right]$

$=\frac{(-1+i)\left(1-i^{13}\right)}{(1-i)}=\frac{-1+i^{13}+i-i^{14}}{(1-i)}$

$=\frac{-1+(i^2)^6.i+i-(i^2)^7}{(1-i)}=\frac{2i+2i^2}{1-i^2}=(i-1)$

28. **(a)** Given roots are real and distinct, then $a^2-4b>0$

$\Rightarrow$ $b<a^2/4$

Again α and β differ by a quantity less than $c (c>0)$

$\Rightarrow$ $|\alpha-\beta|<c$ or $(\alpha-\beta)^2<c^2$

$\Rightarrow$ $(\alpha+\beta)^2-4\alpha\beta<c^2$ or $a^2-4b<c^2$

or $\frac{a^2-c^2}{4}<b$

$\Rightarrow$ $\frac{a^2-c^2}{4}<b<\frac{a^2}{4}$ by (1) and (2)

29. **(c)** When $x<0, |x|=-x$

$\therefore$ Equation is $x^2-x-6=0 \Rightarrow x=-2, 3$

$\because$ $x<0$, $\therefore x=-2$ is the solution.

When $x\geq 0, |x|=x$

$\therefore$ Equation is $x^2+x-6=0 \Rightarrow x=2,-3$

$\because$ $x\geq 0$, $\therefore x=2$ is the solution,

Hence $x=2, -2$ are the solutions and their sum is zero.

30. **(a)** $|z_1|=|z_2|=|z_3|=1$ (given)

Now, $|z_1|=1$ $\Rightarrow |z_1|^2=1 \Rightarrow z_1\overline{z}_1=1$

Similarly, $z_2\overline{z}_2=1$, $z_3\overline{z}_3=1$

Now, $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$ $\Rightarrow |\overline{z}_1+\overline{z}_2+\overline{z}_3|=1$

$\Rightarrow |\overline{z_1+z_2+z_3}|=1$ $\Rightarrow |z_1+z_2+z_3|=1$

31. **(b)** Multiplying the second equation by $\frac{c}{a^3}$,

we get $\frac{b^2c^2}{a^3}x^2-\frac{b^2c}{a^2}x+c=0$

$\Rightarrow$ $a\left(\frac{bc}{a^2}x\right)^2-b\left(\frac{bc}{a^2}\right)x+c=0$

$\Rightarrow$ $\frac{bc}{a^2}x=\alpha,\beta$

$\Rightarrow \quad (\alpha+\beta)\alpha\beta x = \alpha, \beta$

$\Rightarrow x = \dfrac{1}{(\alpha+\beta)\alpha}, \dfrac{1}{(\alpha+\beta)\beta}$

32. **(b)** $(\sin x + i\cos x)^3$
$= \sin^3 x + (i)^3 \cos^3 x + 3i(\sin x)(\cos x)(\sin x + i\cos x)$
$= \sin^3 x - i\cos^3 x + 3i\sin^2 x\cos x - 3\sin x\cos^2 x$
$= \sin^3 x - 3\sin x\cos^2 x - i\cos x(\cos^2 x + 3\sin^2 x)$
$= \sin x(\sin^2 x - 3\cos^2 x) + i\cos x$
Real part of $(\sin x + i\cos x)^3$
$= \sin x(\sin^2 x - 3\cos^2 x)$
$= \sin x[\sin^2 x - 3(1-\sin^2 x)]$
$= \sin x[4\sin^2 x - 3]$
$= 4\sin^3 x - 3\sin x$
$= -(3\sin x - 4\sin^3 x)$
$= -\sin 3x$

33. **(b)** Let the discriminant of the equation $x^2 + px + q = 0$ by D_1, then $D_1 = p^2 - 4q$ and the discriminant D_2 of the equation $x^2 + rx + s = 0$ is $D_2 = r^2 - 4s$

$\therefore D_1 + D_2 = p^2 + r^2 - 4(q+s) = p^2 + r^2 - 2pr$ [from the given relation]

$\therefore D_1 + D_2 = (p-r)^2 \geq 0$
Clearly at least one of D_1 and D_2 must be non-negative consequently at least one of the equation has real roots.

34. **(b)** $z = 1 + i\tan\alpha = r(\cos\theta + i\sin\theta)$
$\Rightarrow r\cos\theta = 1, r\sin\theta = \tan\alpha \Rightarrow r^2 = \sec^2\alpha$

$\Rightarrow r = |\sec\alpha| = \dfrac{1}{|\cos\alpha|}$

Since, $-\pi < \alpha < -\dfrac{\pi}{2}$

$\Rightarrow \cos\alpha < 0 \Rightarrow |\cos\alpha| = -\cos\alpha$

$\therefore \quad r = \dfrac{1}{-\cos\alpha}$. Further, we get

$\cos\theta = -\cos\alpha = \cos(\pi+\alpha)$

Now, $-\pi < \alpha < -\dfrac{\pi}{2} \Rightarrow \pi - \pi < \pi + \alpha < \pi - \dfrac{\pi}{2}$

$\Rightarrow 0 < \pi + \alpha < \dfrac{\pi}{2}$ [Converted to principal value]

$\therefore \cos\theta = \cos(\pi+\alpha) \Rightarrow \theta = \pi + \alpha$

Hence, $z = \dfrac{1}{-\cos\alpha}[\cos(\pi+\alpha) + i\sin(\pi+\alpha)]$

35. **(a)** If α, β be the roots then

$\alpha + \beta = -\dfrac{b}{a}, \alpha\beta = \dfrac{c}{a}$

Now the roots of $\ell x^2 + mx + n = 0$ are $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$

$\therefore \dfrac{1}{\alpha} + \dfrac{1}{\beta} = -\dfrac{m}{\ell}$ and $\dfrac{1}{\alpha}.\dfrac{1}{\beta} = \dfrac{n}{\ell}$

or $\dfrac{\alpha+\beta}{\alpha\beta} = -\dfrac{m}{\ell}$ and $\dfrac{a}{c} = \dfrac{n}{\ell}$

or $-\dfrac{b}{c} = -\dfrac{m}{\ell}$ and $\dfrac{a}{c} = \dfrac{n}{\ell}$

or $\dfrac{a}{n} = \dfrac{b}{m} = \dfrac{c}{\ell}$. $\therefore a : b : c = n : m : \ell$

36. **(b)** For the nonreal roots of the equation

$2z^2 + 2z + \lambda = 0$ ...(i)

discriminant < 0.

That is $4 - 8\lambda < 0 \Rightarrow \lambda > \dfrac{1}{2}$ ...(ii)

Let the roots of (i) be z_1 & z_2

Then $z_1 + z_2 = -\dfrac{2}{2} = -1, \quad z_1 z_2 = \dfrac{\lambda}{2}$

z_1 and z_2 with origin form equilateral triangle if

$z^2 + z_2^2 - z_1 z_2 = 0$

$\Rightarrow (z_1 + z_2)^2 = 3z_1 z_2$

$\Rightarrow (-1)^2 = 3\dfrac{\lambda}{2} \Rightarrow \lambda = \dfrac{2}{3}$

$\lambda = \dfrac{2}{3}\left(> \dfrac{1}{2}\right)$ satisfies the condition (ii). Hence it is the required result.

37. **(b)** Here, $\alpha + \beta = p, \alpha\beta = q$
$\alpha_1 + \beta_1 = q, \alpha_1\beta_1 = p$

Sum of given roots

$= \left(\dfrac{1}{\alpha_1\beta} + \dfrac{1}{\alpha\beta_1}\right) + \left(\dfrac{1}{\alpha\alpha_1} + \dfrac{1}{\beta\beta_1}\right)$

$= \dfrac{\alpha\beta_1 + \alpha_1\beta + \beta\beta_1 + \alpha\alpha_1}{\alpha\beta\alpha_1\beta_1}$

$= \dfrac{(\alpha+\beta)(\alpha_1+\beta_1)}{(\alpha\beta)(\alpha_1\beta_1)} = \dfrac{pq}{qp} = 1$

Product of given roots

$$=\left(\frac{1}{\alpha_1\beta}+\frac{1}{\alpha\beta_1}\right)\left(\frac{1}{\alpha\alpha_1}+\frac{1}{\beta\beta_1}\right)$$

$$=\frac{(\alpha\beta_1+\alpha_1\beta)(\alpha\alpha_1+\beta\beta_1)}{\alpha^2\beta^2\alpha_1^2\beta_1^2}$$

$$=\frac{\alpha\beta(\alpha_1^2+\beta_1^2)+\alpha_1\beta_1(\alpha^2+\beta^2)}{\alpha^2\beta^2\alpha_1^2\beta_1^2}$$

$$=\frac{\alpha\beta\left[(\alpha_1+\beta_1)^2-2\alpha_1\beta_1\right]+\alpha_1\beta_1\left[(\alpha+\beta)^2-2\alpha\beta\right]}{(\alpha\beta)^2(\alpha_1\beta_1)^2}$$

$$=\frac{q\left(q^2-2p\right)+p\left(p^2-2q\right)}{q^2p^2}=\frac{p^3+q^3-4pq}{p^2q^2}$$

Hence, the required equation is

$$(p^2q^2)x^2-(p^2q^2)x+p^3+q^3-4pq=0$$

38. **(c)** Here m and n are the roots of equation.

$(x+p)(x+q)-k=0$

$x^2+x(p+q)+pq-k=0$ ... (i)

If m and n are the roots of equation, then

$(x-m)(x-n)=0$

$\therefore$ $x^2-(m+n)x+mn=0$ ... (ii)

Now equation (i) should be equal to equation (ii),

$(m+n)=-(p+q)$ and $mn=pq-k$

Now, we have to find roots of $(x-m)(x-n)+k=0$

$x^2-(m+n)x+mn+k=0$

$x^2+(p+q)x+(pq-k)+k=0$

$x^2+(p+q)x+pq=0$

$x^2+px+qx+pq=0$

$x(x+p)+q(x+p)=0$

$\therefore$ $x+q=0$ or $x+p=0$

$\therefore$ $x=-q$ and $x=-p$

39. **(d)** Given complex number is

$(1-\sin\theta)+i\cos\theta\equiv a+ib$

Argument $\equiv\tan\phi=\dfrac{b}{a}$

$$\Rightarrow\tan\phi=\frac{\cos\theta}{1-\sin\theta}$$

$$=\frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$=\frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}{\left(\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right)^2}$$

$$=\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$$

$$=\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}=\frac{\tan\frac{\pi}{4}+\tan\frac{\theta}{2}}{1-\tan\frac{\pi}{4}\tan\frac{\theta}{2}}$$

$$\tan\phi=\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$$

Hence, argument $=\dfrac{\pi}{4}+\dfrac{\theta}{2}$

40. **(c)** As given $\omega=\dfrac{z}{z-\frac{1}{3}i}\Rightarrow|\omega|=\dfrac{|z|}{|z-\frac{1}{3}i|}=1$

$$\Rightarrow|z|=\left|z-\frac{1}{3}i\right|$$

$\Rightarrow$ distance of z from origin and point $\left(0,\frac{1}{3}\right)$ is same hence z lies on bisector of the line joining points (0, 0) and (0, 1/3).

Hence, z lies on a straight line.

41. **(b)** $z_1+z_2=-1,\ z_1z_2=\dfrac{b}{3}$

$0^2+z_1{}^2+z_2{}^2=0\times z_1+0\times z_2+z_1z_2$

$\Rightarrow$ $(z_1+z_2)^2-2z_1z_2=z_1z_2\Rightarrow 1=3z_1z_2=3\dfrac{b}{3}$

$\Rightarrow b=1$

42. **(c)** Let the vertices be $z_0, z_1, ...z_5$ w.r.t centre O as origin $|z_0|=1$,

$A_0A_1=|z_1-z_0|=|z_0e^{i\theta}-z_0|$

$\therefore A_0A_1=|z_0||\cos\theta+i\sin\theta-1|$

$=1.\sqrt{(\cos\theta-1)^2+\sin^2\theta}=\sqrt{2(1-\cos\theta)}$

$$\therefore A_0A_1=\sqrt{2.2\sin^2\frac{\theta}{2}}=2\sin\frac{\theta}{2}$$

Where $\theta = \frac{2\pi}{6} = \frac{\pi}{3}$. Replacing θ by 2θ and 4θ, we get, $A_0A_2 = 2\sin\frac{2\theta}{2} = 2\sin\theta$ &

$A_0A_4 = 2\sin\frac{4\theta}{2} = 2\sin 2\theta$

$\therefore (A_0A_1)(A_0A_2)(A_0A_4)$

$= 8\sin\frac{\pi}{6}\sin\frac{\pi}{3}\sin\frac{2\pi}{3}$

$= 8\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = 3$

43. **(a)** Let α be the common root to the equations: $x^2 - kx - 21 = 0$ and $x^2 - 3kx + 35 = 0$

$\therefore$ 'α' satisfies both the equations

$\therefore \quad \alpha^2 - k\alpha - 21 = 0 \quad$ ...(i)

and $\alpha^2 - 3k\alpha + 35 = 0 \quad$ ...(ii)

From (i) and (ii),

$\alpha^2 - 21 = \frac{\alpha^2 + 35}{3}$

$\Rightarrow 3\alpha^2 - 63 = \alpha^2 + 35$

$\Rightarrow \alpha^2 = 49 \Rightarrow \alpha = \pm 7$

Now, again by eliminating α^2 from (i) and (ii), we get

$k\alpha + 21 = 3k\alpha - 35$

$\Rightarrow 2k\alpha = 56 \Rightarrow k = \frac{56}{2\alpha}$

When $\alpha = 7$ then $k = 4$

When $\alpha = -7$ then $k = -4$

Hence, $k = \pm 4$

44. **(b)** $\alpha^2 + \beta^2 = \frac{1}{a^2}$ and $\alpha^2\beta^2 = \frac{1-a^2}{a^2}$

$\Rightarrow \alpha^2 + \beta^2 - 1 = \alpha^2\beta^2 \Rightarrow (\alpha^2 - 1)(\beta^2 - 1) = 0$

$\because \beta^2 \neq 1 \Rightarrow \alpha^2 = 1$, so, $\beta^2 = \frac{1-a^2}{a^2}$

45. **(d)** If $|z-1| > |z+1|$, then max $\{|z-1|, |z+1|\} = |z-1|$

$\Rightarrow$ If $|z|^2 + 1 - z - \bar{z} > |z|^2 + 1 + z + \bar{z}$ then $|z| = |z-1|$

$\Rightarrow$ If $z + \bar{z} < 0$ then $|z|^2 = |z|^2 + 1 - z - \bar{z}$

$\Rightarrow$ If $z + \bar{z} < 0$ then $z + \bar{z} = 1$, which is not possible.

Again If $|z+1| > |z-1|$ then max $\{|z-1|, |z+1|\} = |z+1|$

$\Rightarrow$ If $|z|^2 + 1 + z + \bar{z} > |z|^2 + 1 - z - \bar{z}$ then $|z| = |z+1|$

$\Rightarrow$ If $z + \bar{z} > 0$ then $|z|^2 = |z|^2 + 1 + z + \bar{z}$

$\Rightarrow$ If $z + \bar{z} > 0$ then $z + \bar{z} = -1$ Not possible again.

Therefore the given result cannot hold.

46. **(b)** If α, β be the roots of the equation then

$\alpha + \beta = a - 2, \quad \alpha\beta = -\frac{a+1}{2}$

Sum of square of roots

$S = \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (a-2)^2 + (a+1) = a^2 - 3a + 5$

$S = a^2 - 3a + \frac{9}{4} + \frac{11}{4}$

$S = \left(a - \frac{3}{2}\right)^2 + \frac{11}{4}$

Clearly S is least when

$a - \frac{3}{2} = 0 \Rightarrow a = \frac{3}{2}$

47. **(c)** $x^2 + 6x - 7 < 0$

$\Rightarrow (x+7)(x-1) < 0$

$\Rightarrow x = (-7, 1)$

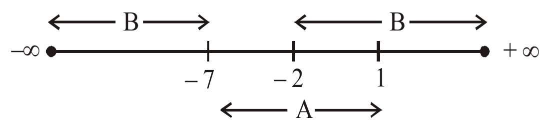
$\Rightarrow A = \{-7, -6, -5, -4, -3, -2, -1, 0, 1\}$

$\Rightarrow x^2 + 9x + 14 > 0$

$\Rightarrow (x+7)(x+2) > 0$

$\Rightarrow x = (-\infty, -7) \cup (-2, \infty)$

$\Rightarrow B = R - \{-7, -6, -5, -4, -3, -2\}$



So $A \cap B = (-2, 1)$

$A|B = (-7, -2)$.

48. **(b)** Given equation is $x^2 + px + q = 0$

Sum of roots = tan30° + tan15° = – p

Product of roots = tan30° . tan15° = q

$\tan 45° = \frac{\tan 30° + \tan 15°}{1 - \tan 30°.\tan 15°} = \frac{-p}{1-q} = 1$

$\Rightarrow \quad -p = 1 - q \Rightarrow q - p = 1$

$\therefore 2 + q - p = 3$

49. **(a)** We have $|z+1| = |z+4-3|$ ...(i)

Now $|z+4-3| \le |z+4| + |-3| \le 3+3 = 6$

[Given $|z+4| \le 3$ & $|-3| = 3$] $\therefore |z+1| \le 6$

Again $|z+1| \ge 0$ [modulus is always non-negative]

$\therefore$ Least value of $|z+1|$ may be zero, which occurs when $z = -1$, For $z = -1$, $|z+4| = |-1+4| = 3$

Which satisfies the given condition that $|z+4| \ge 3$

Hence, the least and the greatest values of $|z+1|$ are 0 and 6

50. **(a)** Consider $|\bar{z}\omega| = |\bar{z}||\omega| = |z||\omega| = |z\omega| = 1$

Consider

$\text{Arg}(\bar{z}\omega) = \arg(\bar{z}) + \arg(\omega) = -\arg(z) + \arg\omega$

$= -\dfrac{\pi}{2} \quad \therefore \bar{z}\omega = -i$

51. **(c)** $\dfrac{\alpha}{a} + \dfrac{\beta}{b} + \dfrac{\gamma}{c} = 1 + i$ squaring

$$\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} + \frac{\gamma^2}{c^2} + 2\left(\frac{\alpha\beta}{ab} + \frac{\beta\gamma}{bc} + \frac{\gamma\alpha}{ac}\right) = 2i$$

or $$\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} + \frac{\gamma^2}{c^2} + \frac{2\alpha\beta\gamma}{abc}\left(\frac{c}{\gamma} + \frac{a}{\alpha} + \frac{b}{\beta}\right) = 2i$$

$$\therefore \quad \frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} + \frac{\gamma^2}{c^2} = 2i$$

52. **(d)** $\displaystyle\sum_{k=33}^{65}\left(\sin\frac{2k\pi}{8} - i\cos\frac{2k\pi}{8}\right)$

$$= \left[\sin\frac{33\pi}{4} + \sin\frac{34\pi}{4} + \ldots + \sin\frac{65\pi}{4}\right]$$

$$- i\left[\cos\frac{33\pi}{4} + \cos\frac{34\pi}{4} + \ldots + \cos\frac{65\pi}{4}\right]$$

$$= \sin\frac{\pi}{4} - i\cos\frac{\pi}{4}$$

$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots + \sin[\alpha+(n-1)\beta]$

$$= \frac{\sin\left\{\alpha + (n-1)\frac{\beta}{2}\right\} \cdot \sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}$$

and $\cos(\alpha) + \cos(\alpha+\beta) + \ldots + \cos(\alpha + (n-1)\beta)$

$$= \frac{\cos\left\{\alpha + (n-1)\frac{\beta}{2}\right\}\sin\left(\frac{n\beta}{2}\right)}{\sin\frac{\beta}{2}} = -\left(\frac{1+i}{\sqrt{2}}\right) = \frac{1-i}{\sqrt{2}}$$

53. **(d)** $$\left(\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}\right)^2 = \frac{\alpha^2+\beta^2}{\alpha\beta} + 2 = \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{\left(-\frac{b}{a}\right)^2}{\left(\frac{b}{a}\right)} = \frac{b}{a}$$

$$\therefore \quad \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \sqrt{\frac{b}{a}} \qquad [\because \alpha, \beta \text{ are real}]$$

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}} = 2\sqrt{\frac{b}{a}}$$

54. **(c)** $x + \dfrac{1}{x} = 1$ or $x^2 - x + 1 = 0$

$$\therefore \quad x = \frac{1}{2} \pm i\frac{\sqrt{3}}{2} \text{ or } x = e^{\frac{i\pi}{3}}$$

$$\therefore \quad x^a + x^{-a} = e^{\frac{ia\pi}{3}} + e^{\frac{-ia}{3}} = 2\cos\frac{a\pi}{3}$$

Hence, $\cos\dfrac{a\pi}{3} + \cos\dfrac{b\pi}{3} + \cos\dfrac{c\pi}{3} = 0$

$a, b, c \in I \;\therefore\; a+b+c|_{min} = (1+3+5) = 9$

55. **(d)** $z + |z| = 8 + 12i$

$\Rightarrow x + iy + \sqrt{x^2+y^2} = 8 + 12i$

$\Rightarrow x + \sqrt{x^2+y^2} = 8$ ...(i) & $y = 12$ ...(ii)

$(x = -5)$ So, $z = -5 + 12i$

$\Rightarrow |z| = \sqrt{25+144} = 13$

$\Rightarrow |z^2| = |z|^2 = 169$

56. **(b)** Let α be the common root of given equations, then

$\alpha^2 + b\alpha - 1 = 0$ ...(i)

and $\alpha^2 + \alpha + b = 0$ ...(ii)

Subtracting (ii) from (i), we get

$(b-1)\alpha - (b+1) = 0$ or $\alpha = \dfrac{b+1}{b-1}$

Substituting this value of α in equation (i), we get

$$\left(\frac{b+1}{b-1}\right)^2 + b\left(\frac{b+1}{b-1}\right) - 1 = 0$$

or $b^3 + 3b = 0 \Rightarrow b = 0, i\sqrt{3}, -i\sqrt{3}$

57. **(b)** For $x \geq -2$, $x^2 - x - 2 + x > 0$

$\Rightarrow x^2 > 2 \Rightarrow x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$\Rightarrow x \in [-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ For $x < -2$

$x^2 + x + 2 + x > 0$ or $x^2 + 2x + 2 > 0$

which is true for all x.

Hence $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

58. **(d)** $z^n = (z+1)^n \Rightarrow |z|^n = |z+1|^n$

or $|z| = |z+1|$.

So the distance of point z remain same from (0, 0) and (–1, 0).

So, z lies on perpendicular bisector of line joining (0, 0) and (–1, 0) that is on $x = -\frac{1}{2}$

59. **(c)** Given equation is

$$x^2 + (2+\lambda)x - \frac{1}{2}(1+\lambda) = 0$$

So $\alpha + \beta = -(2+\lambda)$ and $\alpha\beta = -\left(\frac{1+\lambda}{2}\right)$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$$\Rightarrow \alpha^2 + \beta^2 = [-(2+\lambda)]^2 + 2\frac{(1+\lambda)}{2}$$

$$\Rightarrow \alpha^2 + \beta^2 = \lambda^2 + 4 + 4\lambda + 1 + \lambda = \lambda^2 + 5\lambda + 5$$

Which is minimum for $\lambda = -5/2$.

60. **(d)** $\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1}$; $\frac{z-1}{z+1} = \frac{x^2+y^2-1+2iy}{x^2+y^2+2x+1}$

$$\Rightarrow \text{Re}\left(\frac{z-1}{z+1}\right) = \frac{x^2+y^2-1}{x^2+y^2+2x+1} = 0$$

$$\Rightarrow x^2 + y^2 - 1 = 0 \quad \Rightarrow x^2 + y^2 = 1$$

Also, $z\bar{z} = x^2 + y^2 = 1$

and $z\bar{z} = |z|^2 \Rightarrow |z|^2 = 1$

$\Rightarrow |z| = 1$

61. **(a)** Given quadratic equation

is $ax^2 + bx + c = 0$ whose one root is $\frac{1}{2-\sqrt{-2}}$

Consider $\frac{1}{2-\sqrt{-2}} = \frac{1}{2-\sqrt{2}i} \times \frac{2+\sqrt{2}i}{2+\sqrt{2}i}$

$$= \frac{2+\sqrt{2}i}{4+2} = \frac{2+\sqrt{2}i}{6}$$

$\therefore$ Another root will be $\frac{2-\sqrt{2}i}{6}$

($\because$ complex roots always occurs in pairs)

Thus, sum of roots $= \frac{2+\sqrt{2}i}{6} + \frac{2-\sqrt{2}i}{6} = \frac{4}{6}$

and product of roots $= \left(\frac{2+\sqrt{2}i}{6}\right)\left(\frac{2-\sqrt{2}i}{6}\right)$

$$= \frac{4+2}{36} = \frac{1}{6}$$

$\therefore$ Required equation is

x^2 – (sum of roots) x + (product of roots) = 0

$$x^2 - \frac{4}{6}x + \frac{1}{6} = 0$$

$$\Rightarrow \quad 6x^2 - 4x + 1 = 0$$

Thus, the values of *a, b, c* are 6, – 4, 1 respectively

62. **(a)** Given $2x = 3 + 5i \quad \Rightarrow x = \frac{3+5i}{2}$

Consider $x^3 = \frac{27 + 125i^3 + 225i^2 + 135i}{8}$

$$= \frac{27 - 125i - 225 + 135i}{8} \qquad \left(\because \begin{matrix} i^2 = -1 \\ i^3 = -i \end{matrix}\right)$$

$$= \frac{-198 + 10i}{8} = \frac{-99 + 5i}{4}$$

and $x^2 = \frac{9 + 25i^2 + 30i}{4}$

$$= \frac{9 - 25 + 30i}{4} = \frac{-8 + 15i}{2}$$

Now, Consider $2x^3 + 2x^2 - 7x + 72$

$$= \left(\frac{-99+5i}{2}\right) + (-8 + 15i) - \frac{7(3+5i)}{2} + 72$$

$$= -\frac{99}{2} + \frac{5i}{2} - 8 + 15i - \frac{21}{2} - \frac{35}{2}i + 72$$

$$= \left(-\frac{99}{2} - 8 - \frac{21}{2} + 72\right) + \left(\frac{5}{2} + 15 - \frac{35}{2}\right)i$$

$$= \frac{-99 - 16 - 21 + 144}{2} = \frac{8}{2} = 4$$

63. **(c)** Let $y = \frac{x^2 - bc}{2x - b - c}$

$\Rightarrow x^2 - 2yx + (b+c)y - bc = 0$

$\because x \in R$, so $4y^2 - 4(b+c)y + 4bc \ge 0$

$\Rightarrow \quad x \le b$ or $x \ge c \qquad (\because b < c)$

64. **(b)** Consider both equations

$px^2 + 2qx + r = 0$ ...(i)

and $qx^2 - 2\sqrt{pr}.x + q = 0$ ...(ii)

Since, both the equations are quadratic and have real roots, therefore from equation (1), we have

$\therefore\ 4q^2 - 4pr \ge 0$ (using discriminant)

$\Rightarrow\ q^2 \ge pr$ ...(iii)

and from second equation $4pr - 4q^2 \ge 0$

$\Rightarrow\ pr \ge q^2$ ...(iv)

From eqs. (iii) and (iv) we get $q^2 = pr$.

65. **(b)** $\frac{x^2 - 3x + 4}{x+1} > 1 \Rightarrow \frac{x^2 - 3x + 4}{x+1} - 1 > 0$

$\Rightarrow \frac{x^2 - 4x + 3}{x+1} > 0 \Rightarrow \frac{(x+1)(x-1)(x-3)}{(x+1)^2} > 0$

$\Rightarrow (x+1)(x-1)(x-3) > 0$ and $x \ne -1$

Using method of interval, we get,

$x \in (-1, 1) \cup (3, \infty)$

66. **(b)** Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

Given $\text{Re}(z_1) = |z_1 - 2|$, $\text{Re}(z_2) = |z_2 - 2|$

$\therefore\ y_1^2 - 4x_1 + 4 = 0$ and $y_2^2 - 4x_2 + 4 = 0$

So that $\frac{y_1 - y_2}{x_1 - x_2} = \frac{4}{y_1 + y_2}$ ...(i)

Given $\arg(z_1 - z_2) = \pi/\sqrt{3}$

$\Rightarrow\ \frac{y_1 - y_2}{x_1 - x_2} = \sqrt{3}$ ...(ii)

From (i) and (ii) $\Rightarrow y_1 + y_2 = \frac{4}{\sqrt{3}}$

67. **(d)** $z_1 + i\bar{z}_2 = 0 \Rightarrow \overline{(z_1 + i\bar{z}_2)} = 0$

$\Rightarrow\ i\bar{z}_1 + z_2 = 0 \Rightarrow z_2 = -i\bar{z}_1$

Further, $\arg(\bar{z}_1 z_2) = \frac{\pi}{3} \Rightarrow \arg(\bar{z}_1(-i\bar{z}_1)) = \frac{\pi}{3}$

$\Rightarrow\ \arg(\bar{z}_1) + \arg(-i) + \arg(\bar{z}_1) = \frac{\pi}{3}$

$\Rightarrow\ 2\arg(\bar{z}_1) - \frac{\pi}{2} = \frac{\pi}{3} \Rightarrow \arg(\bar{z}_1) = \frac{5\pi}{12}$

68. **(d)** We have,

$|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and

$|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$

Since, we know $|z|^2 = z\bar{z}$

Now, $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$

$\Rightarrow \left| |z_3|^2 z_1z_2 + |z_2|^2 z_1z_3 + |z_1|^2 z_2z_3 \right| = 12$

$\Rightarrow \left| z_3\bar{z}_3 z_1z_2 + z_2\bar{z}_2 z_1z_3 + z_1\bar{z}_1 z_2z_3 \right| = 12$

$\Rightarrow \left| z_1z_2z_3(\bar{z}_3 + \bar{z}_2 + \bar{z}_1) \right| = 12$

$\Rightarrow |z_1z_2z_3||\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 12$

$\Rightarrow |z_1||z_2||z_3||\bar{z}_1 + \bar{z}_2 + \bar{z}_2| = 12$

$\Rightarrow 1 \times 2 \times 3|\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 12$

$\Rightarrow |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 2$

$\Rightarrow |z_1 + z_2 + z_3| = 2$

69. **(d)** $z = \sqrt{3} + i$

$z^2 e^{z-i} = (\sqrt{3}+i)^2 e^{\sqrt{3}+i-i} = (3 - 1 + 2\sqrt{3}i)e^{\sqrt{3}} =$

$(2 + 2\sqrt{3}i)e^{\sqrt{3}}$

$\Rightarrow$ argument $= \tan^{-1}\frac{2\sqrt{3}}{2} = \tan^{-1}\sqrt{3}$

$\therefore\ \arg(z^2 e^{z-i}) = \pi/3$

70. **(d)** Given equation, $z^2(1 - z^2) = 16$, $z \in \mathbf{C}$

Now, let $z^2 = w = r(\cos\theta + i\sin\theta\}$ where $r > 0$.

$\therefore\ 1 - z^2 = \frac{16}{z^2} \Rightarrow z^2 + \frac{16}{z^2} = 1$

Modulas of $z^2 \Rightarrow r$

$\therefore$ Modulus of $\frac{16}{z^2} \Rightarrow \frac{16}{r}$

$$\Rightarrow \left(r+\frac{16}{r}\right)\cos\theta + i\left(r-\frac{16}{r}\right)\sin\theta = 1$$

On comparing real and imaginary parts, we get

$$\left(r+\frac{16}{r}\right)\cos\theta = 1 \text{ and} \left(r-\frac{16}{r}\right)\sin\theta = 0$$

$\cos\theta = 1 \Rightarrow r+\frac{16}{r} = 1$ (not possible)

or $r-\frac{16}{r} = 0 \Rightarrow r^2 = 16 \Rightarrow r = 4$

$\Rightarrow$ Modulus of $z^2 = |z|^2 = r = 4 \;\Rightarrow |z| = 2$

71. **(c)** $|z|^2 w - |w|^2 z = z - w$

$\Rightarrow z\bar{z}.w - w\bar{w}z = z - w$

$\Rightarrow zw(\bar{z}-\bar{w}) = z - w$ ...(i)

$\Rightarrow \overline{[zw(\bar{z}-\bar{w})]} = \overline{(z-w)}$

$\Rightarrow \bar{z}\bar{w}(z-w) = \bar{z}.\bar{w}$ ...(ii)

Multiply equations (i) and (ii)

$\Rightarrow |z|^2 |w|^2 |z-w|^2 = |z-w|^2$ [$\therefore$ |w| =]

$\Rightarrow |z\bar{w}|^2 = 1 \Rightarrow |z\bar{w}| = 1$

72. **(b)** Given equation $(x^2 + 5x + 5)^{x+5} = 1$

If $x + 5 = 0 \Rightarrow x = -5$

and $x^2 + 5x + 5 = 1$

or $x^2 + 5x + 4 = 0$

$\Rightarrow x = -1, -4$

$x = -1, -4$ and -5 are the three integers satisfying given equation.

73. **(a)** Given equation can be reduced to a quadratic equation.

$$\therefore 2x^2 + x - 11 + \frac{1}{x} + \frac{2}{x^2} = 0$$

$$\Rightarrow 2\left(x^2+\frac{1}{x^2}\right) + \left(x+\frac{1}{x}\right) - 11 = 0$$

Put $x+\frac{1}{x} = y$

$2(y^2 - 2) + y - 11 = 0 \Rightarrow 2y^2 + y - 15 = 0$

$\Rightarrow y = -3$ and $\frac{5}{2}$

$\Rightarrow x+\frac{1}{x} = -3, x+\frac{1}{x} = \frac{5}{2}$

$\Rightarrow x^2 + 3x + 1 = 0, 2x^2 - 5x + 2 = 0$

Only 2nd equation has rational roots as D $= 9$ and roots are $\frac{1}{2}$ and 2.

74. **(d)** The given equation is

$pqx^2 - (p+q)^2 x + (p+q)^2 = 0$

$$x = \frac{(p+q)^2 \pm \sqrt{(p+q)^4 - 4pq(p+q)^2}}{2pq}$$

$$x = \frac{(p+q)^2 \pm (p^2 - q^2)}{2pq}$$

Now, taking (+ve) sign

$x = \frac{p+q}{q}$ and taking (-ve) sign

$x = \frac{p+q}{p}$

$\therefore$ Solution set is $\left\{\frac{p+q}{p}, \frac{p+q}{q}\right\}$

75. **(d)** Given equation is

$|2x-1|^2 - 3|2x-1| + 2 = 0$

Let $|2x-1| = t$

$\therefore t^2 - 3t + 2 = 0$

$\Rightarrow (t-1)(t-2) = 0 \Rightarrow t = 1, 2$

$\Rightarrow |2x-1| = 1$ and $|2x-1| = 2$

$\Rightarrow 2x - 1 = \pm 1$ and $2x - 1 = \pm 2$

$\Rightarrow x = 1, 0$ and $x = \frac{3}{2}, -\frac{1}{2}$

76. **(b)** $\frac{m}{4n} = \frac{\ell}{m} \Rightarrow m^2 = 4n\ell \;\Rightarrow m^2 - 4n\ell = 0$

$\Rightarrow$ Discriminant of equation = 0

$\Rightarrow$ Roots are equal $\Rightarrow$ Both roots $= \frac{9}{2}$

$\therefore$ Equation can be written as $\left(x-\frac{9}{2}\right)\left(x-\frac{9}{2}\right) = 0$

$\Rightarrow 4x^2 - 36x + 81 = 0$

comparing with $\ell x^2 + mx + n = 0$

$\ell = 4, m = -36, n = 81 \Rightarrow \ell + n = 4 + 81 = 85$

77. **(a)** Since, roots are equal

$\therefore \{2(bc + ad)\}^2 = 4(a^2 + b^2)(c^2 + d^2)$

$\Rightarrow 4b^2c^2 + 4a^2d^2 + 8abcd$

$= 4a^2c^2 + 4a^2d^2 + 4b^2c^2 + 4b^2d^2$

$\Rightarrow 4a^2d^2 + 4b^2c^2 - 8abcd = 0$

$\Rightarrow 4(ad - bc)^2 = 0$

$\Rightarrow ad = bc \quad \Rightarrow \dfrac{a}{b} = \dfrac{c}{d}$

78. **(b)** $\left[\dfrac{x + i(x-2)}{3+i}\right] - i = \dfrac{2y + i(1-3y)}{i-3}$

$\Rightarrow (i-3)[x + i(x-2) - 3i + 1] = (3+i)[2y + i(1-3y)]$

$\Rightarrow i(-2x+16) + (-4x+2) = (9y-1) + i(3-7y)$

Now equating real and imaginary parts we have

$-4x + 2 = 9y - 1$

$\Rightarrow 4x + 9y = 3$ ...(i)

$-2x + 16 = 3 - 7y$

$\Rightarrow 2x - 7y = 13$ ...(ii)

$\Rightarrow x = 3$ & $y = -1$

$\therefore x + y = 3 - 1 = 2$

$\Rightarrow$ Option (b) is true.

79. **(d)** $\left(\dfrac{1-i}{1+i}\right)^{2022} + \left(\dfrac{1+i}{1-i}\right)^{2021}$

$= \left(\dfrac{(1-i)^2}{1-i^2}\right)^{2022} + \left(\dfrac{(1+i)^2}{1-i^2}\right)^{2021}$

$= \left(\dfrac{-2i}{2}\right)^{2022} + \left(\dfrac{2i}{2}\right)^{2021} = (-i)^{2022} + (i)^{2021}$

$= (i)^{2022} + (i)^{2021} = (i)^{4\times505+2} + (i)^{4\times505+1}$

$= (i)^2 + (i)^1 = -1 + i = i - 1$

80. **(a)** $\dfrac{1 - 10i\cos\theta}{1 - 10\sqrt{3}i\sin\theta} \times \dfrac{1 + 10\sqrt{3}i\sin\theta}{1 + 10\sqrt{3}i\sin\theta}$

(Rationalize numerator and denominator)

$= \dfrac{1 + 10\sqrt{3}i\sin\theta - 10i\cos\theta + 100\sqrt{3}\sin\theta\cos\theta}{1 + 300\sin^2\theta}$

$= \dfrac{1 + 100\sqrt{3}\sin\theta\cos\theta}{1 + 300\sin^2\theta} + i\dfrac{10\sqrt{3}\sin\theta - 10\cos\theta}{1 + 300\sin^2\theta}$

$\therefore$ It is purely real imaginary part = 0

$\Rightarrow \dfrac{10\sqrt{3}\sin\theta - 10\cos\theta}{1 + 300\sin^2\theta} = 0$

$\Rightarrow 1 + 300\sin^2\theta \neq 0$

$\therefore 10\sqrt{3}\sin\theta - 10\cos\theta = 0$

$\therefore \tan\theta = \dfrac{10}{10\sqrt{3}}$

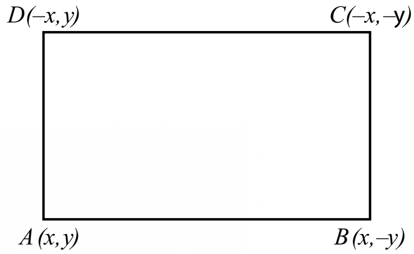
$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}} = \tan\left(\dfrac{\pi}{6}\right) \Rightarrow \theta = \dfrac{\pi}{6}$

81. **(a)** Let a complex number, $z = x + iy$

$\Rightarrow \bar{z} = \bar{x} - iy$

Then, vertices of rectangle for $z, \bar{z}, -z - \bar{z}$ are (x, y),

$(x, -y)(-x, -y)$, $(-x, y)$.



Now, Area of rectangle = $(2x)(2y) = 4xy$

It is given that,

Area $= 2\sqrt{3} = 4xy \Rightarrow 2xy = \sqrt{3}$

$\therefore x = \dfrac{1}{2}, y = \sqrt{3} \quad \therefore z = \dfrac{1}{2} + \sqrt{3}i$

82. **(a)** Given C is a complex number

$= \dfrac{1+iz}{1-iz} = \dfrac{1+a+ib-c}{1+a-ib+c} = \dfrac{(1+a-c)+ib}{(1+a+c)-ib}$

$= \dfrac{(1+a-c)+ib}{(1+a+c)-ib} \times \dfrac{(1+a+c)+ib}{(1+a+c)+ib} \left\{\text{Given, } z = \dfrac{b+ic}{1+a}\right\}$

Here $iz = \dfrac{ib-c}{1+a} \quad \Rightarrow -iz = \dfrac{-ib+c}{1+a}$

$= \dfrac{(1+a-c)(1+a+c) + (1+a-c)bi + (1+a+c)bi - b^2}{(1+a+c)^2 + b^2}$

$= \dfrac{(1+a+c+a+a^2+ac-c-ac-c^2-b^2) + ib(1+a-c+1+a+c)}{(1+a+c)^2+b^2}$

$= \dfrac{1+2a+a^2-(1-a^2)+2(1+a)ib}{1+a^2+c^2+2a+2ac+2c+b^2}$

$$= \frac{2a(1+a)+2(1+a)ib}{2+2a+2ac+2c} = \frac{2(1+a)(a+ib)}{2(1+a)+2c(1+a)}$$

$$= \frac{2(1+a)(a+ib)}{2(1+a)(1+c)} = \frac{a+ib}{1+c}$$

83. **(b)** It is given that,

Conjugate of $(x+iy)(1-2i)$ is $1 + i$.

i.e. $(x-iy)(1+2i) = 1+i \Rightarrow x-iy = \frac{1+i}{1+2i}$

Taking conjugate on both the sides, we get

$$x+iy = \frac{1-i}{1-2i}$$

84. **(d)** Given $a+bi = \frac{i}{1-i}$

$$\Rightarrow a+bi = \frac{i(1+i)}{2} = -\frac{1}{2}+\frac{i}{2} \Rightarrow a = -\frac{1}{2}, b = \frac{1}{2}$$

85. **(c)** $\left(\sum_{m=1}^{2n+1} i^{2m}\right)^{\sum_{k=1}^{k=107} i^k}$

$$= \begin{Bmatrix}(-1)+(-1)^2+(-1)^3+\ldots+(-1)^{2n+1} \\ (\text{i.e., } 2n+1 \text{ terms})\end{Bmatrix}^{(i+i^2+i^3+\ldots+i^{107})}$$

$$= (-1)^{i-1-i} = (-1)^{-1} = \frac{1}{(-1)^1} = -1$$

86. **(d)** **87.** **(d)**

88. **(d)** Let $z = x + iy$ then

Now, $\frac{\bar{z}-1}{\bar{z}-i} = \frac{(x-1)-iy}{x-i(y+1)} \times \frac{x+i(y+1)}{x+i(y+1)}$

$$= \frac{[x(x-1)+y(y+1)]+i[(y+1)(x-1)-xy]}{x^2+(y+1)^2}$$

$$\therefore \text{Im}\left(\frac{\bar{z}-1}{\bar{z}-i}\right) = \frac{xy-y+x-1-xy}{x^2+(y+1)^2} = 1 \quad \text{(given)}$$

$$\Rightarrow x^2+y^2-x+3y+2 = 0, (x, y) \neq (0, -1)$$

Hence, option (d) is correct.

89. **(b)** Given the point A (–2, 1), B (3, –4)

& $\frac{AC}{BC} = \frac{1}{2}$

So, $C \equiv \left(\frac{1\times 3-4}{3}, \frac{-4+2}{3}\right) = \left(\frac{-1}{3}, \frac{-2}{3}\right)$

Now, argument of C $= \tan^{-1}\left(\frac{\frac{-2}{3}}{\frac{-1}{3}}\right) - \pi$

$= \tan^{-1}(2) - \pi$

90. **(d)** $z_1 + i\bar{z}_2 = 0 \Rightarrow (\overline{z_1 + i\bar{z}_2}) = 0$

$\Rightarrow i\bar{z}_1 + z_2 = 0 \Rightarrow z_2 = -i\bar{z}_1$

Further, $\arg(\bar{z}_1 z_2) = \frac{\pi}{3}$

$\Rightarrow \arg(\bar{z}_1(-i\bar{z}_1)) = \frac{\pi}{3}$

$\Rightarrow \arg(\bar{z}_1) + \arg(-i) + \arg(\bar{z}_1) = \frac{\pi}{3}$

$\Rightarrow 2\arg(\bar{z}_1) - \frac{\pi}{2} = \frac{\pi}{3} \Rightarrow \arg(\bar{z}_1) = \frac{5\pi}{12}$

91. **(a)** We have,

$a > 0, z = x + iy$

$\log_{\cos^2\theta}|z-a| > \log_{\cos^2\theta}|z-ai|$

We know that, ;$0 < \cos^2\theta < 1$

So, $|z-a| < |z-ai|$

$(x-a)^2 + y^2 < x^2 + (y-a)^2$

$x^2 + a^2 - 2ax + y^2 < x^2 + y^2 + a^2 - 2ay$

$-2ax < -2ay \Rightarrow x > y$

92. **(d)** $z = \sqrt{3} + i$

$z^2 e^{z-i} = (\sqrt{3}+i)^2 e^{\sqrt{3}+i-i}$

$= (3-1+2\sqrt{3}i)e^{\sqrt{3}}$

$= (2+2\sqrt{3}i)e^{\sqrt{3}}$

$\Rightarrow$ argument $= \tan^{-1}\frac{2\sqrt{3}}{2} = \tan^{-1}\sqrt{3}$

$\therefore \arg(z^2 e^{z-i}) = \pi/3$

93. **(d)** Given equation, $z^2(1-z^2) = 16$, $z \in \mathbf{C}$

Now, let $z^2 = w = r(\cos\theta + i\sin\theta\}$ where $r > 0$.

$\therefore \quad 1-z^2 = \frac{16}{z^2} \Rightarrow z^2 + \frac{16}{z^2} = 1$

Modulas of $z^2 \Rightarrow r$

$\therefore$ Modulus of $\frac{16}{z^2} \Rightarrow \frac{16}{r}$

$\Rightarrow \left(r+\frac{16}{r}\right)\cos\theta + i\left(r-\frac{16}{r}\right)\sin\theta = 1$

On comparing real and imaginary parts, we get

$\left(r+\frac{16}{r}\right)\cos\theta = 1$ and $\left(r-\frac{16}{r}\right)\sin\theta = 0$

$\cos\theta = 1 \Rightarrow r+\frac{16}{r} = 1$ (not possible)

or $r-\frac{16}{r} = 0 \Rightarrow r^2 = 16 \Rightarrow r = 4$

$\Rightarrow$ Modulus of $z^2 = |z|^2 = r = 4 \Rightarrow |z| = 2$

94. **(a)** Given $z_1 = 2 - i$, $z_2 = 6 + 3i$

$\text{amp}\left(\frac{z_1 - z_2}{z_1 + z_2}\right) = \text{amp}\left(\frac{-4-4i}{8+2i}\right)$

$= \text{amp}(-4 - 4i) - \text{amp}(8 + 2i)$

$= \left[\tan^{-1}(1) - \pi\right] - \left[\tan^{-1}\left(\frac{2}{8}\right)\right]$

$= \left(\frac{\pi}{4} - \pi\right) - \tan^{-1}\left(\frac{1}{4}\right) = -\frac{3\pi}{4} - \tan^{-1}\left(\frac{1}{4}\right)$

95. **(b)** Given equation $(x^2 + 5x + 5)^{x+5} = 1$

If $x + 5 = 0 \Rightarrow x = -5$ and $x^2 + 5x + 5 = 1$

or $x^2 + 5x + 4 = 0 \Rightarrow x = -1, -4$

$x = -1, -4$ and -5 are the three integers satisfying given equation.

96. **(a)** Since, roots are equal

$\therefore \{2(bc + ad)\}^2 = 4(a^2 + b^2)(c^2 + d^2)$

$\Rightarrow 4b^2c^2 + 4a^2d^2 + 8abcd$

$= 4a^2c^2 + 4a^2d^2 + 4b^2c^2 + 4b^2d^2$

$\Rightarrow 4a^2d^2 + 4b^2c^2 - 8abcd = 0$

$\Rightarrow 4(ad - bc)^2 = 0 \Rightarrow ad = bc \Rightarrow \frac{a}{b} = \frac{c}{d}$

97. **(d)** The given equation is

$pqx^2 - (p + q)^2 x + (p + q)^2 = 0$

$$x = \frac{(p+q)^2 \pm \sqrt{(p+q)^4 - 4pq(p+q)^2}}{2pq}$$

$$x = \frac{(p+q)^2 \pm (p^2 - q^2)}{2pq}$$

Now, taking (+ve) sign

$x = \frac{p+q}{q}$ and taking (-ve) sign $x = \frac{p+q}{p}$

$\therefore$ Solution set is $\left\{\frac{p+q}{p}, \frac{p+q}{q}\right\}$

98. **(c)** Here we are given that $3p^3x^3 + px^2 + qx + 3 = 0$

When p = 1 and q = –7

Now $(x - 1)(3x^2 + 4x - 3) = 0$

$\Rightarrow x = 1, x = \frac{2 \pm \sqrt{13}}{3}$

$\Rightarrow \alpha = \frac{2 \pm \sqrt{13}}{3}, \ \beta = \frac{2 - \sqrt{13}}{3}$

$\Rightarrow |\alpha - \beta| = \left|\frac{2 + \sqrt{13} - 2 + \sqrt{13}}{3}\right| = \frac{2\sqrt{13}}{3}$

99. **(c)** Given equation is

$(p - q)x^2 + (q - r)x + (r - p) = 0$

$$\Rightarrow x = \frac{(r-q) \pm \sqrt{(q-r)^2 - 4(r-p)(p-q)}}{2(p-q)}$$

$$= \frac{(r-q) \pm \sqrt{q^2 + r^2 - 2qr - 4(rp - rq - p^2 + pq)}}{2(p-q)}$$

$\Rightarrow x = \frac{(r-q) \pm (q + r - 2p)}{2(p-q)} \quad \Rightarrow x = \frac{r-p}{p-q}, 1$

100. **(d)** Given that $3 + i\sqrt{6}$ is one roots therefore $3 - i\sqrt{6}$ is also a root. Let α and β are other two real roots

$\therefore$ product of roots $= \frac{-45}{4}$

$\Rightarrow \alpha.\beta = (3+i\sqrt{6})(3-i\sqrt{6}) = \frac{-45}{4}$

$\Rightarrow \alpha.\beta = (9+6) = \frac{-45}{4} \Rightarrow \alpha.\beta = \frac{-3}{4}$

101. **(c)** Since α, β, γ are the roots of the equation $x^3 - ax^2 + bx - c = 0$ ∴ α + β + γ = a

αβ + βγ + γα = b αβγ = c

Now, $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2} = \frac{\beta^2\gamma^2+\alpha^2\gamma^2+\alpha^2\beta^2}{\alpha^2\beta^2\gamma^2}$

$= \frac{(\alpha\beta+\beta\gamma+\gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha+\beta+\gamma)}{(\alpha\beta\gamma)^2}$

$= \frac{b^2 - 2\times c\times a}{c^2} = \frac{b^2-2ca}{c^2}$

102. **(b)** Let $f(x) = x^2 + 7x + 3 = 0$

Roots are $\frac{\alpha}{\alpha+1}$ and $\frac{\beta}{\beta+1}$.

Equation having roots α and β is

$f\left(\frac{x}{x+1}\right) = 0 \quad \left(\frac{x}{x+1}\right)^2 + 7\left(\frac{x}{x+1}\right) + 3 = 0$

$\Rightarrow x^2 + 7x(x+1) + 3(x+1)^2 = 0$

$\Rightarrow x^2 + 7x^2 + 7x + 3(x^2 + 2x + 1) = 0$

$\Rightarrow 11x^2 + 13x + 3 = 0$

103. **(d)** Given equation $x^2 - 2\sqrt{3}x + 4 = 0$ with α & β roots of the equation.

$\alpha+\beta = \frac{-b}{a} = 2\sqrt{3}$ ...(i)

$\alpha\beta = \frac{c}{a} = 4$ ...(ii)

Now, $(\alpha^6 + \beta^6) = (\alpha^2)^3 + (\beta^2)^3$

$= (\alpha^2 + \beta^2)\,[\alpha^4 + \beta^4 - \alpha^2\beta^2]$

$= (\alpha^2 + \beta^2)\,[(\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 - \alpha^2\beta^2]$

$= (\alpha^2 + \beta^2)\,[(\alpha^2 + \beta^2)^2 - 3\alpha^2\beta^2]$ ...(iii)

From (i), take square both sides,

$\alpha^2 + \beta^2 + 2\alpha\beta = 12$

$\alpha^2 + \beta^2 = 12 - 2\times 4 = 12 - 8 = 4$

Put the values in eq. (iii),

$\alpha^6 + \beta^6 = (4)\,[(4)^2 - 3(4)^2]$

$= 4[16 - 48] = 4\times(-32) = -128$

So, option (d) is correct.

104. **(c)** ∵ $(\alpha+\beta+\gamma)^3 = (\alpha^3+\beta^3+\gamma^3) + 3(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$

∵ $\alpha+\beta+\gamma = -1;\ \alpha\beta+\beta\gamma+\gamma\alpha = 1;\ \alpha\beta\gamma = -r$

∴ $-1 = 5 + 3(-1-\gamma)(\beta+\gamma)(\gamma+\alpha)$

⇒ $2 = 1 + \gamma^2 + \gamma + \gamma^3 \Rightarrow \gamma^3 + \gamma^2 + \gamma - 1 = 0$

⇒ $r = -1$

105. **(b)** It is given that α, β and γ are the roots of $x^3 - 6x^2 + 11x + 6 = 0$

Then we know that α + β + γ = 6, αβ + βγ + γα = 11 and αβγ = – 6 ; $\Sigma\alpha^2\beta + \Sigma\alpha\beta^2$

$= \alpha^2\beta + \beta^2\gamma + \gamma^2\alpha + \alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2$

$= \alpha^2\beta + \alpha\beta\gamma + \alpha^2\gamma + \alpha\beta^2 + \beta^2\gamma + \alpha\beta\gamma + \alpha\beta\gamma + \beta\gamma^2 + \gamma^2\alpha - 3\alpha\beta\gamma$

$= (\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha) - 3\alpha\beta\gamma$

$= (6)(11) - 3(-6) = 66 + 18 = 84$

106. **(a)** $(\alpha+\beta)^2 + (\alpha-\beta)^2 = (\alpha+\beta)^2 + (\alpha+\beta)^2 - 4\alpha\beta$

$= 2(-p)^2 - 4\left(\frac{p^2}{6}\right) = \frac{4p^4}{3}$

and $(\alpha+\beta)^2\times(\alpha-\beta)^2 = (\alpha+\beta)^2\,[(\alpha+\beta)^2 - 4\alpha\beta]$

$= (-p)^2\left\{(-p)^2 - \frac{4p^2}{6}\right\} = \frac{p^4}{3}$

∴ Required quadratic equation is

$x^2 - \frac{4p^2}{3}x + \frac{p^4}{3} = 0$

or $3x^2 - 4p^2x + p^4 = 0$.

107. **(a)** We have, $ix^2 + (i-1)x - \frac{1}{2} - i = 0$

$\Rightarrow x = \frac{-(i-1) \pm \sqrt{(i-1)^2 - 4(i)\left(-\frac{1}{2}-i\right)}}{2i}$

$= \frac{-(i-1) \pm \sqrt{-4}}{2i}$

Thus, roots are imaginary. Also, we have $b^2 - 4ac = -4 < 0$ and it is the correct reason for which roots are imaginary.

108. (a) By definition, both the statements are correct.

109. (c) We have, $\arg(\bar{z}_1) = \frac{\pi}{5}, \arg(z_2) = \frac{\pi}{3}$

$\therefore \arg(z_1 z_2) = \arg(z_2) - \arg(\bar{z}_1) = \frac{\pi}{3} - \frac{\pi}{5} = \frac{2\pi}{15}$

$[\because \arg(\bar{z}) = -\arg(z)]$

Statement I is true but Statement II is false.

110. (d) Since $z_1(z_2 \cdot z_3) = z_1 z_2 z_3 \Rightarrow$ Statement 1 is incorrect.

Since $(z_2 + z_3) \cdot z_1 = z_2 z_1 + z_3 z_1 \Rightarrow$ Statement 2 is correct.

111. (4) $z^3 + \frac{3|z|^2}{z} = 0, \Rightarrow z^3 + \frac{3z.\bar{z}}{z} = 0$

$\Rightarrow z^3 + 3\bar{z} = 0$; Let $z = re^{i\theta}$

$\Rightarrow r^3 e^{i3\theta} + 3re^{-i\theta} = 0$

$\Rightarrow e^{i4\theta} = -1 \quad [\because r = \sqrt{3}]$

$\Rightarrow \cos 4\theta + i \sin 4\theta = -1$

$\Rightarrow \cos 4\theta = -1$ ...(i)

Now $0 \le \theta < 2\pi \Rightarrow 0 \le 4\theta < 8\pi$

$\therefore \theta = \pi, 3\pi, 5\pi, 7\pi$

112. (4) $\because z_0$ is a root of quadratic equation $x^2 + x + 1 = 0$

$\therefore z_0 = \omega$ or $\omega^2 \rfloor z_0^3 = 1 \;\therefore z = 3 + 6i z_0^{81} - 3i z_0^{93}$

$= 3 + 6i((z_0)^3)^{27} - 3i((z_0)^3)^{31}$

$= 3 + 6i - 3i = 3 + 3i$

$\therefore \arg(z) = \tan^{-1}\left(\frac{3}{3}\right) = \frac{\pi}{4}$

113. (2) $z = 1 + 2i \Rightarrow |z| = \sqrt{1+4} = \sqrt{5}$

$\therefore f(z) = \frac{7-z}{1-z^2} = \frac{7-1-2i}{1-(1+2i)^2}$

$= \frac{6-2i}{1-(1-4+4i)} = \frac{6-2i}{4-4i} = \frac{3-i}{2-2i}$

$\Rightarrow |f(z)| = \left|\frac{3-i}{2-2i}\right| = \frac{|3-i|}{|2-2i|}$

$= \frac{\sqrt{9+1}}{\sqrt{4+4}} = \frac{\sqrt{5}}{2} = \frac{|z|}{2}$

114. (3) Since, $|z| + z = 3 + i$

Let $z = a + ib$, then

$|z| + z = 3 + i \Rightarrow \sqrt{a^2+b^2} + a + ib = 3 + i$

Compare real and imaginary coefficients on both sides

$b = 1, \sqrt{a^2+b^2} + a = 3 \quad \sqrt{a^2+1} = 3 - a$

$a^2 + 1 = a^2 + 9 - 6a$; $6a = 8$

$a = \frac{4}{3}$ Then, $|z| = \sqrt{\left(\frac{4}{3}\right)^2 + 1} = \sqrt{\frac{16}{9} + 1} = \frac{5}{3}$

115. (2) Let $t = \frac{z-\alpha}{z+\alpha}$

$\because t$ is purely imaginary number.

$\therefore \quad t + \bar{t} = 0 \Rightarrow \frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\alpha}{\bar{z}+\alpha} = 0$

$\Rightarrow (z-\alpha)(\bar{z}+\alpha) + (\bar{z}-\alpha)(z+\alpha) = 0$

$\Rightarrow \bar{z}z - \alpha^2 + z\bar{z} - \alpha^2 = 0$

$\Rightarrow z\bar{z} - \alpha^2 = 0 \Rightarrow |z|^2 - \alpha^2 = 0$

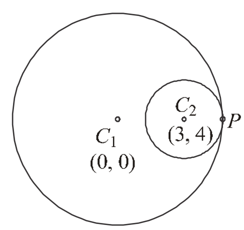
$\Rightarrow \alpha^2 = 4 \quad \Rightarrow \alpha = \pm 2$

116. (0) $|z_1| = 9, |z_2 - 3 - 4i| = 4$

z_1 lies on a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$

z_2 lies on a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$

So, minimum value of $|z_1 - z_2|$ is zero at point of contact (i.e. A)



117. (4) The given quadratic equation is $x^2 - 2x + 2 = 0$

Then, the roots of the this equation are

$\frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$ Now, $\frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = i$

or $\frac{\alpha}{\beta}=\frac{1-i}{1+i}=\frac{(1-i)^2}{1-i^2}=i$ So, $\frac{\alpha}{\beta}=\pm i$

Now, $\left(\frac{\alpha}{\beta}\right)^n=1$ $(\pm i)^n=1$

n must be a multiple of 4.
Hence, the required least value of $n = 4$.

118. (1) Let $z \in S$ then $z=\frac{\alpha+i}{\alpha-i}$

Since, z is a complex number and let $z = x + iy$

Then, $x+iy=\frac{(\alpha+i)^2}{\alpha^2+1}$ (by rationalisation)

$\Rightarrow x+iy=\frac{(\alpha^2-1)}{\alpha^2+1}+\frac{i(2\alpha)}{\alpha^2+1}$

Then compare both sides

$x=\frac{\alpha^2-1}{\alpha^2+1}$ ...(i)

$y=\frac{2\alpha}{\alpha^2+1}$ ...(ii)

Now squaring and adding equations (i) and (ii)

$\Rightarrow x^2+y^2=\frac{(\alpha^2-1)^2}{(\alpha^2+1)^2}+\frac{4\alpha^2}{(\alpha^2+1)^2}=1$

119. (2) $x^2 - (5 + i)x + (18 - i) = 0$;
Here $a = 1$, $b = -(5 + i)$, $c = 18 - i$

$\therefore$ D $= b^2 - 4ac = \{-(5 + i)\}^2 - 4(1)(18 - i)$
$= 25 + i^2 + 10i - 72 + 4i$
$= 25 - 1 + 14i - 72 = -48 + 14i$
$= -49 + 1 + 14i = (7i + 1)^2$

$\therefore\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{(5+i)\pm\sqrt{(7i+1)^2}}{2(1)}$

$=\frac{(5+i)\pm(7i+1)}{2}=\frac{6+8i}{2},\frac{4-6i}{2}=3+4i, 2-3i$

120. (5) We have, $z_1(z_1^2-3z_2^2)=2$ ...(i)

and $z_2(3z_1^2-z_2^2)=11$ ...(ii)

multiplying Eq. (ii) by $i(\sqrt{-1})$ and then adding in Eq. (i), we get

$z_1^3-3z_1z_2^2+i(3z_1^2z_2-z_2^3)=2+11i$

$\Rightarrow (z_1+iz_2)^3=2+11i$ ...(iii)

Again, multiplying Eq. (ii) by $(-i)$ and then adding in Eq. (i), we get

$z_1^3-3z_1z_2^2-i(3z_1^2z_2-z_2^3)=2-11i$

$\Rightarrow (z_1-iz_2)^3=2-11i$ ...(iv)

Now, on multiplying Eqs. (iii) and (iv), we get

$(z_1^2+z_2^2)^3=4+121=125=5^3$ $\therefore z_1^2+z_2^2=5$