BITSAT SOLVED PAPER 2023 SESSION-II

(memory based)

INSTRUCTIONS

- *This question paper contains total 130 questions divided into four parts:*

  Part I : Physics Q. No. 1 to 30

  Part II : Chemistry Q. No. 31 to 60

  Part III : (A) English Proficiency Q. No. 61 to 70

  (B) Logical Reasoning Q. No. 71 to 90

  Part IV : Mathematics Q. No. 91 to 130
- *All questions are multiple choice questions with four options, only one of them is correct.*
- *Each correct answer awarded 3 marks and –1 for each incorrect answer.*
- *Duration of paper-3 Hours*

PART - I : PHYSICS

1. On a temperature scale 'X'. The boiling point of water is 65° X and the freezing point is –15° X. Assume that the X scale is linear. The equivalent temperature corresponding to –95° X on the Farenheit scale would be:

(a) –63° F (b) –112° F

(c) –48° F (d) –148° F

2. In a cuboid of dimension $2L \times 2L \times L$, a charge q is placed at the centre of the surface 'S' having area of $4\ L^2$. The flux through the opposite surface to 'S' is given by

(a) $\frac{q}{12\varepsilon_0}$ (b) $\frac{q}{3\varepsilon_0}$

(c) $\frac{q}{2\varepsilon_0}$ (d) $\frac{q}{6\varepsilon_0}$

3. Ratio of thermal energy released in two resistor R and 3R connected in parallel in an electric circuit is :

(a) 3 : 1 (b) 1 : 1

(c) 1 : 3 (d) 1 : 27

4. The frequency (v) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as : $v = r^a \rho^b s^c$. The values of a, b and c respectively are

(a) $\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right)$ (d) $\left(-\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$

5. A passenger sitting in a train A moving at 90 km/h observes another train B moving in the opposite direction for 8 s. If the velocity of the train B is 54 km/h, then length of train B is :

(a) 80 m (b) 200 m

(c) 120 m (d) 320 m

6. A projectile is projected at 30° from horizontal with initial velocity 40 ms^{-1}. The velocity of the projectile at t = 2 s from the start will be:

(Given $g = 10\ m/s^2$)

(a) $20\sqrt{3}\ ms^{-1}$ (b) $40\sqrt{3}\ ms^{-1}$

(c) $20\ ms^{-1}$ (d) Zero

7. The magnetic moments associated with two closely wound circular coils A and B of radius $r_A = 10$ cm and $r_B = 20$ cm respectively are equal if: (Where N_A, I_A and N_B, I_B are number of turn and current of A and B respectively)

(a) $2N_AI_A = N_BI_B$ (b) $N_A = 2N_B$

(c) $N_AI_A = 4N_BI_B$ (d) $4N_AI_A = N_BI_B$

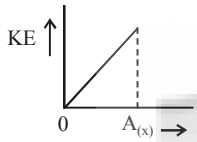
8. A 12 V battery connected to a coil of resistance 6 Ω through a switch, drives a constant current in the circuit. The switch is opened in 1 ms. The emf induced across the coil is 20 V. The inductance of the coil is :

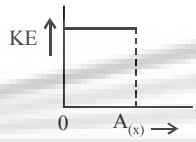
(a) 5 mH (b) 12 mH
(c) 8 mH (d) 10 mH

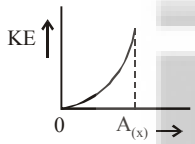
9. Two bodies are having kinetic energies in the ratio 16 : 9. If they have same linear momentum, the ratio of their masses respectively is:

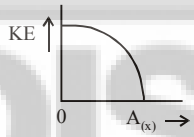
(a) 4 : 3 (b) 3 : 4
(c) 16 : 9 (d) 9 : 16

10. The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by

(a) 

(b) 

(c) 

(d) 

11. Two identical particles each of mass 'm' go round a circle of radius *a* under the action of their mutual gravitational attraction. The angular speed of each particle will be:

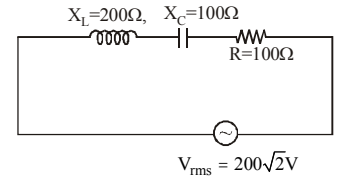
(a) $\sqrt{\frac{Gm}{2a^3}}$ (b) $\sqrt{\frac{Gm}{8a^3}}$

(c) $\sqrt{\frac{Gm}{4a^3}}$ (d) $\sqrt{\frac{Gm}{a^3}}$

12. Two resistances are given as $R_1 = (10 \pm 0.5)\Omega$ and $R_2 = (15 \pm 0.5)\Omega$. The percentage error in the measurement of equivalent resistance when they are connected in parallel is

(a) 6.33 (b) 2.33 (c) 4.33 (d) 5.33

13. In the given circuit, rms value of current (Irms) through the resistor R is :



(a) 2A (b) $\frac{1}{2}$A

(c) 20 A (d) $2\sqrt{2}$A

14. If $\vec{E}$ and $\vec{K}$ represent electric field and propagation vectors of the EM waves in vacuum, then magnetic field vector is given by : (ω - angular frequency) :

(a) $\frac{1}{\omega}(\vec{K}\times\vec{E})$ (b) $\omega(\vec{E}\times\vec{K})$

(c) $\omega(\vec{K}\times\vec{E})$ (d) $\vec{K}\times\vec{E}$

15. A thin prism P_1 with an angle 6° and made of glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without average deviation. The angle of prism P_2 is :

(a) 6° (b) 1.3° (c) 7.8° (d) 4.5°

16. The torque of a force $5\hat{i}+3\hat{j}-7\hat{k}$ about the origin is τ. If the force acts on a particle whose position vector is $2\hat{i}+2\hat{j}+\hat{k}$, then the value of τ will be :

(a) $11\hat{i}+19\hat{j}-4\hat{k}$

(b) $-11\hat{i}+9\hat{j}-16\hat{k}$

(c) $-17\hat{i}+19\hat{j}-4\hat{k}$

(d) $17\hat{i}+9\hat{j}+16\hat{k}$

17. If V is the gravitational potential due to sphere of uniform density on it's surface, then it's value at the center of sphere will be:

(a) $\frac{3V}{2}$ (b) V (c) $\frac{4}{3}$V (d) $\frac{V}{2}$

18. Young's moduli of the material of wires A and B are in the ratio of 1 : 4, while its area of cross sections are in the ratio of 1 : 3. If the same amount of load is applied to both the wires, the amount of elongation produced in the wires A and B will be in the ratio of
[Assume length of wires A and B are same]
(a) 36 : 1 (b) 12 : 1 (c) 1 : 36 (d) 1 : 12

19. A proton and an α-particle are accelerated from rest by 2V and 4V potentials, respectively. The ratio of their de-Broglie wavelength is:
(a) 4 : 1 (b) 2 : 1 (c) 8 : 1 (d) 16 : 1

20. The angular momentum for the electron in Bohr's orbit is L. If the electron is assumed to revolve in second orbit of hydrogen atom, then the change in angular momentum will be :
(a) $\frac{L}{2}$ (b) zero (c) L (d) 2L

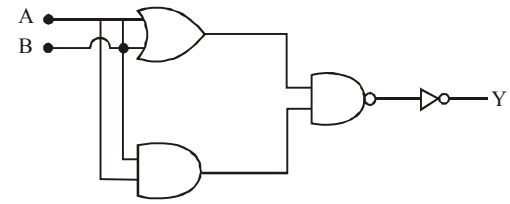
21. A hypothetical gas expands adiabatically such that its volume changes from 08 litres to 27 litres. If the ratio of final pressure of the gas to initial pressure of the gas is $\frac{16}{81}$. Then the ratio of $\frac{C_P}{C_V}$ will be.
(a) $\frac{4}{3}$ (b) $\frac{3}{1}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

22. A flask contains hydrogen and oxygen in the ratio of 2 : 1 by mass at temperature 27°C. The ratio of average kinetic energy per molecule of hydrogen and oxygen respectively is :
(a) 2 : 1 (b) 1 : 1 (c) 1 : 4 (d) 4 : 1

23. A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2 m long and 0.2 m in diameter. The magnetic intensity at the center of the solenoid when a current of 2 A flows through it is:
(a) $2.4 \times 10^3\ A\,m^{-1}$ (b) $1.2 \times 10^3\ A\,m^{-1}$
(c) $1\ A\,m^{-1}$ (d) $2.4 \times 10^{-3}\ A\,m^{-1}$

24. Two radioactive elements A and B initially have same number of atoms. The half life of A is same as the average life of B. If λ_A and λ_B are decay constants of A and B respectively, then choose the correct relation from the given options.
(a) $\lambda_A = \lambda_B$ (b) $\lambda_A = 2\lambda_B$
(c) $\lambda_A = \lambda_B \ln 2$ (d) $\lambda_A \ln 2 = \lambda_B$

25. The logic operations performed by the given digital circuit is equivalent to:



(a) AND (b) NOR
(c) OR (d) NAND

26. Eight equal drops of water are falling through air with a steady speed of 10 cm/s. If the drops coalesce, the new velocity is:
(a) 10 cm/s (b) 40 cm/s
(c) 16 cm/s (d) 5 cm/s

27. A bowl filled with very hot soup cools from 98°C to 86°C in 2 minutes when the room temperature is 22°C. How long it will take to cool from 75°C to 69°C ?
(a) 2 minutes (b) 1.4 minutes
(c) 0.5 minute (d) 1 minute

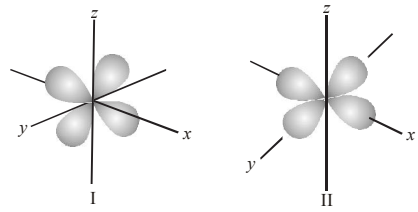
28. A capacitor of capacitance C is charged to a potential V. The flux of the electric field through a closed surface enclosing the positive plate of the capacitor is:
(a) $\frac{CV}{2\varepsilon_0}$ (b) $\frac{2CV}{\varepsilon_0}$
(c) $\frac{CV}{\varepsilon_0}$ (d) Zero

29. A single slit of a width *a* is illuminated by a monochromatic light of wavelength 600 nm. The value of 'a' for which first minimum appears at $\theta = 30°$ on the screen will be :
(a) 0.6 μm (b) 1.2 μm
(c) 1.8 μm (d) 3 μm

30. The engine of a train moving with speed $10 ms^{-1}$ towards a platform sounds a whistle at frequency 400Hz. The frequency heard by a passenger inside the train is (neglect air speed. Speed of sound in air $330 ms^{-1}$)
(a) 200Hz (b) 400Hz
(c) 412 Hz (d) 388 Hz

PART - II : CHEMISTRY

31. Observe the given boundary surface diagrams of two orbitals I and II and choose the correct option.



(a) $I - d_{x^2-y^2}$, $II - d_{yz}$

(b) $I - d_{yz}$, $II - d_{x^2-y^2}$

(c) $I - d_{xz}$, $II - d_{z^2}$

(d) $I - d_{xy}$, $II - d_{xz}$

32. How many elements are there in 6th period of periodic table?

(a) 18 (b) 8

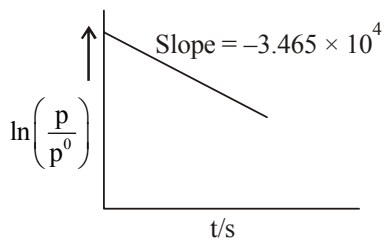
(c) 30 (d) 32

33. If the radius of the 3rd Bohr's orbit of hydrogen atom is r_3 and the radius of 4th Bohr's orbit is r_4. Then :

(a) $r_4 = \frac{9}{16} r_3$ (b) $r_4 = \frac{16}{9} r_3$

(c) $r_4 = \frac{3}{4} r_3$ (d) $r_4 = \frac{4}{3} r_3$

34. For the decomposition of azomethane, $CH_3N_2CH_3(g) \rightarrow CH_3CH_3(g) + N_2(g)$ a first order reaction, the variation in partial pressure with time at 600 K is given as



The half life of the reaction is .

(a) 2×10^{-5}s (b) 2×10^{-4}s

(c) 4×10^{-4}s (d) 2×10^{-3}s

35. In a solid lattice, the cation has left a lattice site and is located at an interstitial position, the lattice defect is :

(a) Interstitial defect

(b) Valency defect

(c) Frenkel defect

(d) Schottky defect

36. Energy required to completely separate one mole of a solid ionic compound into gaseous constituent ions is called ..... .

(a) Ionisation enthalpy

(b) Electron gain enthalpy

(c) Bond dissociation enthalpy

(d) Lattice enthalpy

37. Which of the following solutions of KCl will have the highest value of specific conductance?

(a) 1.0 N

(b) 0.1 N

(c) 1.0×10^{-2}N

(d) 1.0×10^{-3}N

38. In van der Waal's equation of state for a non-ideal gas, the term that accounts for intermolecular forces is

(a) $(V - b)$ (b) RT

(c) $\left(P + \frac{a}{V^2}\right)$ (d) $(RT)^{-1}$

39. Photochemical smog formed in congested metropolitan cities mainly consists of

(a) ozone, peroxyacetyl nitrate and NO_x

(b) smoke, peroxyacetyl nitrate and SO_2

(c) hydrocarbons, SO_2 and CO_2

(d) hydrocarbons, ozone and SO_x

40. Point out the false statement :

(a) The colloidal solution of a liquid in liquid is called gel

(b) Hardy Schulze rule is related with coagulation

(c) Brownian movement and Tyndall effect are shown by colloidal system

(d) Gold number is a measure of the protective power of lyophilic colloid

41. Given that bond energies of H – H and Cl – Cl are 430 kJ mol^{-1} and 240 kJ mol^{-1} respectively and ΔH_f for HCl is – 90 kJ mol^{-1}, bond enthalpy of HCl is

(a) 380 kJ mol^{-1} (b) 425 kJ mol^{-1}

(c) 245 kJ mol^{-1} (d) 290 kJ mol^{-1}

42. Which of the following acts as the stationary phase in paper chromatography ?

(a) Water

(b) Alumina

(c) Silica gel

(d) None of these

43. Thiamine and pyridoxine are also known respectively as :

(a) Vitamin B_2 and Vitamin E

(b) Vitamin E and Vitamin B_2

(c) Vitamin B_6 and Vitamin B_2

(d) Vitamin B_1 and Vitamin B_6

44. Which of the following catalyst is used in preparation of high density polythene ?

(a) Peroxide catalyst

(b) Ziegler - Natta catalyst

(c) Wilkinson's catalyst

(d) Pd - catalyst

45. Magnesium burns in CO_2 to form

(a) MgO + C (b) MgO + CO

(c) $MgCO_3$ (d) MgO.

46. Which one of the following exhibits geometrical isomerism?

(a) 1, 2-Dibromopropene

(b) 2, 3-Dimethylbut-2-ene

(c) 2, 3-Dibromobut-2-ene

(d) Both (a) and (c)

47. In reaction

$BF_3 + 3LiBH_4 \rightarrow 3LiF + X$; X is

(a) B_4H_{10} (b) B_2H_6

(c) BH_3 (d) B_3H_8

48. Which of the following is a hypnotic drug?

(a) luminal (b) salol

(c) catechol (d) chemisol

49. Primary amines can be distinguished from secondary and tertiary amines by reacting with

(a) Chloroform and alcoholic KOH

(b) Methyl iodide

(c) Chloroform alone

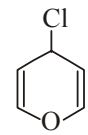
(d) Zinc dust

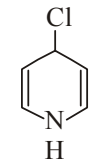
50. Which of the following can be used to prevent the decomposition of H_2O_2?

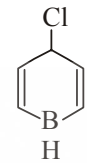
(a) Urea (b) Formaldehyde

(c) Formic acid (d) Ethanol

51. Identify correct reactivity order for S_N1 reaction

(i) 

(ii) 

(iii) 

(a) (i) > (ii) > (iii) (b) (ii) > (iii) > (i)

(c) (i) > (iii) > (ii) (d) (ii) > (i) > (iii)

52. Which of the following oxo acid of sulphur has O–O bond ?

(a) $H_2S_2O_7$ (b) $H_2S_2O_8$

(c) $H_2S_2O_6$ (d) $H_2S_2O_5$

53. The starting material for the manufacture of $KMnO_4$ is

(a) pyrolusite (b) manganite

(c) magnatite (d) haematite

54. Given that $E^{\circ}_{O_2/H_2O} = +1.23$ V;

$E^{\circ}_{S_2O_8^{2-}/SO_4^{2-}} = 2.05$ V ; $E^{\circ}_{Br_2/Br^-} = +1.09$ V

$E^{\circ}_{Au^{3+}/Au} = +1.4$ V

The strongest oxidising agent is :

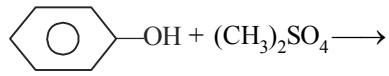
(a) Au^{3+} (b) O_2

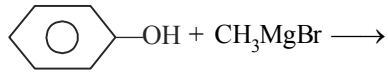
(c) $S_2O_8^{2-}$ (d) Br_2

55. Which will not result in the formation of anisole ?

(a) $PhOH + CH_3Cl \xrightarrow{OH^-}$

(b) $Ph—Cl + CH_3ONa \xrightarrow[300\ atm]{623K}$

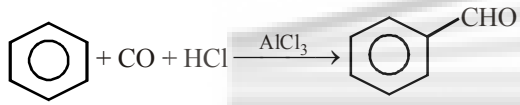
(c)  $+ (CH_3)_2SO_4 \longrightarrow$

(d)  $+ CH_3MgBr \longrightarrow$

56. The compounds $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$ constitutes a pair of
(a) coordination isomers
(b) linkage isomers
(c) ionization isomers
(d) optical isomers

57. Which of the following molecules acts as a Lewis acid ?
(a) $(CH_3)_2O$ (b) $(CH_3)_3P$
(c) $(CH_3)_3N$ (d) $(CH_3)_3B$

58. The reaction



(a) Rosenmund's reaction
(b) Stephen's reaction
(c) Cannizzaro's reaction
(d) Gatterman-Koch reaction

59. A 0.5 molal solution of ethylene glycol in water is used as coolant in a car. If the freezing point constant of water be 1.86°C per mole, the mixture shall freeze at
(a) 0.93°C (b) –0.93°C
(c) 1.86°C (d) –1.86°C

60. Identify the incorrect statement.
(a) Gangue is an ore contaminated with undesired materials.
(b) The scientific and technological process used for isolation of the metal from its ore is known as metallurgy.
(c) Minerals are naturally occurring chemical substances in the earth's crust.
(d) Ores are minerals that may contain a metal.

PART - III (A): ENGLISH PROFICIENCY

DIRECTIONS (Qs. 61-62): *In the following questions, out of the four alternatives, select the word similar in meaning to the word given.*

61. Eloquent
(a) Fluent (b) Ignorant
(c) Rude (d) Significant

62. Nefarious
(a) Iniquitous (b) Purposeful
(c) Suspicious (d) Virtuous

DIRECTIONS: *Pick out the opposite meaning or antonym of the word given below:*

63. CORPULENT
(a) sallow (b) co-operative
(c) enterprising (d) emaciated

DIRECTIONS (Qs. 64-65): *Pick out the most effective word(s) from the given words to fill in the blank to make the sentence meaningfully complete.*

64. Choose the correct word to complete the sentence :
She never _____ with him.
(a) agrees (b) agreeing
(c) was agreed (d) is agree

65. Choose the correct word to fill in the blank:
We____ in the London, but now we live in Delhi.
(a) lived (b) used to live
(c) were (d) happen to live

DIRECTIONS (Qs. 66-67) : *In the following items, some parts of the sentence have been jumbled up. You are required to re-arrange these parts which are labelled P, Q, R, and S to produce the correct sentence. Choose the proper sequence and mark accordingly.*

66. A school of psychology argues that

P : is one of the manifestations of impulse control disorder, a condition in which

Q : an act harmful for oneself or others

R : motorcycling — like gambling or skydiving –

S : an individual cannot resist the impulse or temptation to perform

Which one of the following is the correct sequence?

(a) R - P - S - Q (b) Q - S - P - R

(c) R - S - P - Q (d) Q - P - S - R

67. With six of its neighbours

P : there is a renewed warning for India

Q : and safeguard its own strategic interests

R : ranking high on global roster of failed states

S : to reassess its policy towards them

Which one of the following is the correct sequence?

(a) P - R - S - Q (b) R - P - Q - S

(c) P - R - Q - S (d) R - P - S - Q

DIRECTIONS (Qs. 68-70): *Read the given passage carefully and select the best answer to each question out of the four given alternatives.*

He was a vendor of sweets. He had his own peculiar method of advertising and doing business. He never depended upon others for help and worked hard all alone. I speak of Murali- the man who sold sweets. His customers were children, the future citizens of the world. At the stroke of nine in the morning, Murali would stand in front of the school with his tray of sweets. Till about eleven, the sale would be brisk. After that he moved off to other places. Even when the sweets became sticky in the heat, his business never slackened. There was depression in his business when the holidays came.

68. Who was Murali?

(a) a businessman

(b) a vendor of sweets

(c) a job seeker

(d) a student

69. Who were his customers?

(a) the children

(b) the adults

(c) the office goers

(d) the housewives

70. What time would he go to the school?

(a) at 10 in the morning

(b) at 8 in the morning

(c) at 3 in the afternoon

(d) at 9 in the morning

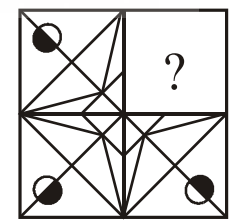
PART - III (B) : LOGICAL REASONING

71. On the following question, Select the releted letters/word from the alternative.

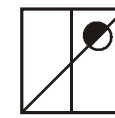
XXIV : 48 :: XIV : ?

(a) 28 (b) 24 (c) 14 (d) 30

72. Which answer figure will complete the pattern in the following question figure?



(a) 

(b) 

(c) 

(d) 

73. In a certain code language 'do re me' means 'he is late', 'fa me la' means `she is early' and 'so ti do' means 'he leaves soon'. Which code in that language means 'late'?

(a) la
(b) do
(c) me
(d) re

74. P is father of J. S is mother of N who is brother of J. B is son of S. C is sister of B. How J is related to C?

(a) Data inadequate
(b) Cousin
(c) Brother
(d) Sister

75. Fine next term of the series
22,45,91,183, ?

(a) 365 (b) 267
(c) 357 (d) 367

76. In the following question, select the number which can be placed at the sign of question mark (?) from the given alternatives.

1	$\frac{1}{2}$	$\frac{3}{2}$
2	$\frac{2}{3}$	$\frac{8}{3}$
3	?	$\frac{19}{5}$

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$
(c) $\frac{3}{4}$ (d) $\frac{4}{5}$

77. Arrange the following word as per Ligical Order:

1. Serve
2. Vegetable
3. Package
4. Prepare
5. Store
6. Cut

(a) 2 4 6 5 3 1
(b) 6 4 5 1 3 2
(c) 2 6 4 3 5 1
(d) 6 4 5 1 2 3

78. Three statements are given, followed by three conclusions numbered I, II and III. Assuming the statements to be true, even if they seem to be at variance with commonly known facts, decide which of the conclusions logically follow (s) from the statements.

All knifes are axes.
All axes are swords .
All saws are swords.

Conclusions :

I. All swords are knifes.
II. Some axes are saws.
III. No axe is saw.

(a) Either conclusion II or III follows.
(b) Only conclusion I follows.
(c) All the conclusions follow.
(d) Only conclusions I and II follow

79. From the given alternatives, select the word which can be formed using the letters of the given word.
REMEMBERING

(a) NEGRO (b) AGREE
(c) RAIN (d) GREEN

80. Find the next term of the series:
BDCA, FHGE, JLKI, ?

(a) ONMP (b) MNPO
(c) NPOM (d) MONP

81. Select the option in which the numbers are related in the same way as are the numbers of the following set.
(7, 52, 346)

(a) (8, 67, 515) (b) (4, 19, 70)
(c) (6, 39, 217) (d) (5, 25, 128)

82. Choose the correct alternative to complete the series.
Lily, Daisy, Datura, __?__

(a) Sun Flower (b) Hibiscus
(c) Marigold (d) Jasmine

83. On the following questions, select the related numbers for the attractive

142 :: 15 :: 234 : ?

(a) 45 (b) 39

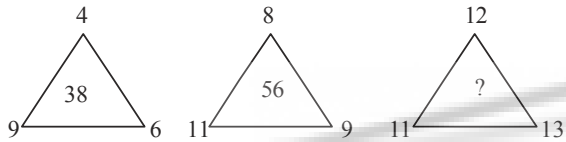
(c) 33 (d) 47

84. Select the odd word from the given alternatives.

(a) 2132 – 161 (b) 2678 – 672

(c) 4325 – 120 (d) 6931 – 162

85. In the following question, select the number which can be placed at the sign of question mark (?) from the given alternatives.



(a) 72 (b) 78

(c) 108 (d) 90

86. Find the odd one out

(a) 37,4 (b) 24,7

(c) 42,4 (d) 14,12

87. If '÷' stands for 'greater than', '×' stands for 'addition', '+' stands for 'division', '–' stands for 'equal to', '>' stands for 'multiplication', '=' stands for 'less than' and '<' stands for 'minus', then which of the following alternative is correct?

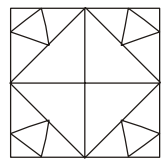
(a) $5 > 2 < 1 - 3 \times 4 \times 1$

(b) $5 < 2 \times 1 + 3 > 4 \times 1$

(c) $5 > 2 \times 1 - 3 > 4 < 1$

(d) $5 + 2 \times 1 = 3 + 4 > 1$

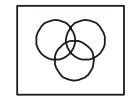
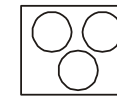
88. How many triangles are there in this figure ?

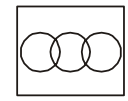
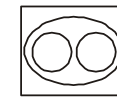


(a) 24 (b) 26

(c) 28 (d) 20

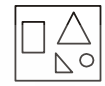
89. Select the Venn diagram that best represents the relationship between the following words Sports, Cricket, Football

(a)  (b) 

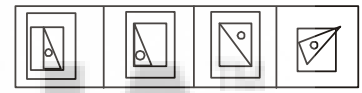
(c)  (d) 

90. Which of the answer figures is formed from the shapes given the problem figure?

Problem Figure :



Answer Figure :



(a) C (b) A

(c) B (d) D

PART - IV : MATHEMATICS

91. Minimum value of Z = 3x + 5y subject to constraints $x + y \geq 2$, $x + 3y \geq 3$, $x, y \geq 0$

(a) 6 (b) 7 (c) 8 (d) 9

92. The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$ is logically equivalent to

(a) $p \Rightarrow q$ (b) $q \Rightarrow p$

(c) $\sim (p \Rightarrow q)$ (d) $\sim (q \Rightarrow p)$

93. A vector perpendicular to the plane containing the vectors $\hat{i} - 2\hat{j} - \hat{k}$ and $3\hat{i} - 2\hat{j} - \hat{k}$ is inclined to the vector $\hat{i} + \hat{j} + \hat{k}$ at an angle

(a) $\tan^{-1}\sqrt{14}$ (b) $\sec^{-1}\sqrt{14}$

(c) $\tan^{-1}\sqrt{15}$ (d) None of these

94. The slope of the tangent to the curve $x = 3t^2 + 1$, $y = t^3 - 1$ at $x = 1$ is:

(a) $\frac{1}{2}$ (b) 0 (c) -2 (d) ∞

95. General solution of $\tan 5\theta = \cot 2\theta$ is

(a) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$ (b) $\theta = \frac{n\pi}{7} + \frac{\pi}{5}$

(c) $\theta = \frac{n\pi}{7} + \frac{\pi}{2}$ (d) $\theta = \frac{n\pi}{7} + \frac{\pi}{3}$

96. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ?

(a) 16 (b) 36 (c) 60 (d) 180

97. If the co-ordinates of the point in which the line joining the points (3, 5, –7) and (–2, 1, 8) is intersected by the plane yz is $\left[a, \frac{13}{b}, c\right]$, then $(a + b - c) =$

(a) 2 (b) 3 (c) 4 (d) 5

98. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to

(a) –5 (b) 5

(c) 2 (d) –2

99. If $f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \le x < 3 \end{cases}$, then the quadratic equation whose roots are $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$ is

(a) $x^2 - 6x + 9 = 0$

(b) $x^2 - 7x + 8 = 0$

(c) $x^2 - 14x + 49 = 0$

(d) $x^2 - 10x + 21 = 0$

100. The area of the region

$\mathbf{R} = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is :

(a) $11\sqrt{3}$ square units

(b) $12\sqrt{3}$ square units

(c) $9\sqrt{3}$ square units

(d) $6\sqrt{3}$ square units

101. If $n(A) = 1000$, $n(B) = 500$ and if $n(A \cap B) \ge 1$ and $n(A \cup B) = p$, then

(a) $500 \le p \le 1000$

(b) $1001 \le p \le 1498$

(c) $1000 \le p \le 1498$

(d) $1000 \le p \le 1499$

102. If n is a positive integer, then $2 \cdot 4^{2n+1} + 3^{3n+1}$ is divisible by :

(a) 2 (b) 7 (c) 11 (d) 27

103. The roots of the given equation $(p - q)x^2 + (q - r)x + (r - p) = 0$ are :

(a) $\frac{p-q}{r-p}, 1$ (b) $\frac{q-r}{p-q}, 1$

(c) $\frac{r-p}{p-q}, 1$ (d) None of these

104. $\int \tan^{-1}\sqrt{x}\, dx$ is equal to

(a) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(b) $x\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(c) $\sqrt{x} - x\tan^{-1}\sqrt{x} + C$

(d) $\sqrt{x} - (x+1)\tan^{-1}\sqrt{x} + C$

105. If $(-4, 5)$ is one vertex and $7x - y + 8 = 0$ is one diagonal of a square, then the equation of second diagonal is

(a) $x + 3y = 21$ (b) $2x - 3y = 7$

(c) $x + 7y = 31$ (d) $2x + 3y = 21$

106. The function $f(x) = \log(1 + x) - \frac{2x}{2 + x}$ is increasing on

(a) $(0, \infty)$ (b) $(-\infty, 0)$

(c) $(-\infty, \infty)$ (d) None of these

107. The domain of the function $f(x) = \sqrt{x - \sqrt{1 - x^2}}$ is

(a) $\left[-1, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, 1\right]$

(b) $[-1, 1]$

(c) $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{\sqrt{2}}, +\infty\right)$

(d) $\left[\frac{1}{\sqrt{2}}, 1\right]$

108. The sum to infinite term of the series

$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots$ is

(a) 3 (b) 4 (c) 6 (d) 2

109. Consider the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$, then f^{-1} is equal to

(a) $\frac{3+4x}{6x-4}$ (b) $\frac{6x-4}{3+4x}$

(c) $\frac{3-4x}{6x-4}$ (d) $\frac{9+2x}{6x-4}$

110. The vertices of the hyperbola

$9x^2-16y^2-36x+96y-252=0$ are

(a) $(6,3),(-2,3)$ (b) $(6,3),(-6,3)$

(c) $(-6,3),(-6,-3)$ (d) $(2,3),(-2,3)$

111. If $y^x = e^{y-x}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1+\log y}{y\log y}$ (b) $\frac{(1+\log y)^2}{y\log y}$

(c) $\frac{1+\log y}{(\log y)^2}$ (d) $\frac{(1+\log y)^2}{\log y}$

112. If $A=\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B=\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, then AB is equal to

(a) B (b) A (c) O (d) I

113. The coordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$ are

(a) (–6, 5) (b) (5, 6)

(c) (–5, 6) (d) (6, 5)

114. If A and B are mutually exclusive events and if $P(B) = \frac{1}{3}$, $P(A \cup B) = \frac{13}{21}$, then P(A) is equal to

(a) 1/7 (b) 4/7 (c) 2/7 (d) 5/7

115. A coin is tossed twice. Then, the probability that atleast one tail occurs is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{3}{4}$

116. The solution set of $(x-2)^{x^3-6x+8} > 1$ is

(a) $(2,\infty)$
(b) $(2,3)\cup(4,\infty)$
(c) $(4,5)\cup(5,\infty)$
(d) $(2,3)\cup(4,5)$

117. If $f(x)=\sqrt{1+\cos^2\left(x^2\right)}$, then the value of $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is

(a) $\frac{\sqrt{\pi}}{6}$ (b) $-\sqrt{\frac{\pi}{6}}$ (c) $\frac{1}{\sqrt{6}}$ (d) $\frac{\pi}{\sqrt{6}}$

118. The integrating factor of $\frac{xdy}{dx}-y=x^4-3x$ is

(a) x (b) log x (c) $\frac{1}{x}$ (d) –x

119. If E and F are events such that $0 < P(F) < 1$, then

(a) $P(E \mid F)+P\left(\overline{E} \mid F\right)=1$

(b) $P(E \mid F)+P\left(E \mid \overline{F}\right)=1$

(c) $P\left(\overline{E} \mid F\right)+P\left(E \mid \overline{F}\right)=1$

(d) $P\left(E \mid \overline{F}\right)+P\left(\overline{E} \mid \overline{F}\right)=0$

120. The shortest distance between the lines $x = y + 2 = 6z - 6$ and $x + 1 = 2y = -12z$ is

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{3}{2}$

121. The value of $\cos\left(2\cos^{-1}x+\sin^{-1}x\right)$ at $x=\frac{1}{5}$ is

(a) $-\frac{2\sqrt{6}}{5}$ (b) $-2\sqrt{6}$

(c) $-\frac{\sqrt{6}}{5}$ (d) None of these

122. $\int_0^{\pi/4}\cos x e^{\sin x}dx$ is equal to

(a) $e+1$ (b) $e-1$
(c) e (d) $-e$

123. If x^{18} occurs in the rth term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then what is the value of r ?

(a) 3 (b) 5 (c) 7 (d) 9

124. Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$.

(a) $\pm\frac{\sqrt{19}}{5}i$ (b) $\pm\frac{\sqrt{19}i}{2}$

(c) $\frac{-1\pm\sqrt{19}i}{2\sqrt{5}}$ (d) $\frac{-1\pm\sqrt{19}i}{\sqrt{5}}$

125. If vertex of a parabola is (2, –1) and the equation of its directrix is $4x - 3y = 21$, then the length of its latus rectum is

(a) 2 (b) 8
(c) 12 (d) 16

126. The modulus of $\frac{\left(1+i\sqrt{3}\right)(2+2i)}{\left(\sqrt{3}-i\right)}$ is

(a) 2 (b) 4
(c) $3\sqrt{2}$ (d) $2\sqrt{2}$

127. The equation of the circle with centre (0, 2) and radius 2 is $x^2 + y^2 - my = 0$. The value of m is

(a) 1 (b) 2 (c) 4 (d) 3

128. If $f(x) = \cos x$, $g(x) = \cos 2x$, $h(x) = \cos 3x$ and $I(x) = \tan x$, then which of the following option is correct?

(a) f(x) and g(x) are strictly decreasing in $(0, \pi/2)$
(b) h(x) is neither increasing nor decreasing in $(0, \pi/2)$
(c) I(x) is strictly increasing in $(0, \pi/2)$
(d) All are correct

129. Value of $\int\frac{dx}{\sqrt{x(a-x)}}$ is

(a) $2\sin^{-1}\sqrt{\frac{x}{a}}+c$

(b) $-2\sin^{-1}\sqrt{\frac{x}{a}}+c$

(c) $2\sin^{-1}\frac{\sqrt{x}}{a}+c$

(d) None of these

130. The standard deviation of 5 scores 1, 2, 3, 4, 5 is $\sqrt{a}$. The value of 'a' is

(a) 2 (b) 3 (c) 5 (d) 1

SOLUTIONS

PART - I : PHYSICS

1. **(d)** Given,
Boiling point of water, = UFP 65°
Freezing point of water, = LFP – 15°

$$\frac{X-LFP}{UFP-LFP}=\frac{T_F-32}{212-32}$$

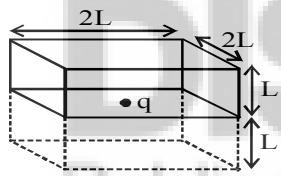
$$\Rightarrow \frac{-95-(-15)}{65-(-15)}=\frac{T_F-32}{180}$$

$$\Rightarrow \frac{-80}{80}=\frac{T_F-32}{180} \Rightarrow T_F=-180+32$$

$$\Rightarrow T_F=-148^\circ F$$

2. **(d)** After placing similar cubic at the bottom, we get cube of side 2L having change q at its centre.

From the Gauss's law electric flux through whole cube, $\phi=\frac{q}{\varepsilon_0}$



Flux passing through shaded face

$$\phi=\frac{q/\varepsilon_0}{6}=\frac{q}{6\varepsilon_0}$$

3. **(a)** Thermal energy is given by

$$H=P\times t=\frac{V^2}{R}\times t$$

Here, voltage V is same.

$$\therefore \quad \frac{H_1}{H_2}=\frac{\frac{V^2t}{R}}{\frac{V^2t}{3R}}=3:1$$

4. **(a)** $\nu=r^a\rho^b s^c \Rightarrow [\nu]=[r]^a[\rho]^b[s]^c$

$$\Rightarrow \quad [T^{-1}]=[L^1]^a[M^1L^{-3}]^b\left[\frac{MLT^{-2}}{L}\right]^c$$

$$\Rightarrow \quad T^{-1}=M^{b+c}.L^{a-3b}.T^{-2c}$$

$$\Rightarrow \quad b+c=0, a-3b=0, -2c=-1$$

$$\Rightarrow \quad c=\frac{1}{2}, b=-\frac{1}{2}, a-3b=0;$$

$$a+\frac{3}{2}=0 \quad \Rightarrow \quad a=-\frac{3}{2}$$

5. **(d)** Velocity of train B w.r.t. train A = $\vec{V}_B-\vec{V}_A$
= 54 – (–90) = 144 km/h

$$=\frac{144\times 5}{18}=40 \text{ m/s}$$

$$\text{Time of crossing}=\frac{\text{length of train}}{\text{relative velocity}} \Rightarrow (8)=\frac{\ell}{40}$$

$\therefore \ell = 8\times 40 = 320$ m

6. **(a)** Given,
Initial velocity of projectile, u = 40 m/s
Angle, $\theta = 30^\circ$
Time of flight

$$T=\frac{2u\sin\theta}{g}=\frac{2\times 40\times 1}{10\times 2}=45 \quad (\because g=10 \text{ m/s}^2)$$

It means projectile is at maximum height at t = 2 s. At maximum height vertical component of velocity is zero.
Velocity at t = 2s = $V_x = u\cos\theta = 40\cos 30^\circ$

$$=20\sqrt{3}\text{ms}^{-1}.$$

7. **(c)** The magnetic moment associated with circular coil,
M = NIA
$M_A = M_B$
$\therefore N_A I_A A_A = N_B I_B A_B$
$\therefore N_A I_A \pi(0.1)^2 = N_B I_B \pi(0.2)^2 (\because A=\pi r^2)$
$\therefore N_A I_A = 4N_B I_B$

8. **(d)** Induced emf, $\varepsilon = -L\frac{dI}{dt} \Rightarrow 20 = -L\frac{(0-2)}{10^{-3}}$

Hence, inductance of the coil.
$L = 10\,mH$

9. **(d)** $K.E. = \frac{P^2}{2m}$

$$\therefore \frac{K_1}{K_2} = \frac{p_1^2}{2m_1} \times \frac{2m_2}{p_2^2} = \frac{m_2}{m_1} = \frac{16}{9}$$

$$\therefore \frac{m_1}{m_2} = \frac{9}{16}$$

10. **(d)** $T.E = K.E + P.E$
$\Rightarrow K.E = T.E - K.E$
$= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$\Rightarrow KE = \frac{1}{2}m\omega^2\ (A^2 - x^2)\ \ [\because k = m\omega^2]$

KE is maximum at $x = 0$ & K.E is zero when $x = A$
$\therefore$ K.E vs x graph is parabola.

11. **(c)** Radius of circle, $r = a$
Centrifugal force, $F = m\omega^2 r$

$$\Rightarrow \frac{Gmm}{(2a)^2} = m\omega^2\ a$$

$$\left(\because F = \frac{G\,M_1M_2}{d}\right)$$

Here $M_1 = m$; $M_2 = m$ and $d = 2a$

$\Rightarrow$ angular speed, $\omega = \sqrt{\frac{Gm}{4a^3}}$

12. **(c)** Equivalent resistance in parallel combination is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{10 \times 15}{10 + 15} = 6$$

Differentiating both sides, we get

$$\frac{\Delta R_{eq}}{R^2_{eq}} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Rightarrow \frac{\Delta R_{eq}}{R_{eq}} = \left(\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right) R_{eq}$$

$$= \left(\frac{0.5}{100} + \frac{0.5}{225}\right) 6 = \left(\frac{6 \times 0.5}{25}\right)\left(\frac{1}{4} + \frac{1}{9}\right) = \frac{13}{300}$$

$$\frac{\Delta R_{eq}}{R_{eq}} \times 100 = \frac{13}{3} = 4.33\%$$

13. **(a)** Given,
Capacitive reactance, $X_C = 100\,\Omega$
Inductive reactance, $X_L = 200\,\Omega$
Resistance, $R = 100\,\Omega$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = \sqrt{100^2 + (200 - 100)^2} = 100\sqrt{2}\ \Omega$
RMS value of current,

$$i_{rms} = \frac{V_{rms}}{Z} = \frac{200\sqrt{2}}{100\sqrt{2}} = 2A$$

14. **(a)** For an EM wave

$$\vec{B} = \frac{1}{\omega}\left(\vec{K} \times \vec{E}\right)$$

15. **(d)** Angle of deviation for first prism
$\delta_1 = A_1(\mu_1 - 1) = 6(1.54 - 1)$
Angle of deviation for second prism
$\delta_2 = A_2(\mu_2 - 1)$
$= A_2(1.72 - 1)$
For dispersion without deviation
$\delta_1 = \delta_2$
$\Rightarrow 6^\circ(1.54 - 1) = A_2(1.72 - 1)$

$$\Rightarrow A_2 = \frac{6^\circ \times 0.54}{0.72} = \frac{18^\circ}{4} = 4.5^\circ$$

16. **(c)** Given that torque, $\vec{\tau} = 5\hat{i} + 3\hat{j} - 7\hat{k}$

Position vector, $\vec{r} = 2\hat{i} + +2\hat{j} + \hat{k}$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ 2 & 2 & 1 \\ 5 & 3 & -7 \end{vmatrix}$$

$$= \hat{i}(-14-3) - \hat{j}(-14-5) + \hat{k}(6-10)$$

$$= -17\hat{i} + 19\hat{j} - 4\hat{k}$$

17. (a) Potential due to sphere

$$V = \frac{GM}{2R^3}\left(3R^2 - r^2\right)$$

$$\text{At surface } r = R \Rightarrow V = \left(\frac{GM}{R}\right)$$

At centre, r = 0

$$\therefore V_0 = \frac{3GM}{2R} = \left(\frac{3V}{2}\right)$$

18. (b) Elongation produced in the wire is given by

$$\Delta L = \frac{FL}{AY}$$

Here, L = length of wire
A = area of cross-section of wire
Y = Young's moduli of material of wire.
$\because$ F and L is same and

$$\frac{Y_A}{Y_B} = \frac{1}{4} \quad \text{(given)}$$

$$\frac{A_A}{A_B} = \frac{1}{3} \quad \text{(given)}$$

$$\therefore \frac{\Delta L_A}{\Delta L_B} = \frac{A_B}{A_A}\frac{Y_B}{Y_A} = \frac{3}{1} \times \frac{4}{1} = 12$$

19. (a) De-Broglie wavelength,

$$\lambda = \frac{h}{m\upsilon} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mq\Delta V}}$$

$$\therefore \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{2m_p V_p q_p}{2m_\alpha V_\alpha q_\alpha}}$$

$\because V_p = 2V$ and $V_\alpha = 4V$ (given)

$m_\alpha = 4m_p$ and $q_\alpha = 2q_p$

$$\therefore \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{1\times 2\times 1}{4\times 4\times 2}} = \frac{1}{4} \Rightarrow \lambda_p : \lambda_\alpha = 4:1$$

20. (c) From the Bohr's quantization rule

$$\text{Angular momentum } L = \frac{nh}{2\pi}, L_1 = \frac{1.h}{2\pi} = L$$

(n = 1 for first orbit)

$$\text{in (n = 2 for second orbit) } L_2 = \frac{2h}{2\pi} = 2L$$

Hence, change in angular momentum $L_2 - L_1 = 2L - L = L$

21. (a) In adiabatic process

PV^γ = constant

$$\Rightarrow \quad P_1V_1^\gamma = P_2V_2^\gamma$$

$$\Rightarrow \quad \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma \Rightarrow \frac{81}{16} = \left(\frac{27}{8}\right)^\gamma$$

$$\Rightarrow \quad \left(\frac{9}{4}\right)^2 = \left(\frac{9\times 3}{4\times 2}\right)^\gamma \Rightarrow \left(\frac{9}{4}\right)^{2-\gamma} = \left(\frac{3}{2}\right)^\gamma$$

$$\Rightarrow \quad \left(\frac{9}{4}\right)^{2-\gamma} = \left(\frac{9}{4}\right)^{\gamma/2} \Rightarrow 2 - \gamma = \frac{\gamma}{2}$$

$$\Rightarrow \quad 2 = \frac{3}{2}\gamma \Rightarrow \gamma = \frac{4}{3}$$

22. (b) Average kinetic energy for diatomic gases

$$K_{av} = \frac{5}{2}kT$$

$$\therefore \frac{(K_{av})_H}{(K_{av})_O} = \frac{(27+273)}{(27+273)} = 1$$

23. (b) Given that number of turns, n = 1200
Current, I = 2A
Magnetic field at centre inside the solenoid is given by,
$B = \mu_0 nI$
So magnetic intensity at centre of the solenoid,

$$H = \frac{B}{\mu_0} = nI = \left(\frac{1200}{2}\right)(2) \qquad (\because B = \mu_0 H)$$

$H = 1.2 \times 10^3\ Am^{-1}$

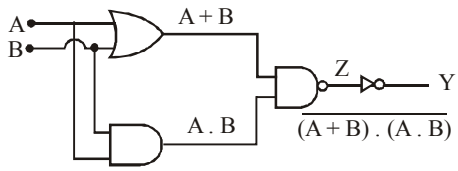
24. **(c)** It is given that

$$\left(\frac{T_1}{2}\right)_A = (T_{av})_B$$

$$\because T_{1/2} = \frac{\ell n 2}{\lambda} \text{ and } T_{av} = \frac{1}{\lambda}$$

$$\therefore \frac{\ell n 2}{\lambda_A} = \frac{1}{\lambda_B} \Rightarrow \lambda_A = \lambda_B \ell 2$$

25. **(a)**



$$Y = \overline{Z} = (A+B).(A.B) = A.A.B + AB.B$$

$$Y = AB + AB = A.B$$

$\therefore$ It is an AND Gate.

26. **(b)** Let r be the radius of small drops of water.
R = radius of big drop formed
as volume remain same.

$$\therefore 8.\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R^3 \Rightarrow R = 2r$$

Terminal velocity,

$$v_T = \frac{2}{9\eta}(\rho - \sigma) r^2 g$$

$$\therefore v_T \propto r^2$$

$$\therefore \frac{v_1}{v_2} = \left(\frac{r}{R}\right)^2$$

$$\Rightarrow \frac{10}{v_2} = \left(\frac{1}{2}\right)^2 \quad (\because v_1 = 10 \text{ cm/s given})$$

$$\Rightarrow v_2 = 40 \text{ cm/s}$$

27. **(b)** The rate of heat flow is given by

$$\frac{\Delta Q}{\Delta t} = -K(T - T_0) \Rightarrow \frac{\Delta Q}{\Delta t} = -K(T_{avg} - T_0)$$

Initially $T_{avg} = \frac{T_1 + T_2}{2}$

Here, $T_1 = 98°C$
$T_2 = 86°C$

(i) $$\frac{ms \times 12}{2} = -K\left(\frac{98+86}{2} - 22\right)$$

$$6 = -\frac{K}{ms}\left[\frac{98+86}{2} - 22\right] \qquad (\because \Delta Q = ms\Delta T)$$

$$6 = -\frac{K}{ms}[70] \qquad \text{....(i)}$$

Now, cool from $T_1' = 75° C$
$T_2' = 69° C$

$$T_{avg} = \frac{T_1' + T_2'}{2} = \frac{75+69}{2}$$

(ii) $$\frac{ms \times 6}{\Delta t} = -K\left(\frac{75+69}{2} - 22\right)$$

$$\frac{6}{\Delta t} = -\frac{K}{ms}(50) \qquad \text{...(ii)}$$

Divide equation (ii) by (i), we have

$$\frac{6}{\Delta t(6)} = \frac{50}{70}$$

$$\Delta t = \frac{7}{5} = 1.4 \text{ min}$$

28. **(c)** From the Gauss's law

$$\phi = \frac{q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0} \quad \because Q = CV$$

$$\therefore \phi = \frac{CV}{\epsilon_0}$$

29. **(b)** Wavelength of monochromatic light,
$\lambda = 600 \times 10^{-9}$ m
As fir first minima a $\sin\theta = \lambda$
$\Rightarrow a \sin 30° = 600 \times 10^{-9}$
$\Rightarrow a = 1200 \times 10^{-9}$ m $= 1.2$ μm

30. **(b)** The relative velocity of a passenger with respect to source of sound (engine of train) is 0. So there will be no Doppler's effect. So frequency heard is 400 Hz.

PART - II : CHEMISTRY

31. **(b)** In boundary surface diagram (1) the four lobes lie between *y* and *z*-axis (d_{yz}) whereas, in boundary surface diagram (2) the four lobes lie on the *x* and *y*-axis ($d_{x^2-y^2}$).

32. **(d)** 6th period consists of 32 elements.

33. **(b)** $r = 0.529 \times \frac{n^2}{Z}$ Å

$$r_3 = 0.529 \times \frac{3^2}{1} \Rightarrow r_4 = 0.529 \times \frac{4^2}{1}$$

$$\frac{r_4}{r_3} = \frac{4^2}{3^2} = \frac{16}{9} \Rightarrow r_4 = \frac{16 r_3}{9}$$

34. **(a)** For first order reaction

$$k = \frac{1}{t} \ln\left(\frac{P_\circ}{P}\right) \Rightarrow \text{In}\left(\frac{P_\circ}{P}\right) = kt$$

$$\Rightarrow \quad \ln\left(\frac{P}{P_\circ}\right) = -kt$$

On comaring with straight line equation $y = mx$

$k = \text{slope} = 3.465 \times 10^4$

$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{3.465 \times 10^4} = 2 \times 10^{-5}\ \text{s}$$

35. **(c)** Frenkel defect is due to dislocation of ion from its usual lattice site to interstitial position.

36. **(d)** Lattice enthalpy is required to completely separate one mole of a solid ionic compound into gaseous constituent ions.

37. **(a)** The specific conductance increases with concentration. The number of ions per cm^3 increase with increase of concentration.

38. **(c)** $\left(P + \frac{a}{V^2}\right)(V - b) = RT$; Here $\left(P + \frac{a}{V^2}\right)$ represents the intermolecular forces.

39. **(a)** Photochemical smog $\Rightarrow O_3$, PAN, NO_x

40. **(a)** Colloid of liquid in liquid is called emulsion. Colloid of liquid in solid is gel.

41. **(b)** $\frac{1}{2}H_2 + \frac{1}{2}Cl_2 \longrightarrow HCl$

$$\Delta H_{HCl} = \sum \text{B.E. of reactant} - \sum \text{B.E. of products}$$

$$-90 = \frac{1}{2} \times 430 + \frac{1}{2} \times 240 - \text{B.E. of HCl}$$

$$\therefore \quad \text{B.E. of HCl} = 215 + 120 + 90 = 425\ \text{kJ mol}^{-1}$$

42. **(a)** Chromatography paper contains water trapped in it, which acts as the stationary phase.

43. **(d)** Vitamin B_1 - Thiamin
Vitamin B_6 - Pyridoxine

44. **(b)** High density polythene is formed when addition polymerisation of ethene takes place in a hydrocarbon solvent in presence of catalyst such as Ziegler-Natta catalyst.

45. **(a)** Mg burns in CO_2 to give MgO and C.

46. **(d)** Alkenes having double bonds with two different groups on each end of the double bond show geometrical isomerism. $A_2b_2c_2$, A_2b_2cd, A_2bcde.

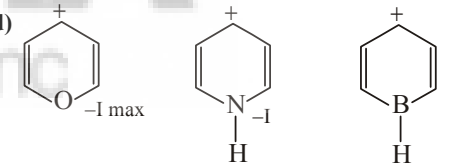
47. **(b)** $2BF_3 + 3LiBH_4 \rightarrow 2B_2H_6 + 3LiF$

48. **(a)** These drugs induce sleep and are habit forming common example of hypnotic drugs are luminal and saconal.

49. **(a)** 1° amines (aliphatic and aromatic) react with $CHCl_3$/KOH to yield isocyanide (foul smelling) This is known as carbylamine test which is not given by 2° and 3° amines.

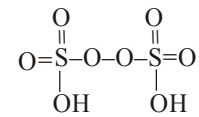
50. **(a)** Urea acts as stablising agent. It prevents the decomposition of H_2O_2.

51. **(d)**



Aromatic Aromatic Anti-aromatic

52. **(b)**



Peroxodisulphuric acid
($H_2S_2O_8$)

53. **(a)** Pyrolusite (It is MnO_2)

54. **(c)** Reduction potential is maximum for $S_2O_8^{2-}$, therefore, it is the strongest oxidising agent amongst the given species.

55. **(d)** In last case CH_4 is produced.

56. **(c)** $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$ are ionisation isomers.

57. **(d)** $(CH_3)_3$ B - is an electron deficient, thus behave as a lewis acid.

58. **(d)** This reaction proceeds in the presence of anhydrous $AlCl_3$ or CuCl.

59. **(b)** $\Delta T_f = K_f \times m = 1.86 \times 0.5 = 0.93°C$;

$T_f = -0.93°C$

60. **(a)** Gangue is the commercially worthless material which contaminates the ore.

PART - III (A): ENGLISH PROFICIENCY

61. **(a)** 'Eloquent' means fluent or persuasive in speaking or writing.

62. **(a)** 'Nefarious' means morally bad in principles or practice.

63. **(d)** If a person is very fat, you euphemistically call him or her corpulent. Emaciated, on the other hand, means extremely thin or weak because of illness or lack of food.

64. **(a)** The sentence has a general tone which is in simple past tense with the usages of verb+s because the subject is singular.

65. **(b)** The mention of 'now we live in Delhi' suggests that they lived in London in the past. So for that we use 'used to'.

66. **(a)** A school of psychology argues that motorcycling – like gambling or skydiving – is one of the manifestations of impulse control disorder, a condition in which an individual cannot resist the impulse or temptation to perform an act harmful for oneself or others.

67. **(d)** With six of its neighbours ranking high on global roster of failed states there is a renewed warning for India to reassess its policy towards them and safeguard its own strategic interests.

68. **(b)** Murali was a vendor of sweets.

69. **(a)** Murali's main customers were children.

70. **(d)** At the stroke of nine in the morning, Murali would stand in front of the school with his tray of sweets.

PART - III (B) : LOGICAL REASONING

71. **(a)** XXIV×2

$\Rightarrow 24 \times 2 = 48$

Similarly,

$XIV \times 2 = 14 \times 2 = 28$

72. **(c)** Answer figure (c) is the best fit to the question figure.

73. **(d)** (do) re [me] → (he) [is] late

fa [me] la → she [is] early

so ti (do) → (he) leaves soon

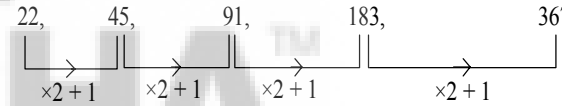
Hence re → Late

74. **(a)** P(+) ⇔ S (–)

J – N(+) B(+) – C(–)

The gender of J is unknown, therefore J may be brother or sister of C.

75. **(d)** Given Series is:



76. **(d)** $\left(\frac{3}{2}-1\right)=\frac{1}{2} \Rightarrow 1^{st}$ row

$\left(\frac{8}{3}-2\right)=\frac{2}{3} \Rightarrow 2^{nd}$ row

$\left(\frac{19}{5}-3\right)=\frac{4}{5} \Rightarrow 3^{rd}$ row

77. **(c)** Meaningful order of words:

2. Vegetable

↓

6. Cut

↓

4. Prepare

↓

3. Package

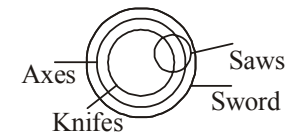
↓

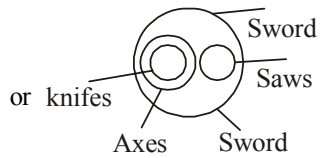
5. Store

↓

1. Serve

78. (a)

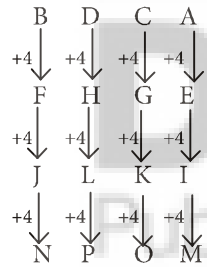




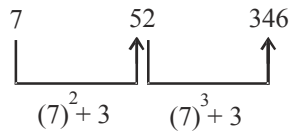
Hence, either 'some axes are saws' or 'no axes are saws

79. (d) GREEN

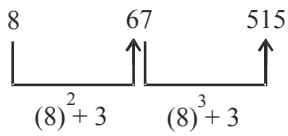
80. (c)



81. (a) As,



Similarly,



82. (d) Lily, Daisy, Datura all have outer part white and inner part yellow.

Jasmine also has outer part white and inner part yellow.

83. (c) $142 : 15 \Rightarrow (1\times4\times2)+(1+4+2)=15$

$234 : ? \Rightarrow (2\times3\times4)+(2+3+4)=33$

84. (a) $2678 \Rightarrow 2\times6\times7\times8=672$

$4325 \Rightarrow 4\times3\times2\times5=120$

$6931 \Rightarrow 6\times9\times3\times1=162$

$2132 \Rightarrow 2\times1\times3\times1=161$

85. (a) As, $(4+6+9)\times2=38$

$(8+9+11)\times2=56$

Similarly,

$(12+13+11)\times2=72$

86. (a) From options

(a) $37, 4 \rightarrow 37\times4=148$ {odd one}

(b) $24, 7 \rightarrow 24\times7=168$

(c) $42, 4 \rightarrow 42\times4=168$

(d) $14, 12 \rightarrow 14\times12=168$

87. (c)

$\div \rightarrow >$	$\times \rightarrow +$
$+ \rightarrow \div$	$- \rightarrow =$
$> \rightarrow \times$	$= \rightarrow <$
$< \rightarrow -$	

$5>2\times1-3>4<1$

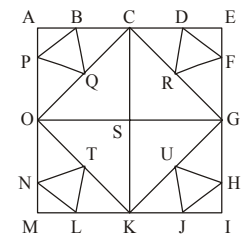
Using the proper notations in (c), we get the statement as:

$5\times2+1=3\times4-1$

$10+1=12-1$

$11=11$

88. (c)

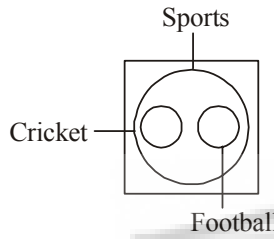


The triangles are:

$\Delta ABP; \Delta BQP; \Delta PQO; \Delta BCQ$

$\Delta ACO; \Delta CSO; \Delta CDR; \Delta DRF;$
$\Delta DEF; \Delta RFG; \Delta CSG; \Delta ECG;$
$\Delta IJH; \Delta JUH; \Delta GHU; \Delta JKU$
$\Delta GIH; \Delta SGK; \Delta ONT; \Delta NTL;$
$\Delta NML; \Delta TLK; \Delta MOK; \Delta SOK;$
$\Delta CGO; \Delta GKC; \Delta KGO; \Delta COK$

89. (d) The Venn diagrams best represent the relationship between - Sports, Cricket, Football figures are shown below.



Both Cricket and Football are comes under the category of sport but the cricket and football are different types of sports so they are separated here.

Hence, correct answer is option (d).

90. (b) It will appear line option (b), when unfolded .

PART - IV : MATHEMATICS

91. (b) Z is 7 minimum at $\left(\frac{3}{2}, \frac{1}{2}\right)$

92. (c) Given boolen expression are
$\sim p \vee q \equiv p \to q$ and $\sim q \wedge p \equiv \sim (p \to q)$
Negation of $\sim (p \to q) \to (p \to q)$
is $\sim (p \to q) \wedge (\sim (p \to q))$ i.e., $\sim (p \to q)$

93. (a) A vector perpendicular to the plane is

$$(\hat{i} - 2\hat{j} - \hat{k}) \times (3\hat{i} - 2\hat{j} - \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -1 \\ 3 & -2 & -1 \end{vmatrix} = -2\hat{j} + 4\hat{k}$$

$$\Rightarrow \text{ unit vector } \hat{a} = \frac{-2\hat{j} + 4\hat{k}}{\sqrt{4+16}} = \frac{-2\hat{j} + 4\hat{k}}{2\sqrt{5}}$$

Angle between the unit vector and $\vec{r} = \hat{i} + \hat{j} + \hat{k}$

$$= \cos^{-1} \frac{\vec{r} \cdot \hat{a}}{|\vec{r}| . |\hat{a}|} = \cos^{-1} \frac{1}{\sqrt{15}} = \tan^{-1} \sqrt{14}$$

94. (b) Given curve is $x = 3t^2 + 1$ …(i)

$$\therefore \frac{dx}{dt} = 6t$$

Second curve is $y = t^3 - 1$ …(ii)

$$\therefore \frac{dy}{dt} = 3t^2$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 3t^2 \times \frac{1}{6t} = \frac{t}{2}$$

But from (i) when x = 1
we have $1 = 3t^2 + 1 \Rightarrow 3t^2 = 0 \Rightarrow t = 0$

$$\therefore \text{ When } x = 1 \text{ then } t = 0 \ \therefore \ \frac{dy}{dx} = 0$$

Hence, slope of the tangent to the curve = 0

95. (a) We have $\tan 5\theta = \cot 2\theta$

$$\Rightarrow \tan 5\theta = \tan\left(\frac{\pi}{2} - 2\theta\right) \ldots \left[\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta\right]$$

$$\Rightarrow 5\theta = n\pi + \frac{\pi}{2} - 2\theta \Rightarrow 7\theta = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{n\pi}{7} + \frac{\pi}{14}.$$

96. (c) X - X - X - X - X. The four digits 3, 3, 5,5 can be arranged at (-) places in $\frac{4!}{2!2!} = 6$ ways.

The five digits 2, 2, 8, 8, 8 can be arranged at (X) places in $\frac{5!}{2!3!}$ ways = 10 ways

Total no. of arrangements = $6 \times 10 = 60$ ways

97. **(b)** Ratio $= -\left(\frac{3}{-2}\right) = \frac{3}{2}$

$\therefore$ Required co-ordinates of the points are

$$\left[\frac{6-6}{5}, \frac{10+3}{5}, \frac{-14+24}{5}\right] = \left(0, \frac{13}{5}, 2\right).$$

$\therefore$ a + b – c = 0 + 5 – 2 = 3

98. **(a)** When the two lines intersect then shortest distance between them is zero i.e.

$$\frac{(\vec{a}_2 - \vec{a}_1)\cdot \vec{b}_1 \times \vec{b}_2}{\left|\vec{b}_1 \times \vec{b}_2\right|} = 0$$

$$\Rightarrow \quad (\vec{a}_2 - \vec{a}_1)\cdot \vec{b}_1 \times \vec{b}_2 = 0$$

where $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b}_1 = k\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$, $\hat{b}_2 = 3\hat{i} + k\hat{j} + 2\hat{k}$

$$\Rightarrow \quad \begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$$\Rightarrow \quad 1(4-3k) - 1(2k-9) - 2(k^2-6) = 0$$

$$\Rightarrow \quad -2k^2 - 5k + 25 = 0 \Rightarrow k = -5 \text{ or } \frac{5}{2}$$

99. **(d)** Since, $f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \le x < 3 \end{cases}$

Now, $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} (x^2 - 1)$

$= \lim\limits_{h \to 0} [(2-h)^2 - 1] = \lim\limits_{h \to 0} [4 + h^2 - 4h - 1]$

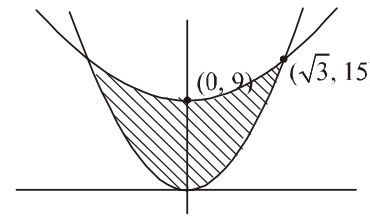
$= \lim\limits_{h \to 0} \left[h^2 - 4h + 3\right] = 0 - 0 + 3 = 3$

and $\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} (2x + 3)$

$= \lim\limits_{h \to 0} [2(2+h) + 3] = \lim\limits_{h \to 0} [4 + 2h + 3] = 7$

Hence the quadratic equation whose roots are 3 and 7 is given by $x^2 - (3+7)x + (3+7) = 0 \Rightarrow x^2 - 10x + 21 = 0$

100. **(b)**



$$\text{Required area} = 2\int_0^{\sqrt{3}} (2x^2 + 9 - 5x^2)\,dx$$

$$= 2\left[9x - x^3\right]_0^{\sqrt{3}} = 2[9\sqrt{3} - 3\sqrt{3}] = 12\sqrt{3}.$$

101. **(d)** n(A) = 1000, n(B) = 500, $n(A \cap B) \ge 1$, $n(A \cup B) = p$; $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$p = 1000 + 500 - n(A \cap B) \Rightarrow 1 \le n(A \cap B) \le 500$

Hence $p \le 1499$ and $p \ge 1000 \Rightarrow 1000 \le p \le 1499$

102. **(c)** Let $P(n) = 2 \cdot 4^{2n+1} + 3^{3n+1}$

Then $P(1) = 2 \cdot 4^3 + 3^4 = 209$, which is divisible by 11 but not divisible by 2, 7 or 27.

Further, let $P(k) = 2 \cdot 4^{2k+1} + 3^{3k+1}$ is divisible by 11, that is,

$2.4^{2k+1} + 3^{3k+1} = 11q$ for some integer q. Now

$P(k+1) = 2 \cdot 4^{2k+3} + 3^{3k+4}$

$= 2 \cdot 4^{2k+1} \cdot 4^2 + 3^{3k+1} \cdot 3^3$

$= 16 \cdot 2.4^{2k+1} + 27.3^{3k+1}$

$= 16 \cdot 2 \cdot 4^{2k+1} + (16+11) \cdot 3^{3k+1}$

$= 16[2 \cdot 4^{2k+1} + 3^{3k+1}] + 11 \cdot 3^{3k+1}$

$= 16 \cdot 11q + 11 \cdot 3^{3k+1}$

$= 11(16q + 3^{3k+1}) = 11m$

where $m = 16q + 3^{3k+1}$ is another integer.

$\therefore$ P(k + 1) is divisible by 11.

103. **(c)** Given equation is

$(p-q)x^2 + (q-r)x + (r-p) = 0$

By using formula for finding the roots

viz : $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we get

$$x = \frac{(r-q) \pm \sqrt{(q-r)^2 - 4(r-p)(p-q)}}{2(p-q)}$$

$$\Rightarrow \quad x = \frac{(r-q) \pm (q+r-2p)}{2(p-q)} = \frac{r-p}{p-q}, 1$$

104. (a) We have, $I = \int 1.\tan^{-1}\sqrt{x}\, dx$

Using by parts,

$$I = \tan^{-1}\sqrt{x}.(x) - \int \frac{1}{1+x} \times \frac{1}{2\sqrt{x}} \times x\, dx$$

$$= x\tan^{-1}\sqrt{x}. - \int \frac{x}{(1+x)2\sqrt{x}} dx$$

$$= x\tan^{-1}\sqrt{x} - \int \left(\frac{1+x}{(1+x)2\sqrt{x}} - \frac{1}{(1+x)2\sqrt{x}} \right) dx$$

$$= x\tan^{-1}\sqrt{x} - \int \frac{dx}{2\sqrt{x}} + \int \frac{dx}{2\sqrt{x}(1+x)}$$

$$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$$

$$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$$

105. (c) One vertex of square is (– 4, 5) and equation of one diagonal is 7x – y + 8 = 0

Diagonal of a square are perpendicular and bisect each other

Let the equation of the other diagonal be y = mx + c where m is the slope of the line and c is the y-intercept.

Since this line passes through (– 4, 5)

$\therefore \quad 5 = -4m + c$ ... (i)

Since this line is at right angle to the line

$7x - y + 8 = 0$ or $y = 7x + 8$, having slope = 7,

$\therefore \quad 7 \times m = -1 \quad \text{or} \quad m = \frac{-1}{7}$

Putting this value of m in equation (i) we get

$$5 = -4 \times \left(\frac{-1}{7} \right) + c$$

$$\text{or } 5 = \frac{4}{7} + c \quad \text{or} \quad c = 5 - \frac{4}{7} = \frac{31}{7}$$

Hence equation of the other diagonal is

$$y = -\frac{1}{7}x + \frac{31}{7} \text{ or } 7y = -x + 31$$

or $\quad x + 7y - 31 = 0 \quad \text{or} \quad x + 7y = 31.$

106. (a) Given $f(x) = \log(1+x) - \frac{2x}{2+x}$

$$f'(x) = \frac{1}{1+x} - \frac{(2+x)(2) - 2x}{(2+x)^2}$$

$$= \frac{1}{1+x} - \frac{4}{(2+x)^2}$$

$$= \frac{(2+x)^2 - 4 - 4x}{(1+x)(2+x)^2} = \frac{x^2}{(1+x)(2+x)^2} > 0 \text{ for all}$$

$x \in (0, \infty)$

Thus, given function f(x) is increasing on (0, ∞).

107. (d) For $f(x)$ to be defined, we must have

$$x - \sqrt{1-x^2} \geq 0 \text{ or } x \geq \sqrt{1-x^2} > 0 \text{ or } x^2 \geq 1 - x^2$$

or $\quad x^2 \geq \frac{1}{2}.$

Now, $x^2 \geq \frac{1}{2} \Rightarrow \left(x - \frac{1}{\sqrt{2}} \right) \left(x + \frac{1}{\sqrt{2}} \right) \geq 0$

$$\Rightarrow x \leq -\frac{1}{\sqrt{2}} \quad \text{or} \quad x \geq \frac{1}{\sqrt{2}}$$

Also, $1 - x^2 \geq 0$ or $x^2 \leq 1$.

Also, $x^2 \leq 1 \Rightarrow (x-1)(x+1) \leq 0 \Rightarrow -1 \leq x \leq 1$

Thus, $x > 0$, $x^2 \geq \frac{1}{2}$ and $x^2 \leq 1 \Rightarrow x \in \left[\frac{1}{\sqrt{2}}, 1 \right]$

108. (a) We have

$$S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots\ldots\infty \quad \ldots(i)$$

Multiplying both sides by $\frac{1}{3}$, we get

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \ldots\ldots\infty \quad \ldots(ii)$$

Subtracting eqn. (ii) from eqn. (i), we get

$$\frac{2}{3}S = 1+\frac{1}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\frac{4}{3^4}+.....\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\frac{4}{3^4}+.....\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1-\frac{1}{3}} = \frac{4}{3}\times\frac{3}{2} \Rightarrow S = 3$$

109. (a) Given $f(x) = \frac{4x+3}{6x-4}$

Let $y = \frac{4x+3}{6x-4}$,

$\Rightarrow 6xy - 4y = 4x + 3 \Rightarrow x(6y-4) = 3 + 4y$

$\Rightarrow x = \frac{3+4y}{6y-4}$

$f^{-1}(x) = \frac{3+4x}{6x-4}$

110. (a) Given hyperbola is

$9x^2 - 16y^2 - 36x + 96y - 252 = 0$

$\Rightarrow 9(x^2-4x) - 16(y^2-6y) = 252$

$\Rightarrow 9(x^2-4x+4) - 16(y^2-6y+9)$

$= 252 + 36 - 144$

$\Rightarrow 9(x-2)^2 - 16(y-3)^2 = 144$

$\Rightarrow \frac{(x-2)^2}{16} - \frac{(y-3)^2}{9} = 1$ ...(i)

Put $x = X + 2$ and $y = Y + 3$

$\therefore$ Equation (i) becomes $\frac{X^2}{16} - \frac{Y^2}{9} = 1$

Now, vertices are $X = \pm a$ wehre $a = 4$ and $Y = 0$

Hence, vertices are (6, 3), (– 2, 3).

111. (d) Here, $y^x = e^{y-x}$

Taking log on both sides, we get

$\log y^x = \log e^{y-x}$

$\left(\because \log a^b = b\log a \text{ and } \log e = 1\right)$

$\Rightarrow x\log y = (y-x)\log e \Rightarrow x\log y = y - x$ ...(i)

On differentiating w.r.t. x, we get

$\frac{d}{dx}(x\log y) = \frac{d}{dx}(y-x)$

(using product rule)

$\Rightarrow x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y(1) = \frac{dy}{dx} - 1$

$\Rightarrow \frac{dy}{dx}\left(\frac{x}{y} - 1\right) = -1 - \log y$

$\Rightarrow \frac{dy}{dx}\left[\frac{y}{(1+\log y)y} - 1\right] = -(1+\log y)$

$\left[\because \text{from eq.(i)}, x = \frac{y}{(1+\log y)}\right]$

$\Rightarrow \frac{dy}{dx}\left[\frac{1-1-\log y}{1+\log y}\right] = -(1+\log y)$

$\Rightarrow \frac{dy}{dx} = -\frac{(1+\log y)^2}{-\log y}$

$\Rightarrow \frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$

112. (c) $AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$AB = \begin{bmatrix} abc-abc & b^2c-b^2c & bc^2-bc^2 \\ -a^2c+a^2c & -abc+abc & -ac+ac \\ a^2b-a^2b & ab^2-ab^2 & abc-abc \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$

113. (b) Let (h, k) be the coordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$. Then, the slope of the perpendicular line is $\frac{k-3}{h-2}$. Again the slope of the given line $x + y - 11 = 0$ is -1

Using the condition of perpendicularity of lines, we have

$$\left(\frac{k-3}{h-2}\right)(-1) = -1 \text{ or } k - h = 1 \quad ...(i)$$

Since (h, k) lies on the given line, we have,

$h + k - 11 = 0$ or $h + k = 11$ ...(ii)

Solving (i) and (ii), we get $h = 5$ and $k = 6$. Thus (5, 6) are the required coordinates of the foot of the perpendicular.

114. (c) For mutually exclusive events

$$P(A \cup B) = P(A) + P(B) \Rightarrow P(A) = \frac{2}{7}$$

115. (d) The sample space is S = {HH, HT, TH, TT}

Let E be the event of getting atleast one tail

$\therefore$ E = {HT, TH, TT}

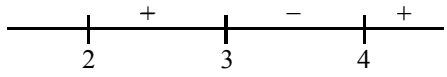
$\therefore$ Required probability p

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{n(E)}{n(S)} = \frac{3}{4}$$

116. (b) Clearly $x > 2$. Write the given inequality as

$(x^2 - 6x + 8)\log(x-2) > 0 \Leftrightarrow (x-2)(x-4)\log(x-2) > 0$

$\Leftrightarrow (x-4)\log(x-2) > 0 \quad [\because x > 2]$



117. (b) We have, $f(x) = \sqrt{1 + \cos^2(x^2)}$ ...(i)

On differentiating (i) w.r.t.x, we get

$$f'(x) = \frac{-2\sin x^2 \cos x^2}{\sqrt{1+\cos^2 x^2}}(x)$$

$$\Rightarrow f'(x) = \frac{-\sin 2x^2}{\sqrt{1+\cos^2 x^2}}(x) \quad ...(ii)$$

Put, $x = \frac{\sqrt{\pi}}{2}$ in (ii), we get

$$f'\left(\frac{\sqrt{\pi}}{2}\right) = -\frac{\sqrt{\pi}}{2} \cdot \frac{\sin 2\left(\frac{\pi}{4}\right)}{\sqrt{1+\frac{1}{2}}}$$

$$= -\frac{\sqrt{\pi}}{2} \cdot \frac{\sin\frac{\pi}{2}}{\sqrt{\frac{3}{2}}} = -\sqrt{\frac{\pi}{6}}$$

118. (c) Since $x\frac{dy}{dx} - y = x^4 - 3x$

$$\therefore \quad \frac{dy}{dx} - \frac{y}{x} = x^3 - 3$$

Hence $IF = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$

119. (a) $P(E \mid F) + P(\overline{E} \mid F)$

$$= \frac{P(E \cap F) + P(\overline{E} \cap F)}{P(F)} = \frac{P((E \cup \overline{E}) \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$$

120. (b) The lines are $\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$

and $\frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}$

Here,

$\vec{a}_1 = -2\hat{j} + \hat{k}$, $b_1 + 6\hat{i} + 6\hat{j} + \hat{k}$, $\vec{a}_2 = -\hat{i}$,

$\vec{b}_2 = 12\hat{i} + 6\hat{j} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 6 & 6 & 1 \\ 12 & 6 & -1 \end{vmatrix} = -12\hat{i} + 18\hat{j} - 36\hat{k}$$

$$\text{Shortest distance} = \frac{\left|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 - \vec{b}_2)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|}$$

$$= \frac{\left|\left(-\hat{i}+2\hat{j}-\hat{k}\right).\left(-12i+18\hat{j}-36\hat{k}\right)\right|}{\sqrt{(-12)^2+(18)^2+(-36)^2}}$$

$$= \frac{|+12+36+36|}{\sqrt{1764}} = \frac{84}{42} = 2$$

121. (a) $\cos\left[2\cos^{-1}x+\sin^{-1}x\right]$

$$=\cos\left[\cos^{-1}x+\cos^{-1}x+\sin^{-1}x\right]$$

$$=\cos\left[\cos^{-1}x+\pi/2\right]=-\sin[\cos^{-1}x]$$

$$=-\sin\left[\sin^{-1}\sqrt{1-x^2}\right]=-\sqrt{1-x^2}$$

$$=-\sqrt{1-\left(\frac{1}{5}\right)^2}=-\sqrt{\frac{24}{25}}=-\frac{2\sqrt{6}}{5}$$

122. (b) Suppose, $I=\int_0^{\pi/2}\cos x\, e^{\sin x}dx$

Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$x \to 0 \Rightarrow t \to 0$

and $x \to \frac{\pi}{2} \Rightarrow t \to 1$

So $I=\int_0^1 e^t dt=\left[e^t\right]_0^1 = e-1$

123. (c) In the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, let T_r is the r^{th} term

$$T_r = 15_{C_{r-1}}(x^4)^{15-r+1}\left(\frac{1}{x^3}\right)^{r-1}$$

$$= 15_{C_{r-1}}x^{64-4r-3r+3} = 15_{C_{r-1}}x^{67-7r}$$

x^{18} occurs in this term

$\Rightarrow 18 = 67-7r \Rightarrow 7r = 49 \Rightarrow r = 7.$

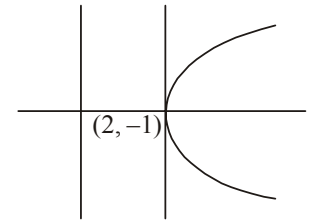
124. (c) Here, $b^2 - 4ac = 1^2 - 4 \times \sqrt{5} \times \sqrt{5}$

$= 1 - 20 = -19$

Therefore, the solutions are

$$\frac{-1\pm\sqrt{-19}}{2\sqrt{5}} = \frac{-1\pm\sqrt{19}i}{2\sqrt{5}}$$

125. (b)



We are given that directrix of the parabola is $4x - 3y = 21$

and vertex of the parabola is (2, –1)

Now $a = \frac{|8+3-21|}{5} = \frac{10}{5} = 2$

$\therefore$ latus rectum of the parabola = $4a = 8$

126. (d) $\frac{\left(1+i\sqrt{3}\right)(2+2i)}{\sqrt{3}-i} = \frac{2+2\sqrt{3}i+2i-2\sqrt{3}}{\sqrt{3}-i}$

$$= \frac{\left(2-2\sqrt{3}\right)+\left(2\sqrt{3}+2\right)i}{\sqrt{3}-i}\times\frac{\sqrt{3}+i}{\sqrt{3}+i}$$

$$= \frac{2\sqrt{3}-6+2i-2\sqrt{3}i+6i+2\sqrt{3}i-2\sqrt{3}-2}{3+1}$$

$$= \frac{8i-8}{4} = -2+2i$$

$\therefore$ Modulus $= \sqrt{(-2)^2+(2)^2} = 2\sqrt{2}.$

127. (c) Here $h = 0$, $k = 2$ and $r = 2$. Therefore, the required equation of the circle is :

$(x-0)^2 + (y-2)^2 = (2)^2$

or $x^2 + y^2 - 4y + 4 = 4$ or $x^2 + y^2 - 4y = 0$

128. (d) (a) Let $f(x) = \cos x$, then $f'(x) = -\sin x$.

In interval $\left(0, \frac{\pi}{2}\right)$, $f'(x) < 0$

Therefore, f(x) is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$

(b) Let $f(x) = \cos 2x \Rightarrow f'(x) = -2\sin x\, 2x$

In interval $\left(0, \frac{\pi}{2}\right)$, $f'(x) < 0$

Because sin 2x will either lie in the first or second quadrant which will give a positive value.

Therefore, f (x) is strictly decreasing on $\left(0,\frac{\pi}{2}\right)$

(c) Let $f'(x) = \cos 3x$

$\Rightarrow$ $f'(x) = -3\sin 3x$. In Interval $\left(0,\frac{\pi}{3}\right)$, $f'(x) < 0$

Because sin 3x will either lie in the first or second quadrant which will give a positive value.

Therefore, f (x) is strictly decreasing on $\left(0,\frac{\pi}{3}\right)$.

When $x \in \left(\frac{\pi}{3},\frac{\pi}{2}\right)$, then $f'(x) > 0$

Because sin3x will lie in the third quadrant.
Therefore, f (x) is not strictly decreasing on $\left(0,\frac{\pi}{2}\right)$

(d) Let $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$.

In Interval $x \in \left(0,\frac{\pi}{2}\right)$, $f'(x) > 0$

Therefore, f(x) is not strictly decreasing on $\left(0,\frac{\pi}{2}\right)$

129. (a) Let $x = a\sin^2\theta$
then $dx = 2a \sin\theta \cos\theta\, d\theta$

$$\therefore \quad I = \int \frac{2a\sin\theta.\cos\theta}{\sqrt{a\sin^2\theta.a\cos^2\theta}}d\theta$$

$$= 2\int d\theta = 2\theta + c$$

$$= 2\sin^{-1}\left(\sqrt{x/a}\right) + c$$

130. (a) Mean $(\bar{x}) = \frac{1+2+3+4+5}{5} = 3$

$$S.D = \sigma = \sqrt{\frac{1}{5}(1+4+9+16+25) - 9} = \sqrt{11-9}$$

$$= \sqrt{2}$$