

























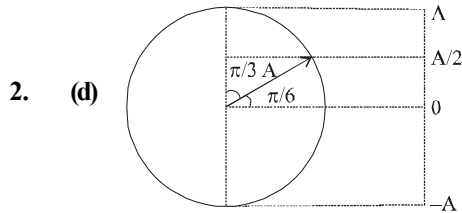




# SOLUTIONS

## PART - I : PHYSICS

1. (a)  $As, F_{\text{air}} = F_{\text{mgd}}$   
 $\Rightarrow \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d_{\text{air}}^2} = \frac{1}{K} \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d^2} \Rightarrow d_{\text{air}} = d\sqrt{K}$



Let time from 0 to  $A/2$  is  $t_1$   
 and from  $A/2$  to  $A$  is  $t_2$   
 From the standard equation of SHM,  
 $x = A_0 \sin(\omega t)$

$$\Rightarrow \frac{A}{2} = A \sin(\omega t_1)$$

$$\Rightarrow \omega t_1 = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \quad \dots(i)$$

then  $\omega t_1 = \pi/6$   
 Using  $x = A_0 \sin \omega t$  again  
 $A = A \sin \omega(t_1 + t_2)$

$$\omega(t_1 + t_2) = \sin^{-1}(1) = \frac{\pi}{2}$$

Using (i)

$$\omega t_2 = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3} \quad \dots(ii)$$

Dividing equation (i) by (ii) we get

$$\frac{t_1}{t_2} = \frac{1}{2}$$

$$\Rightarrow t_2 = 2t_1 = 2 \times 2 = 4 \text{ sec}$$

3. (a) Let

Initial length =  $l_1$

Final length =  $l_2$

Initial area =  $A_1$

Final area =  $A_2$

$\therefore$  Volume remains same

$$\therefore A_1 l_1 = A_2 l_2 \Rightarrow A_1 l_1 = A_2 \frac{l_1}{4}$$

$$\Rightarrow 4A_1 = A_2$$

Initial resistance,  $R_1 = \frac{\rho l_1}{A_1} = 160\Omega$  (given)

Final resistance,  $R_2 = \frac{\rho l_2}{A_2}$

$$\therefore \frac{R_2}{R_1} = \frac{l_2 A_1}{A_2 l_1} = \frac{l_1}{4} \frac{A_1}{4 A_1 l_1}$$

$$\Rightarrow R_2 = \frac{1}{16} R_1 = \frac{1}{16} \times 160 = 10\Omega$$

4. (c) Heat,  $Q = mL$  where,  $L =$  latent heat

$$\therefore L = \frac{Q}{m} = \frac{ML^2 T^{-2}}{M} = M^0 L^2 T^{-2}$$

5. (c) Electric potential at a point P due to a point charge, ( $\because K = 9 \times 10^9$ )

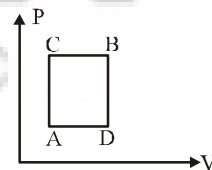
$$V_P = \frac{KQ}{r}$$

$$\Rightarrow 50 = \frac{9 \times 10^9 \times 5 \times 10^{-9}}{r}$$

$$\Rightarrow r = \frac{45}{50} = \frac{9}{10} = 0.9\text{m} = 90\text{cm}$$

6. (a)  $\Delta U$  remains same for both paths ACB and ADB

$$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$$



$$\Rightarrow 60 \text{ J} = 30 \text{ J} + \Delta U_{ACB} \Rightarrow U_{ACB} = 30 \text{ J}$$

As change in internal energy depends only on initial and final point

$$\therefore \Delta U_{ADB} = \Delta U_{ACB} = 30 \text{ J}$$

$$\Delta Q_{ADB} = \Delta U_{ADB} + \Delta W_{ADB} = 10 \text{ J} + 30 \text{ J} = 40 \text{ J}$$

7. (c) Magnetic field due to straight wire,

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic field at  $\frac{a}{2}$  is,

- $B_{a/2} = \frac{\mu_0 I}{2\pi(a/2)}$   
 $\therefore \frac{B_{a/2}}{B_{2a}} = \frac{1}{1} = 1:1$
8. (c) Displacement,  $x = t^3 - 6t^2 + 20t + 15$   
 $\therefore$  Velocity,  $v = \frac{dx}{dt} = 3t^2 - 12t + 20$   
 $\therefore$  Acceleration,  $a = \frac{dv}{dt} = 6t - 12$   
 When  $a = 0$   
 $\Rightarrow 6t - 12 = 0 \Rightarrow t = 2$  s  
 At  $t = 2$  s,  $v = 3(2)^2 - 12(2) + 20 = 8$  m/s
9. (d) The rate of mutual inductance is given by  
 $M = \mu_0 n_1 n_2 \pi r_1^2 \dots(i)$   
 The rate of self inductance is given by  
 $L = \mu_0 n_1^2 \pi r_1^2 \dots(ii)$   
 Dividing (i) by (ii)  
 $\Rightarrow \frac{M}{L} = \frac{n_2}{n_1}$
10. (d)  $v = \frac{C}{\sqrt{\mu_r \epsilon_r}}$   
 $\Rightarrow 1.5 \times 10^8 = \frac{3 \times 10^8}{\sqrt{2 \times \epsilon_r}}$   
 $\Rightarrow 2 \epsilon_r = 4 \Rightarrow \epsilon_r = 2$
11. (c) Range of projectile  
 $R = \frac{v^2 \sin 2\theta}{g}$  ( $\because R \propto \sin(2\theta)$ )  
 $\frac{R_1}{R_2} = \frac{\sin(2\theta_1)}{\sin(2\theta_2)} = \frac{\sin(2 \times 15)}{\sin(2 \times 45)} = \frac{\sin 30^\circ}{\sin 90^\circ}$   
 $\Rightarrow \frac{50}{R_2} = \frac{1}{2} \Rightarrow R_2 = 100$  m
12. (c) Impedance in LCR circuit  
 $Z = \sqrt{(X_L - X_C)^2 + R^2} \quad \because X_L = X_C = R$   
 $\therefore Z = R$
13. (c) Given lights are of same wavelength and stopping potential is independent on intensity. Hence stopping potential will remain same. Intensity  $I_2 > I_1$ , hence saturation current corresponding to  $I_2$  will be greater than that corresponding to  $I_1$ .

14. (a) Acceleration is given as:

$$a = \left( \frac{m_2 - m_1}{m_1 + m_2} \right) g$$

$$\Rightarrow \frac{g}{\sqrt{2}} = \left( \frac{m_2 - m_1}{m_1 + m_2} \right) g$$

$$\Rightarrow \sqrt{2}(m_2 - m_1) = m_1 + m_2$$

$$\Rightarrow \frac{m_1}{m_2} = \left( \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right)$$

15. (a) Wavelength of H-atom is

$$\frac{1}{\lambda} = RZ^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Shortest wavelength for Balmer series:

$$\frac{1}{\lambda_B} = RZ^2 \left( \frac{1}{2^2} - \frac{1}{\infty} \right) \dots(i)$$

Shortest wavelength for Lyman series:

$$\frac{1}{\lambda_L} = RZ^2 \left( \frac{1}{1^2} - \frac{1}{\infty} \right) \dots(ii)$$

Dividing eq. (i) and (ii),

$$\lambda_B : \lambda_L = 4 : 1$$

16. (b) Mass of block,  $m = 1$  kg  
 Force of parallel inclined surface,  $F = 10$  N

Work done against frictional force  
 $= \mu_s N \times 10 = \mu_s Mg \times 10 = 0.1 \times 5 \times 10 = 5$  J

17. (a)  $\because f_0 + f_e = 30$

And magnification,  $m = \frac{f_0}{f_e}$

$$2 = \frac{f_0}{f_e} \Rightarrow f_0 = 2f_e \Rightarrow f_0 + \frac{f_0}{2} = 30 \quad \therefore f_0 = 20 \text{ cm}$$

18. (c)  ${}_6C^{13} + \text{Energy} \rightarrow {}_6C^{12} + {}_0n^1$

Mass defect,  $\Delta m = (12.000000 + 1.008665) - 13.003354$

$$= -0.00531 \text{ u}$$

$\therefore$  Energy required  $= \Delta m \times 931.5 = 0.00531 \times 931.5$   
 MeV = 4.95 MeV

19. (a) Angular momentum,  $L = mvH = mu \cos 30^\circ H$

$$= mu \cos 30^\circ \times \frac{u^2 \sin^2 \theta}{2g} \left[ \because H = \frac{u^2 \sin^2 \theta}{2g} \right]$$

$$= \frac{mu^3}{2g} \times \frac{\sqrt{3}}{2} \times \left( \frac{1}{2} \right)^2 = \frac{\sqrt{3}mu^3}{16g}$$

20. (c) Energy is,  $E = \frac{hc}{\lambda}$   
 $E = \frac{1242 \text{ nm} \cdot \text{eV}}{\lambda} \Rightarrow \lambda = \frac{1242}{6} = 207 \text{ nm}$
21. (d) Basically 1 year is equal to time period of earth revolution around sun.

So,  $T_i = 1 \text{ year}$

Now,

$$T^2 \propto R^3$$

$$\therefore \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

$$\Rightarrow T_2 = \left(\frac{R_2}{R_1}\right)^{3/2} \cdot T_1 = \left(\frac{3R}{R}\right)^{3/2} \times 1$$

$$= 3\sqrt{3} \text{ years.}$$

22. (b) Output  $Y = \overline{\overline{A \cdot B}} = \overline{\overline{A + B}}$  (By De-Morgan Law)

$$\therefore Y = A + B$$

This Boolean expression represents OR gate.

23. (d) RMS speed,

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow V_{\text{rms}} \propto \sqrt{T}$$

$$\text{Here, } T_{\text{initial}} = 200 \text{ K}$$

$$T_{\text{final}} = 800 \text{ K}$$

$$\text{Initial RMS speed} = v_0$$

$$\therefore \frac{v_0}{v_{\text{rms}}} = \sqrt{\frac{200}{800}} \Rightarrow v_{\text{rms}} = 2v_0$$

24. (b) Young's modulus,  $Y = \frac{\text{Stress}}{\text{Strain}}$

If the temperature increases, strain also increases.

Hence young's modulus decreases.

25. (b) Given 1 Main scale division = m  
 n MSD = (n + 1) VSD

$$\Rightarrow 1 \text{ VSD} = \frac{n}{n+1} \text{ MSD}$$

Least count of vernier caliper = 1 MSD - 1 VSD

$$\Rightarrow \text{L.C} = m - m \left(\frac{n}{n+1}\right) = \left(1 - \frac{n}{n+1}\right)m$$

$$= m \left(\frac{n+1-n}{n+1}\right) = \left(\frac{1}{n+1}\right)m \Rightarrow \text{L.C} = \left(\frac{m}{n+1}\right)$$

26. (c) According to question, pressure inside, 1st soap bubble,

$$\Delta P_1 = P_1 - P_0 = 0.01 = \frac{4T}{R_1} \quad \dots(i)$$

$$\text{And } \Delta P_2 = P_2 - P_0 = 0.02 = \frac{4T}{R_2} \quad \dots(ii)$$

Dividing, equation (ii) by (i),

$$\frac{1}{2} = \frac{R_2}{R_1} \Rightarrow R_1 = 2R_2$$

$$\text{Volume } V = \frac{4}{3}\pi R^3 \Rightarrow \frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{8R_2^3}{R_2^3} = 8$$

27. (a) Condition for minima is  $d \sin \theta = n\lambda$   
 For 1st minima  $d \sin \theta = \lambda$

$$\sin \theta = \frac{\lambda}{a} = \frac{2}{4} = \frac{1}{2} \quad \therefore \theta = 30^\circ$$

$$\text{Angular spread} = 2\theta = 60^\circ$$

28. (b) We have  $\Delta V = V_0 \gamma \Delta T$

$$\Delta V = a^3 \cdot (3\alpha) \Delta T$$

$$\text{Now, } 6a^2 = 24 \quad [\because \text{Total surface area of cube} = 6a^2]$$

$$\Rightarrow a^2 = 4 \Rightarrow a = 2$$

$$\text{So, } \Delta V = 2^3(3 \times 5 \times 10^{-4}) \times 10 = 1200 \times 10^{-4} \text{ m}^3 = 1200 \times 10^2 \text{ cm}^3 = 1.2 \times 10^5 \text{ cm}^3$$

29. (a) Position of COM of a mass - system is given as,

$$\vec{r}_{\text{com}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} = \frac{1(\hat{i} + 2\hat{j} + \hat{k}) + 3(-3\hat{i} - 2\hat{j} + \hat{k})}{1 + 3}$$

$$= -2\hat{i} - \hat{j} + \hat{k}$$

$$|\vec{r}_{\text{com}}| = |-2\hat{i} - \hat{j} + \hat{k}| = \sqrt{(2)^2 + (1)^2 + (1)^2} = \sqrt{6}$$

Only option (a) magnitude is

$$\sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$$

So option (a) is correct.

30. (a) At STP

$$\text{Temperature, } T = 273 \text{ K}$$

$$\text{Molecular mass of oxygen, } M = 32 \times 10^{-3} \text{ kg}$$

Speed of sound is given by

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.3 \times 273}{32 \times 10^{-3}}}$$

$$= 314.8541 \approx 315 \text{ m/s}$$

## PART - II : CHEMISTRY

31. (b) 1 M = mole of solute in 1 L solution.

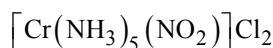




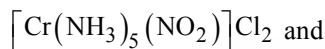
45. (b) Ferrocene is  $\text{Fe}(\eta^5 - \text{C}_5\text{H}_5)_2$   
 Perxenate ion is  $[\text{XeO}_6]^{4-}$  which is octahedral.
46. (d) Sodium stearate is used in soap and sodium lauryl sulphate

$\text{CH}_3(\text{CH}_2)_{10}\text{CH}_2 - \text{OSO}_3^- \text{Na}^+$  is an anionic detergent.

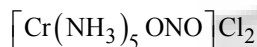
47. (c)  $\text{A} + \text{B}^{2+} \rightleftharpoons \text{A}^{2+} + \text{B}$   
 $G^\circ = -RT \ln K_c$   
 $= -8.314 \times 298 \times 2.303 \times \log 10^{12} = 68.47 \text{ kJ/mol}$
48. (b) The chemical formula of nitropenta ammine chromium (III) chloride is:



It can exist in following two structures



Pentaamminenitrochromium (III) chloride



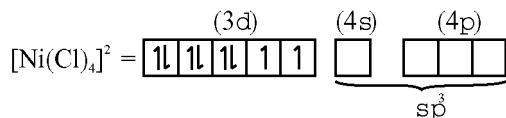
Pentaamminenitrochromium (III) chloride

Therefore, the type of isomerism found in this compound is linkage isomerism as  $\text{NO}_2$  group is linked through N as  $-\text{NO}_2$  or through O as  $-\text{ONO}$ .

49. (c) The stability of hydrides decreases from  $\text{NH}_3$  to  $\text{BiH}_3$  but their reducing character increases down the group.
50. (d)

(a) Ni in  $[\text{Ni}(\text{Cl})_4]^{2-}$  exist as  $\text{Ni}^{2+}$  ion.  
 Cl is a weak field ligand (high spin). It will not cause pairing of electrons.

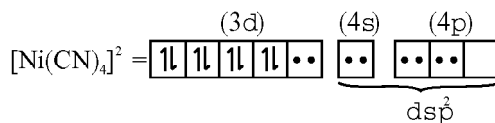
Hence, configuration of  $[\text{Ni}(\text{Cl})_4]^{2-}$  is



Tetrahedral with two unpaired electrons (i.e., paramagnetic) and has  $sp^3$  hybridisation.

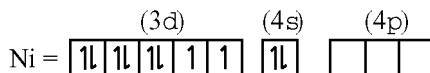
(b) In  $[\text{Co}(\text{C}_2\text{O}_4)_3]^{3-}$ ,  $(\text{C}_2\text{O}_4)^{2-}$  is a bidentate ligand thus, give octahedral structure.

(c) In  $[\text{Ni}(\text{CN})_4]^{2-}$ , Ni exist as  $\text{Ni}^{2+}$  ion.  
 $\text{CN}^-$  is a strong field ligand (low spin). Causes pairing of electrons of  $3d$  orbital.

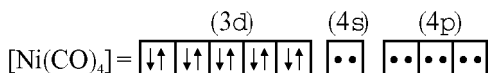


Hence, the structure of is square planar and it is diamagnetic.

(d) In  $[\text{Ni}(\text{CO})_4]$ , Ni has zero oxidation state. i.e.,



CO is a strong field ligand causes rearrangement and pairing of electrons of  $3d$  and  $4s$  orbital.



Structure of  $\text{Ni}(\text{CO})_4$  is tetrahedral with  $sp^3$  hybridisation and it is diamagnetic.

Hence, (d) is the correct answer.

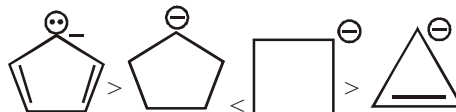
51. (d)

52. (c)

$$\frac{\text{Mass of metal}_1 \text{ deposited}}{\text{Mass of metal}_2 \text{ deposited}} = \frac{\text{Eq. wt. of metal}_1}{\text{Eq. wt. of metal}_2}$$

The equivalent weight of silver is highest among the given options.

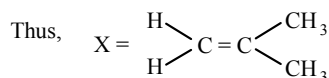
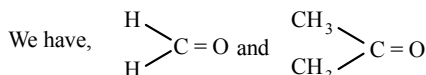
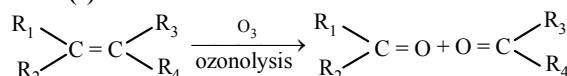
53. (d) As we know compound (4) is aromatic and the compound (1) is anti-aromatic. Hence compound (4) is most stable and compound (1) is least stable among these in compound (2) and (3) carbon atom having charge is  $sp^3$  hybridised. On the basis of angle strain theory compound (3) is more stable than compound (2).



54. (a) *cis*-polyisoprene is natural rubber.

55. (d) Grignard reagent =  $\text{R} - \overset{\delta+}{\text{Mg}} \overset{\delta-}{\text{X}}$

56. (b)



57. (d) The reaction  $P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 2S_2Cl_2 + 4SO_2$  involves change of oxidation state of P from 0 to +3 and that of S from +4 to +2. Thus, it is a redox reaction but not a disproportionation reaction.
58. (b) Le-Chatelier principle is not applicable to pure solids and liquids because they experience negligible change in concentration during chemical equilibrium.
59. (a) no. of moles of  $CH_4$  ( $n_1$ ) =  $\frac{4}{16} = 0.25$  mol

$$\text{no. of moles of } CO_2 (n_2) = \frac{4.4}{44} = 0.1 \text{ mol}$$

$$\text{Total no. of moles } (n_T) = n_1 + n_2 = 0.35 \text{ mol}$$

$$\therefore P = \frac{0.35 \times 0.082 \times 300}{1} = 8.6 \text{ atm}$$

60. (b)  $E_n = -13.6 \left[ \frac{z^2}{n^2} \right] \text{ eV}$

$$\Rightarrow E_1 = -13.6 \left[ \frac{3^2}{1^2} \right] \text{ eV} = -122.4 \text{ eV}$$

$$= -122.4 \times 1.602 \times 10^{-19} \text{ J} = -1.962 \times 10^{-17} \text{ J}$$

**PART - III (A): ENGLISH PROFICIENCY**

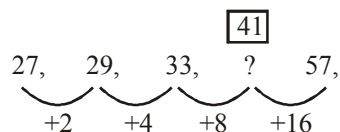
61. (a) 62. (a) 63. (b)
64. (a) India deserves a far bigger share of world trade considering its vast resources.
65. (a)
66. (d) The garbage was first sundried for one to three days to bring down the moisture level.
67. (a) In Voice change, one can transform the sentence from Active to Passive or vice versa, but can't change the sense. That is, one can't remove the information conveyed through the sentence or add any additional information. That is, transformation must be done keeping the information intact. This is the reason behind selecting (a) as the answer.
68. (c)
69. (c) with the debasing of the coinage than
70. (a) Every where in the passage we find that the author favours India gaining an edge over China. The author, throughout the

passage, is highlighting China's own prospective while they are helping the Africans.

**PART - III (B) : LOGICAL REASONING**

71. (c) 72. (c)

73. (b)

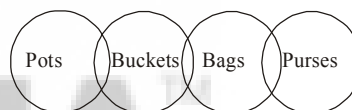


74. (d) The male members in the family are:-

- (i) The man himself
- (ii) his four sons; and
- (iii) his  $(3 \times 4) = 12$  grandsons.

$$\text{Hence total number of male members} = 1 + 4 + 12 = 17$$

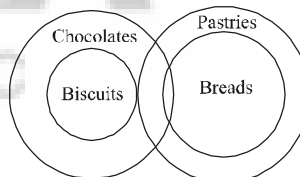
75. (b) Venn-Diagram Representation:



**Conclusions:**

- I. False
- II. False

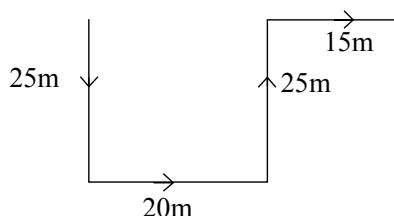
76. (c) Venn-Diagram Representation:



**Conclusions:**

Only II follows.

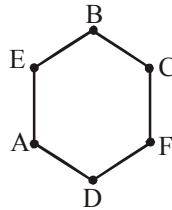
77. (a)



$\therefore$  Rohit is 35 m east.

**For Sol. (78-80)**

The following hexagonal arrangement is possible.



78. (c) The other neighbour of A is E.  
 79. (a) F is placed opposite to E.  
 80. (b) Clearly, C is the required person.

81. (b)

$+\Rightarrow\div$	$-\Rightarrow\times$
$\div\Rightarrow+$	$\times\Rightarrow-$

$$63 \times 24 + 8 \div 4 + 2 - 3 = ?$$

$$\text{or, } ? = 63 - 24 \div 8 + 4 \div 2 \times 3$$

$$\text{or, } ? = 63 - 3 + 2 \times 3$$

$$\text{or, } ? = 66$$

82. (d) Arrangement as per height:  
 $C > D > A > E > B$   
 (Middle)
83. (a) A successful completion of 'Education' equips one with 'Diploma'. Similarly, a successful completion in 'Sports' equips one with 'Trophy'.
84. (b)
85. (c) According to english alphabet, resultant group will be as follows:

P R I N C E  
 C E I N P R

Only two letters 'I and N' will remain unchanged.

86. (d)

N	A	T	I	O	N
4	6	7	2	3	4

E	A	R	N
↓	↓	↓	↓
1	6	5	4

A	T	T	E	N	T	I	O	N
6	7	7	1	4	7	2	3	4

87. (b) 12, 18, 30 is multiple of 6.  
 16, 32, 40 is multiple of 8.  
 36, 18, 27 is multiple of 9.
88. (c) Required area common to  $\triangle$ ,  $\circ$ ,  $\square$ .  
 i.e. 9
89. (c) The series is bbb/bbb/bbbb/bbbb.  
 Thus, in each sequence, 'a' moves one step forward and 'b' takes its place and finally in the fourth sequence, it is eliminated.
90. (a) The colour of clean sky is blue and blue means green. Hence, the colour of clean sky is green.

**PART - IV : MATHEMATICS**

91. (c)
- $$A = \frac{1}{1-r^a} \Rightarrow 1-r^a = \frac{1}{A} \Rightarrow r^a = 1 - \frac{1}{A} = \frac{A-1}{A}$$
- $$B = \frac{1}{1-r^b} \Rightarrow 1-r^b = \frac{1}{B} \Rightarrow r^b = 1 - \frac{1}{B} = \frac{B-1}{B}$$
- $$\therefore a \log r = \log\left(\frac{A-1}{A}\right)$$
- $$\text{and } b \log r = \log\left(\frac{B-1}{B}\right)$$
- $$\therefore \frac{a}{b} = \frac{\log\left(\frac{A-1}{A}\right)}{\log\left(\frac{B-1}{B}\right)} = \log_{\frac{B-1}{B}}\left(\frac{A-1}{A}\right)$$

92. (c) Let  $S = 1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \dots$  ... (i)
- $$\frac{S}{6} = \frac{1}{6} + \frac{5}{6^2} + \frac{12}{6^3} + \frac{22}{6^4} + \dots$$
- ... (ii)
- On subtracting (i) from (ii)
- $$\frac{5}{6}S = 1 + \frac{4}{6} + \frac{7}{6^2} + \frac{10}{6^3} + \frac{13}{6^4} + \dots$$
- $$\frac{5}{36}S = \frac{1}{6} + \frac{4}{6^2} + \frac{7}{6^3} + \frac{10}{6^4} + \frac{13}{6^5} + \dots$$

on subtraction

$$\frac{25}{36}S = 1 + \frac{3}{6} + \frac{3}{6^2} + \frac{3}{6^3} + \dots = \frac{8}{5};$$

$$S = \frac{8}{5} \times \frac{36}{25} = \frac{288}{125}$$

93. (a) Let a complex number,  $z = x + iy$

$$\Rightarrow \bar{z} = \bar{x} - iy$$

Then, vertices of rectangle for  $z, \bar{z}, -z, -\bar{z}$  are  $(x, y), (x, -y), (-x, -y), (-x, y)$ .



Now, Area of rectangle =  $(2x)(2y) = 4xy$

It is given that,

$$\text{Area} = 2\sqrt{3} = 4xy \Rightarrow 2xy = \sqrt{3}$$

$$\therefore x = \frac{1}{2}, y = \sqrt{3} \therefore z = \frac{1}{2} + \sqrt{3}i$$

94. (d)  $(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$$\Rightarrow 2\sin \frac{5x}{2} \cdot \cos \frac{3x}{2} + 2\sin \frac{5x}{2} \cdot \cos \frac{x}{2} = 0$$

$$\Rightarrow 2\sin \frac{5x}{2} \left\{ \cos \frac{3x}{2} + \cos \frac{x}{2} \right\} = 0$$

$$\Rightarrow 2\sin \frac{5x}{2} \left\{ 2\cos x \cos \frac{x}{2} \right\} = 0$$

$$2\sin \frac{5x}{2} = 0 \Rightarrow \frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, \dots$$

$$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$$

$$\cos \frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi;$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

So, sum =  $6\pi + \pi + 2\pi = 9\pi$

95. (b)  $p \rightarrow (q \vee r)$ . If 2 is even number then 2 is a prime number or  $2 + 2 = 2^2$ .

96. (a) 
$$f(-x) = \frac{\cos(-x)}{\left[-\frac{2x}{\pi}\right] + \frac{1}{2}} = \frac{\cos x}{-1 - \left[\frac{2x}{\pi}\right] + \frac{1}{2}}$$

(As  $x$  is not an integral multiple of  $\pi$ )

$$= -\frac{\cos x}{\left[\frac{2x}{\pi}\right] + \frac{1}{2}} = -f(x)$$

Therefore,  $f(x)$  is an odd function.

97. (c)  $|z_1| = |z_2| = |z_3| = \dots = |z_n| = 1$

$$\Rightarrow z_1 \bar{z}_1 = z_2 \bar{z}_2 = z_3 \bar{z}_3 = \dots = z_n \bar{z}_n = 1$$

$$\bar{z}_1 = \frac{1}{z}, \bar{z}_2 = \frac{1}{z_2}, \bar{z}_3 = \frac{1}{z_3}, \dots, \bar{z}_n = \frac{1}{z_n}$$

Now,  $|z_1 + z_2 + z_3 + \dots + z_n|$   
 $= |\bar{z}_1 + \bar{z}_2 + \dots + \bar{z}_n|$

$$|\bar{z}_1 + \bar{z}_2 + \bar{z}_3 + \dots + \bar{z}_n| = \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \dots + \frac{1}{z_n} \right|$$

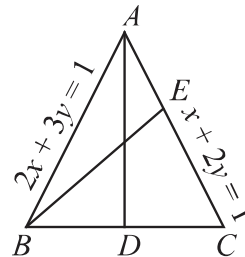
98. (c) Here, point  $A$  is the intersection of line  $AB$  and  $AC$  so equation of line passing through  $A$ .  
 $(x + 2y - 1) + \lambda(2x + 3y - 1) = 0 \dots (i)$

This line passes through the orthocentre  $(0, 0)$ , then

$$-1 - \lambda = 0$$

$$\Rightarrow \lambda = -1$$

On substituting  $\lambda = -1$  in Eq. (i), we get  $x + y = 0$  as the equation of  $AD$ . Since  $AD \perp BC$ , therefore



$$-1 \times -\frac{a}{b} = -1$$

$$\Rightarrow a + b = 0$$

... (ii)

Similarly, by applying the condition that  $BE$  is perpendicular to  $CA$ , we get  $a + 2b = 8$

... (iii)

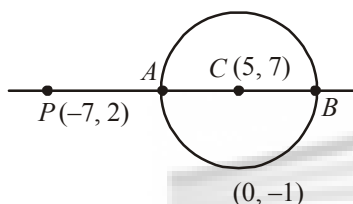
Now, solving Eqs. (ii) and (iii), we get  $a = -8$ ,  
 $b = 8$

99. (a) The centre  $C$  of the circle =  $(5, 7)$  and the radius

$$= \sqrt{5^2 + 7^2 + 51} = 5\sqrt{5}$$

$$PC = \sqrt{12^2 + 5^2} = 13 \Rightarrow q = PA = 13 - 5\sqrt{5}$$

$$\text{and } p = PB = 13 + 5\sqrt{5}$$



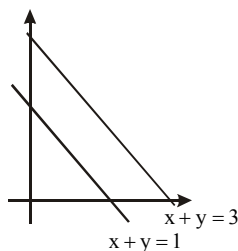
$\therefore$  G.M. of  $p$  and  $q$

$$= \sqrt{pq} = \sqrt{(13 - 5\sqrt{5})(13 + 5\sqrt{5})}$$

$$= \sqrt{169 - 125} = 2\sqrt{11}$$

100. (a) Coordinate of focus will be  $(h + 1, k)$

Now focus should lie to the opposite side of origin with respect to line  $x + y - 1 = 0$  and same side as origin with respect to line  $x + y - 3 = 0$



Hence  $h + k > 0$  and  $h + k < 2$ .

101. (d)  $\lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} \sqrt{\{-h\} \cot\{-h\}}$

$$= \lim_{h \rightarrow 0} \sqrt{(1-h) \cot(1-h)} = \sqrt{\cot 1}$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} \frac{\tan^2\{h\}}{h^2 - [h]^2} = \lim_{h \rightarrow 0} \frac{\tan^2 h}{h^2} = 1$$

$\therefore \lim_{x \rightarrow 0} f(x)$  does not exist.

102. (a) Given equation can be reduced to a quadratic equation.

$$\therefore 2x^2 + x - 11 + \frac{1}{x} + \frac{2}{x^2} = 0$$

$$\Rightarrow 2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 11 = 0$$

Put  $x + \frac{1}{x} = y$

$$2(y^2 - 2) + y - 11 = 0$$

$$\Rightarrow 2y^2 + y - 15 = 0$$

$$\Rightarrow y = -3 \text{ and } \frac{5}{2}$$

$$\Rightarrow x + \frac{1}{x} = -3, x + \frac{1}{x} = \frac{5}{2}$$

$$\Rightarrow x^2 + 3x + 1 = 0, 2x^2 - 5x + 2 = 0$$

Only 2nd equation has rational roots as  $D = 9$

and roots are  $\frac{1}{2}$  and 2.

103. (c)  ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11 : 52$

$$\text{or } (21-r)(20-r)(19-r) = 52 \times 2 \times 21$$

$$\Rightarrow (21-r)(20-r)(19-r) = 14 \times 13 \times 12$$

$$\Rightarrow (21-r)(20-r)(19-r) = (21-7)(20-7)(19-7)$$

$$\Rightarrow r = 7$$

$$\Rightarrow x + \frac{1}{x} = -3, x + \frac{1}{x} = \frac{5}{2}$$

$$\Rightarrow x^2 + 3x + 1 = 0, 2x^2 - 5x + 2 = 0$$

Only 2nd equation has rational roots as  $D = 9$

and roots are  $\frac{1}{2}$  and 2.

104. (a) Total contested candidates = 12

4 candidates are to be elected and voter votes for at least one candidate then total number of

ways of selections.  
 $= {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + {}^{12}C_4$   
 $= 12 + \frac{12 \times 11}{2 \times 1} + \frac{12 \times 11 \times 10}{3 \times 2 \times 1} + \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}$   
 $= 12 + 66 + 220 = 495 = 793$

**105. (c)** Since  $(x + \sqrt{x^2 - 1})^8 + (x - \sqrt{x^2 - 1})^8$   
 $= 2 \left\{ {}^8C_0 x^8 + {}^8C_2 x^6 (x^2 - 1) + {}^8C_4 x^4 (x^2 - 1)^2 \right.$   
 $\left. + {}^8C_6 x^2 (x^2 - 1)^3 + {}^8C_8 x^0 (x^2 - 1)^4 \right\}$

So coefficient of highest power of  $x$

$$= 2 \left\{ {}^8C_0 + {}^8C_2 + {}^8C_4 + {}^8C_6 + {}^8C_8 \right\}$$

$$= (1+1)^8 + (1-1)^8 = 2^8 = 256$$

**106. (a)** Observations : 2, 3, 5, 8, 12

$$\text{Mean} = \frac{2+3+5+8+12}{5} = 6$$

$$\therefore \sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n} = \frac{16+9+1+4+36}{5} = \frac{66}{5}$$

$$\therefore \sigma^2 = 13.2$$

$$\text{Median} = 5 = m$$

$\therefore$  Mean deviation about median

$$\Rightarrow \frac{\sum |x_i - m|}{n} = \frac{3+2+0+3+7}{5} = 3$$

$$M = 3 \Rightarrow \sigma^2 - M = 13.2 - 3 = 10.2.$$

**107. (d)** Let  $S$  denote the set of all triangles in a plane.

Let  $R$  be the relation on  $S$  defined by  $(\Delta_1, \Delta_2) \in R$

$\Rightarrow$  triangle  $\Delta_1 \cong \Delta_2$ :

(i) Let any triangle  $\Delta \in S$ , we have

$\Delta_1 \cong \Delta_2 \Rightarrow (\Delta, \Delta) \in R \forall \Delta \in S \Rightarrow R$  is reflexive on  $S$ .

(ii) Let  $\Delta_1, \Delta_2 \in S$ , such that  $(\Delta_1, \Delta_2) \in R$ , then  $\Delta_1 \cong \Delta_2 \Rightarrow \Delta_2 \cong \Delta_1 \Rightarrow (\Delta_2, \Delta_1) \in R \Rightarrow R$  is symmetric

(iii) Again, let  $\Delta_1, \Delta_2, \Delta_3 \in S$  such that  $(\Delta_1, \Delta_2) \in R$  and

$$(\Delta_2, \Delta_3) \in R \therefore \Delta_1 \cong \Delta_2 \cong \Delta_3$$

$$\therefore (\Delta_1, \Delta_3) \in R$$

$\Rightarrow R$  is transitive.

$\therefore R$  is an equivalence relation.

**108. (c)** Since  $XY = (AB + BA)(AB - BA)$   
 $= (AB)AB + (BA)(AB) - (AB)(BA) - (BA)(BA)$   
 Now  $(XY)^T = ((AB).(AB))^T + (BA.AB)^T - (AB.BA)^T$   
 $= (AB)^T.(AB)^T + (AB)^T.(BA)^T - (BA)^T(AB)^T - (BA)^T(BA)^T$

$$= (B^T.A^T)(B^T.A^T) + (B^T.A^T).(A^T.B^T)$$

$$- (A^T.B^T)(B^T.A^T) - (A^T.B^T)(A^T.B^T)$$

Since,  $A$  &  $B$  are symmetric matrix.

$$= (BA)(BA) + (BA)(AB) - (AB)(BA)$$

$$- (AB)(AB)$$

$$= (BA - AB)(BA + AB) = -YX$$

**109. (c)** Expanding the two determinants, we get

$$(1 - 3x^2 + 2x^3) + (3x^2 - x^3) = 0$$

$$\Rightarrow x^3 + 1 = 0$$

$$\Rightarrow x = -\omega, -\omega^2, -1$$

$$x^{2007} + x^{-2007} = -1 - 1 = -2$$

**110. (c)** Given that,  $f(x) = \frac{x}{\sqrt{1+x^2}}$

**For injective:** Let  $x_1, x_2 \in \mathbf{R}$  such that

$$f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1}{\sqrt{1+x_1^2}} = \frac{x_2}{\sqrt{1+x_2^2}} \Rightarrow \frac{x_1^2}{1+x_1^2} = \frac{x_2^2}{1+x_2^2}$$

$$\Rightarrow x_1^2 + x_1^2 x_1^2 = x_2^2 + x_1^2 x_2^2 \Rightarrow x_1^2 = x_2^2$$

$$\Rightarrow x_1 = x_2$$

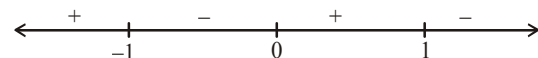
So,  $f(x)$  is injective.

**For surjective:** Let  $y = \frac{x}{\sqrt{1+x^2}}$

$$\Rightarrow y^2(1+x^2) = x^2 \Rightarrow y^2 + y^2 x^2 = x^2$$

$$\Rightarrow x^2(1-y^2) = y^2 \Rightarrow x = \sqrt{\frac{y^2}{1-y^2}}$$

$$\Rightarrow \frac{y^2}{1-y^2} \geq 0$$



$$\therefore y \in (-1, 1)$$

So,  $f(x)$  is not surjective.

111. (d) Given  $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \dots$

$$+ \tan^{-1}\left(\frac{1}{1+n(n+1)}\right) = \tan^{-1}(x)$$

$$\Rightarrow \tan^{-1}\left(\frac{2-1}{1+1.2}\right) + \tan^{-1}\left(\frac{3-2}{1+2.3}\right) + \dots$$

$$+ \tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right) = \tan^{-1}(x)$$

$$\Rightarrow \tan^{-1}(2) - \tan^{-1}(1) + \tan^{-1}(3) - \tan^{-1}(2) + \dots$$

$$+ \tan^{-1}(n+1) - \tan^{-1}(n) = \tan^{-1}(x)$$

$$\Rightarrow \tan^{-1}(n+1) - \tan^{-1}(1) = \tan^{-1}(x)$$

$$\left\{ \because \tan^{-1}(A) - \tan^{-1}(B) = \tan^{-1}\left(\frac{A-B}{1+AB}\right) \right\}$$

$$\Rightarrow \tan^{-1}\left(\frac{n}{n+2}\right) = \tan^{-1}(x) \Rightarrow x = \frac{n}{n+2}$$

112. (c)  $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{x^2 \log(\cos x)}{\log(1+x)}$

$$= \lim_{x \rightarrow 0} \frac{x \cdot \log(\cos x)}{\frac{\log(1+x)}{x}}$$

$$= \lim_{x \rightarrow 0} x \cdot \log(\cos x) = 0 \cdot \log 1 = 0$$

$$\because \lim_{x \rightarrow 0} f(x) = f(0)$$

$$\therefore f(x) \text{ is continuous at } x = 0$$

$$\text{Now, } \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2 \log \cos h - 0}{\log(1+h)}$$

$$\lim_{h \rightarrow 0} \frac{h^2 \log(\cos h)}{1} = 0$$

$$\therefore f(x) \text{ is differentiable at } x = 0$$

113. (b) Given,  $f(x) = 5x - 3$  and  $g(x) = x^2 + 3$

$$\text{Let, } y = f(x), \therefore y = 5x - 3$$

$$y + 3 = 5x \Rightarrow x = \frac{y+3}{5}$$

$$\therefore f^{-1}(y) = \frac{y+3}{5} \Rightarrow f^{-1}(x) = \frac{x+3}{5}$$

$$\text{Now, } g(x) = x^2 + 3;$$

$$\text{So, } g \circ f^{-1}(3) = g[f^{-1}(3)]$$

$$= g\left(\frac{3+3}{5}\right) = g\left(\frac{6}{5}\right) = \left(\frac{6}{5}\right)^2 + 3 = \frac{36}{25} + 3 = \frac{111}{25}$$

114. (a) Given that,

$$\cot(\cos^{-1} x) = \sec\left\{\tan^{-1}\left(\frac{a}{\sqrt{b^2 - a^2}}\right)\right\}$$

$$\text{Since, } \cos^{-1} x = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\text{and } \tan^{-1} x = \sec^{-1}\left(\sqrt{1+x^2}\right)$$

$$\Rightarrow \cot\left(\cot^{-1}\frac{x}{\sqrt{1-x^2}}\right) =$$

$$\sec\left\{\sec^{-1}\sqrt{1+\left(\frac{a}{\sqrt{b^2 - a^2}}\right)^2}\right\}$$

$$\Rightarrow \frac{x}{\sqrt{1-x^2}} = \sqrt{\frac{b^2 - a^2 + a^2}{b^2 - a^2}}$$

$$\Rightarrow \frac{x}{\sqrt{1-x^2}} = \frac{b}{\sqrt{b^2 - a^2}}$$

On squaring both the sides, we get

$$\Rightarrow \frac{x^2}{1-x^2} = \frac{b^2}{b^2 - a^2}$$

$$\Rightarrow x^2 b^2 - x^2 a^2 = b^2 - b^2 x^2$$

$$x^2 b^2 + b^2 x^2 - x^2 a^2 = b^2$$



$$\Rightarrow 2x^2b^2 - x^2a^2 = b^2 \Rightarrow x^2(2b^2 - a^2) = b^2$$

$$\Rightarrow x = \frac{b}{\sqrt{2b^2 - a^2}}$$

115. (b) Given equations,  $x - y + 2z = 4$

$$3x + y + 4z = 6$$

$$x + y + z = 1$$

$$\text{Let } \Delta = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 1(1-4) + 1(3-4) + 2(3-1) \\ = -3 - 1 + 4 = 0$$

$$\text{and } \Delta_1 = \begin{vmatrix} 4 & -1 & 2 \\ 6 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 4(1-4) + 1(6-4) + 2(6-1) \\ = -12 + 2 + 10 = 0$$

Now,  $\Delta = 0$  and  $\Delta_1 = 0$

$\therefore$  These equations have infinitely many solutions.

116. (a)  $\frac{d}{dx}(\tan^{-1}(\cos\sqrt{x}) + \sec^{-1}(e^x))$

$$= \frac{(-\sin\sqrt{x})}{1 + \cos^2\sqrt{x}} \cdot \left(\frac{1}{2\sqrt{x}}\right) + \frac{1}{e^x\sqrt{e^{2x}-1}} \cdot e^x$$

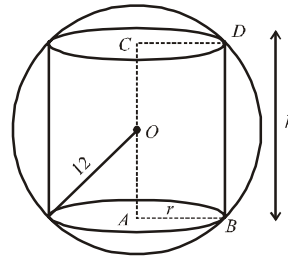
but when  $x = \frac{\pi^2}{4}$

$$= \frac{-\sin\frac{\pi}{2}}{1 + \cos^2\frac{\pi}{2}} \left(\frac{1}{2}\right) \left(\frac{2}{\pi}\right) + \frac{1}{\sqrt{e^{\pi^2/2}-1}}$$

$$= -\frac{1}{\pi} + \frac{1}{\sqrt{e^{\pi^2/2}-1}}$$

117. (b)  $12^2 = r^2 + \left(\frac{h}{2}\right)^2 \Rightarrow V = \pi r^2 h$

$$\Rightarrow V = \pi \left(144 - \frac{h^2}{4}\right) h$$



$$\Rightarrow V = 144\pi h - \frac{\pi}{4}h^3 \Rightarrow \frac{dV}{dh} = 144\pi - \frac{3\pi}{4}h^2$$

$$\Rightarrow \frac{dV}{dh} = 0 \Rightarrow 144\pi = \frac{3\pi}{4}h^2$$

$$\Rightarrow h^2 = 48 \times 4 \Rightarrow h = 8\sqrt{3}$$

$$\therefore 12^2 = r^2 + 48 \Rightarrow r^2 = 96$$

$$\text{Volume} = \pi r^2 h = \pi \times 96 \times 8\sqrt{3} = 768\sqrt{3}\pi \text{ cm}^3.$$

118. (d)  $I = \int \left(\frac{x^3-1}{x^3+x}\right) dx = \int \left(1 - \frac{x+1}{x^3+x}\right) dx$

$$\Rightarrow I = \int 1 \cdot dx - \int \frac{(x+1)}{x^3+x} dx$$

$$= x - \int \frac{(x+1)}{x(x^2+1)} dx$$

$$\text{Let } \frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow (x+1) = A(x^2+1) + (Bx+C)x$$

$$\Rightarrow (x+1) = (A+B)x^2 + Cx + A$$

On comparing coefficient, we get

$$A+B=0, C=1, A=1$$

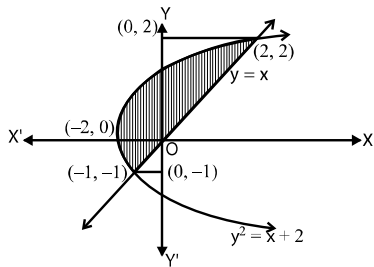
$$\Rightarrow B=-1$$

$$\therefore I = x - \int \frac{1}{x} dx - \int \frac{(1-x)}{x^2+1} dx$$

$$\Rightarrow I = x - \log|x| - \int \frac{1}{x^2+1} dx + \frac{1}{2} \int \frac{2x}{x^2+1} dx$$

$$\Rightarrow I = x - \log|x| - \tan^{-1}x + \frac{1}{2} \log(x^2+1) + c$$

119. (c) Given,  $x = y^2 - 2$  and  $x = y$ .



On solving,  $x = y^2 - 2$  and  $x = y$ , we get  $(-1, -1)$  and  $(2, 2)$ .

Area of the shaded region,

$$\begin{aligned} A &= \int_{-1}^2 y \, dy - \int_{-1}^2 (y^2 - 2) \, dy \\ &= \left[ \frac{y^2}{2} - \frac{y^3}{3} + 2y \right]_{-1}^2 = \left( \frac{4}{2} - \frac{8}{3} + 4 \right) - \left( \frac{1}{2} + \frac{1}{3} - 2 \right) \\ &= \frac{10}{3} + \frac{7}{6} = \frac{27}{6} = \frac{9}{2}. \end{aligned}$$

120. (a)  $\frac{dy}{dx} - y \log_e 2 = 2^{\sin x} (\cos x - 1) \log_e 2$

This is linear differential equation

$$\text{I.F.} = e^{-\log_e 2 \int dx} = e^{-x \log_e 2} = 2^{-x}$$

then general Solution is

$$y 2^{-x} = \int 2^{-x} 2^{\sin x} (\cos x - 1) \log_e 2 \, dx + c$$

Now let  $\sin x - x = t \Rightarrow (\cos x - 1) dx = dt$

$$\therefore y 2^{-x} = \log_e 2 \int 2^t dt + c$$

$$\therefore y 2^{-x} = 2^t + c$$

$$\therefore y = 2^{x+t} + c 2^x$$

$$\therefore y = 2^{\sin x} + c 2^x$$

121. (d) Here we are given that

$$\overline{OA} = \vec{a}, \overline{OB} = \vec{b}, \overline{OC} = \vec{c}$$

$$\text{Now P.V of D i.e } \overline{OD} = \frac{1 \times \overline{OB} + 3 \times \overline{OC}}{1+3}$$

$$\overline{OD} = \frac{\vec{b} + 3\vec{c}}{4}$$

$$\overline{OE} = \frac{4\overline{OD} + \overline{OA}}{4+1} = \frac{4(\vec{b} + 3\vec{c}) + \vec{a}}{5}$$

$$\Rightarrow \overline{OE} = \frac{\vec{a} + \vec{b} + 3\vec{c}}{5}$$

$$\text{Now, } \overline{OE} = \frac{2\overline{OB} + 3\overline{OF}}{2+3}$$

$$\Rightarrow \overline{OF} = \frac{5\overline{OE} - 2\overline{OB}}{3}$$

$$\Rightarrow \overline{OF} = \frac{5(\vec{a} + \vec{b} + 3\vec{c}) - 2\vec{b}}{3}$$

$$\Rightarrow \overline{OF} = \frac{\vec{a} - \vec{b} + 3\vec{c}}{3} \Rightarrow \text{P.V. of F is } \frac{\vec{a} - \vec{b} + 3\vec{c}}{3}$$

122. (b) The given equations are

$$3l + m + 5n = 0 \quad \dots(i)$$

$$\text{and } 6m - 2n + 5l = 0 \quad \dots(ii)$$

From (i), we have  $m = -3l - 5n$ .

Putting  $m = -3l - 5n$  in (ii),

$$\text{we get } 6(-3l - 5n) - 2n + 5l = 0$$

$$\Rightarrow (n + l)(2n + l) = 0$$

$\Rightarrow$  either  $l = -n$  or  $l = -2n$ .

If  $l = -n$ , then putting  $l = -n$  in (i), we obtain  $m = -2n$ .

If  $l = -2n$ , then putting  $l = -2n$  in (i), we obtain  $m = -n$ .

Thus, the direction ratios of two lines are  $-n, -2n, n$  and  $-n, -n, n$  i.e.,  $1, 2, -1$  and  $-2, 1, 1$ .

Hence, the direction cosines are

$$\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}} \text{ or } \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}. \text{ The angle } \theta$$

between the lines is given by

$$\cos \theta = \frac{1}{\sqrt{6}} \times \frac{-2}{\sqrt{6}} + \frac{2}{\sqrt{6}} \times \frac{1}{\sqrt{6}} + \frac{-1}{\sqrt{6}} \times \frac{1}{\sqrt{6}} = \frac{-1}{6}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{-1}{6}\right).$$

123. (a) Let

$$I = \int_5^9 \frac{\log 3x^2 dx}{\log 3x^2 + \log (588 - 84x + 3x^2)}$$

$$= \int_5^9 \frac{\log 3x^2 dx}{\log 3x^2 + \log 3(14 - x)^2}$$

$$= \int_5^9 \frac{\log 3(14 - x)^2 dx}{\log 3(14 - x)^2 + \log 3(14 - (14 - x))^2}$$

$$\left[ \int_a^b f(x) dx = \int_a^b f(a + b - x) dx \right]$$

$$I = \int_5^9 \frac{\log 3(14 - x)^2 dx}{\log 3(14 - x)^2 + \log 3x^2} \quad \dots(ii)$$

Adding Eqs. (i) and (ii), we get

$$2I = \int_5^9 \frac{\log 3x^2 + \log 3(14 - x)^2}{\log 3(14 - x)^2 + \log 3x^2} dx$$

$$2I = \int_5^9 dx = 9 - 5 = 4 \Rightarrow I = 2$$

124. (d)  $P$  (The electronic component fails when first used) =  $P(F) = 0.10$

$$\therefore P(\bar{F}) = 1 - P(F) = 0.90$$

Let  $E$  be the event that a new component will last for one year, then

$$P(E) = P(F) \cdot P\left(\frac{E}{F}\right) + P(\bar{F}) \cdot P\left(\frac{E}{\bar{F}}\right)$$

[Total probability theorem]

$$= 0.10 \times 0 + 0.90 \times 0.99 = 0.891$$

125. (b)  $\hat{i} \times (\bar{a} \times \hat{i}) = (\hat{i} \cdot \hat{i})\bar{a} - (\hat{i} \cdot \bar{a})\hat{i} = \hat{j} + 2\hat{k}$

$$\text{Similarly, } \hat{j} \times (\bar{a} \times \hat{j}) = 2\hat{i} + 2\hat{k},$$

$$\hat{k} \times (\bar{a} \times \hat{k}) = 2\hat{i} + \hat{j}$$

$$\Rightarrow \therefore |\hat{j} + 2\hat{k}|^2 + |2\hat{i} + 2\hat{k}|^2 + |2\hat{i} + \hat{j}|^2$$

$$= 5 + 8 + 5 = 18.$$

126. (d) For a distribution of random variable  $x$ ,

$$\alpha = P(X^6 < 3) = P(X^6 = 1) + P(X^6 = 2)$$

$$= \lambda + 2\lambda = 3\lambda$$

$$\text{and } \beta = P(X^6 < 2) = P(X^6 = 3) + P(X^6 = 4)$$

$$= 3\lambda + 4\lambda = 7\lambda$$

$$\therefore \alpha : \beta = 3 : 7$$

127. (b) Given that  $P_1 : x - 2y - 2z + 1 = 0$

$$P_2 : 2x - 3y - 6z + 1 = 0$$

Equation of plane bisectors

$$\left| \frac{x - 2y - 2z + 1}{\sqrt{1 + 4 + 4}} \right| = \left| \frac{2x - 3y - 6z + 1}{\sqrt{2^2 + 3^2 + 6^2}} \right|$$

$$\frac{x - 2y - 2z + 1}{3} = \pm \frac{2x - 3y - 6z + 1}{7}$$

Since  $a_1 a_2 + b_1 b_2 + c_1 c_2 = 20 > 0$

$\therefore$  Negative sign will be taken for acute bisector.

$$\Rightarrow 7x - 14y - 14z + 7 = -[6x - 9y - 18z + 3]$$

$$\Rightarrow 13x - 23y - 32z + 10 = 0$$

$$\left(-2, 0, -\frac{1}{2}\right) \text{ satisfy it}$$

128. (c) We note that

$$\frac{d}{dx}(x \tan x + 1) = x \sec^2 x + \tan x$$

$\therefore$  integrating by parts with  $x^2$  as first function,

we get

$$I = \int x^2 \frac{x \sec^2 x + \tan x}{(x \tan x + 1)^2} dx$$

$$= x^2 \left( -\frac{1}{x \tan x + 1} \right) - \int 2x \left( -\frac{1}{x \tan x + 1} \right) dx$$

$$= -\frac{x^2}{x \tan x + 1} + 2 \int \frac{x \cos x}{x \sin x + \cos x} dx$$

$$= -\frac{x^2}{x \tan x + 1} + 2 \log_e |x \sin x + \cos x| + c$$

$$\left( \because \frac{d}{dx}(x \sin x + \cos x) = x \cos x \right)$$

129. (c) Given equation of curve  
 $x = 12(t + \sin t \cos t), y = 12(1 + \sin t)^2$   
 Differentiate w.r.t 't',

$$\frac{dx}{dt} = 12(1 + \cos^2 t - \sin^2 t)$$

$$\frac{dx}{dt} = 12(1 + \cos 2t) \text{ and } \frac{dy}{dt} = 24(1 + \sin t) \cos t$$

$$\frac{dy}{dx} = \frac{2(1 + \sin t) \times \cos t}{1 + \cos 2t}$$

$$\Rightarrow \frac{2(1 + \sin t) \cos t}{2 \cos^2 t} = \sqrt{3}$$

$$\Rightarrow t = \frac{\pi}{6}, y_0 = 27$$

130. (b) Negation of given statement is  
 $\sim ((A \wedge C) \rightarrow B)$   
 $\sim (\sim (A \wedge C) \vee B)$ . Using De-Morgan's law,  
 So,  $(A \wedge C) \wedge (\sim B)$

